THE UNIVERSITY OF ALBERTA

An Experimental and Theoretical Investigation of the Growth of Icicles

by

© Kent A. Johnson

A THESIS

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled

"An Experimental and Theoretical Investigation of the Growth of Icicles"

submitted by Kent A. Johnson in partial fulfilment of the requirements for the degree of Master of Science.

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Date: 28 August 1987

ABSTRACT

The icicle is one of the most common crystals found in nature. The objective of this research is to develop a model for the formation and growth of icicles. The processes responsible for icicle formation are discussed and their roles in the growth process determined. Qualitative observations and photographs of naturally occurring icicles are used in developing the theory. Quantitative observations are made through experimental simulation in the laboratory, and these are used to calibrate and verify the theory.

The growth of an icicle is the result of two different growth processes. The tip of the icicle grows through the freezing of a dripping water supply. Here, the heat transfer takes place from a "pendant drop", resulting in growth in length. The icicle grows in diameter through the freezing of water on its surface. Each of these freezing processes is treated separately, and their results are combined in order to predict the overall growth.

The experiments are performed under a range of temperatures from -18 to -5 °C and windspeeds from 1.0 to 4.0 ms^{-1}. These are typical of conditions in which icicles form naturally.

The icicle growth is found to be very dependent on the surrounding

windspeed and temperature. The relative humidity has only a small effect on the modelled growth while growth is found to be very sensitive to the flowrate or driprate from the tip of the icicle. This fact implies that supercooling of water at the tip, which is neglected in the model, is significant. For this reason, the model predicts an upper limit to the actual growth of icicles.

One set of experiments is used to determine an icicle "freezing fraction", the ice to liquid ratio of the icicle tip as it grows. This freezing fraction is then used to compare the theory to another set of experiments. In this manner, it is found that the icicle model does predict the maximum possible length growth rate of icicles.

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List of Symbols

A_{cyl}	surface area of cylindrical slice
A_{sph}	surface area of spherical pendant drop
c_p	specific heat of air
D	diameter
D_{cyl}	diameter of cylindrical slice
D_{sph}	diameter of spherical pendant drop
D_{wa}	diffusivity of water vapour in air
e_s	surface vapour pressure
e_a	ambient vapour pressure
e_o	reference vapour pressure
F	freezing fraction
h_{cyl}	cylindrical heat transfer coefficient
h_{sph}	spherical heat transfer coefficient
$h_{d,cyl}$	cylindrical mass transfer coefficient
$h_{d,sph}$	spherical mass transfer coefficient
k	thermal conductivity of air
L	length
L_o	length of cylindrical segment
l_v	latent heat of vapourization
m	mass
Δm	mass frozen in one timestep
Nu_{cyl}	cylindrical Nusselt number
Nu_{sph}	spherical Nusselt number
p	pressure
Pr	Prandtl number
q_c	heat transfer due to convection
q_{es}	heat transfer due to evaporation (or sublimation)
q_f	heat transfer due to latent heat of freezing
q_r	heat transfer due to radiation

q_{tip}	total tip heat transfer
q_{cyl}	total cylindrical heat transfer
r	relative humidity
R_a	gas constant for air
R_v	gas constant for water vapour
Re_{cyl}	cylindrical Reynolds numebr
Re_{sph}	spherical Reynolds number
Sc	Schmidt number
Sh_{cyl}	cylindrical Sherwood number
Sh_{sph}	spherical Sherwood number
T_a	air temperature
T_o	reference temperature
T_s	surface temperature
Δt	time for one drop to freeze
u	windspeed
W_d	driprate from tip
W_s	water supply rate
ε	0.622
E	emissivity of icicle surface
μ	dynamic viscosity
ρ_a	air density
σ	Stefan-Boltzmann constant
υ	kinematic viscosity of air

1 Introduction

The icicle is something with which virtually everyone, outside of the tropics and subtropics, is familiar. During winter, icicles can often be seen hanging from buildings and other structures and can range in size from a few centimetres to a few metres long. Icicles often attract human attention due to their beauty and varying shape. However, the exact physical processes responsible for icicle formation are rarely contemplated.

Laudise and Barns (1979) have conducted extensive studies into the crystalline properties of icicles and, from these, have suggested a possible growth mechanism. Knight (1980) has examined several aspects of icicles including growing conditions, crystal structure and composition, shapes and air bubbles. Lenggenhager (1978) and Geer (1981) have studied icicles occurring in nature, noticing several features common to all icicles as well as several properties which occur in only a few cases. One such feature, the horizontal bumps or ridges on icicles has been further observed by Hatakeyama and Nemoto (1958) in both natural and laboratory surroundings. Unusual, horizontal icicles have been discussed by Burt (1982). The problem of icicles forming in railway tunnels and how to eliminate them has been examined by Shinojima (1973). The most detailed study of icicles has been provided by Maeno and Takahashi (1984a, 1984b). Laboratory experiments have been performed in a coldroom to study the growth of icicles and a possible mechanism for this growth has been suggested (1984a). As well, several unusual feature were studied in nature and in the laboratory (1984b). These features include horizontal ridges, spiked and bent icicles.

Icicles have been shown to be a significant contributor to both marine and power line ice accretion. Before an attempt can be made to determine this contribution, the underlying process must be first examined. A mathematical model for the formation and growth of icicles can then be developed.

Icicles form when a source of liquid water with supply rate, W_s, drips in surroundings which have an ambient temperature, T_a, below the freezing point. Figure 1.1 is a schematic of an icicle showing these parameters as well as the length, L, the diameter, D, and the driprate from the tip, W_d. Figure 1.2 shows several examples of icicles occurring in nature. All exhibit some degree of similarity. All of the icicles are nearly vertical and appear to contain some air bubbles, although the bubbles cannot be resolved in some cases. Also, the general shape of all the icicles is similar, all are somewhat conical in shape. These factors suggest that there is some common physical process responsible for their formation. However, it is apparent that all icicles do not acquire exactly the same shape. Notice the irregular bumps and wide sections in Figure 1.2b. It even appears, in a few cases, that several icicles have ammalgamated into one thicker column of ice.

There are seemingly two modes of icicle growth. The simpler of the two occurs when all of the incident water freezes. In this case, ($W_d = 0$) referred to as "dry growth", the icicle grows only in diameter and the mass growth rate equals the supply rate of the liquid water. From the point of view of power line and marine ice accretion the mass growth rate is an important factor to be determined and its value is trivially determined for dry growth. However, from an observers point of view, dry growth can be very interesting as transitions between dry and <u>wet</u> growth can produce unusually shaped icicles. The interesting case, in terms of ice accretion, is that where a surplus water supply is present. In this situation, water is continuously dripping from the tip of the icicle. This case ($W_d > 0$) is referred to as "wet growth" and the growth rates in both length and diameter must be found in order to determine the icing rate.

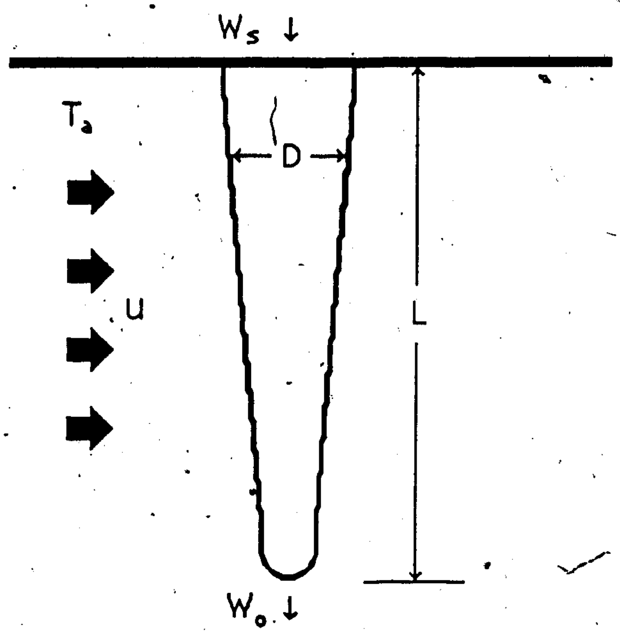


Figure 1.1: Simple sketch of an icicle with length (L), diameter (D), water supply rate (W_s), driprate (W_d), in an airstream with windspeed, u, and temperature, T_a.

Figure 1.2a

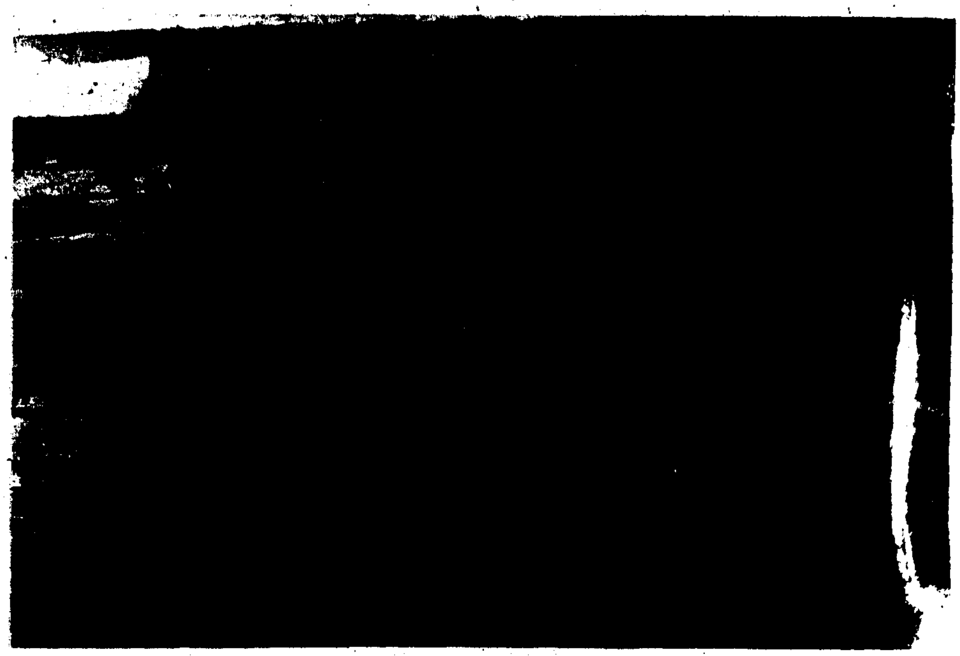


Figure 1.2b

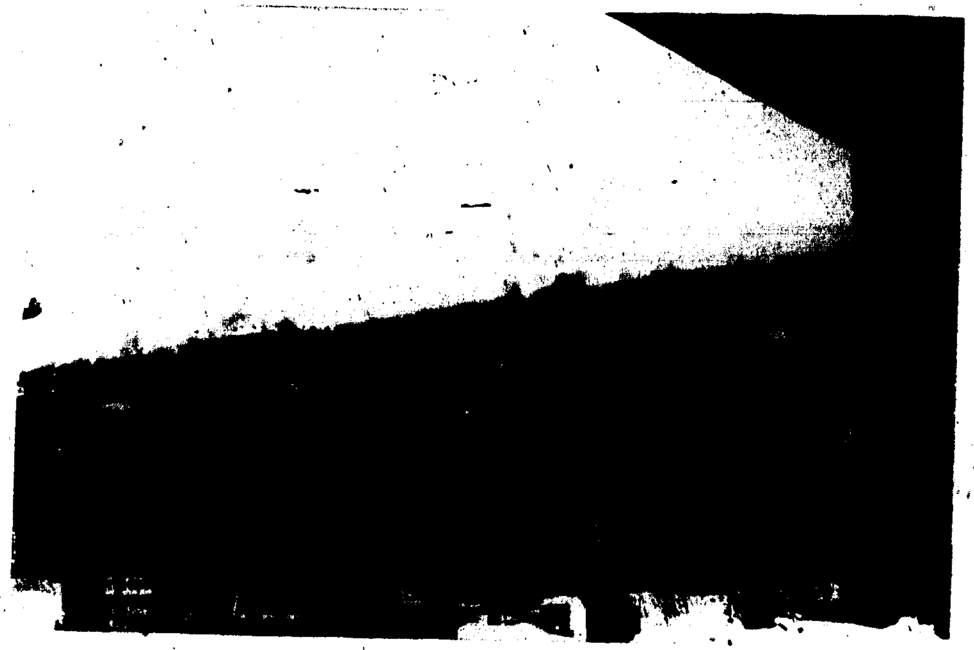
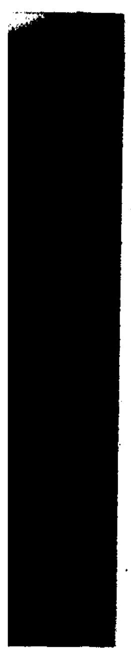


Figure 1.2c

Figure 1.2d



2. A Theory of Icicle Growth

2.1 An Overview of the Icicle Growth Process

In order to understand and model the growth of an icicle, all of the physical processes which can occur must be examined. The situation is that shown in Figure 1.1. A water or ice surface is present in an environment which has a temperature below the freezing point. In such a case several processes can occur simultaneously. Most of these, including evaporation, sublimation, convection and radiation, occur as interactions between the icicle surface and the surroundings. Internal conduction will occur in the body of the icicle if there are any temperature gradients and freezing takes place as heat is removed from the icicle. The combined effects of all these processes must be incorporated into a model, in order that a theory for icicle growth can be formulated. The process is, to say the least, a complex one. Several simplifying assumptions are made in order that the model can be formulated. These assumptions will be mentioned briefly in the description of the model and will be discussed in detail following the mathematical formulation.

2.2 Heat Balance

As is the case with most icing problems, an overall heat balance is the basis of the model. This heat balance must be applied to the pendant drop at the tip as well as to the body of the icicle. The release of latent heat of freezing, q_f, is the dominant supply of heat to the icicle. Significant heat losses are those due to convection, q_c, evaporation and sublimation, q_{es}, and radiation, q_r. Internal conduction in the icicle is assumed to be negligible in this balance. The overall heat balance is simply

$$q_c + q_{es} + q_r + q_f = 0 \tag{2.1}$$

The determination of q_c, q_{es} and q_r enables the deduction of q_f, and thus the mass rate of freezing of the incident water.

The task is then to apply such a heat balance. How can a steady-state heat balance be applied to a process which is certainly time dependent? In order to accomplish this, the heat balance must be applied in a stepwise fashion in time. Under steady conditions the amount of water which can be frozen in a given timestep is calculated, this amount is then added to the value at the previous time, and the process is repeated. If the icicle is segmented into cylindrical slices, each slice can be treated in this manner and growth in diameter can be determined. However, an explanation for growth in length must still be found.

2.3. Tip Processes

Icicle growth is expected to be very dependent on the processes occuring at the tip. All elongation must be a direct result of these processes. In some way a dripping water supply results in the formation of an icicle and, as long as wet growth persists, the dripping from the tip will continue. The successful modelling of the tip process should lead to an effective model of icicle growth. Once the elongation of the icicle has been modelled the radial growth can be incorporated.

There are many parameters which can or will affect the icicle growth process. Such a list includes pressure, temperature, windspeed, relative humidity and the rate at which water is supplied to the icicle. The influence of these factors on the growth of the icicle must be determined.

Regardless of the size of the icicle, if all parameters remain unchanged the tip process will remain the same. For this reason the length growth rate should be independent of time. As will be seen later, one such parameter is the flowrate or driprate from the tip of the icicle. It is not realistic to expect this value to remain constant as the icicle grows. Even if the supply rate is constant, as the icicle grows, more heat transfer occurs and more of the supply water freezes, thus reducing the driprate from the tip. Nonetheless, for some length of time this driprate will be approximately constant and the growth rate in length is expected to vary quite slowly in time. The problem is thus to determine this growth rate.

Consider the origin of an icicle. Initially, a dripping water supply is present and the heat transfer takes place from a single water droplet as in Figure 2.1a. Even though the water is constantly dripping, eventually, the equivalent of one water drop freezes. (Figure 2.1b) If there is no supercooled water or ice shed in the falling drops, this time will be that calculated for a single drop to freeze. In other words, after some time, regardless of the exact process involved, a mass equal to that of one water drop freezes. This time is the factor which determines the length growth rate of the icicle.

The heat transfer from the tip drops can be calculated by treating them as spheres. Even though the shape of the pendant drop is continually changing, some multiple of the surface area of the pendant sphere can be used to represent an average area in the heat transfer calculations of drop diameter can be used in the heat transfer calculations. A more detailed look at the tip of the icicle as it grows is undertaken to determine this parameter. Figures 2.2a and 2.2b show several photographs of the tip of a growing icicle. It is seen that a pendant drop is generally present at the tip. The drop originates as somewhat hemispherical in shape (Figure 2.2a) and grows to an elongated sphere before falling. (Figure 2.2b). Almost immediately after one drop falls, another hemisphere forms, except in cases where the supply rate is extremely low. Based on this evidence, heat

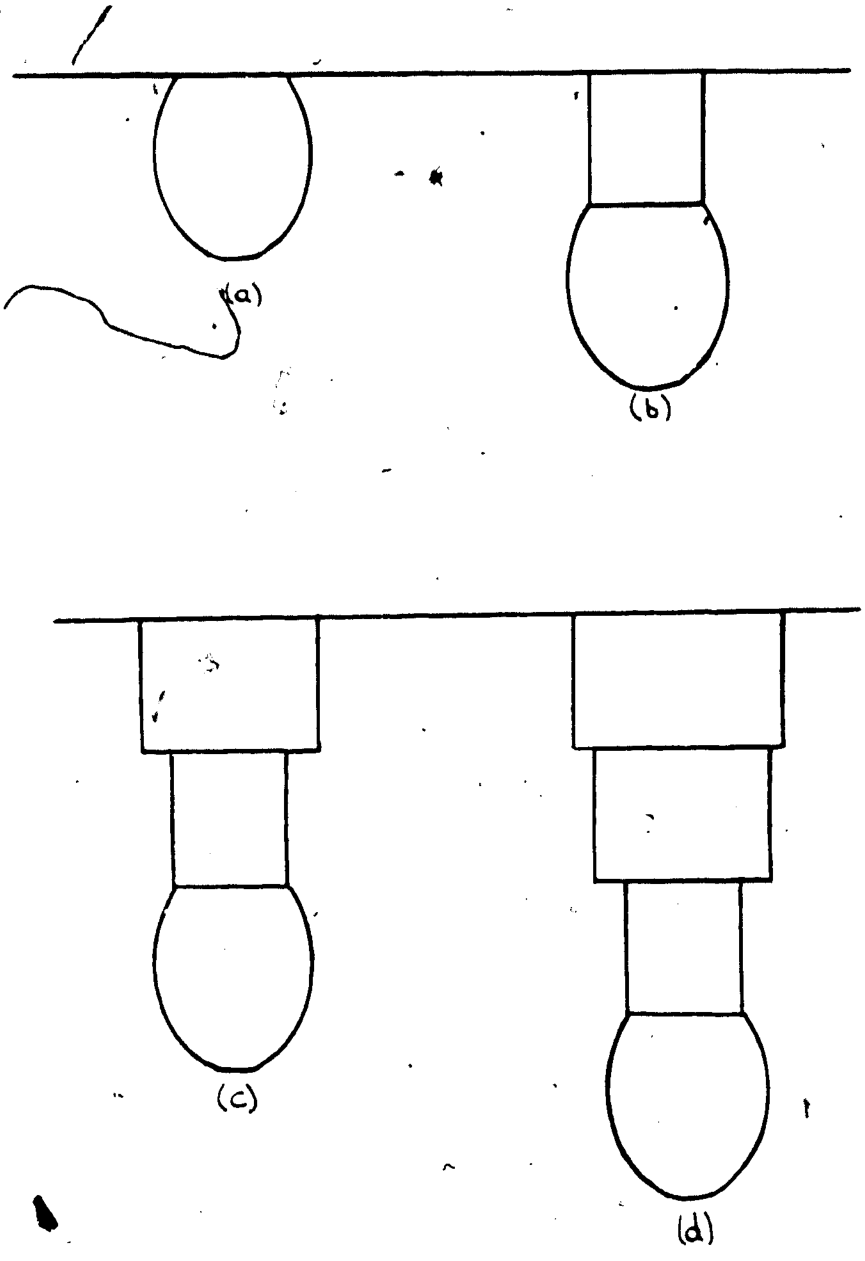


Figure 2.1: Schematic of icicle growth model after 0 (a); 1 (b); 2 (c) and 3 (d) timesteps.

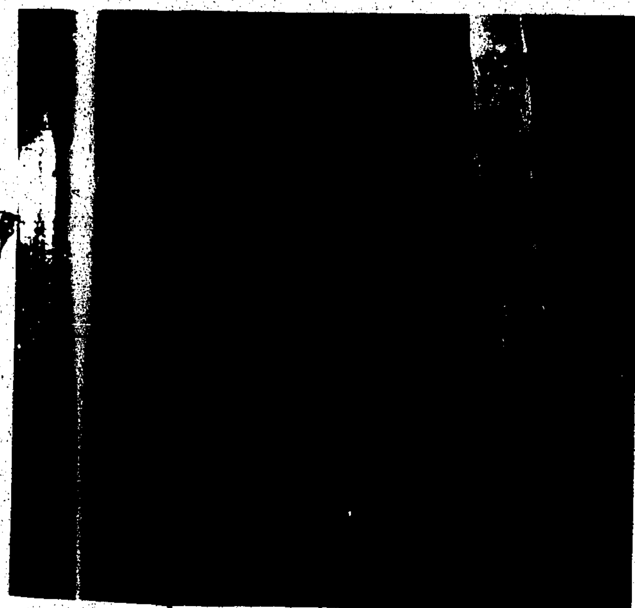


Figure 2.2a

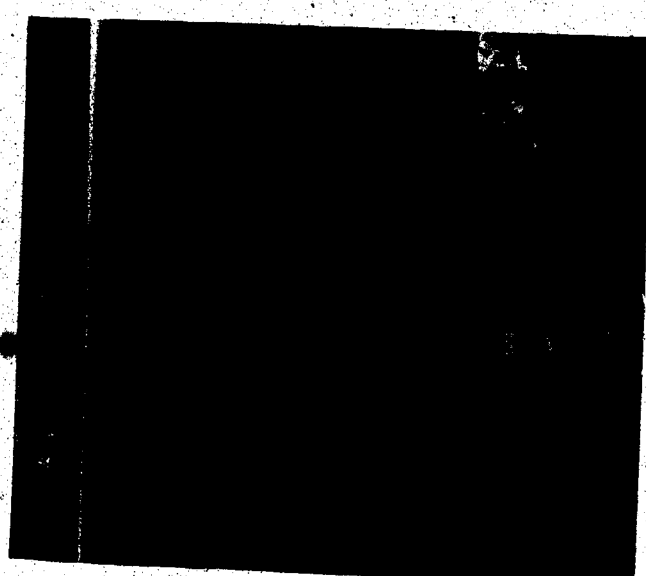


Figure 2.2b

transfer from a spherical pendant drop is chosen to represent the average value. Initially, it might be expected that such an assumption will overestimate the heat transfer from the pendant drop. However, during a single timestep, ice forms, and this ice must remain part of the pendant drop calculation until the next timestep, when that ice grows radially. Thus, the area of the pendant sphere is expected to provide an appropriate average area value.

The "spongy" nature of the ice must also be taken into consideration. As the icicle grows, a certain fraction of the liquid water remains trapped in the ice for some time. This can be seen by observing icicles as they grow. Bubbles can be seen rising in the liquid water in the interior of the bottom 4 to 6 centimetres of the icicle. These bubbles can be seen in Figure 2.2b. A "freezing fraction" must therefore be introduced into the freezing rate calculations. The numerical value of the freezing fraction, F, is not known for icicle growth, and is thus treated as an unknown parameter. Conclusions regarding this freezing fraction will be made following the experiments.

Throughout this discussion reference is made to the "tip drop freezing". As mentioned, only a fraction of the drop actually freezes initially as water is trapped in the icicle. However, for convenience, the process will be referred to as freezing. Now, using various values of F, the time required for one drop to freeze can be calculated.

Once the equivalent drop has frozen what happens to its shape? Obviously, icicles do not maintain the spherical shape as they grow radially. Instead, from natural examples, larger icicles appear to be conical in shape while some smaller icicles even appear as thin cylinders. (see Figures 1.2) Thus, the frozen drop is assumed to acquire the shape of a cylinder, with equal length and diameter, and the same mass as the spherical drop. The icicle model proceeds in a stepwise fashion as shown in Figure 2.1. This length divided by the time required for a single drop to freeze yields the length growth rate.

2.4. Mathematical Formulation

2.4.1 Tip Drop Freezing - Growth in Length

The time required for one drop to freeze is calculated using the following heat and mass transfer correlations for small spheres:

$$q_{c,sph} = -A_{sph}h_{sph}(T_s - T_a) \tag{2.2}$$

$$q_{es,sph} = -A_{sph}h_{d,sph}\,\varepsilon l_v/[R_a T_a(e_s - re_a)] \tag{2.3}$$

$$q_{r,sph} = -A_{sph}\,\sigma E(T_s^4 - T_a^4) \tag{2.4}$$

where

A_{sph} = surface area of the pendant drop

l_v = latent heat of vapourization = 2.50×10^6 Jkg^{-1}

$\varepsilon = 0.622$

r = relative humidity

e_s = surface saturation vapour pressure

e_a = environmental saturation vapour pressure

R_a = gas constant for dry air = 287 $Jkg^{-1}K^{-1}$

T_s = water drop temperature

T_a = surrounding air temperature

σ = Stefan-Boltzmann constant = 5.67×10^{-8} $Wm^{-2}K^{-4}$

E = emmissivity of the water surface

h_{sph} = heat transfer coefficient

$h_{d,sph}$ = mass transfer coefficient .

The air density, ρ_a, is calculated using the ideal gas law:

$$\rho_a = p/R_aT_a \tag{2.5}$$

where p = environmental pressure.

The vapour pressures are calculated using an integrated form of the Clausius-Clapeyron equation (Rogers, 1979):

$$e(T_a) = e_o \exp[l_v/R_v(1/T_o - 1/T_a)] \qquad (2.6)$$

where

$e_o = 611$ Pa

$T_o = 273$ K

R_v = gas constant for water vapour = 461 $Jkg^{-1}K^{-1}$.

The heat and mass transfer coefficients are given in terms of the Nusselt and Sherwood numbers for the spherical tip droplet, respectively;

$$Nu_{sph} = h_{sph}D_{sph}/k = 0.6Pr^{1/3}\, Re_{sph}^{\ 1/2} + 2 \qquad (2.7)$$

$$Sh_{sph} = h_{d,sph}D_{sph}/D_{wa} = 0.6Sc^{1/3}\, Re_{sph}^{\ 1/2} + 2 \qquad (2.8)$$

where

k = thermal conductivity

Pr = Prandtl number

D_{wa} = diffusivity of water vapour in air

Sc = Schmidt number

Re_{sph} = Reynolds number for the spherical pendant drop.

The Reynolds number is given by;

$$Re_{sph} = uD_{sph}/\upsilon \qquad (2.9)$$

where

u = windspeed

υ = kinematic viscosity .

The Prandtl and Schmidt numbers are;

$$Pr = c_p\mu/k \qquad (2.10a)$$

$$Sc = \upsilon/D_{wa} \qquad (2.10b)$$

where

c_p = specific heat of air = 1004 $Jkg^{-1}K^{-1}$

μ = dynamic viscosity.

