

PROJECT REPORT 2001-25

FINAL PROJECT REPORT

sustainable
forest
management
network

réseau
sur la
gestion durable
des forêts



A Decomposition Approach to Modeling Overlapping Tenures in Alberta: A Case Study

A Network of Centres of Excellence
Un réseau de centres d'excellence 

Grant K. Hauer and David M. Nanang

For copies of this or other SFM publications contact:

Sustainable Forest Management Network
G208 Biological Sciences Building
University of Alberta
Edmonton, Alberta, T6G 2E9
Ph: (780) 492 6659
Fax: (780) 492 8160
<http://www.ualberta.ca/sfm>

ISBN 1-55261-135-3

A Decomposition Approach to Modeling Overlapping Tenures in Alberta: A Case Study

**SFM Network Project: Optimization-based Forest Planning Tools for
Sustainable Forest Management**

by

Grant K. Hauer and David M. Nanang

Department of Rural Economy
University of Alberta, Edmonton, Alberta

July 2001

ABSTRACT

The purpose of this study was to demonstrate how a decomposition approach to solving large spatially detailed forest management scheduling models could be applied to the problem of overlapping tenures in Alberta. A Model II forest scheduling model that maximized net present value subject to mill capacity, multiple mill and product demands, regeneration, area, overlapping tenure, and even-flow constraints was specified. The resulting formulation is extremely large with over 5 million decision variables and at least 100,000 constraints. The decomposition approach was able to solve this formulation in about 30 minutes on a computer with a Pentium III processor. This shows that the method used has a potential of being applied in practice to investigate long-term timber supply and demand situations where spatial detail is required.

This overlapping tenure application of the model showed that constraints imposed by overlapping tenures could lead to inefficiencies in wood allocation. The results showed that marginal costs of the overlapping tenure constraints were positive and that the marginal cost of producing wood products increased when overlapping tenure constraints were present. The model also provides important shadow price information (marginal cost or marginal value) that is useful for determining how various constraints affect each demand location in the model. For example, the results in this report show that relaxation of overlapping tenure constraints may lead to gains for some mills and losses for others although the overall effect of removing constraints is positive.

ACKNOWLEDGEMENTS

We would like to thank Sustainable Forest Management Network for providing the financial support needed to carry out this study. Neil Stevens and Mark Messmer of Weyerhaeuser Canada have also provided valuable support with the provision of forest inventory data, yield information and other management information. We would also like to thank our programmers Ryan Thompson and Jackie Lau for their valuable contributions.

INTRODUCTION

Increasing demands for wood products and for non-timber goods and services in forests place ever-greater requirements on scheduling and planning of management activities on forested areas in Canada. The problem is extremely complicated because it requires scheduling the harvest and other management activities on many forest classes over time and space and must simultaneously consider the forest-wide constraints on wood demand (usually from several locations) and other non-timber goods and services. The fact that wood demand usually arises from several locations and from several different companies with different use rights to wood or forest tenures over the same land base complicates planning even further. The purpose of this research was to develop a forest management scheduling model based on economic optimization principles and at the same time capable of handling a large amount of spatial detail, both in terms of representation of forest supply locations and alternative mill demand locations. The basic framework was developed so that the model's capabilities could be expanded to incorporate spatial models of non-timber forest user behavior and a strategic forest access model. In addition, the model was used in a case study to examine the consequences and costs of having several companies with use rights for wood over the same land base (overlapping tenures).

Mathematical programming is a widely used technique in planning the management of forests because of its adaptability to the wide range of problems encountered in forest management. Due to the large spatial and temporal dimensions considered, harvest scheduling usually involves very large linear programming models, including, in some cases, hundreds of thousands of choice variables and thousands of constraints. Current forest management scheduling models used in Alberta for allowable cut calculations, for example, incorporate large amounts of spatial detail, at least for the first harvest in the planning horizon, with some even using the stand polygon as a spatial unit (M. Messmer, Weyerhaeuser Canada Ltd., Edmonton, Alberta, pers. comm.). However, these models tend to be simple "maximize volume over time" objective functions and tend to be quite constrained in terms of the number of alternatives they consider (e.g. the number of regeneration alternatives is usually restricted to one). Even though the models consider relatively few options given the large spatial detail represented and with recent advances in computer technology the resulting models can be cumbersome to use and may take many hours or days to solve.

While incorporating economic criteria into a forest management-scheduling model does not complicate matters in principle, incorporating the overlapping tenure considerations into models does. Overlapping tenures in Alberta impose several types of constraints on woodland operations. To understand these constraints a short description of overlapping tenures is warranted. Overlapping tenures are areas of land where harvesting rights are allocated to more than one firm on the same piece of land. The usual configuration of overlapping tenures in Alberta is that one firm has an area based tenure with rights to harvest over the whole area of the forest as well as rights to harvest all or most of the tree species on the areas. Along with these

rights there are obligations to regenerate harvested areas and to plan for the sequencing of harvests over the land base and to ensure that harvesting operations are sustainable. The full set of rights and obligations are set out in a Forest Management Agreement (FMA). The term overlapping tenure comes about when within the FMA areas there are embedded volume based tenures, which are held by other firms. Although the volume based tenures or quotas are not “area-based” like FMAs there are usually restrictions on which areas within the FMA boundaries that harvests may take place. In addition, volume based tenures are usually species specific. Only certain species (deciduous, conifer) may be harvested by the quota holders. To complicate matters further land is usually classified on the basis of the predominant species and the harvesting rights of volume quota holders are usually restricted to the land base for which species specified by the quota predominates. For example, if the volume quota is for conifer the quota holder may be restricted to harvesting off the conifer land base. In some cases quota holders may also have rights to incidental volumes harvested off other land bases.

While the above sets out the main constraints implied by overlapping tenures, these constraints also interact with other regulations. First, regeneration standards are specified to return forest stands to approximately the same species composition that existed before harvest. Hence, conifer stands are regenerated to return to conifer, deciduous to deciduous and mixed to mixed. This may in some cases prevent stands from being regenerated most cost effectively from the FMA holder’s perspective. Second, there are often implicit sustainability constraints applied to the landbases, embedded within the larger FMA landbase, from which quota holders draw their wood supply. Finally, it is usually unclear as to who has rights to increased allowable cuts that may be obtained by increasing silvicultural input into the forest.

Overlapping tenures have emerged in Alberta as a significant issue as the number of Forest Management Agreement Areas (FMAs) has increased. Presently, there are 18 Forest Management Agreement Areas (FMAs) and 92% of all quotas in Alberta are embedded in these FMAs.

Incorporating the constraints implied by overlapping tenures complicates model formulations in several ways. First, to properly represent the overlapping tenure situation more than one demand location must be represented so that the different tenure holders may be modelled. To fully capture the costs of constraints the transport costs from each supply location to each demand location must be considered. Hence, supply locations or the spatial detail in the original forest representation must be maintained throughout the model’s planning horizon. Second, since overlapping tenures impose restrictions on where and on what kind of land class from which wood can be taken, models must keep track of which land classes and which locations desired volumes are being taken from. Third, if implied sustainability constraints on subsets of land within FMA areas are to be represented, these constraints must be added to the model. Finally, if the cost of regeneration constraints and the interaction of these constraints with overlapping tenure is to be modeled several regeneration options for each land class must be incorporated.

The formulation of the basic underlying forest management scheduling model in combination with the overlapping tenure constraints results in a very large linear programming formulation. As observed earlier standard solution approaches to linear programming problems will be difficult to implement and/or take long solution times. One possibility is to use a simulation model to find approximate solutions (see Cumming and Armstrong 1999). A simulation model has the advantage of being able to incorporate spatial and temporal detail with relative ease. However, one problem with simulation models is that the analyst does not know how close the latest simulation run is to the best solution. In addition, most simulation approaches employ short term scheduling heuristics which do not lend themselves to intertemporally optimized harvest schedules nor do they provide the shadow price information on any of the constraints incorporated in the formulation.

The introduction of an alternative “simulation approach” by Hoganson and Rose (1984) and the augmented lagrangian method by Gunn and Rai (1987, 1988) which are based on a dual decomposition techniques are not only based on optimization procedures but are also capable of incorporating a large amount of spatial and temporal detail. These decomposition methods take advantage of the presence of special structure found in mathematical programming problems to break the larger problem down into easier to solve sub-problems. The model developed in this study uses a dual decomposition technique based on the interpretation of the dual side of a linear or non-linear programming formulation of a Model II forest-management scheduling model. The main advantage of this approach is its ability to include a large amount of spatial and temporal detail in the models. A second advantage is that the procedure focuses on shadow prices on the constraints. This means that shadow prices in terms of \$m⁻³ or \$ ha⁻¹ for each of the constraints in the model can be obtained. The shadow prices from the estimated output constraints (output prices) provide useful information. For example, shadow prices on demand constraints can be interpreted as the marginal cost of production. The simulation approach also allows for optimal scheduling and balancing of multiple products from pure and mixed forest stands. Because the method optimizes over time and across stands, it is ideal for mixed wood management because demands can be specified for each species and product and optimized simultaneously. This makes the model ideal for estimating the marginal costs of land use constraints incorporated into overlapping tenures. The optimization procedure puts appropriate weight on each species in the stand. In addition, it is possible to model multiple markets and demands in different locations.

