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DETERMINATION OF BOTTOM-HOLE PRESSURE IN FLOWING GAS WELLS

by

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A THESIS
SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH
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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled "DETERMINATION OF BOTTOM-HOLE PRESSURE IN FLOWING GAS WELLS", submitted by Jamal Hussein Abou-Kassem, in partial fulfilment of the requirements for the degree of Master of Science in Petroleum Engineering.

....
Supervisor

D L Flock
....

A E Mather
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Date ....April 10, 1975....

TO THE PEOPLE OF PALESTINE

WITH DEEP RESPECT

ABSTRACT

An existing mathematical model for calculating bottom-hole pressure in flowing single-phase gas wells has been modified. The modification improves the various correlations employed in the model. Furthermore, it extends the application to sour natural gas wells. The modified model was tested against field data. The discrepancies between the modified model predictions and the pressure data are traced to a variety of factors.

Several side benefits have resulted from this investigation. These are:

1. A new mathematical correlation was developed for the ideal isobaric heat capacity, for sweet natural gases over the range 32 - 2240°F and for sour natural gases over the range 32 - 300°F.
2. A new mathematical generalized correlation was developed for the heat capacity departure over a range of reduced pressure of 0 - 15 and reduced temperature of 1.05 - 3.0.
3. Mathematical expressions were developed for the partial derivatives of reduced densities and compressibility factors of natural gases with respect to reduced pressure and temperature.
4. A FORTRAN subroutine was developed that is capable of regenerating the Moody friction factor chart for

relative roughness up to 0.05 and Reynolds numbers greater than 4000.

A method is proposed for the estimation of absolute roughness of tubing. The method is essentially another modification of the existing model.

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INTRODUCTION

Flowing bottom-hole pressure is of importance in studying reservoir behavior. Many methods have been developed for the calculation of flowing bottom-hole pressure from surface measurements for single-phase gas flow (1,2,3,4,5,6,7,8). These methods use the Force-Momentum-Balance differential equation and employ various simplifying assumptions.

In 1964, Dranchuk and Quon (9) proposed a general solution for the equations describing steady-state turbulent single-phase gas flow in circular conduits. This method has an advantage over the others in that it permits the simultaneous calculation of the pressure and temperature profiles in the flowing fluid.

Inspection of the Dranchuk and Quon method shows that it employs a constant step-size integrating scheme and approximate formulae for evaluating the various required terms such as: the gas compressibility factor, its partial derivatives with respect to reduced pressure and temperature, the Prandtl number, the gas-film heat-transfer coefficient, and friction factor. Furthermore it handles only the flow of sweet natural gas and requires that the gas composition be known.

The objectives of this study were to

1. Modify the Dranchuk and Quon mathematical model so as to

permit the estimation of absolute roughness when flowing bottom-hole pressure is provided in addition to various surface measurements.

2. Extend the applicability of both the original and modified models so as to handle sour natural gas and flow in the region of partially developed turbulence.
3. Study, test if possible, and choose more sophisticated computer adaptable methods for the evaluation of the gas compressibility factor, its partial derivatives, isobaric heat capacity, friction factor, overall heat-transfer coefficient, gas-film heat-transfer coefficient, Prandtl number, thermal conductivity, and viscosity.
4. Introduce necessary modifications to both the original and modified models so as to permit their use to flow problems where the gas gravity is the only known property of the gas and where gas composition is not available.
5. Compare the calculated bottom-hole pressure values by means of the extended model with field data.

LITERATURE REVIEW

Flowing Bottom-Hole Pressure

A variety of methods has been developed for calculating flowing bottom-hole pressure in dry gas wells (10,11,12, 13,14,15,16,17). Aziz (18) has reviewed most of these methods. These methods utilize the Force-Momentum-Balance differential equation and employ simplifying assumptions such as neglecting the kinetic energy term, assuming constant fluid temperature, etc. Dranchuk and Quon (19) suggested an alternative approach by simultaneously solving the Force-Momentum-Balance and the Total Energy Balance differential equations. Young (20) assumed a linear fluid temperature profile in order to numerically integrate the Force-Momentum-Balance differential equation. Recently Dranchuk and McFarland (21) have studied Young's method and as a result have proposed a modified solution for the same differential equation. The latter two methods assume a constant friction factor as suggested by Cullender and Smith (22).

Compressibility Factor of Natural Gases

The gas compressibility factor is a coefficient which corrects for the non-ideal behaviour of real gases. This factor is necessary in any calculation involving volumes of gaseous mixtures. Standing and Katz (23) presented a graphical correlation for the compressibility factor of dry and sweet natural gases. Elfrink, et al. (24) proposed another graphical correlation for the same factor for sweet and two-phase as well as one-phase natural gases. The former correlation has been widely accepted by the petroleum industry. Accordingly several fitting and interpolation techniques (25,26,27) have sought a mathematical representation for the Standing and Katz correlation.

In the last 35 years many equations of state have been developed to handle hydrocarbon gases and their mixtures (28,29,30,31). Wichert and Aziz (32,33) presented a brief review of the various equations of state used for calculating natural gas compressibility factors. Among these equations those of Redlich-Kwong (34) and Benedict-Webb-Rubin (35) have received wide interest. Several investigators (36,37,38) have introduced a few modifications to the B-W-R equation to make it applicable at very low temperatures and useful in handling complex hydrocarbon mixtures. Others (39,40,41) have fitted the generalized forms of various equations of state to the tabulated Standing and Katz correlation (42) in attempts to predict the volumetric behavior of sweet natural gases.

In order to predict the volumetric behavior of dry sour natural gases, Robinson, et al. (43) developed a correction factor correlation to be used with the Standing and Katz correlation. Wichert and Aziz (44,45,46) presented an alternative approach which employs modified pseudocriticals of sour natural gas.

Isobaric Heat Capacity of Natural Gases

The isobaric heat capacity of gases is usually calculated by evaluating the ideal isobaric heat capacity and the heat capacity departure from the ideal state at high pressures.

Kobe, et al. (47) and Thinh, et al. (48) presented the ideal isobaric heat capacity of pure gases as a function of temperature by means of third degree polynomials. Kothari and Doraiswamy (49), through dimensional analysis, showed that the ideal isobaric heat capacity for pure gases could be expressed as a function of molecular weight and reduced temperature. On the other hand the ideal isobaric heat capacity of a gas mixture may be expressed by means of the molal average (50).

The heat capacity departure from the ideal state at high pressures has been studied by many investigators. Dodge (51) developed a generalized correlation for the ratio $\frac{C_p}{C_p^\circ}$. Watson and Smith (52), Lydersen, et al. (53), Edmister (54), Weiss and Joffe (55), and Sherwood (56) presented either graphical or tabulated generalized correlations for the difference $(C_p - C_p^\circ)$. Comings and Nathan (57) showed that the difference $(C_p - C_p^\circ)$ and not the ratio $\frac{C_p}{C_p^\circ}$ is a function of reduced pressure and temperature. As a result, they recommended the use of the difference $(C_p - C_p^\circ)$ as a generalized correlation. Furthermore they pointed out that the

Watson and Smith generalized correlation might give errors as high as 50 per cent. The Edmister generalized correlation may be in error by as much as 50 to 100 per cent (58). Sledjeski (59), Seifarth and Joffe (60) used the B-W-R equation of state for predicting $(C_p - C_p^\circ)$ of propane and methane. They reported that satisfactory agreement with experimental data was obtained for temperatures greater than 150°F.

The Hydraulic Friction Factor

Under normal conditions of fluid flow in pipe (normal velocities, single-phase flow, confined conduits, no temperature difference between the fluid and the boundary wall) the friction factor is a function of Reynolds number and relative roughness (61,62). For commercial pipes, the friction factor decreases at a decreasing rate as Reynolds number increases, and once the boundary Reynolds number (R_{e_b}) has been reached, the friction factor remains constant (63,64).

Several investigators (65,66) have developed different mathematical expressions for the friction factor in both the rough and smooth pipe regions. The friction factor in the transition region was controversial until Colebrook (67) bridged the gap between the smooth and rough pipe regions with his particular transition law. Later, Moody (68) constructed a friction factor chart in which he employed the Von Kármán formulae for the smooth and rough pipe regions and the Colebrook transition law for the transition region.

Cullender and Smith (69) assumed that gas flow through tubing in gas wells is fully turbulent. Consequently they suggested a constant friction factor as a function of tubing diameter only.

Absolute Roughness of Gas Well Tubing

It is well known that absolute roughness changes with time due to pipe corrosion. Colebrook (70) found that the rate of deterioration of pipe depends on time, type of flow, lining material, size of pipe, and the character of the flowing fluid.

Cullender and Binckley (71) reported an absolute roughness of 0.0006 inch for gas well casing. Smith, et al. (72), from their own experimental data, reported a value of 0.00065 inch for the absolute roughness of gas well tubing. A range of absolute roughness of 0.00055-0.0019 inch for experimental gas pipe lines has been reported by others (73).

Overall Heat-Transfer Coefficient

Heat-transfer between a flowing fluid in a pipe and a solid separated by one or more solid layers can be conveniently handled by utilizing the overall heat-transfer coefficient concept. Expressions for the overall heat-transfer coefficient for uniform geometries may be found in any heat-transfer reference (74,75). Holst (76) studied this coefficient for the case of steam injection down a well assuming simple and uniform geometries. Dranchuk and Quon (77) suggested an expression which requires a trial-and-error procedure for the same coefficient in a gas well system.

The various expressions for the overall heat-transfer coefficient require knowledge of the gas-film heat-transfer coefficient. The latter coefficient has been exhaustively investigated. Simple, semi-empirical, empirical and sophisticated models have been developed for the estimation of the gas-film heat-transfer coefficient (78,79,80,81,82,83).

Thermal Conductivity of Gas Mixtures

Reid and Sherwood (84) have reviewed most of the existing methods for the estimation of the thermal conductivity of pure gases and their mixtures. Among these methods, that of Misic and Thodos (85,86) for pure gases may be efficiently adapted to computer calculations. Lindsay and Bromley (87) proposed a generalized equation for the thermal conductivity of gas mixtures at atmospheric pressure. Stiel and Thodos (88) studied the effect of pressure on the thermal conductivity of pure gases.

Viscosity of Pure and Natural Gases

The viscosity of pure gases and their mixtures has been studied by numerous investigators. Reid and Sherwood (89) gave an excellent review on the subject. Herning and Zipperer (90) suggested an empirical formula for estimating the viscosity of a gas mixture from those of the individual components. Bicher and Katz (91) presented a graphical method for natural gas viscosity as a function of specific gravity, temperature, and pressure. Carr, et al. (92) developed another graphical correlation, which utilized the law of corresponding state. Lee, et al. (93,94) studied and developed empirical formulae for the viscosity of sweet natural gases as well as pure hydrocarbons.

MODEL DEVELOPMENT

Four main models were developed to meet part of the objectives of this study. The objective, development, and requirements of each main model are as follows:

Main Model A

The objective of this model is to estimate the absolute roughness of tubing.

The development of this model is presented in detail in Appendix A. It can be briefly described as follows:

- The Force-Momentum-Balance differential equation for a fluid element contained in a pipe segment may be written as:

$$\frac{V}{M_w} dP + \frac{g}{g_c} dz + \frac{\langle u\rangle d\langle u\rangle}{\alpha g_c} + \frac{f\langle u\rangle^2}{2g_c D} dL = 0 \qquad (1)$$

where α is the inverse of the momentum correction factor. (The value of the momentum correction factor for turbulent flow is closer to unity than for laminar flow (95).)

- The Total Energy Balance differential equation over the same pipe segment may be expressed as:

$$\delta q - \frac{dW}{J} = \frac{\pi D^2}{4} \left(\frac{G}{M_w}\right) \left[dh + \frac{\langle u\rangle d\langle u\rangle}{\alpha J g_c} + \frac{g}{g_c J} dz\right] \qquad (2)$$

- The equation of state:

$$PV = ZRT \tag{3}$$

- Equations 1, 2, and 3 were rearranged to yield the following:

$$A^* \frac{dP_r}{dL} + B^* \frac{dT_r}{dL} = C^* \tag{4}$$

and

$$D^* \frac{dP_r}{dL} + E^* \frac{dT_r}{dL} = F^* \tag{5}$$

From which

$$\frac{dP_r}{dL} = \frac{C^* \cdot E^* - B^* \cdot F^*}{A^* \cdot E^* - B^* \cdot D^*} \tag{6}$$

and

$$\frac{dT_r}{dL} = \frac{A^* \cdot F^* - C^* \cdot D^*}{A^* \cdot E^* - B^* \cdot D^*} \tag{7}$$

where

$$\begin{aligned}
A^* &= fcn(T_r, P_r, T_c, P_c, M_w, Q_o, D) \\
B^* &= fcn(T_r, P_r, T_c, P_c, M_w, Q_o, D) \\
C^* &= fcn(T_r, P_r, T_c, P_c, M_w, Q_o, D, \tfrac{\varepsilon}{d}, S) \\
D^* &= fcn(T_r, P_r, T_c, P_c, M_w, Q_o, D) \\
E^* &= fcn(T_r, P_r, T_c, P_c, M_w, Q_o, D) \\
\text{and} \quad F^* &= fcn(T_r, P_r, T_c, P_c, M_w, Q_o, D, U)
\end{aligned} \tag{8}$$

In order to obtain a solution, $\frac{dP_r}{dL}$ and $\frac{dT_r}{dL}$ must be finite.

Equations 6, 7, and 8 imply that

$$\frac{dP_r}{dL} = fcn(T_r, P_r, T_c, P_c, M_w, Q_o, D, \frac{\varepsilon}{d}, \mathcal{S}, U) \tag{9}$$

and

$$\frac{dT_r}{dL} = fcn(T_r, P_r, T_c, P_c, M_w, Q_o, D, \frac{\varepsilon}{d}, \mathcal{S}, U) \tag{10}$$

- For a specific flow problem, that is, when pipe dimensions and position in space ($L^*, D, \mathcal{S}$), flow rate (Q_o), and gas properties (T_c, P_c, M_w) are given; Equations 9 and 10 reduce to

$$\frac{dP_r}{dL} = fcn(T_r, P_r \frac{\varepsilon}{d}, U) \tag{11}$$

and

$$\frac{dT_r}{dL} = fcn(T_r, P_r, \frac{\varepsilon}{d}, U) \tag{12}$$

- The overall heat-transfer coefficient, U, as suggested by Dranchuk and Quon (96) is:

$$\frac{1}{U} = \frac{1}{h_o} + R^* \tag{13}$$

where h_o = gas-film heat-transfer coefficient

$$= fcn(\overline{P}_r, R_e, \frac{\varepsilon}{d})$$

$$= fcn(T_r, P_r, \frac{\varepsilon}{d}) \qquad (14)$$

$\overline{P}_r$ = Prandtl number

R_e = Reynolds number

and R^* = average thermal resistance of the surroundings, ft.^{2} sec. °R/Btu

- Combining Equations 11, 12, 13, and 14 yields,

$$\frac{dP_r}{dL} = fcn(T_r, P_r, \frac{\varepsilon}{d}, R^*) \qquad (15)$$

and

$$\frac{dT_r}{dL} = fcn(T_r, P_r, \frac{\varepsilon}{d}, R^*) \qquad (16)$$

- It is assumed that $\frac{\varepsilon}{d}$ and R^* are constants but have unknown values which have to be determined.

Thus the main model A, which describes single-phase gas flow in pipes, consists of two non-linear differential equations, Equations 6 and 7 or 15 and 16, and two-parameter estimations namely $\frac{\varepsilon}{d}$ and R^*. This system of differential equations has to be solved simultaneously for the boundary conditions:

At $L = 0$ $P_{(0)} = P_1$ and $T_{(0)} = T_1$

At $L = L^*$ $P_{(L^*)} = P_2$ and $T_{(L^*)} = T_2$

This model requires algebraic expressions for the gas compressibility factor, its partial derivatives, friction factor, the Prandtl number, gas-film heat-transfer coefficient, isobaric heat capacity, viscosity and thermal conductivity.

Data required are: gas composition, flow rate, tubing head pressure and temperature, bottom-hole pressure and temperature, length and inner diameter of tubing, and ground surface temperature.

Main Model AA (Method 2)

The objective of this model is to calculate the flowing bottom-hole pressure.

This model is the original Dranchuk and Quon model. It is a special case of main model A where $\frac{\varepsilon}{d}$ is assumed to be $\frac{0.0006}{d}$ as suggested by Cullender and Smith (97). Thus, this model consists of two non-linear differential equations, Equations 6 and 7, and one parameter estimation namely R*. This system of differential equations has to be solved simultaneously for the boundary conditions:

At $L = 0 \quad T_{(0)} = T_1$

At $L = L^* \quad P_{(L^*)} = P_2$ and $T_{(L^*)} = T_2$

This model requires algebraic expressions for the gas compressibility factor, its partial derivatives, friction factor, the Prandtl number, gas-film heat-transfer coefficient, isobaric heat capacity, viscosity and thermal conductivity.

Data required are: gas composition, flow rate, tubing head pressure and temperature, bottom-hole temperature, length and inner diameter of tubing, and ground surface temperature.

Main Model B

The objective of this model is to estimate the absolute roughness of tubing.

This model differs from main model A in that it assumes the overall heat-transfer coefficient constant at some average value; hence

$$\frac{1}{U} = R^{**} = \text{constant} \tag{17}$$

Consequently when Equations 11, 12, and 17 are combined,

$$\frac{dP_r}{dL} = fcn(T_r, P_r, \frac{\varepsilon}{d}, R^{**}) \tag{18}$$

and

$$\frac{dT_r}{dL} = fcn(T_r, P_r, \frac{\varepsilon}{d}, R^{**}) \tag{19}$$

The same boundary conditions as those in main model A are employed.

This model requires algebraic expressions for the gas compressibility factor, its partial derivatives, friction factor, isobaric heat capacity, and viscosity.

Data required are: flow rate, tubing head pressure and temperature, bottom-hole pressure and temperature, length and inner diameter of tubing, ground surface temperature, and either the gas composition or the gas gravity along with the mole fractions of N_2, CO_2, and H_2S.

Main Model BB (Method 2)

The objective of this model is to calculate the flowing bottom-hole pressure.

This model is a special case of main model B where $\frac{\varepsilon}{d}$ is assumed to be $\frac{0.0006}{d}$. Thus this model consists of two non-linear differential equations, Equations 6 and 7, and one parameter estimation namely R**.

The same boundary conditions as those in main model AA are employed.

This model requires algebraic expressions for the gas compressibility factor, its partial derivatives, friction factor, isobaric heat capacity, and viscosity.

Data required are: flow rate, tubing head pressure and temperature, bottom-hole temperature, length and inner diameter of tubing, ground surface temperature, and either the gas composition or the gas gravity along with the mole fractions of N_2, CO_2, and H_2S.

Special Cases of Main Models AA and BB

In the following special cases both main models AA and BB reduce to one model:

1. Adiabatic Flow (Method 1):

There is no heat-transfer between the flowing fluid and the surrounding; that is,

$$U = 0 \tag{20}$$

This method requires solving simultaneously Equations 6 and 7 for the boundary conditions.

At $L = 0$ $T_{(0)} = T_1$

and

At $L = L^*$ $P_{(L^*)} = P_2$

2. Complete Solution of the Force-Momentum-Balance Differential Equation (Method 3):

This method assumes a linear temperature profile in the flowing fluid. It requires solving Equation 4 for the boundary condition:

$L = L^*$ $P_{(L^*)} = P_2$

Equation 4 may be written as:

$$\frac{dP_r}{dL} = (C^* - B^* \frac{dT_r}{dL})/D^* \tag{21}$$

where

$$\frac{dT_r}{dL} = \left(\frac{T_{(L^*)} - T_{(0)}}{L^*}\right)/{}_pT_c \qquad (22)$$

3. Method 4:

This method is a special case of Method 3 where the friction factor is assumed constant as suggested by Cullender and Smith (98).

Method 4 is equivalent to that of Dranchuk and McFarland (99).

The main models require algebraic expressions (models) for the gas compressibility factor, its partial derivatives with respect to reduced temperature and pressure, isobaric heat capacity, friction factor, overall heat-transfer coefficient, gas-film heat-transfer coefficient, the Prandtl number, thermal conductivity, and viscosity.

The various mathematical expressions for these terms are the subject of the rest of this section.

Compressibility Factor of Natural Gases

The compressibility factor of dry sweet natural gas may be expressed analytically by many methods. In this study the reduced form of the B-W-R equation of state as suggested by Purvis (100) and Dranchuk, et al. (101) was chosen to represent the compressibility factor, where

$$Z = \frac{Z_c P_r}{\rho_r T_r} = 1 + \left(A_1 + \frac{A_2}{T_r} + \frac{A_3}{T_r^{\,3}}\right)\rho_r + \left(A_4 + \frac{A_5}{T_r}\right)\rho_r^2$$

$$+ \frac{A_5 A_6}{T_r}\,\rho_r^5 + \frac{A_7}{T_r^{\,3}}\,\rho_r^2 (1 + A_8 \rho_r^2) exp(-A_8 \rho_r^2) \qquad (23)$$

and

A_1 = 0.31506237

A_2 = -1.0467099

A_3 = -0.57832727

A_4 = 0.53530771

A_5 = -0.61232032

A_6 = -0.10488813

A_7 = 0.68157001

A_8 = 0.68446549

Equation 23 may be extended to sour gas by applying modified pseudocritical pressure and temperature, when calculating the pseudoreduced pressure and temperature, as suggested by Wichert and Aziz (102), where

$$\varepsilon_3 = 120\left[(Y_{CO_2} + Y_{H_2S})^{0.9} - (Y_{CO_2} + Y_{H_2S})^{1.6}\right]$$

$$+ \; 15\left[Y_{H_2S}^{0.5} - Y_{H_2S}^{4}\right] \qquad (24)$$

$$T_c' = T_c - \varepsilon_3$$

and

$$P_c' = \frac{P_c T_c'}{\left[T_c + Y_{H_2S}(1 - Y_{H_2S})\varepsilon_3\right]}$$

Partial Derivatives of Compressibility Factor

The partial derivatives of Z with respect to reduced temperature and pressure may be calculated by employing a numerical technique. However mathematical expressions for these derivatives may be derived from Equation 23 as shown in Appendix B. These mathematical expressions may be expressed as:

$$\left(\frac{\partial Z}{\partial T_r}\right)_{P_r} = Z\left[-\frac{1}{T_r} - \frac{1}{\rho_r}\left(\frac{\partial \rho_r}{\partial T_r}\right)_{P_r}\right] \qquad (25)$$

and

$$\left(\frac{\partial Z}{\partial P_r}\right)_{T_r} = Z\left[\frac{1}{P_r} - \frac{1}{\rho_r}\left(\frac{\partial \rho_r}{\partial P_r}\right)_{T_r}\right] \qquad (26)$$

The partial derivatives of reduced density with respect to reduced temperature and pressure, $\left(\frac{\partial \rho_r}{\partial T_r}\right)_{P_r}$ and $\left(\frac{\partial \rho_r}{\partial P_r}\right)_{T_r}$, may be written as:

$$\left(\frac{\partial \rho_r}{\partial T_r}\right)_{P_r} = \frac{-\left[\rho_r + \left(A_1 - \frac{2A_3}{T_r^{\,3}}\right)\rho_r^2 + A_4\rho_r^3 - \frac{2A_7}{T_r^{\,3}}\,\rho_r^3(1+A_8\rho_r^2)\,exp(-A_8\rho_r^2)\right]}{\left[\begin{array}{l} T_r + 2\left(A_1T_r + A_2 + \frac{A_3}{T_r^{\,2}}\right)\rho_r + 3(A_4T_r + A_5)\rho_r^2 + \\ 6A_5A_6\rho_r^5 + \frac{A_7}{T_r^{\,2}}\,\rho_r^2(3 + 3A_8\rho_r^2 - 2A_8^2\rho_r^4)\,exp(-A_8\rho_r^2) \end{array}\right]}$$

(27)

and

$$\left(\frac{\partial \rho_r}{\partial P_r}\right)_{T_r} = \frac{Z_c}{\begin{bmatrix} T_r + 2(A_1 T_r + A_2 + \frac{A_3}{T_r^3})\rho_r + 3(A_4 T_r + A_5)\rho_r^2 + \\ 6A_5 A_6 \rho_r^5 + \frac{A_7}{T_r^2}\rho_r^2(3 + 3A_8\rho_r^2 - 2A_8^2\rho_r^4)exp(-A_8\rho_r^2) \end{bmatrix}} \quad (28)$$

Isobaric Heat Capacity of Natural Gases

The isobaric heat capacity of natural gas may be represented by:

$$C_{p_m} = C^\circ_{p_m} + (C_{p_m} - C^\circ_{p_m}) \quad (29)$$

where C_{p_m} = isobaric heat capacity of natural gases, Btu/lb mole. °R.

$C^\circ_{p_m}$ = ideal isobaric heat capacity of natural gases, Btu/lb mole. °R.

$(C_{p_m} - C^\circ_{p_m})$ = heat capacity departure from the ideal state of natural gases, Btu/lb mole. °R.

1. Ideal Isobaric Heat Capacity:

Two models may be used to represent the ideal heat capacity of natural gas.

a. Model $C^\circ_{p_m}$ 1:

In this model, $C^\circ_{p_m}$ is taken as the molal average; that is,

$$C^{\circ}_{p_m} = \sum_{i=1}^{n} C^{\circ}_{p_i} Y_i \qquad (30)$$

where

$$C^{\circ}_{p_i} = \sum_{j=0}^{k} a_j T^j \qquad (31)$$

This model requires that the gas composition be known.

b. Model $C^{\circ}_{p_m}$ 2:

This model was developed in this study to represent the ideal isobaric heat capacity of sweet natural gas as a function of gas gravity (G) and pseudoreduced temperature (T_r); that is,

$$C^{\circ}_{p_{m_{H.C.}}} = A(G) + B(G)T_r \qquad (32)$$

where the subscript $m_{H.C.}$ means a mixture of hydrocarbons. This equation can be extended to sour natural gas by applying corrections to account for the presence of N_2, CO_2, and H_2S as follows:

$$C^{\circ}_{p_m} = \frac{C^{\circ}_{p_{m_{H.C.}}}}{F_{CO_2} \cdot F_{N_2} \cdot F_{H_2S}} \qquad (33)$$

$$F_{N_2} = fcn(Y_{N_2}, T) \qquad (34)$$

$$F_{CO_2} = fcn(Y_{CO_2}, T) \qquad (35)$$

and

$$F_{H_2S} = fcn(Y_{H_2S}, T) \qquad (36)$$

where T = temperature of gas, °R.

2. Heat Capacity Departure:

A heat capacity departure model was developed by employing the reduced form of the B-W-R equation of state and utilizing thermodynamic properties as shown in Appendix C. The heat capacity departure may be expressed as:

$$\frac{C_{p_m} - C^{\circ}_{p_m}}{R} = \left[-1 - \frac{6A_3}{T_r^{\,3}}\rho_r - \frac{6A_7}{A_8T_r^{\,3}} + \left(\frac{6A_7}{A_8T_r^{\,3}} + \frac{3A_7}{T_r^{\,3}}\rho_r^2\right)exp(-A_8\rho_r^2)\right]$$

$$+ \frac{T_r\left[1 + \left(A_1 - \frac{2A_3}{T_r^{\,3}}\right)\rho_r + A_4\rho_r^2 - \frac{2A_7}{T_r^{\,3}}\rho_r^2(1+A_8\rho_r^2)exp(-A_8\rho_r^2)\right]^2}{\left[\begin{array}{l} T_r + 2\left(A_1T_r + A_2 + \frac{A_3}{T_r^{\,2}}\right)\rho_r + 3(A_4T_r + A_5)\rho_r^2 + \\ 6A_5A_6\rho_r^5 + \frac{A_7}{T_r^{\,2}}\rho_r^2(3 + 3A_8\rho_r^2 - 2A_8^2\rho_r^4)exp(-A_8\rho_r^2) \end{array}\right]} \qquad (37)$$

Although this equation was developed for sweet natural gases, it may be applied to sour natural gases as will be shown later.

