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A REFERENCE POINT THEORY OF DECISION-MAKING UNDER UNCERTAINTY

by

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DEDICATION

To my mother Kam-Oi Seto and my beloved wife Lillian

ABSTRACT

The most widely used decision model for choice under uncertainty is the Expected Utility (EU) model developed by von Neumann and Morgenstern (1944). In spite of its popularity, the predictions of the EU model have often been found to be inconsistent with observed behaviour, in both experimental research and field studies. The systematic and persistent nature of these anomalies has undermined the reliability of the EU model.

Research in cognitive psychology has recognized that the decision process under uncertainty is very complicated. It may involve several phases like editing, framing, and evaluation (Kahneman and Tversky, 1979). The notion of bounded rationality recognizes the limited capacity of information processing. Therefore, in making decisions, people often use heuristics to simplify the task. The findings in field studies and experimental research suggest that a commonly used heuristic for decision-making is the adoption of a reference point. A reference point model also acknowledges the endowment effect and the status quo bias in reaching a decision. Furthermore, a reference point model can capture the framing effect when the context of the problem will influence the selection of the reference point.

The reference point model developed in this research is similar to the skew-symmetric-bilinear (SSB) utility function (Fishburn, 1982) and regret theory (Bell, 1982; and Loomes and Sugden, 1982). The reference point model will use a stationary anchor, instead of random pairwise comparison between alternatives in the SSB utility function

and regret theory. Consequently, transitivity is preserved in the reference-specific preference orderings. The research shows that reference point theory can accomodate various anomalies inconsistent with the EU predictions. The isolation effect, the response mode effect, and the preference reversal phenomenon can be predicted by switching reference points. When reference point is used as heuristic and editing is involved in the decision-making process, reference point theory can explain the common consequence effect, the common ratio effect, and the reflection effect.

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CHAPTER 1 INTRODUCTION

The study of decision-making under uncertainty is dominated by a single approach, namely the Expected Utility (EU) model. It was first introduced in the eighteenth century to resolve the paradox between the St. Petersburg gamble and the expected value approach to evaluating risky prospects. The axiomatization of the von Neumann-Morgenstern (VNM) utility function in 1944 transformed the EU model from a descriptive model into a predictive model. With a set of axioms that governs the operation of the VNM utility function, the EU model became the standard approach to modeling decision-making.

Many theories which analyze economic behaviour under uncertainty are built upon the analytics of the EU model and follow the axioms in the VNM utility function. In financial economics, the capital asset pricing model (Markowitz, 1952a), the demand for money and liquidity preference (Tobin, 1958), and the competitive equilibrium in the stock market (Grossman and Hart, 1979) are several examples which assume the decision makers will maximize their expected utility in reaching the optimal portfolio. The theory of insurance (Mossin, 1968) and its extensions including the problems of adverse selection (Rothschild and Stiglitz, 1976) and moral hazard (Shavell, 1979) also make use of the VNM utility function and the EU framework. Besides adverse selection and moral hazard, other areas in the economics of information (incomplete information) like the principal-agent problem (Rees 1985a, 1985b) and the efficiency wage model (Akerlof and Yellen, 1986) are built on the same proposition that individuals optimize by maximizing their expected utilities. The EU approach thus forms a paradigm for the understanding of economic behaviour under uncertainty.

The Expected Utility model provides an analytical framework within which choice can be studied; yet, it also produces many choice paradoxes or anomalies where it fails to predict empirically observed behaviour and experimental results. Ever since the discussion of the gambling-insurance paradox in 1948 (Friedman and Savage), the validity of the EU model (and its predictive capacity) has consistently been under attack due to the discovery of a string of paradoxes. Laboratory studies have challenged each of the four major axioms in the VNM utility function. In particular, the transitivity axiom and the independence axiom attract most of the criticism. Results from field studies show that individuals do not optimize according to the EU decision-making process.

In light of these anomalies, decision science has been filled with new theories in the past fifteen years. Broadly speaking, the holistic judgment models seek to harmonize the anomalies with the EU model by adjusting the probability distributions of the prospects. The adjustment that takes place in the probability distribution reconciles the anomalies with the theories by dropping the independence axiom. The non-holistic models, on the other hand, attempt to change the evaluation of outcomes envisaged in the EU model. These models introduce heuristics or other evaluation criteria into the decision- making process. These models will usually introduce intransitivity into the preference ordering of the alternative prospects.

This study contributes to the understanding of choice under uncertainty by devising a new theory, Reference Point theory, to model the decision-making process. The new theory allows asymmetry in the evaluation of gains and losses with respect to a reference point. The disparity in the treatment of gains and losses has been found by Kahneman, Knetsch, and Thaler (1990), Kahneman and Tversky (1979), Samuelson and Zeckhauser (1988), Thaler (1980), and Tversky and Kahneman (1991). A discrepancy between the willingness to pay (the evaluation of gains) and the willingness to accept (the evaluation of losses) is also reported in many experiments involving riskless alternatives (Coursey, Hovis, and Schulze, 1987, and Knetsch and Sinden, 1984). The asymmetry in gains and losses is confirmed in different types of experiments and field studies of evaluating public and private goods alike. This characteristic in ranking alternatives is captured in the new theory. Based on the previous findings in experimental studies, the new theory uses a reference point to reflect the asymmetry in evaluating gains and losses.

The evaluation of a prospect relative to a reference point requires appropriate partitioning of the probability distributions in order to fully reflect rejoice and regret in dscision-making. This procedure, however, complicates the decision-making process and a specific editing rule is introduced. As a result, using reference point together with editing may cause biases in decision-making as documented in Tversky and Kahneman (1974).

The remainder of the thesis is organized as follows. Chapters 2 and 3 provide a literature review related to the EU model. Chapter 2 describes the formation and the axiomatization of the EU model. It analyzes the implications of these axioms for decision-making. Related issues in decision-making about defining and measuring risk are also covered in Chapter 2. The next chapter then reviews the anomalies surrounding the EU model. The first section in Chapter 3 examines the results of various laboratory experiments which were inconsistent with EU's predictions. The anomalies found in the field studies are examined in Section 3.2. After gaining some understandings of the nature of the paradoxes, the chapter surveys different models that suggest alternative ways to accommodate the paradoxes. Section 3.3 reviews the holistic models and Section 3.4 reviews the non-holistic models. The literature review will hopefully provide a basic understanding of the problem and the different suggestions offered in the alternative decision theories.

Many models surveyed in Chapter 3 recognize the importance of the psychological dimension of decision-making. Chapter 4 begins by examining the Regret theory which explicitly models the rejoice and the regret generated in the decision-making process. The preference ordering in Regret theory, however, is plagued by intransitivity when more than two prospects are considered. Section 4.2 shows that the solutions offered by the authors of Regret theory and the similar Skew-Symmetric Bilinear (SSB) Utility theory do not successfully restore transitivity in preference. My new model, following the same structure as Regret theory, hypothesizes that rejoice and regret in decision-making are derived from a reference point. Hence, the new theory is called Reference Point theory. By incorporating a stable reference point in the valuation function, the preference in Reference Point theory is transitive. It is an important feature of the new theory since transitivity is believed to be essential in defining rational choice. Section 4.4 shows that, given a specific reference point, the valuation function of Reference Point theory satisfies the same axioms as the VNM utility function in the EU model. In other words, the preference ordering given by the valuation function is reference-specific.

Chapter 5 continues the development of Reference Point theory by examining the elements of the new theory. The functional form of the valuation function is specified in light of the evidence related to the psychological dimension of decision-making. Section 5.1 reviews the results of experiments showing the impact of psychology in decision-making. In particular, the issue of the disparity in evaluating gains and losses is addressed. The rejoice-regret function of Reference Point theory is developed in Section 5.2. It is specified according to the hypothesis that losses command a larger impact on utility (disutility) than gains. Several functional forms are considered for the rejoice-regret function and the convex-concave function provides the best representation of the above hypothesis. Such a choice differentiates Reference Point theory from Regret theory which assumes a concave-convex and symmetrical function for its rejoice-regret function. Section 5.3 combines the VNM utility function and the rejoice-regret function derived in Section 5.2 to form a modified utility function. The final section in Chapter 5 hypothesizes the rule for determining the reference point.

Chapters 6 and 7 use the Reference Point theory to determine choice under uncertainty. Given a riskless reference point, Section 6.1 derives the valuation index of the RPT from a modified utility curve. Section 6.2 shows that the modified utility function is affected by the choice of reference point. Thus, decisions are also influenced by the choice of reference point. The section continues to examine the effect of an increasing reference point on the degree of risk aversion. Section 6.3 describes the analytics of using an uncertain reference point in the decision-making process. Section 6.4 studies the demand for insurance decision.

In Chapter 7, several anomalies are reexamined according to the decision-making process described in Reference Point theory. The paradoxical choice patterns revealed in some of the anomalies are consistent with different reference points associated with the decision frame. In other words, since the preference ordering is reference-specific, a new reference point will lead to a different preference ordering. Reference-specific preference is important in explaining the response mode effect, the reflection effect, and the preference reversal phenomenon. In these three categories of anomalies, a different choice pattern is elicited for seemingly identical prospects when they are framed in different problem contexts. Reference point theory suggests that the differences in the context of the problem will lead to different reference points in the evaluation process; consequently, different preference orderings are derived. This feature allows Reference Point theory to accommodate these anomalies which cannot be explained by the Generalized Expected Utility theory.

For the common ratio effect, the common consequence effect, and the isolation effect, which are related to the violation of the independence axiom, the difference in reference points does not account for the anomalous choice pattern. This is probably due to the fact that the preference in Reference Point theory also satisfies the independence axiom. In order to resolve these anomalies, an editing phase is introduced to simplify the evaluation process. The editing phase causes the decision-making process to be procedure dependent which eliminates independence in the preference.

The characteristics of Reference Point theory and its relation to other non-expected utility theories are summarized in Chapter 8. It concludes the study by pointing out the differences in methodology between the existing theories in decision science and Reference Point theory. The concluding remarks compare the limitations of the two different approaches.

CHAPTER 2 LITERATURE REVIEW OF THE EXPECTED UTILITY MODEL

This chapter provides a summary of the literature related to decision-making under uncertainty. The first section (Section 2.1) reviews the historical background that led to the development of the Expected Utility (EU) model. At this stage, the EU model is a descriptive model in the sense that it provides an explanation to resolve the choice paradox found in the St. Petersburg gamble. Section 2.2 covers the axiomatization of the von Neumann-Morgenstern utility function in the EU model. The EU model was transformed into a predictive model after von Neumann-Morgenstern axiomatized the preference ordering. It soon became the most popular model used to guide decision-making under uncertainty. Section 2.3 discusses the issues related to defining risk; it also examines different methods of measuring risk. Section 2.4 examines each of the axioms in the EU model and studies their implications for decision-making. Through this literature review, a better understanding of the EU model will provide some important clues for investigating the anomalies which surround the EU model. These anomalies, together with alternatives to the EU model, will be covered in Chapter 3.

2.1 HISTORICAL DEVELOPMENT OF THE EXPECTED UTILITY MODEL

The introduction of modern probability theory in the seventeenth century sparked the development of scientific theory towards risk-bearing.[1] The attractiveness of a gamble offering payoffs $(x_1, \ldots, x_n)$ with probabilities $(p_1, \ldots, p_n)$ is determined in part by its expected value $x = \sum_{i=1}^{n} p_i x_i$ (Machina, 1987a, p.122). The expected value (EV) model explicitly recognizes two basic elements in reaching a decision among risky prospects; namely, the value of each payoff and the probability of obtaining it. This model provides a simple structure to assess the relationship between risk and payoff systematically.

The discovery of "anomalous" behaviour not predicted by the EV model marked the beginning of subsequent development of different decision-making models. The first anomaly dates back to 1728 when Nicholas Bernoulli formulated the St. Petersburg gamble. This gamble offers a prize of ($\$2^{n-1}$) when "head" first appears in the n^{th} coin toss; the gamble stops when the first "head" appears.[2] The EV of the gamble is given by

$$EV = \sum_{n=1}^{\infty} (1/2)^n \cdot (\$2^{n-1}) \tag{2.1}$$

where $(1/2)^n$ gives the probability of having "head" first appears in the n^{th} toss, which is multiplied by the corresponding award for different values of n. The above series in (2.1) is equivalent to an infinite series with $1/2 in each term and therefore EV approaches infinity if n does. According to its EV, this gamble should be worth more than any finite dollar amount. Yet, Nicholas Bernoulli observed that people would only forgo a moderate amount for this gamble. It appears that people do not follow the EV decision rule for choice among risky prospects.

Independently, Gabriel Cramer and Daniel Bernoulli resolved the St. Petersburg paradox by adopting the notion of expected utility (EU). They reasoned that a prize of $200 was not necessarily "worth" twice as much as a prize of $100. Instead of using the EV, they transformed the value of a possible prize into a utility measurement and calculated the EU of the St. Petersburg gamble as follows:

$$EU = \sum_{n=1}^{\infty} (1/2)^n \cdot U(\$2^{n-1}) \tag{2.2}$$

where $U(\cdot)$ is the function that transforms monetary value into utility measurement. A strictly concave $U(\cdot)$ which is bounded above will lead to a finite series for EU expressed in (2.2). A twice-differentiable concave function implies that $U(x) > 0$, $U'(x) > 0$, and $U''(x) < 0$ for $x > 0$. Bernoulli proposed to use a logarithmic utility function $U(x) = b \log[(\alpha+x) / \alpha]$ where x is the monetary value of a possible prize, α is the initial wealth, and b is a scalar. With this specification for $U(\cdot)$, he showed that EU is indeed finite.

The utility transformation has further significance than just successfully resolving the St. Petersburg gamble; the adoption of a utility function also added a third element, namely, the risk attitude of the decision maker, to the decision-making process. The shape of the utility function will reflect the attitude of a decision maker towards risk (Machina and Rothschild, 1987, p.202)[3]. A strictly concave utility function indicates a risk-averse attitude and a strictly convex utility function reflects a risk-loving attitude. A risk-averse individual, with a strictly concave utility function, will consider a 50% chance to win $200 less desirable than a 100% chance to win $100, although both prospects promise the same expected monetary value. Therefore, he will demand an increase in expected monetary value to the first prospect in order to bear the additional risk.

Figure 2.1 shows a utility function of a risk-averse individual. The individual will always assign a higher utility index to a riskless prospect than any risky prospect that generates the same expected monetary value. $U(\cdot)$ is a strictly concave function if $U[\alpha x_1 + (1-\alpha)x_2] > \alpha U(x_1) + (1-\alpha)U(x_2)$ for $0 < \alpha < 1$. The EU of two possible outcomes is less than the utility of their expected value; hence, the St. Petersburg paradox is resolved by assuming risk-averse behaviour on the part of the decision maker. A strictly convex utility function will lead to the opposite conclusion where the EU index increases as risk increases.

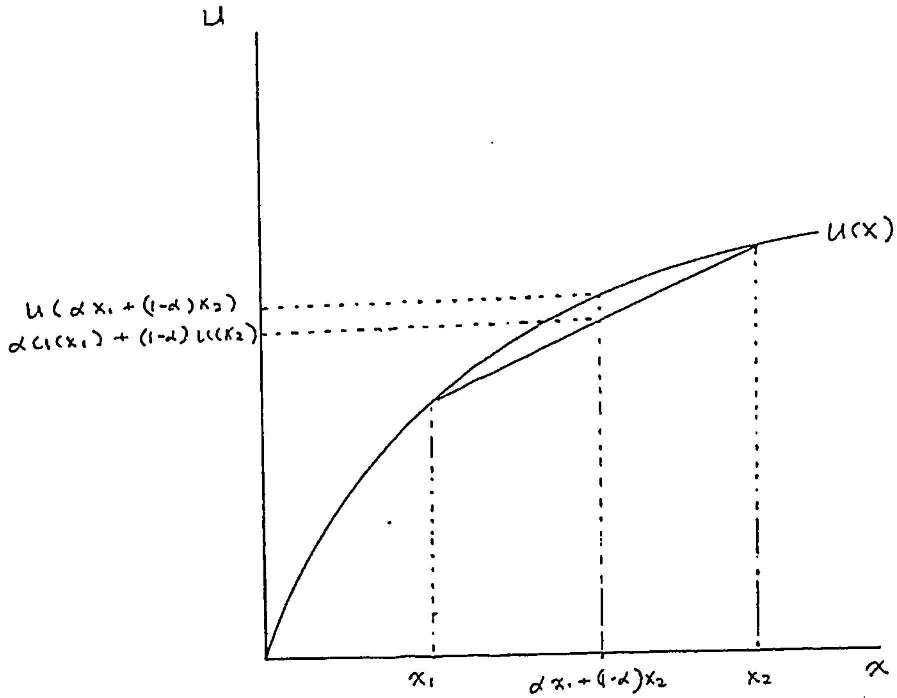


FIGURE 2.1 RISK AVERSION AND STRICTLY CONCAVE UTILITY FUNCTION

2.2 THE AXIOMATIZATION OF THE EXPECTED UTILITY MODEL

The EU model which was introduced by Bernoulli expands the scope of decision-making by incorporating the degree of risk aversion into the model. Bernoulli successfully resolved the St. Petersburg paradox with this feature. Decision making regarding risky prospects entered a new era when von Neumann and Morgenstern axiomatized the EU model. In their seminal work *Theory of Games and Economic Behavior* (1944), they used a set of basic axioms to construct a preference ordering function $V(\cdot)$ for risky prospects. The preference function takes the form

$$V(\tilde{x}) = \sum_{i=1}^{n} p_i U(x_i) \qquad (2.3)$$

where $\tilde{x}$ is a risky prospect that contains x_i as outcome for state i with n possible states,

p_i is the probability for the i^{th} state of nature to occur,

$U(\cdot)$ is the von Neumann-Morgenstern (VNM) utility function.

$U(\cdot)$ is a function that assigns a utility index for some outcome x_i; in consumer theory $U(x_i) > U(x_j) \Leftrightarrow x_i \succ x_j$ where "$\succ$" stands for "is strictly preferred to" (Fishburn, 1982, p.2). In ordinary consumer theory $U(\cdot)$ can be subjected to any positive monotonic transformation where the preference orderings among the x's are still preserved; in this sense $U(\cdot)$ represents ordinal preference orderings. For consumer theory regarding risky prospects, only positive linear transformations can be performed on $U(\cdot)$ without altering the preference ordering described by V(·). In other words, U(·) represents cardinal preference over x_i. V(·), however, gives ordinal rankings among different risky prospects (Schoemaker, 1982, p.533).

von Neumann and Morgenstern constructed a set of axioms that specifies the rules of operation for U(·); hence, it is named the von Neumann and Morgenstern (VNM) utility function. They showed that if individual preference satisfies these axioms, the individual will maximize his EU by selecting the risky prospect that yields the highest value of V(·). Before von Neumann's and Morgenstern's contribution, the EU model of Bernoulli was mostly a descriptive model because he did not address the issue of how to measure utility nor why his expectation principle would be rational (Schoemaker, 1982, p.531). The axiomatization of the VNM utility function transforms the EU model into a normative tool to guide decision-making. Within a few decades, it became a popular model for economic analysis regarding risky alternatives.

2.3 THE AXIOMS OF THE EXPECTED UTILITY MODEL

In von Neumann and Morgenstern (1953, p.26), three categories of axioms were used; viz. (1) complete ordering, (2) algebra of combining, and (3) ordering and combining. These axioms define the rules of operation of the cardinal utility function U(·) in order to give a consistent preference ordering by V(·) over possible outcome x's.[4] Fishburn (1982b, 1983), in his theoretical models, defined preference over a set of probability distributions associated with the risky prospects. Machina (1987a, 1987b) also described the consumer problem as choosing the most desirable probability distribution when the individual is presented with risky alternatives. It is clear that a lottery (or risky prospect) consists of two components, the outcome vector and the probability distribution. With a finite set of outcomes for each lottery, a common outcome set for all the lotteries can be formulated. One can then redefine all the risky alternatives according to this common outcome set and the individual is in effect choosing a probability distribution to maximize utility. As shown later in Section 2.3.2, the algebra of combining axioms becomes redundant when the axioms are defined over probability distributions.

2.3.1 Complete Ordering Axioms

The first category — complete ordering — includes two separate axioms stating that preferences are complete (A1) and transitive (A2).

(A1) For any two outcomes x_1, x_2; one and only one of the three following relations holds:

$x_1 \succ x_2$, $x_2 \succ x_1$, or $x_1 \sim x_2$

where "$\succ$" stands for "is strictly preferred to", and "$\sim$" stands for "is indifferent to".

(A2) $x_1 \succ x_2$, $x_2 \succ x_3$ imply $x_1 \succ x_3$.

These two axioms are indispensable for consumer theory under both riskless and risky situations; hence, one can apply the axiom to $\tilde{x}$'s or x's. The completeness axiom (A1) says that any two outcomes can be compared in such a way that either one is preferred to the other or they are equally preferable. There is one definite preference relationship between two alternative outcomes. The transitivity axiom (A2) is required to ensure that an optimal choice will be reached (Varian, 1984, p.112). The completeness axiom in (A1) and (A2) can be applied to riskless or risky alternatives.

Schoemaker (1982, p.531) describes these two axioms in terms of lotteries (the entire risky prospect) instead of using the events in a lottery. In other words, the completeness axiom now implies that an individual will prefer one lottery to another or he is indifferent between the two lotteries. When these two axioms are written in terms of probability distributions, they become

(A1′) For any two distributions F and G, either $F \succ G$, $G \succ F$, or $F \sim G$.

(A2′) $F \succ G$, $G \succ H$ imply $F \succ H$.

(Machina, 1987a, 1987b uses weak preference instead of strict preference.)

2.3.2 Algebra of combining

The second category of axioms — regarding the algebra of combining — contains the following two assumptions:

(A3) $\alpha x_1 + (1-\alpha)x_2 \sim (1-\alpha)x_2 + \alpha x_1$

(A4) $\alpha[\beta x_1 + (1-\beta)x_2] + (1-\alpha)x_2 \sim \mu x_1 + (1-\mu)x_2$ where $\mu = \alpha\beta$.

(A3) states that the order of the combination is irrelevant to the determination of preference for a lottery. In Machina (1987a, 1987b), this axiom is not included in the list of axioms. When the preference ordering is defined over probability distributions, it is apparent that (A3) is unnecessary. The rules of mathematical operation embedded in the probability theory yield the same implication as (A3). When the mixture of two probability distributions $\alpha F + (1-\alpha)G$ (where $F = (f_1, f_2, \ldots, f_n)$ and $G = (g_1, g_2, \ldots, g_n)$) defines the probability distribution of the common outcome set $(x_1, x_2, \ldots, x_n)$, the probability for x_i is given by $\alpha f_i + (1-\alpha)g_i$, or equivalently $(1-\alpha)g_i + \alpha f_i$; for all i. It is obvious that the two mixtures of probabilities are equal and therefore $\alpha F + (1-\alpha)G \sim (1-\alpha)G + \alpha F$.

(A4), also known as the reduction principle, states that an individual is indifferent between a compound lottery and a simple lottery as long as the two have the same probability distribution for each outcome. In other words, the decision maker is able to "see through" a compound lottery and is only concerned about the overall probabilities for each outcome. The probability theory that guarantees linearity in probabilities has the same implication as this axiom.

2.3.3 Ordering And Combining

The third category "ordering and combining" in von Neumann and Morgenstern (1953, p.26) also consists of two separate axioms: the independence axiom and the continuity (or mixture continuity) axiom. For the independence axiom, two assumptions are given as follows:

(A5-a) $x_2 \succ x_1$ implies that $\alpha x_1 + (1-\alpha)x_2 \succ x_1$, for $0 < \alpha < 1$

(A5-b) $x_1 \succ x_2$ implies that $\alpha x_1 + (1-\alpha)x_2 \succ x_2$, for $0 < \alpha < 1$.

(A5-a) is the dual to (A5-b) with opposite preference orderings among the outcomes x's. The intuitive meaning of this axiom, given in von Neumann and Morgenstern (1953, p.27), is to assume that "any kind of complementarity (or the opposite) has been excluded" between the outcomes or the lotteries. In other words, the utilities generated from the outcomes x's are independent. Hence, it is named the independence axiom. In the context of choice under uncertainty, Savage's sure-thing principle carries the same implication as the independence axiom. It says that for two prospects $\tilde{x}_i$ and $\tilde{x}_k$, if $\tilde{x}_i \succ \tilde{x}_k$ in state 1 and $\tilde{x}_i \sim \tilde{x}_k$ in state 2; then $\tilde{x}_i \succ \tilde{x}_k$ regardless of which state will occur (Savage, 1954, p.21). Thus, when two acts are considered, one needs only to consider the states where the two acts lead to different consequences (Schmeidler and Wakker, 1987).

Schoemaker (1982) and Machina (1987a, 1987b), respectively, restate the independence axiom as (A5) and (A5′) below:

(A5) For outcomes x_1, x_2, and x_3, if $x_1 \succ x_2$, then

$\alpha x_1 + (1-\alpha)x_3 \succ \alpha x_2 + (1-\alpha)x_3$, $0 < \alpha \leq 1$.

Or

(A5′) For probability distributions F, G, and H, if $F \succ G$, then

$$\alpha F + (1-\alpha)H \succ \alpha G + (1-\alpha)H, \quad 0 < \alpha \leq 1.$$

The equivalence between (A5) and (A5-a) (or (A5-b)) is based on the condition that $x_1 \sim \alpha x_1 + (1-\alpha)x_1$. Fishburn (1982a, pp.14-5) proved that $x_1 \sim \alpha x_1 + (1-\alpha)x_1$ can be derived from the mixture-set axioms (M1), (M2), and (M3):

(M1) $1x_1 + 0x_2 \sim x_1$

(M2) $\lambda x_1 + (1-\lambda)x_2 \sim (1-\lambda)x_2 + \lambda x_1$

(M3) $\lambda[\mu x_1 + (1-\mu)x_2] + (1-\lambda)x_2 \sim \lambda\mu x_1 + (1-\lambda\mu)x_2$.

(M1) establishes the equivalence between expressing an event in a lottery (risky) format with a 100% chance of getting x_1 and a certain event of getting x_1. (M2) and (M3) are identical to the axioms (A3) and (A4) in the "algebra and combining" category of VNM's original categorization. Similar to the discussion in Section 2.4.2, the mixture-set axioms are contained in the axioms of probability theory; no additional condition is added to derive (A5) from (A-5a) or (A5-b).

Independent utilities require that the preference function is strongly additive (Green, 1976, p.91, Henderson and Quandt, 1980, pp.39-40). This determines the fundamental structure of the preference function $V(\cdot)$ of the expected utility model stated in Equation (2.3). With the independence axiom, the preference function $V(\cdot)$ is restricted to be linear in the probabilities (Machina, 1987b, p.233). This feature is apparent when $V(\cdot)$ is defined over the probability distributions associated with the common outcome set. Consequently, the independence axiom implies that the indifference curves in the probability space are linear and parallel to each other (Fishburn, 1983, p.296; and Machina, 1987a, p.125). Figure 2.2 below depicts several linear indifference curves. When $x_1 \succ x_2$, the linear indifference curves imply that $\alpha x_1 + (1-\alpha)x_3 \succ \alpha x_2 + (1-\alpha)x_3$ for $0 < \alpha \leq 1$.

The independence axiom is usually defended by appealing to a two-stage decision-making process. The choice between $\alpha x_1 + (1-\alpha)x_3$ and $\alpha x_2 + (1-\alpha)x_3$ is assumed to be equivalent to being presented with a coin that has probability $(1-\alpha)$ of landing tail and being rewarded x_3 and being asked before the flip whether one would rather win x_1 or x_2 in the event of a head (Machina, 1987b, p.236). The independence axiom assumes that the common term in each risky prospect will not influence the choice between the different terms. All three axioms (A3), (A4), and (A5) are related to the linearity property in probability theory. Many researchers found that this assumption is rejected by laboratory experiments; this topic will be discussed in the following section about experimental anomalies.

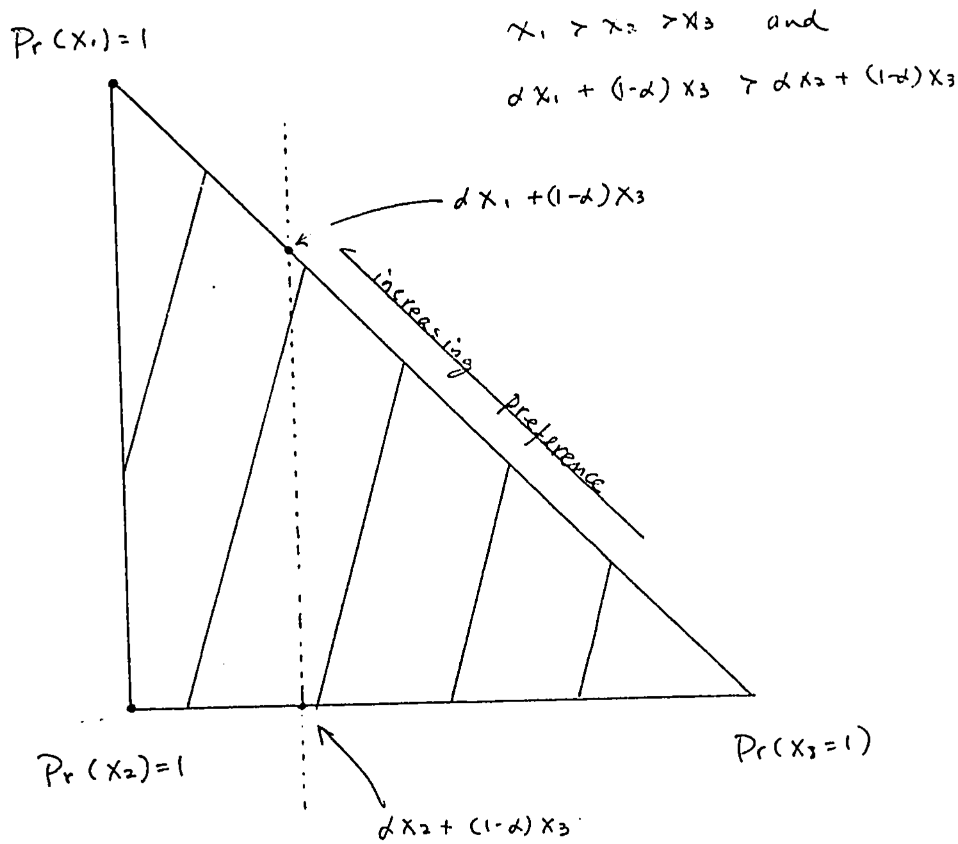


FIGURE 2.2 THE INDEPENDENCE AXIOM AND LINEAR INDIFFERENCE CURVES

This axiom is also regarded as the most restrictive assumption in the EU model. Many alternative theories try to relax this assumption in an attempt to resolve the observed anomalies. To quote Machina (1987a, p.125) on this issue:

> "However, the strongest implication of the expected utility hypothesis stems from the form of the expected utility maximand or preference function $\sum U(x_i)p_i$. Although this preference function generalizes the expected value form $\sum x_i p_i$ by dropping the property of linearity in the payoffs (the x_i's), it retains the other key property of this form, namely linearity in the probabilities."

The second axiom in the ordering and combining category is the continuity axiom; von Neumann and Morgenstern (1953, p.26) expressed this axiom as a pair of dual assumptions:

(A6-a) $x_3 \succ x_2 \succ x_1$ implies the existence of an $\alpha \in (0,1)$ such that

$$x_2 \succ \alpha x_1 + (1-\alpha)x_3,$$

(A6-b) $x_1 \succ x_2 \succ x_3$ implies the existence of an $\alpha \in (0,1)$ such that

$$\alpha x_1 + (1-\alpha)x_3 \succ x_2.$$

In (A6-a), when $x_3 \succ x_2 \succ x_1$, there exists a mixture $\alpha x_1 + (1-\alpha)x_3$ that is close enough to x_1 (with small weight on x_3) which is inferior to x_2. (A6-b) contains the same assumption with the opposite preference orderings between x_1, x_2, and x_3. These two conditions imply that the preference ordering is continuous (Varian, 1984, p.112)[5]. (A6-a) and (A6-b) are combined into (A6) in Schoemaker (1982) and Machina (1987a), respectively, as follows:

(A6) For $x_3 \succ x_2 \succ x_1$, there exists some $\beta \in [0,1]$ such that

$$\beta x_1 + (1-\beta)x_3 \sim x_2.$$

Or

(A6′) For $F \succ G \succ H$, there exists some $\beta \in [0,1]$ such that

$$\beta F + (1-\beta)H \sim G.$$

One can see the close resemblance between (A6-a) and (A6-b) and (A6) by changing $\succ$ into $\sim$. The continuity axiom in (A6) brings out the concept of certainty equivalence. It states that there exists a riskless amount x_2 which is as attractive as a lottery $\beta x_1 + (1-\beta)x_3$.

2.3.4 Other Axioms

Some authors (Schoemaker, 1982, Fishburn, 1983) also include the dominance axiom

(A7) For $x_1 \succ x_2$, $\alpha x_1 + (1-\alpha)x_2 \succ \beta x_1+(1-\beta)x_2$ iff $\alpha > \beta$

in the list of axioms. (A7) is equivalent to the notion of first degree stochastic dominance (FSD) in the theorem of stochastic dominance (Levy, 1987, p.500). FSD requires that $U'(\cdot) > 0$. This condition for FSD is in turn equivalent to the axiom of non-satiation in standard consumer theory (Green, 1976, p.34). Hence, (A7) only expresses a widely held assumption in consumer theory in a stochastic setting. The decision criterion of the EU model, selecting the prospect which yields the highest EU index in the value function $V(\cdot)$, has incorporated this axiom into the decision criterion.

To conclude, the VNM $U(\cdot)$ will obey the four axioms (A1) Completeness, (A2) Transitivity, (A5) Independence, and (A6) Mixture Continuity. Together they define the mathematical operation permissible on $U(\cdot)$ and impose a specific structure on the preference function $V(\cdot)$ such that

$$V(F) = V(f_1, f_2, \ldots, f_n) \equiv \sum_i f_i U(x_i). \tag{2.4}$$

In order to satisfy this particular functional form of $V(\cdot)$ and preserve the preference ordering, $U(\cdot)$ must be cardinal because it can only be subjected to positive linear transformations (Machina, 1987a, p.123). $V(\cdot)$ is ordinal in that any form of positive monotonic transformation will preserve the initial orderings among alternative risky prospects found in $V(\cdot)$ before the transformation. The ranking of the prospects based on the EU indices remains unchanged after the transformation.

2.4 STUDIES RELATED TO DEFINING RISK

The introduction of attitudes toward risk in decision-making sparked the study of the characteristics of risk. Relevant studies in this area include (i) finding a suitable measurement of risk, (ii) obtaining a definition for increasing risk, and (iii) measuring the degree of risk aversion.

2.4.1 Measurement of Risk

The first issue concerns measuring risk. The most popular univariate measure of the riskiness of a random variable is its standard deviation; a statistical index to summarize the weighted deviation of each possible outcome from the mean. For two random variables that have the same mean, the one which has a smaller standard deviation is said to be less risky and therefore should be preferred by any risk-averse individual. The mean-standard deviation analysis gained its popularity in the 1950s and 1960s due to its sound statistical foundation and its applicability. In particular, the development of modern portfolio theory (for example, Markowitz, 1952a, 1959; and Tobin 1958) in financial economics is an outstanding example and shows the usefulness of standard deviation in characterizing the riskiness of a random variable. However, the mean-standard deviation approach has its drawbacks.

The first objection to the mean-standard deviation approach is due to the restrictiveness it imposes on the utility function. In order to be consistent with the mean-standard deviation analysis, the VNM $U(\cdot)$ must be a quadratic function $U(x) = ax + bx^2$. It can be shown that, with the quadratic function, $EU = b \cdot Var(x) + E(x)[a+bE(x)]$ or in other words expected utility is only a function of the mean and variance of x. Given that $U(x) = ax + bx^2$, the parameter "b" in the utility function must be negative to obtain $U''(\cdot) < 0$ and risk-averse attitude. With these restrictions on $U(\cdot)$ and b, the individual's utility will decrease as wealth increases beyond $-a/2b$ and the individual will be more averse to constant additive risk at high wealth levels than at low wealth levels. The empirical findings did not agree with this implication (Pratt, 1964).

The second attack on the mean-standard deviation approach was launched by Borch (1969). He showed that for two random variables $\tilde{x}_1$ and $\tilde{x}_2$ that are ranked as indifferent with different mean-standard deviation combinations (one promises a higher expected return associated with higher standard deviation), it is possible for $\tilde{x}_2$ to dominate $\tilde{x}_1$ in the sense of first degree stochastic dominance. (For a definition of first degree stochastic dominance, see Levy, 1987.) Any individual with an increasing VNM utility function would strictly prefer $\tilde{x}_2$ to $\tilde{x}_1$. Borch's argument is repeated in Appendix A. These criticisms led to new efforts to find a satisfactory measure of risk.

2.4.2 Definition for Increasing Risk

The most important research on replacing the mean-standard deviation approach is found in Hadar and Russell (1969), Hanoch and Levy (1969), and Rothschild and Stiglitz (1970, 1971). They developed an alternative characterization of risk by the notion of increasing risk. Rothschild and Stiglitz (1970, pp.225-6) showed that the following three definitions for increasing risk are equivalent to each other:

For two random variables $\tilde{x}_1$ and $\tilde{x}_2$ which have the same mean, $\tilde{x}_2$ is riskier than $\tilde{x}_1$ if

(1) $\tilde{x}_2$ is equal to $\tilde{x}_1$ plus noise.

Formally, $\tilde{x}_2 =_d \tilde{x}_1 + \varepsilon$, where "$=_d$" means "has the same distribution as" and ε is a random variable with the property that $E(\varepsilon|\tilde{x}_1) = 0$.

(2) Every risk averter prefers $\tilde{x}_1$ to $\tilde{x}_2$.

In other words, for any strictly concave VNM $U(\cdot)$, $E[U(\tilde{x}_1)] \geq E[U(\tilde{x}_2)]$.

(3) $\tilde{x}_2$ has more weight in the tails than $\tilde{x}_1$.

This definition introduces the concept of "mean preserving spread". Intuitively, a "mean preserving spread" consists of moving probability mass from the centre of a probability distribution to its tails in a manner that will preserve the expected value of the distribution. With higher probabilities assigned to the extreme values in the two ends, or more weight in the tails of the distribution, the risk increases.

This approach to define risk, unlike the univariate measure, is not always able to rank any pairs of random variables; hence, only partial orderings are derived (Laffont 1989, p.26).

2.4.3 Measurement of Risk Aversion

Another important concept in the characterization of risk attitude is the degree of risk aversion. Arrow (1965) and Pratt (1964) used the VNM $U(\cdot)$ to derive a measure of degree of risk aversion. The

Arrow-Pratt index of absolute risk aversion, $r(x) = \frac{-U''(x)}{U'(x)}$, measures risk aversion using the degree of concavity of the utility function (Machina 1987b, p.235). A utility function which exhibits a greater degree of concavity implies that the index r(x) increases at every level of x. Its economic implication is that such a utility function is associated with a greater degree of risk aversion. This can be illustrated by the concept of certainty equivalent, a riskless outcome that is ranked as equivalent to a risky prospect. Mathematically, CE(a) is the certainty equivalent to a risky prospect $\tilde{x}$ for an individual with utility function $U_a(\cdot)$ if $U_a(CE(a)) = E[U_a(\tilde{x})]$. When the concavity of the utility function increases, it is equivalent to say that $U_b(\cdot)$ is a concave transformation of $U_a(\cdot)$ where $U_b = F(U_a)$, $F'(\cdot) > 0$ and $F''(\cdot) < 0$. The certainty equivalent to $\tilde{x}$ for an individual exhibiting utility $U_b(\cdot)$ is given by $U_b(CE(b)) = E[U_b(\tilde{x})]$. $CE(b) < CE(a)$ because $U_b(\cdot)$ has a greater concavity than $U_a(\cdot)$. The more concave utility function $U_b(\cdot)$ exhibits a higher degree of risk aversion and the individual will forgo a larger amount to avoid the risky prospect. In other words, he or she will tolerate a larger risk premium (expected monetary value of the risky prospect minus certainty equivalent, $E[\tilde{x}]-CE(b)$) in order to eliminate the risk.

The Arrow-Pratt index is also able to describe the risk attitude as wealth changes. An individual is said to exhibit decreasing (increasing) risk aversion when r(x) is a decreasing (increasing) function of wealth. That is, as wealth increases, this individual is willing to pay a lower (higher) premium when he exhibits decreasing (increasing) risk aversion. When the degree of risk aversion is independent of x, he exhibits constant risk aversion. That is, he will have the same certainty equivalent for the same risky prospect regardless of his wealth. It is commonly asserted that an individual will tolerate more risk when his wealth increases; that is decreasing absolute risk aversion.

The degree of risk aversion as defined in the Arrow-Pratt index will influence the slope of the indifference curves in the triangular probability space. Consider a risky prospect $\tilde{x}$ which has 50% chance to win $1,000 and a riskless prospect $\tilde{y}$ which has 100% chance to win $500. The expected value of $\tilde{x}$ is equal to the expected value of $\tilde{y}$. When an individual is neutral to risk, only the expected outcome matters in his choice. Therefore, he is indifferent between $\tilde{x}$ and $\tilde{y}$ since both prospects have the same expected value. The risk-neutral indifference curve in Figure 2.3 is a straight line that joins between $\tilde{x}$ and $\tilde{y}$ in the probability triangle. To a risk-averse individual who has to be compensated for exposure to risk, the riskless prospect $\tilde{y}$ is preferred to the risky prospect $\tilde{x}$. Consequently, in Figure 2.3, the indifference curve which exhibits risk-aversion is steeper than the risk-neutral indifference curve. On the other hand, a risk-seeking individual will always choose the risky prospect $\tilde{x}$; his indifference curve is flatter than the risk-neutral indifference curve.

The EU model became a popular decision model for analyzing economic behaviour under uncertainty. Yet, some findings from feild studies and laboratory experiments contradict the EU predictions. These anomalies surrounding the EU model are discussed in Chapter 3.

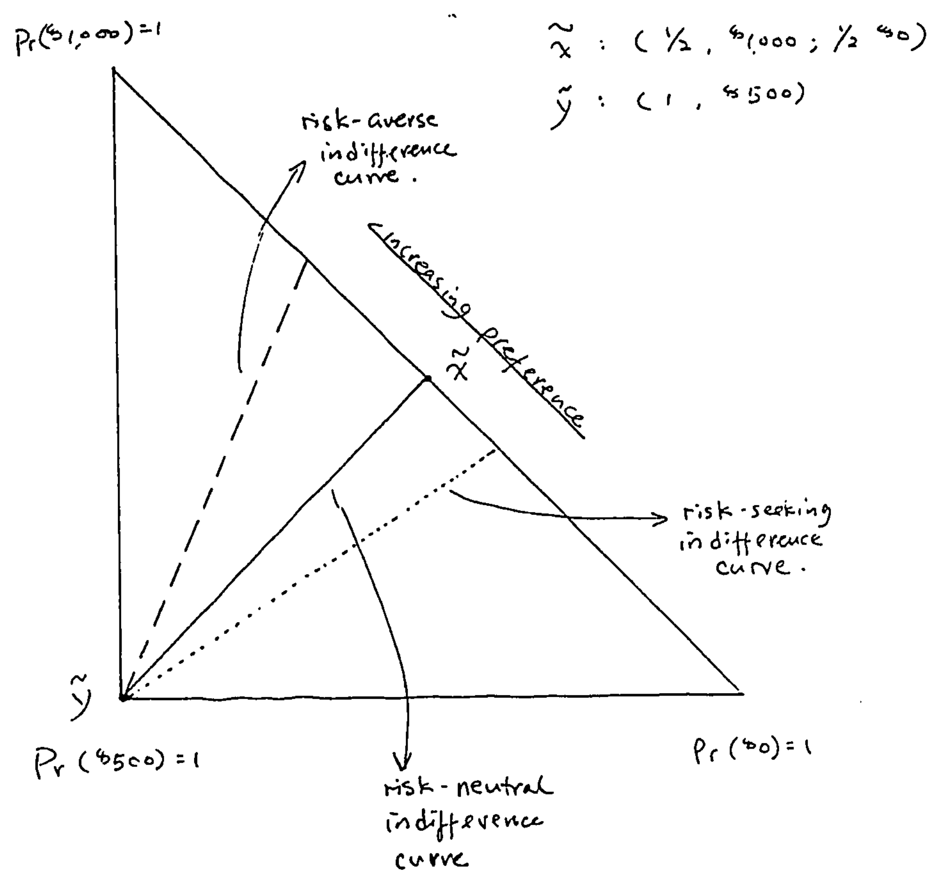


FIGURE 2.3 RISK AVERSION INCREASES THE SLOPE OF THE INDIFFERENCE CURVES

CHAPTER 3 LITERATURE REVIEW OF THE ANOMALIES SURROUNDING THE EXPECTED UTILITY MODEL AND THE ALTERNATIVE THEORIES

The axiomatization of the VNM utility function allows the EU model to predict choice patterns under uncertainty. Not long after the seminal work of von Neumann and Morgenstern, the first choice paradox was discovered by Allais in (1953). Further paradoxical findings were discovered since the Allais paradox. These anomalies which were found in laboratory experiments are discussed in Section 3.1. Section 3.2 reports some findings from field studies which show that the EU model fails to predict individual behaviour under uncertainty. Together, the laboratory experiments and the field studies question the predictive power of the EU model. The legitimacy of the EU model as a predictive decision model is undermined; this leads to a search for alternative decision models. Sections 3.3 and 3.4, respectively, survey various holistic and non-holistic models which can accommodate the anomalies in the EU model.

3.1 EXPERIMENTAL ANOMALIES

As in the case of the EV model, researchers discovered various kinds of anomalous behaviour which were inconsistent with the EU model. This section will focus on anomalies revealed in laboratory experiments - many of them intended to test a particular axiom of the

EU model. The next section (3.2) will discuss the anomalies observed in real life economic decisions.

The remainder of this section will be divided into seven subsections examining several anomalies found in the experimental studies. These sub-sections cover (i) the common ratio effect, (ii) the common consequence effect, (iii) the response mode effect, (iv) the isolation effect, (v) the reflection effect, (vi) the preference reversal (PR) phenomenon, and (vii) other anomalies related to the axioms in the EU model.

The survey in these subsections will focus on the specific cause accountable for behaviour which deviates from the EU prediction. The two axioms tested most rigorously are the transitivity axiom (A2) and the independence axiom (A5). Most of the studies showed that the independence axiom of the EU model imposes too much rigidity in preferences which leads to the common ratio effect and the common consequence effect. The response mode effect, the isolation effect, and the reflection effect are likely caused by the framing effect or the violation of procedure invariance. Various studies have shown that the PR phenomenon may be caused by the violation of transitivity, the violation of the independence axiom, or the violation of procedure invariance. These studies are summarized in Section 3.1.6. Experiments designed to test one of the 4 axioms in the EU model are discussed in Section 3.1.7.

3.1.1 The Common Ratio Effect

The common ratio effect was first discovered by Allais (1953). Consider two pairs of gambles used in Kahneman and Tversky's [1] experiment (1979, pp.266-7, Problems 3 and 4):

PROBLEM 3: (N = 95)

Choose between

$\tilde{a}_1$: 0.80 chance of $4,000; [20%]

$\tilde{a}_2$: 1.00 chance of $3,000. [80%]

PROBLEM 4: (N = 95)

Choose between

$\tilde{a}_3$: 0.20 chance of $4,000; [65%]

$\tilde{a}_4$: 0.25 chance of $3,000. [35%]

$\tilde{a}_3$ can be formed as a combinations of $\tilde{a}_1$ and a $0 payoff, $\tilde{a}_5$; and similarly for $\tilde{a}_4$ as a combinations of $\tilde{a}_2$ and $\tilde{a}_5$ as follows:

$\tilde{a}_3 = \alpha\tilde{a}_1 + (1-\alpha)\tilde{a}_5$, and

$\tilde{a}_4 = \alpha\tilde{a}_2 + (1-\alpha)\tilde{a}_5$ where $\tilde{a}_5 = \$0$ and $\alpha = 0.25$.

Hence, according to the independence axiom, $\tilde{a}_1 \succ \tilde{a}_2$ iff $\tilde{a}_3 \succ \tilde{a}_4$. It is called the common ratio effect because both gambles have the same ratio of likelihood to win the higher prize relative to the lower prize.[2]

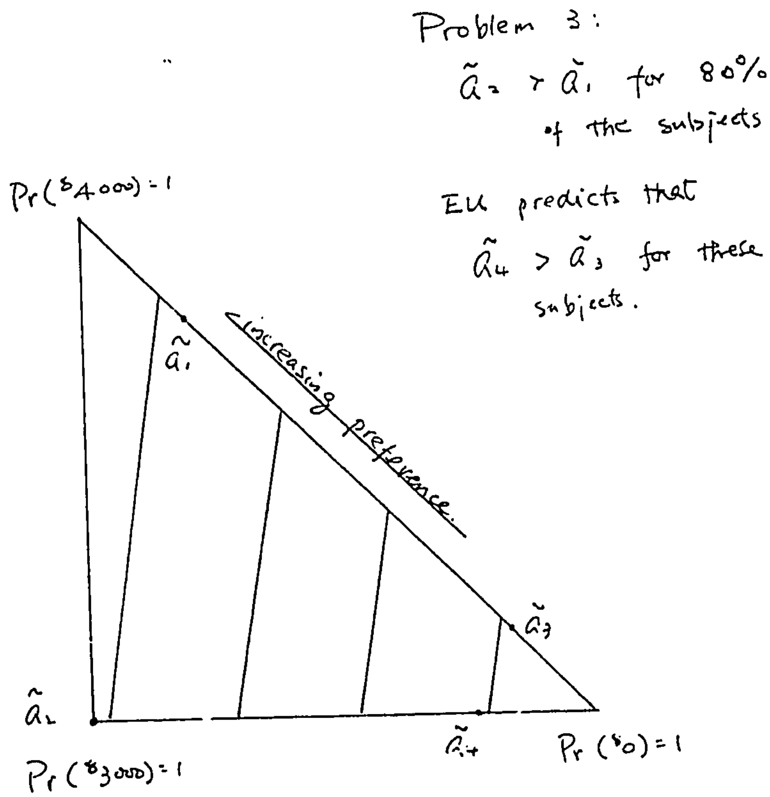


FIGURE 3.1 LINEAR INDIFFERENCE CURVES OF THE EU MODEL AND THE COMMON RATIO EFFECT

Experimental results showed that subjects frequently choose $\tilde{a}_2$ over $\tilde{a}_1$ but preferred $\tilde{a}_3$ to $\tilde{a}_4$. This choice pattern violates the independence axiom where the common term $(1-\alpha)\tilde{a}_5$ in both $\tilde{a}_3$ to $\tilde{a}_4$ will not influence the choice between the two prospects. The linear indifference curves of the EU model will reflect the same preference rankings of $\tilde{a}_1$ and $\tilde{a}_3$ vis-á-vis $\tilde{a}_2$ and $\tilde{a}_4$. Figure 3.1 above shows the preferences of a risk-averse individual where $\tilde{a}_1 \prec \tilde{a}_2$ and consequently $\tilde{a}_3 \prec \tilde{a}_4$. The majority of the subjects ˙ deed chose $\tilde{a}_2$ over $\tilde{a}_1$ but preferred $\tilde{a}_3$ to $\tilde{a}_4$. The linear indifference curves are clearly incompatible with the observed choice pattern.

Loomes (1991) presented new evidence of violation of the independence axiom. He asked the subjects to allocate a fixed amount of money (£20.00) to two alternative payoffs A and B for some given probabilities such that Pr(A) > Pr(B). Among the 12 problems in the experiment, 10 of them constitute 5 pairs where the ratio of probabilities is the same for both problems in each pair. Problems i and j will form such pair when $\frac{Pr(A_i)}{Pr(B_i)} = \frac{Pr(A_j)}{Pr(B_j)}$; this experimental design will elicit the common ratio effect. Given $Pr(A_i)$ and $Pr(B_i)$ and the subject allocates £20.00 according to the ratio $(A_i)/(B_i)$, the independence axiom implies that the subject should follow the same allocation for $Pr(A_j)$ and $Pr(B_j)$. In other words, the independence axiom entails that $\frac{(A_i)}{(B_i)} = \frac{(A_j)}{(B_j)}$ when $\frac{Pr(A_i)}{Pr(B_i)} = \frac{Pr(A_j)}{Pr(B_j)}$. Loomes' subjects, however, consistently deviated from this proposition and violated the independence axiom.

3.1.2 The Common Consequence Effect

The common consequence effect was also discovered by Allais (1953) and it is commonly known as the Allais paradox. The choices of the subjects in the common consequence effect experiment consistently deviated from the predictions given by the EU model. The discovery of the Allais paradox posed a major challenge to the EU model as a prescriptive model. It raised questions about the cohesiveness of the predictions generated by the EU model.

In an experiment, subjects are asked to choose from two pairs of gambles as follows (Kahneman and Tversky, 1979, pp.265-6):

PROBLEM 1: (N = 72)

Choose between

$\tilde{b}_1$: $2,500 with probability .33,
$2,400 with probability .66,
$ 0 with probability .01; [18%]

$\tilde{b}_2$: $2,400 with certainty. [82%]

PROBLEM 2: (N = 72)

Choose between

$\tilde{b}_3$: $2,500 with probability .33,
$0 with probability .67; [83%]

$\tilde{b}_2$: $2,400 with probability .34,
$0 with probability .66. [17%]

Assuming $0 gives no utility, that is U($0) = 0, the expected utilities for each alternative are:

$$EU(\tilde{b}_1) = 0.33 \cdot U(2,500) + 0.66 \cdot U(2,400) + 0.01 \cdot U(0)$$
$$EU(\tilde{b}_2) = U(2,400)$$
$$= 0.34 \cdot U(2,400) + 0.66 \cdot U(2,400),$$
$$EU(\tilde{b}_3) = 0.33\ U(2,500) + 0.66 \cdot U(0) + 0.01 \cdot U(0),$$
$$EU(\tilde{b}_4) = 0.34\ U(2,400) + 0.66 \cdot U(0).$$

Because of the independence axiom, the common terms in $\tilde{b}_1$ and $\tilde{b}_2$, and $\tilde{b}_3$ and $\tilde{b}_4$ would be ignored; consequently, the EU model predicts that if $\tilde{b}_1 \succ \tilde{b}_2$, then $\tilde{b}_3 \succ \tilde{b}_4$ or vice versa.

It is clear that the linear indifference curves of the EU model implies that $\tilde{b}_1 \succ \tilde{b}_2$ will always lead to $\tilde{b}_3 \succ \tilde{b}_4$ for individuals exhibiting risk-seeking, risk-neutral, or very modest risk-averse preferences. For individuals exhibiting a stronger degree of risk-aversion the slope of the linear indifference curves increases; thus, $\tilde{b}_1 \prec \tilde{b}_2$ and $\tilde{b}_3 \prec \tilde{b}_4$. (See Section 2.4.3 and Figure 2.3 for the discussion on the impact of the attitudes toward risk on the slope of the indifference curve.) Figure 3.2 below shows the preferences of the risk-averse individual where $\tilde{b}_1 \prec \tilde{b}_2$ and $\tilde{b}_3 \prec \tilde{b}_4$. The majority of the subjects, however, chose $\tilde{b}_2$ over $\tilde{b}_1$ but preferred $\tilde{b}_3$ to $\tilde{b}_4$.

