UNIVERSITY OF ALBERTA

BENDING STRESS MINIMIZATION OF CORRUGATED CLADDING

BY ©

LEE, CHUNG-LOK

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Abstract

In cladding design, it is always desirable to achieve optimal utilization of material while the imposed design constraints are satisfied. This optimization problem is usually solved by designing cladding profiles that require the minimum amount of material to cover a given width or by using profiles that provide maximum strength for a given amount of material.

A numerical procedure, the method of local variation, is developed and tested to search for optimal cladding profiles that have the maximum resistance to bending stress and satisfy the coverage requirement for a given amount of material. The loading condition on the claddings is assumed to be pure bending and the maximum compressive stress on the cladding cross-section is assumed to be less than the critical buckling load. Thus the objective of the optimization is to obtain cladding profiles that have the maximum section modulus while satisfying the constraints.

Three different cladding configurations including the doubly symmetrical corrugated, sandwich and non-symmetrical are considered. Sample results of each case are discussed and the characteristics of the claddings related to some design parameters are also presented.

Acknowledgement

The author would like to express his deep appreciation to Dr. M.G.Faulkner and Dr. A.Mioduchowski, who both supervised this thesis, for their invaluable advice, guidance and suggestion of the topic. Thank is also due to his parents, family members and friends who have been supporting the author in many ways.

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c = distance between the centroidal axis and the extreme point
$\Delta\theta$ = angle of variation
$\Delta\theta_0$ = initial angle of variation
Δu = segment length
Δu_0 = initial segment length
I_x = area moment of inertia
l = coverage
L = total coverage of a cladding
m = total number of segment
M_x = applied bending moment parallel to x axis
n = current segment number
R = contour length to coverage ratio
R_t = panel to cladding thickness ratio
σ = bending stress along the cladding cross-section
t_c = cladding thickness
t_p = panel thickness
V = material volume
y_c = y location of the centroidal axis
Z = section modulus

Chapter 1 Introduction

Because of its strength to weight ratio and ease of erection, corrugated cladding has been widely used for roofing and siding in building since the late eighteenth century. In recent years its usage has been extended to flooring system for buildings and bridge construction[1].

In the above applications, the loadings on the cladding can be due to its own weight, waterproofing, insulation, snow, rain, wind as well as loading during construction or maintenance. While the last two types of loading are concentrated, the former are usually considered as uniformly distributed. The general analyses of corrugated claddings can use either folded plate theory or orthotropic plate theory, however, for simplicity and design purposes the cladding can be considered simply as a series of linked beams with each trough acting as a single beam[2]. As the cladding used to support these loads is long compared to the cross-sectional dimensions, the major internal stresses are due to bending.

In [3], the cross-sectional profile of a doubly symmetric thin-walled beam under pure bending was considered. Using a variational formulation, profiles that yield the maximum section moduli and satisfy a dimensional constraint in width were obtained. Although these profiles were intended for thin-walled beams, they can be adopted to cladding design if the cladding has a doubly symmetric profile and is under only pure bending. This analysis did not consider the possibility of buckling.

Seaburg[4] developed a computer program to determine cladding profiles that require a minimum amount of material(minimum weight) to cover a given width while the cross section also satisfied other constraints such as a maximum permissable stress. Optimization was accomplished by combining a search technique with non-linear programming based on the provisions given in the American Iron and Steel Institute(AISI) Cold-Formed Steel Design Manual[5]. In this analysis post-buckling behaviour of the cladding was considered as in the AISI specification which incorporates the concept of effective width design for flat compression elements. In the specifications the effective width is formulated as a function of the actual compressive stress acting on the cladding and cladding dimensions.

The goal of both [3] and [4] was to achieve maximum utilization of material by using the appropriate cladding profiles. The objective of optimization in [3] was maximization of beam resistance to bending stress and was weight minimization in [4]. Both of these authors achieved optimization by means of mathematical modelling or mathematical programming.

The objective of this study is to minimize bending stress due to a pure bending moment acting on the cladding by designing the appropriate profiles which also satisfy the imposed dimensional constraints. However, instead of using an analytical approach to obtain the profiles, a numerical procedure, termed the local variation method, is to be used. The basic principal of this technique is drawn from the paper by Chernous'Ko[6] and is essentially a procedure in which the geometry of an initial profile is perturbed in a systematic way. Any profile that satisfies the required constraints and provides a higher(or lower) value of the desired property is retained and the perturbation and calculations are continued on the retained profile. This process of alternating the geometry is continued until no further improvement can be made. In this way mathematical modelling of the problem is not necessary as the actual geometry and its properties are generated and evaluated in an approximate manner.

In [4] the hat-shaped cladding profile was defined by circular arcs and tangent lines. With this approach, only limited types of profile geometry can be described and may not be a good approximation of the optimal shape. Because the local variation method allows the cladding be perturbed to any shape as long as the imposed constraints are met, the final solution will not be limited to a specific type of geometry.

In this study, the cladding is assumed to be under pure bending and the magnitude of the maximum compressive stress is assumed always less than the buckling load. Instead of considering the whole cladding, a single trough of the cladding as shown in figure 1-1 is considered and treated as a simply supported beam. Although the post-buckling properties are not considered in this technique, they could be determined with the AISI provisions.

In the following sections, details and the computer algorithm regarding the local variation method are presented. The numerical procedures employed are tested by comparing results with those shown in [3] as well as with some limiting cases. Three different cladding configurations, including the corrugated, sandwich and non-symmetrical as shown in figure 1-2, 1-3 and 1-4 are investigated and the general results concerning the trends shown are discussed.

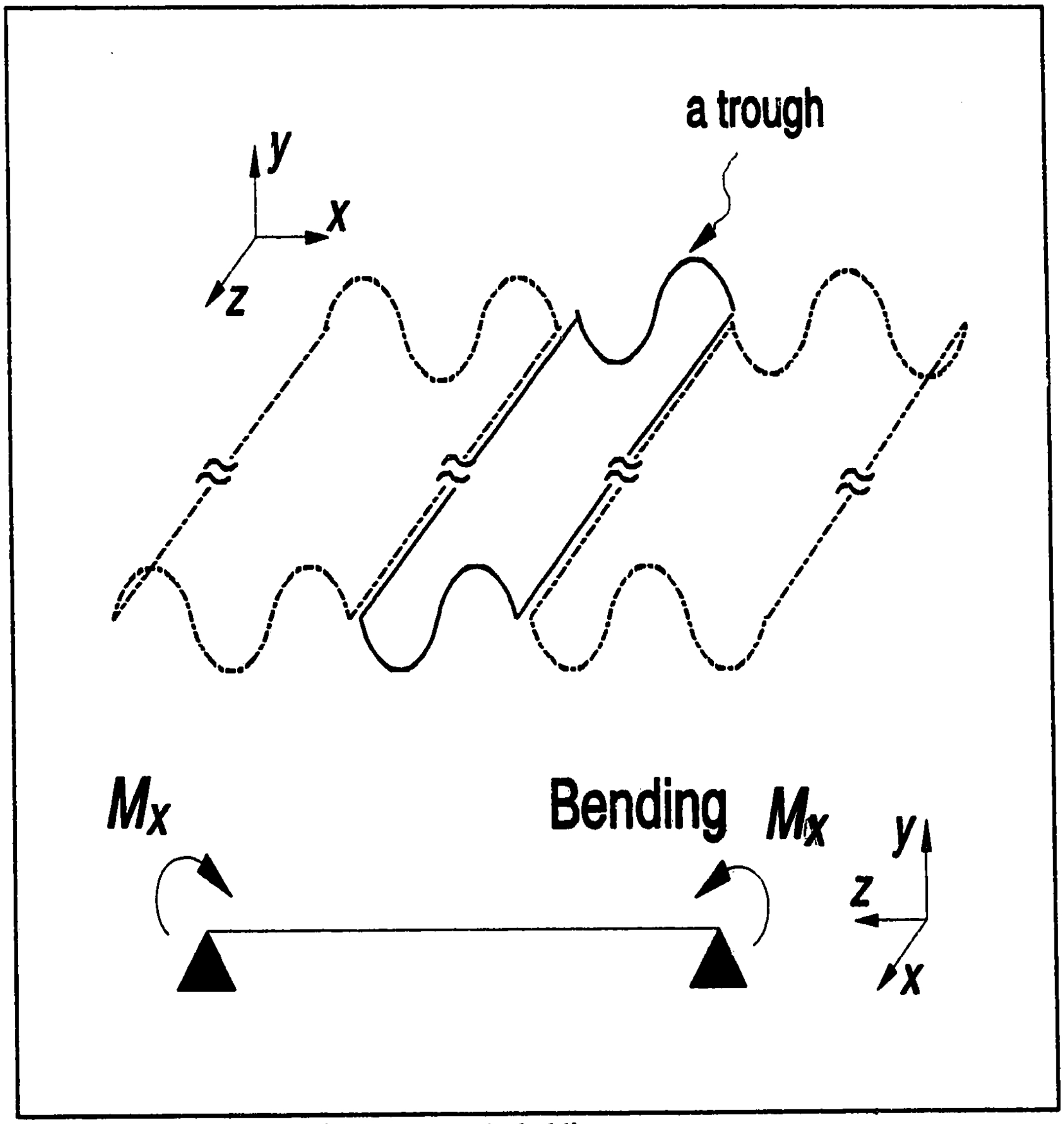

Figure 1-1: A trough of a corrugated cladding

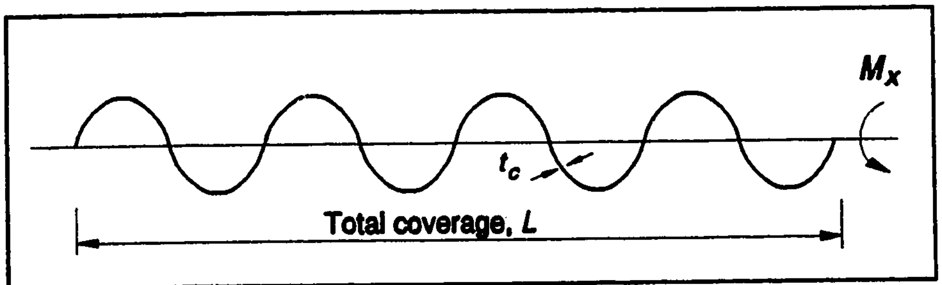

Figure 1-2: A general corrugated cladding

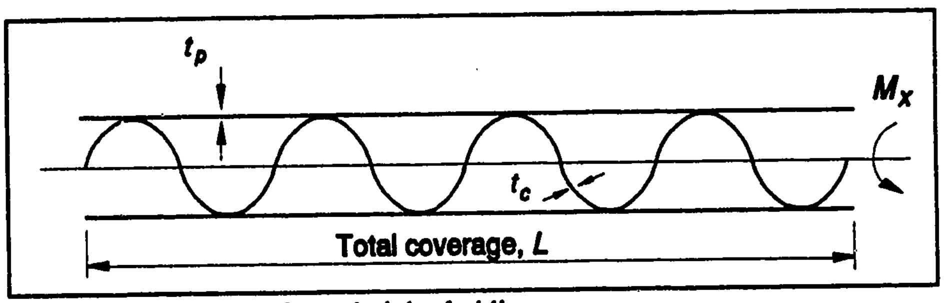

Figure 1-3: A general sandwich cladding

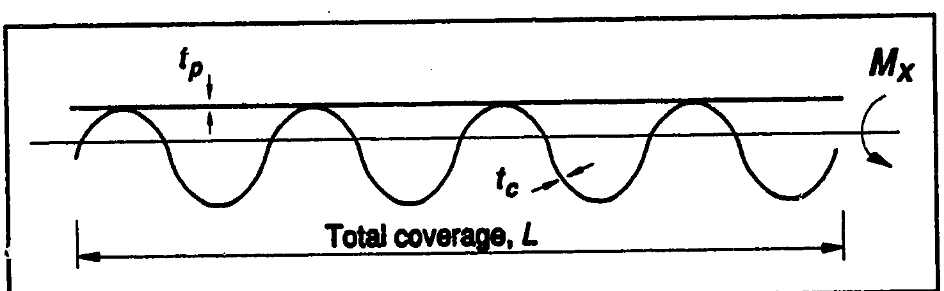

Figure 1-4: A general non-symmetrical cladding

Chapter 2 Numerical Procedure Development

In this chapter, details describing the algorithm of the local variation method are presented. Findings on verification and characteristics of the numerical procedure are also discussed.

2.1 Computer Program Development

Consider a cross-section in the x-y plane under pure bending M_x parallel to the x axis which coincides with the centroidal axis of the cladding as shown in figure 2-1. The maximum stress, σ, acting on the cross-section due to the bending is:

$$\sigma = M_x \frac{c}{I_x} \tag{2-1}$$

where σ = maximum compressive/tensile stress along the z-axis
I_x = area moment of inertia of the cross-section about the centroidal axis
c = maximum y distance measured from the centroidal axis to the extreme point

From equation (2-1), the magnitude of σ is proportional to the geometric ratio c/I_x of the cross-section. Thus to minimize σ for a constant M_x, it is necessary to maximize the section modulus $Z=I_x/c$.

In designing a cladding profile of constant thickness t_c, with coverage l and a contour length u_c, it is often desirable to use the optimal profile which has the highest section modulus while other imposed dimensional constraints such as height are satisfied. While there are several techniques which can be used to maximize Z, a numerical procedure termed the local variational method is employed.

The basic principal of the local variation method is to systematically construct numerous profiles which satisfy the constraints and the one with the highest section modulus is selected to be the optimal profile. To obtain the optimal profile, the local variation method is divided into two steps. At first an approximate profile, the 0 solution, is obtained and based on it the optimal profile is then obtained.

Consider the upper half of an anti-symmetrical profile shown in figure 2-2 which consists of m(4 in this case) straight segments of equal length Δu with its centroidal axis, the axis of symmetry, coincident with the x-axis. Any geometric changes of the upper half will result in similar changes in the lower portion so that the centroidal axis is always the axis of symmetry. Without changing the contour length u_c, new profiles can be constructed as shown in figure 2-3 by changing the angle θ of each segment to $\theta+\Delta\theta$ or θ-$\Delta\theta$ where $\Delta\theta=\Delta\theta_0$ or keeping θ at its original value. Thus including the original position, each segment will have 3 possible positions.

The theoretical number of new profiles that can be constructed depends on the number of segments employed and the end condition imposed on the left end of the first segment while the end of the last segment is fixed. Referring to figure 2-4, for 4 segments and including the original profile, the maximum number of new profiles are, $3^4=81$ (free-end), $2\text{x}3^3=54$ (slide-end) or $2\times3^2=18$ (fixed-end). However, as the coverage constraint must always be satisfied, only the slide-end or fixed-end condition is allowed. Thus the maximum number of new profiles will either be 54 or 18. For 8 segments, the corresponding number of new profiles are $3^8=6551$, $2\text{x}3^7=4374$ and $2\text{x}3^6=1458$. The actual number of new profiles depends on the magnitude of $\Delta\theta_0$ chosen and could be less than the theoretical values when a relatively large $\Delta\theta_0$ is used.

As seen in figure 2-4 for the slide-end condition the coordinate values of point 1 are calculated by:

$$y_1 = y_2 \pm \sqrt{\Delta u^2 - (x_2 - x_1)^2} \qquad (2\text{-}2)$$

where x_1 = constant.

For the fixed-end condition the coordinate values of point 2 are found, depending on which of the two situations shown in figure 2-5 and 2-6(case 1 and 2 respectively) that the segments are in, from:

case 1

$$x_2 = \bar{x}_{13} \pm h \times \sin\beta \qquad (2\text{-}3)$$

$$y_2 = \bar{y}_{13} \pm h \times \cos\beta \qquad (2\text{-}4)$$

case 2

$$x_2 = \bar{x}_{13} \pm h \times \sin\beta \qquad (2\text{-}5)$$

$$y_2 = \bar{y}_{13} \mp h \times \cos\beta \qquad (2\text{-}6)$$

where

$$\bar{x}_{13} = \frac{x_1 + x_3}{2} \qquad (2\text{-}7)$$

$$\bar{y}_{13} = \frac{y_1 + y_3}{2} \qquad (2\text{-}8)$$

$$h = \sqrt{\Delta u^2 - \frac{(x_1 - x_3)^2 + (y_1 - y_3)^2}{4}} \qquad (2\text{-}9)$$

Equations (2-3) to (2-6) are valid only if $\Delta u_1 = \Delta u_2$. More general equations (2-10) and (2-11) which equate the length of Δu_1 and Δu_2 may be used to solve for the coordinate values, x_2 and y_2 of point 2. These are:

$$\Delta u_1 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \tag{2-10}$$

$$\Delta u_2 = \sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2} \tag{2-11}$$

However, only one of the two possible locations of point 2 may be found if (2-10) and (2-11) are solved numerically. On the contrary equations (2-3) and (2-4) or (2-5) and (2-6) guarantee that both of the possible locations of 2 are obtained as long as Δu is prescribed to be the same for both segments. Coordinate values of other points are calculated using:

$$x_n = x_{n+1} + \Delta u \times \cos(\theta_n + j \times \Delta\theta) \tag{2-12}$$

$$y_n = y_{n+1} + \Delta u \times \sin(\theta_n + j \times \Delta\theta) \tag{2-13}$$

$$j = 0, \pm 1$$

At every local variation procedure, sets of maximum 54 or 18 profiles are constructed for each 4 consecutive segments considered. The profile on which the variation is based, is replaced by the intermediate profile which has the maximum section modulus among the set. Based on the intermediate profile, another set of profiles is constructed with the same 4 segments and the process of replacing and constructing profiles is continued until no profile with higher section modulus can be found. The procedure is shown in figure A-1 in the appendix.

To determine the section modulus Z, the area moment of inertia I_x and the maximum distance c must be determined. In this study I_x is approximated by summing the area moment of inertia of each segment with respect to the centroidal axis. I_x is calculated from:

$$I_x = \sum_{n=1}^{m} \Delta I_{x_n} \tag{2-14}$$

$$\Delta I_{x_n} = \frac{t_c \Delta u^3}{12} \sin^2\theta_n + t_c \Delta u \bar{y}^2 \tag{2-15}$$

$$\bar{y} = \frac{y_n + y_{n+1}}{2} \tag{2-16}$$

where m = total number of segments

ΔI_{x_n} = area moment of inertia of a segment above the centroidal axis.