Previous studies that have estimated the costs associated with institutional and /or overlapping tenure constraints are Cumming and Armstrong (1999) and Avalapati and Luckert (1997). The former study compared existing tenure arrangements with a global policy where a single agent is responsible for forest management and for supplying all mills with timber. The authors concluded that the costs of overlapping tenures and divided land bases are substantial enough to warrant a thorough examination of forest policy in Alberta. Alavalapati and Luckert (1997) modeled the short-run timber supply of quota holders in Alberta in the face of institutional constraints and fixed stumpage prices using dynamic optimization techniques. The shadow prices of mill processing capacity and allowable cut restrictions were estimated for large,

medium and small tenure holders to reflect the different cost structures of different sized firms. The results indicated all categories of quota holders studied incurred substantial costs due to these two institutional constraints, and that simultaneous elimination of both constraints leads to more cost reduction than the combined savings from eliminating each constraint individually. The focus of that study was on quota holders, whilst we model both FMA and quota holders and include a lot of spatial detail, that requires the use of a decomposition technique.

The results presented in this report demonstrate the utility of the dual decomposition approach by modeling two hypothetical, but plausible timber, supply problems. While the timber supply model is formulated for two real land bases we have modified mill demands and constraints slightly. Hence, while the description of the land base and the overall overlapping tenure situation are meant to be realistic, the model implementation is modified enough to be less realistic in terms of the magnitude of the direction of changes, the wood demands represented and the number of mills represented. A more detailed examination of the costs implied by overlapping tenures will be left for discussion in a future paper.

DATA ANALYSIS

Model Formulation

Two Weyerhaeuser FMA areas were used in this study. A forest management scheduling model was formulated as a mathematical programming problem with an objective function that maximizes the net present value from wood products subject to regeneration, evenflow, multiple mill demands and capacities, and overlapping tenure constraints. The two FMAs used in this study are those in Edson and Drayton Valley with a total productive forest area of 550,000 ha spread over approximately 145 townships. These two FMAs supply two oriented strand board (OSB) mills in Edson and Drayton Valley with rated capacities of 415 and 445 million square feet (3/8" basis)/annum respectively and one sawmill in Drayton Valley (with a rated capacity of 120 million board feet of lumber annually) with logs for their operations. In addition, there are timber quotas on the FMAs of Weldwood Canada, Miller Western, Blue Ridge and Sunpine timber companies. Weyerhaeuser also buys wood from private landowners to cater for any shortfalls in their wood requirements that are not met from these two FMAs and quotas. On the Drayton Valley FMA, there are two firms that have volume quotas, whilst there are five quota holders on the Edson FMA. The operations on both FMAs therefore constitute a good example to investigate overlapping tenures.

Mathematically, the timber supply problem for the two Forest Management Agreement (FMA) areas can be described by the set of equations (P) given in the Appendix. The formulation is an extension of the model II formulation given in Johnson and Scheurman (1977). The objective function maximizes the net benefit of timber products from all mills (firms) with rights to timber on the two FMAs in Edson and Drayton Valley subject to model II age class constraints, demand constraints, sustained yield constraints, and overlapping tenure constraints.

The benefits are the net returns of the value of wood products minus costs of regeneration, harvesting, transportation, and wood purchased by Weyerhaeuser from private landowners.

Data Descriptions

This section provides a description of the data and their sources, as well as methods used to derive some of the variables. As indicated earlier, the objectives of the study are to investigate forest management scheduling problems in Alberta, using the above-specified model. To accomplish this task, we need to classify the two FMA areas into stands, project the growth and yield of these stands, determine the value of ending inventory, compute the soil expectation values of the stands for the different prescriptions, and schedule the various stands for harvesting. These procedures required large amounts of data from many different sources. The different types of data include inventory, growth and yield, transport costs, mill locations and outputs, shares of allowable cut for the various firms, regeneration prescriptions, etc. These data types are briefly described below together with their sources.

Forest type classification

The total area of the two FMAs was classified into forest types based on the Alberta Vegetation Inventory (AVI). Forest types are defined based on the cover type (conifer, deciduous, conifer deciduous, or deciduous conifer), dominant species, and the timber productivity rating. Cover type measures species composition of the stands based on crown closure. Species composition in the AVI shows the percentage of each species to the nearest 10%. The AVI identifies five timber productivity ratings (TPR). The TPR is the potential timber productivity of a stand based on height and age of dominant and co-dominant trees of the leading species. The four TPR codes G, M, F, and U are interpreted as good, medium, fair and unproductive sites. Based on this classification, there were 32 forest types.

Because transportation costs play an important role in this study, the classification of the forest into forest types and age combinations was extended into supply location/forest type/age class combination. For the purposes of the example in this report, a supply location is one-quarter of a township and therefore has a total area of 25,000 ha. Given that there are 145 townships in the study area, there were 579 locations identified. Therefore, a supply location and forest type combination will be referred to as a stand type, whilst a stand type and age class combination within a supply location will be called an analysis area. The classification used resulted in 6,741 stand types and 29,885 analysis areas.

Regeneration prescriptions

Two types of regeneration treatment prescriptions were defined for existing stands and regenerated stands, according to forest type. In general, however, these included natural regeneration and combinations of seeding, planting, tending and pre-commercial thinning. For existing stands, only one tending operation was prescribed. Three prescriptions were assigned to regenerated (bare land) stands. These were natural regeneration, basic planting and planting with tending operations. Natural regeneration involves allowing the stands to regenerate naturally with little intervention by the firm. Basic planting involves planting of seedlings with very little

tending operations. The most intensive prescription is the planting with tending. The tending operations considered are herbicide applications and spacing of stands. All conifer and deciduous land bases were prescribed to regenerate into conifers and hardwoods respectively.

Growth and yield

For each analysis area and regenerated stands, net merchantable volumes for the three species types were projected using yield curves developed by staff of Weyerhaeuser. The three species types are pine, spruce (white and black), and aspen. Yield curves were developed for each cover type, dominant species, timber productivity rating, and crown density. Crown density is classified in the Alberta Vegetation Inventory (AVI) into four groups; A, B, C and D, from the lowest to the highest density. Crown closure measures the percentage of ground area covered by the vertical projection of tree crowns onto the ground. In terms of percentages of crown cover, the codes A, B, C and D represent respectively, 6-30%, 31-50%, 51-70% and 71-100%. Based on this classification, there were 128 yield curves for the two FMAs.

Since tree size affects both processing and harvesting costs, it was considered important to sort tree products. Product sorting was limited to two classes: sawtimber and merchantable volumes. Therefore, there were six tree product types used in this study.

Demand locations

Wood from the two FMAs are delivered to nine demand locations in Edson, Drayton Valley, and Whitecourt. Table 1 shows the five sawmills, two oriented strand board (OSB) mills, and two pulp/paper mills and their locations. Pulp/paper mills can either produce their own chips, or purchase chips from the sawmills, which produce chips as by-products of lumber production. It should also be noted that wood from other FMAs and quotas held by Weyerhaeuser and the other timber firms is supplied to these mills, but are not considered in this study. Due to the small quotas for mills 2, 5, and 9, (Table 1) and due to the presence of other larger mills producing the same products in the same locations, the demands for these mills were added to the demands for the larger mills.

Harvesting and transportation costs

Transportation cost of wood from each supply location to each demand location was calculated along the shortest distance possible in the road network. Since each supply location is 5 x 5 km, shortest distances were calculated from the center of each supply location to each mill. Harvesting costs per cubic metre was estimated based on the tree-to-truck cost equation developed by Beck et al. (1987). This method of determining harvesting costs recognizes that costs decrease with increasing tree age (size). Therefore, a variable cost structure was used for each stand type.

Table 1. Demand locations, product types and share of AAC in each FMA that goes to each mill (conifer/deciduous).