The Hydraulic Friction Factor

The hydraulic friction factor as presented by Moody (103) in chart form has been widely accepted and used for estimating pressure drop due to friction in both gas and oil pipe lines. This chart was developed by using the following equations for the various regions:

1. The Smooth Pipe Region:

$$\frac{1}{\sqrt{f}} = 2\ \log\left(\frac{R_e\sqrt{f}}{2.51}\right)\ ; \quad \varepsilon^* < 3 \qquad (38)$$

where f = friction factor.

R_e = Reynolds number

$= 20014.855\ \frac{Q_o G}{\mu d}$ (field units)

G = gas gravity, (air = 1).

$= \frac{M_w}{M_{air}} = \frac{M_w}{28.97}$

μ = gas viscosity, c.p.

Q_o = gas flow rate, MMSCF/Day.

$\varepsilon^* = \sqrt{\frac{f}{8}}\ R_e\ \frac{\varepsilon}{d}$

ε = absolute roughness, inch.

and d = inner diameter of tubing, inch.

2. The Transition Region:

This region is sometimes referred to as the region of partially developed turbulence. The friction factor in this region was represented by the Colebrook transition law (104), where

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/d}{3.7} + \frac{2.51}{R_e\sqrt{f}}\right) ; \quad 3 \leq \varepsilon^* \leq 60 \qquad (39)$$

3. The Rough Pipe Region:

This region is sometimes referred to as the fully developed turbulent region. The Von Kármán rough pipe law was employed to represent the friction factor in this region, where

$$\frac{1}{\sqrt{f}} = 2\log\left(\frac{3.7}{\varepsilon/d}\right) ; \quad \varepsilon^* > 60 \qquad (40)$$

Rather than using ε^* to express the boundary between the transition and rough pipe regions one may use the boundary Reynolds number (R_{e_b}) technique proposed by Moody (105), where

$$R_{e_b} = \frac{3200}{\varepsilon/d} \qquad (41)$$

This method is simpler and easier to use than ε^* and was, therefore, chosen in this study.

Equation 39 is too cumbersome for hand calculation since it requires trial-and-error. For this reason Cullender and Binckley (106) suggest a simplified friction factor formula as a function of flow rate, viscosity, gas gravity, and pipe diameter. On the other hand Cullender and Smith (107) suggested two friction factor formulae for fully developed turbulent flow (rough pipe region), both of which are functions of pipe diameter only. Both groups assumed an absolute roughness of 0.0006 inch in the development of their approximate formulae.

Moody's chart was employed (in this investigation) to represent the hydraulic friction factor for the following reasons:

1. Main models A and B require that the friction factor be a function of relative roughness in order to give an estimate of absolute roughness of the tubing.

2. Flow of natural gas in gas wells is usually turbulent and falls in either the transition or rough pipe region. However, during "Back Pressure Tests", flow is more likely to be in the transition region, especially at low flow rates. Therefore, the Cullender and Smith friction factor equations may not hold.

3. Moody's chart has been accepted as applicable to pipes with relative roughness of 0.0 - 0.05 , while the other formulae are applicable only for pipes with absolute roughness of 0.0006 inch.

Overall Heat-Transfer Coefficient

The heat-transfer from a flowing fluid through pipe to the surroundings over a pipe segment (dL) may be expressed as:

$$- \delta q = - \pi D \, dL \, U(T_s - T) \tag{43}$$

For a composite cylindrical wall, the overall heat-transfer coefficient (U) may be represented by three models:

1. Model U_1:

This model can be found in any heat-transfer reference (108). The model that applies to gas wells may be written as:

$$\frac{1}{U} = \frac{1}{h_o} + \frac{D_o}{2} \sum_{i=1}^{n} \frac{ln(D_i/D_{i-1})}{k_{i-1,i}} \tag{44}$$

2. Model U_2:

This model was suggested by Dranchuk and Quon (109) for gas well systems. It assumes a constant thermal resistivity of the surroundings, where

$$\frac{1}{U} = \frac{1}{h_o} + R^* \tag{45}$$

3. Model U_3:

This model is proposed in this study for gas well systems. It assumes that the overall heat-transfer coefficient is constant at some average value; that is,

$$\frac{1}{U} = R^{**} = \text{constant} \qquad (46)$$

Gas-Film Heat-Transfer Coefficient

As can be seen from the previous section, models U_1 and U_2 require an estimate for the gas-film heat-transfer coefficient, h_o. Such an estimate, in turn, requires an estimate of the Stanton number.

Many investigators have developed various models for the Stanton number, which is related to the gas-film heat-transfer coefficient by (110):

$$S_t = \frac{h_o}{\rho \langle u \rangle c_p} \qquad (47)$$

where S_t = Stanton number.

ρ = gas density, lbm/ft^3.

$\langle u \rangle$ = mean gas velocity, ft/sec.

and c_p = isobaric heat capacity, Btu/lbm.°R.

The Gowen and Smith (111) model is considered the most sophisticated in addition to being the best of all in reproducing experimental data. This model may be expressed as:

$$S_t = \frac{\sqrt{\frac{f}{8}}}{\chi - 4.5} \qquad (48)$$

where

$$X = P_r^{0.5} \left[0.155 \left(\frac{f}{8}\right)^{0.27} R_e^{0.54} + \sqrt{\frac{8}{f}} \right] \tag{49}$$

R_e = Reynolds number.

P_r = Prandtl number.

$$= \frac{C_p \mu}{M_w k} \tag{50}$$

f = friction factor.

k = thermal conductivity, cal/cm. sec. K.

μ = viscosity, gm/sec. cm or poise.

C_p = isobaric heat capacity, cal/gm mole. K.

M_w = molecular weight, gm/gm mole.

Thermal Conductivity of Gas Mixtures

Thermal conductivity is required for the calculation of Prandtl number, which, in turn, is necessary for the calculation of the Stanton number. Thermal conductivity of gas mixtures may be expressed as:

$$k_m = k_m^{\circ} + (k_m - k_m^{\circ}) \tag{51}$$

where k_m = thermal conductivity of gas mixtures.

k_m° = thermal conductivity of gas mixtures at one atmosphere.

$(k_m - k_m^{\circ})$ = thermal conductivity departure of gas mixtures.

The Lindsay and Bromley generalized equation (112) can be used for the estimation of the thermal conductivity of gas mixtures at one atmosphere, where

$$k_m = \sum_{i=1}^{n} \frac{k_i^\circ}{\left[1 + \sum_{\substack{j=1 \\ j \neq i}}^{n} A_{ij} \frac{Y_j}{Y_i}\right]} \qquad (52)$$

$$A_{ij} = 0.25 \left\{ 1 + \left[\frac{\mu_i^\circ}{\mu_j^\circ} \left(\frac{M_{w_j}}{M_{w_i}}\right)^{0.75} \frac{\left(1+\frac{S_i}{T}\right)}{\left(1+\frac{S_j}{T}\right)} \right]^{0.5} \right\}^2 \left[\frac{1+\frac{S_{ij}}{T}}{1+\frac{S_i}{T}} \right] \qquad (53)$$

$$S_i = 1.5\, T_{b_i} \qquad (54)$$

$$S_{ij} = \sqrt{S_i S_j} \qquad (55)$$

S_i = the Sutherland constant for ith component, K.

μ_i° = low pressure viscosity of ith component, c.p.

M_{w_i} = molecular weight of ith component, gm/gm mole

T_{b_i} = normal boiling point temperature of ith component, K.

and k_i° = thermal conductivity of ith component at one atmosphere, cal/sec. cm. K.

The thermal conductivity of pure components at one atmosphere can be represented as suggested by Misic and Thodos (113, 114) where

For Hydrocarbons:

$$k_i^\circ = \frac{C_{p_i}^\circ}{10^6 \gamma_i} \left(14.52\, T_{r_i} - 5.14\right)^{\frac{2}{3}} \quad ; \quad T_{r_i} \geq 1 \tag{56}$$

For Non-hydrocarbons:

$$k_i^\circ = \frac{C_{p_i}^{\circ\;0.75}}{10^6 \gamma_i} \left[(195 Z_{c_i} - 31.94) T_{r_i} + (16.83 - 82.5 Z_{c_i}) \right]^{(1.524 - 2.8 Z_{c_i})} ;$$

$$1 \leq T_{r_i} \leq 3 \tag{57}$$

and

$$\gamma_i = \frac{T_{c_i}^{\frac{1}{6}} \, M_{w_i}^{\frac{1}{2}}}{P_{c_i}^{\frac{2}{3}}} \tag{58}$$

with T_{c_i} and P_{c_i} in degrees Kelvin and atmospheres respectively.

The thermal conductivity departure can be represented as suggested by Stiel and Thodos (115), where

$$(k_m - k_m^\circ) = \frac{14.0\left[exp(0.534\rho_r) - 1\right]}{10^8\ \gamma\ Z_c}; \qquad \rho_r \le 0.5 \qquad (59)$$

$$= \frac{13.1\left[exp(0.67\ \rho_r) - 1.069\right]}{10^8\ \gamma\ Z_c}; \qquad 0.5 < \rho_r \le 2.0 \qquad (60)$$

$$= \frac{2.976\left[exp(1.155\ \rho_r) - 2.016\right]}{10^8\ \gamma\ Z_c}; \qquad 2.0 < \rho_r \le 2.8 \qquad (61)$$

<u>Viscosity of Pure and Natural Gases</u>

The viscosity of natural gases is required for the calculation of the Reynolds and Prandtl numbers. It can be evaluated by means of the Lee, et al. correlation (116), where

$$\mu_m = K\ exp\ (X\ \rho^Y)$$

$$K = \frac{(9.4 + 0.02\ M_w)T^{1.5}}{(209 + 19\ M_w + T)}$$

$$X = 3.5 + \frac{986}{T} + 0.01\ M_w \qquad (62)$$

$$Y = 2.4 - 0.2\ X$$

where μ_m = natural gas viscosity, micropoise.

ρ = natural gas density, gm/cm^3.

T = temperature, °R.

and M_w = average molecular weight, gm/gm mole.

They recommend that the density be calculated using the Standing and Katz compressibility factor correlation.

The viscosity of pure components at low pressure is needed for the calculation of thermal conductivity at low pressure. It can be expressed by using the modified rigid interacting sphere model (117), where

$$\mu_i^o = 0.0026693 \frac{\sqrt{M_{w_i} T}}{\sigma_i^2 \, \Omega_{v_i}} \tag{63}$$

$$\frac{1}{\Omega_{v_i}} = 0.697 \left(1 + 0.323 \, ln \, T_i^*\right) \quad (118) \tag{64}$$

μ_i^o = viscosity of ith component, c.p.

$T_i^* = \dfrac{T}{\left(\frac{\varepsilon_o}{K}\right)_i}$

T = temperature, K.

ε_o = maximum energy of attraction for Lennard-Jones potential, ergs.

K = Boltzmann constant, 1.3805×10^{-16} ergs/ K.

and σ = collision diameter for Lennard-Jones potential, Å.

Values of σ and $\left(\frac{\varepsilon_o}{K}\right)$ are available in the literature (119,120). Alternatively, the Stiel and Thodos empirical equations (121) may be used.

If $T_{r_i} < 1.5$ or 2.0, Reid and Sherwood (122) recommend the modification of the value of μ°_i according to the following relation:

$$\mu^\circ_{1_i(T_{r_i}<2)} = \mu^\circ_{2_i(T_{r_i}=2)} \frac{\left(\frac{T_1^*}{T_2^*}\right)_i}{\left(\frac{\Omega_{v_1}}{\Omega_{v_2}}\right)_i} \tag{65}$$

where μ°_2 is chosen arbitrarily to correspond to that at $T_{r_i} = 2$.

Pseudocritical Properties of Natural Gases

The gas compressibility factor, its partial derivatives, heat capacity and thermal conductivity departure models employ the law of corresponding states. Consequently an estimate of the pseudocritical properties is necessary to calculate the reduced pressure and temperature. These pseudocriticals may be taken as the molal average as suggested by Kay (123) when the gas composition is available. However when the gas composition is not available and where the gas gravity (*G*) is the only known property of the gas, the pseudocriticals may be estimated from the Brown, et al. correlation (124). Flores (125) presented the correlation in an algebraic form as

$$p^{T}c_{H.C.} = 171.137 + 313.725\ G \qquad (66)$$

$$p^{P}c_{H.C.} = 695.100 - 40.000\ G\ ; \quad G \leq 0.85 \qquad (67)$$

$$= 704.396 - 51.724\ G\ ; \quad G > 0.85 \qquad (68)$$

Carr, et al. (126) have introduced corrections in order to make this correlation applicable to sour natural gases if the mole fractions of N_2, CO_2 and H_2S are known. Hence the pseudocriticals of sour natural gases may be estimated from:

$$p^{T}c = p^{T}c_{H.C.} - 250\ Y_{N_2} - 80\ Y_{CO_2} + 130\ Y_{H_2S} \qquad (69)$$

and

$$p^{P}c = p^{P}c_{H.C.} - 170\ Y_{N_2} + 440\ Y_{CO_2} + 600\ Y_{H_2S} \qquad (70)$$

PRELIMINARY MODEL TESTING

The various algebraic expressions selected in the previous section to represent compressibility factor, its partial derivatives, isobaric heat capacity, friction factor, and viscosity were tested by comparing with experimental data or correlations available in the literature. The results of these tests are presented in this section.

Compressibility Factor of Natural Gases

Table 1 presents a comparison of the compressibility factor model as expressed by Equation 23 with 1350 experimental data points (127,128,129). These experimental data are for sweet as well as sour natural gases with up to 73.85% H_2S and 54.46% CO_2.

Partial Derivatives of Compressibility Factor

The partial derivatives of the compressibility factor with respect to reduced pressure and temperature were compared to values determined graphically. This was achieved by using experimental compressibility factors (130,131). First, a few isobars and isotherms were plotted, then the slopes of these curves at specific P_r and T_r were calculated. Finally these results were compared to the partial derivatives estimated by the algebraic expressions. It was found

TABLE 1

COMPARISON OF CALCULATED Z-FACTORS
WITH EXPERIMENTAL DATA

Source of Experimental Data	No. of Points	Average Absolute Deviation from Experimental Values
Reference (127)	58	1.030%
Reference (128)	207	1.202%
Reference (129)	1085	0.976%
All Data Points	1350	1.013%

that Equation 25 gives an average absolute deviation of 5.84 per cent when compared with 5 points, while Equation 26 deviates by 1.96 per cent when compared with 16 points.

Isobaric Heat Capacity of Natural Gases

1. Ideal Isobaric Heat Capacity

The isobaric heat capacity of sweet natural gases which have no nitrogen may be represented by Equation 32 as suggested in Model Development. This is an equation of a family of straight lines, the slopes and intercepts of which are functions of the gas gravity.

To test the model represented by Equation 32, the compositions of 72 gases were collected from the literature and the equivalent pure hydrocarbon compositions were calculated. Equations 30 and 31 were then employed to calculate the ideal isobaric heat capacities over a temperature range of 30 to 300°F at 10°F intervals, thus producing 2016 data points. Data points for each gas mixture were plotted as shown in Figure 1. Each gas mixture showed a straight line relationship expressed by Equation 32:

$$\overset{\circ}{Cp}_{m_{H.C.}} = A(G) + B(G)T_r \tag{32}$$

A(*G*) and B(*G*) were found to be adequately represented by means of second degree polynomials in *G* as shown in Figure 2; that is,

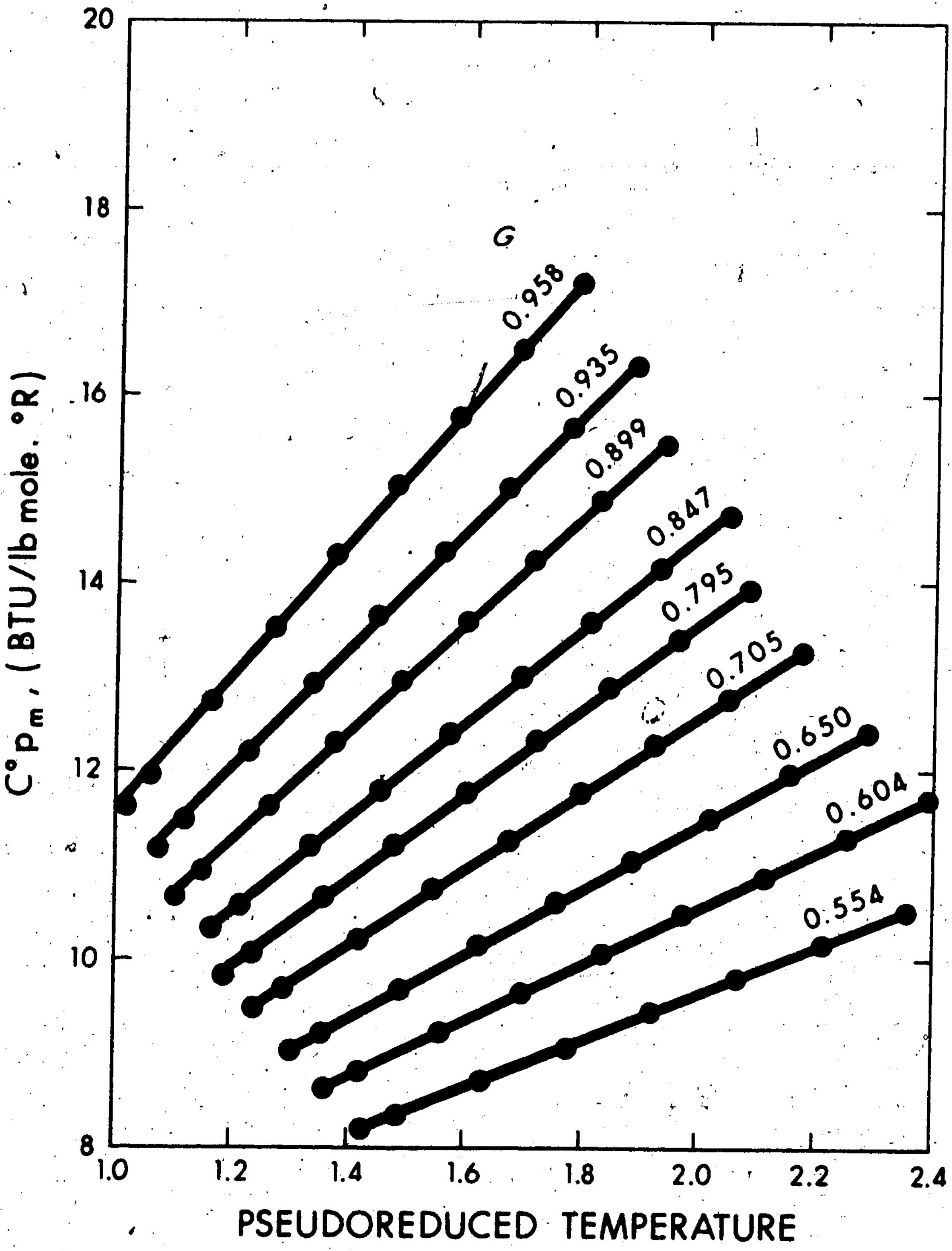

FIGURE 1. IDEAL ISOBARIC HEAT CAPACITY OF NATURAL GASES

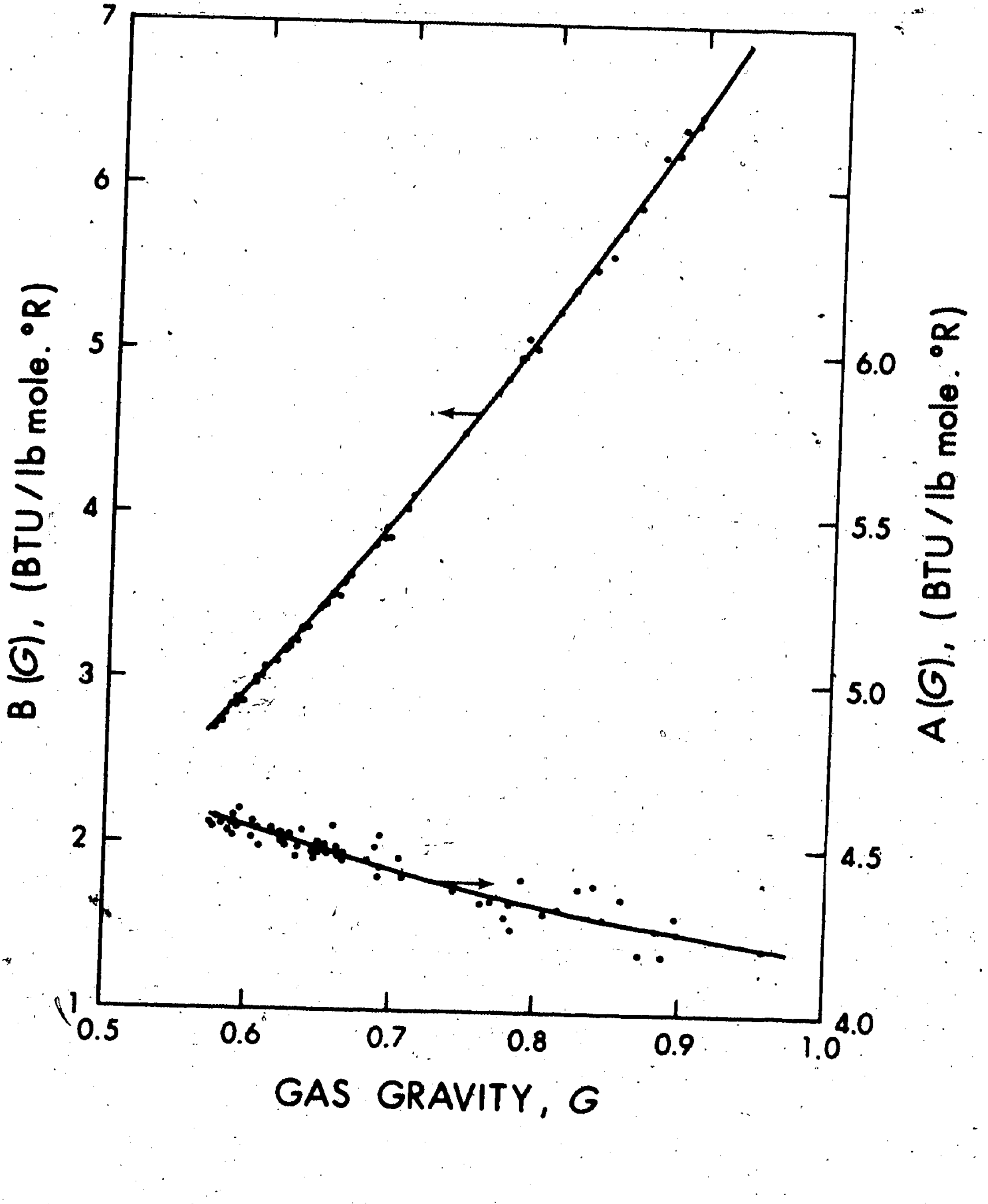

FIGURE 2. THE SLOPES AND INTERCEPTS IN EQUATION 32 AS FUNCTIONS OF GAS GRAVITY.

$$A(G) = B_1 + B_2G + B_3G^2 \tag{71}$$

and

$$B(G) = B_4 + B_5G + B_6G^2 \tag{72}$$

The coefficients in Equations 71 and 72 were obtained by minimizing the sum of squares of errors in A(G) and B(G) respectively. These coefficients (will be referred to as the set of coefficients No. 0) are:

$B_1 = 5.596695$ $B_2 = -2.233480$ $B_3 = 0.807265$

$B_4 = -1.003900$ $B_5 = 3.141600$ $B_6 = 5.758700$

Equations 32, 71 and 72 give an average absolute deviation of 0.079 per cent and a standard deviation of 0.00119 with respect to the 2016 data points used to determine the coefficients.

Studying the functional forms of Equations 34 through 36 individually showed that they can be expressed as:

$$F_{CO_2} = 1 + Y_{CO_2}\,(B_7 + B_8T) \tag{73}$$

$$F_{H_2S} = 1 + Y_{H_2S}\,(B_9 + B_{10}T) \tag{74}$$

and

$$F_{N_2} = 1 + (B_{11}Y_{N_2} + B_{12}Y_{N_2}^2) + (B_{13}Y_{N_2} + B_{14}Y_{N_2}^2)T \tag{75}$$

where T = temperature, °R.

The coefficients B_7 through B_{14} were obtained by minimizing the sum of squares of errors in $C^\circ_{p_m}$ (as expressed by Equation 33) and were found to be:

$B_7 = 0.4258600$ $B_8 = 1.24323 \times 10^{-3}$ $B_9 = -0.0405600$

$B_{10} = 1.00889 \times 10^{-3}$ $B_{11} = 0.3623622$ $B_{12} = -0.4661581$

$B_{13} = 0.97570 \times 10^{-3}$ $B_{14} = 2.708217 \times 10^{-3}$

Equations 33, 73, 74, and 75 give an average absolute deviation of 0.629 per cent and standard deviation of 0.00989 with respect to 2016 data points used to determine the coefficients B_7 through B_{14}.

Equations 32, 71, and 72 are applicable over a temperature range of 30-300°F to sweet natural gas which has no nitrogen. The same equations can be used over a broader temperature range; however, it is recommended that other sets of coefficients depending on temperature range of operation be used. The quality of fit of the new sets of coefficients is shown in Table 2.

2. Heat Capacity Departure:

The heat capacity departure model as expressed by Equation 37 was compared with the tabulated Weiss and Joffe generalized correlation (132). The model gives an average absolute deviation of 5.91 per cent when compared with 150 points.

TABLE 2

QUALITY OF FIT

	Set of Coefficients No. 1	Set of Coefficients No. 2
Temperature Range	80 ≤ T < 1160°F	1160 ≤ T ≤ 2240°F
No. of Test Points	7848 at 10°F intervals	
Average Deviation	-0.0103%	-0.00873%
Average Absolute Deviation	0.7709%	0.8024%
Standard Deviation	0.009586	0.009417
Maximum Deviation	4.097%	2.430%
No. of Points Having Deviations ≥ 2%	229	160

Set of Coefficients No. 1

$B_1 = 4.72288190$ $B_2 = -1.72132560$ $B_3 = 3.21713790$

$B_4 = -0.91474246$ $B_5 = 4.37040250$ $B_6 = 3.04884500$

Set of Coefficients No. 2

$B_1 = 6.31632700$ $B_2 = -0.063316262$ $B_3 = 11.54552600$

$B_4 = -0.97057393$ $B_5 = 4.75787240$ $B_6 = -0.80656836$

Tables 3 and 4 present comparisons of the various heat capacity departure correlations and the proposed generalized correlation as expressed by Equation 37 with the experimental data of Workman (133), Krase and Mackey (134). Table 5 presents comparisons of Equation 37 and the various $(C_p - C_p^\circ)$ generalized correlations with that of Weiss and Joffe. Table 6 present comparisons of Equation 37 with the experimental data of Sage, et al. (135) for two natural gases. It also presents the effect of any errors in $(C_p - C_p^\circ)$ on the isobaric heat capacity (C_p).

The Hydraulic Friction Factor

Table 7 presents comparisons between the values of friction factors as suggested by Moody (136) and Cullender and Smith (137). This comparison is carried out for the same 12 test-points used to test main models AA and BB.

Viscosity of Natural Gases

The Lee, et al. viscosity correlation (138) was tested with their 111 viscosity data points. This correlation gives an average absolute deviation of 3.71 per cent from experimental data and a standard deviation of 0.052527. The maximum deviation encountered is 16.38 per cent.

