THE UNIVERSITY OF ALBERTA

RELIABILITY MODELLING OF A LARGE COMPUTER SYSTEM

by

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This thesis is dedicated to my mother and the memory of my father.

RELIABILITY MODELLING OF A LARGE COMPUTER SYSTEM

ABSTRACT

This thesis presents a new 30 state generalized Markov state space model that can be used to predict the frequency and duration of the operating and failed states that occur in a computer system and the performance characteristics of the distinctive cyclic patterns of operation and failure that occur in a given computer system. Based on the generalized model, the evolution equations for the probability of the various system failed states caused by hardware, software, operator and analyst errors exhibited by the Alberta Government's Computer Center's VM-based system are developed and presented in detail. The proposed models can be used to predict the cyclic transitions between the failure and operational states of a computer system and the processes it controls, given the outage statistics of a given computer system.

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CHAPTER I
INTRODUCTION

Society is dependent upon the use of computerized technology to perform many tasks in its personal, commercial and industrial sectors (e.g., can you imagine a bank today operating without a computer system?). Society's use and dependence on digital computer systems is increasing (e.g., total computerized flight of Air France's A320 airbus). Digital computer systems are required to operate as economically as possible and provide society with assurances of continuity and quality of service. Failure to meet these assurances can quickly eradicate the economic benefits of computer technology. These assurances pose many questions for society when a computer system's performance is curtailed. The questions often posed are:

(a) What level of computer system reliability is adequate?

(b) How does a computer system fail?

(c) What are the causes of a computer system's failure?

(d) How long will the computer system be down?

(e) How often does the computer system go down?

(f) What is the cost of a computer failure to society?

(g) Can the cost of computerized technology be justified?

To answer some of these questions, it is necessary to define what is a "computer system".

1.1 Definition of a computer system

A computer system consists of many physical devices such as the central processing unit (CPU), memory, printers, terminals, direct access storage devices (DASD), keyboards, cables, etc. that are interconnected to form a "system". These physical devices are collectively labelled as "hardware". The "software" of a computer system are the instructions (written in various computer languages) that form various computer programs which link the various hardware devices together to form an operating system and permit the use of other application computer programs to perform various tasks as dictated by the users of the system.

1.2 Computer system failures

A computer system failure occurs when the system is not available to the user. This failure is primarily due to hardware or software failures [1,2,3] that curtail or limit the performance of the computer system. A hardware failure of a computer system occurs when a problem (e.g., main storage unit (MSU) thermal check) arises in the CPU and/or its support systems (e.g., the MSU and/or the CPU power supply) so that the CPU does not operate correctly or at all, resulting in a computer interruption. A software failure of a computer system occurs when a problem (e.g., a computer system gets "hung up" in a wait state) arises in the operating system resulting in the computer system not completing what the user requested of it. A computer system could also fail due to a problem with the system's power supply, however, this is rarely a problem today as most large computer systems have uninterruptible power supplies (UPS) to ensure continuous power flow to their system.

When a computer system fails, there are various means of restoring the system to a full or partial operating state. For example, when the operating system detects a problem (e.g., an abnormal program end) in an application program, it may be able to perform a "restart" from a known set of conditions so that the program will run to completion. However, if the problem originates within the operating system, it may be necessary to perform an initial program load (IPL) which involves the process of reloading the operating system to return the computer system to an operational state. The complexity of these restoration activities has a significant impact on the duration of computer system interruptions.

The reliability of a computer system is defined as the probability that the system will perform adequately for the desired period of time when operating in a given environment. One of the key reliability random variables that characterize each computer system mode of failure (e.g., hardware, software and the people who operate and program the system) is the hazard rate. This is defined as the number of observed failures caused by that particular mode of computer system failure per unit of operating time. For example, a computer system's hardware is composed of many electronic devices (e.g., integrated circuits) and mechanical devices (e.g., motors). Studies on these components have shown that their hazard rate varies with time in a pattern that is somewhat shaped like a bathtub as illustrated in Figure 1.1. When the hazard rate is constant, it is called a "failure rate"[1].

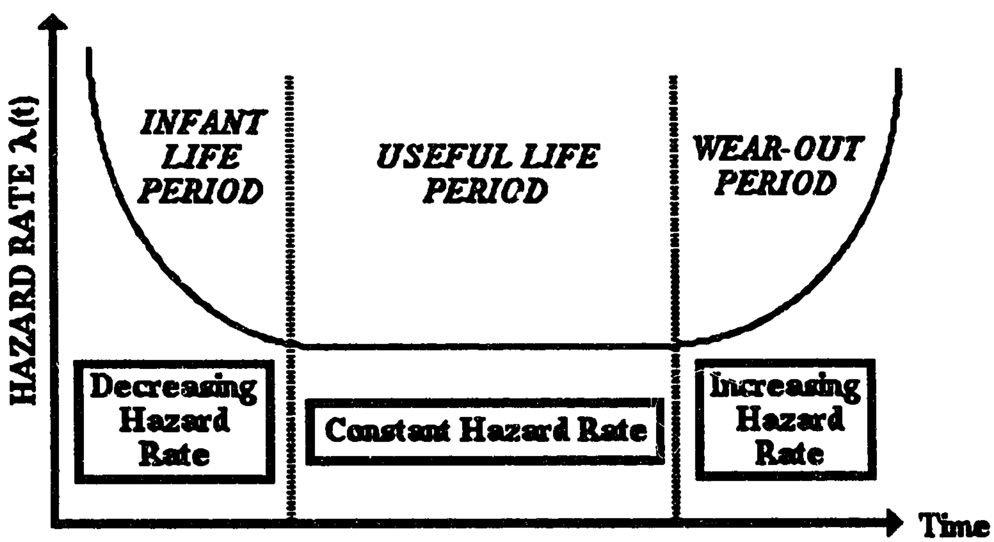

Figure 1.1 Hazard rate λ (t) for typical computer hardware.

The initial period in the life cycle of a computer system's hardware has a high hazard rate and is classified as the infant life period or burn-in period of the life cycle of electronic components and the "bedding-in" period for mechanical devices. This high initial hazard rate is usually due to manufacturing and assembly defects. The initial period is followed by a constant failure rate period which is referred to as the "useful" life period. Then, the wear-out period follows where the failure rate increases significantly with time.

The constant failure rate period for a computer system may not be achieved for some years after the installation of a system due to problems with the initial design concepts not being completely correct or due to an unexpectedly long burn-in period where the system defects are not completely identified and remedied. The constant failure rate period may be far from constant with variations caused by different usage of the system or other disturbance factors (e.g., stress levels on IC's, environmental changes, etc.). These factors are very difficult to incorporate into any computer system performance model.

When a computer system is designed, the manufacturer estimates the reliability of the hardware devices used. The evaluation process is based on the hazard rates obtained from field data of the individual devices (e.g., CPU, printer) and the individual components of these devices. The individual component hazard rates take into account stress factors specified by the design parameters on ambient temperature, voltage, and power ratings derived from the environment in which the devices will work. Other factors that computer manufacturers consider in calculating hardware reliability of a computer system are the known deficiencies in the design, production, use and maintenance of the computer system. The reliability of a computer component can be greatly affected by the storage time. The storage time is the amount of time that a component is not in use (e.g., sitting on a shelf). Some IC's have a significant failure rate when they are not in use. The modelling techniques required to evaluate the performance of a computer system consisting of millions of components is an extremely difficult and involved process.

The evaluation of computer software reliability involves a different set of considerations than hardware reliability (e.g., the hazard rate "bathtub" curve). First, software failures are primarily due to design errors with production, use, and maintenance factors being negligible. An exception to this statement is corrections made to a computer program which may cause future computer system interruptions. The concept of repair is not directly applicable to correcting erroneous software. Instead, software is redesigned to improve reliability only if it removes the original error(s) and introduces no others. The software reliability is a function of the effort put into detecting and correcting errors which can lead to computer system failures when a program step or path which has an error in it is executed. The external environment does not affect software reliability except where it affects program input and output devices (e.g., faulty user terminal, printers stalling, etc.).

The reliability of the personnel who operate and program computer systems is a difficult parameter to evaluate but can be estimated empirically from the number of computer system interruptions they cause. The reliability of these personnel and thus the computer system reliability depends on the amount and level of training the people have received.

1.3 Computer cyclic patterns of operation

When all the causes of computer system interruptions are considered individually or collectively, a computer system will exhibit distinctive cyclic patterns of performance. The system will be in an operational state (i.e., up state) for certain period of time (e.g., "m" hours) and then, as a result of a failure, be in a down state for another period of time (e.g., "r" hours). The computer system is then restored to an operational state following remedial actions to remove the effects of the failure. These cyclic patterns are illustrated in Figure 1.2.

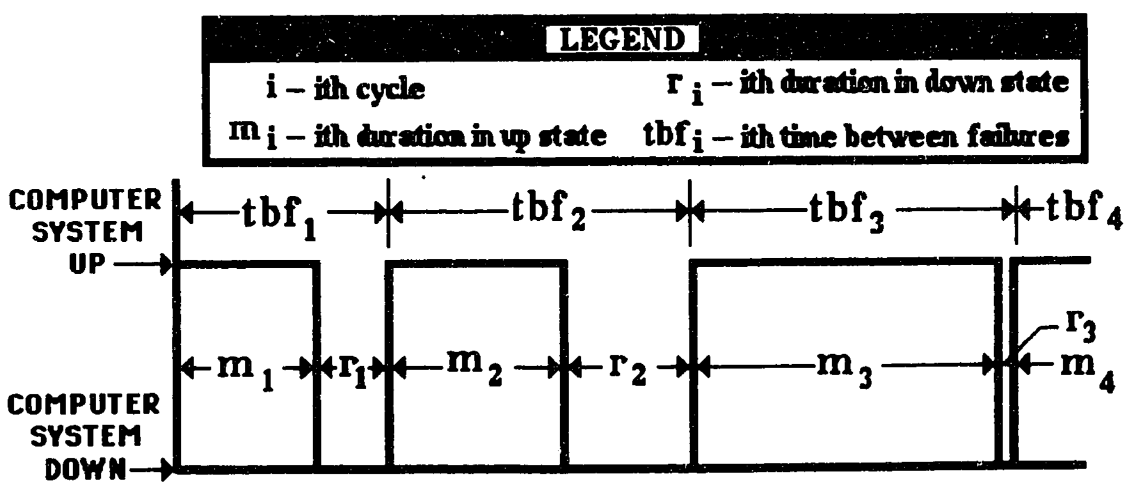

Figure 1.2. Computer system cyclic operational states

The mean up time after restoration (i.e., **MUT**) of a computer system is defined as:

$$\mathbf{MUT} = \frac{1}{n} \sum_{i=1}^{n} m_i \tag{1}$$

The mean down time (i.e., **MDT**) of a computer system is defined as:

$$\mathbf{MDT} = \frac{1}{n} \sum_{i=1}^{n} r_i \tag{2}$$

There is almost a denumerably infinite number of factors that affect the performance of a computer system which cannot be incorporated into any practical model without an extremely sophisticated and detailed outage reporting system. These are not maintained in the computer industry today. In order to develop practical computer system performance models, it is necessary to study in detail the computer interruption patterns as captured by existing computer system outage reporting systems. The unique causes of a given computer system's interruptions can be defined by analysing the restoration - repair processes that were required to restore a particular computer system to a fully or partially operating state. A knowledge of these computer system cyclic failure - renewal processes can be obtained from a detailed statistical analysis of existing interruption-outage reporting systems which log the failure and operating states of a given computer system as a function of time. A detailed statistical analysis of a computer system's primary performance variables (e.g. MUT, MDT, "failure rates","time between failures", individual "down" times and "up" times, etc. .) provides a knowledge base for the development of detailed computer system performance models. Such models are the subject of this thesis.

1.4 Thesis Objective

This thesis is directed at developing a generalized model of the performance of a large digital computer system that accounts for the frequency and duration of complete computer system failures (i.e., requiring an IPL to restore the system to a fully operational state). Problems resulting in restricted computer operational states are not included in the generalized model. The interruption reports of any computer system can in many cases provide the necessary statistical characteristics of the state variables necessary to evaluate computer performance. This thesis will present a new 30 state generalized Markov state space model based on the information/management data bases of the Alberta Government Computer Center's VM-based system. The evolution equations for the probability of the various system failed states caused by hardware, software, operator and analyst errors exhibited by the Alberta Government Computer Center's VM-based system will be developed and presented in detail. The proposed models can be used to predict the cyclic transitions between total system failures and the operational states of a given computer system and the processes it controls. The methodology presented in this thesis will provide a basis for custom modelling of any computer system's performance based on its interruption reports.

1.5 Scope of Thesis

Chapters II and III will describe the data base in detail and present the results of preliminary statistical analysis of the key performance variables to determine if the computer system exhibits distinctive patterns of operation. In Chapter IV a general model based on the results of the statistical analysis will be presented. Chapters V to VIII will present the individual Markov models for each computer failure mode (i.e., hardware, software, unknown, analyst and operator). Chapter IX will present the Markov model for the assessment of the overall reliability performance of the Alberta

Government Computer Center's VM-based system. Chapter X will present the conclusions and discussions of the thesis.

CHAPTER II

COMPUTER SITE DATA BASE

2.1 Description of information/management computer data base

The computer system that was studied in this thesis was the Alberta Government Central Computer Center VM operating system computer. This computer system became operational in September 1983 and utilized an Amdahl 470/V8 CPU. On October 31, 1986 the CPU was updated to an IBM 3081K CPU. The management of the computer system implemented an information/management data base to keep track of the system operation. The information/management data base of the computer system had two components, a change form report and a problem report. The change form reports covered any proposed changes to the system configuration, hardware or software, or system operation and the implementation and results of the change. The problem reports covered any deviation in equipment or computer system operation from what is normally expected and what was done to identify and correct the problem if possible. The problem reports contained the information required to evaluate the computer system's perfomance.

2.2 Computer system problem reports

The problem reports were set up to provide the management or other interested people with the maximum amount of information on what was happening on the computer system at a glance. The heading of the report includes the record identification number and a brief problem description. The first page of the report has the data divided into five categories:

(1) ***Problem Reporter Data:*** this includes the person reporting, date, time, and nature of the problem and its impact on the system.

(2) ***Problem Status Data:*** this includes who is assigned to the problem and their progress on the problem.

(3) ***Problem Close Data:*** this covers the resolution of the problem.

(4) ***Interested Privilege Classes:*** who has access to the problem reports

(5) ***Problem Supplemental Data:*** Other information on the problem.

The second page of the report contains the:

1) ***Detail Data:*** which includes the Description Text of the problem and the Resolution Text of the problem if it was solved.

2) ***Journalized Problem Data:*** a summary of the problem report.

2.3 Identification of the causes of computer system problems

The problem reports were analyzed in some detail and it was decided that the problem report data base would be partitioned on the basis of what caused the problem rather than the type of problem. This decision would yield more information about the operation of the computer system. For example, a problem caused by an incorrect system operator action is more descriptive of system operation than saying that there was a software problem. The reports were separated into the categories based on what caused the problem. the five categories chosen were:

(1) Hardware: any problem caused by the hardware of the computer system.

(2) Software: any problem caused by the software of the computer system.

(3) Operator: any problem caused by the operators of the computer system.

(4) Analyst: any problem caused by the computer system analysts.

(5) Unknown: any problem for which the cause was not known.

A category for problems caused by power system was not included because this computer system has an UPS (i.e., an uninterruptible power supply) and there were no problems recorded to date.

The problem report data base was further analyzed to determine the means of restoration following a computer system problem. The majority of problems have no major impact on the performance of the computer system (e.g., console lights on, system still operational) while other problems resulted in restricted computer system operation. The problem report data base was partitioned into two distinct regions to identify those problems which caused a computer system failure and those which caused restricted system operation and/or were minor problems. The number of problems and computer system failures (i.e., requiring an IPL) are shown in Table 2.1.

Table 2.1 Number of computer problems versus means of restoration (1985-1986)

NUMBER OF PROBLEM REPORTS	NUMBER OF IPL'S	NUMBER OF NON-IPL'S
224	**44**	**180**[1]

NOTE: (1) Non-IPL's includes all minor problems and problems whose occurrence results in restricted computer operation.

CHAPTER III
STATISTICAL METHODOLOGY

3.1 Introduction

The key parameters necessary to model the reliability of a computer system are the "time to failure" and the "restoration time" variables for each problem category. A detailed study of these variables and the cyclic failure-renewal processes exhibited by a computer system will define the statistical characteristics of the system's operational and failed states which are necessary for performance modelling.

3.2 Statistical representation of the "time to failure" variable with known statistical distributions

An initial statistical objective of this thesis was to represent the computer system reliability variable " time to failure" by a known statistical distribution (e.g., exponential, normal, Weibull, etc.). If the "time to failure" variable for each problem category can be represented by known statistical probability density functions, then the performance of a computer system can be modelled by existing reliability models. Frequency histograms of the "time to failure" variable for the major problem areas, namely; system, hardware, software and unknown causes are shown in Figures 3.1 to 3.4, respectively, to provide a visual basis for possible pattern matching with known statistical distributions.

An examination of the clustering patterns exhibited in all the frequency histograms revealed that the problem report data base could be stratified into five distinct periods of continuous computer operation following a distinctive failure mode. The selected periods or operational time zones are defined in Table 3.1. The computer system operational time zone boundaries were selected to accommodate all modes of computer system failures (i.e., hardware failures, software failures, unknown failures, analyst errors and operator errors) and are shown in Figures 3.1 to 3.4.

Table 3.1 Definition of computer system distinctive operating periods in weeks.

TIME ZONE NUMBER	OPERATIONAL TIME ZONE BOUNDARIES IN WEEKS
1	0 - 4
2	4 - 8
3	8 - 16
4	16 - 32
5	32+

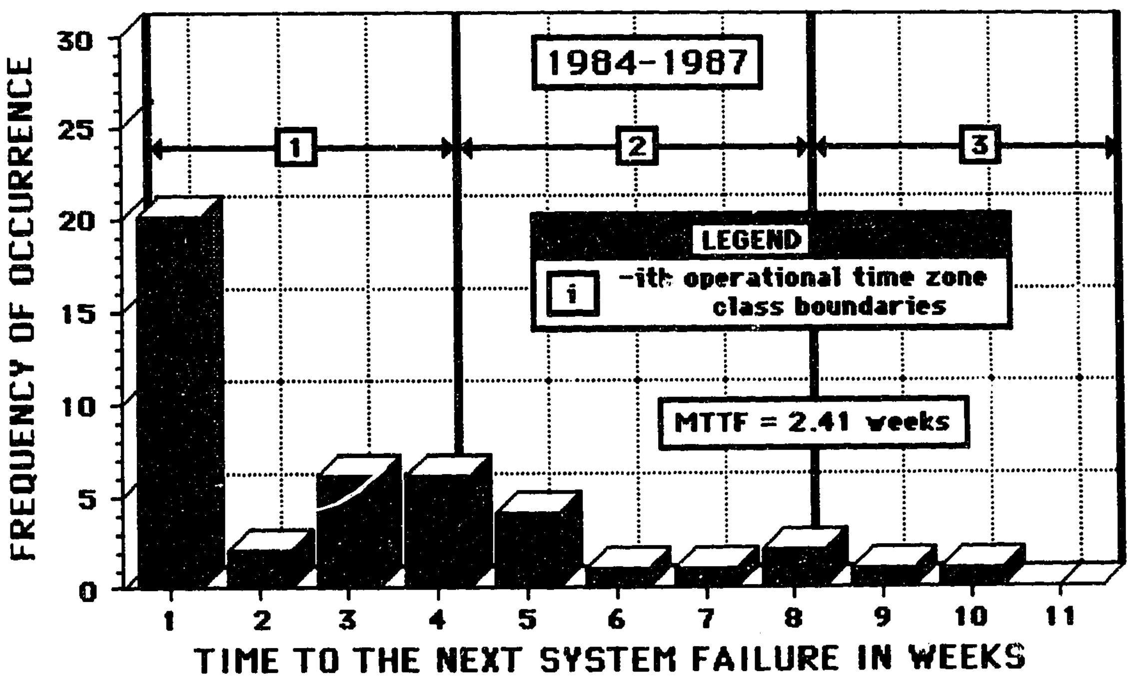

Figure 3.1 Frequency histogram of the time to the next system failure caused by all problems

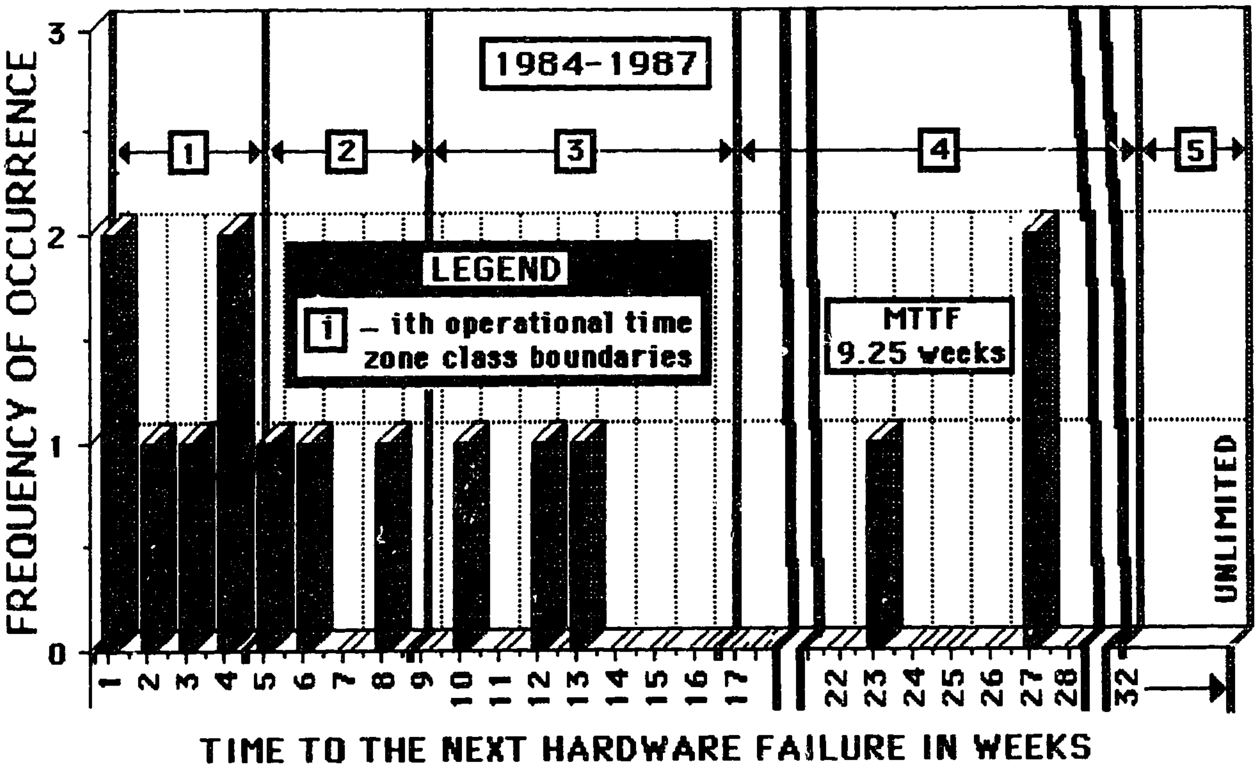

Figure 3.2 Frequency histogram of time to the next system failure caused by hardware problems.

