University of Alberta

Transmit Antenna Selection Using Orthogonal Space-Time Block Codes

by

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to my parents, Tajedin Kaviani and Zahra Khoshnoudrad, for their endless encouragement, love and support

Abstract

Multiple-input multiple-output (MIMO) systems that use multiple antennas can significantly improve the reliability and data rate of wireless communications. MIMO systems use space-time coding techniques such as orthogonal space-time block codes (OSTBCs). However, the use of multiple antenna increases the complexity of both the transmitter and the receiver. Antenna selection is a scheme to reduce the system complexity and cost and uses a low-rate feedback channel from receiver to transmitter to improve the performance.

In this thesis, we derive the bit error rate (BER) of OSTBCs with antenna selection for independent and receive correlated Rayleigh channels. Pulse amplitude modulation (PAM), quadrature amplitude modulation (QAM), and pulse shift keying (PSK) constellations are used in OSTBCs. We provide a novel analytical framework for the diversity analysis and approximation of BER to show approximations for BER expressions. As a conclusion the system achieves full diversity order when transmit antenna selection scheme is used.

Acknowledgements

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Again, Most of all thanks to God the Divine who continues to make the impossible possible.

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List of Notations

a	the scalar a
$\mathbf{a}$	the vector $\mathbf{a}$
$\mathbf{A}$	the matrix $\mathbf{A}$
$\text{vec}(\mathbf{A})$	the operation stacks columns of $\mathbf{A}$ to one column vector
$(\cdot)^H$	the conjugate transpose (Hermitian)
$\|\cdot\|$	the vector Euclidean norm
$\|\cdot\|_F$	the matrix Frobenius norm
$(\cdot)^*$	the complex conjugate
$\mathbf{A}^{1/2}$	the Hermitian square root of $\mathbf{A}$
$\mathcal{CN}(\mu, \sigma^2)$	the circularly symmetric complex Gaussian variable with mean μ and variance σ^2
$\mathcal{N}(\mu, \sigma^2)$	the Gaussian random variable with mean μ and variance σ^2
$\log_2(x)$	the base-2 logarithm of x
$\log(x)$	the natural logarithm of x
$\binom{j}{i_1, \ldots, i_n}$	$\frac{j!}{i_1! \cdots i_n!}$
$\binom{j}{i}$	$\frac{j!}{i!(j-i)!}$
$\otimes$	the Kronecker product
$\mathbb{C}^m$	the m-dimensional complex vector space
$\mathbb{C}^{m \times n}$	the set of $m \times n$ complex matrices
$\mathcal{E}_y[\cdot]$	statistical expectation over y
L_t	the number of transmit antennas
L_r	the number of receive antennas
N	the number of transmit antennas selected
R_s	the code rate
Q	the number of input symbols
T	the number of time slots
N_0	noise variance
ρ	the signal to noise ratio (SNR) per symbol
E_b	the bit energy
E_s	the symbol energy

P_b	the bit error probability
G_c	the coding gain
G_d	the diversity order
$\mathcal{Q}(\cdot)$	the Gaussian $\mathcal{Q}$-function
$\mathcal{Q}_m(\alpha, \beta)$	generalized Marcum's $\mathcal{Q}$ function
$\mathrm{erf}(\cdot)$	the Gaussian error function
$f_x(x)$	the probability density function (pdf) of x
$F_x(x)$	the cumulative density function (cdf) of x
$\Phi_x(s)$	the moment generating function (MGF) of x
γ_b	the SNR per bit
M	the size of the signal constellation
$\mathcal{S}$	the symbol constellation
$\mathbf{I}$	the identity matrix
$\delta_{i,j}$	the Kronecker delta function is zero except when $i = j$ equal to 1
$\mathbf{R}_r$	the receive correlation matrix
$\mathbf{H}$	the channel matrix
λ	the eigenvalue of a matrix
C	the channel capacity
$\Gamma(\cdot)$	the gamma function
$\arg\max$	the argument of the maximum
$\mathcal{L}_x[f(x)](s)$	the Laplace transform of $f(x)$
$\mathcal{L}_x^{-1}[\Phi(s)](x)$	the inverse Laplace transform of $\Phi(s)$

Abbreviations

ADC	analog to digital converter
AWGN	additive white Gaussian noise
BER	bit error rate
BPSK	binary pulse shift keying
CSI	channel-side information
cdf	cumulative density function
DAC	Digital to Analog converter
EGC	equal gain combining
GSC	generalized selection combining
iid	independent and identically distributed
LNA	low noise amplifier
LOS	line of sight
MGF	moment generating function
MIMO	multiple-input multiple-output
ML	maximum likelihood
MMSE	minimum mean square error
MRC	maximal ratio combining
NLOS	non line of sight
OSTBC	orthogonal space-time block code
PAM	pulse amplitude modulation
pdf	probability density function
PSK	pulse shift keying
QAM	quadrature amplitude modulation
RF	radio frequency
SER	symbol error rate
SISO	single input single output
SNR	signal to noise ratio
STBC	space-time block code
V-BLAST	vertical bell laboratories layered space-time
ZF	zero-forcing

Chapter 1

Introduction

Section 1.1 gives a general introduction to multiple-input multiple-output (MIMO) wireless systems. Antenna selection techniques are briefly summarized in Section 1.2. The contributions and the organization of the remaining chapters can be found in Section 1.3.

1.1 MIMO Wireless Systems

The increasing demand for high data rates due to emerging new technologies makes wireless communications an exciting and challenging field. Wireless multiple-input multiple-output (MIMO) systems, which employ multiple antennas at both the transmitter and receiver, improve the reliability and achieve larger data rates in comparison to systems that employ single antennas at the transmitter and receiver ends [1], [2].

Wireless links are impaired by random signal fluctuations known as *fading*. *Diversity* provides the receiver with multiple (ideally independent) fading replicas of the transmitted signal and is therefore a powerful solution to combat fading. Diversity may be achieved using multiple transmit and/or receive antennas.

Space-time codes [3–6] are capable of extracting spatial diversity order in MIMO systems without requiring channel knowledge at the transmitter. Orthogonal space-time block codes (OSTBCs) [4], [5], [7], are particularly attractive since they yield the maximum spatial diversity order and, at the same time, decouple the MIMO signal vector detection into a number of equivalent Single-Input Single-Output (SISO) systems with scalar detection, thereby significantly reducing decoding complexity (at the expense of spatial transmission rate).

1.2 Antenna Selection

For practical applications, the cost and the complexity of MIMO systems are significant because of the large number of radio frequency (RF) chains required for every active transmit/receive antenna pair. An RF chain comprises low noise amplifiers, frequency down/up converters, a power amplifier, analog-to-digital/digital-to-analog converters, and several filters, all of which clearly increases the implementation costs. This increase has hindered the wide deployment of MIMO systems. For example, the third-generation cellular system (3GPP) supports the Alamouti transmit diversity scheme [4, 8] with only two transmit and one receive antenna as an option. Also in the IEEE 802.16 standard, known as WiMax [9], only the Alamouti scheme is offered as an option.

In the next-generation of wireless standards, where MIMO adoption is needed for higher data rates, complexity issues have led many researchers to develop methods that can reduce the implementation cost and retain the benefits of MIMO systems. Antenna selection, which seeks the utilization of a subset of all available antennas at the transmitter and/or receiver, is such a technique [10] and [11]. Selecting a subset of antennas at the transmitter or the receiver is called *transmit antenna selection* or *receive antenna selection*, respectively.

1.3 Contribution of this thesis

In this thesis we provide a general, exact closed-form BER analysis of transmit antenna selection and OSTBCs for Independent and correlated fading channels.

- The exact BER for M-ary PAM and QAM is derived for arbitrary $N \geq 2$ transmit antenna selection employing OSTBCs. An approximate BER expression for M-PSK is also derived.

- The moment generating function (MGF) of N largest instantaneous signal-to-noise ratios (SNRs) is derived in [12] but for the generalized selection combining (GSC) scheme. Using this MGF for the transmit antenna selection scheme, we derive the exact and approximate BER expressions.

- Using the asymptotic analysis, the diversity and coding gain of the system are derived. It is shown that full diversity order is achieved through transmit antenna selection using OSTBCs.

- The closed-form results and approximations can be computed much faster than computer simulations and numerical methods of analysis of performance.

- The performance problem is complicated because the analysis requires the statistics of the ordered random variables. Although mathematical and engineering literature has several thousands papers on order statistics of independent random variables, very few results are available on the order statistics of correlated random variables. As a result no performance analysis is available on transmit antenna selection over correlated channels. However, we analyze the performance for receive antenna selection over correlated channels.

Chapter 2 reviews the background and preliminaries for the whole thesis. Chapter 3 presents the antenna selection criteria. The performance of transmit antenna selection using OSTBCs in the independent Rayleigh fading channels is discussed in chapter 4. In Chapter 5, the performance of OSTBCs with transmit antenna selection in receive correlated Rayleigh fading channels is analyzed. Chapter 6 concludes the thesis.

Chapter 2

Background

Section 2.1 overviews the mathematical notation used throughout the thesis. The MIMO system model is presented in Section 2.2. Fundamentals of digital communications are reviewed in Section 2.3. Section 2.6 overviews OSTBCs. Spatial multiplexing is discussed in Section 2.7. Section 2.4 reviews the Performance analysis methods for different M-ary signal constellations in MIMO systems. Diversity order and Coding gains are introduced in Section 2.5 for subsequent asymptotic performance analysis. Section 2.8 provides an overview on channel capacity. Finally, some useful probability preliminaries are presented in Section 2.9.

2.1 Notation

This thesis uses $\|\cdot\|_F$ for the matrix Frobenius norm (i.e. $\|\mathbf{H}\|_F = \sum_k \sum_l |h_{k,l}|^2$). The Euclidean norm for vector $\mathbf{h}$ of length n is $\|\mathbf{h}\| = (h_1^2 + ... + h_n^2)^{1/2}$. $\mathbb{C}^m$ and $\mathbb{C}^{m \times n}$ are used to refer to the m-dimensional complex vector space and the set of $m \times n$ complex matrices, respectively. The conjugate transposition operator is given by ^{H} which is also known as Hermitian operator. The determinant is represented by $\det(\cdot)$. The operator $\mathrm{vec}(\cdot)$ stacks the columns of a matrix into one column vector. The operator $\otimes$ is the Kronecker product.

A circularly symmetric complex Gaussian variable with mean μ and variance σ^2 is denoted by $z \sim \mathcal{CN}(\mu, \sigma^2)$. $\mathcal{E}_y[\cdot]$ is used to denote expectation with respect to y. The floor of a number is returned by $\lfloor \cdot \rfloor$. δ_{ij} is the Kronecker delta function.

2.2 System Model

We consider a wireless communication system with L_t transmit and L_r receive antennas. Let $\mathbf{H} \in \mathbb{C}^{L_r \times L_t}$ be the channel matrix. The quasi-static flat Rayleigh fading MIMO channel for this system can be represented as [6]

$$\mathbf{H} = \begin{bmatrix} h_{1,1} & h_{1,2} & \cdots & h_{L_t,1} \\ h_{2,1} & & & \\ \vdots & & \ddots & \vdots \\ h_{L_r,1} & h_{L_r,2} & \cdots & h_{L_r,L_t} \end{bmatrix} \tag{2.1}$$

where $h_{i,j}$ is the path gain between transmit antenna j and receive antenna i. The entries of $\mathbf{H}$ are $\mathcal{CN}(0,1)$ with positive semi-definite autocorrelation given by $\mathbf{R} = \mathcal{E}\{\text{vec}(\mathbf{H})\,\text{vec}^H(\mathbf{H})\}$ of size $L_tL_r \times L_tL_r$. The channel $\mathbf{H}$ is known at the receiver while it is unknown at the transmitter. A limited-rate feedback channel from the receiver to transmitter is available. The receiver uses this channel to inform the transmitter about the selected antennas. N transmit antennas out of L_t are selected and activated for the transmission of OSTBC signal matrices, while the rest are inactive.

In the presence of a line of sight (LOS) component between the transmitter and receiver, the MIMO channel can be modeled as sum of a zero mean complex gaussian channel and a fixed component [6],

$$\mathbf{H} = \sqrt{\frac{K}{1+K}}\bar{\mathbf{H}} + \sqrt{\frac{1}{1+K}}\mathbf{H}_w \tag{2.2}$$

where $\sqrt{K/(1+K)}\bar{\mathbf{H}} = \mathcal{E}[\mathbf{H}]$ is the fixed LOS component of the channel and $\sqrt{1/(1+K)}\mathbf{H}_w$ is the fading component where $\mathbf{H}_w$ is spatially white complex Gaussian channel matrix. The entries of $\bar{\mathbf{H}}$ are assumed to have unit power.

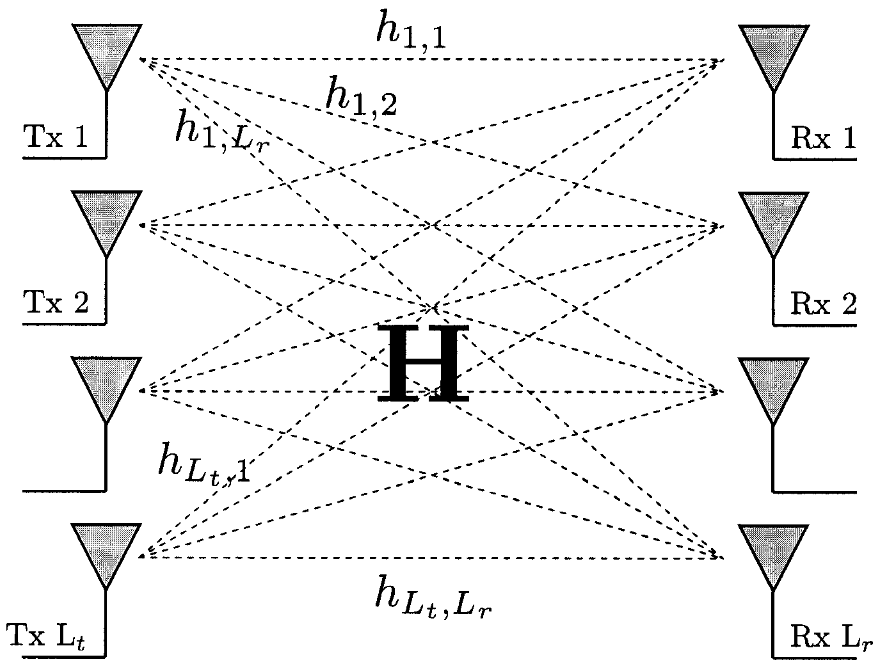

Figure 2.1: MIMO Channel Model

K is the Rician K-factor of the MIMO system. When $K = 0$, the channel reduces to the conventional Rayleigh fading channel.

In the Kronecker model for the correlated channels, the channel matrix is modeled as the product of the receive correlation matrix, an independent identically distributed (i.i.d.) complex Gaussian matrix, and the transmit correlation matrix [13]

$$\mathbf{H} = \mathbf{R}_r^{1/2}\mathbf{H}_w\mathbf{R}_t^{1/2} \tag{2.3}$$

where $\mathbf{H}_w$ is a $L_r \times L_t$ matrix with i.i.d. circular complex Gaussian elements with mean zero and variance one, $\mathbf{R}_t$ and $\mathbf{R}_r$ are the transmit and receiver correlation matrices $\mathbf{R}_r = \mathcal{E}\left\{\mathbf{h}_i\mathbf{h}_i^H\right\}$ $(i = 1, 2, ..., L_t)$ [6]. Thus total correlation matrix $\mathbf{R}$ is given by

$$\mathbf{R} = \mathbf{R}_t^T \otimes \mathbf{R}_r. \tag{2.4}$$

The Kronecker model has been verified for non Line of sight (NLOS) [14], [15]. However, the accuracy of model has been questioned recently in large antenna arrays [16].

Suppose an OSTBC is transmitted over the channel given in (2.1). The received signals are expressed as

$$\mathbf{Y} = \sqrt{\frac{E_s}{N}}\tilde{\mathbf{H}}\mathbf{X} + \mathbf{V} \tag{2.5}$$

where the matrix $\mathbf{Y} \in \mathbb{C}^{L_r \times T}$ is the complex received matrix, $\tilde{\mathbf{H}}$ is a submatrix of $\mathbf{H}$, $\mathbf{X} \in \mathbb{C}^{N \times T}$ is the complex transmitted matrix and $\mathbf{V} \in \mathbb{C}^{L_r \times T}$ is the additive noise matrix with independent and identical distributed entries of $\mathcal{CN}(0, N_0)$. The coefficient $\sqrt{E_s/N}$ ensures that the total transmitted power in each channel use is E_s and independent of number of transmit antennas. Since T symbol periods are necessary to transmit Q symbols, the symbol rate R_s of the STBC is defined as $R_s = Q/T$.

2.3 Fundamentals of Digital Communications

Fig. 2.2 shows the generic transmitter. The original message is in form of a digital sequence of bits which is generated by an information source. The channel encoder adds redundant bits in order to detect/correct transmission errors.

The digital modulator maps the message bits into symbols from a constellation $\mathcal{S}$. Figures 2.3, 2.4, and 2.5 illustrate several popular signal constellations.

M-ary *pulse amplitude modulation* (M-PAM) can be expressed as

$$s(t) = A_M \cos 2\pi f_c t, \quad 0 \le t \le \tau \tag{2.6}$$

where A_M is the signal amplitude of the in-phase component, f_c is the carrier frequency, and τ is the symbol time duration. In M-PAM, $\log_2 M$ bits of data

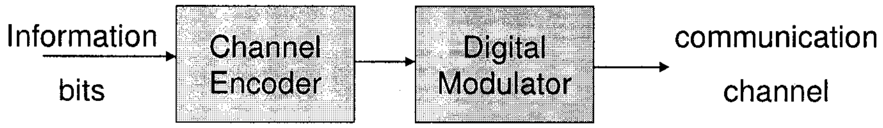

Figure 2.2: Generic transmitter

are used to select from the set $\{\pm d, \pm 3d, \dots, \pm(M-1)d\}$, where $2d$ is the minimum distance [17] and is related to the bit energy E_b as

$$d = \sqrt{\frac{3 \log_2 M \cdot E_b}{(M^2 - 1)}}. \tag{2.7}$$

M-ary *Quadrature Amplitude Modulation* signals are represented as

$$s(t) = A_M \cos 2\pi f_c t - \hat{A}_M \sin 2\pi f_c t, \quad 0 \leq t \leq \tau \tag{2.8}$$

where $A_M + j\hat{A}_M$ is the corresponding signal point as it is shown for the 16-QAM in Figure 2.4. The in-phase amplitude A_M and the quadrature amplitude $\bar{A}_M$ are selected from the set $\{\pm d, \pm 3d, \dots, \pm(M-1)d\}$, where $2d$ is the minimum distance between signal and is related to the bit energy E_b same as (2.7) for the M^2-QAM signal constellation.