The values of υ, D_{wa} and k vary with temperature and are calculated using a linear interpolation between minus twenty and zero degrees Celsius as follows. (Smithsonian Institution, 1963) All values are calculated for a reference pressure of 93 kPa, a reasonable mean value for the location at which the experiments were performed.

$$\upsilon = 1.45X10^{-5} + 9.0x10^{-8} \times T_a \quad (m^2s^{-1}) \tag{2.11a}$$

$$D_{wa} = 2.13x10^{-5} + 1.4x10^{-7} \times T_a \quad (m^2s^{-1}) \tag{2.11b}$$

$$k = 2.43x10^{-2} + 7.5x10^{-5} \times T_a \quad (Wm^{-1}K^{-1}) \tag{2.11c}$$

Equations 2.11a to 2.11c are accurate to within 0.5 percent and valid for T_a in degrees Celsius in the range $-20^oC \leq T_a \leq 0^oC$.

Note that equations 2.11a and 2.11b are only valid near a pressure of 93 kPa. For other pressures the values can be calculated using (Smithsonian Institution, 1963);

$$\upsilon = \mu/\rho_a \tag{2.12a}$$

$$D_{wa} = D_{wa,o} (T_a/T_o)^{1.81} p_o/p \tag{2.12b}$$

where μ is the dynamic viscosity and $D_{wa,o}$ is a reference value at temperature T_0 and pressure p_0. Computed values of υ are shown along with values of the diffusivity of water vapour in air, for various temperatures and pressures, in Tables 2.1a, b and c.

Note that Equations 2.11 were used in all theoretical calculations in order that the theory could be compared with experimental results; however, a set of equations similar to (2.11) could be derived for any pressure value by calculating values similar to those in Tables 2.1. Values of the Prandtl number for the different temperatures, from Equation 2.10a, are also shown in Table 2.1c. Note that the Prandtl number does not vary significantly in the given temperature range. Thus, a value of Pr = 0.71 will be used in all calculations. Such a result is accurate to within 0.5 percent from -20 to 0 degrees Celsius. Similarly, and with equal accuracy, the Schmidt number is equal to 0.60 for the indicated temperature range.

p \ T	-20	-10	0
92	1.27	1.37	1.47
96	1.23	1.31	1.40
100	1.17	1.26	1.35
104	1.13	1.22	1.29

Table 2.1a: Values of the kinematic viscosity (m^2s^{-1}) for several sets of temperature and pressure. ($\upsilon \times 10^5$)

p \ T	-20	-10	0
92	2.14	2.29	2.46
96	2.05	2.20	2.35
100	1.97	2.11	2.26
104	1.89	2.03	2.17

Table 2.1b: Values of the diffusivity of water (m^2s^{-1}) vapour in air for several sets of temperature and pressure. ($D_{wa} \times 10^5$)

T	Pr
-20	0.713
-10	0.710
0	0.711

Table 2.1c: Values of the Prandtl number for different temperatures.

The latent heat of freezing term is given by;

$$q_{f,sph} = Fm_o l_f/\Delta t \tag{2.13}$$

where

F = freezing fraction

m_o = mass of spherical pendant drop

l_f = latent heat of freezing

Under conditions favourable for icicle formation, it is quickly seen that the radiative term is small compared with the other two heat loss terms. Hence, the radiative term will be neglected. Representative values of the heat loss terms are shown later in Table 2.3. Balancing the remaining three terms, using (2.1) to (2.13), yields the time Δt, during which the equivalent of one drop freezes. The mass of this drop is then assumed to acquire the shape of a cylinder. The embryo icicle is assumed to have the density of pure ice; however, if the freezing fraction is small, the density of water would be more appropriate. Nevertheless, the density of ice is chosen as, eventually, after several timesteps, all the water will be frozen. As previously mentioned, the length of the equivalent cylinder divided by this time yields the length growth rate. Figures 2.3 to 2.5 display this growth rate as a function of temperature, windspeed and humidity for several freezing fractions. All other parameters are chosen to be typical of conditions favourable for icicle growth. The values of all parameters are shown on the figures. It is now necessary to implement radial growth of the icicle as it grows in length.

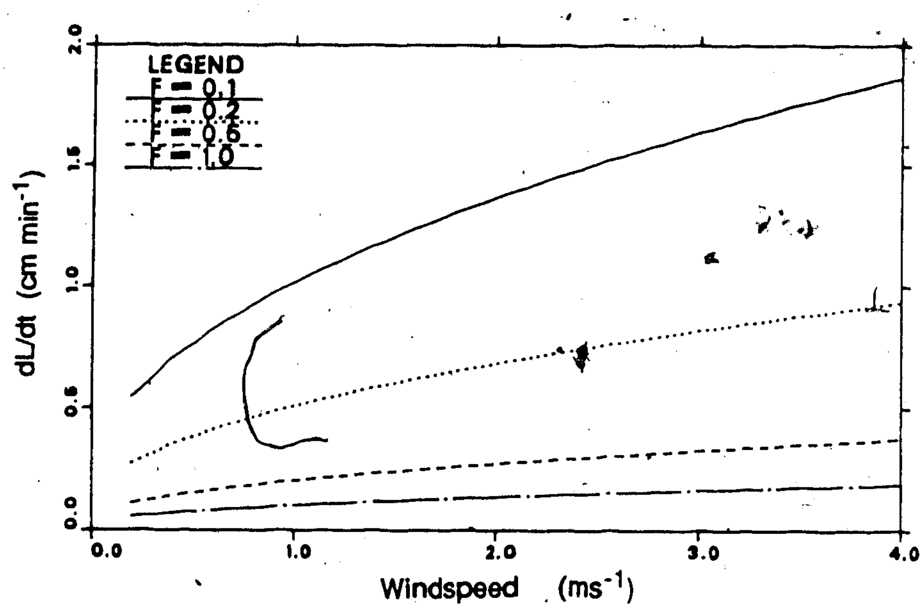


Figure 2.3: Length growth rate versus windspeed for various values of the freezing fraction, F. (T = -12° C, r = 1.0)

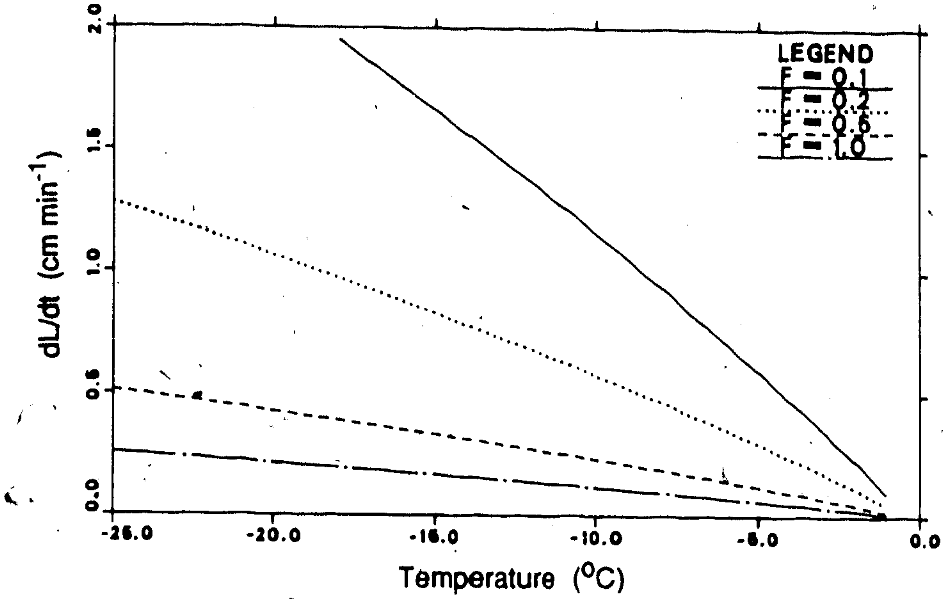


Figure 2.4: Length growth rate versus temperature for various values of the freezing fraction, F. ($u = 2.0\ ms^{-1}$, $r = 1.0$)

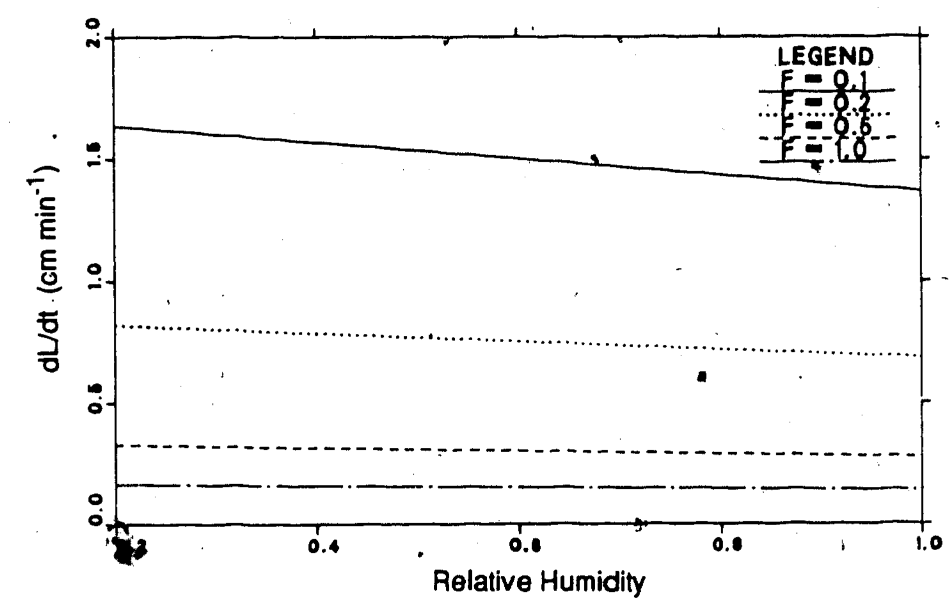


Figure 2.5: Length growth rate versus relative humidity, r, for various values of the freezing fraction, F. ($T = -12^{o}$ C, $u = 2.0\ ms^{-1}$)

2.4.2 Radial Growth

A timestep must now be chosen in order to model the icicle growth. The time required for one drop to freeze appears to be a natural choice. Thus, during the first timestep the equivalent cylinder, representing the mass of one drop, freezes. During the second timestep the initial cylinder grows radially while another drop freezes, creating a new cylindrical segment at the bottom. In each succesive timestep all existing cylinders grow radially while a new drop freezes into a cylinder at the tip. Figures 2.1a to 2.1d show this process. The heat transfer equations are the same as those for the sphere with the transfer coefficients and areas replaced by those for a cylinder. This amounts to replacing the "sph" subscripts with "cyl" in equations 2.2 to 2.13. The convective heat transfer term for the cylinder is:

$$q_{c,cyl} = h_{cyl} A_{cyl} (T_a - T_w) \tag{2.14}$$

where the heat transfer coefficient, h_{cyl}, is given by (Incropera and DeWitt, 1984);

$$Nu_{cyl} = h_{cyl} D_{cyl}/k = 0.6 Pr^{1/3} Re_{cyl}^{1/2} . \tag{2.15}$$

This correlation is expected to be accurate within ± thirty percent in the following range:

$$0.67 \leq Pr \leq 300$$

$$10 \leq Re_{cyl} \leq 10^5$$

For the Reynolds number as defined previously and using typical values of $\upsilon = 1.35 \times 10^{-5}$ ($m^2 s^{-1}$) and $D_{cyl} = 0.005$ (m) the range of Reynolds numbers converts to a range of windspeeds of

$$0.025 \; ms^{-1} \leq u \leq 250 \; ms^{-1}.$$

Only in carefully controlled laboratory situations would windpeeds of less than the lower limiting value be incurred. The given range of windspeeds certainly encompasses the range in which icicles occur in nature.

The heat transfer due to evaporation is:

$$q_{es,cyl} = A_{cyl} \, h_{d,cyl} \, l_v \varepsilon / [R_a T_a \, (re_a - e_s)] \qquad (2.16)$$

where A_{cyl} = area of cylindrical slice.

The mass transfer coefficient for cylinders, $h_{d,cyl}$, is given by the following correlation for the Sherwood number:

$$Sh = h_{d,cyl} \, D_{cyl} / D_{wa} = 0.6 Sc^{1/3} \, Re_{cyl}^{\;1/2} \qquad (2.17)$$

The heat balance, equation 2.1, is applied to obtain:

$$- (q_{c,cyl} + q_{es,cyl}) = q_{f,cyl} = \Delta m l_f / \Delta t \qquad (2.18)$$

where

Δm = mass frozen on cylinder in time Δt .

This increase in mass, Δm, is added to the previous mass of the cylinder, the new diameter is calculated, and the process is repeated. The ice freezing on the outside of the icicle is assumed to be pure; that is, the freezing fraction is unity. Each cylindrical slice of the icicle is grown in this manner for the desired length of time. After each time step another cylinder is added to the tip of the icicle. Equations 2.15 to 2.18 are used to calculate the increase in mass (and hence diameter) of each cylindrical segment for each timestep.

2.4.3 Model Implementation

It is readily noted that a type of self-similarity in time exists using this procedure. Any segment of the icicle is identical to that immediately above it at the previous timestep. That is:

$$D_n(t) = D_{n-1}(t-\Delta t) \qquad (2.19)$$

After n timesteps the icicle will appear as in Figure 2.6.

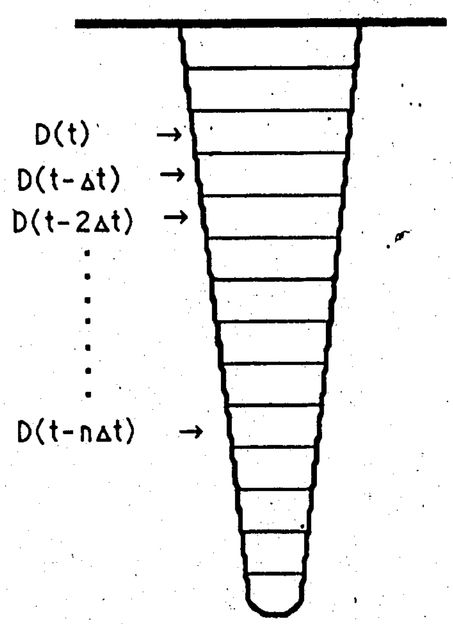


Figure 2.6: Schematic of icicle segment time symmetry for any time t.

For this reason only the initial (root) segment needs to be grown in order to produce a complete profile of the icicle. A simple computer routine which accomplishes this is shown in Appendix A. Knowing the shape of the icicle as it evolves, several properties of interest can be discerned. These include the mass growth in time, the change in surface area with time and the diameter as a function of the distance from the tip.

2.4.4 Choice of Timestep

The basis of the model is that in a certain time, the equivalent of one drop freezes. If, instead, a specific tip diameter is chosen, then, for any timestep, the corresponding length increase can be calculated. A value of 0.45 cm is chosen based on the previous theory and observations. Using this modification, the model can be applied for any choice of timestep. Table 2.2 displays the differences in the modelled icicle growth for several values of the timestep under conditions typical for icicle formation.

Table 2.2: Diameter at 40 cm from icicle tip for several different choices of timestep.

Conditions: T = -12°C, u = 2.0 ms^{-1}, F = 0.2, r = 1.0

Time Δt: 30 sec (time for one drop to freeze)

Timestep (sec)	Diameter 40 cm from Tip (cm)
240	2.06
120	2.15
60	2.19
30	2.21
15	2.23
7.5	2.23
4	2.23

It is readily seen that the model is not significantly dependent upon the choice of the timestep. Any value is possible provided that it is not significantly larger than the time required for one drop to freeze. This is not a surprise since the crux of the model is the assumption that the equivalent drop freezes, in a time Δt, into a cylinder of equal length and diameter. Thus, no matter what the choice of timestep, the growth rate in length will remain the same. The time, Δt, is therefore used as the timestep for the growth model. Any significantly larger timestep will result in the modelled icicles being thinner than those with a more precise timestep, due to the fact that the tip is initially allowed to grow in length for one timestep before radial growth is initiated.

2.5. Assumptions

Many simplifying assumptions have been made in order to solve the icicle heat balance. Several of these have been mentioned previously but have not been discussed or supported. The following section details the major assumptions and their likely effect on the growth of the icicle.

2.5.1 Neglection of Radiation

One of the first assumptions is to neglect the radiative heat transfer term. Equation 2.4 computes the radiative heat transfer for the spherical tip droplet. This amount is assumed to be negligible compared to the convective and evaporative terms, 2.2 and 2.3 respectively. The radiative term, 2.4, is:

$$q_r = -AE\sigma(T_w^4 - T_a^4) \quad (2.4$$

Dividing through by the area in 2.2 to 2.4 produces a heat transfer value per unit area, in this case Watts per square metre (Wm^{-2}). Values for the three heat loss terms for some representative conditions for icicle growth are shown in Table 2.3.

T (°C)	u (ms^{-1})	q_c (Wm^{-2})	q_{es} (Wm^{-2})	q_r (Wm^{-2})
-5	0.5	-210	-130	-22
-5	2.0	-390	-230	-22
-5	4.0	-530	-310	-22
-10	0.5	-440	-230	-43
-10	2.0	-780	-420	-43
-10	4.0	-1060	-570	-43
-15	0.5	-650	-310	-64
-15	2.0	-1170	-550	-64
-15	4.0	-1600	-750	-64
-20	0.5	-870	-360	-83
-20	2.0	-1550	-640	-83
-20	4.0	-2120	-870	-83

Table 2.3: Values of the convective (q_c), evaporative (q_{es}) and radiative (q_r) heat loss terms for the spherical pendant drop for different values of temperature (T_a) and windspeed (u).

For all of the above instances the emissivity, E, is set equal to one; that is, the maximum radiative heat loss is calculated. Also, the humidity, r, is set equal to unity producing the minimum value of the evaporative term. Only at very low windspeeds does the radiative term become greater than five percent of the sum of q_{es} and q_c. Thus, in almost all cases, radiation accounts for less than five percent of the total heat loss, a small amount considering the expected uncertainty of the heat transfer correlations.

2.5.2 Supercooling at the Tip

The constraint that no supercooled water or ice is shed from the tip of the icicle may not be fully satisfied. If this occurs, another term must be included in the heat balance. If the supply rate is sufficiently strong, it is expected that the rapid flowrate will prevent any ice formation. Open channels in lakes and ice-free, rapidly flowing rivers are analogous to this situation. Undoubtedly, there will be some supercooling of the water, although the error induced may be small due to the large ratio of latent to specific heat for water. However, for a strong supply rate, all of the heat loss may be accounted for by a small degree of supercooling. If the supercooling is significant or if any ice is shed in the dripping water, the observed growth is expected to be slower than the modelled growth. For this reason, the model yields an upper limit to the growth rate. The observed growth rates are expected to approach but not exceed this maximum value.

2.5.3 Water Supply Continuity

For radial growth it is assumed that the water supply rate is continuous in time and evenly distributed about the circular circumference of the icicle. However, from observations of growing icicles, both in nature and in the laboratory, the water supply has surges and lulls in its rate. Nonetheless, provided that a liquid water film is maintained about the icicle, wet growth will persist and the variation in the water supply rate can be neglected for radial growth. Observations show that such a film of water is present, except when icicles become large. In some of these cases, the liquid water will not be evenly distributed about the icicle. Instead, preferred paths develop. An uneven temperature distribution will result as shown in Figure 2.7

If preferred paths do exist, the icicle may no longer have its entire surface at zero degrees. In order to fully accommodate this situation, the model must account for the surface temperature distribution as well as conduction in the interior of the ice, creating a very complex situation, to say the least. However, if the temperature difference across the icicle is small enough, the temperature distribution can be neglected, and the total heat transfer is unchanged. It has been suggested that internal conduction in "spongy" ice is very slow (Szilder, 1987). But, if liquid is present in the interior, it is unlikely that the outside surface could cool significantly below freezing. Once the liquid has entirely frozen, the higher conductivity of the ice will maintain the opposite surface at or near the freezing point. It is therefore assumed that when this situation does occur, the effect of the temperature distribution is negligible.

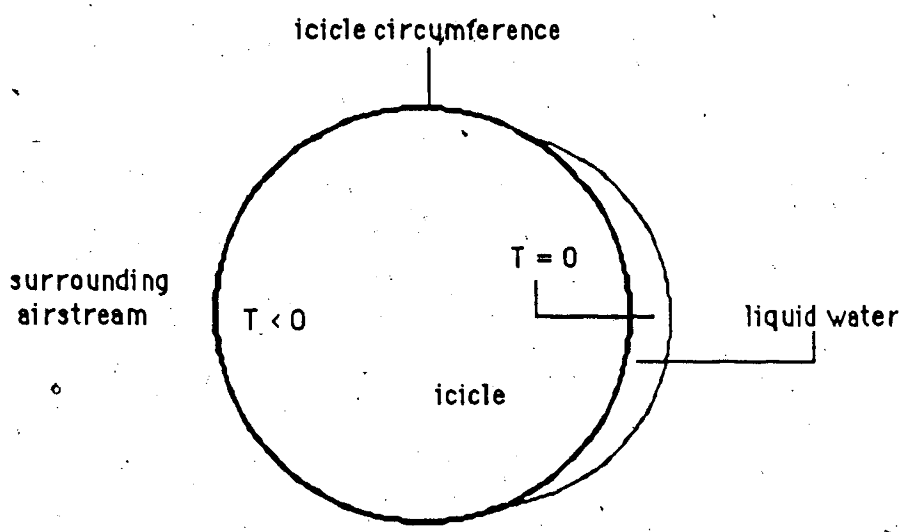


Figure 2.7: Cross-section of an icicle where the entire surface is no longer experiencing wet growth.

2.5.4 Non-circular Cross-section

A result of these preferred paths is that the icicle may not posess a circular cross section. As the icicle becomes larger, the entire surface of the icicle may not remain wet. If this is the case, the side with the water supply in Figure 2.7 will grow while the opposite side will not. Also, the fact that the heat transfer from a cylinder in cross flow varies about the circumference (Lozowski et al., 1983) contributes to a non-circular cross-section. In order to deal with non-circular cross-sections an elliptical-cross-section could be assumed. Incorporating the circumference of an ellipse into the area used in the heat transfer calculations would be a difficult task due to the difficulty in computing such a circumference. Also, the lack of heat transfer correlations for such shapes encourages the use of the simpler circular cross-section which is used in the model.

2.5.5 Radial Conduction

As mentioned earlier, liquid water remains trapped in the icicle near the tip, as it grows. This water ultimately freezes by conduction of heat away from the liquid water. Conduction radially outward is very small due to the fact that there is only a very small temperature difference across the ice-water interface. Thus, the rate of heat transfer through the walls is small compared with that from the surface to the airstream and is neglected. Any further heat transfer from the liquid interior must occur along the axis of the icicle. (Makkonen, 1987)

2.5.6 Shape Assumption at the Tip

Possibly the most crucial assumption revolves around the tip process itself. As has been explained previously, the heat transfer from the tip is calculated as that from a sphere the size of the falling water drops. The only support for this assumption is through observation, as is the case for the assumption that the "equivalent drop" freezes into a cylinder with equal length and diameter. These assumptions are, at best, crude approximations but, as will be mentioned in the results, seem to be a reasonable choice.

These are the major assumptions made in deriving the model. Little evidence has been provided to support them at this point. The principal method for supporting them is through experiment. The approximations and their possible ramifications for the model will be discussed after observing icicle growth through experimental simulation.

2.6. Summary

Figures 2.8 to 2.13 display the model relationships for varying values of the freezing fraction, F, and relative humidity, r, under a typical set of temperature and windspeed values. The mass of the icicle as a function of time is shown in Figures2.8 and 2.9; the icicle surface area as a function of time in Figures 2.10 and 2.11; and, the diameter versus the distance from the tip in Figures 2.12 and 2.13.

Notice that as the icicle grows, the rate of increase in diameter decreases.

But, the rate of increase in area increases with time as does the rate of mass increase. This is not surprising as the heat transfer, on which the increase in mass depends, is directly proportional to the surface area of the icicle. Hence, these relations are directly related.

It is also noted that, for constant freezing fraction, the diameter to distance from tip relationship does not vary appreciably with temperature, pressure or humidity. Figures 2.13 to 2.15 display the effect of relative humidity, temperature and windspeed on this relation. Notice that the difference in shape with changing temperature or humidity cannot even be seen in Figures 2.13 and 2.14. Figure 2.15 shows the prediction that icicles will be slightly thicker for larger windspeeds. The length to diameter ratio of the icicle is related to the ratio of the total heat transfer at the tip, q_{tip}, to that from the sides, q_{cyl}.

$$L/d \propto q_{tip}/q_{cyl} \tag{2.19}$$

Using the heat transfer equation for the convective and evaporative terms, it can be found that the expressions for q_{tip} and q_{cyl} posess terms of the same form except that q_{tip} has two extra terms due to the extra factor of two in the spherical heat and mass transfer correlations. Through some algebra these terms can be shown to be inversely proportional to the windspeed, u. Nevertheless, considering errors associated with measuring these values, the diameter relationship for icicles does not vary significantly with any of these parameters. In other words, with other conditions constant, for any set of temperature, windspeed and humidity, all resulting icicles should be similar in shape.

