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Ernst Mally's Anticipation of Encoding

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Ernst Mally's *Gegenstandstheoretische Grundlagen der Logik und Logistik* (1912) proposes that the abstract object "the circle" does not satisfy the properties of circles, but instead "determines" the class of circles. In this he anticipates the notion of "encoding" that Edward Zalta proposes for his theory of Abstract Objects. It is argued that Mally did anticipate the notion of "encoding", but sees it as a way of taking the concept as the subject of a proposition, rather than as a primitive notion in the theory of a new ontological category of abstract objects, as Zalta does.

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The Austrian philosopher Ernst Mally discusses a notion of "determination" (*Determinierung*) in his monograph "Object-theoretic foundations of logic and logistic" (*Gegenstandstheoretische Grundlagen der Logik und Logistik*) published in 1912.[1] The distinction Mally makes there between determination and "satisfaction" (*Erfüllung*), the two ways in which an object may have a property, has been developed philosophically and presented formally by Edward Zalta as the basis of his "Object Theory" in a series of papers and books since the 1980s.[2] This paper is an investigation of the distinction that Mally makes, and an examination of the precise manner in which it is an anticipation of Zalta's distinction.

Mally's approach to formal logic is in the tradition of Ernst Schröder's *Algebra der Logik* (1890), and as such, the formal presentation of determination is part of an account of a logic that is capable of distinct interpretations. In particular, the system can be interpreted as, on the one hand, a logic of propositions and predication, and on the other, a logic of classes. There is, however, no explicit reading of the logic as involving individuals and predicates of individuals, in the fashion of Frege's logic. The notion of determination in the sense in which it anticipates encoding, consequently, must be yet another interpretation of Schröder's formal calculus. The difference between these interpretations depends on two ways of taking abstract objects, as "objectives" (*Objektive*) and "objecta" (*Objekte*).[3] The most prominent use of "determination", in the work, is to represent restrictions in the theory of classes, as in "the *F*s which are *G*", which restricts the *F*s to a subset of their number. The interpretation of determination as a new form of predication is only briefly introduced in a philosophical section about the use of the formal calculus presented in the first part of Mally's book. The conclusion of this investigation then, is that Mally does anticipate the notion of "encoding" with his "determination", but that he only proposes it as one interpretation of the formal system he has presented. He does not present any argument that this is indeed an alternative interpretation of the system, nor does he present any of the sort of formal development specific to the encoding interpretation of the notion of determination. This is indeed an anticipation of the notion of encoding, but one which only is suggested, and not further motivated or developed. It is an anticipation in much the same way that Schröder himself had anticipated the theory of types as the conclusion of the very argument that Frege and others identified as relying fatally on a confusion of the subset and membership relations.[4]

The distinction between the two modes of predication, namely, that an object can "have" a property in two different ways, is central to Zalta's Neo-Meinongian theory of abstract objects.[5] Thus the notorious "round square" can be both round and square in the encoding sense of "be," but not both round and square in the exemplification sense. The latter, familiar, sense is expressed with a predicate letter F followed by a singular term, a, as in Fa. The sense in which the round square is round, by encoding roundness, is expressed with the reverse order: aF. This basic notion is developed within an axiomatic system that is the basis for a wide ranging theory of various entities such as possible worlds, numbers and other mathematical entities, universals, and so on. In particular, it is possible to use the notion of encoding to provide answers to the objections to Meinong's theory of non-existent objects which Russell posed in his reviews in 1905 and 1907. To the objection that objects like the round square conflict with the easily established mathematical fact that nothing can be round and square, the response is that while the abstract object "the round square" encodes the properties of being round and square, it does not exemplify those properties. Instead it only exemplifies, for the most part, rather generic properties of abstract objects, namely being non-spatiotemporal, colorless, etc. The round square can be related to ordinary objects, however, and that is how abstract objects enter into their important role as objects of thought. Russell raised further difficulties for Meinongian objects. Existence will also be a property that is exemplified by some objects and not others. Russell objected that "the existing round square", for example, must exist, contradicting the obvious truth that there is no such thing. On Zalta's theory, however, there is an object that encodes, among others, the property of existence, but it will not exemplify it. Thus "the existing round square" does not exist, although it encodes the property of existing. Non-existent objects often appear to be contradictory and incomplete, such as "the present King of France", who would appear to be neither bald nor not bald. In the sense of exemplification, every object, abstract or not, is complete. Abstract objects, being non-spatiotemporal, certainly are not in the extension of "bald" and so are not bald. What properties an abstract object encodes, however, will determine its nature. Thus there is an abstract object which simply encodes the property of being the present king of France and nothing else. It does not encode being bald, nor being not bald. It is indeterminate. Similarly there will be an abstract object which simply encodes the property of being triangular, which we might well name "the triangle". Nothing can exemplify the property of being a triangle without also exemplifying the properties which logically follow from that; being a three-sided plane figure, for which various theorems hold, etc. From the fact that "the triangle" encodes the one property of being a triangle, it does not follow that it encodes any other properties whatsoever.[6] The details and applications of this theory have been worked out by Zalta in his books and with others in a number of papers.

Following the discussion in J.N. Findlay's 1963 book, which originated as his doctoral thesis under Mally, Zalta gives credit to Mally for originating this distinction.[7] An examination of Mally's 1912 essay *Gegenstandstheoretische Grundlagen der Logik und Logistik* shows that Mally does distinguish two ways in which an object can be said to possess a property, and so the basis of this credit to Mally is correct. Mally, however, does not mention Russell's objections, and does not present his own account as responding to difficulties that others had found in a theory of non-existent objects.