TABLE 3

COMPARISON OF EQUATION 37 AND VARIOUS $(C_p - C_p^\circ)$ GENERALIZED CORRELATIONS WITH THE EXPERIMENTAL DATA OF KRASE AND MACKEY FOR NITROGEN

T_r	P_r	$(C_p - C_p^\circ)_{exp.}$ Btu/lb mole.°R	% Deviation from Experimental Values			
			Equation 37	Weiss and Joffe	Edmister	Hougen and Watson
2.37	5.97	1.47	14	21	-1	77
2.40	14.91	2.30	-6	9	*	*
2.56	14.91	2.00	-3	13	*	*
2.96	14.91	1.46	1	10	*	*
3.36	14.91	1.12	*	13	*	*

* Outside the range of correlation.

TABLE 4

COMPARISON OF EQUATION 37 AND VARIOUS (C_p - C_p°) GENERALIZED CORRELATIONS WITH THE EXPERIMENTAL DATA OF WORKMAN FOR NITROGEN AND OXYGEN

P_r	$(C_p - C_p^\circ)_{exp.}$ Btu/lb mole.°R	% Deviations from Experimental Values				
		Equation 37	Weiss and Joffe	Edmister	Hougen and Watson	Lydersen, et al.
			Oxygen (T_r = 1.938)			
0.119	0.1116	2	*	-1	59	*
0.597	0.3630	3	-1	-4	46	-2
0.996	0.6140	-2	1	-2	45	9
1.392	0.8750	-1	-1	-4	39	40
1.792	0.1410	-1	-2	-4	38	31
2.390	0.1410	8	9	3	49	24
2.580	1.6800	-2	2	-6	34	7
			Nitrogen (T_r = 2.64)			
0.295	0.0671	5	13	-15	49	*
0.895	0.2230	-3	2	-23	75	*
1.478	0.3820	-6	-1	-26	68	-48
2.060	0.5400	-8	-4	-30	54	-41
2.660	0.6860	-8	-2	-30	53	-37
3.240	0.8260	-8	-2	-30	46	-37
3.840	0.9620	-8	-1	-28	40	-39

* Outside the range of correlation.

TABLE 5

COMPARISON OF EQUATION 37 AND OTHER (C_p - C_p^o) GENERALIZED CORRELATIONS WITH THE WEISS AND JOFFE CORRELATION

	% Deviation from the Weiss and Joffe Values			
Correlation	T_r = 1.0	1.2	1.5	2.5
Hougen and Watson	59 to 62	26 to 55	33 to 64	43 to 63
Edmister	11 to 37	-1 to -11	-4 to -8	-22 to -23
Lydersen, et al.	-9 to 35	5 to 29	-61 to 20	-30 to -108
Sherwood	13 to 35	1 to 35	-25 to 5	8 to 66
Equation 37	*	-6 to 6	-17 to 0	-4 to -13

* Outside the range of correlation.

TABLE 5

COMPARISON OF $(C_p - C_p^{\circ})$ ESTIMATED BY EQUATION 37 WITH THE SAGE, et al. EXPERIMENTAL VALUES FOR TWO NATURAL GASES

	Temperature, °F														
	70			130			190			250			310		
Pressure (psia)	$(C_p-C_p^{\circ})_{exp.}$	% Error in $(C_p-C_p^{\circ})_{calc.}$*	% Error in $C_{p_{calc.}}$	$(C_p-C_p^{\circ})_{exp.}$	% Error in $(C_p-C_p^{\circ})_{calc.}$*	% Error in $C_{p_{calc.}}$	$(C_p-C_p^{\circ})_{exp.}$	% Error in $(C_p-C_p^{\circ})_{calc.}$*	% Error in $C_{p_{calc.}}$	$(C_p-C_p^{\circ})_{exp.}$	% Error in $(C_p-C_p^{\circ})_{calc.}$*	% Error in $C_{p_{calc.}}$	$(C_p-C_p^{\circ})_{exp.}$	% Error in $(C_p-C_p^{\circ})_{calc.}$*	% Error in $C_{p_{calc.}}$
	Natural Gas A														
200	0.3932	41.0	1.6	0.3324	15.0	0.5	0.2918	-4.8	-0.1	0.2594	-19.0	-0.4	0.2310	-29.0	-0.5
400	0.8025	51.0	3.3	0.6769	19.8	1.2	0.5958	-2.9	-0.2	0.5391	-19.7	-0.9	0.4823	-30.5	-1.2
500	0.0113	57.0	5.3	0.8572	21.8	1.7	0.7539	-2.1	-0.1	0.6809	-19.5	-1.1	0.6120	-30.9	-1.5
	Natural Gas B														
200	0.3847	29.1	1.1	0.3326	3.5	0.1	0.2805	-10.5	-0.3	0.2445	-22.0	-0.5	0.2124	-29.8	-0.5
400	0.8055	33.4	2.5	0.6853	5.9	0.4	0.5731	-9.1	-0.5	0.4929	-20.5	-0.9	0.4328	-29.7	-1.1
600	1.2864	36.0	4.2	1.0820	5.9	0.6	0.8816	-[illegible].2	-0.6	0.7414	-18.9	-1.2	0.6492	-28.5	-1.5

* $(C_p - C_p^{\circ})$ calculated by using Equation 37, Btu/lb mole.°R.

TABLE 7

COMPARISONS BETWEEN THE MOODY AND CULLENDER AND SMITH FRICTION FACTORS

Test-Point	R_{e_b} (10^6)	R_e (10^6)		Friction Factor			% Deviation of Cullender and Smith Values from Moody's	
				Moody$^+$		Cullender and Smith		
		W	S	W	S		W	S
A_1	13.018	0.280	0.275	0.016719	0.016749	0.014319	-14.36	-14.51
A_2	13.018	0.563	0.543	0.015693	0.015735	0.014319	-8.76	-9.00
A_3	13.018	0.798	0.761	0.015340	0.015383	0.014319	-6.66	-6.92
A_4	13.018	1.100	1.032	0.015091	0.015133	0.014319	-5.12	-5.38
A_5	13.018	1.744	1.608	0.014825	0.014864	0.014319	-3.41	-3.67
A_6	13.018	1.816	1.673	0.014807	0.014845	0.014319	-3.30	-3.54
B_1	15.952	1.326	1.286	0.014479	0.014500	0.013681	-5.51	-5.65
B_2	15.952	2.016	1.906	0.014243	0.014270	0.013681	-3.95	-4.13
B_3	15.952	2.635	2.414	0.014131	0.014165	0.013681	-3.18	-3.42
B_4	15.952	3.203	2.880	0.014065	0.014099	0.013681	-2.73	-2.97
C	15.952	4.076	3.515	0.013997	0.014037	0.013681	-2.26	-2.54
D	15.952	4.279	3.677	0.013985	0.014024	0.013681	-2.17	-2.45

where W indicates well head conditions
S indicates bottom-hole conditions
+ indicates absolute roughness of 0.0006 inch.

DATA AND RESULTS

Back Pressure Data

Back pressure data were used to test main models AA and BB. The test consists of 12 test-points taken from four back pressure tests for three natural gas wells. Table 8 presents a summary of these data. The pressure and temperature data are shown in Table 9. Appendix F is a tabulation of the source data and gas properties upon which this study was based.

Accuracy of Field Data

1. Flow Rate:

The flow rate was measured using either critical flow provers or orifice meters, and was reported at the standard conditions of 14.65 psia and 60°F. The accuracy of the flow measuring devices is not available.

2. Length of Tubing:

The reported values of length of tubing are those from Kelly Bushing (K.B.) to the mid-point of perforations (M.P.P.). The depth at which the pressure measuring device was located is not stated. The length of tubing of interest in this study is that between the well head and the bottom-hole pressure measuring device. This may be different from

TABLE 8

SUMMARY OF FIELD DATA

Length of Flow String, ft.	4753 - 11029
Flow Rate, MMSCF/Day	0.833 - 17.359
Gas/Condensate Ratio, MMSCF/bbl.	0.208 - ∞
Tubing-Head Temperature, °F	47 - 170
Bottom-Hole Temperature, °F	115 - 246
Tubing-Head Pressure, PSIA	985 - 3250
Bottom-Hole Pressure, PSIA	1104 - 4250
Water/Gas Ratio, bbl/MMSCF	0.0 - 5.2
Y_{CO_2}, % mole	0.56 - 2.88
Y_{H_2S}, % mole	0.0 - 18.89
Y_{N_2}, % mole	1.05 - 4.56

TABLE 9

PRESSURE AND TEMPERATURE DATA

Test-Point	Q_o (MMSCF/Day)	Tubing-Head		Bottom-Hole		$\Delta P_{meas.}$ (PSIA)	W./G. Ratio (bbl/MMSCF)	G./Cond. Ratio (MMSCF/bbl)
		Temp. (°F)	Press. (PSIA)	Temp. (°F)	Press. (PSIA)			
A_1	0.883	47	1660.0	115	1928.7	268.7	0.0	0.2388
A_2	1.606	53	1566.0	115	1808.4	242.4	0.0	0.3071
A_3	2.186	57	1473.0	115	1701.8	228.8	0.0	0.2081
A_4	2.910	58	1360.0	115	1567.5	207.5	0.0	0.5569
A_5	4.043	69	862.0	115	1048.0	186.0	0.0	0.3111
A_6	4.291	68	895.0	115	1103.7	208.7	0.0	2.4655
B_1	6.527	132	3249.4	246	4249.6	1000.2	1.5	∞
B_2	9.598	140	3168.4	246	4205.8	1037.4	1.5	∞
B_3	12.048	153	3078.4	246	4163.2	1084.8	1.5	∞
B_4	14.277	158	2990.4	246	4123.2	1132.8	1.5	∞
C	15.606	155	2314.5	243	3295.5	981.0	5.2	∞
D	17.359	170	2599.5	243	3690.4	1090.9	1.3	∞

the values reported in Appendix F for length of tubing.

3. Pressure Data:

The tubing head pressure was measured with a dead weight gauge or a Barton gauge. The bottom-hole pressure was measured with an Amerada, tester type, gauge, or the Sperry Sun gauge. The accuracy of these pressure devices is shown in Table 10 (139). The accuracy of the pressure data, is presented in Table 11.

TABLE 10

ACCURACY OF PRESSURE MEASURING DEVICES

Measuring Devices	Accuracy, % of Rated Pressure
Dead Weight Gauge	±0.0
Barton Gauge	±0.5* to ± 0.75**
Sperry Sun Gauge	±0.1
Amerada or Tester Type Gauge	±0.2* to ± 0.25**

* For rated pressure less than or equal to 2000 psi.
** For rated pressure greater than 2000 psi.

Results

Computer FORTRAN programs were written for main models AA and BB (method 2) and methods 1, 3, and 4.

Table 12 presents comparisons between the measured and

TABLE 11

ACCURACY OF PRESSURE DATA

Test-Point	Tubing-Head Press.		Bottom-Hole Press.		Accuracy of Press. Drop (PSI)
	Rated Press. of Gauge (PSI)	Accuracy (PSI)	Rated Press. of Gauge (PSI)	Accuracy (PSI)	
A_1	2000	±10.0	2000	± 4.0	±14.0
A_2	2000	±10.0	2000	± 4.0	±14.0
A_3	2000	±10.0	2000	± 4.0	±14.0
A_4	2000	±10.0	2000	± 4.0	±14.0
A_5	2000	±10.0	2000	± 4.0	±14.0
A_6	2000	±10.0	2000	± 4.0	±14.0
B_1	4000	±30.0	5650	±11.3	±41.3
B_2	4000	±30.0	5650	±11.3	±41.3
B_3	4000	±30.0	5650	±11.3	±41.3
B_4	4000	±30.0	5650	±11.3	±41.3
C	*	± 0.0	5650+	±11.3	±11.3
D	*	± 0.0	5650+	±11.3	±11.3

* Dead Weight Gauge was used.
\+ Assumed.

calculated flowing bottom-hole pressures for all test-points. The test-points are presented in order of increasing flow rate. Table 13 presents the deviations of the calculated from the measured flowing bottom-hole pressures, while Table 14 exhibits the per cent deviations of the calculated from the measured pressure drops. Table 15 presents the various flowing bottom-hole pressures calculated by main models AA and BB.

TABLE 12

COMPARISONS BETWEEN THE MEASURED AND THE CALCULATED FLOWING BOTTOM-HOLE PRESSURES

Test-Point	$B.H.P._{meas.}$ (PSIA)	$B.H.P._{calc.}$, (PSIA) Method 1	Method 2 (Main Model BB)*	Method 3	Method 4
A_1	1928.7	1888.4	1911.8	1916.0	1915.8
A_2	1808.4	1782.5	1799.5	1803.6	1803.1
A_3	1701.8	1682.5	1695.8	1699.6	1698.9
A_4	1567.5	1557.5	1567.7	1570.7	1569.8
A_5	1048.0	1029.6	1031.1	1031.9	1029.9
A_6	1103.7	1069.4	1071.1	1071.8	1069.8
B_1	4249.6	4138.4	4210.5	4235.4	4233.1
B_2	4205.8	4095.1	4152.2	4173.6	4169.9
B_3	4163.2	4046.2	4085.3	4101.8	4096.9
B_4	4123.2	4007.6	4037.5	4051.2	4045.2
C	3295.5	3277.9	3293.0	3301.6	3294.4
D	3690.4	3675.2	3686.1	3692.8	3684.9

* G, ${}_pT_c$, ${}_pP_c$, Y_{N_2}, Y_{CO_2}, and Y_{H_2S} were supplied to the computer program as data.

TABLE 13

DEVIATIONS OF THE CALCULATED FLOWING BOTTOM-HOLE PRESSURES FROM THE MEASURED VALUES

Test-Point	Accuracy of B.H.P.$_{calc.}$ (PSI)	Accuracy of B.H.P.$_{meas.}$ (PSI)	Deviation of B.H.P.$_{calc.}$ from B.H.P.$_{meas.}$, (PSI)			
			Method 1	Method 2 (Main Model BB)	Method 3	Method 4
A_1	±10.0	± 4.0	- 40.3	-16.9	-12.7	-12.9
A_2	±10.0	± 4.0	- 25.9	- 8.9	- 4.8	- 5.3
A_3	±10.0	± 4.0	- 19.3	- 6.0	- 2.2	- 2.9
A_4	±10.0	± 4.0	- 10.0	+ 0.2	+ 3.2	+ 2.3
A_5	±10.0	± 4.0	- 18.4	-16.9	-16.1	-18.1
A_6	±10.0	± 4.0	- 34.3	-32.6	-31.9	-33.9
B_1	±30.0	±11.3	-111.2	-39.1	-14.2	-16.5
B_2	±30.0	±11.3	-110.7	-53.6	-32.2	-35.9
B_3	±30.0	±11.3	-117.0	-77.9	-61.4	-66.3
B_4	±30.0	±11.3	-115.6	-85.7	-72.0	-78.0
C	± 0.0	±11.3	-17.6	- 2.5	+ 6.1	- 1.1
D	± 0.0	±11.3	-15.2	- 4.3	+ 2.4	- 5.5

TABLE 14

DEVIATIONS OF THE CALCULATED PRESSURE DROPS FROM THE MEASURED VALUES

Test-Point	Accuracy of $\Delta P_{meas.}$ in (% of $\Delta P_{meas.}$)	% Deviation of $\Delta P_{calc.}$ from $\Delta P_{meas.}$			
		Method 1	Method 2 (Main Model BB)*	Method 3	Method 4
A_1	±4.9	-15.0	- 6.3	- 4.7	- 4.8
A_2	±6.1	-10.7	- 3.7	- 2.0	- 2.2
A_3	±6.7	- 8.4	- 2.6	- 1.0	- 1.3
A_4	±7.5	- 4.8	+ 0.1	+ 1.5	+ 1.0
A_5	±6.7	- 9.9	- 9.1	- 8.7	- 9.7
A_6	±7.5	-16.4	-15.6	-15.3	-16.2
B_1	±4.4	-11.1	- 3.9	- 1.4	- 1.7
B_2	±4.3	-10.7	- 5.2	- 3.1	- 3.5
B_3	±4.1	-11.2	- 7.4	- 5.9	- 6.3
B_4	±3.9	-10.2	- 7.6	- 6.4	- 6.9
C	±1.0	- 1.8	- 0.3	+ 0.6	- 0.1
D	±0.9	- 1.4	- 0.4	+ 0.2	- 0.5

* G, ${}_pT_c$, ${}_pP_c$, Y_{N_2}, Y_{CO_2}, and Y_{H_2S} were supplied to the computer program as data.

TABLE 15

A COMPARISON OF CALCULATED BOTTOM-HOLE PRESSURES AND COMPUTATIONAL COSTS FOR MAIN MODELS AA AND BB

	Main Model BB						Main Model AA	
	a		b		c		c	
Test-Point	B.H.P.cal. (PSIA)	Cost of Comput. ($)	B.H.P.cal. (PSIA)	Cost of Comput. ($)	B.H.P.cal. (PSIA)	Cost of Comput. ($)	B.H.P.cal. (PSIA)	Cost of Comput. ($)
A_1	1911.8	0.55	1911.8	0.55	1911.8	0.75	1911.8	10.83
A_2	1799.5	0.46	1799.5	0.46	1799.4	0.69	1799.5	7.34
A_3	1695.8	0.40	1695.8	0.40	1695.8	0.53	1695.8	5.56
A_4	1567.7	0.42	1567.7	0.42	1567.7	0.51	1567.7	4.19
A_5	1031.1	0.35	1031.1	0.36	1031.1	0.44	1031.1	3.18
A_6	1071.0	0.31	1071.1	0.31	1071.0	0.45	1071.1	3.24
B_1	4192.4	0.41	4210.5	0.41	*	*	*	*
B_2	4135.9	0.43	4152.2	0.40	*	*	*	*
B_3	4071.3	0.40	4085.3	0.36	*	*	*	*
B_4	4025.2	0.35	4037.5	0.42	*	*	*	*
C	3289.1	0.42	3293.0	0.39	*	*	*	*
D	3678.3	0.32	3686.1	0.36	*	*	*	*

* Gas analysis is not available.

a G, Y_{CO_2}, Y_{N_2}, and Y_{H_2S} were supplied to the program as data.

b G, pT_c, pp_c, Y_{CO_2}, Y_{N_2}, and Y_{H_2S} were supplied to the program as data.

c The gas composition was supplied to the program as data.

DISCUSSION

Compressibility Factor of Natural Gases

The natural gas compressibility factor model as expressed by Equation 23, which is an equation of state, can predict Z with an average deviation of 1.013 per cent from experimental data as shown in Table 1. Such accuracy is considered acceptable for engineering applications.

Partial Derivatives of Compressibility Factor

The partial derivatives of the gas compressibility factor may be calculated by employing a numerical scheme or by using Equations 25 and 26. However, a numerical technique is cumbersome and costly. These equations are expected to work well since they were derived from an equation of state, Equation 23. Robinson (140) states that the B-W-R equation of state gives good predictions for the partial derivatives of compressibility factor.

Isobaric Heat Capacity of Natural Gases

1. Ideal Isobaric Heat Capacity:

The proposed model for ideal isobaric heat capacity of natural gases as expressed by Equations 32, 71, and 72 gives excellent results as was shown earlier in Preliminary Model Testing. However, it is limited in its application to a specific temperature range and natural gases which consist of pure hydrocarbons. Furthermore, it requires that the hydrocarbon gas mixtures be rich in methane.

The ideal isobaric heat capacity of sweet natural gas may be expressed by the same equations over a wider range of temperature; however, new sets of coefficients, B_1 through B_6, should be used. The set of coefficients No. 1 covers the range of temperature of $80 \leq T < 1160°F$, while the set of coefficients No. 2 covers the range of temperature of $1160 \leq T \leq 2240°F$. The temperature of 1160°F was chosen in a way that the standard deviation given by the model is almost the same when either set is used over its range as may be seen from Table 2.

It should be noted that the pseudoreduced temperature term appearing in Equation 32 is defined as $\frac{T}{{}_pT_c}$. The pseudocritical temperature, ${}_pT_c$, should be calculated using the molal average not Equations 66 and 69. However if the gas composition is not available Equations 66 and 69 may be used but higher errors should be expected.

Both sets of coefficients No. 1 and 2 were obtained by employing the Thinh, et al. equations (141) in obtaining the data points. These sets of coefficients could be used with Equations 32, 71, and 72 for natural gases operating at high temperatures.

Equations 33, 73, 74 and 75 were developed to take into account the effect of presence of N_2, CO_2, and H_2S. These correction equations give good results as was indicated earlier in Preliminary Model Testing; however, their use is subject to the following restrictions.

i. $32 \leq T \leq 300°F$

ii. $0.0 \leq Y_{H_2S} \leq 0.40$ mole fraction

iii. $0.0 \leq Y_{CO_2} \leq 0.20$ mole fraction

iv. $0.0 \leq Y_{N_2} \leq 0.25$ mole fraction

v. They may only be applied when the set of coefficients No. 0 is used in Equations 32, 71, and 72.

2. Heat Capacity Departure:

The proposed heat capacity departure model as expressed by Equation 37 was compared with the Weiss and Joffe (142) generalized correlation as shown in Table 5. When the various $(C_p - C_p^\circ)$ generalized correlations (143,144) were compared with the Weiss and Joffe correlation, the proposed model gave the least deviation as shown in Table 5. This is probably due to the fact that both correlations use the B-W-R equation of state.

The Weiss and Joffe correlation was developed for pure hydrocarbons, then used to estimate the heat capacity departure of oxygen and nitrogen. It gave acceptable results and as a result was considered as a generalized correlation (145). In a similar manner the proposed heat capacity departure model was developed from an equation of state which describes the volumetric behavior of sweet natural gases, Equation 23, then was tested by comparing with the experimental data of Workman (146), Krase and Mackey (147) for oxygen and nitrogen. This model gives satisfactory agreement with experimental data as shown in Tables 3 and 4. This check was performed in order to extend the applicability of the proposed model to non-hydrocarbon gases and thus to sour natural gases.

The various comparisons and tests performed on the proposed model suggest that it could be used as a generalized correlation for heat capacity departure. This is in spite of the fact that the coefficients of Equation 37 were originally developed using only sweet natural gas compressibility factor data.

It should be mentioned that although errors in (C_p - C_p°) may at times be large, the effect on overall accuracy of C_p is usually much smaller. Since the value of C_p is the quantity desired, it is the error in $\left(\frac{C_p - C_p^\circ}{C_p}\right)$ which is of importance not the error in (C_p - C_p°) (148) as shown in Table 6.

The proposed (C_p - C_p°) generalized correlation has an advantage over the others in that it is adaptable to computer usage.

The Hydraulic Friction Factor

The friction factor in the rough pipe region is represented by the Von Kármán formula. Colebrook (149) derived an approximation for the friction factor in this region when the absolute roughness of pipe is known. The friction factor equations suggested by Cullender and Smith (150) are but the Colebrook approximation when the absolute roughness is 0.0006 inch.

The assumption of fully developed turbulent flow of gas through tubing may be in error. As for example, for the 12 test-points used in testing main models AA and BB, flow was in the transition zone as the comparison between Reynolds number (R_e) and the boundary Reynolds number (R_{e_b}) in Table 7 may indicate. As a result the Cullender and Smith equations, which employ the assumption of fully turbulent flow, may give errors in friction factors estimated ranging from -2.17 to -14.51 per cent as shown in Table 7. This order of error may produce errors of -7.9 to -0.2 psi in the calculated flowing bottom-hole pressure as the comparison between methods 3 and 4 in Table 12 may indicate.

Overall Heat-Transfer Coefficient

In the development of the U_1 model it was assumed that the various solid layers were tightly fitted together with no intervening "air spaces". If the layers touch each other only at several points, the resistance to heat flow will increase remarkably (151). This suggests that the thermal resistance of the surroundings in gas well systems will have a comparatively large value due to the presence of the annular space.

In shallow gas wells where there exists one set of casing only, the composite cylindrical wall consists of tubing, annular space, casing, and cement. In deep wells the geometry of the composite wall is so complicated due to several factors such as: the presence of more than one set of casing, eccentricity of both tubing and casing sets, lack of cement at some sections, inevitable reduction in well diameter as drilling advances, caving at some shale sections and enlargement at others, presence of fluid such as water in contact with either casing or cement at some unknown sections along the well course, corrosion, etc.. Due to such complexity and lack of information, it is reasonable to assume that the system has an average thermal resistance, R*, which will be determined by a trial-and-error procedure. Furthermore, this complexity and lack of information show it is not possible to employ model U_1 in gas well systems.

In model U_2, the term $\frac{1}{h_o}$ is usually small, costly to compute, and of the order of 1 to 5 per cent of R*. This suggests that model U_3 is preferable to U_2 since it requires considerably less computing time and does not affect the final results.

1. Stanton Number:

Comparison of the values of the Stanton number estimated using the various models indicates that there is little difference for low Prandtl Number fluids like air (152). However, the best expression that reproduces experimental data is that of Gowen and Smith (153). The reliability of this model in predicting the Stanton numer lies in an implicit assumption that the friction factor and hence the equivalent sand relative roughness $(\frac{\epsilon_s}{d})$ completely characterizes the effect of roughness on temperature profile.

2. Prandtl Number of Natural Gases:

The Prandtl number, which is necessary to calculate the Stanton number, is defined as in Equation 50. For a gas mixture, it is recommended that the Prandtl number be calculated using Equation 50; however, the various terms appearing in the definition should be estimated separately (154).

Thermal Conductivity of Gas Mixtures

Equation 56 applies to paraffins giving an average deviation of 2.6 per cent from the experimental data (155), while Equation 57 applies to non-hydrocarbons giving an average deviation of 2.2 per cent from the experimental data (156). The Sutherland constant estimated from boiling point temperature as expressed by Equation 54 may not be reliable; however, an error of 20 per cent in the Sutherland constant produces only an error of 1 per cent in the estimated gas mixture thermal conductivity at atmospheric pressure (157). Hence the simplification introduced by Equation 54 (due to necessity) is justified.

Equations 59, 60, and 61 for the estimation of the thermal conductivity departure, are usually applicable to pure non-polar gases. However, Reid and Sherwood (158) have recommended the use of these equations for gas mixtures. Their recommendation was based on an assumption that a gas mixture can be considered as a hypothetical pure gas with pseudocritical properties. This suggestion has been accepted because few experimental data for thermal conductivity of high-pressure gas mixtures are available.

The gas mixture thermal conductivity model is expected to give errors of less than 5 per cent for simple non-polar gas mixtures for $T_r < 1.3$. The same model may be used for polar and non-polar polyatomic gas mixtures, but larger errors may occur (159).

Viscosity of Pure and Natural Gases

Equation 63 was originally developed for monatomic gases. However, it is found to be remarkably good for polyatomic gases as well (160).

Equation 64 is an approximation for the collision integral function, Ω_v, for non-polar gases. It deviates 1 to 2 per cent from the tabulated function (161). It may also be applied to polar gases; however, errors up to 2 per cent may occur (162). The collision integral function is required for the estimation of pure gas viscosities at atmospheric pressure as given by Equation 53.

The Lee, et al. correlation (163) for calculating the viscosity of sweet natural gases gives errors ranging from -11.05 to +16.38 per cent from their experimental data. The presence of N_2, CO_2, and H_2S tends to increase the viscosity of natural gases (164). As a result the Lee, et al. correlation may give errors larger or smaller than those mentioned when applied to sour natural gases.

Flowing Bottom-Hole Pressure

As may be seen from Tables 12 and 13, method 1 always underestimates the bottom-hole pressure. Methods 2 and 4 underestimate 11 of 12 test-points, while method 3 underestimates 9 of 12 test-points. It can also be seen that method 3 deviates the least from the measured bottom-hole pressure for 10 of 12 test-points.