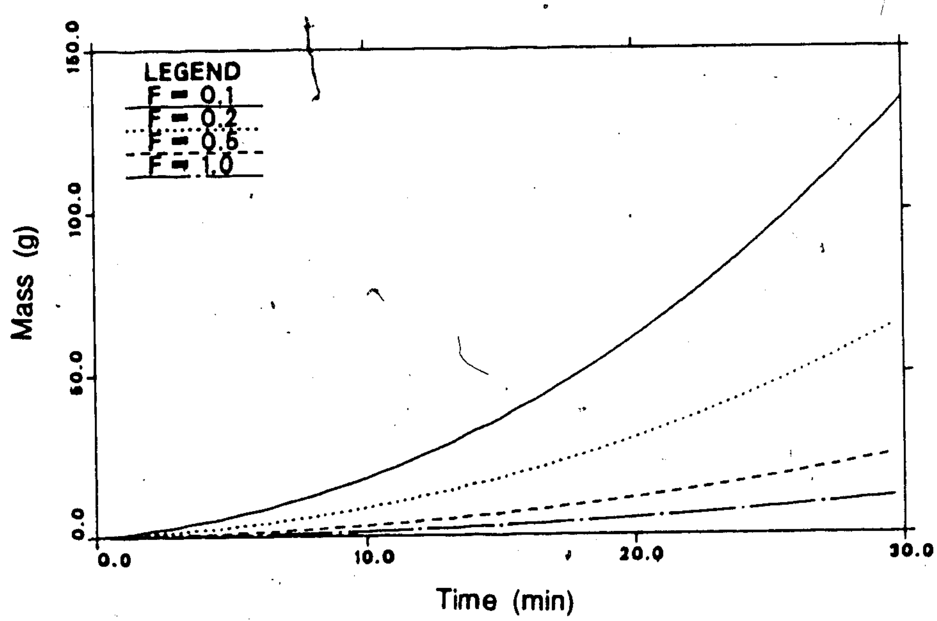


Figure 2.8: Icicle Mass versus time for various values of the freezing fraction. (u = 2.0 ms^{-1}, T = -12°C, r = 1.0)

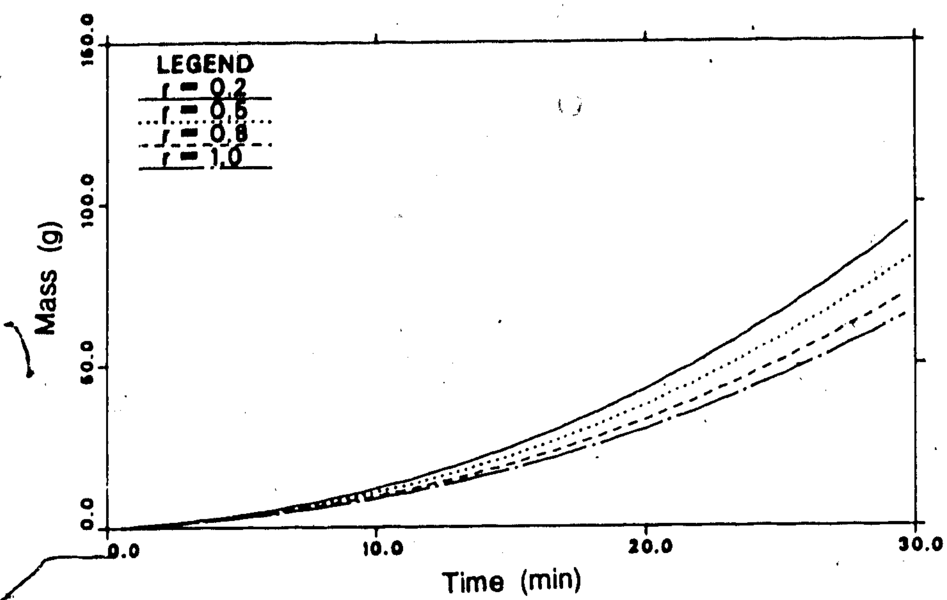


Figure 2.9: Icicle mass versus time for various values of the relative humidity. (u = 2.0 ms^{-1}, T = -12°C, F = 0.2)

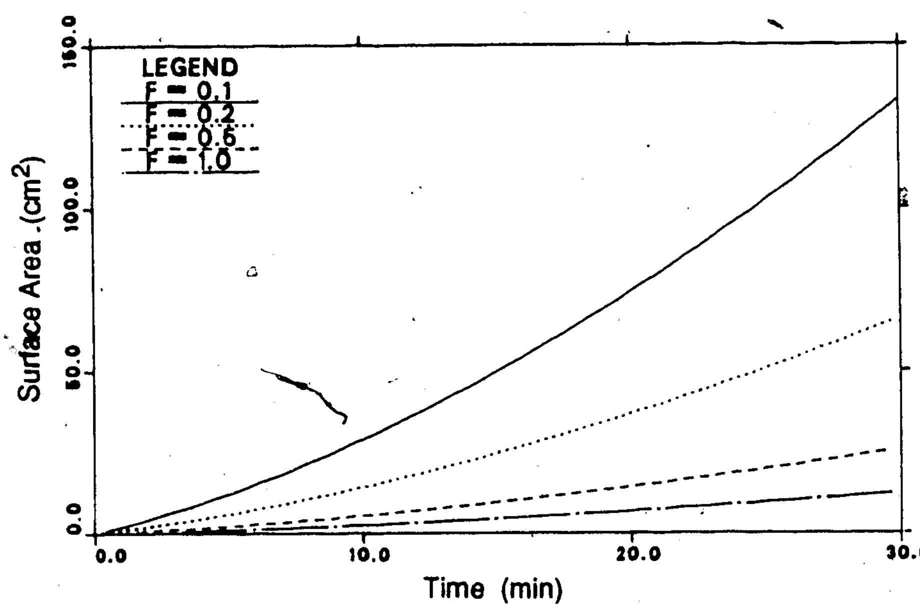


Figure 2.10: Icicle surface area versus time for various values of the freezing fraction. (u = 2.0 ms^{-1}, T = -12°C, r =1.0)

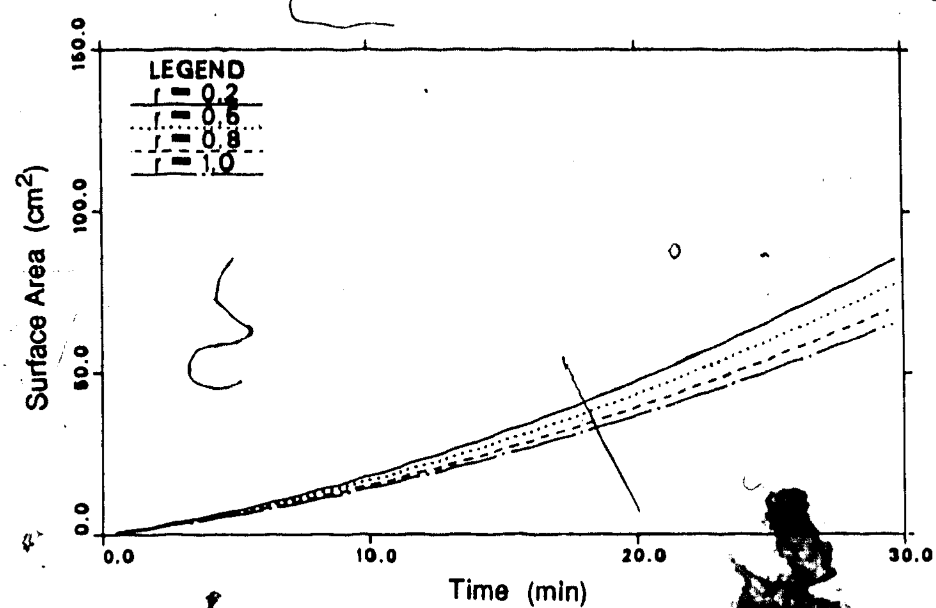


Figure 2.11: Icicle surface area versus time for various v[illegible] of the relative humidity. (u = 2.0 ms^{-1}, T = -12°C, F = 0.2)

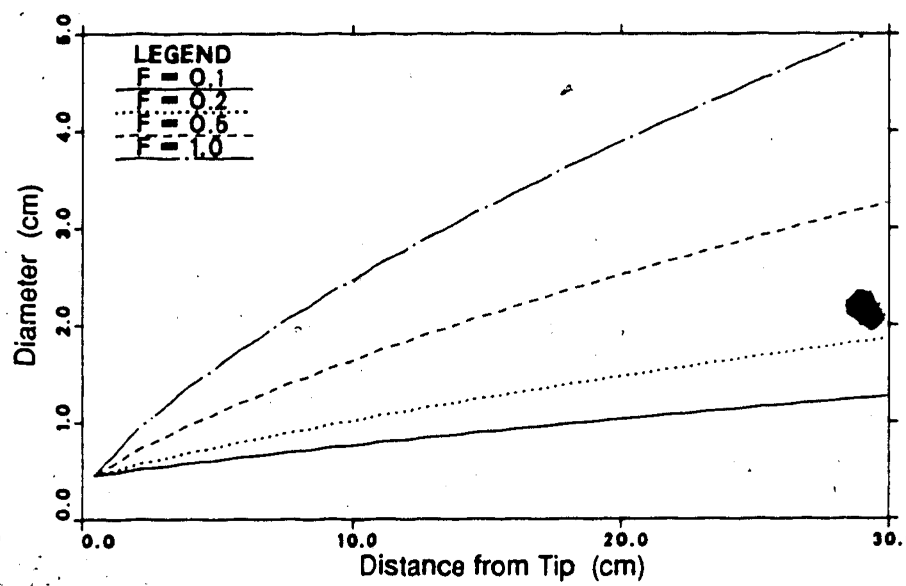


Figure 2.12: Diameter versus distance from tip for various values of the freezing fraction. (u = 2.0 ms^{-1}, T = -12°C, r = 1.0)

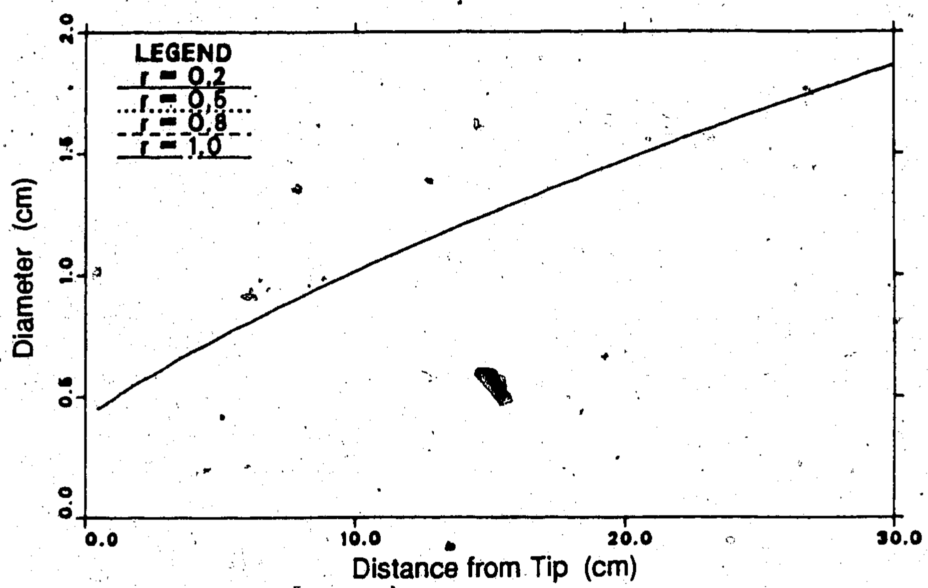


Figure 2.13: Diameter versus distance from tip for various values of the relative humidity. ($u = 2.0\ ms^{-1}$, $T = -12^{o}C$, $F = 0.2$)

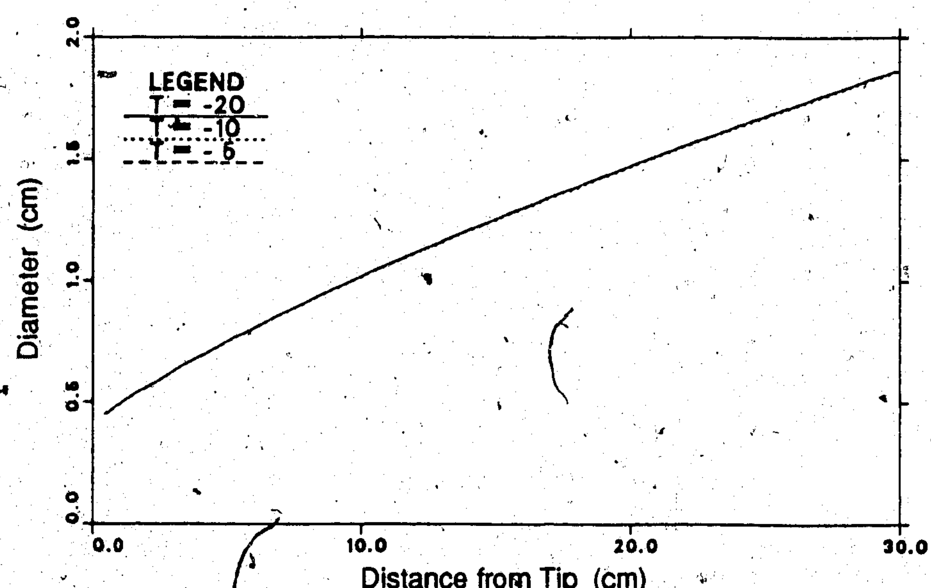


Figure 2.14: Icicle diameter versus distance from tip for various temperature values. ($u = 2.0\ ms^{-1}$, $F = 0.2$, $r = 1.0$)

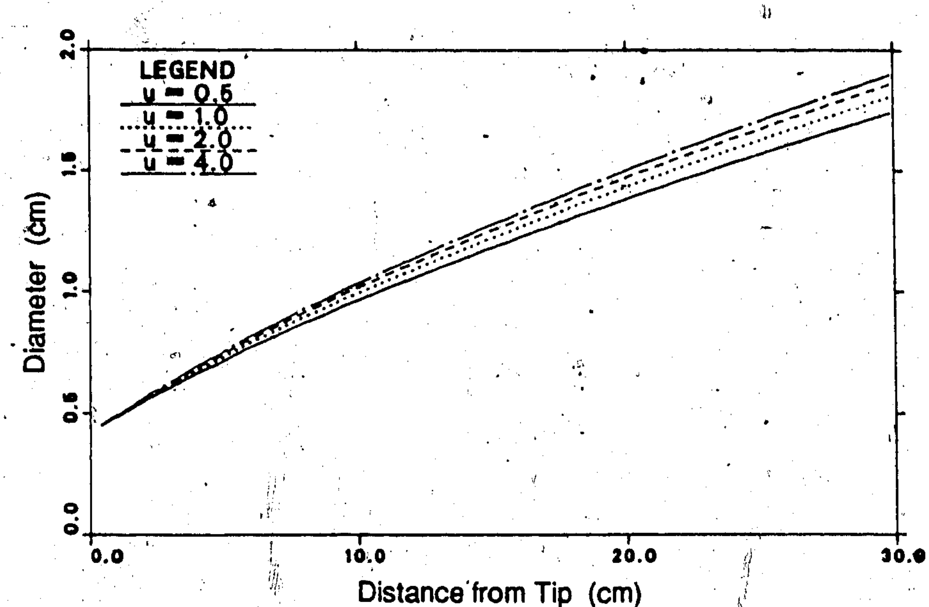


Figure 2.15: Icicle diameter versus distance from tip for several different windspeeds. (T = -12°C, F = 0.2, r = 1.0)

The icicle model can now predict growth under any set of environmental conditions. The only manner in which the model can be tested is through experiments. Comparison with experimental results will provide more insight into the actual processes which occur during the formation of an icicle. Quantitative as well as qualitative observations will be made in order to determine the validity of the model.

3 Icicle Growth Experiments

Icicles, seen to occur frequently in nature, are found to be very difficult to reproduce in the laboratory. In order to produce an icicle in the laboratory, a supply of water at the freezing point is required. Also, the ability to regulate and control three main parameters, temperature, windspeed and flowrate, is desired. Such stipulations must all be met in order that any quantitative results be obtained.

3.1 Laboratory Simulation: Method One

One method of producing icicles is by simulating the process occurring in nature. The most commonly observed situation involves solar radiation incident on a snow or ice surface with the ambient air temperature below freezing. The radiation melts the snow or ice and the meltwater flows and drips from some collection point. The icicle generally forms in a location shaded from the sunlight, where the meltwater drips. An experimental simulation of this process is shown in Figure 3.1. A floodlamp was used as a radiant heat source incident upon a vessel of crushed ice. The melted water flowed out through a hole in the bottom of the vessel, beneath which the icicle formed on a plastic rod. The plastic rod (5/8 inch diameter) was machined to a hemisphere at one end in order to force the dripping water to collect at one point. If performed in appropriate surroundings, such as a coldroom or windtunnel, this process allows for the control of temperature and windspeed. However, the water flowrate or driprate proved to be very difficult to control. The crushed ice tended to fuse together and the meltwater was produced in surges. In some cases these surges were sufficiently strong to fully retard the elongation of the icicle. Careful attention must be paid to this factor when extracting quantitative results with this procedure. This type of apparatus was only used for qualitative observations in this study. Such a process simulates nature with some degree of accuracy; but it is not a good simulator of all atmospheric icing conditions. Icicles, such as those which form on power lines and ships, are formed in a different manner.

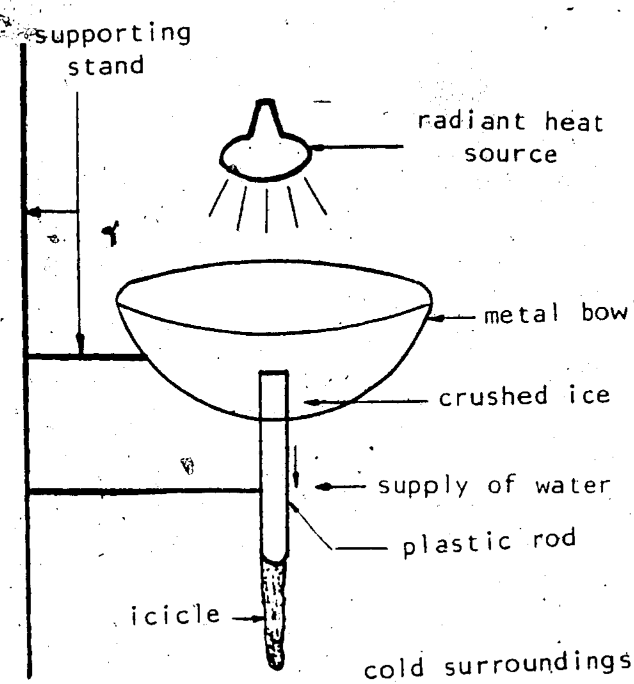


Figure 3.1: Schematic of one method of icicle growth.

3.2 Laboratory Simulation: Method Two

A surplus of liquid water, sometimes supercooled, is believed to be responsible for the formation of icicles on power lines and ships. This water drips from structures on which it is incident. Such a situation can be simulated, but, as before, a dripping water supply at zero degrees Celsius is required. The supercooling is ignored for now as a controllable, supercooled water supply is very difficult to produce and, when water and ice coexist, the temperature is very near to the freezing point.

The apparatus used to create a regulated supply of water at zero degrees is shown in Figure 3.2. A control valve must be located in an ice bath. (or zero degree surroundings) In the case shown the water flows out of the vessel directly into the cold surroundings where the icicle is again formed on the plastic rod. Given the proper surroundings this process allows for controlling of the water supply rate from within the cold region. In order for icicles to be produced for quantitative study, such a water supply must drip into a region where temperature and windspeed can be controlled and monitored.

A coldroom with a fan-forced airstream was first used to create measurable conditions. However, these conditions were found not to be very consistent in space or time. Therefore, in order to achieve more accurate windspeeds and temperatures, an icing wind tunnel was used.

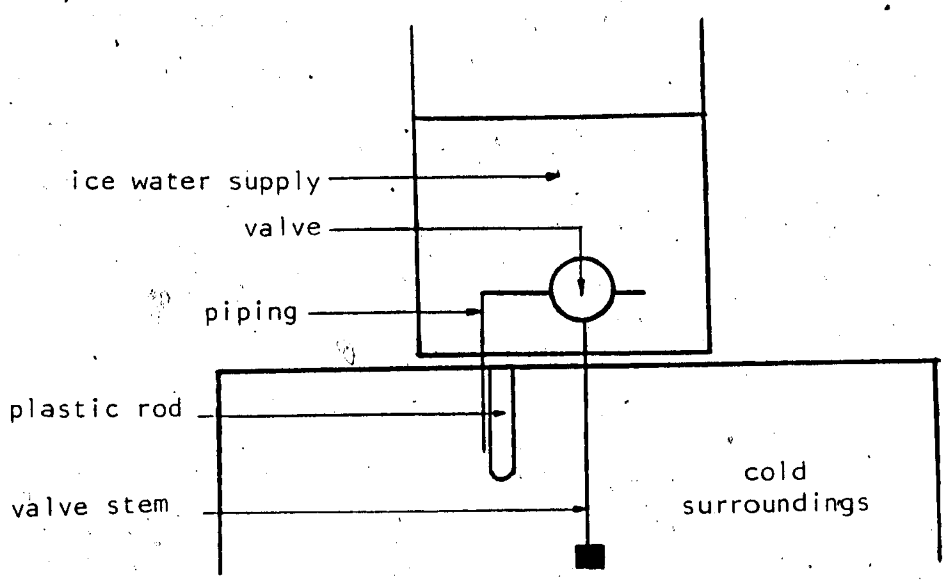


Figure 3.2 Schematic of water supply apparatus.

The tunnel used was the FROST (Fundamental Research on Solidification and Thawing) tunnel located in the Mechanical Engineering Department at the University of Alberta. It is a closed-loop wind tunnel designed to produce windspeeds of 10 to 40 ms^{-1} in the test section. Narten (1985) describes the FROST tunnel in more detail. Experiments were performed in the settling chamber, the section just before the contraction, where windspeeds of between zero and three ms^{-1} are attainable. Figure 3.3 shows a photograph of the wide section of the FROST tunnel while Figure 3.4 shows the experimental apparatus in use on the inside. All numerical results were obtained using the FROST Tunnel.

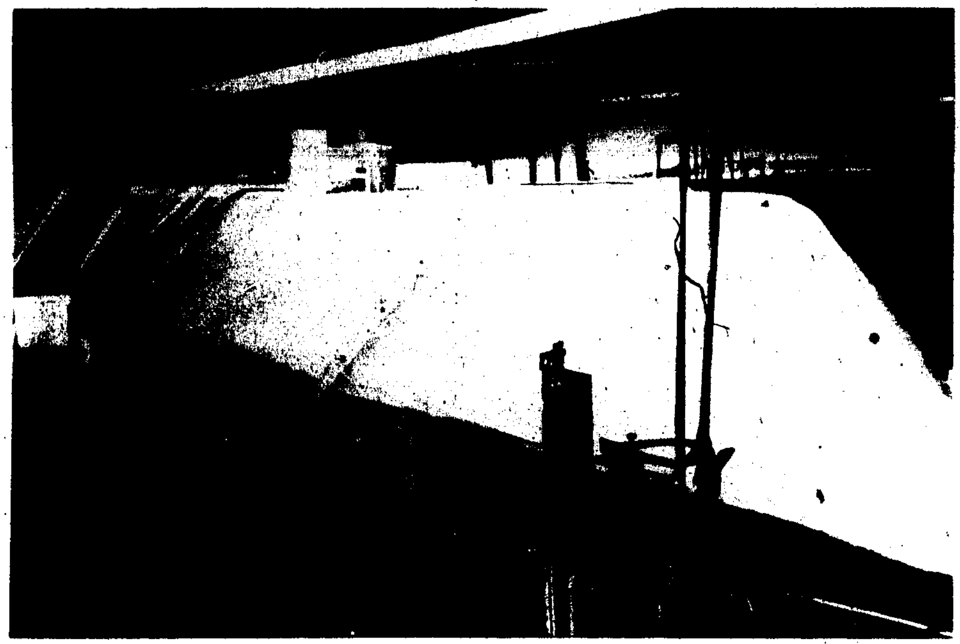


Figure 3.3: Photograph of the wide section of the FROST Tunnel in which experiments are performed.

The windspeed was measured using a Cassela handheld anemometer calibrated with a low velocity flow analyzer (DISA model 54N50). Repeated windspeed measurements demonstrated an uncertainty of plus or minus 0.1 metres per second for the entire range of windspeeds. The temperature was monitored using a Copper-Constantan thermocouple with an ice bath as the reference junction. The thermocouple voltage was measured using a Fluke model 8062A multimeter. The thermocouple temperatures are expected to be accurate to within 0.5 degrees Celsius. The thermocouple was checked with an ordinary mercury bulb thermometer which was accurate to within ±0.2 °C. An ice bulb temperature was also measured to determine an approximate value of the relative humidity in the tunnel. Atmospheric pressure values were extracted from the hourly synoptic observations at the Edmonton Municipal Airport, approximately three kilometres from the experimental site.

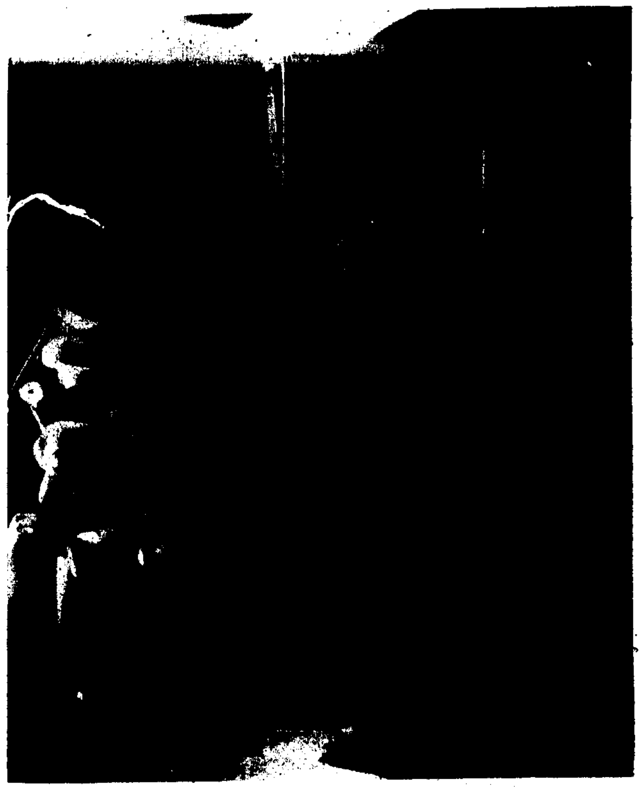


Figure 3.4: Photograph of the icicle apparatus in operation inside the FROST Tunnel.

The most difficult problem to overcome related to the creation of the water supply at zero degrees Celsius. If the region around the piping was too cold, the aperture froze, while if too warm, the supply water acquired a temperature above freezing and icicles did not form. Freezing was prevented by allowing a small amount of room air to enter the hole through which the piping passes. In this way, by trial and error, the water was kept reasonably close to the freezing point. The dripping water temperature was checked by inserting a thermocouple into the piping through which the water flowed.

Icicles may have formed even if the water at the end of the plastic rod was slightly above freezing. It is expected that this was the case and for this reason, the first few centimetres of the icicle were never measured quantitatively. Once a short length of icicle has formed, then the water was surely at the freezing point as solid and liquid coexisted near the surface of the icicle.

3.3 Procedures

The experiments were performed in two ways. Procedure 1 was used for most of the trials, during which an icicle was grown while periodically adjusting the water supply rate. In this way, a slow driprate persists from the tip of the icicle Thus optimizing the growth rate. In order to achieve this, the water supply rate must be adjusted periodically as the icicle becomes larger. In this case the length was recorded as a function of time while the times when the supply rate was increased were noted as well. If the supply rate were not increased periodically, dry growth would have begun shortly after the icicle began to grow, provided that the supply rate was not too fast. If the supply rate was very fast, it was found that no icicle formed at all.

Procedure 2 was used for some of the trials, in which the supply rate was not adjusted during the growth process. Instead, a fast supply was used initially, and, as the icicle grew, the freezing rate increased until, eventually, the freezing rate equalled the supply rate and the onset of dry growth was imminent. In this case, the length and driprate from the tip were recorded as functions of time. This method provides a manner in which the dependence of growth rate on driprate can be determined. The validity of the assumption of no supercooled water or ice shed from the tip can also be determined. In order that wet growth persists for a sufficient length of time, the supply rate must be adjusted at the start to be considerably faster than that of Procedure 1. The driprate at the beginning of this process was adjusted to between one and two drops per second. (60 - 100 min^{-1}) A slower rate resulted in an icicle which grew for only a short period of time while a faster rate produced icicles far too slowly.

3.4 Observations

Observations of icicles were obtained in two ways. The shapes of the icicles were measured using a set of vernier calipers. For these measurements, the diameter of the icicle was recorded as a function of the distance from the tip. Because icicles are too fragile to be measured with calipers during their growth, these results were all obtained after the icicle had finished growing. The length of the icicle as a function of time was measured using an ordinary ruler. These results are accurate to within ±0.1 cm. The driprate from the tip was evaluated by simply counting drops in a given time interval. The theoretical size of the tip drops was verified by collecting a quantity of drops and measuring the total mass. This mass didided by the number of drop yielded the size of the pendant drops.

The second method of observation used was photography. Several different lens and camera combinations were used. A Canon F-1 camera was used to take photographs through a microscope. The microscope is a Wild M3 with magnification of 6.4, 16 and 40. Most pictures were taken using a Pentax SP 500 camera with a Pentax 50mm, f4 macro lens. A close-up ring was also used for several pictures. Kodak 400 ASA film was used for all but a few photos which were taken with ASA 1000 film. The best results were achieved using object illumination with a dark background. The object was illuminated from the side using a microscope lamp. The shutter speed and stop were optimized by trial and error. The specific values were recorded for each photograph for future reference.

Growing icicles under controlled conditions in the laboratory is a tedious process, consuming much time before any useful results are achieved. Any adjustment in the wind tunnel or coldroom conditions requires modification of the apparatus. Nonetheless, once some experience is has been gained, the procedure becomes easier to implement and the conditions easier to control.