Because cladding profiles are approximated by straight line segments, the value c is defined as the maximum distance in y direction measured from the centroidal axis to the end points of the line segments plus material thickness which is half of the cladding thickness in this case.

2.3 Determination of the Approximate Optimal Solution

To find any so-called 0 solution, all the segments must be involved in the variation procedure. For example, given the profile shown previously in figure 2-3 as the starting profile and not constraining the height of the profiles, a slide-end variation procedure involving 4 segments with $\Delta\theta = \Delta\theta_0$ at an arbitrary value is used to find the profile with the highest possible section modulus.(An 8-segment slide-end procedure would be used if the starting profile has 8 segments.) When no further improvement on

Z can be made, $\Delta\theta$ is halved($\Delta\theta=0.5\Delta\theta_0$) and the variation procedure is repeated. The last intermediate profile found, after $\Delta\theta$ has been decreased, is the 0 solution. A flow chart concerning the procedure of obtaining the approximation is shown in figure A-2.

Ideally, the more segments used in finding the 0 solution the closer it will be to the optimal solution as more profiles can be considered. However, computation time increases dramatically as the number of segments is increased.

2.4 Determination of Optimal Profile

To determine a better approximation to the optimal solution, the 0 solution is subdivided into more segments. For example, as shown in figure 2-7, if a 4-segment 0 solution is used, it is doubled to 8 segments. Thus m, the total number of segments becomes 8, and the length of each segment is reduced from its original length Δu_0 to $0.5\Delta u_0$ by dividing each segment at its mid-point.

Instead of involving all 8 segments in each variation as was done to obtain the 0 solution, only 4 consecutive segments are used. With $\Delta\theta$ now at 1/2 of its original value $\Delta\theta_0$, the variation procedure is first performed on segment $n=1$ including segment 2, 3 and 4 then in sequence on $n=2,3...m-3$. In this manner the profile geometry is changed locally at each variation because only 4 segments are involved at each time and the others are left unchanged. In addition, the procedure is repeated in sequence on segments $n=1,2,3...n-1$ after it was performed on segment n because a better profile may not be able to be constructed until geometry of segments n, $n+1$, $n+2$ and $n+3$ have been changed. Following this, $\Delta\theta$ is again decreased now to $0.25\Delta\theta_0$ and the above local variation procedure is repeated until there is no improvement in section modulus when variation procedure has been performed on all m segments. Calculation was terminated after 32 segments have been used as no significant improvement ($<0.01\%$) in Z resulted. The flow charts which show the procedure employed to obtain the optimal solution and the complete local variation method are shown in figures A-3 and A-4.

2.5 Verification of the Numerical Procedure

The local variation method was tested by comparing optimal profiles given in [3] and shown in figures 2-8 and 2-9 which were obtained using variational calculus. The axes $\overline{X}$ and $\overline{Y}$ are non-dimensional with respect to the contour length, $u_c=1$. The profiles shown are an approximation of the plots of the profiles published in [3] because a numerical description of the profiles is not available. Heights of the profiles in [3] were not specified prior to calculation but were determined in the process of finding the optimal profiles.

Figure 2-10 and 2-11 are profiles obtained using the numerical procedure which satisfy the same dimensional constraints imposed in figures 2-8 and 2-9. They again represent a quarter of a trough of a doubly symmetrical cladding. The section moduli of the profiles are also listed and are calculated with respect to their centroidal axes which, due to symmetry, is the base line of each figure. For each of the profiles obtained, the starting profile used was a straight line consisting of 4 segments, and the initial angle of variation $\Delta\theta_0$ was 0.1 rads. In terms of section modulus, the results show that the numerically obtained profiles match closely with the profiles given in [3] for all values of coverage l and also generally match in terms of the predicted shape for cases including $l=0.25$ and $l=0.43$. Profiles 3, 4 and 5 in figure 2-10 have a section modulus at most 9% higher than the given profiles shown in figure 2-8. This discrepancy may be due to inaccurate reproduction of the profiles in [3]. While profile 1 in figure 2-9 with u_c=5.33 and l=1 cannot be reproduced by the numerical procedure; the trend can still be seen in figure 2-11. The two solution differ by only 3% in numerical value.

2.6 Characteristics of the Numerical Procedure

In using the technique outlined above to determine optimal profiles, only a few parameters must be specified to begin. These include the initial angle of variation, $\Delta\theta_0$, number of segments used in the starting profile and an initial shape. It is of interest to ascertain the effect which these initial parameters have on the final answer.

To investigate the effect of $\Delta\theta_0$, all profiles in figure 2-12 were obtained with the same contour length, coverage, and starting profile but with different $\Delta\theta_0$. Although an unique profile was not obtained as it was expected, the section moduli of the final profiles were very close. Except for the one obtained with $\Delta\theta_0$=0.6 rads, about 34°, the maximum discrepancy in Z among the profiles is less than 0.5% while the maximum discrepancy in height is 8%.

The insensitivity of section modulus to profile height can be explained with reference to equation (2-1) since I_x increases as c increases. Thus the overall effect on section modulus, I_x/c, will be small. Similar characteristics were observed for other values of coverage. This implies that a considerably large tolerance is allowed in reproducing the optimal shape during fabrication as profiles which are in the vicinity of the optimal profiles have similar section moduli.

Profiles shown in figure 2-13 were obtained to determine the effect of the number of segments. Profiles *a* and *b* which have a coverage of 0.58 were obtained with a straight line starting profile with 4 and 8 segments respectively. Their section moduli are within 0.01% although their geometries do not look similar. Profiles *d* and *e* were obtained by conducting the similar test with l=0.25. Their relative difference in Z is 1.6% and profile *d* which was obtained with a 4 segment starting profile has a higher section modulus then profile *e*. In this case the fact that a 4-segment starting profile yielded better results than an 8-segment starting profile is due to the shape of the optimal solution accidentally having a coverage to height ratio of 1:3.

The effect of shape of the starting profile is not significant as long as the approximate optimal profile are used before calculation of the final profile begins. Failure to do this can result in determining only a locally optimal solution rather than the global one desired.

To conclude, it has been determined that the ability of the numerical procedure to find the optimal profile depends mainly on the number of segments used in the starting profile to generate the so-called 0 solution profile. In general, the use of a 4-segment and an 8-segment starting profile will both yield optimal profiles that have a very similar section modulus except when there are discontinuities (abrupt changes in geometry) in the optimal solution. As the shape of the optimal profiles cannot be foreseen for many cases, caution must be taken when using the numerical procedure and the genuineness of any calculated optimal profiles should be tested by using starting profiles with different numbers of segments to check the final result.