Table 14 indicates that methods 2, 3, and 4 estimate pressure drops for test-points A_2, A_3, A_4, B_1, C, and D with accuracy comparable to those of the measured values. The various methods estimate flowing bottom-hole pressures in excellent agreement with the measured values for test-points A_4, C, and D as shown in Table 14. This is probably due to using accurate surface pressure gauges (Dead Weight Gauges) in test-points C and D.

The analysis of Table 14 exhibits what may be a systematic error in the calculated bottom-hole pressure as the flow rate increases as is shown in Figure 3. This behavior may explain the excellent agreement of the various methods with test-point A_4.

Figure 4 shows examples of estimated (methods 1 and 2) and assumed (methods 3 and 4) temperature profiles. It shows that the fluid temperature at every depth is decreasing in the following order: methods 1, 2, and either 3 or 4. It can also be seen that the fluid temperature is always greater than or equal to the earth temperature. This suggests

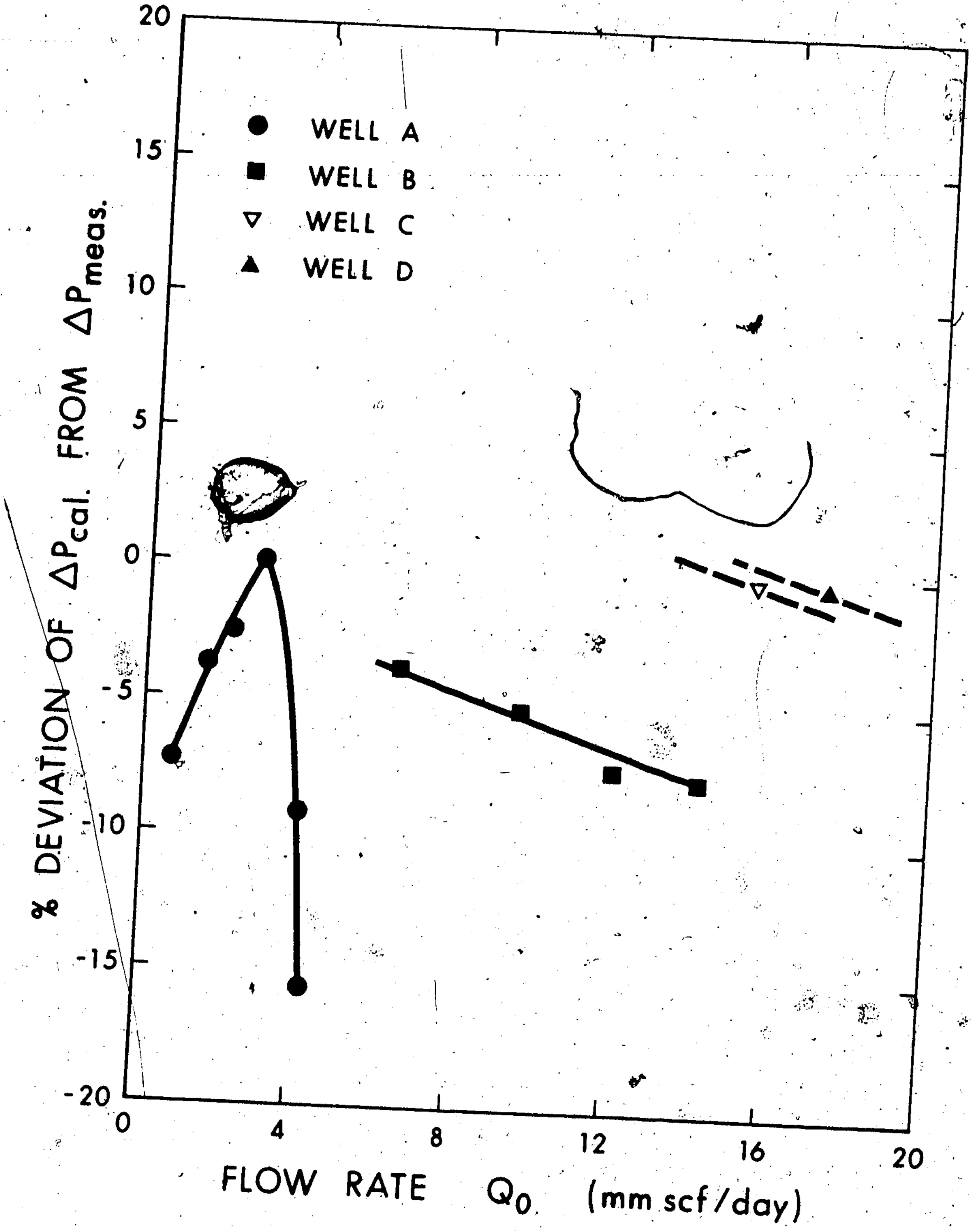

FIGURE 3. DEVIATION OF PRESSURE DROPS CALCULATED BY METHOD 2 FROM MEASURED VALUES FOR ALL TEST POINTS

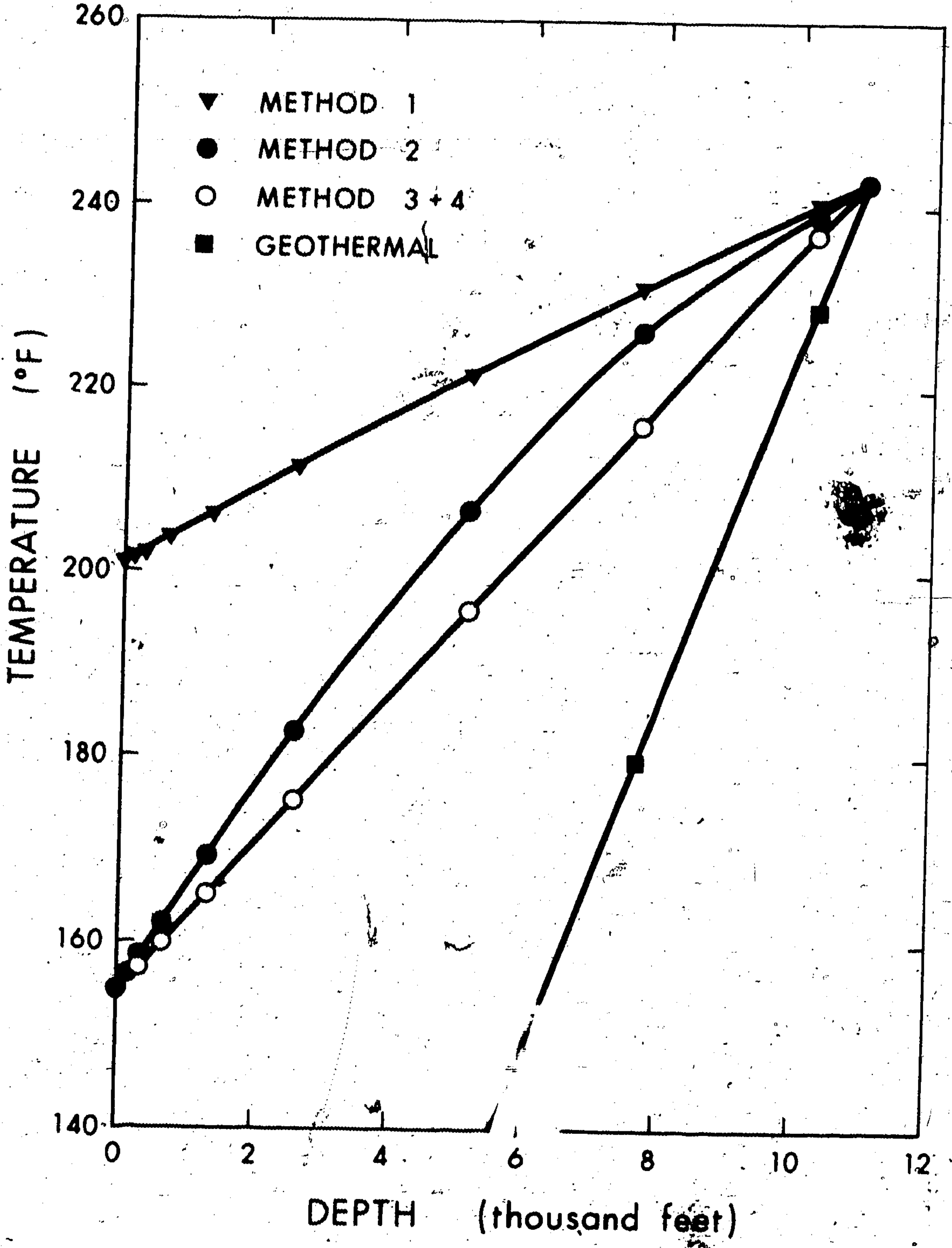

FIGURE 4. FLUID TEMPERATURE PROFILES CALCULATED BY METHODS 1, 2, 3, AND 4 FOR TEST-POINT C

that in these examples heat is being transferred from the flowing fluid to the surroundings. Furthermore, it suggests that methods 3 and 4 are equivalent to a hypothetical case where heat-transfer is such as to bring the fluid temperature profile to a straight line.

The analysis of Table 12 indicates that for any test-point, method 1 predicts the lowest flowing bottom-hole pressure followed by methods 2, 4, and finally method 3. This behavior is consistent in all test-points for methods 1, 2, and 3. However at three test-points, namely A_5, A_6, and D, methods 2 and 4 switch positions. This may be due to the use of the Cullender and Smith friction factor equations (which deviate from the Von Kármán rough pipe law by -0.04%, -0.04%, -0.41% for test-points A_5, A_6, and D respectively) in method 4.

Reasons for Deviations of Calculated from Measured Bottom-Hole Pressures

This discrepancy between measured flowing bottom-hole pressures and those calculated by method 2 may be attributed to a combination of the following factors:

1. The Existence of Two-Phase Flow:

The presence of condensate in the flowing gas has been dealt with according to Standing (165) when the gas/condensate ratio is greater than 40,000 SCF/bbl. He suggests that the gravity of the flowing stream (G_f) be used instead of that of dry gas (G) according to:

$$G_f = \frac{G + 4.591 \frac{\rho_{cond.}}{R}}{1 + \frac{1123}{R}} \tag{71}$$

where R = Gas/Condens Ratio, SCF/bbl.

and $\rho_{cond.}$ = density of condensate, gm/cm^3.

The pseudocritical pressure and temperature of the flowing stream may then be estimated using G_f.

This method of dealing with two-phase flow does not account for the effect of the liquid phase presence on the isobaric heat capacity, viscosity, thermal conductivity, and friction factor.

2. The Presence of Water Vapor in the Flowing Fluid:

The presence of water vapor in the flowing stream has not been accounted for. Although the water/gas ratio is small (1.5 bbl/MMSCF) except for test-point C (5.2 bbl/MMSCF), water is highly polar and as a result the behavior of the water-gas mixture system may deviate from the law of corresponding states. This behaviour may not be estimated accurately by the correlations employed and as a result errors in the estimated pressure may be introduced. However, it is surprising to find that test-point C is in very good agreement for the various methods.

3. Well Bore Deviation from Vertical:

Due to lack of information to the contrary all calculathat the gravity of the flowing stream (G_f) be used instead of that of dry gas (G) according to:

$$G_f = \frac{G + 4.591 \frac{\rho_{cond.}}{R}}{1 + \frac{1123}{R}} \qquad (71)$$

where R = Gas/Condensate Ratio, SCF/bbl.

and $\rho_{cond.}$ = density of condensate, gm/cm^3.

The pseudocritical pressure and temperature of the flowing stream may then be estimated using G_f.

This method of de[illegible] with two-phase flow does not account for the effect [illegible] the liquid phase presence on the isobar[illegible] heat capacity, viscosity, thermal conductivity, and friction factor.

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3. Well Bore Deviation from Vertical:

Due to lack of information to the contrary all calculations of flowing bottom-hole pressure were made assuming that the tubing was situated in the vertical position. However, in practice well bore and therefore tubing segments may deviate from the vertical as is shown in Figure 5. Such deviations may result in a decrease in pressure drop, in which case the calculated flowing bottom-hole pressure may show positive deviations.

4. Unreliable Estimates of the Viscosity of Sour Gases:

The presence of appreciable amounts of CO_2, N_2, and H_2S tends to increase the viscosity of hydrocarbon mixtures (167). The Lee, et al. (168) viscosity correlation, therefore, may underestimate or overestimate the viscosity of sour natural gases depending on the sign and magnitude of errors produced by this correlation when applied to hydrocarbon mixtures. Consequently Reynolds number and hence the friction factor may be in error. Such errors may be significant especially at relatively low Reynolds number.

5.- Uncertainty of the Length of Flow String:

The measured flowing bottom-hole pressures were reported at the mid-point-of-perforations (MPP). It is not known from the available data whether the measuring device was located at that point or whether in fact the tubing extended to the mid-point of perforations. If the tubing does not extend to the mid-point-of-perforations, method 2 tends to under-

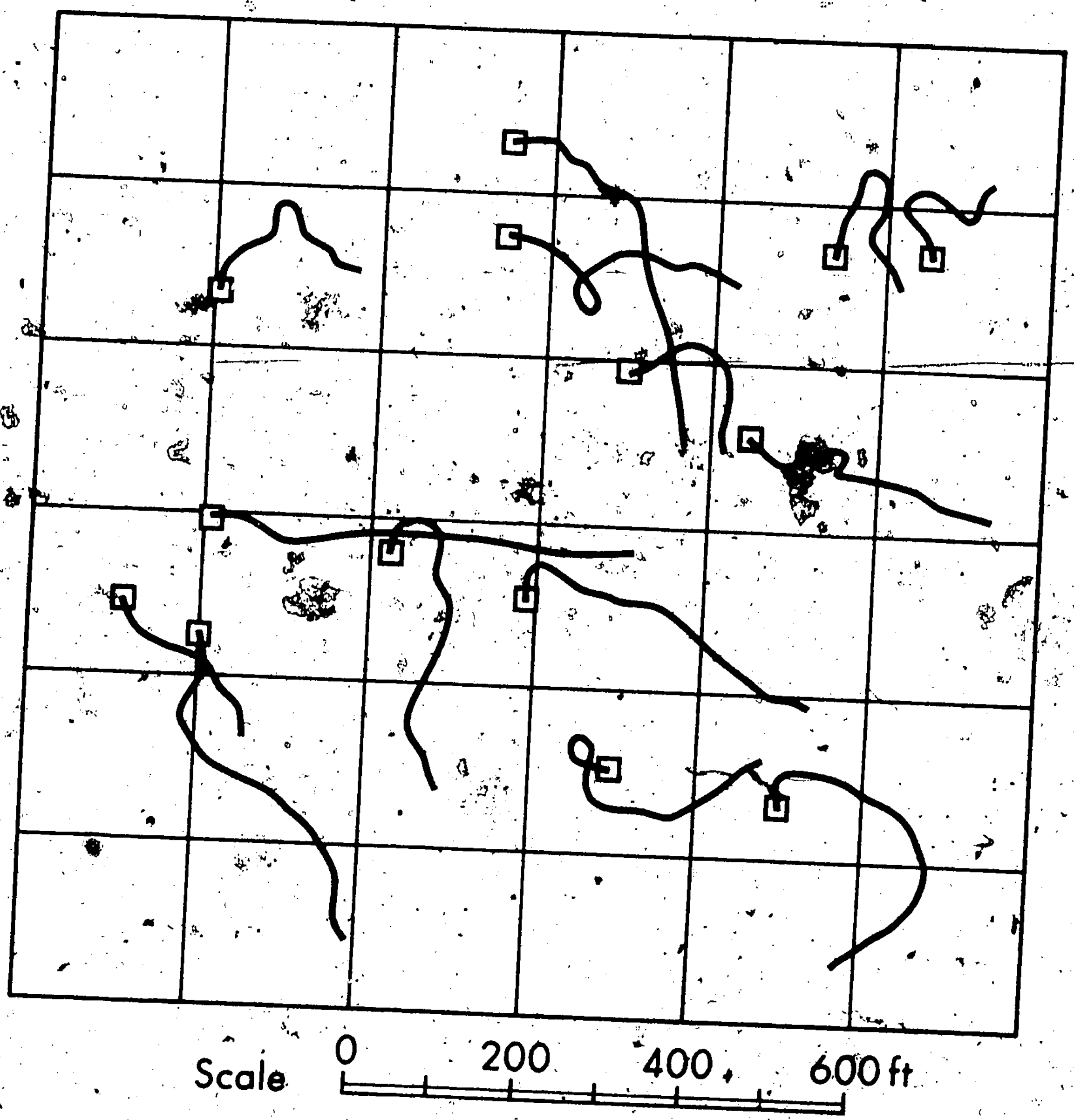

FIGURE 5. PLAN VIEW OF 14 VERTICAL WELLS DRILLED TO 6000 FEET (166)

estimate the flowing bottom-hole pressure.

6. Variation of the Overall Heat-Transfer Coefficient:

Models U_2 and U_3 (due to necessity) were employed as a means for estimating the overall heat-transfer coefficient for complex geometries such as those encountered in gas wells. However, these approximations may not be reasonable from a physical standpoint. Consequently the heat-transfer between the flowing fluid and the surroundings may be in error which may affect the calculated flowing bottom-hole pressure.

Main models AA and BB employ models U_2 and U_3 respectively. It has been found that these main models yield calculated pressure and temperature profiles which are essentially the same; however, the cost of computation may be reduced by about a factor of 10 when using main model BB instead of AA as may be seen from Table 15. For example in six test-points the range of cost for the calculations when the computer programs were stored on disks, was:

Main model AA	\$3.18 - \$10.83
Main model BB	\$0.44 - \$ 0.75

7. Uncertainty of Absolute Roughness of Tubing:

This is probably one of the most important sources of error. The absolute roughness of tubing was taken as 0.0006 inch. This value is that shown by Cullender and Binckley (169) for new tubing. This value would in all likelihood be too low for old pi[illegible] as indicated by Colebrook (170). He reported that an increase in roughness may cause 20 to 30 per cent reduction in the carrying capacity of pipes while the reduction in the area perpendicular to flow due to such increase in roughness may cause a reduction of only 2 to 3 per cent of the pipe carrying capacity. This implies that the same increase in roughness may cause an increase of 25 to 43 per cent of pressure loss due to friction if the pipe carrying capacity were kept at its initial value. Furthermore, Ippen (171) has reported a practical example where the absolute roughness was doubled within three years and the associated friction factor was increased by 20 per cent.

This suggests that flowing bottom-hole pressure calculated using the value of roughness of 0.0006 inch will yield values which are usually low. The 12 test-points presented here tend to support this.

CONCLUSIONS

As a result of this investigation, the following conclusions can be made:

1. The assumption of adiabatic flow in gas wells is unrealistic and yields the lower bound estimate for flowing bottom-hole pressure.
2. The apparent failure of main models AA and BB may be attributed to unreliable data and errors [illegible] different terms such as viscosity and absolu[illegible]ss, etc...
3. The complete solution of the Force-Momentum-Balance differential equation, as suggested in this study, yields the closest estimates to the measured flowing bottom-hole pressures.
4. The assumption of absolute roughness of 0.0006 inch for old and new tubing may be in error. In all likelihood this value is too small.

RECOMMENDATIONS

The following recommendations are made:

1. The Lee, et al. (172) viscosity correlation should be replaced by a more accurate one capable of representing the viscosity of sour as well as sweet natural gases. In this regard it is recommended that the Carr, et al. (173), Dean and Stiel (174) viscosity correlations be examined.
2. The parameters in the equations giving the pseudocriticals of natural gases as a function of gas gravity should be re-estimated by using data for real natural gases rather than fitting the Brown, et al. (175) correlation.
3. The absolute roughness of tubing used in gas wells should be re-estimated. This can be done by employing accurate surface and bottom-hole measurements for large number of dry gas wells and applying main model A or B. The values of absolute roughness estimated may then be correlated to age of tubing, the degree of sourness of flowing gas, etc.

NOMENCLATURE

A, B = functions of specified arguments or constants depending upon usage.

B^* = a function of specified argument.

c_p = isobaric heat capacity, Btu/lbm.°R.

C^* = a function of specified argument.

$\overset{\circ}{Cp}$ = molal ideal isobaric heat capacity, Btu/lb mole.°R.

$\overset{\circ}{Cp}_m$ = molal ideal isobaric heat capacity of gas mixtures, Btu/lb mole.°R.

$\overset{\circ}{Cp}_{m_{H.C.}}$ = molal ideal isobaric heat capacity of pure hydrocarbon mixtures, Btu/lb mole.°R.

C_v° = molal specific heat at infinite volume, Btu/lb mole.°R.

Cp = molal isobaric heat capacity, Btu/lb mole.°R.

Cp_m = molal isobaric heat capacity of gas mixtures, Btu/lb mole.°R.

C_v = molal constant-volume specific heat, Btu/lb mole.°R.

d = inner pipe diameter, inches.

D = inner pipe diameter, feet.

D^* = a function of specified argument.

D_o = inner pipe diameter, feet.

exp = exponential.

E^* = a function of specified argument.

E_p = estimated truncation error in calculated pressure, lb/in^2.

E_T = estimated truncation error in calculated temperature, °F.

f = Moody's friction factor, Dimensionless.

f_1, f_2, fcn = functions [illegible]

F^*, F_{CO_2}, F_{H_2S}, F_{N_2} = functions [illegible]pecified arguments.

g = local gravitational acceleration, 32.174 ft/sec^2.

g_c = conversion factor, 32.174 lbm.ft/lb$_f$.sec^2.

G = molal flow rate per unit area, lb mole/ft^2.sec.

$\mathcal{G}$ = gas gravity, (air = 1).

$\mathcal{G}_f$ = gravity of flowing stream, (air = 1).

h = enthalpy, Btu/lbm.

h_o = gas-film heat-transfer coefficient, Btu/ft^2.sec.°R.

h_k = trial step size at kth interval, feet.

J = conversion factor, 778.16 lb$_f$.ft/Btu.

k = upper limit integer.

k = thermal conductivity, cal/cm.sec.K.

$k°$ = thermal conductivity at atmospheric pressure, cal/cm.sec.K.

$k_m^°$ = thermal conductivity of gas mixtures, at atmospheric pressure, cal/cm.sec.K.

k_m = thermal conductivity of gas mixtures, cal/cm.sec.K.

k_{P_1}, k_{P_2}, k_{P_3}, k_{P_4}, k_{P_5}, k_{T_1}, k_{T_2}, k_{T_3}, k_{T_4}, k_{T_5} = functions of specified arguments.

K = a function of specified argument.

ln = natural logarithm.

log = logarithm, base 10.

L = length variable, feet.

L^* = length of tubing, feet.

M_W = molecular weight or average molecular weight depending upon usage, lbm/lb mole.

n = upper limit integer.

P = pressure variable, lb_f/ft^2 absolute.

P_c = critical pressure or pseudocritical pressure depending upon context, lb_f/in^2 absolute.

p_{P_c} = pseudocritical pressure, lb_f/in^2 absolute.

$p_{P_{c_{H.C.}}}$ = pseudocritical pressure of pure hydrocarbon mixtures, lb_f/in^2 absolute.

P_r = reduced or pseudocritical pressure depending upon context, Dimensionless.

$\overline{P}_r$ = Prandtl Number, Dimensionless.

q = heat flux, Btu/sec.

Q_o = flow rate at standard conditions, MMSCF/Day.

R = universal gas constant, 1545 $lb_f \cdot$ ft/lb mole °R.

R = Gas/Condensate Ratio, SCF/bbl.

R^* = average thermal resistance of the surrounding as defined by Equation 13, ft^2.sec.°F/Btu.

R^{**} = average overall heat-transfer resistance, ft^2.sec.°F/Btu.

R_e = Reynolds Number, Dimensionless.

R_{e_b} = Reynolds Number at the boundary between the transition and rough pipe regions, Dimensionless.

S = Sutherland Constant, °K.

S = sin θ, dimensionless.

T = temperature variable, °R.

T_b = normal boiling point temperature, K.

T_c = critical or pseudocritical temperature depending upon context, °R.

${}_pT_c$ = pseudocritical temperature, °R.

${}_pT_{c_{H.C}}$ = pseudocritical temperature of pure hydrocarbon mixtures, °R.

T_r = reduced or pseudoreduced temperature, Dimensionless.

T_s = temperature of surroundings, °R.

T_{sr} = $\frac{T_s}{T_c}$, Dimensionless.

u = velocity of flow, ft/sec.

$\langle u \rangle$ = average velocity of flow, ft/sec.

U = overall heat-transfer coefficient, Btu/ft^2.sec.°F.

V = molal volume variable, ft^3/lb-mole.

W = shaft work done by gas, lb$_f$.ft.

X, Y = functions of specified arguments.

Y = mole fraction, Dimensionless.

Y_{CO_2} = mole fraction of carbon dioxide.

Y_{H_2S} = mole fraction of hydrogen sulphide.

Y_{N_2} = mole fraction of nitrogen.

z = height above datum, feet.

Z = compressibility factor, Dimensionless.

Z_c = critical compressibility factor, Dimensionless.

Greek

α = inverse of the momentum correction factor.

γ = a parameter defined by Equation 58.

δ = a change in the value of a variable.

δ_1, δ_2 = tolerances used as convers[illegible]ia.

ΔP = pressure drop, lb_f/in^2

ε = absolute roughness, inches.

ε^* = $\sqrt{\frac{f}{8}} \cdot R_e \cdot \frac{\varepsilon}{d}$ = Shear Reynolds number x relative roughness.

ε_0 = maximum energy of attraction for the Lennard-Jones potential, erg.

ε_3 = critical temperature correction as defined by Equation 24, °F.

ε_s = equivalent sand roughness, inches.

θ = dip angle of pipe, degrees.

μ = viscosity, c.p.

μ° = viscosity at one atmosphere, c.p.

μ_m = viscosity of gas mixtures, c.p.

ρ = density variable, lbm/ft^3.

ρ_c = critical density, lbm/ft^3.

ρ_{cond} = density of condensate, gm/cm^3.

ρ_r = reduced density, Dimensionless.

σ = collision diameter for the Lennard-Jones Potential, Å.

Σ = summation operator.

χ = a function of specified argument as defined by Equation 49.

Ω_v = collision integral.

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APPENDIX A

DEVELOPMENT OF A MATHEMATICAL MODEL FOR STEADY STATE COMPRESSIBLE FLOW OF GAS IN PIPES

For the steady state flow of gas through a differential length of pipe of known dimensions and position in space, the Total Energy Balance Equation and the Force Momentum Balance Equation, based on the fluid element contained in a differential length (dL) may be written.

The Total Energy Balance Equation:

✓ $$\delta q - \frac{dW}{J} = \frac{\pi D^2}{4}\frac{G}{M_w}\left[dh + \frac{\langle u\rangle d\langle u\rangle}{\alpha g_c J} + \frac{g}{g_c}\frac{dz}{J}\right] \qquad \text{(A-1)}$$

The Force Momentum Balance Equation:

$$\left(\frac{V}{M_w}\right)dP + \frac{\langle u\rangle d\langle u\rangle}{\alpha g_c} + \frac{g}{g_c}\,dz + \frac{f\langle u\rangle^2\,dL}{2g_c D} = 0 \qquad \text{(A-2)}$$

where δq = the transferred heat from the surroundings to the fluid element, Btu.

W = shaft work, $lb_f \cdot ft$.

G = molal flow rate per unit area, lb mole/sec. $ft.^2$

h = enthalpy, Btu/lbm.

$\langle u\rangle$ = mean velocity of fluid, ft/sec.

P = pressure, lb_f/ft^2 absolute.

V = molal volume, ft^3/lb mole.

f = Moody's hydraulic friction factor.

z = elevation above datum, ft.

D = I.D. of pipe, ft.

M_w = the molecular weight of gas, lbm/lb mole.

α = the inverse of the momentum correction factor

$= \frac{\langle u \rangle^2}{u^2} \simeq 1.0$

J = conversion factor.

