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PERMANENT ADDRESS:

*25, 10640-123 St.  
EDMONTON, ALBERTA  
CANADA T5N 1P2*

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A STATISTICAL ANALYSIS OF STEADY GAS FLOW AND THE  
DEVELOPMENT OF A MATHEMATICAL MODEL FOR TRANSIENT  
RADIAL GAS FLOW THROUGH POROUS MEDIA

by



JORGE FLORES

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH  
IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE  
OF DOCTOR OF PHILOSOPHY IN PETROLEUM ENGINEERING

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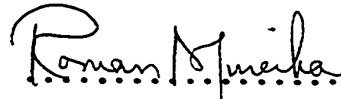
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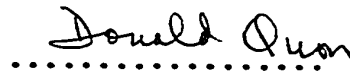
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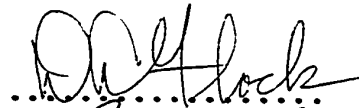
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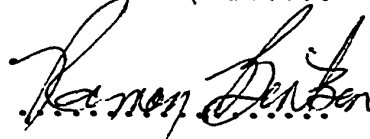
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Supervisor

  
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.....  
External Examiner

Date May 7, 1973 .....

## ABSTRACT

Recent studies of steady gas flow through porous media have shown that slippage and inertial effects can occur simultaneously and that, under certain conditions, both effects can be significant. On the basis of these studies, four mechanistic models, three of them in integrated form and one in differential form, have been formulated to describe steady isothermal radial gas flow.

For these four mathematical models experimental data have been obtained for a wide range of rock permeabilities, and a statistical analysis has been performed in order to ascertain their region of applicability and to make explicit the relative importance of the design parameters when dealing with different types of rocks. Specifically, the Bayesian procedure was applied in the area of model discrimination.

A similar analysis has been done of the steady linear gas flow using three mechanistic models given in algebraic form.

Theories describing transient gas flow through porous media have also shown that slippage and inertial effects are still significant even though flow were transient. However, the majority of this work has been done on linear systems, and systems which were only valid for viscous flow.



In this study, a theoretical investigation of transient flow has been carried out on a radial system exhibiting slippage and inertial effects. Specifically, a mathematical model has been developed which not only describes transient visco-inertial gas flow when slippage is present, but also considers rock compressibility and takes into account changes of gas properties with pressure. Solutions have been obtained for the homogeneous finite radial system in general and four sets of boundary conditions in particular.

Results for steady flow are presented to show the importance of the slippage effect for tight reservoirs and to make explicit the prediction of each model when subject to the absolute performance test. Transient flow results indicate that under certain conditions slip, inertial and rock compressibility effects appreciably influence both pressure and mass flux distributions.

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## INTRODUCTION

Recent examinations of the physics of the steady flow of gases through porous media have indicated that not only can viscous, inertial and slippage effects be operative simultaneously, but that a single mathematical equation relating these three physical parameters can be used to describe such flow (30, 59). However, for steady radial gas flow, there still remains unknown the solution of the most general equation from which the other simplified models can be derived. This general model is represented by a first order non-linear ordinary differential equation, whose analytical solution does not appear possible. Moreover, in mathematical modelling programs a stage is often reached where the parameters, either empirical or theoretical constants, must be estimated from experimental observations. This is sometimes called a regression problem. In some cases where data are available, a regression analysis may indicate that the parameters cannot be precisely estimated from these data. Thus, the collection of further data may be required. In other cases, no data may have yet been collected. In both of these circumstances some form of experimental plan is necessary which will yield precise estimates of the parameters (11, 45).

On the other hand, theories describing transient linear gas flow through porous media have also shown

that the transient phenomenon is also affected by slippage and inertial effects (3, 18, 20). Nevertheless, it appears that for transient radial flow no solutions are yet available considering the effect of the three physical parameters. The major difficulty arises from non-linearity of the partial differential equations which describe such flow. In fact, second order non-linear partial differential equations often arise as a result of mass, energy, or momentum balances written for a differential volume where the flux entering or leaving the "control volume" is proportional to some potential or driving force. However, with the use of high speed computers and the development of efficient numerical methods, many unresolved problems can now be handled and more rigorous mathematical models can be developed.

One of the objectives of this investigation is first to solve the rigorous model for steady radial gas flow and then to perform a statistical analysis of the radial and linear gas flow. Experimental data were obtained for a wide range of rock permeabilities in order to ascertain the region of applicability of the various models, to determine their absolute performance, and to obtain more precise estimates of the parameters using the sequential design procedure.

A second objective of this work is the development of a mathematical model which will not only describe transient radial gas flow through porous media under conditions in which viscous, inertial, and slippage effects are simultaneously operative, but it will also take into account changes of gas properties with pressure and rock compressibility.

## CHAPTER 1

### GENERAL BACKGROUND IN MODELLING

#### 1.1 Theory in Model Building

Physical and mathematical models are used mainly because the cost of testing different alternatives on the models are much less than the same tests on a full-sized system.

The scientist seeks to discover the mechanism behind a particular phenomenon in which he is interested. He therefore formulates a hypothesis and tries to reproduce the mechanism in a laboratory including the most important characteristics of the phenomenon.

On the other hand, the engineer is most interested in the performance of a process, rather than in establishing the true mechanism. Obviously, this implies that the engineer might very well use different models for the same phenomenon, depending upon the performance criterion.

Figure 1-1 shows a further classification of the physical and mathematical models (60). Briefly, the physical models are defined as follows:

Iconic models closely resemble the original, and as they grow in size and become more complex they merge into the real thing. Typical examples of this type of model are the pilot plants for oil secondary recovery.



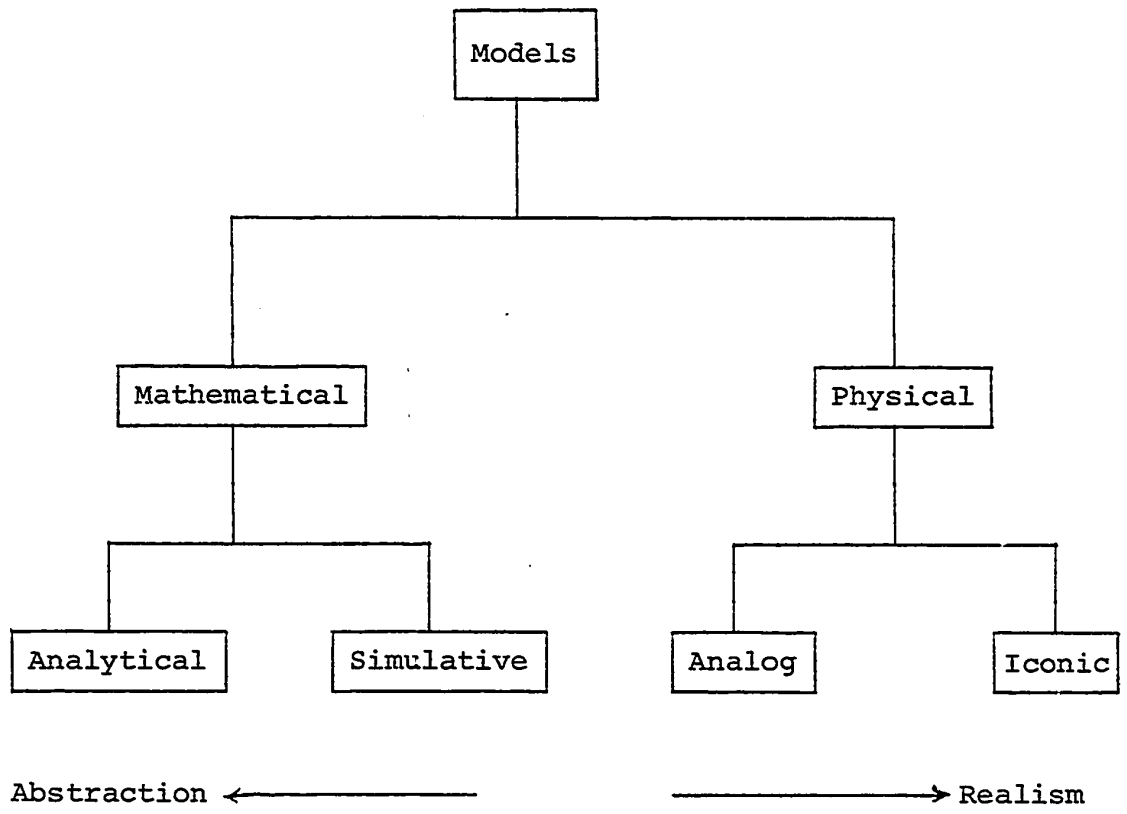


Figure 1-1. Classification of Models

Analog models, unlike iconic models, may bear no resemblance to the original, but because they obey similar laws the performance of the original may be predicted from the behaviour of the analog. Examples of this type of model are the analog computer and the resistance capacitance reservoir analyzer.

## 1.2 Mathematical Models to Describe Physical Phenomena

As can be seen in Figure 1-1, mathematical models can be further classified into analytical models and simulation models. The former are solved either exactly or approximately, whereas the latter are nearly always run on a computer.

Mathematical models involve systems in which there is a dependent variable related to a set of independent variables. For example, the experimenter might wish to study the change of pressure in a flow system as a function of flow rate, time and temperature. Therefore, if the dependent variable is considered as  $y$  and the set of independent variables is represented by  $\underline{x} = (x_1, x_2, \dots, x_m)$ ; the mathematical model can be formulated as

$$y = \phi(\underline{x}, \underline{a}) \quad (1-1)$$

where  $\underline{a} = (a_1, a_2, \dots, a_q)^T$  are the  $q$  parameters defined as physical constants of the system under study.

Most of the time, however, only a mechanistically based mathematical representation is possible, since a mathematical model in terms of its exact mechanism is seldom obtainable. This approach via a mechanistic model is particularly useful in obtaining a meaningful extrapolation.

On the other hand, mathematical models do not necessarily have to be mechanistic models. In fact, an empirical relationship might be entirely satisfactory depending upon the purpose of the investigation. For example, if the only purpose is to have a good prediction in a given region of interest, an empirical model can well be used, the only limitation being that it will provide little basis for extrapolation.

From the foregoing, a mathematical model usually incorporates some, but not all the characteristics of the real system. Unfortunately, it is not always possible to tell which characteristics are the most important.

In many engineering problems, it is not always possible to find closed form solutions for the equation used to describe the mathematical model. Therefore, one has to resort to the application of numerical techniques. Moreover, when more realism is needed, simulation studies can be undertaken with the implication of a more detailed model on which the engineer performs his experiment with

the aid of a computer. These simulation models, however, may have the disadvantage of being very costly.

Two primary requirements which a mathematical model must satisfy have been pointed out in the literature (60),

a) "It must approximate reality with a reasonable degree of accuracy. The term reasonable must be judged in the context of what we want the model to do, in the light of the objective function which has been formulated."

b) "It must be solvable on a practical basis, i.e. the benefits resulting from its solution must obviously be greater than the costs of the solution."

Summarizing, mathematical models provide a simple way of describing a real system. Furthermore, with the advent of high speed computers, simulation models can be run in order to examine the effect of varying certain variables or parameters and test alternatives for a given project. This is particularly important in the area of petroleum reservoir simulation where it is necessary to use history matching in order to predict the future performance of the reservoir.

### 1.3 Criteria for Model Discrimination

Quite often, it is desired to choose from one or more physical mechanisms, the model which more closely represents the true phenomenon. For this achievement, several criteria have been described in the literature (10, 37, 69). Some of them were applied to empirical models commonly expressed as polynomials in the independent variables and linear in the parameters. Mechanistic models, however, are somewhat more complicated mathematically and are usually non-linear in both the dependent variable and the parameters.

The earliest criterion used for model performance is the minimum sum of the squares of the errors which finds its justification in the maximum likelihood criterion. In fact, if the errors are considered to be normally distributed, the least squares estimates are also the maximum likelihood estimates.

Another approach is based on the expected decrease in entropy (10), and is actually a measure of the randomness in a given situation. This criterion gives a zero value for entropy when one model is certainly the correct one and a maximum for entropy when all models have equal probability.

Recent studies, however, have shown that a better criterion may be considered to be the Bayesian expected likelihood procedure (69). This criterion is essentially based on a Bayesian interpretation of probability and it will be used in this work.

In general, let us consider that the true model, in differential or algebraic form, is a member of the following metamodel which comprises k models:

$$\begin{aligned}
 y &= \phi_1(\underline{x}, \underline{a}_1) + \varepsilon_1 \\
 &: \\
 y &= \phi_g(\underline{x}, \underline{a}_g) + \varepsilon_g \\
 &: \\
 y &= \phi_k(\underline{x}, \underline{a}_k) + \varepsilon_k
 \end{aligned}
 \tag{1-2}$$

where  $\underline{x}$  is the set of independent variables,  $\underline{a}_g$  is the vector of design parameters for the  $g^{\text{th}}$  model, and  $y$  is the state variable.

The problem of model uncertainty can be handled by assigning probabilities to the models. Let  $p_g$  be the probability that the  $g^{\text{th}}$  model is correct, therefore, according to the axioms of probability we can write:

$$\begin{aligned}
 0 &\leq p_g \leq 1 \\
 \sum_{g=1}^k p_g &= 1 \quad .
 \end{aligned}
 \tag{1-3}$$

The main characteristic of the Bayesian approach lies in the fact that values assigned to the  $p_g$ 's are subjective probabilities and not objective probabilities as in classical statistics. These prior probabilities will be modified in the light of experimental evidence, to obtain the posterior probability of each model according to Bayes' theorem given by the following expression:

$$p_g'' = \frac{p_g' L(\underline{y}/\phi_g)}{\sum_{r=1}^k p_r' L(\underline{y}/\phi_r)} \quad (1-4)$$

where the single prime represents prior probabilities (before observing the data), and the double prime represents posterior probabilities (after observing the data). The term  $L(\underline{y}/\phi_g)$  stands for the likelihood which designates the probability of observing the data given that the model  $\phi_g$  is true.

When no information is available about the behaviour of the models, it is common to consider the prior probabilities all equal. Similarly, in the absence of information about the parameters, their values can be considered to be uniformly distributed over the range  $-\infty$  to  $+\infty$ , that is, an improper uniform distribution may be assumed.

Moreover, it should be pointed out that the model  $\phi_g$  involves a set of parameters which are uncertain and therefore can be considered as random variables in the Bayesian sense. Consequently,  $L(\underline{y}/\phi_g)$  in Equation (1-4) should be considered as the unconditional likelihood calculated by means of the following expression:

$$L(\underline{y}/\phi_g) = \int_{\underline{a}_g} L(\underline{y}/\phi_g, \underline{a}_g) f(\underline{a}_g) d\underline{a}_g \quad . \quad (1-5)$$

This equation requires a definition of the joint parameter density function  $f(\underline{a}_g)$ . In this regard, some investigators have used the prior parameter density,  $f'(\underline{a}_g)$ . However, Singh (69) has shown that the discrimination procedure is less sensitive to the uncertainty in the parameter distribution, if the posterior rather than the prior parameter distribution is used in Equation (1-5). Consequently, the posterior density will be used for calculating the expected likelihood.

Applying Bayes' theorem, the posterior parameter density is obtained using the expression:

$$f''(\underline{a}_g) = \frac{f'(\underline{a}_g)L(\underline{y}/\underline{a}_g)}{\int_{\underline{a}_g} L(\underline{y}/\underline{a}_g)f'(\underline{a}_g)d\underline{a}_g} \quad . \quad (1-6)$$



If the models are linear in the parameters and provided the posterior parameter distribution is multivariate normal, the calculations to determine the posterior probability of each model are relatively simple.

When dealing with non-linear models the problem is a little more involved. However, if the prior joint parameter distribution is assumed to be improper uniform, the posterior probabilities may be calculated in the following manner (60).

For the  $g^{\text{th}}$  model with  $q$  parameters, let

$$\underline{\phi}_g = \{\phi_g(\underline{x}_i, \underline{a}_g); i=1, \dots, n\}, \quad n \text{ data points} \quad (1-7)$$

$$S_g^2 = [\underline{y} - \underline{\phi}_g]^T [\underline{y} - \underline{\phi}_g] \quad (1-8)$$

$$\hat{S}_g^2 = \min_{\underline{a}_g} S_g^2; \text{ minimum sum of the squares of the errors} \quad (1-9)$$

$$\hat{\underline{a}}_g = \text{parameter vector which minimizes } S_g^2 \quad (1-10)$$

$$\underline{X}_g = \{x_{ijk}; i=1, \dots, n; j=1, \dots, q\}, \quad n \times q \text{ design matrix}$$

$$x_{ijk} = \left( \frac{\partial \phi_g(\underline{x}_i, \underline{a}_g)}{\partial a_{jk}} \right)_{\underline{a}_g = \hat{\underline{a}}_g} \quad (1-11)$$

$$\underline{A}_g = \underline{X}_g^T \underline{X}_g, \quad \text{a symmetric and positive definite matrix } (q \times q) \quad (1-12)$$

$$\sigma_g^2 = \frac{\hat{S}_g^2}{n-q} \quad , \quad \text{the unbiased estimate of error variance} \quad (1-13)$$

$$\underline{C}_g = \sigma_g^2 \underline{A}_g^{-1} \quad , \quad \text{the variance-covariance matrix of} \\ \text{the parameters } (q \times q) \quad (1-14)$$

The diagonal elements of  $\underline{C}_g$  represent the variance of the corresponding parameters. If the elements of this variance-covariance matrix are normalized in such a way that the diagonal elements are unity, we obtain the correlation matrix  $\underline{R}$  whose elements are defined as

$$r_{ij} = \frac{c_{ij}}{\sqrt{c_{ii}} \sqrt{c_{jj}}} \quad .$$

As has been previously mentioned, when an improper uniform prior distribution of the parameters is assumed, the posterior distribution is the multivariate normal given by

$$f''(\underline{a}_g) = f_N^q(\underline{a}_g/\hat{\underline{a}}_g, \underline{C}_g) = (2\pi)^{-q/2} |\underline{C}_g|^{-1/2} \\ \times \exp\left\{-\frac{1}{2}(\underline{a}_g - \hat{\underline{a}}_g)^T \underline{C}_g^{-1} (\underline{a}_g - \hat{\underline{a}}_g)\right\} \quad . \quad (1-15)$$

This equation is the required expression for use in Equation (1-5) in order to calculate the expected or

unconditional likelihood.

One of the limitations of the Bayesian discrimination procedure is the large amount of computation needed to evaluate the unconditional likelihood. Singh (69) has shown that only the sufficient statistics of the data is needed to evaluate the term  $L(\underline{y}/\phi_g)$  in Equation (1-5). The required expression reads as follows:

$$L(\underline{y}/\phi_j) = (2\pi\sigma_g^2)^{-n/2} \exp\left[-\frac{\hat{S}_g^2}{2\sigma_g^2}\right] \times \left(\frac{1}{2}\right)^{q/2} \quad (1-16)$$

Once the expected likelihood for all the models is obtained, the posterior probabilities of each model can be calculated using Equation (1-4).

It should also be mentioned that the posterior probability of success for a model by no means implies that the model is an adequate one, since it only gives a relative capability of the model to describe the phenomenon, and any model can have the highest

probability of success if the other models are even poorer. Consequently, the absolute performance of a model might be determined using the following criteria:

- a) The actual errors or residuals should be uncorrelated over the entire region of investigation; any correlation with the independent variable would imply systematic model errors.
- b) The error variance and the values of the design parameters should not change drastically when different sets of data are used or when additional data are incorporated. The existence of large differences in the values for the error variance and in the parameters would indicate large modelling errors and inadequacy of the model.

## CHAPTER 2

### MATHEMATICAL MODELLING OF STEADY GAS FLOW

#### 2.1 Literature Review and Theory

The phenomenon of gas flow through porous media has been a subject of investigation for quite some time. Numerous investigators (28, 30, 31, 52, 53, 59, 71) have carried out experiments to verify the validity of Darcy's equation, the majority of this work being on linear systems.

In the process of this development, it was realized that two distinct phenomena which were previously neglected; significantly affected the physics of isothermal gas flow through porous media. These were slippage and inertial effects. In fact, in 1941 Klinkenberg (52) observed that air permeabilities are higher than liquid permeabilities in the same porous medium as calculated from Darcy's law. Indeed, this is an indication that Darcy's law is not valid for gases. As far as a characterization of the point when Darcy's law becomes no longer valid for gas flow is concerned, Calhoun and Yuster (14) summarized the facts by stating that Darcy's equation breaks down if the pore diameters become comparable with, or less than, the mean free path of the gas molecules. One can again

observe an analogy between this breakdown of Hagen-Poiseuille's law in capillaries, i.e. if the radius of the capillary is made smaller and smaller, the originally viscous flow of the gas undergoes a transition to slip flow and thence to molecular streaming what is called Knudsen flow.

On the other hand, a number of investigators (30, 33, 36, 39) have agreed that there exist a certain "seepage" velocity, above which Darcy's law would no longer be valid. It is well known that, even in tubes, the dependence of the pressure drop on flow velocity becomes non-linear as soon as the inertia terms in the Navier-Stokes equations become important. This occurs in curved tubes long before the onset of turbulence. Since the curvature of the channels is being neglected in hydraulic radius theories, the possible non-linearity in laminar flow is completely overlooked. Attempts to give a universal characterization of the certain seepage velocity by introducing a Reynolds number have failed because the critical Reynolds number was found to vary erratically within a factor 750 for various porous media and fluids (68), p.159. Even at that, the critical Reynolds number would be at least about 25 times smaller than would be expected if a porous medium were equivalent to an assemblance of straight tubes because of the different types of restrictions encountered in the flow system. Therefore,

the breakdown of Darcy's law at high flow velocities is probably primarily due not to the onset of turbulence, but rather to the emergence of inertial effects in laminar flow.

Consequently the theory of strictly viscous flow through homogeneous porous media is based on the experiment performed by Darcy (21) in 1856. Hubbert (43) quotes Henry Darcy to have experimentally established that the downward flow of water through sand filters was governed by the following expression

$$Q = - C \frac{(p_2^* - p_1^*)}{H} \quad (2-1)$$

which subsequently was called Darcy's law. In Equation (2-1)  $Q$  is the total volume of fluid percolating in unit time,  $C$  is a constant depending on the properties of the fluid and of the porous medium,  $H$  is the height of the filter bed, and  $p_2^* - p_1^*$  is the difference in readings of open manometer tubes separated a distance  $H$ . Implicit generalizations of the observations give the following equation:

$$Q = - \frac{kA}{\mu} \left( \frac{p_2^* - p_1^*}{H} \right) \quad (2-2)$$

where  $k$  is the absolute permeability of the porous medium,  $\mu$  is the viscosity of the incompressible fluid, and  $A$  is the cross section area.

Since  $p^*$  represents a potential given by  $p + \rho g z$  at any point, Equation (2-2) can be written in terms of pressure to yield

$$Q = - \frac{KA}{\mu H} (\Delta p - \rho_L g \Delta z) \quad (2-3)$$

where

$$\Delta p = p_2 - p_1$$

$g$  = gravity acceleration

$\rho_L$  = liquid density

$$\Delta z = z_1 - z_2 (= H) .$$

Therefore, if we consider linear horizontal flow, i.e.  $g = 0$ , Darcy's law, in differential form, is given by

$$- \frac{dp}{dx} = \frac{\mu}{k} q \equiv \alpha \mu q \quad (2-4)$$

where  $q$  is the seepage or superficial velocity, and  $\alpha$  is the viscous resistance coefficient equal to the reciprocal of permeability. Hubbert (43) has shown the theoretical soundness of this experimental proposition by deriving it from the fundamental Navier-Stokes equation.

As pointed out before, Klinkenberg (52) in a study especially dedicated to the flow of gas through some porous media which were considered as a bundle of capillaries, one third of each was oriented in each of the coordinate directions, developed a dependence of the permeability upon the mean pressure. The relationship is given by



$$k_a = k \left( 1 + \frac{b}{p_{avg}} \right) \quad (2-5)$$

where  $p_{avg}$  is the mean of the inlet and outlet pressures,  $b$  is a lumped constant defined as the slippage coefficient. It is a characteristic of the porous medium and the particular flowing gas.

Later on, it was stated (28, 30) that the assumption made when considering  $p_{avg}$  not a function of  $p$  to integrate Darcy's equation might be questionable and it was suggested to use the apparent permeability defined as

$$k_a = k \left( 1 + \frac{b}{p} \right) \quad (2-5a)$$

On the other hand, to take care of the deviations of Darcy's law observed when the flow rate was continuously increased, Forchheimer (36) suggested a modification of Darcy's law and proposed an equation of the form

$$-\frac{dp}{dx} = A q + B q^2 \quad (2-6)$$

where  $A$  and  $B$  are considered to be constants. This model was obtained on the basis of empirical reasoning and curve fitting technique applied to various experimental data then available.

Chwyl (18) adopting an approach similar to that of Hubbert (43) derived Forchheimer's equation from the fundamental Navier-Stokes equation, thereby giving to Equation (2-6) a sound theoretical basis.

Further investigation (7, 33, 38, 39, 71) confirmed the validity of the following expression for determining pressure gradient for both low and high speed horizontal linear flow through porous media,

$$-\frac{dp}{dx} = \alpha \mu q + \beta \rho q |q| \quad (2-7)$$

where  $\beta$  is defined as the inertial resistance coefficient.

Equation (2-7) can be written in terms of the mass velocity per unit area  $\omega/A = \rho q$ , because the mass velocity remains constant in steady-state flow even though the gas may be expanding. Accordingly, Equation (2-7) after using the modified gas law and assuming average constant value of the compressibility factor and viscosity, can be integrated as (53)

$$\frac{M(p_1^2 - p_2^2)}{2\mu_{avg} z_{avg} T_{avg} L \omega/A} = \frac{1}{k} + \frac{\omega}{A} \frac{\beta}{\mu_{avg}} \quad (2-8)$$

The rigorous mathematical model for linear steady gas flow where slippage is also included, is stated in section 2.3.

Tek (71) derived a generalized Darcy equation in the form of friction factor - Reynolds number correlation, to obtain

$$-\frac{dp}{dx} = \frac{1}{k} \mu q + \lambda_f \frac{\delta s^{\rho}}{\phi k} q^2 \quad (2-9)$$

where  $\ell_f$  is a "lithology factor" representing the particular porous medium, and  $\delta_s$  is the mean particle diameter. Comparing Equation (2-7) with Equation (2-9) we can infer that

$$\beta = \ell_f \frac{\delta_s}{\phi k} \quad .$$

Blick (7) working on a capillary orifice model for high speed flow through porous media and applying momentum theory to a control volume, obtained for horizontal isothermal gas flow the following expression for the pressure gradient:

$$-\frac{dp}{dx} = \frac{2 c_f N_{RE}}{\delta^2 \phi B} \mu q + \frac{c_d}{2 \delta \phi^2 B} \rho q^2 \quad , \quad (2-10)$$

where

$$c_f = \frac{16}{N_{RE}} \quad ; \quad (\text{for viscous flow})$$

$N_{RE}$  = Reynolds number

$\delta$  = Average pore diameter

$c_d$  = Drag coefficient of orifice plate

$\phi$  = Porosity, fraction

$$B = 1 - \frac{\alpha^2}{\phi} \left( \frac{M}{ZRT} \right)$$

Equation (2-10) suggests the following:

$$\alpha = \frac{2 c_f N_{RE}}{\delta^2 \phi B} = \frac{1}{k}$$

$$\beta = \frac{c_d}{2 \delta \phi^2 B}$$

For steady radial flow, Tek (71) integrated Forchheimer equation under the assumption of constant gas properties and no slippage to obtain the following radial gas flow equation with field units,

$$p_e^2 - p_w^2 = \frac{1,424 \mu_{avg} T_{avg} z_{avg} Q_g}{hk} \ln \left( \frac{r_e}{r_w} \right) + \frac{3.1602 \times 10^{-12} \beta G z_{avg} T_{avg} Q_g^2}{h^2} \left( \frac{1}{r_w} - \frac{1}{r_e} \right) \quad (2-11)$$

Piplasure (59) generalized the above equation by considering slippage in the development of the model.

He obtained the following equation:

$$p_e^2 - p_w^2 = \frac{1,424 \mu_{avg} T_{avg} z_{avg} Q_g}{hk} \ln \left( \frac{r_e}{r_w} \right) + \frac{3.1602 \times 10^{-12} \beta G T_{avg} z_{avg} Q_g^2}{h^2} \left( \frac{1}{r_w} - \frac{1}{r_e} \right) - 2b(p_e - p_w) \quad (2-12)$$

The above equation contains the extra term  $2b(p_e - p_w)$  as compared with Tek's equation. However, Equation (2-12) represents a simplified model describing steady radial gas flow. The rigorous mathematical model is stated in the following section.

## 2.2 Statement of the Mechanistic Models for Steady Radial Gas Flow

It has been shown in the previous section, that steady radial horizontal gas flow under the conditions of no slippage, can be described by the following equation:

$$-\frac{dp}{dr} = \frac{\mu q}{k} + \beta \rho q |q| \quad (2-13)$$

The above equation considers the absolute value of  $q$  so that it can be used either for injection ( $q$  is positive) or for production ( $q$  is negative). Therefore, if the problem is restricted to production, Equation (2-13) can be written as

$$\frac{dp}{dr} = \frac{\mu q}{k} + \beta \rho q^2 \quad (2-14)$$

which implies  $q$  being a positive quantity.

The following models are formulated using field units.

### Model 1

If in Equation (2-13) inertial effects are neglected and correction for slippage is made using Equation (2-5a), the integration gives the following two-parameter model:

$$p_e^2 - p_w^2 = \frac{1,424 \mu_{avg} T_{avg} z_{avg} Q_g}{hk} \ln\left(\frac{r_e}{r_w}\right) - 2b(p_e - p_w) , \quad (2-15)$$

the design parameters being  $k$  and  $b$ .

### Model 2

If slippage effects are negligible, the integration of Equation (2-14) after using the modified gas law, yields the following two-parameter model as given by Tek et al. (71):

$$p_e^2 - p_w^2 = \frac{1,424 \mu_{avg} T_{avg} z_{avg} Q_g}{hk} \ln\left(\frac{r_e}{r_w}\right) + \frac{3.1602 \times 10^{-12} \beta G T_{avg} z_{avg} Q_g^2}{h^2} \left(\frac{1}{r_w} - \frac{1}{r_e}\right) , \quad (2-16)$$

the design parameters being  $k$  and  $\beta$ .

### Model 3

If both slippage and inertial effects are considered simultaneously, Equation (2-14) becomes

$$\frac{dp}{dr} = \frac{\frac{c_1 r}{k} + C_2 \beta \left(1 + \frac{b}{p}\right)}{r \left(1 + \frac{b}{p}\right) r^2} \quad (2-17)$$

where

$$C_1 = \frac{\mu_{avg} T_{avg} z_{avg} p_o Q_o}{T_o (2\pi h)} \quad (2-18)$$

$$C_2 = \left(\frac{p_o M}{R T_o}\right)^2 \left(\frac{z_{avg} T_{avg} R}{M}\right) \left[\frac{Q_o}{(2\pi h)^2}\right] \quad (2-19)$$

and  $p_o$ ,  $T_o$ ,  $Q_o$  refer to standard conditions.

In order to integrate Equation (2-17), Piplapure (59) assumed the term  $b/p$  sufficiently small in the numerator to obtain the following simplified model:

$$\frac{dp}{dr} = \frac{\frac{c_1 r}{k} + C_2 \beta}{p(1 + \frac{b}{p})r^2} \quad (2-20)$$

The approximate Eq. (2-20) is now separable and the integration gives:

$$\begin{aligned} p_e^2 - p_w^2 &= \frac{1,424 \mu_{avg} T_{avg} z_{avg} Q_g}{hk} \ln\left(\frac{r_e}{r_w}\right) \\ &+ \frac{3.1602 \times 10^{-12} \beta G T_{avg} z_{avg} Q_g^2}{h^2} \left(\frac{1}{r_w} - \frac{1}{r_e}\right) \\ &- 2b(p_e - p_w) \quad (2-21) \end{aligned}$$

The above equation represents a simplified model with design parameters  $k$ ,  $b$ , and  $\beta$ .

#### Model 4

The fourth mathematical model is given by the non-linear ordinary differential equation stated as Equation (2-17). Using field units, the equation can be written as

$$\frac{dp}{dr} = \frac{712 \mu_{avg} T_{avg} Z_{avg} Q_g}{hkr(p+b)} + \frac{1.5801 \times 10^{-12} \beta G T_{avg} Z_{avg} Q_g^2}{h^2 r^2 p} \quad (2-22)$$

or

$$\frac{dp}{dr} = \frac{D_1}{kr(p+b)} + \frac{D_2 \beta}{r^2 p} \quad (2-23)$$

where

$$D_1 = \frac{712 \mu_{avg} T_{avg} Z_{avg} Q_g}{h} \quad (2-24)$$

$$D_2 = \frac{1.5801 \times 10^{-12} G T_{avg} Z_{avg} Q_g^2}{h^2} \quad (2-25)$$

Equation (2-22) is a non-linear ordinary differential equation not only in the dependent variable but also in the parameters  $k$ ,  $b$ , and  $\beta$ . It represents the rigorous equation for steady radial gas flow for which no analytical solution appears possible.

The field units used in the above mathematical models are defined as follows:

$$\mu = \text{cp} \quad h, r_w, r_e = \text{ft}$$

$$k = \text{millidarcies} \quad Q_g = \text{MSCF/day}$$

$$T_o = 520^\circ\text{R} \quad p_o = 14.697 \text{ psia.}$$



### 2.3 Statement of the Mechanistic Models for Linear Flow

When taking the experimental data for the radial flow, the core sample was only confined in the vertical direction. Hence, it was thought there might be a possibility that some of the results obtained for the steady radial gas flow might have been affected by distortion of the rock in the radial direction.

Therefore, in order to be more confident of the analysis done for the radial case, a complementary analysis was performed of the steady linear gas flow.

In this case the core sample was properly confined in both directions and the possibility of rock distortion was eliminated.

The Forchheimer equation for linear gas flow, after including slippage and using the modified gas law, can be formulated as (53)

$$-\frac{dp}{dx} = \frac{\mu_{avg} T_{avg} z_{avg} R \omega/A}{Mk(p+b)} + \frac{3z_{avg} T_{avg} R(\omega/A)^2}{g_c Mp} \quad (2-26)$$

On the basis of the above equation the following mechanistic models given in consistent units were analyzed.

#### Model A

When slippage is the only effect considered, i.e.  $\beta = 0$ , the following two-parameter model can be derived from Equation (2-26)

$$p_e = [p_w^2 - 2b(p_e - p_w) + \frac{2 A^* R/M L W \mu_{avg}}{k}]^{0.5} \quad (2-27)$$

with design parameters  $k$  and  $b$ .

### Model B

If inertial effect is considered and slippage is neglected, i.e.  $b=0$ , Equation (2-26) yields the following two-parameter model

$$p_e = [p_w^2 + 2(\frac{1}{k} + \beta W) A^* R/M L W \mu_{avg}]^{0.5} \quad (2-28)$$

the design parameters being  $k$  and  $\beta$ .

### Model C

The integration of Equation (2-26) yields the following expression (53)

$$\frac{M(p_e^2 - p_w^2)}{2 \mu_{avg} T_{avg} Z_{avg} R L \omega/A} = \frac{\gamma^3}{\gamma^2 - \frac{2ab^2}{k(p_e^2 - p_w^2)} \ln \frac{ab+p_e\gamma}{ab+p_w\gamma} + \frac{2b\gamma}{k(p_e + p_w)}} \quad (2-29)$$

where

$$\gamma \equiv \frac{1}{k} + \alpha$$

$$\alpha \equiv \frac{\beta \omega}{\mu_{avg} A}$$

Equation (2-29) represents the rigorous mathematical model for the linear case. The parameter estimation and statistical analysis was performed after solving Equation (2-29) for the external pressure,  $p_e$ . Hence, the following predictive model was obtained:

$$p_e = \left[ p_w^2 + \frac{2 \left(\frac{1}{k} + \beta W\right)^3 A R/M L W \mu_{avg}}{\left(\frac{1}{k} + \beta W\right)^2 - \frac{2\beta W b^2}{k(p_e^2 - p_w^2)} \ln \frac{\beta W b + p_e \left(\frac{1}{k} + \beta W\right)}{\beta W b + p_w \left(\frac{1}{k} + \beta W\right)} + \frac{2b \left(\frac{1}{k} + \beta W\right)}{k(p_e + p_w)} \right]^{0.5} \quad (2-30)$$

the design parameters being now  $k$ ,  $b$ , and  $\beta$ .

#### Model D

A fourth mathematical model is given by Equation (2-26). This model is the same rigorous model but expressed in differential form, and it was considered in order to determine the accuracy of the technique for parameter estimation of non-linear ordinary differential equations. This is possible because the analytical solution is known and given by Equation (2-30).

## CHAPTER 3

### STATISTICAL ANALYSIS OF THE STEADY GAS FLOW

#### 3.1 Experimental Work and Techniques Used to Perform the Analysis

Parameter estimation and model discrimination based on the mathematical models described in the previous chapter, were performed using experimental data obtained from four limestone core samples for the radial flow case and six sandstone core samples for the linear flow case.

The following table shows the characteristics of the core samples used for steady radial flow:

Table 3-1

Core Samples Used for Steady Radial Gas Flow

<u>Core sample No.</u>	<u>Outer radius (ft)</u>	<u>Inner radius (ft)</u>	<u>Thickness (ft)</u>	<u>Porosity</u>
1	0.49990	0.00540	0.07500	0.13210
2	0.49100	0.02125	0.07933	0.10950
111	0.48835	0.08587	0.12575	0.14430
112	0.48869	0.08551	0.12133	0.13470

Procedure detail for conducting flow tests using a radial gas flow cell is outlined in Reference (59). Only slight modifications were necessary in order to obtain more accurate readings for the inlet and outlet pressures, when data for the six runs were taken using core sample No. 1. Data for core samples No. 2, 111, and 112 were obtained from the literature (59).

Prior to flow tests, the porosity of core sample No. 1 was determined using a Boyle's law porosimeter. The flow tests were conducted in the following manner:

- a) The confining pressure was increased with each flow rate increase so that the difference between the confining pressure and the arithmetic mean flowing pressure (net confining pressure) was held constant at some preselected value, i.e. 600 psig. This confinement was only done in the vertical direction, since total confinement was impossible because of the design of the flow cell.
- b) Flow rate was changed by raising the upstream pressure while holding the downstream at atmospheric pressure.
- c) Flow rate was changed by raising the upstream pressure while holding the downstream pressure at a preselected value.

The following mean flowing pressures and flow rate ranges were used in the flow tests for radial flow.

Table 3-2

Runs Performed on Core Samples for the Radial Gas Flow

<u>Core Sample No.</u>	<u>Run No.</u>	<u>Number of Data Points</u>	<u>Mean Pressure Range, psia</u>	<u>Flow Rate Range, Mscf/day</u>
1	1	15	14.5 - 38.7	$0.710 \times 10^{-3} - 0.418 \times 10^{-1}$
1	2	15	19.4 - 58.8	$0.570 \times 10^{-2} - 0.976 \times 10^{-1}$
1	3	23	19.4 - 79.3	$0.570 \times 10^{-2} - 0.168$
1	4	30	19.4 - 115.4	$0.570 \times 10^{-2} - 0.304$
1	5	14	56.0 - 158.6	$0.690 \times 10^{-2} - 0.413$
1	6	18	38.5 - 198.6	$0.895 \times 10^{-2} - 0.416$
2	1	14	18.6 - 113.7	$0.675 \times 10^{-3} - 0.615 \times 10^{-1}$
111	1	16	19.5 - 133.0	$0.260 \times 10^{-2} - 0.957$
112	1	11	14.3 - 64.5	$0.510 \times 10^{-2} - 0.714$

For the linear flow, experimental data were obtained from Reference (29). The core samples used for the analysis are given in the following table.

Table 3-3

## Core Samples Used for the Steady Linear Gas Flow

<u>Core Sample</u>	<u>Run No</u>	<u>Length (cm)</u>	<u>Diameter (cm)</u>	<u>Sleeve Pressure (psig)</u>
2	7	7.1837	2.5203	4100.0
5	11	7.2704	2.5227	3935.0
7	2	6.8636	2.5213	450.0
9	18	7.2217	2.5078	3900.0
11	5	7.1267	2.5215	4337.0
15	4	7.1950	2.5210	428.0

Experimental data for the linear flow comprised ten data points for each run and, unlike the radial case, the rock samples were totally confined. This analysis was performed in order to avoid any suspicion of rock distortion effect on the analysis of radial flow.

Flow tests were conducted in a way similar to that of radial gas flow.

Estimation of the parameters for the algebraic forms was performed using Marquardt's algorithm for non-linear parameter estimation (55). A computer program was developed using this algorithm and a copy is given in Appendix A. Parameter estimation for the differential

model was carried out applying a quasilinearization technique with data perturbation (24). The program given by Donnelly (23) was slightly modified so that it could be fitted to solve the first order non-linear ordinary differential equations stated in Chapter 2. An extension of the program was done so as to calculate the reliability of the parameters and the expected likelihood of the model. A summary of the algorithm and a copy of the computer program are given in Appendix B.

The model discrimination was carried out applying the Bayesian procedure as discussed in Chapter 1.

### 3.2 Analysis of the Steady Radial Flow

In doing this analysis, six different runs were performed using core sample No. 1. The main objectives of these runs are summarized as follows:

- a) To test the absolute performance of each model,
- b) To use the sequential design procedure in order to obtain the best estimates of the parameters, and
- c) To find regions of maximum discrimination among the rival models by calculating the posterior probability of each model.

The relevant expressions necessary for parameter estimation, together with input data and results obtained



from each model are given in Appendix C. From these results, the reliability of the parameters and the posterior probability of each model were calculated.

The following tables show values of the least square parameters with their 95 % confidence limit, and the posterior probability of each model, for the six runs corresponding to core sample No. 1.

Table 3-4

Reliability of the Parameters and Posterior Probability  
of Each Model

Core Sample No. 1 - Run 1

k = permeability, millidarcies (md.)

b = slippage coefficient, psia.

$\beta$  = inertial coefficient,  $\text{ft}^{-1}$  in  $10^{10}$ .

Model	Parameters and 95 % confidence	$\sigma^2$ (psia <sup>2</sup> )	<L>	p"
1	k= 6.997±0.190 b= 9.768±1.071	0.037	0.194×10 <sup>2</sup>	0.000
2	k=10.468±0.097 $\beta$ = 0.737±0.526	0.021	0.089×10 <sup>3</sup>	0.000
3	k= 8.668±0.475 b= 4.177±1.312 $\beta$ = 0.446±0.100	0.005	0.299×10 <sup>8</sup>	0.201
4	k= 8.756±0.496 b= 3.993±1.350 $\beta$ = 0.408±0.106	0.004	0.119×10 <sup>9</sup>	0.799

Table 3-5

Reliability of the Parameters and Posterior Probability  
of Each Model

Core Sample No. 1 - Run 2

<u>Model</u>	<u>Parameters and</u> <u>95 % confidence</u>	$\sigma^2$ <u>(psia<sup>2</sup>)</u>	<u>&lt;L&gt;</u>	<u>p''</u>
1	k=5.772±0.257 b=18.623±2.802	0.493	0.274×10 <sup>-6</sup>	0.000
2	k=10.012±0.113 β= 0.545±0.148	0.081	0.822×10 <sup>-1</sup>	0.000
3	k= 8.265±0.214 b= 5.600±0.811 β= 0.403±0.023	0.005	0.314×10 <sup>8</sup>	0.154
4	k= 8.480±0.201 b= 5.038±0.750 β= 0.382±0.024	0.004	0.173×10 <sup>9</sup>	0.846

Table 3-6

Reliability of the Parameters and Posterior Probability  
of Each Model

Core Sample No. 1 - Run 3

<u>Model</u> -	<u>Parameters and</u> <u>95 % confidence</u>	$\sigma^2$ <u>(psia<sup>2</sup>)</u>	<u>&lt;L&gt;</u> -	<u>p"</u> -
1	k= 5.071±0.225 b=26.593±3.661	1.961	0.144×10 <sup>-16</sup>	0.000
2	k= 9.593±0.125 β= 0.443±0.071	0.301	0.132×10 <sup>-7</sup>	0.000
3	k= 7.683±0.236 b= 7.886±1.179 β= 0.334±0.019	0.030	0.121×10 <sup>4</sup>	0.050
4	k= 7.901±0.217 b= 7.288±1.084 β= 0.313±0.019	0.020	0.228×10 <sup>5</sup>	0.950

Table 3-7

Reliability of the Parameters and Posterior Probability  
of Each Model

Core Sample No. 1 - Run 4

<u>Model</u>	<u>Parameters and 95 % confidence</u>	<u><math>\sigma^2</math> (psia<sup>2</sup>)</u>	<u>&lt;L&gt;</u>	<u>p''</u>
1	k= 4.273±0.251 b=40.266±6.464	12.154	0.237×10 <sup>-34</sup>	0.000
2	k= 9.297±0.110 β= 0.397±0.028	0.626	0.504×10 <sup>-15</sup>	0.000
3	k= 7.657±0.237 b= 8.100±1.365 β= 0.338±0.012	0.104	0.384×10 <sup>-3</sup>	0.397
4	k= 7.868±0.209 b= 7.590±1.284 β= 0.316±0.013	0.100	0.580×10 <sup>-3</sup>	0.603

Table 3-8

Reliability of the Parameters and Posterior Probability  
of Each Model

Core Sample No. 1 - Run 5

<u>Model</u> -	<u>Parameters and</u> <u>95 % confidence</u>	$\sigma^2$ <u>(psia<sup>2</sup>)</u>	<u>&lt;L&gt;</u> -	<u>p''</u> -
1	k= 2.468±0.104 b=166.412±12.005	1.718	0.725×10 <sup>-10</sup>	0.000
2	k= 8.376±0.211 β= 0.306±0.031	1.483	0.203×10 <sup>-9</sup>	0.000
3	k= 3.883±0.233 b=81.135±8.999 β= 0.164±0.017	0.052	0.485×10	0.000
4	k= 4.724±0.278 b=56.829±7.326 β= 0.148±0.016	0.017	0.108×10 <sup>5</sup>	1.000

Table 3-9

Reliability of the Parameters and Posterior Probability  
of Each Model

Core Sample No. 1 - Run 6

<u>Model</u> -	<u>Parameters and</u> <u>95 % confidence</u>	$\sigma^2$ <u>(psia<sup>2</sup>)</u>	<u>&lt;L&gt;</u> -	<u>p"</u> -
1	k= 2.990±0.222 b=102.696±15.174	12.585	0.139×10 <sup>-20</sup>	0.000
2	k= 8.706±0.177 β= 0.334±0.026	1.440	0.412×10 <sup>-12</sup>	0.000
3	k= 5.983±0.540 b=25.462±7.208 β= 0.264±0.023	0.353	0.213×10 <sup>-6</sup>	0.003
4	k= 6.532±0.506 b=20.696±6.107 β= 0.245±0.025	0.234	0.615×10 <sup>-4</sup>	0.997

A summary of the posterior probabilities obtained from the above analysis using core sample No. 1 is given in the following table.

Table 3-10

Values of Posterior Probability when Discriminating among  
the Four Models for Radial Flow

Core Sample No. 1

<u>Number of Observations</u>	<u>Run No.</u>	<u>Mechanistic Models</u>			
		<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
0	-	0.250	0.250	0.250	0.250
15	1	0.000	0.000	0.201	0.799
15	2	0.000	0.000	0.154	0.846
23	3	0.000	0.000	0.050	0.950
30	4	0.000	0.000	0.397	0.603
14	5	0.000	0.000	0.000	1.000
18	6	0.000	0.000	0.003	0.997

In order to ascertain the relative importance of the physical parameters when dealing with different types of rocks, three more core samples were analyzed with permeability ranging from 0.5 to 12.0 millidarcies.

The tables below show the least square parameters and the posterior probability of each model using the other three core samples.

Table 3-11

Reliability of the Parameters and Posterior Probability  
of Each Model

Core Sample No. 2

$\beta$  ; in  $10^{10}$

<u>Model</u> -	<u>Parameters and</u> <u>95 % confidence</u>	$\sigma^2$ <u>(psia<sup>2</sup>)</u>	<u>&lt;L&gt;</u> -	<u>p"</u> -
1	k=0.666±0.008 b=8.962±0.978	0.296	0.161×10 <sup>-4</sup>	0.000
2	k=0.812±0.019 $\beta$ =0.136±0.199	1.405	0.300×10 <sup>-9</sup>	0.000
3	k=0.706±0.009 b=6.580±0.560 $\beta$ =0.051±0.010	0.030	0.231×10 <sup>3</sup>	0.215
4	k=0.708±0.009 b=6.507±0.590 $\beta$ =0.049±0.009	0.025	0.838×10 <sup>3</sup>	0.785



Table 3-12

Reliability of the Parameters and Posterior Probability  
of Each Model

Core Sample No. 111

$\beta$  ; in  $10^{10}$

<u>Model</u> -	<u>Parameters and</u> <u>95 % confidence</u>	$\sigma^2$ <u>(psia<sup>2</sup>)</u>	<u>&lt;L&gt;</u> -	<u>p"</u> -
1	k= 2.444±0.047 b=18.411±1.886	0.958	$0.266 \times 10^{-9}$	0.000
2	k= 3.451±0.106 $\beta$ = 0.156±0.270	2.608	$0.878 \times 10^{-13}$	0.000
3	k= 2.737±0.044 b=12.219±0.914 $\beta$ = 0.640±0.082	0.052	$0.590 \times 10$	0.190
4	k= 2.757±0.049 b=11.986±1.019 $\beta$ = 0.607±0.083	0.042	$0.252 \times 10^2$	0.810

Table 3-13

Reliability of the Parameters and Posterior Probability  
of Each Model  
Core Sample No. 112  
 $\beta$  ; in  $10^{10}$

<u>Model</u> -	<u>Parameters and</u> <u>95 % confidence</u>	$\sigma^2$ <u>(psia<sup>2</sup>)</u>	<u>&lt;L&gt;</u> -	<u>p"</u> -
1	k= 8.372±0.548 b=17.591±4.189	1.051	$0.440 \times 10^{-6}$	0.000
2	k=14.552±0.423 $\beta$ = 0.711±0.085	0.260	$0.454 \times 10^{-3}$	0.000
3	k=11.722±0.649 b= 6.229±1.704 $\beta$ = 0.510±0.067	0.041	$0.108 \times 10^2$	0.360
4	k=12.015±0.694 b= 5.686±1.806 $\beta$ = 0.489±0.074	0.030	$0.193 \times 10^2$	0.640

### 3.3 Analysis of the Steady Linear Flow

This analysis was performed using six core samples with permeability values ranging from 0.04 to 288.00 millidarcies.

The necessary expressions for parameter estimation, input data, and results obtained from each model are given in Appendix D. From these results, the 95 % confidence limit of the parameters and the posterior probability of each model were calculated.

The following tables show the reliability of the least square parameters, the expected likelihood and the posterior probability of each model, for all the core samples considered in this analysis.

Table 3-14

Reliability of the Parameters and Posterior Probability  
of Each Model

Core Sample No. 9

$\beta$  ; in  $10^{13}$

Model	Parameters and 95 % confidence	$\sigma^2$ (psia <sup>2</sup> )	<L>	p"
-	-	-	-	-
A	k= 0.042±0.0003 b=28.399±1.3512	0.415	0.198×10 <sup>-25</sup>	0.010
B	k= 0.054±0.0020 $\beta$ = 0.722±0.2690	19.410	0.906×10 <sup>-34</sup>	0.000
C	k= 0.041±0.0009 b=31.558±2.9291 $\beta$ =-0.090±0.0722	0.193	0.157×10 <sup>-23</sup>	0.990

Table 3-15

Reliability of the Parameters and Posterior Probability  
of Each Model

Core Sample No. 5

$\beta$  ; in  $10^{11}$

<u>Model</u>	<u>Parameters and 95 % confidence</u>	<u><math>\sigma^2</math> (psia<sup>2</sup>)</u>	<u>&lt;L&gt;</u>	<u>p"</u>
A	k= 0.524±0.013 b=16.078±3.448	5.198	0.671×10 <sup>-31</sup>	0.000
B	k= 0.645±0.023 $\beta$ = 0.658±0.188	8.031	0.681×10 <sup>-32</sup>	0.000
C	k= 0.527±0.017 b= 9.919±2.403 $\beta$ = 0.319±0.093	0.604	0.455×10 <sup>-26</sup>	1.000
D	k= 0.570±0.015 b=10.124±2.260 $\beta$ = 0.281±0.083	0.639	0.379×10 <sup>-26</sup>	-

Table 3-16

Reliability of the Parameters and Posterior Probability  
of Each Model

Core Sample No. 11

$\beta$  ; in  $10^{10}$

<u>Model</u>	<u>Parameters and</u> <u>95 % confidence</u>	$\sigma^2$ <u>(psia<sup>2</sup>)</u>	<u>&lt;L&gt;</u> <u>-</u>	<u>p"</u> <u>-</u>
A	k=8.040±0.072 b=4.032±0.469	0.033	0.640×10 <sup>-20</sup>	0.000
B	k=9.170±0.171 $\beta$ =0.178±0.050	0.170	0.178×10 <sup>-23</sup>	0.000
C	k=8.358±0.031 b=2.921±0.121 $\beta$ =0.589±0.054	0.001	0.193×10 <sup>-10</sup>	1.000
D	k=8.357±0.032 b=2.921±0.121 $\beta$ =0.586±0.054	0.001	0.146×10 <sup>-10</sup>	-

Table 3-17

Reliability of the Parameters and Posterior Probability  
of Each Model

Core Sample No.15

$\beta$  ; in  $10^{10}$

<u>Model</u>	<u>Parameters and</u> <u>95 % confidence</u>	$\sigma^2$ <u>(psia<sup>2</sup>)</u>	<u>&lt;L&gt;</u>	<u>p"</u>
A	k=8.295±0.119 b=4.165±0.758	0.085	$0.556 \times 10^{-22}$	0.000
B	k=9.520±0.149 $\beta=0.178 \pm 0.039$	0.118	$0.108 \times 10^{-23}$	0.000
C	k=8.817±0.072 b=2.403±0.257 $\beta=0.087 \pm 0.011$	0.002	$0.773 \times 10^{-14}$	1.000

Table 3-18

Reliability of the Parameters and Posterior Probability  
of Each Model

Core Sample No. 7

$\beta$  ; in  $10^9$

<u>Model</u>	<u>Parameters and 95 % confidence</u>	$\sigma^2$ <u>(psia<sup>2</sup>)</u>	<u>&lt;L&gt;</u>	<u>p''</u>
A	k=29.271±1.131 b= 6.524±2.194	0.587	$0.368 \times 10^{-26}$	0.000
B	k=36.390±0.461 $\beta$ = 0.212±0.022	0.066	$0.194 \times 10^{-21}$	0.002
C	k=34.522±0.956 b= 1.648±0.861 $\beta$ = 0.168±0.026	0.023	$0.727 \times 10^{-19}$	0.998
D	k=34.532±0.953 b= 1.656±0.857 $\beta$ = 0.169±0.026	0.023	$0.646 \times 10^{-19}$	-

Table 3-19

Reliability of the Parameters and Posterior Probability  
of Each Model  
Core Sample No. 2  
 $\beta$  ; in  $10^8$

Model	Parameters and 95 % confidence	$\sigma^2$ (psia <sup>2</sup> )	<L>	p <sup>n</sup>
-	-	-	-	-
A	k=220.361±8.381 b= 5.075±1.396	0.066	$0.196 \times 10^{-21}$	0.000
B	k=281.665±3.122 $\beta$ = 0.205±0.019	0.008	$0.579 \times 10^{-17}$	0.498
C	k=287.712±24.094 b= -0.410± 1.584 $\beta$ = 0.221± 0.066	0.009	$0.583 \times 10^{-17}$	0.502

#### 3.4 Test of Goodness of Fit of the Error Distribution to a Normal Distribution

The statistical analysis performed in Sections 3.2 and 3.3 was based on the assumption that the errors are normally distributed with mean zero and known variance  $\sigma^2$ . In fact, the expected likelihood was calculated considering the posterior distribution of the parameters,



$f''(\underline{a})$ , to be multivariate normal. Moreover, in order to use Bayes' theorem, the likelihood of the observation vector must be computed; and this calculation requires a knowledge of the distribution of the error vector.

Consequently, it is necessary to examine the residuals in order to find out the probability of having the distribution of the errors given by

$$f(\varepsilon) = f_N(\varepsilon/0, \sigma^2) \quad (3-1)$$

This analysis is sometimes called "goodness" of fit to a given distribution (56). The null hypothesis is that the given distribution comes from a normal distribution given by Eq. (3-1), and it is to be rejected if the probability is very small.

The computer program given in Appendix E calculates the value of the statistic chi-square,  $\chi^2$  and its probability for the "goodness" of fit to a normal distribution.

The value of the statistic was computed using the following expression:

$$\chi^2 = \sum_{i=1}^k \frac{(f_i - e_i)^2}{e_i} \quad (3-2)$$

where

$f_i$  is the observed frequency

$e_i$  is the expected frequency.

Procedure details for computing the statistic and its probability are included in Appendix E together with a sample calculation.

Values obtained for  $\chi^2$  indicate that the observed distribution of the error vector can be considered as coming from a normal distribution, tested at a level of significance of 0.05.

It should be mentioned that in obtaining the error vector  $\varepsilon_{ji}$  for the  $j$ -th model, different initial value of the parameters were tried in order to confirm that a global minimum was found. The results showed identical least squares parameters with a tolerance equal to  $0.1 \times 10^{-5}$ .

## CHAPTER 4

### MATHEMATICAL MODELLING OF TRANSIENT RADIAL GAS FLOW

#### 4.1 Literature Review and Theory

In recent years a considerable effort has been directed to the theory of isothermal flow of gases through porous media. However, the present state of knowledge is far from being fully developed. The difficulty lies in the non-linearity of partial differential equations which describe both real and ideal gas flow. Solutions which are available are approximate analytical solutions, graphical solutions, analogue solutions, and numerical solutions.

The earliest attempt to solve this problem involved the method of successions of steady states proposed by Muskat (57). Approximate analytical solutions (17, 32, 67) were obtained by linearizing the flow equation for ideal gas to yield a diffusivity-type equation. Such solutions, though widely used and easy to apply to engineering problems, are of limited value because of idealized assumptions and restrictions imposed upon the flow equation. The validity of linearized equations and the conditions under which their solutions apply have not been fully discussed

in the literature. Approximate solutions are those of Heatherington et al. (40), and MacRoberts (54).

A graphical solution of the linearized equation was given by Cornell and Katz (48). Also, by using the mean value of the time derivative in the flow equation, Rowan and Clegg (67) gave several simple approximate solutions.

Numerical methods using finite difference equations and digital computing techniques have been used extensively for solving both ideal and real gas equations. Aronofsky and Jenkins (5, 47), and Bruce et al. (13) gave numerical solutions for linear and radial transient gas flow. Aronofsky (3) included the effect of slippage on ideal gas flow assuming Darcy flow. Aronofsky and Ferris (4) considered linear flow of a real gas where the gas properties were expressed as linear functions of pressure.

It has already been mentioned in Chapter 2 that as the gas flow velocity increases, departure from Darcy's law occurs. Such flow is termed non-Darcy, or "turbulent" flow and defines a region where the inertial effect is significant. A gas flow equation including a quadratic velocity term to account for the inertial effect has been solved by Swift and Kiel (70) and Tek et al. (71) for ideal gases. Eilerts et al. (32) and Carter

(17) also included non-Darcy flow in their solutions for real gases assuming no slippage. An approximate solution including non-Darcy flow has also been presented by Rowan and Clegg (67).

It should be pointed out that all the solutions for real gas as mentioned above were obtained with the implicit assumption of small pressure gradients, which implies omission of the term  $(\partial p^2 / \partial x)^2$ . Also, a general assumption was always that of constant gas properties. Variation of gas properties with pressure has been neglected because of analytical difficulties, even in approximate analytical solutions. Moreover, all the flow equations result from the assumption of constant porosity in the continuity equation.

The first attempt to obtain solutions involving no assumption of small pressure gradient in the flow of real gases appears to be given by Al-Hussainy (1). He considered the effect of a pressure dependent viscosity and gas deviation factor through the use of a flow equation in terms of a "real gas pseudo pressure". The analysis was done assuming Darcy-flow and neglecting the slippage effect.

Chwyl (18) developed a flow equation for linear gas flow which takes into account slippage and non-Darcy flow but failed to consider changes of gas properties and porosity with pressure.

In summary, the mechanism of fluid flow through a porous medium is governed by the physical properties of the rock, geometry of flow, PVT properties of the fluid, pressure distribution within the flow system, and the different regimes of flow.

#### 4.2 Development of a Mathematical Model

A mathematical model has been developed which is capable of describing transient radial gas flow through porous media under conditions in which viscous, inertial and molecular streaming effects are simultaneously operative. The model also takes into account changes of gas properties and porosity with pressure, and does not imply the use of small pressure gradients.

In deriving the flow equation and obtaining the solutions, the following assumptions were made:

a) The porous medium is homogeneous with respect to permeability, and isotropic.

b) The rock compressibility,  $c_r$ , is defined as

$$c_r = - \frac{1}{V_r} \frac{\partial V_r}{\partial p} \quad (4-1)$$

c) The pressure-dependent permeability, that is the slippage effect, is considered through a modified Klinkenberg equation.

$$k_a = k \left( 1 + \frac{b}{p} \right) \quad . \quad (4-2)$$

- d) Flow is isothermal.
- e) The flowing gas is of constant composition.
- f) The system is plane radial geometry and uniform thickness.
- g) The continuity equation is valid, i.e.

$$\frac{1}{r} \frac{\partial}{\partial r} [r(\rho q)] = - \frac{\partial(\rho \phi)}{\partial t} \quad . \quad (4-3)$$

- h) Forchheimer's equation is applicable, that is Darcy and non-Darcy flows are considered,

$$- \frac{\partial p}{\partial r} = \frac{\mu}{k_a} q + \beta \rho q |q| \quad . \quad (4-4)$$

- i) Gravitational forces are neglected.
- j) Changes of gas properties are computed as function of pressure at a constant reservoir temperature.
- k) Changes in rock porosity only depend on the difference between the internal pressure and the conditions at which porosity was measured.

As it is shown in Appendix F, the Forchheimer equation may be expressed, after Klinkenberg's correction term is introduced, as

$$-\frac{\partial p^2}{\partial r} = \frac{2\mu Z p}{\alpha_1 k(p+b)} \rho q + \frac{2\beta Z}{144 \alpha_1 g_c} (\rho q) (|\rho q|) . \quad (4-5)$$

To describe transient flow behaviour, the equation of continuity is needed, and this is stated as

$$\frac{1}{r} \frac{\partial}{\partial r} [r(\rho q)] = -[c_g + c_r \left(\frac{1}{\phi} - 1\right)] \frac{\phi \alpha_1}{2Z} \frac{\partial p^2}{\partial t} . \quad (4-6)$$

This equation which is developed in Appendix F, takes into account changes of porosity  $\phi$ , with time which amounts to changes of  $\phi$  with pressure.

Converting Equations (4-5) and (4-6) into dimensionless variables as shown in Appendix F, they can be written as

$$-\frac{\partial \bar{u}}{\partial \bar{r}} = \frac{2r_e \mu Z \bar{p}}{k p_f (\bar{p} + \bar{b})} \sqrt{\frac{72 g_c}{\alpha_1}} (\bar{\rho} q) + r_e \beta Z (\bar{\rho} q) (|\bar{\rho} q|) \quad (4-7)$$

and

$$\frac{\partial}{\partial \bar{r}} [\bar{r}(\bar{\rho} q)] = -\frac{\bar{r} p_f}{Z} [c_g + c_r \left(\frac{1}{\phi_0} - 1\right)] \frac{\partial \bar{u}}{\partial \bar{t}} . \quad (4-8)$$

These two equations must be combined according to the procedure also outlined in Appendix F. The result for the case where the rate of change of porosity was assumed to be constant, is the following flow equation which is a second order non-linear parabolic-type partial differential equation



$$\frac{\partial}{\partial \bar{r}} [K(\bar{r}, \bar{u}, \bar{u}_{\bar{r}}) \frac{\partial \bar{u}}{\partial \bar{r}}] = G(\bar{r}, \bar{u}) \frac{\partial \bar{u}}{\partial \bar{t}} \quad (4-9)$$

where  $\bar{u} \equiv \bar{p}^2$ , and the non-linear coefficients are defined as:

$$K(\bar{r}, \bar{u}, \bar{u}_{\bar{r}}) = \frac{\bar{r} k p_f(\bar{p} + \bar{b})}{2r_e \mu z \bar{p} \sqrt{\frac{72 g_c}{\alpha_1}} + r_e \beta k p_f(\bar{p} + \bar{b}) z |\bar{\rho q}|} \quad (4-10)$$

$$G(\bar{r}, \bar{u}) = \frac{\bar{r} p_f}{z} [c_g + c_r (\frac{1}{\phi_o} - 1)] \quad (4-11)$$

It should be noted that the dimensionless mass flux ( $\bar{\rho q}$ ), appears in the non-linear coefficient K. Hence, it must be related to the dimensionless pressure distribution by the Forchheimer equation as expressed in Eq. (4-7), and this further implies a dependence of the coefficient on the value of the derivative  $\partial \bar{u} / \partial \bar{r}$ .

In order to take into account changes of gas properties with pressure, the Benedict, Webb and Rubin equation (BWR) was used to compute values for the gas deviation factor Z, and a relevant equation was derived to calculate the gas compressibility  $c_g$ . Values for viscosity were obtained using the polynomial fit given by Kesting and Wang (49). The pressure used in the computation of these values is shown in Appendix H.

To completely describe isothermal plane radial transient gas flow through a finite porous system, Equations (4-7) and (4-9) must be accompanied by an appropriate set of boundary conditions. The four sets of boundary conditions, corresponding to the constant terminal rate case and constant terminal pressure case, are summarized in the next table with boundary conditions given in real and dimensionless variables.

Table 4-1

Summary of the Boundary Conditions for Transient Radial Gas Flow

DESCRIPTION	CASE I	CASE II	CASE III	CASE IV
Initial Condition	$p(r,0) = P_f$ $\bar{p}(\bar{r},0) = 1.0$	$p(r,0) = P_f$ $\bar{p}(\bar{r},0) = 1.0$	$p(r,0) = P_f$ $\bar{p}(\bar{r},0) = 1.0$	$p(r,0) = P_f$ $\bar{p}(\bar{r},0) = 1.0$
Boundary Condition at the Wellbore	$\rho q(r_w, t) = F_w$ $\overline{\rho q}(\bar{r}_w, \bar{t}) = \bar{F}_w$	$\rho q(r_w, t) = F_w$ $\overline{\rho q}(\bar{r}_w, \bar{t}) = \bar{F}_w$	$p(r_w, t) = 0.5 P_f$ $\bar{p}(\bar{r}_w, \bar{t}) = 0.5$	$p(r_w, t) = 0.5 P_f$ $\bar{p}(\bar{r}_w, \bar{t}) = 0.5$
Boundary Condition at the External Boundary	$p(r_e, t) = P_f$ $\bar{p}(1, \bar{t}) = 1.0$	$\rho q(r_e, t) = 0.0$ $\overline{\rho q}(1, \bar{t}) = 0.0$	$p(r_e, t) = P_f$ $\bar{p}(1, \bar{t}) = 1.0$	$\rho q(r_e, t) = 0.0$ $\overline{\rho q}(1, \bar{t}) = 0.0$

A. Constant Terminal Rate Case:

Case I. With constant pressure at the external boundary

Case II. With a sealed external boundary

B. Constant Terminal Pressure Case:

Case III. With constant pressure at the external boundary

Case IV. With a sealed external boundary

## CHAPTER 5

### SOLUTION OF THE MODEL FOR THE TRANSIENT PROBLEM

#### 5.1 General Discussion of Numerical Methods for Parabolic Equations

One of the most important applications of numerical methods of solution is to non-linear, partial differential equations. Several methods for solving quasi-linear equations (13, 16, 19, 22, 25, 27, 34, 35, 58, 61) have been developed that result in finite difference equations which can be solved by existing algorithms. Until recently, many of these problems were intractable; with the use of high speed computers and the development of efficient numerical methods, many unresolved problems can now be handled.

A typical second order quasi-linear parabolic-type partial differential equation, illustrated here for the two-dimensional case, is as follows:

$$G(x,y,u) \frac{\partial u}{\partial t} = \frac{\partial}{\partial x} [K(x,y,u) \frac{\partial u}{\partial x}] + \frac{\partial}{\partial y} [K(x,y,u) \frac{\partial u}{\partial y}] + S(x,y,u,t) \quad (5-1)$$

where  $G$ ,  $K$ , and  $S$  are known functions, and the problem being that of finding  $u$  throughout a region  $R$  for all  $t$ .

For the linear case, that is when the coefficients  $G$ ,  $K$ , and  $S$  in Eq. (5-1) are not functions of the dependent variable, a discretization of the space variables  $x$  and  $y$  and application of Green's theorem gives rise to a set of  $N$  first order, linear, ordinary differential equations, expressed in matrix notation as follows:

$$\underline{G} \frac{d\underline{u}}{dt} = \underline{A} \underline{u} + \underline{S}(t) \quad (5-2)$$

where  $\underline{G}$  is a diagonal, positive definite matrix,  $\underline{A}$  is a diagonally dominant, symmetric, and negative definite matrix, and  $\underline{S}(t)$  is a vector incorporating all the boundary conditions.

Under these conditions, a closed form solution of Equation (5-2) has been obtained by Darsi and Quon (22). However, it was stated that for large matrices, Equation (5-2) should be integrated numerically. In doing that, the left hand side of the equation is integrated with respect to time using a simple first order correct forward difference formula. On the right hand side, the matrix  $\underline{A}$  is split into two parts, one part pre-multiplies the vector  $\underline{u}_m$  at time level  $(m\Delta t)$ , the other pre-multiplies  $\underline{u}_{m+1}$  at time level  $(m+1)\Delta t$ .

Although a number of methods of splitting the matrix  $\underline{A}$  have been proposed, the following are the most common: the forward difference explicit, the backward difference implicit, the Crank-Nicolson, the alternating direction implicit procedure, and the alternating direction explicit procedure. These methods have been extensively discussed in the literature (2, 16, 19, 34, 35, 42, 61).

For the case where the coefficients are functions of the dependent variable  $u$ , the matrix  $\underline{A}$  in Equation (5-2) will no longer be constant but will be a function of  $u$ . Obviously, the criteria for stability known at the present for the linear case will not necessarily work. However, the transformation of variables suggested by Carslaw and Jeager (15) can be applied to simplify the problem (60).

Let

$$K(x, y, u) = \alpha(x, y) \beta(u) \quad (5-3)$$

then Equation (5-1) becomes

$$\begin{aligned} G(x, y, u) \frac{\partial u}{\partial t} &= \frac{\partial}{\partial x} [\alpha(x, y) \beta(u) \frac{\partial u}{\partial x}] + \frac{\partial}{\partial y} [\alpha(x, y) \beta(u) \frac{\partial u}{\partial x}] \\ &+ S(x, y, t) \quad . \end{aligned} \quad (5-4)$$

Moreover, let

$$\omega = \frac{1}{\beta(u_0)} \int_{u_0}^u \beta(\sigma) d\sigma \quad (5-5)$$

where  $\sigma$  is a dummy variable. Equation (5-4) now becomes

$$\frac{G(x,y,u)}{\beta(u)} \frac{\partial \omega}{\partial t} = \frac{\partial}{\partial x} [\alpha(x,y) \frac{\partial \omega}{\partial x}] + \frac{\partial}{\partial y} [\alpha(x,y) \frac{\partial \omega}{\partial y}] + S(x,y,t) . \quad (5-6)$$

The right hand side of the above equation is now linear, and if space variables are discretized as before, then Equation (5-6) yields

$$\underline{f}(\omega) \frac{\partial \omega}{\partial t} = \underline{B} \omega + \underline{S}(t) . \quad (5-7)$$

Although the diagonal matrix  $\underline{f}(\omega)$  is a function of  $\omega$  and hence varies, it always remains positive definite;  $\underline{B}$  is a constant, symmetric diagonally dominant matrix. The conditions for stability of both ADIP and ADEP are therefore met.

It should be pointed out that most of the time it is difficult to split the non-linear coefficient as a product of two functions such as in Equation (5-3). Moreover, if the non-linear coefficient is a function of the space derivative, the transformation is no longer applicable, and this method of linearization fails.

A general finite difference equation to solve quasi-linear equations is the Crank-Nicolson equation in conjunction with some techniques of handling the

the non-linear coefficients. To discuss the various methods for numerically solving quasi-linear parabolic equations, let us consider the simplest equation of this type, having only one non-linear coefficient  $a(u)$ ,

$$a(u) \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} . \quad (5-8)$$

The Crank-Nicolson analogs to the derivatives are centered about the time level  $t_{m+\frac{1}{2}}$ . An analog to the non-linear coefficient,  $a(u)$ , is required at this time level; and, if the resulting finite difference equations are to be linear, this analog must not contain values of  $u$  at the time level  $t_{m+1}$ . The simplest such analog is obtained by evaluating  $a(u)$  at the old time level  $m$ , and using  $a(u_m)$  for  $a(u_{m+\frac{1}{2}})$ . If the function  $a(u)$  does not change very rapidly with  $u$ , the solution to the resulting finite difference equations should be fairly near the correct solution. These values can be improved by next evaluating  $a(u_{m+\frac{1}{2}})$  as  $a(\frac{u_m + u_{m+1}^{(1)}}{2})$ , where  $u_{m+1}^{(1)}$  is the solution obtained when  $a(u_m)$  was used for  $a(u_{m+\frac{1}{2}})$ . The result of the continuation of this procedure is the following iterative equations:

$$\left[ a\left(\frac{u_{i,m} + u_{i,m+1}^{(k)}}{2}\right) \right] \frac{1}{2} \Delta_x^2 (u_{i,m} + u_{i,m+1}^{(k+1)}) = \frac{u_{i,m+1}^{(k+1)} - u_{i,m}}{\Delta t} \quad (5-9)$$



where

$$\Delta_x^2 \equiv \frac{u_{i+1,m} - 2u_{i,m} + u_{i-1,m}}{(\Delta x)^2}$$

and

$$u_{i,m+1}^{(0)} = u_{i,m} \cdot$$

It should be pointed out that in Equation (5-9) the unknown values are  $u_{i,m+1}^{(k+1)}$ , and values of  $u_{i,m+1}^{(k)}$  are completely known. Therefore, the resulting finite difference equations are still linear. Iteration is continued until  $u_{i,m+1}^{(k+1)} - u_{i,m+1}^{(k)} \leq \epsilon$ , a predetermined tolerance.

The coefficient matrix for the set of linear algebraic equations is tridiagonal, hence the Thomas algorithm outlined in Appendix G can be used for the solution. This method might converge in four or five iterations. However, it is worth noting that since the first analog to  $a(u_{i,m+\frac{1}{2}})$  uses an old value, there will be some limitations on the size of the time increment to ensure stability.

Douglas (25) has devised a method for projecting the value of  $u_i$  to the half-time level for use in the non-linear coefficients. This method converges more rapidly than the method described above, but still has some restrictions for stability which besides are rather

complex. In this method the value of the dependent variable at the half-time level is obtained from a truncated Taylor series as follows:

$$u_{i,m+\frac{1}{2}} = u_{i,m} + \left(\frac{\partial u}{\partial t}\right)_{i,m} \left(\frac{\Delta t}{2}\right) + \left(\frac{\partial^2 u}{\partial t^2}\right)_{i,m} \frac{1}{2!} \left(\frac{\Delta t}{2}\right)^2 + \dots + \quad (5-10)$$

The series in Equation (5-10) is truncated after the second term to obtain a second order correct analog to  $u_{i,m+\frac{1}{2}}$ . The time derivative in this analog is then obtained from Equation (5-8). The resulting finite difference analog for  $u_{i,m+\frac{1}{2}}$  to be used in the non-linear coefficient is then

$$u_{i,m+\frac{1}{2}} = u_{i,m} + \frac{\Delta t}{2} [a(u_{i,m}) \Delta_x^2 u_{i,m}] . \quad (5-11)$$

This value is then used in evaluating  $a(u)$  for use in the Crank-Nicolson analog to (5-8). The resulting finite difference equation can be written as

$$a\left[u_{i,m} + \frac{\Delta t}{2} a(u_{i,m}) \Delta_x^2 u_{i,m}\right] \frac{1}{2} \Delta_x^2 (u_{i,m} + u_{i,m+1}) = \frac{u_{i,m+1} - u_{i,m}}{\Delta t} \quad (5-12)$$

The values of  $u_{i,m+1}$  which result from the application of (5-12) can be corrected by an iteration procedure similar to that described previously. However, sometimes

it might be more advisable to decrease the time step rather than to iterate.

The actual application of (5-9) is performed in two steps. First the values of  $u_{i,m+\frac{1}{2}}$  are determined explicitly from Equation (5-11) and used to evaluate  $a(u_{i,m+\frac{1}{2}})$ . These values are then used to compute the elements of the coefficient matrix for (5-12), and this system of linear equations can be again solved using the Thomas algorithm. Such a procedure has been proved to be efficient for the numerical solution of a number of quasi-linear, partial differential equations.

Another analog to  $u_{i,m+\frac{1}{2}}$  for use in the non-linear coefficients can also be obtained from a truncated backward Taylor series projection written about the time level  $t_{m+\frac{1}{2}}$ . In this case, the Taylor series is

$$u_{i,m} = u_{i,m+\frac{1}{2}} - \left(\frac{\partial u}{\partial t}\right)_{i,m+\frac{1}{2}} \frac{\Delta t}{2} + \left(\frac{\partial^2 u}{\partial t^2}\right)_{i,m+\frac{1}{2}} \frac{1}{2!} \left(\frac{\Delta t}{2}\right)^2 - \dots - \quad (5-13)$$

Again, the Taylor series is truncated after the second term to obtain a second order correct analog, and the time derivative is obtained from Equation (5-8). In this case, the time derivative should be evaluated at the time level  $t_{m+\frac{1}{2}}$ . Moreover, if the space derivative is evaluated at  $t_{m+\frac{1}{2}}$  and the non-linear coefficient is evaluated at  $t_m$ , the resulting finite difference equations

could be written as

$$a(u_{i,m}) \Delta_x^2 u_{i,m+\frac{1}{2}} = \frac{u_{i,m+\frac{1}{2}} - u_{i,m}}{\Delta t/2} \quad (5-14)$$

This equation is not explicit as is (5-11), but the resulting coefficient matrix is tridiagonal. The values of  $u_{i,m+\frac{1}{2}}$  are therefore readily obtained, and these are used in a Crank-Nicolson analog to Equation (5-8) which is

$$a(u_{i,m+\frac{1}{2}}) \frac{1}{2} \Delta_x^2 (u_{i,m} + u_{i,m+1}) = \frac{u_{i,m+1} - u_{i,m}}{\Delta t} \quad (5-15)$$

The values obtained from the two-step process using (5-14) and then (5-15) can also be improved by an iteration technique described before. It is obvious that a centered or Crank-Nicolson type of evaluation of  $u_{i,m+\frac{1}{2}}$  would be more convenient to use with Equation (5-15) than is (5-14).

The centered Taylor series projection for  $u_{i,m+\frac{1}{2}}$  can be obtained from the Taylor series for  $u_{i,m+\frac{1}{2}}$  and  $u_{i,m}$  written about the level  $t_{m+\frac{1}{2}}$ . In this case  $a(u_{i,m})$  is used for  $a(u_{i,m+\frac{1}{2}})$ , and the resulting finite difference equation is

$$a(u_{i,m}) \frac{1}{2} \Delta_x^2 (u_{i,m} + u_{i,m+\frac{1}{2}}) = \frac{u_{i,m+\frac{1}{2}} - u_{i,m}}{\Delta t/2} \quad (5-16)$$

The values of  $u_{i,m+\frac{1}{2}}$  obtained from this implicit equation are then used in (5-15) to obtain the values of  $u_{i,m+1}$ . It is important to note that the solution to Equation(5-16) can be considered as the first iterative value of (5-15) rather than as a value of the dependent variable at the half time level  $t_{m+\frac{1}{2}}$ . This method is the so-called predictor-corrector method (25,27).

While a large fraction of the problems encountered by an engineer can be described by quasi-linear equations, there are a significant number in which other types of non-linearities arise. Douglas (25) has proposed an adaptation of the methods described above which will apply to non-linear equations. This class of equations is defined by

$$\frac{\partial^2 u}{\partial x^2} + g_1(x,t,u,u_x) \frac{\partial u}{\partial x} = g_2(x,t,u,u_x) \frac{\partial u}{\partial t} \quad (5-17)$$

where

$$u_x \equiv \frac{\partial u}{\partial x} .$$

To solve this equation numerically, one must first obtain  $u_{i,m+\frac{1}{2}}$  for all values of  $i$ . These values may be obtained by any of the methods discussed above. The actual finite difference equations are then written according to the Crank-Nicolson technique with

$$g_1 = g_1 \left( x_i, t_{m+\frac{1}{2}}, u_{i,m+\frac{1}{2}}, \frac{u_{i+1,m+\frac{1}{2}} - u_{i-1,m-\frac{1}{2}}}{2(\Delta x)} \right) \quad (5-18)$$

and  $g_2$  evaluated similarly. This method of evaluating  $g_1$  and  $g_2$  for use in the Crank-Nicolson equation results in linear finite difference equations which can be solved readily in a fairly routine fashion. Furthermore, Douglas (25) has shown that this method preserves the unconditional stability of the Crank-Nicolson equations, though the time analog has a truncation error which is of the order of  $(\Delta t)^{1.5}$ .

This method can be used for any type of non-linearity for which the coefficients of the derivatives are functions of the dependent variable,  $u$ , and of its first space derivative,  $\partial u / \partial x$ . This type of non-linearity was found when developing the mathematical model for transient radial gas flow, and it is completely discussed in Appendix G.

In all the methods discussed before, the dependent variable  $u$  was first evaluated at time level  $t_{m+\frac{1}{2}}$  by a truncated Taylor series, to obtain  $u_{m+\frac{1}{2}}$ . The value of any function of  $u$ , i.e.,  $G(u)$  was then computed using the value of  $u_{m+\frac{1}{2}}$ , from the exact relationship between  $G$  and  $u$  to obtain  $G_{m+\frac{1}{2}}$ . Alternatively, for cases where the rate of change of the derivatives of the function with respect to time does not show drastic variations; or where the

initial guess and final values of the function after achieving convergence are reasonably close; the value at the half time level of the function may be obtained using the following expression:

$$G_{m+\frac{1}{2}} = \frac{G(u_m) + G(u_{m+1})}{2} \quad (5-19)$$

It is clear that if Eq. (5-19) does not provide a value which is close to that given by  $G(u_{m+\frac{1}{2}})$  serious problems of convergence might be expected. Whenever possible, however, Eq. (5-19) is generally preferred because of its simplicity.

A similar situation is found when it is necessary to evaluate functions halfway between the  $\ell^{\text{th}}$  and  $(\ell^{\text{th}}+1)$  element, say,  $G_{\ell+\frac{1}{2}}$ .

## 5.2 Procedure Employed to Solve the Model

The mathematical model developed in Chapter 4 can be classified as a second order, non-linear, parabolic-type partial differential equation. Mainly because of its non-linearity, this equation does not lend itself to analytical solution and consequently must be solved by means of a numerical technique. The technique must be stable and it must converge to solutions that can be

considered correct. Since the non-linear coefficient  $K$  in Equation (4-9) depends on the space derivative  $\partial \bar{u} / \partial \bar{r}$ , this equation has to be coupled with Equation (4-7).

Finite difference approximation procedures are classified as either explicit or implicit (74). Explicit procedures yield solutions directly in terms of known quantities, but stability problems arise if the mesh size ratio is not properly chosen. Implicit procedures involve increased computational complexity but overcome this stability problem. In order to minimize problems of stability, implicit procedures were selected and the Crank-Nicolson procedure was chosen in particular so that a general program could be written to handle the four different boundary conditions given in Table 4-20.

The following discrete approximations were employed in the numerical technique:

- a) A second order difference approximation of the space derivative shown as coefficient of  $K$  in Equation (4-9), evaluated at points halfway between the  $l^{th}$  element and the  $(l^{th} + 1)$  element.
- b) A second order difference approximation of the time derivative evaluated at the half time level.
- c) A second order difference approximation of the space derivative in the Forchheimer equation, evaluated



halfway between the  $\ell^{\text{th}}$  and  $(\ell^{\text{th}}+1)$  element and at the half time level.

d) A second order difference approximation of all the relevant boundary conditions.

Since the non-linear coefficient  $K$  in the parabolic equation is a function of the space derivative and was evaluated at  $\bar{x}_{i+\frac{1}{2}}$  and  $t_{m+\frac{1}{2}}$ , the space derivative was approximated at these same points. Evaluation of the functions at these points were performed using Equation (5-19).

The system of non-linear algebraic equations obtained after discretization can be written in matrix notation as

$$\underline{M}_{m+\frac{1}{2}} \bar{u}_{m+1} = \underline{b}_m \quad (5-20)$$

The Thomas algorithm outlined in Appendix G, was applied to obtain the solution vector  $\bar{u}_{m+1}$  at each iteration. Use of this algorithm was possible because the coefficient matrix  $\underline{M}$  is tridiagonal.

Changes of gas properties as a function of pressure were computed at each grid point by calling the subroutine "FPRESS". Values of the gas deviation factor  $Z$ , were calculated using the Benedict, Webb, and Rubin equation (BWR) as cited in reference (8). A particular expression was derived in order to calculate the gas compressibility

$c_g$ , and values of viscosity  $\mu$ , were obtained using the polynomial fit given by Kesting and Wang (49).

The appropriate boundary conditions corresponding to the four cases considered in this work, were set through the subroutine "BCOND".

Convergence of the solution at each time step was achieved when the following error criterion was satisfied at the  $(k^{th}+1)$  iteration

$$p_{\ell}^{(k+1)} - p_{\ell}^{(k)} \leq 0.001 \quad 1 \leq \ell \leq LL \quad . \quad (5-21)$$

Fraction of gas produced at different time levels was also calculated according to the procedure indicated in Appendix G.

Copies of the computer programs developed for the four cases are included in Appendix K.

### 5.3 Stability and Convergence

The accuracy of a finite difference solution to a partial differential equation is conveniently discussed in terms of the "convergence" and "stability" of the difference scheme.