Mill #	Location	Mill Ownership	Mill type	End Product	Edson (%)	Drayton Valley (%)
1	Drayton Valley	Weyerhaeuser	Sawmill	Lumber	38.27/0.00	90.08/0.00
2	Drayton Valley	Tallpine	Sawmill	Lumber	0.00/0.00	7.64/0.00
3	Edson	Edson Timber Pdts.	Sawmill	Lumber	10.25/0.00	0.00/0.00
4	Whitecourt	Blue Ridge	Sawmill	Lumber	11.55/0.00	0.00/0.00
5	Whitecourt	Millar Western	Sawmill	Lumber	0.57/0.00	0.00/0.00
6	Drayton Valley	Weyerhaeuser	OSB mill	OSB	0.00/0.00	0.00/100
7	Edson	Weyerhaeuser	OSB mill	OSB	0.00/90.56	0.00/0.00
8	Whitecourt	ANC Timber Ltd.	Pulp mill	Chips	26.41/0.00	0.00/0.00
9	Whitecourt	Millar Western	Pulp mill	Chips	0.57/0.00	0.00/0.00

Model Scenarios and Overlapping Tenure Constraints

In this section we outline two hypothetical scenarios. The first scenario, which is called the Base Run is meant to represent a case where overlapping tenure constraints are present. The second scenario is one in which overlapping tenure constraints have been removed. The two scenarios are shown in detail in Table 2. Neither scenario is meant to represent exactly what is actually happening on the two FMA areas. Rather the scenarios are meant to be representative of the types of constraints found when overlapping tenures are present on this area or other FMA areas.

The two runs are designed to reveal the effect of only two of the constraints implied by overlapping tenure discussed above. The two runs are also designed to reveal the effect of restrictions on where tenure holders may harvest. The second column in Table 4 identifies the demand location of which there are six. The demand locations are Drayton Valley (DV), Edson (ED), Whitecourt (WC). The demand locations also specify the type of demands. In this example there are 3 types of demand locations (Sawmills, OSB mills, and Pulp mills). The third column shows an estimate of the maximum price in terms of \$/m³ of roundwood that could be paid at the mill gate. These include constraints on allowed locations and on stand types from which wood may be harvested. The fourth column shows the maximum volumes that can be consumed by each mill on an annual basis. In this set of runs demands for hardwoods and softwoods add up approximately to the total volumes of these species harvested on the Edson and Drayton Valley FMA areas during the last 5 years. Hence, the maximum volumes that can be

Table 2. Summary demands and overlapping tenure constraints.

Types of constraints						
	Demand (thousands of cubic metres per year)				Restrictions on areas that mills allowed to harvest	
Model run	Demand Locations	Mill price \$/m3 *	Max 000s m3/yr	Wood type	Allowed locations	Allowed stand types
BaseRun 1	1 Sawmill (DV)	200	70	SW	Drayton, All	SW, HW
	2 Sawmill (ED)	200	70	SW	Edson, E1, E2, W6	SW, HW
	3 Sawmill (WC)	200	70	SW	Edson, E1	SW
	4 OSB (DV)	100	160	HW	Edson, W6	SW
	5 OSB (ED)	100	200	HW	All	HW, SW
	6 Chips (Pulpmill) (WC)	80	70	SW	All	HW, SW
Scenario 1	1 Sawmill (DV)	100	70	SW	Edson, W6	SW
	2 Sawmill (ED)	100	70	SW	All	All
	3 Sawmill (WC)	100	70	SW	All	All
	4 OSB (DV)	100	160	HW	All	All
	5 OSB (ED)	100	200	HW	All	All
	6 Chips (Pulpmill) (WC)	80	70	SW	All	All

* This is cubic meters of final product.

harvested in each area represent a combination of mill capacities and allowable cuts. In fact, the maximum volumes add up approximately to the allowable cuts for hardwood and softwoods. For this reason we do not impose allowable cut constraints because these constraints are redundant or nearly so in the presence of the maximum harvest levels shown in the fourth column of Table 2. Imposition of the allowable cut constraints in this formulation requires a modification of the demand configurations, which is being pursued in current research on the topic of overlapping tenures.

The last two columns in Table 2 summarise the overlapping tenure constraints. The second to last column shows the locations from which each mill is allowed to harvest while the last column shows the cover types from which each mill is allowed to harvest. The sawmill in Drayton Valley and the two OSB mills represents the FMA tenure holder's demand locations. The area restrictions for the Drayton Valley sawmill allow wood to be taken from the Drayton Valley FMA but not from Edson but there are no restrictions on where the wood may come from within the Drayton Valley FMA. Both OSB mills may harvest wood from anywhere in the Edson and the Drayton Valley FMAs. However, the sawmills and the pulp mill may harvest wood only from Edson. In addition, they are limited to harvest only from certain areas or FMUs within the Edson FMA and from the softwood cover type. In the Scenario Onr these constraints are

eliminated entirely so that wood may flow to any mill from any location. In this scenario wood is optimally allocated solely on the basis of maximising net returns.

Model Size

The major drawback of the problem defined above and in the appendix is its extremely large size. The total number of analysis areas (forest types/locations/age class combinations) is 29,885 for this management problem with 13,057 in Drayton Valley and the remaining 16,828 in Edson. We define the number of decision variables and constraints to the overlapping tenure problem using the following assumptions:

1. a planning horizon of 100 years with 10 planning periods
2. a minimum rotation of 40 years
3. three regeneration prescriptions per stand (natural regeneration, basic, and intensive silviculture)
4. a total of 6,741 stand types
5. approximately 10 shipping alternatives for each stand. This is based on 3 species with 2 size classes for each species, 4 possible destinations for softwood species, two possible destinations for hardwood species and the assumption that only half of these alternatives would be available for each stand on average.

With these assumptions, the resulting model has approximately 5,000,000 decision variables and 97,000 constraints (Tables 3 and 4). The number of constraints is conservative. If we also consider the overlapping tenure constraints, the number of constraints is probably in the range of 120,000. It is obvious that an attempt to solve a problem of this size with conventional linear programming techniques such as the simplex method is impractical.

Solution Approach

The model was solved using a variant of the dual decomposition algorithm proposed by Hoganson and Rose (1984). The principles behind this method are extremely simple. Using duality theory from mathematical programming theory, the original programming formulation can be viewed as a series of individual stand level decision problems. The stand level decisions include harvest timing for initial and subsequent harvests, mill destination for each timber type, and regeneration options. All the possible stand-level decisions are evaluated with a stand level objective function that is linked to the forest level objectives via shadow prices on the forest wide constraints. The solution to the stand level problem amounts to a stand level benefit-cost analysis. Costs include harvest, regeneration and transport costs. Benefits include the marginal value of timber derived from the forest-level demand constraints. Other costs include shadow costs or marginal costs of forest wide constraints that affect the stand of interest. The stand level decision problems are extremely easy to solve using dynamic programming. The solution is the combination of rotation, regeneration, and transport decisions that yield the highest net present value. The algorithm begins by solving each stand level problem using initial

Table 3. Calculation of the number of constraints for Problem, not including overlapping tenure constraints.

Eqn #	Constraint type	Constraint calculation		Number of constraints
1	Sawmill demand	3 sawmills	x10 periods	30
2	OSB mill demand	2 OSB mills	x10 periods	20
3	Pulpmill demand	2 chipmills	x10 periods	20
4	Sawmill chip production	3 sawmills	x10 periods	30
5	Initial area constraints	29,885 analysis areas		29,885
6	Area harvested = area regenerated	6741 stand types	x10 periods	67,410
Total				97,395

Table 4. Calculation of the number of decision variables for Problem P.

Variable Types	Birth period	Number of periods	Number of shipping alternatives	Number of prescriptions	Number of stand types or analysis areas	Number of decision variables
Initial Stands						
Harvesting variables		6	9	1	29,885	1,793,100
Ending inventory		1		1	29,885	29,885
Regeneration stands						
Harvest and regeneration variables	1	(10-4-1)	9	3	6741 stand types	1,011,150
	2	(10-4-2)	9	3	6741 stand types	808,920
	3	(10-4-3)	9	3	6741 stand types	606,690
	4	(10-4-4)	9	3	6741 stand types	404,460
	5	(10-4-5)	9	3	6741 stand types	202,230
	6	0				
	7	0				
	8	0				
	9	0				
	10	0				
Ending inventory		10		3	6741 stand types	202,230
Total						5,058,665

guesses at the shadow prices for each forest wide constraint. After all the stand level problems are solved the volume flows implied by the harvest timing and transport options are added up and compared to the demand and constraint levels. If the flows deviate from the constraint levels and mill demand levels then the shadow prices are adjusted using simple intuitive shadow price adjustment procedures. For example, if the harvest area restriction for a mill is violated (that is wood is delivered from a supply area to a mill when the mill is not allowed to harvest from that area) then the shadow price on the constraint is increased. This has the effect of imposing a cost penalty on transport options that violate the harvest area restriction. A second example, is when wood is oversupplied to a mill according to the mill demand constraint then the shadow price on the mill constraint will be decreased. Once the shadow prices have been adjusted, the stand level problems are solved again. This process is continued until the flows converge and all constraints are satisfied within a reasonable tolerance and there is no systematic deviation of constraints over time. The most important aspect of this approach is the ability to re-estimate the dual prices using previous price estimates. The various methods of adjusting the prices on the constraints are discussed in detail in previous studies (e. g., Hoganson and Rose 1984, Hauer 1993).

