Block Caving Production Layout Optimization Considering Uncertainty in Grade Models

by

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Abstract

The mining industry of today demands large-scale extraction methods, and caving has become the preferred underground mining technique because of high production rates, low mining costs, and low waste production. Moreover, there is a current growth of concerns about the effects of uncertainty and risk in mine design, safety, and production schedules. Companies are then moving away from traditional approaches to adopt techniques that quantify uncertainty and optimize critical processes to succeed. One key engineering factor for the success of a block caving is the drawpoint spacing layout, that is designed prior to operation. The current guides to design this important element remains controversial due to uncertainty influenced by different aspects of caving, but most importantly by the block model.

Grade modeling is the basis of mine design and planning and plays an important role in the prediction of financial outcomes. Conventional layout design calculates layouts based on deterministic modeling that generates a single model of economic value and therefore is not capable of accounting for uncertainty. In contrast, stochastic orebody modeling allows for quantifying uncertainty by generating multiple equiprobable models which can be integrated into an optimization process. There are currently well-studied procedures to optimize mine designs using orebody uncertainty in open-pit mines, but little work is done in block caving.

This research provides a methodology in which uncertainty from the block model is assessed with Sequential Gaussian simulation (SGS) to determine the optimal drawpoint spacing and level of extraction. Multiple models are used in a transfer function to provides a summary of responses and then select the optimal option. The approach is supported by block caving definitions and explained with conceptual examples. A copper-gold caving project is used to demonstrate the methodology using Gaussian simulation and a signed distance function. Data for the study is from 37 drill holes and assays is composited to 10 m with copper-gold mineralization. Fifteen potential layouts are selected. Optimizing the drawpoint spacing gives 30 × 13 layout as the optimal one, and the optimal extraction level is 440 m.

Tonnage uncertainty is added to the case study. The variability of domain boundaries is incorporated into the work. A set of twenty implicit models with simulated grades are used to evaluate boundary uncertainty. The tonnage uncertainty confirms that the 31 × 14 layout and 31 × 15 present lower risk compared to the optimal 30 x 13 layout.

Dedication

"To my lovely mother, siblings , wife and specially to my wonderful kids Mathew and Hannah, both are the engine of my life."

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List of Symbols

Symbol	Description
$E[\quad]$	Expected value
g_i	Block grade at location i
g_z	Economic cut-off grade
IR	Interest rate
i	Indexer for summation
L	Total number of realizations
R	Risk
S	Total number of potential layouts

List of Abbreviations

Abbreviation	Description
2-D	Two-dimensional
3-D	Three-dimensional
HPO	Heuristic pit optimizer
AGRM	Active geological risk management
BHOD	Best height of draw
CDF	cumulative distribution function
FF	Footprint Finder
FSD	First-degree stochastic dominance
GSLIB	Geostatistical software library
HIZ	Height of interaction zone
IDW	Inverse distance weighting
IDZ	Isolated draw zone
LHD	Load-haul-dump
LVA	Local varying anisotropy
MCS	Monte Carlo simulation
MRMR	Mining rock mass rating
NPV	Net present value
OK	Ordinary Kriging
SK	Simple Kriging
PCBC	Personal computer block caving
PRC	Production rate curve
RBF	Radial basis function
REBOP	Rapid emulator based on particle flow code
SDF	Signed distance function
SGS	Sequential Gaussian simulation
SSD	Second-degree stochastic dominance
TF	Transfer Function
OUU	Optimize under uncertainty

Chapter 1

Introduction

This chapter provides a general overview of the research. The background, problem statement, literature review, scope of the research and thesis outline are included in this chapter.

1.1 Background

Block caving is a low-cost underground mining method for undermining large and low-grade orebodies (Figure 1.1). Fractured rock is allowed to collapse, and then fragments are extracted from drawpoints at production levels. This mining approach is increasingly being suggested for deposits worldwide, and several existing large open-pit mines are planning to extend their operations underground to maintain their profitability (Hem & Caldwell, 2012). However, the costly capital investment and the range of operational and economic risks can potentially neutralize these caving advantages; decision making on any caving process, such as the design of the extraction level layout, cannot be taken lightly and must be anticipated prior to mining (Brown & Chitombo, 2007a).

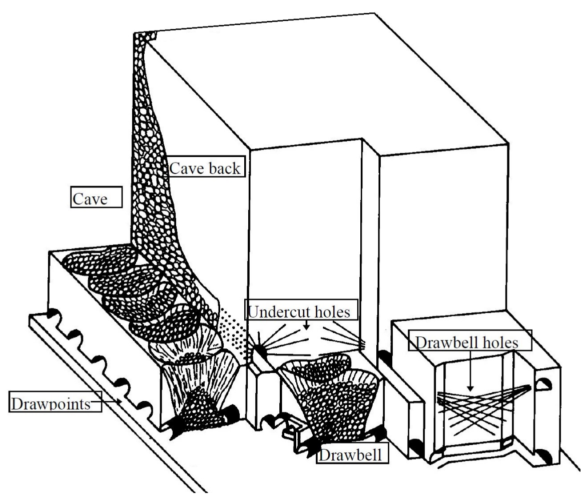

Figure 1.1: Graphic representation of block caving, (Laubscher, 2000)

Block caving is highly sensitive to the initial layout design and there is limited flexibility to change the spacing between drawpoints once development is started (Laubscher, 1994). The selection of an optimal drawpoint spacing is truly essential to allow for the most economical extraction of the ore, and therefore the success of a caving project.

Over the last two decades, various researchers includung Julin and Tobie (1992), Hustrulid (2000), Laubscher (1994, 2000), Kvapil (2008), Castro et al. (2009), Trueman et al. (2008), Pierce (2010), and Castro et al. (2012) proposed several approaches aiming to design the production level layout with a feasible drawpoint spacing. These methodologies are done based on many years of empirical knowledge and improved understanding of gravity flow through physical modeling. However, the extraction layout design remains the most arguable of caving aspects (Chitombo, 2010; Diering, 2013) and the Laubscher's guideline (1994) for layout design has been preferred over more plausible solutions.

Current practice for determining the drawpoint spacing of the production level layout is based on an empirical approach that considers certain geotechnical aspects and mining constraints. This traditional approach also include the use of a single estimated grade model. However, the optimization of drawpoints spacing considering orebody uncertainty is actually not possible.

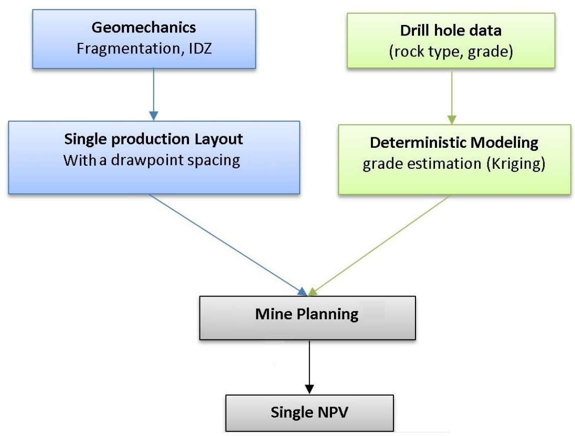

Figure 1.2: Common workflow for layout optimization in today's caving projects

Sequential Gaussian simulation (SGS), which is useful to quantify grade uncertainty, represents an ideal alternative to traditional estimation-based approaches to solve this caving aspect. The drawpoint spacing alongside the elevation of production level layout can be optimized considering uncertainty. The statistical moments of the response variables, Net present value (NPV), are assessed within a stochastic framework and a subsequent decision-making, based on a risk-and-reward principle, is then possible. The main purpose of this thesis is to develop and implement a practical methodology using SGS and innovative decision-making tools to Optimize under uncertainty (OUU) the block-cave production level layout.

1.2 Summary of Literature Review

Several researchers have developed methodologies attempting to design a feasible drawpoint spacing of the production level layout. Most of these available design methods are deterministic and empirical. They have been developed by experimental studies utilizing production data from worldwide caving mines during many years. For instance, Henriquez (1989) contributed with a methodology that uses a set of development costs for a range of potential layouts to select the drawpoint spacing. The Henriquez's method has been implemented in the Andina division at Codelco, Chile.

In 1992, Julin and Tobie claimed that the broken ore is extracted from the production level whilst the orebody above continues to break by gravity, thus they affirmed that comparing the rock fragmentation of a project to other successful caving mines is the best guideline. Therefore, a table to estimate drawpoint spacing according to rock fragmentation from caving experiences worldwide is presented by Julin and Tobie (1992). Eight years later, an empirical guideline to determine the radius and diameter of the Isolated draw zone (IDZ) based on the mean fragment size is developed by Hustrulid (2000). His guideline is based on the approach by Julin and Tobie (1992) and others. Hustrulid argues that the IDZ radius is from 8 to 12 times the fragment size.

Laubscher (1994, 2000) is the most active author on the subject. He has several important publications, including a state-of-the-art paper (1994) and a block caving manual (2000). In terms of layout design, he recommends that the relationship between the average fragment size of the caved rock with the IDZ diameter and the related spacing should be used to determine the best ore draw. This approach is supported by Laubscher 's experience related to gravity-flow characteristics of the broken rock and the height of interaction between adjacent drawpoints. After Laubscher's publications , the relationship between drawpoint spacing and the diameter of the IDZ, which has been developed based on sand investigations, is widely accepted by caving operations. Therefore, a graph to determine the IDZ diameter as a function of the fragmental size of several types of rocks is presented by Kvapil (2008). His contribution is based on several years of studies on sand models in where dilution is minimized and ore recovery is maximized, provided that the uniform flow of caved rock is achieved. Susaeta et al. (2008) suggest a method for layout design using the rock types and a required draw strategy for minimizing dilution that leads to improving ore recovery.

Publications about contemporary design of block caving, based on larger experiments using predictive models for gravity flow are also available. The gravity-flow models depend on the properties of the caved rock for predicting movement and extraction in caving mines (Pierce, 2010). For instance, Castro et al.(2009) used a flow simulator to investigate the influence of gravity-flow from cohesionless rock, and then determine the dilution entry related to the IDZ and drawpoint spacing. He concluded that the mass which is drawn and the height of draw affect the geometry of the flow zone while the fragment size has lesser influence; his conclusions gave him a better understanding of the mechanism related to the IDZ for predicting the drawpoint spacing. Information from the Inca Oeste mine and Esmeralda mine located in Chile is used to validate the approach suggested by Castro et al. (2009). Another predictive model for gravity flow of fragmented rock is presented by Pierce (2010). He highlighted the benefits of the Rapid emulator based on particle flow code (REBOP) which is used to control the draw of caved material from caving operations. Pierce claimed that a good understanding of the interaction of adjacent flow zones under caving conditions in a specific mine is relevant, thus he used REBOP to evaluate the properties of flowing ore material for assisting in the design of adequate drawpoint spacing and production schedules in caving mines.

Most recently, a methodology to determine the optimal drawpoint spacing considering technical and economic aspects is suggested by Castro et al. (2012). Their work is based on the subsequent analysis of the primary recovery of mineral reserves under the concept of interaction of adjacent production drawpoints, this concept has been widely studied by Trueman et al. (2008) using a large gravel model. According to Castro et al. (2012), a model of flow that occurs near the reference point and the associated development cost is used to obtain the best spacing between draw zones. This method has been validated with mined rock data of the El Teniente mine; the results show that the primary recovery depends on the variability of the Height of interaction zone (HIZ) related to the angle of friction of the caved rock and also the drawpoint spacing. They also suggest a procedure to estimate the spacing between drawpoints considering different mining costs and metals prices.

Although Julin and Tobie (1992), Laubscher (1994), Pierce (2010) and other authors made important contributions to determine the best drawpoint spacing , this mining aspect remain questionable (Chitombo, 2010). In addition, the traditional layout design and current selection of drawpoint spacing depend mostly on deterministic decisions.

Optimization under uncertainty is becoming increasing recognized for decision-making in mining processes, and has emerged as the framework through which several solutions are achieved. Multiple passive and active approaches to the management of risk for decision-making in open-pit mining projects exist (Acorn, 2017). However, their counterparts in block caving are less common. There is not a practical method, in the available block-cave literature, that considers the orebody uncertainty to actively manage risk and therefore determine the best possible drawpoint spacing alongside the elevation of extraction aimed to design an optimal production level layout.

1.3 Problem Statement

During the initial stage of a caving project, one basic concern for the mine planning is to decide which is the drawpoint spacing for the extraction layout that will help to maximize the project value. Evidently, this is not a trivial problem because drawpoint spacing might control certain mining aspects including , capital cost, ore recovery, etc., while being constrained by other factors. For instance, if the spacing is too wide, pillars of material are generated between draw points and thus generating ore loss due to ore trapping. Conversely, too close spacing generates a larger number of drawpoints and therefore the capital cost is increased due to excessive mine development. Keeping these things in mind, an optimal drawpoint spacing for the block-cave production layout is required.

As mentioned in Section 1.2, the current methods for determining the drawpoint spacing and therefore the production level in caving projects includes the use of mining aspects related to geomechanics and gravity-flow characteristics of the caved rock. This guideline has being conducted empirically and is widely accepted and used for decades. The layout determined empirically is used over a unique estimated grade model for mine planning, providing sub-optimal predictions.

However, the inherent orebody uncertainty , which is acknowledged to cause planning deviations, is not considered in this process (Figure 1.2). The uncertainty, including geological boundary and grade, can lead to differences between the true profit and expected predictions of a mining project (Osanloo et al., 2008). Therefore, a different layout design procedure in which uncertainty is included should be implemented.

The optimal caving extraction layout has to contain a drawpoint configuration that aims for high ore recovery whilst the capital development cost is kept reasonably low, and it must be determined considering orebody uncertainty. Hence, this research is driven by the following thesis statement:

" The production level layout which is determined considering orebody uncertainty is optimal and can maximize the value of a caving project."

1.4 Objectives of the Research

The main objective of this research is to develop a methodology that implements SGS to determine an optimal design of the production level layout, and therefore maximize the economic value of a caving project. The first step toward achieving the objectives is to determine the optimal drawpoint spacing. As a result, the determination of the elevation of extraction is possible.

Assessing uncertainty in the geological boundaries for knowing the tonnage uncertainty and the effects on predictions of ore reserves is important. Therefore, the evaluation of uncertainty in the orebody boundaries is included as the secondary objective of this thesis.

1.5 Scope of the Research

The scope of this research is defined by taking into account some important concepts and statements (Section 1.1; Chapter 2;). For instance, the production level layout presents limited flexibility once mine development is started; and most importantly, the optimization of a layout design is essential for the economic success of a caving project (Laubscher, 1994). The research is then concerned to design a drawpoint layout for predicting the best-expected return using information prior to development or production.

The assessment of uncertainty is recognized for solving optimization problems in the mining industry. However, methods for maximizing profit in caving mines through optimization of extraction layouts considering grade uncertainty is not currently discussed nor published in block caving literature. The common and simplistic approach for determining the drawpoint spacing does not results in optimal layout due to various factors. This traditional optimization is explained in Section 1.2 and illustrated in Figure 1.3 (alternative 1). Similarly in the same Figure, alternative 2 considers optimizing the production layout on one estimated model in the same way as alternative 1 but evaluates profit uncertainty and risk using a set of grade realizations; the resulting risk assessment and uncertainty calculation are sub-optimal because the realizations are never used directly in the optimization of the layout, but this is commonly done to try to evaluate the risk associated with a plan created from an estimated model.

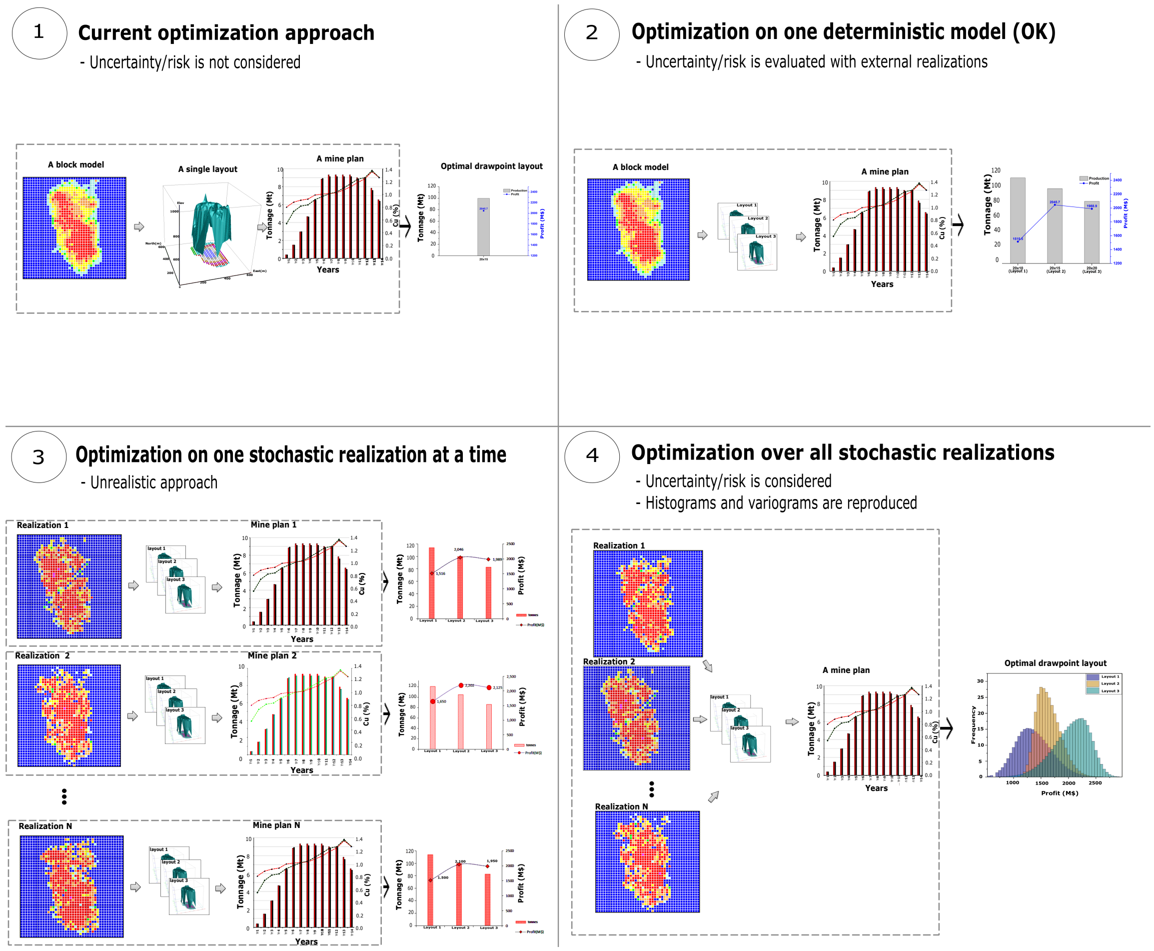

Figure 1.3: Four alternatives to determine the drawpoint spacing. Alternative 4 is the approach proposed in this thesis

Despite SGS provides models that are equally probable and allow for quantifying grade uncertainty, an unrealistic practice (Alternative 3, Figure 1.3) can arise where layouts are optimized on individual realizations. Using one realization for optimizing layouts is unrealistic and is never recommended. Optimizing the layout over all realizations simultaneously and considering these through all production decisions is advisable; however, the use of multiple models in mine planning is a difficult optimization problem but is the approach proposed herein; one layout design that is optimal over all realizations is generated through optimization of drawpoint spacing in presence of risk (Alternative 4, Figure 1.3) .

The research considers using the Monte Carlo simulation (MCS) framework to optimize the drawpoint spacing and level of extraction while being constrained by mining factors. SGS is devised to provide a set of equally-probable realizations which pass through a Transfer Function (TF) coded in Personal computer block caving (PCBC) to generate a large number of responses, and therefore the risk is managed actively for decision making. The mining and operational constraints such as fragmentation, mining sequence, mining cost, development cost, and other factors are included in the proposed optimization framework. However, ”pillar safety factors” and ”level of dilution” are not explicitly considered in this study.

It is worth noting that the intention of this research is to evaluate stochastic models and then provide risk assessments from a set of mine designs based on the economic return (NPV). yet, all sources of block-caving uncertainty are not included. The software used to develop the research are from the Geostatistical software library (GSLIB) library (Deutsch & Journel, 1998); and PCBC (Diering, Richter, & Villa, 2010).

1.6 Thesis Outline

This thesis is organized in five chapters. ***Chapter 1*** is the introduction to the study, where the background, a summary of available literature on drawpoint spacing, problem statement, the objectives of the thesis, scope of the research, and thesis outline are presented. An overall explanation about block-cave mining and the importance of using uncertainty assessment to manage risk in order to optimize mine designs is highlighted. The problem statement, the objectives and the scope of the research are carefully organized for explaining the significance of a proposed optimization approach.

Chapter 2 is the section where a number of important definitions and terminology are reviewed. To achieve the objectives of the research, some relevant concepts need to be understood. This chapter provides background information about block caving factors such as cavebility, undercutting, fragmentation, and drawpoint spacing. Moreover, the economic envelope and production scheduling made in PCBC are explained. Relevant concepts about orebody modeling; traditional and modern geostatistical techniques including Ordinary Kriging (OK) and SGS; uncertainty and risk-based decisions are also explained.

Chapter 3 describes a proposed methodology for optimizing the production level layout determining the optimal drawpoint spacing and the optimal level of extraction for a block caving project. The proposed approach is an implementation of stochastic modeling with SGS, using a sophisticated multi-stage algorithm (PCBC) as the transfer function to calculate the production schedules. The responses are used to perform the optimization of the production level layout within a stochastic framework, decision making for determining the optimal drawpoint spacing to maximize the profit is also discussed.

Chapter 4 presents a case study that is shown in two parts. The first part demonstrates the complete application of the method proposed in Chapter 3 to optimize the drawpoint spacing layout. The selection of the optimal layout considering grade uncertainty is illustrated in this section. Fifteen proposed layouts are evaluated over a set of twenty stochastic realizations to determine the optimal drawpoint spacing. The proposed methodology discloses a potential area for the usage equally probable models and the related uncertainty for optimizing the mining design in the block caving context.

The second part of the case study presents the implementation of an implicit modeling technique based on the Signed distance function (SDF). This approach is developed to asset the boundary uncertainty of a block-cave project. The effects of the tonnage uncertainty in the decision-making process of an optimal layout design are explored.

Chapter 5 finally summarizes the results of the research explaining more in detail about the contributions and limitations of the study. Further research is suggested for extending the proposed optimization methodology to improve the understanding of stochastic solutions in the block-cave context. Limitations of the research and possible improvements are suggested for future work.

Chapter 2

Concepts and Terminology

This chapter provides brief descriptions of relevant concepts and terminology. The block-cave mining method, factors that affect block caving, grade modeling, optimization approach, mine planning and scheduling in PCBC, decision making under uncertainty and boundary uncertainty are explained here.