Let  $D$  represent the exact solution of the partial differential equation,  $\Delta$  represent the exact solution of the partial difference equation, and  $N$  represent

the numerical solution of the partial difference equation. Hence,  $(D-\Delta)$  is the truncation error; it arises because of the finite distance between points of the difference mesh. Normally, a numerical solution can be termed "convergent" if it approaches the exact solution as the grid spacing approaches zero.

On the other hand,  $(\Delta-N)$  represents the numerical error. If a faultless computer working to an infinite number of decimal places were employed, the numerical error would be zero. Although  $(\Delta-N)$  may consist of several kinds of errors, it is usually considered limited to round-off errors. A numerical procedure can be termed "stable" if an error introduced at the outset of an iteration scheme does not magnify itself as iteration continues.

Whether a given finite difference scheme satisfies the criteria for convergence and stability (difference-scheme is convergent/divergent and stable/unstable) depends upon the form of the  $\Delta$ -equation and upon the initial and boundary conditions. If the  $\Delta$ -equation is linear, stability (and usually convergence also) will not depend on the initial and boundary conditions. However, for most problems,  $D$  and  $\Delta$  are unavailable. Therefore, the principal problem in the numerical solution of partial differential equations is to determine  $N$

such that  $(D-N)$  is smaller than some preassigned allowable error throughout the whole region considered. The following expression can be stated:

$$\text{Total error} = \text{Discretization error} + \text{round-off error} ,$$

or, mathematically

$$(D-N) = (D-\Delta) + (\Delta-N) . \quad (5-22)$$

Whenever  $(D-N)$  is large, we shall ask whether lack of convergence or lack of stability is chiefly responsible for the discrepancy. It happens that very often in such cases, the discretization error overshadows the numerical or round-off error.

In the numerical technique employed no instability problems were encountered because of the use of implicit formulae. However, loss of accuracy in the solution was observed when using large values of  $\Delta \bar{t}$  especially for large values of permeability where the transient response is fast and yet the flux must be constant at the wellbore. This was manifested by oscillations in the wellbore pressure, but nevertheless continuously decreasing towards the steady state condition. A decrease in time step was always the solution to this type of oscillations at the wellbore.

Since no analytical solution can be obtained to the problem, convergence was assumed when the results obtained at one grid spacing agreed with a tolerance of 0.003 to those obtained at a grid spacing that was one-half the value of the previous grid spacing. Both the dimensionless space increment  $\Delta\bar{r}$ , and the dimensionless time increment  $\Delta\bar{t}$  were checked. Values of  $\Delta\bar{r}$  close to the wellbore, where the largest pressure changes take place, were as small as 0.0010 for the first 51 grid points. The remaining 29 points were considered with a  $\Delta\bar{r}$  equal to 0.0327. An increase in the number of grid points having  $\Delta\bar{r} = 0.0010$  did not have any effect on the solutions obtained when only 51 grid points were considered with  $\Delta\bar{r} = 0.0010$ .

Values of the dimensionless time step for large values of permeability had to be smaller than those used for low values of permeability, in order to avoid the oscillations observed at the wellbore and to satisfy the requirement of 0.003 as a maximum difference in solutions for a smaller time step. Specifically, the following set of values were used for the two different rock permeabilities considered in this work:

500.0 md. :

$$\Delta\bar{t}_{1-2} = 0.25 ; \Delta\bar{t}_{3-4} = 0.50 ; \Delta\bar{t}_{5-6} = 1.00 ;$$

$$\Delta \bar{t}_{7-8} = 2.50 ; \Delta \bar{t}_{9-10} = 5.00 ; \Delta \bar{t}_{11-12} = 7.50 ;$$

$$\Delta \bar{t}_{13-6} = 10.00 .$$

10.0 md. :

$$\Delta \bar{t}_1 = 0.50 ; \Delta \bar{t}_2 = 1.00 ; \Delta \bar{t}_3 = 2.00 ; \Delta \bar{t}_4 = 5.00$$

$$\Delta \bar{t}_5 = 10.00 ; \Delta \bar{t}_6 = 15.00 ; \Delta \bar{t}_7 = 20.00 ; \Delta \bar{t}_8 = 25.00$$

$$\Delta \bar{t}_9 = 30.00 ; \Delta \bar{t}_{10} = 35.00 ; \Delta \bar{t}_{11} = 40.00 ; \Delta \bar{t}_{12} = 45.00$$

$$\Delta \bar{t}_{13} = 50.00 ; \Delta \bar{t}_{14} = 60.00 ; \Delta \bar{t}_{15} = 70.00 ; \Delta \bar{t}_{16} = 80.00$$

Values of the dimensionless distance  $\Delta \bar{r}$  and the dimensionless time  $\Delta \bar{t}$  stated above were considered as the optimum grid size at which solutions were obtained.

Unfortunately, the criterion used for convergence might not imply an absolute check since the numerical results can still differ from the true solution depending on the rate of convergence. Nevertheless, under the criterion used the numerical solution can be considered "convergent" and "stable".

## CHAPTER 6

### DISCUSSION OF RESULTS

#### 6.1 Discussion of the Results Concerning the Statistical Analysis

The results relevant to the analysis of radial flow were summarized in Section 3.2. The results obtained using core sample No. 1 will be discussed first.

Table 3-4 corresponds to a set of 15 data points which were taken in a relatively narrow flow rate range with mean flowing pressure varying only from 14.5 to 38.7 psia. This implies that the experimental data are most likely in the laminar flow region and therefore slippage effect must be more important than inertial effect. In fact, Model 2 shows a poor confidence of the inertial parameter  $\beta$ , whereas Model 1 shows a very good confidence of the slippage parameter  $b$ . It may be seen, however, that the three-parameter models suggest that both effects are simultaneously operative. Moreover, the only rival models are Models 3 and 4, since the posterior probability of Models 1 and 2 is zero.

Results given in Tables 3-5 to 3-7 indicate a continuous improvement in the confidence of the parameters for Models 2, 3, and 4. This improvement is associated with an increase in the number of data points

and with a large change in the value of the determinant of the matrix  $\underline{A} = \underline{X}^T \underline{X}$ . Specifically, results obtained from run 1 and run 4 using Model 4, shows that the value of the determinant changed by a factor  $10^5$ .

On the other hand, Model 1 shows a drastic change in the value of the parameter estimates and a very low value of the expected likelihood which imply a poor absolute performance of the model.

Parameter values corresponding to  $k$  and  $b$  for Models 3 and 4 show a trend to converge toward values of  $k$  and  $b$  as given in Table 3-4 using Model 1. This is what should be expected since those values for run 1 represent the most confident values because they were obtained with data corresponding to the laminar flow region, that is with flow rates as low as possible.

Posterior probabilities of Models 1 and 2 have always been zero for these four runs. On the other hand, the expected likelihood for Model 2 has always been greater than that corresponding to Model 1; and this difference has been magnified in Table 3-7 where most of the data are in the visco-inertial region. With regard to the posterior probability of the three-parameter models, it is observed in Table 3-6 that Model 4 is clearly superior to Model 3; whereas in Table 3-7 where the results were obtained using the



largest information from the rock sample, Model 3 has a posterior probability comparable to that of Model 4. This might be an indication that the information obtained from the core sample in run 3 involves a region where the slippage term  $b/p$  cannot yet be neglected; whereas in run 4 the use of high values of flow rate involving high values of mean pressure makes the slippage effect insignificant and this validates the assumption of partially neglecting the term  $b/p$  in the rigorous Model 4 to derive the three-parameter simplified Model 3.

Parameter estimates using Models 3 and 4 are very close each other and when enough data in the viscous and visco-inertial region are available, Model 3 should be preferred because of its simplicity and the saving in computer time.

In order to make a further analysis of the absolute performance of the models runs 5 and 6 were set up. Run 5 was taken with a constant outlet pressure of 40 psig and run 6 with a constant outlet pressure of 20 psig. This permitted an examination of the effect of having low flow rates with high values of mean flowing pressure. Results for the former were summarized in Table 3-8 and for the latter in Table 3-9.

Values of the parameter estimates obtained from run 5 changed drastically not only for Model 1, but also for the three parameter Models 3 and 4. This left Model 2 as the only model whose behaviour so far satisfied the criteria of absolute performance with regard to that of having relatively unchanging parameter estimates.

A similar behaviour was observed in Table 3-9 corresponding to run 6. Therefore, it is evident that Model 2 is not only keeping the parameter values pretty well within the 95 % confidence but also shows an error variance whose maximum value is only 1.48 psia<sup>2</sup>. This performance suggests that the slippage parameter  $b$ , is a third parameter which can only be used to obtain a better prediction in the region where the parameters were obtained but certainly no extrapolation is possible.

For runs 5 and 6, the posterior probabilities of Model 4 were very close to unity. However, as pointed out above, the inadequacy of the model was evident.

Also, values of the parameter estimates obtained from the three-parameter models are no longer close to each other. In Table 3-8, Model 3 estimates a value of  $b = 81.1$  psia and Model 4 gives a value of  $b = 56.8$  psia.

Table 3-10 shows the posterior probability of the models for the different regions examined. It is clear that the sequential design of experiments shows that the region of maximum discrimination between Models 3 and 4 is that shown in run 5 which involves high flow rates with mean pressure values higher than 60.0 psia.

Tables 3-11 to 3-13 show the analysis performed using different rock samples. It may be seen in Table 3-13 that only for large values of permeability, the posterior probability of the simplified three-parameter model (Model 3) becomes comparable to that of the differential model (Model 4). This is expected, since for large values of permeability the effect of the slippage term  $b/p$  is less significant, and therefore the assumption introduced in deriving Model 3 is valid. On the other hand, the results given in Tables 3-11 and 3-12 show that only for rock samples with low permeability the performance of Model 1 is much better than Model 2 as indicated by the expected likelihood. This essentially is a result of the low values of flow rates which most likely occur in rock samples with low permeability. In fact, this is best shown looking at Model 2 which gives a confidence limit for the inertial parameter  $\beta$ , that is higher than the value of the parameter itself.

The analysis of the linear flow was summarized in Tables 3-14 to 3-19. These results essentially show much more clearly what has already been stated for the radial flow.

The results using the core sample with the lowest value of permeability is given in Table 3-14. Model C estimates a negative value for the inertial parameter  $\beta$  which has no physical meaning at all. On the other hand, the high confidence of the parameters  $k$  and  $b$  given by Model A shows that slippage effect is indeed controlling the nature of flow. This better performance of Model A is again associated with the lowest values of flow rates obtained using this core sample even though the final inlet pressure was already about 500.0 psia.

Tables 3-15 to 3-19 show clearly the improvement in the confidence of the parameter  $\beta$  as higher flow rates are obtained using core samples with large values of permeability. This is best shown in Table 3-19 which corresponds to a rock sample having approximately 288.0 millidarcies. High confidence of the parameter  $\beta$  is clear and Model B has now almost the same posterior probability as the three-parameter Model C, showing the minimum effect of the slippage parameter.

It is interesting to note that for low values of permeability the value of the inertial coefficient  $\beta$ , is very large but its effect is insignificant because of the low flow rates encountered in tight reservoirs.

The correlation matrices given in Appendices C and D show that parameters  $k$  and  $\beta$  are highly positive correlated, whereas the correlation between  $k$  and  $b$  and the correlation between  $b$  and  $\beta$  are both highly negative correlated.

## 6.2 Discussion of Results for the Constant Terminal Rate Case: Case I

Solutions for this case were obtained choosing a constant dimensionless mass flux of  $-0.4 \times 10^{-4}$  at the wellbore and a gas reservoir having an initial pressure of 2,000.0 psia, a drainage radius of 500.0 ft., and a wellbore radius of 6.0 in.

Figures 6-1 to 6-7 inclusive show solutions using permeability values of 500.0 and 10.0 millidarcies. For a permeability value equal to 500.0, slippage and inertial coefficients were considered to be 10.0 psia and  $0.1 \times 10^9 \text{ ft}^{-1}$  respectively; whereas for a permeability equal to 10.0 millidarcies the corresponding values for the other two parameters were 50.0 psia and  $0.1 \times 10^{10} \text{ ft}^{-1}$ . These average values of the parameters were considered on the basis of the values obtained in the steady gas flow. On the other hand, since the mathematical model involves values for rock porosity and rock compressibility, these were assumed to be 0.132 (measured at surface conditions) and  $0.6 \times 10^{-5} \text{ psia}^{-1}$  respectively.

Figures 6-1 to 6-3 correspond to solutions with rock permeability equal to 500.0 millidarcies. Figure 6-1 is a straight plot of the dimensionless pressure-squared distribution for different dimensionless time. Figure 6-2 shows the effect of the inertial coefficient as compared with the solutions obtained assuming only Darcy flow. Deviations from the Darcy-solution are more pronounced in the area close to the wellbore giving larger pressure drops because of the inertial phenomenon which has to be overcome in order to maintain the constant mass flux at the wellbore. This pressure drop in the reservoir creates, in turn, higher mass fluxes in the vicinity of the wellbore as shown in Figure 6-3. Moreover, since fluxes are coming from relative large areas inside the reservoir, the wellbore pressure does not show a significant drop.

The effect of slippage was negligible as can be seen in Table 6-1 for a permeability equal to 500.0 md.

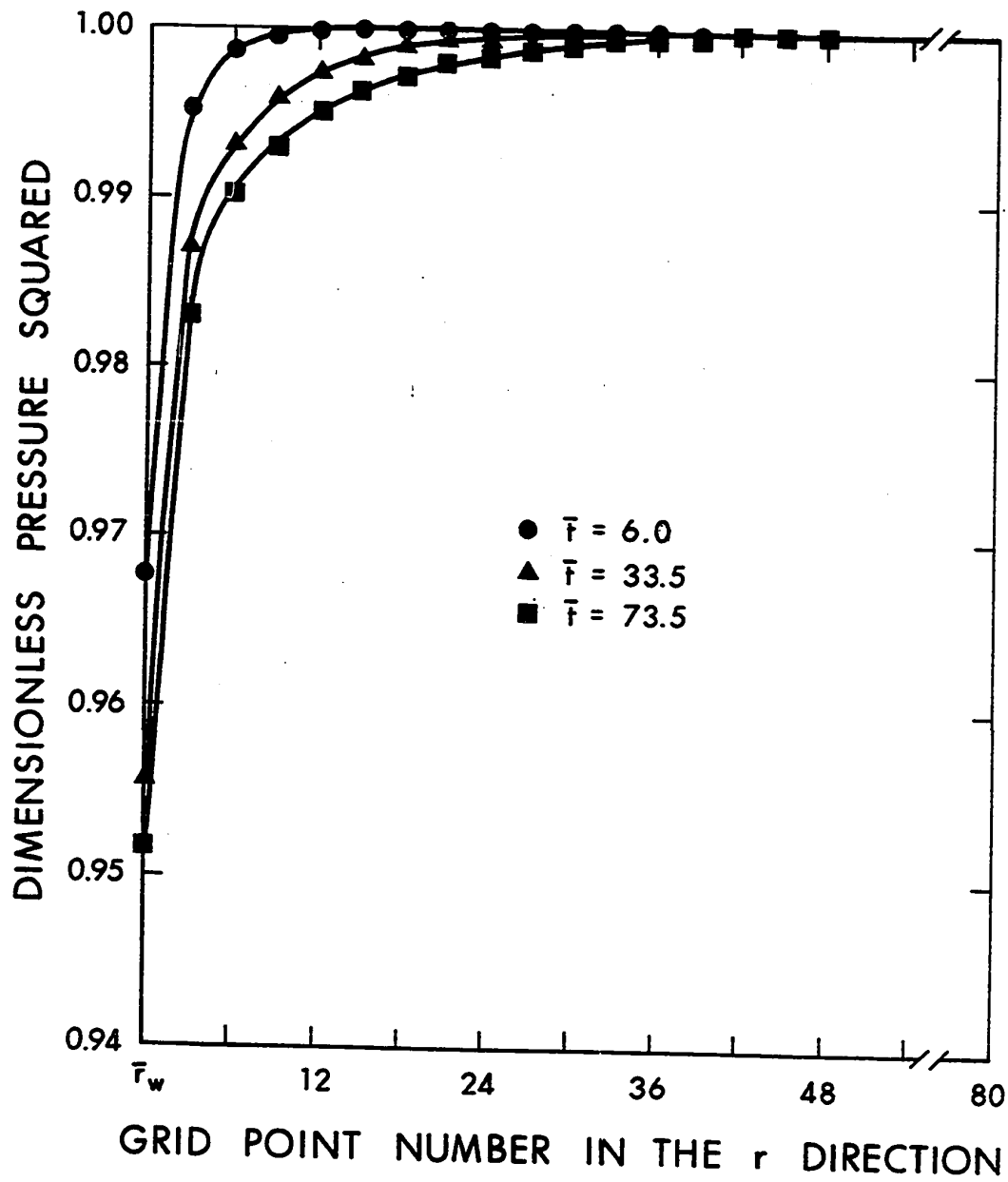


Figure 6-1. Dimensionless Pressure-Squared Distribution -  
Case I:  $k=500.0$  md.,  $b=10.0$  psia,  $\beta=0.1 \times 10^9$  ft<sup>-1</sup>

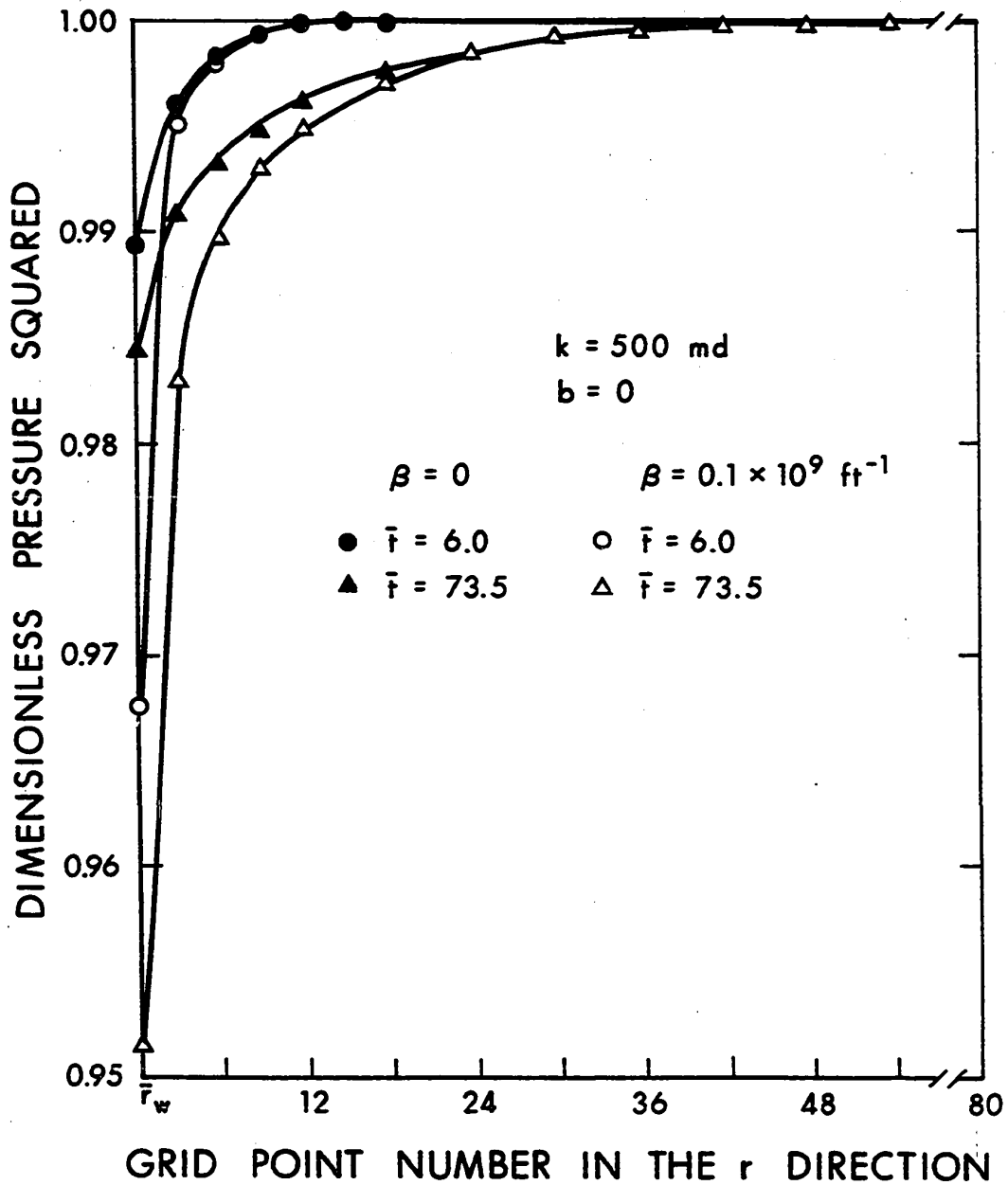


Figure 6-2. Dimensionless Pressure-Squared Distribution  
 Showing the Effect of the Inertial Parameter  
 $\beta$  - Case I.



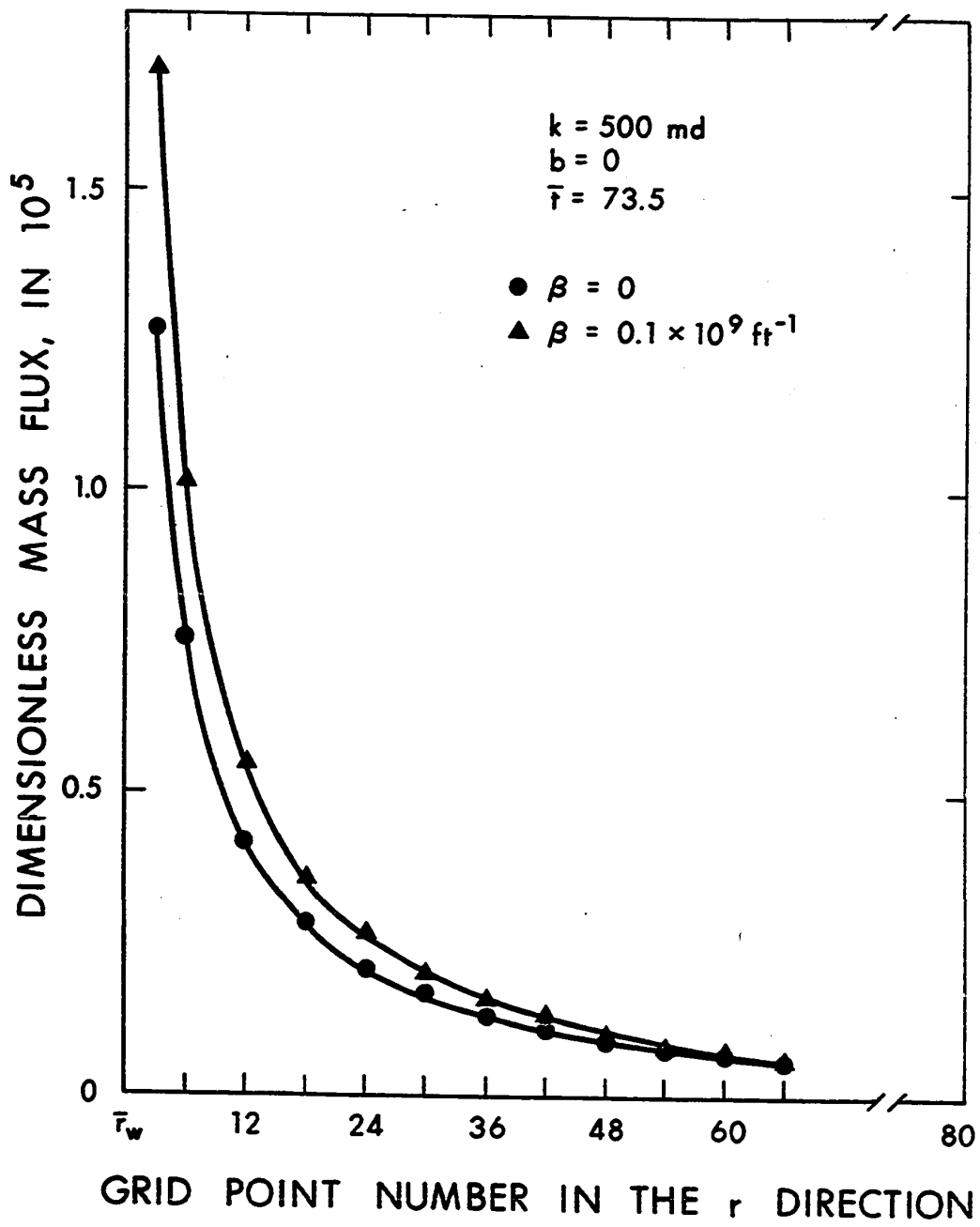


Figure 6-3. Dimensionless Mass Flux Distribution Showing The Effect of the Inertial Parameter  $\beta$  - Case I.

Table 6-1

Values of Dimensionless Pressure-Squared at the Wellbore  
Showing the Effect of the Slippage Parameter for Different  
Rock Permeabilities

$$\beta = 0.0$$

<u>k</u> (md)	<u>b</u> (psia)	<u>1.50</u>	<u>18.50</u>	<u>33.5</u>
500.0	100.0	0.993	0.987	0.986
	50.0	0.993	0.987	0.986
	10.0	0.992	0.987	0.986
	0.0	0.992	0.987	0.986
10.0	100.0	0.953	0.706	0.613
	50.0	0.952	0.690	0.608
	10.0	0.952	0.688	0.608
	0.0	0.951	0.688	0.608
0.1	100.0	0.933	0.330	0.066
	0.0	0.936	0.356	0.094

Similarly, the effect of rock compressibility for different combinations of the other three parameters was found to have no effect at all as shown in Table 6-2.

Table 6-2

Values of Dimensionless Pressure-Squared at the Wellbore  
Showing Effect of Rock Compressibility

k = 500.0 md.

$c_r$ (psia <sup>-1</sup> )	b (psia)	$\beta$ (ft <sup>-1</sup> )	$\bar{t}$		
			1.5	18.5	73.5
0	0	0	0.992	0.987	0.985
$0.6 \times 10^{-5}$	0	0	0.992	0.987	0.985
0	10	0	0.992	0.987	0.985
$0.6 \times 10^{-5}$	10	0	0.992	0.987	0.985
0	0	$0.1 \times 10^9$	0.980	0.959	0.951
$0.6 \times 10^{-5}$	0	$0.1 \times 10^9$	0.980	0.959	0.952
0	10	$0.1 \times 10^9$	0.980	0.959	0.951
$0.6 \times 10^{-5}$	10	$0.1 \times 10^9$	0.980	0.959	0.951

It is then quite clear that the effects of slippage and rock compressibility on the pressure distribution for a permeability value of 500.0 md., are indeed negligible.

Figures 6-4 to 6-7 show solutions with the set of parameters corresponding to a permeability value of 10.0 millidarcies. Figure 6-4 is a straight plot of the

dimensionless pressure-squared distribution for different dimensionless time.

By comparing Figure 6-1 with Figure 6-4, it may be observed that a marked difference occurs in the extent of the transient affecting the area inside the reservoir. In fact for a rock permeability of 500.0 millidarcies the transient was able to affect the 48th grid point at the end of  $\bar{t} = 73.5$ , whereas for a value of 10.0 millidarcies the transient only reached the 21st grid point at the end of  $\bar{t} = 278.0$ . Consequently, in order to have a constant mass flux at the producing face, the pressure drop at the wellbore was very large giving a value of  $\bar{p}_w^2 = 0.0304$  at  $\bar{t} = 278.0$ , as shown in Figure 6-4.

The effect of the inertial coefficient  $\beta$  is shown in Figure 6-5 where, rather surprisingly, except at the wellbore, lower pressure drops inside the reservoir are obtained as compared with Darcy flow. This behaviour is explained in Figure 6-6 where the flux in the vicinity of the wellbore is smaller compared to those from Darcy flow, and therefore lower pressure drops should be expected. As it is shown in Appendix I, the solution corresponding to the set  $k = 10.0$  md.,  $b = 0.0$ , and  $\beta = 0.1 \times 10^{10} \text{ ft}^{-1}$  shows at the end of  $\bar{t} = 338.0$  a value for  $\bar{p}_w^2 = 0.005$  and therefore production was terminated.

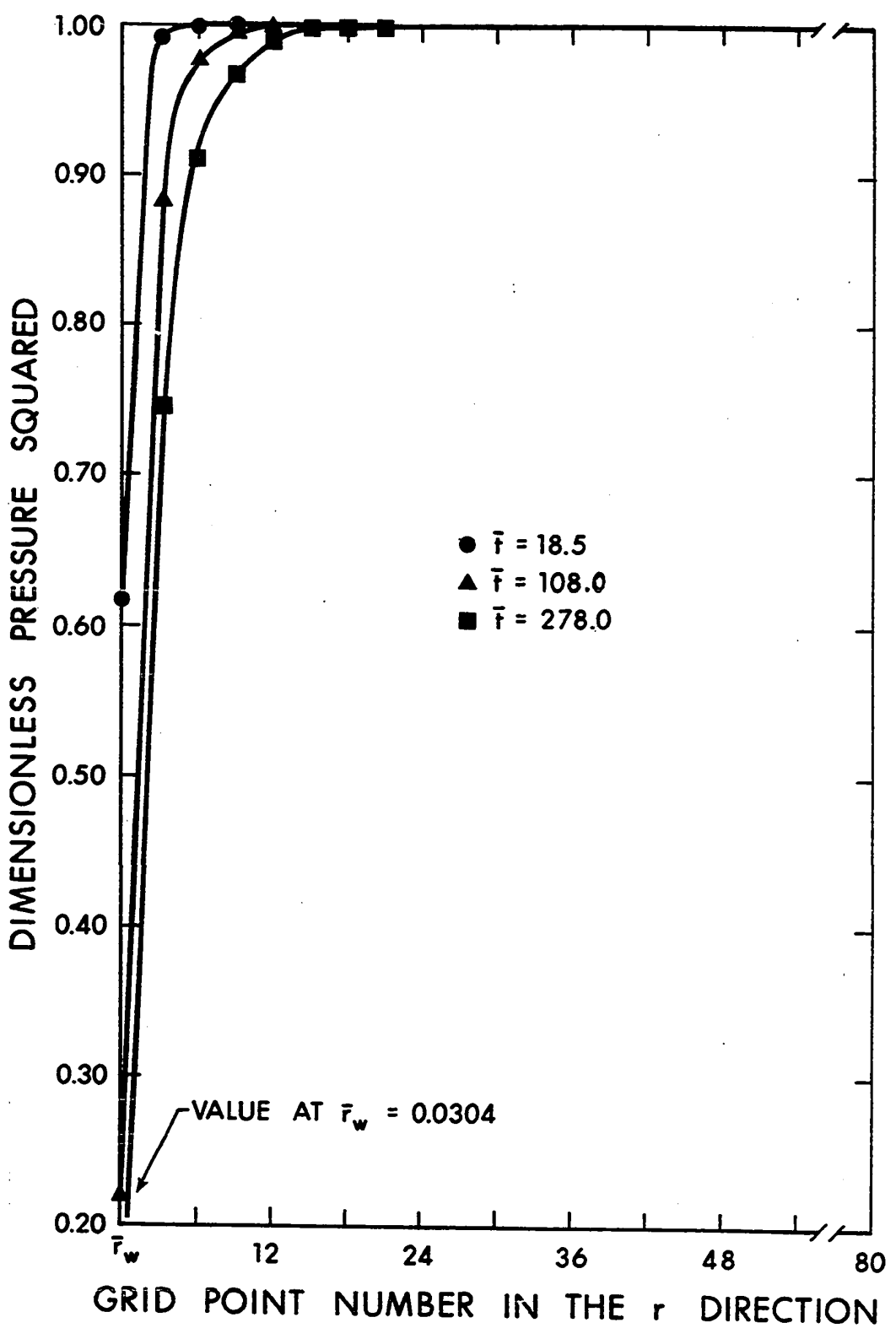


Figure 6-4. Dimensionless Pressure-Squared Distribution -  
Case I:  $k=10.0$  md.,  $b=50.0$  psia,  $\beta=0.1 \times 10^{10} \text{ ft}^{-1}$ .

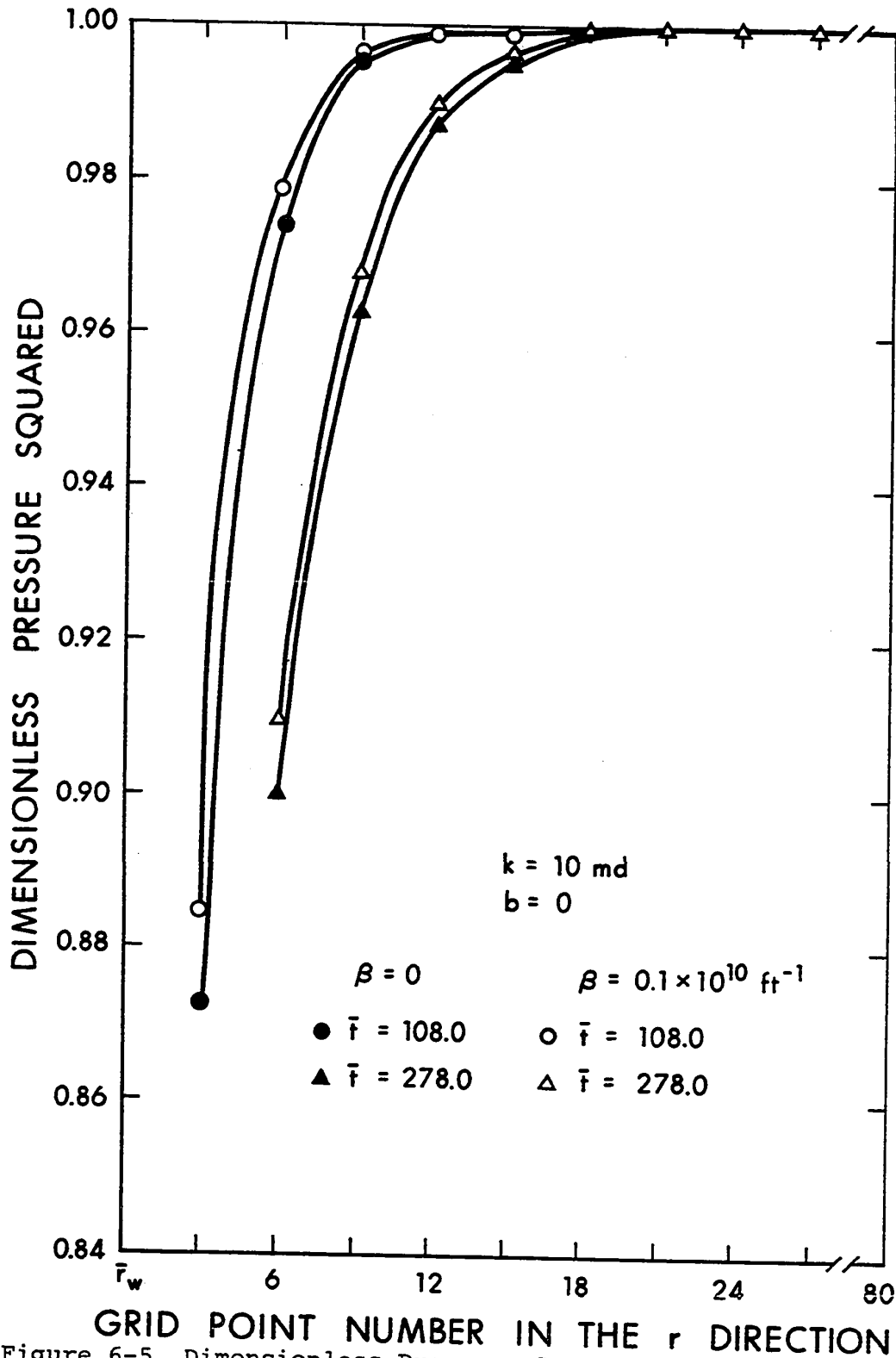


Figure 6-5. Dimensionless Pressure-Squared Distribution  
 Showing the Effect of the Inertial Parameter  $\beta$  -  
 Case I.

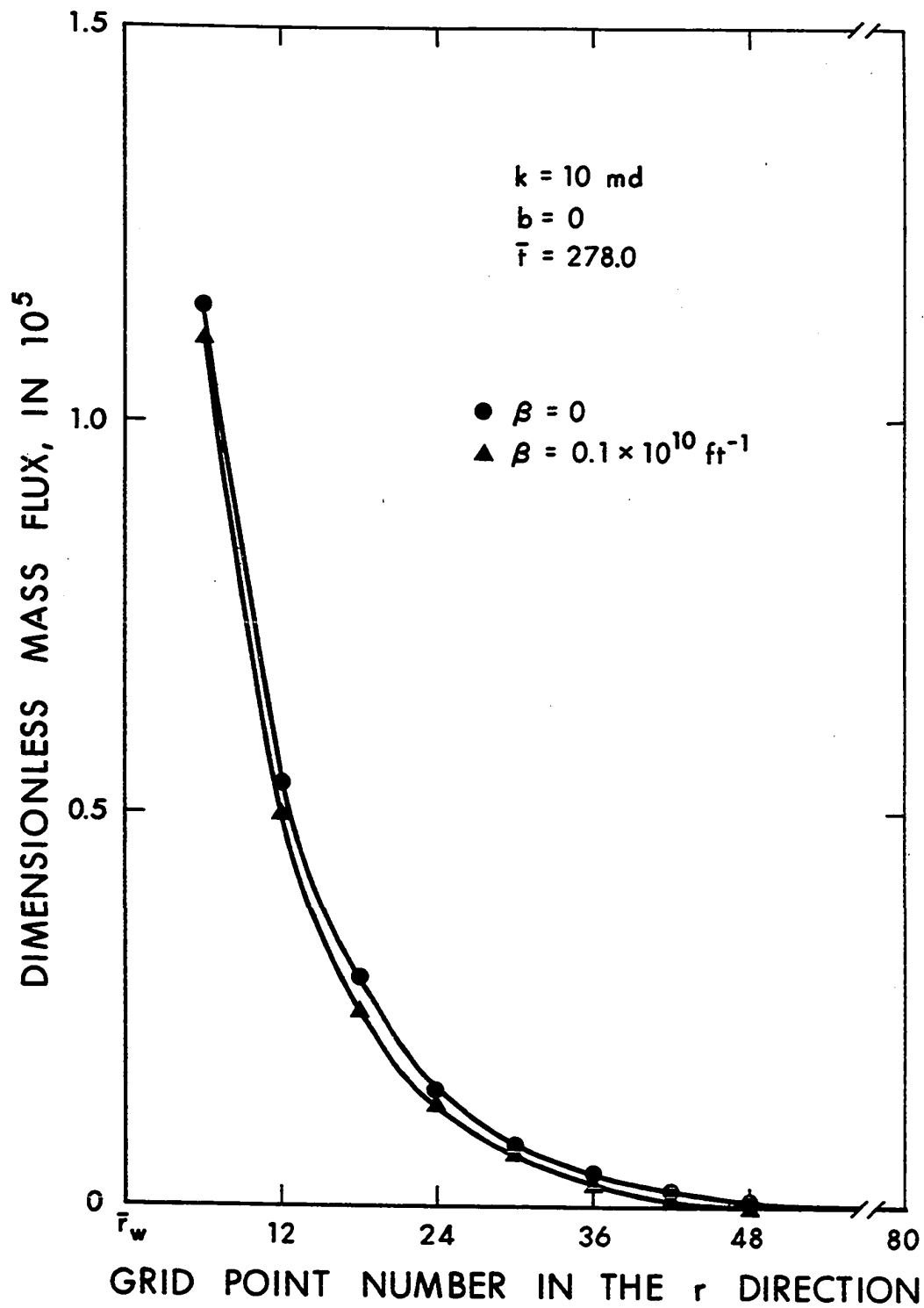


Figure 6-6. Dimensionless Mass Flux Distribution Showing the Effect of the Inertial Parameter  $\beta$  - Case I.

The effect of slippage is also shown in Table 6-1. It may be seen that the slippage coefficient showed a small effect on the solutions. However, it is interesting to note that this effect acts in two directions depending on the value of the slippage coefficient  $b$ , and on the rock permeability.

Figure 6-7 shows that for values of the dimensionless time less than 33.5 with  $b = 50.0$  psia, the slippage gave a smaller pressure drop at the wellbore as compared with Darcy flow. However, after  $\bar{t} = 33.5$  slippage becomes more significant because wellbore pressure is decreasing and since the transient does not move fast in tight reservoirs, the depletion rate has to be increased in the vicinity of the wellbore to maintain a constant mass flux at the producing face. Obviously, this implies larger pressure drops. For larger values of  $b$ , this phenomenon was also observed but at a later time. Specifically, for  $b = 100.0$  psia it occurred at  $\bar{t} = 143.0$ .

To further show the effect of the slippage phenomenon in tight reservoirs, two solutions were obtained for a rock permeability of 0.1 millidarcies. Results showing this effect for values of  $b = 0.0$  and  $b = 100.0$  are given in Table 6-1. It is evident that this effect is magnified as the rock permeability



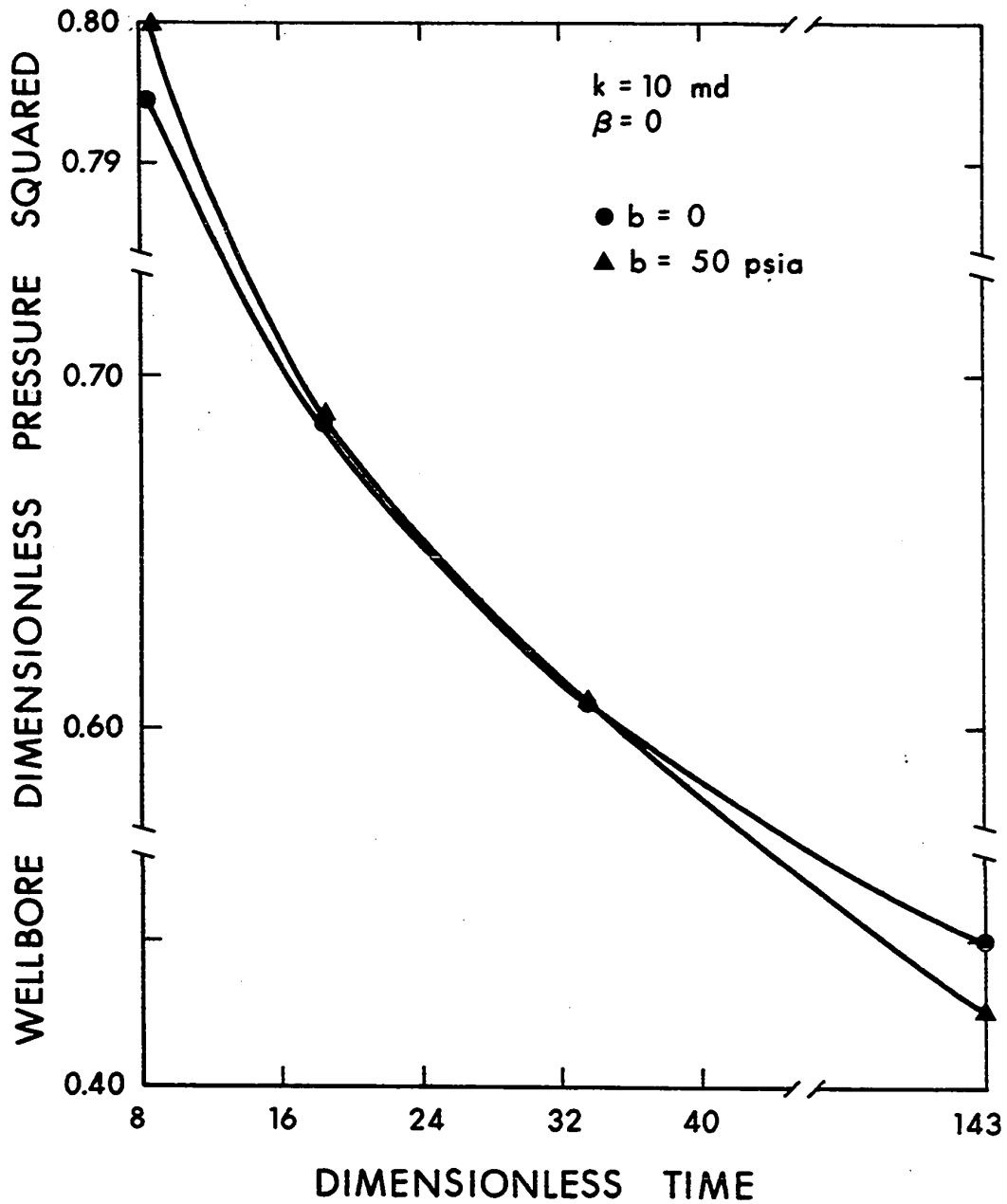


Figure 6-7. Effect of the Slippage Parameter on the Wellbore Dimensionless Pressure-Squared as Function of Time - Case I.

decreases and as time increases. Moreover, for this case the slippage only acted in one direction, that is, the pressure drop at the wellbore was always larger than that obtained with  $b = 0.0$ .

Table 6-3 shows the influence of the slippage coefficient in the presence of inertial effects. It may be observed that when slippage was present, the pressure drops were even higher than those obtained when only the inertial coefficient was present. Furthermore, this effect is magnified at the wellbore as time increases and dies out as the radial distance increases. At first glance, this might be thought as contrary to the theory since inclusion of slippage should imply a smaller pressure drop. However, the fact that slippage is now associated with inertial effects makes that the region close to the wellbore, where slippage is more significant, be depleted at a higher rate to satisfy the constraint of constant mass flux at the producing face, and therefore larger pressure drops occurred.

Table 6-3

Values of Dimensionless Pressure-Squared Showing  
Influence of the Slippage Parameter in the Presence  
of Inertial Effects

$$k = 10.0 \text{ md.}, \quad b = 50.0 \text{ psia}, \quad \beta = 0.1 \times 10^{10} \text{ ft}^{-1}$$

$\bar{t} \backslash \bar{r}$	$\bar{r}_w$	3	6	9	12	15
1.5	0.943 <sup>†</sup>	1.000	1.000	1.000	1.000	1.000
	0.943 <sup>*</sup>	1.000	1.000	1.000	1.000	1.000
18.5	0.618	0.993	0.999	1.000	1.000	1.000
	0.617	0.993	0.999	1.000	1.000	1.000
53.5	0.381	0.951	0.996	1.000	1.000	1.000
	0.380	0.951	0.996	1.000	1.000	1.000
143.0	0.160	0.849	0.964	0.993	0.999	1.000
	0.159	0.848	0.964	0.992	0.999	1.000
278.0	0.033	0.749	0.910	0.968	0.990	0.997
	0.030	0.747	0.909	0.967	0.989	0.997

† Solutions with parameters  $k$  and  $\beta$ .

\* Solutions with parameters  $k$ ,  $b$ , and  $\beta$ .

The effect of rock compressibility is shown in Table 6-4. Deviations from Darcy and non-Darcy flow were observed. In fact, it was found that inclusion of the rock compressibility always results in smaller pressure drops. This suggests that more flux is expected when the rock compressibility is considered, but since the mass flux is kept constant at the wellbore the net effect is a restriction of the mass flux inside the reservoir which implies a smaller pressure drop.

It should be pointed out that for a permeability of 500.0 millidarcies there was no appreciable effect of the rock compressibility. This merely indicated that the extra flux provided by the effect of the rock compressibility was indeed insignificant as compared with the total mass flux moving inside the reservoir.

The solutions obtained for Case I are presented in Appendix I.

Table 6-4

Values of Dimensionless Pressure-Squared at the Wellbore  
Showing Effect of Rock Compressibility

$k = 10.0$  md.

$c_r^{-1}$ (psia <sup>-1</sup> )	b (psia)	$\beta$ (ft <sup>-1</sup> )	$\bar{t}$		
			1.5	143.0	278.0
0	0	0	0.948	0.392	0.297
$0.6 \times 10^{-5}$	0	0	0.951	0.402	0.307
0	50.0	0	0.949	0.391	0.295
$0.6 \times 10^{-5}$	50.0	0	0.952	0.401	0.305
0	0	$0.1 \times 10^{10}$	0.939	0.148	0.025
$0.6 \times 10^{-5}$	0	$0.1 \times 10^{10}$	0.943	0.160	0.033
0	50.0	$0.1 \times 10^{10}$	0.939	0.147	0.022
$0.6 \times 10^{-5}$	50.0	$0.1 \times 10^{10}$	0.943	0.159	0.030

### 6.3 Discussion of Results for the Constant Terminal Pressure Case: Case IV

The solutions for this Case have been obtained with a constant dimensionless wellbore pressure equals to 0.5. The corresponding values of the parameters and other relevant data have been considered the same as in Case I.

Figures 6-8 to 6-10 inclusive correspond to solutions with rock permeability equals to 500.0 millidarcies. Figure 6-8 is a straight plot of the dimensionless pressure-squared distribution for different dimensionless time. Figure 6-9 shows the effect of the inertial coefficient as compared to Darcy flow. By comparing Figure 6-9 and Figure 6-2 corresponding to Case I, it is quite clear that for the constant terminal pressure case, the solution for non-Darcy flow shows now a larger departure from the Darcy flow. Moreover, the effect has been reversed in the sense that now the inclusion of inertial effects provides a smaller pressure drop. This becomes quite obvious in Figure 6-10 where the mass flux is very small because of the inertial effects and the constraint of constant pressure at the wellbore.

As in Case I, slippage and rock compressibility did not show any appreciable effect, as can be observed

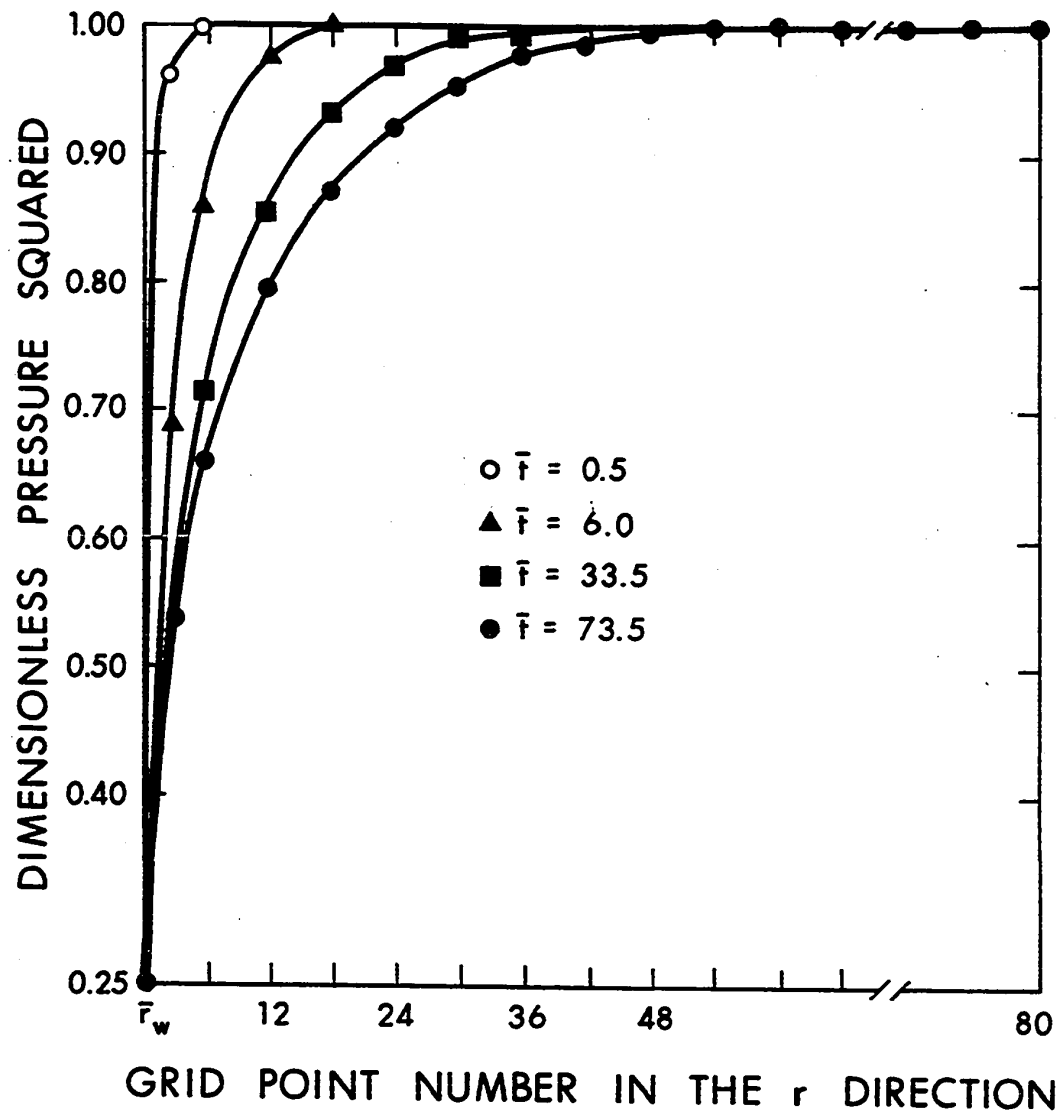


Figure 6-8. Dimensionless Pressure-Squared Distribution -  
 Case IV:  $k=500.0$  md.,  $b=10.0$  psia,  $\beta = 0.1 \times 10^9$  ft<sup>-1</sup>.

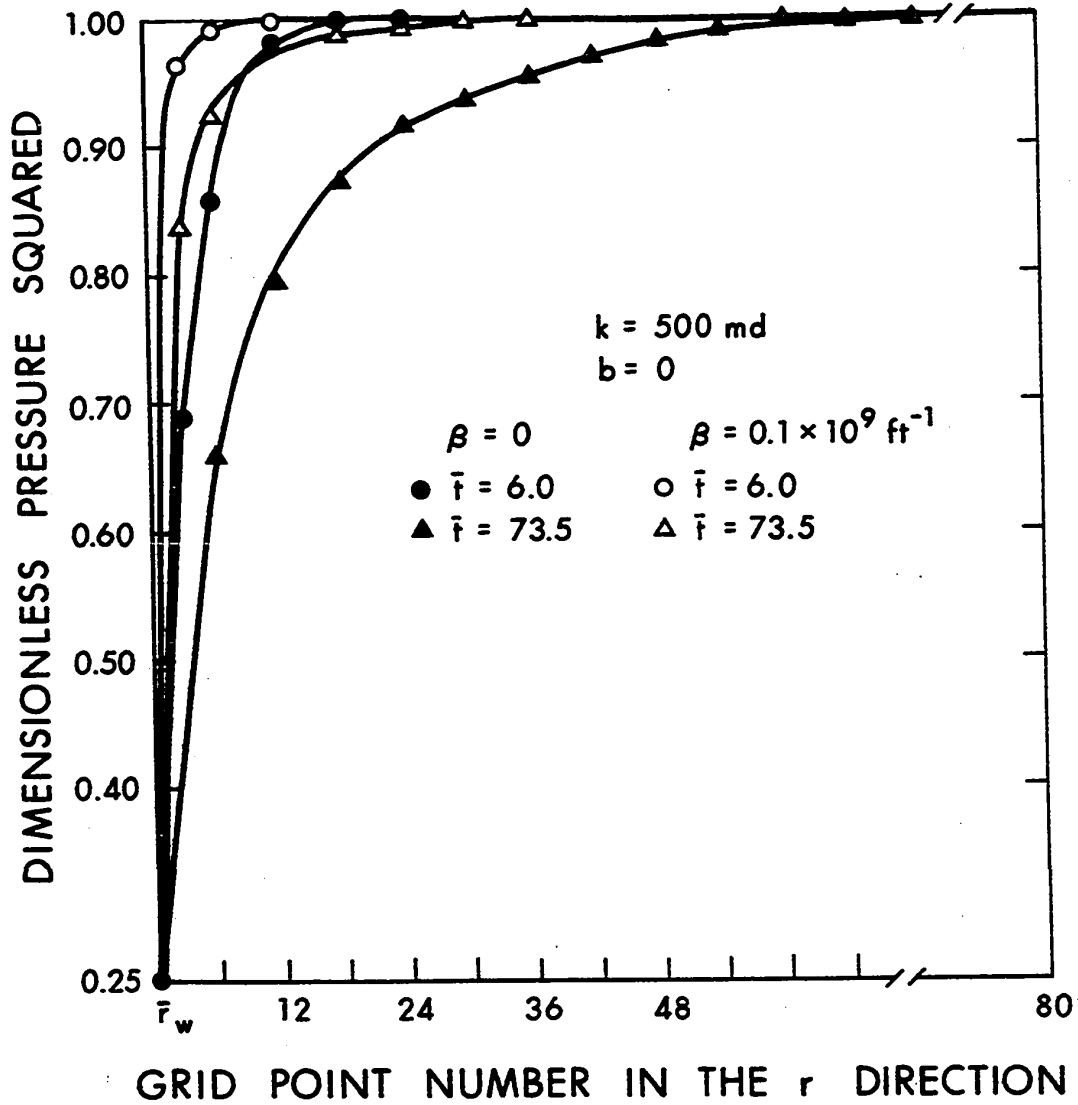


Figure 6-9. Dimensionless Pressure-Squared Distribution Showing the Effect of the Inertial Parameter  $\beta$  - Case IV.



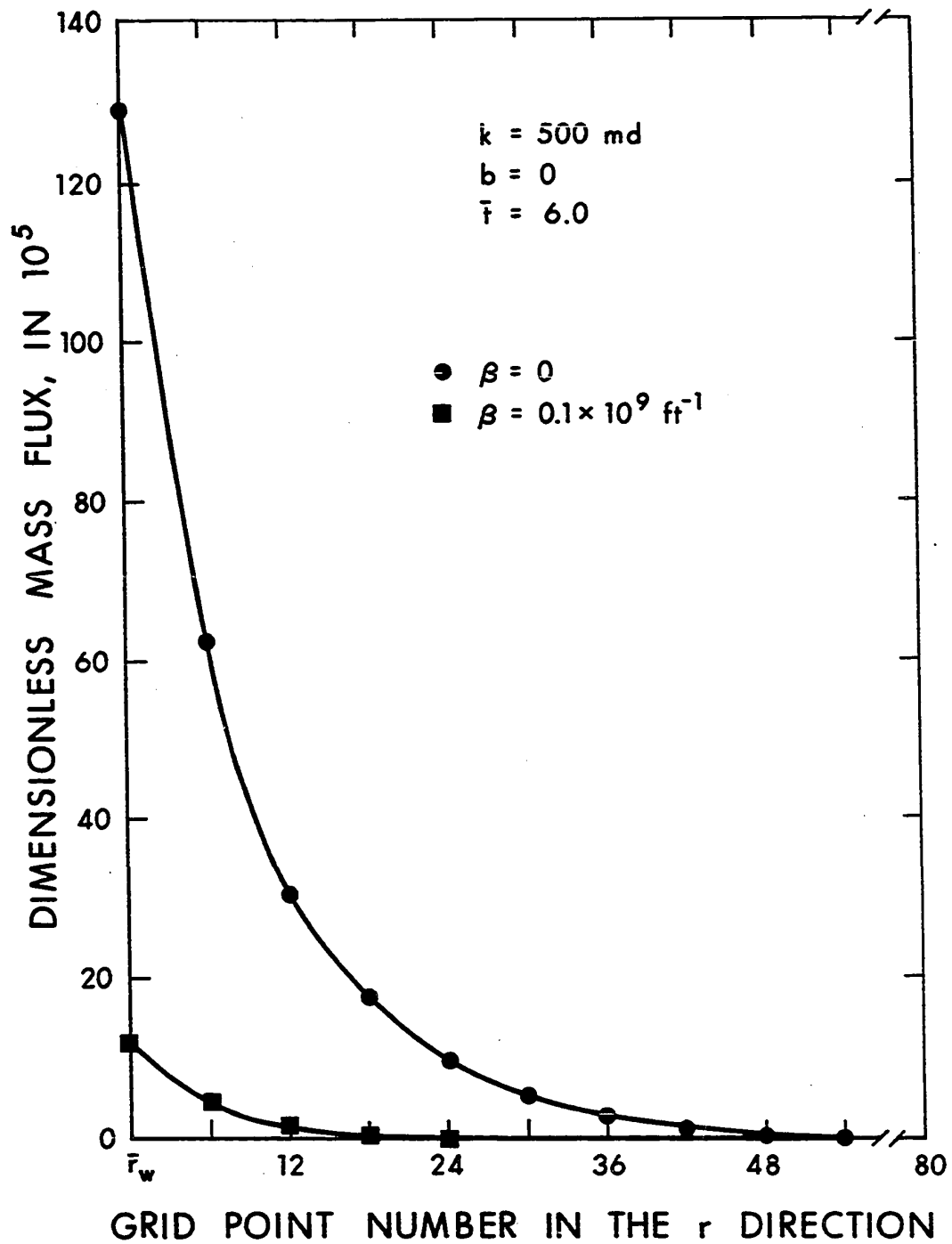


Figure 6-10. Dimensionless Mass Flux Distribution Showing the Effect of the Inertial Parameter  $\beta$  - Case IV.

in the Tables presented in Appendix J.

The solutions with the set of parameters corresponding to a permeability value of 10.0 millidarcies are shown in Figures 6-11 to 6-13. Figure 6-11 once again shows that for tight reservoirs the extent of the transient affecting the area inside the reservoir is relatively small. Figure 6-12 shows again that inertial effects restrict the mass flux inside the reservoir and therefore a smaller pressure drop is obtained. Moreover, the departure from Darcy flow now is not so impressive as shown in Figure 6-9 for a permeability value of 500.0 millidarcies. This further indicates that for tight reservoirs inertial effects are less significant because of the small fluxes in the reservoir.

Figure 6-13 shows in fact that the difference in fluxes for Darcy and non-Darcy flow is quite small as compared with Figure 6-10.

The small effect of slippage at the wellbore may also be seen in Figure 6-13. It may be observed that slippage is most significant at the wellbore and modifies Darcy flow by giving a larger flux for a constant wellbore pressure. It should be mentioned that for tight reservoirs values of the slippage coefficient  $b$ , might be easily larger than 50.0 psia; in which case the above effect would be magnified. Table 6-5 shows clearly the effect of the slippage on the mass flux

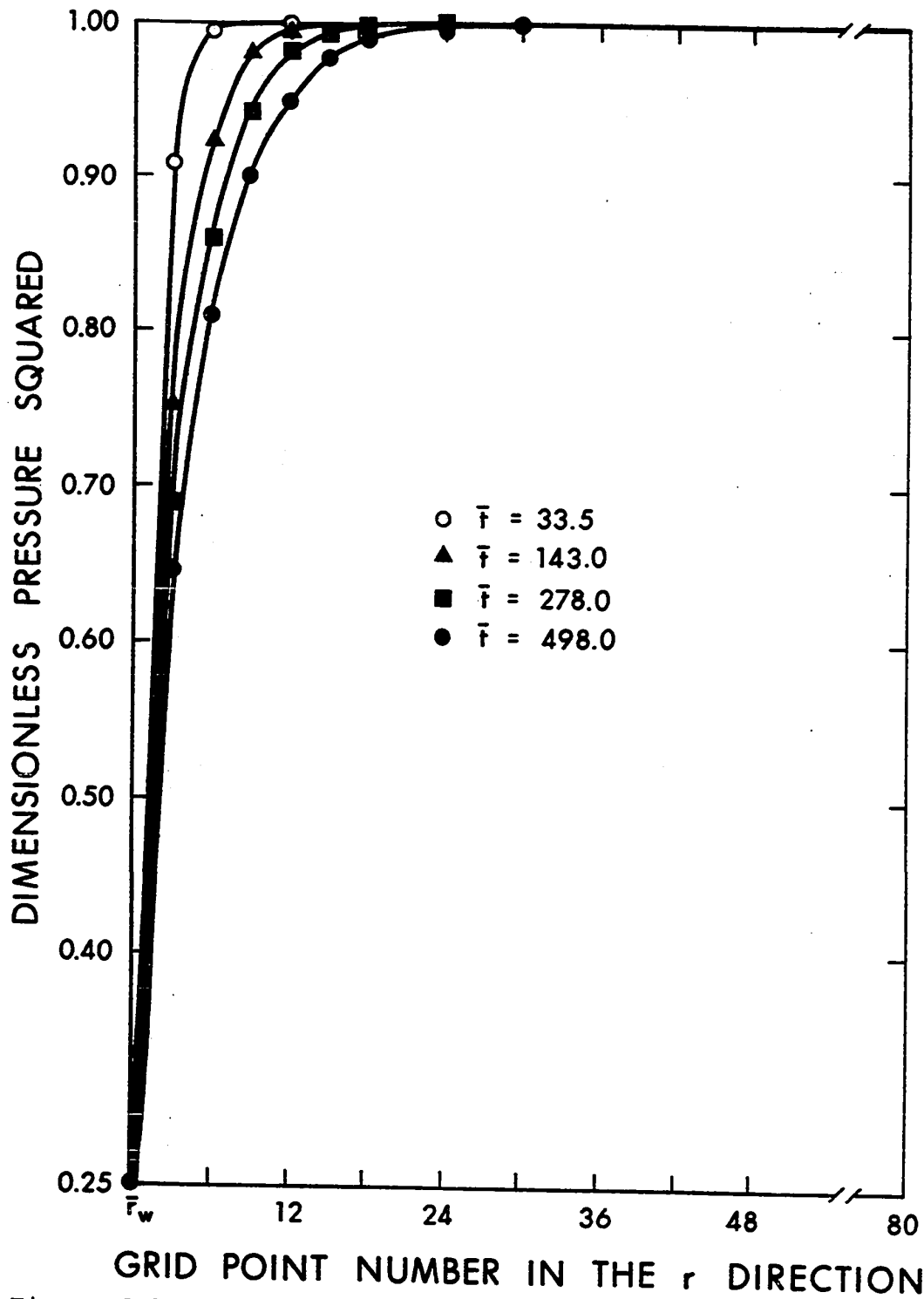


Figure 6-11. Dimensionless Pressure-Squared Distribution -  
 Case IV:  $k=10.0$  md.,  $b=50.0$  psia,  $\beta=0.1 \times 10^{10}$   
 $\text{ft}^{-1}$ .

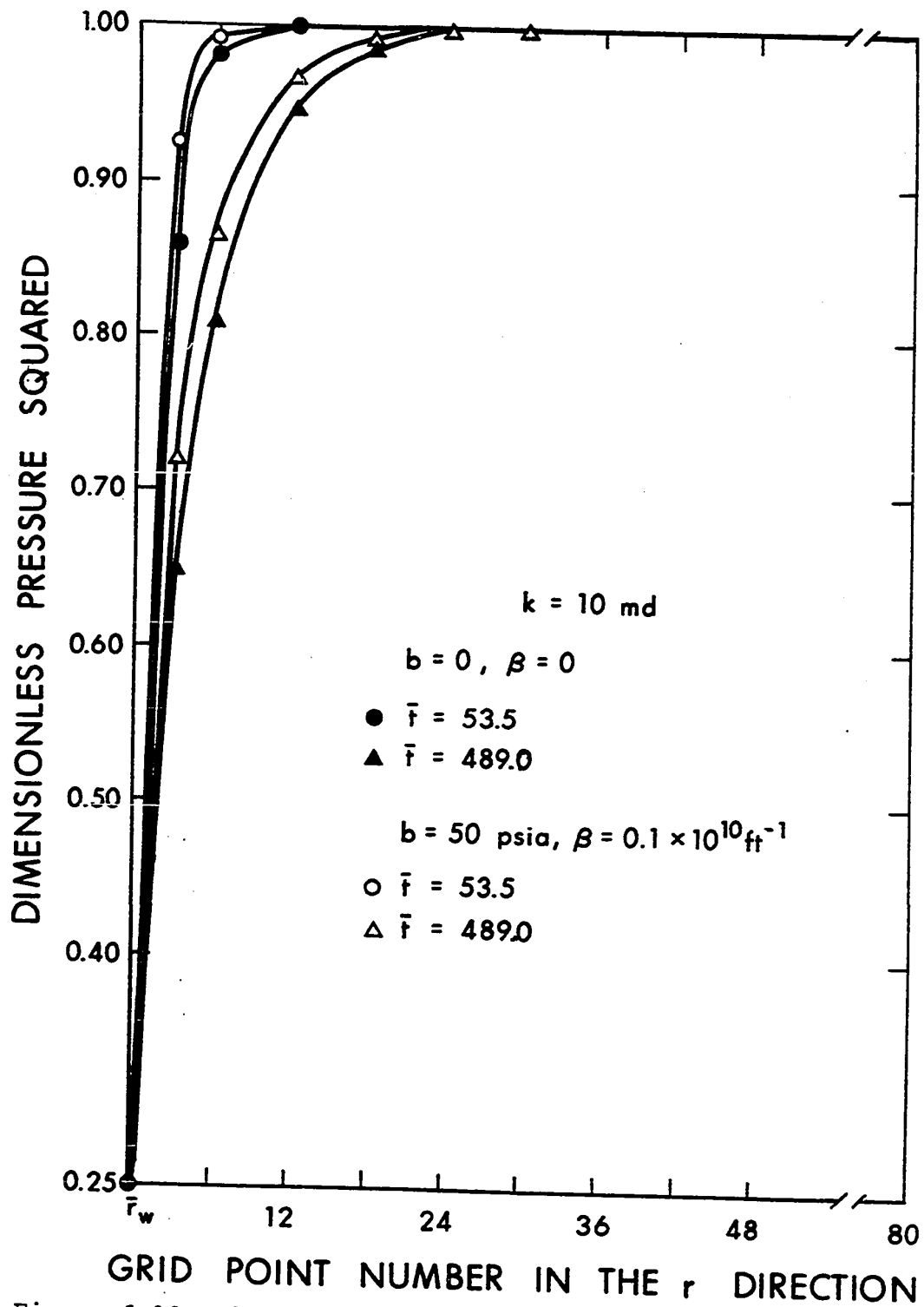


Figure 6-12. Dimensionless Pressure-Squared Distribution Showing the Effect of the Slippage and Inertial Parameters - Case IV.

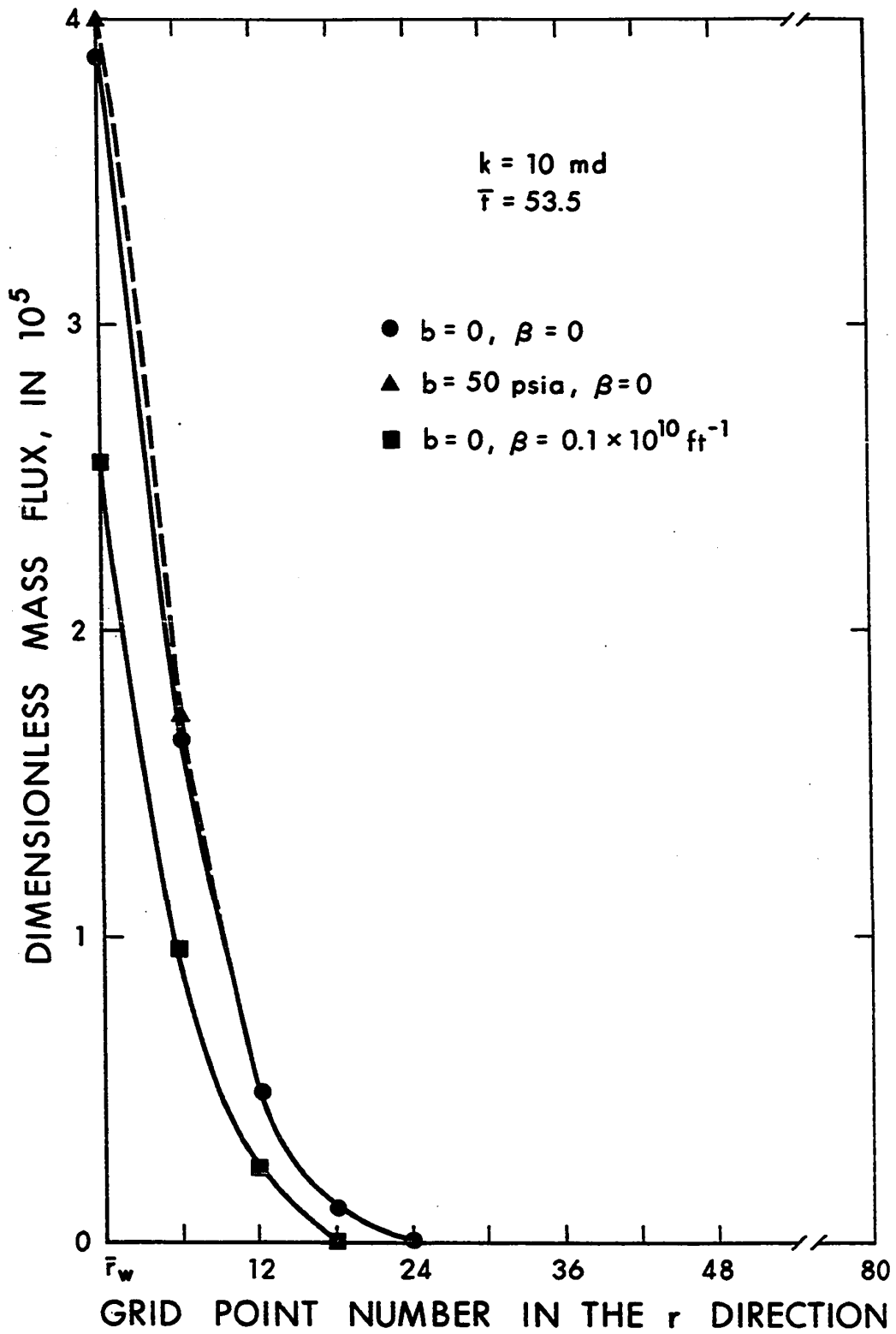


Figure 6-13. Dimensionless Mass Flux Distribution Showing the Effect of the Slippage and Inertial Parameters - Case IV.

distribution. The corresponding dimensionless pressure-squared distribution given in Table 6-6 shows a larger pressure drop associated with the larger fluxes.

It should be pointed out that the same effect of the slippage on the dimensionless flux was observed even in the presence of inertial effects, although the difference in mass fluxes was somewhat smaller.

The effect of rock compressibility on the mass flux and pressure distributions is shown in Table 6-7 and Table 6-8. It can be observed that the effect of the rock compressibility is to give slightly larger mass fluxes in the reservoir with a smaller pressure drop. This is in agreement with the constraint of constant pressure at the wellbore.

Table 6-5

Values of Dimensionless Mass Flux Showing Effect of  
the Slippage Parameter

$k = 10.0 \text{ md.}$

$\overline{\rho q}$  ; in  $10^{-4}$

$\bar{t} \backslash \bar{r}$	$\bar{r}_w$	3	6	9	12
18.5	-0.570 <sup>†</sup>	-0.422	-0.162	-0.052	-0.014
	-0.590*	-0.434	-0.168	-0.055	-0.015
78.5	-0.345	-0.277	-0.154	-0.094	-0.057
	-0.356	-0.285	-0.159	-0.097	-0.060
143.0	-0.293	-0.238	-0.138	-0.091	-0.063
	-0.302	-0.245	-0.142	-0.094	-0.065
338.0	-0.240	-0.197	-0.117	-0.081	-0.060
	-0.248	-0.203	-0.120	-0.083	-0.062

<sup>†</sup>Solutions with  $b = 0.0$

\* Solutions with  $b = 50.0 \text{ psia.}$

Table 6-6

Values of Dimensionless Pressure-Squared Showing Effect  
of the Slippage Parameter

$k = 10.0$  md.

$\bar{t}$ \ / \ $\bar{r}$	$\bar{r}_w$	3	6	9	12
18.5	0.250 <sup>†</sup>	0.962	0.999	1.000	1.000
	0.250 <sup>*</sup>	0.960	0.999	1.000	1.000
78.5	0.250	0.822	0.968	0.996	1.000
	0.250	0.818	0.966	0.996	1.000
143.0	0.250	0.759	0.925	0.981	0.996
	0.250	0.755	0.922	0.980	0.996
338.0	0.250	0.681	0.847	0.931	0.971
	0.250	0.677	0.844	0.928	0.970

<sup>†</sup>Solutions with  $b = 0.0$ .

<sup>\*</sup>Solutions with  $b = 50.0$  psia.



Table 6-7

Values of Dimensionless Mass Flux Showing Effect of the  
Rock Compressibility

$$k = 10.0 \text{ md.}$$

$$b = 50.0 \text{ psia}$$

$$\beta = 0.1 \times 10^{10} \text{ ft}^{-1}$$

$$\overline{\rho q} ; \text{ in } 10^{-4}$$

$\bar{t}$ \ / \ $\bar{r}$	$\bar{r}_w$	3	6	9	12
18.5	-0.311 <sup>†</sup>	-0.247	-0.079	-0.026	-0.008
	-0.314 <sup>*</sup>	-0.249	-0.077	-0.024	-0.007
143.0	-0.209	-0.175	-0.098	-0.062	-0.041
	-0.211	-0.177	-0.098	-0.062	-0.041
278.0	-0.186	-0.157	-0.091	-0.062	-0.044
	-0.188	-0.158	-0.092	-0.062	-0.044
489.0	-0.170	-0.144	-0.085	-0.059	-0.044
	-0.172	-0.146	-0.086	-0.060	-0.044

<sup>†</sup>Solutions with  $c_r = 0.0$ .

<sup>\*</sup>Solutions with  $c_r = 0.6 \times 10^{-5} \text{ psia}^{-1}$ .

Table 6-8

Values of Dimensionless Pressure-Squared Showing Effect  
of the Rock Compressibility

$$k = 10.0 \text{ md.}$$

$$b = 50.0 \text{ psia}$$

$$\beta = 0.1 \times 10^{10} \text{ ft}^{-1}$$

$\bar{t}$ \ / \ $\bar{r}$	$\bar{r}_w$	3	6	9	12
18.5	0.250 <sup>†</sup>	0.979	1.000	1.000	1.000
	0.250 <sup>*</sup>	0.982	1.000	1.000	1.000
143.0	0.250	0.827	0.951	0.988	0.997
	0.250	0.834	0.956	0.990	0.998
278.0	0.250	0.763	0.903	0.961	0.986
	0.250	0.770	0.909	0.965	0.988
489.0	0.250	0.714	0.856	0.926	0.963
	0.250	0.721	0.862	0.931	0.967

<sup>†</sup>Solutions with  $c_r = 0.0$ .

<sup>\*</sup>Solutions with  $c_r = 0.6 \times 10^{-5} \text{ psia}^{-1}$ .

## CHAPTER 7

## CONCLUSIONS AND RECOMMENDATIONS

7.1 Conclusions with Regard to Steady Gas Flow

As a result of the statistical analysis performed on the mathematical models describing steady gas flow, the following conclusions can be formulated:

1. Out of the four mathematical models considered for radial gas flow through porous media, the differential model which corresponds to the most general equation gives the best prediction associated with the highest posterior probability. However, this differential model failed to pass the absolute performance test since, for some runs, values of the parameter estimates changed rather drastically well outside of the 95 % confidence limit, when compared with the best estimates obtained using the largest information from the core sample.
2. The three-parameter simplified model appears to describe reasonably well radial gas flow through porous media, since its solutions are quite close to those obtained from the differential model. However, this is true as long as information from the core sample includes data points in the viscous region with low values of mean flowing pressure; otherwise it behaves like the

differential model showing a drastic change in the parameter estimates.

3. The two-parameter model with parameters  $k$  and  $\beta$  was the only model whose behaviour satisfied the criteria of absolute performance especially for the relatively unchanging parameter estimates over the entire operating space.

4. The information used from core samples with permeability ranging from 0.04 to 288.00 millidarcies shows that for low values of permeability, even with inlet pressure as high as 500.00 psig, the slippage effect is extremely important because of the low flow rates developed in tight reservoirs. However, the better performance of the two-parameter model- $k$  and  $b$ , is considered as a local approximation because of the larger variance obtained when additional data are incorporated.

5. For large values of permeability inertial effects are the most important. This of course, is because of the larger flow rates taking place even at low values of inlet pressure.

6. The sequential design procedure provides a means to estimate the design parameters with a better confidence. Specifically, for core sample No. 1 when using 15 data points instead of 30, the percent error in the

parameters, namely; permeability, slippage and inertial were 7.8, 33.6, and 20.9 respectively.

7. The chi-squared statistic indicates that the actual errors obtained from the models, can be considered as coming from a normal distribution, tested at a level of significance of 0.05.

## 7.2 Conclusions Concerning Transient Gas Flow

The theoretical study undertaken in this investigation and the results that were obtained, provide the following conclusions:

1. A mathematical model has been developed which is capable of describing visco-inertial transient radial gas flow through porous media when slippage and rock compressibility are considered. The model is quite general in the sense that changes of gas properties with pressure are accounted for, and it does not require the assumption of small pressure gradients in the entire flow system.
2. The mathematical model is not only able to describe flow when all four effects occur simultaneously, but it can be used to describe and compare flow when any combination of these effects is present.
3. This theoretical investigation has shown that the model can be solved numerically for all the four sets of boundary conditions. Despite the fact that the solutions were obtained at an optimum grid size, no

statement can be made concerning the accuracy with which the model predicts actual flow as the results obtained were not compared with experimental observations.

4. An examination of the numerical results suggests that slippage and rock compressibility appreciably alter the pressure and mass flux distributions in typically tight reservoirs. For large values of permeability these effects are indeed negligible.

5. Further examination of the results obtained for the constant terminal rate case indicates that inertial effects modify the solutions in a manner which depends on the rock permeability. For large values of permeability, the inertial effect is but another resistance to flow such that larger pressure drops occur in the reservoir. However, for tight reservoirs larger pressure drop was only observed at the wellbore as a result of the constant mass flux imposed at that boundary.

6. For the constant terminal pressure case, inertial effects always restrict the mass flux in the reservoir and therefore smaller pressure drops were observed in the reservoir; the effect being more significant for large values of permeability.

7. For tight reservoirs, the inclusion of slippage in the presence of inertial effect shows that these two effects do not tend to cancel each other, but rather a larger pressure drop was observed when slippage was considered.

### 7.3 Recommendations

The following recommendations are made:

1. A modification in the term including the slippage coefficient should be attempted, since this parameter had the largest variation. This might be achieved by modifying the assumptions made by Klinkenberg (52) when developing the slippage term.
2. Further experimental data for linear gas flow taken under conditions similar to those given in runs 5 and 6 for radial flow, could be used in order to verify in a more rigorous way the absolute performance of the models for steady gas flow.
3. An expression for the expected likelihood of the models developed assuming an unknown error variance  $\sigma^2$ , could be used in order to be more realistic. In this case, the posterior parameter-variance joint distribution is approximately multivariate normal-gamma-2.
4. A relationship between rock compressibility and porosity should be developed so that the assumption of constant rock compressibility could be dropped.
5. The subroutine "FPRESS" which calculates nitrogen properties as a function of pressure should be extended so as to include an alternative to evaluate natural gas properties. This can be done using correlations already available in the literature.

6. The numerical results obtained for the transient radial flow should be compared with experimental observations so that the accuracy with which the model predicts actual flow could be known.



## NOMENCLATURE

- A - a symmetric and positive definitive matrix ( $q \times q$ )  
defined as  $\underline{\underline{X}}^T \underline{\underline{X}}$  .
- |A| - determinant of A.
- A\* - group of variables defined as  $\mu_{avg} T_{avg} Z_{avg}$  in  
the radial case, and  $\mu_{avg} T_{avg} Z_{avg}/g_c$  in the  
linear case.
- A - cross sectional area.
- a - parameter vector.
- $\hat{a}$  - Parameter vector which gives  $\hat{S}^2$ .
- b - slippage coefficient.
- C - variance-covariance matrix.
- $c_g$  - gas compressibility.
- $c_r$  - rock compressibility.
- $D_1$  - group of variables defined as  $712.0 \mu_{avg} T_{avg} Z_{avg} Q_g/h$
- $D_2$  - group of variables defined as  $1.5801 \times 10^{-12} \times$   
 $G T_{avg} Z_{avg} Q_g^2/h^2$ .
- $\bar{F}_w$  - dimensionless mass flux at the wellbore.
- $f'(\underline{a})$  - prior parameter density.
- $f''(\underline{a})$  - posterior parameter density.
- G - gas specific gravity.
- $g_c$  - conversion factor.

- $h$  - reservoir thickness  
 $k$  - absolute rock permeability  
 $\langle L \rangle$  - expected likelihood of  $y$  given the model is true  
 $L$  - core length  
 $\ln$  - natural logarithm  
 $\underline{M}$  - coefficient matrix after discretizing the parabolic equation  
 $M$  - gas molecular weight  
 $n$  - number of data points  
 $p$  - pressure  
 $p_f$  - initial (formation) reservoir pressure  
 $\bar{p}$  - dimensionless pressure  
 $p_1, p_e$  - external or inlet pressure  
 $p_2, p_w$  - wellbore or outlet pressure  
 $p'$  - prior probability of a model being true  
 $p''$  - posterior probability of a model being true  
 $Q_g$  - gas flow rate  
 $q$  { - number of parameters in a given model  
- seepage or superficial velocity  
 $\underline{R}$  - correlation matrix  
 $R$  - universal gas constant

- $r$  - independent variable for radial flow  
 $\bar{r}$  - dimensionless radial distance  
 $r_e$  - external radius  
 $r_w$  - wellbore radius  
 $\hat{S}^2$  - the minimum sum of the squares of the errors  
 $T$  - temperature  
 $t$  - time  
 $\bar{t}$  - dimensionless time  
 $\bar{u}$  - dimensionless pressure-squared  
 $\bar{u}_r$  - equivalent to  $\partial\bar{u}/\partial\bar{r}$   
 $W$  - group of variables defined as  $\omega/\mu_{avg} A$  in the linear gas flow  
 $\underline{X}$  - the design matrix ( $n \times q$ ) with elements defined as partial derivatives of the function with respect to the parameters.  
 $x$  - independent variable  
 $y$  - dependent variable  
 $Z$  - gas compressibility factor

#### Greek Letters

- $\alpha$  - viscous resistance coefficient =  $1/k$ .  
 $\alpha_1$  - group of variables defined as  $M/RT$   
 $\beta$  - inertial coefficient

- $\omega$  - mass rate
- $\sigma^2$  - unbiased error variance
- $\mu$  - gas viscosity
- $\rho$  - gas density
- $\rho_r$  - rock density
- $\phi$  - porosity, fraction
- $\phi( )$  - function

#### Superscripts

- (k) - iterative parameter
- T - transpose of a matrix
- - denotes a dimensionless quantity

#### Subscripts

- avg - average value
- i,l - index for grid points in the  $\bar{r}$  direction
- m - time index
- - denotes matrices or vectors

7

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APPENDIX A

LEAST-SQUARES ESTIMATION OF NON-LINEAR  
PARAMETERS USING MARQUARDT'S ALGORITHM

The computational aspects of finding the least-squares estimation of non-linear parameters can be summarized as follows.