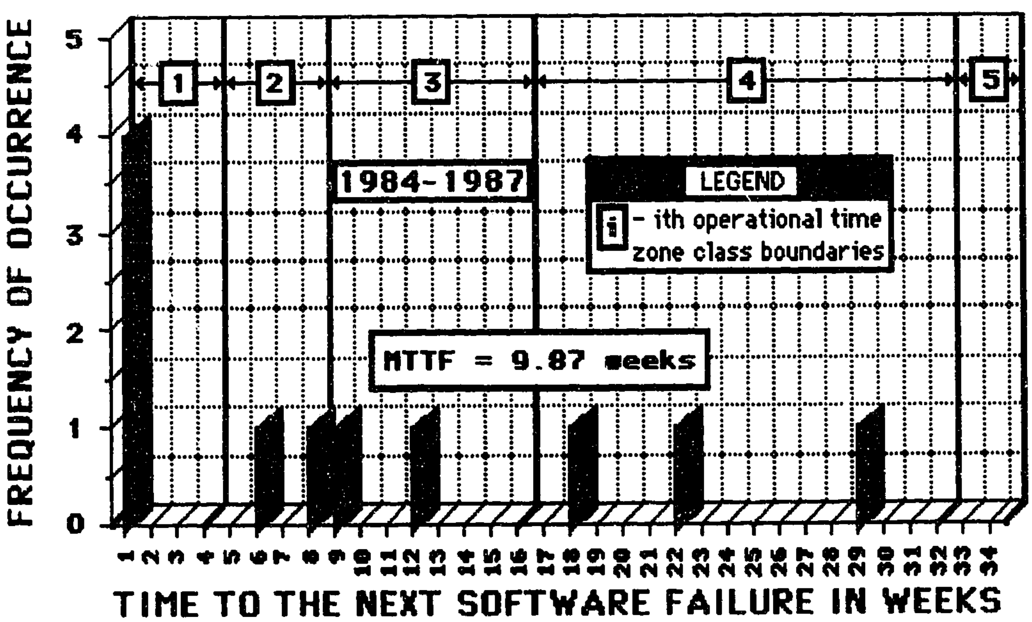

Figure 3.3 Frequency histogram of the time to the next system failure caused by software problems.

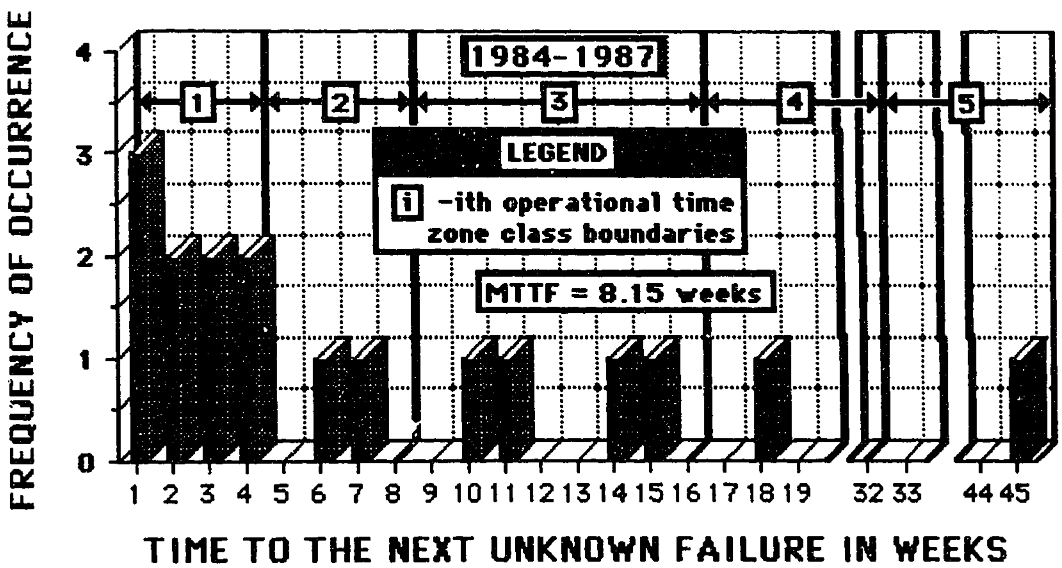

Figure 3.4 Frequency histogram of the time to the next system failure caused by unknown problems.

The expected value of the time to failure variable (i.e., MTTF or mean time to failure) was calculated for each problem category and represents the time the computer system was operational until the next defined problem failure occured. For example, the expected value of computer system operation following each hardware failure was approximately 9.25 weeks (i.e., 1533.84 hours).

Statistical analysis of the computer system's operational states to obtain the expected value of the duration highlighted a major problem.. The MTTF shown in Figures 3.1 to 3.4 did not adequately characterize the central tendency of the data due to the data's large variance and numerous modes over the range monitored. This range of data precluded known statistical distributions (e.g., Weibull, log normal, normal, etc.) from being used to represent the data. The initial attempts to represent the variables by traditional statistical probability density functions failed due to the multimodal nature of the time to failure variable.

3.2 Search for time dependent failure patterns in problem report data base

The statistical analysis of the Alberta Government's computer center's VM based system was then directed at examining the various computer system problems to determine whether they exhibited distinctive failure patterns as a function of time (i.e., time dependency of the time to failure data). The first time of occurrence stratification of the problem data base was by the hour of the day. A frequency histogram of the number of problems that occurred in 1985 and 1986 versus the time of day the events occurred are shown in Figures 3.5 and 3.6, respectively.

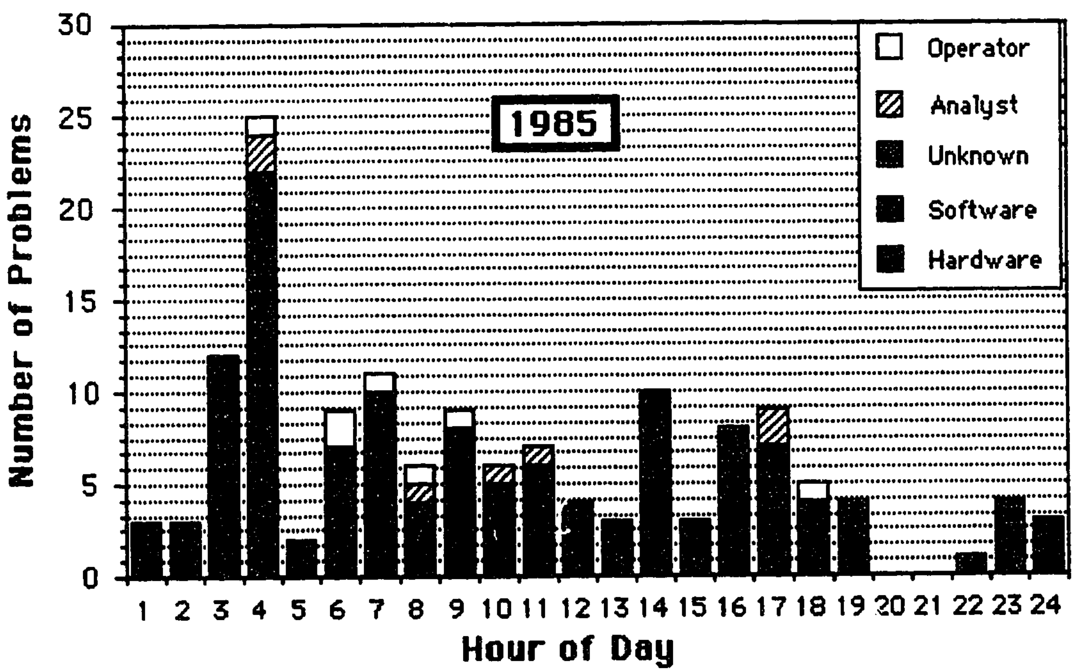

Figure 3.5 Number of problems that occurred in 1985 versus the hour of the day in which they occurred.

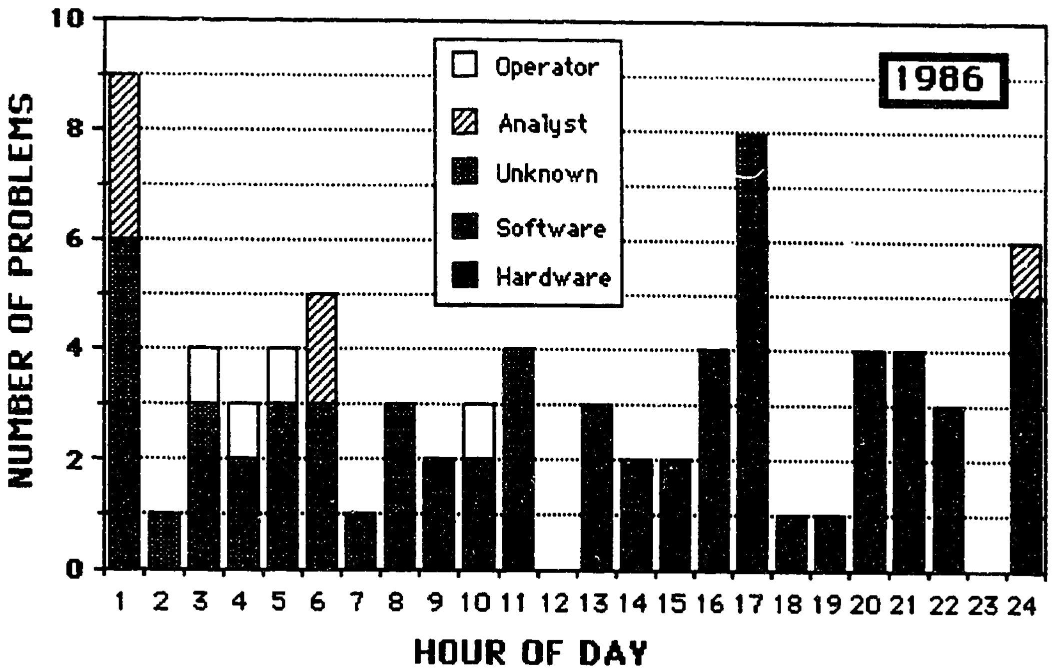

Figure 3.6 Number of problems that occurred in 1986 versus the hour of the day in which they occurred.

An examination of Figures 3.5 and 3.6 reveals that the number of problems that curtail the performance of the computer system vary significantly from year to year and from hour to hour. The frequency distributions of the various categories of problems exhibited multimodal tendencies which are extremely difficult to model statistically. No readily visible patterns of occurrence are evident when the various failure modes of the computer system are correlated with the hour of the day in which the failure mode occurred.

The second stratification of the data base was by the day of the week in which the problem occurred. A frequency histogram of the number of categorized problems that occurred in 1985 and 1986 versus the day of the week are shown in Figures 3.7 and 3.8, respectively.

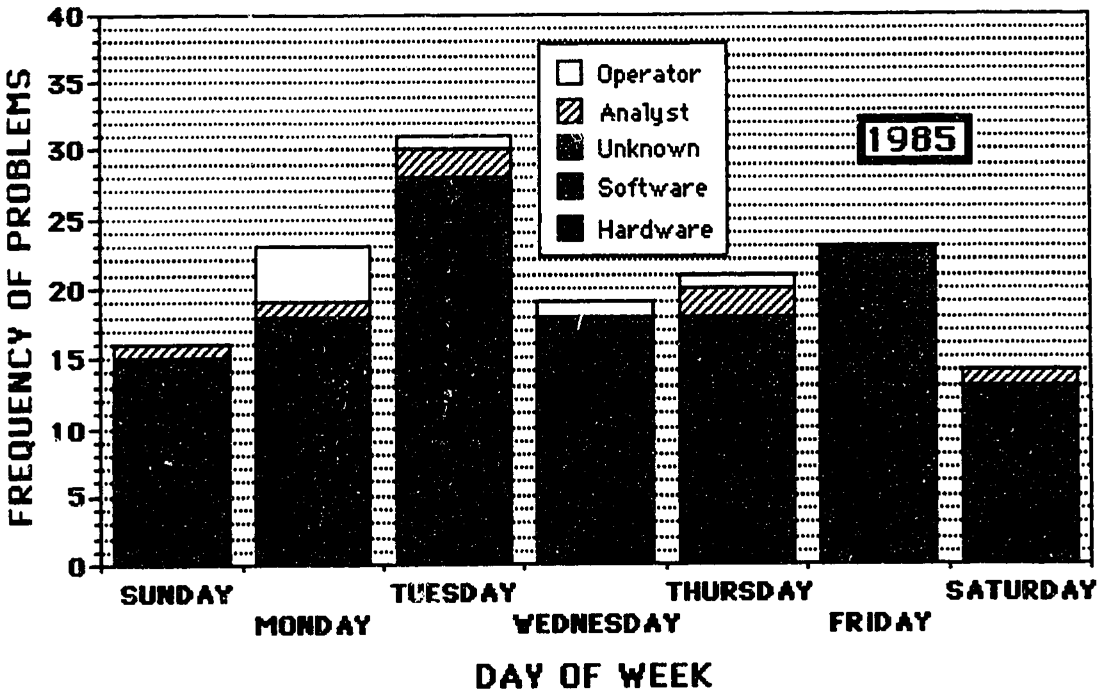

Figure 3.7 Number of problems that occurred in 1985 versus the day of week in which they occurred.

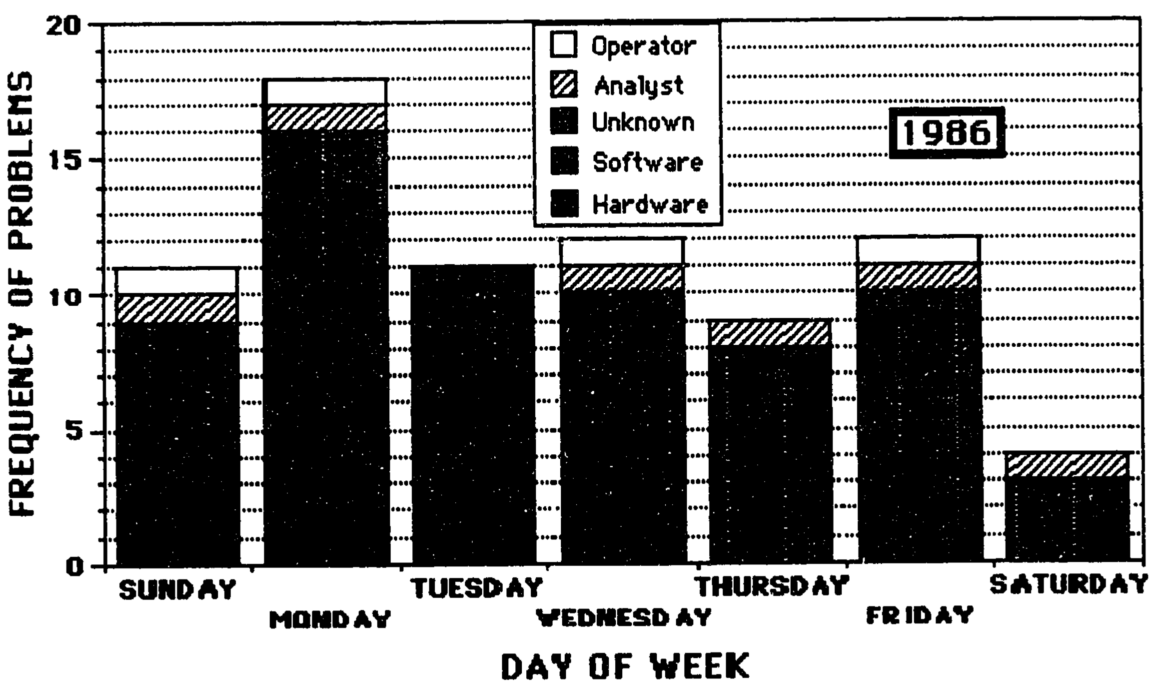

Figure 3.8 Number of problems that occurred in 1986 versus the day of week in which they occurred.

The stratification of the problem report data base by the hour of the day and the day of the week in which the problem occurred provided no distinctive statistical patterns of occurrence over the two year period (i.e., 1985-1986).

3.3 Chronological order arrangement of time to failure variable

In the development of histograms, no attention was given to the order in which events occurred (e.g., data clustered into class intervals). The statistical question that was addressed next was: "Does the "time to failure" variable exhibit distinctive operating cycles?" To answer this question, the time to the next problem category was plotted as a function of its chronological order of occurrence for hardware, software, system and unknown problems and is shown in Figures 3.9 to 3.12, respectively.

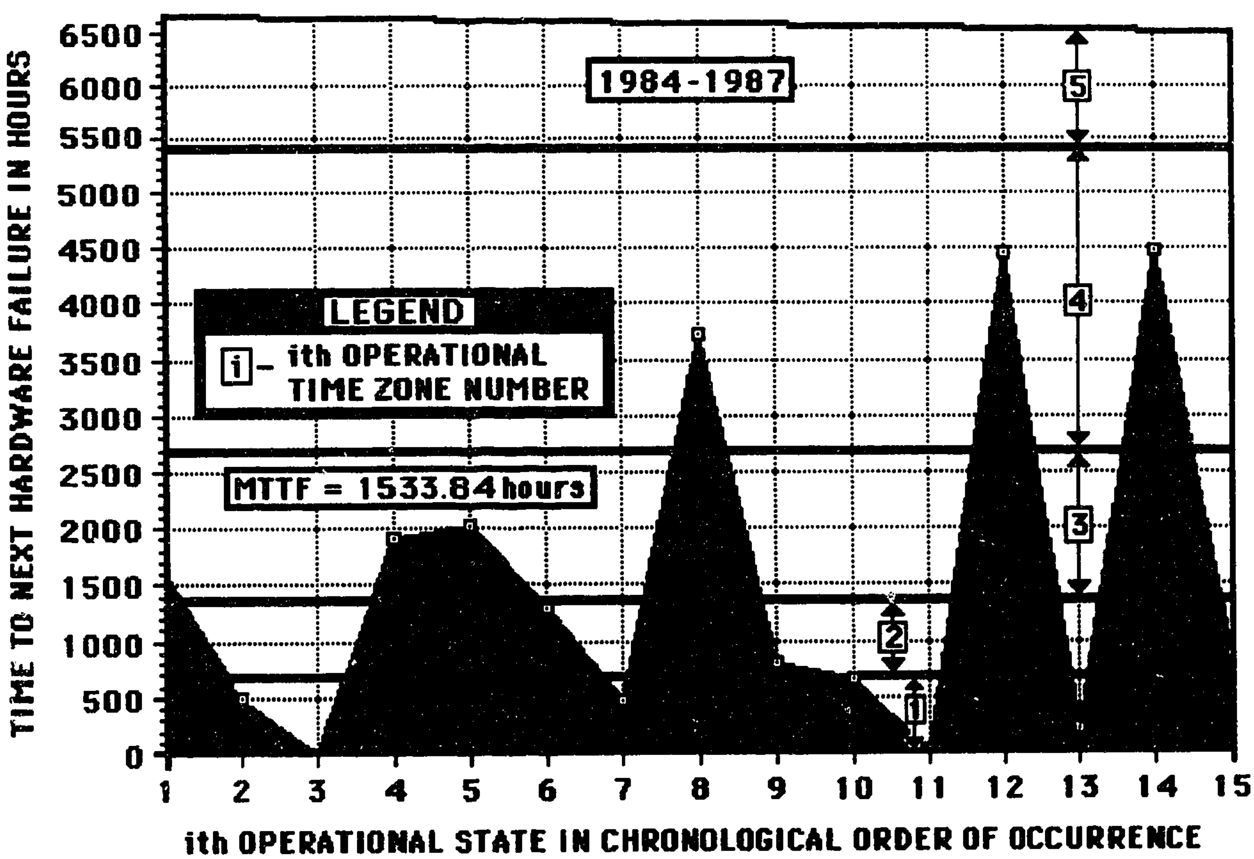

Figure 3.9 Time to the next system failure caused by hardware problems.

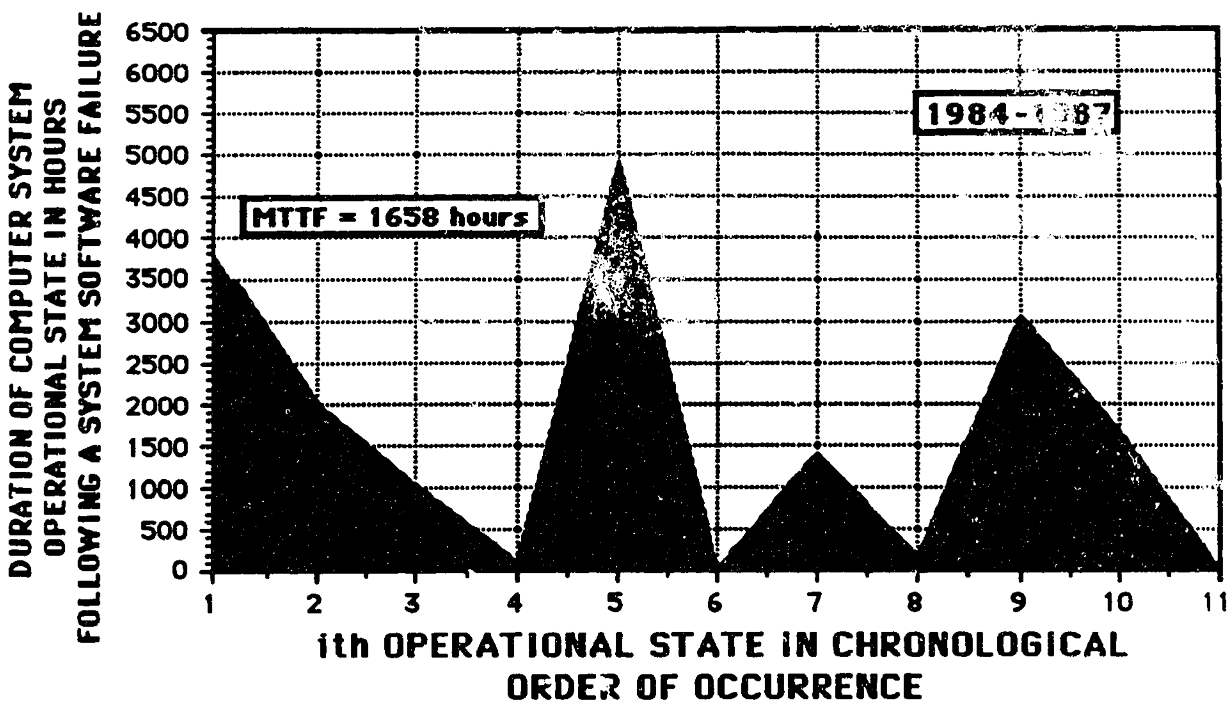

Figure 3.10 Time to the next system failure caused by software problems.

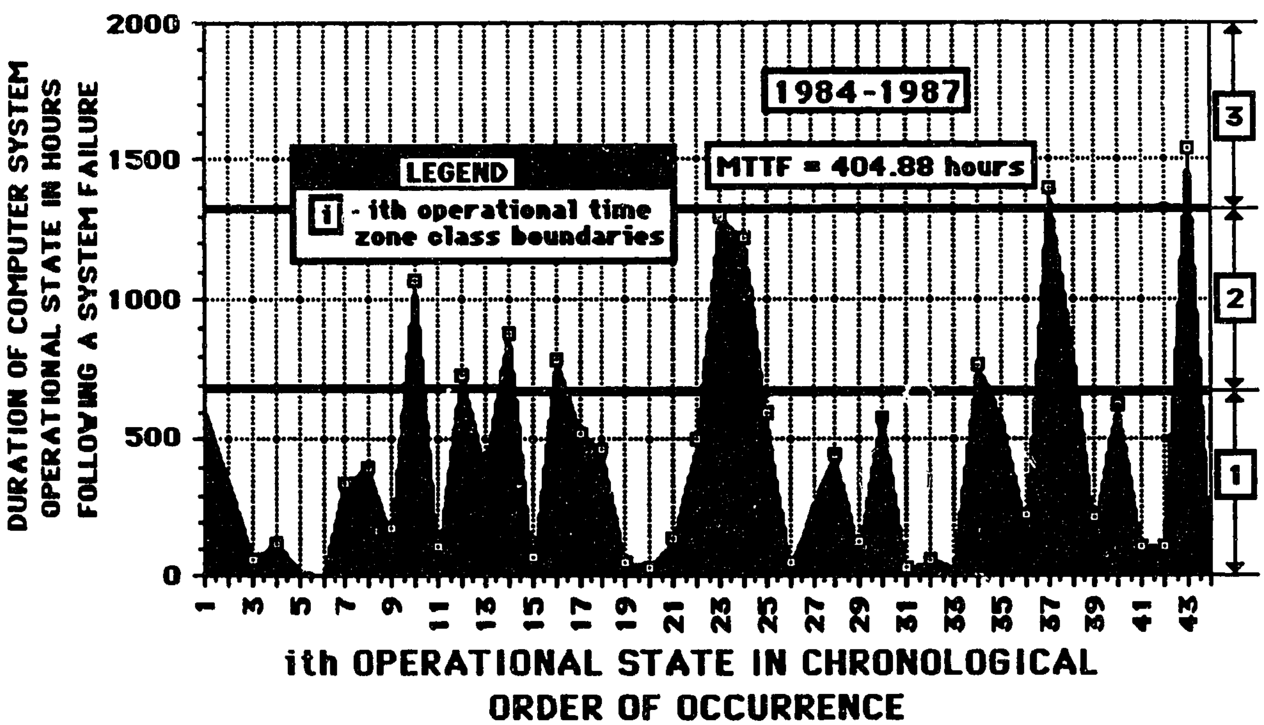

Figure 3.11 Time to the next system failure caused by all problems.