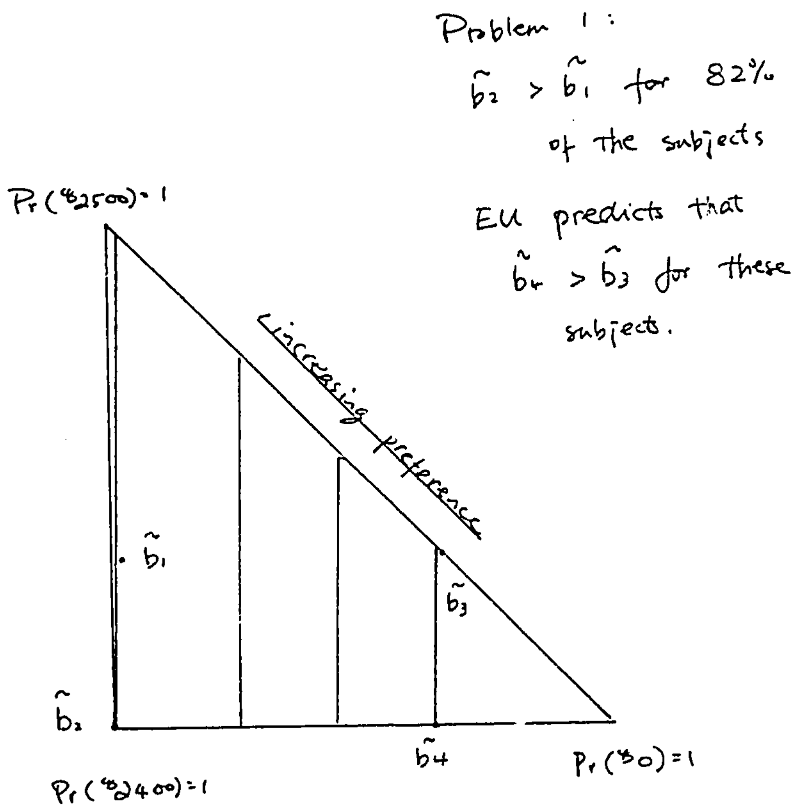


FIGURE 3.2 LINEAR INDIFFERENCE CURVES OF THE EU MODEL AND THE COMMON CONSEQUENCE EFFECT

The Allais paradox is called the common consequence effect (Machina 1987a, p.129) because of the common term $(0.66 \cdot U(2,400))$ in $\tilde{b}_1$ and $\tilde{b}_2$. The common consequence effect has been observed in many other studies, for example, Morrison (1967), Raiffa (1968), Slovic and Tversky (1974), MacCrimmon and Larsson (1979), Kahneman and Tversky (1979), and Chew and Waller (1986).

Ellsberg (1961) performed an experiment in a similar pattern; instead of known probability, he introduced uncertainty (ambiguity) into the chance of receiving a particular payoff. In his experiment, he told the subjects that an urn contains 30 red balls and 60 black and yellow balls in unknown proportion. He then asked the subjects to choose between

I: receiving $100 when a red ball is drawn from the urn

and

II: receiving $100 when a black ball is drawn from the urn.

The majority response was action I preferred to II. He then asked the subjects to choose again between

III: receiving $100 when a red or yellow ball is drawn from the urn

and

IV: receiving $100 when a black or yellow ball is drawn from the urn.

The majority response was action IV preferred to III. The payoff in the event of drawing the yellow ball is identical in each pair of actions and therefore it should not affect one's choice according to Savage's sure-thing principle. Yet the subjects' responses clearly indicated otherwise. This phenomenon is named after him as the Ellsberg paradox. Clearly, the violation of the sure-thing principle observed by Ellsberg also casts doubt on the independent nature of the decision-making process postulated in the EU model. These results showed that subjects would not simply eliminate the common outcome across different prospects. In other words, each outcome in the prospect is evaluated vis-á-vis other terms in the prospect and it is not treated independently.

Conlisk (1989) restructured the Allais paradox in a three-stage form in order to bring out the intuition of the independence axiom explicitly for the subjects. Conlisk found that violation of the independence axiom was reduced; nevertheless, his result still rejected the assumption of linearity in probabilities.

The most important implication of the independence axiom is the fact that the indifference curves in the probability space are linear and parallel to each other (Fishburn, 1983, p.296; and Machina, 1987a, p.125). The observed choice pattern of the subjects, on the other hand, indicated that the indifference curves are fanning-out instead of linear and parallel (See Figure 3.3). Intuitively, the violations of the independence axiom implied that individuals do not treat the alternative payoffs in a gamble independently.

The studies cited above repeatedly repudiate the validity of the independence axiom. All the experiments suggested that there is some inter-dependency among the payoffs in the prospects. Kahneman and Tversky (1979) grouped the common ratio effect and the common consequence effect together as the certainty effect. They reasoned that these two types of violation of the independence axiom were caused by overweighing the outcomes that were considered as certain

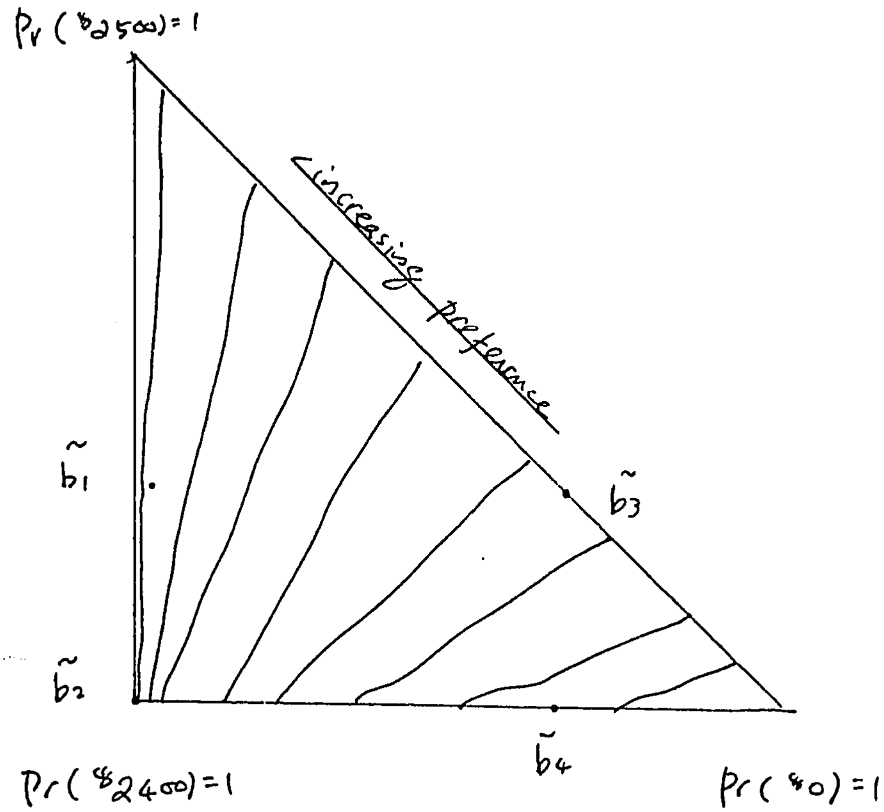


FIGURE 3.3 THE COMMON CONSEQUENCE EFFECT IS CONSISTENT WITH FANNING-OUT INDIFFERENCE CURVES

relative to outcomes which were merely probable (p.265). Other holistic models surveyed in Section 3.3 also try to avoid the independence axiom and linearity in probability. Most of the non-holistic models (surveyed in Section 3.4) utilize sequential elimination or pairwise comparison in the decision-making process; these models do not formulate a ranking index based solely on the outcomes of a prospect.

Moreover, Loomes' (1991) results suggested that there is more than one pattern of fanning-out and some of the generalized models (discussed in Section 3.3) could not accommodate some of the choice patterns observed. This may suggest that the inter-dependency is very complicated. These results might confirm the various notions of alternative rationalities in which people use more than one decision "frame" (Loomes, 1991, p.105).

3.1.3 The Response Mode Effect

Rationality according to the EU model requires that the preference ordering between the same prospects should not be reversed with changes of context. Yet, various studies (for example, Kahneman, Knetsch, and Thaler (1991), Kahneman and Tversky (1979, 1984), and Tversky and Kahneman (1981)) have showed that decisions and choices were influenced by the context of problems. The "decision frame" would affect the perception of the prospects and the resulting preferences are not stable.

Consider the following problems in Kahneman and Tversky (1979, p.273):

PROBLEM 11: (N = 70)

In addition to whatever you own, you have been given 1,000. You are now asked to choose between

$\tilde{c}_1$: 0.50 chance of 1,000; [16%]

$\tilde{c}_2$: 1.00 chance of 500. [84%]

PROBLEM 12: (N = 70)

In addition to whatever you own, you have been given 2,000. You are now asked to choose between

$\tilde{c}_3$: 0.50 chance of -1,000; [69%]

$\tilde{c}_4$: 1.00 chance of -500. [31%]

Although $\tilde{c}_1$ is identical to $\tilde{c}_3$ and $\tilde{c}_2$ is identical to $\tilde{c}_4$ in terms of probability distributions and outcomes, a substantial fraction of subjects reversed their preferences between the two seemingly identical prospects. 84% of the subjects exhibited risk-aversion when they had to choose between positive prospects. 69% of the subjects became risk-seeking in the problem involving negative prospects. The reversal in preference between gains and losses was documented in other studies cited above. Note that the "fanning-out" indifference curves will not predict this kind of choice pattern. In the probability triangle $\tilde{c}_1$ and $\tilde{c}_2$ are located on the same position as $\tilde{c}_3$ and $\tilde{c}_4$, respectively. In other words, the anomaly caused by framing is not related to the validity of the independence axiom.

3.1.4 The Isolation Effect

Similar to the response mode effect, the isolation effect also indicates changes in preference for seemingly identical prospects when the structure of the gambles changes. Unlike the response mode effect, the anomalous behaviour in the isolation effect is discovered over positive prospects. Kahneman and Tversky (1979, p.271) reported the isolation effect as follows:

PROBLEM 4: (N = 95)

Choose between

$\tilde{a}_3$:	0.20 chance of \$4,000;	[65%]
$\tilde{a}_4$:	0.25 chance of \$3,000.	[35%]

PROBLEM 10: (N = 141)

Choose between

$\tilde{a}'_3$:	0.25 chance to win $\tilde{a}_1$ (0.80 chance of \$4,000);	[22%]
$\tilde{a}'_4$:	0.25 chance to win $\tilde{a}_2$ (1.00 chance of \$3,000).	[78%]

The two-stage gambles in Problem 10 generated a preference ordering which was different from the ranking in Problem 4 although $\tilde{a}_3$ and $\tilde{a}_4$ are identical to $\tilde{a}'_3$ and $\tilde{a}'_4$, respectively. Examining the reversal with the probability triangle diagram indicates that the isolation effect is not related to the independence axiom. The "fanning-out" indifference curves cannot explain this anomaly. The variations in the decision frames invoke different decision-making processes which lead to "unstable" preference. Prospects $\tilde{a}_3$, $\tilde{a}_4$, $\tilde{a}'_3$, and $\tilde{a}'_4$ are depicted in the probability triangle in Figure 3.4.

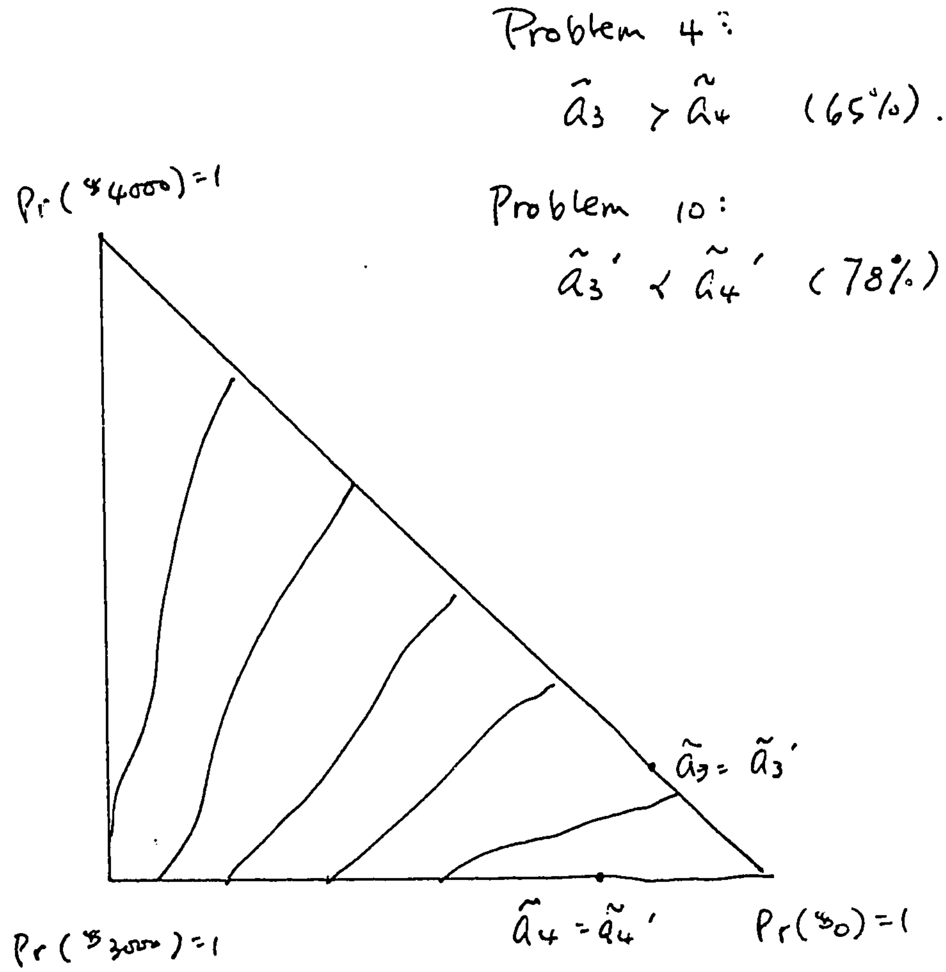


FIGURE 3.4 ANOMALIES RELATED TO DECISION FRAMES ARE NOT CONSISTENT WITH FANNING-OUT INDIFFERENCE CURVES

3.1.5 The Reflection Effect

Kahneman and Tversky (1979, p.268) observed that the choice pattern was reversed if the prospects involved losses instead of gains. The preferences over positive prospects exhibited risk-aversion while preferences over negative prospects exhibited risk-seeking attitudes. Consider $\tilde{d}_1$, $\tilde{d}_2$, $\tilde{d}_3$, and $\tilde{d}_4$ which are the negative counterparts of $\tilde{a}_1$, $\tilde{a}_2$, $\tilde{a}_3$, and $\tilde{a}_4$, respectively.

PROBLEM 3′: (N = 95)

Choose between

$\tilde{d}_1$:	0.80 chance of -$4,000;	[92%]
$\tilde{d}_2$:	1.00 chance of -$3,000.	[8%]

PROBLEM 4′: (N = 95)

Choose between

$\tilde{d}_3$:	0.20 chance of -$4,000;	[42%]
$\tilde{d}_4$:	0.25 chance of -$3,000.	[58%]

The results showed a reversal in preferences between gains and losses. ALthough the choice patterns $\tilde{a}_2 \} \tilde{a}_1$ and $\tilde{a}_3 \} \tilde{a}_4$ were obtained under the common ratio effect, the rankings became $\tilde{d}_1 \} \tilde{d}_2$ and $\tilde{d}_4 \} \tilde{d}_3$. Note that the common ratio effect also extended to the negative prospects. The reflection effect was different from the response mode effect. The response mode effect created the image of negative prospects by changing the decision frames; the prospects in the two problems still yielded the same positive outcomes. Under the reflection effect, the positive prospects were transformed into negative prospects.

3.1.6 The Preference Reversal Phenomenon

Another widely observed anomalous behaviour that reflects intransitive pref[illegible] is the preference reversal (PR) phenomenon. This phenomenon i[illegible] from experiments offering lottery $\tilde{A}$ with high probability t[illegible] modest amount (called the P-bet) and lottery $\tilde{B}$ with considerably lower probability to win a large amount (called the \$-bet). In the experiments, subjects often choose the P-bet over the \$-bet; yet, they would assign a higher monetary value to the latter. The problem can be structured as the following:

$V(\tilde{A}) > V(\tilde{B})$ - since individual chooses P-bet over \$-bet.

$V(\tilde{A}) = EU(\tilde{A}) = U(CE(\tilde{A}))$ - individual assigns a monetary value to $\tilde{A}$ which is $CE(\tilde{A})$, the certainty equivalence to lottery $\tilde{A}$.

$V(\tilde{B}) = EU(\tilde{B}) = U(CE(\tilde{B}))$ - individual assigns a monetary value to $\tilde{B}$ which is $CE(\tilde{B})$, the certainty equivalence to lottery $\tilde{B}$.

$CE(\tilde{A}) < CE(\tilde{B}) \Leftrightarrow V(\tilde{A}) < V(\tilde{B})$ because $U(\cdot)$ is a monotonic increasing function.

Lichtenstein and Slovic (1971) first discovered the reversals between bids and choices in gambling experiments. Many later studies introduced variations in the experimental design to increase motivation and reduce the possibility of confusion and errors. These studies hypothesized that (i) if the stakes are large enough, people will get it right, (ii) people need time to learn, and (iii) competition and arbitrage will eliminate the irrational agents (Thaler, 1987).

Lindman (1971) tried to eliminate the contextual factors that might have contributed to the reversals. Lichtenstein and Slovic (1973) conducted their experiment in a casino in downtown Las Vegas, hoping that it would replicate the real world scenario and would improve the motivation to reveal true preferences. Grether and Plott (1979) explicitly identified 13 economic and psychological factors that might cause explain PR. However, they failed in each attempt to eliminate the PR phenomenon.

Pommerehne, Schneider, and Zweifel (1982) replicated the experiments in Grether and Plott (1979) with 3 variations aimed at increasing the incentive of the subjects. They increased the face value of the stakes and rewarded the subjects on a pro rata basis.[3] They also increased the differences in the expected values between the P-bet and the $-bet, with the P-bet having distinctly higher expected payoffs in some gambles, and the $-bet in others. Finally, they rewarded subjects based on their cumulative performance on a pro rata basis; they hypothesized that PR will be reduced in later runs because of learning. Furthermore, given that they were rewarded on a pro rata basis, subjects would have higher incentive to improve on their positions in the later runs. Their experimental results showed a lower percentage of PR when compared to Grether and Plott's results (1979). Yet, the percentage of choices that indicated PR remained persistently high at 50%. Reilly (1982) studied the effect of providing participants with more information about the gambles (their expected values). The group with more information reported slightly more consistent choices than the other group. A substantial percentage of PR, however, remained in both groups (p.581, Table 2).

Most of the these studies in the 70s and the early 80s were motivated by a skepticism about PR. Their major emphasis was to eliminate PR by offering "adequate" incentive and controlling for other factors. As these efforts failed to "correct" the subjects' decision-making process, researchers started to recognize PR as a consistent preference pattern. Researchers rejoined the search for the cause(s) of PR; they tried to develop alternative models which could accommodate PR. The discussion of these alternative models is covered in Sections 3.3 and 3.4 below which explore the development in decision theory after the EU model.

The simplest explanation for the PR phenomenon is that it reflects irrational (albeit consistent) behaviour. However, the idea of irrationality is very foreign to, and is usually rejected by, economists; since economists always assume human behaviour as rational (Simon, 1987, p.25).

> "(Economists) are prepared to make whatever auxiliary empirical assumptions are necessary in order to preserve the utility-maximization postulate, even when the empirical assumptions are

unverified (Simon, 1987, p.38.)"

Therefore, PR requires modification of the definition of rationality to reconcile decision theory and the observed data.

March (1978, pp.591-3) identified seven alternative definitions for rationality. Each of these concepts emphasizes a specific aspect of the decision-making process.[4] He justified these alternative definitions for rationality by examining the notion of rationality under the context of uncertainty.

> "Rational choice involves two kinds of guesses: guesses about future consequences of current actions and guesses about future preferences for those consequences" (March, 1978, p.589.)

When decision is reached following the holistic approach, every alternative is assigned a utility index and the alternative that yields the highest value will be selected. Consequently, the evaluation function in a holistic model explicitly considers all the possible consequences of each action. The resulting preference orderings among the alternatives are assumed to be complete. Furthermore, holistic judgment models assume that future consequences are exogenous, stable, and precise; hence, the nature of uncertainty in the second guess is ignored. In doing so, consistency is built into an overall scheme of decision criteria. This characteristic of holistic models is called procedure invariance in the literature (see, for example, Tversky, Slovic, and Kahneman, 1990). Procedure invariance assumes that preference orderings depend solely on the contents of the prospects whereas the context of the problems or the sequence of evaluation should not matter. Procedure invariance is possible only if the evaluation function is unique, stable, and consistent; such an evaluation function is the characteristic of holistic judgment models.

Alternative rationalities, on the other hand, acknowledge the impreciseness in preference due to information processing constraints (March, 1978) and assume that the decision-making process is procedure dependent. Procedure dependence usually occurs in a non-holistic model which does not assume a unique evaluation function. This impreciseness in the evaluation process may be caused by response mode bias (the framing effect) or binary comparison. Response mode bias arises when the individual has to respond to the decision problem in different contexts. Therefore, the evaluation procedure is dependent on the problem context. The decision-making process in binary comparisons is also procedure dependent because the evaluation function is dependent on the two prospects under consideration; replacing one of the two prospects will lead to a different evaluation function. Furthermore, binary comparison in lexicographic models also tends to ignore some aspects of the prospects in the sequential elimination process.

When the decisions about judgment (for example, elicitation of selling price) and about choices (selecting the preferred option) are considered as two distinct decision processes, evaluation is subjected to the response mode bias. In other words, "the way in which an individual has to respond to the decision problem is an important aspect of framing" (Slovic, Fischhoff, and Lichtenstein, 1982, p.28). As a result of this hidden bias, PR may occur.

Tversky and Kahneman (1981), for example, studied the impact of framing on preferences. They suggested that, in some cases, the context (framing) of the problem will lead to PR in spite of identical (economic or social) consequences in both prospects. More discussions and empirical studies can be found in Hogarth (1975); Slovic, Fischhoff, and Lichtenstein (1982), Hershey and Schoemaker (1985), MacCrimmon and Wehrung (1986), and Tversky, Slovic, and Kahneman (1990).

An alternative explanation for PR was investigated by Holt (1986), Karni and Safra (1987), and Segal (1988).[5] They suggested that the apparent violations of transitivity revealed in PR might be caused by shortcomings in the experimental design to elicit selling prices for the two lotteries. Each paper examined the possibility that PR constitutes a violation of the independence axiom of the EU model. Karni and Safra (1987) pointed out an important link overlooked by previous studies, and this mistake led to the conclusion that PR was caused by intransitivity. They examined the validity of the unnoticed and unchallenged assumption that the certainty equivalent of a lottery is equal to its elicited selling price. They proved that, under reasonable restrictions on the set of preference relations over lotteries,[6] the certainty equivalent and the elicited selling price of a lottery are identical if and only if the independence axiom is satisfied (Karni, 1987, p.941). In other words, it is possible to find some pair of lotteries which generate PR when the independence axiom is violated (while other EU axioms are intact, in particular, the transitivity axiom).

Holt (1986), in experiments using the random lottery selection method to control for income effects, also found that violation of the independence axiom would lead to PR. With this method, subjects were presented with two lotteries (P-bet and \$-bet); they were asked to choose one of the two lotteries and to give the minimum selling prices for both lotteries. Subjects knew that their payoffs were determined by a subsequent gamble with 1/3 chance to be awarded the lottery of their preferred choice, the P-bet selling price, or the \$-bet selling price. Each subject was faced with a compound lottery decision to choose between (1/3, P-bet; 1/3, R_P; 1/3, $R_\$$) and (1/3, \$-bet; 1/3, R_P; 1/3, $R_\$$) where R_P and $R_\$$ are the reservation prices for the P-bet and the \$-bet, respectively. The compound lottery decision would be identical to the single-stage decision if the subject's preferences satisfy the independence axiom. In addition, the preferred lottery would command a higher reservation price. However, Holt showed that if the independence axiom is violated, the lottery choice and the elicited selling price decisions in the second-stage gamble are not separable (p.511). In this case, an individual who assigned a higher reservation price to the \$-bet might prefer the P-bet in the compound lottery decision (Holt, 1986, p.514). Holt asserted that this pattern is similar to the common consequence effect with negative prospects in Kahneman and Tversky (1979); the elicitation of selling prices would add a common term to the compound lotteries. Thus, PR occurs as a result of nonindependent instead of non-transitive preferences, for the common consequence effect is undoubtedly caused by the violation of the independence axiom.

Instead of reducing the evaluation and the selection into a compound lottery that contains the elicited selling prices and one of the P-bet or $-bet, Segal (1988) characterized the task of choice and the task of valuation as a two-stage lottery. The first stage involves the uncertain prospect that the elicited selling price is greater than the random offer. If the random offer is believed to be less than the elicited selling price, the second-stage gamble becomes playing out the lottery. Otherwise, the individual will take the chance to receive the yet to be known random offer as the second-stage lottery. Segal showed by a numerical example (p.235) that an individual who violates the reduction principle may report a higher selling price for the $-bet and still prefer the P-bet. Segal's theory concentrated on the violation of the reduction principle while restricting the independence axiom to multi-stage lotteries. If the conventional definition of independence was used, Segal's work showed that the PR phenomenon may be caused by violation of the independence axiom (Loomes, Starmer, and Sugden, 1991, p.427).

Recent studies have tended to argue that PR is caused by factors other than the violation of the independence axiom. Unlike Holt (1986), Karni and Safra (1987), and Segal (1988), Cox and Epstein (1989) designed experiments which were strictly a one-stage lottery to elicit preferences and selling prices. Hence, the possibility of violating the reduction principle or the independence axiom was eliminated. They found that more than 30% of 540 decisions in their two experiments indicated PR. They carefully identified the wealth effect, the outcome effect, and the framing effect and examined their impacts on the preferences. They concluded from a logit analysis that these effects could not account for the choice reversals. They suggested that "the choice reversals in our experiments are violation of the asymmetry axiom,[7] which is even more fundamental than transitivity (Cox and Epstein, 1989, p.422)."

Loomes, Starmer, and Sugden (1989, 1991) studied PR in the context of Regret theory. (Regret theory is reviewed in Section 3.4.) Their experiments were designed to elicit the choices in pairwise comparisons between 3 prospects. Instead of eliciting selling prices for the P-bet and the $-bet, the authors used different specified amounts for the certainty offer. In this way, the potential problems associated with compound lotteries[8] and information processing[9] were eliminated. This experimental design would identify factors other than the violation of the independence axiom and information processing effects. It should be noted that their results did not eliminate the potential problems caused by the two controlled factors; instead their experiment highlighted the non-transitive nature of choice under uncertainty. They demonstrated that a particular preference cycle[10] is consistent with decisions generated by Regret theory if the $-bet's payoff is not worse than the P-bet's payoff in the unfavorable state (Loomes, Starmer, and Sugden, 1989, p.142)[11]. Their results showed that the majority of PR instances belonged to the "predicted" preference cycle. In one experiment, among 283 subjects' decisions, 26 out of 29 observed PR were consistent with the prediction of Regret theory (Loomes, Starmer, and Sugden, 1989, p.147, Table 1). Other experiments indicated the same pattern in favour of Regret theory (see Loomes, Starmer, and Sugden 1989, pp.147-9, Tables 2 and 4, and Loomes, Starmer, and Sugden 1991, p.437, Tables 4 and 5).

Tversky, Slovic, and Kahneman (1990) identified three possible causes for PR: (i) violation of the independence axiom or the reduction principle, (ii) violation of the transitivity axiom, and (iii) violation of procedure invariance. Procedure invariance assumes that the context of the problem or the method of evaluation will not alter the ranking of the prospects. Their study showed that PR cannot be explained by violation of the independence axiom or the reduction principle (p.214). They found that the two-stage elicitation procedure which requires the reduction principle (or the independence axiom) generates similar frequency of PR cases as the one-stage elicitation procedure. The argument put forward by Holt (1986), Karni and Safra (1987), and Segal (1988) was dismissed as a valid explanation for PR. Secondly, the diagnostic procedure in Tversky, Slovic, and Kahneman (1990) indicated that 90% of the observed PR could be attributed to a failure of procedure invariance while only 10% of PR were caused by violation of intransitivity (Tversky, Slovic, and Kahneman, 1990, p.210, Table 3). The failure in procedure invariance (biases in the evaluation procedure) was mainly due to overpricing of the \$-bet (the other form of procedure invariance is underpricing of the P-bet). 83.9% of the results indicated this kind of procedure dependence (Tversky, Slovic, and Kahneman, 1990, p.210,

Table 3). They reasoned that this consistent pattern in overpricing the $-bet was caused by a response mode effect called scale compatibility. By scale compatibility, they meant "the prices and the payoffs are expressed in the same units (Tversky, Slovic, and Kahneman, 1990, p.214)." Consequently, payoffs were weighted more heavily in pricing which led to overpricing of the $-bet although subjects preferred the P-bet.

The conclusion in Tversky, Slovic, and Kahneman (1990) is similar to the concept of alternative rationality (March, 1978; and Simon, 1987) discussed above. It is interesting to note that among 13 economic or psychological explanations examined in Grether and Plott (1979, p.625, Table 1), response mode bias is the only explanation which accounts for all the PR cases in the then available experimental studies.

3.1.7 Other Experimental Anomalies

The validity of the axioms of the EU model were tested in laboratory experiments. Mosteller and Nogee's (1951) experiment did not support the completeness axiom. Coombs (1975) found that nearly half of his subjects' preferences violated the mixture-continuity axiom. Tversky (1969) observed systematic and predictable violations of transitivity. McCord and de Neufville (1983, 1984) found that the certainty equivalents revealed by the subjects were not consistent with the independence axiom.

The completeness axiom states that an individual is able to give a definite ranking between two alternatives. Any decision model will adopt some kind of selection criterion which implicitly recognizes the decision maker's ability to give consistent preference orderings. Mosteller and Nogee (1951), who were among the first to test the EU model, reported inconsistent preference orderings in repeated experiments. The design of their experiments was a variation of poker dice. The subjects were shown a stimulus card in each play which contained a "hand" and the payoff to be won for a 5¢ wager. The subject won the payoff if the result of his "hand" from rolling five dice beat the "hand" in the stimulus card; he lost 5¢ otherwise. There were seven different payoff amounts for each of the seven "hands". One of 49 possible offers was to be shown for each play. The subjects could accept or reject to play each time a stimulus card was shown. When increasing the payoff of a bet gradually and keeping the probability to win constant, they did not observe a value-threshold where subjects always rejected a bet offering less than this threshold payoff value and always accepted a bet offering more than this threshold. Instead, they observed that the decision maker will exhibit a vacillation in determining his preference. "(A)s the offer increases, ..., the bet is taken occasionally, then more and more often, until, finally, the bet is taken nearly all the time. There is not a sudden jump from no acceptance to all acceptances at a particular offer ... (p.374.)" They found that subjects gradually increased the frequency of risks taken as the value of the risk increased (p.404). These findings were inconsistent with the all-or-none assumption implied by the completeness axiom.

The mixture-continuity axiom was tested by Coombs (1975). He asked subjects to rank three prospects A, B, and C; where C is a probability mixture of A and B. Either A ≻ C ≻ B or B ≻ C ≻ A is consistent with the mixture-continuity axiom. The results show that only about 54% of the subjects reported either one of the above orderings. In other words, the preference orderings of more than 46% of the subjects violated this axiom. Similar results had been observed by Becker, Morris, and Marschak (1963).

The inherent variability and momentary fluctuation in the decision-making process as shown in Mosteller and Nogee (1951) suggests that preference should be defined in probabilistic fashion, at least for preference assigned to uncertain prospects. The notion of stochastic transitivity was introduced by Tversky (1969). The preference between two risky alternatives x_1 and x_2 is defined as

$$x_1 \succsim x_2 \Leftrightarrow Pr(x_1, x_2) \geq 1/2 \qquad (3.1)$$

where $Pr(x_1, x_2)$ is the probability of choosing x_1 over x_2, and

$$Pr(x_1, x_2) + Pr(x_2, x_1) = 1.$$

This is called weak stochastic transitivity (Tversky, 1969, p.31; see also Becker, DeGroot, and Marschak, 1964).

Tversky (1969) assessed the nature of systematic and predictable violations of weak stochastic transitivity. He observed that subjects using a lexicographic semi-order decision rule[12] consistently revealed intransitive preference orderings (Tversky, 1969, p.35, Table 2). Yet, in follow-up interviews, the subjects indicated that they believed their choices would be transitive (p.36)!

McCord and de Neufville (1983, 1984) examined the utility evaluation effect in a study trying to "recover" the utility functions of the subjects. The process of eliciting the certainty equivalents of lotteries will generate information about the utility curve. Consider the following three lotteries and their certainty equivalents CE_{ij}, j = 1, 2, 3 and i denotes probability distribution $(p_i, 1-p_i)$:

$$U(CE_{11}) = p_1U(M) + (1-p_1)U(0),$$
$$U(CE_{12}) = p_1U(CE_{11}) + (1-p_1)U(0), \text{ and}$$
$$U(CE_{13}) = p_1U(M) + (1-p_1)U(CE_{11}).$$

Together with the independence axiom, all three lotteries can be expressed in terms of probability distributions with the common outcome space (M,0) as follows:

$$U(CE_{11}) = p_1U(M) + (1-p_1)U(0),$$
$$U(CE_{12}) = (p_1)^2U(M) + [1-(p_1)^2]U(0),$$
$$U(CE_{13}) = p_1[1+(1-p_1)]U(M) + (1-p_1)^2U(0).$$

If the utility index of U(M) is normalized to equal to 1 and U(0) = 0, then $U(CE_{11}) = p_1$, $U(CE_{12}) = (p_1)^2$, and $U(CE_{13}) = p_1[1+(1-p_1)]$. For some $p_i = p_1$, subjects were asked to give estimates of utility for CE_{11}, CE_{12}, and CE_{13}. Consequently, three pairs of co-ordinates $\{CE_{11}, p_1\}$, $\{CE_{12}, (p_1)^2\}$, and $\{CE_{13}, p_1[1+(1-p_1)]\}$ will recover three points on the utility curve. This is shown in Figure 3.5. As p_i varies, a new set of certainty equivalents will be obtained. Three pairs of co-ordinates $\{CE_{21}, p_2\}$, $\{CE_{22}, (p_2)^2\}$, and

$\{CE_{23}, p_2[1+(1-p_2)]\}$ are derived with $p_1 = p_2$ where $p_2 > p_1$. These points are also shown in Figure 3.5. Such an exercise will generate enough information to construct the utility curves of the subjects.

McCord and de Neufville (1983, 1984), however, found that for different values of p_1 the subjects formed different utility curves. They discovered that increasing the likelihood of a prospect will move the entire utility function to a higher position, instead of moving along one particular utility function when the chance to win increases (see Machina, 1987a, pp. 130-1). For example, when $p_2 = p_1[1+(1-p_1)]$; CE_{21} is less than CE_{13} (instead of being equal as implied by the independence axiom). For a set of lotteries that has a higher likelihood to be won, subjects would assign to them higher utility indices. This phenomenon is shown in Figure 3.6. This evid ~e supported ˙ ıe certainty effect where individuals overweight ˙ ..ospect which they considered as more certain than other prospects.

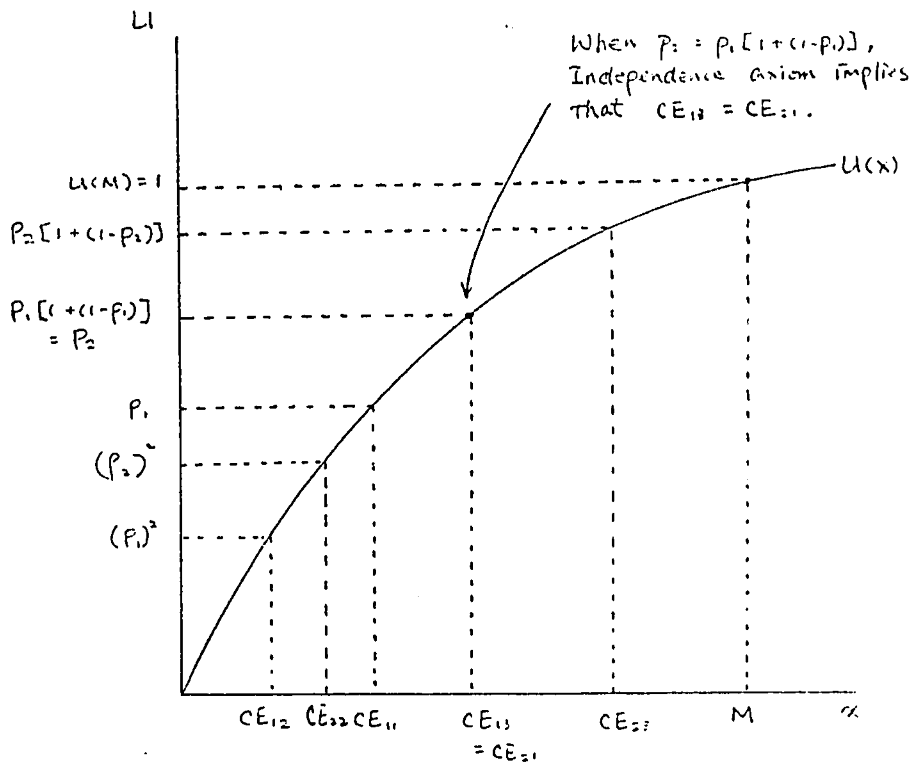


FIGURE 3.5 RECOVERING UTILITY CURVE WITH THE INDEPENDENCE AXIOM

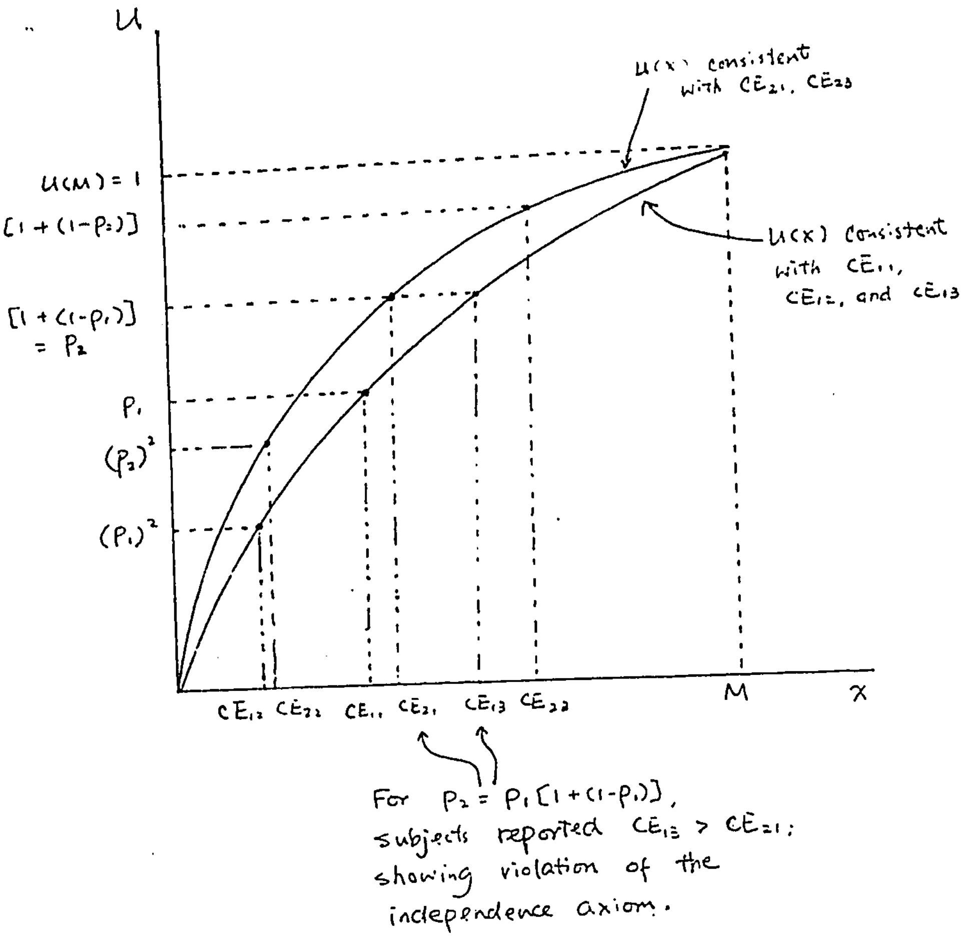


FIGURE 3.6 ACTUAL UTILITY CURVES RECOVERED CONFIRMING THE VIOLATION OF THE INDEPENDENCE AXIOM

3.2 ANOMALIES FOUND IN FIELD STUDIES

This section will discuss some of the anomalies found in "real world" economic decisions. The maximization of expected utility as the decision criterion for risky alternatives has been repeatedly rejected by empirical evidence ever since it was introduced by Bernoulli.

3.2.1 Co-existence of Gambling and Insurance

The existence of gambling activity posed a serious problem for the EU model. The then prevailing belief in diminishing marginal utility by the classical economists rendered the problem unsolvable (Friedman and Savage, 1948, p.280). Diminishing marginal utility implies that the utility function is strictly concave and $U''(\cdot) < 0$; an individual always exhibits risk-averse behaviour. Hence, the belief in diminishing marginal utility implied that an individual will never participate in a gamble even if it is actuarially fair. It was recognized later that diminishing marginal utility is not a necessary assumption to explain riskless consumer choices (see Green, 1976, pp.90-1). This finding implied that participation in "unfair" gambles characterized risk-seeking attitudes.

The usefulness of the EU model, however, remains unclear. This is exemplified by gambling and insurance phenomena. The EU model does not account for the co-existence of risk-seeking (gambling) and risk-avoiding (buying insurance) behaviour. In a large survey of 1075 farm managers, Johnson et. al. (1961) documented that some of the respondents accepted both actuarially unfair gambles and actuarially unfair insurance schemes (see Schoemaker, 1980, p.22). Friedman and Savage (1948) proposed an inverted S-shape utility function to harmonize the coexistence of gambling and buying insurance (Figure 3.7(a)). Yet, this inverted S-shape utility curve implies that an individual would prefer any fair gamble offering a large prize; it fails to address the St. Petersburg paradox. Consequently, they modified their proposed utility curve by imposing a terminal concave section to resolve the St. Petersburg paradox. Therefore, the suggested utility function contains one convex and two concave segments. (see Figure 3.7(b)). Friedman and Savage regarded the two concave segments as corresponding to qualitatively different socioeconomic levels, and the convex segment to the transition between the two levels. A gamble which promises a large prize with a small probability) will promote the individual into the next socioeconomic level; despite risk aversion, he still has great aspiration to take such risk.

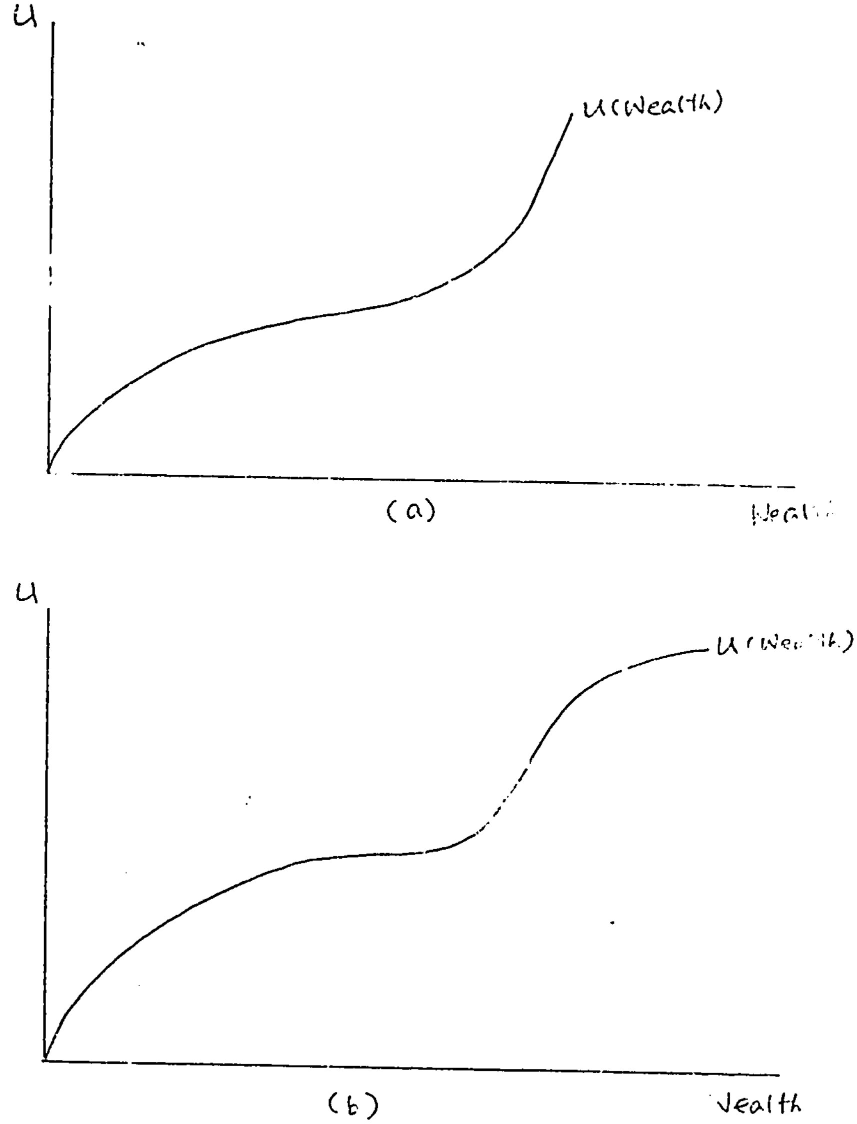


FIGURE 3.7 FRIEDMAN-SAVAGE UTILITY FUNCTION

3.2.2 The Problems of Over-Insurance

In a flight insurance study, Eisner and Strotz (1961) found that travellers' strong demand for air-trip insurance was not consistent with the EU model. They reasoned that a general coverage (life) insurance against death from all causes (accidental, of course) should be better than a specific policy; especially when the latter is more expensive than the former (p.365) Assuming that individuals were EU maximizers and already had the optimum coverage, the widespread phenomenon of purchasing air-travel insurance violated the EU model's prediction. They offered three explanations for this anomaly (pp.365-6). First, they suggested that people might under-estimate the probability of general risks; relatively, they over-estimated the probability of a specific risk. Therefore, they would regard air-travel insurance as substantially cheaper than general life insurance.[13] The second explanation they offered was that people were under-insured and air-travel insurance was a convenient way to provide temporary coverage. They resorted to the Friedman-Savage utility function as the third explanation for the anomaly they observed.

Katona (1965) studied the effects of private pension plans on discretionary savings and discovered behaviour that was inconsistent with the EU model. Traditional economic theory hypothesized that private pension plans (as a form of forced savings) would displace discretionary savings.[14] Katona, on the contrary, discovered that the average saving ratio of people covered by pensions was higher than people uncovered (p.6). He attributed this phenomenon to psychological factors. He quotes from Cagan (1963, p.23): "Realization of retirement needs and of the opportunities for financial independence opened up by pension stimulate the motivation to save." This aspiration level hypothesis acknowledges factors unaccounted for in the EU model.

Pashigian et. al. (1966) discovered a strong preference for expensive low deductible automobile insurance. Using a quadratic utility function, they constructed a testable condition according to the EU hypothesis to examine this phenomenon. If EU maximization truly reflects the insured's decision-making process, then the following conditions should be satisfied (Pashigian et. al., 1966, p.39, Equation 9):

$$p^* \le \frac{-\partial R}{\partial D} \le \frac{(R + D)\cdot p^*}{R + Dp^* + \bar{L}} \qquad (3.2)$$

where D is the size of the deductible, R = R(D) is the annual premium, p^* is the probability that a loss in excess of D will occur, and $\bar{L}$ is the mean loss for losses less than D.

A risk-neutral individual will set $p^* = \frac{-\partial R}{\partial D}$; such an individual is willing to buy additional coverage (lower D) only when it is offered at the actuarially fair rate (p^*). An extremely risk-averse individual will set $\frac{-\partial R}{\partial D} = \frac{(R + D)\cdot p^*}{R + Dp^* + \bar{L}}$.[15] This extremely risk-averse individual is willing to increase coverage as long as the marginal cost of increasing coverage is less than $\frac{(R + D)\cdot p^*}{R + Dp^* + \bar{L}}$, the ratio of expected cost when the loss is greater than D to the overall expected cost when there is an accident. The aggregate data (14 areas in the State of Missouri) clearly rejected the inequalities stated above for all three classes of deductibles (D = $50, $100, or $250) (Pashigian et. al., 1966, p.40, Table 2). Pashigian et. al. derived the average premium (total written premium divided by total number of automobiles insured) and found that the above inequalities were satisfied for the D = $250 class only (Pashigian et. al., 1966, p.41, Table 4). Yet, more than 99% of the policies had $50 or $100 deductibles (Pashigian et. al., 1966, p.41, Table 3). These results showed that if the quadratic specification for the utility function is a good approximation. the EU model failed to explain the choice of deductible in the automobile insurance market.

3.3.3 The Problem of Under-Insurance

Another anomaly was revealed in data on insurance against flood and earthquake disasters. Kunreuther (1976, p.231) found that homeowners in high risk regions were uninsured in spite of Federal subsidies of up to 90%. (See also Anderson (1974) on flood insurance and Kunreuther et. al. (1978) for a national study on flood and earthquake insurance.) From the results of face-to-face interviews which involved 2055 flood plains homeowners and 1006 homeowners living in earthquake regions, Kunreuther solicited their subjective estimates for the important decision variables like premium, probability of a disaster, and the magnitude of the loss. By design, about half of the respondents in each catastrophe category was insured (Kunreuther, 1976, p.233, Table 1). The survey showed that 68% of the uninsured homeowners in flood plains and 76% of the uninsured homeowners in earthquake areas were unable to estimate the cost of insurance policies. An even higher percentage of these homeowners, 82% and 85% respectively, were unable to estimate the deductible amount (Kunreuther, 1976, p.234, Table 2). Nine percent of uninsured flood plain homeowners could not provide an estimate for expected damage and 29% expected no damage from a severe flood. The corresponding figures obtained from the earthquake area homeowners were 8% and 12%, respectively (Kunreuther, 1976, p.234, Table 3). Fifteen percent of uninsured flood plains homeowners and 10% of uninsured earthquake area homeowners were not able to estimate subjectively the probability of a catastrophe in their regions. Hence, these homeowners were not able to use the EU model to evaluate the attractiveness of insurance. Kunreuther also found that a smaller, but still significant, proportion of the insured homeowners had problems in providing usable estimates as well.

To evaluate the EU model, Kunreuther constructed the contingency-price ratio; $k = \frac{(1-p)c}{p(1-c-t)}$ where p is the probability of a disaster, c is the per dollar cost of insurance, t is the tax write off on uninsured losses and it is assumed to be equal to 30%. The contingency-price ratio, k, measures "the expected cost of insurance should a disaster not occur [(1-p)c] to the expected net gain in assets from insurance [(1-c-t)] (Kunreuther, 1976, p.230)." According to the EU model, homeowners would adjust the amount of protection such that $\frac{U'(\text{Disaster State})}{U'(\text{Non-disaster State})} = k$ for optimizing behaviour. In other words, the ratio of marginal utility for the last dollar spent in insurance coverage is equal to the contingency-price ratio. Based on usable estimates from the interviews, Kunreuther classified these samples into 6 categories according to the derived values of k. Only 37% of the flood-insured homeowners had estimates of p and c that were clearly consistent with the EU model (Kunreuther, 1976, p.238, Table 6).

Kunreuther's findings showed that the EU model was not a good decision model for low probability events. The recognition of limited information processing capacity of decision makers led him to recommend a sequential choice process for insurance decisions. In the sequential model, individuals will go through four separate stages before reaching a purchasing decision. This model limits the amount of information processing in each stage and recognizes that individuals may fail to reach a plausible decision if they do not have enough information (or inadequate awareness of the issue) in the preliminary stages (Kunreuther, 1976, p.245, Figure 1).

3.3 ALTERNATIVE HOLISTIC JUDGMENT MODELS

The dubious predictive power of the EU model sparked the search for a better alternative. The models surveyed in this section are listed in Table 3.1; they closely resemble the structure of the EU model in Equation (2.3). They are usually called holistic judgment models. According to the holistic approach, every alternative is assigned a utility index and the alternative that yields the highest value will be selected.[16] The non-holistic models are covered in Section 3.4.

TABLE 3.1

HOLISTIC JUDGMENT MODELS FOR CHOICE UNDER RISK AND UNCERTAINTY

Subjective Expected Utility	(Edwards, 1955)
$$\sum_{j=1}^{n} f(p_j) \cdot U(x_j)$$	
Optimism / Pessimism	(Hey, 1984b)
$$\sum_{j=1}^{n} f(p_j; \tilde{x}) \cdot U(x_j)$$	
Prospective Reference Theory	(Viscusi, 1989)
$$\sum_{j=1}^{n} p_j^* \cdot v(x_j) \text{ where } p_j^* = \frac{\gamma q_j + \xi p_j}{\gamma + \xi}$$	
Subjective Weighted Utility	(Karmarkar, 1978)
$$\sum_{j=1}^{n} \frac{\pi(p_j)}{\sum_i \pi(p_i)} \cdot U(x_j)$$	
Expected Utility with Rank Dependent Probabilities	(Quiggin, 1982; Yarri, 1987)
$$\sum_{j=1}^{n} [g(\sum_{i=1}^{j} p_i) - g(\sum_{i=1}^{j-1} p_i)] \cdot U(x_j)$$	

Table 3.1 continues ...

Choquet Expected Utility[1] (Schmeidler, 1987)

$$\sum_{j=1}^{n} [V(U^{j}) - V(U^{j-1})] \cdot U(x_j)$$

Ordinal Independence (Chew, 1984; Green and Julien, 1988; Segal, 1984)

$$\sum_{j=1}^{n} [\psi(x_j; \sum_{i=1}^{j} p_i) - \psi(x_j; \sum_{i=1}^{j-1} p_i)]$$

Cumulative Utility[1] (Chew and Wakker, 1991)

$$\sum_{j=1}^{n} [W(x_j; U^{j}) - W(x_j; U^{j-1})]$$

Generalized Expected Utility (Machina, 1982a)

$$\sum_{j=1}^{n} p_j \cdot v(x_j) + \left[\sum_{j=1}^{n} p_j \cdot \tau(x_j)\right]^2$$

Mixture Symmetric Utility (Chew, Epstein, and Segal, 1991)

$$\sum_{i=1}^{n} \sum_{j=1}^{n} p_i p_j \cdot T(x_i; x_j)$$

Weighted Utility (Chew, 1983; Chew and MacCrimmon, 1979a,b; Fishburn, 1983)

$$\sum_{j=1}^{n} \frac{\tau(x_j)p_j}{\sum_i \tau(x_i)p_i} \cdot U(x_j)$$

1 U^j denotes union of events 1 to j.

3.3.1 Models with Subjective Probabilities and Decision Weights

In the EU theory, probabilities (p_i) are assumed to be objectively given and obey the axioms of probability theory (for example, $0 \le p_i \le 1$, and $\sum_i p_i = 1$). Several theories have replaced the objective probabilities in the EU model by subjective probabilities, Bayesian probabilities, and various forms of decision weights. In general, subjective probabilities or decision weights do not necessarily conform to the axioms of probability theory. Consequently, due to this relaxation, these models will accommodate the observed anomalous behaviour.

3.3.1.1 The Subjective Expected Utility theory

The Subjective Expected Utility (SEU) theory assumes that decision makers assign probabilities to different states of nature subjectively. Therefore, the SEU theory maintains that individual choices of prospects (alternatives) are rational; any apparent conflicts between the observed evidence and the EU model will disappear if the "appropriate" probabilities are used (Hey, 1984a). Edwards (1955, p.204) derived a scheme to measure subjective probabilities $f(p_i)$ when a utility scale of all the outcomes was determined. For example, if a subject revealed that he was indifferent between a sure gain of \$2.40 and a 95% chance of winning \$2.50, then $f(0.95)\ U(\$2.50) + f(0.05)\ U(\$0) = f(1)\ U(\$2.40)$. Using the assumptions that $f(1) = 1$ and $U(\$0) = 0$, the subjective probability for $p_i = 0.95$ was obtained from the ratio of the two utilities indices: $f(0.95) = U(\$2.40)/U(\$2.50)$.

The major axiomatic difference between subjective and objective probabilities is that $f(p_i)$ is allowed to be greater than 1.[17] Edwards (1955) indeed found this phenomenon in his experiments. Other than this, subjective and objective probabilities have similar axioms like $f(0) = 0$, $f(1) = 1$, and

$\sum_i f(p_i) = 1$.

The SEU theory maintains the axiom of stochastic dominance but it is defined with respect to the subjective probabilities. The axiom of independence is not compatible with the SEU theory. The following two gambles illustrate the incompatibility.

$\tilde{e}_1$: 81% chance to receive \$1 million,
19% chance to receive \$0; or

$\tilde{e}_2$: 20% chance to receive \$5 million,
60% chance to receive \$1 million,
20% chance to receive \$0.

and

$\tilde{e}_3$: 21% chance to receive \$1 million,
79% chance to receive \$0; or

$\tilde{e}_4$: 20% chance to receive \$5 million,
80% chance to receive \$0.