4 Results and Discussion

4.1 Discussion of Experiments

The first experimental results consist of photographs taken of growing icicles. Many of these provide insight into the processes resulting in icicle formation. The most crucial assumption regarding icicle growth was that the spherical tip droplet freezes into the shape of a cylinder. Notice the spherical pendant drop in Figure 4.1. This was taken of the tip section of an icicle as it grew. Although the pendant sphere disappears for a short period of time after dripping, as shown earlier in Figure 2.1b, the theory of a mean spherical drop seems reasonable. Notice, in Figure 4.1, that the frozen portion, above the pendant drop, does appear cylindrical and has a slightly smaller diameter than the pendant drop. The pendant drop diameter of 5.0 millimetres was checked experimentally as documented in Appendix B. These results produce an approximate diameter of 4.98 millimeters, sufficiently close to 5.0 for the expected precision of these experiments. Once the equivalent of the single droplet freezes, it is expected to take the shape of a cylinder with equal length and diameter. This assumption is supported by the photograph in Figure 4.1 and those shown earlier in Figures 2.1. A simple calculation shows that a water drop of the above size freezing into the shape of a cylinder will acquire an equidistant length and diameter of approximately 4.5 millimeters, slightly smaller than the diameter of the spherical pendant drop.

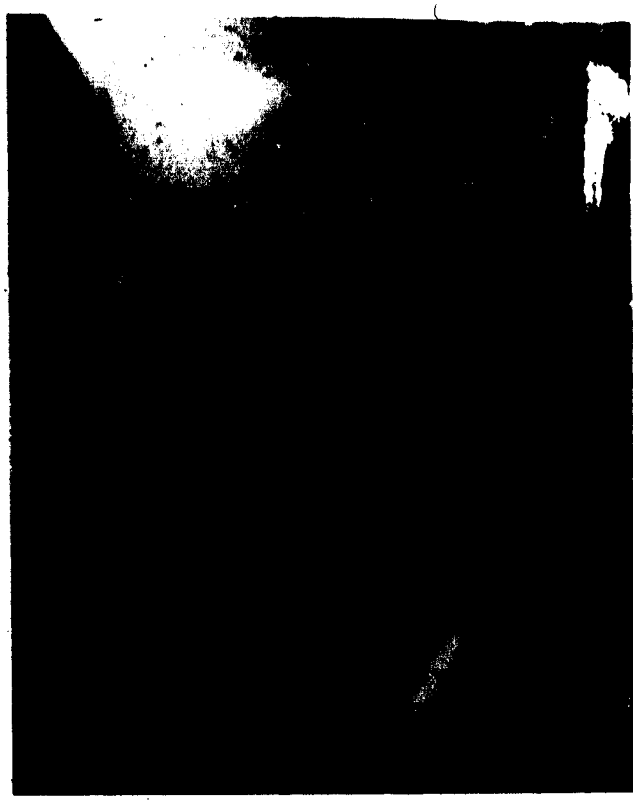


Figure 4.1: Photomicrograph (magnification of 6.4) of the tip of a growing icicle. (T_a = -11°C in coldroom)

The detailed experimental results are shown in Appendix B. The relative humidity in the coldroom was assumed to be one hundred percent, based on the fact that no measurable ice-bulb depression was found. The presence of frost on objects in the coldroom also confirmed that a value of 1.0 was appropriate. The 100 percent value was also used for the wind tunnel, as the operating temperature was always below the dewpoint temperature of the surrounding room from which the air originated. Also, identical values of dry and ice-bulb temperatures were measured using the psychrometer.

For temperatures above -18°C; this corresponds to a relative humidity of greater than 80% based on the uncertainties of the thermometers. Due to the relatively weak dependence of growth rate on relative humidity (see Figure 2.5), the value of 1.0 is reasonable.

As mentioned in Section 3, the windspeed and temperature were found to be essentially constant in space and time in the FROST Tunnel. These values were measured periodically and were found to vary less than the expected uncertainties of 0.1 ms^{-1} and 0.5°C, respectively. Hence, these values represent the expected errors in the measurements. The length values, measured with a ruler, are accurate to within ±0.1 cm. The only instance where these errors become large, in a relative sense, is when the length growth is found through the small difference between two of the length values.

The first section of results is from trials where the driprate from the tip was maintained at less than one drop per second. In these cases the supply rate was increased periodically as the icicle grew and was able to freeze more of the incident water. After the driprate had slowed to less than five drops per minute, it was increased to thirty or forty per minute. The times at which the supply rate was increased are indicated in the results by an asterisk. (*) The dashed vertical lines on the graphs also represent these times. Trials 1 through 17 are this type of experiment.

The supply rate was not adjusted during the second set of experiments. These results are those in the second section - trials 18 through 25. In these cases the driprate from the tip decreased gradually until all the supply water was frozen and wet growth ceased. The graphs in the results display length against time and length growth rate against flowrate, or driprate from the tip. It must be kept in mind that, although the supply valve was not adjusted, the supply rate was not always constant. It was noticed that, even with no icicle growing, the driprate sometimes decreased with time. This was likely due to the fact that, as the ice water supply was dimished, the supply rate decreased somewhat due to the reduced downward pressure of the water and the build up of ice in the aperture through which the water flows to the icicle. However, this fact does not affect the results which are presented.

4.2 Analysis of the Data

For trials 1 through 17, the driprate from the tip was not closely monitored. It is readily observed from the results that the growth rate is not constant in time, (see for example, trial 10) and, hence, the assumption that no supercooled water or ice is shed must not be satisfied. Obviously, the length growth rate is dependent upon the driprate from the tip. For most of these trials, the same pattern appears, particularly in the graphical data. The growth rate increases with time until the supply increases, at which time the growth rate suddenly decreases. This pattern repeats itself throughout the growth process. All of the trials which ran for a sufficient length of time show some resemblance to the pattern shown in Figure 4.2.

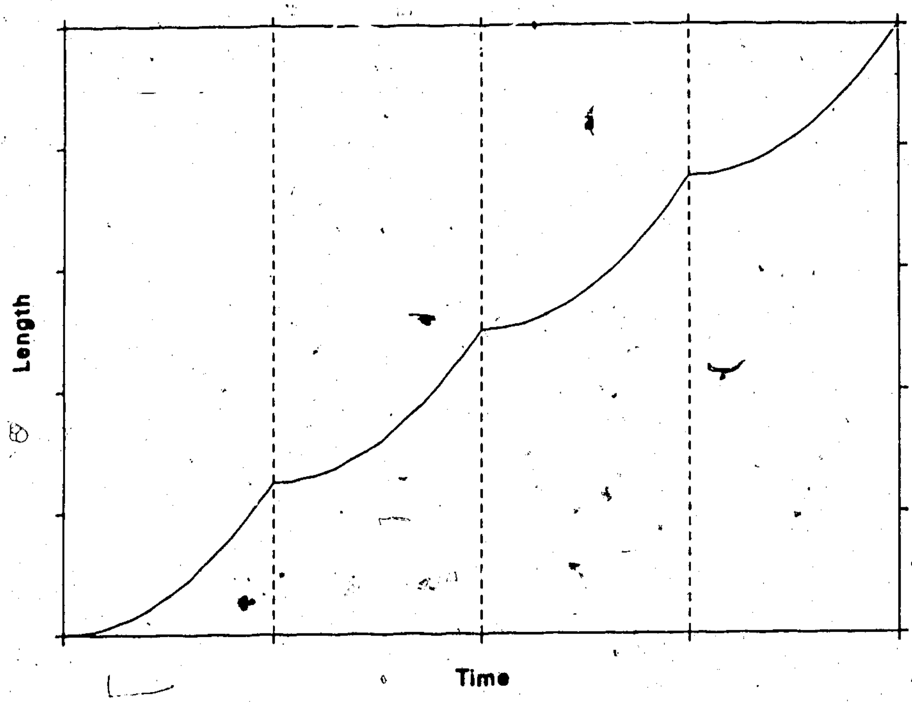


Figure 4.2: Expected dependence of length on time with periodic increases of the driprate from the tip. (dashed vertical lines represent times at which the driprate was increased)

As the driprate decreases, the growth rate increases toward some asymptotic limit. This limit is expected to be that predicted by the theory with the appropriate freezing fraction. For this reason, the result of interest in trials 1 through 17 is this maximum value or upper limit. The maximum growth rate in centimetres per minute is included in the data.

Trials 18 through 25 show the dependence of elongation rate on driprate from the tip. It should be noted that initially, as the driprate decreases, the growth rate increases. However, as time goes on and the driprate nears zero, the growth rate decreases slightly. This feature does not appear in Figure 4.2, as in these trials, the driprate was not allowed to become small enough. The experimental length versus time relations all exhibit a similar pattern to that of trial 24. Figure 4.3 shows the typical pattern observed.

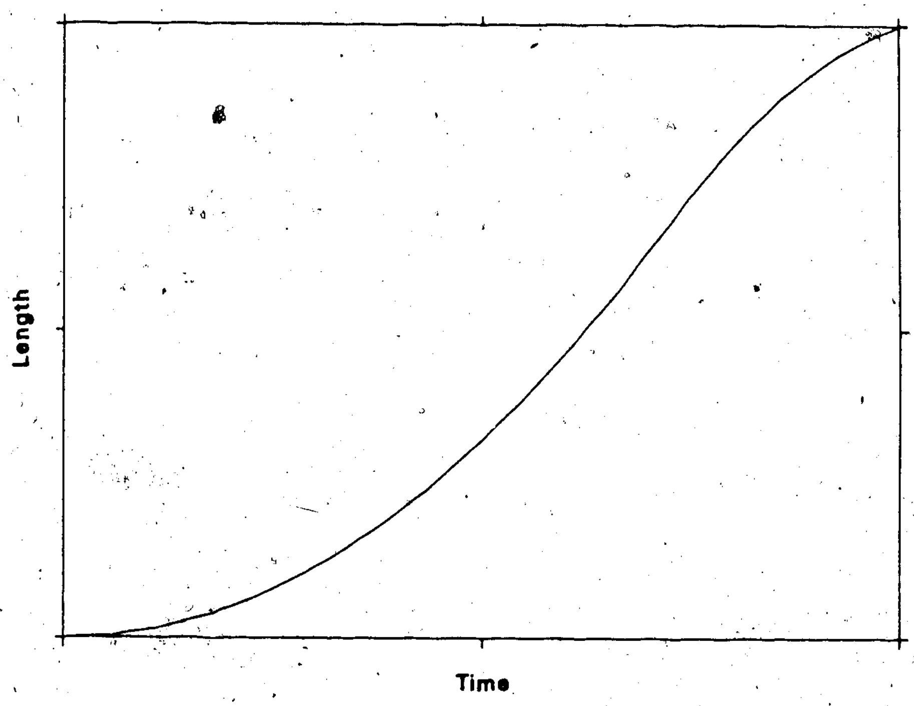


Figure 4.3: Typical length versus time graph for trials 18 through 25.

The graphs of length growth rate, dL/dt, versus time also show this pattern, (see Trials 18 - 25 in Appendix B) The pattern is not as evident due to the fact that the dL/dt values were obtained by taking the difference between two values with reasonably large associated errors. Figure 4.4 shows an idealized representation of this pattern. Trial 19 is the best example of this pattern.

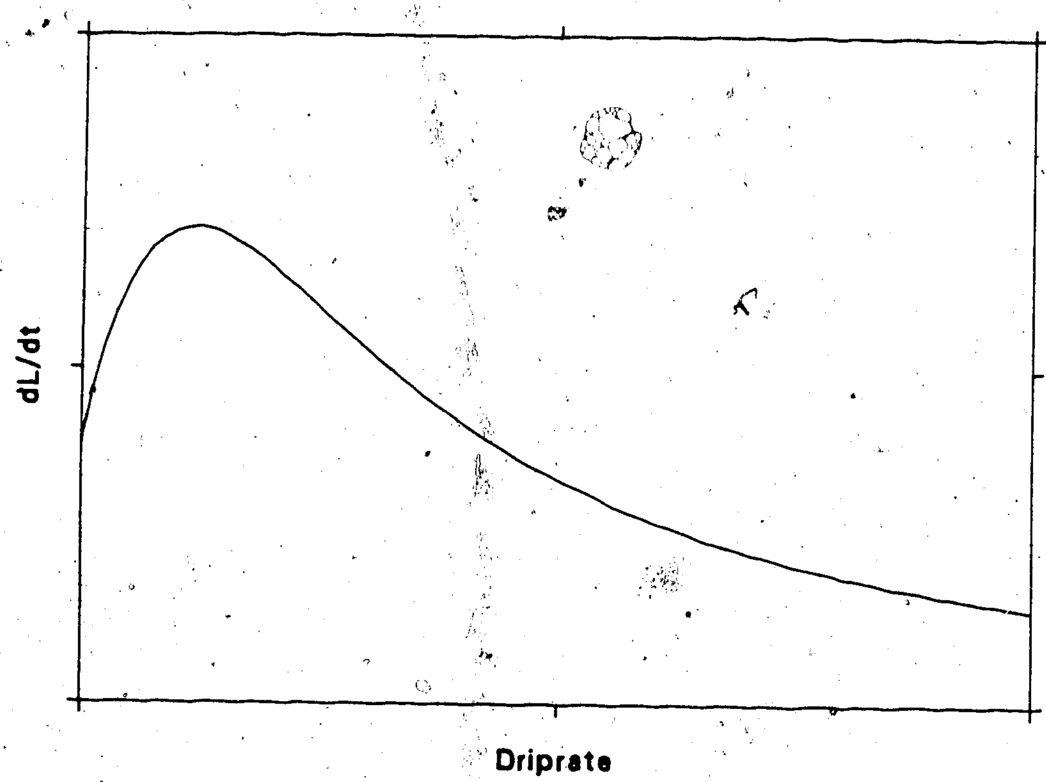


Figure 4.4: Typical length growth rate versus driprate from tip

A maximum value of the growth rate is observed at a driprate approaching zero. The explanation for such a maximum is as follows. At very large driprates, dL/dt is quite small, as seen in all of the examples. As the driprate decreases, the value of dL/dt increases for the reason explained in the theory section. That is, a faster flowrate from the tip of the icicle inhibits the freezing process. For a sufficiently fast supply, the drops at the tip will only exist for a short period of time before falling. Each drop will cool very slightly, not enough to initiate any freezing. Thus, a small amount of supercooling accounts for the heat transfer instead of freezing.

Because the water flow is indeed a dripping process, as the driprate becomes very small, it takes a longer time for the pendant drop to reform after falling. Thus, the heat transfer is reduced and the amount frozen, or the growth rate is decreased. The model, however, is based upon the assumption of a continuous water supply with no supercooled water shed from the tip. This represents an upper limit of the growth. Since it can be inferred from the experiments that there certainly is supercooled water shed, the model result would be equivalent to the situation where the supply rate equals the freezing rate. In other words, the model assumes no supercooling at the tip yet it assumes wet growth. This implies that there is no dripping from the tip but also, there is no water deficiency at slow driprates as described above. It is not realistic to expect such a case to occur naturally, but this case provides valuable information regarding the maximum growth of icicles. The upper limit which the model predicts, can be compared to the upper limit determined from these results.

The value of the length growth rate at a driprate of zero cannot be defined, but the limit as the driprate approaches zero can be extrapolated from the graphs as shown in Figure 4.5.

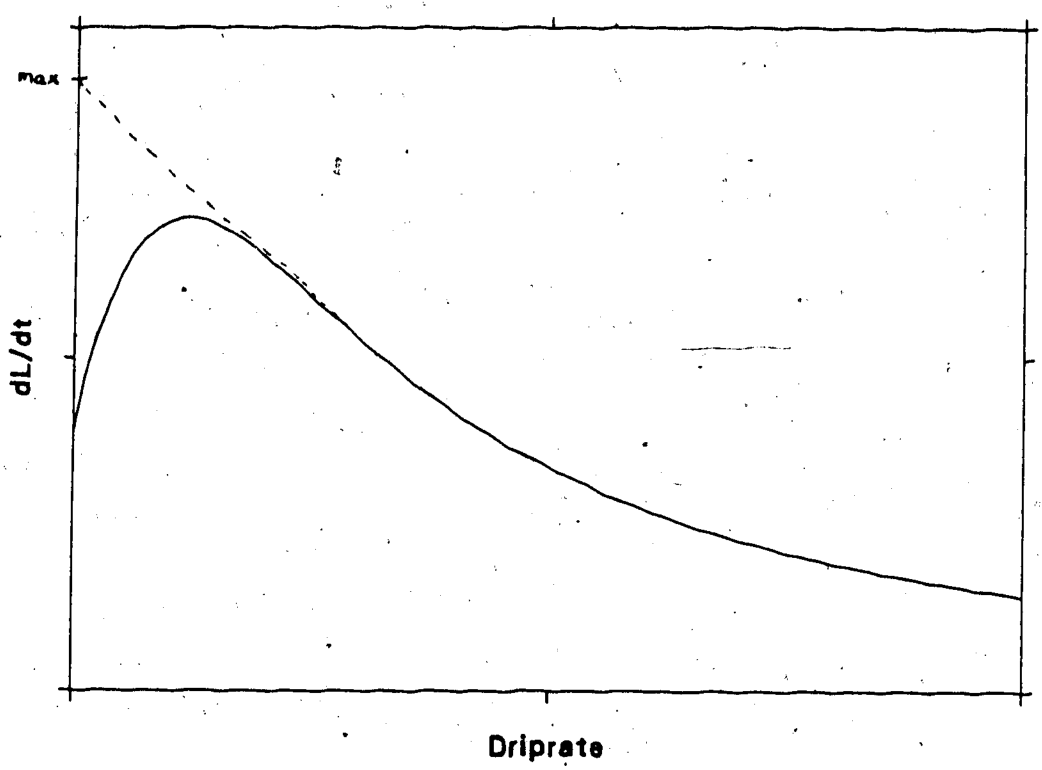


Figure 4.5: Method of extracting modelled maximum growth rate

Taking this value and putting it into the model will yield an estimate of the freezing fraction. If the model does indeed predict the upper limit of length growth rate, the modelled growth rates should never be exceeded by the maximum growth rates in trials 1 through 17.

4.3 Experimental Results

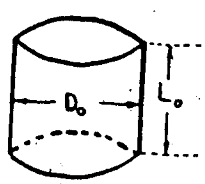
Using the aforementioned extrapolation technique, the following maximum growth rates and corresponding freezing fraction, F, can be found for each of trials 18 through 27. These results are shown in Table 4.1.

Table 4.1: Maximum growth rates and corresponding freezing fractions from experimental data.

Trial	Temperature	Windspeed	dL/dt (max)	F
18	-13.0 °C	2.9 ms^{-1}	1.8 cm min^{-1}	0.10
19	-13.0 °C	2.9 ms^{-1}	1.8 cm min^{-1}	0.10
20	-13.0 °C	2.9 ms^{-1}	1.7 cm min^{-1}	0.10
21	-13.0 °C	2.9 ms^{-1}	1.9 cm min^{-1}	0.09
22	-15.5 °C	1.9 ms^{-1}	1.6 cm min^{-1}	0.11
23	-16.0 °C	1.9 ms^{-1}	1.8 cm min^{-1}	0.10
24	-16.0 °C	1.9 ms^{-1}	1.8 cm min^{-1}	0.10
25	- 9.5 °C	2.9 ms^{-1}	1.1 cm min^{-1}	0.12
26	-10.5 °C	2.9 ms^{-1}	1.4 cm min^{-1}	0.10
27	-10.5 °C	2.9 ms^{-1}	1.3 cm min^{-1}	0.11

The previous table shows that there does indeed seem to be a constant freezing fraction for different conditions. A value of 0.10 appears to be the best choice from the given results. It is expected that the freezing fraction represents the minimum amount of ice necessary in order that the icicle maintain its shape and mechanical integrity.

Observations suggest that the icicle initially freezes on the outside, with liquid water trapped within it. This freezing fraction can be used to compute the thickness of such an ice shield about the tip of an icicle, if it is assumed that the icicle does indeed freeze in such a fashion. If this is the case, then 10 percent of the cross-sectional area of the tip of the icicle is ice while the remainder is water. The calculation is as follows:



$$L_o = D_o = 0.45 \text{ cm}$$

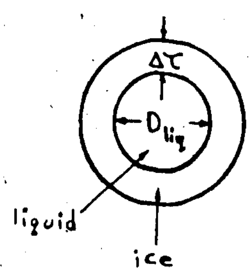
if $F = 0.10$ then

$$0.90 \times \pi D_o^2 = \pi D_{liq}^2$$

or, $D_{liq} = (0.90 D_o^2)^{1/2}$

$$D_{liq} = [0.90 \times (0.45)^2]^{1/2}$$

$$= 0.43 \text{ cm}$$



Thus, $\Delta\tau = (D_o - D_{liq})/2 = 0.010 \text{ cm}$

$$= 100\ \mu\text{m}$$

Hence, if the ice freezes as a thin shield, it will have a thickness of approximately 100 microns if the freezing fraction is equal to 0.10.

This value of 0.10 can then be used in the theory to calculate the upper limit of the growth rate for trials 1 through 17. These results are given in Table

4.2. The theory for a freezing fraction of 0.10 is compared to the observed maximum growth rates from Trials 1 to 17. It is readily noted that the experimental maximum exceeds the theoretical upper limit in only two cases, trials 11 and 17. In both cases, systematic errors are more than able to account for the magnitude of the exceedance. In several other cases, for example, trials 8 and 15, the observed maximum growth rate is near to but slightly less than the theoretical growth rate. Also, several cases show an observed growth rate far less than the theoretical result as in trials 2 and 4.

Another explanation for the growth rates exceeding the theory is as follows. As water freezes it does so as dendrites or thin columns (Knight, 1979). In some cases, if these columns orient themselves along the axis of the icicle, there can be a sudden quick increase in length. In this case the icicle does not freeze as a cylinder as in the theory, but instead posesses a thinner cross section. This thinner cross section is evident near the tip of several icicles. The diameter data for trials 10, 20 and 27 display this feature. Note that near the tip, for these trials, the diameter was significantly less than the theoretical value of 0.45 cm. If the icicle does not maintain the circular cross section, the length growth could vary from the theoretical value. Most of the freezing could conceivably take place from a dendritic column of the icicle, not from a cylinder, as modelled. This was only observed in a few cases, although it could have been camouflaged by the driprate from the tip in others. The model does indeed seem to provide an upper limit to the length growth rates of the icicles.

Table 4.2: Summary of maximum growth rates for trials 1 through 17, and the theoretical upper limit of the growth rate.

Trial	T (°C)	u (ms^{-1})	observed dL/dt (max, cm min^{-1})	theoretical dL/dt (cm min^{-1})
1	-13.5	2.3	0.8	1.6
2	-13.0	2.5	0.7	1.6
3	-17.5	1.3	1.5	1.6
4	-14.0	2.8	0.9	1.8
5	-13.0	2.7	1.4	1.7
6	-11.0	2.7	0.8	1.4
7	-14.0	3.2	1.3	1.9
8	- 8.0	3.4	1.1	1.2
9	- 8.0	2.5	0.6	1.0
10	-12.0	3.5	1.4	1.8
11	- 5.0	3.2	0.8	0.7
12	- 6.0	3.1	0.6	0.9
13	-15.0	2.7	1.3	1.9
14	-12.0	2.8	1.3	1.6
15	- 8.5	2.7	0.9	1.1
16	-16.0	2.1	1.3	1.8
17	-10.5	2.1	1.3	1.2

A further method of verification of the model does not take into account the growth rate, but instead, the diameter of the icicle as a function of the distance from the tip. Figure 2.8 displays the theoretical relationship between these parameters. It has been shown that the shedding of supercooled water retards the growth rate significantly. However, the radial growth is not affected by this phenomenon. With ice and liquid water in equilibrium, the surface will always remain at the freezing point. Hence, the amount of water frozen in a given time is the same regardless of the flowrate. Now, if the elongation is retarded while the radial growth is not, the icicle will be thicker than that predicted by the model. As the driprate from the tip nears zero, the growth rate in length becomes less and less retarded, and the length to diameter ratio approaches that predicted by the theory. In the second set of experiments, the flowrate from the tip, W_d, was initially large and decreased to zero as the icicles grew. Thus, the top section of the icicle, being formed earlier with the higher driprate, should be thicker than the prediction of the model. The section near the tip, on the other hand, was formed with a low driprate and should approach the predicted shape. The trials in which such a comparison can be made are trials 19,20,21,24,26,27. Figures 4.6 to 4.11 show the experimental and theoretical diameter to length relations. The icicles, as mentioned earlier do not usually possess a circular cross section, but instead, are approximately elliptical in cross section. The diameter used in the following graphs is an average of the major and minor axes. All the graphs show that the tip section of the icicle falls near to or even slightly below the theoretical curve. With the exception of Figure 4.6, the experimental diameter values begin to more and more exceed the theoretical curve, further from the tip, as expected due to the retarding of the elongation rate. Figures 4.7 and 4.9 are the most distinct examples of this fact.

For all of the previous discussion it must be kept in mind that there are many unpredictable features that any individual icicle may posess. Bumps and ridges appear almost always (see Figure 4.12) and bends or corners even occur occasionally. (see Figure 4.13) For this reason all measurements are approximate and no two icicles, even if grown under identitions, are identical.

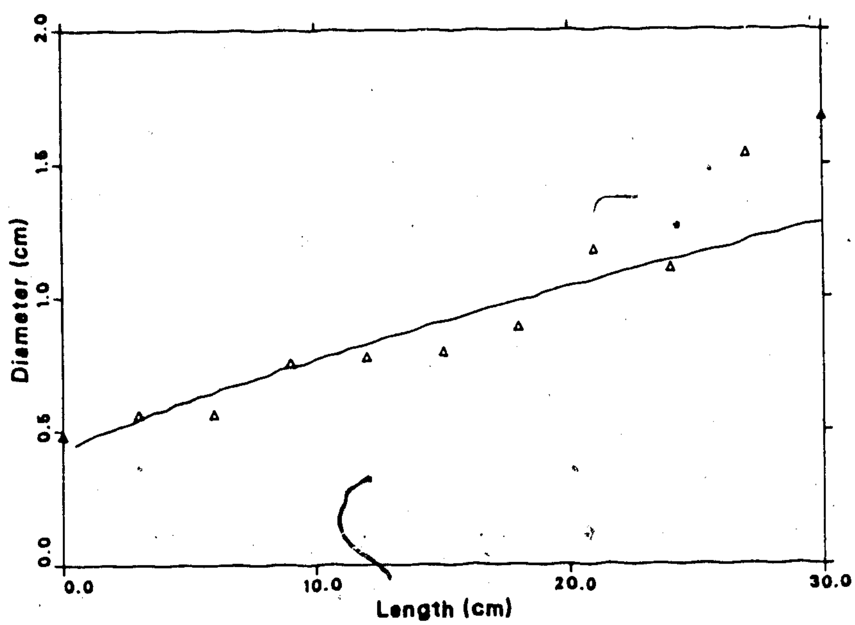


Figure 4.6: Diameter vs distance from tip theory (solid line) and experiment (symbols) for Trial 19.

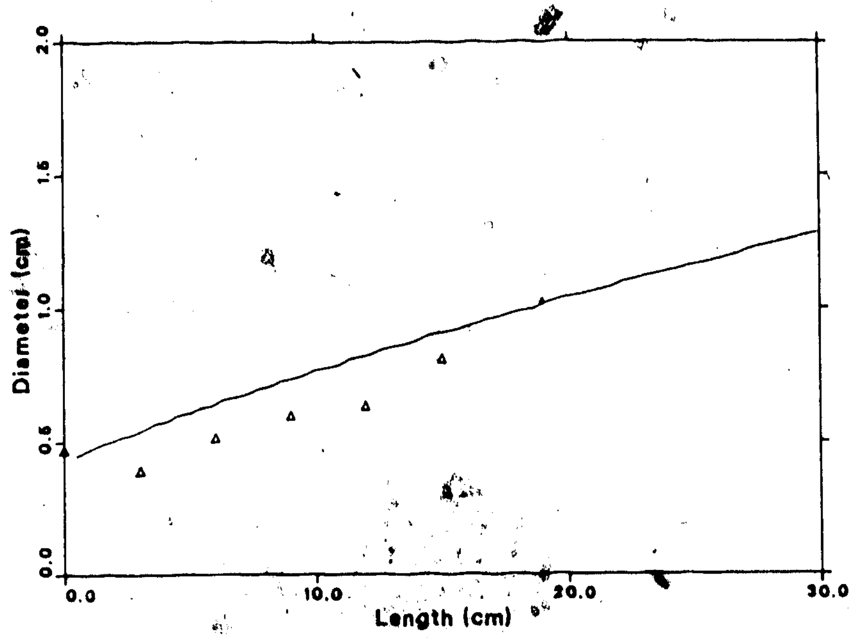


Figure 4.7: Diameter vs distance from tip theory (solid line) and experiment (symbols) for Trial 20.