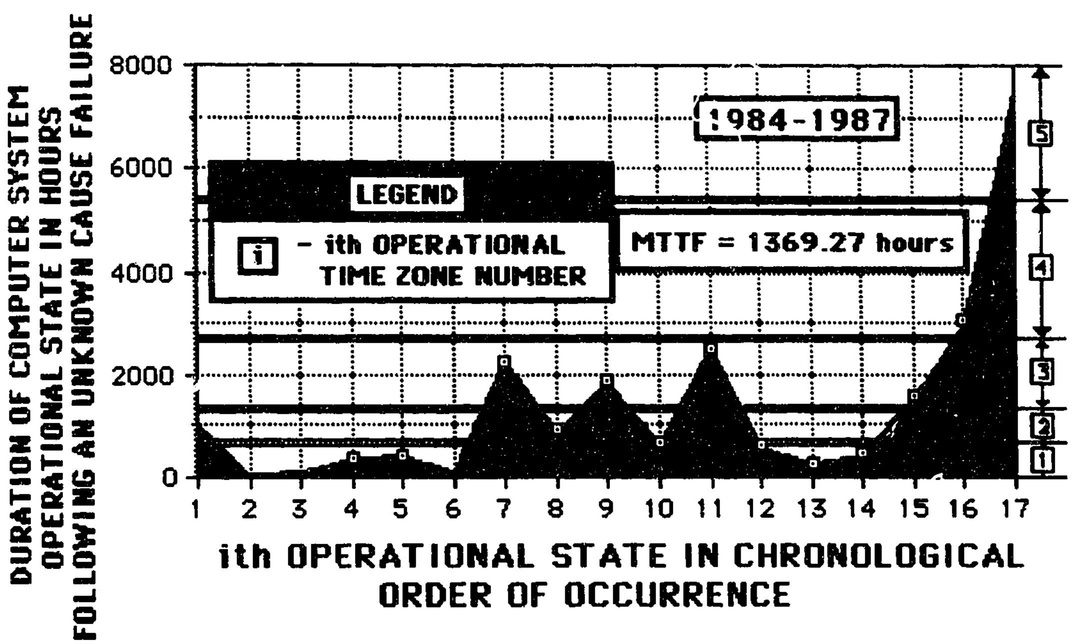

Figure 3.12 Time to the next system failure caused by unknown problems.

An examination of Figures 9 to 12 reveals distinctive operating patterns for all problem categories. It appears that once this computer system has been operational for a specified duration (e.g., x hours) before a given problem curtails its performance, the next sequential computer system operational period (e.g., y hours) after the computer system restoration activities have been completed, will be significantly less than the previous computer operational period (i.e., y << x). An operating cycle of a long operational period followed by one or more operational periods of shorter duration was consistently observed during the study period.

These observations are characteristic of a Markov process in which the computer system resides within an operational state for a certain period of time, then fails and is restored to another operating state of another time period. This cyclic performance continues with distinct probabilistic transitions between operating states and failed states. Once the computer system fails, then the primary questions that must be answered are:

(1) what is the duration of the failed state?

(2) are there distinctive patterns associated with computer system restoration activities?

(3) can the duration of the failed states be represented by known statistical distributions?

Answers to these questions will provide a basis for the development of a Markov model of the performance of the Government of Alberta's computer system.

3.4 Statistical representation of the "restoration time" variable with known statistical distributions

Frequency histograms of the "restoration time" variable for system, hardware, software and unknown causes are shown in Figures 3.13 to 3.16, respectively, to provide a visual basis for possible pattern matching with known statistical distributions.

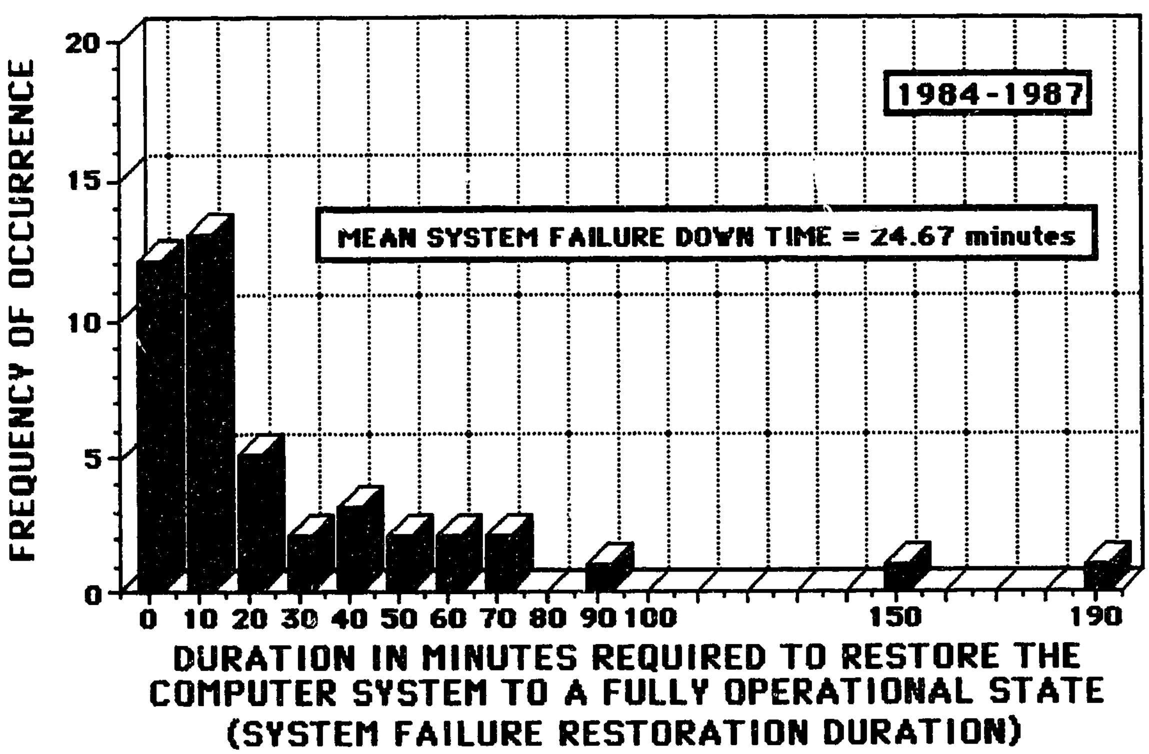

Figure 3.13 Computer system restoration duration following any failure category

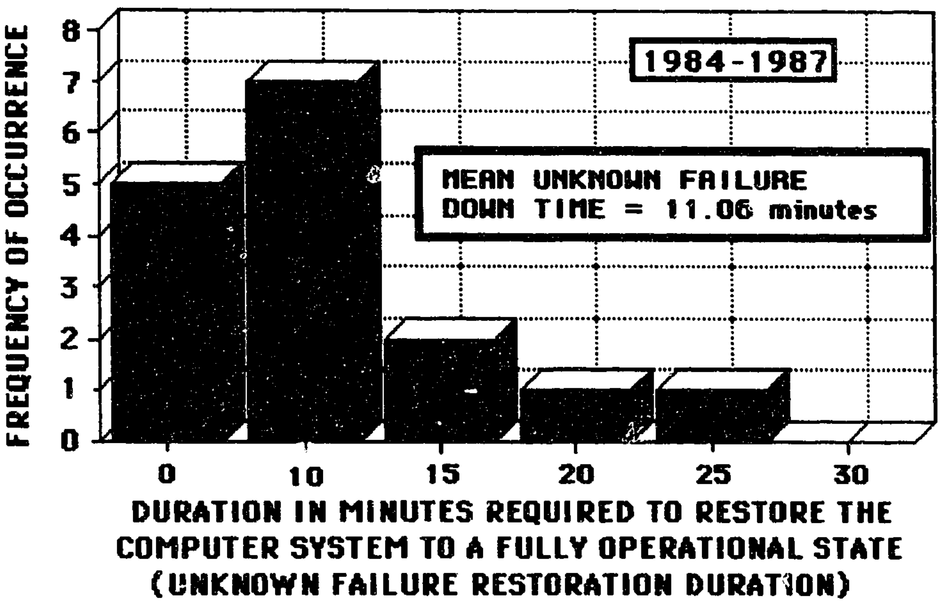

Figure 3.14 Computer system restoration duration following an unknown failure

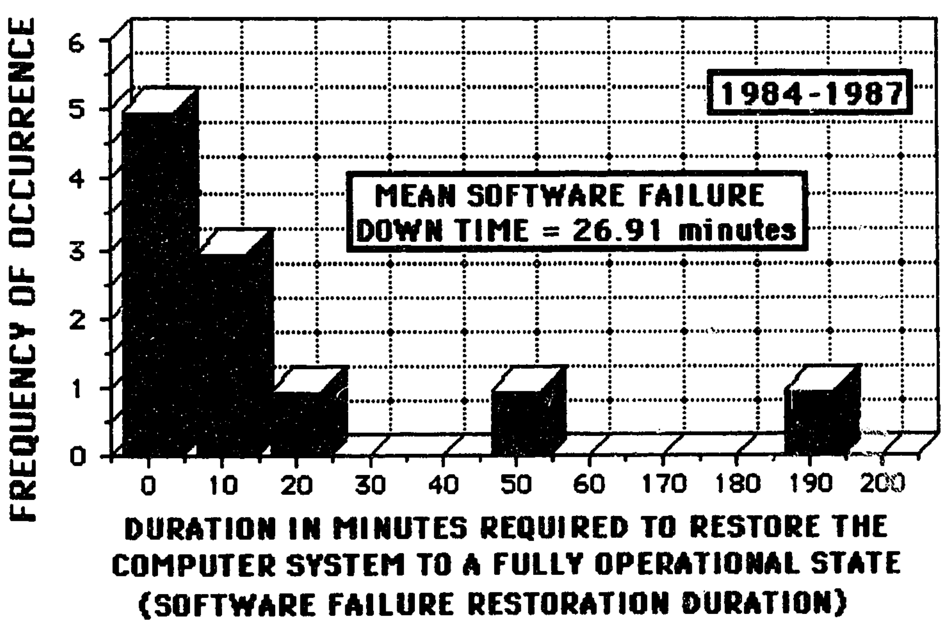

Figure 3.15 Computer system restoration duration following a software failure

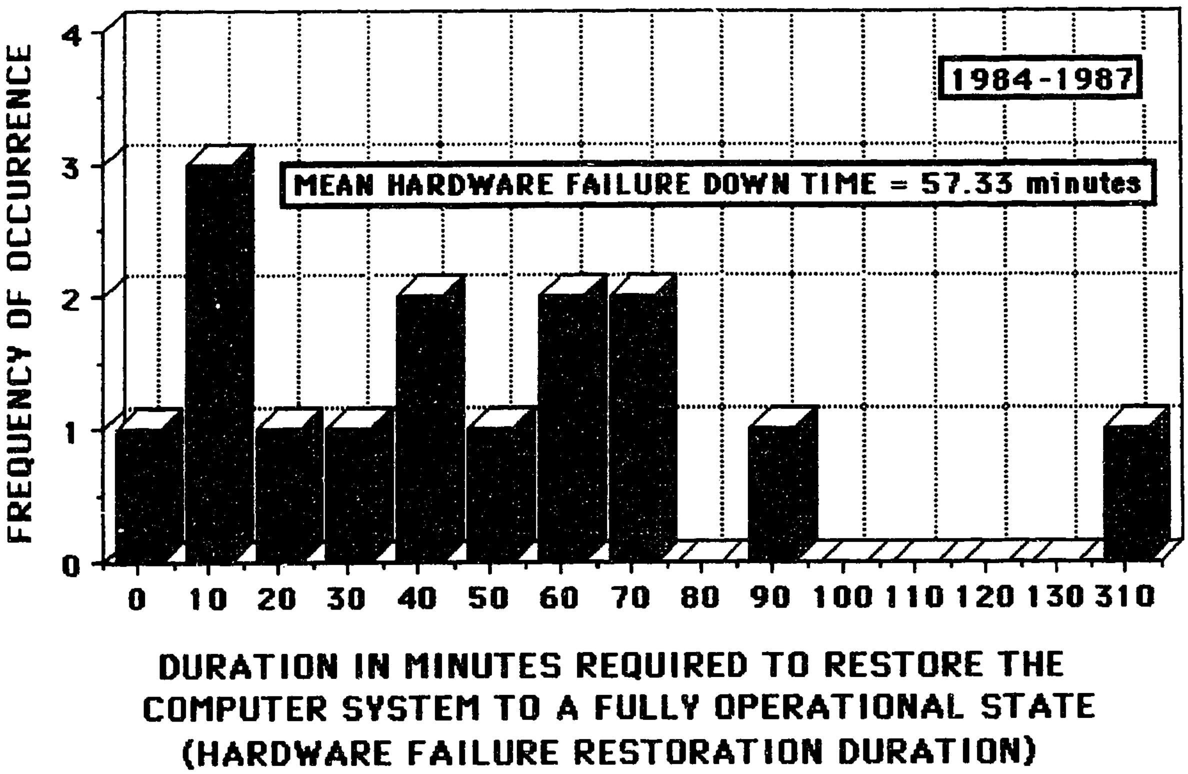

Figure 3.16 Computer system restoration duration following a hardware failure

The key statistical characteristic exhibited by the "restoration time" variables was that the expected duration of restoration activities were orders of magnitude less than the "time to failure" variable for each problem category. The "restoration time" variables were positively skewed with a number of observations significantly removed from the distribution mean and this characteristic is difficult to represent by known statistical distributions and techniques (e.g., transformation of variables).

3.6 Discussion of statistical results

The key reliability variables for all problem categories "time to failure" and "restoration time" could not be represented adequately by known statistical distributions due to the problematic nature of the data. The primarily multimodal nature of these variables prevented the use of existing reliability models which are based on known statistical distributions.

The "time to failure" variables exhibited distinctive operating cycles which can be modelled by a Markov model where the transition rates between operating states can reflect these statistical characteristics. The expected values of the "restoration time" were confined to a short interval (e.g., 0 - 60 minutes) compared with the "time to failure" variable (e.g., 371 to 1658 hours) and these statistical characteristics can be included in a Markov model, the subject of the next . .pter.

CHAPTER IV

GENERALIZED MARKOV MODEL

4.1 Introduction

The statistical analysis of the Alberta Government's computer center's VM based system's problem report data base revealed a distinctive operating pattern for the computer system. An operating cycle of a long operating period followed by one or more shorter operating periods of shorter duration was observed in the monitored data. These observations formed the basis of a generalized failure Markov model.

4.2 State space diagram

The state of the computer system at any given point in time was assumed to be characterized by a set of "operational" states of varying durations and another set of repair/restoration states in which the computer system is in a "failed" state. A set of five operational states was defined. The number assigned to each operational state (i.e., its time zone number as shown in Table 4.1) defines the duration of computer system operation. For example, if the computer system resides in state 1, then the computer system will be operational for only a period ranging from 0 to 4 weeks; in state 2, the computer system will be operational for a period ranging from 4 to 8 weeks, etc. The higher the operational state number, the longer the duration of computer system operation.

Table 4.1 Definition of computer system operating intervals in weeks and their corresponding time zone number

TIME ZONE NUMBER	OPERATIONAL TIME ZONE BOUNDARIES IN WEEKS
1	0 - 4
2	4 - 8
3	8 - 16
4	16 - 32
5	32+

The possible cyclic state transitions from a given operational state to a failed state and then to a another operational state is shown in the generalized state space diagram of a computer system in Figure 4.1.

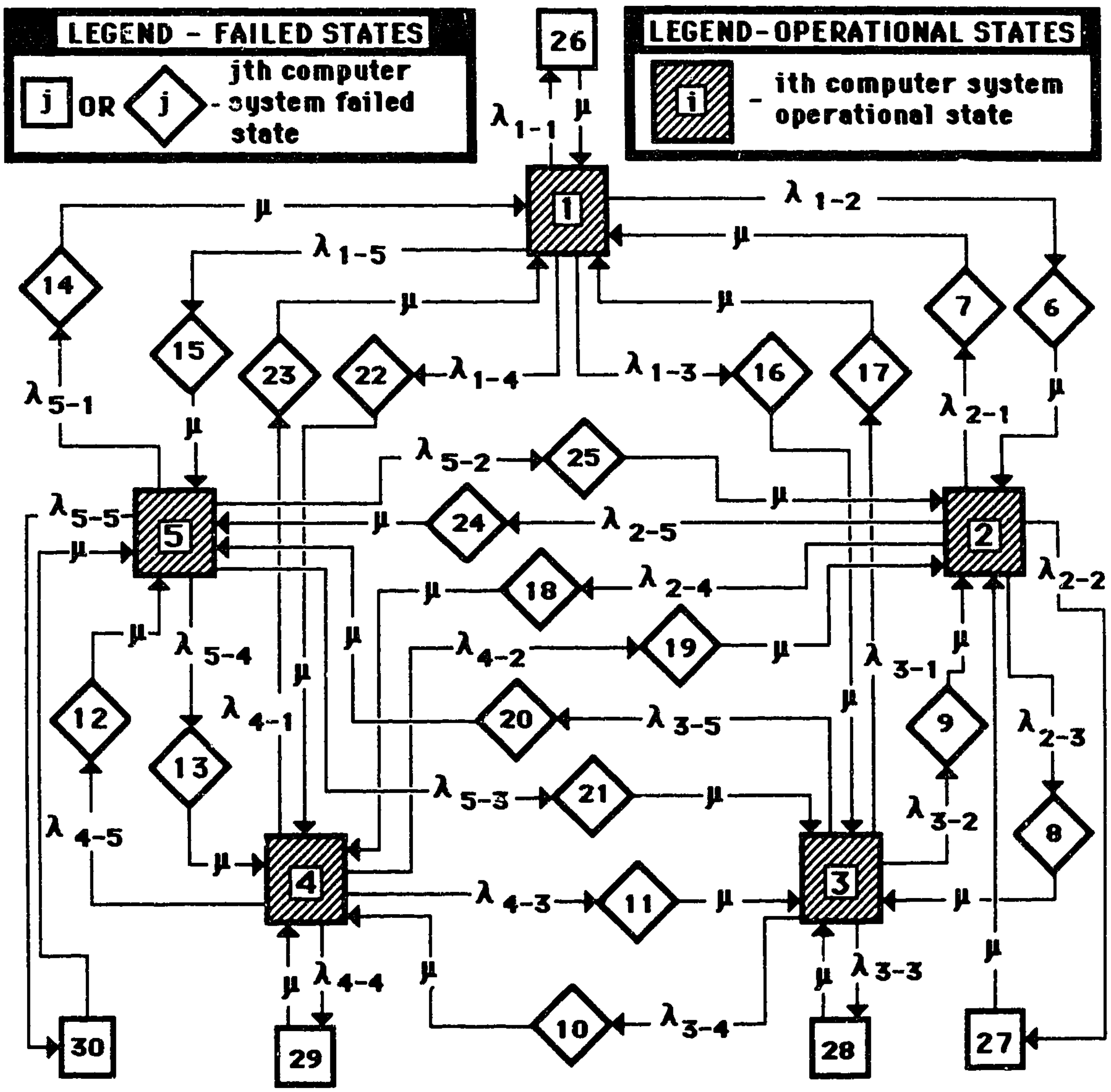

Figure 4.1 State space diagram for generalized Markov model.

The repair/restoration rate μ value was defined from the inverse of the average of the duration of all the failures. The repair/restoration states were characterized by an average repair/restoration rate for all failed states because the duration in the repair/restoration state was significantly smaller than the duration in an operational state. If the duration of the failed states were of the same magnitude as the operational states, then each failed state would be characterized by a unique repair/restoration rate μ_i as opposed to a single value μ.

The transition rates in the general model are polarized (i.e., giving the direction to the next operating state in the life cycle of the computer system). The transition rates shown in Figure 4.1 are rigidly defined in the following manner:

i - the **ith** computer system operating state prior to a computer system failure

j - the **jth** computer system operating state immediately following a repair/restoration state which was entered into from state **i.**

The transition between operating states must always pass through a repair/restoration state.

The departure rate $\lambda_{i\text{-}j}$ from a particular up state was defined as the number of transitions that occur from the source state to the destination state divided by the total number of transitions that originate from that state and this is multiplied by the empirically determined failure rate for the state.

The steady state probabilities of occupying each state in closed form for the 30 state generalized model can be solved by a frequency balance approach where the sum of the frequencies of departure from a given state must equal the sum of the frequencies of entry into the given state[1]. The frequency of departure from a given state is equal to the steady state probability of the state, times the sum of the transition rates departing from that state. The frequency of entry into a given state from another state is equal to the probability of the other state, times the directional transitional rate linking the given state (i.e., the process in time goes from the other state to the given state). The total frequency of entry into a given state is equal to the sum of all frequencies entering the given state. For example, the frequency balance equation for state 1 is given by the following expression:

$$P_1(\lambda_{1\text{-}1} + \lambda_{1\text{-}2} + \lambda_{1\text{-}3} + \lambda_{1\text{-}4} + \lambda_{1\text{-}5}) = \mu\,(P_7 + P_{14} + P_{17} + P_{23} + P_{26}) \qquad (1)$$

For each state in the state space diagram shown in Figure 4.1, a frequency balance equation can be expressed resulting in a set of 30 frequency balance equations. In order to solve for the steady state probabilities of occupying each state an additional constraining equation is required. This constraining equation expresses the fundamental relationship that the sum of all the state probabilities must be equal to one.

The set of 30 frequency balance equations are shown in Appendix A. The generalized solution of these equations in closed form is presented in the Appendix A due to the length of the closed form solutions for each state probability.

4.3 Application of the generalized Markov model

The generalized solution for the steady state probabilities for all 30 states contains all the possible states of computer system operation and the transitions between these states through restoration states. The transition rates between the various operational states will be evaluated for each computer system failure mode. The number of transitions between the various operational states for a given failure mode is significantly less than the generalized state space diagram. The remaining chapters (i.e., Chapters V to IX) of this thesis will focus on presenting the state space diagrams, the frequency balance equations and the closed form solutions of the steady state probabilities of the operational states for each failure mode (i.e., hardware, software, unknown, analyst and operator and the aggregate of all failure modes) of the Alberta Government Computer Center's VM-based system.

CHAPTER V

HARDWARE FAILURE MARKOV MODEL

5.1 Introduction

The analysis of the computer system hardware failure data contained in the problem report data base was directed at separating those hardware problems which resulted in restricted computer system operation and those which curtailed the performance of the system completely. To demonstrate the difference between problems that caused restricted system operation and those that caused a computer system failure, a sample of problems that have occurred are tabulated in Table 5.1

Table 5.1 Sample of hardware problems and their impact on computer system performance

DESCRIPTION OF HARDWARE PROBLEM	DESCRIPTION OF COMPUTER SYSTEM OPERATION
1. equipment check on 718 drum	**restricted**
2. keyboard locked out	**restricted**
3. error light 2 on C48 controller	**restricted**
4. soft machine check recording disabled	**restricted**
5. terminal has a blank screen	**restricted**
6. interfaces not disabled on bad director	**restricted**
7. machine check/supervisor damage on Amdahl V8	**system failure**
8. power check on V8	**system failure**
9. main storage unit thermal check on V8	**system failure**
10. locked out of VM	**system failure**

5.2 State space diagram for hardware failures only

Only those hardware failures that caused computer system failures to occur are considered in the state space diagram. The actual time of occurrence of these failures, the duration of the outage and the time to the next hardware failure (i.e., TTF) are shown in Table 5.2.