Let the experimental model be

$$y = \phi(\underline{x}, \underline{a}) + \epsilon \quad (A-1)$$

where  $\underline{x}$  is the set of independent variables,  $\{x_1, x_2, \dots, x_r\}$ ,  $\underline{a}$  is the parameter vector,  $[a_1, a_2, \dots, a_q]$  and  $\epsilon$  is an error vector which includes both experimental and modelling errors.

For a given set of data, say  $n$  data points, we define the set

$$\{y_i, x_{1i}, x_{2i}, \dots, x_{ri} : i=1, 2, \dots, n\} .$$

The problem is to find the parameter vector  $\underline{a}$  for which

$$s^2 = \sum_{i=1}^n \{y_i - \phi(\underline{x}_i, \underline{a})\}^2 = \sum_i \epsilon_i^2 \quad (A-2)$$

is a minimum. Let this minimum be  $\hat{s}^2$ . Hence,

$$\hat{s}^2 = \min_{\underline{a}} \sum_i \{y_i - \phi(\underline{x}_i, \underline{a})\}^2 . \quad (A-3)$$

The basis of most methods is some kind of linearization of the problem, that is a succession of linearizations which will hopefully lead us to the true solution.

There are two common approaches on which most algorithms for the least-squares estimation of non-linear parameters are based.

1. The model may be expanded as a Taylor series and corrections to the several parameters calculated at each iteration on the assumption of local linearity. If the expansion is performed about some trial value of  $\underline{a}$ , say  $\underline{a}^{(0)}$ , we can write:

$$\phi^* (\underline{x}_i, \underline{a}^{(0)} + \underline{\delta}_T) = \phi (\underline{x}_i, \underline{a}^{(0)}) + \sum_{j=1}^q \left( \frac{\partial \phi_i}{\partial a_j} \right)_{\underline{a}=\underline{a}^{(0)}} (\delta_T)_j + \dots$$

$$i = 1, 2, \dots, n \quad (\text{A-4})$$

or in matrix notation

$$\underline{\phi}^* = \underline{\phi}_0 + \underline{X} \underline{\delta}_T \quad (\text{A-5})$$

where  $\underline{\phi}^*$  is used to distinguish predictions based upon the linearized model and the actual non-linear model,  $\underline{X}$  is a matrix of partial derivatives with respect to the parameters, called the design matrix. Hence,



$$\underline{X}^{(0)}_{(n \times q)} = \begin{bmatrix} \frac{\partial \phi_1}{\partial a_1} & \dots & \frac{\partial \phi_1}{\partial a_q} \\ \frac{\partial \phi_2}{\partial a_1} & & \frac{\partial \phi_2}{\partial a_q} \\ \vdots & & \vdots \\ \frac{\partial \phi_n}{\partial a_1} & & \frac{\partial \phi_n}{\partial a_q} \end{bmatrix}_{\underline{a}=\underline{a}^{(0)}} \quad (\text{A-6})$$

The correction vector  $\underline{\delta}_T$  appears linearly in equation (A-5). Therefore, it can be found by the standard least-squares method of setting  $(\partial s^{*2} / \partial \delta_T) = 0$ , for all  $j$ .

The vector  $\underline{\delta}_T$  is then found by solving

$$\underline{A} \underline{\delta}_T = \underline{g} \quad (\text{A-7})$$

where  $\underline{A}_{(q \times q)}$  is defined as  $\underline{X}^T \underline{X}$ , and it is a symmetric and positive definite matrix.  $\underline{g}$  is a vector defined as  $\underline{X}^T (\underline{y} - \underline{\phi}^{(0)})$ .  $\underline{\phi}^{(0)}$  is defined as the following vector

$$\underline{\phi}^{(0)} = \begin{bmatrix} \phi(\underline{x}_1, \underline{a}^{(0)}) \\ \phi(\underline{x}_2, \underline{a}^{(0)}) \\ \vdots \\ \phi(\underline{x}_n, \underline{a}^{(0)}) \end{bmatrix}$$

If the function was originally linear, or if the approximation given by the truncated Taylor series is valid, then the least-squares estimators are given by

$$\hat{\underline{a}} = \underline{a}^{(0)} + \hat{\underline{\delta}}_T . \quad (\text{A-8})$$

But because of truncation error, we do not get to the true answers, but hopefully an improved set of values

$$\underline{a}^{(1)} = \underline{a}^{(0)} + \underline{\delta}_T^{(0)} . \quad (\text{A-9})$$

In practice, if the approximation is grossly in error,  $s^2(\underline{a}^{(1)})$  may not show an improvement over  $s^2(\underline{a}^{(0)})$ . This would indicate that the extrapolation was beyond the region where  $\phi$  can be adequately represented by the linearized model in (A-5). One remedy is to modify the step size in  $\underline{a}$  to a value  $k_T \underline{\delta}_T$ ,  $0 < k \leq 1$ , once the direction has been specified by  $\underline{\delta}_T$ . Even so, failure to converge is not uncommon.

2. The other approach is the so-called gradient method; by which we find the slope of the function to be minimized by taking partial derivatives, and then moving in the direction of the negative gradient. Thus, from some current point  $\underline{a}^{(0)}$ , the following is computed:

$$\left\{ \frac{\partial s^2}{\partial a_1}, \frac{\partial s^2}{\partial a_2}, \dots, \frac{\partial s^2}{\partial a_q} \right\}_{\underline{a}=\underline{a}^{(0)}} .$$

Then

$$\underline{\delta}_G^{(0)} = -k_G^{(0)} \left( \frac{\partial s^2}{\partial a_1}, \dots, \frac{\partial s^2}{\partial a_q} \right)_{\underline{a}=\underline{a}^{(0)}}^T \quad (\text{A-10})$$

where  $k_G$  is the step size which indeed has to be controlled carefully once the direction of the correction vector has been established. Even so, convergence is not guaranteed and if it does, slow convergence is the rule rather than the exception.

Marquardt's algorithm (55) generalizes and modifies Equation (A-7) as

$$[\underline{A}^{(0)} + \lambda \underline{I}] \underline{\delta}_0 = \underline{g} \quad (\text{A-11})$$

where  $\lambda \geq 0$ . There are 3 theorems which are proved in the algorithm.

#### Theorem 1

Let  $\underline{\delta}_0$  satisfy Equation (A-11), then  $\underline{\delta}_0$  minimizes  $s^{*2}$  on the sphere whose radius  $||\underline{\delta}||$  is equal to  $||\underline{\delta}_0||$ . The asterisk refers to the linearized model.

#### Theorem 2

Let  $\underline{\delta}_0(\lambda)$  be the solution of Equation (A-11) for a given value of  $\lambda$ . Then  $||\underline{\delta}_0(\lambda)||$  is a continuous decreasing function of  $\lambda$  such that as  $\lambda \rightarrow \infty$ ,  $||\underline{\delta}_0(\lambda)||^2 \rightarrow 0$ .

#### Theorem 3

Let  $\gamma$  be the angle between  $\underline{\delta}_0$  and  $\underline{\delta}_G$ . Then  $\gamma$  is a continuous monotone decreasing function of  $\lambda$  such that

as  $\lambda \rightarrow \infty$ ,  $\gamma \rightarrow 0$ . Since  $\underline{\delta}_G$  is independent of  $\lambda$ , it follows that  $\underline{\delta}_O$  rotates towards  $\underline{\delta}_G$  as  $\lambda \rightarrow \infty$ .

In order to improve the numerical aspects of computing procedures, Marquardt suggests that the  $\underline{A}$  matrix should be normalized as follows. Let  $\underline{A}^{(o)} = \{a_{ij}^{(o)}\}$  and define

$$\underline{D}^{(o)} \equiv \begin{pmatrix} \frac{1}{\sqrt{a_{11}^{(o)}}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{a_{22}^{(o)}}} & \\ & & \ddots \\ 0 & & & \frac{1}{\sqrt{a_{qq}^{(o)}}} \end{pmatrix} \quad (\text{A-12})$$

Hence, we can write

$$\underline{\tilde{\delta}}_T^{(o)} = [\underline{D}^{-1}]^{(o)} \underline{\delta}_T^{(o)} \quad (\text{A-13})$$

$$\underline{\tilde{g}}_T^{(o)} = \underline{D}^{(o)} \underline{g}_T^{(o)} \quad (\text{A-14})$$

$$\underline{\tilde{A}}^{(o)} = \underline{D}^{(o)} \underline{A}^{(o)} \underline{D}^{(o)} \quad (\text{A-15})$$

Then, the correction vector for the Taylor series can be obtained from

$$\underline{\tilde{A}}^{(o)} \underline{\tilde{\delta}}_T^{(o)} = \underline{\tilde{g}}_T^{(o)} \quad (\text{A-16})$$

Similarly, the general Equation (A-11) at the  $r^{\text{th}}$  iteration is written as

$$(\underline{\tilde{A}}(r) + \lambda(r) \underline{I}) \underline{\tilde{\delta}}(r) = \underline{\tilde{g}}(r) . \quad (\text{A-17})$$

After solving for  $\underline{\tilde{\delta}}^{(0)}$ , the actual correction vector is obtained from

$$\underline{\delta}_T^{(0)} = \underline{D}^{(0)} \underline{\tilde{\delta}}^{(0)} \quad (\text{A-18})$$

and

$$\underline{a}^{(1)} = \underline{a}^{(0)} + \underline{\delta}_T^{(0)} . \quad (\text{A-19})$$

Conceptually, we see that Marquardt's method reduces to Taylor's method (sometimes called Gauss-Newton) as  $\lambda \rightarrow 0$ , and to the gradient method as  $\lambda \rightarrow \infty$ . A finite value of  $\lambda$  interpolates between the two. Computationally, we use a small value of  $\lambda$  when we are at a point such that Gauss-Newton converges, since the method gives rapid convergence when it does converge. We may have to use a large value of  $\lambda$  in the first few steps to make certain that we are going in the right direction. It follows from the theorem, that unless the initial guess  $\underline{a}^{(0)}$ , is at a minimum, there must exist a  $\lambda$  such that  $s^{2(1)} < s^{2(0)}$ .

The algorithm is summarized as follows:

1. Choose  $v > 1$ ; a recommended choice is  $v = 10.0$ .
2. Let  $\lambda^{(r)}$  denote the value of  $\lambda$  at the  $r^{\text{th}}$  iteration. To get started, we might choose  $\lambda^{(0)} = 0.01$ .
3. Compute  $s^2(\lambda^{(r)})$  say  $s^2(\lambda^{(r)}/v)$ 
  - a) If  $s^2(\frac{\lambda^{(r)}}{v}) \leq s^2(\lambda^{(r)})$ , let  $\lambda^{(r+1)} = \frac{\lambda^{(r)}}{v}$
  - b) If  $s^2(\frac{\lambda^{(r)}}{v}) > s^2(\lambda^{(r)})$ , let  $\lambda^{(r+1)} = \lambda^{(r)}$
  - c) If both  $s^2(\lambda^{(r)})$  and  $s^2(\frac{\lambda^{(r)}}{v})$  are greater than  $s^2(\lambda^{(r)})$  increase  $\lambda$  by successive multiplication by  $v$  until  $s^2(\lambda^{(r)} v^\omega) \leq s^2(\lambda^{(r)})$  where  $\omega$  is the smallest positive integer that will satisfy this requirement.
4. The criterion for convergence is given by

$$\frac{|\delta_j^{(r)}|}{\tau + |a_j^{(r)}|} < \tilde{\epsilon} .$$

for all  $j$  and for some suitable  $\tilde{\epsilon} > 0$ , say  $10^{-5}$  and some suitably  $\tau$ , say  $10^{-3}$ .

For cases where the parameters are highly correlated ( $> 0.99$ ) it can happen that  $\lambda$  will be increased to unreasonably large values, therefore it is suggested that an improved parameter vector be found using the following expression

$$\underline{a}^{(r+1)} = \underline{a}^{(r)} + k^{(r)} \underline{\delta}^{(r)} . \quad (\text{A-20})$$

where  $k^{(r)}$  is equal to 1 if  $\gamma^{(r)} \geq \gamma_0$ , otherwise  $k^{(r)}$  should be sufficiently small so that  $s^{2(r+1)} < s^{2(r)}$ . This can always be done since  $\gamma^{(r)} < \gamma_0 < \pi/2$ . A suitable choice for the criterion angle is  $\gamma_0 = \pi/4$ .

A copy of the computer program using Marquardt's algorithm including the feature for parameters which are highly correlated is given in the following pages. The program is for the case of steady radial gas flow, and it can handle two- and three-parameter models.

## MAIN LINE

```

C *****
C
C STEADY ISOTHERMAL RADIAL GAS FLOW
C
C NON-LINEAR PARAMETER ESTIMATION FOR
C INTEGRATED FORMS
C
C THIS PROGRAM COMPUTES THE LEAST SQUARE ESTIMATES OF
C NON-LINEAR PARAMETERS. FOR A NORMALLY DISTRIBUTED SET
C OF INDEPENDENT AND DEPENDENT VARIABLES THESE
C PARAMETERS ARE ALSO THE MAXIMUM LIKELIHOOD ESTIMATES.
C THE PROGRAM IS GENERAL BUT REQUIRES THE EXPLICIT
C DEFINITION OF THE OBJECTIVE FUNCTION AND OF ITS
C DERIVATIVES WITH RESPECT TO THE PARAMETERS. THIS
C PROGRAM ALSO COMPUTES THE RELIABILITY OF THE
C PARAMETERS AND THE EXPECTED LIKELIHOOD OF THE MODEL.
C IT ALSO HANDLES CASES WHERE THE PARAMETERS ARE HIGHLY
C CORRELATED.
C
C INPUT DATA
C
C ICONT - CODE TO PERFORM PARAMETER ESTIMATION
C         ICONT=0 PARAMETERS ARE ESTIMATED FOR
C           TWO AND THREE PARAMETER MODELS
C         ICONT=1 PARAMETER ESTIMATION IS ONLY
C           DONE FOR A THREE-PARAMETER MODEL
C
C M - AN INTEGER TO INDICATE NUMBER OF
C     PARAMETERS
C
C N - AN INTEGER TO INDICATE NUMBER OF
C     DATA POINTS
C
C B - INITIAL GUESS OF THE PARAMETER VECTOR
C     B(1) PERMEABILITY, MILLIDARCIES
C     B(2) SLIPPAGE COEFF., PSIA
C     B(3) INERTIAL COEFF., 1/FT
C
C LMD1 - STARTING VALUE OF LAMBDA
C
C W - A CONSTANT WHICH MAY BE USED TO
C     INCREASE THE RADIUS OF THE SPHERE
C     WHERE A SEARCH IS BEING CONDUCTED
C
C T - A CONSTANT CONSIDERED IN THE EXPRESSION
C     FOR CONVERGENCE
C
C RE - EXTERNAL RADIUS, FT.
C
C RW - WELLBORE RADIUS, FT.
C
C H - THICKNESS OF ROCK SAMPLE, FT.
C
C PW - SET OF PRESSURES AT RW, PSIA
C
C PE - SET OF PRESSURES AT RE, PSIA
C
C ZZ - SET OF VARIABLES DEFINED FOR EACH
C     RUN AS THE PRODUCT OF VISCOSITY,
C     TEMPERATURE, AND COMPRESSIBILITY
C     FACTOR

```



## MAIN LINE ... (CONT'D)

VIS - SET OF AVERAGE VALUES OF VISCOSITY  
FOR EACH RUN, CP.  
Q - SET OF VALUES FOR GAS FLOW RATE,  
MSCF/DAY

## REMARKS

1. OBJECTIVE FUNCTION AND ITS DERIVATIVES WITH RESPECT TO THE PARAMETERS ARE EXPLICITLY DEFINED BY FORTRAN FUNCTION STATEMENTS
2. VALUES OF VISCOSITY FOR EACH RUN WERE OBTAINED AT THE ARITHMETIC MEAN TEMPERATURE AND PRESSURE OF THE FLOWING GAS USING THE RELATIONSHIP SUGGESTED BY KESTING AND WANG. VALUES FOR THE COMPRESSIBILITY FACTOR WERE DETERMINED BY FIVE-POINT LAGRANGIAN INTERPOLATION (REF. 59)

## SUBROUTINES REQUIRED

## MINV

\*\*\*\*\*

```

REAL*8 P(90,3),PT(3,90),A(3,3),AIN(3,3),AIN1(3,3)
1,AIN2(3,3),AN(3,3),AN1(3,3),AN2(3,3),AA1(3,3),AA2(3,3)
2,B(3),B1(3),B2(3),Y(90),PE(90),PW(90),D1(3),D2(3),
3G(3),F,FK,FBB,FBK,K,BB,BK,AA,QQ,VV,PPE,PPW,ZZ(90),
4Q(90),VIS(90),DN1,DN2,DABS,DSQRT,LMD1,LMD2,NU,SQ,SQ1,
5SQ2,T,DMAX,D(3),DLOG,RE,RW,H,ST(90),SQPE
REAL*8 Z(4,4),ZI(4,4),XX(4,4),C(4,4),L95(4),X(16),
1DD,ERR(90),S(4,4),AN5(4,4),AN6(4,4),ERROR(90),PEE(90)
2,VAR,DEXP,PROD1,PROD2,GSUM,SUM1,SUM2,DNORM1,DNORM2,
3GNORM,COSG1,COS21,COS22
DIMENSION L(4),MV(4)

```

FUNCTION AND DERIVATIVES FOR THE TWO-PARAMETER  
MODEL (K,B)

```

F(K,BK,AA,QQ,VV,PPE,PPW)=(PPW**2+1424.0*AA*QQ*DLOG(RE
#/RW)/(H*K)-
12.0*BK*(PPE-PPW))**.5
FK(K,BK,AA,QQ,VV,PPE,PPW)=0.5*(PPW**2+1424.0*AA*QQ
**DLOG(RE/RW)/(H*
IK)-2.0*BK*(PPE-PPW))**(-0.5)*(-1424.0*AA*QQ*DLOG(RE
#/RW)/(H*K**2))
FBB(K,BK,AA,QQ,VV,PPE,PPW)=0.5*(PPW**2+1424.0*AA*QQ
**DLOG(RE/RW)/

```

## MAIN LINE ... (CONT'D)

$$1(H*K)-2.0*BK*(PPE-PPW)**(-0.5)*(-2.0*(PPE-PPW))$$

C  
C  
C  
FUNCTION AND DERIVATIVES FOR THE TWO-PARAMETER  
MODEL (K,BETA)

$$F(K, BB, AA, QQ, VV, PPE, PPW) = (PPW**2 + 1424.0 * AA * QQ * DLOG(RE$$

$$*/RW) / (H*K) +$$

$$13.1602D-12 * BB * 0.9672 * AA / VV * QQ**2 * (1.0/RW - 1.0/RE) / (H*$$

$$**2)**0.5$$

$$FK(K, BB, AA, QQ, VV, PPE, PPW) = 0.5 * (PPW**2 + 1424.0 * AA * QQ$$

$$**DLOG(RE/RW) / (H*$$

$$1K) + 3.1602D-12 * BB * 0.9672 * AA / VV * QQ**2 * (1.0/RW - 1.0/RE) / (H$$

$$**2)**(-0.5$$

$$2) * (-1424.0 * AA * QQ * DLOG(RE/RW) / (H*K**2))$$

$$FBB(K, BB, AA, QQ, VV, PPE, PPW) = 0.5 * (PPW**2 + 1424.0 * AA * QQ$$

$$**DLOG(RE/RW) /$$

$$1(H*K) + 3.1602D-12 * BB * 0.9672 * AA / VV * (1.0/RW - 1.0/RE) / (H*$$

$$**2)**(-0.5)*$$

$$2(3.1602D-12 * 0.9672 * AA / VV * QQ**2 * (1.0/RW - 1.0/RE) / H**2)$$

C  
C  
C  
FUNCTION AND DERIVATIVES FOR THE THREE-PARAMETER  
SIMPLIFIED MODEL (K,B,BETA)

$$F1(K, BK, BB, AA, QQ, VV, PPE, PPW) = (PPW**2 + 1424.0 * AA * QQ$$

$$**DLOG(RE/RW) / (H*$$

$$1K) + 3.1602D-12 * BB * 0.9672 * AA / VV * QQ**2 * (1.0/RW - 1.0/RE) / H*$$

$$**2 - 2.0 * BK *$$

$$2(PPE - PPW)**0.5$$

$$FK1(K, BK, BB, AA, QQ, VV, PPE, PPW) = 0.5 * (PPW**2 + 1424.0 * AA * QQ$$

$$**DLOG(RE/RW)$$

$$1 / (H*K) + 3.1602D-12 * BB * 0.9672 * AA / VV * QQ**2 * (1.0/RW - 1.0$$

$$*/RE) / H**2 - 2.0 * B$$

$$2K * (PPE - PPW)**(-0.5) * (-1424.0 * AA * QQ * DLOG(RE/RW) / (H*K*$$

$$**2))$$

$$FBK(K, BK, BB, AA, QQ, VV, PPE, PPW) = 0.5 * (PPW**2 + 1424.0 * AA * QQ$$

$$**DLOG(RE/RW)$$

$$1 / (H*K) + 3.1602D-12 * BB * 0.9672 * AA / VV * QQ**2 * (1.0/RW - 1.0$$

$$*/RE) / H**2 - 2.0 *$$

$$2BK * (PPE - PPW)**(-0.5) * (-2.0 * (PPE - PPW))$$

$$FBB1(K, BK, BB, AA, QQ, VV, PPE, PPW) = 0.5 * (PPW**2 + 1424.0 * AA$$

$$**QQ * DLOG(RE/RW$$

$$1) / (H*K) + 3.1602D-12 * BB * 0.9672 * AA / VV * QQ**2 * (1.0/RW - 1.0$$

$$*/RE) / H**2 - 2.0$$

$$2 * BK * (PPE - PPW)**(-0.5) * (3.1602D-12 * 0.9672 * AA / VV * QQ**2$$

$$** (1.0/RW - 1.0$$

$$3/RE) / H**2)$$

## MAIN LINE ... (CONT'D)

```

        READ(5,693) ICONT,M,N
693  FORMAT(3I6)
999  READ(5,100) (B(I),I=1,M)
100  FORMAT(4D16.6)
        WRITE(6,109) (B(I),I=1,M)
109  FORMAT(/,2X,'INITIAL VALUE OF THE PARAMETERS',
1/,4X,4D16.6)
        READ(5,102) LMD1,NU,T
102  FORMAT(3F10.4)
        IF(ICONT.EQ.1) GO TO 965
        READ(5,190) RE,RW,H
190  FORMAT(3D13.5)
        WRITE(6,975) RE,RW,H
975  FORMAT(/,2X,'CORE DIMENSIONS ',/,4X,'RE=',F8.4,/,4X,
1'RW=',F8.4,/,4X,'H=',F8.4)
        READ(5,101) (PE(I),PW(I),ZZ(I),VIS(I),Q(I),I=1,N)
101  FORMAT(5D13.6)
        WRITE(6,987)
987  FORMAT(/,2X,'SET OF VARIABLES')
        WRITE(6,988)
988  FORMAT(/,10X,'PE',11X,'PW',6X,'MU#T#Z',10X,'MU'
1,11X,'Q')
        WRITE(6,105) (PE(I),PW(I),ZZ(I),VIS(I),Q(I),I=1,N)
105  FORMAT(/,4X,5D13.6)
        DO 104 I=1,N
104  Y(I)=PE(I)
        EX=0.
965  ISTORE=1
        LMD2=LMD1/NU
52  IF(ICONT.EQ.1) GO TO 521
        DO 4 I=1,N
        DO 5 J=1,M
        IF(J.GT.1) GO TO 6
        P(I,J)=FK(B(1),B(2),ZZ(I),Q(I),VIS(I),PE(I),PW(I))
        GO TO 5
6  P(I,J)=FBB(B(1),B(2),ZZ(I),Q(I),VIS(I),PE(I),PW(I))
5  CONTINUE
4  CONTINUE
        GO TO 545
521  DO 444 I=1,N
        DO 445 J=1,M
        IF(J.GT.1) GO TO 666
        P(I,J)=FK1(B(1),B(2),B(3),ZZ(I),Q(I),VIS(I),PE(I)
        *,PW(I))
        GO TO 445
666  IF(J.GT.2) GO TO 955
        P(I,J)=FBK(B(1),B(2),B(3),ZZ(I),Q(I),VIS(I),PE(I)
        *,PW(I))
        GO TO 445
955  P(I,J)=FBB1(B(1),B(2),B(3),ZZ(I),Q(I),VIS(I),PE(I)

```

## MAIN LINE ... (CONT'D)

```

      *,PW(I))
445  CONTINUE
444  CONTINUE

C    CONSTRUCT THE TRANSPOSE PT OF P AND COMPUTE A=PT#P
C
545  DO 7 I=1,N
      DO 8 J=1,M
          PT(J,I)=P(I,J)
6    CONTINUE
7    CONTINUE
      DO 9 J=1,M
          DO 10 JJ=1,M
              A(J,JJ)=0.
          DO 11 I=1,N
              A(J,JJ)=A(J,JJ)+PT(J,I)*P(I,JJ)
11   CONTINUE
10   CONTINUE
9    CONTINUE

C    COMPUTE VECTOR G GIVEN BY PT*(Y-F) EVALUATED
C    AT THE PRESENT VALUE OF PARAMETER VECTOR B
      DO 12 J=1,M
          G(J)=0.
          DO 13 I=1,N
              IF(ICONT.EQ.1) GO TO 449
              G(J)=G(J)+PT(J,I)*(Y(I)-F(B(1),B(2),ZZ(I),Q(I),VIS(I)
              *,PE(I),PW(I))
              1)
              GO TO 13
449  G(J)=G(J)+PT(J,I)*(Y(I)-F1(B(1),B(2),B(3),ZZ(I),Q(I)
              *,VIS(I),PE(I),
              1PW(I)))
13   CONTINUE
12   CONTINUE
      IF(ISTORE.EQ.M) GO TO 550

C    NORMALIZE A AND G
      DO 14 I=1,M
          DO 15 J=1,M
              IF(I.EQ.J) GO TO 16
              IF(I.GT.J) GO TO 17
              AN(I,J)=A(I,J)/(DSQRT(A(I,I))*DSQRT(A(J,J)))
              GO TO 15
16   AN(I,J)=1.
              GO TO 15
17   AN(I,J)=AN(J,I)

```

## MAIN LINE ... (CONT'D)

```

15  CONTINUE
    G(I)=G(I)/DSQRT(A(I,I))
14  CONTINUE

C   COMPUTE AN+LMD*I FOR BOTH VALUES OF LAMBDA
C

42  DO 21 I=1,M
    DO 22 J=1,M
    IF(I.LT.J.OR.I.GT.J) GO TO 23
    AN1(I,J)=AN(I,J)+LMD1
    AN2(I,J)=AN(I,J)+LMD2
    GO TO 22
23  AN1(I,J)=AN(I,J)
    AN2(I,J)=AN(I,J)
22  CONTINUE
21  CONTINUE
    DO 976 I=1,M
    DO 976 J=1,M
    AN5(I,J)=AN1(I,J)
976 AN6(I,J)=AN2(I,J)

C   SINCE (AN+LMD*I) IS STILL SYMMETRIC AND POSITIVE
C   DEFINITE ONE CAN USE BROWN'S TECHNIQUE TO FIND THE
C   INVERSE
C

24  INDEX=1
    DMAX=0.

C   SELECT THE ALGEBRAICALLY LARGEST DIAGONAL ELEMENT
C

    DO 25 I=1,M
    IF(AN1(I,I).LT.DMAX) GO TO 25
    DMAX=AN1(I,I)
    II=I
25  CONTINUE
    AA1(II,II)=-1./AN1(II,II)
    AA2(II,II)=-1./AN2(II,II)

C   ALTER THE II(TH) ROW
C

    DO 18 J=1,M
    IF(II.EQ.J) GO TO 18
    AA1(II,J)=-AN1(II,J)*AA1(II,II)
    AA2(II,J)=-AN2(II,J)*AA2(II,II)

C   USE SYMMETRY TO GET THE SYMMETRIC PART OF THE II(TH)

```

## MAIN LINE ... (CONT'D)

```

C     COLUMN
C
      AA1(J,II)=AA1(II,J)
      AA2(J,II)=AA2(II,J)
18    CONTINUE

C     ALTER ALL THE OTHER ROWS IN THE UPPER TRIANGLE OF THE
C     MATRIX
C
      DO 19 I=1,M
      IF(I.EQ.II) GO TO 19
      DO 20 J=1,M
      IF(J.EQ.II) GO TO 20
      IF(J.LT.I) GO TO 20
      AA1(I,J)=AN1(I,J)-AA1(I,II)*AN1(J,II)
      AA2(I,J)=AN2(I,J)-AA2(I,II)*AN2(J,II)
20    CONTINUE
19    CONTINUE

C     USE SYMMETRY TO GENERATE THE LOWER PART OF THE MATRIX
C
      DO 26 I=1,M
      IF(I.EQ.II) GO TO 26
      DO 27 J=1,M
      IF(J.EQ.II) GO TO 27
      IF(J.GE.I) GO TO 27
      AA1(I,J)=AA1(J,I)
      AA2(I,J)=AA2(J,I)
27    CONTINUE
26    CONTINUE
      IF(INDEX.EQ.M) GO TO 28
      INDEX=INDEX+1

C     RESET AN1=AA1 AND AN2=AA2
C
      DO 29 I=1,M
      DO 30 J=1,M
      AN1(I,J)=AA1(I,J)
      AN2(I,J)=AA2(I,J)
30    CONTINUE
29    CONTINUE
      GO TO 24
28    DO 31 I=1,M
      DO 32 J=1,M
      AIN1(I,J)=-AA1(I,J)
      AIN2(I,J)=-AA2(I,J)

```

## MAIN LINE ... (CONT'D)

```

32 CONTINUE
31 CONTINUE

C COMPUTE NORMALIZED DELTAS USING THE INVERSES AIN1
C AND AIN2
C

DO 33 I=1,M
D1(I)=0.
D2(I)=0.
DO 34 J=1,M
D1(I)=D1(I)+AIN1(I,J)*G(J)
D2(I)=D2(I)+AIN2(I,J)*G(J)
34 CONTINUE
33 CONTINUE

C OBTAIN ACTUAL DELTAS

DO 35 I=1,M
D1(I)=D1(I)/DSQRT(A(I,I))
D2(I)=D2(I)/DSQRT(A(I,I))
G(I)=G(I)*DSQRT(A(I,I))
35 CONTINUE

C OBTAIN THE SUM OF SQUARES OF THE ERRORS
C AT PRESENT VALUE OF THE PARAMETER VECTOR
C

SQ=0.
DO 36 I=1,N
IF(ICONT.EQ.1) GO TO 366
SQ=SQ+(Y(I)-F(B(1),B(2),ZZ(I),Q(I),VIS(I),PE(I)
*,PW(I)))**2
GO TO 36
366 SQ=SQ+(Y(I)-F1(B(1),B(2),B(3),ZZ(I),Q(I),VIS(I),PE(I)
*,PW(I)))**2
36 CONTINUE

C COMPUTE NEW BETAS CONSIDERING THE ALTERNATIVE
C FOR PARAMETERS HIGHLY CORRELATED
C

PROD1=0.
PROD2=0.
GSUM=0.
SUM1=0.
SUM2=0.
DO 766 I=1,M
SUM1=SUM1+D1(I)**2
SUM2=SUM2+D2(I)**2

```

## MAIN LINE ... (CONT'D)

```

GSUM=GSUM+G(I)**2
DNORM1=DSQRT(SUM1)
DNORM2=DSQRT(SUM2)
GNORM=DSQRT(GSUM)
PROD1=PROD1+D1(I)*G(I)
PROD2=PROD2+D2(I)*G(I)
766 CONTINUE
COSG1=2.0**0.5/2.0
COSG21=PROD1/(DNORM1*GNORM)
COSG22=PROD2/(DNORM2*GNORM)
DO 37 I=1,M
B1(I)=B(I)+D1(I)
B2(I)=B(I)+D2(I)
37 CONTINUE
768 SQ1=0.
SQ2=0.
DO 38 I=1,N
IF(ICONT.EQ.1) GO TO 388
SQ1=SQ1+(Y(I)-F(B1(1),B1(2),ZZ(I),Q(I),VIS(I),PE(I)
*,PW(I)))**2
SQ2=SQ2+(Y(I)-F(B2(1),B2(2),ZZ(I),Q(I),VIS(I),PE(I)
*,PW(I)))**2
GO TO 38
388 SQ1=SQ1+(Y(I)-F1(B1(1),B1(2),B1(3),ZZ(I),Q(I),VIS(I)
*,PE(I),PW(I)))
1**2
SQ2=SQ2+(Y(I)-F1(B2(1),B2(2),B2(3),ZZ(I),Q(I),VIS(I)
*,PE(I),PW(I)))
1**2
38 CONTINUE
C CHECK FOR NEXT VALUES OF LMD1 AND LMD2
C
IF(SQ2.LE.SQ) GO TO 39
IF(SQ2.GT.SQ.AND.SQ1.LE.SQ) GO TO 40
IF(SQ2.GT.SQ.AND.SQ1.GT.SQ) GO TO 41
39 LMD1=LMD2
LMD2=LMD1/NU
GO TO 43
41 IF(COSG21.LT.COSG1) GO TO 767
LMD2=NU*LMD1
LMD1=NU*LMD2
EX=EX+2.
GO TO 42
767 B1(I)=B(I)+0.4D 00*D1(I)
B2(I)=B(I)+0.4D 00*D2(I)
GO TO 768
40 DO 44 I=1,M
D(I)=D1(I)

```



## MAIN LINE ... (CONT'D)

```

44  CONTINUE
    GO TO 46

C   TEST FOR CONVERGENCE
C

43  DO 45 I=1,M
    D(I)=D2(I)
45  CONTINUE
46  DO 47 I=1,M
    DN1=T+DABS(B(I))
    DN2=DABS(D(I))
    IF((DN2/DN1).GT.1.0D-06) GO TO 48
    ISTORE=I
47  CONTINUE
48  IF(SQ2.LE.SQ) GO TO 49
    DO 50 I=1,M
    B(I)=B1(I)
50  CONTINUE
    GO TO 52
49  DO 53 I=1,M
    B(I)=B2(I)
53  CONTINUE
    GO TO 52
550 IF(ICONT.EQ.1) GO TO 250
    WRITE(6,690)
690  FORMAT(1H1,/,20X,'THE SOLUTION FOR THE 2 PARAMETERS
    * SIMPLIFIED MO
    IDEL IS GIVEN BELOW')
    WRITE(6,655) (B(I),I=1,M)
655  FORMAT(/,15X,'LEAST SQUARE PARAMETERS ESTIMATES ARE
    * K=',D15.6,15
    1X,'BETA=',D15.6)
    GO TO 656
250  WRITE(6,60)
60  FORMAT(1H1,/,20X,'THE SOLUTION FOR THE 3-PARAMETERS
    * SIMPLIFIED MO
    IDEL IS GIVEN BELOW')
    WRITE(6,65) (B(I),I=1,M)
65  FORMAT(/,15X,'LEAST SQUARE PARAMETER ESTIMATES
    * ARE K=',D15.6,
    15X,'B=',D15.6,5X,'BETA=',D15.6)
656  SQPE=0.
    DO 199 I=1,N
    IF(ICONT.EQ.1) GO TO 1990
    ST(I)=F(B(1),B(2),ZZ(I),Q(I),VIS(I),PE(I),PW(I))
    GO TO 657
1990 ST(I)=F1(B(1),B(2),B(3),ZZ(I),Q(I),VIS(I),PE(I),PW(I))
657  PEE(I)=ST(I)
    ERROR(I)=PE(I)-PEE(I)

```

## MAIN LINE ... (CONT'D)

```

SQPE=SQPE+ERROR(I)**2
199 CONTINUE
WRITE(6,805)
805 FORMAT(///,5X,'OBSERVED PE',4X,'CALCULATED PE',6X
*,'ERROR',14X,'THE
DESIGN MATRIX P')
DO 809 I=1,N
809 WRITE(6,803) PE(I),PEE(I),ERROR(I),(P(I,J),J=1,M)
803 FORMAT(/,3X,6D15.6)
VAR=SQPE/(N-M)
VAR1=SQ/(N-M)
STDV=VAR**0.5
WRITE(6,8000) SQPE, SQ
8000 FORMAT(//,2X,'THE SUM OF THE SQUARE OF THE ERRORS FOR
* PREDICTING P
IE IS ', 2D16.8)
WRITE(6,91) VAR
91 FORMAT(//,2X,'VARIANCE=',F16.4)
WRITE(6,68) STDV
68 FORMAT(//,2X,'STANDARD DEVIATION=',F10.4)
DO 907 I=1,N
907 ERR(I)=Y(I)-ST(I)
WRITE(6,74)
74 FORMAT(1H1,//,20X,'THE A-MATRIX =PT*P IS AS FOLLOWS')
DO 75 I=1,M
WRITE(6,76)(A(I,J),J=1,M)
76 FORMAT(//,10X,4D22.15)
75 CONTINUE
JJ=0
DO 500 J=1,M
DO 500 I=1,M
JJ=JJ+1
500 X(JJ)=A(I,J)
IF(ICONT.EQ.1) GO TO 904
CALL MINV(X,2,DD,L,MV)
GO TO 906
904 CALL MINV(X,3,DD,L,MV)
906 WRITE(6,903) DD
903 FORMAT(//,8X,'THE DETERMINANT OF A-MATRIX IS',D15.6)
IF(ICONT.EQ.1) GO TO 7011
KK=0
DO 2000 J=1,M
DO 2000 I=1,M
KK=KK+1
2000 S(I,J)=X(KK)
GO TO 7022
7011 KK=0
DO 7012 J=1,M
DO 7012 I=1,M
KK=KK+1

```

## MAIN LINE ...(CONT'D)

```

7012 S(I,J)=X(KK)
7022 DO 701 J=1,M
      DO 701 I=1,M
701  C(I,J)=VAR1*S(I,J)
      WRITE(6,702)
702  FORMAT(/,20X,'THE VARIANCE COVARIANCE MATRIX IS GIVEN
      * BELOW')
      DO 703 I=1,M
703  WRITE(6,704) (C(I,J),J=1,M)
704  FORMAT(/,6X,4(3X,D15.6))
      IF(ICONT.EQ.1) GO TO 1077
      WRITE(6,107) (B(I),I=1,M)
107  FORMAT(/,2X,'PARAMETERS=',12X,'K=',D15.6,3X,'BETA='
      *,D15.6)
      GO TO 1080
1077 WRITE(6,1078) (B(I),I=1,M)
1078 FORMAT(/,2X,'PARAMETERS=',12X,'K=',D15.6,3X,'B='
      *,D15.6,3X,'BETA='
      1,D15.6)
1080 DO 108 I=1,M
      108  L95(I)=2.0*C(I,I)**0.5
          WRITE(6,209) (L95(I),I=1,M)
209  FORMAT(/,2X,'95 PERCENT CONFIDENCE',4X,D15.6,5X,D15.6
      *,6X,D15.6)

```

## C CALCULATING THE LIKELIHOOD OF THE MODELS

```

      XLIKE=(2.0*3.14159*VAR)**(-N/2)*DEXP(-SQPE/(2.0*VAR))
      **0.5
      1**(M/2)
      IF(ICONT.EQ.1) GO TO 8766
      WRITE(6,8755) XLIKE
8755 FORMAT(/,2X,'LIKELIHOOD FOR THE 2 PARAMETERS MODEL
      * IS ',E15.6)
      GO TO 8799
8766 WRITE(6,876) XLIKE
876  FORMAT(/,2X,'LIKELIHOOD FOR THE 3 PARAMETERS MODEL
      * IS ',E15.6)
8799 ICONT=ICONT+1
      WRITE(6,8800) LMD1,LMD2
8800 FORMAT(/,2X,'FINAL VALUES OF LAMBDA=',2D15.6)
      IF(ICONT.GT.1) GO TO 77
      M=3
      GO TO 999
77  STOP
      END

```

## SUBROUTINE MINV

```

C *****
C THIS SUBROUTINE FINDS THE INVERSE OF MATRIX A AND ITS
C DETERMINANT D.
C
C REMARKS
C
C 1. MATRIX A MUST BE GIVEN AS AN INPUT VECTOR
C COLUMNWISE
C 2. MATRIX A IS DESTROYED AND REPLACED BY THE
C INVERSE
C *****

```

```

SUBROUTINE MINV(A,N,D,L,M)

```

```

REAL*8 A(16),D,BIGA,HOLD,DABS
DIMENSION L(4),M(4)

```

```

C SEARCH FOR LARGEST ELEMENT
C

```

```

D=1.
NK=-N
DO 80 K=1,N
NK=NK+N
L(K)=K
M(K)=K
KK=NK+K
BIGA=A(KK)
DO 20 J=K,N
IZ=N*(J-1)
DO 20 I=K,N
IJ=IZ+I
10 IF(DABS(BIGA)-DABS(A(IJ))) 15,20,20
15 BIGA=A(IJ)
L(K)=I
M(K)=J
20 CONTINUE

```

```

C INTERCHANGE ROWS
C

```

```

J=L(K)
IF(J-K) 35,35,25
25 KI=K-N
DO 30 I=1,N
KI=KI+N

```

## SUBROUTINE MINV ... (CONT'D)

```
HOLD=-A(KI)
JI=KI-K+J
A(KI)=A(JI)
30 A(JI)=HOLD

C INTERCHANGE COLUMNS
C

35 I=M(K)
IF(I-K) 45, 45, 38
38 JP=N*(I-1)
DO 40 J=1,N
JK=NK+J
JI=JP+J
HOLD=-A(JK)
A(JK)=A(JI)
40 A(JI)=HOLD

C DIVIDE COLUMN BY MINUS PIVOT(VALUE OF PIVOT ELEMENT IS
C CONTAINED IN BIGA)
C

45 IF(BIGA) 48,46,48
46 D=0.
RETURN
48 DO 55 I=1,N
IF(I-K) 50,55,50
50 IK=NK+I
A(IK)=A(IK)/(-BIGA)
55 CONTINUE

C REDUCE MATRIX
C

DO 65 I=1,N
IK=NK+I
HOLD=A(IK)
IJ=I-N
DO 65 J=1,N
IJ=IJ+N
IF(I-K) 60,65,60
60 IF(J-K) 62,65,62
62 KJ=IJ-I+K
A(IJ)=HOLD*A(KJ)+A(IJ)
65 CONTINUE

C DIVIDE ROW BY PIVOT
C

KJ=K-N
```

## SUBROUTINE MINV ... (CONT'D)

```
DO 75 J=1,N
  KJ=KJ+N
  IF(J-K) 70,75,70
70  A(KJ)=A(KJ)/BIGA
75  CONTINUE

C    PRODUCT OF PIVOTS
C

  D=D*BIGA
  A(KK)=1./BIGA
80  CONTINUE
  K=N
100 K=K-1
  IF(K) 150,150,105
105  I=L(K)
  IF(I-K) 120,120,108
108  JQ=N*(K-1)
  JR=N*(I-1)
  DO 110 J=1,N
  JK=JQ+J
  HOLD=A(JK)
  JI=JR+J
  A(JK)=-A(JI)
110  A(JI)=HOLD
120  J=M(K)
  IF(J-K) 100,100,125
125  KI=K-N
  DO 130 I=1,N
  KI=KI+N
  HOLD=A(KI)
  JI=KI-K+J
  A(KI)=-A(JI)
130  A(JI)=HOLD
  GO TO 100
150  RETURN
  END
```

## APPENDIX B

### SALIENT POINTS OF THE QUASILINEARIZATION AND DATA PERTURBATION TECHNIQUE FOR PARAMETER ESTIMATION IN SYSTEMS OF ORDINARY DIFFERENTIAL EQUATIONS

A very powerful way of attacking the problem of computing the "best fit" parameters for a set of non-linear ordinary differential equations uses an adaptation of the Newton-Raphson-Kantorovich procedure, called quasilinearization, which regards the non-linear problem as a limit of a sequence of linear problems. Starting from an initial trial solution, convergence if it does occur, occurs rapidly; further, convergence is assured if the initial guess is "close enough" to the true solution. The difficulty of making a good initial guess, a serious limitation of the method in the past, has been in principle overcome by the algorithm presented in References (23, 24). When a given vector may not be within the domain of convergence of the original problem, it must be within the domain of convergence of some other derived problem. The latter may then be perturbed towards the original problem in a finite number of steps. In this case, new data points are derived; these are subsequently adjusted until they coincide with the original data.

The computer program given in Reference (23) has been slightly modified in order to compute the parameters working with the rigorous mathematical model for steady radial gas flow. It was also extended to calculate the reliability of the parameters and the expected likelihood of the model. A brief theory and a summary of the algorithm is given below.

Assume that the behavior of a physical system can be adequately approximated by the following vector differential equation

$$\frac{d\underline{y}}{dr} = \underline{f}(\underline{y}, \underline{a}, r) \quad (B-1)$$

where

$\underline{y}$  = vector of state variables of order  $s$ ;

$\underline{a}$  = vector of constant system parameters of order  $q$ ;

$r$  = independent variable, for example, radial distance.

Suppose that we have  $n$  different boundary conditions -  $\underline{y}_j(r_0)$ ,  $j=1, \dots, n$  - for each of which is available observed data of some or all of the elements of  $\underline{y}$  at some discrete values of  $r$ .

The set of vector differential equations now becomes:



$$\frac{dy_j}{dr} = f_j(y_j, \underline{a}, r) \quad (\text{B-2})$$

B.C.  $y_j(r_0)$  is specified

$$j = 1, \dots, n$$

Let  $\hat{y}_{ijk}$  be the observed value of the  $i^{\text{th}}$  dependent or state variable at a radius  $r_{jk}$  where the latter corresponds to the  $k^{\text{th}}$  discrete value of the radial distance for the  $j^{\text{th}}$  initial condition.

By taking the parameter vector  $\underline{a}$  to be the solution of

$$\frac{d\underline{a}}{dr} = \underline{0} \quad , \quad (\text{B-3})$$

the problem can then be formulated as a boundary value problem because of the missing boundary condition  $\underline{a}(r_0)$ . Hence, the system becomes

$$\frac{dy_j}{dr} = f_j(y_j, \underline{a}, r) \quad j=1, \dots, n \quad (\text{B-4})$$

$$\frac{d\underline{a}}{dr} = \underline{0} \quad \underline{0} \text{ is of order } q \times 1$$

B.C.  $y_j(r_0)$  is specified for  $j=1, 2, \dots, n$

$\underline{a}(r_0)$  is unknown.

Quasilinearization treats the non-linear problem as a limit of a sequence of linear problems, each of which

can then be handled by conventional means. Therefore, if

$$\lim_{k \rightarrow \infty} \underline{a}^{(k)} = \underline{a} \quad (\text{B-5})$$

where  $\underline{a}^{(k)}$  represents the solution of the  $k^{\text{th}}$  linear problem in the sequence and  $\underline{a}$  is the true solution, then hopefully a feasible method of approach would be obtained.

Let

$$\underline{z} \equiv \begin{pmatrix} \underline{y}_1 \\ \underline{y}_2 \\ \vdots \\ \underline{y}_n \\ \underline{a} \end{pmatrix}$$

and

$$\underline{g}(\underline{z}, r) \equiv \begin{pmatrix} \underline{f}_1(\underline{z}_1, \underline{a}, r) \\ \underline{f}_2(\underline{z}_2, \underline{a}, r) \\ \vdots \\ \underline{f}_n(\underline{z}_n, \underline{a}, r) \\ \underline{0} \end{pmatrix}$$

Then, the system (B-4) becomes:

$$\frac{d\underline{z}}{dr} = \underline{g}(\underline{z}, r) \quad (\text{B-6})$$

where

$\underline{z}$  is a vector of order  $\omega \times 1$

and

$$\omega = ns + q .$$

For the  $(k+1)^{\text{th}}$  iteration,

$$\frac{dz^{(k+1)}}{dr} = g^{(k+1)}(z, r) \equiv \underline{g}^{(k+1)} . \quad (\text{B-7})$$

Using a Taylor's series expansion truncated after the first derivative,  $\underline{g}^{(k+1)}$  is approximated by:

$$\underline{g}^{(k+1)} = \underline{g}^{(k)} + \underline{J}^{(k)} [\underline{z}^{(k+1)} - \underline{z}^{(k)}]$$

or

$$\underline{g}^{(k+1)} = \underline{J}^{(k)} \underline{z}^{(k+1)} + [\underline{g}^{(k)} - \underline{J}^{(k)} \underline{z}^{(k)}] \quad (\text{B-8})$$

where  $\underline{J}^{(k)}(r)$  is a Jacobian matrix of partial derivatives, that is

$$\underline{J}^{(k)} = \begin{pmatrix} \frac{\partial g_1}{\partial z_1} & \frac{\partial g_1}{\partial z_2} & \cdots & \frac{\partial g_1}{\partial z_\omega} \\ \frac{\partial g_2}{\partial z_1} & \frac{\partial g_2}{\partial z_2} & \cdots & \frac{\partial g_2}{\partial z_\omega} \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ \frac{\partial g_\omega}{\partial z_1} & \frac{\partial g_\omega}{\partial z_2} & \vdots & \vdots & \frac{\partial g_\omega}{\partial z_\omega} \end{pmatrix}^{(k)} \quad (\text{B-9})$$

Replacing Eq. (B-8) in (B-7) the following recursive equation is obtained

$$\frac{d\underline{z}^{(k+1)}}{dr} = \underline{J}^{(k)} \underline{z}^{(k+1)} + [\underline{g}^{(k)} - \underline{J}^{(k)} \underline{z}^{(k)}] \quad (\text{B-10})$$

Equation (B-10) is now a non-homogeneous linear ordinary differential equation which falls in the category of linear boundary value problem. The general solution may be written as a linear combination of independent solutions corresponding to the complementary homogeneous equation

$$\frac{d\underline{z}^{(k+1)}}{dr} = \underline{J}^{(k)} \underline{z}^{(k+1)} \quad , \quad (\text{B-11})$$

plus one particular solution corresponding to Equation (B-10). Hence,

$$\begin{aligned} \underline{z}^{(k+1)} = c_1^{(k+1)} \underline{z}_1^{(k+1)} + c_2^{(k+1)} \underline{z}_2^{(k+1)} + \dots + c_\omega^{(k+1)} \underline{z}_\omega^{(k+1)} \\ + \underline{u}^{(k+1)} \end{aligned} \quad (\text{B-12})$$

or in matrix notation

$$\underline{z}^{(k+1)}(r) = \underline{Y}^{(k+1)}(r) \underline{c}^{(k+1)} + \underline{u}^{(k+1)}(r) \quad (\text{B-13})$$

where  $\underline{Y}^{(k+1)}(r)$  is the matrix  $(\omega \times \omega)$  of independent solutions with the initial condition  $\underline{Y}^{(k+1)}(r_0) = \underline{I}_{(\omega \times \omega)}$ . The vector  $\underline{u}^{(k+1)}(r)$  is the particular solution with the initial condition  $\underline{u}^{(k+1)}(r_0) = \underline{0}_{(\omega \times 1)}$ . The expression given in Equation (B-12) is in a form where the conventional

least-squares analysis can be applied. The vector  $\underline{c}^{(k+1)}$  represents the coefficients that are to be determined such that the sum of the square of the errors between the observed value and the value predicted by equation (B-2) is minimized. The elements of the parameter vector  $\underline{a}^{(k+1)}$  are contained in  $\underline{c}^{(k+1)}$ . The iterative procedure continues with Equation (B-10) until convergence is reached.

A more efficient computational scheme presented by Donnelly and Quon (24) takes advantage of the structure of the Jacobian  $\underline{J}^{(k)}$  which contains many zero entries.

Writing Equation (B-11) in expanded form we obtain:

$$\frac{d}{dr} \begin{pmatrix} \underline{y}_1 \\ \vdots \\ \underline{y}_j \\ \vdots \\ \underline{y}_n \\ \underline{a} \end{pmatrix}^{(k+1)} = \begin{pmatrix} \underline{J}_{y_1} & \underline{0} & \underline{J}_{a_1} \\ \vdots & \vdots & \vdots \\ \underline{J}_{y_j} & \underline{0} & \underline{J}_{a_j} \\ \vdots & \vdots & \vdots \\ \underline{0} & \underline{J}_{y_n} & \underline{J}_{a_n} \\ \underline{0} & \underline{0} & \underline{0} \end{pmatrix}^{(k)} \begin{pmatrix} \underline{y}_1 \\ \vdots \\ \underline{y}_j \\ \vdots \\ \underline{y}_n \\ \underline{a} \end{pmatrix}^{(k+1)} \quad (\text{B-14})$$

where

$$\underline{J}_{-y_j}^{(k)} = \begin{pmatrix} \frac{\partial f_{ij}}{\partial y_{ij}} & \dots & \frac{\partial f_{ij}}{\partial y_{sj}} \\ \vdots & & \\ \frac{\partial f_{sj}}{\partial y_{ij}} & \dots & \frac{\partial f_{sj}}{\partial y_{sj}} \end{pmatrix}^{(k)} \quad j=1,2,\dots,s \quad (\text{B-15})$$

is a Jacobian matrix of partial derivatives with respect to the state variables and

$$\underline{J}_{-a_j}^{(k)} = \begin{pmatrix} \frac{\partial f_{ij}}{\partial a_1} & \dots & \frac{\partial f_{ij}}{\partial a_q} \\ \vdots & & \\ \frac{\partial f_{sj}}{\partial a_1} & \dots & \frac{\partial f_{sj}}{\partial a_q} \end{pmatrix}^{(k)} \quad j=1,2,\dots,q \quad (\text{B-16})$$

is a Jacobian matrix of partial derivatives with respect to the parameters.

Since each vector  $\underline{y}_j^{(k+1)}$  is coupled only to the parameter vector  $\underline{a}^{(k)}$  and not to any other state variable vector with a different initial condition, the general solution in Equation (B-13) can be written as

$$\begin{pmatrix} \underline{y}_1 \\ \vdots \\ \underline{y}_j \\ \vdots \\ \underline{y}_n \\ \underline{a} \end{pmatrix}^{(k+1)} = \begin{pmatrix} \underline{F}_{y1} & \underline{0} & \underline{G}_{a1} \\ \underline{0} & \underline{F}_{yj} & \underline{G}_{aj} \\ & & \underline{F}_{yn} & \underline{G}_{an} \\ \underline{0} & & \underline{0} & \underline{F}_a \end{pmatrix}^{(k+1)} \begin{pmatrix} \underline{c}_1 \\ \vdots \\ \underline{c}_j \\ \vdots \\ \underline{c}_n \\ \underline{c}_a \end{pmatrix}^{(k+1)} + \begin{pmatrix} \underline{e}_1 \\ \vdots \\ \underline{e}_j \\ \vdots \\ \underline{e}_n \\ \underline{e}_a \end{pmatrix}^{(k+1)} \quad (\text{B-17})$$

where the first term on the right hand side represents the solutions to the homogeneous equation and the last term a particular solution.

The diagonal matrices  $\underline{F}_{-y_j}^{(k+1)}$  ( $j=1,2,\dots,n$ ) are generated from the homogeneous equation

$$\frac{d\underline{F}_{-y_j}^{(k+1)}}{dr} = \underline{J}_{-y_j}^{(k)} \underline{F}_{-y_j}^{(k+1)} \quad j=1,2,\dots,n \quad (\text{B-18})$$

with

$$\underline{F}_{-y_j}^{(k+1)}(r_0) = \underline{I}_{(s \times s)} \quad .$$

The matrix  $\underline{F}_{-a}^{(k+1)}$  is generated from

$$\frac{d\underline{F}_{-a}^{(k+1)}}{dr} = \underline{0} \quad (\text{B-19})$$

with

$$\underline{F}_{-a}^{(k+1)}(r_0) = \underline{I}_{(q \times q)} \quad .$$

The solution to Equation (B-19) is by inspection:

$$\underline{F}_{-a}^{(k+1)}(r) = \underline{I} \quad . \quad (\text{B-20})$$

The matrices  $\underline{G}_{-a_j}$  ( $j=1,2,\dots,n$ ) are generated from

$$\begin{aligned} \frac{d\underline{G}_{-a_j}^{(k+1)}}{dr} &= \underline{J}_{-y_j}^{(k)} \underline{G}_{-a_j}^{(k+1)} + \underline{J}_{-a_j}^{(k)} \underline{F}_{-a}^{(k+1)} \\ &= \underline{J}_{-y_j}^{(k)} \underline{G}_{-a_j}^{(k+1)} + \underline{J}_{-a_j}^{(k)} \end{aligned} \quad (\text{B-21})$$

with

$$\underline{G}_{aj}^{(k+1)}(r_0) = \underline{0}_{(s \times q)} \quad .$$

The particular solution  $\underline{u}$  is generated from the non-homogeneous equation

$$\frac{d\underline{u}^{(k+1)}}{dr} = \underline{J}^{(k)} [\underline{u}^{(k+1)} - \underline{u}^{(k)}] + \underline{q}^{(k)} \quad . \quad (B-22)$$

The general solution in Eq. (B-17) can be written in terms of each state variable vector  $\underline{y}_j^{(k+1)}$  as follows:

$$\underline{y}_j^{(k+1)} = \underline{F}_{y_j}^{(k+1)} \underline{c}_j^{(k+1)} + \underline{G}_{aj}^{(k+1)} \underline{a}^{(k+1)} + \underline{u}_j^{(k+1)} \quad . \quad (B-23)$$

If we define a vector

$$\underline{\alpha}_j^{(k+1)} \equiv \underline{F}_{y_j}^{(k+1)} \underline{c}_j^{(k+1)} + \underline{u}_j^{(k+1)} \quad , \quad (B-24)$$

Equation (B-23) becomes

$$\underline{y}_j^{(k+1)} = \underline{\alpha}_j^{(k+1)} + \underline{G}_{aj}^{(k+1)} \underline{a}^{(k+1)} \quad . \quad (B-25)$$

Moreover, the problem has now been formulated in such a way that

$$\underline{c}_j = \underline{y}_j(r_0) \quad . \quad (B-26)$$

In many problems we can assume that  $\underline{y}_j(r_0)$  is exact, then we have a solution for  $\underline{c}_j$ , and the need for



integrating Equation (B-18) is no longer necessary since  $\underline{\alpha}_j^{(k+1)}$  in Equation (B-24) which is the particular solution starting with the known boundary condition, can be found by means of the following expression:

$$\frac{d\underline{\alpha}_j^{(k+1)}}{dr} = \underline{J}_{\underline{y}_j}^{(k)} [\underline{\alpha}_j^{(k+1)} - \underline{y}_j^{(k)}] - \underline{J}_{\underline{a}_j}^{(k)} \underline{a}^{(k)} + \underline{f}_j^{(k)} \quad (\text{B-27})$$

with

$$\underline{\alpha}_j^{(k+1)}(r_0) = \underline{y}_j(r_0) .$$

Therefore, Equation (B-25) is now expressed linearly in terms of the parameter vector  $\underline{a}^{(k+1)}$  alone and the least-square analysis can be carried on as before.

#### Data Perturbation

The standard procedure is to guess an initial trial vector  $\underline{a}^{(0)}$ . With this, Equation (B-4) can be integrated to obtain a predicted value  $y_{ijk}^{(0)}$  for all points at which observed data  $\hat{y}_{ijk}$  are available. If the initial trial vector  $\underline{a}^{(0)}$  results in a set of  $y_{ijk}^{(0)}$  "close enough" to  $\hat{y}_{ijk}$  then the quasilinearization technique will converge. If not, then the original problem is modified by perturbing the data and using pseudo-boundary conditions in the following way:

$$y_{ijk}^* = y_{ijk}^{(0)} + \hat{R}(\hat{y}_{ijk} - y_{ijk}^{(0)}) \quad 0 < \hat{R} \leq 1.0 \quad . \quad (\text{B-28})$$

The perturbed data thus fall between the observed data and the predicted data with  $y_{ijk}^*$  being the derived  $i^{\text{th}}$  state variable at a distance  $r_{jk}$  where the latter corresponds to the  $k^{\text{th}}$  discrete value of the radial distance for which the state variable was observed for the  $j^{\text{th}}$  initial condition. The choice of  $\hat{R} = 1.0$  forces  $y_{ijk}^*$  to be identical with the observed value  $\hat{y}_{ijk}$  and is equivalent to using quasilinearization directly.

Once the derived problem is solved, the solution  $y^*$  may be used as the initial guess  $y^{(0)}$  and the procedure may be repeated a finite number of steps until  $y^{(0)}$  is sufficiently close to the actual solution so that the actual boundary values  $\hat{y}_{ijk}$  may be used in place of the artificial set of boundary conditions.

A copy of the modified computer program to solve the non-linear ordinary differential equation which represents the rigorous mathematical model for steady radial gas flow, is given in the following pages.

## MAIN LINE

\*\*\*\*\*

STEADY ISOTHERMAL RADIAL GAS FLOW IN POROUS MEDIA

PARAMETER ESTIMATION IN SYSTEM OF FIRST ORDER  
NON-LINEAR DIFFERENTIAL EQUATIONS

THIS PROGRAM CARRIES OUT PARAMETER ESTIMATION FOR A SET OF FIRST ORDER NONLINEAR DIFFERENTIAL EQUATIONS. PHASE I OF THE PROGRAM OBTAINS A GOOD STARTING VALUE FOR THE PARAMETERS THRU DATA PERTURBATION AND PHASE II EMPLOYS QUASILINEARIZATION SCHEME FOR THE FINAL CONVERGENCE. FOURTH ORDER RUNGE-KUTTA IS USED FOR INTEGRATION OF THE MATRIX DIFFERENTIAL EQUATIONS. THE PROGRAM ALSO COMPUTES THE RELIABILITY OF THE PARAMETERS AND THE EXPECTED LIKELIHOOD OF THE MODEL.

## INPUT DATA

NE - NUMBER OF STATE VARIABLES.  
 NK - NUMBER OF PARAMETERS.  
 NDS - NUMBER OF DATA SETS EACH OVER A RANGE OF INDEPENDENT VARIABLE.  
 TC - VALUES OF THE INDEPENDENT VARIABLE AT WHICH DATA IS AVAILABLE. IN THIS CASE, THE EXTERNAL RADIUS RE, FT.  
 NPIJ - NUMBER OF DATA POINTS FOR EACH VALUE OF THE INDEPENDENT VARIABLE  
 XO - INITIAL CONDITIONS FOR THE STATE VARIABLES. IN THIS CASE, THE WELLBORE PRESSURE PW, PSIA  
 XB1 - DATA MATRIX OF VALUES OF THE STATE VARIABLES CORRESPONDING TO THE VALUES OF THE INDEPENDENT VARIABLE TC. IN THIS CASE THE EXTERNAL PRESSURE PE, PSIA  
 RMAX - UPPER LIMIT ON THE ITERATIONS EXPECTED IN THE QUASILINEARIZATION ROUTINE  
 NC - NUMBER OF INTERPOLATED POINTS FOR EACH DATA SET, =600  
 KSI - REFERENCE INDEX FOR THE PARAMETERS E.G. 1,2,...ETC.  
 CHMAX - MAXIMUM CHANGE ALLOWED IN THE PARAMETERS  
 CHMIN - MINIMUM CHANGE ALLOWED IN THE PARAMETERS  
 ERROR - CRITERION FOR CONVERGENCE  
 RMAXI - INITIAL STEP SIZE  
 CO - INITIAL VALUES OF THE PARAMETER VECTOR  
 CO(1) PERMEABILITY, MILLIDARCIES  
 CO(2) SLIPPAGE COEFF., PSIA.  
 CO(3) INERTIAL COEFF., 1/FT.



## MAIN LINE      ... (CONT'D)

```

DIMENSION L(4), MV(4)
COMMON XB1,LTAR,VAR,AMAT
COMMON /AB8/X0,I,NDS/AB3/C,NE,NK,NO/AB4/UB,X,P,H,T
1,NCJ,LL/AB10/TC,XB,ERROR,CHMAX,RMAXI,KSI,NPIJ,NC,NP,
2NPTOT,RMAX,NPTOT,R,J,M1,M2,II,JJ,S,ICHECK/AB4/
3LKJM,LKJI/AB6/CO,ML
  READ(5,1) NE,NK,RMAX,NDS,LL,ICUTM,LKJM
  WRITE(6,9)
9  FORMAT(/,2X,'CONTROL CARD FOR NUMBER OF STATE
  * VARIABLES, PARAMETER
  1S, ETC')
  WRITE(6,1) NE,NK,RMAX,NDS,LL,ICUTM,LKJM
  RMAXS=RMAX
  LKJS=LKJM
  ICUTS=ICUTM
  NO=1
  READ(5,11) (CO(II),II=1,NK)
  WRITE(6,10)
10 FORMAT(/,2X,'INITIAL VALUE OF THE PARAMETERS')
  WRITE(6,11)(CO(II),II=1,NK)
  READ(5,11) ERROR,RMAXI,CHMAX,CHMIN
  WRITE(6,12)
12 FORMAT(/,2X,'CONTROL CARD FOR ERROR CRITERION,INITIAL
  * STEP SIZE')
  WRITE(6,11)ERROR,RMAXI,CHMAX,CHMIN
11  FORMAT(5E16.6)
  NPTOT=0
  DO 101 I=1,NDS
  READ(5,2)(X0(I,J),J=1,NE)
  WRITE(6,3)
3  FORMAT(/,2X,'INITIAL VALUE OF THE STATE VARIABLE')
  WRITE(6,2)(X0(I,J),J=1,NE)
  READ(5,1) NP(I)
  WRITE(6,5)
5  FORMAT(/,2X,'NUMBER OF DATA POINTS IN EACH SET')
  WRITE(6,1)NP(I)
  M1=NP(I)
  READ(5,1) (NPIJ(I,J),J=1,M1)
  WRITE(6,13)
13 FORMAT(/,2X,'NUMBER OF DATA POINTS AT EACH VALUE OF
  * THE INDEPENDEN
  IT VARIABLE')
  WRITE(6,1)(NPIJ(I,J),J=1,M1)
  READ(5,2) (TC(I,J),J=1,M1)
  WRITE(6,90)
90 FORMAT(/,2X,'VALUES OF THE INDEPENDENT VARIABLE AT
  * WHICH VALUES OF
  1THE STATE VARIABLE ARE KNOWN')
  WRITE(6,2)(TC(I,J),J=1,M1)
  READ(5,1) (NC(I,J),J=1,M1)

```

## MAIN LINE      ... (CONT'D)

```

WRITE(6,91)
91  FORMAT(/,2X,'NUMBER OF INTERPOLATED POINTS')
    WRITE(6,1)(NC(I,J),J=1,M1)
    NPITOT(I)=0
    DO 100 J=1,M1
    M2=NPIJ(I,J)
    NPITOT(I)=NPITOT(I)+M2
    READ(5,1) (KSI(I,J,K),K=1,M2)
    WRITE(6,92)
92  FORMAT(/,2X,'INDEX TO CONTROL NUMBER OF STATE
* VARIABLES')
    WRITE(6,1)(KSI(I,J,K),K=1,M2)
    READ(5,2) (XB1(I,J,K),K=1,M2)
    WRITE(6,93)
93  FORMAT(/,2X,'VALUES OF THE STATE VARIABLES AT
* CORRESPONDING VALUES
1 OF INDEPENDENT VARIABLE')
100 WRITE(6,2)(XB1(I,J,K),K=1,M2)
101 NPTOT=NPTOT+NPITOT(I)
    WRITE(6,22) NPTOT
22  FORMAT('OTHE TOTAL NO. OF DATA POINTS IS',I5)
1  FORMAT(16(1X,I4))
2  FORMAT(5(1X,F13.5))
    IF(NPTOT.LE.500)GO TO 120
    WRITE(6,21)
21  FORMAT('ONO. OF DATA POINTD EXCEEDS 500')
    STOP
120 ICUTI=0
114 ICHECK=1
    WRITE(6,18)
18  FORMAT('OBEGIN PHASE I')
    LKJM=LKJS
    ICUTM=ICUTS
    RMAX=RMAXS
110 RP=0.0
    DO 107 I=1,NK
107  C(I)=CO(I)
    SUMB=0.0
    DO 103 I=1,NDS
    T=0.08551
    ML=I
    DO 104 II=1,NE
104  X(II)=XO(I,II)
    M1=NP(I)
    DO 103 J=1,M1
    NCJ=NC(I,J)
    H=(TC(I,J)-T)/NCJ
    CALL GENER(ML)
    M2=NPIJ(I,J)
    DO 103 K=1,M2

```

## MAIN LINE ... (CONT'D)

```

S=KSI(I,J,K)
XBAR(I,J,K)=X(S)
SUMA=ABS(XB1(I,J,K)-XBAR(I,J,K))
SUMB=SUMB+SUMA**2
103 IF(SUMA.GT.RP)RP=SUMA
ICUTI=ICUTI+1
WRITE(6,14) ICUTI
14 FORMAT('0',10X,'BEGIN STEP',I5)
WRITE(6,25) (I,CO(I),I=1,NK)
WRITE(6,19) SUMB
IF(ICUTI.LT.ICUTM) GO TO 119
WRITE(6,20)
20 FORMAT('0STEP COUNT EXCEEDED')
STOP
119 RATIO=1.0-RMAXI/RP
IF(RP.GT.RMAXI) GO TO 111
WRITE(6,17)
17 FORMAT('1BEGIN PHASE II')
RMAX=10
RATIO=0.0
LKJM=IABS(LKJM)
ICUTM=-ICUTM
111 DO 106 I=1,NDS
M1=NP(I)
DO 106 J=1,M1
M2=NPIJ(I,J)
DO 106 K=1,M2
XB(I,J,K)=XB1(I,J,K)+RATIO*(XBAR(I,J,K)-XB1(I,J,K))
106 CONTINUE
CALL QUASI
IF(ICHECK.LT.0) GO TO 114
IF(ICUTM.LT.0)GO TO 112
SUMA=0.0
DO 118 I=1,NK
118 SUMA=SUMA+ABS((C(I)-CO(I))/C(I))
IF(SUMA.LT.CHMIN) RMAXI=3.0*RMAXI
GO TO 110
112 DO 115 I=1,NK
115 C(I)=CO(I)
IF(LTAR.EQ.1) GO TO 1127
SUMB=0.0
DO 116 I=1,NDS
ML=I
T=0.08551
DO 117 II=1,NE
117 X(II)=XO(I,II)
M1=NP(I)
DO 116 J=1,M1
NCJ=NC(I,J)
H=(TC(I,J)-T)/NCJ

```

## MAIN LINE ... (CONT'D)

```

CALL GENER(ML)
M2=NPIJ(I,J)
DO 116 K=1,M2
S=KSI(I,J,K)
116 SUMB=SUMB+(XB1(I,J,K)-X(S))**2
WRITE(6,19) SUMB
19 FORMAT('0',9X,'THE SUM OF THE SQUARES OF THE ERRORS
* IS'
1,E14.4)
1127 IF(ICHECK.EQ.0) WRITE(6,23)
23 FORMAT('0','CONVERGENCE TO THE DESIRED ACCURACY WAS',
1'NOT OBTAINED AFTER 10 ITERATIONS IN PHASE II')
WRITE(6,24)
24 FORMAT('0THE FINAL SOLUTION OBTAINED')
WRITE(6,25) (I,CO(I),I=1,NK)
25 FORMAT('0 K',I2,'=',E16.6)
16 FORMAT('0 AFTER',I5,' STEPS')
JJ=0
DO 8000 J=1,NK
DO 8000 I=1,NK
JJ=JJ+1
8000 XDET(JJ)=AMAT(I,J)
CALL MINV(XDET,3,DD,L,MV)
WRITE(6,903) DD
903 FORMAT('//,8X,'THE DETERMINANT OF A-MATRIX IS',D15.6)
KK=0
DO 200 J=1,NK
DO 200 I=1,NK
KK=KK+1
200 SS(I,J)=XDET(KK)
DO 701 J=1,NK
DO 701 I=1,NK
701 VARC(I,J)=VAR*SS(I,J)
WRITE(6,702)
702 FORMAT(1H1,//,20X,'THE VARIANCE COVARIANCE MATRIX IS
* GIVEN BELOW')
DO 703 I=1,NK
703 WRITE(6,704) (VARC(I,J),J=1,NK)
704 FORMAT('//,6X,4(3X,D15.6))
WRITE(6,907) (CO(I),I=1,NK)
907 FORMAT('//,2X,'PARAMETERS ',12X,'K=',E15.6,3X,'B='
*,E15.6,3X,'BETA='
1,E15.6)
DO 108 I=1,NK
108 L95(I)=2.0*VARC(I,I)**0.5
WRITE(6,209) (L95(I),I=1,NK)
209 FORMAT('/',2X,'95 PERCENT CONFIDENCE',4X,D15.6,5X,D15.6
*,6X,D15.6)
STOP
END

```



## SUBROUTINE QUASI

```

C *****
C
C THIS SUBROUTINE WORKS OUT THE PHASE II TOWARDS FINAL
C CONVERGENCE BY QUASILINEARIZATION AFTER GOOD INITIAL
C ESTIMATES OF THE PARAMETERS HAVE BEEN OBTAINED FROM
C THE DATA PERTURBATION SCHEME.
C *****
C
SUBROUTINE QUASI

REAL*4 UB(05,05),X(05),P(05),TC(30,15),XD(30,05)
1, XB(30,15,05),T,H,C(05),CR(05),ERROR,CHMAX,RMAXI
REAL*4 BST(500,05),UST(500),YOB(500),EXTER
REAL*8 B(500,05),Q(500),AMAT(4,4),VARC(4,4)
INTEGER KSI(30,15,05),NPIJ(30,15),NC(30,15),NP(30),
INPITOT(30),NE,NK,RMAX,NDS,NPTOT,R,I,J,M1,M2,INDEX,
2ICOUNT,II,JJ,S,LL,NO,LKJM,LKJI,ICHECK
COMMON XB1(30,15,05),LTAR,VAR,AMAT
COMMON /AB6/CR/AB1/B,Q/AB8/XD,I,NDS/AB3/C,NE,NK,NO/
1AB4/UB,X,P,H,T,NCJ,LL/AB10/TC,XB,ERROR,CHMAX,RMAXI
2,KSI,NPIJ,NC,NP,NPITOT,RMAX,NPTOT,R,J,M1,M2,II,JJ,S,
3ICHECK/AB4/LKJM,LKJI
LKJI=0
SUMBB=1.0E 08
DO 103 R=1,RMAX
LKJI=LKJI+1
INDEX=0
DO 104 I=1,NDS
ML=I
S=R-1
IF (LKJI.NE.LKJM)GO TO 113
WRITE(6,5)S,I,NPITOT(I)
DO 112 II=1,NE
112 WRITE(6,6) II,XD(I,II)
WRITE(6,8) (C(II),II=1,NK)
WRITE(6,7)
5 FORMAT(15H1FOR ITERATION ,I4,11H DATA SET ,I4,' WITH '
1,I4,'POINTS'/'OTHE INITIAL CONDITION IS'/)
6 FORMAT(10X,1HX,I2,2H =,F14.6)
7 FORMAT(1H0,14X,1HT,8X,2HX1,8X,2HX2,8X,2HX3,8X,2HX4,8X,
12HX5,8X,2HX6,8X,2HX7,8X,2HX8,8X,2HX9,8X,3HX10/)
8 FORMAT(19H0THE CONSTANTS ARE /10X,8E16.6)
113 T=0.08551
DO 105 II=1,NE
DO 106 JJ=1,NK
106 UB(-II,JJ)=0.0
X(II)=XD(I,II)

```

## SUBROUTINE QUASI ... (CONT'D)

```

105  P(II)=X0(I,II)
      M1=NP(I)
      DO 104 J=1,M1
      NGJ=NC(I,J)
      H=(TC(I,J)-T)/FLOAT(NCJ)
      CALL INTE(R,ML)
      IF(LKJI.NE.LKJM)GO TO 114
      WRITE(6,9) T,(X(II),II=1,NE)
9     FORMAT(6X,1H#,10F12.6)
114  M2=NPIJ(I,J)
      DO 110 K=1,M2
      ICOUNT=K+INDEX
      S=KSI(I,J,K)
      Y0B(ICOUNT)=XB(I,J,K)
      UST(ICOUNT)=P(S)
      Q(ICOUNT)=Y0B(ICOUNT)-UST(ICOUNT)
      DO 110 JJ=1,NK
      BST(ICOUNT,JJ)=UB(S,JJ)
110  B(ICOUNT,JJ)=UB(S,JJ)
104  INDEX=INDEX+M2
      SUMB=0.0
      DO1116 I=1,NDS
      ML=I
      T=0.08551
      DO1117 II=1,NE
      M1=NP(I)
1117 X(II)=X0(I,II)
      DO1116 J=1,M1
      NCJ=NC(I,J)
      H=(TC(I,J)-T)/NCJ
      CALL GENER(ML)
      M2=NPIJ(I,J)
      DO1116 K=1,M2
      S=KSI(I,J,K)
1116 SUMB=SUMB+(XB1(I,J,K)-X(S))**2
      WRITE(6,19) SUMB
19   FORMAT('0',9X,'THE SUM OF THE SQUARES OF THE ERRORS
      * IS'
      1,E14.4)
      IF(R.EQ.10) GO TO 1009
      IF(LKJM.EQ.LKJI) LKJI=0
      CALL LEAST(NK,NPTOT)
      SUMA=0.0
      DO 108 II=1,NK
108  SUMA=SUMA+ABS(C(II)-CR(II))/(ABS(C(II))+0.1)
      IF(SUMB.GE.SUMBB)GO TO 1007
      SUMBB = SUMB
      1007 IF(SUMA.GT.CHMAX) GO TO 116
      DO 107 II=1,NK
107  C(II)=CR(II)

```

## SUBROUTINE QUASI ... (CONT'D)

```

IF(SUMA.GE.ERROR)GO TO 119
GO TO 3011
1009 CONTINUE
IF(RMAX.LT.10) GO TO 115
GO TO 119
3011 WRITE(6,12)
DO 120 II=1,NPTOT
EXTER=YOB(II)-UST(II)
DO 121 JJ=1,NK
121 EXTER=EXTER-BST(II,JJ)*C(JJ)
120 WRITE(6,11) EXTER,YOB(II),UST(II),(BST(II,JJ),JJ=1,NK)
11 FORMAT(1X,8E16.6)
12 FORMAT('1COEFFICIENTS OF THE DESIGN EQUATION'//6X
*, 'Y=BK+U+E
1 WHERE '//7X, 'Y IS THE OBSERVATION VECTOR'/7X, 'B IS
* THE DESIGN MA
2TRIX'/7X, 'K IS THE PARAMETER VECTOR'/7X, 'U IS
* APARTICULAR SOLUTION
3'/7X, 'E IS A ERROR VECTOR'//11X, 'E',15X, 'Y',15X, 'U'
*,10X, 'THE DESIG
4N MATRIX B'//)
MM=M1*NE*NDS
VAR=SUMB/(MM-NK)
DO1211 IHS=1,NK
DO1211 JHS=1,NK
AMAT(IHS,JHS)=0.0
DO1211 KHS=1,MM
AMAT(IHS,JHS)=AMAT(IHS,JHS)+B(KHS,IHS)*B(KHS,JHS)
1211 CONTINUE
WRITE(6,1214) VAR
1214 FORMAT('-', 'VARIANCE=', F16.4)

C CALCULATING THE EXPECTED LIKELIHOOD OF THE MODEL

XLIKE=(2.0*3.14159*VAR)**(-MM/2)*EXP(-SUMB/(2.0*VAR))
**0.5**(NK/2)
WRITE(6,876) XLIKE
876 FORMAT('//,2X, 'EXPECTED LIKELIHOOD OF THE RIGOROUS
* MODEL IS ',
1E15.6)
WRITE(6,1215)
1215 FORMAT('-', 'A-MATRIX ')
WRITE(6,1213) ((AMAT(IHS,JHS),JHS=1,NK),IHS=1,NK)
1213 FORMAT('-',3D14.4)
LTAR=1
GO TO 115
119 IF(R.EQ.10) ICHECK=0
IF(R.EQ.10) GO TO 118
103 CONTINUE
R=RMAX+1

```

## SUBROUTINE QUASI ... (CONT'D)

```
WRITE(6,10) RMAX
WRITE(6,8) (C(II),II=1,NK)
10  FORMAT(7HOAFTER ,I5,12H ITERATIONS)
115 RETURN
116 RMAXI=RMAXI#0.5
    DO 117 II=1,NK
117  CR(II)=C(II)
    ICHECK=-1
118 RETURN
    END
```

## SUBROUTINE GENER

```

C *****
C
C THIS SUBROUTINE INTERPOLATES THE DATA FOR USE IN
C QUASI AND INTE BY INTEGRATION OF THE DERIVATIVES
C USING FOURTH ORDER RUNGE-KUTTA INTEGRATION SCHEME.
C *****
C
SUBROUTINE GENER(ML)

REAL*4 UB(05,05),P(05),X(05),XA(05),KP(4,05),G(05),T
*,TA,H
1,JX(05,05),JK(05,05),C(05)
-INTEGER NCJ,NE,NK,LL,NO,N,II,I,ML
COMMON LTAR
COMMON XB1(30,15,05)
COMMON /AB3/C,NE,NK,NO/AB4/UB,X,P,H,T,NCJ,LL/AB5/JX,JK
*,XA,G
DO 100 N=1,NCJ
DO 101 II=1,4
GO TO(102,103,104,105),II
102 TA=T
DO 106 I=1,NE
106 XA(I)=X(I)
GO TO 107
103 TA=T+0.5*H
DO 108 I=1,NE
108 XA(I)=X(I)+0.5*KP(1,I)
GO TO 107
104 DO 109 I=1,NE
109 XA(I)=X(I)+0.5*KP(2,I)
GO TO 107
105 TA=T+H
DO 110 I=1,NE
110 XA(I)=X(I)+KP(3,I)
107 CALL SLOPE(1,TA,-1,ML)
DO 101 I=1,NE
101 KP(II,I)=H*G(I)
T=TA
DO 100 I=1,NE
100 X(I)=X(I)+(KP(1,I)+2.0*(KP(2,I)+KP(3,I))+KP(4,I))/6.0
RETURN
END

```

## SUBROUTINE LEAST

```

C *****
C
C THIS SUBROUTINE SETS UP THE DESIGN MATRIX FOR
C PARAMETER ESTIMATION FOR BOTH DATA PERTURBATION AND
C QUASILINEARIZATION.
C *****
C

```

```

SUBROUTINE LEAST(N,M)

```

```

REAL*4 X(05)
REAL*8 A(500,05),B(05,05),R(500),C(05),SUMA,SUMB
INTEGER I,J,N,M,S
COMMON XB1(30,15,05),LTAR
COMMON /AB2/B,C/AB6/X/AB1/A,R
IF(M.GT.N) GO TO 14
IF(M.EQ.N) GO TO 17
WRITE(6,2)
2  FORMAT(18HJINSUFFICIENT DATA)
STOP
17  DO 15 I=1,N
    C(I)=R(I)
    DO 15 J=1,N
15  B(I,J)=A(I,J)
    GO TO 16
14  DO 10 I=1,N
    DO 12 J=I,N
    SUMA=0.0
    DO 11 S=1,M
11  SUMA=SUMA+A(S,I)*A(S,J)
    B(I,J)=SUMA
12  B(J,I)=SUMA
    SUMA=0.0
    DO 13 S=1,M
13  SUMA=SUMA+A(S,I)*R(S)
10  C(I)=SUMA
16  CALL GAUSS(N)
    RETURN
END

```

## SUBROUTINE GAUSS

```

C *****
C
C THIS SUBROUTINE ESTIMATES THE PARAMETERS OF THE
C LINEARIZED MODEL USING GAUSS-SIDEL ELIMINATION.
C *****
C

```

## SUBROUTINE GAUSS(N)

```

REAL*4 XX(05)
REAL*8 A(05,05),R(05),X(05),S,U
INTEGER II(05),I,J,K,N,L,M,H,MM,T
COMMON XB1(30,15,05), LTAR
COMMON /AB2/A,R/AB6/XX
M=N-1
WRITE(6,32)M,N
32 FORMAT('0',2I4)
DO 10 I=1,N
10 II(I)=I
IF(M.EQ.0) GO TO 24
DO 11 J=1,M
S=0.0
DO 12 I=J,N
DO 12 K=J,N
U=DABS(A(I,J))
IF(U.LE.S) GO TO 12
S=U
L=I
T=K
12 CONTINUE
IF(L.EQ.J) GO TO 19
DO 14 I=J,N
S=A(L,I)
A(L,I)=A(J,I)
14 A(J,I)=S
S=R(L)
R(L)=R(J)
R(J)=S
19 IF(T.EQ.J) GO TO 13
DO 20 I=J,N
S=A(I,T)
A(I,T)=A(I,J)
20 A(I,J)=S
I=II(T)
II(T)=II(J)
II(J)=I
13 IF(DABS(A(J,J)).GT.1.0D-35) GO TO 15
IF(J.EQ.I) GO TO 15

```

## SUBROUTINE GAUSS ... (CONT'D)

```
WRITE(6,3)
STOP
15 MM=J+1
DO 11 I=MM,N
IF(DABS(A(I,J)).LT.1.0D-35) GO TO 11
S=A(J,J)/A(I,J)
A(I,J)=0.0
DO 16 K=MM,N
16 A(I,K)=A(J,K)-S*A(I,K)
R(I)=R(J)-S*R(I)
11 CONTINUE
24 DO 17 K=1,N
I=N+1-K
S=0.0
IF(I.EQ.N) GO TO 17
MM=I+1
DO 18 J=MM,N
18 S=S+A(I,J)*X(J)
17 X(I)=(R(I)-S)/A(I,I)
DO 21 I=1,N
K=II(I)
IF(I.EQ.K) GO TO 21
S=X(K)
X(K)=X(I)
X(I)=S
II(I)=II(K)
II(K)=K
21 CONTINUE
DO 23 I=1,N
23 XX(I)=X(I)
3 FORMAT(16HJMATRIX SINGULAR)
RETURN
END
```



## SUBROUTINE SLOPE

```

C *****
C
C THIS ROUTINE CALCULATES THE DERIVATIVE OF THE
C FUNCTION BOTH WITH RESPECT TO STATE VARIABLES AND THE
C PARAMETERS.
C *****
C

```

```

SUBROUTINE SLOPE(R,T,IJACK,ML)

```

```

REAL*4 X(05),JX(05,05),JK(05,05),G(05),T,C(05),
1X0(30,05)
REAL X1,X2,K1,K2,K3,KE1,KE2,KE3
INTEGER R,NE,NK,NO,II,NDS,IJACK,ML
INTEGER I,J
COMMON XB1(30,15,05),LTAR
COMMON /AB3/C,NE,NK,NO/AB5/JX,JK,X,G/AB8/X0,II,NDS
20 D1=0.277197E 03
D2=0.142567E-11
GO TO 90
21 D1=0.357607E 03
D2=0.238864E-11
GO TO 90
22 D1=0.734392E 03
D2=0.100444E-10
GO TO 90
23 D1=0.113607E 04
D2=0.239352E-10
GO TO 90
24 D1=0.218605E 04
D2=0.882131E-10
GO TO 90
25 D1=0.496934E 04
D2=0.455561E-09
GO TO 90
26 D1=0.111042E 05
D2=0.227560E-08
GO TO 90
27 D1=0.187636E 05
D2=0.648947E-08
GO TO 90
28 D1=0.235629E 05
D2=0.102525E-07
GO TO 90
29 D1=0.297832E 05
D2=0.163408E-07
GO TO 90

```

## SUBROUTINE SLOPE ... (CONT'D)

```
30  D1=0.389726E 05
    D2=0.279409E-07
90  DO 16 I=1,NE
    IF(ABS(X(I)).LE.1.0E 05) GO TO 16
    WRITE(6,17)
    STOP
16  CONTINUE
17  FORMAT('OSTATE VARIABLE BOUND EXCEEDED')
    K3=C(3)
    K2=C(2)
    K1=C(1)
    X1=X(1)
    GO TO (11,12),NO
11  DO 13 I=1,NE
    G(I)=0.0
    DO 14 J=1,NE
14  JX(I,J)=0.0
    DO 13 J=1,NK
13  JK(I,J)=0.0
    NO=2
12  G(1)=D1/(K1*T*(X1+K2))+D2*K3/(T**2*X1)
    IF(IJACK.LT.0) GO TO 15
    JX(1,1)=-D1/(K1*T*(X1+K2)**2)+D2*K3/(T**2*X1**2)
    JK(1,1)=-D1/(K1**2*T*(X1+K2))
    JK(1,2)=-D1/(K1*T*(X1+K2)**2)
    JK(1,3)=D2/(T**2*X1)
15  RETURN
    END
```

## SUBROUTINE INTE

```

C *****
C THIS SUBROUTINE INTEGRATES THE MATRIX DIFFERENTIAL
C EQUATION FOR QUASILINEARIZATION AND OBTAINS A
C PARTICULAR SOLUTION.
C *****
C

```

```

SUBROUTINE INTE(R,ML)

```

```

REAL*4 UB(05,05),P(05),X(05),UBA(05,05),PA(05),
1XA(05),KX(4,05),KUB(4,05,05),KP(4,05),G(05),T,TA,H,
2SUMA,SUMB,JX(05,05),JK(05,05),C(05)
INTEGER NCJ,R,I,J,II,NE,NK,L,LL,S,N,NO,LKJM,LKJI
COMMON XB1(30,15,05),LTAR
COMMON /AB3/C,NE,NK,NO/AB4/UB,X,P,H,T,NCJ,LL/AB5/JX,
1JK,XA,G/AB4/LKJM,LKJI
L=0
DO 100 N=1,NCJ
DO 101 II=1,4
GO TO (102,103,104,105),II
102 TA=T
DO 109 I=1,NE
PA(I)=P(I)
XA(I)=X(I)
DO 109 J=1,NK
109 UBA(I,J)=UB(I,J)
GO TO 110
103 TA=T+0.5*H
DO 113 I=1,NE
PA(I)=P(I)+0.5*KP(1,I)
XA(I)=X(I)+0.5*KX(1,I)
DO 113 J=1,NK
113 UBA(I,J)=UB(I,J)+0.5*KUB(1,I,J)
GO TO 110
104 DO 114 I=1,NE
PA(I)=P(I)+0.5*KP(2,I)
XA(I)=X(I)+0.5*KX(2,I)
DO 114 J=1,NK
114 UBA(I,J)=UB(I,J)+0.5*KUB(2,I,J)
GO TO 110
105 TA=T+H
DO 117 I=1,NE
PA(I)=P(I)+KP(3,I)
XA(I)=X(I)+KX(3,I)
DO 117 J=1,NK
117 UBA(I,J)=UB(I,J)+KUB(3,I,J)
110 CALL SLOPE(R,TA,1,ML)

```

## SUBROUTINE INTE ... (CONT'D)

```

DO 115 I=1,NE
SUMA=G(I)
115 KX(II,I)=H*SUMA
118 DO 120 I=1,NE
SUMA=G(I)
DO 121 J=1,NK
SUMB=JK(I,J)
SUMA=SUMA-JK(I,J)*C(J)
DO 122 S=1,NE
122 SUMB=SUMB+JX(I,S)*UBA(S,J)
121 KUB(II,I,J)=H*SUMB
DO 130 J=1,NE
130 SUMA=SUMA+JX(I,J)*(PA(J)-XA(J))
120 KP(II,I)=H*SUMA
101 CONTINUE
DO 127 I=1,NE
P(I)=P(I)+(KP(1,I)+2.0*(KP(2,I)+KP(3,I))+KP(4,I))/6.0
DO 127 J=1,NK
UB(I,J)=UB(I,J)+(KUB(1,I,J)+2.0*(KUB(2,I,J)+KUB(3,I
*,J))+KUB(4,I,
1J))/6.0
IF(ABS(UB(I,J)).LT.1.0E 50) GO TO 127
WRITE(6,2)
2 FORMAT('OTHE INTEGRATION OF THE FUNDERMENTAL MATRIX
* IS',
1' UNBOUNDED')
STOP
127 CONTINUE
DO 129 I=1,NE
129 X(I)=X(I)+(KX(1,I)+2.0*(KX(2,I)+KX(3,I))+KX(4,I))/6.0
T=TA
L=L+1
IF(L.NE.LL)GO TO 100
L=0
IF(LKJI.NE.LKJM) GO TO 100
IF(N.EQ.NCJ) GO TO 100
WRITE(6,1) T,(X(I),I=1,NE)
1 FORMAT(7X,10F12.6)
100 CONTINUE
RETURN
END

```

## APPENDIX C

### ANALYSIS OF THE STEADY RADIAL GAS FLOW

Out of the four mathematical models listed on this analysis three are given in algebraic form and one in differential form. For both types of non-linear models the derivatives of the function with respect to the parameters must be known. Therefore, the following expressions for each mathematical model were obtained.