Indeed there is little in Mally's published work that shows that he was trying to develop a response to Russell's logical arguments. There is a letter from Mally to Russell from 2 April 1905, which Russell used in preparing his 1905 review. Mally ends the letter of explanation replying that his English is not good enough to reply to the details of the draft of the review that Russell had sent. In a letter of 13 September 1913 Mally asks for permission to translate *Principia Mathematica*.[8] It was Mally's colleague at Graz, Hans Mokre, who eventually (in 1932) published a German translation of the prefaces and Introductions to the two editions of *Principia Mathematica*. In his forward to the translation, Morke mentions the debate between Meinong and Russell that followed the publication of the first reviews, and states that Mally took Meinong's theory into the realm of logistic ("logistische Gebiet"). Mokre thanks Mally as well as other colleagues for their discussions of the material and help with the translations. So Mally had been keenly aware of Russell's logic and of both editions of *Principia Mathematica*. There is no mention of Russell's logic in the *Gegenstandstheoretische Grundlagen* of 1912, however, and all of the symbolic logic is in the system of Schröder. While there is thus no explicit response to Russell in 1912, Mally does discuss the concept of a "existing round square" (*bestehenden runden Vierecke*) and describes it as a "thing" (*Ding*) which is "impossible" (*unmöglich*) (p.71). Mally also says that the object "the circle" does not *satisfy* the property of being a circle but instead is *determined* by it.

Here are the two crucial passages from section §33, following the translation by Zalta and Süßbauer (Zalta 1998). Mally (p.63) here considers the object "the circle" which has only the properties defining a circle, namely, of being a closed line, on a plane, with points equidistant from a single point, etc., but no properties of a particular circle, such as a particular diameter or location:

> „*der Kreis*" (in abstracto) *erfüllt* die im Kreisbegriffe angenommenen Objektive *nicht* ... er ist nicht ein Kreis; er fällt deshalb auch nicht unter den Umfang des Kreisbegriffes, gehört der Klasse der Kreise nicht an, sondern bestimmt sie nur irgendwie und vertritt sie unserem Erfassen gegenüber: als der *Begriffsgegenstand*, nicht als Zielgegenstand des Begriffes.

> "... *the circle*" (in abstraction) *does not satisfy* the hypothesized objectives in the circle concept, ... it is not a circle; therefore it isn't in the extension of the circle concept, it doesn't belong to the class of circles, but determines them in some sense and represents them when we grasp them: as the *concept-object*, not as the intended object [target object] of the concept.

On the next page, (64), we have:

> Wir sagen: der (abstrakte) Gegenstand „Kreis" ist definiert oder determiniert durch die Objektive „eine geschlossene Linie zu sein", „in der Ebene zu liegen" und „nur Punkte zu enthalten, die von *einem* Punkte gleichen Abstand haben"; er ist als *Determinat* dieser Objektive zu bezeichnen, aber nicht als „implizites" (vgl. §30), da er ja die Objektive nicht erfüllt, sondern, wie man vielleicht sagen könnte, als bloß explizites oder als „Formdeterminat" dieser Objektive.

> We say: the (abstract) object "circle" is defined or determined by the objectives "to be a closed line", "to lie in a plane", and "to contain only points which are equidistant from *a single* point"; we should call it the *determinate* of those objectives, but not as an "implicit" one (cf. §30), because it does not satisfy the objectives, but, as one might say, only as an explicit one or as a "formdeterminate" of these objectives.

In this case, presumably, we are to consider "the circle" to be an object which is determined by the properties of being a closed line, and lying in a plane, and containing only points equidistant from a single point, namely the properties that are included in the definition of "circle". Mally's term "Formdeterminat", is a neologism, which is here translated as "formdeterminate".

These notions appear in the passage on page 71, which seems almost directly to respond to Russell's objection about "the existing round square":

> Wir können zum Beispiel den Begriff des „bestehenden runden Viereckes" bilden; was wir darin unmittelbar „bloß denken" is das Formdeterminat des Rundseins, Viereckseins und Bestehens, das aber offenbar seine Determinationen nicht erfüllt, sondern eben bloß als explizite (Formal-)Determinatoren an sich hat. Aber auch was wir durch das Denken dieses Begriffes *meinen*, *besteht nicht*: ein Ding, das die angegebenen Determinationen erfüllte, ist eben in keiner Weise, es ist „unmöglich."

> We can, for example, form the concept of an "existing round square", but what we think of is merely the "formdeterminate" of being round, being square, and existing, that manifestly does not satisfy the determinations, but rather has them as explicit (formal) determinations. What we *intend* with these concepts, *does not exist*: a thing, that satisfies the given determinations, is not in any sense, it is "impossible".

(It appears that Mally describes the relation of an object to a property which determines it as "explicit formal determination", whereas ordinary exemplification is "implicit".)

This discussion of determination for these abstract objects such as "the circle" is only explained in detail in §33 and §34, which are in the second part of the essay, which extends from §26 to §42 occupying pages 51 to 87. This second part is devoted to a development (Ergänzung) of the formal matter in the first 50 pages. In that first, formal part, this notion of determination only occurs in a footnote.

Mally's notation is as follows:

$\alpha, \beta, \gamma \ldots$ are variables for propositions (or propositional functions).

$\alpha \prec \beta$ and $\beta \succ \alpha$ are $\beta \supset \alpha$.

$\alpha \not\succ \beta$ is $\alpha \wedge \beta$, $\alpha \times \beta$ is $\alpha \vee \beta$, $\bar{\alpha}$ is $\sim \alpha$, $\tilde{1}$ is $\perp$, and $\tilde{0}$ is $\top$.

$a, b, \ldots$ are variables for classes, the extensions of $\alpha, \beta, \ldots$ respectively, when those are propositional functions.

$A, B, \ldots$ are variables for members of the classes $a, b, \ldots$.

$a \lessdot b$ and $b \gtrdot a$ are $a \subseteq b$.

$a.b$ (also ab) is $a \cap b$, $a + b$ is $a \cup b$, $\dot{1}$ is V, the universe, and $\dot{0}$ is Λ, the empty set.

$\equiv$ is identity (and, for propositions, equivalence).

The relationship of implication $\alpha \prec \beta$ between propositional functions is read as the universal quantification of the relationship, namely "All β's are α's". When α and β are propositions, this is just material implication ($\alpha \vee \sim \beta =$ the True), as can be seen from §9, 34: $(\alpha \prec \beta) \equiv (\alpha \times \bar{\beta} \equiv \tilde{0})$ (on p.15).