Table 5.2 Problem report data base for hardware failures

DATE OF OCCURRENCE	TIME OF OUTAGE	DURATION (minutes)	TIME OF RESTORATION	TTF (hours)
November 3, 1984	8:00	131	10:11	..
January 8, 1985	7:45	37	8:22	1581.57
January 29, 1985	10:11	12	10:23	505.82
January 29, 1985	18:07	26	18:33	7.73
April 19, 1985	5:00	40	5:40	1906.54
July 13, 1985	1:42	12	1:54	2036.03
September 5, 1985	3:36	60	4:36	1297.78
September 24, 1985	8:55	17	9:12	460.32
February 26, 1986	10:30	14	10:44	3721.30
March 31, 1986	12:24	70	13:34	793.67
April 27, 1986	20:14	66	21:20	654.67
April 28, 1986	21:10	86	22:36	23.83
October 31, 1986	13:08	49	13:57	4454.53
November 9, 1986	7:30	61	8:31	209.55
May 18, 1987	11:21	4	11:25	4490.83
June 23, 23, 1987	10:54	306	16:00	863.48

A graphical representation of the time to the next hardware failure is shown in Figure 5.1. It is evident from the Figure 5.1 that hardware failures follow a distinctive operating pattern. This pattern can be expressed in the form of a state space diagram shown in Figure 5.3 which is based on the generalized state space diagram presented in Chapter 4.

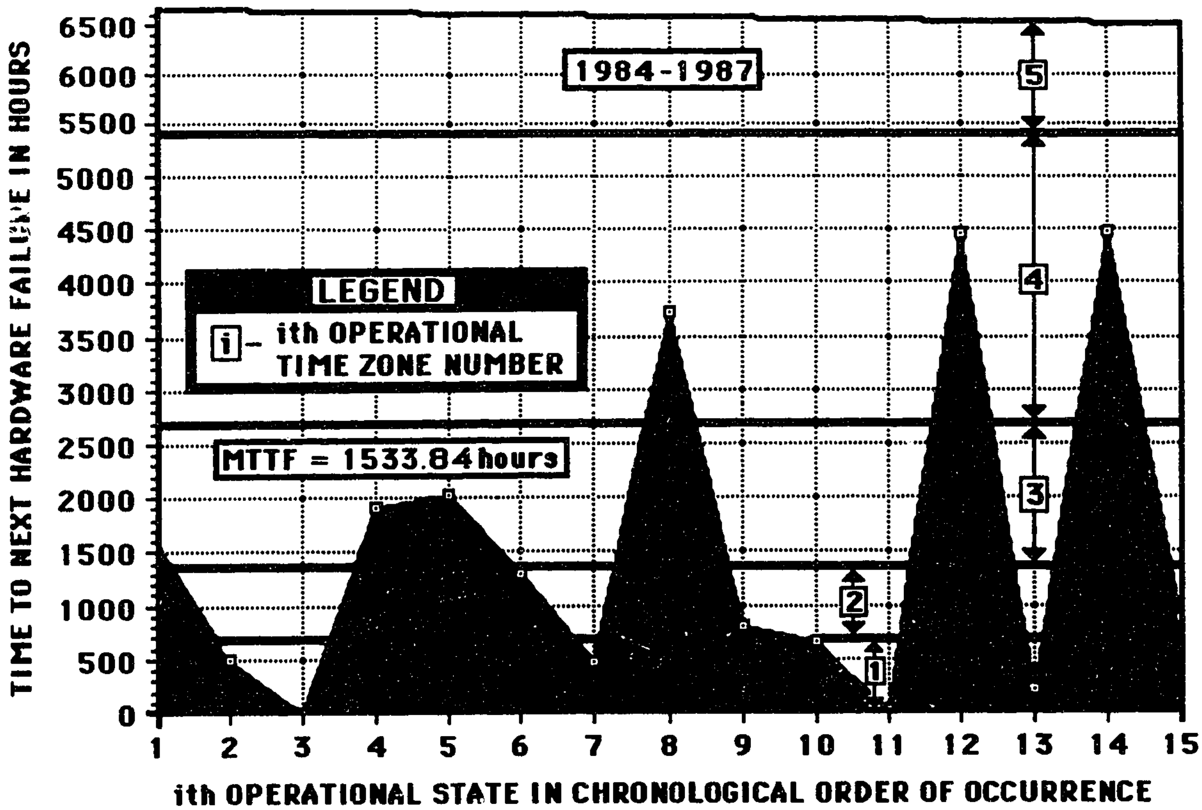

Figure 5.1 Time to the next hardware failure in chronological order.

In an examination of the chronological order of computer system operational times, the computer system operated in the following sequential time zones following each hardware failure:

[3 - 1 - 1 - 3 - 3 - 2 - 1 - 4 - 2 - 1 - 1 - 4 - 1 - 4 - 2]

This sequence defines the transitions between operating states. Note the number of transitions between operating states is significantly less than the generalized model where all possible transitions between all possible operating states is considered. Because many of the transitions of the general model did not occur, a much simpler model can be developed to model hardware failures. Based on the observed transitions between the various operating states, a state space diagram can be constructed to model hardware failures and their impact on computer system performance evaluated.

When the computer system's performance is curtailed by a hardware failure, it is necessary to restore the system to a fully operational state, if possible, by various restoration activities (e.g., repair, replacement, system reconfiguration, etc.). The frequency histogram of the duration the computer is in a failed state following a hardware failure is shown in Figure 5.2. The mean duration of restoration/repair activities is 57.33 minutes and defines the restoration/repair rate.

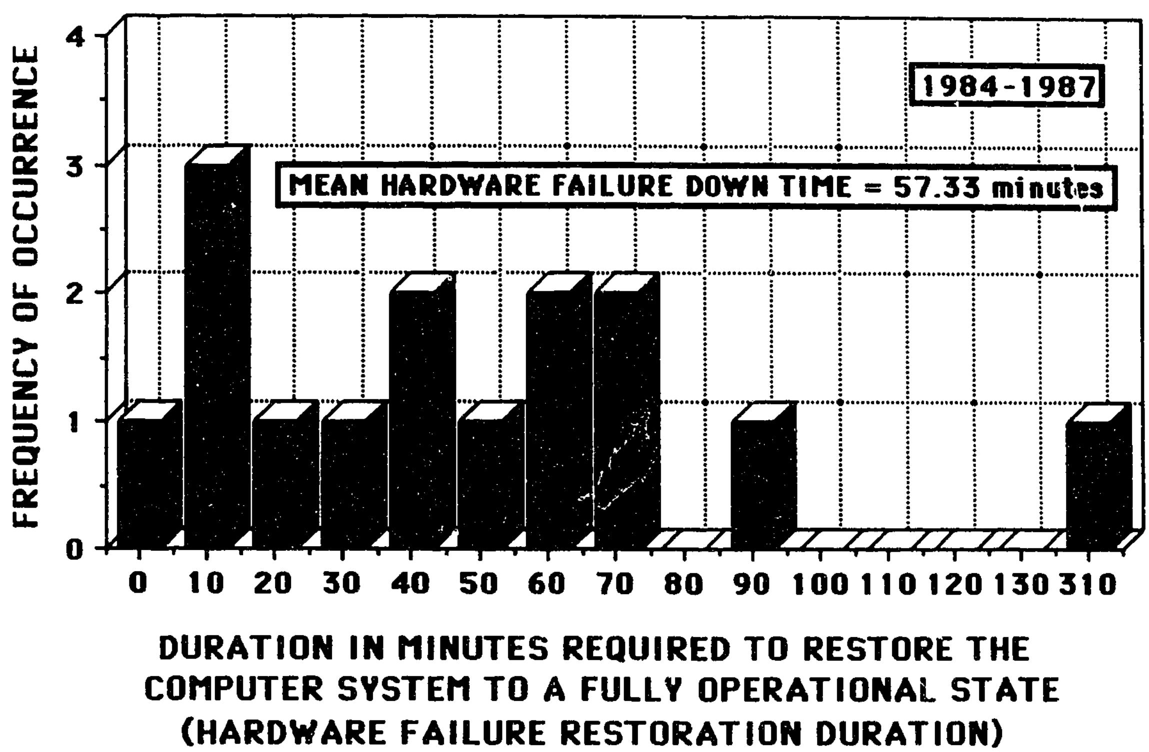

Figure 5.2 Computer system restoration duration following a hardware failure

Based on the observed transitions between operating states and the duration of restoration activities following a hardware failure, a state space diagram of the dynamic process can be constructed as shown in Figure 5.3.

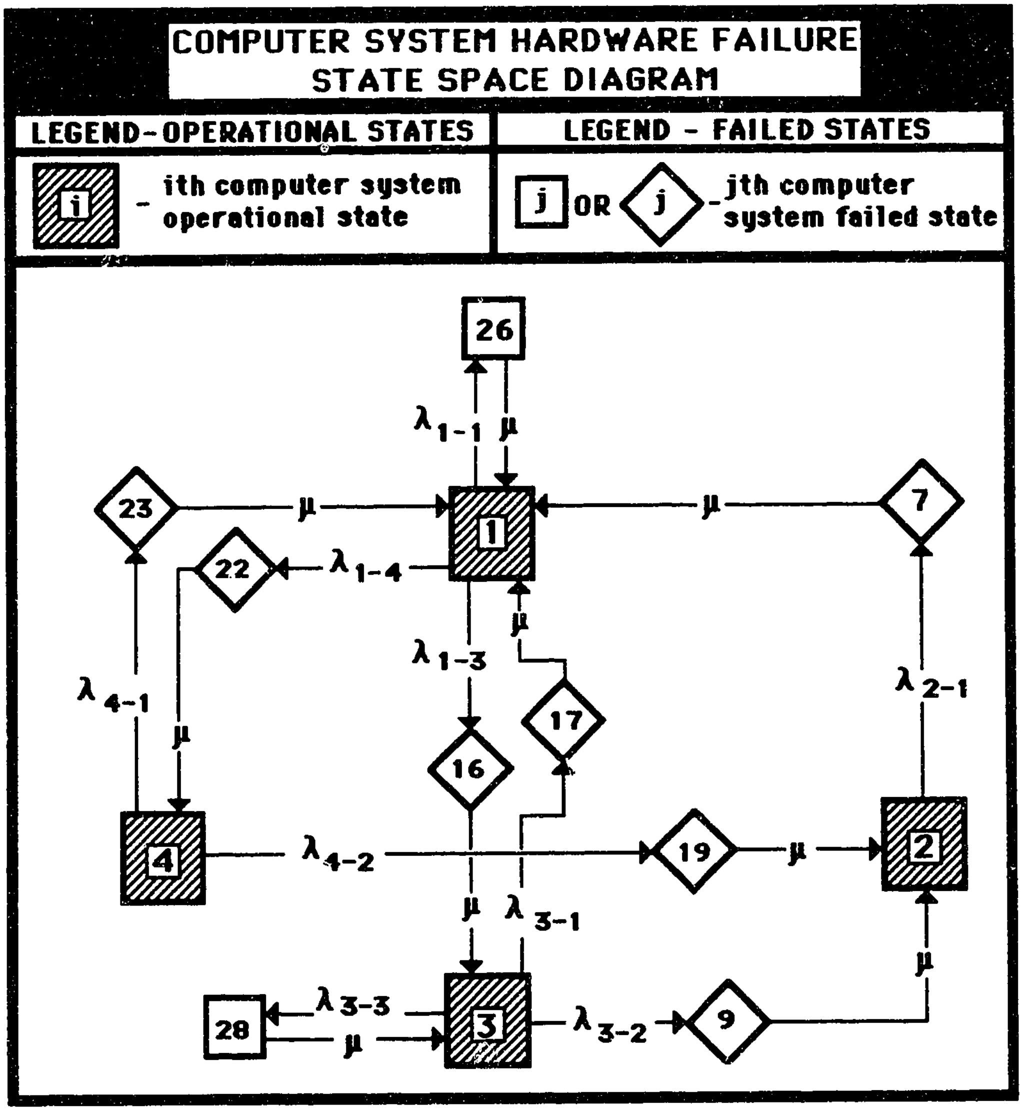

Figure 5.3 State space diagram for hardware failure Markov model.

The required operational states are one through four and the down states are specified from the observed transitions between the operational states. For example, down state number 9 is needed to accommodate the transition from operational state 3 to operational state 2, however, down state 8 in the general model is not required because there is no observed transition from operational state 2 to operational state 3.

5.3 Evaluation of steady state "state" probabilities in a closed form solution

The 13 state Markov model state probabilities can be solved by a frequency balance approach. The initial step is to define the frequency balance equations for each state in the state space diagram. These equations are listed as follows:

$$P_1 (\lambda_{11} + \lambda_{13} + \lambda_{14}) = \mu (P_7 + P_{17} + P_{23} + P_{26}) \qquad (1)$$

$$P_2 (\lambda_{21}) = \mu (P_9 + P_{19}) \qquad (2)$$

$$P_3 (\lambda_{31} + \lambda_{32} + \lambda_{33}) = \mu (P_{16} + P_{28}) \qquad (3)$$

$$P_4 (\lambda_{41} + \lambda_{42}) = \mu (P_{22}) \qquad (4)$$

$$P_7 (\mu) = P_2 \lambda_{21} \qquad (5)$$

$$P_9 (\mu) = P_3 \lambda_{32} \qquad (6)$$

$$P_{16} (\mu) = P_1 \lambda_{13} \qquad (7)$$

$$P_{17} (\mu) = P_3 \lambda_{31} \qquad (8)$$

$$P_{19} (\mu) = P_4 \lambda_{42} \qquad (9)$$

$$P_{22} (\mu) = P_1 \lambda_{14} \qquad (10)$$

$$P_{23} (\mu) = P_4 \lambda_{41} \qquad (11)$$

$$P_{26} (\mu) = P_1 \lambda_{11} \qquad (12)$$

$$P_{28} (\mu) = P_3 \lambda_{33} \qquad (13)$$

An additional equation is required for the solution of the steady state probabilities. This equation states that the sum of all the probabilities of all the states shown in Figure 5.3 is equal to one or in mathematical form can be expressed as:

$$P_1 + P_2 + P_3 + P_4 + P_7 + P_9 + P_{16} + P_{17} + P_{19} + P_{22} + P_{23} + P_{26} + P_{28} = 1.0 \quad (14)$$

The solution of the above equations by various traditional methods (e.g., matrix inversion) is extremely difficult, messy and time consuming particularly when the steady state solutions are expressed in closed form. To overcome these difficulties, equation (14) can be expressed in terms of the ratio of probabilities with the base probability equal to state 1 as follows:

$$P_1(1 + P_2/P_1 + P_3/P_1 + P_4/P_1 + P_7/P_1 + P_9/P_1 + P_{16}/P_1 + P_{17}/P_1 + \ldots$$
$$P_{19}/P_1 + P_{22}/P_1 + P_{23}/P_1 + P_{26}/P_1 + P_{28}/P_1) = 1.0 \quad (15)$$

The individual probability ratios shown in equation (15) can be evaluated from the frequency balance equations (i.e., continual substitution and back substitution until the probability ratio is evaluated). Once the probability ratios of equation (15) have been evaluated, then the probability of state 1 can be evaluated. In order to simplify the final form of the solution for P_1, the following constants will be defined:

$$A = \lambda_{31} + \lambda_{32} \quad (16)$$

$$B = \lambda_{41} + \lambda_{42} \quad (17)$$

$$C = (\lambda_{32}\lambda_{13}/A) + (\lambda_{42}\lambda_{14}/B) \quad (18)$$

$$D = \mu\ A \quad (19)$$

$$E = \mu\ B \quad (20)$$

The final form of the solution for P_1 can be expressed in terms of the above constants and a set of "state probability constants" as defined in Table 5.3.

Table 5.3 State probability constants (A(i))

STATE (i)	STATE PROBABILITY CONSTANT A(i)
2	C/λ_{21}
3	λ_{13}/A
4	λ_{14}/B
7	C/μ
9	$\lambda_{13}\lambda_{32}/D$
16	λ_{13}/μ
17	$\lambda_{13}\lambda_{31}/D$
19	$\lambda_{14}\lambda_{42}/E$
22	λ_{14}/μ
23	$\lambda_{14}\lambda_{41}/E$
26	λ_{11}/μ
28	$\lambda_{13}\lambda_{33}/D$

The steady state probability P_1 is defined by the following equation:

$$P_1 = \frac{1}{1 + \sum_{i=2}^{n} A(i)} \tag{21}$$

The probability of occupying any other state P_i is given by:

$$P_i = P_1 A(i) \tag{22}$$

5.4 Evaluation of Markov model parameters

The key parameters required to evaluate the steady state probabilities of the hardware Markov model are the repair/restoration rate and the state transitional rates as shown in Figure 5.3. The mean time the computer system resides in each operational state and the number of departures from that state are shown in Table 5.4.

Table 5.4 Mean duration of system operation in each state and number of transitions from a given state

STATE NUMBER	MEAN DURATION IN STATE (hours)	NUMBER OF TRANSITIONS FROM THE GIVEN STATE
1	310.32	6
2	984.95	2
3	1841.38	3
4	4222.22	3

The rate of departure from a given state λ_i is defined as follows:

$$\lambda_i = \frac{\text{total number of transitions from state i}}{\text{total duration in state}}$$

$$= 1.0 / \text{mean duration in state i} \quad (23)$$

The individual directional transition rates λ_{ij} from a given state **i** to another operational state **j** through a restoration state is defined as:

$$\lambda_{ij} = \lambda_i P(i,j) \quad (24)$$

where: **P(i,j)** = probability of a transition from state **i** to **j**

P(i,j) is defined as follows:

$$P(i,j) = \frac{\text{number of transitions from state i to j}}{\text{total number of transitions from state i}} \quad (25)$$

The individual transition rates of the hardware state space diagram are listed in Table 5.5

Table 5.5 Individual hardware transition rates between operating states

TRANSITION RATE	VALUE (departures/hour) x 10^{-6}
λ_{11}	1074.160000
λ_{13}	537.080000
λ_{14}	1611.240000
λ_{21}	1015.279900
λ_{31}	181.023650
λ_{32}	181.023650
λ_{33}	181.023650
λ_{41}	78.947409
λ_{42}	157.894810

The restoration rate μ is defined as the reciprocal of the mean time to restore the computer system to a fully operational state and is equal to 1.0465116 restorations per hour for hardware failures.

5.5 Calculation of the frequency and duration of computer system operation

The performance of a computer system is usually quantified by two variables; i.e., the probability that the computer system is operational and the frequency of departures from its operational state. Based on the transition rates between operational states and the restoration rate, the probability of occupying any of the states contained in the state space diagram (i.e., shown in Figure 5.3) can be evaluated from equations 21 and 22. The quantitative results of the steady state "state", probabilities are shown in Table 5.6.

Table 5.6 State probabilities for hardware Markov model

STATE (i)	STATE PROBABILITY P(i)
1	0.094200752
2	0.124579770
3	0.139742340
4	0.640848640
7	0.000120862
9	0.000024172
16	0.000048344
17	0.000024172
19	0.000096689
22	0.000145034
23	0.000048345
26	0.000096689
28	0.000024172

The probability of being in an operational state is equal to the sum of the probabilities of occupying all operational states and can be expressed mathematically as:

$$P(\text{operational}) = P_1 + P_2 + P_3 + P_4 \tag{26}$$

$$= 0.999371510$$

The frequency of departures from the operational states is equal to the sum of the frequency of departure from each individual operating state and can be expressed as follows:

$$f_{down} = P_1(\lambda_{11} + \lambda_{13} + \lambda_{14}) + P_2(\lambda_{21}) + P_3(\lambda_{31} + \lambda_{32} + \lambda_{33}) + \ldots\ldots + P_4(\lambda_{41} + \lambda_{13}) \quad (27)$$

= 0.00065771342 outages or occurrences/hour

= 5.76456 outages caused by hardware failures/year

5.6 Discussion of hardware failure Markov model

Based on the observed hardware failure patterns, this chapter has presented a 13 state Markov model for evaluating the impact of hardware failures on the performance of the Government of Alberta's Central VM-based computer system. The evolution equations of the steady state "state" probabilities in closed form were presented. The detailed frequency balance equations of the Markov model which is required for the solution of the state equations are also presented. The actual state transition rates are presented. Based on the evolution equations and the state transition rates, the probability of the computer system being operational was calculated and the frequency of computer hardware system outages per year was evaluated.

The next chapters VI to IX will present the Markov models for software, unknown, "analyst and operator" and system failures, respectively, based on the generalized Markov model presented in chapter IV.

CHAPTER VI

SOFTWARE FAILURE MARKOV MODEL

6.1 Introduction

The analysis of the computer system software failure data contained in the problem report data base was directed at separating those software problems which resulted in restricted computer system operation and those which curtailed the performance of the system completely. To demonstrate the difference between problems that caused restricted system operation and those that caused a computer system failure, a sample of problems that have occurred are tabulated in Table 6.1

Table 6.1 Sample of software problems and their impact on computer system performance

DESCRIPTION OF SOFTWARE PROBLEM	DESCRIPTION OF COMPUTER SYSTEM OPERATION
1. DISKACNT has noted an error	restricted
2. DIRMAINT not logged	restricted
3. bad return code for SYSDUMP1	restricted
4. Qreader has error 24	restricted
5. number of CCCPROP logfiles is incorrect	restricted
6. system temporary space full	restricted
7. CP abend - console locked	system failure
8. CP abend - system recovered by VSAFE	system failure
9. system failure code DSP001	system failure
10. VM restarted itself and cut a dump	system failure

6.2 State space diagram for software failures only

Only those software failures that caused computer system failures to occur are considered in the state space diagram. The actual time of occurrence of these failures, the time of outage, the duration of the outage and the time to the next software failure (i.e., TTF) are shown in Table 6.2.

Table 6.2 Problem report data base for software failures.

DATE OF OCCURRENCE	TIME OF OUTAGE	DURATION (minutes)	TIME OF RESTORATION	TTF (hours)
December 14, 1984	9:39	2	9:41	-
May 21, 1985	13:06	2	13:08	3795.42
August 14, 1985	16:25	1	16:26	2043.28
September 27, 1985	17:03	5	17:08	1056.62
October 2, 1985	15:51	8	15:59	118.72
April 24, 1986	16:21	7	16:28	4896.37
April 25, 1986	8:01	20	8:21	15.55
June 23, 1986	10:06	51	10:57	1417.75
June 30, 1986	00:00	188	3:08	157.05
November 4, 1986	15:07	12	15:19	3059.98
January 12, 1987	9:01	1	9:02	1649.70
January 13, 1987	9:24	1	9:25	24.37

A graphical representation of the time to the next software failure is shown in Figure 6.1. It is evident from the Figure 6.1 that software failures follow a distinctive operating pattern. This pattern can be expressed in the form of a state space diagram shown in Figure 6.3 which is based on the generalized state space diagram presented in Chapter 4.

In an examination of the chronological order of computer system operational times, the computer system operated in the following sequential time zones following each software failure:

[4 - 3 - 2 - 1 - 4 - 1 - 3 - 1 - 4 - 3 - 1]

This sequence defines the transitions between operating states. Note the number of transitions between operating states is significantly less than the generalized model where all possible transitions between all possible operating states is considered. Because many of the transitions of the general model did not occur, a much simpler model can be developed to model software failures.