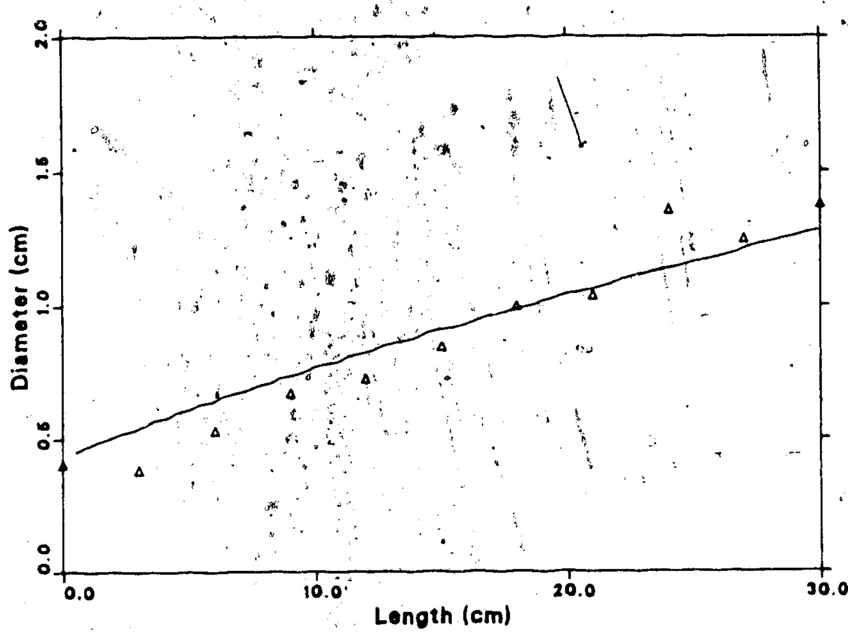


Figure 4.8: Diameter vs distance from tip theory (solid line) and experiment (symbols) for Trial 21.

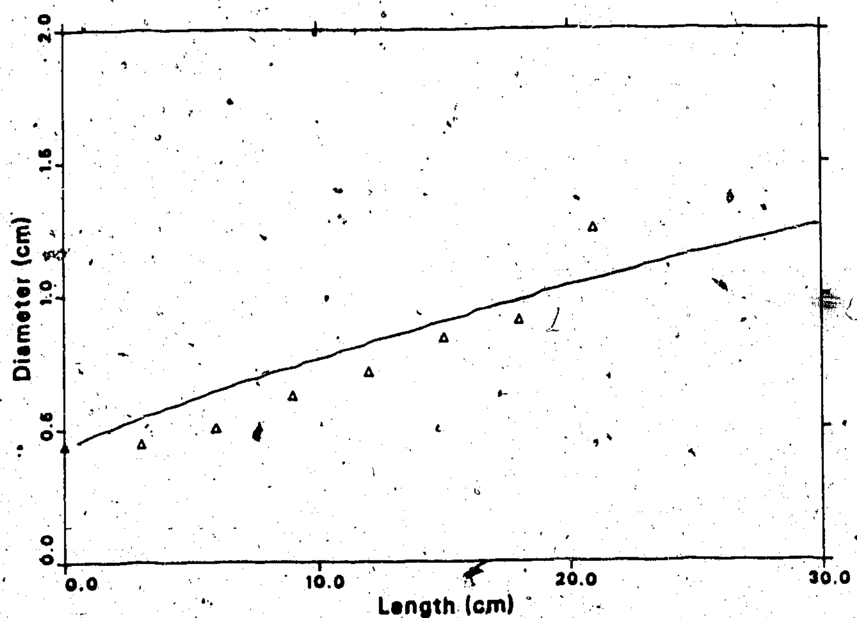


Figure 4.9: Diameter vs distance from tip theory (solid line) and experiment (symbols) for Trial 24.

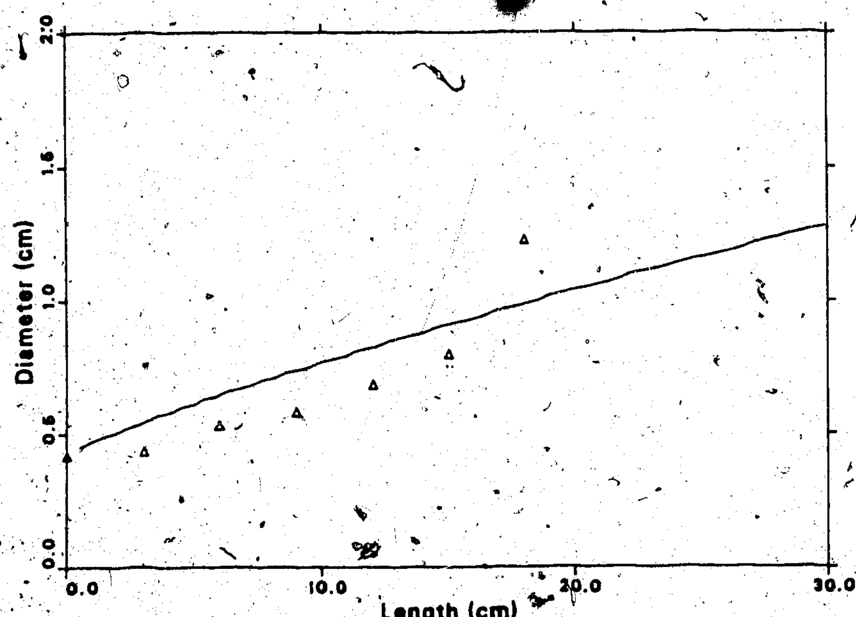


Figure 4.10: Diameter vs distance from tip theory (solid line) and experiment (symbols) for Trial 26.

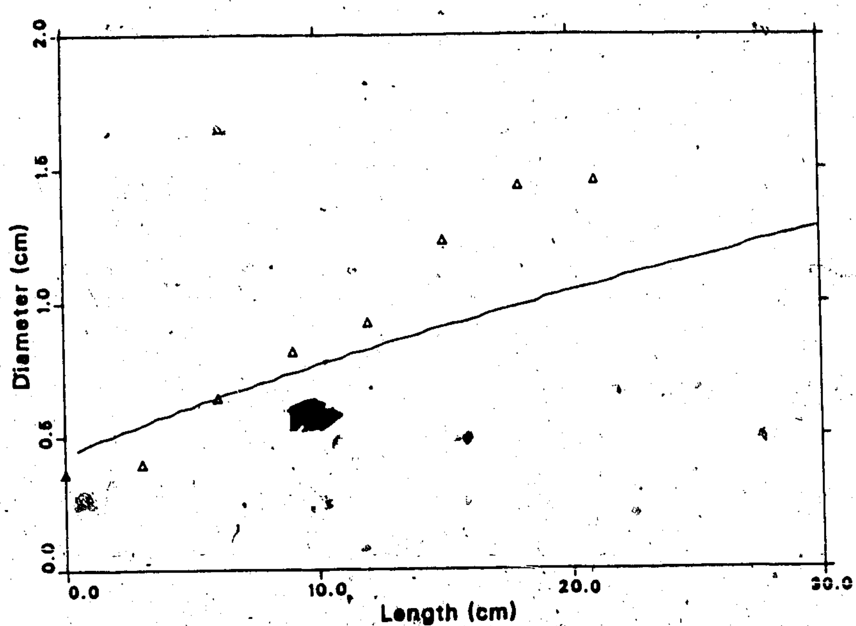


Figure 4.11: Diameter vs distance from tip theory (solid line) and experiment (symbols) for Trial 27.

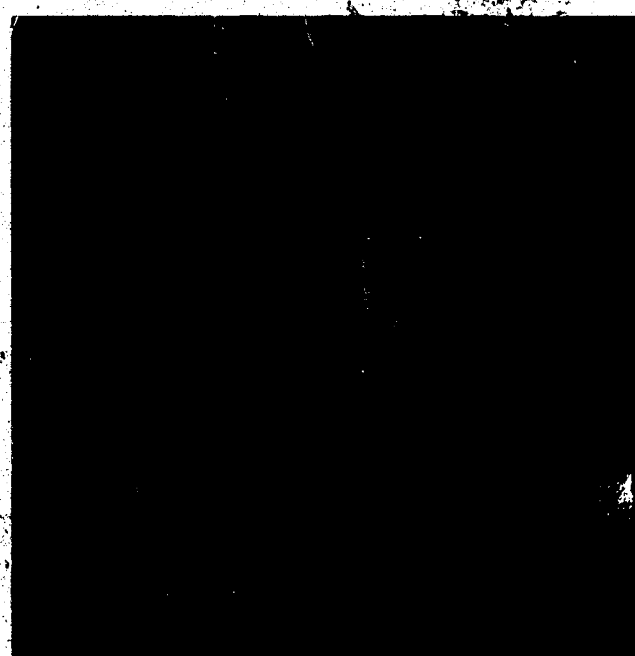


Figure 4.12: Photograph displaying bumps on an icicle.

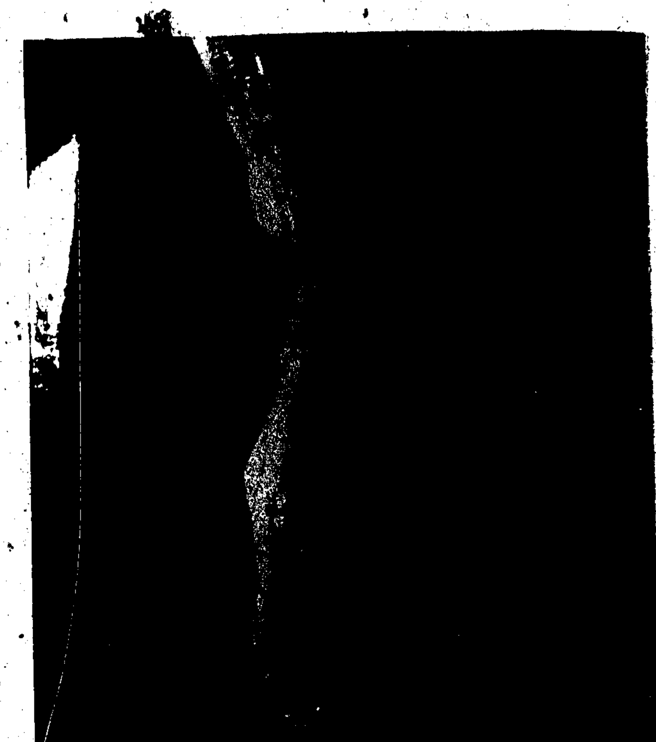


Figure 4.13: Photograph of a "bent" icicle.

5 Conclusions

5.1 Overall Conclusions

5.1.1 Model Restrictions

The icicle model has been designed to predict "wet growth", where the water supply rate to the icicle exceeds the freezing rate and no supercooled water or ice is shed from the tip. However, in Section 4.2, it has been discussed that this was not the case. In effect, the model predicts an optimum case, where the supply rate equals the freezing rate. As mentioned, such a case will not occur naturally, but this case does yield an upper limit to the growth of an icicle. The situation of "dry growth" has not been discussed as, in this case, the icicle grows only in diameter and the freezing rate equals the supply rate. The shape of the icicle would be very different from that of "wet growth", but, in most applications, the icing rate is the critical factor to be determined.

The various correlations and numerical approximations used were tested for accuracy in only a small range of conditions. Temperatures from -20 to 0 degrees Celsius were used along with windspeeds of zero to four meters per second. Such a range should encompass most conditions in which icicles form, but the model could be extended to cover a larger range. Pressure-dependent variables and constants were computed for a surface pressure of 93 kPa, appropriate for the location of the experiments but these could easily be adjusted for any location. In applying the model, these facts must be considered before any results can be computed.

5.1.2 Validity of the Assumptions

The various assumptions made in deriving the model and their likely effects were discussed in Section 2.5. With the exception of the restriction of no ice or supercooled water shed (Section 2.5.2), no evidence was found to dispute these. However, in individual cases, some of these may not be true. For example, the icicle shown in Figure 4.14 does not conform to several of the assumptions.

As has been discussed, the growth in length of the icicle is the most difficult factor to determine. The processes involved appear to be quite complex. The assumption of a single drop freezing into the shape of a cylinder is obviously a crude simplification of the actual process, but, observations support the cylindrical assumption in most cases. Using such an assumption, the maximum growth of an icicle can be predicted with some degree of accuracy.

As previously discussed, the theoretical model yields an upper limit to the growth of an icicle in both length and mass. The major factor responsible for retarding the growth below the theoretical limit is the water flow at the tip of the icicle. As mentioned earlier, the faster the driprate from the tip, the slower the length growth, except where the driprate is very low. (see Section 4.2) At this point the model can only predict the maximum possible icing rate. However, in many design problems, this is the factor of most significance.

In order to apply the model to individual cases, some functi[illegible] be introduced to account for the effect of the driprate from the [illegible]s function may be determined theoretically or experimentally [illegible] a function would produce an expression for the length growth rate as follows:

$$(dL/dt)_{actual} = (dL/dt)_{max} \times F_{drip}(W_d,u,T) \qquad (5.1)$$

where

F_{drip} = function accounting for the driprate from the tip of the icicle.

The domain of F_{drip} would obviously be between zero and one and such a function would conceivably be a function of windspeed and temperature as well. Figure 5.1 shows a composite of the dl/dt versus driprate graphs for trials 18 through 21, all of which were carried out under the same conditions. From the figure it is seen that there is a maximum in dL/dt near W_d = 7 min^{-1}. But, due to the large amount of scatter, discerning a functional form for F_{drip} even for a single set of conditions is difficult.

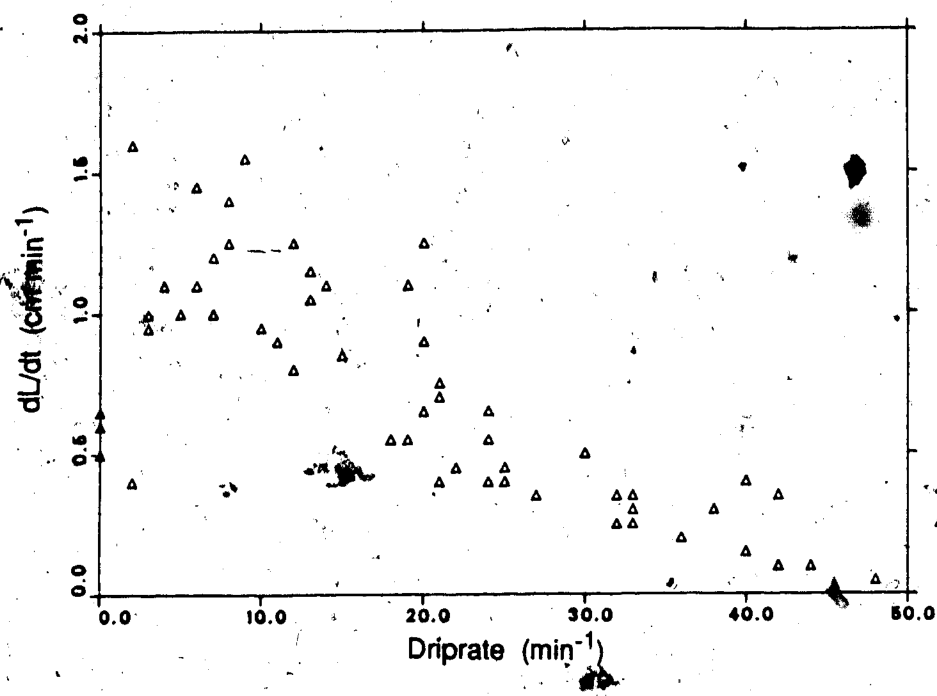


Figure 5.1: Composite dL/dt versus driprate from tip for trials 18-21.

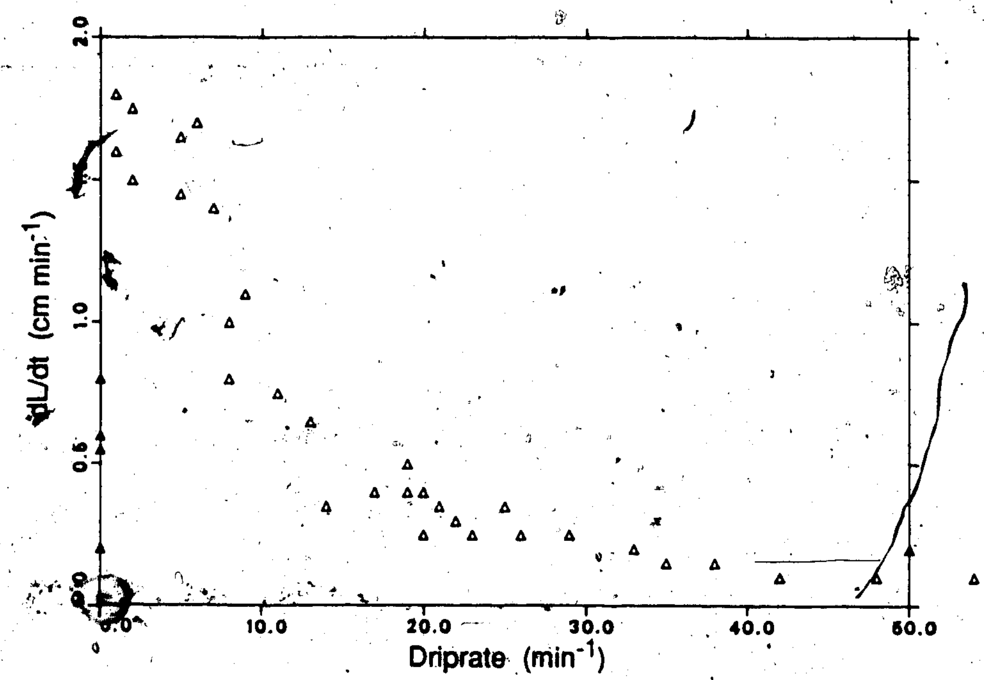


Figure 5.2: Composite dL/dt versus driprate from tip for trials 22-24.

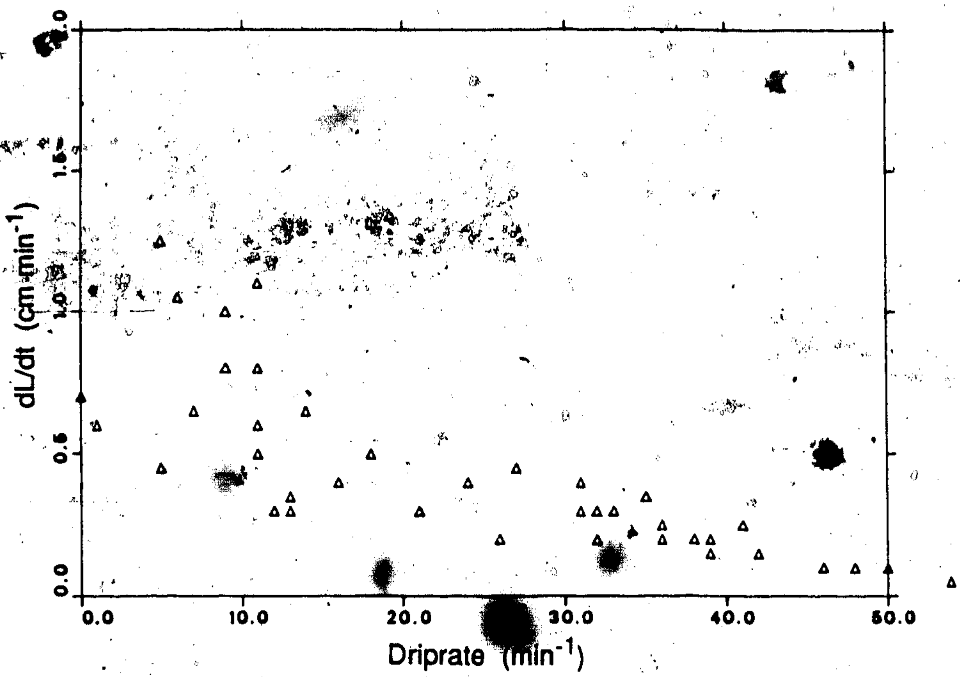


Figure 5.3: Composite dL/dt versus driprate from tip for trials 26-27.

Figure 5.2 displays the same relation for trials 22 to 24. In this case the maximum appears nearer to 5 min^{-1} although the scatter is still evident. Figure 5.3 contains the points for trials 26 and 27. It must be kept in mind that there are large error bars associated with these graphs. Each of the two values which are differenced possess errors of ± 0.1 cm so the dL/dt values are expected to be accurate to within ± 0.2 cm. For any set of conditions, a curve could be fit to the experimental results to deduce a relation for F_{drip}. This will not be done here due to the limited usefulness of such a result. Figures 5.1 and 5.2 suggest that this relation is indeed a function of temperature and or windspeed. However, there is insufficient data available to attempt to determine F_{drip} as a function of the surrounding parameters.

5.1.3 Summary

The icicle is one of many natural phenomena which, initially, appear to be a simple process. This is certainly not the case. Many aspects of icicle growth are not fully understood. The model derived in this presentataion should provide some insight into the growth of icicles and their contribution to certain icing problems. Also, the model provides information on how certain factors affect the growth of an icicle. In this way, the result of controlling one or more of these factors can be determined. The author hopes that this model will provide some groundwork for future investigation of icicles and their growth in icing situations.

5.2 Further Research

There are three major areas where further research could refine the icicle model. These are the tip process, the freezing fraction and the drip function, F_{drip}. Simplifying assumptions have been made in the case of the tip process which very likely affect the conclusions made regarding the other two.

The heat transfer from the tip of the icicle was assumed to take place from a sphere. A sphere was chosen over which to calculate the heat transfer at the tip as discussed in Section 2.3. Further study of this process might lead to a more precise representation of the heat transfer from these drops. Careful studies could be performed to determine the average surface area of the pendant drop. Possibly some multiple of the spherical area might be more accurate.

The freezing fraction and the properties of the ice could also be investigated further. A freezing fraction of 0.10 was obtained through experiment but little theoretical evidence was provided to support this choice. An optical technique could possibly be developed to carefully observe the tip section of the icicle, as it grows. In this way, the properties of the tip may be better understood. Past research has concluded that the ice freezes as dendritic needles, but whether their alignment is parallel to the surface or random is not known. Studying this property is very difficult as it must be accomplished while the icicle grows. Once the growth ceases, all the water freezes the ice-liquid characteristics are lost. A better understanding of the tip properties would enhance the knowledge of icicle growth.

From the experiments carried out, a qualitative relationship between the length growth rate and the driprate from the tip, W_d has been shown. (see figure 4.4) However, no quantitative relation can be found due to the limited availability of experimental data. Extensive experiments could lead to a quantitative empirical relation, F_{drip}, between the driprate and the elongation rate.

Experimentally, the apparatus could be refined to yield more accurate results. An optical or photographic method could be used to measure the length and diameter of the icicle as it grows. Also, a more sophisticated valve or pump would make more accurate regulation of the supply rate possible. These factors would make the experimental results considerably more precise.

In order to utilize more accurate experimental results, the theory would have to be more accurate. The large uncertainties in the heat transfer correlations make such a refinement difficult. Possibly, more accurate could be found which are applicable only to the small range of conditions in which icicles grow. A number of experimental modifications are possible but, unless the theory possesses equal precision, such results will be of little value.

5.3 Extensions of the Model

The icicle model could be extended to include several other factors. These include the effects of airborne spray and salinity. Neither of these factors will be discussed in detail as no experiments were performed to support any theories.

Airborne spray would conceivably have a significant effect on the growth of an icicle, if wet growth was occurring. In most cases, spray is supercooled so an extra term would have to be included in the heat balance. Also, the spray would increase the driprate from the tip, decreasing the growth rate. In the case of dry growth there are two possibilities: Either the growth could remain dry and icing could occur on the existing icicle as if it were a cylinder; or, the spray could be sufficient to produce wet growth an the icicle could elongate as well.

The author has very little knowledge of the properties of salt water so this extension will onlt be mentioned briefly. Presumably, the icicle model could be extended to this case by replacing all properties with those for salt water. However, many factors, including the freezing fraction and the pendant drop size, may be different for salt water. Little, if any, study has been done on marine icicles and this would provide an interesting topic for further observations and research.

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APPENDIX A

The following FORTRAN program computes the time required for one drop to freeze, then, uses this timestep to grow the icicle in length and diameter by growing the initial or root segment. The length growth rate, dL/dt, is also computed from this timestep.

The list of variables and their corresponding program variables is included.

Variable	Symbol	Program Variable
Air Temperature	T_a	TA
Windspeed	u	U
Air density	ρ_a	RHO
Relative humidity	r	R
Pressure	p	P
Freezing fraction	F	F
Kinematic Viscosity	υ	GNU
Dynamic Viscosity	μ	MU
Diffusivity	D_{wa}	DWA
Thermal conductivity	k	K
Prandtl number	Pr	PR
Schmidt number	Sc	SC
Reynolds number	Re	RE
Pendant drop diameter	D_o	DO
Pendant drop mass	m_o	MO
Initial cylinder diameter	L_o	LO
Cylindrical slice diameters	D(i)	D(I)
Cylindrical slice volume	V	V
Heat transfer coefficient	h	H
Mass transfer coefficient	h_d	HD
Evaporative heating term	q_{es}	QES

Variable	Symbol	Program Variable
Convective heating term	q_c	QC
Timestep	Δt	DELTAT
Mass frozen in Δt	Δm	DELTAM
Surrounding vapour pressure	e_a	EA
Length growth rate	dL/dt	DLDT
Icicle area	A	A
Icicle mass	m	M

```
C       This program computes the length growth rate of an icicle and
C       proceeds to grow the icicle.  Surface area and mass are output as
C       functions of time while the diameter is output as a function of the
C       distance from the tip.
C
C
        REAL  TA,P,GNU,PR,U,DO,RE,H,QES,QC,DELTAT,MO,HD,R
        REAL  D(100),EA,V,DELTAM,F,A,M,SC,DWA,K,MU,RHO,DLDT
C
C       Input environmental parameters as well as freezing fraction
C
        READ(2,*) TA,P,U,F,R
C
        MU = 1.72E-5 + 5.E-8*TA
        DWA = 2.13E-5 + 1.4E-7*TA
        K = 0.0243 + 7.5E-5*TA
        RHO = P/(287.*(TA + 273.))
        GNU = MU/GHO
        PR = 0.71
        SC = GNU/DWA
        DO = 0.005
        LO = 0.00453
        MO = 6.54E-5
        EA = 611.*EXP(19.9-5420./(TA + 273.))
        RE = U*DO/GNU
```

```
      H = K/DO*(2. + 0.6*PR**0.3333*RE**0.5)
      HD = DWA/DO*(2. +0.6*SC**0.3333*RE**0.5)
      QES = 17030./(TA + 273.)*DO*DO*HD*(R*EA - 611.)
      QC = 3.14*DO*DO*H*TA
      DELTAT = -MO*F*3.34E5/(QES + QC)
      DLDT = LO/DELTAT*6000.
      WRITE(7,13) DLDT
      V = 3.14*D0**3.*0.1852
C
C     Grow root slice for 100 (or any number of) timesteps
C
      DO 99  I = 1,100
         D(I) = (1.272*V/LO)**0.5
         RE = U*D(I)/GNU
         H = K/D(I)*0.6*RE**0.5*PR**0.3333
         HD = DWA/D(I)*0.6*RE**0.5*SC**0.3333
         QES = 17030.*LO*D(I)*HD/(TA + 273.)*(R*EA - 611.)
         QC = 3.14*LO*D(I)*H*TA
         DELTAM = -(QES + QC)*DELTAT*3.E-6
 99      V = V + 1.1111E-3*DELTAM
      A = 0.0
      M = 0.0
      DO 100  I = 1,100
         A = A + 3.14*D(I)*LO
         M = M + 3.14*D(I)*D(I)*DO*900.
         WRITE(8,13)  I*DELTAT/60. , 10000.*A
         WRITE(9,13)  I*DELTAT/60. , 1000.*M
 100     WRITE(7,13)  100.*I*DO , 100.*D(I)
 13   FORMAT(3F12.5)
      STOP
      END
```

<u>Appendix B</u>

All experimental data follows in this appendix. The first page contains the results of several tests conducted to determine the size of the pendant drops falling from the tip of the icicle. The 27 experimental trials follow. Trials 1 to 17 are described by Procedure 1 in Section 3.3 while Trials 18 to 25 are trials performed using procedure 2.