The notion of determination first appears in §11, Definition 40, is as follows (p.18):

40. (Definition) Aus der Klasse a den (gesamten) Teilbereich auffassen, der (zugleich) dem Geltungsbereiche des Objektivs β angehört, heißt a durch β determinieren. ... Die Determination von a durch β (und ihr Ergebnis) werde angeschrieben in den Zeichen

$$a^{\beta}$$

zu lesen etwas als „a mit der Bestimmung β".

40. (Definition) Comprehending out of the class a the (complete) portion, which (at the same time) belongs to the extension of the concept β, means that a is determined by β. The determination of a through β (and its result) is to be written in symbols

$$a^{\beta}$$

to be read as "a with the determination [Bestimmung] β."

Immediately following this definition is the assertion that $a^{\beta} \equiv ab$. Thus a^{β} is the class of a's which are β, that is the intersection of a and b. This is confirmed by:

41. (Fundamental proposition R) $(\alpha \succ \beta) \equiv (\dot{1}^{\alpha} \lessdot \dot{1}^{\beta})$

This means: the propositional functions α and β stand in the relation $\alpha \succ \beta$ (all αs are βs) if and only if the class of individuals which are α ($\dot{1}^{\alpha}$, which is just a) is a subset of the class of individuals which are β ($\dot{1}^{\beta}$, namely b).

In a footnote after the initial statement of the definition of "determination" on p.18, Mally applies the notion of "determination" to the special representative objects such as "the circle" (as above) or here, "the quadrilateral", which will be prominent in the second half. There is no separate formal treatment of them however:

Was determiniert wird, ist nicht eigentlich die Klasse a (diese wird durch die Determination nur „eingeschränkt", das heißt, man geht von ihr zu einer Artklasse über), auch nicht ein Ding von a, das heißt, irgendein spezielles Etwas, das a ist, sondern „das a", das ist der abstrakte Vertreter aller Dinge der Klasse a (z.B. „das Viereck" zum Parallelogramm, nicht irgendein konkretes Viereck, woran es ja nichts zu determinieren, sondern nur zu prädizieren gibt). Auch sagt man, es werde die Gattung zur Art determiniert ....

What is determined is not actually the class a (it is in fact only "restricted" by the determination, that is, one goes from it to a specific class), i.e. also not an element of a, but rather, rather "the a", the abstract representative of all things in the class a (for example, one goes from "the quadrilateral" to the parallelogram, not some particular quadrilateral, of which nothing can be determined, but only predicated). One also says that the genus is determined to a species . . .

My project is in fact to understand the notion that abstract objects such as "the α" can also be determined by concepts β, and not just classes a. Yet here the possibility is simply asserted rather than explained. What is stated in the definition is simply what it means for the class a to be determined by β.

Here it is an individual object that is said to be determined, although that is not one of the official interpretations of the symbols defined in 40. (It is unfortunate that in the font used by Mally's publisher the lower case Roman letter 'a' which stands for classes, is almost indistinguishable from the lower case Greek 'α' for propositions or propositional functions. The force of my argument depends on having rendered the symbols as Mally intended them, thus attributing occasional typographical errors to the text.)

In fact, in §14, just three pages later (p.21) we get another sense of determination, represented with the same notation:

In §11 ist der Fall, daß ein Objektiv als Determinand auftrete, ausdrücklich ausgeschlossen worden. Nun soll der Begriff der Determination auch auf diesen Fall angewendet werden.

51.(Definition) α^{β} bedeute das Objektiv α mit der Bestimmung β.

Man kann α^{β} lesen als "α, welches β erfüllt," aber auch als Setzung: "α erfülle β".

In § 11 the case has been expressively excluded that an objective should occur appears as a determinator. Now the concept of determination should also be used in this case.

51.(Definition) α^{β} means the objective α with the determination β.

One can read α^{β} as "α, which satisfies β," but also as the formulation: "α which satisfy β".

(The symbols here have it that propositional functions are determined in contrast with classes, but again not abstract *objects*.)

Surely one reason that Mally expresses the relation of determination as a relation of concepts in this way is that the *Grundlagen* is firmly couched in the terminology and concepts of Ernst Schröder's *Algebra der Logik*. That work is an account of the logic of concepts using one primitive notion of "subsumption", and is notorious in that it does not distinguish the subset from membership relations. Both Russell and Frege criticised Schröder's logic for this confusion. This confusion seems to be genuine, as can be seen from the arguments which Schröder presents and which seem to rely on it, as pointed out in Frege's discussion. This confusion in Schröder's logic resurfaces in Mally where he confronts problems in expressing the distinction between modes of predication. It comes, I will argue, from Mally's view that it is the same subject concept, say "Triangle", which is the subject of predications, as in statements about all triangles, and in statements about the Triangle as the "objectum" (*Objekt*) of a proposition.

My suggestion is that Mally associated the new mode of predication with taking what is ordinarily represented by a concept, within a proposition, as an an individual, an objectum *Objekt*. This is a difference in attitude towards the concept, of the sort that would be familiar to a member of the so-called "Graz school" that was founded by Meinong. Thus when Mally talks about taking a concept as an objectum, and so for us, as talking about an abstract object, this is done by literally taking the concept *as* an individual, not by changing our attention from the concept to some arbitrary or representative object which exemplifies it.

There is no separate logic of abstract objects in Mally, much less a comprehension principle or other devices of formalism that were introduced by Zalta for his object theory. In a way, then, one could see Mally's theory as holding that the Triangle as such, what we now might call the abstract object, is simply the concept of Triangle, taken in a certain way. It is taken in that way, namely as an "Objekt," in which it can be said to have properties in a different way.

As evidence for this analysis, consider another important logical notion in Mally's *Grundlagen*, that of "reciprocity", that one set of axioms can be interpreted in different ways, namely as about concepts and about propositions.

First there is a summary below of the logical notions that Mally borrows from Schröder and then Mally's own sketchy account of reciprocity, which will then set the stage for the main thesis.