= 778.16 lb_f.ft/Btu

g = local gravitational acceleration.

= 32.174 ft/sec.^{2} (assumed).

g_c = conversion factor.

$= 32.174 \frac{\text{lbm.ft.}}{\text{lb}_f\text{. sec}^2}$

Equations (A-1) and (A-2) were rearranged into forms appropriate to numerical solution. This task was accomplished as follows:

By definition:

$$G = \frac{\langle u \rangle}{V}$$

Thus

$$\langle u \rangle = GV \tag{A-3}$$

and

$$d\langle u \rangle = G\,dV \tag{A-4}$$

The dip angle of the pipe is θ, where

$$\sin\theta = S = \frac{dz}{dL} \tag{A-5}$$

The equation of state is:

$$PV = ZRT \tag{A-6}$$

From which

$$dV = \frac{ZR}{P}\,dT + \frac{RT}{P}\,dZ - \frac{ZRT}{P^2}\,dP \tag{A-7}$$

and

$$\left(\frac{\partial V}{\partial T}\right)_P = \frac{ZR}{P} + \frac{RT}{P}\left(\frac{\partial Z}{\partial T}\right)_P \tag{A-8}$$

where $Z = fcn(T_r, P_r)$. Thus

$$dZ = \left(\frac{\partial Z}{\partial T_r}\right)_{P_r} dT_r + \left(\frac{\partial Z}{\partial P_r}\right)_{T_r} dP_r$$

or

$$dZ = \left(\frac{\partial Z}{\partial T}\right)_P dT + \left(\frac{\partial Z}{\partial P}\right)_T dP \tag{A-9}$$

The gas does no shaft work; that is,

$$dW = 0 \tag{A-10}$$

The transferred heat to the fluid element from the surroundings may be given by

$$\delta q = \pi DdL\ U(T_s - T) \tag{A-11}$$

where T_s = the temperature of the surroundings, °F.

T = the temperature of the flowing fluid element, °F.

U = the overall heat-transfer coefficient, Btu/ft^2. °F.

It is known that $h = fcn(T,P)$. Thus

$$dh = \left(\frac{\partial h}{\partial T}\right)_P \cdot dT + \left(\frac{\partial h}{\partial P}\right)_T \cdot dP$$

$$dh = c_p \cdot dT + \left(\frac{\partial h}{\partial P}\right)_T \cdot dP \tag{A-12}$$

But $\left(\frac{\partial h}{\partial P}\right)_T = \frac{V - T\left(\frac{\partial V}{\partial T}\right)_P}{JM_w}$ since h is in Btu/lbm units. (A-13)

Equations (A-6), (A-7), (A-12), and (A-13) give

$$dh = c_p \cdot dT - \frac{RT^2}{JPM_w} \left(\frac{\partial Z}{\partial T}\right)_P \cdot dP \tag{A-14}$$

where c_p is in Btu/lbm. °R units.

$$\frac{\langle u\rangle d\langle u\rangle}{\alpha g_c} = \frac{G^2 V dV}{\alpha g_c}$$ by virtue of Equations (A-3) and (A-4).

$$= \frac{G^2 Z^2 R^2 T^2}{\alpha g_c P^2}\left(\frac{dT}{T} + \frac{dZ}{Z} - \frac{dP}{P}\right)$$ with the aid of Equation (A-7). (A-15)

$$\frac{f\langle u\rangle^2\, dL}{2 g_c D} = \frac{f G^2 Z^2 R^2 T^2}{2 g_c D P^2} \cdot dL$$ with the aid of Equation (A-3) (A-16)

$$\frac{V}{M_w}\, dP = \frac{ZRT}{PM_w}\, dP$$ with the aid of Equation (A-6). (A-17)

$$\frac{g}{g_c}\, dz = \frac{g}{g_c}\, S\, dL$$ by virtue of Equation (A-5) (A-18)

Combining Equations (A-1), (A-9), (A-10), (A-11), (A-14), (A-15), and (A-18) and rearranging yield

$$\frac{dP}{dL}\left[\frac{G^2 Z R^2 T^2}{\alpha g_c J P^2}\left(\frac{\partial Z}{\partial P}\right)_T - \frac{RT^2}{JPM_w}\left(\frac{\partial Z}{\partial T}\right)_P - \frac{G^2 Z^2 R^2 T^2}{\alpha g_c J P^3}\right] +$$

$$\frac{dT}{dL}\left[c_P + \frac{G^2 Z^2 R^2 T}{\alpha g_c J P^2} + \frac{G^2 Z^2 R^2 T^2}{\alpha g_c J P^2}\left(\frac{\partial Z}{\partial T}\right)_P\right] =$$

$$\frac{4U(T_s - T)/M_w - \frac{g}{g_c}\, DGS}{DGJ} \qquad \text{(A-19)}$$

Combining Equations (A-2), (A-9), (A-15), (A-16), (A-17) and (A-18) and rearranging yield

$$\frac{dP}{dL}\left[\frac{g_c P^2}{M_w} + \frac{G^2RTP}{\alpha}\left(\frac{\partial Z}{\partial P}\right)_T - \frac{G^2ZRT}{\alpha}\right] +$$

$$\frac{dT}{dL}\left[\frac{G^2ZRP}{\alpha} + \frac{G^2RTP}{\alpha}\left(\frac{\partial Z}{\partial T}\right)_P\right] = -\frac{g}{ZT} - \frac{fG^2ZRTP}{2D} \qquad \text{(A-20)}$$

By employing the definitions of P_r and T_r and multiplying both sides of Equation (A-19) by -1, Equations (A-19) and (A-20) may be written as:

$$\frac{dP_r}{dL}\left[-\frac{G^2ZR^2T^2}{\alpha J g_c P^2}\left(\frac{\partial Z}{\partial P_r}\right)_{T_r} + \frac{RTT_r}{JP_rM_w}\left(\frac{\partial Z}{\partial T_r}\right)_{P_r} + \frac{G^2Z^2R^2T^2}{\alpha g_c JP^2P_r}\right] +$$

$$\frac{dT_r}{dL}\left[-\frac{T}{T_r}c_p - \frac{G^2Z^2R^2T^2}{\alpha g_c JP^2T_r} - \frac{G^2ZR^2T^2}{\alpha J g_c P^2}\left(\frac{\partial Z}{\partial T_r}\right)_{P_r}\right] =$$

$$\frac{\frac{g}{g_c}DGs - 4UJ(T_{sr} - T_r)/M_w}{DGJ} \qquad \text{(A-21)}$$

and

$$\frac{dP_r}{dL}\left[\frac{g_c P^3}{P_rM_w} - \frac{G^2RTZP}{\alpha P_r} + \frac{G^2RTP}{\alpha}\left(\frac{\partial Z}{\partial P_r}\right)_{T_r}\right] +$$

$$\frac{dT_r}{dL}\left[\frac{G^2ZRPT}{\alpha T_r} + \frac{G^2RTP}{\alpha}\left(\frac{\partial Z}{\partial T_r}\right)_{P_r}\right] =$$

$$-\frac{gsP^3}{ZRT} - \frac{fG^2ZRTP}{2D} \qquad \text{(A-22)}$$

Equations (A-21) and (A-22) may be written as:

$$D^* \frac{dP_r}{dL} + E^* \frac{dT_r}{dL} = F^* \tag{A-23}$$

and $A^* \frac{dP_r}{dL} + B^* \frac{dT_r}{dL} = C^*$ respectively (A-24)

where

$$A^* = \left[\frac{g_c P^3}{P_r M_w} - \frac{G^2 RTZP}{\alpha P_r} + \frac{G^2 RTP}{\alpha} \left(\frac{\partial Z}{\partial P_r} \right)_{T_r} \right]$$

$$= \mathit{fcn}(T_r, P_r, T_c, P_c, M_w, Q_o, D) \tag{A-25}$$

$$B^* = \left[\frac{G^2 ZRPT}{\alpha T_r} + \frac{G^2 RTP}{\alpha} \left(\frac{\partial Z}{\partial T_r} \right)_{P_r} \right]$$

$$= \mathit{fcn}(T_r, P_r, T_c, P_c, M_w, Q_o, D) \tag{A-26}$$

$$C^* = \left[-\frac{gSP^3}{ZRT} - \frac{fG^2 ZRTP}{2D} \right]$$

$$= \mathit{fcn}(T_r, P_r, T_c, P_c, M_w, Q_o, D, (\frac{\varepsilon}{d}), R_e, S)$$

But $R_e = \frac{\rho \langle u \rangle D}{\mu} = \mathit{fcn}(T_r, P_r, Q_o, D)$

$$C^* = fcn(T_r, P_r, T_c, P_c, M_w, Q_o, D, (\frac{\varepsilon}{d}), S) \qquad (A\text{-}27)$$

$$D^* = \left[- \frac{G^2 Z R^2 T^2}{\alpha g_c J P^2} (\frac{\partial Z}{\partial P_r})_{T_r} + \frac{R T T_r}{J P_r M_w} (\frac{\partial Z}{\partial T_r})_{P_r} + \frac{G^2 Z^2 R^2 T^2}{\alpha g_c J P^2 P_r} \right]$$

$$= fcn(T_r, P_r, T_c, P_c, M_w, Q_o, D) \qquad (A\text{-}28)$$

$$E^* = \left[- \frac{T}{T_r} c_p - \frac{G^2 Z^2 R^2 T^2}{\alpha g_c J P^2 T_r} - \frac{G^2 Z R^2 T^2}{\alpha g_c J P^2} (\frac{\partial Z}{\partial T_r})_{P_r} \right]$$

$$= fcn(T_r, P_r, T_c, P_c, M_w, Q_o, D) \qquad (A\text{-}29)$$

and

$$F^* = \frac{\frac{g}{g_c} D G S - 4UJ(T_{sr} - T_r)/M_w}{DGJ}$$

$$= fcn(T_r, P_r, T_c, P_c, M_w, Q_o, S, U) \qquad (A\text{-}30)$$

Solving Equations (A-23) and (A-24) simultaneously for $\frac{dP_r}{dL}$ and $\frac{dT_r}{dL}$ gives

$$\frac{dP_r}{dL} = \frac{C^* \cdot E^* - B^* \cdot F^*}{A^* \cdot E^* - B^* \cdot D^*} \qquad (A\text{-}31)$$

and

$$\frac{dT_r}{dL} = \frac{A^* \cdot F^* - C^* \cdot D^*}{A^* \cdot E^* - B^* \cdot D^*} \qquad \text{(A-32)}$$

In order to obtain a solution, $\frac{dP_r}{dL}$ and $\frac{dT_r}{dL}$ must be finite.

Equations (A-31) and (A-32) with the aid of Equations (A-25) through (A-30) may be rewritten as

$$\frac{dP_r}{dL} = \mathit{fcn}(T_r, P_r, T_c, P_c, M_w, Q_0, D, \tfrac{\varepsilon}{d}, U, \mathcal{S}) \qquad \text{(A-33)}$$

and

$$\frac{dT_r}{dL} = \mathit{fcn}(T_r, P_r, T_c, P_c, M_w, Q_0, D, \tfrac{\varepsilon}{d}, U, \mathcal{S}) \qquad \text{(A-34)}$$

For a specific flow case, that is, when the pipe dimensions and position in space ($L^*, D, \mathcal{S}$), flow rate (Q_0) and gas properties (T_c, P_c, M_w) are given, Equations (A-33) and (A-34) reduce to

$$\frac{dP_r}{dL} = \mathit{fcn}(T_r, P_r, (\tfrac{\varepsilon}{d}), U) \qquad \text{(A-35)}$$

and

$$\frac{dT_r}{dL} = \mathit{fcn}(T_r, P_r, (\tfrac{\varepsilon}{d}), U) \qquad \text{(A-36)}$$

Equations (A-35) and (A-36) constitute the mathematical model for a steady-state compressible flow of gas in pipes.

APPENDIX B

DEVELOPMENT OF MATHEMATICAL EXPRESSIONS FOR THE PARTIAL DERIVATIVES OF Z WITH RESPECT TO T_r AND P_r

The reduced Benedict-Webb-Rubin equation of state has the form

$$Z = 1 + \left(A_1 + \frac{A_2}{T_r} + \frac{A_3}{T_r^3}\right)\rho_r + \left(A_4 + \frac{A_5}{T_r}\right)\rho_r^2 + A_5 A_6 \frac{\rho_r^5}{T_r}$$

$$+ A_7 \frac{\rho_r^2}{T_r^3}\left(1 + A_8\rho_r^2\right) exp\left(-A_8\rho_r^2\right) \qquad \text{(B-1)}$$

where

$$Z = \frac{PV}{RT} = \frac{PM_w}{\rho RT} \qquad \text{(B-2)}$$

At the critical point

$$Z_c = \frac{P_c M_w}{\rho_c R T_c} \qquad \text{(B-3)}$$

Dividing Equation (B-2) by Equation (B-3) and rearranging give

$$Z = Z_c \frac{P_r}{\rho_r T_r} \qquad \text{(B-4)}$$

Combining Equations (B-1) and (B-4) produces another form of reduced Bendict-Webb-Rubin equation of state

$$P_r = \frac{1}{Z_c}\left[\begin{array}{l} \rho_r T_r + (A_1 T_r + A_2 + \frac{A_3}{T_r^2})\rho_r^2 + (A_4 T_r + A_5)\rho_r^3 \\ \quad + A_5 A_6 \rho_r^6 + A_7 \frac{\rho_r^3}{T_r^2}(1 + A_8\rho_r^2)exp(-A_8\rho_r^2) \end{array}\right] \qquad \text{(B-5)}$$

Equation (B-4) was employed to derive $(\frac{\partial Z}{\partial T_r})_{P_r}$ and $(\frac{\partial Z}{\partial P_r})_{T_r}$ by partial differentiation as follows:

$$(\frac{\partial Z}{\partial T_r})_{P_r} = \frac{\partial}{\partial T_r}(\frac{Z_c P_r}{T_r \rho_r})_{P_r}$$

$$= Z_c P_r \left[- \frac{1}{\rho_r T_r^2} - \frac{1}{T_r \rho_r^2}(\frac{\partial \rho_r}{\partial T_r})_{P_r}\right]$$

$$= \frac{Z_c P_r}{\rho_r T_r}\left[- \frac{1}{T_r} - \frac{1}{\rho_r}(\frac{\partial \rho_r}{\partial T_r})_{P_r}\right]$$

$$(\frac{\partial Z}{\partial T_r})_{P_r} = Z\left[- \frac{1}{T_r} - \frac{1}{\rho_r}(\frac{\partial \rho_r}{\partial T_r})_{P_r}\right] \qquad \text{(B-6)}$$

And

$$\left(\frac{\partial Z}{\partial P_r}\right)_{T_r} = \frac{\partial}{\partial P_r}\left(\frac{Z_c P_r}{T_r \rho_r}\right)_{T_r}$$

$$= \frac{Z_c}{T_r}\left[\frac{1}{\rho_r} - \frac{P_r}{\rho_r^2}\left(\frac{\partial \rho_r}{\partial P_r}\right)_{T_r}\right]$$

$$= \frac{Z_c P_r}{\rho_r T_r}\left[\frac{1}{P_r} - \frac{1}{\rho_r}\left(\frac{\partial \rho_r}{\partial P_r}\right)_{T_r}\right]$$

$$\left(\frac{\partial Z}{\partial P_r}\right)_{T_r} = Z\left[\frac{1}{P_r} - \frac{1}{\rho_r}\left(\frac{\partial \rho_r}{\partial P_r}\right)_{T_r}\right] \qquad (B-7)$$

Equations (B-6) and (B-7) imply that the partial derivatives of Z with respect to T_r and P_r can be evaluated once the partial derivatives of ρ_r with respect to T_r and P_r are known.

Now $$\left(\frac{\partial \rho_r}{\partial P_r}\right)_{T_r} = \frac{1}{\left(\frac{\partial P_r}{\partial \rho_r}\right)_{T_r}} \quad ; \text{ provided } \left(\frac{\partial P_r}{\partial \rho_r}\right)_{T_r} \neq 0 \qquad (B-8)$$

$$\left(\frac{\partial \rho_r}{\partial T_r}\right)_{P_r} = -\left(\frac{\partial P_r}{\partial T_r}\right)_{\rho_r} \Big/ \left(\frac{\partial P_r}{\partial \rho_r}\right)_{T_r} \qquad (B-9)$$

$(\frac{\partial P_r}{\partial T_r})_{\rho_r}$ and $(\frac{\partial P_r}{\partial \rho_r})_{T_r}$ were obtained by performing partial differentiation on Equation (B-5)

$$(\frac{\partial P_r}{\partial T_r})_{\rho_r} = \frac{1}{Z_c}\left[\begin{array}{l}\rho_r + (A_1 - \frac{2A_3}{T_r^3})\rho_r^2 + A_4\rho_r^3 \\ \\ \quad - 2A_7 \frac{\rho_r^3}{T_r^3}(1 + A_8\rho_r^2)exp(-A_8\rho_r^2)\end{array}\right] \qquad \text{(B-10)}$$

$$(\frac{\partial P_r}{\partial \rho_r})_{T_r} = \frac{1}{Z_c}\left[\begin{array}{l}T_r + 2(A_1T_r + A_2 + \frac{A_3}{T_r^2})\rho_r \\ \\ \quad + 3(A_4T_r + A_5)\rho_r^2 + 6A_5A_6\rho_r^5 \\ \\ \quad + A_7 \frac{\rho_r^2}{T_r^2} exp(-A_8\rho_r^2)(3 + 3A_8\rho_r^2 - 2A_8^2\rho_r^4)\end{array}\right] \qquad \text{(B-11)}$$

Substituting Equation (B-11) into Equation (B-8) gives

$$\left(\frac{\partial \rho_r}{\partial P_r}\right)_{T_r} = \frac{Z_c}{\left[\begin{array}{l} T_r + 2(A_1T_r + A_2 + \frac{A_3}{T_r^2})\rho_r \\ \\ \quad + 3(A_4T_r + A_5)\rho_r^2 + 6A_5A_6\rho_r^5 \\ \\ \quad + A_7 \frac{\rho_r^2}{T_r^2} \, exp(-A_8\rho_r^2)(3 + 3A_8\rho_r^2 - 2A_8^2\rho_r^4) \end{array}\right]} \qquad \text{(B-12)}$$

Substituting Equations (B-10) and (B-11) into Equation (B-9) gives

$$\left(\frac{\partial \rho_r}{\partial T_r}\right)_{P_r} = - \frac{\left[\begin{array}{l} \rho_r + (A_1 - \frac{2A_3}{T_r^3})\rho_r^2 + A_4\rho_r^3 \\ \\ \quad - 2A_7 \frac{\rho_r^3}{T_r^3} (1 + A_8\rho_r^2) exp(-A_8\rho_r^2) \end{array}\right]}{\left[\begin{array}{l} T_r + 2(A_1T_r + A_2 + \frac{A_3}{T_r^2})\rho_r \\ \\ \quad + 3(A_4T_r + A_5)\rho_r^2 + 6A_5A_6\rho_r^5 \\ \\ \quad + A_7 \frac{\rho_r^2}{T_r^2} \, exp(-A_8\rho_r^2)(3 + 3A_8\rho_r^2 - 2A_8^2\rho_r^4) \end{array}\right]} \qquad \text{(B-13)}$$

Equations (B-6) and (B-7) with the aid of Equations (B-12) and (B-13) made it possible to estimate $(\frac{\partial Z}{\partial T_r})_{P_r}$ and $(\frac{\partial Z}{\partial P_r})_{T_r}$ at T_r and P_r analytically.

APPENDIX C

DEVELOPMENT OF A MATHEMATICAL EXPRESSION FOR HEAT CAPACITY DEPARTURE

The reduced form of the Benedict-Webb-Rubin equation of state is

$$P_r = \frac{1}{Z_c}\left[\rho_r T_r + (A_1 T_r + A_2 + \frac{A_3}{T_r^2})\rho_r^2 + (A_4 T_r + A_5)\rho_r^3 + A_5 A_6 \rho_r^6 + A_7 \frac{\rho_r^3}{T_r^2}(1 + A_8\rho_r^2)exp(-A_8\rho_r^2)\right] \qquad \text{(C-1)}$$

The heat capacity departure from the ideal state is defined as $(C_p - C_p^\circ)$.

For a gas in the ideal state

$$C_p^\circ = C_v^\circ + R \qquad \text{(C-2)}$$

Thus

$$(C_p - C_p^\circ) = C_p - (C_v^\circ + R) \qquad \text{(C-3)}$$

By adding and subtracting C_v from the left hand side of Equation (C-3) and rearranging, one may get

$$(C_p - C_p^\circ) = (C_p - C_v) + (C_v - C_v^\circ) - R \qquad \text{(C-4)}$$

where C_p = isobaric heat capacity at pressure P and temperature T, Btu/lb mole.°R.

C_p° = isobaric heat capacity at zero pressure and temperature T, Btu/lb mole.°R.

C_v = constant-volume heat capacity at pressure P and temperature T, Btu/lb mole.°R.

C_v° = constant-volume specific heat capacity at zero pressure or at infinite volume and temperature T, Btu/lb mole.°R.

R = universal gas constant, 1.98654 Btu/lb mole.°R.

Expressions for $(C_p - C_v)$ and $(C_v - C_v^\circ)$ may be obtained from Equation (C-1) by applying the following thermodynamic relationships (176):

$$\left(\frac{\partial C_v}{\partial V}\right)_T = T\left(\frac{\partial^2 P}{\partial T^2}\right)_V \qquad \text{(C-5)}$$

$$(C_p - C_v) = -T\left(\frac{\partial P}{\partial T}\right)_V^2 / \left(\frac{\partial P}{\partial V}\right)_T \qquad \text{(C-6)}$$

Equations (C-5) and (C-6) may be rewritten as:

$$\left(\frac{\partial C_v}{\partial \rho_r}\right)_{T_r} = -RZ_c \frac{T_r}{\rho_r^2} \left(\frac{\partial^2 P_r}{\partial T_r^2}\right)_{\rho_r} \qquad \text{(C-7)}$$

$$(C_p - C_v) = RZ_c \frac{T_r}{\rho_r^2} \left(\frac{\partial P_r}{\partial T_r}\right)_{\rho_r}^2 / \left(\frac{\partial P_r}{\partial \rho_r}\right)_{T_r} \qquad \text{(C-8)}$$

By performing partial differentiation on Equation (C-1), the following expressions were obtained:

$$\left(\frac{\partial P_r}{\partial T_r}\right)_{\rho_r} = \frac{1}{Z_c}\left[\begin{matrix}\rho_r + (A_1 - \frac{2A_3}{T_r^3})\rho_r^2 + A_4\rho_r^3 \\ - 2A_7\frac{\rho_r^3}{T_r^3}(1 + A_8\rho_r^2)exp(-A_8\rho_r^2)\end{matrix}\right] \qquad (C\text{-}9)$$

$$\left(\frac{\partial^2 P_r}{\partial T_r^2}\right)_{\rho_r} = \frac{1}{Z_c}\left[6A_3\frac{\rho_r^2}{T_r^4} + 6A_7\frac{\rho_r^3}{T_r^4}(1 + A_8\rho_r^2)exp(-A_8\rho_r^2)\right] \qquad (C\text{-}10)$$

$$\left(\frac{\partial P_r}{\partial \rho_r}\right)_{T_r} = \frac{1}{Z_c}\left[\begin{matrix}T_r + 2(A_1T_r + A_2 + \frac{A_3}{T_r^2})\rho_r \\ + 3(A_4T_r + A_5)\rho_r^2 + 6A_5A_6\rho_r^5 \\ + A_7\frac{\rho_r^2}{T_r^2}exp(-A_8\rho_r^2)(3 + 3A_8\rho_r^2 - 2A_8^2\rho_r^4)\end{matrix}\right] \qquad (C\text{-}11)$$

Substituting Equations (C-9), and (C-11) into Equation (C-7) gives

$$(C_p - C_v) = \frac{RT_r\left[1 + (A_1 - \frac{2A_3}{T_r^{\,3}})\rho_r + A_4\rho_r^2 - 2A_7\frac{\rho_r^2}{T_r^{\,3}}(1 + A_8\rho_r^2)exp(-A_8\rho_r^2)\right]^2}{\left[T_r + 2(A_1T_r + A_2 + \frac{A_3}{T_r^{\,2}})\rho_r + 3(A_4T_r + A_5)\rho_r^2 + 6A_5A_6\rho_r^5 + A_7\frac{\rho_r^2}{T_r^{\,2}}exp(-A_8\rho_r^2)(3 + 3A_8\rho_r^2 - 2A_8^2\rho_r^4)\right]} \qquad (C\text{-}12)$$

Substituting Equation (C-10) into Equation (C-7) gives

$$\left(\frac{\partial C_v}{\partial \rho_r}\right)_{T_r} = -R\left[\frac{6A_7}{T_r^{\,3}} + 6A_7\frac{\rho_r}{T_r^{\,3}}(1 + A_8\rho_r^2)exp(-A_8\rho_r^2)\right] \qquad (C\text{-}13)$$

Boundary Conditions:

At $\rho_r = 0$, $C_v = C_v^\circ$

$\rho_r = \rho_r$, $C_v = C_v$

By separating variables in Equation (C-13), integrating, and applying the above Boundary Conditions, one may obtain:

$$\int_{C_v^\circ}^{C_v} (\partial C_v)_{T_r} = -R\left[\int_0^{\rho_r} \left(\frac{6A_7}{T_r^3}\right)_{T_r} d\rho_r + \int_0^{\rho_r} \left(6A_7 \frac{\rho_r}{T_r^3}\, exp(-A_8\rho_r^2)\right)_{T_r} d\rho_r + \int_0^{\rho_r} \left(6A_7A_8 \frac{\rho_r^3}{T_r^3}\, exp(-A_8\rho_r^2)\right)_{T_r} d\rho_r\right]$$

$$(C_v - C_v^\circ) = -R[X + Y + Q] \qquad \text{(C-14)}$$

where

$$X = 6A_3 \frac{\rho_r}{T_r^3} \qquad \text{(C-15)}$$

$$Y = -\frac{3A_7\ exp(-A_8\rho_r^2)}{A_8T_r^3} + \frac{3A_7}{A_8}\frac{1}{T_r^3} \qquad \text{(C-16)}$$

$$Q = -3A_7 \frac{\rho_r^2}{T_r^3}\, exp(-A_8\rho_r^2) - \frac{3A_7}{A_8}\frac{exp(-A_8\rho_r^2)}{T_r^3} + \frac{3A_7}{T_r^3} \qquad \text{(C-17)}$$

or

$$(C_v - C_v^\circ) = R\left[-6A_3 \frac{\rho_r}{T_r^3} - \frac{6A_7}{A_8T_r^3} + \left(\frac{6A_7}{A_8T_r^3} + 3A_7 \frac{\rho_r^2}{T_r^3}\right) exp(-A_8\rho_r^2)\right]$$

$$\text{(C-18)}$$

Substituting Equations (C-12) and (C-18) into Equation

(C-4) results in the final form of heat capacity departure equation, which may be written as:

$$(C_p - C_p^\circ) = R \left[\begin{aligned} & - 1 - 6A_3 \frac{\rho_r}{T_r^{\,3}} - \frac{6A_7}{A_8 T_r^{\,3}} \\ & + \left(\frac{6A_7}{A_8 T_r^{\,3}} + 3A_7 \frac{\rho_r^2}{T_r^{\,3}}\right) exp(-A_8 \rho_r^2) \\ & + \frac{T_r \left[\begin{aligned} & 1 + \left(A_1 - \frac{2A_3}{T_r^{\,3}}\right)\rho_r + A_4 \rho_r^2 \\ & - 2A_7 \frac{\rho_r^2}{T_r^{\,3}} (1 + A_8 \rho_r^2) exp(-A_8 \rho_r^2) \end{aligned} \right]^2}{\left[\begin{aligned} & T_r + 2\left(A_1 T_r + A_2 + \frac{A_3}{T_r^{\,2}}\right)\rho_r \\ & + 3(A_4 T_r + A_5)\rho_r^2 + 6A_5 A_6 \rho_r^5 \\ & + A_7 \frac{\rho_r^2}{T_r^{\,2}} exp(-A_8 \rho_r^2)(3 + 3A_8 \rho_r^2 - 2A_8^2 \rho_r^4) \end{aligned} \right]} \end{aligned} \right]$$

(C-19)

APPENDIX D

THE HYDRAULIC FRICTION FACTOR IN BOTH THE TRANSITION AND THE ROUGH PIPE REGIONS

Transition Region

This region is characterized by $4000 \leq R_e \leq \frac{3200}{(\varepsilon/d)}$. The hydraulic friction factor is given by the Colebrook transition law (177).

$$f = \left[- \frac{1}{2 \, log(\frac{\varepsilon/d}{3.7} + \frac{2.51}{R_e\sqrt{f}})} \right]^2 \qquad \text{(D-1)}$$

where f = Moody's friction factor.