#### Model 1

When slippage is considered and inertial effect is neglected, i.e.  $\beta$  is zero, the following model was derived:

$$p_e = \left[ p_w^2 + \frac{1,424 \mu_{avg} T_{avg} z_{avg} Q_g}{hk} \ln\left(\frac{r_e}{r_w}\right) - 2b(p_e - p_w) \right]^{0.5} \quad (C-1)$$

or

$$p_e = \gamma(Q_g, \underline{a})$$

where

$$\underline{a} = \begin{bmatrix} k \\ b \end{bmatrix} .$$

Therefore, if the expression in brackets in Equation (C-1) is called "VAR 1"; the expressions for the derivatives can be written as

$$a) \quad \frac{\partial Y}{\partial k} = -0.5 (\text{VAR } 1)^{-0.5} \left[ \frac{1,424 A^* Q_g}{hk^2} \ln \left( \frac{r_e}{r_w} \right) \right] \quad (\text{C-2})$$

$$b) \quad \frac{\partial Y}{\partial b} = -0.5 (\text{VAR } 1)^{-0.5} [2(p_e - p_w)] \quad (\text{C-3})$$

### Model 2

When slippage effect is neglected and inertial effect is considered, we have the following model:

$$p_e = \left[ p_w^2 + \frac{1,424 \mu_{\text{avg}} T_{\text{avg}} Z_{\text{avg}} Q_g}{hk} \ln \left( \frac{r_e}{r_w} \right) + \frac{3.1602 \times 10^{-12} \beta G T_{\text{avg}} Z_{\text{avg}} Q_g^2}{h^2} \left( \frac{1}{r_w} - \frac{1}{r_e} \right) \right]^{0.5} \quad (\text{C-4})$$

or

$$p_e = \phi(Q_g, \underline{a})$$

where

$$\underline{a} = \begin{bmatrix} k \\ \beta \end{bmatrix} \quad .$$

Let the expression in brackets in Equation (C-4) be called "VAR 2". Hence, the expressions for the derivatives are:

$$a) \quad \frac{\partial \phi}{\partial k} = -0.5 (\text{VAR } 2)^{-0.5} \left[ \frac{1,424 A^* Q_g}{hk} \ln \left( \frac{r_e}{r_w} \right) \right] \quad (\text{C-5})$$

$$b) \quad \frac{\partial \phi}{\partial \beta} = 0.5 (\text{VAR } 2)^{-0.5} \left[ \frac{3.1602 \times 10^{-12} G T_{\text{avg}} Z_{\text{avg}} Q_g^2}{h^2} \times \left( \frac{1}{r_w} - \frac{1}{r_e} \right) \right] \quad (\text{C-6})$$

Model 3

The simplified three-parameter model was stated as

$$p_e = \left( p_w^2 - 2b(p_e - p_w) + \frac{1,424 \mu_{avg} T_{avg} z_{avg} Q_g}{hk} \ln\left(\frac{r_e}{r_w}\right) + \frac{3.1602 \times 10^{-12} \beta G T_{avg} z_{avg} Q_g^2}{h^2} \left(\frac{1}{r_w} - \frac{1}{r_e}\right) \right)^{0.5} \quad (C-7)$$

or

$$p_e = \theta(Q_g, \underline{a})$$

where

$$\underline{a} = \begin{pmatrix} k \\ b \\ \beta \end{pmatrix} .$$

Let the expression in brackets in Equation (C-7) be called "VAR 3". Hence:

$$a) \quad \frac{\partial \theta}{\partial k} = -0.5 (\text{VAR } 3)^{-0.5} \left[ \frac{1,424 A^* Q_g}{hk^2} \ln\left(\frac{r_e}{r_w}\right) \right] \quad (C-8)$$

$$b) \quad \frac{\partial \theta}{\partial b} = (\text{VAR } 3)^{-0.5} (p_e - p_w) \quad (C-9)$$

$$c) \quad \frac{\partial \theta}{\partial \beta} = -0.5 (\text{VAR } 3)^{-0.5} \left[ \frac{3.1602 \times 10^{-12} G T_{avg} z_{avg} Q_g^2}{h^2} \times \left(\frac{1}{r_w} - \frac{1}{r_e}\right) \right] . \quad (C-10)$$

Model 4

The rigorous three-parameter mathematical model was derived as

$$\frac{dp}{dr} = \frac{712 \mu_{avg} T_{avg} z_{avg} Q_g}{r h k (p+b)} + \frac{1.5801 \times 10^{-12} \beta G T_{avg} z_{avg} Q_g^2}{h^2 r^2 p} \quad (C-11)$$

or

$$\frac{dp}{dr} = \frac{D_1}{r k (p+b)} + \frac{D_2 \beta}{r^2 p}$$

which can be written as

$$\frac{dp}{dr} = \phi(Q_g, p, r; \underline{a})$$

where

$$\underline{a} = \begin{pmatrix} k \\ b \\ \beta \end{pmatrix} .$$

Thus, the expressions for the derivatives are given by

$$a) \quad \frac{\partial \phi}{\partial k} = - \frac{D_1}{r k^2 (p+b)} \quad (C-12)$$

$$b) \quad \frac{\partial \phi}{\partial b} = - \frac{D_1}{r k (p+b)^2} \quad (C-13)$$

$$c) \quad \frac{\partial \phi}{\partial \beta} = \frac{D_2}{r^2 p} \quad (C-14)$$

For this case, the algorithm in addition requires



the derivative of the function with respect to the state variable. Hence,

$$d) \quad \frac{\partial \phi}{\partial p} = - \frac{D_1}{rk(p+b)^2} - \frac{D_2 \beta}{r^2 p^2} \quad (C-15)$$

Input data and results for all the core samples using each model, are presented in the following pages.

TABLE C-1

\*\*\*\*\*

INPUT DATA FOR CORE SAMPLE NO.1  
RUN 1

PE (PSIA)	PW (PSIA)	VIS*T#Z (CP-DEG.R)	VIS (CP)	QG (MSCF/DAY)
15.385	13.617	0.942D 01	0.177D-01	0.710D-03
16.819	13.617	0.943D 01	0.177D-01	0.132D-02
18.943	13.530	0.950D 01	0.178D-01	0.232D-02
25.022	13.729	0.936D 01	0.176D-01	0.570D-02
29.099	13.509	0.941D 01	0.177D-01	0.833D-02
29.580	13.626	0.949D 01	0.177D-01	0.857D-02
33.598	13.626	0.949D 01	0.178D-01	0.115D-01
34.955	13.735	0.935D 01	0.176D-01	0.127D-01
38.565	13.623	0.950D 01	0.178D-01	0.154D-01
42.690	13.633	0.950D 01	0.176D-01	0.189D-01
43.076	13.724	0.934D 01	0.176D-01	0.198D-01
48.656	13.724	0.933D 01	0.176D-01	0.251D-01
53.656	13.753	0.933D 01	0.176D-01	0.304D-01
58.656	13.772	0.933D 01	0.176D-01	0.359D-01
63.656	13.791	0.934D 01	0.176D-01	0.418D-01

\*\*\*\*\*

TABLE C-1A

\*\*\*\*\*

RESULTS USING MODEL 1

LEAST SQUARE PARAMETERS

K=6.997 MD.  
B=9.768 PSIA.

ERROR VECTOR  
(PSIA)

DESIGN MATRIX  
X

\*\*\*\*\*

0.121D 00	-0.384D 00	-0.116D 00
0.217D 00	-0.658D 00	-0.193D 00
0.293D 00	-0.104D 01	-0.290D 00
0.671D-01	-0.188D 01	-0.453D 00
0.114D 00	-0.237D 01	-0.538D 00
0.400D-01	-0.242D 01	-0.540D 00
-0.213D-01	-0.284D 01	-0.594D 00
-0.134D 00	-0.298D 01	-0.605D 00
-0.517D-01	-0.332D 01	-0.646D 00
-0.550D-01	-0.369D 01	-0.680D 00
-0.345D 00	-0.374D 01	-0.676D 00
-0.182D 00	-0.421D 01	-0.715D 00
-0.122D 00	-0.463D 01	-0.742D 00
0.136D 00	-0.502D 01	-0.767D 00
0.330D 00	-0.541D 01	-0.787D 00

SUM OF THE SQUARES OF THE ERRORS=0.485  
 ERROR VARIANCE=0.037  
 EXPECTED LIKELIHOOD=0.194D 02  
 DETERMINANT OF A MATRIX=0.216D 02  
 VARIANCE-COVARIANCE MATRIX

0.907D-02	-0.503D-01
-0.503D-01	0.287D 00

CORRELATION MATRIX

1.000	-0.986
-0.986	1.000

TABLE C-1B

\*\*\*\*\*  
RESULTS USING MODEL 2

LEAST SQUARE PARAMETERS

K =10.468 MD.  
BETA=0.737D 10 1/FT.

ERROR VECTOR  
(PSIA)

DESIGN MATRIX  
X

\*\*\*\*\*

-0.123D 00	-0.169D 00	0.213D-13
-0.157D 00	-0.287D 00	0.737D-13
-0.188D 00	-0.451D 00	0.228D-12
-0.253D 00	-0.827D 00	0.137D-11
-0.100D 00	-0.105D 01	0.293D-11
-0.116D 00	-0.107D 01	0.311D-11
-0.369D-02	-0.127D 01	0.555D-11
-0.450D-01	-0.133D 01	0.679D-11
0.141D 00	-0.149D 01	0.100D-10
0.234D 00	-0.166D 01	0.152D-10
-0.270D-01	-0.168D 01	0.165D-10
0.104D 00	-0.189D 01	0.266D-10
0.414D-01	-0.207D 01	0.389D-10
0.356D-01	-0.224D 01	0.541D-10
-0.150D 00	-0.240D 01	0.734D-10

SUM OF THE SQUARES OF THE ERRORS=0.276  
ERROR VARIANCE=0.021  
EXPECTED LIKELIHOOD=0.989D 03  
DETERMINANT OF A MATRIX=0.102D-18  
VARIANCE COVARIANCE MATRIX

0.235D-02	0.109D 09
0.109D 09	0.693D 19

CORRELATION MATRIX

1.000	0.854
0.854	1.000

TABLE C-1C

\*\*\*\*\*

## RESULTS USING MODEL 3

## LEAST SQUARE PARAMETERS

K =8.668 MD.  
 B =4.177 PSIA.  
 BETA=0.446D 10 1/FT.

ERROR VECTOR  
 (PSIA)

DESIGN MATRIX  
 X

\*\*\*\*\*

-0.116D-01	-0.248D 00	-0.115D 00	0.865D-12
0.146D-01	-0.423D 00	-0.191D 00	0.274D-11
0.359D-01	-0.666D 00	-0.286D 00	0.755D-11
-0.856D-01	-0.121D 01	-0.450D 00	0.340D-10
0.255D-01	-0.154D 01	-0.536D 00	0.630D-10
-0.149D-01	-0.157D 01	-0.539D 00	0.658D-10
0.230D-01	-0.185D 01	-0.595D 00	0.104D-09
-0.501D-01	-0.194D 01	-0.606D 00	0.121D-09
0.887D-01	-0.217D 01	-0.648D 00	0.163D-09
0.134D 00	-0.242D 01	-0.683D 00	0.223D-09
-0.143D 00	-0.245D 01	-0.679D 00	0.238D-09
-0.128D-01	-0.275D 01	-0.718D 00	0.341D-09
-0.417D-01	-0.302D 01	-0.743D 00	0.452D-09
0.427D-01	-0.327D 01	-0.766D 00	0.577D-09
-0.841D-02	-0.351D 01	-0.783D 00	0.721D-09

SUM OF THE SQUARES OF THE ERRORS=0.063

ERROR VARIANCE=0.005

EXPECTED LIKELIHOOD=0.299D 08

DETERMINANT OF A MATRIX=0.192D-18

VARIANCE-COVARIANCE MATRIX

0.565D-01	-0.155D 00	0.115D 09
-0.155D 00	0.431D 00	-0.311D 09
0.115D 09	-0.311D 09	0.248D 18

CORRELATION MATRIX

1.000	-0.993	0.972
-0.993	1.000	-0.951
0.972	-0.951	1.000

TABLE C-1D

\*\*\*\*\*

## RESULTS USING MODEL 4

## LEAST SQUARE PARAMETERS

K =8.756 MD.  
 B =3.993 PSIA.  
 BETA=0.408D 10 1/FT.

ERROR VECTOR  
(PSIA)DESIGN MATRIX  
X

\*\*\*\*\*

-0.582D-02	-0.193D 00	-0.915D-01	0.883D-12
0.154D-01	-0.335D 00	-0.153D 00	0.283D-11
0.333D-01	-0.539D 00	-0.233D 00	0.793D-11
-0.695D-01	-0.103D 01	-0.383D 00	0.366D-10
0.242D-01	-0.133D 01	-0.457D 00	0.682D-10
-0.105D-01	-0.136D 01	-0.461D 00	0.712D-10
0.223D-01	-0.162D 01	-0.510D 00	0.112D-09
-0.438D-01	-0.170D 01	-0.521D 00	0.131D-09
0.794D-01	-0.192D 01	-0.9553D 00	0.177D-09
0.119D 00	-0.216D 01	-0.580D 00	0.242D-09
-0.134D 00	-0.219D 01	-0.580D 00	0.258D-09
-0.169D 00	-0.249D 01	-0.604D 00	0.368D-09
-0.431D 00	-0.275D 01	-0.618D 00	0.486D-09
0.393D-01	-0.299D 01	-0.627D 00	0.618D-09
-0.107D-02	-0.322D 01	-0.631D 00	0.770D-09

SUM OF THE SQUARES OF THE ERRORS=0.052

ERROR VARIANCE=0.004

EXPECTED LIKELIHOOD=0.119D 09

DETERMINANT OF A MATRIX=0.123D-18

VARIANCE-COVARIANCE MATRIX

0.614D-01	-0.167D 00	0.129D 09
-0.167D 00	0.456D 00	-0.345D 09
0.129D 09	-0.345D 09	0.280D 18

CORRELATION MATRIX

1.000	-0.998	0.984
-0.998	1.000	-0.966
0.984	-0.966	1.000

TABLE C-2

\*\*\*\*\*  
 INPUT DATA FOR CORE SAMPLE NO. 1  
 RUN 2

PE (PSIA)	PW (PSIA)	VIS#T#Z (CP-DEG.R)	VIS (CP)	QG (MSCF/DAY)
25.022	13.729	0.936D 01	0.176D-01	0.570D-02
34.955	13.735	0.935D 01	0.176D-01	0.127D-01
43.076	13.724	0.934D 01	0.176D-01	0.198D-01
48.656	13.724	0.933D 01	0.176D-01	0.251D-01
53.656	13.753	0.933D 01	0.176D-01	0.304D-01
58.656	13.772	0.933D 01	0.176D-01	0.359D-01
63.656	13.791	0.934D 01	0.176D-01	0.418D-01
68.656	13.807	0.935D 01	0.176D-01	0.478D-01
73.656	13.842	0.934D 01	0.176D-01	0.543D-01
78.756	13.880	0.935D 01	0.177D-01	0.612D-01
83.616	13.888	0.934D 01	0.176D-01	0.679D-01
88.456	13.946	0.935D 01	0.177D-01	0.749D-01
93.556	13.994	0.935D 01	0.177D-01	0.826D-01
98.656	14.043	0.936D 01	0.177D-01	0.900D-01
103.456	14.101	0.936D 01	0.177D-01	0.976D-01

TABLE C-2A

\*\*\*\*\*  
RESULTS USING MODEL 1

## LEAST SQUARE PARAMETERS

K=5.772 MD.  
B=18.623 PSIA.

ERROR VECTOR (PSIA)	DESIGN MATRIX X	
0.132D 01	-0.290D 01	-0.477D 00
0.830D 00	-0.448D 01	-0.622D 00
0.971D-01	-0.555D 01	-0.683D 00
-0.101D 00	-0.620D 01	-0.716D 00
-0.419D 00	-0.676D 01	-0.738D 00
-0.522D 00	-0.730D 01	-0.758D 00
-0.719D 00	-0.782D 01	-0.775D 00
-0.561D 00	-0.831D 01	-0.792D 00
-0.624D 00	-0.881D 01	-0.805D 00
-0.524D 00	-0.930D 01	-0.818D 00
-0.286D 00	-0.975D 01	-0.831D 00
-0.599D-01	-0.102D 02	-0.842D 00
0.136D 00	-0.107D 02	-0.852D 00
0.777D 00	-0.111D 02	-0.864D 00
0.114D 01	-0.115D 02	-0.873D 00

SUM OF THE SQUARES OF THE ERRORS=6.412  
ERROR VARIANCE=0.493  
EXPECTED LIKELIHOOD=0.274D-06  
DETERMINANT OF A MATRIX=0.266D 03  
VARIANCE-COVARIANCE MATRIX

0.165D-01	-0.177D 00
-0.177D 00	0.196D 01

## CORRELATION MATRIX

1.000	-0.984
-0.984	1.000



TABLE C-2B

\*\*\*\*\*  
 RESULTS USING MODEL 2

LEAST SQUARE PARAMETERS

K =10.012 MD.  
 BETA=0.545D 10 1/FT.

ERROR VECTOR  
 (PSIA)

DESIGN MATRIX  
 X

\*\*\*\*\*

-0.579D 00	-0.893D 00	0.159D-11
-0.443D 00	-0.144D 01	0.789D-11
-0.367D 00	-0.182D 01	0.192D-10
-0.140D 00	-0.206D 01	0.309D-10
-0.761D-01	-0.226D 01	0.451D-10
0.777D-01	-0.245D 01	0.628D-10
0.906D-01	-0.263D 01	0.852D-10
0.291D 00	-0.280D 01	0.111D-09
0.232D 00	-0.296D 01	0.144D-09
0.212D 00	-0.312D 01	0.182D-09
0.204D 00	-0.326D 01	0.224D-09
0.118D 00	-0.339D 01	0.272D-09
-0.120D 00	-0.353D 01	0.331D-09
-0.537D-01	-0.366D 01	0.393D-09
-0.309D 00	-0.377D 01	0.461D-09

SUM OF THE SQUARES OF THE ERRORS=1.058  
 ERROR VARIANCE=0.081  
 EXPECTED LIKELIHOOD=0.822D-01  
 DETERMINANT OF A MATRIX=0.174D-16  
 VARIANCE-COVARIANCE MATRIX

0.318D-02	0.369D 08
0.369D 08	0.547D 18

CORRELATION MATRIX

1.000	0.885
0.885	1.000

TABLE C-2C

\*\*\*\*\*

## RESULTS USING MODEL 3

## LEAST SQUARE PARAMETERS

K =8.265 MD.  
 B =5.599 PSIA.  
 BETA=0.403D 10 1/FT.

ERROR VECTOR  
(PSIA)DESIGN MATRIX  
X

\*\*\*\*\*

0.560D-01	-0.134D 01	-0.452D 00	0.342D-10
0.472D-01	-0.214D 01	-0.608D 00	0.121D-09
-0.105D 00	-0.269D 01	-0.680D 00	0.239D-09
-0.585D-02	-0.303D 01	-0.718D 00	0.341D-09
-0.622D-01	-0.332D 01	-0.743D 00	0.452D-09
0.463D-02	-0.359D 01	-0.765D 00	0.576D-09
-0.601D-01	-0.385D 01	-0.783D 00	0.720D-09
0.109D 00	-0.409D 01	-0.800D 00	0.874D-09
0.262D-01	-0.433D 01	-0.812D 00	0.105D-08
0.150D-01	-0.457D 01	-0.824D 00	0.125D-08
0.516D-01	-0.477D 01	-0.834D 00	0.145D-08
0.258D-01	-0.498D 01	-0.843D 00	0.166D-08
-0.122D 00	-0.518D 01	-0.849D 00	0.191D-08
0.745D-01	-0.537D 01	-0.858D 00	0.216D-08
-0.409D-01	-0.555D 01	-0.863D 00	0.242D-08

SUM OF THE SQUARES OF THE ERRORS=0.062  
 ERROR VARIANCE=0.005  
 EXPECTED LIKELIHOOD=0.314D 08  
 DETERMINANT OF A MATRIX=0.237D-16  
 VARIANCE-COVARIANCE MATRIX

0.115D-01	-0.431D-01	0.120D 08
-0.431D-01	0.165D 00	-0.440D 08
0.120D 08	-0.440D 08	0.135D 17

## CORRELATION MATRIX

1.000	-0.989	0.963
-0.989	1.000	-0.932
0.963	-0.932	1.000



TABLE C-3

\*\*\*\*\*  
 INPUT DATA FOR CORE SAMPLE NO. 1  
 RUN 3

IN ADDITION TO THE 15 DATA POINTS OBTAINED IN  
 RUN 2, THE FOLLOWING 8 DATA POINTS WERE ADDED.

PE (PSIA)	PW (PSIA)	VIS*T*Z (CP-DEG.R)	VIS (CP)	QG (MSCF/DAY)
108.456	14.169	0.936D 01	0.177D-01	0.106D 00
113.556	14.246	0.936D 01	0.176D-01	0.114D 00
118.694	14.352	0.931D 01	0.176D-01	0.124D 00
123.694	14.448	0.931D 01	0.176D-01	0.132D 00
128.694	14.541	0.932D 01	0.176D-01	0.141D 00
133.694	14.636	0.932D 01	0.176D-01	0.150D 00
138.694	14.748	0.932D 01	0.176D-01	0.159D 00
143.694	14.864	0.932D 01	0.177D-01	0.168D 00

TABLE C-3A

\*\*\*\*\*  
 RESULTS USING MODEL 1

## LEAST SQUARE PARAMETERS

K=5.071 MD.  
 B=26.593 PSIA.

ERROR VECTOR (PSIA)	DESIGN MATRIX X	
0.286D 01	-0.402D 01	-0.509D 00
0.224D 01	-0.606D 01	-0.649D 00
0.113D 01	-0.736D 01	-0.700D 00
0.671D 00	-0.816D 01	-0.728D 00
0.718D-01	-0.884D 01	-0.745D 00
-0.294D 00	-0.949D 01	-0.761D 00
-0.780D 00	-0.101D 02	-0.774D 00
-0.874D 00	-0.107D 02	-0.789D 00
-0.123D 01	-0.113D 02	-0.799D 00
-0.141D 01	-0.119D 02	-0.809D 00
-0.143D 01	-0.125D 02	-0.820D 00
-0.147D 01	-0.130D 02	-0.829D 00
-0.156D 01	-0.136D 02	-0.836D 00
-0.116D 01	-0.141D 02	-0.848D 00
-0.105D 01	-0.146D 02	-0.855D 00
-0.709D 00	-0.151D 02	-0.864D 00
-0.113D 00	-0.156D 02	-0.874D 00
-0.153D 00	-0.162D 02	-0.878D 00
0.197D 00	-0.166D 02	-0.885D 00
0.595D 00	-0.171D 02	-0.891D 00
0.140D 01	-0.176D 02	-0.900D 00
0.182D 01	-0.181D 02	-0.906D 00
0.243D 01	-0.186D 02	-0.912D 00

SUM OF THE SQUARES OF THE ERRORS=41.048  
 ERROR VARIANCE=1.955  
 EXPECTED LIKELIHOOD=0.144D-16  
 DETERMINANT OF A MATRIX=0.235D 04

## TABLE C-3A...(CONT'D)

## VARIANCE-COVARIANCE MATRIX

0.126D-01	-0.201D 00
-0.201D 00	0.335D 01

## CORRELATION MATRIX

1.000	-0.978
-0.978	1.000

TABLE C-3B

\*\*\*\*\*  
 RESULTS USING MODEL 2

## LEAST SQUARE PARAMETERS

K =9.594 MD.  
 BETA=0.443D 10 1/FT.

ERROR VECTOR (PSIA)                      DESIGN MATRIX X  
 \*\*\*\*\*

-0.933D 00	-0.959D 00	0.177D-11
-0.945D 00	-0.154D 01	0.875D-11
-0.918D 00	-0.196D 01	0.213D-10
-0.691D 00	-0.222D 01	0.342D-10
-0.602D 00	-0.244D 01	0.500D-10
-0.403D 00	-0.265D 01	0.696D-10
-0.323D 00	-0.285D 01	0.943D-10
-0.384D-01	-0.303D 01	0.123D-09
0.127D-01	-0.322D 01	0.159D-09
0.123D 00	-0.339D 01	0.201D-09
0.258D 00	-0.355D 01	0.248D-09
0.331D 00	-0.371D 01	0.301D-09
0.284D 00	-0.386D 01	0.366D-09
0.545D 00	-0.401D 01	0.434D-09
0.501D 00	-0.414D 01	0.510D-09
0.568D 00	-0.427D 01	0.595D-09
0.772D 00	-0.440D 01	0.688D-09
0.217D 00	-0.453D 01	0.810D-09
0.328D-01	-0.465D 01	0.928D-09
-0.179D 00	-0.476D 01	0.106D-08
-0.100D 00	-0.486D 01	0.119D-08
-0.463D 00	-0.497D 01	0.134D-08
-0.727D 00	-0.507D 01	0.150D-08

SUM OF THE SQUARES OF THE ERRORS=6.286  
 ERROR VARIANCE=0.299  
 EXPECTED LIKELIHOOD=0.132D-07  
 DETERMINANT OF A MATRIX=0.749D-15

## TABLE C-3B... (CONT'D)

## VARIANCE-COVARIANCE MATRIX

0.388D-02	0.192D 08
0.192D 08	0.126D 18

## CORRELATION MATRIX

1.000	0.868
0.868	1.000



TABLE C-3C

\*\*\*\*\*

## RESULTS USING MODEL 3

## LEAST SQUARE PARAMETERS

K =7.683 MD.  
 B =7.886 PSIA.  
 BETA=0.334D 10 1/FT.

ERROR VECTOR  
(PSIA)DESIGN MATRIX  
X

\*\*\*\*\*

0.274D 00	-0.157D 01	-0.456D 00	0.345D-10
0.181D 00	-0.248D 01	-0.610D 00	0.122D-09
-0.739D-01	-0.312D 01	-0.680D 00	0.239D-09
-0.291D-01	-0.350D 01	-0.718D 00	0.341D-09
-0.134D 00	-0.384D 01	-0.742D 00	0.451D-09
-0.996D-01	-0.415D 01	-0.764D 00	0.575D-09
-0.191D 00	-0.445D 01	-0.718D 00	0.719D-09
-0.269D-01	-0.473D 01	-0.799D 00	0.873D-09
-0.110D 00	-0.501D 01	-0.811D 00	0.105D-08
-0.106D 00	-0.528D 01	-0.823D 00	0.125D-08
-0.382D-01	-0.552D 01	-0.834D 00	0.145D-08
-0.245D-01	-0.576D 01	-0.842D 00	0.166D-08
-0.117D 00	-0.600D 01	-0.849D 00	0.191D-08
0.151D 00	-0.622D 01	-0.859D 00	0.216D-08
0.114D 00	-0.643D 01	-0.865D 00	0.242D-08
0.220D 00	-0.664D 01	-0.871D 00	0.270D-08
0.495D 00	-0.684D 01	-0.873D 00	0.300D-08
0.311D-01	-0.705D 01	-0.879D 00	0.336D-08
-0.523D-01	-0.724D 01	-0.883D 00	0.370D-08
-0.145D 00	-0.743D 01	-0.886D 00	0.405D-08
0.880D-01	-0.759D 01	-0.891D 00	0.439D-08
-0.115D 00	-0.777D 01	-0.893D 00	0.477D-08
-0.194D 00	-0.794D 01	-0.895D 00	0.516D-08

SUM OF THE SQUARES OF THE ERRORS=0.632

ERROR VARIANCE=0.032

EXPECTED LIKELIHOOD=0.121D 04

DETERMINANT OF A MATRIX=0.149D-14

## TABLE C-3C... (CONT'D)

## VARIANCE-COVARIANCE MATRIX

0.139D-01	-0.689D-01	0.107D 08
-0.689D-01	0.348D 00	-0.511D 08
0.107D 08	-0.511D 08	0.899D 16

## CORRELATION MATRIX

1.000	-0.991	0.957
-0.991	1.000	-0.914
0.957	-0.914	1.000

TABLE C-3D

\*\*\*\*\*

## RESULTS USING MODEL 4

## LEAST SQUARE PARAMETERS

K =7.901 MD.  
 B =7.288 PSIA.  
 BETA=0.312D 10 1/FT.

ERROR VECTOR  
(PSIA)DESIGN MATRIX  
X

\*\*\*\*\*

0.229D 00	-0.114D 01	-0.340D 00	0.386D-10
0.183D 00	-0.194D 01	-0.482D 00	0.140D-09
-0.253D-01	-0.252D 01	-0.550D 00	0.275D-09
0.834D-02	-0.288D 01	-0.580D 00	0.390D-09
-0.893D-01	-0.319D 01	-0.599D 00	0.515D-09
-0.673D-01	-0.348D 01	-0.613D 00	0.653D-09
-0.158D 00	-0.376D 01	-0.623D 00	0.811D-09
-0.202D-01	-0.402D 01	-0.629D 00	0.978D-09
-0.104D 00	-0.429D 01	-0.632D 00	0.117D-08
-0.108D 00	-0.454D 01	-0.634D 00	0.138D-08
-0.537D-01	-0.477D 01	-0.633D 00	0.159D-08
-0.447D-01	-0.500D 01	-0.632D 00	0.182D-08
-0.133D 00	-0.523D 01	-0.629D 00	0.209D-08
0.115D 00	-0.544D 01	-0.625D 00	0.235D-08
0.815D-01	-0.564D 01	-0.621D 00	0.262D-08
0.184D 00	-0.584D 01	-0.616D 00	0.291D-08
0.447D 00	-0.603D 01	-0.611D 00	0.322D-08
0.157D-01	-0.624D 01	-0.604D 00	0.360D-08
-0.535D-01	-0.642D 01	-0.598D 00	0.396D-08
-0.132D 00	-0.661D 01	-0.592D 00	0.432D-08
0.967D-01	-0.677D 01	-0.586D 00	0.468D-08
-0.854D-01	-0.694D 01	-0.580D 00	0.508D-08
-0.151D 00	-0.711D 01	-0.574D 00	0.549D-08

SUM OF THE SQUARES OF THE ERRORS=0.484  
 ERROR VARIANCE=0.024  
 EXPECTED LIKELIHOOD=0.228D 05  
 DETERMINANT OF A MATRIX=0.100D-14

## TABLE C-3D... (CONT'D)

## VARIANCE-COVARIANCE MATRIX

0.118D-01	-0.581D-01	0.103D 08
-0.581D-01	0.294D 00	-0.491D 08
0.103D 08	-0.491D 08	0.936D 16

## CORRELATION MATRIX

1.000	-0.986	0.980
-0.986	1.000	-0.936
0.980	-0.936	1.000

TABLE C-4

\*\*\*\*\*

## INPUT DATA FOR CORE SAMPLE NO. 1

RUN 4

THE FOLLOWING 7 DATA POINTS WERE ADDED TO  
THOSE OBTAINED IN RUNS 2 AND 3.

PE (PSIA)	PW (PSIA)	VIS#T#Z (CP-DEG.R)	VIS (CP)	QG (MSCF/DAY)
153.446	14.839	0.947D 01	0.178D-01	0.184D 00
163.578	15.241	0.949D 01	0.178D-01	0.202D 00
173.578	15.551	0.949D 01	0.178D-01	0.221D 00
183.578	15.902	0.949D 01	0.178D-01	0.241D 00
193.578	16.305	0.948D 01	0.178D-01	0.262D 00
203.578	16.711	0.945D 01	0.178D-01	0.282D 00
213.578	17.185	0.945D 01	0.178D-01	0.304D 00

TABLE C-4A

\*\*\*\*\*  
RESULTS USING MODEL 1

## LEAST SQUARE PARAMETERS

K=4.273 MD.  
B=40.266 PSIA.

ERROR VECTOR (PSIA)	DESIGN MATRIX X	
0.628D 01	-0.669D 01	-0.602D 00
0.553D 01	-0.949D 01	-0.721D 00
0.380D 01	-0.111D 02	-0.747D 00
0.294D 01	-0.121D 02	-0.764D 00
0.191D 01	-0.129D 02	-0.771D 00
0.114D 01	-0.137D 02	-0.780D 00
0.215D 00	-0.145D 02	-0.786D 00
-0.249D 00	-0.152D 02	-0.796D 00
-0.104D 01	-0.160D 02	-0.801D 00
-0.164D 01	-0.167D 02	-0.807D 00
-0.203D 01	-0.174D 02	-0.814D 00
-0.246D 01	-0.181D 02	-0.820D 00
-0.297D 01	-0.188D 02	-0.824D 00
-0.291D 01	-0.195D 02	-0.833D 00
-0.318D 01	-0.202D 02	-0.838D 00
-0.320D 01	-0.208D 02	-0.844D 00
-0.293D 01	-0.215D 02	-0.853D 00
-0.341D 01	-0.221D 02	-0.855D 00
-0.344D 01	-0.228D 02	-0.859D 00
-0.341D 01	-0.234D 02	-0.864D 00
-0.291D 01	-0.240D 02	-0.872D 00
-0.287D 01	-0.246D 02	-0.876D 00
-0.260D 01	-0.252D 02	-0.881D 00
-0.163D 01	-0.264D 02	-0.894D 00
0.101D 00	-0.275D 02	-0.907D 00
0.162D 01	-0.286D 02	-0.919D 00
0.284D 01	-0.297D 02	-0.928D 00
0.421D 01	-0.308D 02	-0.936D 00
0.597D 01	-0.319D 02	-0.946D 00
0.754D 01	-0.329D 02	-0.953D 00

SUM OF THE SQUARES OF THE ERRORS=340.312  
 ERROR VARIANCE=12.154  
 EXPECTED LIKELIHOOD=0.237D-34  
 DETERMINANT OF A MATRIX=0.163D 05

## TABLE C-4A... (CONT'D)

## VARIANCE-COVARIANCE MATRIX

0.158D-01	-0.395D 00
-0.395D 00	0.104D 02

## CORRELATION MATRIX

1.000	-0.974
-0.974	1.000

TABLE C-4B

\*\*\*\*\*  
 RESULTS USING MODEL 2

K =9.297 MD.  
 BETA=0.397D 10 1/FT.

ERROR VECTOR  
 (PSIA)

DESIGN MATRIX  
 X

\*\*\*\*\*

-0.121D 01	-0.101D 01	0.187D-11
-0.136D 01	-0.162D 01	0.924D-11
-0.141D 01	-0.207D 01	0.225D-10
-0.121D 01	-0.234D 01	0.361D-10
-0.114D 01	-0.257D 01	0.528D-10
-0.946D 00	-0.279D 01	0.734D-10
-0.860D 00	-0.301D 01	0.996D-10
-0.561D 00	-0.320D 01	0.130D-09
-0.483D 00	-0.340D 01	0.168D-09
-0.337D 00	-0.359D 01	0.212D-09
-0.156D 00	-0.376D 01	0.261D-09
-0.301D-01	-0.393D 01	0.317D-09
-0.107D-01	-0.410D 01	0.385D-09
0.323D 00	-0.426D 01	0.457D-09
0.358D 00	-0.440D 01	0.537D-09
0.514D 00	-0.455D 01	0.627D-09
0.814D 00	-0.468D 01	0.724D-09
0.390D 00	-0.483D 01	0.854D-09
0.323D 00	-0.496D 01	0.977D-09
0.235D 00	-0.509D 01	0.111D-08
0.439D 00	-0.520D 01	0.125D-08
0.217D 00	-0.532D 01	0.141D-08
0.990D-01	-0.543D 01	0.157D-08
0.502D 00	-0.565D 01	0.188D-08
0.101D 01	-0.584D 01	0.225D-08
0.102D 01	-0.602D 01	0.268D-08
0.510D 00	-0.620D 01	0.318D-08
-0.184D 00	-0.636D 01	0.375D-08
-0.616D 00	-0.615D 01	0.433D-08
-0.156D 01	-0.666D 01	0.500D-08

SUM OF THE SQUARES OF THE ERRORS=17.515  
 ERROR VARIANCE=0.626  
 EXPECTED LIKELIHOOD=0.504D-15  
 DETERMINANT OF A MATRIX=0.194D-13



## TABLE C-4B... (CONT'D)

## VARIANCE-COVARIANCE MATRIX

0.004D-02	0.641D 07
0.641D 07	0.201D 17

## CORRELATION MATRIX

1.000	0.820
0.820	1.000

TABLE C-4C

\*\*\*\*\*

## RESULTS USING MODEL 3

## LEAST SQUARE PARAMETERS

K =7.657 MD.  
 B =8.096 PSIA.  
 BETA=0.338D 10 1/FT.

ERROR VECTOR  
 (PSIA)

DESIGN MATRIX  
 X

\*\*\*\*\*

0.329D 00	-0.158D 01	-0.457D 00	0.346D-10
0.241D 00	-0.250D 01	-0.611D 00	0.122D-09
-0.201D-01	-0.314D 01	-0.681D 00	0.239D-09
0.188D-01	-0.353D 01	-0.718D 00	0.341D-09
-0.938D-01	-0.387D 01	-0.742D 00	0.451D-09
-0.675D-01	-0.418D 01	-0.764D 00	0.576D-09
-0.168D 00	-0.448D 01	-0.781D 00	0.719D-09
-0.138D-01	-0.476D 01	-0.799D 00	0.873D-09
-0.108D 00	-0.504D 01	-0.811D 00	0.105D-08
-0.116D 00	-0.531D 01	-0.823D 00	0.125D-08
-0.601D-01	-0.555D 01	-0.833D 00	0.145D-08
-0.588D-01	-0.579D 01	-0.842D 00	0.166D-08
-0.166D 00	-0.604D 01	-0.849D 00	0.191D-08
0.895D-01	-0.626D 01	-0.858D 00	0.216D-08
0.379D-01	-0.647D 01	-0.864D 00	0.242D-08
0.130D 00	-0.668D 01	-0.870D 00	0.270D-08
0.390D 00	-0.688D 01	-0.877D 00	0.299D-08
-0.938D-01	-0.709D 01	-0.878D 00	0.336D-08
-0.194D 00	-0.728D 01	-0.882D 00	0.370D-08
-0.304D 00	-0.747D 01	-0.885D 00	0.404D-08
-0.877D-01	-0.763D 01	-0.890D 00	0.438D-08
-0.310D 00	-0.781D 01	-0.892D 00	0.477D-08
-0.407D 00	-0.798D 01	-0.894D 00	0.515D-08
0.187D-01	-0.831D 01	-0.903D 00	0.582D-08
0.652D 00	-0.860D 01	-0.910D 00	0.659D-08
0.826D 00	-0.887D 01	-0.915D 00	0.744D-08
0.507D 00	-0.914D 01	-0.916D 00	0.836D-08
0.511D-01	-0.939D 01	-0.916D 00	0.934D-08
-0.120D 00	-0.963D 01	-0.917D 00	0.103D-07
-0.763D 00	-0.985D 01	-0.916D 00	0.114D-07

SUM OF THE SQUARES OF THE ERRORS=2.818  
 ERROR VARIANCE=0.104  
 EXPECTED LIKELIHOOD=0.384D-03  
 DETERMINANT OF A MATRIX=0.391D-13

TABLE C-4C... (CONT'D)

VARIANCE-COVARIANCE MATRIX

0.141D-01	-0.798D-01	0.684D 07
-0.798D-01	0.466D 00	-0.369D 08
0.684D 07	-0.369D 08	0.374D 16

CORRELATION MATRIX

1.000	-0.984	0.942
-0.984	1.000	-0.884
0.942	-0.884	1.000

TABLE C-4D

\*\*\*\*\*

## RESULTS USING MODEL 4

## LEAST SQUARE PARAMETERS

K =7.868 MD.  
 B =7.590 PSIA.  
 BETA=0.316D 10 1/FT.

ERROR VECTOR  
 (PSIA)

DESIGN MATRIX  
 X

\*\*\*\*\*

0.292D 00	-0.114D 01	-0.335D 00	0.388D-10
0.260D 00	-0.194D 01	-0.478D 00	0.141D-09
0.488D-01	-0.253D 01	-0.546D 00	0.276D-09
0.760D-01	-0.289D 01	-0.576D 00	0.393D-09
-0.298D-01	-0.320D 01	-0.595D 00	0.517D-09
-0.177D-01	-0.349D 01	-0.609D 00	0.656D-09
-0.120D 00	-0.378D 01	-0.619D 00	0.814D-09
0.542D-02	-0.404D 01	-0.625D 00	0.982D-09
-0.922D-01	-0.430D 01	-0.628D 00	0.117D-08
-0.111D 00	-0.456D 01	-0.630D 00	0.138D-08
-0.711D-01	-0.479D 01	-0.630D 00	0.160D-08
-0.775D-01	-0.502D 01	-0.628D 00	0.183D-08
-0.183D 00	-0.525D 01	-0.625D 00	0.209D-08
0.482D-01	-0.547D 01	-0.622D 00	0.235D-08
-0.163D-02	-0.567D 01	-0.617D 00	0.263D-08
0.832D-01	-0.587D 01	-0.613D 00	0.292D-08
0.328D 00	-0.606D 01	-0.608D 00	0.322D-08
-0.124D 00	-0.627D 01	-0.600D 00	0.361D-08
-0.213D 00	-0.645D 01	-0.594D 00	0.396D-08
-0.310D 00	-0.663D 01	-0.588D 00	0.432D-08
-0.101D 00	-0.680D 01	-0.583D 00	0.468D-08
-0.304D 00	-0.697D 01	-0.576D 00	0.508D-08
-0.390D 00	-0.714D 01	-0.570D 00	0.549D-08
0.253D-01	-0.746D 01	-0.563D 00	0.621D-08
0.647D 00	-0.775D 01	-0.552D 00	0.703D-08
0.822D 00	-0.802D 01	-0.541D 00	0.795D-08
0.517D 00	-0.829D 01	-0.530D 00	0.894D-08
0.727D-01	-0.855D 01	-0.519D 00	0.100D-07
-0.105D 00	-0.879D 01	-0.510D 00	0.111D-07
-0.744D 00	-0.902D 01	-0.500D 00	0.123D-07

SUM OF THE SQUARES OF THE ERRORS=2.705  
 ERROR VARIANCE=0.100  
 EXPECTED LIKELIHOOD=0.580D-03  
 DETERMINANT OF A MATRIX=0.342D-13

TABLE C-4D... (CONT'D)

## VARIANCE-COVARIANCE MATRIX

0.109D-01	-0.654D-01	0.638D 07
-0.654D-01	0.412D 00	-0.364D 08
0.638D 07	-0.364D 08	0.399D 16

## CORRELATION MATRIX

1.000	-0.976	0.967
-0.976	1.000	-0.898
0.967	-0.898	1.000

TABLE C-5

\*\*\*\*\*

INPUT DATA FOR CORE SAMPLE NO. 1  
RUN 5

PE (PSIA)	PW (PSIA)	VIS* <sup>T</sup> *Z (CP-DEG.R)	VIS (CP)	QG (MSCF/DAY)
58.520	53.420	0.943D 01	0.177D-01	0.690D-02
63.520	53.720	0.944D 01	0.177D-01	0.130D-01
73.520	53.620	0.945D 01	0.177D-01	0.273D-01
83.520	53.120	0.947D 01	0.178D-01	0.426D-01
93.520	53.420	0.948D 01	0.178D-01	0.578D-01
103.520	53.520	0.948D 01	0.178D-01	0.737D-01
113.520	53.520	0.949D 01	0.178D-01	0.904D-01
128.520	53.920	0.949D 01	0.178D-01	0.117D 00
143.516	53.666	0.950D 01	0.178D-01	0.145D 00
163.516	53.516	0.951D 01	0.178D-01	0.185D 00
183.516	53.716	0.951D 01	0.178D-01	0.225D 00
203.539	53.539	0.938D 01	0.177D-01	0.272D 00
243.539	54.339	0.941D 01	0.178D-01	0.364D 00
263.539	53.589	0.941D 01	0.178D-01	0.413D 00

TABLE 6-5A

\*\*\*\*\*  
 RESULTS USING MODEL 1

## LEAST SQUARE PARAMETERS

K=2.468 MD.  
 B=166.412 PSIA.

ERROR VECTOR (PSIA)	DESIGN MATRIX X	
0.287D-01	-0.784D 01	-0.872D-01
0.110D 01	-0.139D 02	-0.157D 00
0.115D 01	-0.252D 02	-0.275D 00
0.131D 01	-0.346D 02	-0.370D 00
0.942D 00	-0.417D 02	-0.433D 00
0.879D 00	-0.480D 02	-0.487D 00
0.641D 00	-0.536D 02	-0.532D 00
-0.107D 01	-0.605D 02	-0.576D 00
-0.133D 01	-0.671D 02	-0.620D 00
-0.169D 01	-0.748D 02	-0.666D 00
-0.142D 01	-0.816D 02	-0.702D 00
-0.979D 00	-0.880D 02	-0.733D 00
0.337D 00	-0.993D 02	-0.778D 00
0.233D 01	-0.105D 03	-0.804D 00

SUM OF THE SQUARES OF THE ERRORS=20.605  
 ERROR VARIANCE=1.718  
 EXPECTED LIKELIHOOD=0.725D-10  
 DETERMINANT OF A MATRIX=0.277D 04  
 VARIANCE-COVARIANCE MATRIX

0.273D-02	-0.312D 00
-0.312D 00	0.360D 02

## CORRELATION MATRIX

1.000	-0.995
-0.995	1.000

TABLE C-5B

\*\*\*\*\*  
 RESULTS USING MODEL 2

## LEAST SQUARE PARAMETERS

K =8.376 MD.  
 BETA=0.306D 10 1/FT.

ERROR VECTOR (PSIA)	DESIGN MATRIX X	
*****	*****	*****
-0.884D 00	-0.671D 00	0.309D-11
-0.107D 01	-0.116D 01	0.110D-10
-0.161D 01	-0.210D 01	0.482D-10
-0.167D 01	-0.290D 01	0.117D-09
-0.141D 01	-0.353D 01	0.214D-09
-0.917D 00	-0.410D 01	0.346D-09
-0.421D 00	-0.461D 01	0.519D-09
-0.859D-01	-0.530D 01	0.865D-09
0.543D 00	-0.591D 01	0.132D-08
0.115D 01	-0.661D 01	0.211D-08
0.172D 01	-0.721D 01	0.310D-08
0.114D 01	-0.773D 01	0.447D-08
-0.105D 00	-0.860D 01	0.780D-08
-0.125D 01	-0.898D 01	0.992D-08

SUM OF THE SQUARES OF THE ERRORS=17.805  
 ERROR VARIANCE=1.218  
 EXPECTED LIKELIHOOD=0.203D-09  
 DETERMINANT OF A MATRIX=0.263D-13  
 VARIANCE-COVARIANCE MATRIX

0.111D-01	0.138D 08
0.138D 08	0.248D 17

## CORRELATION MATRIX

1.000	0.832
0.832	1.000



TABLE C-5C

\*\*\*\*\*  
 RESULTS USING MODEL 3

## LEAST SQUARE PARAMETERS

K =3.883 MD.  
 B =81.135 PSIA.  
 BETA=0.164D 10 1/FT.

ERROR VECTOR  
 (PSIA)

DESIGN MATRIX  
 X

\*\*\*\*\*

-0.387D 00	-0.315D 01	-0.866D-01	0.213D-10
0.706D-01	-0.551D 01	-0.154D 00	0.703D-10
-0.120D 00	-0.999D 01	-0.270D 00	0.268D-09
-0.297D-01	-0.138D 02	-0.364D 00	0.575D-09
-0.435D-01	-0.167D 02	-0.429D 00	0.944D-09
0.197D 00	-0.193D 02	-0.484D 00	0.139D-08
0.343D 00	-0.216D 02	-0.530D 00	0.191D-08
-0.329D 00	-0.246D 02	-0.579D 00	0.282D-08
-0.162D 00	-0.274D 02	-0.625D 00	0.388D-08
-0.843D-01	-0.305D 02	-0.672D 00	0.550D-08
0.264D 00	-0.333D 02	-0.708D 00	0.731D-08
0.756D-01	-0.358D 02	-0.737D 00	0.956D-08
-0.169D 00	-0.400D 02	-0.776D 00	0.143D-07
0.556D-01	-0.420D 02	-0.797D 00	0.170D-07

SUM OF THE SQUARES OF THE ERRORS=0.577  
 ERROR VARIANCE=0.052  
 EXPECTED LIKELIHOOD=0.485D 01  
 DETERMINANT OF A MATRIX=0.332D-14  
 VARIANCE-COVARIANCE MATRIX

0.135D-01	-0.523D 00	0.957D 07
-0.523D 00	0.202D 02	-0.367D 09
0.957D 07	-0.367D 09	0.703D 16

CORRELATION MATRIX

1.000	-1.000	0.982
-1.000	1.000	-0.974
0.982	-0.974	1.000

TABLE C-5D

\*\*\*\*\*

## RESULTS USING MODEL 4

## LEAST SQUARE PARAMETERS

K =4.724 MD.  
 B =56.829 PSIA.  
 BETA=0.148D 10 1/FT.

ERROR VECTOR  
(PSIA)DESIGN MATRIX  
X

\*\*\*\*\*

-0.171D 00	-0.108D 01	-0.454D-01	0.222D-10
0.202D-01	-0.196D 01	-0.803D-01	0.748D-10
-0.681D-01	-0.381D 01	-0.149D 00	0.299D-09
-0.360D-01	-0.553D 01	-0.208D 00	0.665D-09
-0.247D-01	-0.699D 01	-0.252D 00	0.111D-08
0.121D 00	-0.836D 01	-0.290D 00	0.166D-08
0.213D 00	-0.965D 01	-0.322D 00	0.231D-08
-0.161D 00	-0.115D 02	-0.360D 00	0.344D-08
-0.898D-01	-0.131D 02	-0.390D 00	0.476D-08
-0.715D-01	-0.152D 02	-0.419D 00	0.672D-08
0.199D 00	-0.169D 02	-0.438D 00	0.886D-08
-0.573D-01	-0.186D 02	-0.450D 00	0.115D-07
-0.874D-01	-0.216D 02	-0.461D 00	0.167D-07
0.617D-01	-0.230D 02	-0.463D 00	0.197D-07

SUM OF THE SQUARES OF THE ERRORS=0.191

ERROR VARIANCE=0.017

EXPECTED LIKELIHOOD=0.108D 05

DETERMINANT OF A MATRIX=0.390D-15

VARIANCE-COVARIANCE MATRIX

0.193D-01	-0.508D 00	0.109D 08
-0.508D 00	0.134D 02	-0.286D 09
0.109D 08	-0.286D 09	0.624D 16

CORRELATION MATRIX

1.000	-0.999	0.993
-0.999	1.000	-0.989
0.993	-0.989	1.000

TABLE C-6

\*\*\*\*\*  
 INPUT DATA FOR CORE SAMPLE NO. 1  
 RUN 6

PE (PSIA)	PW (PSIA)	VIS*T*Z (CP-DEG.R)	VIS (CP)	QG (MSCF/DAY)
43.366	33.586	0.941D 01	0.177D-01	0.895D-02
53.363	33.228	0.942D 01	0.177D-01	0.199D-01
63.553	33.466	0.943D 01	0.177D-01	0.317D-01
73.553	33.547	0.946D 01	0.177D-01	0.444D-01
83.253	33.567	0.946D 01	0.177D-01	0.580D-01
93.253	33.538	0.946D 01	0.178D-01	0.730D-01
103.153	33.451	0.947D 01	0.178D-01	0.884D-01
112.347	33.503	0.947D 01	0.178D-01	0.103D 00
123.547	33.532	0.948D 01	0.178D-01	0.123D 00
133.547	33.609	0.948D 01	0.178D-01	0.141D 00
143.797	33.561	0.948D 01	0.178D-01	0.160D 00
153.547	33.483	0.949D 01	0.178D-01	0.179D 00
173.632	33.584	0.941D 01	0.177D-01	0.216D 00
193.626	33.640	0.941D 01	0.178D-01	0.256D 00
213.626	33.636	0.942D 01	0.178D-01	0.302D 00
233.626	33.602	0.943D 01	0.178D-01	0.347D 00
253.620	33.538	0.944D 01	0.178D-01	0.392D 00
263.624	33.619	0.944D 01	0.178D-01	0.416D 00

TABLE C-6A

\*\*\*\*\*  
 RESULTS USING MODEL 1

LEAST SQUARE PARAMETERS  
 K=2.990 MD.  
 B=102.696 PSIA.

ERROR VECTOR (PSIA)	DESIGN MATRIX X	
0.413D 01	-0.103D 02	-0.249D 00
0.482D 01	-0.186D 02	-0.415D 00
0.420D 01	-0.242D 02	-0.507D 00
0.310D 01	-0.286D 02	-0.568D 00
0.159D 01	-0.322D 02	-0.608D 00
0.574D-02	-0.356D 02	-0.640D 00
-0.101D 01	-0.386D 02	-0.669D 00
-0.160D 01	-0.412D 02	-0.692D 00
-0.351D 01	-0.441D 02	-0.708D 00
-0.395D 01	-0.466D 02	-0.727D 00
-0.436D 01	-0.491D 02	-0.744D 00
-0.530D 01	-0.513D 02	-0.756D 00
-0.202D 01	-0.557D 02	-0.797D 00
-0.128D 01	-0.598D 02	-0.821D 00
-0.260D 00	-0.638D 02	-0.842D 00
0.118D 01	-0.676D 02	-0.861D 00
0.401D 01	-0.713D 02	-0.882D 00
0.541D 01	-0.730D 02	-0.891D 00

SUM OF THE SQUARES OF THE ERRORS=201.301  
 ERROR VARIANCE=12.585  
 EXPECTED LIKELIHOOD=0.139D-20  
 DETERMINANT OF A MATRIX=0.923D 04  
 VARIANCE-COVARIANCE MATRIX

0.123D-01	-0.830D 00
-0.830D 00	0.576D 02

CORRELATION MATRIX

1.000	-0.986
-0.986	1.000

TABLE C-6B

\*\*\*\*\*

## RESULTS USING MODEL 2

## LEAST SQUARE PARAMETERS

K =8.706 MD.  
 BETA=0.334D 10 1/FT.

ERROR VECTOR  
 (PSIA)

DESIGN MATRIX  
 X

\*\*\*\*\*

-0.106D 01	-0.107D 01	0.500D-11
-0.163D 01	-0.193D 01	0.247D-10
-0.160D 01	-0.260D 01	0.623D-10
-0.138D 01	-0.317D 01	0.122D-09
-0.118D 01	-0.368D 01	0.207D-09
-0.974D 00	-0.415D 01	0.328D-09
-0.578D 00	-0.457D 01	0.479D-09
-0.709D-01	-0.492D 01	0.649D-09
-0.186D 00	-0.534D 01	0.920D-09
0.199D 00	-0.567D 01	0.120D-08
0.426D 00	-0.598D 01	0.153D-08
0.137D 00	-0.627D 01	0.192D-08
0.201D 01	-0.672D 01	0.278D-08
0.190D 01	-0.717D 01	0.390D-08
0.118D 01	-0.758D 01	0.528D-08
0.391D-02	-0.794D 01	0.692D-08
-0.816D 00	-0.825D 01	0.877D-08
-0.146D 01	-0.839D 01	0.979D-08

SUM OF THE SQUARES OF THE ERRORS=23.010

ERROR VARIANCE=1.440

EXPECTED LIKELIHOOD=0.412D-12

DETERMINANT OF A MATRIX=0.516D-13

VARIANCE-COVARIANCE MATRIX

0.783D-02      0.940D 07

0.940D 07      0.164D 17

CORRELATION MATRIX

1.000      0.830

0.830      1.000

TABLE C-6C

RESULTS USING MODEL 3

\*\*\*\*\*

LEAST SQUARE PARAMETERS

K =5.983 MD.  
B =25.462 PSIA.  
BETA=0.264D 10 1/FT.

ERROR VECTOR  
(PSIA)

DESIGN MATRIX  
X

\*\*\*\*\*

0.348D 00	-0.235D 01	-0.227D 00	0.491D-10
0.199D 00	-0.423D 01	-0.379D 00	0.197D-09
0.174D 00	-0.566D 01	-0.475D 00	0.418D-09
0.150D 00	-0.686D 01	-0.545D 00	0.709D-09
-0.237D-01	-0.790D 01	-0.597D 00	0.107D-08
-0.236D 00	-0.886D 01	-0.639D 00	0.151D-08
-0.185D 00	-0..72D 01	-0.675D 00	0.200D-08
0.405D-01	-0.104D 02	-0.702D 00	0.250D-08
-0.551D 00	-0.113D 02	-0.725D 00	0.323D-08
-0.409D 00	-0.119D 02	-0.746D 00	0.390D-08
-0.387D 00	-0.126D 02	-0.765D 00	0.467D-08
-0.898D 00	-0.132D 02	-0.777D 00	0.548D-08
0.120D 01	-0.142D 02	-0.812D 00	0.715D-08
0.114D 01	-0.152D 02	-0.831D 00	0.908D-08
0.648D 00	-0.160D 02	-0.845D 00	0.112D-07
-0.110D 00	-0.168D 02	-0.856D 00	0.136D-07
-0.282D 00	-0.175D 02	-0.867D 00	0.160D-07
-0.562D 00	-0.178D 02	-0.871D 00	0.172D-07

SUM OF THE SQUARES OF THE ERRORS=5.295  
ERROR VARIANCE=0.353  
EXPECTED LIKELIHOOD=0.213D-06  
DETERMINANT OF A MATRIX=0.150D-13  
VARIANCE-COVARIANCE MATRIX

0.730D-01	-0.969D 00	0.293D 08
-0.969D 00	0.130D 02	-0.380D 09
0.293D 08	-0.380D 09	0.128D 17

TABLE C-6C... (CONT'D)

## CORRELATION MATRIX

1.000	-0.995	0.959
-0.995	1.000	-0.932
0.959	-0.932	1.000

TABLE C-6D

\*\*\*\*\*  
 RESULTS USING MODEL 4

## LEAST SQUARE PARAMETERS

K =6.532 MD.  
 B =20.696 PSIA.  
 BETA=0.245D 10 1/FT.

ERROR VECTOR  
 (PSIA)

DESIGN MATRIX  
 X

\*\*\*\*\*

0.263D 00	-0.133D 01	-0.147D 00	0.523D-10
0.214D 00	-0.255D 01	-0.260D 00	0.219D-09
0.235D 00	-0.357D 01	-0.335D 00	0.474D-09
0.234D 00	-0.448D 01	-0.390D 00	0.813D-09
0.978D-01	-0.529D 01	-0.429D 00	0.123D-08
-0.918D-01	-0.606D 01	-0.459D 00	0.173D-08
-0.887D-01	-0.675D 01	-0.479D 00	0.229D-08
0.637D-01	-0.734D 01	-0.491D 00	0.285D-08
-0.466D 00	-0.804D 01	-0.502D 00	0.365D-08
-0.383D 00	-0.860D 01	-0.506D 00	0.439D-08
-0.403D 00	-0.914D 01	-0.509D 00	0.521D-08
-0.875D 00	-0.966D 01	-0.509D 00	0.608D-08
0.891D 00	-0.105D 02	-0.503D 00	0.782D-08
0.855D 00	-0.113D 02	-0.494D 00	0.984D-08
0.496D 00	-0.121D 02	-0.483D 00	0.121D-07
-0.985D-01	-0.128D 02	-0.470D 00	0.145D-07
-0.163D 00	-0.134D 02	-0.458D 00	0.170D-07
-0.364D 00	-0.137D 02	-0.452D 00	0.183D-07

SUM OF THE SQUARES OF THE ERRORS=3.516

ERROR VARIANCE=0.234

EXPECTED LIKELIHOOD=0.615D-04

DETERMINANT OF A MATRIX=0.680D-14

VARIANCE-COVARIANCE MATRIX

0.640D-01	-0.766D 00	0.316D 08
-0.766D 00	0.932D 01	-0.372D 09
0.316D 08	-0.372D 09	0.161D 17



TABLE C-6D... (CONT'D)

## CORRELATION MATRIX

1.000	-0.992	0.984
-0.992	1.000	-0.960
0.984	-0.960	1.000

TABLE C-7

```

*****
INPUT DATA FOR CORE SAMPLE NO. 2
*****
PE          PW          VIS*T#Z      VIS          QG
*****
23.610     13.610     0.934D 01   0.176D-01   0.675D-03
29.620     13.620     0.936D 01   0.176D-01   0.121D-02
33.625     13.625     0.933D 01   0.176D-01   0.163D-02
35.610     13.610     0.934D 01   0.176D-01   0.184D-02
41.625     13.625     0.933D 01   0.176D-01   0.254D-02
53.610     13.610     0.934D 01   0.176D-01   0.427D-02
73.610     13.610     0.933D 01   0.176D-01   0.809D-02
93.654     13.655     0.931D 01   0.176D-01   0.128D-01
113.654    13.655     0.932D 01   0.176D-01   0.186D-01
133.654    13.656     0.933D 01   0.177D-01   0.254D-01
153.654    13.657     0.933D 01   0.177D-01   0.329D-01
173.654    13.657     0.934D 01   0.177D-01   0.417D-01
193.654    13.658     0.935D 01   0.177D-01   0.511D-01
213.654    13.658     0.936D 01   0.177D-01   0.615D-01

```

TABLE C-7A

\*\*\*\*\*  
 RESULTS USING MODEL 1

## LEAST SQUARE PARAMETERS

K=0.666 MD.  
 B=8.962 PSIA.

ERROR VECTOR (PSIA)	DESIGN MATRIX X	
0.931D 00	-0.172D 02	-0.431D 00
0.403D 00	-0.245D 02	-0.548D 00
0.226D 00	-0.289D 02	-0.599D 00
0.293D 00	-0.309D 02	-0.623D 00
0.546D 00	-0.366D 02	-0.682D 00
0.322D 00	-0.475D 02	-0.751D 00
-0.531D 00	-0.646D 02	-0.809D 00
-0.508D 00	-0.806D 02	-0.850D 00
-0.710D 00	-0.963D 02	-0.874D 00
-0.733D 00	-0.112D 03	-0.893D 00
-0.198D 00	-0.127D 03	-0.910D 00
-0.219D 00	-0.142D 03	-0.920D 00
0.307D 00	-0.157D 03	-0.931D 00
0.967D 00	-0.172D 03	-0.940D 00

SUM OF THE SQUARES OF THE ERRORS=3.552  
 ERROR VARIANCE=0.296  
 EXPECTED LIKELIHOOD=0.164D-04  
 DETERMINANT OF A MATRIX=0.160D 06  
 VARIANCE-COVARIANCE MATRIX

0.160D-04	-0.181D-02
-0.181D-02	0.239D 00

## CORRELATION MATRIX

1.000	-0.926
-0.926	1.000

TABLE C-7B

\*\*\*\*\*

## RESULTS USING MODEL 2

## LEAST SQUARE PARAMETERS

K =0.812 MD.  
 BETA=0.136D 12 1/FT.

ERROR VECTOR  
(PSIA)DESIGN MATRIX  
X

\*\*\*\*\*

-0.136D 01	-0.108D 02	0.210D-14
-0.155D 01	-0.155D 02	0.672D-14
-0.169D 01	-0.184D 02	0.123D-13
-0.162D 01	-0.198D 02	0.157D-13
-0.128D 01	-0.236D 02	0.297D-13
-0.994D 00	-0.312D 02	0.839D-13
-0.768D 00	-0.434D 02	0.301D-12
0.117D 00	-0.547D 02	0.757D-12
0.565D 00	-0.657D 02	0.159D-11
0.836D 00	-0.762D 02	0.295D-11
0.123D 01	-0.862D 02	0.496D-11
0.619D 00	-0.962D 02	0.792D-11
0.274D-01	-0.106D 03	0.119D-10
-0.979D 00	-0.115D 03	0.172D-10

SUM OF THE SQUARES OF THE ERRORS=16.856

ERROR VARIANCE=1.405

EXPECTED LIKELIHOOD=0.297D-09

DETERMINANT OF A MATRIX=0.833D-17

VARIANCE-COVARIANCE MATRIX

0.903D-04      0.809D 09

0.809D 09      0.987D 22

CORRELATION MATRIX

1.000      0.857

0.857      1.000

TABLE C-7C

\*\*\*\*\*

## RESULTS USING MODEL 3

## LEAST SQUARE PARAMETERS

K =0.706 MD.  
 B =6.580 PSIA.  
 BETA=0.507D 11 1/FT.

ERROR VECTOR  
 (PSIA)

DESIGN MATRIX  
 X

\*\*\*\*\*

0.646D-02	-0.151D 02	-0.424D 00	0.112D-12
0.357D-02	-0.215D 02	-0.540D 00	0.285D-12
-0.139D 00	-0.255D 02	-0.592D 00	0.457D-12
-0.604D-01	-0.273D 02	-0.617D 00	0.552D-12
0.246D 00	-0.324D 02	-0.677D 00	0.902D-12
0.209D 00	-0.422D 02	-0.749D 00	0.198D-11
-0.301D 00	-0.578D 02	-0.812D 00	0.513D-11
-0.295D-01	-0.722D 02	-0.854D 00	0.102D-10
-0.773D-01	-0.863D 02	-0.879D 00	0.176D-10
-0.823D-01	-0.100D 03	-0.897D 00	0.278D-10
0.311D-01	-0.113D 03	-0.913D 00	0.409D-10
-0.233D-01	-0.127D 03	-0.921D 00	0.577D-10
-0.498D-02	-0.140D 03	-0.929D 00	0.780D-10
-0.589D-01	-0.152D 03	-0.936D 00	0.102D-09

SUM OF THE SQUARES OF THE ERRORS=0.332

ERROR VARIANCE=0.030

EXPECTED LIKELIHOOD=0.231D 03

DETERMINANT OF A MATRIX=0.156D-15

VARIANCE-COVARIANCE MATRIX

0.187D-04	-0.115D-02	0.200D 08
-0.115D 02	0.784D-01	-0.114D 10
0.200D 08	-0.114D 10	0.242D 20

CORRELATION MATRIX

1.000	-0.950	0.940
-0.950	1.000	-0.828
0.940	-0.828	1.000

TABLE C-7D

\*\*\*\*\*  
 RESULTS USING MODEL 4

## LEAST SQUARE PARAMETERS

K =0.708 MD.  
 B =6.507 PSIA.  
 BETA=0.490D 11 1/FT.

ERROR VECTOR  
 (PSIA)

DESIGN MATRIX  
 X

\*\*\*\*\*

0.619D-02	-0.118D 02	-0.332D 00	0.123D-12
0.391D-02	-0.176D 02	-0.442D 00	0.318D-12
-0.115D 00	-0.213D 02	-0.499D 00	0.514D-12
-0.493D-01	-0.230D 02	-0.522D 00	0.622D-12
0.216D 00	-0.278D 02	-0.578D 00	0.102D-11
0.193D 00	-0.374D 02	-0.661D 00	0.222D-11
-0.267D 00	-0.528D 02	-0.742D 00	0.571D-11
-0.203D-01	-0.671D 02	-0.785D 00	0.112D-10
-0.705D-01	-0.811D 02	-0.812D 00	0.192D-10
-0.829D-01	-0.948D 02	-0.827D 00	0.300D-10
0.285D 00	-0.108D 03	-0.836D 00	0.437D-10
-0.352D-01	-0.121D 03	-0.841D 00	0.613D-10
-0.119D-02	-0.134D 03	-0.842D 00	0.824D-10
-0.482D-01	-0.147D 03	-0.840D 00	0.107D-09

SUM OF THE SQUARES OF THE ERRORS=0.276  
 ERROR VARIANCE=0.025  
 EXPECTED LIKELIHOOD=0.838D 03  
 DETERMINANT OF A MATRIX=0.104D-15  
 VARIANCE-COVARIANCE MATRIX

0.201D-04	-0.126D-02	0.202D 08
-0.126D-02	0.871D-01	-0.117D 10
0.202D 08	-0.117D 10	0.222D 20

CORRELATION MATRIX

1.000	-0.952	0.956
-0.952	1.000	-0.841
0.956	-0.841	1.000

TABLE C-8

\*\*\*\*\*

## INPUT DATA FOR CORE SAMPLE NO. 111

PE (PSIA)	PW (PSIA)	VIS*T#Z (CP-DEG.R)	VIS (CP)	QG (MSCF/DAY)
20.796	18.070	0.922D 01	0.175D-01	0.260D-02
25.069	20.840	0.923D 01	0.175D-01	0.445D-02
26.783	21.975	0.922D 01	0.175D-01	0.530D-02
20.486	13.610	0.914D 01	0.175D-01	0.610D-02
22.038	13.689	0.919D 01	0.175D-01	0.757D-02
46.987	13.487	0.922D 01	0.175D-01	0.429D-01
58.477	13.477	0.922D 01	0.175D-01	0.645D-01
70.973	13.473	0.924D 01	0.176D-01	0.937D-01
80.473	13.473	0.925D 01	0.176D-01	0.118D 00
93.973	13.473	0.925D 01	0.176D-01	0.158D 00
105.521	13.522	0.923D 01	0.176D-01	0.196D 00
134.521	13.522	0.924D 01	0.176D-01	0.303D 00
164.521	13.522	0.925D 01	0.176D-01	0.443D 00
195.521	13.522	0.925D 01	0.176D-01	0.604D 00
223.521	13.522	0.926D 01	0.176D-01	0.767D 00
252.521	13.522	0.927D 01	0.177D-01	0.957D 00

TABLE C-8A

\*\*\*\*\*

## RESULTS USING MODEL 1

## LEAST SQUARE PARAMETERS

K=2.444 MD.  
B=18.411 PSIA.

ERROR VECTOR  
(PSIA)

DESIGN MATRIX  
X

\*\*\*\*\*

0.326D 00	-0.193D 01	-0.133D 00
0.357D 00	-0.275D 01	-0.171D 00
0.341D 00	-0.304D 01	-0.182D 00
0.965D 00	-0.471D 01	-0.352D 00
0.106D 01	-0.546D 01	-0.398D 00
0.813D 00	-0.141D 02	-0.726D 00
0.881D 00	-0.170D 02	-0.781D 00
0.911D-02	-0.201D 02	-0.810D 00
-0.252D 00	-0.223D 02	-0.830D 00
-0.655D 00	-0.254D 02	-0.851D 00
-0.102D 01	-0.280D 02	-0.864D 00
-0.777D 00	-0.341D 02	-0.894D 00
-0.170D 01	-0.406D 02	-0.908D 00
-0.775D 00	-0.470D 02	-0.927D 00
0.640D 00	-0.525D 02	-0.942D 00
0.188D 01	-0.583D 02	-0.954D 00

SUM OF THE SQUARES OF THE ERRORS=13.406  
 ERROR VARIANCE=0.958  
 EXPECTED LIKELIHOOD=0.266D-09  
 DETERMINANT OF A MATRIX=0.152D 05  
 VARIANCE-COVARIANCE MATRIX

0.543D-03	-0.206D-01
-0.206D-01	0.889D 00

CORRELATION MATRIX

1.000	-0.938
-0.938	1.000



TABLE C-8B

\*\*\*\*\*

RESULTS USING MODEL 2

LEAST SQUARE PARAMETERS

K =3.451 MD.  
 BETA=0.156D 11 1/FT.

ERROR VECTOR  
 (PSIA)

DESIGN MATRIX  
 X

\*\*\*\*\*

-0.725D 00	-0.919D 00	0.262D-13
-0.810D 00	-0.132D 01	0.772D-13
-0.818D 00	-0.146D 01	0.108D-12
-0.196D 01	-0.205D 01	0.144D-12
-0.215D 01	-0.238D 01	0.222D-12
-0.265D 01	-0.657D 01	0.674D-11
-0.184D 01	-0.815D 01	0.148D-10
-0.149D 01	-0.987D 01	0.300D-10
-0.930D 00	-0.111D 02	0.463D-10
-0.208D 00	-0.128D 02	0.787D-10
0.297D 00	-0.142D 02	0.116D-09
0.197D 01	-0.175D 02	0.253D-09
0.160D 01	-0.208D 02	0.486D-09
0.151D 01	-0.238D 02	0.818D-09
0.677D 00	-0.263D 02	0.128D-08
-0.185D 01	-0.288D 02	0.174D-08

SUM OF THE SQUARES OF THE ERRORS=36.509  
 ERROR VARIANCE=2.608  
 EXPECTED LIKELIHOOD=0.878D-13  
 DETERMINANT OF A MATRIX=0.507D-14  
 VARIANCE-COVARIANCE MATRIX

0.282D-02	0.615D 08
0.615D 08	0.182D 19

CORRELATION MATRIX

1.000	0.858
0.858	1.000

TABLE C-8C

\*\*\*\*\*

## RESULTS USING MODEL 3

## LEAST SQUARE PARAMETERS

K =2.737 MD.  
 B =12.219 PSIA.  
 BETA=0.641D 10 1/FT.

ERROR VECTOR  
(PSIA)DESIGN MATRIX  
X

\*\*\*\*\*

0.807D-02	-0.151D 01	-0.131D 00	0.158D-12
0.170D-01	-0.216D 01	-0.169D 00	0.389D-12
0.105D-01	-0.240D 01	-0.180D 00	0.511D-12
0.329D-01	-0.358D 01	-0.336D 00	0.884D-12
0.386D-01	-0.415D 01	-0.380D 00	0.127D-11
-0.999D-01	-0.110D 02	-0.711D 00	0.190D-10
0.279D 00	-0.134D 02	-0.773D 00	0.349D-10
-0.131D 00	-0.160D 02	-0.809D 00	0.603D-10
-0.904D-01	-0.178D 02	-0.832D 00	0.847D-10
-0.961D-01	-0.204D 02	-0.856D 00	0.129D-09
-0.167D 00	-0.225D 02	-0.870D 00	0.177D-09
0.506D 00	-0.275D 02	-0.903D 00	0.335D-09
-0.373D 00	-0.327D 02	-0.916D 00	0.581D-09
-0.320D-01	-0.376D 02	-0.931D 00	0.910D-09
0.315D 00	-0.418D 02	-0.941D 00	0.128D-08
-0.158D 00	-0.461D 02	-0.946D 00	0.177D-08

SUM OF THE SQUARES OF THE ERRORS=0.674  
 ERROR VARIANCE=0.052  
 EXPECTED LIKELIHOOD=0.590D 01  
 DETERMINANT OF A MATRIX=0.507D-14  
 VARIANCE-COVARIANCE MATRIX

0.493D-03	-0.987D-02	0.868D 07
-0.987D-02	0.209D 00	-0.164D 09
0.868D 07	-0.164D 09	0.169D 18

## CORRELATION MATRIX

1.000	-0.972	0.951
-0.972	1.000	-0.873
0.951	-0.873	1.000

TABLE C-8D

\*\*\*\*\*

RESULTS USING MODEL 4

LEAST SQUARE PARAMETERS

K =2.757 MD.  
B =11.986 PSIA.  
BETA=0.607D 10 1/FT.

ERROR VECTOR  
(PSIA)

DESIGN MATRIX  
X

\*\*\*\*\*

0.179D-01	-0.944D 00	-0.829D-01	0.163D-12
-0.101D 00	-0.148D 01	-0.117D 00	0.401D-12
0.202D-01	-0.163D 01	-0.124D 00	0.530D-12
0.119D-01	-0.222D 01	-0.211D 00	0.972D-12
0.279D-01	-0.265D 01	-0.245D 00	0.141D-11
-0.748D-01	-0.866D 01	-0.565D 00	0.225D-10
0.245D 00	-0.110D 02	-0.631D 00	0.409D-10
-0.108D 00	-0.135D 02	-0.683D 00	0.702D-10
-0.352D-01	-0.153D 02	-0.711D 00	0.977D-10
-0.635D-01	-0.178D 02	-0.741D 00	0.147D-09
-0.157D 00	-0.198D 02	-0.759D 00	0.200D-09
0.449D 00	-0.248D 02	-0.788D 00	0.374D-09
-0.363D 00	-0.299D 02	-0.801D 00	0.635D-09
-0.411D-01	-0.348D 02	-0.804D 00	0.982D-09
0.274D 00	-0.390D 02	-0.801D 00	0.137D-08
-0.124D 00	-0.432D 02	-0.794D 00	0.187D-08

SUM OF THE SQUARES OF THE ERRORS=0.539  
ERROR VARIANCE=0.042  
EXPECTED LIKELIHOOD=0.252D 02  
DETERMINANT OF A MATRIX=0.148D-14  
VARIANCE-COVARIANCE MATRIX

0.608D-03	-0.122D-01	0.992D 07
-0.122D-01	0.260D 00	-0.191D 09
0.992D 07	-0.191D 09	0.174D 18

CORRELATION MATRIX

1.000	-0.970	0.964
-0.970	1.000	-0.898
0.964	-0.898	1.000

TABLE C-9

\*\*\*\*\*

## INPUT DATA FOR CORE SAMPLE NO. 112

PE (PSIA)	PW (PSIA)	VIS*T*Z (CP-DEG.R)	VIS (CP)	QG (MSCF/DAY)
15.349	13.320	0.926D 01	0.176D-01	0.510D-02
15.992	13.438	0.922D 01	0.175D-01	0.661D-02
18.399	13.438	0.923D 01	0.175D-01	0.136D-01
20.702	13.433	0.926D 01	0.176D-01	0.209D-01
26.051	13.330	0.929D 01	0.176D-01	0.401D-01
37.330	13.330	0.929D 01	0.176D-01	0.911D-01
56.330	13.359	0.929D 01	0.176D-01	0.204D 00
75.330	13.399	0.929D 01	0.176D-01	0.344D 00
85.433	13.531	0.928D 01	0.176D-01	0.433D 00
97.580	13.477	0.929D 01	0.176D-01	0.546D 00
115.329	13.590	0.930D 01	0.176D-01	0.714D 00

\*\*\*\*\*

TABLE C-9A

\*\*\*\*\*

## RESULTS USING MODEL 1

## LEAST SQUARE PARAMETERS

K=8.372 MD.  
B=17.591 PSIA.

ERROR VECTOR  
(PSIA)

DESIGN MATRIX  
X

\*\*\*\*\*

0.466D 00	-0.463D 00	-0.136D 00
0.512D 00	-0.574D 00	-0.165D 00
0.740D 00	-0.103D 01	-0.281D 00
0.759D 00	-0.142D 01	-0.364D 00
0.745D 00	-0.215D 01	-0.503D 00
-0.120D 00	-0.330D 01	-0.641D 00
-0.103D 01	-0.481D 01	-0.749D 00
-0.917D 00	-0.612D 01	-0.812D 00
-0.965D 00	-0.678D 01	-0.832D 00
-0.522D 00	-0.755D 01	-0.857D 00
0.205D 01	-0.855D 01	-0.898D 00

SUM OF THE SQUARES OF THE ERRORS=9.469

ERROR VARIANCE=1.052

EXPECTED LIKELIHOOD=0.440D-06

DETERMINANT OF A MATRIX=0.614D 02

VARIANCE-COVARIANCE MATRIX

0.750D-01	-0.558D 00
-0.558D 00	0.439D 01

CORRELATION MATRIX

1.000	-0.972
-0.972	1.000

TABLE C-9B

\*\*\*\*\*  
 RESULTS USING MODEL 2

## LEAST SQUARE PARAMETERS

K =14.549 MD.  
 BETA=0.711D 10 1/FT.

ERROR VECTOR  
 (PSIA)

DESIGN MATRIX  
 X

\*\*\*\*\*

-0.273D 00	-0.146D 00	0.156D-12
-0.336D 00	-0.180D 00	0.262D-12
-0.520D 00	-0.320D 00	0.109D-11
-0.651D 00	-0.438D 00	0.259D-11
-0.663D 00	-0.674D 00	0.938D-11
-0.498D 00	-0.108D 01	0.466D-10
0.191D 00	-0.163D 01	0.216D-09
0.722D 00	-0.207D 01	0.567D-09
0.366D 00	-0.228D 01	0.856D-09
-0.201D 00	-0.251D 01	0.129D-08
-0.250D 00	-0.278D 01	0.206D-08

SUM OF THE SQUARES OF THE ERRORS=2.363  
 ERROR VARIANCE=0.263  
 EXPECTED LIKELIHOOD=0.454D-03  
 DETERMINANT OF A MATRIX=0.414D-16

0.447D-01      0.795D 08

0.795D 08      0.179D 18

## CORRELATION MATRIX

1.000	0.889
0.889	1.000

TABLE C-9C

\*\*\*\*\*  
 RESULTS USING MODEL 3

## LEAST SQUARE PARAMETERS

K =11.722 MD.  
 B =6.229 PSIA.  
 BETA=0.510D 10 1/FT.