The notion is introduced in §7 (p.6):

> **§7. Die Reziprozität. 8. (Grundsatz R.)** Besteht zwischen zwei Objektiven die Folgebeziehung: wenn α gilt, so gilt β, so besteht auch: in jedem Falle, wo α gilt, gilt β oder: jeder Fall von α ist ein Fall von β, daher auch: jeder Gegenstand (A), der α erfüllt, ist ein Gegenstand (B), der β erfüllt.
>
> Aus $\alpha \succ \beta$ folgt also $\alpha \lessdot \beta$ und zugleich $A \lessdot B$ und daher $a \lessdot b$ für beide möglichen Deutungen von a und b.
>
> Verfolgt man die angegebene Umformung im umgekehrten Sinne, so zeigt sich, daß auch die Umkehrung gilt:
>
> Aus $a \lessdot b$ folgt $A \lessdot B$ und $\alpha \lessdot \beta$ und daraus $\alpha \succ \beta$.
>
> Es besteht also einerseits $(\alpha \succ \beta) \succ (a \lessdot b)$ und andererseits $(a \lessdot b) \succ (\alpha \succ \beta)$, das heißt $R \quad (\alpha \succ \beta) \equiv (a \lessdot b)$.

> **§7. Reciprocity. 8. (Fundamental proposition R.)** If the consequence relation obtains between two objectives: when α holds, so does β, then this also obtains: in every case, where α holds, β holds or: every instance of α is an instance of β, hence also: every object (A), that satisfies α, is an object (B), which satisfies β.
>
> From $\alpha \succ \beta$ it follows that $\alpha \lessdot \beta$ and also $A \lessdot B$ and therefore $a \lessdot b$ for both possible interpretations of a and b.
>
> If one looks at the the transformation in question in the reversed sense, it emerges that the converse also holds:
>
> From $a \lessdot b$ it follows that $A \lessdot B$ and $\alpha \lessdot \beta$, and hence $\alpha \succ \beta$.
>
> So it holds, on the one hand, that $(\alpha \succ \beta) \succ (a \lessdot b)$ and on the other that $(a \lessdot b) \succ (\alpha \succ \beta)$, that is $R \quad (\alpha \succ \beta) \equiv (a \lessdot b)$.

Thus Mally asserts that sentences of the form $(\alpha \succ \beta)$ expressing implications, themselves imply those of the form $\alpha \lessdot \beta$ (a seemingly ill-formed use of the subset symbol with class concepts) and those in turn imply $a \lessdot b$, and vice versa, so $\alpha \succ \beta$ and $a \lessdot b$ are logically equivalent. Mally extends this in an insufficiently precise way to a wide ranging correspondence between sentences which use the two primitives. He describes this as a sort of duality, which he calls "reciprocity."[9] Here is the crucial paragraph from §10 (p.17) of Mally (1912):

> **39. (Satz.)** Es bezeichne $\varphi_1(\neq, \times, \succ)$ eine Einschließungsbeziehung zwischen additiven und multiplikativen Verknüpfungen von Objektiven, $\varphi_2(\neq, \times, \succ)$ eine Beziehung derselben Art. Besteht nun zwischen beiden eine Beziehung Φ von der unten angegebenen Form, so gibt es dazu, nach R, eine *reziproke*, F, von der Art der daneben verzeichneten, worin (gegenüber Φ) $\neq$ durch ., $\times$ durch $+$ und $\succ$ durch $\lessdot$ ersetzt ist.
>
> $\Phi \ldots \varphi_1(\neq, \times, \succ) \succ \varphi_2(\neq, \times, \succ) \qquad F \ldots f_1(., +, \lessdot) \lessdot f_2(., +, \lessdot)$.
>
> Wegen der *formalen* Übereinstimmung der Definitionen für $\neq$ und $+$, für $\times$ und ., für $\succ$ und $\gtrdot$ (vgl. 26, Anm.) gilt aber auch die Beziehung Φ', die man aus F erhält, wenn man in den „primären" Relationen f die Verknüpfungen und Beziehungen durch ihre formalen Gegenstücke ersetzt, während die „sekundäre" Beziehung $\lessdot$, die zwischen den primären f_1 und f_2 besteht (sowie etwa auftretende sekundäre Verknüpfungen, d.h. Verknüpfungen zwischen Einschließungsbeziehungen), eben wegen der genannten formalen ÜĘbereinstimmung auch zwischen den formalen Gegenstücken jener Primärrelationen (den φ'_1 und φ'_2) erhalten bleibt, also durch die ihr *äquivalente* Beziehung $\succ$ ausgedrückt werden kann. Zu der so gefundenen Beziehung Φ' besteht dann wieder die reziproke, F'. Man hat

also
$\Phi' \ldots \varphi'_1(\times, \nsim, \not\prec) \not\succ \varphi'_2(\times, \nsim, \not\prec) \quad F' \ldots f'_1(+, ., \geqslant) \leqslant f'_2(+, ., \geqslant)$.

Man nennt die Beziehungen φ und φ' einander *dual* entsprechend, ebenso f und f'; in einem weiteren Sinne können auch Φ' und F' duale Gegenstücke zu Φ und F heißen. Jeder Satz von der Art Φ (oder F, Φ', F') vertritt demnach eine Vierzahl von Sätzen, Φ, F, Φ', F', — eine Tatsache, die sich eine systematische Darstellung der symbolischen Logik entsprechend zunutze zu machen hätte.

39. (Proposition.) Let $\varphi_1(\nsim, \times, \not\succ)$ indicate a relation of inclusion between additive and multiplicative combinations of objectives, $\varphi_2(\nsim, \times, \not\succ)$ a relation of the same kind. If a relation Φ of the form indicated below obtains between these two, then there is, according to R, a *reciprocal* relation, F, of the sort indicated next to it, in which (with respect to Φ) $\nsim$ is replaced by ., $\times$ by $+$ and $\not\succ$ by $\leqslant$.
$\Phi \ldots \varphi_1(\nsim, \times, \not\succ) \not\succ \varphi_2(\nsim, \times, \not\succ) \qquad F \ldots f_1(., +, \leqslant) \leqslant f_2(., +, \leqslant)$.