ε = absolute roughness, inch.

d = I.D. of tubing, inch.

R_e = Reynolds number.

By taking the square root of both sides of Equation (D-1) and by letting $x = \sqrt{f}$, one may obtain

$$x = - \frac{1}{2 \, log(\frac{\varepsilon/d}{3.7} + \frac{2.51}{R_e x})} \qquad \text{(D-2)}$$

In order to solve for x in Equation (D-2), the Newton-

Raphson iterative method was used. The iterative equation has the following form:

$$x_{i+1} = x_i - \frac{g(x_i)}{g'(x_i)} \text{ ; providing } g'(x_i) \neq 0 \qquad \text{(D-3)}$$

where $g(x_i)$ is obtained from Equation (D-2) as:

$$g(x_i) = 1 + 2x_i \, log\left(\frac{(\varepsilon/d)}{3.7} + \frac{2.51}{R_e x_i}\right) \qquad \text{(D-4)}$$

Thus,

$$g'(x_i) = 2 \, log\left(\frac{(\varepsilon/d)}{3.7} + \frac{2.51}{R_e x_i}\right) - \frac{5.02 \, log \, e}{R_e x_i\left(\frac{(\varepsilon/d)}{3.7} + \frac{2.51}{R_e x_i}\right)} \qquad \text{(D-5)}$$

Combining Equations (D-3), (D-4), and (D-5) provides the final form of the iterative equation for x,

$$x_{i+1} = x_i - \frac{\left[1 + 2x_i \, log\left(\frac{(\varepsilon/d)}{3.7} + \frac{2.51}{R_e x_i}\right)\right]}{\left[2 \, log\left(\frac{(\varepsilon/d)}{3.7} + \frac{2.51}{R_e x_i}\right) - \frac{5.02 \, log \, e}{R_e x_i\left(\frac{(\varepsilon/d)}{3.7} + \frac{2.51}{R_e x_i}\right)}\right]} \qquad \text{(D-6)}$$

Once Equation (D-6) is satisfied after n iterations

$$f = x_n^2$$

The Fully Rough Pipe Region

Flow of fluid falls in this region when $R_e > \frac{3200}{(\varepsilon/d)}$. The Van-Kármán equation was employed to estimate the hydraulic friction factor in this region.

$$f = \left[- \frac{1}{2\, log(\frac{(\varepsilon/d)}{3.7})} \right]^2 \qquad \text{(D-7)}$$

APPENDIX E

AN ALGORITHM FOR SOLVING THE SYSTEM OF DIFFERENTIAL EQUATIONS IN MAIN MODELS AA AND BB

The system of differential equations that describes the flow problem is as follows

$$\frac{dP_r}{dL} = f_1(T_r, P_r, R^*, \varepsilon/d) \qquad \text{(E-1)}$$

$$\frac{dT_r}{dL} = f_2(T_r, P_r, R^*, \varepsilon/d) \qquad \text{(E-2)}$$

$$\frac{dR^*}{dL} = 0 \qquad \text{(E-3)}$$

The Boundary Conditions are:

At $L = 0$: $T = T_{(L=0)} = T_1$

$L = L^*$: $T = T_{(L=L^*)} = T_2$ and $P = P_{(L=L^*)} = P_2$

The modified Runge-Kutta-Merson algorithm as suggested by Niesse (178), Chai and Burgin (179) was utilized to solve this system of differential equations as follows:

(1) Let $(\varepsilon/d) = 0.0006/d$ (for new tubing)

$R^* = 7000$; $1000 < R^* \leq 7000$

(2) For a trial step size (h_k), compute k_{P_i} and k_{T_i}, $i=1,5$:

$$k_{P_1} = \frac{h_k}{3} f_1(T_{r_n}, P_{r_n}, R^*, \varepsilon/d) \;;$$

$$k_{T_1} = \frac{h_k}{3} f_2(T_{r_n}, P_{r_n}, R^*, \varepsilon/d)$$

$$k_{P_2} = \frac{h_k}{3} f_1(T_{r_n} + k_{T_1}, P_{r_n} + k_{P_1}, R^*, \varepsilon/d) \;;$$

$$k_{T_2} = \frac{h_k}{3} f_2(T_{r_n} + k_{T_1}, P_{r_n} + k_{P_1}, R^*, \varepsilon/d)$$

$$k_{P_3} = \frac{h_k}{3} f_1(T_{r_n} + \frac{k_{T_1}}{2} + \frac{k_{T_2}}{2}, P_{r_n} + \frac{k_{P_1}}{2} + \frac{k_{P_2}}{2}, R^*, \varepsilon/d)$$

$$k_{T_3} = \frac{h_k}{3} f_2(T_{r_n} + \frac{k_{T_1}}{2} + \frac{k_{T_2}}{2}, P_{r_n} + \frac{k_{P_1}}{2} + \frac{k_{P_2}}{2}, R^*, \varepsilon/d)$$

$$k_{P_4} = \frac{h_k}{3} f_1(T_{r_n} + \frac{3}{8} k_{T_1} + \frac{9}{8} k_{T_3}, P_{r_n} + \frac{3}{8} k_{P_1} + \frac{9}{8} k_{P_3}, R^*, \varepsilon/d)$$

$$k_{T_4} = \frac{h_k}{3} f_2(T_{r_n} + \frac{3}{8} k_{T_1} + \frac{9}{8} k_{T_3}, P_{r_n} + \frac{3}{8} k_{P_1} + \frac{9}{8} k_{P_3}, R^*, \varepsilon/d)$$

$$k_{P_5} = \frac{h_k}{3} f_1(T_{r_n} + \frac{3}{2} k_{T_1} - \frac{9}{2} k_{T_3} + 6k_{T_4}, P_{r_n} + \frac{3}{2} k_{P_1} - \frac{9}{2} k_{P_3} + 6k_{P_4}, R^*, \varepsilon/d)$$

$$k_{T_5} = \frac{h_k}{3} f_2(T_{r_n} + \frac{3}{2} k_{T_1} - \frac{9}{2} k_{T_3} + 6k_{T_4}, P_{r_n} + \frac{3}{2} k_{P_1} - \frac{9}{2} k_{P_3} + 6k_{P_4}, R^*, \varepsilon/d)$$

and calculate

$$P_{r_{n+1}} = P_{r_n} + \frac{3}{2} k_{P_1} - \frac{9}{2} k_{P_3} + 6k_{P_4}$$

$$T_{r_{n+1}} = T_{r_n} + \frac{3}{2} k_{T_1} - \frac{9}{2} k_{T_3} + 6k_{T_4}$$

(3) Compute the estimated truncation error

$$E_P = P_c \left[k_{P_1} - \frac{9}{2} k_{P_3} + 4k_{P_4} - \frac{k_{P_5}}{2} \right]$$

$$E_T = T_c \left[k_{T_1} - \frac{9}{2} k_{T_3} + 4k_{T_4} - \frac{k_{T_5}}{2} \right]$$

(4) Let δ_1 = tolerance in predicted pressure P.

δ_2 = tolerance in predicted temperature T.

If either $E_P > \delta_1$ or $E_T > \delta_2$, halve the trial step size and start over from step (2) until $E_P \leq \delta_1$ and $E_T \leq \delta_2$.

(5) Compute the corrected value of $P_{r_{n+1}}$ and $T_{r_{n+1}}$:

$$P_{r_{n+1}}(\text{corrected}) = P_{r_{n+1}} - \frac{E_P}{P_c}$$

$$T_{r_{n+1}}(\text{corrected}) = T_{r_{n+1}} - \frac{E_T}{T_c}$$

This correction raises the accuracy to the fourth order.

(6) If $E_P < \frac{\delta_1}{16}$ and $E_T < \frac{\delta_2}{16}$, double the trial step size for the next integration step. If not, leave the trial step size as is for the next integration interval, h_{k+1}.

(7) If $\left[L^* - \sum_{i=1}^{k} h_i\right] = 0$, go to step (8).

If $\left[L^* - \sum_{i=1}^{k} h_i\right] \geq h_{k+1}$, go to step (2).

If $\left[L^* - \sum_{i=1}^{k} h_i\right] < h_{k+1}$, let $h_{k+1} = \left[L^* - \sum_{i=1}^{k} h_i\right]$

and go to step (2).

(8) At this stage $T_{(L=0)}$ and $P_{(L=0)}$ are estimated.

If $T_{(L=0)} = T_1$, then $P_1 = P_{(L=0)}$. STOP.

If $T_{(L=0)} \neq T_1$, modify the value of R*.

(9) Let $k = 1$ and start over from step (2).

For main model BB, R** replaces R*.

APPENDIX F

SOURCE DATA

WELL A

FIELD: Dunvegan

WELL NAME: Anderson et al.
Dunvegan 6-29

POOL: Debolt

LOCATION: 6-29-80-3 W6M

PERFORATIONS: 4748'-4785' K.B.

DISTANCE TO M.P.P.: 4753' K.B.

TUBING SIZE: 2.441" I.D.

RESERVOIR TEMPERATURE: 115°F

GAS PROPERTIES:

$G = 0.6402$ $_pP_c = 669.9$ PSIA $_pT_c = 367.2$ °R

MOL.% $CO_2 = 0.56$ MOL.% $N_2 = 1.05$ MOL.% $H_2S = 0.00$

MOL.% $C_1 = 90.33$ MOL.% $C_2 = 4.04$ MOL.% $C_3 = 1.95$

MOL.% $i\text{-}C_4 = 0.31$ MOL.% $n\text{-}C_4 = 0.70$ MOL.% $i\text{-}C_5 = 0.20$

MOL.% $n\text{-}C_5 = 0.25$ MOL.% C_6's $= 0.22$ MOL.% C_7's $= 0.25$

MOL.% $C_8^+ = 0.07$ MOL.% $H_e = 0.07$

Density of Condensate = 0.777 gm/c.c.

WELL OPERATOR: Anderson Exploration Limited

TYPE OF TEST: Modified Isochronal

DATE OF TEST: April 5, 1971

GAS PRODUCED THROUGH: Tubing

WELL A (CONTINUED)

PRODUCTION AND PRESSURE DATA

Total Flow Rate (MMSCF/Day)	Tubing Head		Press. at M.P.P. (PSIA)	Condensate Prod. Rate (bbl/Day)	Water/Gas Ratio (bbl/MMSCF)
	Press. (PSIA)	Temp. (°F)			
0.833	1660.0	47	1928.7	3.48	0.0
1.606	1566.0	53	1808.4	5.22	0.0
2.186	1473.0	57	1701.8	10.44	0.0
2.910	1360.0	58	1567.5	5.22	0.0
4.291	895.0	68	1103.7	1.74	0.0
4.043	862.0	69	1048.0	12.97	0.0

WELL B

FIELD: Bigstone

WELL NAME: Pan Am. HB,G-2

POOL: D-3A

LOCATION: 2-25-60-22 W5M

PERFORATIONS: 11006'-11052' K.B.

DISTANCE TO M.P.P.: 11029' K.B.

TUBING SIZE: 2.992" I.D.

RESERVOIR TEMPERATURE: 246°F

GAS PROPERTIES:

$G = 0.6997$ $\quad {}_pP_c = 801.2$ PSIA $\quad {}_pT_c = 410.9$ °R

MOL.% $CO_2 = 2.88$ $\quad$ MOL.% $N_2 = 4.56$ $\quad$ MOL.% $H_2S = 18.89$

(Complete gas analysis at the time of test is not available.)

WELL OPERATOR: Amoco Canada Petroleum Company Limited

TYPE OF TEST: Multi-Point

DATE OF TEST: September 20, 1970

GAS PRODUCED THROUGH: Tubing

WELL B (CONTINUED)

PRODUCTION AND PRESSURE DATA

Total Flow Rate (MMSCF/Day)	Tubing Head		Press. at M.P.P. (PSIA)	Condensate Prod. Rate (bbl/Day)	Water/Gas Ratio (bbl/MMSCF)
	Press. (PSIA)	Temp. (°F)			
6.527	3249.4	132	4249.6	0.0	1.5
9.598	3168.4	140	4205.8	0.0	1.5
12.048	3078.4	153	4163.2	0.0	1.5
14.277	2990.4	158	4123.2	0.0	1.5

WELL C

FIELD: Bigstone

WELL NAME: Pan Am. HB,C-1

POOL: D-3A

LOCATION: 13-35-60-22 W5M

PERFORATIONS: 10957'-10974' K.B.

DISTANCE TO M.P.P.: 10965' K.B.

TUBING SIZE: 2.992" I.D.

RESERVOIR TEMPERATURE: 243°F

GAS PROPERTIES:

$G = 0.702$ $\quad {}_{p}P_{c} = 798.3$ PSIA $\quad {}_{p}T_{c} = 409.5$ °R

MOL.% $CO_2 = 2.78$ $\quad$ MOL.% $N_2 = 4.56$ $\quad$ MOL.% $H_2S = 18.50$

(Complete gas analysis at the time of test is not available.)

WELL OPERATOR: Amoco Canada Petroleum Company Limited

TYPE OF TEST: Single-Point

DATE OF TEST: October 10, 1972

GAS PRODUCED THROUGH: Tubing

WELL C (CONTINUED)

PRODUCTION AND PRESSURE DATA

Total Flow Rate (MMSCF/Day)	Tubing Head		Press. at M.P.P. (PSIA)	Condensate Prod. Rate (bbl/Day)	Water/Gas Ratio (bbl/MMSCF)
	Press. (PSIA)	Temp. (°F)			
15.606	2314.5*	155	3295.5	0.0	5.21

* Dead Weight Test Data.

WELL D

FIELD: Bigstone

WELL NAME: Pan Am. HB,G-2

POOL: D-3A

LOCATION: 2-25-60-22 W5M

PERFORATIONS: 11006'-11052' K.B.

DISTANCE TO M.P.P.: 11029' K.B.

TUBING SIZE: 2.992" I.D.

RESERVOIR TEMPERATURE: 243°F

GAS PROPERTIES:

$G = 0.702$ $\quad {}_pP_c = 798.3$ PSIA $\quad {}_pT_c = 409.5$ °R

MOL.% $CO_2 = 2.78$ $\quad$ MOL.% $N_2 = 3.52$ $\quad$ MOL.% $H_2S = 18.50$

(Complete gas analysis at the time of test is not available.)

WELL OPERATOR; Amoco Canada Petroleum Company Limited

TYPE OF TEST: Single-Point

DATE OF TEST: October 10, 1972

GAS PRODUCED THROUGH: Tubing

WELL D (CONTINUED)

PRODUCTION AND PRESSURE DATA

Total Flow Rate (MMSCF/Day)	Tubing Head		Press. at M.P.P. (PSIA)	Condensate Prod. Rate (bbl/Day)	Water/Gas Ratio (bbl/MMSCF)
	Press. (PSIA)	Temp. (°F)			
17.359	2.599.5*	170	3690.4	0.0	1.3

* Dead Weight Test Data.

APPENDIX G

COPIES OF THE FORTRAN COMPUTER PROGRAMS FOR MAIN MODELS AA AND BB

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C***********************************************
C********************  MAIN MODEL AA ****************
C***********************************************

C**    THIS MAIN MODEL,'AA', REQUIRES THE FOLLOWING
C**        FUNCTION AND SUBROUTINE PROGRAMS :
C**        F,SEARCH,AMEIN,KASEM,BWR,LEE,MOODY,
C**        QUON & MISIC . THESE PROGRAMS ARE LISTED
C**        IN PAGES TO FOLLOW .

C**    HOW TO USE THIS COMPUTER PROGRAM

C      ITYPE IS A FLAG INTEGER
C            ITYPE=0 : INJECTION PROBLEM
C            ITYPE=1 : PRODUCTION PROBLEM
C      M     IS A FLAG INTEGER
C            M=1 : SUPPLY GAS ANALYSIS
C      MMM   IS A FLAG INTEGER
C            MMM=0 : ONE-PHASE FLOW
C            MMM=1 : TWO-PHASE FLOW

C      COMBINATIONS OF THE FLAG INTEGERS ARE AS FOLLOWS:
C      ITYPE   M       MMM         DESCRIPTION
C        1     1         1  TWO-PHASE FLOW PRODUCTION
C                           PROBLEM, SUPPLY GAS COMPOSITION
C                           AND SG OF FLOWING STREAM
C        1     1         0  ONE-PHASE FLOW PRODUCTION
C                           PROBLEM, SUPPLY GAS COMPOSITION

C      GASUC IS GAS UNIVERSAL CONSTANT
C      GC    IS CONVERSION FACTOR
C      AJ    IS CONVERSION FACTOR
C      SLOPE IS SLOPE OF PIPE SEGMENTS

C**    THE FOLLOWING TERMS BELONG TO PURE GASES
C      YMOLF IS MOLE FRACTION
C      YMW   IS MOLECULAR WEIGHT
C      YMW1  IS MOLECULAR WEIGHT
C      TEMC  IS CRITICAL TEMPERATURE IN DEG. RANKIN
C      TEMC1 IS CRITICAL TEMPERATURE IN DEG. RANKIN
C      PREC  IS CRITICAL PRESSURE IN PSIA
C      PREC1 IS CRITICAL PRESSURE IN PSIA
C      VOLC  IS CRITICAL VOLUME IN CU. FT/LB
C      VOLC1 IS CRITICAL VOLUME IN CU. FT/LB
C      TNBOI IS NORMAL BOILING POINT IN DEG. R
C      TNBO1 IS NORMAL BOILING POINT IN DEG. R
C      SEGM1 IS COLLISION DIAMETER IN ANGESTROMS
C      SEGMA IS COLLISION DIAMETER IN ANGESTROMS
```

```
C      EOK    IS A PARAMETER IN LENNARD-JONES POTENTIAL,
C                 IT IS  (EO /K ) IN THE TEXT
C      EOK1   IS THE SAME AS EOK
C      CPOC   IS THE FOUR COEFFICIENTS OF THE IDEAL
C                HEAT CAPACITY EQ. OF PURE GASES
C                (KOBE ET AL. )
C      CPOC1  IS THE SAME AS CPOC

C      SG     IS GAS GRAVITY
C      PTEMC  IS PSEUDOCRITICAL TEMP. IN DEG. RANKIN
C      PPREC  IS PSEUDOCRITICAL PRESS. IN PSIA
C      DIAM   IS TUBING DIAMETER IN INCHES
C      DEPTH  IS LENGTH OF TUBING IN FEET
C      TSOD   IS GROUND SURFACE TEMP. IN DEG. F
C      TSD    IS TUBING HEAD TEMP. IN DEG. F
C      TDD    IS BOTTOM-HOLE TEMP. IN DEG. F
C      PSD    IS TUBING HEAD PRESSURE IN PSIA
C      Q      IS TOTAL FLOW RATE IN MSCF/DAY

       IMPLICIT REAL*8 (A-H,O-Z)
       EXTERNAL F
       DIMENSION VOLC1(15),TNBO1(15),SEGM1(15),EOK1(15)
       DIMENSION YMW1(15),TEMC1(15),PREC1(15)
       DIMENSION CPOC1(15,4)
       COMMON /BL1/GASUC,GC,AJ,SLOPE/BL2/ITYPE
       COMMON /BL10/TNBOI(15)/BL4/SEGMA(15),EOK(15)
       COMMON /BL5/YMW(15),TC(15),ZC(15),GAMA(15)
       COMMON /BL8/CPOC(15,4)
       COMMON /BL11/TEMC(15),PREC(15),VOLC(15)
       COMMON /BL12/PTEMC,ZTEMC,PPREC,ZPREC/BL18/M
       COMMON /BL13/DIAM/BL14/SG/BL9/YMOLF(15)
       COMMON /BL15/Q,TSD,PSD,TDD,DEPTH,TSOD,DL
       COMMON /BL30/VMASS
       COMMON /BL50/MMM
       DATA  YMW1/0.44013D 02,0.28013D 02,0.34076D 02,
     1            0.16043D 02,0.30070D 02,0.44097D 02,
     2            0.58124D 02,0.58124D 02,0.72151D 02,
     3            0.72151D 02,0.86178D 02,0.86178D 02,
     4            0.10021D 03,0.10021D 03,0.11423D 03/
       DATA TEMC1/0.54757D 03,0.22727D 03,0.67237D 03,
     1            0.34304D 03,0.54976D 03,0.66568D 03,
     2            0.73465D 03,0.76532D 03,0.82877D 03,
     3            0.84537D 03,0.89550D 03,0.91337D 03,
     4            0.95467D 03,0.97247D 03,0.10239D 04/
       DATA PREC1/0.1071D 04,0.4930D 03,0.1306D 04,
     1            0.6678D 03,0.7078D 03,0.6163D 03,
     2            0.5291D 03,0.5507D 03,0.4904D 03,
     3            0.4886D 03,0.4366D 03,0.4369D 03,
     4            0.3965D 03,0.3968D 03,0.3606D 03/
       DATA VOLC1/0.342D-01,0.514D-01,0.459D-01,
     1            0.991D-01,0.788D-01,0.737D-01,
     2            0.724D-01,0.702D-01,0.679D-01,
```

```
     3           0.675D-01,0.681D-01,0.688D-01,
     4           0.673D-01,0.691D-01,0.690D-01/
      DATA TNBO1/0.35037D 03,0.13927D 03,0.38307D 03,
     1           0.20098D 03,0.33219D 03,0.41600D 03,
     2           0.47057D 03,0.49077D 03,0.54179D 03,
     3           0.55659D 03,0.60014D 03,0.61539D 03,
     4           0.65376D 03,0.66884D 03,0.71789D 03/
      DATA SEGM1/0.3996D 01,0.3798D 01,0.3623D 01,
     1           0.3758D 01,0.4443D 01,0.5118D 01,
     2           0.5278D 01,0.4687D 01,0.6051D 01,
     3           0.5784D 01,0.6256D 01,0.5949D 01,
     4           0.7015D 01,0.7004D 01,0.7451D 01/
      DATA  EOK1/0.1900D 03,0.7140D 02,0.3011D 03,
     1           0.1486D 03,0.2157D 03,0.2371D 03,
     2           0.3301D 03,0.5314D 03,0.2694D 03,
     3           0.3411D 03,0.2472D 03,0.3993D 03,
     4           0.2742D 03,0.2882D 03,0.3200D 03/
      DATA CPOC1/0.5316D 01, 0.6903D 01, 0.7070D 01,
     A           0.4750D 01, 0.1648D 01,-0.9660D 00,
     B          -0.1890D 01, 0.9450D 00,-0.2273D 01,
     C           0.1618D 01,-0.9081D 00, 0.1657D 01,
     D          -0.26069D 01, 0.31899D 01, 0.35844D 01,
     E     0.7936111D-02,-0.2085000D-03, 0.1737777D-02,
     F     0.6666666D-02, 0.2291111D-01, 0.4043888D-01,
     G     0.5519999D-01, 0.4929444D-01, 0.6907777D-01,
     H     0.6027777D-01, 0.7843494D-01, 0.7327777D-01,
     I     0.9611055D-01, 0.8315605D-01, 0.9502444D-01,
     J    -0.2580864D-05, 0.5956790D-06, 0.4209876D-06,
     K     0.9351851D-06,-0.4722222D-05,-0.1158950D-04,
     L    -0.1695987D-04,-0.1351851D-04,-0.2190432D-04,
     M    -0.1655864D-04,-0.2396910D-04,-0.2112345D-04,
     O    -0.3089506D-04,-0.2373148D-04,-0.2742592D-04,
     P     0.3058984D-09,-0.1176440D-09,-0.1348936D-09,
     Q    -0.4509602D-09, 0.2983539D-09, 0.1299725D-08,
     R     0.2043895D-08, 0.1433470D-08, 0.2719478D-08,
     S     0.1731824D-08, 0.2848422D-08, 0.2362825D-08,
     T     0.3834362D-08, 0.2614881D-08, 0.3060012D-08/
      GC=0.32174D 02
      GASUC=0.1545D 04
      AJ=0.77816D 03
      SLOPE=0.1D 01
      DO 5 I=1,15
      YMW(I)=YMW1(I)
      TEMC(I)=TEMC1(I)
      PREC(I)=PREC1(I)
      VOLC(I)=VCLC1(I)
      TNBCI(I)=TNBO1(I)
      SEGMA(I)=SEGM1(I)
      EOK(I)=EOK1(I)
      DO 5 J=1,4
    5 CPOC(I,J)=CPOC1(I,J)
      WRITE(6,208)
```

```
      WRITE(6,202)  (YMW(I),TEMC(I),PREC(I),VOLC(I),
     *TNBOI(I),SEGMA(I),EOK(I),I=1,15)
      WRITE(6,213)
      WRITE(6,205)((CPOC(I,J),J=1,4),I=1,15)
      WRITE(6,214)

C     READ 3 CARDS. GAS COMPOSITION.EACH CARD HAS
C         THE MOLE FRACTIONS OF 5 COMPONENTS. THE
C         ORDER OF COMPONENTS IS AS FOLLOWS :
C         CO2,N2,H2S,C1,C2,C3,I-C4,N-C4,I-C5,N-C5,
C         I-C6,N-C6,I-C7,N-C7,N-C8 .
C         IF GAS COMPOSITION IS NOT AVAILABLE,
C         SUPPLY 3 BLANK CARDS .