ERROR VECTOR  
 (PSIA)

DESIGN MATRIX  
 X

\*\*\*\*\*

0.274D-01	-0.230D 00	-0.132D 00	0.898D-12
0.121D-01	-0.284D 00	-0.160D 00	0.144D-11
0.863D-02	-0.507D 00	-0.270D 00	0.527D-11
-0.375D-01	-0.695D 00	-0.350D 00	0.111D-10
0.112D-01	-0.107D 01	-0.489D 00	0.327D-10
-0.107D 00	-0.168D 01	-0.641D 00	0.117D-09
0.294D-01	-0.250D 01	-0.763D 00	0.390D-09
0.314D 00	-0.317D 01	-0.826D 00	0.835D-09
-0.340D-01	-0.350D 01	-0.841D 00	0.116D-08
-0.402D 00	-0.386D 01	-0.858D 00	0.161D-08
0.182D 00	-0.429D 01	-0.884D 00	0.234D-08

SUM OF THE SQUARES OF THE ERRORS=0.309  
 ERROR VARIANCE=0.039  
 EXPECTED LIKELIHOOD=0.108D 02  
 DETERMINANT OF A MATRIX=0.538D-17  
 VARIANCE-COVARIANCE MATRIX

0.105D 00	-0.271D 00	0.102D 09
-0.271D 00	0.726D 00	-0.249D 09
0.102D 09	-0.249D 09	0.111D 18

CORRELATION MATRIX

1.000	-0.982	0.945
-0.982	1.000	-0.877
0.945	-0.877	1.000

## TABLE C-9D

\*\*\*\*\*

## RESULTS USING MODEL 4

## LEAST SQUARE PARAMETERS

K =12.015 MD.  
 B =5.686 PSIA.  
 BETA=0.489D 10 1/FT.

ERROR VECTOR (PSIA)	DESIGN MATRIX X		
0.169D-01	-0.159D 00	-0.956D-01	0.919D-12
0.619D-02	-0.199D 00	-0.117D 00	0.148D-11
0.234D-02	-0.368D 00	-0.205D 00	0.552D-11
-0.328D-01	-0.519D 00	-0.274D 00	0.118D-10
0.850D-02	-0.831D 00	-0.393D 00	0.353D-10
-0.838D-01	-0.139D 01	-0.535D 00	0.128D-09
0.347D-01	-0.215D 01	-0.629D 00	0.421D-09
0.286D 00	-0.279D 01	-0.653D 00	0.888D-09
-0.396D-01	-0.310D 01	-0.652D 00	0.122D-08
-0.387D 00	-0.345D 01	-0.645D 00	0.169D-08
0.182D 00	-0.387D 01	-0.626D 00	0.244D-08

SUM OF THE SQUARES OF THE ERRORS=0.276

ERROR VARIANCE=0.035

EXPECTED LIKELIHOOD=0.193D 02

DETERMINANT OF A MATRIX=0.310D-17

VARIANCE-COVARIANCE MATRIX

0.120D 00	-0.306D 00	0.124D 09
-0.306D 00	0.815D 00	-0.301D 09
0.124D 09	-0.301D 09	0.136D 18

CORRELATION MATRIX

1.000	-0.978	0.971
-0.978	1.000	-0.904
0.971	-0.904	1.000



## APPENDIX D

### ANALYSIS OF THE STEADY LINEAR GAS FLOW

In order to estimate the non-linear parameters for the models stated in Chapter 3, the function and its derivatives with respect to the parameters must be stated explicitly.

#### Model A

This mathematical model corresponds to the following two-parameter model obtained before,

$$p_e = \left( p_w^2 - 2b(p_e - p_w) + \frac{2 \cdot A^* R / M L W \mu_{avg}}{k} \right)^{0.5} \quad (D-1)$$

or

$$p_e = \psi(W, \underline{a})$$

where the parameter vector is

$$\underline{a} = \begin{pmatrix} k \\ b \end{pmatrix}$$

Let the expression in brackets given in Equation (D-1) be called "value 1", hence the derivatives can be written as

$$a) \quad \frac{\partial \psi}{\partial k} = -0.5 (\text{value 1})^{-0.5} \left( \frac{2 A^* R / M L W \mu_{avg}}{k^2} \right) \quad (D-2)$$

$$b) \frac{\partial \psi}{\partial b} = -0.5(\text{value 1})^{-0.5} [2(p_e - p_w)] \quad (D-3)$$

### Model B

When slippage effect was neglected, the following two-parameter model was obtained:

$$p_e = [p_w^2 + 2\left(\frac{1}{k} + \beta W\right) A^* R/M LW \mu_{avg}]^{0.5} \quad (D-4)$$

or

$$p_e = \phi(W, \underline{a})$$

where

$$\underline{a} = \begin{pmatrix} k \\ \beta \end{pmatrix} .$$

Let the expression in brackets in Equation (D-4) be "value 2", then the derivatives are given by

$$a) \frac{\partial \phi}{\partial k} = -(\text{value 2})^{-0.5} \left(\frac{1}{k^2} A^* R/M LW \mu_{avg}\right) \quad (D-5)$$

$$b) \frac{\partial \phi}{\partial \beta} = (\text{value 2})^{-0.5} (A^* R/M LW^2 \mu_{avg}) \quad (D-6)$$

### Model C

For the general three parameter model the function was written as

$$p_e = \left[ p_w^2 + \frac{2 \left( \frac{1}{k} + \beta W \right)^3 A^* R/M L W \mu_{avg}}{\left( \frac{1}{k} + \beta W \right)^2 - \frac{2 \beta W b^2}{k (p_e^2 - p_w^2)} \ln \frac{\beta W b + p_e \left( \frac{1}{k} + \beta W \right)}{\beta W b + p_w \left( \frac{1}{k} + \beta W \right)} + \frac{2b \left( \frac{1}{k} + \beta W \right)}{k (p_e + p_w)} \right]^{0.5}$$

(D-7)

or

$$p_e = f(W, \underline{a})$$

where the design parameter vector is

$$\underline{a} = \begin{bmatrix} k \\ b \\ \beta \end{bmatrix}$$

Let "value 3" be the expression in brackets in Equation (D-7), then, the corresponding derivatives with respect to the parameters are

$$a) \quad \frac{\partial f}{\partial k} = C1(k1 + k2)/C2 \quad (D-8)$$

where

$$C1 = 0.5 (\text{value 3})^{-0.5} \quad (D-9)$$

$$k1 = \left[ \left( \frac{1}{k} + \beta W \right)^2 - \frac{2 \beta W b^2}{k (p_e^2 - p_w^2)} \ln \frac{\beta W b + p_e \left( \frac{1}{k} + \beta W \right)}{\beta W b + p_w \left( \frac{1}{k} + \beta W \right)} + \frac{2b \left( \frac{1}{k} + \beta W \right)}{k (p_e + p_w)} \right] \times$$

$$\left[ - \frac{6}{k^2} (A^* R/M L W \mu_{avg}) \left( \frac{1}{k} + \beta W \right)^2 \right] \quad (D-10)$$

$$k_2 = -2 \left( \frac{1}{k} + \beta W \right)^3 A^* R/M L W \mu_{avg} \left\langle - \frac{2}{k^2} \left( \frac{1}{k} + \beta W \right) - \right. \\ \left. \left[ \frac{2\beta W b^2}{k(p_e^2 - p_w^2)} \left( \frac{p_w}{k^2} [\beta W b + p_e \left( \frac{1}{k} + \beta W \right)] - \frac{p_w}{k^2} [\beta W b + p_w \left( \frac{1}{k} + \beta W \right)] \right) \right] - \right. \\ \left. \frac{2\beta W b^2}{k^2(p_e^2 - p_w^2)} \ln \frac{\beta W b + p_e \left( \frac{1}{k} + \beta W \right)}{\beta W b + p_w \left( \frac{1}{k} + \beta W \right)} - \frac{2b(2 + k\beta W)}{k^3(p_e + p_w)} \right\rangle \quad (D-11)$$

$$c_2 = \left[ \left( \frac{1}{k} + \beta W \right)^2 - \frac{2\beta W b^2}{k(p_e^2 - p_w^2)} \ln \frac{\beta W b + p_e \left( \frac{1}{k} + \beta W \right)}{\beta W b + p_w \left( \frac{1}{k} + \beta W \right)} + \frac{2b \left( \frac{1}{k} + \beta W \right)}{k(p_e + p_w)} \right]^2 \quad (D-12)$$

$$b) \quad \frac{\partial f}{\partial b} = c_1 B_1 / c_1 \quad (D-13)$$

where

$$B_1 = -2 \left( \frac{1}{k} + \beta W \right)^3 A^* R/M L W \mu_{avg} \left\langle - \frac{2\beta W b^2}{k(p_e^2 - p_w^2)} \left( \frac{\beta W \left( \frac{1}{k} + \beta W \right) (p_e - p_w)}{[\beta W b + p_w \left( \frac{1}{k} + \beta W \right)] [\beta W b + p_e \left( \frac{1}{k} + \beta W \right)]} \right) - \right. \\ \left. \frac{4\beta W b}{k(p_e^2 - p_w^2)} \ln \frac{\beta W b + p_e \left( \frac{1}{k} + \beta W \right)}{\beta W b + p_w \left( \frac{1}{k} + \beta W \right)} + \frac{2 \left( \frac{1}{k} + \beta W \right)}{k(p_e + p_w)} \right\rangle \quad (D-14)$$

$$c) \quad \frac{\partial f}{\partial \beta} = c_1 (B B_1 + B B_2) / c_2 \quad (D-15)$$

where

$$\begin{aligned}
 BB1 = & \left( \frac{1}{k+\beta W} \right)^2 - \frac{2\beta W b^2}{k(p_e^2 - p_w^2)} \ln \frac{\beta W b + p_e \left( \frac{1}{k+\beta W} \right)}{\beta W b + p_w \left( \frac{1}{k+\beta W} \right)} + \\
 & \frac{2b \left( \frac{1}{k+\beta W} \right)}{k(p_e + p_w)} \left( 6 A^* R/M L W \mu_{avg} \left( \frac{1}{k+\beta W} \right)^2 \right) \quad (D-16)
 \end{aligned}$$

$$\begin{aligned}
 BB2 = & -2 \left( \frac{1}{k+\beta W} \right)^3 A^* R/M L W \mu_{avg} \left\langle 2 \left( \frac{1}{k+\beta W} \right) W \right. \\
 & + \frac{2bW}{k(p_e + p_w)} - \left. \left( \frac{2\beta W b^2}{k(p_e^2 - p_w^2)} \left( \frac{Wb(p_e - p_w)}{k[\beta W b + p_e \left( \frac{1}{k+\beta W} \right)] [\beta W b + p_w \left( \frac{1}{k+\beta W} \right)]} \right) \right) \right. \\
 & \left. + \frac{2Wb^2}{k(p_e^2 - p_w^2)} \ln \frac{\beta W b + p_e \left( \frac{1}{k+\beta W} \right)}{\beta W b + p_w \left( \frac{1}{k+\beta W} \right)} \right\rangle \quad (D-17)
 \end{aligned}$$

#### Model D

This fourth mathematical model, which represents the same general model for the linear case, was stated as

$$\frac{dp}{dr} = \frac{\mu_{avg} T_{avg} Z_{avg} R \omega/A}{Mk(p+b)} + \frac{\beta (\omega/A)^2 Z_{avg} R T_{avg}}{Mp} \quad (D-18)$$

or

$$\frac{dp}{dr} = \theta (\omega/A, \underline{a})$$

where

$$\underline{a} = \begin{pmatrix} k \\ b \\ \beta \end{pmatrix} .$$

The derivatives with respect to both the parameters and the state variable, are given by the following expressions:

$$a) \quad \frac{\partial \theta}{\partial k} = - \frac{\mu_{avg} T_{avg} Z_{avg} R \omega/A}{Mk^2(p + b)} \quad (D-19)$$

$$b) \quad \frac{\partial \theta}{\partial b} = - \frac{\mu_{avg} T_{avg} Z_{avg} R \omega/A}{Mk(p + b)^2} \quad (D-20)$$

$$c) \quad \frac{\partial \theta}{\partial \beta} = \frac{(\omega/A)^2 Z_{avg} R T_{avg}}{Mp} \quad (D-21)$$

$$d) \quad \frac{\partial \theta}{\partial p} = - \frac{\mu_{avg} T_{avg} Z_{avg} R \omega/A}{Mk(p + b)^2} - \frac{\beta (\omega/A)^2 Z_{avg} R T_{avg}}{Mp^2} \quad (D-22)$$

It is to be noted the simplicity of the derivatives when considering the ordinary differential equation rather than the rigorous model in algebraic form.

Input data and results obtained using each model for all the core samples shown in Table 3-3, are given in the following pages.

TABLE D-1

\*\*\*\*\*

INPUT DATA FOR CORE SAMPLE NO. 9  
 RUN 18

PE, PSFA.  
 PW, PSFA.  
 VIS\* $T^2$ /GC, LBF-SEC/FT\*\*2-DEG. R.  
 VIS, LBM/(FT-SEC).  
 MASS FLUX, LBM/(SEC-FT\*\*2).

PE	PW	VIS* $T^2$ /GC	VIS	MASS FLUX
17941.91	1929.12	0.202D-03	0.120D-04	0.387D-04
22248.90	1930.50	0.202D-03	0.121D-04	0.561D-04
26526.30	1931.11	0.202D-03	0.121D-04	0.771D-04
30803.90	1931.90	0.202D-03	0.121D-04	0.101D-03
38004.21	1931.90	0.203D-03	0.121D-04	0.148D-03
45146.80	1932.51	0.203D-03	0.121D-04	0.203D-03
52346.80	1932.50	0.203D-03	0.121D-04	0.267D-03
59360.81	1933.61	0.203D-03	0.122D-04	0.337D-03
66605.10	1934.70	0.204D-03	0.122D-04	0.420D-03
73805.41	1935.00	0.204D-03	0.122D-04	0.513D-03

TABLE D-1A

\*\*\*\*\*

## RESULTS USING MODEL A

## LEAST SQUARE PARAMETERS

K=0.042 MD.  
B=28.399 PSIA.

ERROR VECTOR  
(PSFA)

DESIGN MATRIX  
X

\*\*\*\*\*

-0.707D 02	-0.278D 20	-0.889D 00
0.317D 02	-0.328D 20	-0.915D 00
-0.228D 02	-0.377D 20	-0.926D 00
-0.315D 02	-0.426D 20	-0.936D 00
0.180D 02	-0.507D 20	-0.950D 00
0.187D 02	-0.587D 20	-0.958D 00
0.217D 02	-0.667D 20	-0.963D 00
0.156D 03	-0.744D 20	-0.970D 00
0.541D 02	-0.826D 20	-0.972D 00
-0.181D 03	-0.909D 20	-0.971D 00

SUM OF THE SQUARES OF THE ERRORS=0.688D 05  
 ERROR VARIANCE=0.860D 04  
 EXPECTED LIKELIHOOD=0.198D-25  
 DETERMINANT OF A MATRIX=0.102D 00  
 VARIANCE-COVARIANCE MATRIX

0.207D-07	-0.922D 04
-0.922D 04	0.456D 00

## CORRELATION MATRIX

1.000	-0.948
-0.948	1.000



TABLE D-1B

\*\*\*\*\*  
 RESULTS USING MODEL B  
 \*\*\*\*\*

## LEAST SQUARE PARAMETERS

K =0.054 MD.  
 BETA=0.722D 13 1/FT.

ERROR VECTOR  
 (PSFA)

DESIGN MATRIX  
 X

\*\*\*\*\*

-0.104D 04	-0.160D 20	0.172D-10
-0.661D 03	-0.193D 20	0.300D-10
-0.410D 03	-0.226D 20	0.482D-10
-0.147D 03	-0.258D 20	0.721D-10
0.267D 03	-0.310D 20	0.127D-09
0.500D 03	-0.360D 20	0.202D-09
0.567D 03	-0.410D 20	0.302D-09
0.570D 03	-0.455D 20	0.422D-09
0.122D 03	-0.502D 20	0.579D-09
-0.713D 03	-0.548D 20	0.770D-09

SUM OF THE SQUARES OF THE ERRORS=0.321D 07  
 ERROR VARIANCE=0.401D 06  
 EXPECTED LIKELIHOOD=0.906D-34  
 DETERMINANT OF A MATRIX=0.175D 00  
 VARIANCE-COVARIANCE MATRIX

0.152D-05	0.150D 10
0.150D 10	0.181D 25

CORRELATION MATRIX

1.000	0.908
0.908	1.000

TABLE D-1C

\*\*\*\*\*  
 RESULTS USING MODEL C

LEAST SQUARE PARAMETERS

K =0.0412 MD.  
 B =31.558 PSIA.  
 BETA=-0.899D 12 1/FT.

ERROR VECTOR (PSFA)                      DESIGN MATRIX X  
 \*\*\*\*\*

0.963D 01	-0.203D 20	-0.613D 00	0.189D-10
0.593D 02	-0.252D 20	-0.664D 00	0.321D-10
-0.649D 01	-0.302D 20	-0.706D 00	0.504D-10
-0.372D 02	-0.352D 20	-0.738D 00	0.743D-10
-0.288D 02	-0.435D 20	-0.779D 00	0.128D-09
-0.451D 02	-0.519D 20	-0.811D 00	0.203D-09
-0.409D 02	-0.603D 20	-0.836D 00	0.303D-09
0.100D 03	-0.684D 20	-0.854D 00	0.425D-09
0.615D 02	-0.770D 20	-0.873D 00	0.586D-09
-0.665D 02	-0.858D 20	-0.890D 00	0.786D-09

SUM OF THE SQUARES OF THE ERRORS=0.278D 05  
 ERROR VARIANCE=0.397D 04  
 EXPECTED LIKELIHOOD=0.157D-23  
 DETERMINANT OF A MATRIX=0.204D-02  
 VARIANCE-COVARIANCE MATRIX

0.225D-06	-0.681D-03	0.166D 09
-0.681D-03	0.214D 10	-0.482D 12
0.166D 09	-0.482D 12	0.130D 24

CORRELATION MATRIX

1.000	-0.981	0.969
-0.981	1.000	-0.912
0.969	-0.912	1.000

TABLE D-2

\*\*\*\*\*

INPUT DATA FOR CORE SAMPLE NO. 5  
RUN 11\*\*\*\*\*  
PE PW VIS\*T\*Z/GC VIS MASS FLUX  
\*\*\*\*\*

5594.50	1958.06	0.201D-03	0.120D-04	0.436D-04
7764.48	1960.85	0.202D-03	0.120D-04	0.844D-04
10673.60	1960.85	0.202D-03	0.120D-04	0.155D-03
13605.00	1961.41	0.202D-03	0.120D-04	0.245D-03
17974.20	1961.41	0.202D-03	0.120D-04	0.419D-03
23705.41	1978.69	0.202D-03	0.121D-04	0.687D-03
30847.80	1990.21	0.202D-03	0.121D-04	0.116D-02
38048.10	1997.41	0.203D-03	0.121D-04	0.172D-02
52246.22	2047.81	0.203D-03	0.121D-04	0.312D-02
66761.40	2122.69	0.204D-03	0.122D-04	0.486D-02

TABLE D-2A

\*\*\*\*\*  
 RESULTS USING MODEL A

LEAST SQUARE PARAMETERS

K=0.524 MD.  
 B=16.078 PSIA.

ERROR VECTOR  
 (PSFA)

DESIGN MATRIX  
 X

\*\*\*\*\*

0.255D 03	-0.699D 18	-0.681D 00
0.193D 03	-0.954D 18	-0.767D 00
0.121D 03	-0.126D 19	-0.826D 00
0.815D 02	-0.155D 19	-0.861D 00
-0.167D 03	-0.198D 19	-0.883D 00
0.836D 02	-0.249D 19	-0.920D 00
-0.353D 03	-0.318D 19	-0.925D 00
-0.397D 03	-0.384D 19	-0.938D 00
-0.295D 03	-0.511D 19	-0.955D 00
0.572D 03	-0.635D 19	-0.977D 00

SUM OF THE SQUARES OF THE ERRORS=0.855D 06  
 ERROR VARIANCE=0.107D 06  
 EXPECTED LIKELIHOOD=0.671D-31  
 DETERMINANT OF A MATRIX=0.225D 00  
 VARIANCE-COVARIANCE MATRIX

0.394D-04	-0.952D-02
-0.952D-02	0.297D 01

CORRELATION MATRIX

1.000	-0.880
-0.880	1.000

TABLE D-2B

\*\*\*\*\*  
RESULTS USING MODEL B

LEAST SQUARE PARAMETERS

K =0.645 MD.  
BETA=0.658D 11 1/FT.

ERROR VECTOR  
(PSFA)

DESIGN MATRIX  
X

\*\*\*\*\*

-0.536D 03	-0.401D 18	0.684D-10
-0.558D 03	-0.572D 18	0.189D-09
-0.487D 03	-0.784D 18	0.476D-09
-0.347D 03	-0.989D 18	0.944D-09
-0.277D 03	-0.130D 19	0.212D-08
0.276D 03	-0.165D 19	0.443D-08
0.221D 03	-0.213D 19	0.959D-08
0.350D 03	-0.258D 19	0.172D-07
0.178D 03	-0.340D 19	0.410D-07
-0.198D 03	-0.413D 19	0.776D-07

SUM OF THE SQUARES OF THE ERRORS=0.135D 07  
ERROR VARIANCE=0.169D 06  
EXPECTED LIKELIHOOD=0.681D-32  
DETERMINANT OF A MATRIX=0.235D 00  
VARIANCE-COVARIANCE MATRIX

0.138D-03	0.967D 08
0.967D 09	0.887D 20

CORRELATION MATRIX

1.000	0.875
0.875	1.000

TABLE D-2C

\*\*\*\*\*

## RESULTS USING MODEL C

## LEAST SQUARE PARAMETERS

K =0.572 MD.  
 B =9.919 PSIA.  
 BETA=0.319D 11. 1/FT.

ERROR VECTOR  
(PSFA)DESIGN MATRIX  
X

\*\*\*\*\*

-0.141D 02	-0.405D 18	-0.473D 00	0.753D-10
-0.358D 02	-0.601D 18	-0.580D 00	0.203D-09
-0.345D 02	-0.850D 18	-0.666D 00	0.500D-09
0.290D 01	-0.109D 19	-0.719D 00	0.975D-09
-0.913D 02	-0.146D 19	-0.778D 00	0.215D-08
0.247D 03	-0.189D 19	-0.805D 00	0.444D-08
-0.153D 02	-0.248D 19	-0.843D 00	0.955D-08
-0.137D 02	-0.304D 19	-0.857D 00	0.171D-07
-0.118D 03	-0.409D 19	-0.866D 00	0.409D-07
0.555D 02	-0.508D 19	-0.855D 00	0.780D-07

SUM OF THE SQUARES OF THE ERRORS=0.894D 05

ERROR VARIANCE=0.128D 05

EXPECTED LIKELIHOOD=0.455D-26

DETERMINANT OF A MATRIX=0.160D-01

VARIANCE-COVARIANCE MATRIX

0.682D-04	-0.931D-02	0.363D 08
-0.931D-02	0.144D 01	-0.456D 10
0.363D 08	-0.456D 10	0.218D 20

CORRELATION MATRIX

1.000	-0.939	0.941
-0.939	1.000	-0.813
0.941	-0.813	1.000

TABLE D-2D

\*\*\*\*\*

## RESULTS USING MODEL D

## LEAST SQUARE PARAMETERS

K =0.570 MD.  
 B =10.124 PSIA.  
 BETA=0.281D 11 1/FT.

ERROR VECTOR  
(PSFA)DESIGN MATRIX  
X

\*\*\*\*\*

-0.104D 02	-0.475D 04	-0.516D 00	0.829D-10
-0.352D 02	-0.701D 04	-0.630D 00	0.222D-09
-0.359D 02	-0.981D 04	-0.717D 00	0.541D-09
0.202D 01	-0.125D 05	-0.769D 00	0.105D-08
-0.101D 03	-0.165D 05	-0.819D 00	0.228D-08
0.253D 03	-0.212D 05	-0.853D 00	0.466D-08
-0.190D 02	-0.276D 05	-0.883D 00	0.994D-08
-0.124D 02	-0.336D 05	-0.907D 00	0.177D-07
-0.112D 03	-0.451D 05	-0.963D 00	0.425D-07
0.521D 02	-0.561D 05	-0.104D 01	0.816D-07

SUM OF THE SQUARES OF THE ERRORS=0.927D 05  
 ERROR VARIANCE=0.132D 05  
 EXPECTED LIKELIHOOD=0.379D-26  
 DETERMINANT OF A MATRIX=0.786D-05  
 VARIANCE-COVARIANCE MATRIX

0.597D-04	-0.816D-02	0.295D 08
-0.816D-02	0.128D 01	-0.363D 10
0.295D 08	-0.363D 10	0.172D 20

## CORRELATION MATRIX

1.000	-0.935	0.923
-0.935	1.000	-0.776
0.923	-0.776	1.000

TABLE D-3

\*\*\*\*\*  
 INPUT DATA FOR CORE SAMPLE NO. 11  
 RUN 5

PE	PW	VIS*T*Z/GC	VIS	MASS FLUX
2690.63	1963.35	0.201D-03	0.120D-04	0.687D-04
3126.43	1962.80	0.201D-03	0.120D-04	0.118D-03
3713.00	1962.24	0.201D-03	0.120D-04	0.195D-03
4288.95	1961.68	0.201D-03	0.120D-04	0.282D-03
4862.18	1960.29	0.201D-03	0.120D-04	0.380D-03
6330.36	1973.86	0.201D-03	0.120D-04	0.680D-03
9230.51	1993.79	0.201D-03	0.120D-04	0.149D-02
12139.60	2058.59	0.201D-03	0.120D-04	0.256D-02
15033.30	2129.75	0.202D-03	0.120D-04	0.390D-02
19366.60	2288.15	0.202D-03	0.120D-04	0.638D-02



TABLE D-3A

\*\*\*\*\*

## RESULTS USING MODEL A

## LEAST SQUARE PARAMETERS

K=8.044 MD.  
B=4.032 PSIA.

ERROR VECTOR  
(PSFA)

DESIGN MATRIX  
X

\*\*\*\*\*

0.112D 02	-0.911D 16	-0.271D 00
0.153D 02	-0.135D 17	-0.374D 00
0.165D 02	-0.188D 17	-0.474D 00
0.177D 02	-0.234D 17	-0.545D 00
0.149D 02	-0.278D 17	-0.599D 00
-0.377D 01	-0.381D 17	-0.688D 00
-0.297D 02	-0.570D 17	-0.781D 00
-0.307D 02	-0.748D 17	-0.828D 00
-0.251D 02	-0.922D 17	-0.857D 00
0.426D 02	-0.118D 18	-0.884D 00

SUM OF THE SQUARES OF THE ERRORS=0.545D 04

ERROR VARIANCE=0.681D 03

EXPECTED LIKELIHOOD=0.640D-20

DETERMINANT OF A MATRIX=0.136D 00

VARIANCE-COVARIANCE MATRIX

0.128D-02	-0.779D-02
-0.779D-02	0.549D-01

CORRELATION MATRIX

1.000	-0.929
-0.929	1.000

TABLE D-3B

\*\*\*\*\*  
 RESULTS USING MODEL B

## LEAST SQUARE PARAMETERS

K =9.170 MD.  
 BETA=0.178D 10 1/FT.

ERROR VECTOR  
 (PSFA)

DESIGN MATRIX  
 X

\*\*\*\*\*

-0.508D 02	-0.685D 16	0.372D-09
-0.610D 02	-0.101D 17	0.948D-09
-0.648D 02	-0.141D 17	0.218D-08
-0.594D 02	-0.177D 17	0.395D-08
-0.520D 02	-0.211D 17	0.633D-08
-0.309D 02	-0.292D 17	0.157D-07
0.232D 02	-0.441D 17	0.518D-07
0.669D 02	-0.580D 17	0.117D-06
0.570D 02	-0.713D 17	0.220D-06
-0.449D 02	-0.900D 17	0.453D-06

SUM OF THE SQUARES OF THE ERRORS=0.280D 05  
 ERROR VARIANCE=0.350D 04  
 EXPECTED LIKELIHOOD=0.178D-23  
 DETERMINANT OF A MATRIX=0.206D 00  
 VARIANCE-COVARIANCE MATRIX

0.734D-02	0.191D 08
0.191D 08	0.628D 17

CORRELATION MATRIX

1.000	0.891
0.891	1.000

TABLE D-3C

\*\*\*\*\*  
RESULTS USING MODEL C

LEAST SQUARE PARAMETERS

K =8.358 MD.  
B =2.921 PSIA.  
BETA=0.589D 09 1/FT.

ERROR VECTOR  
(PSFA)

DESIGN MATRIX  
X

\*\*\*\*\*

-0.275D 01	-0.711D 16	-0.230D 00	0.379D-09
-0.160D 01	-0.107D 17	-0.320D 00	0.967D-09
-0.512D 00	-0.151D 17	-0.410D 00	0.222D-08
0.262D 01	-0.191D 17	-0.477D 00	0.401D-08
0.363D 01	-0.229D 17	-0.530D 00	0.641D-08
-0.152D 01	-0.321D 17	-0.623D 00	0.158D-07
-0.410D 01	-0.493D 17	-0.725D 00	0.517D-07
0.397D 01	-0.656D 17	-0.774D 00	0.117D-06
-0.126D 01	-0.815D 17	-0.804D 00	0.219D-06
-0.360D-01	-0.105D 18	-0.825D 00	0.454D-06

SUM OF THE SQUARES OF THE ERRORS=0.670D 02  
ERROR VARIANCE=0.960D 01  
EXPECTED LIKELIHOOD=0.193D-10  
DETERMINANT OF A MATRIX=0.662D-02  
VARIANCE-COVARIANCE MATRIX

0.247D-03	-0.917D-03	0.402D 06
-0.917D-03	0.364D-02	-0.139D 07
0.402D 06	-0.139D 07	0.722D 15

CORRELATION MATRIX

1.000	-0.966	0.951
-0.966	1.000	-0.863
0.951	-0.863	1.000

TABLE D-3D

\*\*\*\*\*

RESULTS USING MODEL D

LEAST SQUARE PARAMETERS

K =8.357 MD.  
 B =2.922 PSIA.  
 BETA=0.586D 09 1/FT.

ERROR VECTOR  
(PSFA)

DESIGN MATRIX  
X

\*\*\*\*\*

-0.282D 01	-0.771D 02	-0.234D 00	0.387D-09
-0.166D 01	-0.116D 03	-0.328D 00	0.993D-09
-0.528D 00	-0.165D 03	-0.423D 00	0.229D-08
0.270D 01	-0.209D 03	-0.493D 00	0.414D-08
0.373D 01	-0.251D 03	-0.548D 00	0.662D-08
-0.160D 01	-0.352D 03	-0.643D 00	0.163D-07
-0.426D 01	-0.538D 03	-0.745D 00	0.531D-07
0.407D 01	-0.713D 03	-0.793D 00	0.120D-06
-0.123D 01	-0.884D 03	-0.820D 00	0.223D-06
0.700D-02	-0.113D 04	-0.840D 00	0.461D-06

SUM OF THE SQUARES OF THE ERRORS=0.707D 02  
 ERROR VARIANCE=0.101D 02  
 EXPECTED LIKELIHOOD=0.146D-10  
 DETERMINANT OF A MATRIX=0.228D-07  
 VARIANCE-COVARIANCE MATRIX

0.249D-03	-0.920D-03	0.405D 06
-0.920D-03	0.365D-02	-0.141D 07
0.405D 06	-0.141D 07	0.728D 15

CORRELATION MATRIX

1.000	-0.966	0.951
-0.966	1.000	-0.863
0.951	-0.863	1.000

TABLE D-4

```
*****
INPUT DATA FOR CORE SAMPLE NO. 15
RUN 4
*****
      PE      PW      VIS#T#Z/GC      VIS      MASS FLUX
*****
2714.99  1975.61  0.201D-03  0.120D-04  0.708D-04
3139.24  1975.61  0.201D-03  0.120D-04  0.122D-03
3720.50  1975.05  0.201D-03  0.120D-04  0.200D-03
4292.38  1974.77  0.201D-03  0.120D-04  0.288D-03
4883.70  1974.77  0.201D-03  0.120D-04  0.392D-03
6335.24  1974.49  0.201D-03  0.120D-04  0.695D-03
9245.54  1987.22  0.201D-03  0.120D-04  0.153D-02
12140.10 2008.82  0.201D-03  0.120D-04  0.264D-02
15078.30 2016.02  0.202D-03  0.120D-04  0.403D-02
19453.60 2079.99  0.202D-03  0.120D-04  0.659D-02
*****
```

TABLE D-4A

\*\*\*\*\*  
 RESULTS USING MODEL A

## LEAST SQUARE PARAMETERS

K=8.295 MD.  
 B=4.165 PSIA.

ERROR VECTOR  
 (PSFA)

DESIGN MATRIX  
 X

\*\*\*\*\*

0.284D 02	-0.888D 16	-0.275D 00
0.188D 02	-0.132D 17	-0.373D 00
0.199D 02	-0.182D 17	-0.472D 00
0.251D 02	-0.227D 17	-0.543D 00
0.152D 02	-0.272D 17	-0.597D 00
0.101D 02	-0.371D 17	-0.689D 00
-0.419D 02	-0.556D 17	-0.782D 00
-0.638D 02	-0.730D 17	-0.830D 00
-0.334D 02	-0.901D 17	-0.864D 00
0.678D 02	-0.115D 18	-0.896D 00

SUM OF THE SQUARES OF THE ERRORS=0.141D 05  
 ERROR VARIANCE=0.176D 04  
 EXPECTED LIKELIHOOD=0.556D-22  
 DETERMINANT OF A MATRIX=0.134D 00  
 VARIANCE-COVARIANCE MATRIX

0.354D-02	-0.210D-01
-0.210D-01	0.144D 00

CORRELATION MATRIX

1.000	-0.931
-0.931	1.000

TABLE D-4B

\*\*\*\*\*

## RESULTS USING MODEL B

## LEAST SQUARE PARAMETERS

K =9.520 MD.  
 BETA=0.178D 10 1/FT.

ERROR VECTOR  
(PSFA)DESIGN MATRIX  
X

\*\*\*\*\*

-0.360D 02	-0.658D 16	0.397D-09
-0.567D 02	-0.975D 16	0.101D-08
-0.597D 02	-0.136D 17	0.231D-08
-0.499D 02	-0.170D 17	0.416D-08
-0.473D 02	-0.204D 17	0.680D-08
-0.126D 02	-0.281D 17	0.166D-07
0.219D 02	-0.426D 17	0.554D-07
0.446D 02	-0.559D 17	0.126D-06
0.519D 02	-0.688D 17	0.236D-06
-0.373D 02	-0.867D 17	0.485D-06

SUM OF THE SQUARES OF THE ERRORS=0.195D 05  
 ERROR VARIANCE=0.244D 04  
 EXPECTED LIKELIHOOD=0.108D-23  
 DETERMINANT OF A MATRIX=0.206D 00  
 VARIANCE-COVARIANCE MATRIX

0.552D-02	0.129D 08
0.129D 08	0.381D 17

## CORRELATION MATRIX

1.000	0.891
0.891	1.000

TABLE D-4C

\*\*\*\*\*

RESULTS USING MODEL C

LEAST SQUARE PARAMETERS

K =8.817 MD.  
 B =2.403 PSIA.  
 BETA=0.872D 09 1/FT.

ERROR VECTOR  
 (PSFA)

DESIGN MATRIX  
 X

\*\*\*\*\*

0.393D 01	-0.679D 16	-0.236D 00	0.403D-09
-0.718D 01	-0.102D 17	-0.328D 00	0.103D-08
-0.618D 01	-0.143D 17	-0.419D 00	0.234D-08
0.162D 01	-0.180D 17	-0.485D 00	0.421D-08
-0.133D 01	-0.218D 17	-0.540D 00	0.687D-08
0.118D 02	-0.303D 17	-0.631D 00	0.167D-07
-0.112D 01	-0.466D 17	-0.731D 00	0.553D-07
-0.728D 01	-0.619D 17	-0.781D 00	0.125D-06
0.400D 01	-0.769D 17	-0.809D 00	0.235D-06
-0.292D 00	-0.982D 17	-0.826D 00	0.487D-06

SUM OF THE SQUARES OF THE ERRORS=0.320D 03  
 ERROR VARIANCE=0.457D 02  
 EXPECTED LIKELIHOOD=0.773D-14  
 DETERMINANT OF A MATRIX=0.691D-02  
 VARIANCE-COVARIANCE MATRIX

0.129D-02	-0.446D-02	0.184D 07
-0.446D-02	0.165D-01	-0.598D 07
0.184D 07	-0.598D 07	0.292D 16

CORRELATION MATRIX

1.000	-0.965	0.950
-0.965	1.000	-0.861
0.950	-0.861	1.000



TABLE D-5

\*\*\*\*\*  
INPUT DATA FOR CORE SAMPLE NO. 7  
RUN 2

PE	PW	VIS*T*Z/GC	VIS	MASS FLUX
2261.92	1965.03	0.201D-03	0.120D-04	0.103D-03
2687.78	1967.04	0.201D-03	0.120D-04	0.272D-03
3129.50	1969.26	0.201D-03	0.120D-04	0.474D-03
3687.65	1973.29	0.201D-03	0.120D-04	0.767D-03
4870.53	1984.95	0.201D-03	0.120D-04	0.153D-02
6318.85	2002.16	0.201D-03	0.120D-04	0.274D-02
9212.10	2057.09	0.202D-03	0.120D-04	0.599D-02
12144.20	2149.50	0.201D-03	0.120D-04	0.103D-01
15052.80	2283.12	0.202D-03	0.120D-04	0.153D-01
19450.80	2590.78	0.201D-03	0.120D-04	0.247D-01

TABLE D-5A

\*\*\*\*\*

## RESULTS USING MODEL A

## LEAST SQUARE PARAMETERS

K=29.271 MD.  
B=6.524 PSIA.

ERROR VECTOR  
(PSFA)

DESIGN MATRIX  
X

\*\*\*\*\*

0.353D 02	-0.120D 16	-0.133D 00
0.634D 02	-0.268D 16	-0.275D 00
0.787D 02	-0.401D 16	-0.380D 00
0.834D 02	-0.550D 16	-0.476D 00
0.582D 02	-0.824D 16	-0.600D 00
0.115D 01	-0.112D 17	-0.683D 00
-0.122D 03	-0.166D 17	-0.767D 00
-0.174D 03	-0.217D 17	-0.811D 00
-0.447D 02	-0.263D 17	-0.846D 00
0.166D 03	-0.331D 17	-0.874D 00

SUM OF THE SQUARES OF THE ERRORS=0.964D 05

ERROR VARIANCE=0.121D 05

EXPECTED LIKELIHOOD=0.368D-26

DETERMINANT OF A MATRIX=0.120D 00

VARIANCE-COVARIANCE MATRIX

0.320D 00	-0.582D 00
-0.582D 00	0.120D 01

CORRELATION MATRIX

1.000	-0.938
-0.938	1.000

TABLE D-5B

\*\*\*\*\*  
 RESULTS USING MODEL B

## LEAST SQUARE PARAMETERS

K =36.390 MD.  
 BETA=0.212D 09 1/FT.

ERROR VECTOR  
 (PSFA)

DESIGN MATRIX  
 X

\*\*\*\*\*

-0.169D 02	-0.756D 15	0.969D-09
-0.312D 02	-0.167D 16	0.567D-08
-0.357D 02	-0.250D 16	0.148D-07
-0.344D 02	-0.344D 16	0.329D-07
-0.251D 02	-0.524D 16	0.999D-07
-0.117D 02	-0.724D 16	0.246D-06
-0.154D 01	-0.109D 17	0.810D-06
0.104D 01	-0.142D 17	0.182D-05
0.723D 02	-0.171D 17	0.326D-05
-0.353D 02	-0.212D 17	0.650D-05

SUM OF THE SQUARES OF THE ERRORS=0.110D 05  
 ERROR VARIANCE=0.137D 04  
 EXPECTED LIKELIHOOD=0.194D-21  
 DETERMINANT OF A MATRIX=0.196D 00  
 VARIANCE-COVARIANCE MATRIX

0.531D-01	0.229D 07
0.229D 07	0.122D 15

## CORRELATION MATRIX

1.000	0.897
0.897	1.000

TABLE D-5C

\*\*\*\*\*

RESULTS USING MODEL C

LEAST SQUARE PARAMETERS

K =34.521 MD.  
 B =1.648 PSIA.  
 BETA=0.168D 09 1/FT.

ERROR VECTOR  
(PSFA)

DESIGN MATRIX  
X

\*\*\*\*\*

-0.153D 01	-0.760D 15	-0.119D 00	0.975D-09
-0.267D 01	-0.171D 16	-0.244D 00	0.573D-08
-0.362D 01	-0.257D 16	-0.338D 00	0.149D-07
0.371D 01	-0.357D 16	-0.426D 00	0.332D-07
0.802D 01	-0.548D 16	-0.548D 00	0.101D-06
0.688D 01	-0.763D 16	-0.634D 00	0.247D-06
-0.144D 02	-0.116D 17	-0.723D 00	0.809D-06
-0.302D 02	-0.153D 17	-0.757D 00	0.182D-05
0.429D 02	-0.185D 17	-0.761D 00	0.326D-05
-0.117D 02	-0.231D 17	-0.751D 00	0.651D-05

SUM OF THE SQUARES OF THE ERRORS=0.324D 04  
 ERROR VARIANCE=0.462D 03  
 EXPECTED LIKELIHOOD=0.727D-19  
 DETERMINANT OF A MATRIX=0.685D-02  
 VARIANCE-COVARIANCE MATRIX

0.229D 00	-0.198D 00	0.586D 07
-0.198D 00	0.185D 00	-0.477D 07
0.586D 07	-0.477D 07	0.166D 15

CORRELATION MATRIX

1.000	-0.964	0.951
-0.964	1.000	-0.861
0.951	-0.861	1.000

TABLE D-5D

\*\*\*\*\*  
RESULTS USING MODEL D

## LEAST SQUARE PARAMETERS

K =34.533 MD.  
B =1.636 PSIA.  
BETA=0.169D 09 1/FT.

ERROR VECTOR  
(PSFA)

DESIGN MATRIX  
X

\*\*\*\*\*

-0.164D 01	-0.812D 01	-0.119D 00	0.982D-09
-0.290D 01	-0.183D 02	-0.247D 00	0.580D-08
-0.639D 00	-0.277D 02	-0.344D 00	0.152D-07
0.353D 01	-0.386D 02	-0.435D 00	0.338D-07
0.795D 01	-0.593D 02	-0.560D 00	0.102D-06
0.688D 01	-0.825D 02	-0.648D 00	0.252D-06
-0.146D 02	-0.125D 03	-0.733D 00	0.822D-06
-0.304D 02	-0.164D 03	-0.765D 00	0.184D-05
0.436D 02	-0.198D 03	-0.771D 00	0.329D-05
-0.120D 02	-0.247D 03	-0.757D 00	0.656D-05

SUM OF THE SQUARES OF THE ERRORS=0.332D 04  
ERROR VARIANCE=0.473D 03  
EXPECTED LIKELIHOOD=0.646D-19  
DETERMINANT OF A MATRIX=0.213D-06  
VARIANCE-COVARIANCE MATRIX

0.227D 00	-0.197D 00	0.583D 07
-0.197D 00	0.183D 00	-0.474D 07
0.583D 07	-0.474D 07	0.165D 15

## CORRELATION MATRIX

1.000	-0.964	0.957
-0.964	1.000	-0.860
0.957	-0.860	1.000

TABLE D-6

\*\*\*\*\*

INPUT DATA FOR CORE SAMPLE NO. 2  
RUN 7

PE	PW	VIS*T#Z/GC	VIS	MASS FLUX
2109.40	1956.67	0.201D-03	0.120D-04	0.364D-03
2268.84	1956.11	0.201D-03	0.120D-04	0.695D-03
2683.39	1992.11	0.201D-03	0.120D-04	0.182D-02
3119.19	2027.56	0.201D-03	0.120D-04	0.314D-02
3686.47	2099.56	0.201D-03	0.120D-04	0.516D-02
4282.83	2207.56	0.201D-03	0.120D-04	0.745D-02
4849.82	2344.08	0.201D-03	0.120D-04	0.989D-02
6318.91	2819.28	0.201D-03	0.120D-04	0.167D-01
7758.91	3467.28	3.201D-03	0.120D-04	0.246D-01
9213.46	4259.28	0.201D-03	0.120D-04	0.327D-01

TABLE D-6A

\*\*\*\*\*  
 RESULTS USING MODEL A

## LEAST SQUARE PARAMETERS

K=230.361 MD.  
 B=5.075 PSIA.

ERROR VECTOR  
 (PSFA)

DESIGN MATRIX  
 X

\*\*\*\*\*

0.723D 01	-0.827D 14	-0.727D-01
0.506D 02	-0.149D 15	-0.141D 00
0.355D 02	-0.327D 15	-0.261D 00
0.332D 02	-0.485D 15	-0.354D 00
-0.161D 01	-0.667D 15	-0.430D 00
-0.155D 02	-0.827D 15	-0.483D 00
-0.382D 02	-0.964D 15	-0.513D 00
-0.222D 02	-0.126D 16	-0.552D 00
-0.313D 02	-0.151D 16	-0.551D 00
0.528D 02	-0.170D 16	-0.541D 00

SUM OF THE SQUARES OF THE ERRORS=0.109D 05  
 ERROR VARIANCE=0.137D 04  
 EXPECTED LIKELIHOOD=0.196D-21  
 DETERMINANT OF A MATRIX=0.750D-01  
 VARIANCE-COVARIANCE MATRIX

0.176D 02	-0.281D 01
-0.281D 01	0.487D 00

CORRELATION MATRIX

1.000	-0.962
-0.962	1.000

TABLE D-6B

\*\*\*\*\*

## RESULTS USING MODEL B

## LEAST SQUARE PARAMETERS

K =281.665 MD.  
 BETA=0.205D 08 1/FT.

ERROR VECTOR  
(PSFA)DESIGN MATRIX  
X

\*\*\*\*\*

-0.399D 01	-0.504D 14	0.137D-07
0.229D 02	-0.903D 14	0.468D-07
0.590D 01	-0.198D 15	0.268D-06
0.773D 01	-0.295D 15	0.689D-06
-0.849D 01	-0.407D 15	0.157D-05
-0.474D 01	-0.507D 15	0.282D-05
-0.107D 02	-0.594D 15	0.438D-05
0.191D 02	-0.776D 15	0.968D-05
-0.131D 02	-0.924D 15	0.170D-04
0.407D 01	-0.104D 16	0.253D-04

SUM OF THE SQUARES OF THE ERRORS=0.140D 04  
 ERROR VARIANCE=0.174D 03  
 EXPECTED LIKELIHOOD=0.579D-17  
 DETERMINANT OF A MATRIX=0.184D 00  
 VARIANCE-COVARIANCE MATRIX

0.244D 01 0.134D 07

0.134D 07 0.901D 12

## CORRELATION MATRIX

1.000 0.903

0.903 1.000



TABLE D-6C

\*\*\*\*\*  
RESULTS USING MODEL C

## LEAST SQUARE PARAMETERS

K =287.712 MD.  
B =-0.410 PSIA.  
BETA=0.221D 08 1/FT.

ERROR VECTOR  
(PSFA)DESIGN MATRIX  
X

\*\*\*\*\*

-0.525D 01	-0.497D 14	-0.769D-01	0.137D-07
0.209D 02	-0.890D 14	-0.132D 00	0.467D-07
0.290D 01	-0.194D 15	-0.261D 00	0.268D-06
0.488D 01	-0.289D 15	-0.351D 00	0.689D-06
-0.103D 02	-0.398D 15	-0.430D 00	0.157D-05
-0.493D 01	-0.495D 15	-0.475D 00	0.282D-05
-0.951D 01	-0.578D 15	-0.500D 00	0.438D-05
0.225D 02	-0.754D 15	-0.511D 00	0.968D-05
-0.112D 02	-0.896D 15	-0.493D 00	0.170D-04
0.151D 01	-0.100D 16	-0.459D 00	0.253D-04

SUM OF THE SQUARES OF THE ERRORS=0.135D 04  
ERROR VARIANCE=0.192D 03  
EXPECTED LIKELIHOOD=0.583D-17  
DETERMINANT OF A MATRIX=0.175D-02  
VARIANCE-COVARIANCE MATRIX

0.145D 03	-0.945D 01	0.391D 08
-0.945D 01	0.628D 00	-0.250D 07
0.391D 08	-0.250D 07	0.109D 14

## CORRELATION MATRIX

1.000	-0.990	0.982
-0.990	1.000	-0.954
0.982	-0.954	1.000

## APPENDIX E

### "GOODNESS" OF FIT OF A GIVEN DISTRIBUTION TO A NORMAL DISTRIBUTION

The computer program given in the following pages calculates the value of the statistic  $\chi^2$  and its probability for testing the "goodness" of fit of a given distribution to a normal distribution.

The given distribution has to be defined by the number of classes, the array of the class midpoints and the array of the obtained frequencies for each class midpoint. The class midpoints are supposed to have equal distances, i.e. the sizes of all class intervals are supposed to be equal.

Additional input is the value of TMIN, which defines the limit - when classes will be taken together, the mean, and the standard deviation of the hypothetical normal distribution.

Output of the program is the value of the statistic  $\chi^2$ , the degrees of freedom, and the probability associated with the value of  $\chi^2$ .

The null hypothesis is that the given distribution comes from a normal distribution with given mean and standard deviation. The null hypothesis is to be rejected if this probability is very small.

The program requires the subroutines NDTR, CDTR and DLGAM from IBM's Scientific Subroutine Package. The purpose of each subroutine is given in the program.

As an example, the distribution of the error vector obtained from Model 4, core sample No. 1, with 30 data points has been tested as follows.

In order to work with positive numbers, each element of the error vector was increased in 10 units. Next, the error vector was grouped in six classes and the following observed frequencies were obtained:

<u>Classes</u>	<u>Observed Frequencies</u>
9.1- 9.4	1
9.4- 9.7	3
9.7-10.0	13
10.0-10.3	9
10.3-10.6	2
10.6-10.9	2

The normal probabilities are obtained by calling the subroutine NDTR, which accepts the standardized random variable obtained from the general expression

$$z_i = \frac{\epsilon_i - \hat{\epsilon}}{\sigma} \quad (E-1)$$

where  $\hat{\epsilon}$  and  $\sigma$  are the mean and the standard deviation, respectively, of the distribution corresponding to the random variable,  $\epsilon_i$ . The distribution of the random

variable  $z_i$  in Equation (E-1) has zero mean and unit variance.

The expected frequencies,  $e_i$ , are calculated by multiplying each of the corresponding normal probabilities by the total frequency. If the value of any expected frequency is less than TMIN, then this class is added to the next class. If the value of two combined classes is still less than TMIN, then these two classes are added to the next class, and so on.

In order to follow the usual criterion of having the expected frequencies larger than or equal to 5, the value of TMIN was chosen to be 5.

The value of the statistic was computed using the following expression

$$\chi^2 = \sum_{i=1}^k \frac{(f_i - e_i)^2}{e_i} \quad . \quad (E-2)$$

The sampling distribution of this statistic is approximately the chi-square distribution. In general, the number of degrees of freedom for the chi-square test of "goodness" of fit is the number of terms in the formula for  $\chi^2$  minus the number of quantities obtained from the original data which are required to calculate the expected frequencies. In the given example we need to know the mean of the distribution, the standard

deviation and the total frequency to calculate the  $e_i$  values. This implies  $(k-3)$  degrees of freedom, where  $k$  is the number of terms in Equation (E-2).

For the example, the mean of the distribution was 10, and the standard deviation was 0.3333. The output obtained from the computer program gave

Chi-square = 2.1703

Degrees of freedom = 1

Probability = 0.1407

The above results show that the probability of having a value for  $\chi^2 \geq 2.170$  is 0.141. In other words, since the value obtained for the chi-square is less than 3.841, the value of  $\chi^2$  for 1 degree of freedom at a 0.05 level of significance (56), the null hypothesis cannot be rejected and we conclude that the normal distribution provides a good fit.

## MAIN LINE

```

C *****
C
C THIS PROGRAM COMPUTES THE VALUE OF CHI-SQUARE AND ITS
C PROBABILITY FOR TESTING THE GOODNES OF FIT OF A GIVEN
C DISTRIBUTION TO A NORMAL DISTRIBUTION.
C
C INPUT DATA
C
C NCLASS - INTEGER SCALAR FOR THE NUMBER OF CLASSES
C TMIN - SCALAR FOR THE LIMIT THAT DEFINES
C WHEN CLASSES SHOULD BE TAKEN TOGETHER
C CLASMP - CLASS MIDPOINTS OF THE GIVEN
C DISTRIBUTION
C XMEAN - AVERAGE OF THE HIPOTHETICAL NORMAL
C DISTRIBUTION
C SIGMA - STANDARD DEVIATION OF THE HIPOTHETICAL
C NORMAL DISTRIBUTION
C OBSF - OBSERVED FREQUENCY OF THE GIVEN
C DISTRIBUTION
C N - INTEGER EQUALS TO THE NUMBER OF CLASSES
C
C SUBROUTINES REQUIRED
C
C NDTR
C CDTR
C DLGAM
C *****
C
C DIMENSION CLASMP(40),OBSF(40)
C REAL CLASMP
C INTEGER OBSF
C READ(5,91) N
91 FORMAT(10I4)
C READ(5,91) NCLASS
90 READ(5,90) TMIN
C FORMAT(10F6.2)
C READ(5,90) (CLASMP(I),I=1,N)
C READ(5,91) (OBSF(I),I=1,N)
92 READ(5,92) XMEAN, SIGMA
C FORMAT(2F10.4)
C ITOTF=0
C DO 600 I=1,NCLASS
600 ITOTF=ITOTF+OBSF(I)
C FTOT=ITOTF
C UP=(CLASMP(2)-CLASMP(1))*0.5
C NCLASG=0
C PLOLD=0.
C OBSFRQ=0.

```

## MAIN LINE ...(CONT'D)

```

EXPFRQ=0.
CHI=0.
NCMIN=NCLASS-1
DO 500 I=1,NCMIN
UPPERL=CLASMP(I)+UP
T=(UPPERL-XMEAN)/SIGMA
CALL NDTR(T,PL,D)
P=PL-PLOLD
PLOLD=PL
OBSFRQ=OBSFRQ+OBSF(I)
EXPFRQ=EXPFRQ+FTOT*P

C   CHECKING THE MINIMUM NUMBER OF CLASSES TO BE
C   TAKEN TOGETHER

IF(EXPFRQ.LT.TMIN) GO TO 500
CHIADT=(OBSFRQ-EXPFRQ)**2/EXPFRQ
CHI=CHI+CHIADT
OBSOLD=OBSFRQ
EXPOLD=EXPFRQ
OBSFRQ=0.
EXPFRQ=0.
NCLASG=NCLASG+1
500 CONTINUE
P=1.0-PLOLD
OBSFRQ=OBSFRQ+OBSF(NCLASS)
EXPFRQ=EXPFRQ+FTOT*P
IF(EXPFRQ.GE.TMIN) GO TO 200
NCLASG=NCLASG-1
CHI=CHI-CHIADT
OBSFRQ=OBSFRQ+OBSOLD
EXPFRQ=EXPFRQ+EXPOLD
200 NCLASG=NCLASG+1
300 CHI=CHI+(OBSFRQ-EXPFRQ)**2/EXPFRQ

C   CALCULATING THE DEGREES OF FREEDOM AND THE
C   PROBABILITY OF THE STATISTIC

NDF=NCLASG-2
DF=NDF
CALL CDTR(CHI,DF,PL,D,IER)
P=1.0-PL
WRITE(6,800) CHI,NDF,P
800 FORMAT(20X,'GOODNES OF FIT TO NORMAL DISTRIBUTION',//
*,5X,'CHI
1=',F10.4,4X,'DEGREES OF FREEDOM=',I4,4X,'PROBABILITY='
*,F9.5)
STOP
END

```

## SUBROUTINE NDTR

```

C *****
C
C THIS SUBROUTINE COMPUTES Y=P(X) = PROBABILITY THAT
C THE RANDOM VARIABLE U, DISTRIBUTED NORMALLY (0,1),
C IS LESS THAN OR EQUAL TO X. F(X), THE ORDINATE OF
C THE NORMAL DENSITY X, IS ALSO COMPUTED
C
C     PARAMETERS
C
C     X - INPUT SCALAR FOR WHICH P(X) IS COMPUTED
C     P - OUTPUT PROBABILITY
C     D - OUTPUT DENSITY
C
C     REMARKS
C
C     THE METHOD IS BASED ON THE THEORY GIVEN BY
C     C. HASTINGS, APPROXIMATIONS FOR DIGITAL
C     COMPUTERS, PRINCETON UNIV. PRESS, PRINCETON,
C     N. J., 1955. SEE EQUATION 26.2.17, HANDBOOK OF
C     MATHEMATICAL FUNCTIONS, ABRAMOWITZ AND STEGUN,
C     DOVER PUBLICATIONS, INC., N. Y.
C *****
C
C     SUBROUTINE NDTR(X,P,D)
C     AX=ABS(X)
C     T=1.0/(1.0+.2316419*AX)
C     D=0.3989423*EXP(-X*X/2.0)
C     P = 1.0 - D*T*(((1.330274*T - 1.821256)*T +
C 11.781478)*T-0.3565638)*T+0.3193815)
C     IF(X)1,2,2
C 1 P=1.0-P
C 2 RETURN
C     END

```



## SUBROUTINE CDTR

```

C *****
C
C THIS SUBROUTINE COMPUTES P(X) = PROBABILITY THAT THE
C RANDOM VARIABLE U, DISTRIBUTED ACCORDING TO THE
C CHI-SQUARE DISTRIBUTION WITH G DEGREES OF FREEDOM, IS
C LESS THAN OR EQUAL TO X. F(G,X), THE ORDINATE OF THE
C CHI-SQUARE DENSITY AT X, IS ALSO COMPUTED
C
C PARAMETERS
C
C X - INPUT SCALAR FOR WHICH P(X) IS COMPUTED
C G - NUMBER OF DEGREES OF FREEDOM OF THE
C CHI-SQUARE DISTRIBUTION-
C P - OUTPUT PROBABILITY
C D - OUTPUT DENSITY
C IER - RESULTANT ERROR CODE WHERE
C 0--NO ERROR
C -1--AN INPUT PARAMETER IS INVALID
C 1--INVALID OUTPUT. P IS LESS THAN ZERO
C OR GREATER THAN ONE
C
C REMARKS
C
C METHOD REFERS TO R.E. MARGMANN AND S.P. GHOSH,
C STATISTICAL DISTRIBUTION PROGRAMS FOR A COMPUTER
C LANGUAGE, IBM RESEARCH REPORT RC-1094, 1963
C
C SUBROUTINES REQUIRED
C
C DLGAM
C NDTR
C *****

```

```

SUBROUTINE CDTR(X,G,P,D,IER)
DOUBLE PRECISION XX,DLXX,X2,DLX2,GG,G2,DLT3,THETA,THP1
*,
1GLG2,DD,T11,SER,CC,XI,FAC,TLOG,TERM,GTH,A2,A,B,C,DT2
*,DT3,THPI

```

```

C TEST FOR VALID INPUT DATA
C

```

```

IF(G-(.5-1.E-5)) 590,10,10
10 IF(G-2.E+5) 20,20,590
20 IF(X) 590,30,30

```

```

C TEST FOR X NEAR 0.0
C

```

## SUBROUTINE CDTR ... (CONT'D)

```
30 IF(X-1.E-8) 40,40,80
40 P=0.0
   IF(G-2.) 50,60,70
50 D=1.E75
   GO TO 610
60 D=0.5
   GO TO 610
70 D=0.0
   GO TO 610

C       TEST FOR X GREATER THAN 1.E+6
C

80 IF(X-1.E+6) 100,100,90
90 D=0.0
   P=1.0
   GO TO 610

C       SET PROGRAM PARAMETERS
C

100 XX=DBLE(X)
    DLXX=DLOG(XX)
    X2=XX/2.00
    DLX2=DLOG(X2)
    GG=DBLE(G)
    G2=GG/2.00

C       COMPUTE ORDINATE
C

    CALL DLGAM(G2,GLG2,I0K)
    DD=(G2-1.00)*DLXX-X2-G2*.6931471805599453 -GLG2
    IF(DD-1.68D02) 110,110,120
110 IF(DD+1.68D02) 130,130,140
120 D=1.E75
    GO TO 150
130 D=0.0
    GO TO 150
140 DD=DEXP(DD)
    D=SNGL(DD)

C
C       TEST FOR G GREATER THAN 1000.0
C       TEST FOR X GREATER THAN 2000.0
C

150 IF(G-1000.) 160,160,180
160 IF(X-2000.) 190,190,170
170 P=1.0
```

## SUBROUTINE CDTR ... (CONT'D)

```

      GO TO 610
180  A=DLOG(XX/GG)/3.D0
      A=DEXP(A)
      B=2.D0/(9.D0*GG)
      C=(A-1.D0+B)/DSQRT(B)
      SC=SNGL(C)
      CALL NDTR(SC,P,DUMMY)
      GO TO 490

C      COMPUTE THETA
C
190  K= IDINT(G2)
      THETA=G2-DFLOAT(K)
      IF(THETA-1.D-8) 200,200,210
200  THETA=0.D0
210  THP1=THETA+1.D0

C      SELECT METHOD OF COMPUTING T1
C
      IF(THETA) 230,230,220
220  IF(XX-10.D0) 260,260,320

C      COMPUTE T1 FOR THETA EQUALS 0.0
C
230  IF(X2-1.68D02) 250,240,240
240  T1=1.0
      GO TO 400
250  T11=1.D0-DEXP(-X2)
      T1=SNGL(T11)
      GO TO 400

C      COMPUTE T1 FOR THETA GREATER THAN 0.0 AND
C      X LESS THAN OR EQUAL TO 10.0
C
260  SER=X2*(1.D0/THP1-X2/(THP1+1.D0))
      J=+1
      CC=DFLOAT(J)
      DO 270 IT1=3,30
      XI=DFLOAT(IT1)
      CALL DLGAM(XI,FAC,IOK)
      TLOG=XI*DLX2-FAC-DLOG(XI+THETA)
      TERM=DEXP(TLOG)
      TERM=DSIGN(TERM,CC)
      SER=SER+TERM
      CC=-CC
      IF(DABS(TERM)-1.D-9) 280,270,270

```

## SUBROUTINE CDTR ... (CONT'D)

```

270 CONTINUE
    GO TO 600
280 IF(SER) 600,600,290
290 CALL DLGAM(THP1,GTH,IOK)
    TLOG=THETA*DLX2+DLOG(SER)-GTH
    IF(TLOG+1.68D02) 300,300,310
300 T1=0.
    GO TO 400
310 T11=DEXP(TLOG)
    T1=SNGL(T11)
    GO TO 400

C      COMPUTE T1 FOR THETA GREATER THAN 0.0 AND
C      X GREATER THAN 10.0 AND LESS THAN 2000.0
C

320 A2=0.D0
    DO 340 I=1,25
    XI=DFLOAT(I)
    CALL DLGAM(THP1,GTH,IOK)
    T11=- (13.D0*XX)/XI +THP1*DLOG(13.D0*XX/XI) -GTH
    *-DLOG(XI)
    IF(T11+1.68D02) 340,340,330
330 T11=DEXP(T11)
    A2=A2+T11
340 CONTINUE
    A=1.01282051+THETA/156.D0-XX/312.D0
    B=DABS(A)
    C= -X2+THP1*DLX2+DLOG(B)-GTH-3.951243718581427
    IF(C+1.68D02) 370,370,350
350 IF(A) 360,370,380
360 C=-DEXP(C)
    GO TO 390
370 C=0.D0
    GO TO 390
380 C=DEXP(C)
390 C=A2+C
    T11=1.0D0-C
    T1=SNGL(T11)

C      SELECT PROPER EXPRESSION FOR P
C

400 IF(G-2.) 420,410,410
410 IF(G-4.) 450,460,460

C      COMPUTE P FOR G GREATER THAN ZERO AND LESS THAN 2.0
C

420 CALL DLGAM(THP1,GTH,IOK)

```

## SUBROUTINE CDTR ... (CONT'D)

DT2=THETA\*DLXX-X2-THP1\*.6931471805599453 -GTH  
 IF(DT2+1.68D02) 430,430,440

430 P=T1  
 GO TO 490  
 440 DT2=DEXP(DT2)  
 T2=SNGL(DT2)  
 P=T1+T2+T2  
 GO TO 490

C COMPUTE P FOR G GREATER THAN OR EQUAL TO 2.0  
 C AND LESS THAN 4.0  
 C

450 P=T1  
 GO TO 490

C COMPUTE P FOR G GREATER THAN OR EQUAL TO 4.0  
 C AND LESS THAN OR EQUAL TO 1000.0  
 C

460 DT3=0.00  
 DO 480 I3=2,K  
 THPI=DFLOAT(I3)+THETA  
 CALL DLGAM(THPI,GTH,I0K)  
 DLT3=THPI\*DLX2-DLXX-X2-GTH  
 IF(DLT3+1.68D02) 480,480,470  
 470 DT3=DT3+DEXP(DLT3)  
 480 CONTINUE  
 T3=SNGL(DT3)  
 P=T1-T3-T3

C SET ERROR INDICATOR  
 C

490 IF(P) 500,520,520  
 500 IF(ABS(P)-1.E-7) 510,510,600  
 510 P=0.0  
 GO TO 610  
 520 IF(1.-P) 530,550,550  
 530 IF(ABS(1.-P)-1.E-7) 540,540,600  
 540 P=1.0  
 GO TO 610  
 550 IF(P-1.E-8) 560,560,570  
 560 P=0.0  
 GO TO 610  
 570 IF(-(1.0-P)-1.E-8) 580,580,610  
 580 P=1.0  
 GO TO 610  
 590 IER=-1  
 D=-1.E75

## SUBROUTINE CDTR ... (CONT'D)

```
P=-1.E75  
GO TO 620  
600 IER=+1  
P= 1.E75  
GO TO 620  
610 IER=0  
620 RETURN  
END
```

## SUBROUTINE DLGAM

```

C *****
C THIS SUBROUTINE COMPUTES THE DOUBLE PRECISION NATURAL
C LOGARITHM OF THE GAMMA FUNCTION OF A GIVEN DOUBLE
C PRECISION ARGUMENT
C
C PARAMETERS
C
C XX - THE DOUBLE PRECISION ARGUMENT FOR THE
C LOG GAMMA FUNCTION
C DLNG - THE RESULTANT DOUBLE PRECISION LOG GAMMA
C FUNCTION VALUE
C IER - RESULTANT ERROR CODE WHERE
C 0--NO ERROR
C -1--XX IS WITHIN 10**(-9) OF BEING ZERO
C OR XX IS NEGATIVE. DLNG IS SET TO
C -1.0D75
C 1--XX IS GREATER THAN 10**70. DLNG IS SET
C TO +1.0D75
C
C REMARKS
C
C THE METHOD USES THE EULER-MCLAURIN EXPANSION
C TO THE SEVENTH DERIVATIVE TERM, AS GIVEN BY
C M. ABRAMOWITZ AND I.A. STEGUN, 'HANDBOOK OF
C MATHEMATICAL FUNCTIONS', U.S. DEPARTMENT OF
C COMMERCE, NATIONAL BUREAU OF STANDARDS APPLIED
C MATHEMATICS SERIES, 1966, EQUATION 6.1.41
C *****
C
C SUBROUTINE DLGAM(XX,DLNG,IER)
C DOUBLE PRECISION XX,ZZ,TERM,RZ2,DLNG
C IER=0
C ZZ=XX
C IF(XX-1.D10) 2,2,1
C 1 IF(XX-1.D70) 8,9,9
C
C SEE IF XX IS NEAR ZERO OR NEGATIVE
C
C 2 IF(XX-1.D-9) 3,3,4
C 3 IER=-1
C DLNG=-1.D75
C GO TO 10
C
C XX GREATER THAN ZERO AND LESS THAN OR EQUAL TO
C 1.D+10
C

```

## SUBROUTINE DLGAM ... (CONT'D)

```

4 TERM=1.D0
5 IF(ZZ-18.D0) 6,6,7
6 TERM=TERM*ZZ
  ZZ=ZZ+1.D0
  GO TO 5
7 RZ2=1.D0/ZZ**2
  DLNG=(ZZ-0.5D0)*DLOG(ZZ)-ZZ +0.9189385332046727
  *-DLOG(TERM)+
  1(1.D0/ZZ)*(.8333333333333333D-1 -(RZ2
  **(.2777777777777777D-2 +(RZ2*
  2(.7936507936507936D-3 -(RZ2*(.5952380952380952D
  *-3))))))
  GO TO 10

C      XX GREATER THAN 1.D+10 AND LESS THAN 1.D+70
C
8 DLNG=ZZ*(DLOG(ZZ)-1.D0)
  GO TO 10

C      XX GREATER THAN OR EQUAL TO 1.D+70
C
9 IER=+1
  DLNG=1.D75
10 RETURN
  END

```



APPENDIX F

DEVELOPMENT OF A MATHEMATICAL MODEL FOR  
TRANSIENT RADIAL GAS FLOW

The following equations describe transient isothermal gas flow through a plane radial porous system.

1) Forchheimer's quadratic flow equation:

$$-\frac{\partial p}{\partial r} = \frac{\mu}{k_a} q + \frac{\beta \rho q |q|}{144 g_c} \quad (\text{F-1})$$

2) Equation of continuity:

$$\frac{1}{r} \frac{\partial}{\partial r} [r(\rho q)] = - \frac{\partial (\rho \phi)}{\partial t} \quad (\text{F-2})$$

3) Slippage correction term:

$$k_a = k \left( 1 + \frac{b}{p} \right) \quad (\text{F-3})$$

4) Equation of state:

$$\rho = \frac{pM}{ZRT} \equiv \frac{\alpha_1 p}{Z} \quad (\text{F-4})$$

Equation (F-1) implies the following units:

$$p = \text{psia}$$

$$r = \text{ft}$$

$$\mu = \text{psia-sec}$$

$$k_a = \text{ft}^2$$

$$\begin{aligned}
 q &= \text{ft/sec} \\
 \beta &= \text{ft}^{-1} \\
 \rho &= \text{lb}_m/\text{ft}^3 \\
 g_c &= \frac{\text{lb}_m \cdot \text{ft}}{\text{lb}_f \cdot \text{sec}^2} \cdot
 \end{aligned}$$

Multiplying Equation (F-1) through by density and using the identity

$$\frac{\partial p^2}{\partial r} = 2p \frac{\partial p}{\partial r} ,$$

the following equation is obtained

$$-\frac{\partial p^2}{\partial r} = \frac{2\mu Z}{\alpha_1 k_a} \rho q + \frac{2\beta Z}{144 \alpha_1 g_c} (\rho q) (|\rho q|) \quad (\text{F-5})$$

where  $\mu$  and  $Z$  are functions of pressure and the flux ( $\rho q$ ) is negative for production and positive for injection.

Using the slippage correction term to correct apparent permeability for slippage, results in the following expression

$$-\frac{\partial p^2}{\partial r} = \frac{2\mu Z p}{\alpha_1 k(p+b)} (\rho q) + \frac{2\beta Z}{144 \alpha_1 g_c} (\rho q) (|\rho q|) . (\text{F-6})$$

Equations (F-4) and (F-6) may be written in terms of dimensionless variables so that pressure  $p$ , and distance  $r$  are normalized. Let us define the basic dimensionless variables as

$$\bar{p} = \frac{p}{p_f}$$

$$\bar{r} = \frac{r}{r_e}$$

and for convenience

$$\bar{b} = \frac{b}{p_f} .$$

Therefore, Equation (F-6) becomes

$$-\frac{\partial \bar{p}^2}{\partial \bar{r}} = \frac{2 r_e \mu Z \bar{p}}{\alpha_1 k p_f^2 (\bar{p} + \bar{b})} (\rho q) + \frac{2 r_e \beta Z}{144 p_f^2 \alpha_1 g_c} (\rho q) (|\rho q|) . \quad (F-7)$$

Furthermore, if we let  $\bar{p}^2 \equiv \bar{u}$  and from the second term on the right hand side of Equation (F-7) define a dimensionless mass flux  $\bar{\rho q}$  given by

$$(\bar{\rho q})^2 = \frac{2}{144 \alpha_1 p_f^2 g_c} (\rho q)^2$$

or

$$\bar{\rho q} = \frac{1}{p_f \sqrt{72 \alpha_1 g_c}} (\rho q) , \quad (F-8)$$

then, the expression in (F-7) becomes

$$-\frac{\partial \bar{u}}{\partial \bar{r}} = \frac{2 r_e \mu Z \bar{p}}{k p_f (\bar{p} + \bar{b})} \sqrt{\frac{72 g_c}{\alpha_1}} (\bar{\rho q}) + r_e \beta Z (\bar{\rho q}) (|\bar{\rho q}|) \quad (F-9)$$

Expanding the right hand side of the equation of continuity (F-4), we obtain

$$\frac{\partial(\rho\phi)}{\partial t} = \frac{\partial(\rho\phi)}{\partial p} \frac{\partial p}{\partial t} + [\phi \frac{\partial \rho}{\partial p} + \rho \frac{\partial \phi}{\partial p}] \frac{\partial p}{\partial t} \quad (F-10)$$

From equation of state, we have

$$\frac{\partial \rho}{\partial p} = \frac{\alpha_1 Z - \alpha_1 p Z'}{Z^2} \quad (F-11)$$

An approximate expression for  $\partial\phi/\partial p$  is obtained as follows:

The isothermal rock compressibility,  $c_r$  is defined as

$$c_r = \frac{1}{V_r} \left( \frac{\partial V_r}{\partial p} \right)_T \quad (F-12)$$

where  $V_r$  is the rock or solid volume, which can be expressed as

$$V_r = V_b (1 - \phi) \quad (F-13)$$

where

$$V_b = V_r + V_p \quad , \quad (\text{bulk volume}) \quad (F-14)$$

$$V_p = \phi V_b \quad . \quad (\text{pore volume}) \quad (F-15)$$

Replacing Equation (F-13) in Equation (F-12), we obtain

$$c_r = - \frac{1}{V_b(1-\phi)} \left[ \frac{\partial [V_b(1-\phi)]}{\partial p} \right] \quad (F-16)$$

Furthermore, if we assume a negligible change in bulk volume; the above equation can be written as

$$c_r = - \frac{1}{1-\phi} \frac{\partial(1-\phi)}{\partial p} \quad (F-17)$$

Performing the differentiation, the above equation yields

$$c_r = \frac{1}{1-\phi} \frac{\partial \phi}{\partial p} \quad (F-18)$$

or

$$\frac{\partial \phi}{\partial p} = c_r(1-\phi) \quad (F-19)$$

Using Equations (F-11) and (F-19), Equation (F-10) becomes

$$\frac{\partial(\rho\phi)}{\partial t} = \left[ \frac{\phi\alpha_1 z - \phi\alpha_1 p z'}{z^2} + \frac{\alpha_1 p}{z} c_r(1-\phi) \right] \frac{1}{2p} \frac{\partial p^2}{\partial t} \quad (F-20)$$

and the equation of continuity can be written now as

$$\frac{1}{r} \frac{\partial}{\partial r} [r(\rho q)] = - \left[ \frac{\phi\alpha_1 z - \phi\alpha_1 p z'}{z^2} + \frac{\alpha_1 p}{z} c_r(1-\phi) \right] \frac{1}{2p} \frac{\partial p^2}{\partial t}$$

or

$$\frac{1}{r} \frac{\partial}{\partial r} [r(\rho q)] = - \left[ \frac{1}{p} - \frac{z'}{z} + c_r \left( \frac{1}{\phi} - 1 \right) \right] \frac{\phi\alpha_1}{2z} \frac{\partial p^2}{\partial t} \quad (F-21)$$

Recalling that

$$\left(\frac{1}{p} - \frac{1}{Z} \frac{\partial Z}{\partial p}\right)$$

is the gas compressibility  $c_g$ , Equation (F-21) becomes

$$\frac{1}{r} \frac{\partial}{\partial r} [r(\rho q)] = -[c_g + c_r \left(\frac{1}{\phi} - 1\right)] \frac{\phi \alpha_1}{2Z} \frac{\partial p^2}{\partial t} \quad . \quad (F-22)$$

Using the dimensionless variables defined before, the above equation becomes

$$\frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} [\bar{r}(\bar{\rho q})] = - \frac{\phi_0 r_e p_f}{2Z \sqrt{\frac{72 g_c}{\alpha_1}}} [c_g + c_r \left(\frac{1}{\phi} - 1\right)] \frac{\partial \bar{u}}{\partial t} \quad . \quad (F-23)$$

It should be pointed out that in the above equation the rock porosity  $\phi$ , is a function of pressure. However, if we assume that the rate of change of porosity with pressure as given in Equation (F-19) is constant, and only depends on the value of porosity at surface conditions  $\phi_0$ , then the expression for  $\partial \phi / \partial p$  can be written as

$$\frac{\partial \phi}{\partial p} = c_r (1 - \phi_0) \quad (F-19a)$$

and Eq. (F-23) can be approximated as

$$\frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} [\bar{r}(\bar{\rho q})] = - \frac{\phi_0 r_e p_f}{2Z \sqrt{\frac{72 g_c}{\alpha_1}}} [c_g + c_r \left(\frac{1}{\phi_0} - 1\right)] \frac{\partial \bar{u}}{\partial t} \quad . \quad (F-24)$$

Furthermore, if a dimensionless time is defined as

$$\bar{t} = \frac{2 \sqrt{\frac{72 g_c}{\alpha_1}}}{\phi_0 r_e} t \quad , \quad (F-25)$$

then Equation (F-24) becomes

$$\frac{\partial}{\partial \bar{r}} [\bar{r}(\bar{\rho q})] = - \frac{\bar{r} p_f}{z} [c_g + c_r (\frac{1}{\phi_0} - 1)] \frac{\partial \bar{u}}{\partial \bar{t}} \quad . \quad (F-26)$$

The system to solve is therefore given by Equation (F-9) and Equation (F-26).

Multiplying Equation (F-9) through by  $\bar{r}$  and solving for  $\bar{r}(\bar{\rho q})$  we obtain

$$\bar{r}(\bar{\rho q}) = \frac{-\bar{r} k p_f (\bar{p} + \bar{b}) \frac{\partial \bar{u}}{\partial \bar{r}}}{2 r_e \mu z \bar{p} \sqrt{\frac{72 g_c}{\alpha_1}} + r_e \beta k p_f (\bar{p} + \bar{b}) z |\bar{\rho q}|} \quad (F-27)$$

Replacing the above expression in Equation (F-26), we have

$$\begin{aligned} \frac{\partial}{\partial \bar{r}} \left( \frac{\bar{r} k p_f (\bar{p} + \bar{b}) \frac{\partial \bar{u}}{\partial \bar{r}}}{2 r_e \mu z \bar{p} \sqrt{\frac{72 g_c}{\alpha_1}} + r_e \beta k p_f (\bar{p} + \bar{b}) z |\bar{\rho q}|} \right) \\ = \frac{\bar{r} p_f}{z} [c_g + c_r (\frac{1}{\phi_0} - 1)] \frac{\partial \bar{u}}{\partial \bar{t}} \quad . \quad (F-28) \end{aligned}$$

This equation is a second order non-linear parabolic differential equation which can be written in compact form as

$$\frac{\partial}{\partial \bar{r}} [K(\bar{r}, \bar{u}, \bar{u}_{\bar{r}}) \bar{u}_{\bar{r}}] = G(\bar{r}, \bar{u}) \bar{u}_{\bar{r}} \quad (\text{F-29})$$

where the non-linear coefficients are defined as

$$K(\bar{r}, \bar{u}, \bar{u}_{\bar{r}}) = \frac{\bar{r} k p_f (\bar{p} + \bar{b})}{2r_e \mu Z \bar{p} \sqrt{\frac{27 g_c}{\alpha_1}} + r_e \beta k p_f (\bar{p} + \bar{b}) Z |\bar{\rho q}|} \quad (\text{F-30})$$

$$G(\bar{r}, \bar{u}) = \frac{\bar{r} p_f}{Z} [c_g + c_r (\frac{1}{\phi_0} - 1)] \quad (\text{F-31})$$

with parameters  $k$ ,  $b$ ,  $\beta$ , and  $\mu$ ,  $Z$ , and  $c_g$  being functions of pressure.

Since the non-linear coefficient  $K(\bar{r}, \bar{u}, \bar{u}_{\bar{r}})$  contains the dimensionless mass flux  $(\bar{\rho q})$ , the solution to Equation (F-29) must be coupled with the dimensionless Forchheimer equation which accounts for slippage. This equation in terms of the dimensionless variables is given by

$$-\frac{\partial \bar{u}}{\partial \bar{r}} = \frac{2r_e \mu Z \bar{p} \sqrt{\frac{72 g_c}{\alpha_1}}}{k p_f (\bar{p} + \bar{b})} (\bar{\rho q}) + r_e \beta Z (\bar{\rho q}) (|\bar{\rho q}|) \quad (\text{F-32})$$

If the assumption of constant rate of change of porosity with pressure is dropped, integration of Equation (F-19) yields



$$-\left[\ln(1-\phi)\right]_{\phi_0}^{\phi} = c_r \left[\frac{p}{p_0}\right]^P$$

where  $\phi_0$  and  $p_0$  refer to surface conditions. Solving for  $\phi$  we obtain

$$\phi = 1 - (1 - \phi_0) \left[ e^{-c_r (p - p_0)} \right] \quad (F-34)$$

This is the value of porosity which would replace  $\phi_0$  in Equation (F-24). Values of porosity will then be calculated at each grid point using the corresponding pressure. In this case, the dimensionless time would have to be defined as

$$\bar{t} = \frac{2 \sqrt{\frac{72 g_c}{\alpha_1}}}{r_e} t \quad (F-35)$$

since the rate of change of porosity with pressure is no longer a constant involving  $\phi_0$ . Also, the expression for the non-linear coefficient  $G(\bar{r}, \bar{u})$  in Equation (F-31) becomes

$$G(\bar{r}, \bar{u}) = \frac{\phi \bar{r} p_f}{Z} \left[ c_g + c_r \left( \frac{1}{\phi} - 1 \right) \right] \quad (F-36)$$

with the value of  $\phi$  given by Equation (F-34).

An alternative for the case where the rate of change of porosity with pressure is assumed to be a constant, is to use the same Equation (F-36) to evaluate  $G(\bar{r}, \bar{u})$  but with  $\phi_0$  instead of  $\phi$ , and the dimensionless time being defined by Equation (F-35).