Because of the *formal* agreement between the definitions of $\nsim$ and $+$, $\times$ and ., $\not\succ$ and $\geqslant$ (cf. 26, note), however, the relationship Φ' which results from F through replacing the combinations and relations in the "primary" relations f by their formal counterparts also holds, while the "secondary" relationship $\leqslant$, that obtains between the primary f_1 and f_2 (as well as any secondary combinations that may hold, that is, combinations between relations of inclusion) will continue to hold between the formal counterparts of those primary relations (φ'_1 and φ'_2) and hence can be expressed through the relation $\not\succ$ that is equivalent to it. For the relationship Φ', so determined, there is then again the reciprocal, F'. So one has
$\Phi' \ldots \varphi'_1(\times, \nsim, \not\prec) \not\succ \varphi'_2(\times, \nsim, \not\prec) \quad F' \ldots f'_1(+, ., \geqslant) \leqslant f'_2(+, ., \geqslant)$.

One says that φ und φ' are *dual* to each other, and also f and f'; and in a wider sense Φ' and F' can also be considered as dual counterparts to Φ and F. Every proposition of the kind of Φ (or F, Φ', F') accordingly represents a foursome of propositions, Φ, F, Φ', F', a fact, which it would be necessary to take advantage of in a systematic presentation of symbolic logic.

This is none too clear, and Mally himself concludes that a precise account of reciprocity would be necessary in a fully "systematic presentation" of symbolic logic. This seems to suggest, however, that one can fairly freely replace pairs of dual notions, as long as it is done systematically through an entire formula.[10]

A simple statement of reciprocity would be that the axioms for the one primitive notion of Schröder's logic, the "subsumption" relation, can be read as ambiguously true of the subset relation and predication (or membership of an object in a class). But this seems clearly false. There are principles that hold of logic which don't hold of subsets. Consider this example (which comes from Edgar Morscher). Consider the theorem of propositional logic, $(p \supset q) \vee (q \supset p)$. In Mally's symbolism it is:

(1) $\quad (\alpha \not\prec \beta) \times (\beta \not\prec \alpha)$

The corresponding "reciprocal" principle about sets, with a simple replacement of the symbol for implication with that of subset, would seem to be this proposition, $(x \subseteq y) \vee (y \subseteq x)$, which is clearly not true:

(2) $\quad (a \leqslant b) \times (b \leqslant a)$

The proposed principle of "reciprocity" seems to describe the phenomenon of "duality", which occurs in several places in logic, as when it is said that within propositional logic $\vee$ and $\wedge$ are duals. The proper analysis of Morscher's example is obtained by using a translation which produces equivalences. In this case, to get from relations of propositions to relations between sets, we have to translate the propositional connective $p \supset q$ first as $\sim p \vee q$ and then translate $\sim$ with the complement of a set $\bar{x}$ and $\vee$ with the union of sets $x \cup y$, and state a theorem of propositional logic as an identity with the *verum*, $\top$. Formula (1) must be rewritten as $(\sim p \vee q) \vee (\sim q \vee p) \equiv \top$:

(3) $\quad (\bar{\alpha} \times \beta) \times (\bar{\beta} \times \alpha) \equiv \tilde{0}$

The translation of this theorem of propositional logic will yield a truth about sets. (Of course for the complement of a set to exist, and for there to be a universal set, we must restrict the set theoretic interpretation to a set.) In this way the dual of the theorem of propositional logic (1) is a true claim about sets $(\bar{x} \cup y) \cup (\bar{y} \cup x) =$ V, in which $\bar{x}$ is the *complement* of x:

$$(\bar{a} + b) + (\bar{b} + a) = \dot{1} \tag{4}$$

As Mally states the reciprocity (Reziprozität) principle (R) in §10 (p.17), he seems to suggest that one may substitute expressions one by one, in particular $<$ for $-\!\!\!<$, to get a claim about classes that is the dual of a theorem about classes. But, clearly, if the formula about classes has the symbol $<$ within the scope of another $<$, this isn't a possible way to interpret it. One could reinterpret (1) as an assertion about implication, namely, $\sim (p \supset q) \supset (q \supset p)$:

$$-(a -\!\!\!< b) -\!\!\!< (b -\!\!\!< a) \tag{5}$$

but the one for one substitution of subset for implication signs in (5) also doesn't get something that is well-formed as it has an expression with $<$ inside of another expression about $<$. It must be that the internal ("primary") implication must be translated in such a way that it expresses a name for a set, so that the principal connective ("secondary relationship $<$") can really express the subset relation.

This notion of a formalism that is interpretable in two fashions is common in the wider algebraic tradition to logic to which Schröder, and hence, Mally, belonged. Thus Guiseppe Peano lists forty three axioms of logic in his famous "*Arithmetices principia*", then an additional eight for classes, and then says that "Propositions 1-41 still hold if $a, b, \ldots$ denote classes ...", but then he goes on to list additional principles only true of classes. What Peano presents, then, are really two separate calculi, rather than two interpretations of one formal system.[11] Mally, however, doesn't seem to make this distinction.

The argument about determination here does not depend on the extent of Mally's confusion about reciprocity, but it is at least supported by the extent to which Mally thought that a given expression can be simply replaced symbol by symbol to produce a valid proposition about a subject matter for which it was not intended. Thus Mally's formalism of classes could be interpreted as about "formdeterminates" or abstract individuals, and the properties that determine them.

We are now in a position to understand how Mally could express a mode in which an object has a property with the relationship of *determination* between classes.