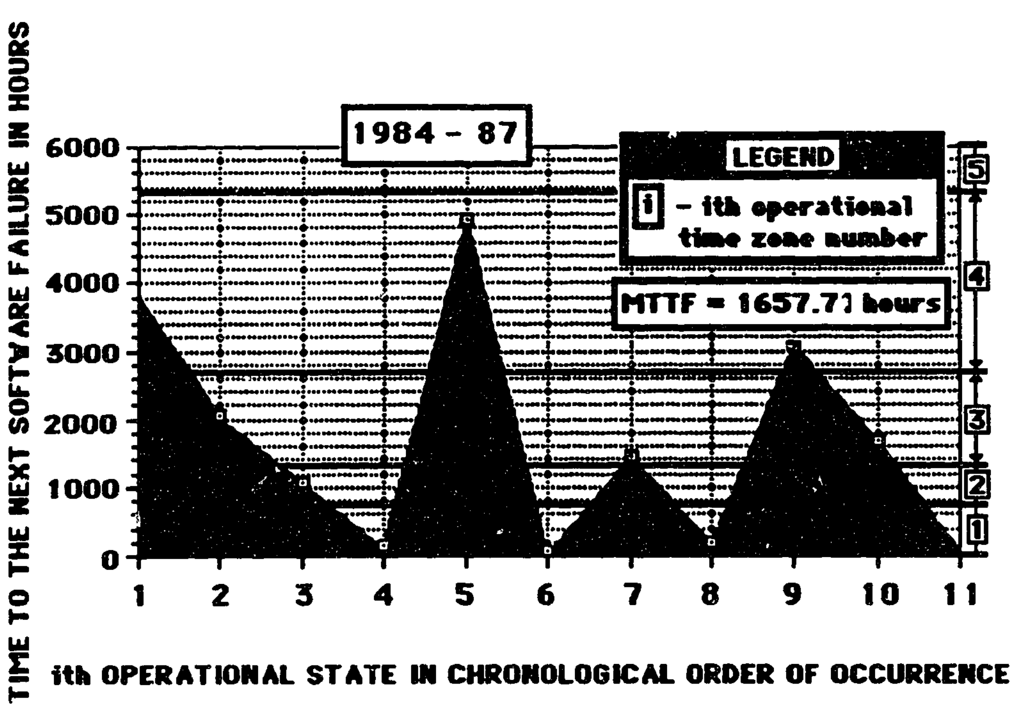

Figure 6.1 Time to the next software failure in chronological order.

Based on the observed transitions between the various operating states, a state space diagram can be constructed to model software failures and their impact on computer system performance evaluated.

When the computer system's performance is curtailed by a software failure, it is necessary to restore the system to a fully operational state, if possible, by various restoration activities (e.g., IPL'ing, using an old version of the software, upgrading the software, etc.). The frequency histogram of the duration the computer is in a failed state following a software failure is shown in Figure 6.2. The mean value of the duration of restoration/repair activities is 26.91 minutes which defines the restoration/repair rate.

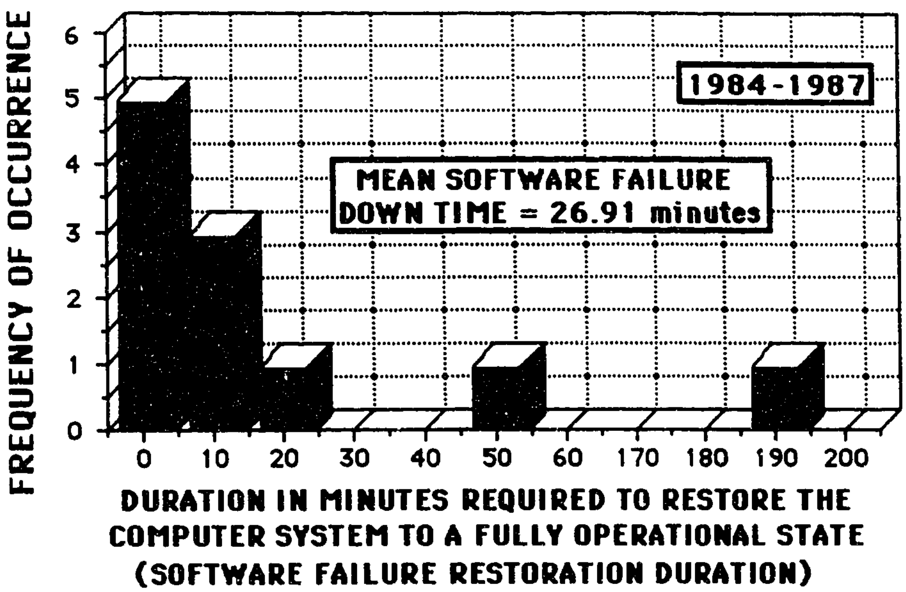

Figure 6.2 Computer system restoration duration following a software failure

Based on the observed transitions between operating states and the duration of restoration activities following a software failure, a state space diagram of the dynamic process can be constructed as shown in Figure 6.3.

The required operational states are one through four and the down states are specified from the observed transitions between the operational states. For example, down state number 9 is needed to accommodate the transition from operational state 3 to operational state 2, however, down state 8 in the general model is not required because there is no observed transition from operational state 2 to operational state 3.

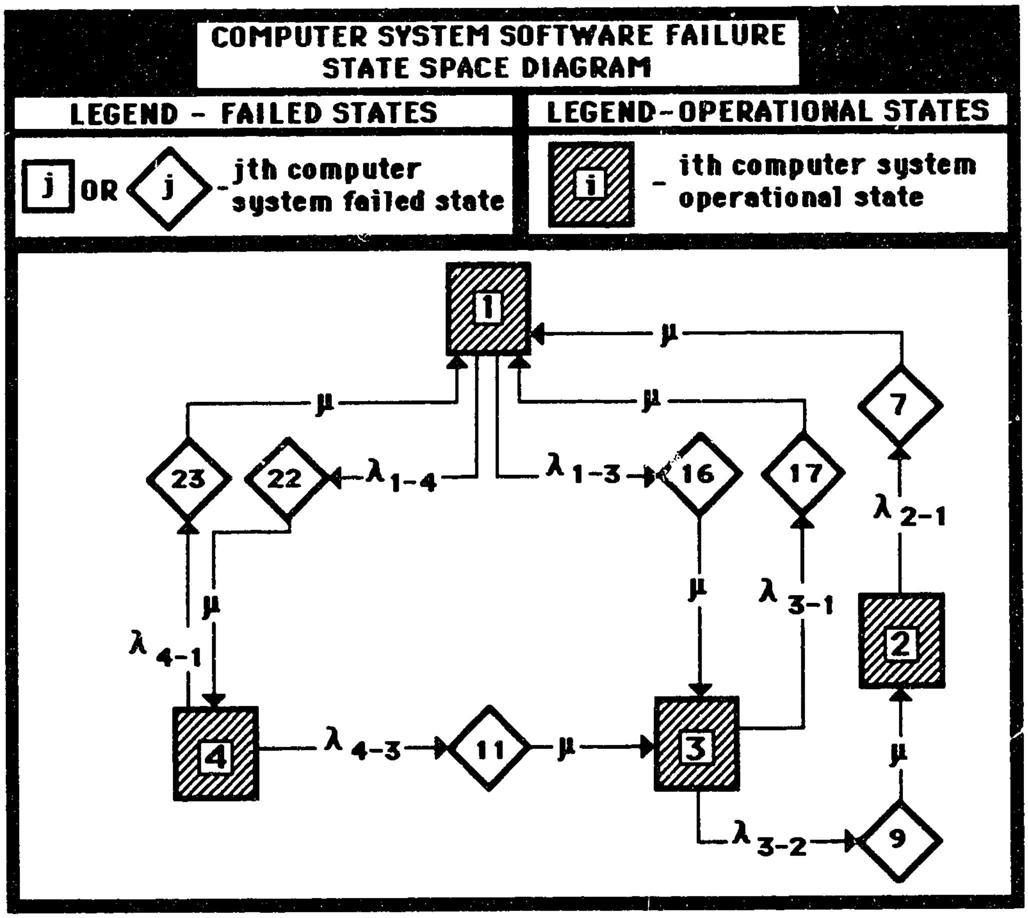

Figure 6.3 State space diagram for software failure Markov model.

6.3 Evaluation of steady state "state" probabilities in a closed form solution

The 11 state Markov model state probabilities can be solved by a frequency balance approach. The initial step is to define the frequency balance equations for each state in the state space diagram. These equations are listed as follows:

$$P_1(\lambda_{13} + \lambda_{14}) = \mu(P_7 + P_{17} + P_{23}) \tag{1}$$

$$P_2(\lambda_{21}) = \mu(P_9) \tag{2}$$

$$P_3(\lambda_{31} + \lambda_{32}) = \mu (P_{11} + P_{16}) \quad (3)$$

$$P_4 (\lambda_{41} + \lambda_{43}) = \mu (P_{22}) \quad (4)$$

$$P_7 (\mu) = P_2 \lambda_{21} \quad (5)$$

$$P_9 (\mu) = P_3 \lambda_{32} \quad (6)$$

$$P_{11} (\mu) = P_4 \lambda_{43} \quad (7)$$

$$P_{16} (\mu) = P_1 \lambda_{13} \quad (8)$$

$$P_{17} (\mu) = P_3 \lambda_{31} \quad (9)$$

$$P_{22} (\mu) = P_1 \lambda_{14} \quad (10)$$

$$P_{23} (\mu) = P_4 \lambda_{41} \quad (11)$$

An additional equation is requireu for the solution of the steady state probabilities. This equation states that the sum of all the probabilities of all the states shown in Figure 6.3 is equal to one or in mathematical form can be expressed as:

$$P_1 + P_2 + P_3 + P_4 + P_7 + P_9 + P_{11} + P_{16} + P_{17} + P_{22} + P_{23} = 1.0 \quad (12)$$

The solution of the above equations by various traditional methods (e.g., matrix inversion) is extremely difficult, messy and time consuming particularly when the steady state solutions are expressed in closed form. To overcome these difficulties, equation (12) can be expressed in terms of the ratio of probabilities with the base probability equal to state 1 as follows:

$$P_1(1 + P_2/P_1 + P_3/P_1 + P_4/P_1 + P_7/P_1 + P_9/P_1 + P_{11}/P_1 + P_{16}/P_1 + \ldots\ldots$$
$$P_{17}/P_1 + P_{22}/P_1 + P_{23}/P_1) = 1.0 \quad (13)$$

The individual probability ratios shown in equation (13) can be evaluated from the frequency balance equations (i.e., continual substitution and back substitution until the probability ratio is evaluated). Once the probability ratios of equation (13) have been evaluated, then the probability of state 1 can be evaluated. In order to simplify the final form of the solution for P_1, the following constants will be defined:

$$A = \lambda_{31} + \lambda_{32} \tag{14}$$

$$B = \lambda_{41} + \lambda_{43} \tag{15}$$

$$C = (\lambda_{13}/A) + (\lambda_{43}\lambda_{14}/AB) \tag{16}$$

$$D = \mu\ B \tag{17}$$

The final form of the solution for P_1 can be expressed in terms of the above constants and a set of "state probability constants" as defined in Table 6.3.

Table 6.3 State probability constants (A(i))

STATE (i)	STATE PROBABILITY CONSTANT A(i)
2	$\lambda_{32} C/ \lambda_{21}$
3	C
4	λ_{14}/ B
7	$\lambda_{32} C/ \mu$
9	$\lambda_{32} C/ \mu$
11	$\lambda_{43}\lambda_{14}/ D$
16	λ_{13}/ μ
17	$\lambda_{31} C/ \mu$
22	λ_{14}/ μ
23	$\lambda_{14}\lambda_{41}/ D$

The steady state probability P_1 is defined by the following equation:

$$P_1 = \frac{1}{1 + \sum_{i=2}^{n} A(i)} \qquad (18)$$

The probability of occupying any other state P_i is given by:

$$P_i = P_1 A(i) \qquad (19)$$

6.4 Evaluation of Markov model parameters

The key parameters required to evaluate the steady state probabilities of the software Markov model are the repair/restoration rate and the state transitional rates as shown in Figure 6.3. The mean time the computer system resides in each operational state and the number of departures from that state are shown in Table 6.4.

Table 6.4 Mean duration of system operation in each state and number of transitions from a given state

STATE NUMBER	MEAN DURATION IN STATE (hours)	NUMBER OF TRANSITIONS FROM THE GIVEN STATE
1	78.92	3
2	1056.62	1
3	1703.58	3
4	3917.26	3

The rate of departure from a given state λ_i is defined as follows:

$$\lambda_i = \frac{\text{total number of transitions from state i}}{\text{total duration in state}}$$

$$= 1.0 \,/\, \text{mean duration in state i} \qquad (20)$$

The individual directional transition rates λ_{ij} from a given state **i** to another operational state **j** through a restoration state is defined as:

$$\lambda_{ij} = \lambda_i P(i,j) \qquad (21)$$

where: **P(i,j)** = probability of a transition from state **i** to **j**

P(i,j) is defined as follows:

$$P(i,j) = \frac{\text{number of transitions from state i to j}}{\text{total number of transitions from state i}} \qquad (22)$$

The individual transition rates of the software state space diagram are listed in Table 6.5

Table 6.5 Individual software transition rates between operating states

TRANSITION RATE	VALUE (departures/hour) x 10^{-6}
λ_{13}	4223.552600
λ_{14}	8447.105200
λ_{21}	946.414030
λ_{31}	391.333520
λ_{32}	195.666760
λ_{41}	85.093564
λ_{42}	170.187120

The restoration rate μ is defined as the reciprocal of the mean time to restore the computer system to a fully operational state and is equal to 2.2297297 restorations per hour for software failures.

6.5 Calculation of the frequency and duration of computer system operation

The performance of a computer system is usually quantified by two variables; i.e., the probability that the computer system is operational and the frequency of departures from its operational state. Based on the transition rates between operational states and the restoration rate, the probability of occupying any of the states contained in the state space diagram (i.e., shown in Figure 6.3) can be evaluated from equations 18 and 19. The quantitative results of the steady state "state" probabilities are shown in Table 6.6.

Table 6.6 State probabilities for software Markov model.

STATE (i)	STATE PROBABILITY P(i)
1	0.018394357000
2	0.063846469000
3	0.308816850000
4	0.608659700000
7	0.000027099784
9	0.000027099784
11	0.000046456773
16	0.000034842579
17	0.000054199568
22	0.000069685159
23	0.000023228386

The probability of being in an operational state is equal to the sum of the probabilities of occupying all operational states and can be expressed mathematically as:

$$\begin{aligned} P(\text{operational}) &= P_1 + P_2 + P_3 + P_4 \\ &= 0.99971738 \end{aligned} \quad (23)$$

The frequency of departures from the operational states is equal to the sum of the frequency of departure from each individual operating state and can be expressed as follows:

$$f_{down} = P_1(\lambda_{13} + \lambda_{14}) + P_2(\lambda_{21}) + P_3(\lambda_{31} + \lambda_{32}) + \ldots\ldots + P_4(\lambda_{41} + \lambda_{43}) \qquad (24)$$

= 0.00063014845 outages or occurrences/hour

= 5.5201004 outages caused by software failures/year

6.6 Discussion of software failure Markov model

Based on the observed software failure patterns, this chapter has presented an 11 state Markov model for evaluating the frequency and duration of software failures of the Government of Alberta's Central Computer Center's VM-based computer system. The software Markov model was characterized by 11 states being observed out of the possible 30 generalized states. The MTTF of software failures was 1657.7 hours and was longer than hardware failures (i.e., 1533.8 hours). However, the mean software restoration duration of 26.91 minutes was significantly less than the mean hardware failure restoration activities of 57.33 minutes. Based on the evolution equations developed in this chapter and the observed state transition rates, the probability of the computer system being operational was calculated and the frequency of computer system software outages per year was evaluated.

CHAPTER VII

UNKNOWN FAILURE MARKOV MODEL

7.1 Introduction

The analysis of the computer system unknown failure data contained in the problem report data base was directed at separating those unknown problems which resulted in restricted computer system operation and those which curtailed the performance of the system completely. To demonstrate the difference between problems that caused restricted system operation and those that caused a computer system failure, a sample of problems that have occurred are tabulated in Table 7.1.

Table 7.1 Sample of unknown problems and their impact on computer system performance

DESCRIPTION OF UNKNOWN PROBLEM	DESCRIPTION OF COMPUTER SYSTEM OPERATION
1. incorrect volser mounted for SYSDUMP1	**restricted**
2. bad return code from AUTOLOG for VMAP	**restricted**
3. shutdown command didn't work for VM	**restricted**
4. VMSMFWEEK tried to run for the second time	**restricted**
5. PASSTHRU link CALVM didn't activate at IPL	**restricted**
6. console log for PASSTHRU missing	**restricted**
7. system in 100% supervision state	**system failure**
8. users unable to logon to VM	**system failure**
9. VM directory unavailable	**system failure**
10. CNVA crashed and re-IPLed itself	**system failure**

7.2 State space diagram for unknown failures only

Only those unknown failures that caused computer system failures to occur are considered in the state space diagram. The actual time of occurrence of these failures, the duration of the outage and the time to the next unknown failure (i.e., TTF) are shown in Table 7.2.

Table 7.2 Problem report data base for unknown failures

DATE OF OCCURRENCE	TIME OF OUTAGE	DURATION (minutes)	TIME OF RESTORATION	TTF (hours)
December 9, 1984	23:20	8	23:28	-
January 22, 1985	10:37	12	10:49	1043.15
January 24, 1985	17:33	1	17:34	54.73
January 29, 1985	16:00	1	16:01	118.43
February 12, 1985	13:00	5	13:05	332.98
February 28, 1985	16:00	6	16:06	386.92
March 5, 1985	15:28	7	15:35	119.37
June 6, 1985	8:58	10	9:08	2225.38
July 12, 1985	16:47	13	17:00	871.65
September 26, 1985	13:30	2	13:32	1820.50
October 21, 1985	13:34	20	13:54	600.03
February 3, 1986	9:44	37	10:21	2515.83
February 27, 1986	10:17	25	10:42	575.93
March 10, 1986	00:42	4	00:46	254.00
March 27, 1986	19:41	10	19:51	426.92
May 30, 1986	17:23	16	17:39	1533.53
September 29, 1986	6:55	8	7:03	2917.27
nothing before August 7, 1987				7480.95+

A graphical representation of the time to the next unknown failure is shown in Figure 7.1. It is evident from the Figure 7.1 that unknown failures follow a distinctive operating pattern. This pattern can be expressed in the form of a state space diagram shown in Figure 7.3 which is based on the generalized state space diagram presented in Chapter 4.

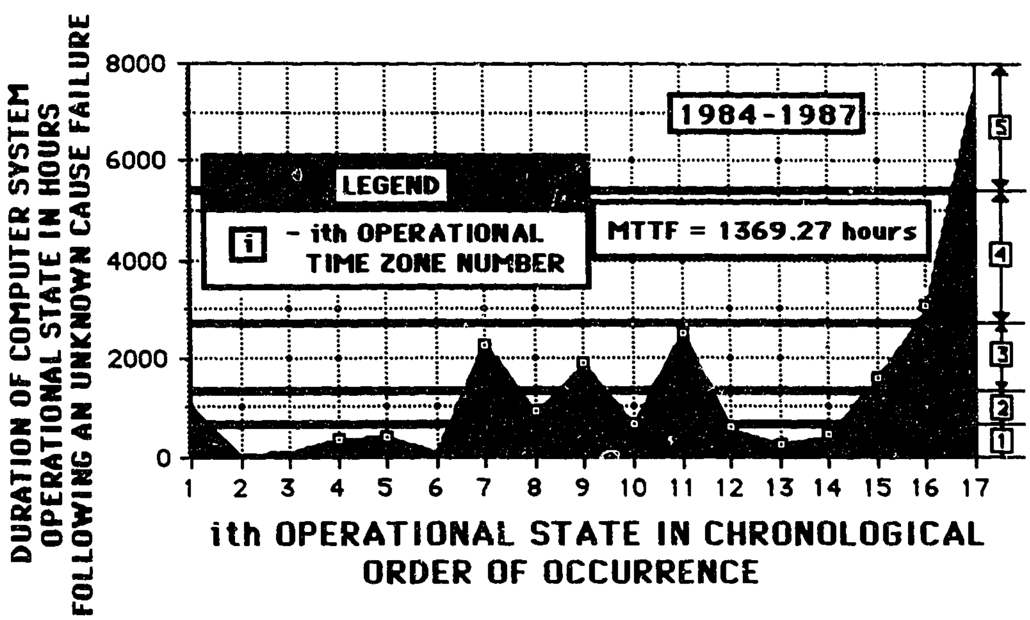

Figure 7.1 Time to the next unknown failure in chronological order.

In an examination of the chronological order of computer system operational times, the computer system operated in the following sequential time zones following each unknown failure:

[2 - 1 - 1 - 1 - 1 - 1 - 3 - 2 - 3 - 1 - 3 - 1 - 1 - 1 - 3 - 4 - 5]

This sequence defines the transitions between operating states. Note the number of transitions between operating states is significantly less than the generalized model where all possible transitions between all possible operating states is considered. Because many of the transitions of the general model did not occur, a much simpler model can be developed to model unknown failures. Based on the observed transitions between the various operating states, a state space diagram can be constructed to model unknown failures and their impact on computer system performance evaluated.

When the computer system's performance is curtailed by an unknown failure, it is necessary to restore the system to a fully operational state, if possible, by various restoration activities (e.g., replacement, system reconfiguration, IPL'ing, using an old version of the software, upgradng the software, etc.). The frequency histogram of the duration the computer is in a failed state following an unknown failure is shown in Figure 7.2. The expected value of the duration of restoration/repair activities is 11.06 minutes and defines the restoration/repair rate.

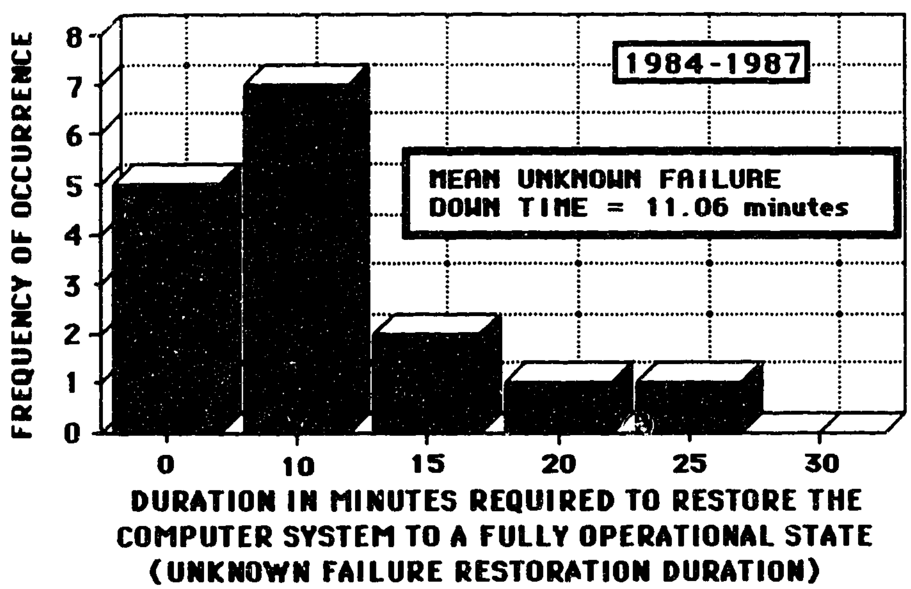

Figure 7.2 Computer system restoration duration following an unknown failure.

Based on the observed transitions between operating states and the duration of restoration activities following an unknown failure and the assumption of a transition from state 5 to state 1 to allow the development of a closed system, a state space diagram of the dynamic process can be constructed as shown in Figure 7.3.