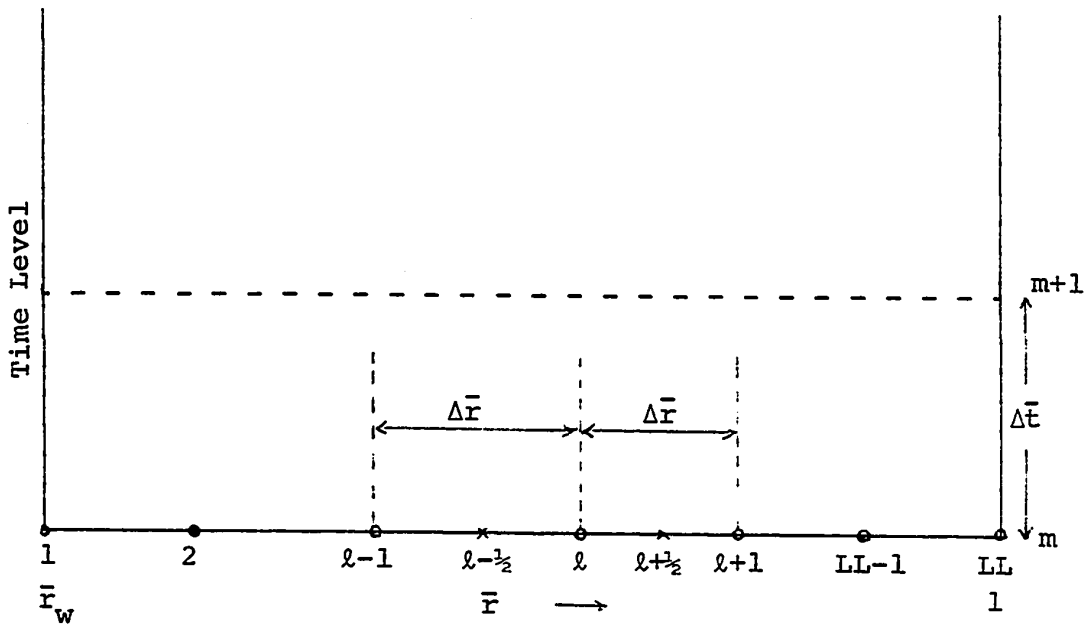
APPENDIX G

DISCRETIZATION OF THE PARTIAL DIFFERENTIAL EQUATIONS

The non-linear second order parabolic differential equation describing plane radial isothermal gas flow through homogeneous and isotropic porous media is re-stated below in compact form:

$$\frac{\partial}{\partial \bar{r}} [K(\bar{r}, \bar{u}, \bar{u}_r) \bar{u}_r] = G(\bar{r}, \bar{u}) \bar{u}_t \quad (G-1)$$

Let the region be defined as follows:



Employing second order correct of the space derivative at points halfway between the  $l^{\text{th}}$  element

and the ( $\ell^{\text{th}}+1$ ) element the following discrete approximation is obtained

$$\left(\frac{\bar{u}}{r}\right)_{\ell+\frac{1}{2}} = \frac{\bar{u}_{\ell+1} - \bar{u}_{\ell}}{\Delta \bar{r}} \quad (G-2)$$

This gives rise to an ordinary differential equation, which approximates the partial differential equation as

$$\frac{K_{\ell+\frac{1}{2}} \left[ \frac{\bar{u}_{\ell+1} - \bar{u}_{\ell}}{\Delta \bar{r}} \right] - K_{\ell-\frac{1}{2}} \left[ \frac{u_{\ell} - u_{\ell-1}}{\Delta \bar{r}} \right]}{\Delta \bar{r}} = G_{\ell} \left( \frac{d\bar{u}}{d\bar{t}} \right)_{\ell} \quad (G-3)$$

The Crank-Nicolson implicit procedure uses a forward second order difference approximation on the time derivative, namely

$$\left( \frac{d\bar{u}}{d\bar{t}} \right)_{\ell, m+\frac{1}{2}} = \left( \frac{\bar{u}_{m+1} - \bar{u}_m}{\Delta \bar{t}} \right)_{\ell}$$

and space derivatives at a corresponding time level of ( $m+\frac{1}{2}$ ). Equation (G-3) is therefore rewritten as

$$\frac{K_{\ell+\frac{1}{2}, m+\frac{1}{2}} \left( \frac{\bar{u}_{\ell+1} - \bar{u}_{\ell}}{\Delta \bar{r}} \right)_{m+\frac{1}{2}} - K_{\ell-\frac{1}{2}, m+\frac{1}{2}} \left( \frac{\bar{u}_{\ell} - \bar{u}_{\ell-1}}{\Delta \bar{r}} \right)_{m+\frac{1}{2}}}{\Delta \bar{r}} =$$

$$G_{\ell, m+\frac{1}{2}} \left( \frac{\bar{u}_{m+1} - \bar{u}_m}{\Delta \bar{t}} \right)_{\ell} \quad (G-4)$$

or

$$\frac{K_{\ell+\frac{1}{2},m+\frac{1}{2}} \frac{1}{2} \left( \frac{\bar{u}_{\ell+1,m+1} - \bar{u}_{\ell,m+1}}{\Delta \bar{r}} + \frac{\bar{u}_{\ell+1,m} - \bar{u}_{\ell,m}}{\Delta \bar{r}} \right) -}{\Delta \bar{r}} - \frac{K_{\ell-\frac{1}{2},m+\frac{1}{2}} \frac{1}{2} \left( \frac{\bar{u}_{\ell,m+1} - \bar{u}_{\ell-1,m+1}}{\Delta \bar{r}} + \frac{\bar{u}_{\ell,m} - \bar{u}_{\ell-1,m}}{\Delta \bar{r}} \right)}{\Delta \bar{r}} =$$

$$G_{\ell,m+\frac{1}{2}} \left( \frac{\bar{u}_{\ell,m+1} - \bar{u}_{\ell,m}}{\Delta \bar{t}} \right) \quad (G-5)$$

where

$$K_{\ell+\frac{1}{2},m+\frac{1}{2}} = \frac{1}{2} [K_{\ell+\frac{1}{2},m+1} + K_{\ell+\frac{1}{2},m}]$$

$$K_{\ell-\frac{1}{2},m+\frac{1}{2}} = \frac{1}{2} [K_{\ell-\frac{1}{2},m+1} + K_{\ell-\frac{1}{2},m}]$$

$$G_{\ell,m+\frac{1}{2}} = \frac{1}{2} [G_{\ell,m+1} + G_{\ell,m}] .$$

Equation (G-5) can be rearranged to become

$$K_{\ell-\frac{1}{2},m+\frac{1}{2}} \bar{u}_{\ell-1,m+1} + (-\rho G_{\ell,m+\frac{1}{2}} - K_{\ell+\frac{1}{2},m+\frac{1}{2}} - K_{\ell-\frac{1}{2},m+\frac{1}{2}}) \bar{u}_{\ell,m+1}$$

$$+ K_{\ell+\frac{1}{2},m+\frac{1}{2}} \bar{u}_{\ell+1,m+1} = -K_{\ell-\frac{1}{2},m+\frac{1}{2}} \bar{u}_{\ell-1,m}$$

$$+ (K_{\ell+\frac{1}{2},m+\frac{1}{2}} + K_{\ell-\frac{1}{2},m+\frac{1}{2}} - \rho G_{\ell,m+\frac{1}{2}}) \bar{u}_{\ell,m}$$

$$- K_{\ell+\frac{1}{2},m+\frac{1}{2}} \bar{u}_{\ell+1,m} \quad (G-6)$$

where

$$\rho \equiv \frac{2(\Delta \bar{r}^2)}{\Delta \bar{t}} .$$

The above Equation (G-6) is a general equation for any grid point ( $l$ ). Since an equation of this form must be written for every grid point where  $\bar{u}_{m+1}$  is unknown, a system of non-linear algebraic equations results which can be concisely represented by

$$\underline{M} \bar{u}_{m+1} = \underline{b}_m \quad (G-7)$$

where  $\underline{M}$  is a tridiagonal matrix,  $\bar{u}_{m+1}$  is the unknown vector, and  $\underline{b}_m$  is a vector evaluated at the previous time.

The size of the system of non-linear algebraic equations depends upon the number of grid points at which  $\bar{u}_{m+1}$  must be evaluated. Therefore, the system depends on the set of boundary conditions to be considered. It should be mentioned that two more unknowns are introduced than there are equations. These are subsequently removed by the use of the two available boundary conditions.

Since the coefficient matrix  $\underline{M}$  in Equation (G-7) is tridiagonal, the Thomas algorithm was used to solve the system of algebraic equations. While the method is equivalent to plain Gaussian elimination, it avoids the

error growth associated with the back solution of the elimination method and also minimizes the storage problems in machine computation.

The method may be summarized as follows. For a system of equations,

$$\begin{aligned} b_1 u_1 + c_1 u_2 &= d_1 \\ a_i u_{i-1} + b_i u_i + c_i u_{i+1} &= d_i \quad 2 \leq i \leq n-1 \quad (G-8) \\ a_n u_{n-1} + b_n u_n &= d_n \end{aligned}$$

Let

$$\begin{aligned} e_1 &= b_1 \\ e_i &= b_i - a_i c_{i-1} / e_{i-1} \quad 2 \leq i \leq n \quad (G-9) \end{aligned}$$

and

$$\begin{aligned} g_1 &= d_1 / e_1 \\ g_i &= (d_i - a_i g_{i-1}) / e_i \quad 2 \leq i \leq n \quad (G-10) \end{aligned}$$

Therefore, the solution vector  $\underline{u}$ , is given by

$$\begin{aligned} u_n &= g_n \\ u_i &= g_i - c_i u_{i+1} / e_i \quad 1 \leq i \leq n-1 \quad (G-11) \end{aligned}$$

Each set of boundary conditions yielded a different sized matrix with different non-linear terms. Hence, the boundary conditions are treated separately.

Case I: The Constant Terminal Rate Case and Constant External Pressure.

a) At the producing face  $l=1$ , we have

$$(\bar{\rho}q)_{1,m+1} = \bar{F}_w \quad 0 < m \leq MM \quad . \quad (G-12)$$

The dimensionless constant mass flux  $\bar{F}_w$ , is related to the pressure distribution through Forchheimer's equation corrected for slippage; namely

$$-(\bar{u}_r)_{1,m+1} = \frac{2r_e \mu Z \bar{p} \sqrt{\frac{72 g_c}{\alpha_1}}}{k p_f (\bar{p} + \bar{b})} \bar{F}_w + r_e \beta Z \bar{F}_w |\bar{F}_w| \quad . \quad (G-13)$$

Using a second order correct finite difference equation at point 1, we obtain

$$\frac{\bar{u}_{0,m+1} - \bar{u}_{2,m+1}}{2\Delta \bar{r}} = \frac{2r_e \mu Z \bar{p}_{1,m+1} \sqrt{\frac{72 g_c}{\alpha}}}{k p_f (\bar{p}_{1,m+1} + \bar{b})} \bar{F}_w + r_e \beta Z \bar{F}_w |\bar{F}_w| \quad . \quad (G-14)$$

Solving for the fictitious point  $\bar{u}_{0,m+1}$ , we obtain

$$\bar{u}_{o,m+1} = \frac{4\Delta\bar{r} r_e \mu Z \bar{p}_{1,m+1} \sqrt{\frac{72 g_c}{\alpha_1}}}{k p_f (\bar{p}_{1,m+1} + \bar{b})} \bar{F}_w + 2\Delta\bar{r} r_e \beta Z \bar{F}_w |\bar{F}_w| + u_{2,m+1} \quad (G-15)$$

Similarly, at the level m the following expression is obtained:

$$\bar{u}_{o,m} = \frac{4\Delta\bar{r} r_e \mu Z \bar{p}_{1,m} \sqrt{\frac{72 g_c}{\alpha_1}}}{k p_f (\bar{p}_{1,m} + \bar{b})} \bar{F}_w + 2\Delta\bar{r} r_e \beta Z \bar{F}_w |\bar{F}_w| + u_{2,m} \quad (G-16)$$

The general equation (G-6) written for the grid point  $l = 1$  yields the following expression after dropping the subscript  $(m+\frac{1}{2})$  from the non-linear coefficients.

$$K_{\frac{1}{2}} \bar{u}_{o,m+1} + (-\rho G_1 - K_{1\frac{1}{2}} + K_{\frac{1}{2}}) \bar{u}_{1,m+1} + K_{1\frac{1}{2}} u_{2,m+1} = -K_{\frac{1}{2}} \bar{u}_{o,m} + (K_{1\frac{1}{2}} + K_{\frac{1}{2}} - \rho G_1) \bar{u}_{1,m} - K_{1\frac{1}{2}} \bar{u}_{2,m} \quad (G-17)$$

If Equation (G-15) is used, the above equation may be written as

$$a(1,1) \bar{u}_{1,m+1} + a(1,2) \bar{u}_{2,m+1} = b(1) \quad (G-18)$$

where



$$a(1,1) = -\rho G_1 - K_{1\frac{1}{2}} - K_{\frac{1}{2}}$$

$$a(1,2) = K_{1\frac{1}{2}} + K_{\frac{1}{2}}$$

$$b(1) = -K_{\frac{1}{2}}(\bar{u}_{0,m+1} - \bar{u}_{2,m+1}) - K_{\frac{1}{2}}\bar{u}_{0,m} \\ + (K_{1\frac{1}{2}} + K_{\frac{1}{2}} - \rho G_1)\bar{u}_{1,m} - K_{1\frac{1}{2}}\bar{u}_{2,m} .$$

b) At the external boundary  $l = LL$ , we have

$$\bar{p}_{LL,m+1} = 1.0 \quad \text{for all times} . \quad (G-19)$$

Therefore, Equation (G-6) for the grid point  $(LL-1) = LLL$  may be written as

$$K_{LLL-\frac{1}{2}}\bar{u}_{LLL-1,m+1} + (-\rho G_{LLL} - K_{LLL+\frac{1}{2}} - K_{LLL-\frac{1}{2}})\bar{u}_{LLL,m+1} \\ + K_{LLL+\frac{1}{2}}\bar{u}_{LL,m+1} = -K_{LLL-\frac{1}{2}}\bar{u}_{LLL-1,m} + (K_{LLL+\frac{1}{2}} + K_{LLL-\frac{1}{2}} - \rho G_{LLL}) \times \\ \times \bar{u}_{LLL,m} - K_{LLL+\frac{1}{2}}\bar{u}_{LL,m} . \quad (G-20)$$

Since  $\bar{u}_{LL,m+1} = \bar{u}_{LL,m} = 1.0$ , the above expression becomes

$$K_{LLL-\frac{1}{2}}\bar{u}_{LLL-1,m+1} + (-\rho G_{LLL} - K_{LLL+\frac{1}{2}} - K_{LLL-\frac{1}{2}})\bar{u}_{LLL,m+1} = \\ -2K_{LLL+\frac{1}{2}} - K_{LLL-\frac{1}{2}}\bar{u}_{LLL-1,m} + (K_{LLL+\frac{1}{2}} + K_{LLL-\frac{1}{2}} - \rho G_{LLL})\bar{u}_{LLL,m} . \quad (G-21)$$

This equation can be rewritten as follows:

$$a(II, JJ-1) \bar{u}_{LLL-1, m+1} + a(II, JJ) \bar{u}_{LLL, m+1} = b(II) \quad (G-22)$$

where

$$a(II, JJ-1) = K_{LLL-\frac{1}{2}}$$

$$a(II, JJ) = (-\rho G_{LLL} - K_{LLL+\frac{1}{2}} - K_{LLL-\frac{1}{2}})$$

$$b(II) = -2K_{LLL+\frac{1}{2}} - K_{LLL-\frac{1}{2}} \bar{u}_{LLL-1, m} \\ + (K_{LLL+\frac{1}{2}} + K_{LLL-\frac{1}{2}} - \rho G_{LLL}) \bar{u}_{LLL, m}$$

Any other element in the tridaigonal matrix  $\underline{M}$  and in the vector  $\underline{b}$  can be computed using the following expression obtained from the general equation (G-6):

$$a(I, J-1) \bar{u}_{\ell-1, m+1} + a(I, J) \bar{u}_{\ell, m+1} + a(I, J+1) \bar{u}_{\ell+1, m+1} = b(I) \quad (G-23)$$

where

$$a(I, J-1) = K_{\ell+\frac{1}{2}, m+\frac{1}{2}}$$

$$a(I, J) = (-\rho G_{\ell, m+\frac{1}{2}} - K_{\ell+\frac{1}{2}, m+\frac{1}{2}} - K_{\ell-\frac{1}{2}, m+\frac{1}{2}})$$

$$a(I, J+1) = K_{\ell+\frac{1}{2}, m+\frac{1}{2}}$$

$$b(I) = -K_{\ell-\frac{1}{2},m+\frac{1}{2}} \bar{u}_{\ell-1,m} + (K_{\ell+\frac{1}{2},m+\frac{1}{2}} + K_{\ell-\frac{1}{2},m+\frac{1}{2}} - G_{\ell,m+\frac{1}{2}}) \bar{u}_{\ell,m} \\ - K_{\ell+\frac{1}{2},m+\frac{1}{2}} \bar{u}_{\ell+1,m} .$$

The matrix size for Case I is then

$$II = JJ = LL - 1.$$

Case II: The Constant Terminal Rate Case and Sealed External Boundary.

a) At the producing face  $\ell = 1$ , we have

$$(\overline{\rho q})_{1,m+1} = \bar{F}_w \quad \text{for all times} . \quad (G-24)$$

Since this boundary condition is identical to the one encountered in Case I, the matrix elements  $a(1,1)$ ,  $a(1,2)$ , and the vector element  $b(1)$  are defined by Equation (G-18).

b) At the external boundary,  $\ell = LL$

$$(\overline{\rho q})_{LL,m+1} = 0 \quad \text{for all times} . \quad (G-25)$$

This boundary condition can be satisfied by demanding that

$$\bar{u}_{LL+1,m+1} = u_{LL-1,m+1}$$

and

(G-26)

$$\bar{u}_{LL+1,m} = u_{LL-1,m} \cdot$$

When the general equation is written for grid point  $l = LL$ , and being understood the non-linear coefficients are always evaluated at  $(m+\frac{1}{2})$  time level, the following expression is obtained:

$$\begin{aligned} K_{LL-\frac{1}{2}} \bar{u}_{LL-1,m+1} + (-\rho G_{LL} - K_{LL+\frac{1}{2}} - K_{LL-\frac{1}{2}}) \bar{u}_{LL,m+1} + \\ + K_{LL+\frac{1}{2}} \bar{u}_{LL+1,m+1} = -K_{LL-\frac{1}{2}} \bar{u}_{LL-1,m} + (K_{LL+\frac{1}{2}} + K_{LL-\frac{1}{2}} - \rho G_{LL}) \times \\ \times \bar{u}_{LL,m} - K_{LL+\frac{1}{2}} \bar{u}_{LL+1,m} \end{aligned} \quad (G-27)$$

Making use of the condition in Equation (G-26), the above equation becomes

$$a(II, JJ-1) \bar{u}_{LL-1,m+1} + a(II, JJ) \bar{u}_{LL,m+1} = b(II) \quad (G-28)$$

where

$$a(II, JJ-1) = K_{LL-\frac{1}{2}} + K_{LL+\frac{1}{2}}$$

$$a(II, JJ) = (-\rho G_{LL} - K_{LL+\frac{1}{2}} - K_{LL-\frac{1}{2}})$$

$$b(\text{II}) = (-K_{\text{LL}-\frac{1}{2}} - K_{\text{LL}+\frac{1}{2}})\bar{u}_{\text{LL}-1,m} + (K_{\text{LL}+\frac{1}{2}} + K_{\text{LL}-\frac{1}{2}} - \rho G_{\text{LL}})\bar{u}_{\text{LL},m}$$

At any other grid point, the tridiagonal elements of the coefficient matrix and the known vector are defined by Equation (G-23). The size of the coefficient matrix for Case II is

$$\text{II} = \text{JJ} = \text{LL}.$$

Case III: The Constant Terminal Pressure Case and Constant External Pressure

a) At the producing face  $\ell = 1$ , we have

$$\bar{p}_{1,m+1} = \bar{p}_w \quad \text{for all times} \quad (\text{G-29})$$

or

$$\bar{u}_{1,m+1} = \bar{p}_w^2 .$$

Writing Equation (G-6) for the grid point  $\ell=2$ , the following equation is obtained

$$K_{2-\frac{1}{2}}\bar{u}_{1,m+1} + (-\rho G_2 - K_{2+\frac{1}{2}} - K_{2-\frac{1}{2}})\bar{u}_{2,m+1} + K_{2+\frac{1}{2}}\bar{u}_{3,m+1} = \\ -K_{2-\frac{1}{2}}\bar{u}_{1,m} + (K_{2+\frac{1}{2}} + K_{2-\frac{1}{2}} - \rho G_2)\bar{u}_{2,m} - K_{2+\frac{1}{2}}\bar{u}_{3,m} . \quad (\text{G-30})$$

If the boundary condition given by Equation (G-29) is used, the above equation becomes

$$a(1,1)\bar{u}_{2,m+1} + a(1,2)\bar{u}_{3,m+1} = b(1) \quad (G-31)$$

$$a(1,1) = (-\rho G_2 - K_{2+\frac{1}{2}} - K_{2-\frac{1}{2}})$$

$$a(1,2) = K_{2+\frac{1}{2}}$$

$$b(1) = -K_{2-\frac{1}{2}}\bar{u}_{1,m} + (K_{2+\frac{1}{2}} + K_{2-\frac{1}{2}} - \rho G_2)\bar{u}_{2,m} \\ - K_{2-\frac{1}{2}}\bar{u}_{1,m+1} - K_{2+\frac{1}{2}}\bar{u}_{3,m} \quad .$$

It should be remembered that in the expression for  $b(1)$ , the value of  $\bar{u}_{1,m+1}$  is a constant known at all times.

b) At the external boundary  $l = LL$ ,

$$\bar{p}_{LL,m+1} = 1.0 \quad \text{for all times} \quad . \quad (G-32)$$

Since this boundary condition is identical to the condition stated for Case I, the elements  $a(II,J-1)$ ,  $a(II,JJ)$  and  $b(II)$  are given by Equation (G-22). Equation (G-23) gives the remaining elements of the coefficient matrix. The size of the matrix is now

$$II = JJ = (LL - 2) \quad .$$

Case IV: The Constant Terminal Pressure Case and Sealed  
External Boundary

- a) At the producing face  $l = 1$ ,

$$\bar{p}_{1,m+1} = \bar{p}_w \quad \text{for all times} \quad . \quad (G-33)$$

Therefore, the elements  $a(1,1)$ ,  $a(1,2)$  and  $b(1)$  for the grid point  $l = 2$ , are defined by Equation (G-31).

- b) At the external boundary  $l = LL$ ,

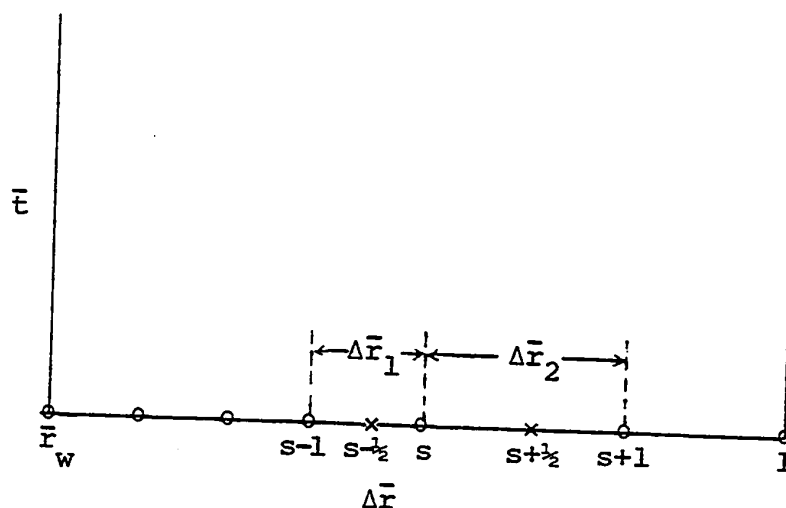
$$(\bar{p}_q)_{LL,m+1} = 0 \quad \text{for all times} \quad . \quad (G-34)$$

This boundary condition is identical to the one in Case II. Hence, the elements  $a(II, JJ-1)$ ,  $a(II, JJ)$ , and  $b(II)$  are obtained using Equation (G-28). The remaining elements are defined by Equation (G-23) and the size of the matrix is

$$II = JJ = (LL - 1) \quad .$$

The above development considers a constant  $\Delta \bar{r}$  throughout the region. However, since the largest pressure drop occurs in the vicinity of the wellbore, it was decided to use two different sizes of the dimensionless radial distance  $\Delta \bar{r}$ , the smallest being in the region close to the wellbore, and set equal to the dimensionless wellbore radius,  $\bar{r}_w$ .

The region now can be represented by the following diagram:



Let  $s$  designate the grid point where a change in the magnitude of  $\Delta \bar{r}$  occurs. This consideration implies the development of a special equation at the grid point  $s$  since now the distances  $(s+\frac{1}{2})$  and  $(s-\frac{1}{2})$  from  $s$ , where fluxes and non-linear coefficients are to be evaluated, are not equal.

Therefore, at grid point  $s$  using Equation (G-5), we have:

$$K_{s+\frac{1}{2}} \frac{1}{2} \left( \frac{\bar{u}_{s+1,m+1} - \bar{u}_{s,m+1} + \bar{u}_{s+1,m} - \bar{u}_{s,m}}{\Delta \bar{r}_2} \right) - \frac{\Delta \bar{r}_2}{2} + \frac{\Delta \bar{r}_1}{2}$$



$$\begin{aligned}
& - K_{s-\frac{1}{2}} \frac{1}{2} \left( \frac{\bar{u}_{s,m+1} - \bar{u}_{s-1,m-1} + \bar{u}_{s,m} - \bar{u}_{s-1,m}}{\Delta \bar{r}_1} \right) \\
& \frac{\Delta \bar{r}_2}{2} + \frac{\Delta \bar{r}_1}{2} = \\
& = G_s \left( \frac{\bar{u}_{s,m+1} - \bar{u}_{s,m}}{\Delta \bar{t}} \right) \quad (G-35)
\end{aligned}$$

After some algebraic manipulations the above equation may be written as

$$\begin{aligned}
& K_{s+\frac{1}{2}} \Delta \bar{r}_1 (\bar{u}_{s+1,m+1} - \bar{u}_{s,m+1} + \bar{u}_{s+1,m} - \bar{u}_{s,m}) \\
& - K_{s-\frac{1}{2}} \Delta \bar{r}_2 (\bar{u}_{s,m+1} - \bar{u}_{s-1,m+1} + \bar{u}_{s,m} - \bar{u}_{s-1,m}) = \\
& = \rho_1 G_s (\bar{u}_{s,m+1} - \bar{u}_{s,m}) \quad (G-36)
\end{aligned}$$

where

$$\rho_1 = \frac{\Delta \bar{r}_2 \times \Delta \bar{r}_1 (\Delta \bar{r}_2 + \Delta \bar{r}_1)}{\Delta \bar{t}} \quad (G-37)$$

Finally, the specific algebraic equation at grid point  $s$  is given by the following expression

$$\begin{aligned}
& K_{s-\frac{1}{2}} \Delta \bar{r}_2 \bar{u}_{s-1,m+1} + (-\rho_1 G_s - K_{s+\frac{1}{2}} \Delta \bar{r}_1 - K_{s-\frac{1}{2}} \Delta \bar{r}_2) \bar{u}_{s,m+1} \\
& + K_{s+\frac{1}{2}} \Delta \bar{r}_1 \bar{u}_{s+1,m+1} = -K_{s-\frac{1}{2}} \Delta \bar{r}_2 \bar{u}_{s-1,m} + (K_{s+\frac{1}{2}} \Delta \bar{r}_1 + K_{s-\frac{1}{2}} \Delta \bar{r}_2 \\
& - \rho_1 G_s) \bar{u}_{s,m} - K_{s+\frac{1}{2}} \Delta \bar{r}_1 \bar{u}_{s+1,m} \quad (G-38)
\end{aligned}$$

At grid points greater than  $s$  the same equations are applied except that the following definition for  $\rho$  must be used

$$\rho_2 = \frac{2\Delta\bar{r}_2^{-2}}{\Delta\bar{t}} \quad (G-39)$$

#### Calculation of the Dimensionless Mass Flux

The non-linear coefficients  $K_{\ell+1/2, m+1/2}$  and  $K_{\ell-1/2, m+1/2}$  contain the dimensionless mass flux  $\bar{\rho q}$ , which is related to the dependent variable  $\bar{u}$  through the modified Forchheimer equation. The term  $\bar{u}_{\bar{r}}$  in this equation was approximated by a second order central finite difference equation evaluated at  $t_{m+1/2}$  and half way between the  $\ell^{\text{th}}$  and  $(\ell^{\text{th}} + 1)$  element. These levels were used because the non-linear coefficients were evaluated at those levels.

Hence, the term  $\bar{u}_{\bar{r}}$  is given by the following expression

$$(\bar{u}_{\bar{r}})_{\ell+1/2, m+1/2} = \frac{(\bar{u}_{\bar{r}})_{\ell+1/2, m} + (\bar{u}_{\bar{r}})_{\ell+1/2, m+1}}{2} \quad (G-40)$$

where

$$(\bar{u}_{\bar{r}})_{\ell+1/2} = \frac{(\bar{u})_{\ell+1} - (\bar{u})_{\ell}}{\Delta\bar{r}} \quad (G-41)$$

The value of the mass flux can then be readily evaluated from Forchheimer's equation and used for evaluating the non-linear coefficients in the parabolic equation.

For the constant terminal pressure case, where it is necessary to predict the flux at the wellbore, a second order correct forward difference equation was used. The selected polynomial was of the general form

$$y = a_0 + a_1x + a_2x^2 \quad . \quad (G-42)$$

The evaluation of the coefficients  $a_0$ ,  $a_1$ , and  $a_2$  was performed in terms of the known functions values at the first three grid points at the locations  $r_w$ ,  $2r_w$ , and  $3r_w$ . Hence, the following simultaneous equations were obtained:

$$\begin{aligned} \bar{u}_1 &= a_0 + a_1\bar{r}_w + a_2\bar{r}_w^2 \\ \bar{u}_2 &= a_0 + a_1(2\bar{r}_w) + a_2(4\bar{r}_w^2) \\ \bar{u}_3 &= a_0 + a_1(3\bar{r}_w) + a_2(9\bar{r}_w^2) \end{aligned} \quad (G-43)$$

or in matrix notation

$$\underline{\bar{u}} = \underline{M} \underline{a} \quad . \quad (G-44)$$

Solving for  $\underline{a}$ , we obtain

$$\underline{a} = \underline{M}^{-1} \underline{\bar{u}} \quad (G-45)$$

where the matrix coefficient  $\underline{M}$  is

$$\underline{M} = \begin{pmatrix} 1 & 0.001 & 0.000001 \\ 1 & 0.002 & 0.000004 \\ 1 & 0.003 & 0.000009 \end{pmatrix} \quad (G-46)$$

Using the general equation (G-42), we obtain

$$[\bar{u}_r]_{\bar{r}=\bar{r}_w} = a_1 + 2a_2\bar{r}_w \quad (G-47)$$

The solution of Equation (G-45) gives

$$a_0 = 3\bar{u}_1 - 3\bar{u}_2 + \bar{u}_3$$

$$a_1 = -2.5 \times 10^3 \bar{u}_1 + 4.0 \times 10^3 \bar{u}_2 - 1.5 \times 10^3 \bar{u}_3 \quad (G-48)$$

$$a_2 = 5.0 \times 10^5 \bar{u}_1 - 1.0 \times 10^6 \bar{u}_2 + 5.0 \times 10^5 \bar{u}_3 \quad .$$

Hence, the expression for the derivative at  $\bar{r}=\bar{r}_w$  is obtained using Equation (G-47),

$$[\bar{u}_r]_{\bar{r}=\bar{r}_w} = -1,500\bar{u}_1 + 2000\bar{u}_2 - 500\bar{u}_3 \quad (G-49)$$

or written as a function of  $\Delta\bar{r}$ , we obtain

$$[\bar{u}_r]_{\bar{r}=\bar{r}_w} = \frac{-3\bar{u}_1 + 4\bar{u}_2 - \bar{u}_3}{2\Delta\bar{r}} \quad (G-50)$$

Calculation of Fraction of Gas Produced

The fraction of gas produced at the end of any step can be evaluated using average fluxes.

The initial mass of gas in the reservoir can be obtained from the modified gas law:

$$p_i \phi_i v_i = \frac{Z_i \omega_i}{M} RT \tag{G-51}$$

or

$$\omega_i = \alpha_1 \frac{p_i \phi_i v_i}{Z_i} \tag{G-52}$$

where

$$\alpha_1 = \frac{M}{RT} .$$

Moreover, since  $v_i = \pi(r_e^2 - r_w^2)h$ , Equation (G-52) becomes

$$\omega_i = \alpha_1 \frac{p_i \phi_i}{Z_i} \pi r_e^2 (1 - \bar{r}_w^2) h . \tag{G-53}$$

On the other hand, the amount of gas produced at any time, can be calculated by the following expression

$$\omega_p = \int_{t_i}^t (\rho q)_{L=1} A_1 dt \tag{G-54}$$

where  $(\rho q)_{L=1}$  is the mass flux at the wellbore.

Using the definitions of the dimensionless mass flux and dimensionless time, we can write:

$$(\rho q) dt = (\overline{\rho q} p_i \sqrt{72 \alpha_1 g_c}) \left( \frac{\phi_o r_e}{2 \sqrt{\frac{72 g_c}{\alpha_1}}} d\bar{t} \right)$$

or

$$(\rho q) dt = \frac{\overline{\rho q} p_i \alpha_1 \phi_o r_e}{2} d\bar{t} .$$

Also,

$$A_1 = 2\pi r_w h .$$

Hence, Equation (G-54) yields

$$\omega_p = p_i \alpha_1 \phi_o r_e^2 \pi h [\bar{r}_w] \int_{\bar{t}_i}^{\bar{t}} (\overline{\rho q})_{L=1} d\bar{t} \quad (G-55)$$

and the fraction of gas produced, FGP, is given by

$$FGP = \frac{\omega_p}{\omega_i} = \frac{z_i \phi_o}{\phi_i (1 - \bar{r}_w^2)} [\bar{r}_w] \int_{\bar{t}_i}^{\bar{t}} (\overline{\rho q})_{L=1} d\bar{t} . \quad (G-56)$$

Values for the integral are found using average values of the mass flux at the wellbore,  $L=1$ , at each time step.

When the dimensionless time is defined as

$$\bar{t} = \frac{2}{r_e} \sqrt{\frac{72 g_c}{\alpha_1}} t , \quad (G-57)$$

The expression for the fraction of gas produced becomes

$$FGP = \frac{z_i}{\phi_i (1 - \bar{r}_w^2)} [\bar{r}_w \int_{\bar{t}_i}^{\bar{t}} (\overline{\rho q})_{L=1} d\bar{t}] \quad (G-58)$$

Should we have any influx term at the external radius,  $r_e$ , the rate of accumulation in the reservoir can be calculated from the continuity equation which reads (mass rate in)<sub>L=LL</sub> - (mass rate out)<sub>L=1</sub> = rate of accumulation.

Mathematically, the amount of gas accumulated is therefore given by the following expression

$$\text{Accumulation} = \frac{z_i \phi_o}{\phi_i (1 - \bar{r}_w^2)} \left[ \int_{\bar{t}_i}^{\bar{t}} (\overline{\rho q})_{L=LL} d\bar{t} - \bar{r}_w \int_{\bar{t}_i}^{\bar{t}} (\overline{\rho q})_{L=1} d\bar{t} \right] \quad (G-59)$$

From the above equation, it is obvious that when the dimensionless flux at L=LL is zero, the second term in the square brackets represents the amount of gas being depleted from the reservoir, i.e., the gas produced at the wellbore.

Copies of the computer programs for the four sets of boundary conditions are included in Appendix K.

## APPENDIX H

### DETERMINATION OF GAS PROPERTIES AS A FUNCTION OF PRESSURE

In the iterative procedure to obtain the solution corresponding to Equation (F-29), values of the deviation factor  $Z$ , gas compressibility  $c_g$ , and viscosity  $\mu$ , must be computed as a function of pressure at each grid point in the region.

Gas in the reservoir was assumed to be nitrogen. This assumption was made on purpose so that a further experimental work in a laboratory might be carried on in order to verify the solutions obtained in this work. Nitrogen properties have been well studied in the literature and summarized in Reference (8).

In this study, Benedict, Webb, and Rubin equation (BWR) was used to calculate the density of nitrogen  $\rho$ , and  $Z$  was obtained using the following expression

$$Z = \frac{P}{\rho RT} \quad (H-1)$$

where

$p$ , atm

$\rho$ , moles/liter

$R$ ,  $\frac{\text{liter-atm}}{^\circ\text{K} - \text{mole gm}}$  (= 0.0825)

$T$ ,  $^\circ\text{K}$



The gas compressibility is by definition,

$$c_g \equiv \frac{1}{p} - \frac{1}{Z} \frac{\partial Z}{\partial p} \quad . \quad (H-2)$$

An expression for  $\partial Z/\partial p$  may be obtained from Equation (H-1). Thus,

$$\frac{\partial Z}{\partial p} = - \frac{p}{RT\rho^2} \frac{\partial \rho}{\partial p} + \frac{1}{RT\rho} \quad . \quad (H-3)$$

It remains to find an expression for  $\partial \rho/\partial p$ . The BWR equation may be written as

$$Z = \frac{p}{\rho RT} = 1 + \left( B_0 - \frac{A_0}{RT} - \frac{C_0}{RT^3} \right) \rho + \left( b - \frac{a}{RT} \right) \rho^2 + \frac{ac}{RT} \rho^5 + \frac{c\rho^2}{RT^3} (1 + \gamma\rho^2) e^{-\gamma\rho^2} \quad . \quad (H-4)$$

For an isothermal process, the above expression becomes

$$\frac{p}{\rho RT} = 1 + A\rho + B\rho^2 + C\rho^5 + D\rho^2(1 + \gamma\rho^2) e^{-\gamma\rho^2} \quad (H-5)$$

where

$$A = B_0 - \frac{A_0}{RT} - \frac{C_0}{RT^3}$$

$$B = b - \frac{a}{RT}$$

$$C = \frac{ac}{RT}$$

$$D = \frac{c}{RT^3} \quad .$$

Solving Equation (H-5) for  $p$ , we obtain

$$p = \rho RT + A_1 \rho^2 + B_1 \rho^3 + C_1 \rho^6 + D_1 \rho^3 (1 + \gamma \rho^2) e^{-\gamma \rho^2} \quad (\text{H-6})$$

where

$$A_1 = A \times RT$$

$$B_1 = B \times RT$$

$$C_1 = C \times RT$$

$$D_1 = D \times RT \quad .$$

Deriving Equation (H-6) with respect to  $p$ , we get

$$\begin{aligned} 1 = RT \frac{\partial \rho}{\partial p} + 2A_1 \rho \frac{\partial \rho}{\partial p} + 3B_1 \rho^2 \frac{\partial \rho}{\partial p} + 6C_1 \rho^5 \frac{\partial \rho}{\partial p} \\ + D_1 \rho^3 (1 + \gamma \rho^2) e^{-\gamma \rho^2} (-2\gamma \rho \frac{\partial \rho}{\partial p}) + D_1 \rho^3 e^{-\gamma \rho^2} (2\gamma \rho \frac{\partial \rho}{\partial p}) \\ + (1 + \gamma \rho^2) e^{-\gamma \rho^2} (3D_1 \rho^2 \frac{\partial \rho}{\partial p}) \quad . \end{aligned} \quad (\text{H-7})$$

From the above equation we solve for  $\partial \rho / \partial p$  to obtain

$$\frac{\partial \rho}{\partial p} = \frac{1}{RT + 2A_1 \rho + 3B_1 \rho^2 + 6C_1 \rho^5 + 2D_1 \gamma \rho^4 e^{-\gamma \rho^2} [1 - (1 + \gamma \rho^2)] + 3D_1 \rho^2 (1 + \gamma \rho^2) \times e^{-\gamma \rho^2}} \quad . \quad (\text{H-8})$$

Once  $\partial \rho / \partial p$  is computed, Equation (H-3) can be used to obtain  $\partial Z / \partial p$ . Finally, the use of Equation (H-2) gives the value for  $c_g$ .

The viscosity of nitrogen was calculated using the polynomial fit given by Kesting and Wang (49),

$$\frac{\mu}{\mu_a} = 1 + 8.958 \times 10^{-4} (p-1) + 6.120 \times 10^{-7} (p-1)^2 + 3.997 \times 10^{-8} (p-1)^3 \quad (\text{H-9})$$

where  $\mu_a$  is the viscosity of  $N_2$  at 1 atm and is equal to  $1.778 \times 10^{-4}$  poise. Equation (H-9) implies the following units:

$\mu$  , poise

$p$  , atm

The temperature correction is considered as  $4.550 \times 10^{-7}$  poise/deg.c. Since Equation (H-9) is at 25°C, the final expression for viscosity is given by

$$\mu = 1.778 \times 10^{-4} [1 + 8.958 \times 10^{-4} (p-1) + 6.120 \times 10^{-7} (p-1)^2 + 3.997 \times 10^{-8} (p-1)^3] + 4.55 \times 10^{-7} (T - 25) \quad (\text{H-10})$$

where T is in °C.

The evaluation of the gas (nitrogen) properties at each grid point are performed through the subroutine "FPRESS" given in Appendix K.

## APPENDIX I

### COMPUTER RESULTS FOR THE CONSTANT TERMINAL RATE CASE

#### CASE I

The following pages show the solution for Case I at the optimum grid size as stated in Section 5.3. The first part corresponds to solutions obtained with a permeability value of 500.0 millidarcies, and the second part to those obtained with a permeability of 10.0 millidarcies. Only two solutions are presented for a permeability of 0.1 millidarcies.

The solutions include results for the dimensionless pressure-squared and dimensionless mass flux distributions which were obtained considering a constant rate of change of porosity with pressure. This was preferred because the difference in the solutions as compared with those obtained considering a variable rate of change of porosity, was negligible, and besides this allowed some saving in computer time. However, the computer program given in Appendix K, Section K.1, can handle any of the conditions for change of porosity depending on the value of the integer "IPOR" as explained in the program. Alternative is also given for the use of  $\bar{\Delta r}$  in the sense that the program can accept a constant  $\bar{\Delta r}$  throughout the region or two different values depending on the value of the integer "ICONT", the switching point being specified by the parameter "IFG".

Solutions for Case II are not included since they were identical to those of Case I due to the fact that the transient did not reach the external boundary for the number of time steps used. A copy of the computer program for Case II is also given in Appendix K, Section K.2.

I.1

SOLUTIONS OBTAINED FOR A PERMEABILITY VALUE  
OF 500.0 MILLIDARCIES



SLIP COEFF(B) = 0.0 INERTIAL COEFF(IDU) = 0.0 PERMEABILITY(K) = 500.000 MD

FLUX DISTRIBUTION FOR THE FIRST 40 GRID POINTS

0.250E 00	-0.400D-04	-0.262D-05	-0.528D-06	-0.124D-06	-0.309D-07	-0.792D-08	-0.207D-08	-0.549D-09	-0.147D-09	-0.397D-10
	-0.108D-10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.500E 00	-0.400D-04	-0.961D-05	-0.268D-05	-0.742D-06	-0.218D-06	-0.647D-07	-0.192D-07	-0.570D-08	-0.169D-08	-0.500D-09
	-0.148D-09	-0.436D-10	-0.129D-10	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.100E 01	-0.400D-04	-0.140D-04	-0.568D-05	-0.236D-05	-0.973D-06	-0.395D-06	-0.159D-06	-0.432D-07	-0.250D-07	-0.984D-08
	-0.386D-08	-0.151D-08	-0.592D-09	-0.231D-09	-0.905D-10	-0.354D-10	-0.136D-10	-0.420D-11	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.150E 01	-0.400D-04	-0.141D-04	-0.728D-05	-0.391D-05	-0.204D-05	-0.102D-05	-0.497D-06	-0.235D-06	-0.168D-06	-0.490D-07
	-0.219D-07	-0.963D-08	-0.420D-08	-0.182D-08	-0.779D-09	-0.333D-09	-0.141D-09	-0.596D-10	-0.251D-10	-0.105D-10
	-0.440D-11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.250E 01	-0.400D-04	-0.143D-04	-0.768D-05	-0.465D-05	-0.289D-05	-0.177D-05	-0.106D-05	-0.616D-06	-0.351D-06	-0.195D-06
	-0.107D-06	-0.579D-07	-0.309D-07	-0.164D-07	-0.859D-08	-0.449D-08	-0.233D-08	-0.121D-08	-0.622D-09	-0.320D-09
	-0.164D-09	-0.843D-10	-0.432D-10	-0.221D-10	-0.113D-10	-0.574D-11	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.350E 01	-0.400D-04	-0.142D-04	-0.799D-05	-0.513D-05	-0.360D-05	-0.236D-05	-0.160D-05	-0.107D-05	-0.694D-06	-0.442D-06
	-0.276D-06	-0.169D-06	-0.102D-06	-0.601D-07	-0.351D-07	-0.203D-07	-0.116D-07	-0.688D-08	-0.370D-08	-0.207D-08
	-0.115D-08	-0.634D-09	-0.348D-09	-0.191D-09	-0.104D-09	-0.569D-10	-0.307D-10	-0.166D-10	-0.894D-11	-0.481D-11
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.600E 01	-0.400D-04	-0.141D-04	-0.808D-05	-0.540D-05	-0.383D-05	-0.279D-05	-0.205D-05	-0.151D-05	-0.110D-05	-0.800D-06
	-0.573D-06	-0.406D-06	-0.284D-06	-0.197D-06	-0.135D-06	-0.918D-07	-0.620D-07	-0.415D-07	-0.277D-07	-0.184D-07
	-0.121D-07	-0.798D-08	-0.524D-08	-0.343D-08	-0.240D-08	-0.146D-08	-0.953D-09	-0.621D-09	-0.404D-09	-0.262D-09
	-0.171D-09	-0.111D-09	-0.720D-10	-0.467D-10	-0.324D-10	-0.197D-10	-0.121D-10	-0.622D-11	-0.540D-11	0.0
0.850E 01	-0.400D-04	-0.138D-04	-0.811D-05	-0.556D-05	-0.409D-05	-0.312D-05	-0.242D-05	-0.189D-05	-0.148D-05	-0.115D-05
	-0.897D-06	-0.692D-06	-0.529D-06	-0.401D-06	-0.301D-06	-0.224D-06	-0.165D-06	-0.121D-06	-0.876D-07	-0.631D-07
	-0.481D-07	-0.320D-07	-0.226D-07	-0.159D-07	-0.111D-07	-0.776D-08	-0.539D-08	-0.373D-08	-0.257D-08	-0.177D-08
	-0.121D-08	-0.630D-09	-0.566D-09	-0.386D-09	-0.263D-09	-0.178D-09	-0.121D-09	-0.818D-10	-0.553D-10	-0.374D-10



0.135E 02	-0.400D-04	-0.137D-04	-0.606D-05	-0.560D-05	-0.420D-05	-0.37D-05	-0.262D-05	-0.212D-05	-0.173D-05	-0.141D-05
	-0.116D-05	-0.947D-06	-0.773D-06	-0.628D-06	-0.508D-06	-0.409D-06	-0.328D-06	-0.261D-06	-0.206D-06	-0.163D-06
	-0.127D-06	-0.992D-07	-0.876D-07	-0.754D-07	-0.647D-07	-0.550D-07	-0.467D-07	-0.393D-07	-0.328D-07	-0.270D-07
	-0.879D-08	-0.661D-08	-0.497D-08	-0.372D-08	-0.279D-08	-0.208D-08	-0.155D-08	-0.116D-08	-0.862D-09	-0.641D-09
0.165E 02	-0.400D-04	-0.135D-04	-0.759D-05	-0.560D-05	-0.425D-05	-0.337D-05	-0.275D-05	-0.228D-05	-0.191D-05	-0.161D-05
	-0.136D-05	-0.115D-05	-0.980D-06	-0.831D-06	-0.703D-06	-0.594D-06	-0.500D-06	-0.419D-06	-0.350D-06	-0.291D-06
	-0.241D-06	-0.199D-06	-0.163D-06	-0.134D-06	-0.109D-06	-0.881D-07	-0.715D-07	-0.575D-07	-0.458D-07	-0.365D-07
	-0.290D-07	-0.230D-07	-0.182D-07	-0.143D-07	-0.113D-07	-0.892D-08	-0.689D-08	-0.538D-08	-0.419D-08	-0.325D-08
0.260E 02	-0.400D-04	-0.133D-04	-0.775D-05	-0.589D-05	-0.426D-05	-0.341D-05	-0.281D-05	-0.236D-05	-0.200D-05	-0.172D-05
	-0.148D-05	-0.128D-05	-0.111D-05	-0.970D-06	-0.843D-06	-0.744D-06	-0.637D-06	-0.553D-06	-0.479D-06	-0.414D-06
	-0.357D-06	-0.307D-06	-0.263D-06	-0.225D-06	-0.192D-06	-0.163D-06	-0.138D-06	-0.116D-06	-0.979D-07	-0.821D-07
	-0.687D-07	-0.573D-07	-0.476D-07	-0.395D-07	-0.327D-07	-0.269D-07	-0.222D-07	-0.182D-07	-0.149D-07	-0.122D-07
0.335E 02	-0.400D-04	-0.131D-04	-0.784D-05	-0.554D-05	-0.426D-05	-0.343D-05	-0.284D-05	-0.240D-05	-0.206D-05	-0.179D-05
	-0.156D-05	-0.137D-05	-0.121D-05	-0.107D-05	-0.949D-06	-0.841D-06	-0.746D-06	-0.662D-06	-0.586D-06	-0.519D-06
	-0.459D-06	-0.409D-06	-0.357D-06	-0.315D-06	-0.276D-06	-0.242D-06	-0.212D-06	-0.185D-06	-0.161D-06	-0.139D-06
	-0.121D-06	-0.104D-06	-0.859D-07	-0.772D-07	-0.667D-07	-0.566D-07	-0.483D-07	-0.412D-07	-0.350D-07	-0.297D-07
0.435E 02	-0.400D-04	-0.130D-04	-0.776D-05	-0.551D-05	-0.424D-05	-0.341D-05	-0.285D-05	-0.243D-05	-0.210D-05	-0.183D-05
	-0.161D-05	-0.143D-05	-0.127D-05	-0.114D-05	-0.107D-05	-0.916D-06	-0.823D-06	-0.741D-06	-0.666D-06	-0.600D-06
	-0.539D-06	-0.485D-06	-0.435D-06	-0.391D-06	-0.350D-06	-0.314D-06	-0.280D-06	-0.250D-06	-0.223D-06	-0.199D-06
	-0.176D-06	-0.157D-06	-0.139D-06	-0.123D-06	-0.108D-06	-0.952D-07	-0.839D-07	-0.756D-07	-0.645D-07	-0.565D-07
0.545E 02	-0.400D-04	-0.129D-04	-0.770D-05	-0.547D-05	-0.422D-05	-0.342D-05	-0.286D-05	-0.244D-05	-0.212D-05	-0.186D-05
	-0.165D-05	-0.147D-05	-0.132D-05	-0.119D-05	-0.107D-05	-0.972D-06	-0.882D-06	-0.801D-06	-0.729D-06	-0.663D-06
	-0.504D-06	-0.450D-06	-0.400D-06	-0.355D-06	-0.314D-06	-0.276D-06	-0.242D-06	-0.210D-06	-0.181D-06	-0.155D-06
	-0.230D-06	-0.200D-06	-0.188D-06	-0.169D-06	-0.153D-06	-0.137D-06	-0.123D-06	-0.111D-06	-0.994D-07	-0.892D-07
0.635E 02	-0.400D-04	-0.128D-04	-0.764D-05	-0.543D-05	-0.420D-05	-0.341D-05	-0.286D-05	-0.245D-05	-0.213D-05	-0.188D-05
	-0.167D-05	-0.149D-05	-0.135D-05	-0.122D-05	-0.111D-05	-0.101D-05	-0.920D-06	-0.842D-06	-0.771D-06	-0.707D-06
	-0.449D-06	-0.395D-06	-0.345D-06	-0.302D-06	-0.261D-06	-0.223D-06	-0.188D-06	-0.159D-06	-0.132D-06	-0.110D-06
	-0.274D-06	-0.251D-06	-0.229D-06	-0.210D-06	-0.191D-06	-0.175D-06	-0.159D-06	-0.145D-06	-0.132D-06	-0.121D-06
0.735E 02	-0.400D-04	-0.127D-04	-0.760D-05	-0.541D-05	-0.418D-05	-0.340D-05	-0.285D-05	-0.245D-05	-0.213D-05	-0.188D-05
	-0.168D-05	-0.151D-05	-0.136D-05	-0.124D-05	-0.113D-05	-0.103D-05	-0.947D-06	-0.870D-06	-0.801D-06	-0.738D-06
	-0.681D-06	-0.629D-06	-0.581D-06	-0.537D-06	-0.496D-06	-0.459D-06	-0.424D-06	-0.392D-06	-0.362D-06	-0.335D-06
	-0.309D-06	-0.286D-06	-0.264D-06	-0.243D-06	-0.224D-06	-0.207D-06	-0.191D-06	-0.176D-06	-0.162D-06	-0.149D-06

FRACTION OF GAS PRODUCED AT LAST FIVE TIME LEVELS

FRACTION OF GAS PRODUCED\* 0.139E-05 0.180E-05 0.222E-05 0.263E-05 0.305E-05



SLIP COEFF (B) = 0.0      INERTIAL COEFF (BB) = 0.0      PERMEABILITY (K) = 500.000 MD

FLUX DISTRIBUTION FOR THE FIRST 40 GRID POINTS

0.250E 00	-0.400D-04	-0.243D-05	-0.487D-06	-0.110D-06	-0.260D-07	-0.637D-08	-0.159D-08	-0.403D-09	-0.103D-09	-0.266D-10
	-0.691E-11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.500E 00	-0.400E-04	-0.936E-05	-0.241D-05	-0.666D-06	-0.188D-06	-0.532D-07	-0.151D-07	-0.428D-08	-0.121D-08	-0.343D-09
	-0.970E-10	-0.274E-10	-0.770E-11	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.100E 01	-0.400E-04	-0.130D-04	-0.547D-05	-0.220D-05	-0.874D-06	-0.342D-06	-0.132D-06	-0.508D-07	-0.194D-07	-0.735D-08
	-0.278E-08	-0.165E-08	-0.396E-09	-0.149E-09	-0.563D-10	-0.212D-10	-0.802D-11	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.150E 01	-0.400E-04	-0.141D-04	-0.715D-05	-0.375D-05	-0.189D-05	-0.920D-06	-0.431D-06	-0.196D-06	-0.873D-07	-0.381D-07
	-0.164E-07	-0.657E-08	-0.293E-08	-0.123E-08	-0.504E-09	-0.208D-09	-0.835D-10	-0.348D-10	-0.141D-10	-0.571E-11
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.250E 01	-0.400E-04	-0.143D-04	-0.761D-05	-0.455D-05	-0.277D-05	-0.166D-05	-0.904D-06	-0.546D-06	-0.301D-06	-0.163D-06
	-0.868E-07	-0.455E-07	-0.236E-07	-0.121D-07	-0.619D-08	-0.314D-08	-0.158D-08	-0.797D-09	-0.400D-09	-0.200E-09
	-0.959E-10	-0.458E-10	-0.248D-10	-0.124D-10	-0.616D-11	0.0	0.0	0.0	0.0	0.0
0.350E 01	-0.400E-04	-0.182D-04	-0.795D-05	-0.506D-05	-0.337D-05	-0.226D-05	-0.151D-05	-0.981D-06	-0.624D-06	-0.387D-06
	-0.235E-06	-0.140E-06	-0.818E-07	-0.471D-07	-0.268E-07	-0.150D-07	-0.835D-08	-0.460D-08	-0.252E-08	-0.137E-08
	-0.738D-09	-0.357E-09	-0.212D-09	-0.113D-09	-0.599E-10	-0.317E-10	-0.167E-10	-0.880E-11	-0.462D-11	0.0
0.600E 01	-0.400E-04	-0.141D-04	-0.807D-05	-0.536D-05	-0.378D-05	-0.272D-05	-0.198D-05	-0.148D-05	-0.103D-05	-0.736D-06
	-0.518E-06	-0.360E-06	-0.247E-06	-0.168E-06	-0.113D-06	-0.750D-07	-0.496D-07	-0.326D-07	-0.213D-07	-0.136E-07
	-0.856E-08	-0.538E-08	-0.320E-08	-0.219D-08	-0.153D-08	-0.982D-09	-0.628D-09	-0.402D-09	-0.257D-09	-0.164D-09
0.850E 01	-0.400E-04	-0.139E-04	-0.811D-05	-0.554D-05	-0.406D-05	-0.307D-05	-0.236D-05	-0.183D-05	-0.142D-05	-0.109D-05
	-0.839E-06	-0.638E-06	-0.481D-06	-0.358D-06	-0.265D-06	-0.193D-06	-0.140D-06	-0.100D-06	-0.715E-07	-0.505D-07
	-0.354E-07	-0.247E-07	-0.171D-07	-0.118D-07	-0.812E-08	-0.555D-08	-0.378D-08	-0.257D-08	-0.174D-08	-0.117E-08
	-0.791E-09	-0.531D-09	-0.356D-09	-0.238D-09	-0.159E-09	-0.106D-09	-0.707D-10	-0.470D-10	-0.312D-10	-0.207D-10

0.135E 02	-0.400E-04	-0.137D-04	-0.807D-05	-0.560D-05	-0.418D-05	-0.325D-05	-0.258D-05	-0.208D-05	-0.168D-05	-0.137D-05
	-0.111E-05	-0.860E-06	-0.727E-06	-0.584D-06	-0.467E-06	-0.371D-06	-0.294D-06	-0.231D-06	-0.180D-06	-0.140D-06
	-0.108E-06	-0.829D-07	-0.634D-07	-0.482D-07	-0.365D-07	-0.276D-07	-0.207D-07	-0.155D-07	-0.116D-07	-0.866E-08
	-0.644E-08	-0.478D-08	-0.354D-08	-0.261D-08	-0.193D-08	-0.142D-08	-0.105D-08	-0.760D-09	-0.564D-09	-0.414D-09
0.185E 02	-0.400E-04	-0.135E-04	-0.801D-05	-0.561D-05	-0.425D-05	-0.336D-05	-0.273D-05	-0.222D-05	-0.181D-05	-0.157D-05
	-0.132E-05	-0.112E-05	-0.941D-06	-0.792E-06	-0.665E-06	-0.556E-06	-0.464D-06	-0.389E-06	-0.318D-06	-0.262D-06
	-0.215E-06	-0.175D-06	-0.142D-06	-0.119D-06	-0.919D-07	-0.735D-07	-0.586D-07	-0.465D-07	-0.367D-07	-0.289D-07
	-0.227E-07	-0.177D-07	-0.138D-07	-0.107D-07	-0.832D-08	-0.643D-08	-0.496D-08	-0.381D-08	-0.293D-08	-0.224D-08
0.240E 02	-0.400E-04	-0.133D-04	-0.753D-05	-0.550D-05	-0.426E-05	-0.340D-05	-0.279D-05	-0.234D-05	-0.193D-05	-0.167D-05
	-0.145E-05	-0.125E-05	-0.108E-05	-0.938E-06	-0.811E-06	-0.701D-06	-0.605D-06	-0.521D-06	-0.448D-06	-0.384D-06
	-0.328E-06	-0.280E-06	-0.238D-06	-0.201D-06	-0.170E-06	-0.143D-06	-0.120D-06	-0.100D-06	-0.832E-07	-0.690E-07
	-0.571E-07	-0.471E-07	-0.387D-07	-0.317E-07	-0.259E-07	-0.211D-07	-0.172D-07	-0.139D-07	-0.113D-07	-0.913D-08
0.335E 02	-0.400E-04	-0.132D-04	-0.786D-05	-0.555D-05	-0.426E-05	-0.342D-05	-0.283D-05	-0.239D-05	-0.205D-05	-0.177D-05
	-0.154E-05	-0.135E-05	-0.119E-05	-0.105E-05	-0.923E-06	-0.814D-06	-0.719D-06	-0.634D-06	-0.558D-06	-0.492D-06
	-0.432E-06	-0.379D-06	-0.332D-06	-0.290D-06	-0.253E-06	-0.220D-06	-0.190D-06	-0.165D-06	-0.142D-06	-0.122D-06
	-0.105E-06	-0.857E-07	-0.765D-07	-0.651D-07	-0.553E-07	-0.468D-07	-0.395D-07	-0.333E-07	-0.280E-07	-0.235D-07
0.435E 02	-0.400E-04	-0.130D-04	-0.778D-05	-0.552D-05	-0.425D-05	-0.343D-05	-0.285D-05	-0.242D-05	-0.209D-05	-0.182D-05
	-0.150E-05	-0.141E-05	-0.126D-05	-0.112E-05	-0.999E-06	-0.895D-06	-0.800D-06	-0.717D-06	-0.642E-06	-0.573D-06
	-0.515D-06	-0.461D-06	-0.412D-06	-0.367D-06	-0.327D-06	-0.291D-06	-0.259D-06	-0.230D-06	-0.203D-06	-0.180D-06
	-0.158E-06	-0.139E-06	-0.122D-06	-0.107D-06	-0.940D-07	-0.821D-07	-0.716D-07	-0.623D-07	-0.542D-07	-0.470D-07
0.535E 02	-0.400E-04	-0.129D-04	-0.772D-05	-0.548D-05	-0.423D-05	-0.347D-05	-0.286D-05	-0.244D-05	-0.211E-05	-0.185D-05
	-0.164E-05	-0.146E-05	-0.130E-05	-0.117E-05	-0.106E-05	-0.950D-06	-0.863D-06	-0.781D-06	-0.708E-06	-0.642D-06
	-0.582E-06	-0.528D-06	-0.479D-06	-0.434D-06	-0.392D-06	-0.350D-06	-0.321D-06	-0.290D-06	-0.261D-06	-0.235D-06
	-0.211E-06	-0.190D-06	-0.170D-06	-0.153D-06	-0.137D-06	-0.122D-06	-0.109D-06	-0.970D-07	-0.864D-07	-0.769D-07
0.635E 02	-0.400E-04	-0.128E-04	-0.766D-05	-0.545D-05	-0.421D-05	-0.341D-05	-0.286D-05	-0.244D-05	-0.213D-05	-0.187D-05
	-0.165E-05	-0.141E-05	-0.134D-05	-0.121E-05	-0.109E-05	-0.993D-06	-0.904D-06	-0.825D-06	-0.753D-06	-0.689D-06
	-0.630E-06	-0.576E-06	-0.527D-06	-0.482D-06	-0.441D-06	-0.403D-06	-0.369D-06	-0.337D-06	-0.307E-06	-0.280E-06
	-0.235E-06	-0.212E-06	-0.211D-06	-0.192D-06	-0.175E-06	-0.158D-06	-0.144D-06	-0.130D-06	-0.118D-06	-0.107D-06
0.735E 02	-0.400E-04	-0.127E-04	-0.762D-05	-0.542D-05	-0.419D-05	-0.340D-05	-0.285D-05	-0.243D-05	-0.213D-05	-0.187D-05
	-0.167E-05	-0.150E-05	-0.136E-05	-0.123E-05	-0.112E-05	-0.102D-05	-0.934D-06	-0.856D-06	-0.786D-06	-0.723D-06
	-0.665E-06	-0.612E-06	-0.564D-06	-0.519D-06	-0.478E-06	-0.441D-06	-0.406D-06	-0.374D-06	-0.344D-06	-0.317E-06
	-0.291E-06	-0.268D-06	-0.246D-06	-0.226D-06	-0.207D-06	-0.190E-06	-0.175D-06	-0.160E-06	-0.147D-06	-0.134D-06

FRACTION OF GAS PRODUCED AT LAST FIVE TIME LEVELS

FRACTION OF GAS PRODUCED= 0.139E-05 0.180E-05 0.222E-05 0.263E-05 0.305E-05





0.136E 02	-0.4000-04	-0.1370-04	-0.6070-05	-0.5610-05	-0.4210-05	-0.3220-05	-0.2620-05	-0.2120-05	-0.1730-05	-0.1420-05
	-0.1100-05	-0.9520-06	-0.7770-06	-0.6320-06	-0.5120-06	-0.4120-06	-0.3300-06	-0.2630-06	-0.2090-06	-0.1640-06
	-0.1290-06	-0.1000-06	-0.7800-07	-0.6030-07	-0.4640-07	-0.3500-07	-0.2720-07	-0.2070-07	-0.1570-07	-0.1190-07
	-0.8970-03	-0.6760-00	-0.5080-08	-0.3810-08	-0.2850-08	-0.2130-08	-0.1590-08	-0.1190-08	-0.8860-09	-0.6590-09
0.185E 02	-0.4000-04	-0.1350-04	-0.8000-05	-0.5610-05	-0.4260-05	-0.3380-05	-0.2750-05	-0.2280-05	-0.1910-05	-0.1610-05
	-0.1370-05	-0.1160-05	-0.9840-06	-0.8350-06	-0.7070-06	-0.5970-06	-0.5030-06	-0.4220-06	-0.3530-06	-0.2940-06
	-0.2840-06	-0.2510-06	-0.1650-06	-0.1350-06	-0.1100-06	-0.8920-07	-0.7210-07	-0.5800-07	-0.4650-07	-0.3710-07
	-0.2950-07	-0.2340-07	-0.1850-07	-0.1460-07	-0.1150-07	-0.9000-08	-0.7040-08	-0.5500-08	-0.4280-08	-0.3330-08
0.260E 02	-0.4000-04	-0.1330-04	-0.7920-05	-0.5590-05	-0.4270-05	-0.3420-05	-0.2810-05	-0.2350-05	-0.2010-05	-0.1720-05
	-0.1490-05	-0.1290-05	-0.1120-05	-0.9730-06	-0.8470-06	-0.7300-06	-0.6400-06	-0.5560-06	-0.4820-06	-0.4140-06
	-0.3590-06	-0.3090-06	-0.2650-06	-0.2270-06	-0.1930-06	-0.1640-06	-0.1390-06	-0.1180-06	-0.9900-07	-0.8310-07
	-0.6900-07	-0.6110-07	-0.4830-07	-0.4010-07	-0.3330-07	-0.2740-07	-0.2230-07	-0.1850-07	-0.1520-07	-0.1240-07
0.335E 02	-0.4000-04	-0.1310-04	-0.7880-05	-0.5590-05	-0.4260-05	-0.3430-05	-0.2890-05	-0.2410-05	-0.2070-05	-0.1790-05
	-0.1470-05	-0.1270-05	-0.1120-05	-0.1070-05	-0.9520-06	-0.8450-06	-0.7490-06	-0.6650-06	-0.5950-06	-0.5220-06
	-0.4620-06	-0.4060-06	-0.3600-06	-0.3170-06	-0.2700-06	-0.2440-06	-0.2130-06	-0.1860-06	-0.1620-06	-0.1410-06
	-0.1220-06	-0.1050-06	-0.9000-07	-0.7810-07	-0.6700-07	-0.5740-07	-0.4900-07	-0.4170-07	-0.3550-07	-0.3010-07
0.435E 02	-0.4000-04	-0.1300-04	-0.7780-05	-0.5510-05	-0.4280-05	-0.3430-05	-0.2800-05	-0.2430-05	-0.2100-05	-0.1840-05
	-0.1620-05	-0.1430-05	-0.1280-05	-0.1140-05	-0.1020-05	-0.9190-06	-0.8270-06	-0.7430-06	-0.6600-06	-0.5820-06
	-0.5420-06	-0.4370-06	-0.4380-06	-0.3930-06	-0.3520-06	-0.3100-06	-0.2820-06	-0.2500-06	-0.2250-06	-0.2000-06
	-0.1780-06	-0.1580-06	-0.1400-06	-0.1240-06	-0.1090-06	-0.9620-07	-0.8470-07	-0.7440-07	-0.6530-07	-0.5720-07
0.535E 02	-0.4000-04	-0.1290-04	-0.7710-05	-0.5480-05	-0.4230-05	-0.3420-05	-0.2800-05	-0.2450-05	-0.2120-05	-0.1820-05
	-0.1650-05	-0.1470-05	-0.1320-05	-0.1190-05	-0.1070-05	-0.9740-06	-0.8840-06	-0.8040-06	-0.7310-06	-0.6650-06
	-0.6360-06	-0.5520-06	-0.5020-06	-0.4570-06	-0.4100-06	-0.3780-06	-0.3440-06	-0.3120-06	-0.2830-06	-0.2560-06
	-0.2320-06	-0.2100-06	-0.1890-06	-0.1710-06	-0.1540-06	-0.1380-06	-0.1250-06	-0.1120-06	-0.1000-06	-0.9010-06
0.635E 02	-0.4000-04	-0.1280-04	-0.7650-05	-0.5440-05	-0.4210-05	-0.3410-05	-0.2860-05	-0.2460-05	-0.2130-05	-0.1860-05
	-0.1670-05	-0.1500-05	-0.1350-05	-0.1220-05	-0.1110-05	-0.1010-05	-0.9220-06	-0.8440-06	-0.7730-06	-0.7090-06
	-0.5510-06	-0.4970-06	-0.4490-06	-0.4040-06	-0.3630-06	-0.3250-06	-0.2900-06	-0.2580-06	-0.2280-06	-0.2010-06
	-0.2760-06	-0.2520-06	-0.2310-06	-0.2110-06	-0.1930-06	-0.1760-06	-0.1610-06	-0.1460-06	-0.1330-06	-0.1220-06
0.735E 02	-0.4000-04	-0.1270-04	-0.7610-05	-0.5410-05	-0.4190-05	-0.3400-05	-0.2850-05	-0.2450-05	-0.2140-05	-0.1860-05
	-0.1680-05	-0.1510-05	-0.1370-05	-0.1240-05	-0.1130-05	-0.1040-05	-0.9500-06	-0.8750-06	-0.8030-06	-0.7400-06
	-0.6430-06	-0.6310-06	-0.5630-06	-0.5390-06	-0.4980-06	-0.4610-06	-0.4260-06	-0.3940-06	-0.3640-06	-0.3370-06
	-0.3110-06	-0.2870-06	-0.2650-06	-0.2450-06	-0.2260-06	-0.2080-06	-0.1920-06	-0.1770-06	-0.1630-06	-0.1500-06

FRACTION OF GAS PRODUCED AT LAST FIVE TIME LEVELS

FRACTION OF GAS PRODUCED= 0.139E-09 0.180L-05 0.222E-08 0.263E-05 0.305E-05





SLIP COEFF(U) = 10.000 INERTIAL COEFF(BB) = 0.0 PERMEABILITY(K) = 500.000 MD

FLUX DISTRIBUTION FOR THE FIRST 40 GRID POINTS

0.250E 00	-0.400D-04	-0.244D-05	-0.490D-06	-0.111D-06	-0.254D-07	-0.648D-08	-0.162D-08	-0.412D-09	-0.106D-09	-0.274D-10
	-0.712D-11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.750E 00	-0.400D-04	-0.940D-05	-0.243D-05	-0.672D-06	-0.190D-06	-0.539D-07	-0.154D-07	-0.437D-08	-0.124D-08	-0.352D-09
	-0.998D-10	-0.282D-10	-0.797D-11	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.100E 01	-0.400D-04	-0.139D-04	-0.549D-05	-0.221D-05	-0.822D-06	-0.346D-06	-0.134D-06	-0.513D-07	-0.197D-07	-0.750D-08
	-0.284D-08	-0.108D-08	-0.407D-09	-0.154D-09	-0.581D-10	-0.220D-10	-0.831D-11	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.150E 01	-0.400D-04	-0.142D-04	-0.718D-05	-0.376D-05	-0.191D-05	-0.928D-06	-0.436D-06	-0.199D-06	-0.887D-07	-0.388D-07
	-0.167D-07	-0.713D-08	-0.300D-08	-0.128D-08	-0.521D-09	-0.215D-09	-0.892D-10	-0.360D-10	-0.147D-10	-0.595D-11
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.250E 01	-0.400D-04	-0.143D-04	-0.763D-05	-0.457D-05	-0.276D-05	-0.167D-05	-0.971D-06	-0.551D-06	-0.305D-06	-0.165D-06
	-0.811D-07	-0.464D-07	-0.241D-07	-0.124D-07	-0.633D-08	-0.322D-08	-0.163D-08	-0.820D-09	-0.412D-09	-0.206D-09
	-0.103D-09	-0.518D-10	-0.258D-10	-0.129D-10	-0.642D-11	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.350E 01	-0.400D-04	-0.142D-04	-0.797D-05	-0.508D-05	-0.338D-05	-0.228D-05	-0.151D-05	-0.788D-06	-0.629D-06	-0.391D-06
	-0.238D-06	-0.142D-06	-0.831D-07	-0.479D-07	-0.273D-07	-0.153D-07	-0.854D-08	-0.472D-08	-0.258D-08	-0.141D-08
	-0.760D-09	-0.409D-09	-0.219D-09	-0.117D-09	-0.622D-10	-0.329D-10	-0.174D-10	-0.918D-11	-0.462D-11	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.600E 01	-0.400D-04	-0.141D-04	-0.808D-05	-0.537D-05	-0.379D-05	-0.273D-05	-0.199D-05	-0.144D-05	-0.104D-05	-0.742D-06
	-0.522D-06	-0.353D-06	-0.250D-06	-0.176D-06	-0.114D-06	-0.761D-07	-0.504D-07	-0.331D-07	-0.217D-07	-0.141D-07
	-0.915D-08	-0.591D-08	-0.381D-08	-0.245D-08	-0.157D-08	-0.101D-08	-0.648D-09	-0.413D-09	-0.264D-09	-0.169D-09
	-0.100D-09	-0.690D-10	-0.441D-10	-0.282D-10	-0.180D-10	-0.116D-10	-0.735D-11	-0.469D-11	0.0	0.0
0.750E 01	-0.400D-04	-0.139D-04	-0.813D-05	-0.555D-05	-0.406D-05	-0.308D-05	-0.237D-05	-0.184D-05	-0.142D-05	-0.110D-05
	-0.644D-06	-0.443D-06	-0.405D-06	-0.302D-06	-0.267D-06	-0.196D-06	-0.142D-06	-0.102D-06	-0.726D-07	-0.513D-07
	-0.160D-07	-0.252D-07	-0.175D-07	-0.121D-07	-0.830D-08	-0.568D-08	-0.388D-08	-0.264D-08	-0.176D-08	-0.121D-08
	-0.814D-09	-0.587D-09	-0.367D-09	-0.244D-09	-0.165D-09	-0.110D-09	-0.733D-10	-0.488D-10	-0.324D-10	-0.215D-10

0.135E 02	-0.4000-04	-0.1370-04	-0.8080-05	-0.85610-05	-0.4190-05	-0.3250-05	-0.2850-05	-0.2080-05	-0.1690-05	-0.1370-05
	-0.1110-05	-0.9040-06	-0.7310-06	-0.5880-06	-0.4700-06	-0.3740-06	-0.2960-06	-0.2330-06	-0.1820-06	-0.1410-06
	-0.1090-06	-0.8400-07	-0.6430-07	-0.4890-07	-0.3710-07	-0.2800-07	-0.2110-07	-0.1580-07	-0.1180-07	-0.8840-08
	-0.6580-08	-0.4880-08	-0.3620-08	-0.2680-08	-0.1900-08	-0.1450-08	-0.1070-08	-0.7800-09	-0.5800-09	-0.4260-09
0.185E 02	-0.4000-04	-0.1350-04	-0.8020-05	-0.85620-05	-0.4450-05	-0.3370-05	-0.2730-05	-0.2260-05	-0.1840-05	-0.1580-05
	-0.1330-05	-0.1120-05	-0.9450-06	-0.7960-06	-0.6680-06	-0.5600-06	-0.4670-06	-0.3880-06	-0.3210-06	-0.2640-06
	-0.2170-06	-0.1770-06	-0.1430-06	-0.1160-06	-0.9310-07	-0.7450-07	-0.5940-07	-0.4720-07	-0.3730-07	-0.2940-07
	-0.2310-07	-0.1800-07	-0.1410-07	-0.1090-07	-0.8490-08	-0.6570-08	-0.5070-08	-0.3900-08	-0.3000-08	-0.2300-08
0.260E 02	-0.4000-04	-0.1340-04	-0.7940-05	-0.8550-05	-0.4270-05	-0.3410-05	-0.2800-05	-0.2340-05	-0.1950-05	-0.1700-05
	-0.1460-05	-0.1260-05	-0.1090-05	-0.9410-06	-0.8140-06	-0.7040-06	-0.6000-06	-0.5240-06	-0.4500-06	-0.3860-06
	-0.3310-06	-0.2820-06	-0.2400-06	-0.2030-06	-0.1710-06	-0.1440-06	-0.1210-06	-0.1010-06	-0.0820-06	-0.6990-07
	-0.3740-07	-0.4700-07	-0.3930-07	-0.3220-07	-0.2640-07	-0.2150-07	-0.1750-07	-0.1420-07	-0.1150-07	-0.9310-08
0.335E 02	-0.4000-04	-0.1320-04	-0.7870-05	-0.8560-05	-0.4270-05	-0.3430-05	-0.2840-05	-0.2400-05	-0.2050-05	-0.1780-05
	-0.1550-05	-0.1360-05	-0.1190-05	-0.1050-05	-0.9260-06	-0.8170-06	-0.7220-06	-0.6370-06	-0.5610-06	-0.4940-06
	-0.4340-06	-0.3810-06	-0.3390-06	-0.2920-06	-0.2540-06	-0.2210-06	-0.1920-06	-0.1680-06	-0.1440-06	-0.1230-06
	-0.1000-06	-0.9070-07	-0.7760-07	-0.6600-07	-0.5600-07	-0.4740-07	-0.4010-07	-0.3380-07	-0.2840-07	-0.2390-07
U.435E 02	-0.4000-04	-0.1310-04	-0.7800-05	-0.8550-05	-0.4250-05	-0.3430-05	-0.2860-05	-0.2430-05	-0.2090-05	-0.1820-05
	-0.1600-05	-0.1420-05	-0.1260-05	-0.1120-05	-0.1000-05	-0.8970-06	-0.8030-06	-0.7200-06	-0.6450-06	-0.5780-06
	-0.5170-06	-0.4630-06	-0.4140-06	-0.3690-06	-0.3290-06	-0.2930-06	-0.2610-06	-0.2310-06	-0.2050-06	-0.1810-06
	-0.1600-06	-0.1410-06	-0.1240-06	-0.1090-06	-0.9800-07	-0.8300-07	-0.7240-07	-0.6310-07	-0.5490-07	-0.4770-07
0.535E 02	-0.4300-04	-0.1290-04	-0.7730-05	-0.8490-05	-0.4230-05	-0.3430-05	-0.2860-05	-0.2440-05	-0.2120-05	-0.1860-05
	-0.1640-05	-0.1460-05	-0.1310-05	-0.1180-05	-0.1060-05	-0.9570-06	-0.8660-06	-0.7840-06	-0.7110-06	-0.6450-06
	-0.5050-06	-0.4500-06	-0.4010-06	-0.3600-06	-0.3240-06	-0.2910-06	-0.2630-06	-0.2370-06	-0.2130-06	-0.1900-06
	-0.2130-06	-0.1910-06	-0.1720-06	-0.1540-06	-0.1380-06	-0.1230-06	-0.1100-06	-0.9800-07	-0.8730-07	-0.7770-07
0.635E 02	-0.4000-04	-0.1280-04	-0.7670-05	-0.8480-05	-0.4210-05	-0.3420-05	-0.2860-05	-0.2450-05	-0.2130-05	-0.1870-05
	-0.1600-05	-0.1440-05	-0.1340-05	-0.1210-05	-0.1100-05	-0.9960-06	-0.9070-06	-0.8270-06	-0.7560-06	-0.6910-06
	-0.6320-06	-0.5780-06	-0.5290-06	-0.4840-06	-0.4430-06	-0.4050-06	-0.3700-06	-0.3380-06	-0.3090-06	-0.2820-06
	-0.2570-06	-0.2340-06	-0.2130-06	-0.1940-06	-0.1780-06	-0.1600-06	-0.1450-06	-0.1310-06	-0.1190-06	-0.1080-06
0.745E 02	-0.4000-04	-0.1200-04	-0.7630-05	-0.8430-05	-0.4200-05	-0.3410-05	-0.2860-05	-0.2450-05	-0.2130-05	-0.1880-05
	-0.1680-05	-0.1500-05	-0.1360-05	-0.1230-05	-0.1120-05	-0.1020-05	-0.9360-06	-0.8600-06	-0.7880-06	-0.7280-06
	-0.6670-06	-0.6140-06	-0.5680-06	-0.5210-06	-0.4800-06	-0.4430-06	-0.4080-06	-0.3760-06	-0.3460-06	-0.3180-06
	-0.2930-06	-0.2690-06	-0.2480-06	-0.2270-06	-0.2090-06	-0.1920-06	-0.1760-06	-0.1610-06	-0.1480-06	-0.1350-06

FRACTION OF GAS PRODUCED AT LAST FIVE TIME LEVELS

FRACTION OF GAS PRODUCED= 0.139E-05 0.180E-05 0.222E-05 0.263E-05 0.305E-05





0.135E 02	-0.4000-04	-0.1380-04	-0.6130-05	-0.5650-05	-0.4220-05	-0.3280-05	-0.2610-05	-0.2110-05	-0.1710-05	-0.1390-05
	-0.1130-05	-0.9220-06	-0.7470-06	-0.6030-06	-0.4840-06	-0.3860-06	-0.3060-06	-0.2420-06	-0.1900-06	-0.1480-06
	-0.1150-06	-0.8850-07	-0.6790-07	-0.5190-07	-0.4360-07	-0.4000-07	-0.2200-07	-0.1700-07	-0.1260-07	-0.9450-08
0.165E 02	-0.7150-06	-0.5330-08	-0.3360-08	-0.2940-08	-0.2180-08	-0.1610-08	-0.1190-08	-0.8900-09	-0.6390-09	-0.4780-09
	-0.4600-04	-0.1360-04	-0.6070-05	-0.5650-05	-0.4260-05	-0.3390-05	-0.2740-05	-0.2280-05	-0.1900-05	-0.1600-05
	-0.1350-05	-0.1140-05	-0.9610-06	-0.8110-06	-0.6820-06	-0.5730-06	-0.4790-06	-0.3990-06	-0.3310-06	-0.2730-06
	-0.2250-06	-0.1040-06	-0.1500-06	-0.1210-06	-0.7770-07	-0.7650-07	-0.6200-07	-0.5000-07	-0.3970-07	-0.3140-07
0.260E 02	-0.2470-07	-0.1940-07	-0.1520-07	-0.1180-07	-0.9220-08	-0.7150-08	-0.5540-08	-0.4280-08	-0.3300-08	-0.2540-08
	-0.4600-04	-0.1340-04	-0.7990-05	-0.5630-05	-0.4400-05	-0.3430-05	-0.2820-05	-0.2350-05	-0.2000-05	-0.1710-05
	-0.1480-05	-0.1270-05	-0.1100-05	-0.9550-06	-0.8280-06	-0.7170-06	-0.6280-06	-0.5350-06	-0.4610-06	-0.3960-06
	-0.3400-06	-0.2930-06	-0.2470-06	-0.2100-06	-0.1780-06	-0.1500-06	-0.1260-06	-0.1060-06	-0.8830-07	-0.7350-07
	-0.6100-07	-0.5050-07	-0.4160-07	-0.3430-07	-0.2810-07	-0.2300-07	-0.1890-07	-0.1530-07	-0.1240-07	-0.1010-07
0.335E 02	-0.4000-04	-0.1330-04	-0.7910-05	-0.5590-05	-0.4290-05	-0.3450-05	-0.2860-05	-0.2420-05	-0.2070-05	-0.1790-05
	-0.1560-05	-0.1370-05	-0.1210-05	-0.1000-05	-0.9390-06	-0.8290-06	-0.7330-06	-0.6430-06	-0.5720-06	-0.5040-06
	-0.4440-06	-0.3900-06	-0.3420-06	-0.3000-06	-0.2620-06	-0.2280-06	-0.1990-06	-0.1720-06	-0.1490-06	-0.1280-06
0.435E 02	-0.1110-06	-0.9480-07	-0.8120-07	-0.6930-07	-0.5900-07	-0.5010-07	-0.4240-07	-0.3690-07	-0.3030-07	-0.2550-07
	-0.4000-04	-0.1310-04	-0.7840-05	-0.5560-05	-0.4280-05	-0.3450-05	-0.2870-05	-0.2440-05	-0.2110-05	-0.1840-05
	-0.1620-05	-0.1430-05	-0.1270-05	-0.1130-05	-0.1010-05	-0.9080-06	-0.8140-06	-0.7300-06	-0.6560-06	-0.5880-06
	-0.5270-06	-0.4720-06	-0.4220-06	-0.3780-06	-0.3370-06	-0.3010-06	-0.2640-06	-0.2380-06	-0.2110-06	-0.1870-06
0.535E 02	-0.1650-06	-0.1460-06	-0.1280-06	-0.1130-06	-0.9900-07	-0.8670-07	-0.7560-07	-0.6620-07	-0.5770-07	-0.5020-07
	-0.3000-04	-0.1300-04	-0.7780-05	-0.5520-05	-0.4220-05	-0.3450-05	-0.2880-05	-0.2460-05	-0.2130-05	-0.1870-05
	-0.1650-05	-0.1470-05	-0.1320-05	-0.1190-05	-0.1070-05	-0.9670-06	-0.8760-06	-0.7940-06	-0.7200-06	-0.6540-06
	-0.5940-06	-0.5390-06	-0.4690-06	-0.4440-06	-0.4920-06	-0.3640-06	-0.2930-06	-0.2490-06	-0.2160-06	-0.1930-06
0.635E 02	-0.2190-06	-0.1970-06	-0.1770-06	-0.1500-06	-0.1430-06	-0.1200-06	-0.1140-06	-0.1020-06	-0.9100-07	-0.8120-07
	-0.4000-04	-0.1290-04	-0.7710-05	-0.5480-05	-0.4240-05	-0.3440-05	-0.2880-05	-0.2460-05	-0.2140-05	-0.1870-05
	-0.1680-05	-0.1500-05	-0.1350-05	-0.1220-05	-0.1110-05	-0.1010-05	-0.9170-06	-0.8370-06	-0.7650-06	-0.7000-06
	-0.6410-06	-0.5870-06	-0.5370-06	-0.4920-06	-0.4510-06	-0.4130-06	-0.3780-06	-0.3450-06	-0.3100-06	-0.2840-06
0.735E 02	-0.2630-06	-0.2400-06	-0.2190-06	-0.1990-06	-0.1810-06	-0.1650-06	-0.1500-06	-0.1360-06	-0.1230-06	-0.1120-06
	-0.4000-04	-0.1280-04	-0.7670-05	-0.5460-05	-0.4220-05	-0.3430-05	-0.2870-05	-0.2460-05	-0.2150-05	-0.1900-05
	-0.1690-05	-0.1520-05	-0.1370-05	-0.1240-05	-0.1130-05	-0.1030-05	-0.9450-06	-0.8670-06	-0.7970-06	-0.7330-06
	-0.6750-06	-0.6220-06	-0.5740-06	-0.5290-06	-0.4840-06	-0.4500-06	-0.4150-06	-0.3810-06	-0.3530-06	-0.3250-06
	-0.2990-06	-0.2760-06	-0.2540-06	-0.2330-06	-0.2140-06	-0.1970-06	-0.1810-06	-0.1660-06	-0.1520-06	-0.1400-06