Meinong's term "*Gegenstandstheorie*" is indeed properly translated as "Object theory", and so the title of Mally's monograph is appropriately represented as "Object theoretic foundations of logic and logistic." The translation of "Gegenstand" as "object" is familar from Frege's distinction between "Begriff" and "Gegenstand" (concept and object), as it appears in the title of his famous essay.[12] For Frege, of course, the distinction between concept and object marked a fundamental and unbridgeable ontological distinction. Not only is everything in Frege's logical ontology one or the other, it is, famously, not even possible to name or refer to a concept with any singular term, which could only be capable of naming an object. Thus "the concept horse," if it is to have a reference, cannot refer to a concept, and Frege observes that we have to accept the paradoxical formulation "The concept horse is not a concept." Frege's logic is based on the notion of concept, which are the reference of predicates, and objects, which correspond to singular terms. Sentences all refer to truth values, which are objects, and so propositional connectives literally refer to functions from objects to objects, in particular truth values to truth values. Quantifiers are in fact second order functions, mapping concepts onto truth values, depending on whether all or some objects fall under the given concept.

Schröder's logic treats objects very differently. The single formal system is based on the notion of *subsumption*, a relation which holds between both concepts and classes, yet symbolized by a single symbol $a \Subset b$. Mally describes these two interpretations as related by "reciprocity." Any reasoning about an object would have to be carried out with respect to its singleton class, or the concept uniquely true of that object. Frege, and others, including Edmund Husserl (1979), and later Russell, explicitly criticize this confusion of an object and its singleton. Norbert Wiener, who wrote his doctoral thesis at Harvard on a comparison of Schröder's logic with that of Russell and Whitehead, claims that there is no confusion on this point, but rather one can be considered as the other for logical purposes.[13] This is in keeping with Mally's notion of reciprocity between the interpretations of the logic on the two interpretations he considers, about classes and about propositions and propositional functions.

Unlike those two interpretations, there is no further interpretation of Mally's logic which makes it about concepts and objects as Frege made the distinction. Logic seems only to deal with classes and propositions for Mally. Classes and propositions are both among the "objectives" (Objektives) which are among the subject matter of "object theory" (Gegenstandstheorie). Mally does have the notion of an objectum (*Objekt*), however, but it does not appear as an interpretation of the formal system. An objectum for Mally is the subject of what we would now call a singular proposition. It is what the proposition is about, and the rest of the proposition, the predicate, expresses some property of this objectum. An objectum (in the narrower sense) is what an idea or *Vorstellung* represents. It simply *is*, it is not true or false (or true or false *of* something) like an objective. Anything in the subject matter of object theory, any of the many objectives it studies, can be the objectum of some proposition. Here is what Mally has to say about objecta (*Objekte*) in the opening section of the *Grundlagen* (p.1):

> **§1. Zwei Arten des Erfassens**
> Wenn ich urteile „7 ist eine Primzahl", so habe ich *über* die Zahl 7 geurteilt oder 7 *beurteilt*, und ich habe den Sachverhalt, daß 7 eine Primzahl ist, *geurteilt*.
>
> Der Gegenstand (7), über den geurteilt wird, oder der beurteilt wird, heißt gewöhnlich *Objekt* des Urteils.
>
> Das, was geurteilt wird (der geurteilte „Sachverhalt") werde als *Objektiv* des Urteils bezeichnet.
>
> **§1. Two kinds of grasping.**
> When I judge "7 is a prime number", then I have *judged of* the number 7, or *judged of* 7, and I have *judged* the state of affairs that 7 is a prime number.
>
> The object (Gegenstand) 7, about which something is judged, or of which I have judged, is usually called the "objectum" (*Objekt*) of the judgement.
>
> That which is judged (the judged "state of affairs") is called the *objective* of the judgement.

Thus to say that 7 is the objectum *of a judgement* is not to say that it is in a certain ontological category. The most general category to which everything belongs is that of "object"(*Gegenstand*). Objectives (*Objektive*) are the range of objects which can be judged, whether propositions or relations of concepts (such as subsumption or coextensiveness) which form the subject matter of logic. The individual objectum (*Objekt*) is found, so to speak, after the fact of asserting or entertaining a proposition, upon the analysis of that "judgment" or "assumption":

> **§2. Objektiv und Objekt**
> Was geurteilt oder angenommen wird, ist in anderer Stellung zum Erfassungsakte als das, worüber geurteilt oder angenommen wird.

Mit Rücksicht auf diese Stellung ist es zunächst als Objektiv des betreffenden Erfassungsaktes bezeichnet worden, zum Unterschiede von dem Objekte oder den Objekten desselben. . . .

Es gibt aber Gegenstände, die niemals als Objektive, sondern immer nur als Objekte von Urteilen oder Annahmen auftreten, die also nicht gesetzt, sondern nur im engeren Sinne des Worte erfaßt werden können: solche nennen wir *Objekte* im *engeren Sinne des Wortes*. Alles, was nich Objektiv ist, gehört offenbar in diese Kategorie.

§2. Objective and *Objectum***.**
What is judged or assumed stands in a different position with respect to the act of apprehension than that about which something is judged or assumed. With respect to this position it has at first been called the objective of the respective act of apprehension, in order to distinguish it from the objectum (*Objekt*) or objecta (*Objekte*) of the same act. . . .

There are, however, objects which can only appear as an objectum and never as an objective in a judgement or assumption, that are not asserted but only grasped in the narrower sense of the word, and these we call "objecta in the narrower sense of the word." Everything which is not an objective clearly belongs in this category.

If it is only in "with respect" (*mit Rücksicht*) to a position that we can find an objectum, then being the objectum does not mean belonging to an independent ontological category, but rather playing the role that an object plays in certain objectives. According to Findlay, in Meinong there is an ontological classification of Gegenstände, or the most general notion of object, into objecta, which can only be known through ideas, and objectives, which can be the objects of judgement or assumption,and which can be true or false. For Mally anything, that is, any Gegenstand or object, can be the objectum in a proposition, even if it is ordinarily an objective or the content of a judgement.[14]

We are now able to makes sense of Mally's two different accounts of determination. First there is the "definition" of determination at paragraph §11, 40, as a^{β} to be read as "a with the determination β", where it immediately follows "that $a^{\beta} \equiv ab$ namely, the a's which are β." This makes determination an operation on classes, the restriction of a class a to its members which are β. This is different from encoding, for, first, it is the class that is determined, rather than an (abstract) object, and secondly, the determination by β amounts simply to exemplifying the concept β.