For *phase shift keying* (PSK) or phase pulse modulation, the signals are

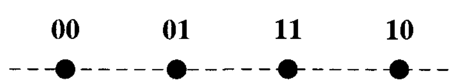

Figure 2.3: 4-PAM signal constellation Gray Mapping.

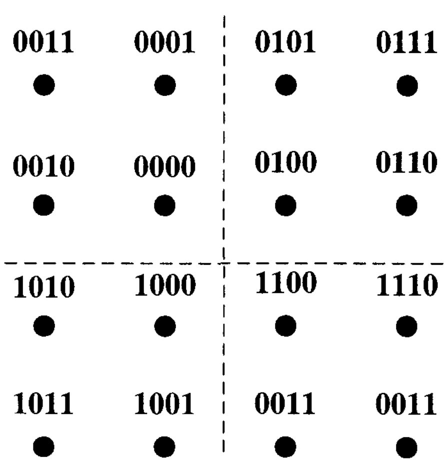

Figure 2.4: 16-QAM signal constellation Gray Mapping.

represented as

$$s(t) = \cos\left(2\pi f_c t + \frac{2\pi}{M}(m-1)\right), \qquad m = 1, 2, ..., M, \quad 0 \le t \le \tau \tag{2.9}$$

where d is the minimum distance between signal points for M-PSK constellation and is related to the bit energy E_b as

$$d = 2\sin\left(\frac{\pi}{M}\right)\sqrt{\log_2 M \cdot E_b}. \tag{2.10}$$

The mapping of $\log_2 M$ data bits to the M possible signals may be done such that the labels of the adjacent signal points differ by one bit. Such a mapping is called *Gray Mapping* and is illustrated in Figures 2.3, 2.4 and 2.5.

Coded modulations based on Gray mapping may employ only one binary component code. The theory of bit-interleaved coded modulation was developed in [18]. The performance of coded modulation over a Rayleigh fading channel can be improved by bit-wise interleaving at the encoder output but not considered in this thesis. However, Gray mapping offers excellent performance in the AWGN channel [18].

With Gray mapping and zero-mean additive white Gaussian noise (AWGN) channel, the exact BER of the n-th bit for M-PAM constellation is given by [19]

$$P_b(n) = \frac{2}{M}\sum_{i=0}^{k_n} B_i Q\left(D_i\sqrt{\gamma_b(\rho)}\right) \tag{2.11}$$

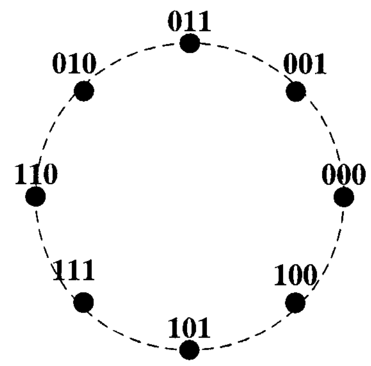

Figure 2.5: 8-PSK signal constellation with Gray Mapping.

where the $\mathcal{Q}$-function is defined as

$$\begin{aligned}\mathcal{Q}(x) =& \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-\frac{t^2}{2}}dt, \quad x \geq 0\\ =& \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp\left(-\frac{x^2}{2\sin^2\theta}\right)d\theta,\end{aligned} \tag{2.12}$$

and the parameters of the sum are given as

$$k_n = (1 - \frac{1}{2^n})M - 1 \tag{2.13a}$$

$$B_i = (-1)^{\left\lfloor \frac{i.2^{n-1}}{M} \right\rfloor}\left(2^{n-1} - \left\lfloor \frac{i.2^{n-1}}{M} + \frac{1}{2} \right\rfloor\right) \tag{2.13b}$$

$$D_i = (2i+1)\sqrt{\frac{6\log_2 M}{M^2 - 1}}. \tag{2.13c}$$

The SNR per symbol, ρ, is defined from the bit energy E_b and the symbol energy E_s as

$$\rho = \frac{E_s}{N_0} = \frac{\log_2 M \cdot E_b}{N_0}. \tag{2.14}$$

The SNR per bit γ_b which depends on the rate of the code used for transmission, will be defined later in this thesis.

Finally, the exact average BER of arbitrary M-PAM can be obtained by adding all bit error probabilities and normalizing by the total bits. That is,

$$P_M^{\text{PAM}} = \frac{1}{\log_2 M}\sum_{n=1}^{\log_2 M} P_b(n). \tag{2.15}$$

A rectangular or square QAM constellations can be decomposed to two independent PAM constellations: I-ary PAM for the in-phase component and J-ary PAM for the quadrature component, where $M = I \times J$. Thus, the exact average BER of M-QAM is given by

$$P_M^{\text{QAM}} = \frac{1}{\log_2(I.J)}\left(\sum_{n=1}^{\log_2 I} P_I(n) + \sum_{m=1}^{\log_2 J} P_J(m)\right). \tag{2.16}$$

A tight approximation for the BER of the coherent M-ary PSK in AWGN channels is given by [20]

$$P_M^{\text{PSK}}(\rho) \simeq \frac{2}{\max(\log_2 M, 2)}\sum_{i=1}^{\max(M/4,1)} \mathcal{Q}\left(\sqrt{2\sin^2\frac{(2i-1)\pi}{M}\gamma_b(\rho)}\right). \tag{2.17}$$

2.4 Performance over MIMO channel

Section (2.3) shows the BER for M-PAM signal constellation using the Gray mapping over an AWGN channel. Since the MIMO channel is a random matrix, we take the expectation with respect to the channel from the BER expression for AWGN in (2.11):

$$\begin{aligned} P_M(n;\rho) =& \frac{2}{M}\sum_{i=0}^{k_n} B_i \mathcal{E}_{\mathbf{H}}\left[\mathcal{Q}\left(D_i\sqrt{\gamma_b(\rho)}\right)\right] \\ =& \frac{2}{M}\sum_{i=0}^{k_n} B_i \mathcal{E}_{\mathbf{H}}\left[\frac{1}{\pi}\int_0^{\pi/2} e^{-\frac{D_i^2}{2\sin^2\theta}\gamma_b} d\theta\right] \\ =& \frac{2}{M}\sum_{i=0}^{k_n} B_i \frac{1}{\pi}\int_0^{\pi/2}\int_0^{\infty} e^{-\frac{D_i^2}{2\sin^2}\gamma_b} f_{\gamma_b}(\gamma_b) d\gamma_b d\theta \\ =& \frac{2}{M}\sum_{i=0}^{k_n} B_i \frac{1}{\pi}\int_0^{\pi/2} \Phi_{\gamma_b}\left(-\frac{D_i^2}{2\sin^2\theta}\right) d\theta \end{aligned} \tag{2.18}$$

where $\mathcal{E}_{\mathbf{H}}[\cdot]$ is expectation over all channel matrices and we have used

$$\mathcal{Q}(x) = \frac{1}{\pi}\int_0^{\pi/2} e^{-\frac{x^2}{2\sin^2\theta}} d\theta \tag{2.19}$$

to obtain the above formula in respect to MGF of γ_b.

The exact average BER of an OSTBC for M-PAM in fading is given by

$$P_M(\rho) = \frac{1}{\log_2 M}\sum_{n=1}^{\log_2 M} P_M(n;\rho). \tag{2.20}$$

Note that a rectangular or square QAM constellations can be decomposed to two independent PAM constellations: I-ary PAM for the in-phase component and J-ary PAM for the quadrature component, where $M = I \times J$. Thus, the exact average BER of M-QAM is given by

$$P_M(\rho) = \frac{1}{\log_2(I.J)}\left(\sum_{n=1}^{\log_2 I} P_I(n;\rho) + \sum_{m=1}^{\log_2 J} P_J(m;\rho)\right). \tag{2.21}$$

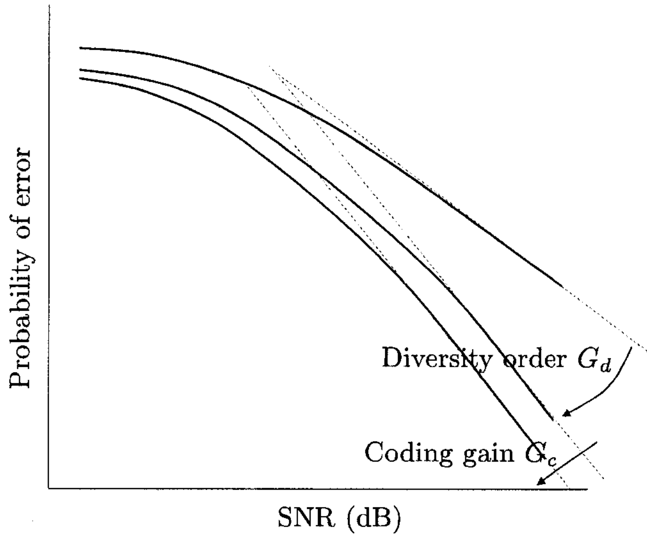

Figure 2.6: Diversity order and Coding gain in the plot of probability of error

2.4.1 Approximate BER for M-ary PSK

By taking the expectation of (2.17) similar to the derivation in (2.18), the approximate BER for M-PSK and over MIMO channel is given by

$$P_M(\rho) \simeq \frac{2}{\max(\log_2 M, 2)} \sum_{i=1}^{\max(M/4,1)} \frac{1}{\pi} \int_0^{\pi/2} \Phi_{\gamma_b}\left(-\frac{\sin^2 \frac{(2i-1)\pi}{M}}{\sin^2\theta}\right) d\theta. \qquad (2.22)$$

2.5 Diversity and Coding Gains

A system is said to have a diversity order (also called diversity gain) of G_d if

$$G_d = -\lim_{E_s/N_0 \to \infty} \frac{\log P_M}{\log(E_s/N_0)}. \qquad (2.23)$$

The average BER at high SNR may be approximated by the expression

$$P_M(\rho) \approx (G_c \cdot \rho)^{-G_d} \qquad (2.24)$$

where G_c is the coding gain, and G_d is the diversity order. The diversity order determines the slope of the average BER curve versus the average SNR ρ at high SNR in a log-log scale, whereas the coding gain (in decibel) determines the shift of the curve in SNR relative to a BER curve given by (ρ^{-G_d}).

In [21], Wang and Giannakis develop a simple and general method to quantify the coding gain and diversity order. They show that the asymptotic performance depends on the behavior of the MGF of the output SNR. We write

$$f(x) = o[g(x)], \quad \text{as } x \to x_0 \tag{2.25}$$

if

$$\lim_{x \to x_0} \frac{f(x)}{g(x)} = 0. \tag{2.26}$$

If the MGF of $\Phi_{\gamma_b}(s)$ can be approximated by a single polynomial term for $s \to \infty$ as

$$|\Phi_{\gamma_b}(s)| = b|s|^{-d} + o(s^{-d}), \tag{2.27}$$

then d is the diversity order of the system at high SNRs and the coding gain can be derived from b [21].

A MIMO system with L_t transmit and L_r receive antennas is said to have *full diversity* when it has used all possible spatial diversity from antennas and therefore has achieved diversity order of $L_t L_r$.

2.6 Orthogonal Space-time Block Codes

Receive diversity, i.e., multiple antennas at the receiver, may not be suitable for the downlink because the placement of multiple antennas on small handsets is expensive and difficult. The multiple antenna burden is preferably placed at the base station. This is called *transmit diversity.* In this case the channel is unknown to the transmitter but known to the receiver. With *space-time coding*, transmit symbols are spread across both space and time [3–7]. In this section, we give a general overview of space-time codes. Of these codes, one particularly interesting structure (namely space-time block code) is the major topic of interest for this thesis.

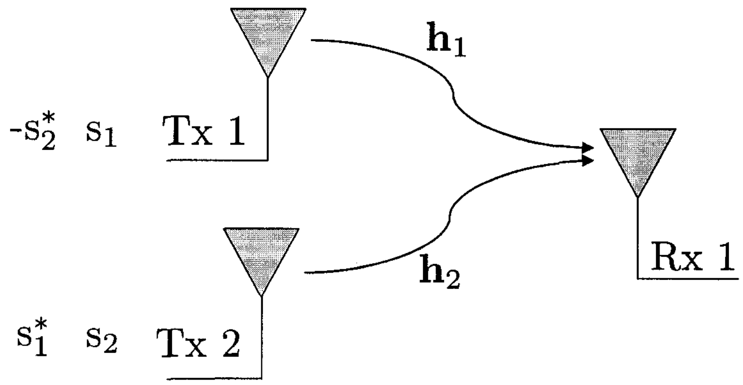

Figure 2.7: Alamouti Scheme with 2 transmit antenna and 1 receive antenna

One of the simplest and most attractive transmit diversity schemes were proposed by Alamouti [4] for the case of two transmit antennas. For transmitting $Q = 2$ complex symbols s_1 and s_2 during two time intervals uses the following transmission matrix

$$\mathbf{X} = \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix}. \tag{2.28}$$

Note that the code rate is $R_s = 2/2 = 1$. Assuming a single receiver, let h_1 and h_2 denote the channel coefficients for transmit antenna 1 and 2 respectively. The channel coefficients are assumed to be constant over two consecutive time intervals. The received signals are given by

$$\begin{aligned} y_1 &= h_1 s_1 + h_2 s_2 + n_1, \\ y_2 &= -h_1 s_2^* + h_2 s_1^* + n_2. \end{aligned} \tag{2.29}$$

Maximum likelihood (ML) detection in (2.28) would be given by

$$\begin{aligned} \tilde{s}_1 &= h_1^* y_1 + h_2 y_2^* = (|h_1|^2 + |h_2|^2) s_1 + \tilde{n_1}, \\ \tilde{s}_2 &= h_2^* y_1 - h_1^* y_2^* = (|h_1|^2 + |h_2|^2) s_2 + \tilde{n_2}. \end{aligned} \tag{2.30}$$

We observe that the transmitted signals are effectively multiplied by $|h_1|^2 + |h_2|^2$. Hence, if one of the paths is in a deep fade, the other may not.

Space-time block codes derive their name from the fact that the encoding is done in both space and time. A space-time block code is defined by the relationship between the Q-tuple input signal $\mathbf{s}$ and the set of signals to be transmitted from L_t antenna over T time intervals. Such a relation is given by $L_t \times T$ transmission matrix $\mathbf{X}$

$$\mathbf{X} = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,T} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,T} \\ \vdots & & \ddots & \vdots \\ x_{L_t,1} & \cdots & & x_{L_t,T} \end{bmatrix} \tag{2.31}$$

where $x_{i,j}$ are functions of Q-tuple input sequence $s_1, s_2, \cdots, s_Q$ and their complex conjugates. At time slot i, $x_{i,j}$ is transmitted from antenna j. Since Q information symbols are transmitted over T time intervals, the rate of the code is defined as $R_s = Q/T$. The receiver use arbitrary number of receive antennas L_r. The design does not depend on the number of receive antennas.

Tarokh et al. [5] extended the Alamouti's 2-transmit diversity scheme to more than two antennas. If

$$\mathbf{X}\mathbf{X}^H = \left(\sum_{i=1}^{Q} |s_i|^2 \right) \mathbf{I} \tag{2.32}$$

where $\mathbf{I}$ is the identity matrix, then the code is called *orthogonal STBC* (OSTBC). If the channel coefficients are constant over a period of T symbols, i.e. $h_{i,j}$ in (2.1) remain constant, then simple linear decoding of OSTBC is possible.

At the receiver, the L_r receive antennas use maximum likelihood (ML) decoding. Assuming perfect channel side-information (CSI), the decoder at antenna j maximizes

$$\sum_{t=1}^{T} \sum_{j=1}^{L_r} \left| y_{j,t} - \sum_{i=1}^{L_t} h_{i,j} x_{i,t} \right|^2 . \tag{2.33}$$

Since the block coding requires only linear processing at the receiver, the decoding can be done efficiently and quickly. Space-time block codes can be constructed for any type of signal constellation and provide full diversity.

In general, OSTBC codeword is created from a set of Q symbols $s_1, s_2, \cdots, s_Q$ all taken from the same signal constellation. An $L_t \times T$ transmission matrix is formulated as

$$\mathbf{X} = \sum_{i=1}^{Q} (s_i \mathbf{A}_i + s_i^* \mathbf{B}_i) \tag{2.34}$$

where $\mathbf{A}_i$ and $\mathbf{B}_i$ are matrices that satisfy several orthogonality constraints [22].

The orthogonality properties of OSTBC allows a simple linear decoding structure [4], [5]. Coding and decoding is performed in such a way that the receive SNR of the data stream is [4], [5]

$$\gamma = \gamma_0 ||\mathbf{H}||_F^2 \tag{2.35}$$

where $\gamma_0 = E_s / L_t N_0$. They also allow for a simple upper bound on the probability of error [10], [7]

$$P_e \leq e^{-\gamma ||\mathbf{H}||_F^2}. \tag{2.36}$$

Equations (2.35) and (2.36) show that maximizing the channel Frobenius norm maximizes SNR as well as minimizes the instantaneous probability of error. This observation is used to develop the selection algorithm.

Several well-known OSTBCs will be used in the thesis and simulations.

For $L_t = 3$, $Q = 3$, $T = 4$ the following code is an OSTBC [23], [24]:

$$\mathbf{X}_3 = \begin{bmatrix} s_1 & 0 & s_2 & -s_3 \\ 0 & s_1 & s_3^* & s_2^* \\ -s_2^* & -s_3 & s_1^* & 0 \end{bmatrix}. \tag{2.37}$$

This code has rate $R_s = 3/4$. An alternative OSTBC for $L_t = 3$, that has the same rate is [23], [24]:

$$\mathbf{X}_3 = \begin{bmatrix} s_1 & -s_2^* & s_3^* & 0 \\ s_2 & s_1^* & 0 & -s_3^* \\ s_3 & 0 & -s_1^* & s_2^* \end{bmatrix} \tag{2.38}$$

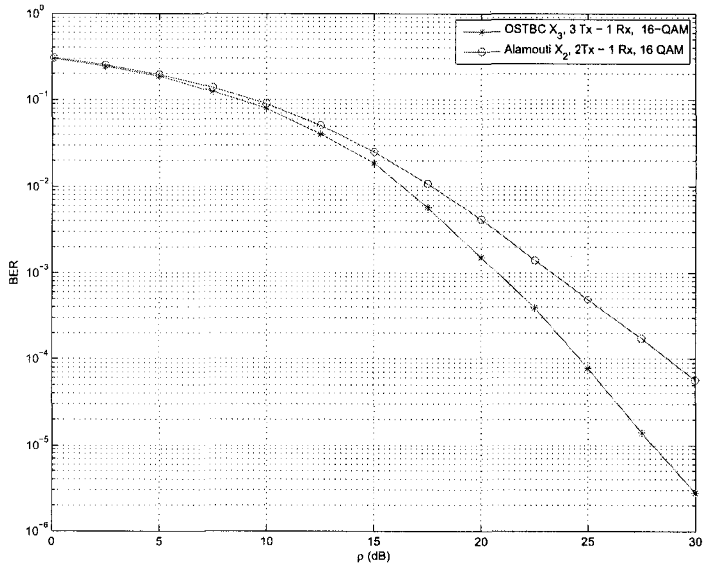

Figure 2.8: BER performance of OSTBCs using 16-QAM.