MODEL RESULTS

In order to implement the model runs, initial price estimates were given for each of the three end products for each demand location. The initial prices used were set to the maximum prices for all three products as shown in Table 4. All models were run on a microcomputer with two Pentium III 500 Mhz microprocessors. The criteria for determining when to stop a run was based on the average percentage deviation of the end product from the target demand by the mill. Average absolute deviations of 5% or less were considered acceptable. Secondly, we observed the changes in the objective function value. In most cases, by the time we achieve the 5% deviation, the objective function value would have stopped changing between iterations. Each iteration of the model takes about 5 seconds per iteration and it takes about 350 iterations for shadow prices and objective function values to converge. Hence, the model takes about 30 minutes to arrive at a solution.

Figures 1 and 2 show the simulated outputs for the final iteration and demand targets for the sawmills and OSB mills for the Base model. The graphs show that the flows are close to the final demands and that the flows are randomly distributed around the demands. The average absolute deviation from a demand constraint for the Baserun was 4.54%, with the highest deviation being 10%.

The shadow prices on each of the constraints provide useful information about the marginal costs of the constraints. The shadow prices on the mill demand constraints for lumber mills for both the base run and scenario 1 are shown in Figure 3. For the purposes of display the shadow prices have been converted into an index for each product type, where the lowest shadow

price has an index value of 100. The correct interpretation of these shadow prices is that these are the marginal costs of regenerating, harvesting, transporting the wood to the millgate, and milling for each mill. There are several patterns of interest in Figure 1.

1. Shadow prices for sawmills 1 and 2 are significantly higher under overlapping tenure constraints. However, for sawmill 3 the marginal costs are actually lower in the first three periods when overlapping tenure constraints are applied. After the fourth period the marginal costs under overlapping tenure constraints actually increase above the marginal costs when there are no overlapping tenure constraints. The general increase in marginal costs of lumber was expected. The overlapping tenure constraints restrict some mill's harvesting to certain locations and land bases, possibly preventing wood allocations to the locations of highest marginal value. Hence, one expects costs to drop once the constraints are removed. Sawmill three's marginal costs are probably lower under the overlapping tenure constraints (in the first three periods) because a) in the short term there is probably good economical wood available in the area that sawmill 3 is allowed to harvest and b) when the constraints are dropped alternative locations (competition) become available for shipping the wood. This result shows that although dropping the constraints results in an overall increase in net returns and reductions in costs individual mills gains are not evenly spread and in some cases gains will be losses.
2. Marginal costs tend to increase over the planning horizon and marginal costs increases tend to be greater when overlapping tenure constraints exist. These increases are due to long term scarcities that emerge as more of the wood is harvested off the FMAs. The marginal costs tend to increase more when overlapping tenure constraints exist because the inefficient wood allocation under these scenarios results in greater long term scarcity and in some cases more silviculture will be required to meet the demand constraints. The long-term scarcity of wood given these scenarios is also reflected in how the age class distribution changes over time. Figure 6 shows that the overall age of the forest gets smaller over time.
3. Marginal costs of sawmill 1, which is owned by an FMA holder, tends to be lower than those for mills 2 and 3 with the exception of mill 3 in the first 4 periods of the baserun. Sawmill 1 costs tend to be lower because the FMA holder has more alternative locations (and possibly lower cost locations) from which to harvest wood.
4. Marginal costs for all sawmills tend to be closer together when overlapping tenure constraints are relaxed than when they are applied. This occurs because the only major difference in costs that can exist in the model once overlapping tenure constraints are removed is in transport costs to the mills. At the margin of each mill's woodshed the value of sending wood to the competing mills will be roughly equivalent. This is just another version of the economic criteria for maximization, which says that at a maximum net present value the marginal returns to each land use will be equalized. Alternatively, under overlapping tenure woodsheds are not determined by economic criteria. Hence,

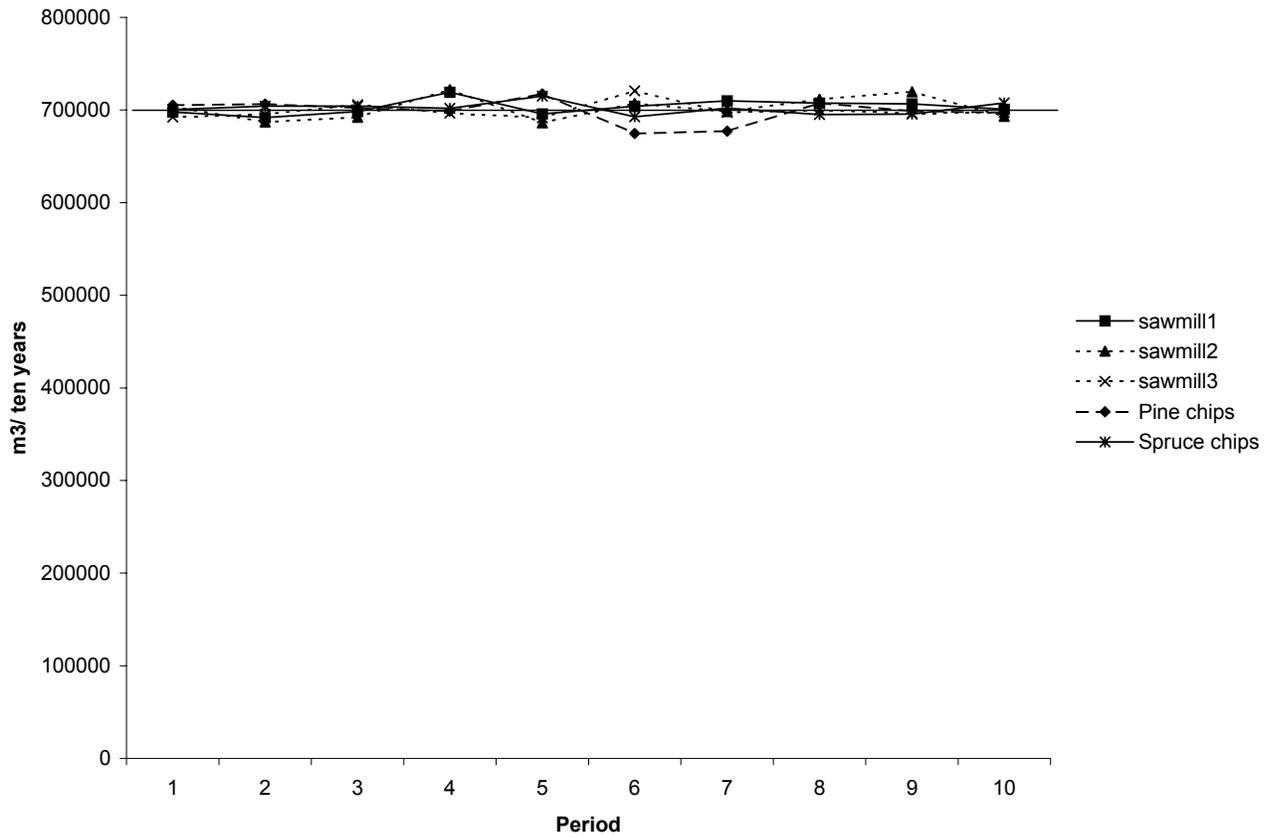


Figure 1. Simulated lumber and chip output for the sawmills and chip mill for the Baserun