Yet already in paragraph §11, 40, and in §33 we have "the circle (in abstraction)" being determined by, but not satisfying, certain properties. That abstract object "circle" is a *formdeterminate*, which is not being an "implicit" determinate, not by falling under, or being in a class restricted by that concept. Instead, I propose, Mally intends determination of an abstract object not to be the relation between classes and concepts expressed by a^{β}, but rather something we would symbolize with an individual variable which ranges over abstract objects, say x, and a class expression β, thus as x^{β}, what Zalta, with F as a predicate variable, could have written as x^{F}, but chose to make into the typographically simpler xF.

Mally does give a way of translating expressions about determination into relations between sets, as when we are told that $a^{\beta} \equiv ab$, that the concept α determined by β is the intersection of the class of α's and the class of βs (ab), but goes on to discuss determination as though it were a primitive notion. My suggestion is that Mally took that primitive notation a^{β} to be subject to two interpretations, and so, presumably, there would be a sort of reciprocity principle available. When an object, such as 7, or more importantly, a concept, is taken as the objectum of a proposition, then one sense of that proposition is to assert that the abstract object which can be taken as the objectum of the proposition, is determined by the predicative part. It can also be read as a simple predication, and in that case it will state something about the subject, namely inclusion if the objectum is a concept. Thus a^{β} taken as about determination, asserts that β determines the abstract object "the α's", which is the class a. Taken in terms of the discussion earlier in the book, it is an expression for a term for "the α's which are β." This is a shift of category, from proposition to subject term, but, in object theory, all of these are objects, and the "theory" of objects will tell us about them. Mally does not examine which principles about determination (as restriction) will carry over to determination (as encoding). In fact if β is a property such as "being F and G", then α^{β} (as restriction) will refer to the α's (the members of a) which are F and which are G, but it does not follow that α^{β} (as encoding) as in "the round and not round thing" will be round and not round. Although Mally does make a stab at describing the relation of reciprocity between propositions and classes, he does absolutely nothing to indicate how to determine which logical principles govern this new relation of determination, not so much as a hint that certain principles about the one can be turned into "reciprocals" about the other. This is described in an "Overview" at the end of Part I of *Grundlagen* (p.50):

§25. Übersicht.

. . . Zwischen objektivischen und objektischen Beziehungen und Verknüpfungen besteht Reziprozität, beziehungsweise formale Entsprechung. Jene können vom Standpunkte des Subjektes aus als Setzungsbeziehungen, beziehungsweise als Setzungsverknüpfungen, diese als Erfassungsbeziehungen, beziehungsweise als Erfassungsverknüpfungen bezeichnet werden, da man den ersteren durch gewisse Verbindungen von Setzungsakten, den letzteren aber durch entsprechende Verbindungen von Erfassungsakten (im engeren Sinne des Wortes) gerecht wird. Die objektivisch-objektischen Beziehungen und Verknüpfungen können als „determinative" gekennzeichnet werden (mit Einschluss der prädikativen); psychisch entsprechen ihnen Akte, in denen durch die Setzung eines Objektivs oder Falles ein Gegenstand als Ding erfaßt wird und die wir ganz allgemein als ein „Bestimmen" zu bezeichnen pflegen (durch Objektive „bestimmen" wir Klassen und Dinge, durch einen Fall ein Ding, indem wir nämlich über sie oder von ihnen Objektive voraussetzen, das heißt annehmen, oder urteilen, prädizieren.)

§25. Overview.

. . . Between relations and connections of objectives and objecta there obtains reciprocity or formal correspondence, respectively. The former can be characterized, from the standpoint of the subject, as relations of positing and connections of positing, respectively, whereas the latter can be characterized by relations of apprehension or connections of apprehension, respectively, since combinations of acts of positing *are appropriate to* the former whereas corresponding combinations of acts of grasping (in the narrower sense of the word) *are* appropriate to the latter. The relations and connections of objectives and objecta can be characterized as "determinative" (including predicative relations and connections). Psychically they correspond to acts in which, through the positing of an objective or instance an object is being grasped as a thing, and which we generally describe as a "determination" (by means of objectives we "determine" classes and things, by means of an instance we determine a thing, in that we presuppose i.e., assume, or judge, predicate, objectives about them or of them).

Mally had the idea that when we treat a concept as an object, namely as the objectum of a proposition, there are two ways of treating predication. One way is to treat it as saying that the concept (as object) exemplifies some further concept. The other way is to treat it as saying that the concept (as object) has the concept in the way similar to that in which one concept is restricted by another. But he says nothing about the logical principles that govern that new relation of predication. That was left for Zalta to develop a formal theory of encoding and exemplification. Mally did anticipate the notion, but that was all, just an anticipation.

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Notes

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[2]Including Zalta (1983) and (1998) among others. The distinction was independently developed by Rapaport (1978), and a similar notion appears in Castaneda (1974).

[3]In what follows "*Gegenstand*" will be rendered as "object", so the obvious translation for "*Objekt*" is taken. I have chosen, therefore to follow J.N.Findlay, and translate "*Objekt*" as "objectum", with the plural form "*Objekte*" as "objecta" (Findlay 1963, 67). In his review, Russell says the following upon the first occurrence of "*Objekt*": "As this word is used in a different sense from *Gegenstand* I shall leave it untranslated, using "object" to translate *Gegenstand*." (Russell, 1905, 597, n.2).

[4]See Church (1976).

[5]This theory has two interpretations of the quantifiers and "existence predicate" (E!). The original, Neo-Meinongian interpretation of the quantifier is as "there is" with no existential commitment, and the predicate is the existence predicate true of those things that there are which also exist. More recently Zalta has preferred the Platonist interpretation on which the quantifier ranges over all existing objects, whether they are abstract or concrete, and the special predicate *E*! is true of those which are concrete.

[6]There will, however, be an abstract object which also does encode all of those properties that follow from being a triangle, like having three interior angles that add up to 180 degrees, etc. It will not, however, be isosceles, for example, nor will it not be isosceles. It will still be incomplete, though possessing all the properties that follow from being a triangle.