For $L_t = 4$, $T = 4$, $Q = 3$ the following code is an OSTBC [23], [24]:

$$\mathbf{X}_4 = \begin{bmatrix} s_1 & 0 & s_2 & -s_3 \\ 0 & s_1 & s_3^* & s_2^* \\ -s_2^* & -s_3 & s_1^* & 0 \\ s_3^* & -s_2 & 0 & s_1^* \end{bmatrix}. \tag{2.39}$$

2.7 Spatial-Multiplexing

This section describes spatial multiplexing, a capacity-achieving MIMO signaling technique.

In spatial-multiplexing [25], a symbol stream $s_1, s_2, \cdots, s_{L_t}$ with $s_i \in \mathcal{S}$ for all i, where $\mathcal{S}$ is the signal constellation, is demultiplexed into a space-time matrix $\mathbf{X}$, which in this case is simply a vector $(T = 1)$

$$\mathbf{X} = \mathbf{s} = \begin{pmatrix} s_1 \\ s_2 \\ \vdots \\ s_{L_t} \end{pmatrix}. \quad (2.40)$$

The received vector is

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{v}. \quad (2.41)$$

where $\mathcal{E}_{\mathbf{s}}[\mathbf{s}\mathbf{s}^*] = (E_s/L_t)\mathbf{I}_{L_t}$ is the power constraint for transmission vector.

In spatial-multiplexing, multiple symbols are transmitted at each channel use. Since simple linear combining and single-dimensional detections are no longer optimal, the detection methods are more complicated. Optimal detection is actually multi-dimensional search where vectors in $\mathcal{S}^{L_t}$ are detected, rather than symbols in $\mathcal{S}$ as was the case for OSTBCs.

2.7.1 ML receiver

The optimal receiver for detecting the symbol vector is the ML receiver. The ML receiver is given by

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}\in\mathcal{S}^{L_t}} \|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2. \quad (2.42)$$

This detection minimizes the average probability of error and requires an exhaustive search over all vector symbols $\mathbf{s} \in \mathcal{S}^{L_t}$. If an M-ary signal constellation $\mathcal{S}$ is chosen, the search is performed over total M^{L_t} possible vector symbols. Thus, the decoding complexity of the ML receiver is exponential in L_t. However, there are fast algorithms like sphere decoding [26], [22].

2.7.2 Sphere decoding

Since signal constellation $\mathcal{S}$ is a discrete alphabet, each $\mathbf{H}\mathbf{s}$ can be considered as a lattice point. The main idea behind sphere decoding is only test the lattice points (defined as $\mathbf{H}\mathbf{s}$) lying inside a hypersphere.

Let the QR decomposition of the channel matrix $\mathbf{H}$ be

$$\mathbf{H} = [\mathbf{Q}_1, \mathbf{Q}_2] \begin{bmatrix} \mathbf{R} \\ \mathbf{0} \end{bmatrix} \tag{2.43}$$

where $\mathbf{R}$ is an $L_t \times L_t$ upper-triangular matrix, $\mathbf{0}$ is the $(L_r - L_t) \times L_t$ all zero matrix, $\mathbf{Q}_1$ is an $L_r \times L_t$ unitary matrix and $\mathbf{Q}_2$ is an $L_r \times (L_r - L_t)$ unitary matrix. Note that here we assume that $L_r \geq L_t$. Thus (2.41) is equivalent to

$$\mathbf{y}' = \mathbf{R}\mathbf{s} + \mathbf{v}' \tag{2.44}$$

where $\mathbf{y}' = \mathbf{Q}_1^T\mathbf{y}$ and $\mathbf{v}' = \mathbf{Q}_1^T\mathbf{v}$ is also an i.i.d. Gaussian vector with mean zero and variance N_0.

The lattice point $\mathbf{Rs}$ lies in a sphere of radius d if, and only if

$$\|\mathbf{y}' - \mathbf{Rs}\|^2 \leq d^2. \tag{2.45}$$

Thus (2.45) can be expanded as

$$\sum_{i=1}^{L_t} \left| y_i' - \sum_{j=i}^{L_t} r_{i,j} s_j \right|^2 \leq d^2 \tag{2.46}$$

where $r_{i,j}$, $(i \leq j)$ are nonzero entries of $\mathbf{R}$. Expanding the left hand side of (2.46) yields

$$(y_{L_t}' - r_{L_t,L_t}s_{L_t})^2 + (y_{L_t-1}' - r_{L_t-1,L_t}s_{L_t} - r_{L_t-1,L_t-1}s_{L_t-1})^2 + \cdots + \sum_{i=1}^{L_t} \left(y_1' - \sum_{j=1}^{L_t} r_{1,j}s_j \right)^2 \leq d^2 \tag{2.47}$$

where the first term depends only on s_{L_t}, the second term on s_{L_t}, s_{L_t-1} and so forth. For $\mathbf{R}s$ to lie within the sphere, it is necessary that each of the terms of left hand side of equation above be less than d^2, which means $(y_{L_t}' - r_{L_t,L_t}s_{L_t})^2 \leq d^2$. This is equivalent to

$$\left\lceil \frac{-d + y_{L_t}'}{r_{L_t,L_t}} \right\rceil \leq s_{L_t} \leq \left\lfloor \frac{d + y_{L_t}'}{r_{L_t,L_t}} \right\rfloor. \tag{2.48}$$

For each candidate of s_{L_t} satisfying the above bound, we subtract value of the first term from both sides of (2.47). The next step is to define necessary condition for s_{L_t-1}

$$(y'_{L_t-1} - r_{L_t-1,L_t}s_{L_t} - r_{L_t-1,L_t-1}s_{L_t-1})^2 \leq d^2_{L_t-1} = d^2 - (y'_{L_t} - r_{L_t,L_t}s_{L_t})^2 \quad (2.49)$$

which leads to the following decision bound for s_{L_t-1}

$$\left\lceil \frac{-d_{L_t-1} + y'_{L_t-1} - r_{L_t-1,L_t}s_{L_t}}{r_{L_t-1,L_t-1}} \right\rceil \leq s_{L_t-1} \leq \left\lceil \frac{d_{L_t-1} + y'_{L_t-1} - r_{L_t-1,L_t}s_{L_t}}{r_{L_t-1,L_t-1}} \right\rceil . \quad (2.50)$$

The sphere decoder chooses a candidate for s_{L_t-1} from the above bound. The same process will be continued for s_{L_t-2} and so on.

2.7.3 V-BLAST

V-BLAST is a simple receiver scheme using sequential nulling and interference cancelation [27]. Nulling is performed by linearly weighting the received symbols to satisfy the zero forcing (ZF) or minimum mean squared error (MMSE) criterion. Zero-Forcing is used as nulling step without detection ordering. Let $\mathbf{h}_i$ be the ith column of the channel matrix $\mathbf{H}$. Thus, the ZF nulling vector $\mathbf{w}_i$, $i = 1, 2, \cdots, L_t$ is chosen such that

$$\mathbf{w}_i^H \mathbf{h}_j = \delta_{ij}. \quad (2.51)$$

Thus, the effect of symbols which have already been detected will be subtracted from the symbols not detected yet. This interference cancelation improves the overall performance when the order in which the components of symbol vector $\mathbf{s}$ are detected is chosen carefully.

Let $s_{(1)}, s_{(2)}, \ldots, s_{(L_t)}$ be the order of the symbols that must be detected, where (i) is an integer between 1 and L_t. Let the received vector $\mathbf{y}$ be $\mathbf{y}_{(1)}$. The first symbol is then detected as

$$\hat{s}_{(1)} = \arg\min_{s \in \mathcal{S}} \left| s - \mathbf{w}_1^H \mathbf{y}_{(1)} \right| . \quad (2.52)$$

The next step is to subtract the interference due to $\hat{s}_{(1)}$ on the other symbols by taking $\mathbf{y}_{(2)} = \mathbf{y}_{(1)} - \hat{s}_{(1)}\mathbf{h}_{(1)}$. Assuming the detection is correct (i.e. $\hat{s}_{(1)} = s_{(1)}$), the next symbol $s_{(2)}$ is then detected using ZF nulling vector $\mathbf{w}_{(2)}$. Same steps are then performed for next symbols by operation in turn on the modified received vectors $\mathbf{y}_{(3)}, \ldots, y_{(L_t)}$.

To find the optimal order of detection of symbols, note that in the kth iteration of V-BLAST algorithm, the signal with maximum post-detection SNR among remaining $L_t - k + 1$ symbols must be detected. The post-detection SNR for the kth detected symbol is given by

$$\rho_{(k)} = \frac{\mathcal{E}\left[|s_k|^2\right]}{N_0 \|\mathbf{w}_k\|^2}. \tag{2.53}$$

The V-BLAST scheme can also be described by the QR decomposition explained in the previous section.

2.8 Channel Capacity

In this section, we provide an overview on the MIMO channel capacity. Channel capacity provides a limit to the amount of information that can be transmitted across the channel with low probability of error.

2.8.1 Channel capacity for Single-user system

Channel capacity of a single user system where the transmitter sends the signal p.d.f. $p_X(x)$ is given by [28]

$$C = \max_{p_X(x)} I(X;Y), \tag{2.54}$$

i.e., channel capacity is mutual information between input and output signals, maximized over all possible distributions of the input. The significance of the channel capacity is that if the information rate R from the source is less than C (i.e. $R < C$), then it is theoretically possible to achieve error free transmission through the channel by appropriate coding. In this case, the length of the channel codes goes to infinity and the error goes to zero. If $R > C$, error free communication is not possible.

Information capacity is the maximum bits of information per channel use that can be transmitted error-free through the communication channel. For an AWGN channel $\mathbf{H}$ which is known and constant, the capacity is

$$C = \max_{p(x)} I(X, Y|H) = \log_2(1+\rho) \quad \text{bits/sec/Hz}, \tag{2.55}$$

where ρ is the SNR.

In wireless communications, the channel gains vary due to fading. The average mutual information, *ergodic capacity*, is defined as

$$C = \mathcal{E}_{\mathbf{H}} \left[\log_2(1+\rho\|\mathbf{h}\|^2)\right] \tag{2.56}$$

for a system with a single transmit and multiple (say, L_r) receive antennas.

2.8.2 Capacity of system with Multiple Antennas

In [2], the capacity of multiple antenna systems is derived. The ergodic capacity for such systems is given by

$$C = \max_{\mathbf{R}_s \geq 0, \mathrm{tr}\mathbf{R}_s = L_t} \mathcal{E}\left[\log\det\left(\mathbf{I}_{L_r} + \frac{\rho}{L_t}\mathbf{H}\mathbf{R}_s\mathbf{H}^H\right)\right] \tag{2.57}$$

where the expectation is taken over the distribution of the random matrix $\mathbf{H}$. The capacity-achieving $\mathbf{s}$ is a zero-mean complex Gaussian vector with covariance matrix $\mathcal{E}\left[\mathbf{s}\mathbf{s}^H\right] = \mathbf{R}_{s,opt}$, where it is the capacity-maximizing covariance matrix. With the optimizing covariance is $\mathbf{R}_{s,opt} = \mathbf{I}_{L_t}$, the capacity becomes

$$C = \mathcal{E}\left[\log\det\left(\mathbf{I}_{L_r} + \frac{\rho}{L_t}\mathbf{H}\mathbf{H}^H\right)\right]. \tag{2.58}$$

2.9 Some useful probability preliminaries

This section gives preliminary material on probability useful for performance analysis.

Gaussian (normal) distribution: The pdf and cdf of a Gaussian random variable, $\mathcal{N}(\mu, \sigma)$ with mean μ and variance σ^2 is

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{2.59}$$

and

$$\begin{aligned} F(x) &= \int_{-\infty}^{x} f(t) dt \\ &= \frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{x-\mu}{\sqrt{2}\sigma}\right) \\ &= \mathcal{Q}\left(\frac{x-\mu}{\sigma}\right) \end{aligned} \tag{2.60}$$

where $\operatorname{erf}(\cdot)$ and $\mathcal{Q}(\cdot)$ are defined as

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{t^2} dt, \tag{2.61}$$

$$\mathcal{Q}(x) = \frac{1}{2} - \frac{1}{2} \operatorname{erf}\left(\frac{x}{\sqrt{2}}\right) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt. \tag{2.62}$$

Complex Gaussian distribution: A circularly symmetric complex Gaussian random variable, $\mathcal{CN}(\mu, \sigma)$, is a random variable $Y = Y_r + jY_i$ where each of Y_r and Y_i are independent Gaussian distributed random variables with variance $\sigma^2/2$.

Chi-square distribution: Let random variable Y defined as

$$Y = \sum_{i=1}^{n} X_i^2 \tag{2.63}$$

where the X_i, $i = 1, 2, ..., n$ are statistically independent and identically distributed Gaussian random variables with mean zero and variance one. The pdf of Y is

$$f_Y(y) = \frac{1}{2^{\frac{n}{2}} \Gamma(\frac{1}{2}n)} y^{\frac{n}{2}-1} e^{-\frac{y}{2}}, \quad y \geq 0, \tag{2.64}$$

where $\Gamma(\cdot)$ is function defined as

$$\Gamma(r) = \int_0^{\infty} t^{r-1} e^{-t} dt, \tag{2.65a}$$

$$\Gamma(r) = (r-1)!, \quad r \in \mathbb{N}, \tag{2.65b}$$

$$\Gamma(r+1) = r\Gamma(r), \tag{2.65c}$$

$$\Gamma(\frac{1}{2}) = \sqrt{\pi}. \tag{2.65d}$$

This pdf is called *Chi-square (or Gamma) pdf with n degrees of freedom.* The cdf of Y is given by

$$F_Y(y) = \int_0^y \frac{1}{2^{\frac{n}{2}}\Gamma(\frac{1}{2}n)} t^{\frac{n}{2}-1} e^{-\frac{t}{2}} dt, \quad y \geq 0, \tag{2.66}$$

which can be expressed in terms of the incomplete gamma function. When n is even integer, assume $m = \frac{1}{2}n$, the cdf is given in closed-form as

$$F_Y(y) = 1 - e^{-\frac{y}{2}} \sum_{k=0}^{m-1} \frac{1}{k!} \left(\frac{y}{2}\right)^k, \quad y \geq 0. \tag{2.67}$$

This case where n is even, is of interest in next chapters. It arises for a sum of square magnitudes of a number of complex Gaussian random variables,

The *non-central chi-square distribution* arises if X_i, $i = 1, 2, ..., n$ in (2.63) are statistically independent and Gaussian distributed random variables with non-zero mean μ_i, $i = 1, 2, ..., n$ and identical variance σ^2. Thus, the pdf of Y in this case is

$$f_Y(y) = \frac{1}{2\sigma^2} \left(\frac{y}{\mu^2}\right)^{\frac{n-2}{4}} e^{-\frac{\mu^2+y}{2\sigma^2}} I_{\frac{n}{2}-1}\left(\frac{\mu\sqrt{y}}{\sigma^2}\right), \quad y \geq 0 \tag{2.68}$$

where, by definition

$$\mu^2 = \sum_{i=1}^{n} \mu_i^2 \tag{2.69}$$

is the *noncentrality chi-square parameter* and $I_\alpha(\cdot)$ is the modified Bessel function of the first kind represented by

$$I_\alpha(x) = \left(\frac{x}{2}\right)^\alpha \sum_{k=0}^{\infty} \frac{x^{2k}}{2^{2k} k! \Gamma(\alpha + k + 1)}. \tag{2.70}$$

For even integers of $m = \frac{1}{2}n$, the pdf in (2.68) can be simplified to

$$f_Y(y) = \frac{1}{(2\sigma^2)^m} y^{m-1} e^{-\frac{\mu^2+y}{2\sigma^2}} \sum_{k=0}^{\infty} \frac{\mu^{2k} y^k}{(2\sigma^2)^k \, k!(m+k-1)!}, \quad y \geq 0. \tag{2.71}$$

The cdf of the noncentral chi-square distribution with an even integer n degrees of freedom can be expressed in terms of the generalized Marcum's $\mathcal{Q}$ function, which is defined as

$$\mathcal{Q}_m(\alpha, \beta) = \frac{1}{\alpha^{m-1}} \int_{\beta}^{\infty} x^m e^{-\frac{x^2+\alpha^2}{2}} I_{m-1}(\alpha x) dx. \tag{2.72}$$

Thus the cdf can be expressed as [17]

$$F_Y(y) = 1 - \mathcal{Q}_m\left(\frac{\mu}{\sigma}, \frac{\sqrt{y}}{\sigma}\right). \tag{2.73}$$

Rayleigh distribution: In cellular radio, the received signal envelope has a distribution that is closely related to the central chi-square distribution. We provide pdf and cdf of generalized Rayleigh random variable R defined as

$$R = \sqrt{\sum_{i=1}^{n} X_i^2} \tag{2.74}$$

where X_i, $i = 1, 2, ..., n$ are i.i.d. Gaussian random variables with mean zero and variance σ^2. The random variable $Y = R^2$ has chi-square distribution. The pdf of the generalized Rayleigh distribution can be obtained as

$$f_R(r) = \frac{r^{n-1}}{2^{\frac{n-2}{2}}\sigma^n\Gamma(\frac{1}{2}n)} e^{-\frac{r^2}{2\sigma^2}}, \quad r \geq 0. \tag{2.75}$$

The Rayleigh cdf for even numbers of $n = 2m$ is

$$F_R(r) = 1 - e^{-\frac{r^2}{2\sigma^2}} \sum_{k=0}^{m-1} \frac{1}{k!}\left(\frac{r^2}{2\sigma^2}\right), \quad r \geq 0. \tag{2.76}$$

Ricean distribution: Ricean-distributed random variable is closely related to non-central chi-square distribution. A Rice random variable R is defined as (2.74) where X_i, $i = 1, 2, ..., n$ are statistically independent Gaussian random variables with mean μ_i and variance σ^2. The mean μ is defined in the same way as noncentrality chi-square parameter in (2.69). Thus, the pdf of R is given by

$$f_R(r) = \frac{r}{\sigma^2} e^{-\frac{r^2+\mu^2}{2\sigma^2}} I_0\left(\frac{rs}{\sigma^2}\right), \quad r \geq 0 \tag{2.77}$$

The cdf of Rice distribution for the special case where $m = \frac{1}{2}n$ is an integer is expressed as

$$F_R(r) = 1 - \mathcal{Q}_m\left(\frac{\mu}{\sigma}, \frac{r}{\sigma}\right), \quad r \geq 0 \tag{2.78}$$

Order statistics distributions: Suppose there are n independent random variables X_i, $i = 1, 2, ..., n$ each having the same pdf $f(x)$ and the cdf $F(x)$. If they are arranged in ascending order of magnitude as

$$X_{(1)} \leq X_{(2)} \leq \cdots \leq X_{(n)}, \tag{2.79}$$

then $X_{(i)}$, $i = 1, 2, ..., n$ is called the ith order statistic. In this section X_i are assumed to be statistically independent and identically distributed. However, when these random variables are correlated the analysis of order statistic is much more difficult. The pdf of the rth order statistic is given by [29]

$$f_{(r)}(x) = \frac{n!}{(r-1)!(n-r)!} f(x) F^{r-1}(x)[1 - F(x)]^{n-r} \tag{2.80}$$

and the cdf is

$$F_{(r)}(x) = \sum_{i=r}^{n} \binom{n}{i} F^i(x)[1 - F(x)]^{n-i}. \tag{2.81}$$

The joint pdf of $X_{(i_1)}, X_{(i_2)}, ..., X_{(i_k)}$ where $1 \leq i_1 \leq \cdots \leq i_k \leq n$ and $1 \leq k \leq n$ is given in [29] as

$$\begin{aligned} f_{i_1, i_2, ..., i_k}(x_1, ..., x_k) = & \frac{n!}{(n_1 - 1)!(n_2 - n_1 - 1)! \cdots (n - n_k)!} \\ & \times F^{n_1 - 1}(x_1) f(x_1)[F(x_2) - F(x_1)]^{i_2 - i_1 - 1} \\ & \times f(x_2) \cdots [1 - F(x_k)]^{n - n_k} f(x_k). \end{aligned} \tag{2.82}$$

Moment Generating Function: The *moment generation function* (MGF) of a random variable X is defined as the statistical average

$$\mathcal{E}_X\left[e^{sX}\right] \equiv \Phi_X(s) = \int_0^{\infty} e^{sx} f(x) dx = \mathcal{L}_x\left[f(x)\right](s) \tag{2.83}$$

where the variable s can be any complex number and is the Laplace transform of the pdf $f(x)$. In the special case when $s = jv$, $j = \sqrt{-1}$ and v is a real number, the MGF is called *characteristic function* and may be described as the Fourier transform of the pdf $f(x)$.