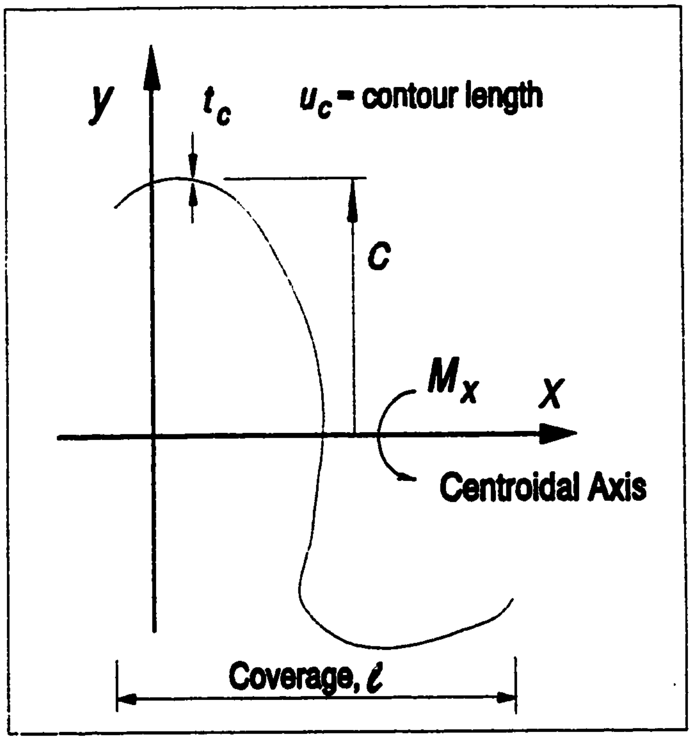

Figure 2-1: A general cladding cross-section

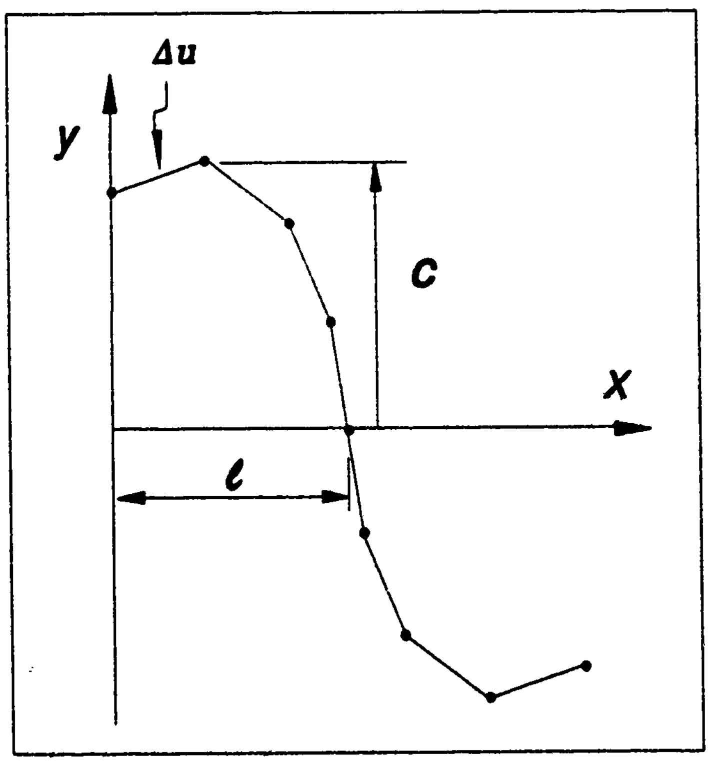

Figure 2-2: A symmetrical cladding

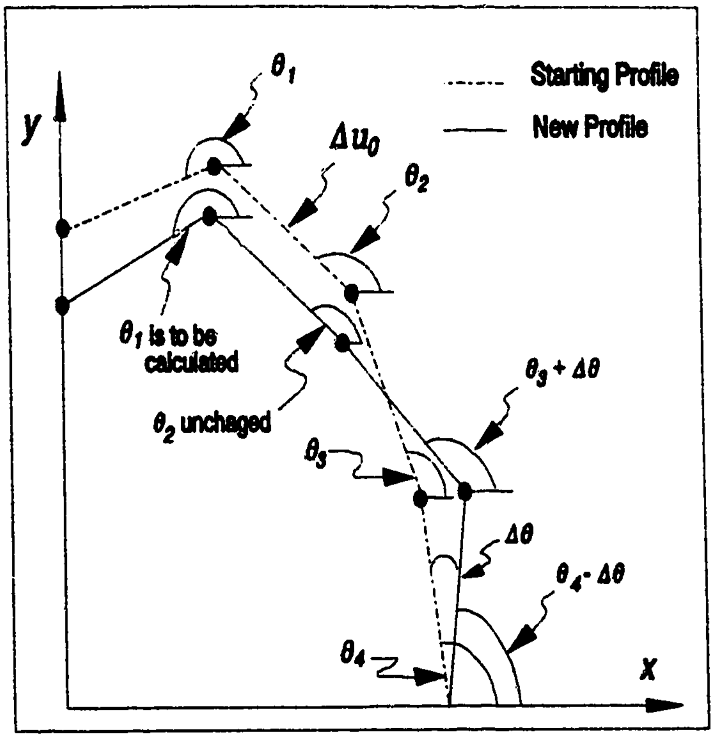

Figure 2-3: General variation procedure

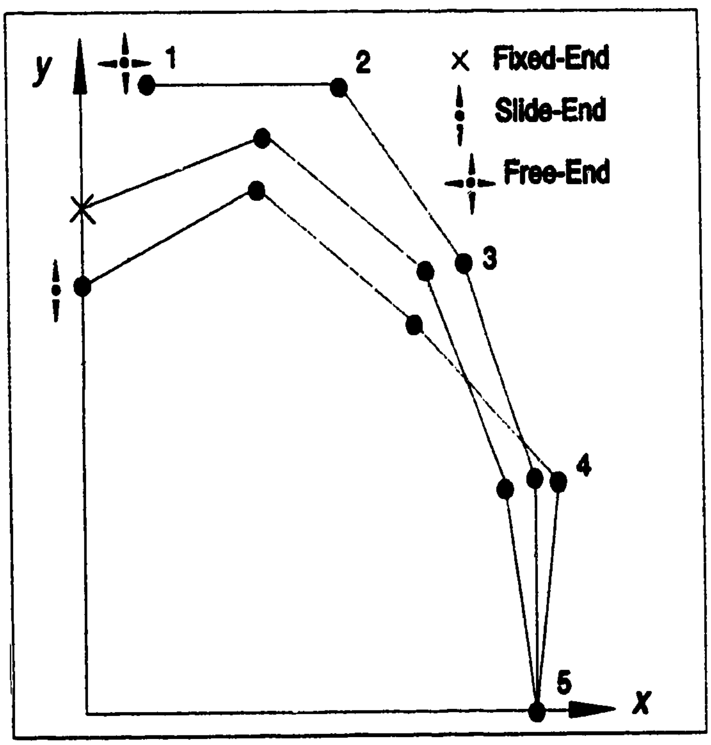

Figure 2-4: Possible end conditions

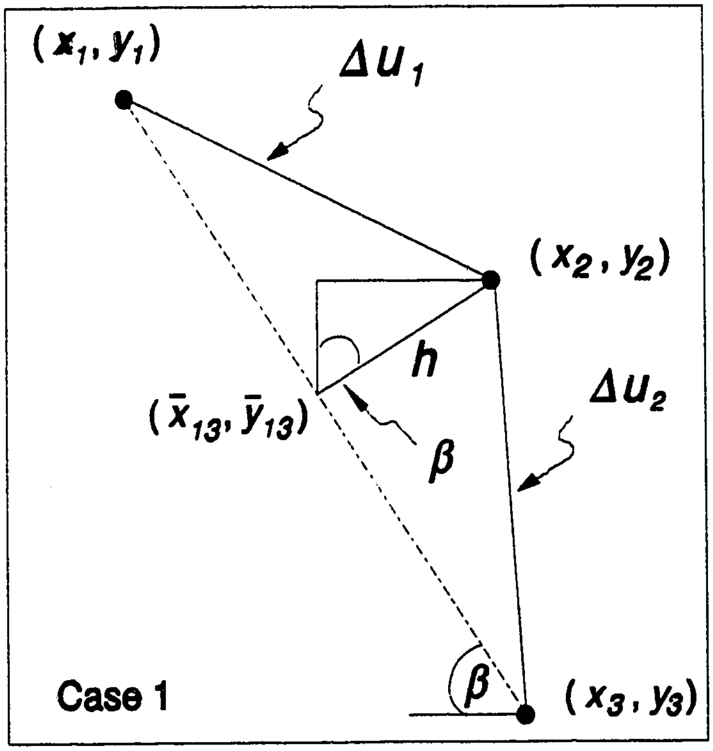

Figure 2-5: Solution method(case 1) for point 2

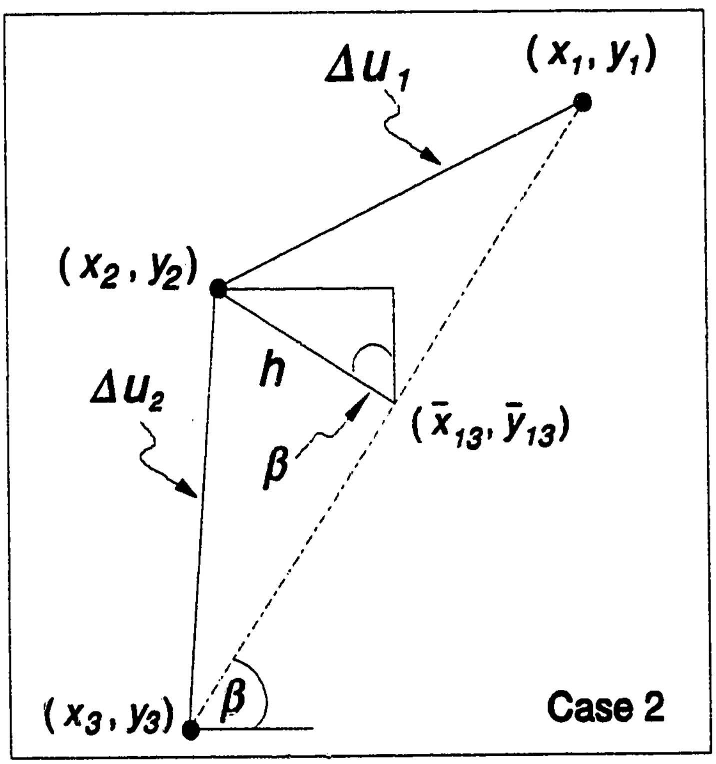

Figure 2-6: Solution method(case 2) for point 2

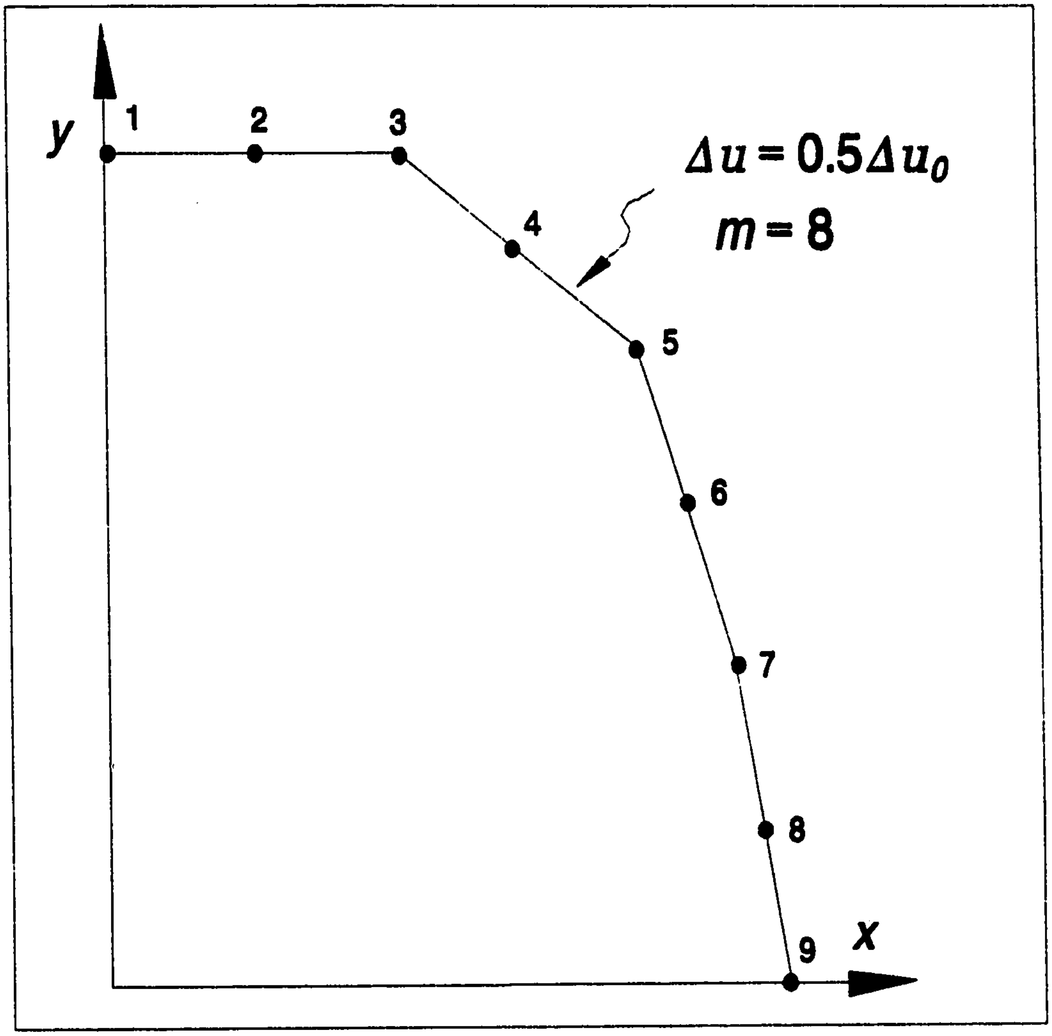

Figure 2-7: Subdivision of starting profile

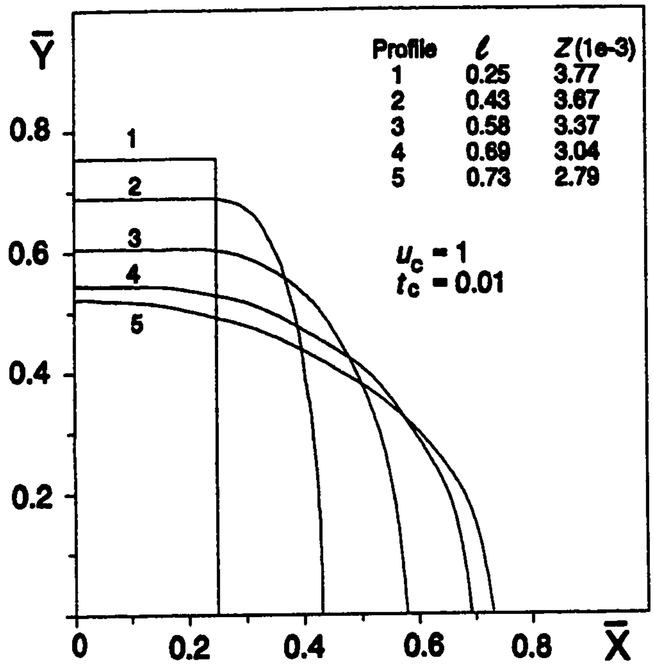

Figure 2-8: Profiles from [3], all u=1

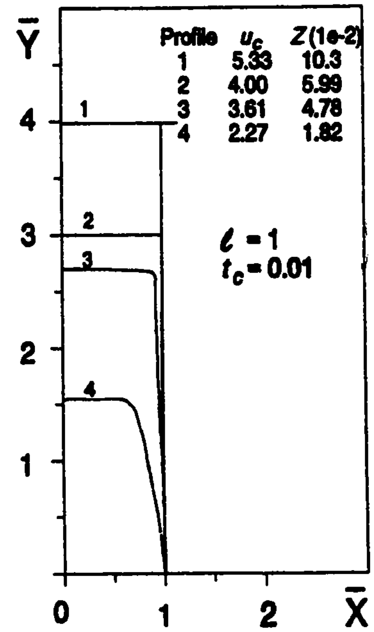

Figure 2-9: Profiles from [3], all l=1

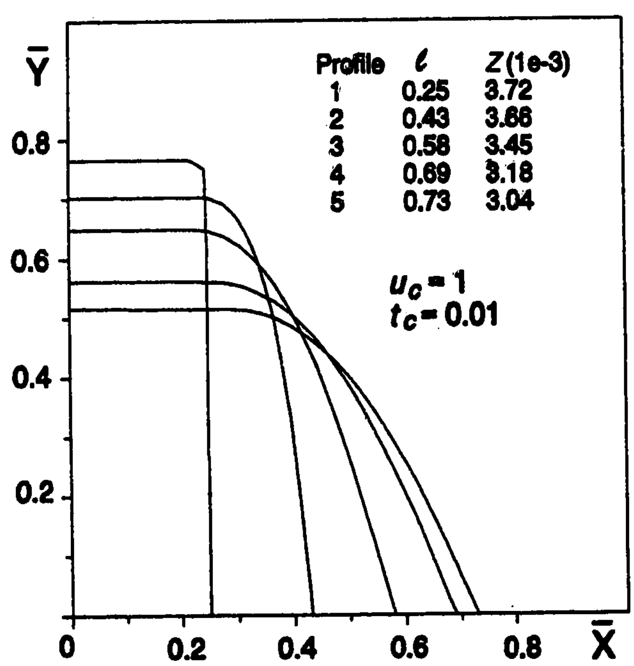

Figure 2-10: Numerically obtained profiles, all u=1

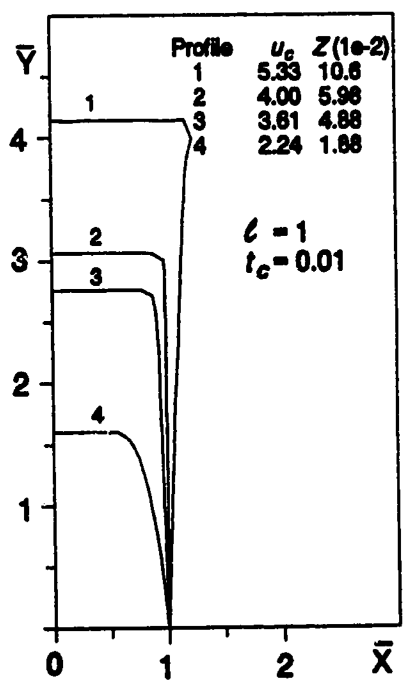

Figure 2-11: Numerically obtained profiles, all l=1

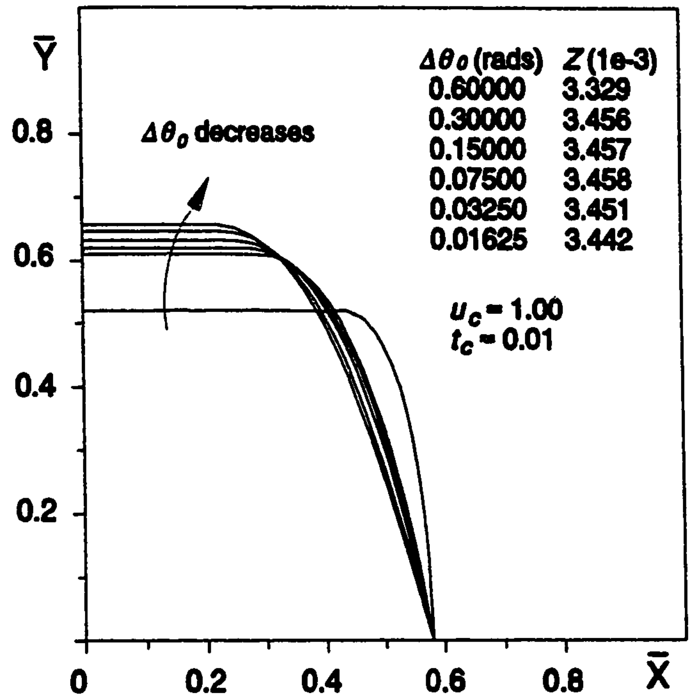

Figure 2-12: Sensitivity of Z to $\Delta\theta_0$

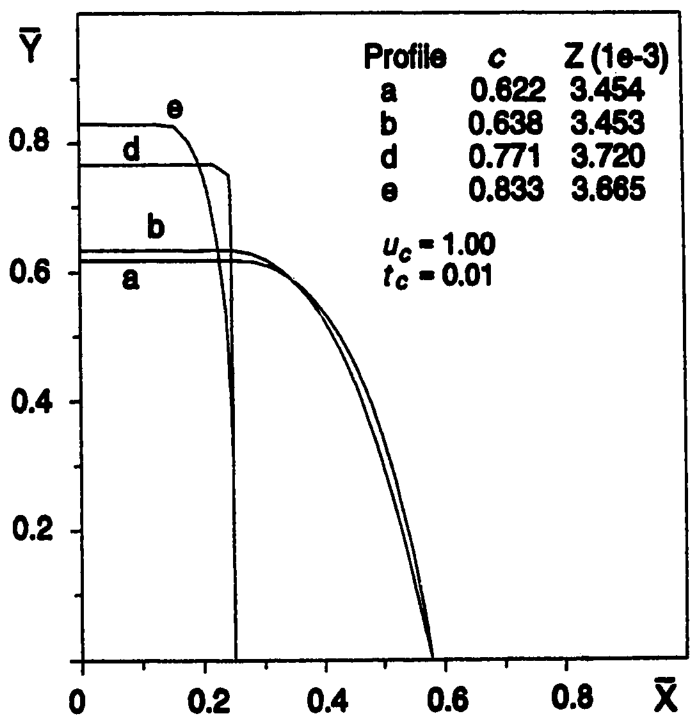

Figure 2-13: Sensitivity of Z to starting profiles

Chapter 3 Symmetrical Claddings

The previous chapter included the description and verification of the local variation method. Solutions for doubly symmetrical corrugated cladding will be completed in this chapter. In addition, application of the numerical procedure to other similar optimization problems including the necessary modifications of the procedure will be presented.

3.1 Numerical Results for Corrugated Configuration

To demonstrate the usefulness of the optimal profiles, figure 3-1 illustrates several simple profiles and the optimal profile which has the same contour length and coverage. The contour length chosen was 1.571 so that a circular profile(3) can also be used for a coverage of 1. Profiles 1, 2 and 4 correspond to a triangular, a rectangular and the optimal profiles respectively. Among the four shown, the triangular and the rectangular profiles have section moduli which are 23% and 18% less than the optimal. The circular shape which is similar to the optimal is only 5% less. Clearly more effective use of material is achieved with the use of the optimal profiles.

Profile 1 in figure 3-2 is the approximate shape of a commercially available corrugated cladding which has been scaled so that its coverage is also 1. Profile 2 is constrained to have the same height as profile 1 and it shows that a 2% higher section modulus can be achieved without changing the height constraint. Profiles 2 and 3 are the optimal profiles obtained with the same contour length and coverage as the commercial profile but which have different constraints. When the height constraint is relaxed, profile 3 yields a 4% gain compared to the commercial profile. Although a 2%

or 4% gain may not seem worthwhile, a considerable saving in material cost is possible in large scale manufacturing.

For a fixed contour length, u_c, the shape of the optimal profile is a function of coverage and height constraints. If the height constraint is relaxed, the profile shape is governed only by a parameter R defined as the ratio of contour length to coverage.

$$R=\frac{u_c}{l} \tag{3-1}$$

Figure 3-3 shows the profiles with various R values for $u_c=1$ and $t_c=0.01$. According to [3] profiles with R equal to or larger than 4 have shapes similar to profiles 1 in figures 2-8 and 2-9 whose ratio of material placed horizontally to material placed vertically is always 1 to 3. All these profiles which have $R\geq 4$ have the same section modulus.

The section moduli of profiles shown in figure 3-3 are plotted against their corresponding R values(up to 4.0) in figure 3-4. For clarity, the corresponding profiles for some R values used in figure 3-4 are not shown in figure 3-3. From figure 3-4, it is noted that section modulus increases until a steady value of approximately 3.720×10^{-3} has been reached. This means, for example, that while for an increase in R from 2.8 to 4.0 the section modulus increases by 0.6% from 3.698×10^{-3} to 3.720×10^{-3}. The corresponding reduction in coverage from 0.35 to 0.25 is 31%. Thus with a large reduction in coverage, little is gained in Z. Obviously for a given contour length, a moderate value of R should be chosen so that a high section modulus is complemented with a relatively large coverage.

3.2 Program Modification and Verification for Sandwich Configuration

The numerical procedure used in optimizing the section modulus of a corrugated cladding can be modified to solve other similar optimization problems. Consider a sandwich corrugated cladding as shown in figure 1-3 which is under pure bending M_x and has two flat panels of thickness t_p welded to the top and bottom of a corrugated cladding, termed the cladding core.

To determine a cladding core profile that produces the maximum section modulus for sandwich cladding, the numerical procedure was modified to account for the effect of the flat panels added to the cladding core's profile.

By adding an additional term to equation (2-14) to account for the moment of inertia contributed by the flat panels yields:

$$Z=\frac{I_x}{y_p+0.5t_p} \tag{3-2}$$

$$I_x=\sum_{n=1}^{m}\Delta I_{x_n}+2t_p l y_p^2 \tag{3-3}$$

$$y_p=y_{max}+0.5(t_c+t_p) \tag{3-4}$$

where y_{max}=maximum y coordinate of segment end points measured from the centroidal axis. The maximum distance between the centroidal axis and the outermost element, c in equation (2-1), in this case is the maximum y coordinate of the segment end points, y_{max}, plus half of the thickness of the cladding core, $0.5t_c$ and the thickness of the panel, t_p. As the thickness of the flat panels are set equal, the axis of symmetry is also the centroidal axis. Again, due to symmetry, only a quarter of the profile is needed to be considered in the following.

The modified procedure was tested for some limiting cases because there are no existing results for verification purposes. Figure 3-5 shows the optimal profiles obtained with the same u_c, t_c and l but different panel thickness, t_p. For clarity, the panels which are on top of the claddings are not shown. For the limiting case when t_p=0, the resulting cladding core profile is expected to be the same as the corrugated claddings, obtained in the previous section, with the same u_c and coverage. As expected, the profile with t_p=0 in figure 3-5 is identical to profile 2 in figure 2-10 and yields the same section modulus.

When t_p increases the profile height also increases as can be seen by considering equation (3-4). The second term becomes dominant as t_p increases and the section modulus is mainly contributed by the panel. Equation (3-2) reduces to approximately

$$Z = 2lt_p y_p \qquad (3\text{-}5)$$

provided that t_p is still small in comparison to y_p. As a result as shown in figure 3-5, the optimal profile will tend to be taller.

3.3 Numerical Results for Sandwich Configuration

It was shown previously that the optimal shapes of the corrugated claddings can be generalized to be a function of the contour length to coverage ratio, R, regardless of cladding thickness. Similar generalization is not possible for the sandwich configuration whose optimal shapes depend on both the thickness ratio, $R_t = t_p/t_c$, and R. To illustrate the effects of these two parameters, a family of sandwich panels constructed with the same volume of material(V), 0.015 cubic units in this case, but with different coverages are shown in figure 3-6. The contour length and thickness of the cladding core were set respectively to 1 and 0.01 so that the panel thickness and the thickness ratio, change with the coverage proportional to $1/R$. Because the span of the panel is assumed to be 1, the panel thickness can be calculated as.

$$t_p = \frac{R(V - t_c u_c)}{2u_c} \tag{3-6}$$

As seen in figure 3-7, the section modulus of the sandwich claddings increases with R, again at a decreasing rate. Unlike the corrugated cladding case in which a constant Z value was obtained with a comparatively low R value, a relatively constant section modulus was not obtained until R approaches 6.0. This implies that one can always increase the section modulus by choosing a bigger R while t_c and u_c are kept constant. However, this may not be justified due to the loss in coverage.

Figure 3-8 illustrates the effectiveness of an optimal cladding core profile by comparing the section moduli obtained based on 5 cladding cores of the same contour length and thickness but of different shapes. Profiles 1 and 2 are a rectangular and a triangular profile while profile 4 and 5 are the commercial available profile and the optimal profile obtained without height constraint(shown in figure 3-2). Profile 3 is the optimal cladding core obtained with the modified procedure. The thickness ratio is 1 in all cases. The rectangular profile and the optimal profile yield the lowest and the highest section modulus at 1.117×10^{-2} and 2.289×10^{-2} respectively so that a significant gain in section modulus results if the optimal cladding core is used.

Although the Z value for the triangular core is only 0.8% less than the optimal value for this case, this may not hold for sandwich cladding with other thickness ratios and contour length to coverage ratios. The relative difference in Z among the optimal and other profiles is a function of the values of R_t and R. A large deficiency in Z may result if the optimal core profile is not used.