2.1 Introduction

Various methodologies exist that enable the use of stochastic modeling in mine design optimization, and most currently available tools are aimed at open-pit operations. However, the purpose of the present research is to provide a methodology for determining the optimal drawpoint spacing of block caving layouts considering ore-body uncertainty. Therefore, a set of relevant concepts and terminology are summarized and carefully organized in this chapter.

2.2 Block-Cave Mining Method

Cave mining refers to mining operations in which fractured rock is allowed to collapse, and then fragments are extracted from drawpoints at production levels (Laubscher, 1994). The use of block caving has increased worldwide and became the preferred underground alternative for mining because of high production rates, low mining costs, and low waste production (Pourrahimian, 2013). The extraction of ore material in block caving is made with the assistance of gravity. Moreover, the suitable ore-handling system is generally dictated by the fragmentation size. For instance, the grizzly system is the most suitable for fine fragmented rock, the slusher system is appropriate for somewhat coarser ore, and the mechanized Load-haul-dump (LHD) system is convenient for coarse material (Rubio, 2002). In order to select the proper system certain parameters should be considered including labor cost, availability, equipment cost , work-force training, planed production rate and other aspects which are particular to each caving mine (Rubio, 2002).

The block caving system is composed by several levels which allow the mining of ore (Figure 2.1). The undercut level, for example, favours the caving of the rock. The production level has the drawpoints in which the broken material is extracted. The haulage level provides facilities and tunnelling systems where trucks and trains collect the ore coming from the production level through ore passes and transport it to the crusher. The ventilation level contains the fans to provide air to the production level (Rubio, 2002).

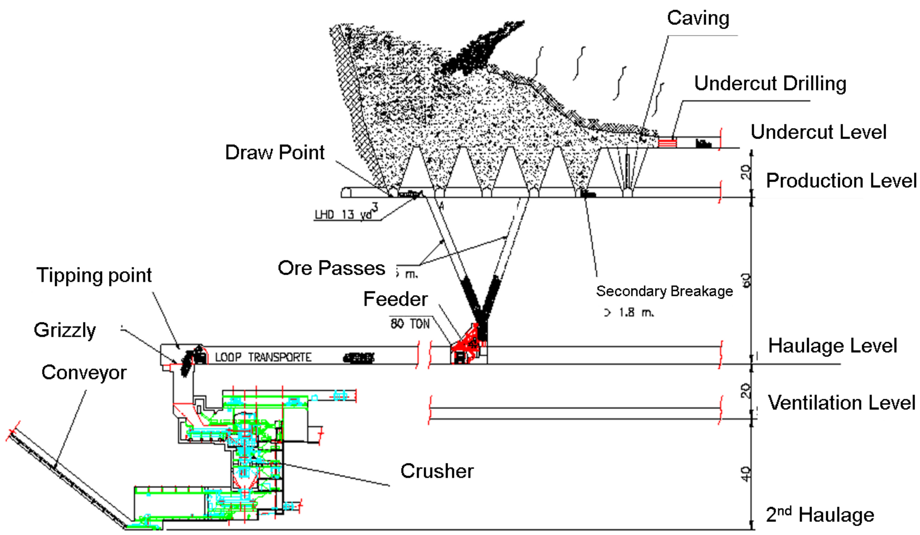

Figure 2.1: Cartoon of the block caving system, after Rubio (2006)

Caving operations are affected and constrained by many factors which are conditioned by the geological characteristics of the deposit and the caving system (Figure 2.1). Cavability, undercutting, rock fragmentation, and drawpoint spacing are the common factors in all caving operations.

2.2.1 Cavability

Cavability is one essential aspect of block caving, because the in-situ rock caves naturally after undercutting and ore is recovered through drawpoints (Laubscher, 1994). Many parameters have been found to control caveability such as rock mass strength, orebody geometry and undercut dimensions (Laubscher, 2000). Cavability is then defined as the continuous failure of the orebody until the void spaces, created by undercutting, is filled with broken rock, as shown in Figure 2.2. Caving operations have shown that two kinds of caving can occur, and they are defined as stress and subsidence caving (Laubscher, 1994). The inability to initiate or sustain caving can represent one of the greatest risk factors for any project in which time consuming and expensive implementations may be required.
The prediction of the orebody cavability can be influential for the control in the mine design, and consequently in the economics of any caving venture Laubscher (1994, 2000). For example, a reliable evaluation of cavability is required to determine the correct undercut dimensions to commence and continuously cave the mineralized rock (Lorig et al., 1995).

Currently, cavability is assessed by empirical methods or numerical modeling. Although several studies demonstrate that numerical modeling has the possibility of providing more fundamental and rigorous assessments for the cave initiation and propagation, a large number of caving operations still use the empirical approach (Sainsbury et al., 2011).

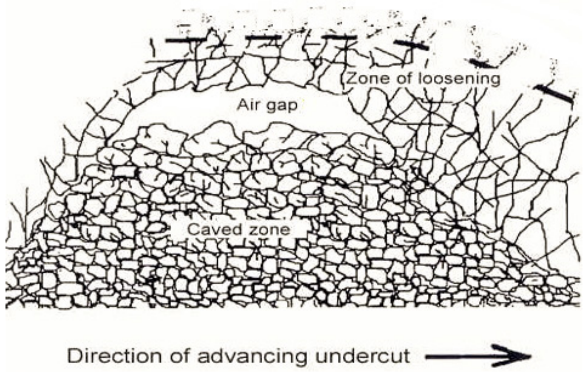

Figure 2.2: Schematic diagram of cavability, after Rubio (2006)

An accurate cavability assessment generally includes the calculation of the hydraulic radius of the undercut at which caving will initiate for an orebody, provided that the geotechnical characteristics including the Mining rock mass rating (MRMR) are estimated (Brown, 2003). Therefore, in order to make a good prediction of cavability, the geotechnical data must be accurate and the variations in the geology of the deposit must be known (Figure 2.3).

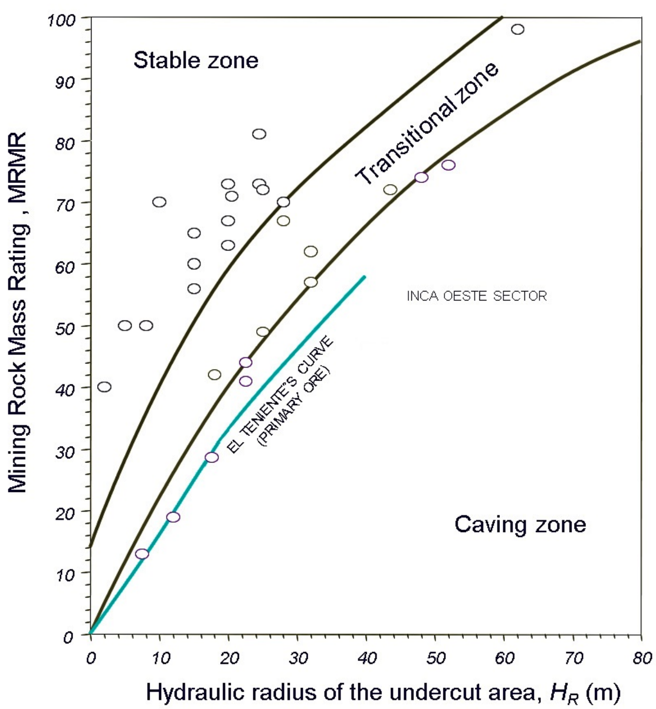

Figure 2.3: The cavability assessment model, after Rubio (2006)

The intensity of the fracture pattern and the cave propagation are two critical factors that determine cavability (Laubscher, 1994). A satisfying caving process needs several sets of fractures which must be ideally combined, vertical fractures along with some horizontal ones are required for a successful gravity breakage of the rock mass (Laubscher, 2000). Cavability depends on cave propagation which is the continuous drawing of fragmented rock, and occurs in a planned and controlled fashion (Brown, 2003). The important aspects to sustain an acceptable cave propagation are the undercut method used, the stresses induced on the cave, the geotechnical characteristics, and the draw control strategy (Laubscher, 2000).

2.2.2 Undercutting

The process of undercutting is an important aspect of cave mining, because a complete and adequate undercut induces caving properly. Block caving experiences worldwide demonstrates that the proper undercut design, planning implementation, and management of the assigned undercut can generate a significant contribution to the success of a block cave venture (Laubscher, 2000). In contrast, poor development of the undercut can risk the economics of the project.

The magnitude of induced stress is influenced by the direction of advance of undercut into the principal stress direction. The undercuts are usually extracted in the direction of the maximum principal stress to reduce stresses in the cave back (Rafiee et al., 2015). The undercut level is located at some distance above the production level, as shown in Figure 2.4.

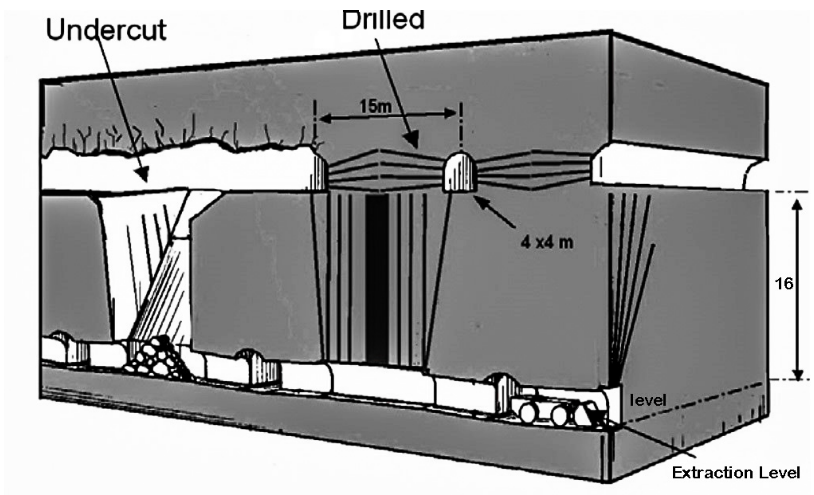

Figure 2.4: Illustration of the undercut and the extraction level (Chitombo, 2010)

In the undercutting process, a series of narrow parallel drifts are made at the undercut level 2.4. Then, long horizontal holes are drilled to blast and extract thin slices of ore (1 - 2m) and the undercutting is generated by removing the layers of ore. As blasted ore is removed, ore caving initiates, thus broken rock above starts to cave and naturally draw by gravity (Halim, 2006).

Several undercutting methods are suggested by Laubscher (1994) and Butcher (1999). They are available and being used to determine the undercutting strategy (Brown, 2003; Brown & Chitombo, 2007b), also known as the undercut sequence. The undercutting methods includes the post-undercutting, pre-undercutting and advance-undercutting, as illustrated in Figure 2.5.

1. Post-undercutting. This undercutting strategy is also known as conventional and consists of mining the undercut after the underlying extraction level and drawpoints are developed.

2. Pre-undercutting. This undercutting strategy consists of mining the undercut before the underlying extraction level and drawpoints are developed.

3. Advance-undercutting. This undercutting strategy consists of mining the undercut slightly ahead of an extraction level that is partially developed and drawpoints have not been excavated.

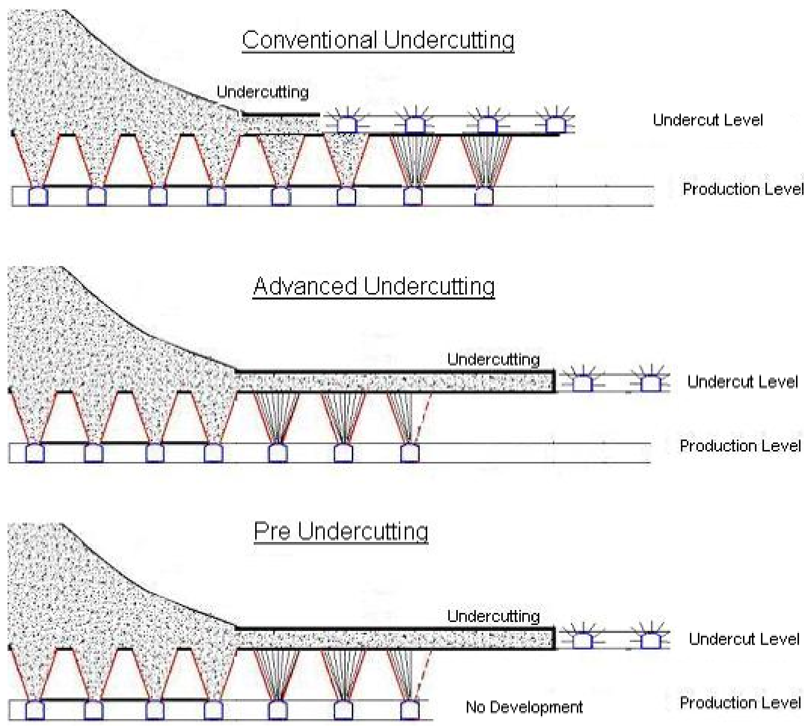

Figure 2.5: Undercutting methods (Rubio, 2006)

In modern operations, the pre- and advance-undercutting options are the most commonly used chitombo2010. They are predominantly applied in mine environments where the induced stresses are relatively high (Rojas et al., 2001). In contrast, post-undercutting development is better suited to lower stress environments. Although the mining abutment stresses are usually related with post-undercutting development, operationally and logistically there is often preference towards applying post-undercutting (Rubio, 2002).

2.2.3 Rock Fragmentation

Rock fragmentation can be defined as the process in which rock particles separate from the cave back and enter the draw column (Laubscher, 2000). The fragmentation, as shown in Figure 2.2, is

Figure 2.6: Fragmentation of caved material at a drawpoint, Rubio (2006)

generated in the orebody during caving. In the ideal project, the orebody breaks into small enough fragments to pass through the drawpoints, and it may control the economic success of a block caving operation (Brown, 2003).

In the initial stage of production, the caving process results in primary fragmentation. Blocks of rock are detached from the cave back as the undercut is mined and caving is initiated . Ore breaking is only done by the action of gravity; therefore, fragmentation is relatively poor (Laubscher, 2000). Secondary fragmentation occurs after primary blocks move down through the drawpoints, so that better fragmentation is extracted as drawing continues (Figure 2.6). According to Laubscher (1994, 2000), these processes are controlled by shearing, crushing, and abrasion between rock blocks, reducing the broken-rock size with depth; and also due to attrition generated by higher stress between orebody fractures.

Although the primary fragmentation is not measured directly, it can be estimated from the broken ore that draws in the early stages of caving (Figure 2.6). Moreover, finer fragment sizes generated by secondary fragmentation can be particularly difficult to measure by conventional methods (Brown, 2003). Several software programs are being developed to determine fragmentation. For example, the Discrete fracture network (DFN) modeling is used to represent the geometrical properties of the fractures (Elmo et al., 2010; Rogers et al, 2010).

In modern caving operations, various parameters are influenced by fragmentation (Laubscher, 2000), such as equipment selection, draw control procedures, drawpoint productivity, secondary breaking costs and drawpoint spacing.

2.2.4 Drawpoint Spacing

The drawpoint spacing is not only one of the essential features of the caving layout, but can also be the most contentious aspect of mine design (Diering, 2013). The more finely ore breaks, the

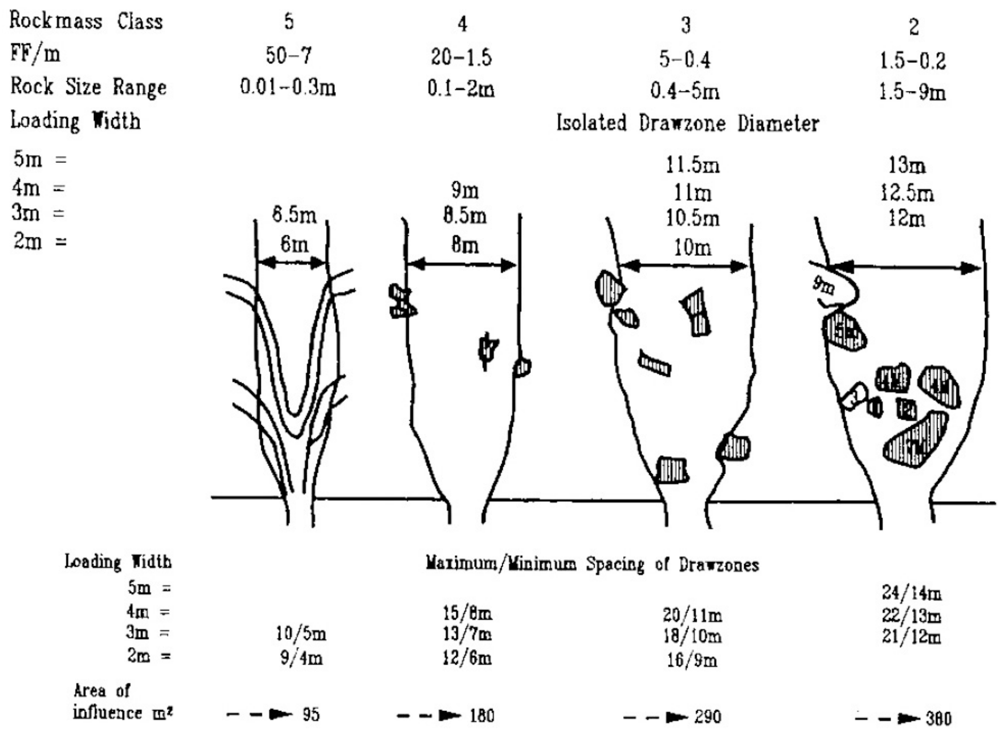

Figure 2.7: Laubscher's drawpoint spacing guideline (Laubscher, 1994)

closer the drawpoints should be, because wider spacing can generate high dilution and ore loss. Therefore, the drawpoint spacing must be determined to aim the recovery and dilution at optimum values (Laubscher, 2000).

The spacing between drawpoints is traditionally determined according to the expected size of the broken rock, and is selected based on experience of caving operations. The drawpoint spacing guidelines proposed by Laubscher (1994, 2000) is then the most accepted in the cave industry. A set of empirical relationships, diagrams, and charts to help with the design process and to predict the cave performance is suggested. The guidelines are mainly based on good geotechnical information related to the isolated drawzone IDZ and expected fragmentation size. For example, the most used chart for drawpoint spacing determination is illustrated in Figure 2.7 and the concept that is referred to as " The Theory of Interactive Draw " is shown in Figure 2.8. Moreover, experiences in various caving operations have shown that drawpoint spacing is a function of the size of the material that is extracted at drawpoints. The IDZ is then determined by the fragment size (Hustrulid, 2000; Julin & Tobie, 1992; Laubscher, 1994).

There are several consequences of having drawpoints at an inappropriate spacing (Richardson, 1981). If the spacing is too wide as illlustrated in Figure 2.9(a), there is not overlapping between the IDZ of drawpoints. Therefore, pillars of material are generated between drawpoints creating ore loss, and the production tunnels can be damaged due to weight concentration. If the spacing is too

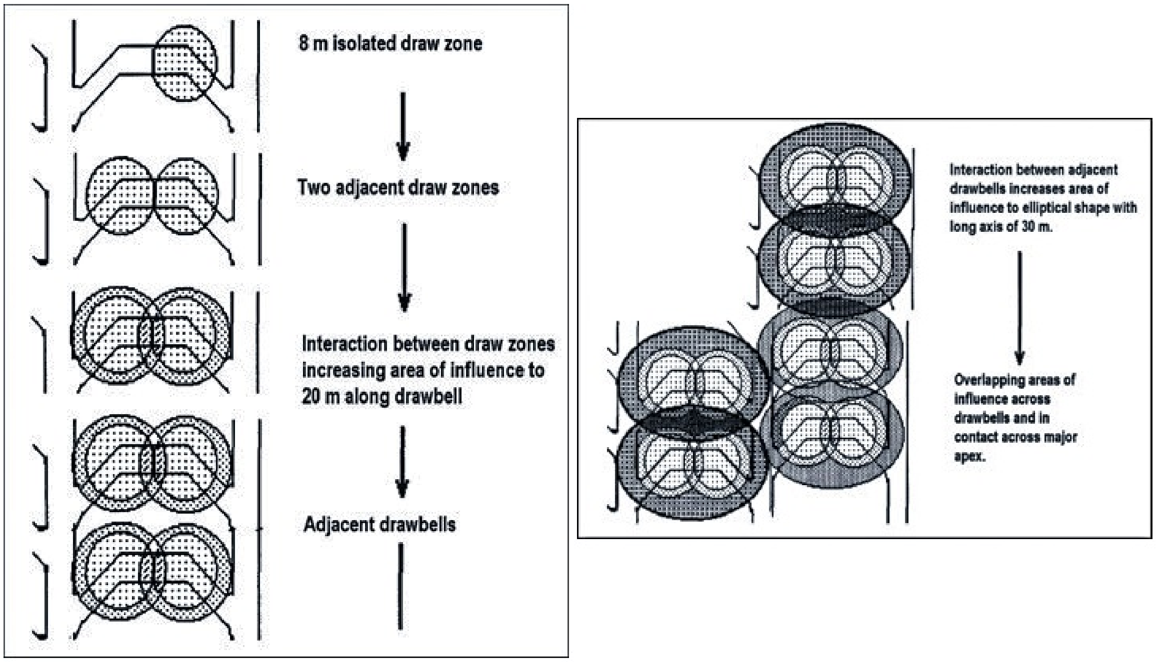

Figure 2.8: The concept Laubscher (2000): Expansion of interacted drawzones

close as shown in Figure 2.9(b), dilution of the ore can occur due to the wide overlapping between the isolated draw envelopes (IDZ), because waste draws downwards through drawzone overlappings, and support problems can occur for having too little pillar between drawpoints. Richardson (1981) claims that the optimal drawpoint spacing is then considered to be the spacing in which the IDZ of each drawpoint slightly overlaps to the surrounding ones.