Assume (2.79) and a random variable is defined from sum of L largest variables as

$$Y = \sum_{i=n-l+1}^{n} X_{(i)}. \tag{2.84}$$

Then the MGF of Y is given by [12]

$$\Phi_Y(s) = l\binom{n}{l}\int_0^{\infty} e^{-sx} f(x)[F(x)]^{n-l}[\phi(s,x)]^{l-1}dx, \quad 1 \leq l \leq n \tag{2.85}$$

where $\phi(s,x) = \int_x^{\infty} e^{-st} f(t)dt$.

2.10 Summary

In this chapter a brief overview of OSTBCs is given. Different receivers such as ML, V-BLAST and Sphere decoders are introduced. Diversity order and coding gain are defined. Useful probability variables and formulas are discussed for subsequent performance analysis.

Chapter 3

Antenna Selection

This chapter discusses antenna selection for multiple-antenna wireless systems. The motivations for antenna selection is discussed in Section 3.1. Antenna selection can be implemented on transmit and/or receiver sides. Receive antenna selection briefly discussed in Section 3.2. Transmit antenna selection is discussed in Section 3.3. Transmit antenna selection criteria based on probability of error minimization and capacity maximization are discussed in Section 3.4.

3.1 Introduction

Multiple antenna wireless communication systems have recently sparked a significant interest due to their higher capacity and better performance compared to SISO systems. The capacity of MIMO system increases linearly with number of receiver or transmit antennas [1,2]. However, the increase in the number of antennas results in high hardware complexity. Since additional antenna elements such as patch or dipole antennas are usually inexpensive, the major cost factor of transmitters and receivers that employ multiple antennas is not actually the number of antennas. An RF chain includes high power amplifier, low-noise amplifier (LNAs), up/downconverter, and digital-to-analog/analog-to-digital converters (DACs/ADCs) and is expensive. Therefore, the number of antennas may be larger than the number of RF chains, and, at any time

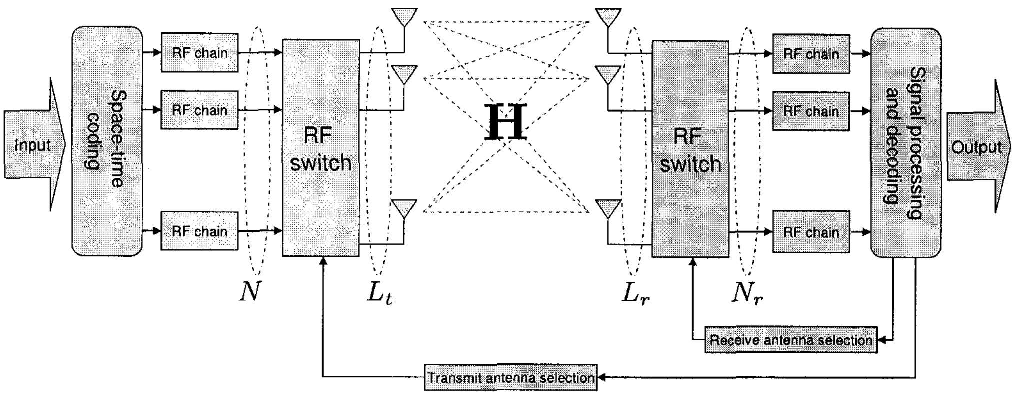

Figure 3.1: MIMO antenna selection system

period, only a subset of antennas via a low-cost RF switch is used for transmission. Thus, antenna selection reduces hardware complexity and costs and yet keeps the advantages of multiple antenna systems. At the receive side, antenna selection reduces the complexity. More interestingly, at the transmit side, antenna selection not only reduces the complexity, but also improves the capacity of the MIMO system [30], [31], [32] at the cost of a minimal amount of feedback. Moreover, full diversity systems are achievable through antenna selection.

Fig. 3.1 shows a MIMO system with transmit and receive antenna selection. An input bit stream is sent through an encoder and modulator. The space-time encoder converts a single bitstream into symbol streams through a proper mapping i.e. Gray mapping and then converts the complex symbol vector into N parallel streams of symbols. Each of these streams is sent through an RF chain to produce signal for transmission through each transmit antennas. However, the number of RF chains are less than transmit antennas (i.e. $N \leq L_t$), thus the RF switch chooses the best N antennas out of L_t. At the receiver, the RF switch chooses best N_r out of L_r receive antennas ($N_r \leq L_r$). The channel seen by the selected subset of transmit and receive antennas, is the sub-matrix $\mathbf{H}_{AS} \in \mathbb{C}^{N_r \times N}$, which is obtained by selecting the rows and columns of the channel matrix $\mathbf{H}$ that correspond to the selected receive and transmit antennas. There are $\binom{L_t}{N}\binom{L_r}{N_r}$ possible sub-matrices of $\mathbf{H}$.

The selection criteria are based either on system capacity [33], or bit error rate (BER) improvement [10]. Optimizing capacity and outage in spatial multiplexing systems is one approach. The other approach is exploring the diversity order and array gain or performance of system for space-time coded systems.

We briefly review previous antenna selection research and show different antenna selection types.

3.2 Receive Antenna Selection

Consider a diversity reception system where several copies of the transmitted signal are received by receiver. Each of these copies experiences different fading gains by a different path. The receiver combines these copies (known as *diversity combining*) in order to maximize the achievable SNR and reduce the complexity of decoder.

There are several types of diversity combining. *Selection diversity* chooses the path with the highest SNR and signal detection based on that specific path. Optimal linear combination of signals received from all the different paths leads to *maximal ratio combining* (MRC) method. *Equal gain combining* (EGC) simply add received signal after being co-phased. *Generalized selection combining* (GSC) [34] is a diversity combining scheme where N_r $(1 \leq N_r \leq L_r)$ branches with the highest SNR are selected from total L_r branches and then combined. Due to its ease of implementation, GSC is known as a widely employed receive diversity technique. However, compared to MRC or EGC, the error performance in fading channels is much poorer when the number of receive antennas L_r is large.

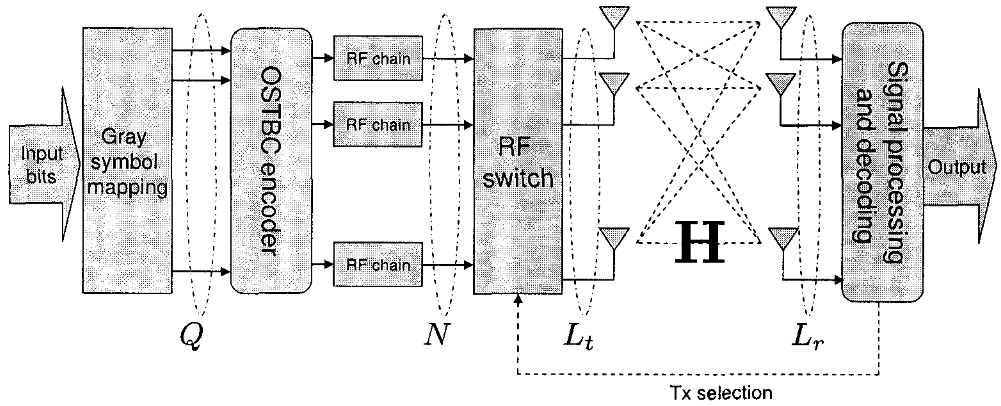

Figure 3.2: Transmit antenna selection scheme

3.3 Transmit Antenna Selection

Transmit antenna selection requires feedback from the receiver side. There are $\binom{L_t}{N}$ combinations of N transmit antennas out of total L_t antennas. Thus $\log_2 \binom{L_t}{N}$ feedback bits are required, which specially for the case of single antenna selection is as low as $\log_2 L_t$ and is feasible for the practical purposes. Aside from the complexity reduction which is the same as receive antenna selection which has discussed before, transmit antenna selection improves capacity and performance of system surprisingly. This is a direct result of having limited feedback from the receiver [30], [31], [32].

Transmit antenna selection has recently been extensively studied; a number on antenna selection algorithms, various selection criteria based on capacity, error rates, and outage and performance analysis techniques have been developed. Capacity of MIMO channels with antenna selection is studied in [35] Joint transmit/receive antenna selection algorithms are presented in [36]. In the multi-mode antenna selection scheme [37], both the number of transmit antennas and mapping of substreams to antennas are dynamically adjusted to get the better performance. However, in this system spatial multiplexing with linear receiver is used over transmit antennas selected. Hybrid selection/maximal-ratio transmission is proposed and studied in [38] where the best N antennas out of total L_t are selected, and then combined to reduce the complexity of system. The outage probability, capacity and SER of antenna selection scheme in MIMO systems are also given in [38].

A method to select optimal set of antennas based on capacity maximization is given in [39]. Gore *et al.* proposed a method for optimal selection for transmit antennas for rank deficient channel matrices based on capacity maximization [40]. These results were further generalized and several fast methods for transmit antenna selection are proposed in [41]. A correlated selection algorithm is presented in [42] where the subset of transmit (receive) antennas are selected in order to maximize the determinant of the transmit (receive) subset selection. This result is based on capacity maximization of antenna selection over correlated channels.

It is shown in [30] that optimum signaling for largest ergodic capacity is generally different with antenna selection than without antenna selection. If the outage probability is used as a performance merit such in wireless local area networks, transmit antenna selection criteria are different. In [43] the active transmit antennas are selected to minimize the total transmit power among users in correlated multiple access channel to achieve a target outage probability and a data rate of each users. Comprehensive surveys on antenna selection can be found in [11, 36].

Transmit antenna selection has recently been studied; a number of antenna selection algorithms, various selection criteria based on capacity, error rates, and outage and performance analysis techniques have been developed. Comprehensive surveys can be found in [11, 36]. Although receive antenna selection with various channel/correlation models have been widely studied (see [44–50]), the analogous results are limited for transmit antenna selection. This thesis will discuss the performance analysis of transmit antenna selection and OSTBCs in fading channels.

The symbol error rate (SER) performance of transmit antenna selection combined with OSTBCs is analyzed in [51]; however, the results are not in closed form. The asymptotic error performance of single transmit antenna selection and receive maximal-ratio (MRC) with generalized selection criterion is investigated in [52]. In [53], the SER formulas of combined single transmit antenna selection and receive generalized selection combining is derived. In this scheme the receiver first selects best receiving antennas for each transmit antenna and then selects the best transmit antenna based on received SNR. Again, the formulas are left as integrals and numerical method are needed. The exact bit error rate (BER) performance of only two transmit antennas with BPSK signals using the Alamouti scheme is derived in [54], [55]. In [56] the asymptotic bit error performance of the Alamouti scheme with transmit antenna selection is investigated for imperfect selection of antenna subset. Transmit antenna selection schemes, most discussed only over independent Rayleigh fading not for the correlated cases and for other kinds of fading.

3.4 Antenna Selection based on Capacity Maximization

Transmit antenna selection is achieved through a feedback from the receiver. However, the transmitter does not know $\mathbf{H}$. The system chooses the best N transmit antennas and sends the symbol vectors through the selected subset. Therefore, the received signal vector can be defined as

$$\mathbf{y} = \sqrt{\frac{E_s}{N}}\mathbf{H}_{AS}\mathbf{s} + \mathbf{v} \tag{3.1}$$

where $\mathbf{s} \in \mathbb{C}^N$ is the symbol vector transmitted from N transmit antennas and selected from arbitrary signal constellation $\mathcal{S}$. E_s is the total transmission power and $\mathbf{v}$ is the noise vector of size L_r which has i.i.d. entries of $\mathcal{CN}(0, N_0)$. The channel model is provided as in (2.3). In fact, the channel seen by the subset is the sub-matrix, $\mathbf{H}_{AS}$, that is obtained by selecting the rows and columns of the channel matrix $\mathbf{H}$ corresponding to the selected antenna subsets. The optimal subset of selected antennas gives the largest mutual information between the transmitter and receiver. The capacity with antenna selection is thus given by [35]

$$C_{AS} = \max_{\mathbf{H}_{AS}} \log \det \left(\mathbf{I}_{L_t} + \frac{E_s}{N_0 L_t} \mathbf{H}_{AS} \mathbf{H}_{AS}^H \right). \tag{3.2}$$

The maximization is over all possible $N_r \times N$ sub-matrices of $\mathbf{H}$. Therefore, the optimal antenna selection channel subset $\mathbf{H}_{AS,opt}$ is given by

$$\mathbf{H}_{AS,opt} = \arg\max_{\mathbf{H}_{AS}} \log \det \left(\mathbf{I}_{L_t} + \frac{\rho}{L_t} \mathbf{H}_{AS} \mathbf{H}_{AS}^H \right) \tag{3.3}$$

A closed form solution for this criteria is quite difficult. For receive antenna selection, lower and upper bound of C_{AS} can be found in [35] and [57].

3.4.1 Antenna Selection for OSTBCs

When OSTBCs are employed with antenna selection, the probability of error can be upper bounded by

$$P_e \leq e^{-\frac{\rho}{N} \|\mathbf{H}_{AS}\|_F^2}. \tag{3.4}$$

Given that the channel subset $\mathbf{H}_{AS}$ is selected, the instantaneous SNR of the received signal γ_{AS} is given by [6]

$$\gamma_{AS} = \frac{\rho}{N} \|\mathbf{H}_{AS}\|_F^2 \tag{3.5}$$

where N is the number of transmit antennas selected. Since above upper bound is enough tight and in a communication system maximizing SNR minimizes the probability of error, thus the optimal rule is to choose $\mathbf{H}_{AS}$ to minimize the probability of error. Therefore, the selected sub-matrix has the largest Frobenius norm of all possible sub-matrices of the channel matrix:

$$\mathbf{H}_{AS,opt} = \arg\max_{\mathbf{H}_{AS}} \|\mathbf{H}_{AS}\|_F^2. \tag{3.6}$$

Transmit antenna selection corresponds to the selection of a set of columns of the channel matrix. Since the Frobenius norm of a matrix is equal to the sum of norms of its columns, transmit antenna selection leads to selecting the number of columns of the channel matrix $\mathbf{H}$ with the largest norms. Assume

$$\mathbf{H} = [\mathbf{h}_1 \mathbf{h}_2 \cdots \mathbf{h}_{L_t}] \tag{3.7}$$

and order the columns of $\mathbf{H}$ with respect to their norms as

$$|\mathbf{h}_{(1)}| \geq |\mathbf{h}_{(2)}| \geq \cdots \geq |\mathbf{h}_{(L_t)}|. \tag{3.8}$$

Then selecting N transmit antennas out of L_t antennas gives the transmit antenna selection channel matrix as

$$\mathbf{H}_{TAS} = \left[\mathbf{h}_{(1)} \mathbf{h}_{(2)} \cdots \mathbf{h}_{(N)}\right]. \tag{3.9}$$

3.5 Summary

The cost and hardware complexity of MIMO wireless systems can be reduced by reducing number of active antennas. This is feasible through antenna selection. Moreover, transmit antenna selection also improves the capacity of the MIMO system and gives better performance. With minimal number of bits of feedback, transmitter chooses the best set of transmit antennas. Transmit antenna selection criteria include maximizing the capacity of system after selection which leads to selecting antennas through exhaustive search algorithm with capacity as an objective function. Minimizing the error rate and therefore maximizing achievable received SNR leads to selecting transmit antennas which have highest norms of the corresponding channel column.

Chapter 4

Performance of OSTBCs over Transmit Antenna Selection

In this chapter, we analyze the performance of transmit antenna selection with OSTBCs over independent Rayleigh fading channels. For this purpose, the MGF of the received SNR is needed. Using order statistics, exact BER expressions are derived in Section 4.3. Section 4.4 presents an asymptotic performance analysis. Diversity and coding gain of this system are derived. Numerical results, simulations and conclusions conclude the chapter.

4.1 System Model

Fig. 4.2 shows a MIMO system with OSTBC with transmit antenna selection. Gray mapping of signal constellation symbols is used. The OSTBC encoder produces an space-time block matrix for transmission over T time slots and over N transmit antennas which are selected based on the criterion of selection to minimize the error rate of the system. Total transmission power is divided by N number of active transmit antennas to fix the total transmission power E_s. The modulated signal waveforms are sent over the Rayleigh fading channel. The channel is assumed to have quasi-static fading which means the channel parameters are constant complex values over T time slots, one block period but may change from one block period to the next T time slots.

In this chapter, we assume that the transmit and receive antennas have no correlation (i.e. $\mathbf{R}_r = \mathbf{I}_{L_r}$, $\mathbf{R}_t = \mathbf{I}_N$). Finally the received signal is processed by the OSTBC decoder. Due to orthogonality property of OSTBCs (2.32), the decoder can detect each symbol separately and treat the MIMO system as an equivalent number of SISO systems. Thus, this eases the decoding complexity.

4.2 The received SNR

With OSTBCs, the MIMO system is equivalent to Q independent single input single output (SISO) systems defined as [5], [3]

$$\tilde{s}_q = \sqrt{\frac{E_s}{N}} \left(\frac{1}{R_s} \|\mathbf{H}_{TAS}\|_F^2 \right) s_q + \nu_q, \quad q = 1, ..., Q \tag{4.1}$$

where $\nu_q \sim \mathcal{CN}\left(0, \frac{1}{R_s}\|\mathbf{H}_{TAS}\|_F^2 N_0\right)$. We conclude that the SNR per bit with an M-ary constellation is

$$\gamma_b(\rho) = \frac{E_s}{N_0} \cdot \frac{1}{R_s N \log_2 M} \|\mathbf{H}_{TAS}\|_F^2 = c\rho \|\mathbf{H}_{TAS}\|_F^2 \tag{4.2}$$

where $\rho = \frac{E_s}{N_0}$ is the SNR per channel and $c = 1/(R_s N \log_2 M)$. Therefore, the antenna selection criterion in (3.6), which selects N transmit antennas, maximizes the instantaneous SNR and thereupon minimizes the error rate.