      READ(5,206)  (YMOLF(I),I=1,15)
      WRITE(6,215)
      WRITE(6,207) (YMOLF(I),I=1,15)

C     READ 1 CARD ( COMBINATION OF FLAG INTEGERS
C         AND SG )

      READ(5,209) ITYPE,M,MMM,SG

C     READ 2 CARDS ( WELL PRODUCTION DATA )

      READ(5,206) DIAM,DEPTH,TSOD
      READ(5,206) Q,TSD,PSD,TDD
      IF(ITYPE) 1,1,2
    1 WRITE(6,218)
      GO TO 3
    2 WRITE(6,219)
      WRITE(6,208)
    3 WRITE(6,216)  DIAM,DEPTH
      WRITE(6,217)  Q,TSOD,TSD,TDD,PSD
      CALL AMEIN
      WRITE(6,227)  SG,PTEMC,PPREC
      WRITE(6,225)
      WRITE(6,226)

C     CALCULATING MOLAL FLOW RATE
      TR=(0.6D 02+0.45967D 03)/ZTEMC
      PR=0.1465D 02/ZPREC
      CALL BWR  (TR,PR,ITER,ZST,DR,CPDM,DZDTR,DZDPR)
      VMASS=0.55744134D-02*Q/(ZST*DIAM**2)

C     READ 1 CARD ( RANGE OF R* )
C         XMIN=1000.0
C         XOUT=1010.0
C         XMAX=7000.0
C     IF CONVERGENCE IS NOT ACHIEVED,THE RANGE
C         MAY BE PUT BETWEEN 200.0 & 1000.0 OR
C         7000.0 & 200000.0
```

```
C**    REMARK ** FOR SAVIN  COMPUTER TIME IN CASE R*
C          IS OUTSIDE THE   ECIFIED RANGE,
C          ALWAYS PUT  XO  =XMIN+10.0

       READ(5,201) XMIN,   T,XMAX
       TEMESP=0.1D 00

C      MINIMIZING FUNCT    F

       CALL SEARCH (F,T  ESP,X,XMIN,XMAX,XOUT)
  201  FORMAT(7F10.4)
  202  FORMAT(10X,7F16
  204  FORMAT(4D15.7)
  205  FORMAT(10X,4D20  )
  206  FORMAT(5F15.8)
  207  FORMAT(18X,F20.5)
  208  FORMAT('1')
  209  FORMAT(3I5,5X,F10.4)
  211  FORMAT(10X,5F15.7)
  213  FORMAT(///////,29X,'A',19X,'B',19X,'C',19X,'D',/)
  214  FORMAT(//,47X,'CPO(I)=A+B*T+C*T**2+D*T**3')
  215  FORMAT('1',/////////,27X,'MOLE FRACTION',/)
  216  FORMAT(//////,10X,'TUBING I. D.',27X,'=',F10.3,
      A '  INCHES',/,
      B10X,'THE LENGTH OF FLOW STRING                   =',
      C F10.3,'  FEET')
  217  FORMAT(10X,'FLOW RATE',30X,'=',F10.3,
      A '  MSCF/DAY',/,
      B10X,'THE AVE. TEMP. OF THE GROUND SURFACE     =',
      C F10.3,'  F',/,
      D10X,'THE TEMP. OF FLOWING STREAM AT SURFACE =',
      E F10.3,'  F',/,
      F10X,'THE TEMP. OF THE PRODUCING FORMATION    =',
      G F10.3,'  F',/,
      H10X,'THE WELL HEAD TUBING PRESSURE            =',
      I F10.3,'  PSIA',/)
  218  FORMAT(10X,'INJECTION  PROBLEM')
  219  FORMAT(10X,'PRODUCTION PROBLEM')
  225  FORMAT(10X,'THE CALCULATED BOTTOM-HOLE PRESSURE',
      1 ' IS THAT WHEN THE ABSOLUTE DIFFERENCE BETWEEN',
      2 /,10X,'EARTH AND CALCULATED FLUID TEMPERATURES',
      3 ' AT THE BOTTOM OF WELL IS WITHIN 0.1 DEG. F .')
  226  FORMAT(/,10X,'FIRST COLUMN IS DEPTH IN FEET .',/,
      1 10X,'SECOND COL. IS EARTH TEMP. IN DEG. F .',/,
      2 10X,'THIRD COL. IS CALC. FLUID TEMP. IN DEG. F .'
      3 ,/,10X,'FOURTH COL. IS CALC. PRESS. IN PSIA .',
      4 /,10X,'FIFTH COL. IS R* .',/)
  227  FORMAT(10X,'GAS GRAVITY, PSEUDO REDUCED TEMP.,',
      1 'AND PSEUDO REDUCED PRESS. ARE :',/,10X,3F10.4)
       STOP
       END
```

```
      FUNCTION F(X)

C**** THIS FUNCTION SUBROUTINE IS TO BE
C****    USED WITH MAIN MODEL AA ONLY

C     F      IS THE FUNCTION TO BE MINIMIZED
C     E      IS ABSOLUTE ROUGHNESS OF TUBING IN INCHES
C     DIAM   IS TUBING DIAMETER IN INCHES
C     DL     IS MONITORED INTEGRATION STEP SIZE IN FEET
C     TGEOG IS GEOTHERMAL GRADIENT , DEG. F/ FOOT
C     PRE    IS PRESSURE IN PSIA
C     TEM    IS TEMPERATURE IN DEG. RANKIN
C     DENS   IS DENSITY OF NAT. GAS IN GM/C.C.
C     REN    IS REYNOLDS NUMBER
C     PRAND IS PRANDTL NUMBER
C     STANT IS STANTON NUMBER
C     GFTCO IS GAS-FILM HEAT-TRANSFER COEFFICIENT
C     OHTCO IS THE OVERALL HEAT-TRANSFER COEFFICIENT

      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION  PRY(5),TRY(5),PRK(5),TRK(5)
      COMMON /BL3/AVEMW/BL13/DIAM/BL14/SG
      COMMON /BL12/PTEMC,ZTEMC,PPREC,ZPREC
      COMMON /BL15/Q,TSD,PSD,TDD,DEPTH,TSOD,DL
      COMMON /BL6/CONST/BL18/M
      COMMON /BL30/VMASS
      COMMON /BL50/MMM
      D1=0.1D-01
      D2=0.1D-01
      DL=0.1D 02
      E=0.6D-03
      RSTAR=X
      RR=E/DIAM
      TGEOG=(TDD-TSOD)/DEPTH

C     THE RUNGE-KUTTA-MERSON INTEGRATION ALGORITHM

      TEM=TSD+0.45967D 03
      PRE=PSD
      TS=TSOD
      TR=TEM/PTEMC
      PR=PRE/PPREC
      TRN=TR
      PRN=PR
      TL=0.0D 00
      WRITE(6,211)  TL,TS,TSD,PRE,RSTAR
    1 DO 14 J1=1,5
      IF(J1-1) 2,2,3
    2 PRY(1)=PRN
      TRY(1)=TRN
      TS=TSOD+TGEOG*TL
    3 IF(J1-2) 11,4,5
```

```
    4 PRY(2)=PRN+PRK(1)
      TRY(2)=TRN+TRK(1)
      TS=TSOD+TGEOG*(TL+DL/0.3D 01)
    5 IF(J1-3) 11,6,7
    6 PRY(3)=PRN+0.5D 00*PRK(1)+0.5D 00*PRK(2)
      TRY(3)=TRN+0.5D 00*TRK(1)+0.5D 00*TRK(2)
      TS=TSOD+TGEOG*(TL+DL/0.3D 01)
    7 IF(J1-4) 11,8,9
    8 PRY(4)=PRN+0.375D 00*PRK(1)+0.1125D 01*PRK(3)
      TRY(4)=TRN+0.375D 00*TRK(1)+0.1125D 01*TRK(3)
      TS=TSOD+TGEOG*(TL+DL/0.2D 01)
    9 IF(J1-5) 11,10,14
   10 ABC1=PRN+0.15D 01*PRK(1)
      ABC2=-0.45D 01*PRK(3)+0.6D 01*PRK(4)
      PRY(5)=ABC1+ABC2
      ABC1=TRN+0.15D 01*TRK(1)
      ABC2=-0.45D 01*TRK(3)+0.6D 01*TRK(4)
      TRY(5)=ABC1+ABC2
      TS=TSOD+TGEOG*(TL+DL)
      PRJAM=PRY(5)
      TRJAM=TRY(5)
   11 PR=PRY(J1)
      TR=TRY(J1)
      TRS=(TS+0.45967D 03)/PTEMC
      PRE=PR*PPREC
      TEM=TR*PTEMC
C     CALCULATING  IDEAL HEAT CAPACITY OF NAT. GAS
      CALL KASEM (TR,TEM,SG,M,CPOM)
      TRC=TEM/ZTEMC
      PRC=PRE/ZPREC
C     CALCULATING COMPRESSIBILITY FACTOR,ITS PARTIAL
C               DERIVATIVES W.R.T. PR & TR ,REDUCED
C               DENSITY, AND HEAT CAPACITY DEPARTURE
C               OF NAT. GAS
      CALL BWR  (TRC,PRC,ITER,Z,DR,CPDM,DZDTR,DZDPR)
      IF(ITER)  22,22,12
   12 DENS=0.149256D-02*AVEMW*PRE/(Z*TEM)
      CPM=CPDM+CPOM
C     CALCULATING VISCOSITY OF NAT. GAS IN C.P.
      CALL LEE  (TEM,AVEMW,DENS,VISM)
      REN=0.20014855D 02*Q*SG/(VISM*DIAM)
C     CALCULATING THE HYDRAULIC FRICTION FACTOR
      CALL MOODY  (RR,REN,F[illegible]
```

```
C     CALCULATING THERMAL COND. OF NAT. GAS

      CALL MISIC (TEM,DR,CONST,THKM)

C     CALCULATING THE PRANDTL NUMBER

      PRAND=CPM*(VISM/0.1D 03)/(AVEMW*THKM)

C     CALCULATING THE STANTON NUMBER AND THE
C        GAS-FILM HEAT-TRANSFER COEFFICIENT

      B1=FF/0.8D 01
      B2=DSQRT(B1)
      ABC3=DSQRT(PRAND)
      ABC4=(B1*REN*REN)**0.27D 00
      PSI=ABC3*(0.155D 00*ABC4+0.1D 01/B2)
      STANT=B2/(PSI-0.45D 01)
      GFTCO=VMASS*CPM*STANT
      U1=0.1D 01/GFTCO+RSTAR
      OHTCO=0.1D 01/U1

C     SOLVING FOR THE RATE OF CHANGE OF REDUCED
C        PRESSURE AND TEMPERATURE WITH LENGTH

      CALL QUON(TEM,PRE,TR,PR,TRS,Z,DZDPR,DZDTR,
     1FF,CPM,OHTCO,VMASS,DPRDL,DTRDL,NASER)
      IF(NASER-1) 22,13,13
   13 PRK(J1)=DL*DPRDL/0.3D 01
      TRK(J1)=DL*DTRDL/0.3D 01
   14 CONTINUE

C     MONITORING THE INTEGRATION STEP SIZE

      ABC5=PRK(1)-0.45D 01*PRK(3)
      ABC6=0.4D 01*PRK(4)-0.5D 00*PRK(5)
      EPR=ABC5+ABC6
      ABC5=TRK(1)-0.45D 01*TRK(3)
      ABC6=0.4D 01*TRK(4)-0.5D 00*TRK(5)
      ETR=ABC5+ABC6
      ETEM=DABS(ETR*PTEMC)
      EPRE=DABS(EPR*PPREC)
      IF(EPRE-D1) 15,15,16
   15 IF(ETEM-D2) 17,17,16
   16 DL=DL/0.2D 01
      GO TO 1
   17 PR=PRJAM-EPR
      TR=TRJAM-ETR
      TI=TL+DL
      IF(EPRE-D1/0.16D 02) 18,20,20
   18 IF(ETEM-D2/0.16D 02) 19,20,20
   19 DL=0.2D 01*DL
   20 PRN=PR
```

```
      TRN=TR
      PRE=PRN*PPREC
      TEM=TRN*PTEMC
      TED=TEM-0.45967D 03
      WRITE(6,211) TL,TS,TED,PRE,RSTAR
      DEL=DEPTH-TL
      IF(DABS(DEL)-0.1D-04) 23,23,25
   25 IF(DEL-DL) 21,1,1
   21 DL=DEL
      GO TO 1
   22 WRITE(6,227)
   23 TED=TEM-0.45967D 03
      TEMTL=TED
      F=TED-TEMTL
  211 FORMAT(10X,5F10.2)
  227 FORMAT(//,10X,'*** THE COURSE OF ITERATIONS IS'
     *,' INTERRUPTED ***',//,7X,'*** THE PROGRAM IS'
     *,' GOING TO ASSUME ANOTHER "RSTAR"***')
      RETURN
      END
```

```
      SUBROUTINE AMEIN

C**** THIS SUBROUTINE CAN BE USED WITH
C****         MAIN MODEL AA ONLY

C     THIS SUBROUTINE CALCULATES THE MOLAL AVERAGE
C         PROPERTIES,THE WICHERT & AZIZ CORRECTED
C         PSEUDOCRITICAL PROPERTIES OF NAT. GAS ,
C         AND OTHER PROPERTIES

C     THE FOLLOWING TERMS BELONG TO PURE GASES

C     S      IS THE SUTHERLAND CONSTANT IN DEG. K
C     VC     IS CRITICAL MOLAL VOLUME, C.C./GM MOLE
C     VCLC   IS CRITICAL VOLUME , CU. FT/ LB.
C     ZC     IS CRITICAL COMPRESSIBILITY FACTOR
C     GAMA   IS CONSTANT DEFINED BY EQ. 58 IN TEXT

C     THE FOLLOWING TERMS BELONG TO NAT. GAS

C     AVEMW IS AVERAGE MOLECULAR WEIGHT
C     SG    IS GAS GRAVITY
C     PPREC IS PSEUDOCRITICCAL PRESSURE IN PSIA
C     PPC   IS PSEUDOCRITICCAL PRESSURE IN ATMOSPHERS
C     PTEMC IS PSEUDOCRITICAL TEMPERATURE,DEG. R
C     PTC   IS PSEUDOCRITICAL TEMPERATURE,DEG. K
C     GAM   IS CONSTANT DEFINED BY EQ. 58 IN TEXT
C     ZTEMC IS CORRECTED PSEUDOCRITICAL TEMPERATURE
C     ZPREC IS CORRECTED PSEUDOCRITICAL PRESSURE

      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION PC(15)
      COMMON /BL3/AVEMW/BL4/SEGMA(15),EOK(15)
      COMMON /BL5/YMW(15),TC(15),ZC(15),GAMA(15)
      CCMMON /BL9/YMOLF(15)/BL10/TNBOI(15)/BL20/S(15)
      COMMON /BL11/TEMC(15),PREC(15),VOLC(15)/BL6/CONST
      COMMON /BL12/PTEMC,ZTEMC,PPREC,ZPREC
      CCMMON /BL14/SG/BL18/M
      COMMON /BL50/MMM
      COST1=0.15D 01/0.18D C1
      CCST2=0.1D 01/C.6D 01
      COST3=0.2D 01/0.3D 01
      Y11=YMOLF(1)
      Y22=YMOLF(2)
      Y33=YMOLF(3)
      IF(M-1) 3,1,1
    1 AVEMW=0.0D 00
      PTEMC=0.0D 00
      PPREC=0.0D 00
      DO 2 I=1,15
      S(I)=COST1*TNBOI(I)
      PC(I)=PREC(I)/0.146964757D 02
```

```
      TC(I)=TEMC(I)/0.18D 01
      VC=0.623826896D 02*YMW(I)*VOLC(I)
      ZC(I)=PC(I)*VC/(0.8206D 02*TC(I))
      Y1=TC(I)**COST2
      Y2=PC(I)**COST3
      GAMA(I)=0.1D 07*Y1*DSQRT(YMW(I))/Y2
      AVEMW=AVEMW+YMOLF(I)*YMW(I)
      PTEMC=PTEMC+YMOLF(I)*TEMC(I)
    2 PPREC=PPREC+YMOLF(I)*PREC(I)
      IF(MMM-1) 10,3,3
   10 SG=AVEMW/0.2897D 02
      GO TO 7
    3 AVEMW=SG*0.2897D 02

C     THE BROWN ET AL AND CARR ET AL CORRELATIONS
C         FOR THE PSEUDOCRITICAL PROPERTIES OF
C         SWEET AND SOUR NAT. GASES

      PTEMC=0.171137D 03+0.313725D 03*SG
      TCCOR=-0.8D 02*Y11-0.25D 03*Y22+0.13D 03*Y33
      PTEMC=PTEMC+TCCOR
      IF(SG-0.85D 00) 4,4,5
    4 PPREC=0.6951D 03-0.4D 02*SG
      GO TO 6
    5 PPREC=0.704396D 03-0.51724D 02*SG
    6 PCCOR=0.44D 03*Y11-0.17D 03*Y22+0.6D 03*Y33
      PPREC=PPREC+PCCOR

C     THE WICHERT & AZIZ CORRECTED PSEUDOCRITICALS
C        DUE TO PRESENCE OF CO2 AND H2S

    7 S1=YMOLF(1)+YMOLF(3)
      S2=YMOLF(3)
      E31=0.12D 03*(S1**0.9D 00-S1**0.16D 01)
      E32=0.15D 02*(DSQRT(S2)-S2**4)
      E3=E31+E32
      S4=PTEMC+S2*(0.1D 01-S2)*E3
      ZTEMC=PTEMC-E3
      ZPREC=PPREC*ZTEMC/S4
      PTC=PTEMC/0.18D 01
      PPC=PPREC/0.14696476D 02
      GAM=PTC**COST2*DSQRT(AVEMW)/PPC**COST3
      CCNST=GAM*0.14348907D 06
      RETURN
      END
```

```
      SUBROUTINE MISIC (TEM,DR,CONST,THKM)

C**** THIS SUBROUTINE CAN BE USED WITH
C****           MAIN MODEL AA ONLY

C     THIS SUBROUTINE CALCULATES THE THERMAL
C          CONDUCTIVITY OF GAS MIXTURES

C**   THE MISIC AND THODOS METHODS ARE USED TO
C     CALCULATE THKO,THKMO,THKMD,AND THKM

C     TEM    IS TEMP. IN DEGREE RANKIN
C     DR     IS REDUCED DENSITY OF GAS MIXTURE
C     THKMO  IS THERMAL COND. OF GAS MIX. AT 0 PRESS.
C     THKMD  IS THERMAL COND. DEPARTURE
C     THKM   IS THERMAL CONDUCTIVITY OF GAS MIXTURE
C               IN CAL/CM SEC K
C     CONST  IS 10**8*0.27**5*GAM

C**   THE FOLLOWING TERMS BELONG TO PURE GASES
C     YMW    IS MOLECULAR WEIGHT
C     TC     IS CRITICAL TEMPERATURE IN DEG. KELVIN
C     ZC     IS CRITICAL COMPRESSIBILITY FACTOR
C     S      IS SUTHERLAND'S CONSTANT IN DEG. KELVIN
C     CPO    IS HEAT CAPACITY AT ZERO PRESSURE
C               IN CAL/GM MOLE K
C     THKO   IS THERMAL COND. AT 0 PRESSURE
C     YMOLF  IS MOLE FRACTION

      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(15,15),SS(15,15),VISO(15),
     * THKO(15),F(2),TRED(15)
      COMMON /BL4/SEGMA(15),EOK(15)
      COMMON /BL5/YMW(15),TC(15),ZC(15),GAMA(15)
      COMMON /BL7/CPO(15)/BL9/YMOLF(15)/BL20/S(15)
      UNITY=0.1D 01
      FRAC=2.0D 00/3.0D 00
      TK=TEM/0.18D 01
      TKK=TK
      DO 8 I=1,15
      TRED(I)=TK/TC(I)
      DO 5 K=1,2
      IFLAG=K
      T=TK/EOK(I)
      IF(T-0.2D 01) 1,2,2
    1 F(K)=0.634D 00*(UNITY+0.475D 00*DLOG(T))
      GO TO 3
    2 F(K)=0.697D 00*(UNITY+0.323D 00*DLOG(T))
    3 IF(TRED(I)-0.2D 01) 4,6,6
    4 TK=0.2D 01*TC(I)
    5 CONTINUE
      Y=DSQRT(YMW(I)*TK)
```

```
    6 VISO(I)=0.26693D-02*Y*F(IFLAG)/SEGMA(I)**2
      IF(IFLAG-1) 8,8,7
    7 TK=TKK
      VISO(I)=VISO(I)*0.5D 00*TRED(I)*F(1)/F(2)
    8 CONTINUE
      DO 11 I=1,15
      IF(I-3) 9,9,10
    9 A1=0.195D 03*ZC(I)-0.3194D 02
      A2=0.1683D 02-0.825D 02*ZC(I)
      A3=0.1524D 01-0.28D 01*ZC(I)
      A4=(A1*TRED(I)+A2)**A3
      THKO(I)=CPO(I)**0.75D 00*A4/GAMA(I)
      GO TO 11
   10 B1=0.1452D 02*TRED(I)-0.514D 01
      THKO(I)=CPO(I)*B1**FRAC/GAMA(I)
   11 CONTINUE
      DO 12 I=1,15
      DO 12 J=1,15
      SS(I,J)=DSQRT(S(I)*S(J))
      C1=(UNITY+SS(I,J)/TK)/(UNITY+S(I)/TK)
      C2=(VISO(I)/VISO(J))*(YMW(J)/YMW(I))**0.75D 00
      C3=(UNITY+S(I)/TK)/(UNITY+S(J)/TK)
      A(I,J)=0.25D 00*C1*(UNITY+DSQRT(C2*C3))**2
   12 CONTINUE
      THKMO=0.0D 00
      DO 15 I=1,15
      IF(YMOLF(I)-0.1D-06) 15,15,25
   25 DUMY=0.1D 01
      DO 14 J=1,15
      IF(YMOLF(J)-0.1D-06) 14,14,26
   26 IF(I-J) 13,14,13
   13 DUMY=DUMY+A(I,J)*YMOLF(J)/YMOLF(I)
   14 CCNTINUE
      THKMO=THKMO+THKO(I)/DUMY
   15 CONTINUE
      IF(DR-0.5D 00) 16,16,17
   16 X1=DEXP(0.535D 00*DR)
      THKMD=0.14D 02*(X1-UNITY)/CONST
      GO TO 20
   17 IF(DR-0.2D 01) 18,18,19
   18 X1=DEXP(0.67D 00*DR)
      THKMD=0.131D 02*(X1-0.1069D 01)/CONST
      GC TO 20
   19 X1=DEXP(0.1155D 01*DR)
      THKMD=0.2976D 01*(X1+0.2016D 01)/CONST
   20 THKM=THKMO+THKMD
      RETURN
      END
```

```
      SUBROUTINE SEARCH(F,EPS,X,YMIN,YMAX,RSTOUT)

C**** THIS SUBROUTINE CAN BE USED WITH
C****    EITHER MAIN MODEL AA OR BB

C     THIS SUBROUTINE SEARCHES FOR THE VALUE OF
C          R* OR R** WHICH MINIMIZES THE FUNCTION 'F'

      IMPLICIT REAL*8 (A-H,O-Z)
      EXTERNAL F
      DIMENSION RST(3),TEST(2),DELTA(3)
      RST(1)=YMIN
      RST(3)=YMAX
      ML=0
   30 IF(ML-17) 1,1,21
   21 WRITE(6,225)
      GO TO 13
    1 RST(2)=(RST(1)+RST(3))/0.2D 01
        IF(RST(2)-RSTOUT) 13,13,2
    2 DO 6 K3=1,3
      IF(ML) 4,4,3
    3 IF(K3-2) 6,4,6
    4 RSTAR=RST(K3)
      X=RSTAR
      DIFF=F(X)
      IF(DABS(DIFF)-EPS) 13,13,5
    5 DELTA(K3)=DIFF
    6 CONTINUE
      TEST(1)=DELTA(1)*DELTA(2)
      TEST(2)=DELTA(3)*DELTA(2)
      DESCN=DELTA(1)*DELTA(3)
      IF(ML) 23,23,24
   23 IF(DESCN) 24,24,25
   25 WRITE(6,226)
      GO TO 13
   24 ML=ML+1
      DO 8 K5=1,2
      IF(TEST(K5)) 7,13,8
    7 ITEST=K5
      GO TO 9
    8 CONTINUE
    9 IF(ITEST-1) 10,10,11
   10 RST(3)=RST(2)
      DELTA(3)=DELTA(2)
      GO TO 30
   11 IF(TEST(2)) 12,13,12
   12 RST(1)=RST(2)
      DELTA(1)=DELTA(2)
      GO TO 30
  225 FORMAT(//,20X,'*** NO SOLUTION IS ',
     *'REACHED WITH 20 ITERATIONS ***')
  226 FORMAT(//,20X,'*** NO SOLUTION IS ',
```

```
   *'PROVIDED TO THIS PROBLEM ***')
13 RETURN
   END
```

```
      SUBROUTINE KASEM(TR,TEM,SG,M,CPOM)
C**** THIS SUBROUTINE CAN BE USED WITH
C****    EITHER MAIN MODEL AA OR BB
C
C     TR     IS REDUCED TEMP.
C     TEM    IS TEMPERATURE IN DEGREE RANKIN
C     SG     IS GAS GRAVITY
C     M      IS A FLAG INTEGER
C            M=0 : SUPPLY SG,H2S,CO2,AND N2
C            M=1 : SUPPLY GAS ANALYSIS
C     CPOM   IS HEAT CAPACITY AT ZERO PRESS. OF NAT. GAS
C     CPC    IS HEAT CAPACITY AT ZERO PRESS. OF PURE GAS
C     YMOLF  IS MOLE FRACTION

      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(14)
      COMMON /BL7/CPO(15)/BL8/CPOC(15,4)/BL9/YMOLF(15)
      DATA A/0.5596695D 01,-0.2233480D 01,
     1       0.807265D 00,-0.100390D 01,0.314160D 01,
     2       0.5758700D 01,0.425860D 00,0.124323D-02,
     3       -0.40560D-01,0.100889D-02,0.3623622D 00,
     4       -0.4661581D 00,0.9757D-03,0.27082167D-02/
      IF(M-1) 4,1,1
    1 CPOM=0.0D 00
      DO 3 I=1,15
      CPO(I)=0.0D 00
      DO 2 J=1,4
    2 CPO(I)=CPO(I)+CPOC(I,J)*TEM**(J-1)
    3 CPOM=CPOM+CPO(I)*YMOLF(I)
      GO TO 5
    4 B=A(1)+A(2)*SG+A(3)*SG**2
      C=A(4)+A(5)*SG+A(6)*SG**2
      CPOM=B+C*TR
      FCO2=0.1D 01+YMOLF(1)*(A(7)+A(8)*TEM)
      FH2S=0.1D 01+YMOLF(3)*(A(9)+A(10)*TEM)
      Y1=A(11)*YMOLF(2)+A(12)*YMOLF(2)**[illegible]
      Y2=A(13)*YMOLF(2)+A(14)*YMOLF(2)**2
      FN2=0.1D 01+Y1+Y2*TEM
      CPOM=CPOM/(FCO2*FN2*FH2S)
    5 RETURN
      END
```

```
      SUBROUTINE BWR(TR,PR,ITER,Z,DR,CPDM,DZDTR,DZDPR)
C**** THIS SUBROUTINE CAN BE USED WITH
C****    EITHER MAIN MODEL AA OR BB
C     TR     IS THE PSEUDO-REDUCED TEMPERATURE
C     PR     IS THE PSEUDO-REDUCED PRESSURE
C     Z      IS THE GAS COMPRESSIBILITY FACTOR
C     DR     IS THE REDUCED DENSITY OF GAS
C     DZDTR  IS THE PARTIAL DERIVATIVE OF Z W.R.T. TR
C     DZDPR  IS THE PARTIAL DERIVATIVE OF Z W.R.T. PR
C     CPDM   IS THE HEAT CAPACITY DEPARTURE
C     ITER   IS AN ITERATION FLAG
C       ITER=0 EITHER OR BOTH TR AND PR ARE
C              OUTSIDE RANGE OF CORRELATION
C       ITER>0 NUMBER OF ITERATIONS
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(8)
      DATA A/0.31506237D 00,1.0467099D 00,
     A   0.57832729D 00,0.53530771D 00,0.61232032D 00,
     B   0.10488813D 00,0.68157001D 00,0.68446549D 00/
      ITER=0
      DR=0.1D 01
      IF(TR-0.105D 01)10,1,1
    1 IF(TR-0.3D 01)2,2,10
    2 IF(PR-0.30D 02)3,3,10
    3 DO 8 ITER=1,30
      DR2=DR*DR
      T1=(A(1)*TR-A(2)-A(3)/TR**2)*DR
      T2=(A(4)*TR-A(5))*DR2
      T3=A(5)*A(6)*DR**5
      T4=A(7)*DR2/TR**2
      T5=A(8)*DR2
      T6=DEXP(-T5)
      P=(TR+T1+T2+T3)*DR+T4*DR*(0.1D 01+T5)*T6
      DP1=TR+0.2D 01*T1+0.3D 01*T2+0.6D 01*T3
      DP2=T4*T6*(0.3D 01+0.3D 01*T5-0.2D 01*T5*T5)
      DP=DP1+DP2
      DR1=DR-(P-0.270D 00*PR)/DP
      IF(DR1)4,4,5
    4 DR1=0.5D 00*DR
    5 IF(DR1-0.22D 01)7,7,6
    6 DR1=DR+0.9D 00*(0.22D 01-DR)
    7 IF(DABS(DR-DR1)-0.1D-04)9,8,8
    8 DR=DR1
    9 Z=0.270D 00*PR/(DR1*TR)
      T7=(A(1)*TR+0.2D 01*A(3)/TR**2)*DR
      T8=A(4)*TR*DR**2
      T9=A(3)*DR/TR**2
      T10=A(7)/A(8)/TR**2
      R1=DP/TR
```

```
      R2=(TR+T7+T8-0.2D 01*T4*T6*(0.1D 01+T5))/TR
      P2=R2*DR
      P3=0.270D 00/DP
      Y1=0.3D 01*T6*(0.2D 01*T10+T4)
      R3=(-TR+0.6D 01*(T9-T10)+Y1)/TR
      P4=P2/DP
      DZDTR=Z*(P4/DR-0.1D 01/TR)
      DZDPR=Z*(0.1D 01/PR-P3/DR)
      CPDM=0.198654284D 01*(R3+R2**2/R1)
   10 RETURN
      END
```

```
      SUBROUTINE LEE(TEM,AVEMW,DENS,VISM)

C**** THIS SUBROUTINE CAN BE USED WITH
C****     EITHER MAIN MODEL AA OR BB

C     THIS SUBROUTINE IS THE LEE,ET AL. VISCOCITY EQ.
C     TEM   IS TEMPERATURE IN DEGREE RANKIN
C     AVEMW IS AVERAGE MOLECULAR WEIGHT
C     DENS  IS THE GAS DENSITY IN GRAM/C.C.
C     VISM  IS THE GAS VISCOCITY IN C.P.