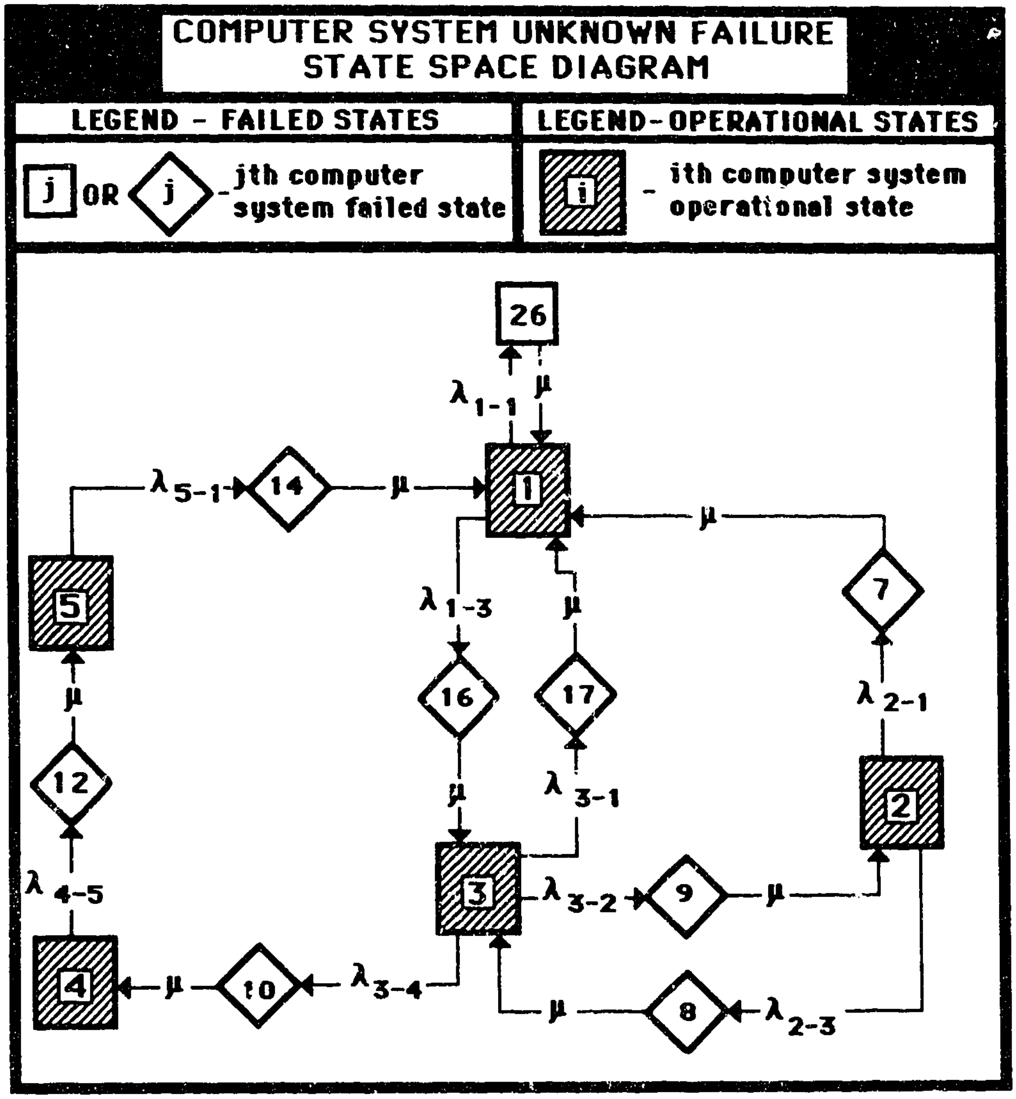

Figure 7.3 State space diagram for unknown failure Markov model.

The required operational states are 1 through 5 and the down states are specified from the observed transitions between the operational states. For example, down state number 10 is needed to accommodate the transition from operational state 3 to operational state 4, however, down state 11 in the general model is not required because there is no observed transition from operational state 4 to operational state 3.

7.3 Evaluation of steady state "state" probabilities in a closed form solution

The 14 state Markov model state probabilities can be solved by a frequency balance approach. The initial step is to define the frequency balance equations for each state in the state space diagram. These equations are listed as follows:

$$P_1 (\lambda_{11} + \lambda_{13}) = \mu (P_7 + P_{14} + P_{17} + P_{26}) \qquad (1)$$

$$P_2 (\lambda_{21} + \lambda_{23}) = \mu (P_9) \qquad (2)$$

$$P_3 (\lambda_{31} + \lambda_{32} + \lambda_{34}) = \mu (P_8 + P_{16}) \qquad (3)$$

$$P_4 (\lambda_{45}) = \mu (P_{10}) \qquad (4)$$

$$P_5 (\lambda_{51}) = \mu (P_{12}) \qquad (5)$$

$$P_7 (\mu) = P_2 \lambda_{21} \qquad (6)$$

$$P_8 (\mu) = P_2 \lambda_{23} \qquad (7)$$

$$P_9 (\mu) = P_3 \lambda_{32} \qquad (8)$$

$$P_{10} (\mu) = P_3 \lambda_{34} \qquad (9)$$

$$P_{12} (\mu) = P_4 \lambda_{45} \qquad (10)$$

$$P_{14} (\mu) = P_5 \lambda_{51} \qquad (11)$$

$$P_{16} (\mu) = P_1 \lambda_{13} \qquad (12)$$

$$P_{17} (\mu) = P_3 \lambda_{31} \qquad (13)$$

$$P_{26} (\mu) = P_1 \lambda_{11} \qquad (14)$$

An additional equation is required for the solution of the steady state probabilities. This equation states that the sum of all the probabilities of all the states shown in Figure 7.3 is equal to one or in mathematical form can be expressed as:

$$P_1 + P_2 + P_3 + P_4 + P_5 + P_7 + P_8 + P_9 + P_{10} + P_{12} + P_{14} + P_{16} + \ldots$$
$$\ldots + P_{17} + P_{26} = 1.0 \qquad (15)$$

The solution of the above equations by various traditional methods (e.g., matrix inversion) is extremely difficult, messy and time consuming particularly when the steady state solutions are expressed in closed form. To overcome these difficulties, equation (15) can be expressed in terms of the ratio of probabilities with the base probability equal to state 1 as follows:

$$P_1(1 + P_2/P_1 + P_3/P_1 + P_4/P_1 + P_5/P_1 + P_7/P_1 + P_8/P_1 + P_9/P_1 + \ldots\ldots$$
$$P_{10}/P_1 + P_{12}/P_1 + P_{14}/P_1 + P_{16}/P_1 + P_{17}/P_1 + P_{26}/P_1) = 1.0 \qquad (16)$$

The individual probability ratios shown in equation (16) can be evaluated from the frequency balance equations (i.e., continual substitution and back substitution until the probability ratio is evaluated). Once the probability ratios of equation (16) have been evaluated, then the probability of state 1 can be evaluated. In order to simplify the final form of the solution for $\mathbf{P_1}$, the following constants will be defined:

$$A = \lambda_{31} + \lambda_{32} + \lambda_{34} \qquad (17)$$

$$B = \lambda_{32} \,/\, (\lambda_{21} + \lambda_{23}) \qquad (18)$$

$$C = \lambda_{13} \,/\, (A - \lambda_{23} B) \qquad (19)$$

$$D = B\,C \qquad (20)$$

The final form of the solution for P_1 can be expressed in terms of the above constants and a set of "state probability constants" as defined in Table 7.3.

Table 7.3 State probability constants (A(i))

STATE (i)	STATE PROBABILITY CONSTANT A(i)
2	D
3	C
4	$C\lambda_{34} / \lambda_{45}$
5	$C\lambda_{34} / \lambda_{51}$
7	$D\lambda_{21} / \mu$
8	$D\lambda_{23} / \mu$
9	$C\lambda_{32} / \mu$
10	$C\lambda_{34} / \mu$
12	$C\lambda_{34} / \mu$
14	$C\lambda_{34} / \mu$
16	λ_{13}/μ
17	$C\lambda_{31} / \mu$
26	λ_{11}/μ

The steady state probability P_1 is defined by the following equation:

$$P_1 = \frac{1}{1 + \sum_{i=2}^{n} A(i)} \qquad (21)$$

The probability of occupying any other state P_i is given by:

$$P_i = P_1 \, A(i) \qquad (22)$$

7.4 Evaluation of Markov model parameters

The key parameters required to evaluate the steady state probabilities of the unknown Markov model are the repair/restoration rate and the state transitional rates as shown in Figure 7.3. The mean time the computer system resides in each operational state and the number of departures from that state are shown in Table 7.4.

Table 7.4 Mean duration of system operation in each state and number of transitions from a given state

STATE NUMBER	MEAN DURATION IN STATE (hours)	NUMBER OF TRANSITIONS FROM THE GIVEN STATE
1	318.81	9
2	957.40	2
3	2023.81	4
4	2917.27	1
5	7480.95	1

The rate of departure from a given state λ_i is defined as follows:

$$\lambda_i = \frac{\textbf{total number of transitions from state i}}{\textbf{total duration in state}}$$

$$= \textbf{1.0 / mean duration in state i} \qquad (23)$$

The individual directional transition rates λ_{ij} from a given state **i** to another operational state **j** through a restoration state is defined as:

$$\lambda_{ij} = \lambda_i \, P(i,j) \qquad (24)$$

where: **P(i,j)** = probability of a transition from state **i** to **j**

P(i,j) is defined as follows:

$$P(i,j) = \frac{\textbf{number of transitions from state i to j}}{\textbf{total number of transitions from state i}} \qquad (25)$$

The individual transition rates of the unknown state space diagram are listed in Table 7.5

Table 7.5 Individual unknown transition rates between operating states.

TRANSITION RATE	VALUE (departures/hour) x 10^{-6}
λ_{11}	1792.367
λ_{13}	1344.275
λ_{21}	522.248
λ_{23}	522.248
λ_{31}	247.059
λ_{32}	123.529
λ_{34}	123.529
λ_{45}	342.786
λ_{51}	133.673

The restoration rate μ is defined as the reciprocal of the mean time to restore the computer system to a fully operational state and is equal to 5.423728814 restorations per hour for unknown failures.

7.5 Calculation of the frequency and duration of computer system operation

The performance of a computer system is usually quantified by two variables; i.e., the probability that the computer system is operational and the frequency of departures from its operational state. Based on the transition rates between operational states and the restoration rate, the probability of occupying any of the states contained in the state space diagram (i.e., shown in Figure 7.3) can be evaluated from equations 21 and 22. The quantitative results of the steady state "state" probabilities are shown in Table 7.6.

Table 7.6 State probabilities for unknown Markov model

STATE (i)	STATE PROBABILITY P(i)
1	0.118039573
2	0.043405150
3	0.367009766
4	0.132258765
5	0.339159970
7	0.000004179
8	0.000004179
9	0.000008359
10	0.000008359
12	0.000008359
14	0.000008359
16	0.000029256
17	0.000016718
26	0.000039008

The probability of being in an operational state is equal to the sum of the probabilities of occupying all operational states and can be expressed mathematically as:

$$P(\text{operational}) = P_1 + P_2 + P_3 + P_4 + P_5 \quad (26)$$

$$= 0.99987322$$

The frequency of departures from the operational states is equal to the sum of the frequency of departure from each individual operating state and can be expressed as follows:

$$f_{down} = P_1(\lambda_{11} + \lambda_{13}) + P_2(\lambda_{21} + \lambda_{23}) + P_3(\lambda_{31} \quad {}_{32} + \lambda_{34}) + \ldots\ldots$$
$$+ P_4(\lambda_{45}) + P_5(\lambda_{51}) \qquad (27)$$

= 0.00068760336 outages or occurrences/hour

= 6.0234055 outages caused by unknown failures/year

7.6 Discussion of unknown failure Markov model

Based on the observed unknown failure patterns, this chapter has presented a 14 state Markov model for evaluating the frequency and duration of unknown failures of the Government of Alberta's Central Computer Center's VM-based computer system. The mean duration of "restoration" activities was 11.06 minutes and was significantly less than the duration required for hardware and software failures. The restoration activities of unknown failure events involved many types of restoration activities (e.g., hardware replacement, software changes, etc.) to restore the computer system to a fully operational state. However, the original cause of the computer system outage was still unknown and classified accordingly.

The evolution equations of the steady state "state" probabilities in closed form were presented. The detailed frequency balance equations of the Markov model for unknown failures were presented and are required for the solution of the state probability equations. Based on these evolution equations and the actual state transition rates, the probability of the computer system being operational was calculated and the frequency of unknown system outages per year was evaluated.

CHAPTER VIII

SYSTEM ANALYSTS' AND SYSTEM OPERATORS' FAILURE MARKOV MODELS

8.1 Introduction

This chapter presents the Markov models for the problems that were caused by the computer system analysts and the computer system operators. There were few observed problems due to these causes and therefore the amount of data is limited.

8.2 State space diagram for system analysts' failures only

Only those system analysts problems that caused computer system failures to occur are considered in the state space diagram. The actual time of occurrence of these failures, the duration of the outage and the time to the next system analysts' failure (i.e., TTF) are shown in Table 8.1.

Table 8.1 Problem report data base for system analysts' failures.

DATE OF OCCURRENCE	TIME OF OUTAGE	DURATION (minutes)	TIME OF RESTORATION	TTF (hours)
November 18, 1984	13:18	53	14:11	-
April 21, 1985	9:00	151	11:31	3690.82
December 14, 1985	16:06	27	16:33	5692.58
February 20, 1987	10:58	30	11:28	10386.42
April 12, 1987	9:00	68	10:08	1221.53

There are too few failures to see if there is a pattern similar to the pattern that was evident for the hardware or software failures. However, the failures can be expressed in the form of a state space diagram shown in Figure 8.1 which is based on the generalized state space diagram presented in Chapter IV.

An examination of the chronological order of computer system operational times, reveals the computer system operated in the following sequential time zones following each system analysts failure: [4 - 5 - 5 - 2]. This sequence defines the transitions between operating states. Note the number of transitions between operating states is significantly less than the generalized model where all possible transitions between all possible operating states is considered. Because many of the transitions of the general model did not occur, a much simpler model can be developed to model system analysts' failures.

Based on the observed transitions between operating states and the duration of restoration activities following a system analysts' failure and the assumption of a transition from state 2 to state 4 to allow the development of a closed system, a state space diagram of the dynamic process can be constructed as shown in Figure 8.1.

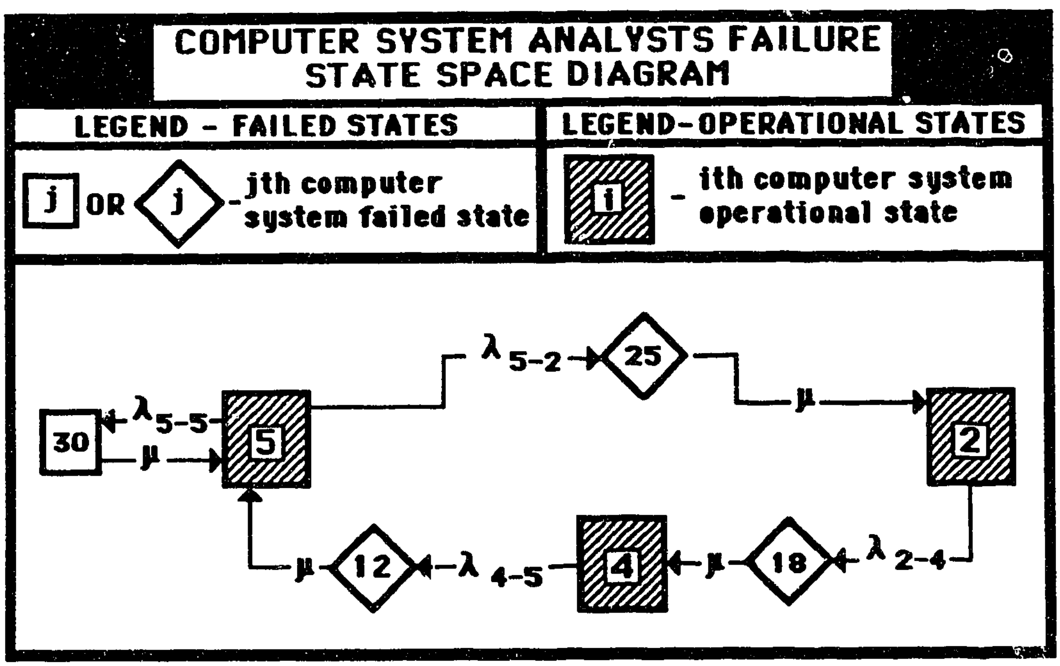

Figure 8.1 State space diagram for system analysts' failure Markov model.

The required operational states are 2, 4 and 5 and the down states are specified from the observed transitions between the operational states. For example, down state number 18 is needed to accommodate the transition from operational state 2 to operational state 4, however, down state 19 in the general model is not required because there is no observed transition from operational state 4 to operational state 2.

8.3 Evaluation of steady state "state" probabilities in a closed form solution

The 7 state Markov model state probabilities can be solved by a frequency balance approach. The initial step is to define the frequency balance equations for each state in the state space diagram. These equations are listed as follows:

$$P_2 (\lambda_{24}) = \mu (P_{25}) \quad (1)$$

$$P_4 (\lambda_{45}) = \mu (P_{18}) \quad (2)$$

$$P_5 (\lambda_{52} + \lambda_{55}) = \mu (P_{12} + P_{30}) \quad (3)$$

$$P_{12} (\mu) = P_4 \lambda_{45} \quad (4)$$

$$P_{18} (\mu) = P_2 \lambda_{24} \quad (5)$$

$$P_{25} (\mu) = P_5 \lambda_{52} \quad (6)$$

$$P_{30} (\mu) = P_5 \lambda_{55} \quad (7)$$

An additional equation is required for the solution of the steady state probabilities. This equation states that the sum of all the probabilities of all the states shown in Figure 8.1 is equal to one or in mathematical form can be expressed as:

$$P_2 + P_4 + P_5 + P_{12} + P_{18} + P_{25} + P_{30} = 1.0 \quad (8)$$

The solution of the above equations by various traditional methods (e.g., matrix inversion) is extremely difficult, messy and time consuming particularly when the steady state solutions are expressed in closed form. To overcome these difficulties, equation (8) can be expressed in terms of the ratio of probabilities with the base probability equal to state 2 as follows:

$$P_2(1 + P_4/P_2 + P_5/P_2 + P_{12}/P_2 + P_{18}/P_2 + P_{25}/P_2 + P_{30}/P_2) = 1.0 \quad (9)$$

The individual probability ratios shown in equation (9) can be evaluated from the frequency balance equations (i.e., continual substitution and back substitution until the probability ratio is evaluated). Once the probability ratios of equation (9) have been evaluated, then the probability of state 2 can be evaluated.

The final form of the solution for $\mathbf{P_2}$ can be expressed in terms of a set of "state probability constants" as defined in Table 8.2.

Table 8.2 State probability constants (A(i))

STATE (i)	STATE PROBABILITY CONSTANT A(i)
4	$\lambda_{24}/\lambda_{45}$
5	$\lambda_{24}/\lambda_{52}$
12	λ_{24}/μ
18	λ_{24}/μ
25	λ_{24}/μ
30	$\lambda_{55}\,\lambda_{24}/\mu\,\lambda_{52}$

The steady state probability $\mathbf{P_2}$ is defined by the following equation:

$$\mathbf{P_2} = \frac{1}{1 + \sum_{i=4}^{n} A(i)} \tag{10}$$

The probability of occupying any other state $\mathbf{P_i}$ is given by:

$$\mathbf{P_i} = \mathbf{P_2}\, A(i) \tag{11}$$

8.4 Evaluation of Markov model parameters

The key parameters required to evaluate the steady state probabilities of the system analysts Markov model are the repair/restoration rate and the state transitional rates as shown in Figure 8.1. The mean time the computer system resides in each operational state and the number of departures from that state are shown in Table 8.3.

Table 8.3 Mean duration of system operation in each state and number of transitions from a given state

STATE NUMBER	MEAN DURATION IN STATE (hours)	NUMBER OF TRANSITIONS FROM THE GIVEN STATE
2	1221.53	1
4	3690.82	1
5	8039.5	2

The rate of departure from a given state λ_i is defined as follows:

$$\lambda_i = \frac{\text{total number of transitions from state i}}{\text{total duration in state}}$$

$$= 1.0 \,/\, \text{mean duration in state i} \qquad (12)$$

The individual directional transition rates λ_{ij} from a given state i to another operational state j through a restoration state is defined as:

$$\lambda_{ij} = \lambda_i P(i,j) \qquad (13)$$

where: $P(i,j)$ - probability of a transition from state i to j

$P(i,j)$ is defined as follows:

$$P(i,j) = \frac{\text{number of transitions from state i to j}}{\text{total number of transitions from state i}} \qquad (14)$$

The individual transition rates of the system analysts state space diagram are listed in Table 8.4.

Table 8.4 Individual system analysts' transition rates between operating states

TRANSITION RATE	VALUE (departures/hour) x 10^{-6}
λ_{24}	818.645460
λ_{45}	270.942500
λ_{52}	62.192922
λ_{55}	62.192922

The restoration rate μ is defined as the reciprocal of the mean time to restore the computer system to a fully operational state and is equal to 0.8695621 restorations per hour for system analysts failures.

8.5 Calculation of the frequency and duration of computer system operation

The performance of a computer system is usually quantified by two variables; i.e., the probability that the computer system is operational and the frequency of departures from its operational state. Based on the transition rates between operational states and the restoration rate, the probability of occupying any of the states contained in the state space diagram (i.e., shown in Figure 8.1) can be evaluated from equations 10 and 11. The quantitative results of the steady state "state" probabilities are shown in Table 8.5.

Table 8.5 State probabilities for system analysts' Markov model

STATE (i)	STATE PROBABILITY P(i)
2	0.058179315000
4	0.175787230000
5	0.765814350000
12	0.000054772468
18	0.000054772468
25	0.000054772468
30	0.000054772468

The probability of being in an operational state is equal to the sum of the probabilities of occupying all operational states and can be expressed mathematically as:

$$P(\text{operational}) = P_2 + P_4 + P_5 \qquad (15)$$
$$= 0.99978091$$

The frequency of departures from the operational states is equal to the sum of the frequency of departure from each individual operating state and can be expressed as follows:

$$f_{down} = P_2(\lambda_{24}) + P_4(\lambda_{45}) + P_5(\lambda_{52} + \lambda_{55}) \qquad (16)$$

= 0.00019051293 outages or occurrences/hour

= 1.6688932 outages caused by system analysts' failures/year

8.6 Discussion of "system analysts" failure Markov model

Based on the observed system analysts' failure patterns, this chapter has presented a Markov model for evaluating the impact of system analysts' failures on the performance of the Government of Alberta's Central Computer Center's VM based computer system. The evolution equations of the steady state "state" probabilities in closed form were presented. The detailed frequency balance equations of the Markov model which is required for the solution of the state equations are also presented. The actual state transition rates are presented. Based on the evolution equations and the state transition rates, the probability of the computer system being operational was calculated and the frequency of computer system analysts' system outages per year was evaluated.

8.7 State space diagram for system operators' failures only

Only those system operators' problems that caused computer system failures to occur are considered in the state space diagram. The actual time of occurrence of these failures, the duration of the outage and the time to the next system operators' failure (i.e., TTF) are shown in Table 8.6.

Table 8.6 Problem report data base for system operators failures.

DATE OF OCCURRENCE	TIME OF OUTAGE	DURATION (minutes)	TIME OF RESTORATION	TTF (hours)
July 17, 1984	1:21	21	1:42	-
August 27, 1986	3:55	5	4:00	18506.22
October 6, 1986	2:12	3	2:15	958.20
nothing before August 7, 1987				7317.75+

There are too few failures to see if there is a pattern similar to the pattern that was evident for the hardware or software failures. However, the failures can be expressed in the form of a state space diagram shown in Figure 8.2 which is based on the generalized state space diagram presented in Chapter 4.

An examination of the chronological order of computer system operational times, reveals the computer system operated in the following sequential time zones following each system operators' failure: [5 - 2 - 5] This sequence defines the transitions between operating states. Note the number of transitions between operating states is significantly less than the generalized model where all possible transitions between all possible operating states is considered. Because many of the transitions of the general model did not occur, a much simpler model can be developed to model system operators' failures.

Based on the observed transitions between operating states and the duration of restoration activities following a system operators' failure, a state space diagram of the dynamic process can be constructed as shown in Figure 8.2.