Another observation from figure 3-8 is that section modulus is higher for profile 4, the commercial corrugated profile, compared to profile 5 which is its optimal cladding profile without sandwich panels with the same contour length and coverage. This is because the added panels have a greater effect in increasing the section modulus with profile 4 due to the fact that the flat panel is at a higher location than it is with profile 5.

Because the thickness ratio R_t affects the shape of the cladding core, it is interesting to note its effect on the section modulus. Suppose a fixed volume of material is provided to construct a quarter of a sandwich cladding having a given coverage. Keeping the cladding thickness t_c constant and depending on the thickness ratio, the contour length of the core u_c is:

$$u_c = \frac{(V-2R_t t_c l)}{t_c} \tag{3-7}$$

when the span is 1. For V=0.0143, l=0.43 and t_c=0.01, figure 3-9 shows profiles obtained with various values of thickness ratio R_t and contour length while figure 3-10 plots their section moduli against R_t. From figure 3-10 , it is seen that the section modulus decreased by 48% with the increase of R_t from 0 to 1.0. Because the panel thickness equals to 0 for R_t=0 and Z is at its highest value when R_t=0, it can be concluded that a simple corrugated configuration provides the highest section modulus of any sandwich configuration for any thickness ratio if a fixed volume of material and fixed cladding thickness are used. From the trend shown in figure 3-10, the implication is that if a sandwich configuration with a set cladding thickness is to be used, the panel thickness should be minimized so that material can be used to increase the contour length instead of the panel thickness.

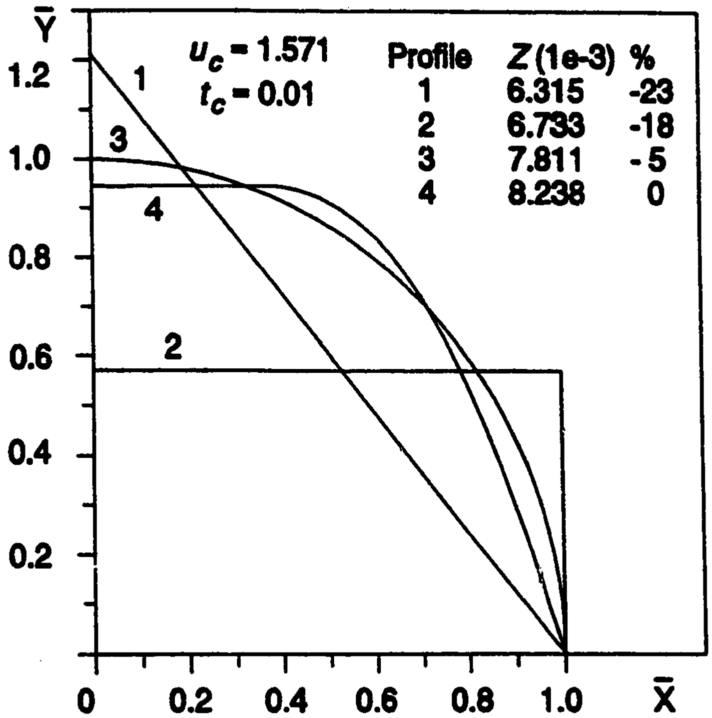

Figure 3-1: Section modulus of various profiles

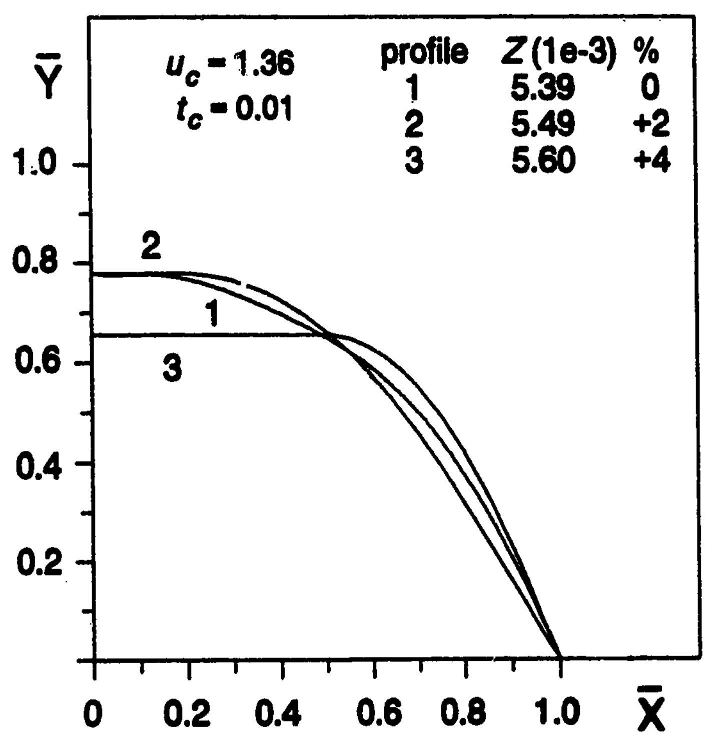

Figure 3-2: Modified commercial profile

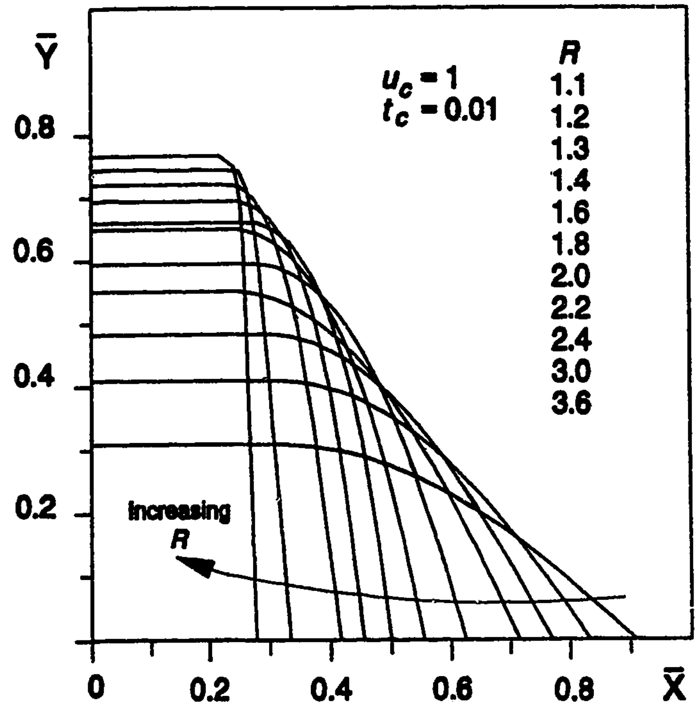

Figure 3-3: Optimal profiles for various *R*

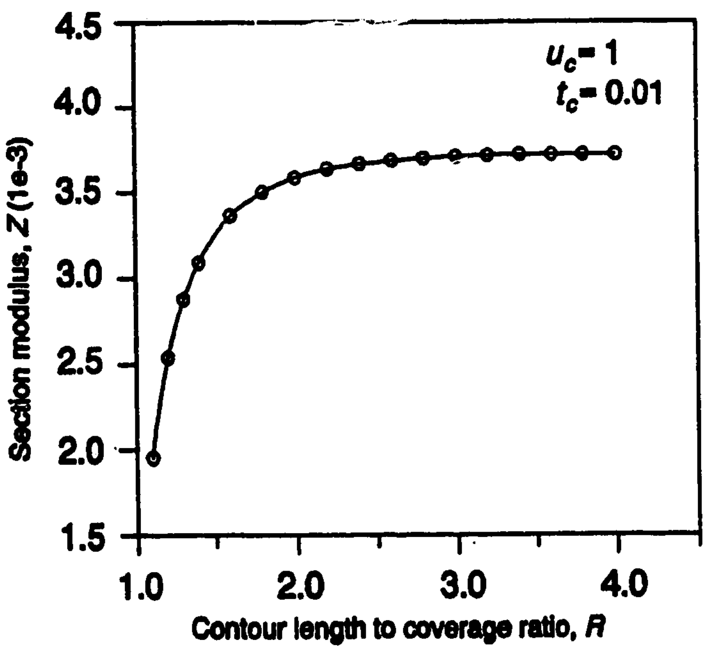

Figure 3-4: Section modulus for various *R*

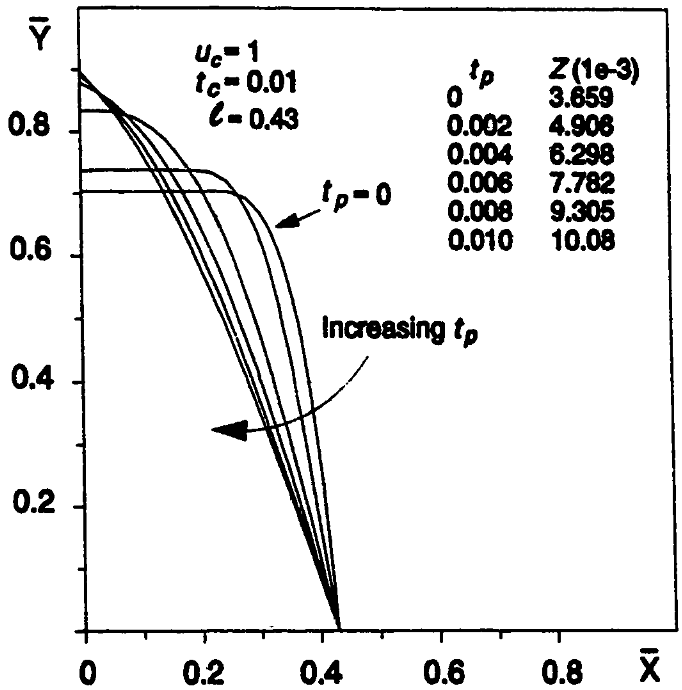

Figure 3-5: Program verification with various t_p

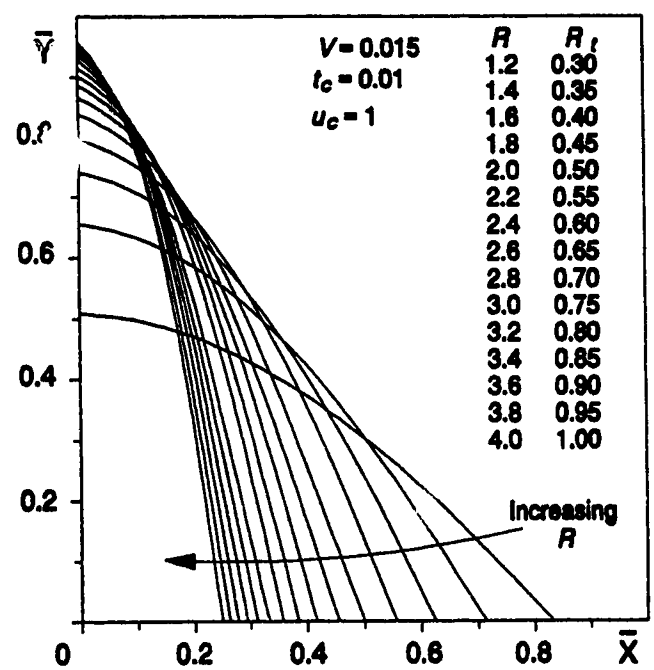

Figure 3-6: Sandwich profiles with the same V

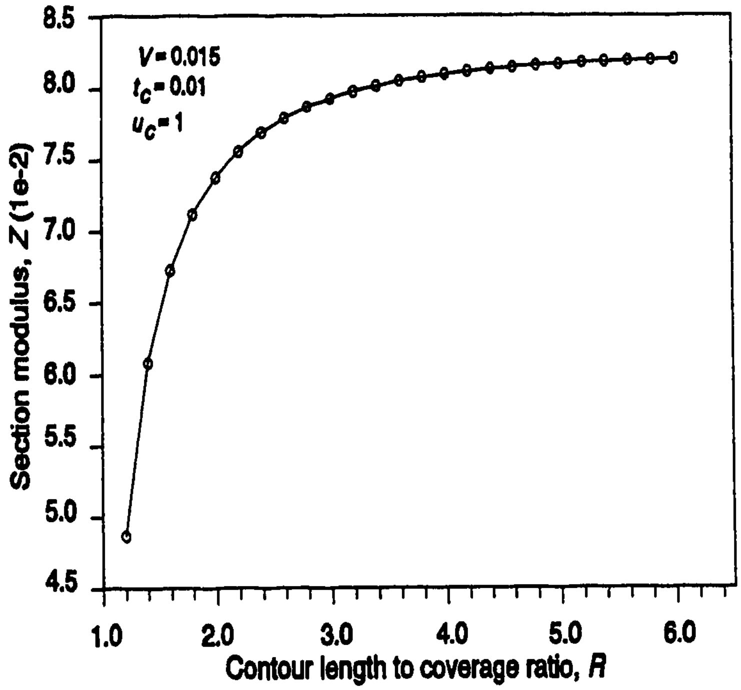

Figure 3-7: Section modulus of sandwich panels with the same *V*

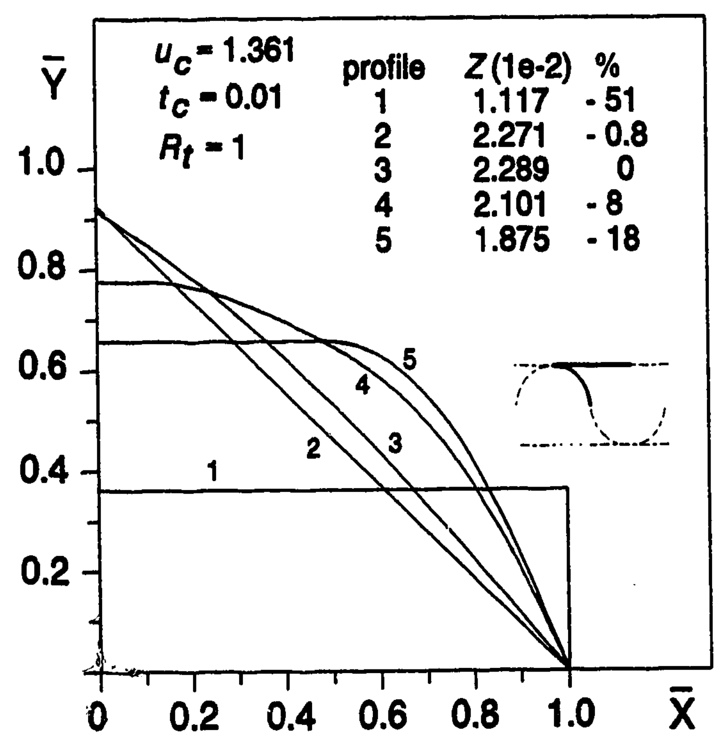

Figure 3-8: Section modulus of various cladding cores

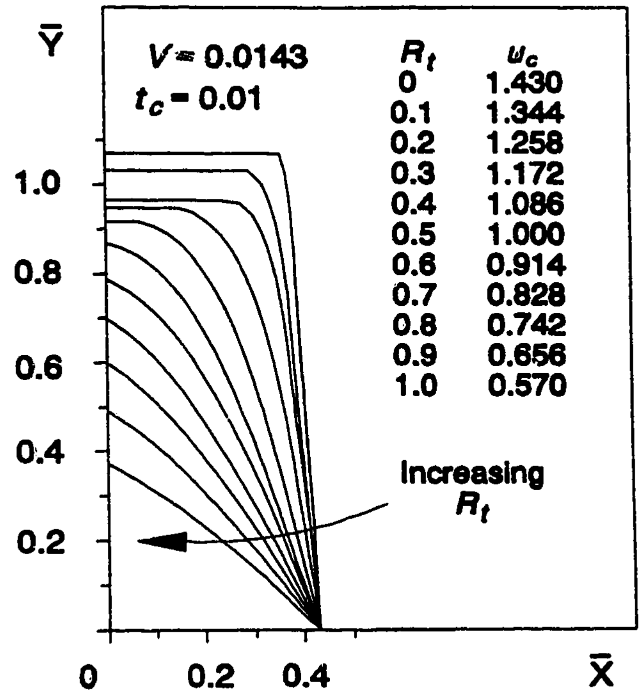

Figure 3-9: Sandwich claddings with various R_t

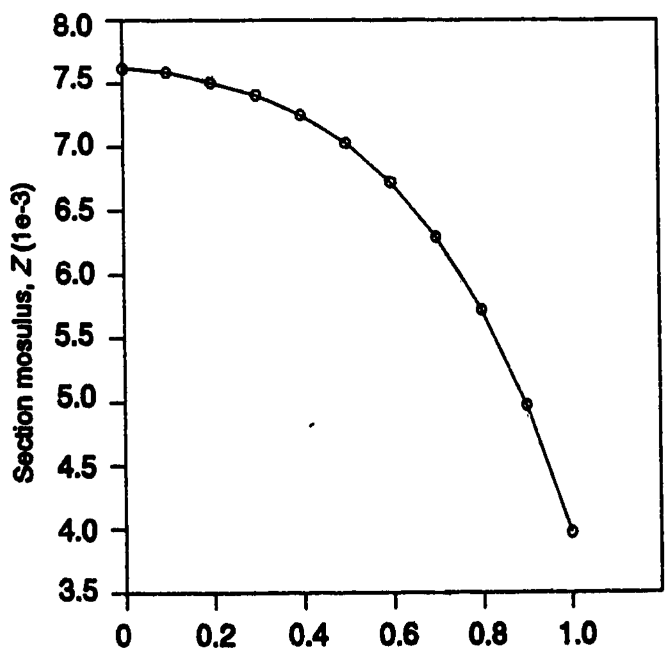

Figure 3-10: Section modulus as a function of $R_t(t_c=0.01)$

Chapter 4 Non-symmetrical claddings

This chapter is devoted to application of the local variation method to problems in which the cladding profile is non-symmetrical about the horizontal axis. For example, if the thickness of the panels for the sandwich configuration shown in figure 1-3 are not equal or only one panel is used as illustrated in figure 1-4, non-symmetrical profiles will result. In this chapter, the later case is considered. Due to this non-symmetrical properties, the location of the centroidal axis can no longer be predicted but has to be determined at every variation. Because there will be only one axis of symmetry for each complete profile, half of the profile will be considered instead of a quarter of the profile as in the doubly symmetrical or sandwich case. The necessary modifications to the numerical procedure for this application will be discussed in the following together with the verification of the modified procedure and some sample results.