FRACTION OF GAS PRODUCED AT LAST FIVE TIME LEVELS

FRACTION OF GAS PRODUCED 0.135E-05 0.165E-05 0.260E-05 0.335E-05 0.435E-05 0.535E-05 0.635E-05 0.735E-05





0.135E 02	-0.4000-04	-0.1390-04	-0.6190-05	-0.5690-05	-0.4260-05	-0.3320-05	-0.2640-05	-0.2140-05	-0.1740-05	-0.1420-05
	-0.1100-05	-0.6440-06	-0.7670-06	-0.6210-06	-0.5000-06	-0.4010-06	-0.3190-06	-0.2530-06	-0.1990-06	-0.1560-06
	-0.1210-06	-0.5410-07	-0.7260-07	-0.5570-07	-0.4200-07	-0.3240-07	-0.2460-07	-0.1960-07	-0.1400-07	-0.1060-07
	-0.7910-08	-0.5920-08	-0.4420-08	-0.3390-08	-0.2450-08	-0.1820-08	-0.1350-08	-0.1000-08	-0.7430-09	-0.5490-09
0.185E 02	-0.4000-04	-0.1370-04	-0.8120-05	-0.5690-05	-0.4320-05	-0.3620-05	-0.2780-05	-0.2200-05	-0.1930-05	-0.1620-05
	-0.1370-05	-0.1160-05	-0.9800-06	-0.8590-06	-0.7490-06	-0.5990-06	-0.4940-06	-0.4120-06	-0.3430-06	-0.2840-06
	-0.2340-06	-0.1020-06	-0.1570-06	-0.1280-06	-0.1040-06	-0.8350-07	-0.6710-07	-0.5360-07	-0.4270-07	-0.3390-07
	-0.2630-07	-0.2110-07	-0.1660-07	-0.1300-07	-0.1020-07	-0.7930-08	-0.6160-08	-0.4780-08	-0.3700-08	-0.2860-08
0.260E 02	-0.4000-04	-0.1350-04	-0.8040-05	-0.5670-05	-0.4330-05	-0.3480-05	-0.2850-05	-0.2490-05	-0.2030-05	-0.1740-05
	-0.1490-05	-0.1290-05	-0.1120-05	-0.9730-06	-0.8440-06	-0.6840-06	-0.5350-06	-0.4540-06	-0.4740-06	-0.4090-06
	-0.3510-06	-0.3010-06	-0.2570-06	-0.2190-06	-0.1860-06	-0.1570-06	-0.1330-06	-0.1110-06	-0.9340-07	-0.7400-07
	-0.6050-07	-0.5400-07	-0.4470-07	-0.3690-07	-0.3030-07	-0.2490-07	-0.2040-07	-0.1670-07	-0.1340-07	-0.1110-07
0.335E 02	-0.4000-04	-0.1330-04	-0.7660-05	-0.5630-05	-0.4320-05	-0.3480-05	-0.2800-05	-0.2440-05	-0.2090-05	-0.1810-05
	-0.1580-05	-0.1390-05	-0.1220-05	-0.1080-05	-0.9540-06	-0.8140-06	-0.6470-06	-0.6610-06	-0.5850-06	-0.5160-06
	-0.4560-06	-0.4010-06	-0.3530-06	-0.3100-06	-0.2710-06	-0.2370-06	-0.2060-06	-0.1800-06	-0.1560-06	-0.1350-06
	-0.1160-06	-0.1000-06	-0.8650-07	-0.7350-07	-0.6270-07	-0.5340-07	-0.4540-07	-0.3850-07	-0.3260-07	-0.2750-07
0.435E 02	-0.4000-04	-0.1320-04	-0.7690-05	-0.5590-05	-0.4310-05	-0.3480-05	-0.2900-05	-0.2460-05	-0.2130-05	-0.1860-05
	-0.1630-05	-0.1450-05	-0.1290-05	-0.1150-05	-0.1030-05	-0.9220-06	-0.8270-06	-0.7430-06	-0.6670-06	-0.5990-06
	-0.4530-06	-0.4030-06	-0.3680-06	-0.3280-06	-0.2970-06	-0.2700-06	-0.2460-06	-0.2240-06	-0.2190-06	-0.1940-06
	-0.1720-06	-0.1520-06	-0.1440-06	-0.1190-06	-0.1040-06	-0.9140-07	-0.8010-07	-0.7010-07	-0.6130-07	-0.5350-07
0.535E 02	-0.4000-04	-0.1310-04	-0.7630-05	-0.5560-05	-0.4290-05	-0.3470-05	-0.2900-05	-0.2490-05	-0.2150-05	-0.1890-05
	-0.1670-05	-0.1490-05	-0.1330-05	-0.1200-05	-0.1080-05	-0.9060-06	-0.8060-06	-0.7320-06	-0.6650-06	-0.6050-06
	-0.4600-06	-0.4050-06	-0.3690-06	-0.3290-06	-0.2940-06	-0.2740-06	-0.2500-06	-0.2270-06	-0.2190-06	-0.1940-06
	-0.2250-06	-0.2040-06	-0.1840-06	-0.1650-06	-0.1480-06	-0.1330-06	-0.1190-06	-0.1070-06	-0.9570-07	-0.8550-07
0.635E 02	-0.4000-04	-0.1300-04	-0.7740-05	-0.5520-05	-0.4270-05	-0.3460-05	-0.2900-05	-0.2480-05	-0.2160-05	-0.1900-05
	-0.1690-05	-0.1510-05	-0.1360-05	-0.1230-05	-0.1120-05	-0.1020-05	-0.9280-06	-0.8440-06	-0.7740-06	-0.7100-06
	-0.4650-06	-0.4070-06	-0.3700-06	-0.3290-06	-0.2920-06	-0.2620-06	-0.2390-06	-0.2160-06	-0.2030-06	-0.1780-06
	-0.2710-06	-0.2470-06	-0.2260-06	-0.2060-06	-0.1880-06	-0.1710-06	-0.1550-06	-0.1410-06	-0.1280-06	-0.1170-06
0.735E 02	-0.4000-04	-0.1290-04	-0.7720-05	-0.5490-05	-0.4250-05	-0.3460-05	-0.2900-05	-0.2480-05	-0.2160-05	-0.1910-05
	-0.1700-05	-0.1530-05	-0.1380-05	-0.1250-05	-0.1140-05	-0.1040-05	-0.9560-06	-0.8780-06	-0.8070-06	-0.7430-06
	-0.4680-06	-0.4120-06	-0.3730-06	-0.3300-06	-0.2970-06	-0.2630-06	-0.2390-06	-0.2160-06	-0.2030-06	-0.1780-06
	-0.3070-06	-0.2830-06	-0.2610-06	-0.2400-06	-0.2210-06	-0.2030-06	-0.1870-06	-0.1720-06	-0.1580-06	-0.1450-06

FRACTION OF GAS PRODUCED AT LAST FIVE TIME LEVELS

FRACTION OF GAS PRODUCED\* 0.1301-05 0.1002-05 0.2727-05 0.2612-05 0.3051-05





SLIP COEFF(B)= 0.0 INERTIAL COEFF(BB)= 0.100D 09 PERMEABILITY(K)= 500.000 MD

FLUX DISTRIBUTION FOR THE FIRST 40 GRID POINTS

0.250E 00	-0.400D-04	-0.653D-05	-0.182D-05	-0.472D-06	-0.120D-06	-0.308D-07	-0.807D-08	-0.214D-08	-0.573D-09	-0.155D-09
	-0.421D-10	-0.115D-10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.500E 00	-0.400D-04	-0.106D-04	-0.364D-05	-0.131D-05	-0.454D-06	-0.151D-06	-0.485D-07	-0.152D-07	-0.472D-08	-0.145D-08
	-0.439E-09	-0.133D-09	-0.399D-10	-0.119D-10	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.100E 01	-0.400D-04	-0.123D-04	-0.483D-05	-0.216D-05	-0.100D-05	-0.456D-06	-0.200D-06	-0.854D-07	-0.355D-07	-0.145D-07
	-0.586D-08	-0.235D-08	-0.932D-09	-0.369D-09	-0.146D-09	-0.574D-10	-0.226D-10	-0.887D-11	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.150E 01	-0.400D-04	-0.138D-04	-0.591D-05	-0.295D-05	-0.157D-05	-0.841D-06	-0.443D-06	-0.227D-06	-0.112D-06	-0.539D-06
	-0.725D-07	-0.116D-07	-0.522D-08	-0.232D-08	-0.102D-08	-0.444D-09	-0.192D-09	-0.824D-10	-0.351D-10	-0.149D-10
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.250E 01	-0.400D-04	-0.149D-04	-0.695D-05	-0.378D-05	-0.220D-05	-0.132D-05	-0.793D-06	-0.474D-06	-0.279D-06	-0.162D-06
	-0.918D-07	-0.510D-07	-0.280D-07	-0.151D-07	-0.810D-08	-0.429D-08	-0.226D-08	-0.118D-08	-0.615D-09	-0.319D-09
	-0.165D-09	-0.849D-10	-0.437D-10	-0.224D-10	-0.115D-10	-0.590D-11	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.350E 01	-0.400D-04	-0.158D-04	-0.784D-05	-0.455D-05	-0.283D-05	-0.182D-05	-0.119D-05	-0.779D-06	-0.508D-06	-0.328D-06
	-0.208E-06	-0.130D-06	-0.801D-07	-0.485D-07	-0.289D-07	-0.170D-07	-0.989D-08	-0.569D-08	-0.324D-08	-0.183D-08
	-0.103D-08	-0.574D-09	-0.318D-09	-0.176D-09	-0.964D-10	-0.527D-10	-0.288D-10	-0.156D-10	-0.847D-11	-0.459D-11
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.600E 01	-0.400D-04	-0.163D-04	-0.860D-05	-0.530D-05	-0.351D-05	-0.241D-05	-0.169D-05	-0.120D-05	-0.858D-06	-0.612D-06
	-0.434D-06	-0.307D-06	-0.215D-06	-0.149D-06	-0.103D-06	-0.704D-07	-0.478D-07	-0.322D-07	-0.216D-07	-0.144D-07
	-0.953D-08	-0.630D-08	-0.415D-08	-0.272D-08	-0.178D-08	-0.117D-08	-0.762D-09	-0.497D-09	-0.323D-09	-0.211D-09
	-0.137E-09	-0.890D-10	-0.579D-10	-0.376D-10	-0.249D-10	-0.159D-10	-0.103D-10	-0.670D-11	-0.436D-11	0.0
0.850E 01	-0.400D-04	-0.167D-04	-0.921D-05	-0.594D-05	-0.412D-05	-0.298D-05	-0.220D-05	-0.165D-05	-0.125D-05	-0.949E-06
	-0.721E-06	-0.546D-06	-0.412D-06	-0.310D-06	-0.231D-06	-0.171D-06	-0.128D-06	-0.922D-07	-0.670D-07	-0.413D-07
	-0.346D-07	-0.246D-07	-0.174D-07	-0.123D-07	-0.862D-08	-0.602D-08	-0.419D-08	-0.290D-08	-0.200D-08	-0.138D-08
	-0.948E-09	-0.650D-09	-0.444D-09	-0.303D-09	-0.206D-09	-0.140D-09	-0.952D-10	-0.645D-10	-0.437D-10	-0.295D-10

0.135E 02	-0.400D-04	-0.169D-04	-0.958D-05	-0.638D-05	-0.459D-05	-0.344D-05	-0.264D-05	-0.206D-05	-0.163D-05	-0.130D-05
	-0.104D-05	-0.829D-06	-0.663D-06	-0.530D-06	-0.423D-06	-0.337D-06	-0.267D-06	-0.211D-06	-0.166D-06	-0.130D-06
	-0.102D-06	-0.789D-07	-0.610D-07	-0.470D-07	-0.361D-07	-0.276D-07	-0.211D-07	-0.160D-07	-0.121D-07	-0.918D-08
	-0.693D-08	-0.521D-08	-0.391D-08	-0.293D-08	-0.220D-08	-0.164D-08	-0.122D-08	-0.912D-09	-0.679D-09	-0.505D-09
0.185E 02	-0.400D-04	-0.170D-04	-0.982D-05	-0.669D-05	-0.492D-05	-0.378D-05	-0.299D-05	-0.241D-05	-0.196D-05	-0.161D-05
	-0.133D-05	-0.111D-05	-0.921D-06	-0.767D-06	-0.638D-06	-0.531D-06	-0.441D-06	-0.366D-06	-0.302D-06	-0.249D-06
	-0.285D-06	-0.168D-06	-0.137E-06	-0.111D-06	-0.902D-07	-0.728D-07	-0.585D-07	-0.469D-07	-0.375D-07	-0.299D-07
	-0.236D-07	-0.187D-07	-0.147D-07	-0.116D-07	-0.909D-08	-0.712D-08	-0.556D-08	-0.433D-08	-0.336D-08	-0.261E-08
0.260E 02	-0.400D-04	-0.170D-04	-0.995D-05	-0.687D-05	-0.514D-05	-0.402D-05	-0.323D-05	-0.265D-05	-0.221D-05	-0.185D-05
	-0.157D-05	-0.133D-05	-0.114D-05	-0.971D-06	-0.811D-06	-0.712D-06	-0.610D-06	-0.523D-06	-0.447D-06	-0.382E-06
	-0.326D-06	-0.278D-06	-0.216D-06	-0.240D-06	-0.170D-06	-0.143D-06	-0.121D-06	-0.101D-06	-0.847D-07	-0.708E-07
	-0.589D-07	-0.490D-07	-0.406D-07	-0.335D-07	-0.276D-07	-0.227D-07	-0.187D-07	-0.153D-07	-0.125D-07	-0.102E-07
0.335E 02	-0.400D-04	-0.170D-04	-0.100D-04	-0.700D-05	-0.529D-05	-0.419D-05	-0.341D-05	-0.284D-05	-0.239D-05	-0.204D-05
	-0.175D-05	-0.151D-05	-0.131D-05	-0.114D-05	-0.998D-06	-0.873D-06	-0.761D-06	-0.668D-06	-0.585D-06	-0.513D-06
	-0.448D-06	-0.392D-06	-0.342E-06	-0.299D-06	-0.260D-06	-0.226D-06	-0.196D-06	-0.170D-06	-0.147D-06	-0.127D-06
	-0.109D-06	-0.939D-07	-0.805D-07	-0.689D-07	-0.588D-07	-0.501D-07	-0.426D-07	-0.361D-07	-0.306D-07	-0.259E-07
0.435E 02	-0.400D-04	-0.170D-04	-0.101D-04	-0.708D-05	-0.538D-05	-0.429D-05	-0.353D-05	-0.296D-05	-0.252D-05	-0.217D-05
	-0.189D-05	-0.163D-05	-0.145D-05	-0.128D-05	-0.113D-05	-0.100D-05	-0.891D-06	-0.793D-06	-0.765D-06	-0.628D-06
	-0.359D-06	-0.498D-06	-0.443E-06	-0.394D-06	-0.350D-06	-0.311D-06	-0.276E-06	-0.245D-06	-0.217E-06	-0.192E-06
	-0.169D-06	-0.149D-06	-0.131D-06	-0.116D-06	-0.101D-06	-0.889D-07	-0.779D-07	-0.681D-07	-0.595D-07	-0.519D-07
0.535E 02	-0.400D-04	-0.170D-04	-0.101D-04	-0.713D-05	-0.545D-05	-0.437D-05	-0.362D-05	-0.306D-05	-0.262D-05	-0.228D-05
	-0.199D-05	-0.176D-05	-0.156D-05	-0.139D-05	-0.124D-05	-0.110D-05	-0.998D-06	-0.898D-06	-0.809D-06	-0.728D-06
	-0.657D-06	-0.583D-06	-0.535E-06	-0.482D-06	-0.435D-06	-0.392D-06	-0.354D-06	-0.319D-06	-0.287D-06	-0.254D-06
	-0.232D-06	-0.208D-06	-0.187D-06	-0.168D-06	-0.150D-06	-0.134D-06	-0.120D-06	-0.107D-06	-0.959D-07	-0.856E-07
0.635E 02	-0.400D-04	-0.170D-04	-0.101D-04	-0.716D-05	-0.549D-05	-0.442D-05	-0.368D-05	-0.312D-05	-0.269D-05	-0.234D-05
	-0.207D-05	-0.184D-05	-0.164D-05	-0.147D-05	-0.132D-05	-0.119D-05	-0.108D-05	-0.978D-06	-0.886D-06	-0.805D-06
	-0.732D-06	-0.666D-06	-0.607E-06	-0.553D-06	-0.503D-06	-0.459D-06	-0.418D-06	-0.381D-06	-0.347D-06	-0.315D-06
	-0.287D-06	-0.261D-06	-0.237D-06	-0.216D-06	-0.196D-06	-0.178D-06	-0.161D-06	-0.146D-06	-0.133D-06	-0.120D-06
0.735E 02	-0.400D-04	-0.170D-04	-0.101D-04	-0.718D-05	-0.552D-05	-0.446D-05	-0.372D-05	-0.317D-05	-0.274D-05	-0.240E-05
	-0.212E-05	-0.189D-05	-0.169D-05	-0.153D-05	-0.138D-05	-0.125D-05	-0.114D-05	-0.104D-05	-0.946D-06	-0.865E-06
	-0.792D-06	-0.725E-06	-0.665D-06	-0.610D-06	-0.559D-06	-0.513D-06	-0.471D-06	-0.433D-06	-0.397E-06	-0.365D-06
	-0.335E-06	-0.307D-06	-0.282D-06	-0.258D-06	-0.237D-06	-0.217D-06	-0.199D-06	-0.183D-06	-0.167D-06	-0.153E-06

FRACTION OF GAS PRODUCED AT LAST FIVE PIPE LEVELS

FRACTION OF GAS PRODUCED= 0.139E-05 0.180E-05 0.222E-05 0.263E-05 0.305E-05





0.135E 02	-0.400D-04	-0.168D-04	-0.981D-05	-0.630D-05	-0.450D-05	-0.335D-05	-0.256D-05	-0.199D-05	-0.156D-05	-0.123D-05
	-0.972D-06	-0.770D-06	-0.610D-06	-0.483D-06	-0.381D-06	-0.300D-06	-0.235D-06	-0.183D-06	-0.142D-06	-0.110D-06
	-0.847D-07	-0.649D-07	-0.495D-07	-0.376D-07	-0.289D-07	-0.215D-07	-0.161D-07	-0.121D-07	-0.904D-08	-0.673D-08
	-0.501D-09	-0.371D-09	-0.275D-09	-0.203D-09	-0.150D-09	-0.110D-09	-0.813D-09	-0.598D-09	-0.439D-09	-0.322D-09
0.185E 02	-0.400D-04	-0.170D-04	-0.979D-05	-0.663D-05	-0.480D-05	-0.373D-05	-0.293D-05	-0.235D-05	-0.190D-05	-0.155D-05
	-0.127D-05	-0.105D-05	-0.865D-06	-0.714D-06	-0.589D-06	-0.485D-06	-0.399D-06	-0.327D-06	-0.268D-06	-0.219D-06
	-0.177D-06	-0.143D-06	-0.116D-06	-0.928D-07	-0.743D-07	-0.592D-07	-0.470D-07	-0.372D-07	-0.293D-07	-0.230D-07
	-0.180D-07	-0.141D-07	-0.109D-07	-0.849D-08	-0.657D-08	-0.508D-08	-0.391D-08	-0.301D-08	-0.231D-08	-0.177D-08
0.265E 02	-0.400D-04	-0.170D-04	-0.933D-05	-0.686D-05	-0.511D-05	-0.395D-05	-0.319D-05	-0.261D-05	-0.216D-05	-0.180D-05
	-0.152D-05	-0.128D-05	-0.108D-05	-0.921D-06	-0.783D-06	-0.665D-06	-0.565D-06	-0.481D-06	-0.408D-06	-0.346D-06
	-0.292D-06	-0.247D-06	-0.208D-06	-0.174D-06	-0.146D-06	-0.122D-06	-0.102D-06	-0.845D-07	-0.700D-07	-0.579D-07
	-0.477D-07	-0.392D-07	-0.321D-07	-0.262D-07	-0.214D-07	-0.174D-07	-0.141D-07	-0.114D-07	-0.922D-08	-0.744D-08
0.335E 02	-0.400D-04	-0.170D-04	-0.100D-04	-0.698D-05	-0.526D-05	-0.416D-05	-0.330D-05	-0.280D-05	-0.235D-05	-0.200D-05
	-0.171D-05	-0.147D-05	-0.127D-05	-0.110D-05	-0.985D-06	-0.830D-06	-0.722D-06	-0.620D-06	-0.547D-06	-0.475D-06
	-0.413D-06	-0.388D-06	-0.311D-06	-0.269D-06	-0.232D-06	-0.200D-06	-0.172D-06	-0.148D-06	-0.127D-06	-0.108D-06
	-0.928D-07	-0.787D-07	-0.668D-07	-0.568D-07	-0.470D-07	-0.403D-07	-0.339D-07	-0.285D-07	-0.239D-07	-0.200D-07
0.435E 02	-0.400D-04	-0.170D-04	-0.101D-04	-0.766D-05	-0.638D-05	-0.427D-05	-0.350D-05	-0.293D-05	-0.249D-05	-0.214D-05
	-0.165D-05	-0.161D-05	-0.141D-05	-0.124D-05	-0.109D-05	-0.964D-06	-0.855D-06	-0.754D-06	-0.668D-06	-0.592D-06
	-0.624D-06	-0.464D-06	-0.411D-06	-0.363D-06	-0.321D-06	-0.283D-06	-0.250D-06	-0.220D-06	-0.193D-06	-0.170D-06
	-0.149D-06	-0.130D-06	-0.114D-06	-0.991D-07	-0.863D-07	-0.750D-07	-0.651D-07	-0.564D-07	-0.488D-07	-0.422D-07
0.535E 02	-0.400D-04	-0.170D-04	-0.101D-04	-0.712D-05	-0.544D-05	-0.435D-05	-0.359D-05	-0.303D-05	-0.259D-05	-0.225D-05
	-0.196D-05	-0.173D-05	-0.153D-05	-0.135D-05	-0.121D-05	-0.108D-05	-0.963D-06	-0.862D-06	-0.774D-06	-0.694D-06
	-0.623D-06	-0.560D-06	-0.503D-06	-0.452D-06	-0.405D-06	-0.364D-06	-0.326D-06	-0.292D-06	-0.265D-06	-0.240D-06
	-0.205D-06	-0.187D-06	-0.166D-06	-0.148D-06	-0.132D-06	-0.117D-06	-0.104D-06	-0.922D-07	-0.817D-07	-0.723D-07
0.635E 02	-0.400D-04	-0.170D-04	-0.101D-04	-0.715D-05	-0.548D-05	-0.441D-05	-0.365D-05	-0.310D-05	-0.267D-05	-0.232D-05
	-0.204D-05	-0.181D-05	-0.161D-05	-0.144D-05	-0.129D-05	-0.116D-05	-0.104D-05	-0.944D-06	-0.854D-06	-0.773D-06
	-0.701D-06	-0.635D-06	-0.576D-06	-0.523D-06	-0.474D-06	-0.430D-06	-0.390D-06	-0.354D-06	-0.321D-06	-0.290D-06
	-0.263D-06	-0.238D-06	-0.215D-06	-0.194D-06	-0.175D-06	-0.158D-06	-0.143D-06	-0.129D-06	-0.116D-06	-0.104D-06
0.735E 02	-0.400D-04	-0.170D-04	-0.101D-04	-0.717D-05	-0.551D-05	-0.444D-05	-0.370D-05	-0.315D-05	-0.272D-05	-0.238D-05
	-0.210D-05	-0.187D-05	-0.167D-05	-0.150D-05	-0.134D-05	-0.122D-05	-0.111D-05	-0.101D-05	-0.917D-06	-0.835D-06
	-0.762D-06	-0.698D-06	-0.636D-06	-0.581D-06	-0.531D-06	-0.486D-06	-0.444D-06	-0.406D-06	-0.371D-06	-0.339D-06
	-0.310D-06	-0.283D-06	-0.259D-06	-0.236D-06	-0.218D-06	-0.196D-06	-0.179D-06	-0.163D-06	-0.149D-06	-0.136D-06

FRACTION CF GAS PRODUCED AT LAST FIVE TIME LEVELS

FRACTION OF GAS PRODUCED# 0.139E-05 0.180E-05 0.224E-05 0.263E-05 0.308E-05



SLIP CCEFF(B)= 10.000 INFRIAL CCEFF(BB)= 0.1000 09 PERMEABILITY(K)= 800.000 MD

FLUX DISTRIBUTION FOR THE FIRST 40 GRID POINTS

0.250E 00	-0.400D-04	-0.655D-05	-0.183D-05	-0.477D-06	-0.121D-06	-0.313D-07	-0.822D-08	-0.219D-08	-0.587D-09	-0.159D-09
	-0.433D-10	-0.119D-10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.500E 00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	-0.400D-04	-0.106D-04	-0.366D-05	-0.131D-05	-0.489D-06	-0.163D-06	-0.493D-07	-0.155D-07	-0.442D-08	-0.148D-08
	-0.451D-09	-0.137D-09	-0.412D-10	-0.124D-10	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.100E 01	-0.400D-04	-0.123D-04	-0.424D-05	-0.217D-05	-0.101D-05	-0.459D-06	-0.203D-06	-0.866D-07	-0.311D-07	-0.148D-07
	-0.598D-08	-0.240D-08	-0.956D-09	-0.379D-09	-0.150D-09	-0.592D-10	-0.233D-10	-0.920D-11	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.150E 01	-0.400D-04	-0.136D-04	-0.592D-05	-0.290D-05	-0.157D-05	-0.645D-06	-0.446D-06	-0.220D-06	-0.113D-06	-0.546D-07
	-0.757D-07	-0.118D-07	-0.533D-08	-0.238D-08	-0.105D-08	-0.467D-09	-0.190D-09	-0.851D-10	-0.364D-10	-0.155D-10
	-0.655D-11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.250E 01	-0.400D-04	-0.149D-04	-0.696D-05	-0.379D-05	-0.220D-05	-0.132D-05	-0.797D-06	-0.477D-06	-0.282D-06	-0.163D-06
	-0.427D-07	-0.517D-07	-0.284D-07	-0.154D-07	-0.826D-08	-0.430D-08	-0.231D-08	-0.121D-08	-0.632D-09	-0.328D-09
	-0.170D-09	-0.677D-10	-0.452D-10	-0.232D-10	-0.119D-10	-0.613D-11	0.0	0.0	0.0	0.0
0.350E 01	-0.400D-04	-0.158D-04	-0.785D-05	-0.456D-05	-0.284D-05	-0.183D-05	-0.119D-05	-0.783D-06	-0.511D-06	-0.330D-06
	-0.210D-06	-0.132D-06	-0.810D-07	-0.491D-07	-0.340D-07	-0.173D-07	-0.101D-07	-0.681D-08	-0.332D-08	-0.188D-08
	-0.108D-08	-0.590D-09	-0.329D-09	-0.181D-09	-0.996D-10	-0.546D-10	-0.298D-10	-0.162D-10	-0.881D-11	-0.477D-11
0.600E 01	-0.400D-04	-0.163D-04	-0.861D-05	-0.531D-05	-0.351D-05	-0.242D-05	-0.170D-05	-0.121D-05	-0.861D-06	-0.615D-06
	-0.437D-06	-0.309D-06	-0.217D-06	-0.151D-06	-0.104D-06	-0.712D-07	-0.484D-07	-0.326D-07	-0.210D-07	-0.146D-07
	-0.970D-08	-0.642D-08	-0.423D-08	-0.278D-08	-0.182D-08	-0.119D-08	-0.781D-09	-0.510D-09	-0.332D-09	-0.216D-09
0.850E 01	-0.400D-04	-0.167D-04	-0.921D-05	-0.595D-05	-0.413D-05	-0.298D-05	-0.221D-05	-0.166D-05	-0.125D-05	-0.952D-06
	-0.723D-06	-0.549D-06	-0.415D-06	-0.312D-06	-0.233D-06	-0.173D-06	-0.127D-06	-0.938D-07	-0.677D-07	-0.489D-07
	-0.351D-07	-0.250D-07	-0.177D-07	-0.125D-07	-0.878D-08	-0.614D-08	-0.427D-08	-0.297D-08	-0.205D-08	-0.141D-08
	-0.972D-09	-0.667D-09	-0.457D-09	-0.312D-09	-0.213D-09	-0.149D-09	-0.983D-10	-0.667D-10	-0.482D-10	-0.300D-10



0.135E 02	-0.400D-04	-0.169D-04	-0.958D-05	-0.639D-05	-0.459D-05	-0.344D-05	-0.265D-05	-0.207D-05	-0.163D-05	-0.130D-05
	-0.104D-05	-0.832D-06	-0.666D-06	-0.533D-06	-0.425D-06	-0.339D-06	-0.269D-06	-0.212D-06	-0.167D-06	-0.131D-06
	-0.102D-06	-0.797D-07	-0.617D-07	-0.476D-07	-0.366D-07	-0.280D-07	-0.214D-07	-0.163D-07	-0.123D-07	-0.934D-08
	-0.705D-08	-0.531D-08	-0.399D-08	-0.299D-08	-0.224D-08	-0.168D-08	-0.125D-08	-0.934D-09	-0.696D-09	-0.518D-09
0.135E 02	-0.403D-04	-0.170D-04	-0.982D-05	-0.665D-05	-0.492D-05	-0.379D-05	-0.299D-05	-0.241D-05	-0.196D-05	-0.162D-05
	-0.134D-05	-0.111D-05	-0.923D-06	-0.769D-06	-0.641D-06	-0.531D-06	-0.443D-06	-0.368D-06	-0.304D-06	-0.251D-06
	-0.206D-06	-0.169D-06	-0.138D-06	-0.112D-06	-0.911D-07	-0.735D-07	-0.592D-07	-0.475D-07	-0.379D-07	-0.302D-07
	-0.240D-07	-0.190D-07	-0.150D-07	-0.110D-07	-0.926D-08	-0.725D-08	-0.566D-08	-0.442D-08	-0.344D-08	-0.267D-08
0.260E 02	-0.400D-04	-0.170D-04	-0.950D-05	-0.639D-05	-0.459D-05	-0.344D-05	-0.265D-05	-0.207D-05	-0.163D-05	-0.130D-05
	-0.157D-05	-0.123D-05	-0.114D-05	-0.973D-06	-0.833D-06	-0.714D-06	-0.612D-06	-0.525D-06	-0.449D-06	-0.384D-06
	-0.328D-06	-0.280D-06	-0.230D-06	-0.202D-06	-0.171D-06	-0.146D-06	-0.122D-06	-0.102D-06	-0.856D-07	-0.715D-07
	-0.596D-07	-0.495D-07	-0.411D-07	-0.340D-07	-0.280D-07	-0.231D-07	-0.189D-07	-0.155D-07	-0.127D-07	-0.104D-07
0.335E 02	-0.400D-04	-0.170D-04	-0.100D-04	-0.701D-05	-0.530D-05	-0.419D-05	-0.342D-05	-0.284D-05	-0.240D-05	-0.204D-05
	-0.176D-05	-0.132D-05	-0.113D-05	-0.115D-05	-0.100D-05	-0.875D-06	-0.765D-06	-0.670D-06	-0.587D-06	-0.514D-06
	-0.450D-06	-0.394D-06	-0.344D-06	-0.300D-06	-0.261D-06	-0.227D-06	-0.198D-06	-0.171D-06	-0.146D-06	-0.120D-06
	-0.110D-06	-0.947D-07	-0.813D-07	-0.696D-07	-0.594D-07	-0.506D-07	-0.431D-07	-0.365D-07	-0.310D-07	-0.262D-07
0.435E 02	-0.400D-04	-0.170D-04	-0.101D-04	-0.708D-05	-0.539D-05	-0.430D-05	-0.353D-05	-0.296D-05	-0.253D-05	-0.216D-05
	-0.189D-05	-0.165D-05	-0.145D-05	-0.128D-05	-0.113D-05	-0.101D-05	-0.893D-06	-0.794D-06	-0.707D-06	-0.630D-06
	-0.561D-06	-0.499D-06	-0.445D-06	-0.396D-06	-0.352D-06	-0.313D-06	-0.274D-06	-0.246D-06	-0.218D-06	-0.193D-06
	-0.170D-06	-0.150D-06	-0.132D-06	-0.116D-06	-0.102D-06	-0.897D-07	-0.786D-07	-0.687D-07	-0.601D-07	-0.524D-07
0.535E 02	-0.400D-04	-0.170D-04	-0.101D-04	-0.713D-05	-0.546D-05	-0.438D-05	-0.362D-05	-0.306D-05	-0.263D-05	-0.228D-05
	-0.200D-05	-0.176D-05	-0.156D-05	-0.139D-05	-0.124D-05	-0.111D-05	-0.100D-05	-0.899D-06	-0.810D-06	-0.730D-06
	-0.650D-06	-0.594D-06	-0.538D-06	-0.484D-06	-0.437D-06	-0.394D-06	-0.355D-06	-0.320D-06	-0.284D-06	-0.259D-06
	-0.233D-06	-0.210D-06	-0.188D-06	-0.169D-06	-0.151D-06	-0.135D-06	-0.121D-06	-0.108D-06	-0.967D-07	-0.864D-07
0.635E 02	-0.400D-04	-0.170D-04	-0.101D-04	-0.716D-05	-0.550D-05	-0.443D-05	-0.366D-05	-0.312D-05	-0.269D-05	-0.235D-05
	-0.207D-05	-0.184D-05	-0.164D-05	-0.147D-05	-0.132D-05	-0.119D-05	-0.106D-05	-0.978D-06	-0.887D-06	-0.804D-06
	-0.734D-06	-0.668D-06	-0.600D-06	-0.540D-06	-0.505D-06	-0.460D-06	-0.419D-06	-0.382D-06	-0.346D-06	-0.317D-06
	-0.288D-06	-0.262D-06	-0.240D-06	-0.217D-06	-0.197D-06	-0.179D-06	-0.162D-06	-0.147D-06	-0.134D-06	-0.121D-06
0.735E 02	-0.400D-04	-0.170D-04	-0.101D-04	-0.718D-05	-0.553D-05	-0.446D-05	-0.372D-05	-0.317D-05	-0.274D-05	-0.240D-05
	-0.212D-05	-0.199D-05	-0.170D-05	-0.153D-05	-0.138D-05	-0.125D-05	-0.114D-05	-0.104D-05	-0.948D-06	-0.867D-06
	-0.791D-06	-0.727D-06	-0.668D-06	-0.611D-06	-0.561D-06	-0.515D-06	-0.473D-06	-0.434D-06	-0.399D-06	-0.368D-06
	-0.336D-06	-0.308D-06	-0.283D-06	-0.260D-06	-0.239D-06	-0.218D-06	-0.200D-06	-0.184D-06	-0.168D-06	-0.154D-06

FRACTION OF GAS PRODUCED AT LAST FIVE TIME LEVELS

FRACTION OF GAS PRODUCED	0.139E-05	0.180E-05	0.222E-05	0.283E-05	0.305E-05
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SLIP COEFF(B)= 10.000 INERTIAL COEFF(BB)= 0.1000 09 PERMEABILITY(K)= 500.000 MD

FLUX DISTRIBUTION FOR THE FIRST 40 GRID POINTS

0.250E 00	-0.400D-04	-0.637D-05	-0.171D-05	-0.422D-06	-0.103D-06	-0.253D-07	-0.634D-08	-0.161D-08	-0.413D-09	-0.107D-09
	-0.278D-10	-0.728D-11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.500E 00	-0.400D-04	-0.104D-04	-0.346D-05	-0.120D-05	-0.400D-06	-0.127D-06	-0.392D-07	-0.118D-07	-0.350D-08	-0.193D-08
	-0.299D-09	-0.858D-10	-0.249D-10	-0.714D-11	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.100E 01	-0.400D-04	-0.121D-04	-0.463D-05	-0.203D-05	-0.914D-06	-0.403D-06	-0.171D-06	-0.703D-07	-0.267D-07	-0.111D-07
	-0.435D-08	-0.167D-08	-0.642D-09	-0.245D-09	-0.938D-10	-0.356D-10	-0.135D-10	-0.514D-11	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.150E 01	-0.400D-04	-0.136D-04	-0.572D-05	-0.280D-05	-0.146D-05	-0.764D-06	-0.391D-06	-0.194D-06	-0.927D-07	-0.430D-07
	-0.195D-07	-0.862D-08	-0.376D-08	-0.161D-08	-0.685D-09	-0.288D-09	-0.120D-09	-0.499D-10	-0.206D-10	-0.844D-11
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.250E 01	-0.400D-04	-0.148D-04	-0.677D-05	-0.363D-05	-0.208D-05	-0.122D-05	-0.725D-06	-0.422D-06	-0.243D-06	-0.137D-06
	-0.794D-07	-0.408D-07	-0.218D-07	-0.115D-07	-0.596D-08	-0.307D-08	-0.157D-08	-0.801D-09	-0.406D-09	-0.205D-09
	-0.103D-09	-0.517D-10	-0.259D-10	-0.130D-10	-0.649D-11	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.350E 01	-0.400D-04	-0.156D-04	-0.768D-05	-0.440D-05	-0.270D-05	-0.172D-05	-0.110D-05	-0.711D-06	-0.455D-06	-0.287D-06
	-0.178D-06	-0.109D-06	-0.653D-07	-0.385D-07	-0.240D-07	-0.128D-07	-0.807D-08	-0.407D-08	-0.226D-08	-0.124D-08
	-0.679D-09	-0.369D-09	-0.189D-09	-0.107D-09	-0.574D-10	-0.306D-10	-0.165D-10	-0.860D-11	-0.454D-11	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.600E 01	-0.400D-04	-0.162D-04	-0.848D-05	-0.517D-05	-0.339D-05	-0.231D-05	-0.160D-05	-0.112D-05	-0.791D-06	-0.556D-06
	-0.389D-06	-0.270D-06	-0.165D-06	-0.127D-06	-0.860D-07	-0.577D-07	-0.384D-07	-0.254D-07	-0.167D-07	-0.109D-07
	-0.711D-08	-0.461D-08	-0.298D-08	-0.192D-08	-0.124D-08	-0.795D-09	-0.510D-09	-0.327D-09	-0.209D-09	-0.134D-09
	-0.856D-10	-0.547D-10	-0.350D-10	-0.224D-10	-0.143D-10	-0.914D-11	-0.594D-11	0.0	0.0	0.0
0.850E 01	-0.400D-04	-0.166D-04	-0.912D-05	-0.584D-05	-0.402D-05	-0.288D-05	-0.211D-05	-0.157D-05	-0.118D-05	-0.843D-06
	-0.663D-06	-0.496D-06	-0.370D-06	-0.274D-06	-0.201D-06	-0.147D-06	-0.107D-06	-0.764D-07	-0.547D-07	-0.387D-07
	-0.273D-07	-0.191D-07	-0.133D-07	-0.920D-08	-0.634D-08	-0.435D-08	-0.297D-08	-0.202D-08	-0.137D-08	-0.911D-09
	-0.628D-09	-0.423D-09	-0.284D-09	-0.191D-09	-0.128D-09	-0.854D-10	-0.570D-10	-0.380D-10	-0.255D-10	-0.166D-10

0.135E 02	-0.400D-04	-0.168D-04	-0.952D-05	-0.631D-05	-0.451D-05	-0.335D-05	-0.255D-05	-0.199D-05	-0.154D-05	-0.123D-05
	-0.975D-06	-0.773D-06	-0.613D-06	-0.485D-06	-0.383D-06	-0.302D-06	-0.237D-06	-0.185D-06	-0.144D-06	-0.111D-06
	-0.856D-07	-0.656D-07	-0.501D-07	-0.381D-07	-0.289D-07	-0.218D-07	-0.164D-07	-0.123D-07	-0.919D-08	-0.686D-08
	-0.510D-09	-0.379D-09	-0.291D-09	-0.209D-09	-0.153D-09	-0.113D-09	-0.833D-09	-0.613D-09	-0.450D-09	-0.331D-09
0.125E 02	-0.400D-04	-0.170D-04	-0.979D-05	-0.659D-05	-0.480D-05	-0.374D-05	-0.294D-05	-0.235D-05	-0.190D-05	-0.155D-05
	-0.128D-05	-0.105D-05	-0.867D-06	-0.716D-06	-0.591D-06	-0.487D-06	-0.401D-06	-0.329D-06	-0.269D-06	-0.220D-06
	-0.178D-06	-0.145D-06	-0.117D-06	-0.937D-07	-0.750D-07	-0.598D-07	-0.475D-07	-0.376D-07	-0.297D-07	-0.233D-07
	-0.183D-07	-0.143D-07	-0.111D-07	-0.864D-08	-0.670D-08	-0.518D-08	-0.399D-08	-0.307D-08	-0.236D-08	-0.181D-08
0.269E 02	-0.400D-04	-0.170D-04	-0.954D-05	-0.688D-05	-0.511D-05	-0.399D-05	-0.320D-05	-0.261D-05	-0.210D-05	-0.181D-05
	-0.152D-05	-0.128D-05	-0.109D-05	-0.923D-06	-0.784D-06	-0.668D-06	-0.568D-06	-0.483D-06	-0.410D-06	-0.347D-06
	-0.294D-06	-0.248D-06	-0.209D-06	-0.176D-06	-0.147D-06	-0.123D-06	-0.103D-06	-0.853D-07	-0.767D-07	-0.685D-07
	-0.482D-07	-0.396D-07	-0.325D-07	-0.268D-07	-0.217D-07	-0.176D-07	-0.143D-07	-0.116D-07	-0.938D-08	-0.758D-08
0.335E 02	-0.400D-04	-0.170D-04	-0.100D-04	-0.690D-05	-0.527D-05	-0.416D-05	-0.338D-05	-0.280D-05	-0.239D-05	-0.200D-05
	-0.171D-05	-0.147D-05	-0.127D-05	-0.110D-05	-0.937D-06	-0.832D-06	-0.724D-06	-0.640D-06	-0.547D-06	-0.477D-06
	-0.415D-06	-0.360D-06	-0.312D-06	-0.270D-06	-0.234D-06	-0.202D-06	-0.174D-06	-0.149D-06	-0.128D-06	-0.109D-06
	-0.934D-07	-0.795D-07	-0.675D-07	-0.572D-07	-0.484D-07	-0.408D-07	-0.344D-07	-0.289D-07	-0.242D-07	-0.202D-07
0.435E 02	-0.400D-04	-0.170D-04	-0.101D-04	-0.706D-05	-0.536D-05	-0.427D-05	-0.360D-05	-0.293D-05	-0.249D-05	-0.214D-05
	-0.195D-05	-0.161D-05	-0.141D-05	-0.124D-05	-0.109D-05	-0.966D-06	-0.854D-06	-0.756D-06	-0.670D-06	-0.594D-06
	-0.526D-06	-0.466D-06	-0.413D-06	-0.365D-06	-0.323D-06	-0.285D-06	-0.251D-06	-0.221D-06	-0.195D-06	-0.171D-06
	-0.150D-06	-0.131D-06	-0.115D-06	-0.100D-06	-0.871D-07	-0.757D-07	-0.657D-07	-0.570D-07	-0.493D-07	-0.427D-07
0.535E 02	-0.400D-04	-0.170D-04	-0.101D-04	-0.712D-05	-0.544D-05	-0.435D-05	-0.360D-05	-0.303D-05	-0.260D-05	-0.225D-05
	-0.196D-05	-0.173D-05	-0.153D-05	-0.136D-05	-0.121D-05	-0.108D-05	-0.965D-06	-0.844D-06	-0.775D-06	-0.646D-06
	-0.625D-06	-0.562D-06	-0.505D-06	-0.453D-06	-0.407D-06	-0.365D-06	-0.324D-06	-0.294D-06	-0.263D-06	-0.235D-06
	-0.210D-06	-0.188D-06	-0.167D-06	-0.149D-06	-0.133D-06	-0.118D-06	-0.103D-06	-0.930D-07	-0.824D-07	-0.730D-07
0.635E 02	-0.400D-04	-0.170D-04	-0.101D-04	-0.715D-05	-0.548D-05	-0.441D-05	-0.366D-05	-0.310D-05	-0.267D-05	-0.232D-05
	-0.204D-05	-0.181D-05	-0.161D-05	-0.144D-05	-0.129D-05	-0.116D-05	-0.103D-05	-0.945D-06	-0.855D-06	-0.775D-06
	-0.702D-06	-0.637D-06	-0.578D-06	-0.524D-06	-0.476D-06	-0.432D-06	-0.392D-06	-0.355D-06	-0.323D-06	-0.292D-06
	-0.264D-06	-0.239D-06	-0.216D-06	-0.195D-06	-0.176D-06	-0.159D-06	-0.144D-06	-0.130D-06	-0.117D-06	-0.105D-06
0.735E 02	-0.400D-04	-0.170D-04	-0.101D-04	-0.717D-05	-0.551D-05	-0.445D-05	-0.370D-05	-0.315D-05	-0.272D-05	-0.238D-05
	-0.210D-05	-0.187D-05	-0.167D-05	-0.150D-05	-0.135D-05	-0.122D-05	-0.110D-05	-0.101D-05	-0.918D-06	-0.837D-06
	-0.764D-06	-0.697D-06	-0.637D-06	-0.582D-06	-0.534D-06	-0.487D-06	-0.445D-06	-0.407D-06	-0.373D-06	-0.341D-06
	-0.311D-06	-0.284D-06	-0.260D-06	-0.237D-06	-0.217D-06	-0.198D-06	-0.180D-06	-0.164D-06	-0.150D-06	-0.137D-06

FRACTION OF GAS PRODUCED AT LAST FIVE TIME LEVELS

FRACTION OF GAS PRODUCED\* 0.139E-05 0.180E-05 0.222E-05 0.263E-05 0.305E-05

I.2

SOLUTIONS OBTAINED FOR A PERMEABILITY VALUE  
OF 10.0 AND 0.1 MILLIDARCIES





0.108E 03	-0.400D-04	-0.199D-04	-0.106D-04	-0.633D-05	-0.390D-05	-0.240D-05	-0.144D-05	-0.840D-06	-0.475D-06	-0.260D-06
	-0.138D-06	-0.714D-07	-0.361D-07	-0.179D-07	-0.874D-08	-0.420D-08	-0.190D-08	-0.933D-09	-0.433D-09	-0.200D-09
	-0.912D-10	-0.414D-10	-0.187D-10	-0.840D-11	-0.370D-11	-0.167D-11	-0.743D-12	-0.326D-12	-0.145D-12	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.143E 03	-0.400D-04	-0.201D-04	-0.110D-04	-0.640D-05	-0.446D-05	-0.294D-05	-0.193D-05	-0.124D-05	-0.779D-06	-0.477D-06
	-0.285D-06	-0.166D-06	-0.945D-07	-0.526D-07	-0.297D-07	-0.154D-07	-0.812D-08	-0.423D-08	-0.217D-08	-0.110D-08
	-0.556D-09	-0.277D-09	-0.137D-09	-0.675D-10	-0.320D-10	-0.160D-10	-0.772D-11	-0.371D-11	-0.178D-11	-0.848D-12
	-0.403D-12	-0.191D-12	-0.902D-13	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.183E 03	-0.400D-04	-0.202D-04	-0.112D-04	-0.770D-05	-0.440D-05	-0.270D-05	-0.160D-05	-0.100D-05	-0.100D-05	-0.724D-06
	-0.472D-06	-0.302D-06	-0.160D-06	-0.116D-06	-0.676D-07	-0.411D-07	-0.234D-07	-0.136D-07	-0.766D-08	-0.426D-08
	-0.234D-08	-0.127D-08	-0.643D-09	-0.364D-09	-0.193D-09	-0.101D-09	-0.520D-10	-0.274D-10	-0.141D-10	-0.723D-11
	-0.100D-11	-0.510D-11	-0.260D-12	-0.410D-12	-0.210D-12	0.0	0.0	0.0	0.0	0.0
0.228E 03	-0.400D-04	-0.202D-04	-0.114D-04	-0.740D-05	-0.410D-05	-0.270D-05	-0.162D-05	-0.102D-05	-0.102D-05	-0.909D-06
	-0.674D-06	-0.462D-06	-0.311D-06	-0.206D-06	-0.134D-06	-0.850D-07	-0.540D-07	-0.334D-07	-0.204D-07	-0.124D-07
	-0.728D-08	-0.428D-08	-0.240D-08	-0.143D-08	-0.811D-09	-0.460D-09	-0.250D-09	-0.142D-09	-0.784D-10	-0.431D-10
	-0.235D-10	-0.127D-10	-0.607D-11	-0.360D-11	-0.197D-11	-0.105D-11	-0.537D-12	-0.294D-12	-0.145D-12	-0.815D-13
0.278E 03	-0.400D-04	-0.202D-04	-0.115D-04	-0.740D-05	-0.440D-05	-0.270D-05	-0.162D-05	-0.102D-05	-0.102D-05	-0.120D-05
	-0.874D-06	-0.462D-06	-0.311D-06	-0.216D-06	-0.140D-06	-0.100D-06	-0.665D-07	-0.415D-07	-0.250D-07	-0.150D-07
	-0.170D-07	-0.112D-07	-0.570D-08	-0.261D-08	-0.157D-08	-0.910D-09	-0.545D-09	-0.326D-09	-0.190D-09	-0.100D-09
	-0.110D-09	-0.634D-10	-0.363D-10	-0.207D-10	-0.117D-10	-0.654D-11	-0.370D-11	-0.204D-11	-0.115D-11	-0.636D-12
0.334E 03	-0.400D-04	-0.202D-04	-0.116D-04	-0.791D-05	-0.450D-05	-0.280D-05	-0.170D-05	-0.110D-05	-0.110D-05	-0.141D-05
	-0.197D-06	-0.904D-06	-0.493D-06	-0.441D-06	-0.321D-06	-0.163D-06	-0.116D-06	-0.802D-07	-0.550D-07	-0.764D-08
	-0.173D-07	-0.250D-07	-0.149D-07	-0.108D-07	-0.701D-08	-0.451D-08	-0.287D-08	-0.181D-08	-0.113D-08	-0.764D-09
	-0.434D-09	-0.285D-09	-0.162D-09	-0.870D-10	-0.564D-10	-0.351D-10	-0.209D-10	-0.124D-10	-0.730D-11	-0.449D-11
0.409E 03	-0.400D-04	-0.202D-04	-0.117D-04	-0.793D-05	-0.457D-05	-0.287D-05	-0.174D-05	-0.114D-05	-0.114D-05	-0.161D-05
	-0.126D-06	-0.977D-06	-0.784D-06	-0.677D-06	-0.449D-06	-0.330D-06	-0.200D-06	-0.132D-06	-0.956D-07	-0.617D-08
	-0.600D-07	-0.404D-07	-0.235D-07	-0.162D-07	-0.110D-07	-0.744D-08	-0.497D-08	-0.310D-08	-0.217D-08	-0.170D-08
	-0.142D-08	-0.921D-09	-0.564D-09	-0.380D-09	-0.242D-09	-0.153D-09	-0.940D-10	-0.604D-10	-0.376D-10	-0.234D-10
0.444E 03	-0.400D-04	-0.202D-04	-0.117D-04	-0.803D-05	-0.461D-05	-0.291D-05	-0.181D-05	-0.121D-05	-0.121D-05	-0.179D-05
	-0.114D-06	-0.111D-06	-0.496D-06	-0.710D-06	-0.442D-06	-0.270D-06	-0.163D-06	-0.103D-06	-0.198D-06	-0.140D-06
	-0.112D-06	-0.827D-07	-0.668D-07	-0.443D-07	-0.320D-07	-0.193D-07	-0.119D-07	-0.602D-08	-0.457D-08	-0.310D-08
	-0.310D-08	-0.262D-08	-0.170D-08	-0.120D-08	-0.807D-09	-0.530D-09	-0.337D-09	-0.235D-09	-0.154D-09	-0.101D-09

FRACTION OF GAS PRODUCED# 0.944E-05 0.116E-04 0.141E-04 0.170E-04 0.203F-04

FRACTION OF GAS PRODUCED AT LAST FIVE TIME LEVELS













0.108E 03	-0.403D-04	-0.295D-04	-0.109D-04	-0.653D-05	-0.404D-05	-0.650D-05	-0.181D-05	-0.087D-06	-0.505D-06	-0.279D-06
	-0.149D-06	-0.781D-07	-0.399D-07	-0.200D-07	-0.948D-08	-0.477D-08	-0.108D-08	-0.508D-09	-0.237D-09	
	-0.163D-09	-0.501D-10	-0.229D-10	-0.104D-10	-0.469D-11	-0.211D-11	-0.946D-12	-0.423D-12	-0.180D-12	-0.837D-13
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.143L 03	-0.469D-04	-0.208D-04	-0.113D-04	-0.706D-05	-0.462D-05	-0.306D-05	-0.201D-05	-0.130D-05	-0.823D-06	-0.508D-06
	-0.308D-06	-0.130D-06	-0.103D-06	-0.579D-07	-0.317D-07	-0.173D-07	-0.920D-08	-0.483D-08	-0.251D-08	-0.129D-08
	-0.658D-09	-0.333D-09	-0.168D-09	-0.818D-10	-0.403D-10	-0.158D-10	-0.968D-11	-0.496D-11	-0.226D-11	-0.107D-11
	-0.523D-12	-0.259D-12	-0.119D-12	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.163E 03	-0.403D-04	-0.207D-04	-0.115D-04	-0.743D-05	-0.505D-05	-0.350D-05	-0.243D-05	-0.169D-05	-0.114D-05	-0.766D-06
	-0.503D-06	-0.240D-06	-0.204D-06	-0.126D-06	-0.764D-07	-0.485D-07	-0.241D-07	-0.153D-07	-0.071D-08	-0.489D-08
	-0.271D-08	-0.148D-08	-0.824D-09	-0.434D-09	-0.232D-09	-0.123D-09	-0.646D-10	-0.338D-10	-0.176D-10	-0.911D-11
	-0.469D-11	-0.241D-11	-0.112D-11	-0.626D-12	-0.317D-12	-0.161D-12	0.0	0.0	0.0	0.0
0.228E 03	-0.403D-04	-0.208D-04	-0.117D-04	-0.700D-05	-0.537D-05	-0.394D-05	-0.278D-05	-0.201D-05	-0.148D-05	-0.102D-05
	-0.714D-06	-0.472D-06	-0.334D-06	-0.223D-06	-0.146D-06	-0.848D-07	-0.570D-07	-0.372D-07	-0.229D-07	-0.139D-07
	-0.832D-08	-0.492D-08	-0.289D-08	-0.167D-08	-0.959D-09	-0.586D-09	-0.308D-09	-0.173D-09	-0.962D-10	-0.532D-10
	-0.293D-11	-0.160D-11	-0.071D-11	-0.472D-11	-0.254D-11	-0.137D-11	-0.732D-12	-0.390D-12	-0.207D-12	-0.110D-12
0.278E 03	-0.400D-04	-0.208D-04	-0.119D-04	-0.700D-05	-0.562D-05	-0.412D-05	-0.306D-05	-0.228D-05	-0.170D-05	-0.126D-05
	-0.972D-06	-0.659D-06	-0.479D-06	-0.334D-06	-0.230D-06	-0.152D-06	-0.110D-06	-0.732D-07	-0.402D-07	-0.313D-07
	-0.201D-07	-0.127D-07	-0.764D-08	-0.494D-08	-0.303D-08	-0.164D-08	-0.111D-08	-0.620D-09	-0.392D-09	-0.241D-09
	-0.136D-09	-0.783D-10	-0.452D-10	-0.299D-10	-0.148D-10	-0.841D-11	-0.476D-11	-0.243D-11	-0.150D-11	-0.084D-11
0.338E 03	-0.400D-04	-0.208D-04	-0.120D-04	-0.607D-05	-0.582D-05	-0.444D-05	-0.330D-05	-0.242D-05	-0.193D-05	-0.148D-05
	-0.112D-05	-0.848D-06	-0.634D-06	-0.470D-06	-0.344D-06	-0.249D-06	-0.179D-06	-0.126D-06	-0.803D-07	-0.627D-07
	-0.414D-07	-0.280D-07	-0.164D-07	-0.124D-07	-0.805D-08	-0.511D-08	-0.334D-08	-0.212D-08	-0.144D-08	-0.079D-08
	-0.571D-09	-0.322D-09	-0.157D-09	-0.120D-09	-0.728D-10	-0.434D-10	-0.263D-10	-0.157D-10	-0.934D-11	-0.555D-11
0.409E 03	-0.400D-04	-0.209D-04	-0.121D-04	-0.620D-05	-0.578D-05	-0.443D-05	-0.350D-05	-0.274D-05	-0.215D-05	-0.164D-05
	-0.137D-05	-0.193D-06	-0.796D-06	-0.612D-06	-0.467D-06	-0.353D-06	-0.264D-06	-0.194D-06	-0.144D-06	-0.104D-06
	-0.752D-07	-0.535D-07	-0.370D-07	-0.264D-07	-0.182D-07	-0.129D-07	-0.850D-08	-0.573D-08	-0.363D-08	-0.254D-08
	-0.167D-08	-0.109D-08	-0.710D-09	-0.459D-09	-0.294D-09	-0.187D-09	-0.119D-09	-0.750D-10	-0.471D-10	-0.275D-10
0.489E 03	-0.400D-04	-0.209D-04	-0.121D-04	-0.631D-05	-0.612D-05	-0.469D-05	-0.364D-05	-0.292D-05	-0.233D-05	-0.187D-05
	-0.153D-05	-0.120D-05	-0.954D-06	-0.756D-06	-0.595D-06	-0.447D-06	-0.362D-06	-0.279D-06	-0.213D-06	-0.162D-06
	-0.122D-06	-0.866D-07	-0.671D-07	-0.491D-07	-0.357D-07	-0.257D-07	-0.184D-07	-0.130D-07	-0.916D-08	-0.641D-08
	-0.444D-08	-0.306D-08	-0.209D-08	-0.142D-08	-0.981D-09	-0.644D-09	-0.431D-09	-0.286D-09	-0.195D-09	-0.124D-09

FRACTION OF GAS PRODUCED AT LAST FIVE TIME LEVELS

FRACTION OF GAS PRODUCED AT LAST FIVE TIME LEVELS C.170E-04 C.141E-04 C.170E-04 0.203F-04







0.108E 03	-0.400D-04	-0.200D-04	-0.106D-04	-0.624D-05	-0.376D-05	-0.226D-05	-0.133D-05	-0.755D-06	-0.413D-06	-0.219D-06
	-0.112D-06	-0.562D-07	-0.275D-07	-0.621D-08	-0.200D-08	-0.132D-08	-0.659D-09	-0.269D-09	-0.120D-09	0.0
	-0.529D-10	-0.232D-10	-0.101D-10	-0.441D-11	-0.191D-11	-0.823D-12	-0.353D-12	-0.151D-12	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.143E 03	-0.400D-04	-0.202D-04	-0.110D-04	-0.670D-05	-0.437D-05	-0.284D-05	-0.182D-05	-0.119D-05	-0.703D-06	-0.419D-06
	-0.243D-06	-0.137D-06	-0.757D-07	-0.408D-07	-0.216D-07	-0.112D-07	-0.572D-08	-0.280D-08	-0.144D-08	-0.706D-09
	-0.344D-09	-0.160D-09	-0.798D-10	-0.300D-10	-0.180D-10	-0.845D-11	-0.396D-11	-0.184D-11	-0.856D-12	-0.396D-12
	-0.182D-12	-0.838D-13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.183E 03	-0.400D-04	-0.203D-04	-0.112D-04	-0.716D-05	-0.481D-05	-0.359D-05	-0.224D-05	-0.151D-05	-0.101D-05	-0.655D-06
	-0.417D-06	-0.260D-06	-0.158D-06	-0.942D-07	-0.550D-07	-0.315D-07	-0.177D-07	-0.982D-08	-0.536D-08	-0.289D-08
	-0.154D-08	-0.811D-09	-0.423D-09	-0.219D-09	-0.112D-09	-0.572D-10	-0.290D-10	-0.146D-10	-0.729D-11	-0.363D-11
	-0.180D-11	-0.807D-12	-0.436D-12	-0.213D-12	-0.104D-12	0.0	0.0	0.0	0.0	0.0
0.228E 03	-0.400D-04	-0.203D-04	-0.114D-04	-0.745D-05	-0.514D-05	-0.364D-05	-0.250D-05	-0.184D-05	-0.129D-05	-0.897D-06
	-0.612D-06	-0.411D-06	-0.270D-06	-0.175D-06	-0.111D-06	-0.603D-07	-0.422D-07	-0.254D-07	-0.151D-07	-0.840D-08
	-0.509D-08	-0.290D-08	-0.163D-08	-0.911D-09	-0.504D-09	-0.276D-09	-0.150D-09	-0.810D-10	-0.434D-10	-0.231D-10
	-0.123D-10	-0.645D-11	-0.338D-11	-0.177D-11	-0.917D-12	-0.475D-12	-0.245D-12	-0.126D-12	0.0	0.0
0.278E 03	-0.400D-04	-0.203D-04	-0.116D-04	-0.766D-05	-0.540D-05	-0.392D-05	-0.248D-05	-0.212D-05	-0.155D-05	-0.113D-05
	-0.809D-06	-0.574D-06	-0.401D-06	-0.276D-06	-0.187D-06	-0.126D-06	-0.810D-07	-0.559D-07	-0.337D-07	-0.211D-07
	-0.131D-07	-0.602D-08	-0.485D-08	-0.290D-08	-0.171D-08	-0.100D-08	-0.584D-09	-0.336D-09	-0.192D-09	-0.109D-09
	-0.614D-10	-0.344D-10	-0.191D-10	-0.106D-10	-0.503D-11	-0.320D-11	-0.174D-11	-0.948D-12	-0.513D-12	-0.277D-12
0.338E 03	-0.400D-04	-0.204D-04	-0.117D-04	-0.762D-05	-0.561D-05	-0.415D-05	-0.312D-05	-0.236D-05	-0.179D-05	-0.134D-05
	-0.101D-05	-0.746D-06	-0.546D-06	-0.395D-06	-0.283D-06	-0.199D-06	-0.139D-06	-0.955D-07	-0.646D-07	-0.434D-07
	-0.287D-07	-0.180D-07	-0.121D-07	-0.775D-08	-0.490D-08	-0.307D-08	-0.190D-08	-0.117D-08	-0.713D-09	-0.431D-09
	-0.259D-09	-0.154D-09	-0.915D-10	-0.539D-10	-0.313D-10	-0.184D-10	-0.107D-10	-0.618D-11	-0.353D-11	-0.202D-11
0.409E 03	-0.400D-04	-0.204D-04	-0.118D-04	-0.790D-05	-0.570D-05	-0.415D-05	-0.333D-05	-0.259D-05	-0.200D-05	-0.155D-05
	-0.120D-05	-0.919D-06	-0.700D-06	-0.528D-06	-0.395D-06	-0.292D-06	-0.214D-06	-0.155D-06	-0.111D-06	-0.785D-07
	-0.550D-07	-0.331D-07	-0.261D-07	-0.177D-07	-0.119D-07	-0.791D-08	-0.521D-08	-0.340D-08	-0.220D-08	-0.142D-08
	-0.903D-09	-0.579D-09	-0.360D-09	-0.223D-09	-0.140D-09	-0.862D-10	-0.530D-10	-0.324D-10	-0.197D-10	-0.119D-10
0.489E 03	-0.400D-04	-0.204D-04	-0.118D-04	-0.806D-05	-0.592D-05	-0.451D-05	-0.381D-05	-0.277D-05	-0.219D-05	-0.174D-05
	-0.138D-05	-0.109D-05	-0.853D-06	-0.668D-06	-0.516D-06	-0.396D-06	-0.302D-06	-0.228D-06	-0.171D-06	-0.127D-06
	-0.930D-07	-0.677D-07	-0.488D-07	-0.349D-07	-0.247D-07	-0.173D-07	-0.120D-07	-0.829D-08	-0.567D-08	-0.385D-08
	-0.259D-08	-0.173D-08	-0.115D-08	-0.759D-09	-0.449D-09	-0.324D-09	-0.210D-09	-0.135D-09	-0.868D-10	-0.554D-10

FRACTION UF GAS PRODUCED AT LAST FIVE TIME LEVELS

FRACTION OF GAS PRODUCED\* 0.949E-05 0.116E-04 0.141E-04 0.170E-04 0.203E-04





0.108E 03	-0.4000-04	-0.2040-04	-0.1080-04	-0.6400-05	-0.3900-05	-0.2350-05	-0.1390-05	-0.7900-06	-0.4360-06	-0.2320-06
	-0.1200-06	-0.6060-07	-0.2590-07	-0.1440-07	-0.6560-08	-0.3210-08	-0.1490-08	-0.6790-09	-0.3070-09	-0.1300-09
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.143E 03	-0.4000-04	-0.2060-04	-0.1120-04	-0.6950-05	-0.4500-05	-0.2930-05	-0.1890-05	-0.1190-05	-0.7300-06	-0.4420-06
	-0.3580-06	-0.1470-06	-0.8150-07	-0.4430-07	-0.2360-07	-0.1230-07	-0.6360-08	-0.3220-08	-0.1520-08	-0.8040-09
	-0.3650-09	-0.1920-09	-0.9300-10	-0.4460-10	-0.2130-10	-0.1010-10	-0.4760-11	-0.2240-11	-0.1050-11	-0.4880-12
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.183E 03	-0.4000-04	-0.2070-04	-0.1150-04	-0.7340-05	-0.3300-05	-0.2320-05	-0.1570-05	-0.1050-05	-0.6670-06	-0.3750-06
	-0.4400-06	-0.2760-06	-0.1690-06	-0.1010-06	-0.5950-07	-0.3400-07	-0.1950-07	-0.1090-07	-0.8560-08	-0.3750-08
	-0.1740-08	-0.9250-09	-0.4670-09	-0.2630-09	-0.1410-09	-0.6730-10	-0.3930-10	-0.1740-10	-0.8770-11	-0.4460-11
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.228E 03	-0.4000-04	-0.2080-04	-0.1170-04	-0.7530-05	-0.3520-05	-0.2570-05	-0.1590-05	-0.1340-05	-0.7360-06	-0.4300-06
	-0.4420-06	-0.3300-06	-0.2260-06	-0.1860-06	-0.1190-06	-0.7490-07	-0.4500-07	-0.2700-07	-0.1660-07	-0.2700-08
	-0.5680-08	-0.3260-08	-0.1850-08	-0.1040-08	-0.5300-09	-0.3200-09	-0.1750-09	-0.9530-10	-0.5150-10	-0.2760-10
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.278E 03	-0.4000-04	-0.2090-04	-0.1180-04	-0.7860-05	-0.3540-05	-0.2600-05	-0.1610-05	-0.1170-05	-0.7670-06	-0.4320-06
	-0.4460-06	-0.3020-06	-0.2200-06	-0.1940-06	-0.1340-06	-0.8010-07	-0.4810-07	-0.2700-07	-0.1660-07	-0.2700-08
	-0.1400-08	-0.8920-09	-0.5430-09	-0.3270-09	-0.1950-09	-0.1150-09	-0.6720-10	-0.3500-10	-0.2240-10	-0.1240-10
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.338E 03	-0.4000-04	-0.2100-04	-0.1190-04	-0.8200-05	-0.3700-05	-0.2700-05	-0.1610-05	-0.1170-05	-0.7670-06	-0.4320-06
	-0.4480-06	-0.3140-06	-0.2240-06	-0.2090-06	-0.1420-06	-0.8140-07	-0.4810-07	-0.2700-07	-0.1660-07	-0.2700-08
	-0.1400-08	-0.8920-09	-0.5430-09	-0.3270-09	-0.1950-09	-0.1150-09	-0.6720-10	-0.3500-10	-0.2240-10	-0.1240-10
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.408E 03	-0.4000-04	-0.2090-04	-0.1210-04	-0.8170-05	-0.3700-05	-0.2660-05	-0.1610-05	-0.1170-05	-0.7670-06	-0.4320-06
	-0.4480-06	-0.3140-06	-0.2240-06	-0.2090-06	-0.1420-06	-0.8140-07	-0.4810-07	-0.2700-07	-0.1660-07	-0.2700-08
	-0.1400-08	-0.8920-09	-0.5430-09	-0.3270-09	-0.1950-09	-0.1150-09	-0.6720-10	-0.3500-10	-0.2240-10	-0.1240-10
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

FRACTION OF GAS PRODUCED AT LAST FIVE TIME LEVELS

FRACTION OF GAS PRODUCED 0.949E-08 0.116L-04 0.181E-04 0.170E-04 0.203L-04





0.108E 03	-0.4000-04	-0.2090-04	-0.1110-04	-0.6570-05	-0.4010-05	-0.2430-05	-0.1440-05	-0.0840-06	-0.4570-06	-0.2460-06
	-0.1200-06	-0.6530-07	-0.3280-07	-0.1590-07	-0.7610-08	-0.3600-08	-0.1680-08	-0.7770-09	-0.3550-09	-0.1610-09
	-0.7260-10	-0.3250-10	-0.1480-10	-0.6400-11	-0.2820-11	-0.1240-11	-0.5420-12	-0.2370-12	-0.1030-12	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.143E 03	-0.4000-04	-0.2110-04	-0.1150-04	-0.7140-05	-0.4630-05	-0.3020-05	-0.1960-05	-0.1240-05	-0.7710-06	-0.4650-06
	-0.2740-06	-0.1570-06	-0.8790-07	-0.4810-07	-0.2590-07	-0.1370-07	-0.7100-08	-0.3640-08	-0.1840-08	-0.9240-09
	-0.4580-09	-0.2250-09	-0.1100-09	-0.5330-10	-0.2570-10	-0.1230-10	-0.5860-11	-0.2740-11	-0.1310-11	-0.6180-12
	-0.2900-12	-0.1350-12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.103E 03	-0.4000-04	-0.2120-04	-0.1160-04	-0.7550-05	-0.5000-05	-0.3400-05	-0.2400-05	-0.1530-05	-0.1100-05	-0.7210-06
	-0.4650-06	-0.2930-06	-0.1810-06	-0.1090-06	-0.6670-07	-0.3760-07	-0.2150-07	-0.1210-07	-0.6720-08	-0.3680-08
	-0.1930-08	-0.1070-08	-0.5670-09	-0.2900-09	-0.1560-09	-0.8060-10	-0.4150-10	-0.2120-10	-0.1080-10	-0.5470-11
	-0.2750-11	-0.1380-11	-0.6900-12	-0.3440-12	-0.1710-12	0.0	0.0	0.0	0.0	0.0
0.228E 03	-0.4000-04	-0.2130-04	-0.1200-04	-0.7850-05	-0.5440-05	-0.3870-05	-0.2770-05	-0.1980-05	-0.1400-05	-0.9800-06
	-0.6700-06	-0.4580-06	-0.3090-06	-0.1990-06	-0.1280-06	-0.8080-07	-0.5010-07	-0.3060-07	-0.1840-07	-0.1090-07
	-0.6410-08	-0.3710-08	-0.2130-08	-0.1210-08	-0.6770-09	-0.3770-09	-0.2080-09	-0.1140-09	-0.6230-10	-0.3370-10
	-0.1810-10	-0.9710-11	-0.5170-11	-0.2740-11	-0.1480-11	-0.7620-12	-0.5990-12	-0.2080-12	-0.1090-12	0.0
0.278E 03	-0.4000-04	-0.2140-04	-0.1220-04	-0.8070-05	-0.5710-05	-0.4160-05	-0.3070-05	-0.2270-05	-0.1670-05	-0.1230-05
	-0.8870-06	-0.6350-06	-0.4480-06	-0.3110-06	-0.2130-06	-0.1440-06	-0.9560-07	-0.6260-07	-0.4040-07	-0.2570-07
	-0.1620-07	-0.1000-07	-0.6180-08	-0.3740-08	-0.2240-08	-0.1340-08	-0.7870-09	-0.4600-09	-0.2670-09	-0.1540-09
	-0.8800-10	-0.5010-10	-0.2830-10	-0.1590-10	-0.8900-11	-0.4950-11	-0.2740-11	-0.1510-11	-0.6320-12	-0.4560-12
0.338E 03	-0.4000-04	-0.2140-04	-0.1230-04	-0.8250-05	-0.5930-05	-0.4410-05	-0.3340-05	-0.2630-05	-0.1920-05	-0.1460-05
	-0.1100-05	-0.8180-06	-0.6040-06	-0.4410-06	-0.3190-06	-0.2270-06	-0.1600-06	-0.1110-06	-0.7630-07	-0.5180-07
	-0.3470-07	-0.2300-07	-0.1500-07	-0.9730-08	-0.6240-08	-0.3560-08	-0.2470-08	-0.1560-08	-0.9710-09	-0.5090-09
	-0.3590-09	-0.2170-09	-0.1310-09	-0.7810-10	-0.4640-10	-0.2740-10	-0.1610-10	-0.9440-11	-0.5500-11	-0.3190-11
0.409E 03	-0.4000-04	-0.2150-04	-0.1240-04	-0.8410-05	-0.6120-05	-0.4620-05	-0.3550-05	-0.2760-05	-0.2150-05	-0.1670-05
	-0.1300-05	-0.1000-05	-0.7400-06	-0.5850-06	-0.4410-06	-0.3250-06	-0.2430-06	-0.1780-06	-0.8120-06	-0.9190-07
	-0.6510-07	-0.4560-07	-0.3160-07	-0.2170-07	-0.1480-07	-0.9950-08	-0.6840-08	-0.4390-08	-0.2880-08	-0.1880-08
	-0.1210-08	-0.7790-09	-0.4970-09	-0.3160-09	-0.1980-09	-0.1240-09	-0.7720-10	-0.4780-10	-0.2950-10	-0.1810-10
0.469E 03	-0.4000-04	-0.2150-04	-0.1250-04	-0.8630-05	-0.6270-05	-0.4790-05	-0.3740-05	-0.2960-05	-0.2350-05	-0.1370-05
	-0.1490-05	-0.1160-05	-0.9330-06	-0.7320-06	-0.5710-06	-0.4420-06	-0.3400-06	-0.2590-06	-0.1950-06	-0.1460-06
	-0.1080-06	-0.7970-07	-0.5800-07	-0.4190-07	-0.3000-07	-0.2130-07	-0.1500-07	-0.1040-07	-0.7220-08	-0.4960-08
	-0.3380-08	-0.2290-08	-0.1540-08	-0.1030-08	-0.6830-09	-0.4510-09	-0.2960-09	-0.1930-09	-0.1250-09	-0.8100-10

FRACTION OF GAS PRODUCED AT LAST FIVE TIME LEVELS

FRACTION OF GAS PRODUCED= 0.949E-05 0.116E-04 0.141E-04 0.170E-04 0.203E-04

TRANSIENT RADIAL GAS FLOW

SLIP COEFF(U)= 0.0      INERTIAL COEFF(U3)= 0.1000 10      PERMEABILITY(K)= 10.000 MD  
 POROSITY= 0.132      ROCK COMPRESSIBILITY= 0.0      INITIAL PRESSURE= 0.2000 04

TIME (TD)	R#	PRESSURE SQUARE DISTRIBUTION DIMENSIONLESS DISTANCE								
		DELTA R1=0.0010			DELTA R2=0.032724					
0.500E 00	0.2500	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.150E 01	0.9389	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.350E 01	0.8713	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.850E 01	0.7500	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.185E 02	0.6029	0.9914	0.9949	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.375E 02	0.4730	0.9738	0.9988	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.535E 02	0.3650	0.9459	0.9951	0.9997	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.785E 02	0.2789	0.9122	0.9873	0.9996	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000
	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.108E 03	0.2071	0.8700	0.9752	0.9958	0.9994	0.9999	1.0000	1.0000	1.0000	1.0000
	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.143E 03	0.1775	0.8394	0.9597	0.9969	0.9994	0.9999	1.0000	1.0000	1.0000	1.0000
	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.193E 03	0.0977	0.8037	0.9418	0.9838	0.9961	0.9992	0.9999	1.0000	1.0000	1.0000
	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.238E 04	0.0569	0.7697	0.9224	0.9746	0.9925	0.9981	0.9996	0.9999	1.0000	1.0000
	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.278E 03	0.0249	0.7179	0.9023	0.9638	0.9876	0.9992	0.9999	0.9999	1.0000	1.0000
	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.334E 03	0.0150	0.6990	0.8950	0.9650	0.9950	0.9990	0.9990	0.9990	0.9990	1.0000
	0.0050	0.6090	0.8050	0.9050	0.9550	0.9850	0.9950	0.9990	0.9990	1.0000
	0.0050	0.5090	0.7050	0.8050	0.8550	0.9050	0.9550	0.9850	0.9990	1.0000





0.109E 03	-0.4000-04	-0.1240-04	-0.5700-05	-0.6540-05	-0.3300-09	-0.1970-05	-0.1170-03	-0.0760-06	-0.3810-06	-0.2000-06
	-0.1110-06	-0.5740-07	-0.2710-07	-0.1440-07	-0.7050-08	-0.3500-08	-0.1610-08	-0.7560-09	-0.3510-09	-0.1620-09
	-0.7410-10	-0.3360-10	-0.1920-10	-0.6830-11	-0.3000-11	-0.1300-11	-0.6000-12	-0.2600-12	-0.1180-12	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.143E 03	-0.4000-04	-0.1940-04	-0.1030-04	-0.6170-05	-0.3890-05	-0.2500-05	-0.1600-05	-0.1020-05	-0.6730-06	-0.3060-06
	-0.4330-06	-0.1340-06	-0.7000-07	-0.4230-07	-0.2310-07	-0.6550-08	-0.3410-08	-0.1750-08	-0.0920-09	-0.4970-12
	-0.4400-09	-0.2240-09	-0.1110-09	-0.9400-10	-0.2070-10	-0.1200-10	-0.3010-11	-0.1440-11	0.0	0.0
	-0.3770-12	-0.1550-12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.143E 01	-0.1600-04	-0.2000-04	-0.1070-04	-0.6040-05	-0.4370-05	-0.2900-05	-0.1350-05	-0.0930-05	-0.5950-06	-0.3300-06
	-0.3300-05	-0.2450-06	-0.1530-06	-0.3370-07	-0.5600-07	-0.3310-07	-0.1970-07	-0.1100-07	-0.6100-08	-0.3630-08
	-0.1970-09	-0.1920-09	-0.5510-09	-0.2700-09	-0.1500-09	-0.8170-10	-0.4200-10	-0.2210-10	-0.1140-10	-0.5850-11
	-0.2000-11	-0.1340-11	-0.7000-12	-0.4000-12	-0.1000-12	-0.0700-12	0.0	0.0	0.0	0.0
0.274E 03	-0.3000-04	-0.2000-04	-0.1100-04	-0.4700-05	-0.3100-05	-0.2100-05	-0.1450-05	-0.1170-05	-0.0130-06	-0.4000-06
	-0.4000-06	-0.2500-06	-0.1500-06	-0.1000-06	-0.6500-07	-0.4100-07	-0.2710-07	-0.1650-07	-0.0030-08	-0.4000-08
	-0.4100-09	-0.3000-09	-0.2000-09	-0.1100-09	-0.0500-09	-0.3700-09	-0.1500-09	-0.0900-09	-0.0900-09	-0.3400-10
	-0.1200-10	-0.1000-10	-0.5500-11	-0.2700-11	-0.1500-11	-0.0400-12	-0.4500-12	-0.2300-12	-0.1250-12	0.0
0.274E 01	-0.4000-04	-0.2000-04	-0.1100-04	-0.7740-05	-0.5000-05	-0.3000-05	-0.1700-05	-0.1410-05	-0.1030-05	-0.3000-05
	-0.2000-06	-0.1500-06	-0.3750-06	-0.2200-06	-0.1000-06	-0.1200-06	-0.0400-06	-0.3550-07	-0.2300-07	-0.2000-07
	-0.1600-07	-0.4100-07	-0.3000-08	-0.3900-08	-0.2100-08	-0.1270-08	-0.7000-09	-0.4500-09	-0.2800-09	-0.1500-09
	-0.3000-10	-0.5100-10	-0.2400-10	-0.1070-10	-0.0400-11	-0.3300-11	-0.1070-11	-0.0900-12	-0.0510-12	-0.0510-12
0.330E 03	0.1000-15	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16
	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16
	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16
	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16

FRACTION OF GAS PRODUCED AT LAST FIVE TIME LEVELS

FRACTION OF GAS PRODUCED= 0.095E-05 0.762E-05 0.949E-05 0.116E-04 0.128E-04





0.108E 03	-0.400E-04	-0.194E-04	-0.959D-05	-0.539D-05	-0.315D-05	-0.195D-05	-0.107D-05	-0.599D-06	-0.327D-06	-0.173D-06
	-0.87E-07	-0.444E-07	-0.217D-07	-0.489D-08	-0.227E-08	-0.104D-08	-0.470E-09	-0.211D-09	-0.934D-10	0.0
	-0.414E-10	-0.181E-10	-0.791D-11	-0.343E-11	-0.148D-11	-0.639D-12	-0.274D-12	-0.117E-12	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.143E 03	-0.400E-04	-0.194E-04	-0.102D-04	-0.604D-05	-0.376D-05	-0.237D-05	-0.144D-05	-0.926D-06	-0.542D-06	-0.333D-06
	-0.193E-06	-0.109E-06	-0.557D-07	-0.321D-07	-0.170D-07	-0.840E-08	-0.444D-08	-0.226D-08	-0.112D-08	-0.552D-09
	-0.267E-09	-0.130E-09	-0.621E-10	-0.295E-10	-0.139D-10	-0.655D-11	-0.306E-11	-0.142D-11	-0.659D-12	-0.304D-12
	-0.140E-12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.183E 03	-0.400E-04	-0.201E-04	-0.106E-04	-0.634D-05	-0.428D-05	-0.283D-05	-0.169D-05	-0.125D-05	-0.821E-06	-0.529D-06
	-0.335E-06	-0.207D-06	-0.126D-06	-0.746D-07	-0.434D-07	-0.240D-07	-0.139D-07	-0.710D-08	-0.420D-08	-0.226D-08
	-0.120E-08	-0.633E-09	-0.330E-09	-0.170E-09	-0.672E-10	-0.444D-10	-0.224D-10	-0.113D-10	-0.562D-11	-0.277D-11
	-0.138E-11	-0.689D-12	-0.334D-12	-0.163D-12	0.0	0.0	0.0	0.0	0.0	0.0
0.228E 03	-0.400E-04	-0.201E-04	-0.109E-04	-0.691D-05	-0.445D-05	-0.221D-05	-0.224D-05	-0.156D-05	-0.104D-05	-0.740D-06
	-0.500E-06	-0.333E-06	-0.140E-06	-0.814D-07	-0.540D-07	-0.314D-07	-0.201D-07	-0.119D-07	-0.692D-08	-0.374D-08
	-0.394E-08	-0.227E-08	-0.124E-08	-0.711E-09	-0.392E-09	-0.215D-09	-0.116D-09	-0.627E-10	-0.336D-10	-0.174D-10
	-0.344E-11	-0.499E-11	-0.260D-11	-0.135D-11	-0.702D-12	-0.363D-12	-0.187D-12	-0.958D-13	0.0	0.0
0.278E 03	-0.400E-04	-0.201E-04	-0.111D-04	-0.719D-05	-0.496D-05	-0.352D-05	-0.254D-05	-0.184D-05	-0.132D-05	-0.949D-06
	-0.574E-06	-0.474E-06	-0.329E-06	-0.224D-06	-0.151D-06	-0.100D-06	-0.655D-07	-0.422D-07	-0.269D-07	-0.168D-07
	-0.104E-07	-0.652E-08	-0.341E-08	-0.227D-08	-0.134D-08	-0.785D-09	-0.455D-09	-0.262D-09	-0.149D-09	-0.845D-10
	-0.475E-10	-0.265D-10	-0.147D-10	-0.915D-11	-0.448E-11	-0.244D-11	-0.133D-11	-0.724D-12	-0.391D-12	-0.211D-12
0.338E 03	0.100E-16	0.100E-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16
	0.100E-16	0.100E-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16
	0.100E-16	0.100E-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16
	0.100E-16	0.100E-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16

FRACTION OF GAS PRODUCED AT LAST FIVE TIME LEVELS

FRACTION OF GAS PRODUCED= 0.595E-05 0.762E-05 0.949E-05 0.116E-04 0.120E-04





0.108E 03	-0.4000-04	-0.1970-04	-0.9850-05	-0.5640-05	-0.3370-05	-0.2030-05	-0.1210-05	-0.7060-06	-0.4010-06	-0.2220-06
	-0.1190-06	-0.6240-07	-0.3190-07	-0.1600-07	-0.7910-08	-0.3850-08	-0.1850-08	-0.8750-09	-0.4110-09	-0.1920-09
	-0.8850-10	-0.4060-10	-0.1860-10	-0.8430-11	-0.3810-11	-0.1720-11	-0.7690-12	-0.3440-12	-0.1530-12	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.143E 03	-0.4000-04	-0.2000-04	-0.1040-04	-0.5270-05	-0.3970-05	-0.2560-05	-0.1650-05	-0.1050-05	-0.6610-06	-0.4060-06
	-0.2440-06	-0.1430-06	-0.8220-07	-0.4620-07	-0.2550-07	-0.1380-07	-0.7370-08	-0.3070-08	-0.2010-08	-0.1030-08
	-0.5260-09	-0.2650-09	-0.1330-09	-0.6590-10	-0.3250-10	-0.1590-10	-0.7700-11	-0.3780-11	-0.2180-11	-0.8800-12
	-0.4230-12	-0.2020-12	-0.9660-13	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.163E 03	-0.4000-04	-0.2030-04	-0.1690-04	-0.6760-05	-0.4460-05	-0.3010-05	-0.2030-05	-0.1390-05	-0.9370-06	-0.6220-06
	-0.4060-06	-0.2600-06	-0.1640-06	-0.1010-06	-0.6110-07	-0.3640-07	-0.2130-07	-0.1230-07	-0.6970-08	-0.3910-08
	-0.2170-08	-0.1190-08	-0.6400-09	-0.3480-09	-0.1860-09	-0.9860-10	-0.5190-10	-0.2720-10	-0.1410-10	-0.7320-11
	-0.3770-11	-0.1940-11	-0.9400-12	-0.5040-12	-0.2560-12	-0.1270-12	0.0	0.0	0.0	0.0
0.228E 03	-0.4000-04	-0.2040-04	-0.1120-04	-0.7120-05	-0.4840-05	-0.3390-05	-0.2400-05	-0.1700-05	-0.1200-05	-0.8550-06
	-0.5880-06	-0.4010-06	-0.2710-06	-0.1800-06	-0.1170-06	-0.7650-07	-0.4700-07	-0.2700-07	-0.1830-07	-0.1110-07
	-0.6660-08	-0.3940-08	-0.2310-08	-0.1340-08	-0.7680-09	-0.4370-09	-0.2420-09	-0.1590-09	-0.7710-10	-0.4270-10
	-0.2330-10	-0.1280-10	-0.6990-11	-0.3790-11	-0.2040-11	-0.1100-11	-0.5880-12	-0.3140-12	-0.1670-12	-0.8440-13
0.278E 03	-0.4000-04	-0.2040-04	-0.1130-04	-0.7390-05	-0.5140-05	-0.3670-05	-0.2690-05	-0.1900-05	-0.1450-05	-0.1060-05
	-0.7110-06	-0.5640-06	-0.3940-06	-0.2770-06	-0.1970-06	-0.1320-06	-0.8890-07	-0.5920-07	-0.3090-07	-0.2520-07
	-0.1620-07	-0.1020-07	-0.6410-08	-0.3970-08	-0.2410-08	-0.1400-08	-0.8900-09	-0.5310-09	-0.3150-09	-0.1850-09
	-0.1080-09	-0.6280-10	-0.3620-10	-0.2080-10	-0.1190-10	-0.6790-11	-0.3820-11	-0.2150-11	-0.1210-11	-0.0740-12
0.338E 03	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16
	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16
	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16
	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16	0.1000-16

FRACTION CF GAS PRODUCED AT LAST FIVE TIME LEVELS

FRACTION CF GAS PRODUCED= 0.595E-05 0.762E-05 0.949E-05 0.110E-04 0.128E-04







0.108E 03	-0.400U-04	-0.194D-04	-0.972D-05	-0.549D-05	-0.322D-05	-0.100U-05	-0.111D-05	-0.627D-06	-0.345D-06	-0.184D-06
	-0.957D-07	-0.444D-07	-0.230D-07	-0.116D-07	-0.451D-08	-0.259D-08	-0.120D-08	-0.548D-09	-0.248D-09	-0.117D-09
	-0.498D-10	-0.221D-10	-0.974D-11	-0.427D-11	-0.146D-11	-0.811D-12	-0.347D-12	-0.132D-12	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.143E 03	-0.400D-04	-0.200D-04	-0.104D-04	-0.415D-05	-0.144D-05	-0.244D-05	-0.154D-05	-0.641D-06	-0.248D-06	-0.152D-06
	-0.305D-04	-0.117D-04	-0.644D-07	-0.353D-07	-0.144D-07	-0.949D-08	-0.508D-08	-0.259D-08	-0.130D-08	-0.645D-09
	-0.317D-09	-0.150D-09	-0.748D-10	-0.359D-10	-0.171D-10	-0.813D-11	-0.404D-11	-0.190D-11	-0.944D-12	-0.534D-12
	-0.113D-12	-0.447D-13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.183E 03	-0.400D-04	-0.203D-04	-0.100D-04	-0.406D-05	-0.144D-05	-0.244D-05	-0.154D-05	-0.641D-06	-0.248D-06	-0.152D-06
	-0.153D-04	-0.220D-06	-0.135D-06	-0.807D-07	-0.474D-07	-0.274D-07	-0.155D-07	-0.607D-08	-0.470D-08	-0.259D-08
	-0.130D-08	-0.740D-09	-0.349D-09	-0.203D-09	-0.104D-09	-0.549D-10	-0.275D-10	-0.143D-10	-0.764D-11	-0.353D-11
	-0.170D-11	-0.877D-12	-0.438D-12	-0.214D-12	-0.104D-12	0.0	0.0	0.0	0.0	0.0
0.228E 03	-0.400D-04	-0.204D-04	-0.110D-04	-0.470D-05	-0.174D-05	-0.324D-05	-0.230D-05	-0.161D-05	-0.112D-05	-0.770D-06
	-0.353D-06	-0.351D-06	-0.231D-06	-0.150D-06	-0.943D-07	-0.556D-07	-0.337D-07	-0.228D-07	-0.133D-07	-0.780D-08
	-0.454D-09	-0.200D-09	-0.148D-09	-0.832D-09	-0.403D-09	-0.256D-09	-0.148D-09	-0.762D-10	-0.412D-10	-0.271D-10
	-0.114D-10	-0.676D-11	-0.331D-11	-0.174D-11	-0.910D-12	-0.475D-12	-0.247D-12	-0.124D-12	0.0	0.0
0.270E 03	-0.400D-04	-0.204D-04	-0.113D-04	-0.473D-05	-0.180D-05	-0.340D-05	-0.260D-05	-0.170D-05	-0.944D-06	-0.534D-06
	-0.204D-06	-0.263D-06	-0.246D-06	-0.161D-06	-0.104D-06	-0.710D-07	-0.471D-07	-0.299D-07	-0.148D-07	-0.780D-08
	-0.116D-07	-0.714D-08	-0.435D-08	-0.262D-08	-0.158D-08	-0.919D-09	-0.548D-09	-0.312D-09	-0.179D-09	-0.103D-09
	-0.532D-10	-0.324D-10	-0.134D-10	-0.102D-10	-0.549D-11	-0.314D-11	-0.172D-11	-0.944D-12	-0.514D-12	-0.270D-12
0.338E 03	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16
	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16
	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16
	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16	0.100D-16

FRACTION OF GAS PRODUCED AT LAST FIVE TIME LEVELS

FRACTION OF GAS PRODUCED= 0.598E-05 0.762E-05 0.949E-05 0.116E-04 0.128E-04















## APPENDIX J

### COMPUTER RESULTS FOR THE CONSTANT TERMINAL PRESSURE CASE

#### CASE IV

The solutions obtained at the optimum grid size are also presented in two parts. Solutions for a permeability value of 500.0 millidarcies are given in Section J.1 and those corresponding to a permeability of 10.0 millidarcies, in Section J.2.