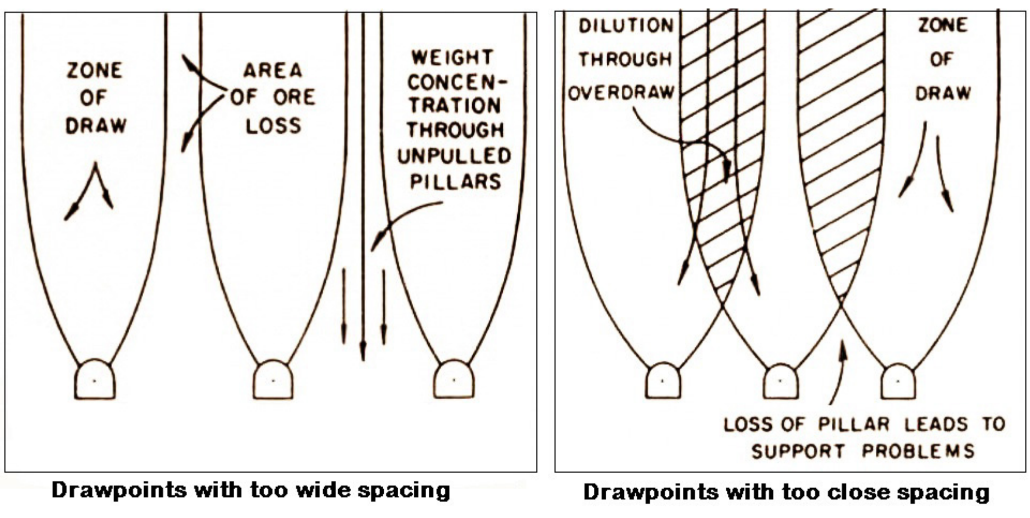

Figure 2.9: Drawpoint spacing (Richardson, 1981): (a) too wide spacing, (b)too close spacing

Among several configurations, the best pattern has a hexagonal arrangement (Richardson, 1981), as illustrated in Figure 2.10. However, this configuration system is unrealistic in practice. In the cave mining system, an extraction level layout requires an extensive amount of quality. A proper

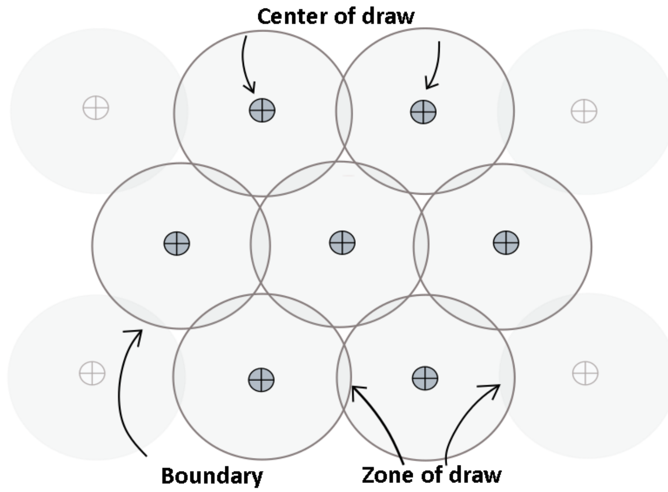

Figure 2.10: Drawpoints of hexagonal pattern (Richardson, 1981)

layout type should be then selected according to three significant factors such as safety, operation and geotechnics (Ahmed et al. 2016). The most common layout types are the El Teniente and the Herringbone layouts, they are illustrated in Figure 2.11.

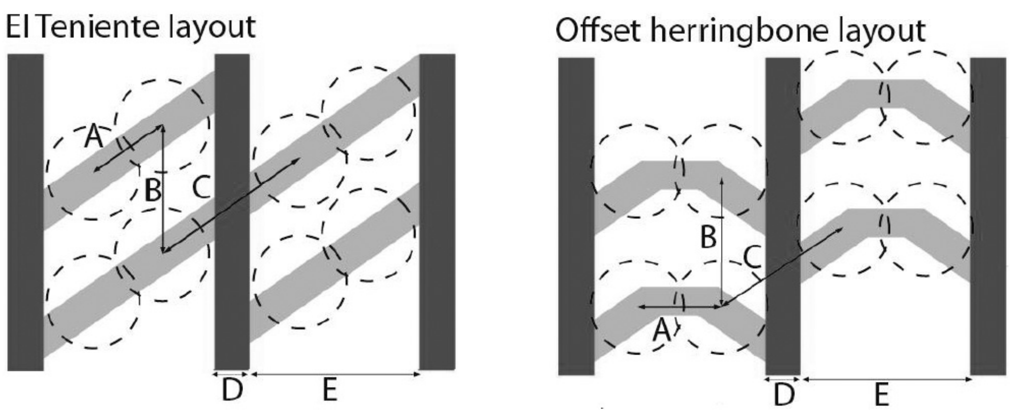

Figure 2.11: Extraction level design parameters (Ahmed et al., 2016): A: distance between drawzone in drawbell, B: distance of drawpoints across minor Apex, C: distance of drawoints across major Apex, D: width of extraction drive, and E: distance between extraction drives

2.3 Block-Cave Mine Planning in PCBC

In underground operations that are exploited by block caving methods, PCBC is the software package of choice. It has been developed over the last 28 years (Figure 2.12) and is used, literally, by all caving mines. The program allows the calculation of the initial level of extraction and the economic envelope of caving projects which are used to calculate production schedules. PCBC incorporates a number of variables, including grade variability and dilution. The variables affect the recoverable resources and therefore the determination of the production schedules. The approach for the production-layout optimization in this thesis is linked to the results of the production schedules.

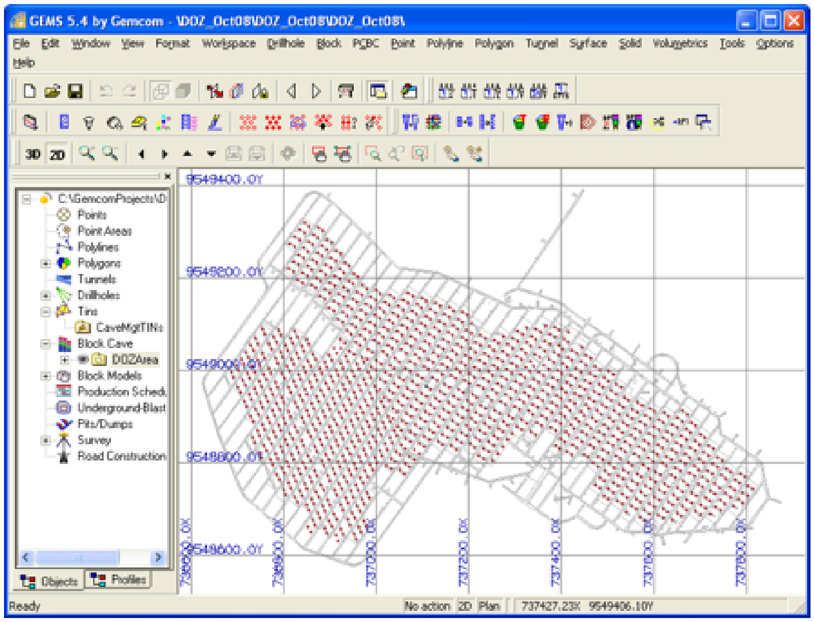

Figure 2.12: Typical interface(view) of PCBC, Diering et al. (2010)

Over the last two decades mine design and scheduling in block caving is conducted based on the PCBC workflow. Hence, this procedure is used as a general guideline in new block-cave operations (Diering et al., 2010). The processes of mine planing, in PCBC, are organized in the following fashion:

1. *Footprint Finder*. This is a procedure that aims to evaluate several elevations of the geological block model to obtain the most profitable production level, it is also named as the block-cave footprint. However, a single deterministic model is used to obtain this best elevation.

2. *Generating the drawpoints*. This stage requires a previous geotechnical evaluation of drawpoint spacing, along with layout type, tunneling orientation, and other design features.

3. *Constructing the slice file*. This is the process in which the geological model is converted into columns and slices. The columns are required to be aligned with the drawpoints.

4. *Calculating the Best height of draw (BHOD)*. This is the process in which each draw column is evaluated to assess the best or highest dollar value considering mining costs, revenue and recovery factors.

5. *Performing the production scheduling*. This process provides tonnage, grade, and NPV forecasts and generates the production schedules for the project.

The principal PCBC processes for mine planning used in this thesis are expanded below.

2.3.1 Footprint Finder

The best level of extraction corresponds to the elevation of the block model where the production layout will be located (Diering et al., 2010). To determine this best level of production, PCBC contains a component called Footprint Finder (FF). The tool uses a single block model as the main input. The model includes grades, mining costs, revenue factors, etc. For each elevation of the block model, the program generates vertical columns and calculates the cumulative sum of benefits. Moreover, a vertical material mix of each column can be applied using an integrated algorithm based on the mixing-method model suggested by Laubscher (1994). Once the elevations of the block model are evaluated, the FF module delivers an output file with the elevations including their total profit and tonnage. Finally, the selected level of extraction is the elevation with the best ratio between maximum benefit and minimum tonnage, as illustrated in Figure 2.13.

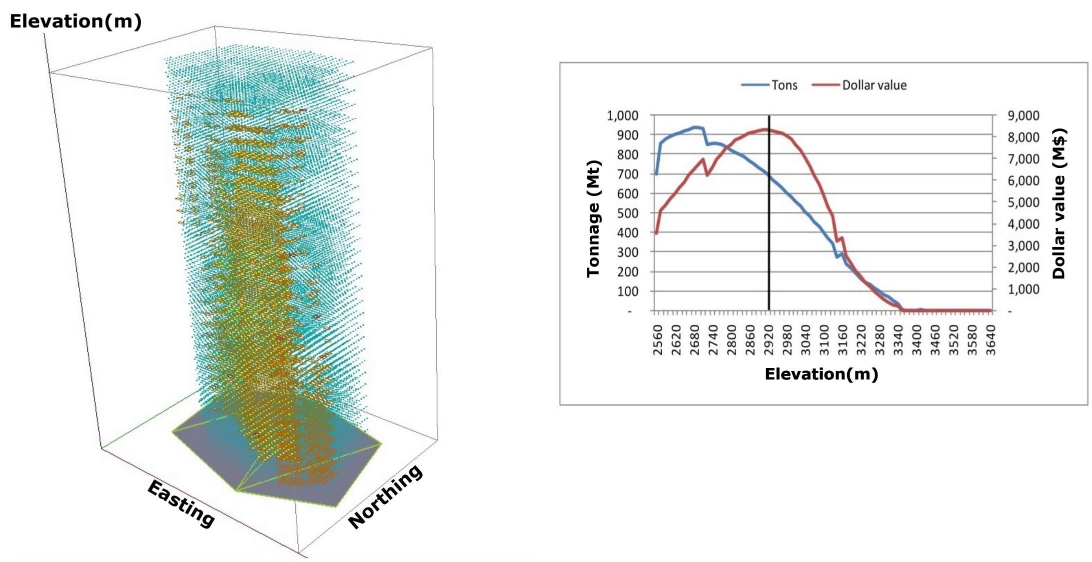

Figure 2.13: Footprint Finder: tonnage and dollar value vs elevation (Diering et al., 2010)

The FF approach is indeed convenient to get an initial insight into the location and outline of the production level. One example of the FF outcome is illustrated in Figure 2.14. A level of extraction has been calculated at the DOZ mine of Freeport McMoRan (Diering et al., 2010), the higher grades of the deposit are in warmer colors.

2.3.2 Economic Envelope

The projection of a set of vertical columns, which form the selected limit of the production level, give rise to an economic envelope. The step by step process to construct the economic envelope is explained as follows.

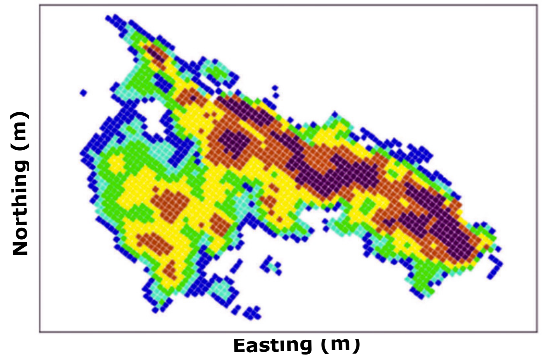

Figure 2.14: Example of selected initial footprint (Diering et al., 2010)

The first step is to define the location of drawpoints at the selected level of extraction. These drawpoints are used to generate the vertical columns (cones). The shape of the cones depends on the geotechnical characteristics of the deposit and the block model. The locations of cones within the production layout must ensure their overlapping to allow a proper mineral extraction. Figure 2.15 summarizes the PCBC approach for calculating the economic block-cave envelope:

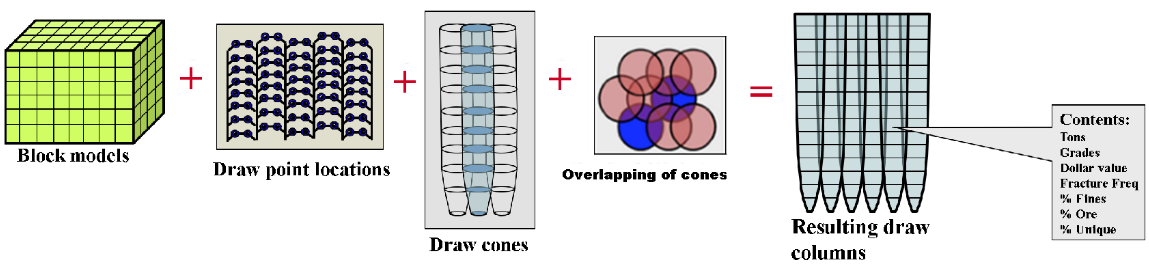

Figure 2.15: Method for calculation of the economic envelope in PCBC (Diering et al., 2010)

The BHOD, that is an specific PCBC function, calculates the cumulative dollar value of each extraction column, as well as their tonnage and average grade to provide an estimate of the recoverable reserves given the level and limit of extraction (Figure 2.16).

2.3.3 Production Scheduling

Mine scheduling is the core of the PCBC system and is performed once the preparation of the economic envelope and the BHOD is generated. The scheduling process in PCBC requires several inputs including the mining sequence of drawpoints along with the undercut development order, and various mining constraints (Diering et al., 2010). Tonnage depletion is scheduled by opening drawpoints according to the selected mining sequence using the assigned Production rate curve (PRC). An illustration of the scheduling process is shown in Figure 2.17.

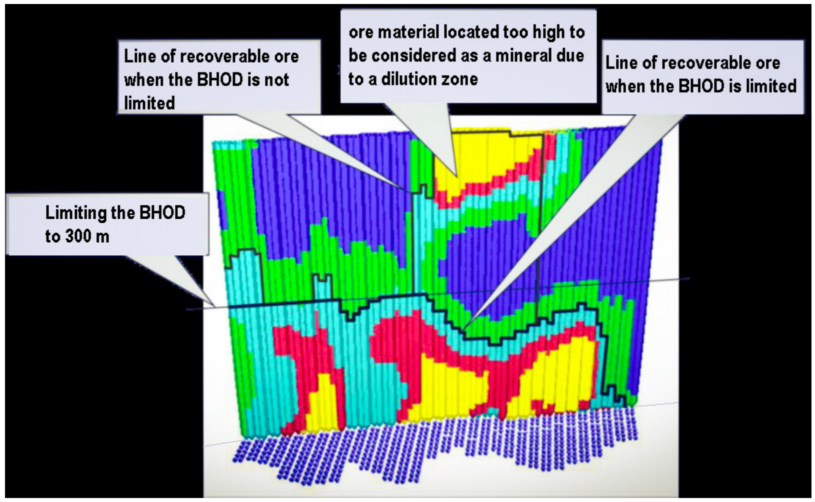

Figure 2.16: The Best Height of Draw (BHOD) given the level and limit of extraction (Rubio, 2006)

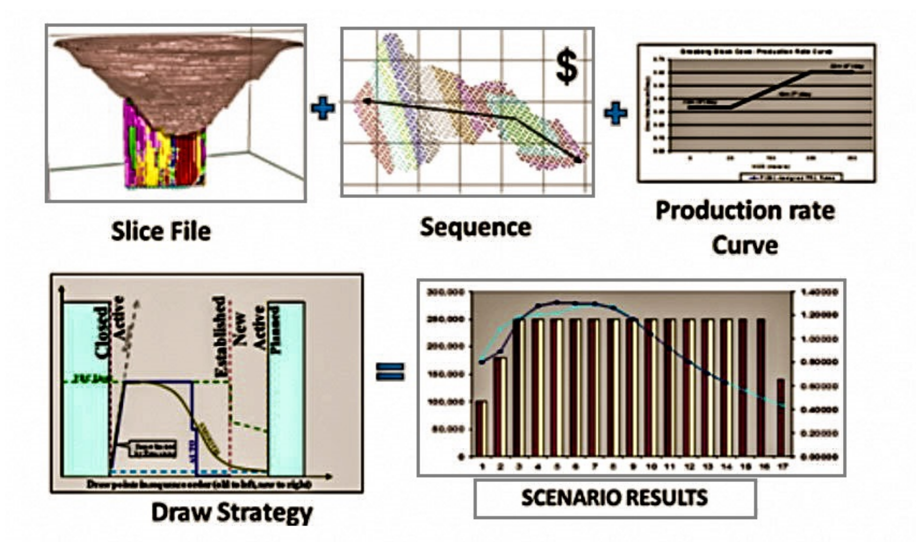

Figure 2.17: Production scheduling components in PCBC (Diering et al., 2010)

2.4 Modeling Boundary Uncertainty

The boundary uncertainty, which is one specific form of uncertainty, is present in the border between two geological units. Even though this uncertainty is commonly ignored in the evaluation of mineral deposits, this geological variation should be assessed. The benefit of knowing the boundary uncertainty for calculation of minable reserves can be significant for decision-making in mine planning.

Interpolating and mapping boundaries is the objective of geological modeling techniques. There are variety of modeling approaches available. However, the method based on the SDF, which is considered in this thesis, is coded to perform the geological modeling and the evaluation of boundary uncertainty. This robust and useful approach not only allows implicit interpolation, but also permits the adjustment of a bandwidth of variability by calibrating two parameters, C and β (Hosseini & Deutsch, 2007; Martin & Boisvert, 2017; Silva & Deutsch, 2012; Wilde & Deutsch, 2012). It is therefore ideal to evaluate the boundary uncertainty in block-cave projects. A Two-dimensional (2-D) location map, with two geological units that are contoured by various boundaries, is illustrated in Figure 2.18.

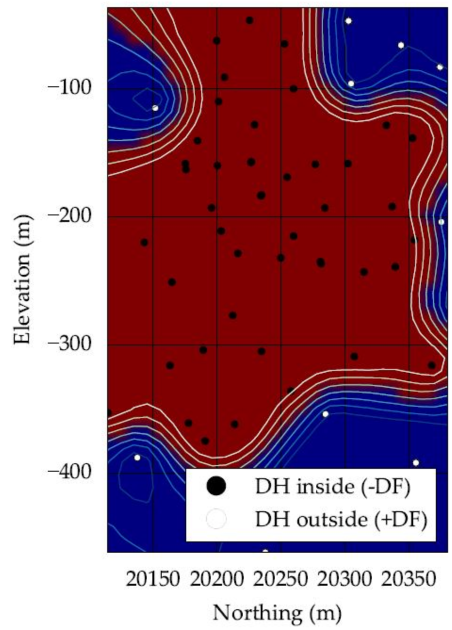

Figure 2.18: 2-D map with drillhole locations inside and outside the domain boundaries, Karpekov (2016)

The SDF is a technique for the geometric and volumetric modeling of categorical data, and is used for boundary modeling by interpolating a distance measure based on the Euclidean distance between a sample and the nearest sample with a different indicator type (Hosseini & Deutsch, 2007; Karpekov, 2016; Martin & Boisvert, 2017; Silva & Deutsch, 2012; Wilde & Deutsch, 2012). The algorithm begins with a binary categorical indicator of the following form (Deutsch & Journel, 1998):

$$i(\mathbf{u}_\alpha) = \begin{cases} 1, & \text{if domain present at } \mathbf{u}_\alpha \\ 0, & otherwise \end{cases}$$

Where $\mathbf{u}_\alpha$ is the sample location. The distance function is defined as positive outside the domain and negative within the domain of interest. Then, the interface that separates the regions in space is determined by the sign of the estimated signed values (Silva & Deutsch, 2012). In presence of anisotropy, the distance is calculated as follows (Karpekov, 2016).

$$distance = \sqrt{\left(\frac{dx}{ax}\right)^2 + \left(\frac{dy}{ay}\right)^2 + \left(\frac{dz}{az}\right)^2}$$

Where d is the separation between the two points in each of the x, y, and z directions, and a is the geometric anisotropy defined for each of the x, y, and z directions.

It is worth noting that the SDF interpolation can be performed by different methods. Inverse distance weighting (IDW) interpolation (Hosseini & Deutsch, 2007); global ordinary kriging (Silva

& Deutsch, 2012; Wilde & Deutsch, 2012); Local varying anisotropy (LVA) kriging and Radial basis function (RBF) interpolators (Martin & Boisvert, 2017). Although all the mentioned interpolation methods are useful, the RBF framework is preferred . Acoording to Cowan et al. (2003), Hillier et al. (2014), and Martin and Boisvert (2017), the RBF algorithm does not draw on first order stationarity and can honor locally variable shapes without special parameterization .Therefore, an artifact free map is guaranteed with RBF framework, it permit further surface generation or extraction of the indicator grids. Interested readers are referred to the manual for implicit modeling by Martin and Boisvert (2017).

2.5 Grade Modeling

Numerical grade modeling for mineral deposits can be classified as deterministic (estimation), when a single model of the deposit is obtained, or stochastic (simulation), when multiple models of the deposit is generated. The optimization method developed in this thesis uses the stochastic approach. However, the principles of both estimation and simulation techniques are reviewed.

2.5.1 Deterministic Modeling

The deterministic modeling approach starts with establishing mineralization controls using geologic domaining, based on geological knowledge. Once the domain is determined the grade interpolation with kriging or another linear estimator is performed (A. G. Journel & Huijbregts, 1978; Rossi & Deutsch, 2013). The common approach for deterministic block modeling is to obtain a value for each block of a single estimated model, generating the best possible prediction (Rossi & Deutsch, 2013). If using some form of kriging technique through estimation, a variogram model must be developed to capture the spatial relationship of the available data. The most frequently used kriging method in practice are Simple Kriging (SK) and OK.

SK forms the basis of several geostatistical techniques including the stochastic modeling approach used in this thesis. This method has the most basic form of kriging in the sense that the model is the simplest in its mathematical formulation. Consider the linear estimation of z_k at location $\mathbf{u}_0$ and scale v given a set of nearby, spatially correlated data of the same data type k:

$$z_k^*(\mathbf{u}_0; v) - m_k(v) = \sum_{i=1}^{n_k} \lambda_i (z_k(\mathbf{u}_i; v) - m_k(v)) \tag{2.1}$$

where $\lambda_i, i = 1, ..., n_k$ are estimation weights assigned to the n_k sample locations with data of type k. The estimation variance $\sigma_{k,E}^2$ of this linear estimator is calculated given the weights and covariances $C_k(v)$ where $C_{i,j}(v)$ is shorthand for the covariance between two locations, $\mathbf{u}_i$ and $\mathbf{u}_j$ with scale v:

$$\sigma_{k,E}^2(\mathbf{u}_0; v) = C_{0,0}(v) - 2\sum_{i=1}^{n_k} \lambda_i C_{i,0}(v) + \sum_{i=1}^{n_k}\sum_{j=1}^{n_k} \lambda_i \lambda_j C_{i,j}(v) \tag{2.2}$$

which is minimized by the simple kriging equations and also known as normal equations (Rossi & Deutsch, 2013):

$$\sum_{j=1}^{n_k} \lambda_j C_{i,j}(v) = C_{i,0}(v), i = 1, ..., n_k \tag{2.3}$$

$$\sigma^2_{k,SK}(\mathbf{u}_0; v) = C_{0,0}(v) - \sum_{i=1}^{n_k} \lambda_i C_{i,0}(v) \tag{2.4}$$

The OK method is preferred as it provides a robust approach for the spatial estimation (Rossi & Deutsch, 2013). However, the generated smooth model is not able to reproduce the local variability, making impossible the determination of the joint uncertainty between multiple locations within the resource model (Neufeld, 2005; Rossi & Deutsch, 2013). This estimation method provides limited mechanisms to integrate the resource estimation with certain mine-process optimizations. Therefore, any layout design or production schedule using OK estimations may fall short in terms of optimization and may generate sub-optimal solutions for complex problems in both open-pit and underground operations.