The SEU theory would not predict that an individual who chooses $\tilde{e}_1$ will choose $\tilde{e}_3$ because $f(0.81)$ does not equal $f(0.60) + f(0.21)$.[18] Therefore, the prediction of the SEU theory can be consistent with the common consequence effect. The SEU theory obeys the axiom of independence only when f(1) is involved in the prospect.[19]

3.3.1.2 The optimism-pessimism model

Hey (1984b) recognized optimistic and pessimistic attitudes as affecting the determination of the subjective probabilities. His model (1984b, p.188) was stated as follows:

$$U(C_i) = \sum_{j=1}^{n} f(p_j; \tilde{x}_i)\, U(x_{ij}). \tag{3.3}$$

C_i denotes choice i which is associated with the vector of possible outcomes $\tilde{x}_i$; x_{ij} is the outcome associated with the jth state (probability p_j) when C_i is selected. $f(p_j; \tilde{x}_i)$ denotes the subjective probability of the jth state adjusted by the individual's attitude to fate. When $f(p_j; \tilde{x}_i) > p_j$, it indicates that the individual is optimistic towards obtaining x_{ij} relative to $\tilde{x}_i$. Pessimism is reflected by $f(p_j; \tilde{x}_i) < p_j$. When optimism or pessimism exists, linearity in probabilities is violated.

The common ratio effect can be interpreted as follows:

$\tilde{a}_1$: 0.80 chance of $4,000;

$\tilde{a}_2$: 1.00 chance of $3,000.

and

$\tilde{a}_3$: 0.20 chance of $4,000;

$\tilde{a}_4$: 0.25 chance of $3,000.

The common ratio effect is observed when $\tilde{a}_2 \succ \tilde{a}_1$ and $\tilde{a}_3 \succ \tilde{a}_4$.

$\tilde{a}_2 \succ \tilde{a}_1 \Rightarrow f(1.0;\ \$0,\$3000)\cdot U(\$3000) > f(0.8;\ \$0,\$4000)\cdot U(\$4000)$ and

$\tilde{a}_3 \succ \tilde{a}_4 \Rightarrow f(0.2;\ \$0,\$4000)\cdot U(\$4000) > f(0.25;\ \$0,\$3000)\cdot U(\$3000)$.

Combining the information of the two inequalities implies that

$$\frac{f(1.0;\ \$0,\$3000)}{f(0.8;\ \$0,\$4000)} > \frac{f(0.25;\ \$0,\$3000)}{f(0.20;\ \$0,\$4000)} .$$

The above inequality does not contain any contradiction if the decision maker views $\tilde{a}_2$ or $\tilde{a}_3$ with optimism and $\tilde{a}_1$ or $\tilde{a}_4$ with pessimism.

3.3.1.3 The Prospective Reference Theory

Viscusi (1989) defined the perceived probability using a Bayesian formulation of posterior probability and derived the Prospective Reference theory (PRT). The posterior probability p_j^* for outcome j is given by

$$p_j^* = \frac{\gamma q_j + \xi p_j}{\gamma + \xi} . \qquad (3.4)$$

The individual prior probability q_j is the reference point probability held by the individual before he takes the lottery. He may derive p_j from the probability stated "objectively" by the lottery or he may observe the lottery ξ times to generate p_j. His posterior probability is a simple weighted average of q_j and p_j where the weights correspond to the fraction of his total information associated with his prior probability and with the information associated with the lottery (Viscusi, 1989, p.239). The ratio ξ/γ determines the importance of the reference point formed by q_j. When γ equals zero or ξ/γ is arbitrarily large, the a priori perception does not affect the formulation of the posterior probability p_j^*. In this case, the decision is reached based on the probabilities given in the lottery itself. When ξ/γ equals zero, the individual completely distrusts the lottery information and is guided by the reference point. Viscusi (1989, pp. 255-6) shows that PRT predicts the common consequence effect in the Allais paradox.

3.3.1.4 The Subjectively Weighted Utility Theory

Another alternative model is the Subjectively Weighted Utility (SWU) theory (Karmarkar, 1978). The SWU theory maps probabilities into subjective weights through the following function:

$$\ln \left[\frac{w_j}{(1-w_j)}\right] = \alpha \ln \left[\frac{p_j}{(1-p_j)}\right], \; 0 < \alpha < \infty \, . \qquad (3.5)$$

Thus for $\alpha = 1$, the subjective weights (w_j) and objective probabilities (p_j) coincide with each other. For $\alpha \neq 1$, the w_j coincide with the p_j at three fixed points: 0, 1/2, and 1. When $\alpha < 1$, w_j overweighs (underweighs) the likelihood of the outcome when p_j is less (greater) than 1/2. When $\alpha > 1$, the reverse is true. Since $0 \leq p_j \leq 1$, the above transformation will guarantee that $0 \leq w_j \leq 1$. However, because decision weights are not governed by the axioms of probability, $\sum_j w_j = 1$ cannot be guaranteed unless the number of states equals 2.[20] In most cases, $\sum_j w_j$ will not equal 1.

The resulting SWU model is given by:

$$SWU = \sum_j \frac{w_j}{\sum_j w_j} U(x_j). \tag{3.6}$$

Because w_j is a function of p_j, we can rewrite the model as

$$SWU = \sum_j \frac{\pi(p_j)}{\sum_j \pi(p_j)} U(x_j) \tag{3.7}$$

where $\pi(\cdot)$ is the weighting function that transforms p_j into w_j. Karmarkar claimed that "[a]ll the axioms underlying the EU model are satisfied (necessarily) by the SWU model except for the axiom of substitutability (1978, p.64)." In his illustration, the axiom of substitutability is the same as the axiom of independence. He presented the following two situations (Karmarkar, 1978, p.64):

$\tilde{z}_1$: 1/2•U($100) + 1/2•U($0); or

$\tilde{z}_2$: U($C) where U($C) is the certainty equivalent to $\tilde{z}_1$;

and

$\tilde{z}_3$: 1/2•U($100) + 1/2•U($C); or

$\tilde{z}_4$: 3/4•U($100) + 1/4•U($0).

By the axiom of independence, we can replace U($C) in $\tilde{z}_3$ by the prospect in $\tilde{z}_1$ and obtain

$\tilde{z}'$: 1/2•U($100) + 1/2•[1/2•U($100) + 1/2•U($0)] or

$\tilde{z}'$: 3/4•U($100) + 1/4•U($0).

Hence, $\tilde{z}_1$ is indifferent to $\tilde{z}_2$ implies that $\tilde{z}_3$ is indifferent to $\tilde{z}_4$ under the EU model. According to the SWU theory, replacing U($C) in $\tilde{z}_3$ by the prospect in $\tilde{z}_1$ we obtain

$\tilde{z}''$: $1/2 \cdot U(\$100) + 1/2 \cdot [1/2 \cdot U(\$100) + 1/2 \cdot U(\$0)]$

$1/2 \cdot U(\$100) + 1/4 \cdot U(\$100) + 1/4 \cdot U(\$0)$.

For $\alpha < 1$, the decision weight function overweighs (underweighs) the likelihood of the outcome for p_j less (greater) than 1/2. Thus, $\pi(3/4) < [\pi(1/2) + \pi(1/4)]$ and $\tilde{z}''$ dominates $\tilde{z}_4$ although $\tilde{z}_1$ is indifferent to $\tilde{z}_2$. When $\alpha > 1$, $\tilde{z}_4$ dominates $\tilde{z}''$.

Fishburn (1978) proved that models which employ decision weights derived from individual probabilities would reduce to the EU model if the axiom of dominance is retained. A heuristic argument is presented in Quiggin (1982, pp. 324-5). For n different possible outcomes each having probability 1/n to get x+ε where ε > 0, we can always find a sufficiently small ε such that

$$U(x) \quad > \quad \sum_{j=1}^{n} \pi(1/n) \cdot U(x+\varepsilon) \tag{3.8}$$

since it is possible to have $\sum_{j=1}^{n} \pi(1/n) < 1$. Hence, violation of dominance is avoided only if $\sum_{j=1}^{n} \pi(1/n) = 1$ which implies that $\pi(1/n) = 1/n$ and the SWU reduces to the EU.

3.3.2 Rank-Dependent Probabilities and the Class of Ω Theories

3.3.2.1 The Anticipated Utility Theory

The Anticipated Utility (AU) theory is designed to sidestep this problem (Quiggin, 1982, pp. 325-6).[21] The decision weights in the AU theory are derived from the entire probability distribution $\bar{p}$. When the outcomes are ordered from worst to best, $x_1 < x_2 < x_3 \ldots < x_n$, the decision weights take the form (Quiggin, 1982, p.329):

$$h_j(\bar{p}) = g(\sum_{i=1}^{j} p_i) - g(\sum_{i=1}^{j-1} p_i) \tag{3.9}$$

where $g(\cdot)$ is a one-element function that defines the behaviour of h (with j-1 independent elements/probabilities). This implies that the outcome vector will influence the decision weights; hence, AU is also named Expected Utility with Rank-Dependent Probabilities (EURDP). For example, consider the common ratio effect again:

$\tilde{a}_1$: 0.80 chance of \$4,000;

$\tilde{a}_2$: 1.00 chance of \$3,000.

and

$\tilde{a}_3$: 0.20 chance of \$4,000;

$\tilde{a}_4$: 0.25 chance of \$3,000.

The Pr(\$0) in $\tilde{a}_1$ and Pr(\$4,000) in $\tilde{a}_3$ are both equal to 20%. Yet, the decision weights associating with them are different because \$0 is the lowest ranking outcome in $\tilde{a}_1$ whereas \$4,000 is the highest ranking outcome in $\tilde{a}_3$. Several properties apply to the transformation in the function h_j:

(i) $h_j(\bar{p}) = 0$ for $p_j = 0$;

(ii) $h_j(\bar{p}) = 1$ for $p_j = 1$;

(iii) $\sum_j h_j(\bar{p}) = 1$, for binary lottery, $h(1/2, 1/2) = (1/2, 1/2)$;

(v) in general, $h_j(\bar{p}) \neq h_i(\bar{p})$ even if $p_j = p_i$.

The last property, illustrated above with the common ratio problem, makes the axiom of independence incompatible with the AU theory.

Having generated $h_j(\bar{p})$ for all j, the anticipated utility function is:

$$AU = \sum_j h_j(\bar{p}) \cdot U(x_j). \tag{3.10}$$

The AU theory is the first in what Karni and Safra (1987, p.681) called the class of Ω theories. According to this class of theories, the probability distribution will influence the decision weight attached to the outcome in each state. This assertion replaces the independence axiom in the EU model by the reduction principle where compound lotteries can be reduced to simple ones by the calculus of probabilities (Karni and Safra, 1987, p.677).

The theories in the Ω class can be divided into three groups of theories: the Expected Utility with Rank-Dependent Probabilities (EURDP), the Generalized Expected Utility (GEU) theory, and the Weighted Utility (WU) theory (Loomes, Starmer, and Sugden, 1989, p.426).

3.3.2.2 The Expected Utility with Rank-Dependent Probabilities

The EURDP theory includes Quiggin's (1982) AU theory and Yarri's (1987) Dual theory.[22] Chew (1984) and Segal (1984) extended these models to general utility functions and probability transformation functions. An adaptation of the EURDP theory for uncertain prospects is given in the Choquet Expected Utility (CEU) theory (Schmeidler, 1989). This generalization derives its name from the use of the Choquet integral. With all the events ordered from the worst to the best, the CEU theory replaces the g function in the AU theory by a capacity function $V(\bigcup_{j=1}^{i} E_j)$ where E_j is the jth event and $\bigcup$ is the union operation of mutually exclusive events in the state space (Chew and Karni, 1991, p.2). The Cumulative Utility (CU) theory is a further generalization of the CEU theory which weakens Savage's sure-thing principle (Chew and Wakker, 1991). The CU theory adopted the notion of comonotonic independence[23] — it requires the sure-thing principle only for acts that impose the same ordering of states of nature in terms of the associated outcomes. The difference between CU theory and CEU theories is that the capacity function and the outcomes are additively separable in the latter; the CU theory allows a more flexible functional form. In a parallel fashion, Green and Jullien (1988) developed a non-linear utility model using the ordinal independence axiom; their model applies to risky prospects. Chew and Wakker (1991, p.10) summarized these various related theories as follows (Table 3.2):

TABLE 3.2

RELATIONSHIP BETWEEN THE CLASS OF Ω THEORIES AND ITS GENERALIZATIONS

RISK		UNCERTAINTY	
EU	$\sum_{j=1}^{n} p_j \cdot U(x_j)$	SEU	$\sum_{j=1}^{n} f(p_j) \cdot U(x_j)$
EURDP	$\sum_{j=1}^{n} [g(\sum_{i=1}^{j} p_i) - g(\sum_{i=1}^{j-1} p_i)] \cdot U(x_j)$	CEU	$\sum_{j=1}^{n} [V(U^j) - V(U^{j-1})] \cdot U(x_j)$
RDU	$\sum_{j=1}^{n} [\psi(x_j; \sum_{i=1}^{j} p_i) - \psi(x_j; \sum_{i=1}^{j-1} p_i)]$	CU	$\sum_{j=1}^{n} [W(x_j; U^j) - W(x_j; U^{j-1})]$

p_j denotes objective probability,

$f(p_j)$ denotes subjective probability for event E_j,

$U(x_j)$ denotes the utility of outcome x_j,

U^j denotes union of events 1 to j,

RDU is rank-dependent utility, it is similar to Green and Jullien (1988) non-linear utility with ordinal independence axiom.

3.3.2.3 <u>The Generalized Expected Utility Theory</u>

The GEU theory proposed by Machina (1982a) utilized a "local utility function" to generate a non-linear preference function. The preference ranking of two probability distributions F and F^* is given by

$$V(F^*) - V(F) = \int U(x;F)[dF^*(x)-dF(x)] + o(\|F^*-F\|) \qquad (3.11)$$

where $V(\cdot)$ is the preference function which is Frêchet differentiable[24] on the space of the choice set with respect to the norm $\|\cdot\|$ and $o(\cdot)$ denotes a function which is zero at zero and of higher order than its argument. The local utility function $U(x;F)$ is defined over outcome x and probability distribution F as follows (Machina, 1982a, p.294):

$$U(x;F) \equiv -\int_0^x h(s;F)ds \qquad (3.12)$$

where $h(\cdot;F)$ is a function that will guarantee $U(\cdot;F)$ to be absolutely continuous and hence differentiable almost everywhere on the choice set (Machina, 1982a, p.294). Intuitively, GEU implies that

> "differential movement from the distribution $F(\cdot)$ to a distribution $F^*(\cdot)$ changes the value of the preference function $V(\cdot)$ by $\int U(x;F)[dF^*(x)-dF(x)]$, that is, by the difference in the expected value of $U(x;F)$ with respect to the distribution $F^*(\cdot)$ and $F(\cdot)$. In other words, in ranking differential shifts from an initial distribution $F(\cdot)$, the individual acts *precisely as would an expected utility maximizer*, with 'local utility function' $U(x;F)$ (Machina, 1982a, p.294)."

A simple example of a nonlinear preference function is given as follows (p.295):

$$V(F) \equiv \int\Theta(x)dF(x) + 1/2[\int\Xi(x)dF(x)]^2 = E_F[\Theta(x)] + 1/2[E_F[\Xi(x)]]^2 \tag{3.13}$$

with local utility function

$$U(x;F) = \Theta(x) + \Xi(x)[\int\Xi(z)dF(z)] = \Theta(x) + \Xi(x)E_F[\Xi(z)] \tag{3.14}$$

where $\Theta(\cdot)$ and $\Xi(\cdot)$ are value functions on x and E_F is the expectation with respect to probability distribution F. Similarly, the GEU preference over discrete events was given as follows: (Machina, 1987a, p.132)

$$V(F) = \sum_{j=1}^{n} p_j\Theta(x_j) + \left[\sum p_j\Xi(x_j)\right]^2 \tag{3.15}$$

While the preference function in the EU theory is linear in the probabilities due to the independence axiom, the GEU theory preference function is quadratic in the probabilities because the valuation of x in the local utility function is dependent on the entire probability distribution.

If V(F) is Gateaux differentiable[25] instead of Frêchet differentiable, where the latter is a stronger notion of differentiability (Chew, Karni, and Safra, 1987, p.373), then the EURDP theory falls into the same framework as the GEU theory (Yarri, 1987, p.112). The above functional form of the GEU theory can be shown to be a special case of the general quadratic form (Machina, 1982a, p.295):

$$1/2\int\int T(x;z)dF(x)dF(z) \tag{3.16}$$

with local utility function $\int T(x;z)dF(z)$ and assuming $T(x;z) \equiv T(z;x)$; or for discrete events:

$$\sum_{i=1}^{n} \sum_{j=1}^{n} p_i p_j T(x_i;x_j) \tag{3.17}$$

A further development of the general quadratic form is found in the Mixture Symmetric Utility theory (Chew, Epstein, and Segal; 1991).

3.3.2.4 The Weighted Utility Theory

The third group in the Ω-class theories is the Weighted Utility (WU) theory. This group includes Chew and MacCrimmon (1979a, 1979b), Chew (1983), and Fishburn (1983). The general functional form for WU theory is given as follows:

$$V(\tilde{x}) = \sum_{j=1}^{n} \frac{\tau(x_j)p_j}{\sum_{i=1}^{n} \tau(x_i)p_i} U(x_j) \tag{3.18}$$

where $\tilde{x}$ is the vector of possible outcomes and $\tau(\cdot)$ is the weighting function. For the outcome in the jth state, the decision weight is

$\dfrac{\tau(x_j)p_j}{\sum_{i=1}^{n} \tau(x_i)p_i}$ which depends on the outcomes in other states. Chew (1983, p.1071 and p.1077) pointed out that when $\tau(\cdot)$ is constant, then quasilinear mean[26] results. Fishburn (1983) derives a WU theory by imposing transitivity on his intransitive skew-symmetric bilinear (SSB) utility theory (Fishburn, 1982). He defined the SSB theory over prospects (instead of separating the probability and the outcome components in the prospect), yielding (Fishburn, 1983, p.297):

$$\tilde{p} \succ \tilde{q} \iff \frac{U(\tilde{p})}{W(\tilde{p})} > \frac{U(\tilde{q})}{W(\tilde{q})} \tag{3.19}$$

where $\tilde{p}$ and $\tilde{q}$ are prospects, $U(\cdot)$ and $W(\cdot)$ are linear, and $W(\cdot)$ is non-negative.[27] Rewriting the transitive SSB theory in terms of probabilities and outcomes, we have

$$V'(\tilde{x}) = \sum_{j=1}^{n} \frac{p_j}{\sum_{i=1}^{n} W(x_i)p_i} U(x_j). \tag{3.20}$$

The above theories attempt to generalize the EU theory by relaxing the axiom of independence. These theories reformulate the decision weights which take into account outcomes and/or probabilities in the other states. Procedures of this kind will accommodate the common consequence effect and common ratio effect. The development towards a generalized version proposes a non-additive separable function which will eliminate linearity between the probabilities and the outcomes.

3.4 NON-HOLISTIC JUDGMENT MODELS

The non-holistic judgment models covered in this section are listed in Table 3.3. Under the non-holistic decision criteria, the optimal choice is usually reached by sequential elimination. Due to this sequential elimination process, the non-holistic models will not generate complete ranking indices for the options under consideration. On the other hand, the holistic models covered in Section 3.3 will always generate these ranking indices for all the options. The sequential elimination process usually involves comparisons vis-à-vis other alternatives; normally, the important attributes are recognized in the early stages of comparison.

TABLE 3.3

NON-HOLISTIC JUDGMENT MODELS FOR CHOICE UNDER RISK AND UNCERTAINTY

Prospect Theory	(Kahneman and Tversky, 1979)
$\pi(p_j) \cdot v(x_j)$	
Regret Theory (Skew-Symmetric Bi-linear Utility)	(Bell, 1982; Fishburn, 1982; Loomes and Sugden, 1982)
$p_j \cdot M(x_{ij};\ x_{kj})$	
Disappointment Theory	(Bell, 1985; Loomes and Sugden, 1986)
$p_j \cdot D(x_{ij};\ \tilde{x}_i)$	
Satisficing Principle	(Simon, 1964)
$x_{ij} \geq x_{sj}$ for all j	
Lexicographic Model	(Fishburn, 1988)
$x_i > x_k$ for min $\{j:\ x_{ij} \neq x_{kj}\}$	
Additive Difference Model	(Tversky, 1969)
$\sum_{j=1}^{n} \phi_j[u_j(x_{ij}) - u_j(x_{kj})]$	

Table 3.3 continues ...

Lexicographic Semi-Order Model (Encarnación, 1988)

$$x_i > x_k \quad \text{for } \min \{ j: V_j(x_i;x_k) > \varepsilon_j^* \}$$

where $V_1 = EU(x_i) - EU(x_k)$;

$V_2 = \sum \{ p_j \mid x_{ij} > x_o \} - \sum \{ p_j \mid x_{kj} > x_o \}$;

$V_3 = \max(x_i) - \max(x_k)$;

and $V_4 = EU(x_i) - EU(x_k)$ and $\varepsilon_4^* = 0$.

The non-holistic models are characterized by sequential elimination or pairwise comparison in the decision-making process. A complete ranking of all the alternatives is usually not available. It is therefore very common for non-holistic models to generate intransitive preference orderings. Models in this category employ heuristics to simplify the evaluation task; an alternative definition of rationality is sometimes required to justify the use of heuristics (see March, 1978).

3.4.1 The Prospect Theory

Kahneman and Tversky (1979) developed Prospect theory in order to provide a viable alternative to the EU model in light of various anomalous findings. In their study, they discovered and categorized several types of behaviour which contradicted the predictions of the EU theory. The results from their questionnaire confirmed the Allais paradox (pp.256-6). They called this the certainty effect which included both the common consequence effect and the common ratio effect. To explain the certainty effect, they argued that "people overweight outcomes that are considered certain, relative to the outcomes which are merely probable (p,265)." Their questionnaire also posed a set of problems twice, with the outcomes changed to losses in the second set. They found that the preference orderings for the positive prospects were reversed when negative prospects were presented. They called this the reflection effect. They suggested that "the reflection of prospects around 0 reverses the preference order (p.268)" and "it appears that certainty increases the aversiveness of losses as well as the desirability of gains (p.269)." Furthermore, they presented evidence that the structure of the prospects affected preference orderings. This observation contradicted the axiom of substitutability (i.e. axiom of independence) of the EU theory. They termed this the isolation effect.

Based on these violations of the predictions of the EU model, Kahneman and Tversky (1979) proposed Prospect theory as an alternative theory for choice under uncertainty. In this theory, the basic equation used to appraise the value of prospects is (p.276):

$$V(x_1, p_1;\ x_2, p_2) = \pi(p_1)v(x_1) + \pi(p_2)v(x_2) \qquad (3.21)$$

where x_1, x_2 are possible outcomes with objective probabilities p_1 and p_2, respectively. $\pi(\cdot)$ is the weighting function which determines the decision weights. It exhibits the following properties (pp.280-4):

(i) $\pi(0) = 0$ and $\pi(1) = 1$,

(ii) subadditivity for small values of objective probability p_j; i.e., $\pi(kp_j) > k\cdot\pi(p_j)$; for $0 < k < 1$,

(iii) overweighting of very low probability; i.e., $\pi(p_j) > p_j$,

(iv) subcertainty: $\pi(p_j) + \pi(1-p_j) < 1$; for all $0 < p_j < 1$, and

(v) Subproportionality: $\dfrac{\pi(p_1p_2)}{\pi(p_1)} \le \dfrac{\pi(p_1p_2p_3)}{\pi(p_1p_3)}$;

for all $0 < p_1,\ p_2,\ p_3 \le 1$.

The value function, $v(\cdot)$, in Prospect theory measures the subjective value of an outcome relative to some reference point (usually, the initial position) while the utility index, $U(\cdot)$, in the EU model is based on the final asset position. Another difference between $v(\cdot)$ and $U(\cdot)$ lies in the area of uniqueness. $U(\cdot)$ is unique up to positive linear transformations whereas $v(\cdot)$ is unique up to positive ratio transformations. Furthermore, conventional $U(\cdot)$ is concave throughout the entire range of outcomes but $v(\cdot)$ is concave for gains and convex for losses to account for risk-taking behaviour for negative prospects and risk avoiding behaviour for positive prospects (pp. 277-280).

Prospect theory also proposed an extra element in the decision process: it is an editing phase used mainly to simplify the prospects before evaluation. Different operations of editing consist of coding, combination, segregation, cancellation, simplification, and the detection of dominance (pp. 274-5). These editing features render the Prospect theory non-holistic because some prospects are eliminated in the early stage of the editing process.

With regard to the four axioms underlying preferences in the EU model, the axiom of transitivity is violated in Prospect theory. Prospect theory allows intransitive preference when triples of prospects are presented (p. 284). When $\pi(p_i) \neq p_i$ and $\pi(p_i) + \pi(1-p_i) < 1$, the axiom of stochastic dominance would be violated, but this is prevented by the operation of detection of dominance where dominated prospects are eliminated based on objective probabilities. As discussed above, models which employ a weighting function will in general violate the axiom of independence; especially when $\pi(\cdot)$ is subadditive. The axiom of continuity is maintained and is named the axiom of solvability (p.289).

3.4.2 The Class of Skew-Symmetric Bilinear Utility Theories

An alternative class of theories which has a common general model suggests the use of a bilinear utility function in place of the usual utility function. The model basically rejects the notion that under each state of world only the individual outcome itself determines utility. These theories argue for a more complex structure of utility determination. There are three different theories which fall into this category — Regret theory, developed independently by Bell (1982) and Loomes and Sugden (1982); Skew-Symmetric Bilinear (SSB) Utility theory put forward by Fishburn (1982); and Disappointment theory also developed independently by Bell (1985) and Loomes and Sugden (1986). The first and the third theories were produced from a conjecture about "intuitive" decision-making which recognizes certain psychological factors influencing choice under uncertainty. The SSB Utility theory on the other hand, was constructed from a set of axioms that determines a coherent preference ordering.

The general model for these theories is given by:

$$\Phi(\tilde{p};\ \tilde{q}) = \sum_{i=1}^{n} \sum_{j=1}^{n} p_i q_j \psi(x_i;\ y_j) \qquad (3.22)$$

$\tilde{p}$ is a risky prospect with probability distribution $(p_1, \ldots, p_n)$ and outcome vector $(x_1, \ldots, x_n)$. $\tilde{q}$ is another risky prospect with

$(q_1, \ldots, q_n)$ and outcome vector $(y_1, \ldots, y_n)$ which is independent of $\tilde{p}$.[28] $\psi(\cdot; \cdot)$ is a SSB function that has the symmetry property $\psi(x_i; y_j) = -\psi(y_j; x_i)$. This will imply that $\Phi(\cdot; \cdot)$ is also a SSB function where $\Phi(\tilde{p}; \tilde{q}) = -\Phi(\tilde{q}; \tilde{p})$. Using the axioms of continuity, dominance, and symmetry, Fishburn (1982) established a set of preference orderings represented by $\Phi(\tilde{p}; \tilde{q})$:

$$\Phi(\tilde{p}; \tilde{q}) \gtreqless 0 \Leftrightarrow \tilde{p} \succsim\!\!\precsim \tilde{q}. \tag{3.23}$$

Regret theory, on the other hand, is concerned with the choice between two actions (instead of two prospects) in the decision-making process. The preference orderings between two actions A_i and A_k is determined by:

$$A_i \succsim\!\!\precsim A_k \Leftrightarrow \sum_j \pi_j \, [M(x_{ij}; x_{kj}) - M(x_{kj}; x_{ij})] \gtreqless 0 \tag{3.24}$$

where $M(x_{ij}; x_{kj})$ is a modified utility function which measures the utility of getting x_{ij} (associated with choosing A_i in the jth state) and foregoing x_{kj} by not choosing A_k.

In order to compare their outcomes consistently, the two actions should have the same probability distribution so that there is a genuine sense of having x_{ij} and missing x_{kj} in the jth state. If the two actions have different probability distributions, the probability terms in the two actions should be partitioned accordingly until a common probability distribution can be applied to both actions. Fishburn and LaValle (1988, p.203) call Regret theory the *States-Additive SSB* (or S^3B) *model* because this partition implies that the actions in Regret theory is "states-additive". $p_i q_j \psi(x_i; y_j)$ in

Equation (3.22) is identical to $\pi_j M(x_{ij};\ x_{kj})$ in Equation (3.24) when π_j is partitioned into $p_i q_j$. Consider a simple example with only two possible states in A_i (p_1, x_{i1}; p_2, x_{i2}) and A_k (q_1, x_{k1}; q_2, x_{k2}) where $p_1 + p_2 = q_1 + q_2 = 1$ but $p_1 \neq q_1$. In order to appropriately analyze rejoice and regret the payoff table should be defined as follows:

	p_1q_1	p_1q_2	p_2q_1	p_2q_2
A_i	x_{i1}	x_{i1}	x_{i2}	x_{i2}
A_k	x_{k1}	x_{k2}	x_{k1}	x_{k2}

Hence, $\sum_{j=1}^{n} \pi_j$ in Equation (3.24) equals $\sum_{a=1}^{2} \sum_{b=1}^{2} p_a q_b$; the outcome sets in both actions should be redefined correspondingly. For two actions A_i (p_1, x_{i1}; ...; p_n, x_{in}) and A_k (q_1, x_{k1}; ...; q_m, x_{km}), Equation (3.24) can be rewritten as

$$A_i \underset{\prec}{\overset{\succ}{\sim}} A_k \quad \Leftrightarrow \quad \sum_{a=1}^{n} \sum_{b=1}^{m} p_a q_b \ [M(x_{iab};\ x_{kba}) - M(x_{kba};\ x_{iab})] \underset{<}{\overset{>}{=}} 0 \tag{3.25}$$

where $x_{iab} = x_{ia}$ and $x_{kba} = x_{kb}$, for $a = 1, \ldots, n$ and $b = 1, \ldots, m$.

A simple specification of $M(\cdot;\ \cdot)$ is given in Loomes and Sugden (1982, p.809) as:

$$M(x_{ij}, x_{kj}) = U(x_{ij}) + R[U(x_{ij}) - U(x_{kj})] \tag{3.26}$$

where $U(\cdot)$ is the basic utility function[29] and $R[\cdot]$ is a rejoice-regret function. Rewriting $[U(x_{ij}) - U(x_{kj})]$ as ξ_{ikj}, $R[\xi_{ikj}] \gtreqless 0 \Leftrightarrow \xi_{ikj} \gtreqless 0$. $R[\cdot]$ is a SSB function since $R[\xi_{ikj}] = -R[-\xi_{ikj}]$ (or $-R[\xi_{kij}]$). One can show that the composite term $[M(x_{iab}; x_{kba}) - M(x_{kba}; x_{iab})]$ in Equation (3.25) is skew-symmetric bilinear when $R[\cdot]$ is SSB. Hence, Regret theory and SSB utility function are identical decision models as $[M(\cdot; \cdot) - M(\cdot; \cdot)]$ in Equation (3.25) equals $\psi(\cdot; \cdot)$ in Equation (3.22). (see Sugden, 1986, p.17).

Loomes and Sugden (1982) showed that the paradoxical results of the EU model found by Kahneman and Tversky (1979) could be accommodated by the Regret theory. The common consequence effect, the common ratio effect, the isolation effect, and the reflection effect are consistent with the Regret theory. They also showed that the preference reversal phenomenon is rational economic behaviour in the context of Regret theory (Loomes and Sugden, 1983). This is largely due to the intransitive nature of this theory.

Disappointment theory (Bell, 1985; Loomes and Sugden, 1986) concerns the comparison of utilities under different states of the world with the same action. In this theory, the rejoice-regret function $R[\cdot]$ is replaced by the elation-disappointment function $D[U(x_{ij}) - U(\bar{x}_i)]$ where $\bar{x}_i$ is the expected value of the possible outcome given action A_i. $D[\cdot]$ has the same SSB property as $R[\cdot]$. The modified utility function becomes:

$$M(x_{ij}; \bar{x}_i) = U(x_{ij}) + D[U(x_{ij}) - U(\bar{x}_i)]. \qquad (3.27)$$

3.4.3 Sequential Elimination Models

Other non-holistic models emphasize the information-processing aspect in which decision-making is viewed as transforming stimuli. Several theoretical stages are outlined in the information-processing system (Klatzky, 1975). They include (i) perception (information presentation), (ii) pattern recognition, (iii) attention, (iv) information storage (requiring short-term memory), (v) information retrieval (requiring long-term memory), and (vi) problem solving. The fact that information is processed suggests that some heuristics are likely to be used in reaching the "optimal" decision. Tversky and Kahneman (1974) examined some common heuristics used in decision-making; they are (i) representativeness, (ii) availability, and (iii) adjustment and anchoring. Researchers (for example Tversky, 1969; Kahneman and Tversky, 1974; Schoemaker, 1980) found that sequential elimination is a commonly used method to implement these heuristics.

A major characteristic of the sequential elimination models is that they are non-compensatory. According to these models, any "surpluses" above the preset criteria at earlier stages of evaluation cannot compensate for "deficiencies" uncovered in the later stage, and vice versa.

Sequential elimination models may take different forms. When the sequential elimination process is structured in terms of comparisons against a disjunctive preset standard, it formulates a disjunctive model. The criteria become conjunctive when they require that all attributes in each prospect meet certain minimum standard. Using a conjunctive criteria will form a conjunctive model. This comparison method conforms to Simon's (1964) satisficing principle where the preset standard (goal) is the constraint in the decision-making process. Alternatives are partitioned into an acceptable or non-acceptable group according to the disjunctive or conjunctive criteria. The elimination process will continue with a different criterion each round until only one alternative remains. This alternative is optimal in the sense that it fulfills all the criteria.

The elimination procedure can be set to compare alternatives against each other. This will lead to a lexicographic model which concerns comparisons within attributes described as follows (Fishburn, 1988, pp.52-53):

$$x_i > x_k \quad \text{for min } \{j: x_{ij} \neq x_{kj}\} \tag{3.28}$$

In Equation (3.28), the decision criterion is formulated as independent evaluation of each attribute in the two alternatives. If two alternatives are different on dimension j = 1 (the most important attribute), the individual will choose the alternative which gives a better value on this dimension. If the two alternatives are identical in dimension 1, then a decision is reached based on dimension 2 only. This procedure continues until all dimensions are exhausted. It is clear that the lexicographic model applies only to multi-attribute alternatives. Because of its non-compensatory nature, lexicographic preference ordering violates the continuity axiom (Varian, 1984, p. 114). Fishburn (1974) provides a detailed survey of lexicographic models.

Similar to the lexicographic decision criterion is the lexicographic semi-order model. Two alternatives are considered to be equal in a dimension i if their difference is equal to or smaller than ε_i (see endnote 12). Encarnación (1988) applied the lexicographic semi-order model[30] to risky prospects in an attempt to resolve some preference paradoxes in the EU model. Encarnación's (1988, pp. 232-235) model is given as follows:

$$
\begin{aligned}
x_i > x_k \quad \text{for } \min \{j\colon V_j(x_i;x_k) > \varepsilon_j^*\} \\
\text{where } V_1 &= EU(x_i) - EU(x_k) \\
V_2 &= p_i - p_k \\
V_3 &= \max(x_i) - \max(x_k) \\
V_4 &= EU(x_i) - EU(x_k) \text{ and } \varepsilon_4^* = 0. \qquad (3.29)
\end{aligned}
$$

The first dimension in his model is the EU index. The second dimension focuses on the probability dimension of the prospects. The third dimension compares the magnitude of the prizes of the two prospects. Finally, the fourth dimension evaluates the EU index again with $\varepsilon = 0$. The model will resolve a "tie" situation if the differences are too small in each of the first three dimensions.

Instead of only comparing the outcomes of two alternatives in each attribute (or state of nature), the additive difference model (Tversky, 1969) also accumulates their differences by an additive difference function. Hence, unlike lexicographic semi-order and lexicographic model, it is a compensatory model. It remains as a non-holistic model because it provides pairwise comparison only.

According to the additive difference model, the preference between x_1 and x_2 is determined as follows (1969, p.41):

$$x_1 \succsim x_2 \Leftrightarrow \sum_{i=1}^{n} \phi_i[U_i(x_{1i}) - U_i(x_{2i})] \geq 0 \qquad (3.30)$$

where ϕ_i is an increasing continuous function

with $\phi_i(-\delta_i) = -\phi_i(\delta_i)$ for all i, and

U_i is the real-valued function for dimension i.

If all the difference functions are linear, $\phi_i(\delta_i) = t_i\delta_i$, and the additive difference model reduces to the additive model where

$$x_1 \succsim x_2 \Leftrightarrow \sum_{i=1}^{n} t_i[U_i(x_{1i}) - U_i(x_{2i})] \geq 0 \qquad (3.31)$$

or

$$x_1 \succsim x_2 \Leftrightarrow \sum_{i=1}^{n} t_iU_i(x_{1i}) \geq \sum_{i=1}^{n} t_iU_i(x_{2i}). \qquad (3.32)$$

The additive model can be applied to risky prospects $\tilde{x}_1$ and $\tilde{x}_2$ where the possible payoffs in different states are considered as attributes in the ith dimension and t_i stands for the probability term. We have

$$\tilde{x}_1 \succsim \tilde{x}_2 \Leftrightarrow \sum_{i=1}^{n} p_iU_i(x_{1i}) \geq \sum_{i=1}^{n} p_iU_i(x_{2i}). \qquad (3.33)$$

When $U_i(\cdot) = U_j(\cdot)$ for $i \neq j$, the additive model becomes the EU model. One can see that linearity in the difference function, $\phi_i(\delta_i) = t_i\delta_i$, restores the linearity in preferences. In other words, a non-linear

$\phi_i(\cdot)$ implies that the independence axiom, as well as some anomalies attributed to this axiom, are avoided. Tversky proved that when $n \geq 3$, that is when alternatives have 3 attributes/dimensions or more, transitive choices are guaranteed under the additive difference model if and only if the ϕ_i's are linear, which is a rather severe restriction. In light of this fact, the additive difference model, like the lexicographic semi-order model, will usually lead to intransitive preferences.

Tversky also pointed out that the lexicographic semi-order model is a limiting case of the additive difference model when one or more of the difference functions approach a step function where $\phi_i(\cdot) = 0$ whenever $\delta_i \leq \varepsilon$ (p. 42).

The above theories recognize and model the intuitive aspect of the decision-making process. All these models are prone to intransitive preference orderings as a result of the non-holistic perspective. Because of preference cycles, intransitive models cannot yield meaningful predictions; there is always a better alternative which is available but not chosen. This fact creates an obstacle to adopting these models in decision-making because in economic theory, rationality is usually based on transitivity.[31] At best, these non-holistic models will provide an explanation for the choices selected. In other words, these models are restricted to serve as descriptive models rather than as normative models.

The research in the following chapters tries to bridge the gap between holistic and non-holistic models. It addresses the psychological aspect of choice under risk and investigates the heuristics used in decision-making. At the same time, the theory will maintain the assumption of transitivity which is fundamental for rational economic behaviour.

CHAPTER 4 THE REJOICE-REGRET MODEL AND REFERENCE POINT THEORY

Some non-holistic judgment models (Prospect theory and Regret theory) reviewed in Section 3.4 conjecture that psychological factors are an important aspect of decision-making. Other non-holistic judgment models suggest that individuals will use heuristics to simplify the decision-making process in a complicated decision problem. These two aspects are usually ignored in the holistic judgment models.

This chapter will develop a decision model based on the structure of Regret theory which explicitly acknowledges the psychological dimension of decision-making in terms of rejoice and regret. Unlike Regret theory, the proposed theory formulates rejoice or regret vis-à-vis a standard action. The adoption of a reference point can be viewed as a heuristic to simplify the task involved. More importantly, the proposed model, named Reference Point theory (RPT), preserves transitivity in the preference rankings through the use of a reference point. Consequently, RPT can provide a complete ordering for all the possible alternatives, an important feature shared by holistic judgment models.

This chapter is divided into four sections. Section 4.1 examines the cause of intransitivity in Regret theory (and SSB utility theory). Section 4.2 discusses the two attempts to restore transitivity by the authors of the original theory. Section 4.3 studies the role of reference points in defining transitive preference ordering. A stable reference point in the rejoice-regret function restores transitivity in the preference. Section 4.4 shows that preference orderings derived from the modified utility function satisfy all 4 axioms in the EU model.

4.1 INTRANSITIVITY IN REGRET THEORY

Loomes and Sugden (1982) hypothesized that the psychology of "what might have been" is important in decision-making. They formulated the Regret theory where each prospect (or action) is evaluated in the context of other prospects. The effect of rejoice or regret in choosing one action vis-à-vis another one will influence the optimal choice of the decision maker. Such a proposition clearly contradicts the independence axiom of the EU model. Moreover, preference rankings are derived from a pairwise comparison of the alternatives being considered; Regret theory is prone to preference cycles.

The structure of Regret theory remains relatively simple; it amends the VNM utility function by a rejoice-regret function. The value index of choosing action A_i over A_k is given as follows:

$$\sum_j p_j \{U(x_{ij}) + R[U(x_{ij}) - U(x_{kj})]\} \qquad (4.1)$$

where $j = 1, \ldots, n$ are the possible states of the world,

p_j is the probability of state j, and

x_{ij} is the outcome from action i in state j.

The rejoice-regret function $R[\cdot]$ can take on different shapes to reflect one's attitude towards "surprise". Nevertheless, Loomes and Sugden (1982, pp.810-815) show that when $R[\cdot]$ is strictly convex (the second derivative $R''[\xi] > 0$ for $\xi > 0$), preferences described by Regret theory can be consistent with the common consequence effect, the common ratio effect, the isolation effect, the reflection effect, and preference reversal (Loomes and Sugden, 1983).

Some studies have confirmed the impact of rejoice-regret on choice under uncertainty. These experimental results are reported in Loomes (1988b, 1988c), Starmer and Sugden (1989), and Sugden (1986). Battalio, Kagel, and Jiranyakul (1990) found some mixed results regarding the predictions of Regret theory, the impact of rejoice-regret was unclear when the lotteries were presented as numerical rather than graphical formulations and when lotteries involved losses (pp.38-40).

Regret theory, which models the decision-making process as a binary choice problem, is plagued with the possibility of intransitive preferences when three or more alternative actions (or prospects in the case of SSB utility theory) are available for selection (Loomes and Sugden, 1982, pp.815-6; Fishburn, 1982, p.33.) The axiom of transitivity, however, is indispensable for utility maximization in consumer theory. Under cyclical preference orderings, for every choice being made, there is always a better alternative which is available but not chosen. With intransitive preference orderings, optimal economic decisions can never be reached.

Before transitivity can be established, possible sources of intransitivity must be identified. This will provide a starting point for solving the problem. Consider three actions A_1, A_2, and A_3; let $A_1 \succ A_2$ and $A_2 \succ A_3$ according to the ranking criterion in Regret theory. That is, if x_{ij} is the outcome of action A_i in the jth state of the world $(j = 1,\ldots,n)$,

$$A_1 \succ A_2 \Leftrightarrow \sum_{j=1}^{n} p_j M(x_{1j}; x_{2j}) > \sum_{j=1}^{n} p_j M(x_{2j}; x_{1j}) \qquad (4.2)$$

where $M(x_{ij}; x_{kj}) = U(x_{ij}) + R[U(x_{ij}) - U(x_{kj})]$ as defined in Equation (3.26) and $R[\cdot]$ is the rejoice-regret function. Similarly,

$$A_2 \succ A_3 \Leftrightarrow \sum_{j=1}^{n} p_j M(x_{2j}; x_{3j}) > \sum_{j=1}^{n} p_j M(x_{3j}; x_{2j}). \qquad (4.3)$$

Given that $A_1 \succ A_2$ and $A_2 \succ A_3$; transitivity implies that $A_1 \succ A_3$, or mathematically

$$\sum_{j=1}^{n} p_j M(x_{1j}; x_{3j}) > \sum_{j=1}^{n} p_j M(x_{3j}; x_{1j}). \qquad (4.4)$$

However, this cannot be guaranteed unless the weak inequality

$$\sum_{j=1}^{n} p_j R[U(x_{2j}) - U(x_{1j})] \geq \sum_{j=1}^{n} p_j R[U(x_{2j}) - U(x_{3j})] \qquad (4.5)$$

is satisfied. Unfortunately, this does not necessarily follow from the information contained in the preference orderings.

4.2 ATTEMPTS TO RESTORE TRANSITIVITY IN REGRET THEORY

Both the authors of Regret theory and SSB utility theory tackled the problem of intransitivity in subsequent papers. Loomes and Sugden (1987a, pp.281-2) conjectured non-pairwise comparison and imposed transitivity by defining preference ranking conditional on a feasible set. They replaced the modified utility function $M(x_{ij}; x_{kj})$ by an alternative function $M^*(\cdot; \cdot) = M^*(x_{ij}; Z_j - \{x_{ij}\})$ where $Z_j = \{ x_{ij} | A_i \in Z \}$ is the set of consequences corresponding to the feasible set of actions in Z if the j^{th} state of the world occurs. $M^*(\cdot; \cdot)$, therefore, yields the utility index for choosing A_i while "missing out on all of the consequences" related to other actions in Z (Loomes and Sugden, 1987a, p.281). The preference ranking between A_i and A_k is now determined as follows:

$$A_i \underset{z}{\succ \atop \sim \atop \prec} A_k \Leftrightarrow \sum_{j=1}^{n} p_j M^*(x_{ij}; Z_j - \{x_{ij}\}) \gtreqless \sum_{j=1}^{n} p_j M^*(x_{kj}; Z_j - \{x_{kj}\}) \qquad (4.6)$$

where $\succ_z$, $\sim_z$, and $\prec_z$ represent the three possible preference orderings conditional on the feasible set Z. Loomes and Sugden (1987a, p.282) reduced (4.6) to

$$A_i \underset{z}{\succ \atop \sim \atop \prec} A_k \Leftrightarrow E(A_i; Z) \gtreqless E(A_k; Z) \qquad (4.7)$$

where E denotes the expectation operation. They claimed that transitivity is reinstated in Regret theory.

Several questions emerge to undermine Loomes and Sugden's approach as a viable solution. First, the second argument in $M^*(\cdot; \cdot)$ is a "composite experience" (according to their terminology); how to measure the utility of this set of possible consequences is left undiscussed. Without clearly defining $U(Z_j-\{x_{ij}\})$, one cannot evaluate $M^*(\cdot;\ \cdot)$ and Regret theory becomes inoperative. Secondly, the simplification of (4.6) to (4.7) is questionable. Using Z to replace the second argument in both $M^*(\cdot;\ \cdot)$ in (4.6) is confusing because these functions actually represent two different sets of actions. In a special case, Loomes and Sugden restricted the feasible set Z to only two actions; this will leave the preference orderings intransitive. This is because the preference $\succ_z$ is only applicable to the two actions included in the feasible set Z; this ordering cannot be generalized to include a third action. When only two actions A_i and A_k are considered, $Z_j-\{x_{ij}\}$ reduces to x_{kj} and, similarly $Z_j-\{x_{kj}\}$ reduces to x_{ij}; $M^*(\cdot;\ \cdot)$ is identical to $M(\cdot;\ \cdot)$ in the original model.[1] The inequalities remain basically the same as before and a transitivity assumption remains unwarranted.

Fishburn (1983) proved that transitivity of the preference ranking in the SSB function $\Phi(\cdot;\ \cdot)$ can be guaranteed if

$$\Phi(\tilde{p};\ \tilde{q}) = U(\tilde{p})W(\tilde{q}) - U(\tilde{q})W(\tilde{p}) \tag{4.8}$$

where $\tilde{p}$ and $\tilde{q}$ are two competing prospects, and $U(\cdot)$ and $W(\cdot)$ are non-negative functions.[2,3] Note that if $W(\cdot)$ is a constant, the model reduces to the EU model. If $\tilde{p} \succ \tilde{q}$ and $\tilde{q} \succ \tilde{r}$, that is $\Phi(\tilde{p};\ \tilde{q}) > 0$ and $\Phi(\tilde{q};\ \tilde{r}) > 0$, respectively; after rearranging terms in (4.8), positive $W(\cdot)$ implies that

$$\frac{U(\tilde{p})}{W(\tilde{p})} > \frac{U(\tilde{q})}{W(\tilde{q})} \tag{4.9}$$

and similarly

$$\frac{U(\tilde{q})}{W(\tilde{q})} > \frac{U(\tilde{r})}{W(\tilde{r})} \quad . \tag{4.10}$$

Equations (4.9) and (4.10) together imply that

$$\frac{U(\tilde{p})}{W(\tilde{p})} > \frac{U(\tilde{r})}{W(\tilde{r})} \tag{4.11}$$

and consequently,

$$U(\tilde{p})W(\tilde{r}) > U(\tilde{r})W(\tilde{p}) \Leftrightarrow \tilde{p} \succ \tilde{r}. \tag{4.12}$$

Therefore, transitivity is established when the bilinear function is separable multiplicatively. (The resulting model is a form of weighted utility theory (Equation 3.18), discussed in sub-section 3.3.2.4.) Unfortunately, this characterization of transitivity cannot be translated into the context of Regret theory. When the outcomes of two prospects are evaluated independently, the rejoice-regret function R[·] loses its characteristics. One can see this clearly if (4.8) is constructed as the difference of two ratios. Hence, when transitivity is imposed, the bilinearity of the two terms in (4.8) can be reduced to a composite function of only one variable. The possibility for rejoice or regret is then completely eliminated from the model.

4.3 TRANSITIVITY IN REFERENCE POINT THEORY

As discussed earlier, transitivity is restored if the weak inequality in (4.5) is satisfied. This can be achieved by restructuring the model as follows:

$$A_i \; {\succsim \atop \precsim} \; A_k \Leftrightarrow \sum_{j=1}^{n} p_j \, M(x_{ij};\, x_{oj}) \; {\geq \atop <} \; \sum_{j=1}^{n} p_j \, M(x_{kj};\, x_{oj}) \qquad (4.13)$$

where x_{oj} is the outcome of a reference action A_o in the j^{th} state of the world.[4] A_o is one of the alternatives available to the individual; it can be A_i, A_k, or any other action in his menu. Equation (4.13) describes the ranking between two actions according to Reference Point theory.

With this alteration to the model, suppose $A_1 \succ A_2$ and $A_2 \succ A_3$; we have

$$\sum_{j=1}^{n} p_j\{U(x_{1j})+R[U(x_{1j})-U(x_{oj})]\} > \sum_{j=1}^{n} p_j\{U(x_{2j})+R[U(x_{2j})-U(x_{oj})]\} \qquad (4.14)$$

and

$$\sum_{j=1}^{n} p_j\{U(x_{2j})+R[U(x_{2j})-U(x_{oj})]\} > \sum_{j=1}^{n} p_j\{U(x_{3j})+R[U(x_{3j})-U(x_{oj})]\}. \qquad (4.15)$$

The right hand side of the inequality in (4.14) is identical to the left hand side of the inequality in (4.15); hence,

$$\sum_{j=1}^{n} p_j\{U(x_{1j})+R[U(x_{1j})-U(x_{oj})]\} > \sum_{j=1}^{n} p_j\{U(x_{3j})+R[U(x_{3j})-U(x_{oj})]\}. \qquad (4.16)$$

In other words, $A_1 \succ A_3$ if $A_1 \succ A_2$ and $A_2 \succ A_3$; that is, transitivity is established.

The above method and that recommended by Loomes and Sugden (1987, pp.281-2) both aim at restricting the second argument in $M(\cdot;\ \cdot)$. Because of the inefficaciousness of using set-specific preference to achieve transitivity, an alternative decision-making mechanism using a reference point is necessary. The present method proposes a "restricted pairwise" comparison. It is restricted in the sense that a stationary reference point is used in the rejoice-regret function when three or more options are available.

Three related issues emerge in the reference point approach. The first issue directly pertains to the validity of the adoption of a reference point in representing the decision process. If in reality people do make decisions based on pairwise comparisons conjectured in Regret theory the present approach will be inappropriate. The second issue concerns the arbitrariness of the choice of the reference point in the new model. This issue becomes important as the optimal solution is conditional on the choice of A_o. How can the analyst be sure that A_o is the reference action adopted by the decision maker? Thirdly, there might be reasons to believe that the restructuring would indirectly eliminate the rejoice-regret element in the model, thus repeating the weakness of the Fishburn (1983) approach.

Data from empirical studies thus far are inadequate for concluding which model better represents the decision-making pattern. In addition to the experiments on preference reversal (Section 3.1.6) tests of Regret theory have also involved binary choice problems. Other studies of human decision-making under uncertainty have usually employed only two alternatives in each question of their experiments. A sample of these studies includes Cohen, Jaffray, and Said (1987);

Hershey, Kunreuther, and Schoemaker (1982); Hershey and Schoemaker (1980b); Kahneman and Tversky (1979); and Schoemaker (1980). The binary choice framework, of course, cannot provide useful information about the comparison process used in the proposed model. Although Schoemaker and Kunreuther (1979) asked their subjects to rank four different policies, their study did not explicate the cognitive process of comparing the various options.

Indirect support for the idea that individuals restrict the number of comparisons they make is, nonetheless, available. Many studies (e.g., Kahneman and Tversky (1979); and Schoemaker and Kunreuther (1979)) demonstrate that individuals have a limited capacity for processing information when making choices under uncertainty. These individuals will use some heuristics to simplify the task (Tversky and Kahneman, 1974). The reference point method is considered to be a popular heuristic used in decision-making. It can be shown that comparatively less complex calculation is involved in RPT than in Regret theory. For n available options, the required number of computations is n for RPT and n(n-1)/2 for Regret theory. The amount of information processing for Regret theory increases at a faster rate than the increase in the number of prospects.

The choice of reference point will affect the selection of the optimal action and the model's predictive power depends on whether this choice successfully mimics the real life situation. Based on the argument in the above paragraph, a limited capacity for processing information may prompt individuals to adopt a "simple" reference point ("simple" in the sense that it requires no additional effort in collecting and processing information). For multi-period decisions like the demand for insurance, the optimal action chosen in the previous period may serve as a good candidate for the reference point in the current period. The choice of reference point and its impact on decision-making is investigated in Chapter 5.

Regarding the bilinearity of RPT, consider the decision rule under the new model:

$$A_i \underset{\prec}{\succ} A_k \Leftrightarrow \sum_{j=1}^{n} p_j M(x_{ij};\ x_{oj}) \underset{<}{\geq} \sum_{j=1}^{n} p_j M(x_{kj};\ x_{oj}) \tag{4.17}$$

It can be summarized by a three-element function as

$$\Upsilon(A_i;\ A_o;\ A_k) = \psi(A_i;\ A_o) - \psi(A_k;\ A_o). \tag{4.18}$$

The pitfall in the Fishburn transitivity condition is alleviated as the rejoice-regret components are retained in the function.

In terms of preference ranking, note that the index given by $\Upsilon(\cdot;\ \cdot;\ \cdot)$ avoids "double-counting" the same emotional factor. In Regret theory, if $U(x_{ij}) > U(x_{kj})$ for some j, both the rejoice factor of choosing A_i over A_k and the regret factor of choosing A_k over A_i are included. In Reference Point theory, if $U(x_{ij}) > U(x_{oj}) > U(x_{kj})$, rejoice from choosing A_i over A_o minus regret from choosing A_k over A_o $\{R[U(x_{ij}) - U(x_{oj})] - R[U(x_{kj}) - U(x_{oj})]\}$ will represent the overall psychological impact of choosing A_i over A_k.[5]

4.4 AXIOMS FOR THE MODIFIED UTILITY FUNCTION IN RPT

Four axioms for preference orderings over lotteries are used in deriving the VNM utility function: they are (A1) completeness, (A2) transitivity, (A5) independence, and (A6) mixture continuity. These axioms were studied in some detail in Section 2.3. This section will check whether these axioms can be applied to the modified utility function of RPT. First, the axioms are repeated as follows:

(A1) The completeness axiom

For any two outcomes x_1 and x_2, one and only one of the three following relations holds:

$x_1 \succ x_2$, $x_1 \prec x_2$, or $x_1 \sim x_2$.

(A2) The transitivity axiom

For outcomes x_1, x_2, and x_3,

$x_1 \succ x_2$ and $x_2 \succ x_3$ imply $x_1 \succ x_3$.

(A5) The independence axiom

For outcomes x_1, x_2, and x_3, if $x_1 \succ x_2$, then

$\alpha x_1 + (1-\alpha)x_3 \succ \alpha x_2 + (1-\alpha)x_3$, $0 < \alpha \leq 1$.