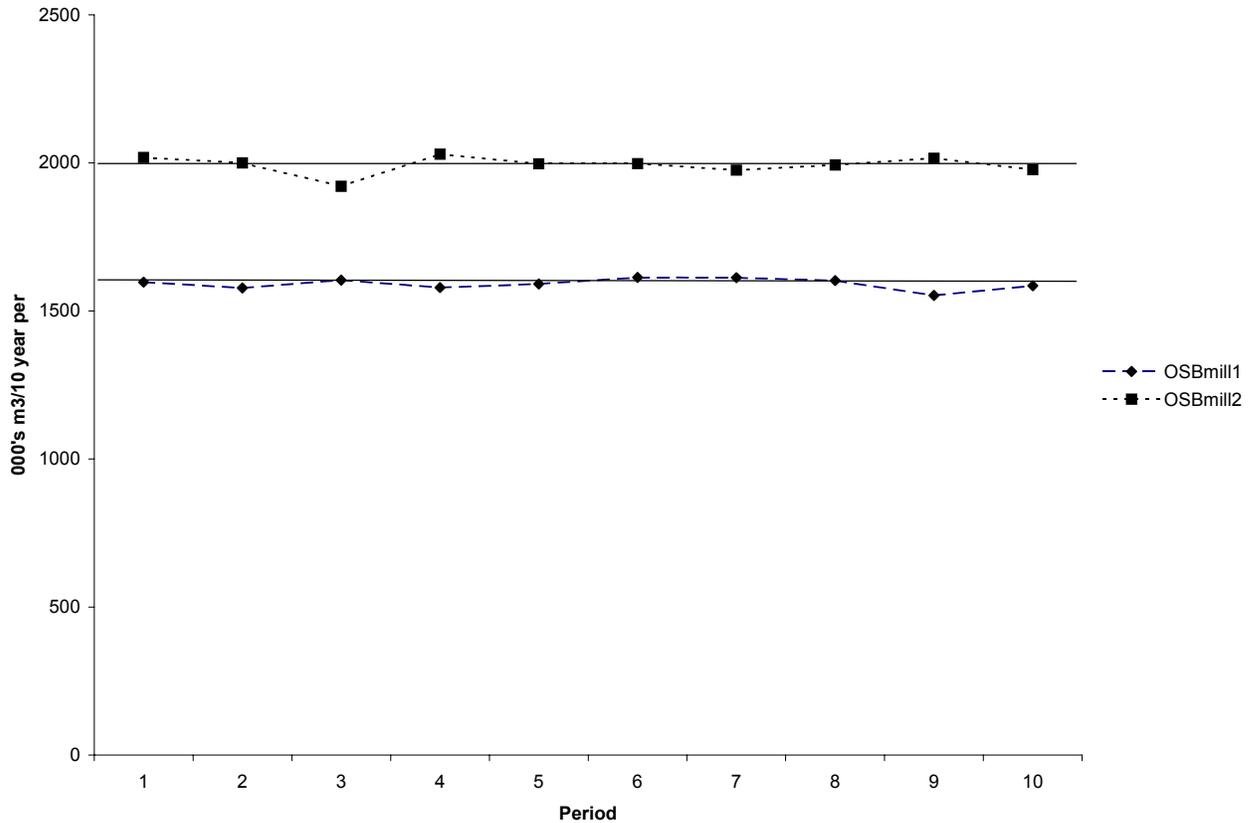


Figure 2. Simulated OSB outputs for the two OSB mills for the Baserun.

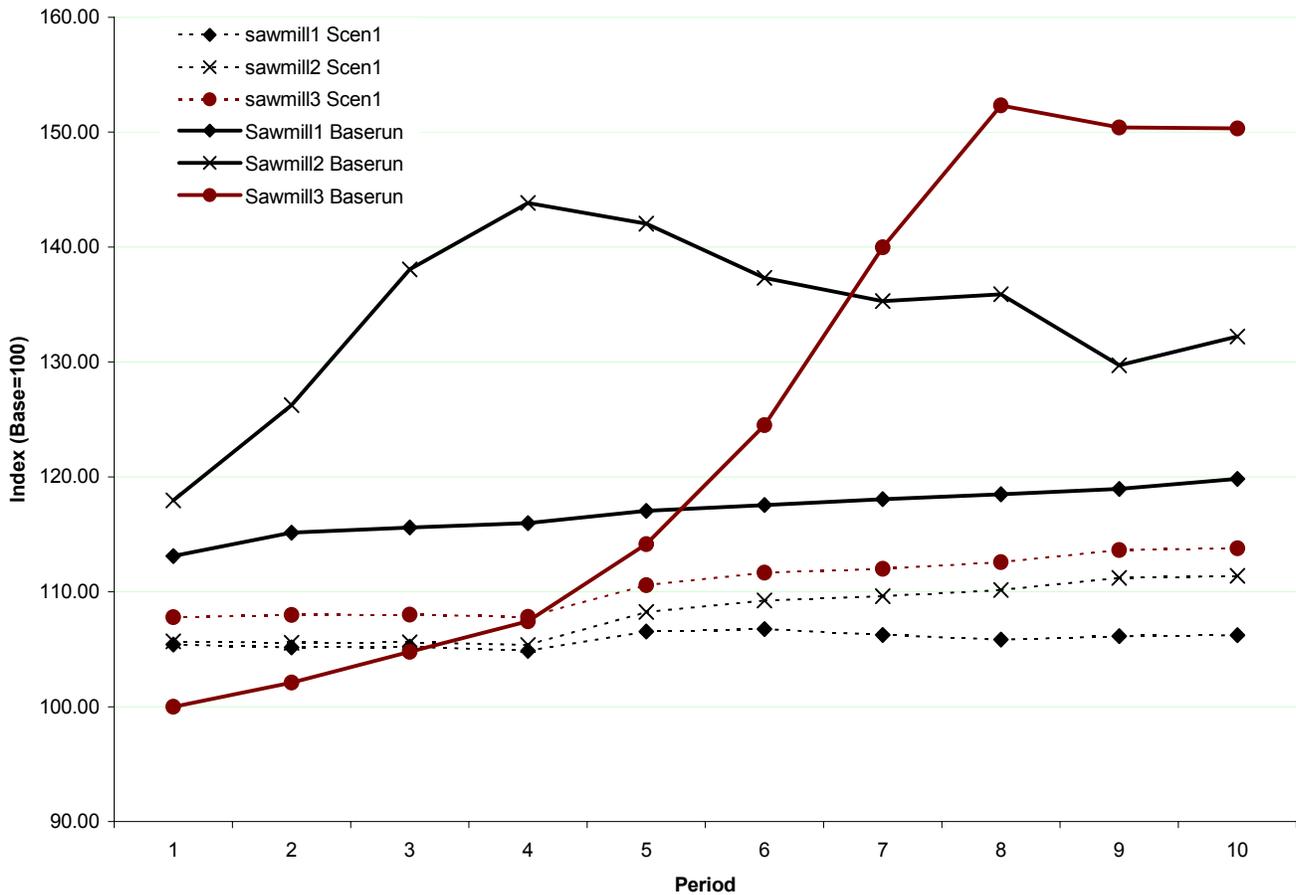


Figure 3. Indexes of shadow prices for lumber mills. Comparison for 3 mills and the baserun and scenario 1.

sawmills 2 and 3 will need to harvest less and less economical stands in order to meet wood requirements.

Shadow prices for OSB mills are shown in Figure 4. For OSB mills, allowing wood to flow across FMA boundaries results in a decrease in marginal costs for both mills. The decrease appears to be greater for the mill 1. Another notable result for the OSB mills is that shadow prices, within a scenario, remain the same or slightly decrease over the planning horizon. This reflects a relative abundance of aspen wood on the two FMAs. Figure 5 shows the shadow prices for pulp mills. The results are similar to those for OSB mills except that the decrease in marginal costs for spruce chips is very large if the overlapping tenure constraints are dropped. This probably reflects a scarcity of spruce wood within the allowed area of harvest.