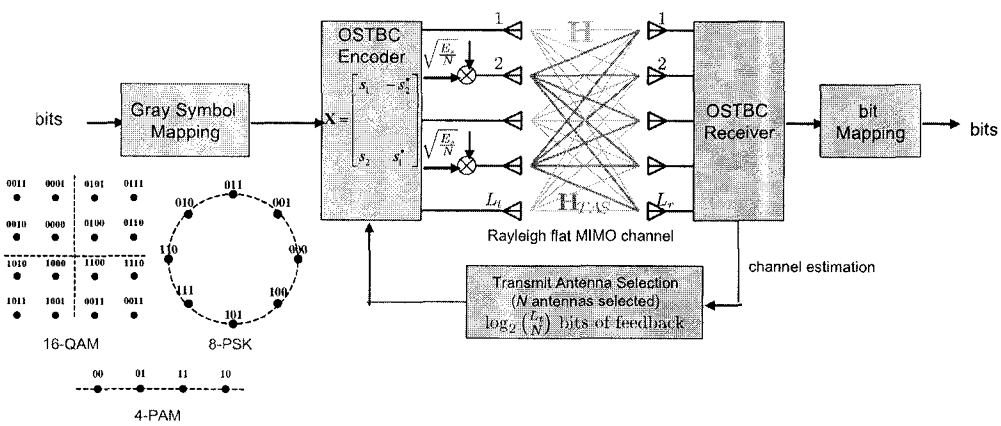

Figure 4.1: MIMO OSTBC with transmit antenna selection.

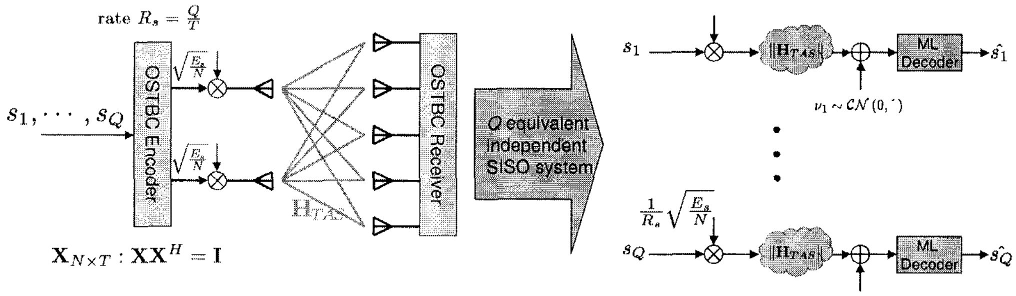

Figure 4.2: OSTBC MIMO system is equivalent to Q independent SISO systems

Thus, as discussed in Chapter 4, the antenna selection criterion involves selecting selects a subset of antennas which minimize the error rate of system and therefore maximizing the SNR of the system. We have

$$\begin{aligned} \mathbf{H}_{TAS,opt} &= \arg\max_{\mathbf{H}_{TAS}} \gamma_b(\rho) \\ &= \arg\max_{\mathbf{H}_{TAS}} c\rho\|\mathbf{H}_{TAS}\|_F^2 \\ &\equiv \arg\max_{\mathbf{H}_{TAS}} \|\mathbf{H}_{TAS}\|_F. \end{aligned} \tag{4.3}$$

Let the scaled norms of the columns of $\mathbf{H}$ be $\gamma_k = c\rho\|\mathbf{h}_k\|^2$, $k = 1, 2, ..., L_t$. Maximizing the SNR is equivalent to maximize the Frobenius norm of the transmit antenna selection sub-channel. The Frobenius norm of a matrix is equal to the sum of the norms of its columns. Thus, optimal transmit antenna selection yields a sub-matrix of the channel matrix $\mathbf{H}$ with the columns with highest norms. In transmit antenna selection (3.6), the best N antennas with the largest γ_k are selected. Assume that

$$\gamma_{(1)} \geq \gamma_{(2)} \geq \cdots \geq \gamma_{(L_t)} \tag{4.4}$$

or equivalently

$$|\mathbf{h}_{(1)}| \geq |\mathbf{h}_{(2)}| \geq \cdots \geq |\mathbf{h}_{(L_t)}|. \tag{4.5}$$

Thus, the received SNR per bit (4.2) is the sum of N largest order statistics

$\gamma_{(i)} = c\rho\|\mathbf{h}_{(i)}\|^2$, $i = 1, 2, ..., L_t$ and can be written as

$$\gamma_b = \sum_{k=1}^{N} \gamma_{(k)}. \tag{4.6}$$

4.3 BER Expressions

Now that we have derived the received SNR, using the relations between MGF of the received SNR and BER in Section 2.4, we will obtain the closed-form expressions of BER for M-PAM, M-QAM and M-PSK signal constellations.

4.3.1 M-ary PAM and QAM

Recall from last section that $\gamma_k = c\rho\|\mathbf{h}_k\|^2$, $k = 1, 2, ..., L_t$, are the scaled norms of the columns of $\mathbf{H}$, which are the received SNR from the kth transmit antenna. In an independent flat Rayleigh MIMO channel, γ_k, $k = 1, 2, ..., L_t$ are i.i.d. chi-squared variables with pdf [17]

$$f_{\gamma_k}(x) = \frac{x^{L_r-1}}{(c\rho)^{L_r}(L_r-1)!} e^{-x/c\rho}, \quad x \geq 0 \tag{4.7}$$

and the cumulative density distribution (cdf) is given by [17]

$$F_{\gamma_k}(x) = 1 - e^{-x/c\rho} \sum_{k=0}^{L_r-1} \frac{1}{k!} \left(\frac{x}{c\rho}\right)^k, \quad x \geq 0 \tag{4.8}$$

The received SNR from the (k)th transmit antenna, $\gamma_{(k)}$, $k = 1, 2, ..., N$, are a set of order statistics (4.4). Thus, the MGF of the sum of N largest order statistics is given by [12]

$$\Phi_{\gamma_b}(s) = N\binom{L_t}{N} \int_0^\infty e^{-sx} f_{\gamma_k}(x)[F_{\gamma_k}(x)]^{L_t-N}[\phi_{\gamma_k}(s,x)]^{N-1}dx, \quad 1 \leq N \leq L_t \tag{4.9}$$

where $\phi_{\gamma_k}(s,x) = \int_x^\infty e^{-st} f_{\gamma_k}(t)dt$. Simply by changing variables in this integral, an other version of the MGF is given by [12]

$$\Phi_{\gamma_b}(s) = (L_t - N)\binom{L_t}{N} \int_0^\infty f_{\gamma_k}(x)[F_{\gamma_k}(x)]^{L_t-N-1}[\phi_{\gamma_k}(s,x)]^N dx, \quad 1 \leq N \leq L_t \tag{4.10}$$

The function $\phi_{\gamma_k}(s,x)$ can be simplified as

$$\begin{aligned}\phi_{\gamma_k}(s,x) =& \int_x^{\infty} e^{-st} f_{\gamma_k}(t)dt \\ =& \frac{1}{(c\rho)^{L_r}(L_r-1)!}\int_x^{\infty} t^{L_r-1}e^{-t\left(s+\frac{1}{c\rho}\right)}dt \\ =& \frac{e^{-x\left(s+\frac{1}{c\rho}\right)}}{(c\rho)^{L_r}\left(s+\frac{1}{c\rho}\right)^{L_r}} \sum_{k=0}^{L_r-1} \frac{x^k\left(s+\frac{1}{c\rho}\right)^k}{k!}.\end{aligned} \tag{4.11}$$

Therefore, substituting these results into (4.9) the MGF of the received SNR for transmit antenna selection is

$$\begin{aligned}\Phi_{\gamma_b}(s) =& N\binom{L_t}{N}\int_0^{\infty} \frac{x^{L_r-1}e^{-x(s+c\rho)}}{(c\rho)^{L_r}(L_r-1)!}\left[1-e^{-x/c\rho}\sum_{k=0}^{L_r-1}\frac{1}{k!}\left(\frac{x}{c\rho}\right)^k\right]^{L_t-N} \\ &\times\left[\frac{e^{-x(s+\frac{1}{c\rho})}}{(c\rho)^{L_r}\left(s+\frac{1}{c\rho}\right)^{L_r}}\sum_{k=0}^{L_r-1}\frac{x^k\left(s+\frac{1}{c\rho}\right)^k}{k!}\right]^{N-1} dx.\end{aligned} \tag{4.12}$$

The second term within the integral above can be expanded as

$$\begin{aligned}&\left[1-e^{-x/c\rho}\sum_{k=0}^{L_r-1}\frac{1}{k!}\left(\frac{x}{c\rho}\right)^k\right]^{L_t-N} \\ =& \sum_{j=0}^{L_t-N}(-1)^j\binom{L_t-N}{j}e^{-\frac{jx}{c\rho}}\left[\sum_{k=0}^{L_r-1}\frac{1}{k!}\left(\frac{x}{c\rho}\right)^k\right]^j \\ =& \sum_{j=0}^{L_t-N}(-1)^j\binom{L_t-N}{j}e^{-\frac{jx}{c\rho}}\sum_{n=0}^{j(L_r-1)} a_n^{(L_r,j)}\left(\frac{x}{c\rho}\right)^n\end{aligned} \tag{4.13}$$

where $a_r^{(m,n)}$ is the coefficient of x^r in expression

$$\left[\sum_{k=0}^{m-1}\frac{x^k}{k!}\right]^n. \tag{4.14}$$

Using this expansion into the MGF expression in (4.12), similar to simplification done in [58] the MGF closed-form expression can be obtained as [59] and [60]

$$\Phi_{\gamma_b}(s) = N\binom{L_t}{N}\frac{1}{(c\rho)^{L_r N}[(L_r-1)!]^N}\sum_{r=0}^{(L_r-1)(N-1)} a_r^{(L_r,N-1)}$$
$$\times \sum_{j=0}^{L_t-N}\binom{L_t-N}{j}(-1)^j\left\{\sum_{l=0}^{j(L_r-1)} a_j^{(L_r,j)}\frac{(l+r+L_r-1)!}{(c\rho)^l}\right.$$
$$\left.\times \frac{1}{\left(\frac{1}{c\rho}+s\right)^{L_r(N-1)-r}}\cdot\frac{1}{\left(\frac{1}{c\rho}(N+j)+Ns\right)^{l+r+L_r}}\right\}. \tag{4.15}$$

Therefore, the BER for the PAM signals can be obtained from (4.15) and (2.18) as [59] and [60].

$$P_M(\rho) = \frac{2N\binom{L_t}{N}}{M\log_2 M}\frac{1}{[(L_r-1)!]^N}\sum_{n=1}^{\log_2 M}\sum_{i=0}^{k_n} B_i \sum_{r=0}^{(L_r-1)(N-1)} a_r^{(L_r,N-1)}$$
$$\times \sum_{j=0}^{L_t-N}\binom{L_t-N}{j}(-1)^j\sum_{l=0}^{j(L_r-1)} a_l^{(L_r,j)}\frac{(l+r+L_r-1)!}{(N+j)^{l+r+L_r}}$$
$$\times I\left(\frac{c\rho D_i^2}{2}, \frac{Nc\rho D_i^2}{2(N+j)}; L_r(N-1)-r, l+r+L_r\right) \tag{4.16}$$

where

$$I(c_1,c_2;m_1,m_2) = \frac{1}{\pi}\int_0^{\pi/2}\left(\frac{\sin^2\theta}{\sin^2\theta+c_1}\right)^{m_1}\left(\frac{\sin^2\theta}{\sin^2\theta+c_2}\right)^{m_2} d\theta$$
$$= \frac{(c_1/c_2)^{m_2-1}}{2(1-c_1/c_2)^{m_1+m_2-1}}\left[\sum_{k=0}^{m_2-1}\left(\frac{c_1}{c_2}-1\right)^k S_k I_k(c_2)\right.$$
$$\left. - \frac{c_1}{c_2}\sum_{k=0}^{m_1-1}\left(1-\frac{c_1}{c_2}\right)^k T_k I_k(c_1)\right]. \tag{4.17}$$

is the integration which has been evaluated in [61]

$$S_k \triangleq \frac{R_k}{\binom{m_1+m_2-1}{k}}, \tag{4.18a}$$

$$T_k \triangleq \sum_{n=0}^{m_2-1}\frac{\binom{k}{n}}{\binom{m_1+m_2-1}{n}}R_n, \tag{4.18b}$$

$$R_k \triangleq (-1)^{m_2-1+k} \frac{\binom{m_2-1}{k}}{(m_2-1)!} \prod_{n=1, n\neq k+1}^{m_2} (m_1+m_2-n), \tag{4.18c}$$

$$I_k(c) = 1 - \sqrt{\frac{c}{1+c}} \left[1 + \sum_{n=1}^{k} \frac{(2n-1)!!}{n! 2^n (1+c)^n} \right]. \tag{4.18d}$$

4.3.2 M-ary PSK

Using the tight approximation for the BER of the coherent M-ary PSK in AWGN channels in (2.17), the BER for M-PSK can be approximated as

$$P_M(\rho) \simeq \frac{2}{\max(\log_2 M, 2)} \sum_{i=1}^{\max(M/4,1)} \frac{1}{\pi} \int_0^{\pi/2} \Phi_{\gamma_b} \left(-\frac{\sin^2 \frac{(2i-1)\pi}{M}}{\sin^2 \theta} \right) d\theta. \tag{4.19}$$

Thus using the same integral simplification in previous section, the exact BER expression is given by

$$\begin{aligned} P_M(\rho) =& \frac{2N\binom{L_t}{N}}{\max(\log_2 M, 2)} \frac{1}{[(L_r-1)!]^N} \sum_{i=1}^{\max(M/4,1)} \sum_{r=0}^{(L_r-1)(N-1)} a_r^{(L_r, N-1)} \\ & \times \sum_{j=0}^{L_t-N} \binom{L_t-N}{j} (-1)^j \sum_{l=0}^{j(L_r-1)} a_l^{(L_r, j)} \frac{(l+r+L_r-1)!}{(N+j)^{l+r+L_r}} \\ & \times I\left(c\rho\delta_i^2, \frac{Nc\rho\delta_i^2}{(N+j)}; L_r(N-1)-r, l+r+L_r \right) \end{aligned} \tag{4.20}$$

where $\delta_i = \sin \frac{(2i-1)\pi}{M}$.

4.4 Asymptotic performance Analysis

In this section, we find asymptotic performance, diversity order and coding gain of the system.

Recall that if the MGF of $\Phi_{\gamma_b}(s)$ can be approximated by a single polynomial term for $s \to \infty$ as

$$|\Phi_{\gamma_b}(s)| = b|s|^{-d} + o(s^{-d}) \tag{4.21}$$

then d is the diversity order of the system at high SNRs and the coding gain can be derived from b [21]. Thus, to obtain the coding gain and diversity order of the system, the approximate MGF is needed as $s \to \infty$. Note that

$$e^{-z} = \sum_{k=0}^{+\infty} \frac{(-z)^k}{k!} \tag{4.22}$$

$$\sum_{k=0}^{L_r} \frac{(-z)^k}{k!} \cdot \sum_{i=0}^{L_r-1} \frac{z^i}{i!} = 1 - \frac{z^{L_r}}{L_r!} + o(z^{L_r}), \quad z \to 0. \tag{4.23}$$

Thus we can approximate $F_{\gamma_b}(x)$ in (4.8) as $x \to 0$

$$\begin{aligned} F_{\gamma_k}(x) =& 1 - e^{-\frac{x}{c\rho}} \sum_{k=0}^{L_r-1} \frac{1}{k!}\left(\frac{x}{c\rho}\right)^k \\ =& \frac{1}{L_r!}\left(\frac{x}{c\rho}\right)^{L_r} + o(x^{L_r}), \end{aligned} \tag{4.24}$$

and note that

$$\int_x^{\infty} e^{-st} f_{\gamma_k}(t)dt = \frac{1}{(s+\frac{1}{c\rho})^{L_r}} \frac{e^{-x(s+\frac{1}{c\rho})}}{(c\rho)^{L_r}} \sum_{k=0}^{L_r-1} \frac{x^k(s+\frac{1}{c\rho})^k}{k!}. \tag{4.25}$$

By substituting these into (4.9), the MGF can be obtained as

$$\begin{aligned} \Phi_{\gamma_b}(s) =& \frac{N\binom{L_t}{N}}{(c\rho)^{L_rN}(L_r-1)!(s+\frac{1}{c\rho})^{L_rN}} \int_0^{\infty} e^{-xN(s+\frac{1}{c\rho})} x^{L_r-1} \\ &\times \left[\frac{1}{L_r!}\left(\frac{x}{c\rho}\right)^{L_r} + o(x^{L_r})\right]^{L_t-N} \left[\sum_{k=0}^{L_r-1} \frac{x^k(s+\frac{1}{c\rho})^k}{k!}\right]^{N-1} dx. \end{aligned} \tag{4.26}$$

To show that the MGF is $o(s^{-G_d})$, we ignore the $o(x^{L_r})$ term in the second term of the integral,

$$\begin{aligned} \Phi_{\gamma_b}(s) \simeq& N\binom{L_t}{N} \frac{L_r}{(c\rho)^{L_rL_t}(L_r!)^{L_t-N+1}(s+\frac{1}{c\rho})^{L_rN}} \\ &\times \left\{ \sum_{j=0}^{(L_r-1)(N-1)} a_j \left(s+\frac{1}{c\rho}\right)^j \int_0^{\infty} e^{-xN(s+\frac{1}{c\rho})} x^{L_r(L_t-N+1)+j-1} dx \right\} \end{aligned} \tag{4.27}$$

in which a_j is the coefficient of x^j, $j = 0, 1, ..., (L_r - 1)(N - 1)$, in the expansion

$$\left[\sum_{k=0}^{L_r-1} \frac{x^k}{k!}\right]^{N-1}. \tag{4.28}$$

Since we have

$$\int_0^{\infty} e^{-ax} P_m(x) dx = \frac{m! p_m}{a^{m+1}} \tag{4.29}$$

where $P_m(x) = \sum_{i=0}^{m} p_i x^i$ is a polynomial of degree m with respect to x,

$$\begin{aligned}\Phi_{\gamma_b}(s) =& \frac{N\binom{L_t}{N} L_r}{(c\rho)^{L_r L_t} (L_r!)^{L_t-N+1} N^{L_r(L_t-N+1)}} \\ &\times \left[\sum_{j=0}^{(L_r-1)(N-1)} \frac{a_j \left[L_r(L_t - N + 1) + j - 1\right]!}{N^j}\right] \\ &\times \frac{1}{(s+\frac{1}{c\rho})^{L_r L_t}} + o(s^{-L_r L_t}), \quad \text{as } s \to \infty\end{aligned} \tag{4.30}$$