(A6) The mixture continuity axiom

For $x_3 \succ x_2 \succ x_1$, there exists some β $(0 < \beta < 1)$ such that

$\beta x_1 + (1-\beta)x_3 \sim x_2$.

4.4.1 Completeness in Reference Point theory

The modified utility function is given by

$$M(x_{1j};\ x_{oj}) = U(x_{1j}) + R[U(x_{1j}) - U(x_{oj})] \tag{4.19}$$

Given the VNM utility function $U(\cdot)$ and a monotonic increasing rejoice-regret function $R[\cdot]$, $U(x_{1j}) > U(x_{2j})$ iff $x_{1j} \succ x_{2j}$. If $U(x_{1j}) > U(x_{2j})$, it implies that $R[U(x_{1j}) - U(x_{oj})] > R[U(x_{2j}) - U(x_{oj})]$ for any given x_{oj}. Consequently, $x_{1j} \succ x_{2j}$ iff $M(x_{1j};\ x_{oj}) > M(x_{2j};\ x_{oj})$ where $M(\cdot;\ \cdot)$ is composed of $U(\cdot)$ and $R[\cdot]$. Similarly, we have $x_{1j} \prec x_{2j} \Leftrightarrow U(x_{1j}) < U(x_{2j})$ and $R[U(x_{1j}) - U(x_{oj})] < R[U(x_{2j}) - U(x_{oj})]$ for any given x_{oj}. Therefore, $x_{1j} \prec x_{2j} \Leftrightarrow M(x_{1j};\ x_{oj}) < M(x_{2j};\ x_{oj})$. When $x_{1j} \sim x_{2j}$, $U(x_{1j}) = U(x_{2j})$ and $R[U(x_{1j}) - U(x_{oj})] = R[U(x_{2j}) - U(x_{oj})]$ for any given x_{oj}. Consequently, $x_{1j} \sim x_{2j} \Leftrightarrow M(x_{1j};\ x_{oj}) = M(x_{2j};\ x_{oj})$. It is clear that when the preference ordering given by the VNM utility function is complete, the modified utility function also satisfies the completeness axiom with a monotonic increasing $R[\cdot]$.

4.4.2 Transitivity in Reference Point Theory

One of the motivations to develop the Reference Point theory is to establish transitivity in Regret theory. Section 4.3 has shown that the preference ordering in Reference Point theory is transitive so long as the same reference point $\tilde{x}_o$ ($\tilde{x}_o$ is a vector composed of x_{oj} for $j = 1, \ldots, n$) is used in the comparison among different outcomes associated with each action. Transitivity is reference-specific in the sense that altering the reference point will lead to another set of preferences. Equations (4.2) to (4.5) show that altering the reference point, as in Regret theory, may lead to intransitivity.

4.4.3 Independence in Reference Point Theory

In Reference Point theory, the value index for x_{1j} is given by $M(x_{1j};\ x_{oj})$. For the composite term $\alpha x_{1j} + (1-\alpha)x_{3j}$, its value index is $[\alpha M(x_{1j};\ x_{oj}) + (1-\alpha)M(x_{3j};\ x_{oj})]$. Similarly, the value index for $\alpha x_{2j} + (1-\alpha)x_{3j}$ is $[\alpha M(x_{2j};\ x_{oj}) + (1-\alpha)M(x_{3j};\ x_{oj})]$. Hence, if $x_{1j} \succ x_{2j}$ then $M(x_{1j};\ x_{oj}) > M(x_{2j};\ x_{oj})$ and it follows that $\alpha x_{1j} + (1-\alpha)x_{3j} \succ \alpha x_{2j} + (1-\alpha)x_{3j}$ (for $0 < \alpha \leq 1$). In other words, the preference in the modified utility function $M(\cdot;\ \cdot)$ is linear in probabilities.

Independence is a much disputed axiom in decision theory and most of the alternatives to the EU model replace this axiom in order to predict the observed anomalies. The modified utility function in RPT retains the independence axiom when the reference point is unchanged. As in the EU model, independence implies that the preference between two actions only depends on their different terms and the identical terms in the two actions will not affect the ranking. Independence follows from the additive structure in Equation (4.13) (repeated here)

$$A_i \underset{\precsim}{\succsim} A_k \Leftrightarrow \sum_{j=1}^{n} p_j\, M(x_{ij};\ x_{oj}) \underset{<}{\geq} \sum_{j=1}^{n} p_j\, M(x_{kj};\ x_{oj})$$

which describes the preference ordering in RPT. If confronted with a new prospect, the decision maker may adopt another reference point. A new reference point will change the modified utility function $M(\cdot;\ \cdot)$ and consequently the evaluation of the outcomes in each action.

Although the identical terms in the two actions still carry no impact in affecting the ranking, the initial preference ordering may not be preserved when the new reference point affects the evaluation of the non-identical terms in the two actions. The analysis in Chapter 8 shows that some of the anomalous behaviour discovered in the experiments can be explained by a shift in reference point.

4.4.4 Mixture Continuity in Reference Point Theory

The modified utility function $M(\cdot;\ \cdot)$, similar to $U(x_{ij})$, is a monotonic increasing function of x_{ij}. Hence, there exists a mixture of $M(x_{1j};\ x_{oj})$ and $M(x_{3j};\ x_{oj})$ that is equal to $M(x_{2j};\ x_{oj})$ where $M(x_{3j};\ x_{oj}) > M(x_{2j};\ x_{oj}) > M(x_{1j};\ x_{oj})$. The reference-specific preference defined in RPT satisfies the axiom of mixture continuity axiom (A6).

CHAPTER 5 THE ELEMENTS OF REFERENCE POINT THEORY

Through the rejoice-regret function, RPT recognizes the psychological dimension in the decision-making process. Section 5.1 discusses the importance of psychological factors in the decision-making process. It reviews the experimental findings of the endowment effect, the status quo bias, and loss aversion. These studies suggest that the negative impact associated with losses outweighs the positive impact of possible gains. Drawing on experimental findings, Section 5.2 determines the appropriate functional form of the rejoice-regret function based on the discussion in Section 5.1. Section 5.3 derives a modified utility curve. Section 5.4 develops the hypothesis to determine the selection of a reference point.

The hypothesis outlined in Section 5.4 suggests that decision-makers apply a continuous function to evaluate the prospects under consideration, the prospect with the highest transformed mean is selected as the reference point. The hypothesized transformation function contains an individual-specfic variable such that each decision-maker derives an unique transformation function to evaluate the prospects. Different reference points will be selected by different individuals. The choice of a reference point has significant bearing on determining the evaluation function. For example, a large reference point will frame expected gain into loss reduction. The analytics of adjusting the reference point will be discussed in Section 6.2.

5.1 THE PSYCHOLOGICAL DIMENSION OF DECISION-MAKING

Research in decision theory and information processing suggests that psychological factors play a significant role in the decision-making process. Experiments reveal that individuals typically demand a large compensation for a reduction in a particular entitlement while they offer to pay a smaller amount for an equal improvement, implying that decisions are reached in relation to a reference point. These experiments show that the perception of the problem by the decision maker is an important aspect of decision-making, as one would expect from cognitive psychology. Research efforts aimed at transforming the decision weights in order to improve on the EU model seem inadequate when research shows that bias in the evaluation process persists even in choices among riskless alternatives. The rest of this section will review the sources of such bias, including the endowment effect, the status quo bias, and loss aversion.

5.1.1 Endowment Effect

The endowment effect, which refers to bias against trading away one's endowment, was widely observed in laboratory experiments pertaining to the study of willingness to pay (WTP) and willingness to accept (WTA).[1] Experimental results by Coursey, Hovis, and Schulze (1987), Kahneman, Knetsch, and Thaler (1990), Knetsch (1989), and Knetsch and Sinden (1984, 1987) demonstrate that subjects exhibited strong attachment to whatever was given to them at the beginning of the experiments. For example, in Knetsch and Sinden (1984), half of the subjects were each endowed with a lottery ticket and the remaining subjects got $2 in cash initially; they were later given the opportunity to switch to either cash or the ticket. They found that substantially more subjects endowed with the ticket wanted to play the lottery than those who got cash. Apparently, most of the subjects who held the ticket expected more than $2 in compensation for giving up the ticket. On the other hand, fewer subjects who got cash wanted to pay $2 to acquire the lottery ticket. The discrepancy in WTA and WTP could be explained by the endowment effect, namely, subjects exhibited strong attachment to their entitlement and were reluctant to trade. In other words, the position of the reference point would influence the utility associated with the lottery ticket.

Another experiment by Knetsch and Sinden (1984) attempted to show that the endowment effect was the major factor accounting for the divergence in WTA and WTP. They introduced two extra groups of subjects whose responsibility was to give advice instead of participating in the trading session. The same pattern emerged for the subjects holding lottery tickets or cash; WTA was larger than WTP. The non-participating subjects, however, responded differently. Twelve of 22 (or 55%) of the non-participating subjects advised the ticket holders to accept the $2 cash offer. Eleven of 23 (or 48%) recommended that the subjects with cash buy the lottery tickets. The discrepancy between WTA and WTP diminished when entitlement was not an issue. These findings supported the hypothesis that entitlement affects the evaluation of a prospect.

5.1.2 Status Quo Bias

One implication of the disparity between WTA and WTP is that individuals are reluctant to switch to a new alternative. They are biased toward the status quo position because the disadvantages of leaving it loom larger than the advantages. Samuelson and Zeckhauser (1988, p.21) found in an experiment that subjects were reluctant to move their businesses to a different building. Only 9% of the subjects would move to a newer building if its rent was 20% higher than the existing facility (and no one would move if the rent increased by more than 30%). The disadvantage of leaving the old building (higher rent) outweighed the advantage of moving into the new building. In contrast, 49% of the subjects demanded a 20% or more reduction in rent in order to relocate from a newer building to an older building. In the same study, similar experiments also found that among different portfolios, subjects were inclined to maintain the portfolio they inherited.

5.1.3 Loss Aversion

Related to the endowment effect and status quo bias, researchers observed that individuals were much more sensitive to losses than gains. They observed that when individuals were confronted with new options, subject's choices were dictated by avoiding losses rather than acquiring improvements. For example, Tversky and Kahneman (1991, p.1045) asked the subjects to choose between two new jobs, A and B:

Job	Contact with others	Commute Time
Present job (A′)	isolated for long stretches	10 min.
Job A	limited contact with others	20 min.
Job B	moderately sociable	60 min.

About 70% of the respondents chose Job A. When the question was set in the following context:

Job	Contact with others	Commute Time
Present job (B′)	much pleasant social contact	80 min.
Job A	limited contact with others	20 min.
Job B	moderately sociable	60 min.

About 67% of the respondents chose Job B. Subjects were more sensitive to the dimension in which they were losing relative to their present job, presumably the reference point in the decision-making process. This situation calls for non-reversible indifference curve where in the first case A is preferred to B but B is preferred to A in the second case (see Knetsch, 1989). The following diagram depicts two sets of indifference curves associated with the two reference points A′ and B′. The preferences shown in Figure 5.1 exhibits loss aversion. A steep indifference curve associated with A′ indicates that giving up the entitlement "commute time saved" requires a large compensation in the other attribute. While a flat indifference curve associated with B′ implies that a small reduction in "social contact" requires a large compensation in "commute time saved".

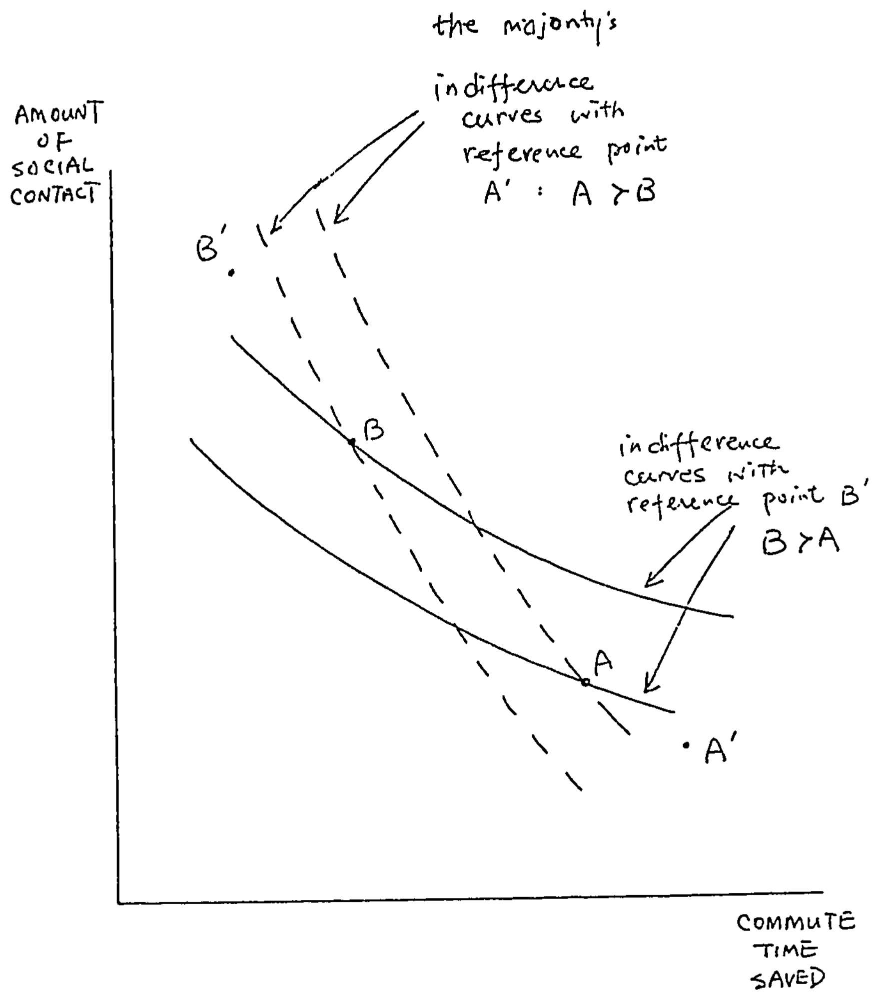


FIGURE 5.1 LOSS AVERSION AND MULTIPLE REFERENCE POINTS

Other research on this topic includes Thaler (1980) and Viscusi, Magat, and Huber (1987). They observed the same response pattern from the subjects. When presented with a possibility that would reduce existing risk, subjects offered a relatively small amount for this option. When they were asked to voluntarily assume additional risk, they demanded a huge compensation. About 77% of the respondents in Viscusi, Magat, and Huber's (1987) experiment refused to assume additional risk. As with the endowment effect, subjects seemed to consider the first scenario as an improvement, and assigned a low WTP value for the gains they would enjoy. On the other hand, they perceived the second scenario as an infringement of their current welfare, and demanded a large WTA as compensation.

Another form of loss aversion is caused by the response mode effect which put the same problem in different contexts. Subjects react differently when the choices are framed in terms of gains instead of losses. Tversky and Kahneman (1981) asked the subjects to choose between two programs to combat a deadly disease that has infected 600 people. The first program will save 200 out of 600 patients and the alternative program will have 1/3 probability to save all 600 patients and 2/3 probability to save none. A majority of the respondents chose the first program. A majority of the respondents chose the second program when the same problem was stated in terms of losses as follows:

> If the first program is adopted 400 patients would die. If the second program is adopted there is 1/3 probability that nobody would die and 2/3 probability that all 600 patients would die.

The contexts of the problems suggest different reference points for the evaluation of the two programs. In the first problem, the context is worded in terms of the number of lives saved. When the choices are stated in terms of gains (the number of lives saved), loss aversion implies that the subjects would avoid the possibility of larger losses in the second program. The decision frame in the second problem suggests that decision is made in relation to the number of death avoided. With reference to this, the subjects tended to ignore the gains of 200 lives in the first program and focused on the benefit of the second program. So, when the choices are stated in terms of the number of patients that would die, the subjects preferred the method that might eliminate losses entirely. The choice pattern revealed that respondents were risk-averse in gains and risk-seeking in losses.

The disparity between WTP and WTA points to the possibility that decision-making involves more than just a comparison of the utility values of the competing choices. The persistence of the disparity challenges the existing notion of utility measurement and preference ranking. New models like Prospect theory and Regret theory include some psychological factors in order to account for this phenomenon. These new models, however, have their shortcomings too.

Prospect theory is often criticized as an ad hoc model without a formal structure (Viscusi, 1989, p.236). The theory's editing process does not have an associated theory that defines the rules of operation. The decision weight function is described by some special features like subadditivity, subcertainty, and subproportionality.

Unlike SWU (Karmarkar, 1978), the decision weight function in Prospect theory lacks a specific transformation function which determines how probabilities are converted into decision weights. The flexibility implanted in the structure allows Prospect theory great freedom to interpret behaviour in a way that is consistent with the theory. This flexibility, however, undermines the predictive power of the theory.

The major problem Regret theory faces is intransitivity. One can easily recognize that intransitivity will render any decision model inappropriate in ranking alternative actions. This issue is discussed in the next section.

5.2 SPECIFICATIONS OF THE REJOICE-REGRET FUNCTION

In light of the discrepancy between WTA and WTP, one may expect that gains and losses of the same magnitude relative to the payoff of the reference point will have different impacts on utility evaluation. Specifically, when the payoff of the reference point is higher than the payoff of an alternative action in a particular state of the world, a decision maker will reduce the value of the alternative action substantially by means of a large regret component. On the contrary, a decision maker will appreciate moderately an alternative action which does better than the reference point; this is equivalent to having a small rejoice component.

Several studies (Kahneman and Tversky, 1979, 1983; and Tversky and Kahneman, 1991) suggest that the ratio between the value of gains and the value of losses is about 2 to 1. More importantly, they found evidence suggesting that the marginal value of both gains and losses decreases with their size (see Figure 5.2). These characteristics of evaluating gains and losses can be applied to the specification of the rejoice-regret function R[·].

The following conditions describe a R[·] that will conform to Figure 5.2. For $\xi_{ij} = U(x_{ij}) - U(x_{oj})$,

(A) Rejoice / Regret Conditions

(i) $R[\xi_{ij}] = 0$ for $x_{ij} = x_{oj}$

(ii) $R[\xi_{ij}] > 0$ for $x_{ij} > x_{oj}$

(iii) $R[\xi_{ij}] < 0$ for $x_{ij} < x_{oj}$

(B) Monotonicity Condition

(iv) $R'[\cdot] > 0$ for $\xi_{ij} \gtrless 0$ where $R'[\cdot] = \left.\dfrac{\partial R[\xi_{ij}]}{\partial \xi_{ij}}\right|_{x_{oj}}$

(C) Strict Concavity for Gains

(v) $R''[\cdot] < 0$ for $\xi_{ij} > 0$ where $R''[\cdot] = \left.\dfrac{\partial^2 R[\xi_{ij}]}{\partial \xi_{ij}^{\,2}}\right|_{x_{oj}}$

(D) Strict Convexity for Losses

(vi) $R''[\cdot] > 0$ for $\xi_{ij} < 0$

(5.1)

The first three conditions in (5.1) define the notion of rejoice and regret. When the payoff of the option is equal to that of the reference point, $U(x_{ij}) - U(x_{oj}) = 0$. In this case, there is neither rejoice nor regret in choosing the alternative option over the

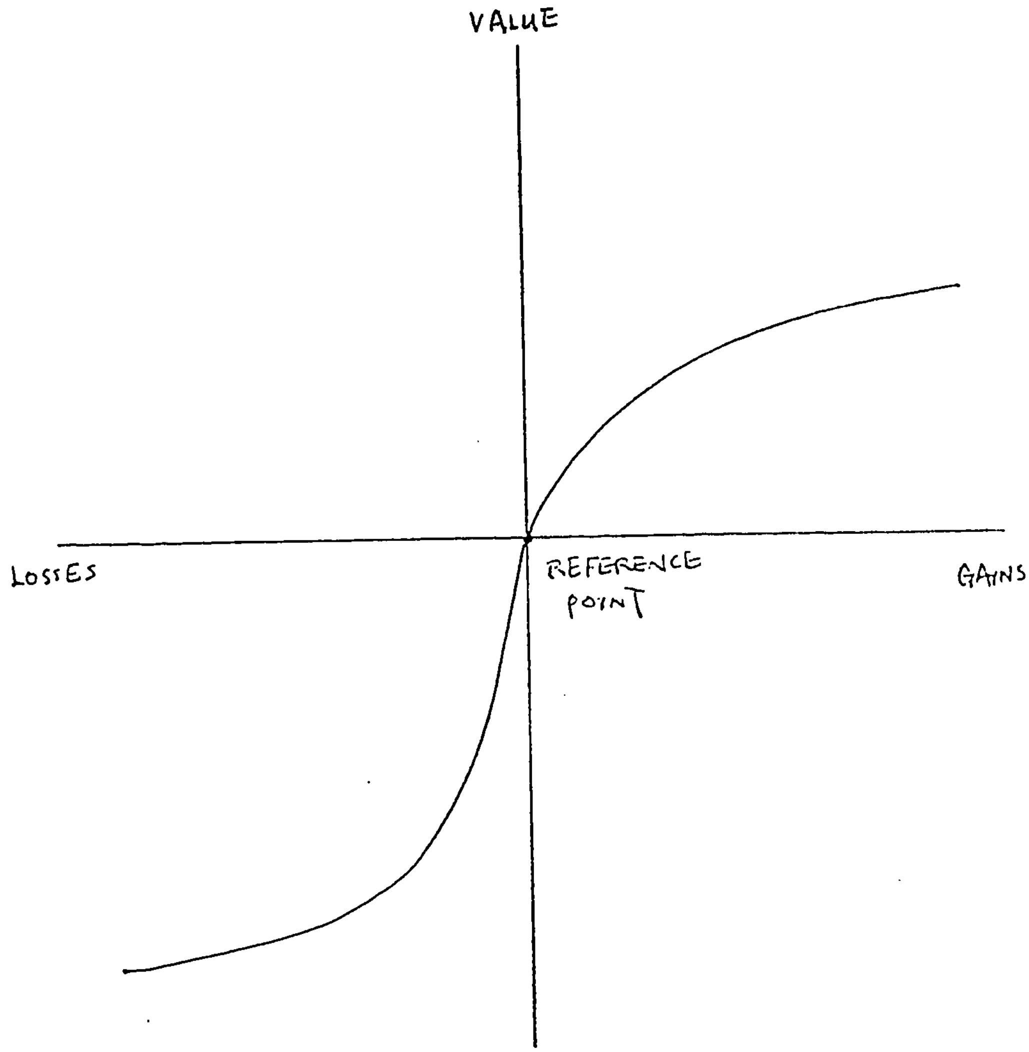


FIGURE 5.2 A TYPICAL VALUE FUNCTION: ASYMMETRY IN THE EVALUATION OF GAINS AND LOSSES

reference point and R[0] = 0. The individual will experience rejoice in choosing the alternative option when $x_{ij} > x_{oj}$. Naturally, $R[U(x_{ij}) - U(x_{oj})] > 0$. Similarly, the individual will experience regret if his choice leads to lower utility than the reference point because $U(x_{ij}) < U(x_{oj})$ when $x_{ij} < x_{oj}$. Consequently, $R[U(x_{ij}) - U(x_{oj})] < 0$.

Condition (iv) implies that $R[\cdot]$ is monotonically increasing with respect to x_{ij} for a given reference point x_{oj}. Conditions (v) and (vi) respectively imply that $R[\cdot]$ is strictly concave when $x_{ij} > x_{oj}$ and strictly convex when $x_{ij} < x_{oj}$. In other words, rejoice (or regret) increases at a decreasing rate as $|U(x_{ij}) - U(x_{oj})|$ increases. Thus, $R[\cdot]$ exhibits diminishing sensitivity to gains and losses.

Assuming a symmetric rejoice-regret function, $R[\xi_{ij}] = -R[-\xi_{ij}]$, a strictly concave VNM $U(\cdot)$ implies that the same amount of monetary gain and monetary loss will derive a larger disutility associated with loss than the utility associated with gain. This will lead to asymmetry in the resulting regret and rejoice. For an example, given two alternative options with payoffs $x_{ij} = x_{oj} + \varepsilon$ and $x_{kj} = x_{oj} - \varepsilon$, where $\varepsilon > 0$, a strictly concave $U(\cdot)$ implies that $\xi_{ij} = U(x_{ij}) - U(x_{oj})$ is less than $-\xi_{kj} = -[U(x_{kj}) - U(x_{oj})]$. It follows that for the same amount of monetary gains and losses ε, the resulting rejoice $R[\xi_{ij}]$ is less than regret $R[-\xi_{kj}]$ for a symmetric $R[\cdot]$. The amount of discrepancy between ξ_{ij} and $-\xi_{kj}$ is positively related to the concavity of the utility function. This relationship is depicted in Figure 5.3.

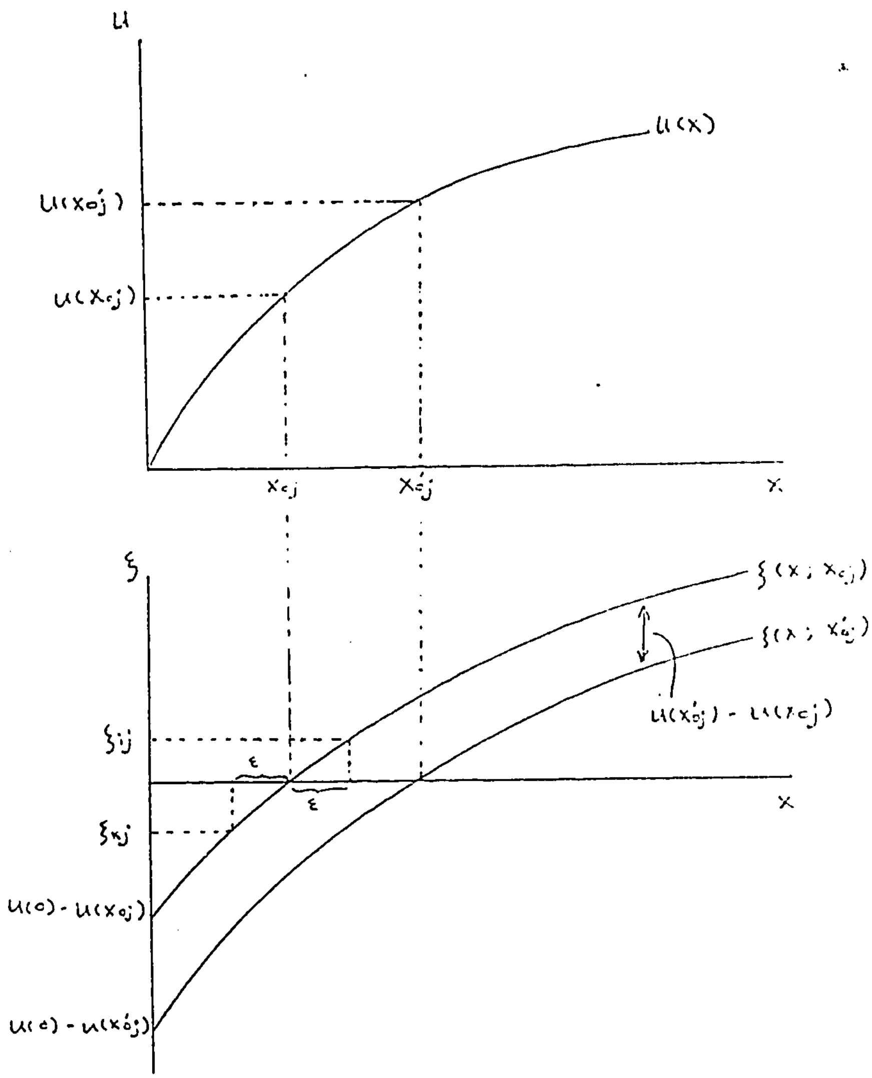


FIGURE 5.3 STRICTLY CONCAVE UTILITY FUNCTION AND ASYMMETRY IN EVALUATING GAINS AND LOSSES

Applying the conditions in (5.1), the resulting functional form of the rejoice-regret function is strictly convex in the negative (or regret) domain and strictly concave in the positive (or rejoice) domain. A convex-concave R[·] function is depicted in Figure 5.4; a typical example of a convex-concave functional form is a cubic function. The degree of convexity in R[·] in the negative domain should be large enough to contain the growth rate of regret when $x_{ij} < x_{oj}$ such that there is diminishing sensitivity to losses.

$$\left.\frac{\partial^2 R[\xi_{ij}]}{\partial x_{ij}^2}\right|_{\bar{x}_{oj}} = R'[\cdot]U''(\cdot) + R''[\cdot][U'(\cdot)]^2 \lessgtr 0.$$

$$\left.\frac{\partial^2 R[\xi_{ij}]}{\partial x_{ij}^2}\right|_{\bar{x}_{oj}} \begin{array}{l} < 0 \quad \text{when } x_{ij} > x_{oj} \\ > 0 \quad \text{when } x_{ij} < x_{oj} \quad \text{iff} \quad \dfrac{R''[\cdot]}{R'[\cdot]} > \dfrac{-U''(\cdot)}{U'(\cdot)^2} \end{array} \tag{5.2}$$

Intuitively, it implies that $R[\xi_{ij}]$ should be curved (greater degree of convexity) enough to contain the increasing growth rate of ξ_{ij} (related to the degree of concavity in U(·)) as $-(x_{ij} - x_{oj})$ grows.

The rejoice-regret function takes a different functional form in Regret theory. Loomes and Sugden showed that a concave-convex R[·] would be able to accommodate anomalies like the common consequence effect, the common ratio effect, the isolation effect, the reflection effect (1982, pp. 811-5) and the preference reversal phenomenon (1983). Hypothesis II in Machina's GEU (1982a, p.301, Figure 4.1(b)) also assumed that the local utility function is concave-convex while the local utility function is strictly concave in Hypothesis I

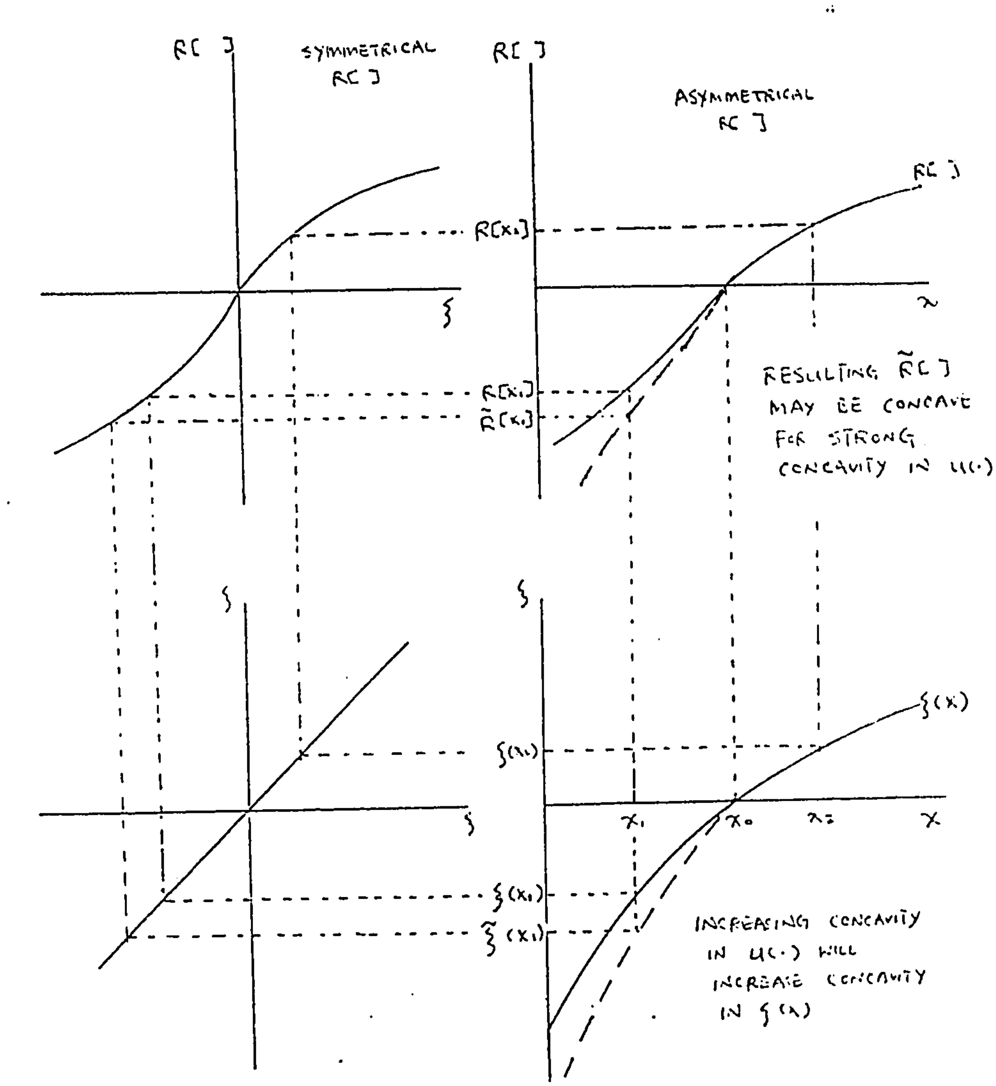


FIGURE 5.4 A CONVEX-CONCAVE REJOICE-REGRET FUNCTION

(Machina, 1982a, p.301, Figure 4.1(a)). Among these alternative functional forms, the convex-concave function in Reference Point theory produces the best replication for Figure 4.1 which reflects the observed disparity in WTP and WTA. With appropriate conditions on the functional form of the rejoice-regret function (Equation (5.2)), RPT is consistent with the hypothesis that an individual derives relatively larger disutility from losses than utility from gains. The experimental findings on the endowment effect, the status quo bias, and loss aversion support this hypothesis. Assuming that the rejoice-regret function takes this functional form, the following section derives a modified utility curve.

5.3 DERIVATION OF MODIFIED UTILITY CURVE

The EU model hypothesizes that the attitude towards risk of the decision maker influences the decision-making process. It shows that risk-averse behaviour, which implies a strictly concave utility function, resolves the St. Petersburg paradox successfully. Reference point theory, in light of the findings of different experimental studies, suggests the incorporation of a new dimension that accounts for the psychological factors in the decision-making process. It replaces the familiar concave utility function by a modified utility function.

The four axioms examined in Section 4.4 ensure that the modified utility is a monotonically increasing and continuous function of x_{ij}, given x_{oj}. One can obtain a modified utility curve by combining a strictly concave U(·) with a convex-concave R[·]. The experience of regret when the chosen alternative is inferior to the reference point may generate a convex segment in the modified utility curve. This situation occurs when the convex segment of R[·] dominates the concavity in U(·). In other words,

$$M''(\cdot,\ x_{oj}) = U''(\cdot) + [U'(\cdot)]^2 R''[\cdot] + U''(\cdot)R'[\cdot] > 0 \text{ when } x_{ij} < x_o$$

or

$$-U''(\cdot) < [U'(\cdot)]^2 R''[\cdot] + U''(\cdot)R'[\cdot] \text{ when } x_{ij} < x_o.$$

The following diagram depicts a convex-concave modified utility function.

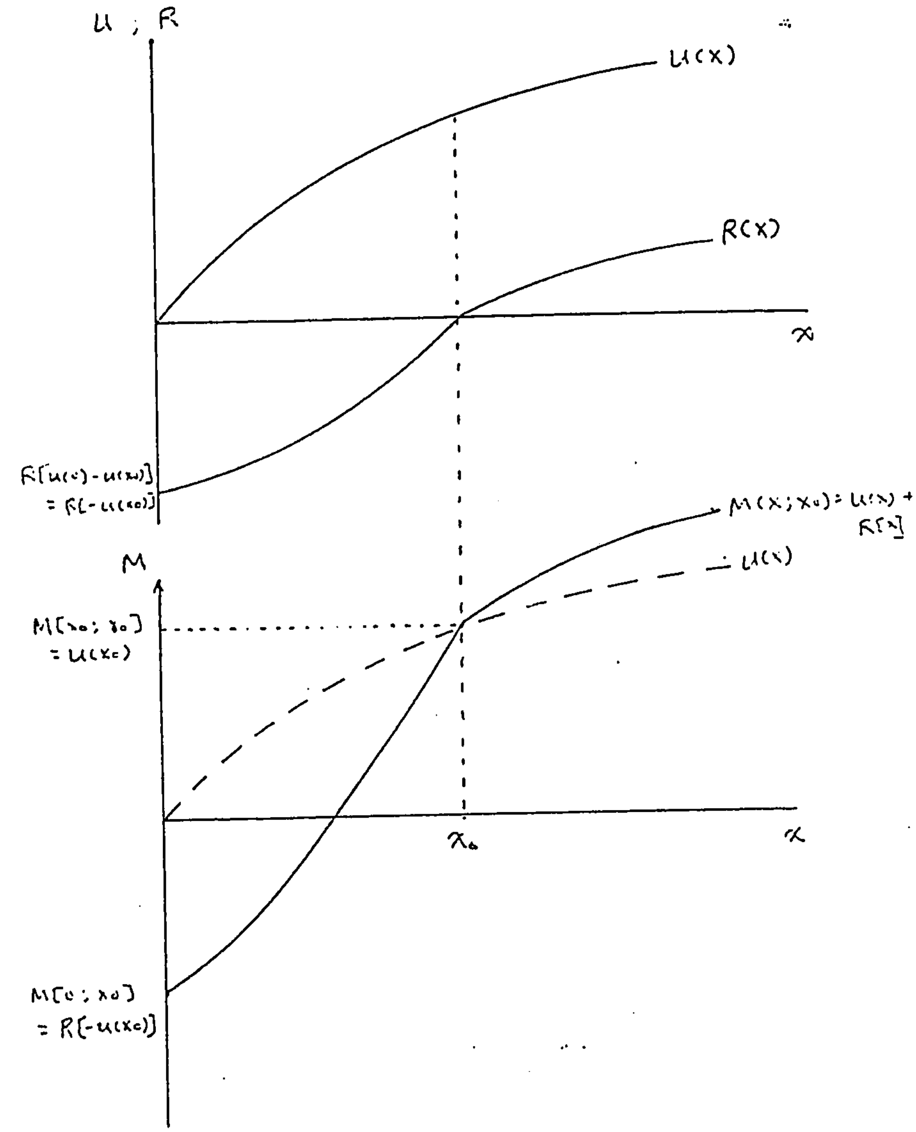


FIGURE 5.5 A MODIFIED UTILITY FUNCTION

The modified utility function depicted in Figure 5.5 is not the first attempt to alter the VNM utility function with its conventional strictly concave utility curve. Friedman and Savage (1948) initially suggested an inverted S-shape (concave-convex) utility to harmonize the coexistence of gambling (risk-seeking) behaviour and buying insurance (risk aversion). The inverted S-shape utility function is similar to the modified utility curve derived from a concave-convex R[·] by assuming that the importance of rejoice and regret increases at an increasing rate. Yet, this inverted S-shape utility curve implies that an individual would prefer any fair gamble with a large prize; it fails to resolve the St. Petersburg Paradox. Consequently, Friedman and Savage (1948) modified their proposed utility curve by imposing a terminal concave section; this function is shown in Chapter 3 (Figure 3.7(b)).

5.4 RULES FOR DETERMINING THE REFERENCE POINT

The modified utility function in Figure 5.5 is dependent on the choice of reference point. The determination of reference point is particularly important because it will influence the evaluation function of Reference Point theory. To highlight the impact of a reference point on decision-making, we compare the decision-making processes of the EU model, a non-EU model, and RPT.

In general, one can identify four major components in the decision-making process of a typical decision theory; they are (i) the prospect of concern, (ii) the evaluation criteria, (iii) the resulting value index, and (iv) the decision based on the ranking of the prospect. Each decision theory will hypothesize a different approach to one or more of these components. For example, the Expected Utility model hypothesizes that the evaluation process is independent of the prospects under consideration; in other words, the valuation function remains the same for different prospects. Consequently, the resulting preference of the EU model will satisfy the independence axiom. The decision-making process in the EU model is described in the following figure:

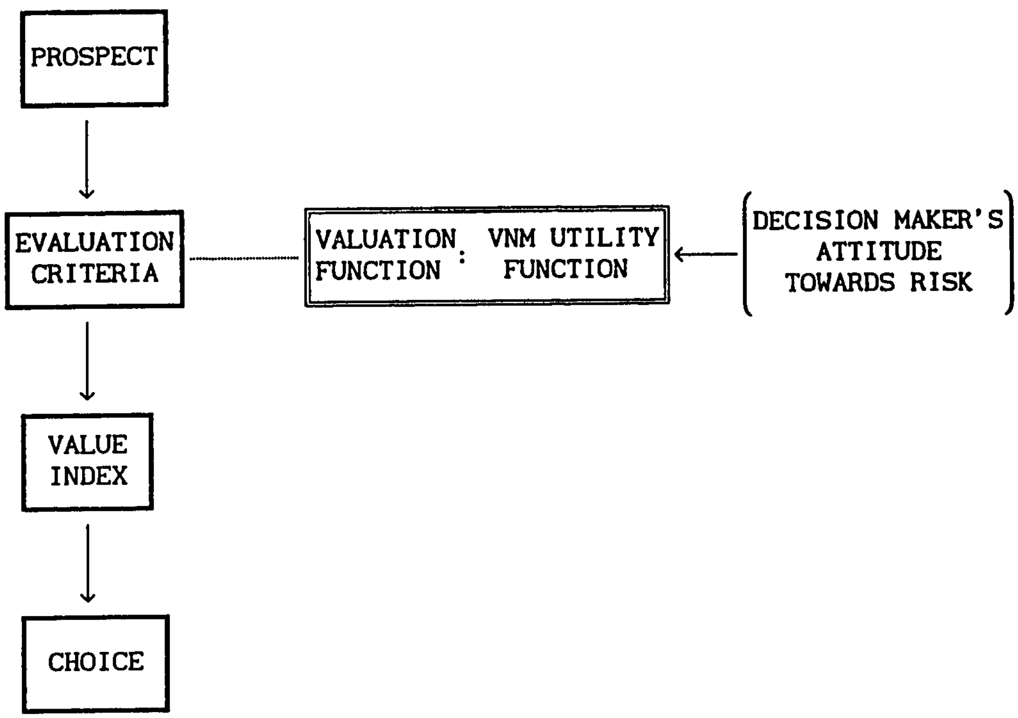


FIGURE 5.6 DECISION-MAKING PROCESS ACCORDING TO THE EU MODEL

The Non-Expected Utility models modify the EU model by relaxing the independence axiom; the valuation functions in these models will vary according to the prospects under consideration. This hypothesis is incorporated into the decision-making process as shown in Figure 5.7 below.

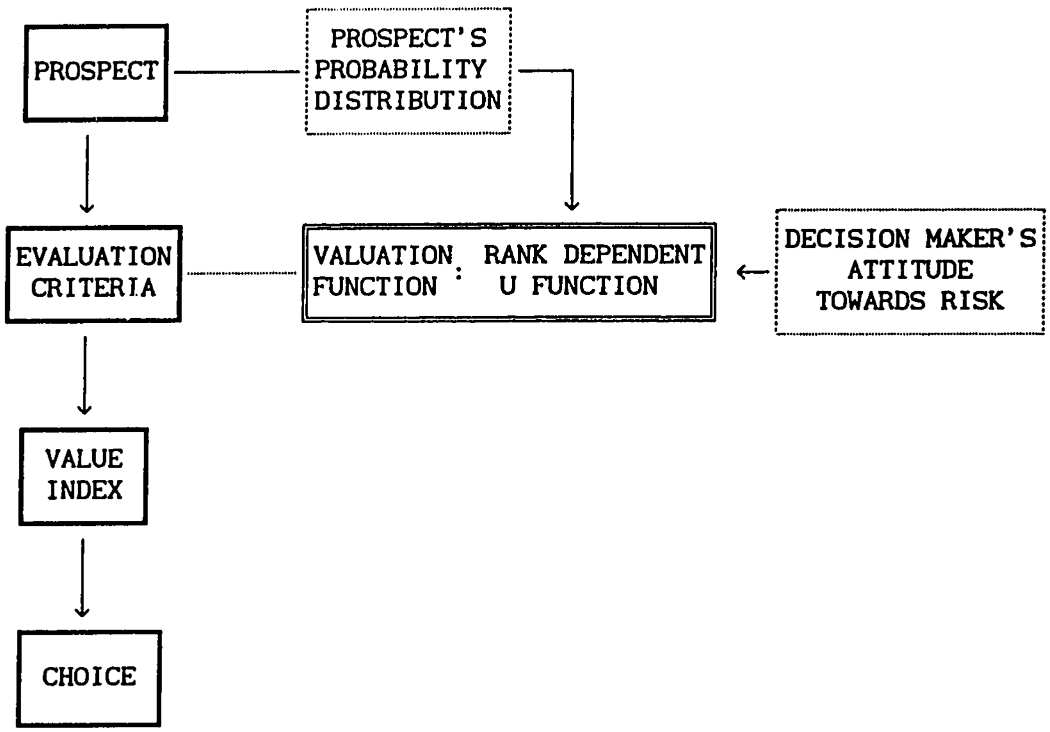


FIGURE 5.7 DECISION-MAKING PROCESS ACCORDING TO EURDP THEORY

The evaluation process in these models assumes that the valuation function is also dependent on the probability distribution of the prospect being considered. The valuation function is determined by the subjective attitude of the decision maker and the objective information contained in the prospects. The decision-making process in this class of decision theory (mainly holistic models including the

EU model) is procedure invariant, the decision maker will assign the same value index to two prospects which have identical probability distributions on the same outcomes. In other words, the context in which the prospects are presented will not influence the evaluation process and the final decision, only the objective information associated with the prospects determines one's choice.

The relaxation of the independence axiom, however, will not accommodate anomalies caused by the response mode effect. That is, the choice of the decision maker is sensitive to the context in which a prospect is presented. The decision maker may prefer Prospect A over Prospect B in one situation while selecting Prospect B and forgoing Prospect A in another situation. According to the anomalous results of these experiments, the decision-making process violated the notion of procedure invariance. The preference reversal phenomenon, the isolation effect, and the response mode effect are typical examples which violate procedure invariance. In these experiments, many decision makers reverse their choices over two prospects under different contexts of elicitation.

In the preference reversal phenomenon, the isolation effect, and the response mode effect, the revealed choice patterns of the subjects suggest that the evaluation criteria are dependent on the contexts of the problems. Reference Point theory attempts to model the changes in the valuation function caused by different decision frames. RPT hypothesizes that framing in decision problems (through manipulating the contexts of the problems or elicitation methods) is going to influence the selection of reference point which in turn will affect the valuation function. The decision-making process in RPT is depicted in Figure 5.8 below.

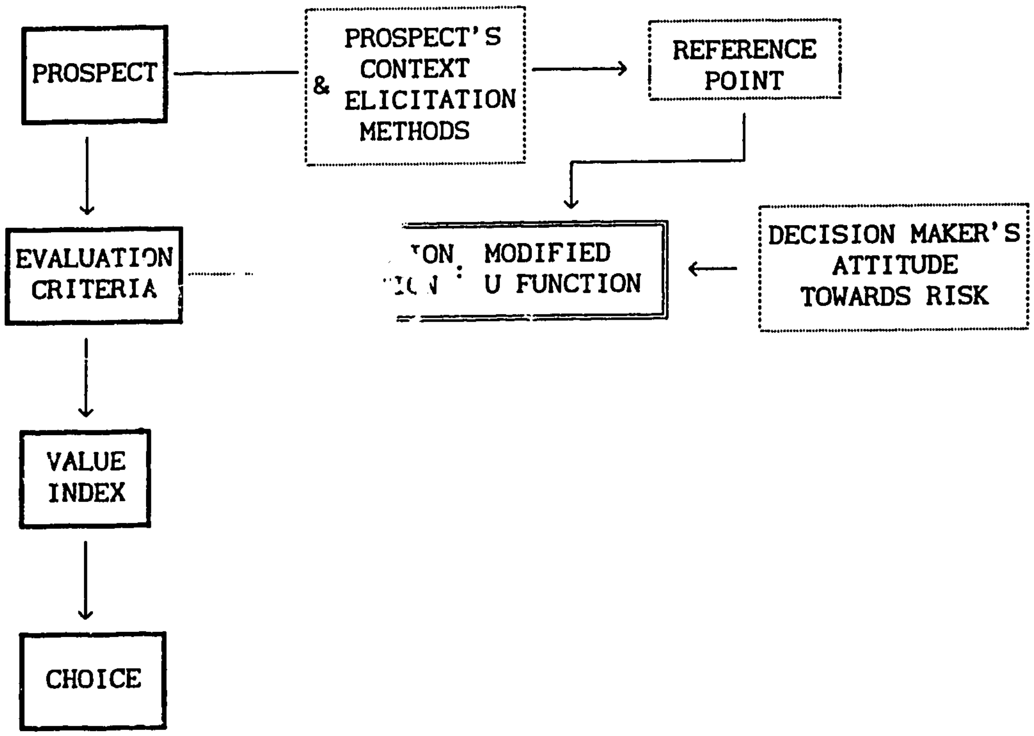


FIGURE 5.8 DECISION-MAKING PROCESS ACCORDING TO RPT

In addition, since Pratt (1964) documented the notion of decreasing risk aversion, it is typical to assume that the amount of wealth will affect the choices of the decision makers. Given that the choice of reference point is critical to the decision-making process, one would expect that the selection of reference point is going to be affected by the decision maker's level of wealth. In other words, the selection of reference point will reflect the decision maker's perception of how well off he is prior to making a choice under uncertainty. The reference point selection process described below will discuss this particular issue in detail.

5.4.1 Risky Reference Prospect: The Reference Point That Involves Positive Prospects

When the decision maker is choosing among prospects with positive outcomes, one of these prospects is likely to be used as the reference point. The reason for choosing one of the available prospects as the reference point is to reduce the amount of information processing required. According to Hypothesis 1 below, the outcomes of the prospects are transformed by $T(\cdot)$ in order to reflect the decision maker's perception of their significance. The transformed mean takes into consideration both the likelihood of the outcomes and the perceived significance of the outcomes. Using transformed mean has the advantage of combining the two most important dimensions of a prospect to formulate the valuation index. Without any prior knowledge about the prospects, it is hypothesized that the prospect with the largest transformed mean value will be selected as the reference point. The hypothesized transformation function used in the selection process is a continuous function.

HYPOTHESIS 1

In order to select a reference point, the decision maker will transform the outcomes of the prospects under consideration by a transformation function $T(\cdot)$. *The prospect with the highest mean transformed value,* $\sum_j p_j T(x_{ij})$, *is selected as the reference point in the decision analysis where* x_{ij} *are the outcomes of prospect* $\tilde{x}_i$.

The hypothesized general transformation function $T(\cdot)$ is assumed to have the following properties:

(i) $T(0) = 0$.

(ii) $T(x_{ij}) > 0$ for $x_{ij} > 0$.

(iii) $T'(x_{ij}) > 0$.

(iv) $T'(k) = 1$ and $1 > T'(x_{ij}) > 0$ for $x_{ij} \neq k$.

(v) $T''(x_{ij}) > 0$ when $x_{ij} < k$ and $T''(x_{ij}) < 0$ when $x_{ij} > k$.

The transformation function $T(x_{ij})$ described above has an inflection point at $x_{ij} = k$; $T(x_{ij})$ is convex for $x_{ij} < k$ and $T(x_{ij})$ is concave for $x_{ij} > k$. Furthermore, $\frac{\partial T(x_{ij})}{\partial x_{ij}}$ reaches its maximum at $x_{ij} = k$. It is assumed that each individual would assign a value to k where k should be greater than 0. It is possible for each decision maker to derive an unique transformation function that he will use consistently to select the reference point.

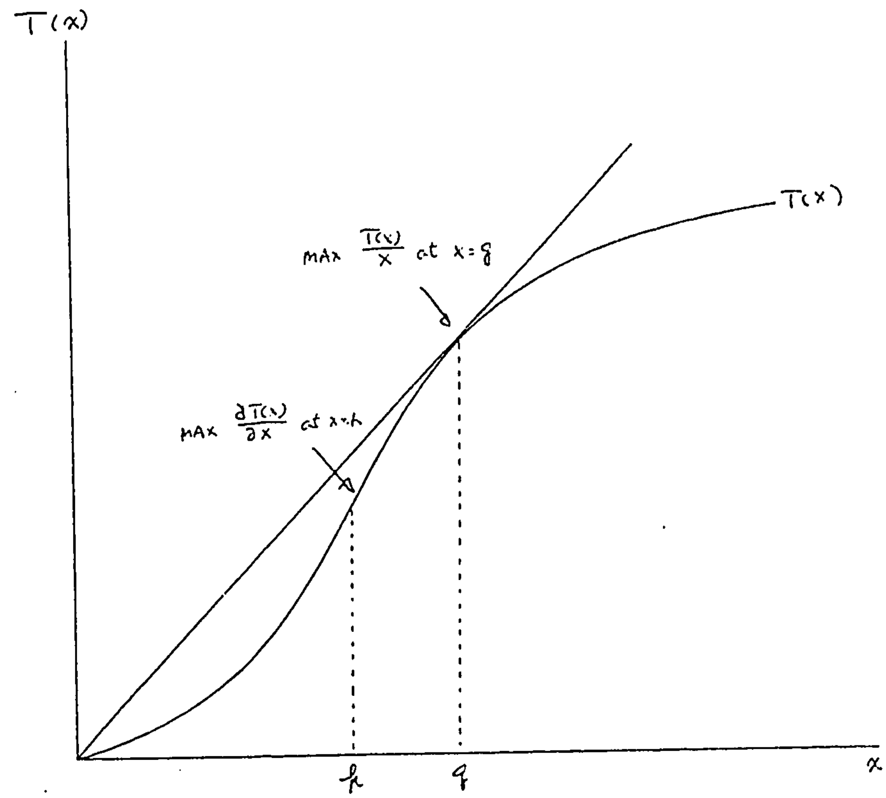


FIGURE 5.9 HYPOTHESIS 1

Hypothesis 1 assumes that each decision maker will customize his transformation function through assigning a particular value to the person-specific parameter, k. The k parameter will capture the heterogenity among the decision makers. A large value in k will increase the overall convexity of T(·), such decision maker will tend to choose a riskier reference point. The choice of k will reflect the degree of risk aversion in selecting tjhe reference point.

Although the selection of a reference point is only an intermediate step in the decision-making process of RPT, the preferred prospect is often identical to the choice of the reference point. This is likely to occur because the modified utility function reflects the status quo bias (see Figure 5.5). The transformation function (in particular the parameter k in T(·)) is very important in the evaluation process of RPT. The impacts of adjusting k on the selection of reference point are discussed in Conjectures 1.1 to 1.6 below.

When each decision maker has a stable transformation function, one can deduce the following conjectures from Hypothesis 1.

Conjecture 1.1

When two prospects (A and B) have the same expected value,

$$E(\text{Prospect A}) = \sum_i p_i x_{Ai} = \sum_j p_j x_{Bj} = E(\text{Prospect B}),$$

and Prospect B is riskier than Prospect A (in the sense of mean preserving spread), Prospect B will be selected as the reference prospect if

$$x_{Ai} \le k \text{ for all } x_{Ai} \text{ and } x_{Bj} \le k \text{ for all } x_{Bj}.$$

The conditions in Conjecture 1.1 imply that all the possible outcomes of each prospect are evaluated on the convex segment of the transformation function. Consequently, the riskier prospect will have the higher mean transformed value.

Conjecture 1.2

When two prospects (A and B) have the same expected value,

$$E(\text{Prospect } A) = \sum_i p_i x_{Ai} = \sum_j p_j x_{Bj} = E(\text{Prospect } B),$$

and Prospect B is riskier than Prospect A (in the sense of mean preserving spread), Prospect A will be selected as the reference prospect if

$$x_{Ai} \geq k \text{ for all } x_{Ai} \text{ and } x_{Bj} \geq k \text{ for all } x_{Bj}.$$

The conditions in Conjecture 1.2, contrary to those in Conjecture 1.1, imply that all the possible outcome of each prospect are evaluated on the concave segment of the transformation function. Consequently, the less risky prospect will have the higher mean transformed value. Conjectures 1.1 and 1.2 together suggest that the selection of reference point is dependent on the parameter value in k.

Conjecture 1.3

Changing the value of the parameter k will influence the choice of the reference point. Increasing the value of k will increase the likelihood of selecting a riskier prospect as the reference prospect. Decreasing the value of k will decrease the likelihood of selecting a riskier prospect as the reference prospect.