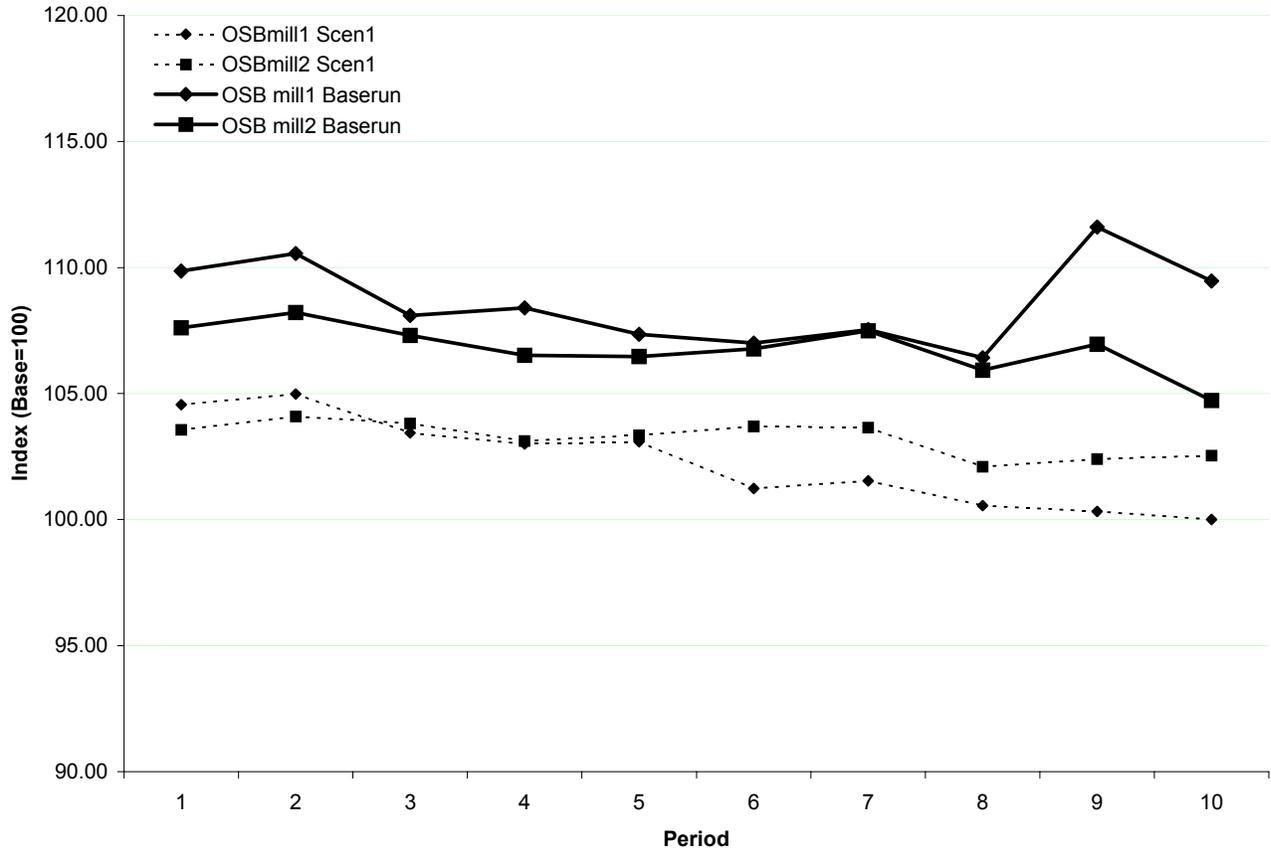


Figure 4. Indexes of shadow prices for OSB mills. Comparison for 2 mills and the baserun and scenario 1.

Table 5 shows the shadow prices or marginal costs of the overlapping tenure constraints. The shadow prices on these constraints represent the marginal reduction in the objective function as result of not allowing a small amount of wood to be harvested from the areas from which the mill is restricted from harvesting. Hence, since the shadow price for Edson sawmill is \$19.92/m³ in the first period the overall objective function would increase \$19.92 for every m³ of wood that it could harvest from outside the area from which it is currently allowed to harvest. The shadow prices for the OSB mills are zero because there are no restrictions on where these mills can harvest. The results in this table also suggest that the cost of the overlapping tenure constraints is large and could increase over time.

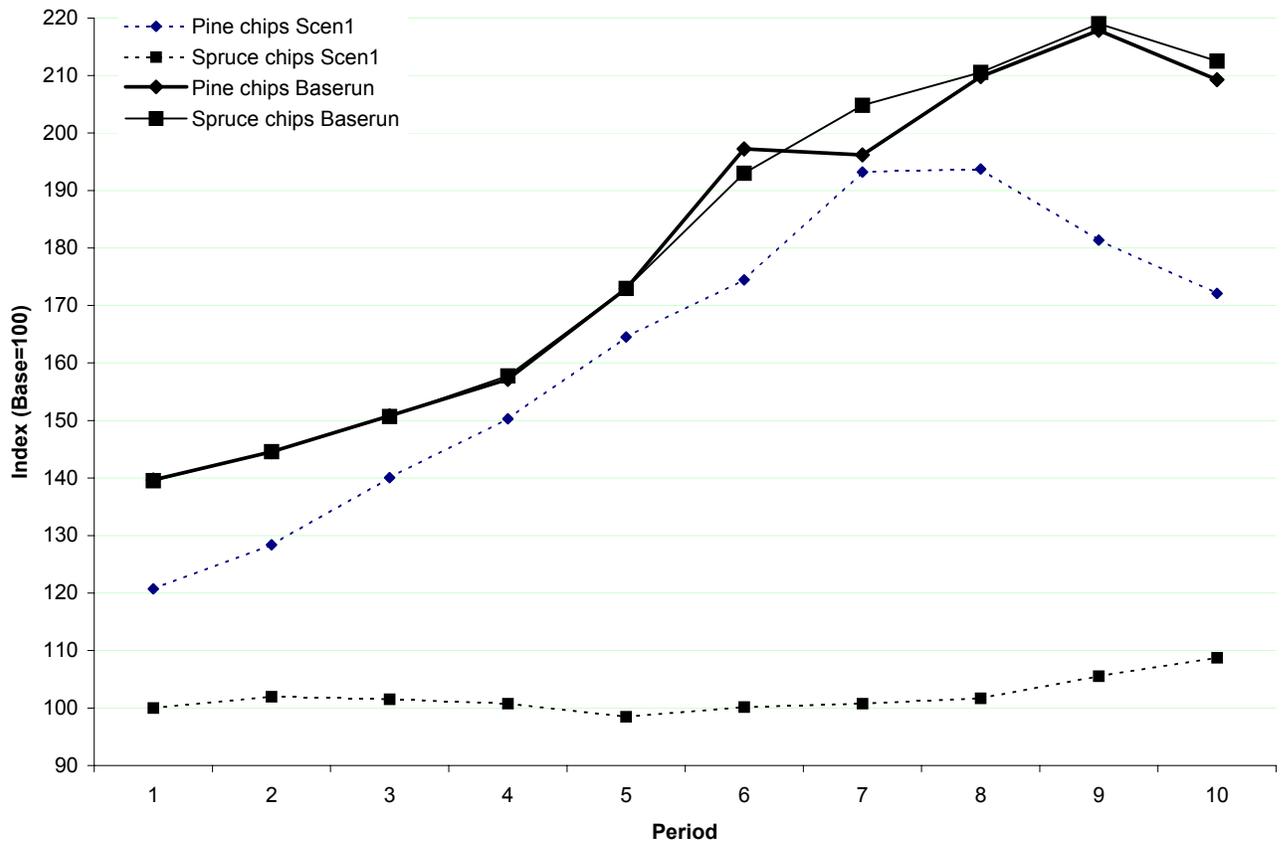


Figure 5. Indexes of shadow prices for chip (pulp) mills. Comparison for 2 mills and the baserun and scenario 1.

Table 5. Final shadow prices of the harvest area and land base restriction by period.

Mill / location	Period									
	1	2	3	4	5	6	7	8	9	10
Sawmill (Drayton Valley)	19.92	20.67	21.35	21.88	22.52	23.41	26.84	29.00	26.26	26.59
Sawmill (Edson)	11.90	14.23	18.76	21.71	21.47	20.98	21.70	25.13	23.46	25.56
Sawmill (Whitecourt)	5.59	7.09	7.32	9.05	12.26	17.21	23.54	29.00	31.98	28.44
OSB Mill (Drayton Valley)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
OSB Mill (Edson)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Chip Mill (Whitecourt)	12.23	13.55	15.31	17.70	21.20	26.30	33.93	35.12	36.32	38.57