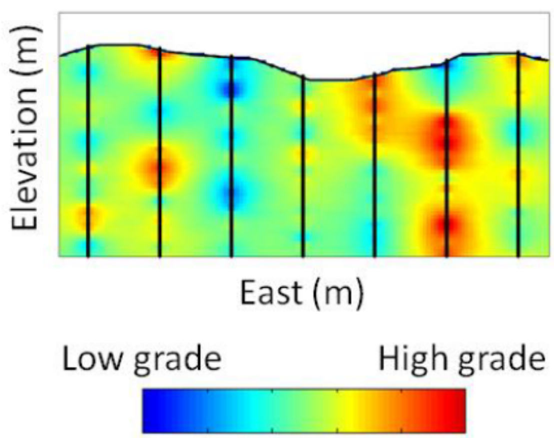

Figure 2.19: Cross-section of a smooth kriged model (Cuba, 2014)

The implementation of deterministic modeling for mining, including the most common methods used to estimate, is well documented theoretically and practically by various authors such as A. G. Journel and Huijbregts (1978); Goovaerts (1997); A. Journel (2007); and Rossi and Deutsch (2013).

2.5.2 Stochastic Modeling

The general idea of simulation is to generate a set of stochastic realizations which are capable to reproduce the histogram and the variogram of the original data. These multiple simulated models can capture the joint uncertainty between multiple locations while reproducing the local variability (Rossi & Deutsch, 2013). Simulation provides a set of possible values which can be integrated into various optimization frameworks in order to solve diverse problems in both open-pit and underground environments. Therefore, these realizations allow for performing not only accurate estimations but also uncertainty assessments to optimize mining processes (Acorn, 2017; Dimitrakopoulos, 1998; Koushavand, 2014; Rossi & Deutsch, 2013).

Various simulation approaches to perform numerical representations through block modeling are used in mining. They are a family of techniques based on the same basic algorithm, known as MCS. However, SGS is the most widely used modeling method for continuous variables (Figure 2.20).

The implementation of SGS is relatively simple, and begins with the transformation of the data to a normal scored distribution. Then, all data values, $z_k^l(\mathbf{u}_i; v), i = 1, ..., n_k, k = 1, ..., K, l = 1, ..., L$ are transformed to Gaussian units, typically with the use of a normal score transform (Deutsch & Journel, 1998). The Gaussian data are enumerated as follows:

$$y_k^l(\mathbf{u}_i; v), i = 1, ..., n_k, k = 1, ..., K, l = 1, ..., L$$

It is worth noting that after univariate transform the data is assumed to be multivariate Gaussian. The model locations are then visited sequentially and the simple kriging equations (2.1, 2.2, 2.3, 2.4) are then applied to calculate the mean and variance. A random residual $R(\mathbf{u}_0)$ is then sampled from a Gaussian distribution with a mean of zero and variance equal to the estimation (kriging) variance. The result is a set of simulated values with the correct variance and conditional covariances. This is added to the following estimate:

$$y_s^*(\mathbf{u}_0; v) = y^*(\mathbf{u}_0; v) + R(\mathbf{u}_0) \tag{2.5}$$

Simulation proceeds sequentially visiting each model location in turn. This process is repeated for each realization, for each variable, and for all locations within the stationary modeling domain D:

$$\{y_k^l(\mathbf{u}_i; v), i = 1, ..., n_D, l = 1, ..., L, k = 1, ..., K, \mathbf{u}\epsilon D\} \tag{2.6}$$

Data is back-transformed to original units and a set of equally probable models that represent the local variability and can capture the joint-grade uncertainty of the deposit is then generated (Figure 2.20).

SGS corrects the smoothness effect generated by OK, preserving the correlation between locations (Acorn, 2017). The standard application for modeling with SGS is well documented in several standart geostatistical books including Goovaerts (1997), Deutsch and Journel (1998), and Rossi and Deutsch (2013).

2.6 Uncertainty and Risk Management

In resource modeling and mine planning, various types of uncertainty are associated with the orebody, including grade, tonnage, volume, mining cost, etc. Therefore, the assessment of these uncertainties and the management of related risk are critical when developing a strategy for mine plan-

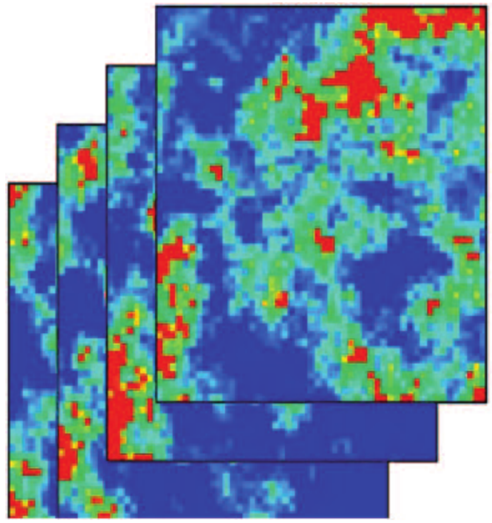

Figure 2.20: Multiple SGS realizations (Rossi & Deutsch, 2013)

ning because this uncertainty can lead to unexpected changes in production plans (Dimitrakopoulos, 2011; Osanloo et al., 2008).

The conventional mine planning, in both open-pit and block caving, starts by using a deterministic block model developed with drillhole information. Each block is later assigned with an economic value based on some deterministic attributes. The feasible mine plan is then determined under assumptions in which grades, metal prices and other attributes are known with certainty. In this context, the best possible mine plan maximizes the net present value. However, the main disadvantage of the deterministic approach is the lack for reproducing the uncertainty and therefore the related risk is practically not included.

Available research suggests that uncertainty assessments should be addressed by the evaluation of the main variables (e.g. grade, rock type, metal price, etc.) at various scenarios (Dimitrakopoulos, 1998; A. G. Journel, 1988). In an open-pit scenario, for instance, Whittle approach is improved to address the uncertainty on metal prices, which is highly variable on time and space. Moreover, the MCS framework using SGS is suggested to deal with the inherent orebody uncertainty. Simulation methods is then used to generate equally-probable models in order to solve diverse optimization problems (Rossi & Deutsch, 2013).

The overall idea of the MCS framework is to produce a distribution of responses (e.g. revenue, profit) rather than a single value of them. For example, the uncertainty associated with a long-term production scheduling can be explored from a distribution by determining a range of expected values and variance. Therefore, the selection of a technically feasible plan, that satisfies operational and technical constraints, can be made by comparing the economic alternatives through the uncertainty assessment and risk-based decisions (Figure 2.21).

A project evaluation, illustrated in Figure 2.22, is performed over a conventional estimated

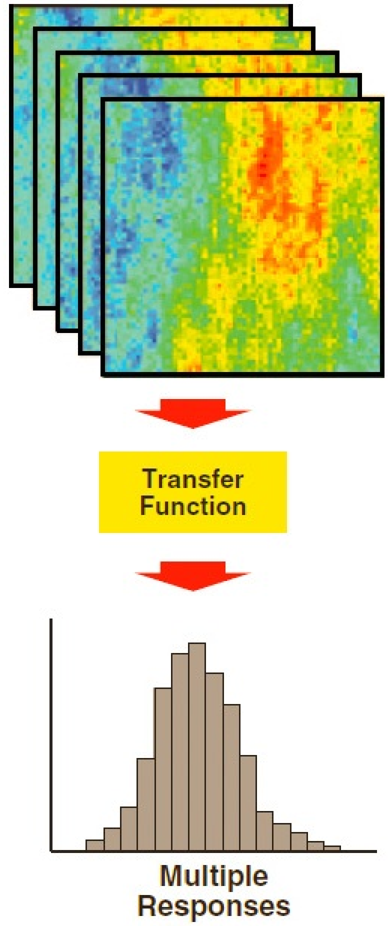

Figure 2.21: Uncertainty assessment framework over responses (Rossi & Deutsch, 2013)

model, as well as using a set of stochastic realizations (Dimitrakopoulos, 2011). The deterministic approach shows overestimation on the NPV results and a lower probability of occurrence compared to the stochastic methodology. Therefore, it can be concluded that the deterministic estimate model lead to the selection of a sub-optimal mine design. In this example, the expected value obtained by the stochastic method is around 17 MUSD, while the deterministic approach generates a value of 25 MUSD , but with a probability of occurrence of 4% (Dimitrakopoulos, 2011).

2.7 Optimization in Presence of Risk

Decision-making in the presence of risk, also known as risk-based decision, is known since the seventeenth century and some demonstrations on the subject is recorded in the available literature (Acorn, 2017; Bernoulli, 1954).

In the mining industry, most of the current effort has gone into optimization methods for open-pit mines to improve traditional processes and solve new problems related to mine planning, production scheduling, and mine designs. These are some of the areas where research has implemented

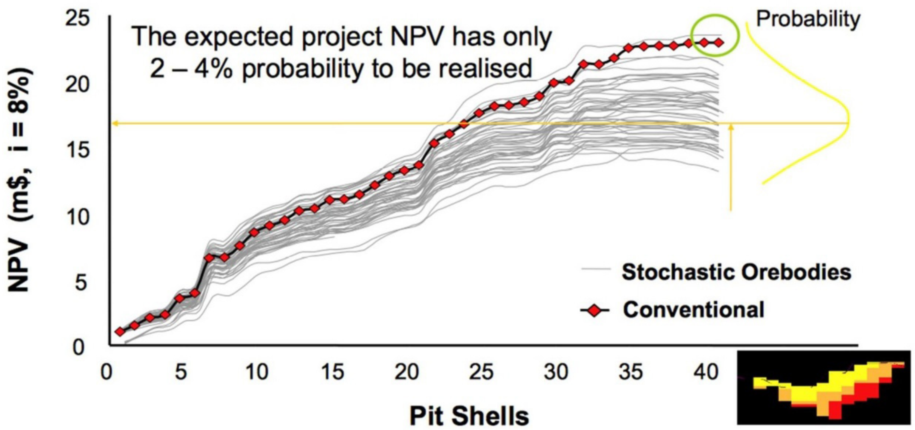

Figure 2.22: Comparison between conventional and stochastic decision under uncertainty for a final open-pit design (Dimitrakopoulos, 2011)

approaches to manage risk for decision-making (Acorn, 2017; Dimitrakopoulos, 1998, 2011; Koushavand, 2014). However, little research has been done to understand and manage risk in block-cave operations (Malaki, 2016). Decision-making related to mine development, production schedules and layout design of caving projects requires, as well, a good understanding of risk management for optimizing processes, and thus overcome potential shortfalls.

A Model for Active Risk Management

Optimization of processes or the determination of the optimal option over a set of feasible results under uncertainty is not always a trivial work. Although different approaches in the areas of engineering and finance provide a premise to manage risk, there is still a lack of robust and practical decision-making models in which various types of uncertainties can be managed in the mining and hydrocarbon sectors (Acorn, 2017; Gallardo & Deutsch, 2017; Malaki, 2016). For this reason, many investigations are currently focused to develop more robust techniques for risk-based decisions in both industries.

In the mining industry,for instance, Acorn (2017) presented an algorithm for Heuristic pit optimizer (HPO), in which an active risk management is implemented. The HPO method is developed based on the portfolio theory of efficient frontier that is proposed by (Markowitz, 1952). Although the HPO is an useful optimizer, the penalization factor recommended for managing risk reduces its robustness.

An innovative decision-making model for Active geological risk management (AGRM) is suggested by Gallardo and Deutsch (2017), for the hydrocarbon sector. The AGRM uses the combination of two selection criteria for decision-making. One is a trade-off between risk and reward, known as the mean-variance approach, based on the efficient-frontier theory (Markowitz, 1959). The second is the stochastic dominance rules (Hadar & Russell, 1969; Hanoch & Levy, 1969). This sequence of decision-making criteria allows for determine rationally the best option among a set of possible outcomes and is consistent with the theory of utility function (Figure 2.23).

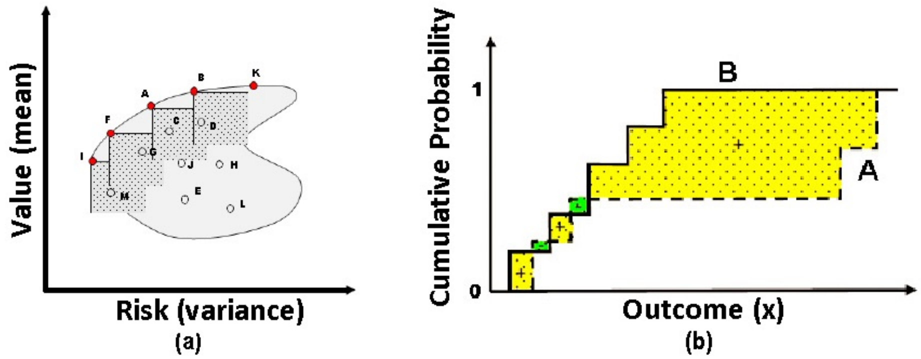

Figure 2.23: (a) mean-variance criterion, after Gallardo and Deutsch (2017), (b) stochastic dominance rules, after (Levy & Sarnat, 1970)Levy and Sarnat (1970)

The utility function is a concept widely used in economics and finance. This theory is according to the preferences of the decision maker, and is classified into three types (Figure 2.24). Risk-averse, risk-neutral and risk-taker (Kochenderfer, 2015), as shown in Figure (2.24). According to Gallardo and Deutsch (2017), there is evidence that investors (e.g. hydrocarbon, mining) show a certain degree of risk aversion, and consequently the AGRM is developed specially for these decision makers.

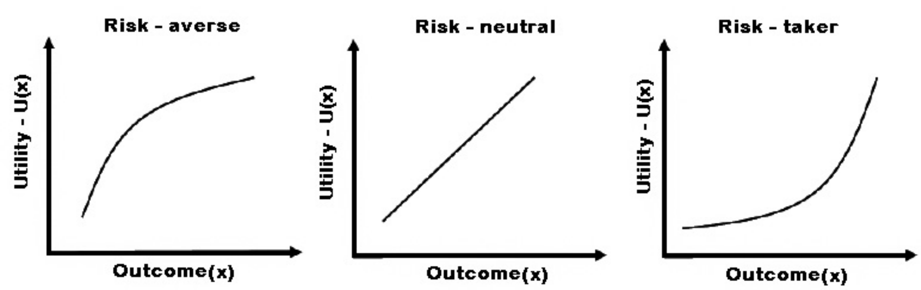

Figure 2.24: Classification of the utility functions; modified after (Kochenderfer, 2015)

The convenient characteristics of the AGRM model make it suitable for the mining industry, especially for block caving (e.g. layout design). The AGRM is therefore adapted for the production layout optimization included in this thesis. The formulation and implementation process of this risk-based model is expanded in Section 3.2.3.1.

Chapter 3

Drawpoint Layout Optimization

This chapter provides information related to a proposed methodology for optimizing the production level layout. The workflow is explained here and then demonstrated in Chapter 4.

3.1 Introduction

As mentioned in Chapter 1, the production level layout is one key engineering element in block caving and there are certain aspects that remain unsolved. A methodology with two important changes to the traditional approach is proposed here. The first modification is to optimize the drawpoint spacing and the second change to optimize the level of production.

The proposed methodology uses SGS to generate a set of equally-probable models that are used to optimize the drawpoint locations while being constrained by geological and mining factors (Figure 3.1).

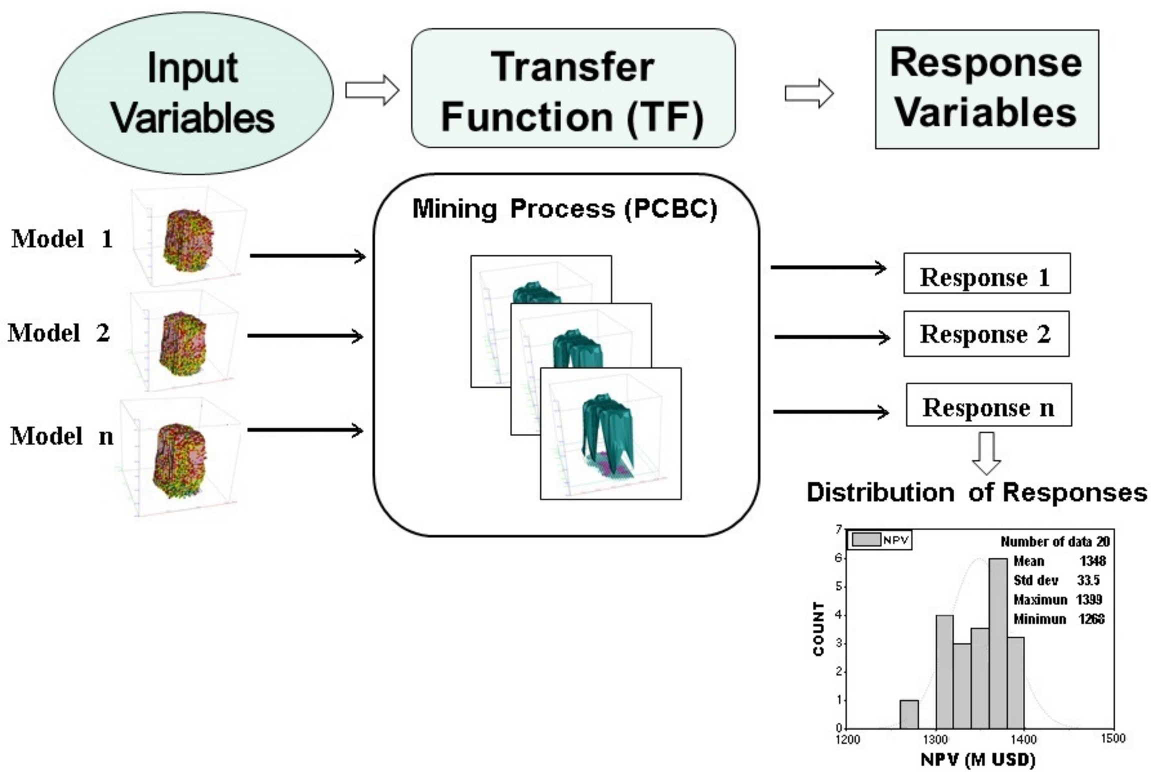

Figure 3.1: Framework for uncertainty assessment of responses

To determine the optimal drawpoint spacing and level of extraction, the optimization framework is linked to the transfer function that provides the response variables for decision making. The proposed method is illustrated graphically in Figure 3.2. These optimization steps are expanded in the following sections.

3.2 Optimization Workflow

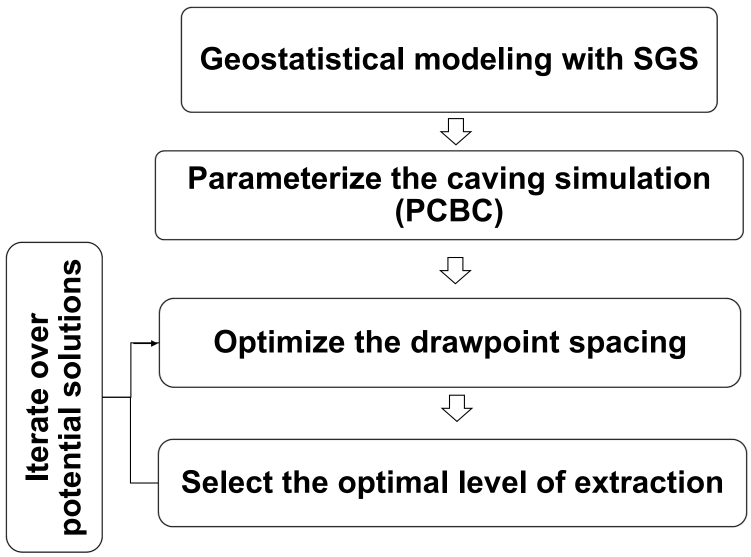

Figure 3.2: Workflow to optimize the production level layout

3.2.1 Stochastic Modeling with SGS

The methodology is based on a stochastic framework that quantifies geological uncertainty and maximizes the economic value of mining by obtaining the optimal production level layout over multiple SGS models. The principal contribution of this method is that is makes it possible to select the drawpoint spacing that is optimal over a set of realizations (Figure 3.3).

Prior to SGS modeling, the boundary delineation of the geological domains must be performed. There are various techniques for modeling geological domains including subjective interpretation based on geological knowledge, stochastic methods, and implicit modeling techniques. Implicit modeling is implemented here to delineate the domain boundaries. A signed distance function (SDF) with implicit modeling can also allow for incorporating boundary uncertainty assessment (Section 2.4). Once the boundary for each domain is defined, SGS is implemented to simulate the continuous variables (Figure 3.3). A comprehensive explanation of concepts and formulation of SGS is described in Section 2.5.

Once SGS realizations are generated, the most important question is how these multiple realizations can be used to solve one specific block caving problem? The answer is to use an stochastic framework to optimize this concern under uncertainty. Hence, SGS realizations should not be sin-

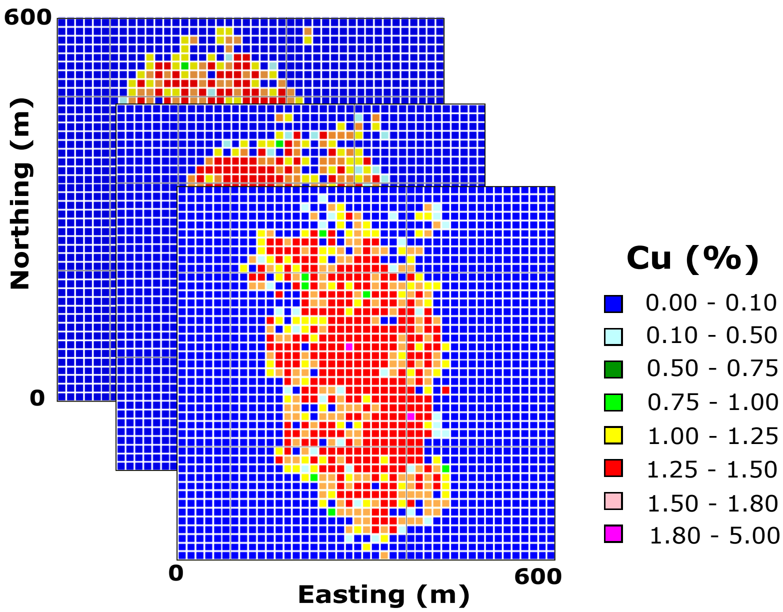

Figure 3.3: Multiple SGS realizations of grades performed in a block caving project

gled out to use in mining processes because optimizing over a single realization is unrealistic, all realizations should be evaluated together (Acorn, 2017; Deutsch, 2015; Rossi & Deutsch, 2013). As explained in Section 1.5 and illustrated in Figure 1.3, optimizing over all realizations is ideal. Figure 3.1 presents the general optimization framework; a set of equally probable models of a deposit is generated to optimize the drawpoint spacing design. In the following section, the transfer function for block caving is discussed, and the parameterization of PCBC is explored.