4.1 Program Modification and Verification for Non-Symmetrical Configuration

The numerical procedure was modified so that the centroidal axis is determined whenever the cladding geometry is altered during the variation process. In this way the section modulus can be determined accurately. The y coordinate of the centroidal axis is found from:

$$y_c = \frac{\sum_{n=1}^{m} \bar{y} \Delta u t_c + y_p t_p l}{\sum_{n=1}^{m} \Delta u t_c + t_p l} \tag{4-1}$$

As was the case for the sandwich claddings, no existing result can be used to verify the modified procedure so that limiting cases were used. The profiles obtained are expected to vary with the thickness ratio R_t, and the coverage. When $t_p=0$, the final profiles should be symmetrical and are the same as those obtained in chapter 3 for the doubly symmetrical claddings for the same contour length to coverage ratio R and same constraints.

Figure 4-1 and 4-2 show the results obtained for the non-symmetrical case using the modified procedure with $u_c=2$ and $t_p=0$ at various R values ranging from 1.2 to 4.0. The starting profile for all the half profiles obtained was a straight line which was divided into 4 segments for figure 4-1 and into 8 segments for figure 4-2. The initial angle of variation $\Delta\theta_0$ was at 0.1 rads for all half profiles and there was no constraint on profile height. Both figures show that all half profiles obtained are symmetric about their centroidal axes which are located at the midpoint of the two extreme points in the y direction. The column 'corrugated' in Table 4-1 lists the section moduli of a half of the doubly symmetrical profiles shown in figure 3-3. It shows that in terms of the section modulus, the profiles obtained with a 4-segment starting profile show larger deficiencies in comparison to results obtained for the corrugated configuration shown previously in figure 3-3 than those obtained with an 8-segment starting profile. To obtain the quarter profiles shown in figure 3-3, a 4-segment profile was used; however this meant that only 2 segments were available for each half of the profile in figure 4-1. This resulted in an insufficient number of degrees of freedom to obtain good approximate optimal solutions. With an 8-segment starting profile which is equivalent to 4 segments per quarter profile, the obtained section moduli are much closer to those shown in figure 3-3. Figure 4-3 shows the bottom half of each of the profiles in figure 4-2. The shape of profiles with low R values are quite comparable to those in figure 3-3, but deviate more and more as the R value increases. Nevertheless, the section modulus is still very comparable with a maximum deficiency of 3.3% at R=2.6.

The profiles in figure 4-4 have the same contour length and coverage but with a different panel thickness. For clarity, the flat panel located at the top of each cladding is not shown. As the panel thickness increases, the cladding profile deviates further from its original symmetrical profile. In fact, a level region develops at the bottom part of the profile as the panel thickness increases. Such phenomena shows that the modified procedure is functioning properly because the maximum distance c in equation (2-1) is kept small while a higher area moment of inertia I_x is attained. The net result is that high section modulus is obtained.

In summary, the validity of the modified procedure to obtain optimal profiles without assuming symmetry appears justified. Instead of a 4-segment starting profile, 8 segments should be used to provide sufficient degrees of freedom.

4.2 Numerical Results for Non-Symmetrical Configuration

To investigate the relationship of a profile's section modulus to its coverage, a family of non-symmetrical profiles, as shown in figure 4-5, which have the same contour length and cladding core thickness but different coverage were generated. A fixed volume of material V, 0.025 cubic units, was used to construct both the cladding and the flat panel. The cladding thickness and contour length are arbitrary chosen at 0.01 and 2 respectively while the coverage is determined by the contour length to coverage ratio R. When span is chosen as 1, the panel thickness can be determined from:

$$t_p = \frac{R(V - t_c u_c)}{u_c} \tag{4-2}$$

Figure 4-6 shows the section modulus as a function of R up to 4.0. It is noted that the rate of increase in section modulus decreases with R and an asymptotic value of approximately 11.20×10^{-3} is attained for $R\geq3.4$ and $R_t\geq0.85$. Thus, it can be concluded that the optimal profile will be essentially a right angle when $R\geq3.4$ and $R_t\geq0.85$. No further improvement in Z can be made by increasing R for the same volume of material when t_c and u_c are kept constant. As an illustration for situations before this limit is reached, figure 4-7 shows the assembled configuration of a non-symmetrical cladding which has $R=2.0$ and $R_t=0.50$.

Figure 4-8 compares the shape and section modulus of an optimal profile with that of three other profiles which are a rectangular(1), a triangular(4) and a symmetrical corrugated profile(3) previously shown in figure 3-3. All the profiles have the same contour length and cladding core thickness while R and panel thickness are chosen at 2 and 0.01. As shown, the optimal profile provides the maximum section modulus which is about 40% higher than the triangular profile and 33% higher than profile 3. Compared to the rectangular profile, the optimal profile is still 5% better. Considering the resulting section modulus of profile 3, it can be seen that symmetrical profiles are no longer optimal for applications in which the centroidal axis does not coincide with the axis of symmetry. This is true especially when thickness ratio is relatively high(0.2). Obviously material is under utilized when the optimal profile is not used.

To investigate the effect of thickness ratio on section modulus, the profiles in figure 4-9 were constructed for sections with the same volume of material but having different thickness ratios. Setting the cladding thickness and coverage constant at 0.01 and 1, the contour length u_c for a thickness ratio R_t, is calculated from:

$$u_c = \frac{V - lR_t t_c}{t_c} \tag{4-3}$$

The result shows that similar to sandwich claddings, increasing the thickness ratio results in a drastic decrease of section modulus when t_c is kept constant(figure 4-10). Because maximum Z is obtained at $R_t=0$, the symmetrical corrugated configuration again proves that it is more efficient than either the sandwich or the non-symmetrical configuration for the same cladding thickness. Should a non-symmetrical cladding of a given t_c be constructed, the panel thickness should be minimized and more material should be used to increase u_c so that highest section modulus can be obtained.

Table 4-1: Comparison of section modului for non-symmetrical profiles obtained with starting profiles with 4 or 8 segments

	Z of a quarter profile (10^{-3})			Relative difference (%) wrt Corrugated	
R	Corrugat-ed	4-segment	8-segment	4-segment	8-segment
1.2	5.074	4.876	5.054	-3.9	-0.4
1.4	6.178	6.090	6.146	-1.4	-0.5
1.6	6.726	6.612	6.698	-1.7	-0.4
1.8	7.002	6.990	6.958	-0.2	-0.6
2.0	7.172	7.088	7.150	-1.2	-0.3
2.2	7.274	7.134	7.060	-1.9	-2.9
2.4	7.330	6.832	7.314	-6.8	-0.2
2.6	7.402	6.856	7.156	-7.4	-3.3
2.8	7.400	6.790	7.240	-8.2	-2.2
3.0	7.402	6.792	7.300	-8.2	-1.4
3.2	7.430	6.824	7.292	-8.2	-1.9
3.4	7.440	6.816	7.238	-8.4	-2.7
3.6	7.436	6.812	7.380	-8.4	-0.8
3.8	7.440	6.800	7.316	-8.6	-1.7
4.0	7.440	6.804	7.324	-8.6	-1.6