To understand Conjecture 1.3, consider two decision makers with different values for k in their respective transformation functions. When a riskier prospect is selected by the individual with a lower k value it must be the case that T(·) exhibits enough degree of convexity over the relevant range. Consequently, the individual with a higher k value, whose T(·) exhibits an even stronger degree of convexity, will also select the same reference point. In other words, if Conjecture 1.1 can be applied to the first individual, then the same conjecture can be applied to the second individual too. However, the contrary is not always true. If the individual with a lower k value selects the less risky prospect as the reference point, it implies that his T(·) exhibits sufficient concavity over the range of possible outcomes. The individual with a higher k value may select otherwise if his T(·) still exhibits sufficient convexity over the relevant range. Similarly, if the individual with a higher k value selects the riskier prospect as the reference point, it does not necessarily imply that the other individual will do the same. Only when the individual with a higher k value selects the less risky prospect as the reference point will the other individual necessarily also select the less risky prospect as the reference point. In this last scenario, when the evaluation takes place on the concave portion of T(·) for the first individual, the individual with a lower k value will also evaluate the prospects on the concave portion of his transformation function.

One can understand the response mode effect by applying Hypothesis 1 to prospects in different contexts. Hershey, Kunreuther, and Schoemaker (1982, p.944, Table 3) observe that the subjects' attitudes towards risk are affected by the context of the decision problem. In the experiment, they construct 18 different problems, each containing one riskless prospect and one risky prospect; the expected value of the risky prospect is always equal to the amount of the sure gain. Their results are listed below.

Question	Prob.	Prize	Sure Amount	% of risk averse subjects (N = 82)	Number of risk averse subjects
1	0.001	10,000	10	47.6%	39
2	0.005	2,000	10	41.5	34
3	0.01	1,000	10	39.0	32
4	0.05	200	10	25.6	21
5	0.10	100	10	23.2	19
6	0.20	50	10	31.7	26
7	0.001	10,000	10	50.0	41
8	0.01	10,000	100	54.9	45
9	0.10	10,000	1,000	69.5	57
10	0.50	10,000	5,000	74.4	61
11	0.90	10,000	9,000	78.0	64
12	0.99	10,000	9,900	70.7	58
13	0.999	10,000	9,990	74.4	61
14	0.01	100	1	15.9	13
15	0.01	1,000	10	35.4	29
16	0.01	10,000	100	59.8	49
17	0.01	100,000	1,000	69.5	57
18	0.01	1,000,000	10,000	80.5	66

The above table shows that the fraction of risk-averse individuals varies from one decision problem to another decision problem. A risk-averse individual will evaluate the risky prospect with a concave valuation function. A concave valuation function is consistent with a riskless reference point which has a smaller outcome than the risky prospect. On the other hand, a risk-seeking individual will select the risky prospect as the reference point. As the number of risk-averse individuals increases, this implies that more individuals use a riskless (or low risk) reference point.

If a decision maker assigns a small value to k, a riskless (or less risky) prospect is more likely to be chosen as the reference point. Combining a riskless reference point with an asymmetric modified utility function, the risky prospect will yield a lower valuation index for the same expected value as the reference point. A small value of k implies risk-aversion. On the other hand, a large value of k implies that a risky prospect with a small chance to win a large prize is selected as the reference point. This risky reference point will generate a substantial amount of regret for choosing the riskless prospect. Consequently, the decision maker becomes risk-seeking.

According to the result in Question 1, there are 39 risk-averse individuals and 43 risk-seeking individuals. These 39 risk-averse individuals should have relatively low values of k and the risk-seeking individuals should have larger values of k in their transformation functions. Switching from Question 1 to Question 2, the riskiness in the risky prospect has been reduced while the expected value of the gamble is unchanged, 5 of the risk-averse individuals in Question 1 become risk-seeking. The following figure depicts three transformation functions. Figure 5.10(i) depicts the transformation function of a typical risk-averse individual where ET(10) > ET(0; 10,000) and ET(10) > ET(0; 2,000). This individual selects the riskless prospect as the reference point in both questions. Figure 5.10(iii) shows that the individual with a large k will always select the risky prospect as the reference point which leads to risk-seeking behaviour. Figure 5.10(ii) depicts the transformation function of an individual who selects the riskless prospect in Question 1 but switches to the risky prospect in Question 2. This individual will exhibit risk aversion in Question 1 and risk-seeking in Question 2. These results in Questions 1 and 2 are consistent with Hypothesis 1 and the assumption that the subjects hold a wide variety of k values.

When the riskiness in the risky prospect is further reduced (moving from Question 2 to Question 3 and further), it is possible to have additional subjects switching from the risk-averse category into the risk-seeking category. For individuals who have already selected a risky prospect, they will never adopt the sure amount as the reference point when the riskiness of the risky prospect is reduced. Note that Hypothesis 1 fails to account for the higher number of risk-averse individuals in Question 6 than in Questions 5 and 4.

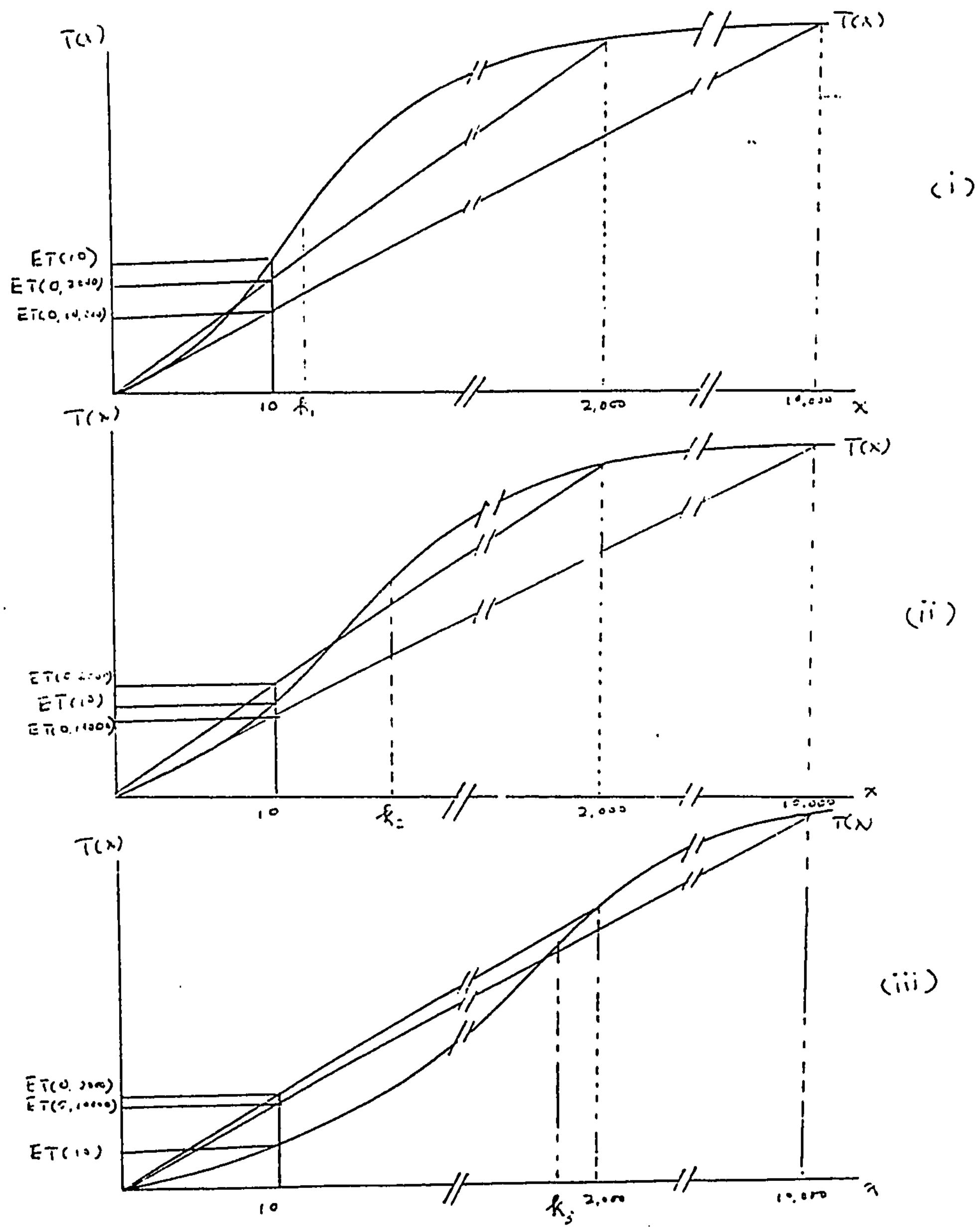


FIGURE 5.10 SWITCHING FROM RISK AVERSION TO RISK SEEKING ACCORDING TO HYPOTHESIS 1

For the seven problems listed in Questions 7 to 13, one can observe that the probability of winning in the risky prospect falls while the prize of the gamble remains constant. Unlike the changes introduced in Questions 1 to 6, these seven problems can be used to investigate the impact of increasing the expected value of the risky prospect on the risk-taking behaviour. As discussed above, subjects with low values of k will select the riskless prospect as the reference point and exhibit risk-averse behaviour (Figure 5.11(i)). Subjects with extremely large values of k will select the risky prospect as the reference point and exhibit risk-seeking behaviour (Figure 5.11(iii)). Some of the subjects with k values in between will switch from risk-seeking to risk-aversion as the expected value of the prospect and the sure amount increase (Figure 5.11(ii)). Again, Hypothesis 1 cannot fully explain all the results in Questions 7 to 13. In particular, less subjects prefer to use the riskless sure amount as the reference point when the probability of winning $10,000 is increased from 0.90 to 0.99 or 0.999 (Questions 11 to 13).

Similar results are found in Questions 14 to 18. In these problems, the probability of winning is being held constant while the prize and the expected value of the risky prospect increase. The results indicate that when the expected value of the risky prospect and the sure gain are increased by the same amount, more subjects become risk-averse in selecting their reference points. According to Hypothesis 1, more subjects are using the convex segment of the transformation to evaluate the prospects (Figure 5.12 (ii) and (iii)) when the expected value of the prospect is low, consequently more

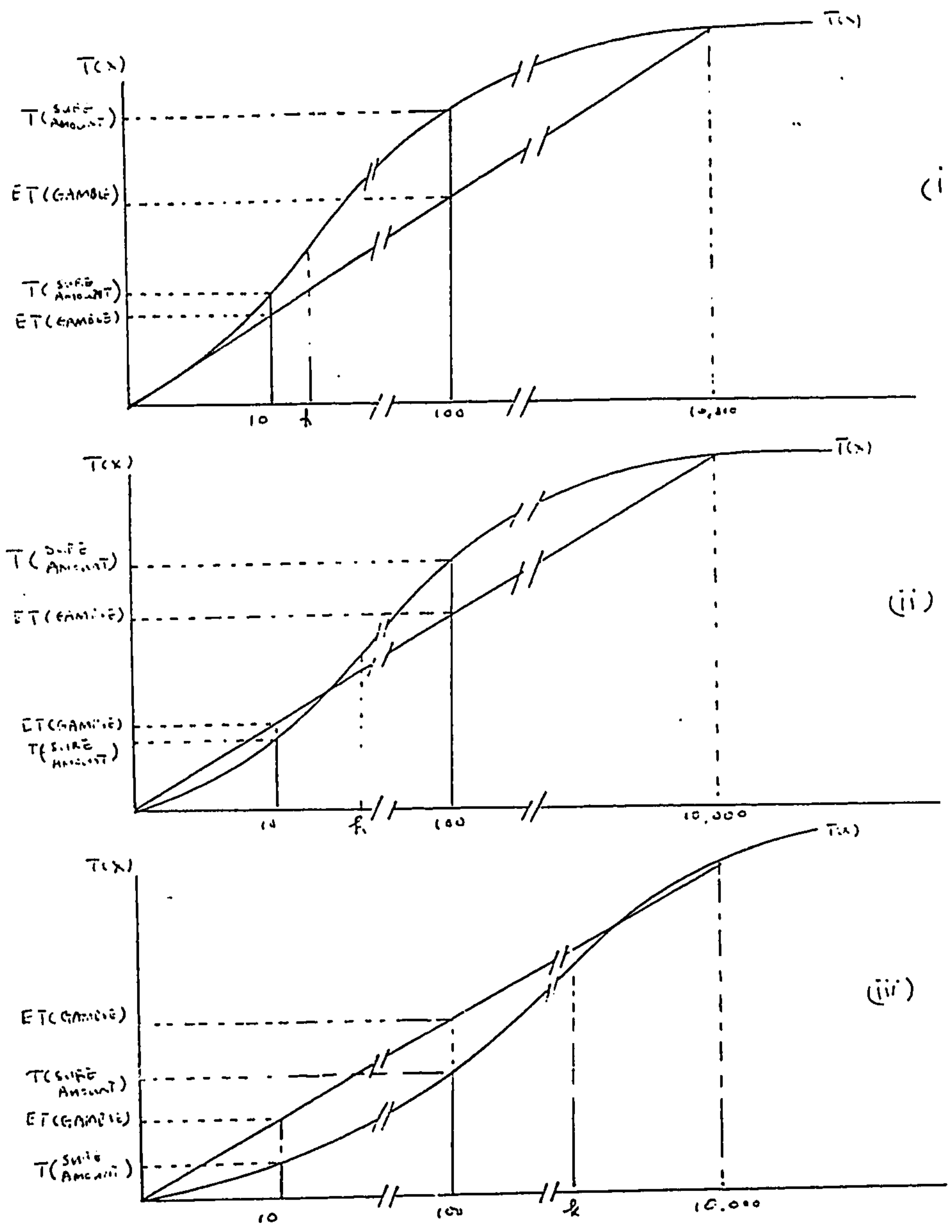


FIGURE 5.11 SWITCHING FROM RISK SEEKING TO RISK AVERSION ACCORDING TO HYPOTHESIS 1

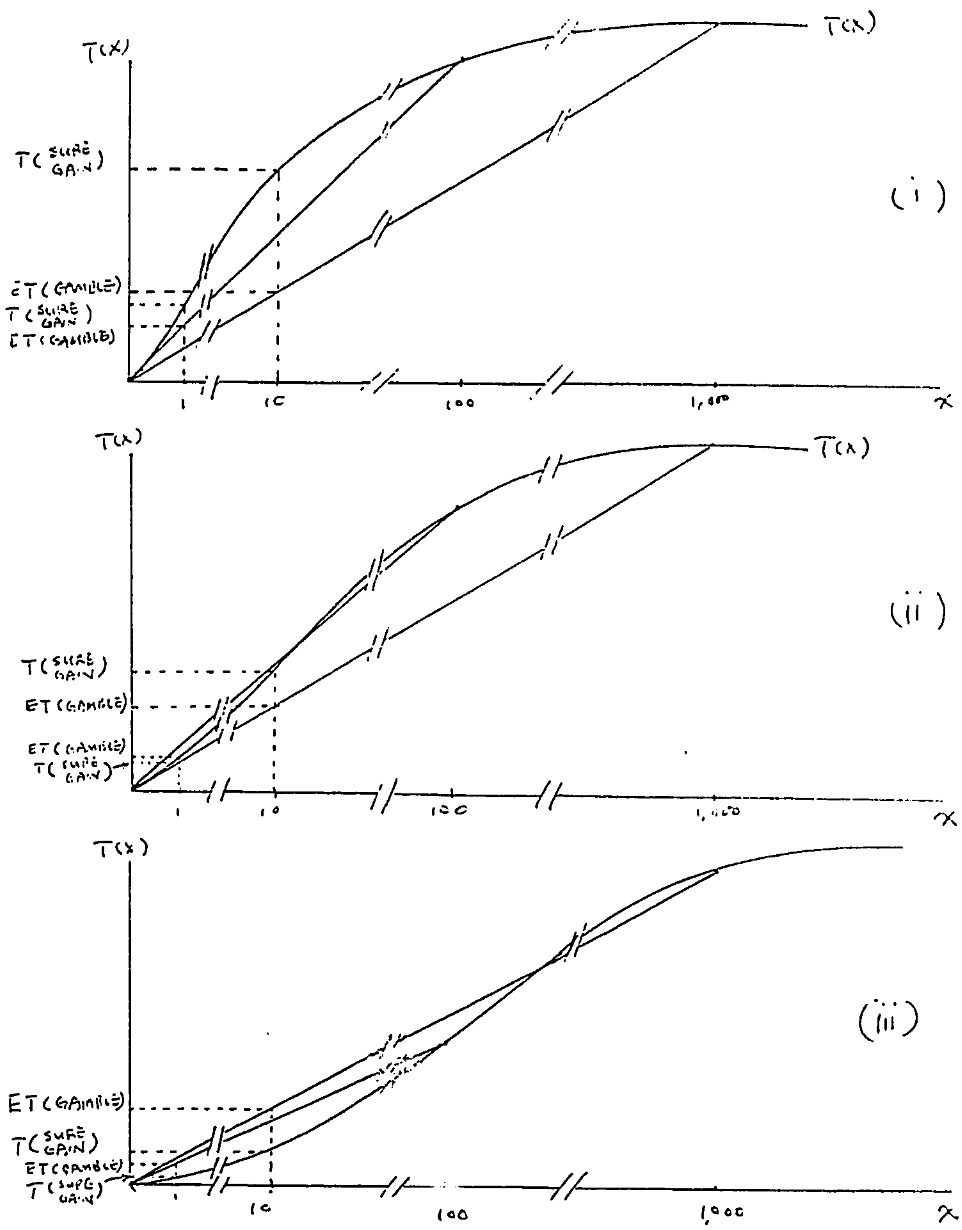


FIGURE 5.12 SWITCHING FROM RISK SEEKING TO RISK AVERSION ACCORDING TO HYPOTHESIS 1

subjects choose the risky prospect as their reference point. As the expected value increases, more individuals utilize the concave segment of the transformation function to evaluate the prospects (Figure 5.12 (i) and (ii)), some subjects switch from the risky reference point and adopt the riskless reference point (Figure 5.12 (ii)).

The following conjectures summarize the above discussion.

Conjecture 1.4

Assuming that the decision makers assign a variety of values to k, which are dependent on some exogeneous factors, fewer (more) decision makers will select the risky prospect as the reference point when the risky prospect becomes more (less) risky according to the definition of mean preserving spread; that is, for the same expected value, there is a smaller (larger) probability to win a larger (smaller) prize.

Conjecture 1.5

Assuming that the decision makers assign a variety of values to k, which are dependent on some exogeneous factors, fewer (more) decision makers will select the risky prospect as the reference point when the expected value of the risky prospect increases (declines) due to a larger (smaller) probability to win a constant prize beyond k.

Conjecture 1.6

Assuming that the decision makers assign a variety of values to k, which are dependent on some exogeneous factors, fewer (more) decision makers will select the risky prospect as the reference point when its prize increases beyond (declines below) k; the expected value of the risky prospect will increase (decrease) by holding the probability distribution unchanged.

It is suggested in Conjectures 1.4 to 1.6 that different individuals will assign different values to k, the possible exogenous factors may include one's degree of risk aversion and / or wealth position. If increasing wealth reduces the degree of risk aversion, individuals are more inclined to select riskier reference point and the riskier prospect is preferred. It implies that the value of k is positively related to one's level of wealth. In addition, two individuals with similar wealth may have different preferences. An individual who is more averse to risk will assign a lower value to k and his choice will reveal a higher degree of risk aversion.

For three pairs of problems in the experiment, Questions 1 & 7, 3 & 15, and 8 & 16, each of these pairs have the same risky prospect and the sure amount in the questions. Yet, there are small differences in terms of the resulting choices between Questions 1 & 7. Questions 3 & 15, and Questions 8 & 16. If the subjects follow reference point theory and Hypothesis 1 in their decision-making, it means that a few subjects have unstable values for parameter k and / or imprecise transformation function. It is also possible that the larger context given in the experiment may influence preferences. The subjects will not only compare the prospects in each problem in isolation, they may compare the prospects in other related problems (having the same sure amount in Questions 1 to 6, the same prize in Questions 7 to 13, and the same probability in Questions 14 to 18). If this is the case, assuming that each decision maker has a stable (and non-changing) transformation function will not be able to account for the macro context effect. Fortunately for the theory, the macro context effect seems to affect only a few subjects. The majority of the subjects exhibit choices that are consistent with a non-changing transformation function in Hypothesis 1.

The tranformation function and the reference point selection mechanism described in Hypothesis 1 is ad hoc and arbitrary since cognition is by and large unobservable. The current hypothesis is adopted for the following reasons. First, the transformation function in Hypothesis 1 predicts that the context of the problem affects the degree of risk aversion; this prediction is consistent with the empirical findings in Hershey, Kunreuther, and Schoemaker (1982). The transformation function provides a systematic framework to account for the response mode effect. Second, the transformation function contains a person-specific parameter to reflect the heterogenity of the decision-makers. It allows some degree of flexibility in modelling the decision-making process. Third, although the model is flexible, it is also testable and refutable. Hypothesis 1 assumes that each individual will use a stable $T(\cdot)$ consistently to select a reference point. One can monitor the choices of the same individual when he faces different sets of problems. The analyst can check to see whether the implied reference point conforms to the assumption of the transformation function in Hypothesis 1. The analyst can also conduct a cross-sectional analysis to compare the choices of different individuals in different problems (see, for example, Conjecture 1.7). Fourth, Hypothesis 1 assumes that T(·) is continuous and the evaluation process focuses in the probability and the outcomes of a prospect. A lexicographic or semi-lexicographic model will adopt different criteria through out the evaluation process. A non-continuous selection function implies that the same prospect will be evaluated according to different criteria depending on the competing alternatives. The advantage of a continuous function is that it allows for a consistent evaluation procedure for small variations in the prospects.

5.4.2 The Context and Framing: The Reference Point That Involves Losses

This section will discuss the role of the prospects' context and the elicitation method in relation to heuristics and the framing effect in decision-making. In particular, the decision frame or the elicitation method may create the impression that the subjects are choosing among prospects with negative outcomes. This response mode effect will transform the evaluation of prospects with positive outcomes into the evaluation of losses. The following discussion extends Hypothesis 1 to formulate rules for selecting the reference points in the negative domain.

The widely observed disparity between the evaluation of gains and the evaluation of losses (even for two prospects with identical probability distributions of the outcomes) suggests that the context of the problem plays an influential role in the decision-making process. The evidence of such disparity is documented in Tversky and Kahneman (1974, 1981, 1987), Kahneman and Tversky (1979 and 1984), and Kahneman, Knetsch, and Thaler (1991). For choice that involves losses (also includes willingness to accept or the elicitation of reservation price), the decision maker's valuation function is convex. A convex valuation function over the domain of losses exhibits risk-seeking behaviour and loss aversion (see Figure 5.2). A concave valuation function for positive prospects exhibits risk-aversion.

The context of the problem will influence the evaluation process due to the framing effect. A choice between two positive prospects can be framed as a choice of losses (or negative prospects) in two ways. First, a relatively large amount (larger than all the possible outcomes in the available prospects) is explicitly introduced in the decision problem as an endowment with certainty. Consequently, the outcomes of the original prospects are converted into negative outcomes by subtracting the amount of endowment from the possible outcomes in each prospect. The negative figures associated with the framed prospects represent the amount of deviations from the endowment point in each possible state of nature.

Second, instead of asking the decision maker to choose between two positive prospects, the question can be framed as eliciting the reservation prices of the prospects. The elicitation process suggests that the decision maker is compensated for giving up the prospects. Under the connotation that he is making a decision regarding losses, the decision maker will select the largest outcome in the prospect under evaluation as the reference point to determine its selling price.

In order to represent aversion against losses, the transformation function should be symmetric where $T(-x) = -T(x)$. An individual who prefers to use the low risk reference point in the positive domain will switch to adopt the high risk reference point in the negative domain. The change in reference point will lead to different attitudes toward risk with respect to gains and losses. Hypothesis 2 is an extension of Hypothesis 1 into the negative domain.

HYPOTHESIS 2

The decision maker will transform the outcome x_i *by a transformation function* $T(x_i)$. *The prospect with the highest mean transformed value,* $\sum_i p_i T(x_i)$, *is selected as the reference point in the decision analysis as in Hypothesis 1. The transformation is symmetric where* $T(-x_i) = -T(x_i)$. *Hence, framing will reverse the ranking of the transformed values. The prospect with a higher* $\sum_i p_i T(x_i)$ *will have a larger negative transformed value given by* $\sum_i p_i T(-x_i)$.

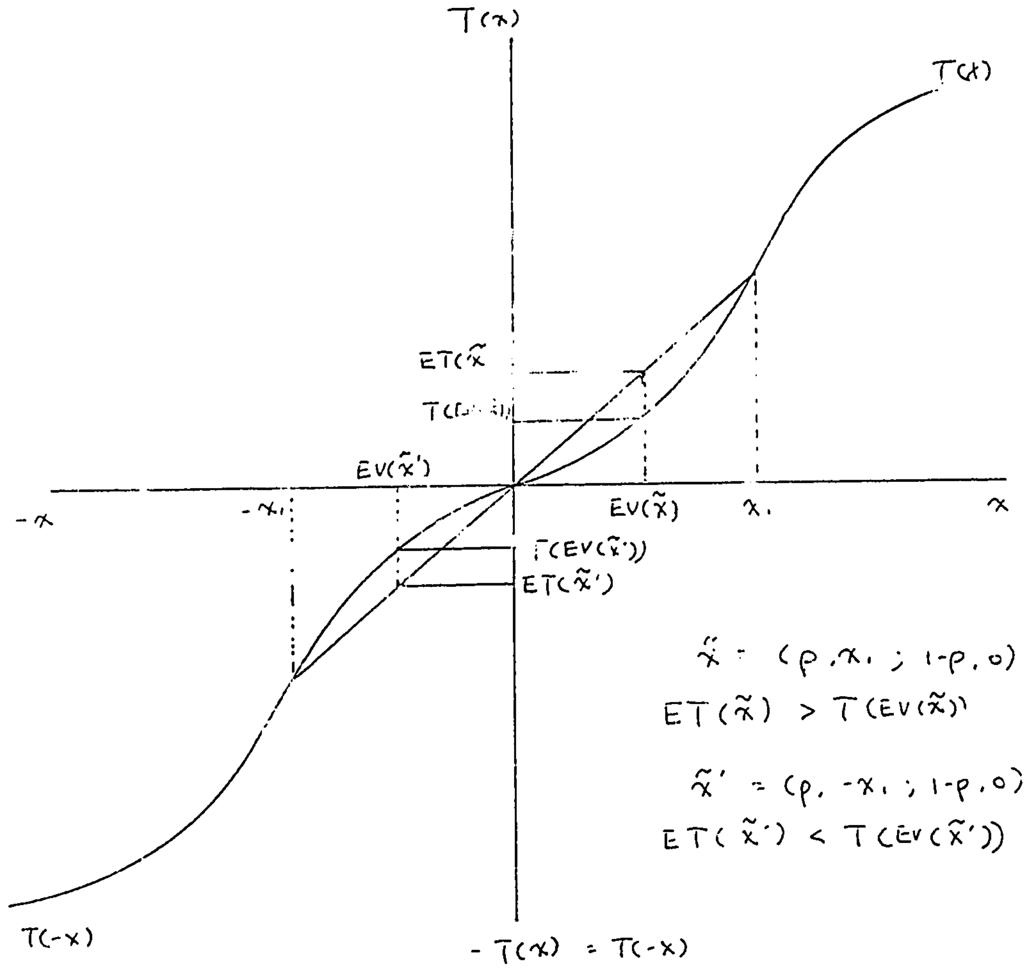


FIGURE 5.13 HYPOTHESES 1 AND 2: THE EFFECT OF FRAMING A DECISION

The effect of framing is depicted in Figure 5.13 according to Hypotheses 1 and 2. For the particular transformation function shown in Figure 5.13, risky prospect with payoff x_1 is evaluated on the convex segment which leads to a higher index than the sure gain. The risky prospect is chosen as the reference point due to convexity. When x_1 is framed as a negative payoff, $-x_1$ will be evaluated on the concave segment of the transformation function according to the conjecture in Hypothesis 2. The resulting reference point becomes the sure gain. The symmetry in the transformation will predict a switch in reference point due to framing.

5.5 SUMMARY

This chapter discusses several important elements of Reference Point theory. The functional form of the rejoice-regret function is specified according to the findings related to the asymmetry in evaluating gains and losses. A modified utility function is then derived based on the appropriate $R[\cdot]$ and the VNM utility function. A hypothesis to determine the reference point is examined in the last section. The hypothesis seems to fit the experimental findings (Hershey, Kunreuther, Schoemaker; 1982) which suggest that the choice of the reference points is sensitive to the context of the problem. The evaluation framework suggested in Reference Point theory will be applied to decision-making under uncertainty in Chapter 6.

CHAPTER 6 CHOICE UNDER UNCERTAINTY WITH REFERENCE POINT THEORY

Chapters 4 and 5 described the decision-making process of Reference Point theory. This chapter applies this process to derive choice under uncertainty. Section 6.1 derives an expected modified utility index (EM index) according to RPT with a simple riskless reference point. Section 6.2 studies the implications of switching reference points on preference and the degree of risk aversion. Section 6.3 discusses the situation where a risky prospect is selected as the reference point. A risky reference point will complicate the derivation of an EM index because the probability distribution of a prospect has to be partitioned according to the events in the reference point. In order to simplify the decision-making process, Hypothesis 3 in Section 6.4 suggests a heuristic which ignores the required partitioning and introduces biases into the decision-making process. Section 6.5 studies the demand for insurance under RPT.

6.1 THE DERIVATION OF VALUATION INDEX IN REFERENCE POINT THEORY

After the decision maker chooses a riskless reference point x_o (a risky reference prospect will be considered later), he can determine the rejoice (or regret) augmented utility of a risky prospect by calculating the expected value of the modified utility index for the outcome in each state. For a risky prospect $\tilde{x}_1$ with probability p_j to obtain x_{1j} in the jth state, the modified utility function (Equation (4.19))

$$M(x_{1j}; x_o) = U(x_{1j}) + R[U(x_{1j}) - U(x_o)]$$

is weighted by the probability distribution to derive the expected modified utility index $EM(\tilde{x}_1; x_o)$ as follows:

$$EM(\tilde{x}_1; x_o) = \sum_{j=1}^{n} p_j M(x_{1j}; x_o)$$

$$EM(\tilde{x}_1; x_o) = \sum_{j=1}^{n} p_j \{U(x_{1j}) + R[U(x_{1j}) - U(x_o)]\} \quad (6.1)$$

The preference ordering between two risky prospects $\tilde{x}_1$ and $\tilde{x}_2$ is determined by the EM indices derived according to Equation (6.1). In particular, we have

$$\tilde{x}_1 \underset{\prec}{\overset{\succ}{\sim}} \tilde{x}_2 \Leftrightarrow EM(\tilde{x}_1; x_o) \underset{<}{\overset{>}{=}} EM(\tilde{x}_2; x_o) \quad (6.2)$$

For three risky prospects $\tilde{x}_1$, $\tilde{x}_2$, and $\tilde{x}_3$, it can be shown that $EM(\tilde{x}_1; x_o) > EM(\tilde{x}_2; x_o)$ and $EM(\tilde{x}_2; x_o) > EM(\tilde{x}_3; x_o)$ implies that $EM(\tilde{x}_1; x_o) > EM(\tilde{x}_3; x_o)$. Thus, $\tilde{x}_1 \succ \tilde{x}_2$ and $\tilde{x}_2 \succ \tilde{x}_3$ implies that $\tilde{x}_1 \succ \tilde{x}_3$; and preference cycles are avoided.

Since Equation (6.1) is linear in probabilities, one can derive the EM index geometrically using a modified utility function. The procedure is similar to the derivation of the EU index with a VNM utility function. Figure 6.1 depicts $EM(\tilde{x}_1; x_o)$ where $\tilde{x}_1$ is a simple two-state prospect with expected value $EV(\tilde{x}_1) = p_1x_{11} + p_2x_{12}$ and $x_{11} > x_o > x_{12}$. The modified utility curve in Figure 5.5 has a kink where $x = x_o$; the curve is concave (convex) for $x > x_o$ $(x < x_o)$. The vertical intercept of the modified utility curve is given by

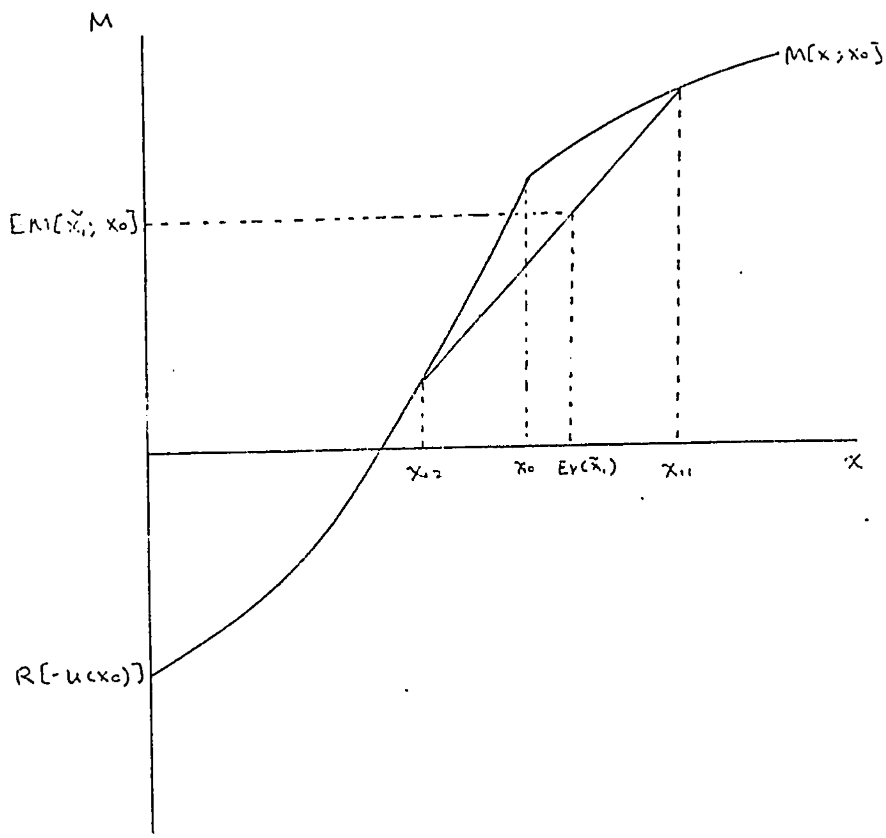


FIGURE 6.1 EM INDEX OF A RISKY PROSPECT

$R[U(0) - U(x_o)]$ or simply $R[-U(x_o)]$. Given this modified utility curve, one can locate $M(x_{11}; x_o)$ and $M(x_{12}; x_o)$ on the curve. According to (6.1) $EM(\tilde{x}_1; x_o)$ is a linear combination of $M(x_{11}; x_o)$ and $M(x_{12}; x_o)$. Hence, the value of $EM(\tilde{x}_1; x_o)$ is located on the straight line joining $M(x_{11}; x_o)$ and $M(x_{12}; x_o)$; the exact location of $EM(\tilde{x}_1; x_o)$ is determined by its expected value $EV(\tilde{x}_1)$ on the x-axis.

6.2 THE IMPLICATIONS OF SWITCHING REFERENCE POINT

According to Equation (4.19) RPT postulates that the reference point is a significant variable in decision-making. The following example shows that the ranking of the EM indices of two prospects can be reversed when different reference points are used in the decision-making process. The example confirms that preference ordering is dependent on the choice of reference point; in other words, preference is reference-specific.

Assume that a risk-averse individual has to choose between a riskless prospect $\tilde{x}_a$ and a risky prospect $\tilde{x}_b$. $EV(\tilde{x}_a) = p_2x_2$ is less than $EV(\tilde{x}_b) = p_1x_1 + p_3x_3$ where $p_2 = p_1 + p_3 = 1$ and $x_1 < x_2 < x_3$. Given $x_o = x_1$, this individual is indifferent between $\tilde{x}_a$ and $\tilde{x}_b$ as shown in Figure 6.2. When the reference point is increased to $x_o^* = x_3$, the individual will use the modified utility curve associated with the new reference point. With the new reference point $x_o^* = x_3$, he will change his preference ordering where $\tilde{x}_a \prec \tilde{x}_b$. The EM indices depicted in Figure 6.2 show that the preference rankings between $\tilde{x}_a$

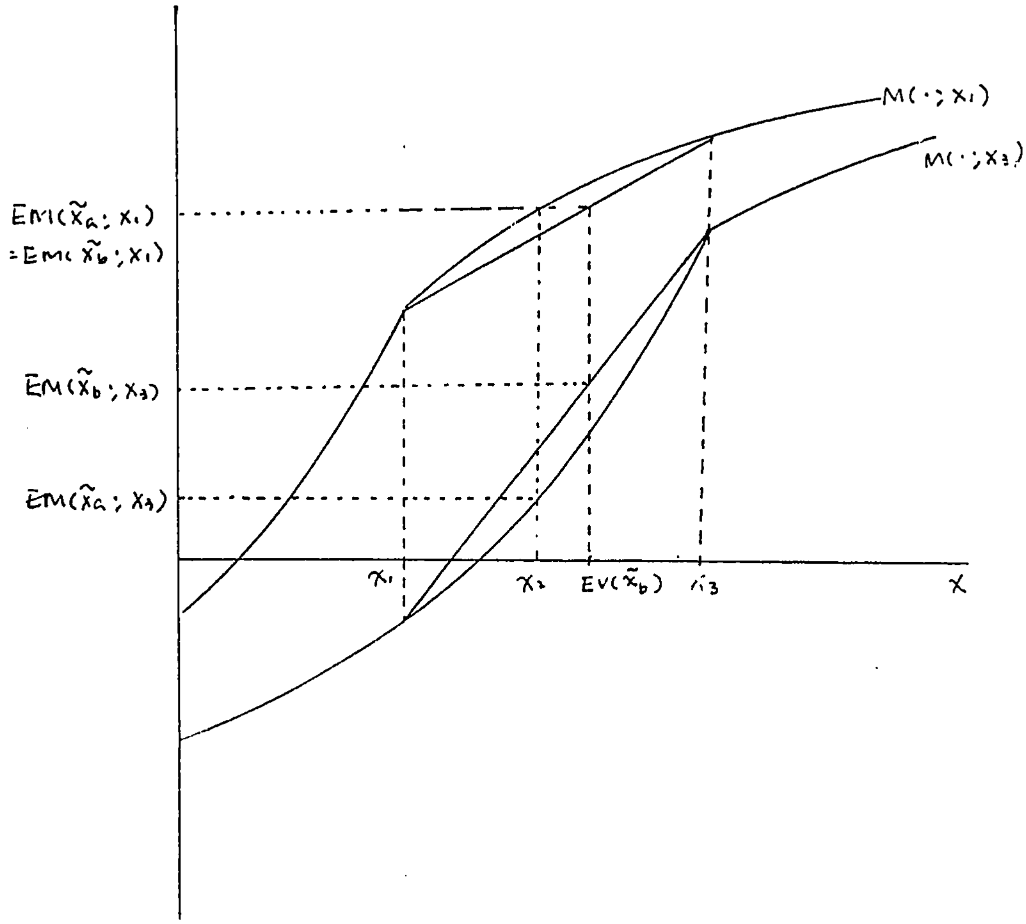


FIGURE 6.2 INCREASING REFERENCE POINT ENCOURAGES RISK-TAKING BEHAVIOUR

and $\tilde{x}_b$ are altered if the decision maker switches from $x_o = x_1$ to $x_o^* = x_3$. This simple example illustrates that the reference point will influence the preference in RPT. Specifically, increasing the reference point encourages risk-taking behaviour.

When preference is dependent on the reference point, it implies that the indifference curves are also dependent on the reference point. For a given reference point, the axiom of independence ensures that the indifference curves in the probability space are linear and parallel to each other (Machina, 1987a, p.125). In order to reflect the change in preference, from $M(\tilde{x}_a; x_o) = M(\tilde{x}_b; x_o)$ to $M(\tilde{x}_a; x_o+dx_o) < M(\tilde{x}_b; x_o+dx_o)$, the slope of the indifference curves decreases. Hence, increasing the reference point will reduce the slope of the indifference curves. As shown in Figures 2.3 and 6.3, flatter indifference curves imply a lower degree of risk aversion.

For a small reference point like $x_o =$ [illegible], the indifference curves in Figure 6.3(a) show that $\hat{x}_a$ [illegible] $\tilde{x}_b$. As the reference point increases to $x_o^* = x_3$, the indifference curves become flatter in Figure 6.3(b) and $\tilde{x}_b \succ \tilde{x}_a$.

It is apparent from the above analysis that the value of the certainty equivalent for the risky prospect $\tilde{x}_b$ is also dependent on the reference point. The conditional certainty equivalent, CCE, (conditioned on the value of the reference point) is an increasing function of the reference point; a larger reference point encourages risk-taking behaviour and increases the value of risky prospects relative to riskless prospects. The CCE for $\tilde{x}_b$ is x_2 for a small reference point x_1 (Figure 6.4(a)) whereas the CCE for $\tilde{x}_b$ becomes

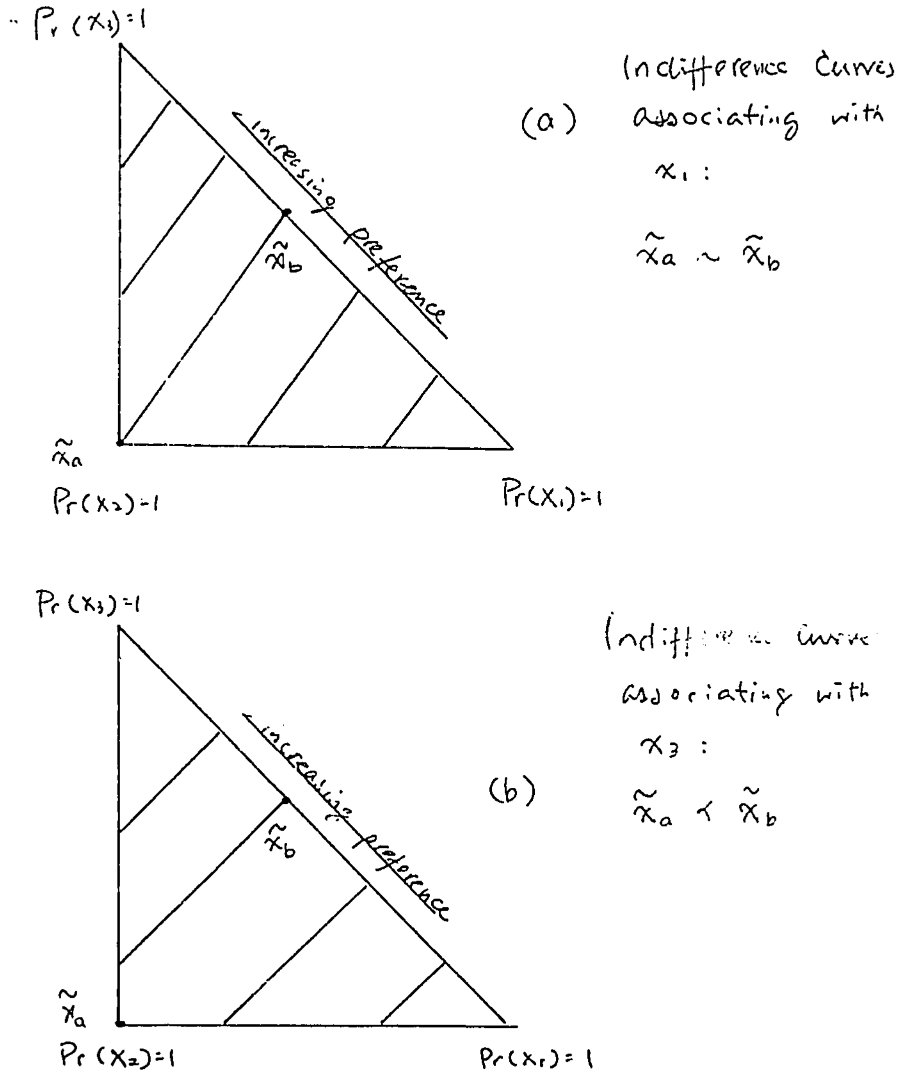


FIGURE 6.3 INCREASING REFERENCE POINT REDUCES THE SLOPE OF THE INDIFFERENCE CURVES

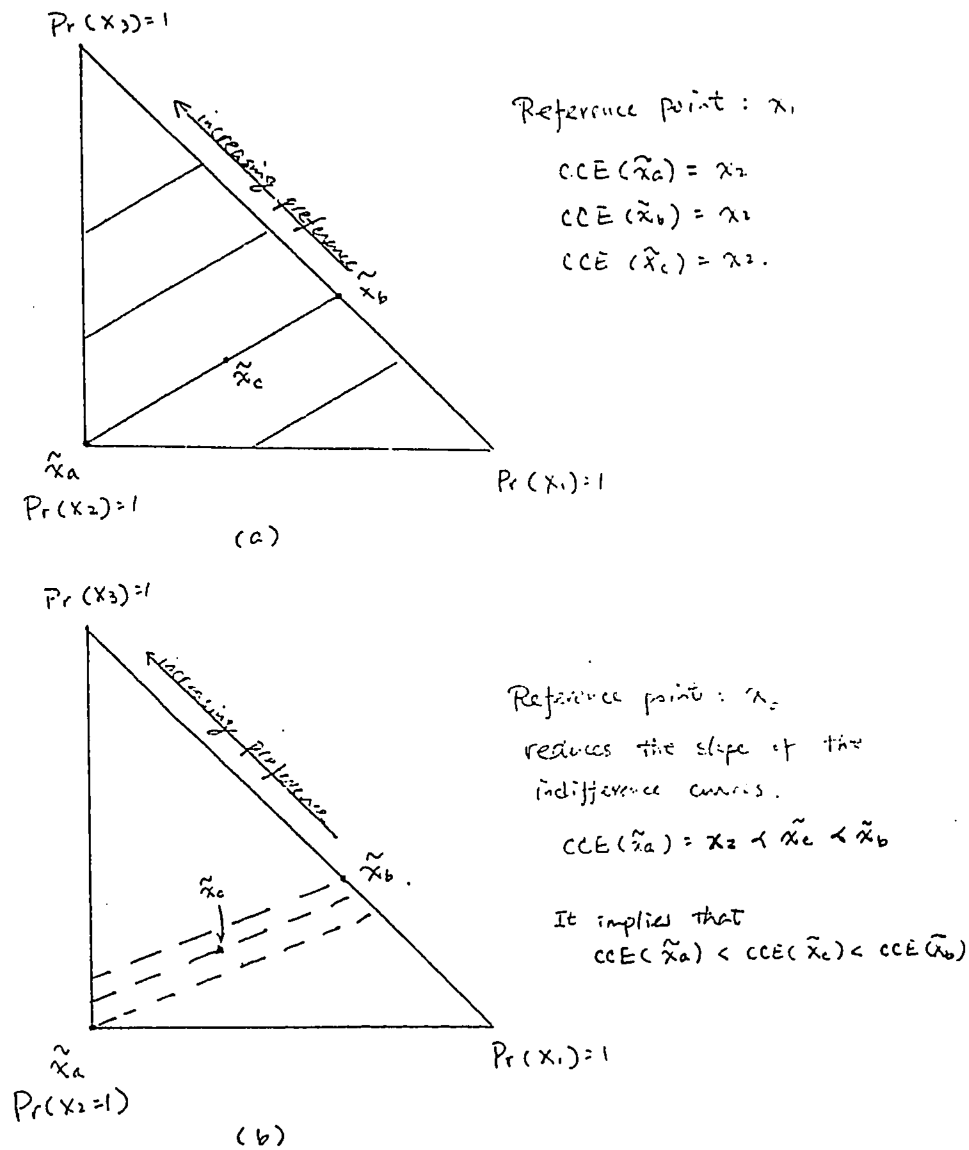


FIGURE 6.4 INCREASING REFERENCE POINT INCREASES THE CONDITIONAL CERTAINTY EQUIVALENT ACCORDING TO THE RISKINESS OF THE PROSPECTS

greater than x_2 for reference point x_3 (Figure 6.4(b)).

The two probability spaces in Figure 6.4 contain three prospects $\tilde{x}_a$, $\tilde{x}_b$, and $\tilde{x}_c$. $\tilde{x}_a$ is a riskless prospect. For reference point $x_o = x_1$, $\tilde{x}_b$ is indifferent to $\tilde{x}_a$. The third prospect $\tilde{x}_c$ is chosen with two properties. First, it is indifferent to $\tilde{x}_a$ and $\tilde{x}_b$. Second, $\tilde{x}_b$ is a mean-preserving spread of $\tilde{x}_c$; that is, $EV(\tilde{x}_b) = EV(\tilde{x}_c)$. $\tilde{x}_b$ contains more risk than $\tilde{x}_c$ because the former prospect has higher probability for extreme values. These three prospects lie on the same indifference curve depicted in Figure 6.4(a). The indifference curves in Figure 6.4(b) are associated with a larger reference point $x_3 > x_1$. With x_3, $\tilde{x}_b \succ \tilde{x}_c \succ \tilde{x}_a$. Consequently, the increase in the CCE for the riskier prospect $\tilde{x}_b$ is larger than the increase for $\tilde{x}_c$. $\tilde{x}_a$, which is a riskless prospect, has the same CCE regardless of the value of the reference point.

The above example shows that loss aversion is associated with the choice of reference point. When a small reference point x_1 is selected, both prospects $\tilde{x}_a$ and $\tilde{x}_b$ are evaluated as gains. Switching to a large reference point x_3, the prospects are evaluated as losses and aversion to losses lead to risk-seeking behaviour. The changes in reference point also suggest the possibility of framing. With a small reference point x_1, the possibility of receiving x_3 in prospect $\tilde{x}_b$ is perceived as gain which is subjected to diminishing marginal utility. When a large reference point is adopted, the possibility of obtaining x_3 in $\tilde{x}_b$ becomes a reduction in loss. Due to loss aversion, $\tilde{x}_b$ is preferred to $\tilde{x}_a$ in the second case but not the first case.

The notion that the utility function is non-stationary was first documented in Markowitz (1952b). He noted that the Friedman-Savage utility function (in Figure 3.7(b)) is not consistent with the observed tendency of individuals of all wealth levels to purchase insurance and lottery tickets. If the same Friedman-Savage utility function applies to individuals with different levels of wealth, the extremely rich (wealth position above the point of inflection of the utility function) will never buy a lottery ticket. Markowitz hypothesized that changes in wealth cause the utility function to shift horizontally so that the level of initial wealth is always located near the inflection point of the utility function (1952b, p. 155).[1] (See Figure 6.5) This notion of the entire utility function being dependent on the level of wealth is compatible with the modified utility function in RPT.

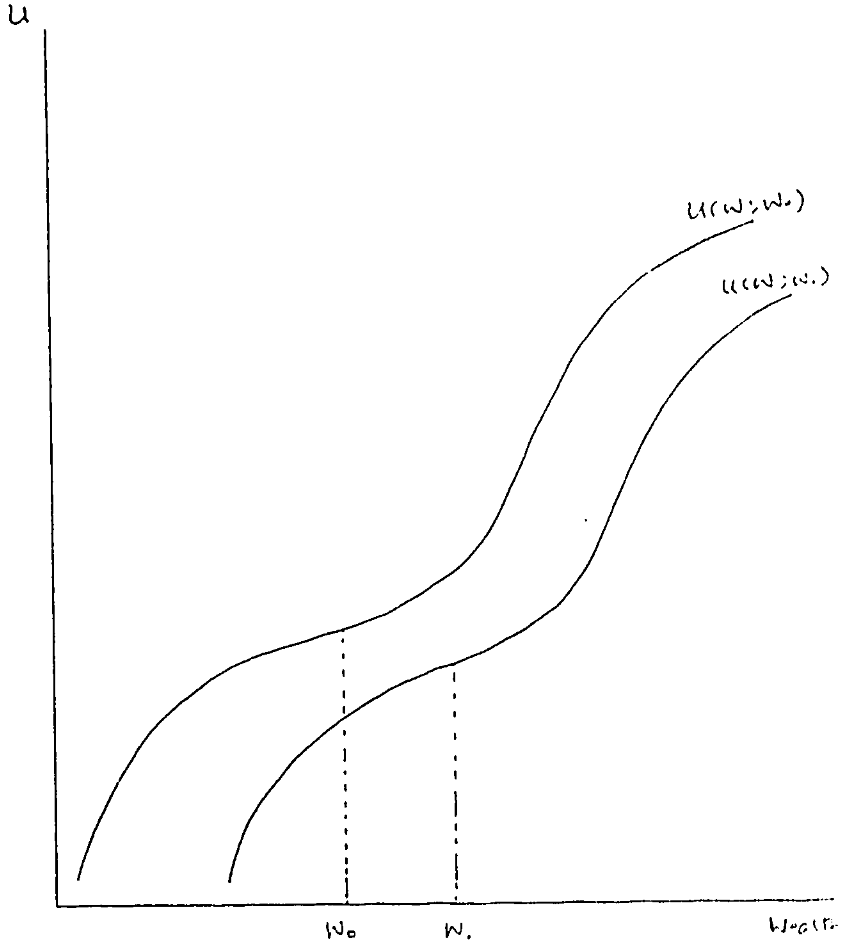


FIGURE 6.5 MARKOWITZ'S HYPOTHESIS OF NON-STATIONARY FRIEDMAN-SAVAGE UTILITY FUNCTION

6.3 DERIVATION OF VALUATION INDEX WITH A RISKY REFERENCE POINT

The analysis in Section 6.1 derived the EM index with a riskless reference point. Decision makers have to choose a risky reference point when a riskless prospect is not available. Furthermore, the decision makers with a large k in their transformation function may choose a risky reference point (see the discussion in Section 5.4 and especially Conjecture 1.3). Given a risky reference point, the decision makers should partition the probability distribution in order to generate the full impact of "what might have been" in the decision-making process. The probability distribution of a prospect should be partitioned such that a common probability distribution can be applied to both the risky reference point and the risky prospect. Consider a risky reference point $\tilde{y}_a$: $(p_1, y_{a1}; p_2, y_{a2})$; a risky prospect $\tilde{y}_b$: $(p_3, y_{b3}; p_4, y_{b4})$ should be partitioned as follows:

$$\tilde{y}_b: (p_1p_3, y_{b3}; p_2p_3, y_{b3}; p_1p_4, y_{b4}; p_2p_4, y_{b4}).$$

Thus, the EM index for $\tilde{y}_b$ with reference point $\tilde{y}_a$ is

$$EM(\tilde{y}_b; \tilde{y}_a) = \begin{array}{l} p_1p_3M(y_{b3}; y_{a1}) + p_2p_3M(y_{b3}; y_{a2}) \\ + p_1p_4M(y_{b4}; y_{a1}) + p_2p_4M(y_{b4}; y_{a2}). \end{array} \qquad (6.3)$$

Expanding Equation (6.3), we have

$$EM(\tilde{y}_b;\ \tilde{y}_a) = \begin{array}{l} p_1p_3\{U(y_{b3}) + R[U(y_{b3})-U(y_{a1})]\} \\ + p_2p_3\{U(y_{b3}) + R[U(y_{b3})-U(y_{a2})]\} \\ + p_1p_4\{U(y_{b4}) + R[U(y_{b4})-U(y_{a1})]\} \\ + p_2p_4\{U(y_{b4}) + R[U(y_{b4})-U(y_{a2})]\}. \end{array} \tag{6.4}$$

The rejoice / regret of having y_{b3} vis-á-vis y_{a1} or y_{a2} and the rejoice / regret of having y_{b4} vis-á-vis y_{a1} or y_{a2} will influence the valuation index of $\tilde{y}_b$. The EM index for the reference point $\tilde{y}_a$ is less complicated since partitioning of the probability is not required;

$$EM(\tilde{y}_a;\ \tilde{y}_a) = p_1M(y_{a1};\ y_{a1}) + p_2M(y_{a2};\ y_{a2}). \tag{6.5}$$

$$EM(\tilde{y}_a;\ \tilde{y}_a) = \begin{array}{l} p_1\{U(y_{a1}) + R[U(y_{a1})-U(y_{a1})]\} \\ + p_2\{U(y_{a2}) + R[U(y_{a2})-U(y_{a2})]\} \end{array}$$

$$= p_1U(y_{a1}) + p_2U(y_{a2}). \tag{6.6}$$

Note that $EM(\tilde{y}_a;\ \tilde{y}_a)$ reduces to $EU(\tilde{y}_a)$ in (6.6) because the reference prospect does not generate any regret nor rejoice relative to itself. The EM index for $\tilde{y}_b$ is derived in Figure 6.6. The first and the third terms in Equation (6.4) equal the expected modified utility of y_{b3} and y_{b4} given outcome y_{a1} in the reference point. This value (denoted as α in Figure 6.6) can be derived from the modified utility curve $M(\cdot;\ y_{a1})$. Similarly, the second and the fourth terms in Equation (6.4) corresponds to the expected modified utility of y_{b3} and y_{b4} in

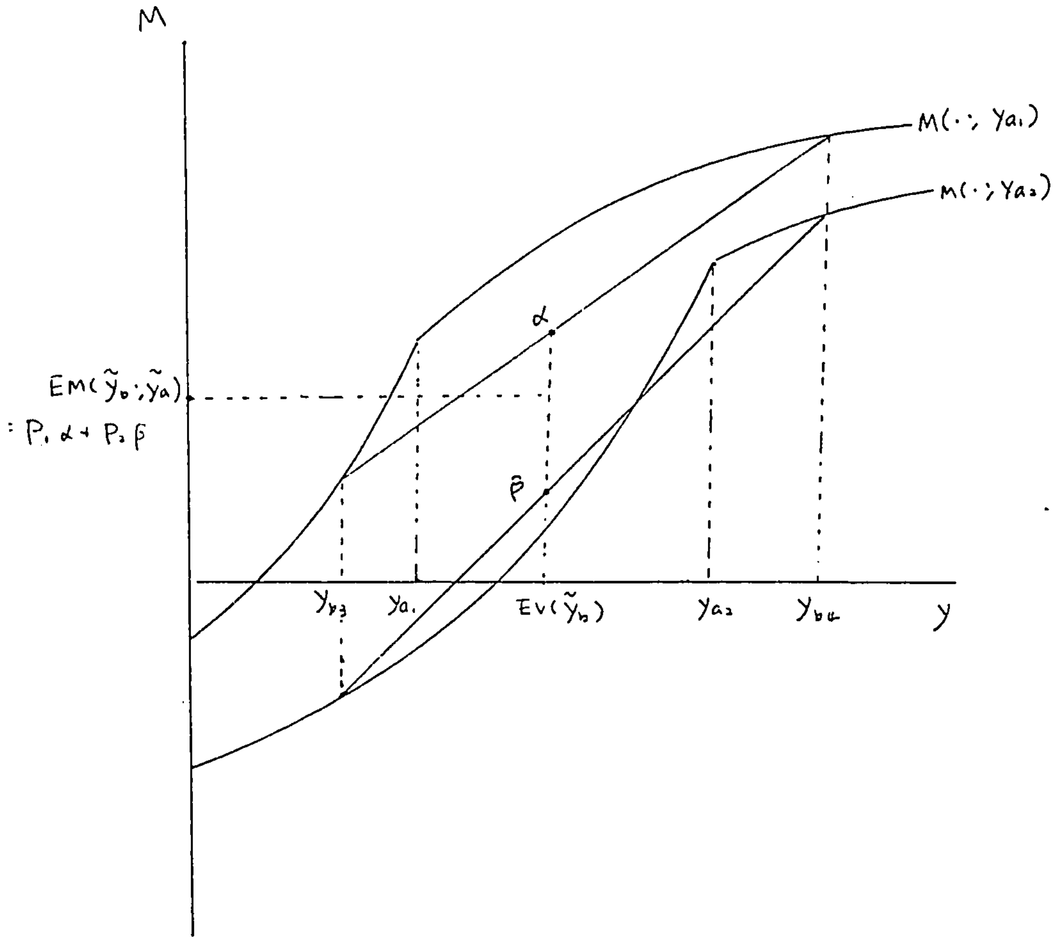


FIGURE 6.6 DERIVATION OF EM INDEX WITH A RISKY REFERENCE POINT

the state that generates y_{a2} in the reference point (denoted as β in Figure 6.6). The two expected modified utilities should be weighted by the probabilities of y_{a1} and y_{a2}, respectively. The EM index for $\tilde{y}_b$ is determined by the linear combination $p_1\alpha + p_2\beta$. Note that α and β are located on a vertical line due to the fact that they have the same expected value $p_3y_{b3} + p_4y_{b4}$. The distance between β and $EM(\tilde{y}_b; \tilde{y}_a)$ equals $p_1 \cdot$(the distance $\alpha-\beta$). When p_1 (p_2) increases, $EM(\tilde{y}_b; \tilde{y}_a)$ moves closer to α (β).