<u>Pendant Drop Size Experiments</u>

	Number of Drops Collected	Mass (g)
	30	1.9
	30	2.0
	40	2.9
	40	2.7
	50	3.1
	50	2.9
	30	1.9
	30	2.1
	30	1.9
	40	2.5
Totals	370	23.9

Average drop mass: 0.0646 g

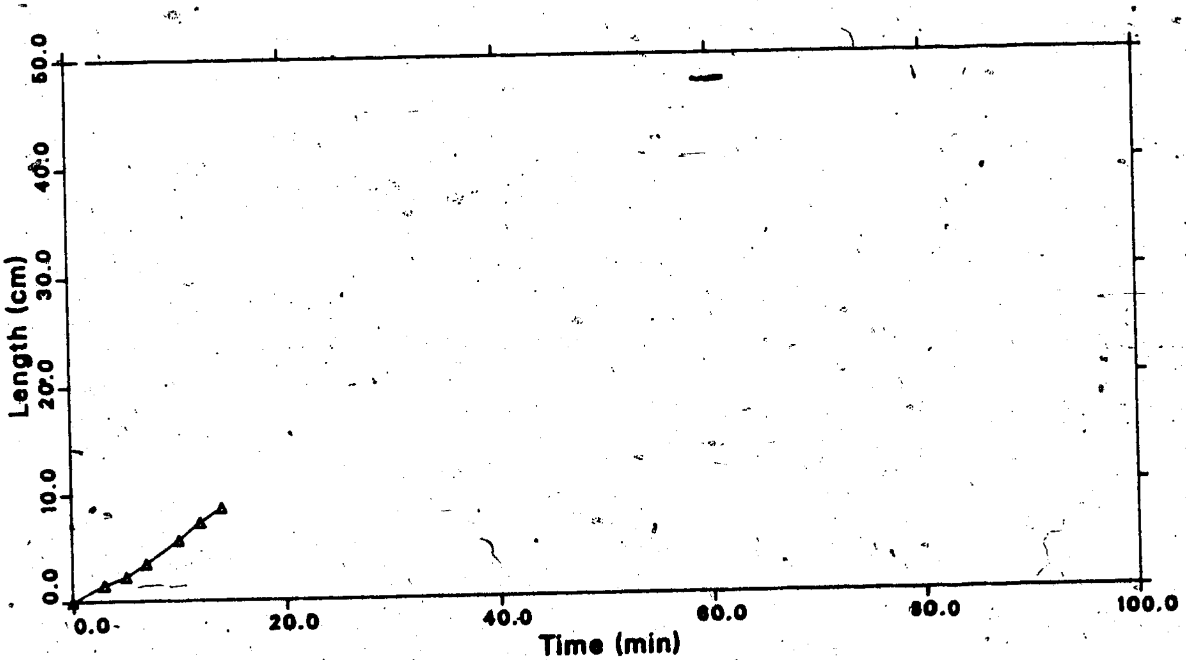
Corresponding diameter: 0.498 cm

Trial: 1 (07-05-01)

Temperature: -13.5 °C
Windspeed: 2.3 ms^{-1}
Pressure: 94.9 kPa

Time (min)	Length (cm)
0	0.0
3	1.5
5	2.3
7	3.5
10	5.7
12	7.3
14	8.7

Average growth rate: 0.65 cm min^{-1}
Maximum growth rate: 0.80 cm min^{-1}

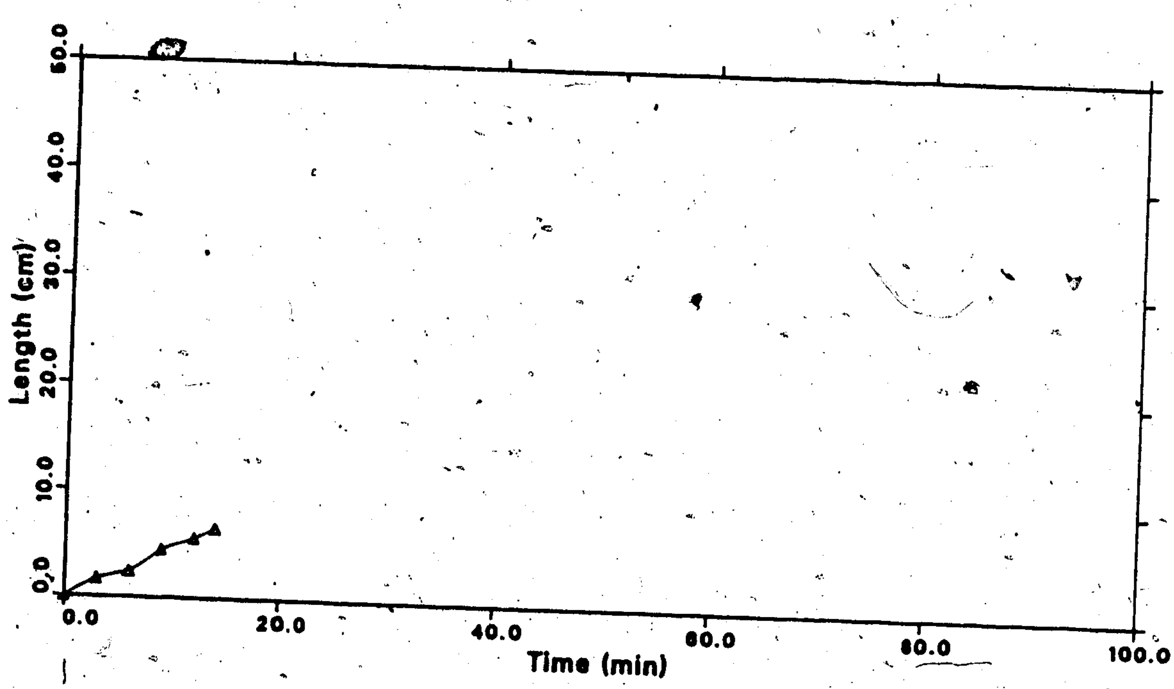


Trial: 2 (07-05-02)

Temperature:	-13.0 °C
Windspeed:	2.5 ms^{-1}
Pressure:	94.9 kPa

Time (min)	Length (cm)
0	0.0
3	1.7
6	2.4
9	4.5
12	5.6
14	6.5

Average growth rate:	0.45 cm min^{-1}
Maximum growth rate:	0.67 cm min^{-1}



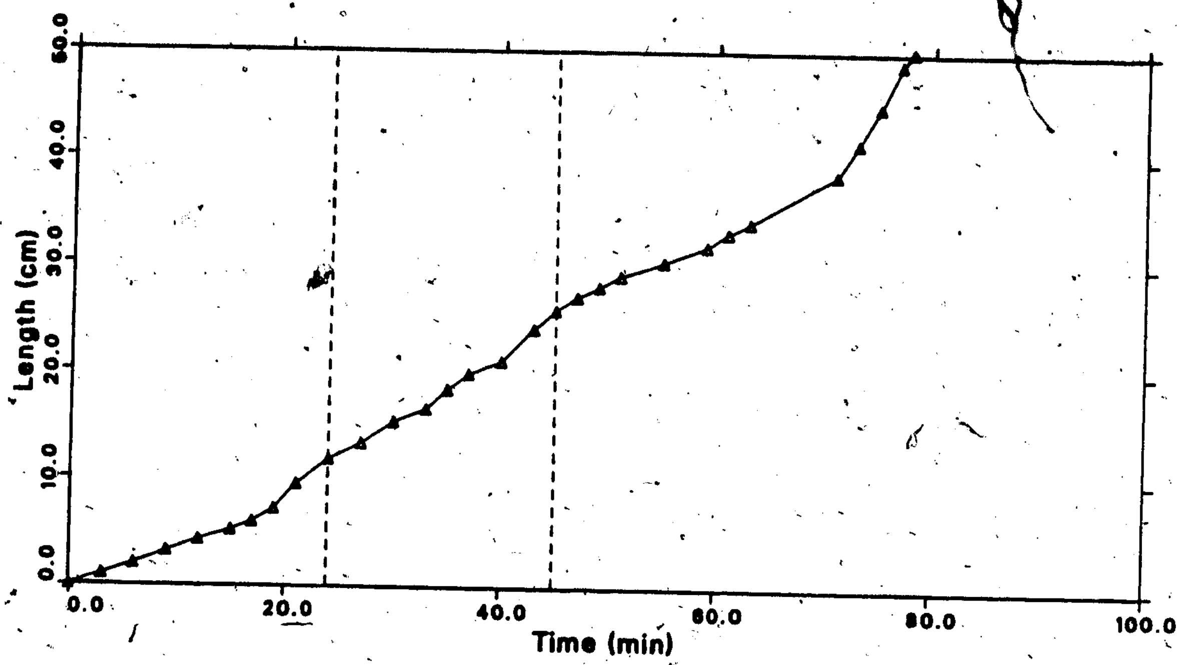
Trial: 3 (07-05-04)

Temperature: -17.5 °C
Windspeed: 1.3 ms^{-1}
Pressure: 93.8 kPa

Time (min)	Length (cm)	Time (min)	Length (cm)
0	0.0	40	21.0
3	1.0	43	24.0
6	2.0	45	25.8
9	3.2	47	27.0
12	4.3	49	28.0
15	5.2	51	29.0
17	6.0	55	30.3
19	7.2	59	31.8
21	9.5	61	33.0
24	11.8	63	34.0
27	13.3	71	38.5
30	15.3	73	41.5
33	16.5	75	45.0
35	18.3	77	49.0
37	19.8	78	50.2

Average Growth Rate: 0.64 cm min^{-1}
Maximum growth rate: 2.0 cm min^{-1}

Trial: 3 (continued)

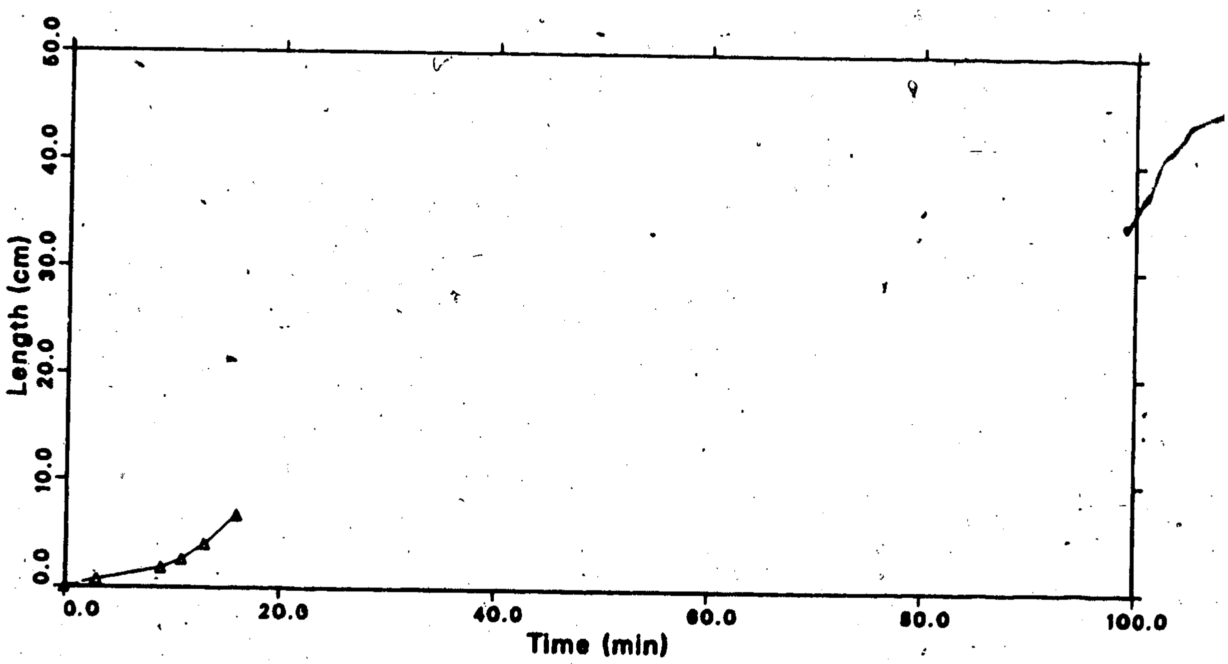


Trial: 4 (08-05-01)

Temperature: -14.0 °C
Windspeed: 2.8 ms^{-1}
Pressure: 93.7 kPa

Time (min)	Length (cm)
0	0.0
3	0.7
9	1.8
11	2.6
13	4.0
16	6.7

Average growth rate: 0.42 cm min^{-1}
Maximum growth rate: 0.90 cm min^{-1}

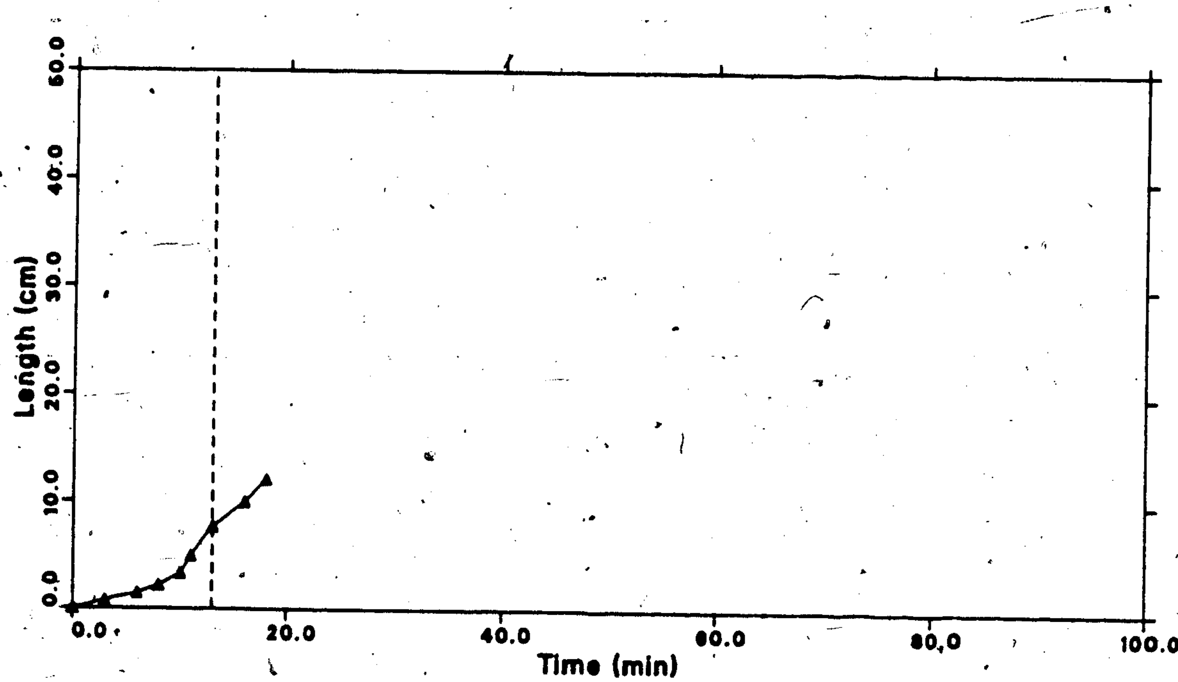


Trial: 5 (08-05-02)

Temperature: -13.0 °C
Windspeed: 2.7 ms^{-1}
Pressure: 93.7 kPa

Time (min)	Length (cm)
0	0.0
3	0.8
6	1.5
8	2.2
10	3.4
11	5.0
13	7.7
16	10.0
18	12.1

Average growth rate: 0.67 cm min^{-1}
Maximum growth rate: 1.60 cm min^{-1}

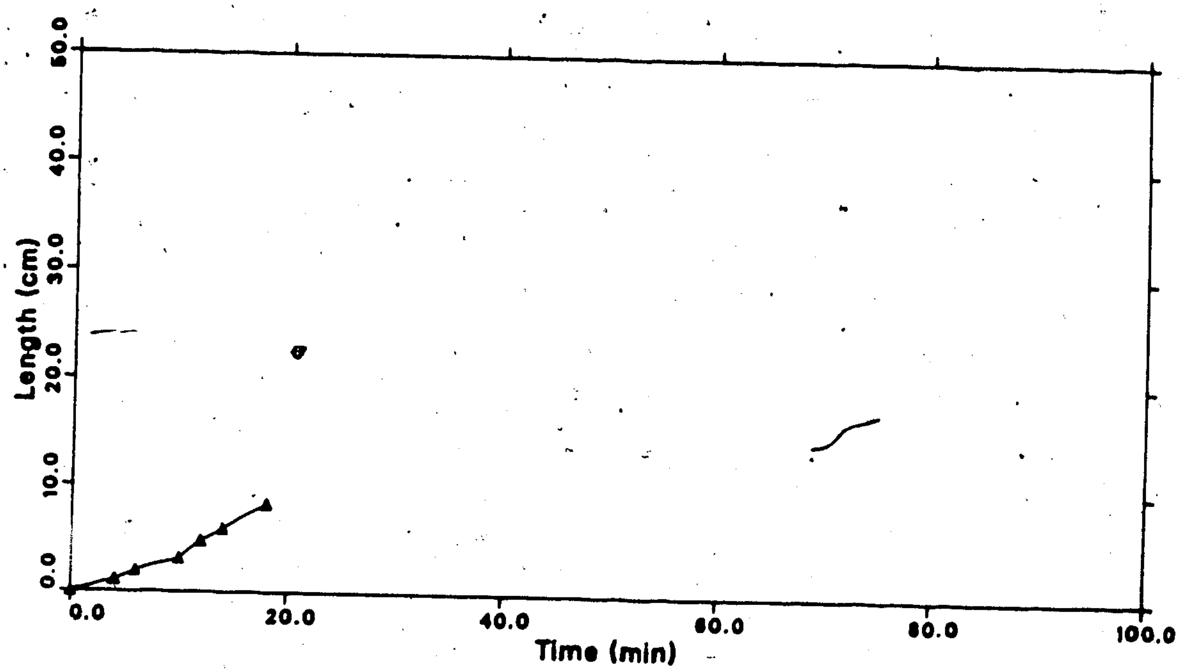


Trial: 6 (08-05-03)

Temperature: -11.0 °C
Windspeed: 2.7 ms^{-1}
Pressure: 94.9 kPa

Time (min)	Length (cm)
0	0.0
4	1.1
6	2.0
10	3.2
12	4.8
14	6.0
18	8.3

Average growth rate: 0.46 cm min^{-1}
Maximum growth rate: 0.80 cm min^{-1}



Trial: 7 (14-05-01)

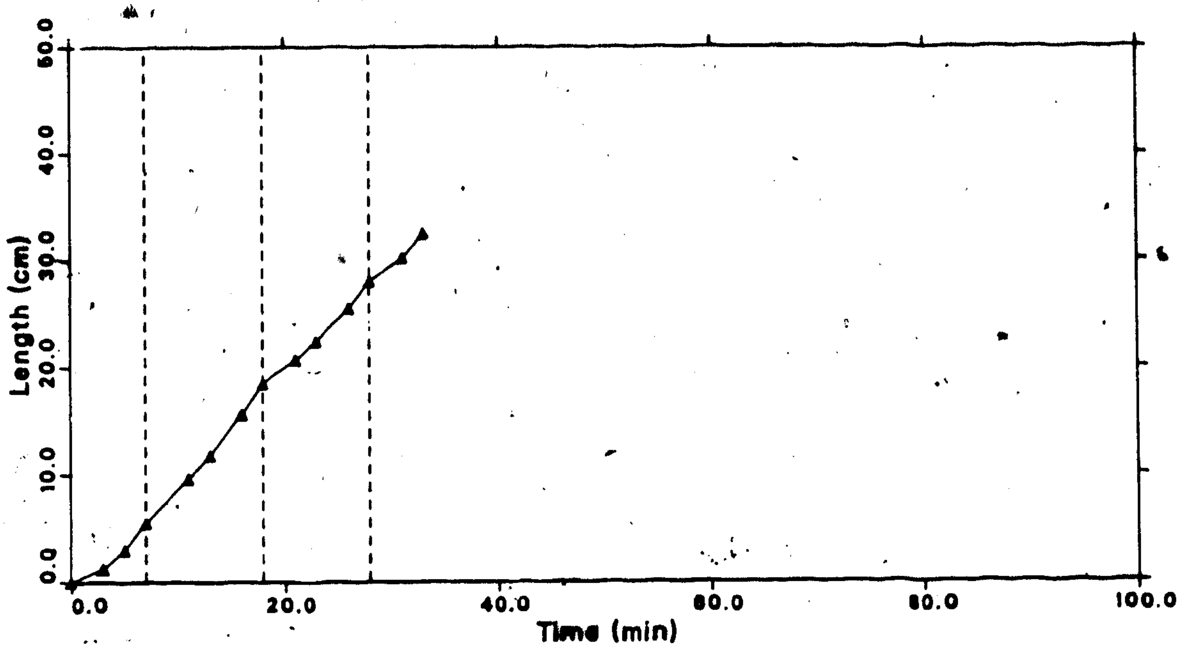
Temperature: -14.0 °C
Windspeed: 3.2 ms^{-1}
Pressure: 92.9 kPa

Time (min)	Length (cm)	Time (min)	Length (cm)
0	0.0	18	18.5
3	1.2	21	20.7
5	3.0	2[illegible]	22.4
7	5.5	26	25.5
11	9.6	28	28.0
13	11.8	31	30.2
16	15.6	33	32.5

Average growth rate: 0.98 cm min^{-1}
Maximum growth rate: 1.30 cm min^{-1}

Distance from Tip (cm)	Minimum Diameter (cm)	Maximum Diameter (cm)
0	0.47	0.47
3	0.42	0.51
6	0.58	0.62
9	0.68	0.78
12	0.78	0.93
15	0.95	1.10
18	0.98	1.32
21	1.12	1.28
24	1.34	1.41
27	1.36	1.40
30	1.51	1.63

Trial: 7 (continued)



Trial: 8 (15-05-01)

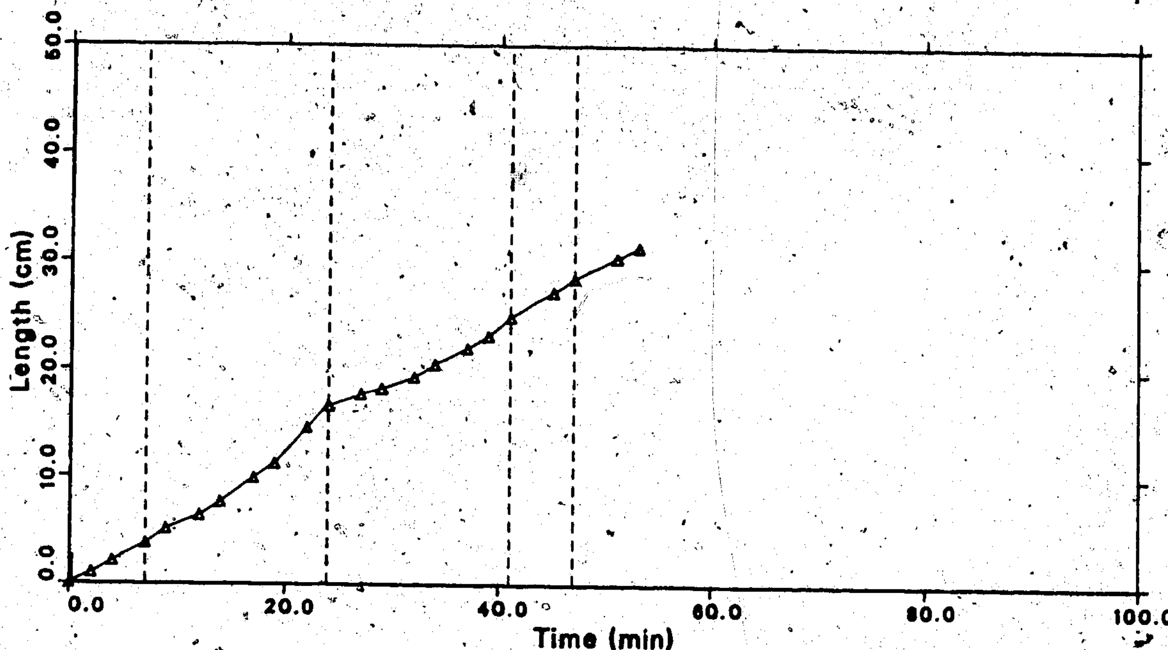
Temperature: - 8.0 °C
Windspeed: 3.4 ms^{-1}
Pressure: 93.6 kPa

Time (min)	Length (cm)	Time (min)	Length (cm)
0	0.0	27	17.7
2	1.0	29	18.2
4	2.1	32	19.3
7 *	3.8	34	20.5
9	5.1	37	22.0
12	6.4	39	23.1
14	7.6	41 *	24.9
17	9.9	45	27.2
19	11.2	47 *	28.5
22	14.6	51	30.4
24 *	16.6	53	31.4

Average growth rate: 0.59 cm min^{-1}
Maximum growth rate: 1.10 cm min^{-1}

Distance from Tip (cm)	Minimum Diameter (cm)	Maximum Diameter (cm)
0	0.39	0.45
3	0.57	0.67
6	0.64	0.73
9	0.72	0.79
12	0.82	0.90
15	1.02	1.33
18	1.04	1.35
21	1.19	1.54
24	1.10	1.57
27	1.23	1.66
30	1.16	1.88

Trial: 8 (continued)



Trial: 9 (15-05-02)