3.2.2 Parameterize the Transfer Function in Block Caving

PCBC (Diering et al., 2010) is used here to simulate block caving operations (section 2.3). PCBC is a mixing–model software in which layout design and production schedules are integrated into a single module. As explained in Section 2.3.3, the basic production schedule in PCBC can be generated once a production layout is assigned and the preparation of ore reserves is completed. Many constraint-driven decisions related to development, draw rate, mine capacity, metal targets and mining sequence need to be made in the mine schedule in which drawpoints are opened based on the selected sequence and depleted following the Production Rate Curve (Diering et al., 2010). Therefore, the amount of tonnage to be mined from the drawpoints along with grades and dollar value are determined in this stage.

To optimize the drawpoint spacing for maximizing economic return, a set of stochastic realizations are required. Each model must contain grades, rock types, and density. These realizations are imported into PCBC.

PCBC was developed based on the work published by Laubscher (1994); therefore, most of the input parameters used in the proposed approach for calculating the production schedules follows his guidelines. Relevant rock-mechanics data, such as the percentage of fines and rock fragmentation of the deposit must be considered. This information should be taken from previous geotechnical studies of the project. First, a model of percentage of fines is a required input in PCBC due to its importance for the mixing process of the block-cave algorithm. Second, the fragment sizes of in-situ rock is also essential for the optimization process. Section 2.2.3 explains that primary fragmentation helps to determine the IDZ, and consequently this relationship helps to determine the potential spacing configurations. This information is selected from Laubscher's guidelines (Figure 2.7).

Determination of the optimal layout design is the principal motivation of the proposed methodology. Therefore, a reasonable number of layouts must be selected for the optimization process. Although practitioners can be tempted to select a large number of configurations to reach the optimal option , the selection of these layouts must be conducted according to accepted design guidelines, Laubscher (1994), and considering experiences from worldwide caving operations. Then, a number of potential drawpoint designs are used in the optimization process. For example, 24 m × 15 m (Chacon et al., 2004); 31 m × 17 m and 30 m × 15 m (Chitombo, 2010); and 34 m × 20 m (Castro et al., 2012) are some possible layouts. An exhaustive search is proposed here to find a reasonable number of layout configurations S. This search is expressed as follows.

$$S = [s_{ij}]_{I \times J} \tag{3.1}$$

$$\begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1J} \\ s_{21} & s_{22} & \cdots & s_{2J} \\ \vdots & \vdots & \vdots & \vdots \\ s_{I1} & s_{I2} & \cdots & s_{IJ} \end{bmatrix} \tag{3.2}$$

I is the number of options for the spacing between production drifts (A), $i = 1, ..., I$; while J is the number of options for the spacing between drawpoints (B), $j = 1, ..., J$ (Figure 3.4).

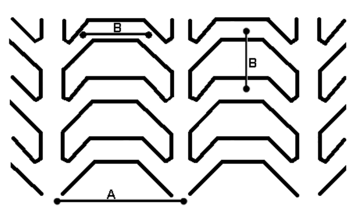

Figure 3.4: A typical herringbone layout, after Chitombo (2010)

In coarse-fragment mines, for example, the production tunnels are spaced each other from 30 m to 34 m long (Castro et al., 2012); and fragments ranges from 5 m^3 to 9 m^3 that corresponds to 13 m of IDZ with wider loading width(5 m), the minimum spacing is then 14 m and a maximum is 24

m (Laubscher, 1994). Therefore, using the exhaustive search, the potential distance between drifts are five, $i = 1, ..., 5$ or 30 m,..., 34 m; and the potential distances between drawpoints are eleven, $j = 1, ..., 11$ or 14 m,..., 24 m (Equation 3.3).

$$S = [s_{ij}]_{5\times 11} \tag{3.3}$$

The searching process generates a set of design configurations (Table 3.1) which are used to optimize the drawpoint spacing of a coarse-fragment project. The exhaustive search for proposed designs is applicable to any fragment size. The most commonly used layouts, Herringbone and El Teniente, are recommended (Figure 2.11); however, any layout type can be considered.

Table 3.1: Block caving layouts based on the fragment size and their correspondent IDZ

	14 m	15 m	16 m	17 m	18 m	19 m	20 m	21 m	22 m	23 m	24 m
30 m	30×14	30×15	30×16	30×17	30×18	30×19	30×20	30×21	30×22	30×23	30×24
31 m	31×14	31×15	31×16	31×17	31×18	31×19	31×20	31×21	31×22	31×23	31×24
32 m	32×14	32×15	32×16	32×17	32×18	32×19	32×20	32×21	32×22	32×23	32×24
33 m	33×14	33×15	33×16	33×17	33×18	33×19	33×20	33×21	33×22	33×23	33×24
34 m	34×14	34×15	34×16	34×17	34×18	34×19	34×20	34×21	34×22	34×23	34×24

Other mining parameters and relevant economic assumptions are required (Table 3.2). This information can be taken from previous engineering and economic studies of the project or perhaps similar operations. Economic data, such as development cost, mining cost, and the interest rate are included. The complete set of input parameters for the transfer function in PCBC is presented in Table 3.2.

Table 3.2: Summary of the main mining parameters and assumptions for block caving. Data is properly set in PCBC prior to optimization

Parameter	Symbol	Units	Description
% of fines	fn	%	Based on a model of fines
Density	ρ	g/cm^3	Average density for the domains
HIZ	-	m	Height for interaction zone
Swell factor	sf	-	Established by experience
HOD MAX	-	m	Maximum height of development
HOD MIN	-	m	Minimum height of development
Initial elevation	iz	m	Initial elevation of extraction
IDZ	-	m	Isolated draw zone diameter
Mining cost	c_m	USD/per ton	Current block mining cost
Capital cost	c_d	MUSD	Development cost
Revenue factor	f_{rev}	USD/%	Unit revenue factor
IR	-	%	Interest rate
Layout type	-	-	Herringbone and El Teniente
Initial elevation	-	m	Initial elevation to initiate scheduling

3.2.3 Optimize the Layout to Maximize NPV

The main purpose of the methodology is to maximize the value of block-cave mines, principally profit by determining the optimal layout. Therefore, the outcomes generated in the transfer function can be presented in terms of (tonnage & grade) or profit (NPV).

Optimization is broken into two steps which are executed within an iterative workflow (Figure 3.13). Step one is to determine the optimal drawpoint spacing, and step two is to select the optimal extraction level.

3.2.3.1 Optimize the Drawpoint Spacing

The simulated models of the mineral resources in a transfer function (scheduling in PCBC)is used to determine the optimal drawpoint spacing. Several layouts are evaluated over all realizations in PCBC to examine multiple scenarios of production schedules in a brute-force approach. This method is "complete enumeration where simply all possible (and valid) values for the decision variables are tested" (Maringer, 2006).

For instance, all possible responses from the set of proposed designs (e.g. fifteen layouts) are calculated in PCBC and the responses are presented here in terms of profit (Figure 3.5).

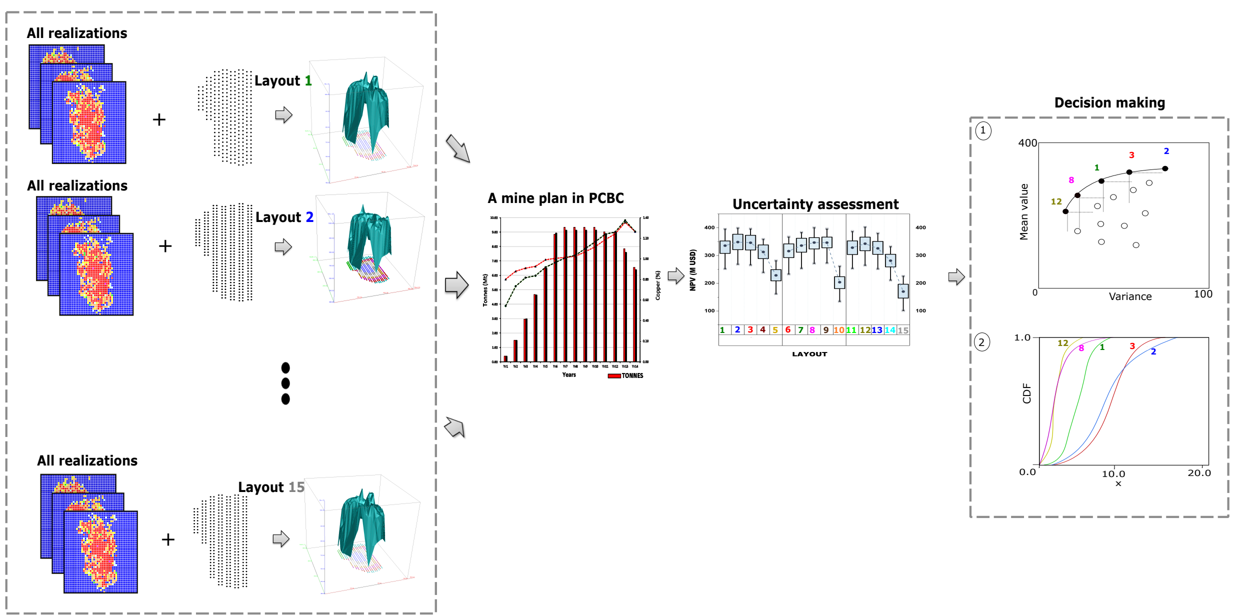

Figure 3.5: Optimization over all stochastic realizations. The transfer function is the mine planning module, and the interest value is profit (NPV)

Realization l, and layout s go into the calculation of the value of interest; considering the layouts as $s = 1, ..., S$ and realizations $l = 1, ..., L$. When the value of interest is profit is denoted as : $P_s\left(l\right)$.

In the hypothetical example of Figure 3.5, $S = 15$, L = all realizations, and the $P_s(l)$ is ranging from 100 to 400 MUSD.

Uncertainty Assessment and Risk-Based Decision

The quantity of responses obtained from PCBC depends on number of proposed layouts and number of realizations. S is the number of proposed layout configurations and L is the number of realizations, the total number of responses is then $S \times L$. The main objective in this step is to organize responses by layout, and therefore generate S distributions to assess uncertainty.

The distribution of each layout, $s = 1, .., S$, is therefore evaluated to provide the expected value(3.4) and variance(3.5). $E[P]_s$ is the expected value for each layout distribution.

$$E[P]_s = \frac{1}{L} \sum_{l=1}^{L} P_s(l) \tag{3.4}$$

The variance of each layout is the measure of risk and is expressed as follows.

$$R_s = \frac{1}{L} \sum_{l=1}^{L} \left(E[P]_s - P_s(l)\right)^2 \tag{3.5}$$

Once the expected value, $E[P]_s$, and measure of risk, R_s, of the S distributions are calculated, the assessment of uncertainty is possible. Therefore, the $E[P]_s$ and R_s of responses on layouts, $s = 1...S$, are used in the decision-making process.

In the example with $S = 15$ (boxplots in Figure 3.6), the options represent the uncertainty in the S distributions. This example includes fifteen layouts and L realizations; and therefore the response values are organized in fifteen distributions (Figure 3.6).

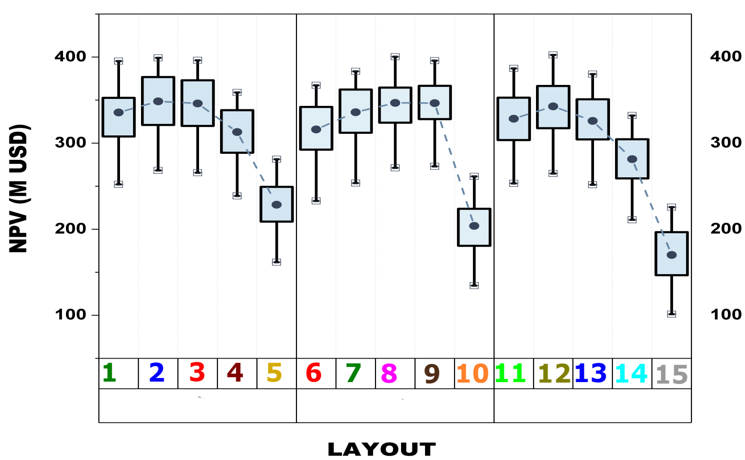

Figure 3.6: Responses of fifteen layouts organized in fifteen distributions of profit (NPV)

The proposed optimization framework uses an active risk-based model. This model contains two decision criteria and is applied here for decision-making, as explained in Section (2.7).

The decision process for the drawpoint spacing over the S layouts starts by using the mean-variance criteria. The options (Figure 3.7) represent the mean-variance relationships from the 15 layout designs

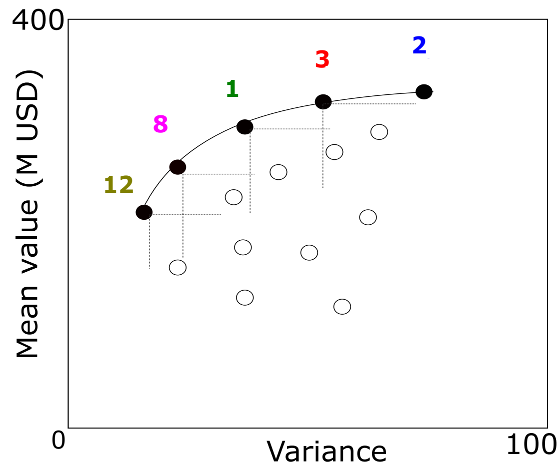

Figure 3.7: Mean-variance criterion for the 15-layout example, modified after Gallardo and Deutsch (2017)

The black dots lay on the efficient frontier curve of Markowitz (1952); 1, 2, 3, 8, and 12 are the best layouts. Each black dot dominates the options in circles located to its southeast (Figure 3.7). In the next step, the decision model is to apply stochastic dominance rules. The First-degree stochastic dominance (FSD) is applied first and illustrated in Figure 3.8a, after that the Second-degree stochastic dominance (SSD) is applied (Figure 3.8b). The FSD indicates that given two cumulative distributions $F(x)$ and $G(x)$, F dominates G, if $F(x) \leq G(x)$, for all values of x; with FSD more value is preferred. FSD decisions are valid for any metric of value. Therefore, layouts 2 and 3 dominate layouts 12, 8, and 1 (Figure 3.8a).

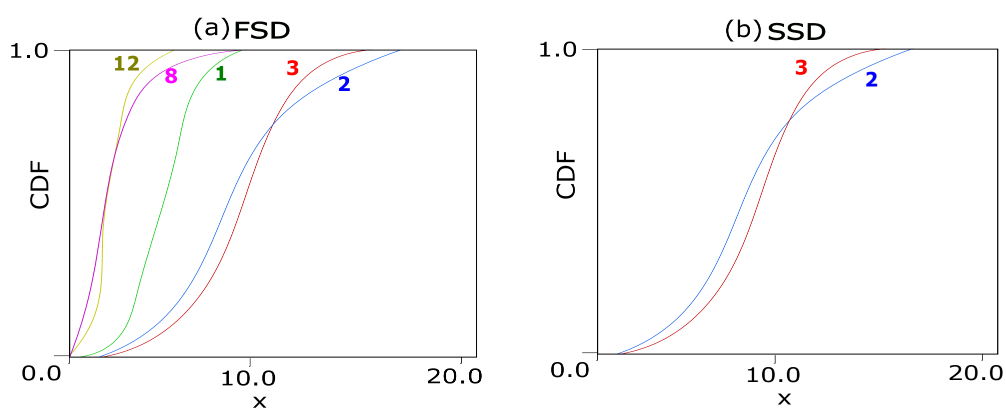

Figure 3.8: The FSD and SSD rules are applied on layouts selected by the efficient frontier (mean-variance), modified after Gallardo and Deutsch (2017)

The SSD explains that the selection criterion of the optimal drawpoint spacing must be led by a well-informed decision according to practitioner's preferences and a notion of economic rationality as explained in Section (2.7). Layout 3 dominates layout 2 if the practitioner is risk-averse; otherwise, the decision maker is risk-taker (Figure 3.8b).

3.2.3.2 Optimize the Level of Extraction

There are well-established approaches to determine the initial level of extraction of block caving. For example, the Footprint Finder in PCBC as described in Section 2.3.1; another method to obtain the level of extraction is proposed by Malaki (2016), this method determines the best elevation of production accounting for grade uncertainty and the maximum discounted profit. However, both the Footprint Finder and Malaki's method did not consider an optimal drawpoint spacing to calculate the optimal recoverable tonnage that has an important influence on the final profit; consequently, the level of extraction is not optimal. Then, an approach to optimize the extraction level is proposed here in which the optimal elevation is found in an iterative process. The optimization of this elevation takes advantage of the previous determination of the optimal drawpoint spacing. The general idea is to optimize the level of extraction over a potential number of elevations starting from an initial elevation (Z_i), as illustrated in Figures 3.9 and 3.10.

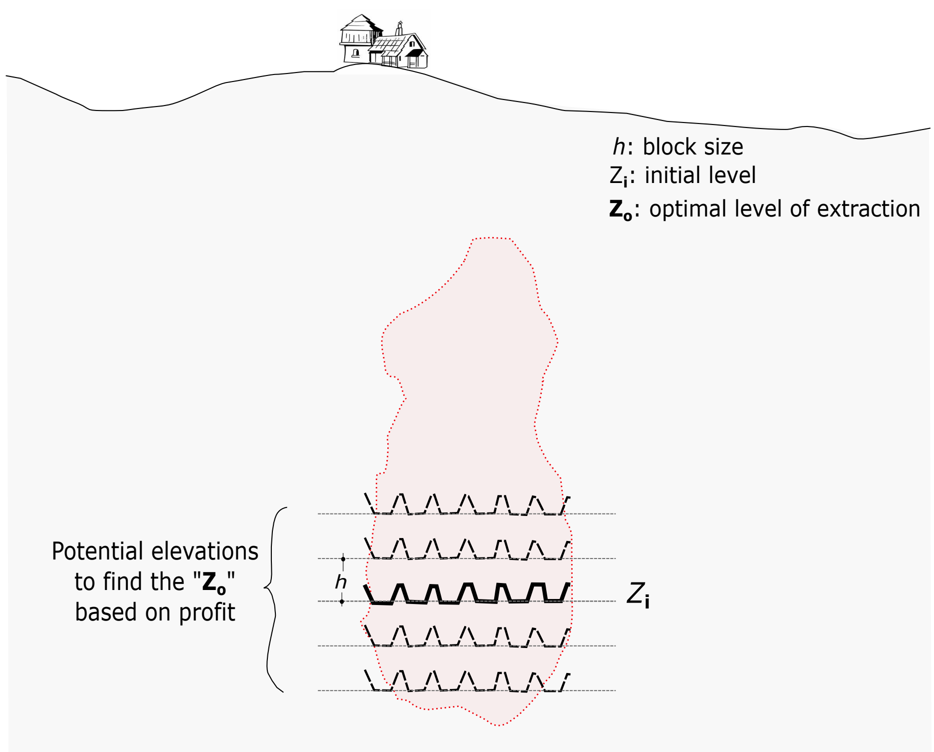

Figure 3.9: Potential elevations to optimize the level of extraction

Optimization of the production level is performed by using PCBC to calculate value over multiple SGS realizations. The optimal drawpoint spacing and several levels (elevations) from above and below the initial level are used in this optimization step (Figure 3.10).

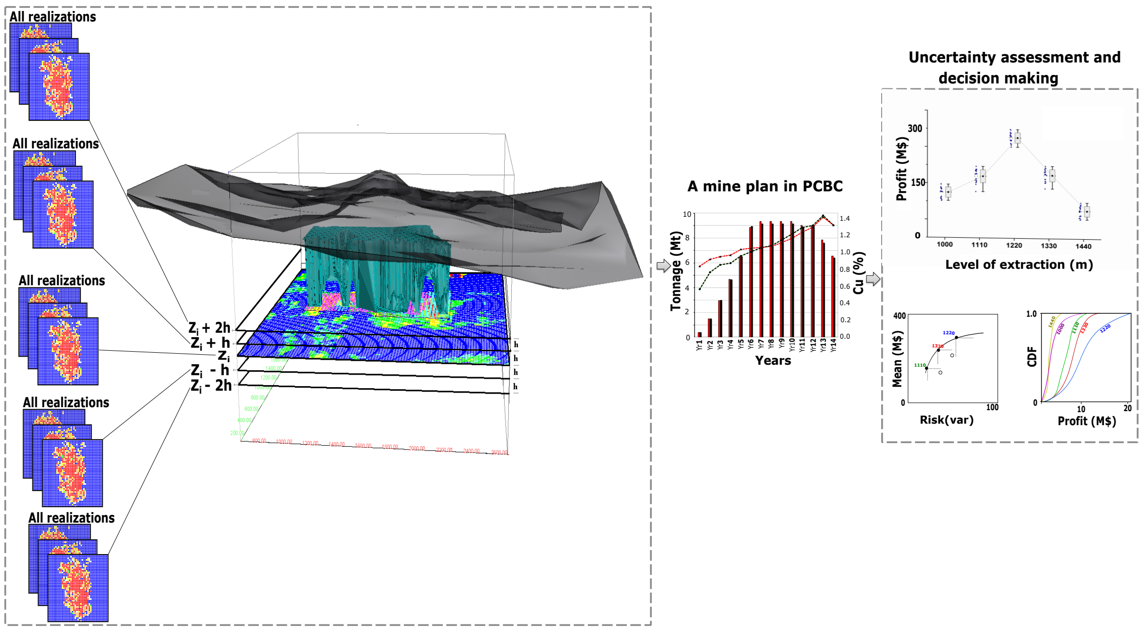

Figure 3.10: Optimization of the level of extraction over all realizations using the optimal drawpoint spacing

The illustration in Figure 3.10 shows the initial level, Z_i, and four selected elevations. The horizontal levels are separated vertically by a distance that is defined according to the length of the vertical block size of the models (h).

The number of responses produced in PCBC, this time, depends on number of proposed elevations and number of realizations. Z is the number of proposed elevations and L is the number of realizations, the total number of responses is then $Z \times L$. The main objective in this step is to organize responses by levels, and therefore generate Z distributions to assess uncertainty.

The values of Z elevation are calculated over L realization (all realizations). A single configuration (optimal drawpoint spacing) is used and the optimal elevation of extraction is then obtained.

The expected value, $E[P]_z$, and measure of risk, R_z, of the Z distributions are calculated. The response distributions on the elevations, $z = 1, ..., Z$, are then assessed for uncertainty (Figure 3.11).