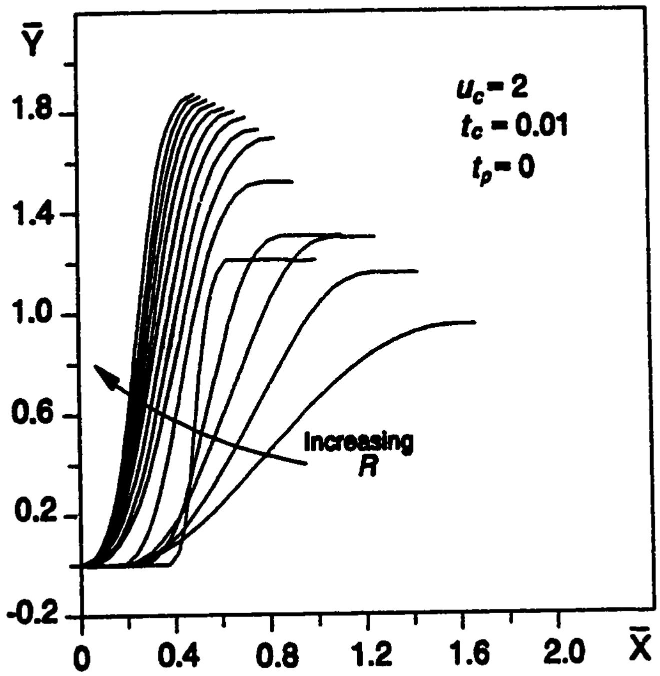

Figure 4-1: Program verification with a 4-segment starting profile

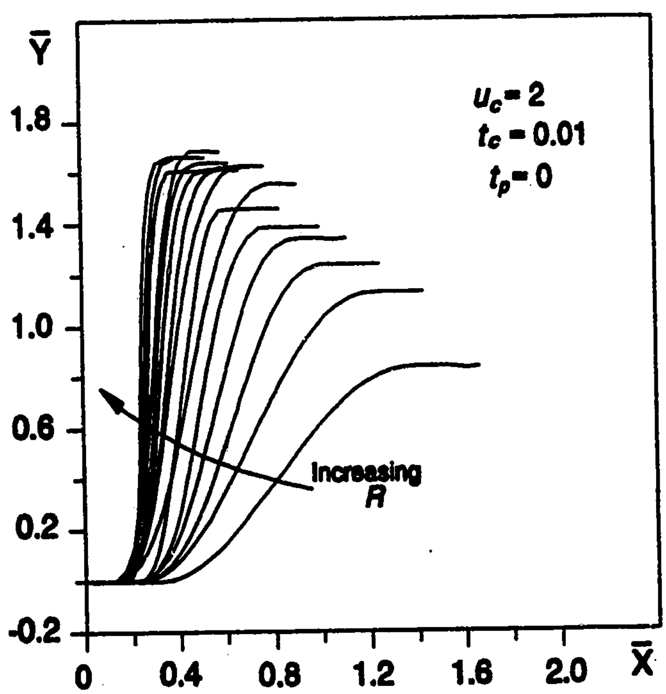

Figure 4-2: Program verification with an 8-segment starting profile

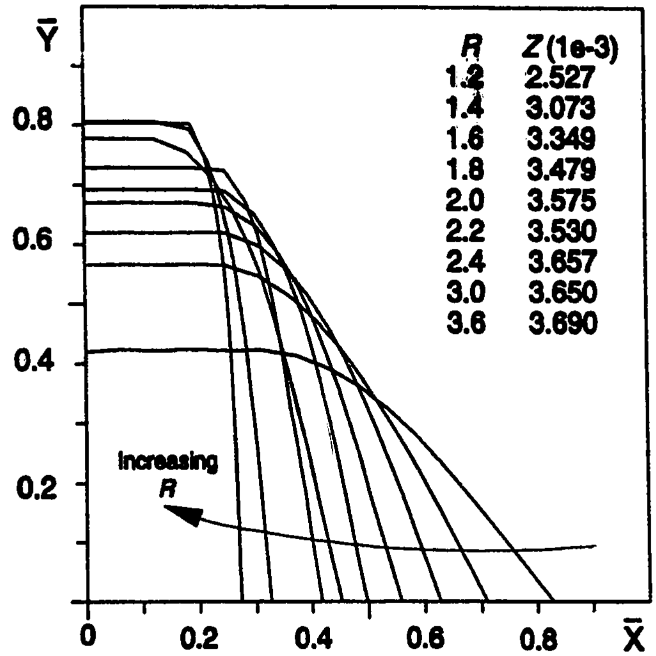

Figure 4-3: Bottom half of profiles in figure 4-2

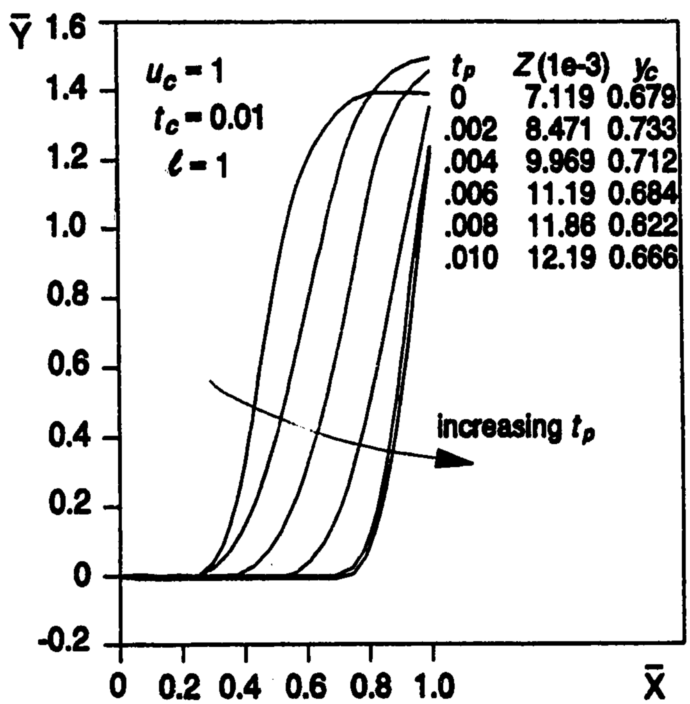

Figure 4-4: Verification with various t_p

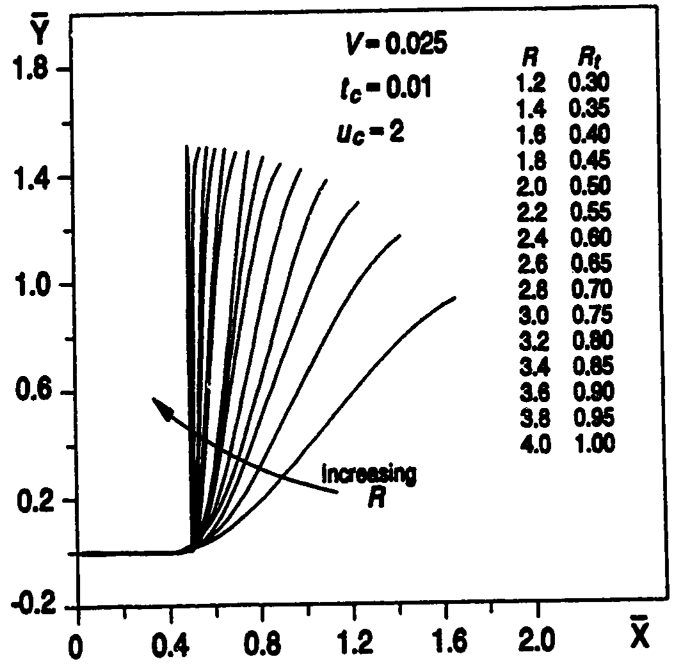

Figure 4-5: Set of non-symmetrical profiles with the same *V*

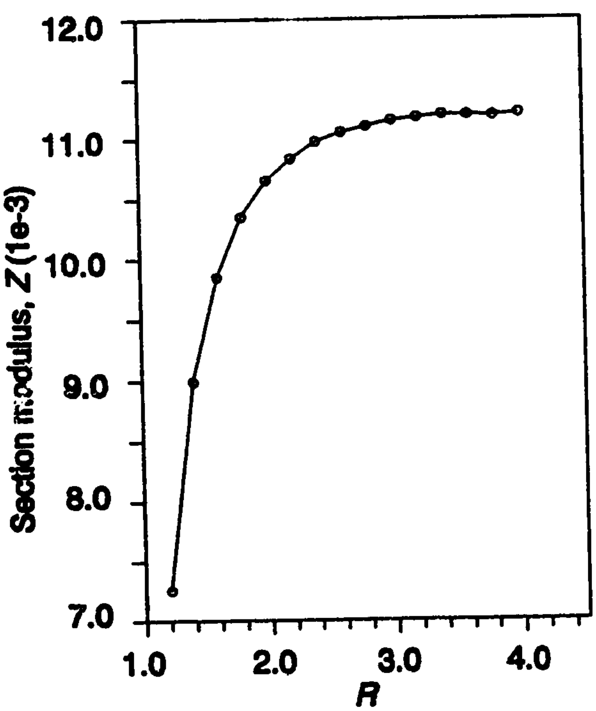

Figure 4-6: Section modulus as a function of *R*

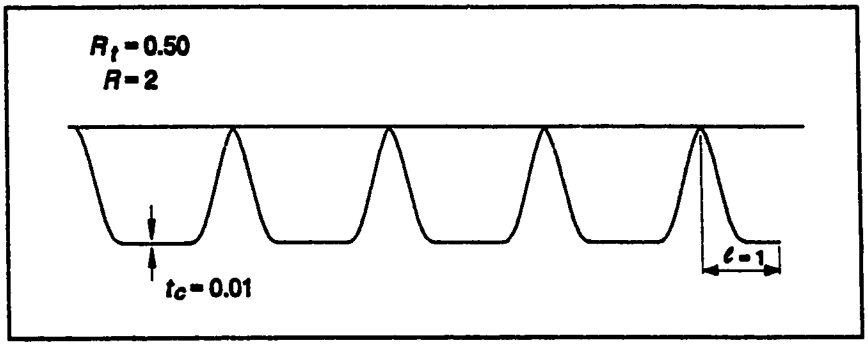

Figure 4-7: An assembled non-symmetric cladding

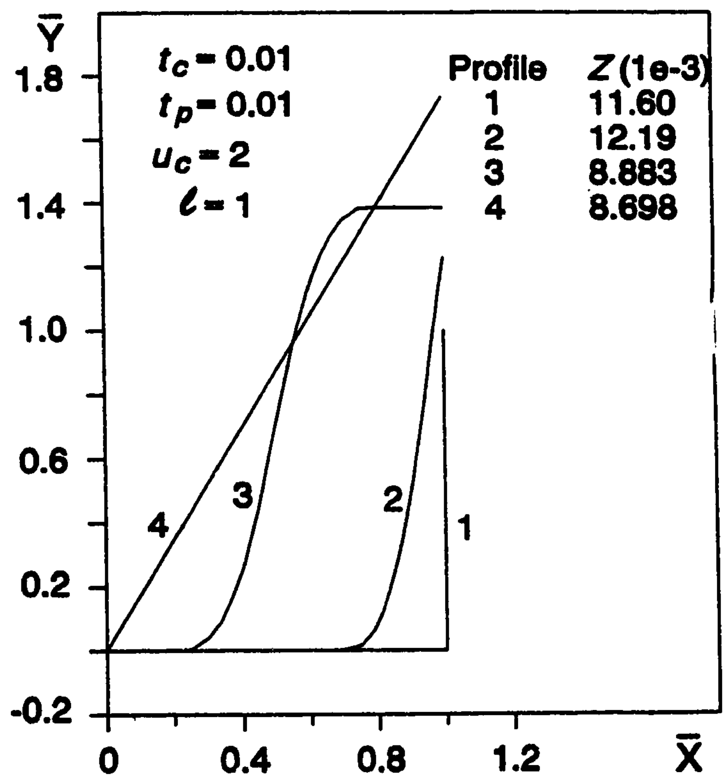

Figure 4-8: Comparison of 4 non-symmetrical profiles

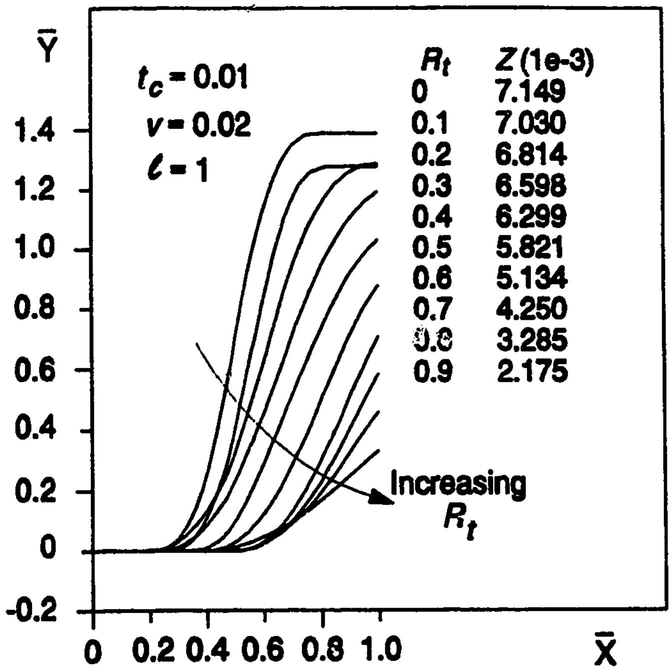

Figure 4-9: Effect of R_t to section modulus

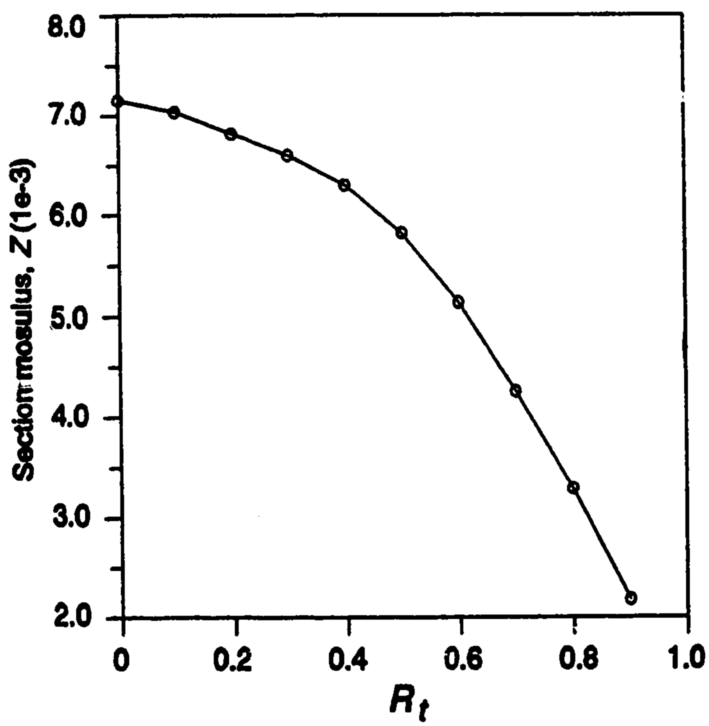

Figure 4-10: Section modulus as a function of R_t

Chapter 5 Conclusion

The method of local variation has been used to obtain optimal profiles of corrugated, sandwich and non-symmetrical cross sections under pure bending. The method is relatively straightforward in this application as it involves simple trigonometry in its formulation. Numerical results were shown to compare favourably with the limited published results which were obtained using a more analytical approach.

The numerical procedure based on this formulation is easily modified to solve for different configurations(i.e. simple corrugated to sandwich cross sections) and to include constraints on the geometry. For example, the consideration of non-symmetrical cases required an additional subroutine which calculated the centroidal axis for each iteration. This is, of course, not required for the symmetric cases.

The fundamental limitation of the local variation method is that global optimal solution is not guaranteed. Solutions obtained could be only locally optimal. However, the possibility of obtaining the global optimal is increased by first obtaining the approximate solution.

Selection of the angle of variation in the procedure is somewhat critical. In all of the sample results shown, the angle of variation $\Delta\theta$ was the same for all the segments at each variation. However during earlier development, it was found that there were cases in which the angle of variation $\Delta\theta$ had to be varied from segment to segment so that the optimal solution could be obtained. If a more refined profile is desired, a variable $\Delta\theta$ may prove advantageous. In general, both the size of the initial value of variation $\Delta\theta$ and the shape of the starting profile do not affect the final solution because a good approximate solution is first obtained.

The numerical procedure as currently formulated is unable to accurately reproduce solutions with very distinct shape discontinuities. This means that sections with a right angle are difficult to obtain as the segment may span the point at which the angle occurs. The number of segments used in the starting profile is also an important consideration. For example, for the doubly symmetrical claddings smaller section modulus resulted when 8 instead of 4 segments were used in the starting profile but better solution were obtained with 8 segments for the non-symmetrical cladding.

The results of the numerical work indicate that of the sections studied that the doubly symmetric corrugated configuration was the most efficient profile when a fixed volume of material is used. In addition, the section moduli are relatively insensitive to small deviations. This implies that large tolerances in profile geometry could be allowed in actual fabrication.

For all of the three configurations evaluated, the section modulus increases at a decreasing rate with the contour to coverage ratio R. For both the corrugated and non-symmetrical claddings when material volume, cladding contour length and cladding thickness are constant, this means that effectively a constant section modulus is reached as R increases. Thus caution must be taken not to use too large an R value as little will be gained in section modulus while the coverage decreases. For the sandwich and non-symmetrical cases with a given cladding thickness, the panel thickness should be kept at a minimum so that as much material as possible can be used to increase the cladding's contour length and obtain a higher section modulus.

Although it has been shown that there are cases that the shape and section moduli of the optimal profiles and some simple profiles such as circular or triangular are all relatively close in section modulus, this is only for certain special cases. For a more efficient use of material, the optimal profiles should be considered as an alternative before employing the simple profiles.