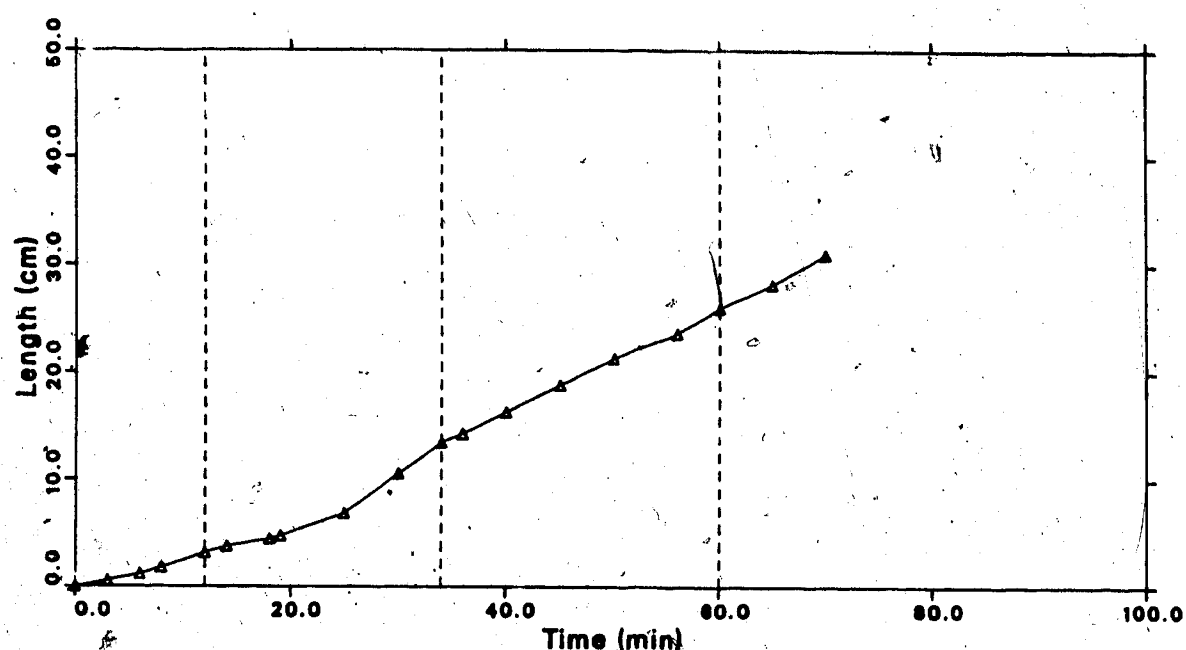
Temperature: - 8.0 °C
Windspeed: 2.5 ms^{-1}
Pressure: 93.6 kPa

Time (min)	Length (cm)	Time (min)	Length (cm)
0	0.0	34 *	13.5
3	0.6	36	14.3
6	1.2	40	16.3
8	1.8	45	18.7
12 *	3.2	50	21.3
14	3.8	56	23.6
18	4.5	60 *	26.0
19	4.9	65	28.2
25	6.9	70	31.0
30	10.6		

Average growth rate: 0.44 cm min^{-1}
Maximum growth rate: 0.60 cm min^{-1}

Distance from Tip (cm)	Minimum Diameter (cm)	Maximum Diameter (cm)
0	0.43	0.43
3	0.40	0.46
6	0.62	0.65
9	0.65	0.85
12	0.65	1.52
15	0.88	1.08
18	1.01	1.13
21	1.13	1.19
24	1.02	1.28
27	1.14	1.46

Trial: 9 (continued)



Trial: 10 (25-05-01)

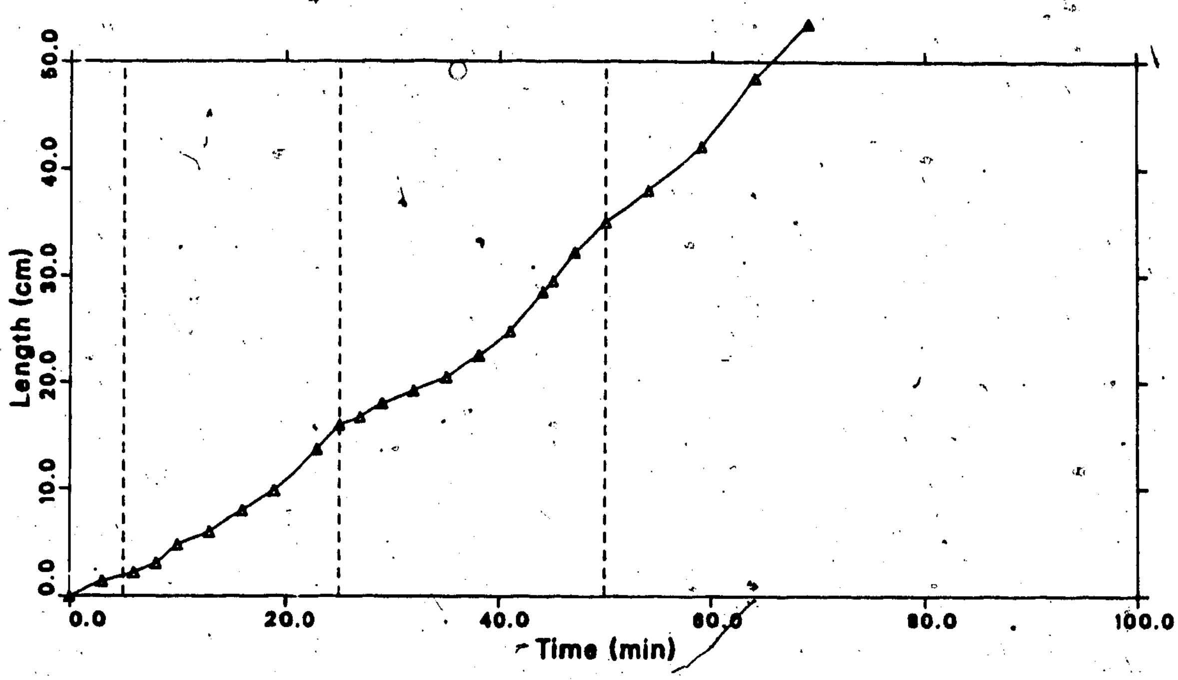
Temperature: -12.0 °C
Windspeed: 3.5 ms^{-1}
Pressure: 93.2 kPa

Time (min)	Length (cm)	Time (min)	Length (cm)
0	0.0	32	19.2
2	1.4	35	20.5
5 *	3.2	38	22.5
8	4.1	41	24.8
10	4.8	44	28.2
13	6.0	45	29.5
16	8.0	47	32.2
19	9.7	50 *	35.0
23	13.7	54	38.0
25 *	16.0	59	42.1
27	16.7	64	46.0
29	18.0	69	51.0

Average growth rate: 0.74 cm min^{-1}
Maximum growth rate: 1.30 cm min^{-1}

Distance from Tip (cm)	Minimum Diameter (cm)	Maximum Diameter (cm)
0	0.29	0.45
5	0.58	0.63
10	0.73	0.84
15	0.87	1.21
20	1.27	1.27
25	1.30	1.57
30	1.31	1.66

Trial: 10 (continued)

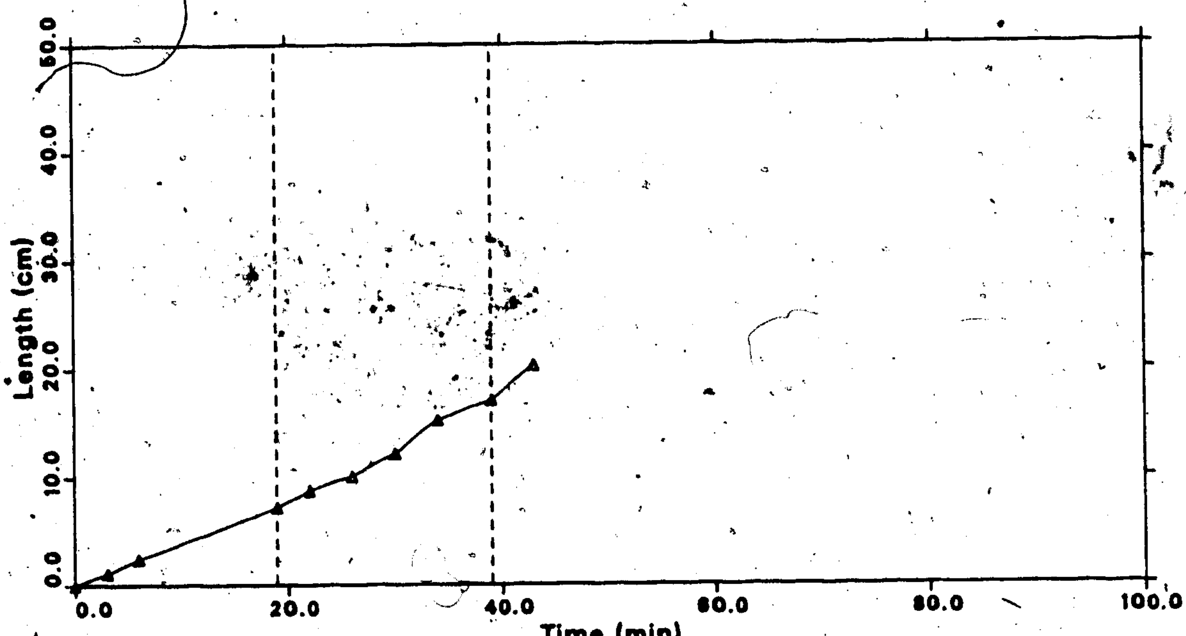


Trial: 11 (25-05-02)

Temperature: - 5.0 °C
Windspeed: 3.2 ms^{-1}
Pressure: 93.2 kPa

Time (min)	Length (cm)	Time (min)	Length (cm)
0	0.0	26	10.1
3	1.1	30	12.1
6	2.4	34	15.3
19 *	7.2	39 *	17.1
22	8.8	43	20.3

Average growth rate: 0.47 cm min^{-1}
Maximum growth rate: 0.80 cm min^{-1}

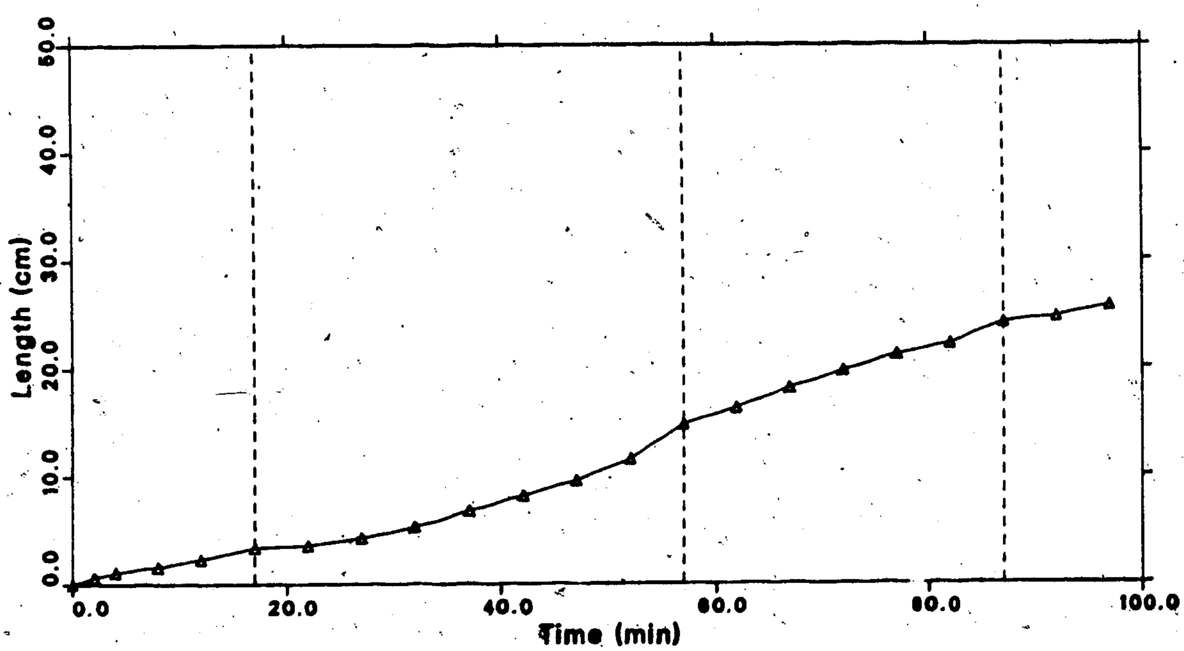


Trial: 12 (27-05-01)

Temperature: - 6.0 °C
Windspeed: 3.1 ms^{-1}
Pressure: 93.1 kPa

Time (min)	Length (cm)	Time (min)	Length (cm)
0	0.0	52	11.6
2	0.6	57 *	14.8
4	1.1	62	16.3
8	1.6	67	18.1
12	2.3	72	19.7
17 *	3.4	77	21.2
22	3.4	82	22.2
27	4.3	87 *	24.2
32	5.3	92	24.7
37	6.8	97	25.7
42	8.2	102	26.9
47	9.6		

Average growth rate: 0.26 cm min^{-1}
Maximum growth rate: 0.64 cm min^{-1}

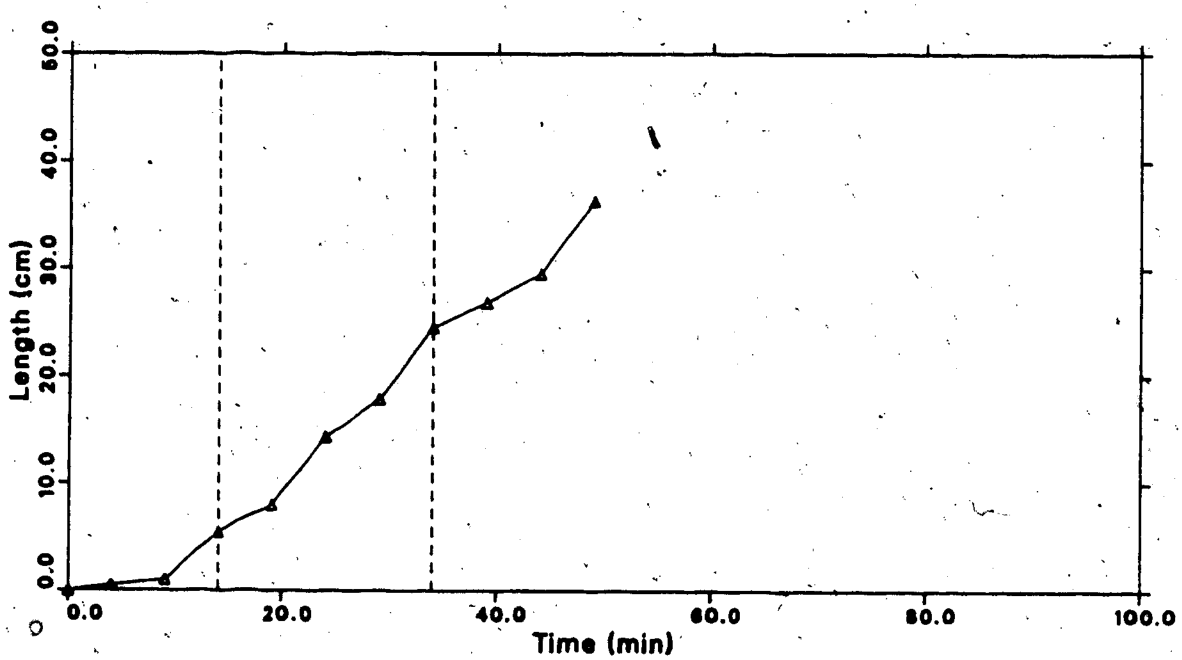


Trial: 13 (28-05-01)

Temperature: -15.0 °C
Windspeed: 2.7 ms^{-1}
Pressure: 93.0 kPa

Time (min)	Length (cm)	Time (min)	Length (cm)
0	0.0	29	17.8
4	0.5	34 *	24.5
9	1.0	39	26.8
14 *	5.5	44	29.5
19	8.0	49	36.3
24	14.3		

Average growth rate: 0.74 cm min^{-1}
Maximum growth rate: 1.30 cm min^{-1}



Trial: 14 (28-05-02)

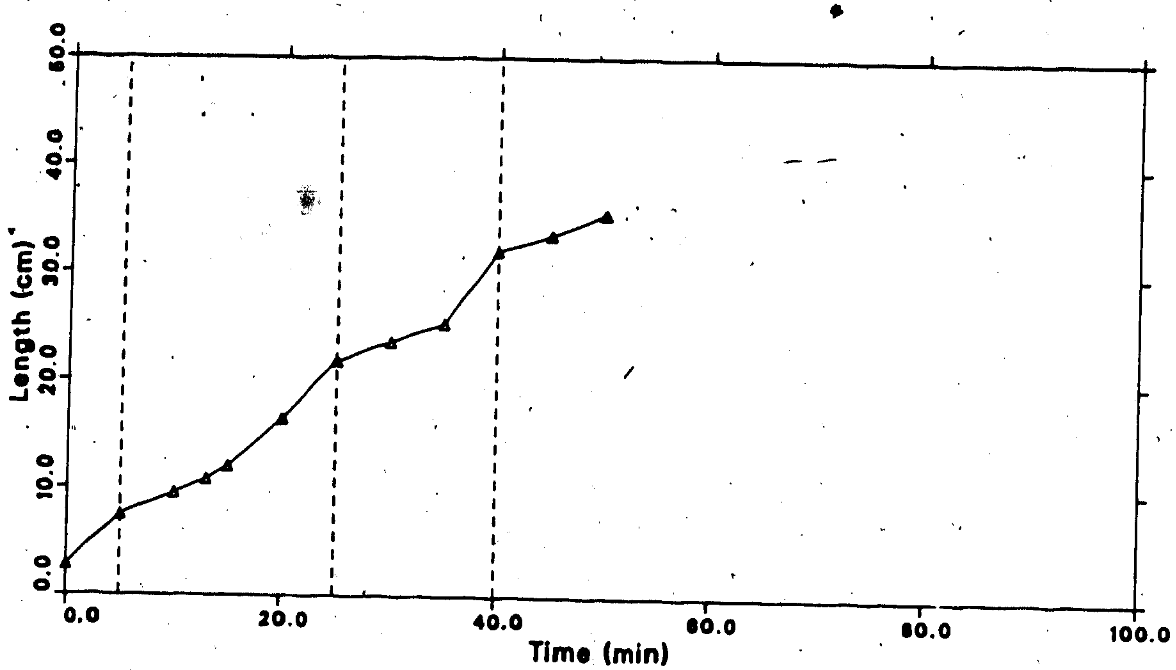
Temperature: -12.0 °C
Windspeed: 2.8 ms^{-1}
Pressure: 93.0 kPa

Time (min)	Length (cm)	Time (min)	Length (cm)
0	0.0	25 *	19.0
5 *	4.7	30	20.8
10	6.7	35	22.5
13	8.0	40 *	29.2
15	9.2	45	30.7
20	13.6	50	32.7

Average growth rate: 0.65 cm min^{-1}
Maximum growth rate: 1.30 cm min^{-1}

Distance from Tip (cm)	Minimum Diameter (cm)	Maximum Diameter (cm)
0	0.43	0.47
3	0.71	0.73
6	0.72	0.82
9	0.93	1.10
12	1.10	1.30
15	1.12	1.40
18	1.18	1.37
21	1.15	1.37
24	0.96	1.74
27	1.08	1.98

Trial: 14 (continued)

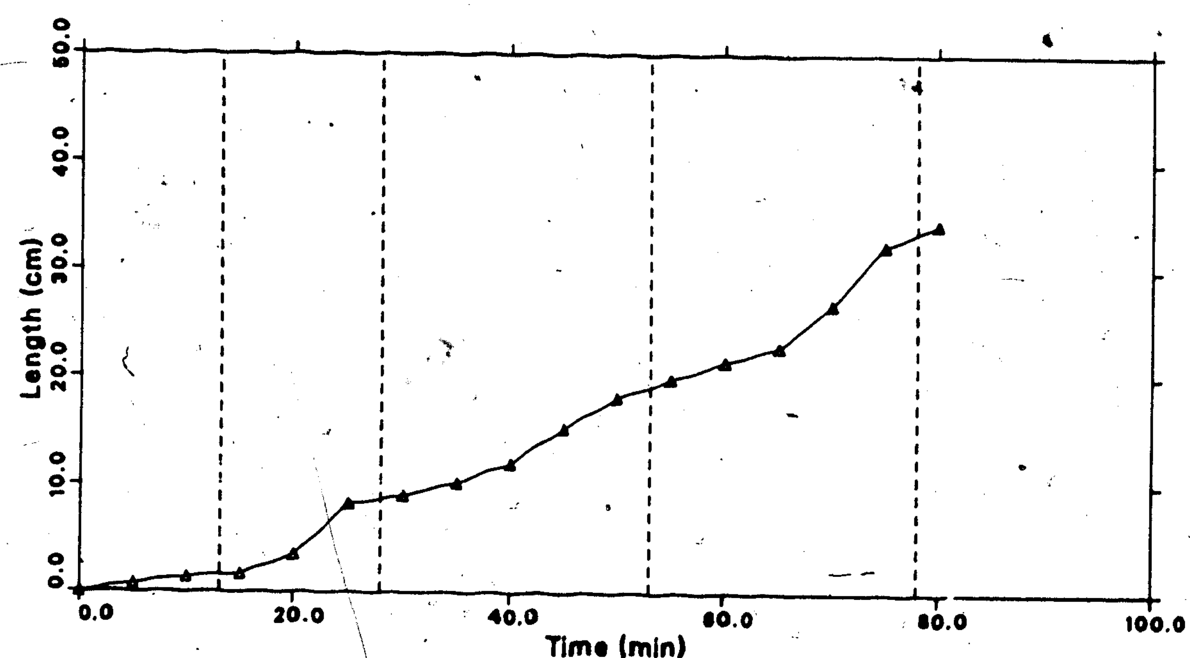


Trial: 15 (28-05-03)

Temperature: - 8.5 °C
Windspeed: 2.7 ms^{-1}
Pressure: 93.0 kPa

Time (min)	Length (cm)	Time (min)	Length (cm)
0	0.0	43	12.2
3	0.2	48	15.5
8	1.0	53 *	18.3
13 *	1.6	58	20.0
18	1.9	63	21.6
23	3.7	68	24.0
28 *	8.4	73	27.0
33	9.2	78 *	31.5
38	10.4	83	34.5

Average growth rate: 0.42 cm min^{-1}
Maximum growth rate: 0.94 cm min^{-1}

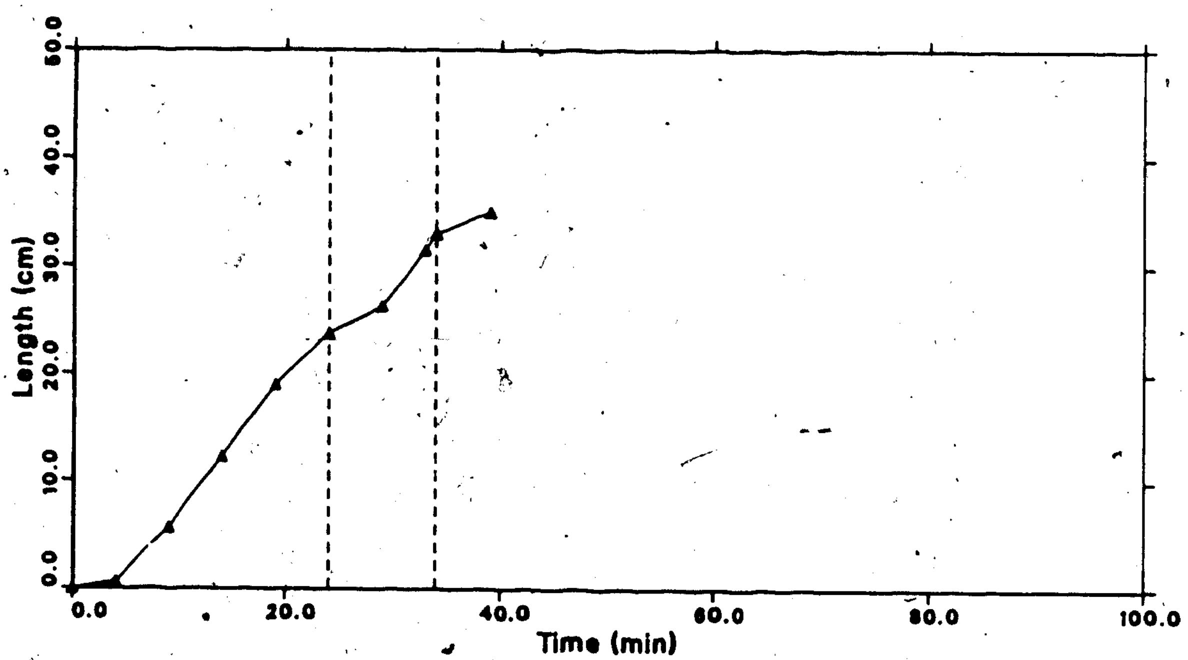


Trial: 16 (14-05-01)

Temperature: -16.0 °C
Windspeed: 2.1 ms^{-1}
Pressure: 92.9 kPa

Time (min)	Length (cm)	Time (min)	Length (cm)
0	0.0	24 *	23.8
4	0.6	29	26.3
9 *	5.7	33	31.5
14 *	12.3	34 *	33.0
19 *	19.0	39	35.0

Average growth rate: 0.90 cm min^{-1}
Maximum growth rate: 1.30 cm min^{-1}

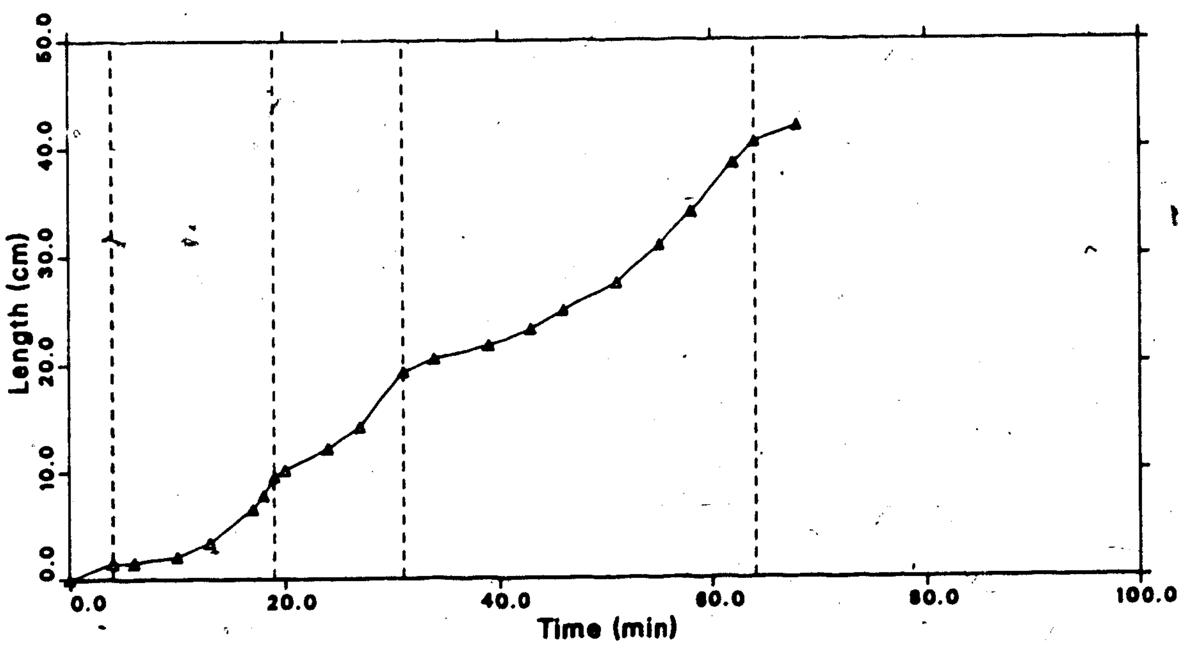


Trial: 17 (29-05-02)

Temperature: -10.5 °C
Windspeed: 2.1 ms^{-1}
Pressure: 92.9 kPa

Time (min)	Length (cm)	Time (min)	Length (cm)
0	0.0	31 *	19.3
4 *	1.5	35	20.5
6	1.5	39	21.7
10	2.1	43	23.2
13	3.4	47	25.0
17	6.5	51	27.5
18	7.8	55	31.0
19 *	9.6	58	34.0
20	10.2	62	38.5
24	12.2	64 *	40.5
27	14.2	68	42.0