As explained in Section (3.2.3.1), the $E[P]_z$ and R_z of the elevations, $z = 1, ..., Z$, are used for decision-making. The decision process for level of extraction over a set of Z elevations (e.g. $Z = 5$) is conducted by using the mean-variance criteria together with the stochastic dominance rules (3.12), similar to the process explained in Section (3.2.3.1). Five box plots showing the uncertainty assessment of responses is illustrated in Figure 3.11.

One example of the decision-making process is shown in Figure 3.12. In the example , the mean-

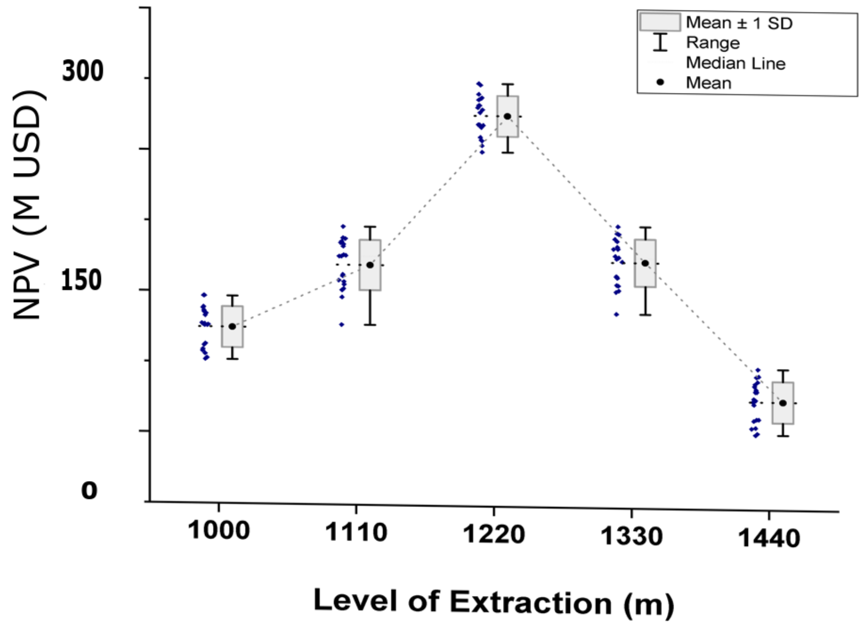

Figure 3.11: Assessment of uncertainty of responses from extraction levels

variance criterion and the stochastic rules are also applied to determine the best option. Notice that, with FSD more profit is preferred. The optimal level of extraction is then 1220m (Figure 3.12).

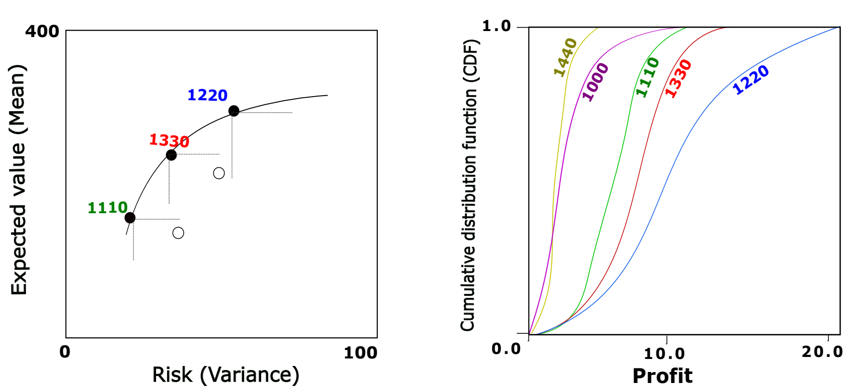

Figure 3.12: Risk-based decision on the set of elevations, 1220 m is the optimal level of extraction

3.2.3.3 Iterative Process to Determine the Optimal Production Layout

This section explains an iterative approach to optimize the production level layout (Figure 3.13). The initial level of extraction is Z_i, and the optimal level of extraction is Z_o.

This iterative system in performed in the following steps:

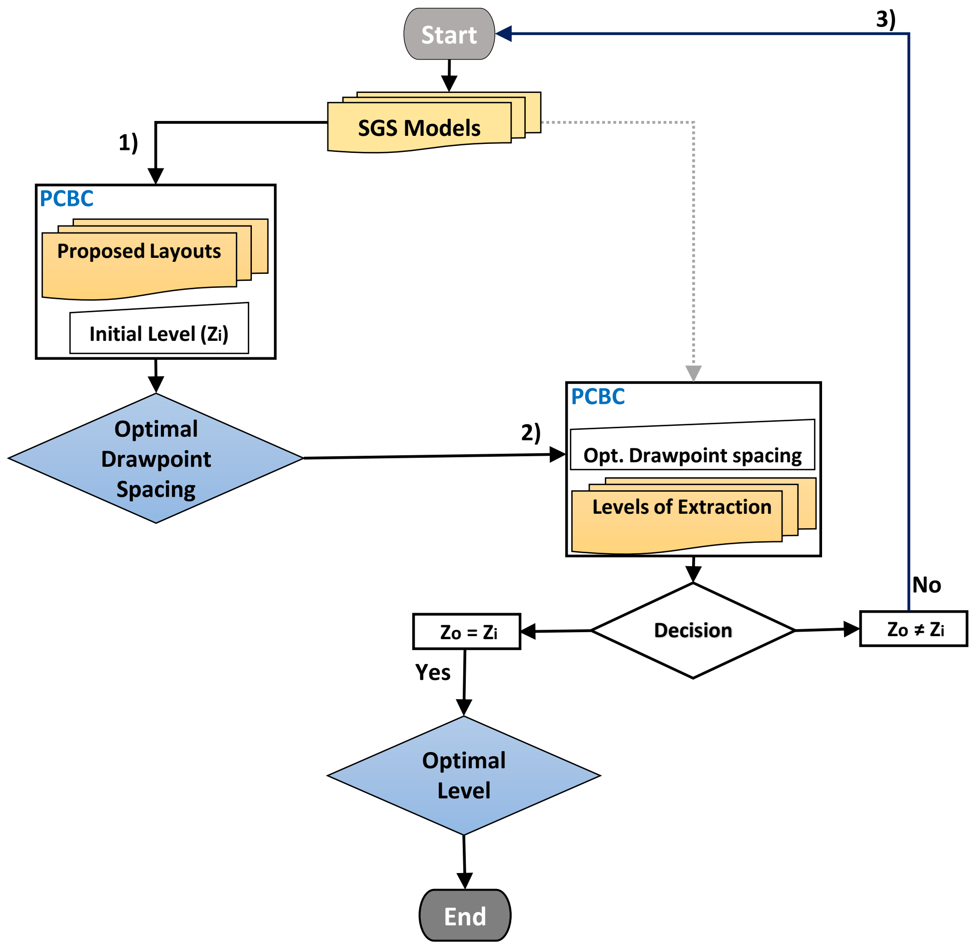

Figure 3.13: Iterative optimization workflow

1. The iterative process starts with the first optimization step, and thus the optimal drawpoint spacing is determined in PCBC over all SGS realizations using several proposed layout configurations at the initial level, Z_i.

2. Once the optimal drawpoint spacing is obtained, the Z_o is determined in PCBC by evaluating the optimal drawpoint spacing at different levels of extraction. If $Z_o = Z_i$, the optimal level, Z_o, is encountered; and therefore, the optimal drawpoint spacing and optimal level,Z_o converge and the optimization process ends.

3. However, if $Z_o \neq Z_i$, a second iteration should start. The best level, Z_o, is now used as the initial level, Z_i, in PCBC, and therefore steps (1) and (2) are repeated sequentially until the optimal drawpoint spacing converges with the optimal level,Z_o, provided that $Z_o = Z_i$.

N successive optimization iterations will end once the optimal drawpoint spacing converges with the optimal level, Z_o. Convergence depend on several factors such as grade variability, number of layout configurations, and proposed levels of extraction.

3.3 Summary

The production layout is a key element in block caving and is currently often determined by empirical methods. The methodology presented, proposes two important contributions to this traditional approach: (1) optimize the drawpoint spacing, and (2) optimize the elevation of production.

The optimization procedure is developed to maximize mine value over many realizations while being constrained by mining considerations (Ugarte, Pourrahimian, & Boisvert, 2017). To determine the optimal drawpoint spacing and level of extraction, the optimization framework includes a transfer function. This provides the response values for decision making.

The proposed approach begins with the delineation of geological domains perhaps with boundary uncertainty. Once the orebody is defined, SGS is implemented to simulate the continuous variables and then generate multiple stochastic realizations that are the main input in the optimization framework. PCBC is used to simulate the block caving operations in which layout design and a transfer function are integrated to generate a set of response values. The transfer function is parametrized in PCBC prior to the optimization process and is then performed considering various mining constraints.

The optimization procedure is broken into two steps. Step one is to optimize the drawpoint spacing, this is the main step of the proposed methodology. A reasonable number of proposed layout configurations are selected to determine a configuration with the optimal drawpoint spacing. The idea at this stage is to use the simulated models in the selected transfer function. Therefore, the layouts are evaluated over all realizations in a brute-force manner. All possible responses are then assessed for uncertainty for determining the optimal drawpoint spacing in presence of risk. Step two is to optimize the extraction level using the optimal drawpoint spacing. Several proposed levels are used to determine the best extraction level.

This optimization method is executed in an integrated iterative fashion in which the optimal drawpoint spacing and the optimal level of extraction,Z_o, are determined. In this way, risk can be assessed and mitigated by selecting the production layout that maximizes mine value but also minimizes variance.

Chapter 4

Case Study: Drawpoint Optimization with Orebody Uncertainty

This chapter provides an illustrative case study with a copper-gold caving project. The approach demonstrates the impact of relevant sources of uncertainty (grade and domain boundaries) for evaluating the layout design. First, the evaluation of grade uncertainty is measured in terms of NPV. Second, domain boundary uncertainty is incorporated. The methodology proposed in Chapter 3 is applied here using Gaussian simulation and a signed distance function for boundary uncertainty.

As mentioned in Chapter 2, cave mining is acknowledged as a complex environment with many sources of uncertainty related to exploration, engineering and economics. The focus of this research is to evaluate the main aspects of geologic uncertainty using stochastic models and then provide risk assessments from a set of mine designs based on the economic return (NPV). However, this work is not intended to provide a comprehensive overview of all sources of uncertainty, such as data collection, resource classification, dilution, geotechnical parameters, mining and metallurgical recovery, metal price, operating cost and so forth.

The data given for the development and implementation is from a block caving project and contains only basic drillhole information. Data with mining parameters and economic assumptions have been extracted from various references and similar block caving projects. Thus, a complete and exhaustive set of data and mining parameters of the project under study is advisable for real implementation.

4.1 Geology and Basic Statistics

The data for the case study is composed of 37 drill holes from a confidential block caving mine. Therefore, an exhaustive detail of the sampling methods, geology and origin of mineralization is not provided in this manuscript. The studied project is interpreted as an intrusive-hosted deposit with copper-gold mineralization trapped in a sub-vertical igneous rock surrounded and covered by sterile host rocks, as shown in Figure 4.1.

Table 4.1: Cu samples statistics by rock type

Lithology	N	% samples	Max(%)	Mean (%)	Std dev.(%)	Coef. Variation
Rock 1	777	52.5	0.099	0.051	0.028	0.55
Rock 2	93	6.3	0.099	0.054	0.028	0.51
Rock 3	609	41.2	3.66	1.37	0.433	0.32

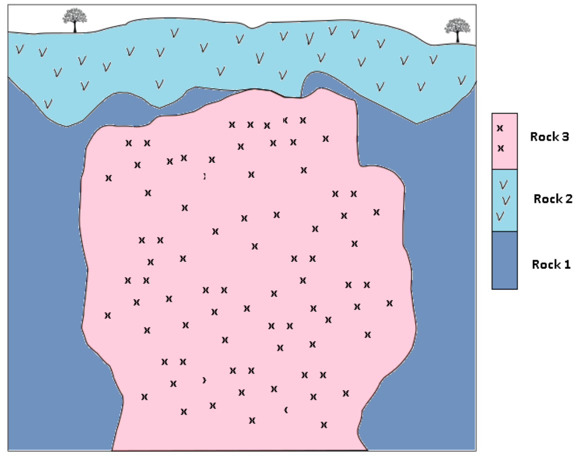

Figure 4.1: Geological interpretation of the project, a section view looking East.

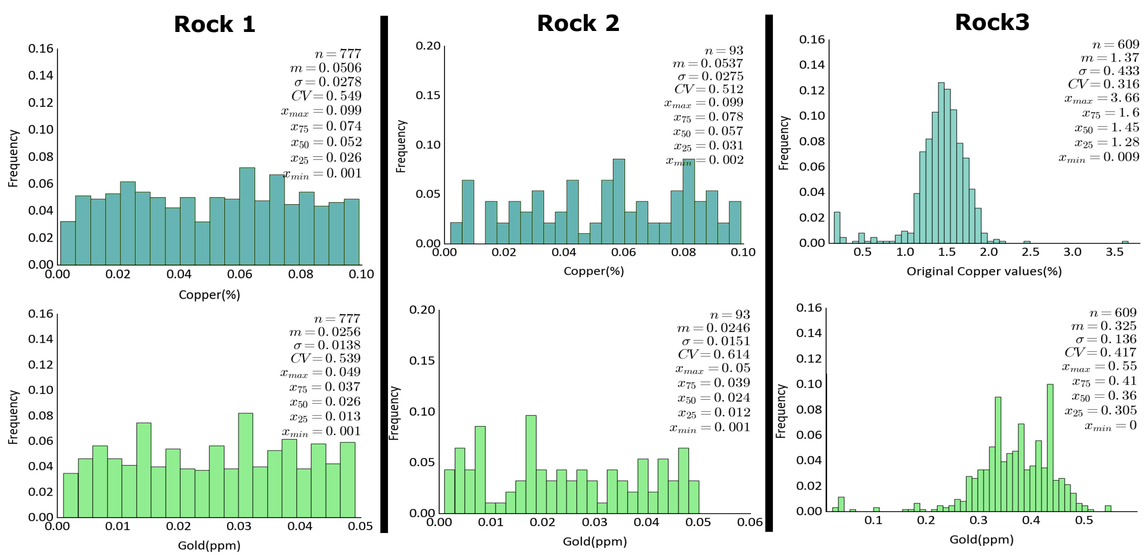

Figure 4.2: Histograms of Cu and Au for all rock types

Table 4.2: Au samples statistics by rock type

Lithology	N	% samples	Max (ppm)	Mean (ppm)	Std dev.(ppm)	Coef. Variation
Rock 1	777	52.5	0.049	0.026	0.014	0.54
Rock 2	93	6.3	0.05	0.025	0.015	0.61
Rock 3	609	41.2	0.55	0.325	0.136	0.42

The basic statistics by lithology type is presented in Figure 4.2. The following conclusions can be drawn from Figure 4.2 and Tables (4.1, 4.2):

- There are a number of samples with very low values in the rock types, and some high-grade outlayers of copper (Cu) and gold (Au) are detected in Rock 3, which are not capped due to the assumption that these were correctly sampled. Grade simulation may not be affected by these grades because SGS reproduces these values better than deterministic techniques.

- There are only 93 samples in Rock 2, which is not enough to form a domain; in addition, the statistics from rock 1 and rock 2 show very close values. Little mineralization is deposited in these host rocks. These sterile units probably need to be grouped in a unique domain for modeling purpose because block caving is concerned for mining ore and the proposed research intends to assess grade uncertainty in the orebody portion of the deposit.

4.2 Part 1: Optimization Considering Grade Uncertainty

This section includes grade uncertainty for optimizing the production level layout and shows the complete implementation of the methodology proposed in Chapter 3.

4.2.1 Domain Definition and Contact Analysis

Common practice in mineral resource-reserve estimation requires the boundary delineation of domains. This criterion is used to spatially delimit areas prior to geostatistical modeling in which the geological homogeneity is assumed (McLennan & Deutsch, 2006).

As explained in Section 4.1, three rock types are defined: rock 1, rock 2, and rock 3 (Figure 4.3). The final estimation domain is constrained by the amount of data available, which requires that some data be grouped.

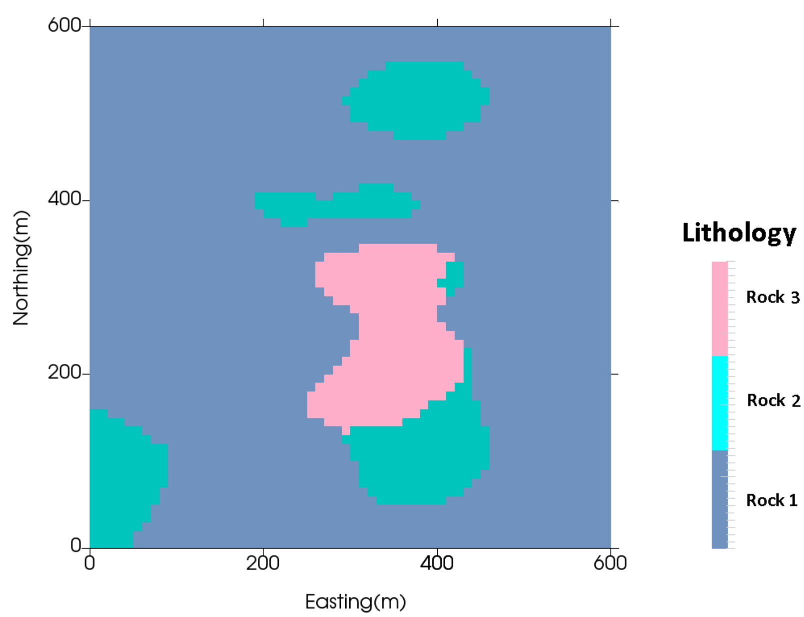

Figure 4.3: Horizontal slice with the lithology: Rock 1, Rock 2 and Rock 3

The analysis of the grade within rock contacts is performed here to help justify the decision of stationarity and to define the simulation approach. Figure 4.4 presents the grade profiles with the average grades of all composites in both sides of the contact by class distance (10 meters). The overall average of grade are different, and they show a clear hard boundary between Rock 3 and Rock 1, also Rock 3 and Rock 2; however, Rock 1 and Rock 2 show a soft boundary.

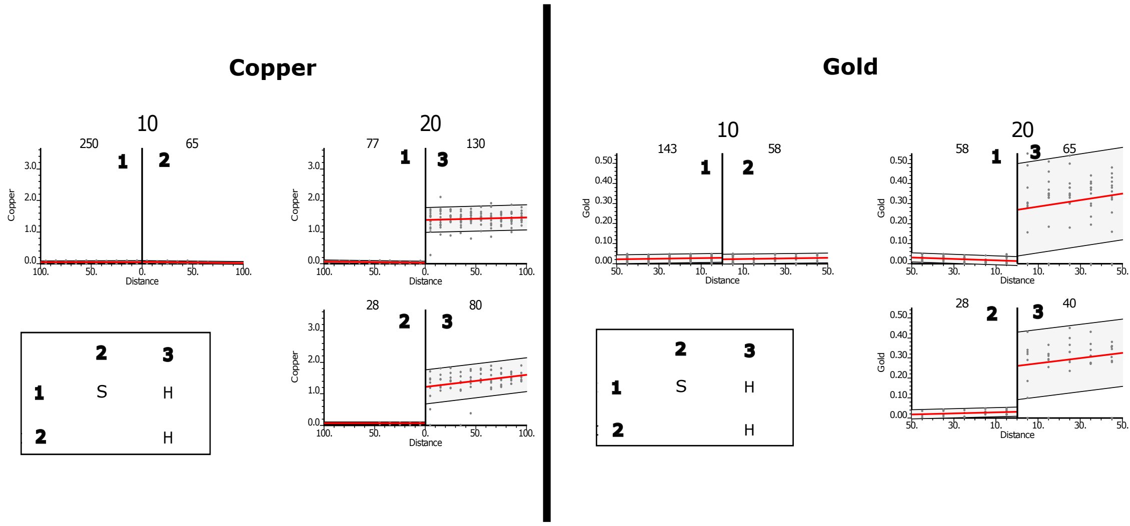

Figure 4.4: Contact analysis of the rocks

The estimation of domains is made based on the combination of lithology and mineralization controls. Two modeling domains are defined: Dom 3 and Dom 1-2 and (Figure 4.5). Dom 3 delineates an intrusive unit where the orebody is located. As illustrated in Figure 4.2 and Tables (4.1, 4.2), the statistics from rock 1 and rock 2 presents very similar values; moreover, the contact analysis between these units shows smooth contacts (Figure 4.4). Therefore, the country rocks, rock 1 and rock 2, are arranged in Dom 1-2, this domain represents the sterile material.

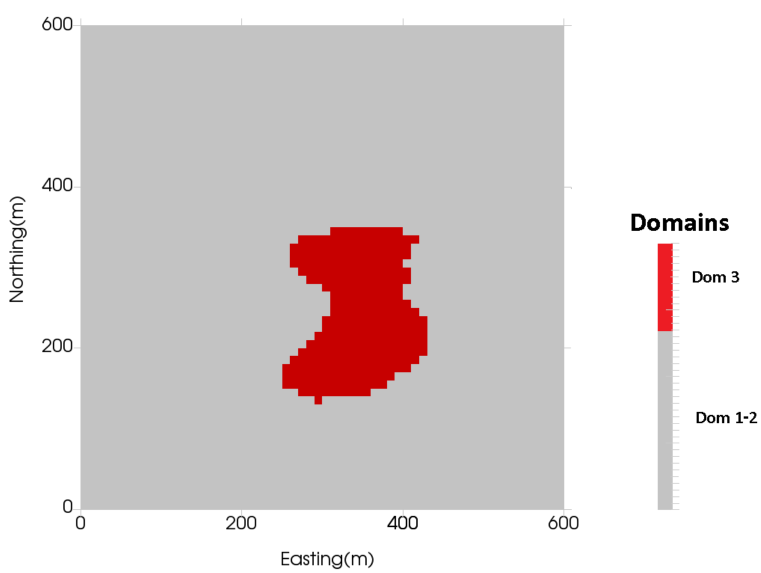

Figure 4.5: Horizontal slice, level 500 m, with the domains: Dom 1-2 and Dom 3

4.2.2 Geostatistical Modeling with SGS

The dataset considers Cu and Au as the continuous variables of interest. The orebody extents in each drill hole are known and the drilling assays have been composited to 10 m. The mean and variance of the original Cu and Au grades for Dom 3 are shown in Figure 4.6. The mean of Cu is 1.36% and mean of Au is 0.325 ppm.