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Appendix

Computer program flow-charts

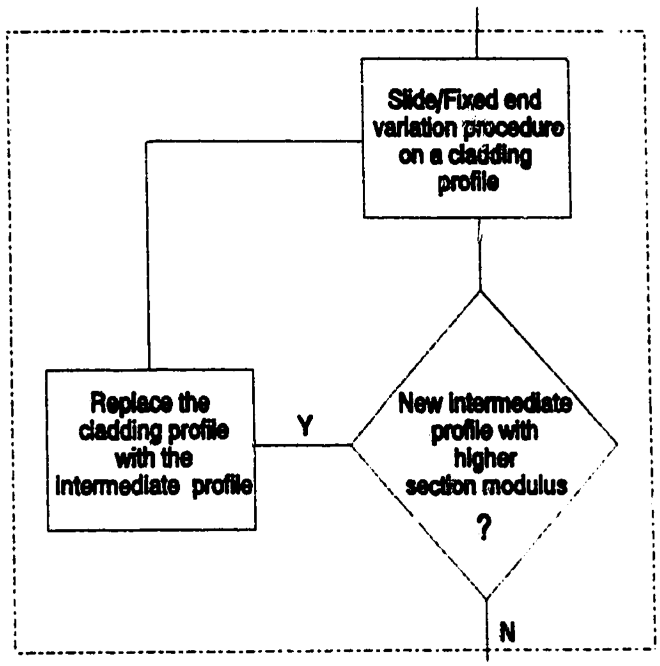

Figure A-1: Flow chart of variation procedure

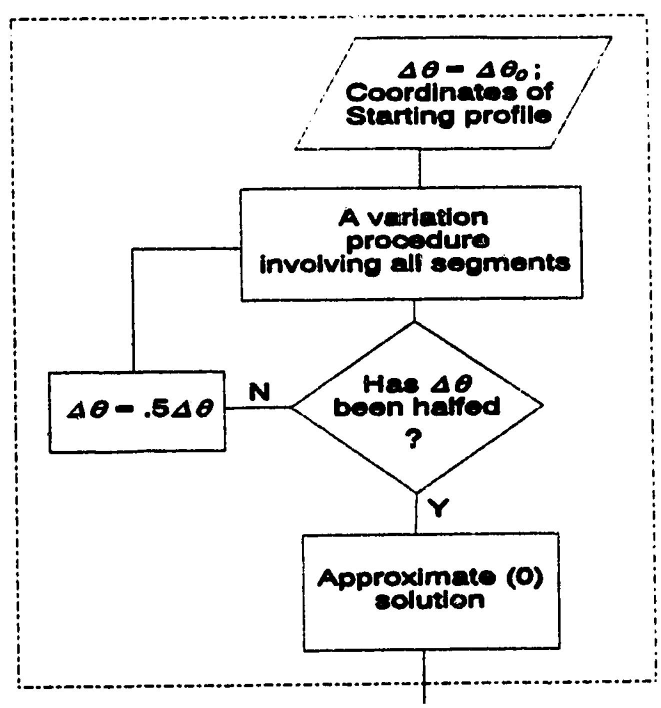

Figure A-2: Flow chart of approximate solution calculation

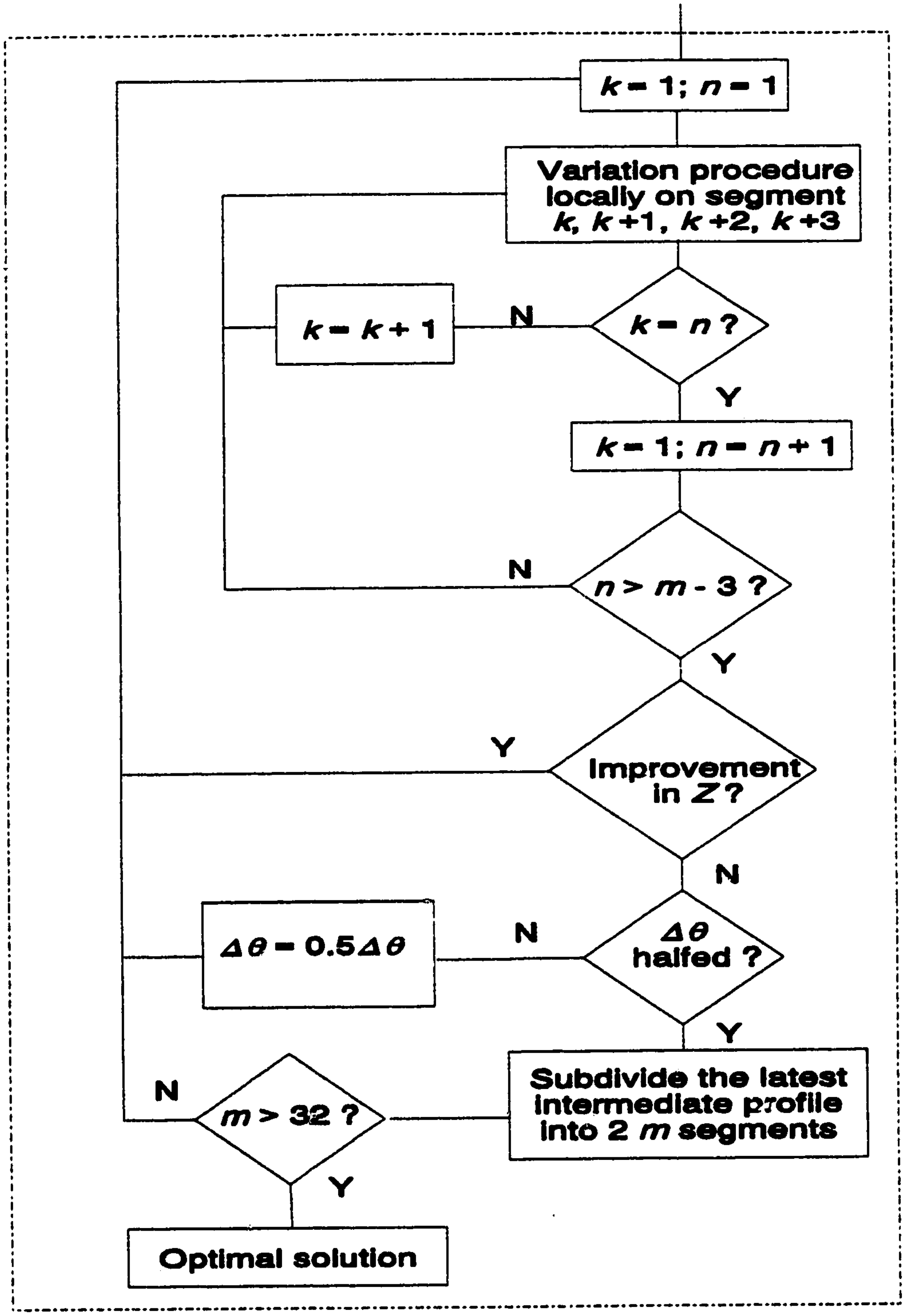

Figure A-3: Flow chart of optimal profile calculation

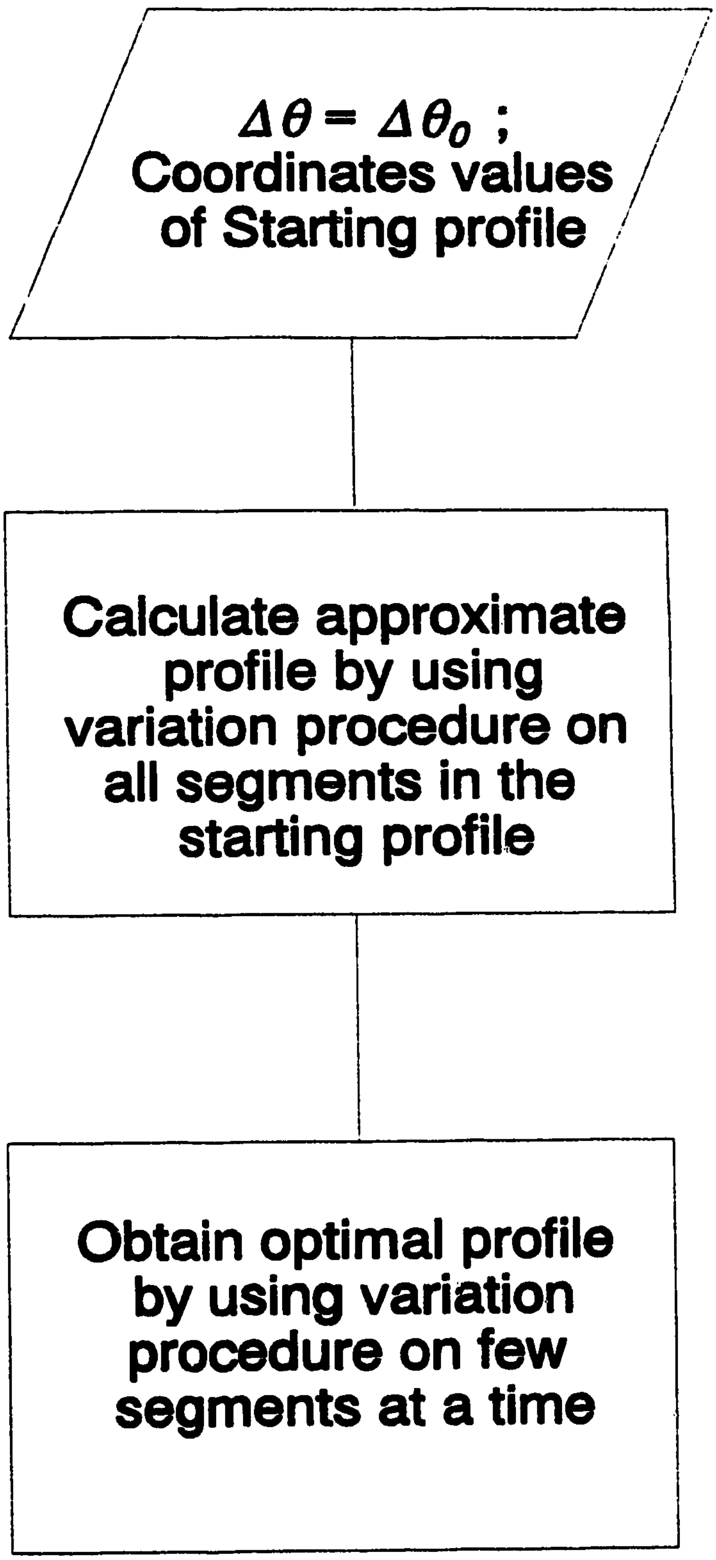

Figure A-4: Major steps of the numerical procedure

Sample computer program

```
'This is the program written in BASIC for the non-symmetrical case
'*********************************************************************

DECLARE SUB anglecheck (x2#, y2#, x1#, y1#, bata#)
DECLARE SUB approxsol (m#(), dtheta#, jnumseg%, maxintz#)
DECLARE SUB cenaxis (m#(), jnumseg%, ycen#, ymax#)
DECLARE FUNCTION dIx# (x1#, y1#, x2#, y2#, ycen#)
DECLARE SUB ducal (m#(), jnumseg%)
DECLARE SUB figgen (m#(), leng#, wid#, jnumseg%, shape#)
DECLARE SUB fixed412 (m#(), jn%, dtheta#, jnumseg%, intz#)
DECLARE SUB optimalsol (m#(), dtheta#, jnumseg%, maxintz#)
DECLARE SUB segdiv (m#(), jnumseg%, jm%)
DECLARE SUB slide41 (m#(), jn%, dtheta#, jnumseg%, intz#)
DECLARE SUB slide44 (m#(), jn%, dtheta#, jnumseg%, intz#)
DECLARE SUB slide81 (m#(), jn%, dtheta#, jnumseg%, intz#)
DECLARE SUB slide88 (m#(), jn%, dtheta#, jnumseg%, intz#)
DECLARE SUB zcal (m#(), jnumseg%, z#)

DEFINT J
DEFDBL A-I, K-Z
DIM cood(33, 4) 'dimension the coordinate array

'define constants and cladding parameters
'*********************************************************************
CONST pi = 3.14159265358979#
CONST length = 2#                'contour length of the cladding
CONST wid = 1#                   'width constraints of the cladding
CONST thickness = .01#           'thickness of the cladding core
CONST panelthickness = .01#      'thickness of the flat panel
CONST xpanel = 1#                'width of the panel
CONST thicknesses = (thickness + panelthickness) / 2
jorder = 2   'number of segments used in the starting profile = 2^jorder
jorderul = 5 'upper limit of jorder
dtheta = .1# 'initial angle of variation
inihigh = 0  'height of the initial profile use 0 for max possible height

jnumseg = 2 ^ jorder

'This subroutine returns the coordinate values which describe the shape of
'the starting profile and stores the values in the coordinate array cood()
CALL figgen(cood(), length, wid, jnumseg, inihigh)

'This sub routine determines the approximate solution and its section modulus
CALL approxsol(cood(), dtheta, jnumseg, maxintz)
```

```
'This DO LOOP determines the optimal profile and its section modulus
DO UNTIL jorder >= jorderul
  jorder = jorder + 1
  jnumseg = 2 ^ jorder
  CALL segdiv(cood(), jnumseg, 2 ^ (jorder - 1))
  CALL optimalsol(cood(), dtheta, jnumseg, maxintz)
LOOP

'The folowing statements stores the coordinate array cood() in a file
'and the optimal profile and its section modulus are shown on screen

OPEN "c:\profile.dat" FOR OUTPUT AS #1
FOR i = 1 to jnumseg+1 STEP 1
  print ""
  FOR j = 1 to 4 STEP 1
    PRINT #1, cood(i,j);
  NEXT j
NEXT i
CLOSE #1
CLS
SCREEN 12
VIEW (130, 20)-(450, 340), , 1
WINDOW (0, 0)-(3, 3)
FOR j = 1 TO jnumseg
  LINE (cood(j, 1), cood(j, 2))-(cood(j + 1, 1), cood(j + 1, 2)), 6
NEXT j
LOCATE 23
PRINT "Section mosulus z = "; maxintz


SUB anglecheck (x2, y2, x1, y1, bata)
'This sub-routine calculates the angle of inclination of a segment with respect
'to point x1, y1
'x1, y1 = the point of reference
'x2, y2 = the second point of a segment
'bata = angle of inclination

IF ABS(x2 - x1) < 1E-16 THEN
  IF y2 - y1 > 0 THEN bata = pi / 2
  IF y2 - y1 < 0 THEN bata = -pi / 2
ELSEIF x2 - x1 <> 0 THEN
  bata = ATN((y2 - y1) / (x2 - x1))
  IF x2 - x1 < 0 THEN
    IF y2 - y1 < 0 THEN bata = -pi + bata
```

```
    IF y2 - y1 > 0 THEN bata = pi + bata
  END IF
END IF
END SUB


SUB approxsol (m(), dtheta, jnumseg, maxintz)
'This sub-routine calculates the approximate solution based on the
'starting profile
'm() = coordinate array which will be changed if an intermediate profile exists
'dtheta = angle of variation
'jnumseg = number of segment used in the starting profile

CALL zcal(m(), jnumseg, maxintz)
tempmaxintz = maxintz
FOR jhalf = 1 TO 2 STEP 1
  dtheta = dtheta / jhalf
  SELECT CASE jnumseg
  CASE 4
    DO
      FOR j = 1 TO 4 STEP 1
        CALL anglecheck(m(j, 1), m(j, 2), m(j + 1, 1), m(j + 1, 2), m(j, 4))
      NEXT j
      CALL slide41(m(), 1, dtheta, jnumseg, tempmaxintz)
      diff = (tempmaxintz - maxintz) / maxintz
      IF diff > 1E-16 THEN maxintz = tempmaxintz
      FOR j = 1 TO 4 STEP 1
        CALL anglecheck(m(j + 1, 1), m(j + 1, 2), m(j, 1), m(j, 2), m(j, 4))
      NEXT j
      CALL slide44(m(), 4, dtheta, jnumseg, tempmaxintz)
      diff = (tempmaxintz - maxintz) / maxintz
      IF diff > 1E-16 THEN maxintz = tempmaxintz
    LOOP UNTIL diff <= 1E-16
  CASE 8
    DO
      FOR j = 1 TO 8 STEP 1
        CALL anglecheck(m(j, 1), m(j, 2), m(j + 1, 1), m(j + 1, 2), m(j, 4))
      NEXT j
      CALL slide81(m(), 1, dtheta, jnumseg, tempmaxintz)
      diff = (tempmaxintz - maxintz) / maxintz
      IF diff > 1E-16 THEN maxintz = tempmaxintz
      FOR j = 1 TO 8 STEP 1
        CALL anglecheck(m(j + 1, 1), m(j + 1, 2), m(j, 1), m(j, 2), m(j, 4))
      NEXT j
```

```
      CALL slide88(m(), 8, dtheta, jnumseg, tempmaxintz)
      diff = (tempmaxintz - maxintz) / maxintz
      IF diff > 1E-16 THEN maxintz = tempmaxintz
    LOOP UNTIL diff <= 1E-16
  END SELECT
NEXT jhalf
END SUB


SUB cenaxis (m(), jnumseg, ycen, ymax)
'This sub-routine determines the location of the centroidal axis(ycen) and the
'maximum value of the y coordinate(ymax)
'm() = coordinate array
'ycen = y location of the centrodial axis W.R.T (0,0)
'jnumseg = number of segment used in the current profile

ymax = m(jnumseg + 1, 2)
FOR j = 1 TO jnumseg STEP 1
  IF ymax < m(j, 2) THEN ymax = m(j, 2)
  sumyda = sumyda + (m(j, 2) + m(j + 1, 2)) / 2 * m(j, 3)
  sumda = sumda + m(j, 3)
NEXT j
sumyda = sumyda * thickness + xpanel * panelthickness * (ymax + thicknesses)
sumda = sumda * thickness + xpanel * panelthickness
ycen = sumyda / sumda
END SUB


FUNCTION dIx (x1, y1, x2, y2, ycen)
'This sub-routine calculates the area moment of inertia of a segment(dIx) wrt
'the calculated centroidal axis(ycen)
'x1, y1, x2, y2 = coordinate values of the segment end points
'ycen = y location of the centroidal axis
  dlsq = (x2 - x1) ^ 2 + (y2 - y1) ^ 2
  sinsqtheta = (y2 - y1) ^ 2 / dlsq
  barysq = (y1 + y2 - 2 * ycen) ^ 2 / 4
  dIx = thickness * dlsq ^ .5 * (dlsq * sinsqtheta / 12 + barysq)
END FUNCTION


SUB ducal (m(), jnumseg)
'This sub-routine calculates the length of each segment
'm() = coordinate array
'jnumseg = number of segments used in the current profile
```

```
FOR j = 1 TO jnumseg STEP 1
  m(j, 3) = SQR((m(j, 1) - m(j + 1, 1)) ^ 2 + (m(j, 2) - m(j + 1, 2)) ^ 2)
NEXT j
END SUB


SUB figgen (m(), leng, wid, jnumseg, shape)
'This sub-routine generates a profile which consists two equal length segments
'and then divides them evenly into more segments using SUB segdiv()
'm() = the coordinate array to be generated which contains the x,y values of
'     the segment end points
'leng = contour length of the profile
'wid = width of the profile
'jnumseg = number of segments used in the starting profile
'shape = specified height of maximum possible height(0 for maximum possible
'height)

height = SQR(leng ^ 2 - wid ^ 2)
IF shape > 0 AND shape < height THEN
  theta = ATN(shape / wid)
  dl = SQR(leng ^ 2 - (shape ^ 2 + wid ^ 2)) / 2
  m(1, 1) = 0
  m(1, 2) = shape
  m(3, 1) = wid
  m(3, 2) = 0
  m(2, 1) = (m(1, 1) + m(3, 1)) / 2 + dl * SIN(theta)
  m(2, 2) = (m(1, 2) + m(3, 2)) / 2 + dl * COS(theta)
ELSE
  m(1, 1) = 0
  m(1, 2) = height
  m(3, 1) = wid
  m(3, 2) = 0
  m(2, 1) = (m(1, 1) + m(3, 1)) / 2
  m(2, 2) = (m(1, 2) + m(3, 2)) / 2
END IF
CALL segdiv(m(), jnumseg, 2)
CALL ducal(m(), jnumseg)
IF jnumseg = 8 THEN
  CALL segdiv(m(), jnumseg, 4)
  CALL ducal(m(), jnumseg)
END IF
END SUB
```

```
SUB fixed412 (m(), jn, dtheta, jnumseg, intz)
'This sub-routine performs the fix-end variation procedure using 4 segments
'by rotating segments 3 and 4
'm() = coordinate array of the profile to be perturbed
'jn = segment number of the first segment of the current 4 segments being perturbed
'dtheta = angle of variation
'jnumseg = number of segment used in the current profile
'intz = section modulus to be returned

DIM coodsol(2), orig(5, 4), jident(2)
jflag = 0
FOR j = 1 TO 5 STEP 1
  orig(j, 1) = m(jn + j - 1, 1)
  orig(j, 2) = m(jn + j - 1, 2)
  orig(j, 3) = m(jn + j - 1, 3)
  orig(j, 4) = m(jn + j - 1, 4)
NEXT j
FOR J1 = -1 TO 1 STEP 1
  m(jn + 3, 1) = m(jn + 4, 1) + orig(4, 3) * COS(orig(4, 4) + J1 * dtheta)
  m(jn + 3, 2) = m(jn + 4, 2) + orig(4, 3) * SIN(orig(4, 4) + J1 * dtheta)
  FOR J2 = -1 TO 1 STEP 1
    m(jn + 2, 1) = m(jn + 3, 1) + orig(3, 3) * COS(orig(3, 4) + J2 * dtheta)
    m(jn + 2, 2) = m(jn + 3, 2) + orig(3, 3) * SIN(orig(3, 4) + J2 * dtheta)
    lengthsq = (m(jn + 2, 1) - m(jn, 1)) ^ 2 + (m(jn + 2, 2) - m(jn, 2)) ^ 2
    lengthsumsq = (m(jn + 1, 3) + m(jn, 3)) ^ 2
    fdiff = lengthsumsq - lengthsq
    xbar = (m(jn, 1) + m(jn + 2, 1)) / 2
    ybar = (m(jn, 2) + m(jn + 2, 2)) / 2
    IF fdiff > 1E-16 THEN
    hsq = m(jn + 1, 3) ^ 2 - lengthsq / 4
      IF hsq < 0 THEN hsq = m(jn, 3) ^ 2 - lengthsq / 4
      h = SQR(hsq)
      IF ABS(m(jn, 1) - m(jn + 2, 1)) < 1E-16 THEN
        IF m(jn, 2) - m(jn + 2, 2) > 0 THEN alpha = pi / 2
        IF m(jn, 2) - m(jn + 2, 2) < 0 THEN alpha = -pi / 2
      ELSE
        alpha = ATN((m(jn, 2) - m(jn + 2, 2)) / (m(jn, 1) - m(jn + 2, 1)))
      END IF
      FOR J3 = 1 TO -1 STEP -2
        IF 0# <= alpha AND alpha <= pi / 2 THEN
          m(jn + 1, 2) = ybar + J3 * h * COS(alpha)
          m(jn + 1, 1) = xbar - J3 * h * SIN(alpha)
        ELSEIF -pi / 2 <= alpha AND alpha <= 0# THEN
          m(jn + 1, 2) = ybar + J3 * h * COS(-alpha)
```

```
            m(jn + 1, 1) = xbar + J3 * h * SIN(-alpha)
          END IF
          CALL zcal(m(), jnumseg, z)
          IF intz < z THEN
            intz = z
            jident(1) = J1
            jident(2) = J2
            coodsol(1) = m(jn + 1, 1)
            coodsol(2) = m(jn + 1, 2)
            jflag = 1
          END IF
        NEXT J3
      END IF
    NEXT J2
  NEXT J1
  IF jflag = 1 THEN
    m(jn + 3, 4) = orig(4, 4) + jident(1) * dtheta
    m(jn + 2, 4) = orig(3, 4) + jident(2) * dtheta
    m(jn + 3, 1) = m(jn + 4, 1) + orig(4, 3) * COS(m(jn + 3, 4))
    m(jn + 3, 2) = m(jn + 4, 2) + orig(4, 3) * SIN(m(jn + 3, 4))
    m(jn + 2, 1) = m(jn + 3, 1) + orig(3, 3) * COS(m(jn + 2, 4))
    m(jn + 2, 2) = m(jn + 3, 2) + orig(3, 3) * SIN(m(jn + 2, 4))
    m(jn + 1, 1) = coodsol(1)
    m(jn + 1, 2) = coodsol(2)
    CALL anglecheck(m(jn + 1, 1), m(jn + 1, 2), m(jn + 2, 1), m(jn + 2, 2),
              m(jn + 1, 4))
    CALL anglecheck(m(jn, 1), m(jn, 2), m(jn + 1, 1), m(jn + 1, 2), m(jn, 4))
  ELSEIF jflag = 0 THEN
    FOR j = 1 TO 5 STEP 1
      m(jn + j - 1, 1) = orig(j, 1)
      m(jn + j - 1, 2) = orig(j, 2)
      m(jn + j - 1, 3) = orig(j, 3)
      m(jn + j - 1, 4) = orig(j, 4)
    NEXT j
  END IF
  END SUB


  SUB optimalsol (m(), dtheta, jnumseg, maxintz)
  'This sub-routine obtains the optimal solution by using the appropriate variation
  'procedure(mainly the fixed-end procedure) to sets of segment of 4
  ' m() = coordinate array
  ' dtheta = angle of variation
  ' jnumseg = number of segments used in the current profile
```

```
'maxintz = section modulus to be returned

FOR jhalf = 1 TO 2 STEP 1
dtheta = dtheta / jhalf
tempmaxintz = maxintz
    FOR j = 1 TO jnumseg STEP 1
      CALL anglecheck(m(j, 1), m(j, 2), m(j + 1, 1), m(j + 1, 2), m(j, 4))
    NEXT j
    FOR jlastseg = 1 TO jnumseg - 3 STEP 1
      FOR jcurrentseg = 1 TO jlastseg STEP 1
        DO
          IF jcurrentseg = 1 THEN
            CALL slide41(m(), jcurrentseg, dtheta, jnumseg, tempmaxintz)
            diff = (tempmaxintz - maxintz) / maxintz
            IF diff > 1E-16 THEN maxintz = tempmaxintz
            CALL fixed412(m(), jcurrentseg, dtheta, jnumseg, tempmaxintz)
            diff = (tempmaxintz - maxintz) / maxintz
            IF diff > 1E-16 THEN maxintz = tempmaxintz
          ELSEIF jcurrentseg > 1 AND jcurrentseg < jnumseg THEN
            CALL fixed412(m(), jcurrentseg, dtheta, jnumseg, tempmaxintz)
            diff = (tempmaxintz - maxintz) / maxintz