6.4 HEURISTICS AND BIASES IN USING RISKY REFERENCE POINT

The evaluation process described in Equations (6.3) and (6.4) requires the decision maker to partition the probability distribution of $\tilde{y}_b$ according to the events in the reference point $\tilde{y}_a$. The resulting index will fully account for the rejoice and the regret of choosing $\tilde{y}_b$ over $\tilde{y}_a$. When partitioning is required, the reference point (which is risky) actually makes the evaluation process more complicated. It is unlikely that decision maker would fully partition the probability distribution given the limitations on information processing.

It seems that people often focus on one dimension, probability or outcomes, at a time (Schoemaker, 1979). It is quite plausible that a decision maker focuses solely on comparing the outcomes of the prospect and the reference point without accessing the probability dimension of the reference point. Using the reference point in this manner will undoubtedly introduce bias into the evaluation of rejoice and regret; nonetheless, heuristics usually lead to bias in judgment (see Kahneman, Slovic, and Tversky, 1982, and Tversky and Kahneman, 1974). With this bias, and assuming that the decision maker would juxtapose the outcomes of the two prospects in the similar states (low-payoff state and high-payoff state), Equation (6.4) becomes

$$EM(\tilde{y}_b; \tilde{y}_a) = \begin{array}{l} p_3\{U(y_{b3}) + R[U(y_{b3})-U(y_{a1})]\} \\ + \; p_4\{U(y_{b4}) + R[U(y_{b4})-U(y_{a2})]\}. \end{array} \qquad (6.7)$$

Equation (6.7) can be understood as a simplification of Equation (6.4) by some form of probability editing. In general, probability editing seeks to transform the objective probabilities by a weighting function. For example, Kahneman and Tversky (1979, 1984); and Tversky and Kahneman (1981) found that subjects tended to overweight low probability and underweight high probability. Without specifying the editing procedure, they assumed that an arbitrary weighting function will replace the probability distribution where the decision weights deviate from the objective probabilities accordingly. Other models with a specific editing procedure like the Subjectively Weighted Utility model (Karmarker, 1978) also focus on the transformation between the decision weights and the objective probabilities. The editing involved in Equation (6.7), however, focuses on the events of the prospects and assumes juxtaposition of similar outcomes. It is summarized in the following hypothesis:

HYPOTHESIS 3

Under the assumption that decision makers have limited information processing capacity, they will not calculate the exact amount of rejoice or regret. Using the reference point as an heuristic, the decision makers will derive rejoice or regret uni-dimensionally by comparing the outcomes of the prospect and the reference point in similar states while the probability distribution of the risky reference point is completely ignored. Only when the reference point or the prospect is risk-free will the decision makers partition the riskless prospect according to the probability distribution of the risky prospect. Rejoice or regret in each state is derived from the difference between the risky and the riskless prospects.

Some of the anomalies (the common ratio effect, the common consequence effect, and the isolation effect) discussed in Section 3.1 will be analyzed in Chapter 7 according to the editing procedure outlined in Hypothesis 3.

6.5 THE DEMAND FOR INSURANCE

This section investigates several issues related to the demand for insurance based on Hypotheses 1 and 2 developed in Section 5.4. This section begins the demand analysis with a simple insurance problem.

The decision maker has to make a binary choice: do not purchase insurance or purchase full coverage. Section 6.5.1 derives the optimal coverage for those individuals who select a riskless reference point and the comparative statics of increasing the premium rate. The results are similar to the EU model. Section 6.5.2 assumes that individuals use a risky reference point to derive the optimal condition. This group of individuals will purchase less than full coverage at the actuarially fair premium rate. It provides one possible explanation for the phenomenon of under-insurance against low-probability disasters. RPT predicts a higher price elasticity of insurance demand when individuals switch from a riskless reference point to a risky reference point.

Reference Point theory assumes that individuals have different values of k in their transformation function. According to Hypothesis 1, the individual with a small k will select the low-risk point (full coverage) as the reference point. The other individual will take a riskier alternative as the reference point. Two transformation functions are depicted in Figure 6.7 to determine their reference points. Facing the probability (p) to lose L in an accident, an individual can purchase insurance to avoid the risk of losing L. If the per \$ cost of insurance c is equal to the probability of having an accident p; the loading factor of the insurance policy is one and the insurance is actuarially fair. In Figure 6.7, it is assumed that the loading factor is greater than one to cover the administrative cost of the insurance company; in this case, $c > p$. To cover a possible loss of L, the premium is cL.

An individual with small k and full coverage as the reference point derives a higher EM index for full coverage than no insurance:

$$EM(W-cL;\ W-cL) > EM(W,\ W-L;\ W-cL) \tag{6.8}$$

where

$$EM(W-cL;\ W-cL) = U(W-cL) \tag{6.9}$$

and

$$EM(W,\ W-L;\ W-cL) = \begin{array}{l}(1-p)\{U(W)+R[U(W)-U(W-cL)]\} \\ +\ p\{U(W-L)+R[U(W-L)-U(W-cL)]\}.\end{array} \tag{6.10}$$

The two EM indices are depicted in Figure 6.8. For an actuarially fair premium pL, the risky prospect ((1-p), W; p, W-L) is a mean-preserving spread of the riskless prospect (W-pL). Due to a concave VNM utility function (which also leads to asymmetry in evaluating gains and losses)[2], we have the inequality in Equation (6.8).

The individual with a sufficiently large k selects the risky prospect as his reference point as shown in Figure 6.7. His EM indices for full coverage and no coverage, respectively, are:

$$EM(W-pL;\ W,\ W-L) = (1-p)M(W-pL;\ W) + pM(W-pL;\ W-L)$$

$$= (1-p)\{U(W-pL)+R[U(W-pL)-U(W)]\} + p\{U(W-pL)+R[U(W-pL)-U(W-L)]\}$$

$$EM(W-pL;\ W,\ W-L) = \begin{array}{l}U(W-cL) + (1-p)R[U(W-pL)-U(W)] \\ \qquad +\ pR[U(W-pL)-U(W-L)]\end{array} \tag{6.11}$$

and

$$EM(W,\ W-L;\ W,\ W-L) = (1-p)U(W) + pU(W-L) \tag{6.12}$$

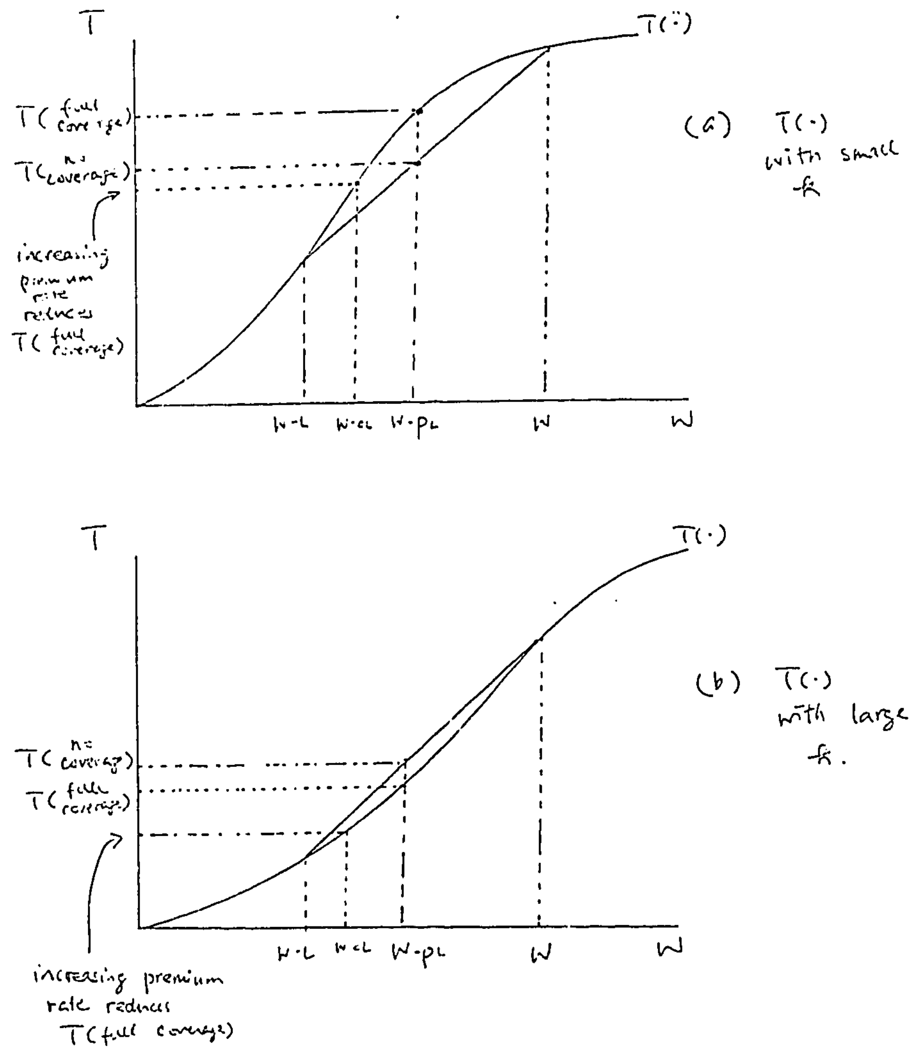


FIGURE 6.7 SELECTING REFERENCE POINTS FOR INSURANCE ANALYSES

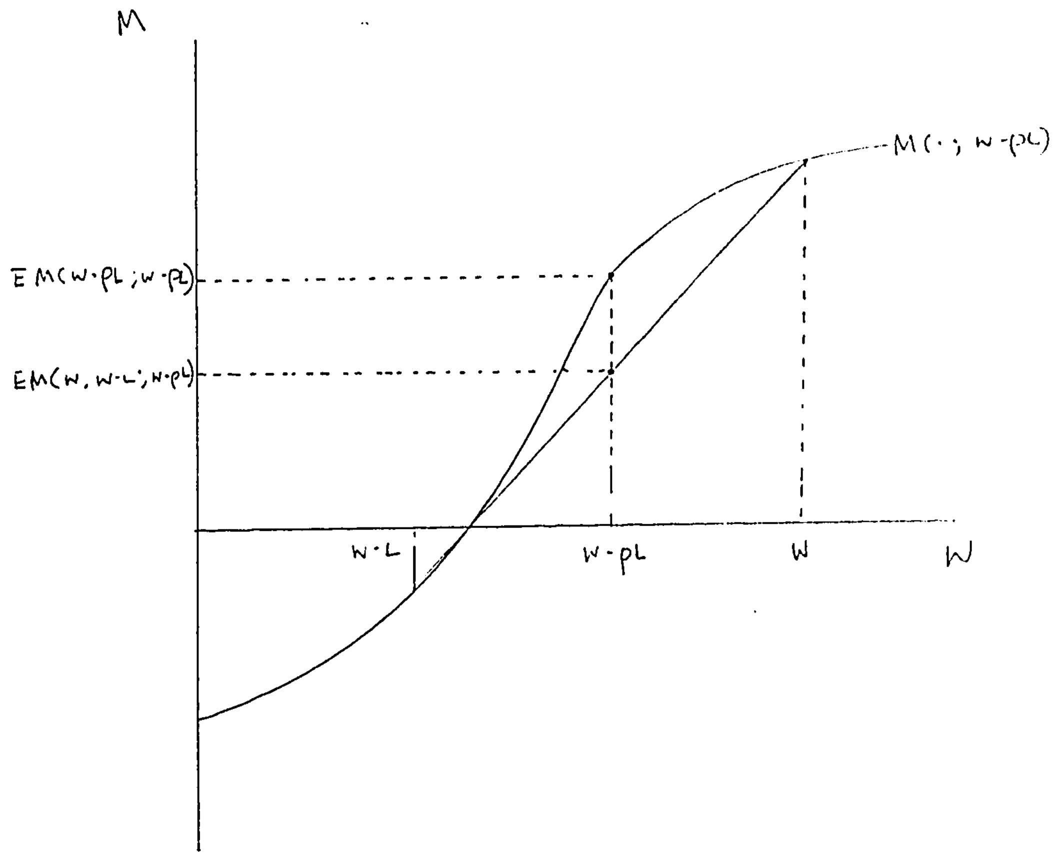


FIGURE 6.8 DEMAND FOR INSURANCE WITH A RISKLESS REFERENCE POINT

From Equations (6.8) to (6.10), after rearranging terms,

$$U(W-cL) - (1-p)R[U(W)-U(W-cL)] - pR[U(W-L)-U(W-cL)] > (1-p)U(W) + pU(W-cL) \quad (6.13)$$

$R[\cdot]$ is symmetrical with respect to ξ, $R[-\xi] = -R[\xi]$.[3] Therefore, Equation (6.13) is identical to

$$U(W-cL) + (1-p)R[U(W-cL)-U(W)] + pR[U(W-cL)-U(W-L)] > (1-p)U(W) + pU(W-cL). \quad (6.14)$$

Thus,

$$EM(W-cL;\ W,\ W-L) > EM(W,\ W-L;\ W,\ W-L) \quad (6.15)$$

and the second individual also prefers to have full coverage than no coverage if insurance is offered at the actuarially fair premium.

6.5.1 Optimal Insurance Coverage With A Riskless Reference Point

To analyze the impact of a premium increase on the demand for insurance, we derive the optimal insurance coverage from the general model of RPT. Note that the following analysis still restricts the choice of reference points to the two extreme scenarios: full coverage or no coverage. The most general approach would allow for the possibility of selecting any reference point between these two polar cases. On the other hand, the restriction regarding the choice of reference points may be desirable in light of limited capacity to process information.

Given wealth (W), the amount of losses (L), the probability of having accident (p), the premium rate per $ insured (c), the reference point (full coverage or W-cL), and coverage (Z), the EM index with full coverage as the reference point is:

$$EM(W-cZ,\ W-L+(1-c)Z;\ W-cL) = \begin{aligned}&(1-p)\{U(W-cZ)+R[U(W-cZ)-U(W-cL)]\}\\ &+ pU(W-L+(1-c)Z)\\ &+ pR[U(W-L+(1-c)Z)-U(W-cL)]\}.\end{aligned} \quad (6.16)$$

The first order condition for optimal coverage Z^* is:

$$\frac{\partial EM}{\partial Z} = \begin{aligned}&-c(1-p)\{U'(S1)+R'[\xi 1]U'(S1)\}\\ &+ (1-c)p\{U'(S2)+R'[\xi 2]U'(S2)\}\end{aligned} = 0 \quad (6.17)$$

where U' and R' denote the first order derivatives of $U(\cdot)$ and $R[\cdot]$, respectively, S1 is W-cZ (the level of wealth in the state without accident), $\xi 1$ is $U(W-cZ)-U(W-cL) \geq 0$ (the amount of rejoice derived from a full coverage reference point in the state without an accident), S2 is W-L+(1-c)Z (the level of wealth in the state with an accident), and $\xi 2$ is $U(W-L+(1-c)Z)-U(W-cL) \leq 0$ (the amount of regret derived from full coverage reference point in the state with an accident). Solving (6.17) to derive the optimal coverage Z^*, we have

$$\frac{c(1-p)}{(1-c)p} = \frac{U'(S2)\{1+R'[\xi 2]\}}{U'(S1)\{1+R'[\xi 1]\}}. \quad (6.18)$$

When c = p for actuarially fair premium, Equation (6.18) implies that

$$U'(S2)\{1+R'[\xi 2]\} = U'(S1)\{1+R'[\xi 1]\} \quad (6.19)$$

which is guaranteed by S1 = S2 and $\xi 1 = \xi 2$; or in other words, $Z^* = L$. We obtain the same result as before.

Increasing the premium rate c, the optimal condition in (6.18) requires that

$$U'(S2)\{1+R'[\xi 2]\} > U'(S1)\{1+R'[\xi 1]\}. \quad (6.20)$$

Due to the fact the $U''(\cdot) < 0$ (concave VNM $U(\cdot)$), increasing S1 relative to S2 reduces $U'(S1)$ relative to $U'(S2)$. In addition, increasing S1 increases $\xi 1$. Since $\xi 1 \geq 0$, larger $\xi 1$ implies a smaller $R'[\xi 1]$ ($R''[\cdot] < 0$ when $\xi > 0$, see condition (v) in (5.1)). Lowering insurance demand from full coverage increases S1 which will reduce $U'(S1)\{1+R'[\xi 1]\}$ in (6.19). On the other hand, this change in insurance coverage reduces S2 and decreases $\xi 2$ and $R'[\xi 2]$ ($R''[\cdot] > 0$ when $\xi < 0$, see condition (vi) in (5.1)). If the curvature in $R[\cdot]$ dominates the concavity in $U(\cdot)$, or

$$U''(\cdot) + R'[\cdot]U''(\cdot) + R''[\cdot][U(\cdot)]^2 > 0 \text{ when } \xi < 0,$$ [4]

lowering the insurance coverage will increase $U'(S2)\{1+R'[\xi 2]\}$ in (6.19). It is clear that lowering insurance coverage satisfies the new optimal condition in Equation (6.20). Thus, increasing the insurance premium rate will reduce the optimal coverage.

6.5.2 Optimal Insurance Coverage With A Risky Reference Point

Given a risky reference point, the EM index becomes:

$$EM(W-cZ,\ W-L+(1-c)Z;\ W,\ W-L) = \begin{aligned}&(1-p)\{U(W-cZ)+R[U(W-cZ)-U(W)]\}\\&+ pU(W-L+(1-c)Z)\\&+ pR[U(W-L+(1-c)Z)-U(W-L)]\}.\end{aligned} \tag{6.21}$$

We obtain similar first order condition for optimal coverage $\hat{Z}$:

$$\frac{\partial EM}{\partial Z} = \begin{aligned}&-c(1-p)\{U'(S1)+R'[\xi 3]U'(S1)\}\\&+ (1-c)p\{U'(S2)+R'[\xi 4]U'(S2)\}\end{aligned} = 0. \tag{6.22}$$

The first order condition is similar to (6.17) except that the rejoice

/ regret terms differ in (6.22). $\xi 3$ is $U(W-cZ)-U(W) \leq 0$ (the amount of regret derived from a no coverage reference point in the state without accident) and $\xi 4$ is $U(W-L+(1-c)Z)-U(W-L) \geq 0$ (the amount of rejoice derived from no coverage reference point in the state with accident). Solving (6.22) to derive the optimal coverage $\hat{Z}$, we have

$$\frac{c(1-p)}{(1-c)p} = \frac{U'(S2)\{1+R'[\xi 4]\}}{U'(S1)\{1+R'[\xi 3]\}}. \tag{6.23}$$

When c = p and premium is actuarially fair, we have

$$U'(S2)\{1+R'[\xi 4]\} = U'(S1)\{1+R'[\xi 3]\}. \tag{6.24}$$

Unlike the optimal condition in (6.19), S1 = S2 will not guarantee that $\xi 3 = \xi 4$. (Note that with no coverage $\xi 3 = U(W-cL)-U(W) < 0$ and $\xi 4 = U(W-cL)-U(W-L) > 0$.) In other words, full coverage (Z = L) may not be the optimal choice for individuals using the risky reference point although full coverage is better than no coverage. Given that $R[\cdot]$ is symmetrical with respect to ξ, $R'[\xi 3]$ equals $R'[\xi 4]$ only if $-\xi 3 = \xi 4$. At full coverage, $-\xi 3 = \xi 4$ is equivalent to $U(W)-U(W-cL) = U(W-cL)-U(W-L)$. In other words, individuals with a risky reference point will demand full coverage at an actuarially fair rate if

$$U(W)-U(W-cL) = U(W-cL)-U(W-L). \tag{6.25}$$

A concave VNM utility function and c = p imply that

$$W-(W-cL) > (W-cL)-(W-L) \Leftrightarrow p > 1/2. \tag{6.26}$$

For low probability accidents (or disasters), these individuals will not purchase full coverage! RPT predicts that they will purchase some insurance coverage because no coverage is not the optimal choice for them (see Equation (6.15)).

Kunreuther (1976) observed that the majority of homeowners living in earthquake regions and flood plains did not purchase subsidized insurance (see review in Section 3.3.3). He suggested that these individuals might fail to reach a plausible decision if they had inadequate awareness of the issue. It is possible to derive higher EM index for no coverage (thus reversing the inequalities in Equations (6.14) and (6.15)) if individuals dismiss (or underestimate) the probability of disasters. They would inflate the no insurance EM index $EM(W, W-L; W, W-L) = (1-p)U(W) + pU(W-pL)$ by overweighing $(1-p)U(W)$. Underestimating the probability of disasters also reduces the EM index of full coverage because regret $(1-p)R[U(W-pL)-U(W)]$ is overvalued.

Assume that when $c = p$, (6.24) is satisfied with some coverage $0 < \hat{Z} < L$. Increasing the premium rate c, the optimal condition in (6.24) should be changed to

$$U'(S2)\{1+R'[\xi 4]\} > U'(S1)\{1+R'[\xi 3]\}. \tag{6.27}$$

Similar to the case with a riskless reference point, increasing S1 and reducing S2 by lowering insurance coverage reduces $U'(S1)$ relative to $U'(S2)$. In addition, increasing S1 increases $\xi 3$. Since $\xi 3 \le 0$, larger $\xi 3$ leads to a larger $R'[\xi 3]$ ($R''[\cdot] > 0$ when $\xi < 0$, see condition (vi) in (5.1)). For the same condition which ensures that regret is diminishing in $M(\cdot; \cdot)$

$$U''(\cdot) + R'[\cdot]U''(\cdot) + R''[\cdot][U(\cdot)]^2 > 0 \text{ when } \xi < 0.$$

Lowering insurance demand from $\hat{Z}$ will reduce $U'(S1)\{1+R'[\xi 3]\}$ in (6.24). On the other hand, this change in insurance coverage reduces S2 and decreases $\xi 4$. The resulting change in $U'(S2)\{1+R'[\xi 4]\}$ is positive. We confirm that increasing the premium rate will reduce the optimal coverage.

Moreover, the increase in the premium rate also affects the selection of the reference point. With the same expected losses, a higher insurance premium may reverse the choice of reference point for those who initially selected the riskless reference point. Individuals who selected the risky reference point will continue to choose the same reference point (see Figure 6.7). For those who initially selected the riskless reference point and demanded full coverage, they will lower their coverage to $\hat{Z}$ due to switching reference point. Then, they will further lower their coverage due to higher cost of insurance. The combined effects of switching reference point and the higher cost of insurance will induce a larger reduction in insurance coverage. Thus, the price elasticity of aggregate demand for insurance increases when decision makers switch to a riskier reference point.

Reference Point theory predicts that individuals with a full coverage reference point will demand full coverage given that the premium is actuarially fair. As the insurance premium increases, the demand for insurance is reduced. These results are similar to the predictions of the EU model. With a no coverage reference point, however, RPT predicts that individuals will demand less than full coverage insurance even though the premium is actuarially fair. This prediction is consistent with the under-insurance phenomenon. Furthermore, RPT predicts that demand for insurance is sensitive to premium adjustment due to switching reference points.

CHAPTER 7 THE ANOMALIES AND REFERENCE POINT THEORY

The EU model was designed to solve the anomaly of its time; namely the St. Petersburg's paradox. Despite the fact that the EU model is a popular model in guiding decisions regarding risky prospects, its predictive power is repudiated by many paradoxical findings. For any decision model to be a successful candidate to replace the EU model, it must resolve these perplexing anomalies. In this chapter, Reference Point theory is examined in light of these anomalies. These anomalies were surveyed in Section 3.1; they are the common ratio effect, the common consequence effect, the response mode effect, the isolation effect, the reflection effect, and the preference reversal phenomenon.

Reference point theory conjectures that the context of the decision problem will influence the selection of the reference point. Besides considering the riskiness of the prospects (through probabilities) and the risk attitude of the decision makers (through a non-linear utility function), the new theory recognizes that the context of the problem is an important dimension in the decision-making process. RPT postulates a scheme where the context of the decision is captured by choosing the appropriate reference point. It is shown below that the context of the problem causes a switch in the reference point in three of the six anomalies, this feature in RPT will account for the response mode effect, the reflection effect, and the preference reversal phenomenon.

For the common ratio effect, the common consequence effect, and the isolation effect, the prospects in the second choice problem are linear transformation of the corresponding prospects in the first choice problem. Thus, the observed choice patterns in these anomalies violate the independence axiom. In other words, a valuation function which is linear in probabilities will not be able to predict these anomalies. According to Hypotheses 1 and 2, subjects will always select the related prospects as the reference point in the respective choice problems due to the fact that the transformation function T(·) is linear in probabilities. Without "switching" reference point, RPT cannot predict these anomalies because the EM index is also linear in probabilities. One can explain these anomalies by the editing process suggested in Hypothesis 3 (see Section 6.4). Hypothesis 3 assumes that when a risky prospect is evaluated against a risky reference point, the probability distribution of the risky reference point will be ignored and the payoffs of the prospects are juxtaposed according to their rank. Based on the assumption in this hypothesis, using a reference point as heuristics will reduce the information processing required. The resulting decision-making process will lead to "biased" decision because the decision makers will ignore some information contained in the prospects. The biases caused by the simplification will account for the anomalies.

The chapter is divided into six sections, each section will cover one type of anomaly found in laboratory experiments. The discussion of the first five anomalies is based on the experiments in Kahneman and Tversky (1979); the number of respondents is denoted by N, and the percentage who choose each option is given in brackets.

7.1 THE COMMON RATIO EFFECT

The common ratio effect was first discovered by Allais (1953). Consider the two pairs of gambles used in Kahneman and Tversky's experiment (1979, pp.266-267, Problems 3 and 4):

PROBLEM 3: (N = 95)

Choose between

$\tilde{a}_1$: 0.80 chance of $4,000; [20%]

$\tilde{a}_2$: 1.00 chance of $3,000. [80%]

PROBLEM 4: (N = 95)

Choose between

$\tilde{a}_3$: 0.20 chance of $4,000; [65%]

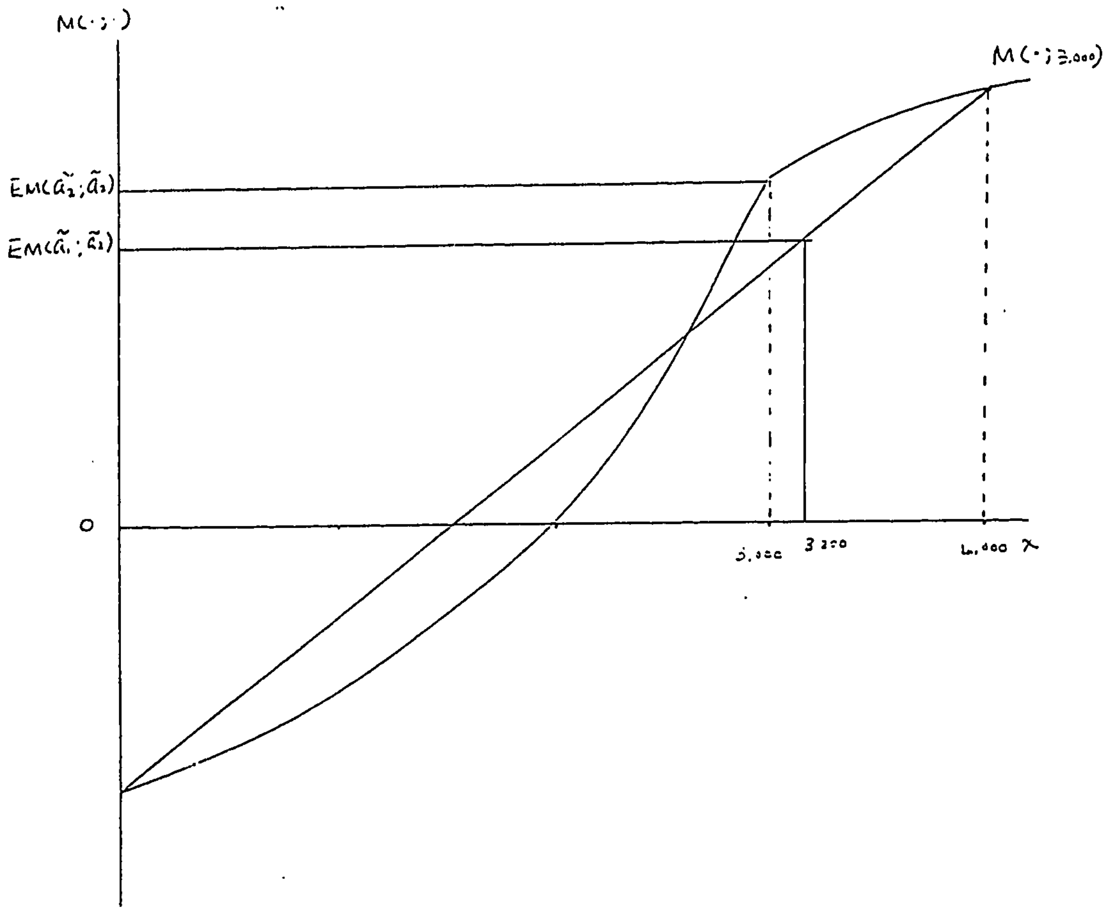
$\tilde{a}_4$: 0.25 chance of $3,000. [35%]

According to the EU model, a large majority of the subjects exhibit a higher degree of risk aversion by choosing $\tilde{a}_2$ over $\tilde{a}_1$ in Problem 3. 20% of the subjects exhibit lower degree of risk aversion or risk-seeking behaviour by choosing the riskier prospect which yields a higher expected return. In Problem 4, the majority of the subjects exhibit lower degree of risk aversion by choosing $\tilde{a}_3$. Note that $\tilde{a}_3$ and $\tilde{a}_4$ are linear transformation of $\tilde{a}_1$ and $\tilde{a}_2$, respectively.

Hence, the independence axiom of the EU model requires that an individual who chooses $\tilde{a}_1$ in Problem 3 will choose $\tilde{a}_3$ in Problem 4. Given that the transformation function and the valuation function in RPT are also linear in probabilities, RPT cannot successfully predict the common ratio effect.

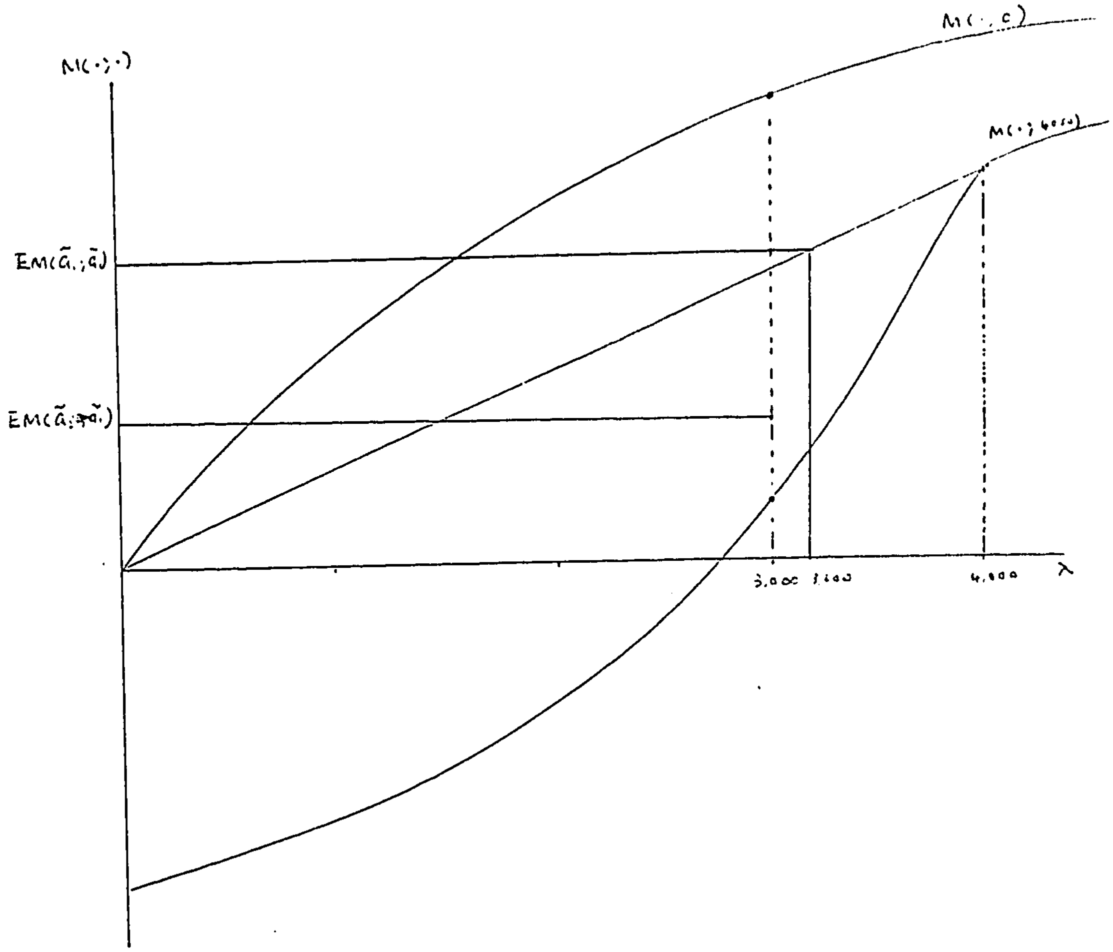
If a decision maker follows the RPT, the first step in his decision-making process is to select a reference point. Subjects with different values in k may choose different reference points. Between $\tilde{a}_1$ and $\tilde{a}_2$, subjects will select riskless prospect $\tilde{a}_2$ as their reference point if their transformation functions exhibit sufficient degree of concavity over the range of payoffs. Such transformation functions are associated with small k. Using this reference point will create a high probability of small rejoice of \$1,000 for choosing prospect $\tilde{a}_1$ over $\tilde{a}_2$ in the state where the outcome in $\tilde{a}_1$ is \$4,000. Even though there is only a relatively small chance for a large regret of \$3,000 when $\tilde{a}_1$ is \$0, decision makers put more emphasis on regret than rejoice. The asymmetry in the rejoice-regret function may lead to status quo bias where decision makers with a \$3,000 reference point will assign a higher EM index to $\tilde{a}_2$. Figure 7.1 depicts the situation where $\tilde{a}_2 \succ \tilde{a}_1$.

For decision makers whose transformation function is associated with a large k, they will evaluate $\tilde{a}_1$ and $\tilde{a}_2$ on the convex segment and consequently select $\tilde{a}_1$ as the reference point. With this reference point, subjects will derive rejoice from choosing $\tilde{a}_2$ over $\tilde{a}_1$ in the state where the outcome in $\tilde{a}_1$ is \$0 but regret when the outcome in $\tilde{a}_1$



Reference Point : ã2

FIGURE 7.1 RISKLESS REFERENCE POINT AND THE COMMON RATIO EFFECT



Reference Point : ã.

FIGURE 7.2 RISKY REFERENCE POINT AND THE COMMON RATIO EFFECT

is \$4,000. Figure 7.2 depicts the case where 0.80[U(4,000)] > 0.20{U(3,000) + R[U(3,000)]} + 0.80{U(3,000) + R[U(3,000) - U(4,000)]} and subjects prefer $\tilde{a}_1$ to $\tilde{a}_2$. The resulting preference once again reflects the asymmetry in evaluating gains and losses. The tendency to choose the reference point as the preferred choice also illustrates the status quo bias (see Kahneman, Knetsch, and Thaler, 1991).

To the subjects who chose $\tilde{a}_2$ as the reference point in Problem 3, T(\$3,000) > 0.8•T(\$4,000) + 0.2•T(\$0). Given T(\$0) = 0, they will also select $\tilde{a}_4$ as the reference point because 0.25•T(\$3,000) > 0.20•T(\$4,000). This result follows from the fact that T(·) is linear in probabilities. Similarly, the remaining subjects who chose $\tilde{a}_1$ as the reference point in Problem 3 will select $\tilde{a}_3$ as the reference point in Problem 4.

When the majority of the subjects continues to pick the low-risk reference point according to Hypothesis 1, the situation where decision makers change from selecting a low-risk reference point to a high-risk reference point does not occur. RPT will continue to assign a higher EM index to the reference point (the low risk reference point) and the status quo bias remains in effect (see Appendix B). Consequently, the common ratio effect cannot be explained (or caused) by switching reference point.

The evaluation procedure outlined in RPT requires the decision maker to partition the probability distribution of $\tilde{a}_3$ according to the events in the reference point $\tilde{a}_4$. The resulting index will fully account for the rejoice and the regret of choosing $\tilde{a}_3$ over $\tilde{a}_4$; $EM(\tilde{a}_3;\ \tilde{a}_4)$ is given as follows:

$$0.20 \cdot 0.25M(4{,}000;\ 3{,}000) + 0.80 \cdot 0.25M(0;\ 3{,}000) + 0.20 \cdot 0.75M(4{,}000;\ 0) + 0.80 \cdot 0.75M(0;\ 0) > 0.25M(3{,}000;\ 3{,}000) + 0.75M(0;\ 0). \quad (7.1)$$

When partitioning is required, the reference point (which is risky) actually makes the evaluation process more complicated. It is unlikely that a decision maker would fully partition the probability distribution given the limitations on information processing. Hypothesis 3 in Section 6.4 assumes that the decision makers will adopt the reference point as a heuristic and juxtapose the outcomes of the two prospects in the similar states. Following the editing process described in Hypothesis 3, Equation (7.1) is simplified as

$$0.20M(4{,}000;\ 3{,}000) + 0.80M(0;\ 0) > 0.25M(3{,}000;\ 3{,}000) + 0.75M(0;\ 0). \quad (7.2)$$

If decision makers follow Hypothesis 3 to derive rejoice or regret, it is possible to generate the common ratio effect. The regret in choosing $\tilde{a}_1$ over $\tilde{a}_2$, 0.2M(0; 3,000) in the left hand side of Equation (7.1), is not present in the choice between $\tilde{a}_3$ and $\tilde{a}_4$ (see Equation (7.2)). Figure 7.3 below depicts the situation where $\tilde{a}_3 \succ \tilde{a}_4$. Starmer and Sugden (1989), in the context of Regret theory, also established some relations between the common ratio effect and the juxtaposition effects.

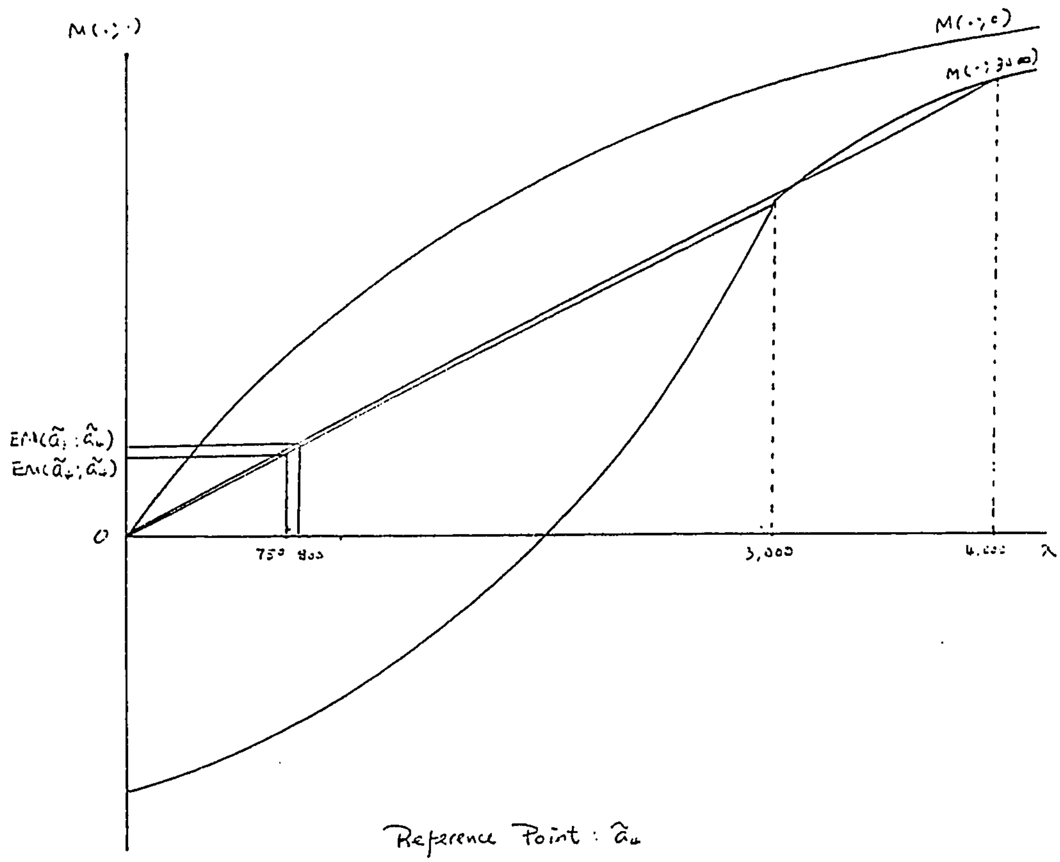


FIGURE 7.3 EDITING IN REFERENCE POINT THEORY AND THE COMMON RATIO EFFECT

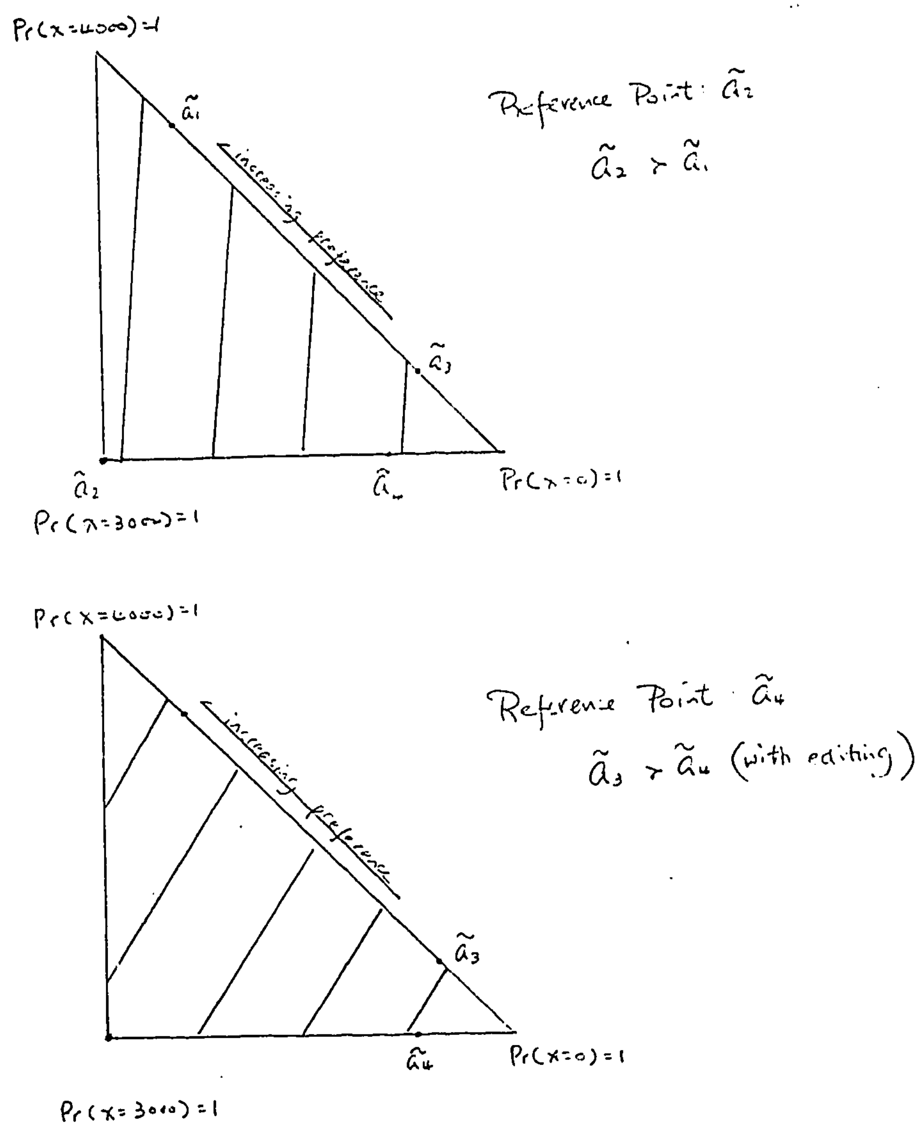


FIGURE 7.4 INDIFFERENCE CURVES ASSOCIATED WITH DIFFERENT REFERENCE POINTS AND THE COMMON RATIO EFFECT

Figure 7.4 above depicts the indifference curves associated with reference points $\tilde{a}_2$ and $\tilde{a}_4$. The indifference curves are linear in probabilities because of the independence axiom. The indifference curves associated with reference point $\tilde{a}_2$ in Figure 7.4 show that $\tilde{a}_2 \succ \tilde{a}_1$; subjects exhibit risk aversion with steep indifference curves. Increasing the riskiness in the reference point will encourage risk-taking behaviour due to the editing process. The editing process will take away the regret in choosing the risky prospect when its outcome is equal to 0 where the outcome of the reference point is positive. Given reference point $\tilde{a}_4$ and editing, the indifference curves show that $\tilde{a}_3 \succ \tilde{a}_4$; the riskier prospect is preferred by the majority of the subjects. These indifference curves are flatter than the indifference curves associated with riskless reference point $\tilde{a}_2$.

Note that the results of Problem 4 indicate that not every subject chose $\tilde{a}_3$. The subjects who selected $\tilde{a}_4$ as the reference point may assign a higher valuation index to $\tilde{a}_4$. It is possible to have $0.20M(4,000;\ 3,000) < 0.25M(3,000;\ 3,000)$ when the individual exhibits a strong degree of risk aversion. This is the case when the concavity in $U(\cdot)$ is large enough to the extent that $0.25U(3,000) - 0.20U(4,000) > 0.20R[U(4,000)-U(3,000)]$. In other words, such an individual is not willing to give up the higher probability to win a positive amount for the possibility of winning a larger amount plus the resulting rejoice factor.

7.2 THE COMMON CONSEQUENCE EFFECT

The common consequence effect is demonstrated by the following pairs of gambles (Problems 1 and 2 in Kahneman and Tversky, 1979, pp. 265-266):

PROBLEM 1: (N = 72)

Choose between

$\tilde{b}_1$: 2,500 with probability .33,
2,400 with probability .66,
0 with probability .01; [18%]

$\tilde{b}_2$: 2,400 with certainty. [82%]

PROBLEM 2: (N = 72)

Choose between

$\tilde{b}_3$: 2,500 with probability .33,
0 with probability .67; [83%]

$\tilde{b}_4$: 2,400 with probability .34,
0 with probability .66; [17%]

According to EU model, subjects who prefer $\tilde{b}_1$ to $\tilde{b}_2$ should also prefer $\tilde{b}_3$ to $\tilde{b}_4$, or vice versa. Yet, Kahneman and Tversky found that 65% of the subjects violate this proposition (p.266). Similar to the common ratio effect, RPT cannot explain the violation of the independence axiom which causes the common consequence effect.

In the context of RPT, subjects with a sufficiently small k will evaluate the payoffs in $\tilde{b}_1$ and $\tilde{b}_2$ over the concave portion of $T(\cdot)$ and $\tilde{b}_2$ will be chosen as the reference point. Choosing $\tilde{b}_1$ in light of the reference point $\tilde{b}_2$ will generate a rejoice of \$100 when the outcome is \$2,500 but a regret of \$2,400 when the outcome is \$0. Due to loss aversion, subjects with reference point $\tilde{b}_2$ may avoid choosing $\tilde{b}_1$ as shown in Figure 7.5 below. In this case, $M(2{,}400;\ 2{,}400) >$ $0.33M(2{,}500;\ 2{,}400) + 0.66M(2{,}400;\ 2{,}400) + 0.01M(0;\ 2.400)$. The EM index for $\tilde{b}_1$ in Figure 7.5 is derived as a linear combination of $0.33M(2{,}500;\ 2{,}400)$ and 0.67θ where θ is a linear combination of $(0.66/0.67)M(2{,}400;\ 2{,}400) + (0.01/0.67)M(0;\ 2.400)$. Hence, θ is located on the strict line connecting $M(2{,}400;\ 2{,}400)$ and $M(0;\ 2.400)$ with expected value 2,364.18. Similarly, $EM(\tilde{b}_1;\ \tilde{b}_2)$ is located on the strict line connecting θ and $M(2{,}500;\ 2{,}400)$ with expected value 2,409.