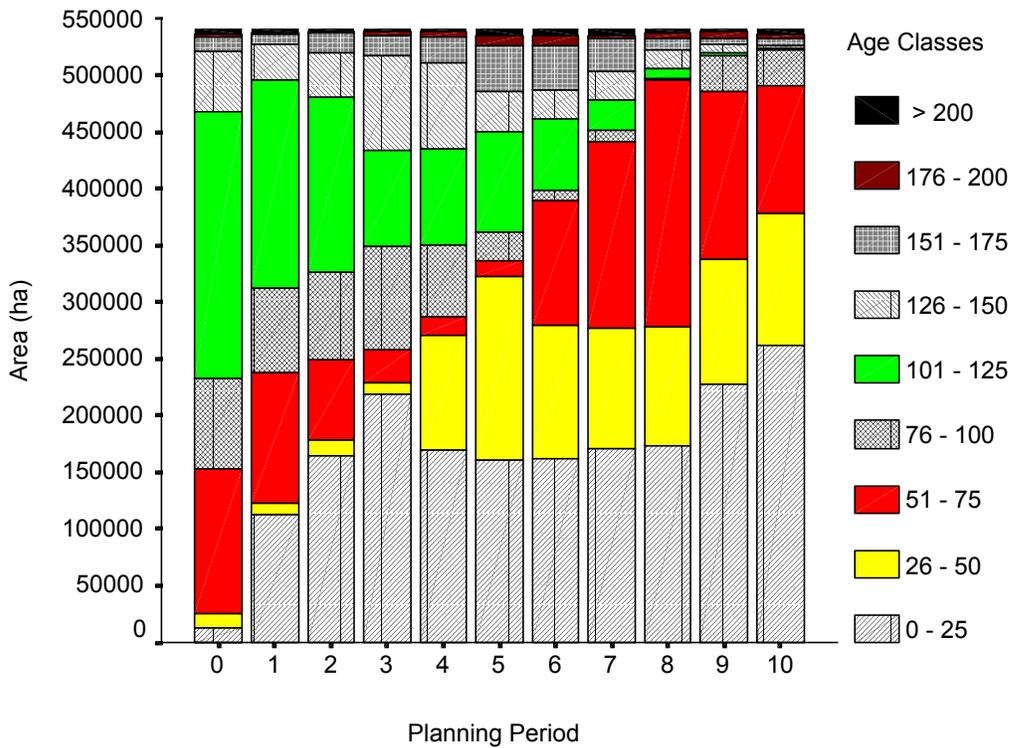


Figure 6. Age class distribution for all species at the beginning of the planning horizon and each planning period for the Baserun.

Potential Management Applications

The management applications of the approach presented are many. Only a few will be explored here. In the context of the current example on overlapping tenures, the model supplies data on the marginal cost of wood and the marginal cost of overlapping tenure constraints. The model shows how inefficiencies in wood allocation, in this case imposed by overlapping tenure constraints, can affect the costs of supplied wood to different mills over time and space. As revealed in our example, the model is also capable of showing that gains (or losses) from policy changes can be uneven. However, the allocation of wood under Scenario One is efficient while the allocation under the Baserun is not. Hence, there must be other allocations that would compensate the losers, which could be identified by making other model runs. This could be invaluable information for the purpose of long term timber supply planning.

A detailed analysis of the harvest schedule was not carried out for this example. However, a more detailed analysis of the harvest schedule would reveal how the transport destinations for wood in each location changes under the two scenarios presented. This analysis would also reveal if there are any differences in which supply locations are harvested, how much they are harvested and at what times they are harvested within the planning horizon. One could

also look at the schedule of regeneration activities and their costs. The increased marginal costs in the overlapping tenure run could be interpreted in a couple of ways. First, the increased marginal costs imply that the marginal value of timber (marginal value = marginal cost) increases in the future, given that the assumptions about the demand in the future are correct. This suggests that prices may become high enough to justify more intensive silviculture. The second and alternative interpretation is that the demand scenario is incorrect and that the implied marginal value or price of timber is too high. In this case the model's demand specification should be reformulated to allow wood harvests to decrease thus decreasing wood harvests in the future. Hence, the model provides a way of tying the wood production to marginal costs and values of timber, which can be compared to expectations of future timber prices. This provides valuable information for supply planning and current planning in silvicultural investment expenditures.

Although the example described here did not include non-timber values there is no reason that non-timber values could not be included. This would require the model to track attributes of the forest that are linked to non-timber values. Constraints on the levels of non-timber value attributes could be added and shadow prices computed for these constraints in a similar manner as the timber value constraints described in this report. Alternatively, in some cases, non-timber values could be incorporated directly into the objective function. For example, recreational forest user utility functions derived from Random Utility Models could be incorporated into this type of framework. Both of these approaches could be extremely useful for evaluating landscape management strategies such as TRIAD. Finally, while the model discussed in this report does not include non-linearities or binary 0-1 variables the framework lends itself to extension in that direction. Specifically, the method is capable of including non-linearities such as; binary 1-0 variables for modeling forest access (whether an area is accessed or not) and non-linear product demands. The incorporation of optimal forest access decisions over time and space will be an important capability for identifying access plans that maximize benefits in terms of both timber values and location of forest reserves under a TRIAD landscape management strategy. Modeling forest access is also important for evaluating the impact of forest access on behavior of non-timber forest users.

CONCLUSIONS

This study applied an optimization approach to forest management scheduling to two forest management problems. The two models investigated were a Base Model, which imposed overlapping tenure constraints and Scenario One, which removed the overlapping tenure constraints. Both models were very large formulations which were solved in a short period of time (approximately 30 min) using a variant of the dual decomposition approach proposed by Hoganson and Rose (1984). Although the model described in this report dealt only with timber supply in the context of overlapping tenure constraints, the model can be extended to consider non-timber values. Other extensions include expanding the spatial detail and including forest

access considerations. In the models reported here each analysis area was included into a supply location defined as a one-quarter township (5 x 5 km). Making the size of the spatial unit smaller might actually improve how the model converges.

A comparison of the results from the Base Model and Scenario One showed that overlapping tenure constraints can be costly. Marginal costs on the overlapping tenure constraints were all positive. In addition marginal costs of wood products at each mill location increased with one exception. The exception shows that costs and benefits of a change in policy can result in gains for some and losses for others. However, further model runs could be made to identify win-win allocations of wood across mills.

REFERENCES

- Beck, J. A., Anderson, R. G., Armstrong, G. W., and Farrow, G. H. 1987. Alberta economic timber supply study: Final report. Canada-Alberta forest resource development agreement, Supply and Services Canada, Ottawa.
- Berk, P., and Bible, T. 1984. Solving and interpreting large-scale harvest scheduling problems by duality and decomposition. *For. Sci.* 30(1): 173-182.
- Dantzig, G. B., and Wolfe, P. 1961. The decomposition algorithm for linear programs. *Econometrica* 29(4): 767-779.
- Fisher, M. L. 1981. The lagrangian relaxation method for solving integer programming problems. *Management Science* 27(1) 1-18.
- Fisher, M. L. 1985. An applications oriented guide to lagrangian relaxation. *Interfaces* 15: 10-21.
- Gunn, E. A., and Rai, A. K. 1987. Modeling and decomposition for planning long-term forest harvesting in an integrated industry structure. *Can. J. For. Res.* 17: 1507-1518.
- Gunn, E. A. Thorburn, M., and Rai, A. K. 1988. A decomposition method based on the augmented lagrangian. *INFOR* 26(2): 91-113.
- Hauer, G. K. 1993. Timber management scheduling with multiple markets and multiple products. M. S. Thesis, University of Minnesota, Minnesota. 122 pp.
- Hoganson, H. M., and Rose, D. W. 1984. A simulation approach to optimal timber management scheduling. *For. Sci.* 30(1): 220-238.
- Johnson, K. L. and Scheurman, H. L. 1977. Techniques for prescribing optimal timber harvest and investment under different objectives-Discussion and Synthesis. *For. Sci. Monograph* 18. 31pp.
- Ladson, L. 1970. Optimization theory for large systems. MacMillan Publishing Company, New York, NY. pp. 200 – 250.
- Luckert, M. K., 1991. Tenures for mixed wood management: a framework for policy analysis. Forest Economics and Policy Analysis Research Unit. University of British Columbia. Working Paper No. 164. 44 pp.

APPENDIX

Model Formulation

$$\begin{aligned}
 P \quad \text{Max} \quad & \sum_{m=1}^M \sum_{t=1}^T \beta^{t-1} \left[\int_0^{M_{mt}} D_{mt}(X) dX \right] + \sum_{i=1}^I \sum_{j=1}^{J_i} \sum_{t=1}^T E_{ins} w_{ins} - \sum_{i=1}^I \sum_{j=1}^{J_i} \sum_{s=-T^0}^{t-z} \sum_{t=1}^T c_{ijst} x_{ijst} \\
 & - \sum_{t=1}^T \sum_{m=1}^{\bar{m}} h_{vt}(V_{vmt}) - \sum_{t=1}^T \sum_{m=1}^{\bar{m}} h_{qt}(V_{qtm})
 \end{aligned} \tag{1.1}$$

subject to

$$\sum_{j=1}^{J_i} \sum_{t=1}^T x_{ijst} + \sum_{j=1}^{J_i} w_{ins} = A_{is} \quad \forall i, s = -T^0, \dots, 0 \tag{1.2}$$

$$\sum_{j=1}^{J_i} \sum_{h=s+z}^T x_{ijsh} + \sum_{j=1}^{J_i} w_{ijt} - \sum_{j=1}^{J_i} \sum_{s=-T^0}^{t-z} x_{ijst} = 0 \quad \forall i, t = 1, \dots, T \tag{1.3}$$