For notational brevity, we define

$$\Xi = \sum_{j=0}^{(L_r-1)(N-1)} \frac{a_j \left[L_r(L_t - N + 1) + j - 1\right]!}{N^j}. \tag{4.31}$$

We substitute the approximated MGF into (2.18) to approximate the BER as

$$\begin{aligned}P_M(\rho) \simeq& N\binom{L_t}{N} \frac{2}{M \log_2 M} \cdot \frac{L_r \Xi}{(L_r!)^{L_t-N+1} N^{L_r(L_t-N+1)}} \\ &\times \sum_{n=1}^{\log_2 M} \sum_{i=0}^{k_n} B_i I_{L_r L_t}\left(D_i^2 c\rho/2\right)\end{aligned} \tag{4.32}$$

where [61, 62]

$$\begin{aligned}I_m(\mu) =& \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left(\frac{\sin^2\theta}{c + \sin^2\theta}\right)^m d\theta \\ =& \frac{1}{2^m}\left(1 - \sqrt{\frac{\mu}{1+\mu}}\right)^m \sum_{k=0}^{m-1} \frac{\binom{m-1+k}{k}}{2^k} \left(1 + \sqrt{\frac{\mu}{1+\mu}}\right)^k.\end{aligned} \tag{4.33}$$

to find the approximation of MGF above, we need to know the limit

$$\lim_{\mu\to\infty} I_m(\mu) = \binom{2m}{m} \frac{1}{2^{2m+1}} \frac{1}{\mu^m}. \tag{4.34}$$

Thus by using this limit, (4.34), we can find

$$\lim_{\rho\to\infty} P_M(\rho) = \frac{2N\binom{L_t}{N}L_r}{M\log_2 M(L_r!)^{L_t-N+1}N^{L_r(L_t-N+1)}}\Xi \times\left(\sum_{n=1}^{\log_2 M}\sum_{i=0}^{k_n}\frac{B_i}{D_i^{2L_rL_t}}\right)\frac{\binom{2L_rL_t}{L_rL_t}}{2^{L_rL_t+1}(c\rho)^{L_rL_t}} \tag{4.35}$$

and compare it to (2.24) to obtain the coding gain and diversity order of system as

$$\begin{aligned} G_c =& \left\{N\binom{L_t}{N}\frac{2}{M\log_2 M}\cdot\frac{L_r\Xi}{(L_r!)^{L_t-N+1}N^{L_r(L_t-N+1)}}\right. \\ &\left.\times\left(\sum_{n=1}^{\log_2 M}\sum_{i=0}^{k_n}\frac{B_i}{D_i^{2L_rL_t}}\right)\frac{\binom{2L_rL_t}{L_rL_t}}{2^{L_rL_t+1}c^{L_rL_t}}\right\}^{-\frac{1}{G_d}} \\ G_d =& L_rL_t. \end{aligned} \tag{4.36}$$

Therefore, if N antennas corresponding to the largest received SNRs are selected, a full diversity order of L_rL_t is obtained.

4.5 Numerical Results

In this section, we consider the orthogonal design proposed in [5], [3], with the rate 3/4 and $3 \leq L_t \leq 6$ transmit antennas. The MIMO system which has been chosen for simulation and numerical results is shown in Fig. 4.3 In order to use this orthogonal space-time block code, we select $N = 3$ transmit antennas. Fig. 4.4 compares the exact expression (4.16), the approximation (4.30) and (4.32), and the Monte Carlo simulation results for the system with $L_r = 2$ receive antennas, all using 16-QAM. Note that (5.45) asymptotically approaches (4.30), which is a tight bound of (4.16) at high SNRs. Fig. 4.5 shows the exact BER for $(L_t; N, L_r)$ systems, where the OSTBC is transmitted over N selected antennas of L_t available transmit antennas. L_r is the number of receive antennas in the system. Again, 16-QAM is used to show the general forms of BER expressions.Therefore, if N antennas corresponding to

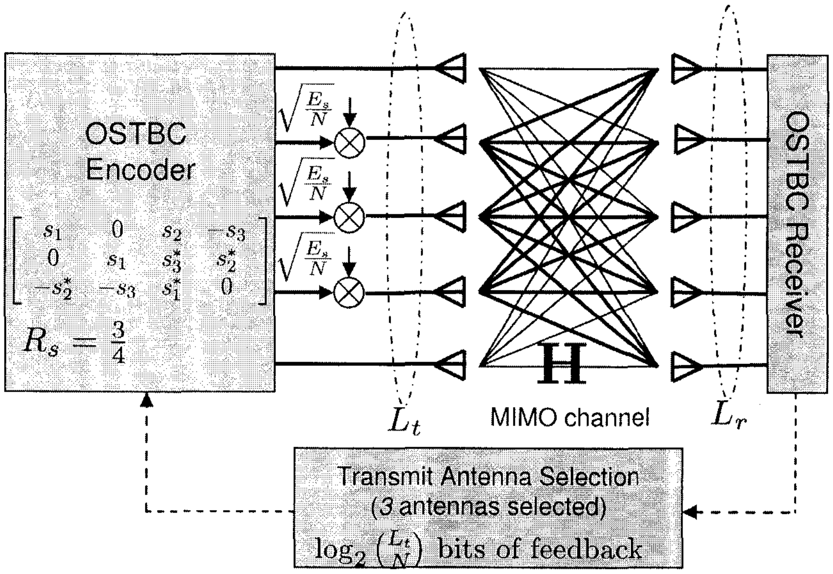

Figure 4.3: MIMO system using OSTBC of rate 3/4 and 3 transmit antenna selected

the largest received SNR, or γ_b, are selected, a full diversity order of $L_r L_t$ is obtained.

4.6 Conclusion

In this chapter, we have derived the performance of transmit antenna selection and OSTBCs. The exact BER expressions for M-PAM and M-QAM and an approximate BER for M-PSK were derived. Our results are sufficiently general to handle an arbitrary number of antennas, unlike the previous results. Moreover, we directly derived the BER, not via the symbol error probability. As expected, we find that this scheme achieves full diversity order asymptotically (i.e., L_t not N), as if all the transmit antennas were used.

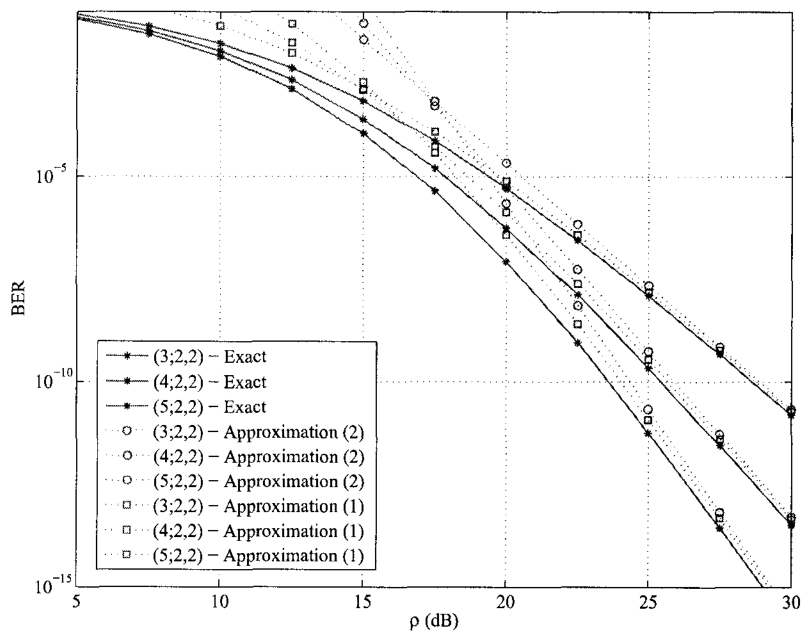

Figure 4.4: Comparison between exact BER for $(L_t, N; L_r)$ systems, selecting optimal N transmit antennas out of L_t with L_r antennas in receiver, using 16-QAM and approximations derived in (4.32) and (4.35) and diversity and coding gain approximated BER in (4.35).

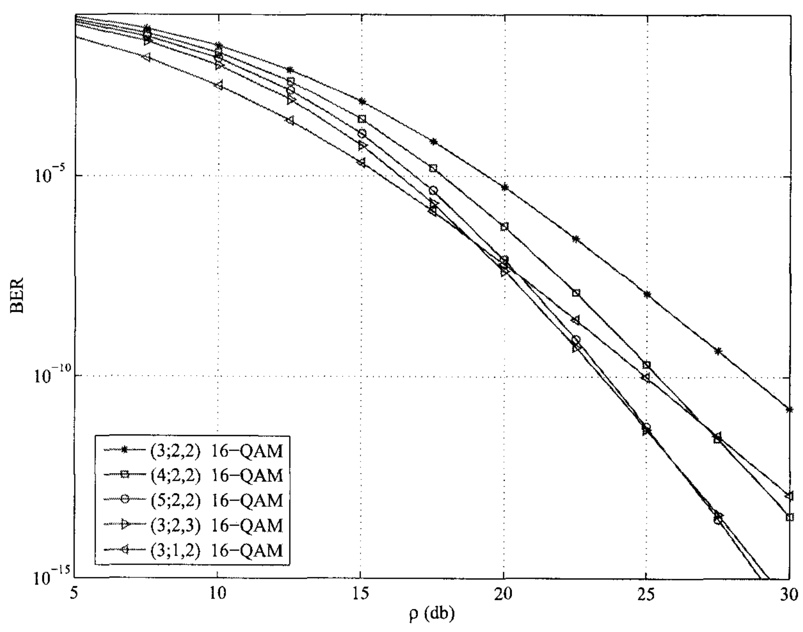

Figure 4.5: Exact BER derived in (5.45) for $(L_t; N, L_r)$ transmit antenna selection systems, using 16-QAM

Chapter 5

Performance of OSTBCs over Transmit Antenna Selection in Receive Correlated Channels

In this chapter, we analyze the performance of transmit antenna selection using OSTBCs in receive correlated Rayleigh fading channels. Using order statistics, we derive the exact expressions for the MGF of the received SNR in Section 5.2 and Exact BER expressions in Section 5.3. Section 5.3 also presents the asymptotic performance analysis. The diversity order and coding gain are obtained.

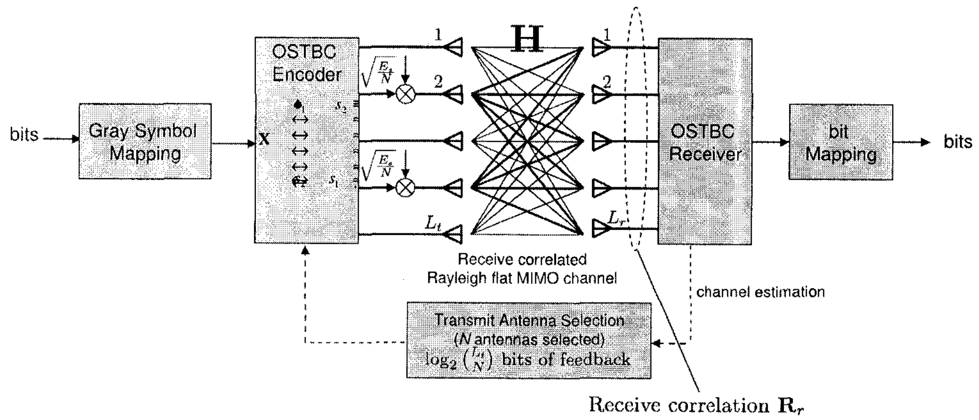

Figure 5.1: MIMO system using OSTBC over Transmit antenna selection in Rayleigh flat fading channel with receive correlation $\mathbf{R}_r$

5.1 System Model

Fig. 5.1 shows transmit antenna selection over receive correlated channels. The OSTBC encoder is used to encode symbols into space-time block matrix. The correlation between receive antennas can be represented by receive correlation matrix $\mathbf{R}_r$ of size $L_r \times L_r$. However, there is no correlation between the transmit antennas. Thus we can treat the received SNR from each transmit antenna independently. Using the Kronecker model (2.3), we find

$$\mathbf{H} = \mathbf{R}_r^{1/2}\mathbf{H}_w \tag{5.1}$$

where $\mathbf{R}_r = \mathcal{E}_{\mathbf{H}}\{\mathbf{h}_i\mathbf{h}_i^H\}$, $i = 1, 2, ..., L_t$ with the assumption that the receive correlation is symmetric between receive antennas. The total correlation matrix has a block structure given by

$$\begin{aligned}\mathbf{R} &= \mathbf{I}_{L_t} \otimes \mathbf{R}_r \\ &= \begin{bmatrix} \mathbf{R}_r & \mathbf{0}_{L_r\times L_r} & \cdots & \mathbf{0}_{L_r\times L_r} \\ \mathbf{0}_{L_r\times L_r} & \mathbf{R}_r & \vdots & \vdots \\ \vdots & \cdots & \mathbf{R}_r & \mathbf{0}_{L_r\times L_r} \\ \mathbf{0}_{L_r\times L_r} & \cdots & \mathbf{0}_{L_r\times L_r} & \mathbf{R}_r \end{bmatrix}_{L_tL_r\times L_tL_r} .\end{aligned} \tag{5.2}$$

Since, there is no transmit correlation, thus the antenna selection criteria still selects the transmit antennas with highest norms of the corresponding columns in the channel matrix $\mathbf{H}$.

5.2 The received SNR

Recall that $\gamma_k = c\rho\|\mathbf{h}_k\|^2$, $k = 1, 2, ..., L_t$, are the scaled norms of the columns of $\mathbf{H}$, which are the received SNR from the kth transmit antenna. Note that there is no correlation between transmit antennas and therefore the received SNR from each transmit antennas, γ_k, are an independent sequence of random variables. Since, the entries of each vector $\mathbf{h}_i$ are correlated with the correlation matrix $\mathbf{R}_r$, then the MGF of γ_k denoted by $\psi_{\gamma_k}(s)$ is given by [6]

$$\psi_{\gamma_k}(s) = \mathcal{E}_{\mathbf{H}}\left\{e^{-s\gamma_k}\right\} = \prod_{i=1}^{m} \frac{1}{1 + sc\rho\lambda_i} \tag{5.3}$$

where $\lambda_1, ..., \lambda_m$ are the non-zero eigenvalues of $\mathbf{R}_r$. Without loss of generality, we assume that $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_m > 0$ and there exist L distinct values ζ_i with multiplicity α_i for $i = 1, 2, ..., L$ in λ_i's. where $\alpha_1 + \alpha_2 + \cdots + \alpha_L = m$. Therefore, the MGF of γ_k can be rewritten as

$$\psi_{\gamma_k}(s) = \frac{1}{A(\rho)} \prod_{q=1}^{L} \left(s + \frac{1}{c\rho\zeta_q}\right)^{-\alpha_q} \tag{5.4}$$

$A(\rho) = \prod_{q=1}^{L}(c\rho\zeta_q)^{\alpha_q}$. Using partial fractions, the MGF can be simplified as

$$\psi_{\gamma_k}(s) = \sum_{q=1}^{L} \sum_{l=1}^{\alpha_q} \beta_{q,l} \left(s + \frac{1}{\sigma_q}\right)^{-l} \tag{5.5}$$

where $\sigma_q = c\rho\zeta_q$ and

$$\beta_{q,l} = \frac{1}{A(\rho)} \cdot \frac{1}{(\alpha_q - 1)!} \left\{ \frac{\partial^{(\alpha_q - 1)}}{\partial s^{\alpha_q - 1}} \left[\prod_{j=1, j \neq q}^{L} \left(s + \frac{1}{\sigma_j}\right)^{-\alpha_j} \right] \right\} \Bigg|_{s = -\frac{1}{\sigma_q}} . \tag{5.6}$$

Therefore, pdf of γ_k can be obtained by the inverse Laplace transform of (5.5) [63]. We must find the pdf as

$$\begin{aligned} f_{\gamma_k}(x) = \mathcal{L}_x^{-1}\left[\psi_{\gamma_k}(s)\right] &= \sum_{q=1}^{L} \sum_{l=1}^{\alpha_q} \beta_{q,l} \frac{x^{l-1}}{(l-1)!} e^{-\frac{x}{\sigma_q}} \\ &= \sum_{q=1}^{L} e^{-\frac{x}{\sigma_q}} P_q(x) \end{aligned} \tag{5.7}$$

where $P_q(x) = \sum_{l=1}^{\alpha_q} \frac{\beta_{q,l}}{(l-1)!} x^{l-1}$ is a polynomial in x of degree $\alpha_q - 1$ and $P_q^{(k)}(x)$ is the k-th derivative of $P_q(x)$ with respect to x. Therefore, the cdf of γ_k would be

$$F_{\gamma_k}(x) = \sum_{q=1}^{L}\sum_{l=1}^{\alpha_q}\beta_{q,l}\sigma_q^l\left[1 - e^{-x/\sigma_q}\sum_{k=0}^{l-1}\frac{1}{k!}\left(\frac{x}{\sigma_q}\right)^k\right]$$
$$= 1 - \sum_{q=1}^{L} e^{-x/\sigma_q}\sum_{k=0}^{\alpha_q-1} P_q^{(k)}(x)\sigma_q^{k+1}$$
$$= 1 - \sum_{q=1}^{L} e^{-x/\sigma_q} Q_q(x) \tag{5.8}$$

where $Q_q(x) = \sum_{k=0}^{\alpha_q-1} P_q^{(k)}(x)\sigma^{k+1}$ is a polynomial in x of degree $\alpha_q - 1$. If there are no multiple eigenvalues, (i.e. $\alpha_q = 1$ for $q = 1, 2, ..., L$), the MGF of γ_k can be rewritten as

$$\psi_{\gamma_k}(s) = \frac{1}{A(\rho)}\prod_{q=1}^{L}\left(s + \frac{1}{\sigma_q}\right)^{-1} = \sum_{q=1}^{L}\beta_q\left(s + \frac{1}{\sigma_q}\right)^{-1} \tag{5.9}$$

where $A(\rho) = \prod_{q=1}^{L}\sigma_q$ and

$$\beta_q = \frac{1}{A(\rho)}\prod_{j=1, j\neq q}^{L}\left(\frac{1}{\sigma_j} - \frac{1}{\sigma_q}\right)^{-1}. \tag{5.10}$$

Therefore, pdf of γ_k can be obtained as

$$f_{\gamma_k}(x) = \sum_{q=1}^{L}\beta_q e^{-\frac{x}{\sigma_q}}, \tag{5.11}$$

and the cdf of that can be found as

$$F_{\gamma_k}(x) = 1 - \sum_{q=1}^{L}\beta_q\sigma_q e^{-\frac{x}{\sigma_q}}. \tag{5.12}$$