Decision makers with a large k, on the other hand, will select $\tilde{b}_1$ as the reference point due to the convexity in $T(\cdot)$. With this reference point, subjects will derive rejoice from choosing $\tilde{b}_2$ over $\tilde{b}_1$ in the state where the outcome in $\tilde{b}_1$ is \$0 but regret when the outcome in $\tilde{b}_1$ is \$2,500. Figure 7.6 depicts the case where

$0.33M(2{,}400;\ 2{,}500) + 0.66M(2{,}400;\ 2{,}400) + 0.01M(2{,}400;\ 0) <$

$0.33M(2{,}500;\ 2{,}500) + 0.66M(2{,}400;\ 2{,}400) + 0.01M(0;\ 0)$

and subjects prefer $\tilde{b}_1$ to $\tilde{b}_2$. The EM index for $\tilde{b}_1$ in Figure 7.6 is derived as a linear combination of $0.33M(2{,}500;\ 2{,}500)$ and 0.67γ where γ is a linear combination of $(0.66/0.67)M(2{,}400;\ 2{,}400) +$ $(0.01/0.67)M(0;\ 0)$. Hence, γ is located on the strict line connecting $M(2{,}400;\ 2{,}400)$ and $M(0;\ 0)$ with expected value 2,364.18. Similarly,

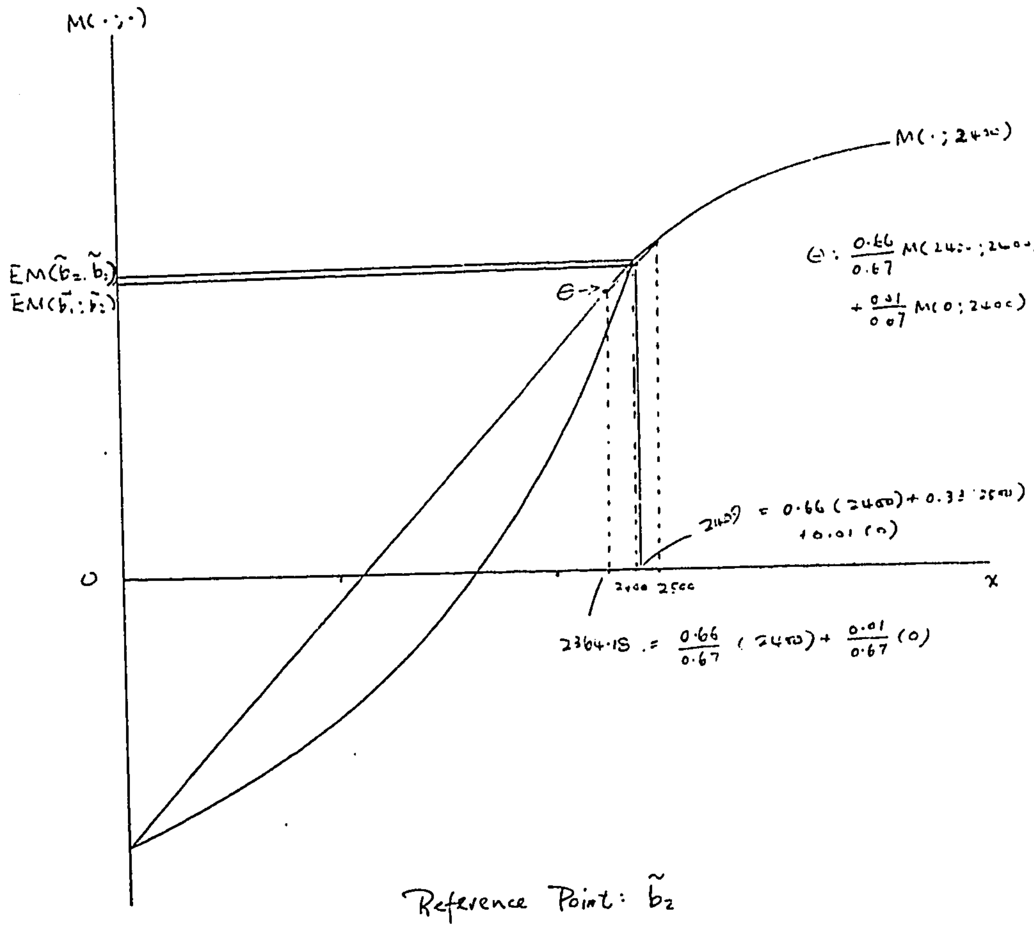


FIGURE 7.5 RISKLESS REFERENCE POINT AND THE COMMON CONSEQUENCE EFFECT

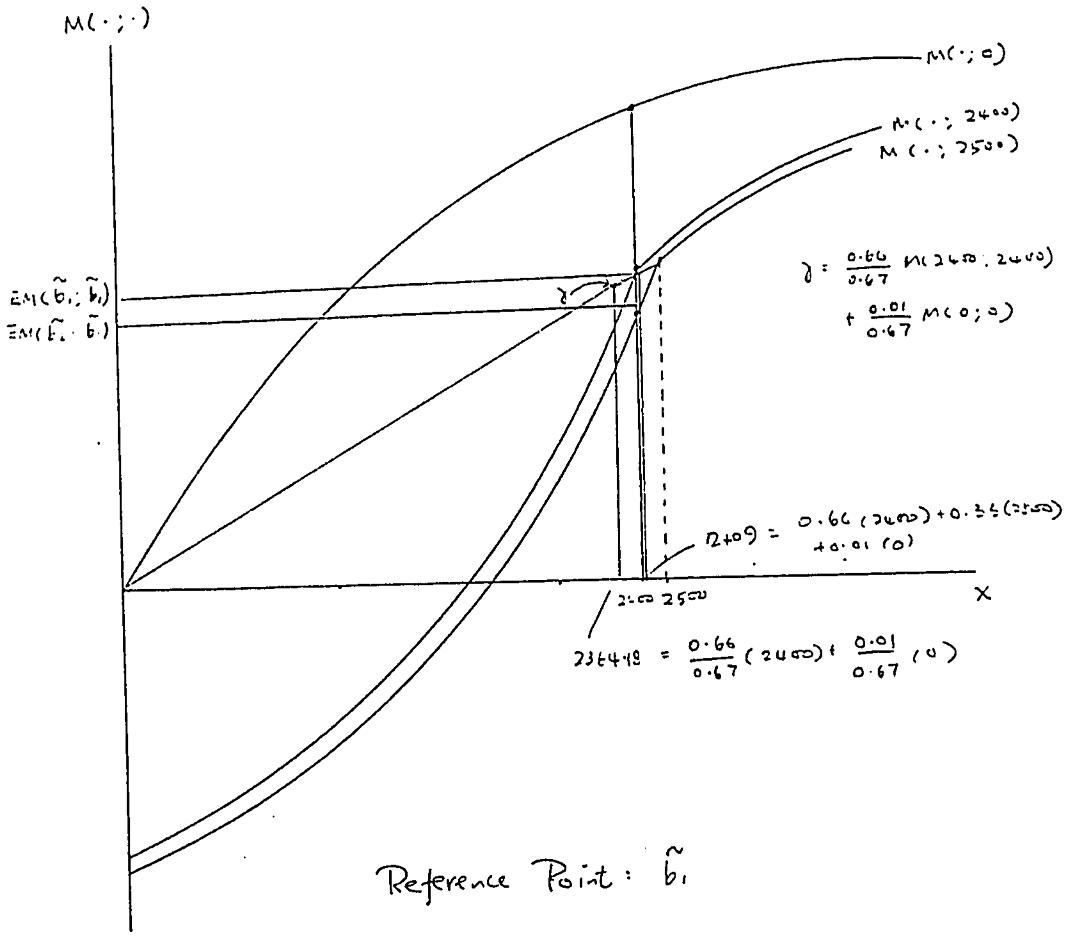


FIGURE 7.6 RISKY REFERENCE POINT AND THE COMMON CONSEQUENCE EFFECT

$EM(\tilde{b}_1; \tilde{b}_1)$ is located on the strict line connecting γ and M(2,500; 2,500) with expected value 2,409. $EM(\tilde{b}_2; \tilde{b}_1)$ is a linear combination of M(2,400; 2,500), M(2,400; 2,400), and M(2,400; 0). Because of the higher probability associating with M(2,400; 2,400), the distance between $EM(\tilde{b}_2; \tilde{b}_1)$ and M(2,400; 2,400) is approximately 1/2 the distance between $EM(\tilde{b}_2; \tilde{b}_1)$ and M(2,400; 2,500). Figures 7.5 and 7.6 show that both groups of decision makers exhibit status quo bias, their respective reference points influence their decisions significantly.

To the subjects who chose $\tilde{b}_2$ as the reference point in Problem 1, $T(2,400) > 0.33 \cdot T(2,500) + 0.66 \cdot T(2,400)$. This group will also select $\tilde{b}_4$ as the reference point because $0.34 \cdot T(2,400) > 0.33 \cdot T(2,500)$. Similarly, the remaining subjects who chose $\tilde{b}_1$ as the reference point in Problem 1 will select $\tilde{b}_3$ as the reference point in Problem 2. RPT predicts that the majority of the subjects (82%) who chose $\tilde{b}_2$ in Problem 1 will prefer $\tilde{b}_4$ to $\tilde{b}_3$ (see Appendix C).

If decision makers follow Hypothesis 3 to derive rejoice or regret without considering the probability distribution of the risky reference prospect $\tilde{b}_4$, the EM indices for $\tilde{b}_3$ and $\tilde{b}_4$ become 0.33M(2,500; 2,400) and 0.34M(2,400; 2,400), respectively. Comparing to the EM index without editing

$$EM(\tilde{b}_3; \tilde{b}_4) = \begin{aligned} &0.33 \cdot 0.34M(2,500;\ 2,400) + 0.67 \cdot 0.34M(0;\ 2,400) \\ &+ 0.33 \cdot 0.66M(2,500;\ 0) \end{aligned} \tag{7.3}$$

the regret component in Equation (7.3) 0.67•0.34R[-U(2,400) disappears due to editing. The rejoice effect of choosing $\tilde{b}_3$ is simplified as

0.33R[U(2,500)-U(2,400)], instead of having two rejoice components 0.33•0.34R[U(2,500)-U(2,400)] and 0.33•0.66R[U(2,500)]. The extent of rejoice is also reduced due to editing. Because of the asymmetrical valuation of regret, the editing process takes away the regret in choosing $\tilde{b}_3$ over $\tilde{b}_4$ and it is possible to generate the common consequence effect when $\tilde{b}_3 \succ \tilde{b}_4$ if

$$0.33M(2,500;\ 2,400) > 0.34M(2,400;\ 2,400). \qquad (7.4)$$

Figure 7.7 below depicts this situation. The modified utility function in Figure 7.7 assumes that the extra $100 in $\tilde{b}_3$ will generate sufficient utility and rejoice to compensate for the slightly lower probability to win $2,500.

Figure 7.8 below depicts the indifference curves associated with reference points $\tilde{b}_2$ and $\tilde{b}_4$. In order to reflect stronger degree of risk aversion where $\tilde{b}_2$ the riskless reference point is also the preferred choice, the indifference curves are steeper. Increasing the riskiness in the reference point will encourage risk-taking behaviour and therefore reduces the slope of the indifference curves.

Note that the results of Problem 2 indicate that not every subject chose $\tilde{b}_3$. With $\tilde{b}_4$ being the reference point, a subject may choose $\tilde{b}_4$ if 0.33M(2,500; 2,400) < 0.34M(2,400; 2,400). It is possible when the concavity in U(·) is large enough to the extent that 0.34U(2,400) - 0.33U(2,500) > 0.33R[U(2,500)-U(2,400)]. This is the case when the subjects exhibit strong degree of risk aversion.

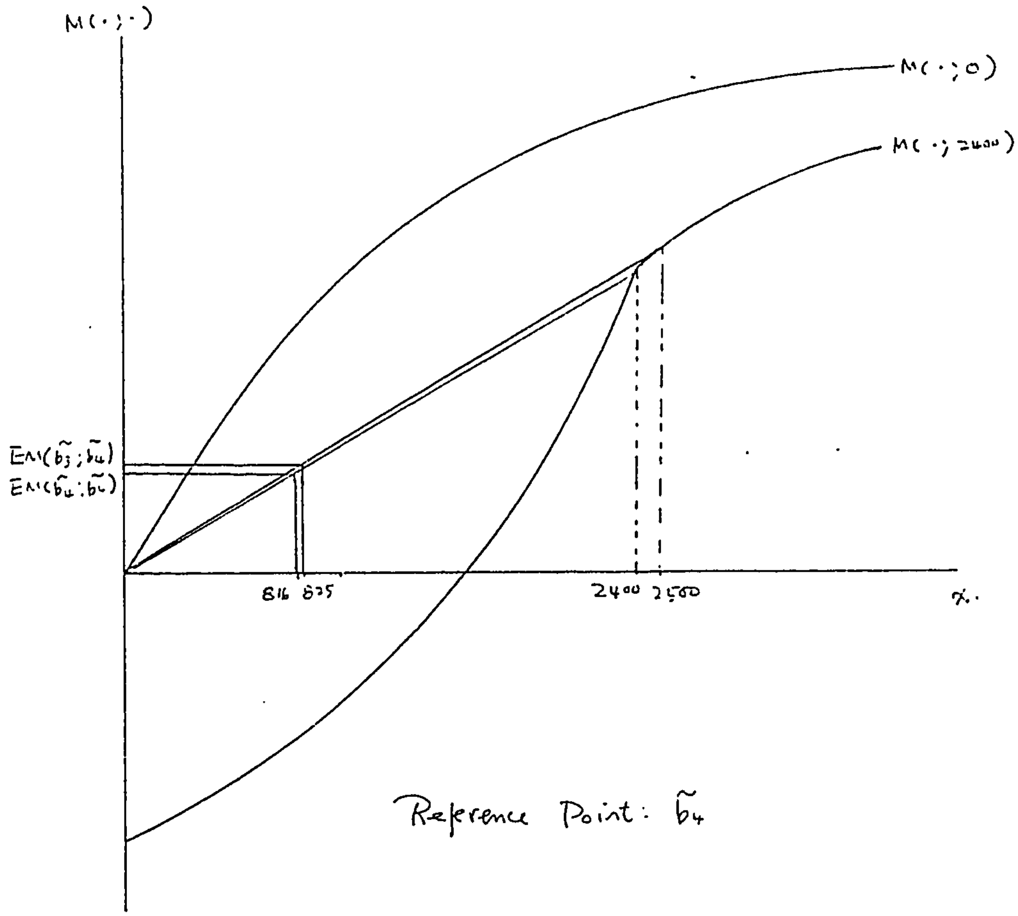


FIGURE 7.7 EDITING IN REFERENCE POINT THEORY AND THE COMMON CONSEQUENCE EFFECT

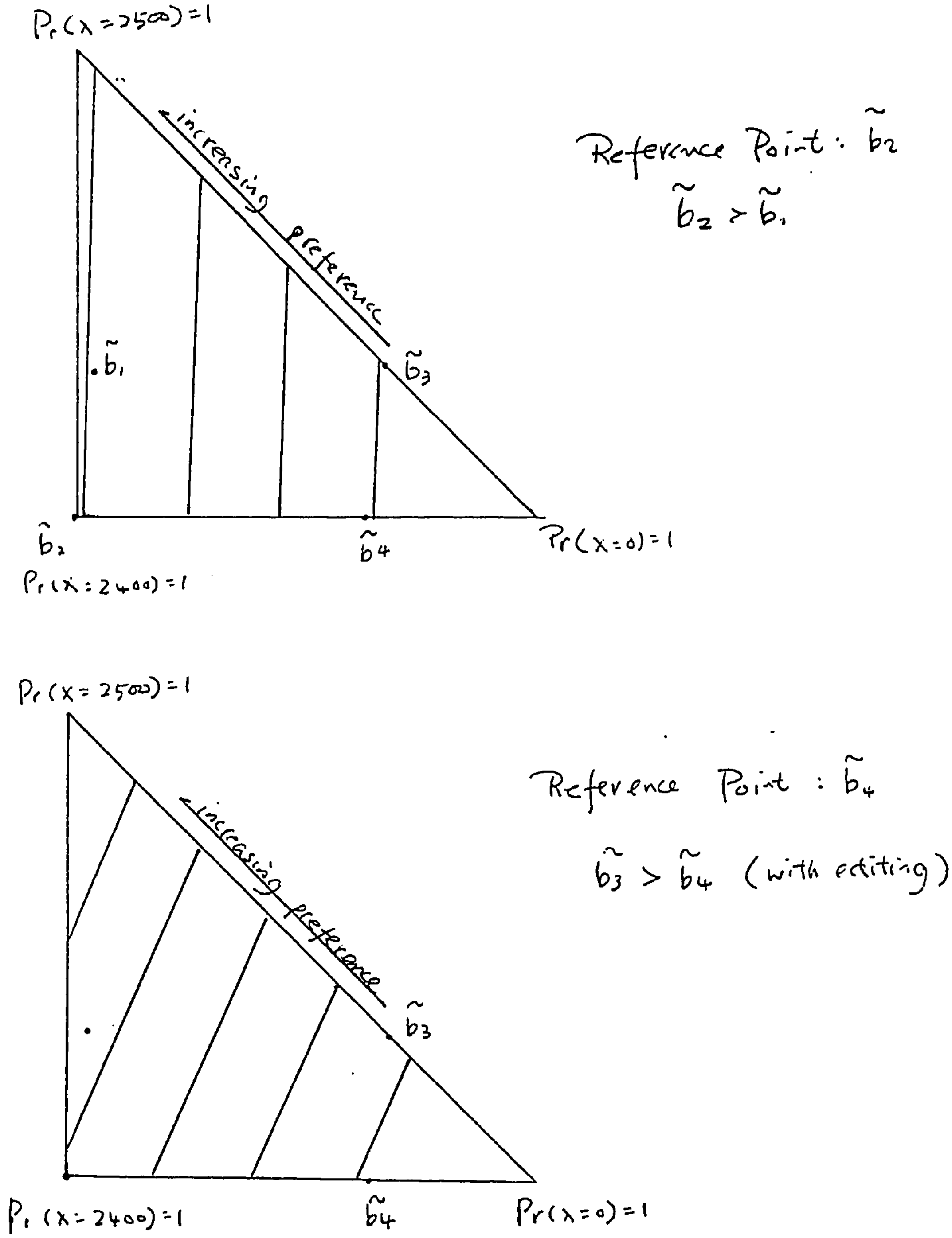


FIGURE 7.8 INDIFFERENCE CURVES ASSOCIATED WITH DIFFERENT REFERENCE POINTS AND THE COMMON CONSEQUENCE EFFECT

7.3 THE RESPONSE MODE EFFECT

Problems 11 and 12 in Kahneman and Tversky (1979, p.273) clearly demonstrate that the decision-making process does not satisfy procedural invariance. The preferences between two prospects which have identical probability to win the same amount are influenced by the context of the problems. In these two problems, their contexts will dictate the choice of the reference points; such response mode effects are accounted for by switching the reference points. This response mode effect further illustrates the importance of loss aversion in the decision-making process. Consider these two problems below:

PROBLEM 11: (N = 70)

In addition to whatever you own, you have been given 1,000. You are now asked to choose between

$\tilde{c}_1$: 0.50 chance of 1000; [16%]

$\tilde{c}_2$: 1.00 chance of 500. [84%]

PROBLEM 12: (N = 68)

In addition to whatever you own, you have been given 2,000. You are now asked to choose between

$\tilde{c}_3$: 0.50 chance of -1,000; [69%]

$\tilde{c}_4$: 1.00 chance of -500. [31%]

Although $\tilde{c}_1$ is identical to $\tilde{c}_3$ and $\tilde{c}_2$ is identical to $\tilde{c}_4$, the majority of the subjects chose $\tilde{c}_2$ instead of $\tilde{c}_1$ but preferred $\tilde{c}_3$ to $\tilde{c}_4$. For those subjects with a small k, a large segment of the transformation function T(·) in Figure 7.9(a) is strictly concave; consequently, the riskless prospect $\tilde{c}_2$ will be selected as the reference point. Figure 7.10 depicts an asymmetrical modified utility function where a negative deviation from the reference point $500 will lead to a large reduction in the modified utility index while the index increases by a smaller amount for a positive deviation. The riskless and relatively small reference point will lead to risk averse behaviour. Figure 7.9(b) illustrates a T(·) with a large k, the subjects will evaluate the prospects on the convex segment and therefore $\tilde{c}_1$ will be selected as the reference point. Using $\tilde{c}_1$ as the reference point, the EM index for $\tilde{c}_1$ is given by the mid-point of line $\alpha\beta$ in Figure 7.11 since $EM(\tilde{c}_1; \tilde{c}_1)$ equals 0.5M(0; 0) + 0.5M(1,000; 1,000). The EM index for $\tilde{c}_2$ is determined by the mid-point of line $\gamma\theta$ in Figure 7.11 since $EM(\tilde{c}_2; \tilde{c}_1)$ equals 0.5M(500; 0) + 0.5M(500; 1,000). $EM(\tilde{c}_1; \tilde{c}_1) > EM(\tilde{c}_2; \tilde{c}_1)$ and risk-taking behaviour follows the selection of a risky reference point.

According to Hypothesis 2 the transformation function T(·) is symmetrical such that $T(-x) = -T(x)$; the rankings of $ET(\tilde{c}_1)$ and $ET(\tilde{c}_2)$ will be reversed in Problem 12 when the outcomes in $\tilde{c}_1$ and $\tilde{c}_2$ becomes losses instead of gains. In Figure 7.9(a), for a symmetric T(·), $ET(\tilde{c}_1) < ET(\tilde{c}_2)$ implies that $ET(\tilde{c}_3) > ET(\tilde{c}_4)$ and $ET(\tilde{c}_1) > ET(\tilde{c}_2)$ implies that $ET(\tilde{c}_3) < ET(\tilde{c}_4)$ in Figure 7.9(b). Consequently, the majority of subjects who are assumed to select $\tilde{c}_2$ in Problem 11 will select $\tilde{c}_3$ as the reference point in Problem 12. The preference of this group will also switch to prefer $\tilde{c}_3$. Similarly, those subjects who select $\tilde{c}_1$ in Problem 11 will select $\tilde{c}_4$ as the reference point and reverse their preference between the two prsopects.

In the probability triangles depicted in Figure 7.12, ($\tilde{c}_1$+\$1,000) and ($\tilde{c}_3$+\$2,000) will take the same position and likewise for ($\tilde{c}_2$+\$1,000) and ($\tilde{c}_4$+\$2,000), the preference shown in the response mode effect is not consistent with a "fanning-out" indifference map. The preference will adjust according to the reference point selected which is affected by the contexts of the problems. The riskless reference point ($\tilde{c}_2$) is associated with the steep indifference curves in Figure 7.12(a) and $\tilde{c}_2 \succ \tilde{c}_1$; the asymmetrical valuation function leads to the status quo bias. Switching the reference point to $\tilde{c}_3$ in Problem 12 as a result of framing will alter the preference. Figure 7.12(b) depicts a set of flatter indifference curves where $\tilde{c}_3 \succ \tilde{c}_4$. This example shows that increasing the riskiness of the reference point will encourage risk-taking behaviour. If the hypothesis of switching reference point is correct, 15% of the subjects who preferred $\tilde{c}_2$ in Problem 11 are not influenced by the different context in Problem 12. These subjects may avoid the response mode effect if they consider both the endowment and the outcomes of the prospects at the same time. To them, prospect $\tilde{c}_1$ is identical to $\tilde{c}_3$ and $\tilde{c}_2$ is identical to $\tilde{c}_4$.

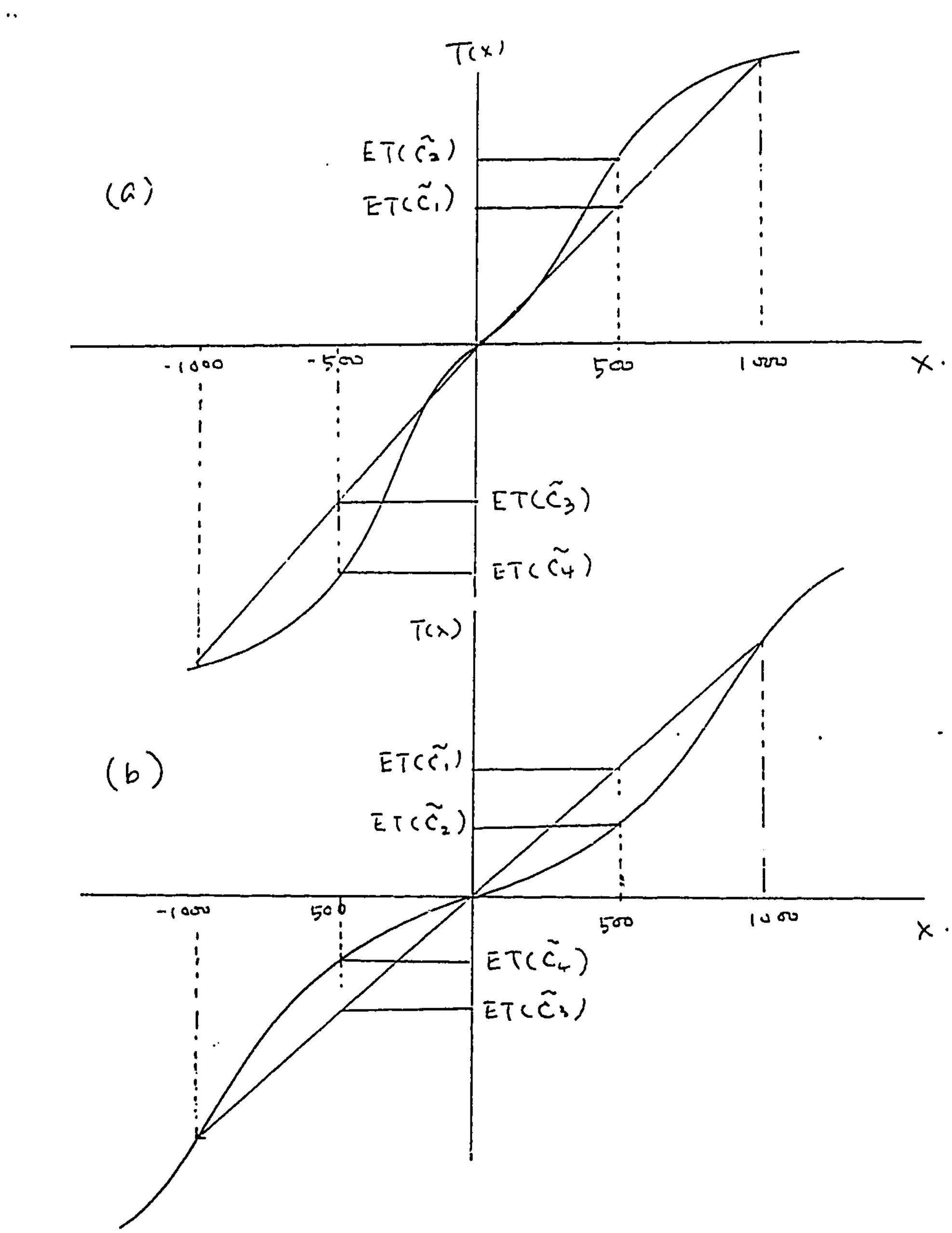


FIGURE 7.9 SELECTING REFERENCE POINTS IN THE RESPONSE MODE EFFECT

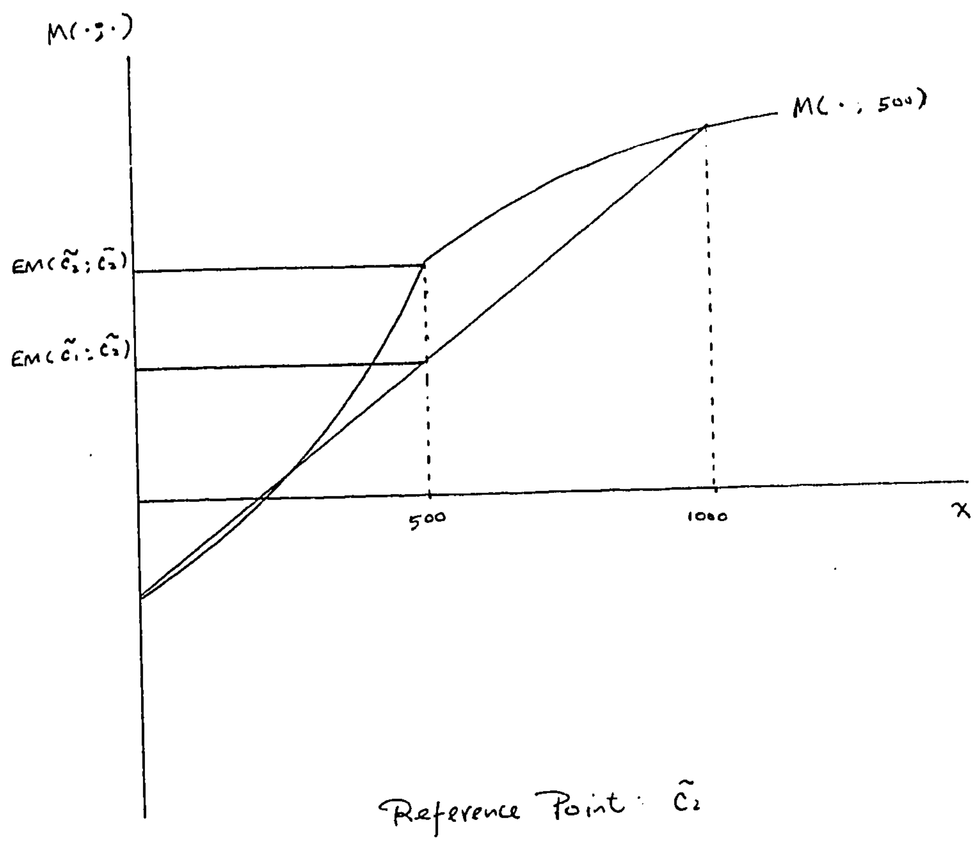


FIGURE 7.10 REFERENCE POINT THEORY AND THE RESPONSE MODE EFFECT RISK AVERSION AND RISKLESS REFERENCE POINT

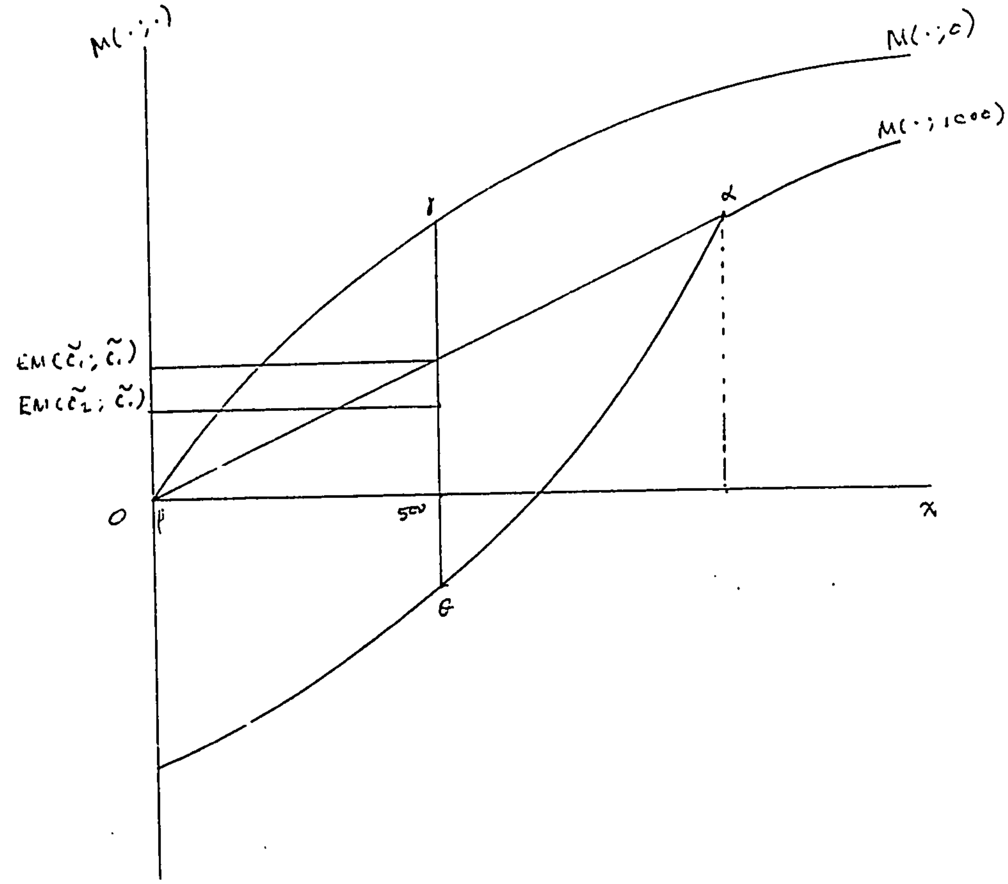


FIGURE 7.11 REFERENCE POINT THEORY AND THE RESPONSE MODE EFFECT

RISKY REFERENCE POINT AND RISK-TAKING BEHAVIOUR

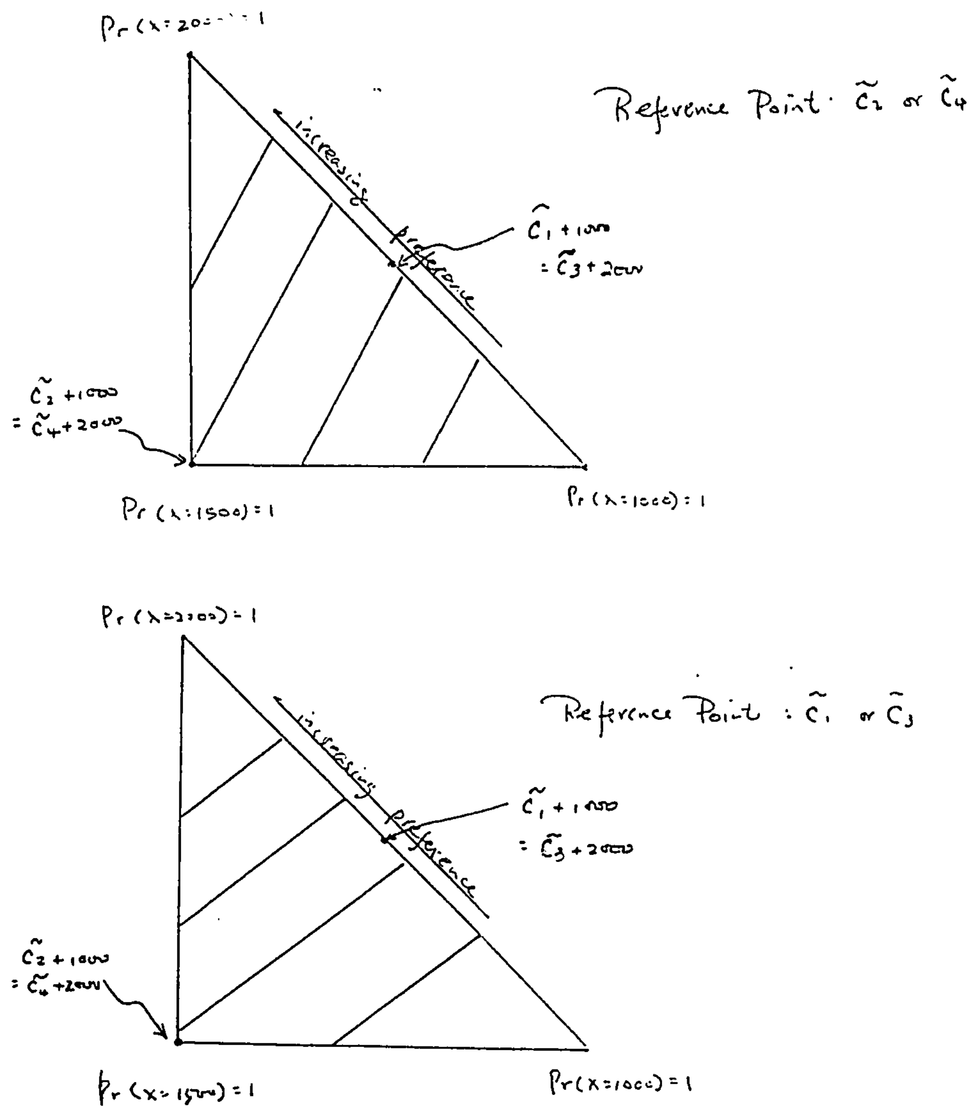


FIGURE 7.12 INDIFFERENCE CURVES AND THE RESPONSE MODE BIAS

7.4 THE ISOLATION EFFECT

The isolation effect is reported in Kahneman and Tversky (1979). They discovered that the preference orderings of two seemingly identical gambles (in terms of odds of winning and the prize) are different when the structure of the gambles changes (comparing Problem 4 in p.266 and Problem 10 in p.271). They concluded that "a pair of prospects can be decomposed into common and distinctive components in more than one way, and different decompositions sometimes lead to different preferences (Kahneman and Tversky, 1979, p.271)." The two problems are reproduced as $\tilde{a}_3$ and $\tilde{a}_4$ (their Problem 4) and $\tilde{a}'_3$ and $\tilde{a}'_4$ (their Problem 10) below:

PROBLEM 4: (N = 95)

Choose between

$\tilde{a}_3$: 0.20 chance of $4,000; [65%]

$\tilde{a}_4$: 0.25 chance of $3,000. [35%]

PROBLEM 10: (N = 141)

Choose between

$\tilde{a}'_3$: 0.25 chance to win $\tilde{a}_1$ (0.80 chance of $4,000); [22%]

$\tilde{a}'_4$: 0.25 chance to win $\tilde{a}_2$ (1.00 chance of $3,000). [78%]

Note that $\tilde{a}_3$ and $\tilde{a}'_3$ have the same odds to win $4,000 while $\tilde{a}_4$ and $\tilde{a}'_4$ have the same odds to win $3,000. Figure 7.3 shows that EM($\tilde{a}_3$; $\tilde{a}_4$) is greater than EM($\tilde{a}_4$; $\tilde{a}_4$) due to editing. Editing simplifies the evaluation process by contrasting the \$3,000 payoff in $\tilde{a}_4$ against the \$4,000 payoff in $\tilde{a}_3$. It will ignore the regret (when the payoffs in $\tilde{a}_3$ and $\tilde{a}_4$ are \$0 and \$3,000, respectively) and the rejoice (when the payoffs in $\tilde{a}_3$ and $\tilde{a}_4$ are \$4,000 and \$0, respectively) of choosing $\tilde{a}_3$ in light of $\tilde{a}_4$.

The first stage of the two-stage gamble in Problem 10 involves a riskless prospect $\tilde{a}_2$. In the evaluation process the decision makers do not have to deal with the complicated probability dimension of the prospects given that one of the prospects is risk free. According to Hypothesis 3, individuals do not need editing to simplify the evaluation process. Although the corresponding prospects in Problems 4 and 10 contain identical probability distributions, the different evaluation procedures (Problem 4 with editing and Problem 10 without editing) generate different preferences and the resulting isolation effect.

The choice between the two-stage gambles is shown in Figure 7.13. The first stage of the gambles in $\tilde{a}'_3$ and $\tilde{a}'_4$ are $\tilde{a}_1$ (0.80 chance of \$4,000) and $\tilde{a}_2$ (1.00 chance of \$3,000), respectively. $\tilde{a}_1$ is denoted as α and $\tilde{a}_2$ is denoted as β in the diagram. For the majority group selecting $\tilde{a}'_4$ as the reference point, the EM indices of the two-stage gambles are $EM(\tilde{a}'_3;\ \tilde{a}'_4) = 0.25EM(\alpha;\ \beta) + 0.75EM(0;\ 0) = 0.25EM(\tilde{a}_1;\ \tilde{a}_2)$ and $EM(\tilde{a}'_4;\ \tilde{a}'_4) = 0.25EM(\beta;\ \beta) + 0.75EM(0;\ 0) = 0.25EM(\tilde{a}_2;\ \tilde{a}_2)$. The diagram shows that $EM(\tilde{a}'_3;\ \tilde{a}'_4)$ is lower than $EM(\tilde{a}'_4;\ \tilde{a}'_4)$ and therefore $\tilde{a}'_4$ is preferred to $\tilde{a}'_3$. This ranking follows directly from the ranking of $\tilde{a}_1$ and $\tilde{a}_2$ in Problem 3; the prospects in the second stage of the two-stage gambles. The evaluation procedures in Problem 3 and Problem

10 assume no editing. The context of Problem 10 separates the probability dimension and the outcome dimension in the reference point $\tilde{a}'_4$. The first stages of the gamble draws the subjects' attention to focus on the outcome dimension; the evaluation process is simplified as the comparison of $\tilde{a}_1$ and $\tilde{a}_2$ (instead of comparing the more complicated $\tilde{a}_3$ and $\tilde{a}_4$ as in Problem 4). The second stage of the gamble adjusts the probability dimension to account for the probability to win $\tilde{a}_1$ or $\tilde{a}_2$. The choice in Problem 10 is made simple by the context design. The isolation effect illustrates the inconsistency in description invariance when the context of the prospects will influence the decision-making process.

The indifference curve diagrams are constructed in Figure 7.14. Since $\tilde{a}_3$ and $\tilde{a}'_3$ have the same odds to win \$4,000, they take the same position on the hypotenuse of the probability triangle where Pr(\$4000) = 0.2 and Pr(0) = 0.8. Similarly, $\tilde{a}_4$ and $\tilde{a}'_4$ occupy the identical position on the bottom of the probability triangle where Pr(\$3,000) = 0.25 and Pr(0) = 0.75. Prospects $\tilde{a}_1$ and $\tilde{a}_2$ of Problem 3 are also shown in the diagrams.

Changing the structures of the prospects will alter the decision-making process; as a result, the indifference curves differ in the two diagrams. Note that the "fanning-out" indifference curves are not able to accommodate the isolation effect because $\tilde{a}'_3$ and $\tilde{a}'_4$ have the same probability distributions as $\tilde{a}_3$ and $\tilde{a}_4$, respectively (Machina, 1982a, p. 308).

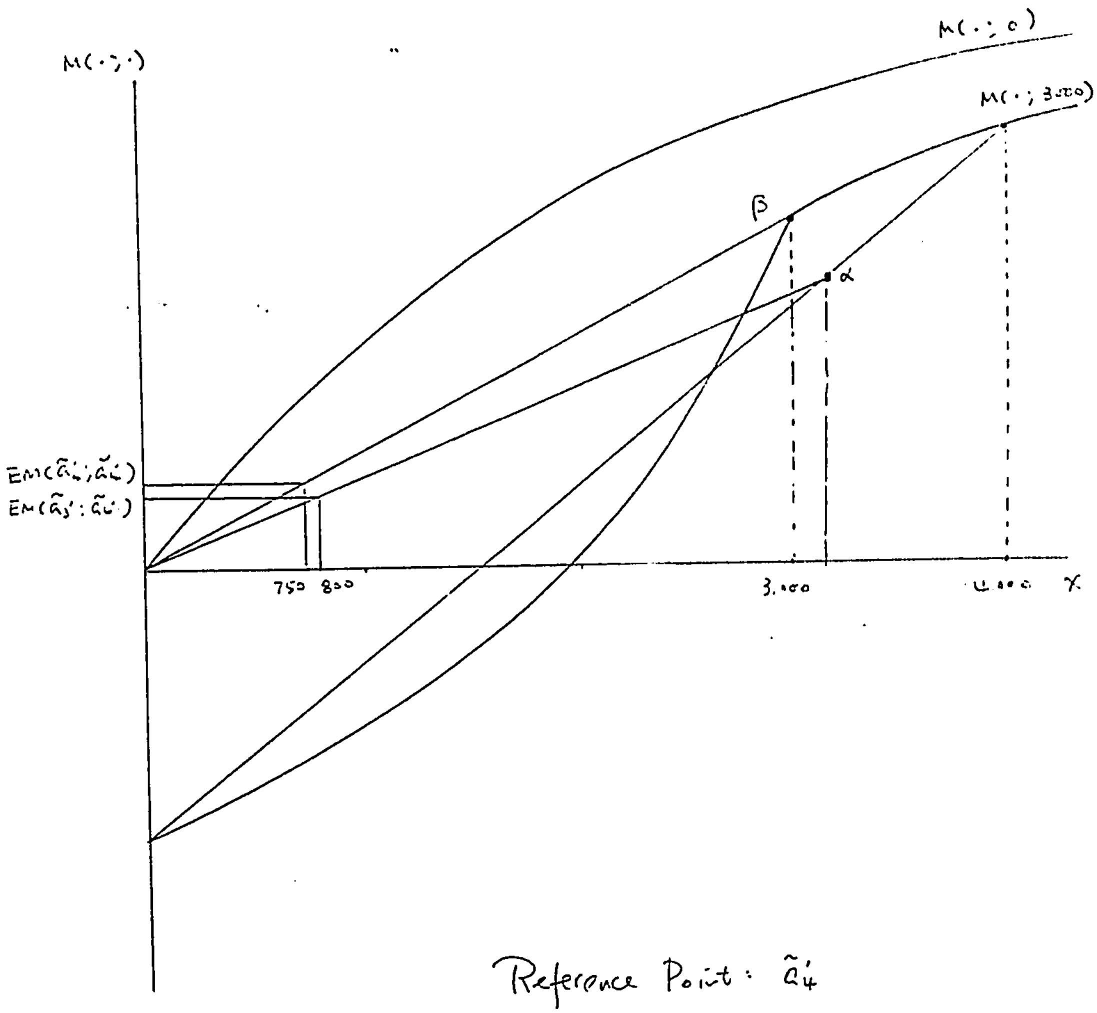


FIGURE 7.13 REFERENCE POINT THEORY AND TWO-STAGE GAMBLES

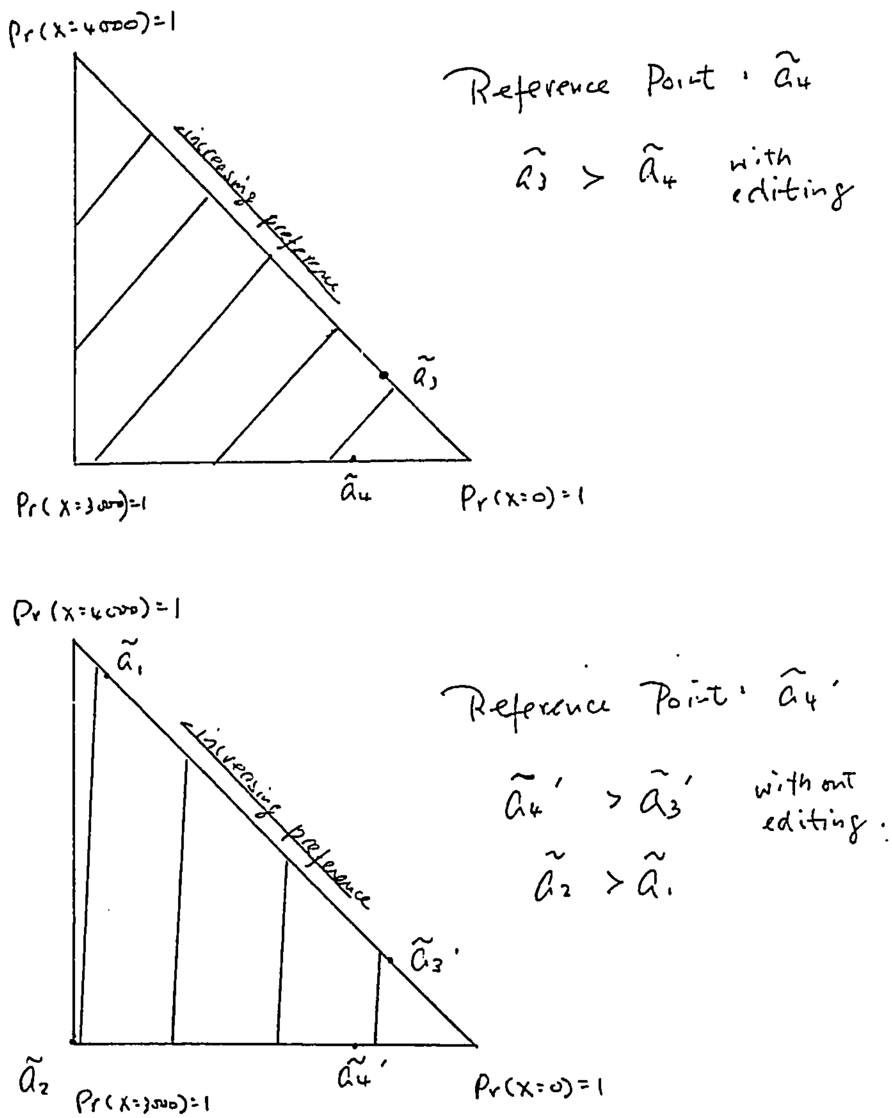


FIGURE 7.14 REFERENCE POINT THEORY AND THE ISOLATION EFFECT

7.5 THE REFLECTION EFFECT

Kahneman and Tversky (1979) observed that the preference pattern was reversed if the prospects involved losses instead of gains. The preferences over positive prospects exhibited risk aversion while the preferences over negative prospects exhibited a risk-seeking attitude. Consider $\tilde{d}_1$, $\tilde{d}_2$, $\tilde{d}_3$, and $\tilde{d}_4$ which are the negative counterparts of $\tilde{a}_1$, $\tilde{a}_2$, $\tilde{a}_3$, and $\tilde{a}_4$, respectively.

PROBLEM 3' (N = 95)

Choose between

$\tilde{d}_1$: 0.80 chance of -\$4,000;	[92%]
$\tilde{d}_2$: 1.00 chance of -\$3,000.	[8%]

PROBLEM 4' (N = 95)

Choose between

$\tilde{d}_3$: 0.20 chance of -\$4,000;	[42%]
$\tilde{d}_4$: 0.25 chance of -\$3,000.	[58%]

The majority of subjects reported that $\tilde{d}_1 \succ \tilde{d}_2$ and $\tilde{d}_4 \succ \tilde{d}_3$ although they also reported that $\tilde{a}_2 \succ \tilde{a}_1$ and $\tilde{a}_3 \succ \tilde{a}_4$ (see Problems 3 and 4 in Section 7.1); this anomaly is labeled as the reflection effect in Kahneman and Tversky (1979, p.268) because "the reflection of the prospects around 0 reverses the preference order." Moreover, the common ratio effect also extended to the negative prospects when $\tilde{d}_1 \succ \tilde{d}_2$ and $\tilde{d}_4 \succ \tilde{d}_3$. According to Hypothesis 3, no editing is needed in the evaluation of $\tilde{d}_1$ and $\tilde{d}_2$ since $\tilde{d}_2$ is a riskless prospect. When two risky prospects $\tilde{d}_3$ and $\tilde{d}_4$ are involved, editing will simplify the decision-making process which will lead to the common ratio effect.

Combining $\tilde{a}_2 \succ \tilde{a}_1$ and $\tilde{d}_1 \succ \tilde{d}_2$ create a new choice paradox; individuals exhibit different attitudes towards risk regarding gains and losses. As discussed in the response mode effect (Section 7.3) above, individuals with a symmetrical transformation function will reverse their choice of reference point between positive and negative prospects. The majority of the subjects who selected $\tilde{a}_2$ as the reference point would pick $\tilde{d}_1$ in Problem 3'. If one assumes that the utility (or disutility) function in the negative domain is symmetrical to the utility function in the positive domain, a strictly concave utility function will lead to a strictly convex disutility function. It implies that a normally risk-averse individual becomes risk-seeking in the negative domain. A strictly convex disutility curve carries a similar intuition as a strictly concave utility curve; the disutility of losing \$200 is less than two times the disutility of losing \$100. Fishburn and Kochenberger (1979) documented the prevalence of risk seeking behaviour involving negative prospects.

With a risky reference point $\tilde{d}_1$ and a symmetrical utility / disutility function, the majority of the subjects would prefer $\tilde{d}_1$ over $\tilde{d}_2$. The majority group ranking indices in Problem 3 are given in Equation (7.1), it is repeated below

$$0.8M(4,000;\ 3,000) + 0.2M(0;\ 3,000) < M(3,000;\ 3,000). \qquad (7.1)$$

Expanding $M(\cdot;\ \cdot)$ into $U(\cdot)$ and $R[\cdot]$, (7.1) becomes

$$0.8\{U(4,000) + R[U(4,000)-U(3,000)]\} + 0.2R[-U(3,000)] < U(3,000). \tag{7.5}$$

If $U(\cdot)$ and $R[\cdot]$ are symmetrical

$$U(-3,000) < 0.8\{U(-4,000) + R[U(-4,000)-U(-3,000)]\} + 0.2R[-U(-3,000)]. \tag{7.6}$$

Rearranging the terms in (7.6), we have

$$0.8\{U(-3,000) - R[U(-4,000)-U(-3,000)]\} + 0.2\{U(-3,000) - R[-U(-3,000)]\} < 0.8U(-4,000). \tag{7.7}$$

Symmetry in $R[\cdot]$ implies that

$$0.8\{U(-3,000) + R[U(-3,000)-U(-4,000)]\} + 0.2\{U(-3,000) + R[U(-3,000)]\} < 0.8U(-4,000). \tag{7.8}$$

Given that the majority group would select $\tilde{d}_1$ as the reference point

$$EM(\tilde{d}_2;\ \tilde{d}_1) = 0.8\{U(-3,000) + R[U(-3,000)-U(-4,000)]\} + 0.2\{U(-3,000) + R[U(-3,000)]\} \text{ and}$$

$$EM(\tilde{d}_1;\ \tilde{d}_1) = 0.8U(-4,000) + 0.2U(0) = 0.8U(-4,000).$$

Thus, by (7.8) $EM(\tilde{d}_2;\ \tilde{d}_1) < EM(\tilde{d}_1;\ \tilde{d}_1)$ although $EM(\tilde{a}_2;\ \tilde{a}_2) > EM(\tilde{a}_1;\tilde{a}_2)$. The majority group of subjects would exhibit the reflection effect if they indeed reverse their reference point between Problem 3 and Problem 3′. Note that in the evaluation of $\tilde{a}_1$, $\tilde{a}_2$, $\tilde{d}_1$, and $\tilde{d}_2$, editing is not required because one of the prospects is riskless. The majority in Problem 3′ increased to 92% of the subjects who preferred $\tilde{d}_1$; the majority in Problem 3 consisted of 80% of 95 subjects. The pattern in switching reference point between the positive and the negative prospects cannot explain the behaviour of the 12% difference between Problem 3 and Problem 3′.

Assuming that editing is involved in making a choice between $\tilde{d}_3$ and $\tilde{d}_4$, $EM(\tilde{d}_4;\ \tilde{d}_3) > EM(\tilde{d}_3;\ \tilde{d}_3)$ if and only if

$$0.25\{U(-3,000) + R[U(-3,000)-U(-4,000)]\} > 0.20U(-4,000). \tag{7.9}$$

$EM(\tilde{d}_3;\ \tilde{d}_3)$ in Equation (7.9) is 1/4 of $EM(\tilde{d}_1;\ \tilde{d}_1)$ in Equation (7.8). The utility component in $EM(\tilde{d}_4;\ \tilde{d}_3)$ is also reduced to 1/4 of the same component in $EM(\tilde{d}_2;\ \tilde{d}_1)$. However, the rejoice-regret component in $EM(\tilde{d}_4;\ \tilde{d}_3)$ is $0.25R[U(-3,000)-U(-4,000)]$ which is larger than 1/4 of $\{0.80R[U(-3,000)-U(-4,000)] + 0.20R[U(-3,000)]\}$ in $EM(\tilde{d}_2;\ \tilde{d}_1)$. The regret (or negative rejoice) $R[U(-3,000)]$ in $EM(\tilde{d}_2;\ \tilde{d}_1)$ becomes rejoice (or negative regret) $R[U(-3,000)-U(-4,000)]$ in $EM(\tilde{d}_4;\ \tilde{d}_3)$. This editing bias will reverse the "less than" inequality in Equation (7.8) into the "greater than" inequality in Equation (7.9). Similar to the choice paradox involving positive prospects, the common ratio effect involving negative prospects where $\tilde{d}_1 \succ \tilde{d}_2$ and $\tilde{d}_4 \succ \tilde{d}_3$ is caused by editing.

The pattern of switching reference points between the positive and the negative prospects will explain the reflection effect in Problems 3′ and 4′. In the discussion of the common ratio effect (Section 7.1), the majority group selected $\tilde{a}_2$ and $\tilde{a}_4$ as the reference points in Problem 3 and Problem 4, respectively. Given that $T(\cdot)$ is symmetric, they would respectively select $\tilde{d}_1$ and $\tilde{d}_3$ as the reference point in Problem 3′ and Problem 4′. In Problem 4 of the common ratio effect, the majority exhibits $EM(\tilde{a}_3;\ \tilde{a}_4) > EM(\tilde{a}_4;\ \tilde{a}_4)$. Equation (7.2) expands the EM indices as follows:

$$0.20M(4,000;\ 3,000) + 0.80M(0;\ 0) > 0.25M(3,000;\ 3,000) + 0.75M(0;\ 0). \tag{7.2}$$

$$0.20\{U(4,000) + R[U(4,000)-U(3,000)]\} > 0.25U(3,000). \tag{7.10}$$

$$0.25U(-3,000) > 0.20\{U(-4,000) + R[U(-4,000)-U(-3,000)]\}. \tag{7.11}$$

$$0.25\{U(-3,000) + R[U(-3,000)-U(-4,000)]\} - 0.05R[U(-3,000)-U(-4,000)] > 0.20U(-4,000). \quad (7.12)$$

$$EM(\tilde{d}_4;\ \tilde{d}_3) - 0.05R[U(-3,000)-U(-4,000)] > EM(\tilde{d}_3;\ \tilde{d}_3). \quad (7.13)$$

The editing process in Problem 4 takes away part of the regret in choosing $\tilde{a}_3$ in light of $\tilde{a}_4$. As discussed above in Equation (7.9), the editing process applied to the negative prospects reverses the regret (negative rejoice) into rejoice (negative regret). Due to the different editing processes, one cannot derive $EM(\tilde{d}_4;\ \tilde{d}_3) > EM(\tilde{d}_3;\ \tilde{d}_3)$ from $EM(\tilde{a}_3;\ \tilde{a}_4) > EM(\tilde{a}_4;\ \tilde{a}_4)$. The extra term in Equation (7.13) reflects the differences in the two editing processes. The majority of the subjects select the less risky prospect $\tilde{a}_4$ in Problem 4 but switch to the more risky prospect $\tilde{d}_3$ in Problem 4'. The switch in reference point will account for the reflection effect when the same editing process is used in Problem 4 and Problem 4'.

For 65% of the subjects in Problem 4, $EM(\tilde{a}_3;\ \tilde{a}_4) > EM(\tilde{a}_4;\ \tilde{a}_4)$. Equation (7.13) should be satisfied among these subjects according to RPT. To these subjects, $EM(\tilde{d}_4;\ \tilde{d}_3) > EM(\tilde{d}_3;\ \tilde{d}_3)$ because $0.05R[U(-3,000)-U(-4,000)] > 0$. Yet, only 58% of the subjects indicated their preference for $\tilde{d}_4$. The pattern of switching reference points between the positive and the negative prospects cannot explain the behaviour of the 7% margin between Problem 4 and Problem 4'.