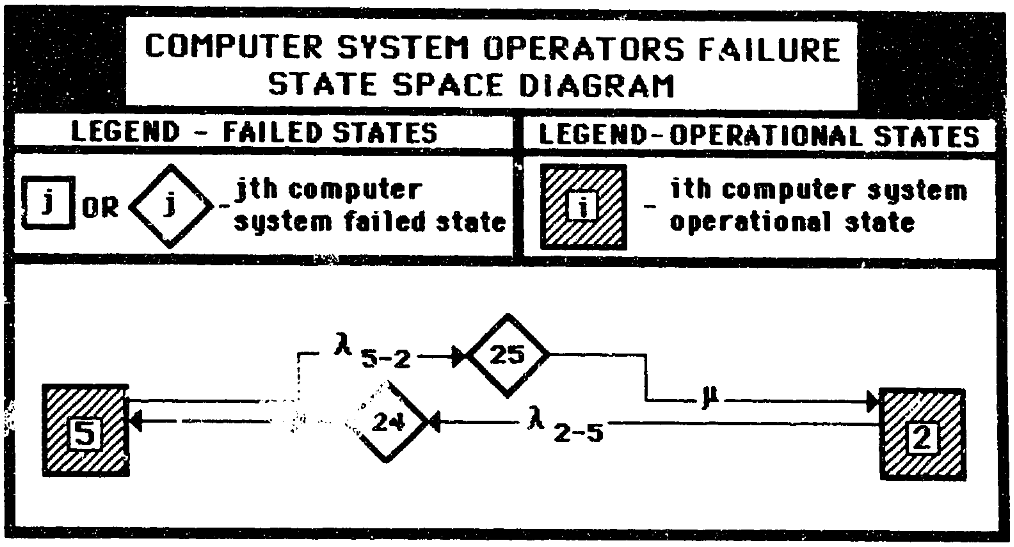

Figure 8.2 State space diagram for system operators failure Markov model.

The required operational states are 2 and 5 and the down states are specified from the observed transitions between the operational states. For example, down state number 24 is needed to accommodate the transition from operational state 2 to operational state 5.

8.8 Evaluation of steady state "state" probabilities in a closed form solution

The 4 state Markov model state probabilities can be solved by a frequency balance approach. The initial step is to define the frequency balance equations for each state in the state space diagram. These equations are listed as follows:

$$P_2(\lambda_{25}) = \mu(P_{25}) \tag{17}$$

$$P_5(\lambda_{52}) = \mu(P_{24}) \tag{18}$$

$$P_{24}(\mu) = P_2\lambda_{25} \tag{19}$$

$$P_{25}(\mu) = P_5\lambda_{52} \tag{20}$$

An additional equation is required for the solution of the steady state probabilities. This equation states that the sum of all the probabilities of all the states shown in Figure 8.2 is equal to one or in mathematical form can be expressed as:

$$P_2 + P_5 + P_{24} + P_{25} = 1.0 \qquad (21)$$

The solution of the above equations by various traditional methods (e.g., matrix inversion) is extremely difficult, messy and time consuming particularly when the steady state solutions are expressed in closed form. To overcome these difficulties, equation (21) can be expressed in terms of the ratio of probabilities with the base probability equal to state 2 as follows:

$$P_2(1 + P_5/P_2 + P_{24}/P_2 + P_{25}/P_2) = 1.0 \qquad (22)$$

The individual probability ratios shown in equation (22) can be evaluated from the frequency balance equations (i.e., continual substitution and back substitution until the probability ratio is evaluated). Once the probability ratios of equation (22) have been evaluated, then the probability of state 2 can be evaluated.

The final form of the solution for P_2 can be expressed in terms of a set of "state probability constants" as defined in Table 8.7.

Table 8.7 State probability constants (A(i)).

STATE (i)	STATE PROBABILITY CONSTANT A(i)
5	$\lambda_{25}/\lambda_{52}$
24	λ_{25}/μ
25	λ_{25}/μ

The steady state probability P_2 is defined by the following equation:

$$P_2 = \frac{1}{1 + \sum_{i=5}^{n} A(i)} \qquad (23)$$

The probability of occupying any other state P_i is given by:

$$P_i = P_2 A(i) \qquad (24)$$

8.9 Evaluation of Markov model parameters

The key parameters required to evaluate the steady state probabilities of the system operators' Markov model are the repair/restoration rate and the state transitional rates as shown in Figure 8.2 The mean time the computer system resides in each operational state and the number of departures from that state are shown in Table 8.8.

Table 8.8 Mean duration of system operation in each state and number of transitions from a given state.

STATE NUMBER	MEAN DURATION IN STATE (hours)	NUMBER OF TRANSITIONS FROM THE GIVEN STATE
2	958.20	1
5	12911.99	1

The rate of departure from a given state λ_i is defined as follows:

$$\lambda_i = \frac{\text{total number of transitions from state i}}{\text{total duration in state}}$$

$$= 1.0 \, / \text{ mean duration in state i} \qquad (25)$$

The individual directional transition rates λ_{ij} from a given state i to another operational state j through a restoration state is defined as:

$$\lambda_{ij} = \lambda_i P(i,j) \qquad (26)$$

where: $P(i,j)$ = probability of a transition from state i to j

P(i,j) is defined as follows:

$$P(i,j) = \frac{\text{number of transitions from state } i \text{ to } j}{\text{total number of transitions from state } i} \quad (27)$$

The individual transition rates of the system operators' state space diagram are listed in Table 8.9.

Table 8.9 Individual system operators' transition rates between operating states.

TRANSITION RATE	VALUE (departures/hour) x 10^{-6}
λ_{25}	1043.623400
λ_{52}	77.447425

The restoration rate μ is defined as the reciprocal of the mean time to restore the computer system to a fully operational state and is equal to 15.0 restorations per hour for system operators' failures.

8.10 Calculation of the frequency and duration of computer system operation

The performance of a computer system is usually quantified by two variables; i.e., the probability that the computer system is operational and the frequency of departures from its operational state. Based on the transition rates between operational states and the restoration rate, the probability of occupying any of the states contained in the state space diagram (i.e., shown in Figure 8.2) can be evaluated from equations 23 and 24. The quantitative results of the steady state "state" probabilities are shown in Table 8.10.

Table 8.10 State probabilities for system operators' Markov model

STATE (i)	STATE PROBABILITY P(i)
2	**0.0690827690000**
5	**0.9309076180000**
24	**0.0000048064265**
25	**0.0000048064265**

The probability of being in an operational state is equal to the sum of the probabilities of occupying all operational states and can be expressed mathematically as:

$$\mathbf{P(operational) = P_2 + P_5 = 0.99999038} \qquad \mathbf{(28)}$$

The frequency of departures from the operational states is equal to the sum of the frequency of departure from each individual operating state and can be expressed as follows:

$$f_{down} = P_2(\lambda_{25}) + P_5(\lambda_{52}) \qquad (29)$$

$= 0.00014419279$ outages or occurrences/hour

$= 1.2631289$ outages caused by system operators failures/year

8.10 Discussion of system operators' failure Markov model

Based on the observed system operators failure patterns, this chapter has presented a 7 state Markov model for evaluating the frequency and duration of system analysts' failures of the Government of Alberta's Central Computer Center's VM-based computer system. The evolution equations of the steady state "state" probabilities in closed form were presented. Based on the evolution equations and the actual state transition rates, the probability of the computer system being operational was calculated and the frequency of computer system operators' system outages per year was evaluated.

In comparison with the other modes of computer system failure, the system analysts' and operator MTTF was significantly greater than the other modes of failure and the frequency of failures significantly less.

CHAPTER IX
COMPUTER SYSTEM MARKOV MODEL

9.1 Introduction

The analysis of the overall computer system performance will be based on the observed statistical failure patterns of all the causes of system failure acting together. Prior to constructing a model of the performance of the computer system, it was necessary to determine if the key reliability variables "time to failure" and "time to restore" characterizing the overall operation of the computer system could be represented by known statistical distributions which would enable the use of existing reliability models.

9.2 Statistical representation of the duration of computer system operational states

A frequency histogram of the duration of computer system operation during the period 1984-1987 is shown in Figure 9.1. The mode of the distribution is the one week

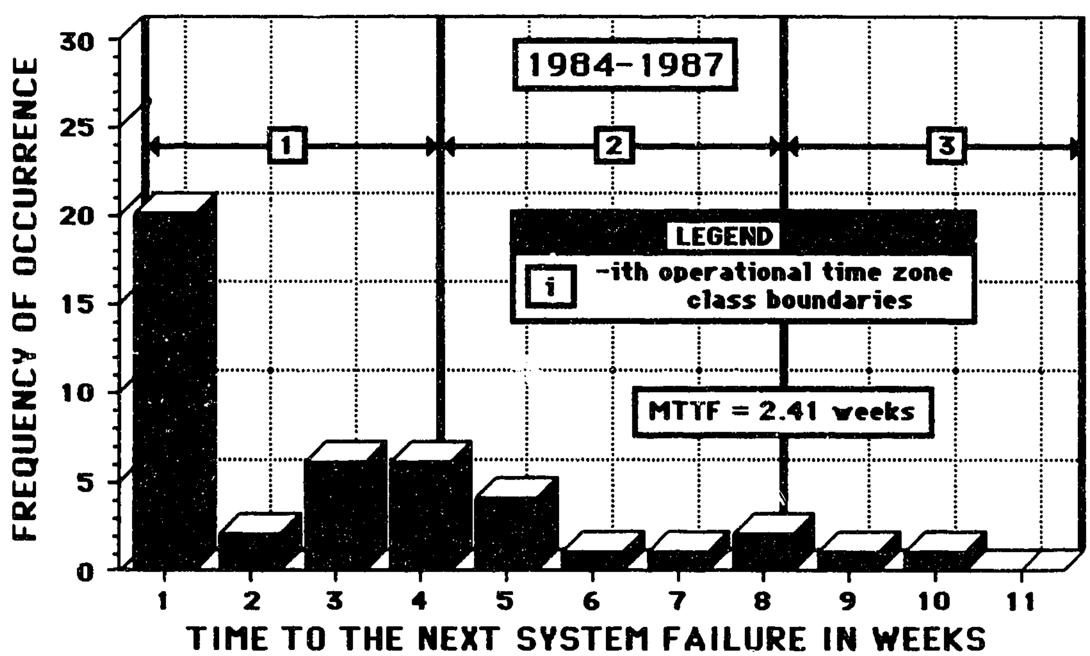

Figure 9.1 Time to the next computer system failure.

interval. The mean time to computer system failure is 2.41 weeks. The distribution is multimodal in nature which cannot be readily represented by known statistical distributions required by existing reliability models.

9.3 Chronological order arrangement of computer system time to failure

In the development of histograms, no attention is given to the order in which events occur (e.g., data clustered into class intervals). The statistical question that was addressed next was: "Does the "time to computer system failure" variable exhibit distinctive operating cycles?" To answer this question, the time to the next computer system's failure was plotted as a function of its chronological order of occurrence as shown in Figure 9.2.

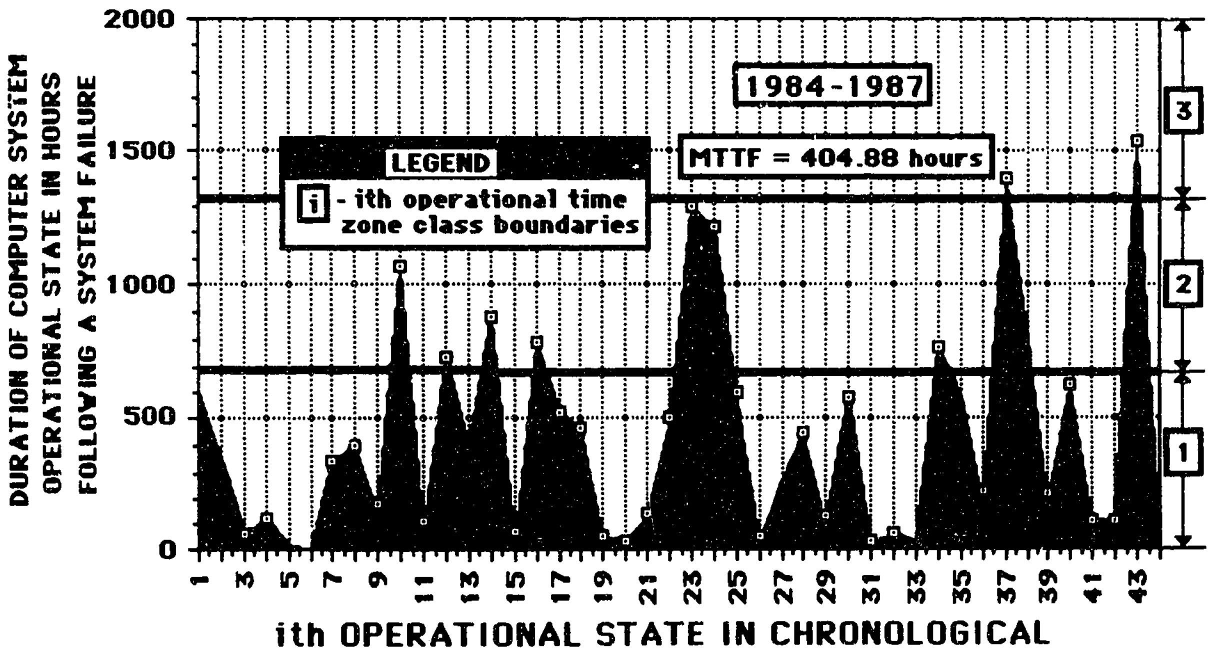

Figure 9.2 Time to the next computer system failure in chronological order.

An examination of Figure 9.2 reveals distinctive operating patterns similar to those presented for hardware, software, unknown failures in the previous chapters. It appears that once this computer system has been operational for a specified duration (e.g., x hours) before a given problem curtails its performance, the next sequential computer system operational period (e.g., y hours) after the computer system restoration activities have been completed, will be significantly less than the previous computer operational period (i.e., y << x). An operating cycle of a long operational period followed by one or more operational periods of shorter duration was consistently observed during the study period.

These observations are characteristic of a Markov process in which the computer system resides within an operational state for a certain period of time, then fails and is restored to another operating state whose duration of existence is significantly different from its predecessor in most cases. This cyclic performance continues with distinct probabilistic transitions between operating states and failed states.

9.4 Statistical representation of the duration of computer system failure states

Once the computer system has failed, then the primary questions that must be answered are:

(1) what is the duration of the failed states of the computer system?

(2) are there distinctive patterns associated with a computer system restoration activities?

(3) can the duration of the failed states be represented by known statistical distributions?

Answers to these questions will provide a basis for the development of a Markov model of the performance of the Government of Alberta's computer system. The frequency histogram of the duration of repair/restoration activities required to restore the computer system to a fully operational state is shown in Figure 9.3.

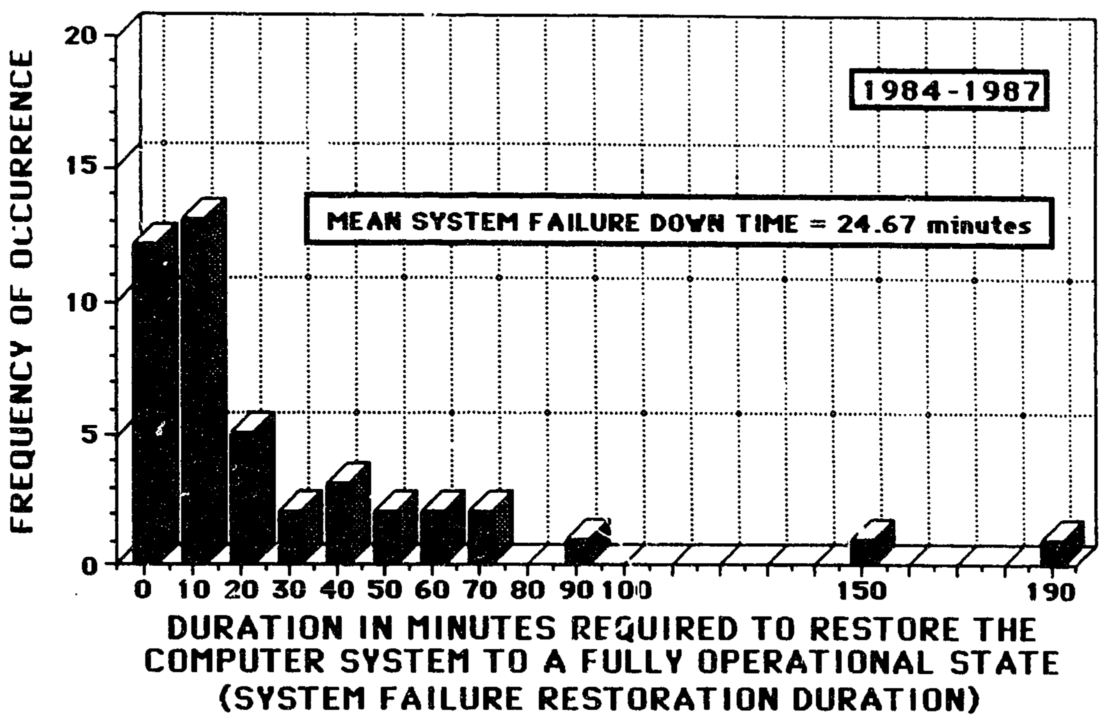

Figure 9.3 Frequency histogram of the duration of repair/restoration activities required to restore the computer system to a fully operational state.

The mean computer system duration in a failed state is 24.67 minutes which is orders of magnitude less than the "time to failure" variable of 404.88 hours.. The "computer system restoration time" variable is positively skewed with a number of observations significantly removed from the distribution mean and this characteristic is difficult to represent by known statistical distributions and techniques (e.g., transformation of variables).

9.5 State space diagram for all computer system failures

Based on the observed transitions between operating states and the duration of restoration activities following a computer system failure, a state space diagram of the dynamic process can be constructed as shown in Figure 9.4.

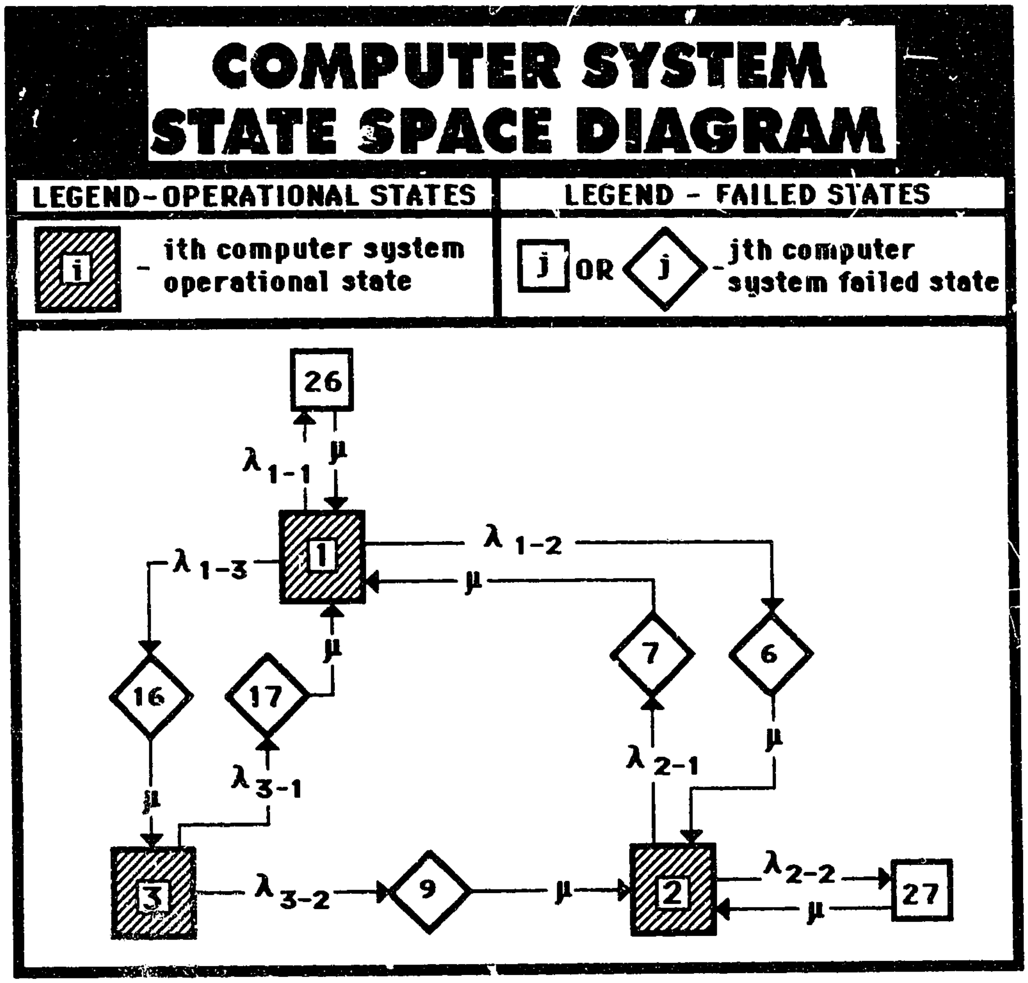

Figure 9.4 State space diagram for computer system failure Markov model.

The observed computer system operational states are operational states 1,2 and 3. and the transitions between operating states pass through failed states. For example, down state number 9 is needed to accommodate the transition from operational state 3 to operational state 2, however, down state 8 in the general model is not required because there was no observed transition from operational state 2 to operational state 3.

9.6 Evaluation of steady state "state" probabilities in a closed form solution

The steady state "state" probabilities of the 9 state Markov model can be solved by a frequency balance approach. The initial step is to define the frequency balance equations for each state in the state space diagram. These equations are listed as follows:

$$P_1(\lambda_{11} + \lambda_{12} + \lambda_{13}) = \mu(P_7 + P_{17} + P_{26}) \quad (1)$$

$$P_2(\lambda_{21} + \lambda_{22}) = \mu(P_6 + P_9 + P_{27}) \quad (2)$$

$$P_3(\lambda_{31} + \lambda_{32}) = \mu(P_{16}) \quad (3)$$

$$P_6(\mu) = P_1 \lambda_{12} \quad (4)$$

$$P_7(\mu) = P_2 \lambda_{21} \quad (5)$$

$$P_9(\mu) = P_3 \lambda_{32} \quad (6)$$

$$P_{16}(\mu) = P_1 \lambda_{13} \quad (7)$$

$$P_{17}(\mu) = P_3 \lambda_{31} \quad (8)$$

$$P_{26}(\mu) = P_1 \lambda_{11} \quad (9)$$

$$P_{27}(\mu) = P_2 \lambda_{22} \quad (10)$$

An additional equation is required for the solution of the steady state probabilities. This equation states that the sum of all the probabilities of all the states shown in Figure 9.4 is equal to one or in mathematical form can be expressed as:

$$P_1 + P_2 + P_3 + P_6 + P_7 + P_9 + P_{16} + P_{17} + P_{26} + P_{27} = 1.0 \quad (11)$$

The solution of the above equations by various traditional methods (e.g., matrix inversion) is extremely difficult, messy and time consuming particularly when the steady state solutions are expressed in closed form. To overcome these difficulties, equation (11) can be expressed in terms of the ratio of probabilities with the base probability equal to state 1 as follows:

$$P_1(1 + P_2/P_1 + P_3/P_1 + P_6/P_1 + P_7/P_1 + P_9/P_1 + P_{16}/P_1 + P_{26}/P_1 + \dots\dots\dots + P_{17}/P_1 + P_{27}/P_1) = 1.0 \qquad (12)$$

The individual probability ratios shown in equation (12) can be evaluated from the frequency balance equations (i.e., continual substitution and back substitution until the probability ratio is evaluated). Once the probability ratios of equation (12) have been evaluated, then the probability of state 1 can be evaluated. The final form of the solution for P_1 can be expressed in terms of the above constants and a set of "state probability constants" as defined in Table 9.1.

Table 9.1 State probability constants (A(i))

STATE (i)	STATE PROBABILITY CONSTANT A(i)
2	$(\lambda_{12} + \lambda_{13}) / \lambda_{21}$
3	$\lambda_{31} / \lambda_{32}$
6	λ_{12} / μ
7	$(\lambda_{12} + \lambda_{13}) / \mu$
9	λ_{13} / μ
16	λ_{13} / μ
17	$\lambda_{31} A(3) / \mu$
26	λ_{11} / μ
27	$A(2) / \mu$

The steady state probability P_1 is defined by the following equation:

$$P_1 = \frac{1}{1 + \sum_{i=2}^{n} A(i)} \tag{13}$$

The probability of occupying any other state P_i is given by:

$$P_i = P_1 A(i) \tag{14}$$

9.7 Evaluation of Markov model parameters

The key parameters required to evaluate the steady state probabilities of the hardware Markov model are the repair/restoration rate and the state transitional rates as shown in Figure 9.4. The mean time the computer system resides in each operational state and the number of departures from that state are shown in Table 9.2.