For the general case, to obtain the MGF, using (4.9), we find that

$$\Phi_{\gamma_b}(s) = (L_t - N)\binom{L_t}{N}\int_0^\infty \sum_{q=1}^{L} e^{-x/\sigma_q}P_q(x)\left(1 - \sum_{q=1}^{L} e^{-x/\sigma_q}Q_q(x)\right)^{L_t-N-1}$$
$$\times\left(\sum_{q=1}^{L} e^{-x(s+\frac{1}{\sigma_q})}\sum_{k=0}^{\alpha_q-1}\frac{P_q^{(k)}(x)}{(s+\frac{1}{\sigma_q})^{k+1}}\right)^N dx \tag{5.13}$$

Note that

$$\int P_m(x)e^{-sx}dx = -\frac{e^{-sx}}{s}\sum_{k=0}^{m}\frac{P^{(k)}(x)}{s^k} \tag{5.14}$$

where $P_m(x)$ is a polynomial in x of degree m and $P^{(k)}(x)$ is the k-th derivative of $P_m(x)$ with respect to x. Thus, we find that

$$\begin{aligned}\Phi_{\gamma_b}(s) =&(L_t - N)\binom{L_t}{N}\int_0^\infty \sum_{q=1}^{L} e^{-x/\sigma_q}P_q(x)\left(1-\sum_{q=1}^{L}e^{-x/\sigma_q}Q_q(x)\right)^{L_t-N-1}\\ &\times\left(\sum_{q=1}^{L}e^{-x(s+\frac{1}{\sigma_q})}\frac{R_q(x;s)}{(s+\frac{1}{\sigma_q})^{\alpha_q}}\right)^N dx\end{aligned} \tag{5.15}$$

where $R_q(x;s) = \sum_{k=0}^{\alpha_q-1}P_q^{(k)}(x)(s+\frac{1}{\sigma_q})^{\alpha_q-k-1}$. By expanding (5.15), we find that

$$\begin{aligned}\Phi_{\gamma_b}(s) =&(L_t - N)\binom{L_t}{N}\sum_{j=1}^{L_t-N-1}(-1)^j\binom{L_t-N-1}{j}\int_0^\infty e^{-Nsx}\sum_{q=1}^{L}e^{-x/\sigma_q}P_q(x)\\ &\times\left(\sum_{\mathbf{i}}\binom{j}{i_1,\cdots,i_L}e^{-x(\frac{i_1}{\sigma_1}+\cdots+\frac{i_L}{\sigma_L})}S_{\mathbf{i}}(x)\right)\\ &\times\left(\sum_{\mathbf{n}}\binom{N}{n_1,\cdots,n_L}e^{-x(\frac{n_1}{\sigma_1}+\cdots+\frac{n_L}{\sigma_L})}\prod_{p=1}^{L}\frac{[R_q(x;s)]^{n_p}}{(s+\frac{1}{\sigma_p})^{n_p\alpha_q}}\right)dx\end{aligned} \tag{5.16}$$

where the sums over $\mathbf{i} = (i_1, i_2, \cdots, i_L)$ and $\mathbf{n} = (n_1, n_2, \cdots, n_L)$ are over all combinations of nonnegative integers such that $\sum_{k=1}^{L} i_k = j$ and $\sum_{k=1}^{L} n_k = N$. We also define

$$S_{\mathbf{i}}(x) = Q_1^{i_1}(x)\cdots Q_L^{i_L}(x), \quad \mathbf{i} = (i_1, ..., i_L) \quad \sum_{k=1}^{L} i_k = j \tag{5.17}$$

Therefore,

$$\Phi_{\gamma_b}(s) = (L_t - N)\binom{L_t}{N}\sum_{j=1}^{L_t-N-1}(-1)^j\binom{L_t-N-1}{j}\sum_{q=1}^{L}\sum_{\mathbf{i}}\sum_{\mathbf{n}}$$
$$\times\left\{\binom{j}{i_1,\cdots,i_L}\binom{N}{n_1,\cdots,n_L}\frac{P_q(0)S_{\mathbf{i}}(0)}{\frac{1}{\sigma_q}+\sum_{k=1}^{L}\frac{i_k}{\sigma_k}+\sum_{k=1}^{L}\frac{n_k}{\sigma_k}+Ns}\right.$$
$$\left.\times\prod_{p=1}^{L}\frac{[R_q(0;s)]^{n_p}}{(s+\frac{1}{\sigma_p})^{n_p\alpha_q}}\right\} \tag{5.18}$$

where $P_q(0) = \beta_{q,1}$, and $S_{\mathbf{i}}(0) = Q_1^{i_1}(0)\cdots Q_L^{i_L}(0)$ where $Q_q(0) = \sum_{k=1}^{\alpha_q}\beta_{q,k}\sigma_q^k = T_q(\sigma_q)$ therefore, $S_{\mathbf{i}}(0) = \prod_{q=1}^{L}T_q^{i_q}(\sigma_q)$, and $R_q(0;s) = \sum_{k=1}^{\alpha_q}\beta_{q,k}(s+\frac{1}{\sigma_q})^{\alpha_q-k}$. Therefore, the MGF has different poles at

$$\sigma_{q,\mathbf{i},\mathbf{n}} = \frac{N}{\left(\frac{1}{\sigma_q}+\sum_{k=1}^{L}\frac{i_k}{\sigma_k}+\sum_{k=1}^{L}\frac{n_k}{\sigma_k}\right)} \quad \text{of order } 1 \tag{5.19a}$$

$$\sigma_q \quad \text{of order } N\alpha_q \tag{5.19b}$$

where $i_1, i_2, \cdots, i_L$ and $n_1, n_2, \cdots, n_L$ are combinations of integers such that $\sum_{k=1}^{L} i_k = j$ and $\sum_{k=1}^{L} n_k = N$. Thus, MGF can be simplified by using the partial expansion as

$$\Phi_{\gamma_b}(s) = \sum_{q=1}^{L}\sum_{j=1}^{L_t-N-1}\sum_{\mathbf{i}}\sum_{\mathbf{n}}\frac{a(q;\mathbf{i};\mathbf{n})}{s+\frac{1}{\sigma_{q,\mathbf{i},\mathbf{n}}}}+\sum_{q=1}^{L}\sum_{l=1}^{N\alpha_q}\frac{b_{q,l}}{(s+\frac{1}{\sigma_q})^l}. \tag{5.20}$$

Note that the coefficients of $b_{q,l}$ and $a(q;\mathbf{i};\mathbf{n})$ can be derived directly from the integral form of MGF (4.9).

5.2.1 Constant Correlation Model

In this case, the receive correlation matrix is

$$\mathbf{R}_r = \begin{bmatrix} 1 & r & \cdots & r \\ r & 1 & \cdots & r \\ \vdots & \vdots & \ddots & \vdots \\ r & r & \cdots & 1 \end{bmatrix}. \tag{5.21}$$

Therefore, $\mathbf{R}_r$ has one eigenvalue of order one equal to $\lambda_1 = r(L_r - 1) + 1$ and one equal to $\lambda_2 = 1 - r$ of order $L_r - 1$. Thus, $\sigma_1 = c\rho\lambda_1$ and $\sigma_2 = c\rho\lambda_2$. Therefore, the MGF of γ_k would be like

$$\begin{aligned}\psi_{\gamma_k}(s) =& \frac{1}{1+s\sigma_1}\left(\frac{1}{1+s\sigma_2}\right)^{L_r-1} \\ =& \frac{\beta_{1,1}}{s+\frac{1}{\sigma_1}} + \frac{\beta_{2,1}}{s+\frac{1}{\sigma_2}} + \cdots + \frac{\beta_{2,L_r-1}}{\left(s+\frac{1}{\sigma_2}\right)^{L_r-1}}.\end{aligned} \tag{5.22}$$

Thus the pdf of the received SNR of the kth transmit antenna, γ_k is obtained by inverse Laplace transformation of the MGF of γ_k (5.22)

$$f_{\gamma_k}(x) = \beta_{1,1}e^{-\frac{x}{\sigma_1}} + \sum_{l=1}^{L_r-1} \beta_{2,l}\frac{x^{l-1}}{(l-1)!}e^{-\frac{x}{\sigma_2}}. \tag{5.23}$$

By integrating from the pdf, we find the cdf as

$$F_{\gamma_k}(x) = \beta_{1,1}\sigma_1(1-e^{-\frac{x}{\sigma_1}}) + \sum_{l=1}^{L_r-1} \beta_{2,l}\sigma_2^l\left(1 - e^{-\frac{x}{\sigma_2}}\sum_{k=0}^{l-1}\frac{1}{k!}\left(\frac{x}{\sigma_2}\right)^k\right) \tag{5.24}$$

where

$$\beta_{1,1} = \beta = \frac{(-1)^{L_r-1}}{\sigma_1\sigma_2^{L_r-1}}\left(\frac{1}{\sigma_1}-\frac{1}{\sigma_2}\right)^{1-L_r}$$

$$\beta_{2,l} = (-1)^l\beta_{1,1}\left(\frac{1}{\sigma_1}-\frac{1}{\sigma_2}\right)^{l-1} = \frac{(-1)^{L_r-l-1}}{\sigma_1\sigma_2^{L_r-1}}\left(\frac{1}{\sigma_1}-\frac{1}{\sigma_2}\right)^{l-L_r}$$

by letting $\mu = \frac{1}{\sigma_1} - \frac{1}{\sigma_2}$ the pdf of γ_k can be simplified as,

$$f_{\gamma_k}(x) = \beta_{1,1}\left\{e^{-\frac{x}{\sigma_1}} - \sum_{l=1}^{L_r-1}\frac{(-\mu x)^{l-1}}{(l-1)!}e^{-\frac{x}{\sigma_2}}\right\} \tag{5.25}$$

thus, the MGF of $\gamma_b = \sum_{k=1}^{N}\gamma_{(k)}$ which is the sum of N largest *independent* order statistics can be obtained from (4.9) and (4.10), [12]

$$\begin{aligned}\Phi_{\gamma_b}(s) =& (L_t - N)\binom{L_t}{N}\sum_{j=1}^{L_t-N-1}(-1)^j\binom{L_t-N-1}{j}\sum_{q=1}^{2}\sum_{i=0}^{j}\sum_{n=0}^{N}\binom{j}{i}\binom{N}{n} \\ &\times \frac{\beta^{1+i+n}\sigma_1^i(1-\beta\sigma_1)^{j-i}}{\frac{1}{\sigma_q}+\frac{i+N-n}{\sigma_1}+\frac{n+j-i}{\sigma_2}+Ns}\frac{1}{\left(s+\frac{1}{\sigma_1}\right)^n}\left[\frac{1}{\sigma_1}\left(\frac{\sigma_1^n}{1+s\sigma_2}\right)^{L_r-1}-\beta\right]^n\end{aligned} \tag{5.26}$$

Note that for deriving the formula above for the MGF we have used the fact that

$$\frac{1}{1+s\sigma_1}\left(\frac{1}{1+s\sigma_2}\right)^{L_r-1} = \frac{\beta}{s+\frac{1}{\sigma_1}} + \sum_{k=1}^{L_r-1}\frac{\beta_{2,k}}{(s+\frac{1}{\sigma_2})^k} \tag{5.27}$$

thus we have

$$\sum_{k=1}^{L_r-1}\frac{\beta_{2,k}}{(s+\frac{1}{\sigma_2})^k} = \frac{1}{s+\frac{1}{\sigma_1}}\left\{\frac{1}{\sigma_1}\left(\frac{1}{1+s\sigma_2}\right)^{L_r-1} - \beta_{1,1}\right\} \tag{5.28}$$

and we can substitute it in (4.9). Setting $s = 0$, we find that

$$\sum_{k=1}^{L_r-1}\beta_{2,k}\sigma_2^k = \sum_{k=1}^{L_r-1}\frac{\beta_{2,k}}{(s+\frac{1}{\sigma_2})^k}\Bigg|_{s=0} = 1-\beta_{1,1}\sigma_1 \tag{5.29}$$

using these simplifications, the final MGF for the constant receive correlation case can be derived easily.

5.2.2 Tridiagonal Correlation Model

We assume that the correlation matrix $\mathbf{R}_r$ is a tridiagonal matrix of the form

$$\mathbf{R}_r = \begin{bmatrix} 1 & r & 0 & & \cdots & 0 \\ r & 1 & r & & & \\ 0 & r & 1 & \ddots & & \vdots \\ \vdots & & \ddots & 1 & r & 0 \\ & & & r & 1 & r \\ 0 & \cdots & & 0 & r & 1 \end{bmatrix} \tag{5.30}$$

where r is a correlation parameter. From (5.3), the MGF of γ_k depends on the eigenvalues of the correlation matrix $\mathbf{R}_r$ for the receive correlated channel. The eigenvalues of a matrix can be found through the characteristic function of the matrix as $\Delta_{L_r}(\lambda) = \det(\mathbf{R}_r - \lambda\mathbf{I}) = 0$. The solutions of this equation for λ are eigenvalues of $\mathbf{R}_r$. We know from the results in [64] that for a tridiagonal matrix of the above form with size $L_r \times L_r$ the characteristic function is

$$\Delta_{L_r}(\lambda) = r^{L_r-1}(1-\lambda)\frac{\sin(l_r+1)\theta}{\sin\theta} \tag{5.31}$$

where $L_r = 2l_r + 1$ is odd and

$$\Delta_{L_r}(\lambda) = r^{L_r} \frac{\sin(l_r + 1)\theta + \sin l_r\theta}{\sin\theta} \tag{5.32}$$

where $L_r = 2l_r$ is even and θ is defined implicitly from λ and the correlation parameter r as

$$(1-\lambda)^2 = 2r^2 + 2r^2\cos\theta = 4r^2\cos^2\frac{\theta}{2} \tag{5.33}$$

which means that

$$\lambda = 1 \pm 2r\cos\frac{\theta}{2} \tag{5.34}$$

Thus, the eigenvalues for an odd L_r are $\lambda_1 = 1$ and the others can be derived from

$$\sin(l_r + 1)\theta = 0 \tag{5.35}$$

where $\sin\theta \neq 0$. Thus

$$\theta = \frac{k\pi}{l_r + 1}, \quad k = 1, 2, ..., l_r \tag{5.36}$$

Therefore, using the definition of θ in (5.33), The rest of eigenvalues of the correlation matrix with odd size are

$$\lambda_{2i}, \lambda_{2i+1} = 1 \pm 2r\cos\frac{i\pi}{2(l_r + 1)}, \quad i = 1, 2, ..., l_r. \tag{5.37}$$

When $L_r = 2l_r$ where l_r is an integer, then

$$\sin(l_r + 1)\theta + \sin l_r\theta = 0, \quad \sin\theta \neq 0 \tag{5.38}$$

where gives us the result that

$$(l_r + 1)\theta = (2k-1)\pi - l_r\theta, \quad k = 1, 2, ..., l_r \tag{5.39}$$

$$\theta = \frac{(2k-1)\pi}{2l_r + 1}, \quad k = 1, 2, ..., l_r \tag{5.40}$$

thus the eigenvalues are

$$\lambda_{2i-1}, \lambda_{2i} = 1 \pm 2r\cos\frac{(2i-1)\pi}{2l_r + 1}, \quad i = 1, 2, ..., l_r \tag{5.41}$$

For the tridiagonal correlation, the eigenvalues are distinct and therefore (5.11) and (5.12) can be used to derive the MGF and the BER expressions.

5.2.3 Exponential Correlation Model

The components of $\mathbf{R}_r$ in the exponential correlation model is given by

$$r_{ij} = r^{|i-j|}, \quad |r| \leq 1 \tag{5.42}$$

where r is the (complex) correlation coefficient of neighboring receive antennas. This is a simple single-parameter model which allows one to study the effects of receive correlation on transmit antenna scheme.

The eigenvalues $\lambda_i, i = 1, 2, ..., L_r$ of the $\mathbf{R}_r$ can be calculated as follows [65]:

$$\lambda_i = \frac{1 - r^2}{1 - 2r\cos\theta_i + r^2}, \quad i = 1, 2, ..., L_r \tag{5.43}$$

where θ_i for $i = 1, 2, ..., L_r$ are solutions of the equations:

$$\begin{aligned} \sin\frac{(L_r+1)\theta}{2} &= r\sin\frac{(L_r-1)\theta}{2}, \\ \cos\frac{(L_r+1)\theta}{2} &= r\cos\frac{(L_r-1)\theta}{2}. \end{aligned} \tag{5.44}$$

The eigenvalues of the exponential receive correlation matrix are distinct and therefore the MGF and BER expressions can be obtained from (5.11) and (5.12).

5.3 BER expressions

In this section, we are finding the BER expressions for transmit antenna selection scheme using OSTBCs over receive correlated Rayleigh fading channels.

5.3.1 M-ary PAM and QAM

Using the closed-form expressions in (5.20) and (2.18), the BER can be obtained as

$$\begin{aligned} P_M(\rho) = \frac{2}{M\log_2 M} \sum_{n=1}^{\log_2 M} \sum_{i=0}^{k_n} B_i \Bigg\{ \sum_{q=1}^{L} \sum_{l=1}^{N\alpha_q} b_{q,l} I_l \left(\frac{D_i^2 \sigma_q}{2} \right) \\ + \sum_{q=1}^{L} \sum_{j=1}^{L_t-N-1} \sum_{\mathbf{i}} \sum_{\mathbf{n}} a(q; \mathbf{i}; \mathbf{n}) I_1 \left(\frac{D_i^2 \sigma_{q,\mathbf{i},\mathbf{n}}}{2} \right) \Bigg\}, \end{aligned} \tag{5.45}$$

where $I_m(\mu)$ has been defined in (4.33).