Average growth rate: 0.62 cm min^{-1}
Maximum growth rate: 1.30 cm min^{-1}

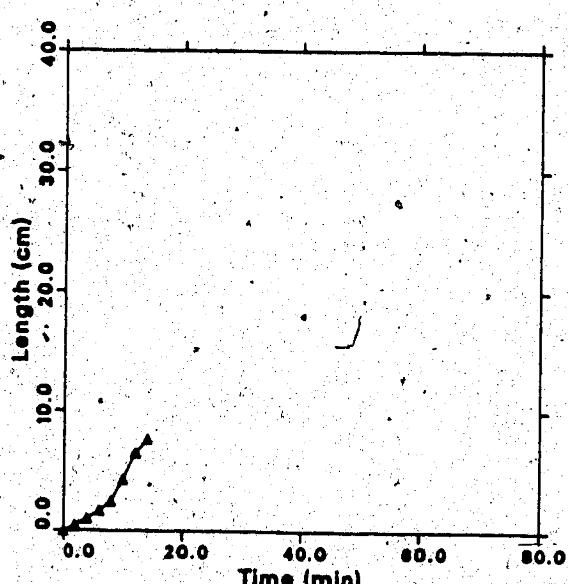


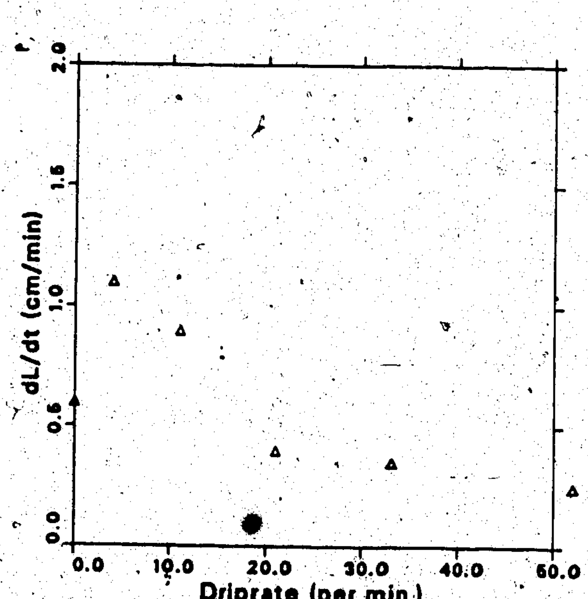
Trial: 18 (19-06-01)

Temperature: -13.0 °C
Windspeed: 2.9 ms^{-1}
Pressure: 93.6 kPa

Time (min)	Length (cm)	Driprate (min^{-1})
0	0.0	52
2	0.5	33
4	1.0	21
6	1.7	11
8	2.5	4
10	4.3	0
12	6.5	0
14	7.7	

Maximum growth rate: 1.10 cm min^{-1}





Trial: 19 (19-06-02)

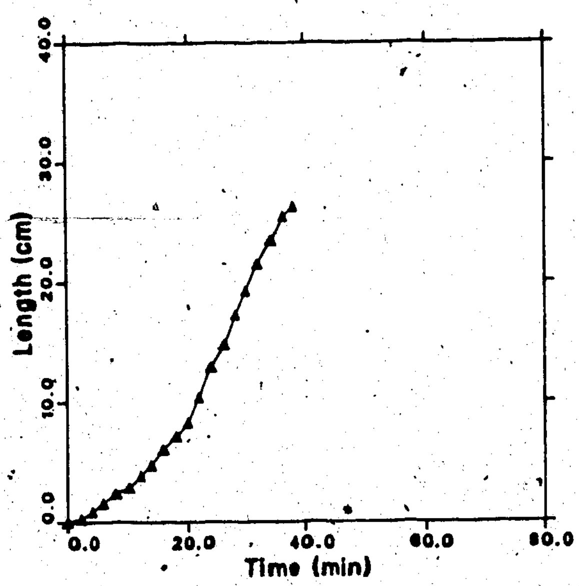
Temperature: -13.0 °C
Windspeed: 2.9 ms^{-1}
Pressure: 93.6 kPa

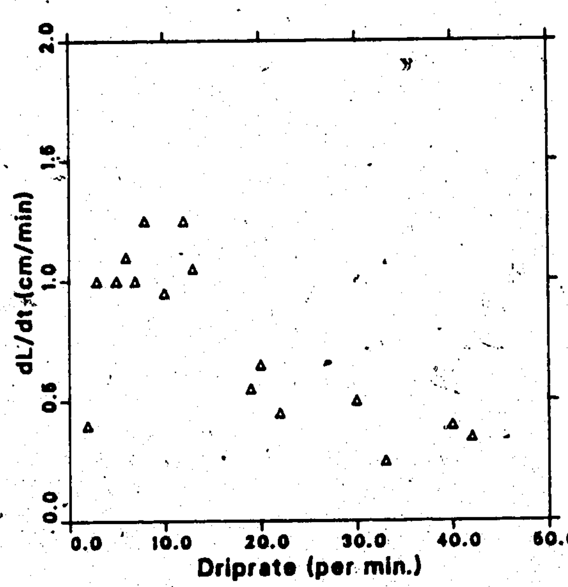
Time (min)	Length (cm)	Driprate (min^{-1})
0	0.0	76
2	0.3	68
4	0.9	42
6	1.6	40
8	2.4	33
10	2.9	30
12	3.9	22
14	4.8	20
16	6.1	19
18	7.2	13
20	8.3	12
22	10.4	10
24	12.9	8
26	14.8	7
28	17.3	6
30	19.3	5
32	21.5	3
34	23.5	2
36	25.5	0
38	26.3	

Maximum growth rate: 1.25 cm min^{-1}

Trial: 19 (continued)

Distance from Tip (cm)	Minimum Diameter (cm)	Maximum Diameter (cm)
0	0.45	0.52
3	0.45	0,70
6	0.54	0.59
9	0.66	0.86
12	0.63	0.96
15	0.60	1.06
18	0.70	1.13
21	1.01	1.38
24	0.77	1.60
27	1.15	2.06
30	1.35	2.09





Trial: 20 (20-06-01)

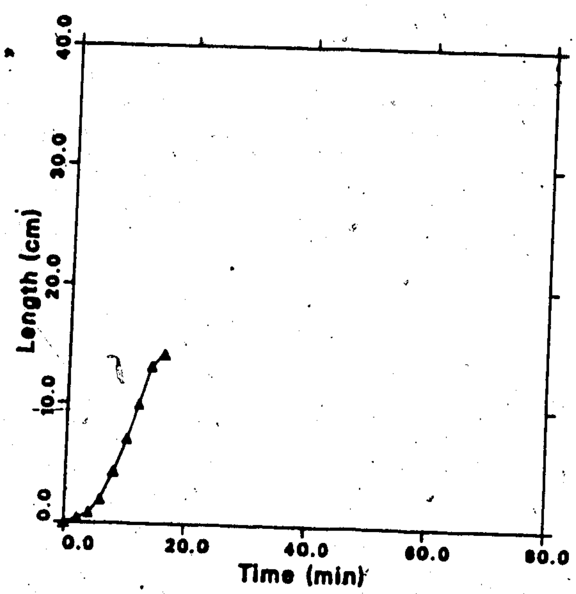
Temperature: -13.0 °C
Windspeed: 2.9 ms^{-1}
Pressure: 92.9 kPa

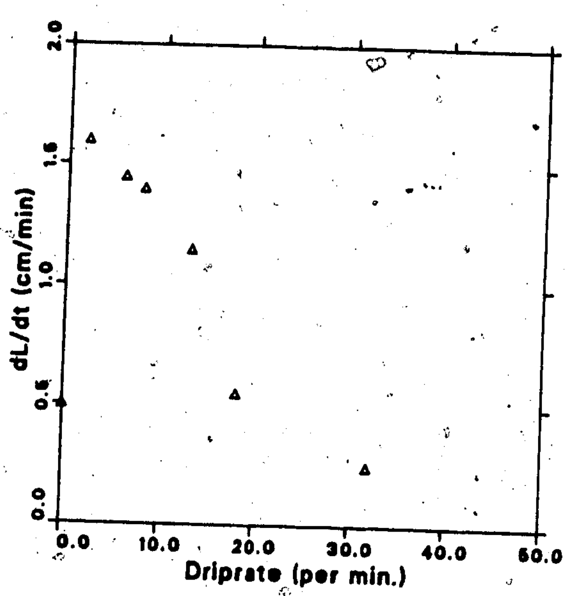
Time (min)	Length (cm)	Driprate (min^{-1})
0	0.0	66
2	0.4	32
4	0.9	18
6	2.0	13
8	4.3	8
10	7.1	6
12	10.0	2
14	13.2	0
16	14.2	

Maximum growth rate: 1.60 cm min^{-1}

Distance from Tip (cm)	Minimum Diameter (cm)	Maximum Diameter (cm)
0	0.42	0.53
3	0.33	0.47
6	0.44	0.61
9	0.55	0.66
12	0.59	0.69
15	0.65	1.02

Trial: 20 (continued)





Trial: 21 (20-06-02)

Temperature: -13.0 °C
Windspeed: 2.9 ms^{-1}
Pressure: 92.9 kPa

Time (min)	Length (cm)	Driprate (min^{-1})
0	0.0	86
2	0.2	72
4	0.5	60
6	0.7	48
8	0.8	44
10	1.0	42
12	1.2	42
14	1.4	42
16	1.6	40
18	1.9	36
20	2.3	32
22	2.8	30
24	3.8	27
26	4.5	25
28	5.3	25
30	6.2	24
32	7.0	38
34	7.6	33
36	8.2	32
38	8.9	24
40	10.0	24
42	11.3	21
44	12.8	21
46	13.6	21
48	15.0	20
50	16.8	20
52	19.3	19

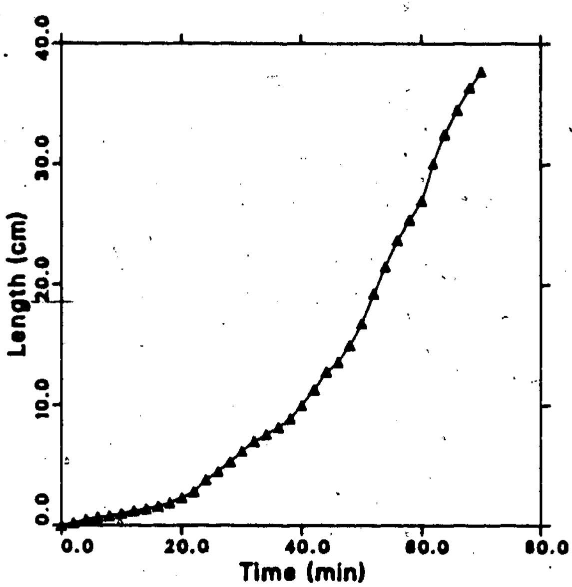
Trial: 21 (continued)

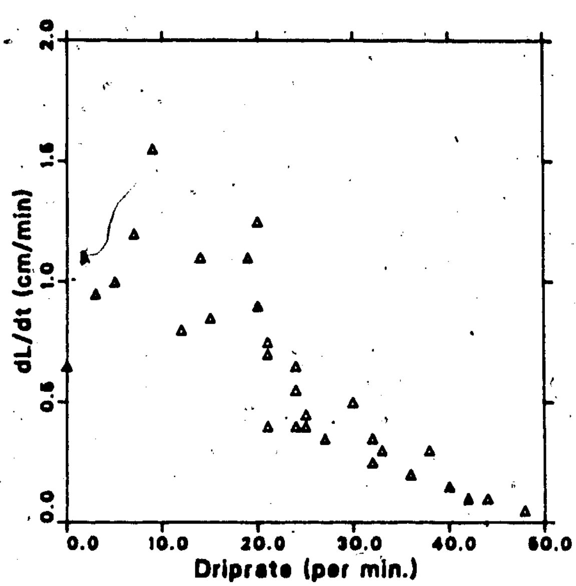
Time (min)	Length (cm)	Driprate (min^{-1})
54	21.5	14
56	23.7	15
58	25.4	12
60	27.0	9
62	30.1	7
64	32.5	5
66	34.5	3
68	36.4	0
70	37.7	

Maximum growth rate: 1.55 cm min^{-1}

Distance from Tip (cm)	Minimum Diameter (cm)	Maximum Diameter (cm)
0	0.40	0.41
3	0.36	0.41
6	0.47	0.60
9	0.63	0.72
12	0.58	0.91
15	0.67	1.07
18	0.83	1.20
21	0.83	1.30
24	1.26	1.46
27	1.02	1.53
30	1.00	1.90
33	1.14	2.24

Trial: 21 (continued)



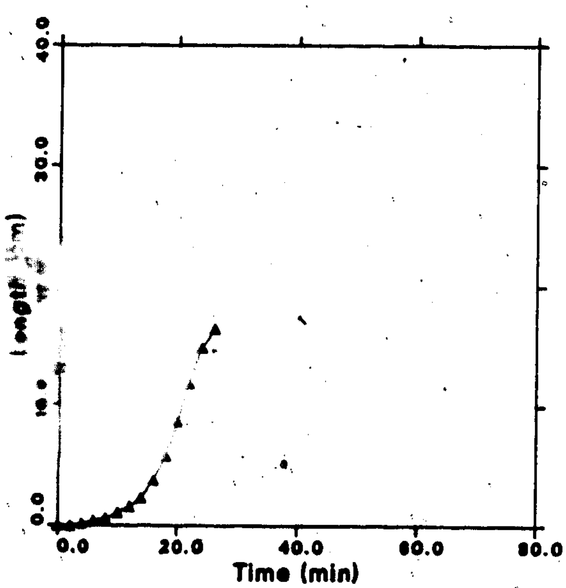


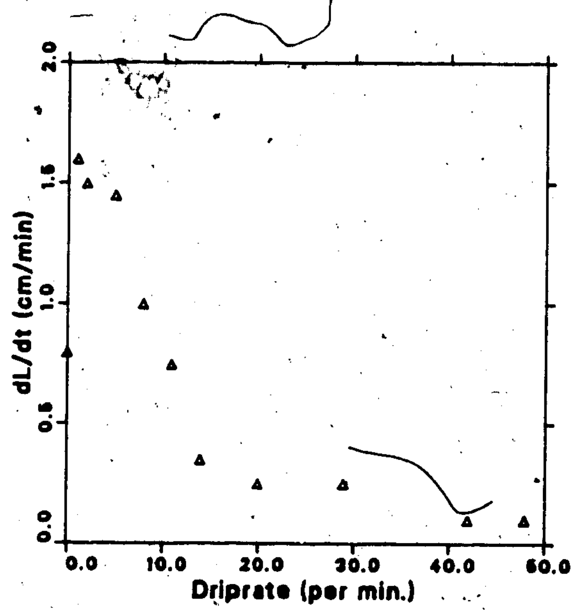
Trial: 22 (20-06-03)

Temperature: -15.5 °C
Windspeed: 1.9 ms^{-1}
Pressure: 92.9 kPa

Time (min)	Length (cm)	Driprate (min^{-1})
0	0.0	58
2	0.0	58
4	0.2	48
6	0.4	42
8	0.6	29
10	1.1	20
12	1.6	14
14	2.3	11
16	3.8	8
18	5.8	5
20	8.7	1
22	11.9	2
24	14.9	0
26	16.5	

Maximum growth rate: 1.60 cm min^{-1}



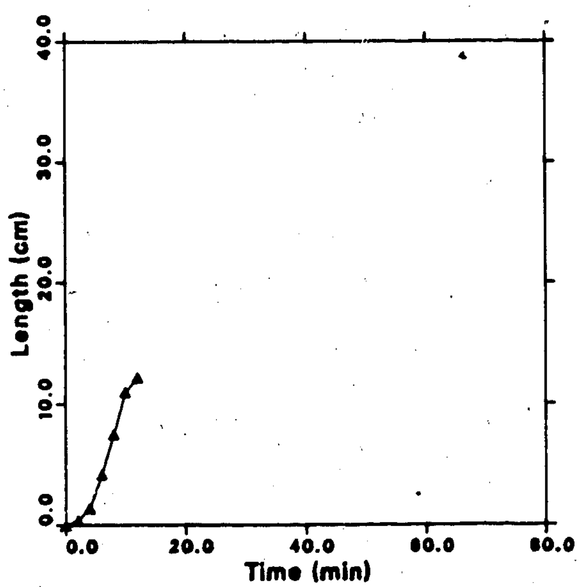


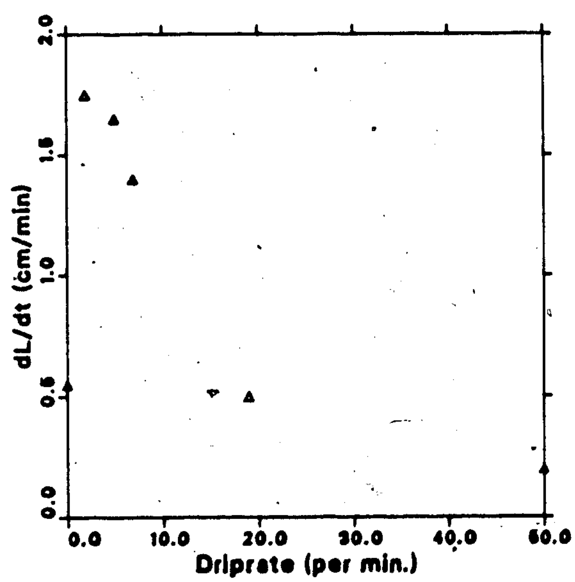
Trial: 23 (20-06-04)

Temperature: -16.0 °C
Windspeed: 1.9 ms^{-1}
Pressure: 92.9 kPa

Time (min)	Length (cm)	Driprate (min^{-1})
0	0.0	50
2	0.4	19
4	1.4	7
6	4.2	5
8	7.5	2
10	11.0	0
12	12.2	

Maximum growth rate: 1.75 cm min^{-1}





Trial: 24 (20-06-05)

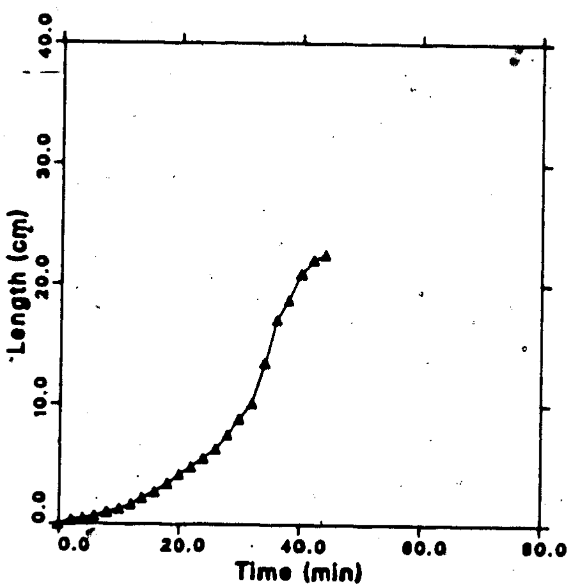
Temperature: -16.0 °C
Windspeed: 1.9 ms^{-1}
Pressure: 92.9 kPa

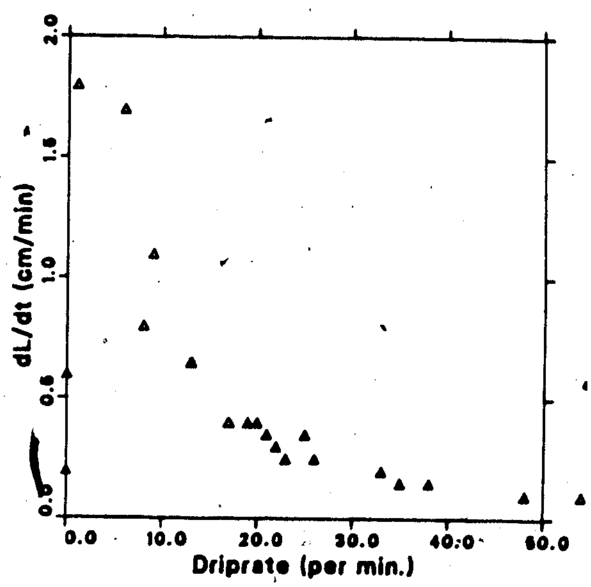
Time (min)	Length (cm)	Driprate (min^{-1})
0	0.0	66
2	0.3	54
4	0.5	48
6	0.7	38
8	1.0	35
10	1.3	33
12	1.7	26
14	2.2	23
16	2.7	25
18	3.4	20
20	4.2	22
22	4.8	21
24	5.5	19
26	6.3	17
28	7.5	13
30	8.8	13
32	10.1	6
34	13.5	1
36	17.1	8
38	18.7	9
40	20.9	0
42	22.1	0
44	22.5	

Maximum growth rate: 1.80 cm min^{-1}

Trial: 24 (continued)

Distance from Tip (cm)	Minimum Diameter (cm)	Maximum Diameter (cm)
0	0.43	0.45
3	0.43	0.46
6	0.48	0.53
9	0.52	0.75
12	0.54	0.94
15	0.63	1.12
18	0.56	1.46
21	0.82	1.90



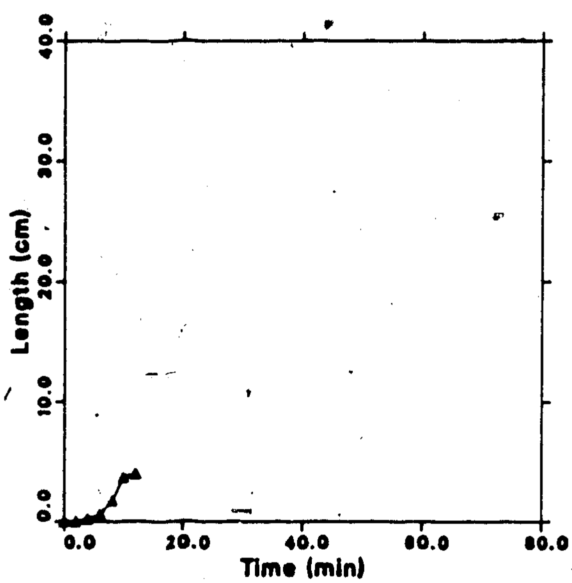


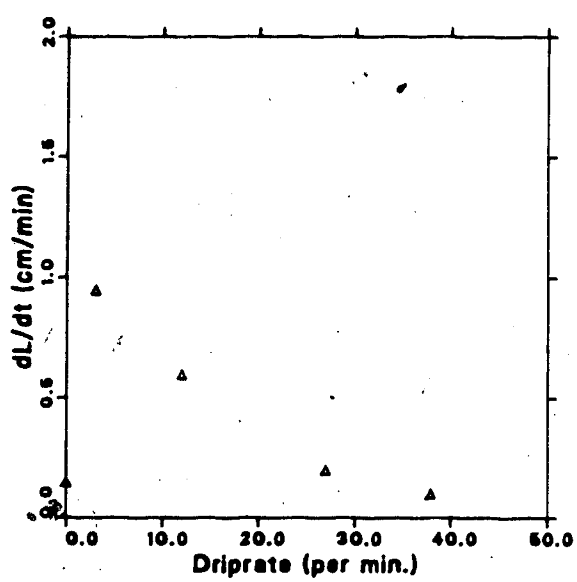
Trial: 25 (20-06-06)

Temperature: - 9.5 °C
Windspeed: 2.9 ms^{-1}
Pressure: 92.9 kPa

Time (min)	Length (cm)	Driprate (min^{-1})
0	0.0	62
2	0.0	38
4	0.2	27
6	0.6	12
8	1.8	3
10	3.7	0
12	4.0	

Maximum growth rate: 0.95 cm min^{-1}





Trial: 26 (20-06-07)

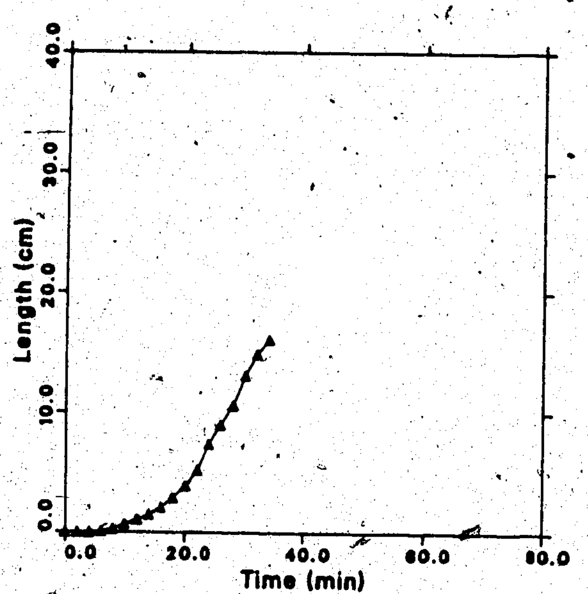
Temperature: -10.5 °C
Windspeed: 2.9 ms^{-1}
Pressure: 92.9 kPa

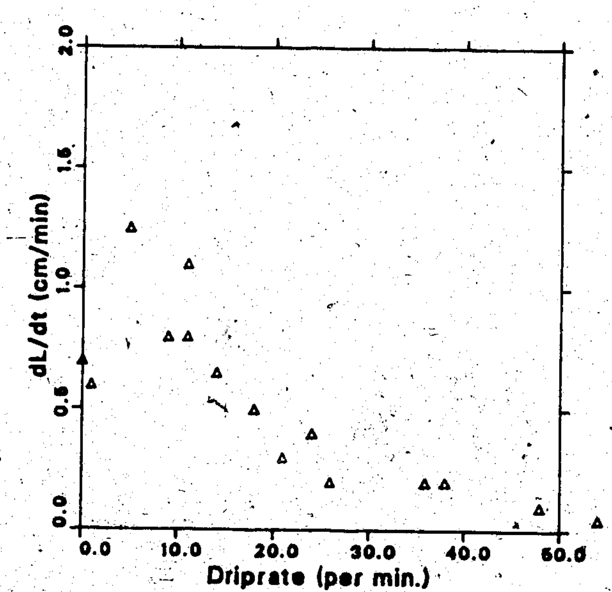
Time (min)	Length (cm)	Driprate (min^{-1})
0	0.0	92
2	0.0	70
4	0.0	54
6	0.1	48
8	0.3	38
10	0.7	36
12	1.1	26
14	1.5	21
16	2.1	24
18	2.9	18
20	3.9	14
22	5.2	11
24	7.4	9
26	9.0	11
28	10.6	5
30	13.1	0
32	14.9	1
34	16.1	

Maximum growth rate: 1.25 cm min^{-1}

Trial: 26 (continued)

Distance from Tip (cm)	Minimum Diameter (cm)	Maximum Diameter (cm)
0	0.42	0.42
3	0.42	0.45
6	0.48	0.59
9	0.52	0.65
12	0.60	0.78
15	0.77	0.83
18	1.12	1.35





Trial: 27 (20-06-08)

Temperature: -10.5 ^oC
Windspeed: 2.9 ms^{-1}
Pressure: 92.9 kPa

Time (min)	Length (cm)	Driprate (min^{-1})
0	0.0	88
2	0.2	72
4	0.5	64
6	0.8	62
8	1.0	58
10	1.2	50
12	1.4	46
14	1.6	42
16	1.9	39
18	2.2	39
20	2.7	41
22	3.2	32
24	3.8	32
26	4.2	36
28	4.7	31
30	5.3	33
32	5.9	31
34	6.7	35
36	7.4	27
38	8.3	16
40	10.1	13
42	10.7	13
44	11.4	12
46	12.0	11
48	13.0	11
50	14.2	9
52	16.2	7

Trial: 27 (continued)

Time (min)	Length (cm)	Driprate (min^{-1})
54	17.5	6
56	19.6	5
58	20.5	

Maximum growth rate: 1.25 cm min^{-1}

Distance from Tip (cm)	Minimum Diameter (cm)	Maximum Diameter (cm)
0	0.32	0.41
3	0.34	0.46
6	0.50	0.83
9	0.75	0.89
12	0.86	1.00
15	1.11	1.37
18	1.14	1.81
21	1.31	1.62