$$\sum_{j \in J_i^c} \left(\sum_{t=1}^T x_{ijst} + w_{ijs} \right) \geq \delta^c \left(\sum_{t=1}^T x_{ijst} \right) \quad \forall i \in I^c, I^{cd}, I^{dc} \tag{1.4}$$

$$\sum_{j \in J_i^d} \left(\sum_{t=1}^T x_{ijst} + w_{ijs} \right) \geq \delta^d \left(\sum_{t=1}^T x_{ijst} \right) \quad \forall i \in I^d \tag{1.5}$$

$$\sum_{i \in I^m} \sum_{j \in J_i^m} \tilde{v}_{ijstm} x_{ijst} \leq 0 \quad \forall s, t \text{ and } m \tag{1.6}$$

$$\sum_{i \in I_k^{AC}} \sum_{j \in J_k^{AC}} \sum_{s=-T^0}^t v_{ijstk} x_{ijst} \leq M_{kt}^{AC} \quad \forall k \tag{1.7}$$

$$M_{kt}^{AC} \leq M_{kt+1}^{AC} \quad \forall k, t = 1, \dots, T-1 \tag{1.8}$$

$$\sum_{i \in I^k} \sum_{j=1}^{J_i} \sum_{s=-T^0}^t v_{ijstm} x_{ijst} \leq \alpha_{mk} M_{kt}^{AC} \quad \forall mt \tag{1.9}$$

$$\sum_{i=1}^I \sum_{j=1}^{J_i} \sum_{s=-T^0}^{t-z} \tilde{v}_{ijstm} x_{ijst} + \tilde{V}_{vmt} + \tilde{V}_{qtm} \geq M_{mt} \quad m = 1, \dots, M \tag{1.10}$$

$$\tilde{v}_{ijst} = \sigma_{ijst} v_{ijst} \quad \forall mt \tag{1.11}$$

$$\tilde{V}_{vmt} = \sigma_{mt} V_{vmt} \quad \forall mt \tag{1.12}$$

$$\tilde{V}_{qtm} = \sigma_{mt} V_{qtm} \quad \forall mt \tag{1.13}$$

$$x_{ijst} \geq 0 \quad \forall ijst \tag{1.14}$$

$$w_{ins} \geq 0 \quad \forall ins \tag{1.15}$$

$$M_{pt} \geq 0 \quad \forall pt \tag{1.16}$$

where:

- $D_{mt}(X)$ = the demand function for wood products from mill m in period t .
- m = counter for mills; $m = 1, \dots, M$;
 $m = 1, \dots, \bar{m}$ for Weyerhaeuser mills, where $\bar{m} < M$
- A_{is} = the number of area unit of stand type i in the first period that were regenerated in period s .
- E_{ins} = the discounted value per unit area of managing stand type i with regeneration prescription n , starting in period s and leaving the stand type as ending inventory
- $w_{ijs} (W_{ijt})$ = area managed stand type i with regeneration prescription j , in period s (period t) and left as ending inventory
- c_{ijst} = the discounted cost per unit area of managing stand type i with regeneration prescription j , starting in period s and final harvest in period t
- $x_{ijst} (x_{ijsh})$ = area managed on stand type i with regeneration prescription and market shipping plan j , starting in period s and final harvest in period t (period h).
- \tilde{v}_{ijstm} = the volume per unit of wood products from mill m , in period t , when stand type i is regenerated in period s and managed with prescription and market shipping plan j .
- v_{ijstk} = the volume harvested of AAC from unit k in period t , when stand type i is regenerated in period s and managed with prescription and market shipping plan j .
- σ_{ijst} = conversion factor of merchantable volume into wood products
- v_{ijst} = the merchantable volume per unit in period t , when stand type i is regenerated in period s and managed with prescription and market shipping plan j .
- M_{mt} = output of mill m in period t
- V_{vmt} = Volume of wood delivered to mill m , from wood acquired by Weyerhaeuser from private landowner v in period t .
- V_{qtm} = Volume of wood delivered to mill m , from wood harvested from quotas owned by Weyerhaeuser in FMA q in period t .
- \tilde{V}_{vmt} = Volume of output of wood products from mill m , from wood acquired by Weyerhaeuser from private landowner v in period t . This is obtained by multiplying the delivered wood volume (V_{vmt}) by the mill conversion factor (σ_{mt}).
- \tilde{V}_{qtm} = Volume of output of wood products from mill m , from wood harvested from quotas owned by Weyerhaeuser in FMA q in period t . This is obtained by multiplying the delivered wood volume (V_{qtm}) by the mill conversion factor (σ_{mt}).
- $h_{vt} (V_{vmt})$ = the cost m^{-3} of wood obtained by Weyerhaeuser from private landowner v in period t
- $h_{qt} (V_{qtm})$ = harvest and transport cost m^{-3} of wood obtained by Weyerhaeuser from its quotas in other FMA area q in period t .
- δ^c = percentage of analysis area that must be regenerated to conifer species
- δ^d = percentage of analysis area that must be regenerated to deciduous species

J_i	= the set of regeneration prescriptions and transport destinations for analysis area i . $J_i = \{(1,1), \dots, (N_i,1); \dots; (1,D), \dots, (N_i, D)\}$. Each pair refers to a prescription and destination combination; where N_i is the number of prescriptions for stand i and D is the number of destinations. It should be noted that wood from any stand i can be sent to more than one destination.
J_i^c	= subset of J_i that includes regeneration prescriptions that meet the conifer standards
J_i^d	= subset of J_i that includes regeneration prescriptions that meet the deciduous standards
\tilde{I}	= set of forest types and locations. This set includes conifer, deciduous, and mixedwood land bases and various site classes of the different land bases. That is, $I^c, I^d, I^{dc}, I^{cd}, I_k^{AC} \subset \tilde{I}$
I	= the number of stand types
L	= number of supply locations
I^m	= the set of locations/forest types that are not available to mill m
J_i^m	= the set of transport/prescriptions that are defined for mill m
I_k^{AC}	= the set of locations/land types that form the basis for calculating AAC on AAC unit k . An AAC unit is defined for the purposes of this study as one of the two FMAs ($k = \text{Edson FMA, Drayton Valley FMA}$).
J_{ik}^{AC}	= the set of management prescriptions/destinations that add into the calculation of AAC on unit k . Note that $J_{ik}^{AC} \subset J_i$
M_{kt}^{AC}	= the volume harvested that forms the basis for AAC on unit k in period t
α_{mk}	= mill m 's share of AAC in unit k and I^k be the set of stands in unit k with wood sent to mill m
z	= minimum time between regeneration and harvest
T	= the number of planning periods in the planning horizon
β	= discount factor

Equation set [1.2] accounts for the forest area regenerated before the planning period (existing stands). Total area harvested during the planning horizon plus area left as ending inventory (at the end of the planning horizon) should equal the initial area (regenerated in period s before planning period).

To account for area regenerated during the planning period. Total area harvested during the planning period plus area left as ending inventory at the end of the planning period should equal area regenerated during the planning period. Equation [1.3] ensures that all harvested areas are regenerated. However, due to the regeneration standards for conifer and deciduous land bases, we need additional constraints. Current provincial regulations require that conifer land bases be regenerated to at least 80% conifer and deciduous land bases to at least 80% deciduous

species. Constraints [1.4] and [1.5] represent these additional regulations. In Equations [1.4] and [1.5], cd (conifer/deciduous) and dc (deciduous/conifer) refer to stands that are defined in the Regeneration Survey Manual as mixed wood stands. These stands are expected to be regenerated to the conifer standards. Only stands originally classified as deciduous are required to be regenerated to the deciduous standards.

Equation 1.6 represents restrictions in FMUs where quota holders are allowed to harvest. In an overlapping tenure situation, restrictions are usually placed on where the quota holders are allowed to harvest.

Equation [1.7] calculates the annual allowable cut for each management unit and time period whilst Equation [1.8] ensures that wood flow (harvested) does not decline over time during the planning horizon.

Since quota holders are usually entitled to a percentage of the AAC allocated to the whole FMA, it means that any silvicultural activities that increase the AAC will be shared amongst the FMA and quota holders. Though the regulation specifies the sharing will be pro-rated, it is not clear how this is done in practice. Consequently, it is not possible to penalize any quota holder who does not contribute to such activities leading to an increase in AAC. Equation [1.9] can be thought of as a disincentive for FMA and quota holders to carry out silvicultural activities over and above the minimum requirements, or as a cost of free-riding to the participating FMA and quota holders.

Equations [1.10] implies that volume of wood products produced from all stands managed in the two FMAs plus volume from quotas owned by all firms outside the two FMA areas, plus volume from private sources should equal the mill demands. It is assumed that mill conversion factors do not differ based on the source of wood, but rather depends on the mill and period of conversion.