            IF diff > 1E-16 THEN maxintz = tempmaxintz
          ELSEIF jcurrentseg = jnumseg THEN
            CALL slide44(m(), jcurrentseg, dtheta, jnumseg, tempmaxintz)
            diff = (tempmaxintz - maxintz) / maxintz
            IF diff > 1E-16 THEN maxintz = tempmaxintz
            CALL fixed412(m(), jcurrentseg, dtheta, jnumseg, tempmaxintz)
            diff = (tempmaxintz - maxintz) / maxintz
            IF diff > 1E-16 THEN maxintz = tempmaxintz
          END IF
        LOOP UNTIL diff <= 1E-16
      NEXT jcurrentseg
    NEXT jlastseg
NEXT jhalf
END SUB


SUB segdiv (m(), jnumseg, jm)
'This sub-routine divides the number of segment from jm segments to 2*(jm)
'segments
'm() = coordinate array
'jm = number of segments being divided

FOR j = jm + 1 TO 2 STEP -1
```

```
  m(2 * j - 1, 1) = m(j, 1)
  m(2 * j - 1, 2) = m(j, 2)
  m(2 * j - 1, 3) = m(j, 3)
  m(2 * j - 1, 4) = m(j, 4)
NEXT j
FOR j = jm + 1 TO 2 STEP -1
  m(2 * j - 2, 1) = (m(2 * j - 1, 1) + m(2 * j - 3, 1)) / 2
  m(2 * j - 2, 2) = (m(2 * j - 1, 2) + m(2 * j - 3, 2)) / 2
NEXT j
CALL ducal(m(), jnumseg)
END SUB


SUB slide41 (m(), jn, dtheta, jnumseg, intz)
'This sub-routine performs the slide-end variation procedure with 4 segments
'by rotating segments 2, 3 and 4
'm() = coordinate array
'jn = the starting segment(segment 1)
'dtheta = angle of variation
'jnumseg = number of segment used in the current profile
'intz = maximum section modulus being returned

DIM jident(4), orig(5, 4),coodsol
jflag = 0
FOR j = 1 TO 5 STEP 1
  orig(j, 1) = m(jn + j - 1, 1)
  orig(j, 2) = m(jn + j - 1, 2)
  orig(j, 3) = m(jn + j - 1, 3)
  orig(j, 4) = m(jn + j - 1, 4)
NEXT j
FOR J4 = 1 TO -1 STEP -1
  m(jn + 3, 1) = m(jn + 4, 1) + m(jn + 3, 3) * COS(m(jn + 3, 4) + J4 * dtheta)
  m(jn + 3, 2) = m(jn + 4, 2) + m(jn + 3, 3) * SIN(m(jn + 3, 4) + J4 * dtheta)
FOR J3 = 1 TO -1 STEP -1
  m(jn + 2, 1) = m(jn + 3, 1) + m(jn + 2, 3) * COS(m(jn + 2, 4) + J3 * dtheta)
  m(jn + 2, 2) = m(jn + 3, 2) + m(jn + 2, 3) * SIN(m(jn + 2, 4) + J3 * dtheta)
FOR J2 = 1 TO -1 STEP -1
  m(jn + 1, 1) = m(jn + 2, 1) + m(jn + 1, 3) * COS(m(jn + 1, 4) + J2 * dtheta)
  m(jn + 1, 2) = m(jn + 2, 2) + m(jn + 1, 3) * SIN(m(jn + 1, 4) + J2 * dtheta)
  IF ABS(m(jn + 1, 1) - m(jn, 1)) <= m(jn, 3) THEN
    FOR J1 = 1 TO -1 STEP -2
      m(jn, 2) = m(jn + 1, 2) + J1 * SQR(m(jn, 3) ^ 2 - (m(jn, 1) - m(jn + 1, 1))
                   ^ 2)
      CALL zcal(m(), jnumseg, z)
```

```
      IF intz < z THEN
        intz = z
        jident(1) = J1
        jident(2) = J2
        jident(3) = J3
        jident(4) = J4
        coodsol = m(jn, 2)
        jflag = 1
      END IF
    NEXT J1
  END IF
NEXT J2
NEXT J3
NEXT J4
IF jflag = 1 THEN
  FOR j = 3 TO 1 STEP -1
    m(jn + j, 4) = m(jn + j, 4) + jident(j + 1) * dtheta
  NEXT j
  FOR j = 3 TO 1 STEP -1
    m(jn + j, 1) = m(jn + j + 1, 1) + m(jn + j, 3) * COS(m(jn + j, 4))
    m(jn + j, 2) = m(jn + j + 1, 2) + m(jn + j, 3) * SIN(m(jn + j, 4))
  NEXT j
  m(jn, 2) = coodsol
  CALL anglecheck(m(jn, 1), m(jn, 2), m(jn + 1, 1), m(jn + 1, 2), m(jn, 4))
ELSEIF jflag = 0 THEN
  FOR j = 1 TO 5 STEP 1
      m(jn + j - 1, 1) = orig(j, 1)
      m(jn + j - 1, 2) = orig(j, 2)
      m(jn + j - 1, 3) = orig(j, 3)
      m(jn + j - 1, 4) = orig(j, 4)
  NEXT j
END IF
END SUB


SUB slide44 (m(), jn, dtheta, jnumseg, intz)
'This sub-routine performs the slide-end variation procedure with 4 segments
'by rotating segments 1, 2 and 3
'm() = the array contains the coordinates of each point
'jn = the starting segment(segment 4)
'dtheta = angle of variation
'jnumseg = number of segment used in the current profile
'intz = section modulus to be returned
```

```
DIM jident(4), orig(5, 4), coodsol
jflag = 0
FOR j = 1 TO 5 STEP 1
  orig(j, 1) = m(jn - 4 + j, 1)
  orig(j, 2) = m(jn - 4 + j, 2)
  orig(j, 3) = m(jn - 4 + j, 3)
  orig(j, 4) = m(jn - 4 + j, 4)
NEXT j
FOR J1 = 1 TO -1 STEP -1
  m(jn - 2, 1) = m(jn - 3, 1) + m(jn - 3, 3) * COS(m(jn - 3, 4) + J1 * dtheta)
  m(jn - 2, 2) = m(jn - 3, 2) + m(jn - 3, 3) * SIN(m(jn - 3, 4) + J1 * dtheta)
FOR J2 = 1 TO -1 STEP -1
  m(jn - 1, 1) = m(jn - 2, 1) + m(jn - 2, 3) * COS(m(jn - 2, 4) + J2 * dtheta)
  m(jn - 1, 2) = m(jn - 2, 2) + m(jn - 2, 3) * SIN(m(jn - 2, 4) + J2 * dtheta)
FOR J3 = 1 TO -1 STEP -1
  m(jn, 1) = m(jn - 1, 1) + m(jn - 1, 3) * COS(m(jn - 1, 4) + J3 * dtheta)
  m(jn, 2) = m(jn - 1, 2) + m(jn - 1, 3) * SIN(m(jn - 1, 4) + J3 * dtheta)
  IF ABS(m(jn, 1) - m(jn + 1, 1)) <= m(jn, 3) THEN
    FOR J4 = 1 TO -1 STEP -2
      m(jn + 1, 2) = m(jn, 2) + J4 * SQR(m(jn, 3) ^ 2 - (m(jn + 1, 1) - m(jn, 1))
                    ^ 2)
      CALL zcal(m(), jnumseg, z)
      IF intz < z THEN
        intz = z
        jident(1) = J1
        jident(2) = J2
        jident(3) = J3
        jident(4) = J4
        coodsol = m(jn + 1, 2)
        jflag = 1
      END IF
    NEXT J4
  END IF
 NEXT J3
 NEXT J2
 NEXT J1
 IF jflag = 1 THEN
   FOR j = 3 TO 1 STEP -1
     m(jn - j, 4) = m(jn - j, 4) + jident(4 - j) * dtheta
   NEXT j
   FOR j = 3 TO 1 STEP -1
     m(jn - j + 1, 1) = m(jn - j, 1) + m(jn - j, 3) * COS(m(jn - j, 4))
     m(jn - j + 1, 2) = m(jn - j, 2) + m(jn - j, 3) * SIN(m(jn - j, 4))
   NEXT j
```

```
  m(jn + 1, 2) = coodsol
  CALL anglecheck(m(jn + 1, 1), m(jn + 1, 2), m(jn, 1), m(jn, 2), m(jn, 4))
ELSEIF jflag = 0 THEN
  FOR j = 1 TO 5 STEP 1
    m(jn - 4 + j, 1) = orig(j, 1)
    m(jn - 4 + j, 2) = orig(j, 2)
    m(jn - 4 + j, 3) = orig(j, 3)
    m(jn - 4 + j, 4) = orig(j, 4)
  NEXT j
END IF
END SUB


SUB slide81 (m(), jn, dtheta, jnumseg, intz)
'This sub-routine performs the slide-end variation procedure with 8 segments
'by rotating segments 2, 3, 4, 5, 6, 7 and 8
'm() = coordinate array
'jn = starting segment(segment 1)
'dtheta = angle of variation
'jnumseg = number of segment used in the current profile
'intz = maximum section modulus being returned

DIM jident(8), coodsol, orig(9, 4)
jflag = 0
FOR j = 1 TO 9 STEP 1
  orig(j, 1) = m(jn + j - 1, 1)
  orig(j, 2) = m(jn + j - 1, 2)
  orig(j, 3) = m(jn + j - 1, 3)
  orig(j, 4) = m(jn + j - 1, 4)
NEXT j
FOR J8 = 1 TO -1 STEP -1
  m(jn + 7, 1) = m(jn + 8, 1) + m(jn + 7, 3) * COS(m(jn + 7, 4) + J8 * dtheta)
  m(jn + 7, 2) = m(jn + 8, 2) + m(jn + 7, 3) * SIN(m(jn + 7, 4) + J8 * dtheta)
FOR J7 = 1 TO -1 STEP -1
  m(jn + 6, 1) = m(jn + 7, 1) + m(jn + 6, 3) * COS(m(jn + 6, 4) + J7 * dtheta)
  m(jn + 6, 2) = m(jn + 7, 2) + m(jn + 6, 3) * SIN(m(jn + 6, 4) + J7 * dtheta)
FOR J6 = 1 TO -1 STEP -1
  m(jn + 5, 1) = m(jn + 6, 1) + m(jn + 5, 3) * COS(m(jn + 5, 4) + J6 * dtheta)
  m(jn + 5, 2) = m(jn + 6, 2) + m(jn + 5, 3) * SIN(m(jn + 5, 4) + J6 * dtheta)
FOR J5 = 1 TO -1 STEP -1
  m(jn + 4, 1) = m(jn + 5, 1) + m(jn + 4, 3) * COS(m(jn + 4, 4) + J5 * dtheta)
  m(jn + 4, 2) = m(jn + 5, 2) + m(jn + 4, 3) * SIN(m(jn + 4, 4) + J5 * dtheta)
FOR J4 = 1 TO -1 STEP -1
  m(jn + 3, 1) = m(jn + 4, 1) + m(jn + 3, 3) * COS(m(jn + 3, 4) + J4 * dtheta)
```

```
  m(jn + 3, 2) = m(jn + 4, 2) + m(jn + 3, 3) * SIN(m(jn + 3, 4) + J4 * dtheta)
FOR J3 = 1 TO -1 STEP -1
  m(jn + 2, 1) = m(jn + 3, 1) + m(jn + 2, 3) * COS(m(jn + 2, 4) + J3 * dtheta)
  m(jn + 2, 2) = m(jn + 3, 2) + m(jn + 2, 3) * SIN(m(jn + 2, 4) + J3 * dtheta)
FOR J2 = 1 TO -1 STEP -1
  m(jn + 1, 1) = m(jn + 2, 1) + m(jn + 1, 3) * COS(m(jn + 1, 4) + J2 * dtheta)
  m(jn + 1, 2) = m(jn + 2, 2) + m(jn + 1, 3) * SIN(m(jn + 1, 4) + J2 * dtheta)
  IF ABS(m(jn + 1, 1) - m(jn, 1)) <= m(jn, 3) THEN
    FOR J1 = 1 TO -1 STEP -2
      m(jn, 2) = m(jn + 1, 2) + J1 * SQR(m(jn, 3) ^ 2 - (m(jn, 1) - m(jn + 1, 1))
                 ^ 2)
      CALL zcal(m(), jnumseg, z)
      IF intz < z THEN
        intz = z
        jident(1) = J1
        jident(2) = J2
        jident(3) = J3
        jident(4) = J4
        jident(5) = J5
        jident(6) = J6
        jident(7) = J7
        jident(8) = J8
        coodsol = m(jn, 2)
        jflag = 1
      END IF
    NEXT J1
  END IF
NEXT J2
NEXT J3
NEXT J4
NEXT J5
NEXT J6
NEXT J7
NEXT J8
IF jflag = 1 THEN
  FOR j = 7 TO 1 STEP -1
    m(jn + j, 4) = m(jn + j, 4) + jident(j + 1) * dtheta
  NEXT j
  FOR j = 7 TO 1 STEP -1
    m(jn + j, 1) = m(jn + j + 1, 1) + m(jn + j, 3) * COS(m(jn + j, 4))
    m(jn + j, 2) = m(jn + j + 1, 2) + m(jn + j, 3) * SIN(m(jn + j, 4))
  NEXT j
  m(jn, 2) = coodsol
  CALL anglecheck(m(jn, 1), m(jn, 2), m(jn + 1, 1), m(jn + 1, 2), m(jn, 4))
```

```
ELSEIF jflag = 0 THEN
  FOR j = 1 TO 9 STEP 1
    m(jn + j - 1, 1) = orig(j, 1)
    m(jn + j - 1, 2) = orig(j, 2)
    m(jn + j - 1, 3) = orig(j, 3)
    m(jn + j - 1, 4) = orig(j, 4)
  NEXT j
END IF
END SUB


SUB slide8S (m(), jn, dtheta, jnumseg, intz)
'This sub-routine performs the slide-end variation procedure with 8 segments
'by rotating segments 1, 2, 3, 4, 5, 6 and 7
'm() = coordinate array
'jn = starting segment(segment 8)
'dtheta = angle of variation
'jnumseg = number of segment used in the current profile
'intz = section modulus to be returned

DIM jident(8), orig(9, 4), coodsol
jflag = 0
FOR j = 1 TO 9 STEP 1
  orig(j, 1) = m(jn - 8 + j, 1)
  orig(j, 2) = m(jn - 8 + j, 2)
  orig(j, 3) = m(jn - 8 + j, 3)
  orig(j, 4) = m(jn - 8 + j, 4)
NEXT j
FOR J1 = 1 TO -1 STEP -1
  m(jn - 6, 1) = m(jn - 7, 1) + m(jn - 7, 3) * COS(m(jn - 7, 4) + J1 * dtheta)
  m(jn - 6, 2) = m(jn - 7, 2) + m(jn - 7, 3) * SIN(m(jn - 7, 4) + J1 * dtheta)
FOR J2 = 1 TO -1 STEP -1
  m(jn - 5, 1) = m(jn - 6, 1) + m(jn - 6, 3) * COS(m(jn - 6, 4) + J2 * dtheta)
  m(jn - 5, 2) = m(jn - 6, 2) + m(jn - 6, 3) * SIN(m(jn - 6, 4) + J2 * dtheta)
FOR J3 = 1 TO -1 STEP -1
  m(jn - 4, 1) = m(jn - 5, 1) + m(jn - 5, 3) * COS(m(jn - 5, 4) + J3 * dtheta)
  m(jn - 4, 2) = m(jn - 5, 2) + m(jn - 5, 3) * SIN(m(jn - 5, 4) + J3 * dtheta)
FOR J4 = 1 TO -1 STEP -1
  m(jn - 3, 1) = m(jn - 4, 1) + m(jn - 4, 3) * COS(m(jn - 4, 4) + J4 * dtheta)
  m(jn - 3, 2) = m(jn - 4, 2) + m(jn - 4, 3) * SIN(m(jn - 4, 4) + J4 * dtheta)
FOR J5 = 1 TO -1 STEP -1
  m(jn - 2, 1) = m(jn - 3, 1) + m(jn - 3, 3) * COS(m(jn - 3, 4) + J5 * dtheta)
  m(jn - 2, 2) = m(jn - 3, 2) + m(jn - 3, 3) * SIN(m(jn - 3, 4) + J5 * dtheta)
FOR J6 = 1 TO -1 STEP -1
```

```
  m(jn - 1, 1) = m(jn - 2, 1) + m(jn - 2, 3) * COS(m(jn - 2, 4) + J6 * dtheta)
  m(jn - 1, 2) = m(jn - 2, 2) + m(jn - 2, 3) * SIN(m(jn - 2, 4) + J6 * dtheta)
FOR J7 = 1 TO -1 STEP -1
  m(jn, 1) = m(jn - 1, 1) + m(jn - 1, 3) * COS(m(jn - 1, 4) + J7 * dtheta)
  m(jn, 2) = m(jn - 1, 2) + m(jn - 1, 3) * SIN(m(jn - 1, 4) + J7 * dtheta)
  IF ABS(m(jn, 1) - m(jn + 1, 1)) <= m(jn, 3) THEN
    FOR J8 = 1 TO -1 STEP -2
      m(jn + 1, 2) = m(jn, 2) + J8 * SQR(m(jn, 3) ^ 2 - (m(jn + 1, 1) - m(jn, 1))
                     ^ 2)
      CALL zcal(m(), jnumseg, z)
      IF intz < z THEN
        intz = z
        jident(1) = J1
        jident(2) = J2
        jident(3) = J3
        jident(4) = J4
        jident(5) = J5
        jident(6) = J6
        jident(7) = J7
        jident(8) = J8
        coodsol = m(jn + 1, 2)
        jflag = 1
      END IF
    NEXT J8
  END IF
NEXT J7
NEXT J6
NEXT J5
NEXT J4
NEXT J3
NEXT J2
NEXT J1
IF jflag = 1 THEN    'change the initial profile to the intermediate profile
  FOR j = 7 TO 1 STEP -1
    m(jn - j, 4) = m(jn - j, 4) + jident(8 - j) * dtheta
  NEXT j
  FOR j = 7 TO 1 STEP -1
    m(jn - j + 1, 1) = m(jn - j, 1) + m(jn - j, 3) * COS(m(jn - j, 4))
    m(jn - j + 1, 2) = m(jn - j, 2) + m(jn - j, 3) * SIN(m(jn - j, 4))
  NEXT j
  m(jn + 1, 2) = coodsol
  CALL anglecheck(m(jn + 1, 1), m(jn + 1, 2), m(jn, 1), m(jn, 2), m(jn, 4))
ELSEIF jflag = 0 THEN
  FOR j = 1 TO 9 STEP 1
```

```
    m(jn - 8 + j, 1) = orig(j, 1)
    m(jn - 8 + j, 2) = orig(j, 2)
    m(jn - 8 + j, 3) = orig(j, 3)
    m(jn - 8 + j, 4) = orig(j, 4)
  NEXT j
END IF
END SUB


SUB zcal (m(), jnumseg, z)
'This sub-routine determines the section modulus Ix/y by finding:
'y location of the centroidal axis(ycen) - SUB cenaxis()
'Area moment of inertia(sumIx) - the first FOR loop
'location of the extreme point(c) - the second FOR loop
'm() = coordinate array
'jnumseg = number of segment used in the current profile
'z = section modulus to be returned

CALL cenaxis(m(), jnumseg, ycen, ymax)
FOR j = 1 TO jnumseg STEP 1
  sumIx = sumIx + dIx(m(j, 1), m(j, 2), m(j + 1, 1), m(j + 1, 2), ycen)
NEXT j
ypal = ymax + thicknesses - ycen
sumIx = sumIx + panelthickness * xpanel * ypal ^ 2
c = ypal + panelthickness / 2
FOR j = 1 TO jnumseg + 1 STEP 1
  IF c < ABS(m(j, 2) - ycen) + thickness / 2 THEN c = ABS(m(j, 2) - ycen) +
thickness / 2
NEXT j
z = sumIx / c
END SUB
```