5.3.2 M-PSK

The tight approximation of M-PSK BER in (2.17) and employing (5.20), (2.18) yield the approximate BER expressions for M-PSK as follows:

$$P_M(\rho) \simeq \frac{2}{\max(\log_2 M, 2)} \sum_{i=1}^{\max(M/4,1)} \left\{ \sum_{q=1}^{L} \sum_{l=1}^{N\alpha_q} b_{q,l} I_l \left(\sigma_q \sin^2 \frac{(2i-1)\pi}{M} \right) \right.$$
$$\left. + \sum_{q=1}^{L} \sum_{j=1}^{L_t-N-1} \sum_{\mathbf{i}} \sum_{\mathbf{n}} a(q;\mathbf{i};\mathbf{n}) I_1 \left(\sigma_{q,\mathbf{i},\mathbf{n}} \sin^2 \frac{(2i-1)\pi}{M} \right) \right\}. \quad (5.46)$$

5.4 Asymptotic Performance Analysis

Using the inverse Laplace transformation of the MGF of γ_k, we obtain another representation for pdf and cdf of γ_k as

$$f_{\gamma_k}(x) = \mathcal{L}_x^{-1}[\psi_{\gamma_k}(s)]$$
$$F_{\gamma_k}(x) = \int_0^x f_{\gamma_k}(t)dt = \mathcal{L}_x^{-1}\left[\frac{\psi_{\gamma_k}(s)}{s}\right]. \quad (5.47)$$

Thus, by inserting inverse Laplace transformation representation of pdf and cdfs into the MGF expressions (4.9) of γ_b, the achievable received SNR of transmit antenna selection system, we obtain

$$\Phi_{\gamma_b}(s) = N\binom{L_t}{N} \int_0^\infty e^{-sx} \mathcal{L}_x^{-1}[\psi_{\gamma_k}(s)]$$
$$\times \left\{ \mathcal{L}_x^{-1}\left[\frac{\psi_{\gamma_k}(s)}{s}\right] \right\}^{L_t-N} \left[\int_x^\infty e^{-st} f_{\gamma_k}(t)dt \right]^{N-1} dx. \quad (5.48)$$

Note that if $f(x)$ can be approximated as a single polynomial [21]

$$f(x) = \alpha x^\beta + o(x^\beta), \quad x \to 0 \quad (5.49)$$

then the Laplace transformation of $f(x)$ also can be approximated as a single polynomial like

$$\psi(s) = \mathcal{L}[f(x)] = \frac{\alpha\beta!}{s^{\beta+1}} + o(s^{-\beta-1}), \quad s \to \infty. \quad (5.50)$$

We use the facts as follows in the approximation process of the MGF and further BER performance of the system.

$$\int_0^\infty e^{-sx} P(x) dx = \frac{q! p_q}{s^{q+1}}, \tag{5.51}$$

$$\int_x^\infty e^{-st} t^Q dt = \frac{e^{-sx}}{s^Q} \sum_{i=0}^{Q} \frac{(sx)^i}{i!}, \tag{5.52}$$

$$\int_0^\infty e^{-sx} x^Q \left[\int_x^\infty e^{-st} P(t) dt \right]^{N-1} dx = \frac{A}{s^B} + o(s^{-B}) \tag{5.53}$$

where $P(x) = p_q x^q + o(x^q)$ as $x \to 0$, is a function of x and $B = Q + 1 + (q + 1)(N - 1)$. Therefore, $o(x^q)$ can be ignored from $P(x)$ in order to approximate the MGF. From (5.3), and the inverse Laplace transformation of the MGF, we have

$$f_{\gamma_k}(x) = \mathcal{L}^{-1} \left[\prod_{i=1}^{m} \frac{1}{1 + sc\rho\lambda_i} \right] = \frac{1}{(c\rho)^m \prod_{i=1}^{m} \lambda_i} \cdot \frac{x^{m-1}}{(m-1)!} + o(x^{m-1}), \tag{5.54}$$

where m is the rank of the receive correlation matrix $\mathbf{R}_r$. Using the fact that $F_{\gamma_k}(x) = \mathcal{L}_x^{-1} \left[\frac{1}{s} \psi_{\gamma_k}(s) \right]$, we also have

$$\begin{aligned} F_{\gamma_k}(x) =& \mathcal{L}^{-1} \left[\frac{1}{s} \cdot \prod_{i=1}^{m} \frac{1}{1 + sc\rho\lambda_i} \right] \\ =& \frac{1}{(c\rho)^{m+1} \prod_{i=1}^{m} \lambda_i} \cdot \frac{x^m}{m!} + o(x^m), \quad x \to \infty. \end{aligned} \tag{5.55}$$

Therefore, using the fact in (5.51), the MGF as $s \to \infty$ is simplified as

$$\begin{aligned} \Phi_{\gamma_b}(s) \simeq& N \binom{L_t}{N} \int_0^\infty e^{-sx} \frac{1}{(c\rho)^m} \cdot \frac{1}{\det \mathbf{R}_r} \frac{x^{m-1}}{(m-1)!} \left\{ \frac{1}{(c\rho)^m} \cdot \frac{1}{\det \mathbf{R}_r} \cdot \frac{x^m}{m!} \right\}^{L_t - N} \\ & \times \left\{ \int_x^\infty e^{-st} \left[\frac{1}{(c\rho)^m \det \mathbf{R}_r} \cdot \frac{t^{m-1}}{(m-1)!} + o(t^{m-1}) \right] dt \right\}^{N-1} dx \end{aligned} \tag{5.56}$$

We ignore the $o(t^{m-1})$ term and use (5.51), thus

$$\begin{aligned} \Phi_{\gamma_b}(s) =& N \binom{L_t}{N} \frac{m^N}{(c\rho)^{L_t m} [\det \mathbf{R}_r]^{L_t} (m!)^{L_t}} \\ & \times \int_0^\infty e^{-Nsx} x^{r(L_t - N + 1) - 1} \frac{1}{s^{m(N-1)}} \left[\sum_{k=0}^{m-1} \frac{(sx)^k}{k!} \right]^{N-1} dx, \end{aligned} \tag{5.57}$$

in which a_j is the coefficient of x^j, $j = 0, 1, ..., (m-1)(N-1)$, in the expansion

$$\left[\sum_{k=0}^{m-1} \frac{(sx)^k}{k!}\right]^{N-1}. \tag{5.58}$$

Thus, inserting the above definition into the MGF expressions, we get

$$\begin{aligned}\Phi_{\gamma_b}(s) =& N\binom{L_t}{N} \frac{m^N}{(c\rho)^{L_t m}[\det \mathbf{R}_r]^{L_t}(m!)^{L_t}} \frac{1}{s^{m(N-1)}} \\ &\times \sum_{j=0}^{(m-1)(N-1)} \int_0^{\infty} e^{-Nsx} a_j s^j x^{m(L_t-N+1)+j-1} dx,\end{aligned} \tag{5.59}$$

and therefore MGF can be approximated as

$$\begin{aligned}\Phi_{\gamma_b}(s) =& N\binom{L_t}{N} \frac{m^N}{(c\rho)^{L_t m}[\det \mathbf{R}_r]^{L_t}(m!)^{L_t} N^{m(L_t-N+1)}} \frac{1}{s^{L_t m}} \\ &\times \sum_{j=0}^{(m-1)(N-1)} \frac{a_j \left[m(L_t - N + 1) + j - 1\right]!}{N^j}\end{aligned} \tag{5.60}$$

For the simplicity of the expression, we define

$$\Lambda = \sum_{j=0}^{(m-1)(N-1)} \frac{a_j \left[m(L_t - N + 1) + j - 1\right]!}{N^j} \tag{5.61}$$

which is a parameter depends on m, N, L_t. The comparison between (5.60) and (4.21) reveals that

$$\begin{aligned}b =& \frac{N\binom{L_t}{N} m^N \Lambda (c\rho)^{-L_t m}}{[\det \mathbf{R}_r]^{L_t}(m!)^{L_t} N^{m(L_t-N+1)}} \\ d =& mL_t\end{aligned} \tag{5.62}$$

If we substitute the approximated MGF into (2.18) then

$$P_M(\rho) \simeq \frac{2N\binom{L_t}{N}}{M \log_2 M} \cdot \frac{m^N \Lambda (c\rho)^{-L_t m}}{[\det \mathbf{R}_r]^{L_t}(m!)^{L_t} N^{m(L_t-N+1)}} \sum_{n=1}^{\log_2 M} \sum_{i=0}^{k_n} B_i I_{mL_t}\left(D_i^2 c\rho/2\right) \tag{5.63}$$

where $I_m(\mu)$ has been defined in (4.33). Note that

$$\lim_{\mu \to \infty} I_p(\mu) = \binom{2p}{p} \frac{1}{2^{2p+1}} \frac{1}{\mu^p} \tag{5.64}$$

Thus

$$\lim_{\rho\to\infty} P_M(\rho) = \frac{2N\binom{L_t}{N}m^N}{M\log_2 M}\cdot\frac{\Lambda}{[\det \mathbf{R}_r]^{L_t}(m!)^{L_t}N^{m(L_t-N+1)}} \times \frac{\binom{2mL_t}{mL_t}}{2^{2mL_t+1}}\left(\sum_{n=1}^{\log_2 M}\sum_{i=0}^{k_n}\frac{B_i}{D_i^{2mL_t}}\right)\frac{1}{(c\rho)^{mL_t}} \tag{5.65}$$

shows that

$$\begin{aligned} G_c =& \left\{\frac{2N\binom{L_t}{N}m^N}{M\log_2 M}\cdot\frac{\Lambda}{[\det \mathbf{R}_r]^{L_t}(m!)^{L_t}N^{m(L_t-N+1)}}\right. \\ & \left.\times\frac{\binom{2mL_t}{mL_t}}{2^{2mL_t+1}c^{mL_t}}\left(\sum_{n=1}^{\log_2 M}\sum_{i=0}^{k_n}\frac{B_i}{D_i^{2mL_t}}\right)\right\}^{-\frac{1}{G_d}}, \\ G_d =& mL_t. \end{aligned} \tag{5.66}$$

are the coding gain and diversity order. Therefore, if N antennas corresponding to the largest received SNR, or γ_b, are selected, and the receive correlation matrix is full rank then full diversity order of L_rL_t is obtained.

5.5 Conclusion

This chapter analyzed the performance of OSTBCs with transmit antenna selection in receive correlated Rayleigh fading channels. The exact BER for M-PAM and M-QAM and an approximate BER for M-PSK were derived. Our results are sufficiently general to handle an arbitrary number of antennas and specially for correlated fading channels. We also derived the BER for the constant receiver correlation. The asymptotic performance of the system has been derived. The diversity order and coding gain of system is found. Thus, full diversity is achieved through transmit antenna selection in receive correlated fading channels.

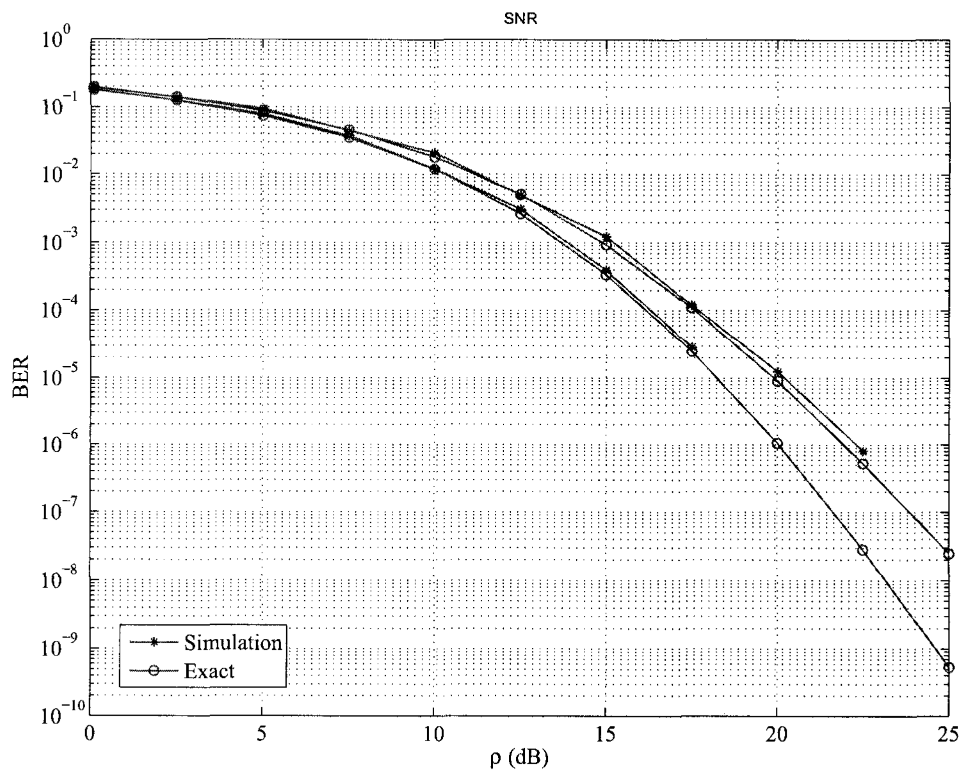

Figure 5.2: Comparison between the exact expression, approximation and simulation for $N = 2$ transmit antenna selection out of $L_t = 3, 4$ with $L_r = 2$ correlated receive antennas, 16-QAM.

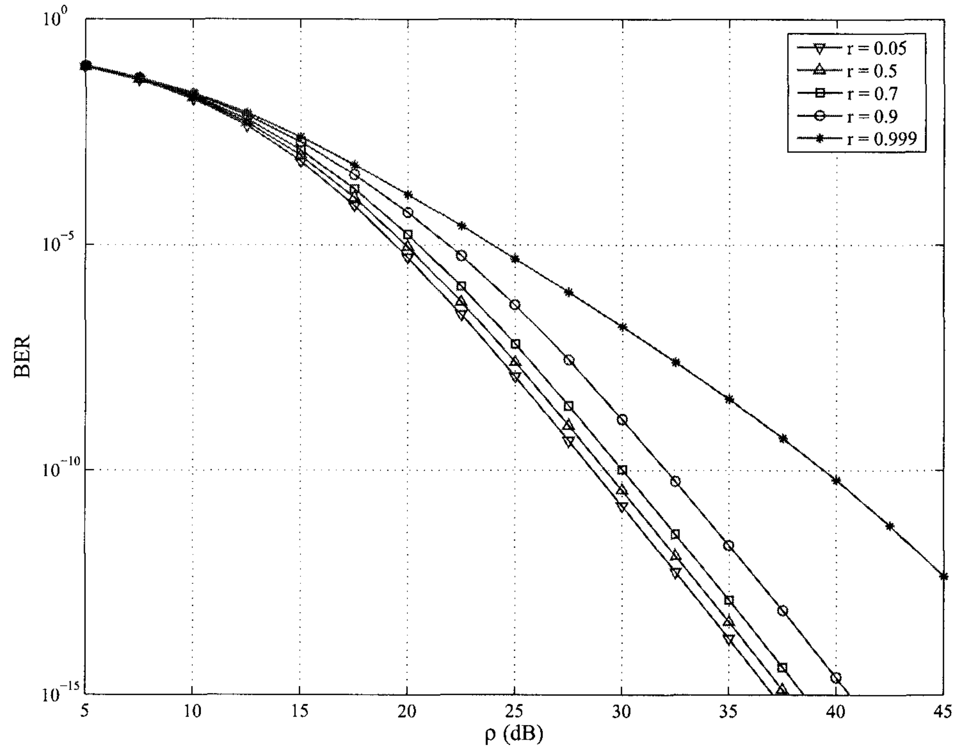

Figure 5.3: Comparison of the performance between different correlation parameter r for $N = 2$ transmit antenna selection out of $L_t = 3$ with $L_r = 2$ correlated receive antennas, 16-QAM.

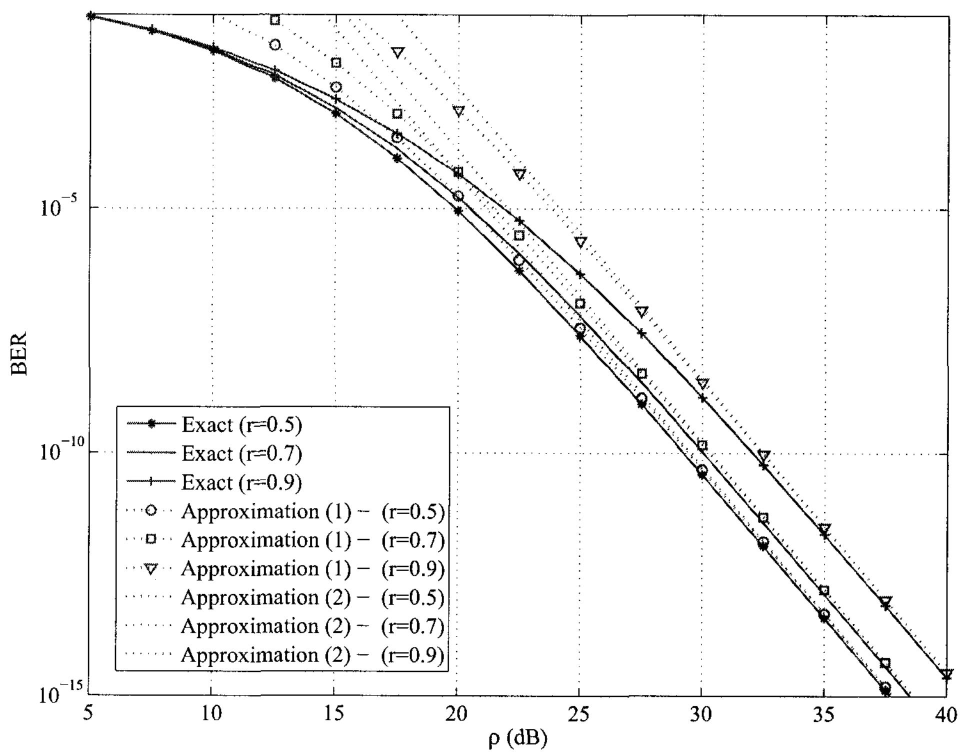

Figure 5.4: Comparison between different correlation parameter r for $N = 2$ transmit antenna selection out of $L_t = 3$ with $L_r = 2$ correlated receive antennas and two approximation of performance, using 16-QAM constellations

Chapter 6

Conclusions and Future Work

The performance of transmit antenna selection with OSTBC in MIMO wireless systems is derived for Rayleigh fading channels without correlation and with receive correlation. Transmit antenna selection gives the better performance and achieves full diversity while reducing the number of RF chains and consequently cost and complexity of system.

In Chapter 2, MIMO wireless systems are introduced. OSTBCs is reviewed and preliminaries for performance analysis of system is presented. In Chapter 3, antenna selection scheme is discussed. The motivations for antenna selection and the recent works are reviewed. There are two main antenna selection criteria, one is based on capacity maximization to achieve the full rate of transmission which leads to spatial multiplexing techniques such as V-BLAST. Minimizing the error rate is another scheme to select transmit antennas. Minimizing probability of error in systems using OSTBCs leads to selecting antennas with the highest received SNR. Maximizing SNR needs selection of antennas which have largest norms of corresponding columns in the MIMO channel matrix.

Chapter 4 studies the performance of OSTBCs with transmit antenna selection in independent Rayleigh fading channels. Exact closed-form BER expressions are derived, which also lead to approximations. The diversity order and coding gain are obtained exactly. We show that a system using any number of transmit antennas and sends OSTBCs over them achieves full diversity.

In Chapter 5, the performance of transmit antenna selection is analyzed for channels with receive correlation. Despite the receive correlation, the received SNRs from transmit antennas are independent. Thus order statistic results can be used in performance analysis of transmit antenna selection. The approximations and coding gain and diversity order of system are derived from the exact form BER expression. Order statistics for correlated variables are not available in the literature at this time thus deriving performance for fading channels with transmit correlation appears impossible.

As a result, transmit antenna selection with OSTBCs achieves full diversity and gives better performance and lower complexity rather than no selection, with the expense of minimal number feedback bits.

Although we derived the performance analysis of transmit antenna selection for receive correlated channels, no performance analysis is available for the transmit correlated case. Studying the performance of transmit antenna selection leads to future system designs to get better performance.

The only known optimum antenna selection in capacity based criteria is exhaustive search. Thus proposing suboptimum and fast selection algorithms with lower complexity is a topic of current and future research.

Transmit antenna selection is only one of the adaptive antenna systems using low rate feedback from receiver to design system and signaling at the transmitter to get better performance. Other adaptive antenna methods are of interest.

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