      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION V(11)
      DATA V/0.94D 01,0.2D-01,0.15D 01,0.209D 03,
     1       0.19D 02,0.35D 01,0.986D 03,0.1D-01,
     2       0.24D 01,0.2D 00,0.1D 05/
      VK1=(V(1)+V(2)*AVEMW)*TEM**V(3)
      VK2=V(4)+V(5)*AVEMW+TEM
      VK=VK1/VK2
      VX=V(6)+V(7)/TEM+V(8)*AVEMW
      VY=V(9)-V(10)*VX
      VY1=VX*DENS**VY
      VISM=VK*DEXP(VY1)/V(11)
      RETURN
      END
```

```
      SUBROUTINE MOODY (RR,REN,FF,I)
C**** THIS SUBROUTINE CAN BE USED WITH
C****     EITHER MAIN MODEL AA OR BB
C      RR    IS THE RELATIVE ROUGHNESS
C      REN   IS REYNOLDS NUMBER
C      RENCR IS THE BOUNDARY BETWEEN THE
C              TRANSITION AND ROUGH PIPE REGIONS
C      FF    IS THE HYDRAULIC FRICTION FACTOR
C      I     IS NO. OF ITERATIONS REQUIRED
C            I=0 ROUGH PIPE REGION
C            I>0 TRANSITION REGION .
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION G(15)
      I=0
      C=DLOG(0.1D 02)
      IF(RR) 2,2,1
    1 RENCR=0.32D 04/RR
      IF(REN-RENCR) 2,6,6
C     THE COLEBROOK TRANSITION LAW
    2 X=0.8D-01
      DO 4 I=1,15
      X1=RR/0.37D 01+0.251D 01/(REN*X)
      X2=DLOG10(X1)
      FX=0.1D 01+0.2D 01*X*X2
      FXD=0.2D 01*X2-0.502D 01/(REN*X*X1*C)
      G(I)=X-FX/FXD
      X=G(I)
      IF(I-1) 4,4,3
    3 IF(DABS(G(I)-G(I-1))-0.1D-05) 5,5,4
    4 CONTINUE
    5 FF=X*X
      GO TO 7
C     VON-KARMAN'S ROUGH PIPE FORMULA
    6 FF=0.1D 01/(0.2D 01*DLOG10(0.37D 01/RR))**2
    7 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE QUON(TEM,PRE,TR,PR,TRS,Z,DZDPR,
     1DZDTR,FF,CPM,OHTCO,VMASS,DPRDL,DTRDL,NASER)
C**** THIS SUBROUTINE CAN BE USED WITH
C****    EITHER MAIN MODEL AA OR BB
C     THIS SUBROUTINE SOLVES FOR THE RATE OF CHANGE
C        OF REDUCED PRESSURE & TEMP. WITH LENGTH
C        (SEE EQUATIONS 6 & 7 IN TEXT )
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /BL1/GASUC,GC,AJ,SLOPE/BL2/ITYPE
      CCMMON /BL3/AVEMW/BL13/DIAM
      NASER=1
      O=SLOPE
      BETA=0.1D 01
      DIAN=DIAM/0.12D 02
      A1=BETA*VMASS**2*TEM*GASUC
      PRF=PRE*0.144D 03
      A2=GC*AJ*PRF**2
      A3=A1/A2
      A4=A3*TEM*GASUC
      A5=Z/PR-DZDPR
      A6=Z/TR+DZDTR
      ASTAR=-A1*A5*PRF+GC*PRF**3/(PR*AVEMW)
      BSTAR=A1*A6*PRF
      CSTA1=-A2*PRF*O/(Z*TEM*AJ*GASUC)
      CSTA2=-A1*Z*PRF*FF/(0.2D 01*DIAN*BETA)
      CSTAR=CSTA1+CSTA2
      DSTAR=A4*A5*Z+TEM*TR*GASUC*DZDTR/(AJ*PR*AVEMW)
      ESTAR=-A4*A6*Z-TEM*CPM/(TR*AVEMW)
      CHRIS=VMASS*DIAN
      FSTA1=CHRIS*O
      FSTA2=0.4D 01*OHTCO*TEM*AJ*(TRS-TR)/(TR*AVEMW)
      FSTAR=(FSTA1-FSTA2)/(CHRIS*AJ)
      A7=ASTAR*ESTAR-BSTAR*DSTAR
      IF(A7) 1,3,1
    1 DPRDL=(CSTAR*ESTAR-BSTAR*FSTAR)/A7
      DTRDL=(ASTAR*FSTAR-CSTAR*DSTAR)/A7
      IF(ITYPE-1) 4,2,2
    2 DPRDL=-DPRDL
      DTRDL=-DTRDL
      GC TO 4
    3 NASER=0
    4 RETURN
      END
```

```
C*********************************************************
C********************* MAIN MODEL BB *******************
C*********************************************************
C**   THIS MAIN MODEL,'BB', REQUIRES THE FOLLOWING
C**        FUNCTION AND SUBROUTINE PROGRAMS:
C**        F & AMEIN (LISTED IN PAGES TO FOLLOW),
C**        SEARCH,KASEM,BWR,LEE,MOODY & QUON (LISTED
C**        IN PREVIOUS PAGES) .

C**   HOW TO USE THIS COMPUTER PROGRAM .

C     ITYPE IS A FLAG INTEGER
C           ITYPE=0 : INJECTION PROBLEM
C           ITYPE=1 : PRODUCTION PROBLEM
C     M     IS A FLAG INTEGER
C           M=0 : SUPPLY SG,H2S,CO2,AND N2
C           M=1 : SUPPLY GAS ANALYSIS
C     MM    IS A FLAG INTEGER
C           MM=0 : DO NOT SUPPLY PSEUDOCRITICALS
C           MM=1 : SUPPLY PSEUDOCRITICALS
C     MMM   IS A FLAG INTEGER
C           MMM=0 : ONE-PHASE FLOW
C           MMM=1 : TWO-PHASE FLOW

C     COMBINATIONS OF THE FLAG INTEGERS ARE AS FOLLOWS:
C     ITYPE  M  MM  MMM        DESCRIPTION
C       1    1   1   1  TWO-PHASE FLOW PRODUCTION
C                       PROBLEM. SUPPLY GAS COMPOSITION
C                       AND SG OF FLOWING STREAM
C       1    1   1   0  ONE-PHASE FLOW PRODUCTION
C                       PROBLEM. SUPPLY GAS COMPOSITION
C       1    0   0   1  TWO-PHASE FLOW PRODUCTION
C                       PROBLEM. SUPPLY SG OF FLOWING
C                       STREAM,CO2,N2 & H2S
C       1    0   0   0  ONE-PHASE FLOW PRODUCTION
C                       PROBLEM. SUPPLY SG,CO2,N2 & H2S
C       1    0   1   0  ONE-PHASE FLOW PRODUCTION
C                       PROBLEM. SUPPLY SG,CO2,N2,H2S
C                       AND THE REPORTED PTEMC & PPREC

C     GASUC IS GAS UNIVERSAL CONSTANT
C     GC    IS CONVERSION FACTOR
C     AJ    IS CONVERSION FACTOR
C     SLOPE IS SLOPE OF PIPE SEGMENTS

C**   THE FOLLOWING TERMS BELONG TO PURE GASES
C     YMOLF IS MOLE FRACTION
C     YMW   IS MOLECULAR WEIGHT
C     YMW1  IS MOLECULAR WEIGHT
C     TEMC  IS CRITICAL TEMPERATURE IN DEG. RANKIN
```

```
C     TEMC1 IS CRITICAL TEMPERATURE IN DEG. RANKIN
C     PREC  IS CRITICAL PRESSURE IN PSIA
C     PREC1 IS CRITICAL PRESSURE IN PSIA
C     CPOC  IS THE FOUR COEFFICIENTS OF THE IDEAL
C              HEAT CAPACITY EQ. OF PURE GASES
C              (KOBE ET AL. )
C     CPOC1 IS THE SAME AS CPOC

C     SG    IS GAS GRAVITY
C     PTEMC IS PSEUDOCRITICAL TEMP. IN DEG. RANKIN
C     PPREC IS PSEUDOCRITICAL PRESS. IN PSIA
C     DIAM  IS TUBING DIAMETER IN INCHES
C     DEPTH IS LENGTH OF TUBING IN FEET
C     TSOD  IS GROUND SURFACE TEMP. IN DEG. F
C     TSD   IS TUBING HEAD TEMP. IN DEG. F
C     TDD   IS BOTTOM-HOLE TEMP. IN DEG. F
C     PSD   IS TUBING HEAD PRESSURE IN PSIA
C     Q     IS TOTAL FLOW RATE IN MSCF/DAY

      IMPLICIT REAL*8 (A-H,O-Z)
      EXTERNAL F
      DIMENSION YMW1(15),TEMC1(15),PREC1(15)
      DIMENSION CPOC1(15,4)
      COMMON /BL1/GASUC,GC,AJ,SLOPE/BL2/ITYPE
      COMMON /BL8/CPOC(15,4)/BL5/YMW(15)
      COMMON /BL11/TEMC(15),PREC(15)
      COMMON /BL12/PTEMC,ZTEMC,PPREC,ZPREC/BL18/M
      COMMON /BL13/DIAM/BL14/SG/BL9/YMOLF(15)
      COMMON /BL15/Q,TSD,PSD,TDD,DEPTH,TSOD,DL
      COMMON /BL30/VMASS/BL31/MM
      COMMON /BL50/MMM
      DATA  YMW1/0.44013D 02,0.28013D 02,0.34076D 02,
     1            0.16043D 02,0.30070D 02,0.44097D 02,
     2            0.58124D 02,0.58124D 02,0.72151D 02,
     3            0.72151D 02,0.86178D 02,0.86178D 02,
     4            0.10021D 03,0.10021D 03,0.11423D 03/
      DATA TEMC1/0.54757D 03,0.22727D 03,0.67237D 03,
     1           0.34304D 03,0.54976D 03,0.66568D 03,
     2           0.73465D 03,0.76532D 03,0.82877D 03,
     3           0.84537D 03,0.89550D 03,0.91337D 03,
     4           0.95467D 03,0.97247D 03,0.10239D 04/
      DATA PREC1/0.1071D 04,0.4930D 03,0.1306D 04,
     1           0.6678D 03,0.7078D 03,0.6163D 03,
     2           0.5291D 03,0.5507D 03,0.4904D 03,
     3           0.4886D 03,0.4366D 03,0.4369D 03,
     4           0.3965D 03,0.3968D 03,0.3606D 03/
      DATA CPOC1/0.5316D 01, 0.6903D 01, 0.7070D 01,
     A           0.4750D 01, 0.1648D 01,-0.9660D 00,
     B          -0.1890D 01, 0.9450D 00,-0.2273D 01,
     C           0.1618D 01,-0.9081D 00, 0.1657D 01,
     D          -0.26069D 01, 0.31899D 01, 0.35844D 01,
     E     0.7936111D-02,-0.2085000D-03, 0.1737777D-02,
```

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     F      0.6666666D-02, 0.2291111D-01, 0.4043888D-01,
     G      0.5518999D-01, 0.4929444D-01, 0.6907777D-01,
     H      0.6027777D-01, 0.7843494D-01, 0.7327777D-01,
     I      0.961[illegible]D-01, 0.8315605D-01, 0.9502444D-01,
     J     -0.2580864D-05, 0.5956790D-06, 0.4209876D-06,
     K      0.9351851D-06,-0.4722222D-05,-0.1158950D-04,
     L     -0.1695987D-04,-0.1351851D-04,-0.2190432D-04,
     M     -0.1655864D-04,-0.23969[illegible]0D-04,-0.2112345D-04,
     O     -0.3089506D-04,-0.2373148D-04,-0.2742592D-04,
     P      0.3058984D-09,-0.11[illegible]D-09,-0.1348936D-09,
     Q     -0.4509602D-09, 0.2985539D-08, 0.1299725D-08,
     R      0.2043895D-08, 0.14334[illegible]D-08, 0.2719478D-08,
     S      0.1731824D-08, 0.2848422D-08, 0.2362825D-08,
     T      0.3834362D-08, 0.2614881D-08, 0.3060012D-08/
      GC=0.32174D 02
      GASUC=0.1545D 04
      AJ=0.77816D 03
      SLOPE=0.1D 01
      DO 5 I=1,15
      YMW(I)=YMW1(I)
      TEMC(I)=TEMC1(I)
      PREC(I)=PREC1(I)
      DO 5 J=1,4
    5 CPOC(I,J)=CPOC1(I,J)
      WRITE(6,208)
      WRITE(6,202) (YMW(I),TEMC(I),PREC(I),I=1,15)
      WRITE(6,213)
      WRITE(6,205) ((CPOC(I,J),J=1,4),I=1,15)
      WRITE(6,214)

C     READ 3 CARDS. GAS COMPOSITION.EACH CARD HAS
C         THE MOLE FRACTIONS OF 5 COMPONENTS. THE
C         ORDER OF COMPONENTS IS AS FOLLOWS :
C         CO2,N2,H2S,C1,C2,C3,I-C4,N-C4,I-C5,N-C5,
C         I-C6,N-C6,I-C7,N-C7,N-C8 .
C         IF GAS COMPOSITION IS NOT AVAILABLE,
C         SUPPLY 3 BLANK CARDS .

      READ(5,206) (YMOLF(I),I=1,15)
      WRITE(6,215)
      WRITE(6,207) (YMOLF(I),I=1,15)

C     READ 1 CARD ( COMBINATION OF FLAG INTEGERS
C        AND SG,PTEMC & PPREC )

      READ(5,209) ITYPE,M,MM,MMM,SG,PTEMC,PPREC

C     READ 2 CARDS ( WELL PRODUCTION DATA )

      READ(5,206) DIAM,DEPTH,TSOD
      READ(5,206) Q,TSD,PSD,TDD
      IF(ITYPE) 1,1,2
```

```
     C F10.3,'  FEET')
 217 FORMAT(10X,'FLOW RATE',30X,'=',F10.3,
     A '  MSCF/DAY',/,
     B10X,'THE AVE. TEMP. OF THE GROUND SURFACE   =',
     C F10.3,'  F',/,
     D10X,'THE TEMP. OF FLOWING STREAM AT SURFACE =',
     E F10.3,'  F',/,
     F10X,'THE TEMP. OF THE PRODUCING FORMATION   =',
     G F10.3,'  F',/,
     H10X,'THE WELL HEAD TUBING PRESSURE          =',
     I F10.3,'  PSIA',/)
 218 FORMAT(10X,'INJECTION  PROBLEM')
 219 FORMAT(10X,'PRODUCTION PROBLEM')
 225 FORMAT(10X,'THE CALCULATED BOTTOM-HOLE PRESSURE',
    1 ' IS THAT WHEN THE ABSOLUTE DIFFERENCE BETWEEN',
    2 /,10X,'EARTH AND CALCULATED FLUID TEMPERATURES',
    3 ', AT THE BOTTOM OF WELL IS WITHIN 0.1 DEG. F .')
 226 FORMAT(/,10X,'FIRST COLUMN IS DEPTH IN FEET .',/,
    1 10X,'SECOND COL. IS EARTH TEMP. IN DEG. F .',/,
    2 10X,'THIRD COL. IS CALC. FLUID TEMP. IN DEG. F .'
    3 ,/,10X,'FOURTH COL. IS CALC. PRESS. IN PSIA .',
    4 /,10X,'FIFTH COL. IS R** .',/)
 227 FORMAT(10X,'GAS GRAVITY, PSEUDO REDUCED TEMP.,',
    1 'AND PSEUDO REDUCED PRESS. ARE :',/,10X,3F10.4)
     STOP
     END
```

```
      FUNCTION F(X)

C**** THIS FUNCTION SUBROUTINE IS TO BE
C****    USED WITH MAIN MODEL BB ONLY

C     F     IS THE FUNCTION TO BE MINIMIZED
C     E     IS ABSOLUTE ROUGHNESS OF TUBING IN INCHES
C     DIAM  IS TUBING DIAMETER IN INCHES
C     DL    IS MCNITORED INTEGRATION STEP SIZE IN FEET
C     TGEOG IS GEOTHERMAL GRADIENT , DEG. F/ FOOT
C     PRE   IS PRESSURE IN PSIA
C     TEM   IS TEMPERATURE IN DEG. RANKIN
C     DENS  IS DENSITY OF NAT. GAS IN GM/C.C.
C     REN   IS REYNOLDS NUMBER
C     OHTCO IS THE OVERALL HEAT-TRANSFER COEFFICIENT

      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION  PRY(5),TRY(5),PRK(5),TRK(5)
      COMMON /BL3/AVEMW/BL13/DIAM/BL14/SG/BL18/M
      CCMMON /BL12/PTEMC,ZTEMC,PPREC,ZPREC
      COMMON /BL15/Q,TSD,PSD,TDD,DEPTH,TSOD,DL
      CCMMON /BL30/VMASS
      COMMON /BL50/MMM
      D1=0.1D-01
      D2=0.1D-01
      DL=0.1D 02
      E=0.6D-03
      RSTAR=X
      RR=E/DIAM
      TGEOG=(TDD-TSOD)/DEPTH

C     THE RUNGE-KUTTA-MERSON INTEGRATION ALGORITHM

      TEM=TSD+0.45967D 03
      PRE=PSD
      TS=TSOD
      TRN=TEM/PTEMC
      PRN=PRE/PPREC
      TL=0.0D 00
      WRITE(6,211) TL,TS,TSD,PRE,RSTAR
    1 DO 14 J1=1,5
      IF(J1-1) 2,2,3
    2 PRY(1)=PRN
      TRY(1)=TRN
      TS=TSOD+TGEOG*TL
    3 IF(J1-2) 11,4,5
    4 PRY(2)=PRN+PRK(1)
      TRY(2)=TRN+TRK(1)
      TS=TSOD+TGEOG*(TL+DL/0.3D 01)
    5 IF(J1-3) 11,6,7
    6 PRY(3)=PRN+0.5D 00*PRK(1)+0.5D 00*PRK(2)
      TRY(3)=TRN+0.5D 00*TRK(1)+0.5D 00*TRK(2)
```

```
      TS=TSOD+TGEOG*(TL+DL/0.3D 01)
    7 IF(J1-4) 11,8,9
    8 PRY(4)=PRN+0.375D 00*PRK(1)+0.1125D 01*PRK(3)
      TRY(4)=TRN+0.375D 00*TRK(1)+0.1125D 01*TRK(3)
      TS=TSOD+TGEOG*(TL+DL/0.2D 01)
    9 IF(J1-5) 11,10,14
   10 ABC1=PRN+0.15D 01*PRK(1)
      ABC2=-0.45D 01*PRK(3)+0.6D 01*PRK(4)
      PRY(5)=ABC1+ABC2
      ABC1=TRN+0.15D 01*TRK(1)
      ABC2=-0.45D 01*TRK(3)+0.6D 01*TRK(4)
      TRY(5)=ABC1+ABC2
      TS=TSOD+TGEOG*(TL+DL)
      PRJAM=PRY(5)
      TRJAM=TRY(5)
   11 PR=PRY(J1)
      TR=TRY(J1)
      TRS=(TS+0.45967D 03)/PTEMC
      PRE=PR*PPREC
      TEM=TR*PTEMC
C       CALCULATING  IDEAL HEAT CAPACITY OF NAT. GAS

      CALL KASEM (TR,TEM,SG,M,CPOM)
      TRC=TEM/ZTEMC
      PRC=PRE/ZPREC

C       CALCULATING COMPRESSIBILITY FACTOR,ITS PARTIAL
C                DERIVATIVES W.R.T. PR & TR ,REDUCED
C                DENSITY, AND HEAT CAPACITY DEPARTURE
C                OF NAT. GAS

      CALL BWR (TRC,PRC,ITER,Z,DR,CPDM,DZDTR,DZDPR)
      IF(ITER) 22,22,12
   12 DENS=0.14925826D-02*AVEMW*PRE/(Z*TEM)
      CPM=CPDM+CPOM

C       CALCULATING VISCOSITY OF NAT. GAS IN C.P.

      CALL LEE (TEM,AVEMW,DENS,VISM)
      REN=0.20014855D 02*Q*SG/(VISM*DIAM)

C       CALCULATING THE HYDRAULIC FRICTION FACTOR

      CALL MOODY (RR,REN,FF,I)
      U1=RSTAR
      OHTCO=0.1D 01/U1

C       SOLVING FOR THE RATE OF CHANGE OF REDUCED
C          PRESSURE AND TEMPERATURE WITH LENGTH
      CALL QUON(TEM,PRE,TR,PR,TRS,Z,DZDPR,DZDTR,
     1FF,CPM,OHTCO,VMASS,DPRDL,DTRDL,NASER)
```

```
      IF(NASER-1) 22,13,13
   13 PRK(J1)=DL*DPRDL/0.3D 01
      TRK(J1)=DL*DTRDL/0.3D 01
   14 CONTINUE
C
C     MONITORING THE INTEGRATION STEP SIZE
C
      ABC5=PRK(1)-0.45D 01*PRK(3)
      ABC6=0.4D 01*PRK(4)-0.5D 00*PRK(5)
      EPR=ABC5+ABC6
      ABC5=TRK(1)-0.45D 01*TRK(3)
      ABC6=0.4D 01*TRK(4)-0.5D 00*TRK(5)
      ETR=ABC5+ABC6
      ETEM=DABS(ETR*PTEMC)
      EPRE=DABS(EPR*PPREC)
      IF(EPRE-D1) 15,15,16
   15 IF(ETEM-D2) 17,17,16
   16 DL=DL/0.2D 01
      GO TO 1
   17 PR=PRJAM-EPR
      TR=TRJAM-ETR
      TL=TL+DL
      IF(EPRE-D1/0.16D 02) 18,20,20
   18 IF(ETEM-D2/0.16D 02) 19,20,20
   19 DL=0.2D 01*DL
   20 PRN=PR
      TRN=TR
      PRE=PRN*PPREC
      TEM=TRN*PTEMC
      TED=TEM-0.45967D 03
      WRITE(6,211) TL,TS,TED,PRE,RSTAR
      DEL=DEPTH-TL
      IF(DABS(DEL)-0.1D-04) 23,23,25
   25 IF(DEL-DL) 21,1,1
   21 DL=DEL
      GO TO 1
   22 WRITE(6,227)
   23 TED=TEM-0.45967D 03
      TEMTL=TED
      F=TED-TEMTL
  211 FORMAT(10X,5F10.2)
  227 FORMAT(//,10X,'*** THE COURSE OF ITERATIONS IS'
     *,' INTERRUPTED ***',//,7X,'*** THE PROGRAM IS'
     *,' GOING TO ASSUME ANOTHER "RSTAR"***')
      RETURN
      END
```

```
      SUBROUTINE AMEIN

C**** THIS SUBROUTINE CAN BE USED WITH
C****          MAIN MODEL BB ONLY

C     THIS SUBROUTINE CALCULATES THE MOLAL AVERAGE
C         PROPERTIES AND THE WICHERT & AZIZ CORRECTED
C         PSEUDOCRITICAL PROPERTIES OF NAT. GAS

C     AVEMW IS AVERAGE MOLECULAR WEIGHT
C     SG    IS GAS GRAVITY
C     PPREC IS PSEUDOCRITICCAL PRESSURE IN PSIA
C     PTEMC IS PSEUDOCRITICAL TEMPERATURE,DEG. R
C     ZTEMC IS CORRECTED PSEUDOCRITICAL TEMPERATURE
C     ZPREC IS CORRECTED PSEUDOCRITICAL PRESSURE

      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /BL3/AVEMW/BL5/YMW(15)/BL9/YMOLF(15)
      COMMON /BL11/TEMC(15),PREC(15)
      COMMON /BL12/PTEMC,ZTEMC,PPREC,ZPREC
      COMMON /BL14/SG/BL18/M/BL31/MM
      COMMON /BL50/MMM
      Y11=YMOLF(1)
      Y22=YMOLF(2)
      Y33=YMOLF(3)
      IF(MMM-1) 10,3,3
   10 IF(M-1) 3,1,1
    1 AVEMW=0.0D 00
      PTEMC=0.0D 00
      PPREC=0.0D 00
      DO 2 I=1,15
      AVEMW=AVEMW+YMOLF(I)*YMW(I)
      PTEMC=PTEMC+YMOLF(I)*TEMC(I)
    2 PPREC=PPREC+YMOLF(I)*PREC(I)
      SG=AVEMW/0.2897D 02
      GO TO 7
    3 AVEMW=SG*0.2897D 02
      IF(MM-1) 11,7,7

C     THE BROWN ET AL AND CARR ET AL CORRELATIONS
C         FOR THE PSEUDOCRITICAL PROPERTIES OF
C         SWEET AND SOUR NAT. GASES

   11 PTEMC=0.171137D 03+0.313725D 03*SG
      TCCOR=-0.8D 02*Y11-0.25D 03*Y22+0.13D 03*Y33
      PTEMC=PTEMC+TCCOR
      IF(SG-0.85D 00) 4,4,5
    4 PPREC=0.6951D 03-0.4D 02*SG
      GO TO 6
    5 PPREC=0.704396D 03-0.51724D 02*SG
    6 PCCOR=0.44D 03*Y11-0.17D 03*Y22+0.6D 03*Y33
      PPREC=PPREC+PCCOR
```

```
C      THE WICHERT & AZIZ CORRECTED PSEUDOCRITICALS
C          DUE TO PRESENCE OF CO2 AND H2S

     7 S1=YMOLF(1)+YMOLF(3)
       S2=YMOLF(3)
       E31=0.12D 03*(S1**0.9D 00-S1**0.16D 01)
       E32=0.15D 02*(DSQRT(S2)-S2**4)
       E3=E31+E32
       S4=PTEMC+S2*(0.1D 01-S2)*E3
       ZTEMC=PTEMC-E3
       ZPREC=PPREC*ZTEMC/S4
       RETURN
       END
```