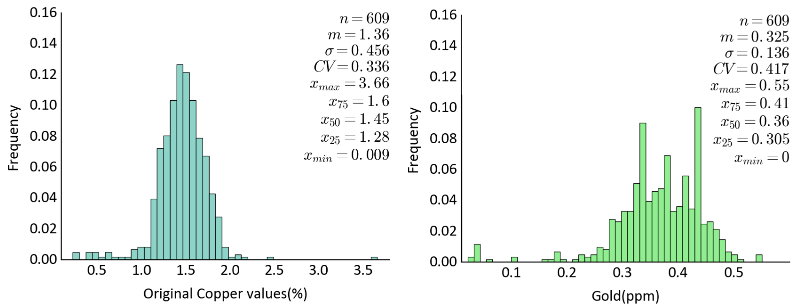

Figure 4.6: (a) Original Cu data, and (b) original Au data for Dom 3

Part of the data are spatially clustered as a result of inclined drilling. Cell declustering (Deutsch & Journel, 1998) is applied to the 10-m composites for estimation to obtain an unbiased prediction of the global mean. After processing the result of various cell sizes , a 150 m x 150 m was chosen as the optimal cell (Figure 4.7).

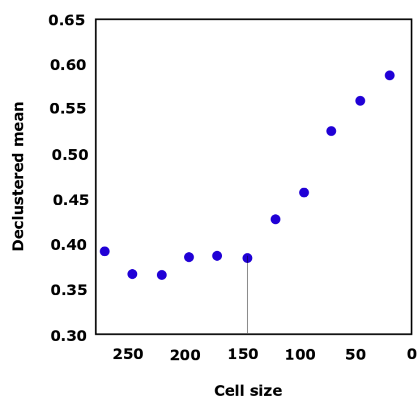

Figure 4.7: Declustering-cell, 150 x 150 is the optimal

The mean of the declustered Cu is 1.233 %, the standard deviation is 0.545 %. The mean of Au is 0.284 ppm, the standard deviation is 0.163 ppm.

The spatial continuity is addressed with variograms for each domain. The variogram model used for SGS is shown with blue lines in Figure 4.9. Twenty stochastic realizations of Cu and Au grades are generated (Figure 4.8).

Finally, the SGS realizations are checked by reviewing the variogram reproduction. The variogram reproduction of Cu and Au for Dom 3 is shown in Figure 4.9. The blue solid line is the horizontal variogram of the samples; the gray lines are the variograms of the realizations and the

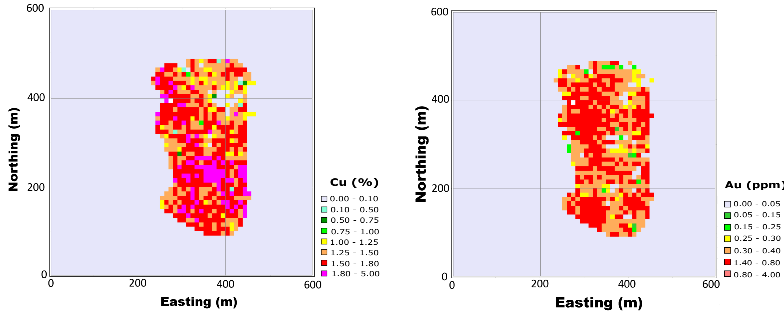

Figure 4.8: Two plots from SGS realizations, level 500 m , showing Cu and Au grades

dashed cyan line is the averaged variogram of the realizations. Note that there is a good reproduction in the variograms of Cu and Au for Dom 3. The histograms of Cu and Au for Dom 3 (Figure 4.10) show a good reproduction in the realizations. Therefore, good input parameters are assumed.

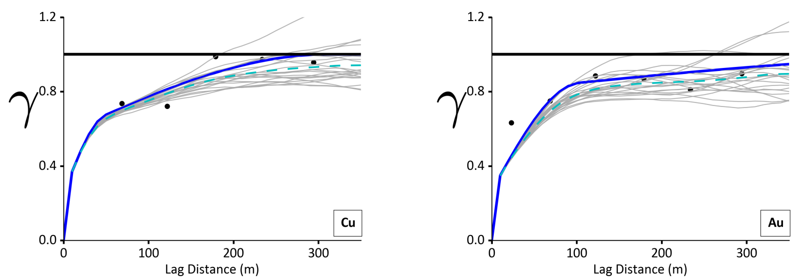

Figure 4.9: Variogram reproductions of Cu and Au for Dom 3. The blue solid line is the horizontal variogram; the gray lines are the variograms of the realizations and the cyan dashed line is the averaged variogram of the realizations

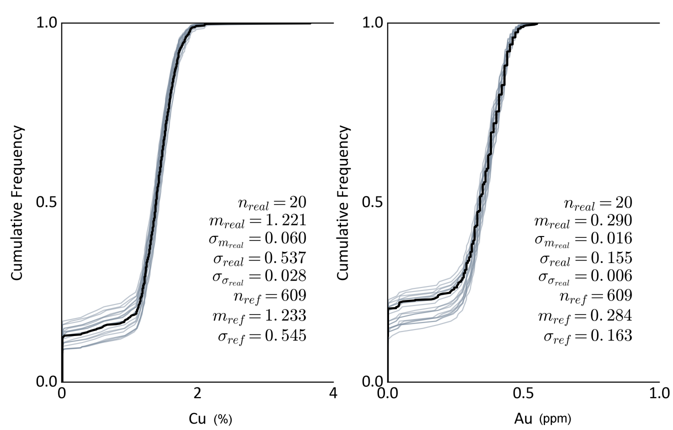

Figure 4.10: Histogram reproductions for Cu and Au for Dom 3. The black solid line is the referential data and the realizations are in gray lines

4.2.3 Mining Parameters and NPV Calculation

The value of parameters used in PCBC for this study have been summarized in Table 4.4. The average fragment size is roughly the size of a moderately fractured rock and has been measured as ranging from 0.5 m^3 to 1.0 m^3 and is classified as a rock mass 3. According to Laubscher (1994), the diameter of the loading width is set to 5 m, which corresponds to an IDZ of 11.5 m. Potential layouts are then proposed in Table 4.4.

Table 4.3: Constraints, mining parameters and assumptions used within PCBC (Ahmed et al., 2016; Chitombo, 2010; Diering, 2013; Diering et al., 2010; Laubscher, 1994)

Parameters and assumptions	Value	Units	Description
% of fines	30	%	Based on a model of fines
Density	2.6	g/cm^3	Average density for the domains
HIZ	100	m	Height for interaction zone
Swell factor	1.2	-	Established by experience
HOD MAX	500	m	Maximum height of development
HOD MIN	50	m	Minimum height of development
Initial elevation	440	m	Initial elevation of extraction
IDZ	11.5	m	Isolated draw zone diameter
Mining cost per ton	16.5	USD	Current block mining cost
IR	10	%	Interest rate
Layout type	-	-	Herringbone

The initial level of extraction is calculated using the FF module (Section2.3.1). The algorithm requires a single block model with grades, rock density, and economic attributes to analyze a wide range of levels where the elevation with the highest value is selected (Diering et al., 2010). The initial level of extraction here is calculated based on the averaged realizations, resulting in an elevation of 440 m.

The Herringbone layout is used here (Figure 4.11) because this layout type is suitable for the block caving project under study. The names of the layout configurations are assigned according to the spacing between drifts (A) and spacing of drawpoints across the minor pillars (B). The distance between drawpoints within the same bell is considered equal to the drawpoints across the minor pillars (B), (Figure 4.11). Fifteen layouts are initially selected; these layouts are chosen based on a specific average of the project's fragment sizes according to Laubscher's guidelines (Laubscher, 1994). The proposed layouts, Table 4.4, reflect the basic drawpoint spacing used in several world-wide mines; for instance, 31 × 17 m in the Herderson mine (Chitombo, 2010), and 30 × 15 m in the Mitchell project (Associates, 2012).

4.2.4 Optimizing the Drawpoint Spacing

Stochastic realizations of Cu and Au are passed through a multi-stage heuristic algorithm within PCBC to generate a set of results of mineable reserves and production schedules. Figure 4.23 shows

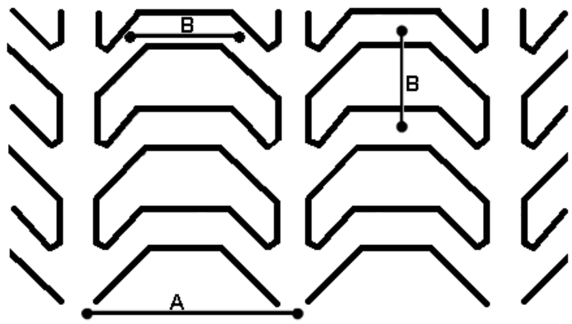

Figure 4.11: A typical herringbone layout, after Chitombo (2010)

Table 4.4: Proposed block caving layouts

A (m)	B (m) 11	B (m) 13	B (m) 14	B (m) 15	B (m) 17
30	30 × 11	30 × 13	30 × 14	30 × 15	30 × 17
31	31 × 11	31 × 13	31 × 14	31 × 15	31 × 17
32	32 × 11	32 × 13	32 × 14	32 × 15	32 × 17

(a) the orebody within a layout configuration (e.g. 30 × 15); (b) one realization of copper (Cu) within the layout, and the economic envelope of mineable reserves where the mine schedule is calculated.

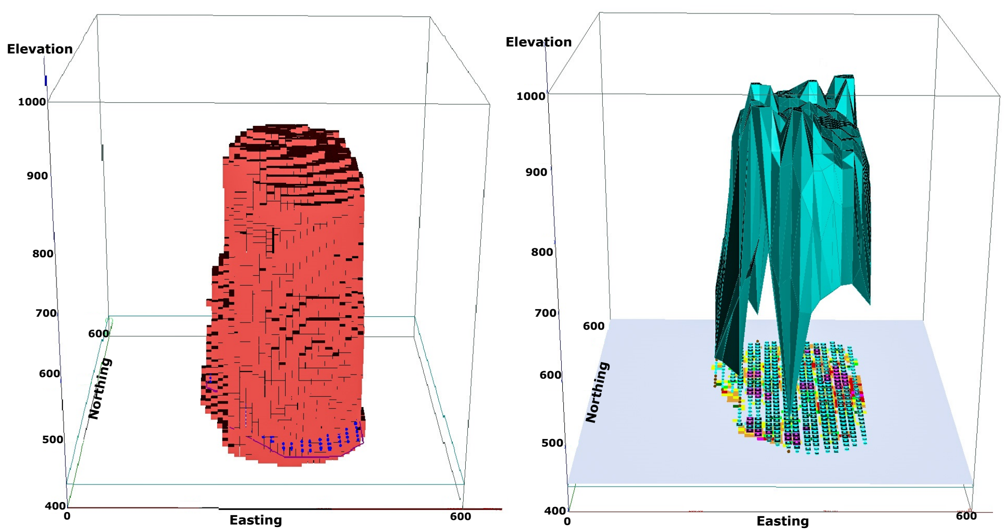

Figure 4.12: Calculation of mineable reserves in PCBC: (a) orebody to be mined; (b) an assigned layout over a SGS model at level of extraction, and the cave envelope calculated from the stochastic model

Production scheduling is required to calculate the NPV for each realization. Scheduled mine plans generate responses in terms of recoverable tonnage and NPV. The responses from the 15 proposed layouts are calculated over all realizations (Figure 4.13). The number of NPV values obtained here is 300. These values are inserted in 20 surfaces (Figure 4.14) which represent the 20 equally-probable models. The responses are grouped into 15 NPV distributions to assess uncertainty (Figure 4.15).

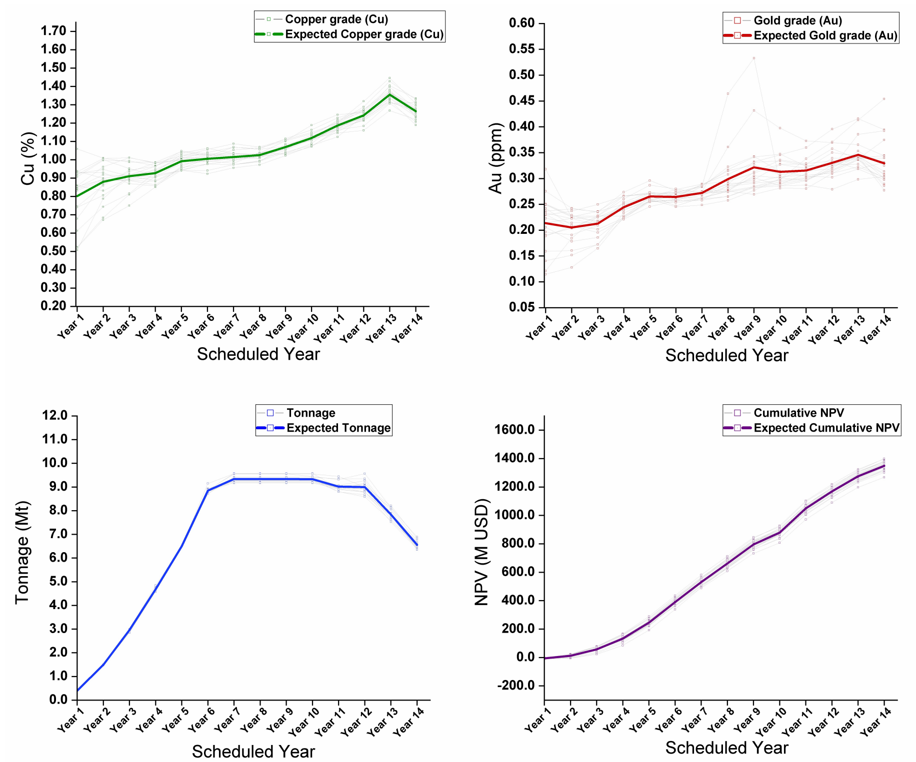

Figure 4.13: Yearly mineable reserves of (a) copper grade, (b) gold grade, and (c) tonnage, and (d) cumulative NPV responses obtained after transfer function (PCBC) calculation

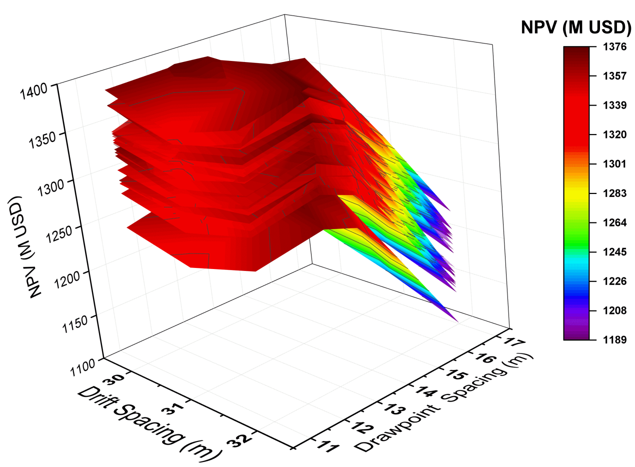

Figure 4.14: 300 NPV responses calculated over 20 realizations is represented by 20 surfaces

The uncertainty in NPV is then quantified to evaluate risk for each layout. The selection of the optimal drawpoint spacing is done by transferring the uncertainty of NPV values to a decision model based on a combination of the mean-variance criterion and the stochastic dominance rules.

The probability density functions (PDFs) of the parameter of interest (NPV) from the proposed layouts are illustrated here as boxplots (Figure 4.15). After the PDF representations are determined

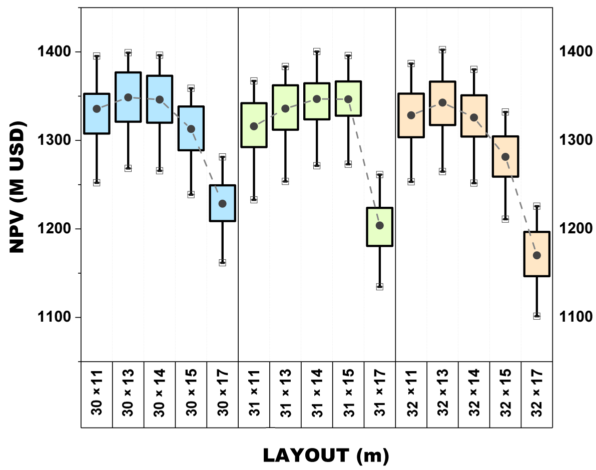

Figure 4.15: Boxplots of 15 distributions, the grey dots are the means of the NPV values of the layout configurations

and the uncertainties are evaluated in the box plots, the mean-variance criterion is applied (4.16). Figure 4.16 shows the relationship between mean and variance for each of the 15 NPV distributions.

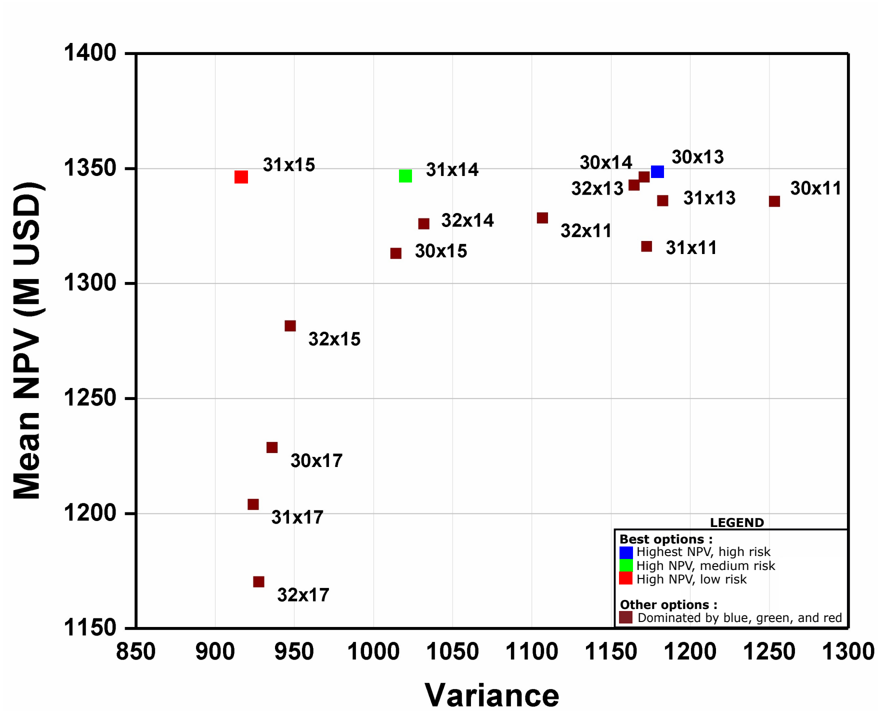

Figure 4.16: Mean-variance relationships. The layouts in blue, green and red squares dominate the layouts in brown squares which are southeast.

Once the layouts are evaluated using a mean-variance criterion, the stochastic dominance rules are applied by calculating the cumulative distribution functions (CDFs) and ranking them, from right to left (Figure 4.17). Three CDF distributions on the right are preferred under the FSD rule.

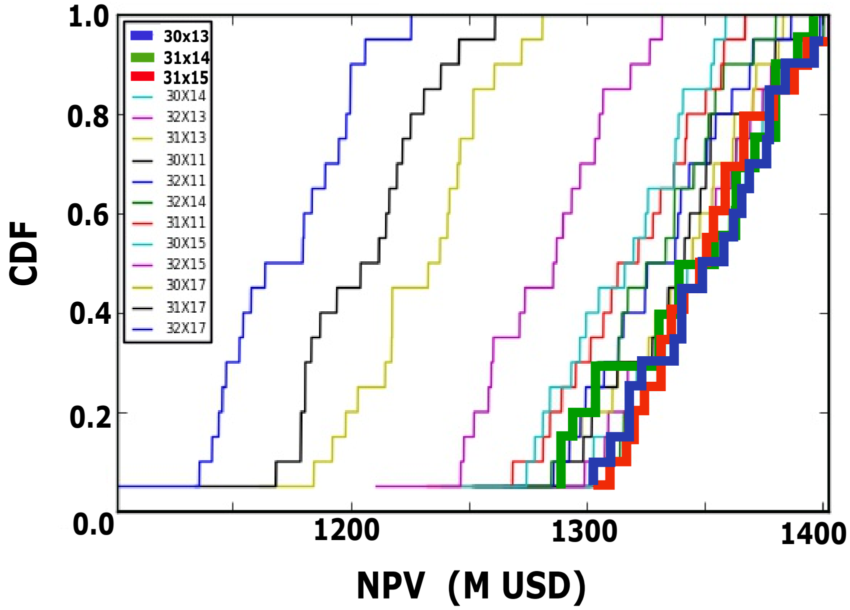

Figure 4.17: 15 CDFs organized from right to left (FSD).

The three best layouts are shown in Figure 4.16. Their mean-variance relations are represented by blue, green and red squares which dominate the others (brown). The layout with the lowest mean and variance can be selected by risk-averse practitioners (31 × 15). However, the layout with the highest mean is often preferred, the 30 × 13 layout is selected as the optimal drawpoint spacing.

4.2.5 Selecting the Optimal Level of Extraction

Once the optimal drawpoint configuration is known, the optimal level of extraction can be determined. The main purpose of this optimization step is to select an optimal level over a set of stochastic realizations and proposed levels using a single layout (30 × 13) which represents the optimal drawpoint spacing.

The selection of proposed levels above and below the initial level Z_i is based on the parameter h which is the vertical block size. Levels [Z_i + h] and [Z_i – h] are selected from above and below, elevations 450 m and 430 m are chosen.

In PCBC , the NPV values of each of the 20 realizations are calculated at the proposed levels of extraction (430 m, 440 m and 450 m), using the 30 × 13 layout. Three NPV distributions are then generated (Figure 4.18).

In Figure 4.18, blue squares are the maximum and minimum NPV values while circles are the NPV means. The blue dot represents the maximum NPV mean. The decision criterion of this section is based on the selection of the distribution with the maximum NPV mean because based on the mean-variance criterion as well as the stochastic dominance rules, the best value is preferred;

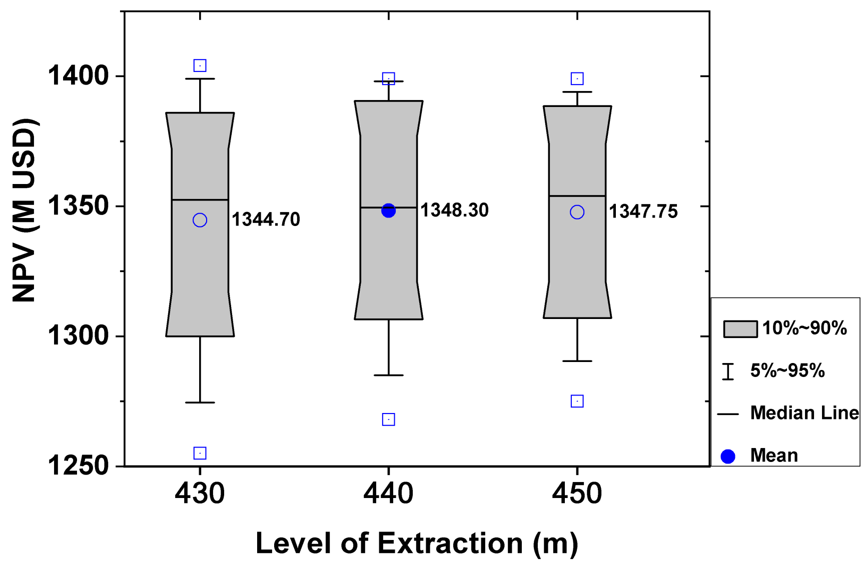

Figure 4.18: Three box plots with 20 NPV values each; they are calculated on three levels of extraction

therefore, the elevation 440 m is the optimal level of extraction. Further refinement can be made by testing more levels above and below. For example, the levels 420 m and 460 m could be evaluated.