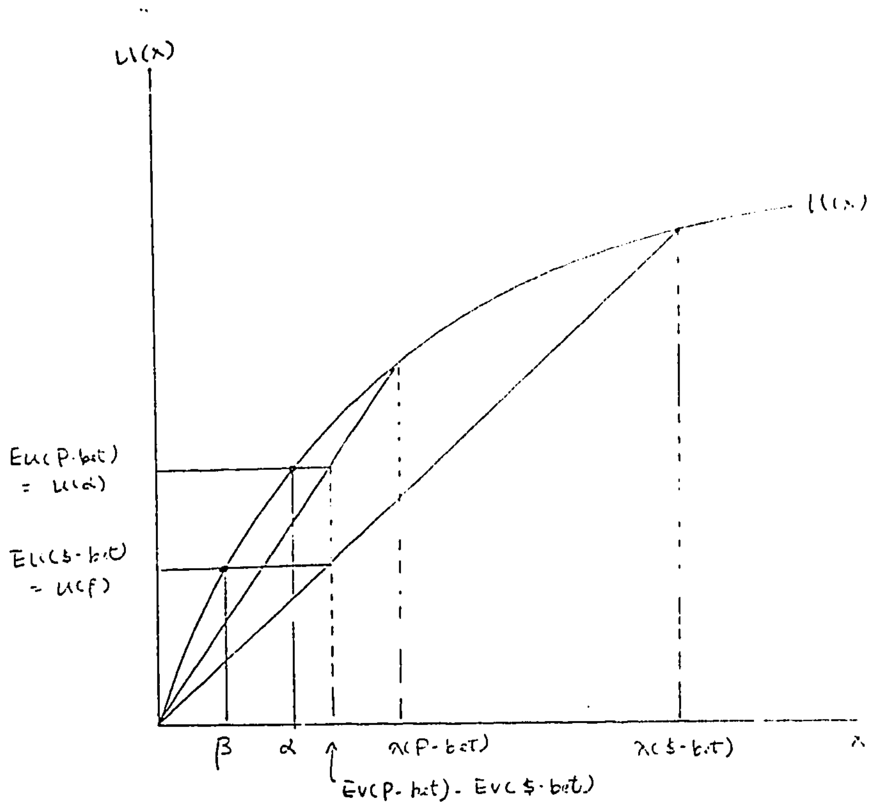
7.6 THE PREFERENCE REVERSAL PHENOMENON

The preference reversal (PR) phenomenon occurs when an individual chooses the P-bet (high probability to win a small amount P) but assigns a higher selling price (certainty equivalent) to the $-bet (small probability to win a large amount $). The experiment for PR implicitly requires the subjects to perform two different tasks. When facing the choice problem, the subjects select either the P-bet or the $-bet to improve utility. On the other hand, the elicitation process induces the subjects to quote the compensations for giving up the P-bet and the $-bet. If different reference points are associated with the choice problem and the elicitation problem, the PR phenomenon may be caused by the reversal in reference points.

According to the EU model, the two procedures of determining preference and eliciting certainty equivalents are fundamentally identical. Procedure invariance follows from the fact that an identical utility function is utilized in both procedures.

In the choice problem, the decision maker will compare the EU indices of the P-bet and the $-bet. In Figure 7.15, the initial wealth of the decision maker is assumed to be 0 and EV(P-bet) is assumed to be equal to EV($-bet) for simplicity. For a strictly concave utility function, EU(P-bet) is greater than EU($-bet) since P-bet contains less risk than $-bet. (According to the definition of increasing risk, $-bet is a mean-preserving spread of P-bet.)

In the elicitation problem, P-bet has the same utility as the cash equivalent α; alternatively, EU(P-bet) is equal to U(α). Similarly,



$EU(\text{P-bet}) > EU(\text{\$-bet})$

$\Rightarrow CE(\text{P-bet}) > CE(\text{\$-bet})$

since $EV(\text{P-bet}) = EV(\text{\$-bet})$.

FIGURE 7.15 PROCEDURE INVARIANCE IN THE EXPECTED UTILITY MODEL AND THE PREFERENCE REVERSAL PHENOMENON

EU(\$-bet) is equal to $U(\beta)$ where β is the cash equivalent to \$-bet. Both α and β can be determined once a utility function is given. Procedure invariance follows from the fact that $EU(P\text{-bet}) > EU(\$\text{-bet}) \Rightarrow U(\alpha) > U(\beta) \Rightarrow \alpha > \beta$; the choice problem and the elicitation problem always lead to identical decisions.

Tversky, Slovic, and Kahneman (1990), however, discovered that the assumption of procedure invariance is violated in 90% of the PR cases in their experiments. Hence, the EU model, which does not differentiate the two procedures, is not an ideal decision model for the PR problem. If the decision makers consider the choice elicitation and the reservation price elicitation as two different problems, the decision-making mechanism in RPT will avoid procedure invariance. They may consider the prospects in the choice elicitation as positive gains. On the other hand, in the price elicitation process the decision makers quote the reservation prices as if they are giving up these prospects; the context connotes a choice of losses. In this case, the prospects are evaluated in the negative domain of the transformation function $T(\cdot)$ as if they are negative prospects.

Figure 7.16 assumes that the decision maker has a relatively low value for k and he selected the P-bet as the reference point in the choice problem. With a symmetrical $T(\cdot)$, the same individual would select the \$-bet in the elicitation problem.

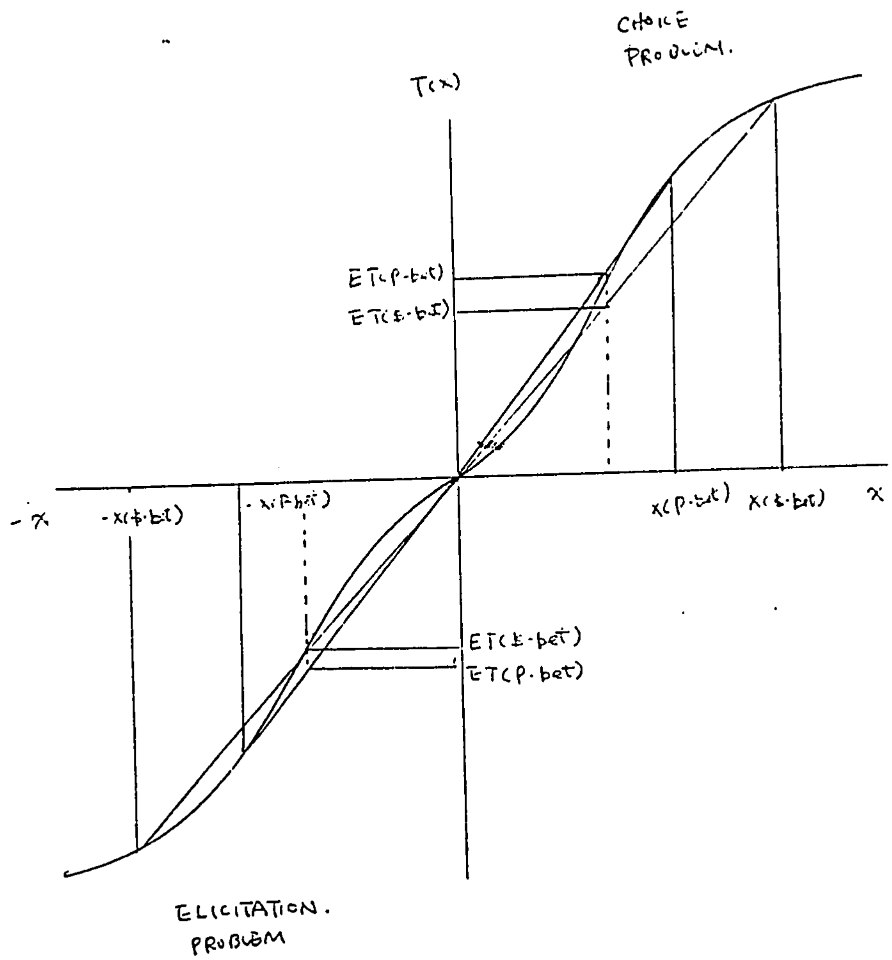


FIGURE 7.16 SELECTING THE REFERENCE POINTS FOR THE CHOICE PROBLEM AND THE ELICITATION PROBLEM

When P-bet is selected as the reference point in the choice problem, the decision maker will assign a higher EM index to the P-bet (see Figure 7.17); such preference exhibits the status quo bias. The ranking also reflects the asymmetry in evaluating gains and losses. Without editing, the EM index for $-bet contains 4 components associated with 4 possible states: winning in both prospects (S1), winning in the P-bet but not in the $-bet (S2), winning in the $-bet but not in the P-bet (S3), and both prospects equal $0 (S4). The weighted modified utilities for the first two states are obtained from $M(\cdot; x(\text{P-bet}))$ (denoted α in Figure 7.17) and the weighted modified utilities for the last two states are obtained from $M(\cdot; \$0)$ (denoted β in Figure 7.17). The regret generated from S2 will dominate the rejoice in S1 and S3.

In the price elicitation problem, the decision makers with small k will switch the reference point to the $-bet; consequently, they will assign a higher EM index to the $-bet (see Figure 7.18). The EM index for P-bet contains 4 components associated with 4 possible states: winning in both prospects (S1), winning in the $-bet but not in the P-bet (S2), winning in the P-bet but not in the $-bet (S3), and both prospects equal $0 (S4). The weighted modified utilities for the first two states are obtained from $M(\cdot; x(\$\text{-bet}))$ (denoted γ in Figure 7.18) and the weighted modified utilities for the last two states are obtained from $M(\cdot; \$0)$ (denoted θ in Figure 7.18). The regret generated from S1 and S2 will dominate the rejoice in S3.

Figure 7.19 depicts the indifference curves assoicated with the $-bet as the reference point. The $-bet is preferred to the P-bet following the EM indices derived in Figure 7.18. Hence, the conditional certainty equivalent or (the price quotation) for $-bet should be larger than the conditional certainty equivalent for the P-bet. The preference reversal phenomenon is caused by the reversal in reference points between the choice problem and the price elicitation problem.

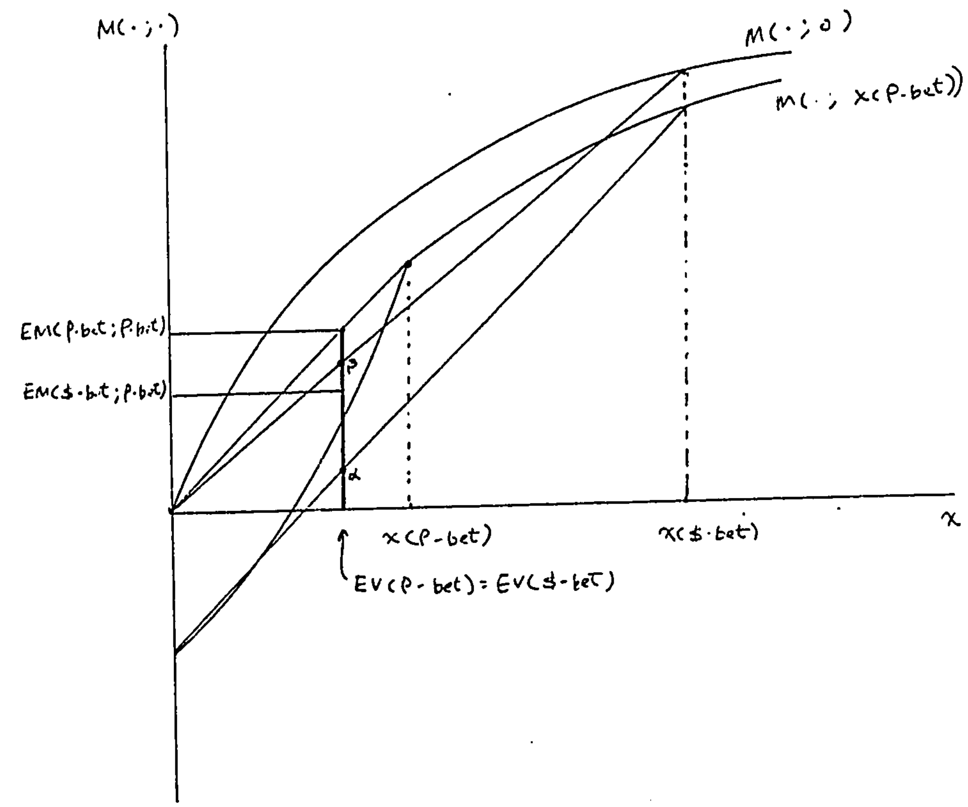


FIGURE 7.17 THE CHOICE PROBLEM IN THE PREFERENCE REVERSAL PHENOMENON WITH THE P-BET AS THE REFERENCE POINT

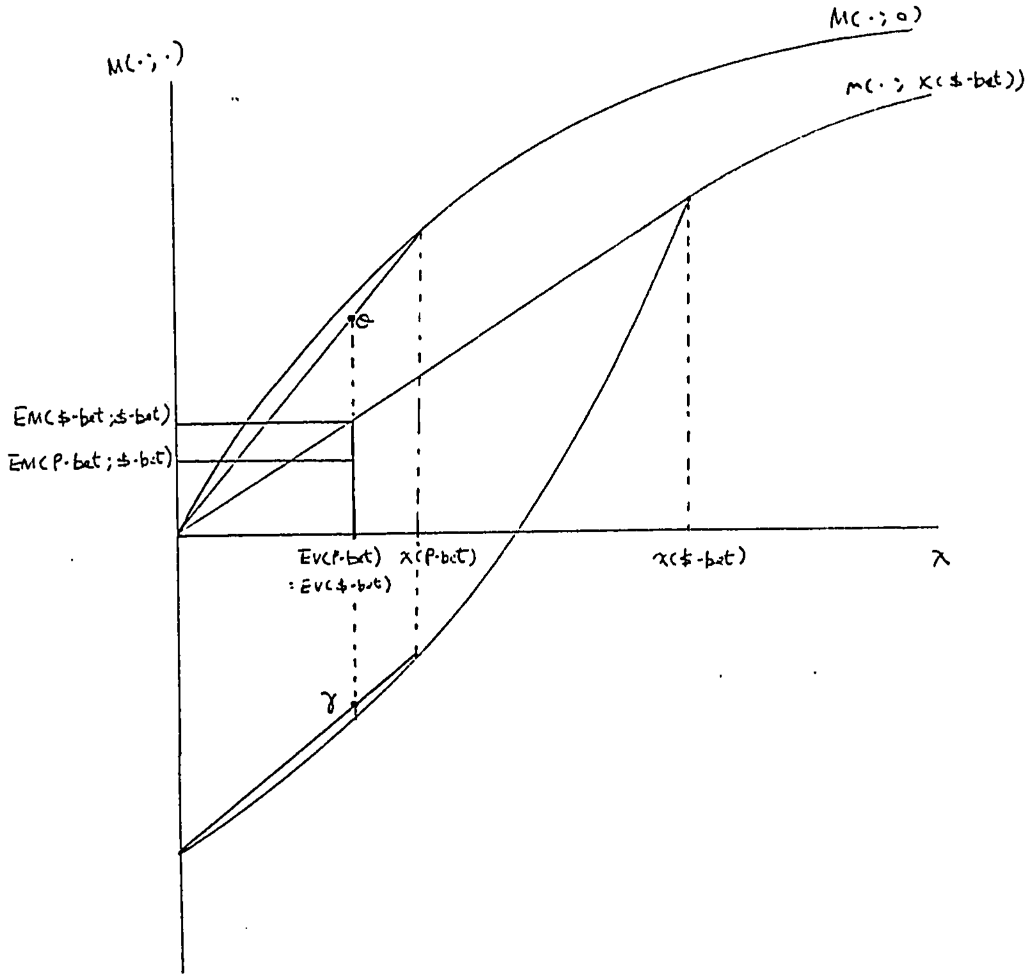


FIGURE 7.18 THE ELICITATION PROBLEM IN THE PREFERENCE REVERSAL PHENOMENON WITH THE $-BET AS THE REFERENCE POINT

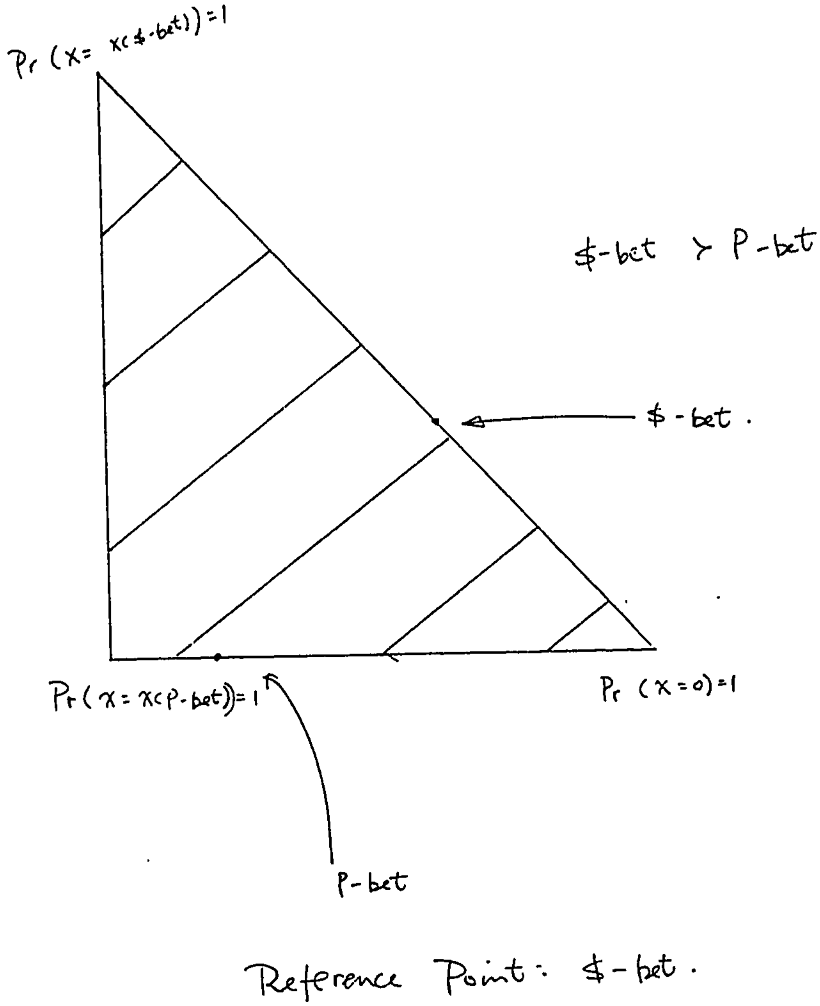


FIGURE 7.19 THE CONDITIONAL CERTAINTY EQUIVALENTS USING THE $-BET AS THE REFERENCE POINT

7.7 SUMMARY

According to RPT, the six categories of anomalies discussed above are caused by two factors. When the context of the problem causes a switch in the reference point, it will account for the response mode effect, the reflection effect, and the preference reversal phenomenon. Reference point theory postulates a scheme where the context of the decision is captured by choosing the appropriate reference point.

The above analyses show that the decision makers do not switch their reference points in the common ratio effect, the common consequence effect, and the isolation effect. Therefore, one cannot assume that these anomalies are caused by switching reference point. These anomalies, howver, can be explained by the editing bias. Using reference points as heuristics, decision makers usually ignore some information contained in the prospects. The heuristics will reduce the information processing required and simplify the decision-making process, but the decision is "biased".

Reference point theory conjectures that the context of the decision problem will influence the selection of the reference point. Besides considering the riskiness of the prospects (through probabilities) and the risk attitude of the decision makers (through a non-linear utility function), the new theory recognizes that the context of the problem is an important dimension in the decision-making process. The analysis in this chapter suggests that the behaviour which contradicts the EU prediction is explicable when the context of the decision is recognized properly.

CHAPTER 8 CONCLUDING REMARKS

Most of the holistic judgment models presented in Table 3.1 are aimed at re-defining the probability term in the EU model. These models, using different decision weight functions, transform the objective probabilities into subjective perceptions regarding the likelihood of the outcomes in the prospects. Through the decision weight functions, these models generate preferences which are non-linear in probabilities and avoid the independence axiom in the EU model. This is especially clear when the transformation involves the probabilities of different states like the anticipated utility theory (Quiggin, 1982) and the class of rank dependent probabilities theories. On the contrary, Reference Point theory leaves the probability term unaltered and concentrates on the value function.

However, four models in this category (ordinal independence model, cumulative utility theory, Generalized Expected Utility model, and mixture symmetric utility theory) modify the EU model beyond just a probability transformation. They utilize the entire probability distribution of a prospect to determine the value indices of the outcomes in each state. The evaluation of each outcome in the prospect is dependent on the entire probability distribution and consequently independence is eliminated. In the cases of the ordinal independence model and cumulative utility theory, the weighing function is also modified.

Among the holistic judgment models, the Generalized Expected Utility theory (GEU) developed by Machina (1982a) and RPT adopt a similar strategy to model the decision-making process. Both of them include extra information to modify the VNM utility function in the EU model. The local utility function in the GEU is conditioned on the probability distribution; it is denoted by U(x; F). The local utility function takes the form

$$\Theta(x_j) + \left[\sum_i p_i \Xi(x_i)\right]^2 .$$

The modified utility function in RPT, on the other hand, is conditioned on the reference point; it is denoted by $M(x_{ij};\ x_{oj})$. The modified utility takes the form

$$U(x_{ij}) + R[U(x_{ij}) - U(x_{oj})].$$

As a result, both theories share a similar structure in modeling the decision-making process. The models differ in terms of the information used in the value functions. In GEU, the outcomes are ranked in an ascending order and each outcome is compared to the outcomes in the same prospect (represented by F in the local utility function). The ranking of the outcomes makes GEU a member of the EURDP category. Furthermore, the local utility function of each prospect utilizes different information sets as F is unique to each prospect. On the other hand, RPT seeks extra information since the reference point may not be identical to the prospect under consideration.

The utility curves derived from the two theories also share some similarities. Hypothesis I in GEU (Machina, 1982a, p.301, Figure

4.1(a)) resembles the case when R[·] is assumed to be strictly concave. These assumptions regarding the functional forms of the local utility function and rejoice-regret function generate strictly concave utility curves. Hypothesis II in GEU (Machina, 1982a, p.301, Figure 4.1(b)) is similar to a concave-convex R[·] and the resulting utility curves have an inverted S-shape. As argued in Section 5.2, RPT adopts the convex-concave R[·] which yields an S-shaped modified utility curve (Figure 5.4).

Based on Hypothesis II, Machina generates the "fanning-out" indifference curves to represent the preferences among risky prospects (1982a, [illegible]orem 5 and Figure 5(b)). As shown in Figure 6 in Machina's [illegible] (1982a, p.307), every prospect with a distinct probab[illegible]ution is associated with a local utility function; a change in the probability distribution will induce the local utility function to shift to a new position. This analysis is similar to Figure 6.2 which shows the effect of changing the reference point in the modified utility curve. Indifference curves in RPT remain linear and parallel to each other; changing the reference point will change the slope of all the indifference curves.

The non-holistic judgment models in Table 3.3 share a common characteristic in modeling decision-making; the value functions of these models emphasize the role of comparison in reaching decisions. The satisficing principle (Simon, 1964) and the lexicographic model (Fishburn, 1988) reach the optimal choice by elimination. All other models in Table 3.3 have value functions containing two elements. Utility (or positive value index) is derived by out-performing the other element in the value function. Although the value function in the Prospect theory has only one element for a possible outcome in the prospect, it represents the change of wealth vis-à-vis a reference point (Kahneman and Tversky, 1979, p.279).

Some elements of the RPT can be identified with different theories in Table 3.3. Its basic structure of amending the VNM utility function with a rejoice-regret function is similar to Regret theory (Loomes and Sugden, 1982) and Disappointment theory (Loomes and Sugden, 1986). Note that the rejoice-regret function in Regret theory is similar to the design of the additive difference model (Tversky, 1969). The choice of a reference point differentiates RPT from Regret theory. Using the expected value of a prospect as the bench-mark for comparison, Disappointment theory yields transitive preference orderings. If the expected value of a prospect does not change, the weighted value index assigned to each prospect will remain unchanged. According to RPT, the value index for each prospect is dependent on the choice of the reference point. In addition, both Regret theory and Disappointment theory assume a concave-convex functional form for the rejoice-regret or elation-disappointment function. Based on experimental findings, RPT, on the other hand, uses a convex-concave function for rejoice and regret.

The value function $V(\cdot)$ in Prospect theory (Kahneman and Tversky, 1979) recognizes the importance of the reference point in the decision- making process. Tversky and Kahneman (1991) elaborate the properties of this value function regarding the reference point heuristics in a reference-dependent model which deals with riskless choice. Some of the properties of this value function (see Figure 5.2) are used in the determination of the functional form for R[·] in RPT (Conditions i to viii in (5.1)). The value function of Prospect theory measures the "changes in wealth or welfare, rather than final states (Kahneman and Tversky, 1979, p.277);" therefore, it carries similar meaning as the rejoice-regret function of RPT. The presence of the VNM utility term in RPT generates the fundamental difference between the two theories. Including this term, RPT recognizes that the expected utility index also plays an important role in determining the value of a prospect. Prospect theory, on the other hand, only concentrates on the differences between the prospect and the reference point. In spite of their similarity, the two models will yield different preference orderings as long as V(·), U(·), and R[·] are nonlinear functions.[1]

The study of various anomalies suggests that the decision-making process is influenced by the choice of the reference point. This fact is derived from the axiomatic properties of the modified utility function; the preference ordering in RPT is reference-specific. The new theory introduces a third dimension, besides expected monetary value and the attitude of the decision maker towards risk, into the analysis of the decision-making process, namely, the reference point. This new element is closely tied to the psychological aspect of decision- making. The variation in the context of the same lotteries (for example, the response mode effect, the isolation effect, and the preference reversal phenomenon) may lead to the adoption of different points for different problems. Through the introduction of different reference points, RPT recognizes the widely accepted hypothesis that the framing of decisions affects choice. RPT hypothesizes that the selection of the reference point is influenced by the decision frame. Thus, the changes in the context of the problems will lead to different preferences associated with different reference points.

The underpinning of the economic approach to rational choice is substantive (or objective); rationality is judged by the outcome of the choice. The approach taken by cognitive psychology emphasizes procedural rationality which concerns the process of reaching a decision. According to procedural rationality, a rational choice is one that is procedurally reasonable in light of the available information and the computational capacity (Simon, 1987, p. 27). The evaluation function consequently is affected by the elicitation procedure and / or framing. In other words, procedural rationality does not assume procedure invariance; ranking between prospects may vary depending on the evaluation process taken. The substantive theory of rationality is apparent in the methodology used in economic analysis; the economic agent derives the optimal choice by maximizing his utility function. Not surprisingly, the economic approach to rationality will assume procedure invariance in decision-making because only the outcome matters in such a definition of rationality. Hence, any choice pattern that violates procedure invariance, like the preference reversal phenomenon, creates a choice paradox. The preference reversal phenomenon is consistent with procedural rationality because the experiment elicits choices by means of two different procedures.

The whole class of economic theories for choice under uncertainty, the group of holistic models, implicitly assumes that rationality is substantive. The EU model is a classic example of substantive rationality. The variations to the EU model which seek to transform the probability distribution into various forms of decision weight functions maintain the same approach. Although new information is included in decision-making (for example, a common strategy of modification is using the information of the entire probability distribution of a prospect to influence the evaluation of a particular outcome in the prospect), the ranking of prospects is independent of the process of evaluation. The significance of this modification is the disposal of the independence axiom. Consequently, these theories are capable of predicting anomalies caused by the violation of the independence axiom, like the common consequence effect and the common ratio effect Yet, these theories are unsuccessful in predicting anomalies which originate from the response mode effect.

The comparison of Reference Point theory and the Generalized Expected Utility (GEU) theory exemplifies the significance of the differences between the two approaches to rationality. The local utility function, $U(\tilde{x}; F)$, in the GEU and the modified utility function in RPT, $M(\tilde{x}; x_o)$, share a similar structure to adjust the evaluation of outcomes in the prospect. The GEU assumes procedure invariance; two prospects with identical probability distributions (on the same outcome set) yield the same ranking. The evaluation process assesses the objective information only; it completely ignores how information is organized and presented. In other words, only the outcome matters in the decision-making process. On the other hand, RPT incorporates the procedural information through the variation in reference points. Preference in RPT is therefore sensitive to this type of information as well as the objective information contained in the prospects. Hence, the indifference curves of RPT change according to the context of the problem while the indifference curves of GEU are stationary.

The independence axiom is maintained in RPT's reference-specific preference ordering. The indifference curves for RPT's preference are consequently linear in probabilities. The independence axiom limits the ability of RPT to predict the common ratio effect and the common consequence effect. RPT fails to accommodate these anomalies when there is no changes in the reference points. In order to explain this type of anomalies, RPT assumes that editing will lead to biases in the evaluation of complex problems.

The emphasis on the process of decision-making renders the decision theories which are based on the procedural theory of rationality more descriptive than predictive. Although these decision theories may successfully identify the "reason" for an anomaly and resolve the anomaly accordingly; they are often criticized on the grounds that their structures are not precise enough to generate meaningful prediction of behaviour. In other words, these procedural decision theories allow unidentified (or not clearly identified) elements to influence the decision-making process. Without exact control on these elements, prediction is not viable.

The understanding of the two different definitions of rationality may provide a better assessment of the two categories of decision theories. Decision models based on substantive rationality aim at predicting behaviour while decision models based on procedural rationality aim at describing behaviour (or the process of decision-making). The predictive theories seek to prescribe economic decisions in light of uncertainty. The descriptive theories, on the other hand, analyzes factors that influence choice in retrospect. It is fair to say that we need both groups of theories to advance our understanding in decision-making.

The purpose of this study is not intended to replace the existing theories by developing a better theory which can resolve the choice paradoxes. Instead, the study develops a new theory which hypothesizes one method to represent the behavioural nature of decision-making. Re-examining the existing anomalies shows that Reference Point theory is a possible candidate to represent the decision-making process. The study of different reference points and their implications on behaviour provides a framework to analyze their role of decision framing.

ENDNOTES FOR CHAPTER 2

1 The distinction between choice under risk and choice under uncertainty is not always clear. A general rule to differentiate them is based on the decision maker's knowledge regarding the probabilities of the payoffs. When the probability distribution of the payoffs is known, it is a case of decision-making under risk. When the decision maker cannot assign actual probabilities to the possible payoffs set, it is a case of uncertainty; he will assign probabilities subjectively instead. (See Machina and Rothschild, 1987, pp. 201-6; and Schmeidler and Wakker, 1987, pp. 229-232.) However, this is not a precise definition due to the complicated nature of probability; mathematicians struggle to refine the definition of objective probability without much success (Schoemaker, 1982, pp. 535-8). Moreover, the consistency axiom of subjective probability theory renders subjective probabilities mathematically indistinguishable from other types of probability (Schoemaker, 1982, p. 537). Although, strictly speaking, the EU model is set up for risky situations with known probabilities (Schmeidler and Wakker, 1987, p. 220), economists use the same approach for theory of choice under uncertainty. For example, Machina (1987, p. 232) does not attempt to distinguish the two situations.

2 See Machina (1987a, p. 123) and Schoemaker (1982, pp. 530-1) for

background information on the St. Petersburg Paradox.

3 See Machina and Rothschild (1987) for a detailed discussion.

4 Schoemaker (1982) describes the complete ordering axioms in von Neumann and Morgenstern's utility function using $\tilde{x}$'s.

5 For any two outcomes, regardless of how small is the difference between them, one can always find a mixture outcome which is preferable to one of the outcomes and at the same time is inferior to the other one. Hence, a continuous spectrum of preference ordering can be established.

ENDNOTES FOR CHAPTER 3

1 Kahneman and Tversky's (1979) results are used in the illustrations because they explicitly reported the percentage of subjects who chose a particular prospect in each problem. Also, their experiments covered six categories of the anomalies examined here.

2 Drèze (1974) and Hagen (1979) observed the Bergen paradox which also demonstrated the common ratio effect in a two-step choice problem. Both $\tilde{a}_3$ and $\tilde{a}_4$ can be structured as two-stage lotteries or compound lotteries. For $\tilde{a}_3$, the first stage of the compound lottery is a 25% chance to win lottery $\tilde{a}_1$. For $\tilde{a}_4$, the first stage of the compound lottery is a 25% chance to win lottery $\tilde{a}_2$.

3 The size of the payoff depended on the success of the subject as compared to the success of other participants. A zero-sum game situation will stimulate the subjects to take the experiments seriously.

4 To repeat March (1978, pp.591-3), these alternative rationalities are:

(i) *limited rationality* which emphasizes the extent to which individuals and groups simplify a decision problem because of the difficulties of anticipating or considering all alternatives and all information.

(ii) *contextual rationality* which emphasizes the extent to which choice behaviour is embedded in a complex of other claims on the attention of actors and other structures of social and cognitive relations.

(iii) *game rationality* which emphasizes the extent to which organization and other social institutions consist of individuals who act in relation to each other intelligently to pursue individual objectives by means of individual calculations of self-interest.

(iv) *process rationality* which emphasizes the extent to which decisions find their sense in attributes of the decision process, rather than in attributes of the decision outcomes.

(v) *adaptive rationality* which emphasizes experiential learning by individuals.

(vi) *selected rationality* which emphasizes the process of selection among individuals or organizations through survival or growth.

(vii) *posterior rationality* which emphasizes the discovery of intentions as an interpretation of action rather than as a prior position.

5 See Loomes, Starmer, and Sugden (1991) for comments on these 3 papers.

6 The restrictions require that preferences over a set of lotteries be complete, transitive, continuous, and monotonic in the sense of first order stochastic dominance (Karni and Safra, 1987, p.681).

7 Cox and Epstein (1989) referred to the asymmetry axiom in their footnote (p.422) "(a)symmetry is more fundamental than transitivity in that it is possible to develop a choice model that does not include the transitivity axiom (Sonnenschein, 1971)."

Sonnenschein (1971) did not explicitly discuss the asymmetry axiom; he tried to replace the transitivity of preferences by the convexity of preferences in proving the existence of a demand function (p.215). Yet, he admitted that "the axioms that we will present will not, in general, be sufficient to prove that competitive equilibria exist, are unbiased, and are optimal (p.215)".

Moreover, he defined the convex set as a circle with (0,0) as the center (p.218 and p.221) to prove that a demand function exists as a result of the convexity of preferences. This unconventional convex set for preferences is problematic. It allows for negative quantities and violates the non-satiation axiom.

8 These problems are the violation of the independence axiom identified by Holt (1986) and Karni and Safra (1987) and the violation of the reduction principle identified by Segal (1988).

9 Without price elicitation, the subjects' only task is to choose between alternatives. So, any PR observed is due to causes other than the information processing effect suggested by Slovic and Lichtenstein (1983).

10 The predicted cycle is $P \succsim \$$, $\$ \succsim C$, and $C \succsim P$ while the opposite $\$ \succsim P$, $P \succsim C$, and $C \succsim \$$ is the unpredicted cycle which is inconsistent with Regret theory. (See Loomes, Starmer, and Sugden, 1991, p.142)

11 The second condition that "the $-bet wins only if the P-bet also wins (Loomes, Starmer, Sugden, 1989, p.142)" is trivial in the sense that (i) P-bet by definition always has a larger probability to win and (ii) one can partition the probabilities in such a way that $p_2 = 0$ (where p_2 is the probability that only prospect A, the $-bet, wins).

In Loomes and Sugden's (1982) original paper on Regret theory (also implicitly in Loomes, Starmer, and Sugden (1989)), they partitioned the probabilities into 4 different states: $p_1 = P_A \cdot P_B$;

$p_2 = P_A(1-P_B)$; $p_3 = (1-P_A)P_B$; $p_4 = (1-P_A)(1-P_B)$ where $\sum_{i=1}^{4} p_i = 1$,

and P_j is the probability of winning in prospect j (j = A or B).

In the experiments in Loomes, Starmer, and Sugden (1989) and more clearly in Loomes, Starmer, and Sugden (1991), they partitioned the probabilities into 3 states: $p_1 = P_A$; $p_2 = 0$; $p_3 = P_B - P_A$;

$p_4 = (1-P_B)$ where $\sum_{i=1}^{4} p_i = 1$. Therefore, p_1 measures the probability that both prospects win, p_3 measures the probability that only prospect B wins, and p_4 measures the probability that both prospects lose.

12 Lexicographic semi-order decision rule:

For two alternatives having more than one dimension, if the difference between the alternatives on Dimension I is greater than ε, choose the alternative that has the higher value on Dimension I. If the difference between the alternatives on Dimension I is less than or equal to ε, choose the alternative that has the higher value on Dimension II (Tversky, 1969, p.32).

This semi-order (or just noticeable difference structure) is imposed on a lexicographic ordering where $\varepsilon = 0$ (Varian, 1984, p.114).

13 The cost of insurance is measured as a cost-benefit ratio as follows (Eisner and Strotz, 1961, p.356):

$$\text{Cost of insurance} = \frac{\text{premium}}{\text{prob} \cdot \text{principal}} = \frac{\text{cost}}{\text{expected benefit}}.$$

14 In a simple two-period model for savings and pensions

$$EU = p_1[U(C_1) + \rho U(C_2)] + (1-p_1)[U(C_1)]$$

where p_1 is the probability to live in period 2,

C_1 is the consumption in period 1; $C_1 = Y - S$, Y and S denote disposable income and savings, respectively,

C_2 is the consumption in period 2; $C_2 = (1+r)S$, r is the interest rate,

ρ is the discount rate for future consumption.

Substituting $C_2 = (1+r)(Y - C_1)$ into the EU equation, we can derive the optimum level of C_1 and consequently the optimum level of savings as follows:

$$U'(C_1) = \rho(1+r)U'[(1+r)(Y - C_1)].$$

If the pension plan (forced savings) is less than discretionary savings (the optimum level of S) and it offers the same interest rate, the pension plan will not induce any change in the optimal level of C_1. Hence, forced savings will displace discretionary savings. If the amount of pension is greater than discretionary savings, it will completely displace the latter.

15 This condition is derived from the assumption of a quadratic utility function $U = A - (B/2)A^2$ where A is the level of wealth and B a scaling (risk aversion) parameter. In the extreme risk averse situation $\partial U/\partial A = 0$ implying that $B = 1/A$. Pashigian et.al. then substituted this case into the optimality condition

$$\frac{-\partial R}{\partial D} = \frac{[1 - B(A-R-D)]p^*}{[1 - B(A-R-Dp^* - \bar{L})]} .$$

When $B = 1/A$, then $\dfrac{-\partial R}{\partial D} = \dfrac{(R+D)p^*}{(R+Dp^* + \bar{L})}$.

16 Fishburn (1988) provided an extensive coverage of non-expected utility theories. His review concentrated on the axiomatization of these theories.

17 This is impossible unless one derives utility from gambling. A one-argument utility function that exhibits a risk-averse attitude

does not account for this possibility.

18 $f(p_i + p_j) = f(p_i) + f(p_j)$ only if $f(p_k) = p_k$ for all k; i.e., subjective probabilities are equal to objective probabilities. If this condition is satisfied, it is not possible to observe the common consequence effect according to SEU.

19 When $p_i = 1$, $f(p_i) = f(p_j) + f(p_k)$ where $p_i = p_j + p_k$ because $\sum_n f(p_n) = 1$ by the axiom of subjective probability.

20 For $\dfrac{w_i}{(1-w_i)} = \left[\dfrac{p_i}{(1-p_i)}\right]^\alpha$, we have

$$w_i = \frac{[p_i/(1-p_i)]^\alpha}{1+[p_i/(1-p_i)]^\alpha} .$$

When n=2, $p_1 + p_2 = 1$ and let $p_1/(1-p_1) = x$, we have

$$w_1 + w_2 = \frac{x^\alpha}{1+x^\alpha} + \frac{x^{-\alpha}}{1+x^{-\alpha}} = 1.$$

21 The underlying structure of the AU theory is intended to avoid the independence axiom of the EU model. Quiggin wrote: "when arguments based upon this (independence) axiom were put to them (the subjects), their answers indicated that they did not accept the validity of the axiom (1982, p.324)."

22 Yarri (1987, pp.112-3) compared his Dual theory to Quiggin's AU theory and showed that AU theory is a generalization of the Dual

theory.

AU theory: $Q(v) = \int \phi(t)d(f \circ G_v)(t) = \int f(G_v(t))d\phi(t)$

Dual theory: $D(v) = \int f(G_v(t))dt$

G_v is the decumulative distribution function (DDF) of random variable v. $G_v(t) = Pr\{ v > t \}$, $0 \le t \le 1$, $G_v(1) = 0$.

$\phi(t)$ is the von Neumann-Morgenstern utility function.

When $\phi(t)$ is linear (Yarri's assumption), AU theory is identical to Dual theory.

Yarri (1987) further commented that Quiggin's approach is perceptional since Quiggin used the empirical observation that "a 50-50 proposition is in fact perceived by decision makers as a 50-50 proposition (p. 113)." Thus, "all risk averse agents in Quiggin's framework must be EU maximizers because the only convex function satisfying f(0) = 0, f(1/2) = 1/2, and f(1) = 1 is the identity (p. 113)."

23 A definition for comonotonic is given in Chew and Wakker (1991, p. 5).

24 A function $V(\cdot)$ is said to be Frêchet differentiable at the point F in the choice set D[0,M] if there exists a continuous linear function $\psi(\cdot;F)$ defined on $\Delta D[0,M]$ such that

$$\lim_{\|F^*-F\|\to 0} \frac{|V(F^*) - V(F) - \psi(F^*-F;F)|}{\|F^* - F\|} = 0.$$

This definition is given in Machina (1982, pp. 293-4.); see also Takayama (1985, p. 81).

25 Chew, Karni, and Safra (1987, p.372, definition 2) give a formal definition for Gateaux differentiable. In relation to Frêchet differentiability where convergence in the limit is required to be uniform in $\|F^* - F\|$, Gateaux differentiability requires that the directional derivative exist for all directions F^*-F and be linear in the direction. The latter, however, does not ensure continuity.

26 The definition of quasilinearity is given in Chew, 1983, p.1068) as follows:

> "Starting with two distributions with the same mean value, quasilinearity requires that mixtures of these distributions with another distribution in the same proportions share the same mean regardless of the distribution that they are mixed with."

Mathematically, for distributions F, G, and H, and functional $M(\cdot)$ that calculates the mean value of a distribution, quasilinearity implies that:

$M(F) = M(G)$, then for $\beta \in (0,1)$, $M(\beta F + (1-\beta)H) = M(\beta G + (1-\beta)H)$.

This is a relaxation to the independence axiom.

27 To serve as the denominator in the weighting function, $W(\cdot)$ cannot be equal to zero. Therefore, $W(\cdot)$ is also a positive function.

28 This assumption is used by Fishburn (1982) in SSB theory where preferences are derived from ranking independent prospects. Regret theory does not require this assumption (Sugden, 1986,

p. 17); preferences in Regret theory are defined over actions.

29 They assume a linear U(·) for simplicity (Loomes and Sugden, 1982, p. 808).

30 Sometimes, researchers view a lexicographic semi-order model as a lexicographic model (for example, Schoemaker, 1980, p. 43). Encarnación (1988) also termed his model lexicographic instead of lexicographic semi-order.

31 See Cook and Levi (1990) and Hogarth and Reder (1987) for collections of papers on intransitivity and rationality.

ENDNOTES FOR CHAPTER 4

1 Loomes and Sugden recognize such a case as a special case in the beginning of their analysis (1987, p.281); yet, they ignore this issue when they put forward the set-specific preference ordering.

2 $\Phi(\tilde{p}, \tilde{q})$ defined on the prospects $\tilde{p}$ and $\tilde{q}$ generates the same preference orderings as $\Sigma_j\ p_j Q(\xi)$ where $\xi = U(x_{ij}) - U(x_{kj})$ and $Q(\xi) = \xi + R[\xi] - R[-\xi]$. Action A_i which leads to different consequences x_{ij} under different states of the world corresponds to the consequences summarized in prospect $\tilde{p}$. Action A_k has the same relation to prospect $\tilde{q}$.

3 $W(\cdot)$ should be a positive function as noted earlier in endnote 27 of Chapter 3.

4 The idea of having a fixed reference point is similar in structure to disappointment theory (Loomes and Sugden, 1986); disappointment theory uses the expected outcome $\tilde{x}$ as the reference point (see Equation (3.27)). In essence, Regret theory (also including RPT) concentrates on the psychological factor of an action vis-à-vis an alternative action. Disappointment theory, on the other hand, tries to capture the psychological factor of an action vis-à-vis its expected outcome. For n alternative actions available, disappointment theory requires the decision-maker to compute n

times the expected outcome $\tilde{x}$ for each action; this is unfavourable when compared to RPT in light of limited information processing capacity.

5 Since $R[\cdot]$ is not a linear function, $R[U(x_{ij}) - U(x_{kj})]$ is not identical to $R[U(x_{ij}) - U(x_{oj})] - R[U(x_{kj}) - U(x_{oj})]$.

ENDNOTES FOR CHAPTER 5

1 Field studies (Bishop and Heberlein, 1979; Brookshire and Coursey, 1987; and Cummings, Brookshire, and Schulze, 1986) attempting to measure the value of public goods yield the same conclusion; namely, a substantial disparity between WTP and WTA.

ENDNOTES FOR CHAPTER 6

1 Markowitz (1952, p. 155) recognized that in some cases, the inflection point should deviate from the current level of wealth. If this happens, it follows that framing will not influence the decision. Consider the following example: an individual is observed buying a lottery ticket with a positive prize \$Y using his current level of wealth as the reference point. Markowitz assumed that the decision is not affected by framing. So, if the individual receives an initial windfall of \$X (\$X > \$Y) and the prize in the lottery ticket becomes (\$Y - \$X) < 0, he will make the same decision if he evaluated the windfall and the negative lottery simultaneously using the same reference point as before. If the presence of the windfall increases the reference point, the negative lottery (\$Y - \$X) will be evaluated on the concave segment and the individual will avoid the lottery. Markowitz uses the notion "customary wealth" to justify ignoring the effect of windfall on the wealth position. However, recent studies (e.g. Kahneman and Tversky, 1979) showed that framing does affect the decision; it is called the response mode effect. This issue is examined further in Section 7.3 with RPT.

2 For the same deviation from a reference point, the disutility of losses associated with a concave VNM is larger than the utility associated with gains. This will generate disparity in deriving

regret and rejoice.

3 R[·] is not symmetrical with respect to outcomes despite that fact that R[ξ] is symmetrical in terms of ξ. See the discussion in Section 5.2.

4 This condition is derived in Section 5.3 as Equation (5.3); it implies that R[·] should be flat enough to dominate the concavity in U(·) in order to generate the desired diminishing regret segment in the modified utility function.

ENDNOTES FOR CHAPTER 8

1 Assuming that two prospects $\tilde{x}_i = (x_{i1}, x_{i2}, \ldots, x_{in})$ and $\tilde{x}_k = (x_{k1}, x_{k2}, \ldots, x_{kn})$ with the same probability distribution (or subjective decision weights) $\tilde{p} = (p_1, p_2, \ldots, p_n)$ are ranked according to Prospect theory,

$$\tilde{x}_i \ \overset{\succ}{\underset{\prec}{\sim}} \ \tilde{x}_k \Leftrightarrow \sum_j p_j V(x_{ij}) \gtrless \sum_j p_j V(x_{kj}).$$

For the value function in Prospect theory that focuses on the change in wealth from a reference point,

$$\tilde{x}_i \ \overset{\succ}{\underset{\prec}{\sim}} \ \tilde{x}_k \Leftrightarrow \sum_j p_j V(x_{ij} - x_{oj}) \gtrless \sum_j p_j V(x_{kj} - x_{oj}).$$

According to Reference Point theory, the EM indices for $\tilde{x}_i$ and $\tilde{x}_k$, respectively are $\sum_j p_j \{U(x_{ij}) + R[U(x_{ij}) - U(x_{oj})]$ and $\sum_j p_j \{U(x_{kj}) + R[U(x_{kj}) - U(x_{oj})]\}$.

RPT will generate the same preference ranking as Prospect theory if and only if $V(\cdot)$, $U(\cdot)$, and $R[\cdot]$ are linear. Besides their difference in the weighting function, reference point theory and prospect theory also take two different approaches in deriving the value functions.

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APPENDIX A

The following material is taken from Borch, 1969, pp. 2-3.

Define three payoffs x, y_1, y_2 as:

$$x = (S_1E_2 - S_2E_1)/(S_1 - S_2)$$

$$y_1 = E_1 + S_1(S_1 - S_2)/(E_1 - E_2)$$

$$y_2 = E_2 + S_2(S_1 - S_2)/(E_1 - E_2)$$

Consider two prospects

$\tilde{x}_1$ with probability (1-p) to obtain x and probability p to obtain y_1

and

$\tilde{x}_2$ with probability (1-p) to obtain x and probability p to obtain y_2

where

$$p = (E_1 - E_2)^2/ [(E_1 - E_2)^2 + (S_1 - S_2)^2],$$

one can show that the mean and standard deviation of $\tilde{x}_i$ are E_i and S_i, respectively, for i = 1, 2. $\tilde{x}_1$ is indifferent to $\tilde{x}_2$ when (E_1, S_1) and (E_2, S_2) lie on the same indifference curve in the mean-standard deviation plane. For a risk averse individual, it is either the case that $E_1 > E_2$ and $S_1 > S_2$ or alternatively $E_2 > E_1$ and $S_2 > S_1$. When $E_1 > E_2$ and $S_1 > S_2$; $y_1 > y_2$ and $\tilde{x}_2$ is inferior to $\tilde{x}_1$ according to the preference function in the EU model. Similarly, when $E_2 > E_1$ and $S_2 > S_1$; $y_2 > y_1$ and $\tilde{x}_1$ is inferior to $\tilde{x}_2$. Hence, the mean-standard deviation approach may fail to distinguish the preference between two prospects while one of them dominates stochastically.

APPENDIX B

If an individual who selected $\tilde{a}_2$ as the reference point also preferred $\tilde{a}_2$ to $\tilde{a}_1$, his ranking indices for the two prospects are given as follows:

$$0.8M(4{,}000;\ 3{,}000) + 0.2M(0;\ 3{,}000) < M(3{,}000;\ 3{,}000). \qquad (B.1)$$

With reference point $\tilde{a}_4$, the same individual would exhibit the common ratio effect if $\tilde{a}_3$ is preferred to $\tilde{a}_4$. In terms of the EM indices,

$$\begin{aligned}&0.20\cdot 0.25M(4{,}000;\ 3{,}000) + 0.80\cdot 0.25M(0;\ 3{,}000) \\ &+ 0.20\cdot 0.75M(4{,}000;\ 0) + 0.80\cdot 0.25M(0;\ 0)\end{aligned} > \begin{aligned}&0.25M(3{,}000;\ 3{,}000) \\ &+ 0.75M(0;\ 0).\end{aligned} \qquad (B.2)$$

Since $M(0;\ 0) = 0$, the above inequality can be simplified as

$$\begin{aligned}&0.20\cdot 0.25M(4{,}000;\ 3{,}000) + 0.80\cdot 0.25M(0;\ 3{,}000) \\ &+ 0.20\cdot 0.75M(4{,}000;\ 0)\end{aligned} > 0.25M(3{,}000;\ 3{,}000). \qquad (B.3)$$

From Equation (B.1), a necessary (but not sufficient) condition for the inequality in Equation (B.3) is

$$\begin{aligned}&0.20\cdot 0.25M(4{,}000;\ 3{,}000) \\ &+ 0.80\cdot 0.25M(0;\ 3{,}000) + 0.20\cdot 0.75M(4{,}000;\ 0)\end{aligned} > \begin{aligned}&0.25\cdot 0.8M(4{,}000;\ 3{,}000) \\ &+ 0.25\cdot 0.2M(0;\ 3{,}000).\end{aligned} \qquad (B.4)$$

Expanding the $M(\cdot;\ \cdot)$ in Equation (B.4) into $U(\cdot)$ and $R(\cdot)$,

$$\begin{aligned}&0.05\{U(4{,}000)+R[U(4{,}000)-U(3{,}000)]\} \\ &+ 0.20\{U(0)+R[U(0)-U(3{,}000)]\} \\ &+ 0.15\{U(4{,}000)+R[U(4{,}000)-U(0)]\}\end{aligned} > \begin{aligned}&0.20\{U(4{,}000) \\ &\quad + R[U(4{,}000)-U(3{,}000)]\} \\ &+ 0.05\{U(0) + R[U(0)-U(3{,}000)]\}.\end{aligned} \qquad (B.5)$$

$$0.20\{U4,000) + R[(U(4,000)-U(3,000)]\} + 0.15\{R[U(4,000)] - R[U(4,000)-U(3,000)]\} + 0.20R[-U(3,000)] > 0.20\{U(4,000) + R[U(4,000)-U(3,000)]\} + 0.05R[-U(3,000)]. \quad (B.6)$$

Simplifying Equation (B.6), we have

$$0.15\{R[U(4,000)] - R[U(4,000)-U(3,000)]\} > -0.15R[-U(3,000)]. \quad (B.7)$$

Due to the fact that R[·] is symmetrical, $R[\xi] = -R[-\xi]$,

$$0.15\{R[U(4,000)] - R[U(4,000)-U(3,000)]\} > 0.15R[U(3,000)]. \quad (B.8)$$

Rearranging Equation (B.8),

$$R[U(4,000)] - R[U(3,000)] > R[U(4,000)-U(3,000)]. \quad (B.9)$$

Concavity in R[·] over the positive domain implies the necessary condition in (B.9) will not be satisfied. Consequently, RPT fails to predict the common ratio effect. On the other hand, Regret theory assumes that R[·] exhibits enough convexity in the positive domain so that R[U(4,000) - R[U(3,000)] is sufficiently larger than R[U(4,000)-U(3,000)] in order to accommodate the common ratio effect. Similarly, GEU also assumes that the local utility function is strictly convex.

APPENDIX C

If an individual who selected $\tilde{b}_2$ as the reference point also preferred $\tilde{b}_2$ to $\tilde{b}_1$, his ranking indices for the two prospects are given as follows:

$$0.33M(2{,}500;\ 2{,}400) + 0.66M(2{,}400;\ 2{,}400) + 0.01M(0;\ 2{,}400) < M(2{,}400;\ 2{,}400). \quad (C.1)$$

Or

$$0.33M(2{,}500;\ 2{,}400) + 0.01M(0;\ 2{,}400) < 0.34M(2{,}400;\ 2{,}400). \quad (C.2)$$

With reference point $\tilde{b}_4$, the same individual would exhibit the common consequence effect if

$$0.33\cdot 0.34M(2{,}500;\ 2{,}400) + 0.67\cdot 0.34M(0;\ 2{,}400) + 0.33\cdot 0.66M(2{,}500;\ 0) + 0.67\cdot 0.66M(0;\ 0) > 0.34M(2{,}400;\ 2{,}400) + 0.66M(0;\ 0). \quad (C.3)$$

Since $M(0;\ 0) = 0$, the above inequality can be simplified as

$$0.33\cdot 0.34M(2{,}500;\ 2{,}400) + 0.67\cdot 0.34M(0;\ 2{,}400) + 0.33\cdot 0.66M(2{,}500;\ 0) > 0.34M(2{,}400;\ 2{,}400). \quad (C.4)$$

From Equation (C.2), a necessary (but not sufficient) condition for the inequality in Equation (C.4) is

$$0.33\cdot 0.34M(2{,}500;\ 2{,}400) + 0.67\cdot 0.34M(0;\ 2{,}400) + 0.33\cdot 0.66M(2{,}500;\ 0) > 0.33M(2{,}500;\ 2{,}400) + 0.01M(0;\ 2{,}400). \quad (C.5)$$

Expanding the $M(\cdot;\ \cdot)$ in Equation (C.5) into $U(\cdot)$ and $R(\cdot)$,

$$
\begin{aligned}
&0.33 \cdot 0.34\{U(2,500)+R[U(2,500)-U(2,400)\} && 0.33\{U(2,500) \\
&+ 0.67 \cdot 0.34R[-U(2,400)] \quad > && \quad + R[U(2,500)-U(2,400)]\} \\
&+ 0.33 \cdot 0.66\{U(2,500)+R[U(2,500)]\} && + 0.01R[-U(2,400].
\end{aligned}
\tag{C.6}
$$

Simplifying Equation (C.6), we have

$$
0.33 \begin{bmatrix} 0.66R[U(2,500)] \\ + 0.34R[U(2,500)-U(2,400)] \\ + 0.66R[-U(2,400)] \end{bmatrix} > 0.33R[U(2,500)-U(2,400)]. \tag{C.7}
$$

$$
0.66\{R[U(2,500)] + R[-U(2,400)]\} > 0.66R[U(2,500)-U(2,400)]. \tag{C.8}
$$

Due to the fact that $R[\cdot]$ is symmetrical, $R[\xi] = -R[-\xi]$,

$$
R[U(2,500)] - R[U(2,400)] > R[U(2,500)-U(2,400)]. \tag{C.9}
$$

Concavity in $R[\cdot]$ over the positive domain implies the necessary condition in Equation (C.9) will not be satisfied. Similar to the common ratio effect, the common consequence cannot be explained (or caused) by switching reference point. On the other hand, Regret theory assumes that $R[\cdot]$ exhibits enough convexity in the positive domain so that R[U(2,500) - R[U(2,400)] is sufficiently larger than R[U(2,400)-U(2,500)] in order to accommodate the common consequence effect. Similarly, GEU also assumes that the local utility function is strictly convex.