[7]See Findlay (1963), 110-12. Findlay uses a slight variation on this terminology. Mally also uses the term "Bestimmung" (perhaps closer to "fixing", but also meaning "determination") for this notion, at least when that is what is attributed to something, a propositional function or concept. Findlay refers to pages 64 and 76 in Mally (1912), however, the very passages quoted below.

[8]Both letters are in the Bertrand Russell Archives.

[9]Schröder's own interest in a much more restricted duality of $\vee$ and $\wedge$ in positive sentences is reported in Kleene (1952, 123).

[10]There is a discussion of the exact nature of the duality of $\vee$ and $\wedge$ in §27 of Kleene (1952).

[11]See van Heijenoort's introduction to Peano, in his (1967), p.90. In *Principles of Mathematics* §25 Russell adjudicates an objection to Schröder by McCall concerning the inference from *pq* implies *r* to (*p* implies *r* or *q* implies *r*) which is valid for propositions, but not for strict implication of propositional functions. It was McCall who didn't make the distinction properly in this case. In *Principia Mathematica* Vol.I, pp121, 209 and 210 it is stated that for certain formulas of propositional logic the corresponding formulas of the calculus of classes are invalid. (Thanks to Edgar Morscher for pointing out this reference.)

[12]Frege (1892).

[13]See Grattan-Guinness (1975).

[14]On page 2 we have: "Als Objekt eines Erfassungsaktes kann dagegen jederlei Gegenstand auftreten." (Any object can occur as the object of some act of apprehension.)

References

Hector-Neri Castañeda. Thinking and the Structure of the World. *Philosophia* 4: 3–40, 1972.

Alonzo Church. Schröder's Anticipation of the Simple Theory of Types. *Erkenntnis* 10: 407–11, 1976.

Ivor Grattan-Guinness. Wiener on the Logics of Russell and Schröder. An Account of his Doctoral Thesis, and of his Discussion of it with Russell, *Annals of Science*, 32, 103–32, 1975.

J.N. Findlay. *Meinong's Theory of Objects and Values*, Oxford: Clarendon Press, 1963.

Gottlob Frege. Über Begriff und Gegenstand, in *Vierteljahrsschrift für wissenschaftliche Philosophie* 16: 192–205. Translated as On Concept and Object, in [Frege 1984]: 182–94, 1892

Gottlob Frege. Kritische Beleuchtung einiger Punkte in E. Schröder's Vorlesungen über die Algebra der Logik, *Archiv für systematische Philosophie*, Vol.1 (1895): 433–56. Translated as A Critical Elucidation of Some Points in E. Schroeder *Vorlesungen über die Algebra der Logik*, in [Frege 1984]: 210–28, 1895.

Gottlob Frege. *Collected Papers on Mathematics, Logic, and Philosophy*. Edited by Brian McGuinness. Oxford: Basil Blackwell, 1984.

Edmund Husserl. Review of Schröder (1890) in *Aufsätze und Rezensionen (1890-1910)*. Edited by Bernhard Rang. *Husserliana XXII*, 1979.

Stephen Cole Kleene. *Introduction to Metamathematics*, New York: Elsevier, 1952.

Ernst Mally. *Gegenstandstheoretische Grundlagen der Logik und Logistik*, Ergänzungsheft zu Band 148, *Zeitschrift für Philosophie und philosophische Kritik*, Leipzig: Barth, 1912. Available online at: www.mally.stanford.edu/mally.html

Alexius Meinong. *Über die Stellung der Gegenstandstheorie im System der Wissenschaften*. R. Voigtländer Verlag: Leipzig, 1907.

Alexius Meinong. *Alexius Meinong Gesamtausgabe*, Vol. II, *Abhandlungen zur Erkenntnistheorie und Gegenstandstheorie*. Edited by R.Haller. Akademische Druck: Graz, 1971.

Giuseppe Peano. *Arithmetices principia*. Translated as: The principles of arithemetic, presented by a new method. In [van Heijenoort 1967]: 83–97, 1889.

William J. Rapaport. Meinongian Theories and a Russellian Paradox. *Noûs* 12 (2): 153–80, 1978.

Bertrand Russell. *The Principles of Mathematics*. Cambridge, Cambridge University Press, 1903.

Bertrand Russell. Review of: A. Meinong, *Untersuchungen zur Gegenstandstheorie und Psychologie*, *Mind, New Series* 14: 530–8, 1905. In *Foundations of Logic 1903-1905*, *The Collected Papers of Bertrand Russell, Volume 4*. Edited by Alasdair Urquhart. London and New York: Routledge, 1994, 596–7. Page references to this edition.

Bertrand Russell. Review of: A. Meinong, *Uber die Stellung der Gegenstandstheorie im System der Wissenschaften*, *Mind, New Series* 16: 436–9, 1907.

Ernst Schröder. *Vorlesungen über die Algebra der Logik (Exacte Logik)*. Leipzig: Tuebner, 1890. Reprinted Bristol: Thoemmes, 2001.

Jean van Heijenoort. *From Frege to Gödel: A Source Book in Mathematical Logic, 1879–1931*. Cambridge: Harvard University Press, 1967.

A.N. Whitehead and B.A. Russell. *Principia Mathematica*, Cambridge, Cambridge University Press, 1910-1913. Second edition, CUP, 1925-27. Preface and Introductions translated by Hans Mokre as *Einführung in die Mathematische Logik (Die Einleitung der „Principia Mathematica")*, München, Berlin: DMV Verlag, 1932.

Edward N. Zalta. *Abstract Objects: An Introduction to Axiomatic Metaphysics*. Dordrecht: Reidel, 1983.

Edward N. Zalta. *Intensional Logic and the Metaphysics of Intentionality*. Cambridge, Mass.: MIT Press, 1988.

Edward N. Zalta. Mally's Determinates and Husserl's Noemata, in Alexander Hieke, editor, *Ernst Mally: Versuch einer Neubewertung*. Sankt Augustin: Academia Verlag, 1998.