Table 9.2 Mean duration of system operation in each state and number of transitions from a given state

STATE NUMBER	MEAN DURATION IN STATE (hours)	NUMBER OF TRANSITIONS FROM THE GIVEN STATE
1	228.54000	34
2	939.78125	8
3	1464.64	2

The rate of departure from a given state λ_i is defined as follows:

$$\lambda_i = \frac{\text{total number of transitions from state i}}{\text{total duration in state}}$$

$$= 1.0 \,/\, \text{mean duration in state i} \tag{15}$$

The individual directional transition rates λ_{ij} from a given state **i** to another operational state **j** through a restoration state is defined as:

$$\lambda_{ij} = \lambda_i P(i,j) \tag{16}$$

where: **P(i,j)** = probability of a transition from state **i** to **j**

P(i,j) is defined as follows:

$$P(i,j) = \frac{\text{number of transitions from state i to j}}{\text{total number of transitions from state i}} \tag{17}$$

The individual transition rates of the computer system state space diagram are listed in Table 9.3.

Table 9.3 Individual computer system transition rates between operating states

TRANSITION RATE	VALUE (departures/hour) x 10^{-6}
λ_{11}	3314.12460
λ_{12}	795.38992
λ_{13}	265.12997
λ_{21}	931.06773
λ_{22}	133.00967
λ_{31}	341.38081
λ_{32}	341.38081

The restoration rate μ is defined as the reciprocal of the mean time to restore the computer system to a fully operational state and is equal to 2.431761825 restorations per hour for computer system failures.

9.8 Calculation of the frequency and duration of computer system operation

The performance of a computer system is usually quantified by two variables; i.e., the probability that the computer system is operational and the frequency of departures from its operational state. Based on the transition rates between operational states and the restoration rate, the probability of occupying any of the states contained in the state space diagram (i.e., shown in Figure 9.4) can be evaluated from equations 13 and 14. The quantitative results of the steady state "state" provabilities are shown in Table 9.4.

Table 9.4 State probabilities for system Markov model

STATE (i)	STATE PROBABILITY P(i)
1	0.418879612
2	0.417479176
3	0.162659316
6	0.000137009
7	0.000159844
9	0.000022835
16	0.000045670
17	0.000022835
26	0.000570870
27	0.000022835

The probability of being in an operational state is equal to the sum of the probabilities of occupying all operational states and can be expressed mathematically as:

$$P(\text{operational}) = P_1 + P_2 + P_3 = 0.999018104 \tag{18}$$

The frequency of departures from the operational states is equal to the sum of the frequency of departure from each individual operating states and can be expressed as follows:

$$f_{down} = P_1(\lambda_{11} + \lambda_{12} + \lambda_{13}) + P_2(\lambda_{22} + \lambda_{21}) + P_3(\lambda_{32}) \qquad (19)$$

= 0.002387737 outages or occurrences/hour

= 20.91657714 computer system outages/year

9.9 Discussion of computer system Markov model

Based on the observed computer system failure patterns caused simultaneously by hardware, software, unknown, and "analyst and operator" failures, this chapter has presented a 10 state Markov model for evaluating the overall performance of the Government of Alberta's Central Computer Center's VM-based computer system. The evolution equations of the steady state "state" probabilities in closed form were presented. Based on the developed evolution equations and the actual state transition rates, the probability of the computer system being operational was calculated and the frequency of computer system "outages per year" evaluated. The mean time to a computer system failure (i.e., 404.88 hours) was significantly less than the individual modes of failure (e.g., hardware failures - 1533.84 hours) contributing to the overall system performance.

CHAPTER X

CONCLUSIONS

The reliability research activities conducted in this thesis were directed at statistically analysing the problem reports of the Alberta Government's Central computer Center VM system and modelling the operational characteristics of this system. The problem reports identified the major modes of computer system failures, namely, hardware, software, unknown, analyst and operator failures. The categories were identified by the management of the system as the major causes of computer system interruptions. For each mode of failure, the time of the occurrence of the event and the duration of restoration activities required to restore the computer system to a fully operational state were recorded.

Initially, the key reliability variables (i.e., "time to failure" and "time to restore") associated with each mode of computer system failure were analysed statistically to determine whether existing probability density functions (e.g., exponential, normal, Weibull, etc.) could be used to represent these variables. If these variables could be statistically represented, then existing reliability models could be used to predict the reliability performance of the central computer system. It was concluded after a detailed analysis that the key reliability variables (i.e., time to failure, restoration duration) could not adequately be represented by known unimodal statistical distributions because the primary reliability variables were multimodal in nature. The primary multimodal nature of these variables prevented the use of existing reliability models which are based on known statistical distributions. It was concluded that the "time to failure" (i.e., operational time) variables were orders of magnitude greater than the "restoration time following a computer interruption" for all modes of failure.

Frequency histograms of the "time to failure" variable for each mode of failure were plotted in their chronological order of their occurrence to determine if the order in which failures occur exhibit any characteristic patterns. The resulting studies revealed distinctive operating patterns for all modes of computer system failure. The transitions of system operation were distinctive, i.e., the transitions between successive operating states were from an operational state whose residence time was long to another operational time whose residence time was significantly less than the previous operating state.

Based on the statistical analysis of the "time to failure" variables for each mode of computer system failure, a generalized 30 state Markov model was developed which had five distinctive operating states and 25 restoration states. All the possible transitions between the five operating states passed through a restoration state which was characterized by a constant rate of departure from that state because the expected values of the "restoration time" were confined to a short interval (e.g., 0 - 60 minutes) compared with the "time to failure" variable (e.g., 371 to 1658 hours). Based on the "frequency balance approach", the closed form solutions of the steady state probabilities for all 30 states containing all the possible states of computer system operation and the transitions between these states through restoration states was evaluated and are presented in detail in Appendix A.

For each mode of computer system failure (i.e., hardware, software, unknown, analyst and operator) the actual direction of the transitions between the five operational states were observed. It was concluded that the actual number of transitions between the five operational states were significantly less than the generalized model during the study period between 1984 and 1987. Reduced order Markov models for each failure mode were developed and were presented in the form state space diagrams. For each model, the closed form steady state probabilities of occupying each state were evaluated by the frequency balance approach and presented in the thesis. The detailed frequency balance equations of each Markov model are also presented. The actual state transition rates are presented. Based on the evolution equations and the state transition rates, the probability of the computer system being operational was calculated and the frequency of computer system outages caused by the various modes of failures per year was evaluated.

A Markov model for the overall computer system reliability performance was developed and presented in the thesis. The results of the model provide a basis for reliability cost-reliability worth studies of any computer system. In these studies the cost of downtime can be used to determine if the reliability of the system can be significantly improved (i.e., optimization of the closed form state probability equations). The individual Markov models of each failure mode provide a basis for assessing the impact of each failure mode on the overall performance of the computer system.

The Markov models presented for each failure mode are valid only for the study period and the observed state transitions that were exhibited by the Government of Alberta's Central computing VM system. The observed state transitions that did not occur were assumed to be zero. However, the generalized Markov model provides the means of readily including these transitions if they are observed and can be applied to any computer system.

The generalized Markov model can be applied to any computer system to evaluate its statistical performance characteristics. The statistical methodology, and the necessary performance variables required for the model are described in detail in the thesis. The frequency balance approach, based on the generalized state space transition diagram and the observed transition rates exhibited by a system, was illustrated to enable the development of custom computerized models for various computer systems and the prediction of steady state frequency and duration of computer system interruptions and operation.

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APPENDIX A

GENERALIZED COMPUTER SYSTEM MARKOV FAILURE MODEL

A.1 Introduction

The generalized solution for the steady state probabilities for all 30 states presented in the generalized Markov state space diagram shown in Figure 4.1 of Chapter IV contains all the possible states of computer system operation and the transitions between these states through restoration states. The transition rates between the various operational states have been evaluated for each computer system failure mode of the Alberta Government Computer Center's VM based system. This section of the thesis will present the frequency balance equations in detail that are required to evaluate the steady state "state" probabilities in closed form. The 30 state generalized state space diagram shown in Figure 4.1 of Chapter IV is repeated in this section as shown in Figure A.1 to readily illustrate the development of the frequency balance equations for each state. This section also presents the closed form solutions for each state of the generalized Markov failure model.

A.2 Evaluation of steady state "state" probabilities in a closed form solution

The 30 state Markov model state probabilities can be solved by a frequency balance approach. The initial step is to define the frequency balance equations for each state in the state space diagram shown in Figure A.1. For each state, the rate of departure from that state must be equal to the rate of entry of all other states contained in the state space diagram. The frequency of departure for a given state is equal to the probability of occupying that state times the sum of the transition rates departing from that state. The frequency of entry into a given state from another state is equal to the probability of the other state linking the given state times the transition rate between the two states. The total frequency of entry into a given state is the sum of the frequencies of all states linking the given state.

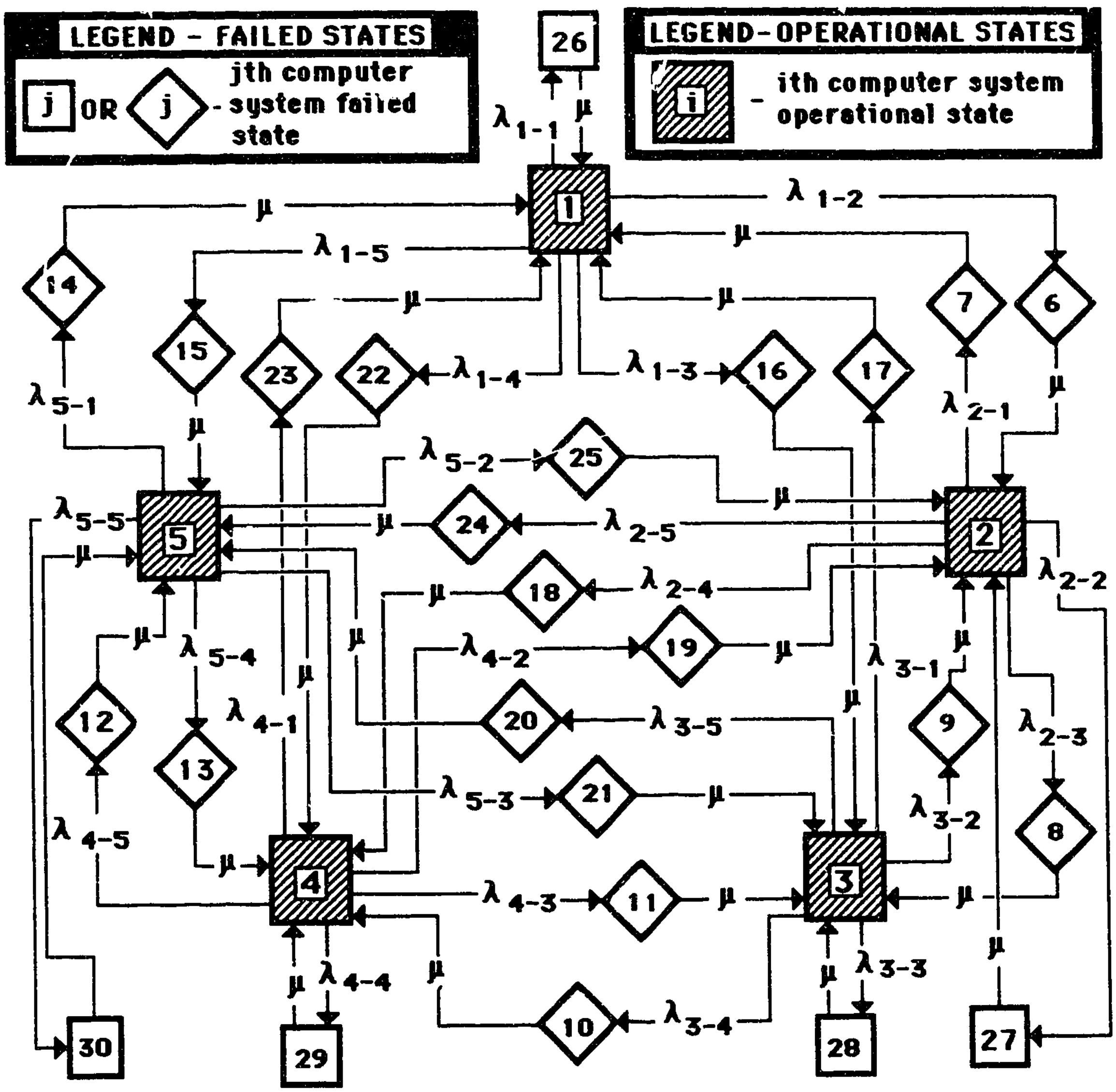

Figure A.1 State space diagram for generalized Markov failure model.

$$P_1 (\lambda_{11} + \lambda_{12} + \lambda_{13} + \lambda_{14} + \lambda_{15}) = \mu (P_7 + P_{14} + P_{17} + P_{23} + P_{26}) \quad (1)$$

$$P_2 (\lambda_{21} + \lambda_{22} + \lambda_{23} + \lambda_{24} + \lambda_{25}) = \mu (P_6 + P_9 + P_{19} + P_{25} + P_{27}) \quad (2)$$

$$P_3 (\lambda_{31} + \lambda_{32} + \lambda_{33} + \lambda_{34} + \lambda_{35}) = \mu (P_8 + P_{11} + P_{16} + P_{21} + P_{28}) \quad (3)$$

$$P_4 (\lambda_{41} + \lambda_{42} + \lambda_{43} + \lambda_{44} + \lambda_{45}) = \mu (P_{10} + P_{13} + P_{18} + P_{22} + P_{29}) \quad (4)$$

$$P_5 (\lambda_{51} + \lambda_{52} + \lambda_{53} + \lambda_{54} + \lambda_{55}) = \mu (P_{12} + P_{15} + P_{20} + P_{24} + P_{30}) \quad (5)$$

$$P_6 (\mu) = P_1 \lambda_{12} \quad (6)$$

$$P_7 (\mu) = P_2 \lambda_{21} \quad (7)$$

$$P_8 (\mu) = P_2 \lambda_{23} \quad (8)$$

$$P_9 (\mu) = P_3 \lambda_{32} \quad (9)$$

$$P_{10} (\mu) = P_3 \lambda_{34} \quad (10)$$

$$P_{11} (\mu) = P_4 \lambda_{43} \quad (11)$$

$$P_{12} (\mu) = P_4 \lambda_{45} \quad (12)$$

$$P_{13} (\mu) = P_5 \lambda_{54} \quad (13)$$

$$P_{14} (\mu) = P_5 \lambda_{51} \quad (14)$$

$$P_{15} (\mu) = P_1 \lambda_{15} \quad (15)$$

$$P_{16} (\mu) = P_1 \lambda_{13} \quad (16)$$

$$P_{17} (\mu) = P_3 \lambda_{31} \quad (17)$$

$$P_{18}(\mu) = P_2 \lambda_{24} \tag{18}$$

$$P_{19}(\mu) = P_4 \lambda_{42} \tag{19}$$

$$P_{20}(\mu) = P_3 \lambda_{35} \tag{20}$$

$$P_{21}(\mu) = P_5 \lambda_{53} \tag{21}$$

$$P_{22}(\mu) = P_1 \lambda_{14} \tag{22}$$

$$P_{23}(\mu) = P_4 \lambda_{41} \tag{23}$$

$$P_{24}(\mu) = P_2 \lambda_{25} \tag{24}$$

$$P_{25}(\mu) = P_5 \lambda_{52} \tag{25}$$

$$P_{26}(\mu) = P_1 \lambda_{11} \tag{26}$$

$$P_{27}(\mu) = P_2 \lambda_{22} \tag{27}$$

$$P_{28}(\mu) = P_3 \lambda_{33} \tag{28}$$

$$P_{29}(\mu) = P_4 \lambda_{44} \tag{29}$$

$$P_{30}(\mu) = P_5 \lambda_{55} \tag{30}$$

An additional equation is required for the solution of the steady state probabilities. This equation states that the sum of all the probabilities of all the states shown in Figure A.1 is equal to one or in mathematical form can be expressed as:

$$P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 + P_8 + P_9 + P_{10} + P_{11} + P_{12} + \dots$$
$$P_{13} + P_{14} + P_{15} + P_{16} + P_{17} + P_{18} + P_{19} + P_{20} + P_{21} + P_{22} + \dots$$
$$P_{23} + P_{24} + P_{25} + P_{26} + P_{27} + P_{28} + P_{29} + P_{30} = 1.0 \tag{31}$$

The solution of the above equations by various traditional methods (e.g., matrix inversion) is extremely difficult, messy and time consuming particularly when the steady state solutions are expressed in closed form. To overcome these difficulties, equation (31) can be expressed in terms of the ratio of probabilities with the base probability equal to state 1 as follows:

$$P_1(1 + P_2/P_1 + P_3/P_1 + P_4/P_1 + P_5/P_1 + P_6/P_1 + P_7/P_1 + P_8/P_1 + \ldots$$

$$P_9/P_1 + P_{10}/P_1 + P_{11}/P_1 + P_{12}/P_1 + P_{13}/P_1 + P_{14}/P_1 + P_{15}/P_1 + \ldots$$

$$P_{16}/P_1 + P_{17}/P_1 + P_{18}/P_1 + P_{19}/P_1 + P_{20}/P_1 + P_{21}/P_1 + P_{22}/P_1 + \ldots$$

$$P_{23}/P_1 + P_{24}/P_1 + P_{25}/P_1 + P_{26}/P_1 + P_{27}/P_1 + P_{28}/P_1 + P_{29}/P_1 + \ldots$$

$$P_{30}/P_1) = 1.0 \qquad (32)$$

The individual probability ratios shown in equation (32) can be evaluated from the frequency balance equations (i.e., continual substitution and back substitution until the probability ratio is evaluated). Once the probability ratios of equation (32) have been evaluated, then the probability of state 1 can be evaluated. In order to simplify the final form of the solution for P_1, the following constants will be defined:

$$A = \lambda_{51} + \lambda_{52} + \lambda_{53} + \lambda_{54} \qquad (33)$$

$$B = \lambda_{41} + \lambda_{42} + \lambda_{43} + \lambda_{45} \qquad (34)$$

$$C = \lambda_{31} + \lambda_{32} + \lambda_{34} + \lambda_{35} \qquad (35)$$

$$D = \lambda_{21} + \lambda_{23} + \lambda_{24} + \lambda_{25} \qquad (36)$$

$$E = (A\,\lambda_{14} + \lambda_{54}\lambda_{15}) \;/\; (AB - \lambda_{54}\lambda_{45}) \qquad (37)$$

$$F = (A\lambda_{24} + \lambda_{54}\lambda_{25}) \ / \ (AB - \lambda_{54}\lambda_{45}) \quad (38)$$

$$G = (A\lambda_{34} + \lambda_{54}\lambda_{35}) \ / \ (AB - \lambda_{54}\lambda_{45}) \quad (39)$$

$$H = \lambda_{13} + E\lambda_{43} + \lambda_{53}\lambda_{15}/A + E\lambda_{53}\lambda_{45}/A \quad (40)$$

$$I = \lambda_{23} + F\lambda_{43} + \lambda_{53}\lambda_{25}/A + F\lambda_{53}\lambda_{45}/A \quad (41)$$

$$J = C - G\lambda_{43} - \lambda_{53}\lambda_{35}/A - G\lambda_{53}\lambda_{45}/A \quad (42)$$

$$K = E + GH/J \quad (43)$$

$$L = F + GI/J \quad (44)$$

$$M = \lambda_{12} + \lambda_{32}H/J + \lambda_{42}K + \lambda_{52}(\lambda_{15}/A + \ldots H\lambda_{35}/AJ + \lambda_{45}K/A) \quad (45)$$

$$N = D - \lambda_{52}\lambda_{25}/A - \lambda_{52}\lambda_{35}I/AJ - \lambda_{52}\lambda_{45}L/A - \ldots \lambda_{32}I/J - \lambda_{45}L \quad (46)$$

$$O = H/J + MI/NJ \quad (47)$$

$$R = E + FM/N + GO \quad (48)$$

$$S = \lambda_{15}/A + \lambda_{25}M/AN + \lambda_{35}O/A + \lambda_{45}R/A \quad (49)$$

The final form of the solution for $\mathbf{P_1}$ can be expressed in terms of the above constants and a set of "state probability constants" as defined in Table A.1.

Table A.1 State probability constants (A(i))

STATE (i)	STATE PROBABILITY CONSTANT A(i)
2	M / N
3	O
4	R
5	S
6	λ_{12} / μ
7	λ_{21} M/Nμ
8	λ_{23} M/Nμ
9	λ_{32} O/μ
10	λ_{34} O/μ
11	λ_{43} R/μ
12	λ_{45} R/μ
13	λ_{54} S/μ
14	λ_{51} S/μ
15	λ_{15} / μ
16	λ_{13} / μ
17	λ_{31} O/μ
18	λ_{24} M/Nμ

Table A.1 State probability constants (A(i)) ... Continued

STATE (i)	STATE PROBABILITY CONSTANT A(i)
19	$\lambda_{42} R/\mu$
20	$\lambda_{35} O/\mu$
21	$\lambda_{53} S/\mu$
22	λ_{14}/μ
23	$\lambda_{41} R/\mu$
24	$\lambda_{25} M/N\mu$
25	$\lambda_{52} S/\mu$
26	λ_{11}/μ
27	$\lambda_{22} M/N\mu$
28	$\lambda_{33} O/\mu$
29	$\lambda_{44} R/\mu$
30	$\lambda_{55} S/\mu$

The steady state probability P_1 is defined by the following equation:

$$P_1 = \frac{1}{1 + \sum_{i=2}^{n} A(i)} \tag{50}$$

The probability of occupying any other state P_i is given by:

$$P_i = P_1 A(i) \tag{51}$$

A.3 Evaluation of Markov model parameters

The key parameters required to evaluate the steady state probabilities of the generalized Markov model are the repair/restoration rate and the state transitional rates as shown in Figure A.1. The rate of departure from a given state λ_i is defined as follows:

$$\lambda_i = \frac{\text{total number of transitions from state i}}{\text{total duration in state}}$$

$$= 1.0 \,/\, \text{mean duration in state i} \tag{52}$$

The individual directional transition rates λ_{ij} from a given state **i** to another operational state **j** through a restoration state is defined as:

$$\lambda_{ij} = \lambda_i P(i,j) \tag{53}$$

where: **P(i,j)** = probability of a transition from state **i** to **j**

P(i,j) is defined as follows:

$$P(i,j) = \frac{\text{number of transitions from state i to j}}{\text{total number of transitions from state i}} \tag{54}$$

The restoration rate μ is defined as the reciprocal of the mean time to restore the computer system to a fully operational state.

The probability of being in an operational state is equal to the sum of the probabilities of occupying all operational states and can be expressed mathematically as:

$$P(\text{operational}) = P_1 + P_2 + P_3 + P_4 + P_5 \tag{55}$$

The frequency of departures from the operational states is equal to the sum of the frequency of departure from each individual operating states and can be expressed as follows:

$$f_{down} = P_1(\lambda_{11} + \lambda_{12} + \lambda_{13} + \lambda_{14} + \lambda_{15}) + P_2(\lambda_{21} + \lambda_{22} + \lambda_{23} + \ldots$$
$$\lambda_{24} + \lambda_{25}) + P_3(\lambda_{31} + \lambda_{32} + \lambda_{33} + \lambda_{34} + \lambda_{35}) + P_4(\lambda_{41} + \ldots$$
$$\lambda_{42} + \lambda_{43} + \lambda_{44} + \lambda_{45}) + P_5(\lambda_{51} + \lambda_{52} + \lambda_{53} + \lambda_{54} + \lambda_{55}) \tag{56}$$