The results show that the optimal production level layout is achieved in the first iteration since the optimal level is the same as the initial level Z_i , Z_o =Z_i . Therefore, there is a convergence between the optimal drawpoint spacing (30 × 13) and the Z_o (440 m).

4.2.6 Comparison of NPV Results Between the Proposed SGS Approach and using OK

The optimal layout using OK is generated for comparison. The main input is a single model with grades of Cu and Au. The OK model is assessed in the same multi-stage algorithm within PCBC to generate a mineable-reserves envelope and the corresponding production scheduling for each proposed layout. Afterwards, 15 scheduled mine plans are generated, giving the responses in terms of grades, tonnage, and NPV for each layout.

The maximum NPV is found with the SGS models and the 30 × 13 layout (Figure 4.19a); whereas, the maximum NPV calculated over a single OK model is the 31 × 14 layout (Figure 4.19b). Tonnage and cumulative NPV results from the SGS and OK plans are provided (Figure 4.20).

In a 14-year planned production, the expected mineable tonnage from year 5 to year 12 using the SGS realizations is greater than tonnage obtained with the OK model; consequently, the cumulative NPV obtained by the SGS plan is slightly higher than that obtained by the OK plan (3.25% greater), as shown in Figure 4.20.

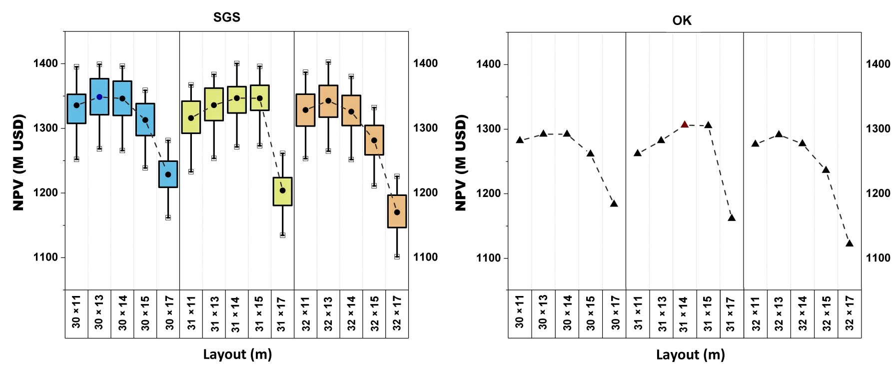

Figure 4.19: (a) The blue dot is the best-expected NPV calculated on the SGS realizations, and the 30 × 13 layout; (b) the solid red triangle is the best NPV generated with the OK model, and the 31 × 14 layout.

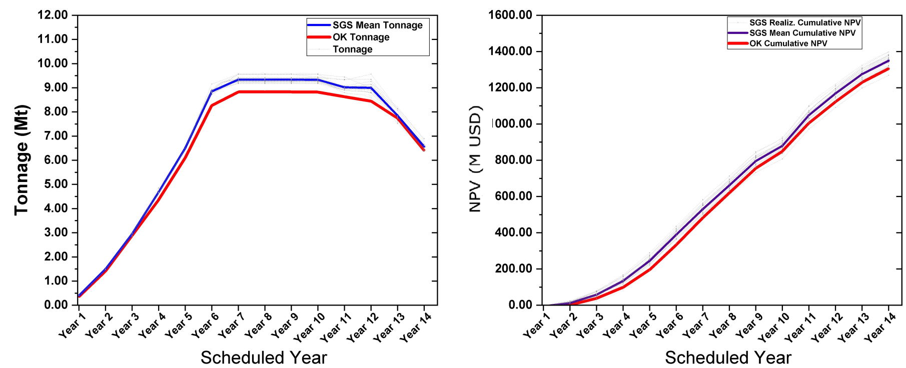

Figure 4.20: (a) Scheduled tonnage (b) cumulative NPV

The difference between the OK and SGS profit is due to the smoothing effect of OK and its inability to reproduce the true block variability of the orebody. Despite there is variability in the whole deposit, much of the variability is located at the boundary of the orebody. Some smoothed blocks generated by OK model are lower than the economic threshold; and therefore these have not been included as minable reserves by PCBC.

4.3 Part 2: Tonnage Uncertainty

Deterministic geological modeling is the method of choice because it provides realistic geological features. However, boundary uncertainty is critical. This section models boundary uncertainty to evaluate minable tonnages. The effect of the boundary uncertainty in the minable tonnage can influence decision making.

4.3.1 Modeling Boundary Uncertainty

Boundary uncertainty is accounted for a set of implicit models based on the distance function (Martin & Boisvert, 2017). As explained in Chapter 2, the domain definition relies on the geological knowledge. These domain models are then built by processing the sampled categorical data of the project.

The tonnage uncertainty for the orebody domain begins with the calibration of the c-parameter and determining the bandwidth as explained in Chapter 2 (Manchuk & Deutsch, 2015; Martin & Boisvert, 2017). The c-parameter can be trained by a jackknife method, expert jugdment is also important (Karpekov, 2016), and is adopted in this case study. The C-parameter selection is according to the smallest spacing between composites and spatial location of the samples giving a value of 20 m.

The bandwidth of uncertainty is limited by (+C):20 and (-C):-20. Multiple geological models can be generated inside the uncertainty bandwidth (Figure 4.21).

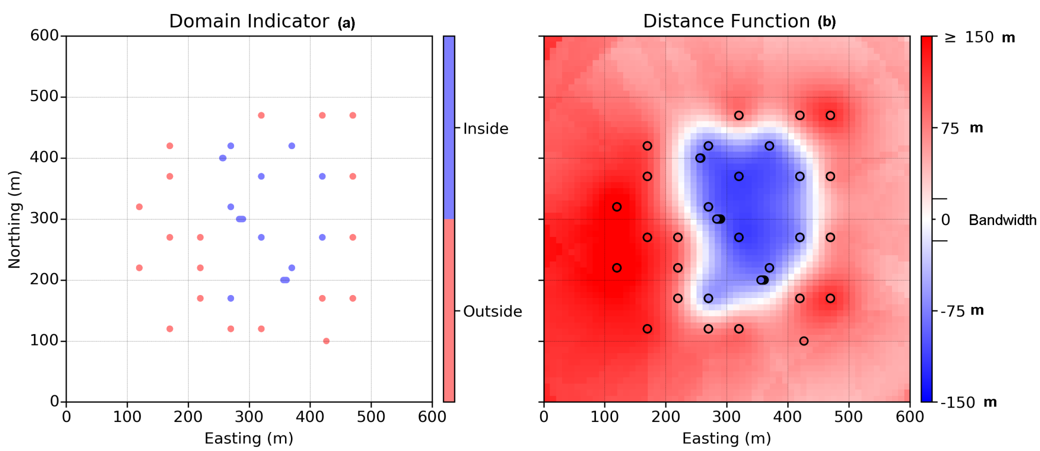

Figure 4.21: Two plan views: locations of the domain indicators are shown in on the left view (a); the interpolation of the SDF values are on the right one (b), the boundary uncertainty zone is between the blue and red domains

4.3.2 Geostatistical Modeling

The same dataset used in part 1 is considered in this section. The orebody extents in each drill hole are known and the drilling assays have been composited to 10 m. Cu and Au are the continuous variables. SGS is also performed in the same manner as in Section 4.2.2 using the twenty implicit models from Section 4.2.1. Twenty stochastic realizations of Cu and Au grades are generated and clipped inside the twenty implicit models (Figure 4.22).

4.3.3 Mining Parameters to Calculate Tonnage

The BHOD is another module of PCBC and is used to calculate the economic envelop (Section 2.3.2) for the twenty models. The parameters used in BHOD is summarized in Table 4.5.

The initial level of extraction used is the elevation of 440 m assuming that this level of extraction is optimal. The herringbone layout and the fifteen proposed layouts are used here, as well.

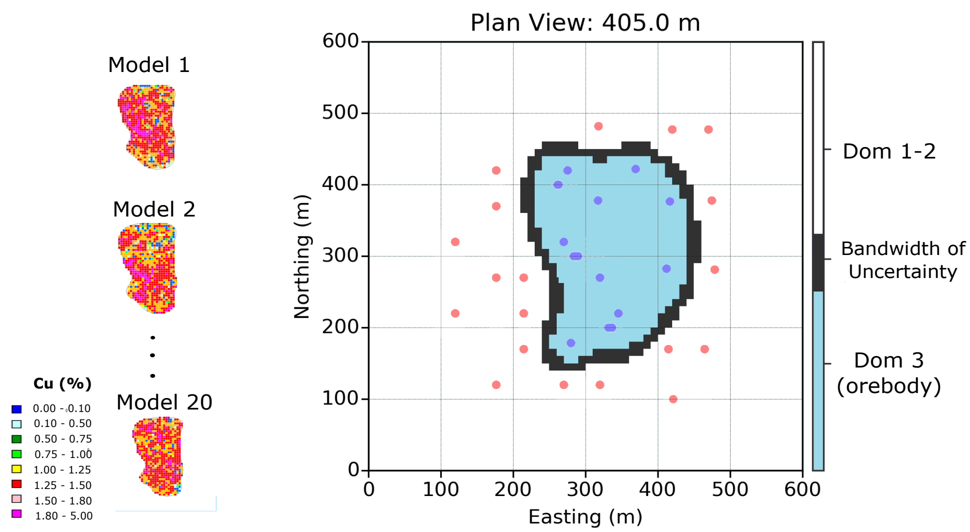

Figure 4.22: Twenty stochastic realizations are clipped into 20 domain models, Cu blocks from the orebody are shown

Table 4.5: Constraints, mining parameters and assumptions used to calculate recoverable reserves within PCBC (Ahmed et al., 2016; Chitombo, 2010; Diering, 2013; Diering et al., 2010; Laubscher, 1994)

Parameters	Value	Units	Description
% of fines	30	%	Based on a model of fines
Density	2.6	g/cm^3	Average density for the domains
HIZ	100	m	Height for interaction zone
Swell factor	1.2	-	Established by experience
HOD MAX	500	m	Maximum height of development
HOD MIN	50	m	Minimum height of development
Initial elevation	440	m	Initial elevation of extraction
Economic cut-off	20.00	USD	Base on the unit revenue per ton of Cu and Au
Shut off value per ton	16.5	USD	Shut off value per ton for drawpoint
Mining cost per ton	16.5	USD	Current block mining cost
Unit revenue per ton (Cu)	45.00	USD	Based on the current cost and selling price
Unit revenue per ton (Au)	7.00	USD	Based on the current cost and selling price
Recovery factor	90	%	Recovery factor Cu and Au
Layout type	-	-	Herringbone

4.3.4 Calculate Minable Tonnage with BHOD

The clipped stochastic realizations are passed through an algorithm of PCBC that calculate the economic envelope (BHOD). A set of results of mineable tonnage is then generated. Figure 4.23 shows (a) the orebody within a layout configuration (e.g. 30 × 15); (b) one realization of copper (Cu) within the layout, and the economic envelope of mineable reserves. The minable-reserve calculation for each model generates a set of responses in terms of tonnage, grade and value. The responses from the 15 proposed layouts are then calculated over all clipped realizations.

The assessment of tonnage uncertainty (Figure 4.25 and 4.24) can help in the decision making to determine the optimal drawpoint spacing. The tonnage responses are grouped into 15 tonnage

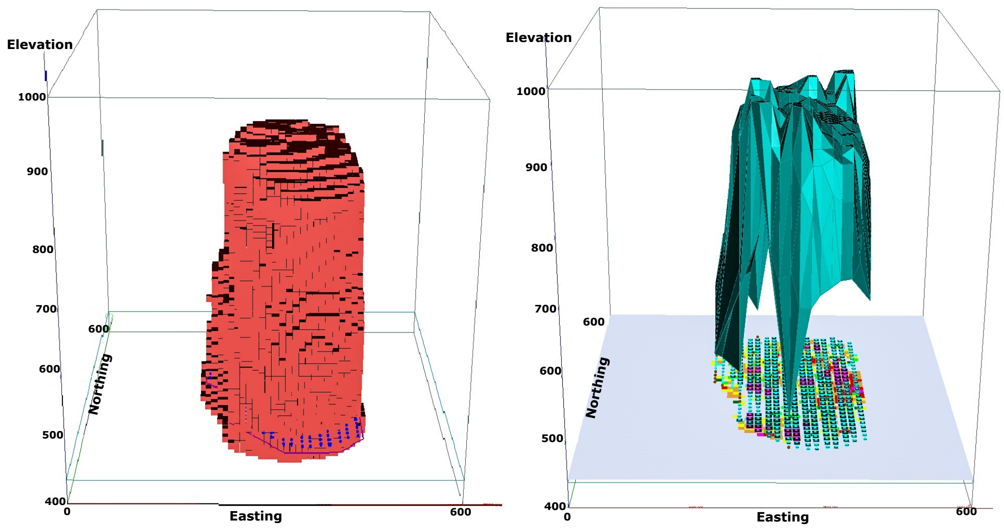

Figure 4.23: Calculation of mineable reserves in PCBC: (a) orebody to be mined; (b) an assigned layout over a SGS model at level of extraction, and the cave envelope calculated from the stochastic model

distributions (layouts) to assess uncertainty and are illustrated here as box plots (Figure 4.24), the means are presented as dots while variances are presented as large colored rectangles. The uncer-

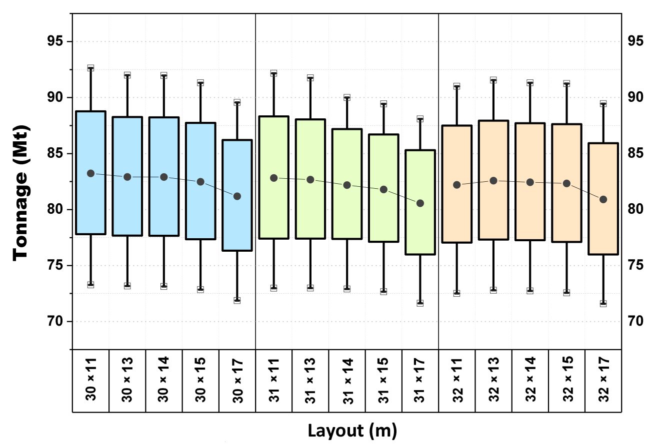

Figure 4.24: 15 boxplots for tonnage uncertainty assessment

tainty in tonnage is then quantified evaluating risk for each layout. Figure 4.25 shows the relationships between mean and risk for the 15 tonnage distributions.

The maximum mean of tonnage is found in the 30 × 11 layout. However, this layout contains the highest risk. After the tonnage uncertainty assessment, the 31 × 14 layout and 31 × 15 layout appear to be more appealing because of their lower risk compared to the optimal layout (30 x 13), previously selected based on profit (Part 1).

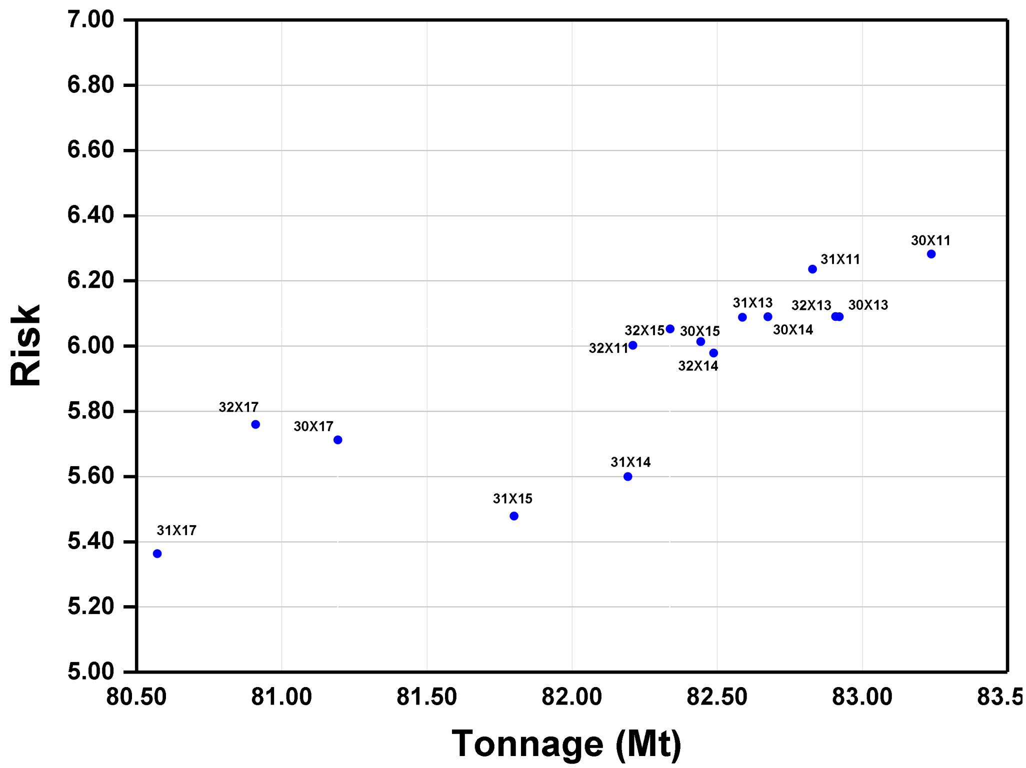

Figure 4.25: risk-mean relationships of tonnage distributions

4.4 Summary

A copper-gold caving project is used to demonstrate the impact of grade and geologic boundaries uncertainties in the optimization of the layout design. The methodology uses Gaussian simulation and a signed distance function.

Data for the study is from 37 drill holes, and assays are composited to 10 m. The project is interpreted as an intrusive-hosted deposit with copper-gold mineralization surrounded by sterile units. Rock types, basic statistics and contact analysis justify the delineations of two domains: Dom 3 and Dom 1-2. The mean of the original Cu and Au grades for Dom 3 is 1.36% and Au is 0.325 ppm respectively. Cell delustering (Deutsch & Journel, 1998) is applied to the composites; then, the mean of Cu is 1.233 %, and for Au the mean is 0.284 ppm. Variogram models are used to assess the spatial continuity and twenty stochastic realizations of Cu and Au grades are generated. Realizations are checked with variograms and histograms and show good reproduction.

A set of mining parameters is summarized and used in the parameterization of the transfer function. The initial level of extraction is calculated with FF, resulting in an elevation of 440 m and the Herringbone layout is used. Fifteen potential layouts are selected. The development cost of each layout configuration is estimated based on the number of drawpoints. Optimizing the drawpoint spacing begins when the realizations are passed through a transfer function (PCBC). A set of results of reserves and schedules are generated to provide responses in tonnage and NPV. Uncertainty is then quantified to evaluate risk for each layout. The selection of the optimal layout is done by transferring the uncertainty of NPV to a decision model. The 30 × 13 layout is the optimal one.

The optimal level of extraction is determined using the optimal layout (30 × 13). Two levels are proposed and selected from above and below the initial level of extraction, 450 m and 430 m are chosen. The responses from the twenty realizations are organized in three respective distributions for uncertainty and risk assessment. The results show that the optimal production level layout is the same as the initial one, level 440 m. A comparison between OK and SGS is performed. This comparison shows differences between the OK and SGS profit that is caused by the smoothing effect of OK that generate sterile blocks in the orebody boundaries.

Boundary uncertainty is critical in the orebody. The effect of the boundary uncertainty in the tonnage can influence decision making. The calibration of the c-parameter (20 m) and the bandwidth is performed. Twenty geological models are generated inside the uncertainty bandwidth. The stochastic realizations of Cu and Au are clipped inside the twenty implicit models. The models are used in the calculation of recoverable tonnes. The maximum tonnage is found in the 30 × 11 layout, but the risk is the highest. The tonnage uncertainty suggests that the 31 × 14 layout and 31 × 15 present lower risk compared to the optimal layout (30 x 13).

Chapter 5

Concluding Remarks

A relevant engineering task for the success of a caving project is the design of the production level layout, and most importantly the drawpoint spacing. Current practice is to perform this important assignment without considering orebody uncertainty. The lack of procedures using multiple geologic models to quantify uncertainty in block-cave design limits the possibility for maximizing the economics of mines. A methodology for optimizing the production layout is proposed, uncertainty from stochastic models are used for decision making rather than a traditional design method based on a single estimated model. An illustrative case study demonstrating the methodology for optimizing the layout design over all realizations and actively managing risk is presented.

5.1 Contributions

Geostatistical techniques are often used in decision making, but the majority of research in mine design has been conducted for open-pit mines, little research in the subject has been done in block caving. Then the research provides several contributions:

- The main contribution of this research is the development of a stochastic optimization methodology in which uncertainty from grade and geology is used to maximize the mine value through optimizing the drawpoint spacing.
- The second contribution is an approach to select the optimal level of production. This approach takes advantage of the optimal drawpoint spacing previously determined and stochastic realizations, and permits to find the best level of extraction.
- The third contribution of this work is related to the implementation of SGS and SDF to generate models that are integrated into a block caving software (PCBC) to solve a controversial element of mine design. Incorporating multiple simulated models into caving schedules delivers a set of responses to determine the optimal layout which maximizes profit in block caving.
- Enforce the concept of actively managing risk to optimize the layout design over all realizations in block caving is the fourth contribution. An innovative risk-management model for decision making is included in the research. This model combines the mean-variance relationship and the stochastic dominance rules.

5.2 Limitations of the Research

The research is concerned with developing and implementing an optimization approach for layout design ; however, there are some limitations that must be considered prior to implementation, and are listed below:

- The computational time can be large when the number of realizations is large. This is generated by the lack of flexibility of the transfer function (PCBC)to process multiple realizations.
- Various sources of uncertainty are acknowledged in block caving. This research provides an overview of risk assessments from geostatistical models, but other sources of uncertainty such as geotechnical and economic are not covered.

5.3 Future Work

Although a practical approach to tackling the optimization of the most arguable caving aspect is developed here, more improvements are possible, and further studies in complementary aspects may be helpful:

- This innovative optimization methodology is the first one of its kind in block caving design. Hence, other sources of uncertainty from geotechnical and economic factors should be integrated into the workflow within future improvements.
- The approach for selecting the proposed layouts should be improved with further studies. For example, adding certain operational aspects such as size of equipment and pillar safety can be important. Each deposit is unique and must be evaluated considering these operational factors.
- Future research should be focused on developing valid algorithms for integrating grade models, layout design and scheduling in block caving in order to have more alternatives of use and comparison with the transfer functions in PCBC.

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