The same considerations stated for Case I were applicable for Case IV. Copies of the computer programs for Case III and Case IV are given in Appendix K, Sections K.3 and K.4, respectively.

J.1

SOLUTIONS OBTAINED FOR A PERMEABILITY VALUE  
OF 500.0 MILLIDARCIES





0.135E 02	-0.106D-02	-0.675D-03	-0.521D-03	-0.366D-03	-0.278D-03	-0.250D-03	-0.178D-03	-0.147D-03	-0.121D-03	-0.101D-03
	-0.141D-04	-0.699D-04	-0.500D-04	-0.479D-04	-0.398D-04	-0.353D-04	-0.262D-04	-0.212D-04	-0.171D-04	-0.136D-04
	-0.110D-04	-0.853D-05	-0.669D-05	-0.522D-05	-0.405D-05	-0.313D-05	-0.241D-05	-0.186D-05	-0.141D-05	-0.107D-05
	-0.816D-06	-0.617D-06	-0.468D-06	-0.351D-06	-0.264D-06	-0.198D-06	-0.148D-06	-0.111D-06	-0.827D-07	-0.615D-07
0.145E 02	-0.993D-03	-0.816D-03	-0.647D-03	-0.495D-03	-0.360D-03	-0.240D-03	-0.174D-03	-0.142D-03	-0.121D-03	-0.105D-03
	-0.901D-04	-0.774D-04	-0.665D-04	-0.571D-04	-0.490D-04	-0.419D-04	-0.357D-04	-0.304D-04	-0.257D-04	-0.217D-04
	-0.102D-04	-0.157D-04	-0.126D-04	-0.104D-04	-0.084D-04	-0.069D-04	-0.057D-04	-0.048D-04	-0.040D-04	-0.033D-04
	-0.243D-05	-0.194D-05	-0.154D-05	-0.123D-05	-0.099D-05	-0.076D-05	-0.061D-05	-0.047D-05	-0.036D-05	-0.028D-05
0.263E 02	-0.436D-03	-0.462D-03	-0.462D-03	-0.437D-03	-0.420D-03	-0.367D-03	-0.316D-03	-0.266D-03	-0.222D-03	-0.180D-03
	-0.919D-04	-0.803D-04	-0.704D-04	-0.618D-04	-0.543D-04	-0.478D-04	-0.420D-04	-0.369D-04	-0.320D-04	-0.280D-04
	-0.243D-04	-0.211D-04	-0.183D-04	-0.158D-04	-0.136D-04	-0.117D-04	-0.097D-04	-0.079D-04	-0.063D-04	-0.049D-04
	-0.514D-05	-0.432D-05	-0.372D-05	-0.301D-05	-0.252D-05	-0.209D-05	-0.173D-05	-0.143D-05	-0.117D-05	-0.092D-05
0.335E 02	-0.892D-03	-0.730D-03	-0.649D-03	-0.540D-03	-0.440D-03	-0.340D-03	-0.260D-03	-0.190D-03	-0.120D-03	-0.080D-03
	-0.921D-04	-0.815D-04	-0.723D-04	-0.644D-04	-0.573D-04	-0.514D-04	-0.459D-04	-0.410D-04	-0.366D-04	-0.327D-04
	-0.291D-04	-0.249D-04	-0.231D-04	-0.200D-04	-0.181D-04	-0.160D-04	-0.141D-04	-0.124D-04	-0.109D-04	-0.095D-04
	-0.631D-05	-0.524D-05	-0.462D-05	-0.400D-05	-0.344D-05	-0.290D-05	-0.240D-05	-0.190D-05	-0.140D-05	-0.100D-05
0.435E 02	-0.850D-03	-0.767D-03	-0.682D-03	-0.601D-03	-0.510D-03	-0.430D-03	-0.350D-03	-0.280D-03	-0.210D-03	-0.150D-03
	-0.913D-04	-0.814D-04	-0.729D-04	-0.655D-04	-0.590D-04	-0.530D-04	-0.470D-04	-0.410D-04	-0.350D-04	-0.300D-04
	-0.324D-04	-0.273D-04	-0.235D-04	-0.200D-04	-0.170D-04	-0.140D-04	-0.110D-04	-0.080D-04	-0.060D-04	-0.040D-04
	-0.113D-04	-0.101D-04	-0.090D-04	-0.080D-04	-0.070D-04	-0.060D-04	-0.050D-04	-0.040D-04	-0.030D-04	-0.020D-04
0.535E 02	-0.927D-03	-0.803D-03	-0.690D-03	-0.590D-03	-0.500D-03	-0.420D-03	-0.340D-03	-0.270D-03	-0.210D-03	-0.160D-03
	-0.907D-04	-0.806D-04	-0.720D-04	-0.640D-04	-0.560D-04	-0.490D-04	-0.430D-04	-0.370D-04	-0.310D-04	-0.260D-04
	-0.347D-04	-0.317D-04	-0.290D-04	-0.260D-04	-0.240D-04	-0.220D-04	-0.200D-04	-0.180D-04	-0.160D-04	-0.150D-04
	-0.140D-04	-0.127D-04	-0.115D-04	-0.104D-04	-0.094D-04	-0.084D-04	-0.074D-04	-0.064D-04	-0.054D-04	-0.044D-04
0.635E 02	-0.804D-03	-0.694D-03	-0.590D-03	-0.500D-03	-0.420D-03	-0.340D-03	-0.270D-03	-0.210D-03	-0.160D-03	-0.110D-03
	-0.890D-04	-0.800D-04	-0.723D-04	-0.640D-04	-0.560D-04	-0.490D-04	-0.430D-04	-0.370D-04	-0.310D-04	-0.260D-04
	-0.360D-04	-0.325D-04	-0.307D-04	-0.283D-04	-0.260D-04	-0.240D-04	-0.220D-04	-0.200D-04	-0.180D-04	-0.170D-04
	-0.160D-04	-0.147D-04	-0.135D-04	-0.124D-04	-0.114D-04	-0.104D-04	-0.094D-04	-0.084D-04	-0.074D-04	-0.064D-04
0.735E 02	-0.785D-03	-0.692D-03	-0.609D-03	-0.527D-03	-0.440D-03	-0.360D-03	-0.280D-03	-0.210D-03	-0.160D-03	-0.110D-03
	-0.870D-04	-0.792D-04	-0.718D-04	-0.650D-04	-0.580D-04	-0.510D-04	-0.450D-04	-0.390D-04	-0.330D-04	-0.280D-04
	-0.390D-04	-0.342D-04	-0.317D-04	-0.295D-04	-0.273D-04	-0.254D-04	-0.236D-04	-0.219D-04	-0.203D-04	-0.187D-04
	-0.175D-04	-0.162D-04	-0.150D-04	-0.139D-04	-0.129D-04	-0.120D-04	-0.111D-04	-0.102D-04	-0.094D-04	-0.087D-04

FRACTION OF GAS PRODUCED AT LAST FIVE TIME LEVELS

FRACTION OF GAS PRODUCED: 0.380E-04 0.479E-04 0.566E-04 0.651E-04 0.733E-04







0.135E 02	-0.1690-02	-0.18970-03	-0.5280-03	-0.3710-03	-0.2810-03	-0.2220-03	-0.1790-03	-0.1470-03	-0.1210-03	-0.9990-04
	-0.3260-04	-0.6310-04	-0.5600-04	-0.4590-04	-0.3730-04	-0.3020-04	-0.2430-04	-0.1940-04	-0.1540-04	-0.1210-04
	-0.9470-05	-0.7370-05	-0.5700-05	-0.4380-05	-0.3350-05	-0.2850-05	-0.2400-05	-0.1980-05	-0.1600-05	-0.0850-06
	-0.6170-06	-0.4900-06	-0.3420-06	-0.2540-06	-0.1890-06	-0.1390-06	-0.1030-06	-0.7570-07	-0.5570-07	-0.4100-07
0.175E 02	-0.1000-02	-0.4040-03	-0.4930-03	-0.3490-03	-0.2670-03	-0.2130-03	-0.1750-03	-0.1460-03	-0.1230-03	-0.1050-03
	-0.8940-04	-0.7540-04	-0.6530-04	-0.5570-04	-0.4740-04	-0.4030-04	-0.3410-04	-0.2870-04	-0.2410-04	-0.2010-04
	-0.1670-04	-0.1380-04	-0.1130-04	-0.9230-05	-0.7500-05	-0.6070-05	-0.4800-05	-0.3710-05	-0.3120-05	-0.2640-05
	-0.1960-05	-0.1540-05	-0.1210-05	-0.9480-06	-0.7390-06	-0.5750-06	-0.4460-06	-0.3450-06	-0.2660-06	-0.2050-06
0.260E 02	-0.7880-03	-0.7080-03	-0.6310-03	-0.5550-03	-0.4850-03	-0.4200-03	-0.3600-03	-0.3030-03	-0.2540-03	-0.2110-03
	-0.1770-04	-0.1590-04	-0.1400-04	-0.1210-04	-0.1030-04	-0.0850-04	-0.0690-04	-0.0540-04	-0.0400-04	-0.0260-04
	-0.2300-04	-0.1980-04	-0.1700-04	-0.1450-04	-0.1240-04	-0.1050-04	-0.0890-04	-0.0750-04	-0.0630-04	-0.0520-04
	-0.4410-05	-0.3670-05	-0.3040-05	-0.2510-05	-0.2060-05	-0.1690-05	-0.1370-05	-0.1110-05	-0.0820-05	-0.0640-05
0.335E 02	-0.9020-03	-0.7450-03	-0.6460-03	-0.5670-03	-0.4970-03	-0.4340-03	-0.3760-03	-0.3220-03	-0.2710-03	-0.2210-03
	-0.7220-04	-0.6140-04	-0.5210-04	-0.4400-04	-0.3690-04	-0.3080-04	-0.2510-04	-0.2010-04	-0.1560-04	-0.1170-04
	-0.2090-04	-0.1740-04	-0.1490-04	-0.1240-04	-0.1030-04	-0.0850-04	-0.0700-04	-0.0570-04	-0.0460-04	-0.0360-04
	-0.7420-05	-0.6000-05	-0.5510-05	-0.4720-05	-0.4040-05	-0.3450-05	-0.2930-05	-0.2490-05	-0.2100-05	-0.1780-05
0.435E 02	-0.8640-03	-0.7150-03	-0.6200-03	-0.5350-03	-0.4590-03	-0.3910-03	-0.3290-03	-0.2720-03	-0.2200-03	-0.1740-03
	-0.3170-04	-0.2810-04	-0.2510-04	-0.2210-04	-0.1910-04	-0.1610-04	-0.1310-04	-0.1010-04	-0.0710-04	-0.0410-04
	-0.3150-04	-0.2740-04	-0.2430-04	-0.2130-04	-0.1830-04	-0.1530-04	-0.1230-04	-0.0930-04	-0.0630-04	-0.0330-04
	-0.1340-04	-0.0920-04	-0.0720-04	-0.0520-04	-0.0320-04	-0.0120-04	-0.0020-04	-0.0020-04	-0.0020-04	-0.0020-04
0.535E 02	-0.8340-03	-0.6900-03	-0.5900-03	-0.5130-03	-0.4430-03	-0.3780-03	-0.3180-03	-0.2630-03	-0.2130-03	-0.1680-03
	-0.3060-04	-0.2710-04	-0.2410-04	-0.2110-04	-0.1810-04	-0.1510-04	-0.1210-04	-0.0910-04	-0.0610-04	-0.0310-04
	-0.3490-04	-0.3100-04	-0.2710-04	-0.2320-04	-0.1930-04	-0.1540-04	-0.1150-04	-0.0760-04	-0.0370-04	-0.0080-04
	-0.1310-04	-0.0920-04	-0.0720-04	-0.0520-04	-0.0320-04	-0.0120-04	-0.0020-04	-0.0020-04	-0.0020-04	-0.0020-04
0.635E 02	-0.8120-03	-0.6710-03	-0.5700-03	-0.4870-03	-0.4120-03	-0.3420-03	-0.2770-03	-0.2170-03	-0.1620-03	-0.1120-03
	-0.3230-04	-0.2840-04	-0.2450-04	-0.2060-04	-0.1670-04	-0.1280-04	-0.0890-04	-0.0500-04	-0.0110-04	-0.0020-04
	-0.3560-04	-0.3170-04	-0.2780-04	-0.2390-04	-0.2000-04	-0.1610-04	-0.1220-04	-0.0830-04	-0.0440-04	-0.0050-04
	-0.1520-04	-0.1130-04	-0.0740-04	-0.0350-04	-0.0060-04	-0.0060-04	-0.0060-04	-0.0060-04	-0.0060-04	-0.0060-04
0.735E 02	-0.7940-03	-0.6560-03	-0.5550-03	-0.4720-03	-0.3970-03	-0.3270-03	-0.2620-03	-0.2020-03	-0.1470-03	-0.0970-03
	-0.3240-04	-0.2850-04	-0.2460-04	-0.2070-04	-0.1680-04	-0.1290-04	-0.0900-04	-0.0510-04	-0.0120-04	-0.0030-04
	-0.3570-04	-0.3180-04	-0.2790-04	-0.2400-04	-0.2010-04	-0.1620-04	-0.1230-04	-0.0840-04	-0.0450-04	-0.0060-04
	-0.1680-04	-0.1290-04	-0.0900-04	-0.0510-04	-0.0120-04	-0.0120-04	-0.0120-04	-0.0120-04	-0.0120-04	-0.0120-04

FRACTION OF GAS PRODUCED AT LAST FIVE TIME LEVELS

FRACTION OF GAS PRODUCED= 0.393E-04 0.685E-04 0.574E-04 0.659E-04 0.743E-04



SLIP CUEFF(U)= 10.000 INERTIAL CCEFF(UB)= 0.0 PERMEABILITY(K)= 500.000 MD

FLUX DISTRIBUTION FOR THE FIRST 40 GRID POINTS

0.25CE 00	-0.1810-02	-0.1260-02	-0.2250-03	-0.5980-04	-0.1490-04	-0.3830-05	-0.1000-05	-0.2670-06	-0.7160-07	-0.1940-07
	-0.5280-08	-0.1450-08	-0.3580-09	-0.1100-09	-0.3040-10	-0.8430-11	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.50CE 00	-0.3120-02	-0.2240-02	-0.7210-03	-0.2320-03	-0.7380-04	-0.2310-04	-0.7170-05	-0.2200-05	-0.6700-06	-0.2020-06
	-0.6110-07	-0.1830-07	-0.5480-08	-0.1630-08	-0.4860-09	-0.1440-09	-0.4260-10	-0.1260-10	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.100E 01	-0.2130-02	-0.1680-02	-0.6610-03	-0.4190-03	-0.1920-03	-0.8450-04	-0.3600-04	-0.1500-04	-0.6110-05	-0.2470-05
	-0.4880-06	-0.3930-06	-0.1560-06	-0.6150-07	-0.2420-07	-0.9540-08	-0.3750-08	-0.1480-08	-0.5810-09	-0.2290-09
	-0.9000-10	-0.3550-10	-0.1400-10	-0.5510-11	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.150E 01	-0.1780-02	-0.1420-02	-0.7800-03	-0.4700-03	-0.2840-03	-0.1590-03	-0.8390-04	-0.4240-04	-0.2070-04	-0.9110-05
	-0.4540-06	-0.2680-06	-0.9240-06	-0.4030-06	-0.1730-06	-0.7760-07	-0.3340-07	-0.1430-07	-0.6050-08	-0.2560-08
	-0.1090-08	-0.4850-09	-0.1520-09	-0.8050-10	-0.3360-10	-0.1400-10	-0.5820-11	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.25CE 01	-0.1580-02	-0.1270-02	-0.7250-03	-0.4640-03	-0.3080-03	-0.2030-03	-0.1300-03	-0.8090-04	-0.4860-04	-0.2840-04
	-0.1620-04	-0.9030-05	-0.4560-05	-0.2680-05	-0.1440-05	-0.7620-06	-0.4010-06	-0.2100-06	-0.1090-06	-0.5680-07
	-0.2490-07	-0.1510-07	-0.7800-08	-0.4010-08	-0.2080-08	-0.1060-08	-0.5430-09	-0.2710-09	-0.1430-09	-0.7320-10
	-0.3780-10	-0.1920-10	-0.5570-11	-0.5000-11	0.0	0.0	0.0	0.0	0.0	0.0
0.350E 01	-0.1400-02	-0.1140-02	-0.6650-03	-0.4480-03	-0.3150-03	-0.2250-03	-0.1610-03	-0.1130-03	-0.7760-04	-0.5200-04
	-0.3300-04	-0.2160-04	-0.1370-04	-0.0240-04	-0.4970-05	-0.2930-05	-0.1710-05	-0.9900-06	-0.5670-06	-0.3210-06
	-0.1010-06	-0.1010-06	-0.5420-07	-0.3110-07	-0.1710-07	-0.5380-08	-0.5120-08	-0.2790-08	-0.1510-08	-0.5420-09
	-0.4430-09	-0.2390-09	-0.1240-09	-0.6960-10	-0.3700-10	-0.1680-10	-0.1060-10	-0.5630-11	0.0	0.0
0.600E 01	-0.1270-02	-0.1040-02	-0.6130-03	-0.4200-03	-0.3070-03	-0.2310-03	-0.1740-03	-0.1130-03	-0.1010-03	-0.7530-04
	-0.5870-04	-0.4060-04	-0.2530-04	-0.2180-04	-0.1490-04	-0.1010-04	-0.6960-05	-0.4740-05	-0.3200-05	-0.2150-05
	-0.1440-05	-0.9540-06	-0.6310-06	-0.4160-06	-0.2740-06	-0.1600-06	-0.1180-06	-0.7690-07	-0.4500-07	-0.3270-07
	-0.2130-07	-0.1390-07	-0.9040-08	-0.5870-08	-0.3930-08	-0.2490-08	-0.1620-08	-0.1090-08	-0.6660-09	-0.4970-09
0.650E 01	-0.1160-02	-0.9490-03	-0.5630-03	-0.3930-03	-0.2950-03	-0.2400-03	-0.1820-03	-0.1450-03	-0.1170-03	-0.9330-04
	-0.7440-04	-0.5080-04	-0.4420-04	-0.3890-04	-0.2700-04	-0.2100-04	-0.1590-04	-0.1140-04	-0.8720-05	-0.6390-05
	-0.4040-05	-0.3340-05	-0.2390-05	-0.1700-05	-0.1120-05	-0.8440-06	-0.6190-06	-0.4150-06	-0.2980-06	-0.2000-06
	-0.1380-06	-0.9490-07	-0.6520-07	-0.4470-07	-0.3050-07	-0.2080-07	-0.1420-07	-0.9640-08	-0.6650-08	-0.4440-08

0.135E 02	-0.1070-02	-0.6800-03	-0.5240-03	-0.3690-03	-0.2900-03	-0.2220-03	-0.1790-03	-0.1480-03	-0.1220-03	-0.1020-03
	-0.6470-04	-0.7050-04	-0.5650-04	-0.4940-04	-0.3900-04	-0.3260-04	-0.2660-04	-0.2150-04	-0.1730-04	-0.1300-04
	-0.1100-04	-0.0660-05	-0.6660-05	-0.5310-05	-0.4130-05	-0.3200-05	-0.2400-05	-0.1890-05	-0.1440-05	-0.1100-05
	-0.6330-06	-0.6330-06	-0.4760-06	-0.3600-06	-0.2710-06	-0.2030-06	-0.1520-06	-0.1140-06	-0.0820-06	-0.0630-06
0.125E 02	-0.9970-03	-0.8210-03	-0.44500-03	-0.3470-03	-0.2650-03	-0.2130-03	-0.1750-03	-0.1470-03	-0.1240-03	-0.1040-03
	-0.2060-04	-0.7000-04	-0.6700-04	-0.5760-04	-0.4440-04	-0.4230-04	-0.3610-04	-0.3070-04	-0.2600-04	-0.2190-04
	-0.1240-04	-0.1540-04	-0.1260-04	-0.1060-04	-0.8720-05	-0.7150-05	-0.5830-05	-0.4740-05	-0.3830-05	-0.3090-05
	-0.2240-05	-0.1920-05	-0.1560-05	-0.1250-05	-0.9920-06	-0.7820-06	-0.6100-06	-0.4830-06	-0.3780-06	-0.2860-06
0.260E 02	-0.9420-03	-0.7770-03	-0.4630-03	-0.3290-03	-0.2530-03	-0.2040-03	-0.1690-03	-0.1420-03	-0.1230-03	-0.1060-03
	-0.9250-04	-0.8090-04	-0.7690-04	-0.6230-04	-0.5470-04	-0.4800-04	-0.4220-04	-0.3700-04	-0.3230-04	-0.2420-04
	-0.2460-04	-0.2140-04	-0.1850-04	-0.1600-04	-0.1330-04	-0.1100-04	-0.1010-04	-0.0610-05	-0.0730-05	-0.0400-05
	-0.5230-05	-0.4400-05	-0.3650-05	-0.3000-05	-0.2570-05	-0.2140-05	-0.1770-05	-0.1460-05	-0.1210-05	-0.0940-05
0.335E 02	-0.3960-03	-0.7400-03	-0.4430-03	-0.3150-03	-0.2430-03	-0.1970-03	-0.1640-03	-0.1400-03	-0.1210-03	-0.1050-03
	-0.4270-04	-0.3200-04	-0.7290-04	-0.5690-04	-0.5790-04	-0.5180-04	-0.4630-04	-0.4140-04	-0.3700-04	-0.3300-04
	-0.2940-04	-0.2620-04	-0.2230-04	-0.2070-04	-0.1830-04	-0.1620-04	-0.1430-04	-0.1260-04	-0.1100-04	-0.0950-05
	-0.6430-05	-0.7340-05	-0.6390-05	-0.5520-05	-0.4770-05	-0.4120-05	-0.3540-05	-0.3040-05	-0.2600-05	-0.2220-05
0.435E 02	-0.4620-03	-0.7110-03	-0.4260-03	-0.3030-03	-0.2360-03	-0.1900-03	-0.1590-03	-0.1360-03	-0.1140-03	-0.1040-03
	-0.4130-04	-0.3130-04	-0.7340-04	-0.5630-04	-0.5530-04	-0.5270-04	-0.4460-04	-0.4000-04	-0.3690-04	-0.3010-04
	-0.3270-04	-0.2660-04	-0.2670-04	-0.2420-04	-0.2180-04	-0.1970-04	-0.1770-04	-0.1590-04	-0.1430-04	-0.1230-04
	-0.1130-04	-0.1020-04	-0.9120-05	-0.9120-05	-0.7230-05	-0.6400-05	-0.5670-05	-0.5030-05	-0.4430-05	-0.3910-05
0.535E 02	-0.6420-03	-0.8170-03	-0.4110-03	-0.2930-03	-0.2270-03	-0.1840-03	-0.1530-03	-0.1300-03	-0.1140-03	-0.1020-03
	-0.9070-04	-0.8130-04	-0.7330-04	-0.6630-04	-0.6020-04	-0.5480-04	-0.4960-04	-0.4470-04	-0.4010-04	-0.3620-04
	-0.3490-04	-0.3200-04	-0.2530-04	-0.2480-04	-0.2450-04	-0.2240-04	-0.2070-04	-0.1870-04	-0.1700-04	-0.1550-04
	-0.1410-04	-0.1330-04	-0.1170-04	-0.1060-04	-0.0900-04	-0.0800-04	-0.0700-04	-0.0610-04	-0.0540-04	-0.0490-04
0.635E 02	-0.9090-03	-0.6690-03	-0.4000-03	-0.2460-03	-0.2210-03	-0.1800-03	-0.1510-03	-0.1300-03	-0.1130-03	-0.1000-03
	-0.1450-04	-0.0600-04	-0.7280-04	-0.5620-04	-0.6030-04	-0.4520-04	-0.4060-04	-0.3650-04	-0.3290-04	-0.2940-04
	-0.3630-04	-0.3350-04	-0.3060-04	-0.2850-04	-0.2630-04	-0.2430-04	-0.2240-04	-0.2060-04	-0.1900-04	-0.1750-04
	-0.1610-04	-0.1490-04	-0.1370-04	-0.1250-04	-0.1130-04	-0.1060-04	-0.0940-04	-0.0890-04	-0.0810-04	-0.0740-04
0.735E 02	-0.7910-03	-0.6630-03	-0.3510-03	-0.2160-03	-0.1760-03	-0.1460-03	-0.1240-03	-0.1120-03	-0.1040-03	-0.0990-03
	-0.5840-04	-0.7970-04	-0.7230-04	-0.6590-04	-0.6010-04	-0.5530-04	-0.5090-04	-0.4700-04	-0.4340-04	-0.4020-04
	-0.3720-04	-0.3450-04	-0.3200-04	-0.2970-04	-0.2760-04	-0.2560-04	-0.2360-04	-0.2210-04	-0.2050-04	-0.1900-04
	-0.1770-04	-0.1640-04	-0.1520-04	-0.1410-04	-0.1310-04	-0.1210-04	-0.1120-04	-0.1040-04	-0.0950-04	-0.0840-04

FRACTION CF GAS PRODUCED AT LAST FIVE TIME LEVELS

FRACTION CF GAS PRODUCED= 0.350E-04 0.402E-04 0.470E-04 0.653E-04 0.739E-04



SLIP COEFF (S) = 10.000 INERTIAL COEFF (SU) = 0.0 PERMEABILITY (K) = 500.000 HD

FLUX DISTRIBUTION FOR THE FIRST 40 GRID POINTS

0.250E 00	-0.194E-02	-0.127D-02	-0.248D-03	-0.556D-04	-0.132D-04	-0.325D-05	-0.814D-06	-0.207D-06	-0.531D-07	-0.137D-07
	-0.347E-08	-0.935E-09	-0.246E-09	-0.647D-10	-0.171D-10	-0.454D-11	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.500E 00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	-0.118D-02	-0.224D-02	-0.711D-03	-0.220D-03	-0.670D-04	-0.201D-04	-0.596D-05	-0.175D-05	-0.509D-06	-0.147D-06
	-0.424E-07	-0.122E-07	-0.347D-08	-0.997D-09	-0.281D-09	-0.796D-10	-0.225D-10	-0.636D-11	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.100E 01	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	-0.218D-02	-0.171D-02	-0.865D-03	-0.410D-03	-0.182D-03	-0.771D-04	-0.316D-04	-0.127D-04	-0.498D-05	-0.194D-05
	-0.748D-06	-0.287D-06	-0.109D-06	-0.416D-07	-0.158D-07	-0.601D-08	-0.228D-08	-0.865D-09	-0.329D-09	-0.125D-09
	-0.474D-10	-0.180D-10	-0.605D-11	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.150E 01	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	-0.141D-02	-0.145D-02	-0.731D-03	-0.470D-03	-0.277E-03	-0.150D-03	-0.767D-04	-0.274D-04	-0.176D-04	-0.805D-05
	-0.339E-05	-0.157D-05	-0.680E-06	-0.290D-06	-0.120D-06	-0.512D-07	-0.212D-07	-0.877D-08	-0.350D-08	-0.147D-08
	-0.600E-09	-0.244D-09	-0.986D-10	-0.398D-10	-0.160D-10	-0.644D-11	0.0	0.0	0.0	0.0
0.250E 01	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	-0.159D-02	-0.120D-02	-0.714D-03	-0.457D-03	-0.306D-03	-0.198D-03	-0.124D-03	-0.751D-04	-0.439D-04	-0.249D-04
	-0.130E-08	-0.745E-05	-0.317D-05	-0.204D-05	-0.101D-05	-0.558D-06	-0.245D-06	-0.145D-06	-0.735E-07	-0.371D-07
	-0.186D-07	-0.015D-08	-0.469D-09	-0.235D-09	-0.117D-09	-0.596D-09	-0.293D-09	-0.145D-09	-0.729D-10	-0.364D-10
	-0.102E-10	-0.909E-11	-0.453D-11	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.150E 01	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	-0.143D-02	-0.116D-02	-0.674D-03	-0.452D-03	-0.314D-03	-0.223D-03	-0.157D-03	-0.103D-03	-0.729D-04	-0.476D-04
	-0.302E-04	-0.148D-04	-0.114D-04	-0.677D-05	-0.394D-05	-0.240D-05	-0.124D-05	-0.724D-06	-0.404D-06	-0.222D-06
	-0.122D-06	-0.662D-07	-0.354D-07	-0.172D-07	-0.103D-07	-0.592D-08	-0.292D-08	-0.155D-08	-0.817D-09	-0.431D-09
	-0.226E-09	-0.113D-09	-0.622D-10	-0.335D-10	-0.170D-10	-0.805D-11	-0.467D-11	0.0	0.0	0.0
0.600E 01	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	-0.159D-02	-0.106D-02	-0.622D-03	-0.425D-03	-0.309E-03	-0.211D-03	-0.174D-03	-0.131D-03	-0.978D-04	-0.714D-04
	-0.522E-04	-0.374E-04	-0.264D-04	-0.184D-04	-0.127D-04	-0.861D-05	-0.579D-05	-0.387D-05	-0.256D-05	-0.168D-05
	-0.110D-05	-0.170D-06	-0.443D-06	-0.301D-06	-0.198D-06	-0.125D-06	-0.802D-07	-0.545D-07	-0.330D-07	-0.211D-07
	-0.135E-07	-0.665D-08	-0.553D-08	-0.354D-08	-0.226D-08	-0.145D-08	-0.724D-09	-0.531D-09	-0.378D-09	-0.242D-09
0.950E 01	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	-0.117D-02	-0.463D-03	-0.571D-03	-0.398D-03	-0.297D-03	-0.230D-03	-0.182D-03	-0.144D-03	-0.115D-03	-0.511D-04
	-0.718E-04	-0.561D-04	-0.434D-04	-0.322D-04	-0.252D-04	-0.183D-04	-0.139D-04	-0.102D-04	-0.780D-05	-0.522E-05
	-0.379D-05	-0.268D-05	-0.189D-05	-0.131D-05	-0.913D-06	-0.640D-06	-0.433D-06	-0.287D-06	-0.202D-06	-0.134D-06
	-0.933E-07	-0.630E-07	-0.427D-07	-0.286D-07	-0.192D-07	-0.129D-07	-0.860D-08	-0.574D-08	-0.383D-08	-0.255D-08

0.135K 02	-0.108B-02	-0.892D-03	-0.531D-03	-0.373D-03	-0.283B-03	-0.223D-03	-0.180D-03	-0.148D-03	-0.122D-03	-0.101D-03
	-0.832E-04	-0.687D-04	-0.566D-04	-0.463D-04	-0.377D-04	-0.306D-04	-0.246D-04	-0.195D-04	-0.156D-04	-0.123D-04
	-0.962D-05	-0.747D-05	-0.590D-05	-0.446D-05	-0.342D-05	-0.260D-05	-0.198D-05	-0.149D-05	-0.113D-05	-0.845D-06
	-0.633E-06	-0.472E-06	-0.352D-06	-0.261D-06	-0.194D-06	-0.143D-06	-0.106D-06	-0.781D-07	-0.576D-07	-0.424D-07
0.185E 02	-0.101D-02	-0.831D-03	-0.496D-03	-0.351D-03	-0.268D-03	-0.214D-03	-0.176D-03	-0.147D-03	-0.124D-03	-0.106D-03
	-0.901E-04	-0.770E-04	-0.659D-04	-0.562D-04	-0.479D-04	-0.407D-04	-0.344D-04	-0.290D-04	-0.243D-04	-0.203D-04
	-0.169D-04	-0.134D-04	-0.115D-04	-0.937D-05	-0.762E-05	-0.617D-05	-0.497D-05	-0.380D-05	-0.278D-05	-0.203D-05
	-0.200E-05	-0.159E-05	-0.124D-05	-0.971D-06	-0.753D-06	-0.590D-06	-0.459D-06	-0.354D-06	-0.274D-06	-0.211D-06
0.240E 02	-0.591D-03	-0.786D-03	-0.470D-03	-0.331D-03	-0.254D-03	-0.206D-03	-0.171D-03	-0.144D-03	-0.123D-03	-0.106D-03
	-0.423E-04	-0.805E-04	-0.701D-04	-0.614D-04	-0.537D-04	-0.469D-04	-0.409D-04	-0.357D-04	-0.310D-04	-0.264D-04
	-0.232E-04	-0.209D-04	-0.172D-04	-0.147D-04	-0.126D-04	-0.107D-04	-0.904D-05	-0.783D-05	-0.681D-05	-0.537D-05
	-0.449E-05	-0.373E-05	-0.309D-05	-0.256D-05	-0.211D-05	-0.173D-05	-0.143D-05	-0.116D-05	-0.943D-06	-0.767D-06
0.335E 02	-0.901D-03	-0.749D-03	-0.440D-03	-0.319D-03	-0.246D-03	-0.199D-03	-0.165D-03	-0.141D-03	-0.121D-03	-0.106D-03
	-0.295E-04	-0.740E-04	-0.726D-04	-0.645D-04	-0.573D-04	-0.510D-04	-0.454D-04	-0.404D-04	-0.359D-04	-0.319D-04
	-0.283D-04	-0.251D-04	-0.221D-04	-0.195D-04	-0.172D-04	-0.151D-04	-0.132D-04	-0.115D-04	-0.100D-04	-0.870D-05
	-0.751E-05	-0.650E-05	-0.560D-05	-0.480D-05	-0.411D-05	-0.351D-05	-0.298D-05	-0.253D-05	-0.215D-05	-0.181D-05
0.435E 02	-0.872D-03	-0.719D-03	-0.431D-03	-0.306D-03	-0.237D-03	-0.192D-03	-0.161D-03	-0.137D-03	-0.119D-03	-0.104D-03
	-0.223E-04	-0.821E-04	-0.734D-04	-0.658D-04	-0.591D-04	-0.533D-04	-0.480D-04	-0.433D-04	-0.391D-04	-0.352E-04
	-0.318E-04	-0.237D-04	-0.258D-04	-0.232D-04	-0.203E-04	-0.187D-04	-0.167D-04	-0.149D-04	-0.133D-04	-0.119D-04
	-0.105D-04	-0.935D-05	-0.828E-05	-0.732D-05	-0.645D-05	-0.568D-05	-0.499D-05	-0.437D-05	-0.383D-05	-0.334D-05
0.535E 02	-0.841D-03	-0.694D-03	-0.416D-03	-0.295D-03	-0.224D-03	-0.186D-03	-0.156D-03	-0.134D-03	-0.117D-03	-0.103D-03
	-0.912E-04	-0.816E-04	-0.738D-04	-0.663D-04	-0.601D-04	-0.548D-04	-0.498D-04	-0.452D-04	-0.412D-04	-0.376D-04
	-0.143D-04	-0.313E-04	-0.285D-04	-0.260D-04	-0.237E-04	-0.216D-04	-0.195D-04	-0.174D-04	-0.162D-04	-0.147D-04
	-0.131D-04	-0.120E-04	-0.108D-04	-0.977D-05	-0.880D-05	-0.792D-05	-0.711D-05	-0.638D-05	-0.572D-05	-0.512D-05
0.635E 02	-0.814D-03	-0.675D-03	-0.405D-03	-0.289D-03	-0.221D-03	-0.182D-03	-0.153D-03	-0.131D-03	-0.114D-03	-0.101D-03
	-0.901E-04	-0.809D-04	-0.731D-04	-0.663D-04	-0.604D-04	-0.551D-04	-0.504D-04	-0.462D-04	-0.424D-04	-0.390D-04
	-0.358E-04	-0.330D-04	-0.303D-04	-0.279D-04	-0.256E-04	-0.236D-04	-0.217D-04	-0.199D-04	-0.183D-04	-0.167D-04
	-0.154E-04	-0.141E-04	-0.129E-04	-0.119D-04	-0.107D-04	-0.980D-05	-0.894D-05	-0.815D-05	-0.742D-05	-0.676D-05
0.735E 02	-0.709D-03	-0.660D-03	-0.395D-03	-0.282D-03	-0.218D-03	-0.178D-03	-0.150D-03	-0.129D-03	-0.113D-03	-0.996D-04
	-0.990E-04	-0.801E-04	-0.726D-04	-0.661D-04	-0.604D-04	-0.553D-04	-0.508D-04	-0.468D-04	-0.432D-04	-0.399D-04
	-0.368E-04	-0.341D-04	-0.315D-04	-0.292D-04	-0.270D-04	-0.250D-04	-0.232D-04	-0.214D-04	-0.198D-04	-0.183D-04
	-0.169E-04	-0.157E-04	-0.145D-04	-0.134D-04	-0.123D-04	-0.114D-04	-0.105D-04	-0.965D-05	-0.889D-05	-0.819D-05

FRACTION OF GAS PRODUCED AT LAST FIVE FIRE LEVELS

FRACTION OF GAS PRODUCED= 0.396E-04 0.468E-04 0.577E-04 0.664E-04 0.748E-04





SLIP COEFF(B)= 0.0 INERTIAL CCEFF(BR)= 0.1000 09 PERMEABILITY(K)= 500.000 MD

FLUX DISTRIBUTION FOR THE FIRST 40 GRID POINTS

0.250E 00	-0.928D-04	-0.769D-04	-0.264D-04	-0.951D-05	-0.314D-05	-0.932D-06	-0.259D-06	-0.698D-07	-0.168D-07	-0.508D-08
	-0.138D-00	-0.377D-09	-0.103D-05	-0.285D-10	-0.786D-11	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.500E 00	-0.132D-03	-0.110D-03	-0.354D-04	-0.151D-04	-0.585D-05	-0.224D-05	-0.859D-06	-0.305D-06	-0.103D-06	-0.334D-07
	-0.106D-07	-0.332D-08	-0.102D-08	-0.313D-09	-0.958D-10	-0.287D-10	-0.864D-11	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.100E 01	-0.131D-03	-0.109D-03	-0.409D-04	-0.164D-04	-0.754D-05	-0.305D-05	-0.171D-05	-0.617D-06	-0.375D-06	-0.166D-06
	-0.709D-07	-0.294D-07	-0.122D-07	-0.493D-08	-0.194D-08	-0.791D-09	-0.314D-09	-0.125D-09	-0.493D-10	0.0
	-0.767D-11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.150E 01	-0.129D-03	-0.107D-03	-0.424D-04	-0.184D-04	-0.911D-05	-0.476D-05	-0.259D-05	-0.143D-05	-0.777D-06	-0.410D-06
	-0.209D-08	-0.103D-08	-0.444D-07	-0.231D-07	-0.106D-07	-0.475D-08	-0.211D-08	-0.928D-09	-0.404D-09	-0.174D-09
	-0.748D-10	-0.319D-10	-0.136D-10	-0.573D-11	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.250E 01	-0.137D-03	-0.106D-03	-0.441D-04	-0.209D-04	-0.110D-04	-0.618D-05	-0.366D-05	-0.222D-05	-0.136D-05	-0.825D-06
	-0.495D-06	-0.292D-06	-0.168D-06	-0.952D-07	-0.529D-07	-0.290D-07	-0.156D-07	-0.837D-08	-0.443D-08	-0.233D-08
	-0.122D-08	-0.636D-09	-0.330D-09	-0.171D-09	-0.810D-10	-0.454D-10	-0.233D-10	-0.120D-10	-0.616D-11	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.350E 01	-0.124D-03	-0.104D-03	-0.469D-04	-0.232D-04	-0.129D-04	-0.771D-05	-0.483D-05	-0.312D-05	-0.204D-05	-0.135D-05
	-0.893D-06	-0.577D-06	-0.371D-06	-0.239D-06	-0.140D-06	-0.895D-07	-0.548D-07	-0.370D-07	-0.160D-07	-0.109D-07
	-0.625D-08	-0.355D-08	-0.201D-08	-0.112D-08	-0.620D-09	-0.347D-09	-0.191D-09	-0.169D-09	-0.572D-10	-0.212D-10
	-0.170D-10	-0.919D-11	-0.447D-11	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.600E 01	-0.120D-03	-0.101D-03	-0.477D-04	-0.258D-04	-0.153D-04	-0.971D-05	-0.644D-05	-0.441D-05	-0.306D-05	-0.214D-05
	-0.159D-05	-0.111D-05	-0.788D-06	-0.599D-06	-0.393D-06	-0.275D-06	-0.191D-06	-0.131D-06	-0.894D-07	-0.665D-07
	-0.407D-07	-0.273D-07	-0.181D-07	-0.120D-07	-0.794D-08	-0.533D-08	-0.343D-08	-0.250D-08	-0.147D-08	-0.961D-09
	-0.627D-09	-0.409D-09	-0.260D-09	-0.173D-09	-0.113D-09	-0.734D-10	-0.473D-10	-0.310D-10	-0.202D-10	-0.131D-10
0.650E 01	-0.116D-03	-0.934D-04	-0.454D-04	-0.284D-04	-0.171D-04	-0.118D-04	-0.821D-05	-0.507D-05	-0.429D-05	-0.316D-05
	-0.238D-05	-0.179D-05	-0.136D-05	-0.102D-05	-0.770D-06	-0.570D-06	-0.435D-06	-0.321D-06	-0.237D-06	-0.174D-06
	-0.126D-08	-0.914D-07	-0.657D-07	-0.468D-07	-0.319D-07	-0.235D-07	-0.165D-07	-0.110D-07	-0.806D-08	-0.559D-08
	-0.387D-08	-0.267D-08	-0.184D-08	-0.126D-08	-0.842D-09	-0.589D-09	-0.402D-09	-0.273D-09	-0.186D-09	-0.126D-09

0.135E 02	-0.1120-03	-0.9370-04	-0.5040-04	-0.3060-04	-0.2010-04	-0.1400-04	-0.1010-04	-0.7490-05	-0.5670-05	-0.4370-05
	-0.3400-05	-0.2670-05	-0.2110-05	-0.1670-05	-0.1330-05	-0.1060-05	-0.8390-06	-0.6650-06	-0.5260-06	-0.4150-06
	-0.3260-06	-0.2500-06	-0.1990-06	-0.1550-06	-0.9250-07	-0.7110-07	-0.5440-07	-0.4150-07	-0.3160-07	-0.2430-07
	-0.2400-07	-0.1910-07	-0.1370-07	-0.1030-07	-0.7740-08	-0.5900-08	-0.4350-08	-0.3250-08	-0.2430-08	-0.1810-08
0.185E 02	-0.1040-03	-0.9290-04	-0.5160-04	-0.2230-04	-0.2220-04	-0.1590-04	-0.1190-04	-0.9080-05	-0.7680-05	-0.5600-05
	-0.4490-05	-0.3630-05	-0.2910-05	-0.1910-05	-0.1910-05	-0.1380-05	-0.1110-05	-0.9120-06	-0.7510-06	-0.5370-06
	-0.4170-06	-0.3570-06	-0.4150-06	-0.3300-06	-0.2760-06	-0.2240-06	-0.1810-06	-0.1460-06	-0.1170-06	-0.8650-06
	-0.7430-07	-0.5950-07	-0.4720-07	-0.3730-07	-0.2940-07	-0.2320-07	-0.1820-07	-0.1420-07	-0.1110-07	-0.6750-07
0.260E 02	-0.1150-03	-0.9070-04	-0.5110-04	-0.3340-04	-0.3310-04	-0.1750-04	-0.1340-04	-0.1050-04	-0.8440-05	-0.6750-05
	-0.5520-05	-0.4560-05	-0.3790-05	-0.3170-05	-0.2660-05	-0.2240-05	-0.1940-05	-0.1610-05	-0.1360-05	-0.1150-05
	-0.4790-06	-0.4100-06	-0.3700-06	-0.3300-06	-0.3030-06	-0.2850-06	-0.2690-06	-0.2520-06	-0.2410-06	-0.2310-06
	-0.1760-06	-0.1470-06	-0.1220-06	-0.1010-06	-0.8390-07	-0.6920-07	-0.5700-07	-0.4690-07	-0.3850-07	-0.3150-07
0.335E 02	-0.1070-03	-0.8470-04	-0.5090-04	-0.3420-04	-0.2470-04	-0.1870-04	-0.1460-04	-0.1160-04	-0.9450-05	-0.7730-05
	-0.5470-05	-0.4430-05	-0.3790-05	-0.3090-05	-0.3330-05	-0.2860-05	-0.2460-05	-0.2120-05	-0.1830-05	-0.1590-05
	-0.1370-05	-0.1190-05	-0.1030-05	-0.8930-06	-0.7730-06	-0.6620-06	-0.5790-06	-0.5030-06	-0.4320-06	-0.3720-06
	-0.3200-06	-0.2750-06	-0.2360-06	-0.2020-06	-0.1730-06	-0.1470-06	-0.1250-06	-0.1070-06	-0.9050-07	-0.7670-07
0.435E 02	-0.1010-03	-0.8710-04	-0.5070-04	-0.3450-04	-0.2540-04	-0.1930-04	-0.1550-04	-0.1260-04	-0.1040-04	-0.8650-05
	-0.7200-05	-0.6200-05	-0.5310-05	-0.4590-05	-0.3940-05	-0.3400-05	-0.3000-05	-0.2630-05	-0.2300-05	-0.2020-05
	-0.1780-05	-0.1560-05	-0.1400-05	-0.1310-05	-0.1270-05	-0.1230-05	-0.1200-05	-0.1180-05	-0.1160-05	-0.1140-05
	-0.4470-06	-0.4310-06	-0.4180-06	-0.4100-06	-0.4020-06	-0.3940-06	-0.3870-06	-0.3800-06	-0.3730-06	-0.3660-06
0.535E 02	-0.9930-04	-0.8590-04	-0.5040-04	-0.3470-04	-0.2590-04	-0.2020-04	-0.1620-04	-0.1330-04	-0.1110-04	-0.9340-05
	-0.7200-05	-0.6480-05	-0.5960-05	-0.5190-05	-0.4540-05	-0.3990-05	-0.3510-05	-0.3110-05	-0.2750-05	-0.2440-05
	-0.2170-05	-0.1930-05	-0.1720-05	-0.1540-05	-0.1370-05	-0.1230-05	-0.1100-05	-0.9800-06	-0.8750-06	-0.7820-06
	-0.4190-06	-0.3940-06	-0.3770-06	-0.3610-06	-0.3440-06	-0.3280-06	-0.3130-06	-0.2980-06	-0.2840-06	-0.2700-06
0.635E 02	-0.9710-04	-0.8690-04	-0.5010-04	-0.3440-04	-0.2620-04	-0.2040-04	-0.1670-04	-0.1380-04	-0.1160-04	-0.9470-05
	-0.5330-05	-0.4600-05	-0.4060-05	-0.3670-05	-0.3300-05	-0.2930-05	-0.2570-05	-0.2210-05	-0.1850-05	-0.1500-05
	-0.2510-05	-0.2250-05	-0.2030-05	-0.1820-05	-0.1640-05	-0.1480-05	-0.1340-05	-0.1210-05	-0.1090-05	-0.9930-06
	-0.8880-06	-0.8620-06	-0.8400-06	-0.8190-06	-0.7990-06	-0.7800-06	-0.7610-06	-0.7430-06	-0.7250-06	-0.7070-06
0.735E 02	-0.9720-04	-0.8410-04	-0.4990-04	-0.3480-04	-0.2630-04	-0.2050-04	-0.1700-04	-0.1420-04	-0.1200-04	-0.1040-04
	-0.6940-05	-0.6210-05	-0.5680-05	-0.5170-05	-0.4700-05	-0.4290-05	-0.3940-05	-0.3640-05	-0.3380-05	-0.3140-05
	-0.2200-05	-0.2030-05	-0.1890-05	-0.1770-05	-0.1660-05	-0.1560-05	-0.1470-05	-0.1380-05	-0.1300-05	-0.1230-05
	-0.1060-05	-0.9680-06	-0.9070-06	-0.8480-06	-0.7920-06	-0.7370-06	-0.6840-06	-0.6330-06	-0.5850-06	-0.5410-06

FRACTION OF GAS PRODUCED AT LAST FIVE TIME LEVELS

FRACTION OF GAS PRODUCED\* 0.308E-05 0.494E-05 0.479E-05 0.700E-05 0.802E-05



SLIP COEFF (U) = 0.0 INERTIAL COEFF (DB) = 0.1000 09 PERMEABILITY (K) = 500.000 MD

FLUX DISTRIBUTION FOR THE FIRST 40 GRID POINTS

0.250E 00	-0.933D-04	-0.772D-04	-0.262D-04	-0.898D-05	-0.282D-05	-0.799D-06	-0.210D-06	-0.542D-07	-0.139D-07	-0.360D-08
	-0.934D-09	-0.244E-09	-0.638D-10	-0.168D-10	-0.442D-11	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.500E 00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	-0.133E-03	-0.110D-03	-0.386D-04	-0.143D-04	-0.539D-05	-0.202D-05	-0.710D-06	-0.247D-06	-0.793D-07	-0.246D-07
	-0.745E-08	-0.222E-08	-0.554D-09	-0.191D-09	-0.554D-10	-0.160D-10	-0.459D-11	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.100E 01	-0.132E-03	-0.109D-03	-0.401D-04	-0.152D-04	-0.702D-05	-0.324D-05	-0.152D-05	-0.702D-06	-0.310D-06	-0.132D-06
	-0.544E-07	-0.217D-07	-0.857D-08	-0.339D-08	-0.129D-08	-0.497D-09	-0.190D-09	-0.727D-10	-0.277D-10	-0.106D-10
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.150E 01	-0.130E-03	-0.168D-03	-0.416D-04	-0.180D-04	-0.859D-05	-0.440D-05	-0.235D-05	-0.127D-05	-0.569D-06	-0.341D-06
	-0.164E-06	-0.796E-07	-0.367D-07	-0.165D-07	-0.726D-08	-0.315D-08	-0.135D-08	-0.571D-09	-0.239D-09	-0.965D-10
	-0.412E-10	-0.169E-10	-0.694E-11	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.250E 01	-0.127E-03	-0.106D-03	-0.434D-04	-0.202D-04	-0.104D-04	-0.578D-05	-0.337D-05	-0.201D-05	-0.121D-05	-0.717D-06
	-0.391E-06	-0.240E-06	-0.135D-06	-0.710D-07	-0.398D-07	-0.211D-07	-0.111D-07	-0.404D-07	-0.233D-07	-0.749D-08
	-0.768E-09	-0.348E-09	-0.196D-09	-0.945D-10	-0.494E-10	-0.244D-10	-0.124D-10	-0.620D-11	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.350E 01	-0.125D-03	-0.165D-03	-0.453D-04	-0.225D-04	-0.123D-04	-0.727D-05	-0.450D-05	-0.207D-05	-0.115D-05	-0.120D-05
	-0.274E-06	-0.494E-06	-0.310E-06	-0.191D-06	-0.116D-06	-0.690D-07	-0.400D-07	-0.233D-07	-0.113D-07	-0.749D-08
	-0.410E-04	-0.231D-04	-0.131D-04	-0.699D-05	-0.373E-05	-0.201D-05	-0.108D-05	-0.575D-06	-0.306D-06	-0.162D-06
	-0.857E-11	-0.452E-11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.600E 01	-0.121D-03	-0.102D-03	-0.473D-04	-0.252D-04	-0.147D-04	-0.924D-05	-0.607D-05	-0.411D-05	-0.286D-05	-0.193D-05
	-0.140E-05	-0.493D-06	-0.649D-06	-0.441D-06	-0.332D-06	-0.228D-06	-0.155D-06	-0.104D-06	-0.599D-07	-0.464D-07
	-0.306E-07	-0.200E-07	-0.131D-07	-0.640D-08	-0.580D-08	-0.355D-08	-0.239D-08	-0.147D-08	-0.944D-09	-0.606E-09
	-0.348E-09	-0.244E-09	-0.153D-09	-0.102D-09	-0.649D-10	-0.415D-10	-0.265D-10	-0.169E-10	-0.108E-10	-0.632E-11
0.850E 01	-0.117E-03	-0.991D-04	-0.493D-04	-0.277D-04	-0.173D-04	-0.114D-04	-0.781D-05	-0.553D-05	-0.401D-05	-0.294D-05
	-0.218E-05	-0.161D-05	-0.122D-05	-0.497D-06	-0.674E-06	-0.499D-06	-0.360D-06	-0.260D-06	-0.195D-06	-0.140D-06
	-0.140E-05	-0.713D-07	-0.543D-07	-0.352D-07	-0.245E-07	-0.170D-07	-0.106D-07	-0.406D-08	-0.551E-08	-0.375E-08
	-0.255E-08	-0.173E-08	-0.117D-08	-0.786D-09	-0.528D-09	-0.354D-09	-0.237D-09	-0.158D-09	-0.106D-09	-0.704E-10

0.175E 02	-0.113D-03	-0.963D-04	-0.502D-04	-0.302D-04	-0.196D-04	-0.135D-04	-0.067D-05	-0.713D-05	-0.536D-05	-0.410D-05
	-0.317E-05	-0.247E-05	-0.194D-05	-0.152D-05	-0.120D-05	-0.943D-06	-0.741D-06	-0.581D-06	-0.455D-06	-0.354D-06
	-0.275E-06	-0.213D-06	-0.164D-06	-0.125D-06	-0.095D-06	-0.728D-07	-0.551D-07	-0.416D-07	-0.313D-07	-0.239D-07
	-0.175E-07	-0.131E-07	-0.097D-07	-0.722E-08	-0.335D-08	-0.395D-08	-0.292D-08	-0.215D-08	-0.159E-08	-0.117D-08
0.185E 02	-0.109D-03	-0.935D-04	-0.509D-04	-0.320D-04	-0.217D-04	-0.155D-04	-0.115D-04	-0.872D-05	-0.676D-05	-0.522D-05
	-0.423E-05	-0.347D-05	-0.275D-05	-0.223E-05	-0.162D-05	-0.149D-05	-0.121D-05	-0.943D-06	-0.812D-06	-0.662E-06
	-0.539E-06	-0.438D-06	-0.355D-06	-0.286D-06	-0.231E-06	-0.195D-06	-0.148D-06	-0.110D-06	-0.933D-07	-0.737D-07
	-0.581E-07	-0.453D-07	-0.357E-07	-0.279D-07	-0.217D-07	-0.164D-07	-0.130D-07	-0.104D-07	-0.775D-08	-0.596D-08
0.250E 02	-0.106D-03	-0.932D-04	-0.511D-04	-0.337D-04	-0.233D-04	-0.171D-04	-0.130D-04	-0.101D-04	-0.802D-05	-0.645D-05
	-0.525D-05	-0.431E-05	-0.357D-05	-0.297D-05	-0.249D-05	-0.200D-05	-0.155D-05	-0.147D-05	-0.124D-05	-0.104D-05
	-0.877D-06	-0.734D-06	-0.621D-06	-0.521D-06	-0.437E-06	-0.365D-06	-0.305D-06	-0.254D-06	-0.211D-06	-0.175D-06
	-0.145E-06	-0.119E-06	-0.981E-07	-0.805E-07	-0.658D-07	-0.537D-07	-0.446D-07	-0.356D-07	-0.299D-07	-0.234D-07
0.335E 02	-0.104D-03	-0.892D-04	-0.510D-04	-0.340D-04	-0.244D-04	-0.184D-04	-0.143D-04	-0.113D-04	-0.915D-05	-0.753D-05
	-0.621E-05	-0.537E-05	-0.436D-05	-0.369D-05	-0.314D-05	-0.260D-05	-0.229D-05	-0.197E-05	-0.169D-05	-0.145E-05
	-0.125E-05	-0.103D-05	-0.828D-06	-0.749D-06	-0.648D-06	-0.591D-06	-0.508D-06	-0.436D-06	-0.373D-06	-0.319D-06
	-0.272E-06	-0.231E-06	-0.197D-06	-0.167D-06	-0.142D-06	-0.120D-06	-0.101E-06	-0.849D-07	-0.713D-07	-0.599D-07
0.435E 02	-0.102E-03	-0.876D-04	-0.509D-04	-0.345D-04	-0.252D-04	-0.193D-04	-0.152D-04	-0.123D-04	-0.101D-04	-0.838D-05
	-0.734D-05	-0.596D-05	-0.509D-05	-0.436D-05	-0.376D-05	-0.325D-05	-0.281D-05	-0.240D-05	-0.215D-05	-0.189D-05
	-0.144D-05	-0.144D-05	-0.126D-05	-0.110E-05	-0.978E-06	-0.844D-06	-0.744D-06	-0.651D-06	-0.579D-06	-0.519D-06
	-0.436E-06	-0.380E-06	-0.333E-06	-0.293D-06	-0.251D-06	-0.218D-06	-0.190D-06	-0.164D-06	-0.142D-06	-0.121D-06
0.535E 02	-0.598E-04	-0.862D-04	-0.505D-04	-0.347D-04	-0.258D-04	-0.200D-04	-0.160D-04	-0.131D-04	-0.109D-04	-0.914D-05
	-0.777D-05	-0.666E-05	-0.574D-05	-0.481D-05	-0.434D-05	-0.390D-05	-0.337D-05	-0.294D-05	-0.259D-05	-0.229D-05
	-0.203E-05	-0.189D-05	-0.169D-05	-0.149D-05	-0.129D-05	-0.113D-05	-0.977D-06	-0.846E-06	-0.748D-06	-0.701D-06
	-0.633E-06	-0.563D-06	-0.491E-06	-0.435D-06	-0.386D-06	-0.342D-06	-0.303D-06	-0.268D-06	-0.237D-06	-0.210D-06
0.635E 02	-0.986D-04	-0.852D-04	-0.502D-04	-0.348D-04	-0.261D-04	-0.205D-04	-0.165D-04	-0.136D-04	-0.114D-04	-0.971D-05
	-0.172E-05	-0.719E-05	-0.625D-05	-0.547D-05	-0.481D-05	-0.428D-05	-0.375D-05	-0.331D-05	-0.297D-05	-0.265D-05
	-0.236E-05	-0.211D-05	-0.193D-05	-0.170D-05	-0.152D-05	-0.137D-05	-0.123D-05	-0.110D-05	-0.992D-06	-0.892D-06
	-0.892E-06	-0.721E-06	-0.642E-06	-0.572E-06	-0.521D-06	-0.469D-06	-0.421D-06	-0.378D-06	-0.349D-06	-0.305D-06
0.735E 02	-0.976D-04	-0.844D-04	-0.491D-04	-0.340D-04	-0.263D-04	-0.208D-04	-0.169D-04	-0.140D-04	-0.119D-04	-0.101D-04
	-0.875E-05	-0.761E-05	-0.667D-05	-0.597D-05	-0.519D-05	-0.465D-05	-0.411D-05	-0.367D-05	-0.329D-05	-0.293D-05
	-0.265E-05	-0.239D-05	-0.216D-05	-0.195D-05	-0.176E-05	-0.159D-05	-0.144D-05	-0.130D-05	-0.118D-05	-0.107D-05
	-0.970E-06	-0.879E-06	-0.798D-06	-0.723D-06	-0.656D-06	-0.595D-06	-0.539D-06	-0.489D-06	-0.444D-06	-0.403D-06

FACTION OF GAS PRODUCED AT LAST FIVE TIME LEVELS

FRACTION OF GAS PRODUCED= 0.390E-05 0.497E-05 0.602E-05 0.705E-05 0.807E-05



SLIP COEFF(B)= 10.000 INERTIAL COEFF(HU)= 0.1000 0? PERMEABILITY(K)= 500.000 MD

FLUX DISTRIBUTION FOR THE FIRST 40 GRID POINTS

0.250E 00	-0.729D-04	-0.759D-04	-0.268D-04	-0.959D-05	-0.316D-05	-0.943D-06	-0.202D-06	-0.710D-07	-0.192D-07	-0.520D-08
	-0.142D-08	-0.388D-09	-0.107D-09	-0.293D-10	-0.816D-11	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.500E 00	-0.132D-03	-0.110D-03	-0.345D-04	-0.151D-04	-0.589D-05	-0.230D-05	-0.667D-06	-0.309D-06	-0.104D-06	-0.341D-07
	-0.107D-07	-0.340D-08	-0.105D-08	-0.324D-09	-0.986D-10	-0.294D-10	-0.900D-11	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.100E 01	-0.131D-03	-0.109D-03	-0.409D-04	-0.170D-04	-0.759D-05	-0.357D-05	-0.173D-05	-0.424D-06	-0.236D-06	-0.162D-06
	-0.721D-07	-0.202D-07	-0.124D-07	-0.505D-08	-0.203D-08	-0.814D-09	-0.324D-09	-0.129D-09	-0.510D-10	-0.292D-10
	-0.798D-11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.150E 01	-0.129D-03	-0.107D-03	-0.424D-04	-0.188D-04	-0.914D-05	-0.477D-05	-0.260D-05	-0.144D-05	-0.783D-06	-0.414D-06
	-0.212D-06	-0.178D-06	-0.502D-07	-0.235D-07	-0.100D-07	-0.487D-08	-0.217D-08	-0.956D-09	-0.417D-09	-0.189D-09
	-0.776D-10	-0.332D-10	-0.141D-10	-0.598D-11	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.250E 01	-0.127D-03	-0.106D-03	-0.442D-04	-0.210D-04	-0.110D-04	-0.620D-05	-0.367D-05	-0.223D-05	-0.136D-05	-0.831D-06
	-0.599D-06	-0.295D-06	-0.179D-06	-0.965D-07	-0.538D-07	-0.294D-07	-0.160D-07	-0.655D-08	-0.454D-08	-0.237D-08
	-0.125D-08	-0.653D-09	-0.340D-09	-0.176D-09	-0.872D-10	-0.470D-10	-0.242D-10	-0.125D-10	-0.641D-11	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.350E 01	-0.124D-03	-0.104D-03	-0.459D-04	-0.232D-04	-0.129D-04	-0.773D-05	-0.484D-05	-0.313D-05	-0.205D-05	-0.135D-05
	-0.390D-06	-0.541D-06	-0.374D-06	-0.237D-06	-0.148D-06	-0.908D-07	-0.541D-07	-0.326D-07	-0.132D-07	-0.111D-07
	-0.340D-04	-0.364D-04	-0.206D-04	-0.116D-04	-0.645D-05	-0.358D-05	-0.198D-05	-0.109D-05	-0.587D-06	-0.225D-06
	-0.177D-10	-0.493D-11	-0.513D-11	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.600E 01	-0.120D-03	-0.101D-03	-0.478D-04	-0.259D-04	-0.153D-04	-0.972D-05	-0.645D-05	-0.442D-05	-0.309D-05	-0.219D-05
	-0.156D-05	-0.111D-05	-0.753D-06	-0.562D-06	-0.396D-06	-0.277D-06	-0.193D-06	-0.133D-06	-0.965D-07	-0.614D-07
	-0.414D-07	-0.277D-07	-0.185D-07	-0.123D-07	-0.910D-08	-0.534D-08	-0.351D-08	-0.230D-08	-0.151D-08	-0.878D-09
	-0.644D-09	-0.420D-09	-0.274D-09	-0.174D-09	-0.116D-09	-0.759D-10	-0.493D-10	-0.321D-10	-0.269D-10	-0.136D-10
0.850E 01	-0.119D-03	-0.986D-04	-0.694D-04	-0.284D-04	-0.174D-04	-0.119D-04	-0.652D-05	-0.456D-05	-0.429D-05	-0.318D-05
	-0.399D-05	-0.190D-05	-0.136D-05	-0.103D-05	-0.774D-06	-0.592D-06	-0.439D-06	-0.328D-06	-0.239D-06	-0.174D-06
	-0.128D-05	-0.924D-07	-0.668D-07	-0.476D-07	-0.338D-07	-0.239D-07	-0.168D-07	-0.118D-07	-0.820D-08	-0.572D-08
	-0.296D-09	-0.273D-09	-0.168D-09	-0.124D-09	-0.846D-09	-0.606D-09	-0.414D-09	-0.282D-09	-0.192D-09	-0.130D-09

0.135E 02	-0.112U-03	-0.957U-04	-0.504U-04	-0.306U-04	-0.201U-04	-0.140U-04	-0.101U-04	-0.742U-05	-0.568U-05	-0.434U-05
	-0.441U-03	-0.258U-05	-0.168U-05	-0.133U-05	-0.106U-05	-0.843U-06	-0.690U-06	-0.529U-06	-0.410U-06	-0.321U-07
	-0.320U-06	-0.258U-06	-0.201U-06	-0.157U-06	-0.121U-06	-0.936U-07	-0.720U-07	-0.552U-07	-0.424U-07	-0.315U-08
	-0.243U-07	-0.184U-07	-0.139U-07	-0.105U-07	-0.781U-08	-0.592U-08	-0.444U-08	-0.320U-08	-0.224U-08	-0.165U-08
0.185E 02	-0.109U-03	-0.930U-04	-0.510U-04	-0.324U-04	-0.222U-04	-0.152U-04	-0.119U-04	-0.909U-05	-0.709U-05	-0.561U-05
	-0.441U-03	-0.263U-05	-0.166U-05	-0.124U-05	-0.140U-05	-0.130U-05	-0.111U-05	-0.916U-06	-0.755U-06	-0.610U-06
	-0.321U-06	-0.258U-06	-0.201U-06	-0.157U-06	-0.121U-06	-0.936U-07	-0.720U-07	-0.552U-07	-0.424U-07	-0.315U-08
	-0.243U-07	-0.184U-07	-0.139U-07	-0.105U-07	-0.781U-08	-0.592U-08	-0.444U-08	-0.320U-08	-0.224U-08	-0.165U-08
0.260E 02	-0.104U-03	-0.707U-04	-0.511U-04	-0.335U-04	-0.233U-04	-0.173U-04	-0.134U-04	-0.105U-04	-0.833U-05	-0.627U-05
	-0.441U-03	-0.263U-05	-0.166U-05	-0.124U-05	-0.140U-05	-0.130U-05	-0.110U-05	-0.916U-06	-0.755U-06	-0.610U-06
	-0.321U-06	-0.258U-06	-0.201U-06	-0.157U-06	-0.121U-06	-0.936U-07	-0.720U-07	-0.552U-07	-0.424U-07	-0.315U-08
	-0.243U-07	-0.184U-07	-0.139U-07	-0.105U-07	-0.781U-08	-0.592U-08	-0.444U-08	-0.320U-08	-0.224U-08	-0.165U-08
0.335E 02	-0.101U-03	-0.897U-04	-0.510U-04	-0.342U-04	-0.247U-04	-0.187U-04	-0.146U-04	-0.110U-04	-0.946U-05	-0.774U-05
	-0.441U-03	-0.263U-05	-0.166U-05	-0.124U-05	-0.140U-05	-0.130U-05	-0.110U-05	-0.916U-06	-0.755U-06	-0.610U-06
	-0.321U-06	-0.258U-06	-0.201U-06	-0.157U-06	-0.121U-06	-0.936U-07	-0.720U-07	-0.552U-07	-0.424U-07	-0.315U-08
	-0.243U-07	-0.184U-07	-0.139U-07	-0.105U-07	-0.781U-08	-0.592U-08	-0.444U-08	-0.320U-08	-0.224U-08	-0.165U-08
0.435E 02	-0.101U-03	-0.872U-04	-0.567U-04	-0.346U-04	-0.254U-04	-0.194U-04	-0.155U-04	-0.126U-04	-0.104U-04	-0.865U-05
	-0.441U-03	-0.263U-05	-0.166U-05	-0.124U-05	-0.140U-05	-0.130U-05	-0.110U-05	-0.916U-06	-0.755U-06	-0.610U-06
	-0.321U-06	-0.258U-06	-0.201U-06	-0.157U-06	-0.121U-06	-0.936U-07	-0.720U-07	-0.552U-07	-0.424U-07	-0.315U-08
	-0.243U-07	-0.184U-07	-0.139U-07	-0.105U-07	-0.781U-08	-0.592U-08	-0.444U-08	-0.320U-08	-0.224U-08	-0.165U-08
0.515E 02	-0.993U-04	-0.859U-04	-0.564U-04	-0.344U-04	-0.259U-04	-0.202U-04	-0.162U-04	-0.131U-04	-0.111U-04	-0.943U-05
	-0.441U-03	-0.263U-05	-0.166U-05	-0.124U-05	-0.140U-05	-0.130U-05	-0.110U-05	-0.916U-06	-0.755U-06	-0.610U-06
	-0.321U-06	-0.258U-06	-0.201U-06	-0.157U-06	-0.121U-06	-0.936U-07	-0.720U-07	-0.552U-07	-0.424U-07	-0.315U-08
	-0.243U-07	-0.184U-07	-0.139U-07	-0.105U-07	-0.781U-08	-0.592U-08	-0.444U-08	-0.320U-08	-0.224U-08	-0.165U-08
0.635E 02	-0.911U-04	-0.849U-04	-0.561U-04	-0.348U-04	-0.262U-04	-0.206U-04	-0.167U-04	-0.134U-04	-0.116U-04	-0.937U-05
	-0.441U-03	-0.263U-05	-0.166U-05	-0.124U-05	-0.140U-05	-0.130U-05	-0.110U-05	-0.916U-06	-0.755U-06	-0.610U-06
	-0.321U-06	-0.258U-06	-0.201U-06	-0.157U-06	-0.121U-06	-0.936U-07	-0.720U-07	-0.552U-07	-0.424U-07	-0.315U-08
	-0.243U-07	-0.184U-07	-0.139U-07	-0.105U-07	-0.781U-08	-0.592U-08	-0.444U-08	-0.320U-08	-0.224U-08	-0.165U-08

FRACTION OF GAS PRODUCED AT LAST FIVE TIME LEVELS

0.388E-05    0.494E-05    0.700E-05    0.802E-05





SLIT COEFF (B) = 10.000 INERTIAL COEFF (ND) = 0.100D 09 PERMEABILITY (K) = 500.000 MD

FLUX DISTRIBUTION FOR THE FIRST 40 GRID POINTS

0.250E 00	-0.923E-04	-0.712E-04	-0.262E-04	-0.002E-05	-0.206E-05	-0.809E-06	-0.214E-06	-0.552E-07	-0.142E-07	-0.369E-08
	-0.529E-09	-0.251E-09	-0.059E-10	-0.179E-10	-0.460E-11	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.500E 00	-0.133E-03	-0.110E-03	-0.386E-04	-0.140E-04	-0.542E-05	-0.204E-05	-0.739E-06	-0.251E-06	-0.807E-07	-0.251E-07
	-0.763E-08	-0.229E-08	-0.674E-09	-0.198E-09	-0.575E-10	-0.166E-10	-0.678E-11	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.100E 01	-0.122E-03	-0.119E-03	-0.410E-04	-0.162E-04	-0.705E-05	-0.326E-05	-0.153E-05	-0.708E-06	-0.314E-06	-0.134E-06
	-0.550E-07	-0.221E-07	-0.976E-08	-0.393E-08	-0.133E-08	-0.512E-09	-0.197E-09	-0.753E-10	-0.248E-10	-0.110E-10
	-0.414E-11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.150E 01	-0.130E-03	-0.148E-03	-0.517E-04	-0.191E-04	-0.861E-05	-0.442E-05	-0.236E-05	-0.127E-05	-0.624E-06	-0.345E-06
	-0.170E-06	-0.441E-07	-0.174E-07	-0.105E-07	-0.744E-08	-0.323E-08	-0.139E-08	-0.583E-09	-0.247E-09	-0.103E-09
	-0.428E-10	-0.176E-10	-0.724E-11	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.250E 01	-0.127E-03	-0.165E-03	-0.430E-04	-0.202E-04	-0.108E-04	-0.580E-05	-0.318E-05	-0.202E-05	-0.121E-05	-0.722E-06
	-0.423E-06	-0.243E-06	-0.139E-06	-0.746E-07	-0.405E-07	-0.215E-07	-0.113E-07	-0.591E-08	-0.303E-08	-0.155E-08
	-0.790E-09	-0.400E-09	-0.202E-09	-0.102E-09	-0.512E-10	-0.257E-10	-0.129E-10	-0.646E-11	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.350E 01	-0.125E-03	-0.169E-03	-0.453E-04	-0.235E-04	-0.124E-04	-0.729E-05	-0.451E-05	-0.288E-05	-0.180E-05	-0.121E-05
	-0.774E-06	-0.457E-06	-0.213E-06	-0.134E-06	-0.117E-06	-0.700E-07	-0.411E-07	-0.234E-07	-0.136E-07	-0.764E-08
	-0.428E-10	-0.237E-10	-0.140E-10	-0.710E-11	-0.405E-11	-0.205E-11	-0.112E-11	-0.547E-12	-0.318E-12	-0.163E-12
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.600E 01	-0.121E-03	-0.162E-03	-0.473E-04	-0.252E-04	-0.148E-04	-0.925E-05	-0.608E-05	-0.412E-05	-0.295E-05	-0.199E-05
	-0.140E-05	-0.561E-06	-0.654E-06	-0.464E-06	-0.335E-06	-0.236E-06	-0.157E-06	-0.106E-06	-0.708E-07	-0.471E-07
	-0.311E-07	-0.264E-07	-0.133E-07	-0.678E-08	-0.362E-08	-0.234E-08	-0.151E-08	-0.959E-09	-0.622E-09	-0.422E-09
	-0.398E-09	-0.258E-09	-0.164E-09	-0.105E-09	-0.671E-10	-0.429E-10	-0.274E-10	-0.176E-10	-0.112E-10	-0.712E-11
0.850E 01	-0.117E-03	-0.162E-03	-0.491E-04	-0.279E-04	-0.173E-04	-0.110E-04	-0.705E-05	-0.550E-05	-0.401E-05	-0.242E-05
	-0.214E-05	-0.163E-05	-0.122E-05	-0.911E-06	-0.678E-06	-0.503E-06	-0.370E-06	-0.271E-06	-0.192E-06	-0.142E-06
	-0.105E-06	-0.221E-07	-0.519E-07	-0.198E-07	-0.240E-07	-0.173E-07	-0.120E-07	-0.822E-08	-0.563E-08	-0.384E-08
	-0.261E-08	-0.177E-08	-0.120E-08	-0.868E-09	-0.543E-09	-0.465E-09	-0.345E-09	-0.261E-09	-0.199E-09	-0.142E-09

0.135E 02	-0.113E-03	-0.964E-04	-0.592E-04	-0.302E-04	-0.197E-04	-0.135E-04	-0.968E-05	-0.714E-05	-0.537E-05	-0.411E-05
	-0.318E-05	-0.248E-05	-0.194E-05	-0.153E-05	-0.120E-05	-0.947E-06	-0.745E-06	-0.585E-06	-0.450E-06	-0.357E-06
	-0.277E-06	-0.215E-06	-0.165E-06	-0.127E-06	-0.969E-07	-0.737E-07	-0.559E-07	-0.422E-07	-0.319E-07	-0.231E-07
	-0.178E-07	-0.133E-07	-0.992E-08	-0.737E-08	-0.546E-08	-0.404E-08	-0.299E-08	-0.221E-08	-0.153E-08	-0.120E-08
0.1E5E 02	-0.109E-03	-0.935E-04	-0.509E-04	-0.320E-04	-0.210E-04	-0.150E-04	-0.115E-04	-0.873E-05	-0.676E-05	-0.532E-05
	-0.424E-05	-0.340E-05	-0.275E-05	-0.224E-05	-0.182E-05	-0.149E-05	-0.122E-05	-0.984E-06	-0.816E-06	-0.664E-06
	-0.542E-06	-0.441E-06	-0.357E-06	-0.285E-06	-0.233E-06	-0.190E-06	-0.149E-06	-0.119E-06	-0.944E-07	-0.747E-07
	-0.555E-07	-0.463E-07	-0.362E-07	-0.283E-07	-0.220E-07	-0.171E-07	-0.133E-07	-0.103E-07	-0.790E-08	-0.608E-08
0.260E 02	-0.106E-03	-0.912E-04	-0.511E-04	-0.332E-04	-0.231E-04	-0.171E-04	-0.135E-04	-0.101E-04	-0.793E-05	-0.645E-05
	-0.526E-05	-0.432E-05	-0.357E-05	-0.297E-05	-0.249E-05	-0.200E-05	-0.175E-05	-0.147E-05	-0.124E-05	-0.105E-05
	-0.691E-06	-0.562E-06	-0.454E-06	-0.360E-06	-0.290E-06	-0.230E-06	-0.187E-06	-0.151E-06	-0.123E-06	-0.101E-06
	-0.141E-06	-0.121E-06	-0.993E-07	-0.814E-07	-0.667E-07	-0.545E-07	-0.444E-07	-0.361E-07	-0.293E-07	-0.237E-07
0.355E 02	-0.104E-03	-0.852E-04	-0.510E-04	-0.340E-04	-0.245E-04	-0.180E-04	-0.143E-04	-0.113E-04	-0.916E-05	-0.757E-05
	-0.621E-05	-0.519E-05	-0.437E-05	-0.370E-05	-0.314E-05	-0.268E-05	-0.230E-05	-0.197E-05	-0.169E-05	-0.144E-05
	-0.176E-05	-0.140E-05	-0.110E-05	-0.090E-05	-0.071E-05	-0.059E-05	-0.051E-05	-0.043E-05	-0.037E-05	-0.032E-05
	-0.274E-06	-0.234E-06	-0.199E-06	-0.169E-06	-0.143E-06	-0.121E-06	-0.102E-06	-0.085E-06	-0.072E-06	-0.060E-06
0.435E 02	-0.102E-03	-0.876E-04	-0.508E-04	-0.345E-04	-0.252E-04	-0.193E-04	-0.157E-04	-0.123E-04	-0.101E-04	-0.839E-05
	-0.703E-05	-0.577E-05	-0.493E-05	-0.437E-05	-0.377E-05	-0.324E-05	-0.281E-05	-0.247E-05	-0.215E-05	-0.184E-05
	-0.165E-05	-0.144E-05	-0.126E-05	-0.117E-05	-0.972E-06	-0.852E-06	-0.747E-06	-0.655E-06	-0.573E-06	-0.502E-06
	-0.418E-06	-0.383E-06	-0.334E-06	-0.291E-06	-0.253E-06	-0.220E-06	-0.191E-06	-0.166E-06	-0.144E-06	-0.124E-06
0.535E 02	-0.998E-04	-0.803E-04	-0.505E-04	-0.347E-04	-0.248E-04	-0.200E-04	-0.160E-04	-0.131E-04	-0.109E-04	-0.915E-05
	-0.777E-05	-0.668E-05	-0.575E-05	-0.490E-05	-0.415E-05	-0.360E-05	-0.314E-05	-0.294E-05	-0.260E-05	-0.230E-05
	-0.293E-05	-0.180E-05	-0.160E-05	-0.142E-05	-0.126E-05	-0.112E-05	-0.100E-05	-0.090E-05	-0.081E-05	-0.074E-05
	-0.626E-06	-0.556E-06	-0.494E-06	-0.439E-06	-0.395E-06	-0.344E-06	-0.305E-06	-0.270E-06	-0.239E-06	-0.212E-06
0.635E 02	-0.920E-04	-0.752E-04	-0.502E-04	-0.348E-04	-0.241E-04	-0.205E-04	-0.165E-04	-0.136E-04	-0.114E-04	-0.971E-05
	-0.633E-05	-0.540E-05	-0.468E-05	-0.405E-05	-0.342E-05	-0.297E-05	-0.276E-05	-0.247E-05	-0.227E-05	-0.204E-05
	-0.237E-05	-0.212E-05	-0.190E-05	-0.170E-05	-0.153E-05	-0.137E-05	-0.123E-05	-0.110E-05	-0.996E-06	-0.955E-06
	-0.565E-06	-0.524E-06	-0.481E-06	-0.438E-06	-0.395E-06	-0.342E-06	-0.319E-06	-0.281E-06	-0.242E-06	-0.207E-06
0.735E 07	-0.576E-04	-0.485E-04	-0.349E-04	-0.263E-04	-0.203E-04	-0.169E-04	-0.143E-04	-0.119E-04	-0.100E-04	-0.101E-04
	-0.671E-05	-0.572E-05	-0.467E-05	-0.380E-05	-0.320E-05	-0.267E-05	-0.229E-05	-0.197E-05	-0.167E-05	-0.144E-05
	-0.266E-05	-0.234E-05	-0.216E-05	-0.195E-05	-0.176E-05	-0.159E-05	-0.144E-05	-0.131E-05	-0.118E-05	-0.107E-05
	-0.974E-06	-0.883E-06	-0.801E-06	-0.726E-06	-0.659E-06	-0.598E-06	-0.542E-06	-0.492E-06	-0.447E-06	-0.405E-06

FRACTION OF GAS PRODUCED AT LAST FIVE TIME LEVELS

FRACTION OF GAS PRODUCED= 0.390E-05 0.457E-05 0.602E-05 0.705E-05 0.807E-05

J.2

SOLUTIONS OBTAINED FOR A PERMEABILITY VALUE  
OF 10.0 MILLIDARCIES





0.106E 03	-0.3090-04	-0.2510-04	-0.1430-04	-0.9250-05	-0.6150-05	-0.4070-05	-0.2630-05	-0.1640-05	-0.9860-06	-0.5700-06
	-0.3180-06	-0.1720-06	-0.8990-07	-0.4600-07	-0.2300-07	-0.1130-07	-0.5460-08	-0.2600-08	-0.1230-08	-0.5720-09
	-0.2640-09	-0.1210-09	-0.5510-10	-0.2500-10	-0.1120-10	-0.5030-11	-0.2250-11	-0.0900-12	-0.4420-12	-0.1950-12
0.143E 03	-0.8610-13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	-0.2980-04	-0.2340-04	-0.1360-04	-0.9000-05	-0.6250-05	-0.4380-05	-0.3040-05	-0.2070-05	-0.1380-05	-0.6890-06
	-0.5580-06	-0.3390-06	-0.2010-06	-0.1160-06	-0.6510-07	-0.3590-07	-0.1940-07	-0.1030-07	-0.5400-08	-0.2790-08
	-0.1430-08	-0.7200-09	-0.3610-09	-0.1750-09	-0.8830-10	-0.4320-10	-0.2100-10	-0.1020-10	-0.4900-11	-0.2350-11
0.163E 03	-0.1120-11	-0.5350-12	-0.2540-12	-0.1200-12	0.0	0.0	0.0	0.0	0.0	0.0
	-0.2710-04	-0.2210-04	-0.1290-04	-0.8740-05	-0.6240-05	-0.4530-05	-0.3100-05	-0.2380-05	-0.1690-05	-0.1180-05
	-0.8050-06	-0.5360-06	-0.3490-06	-0.2220-06	-0.1330-06	-0.8700-07	-0.5000-07	-0.2930-07	-0.1680-07	-0.9450-08
	-0.5340-08	-0.2950-08	-0.1610-08	-0.8700-09	-0.4660-09	-0.2870-09	-0.1300-09	-0.6810-10	-0.3540-10	-0.1830-10
0.228E 03	-0.9400-11	-0.4810-11	-0.2450-11	-0.1240-11	-0.6270-12	-0.3160-12	-0.1580-12	0.0	0.0	0.0
	-0.2570-04	-0.2110-04	-0.1240-04	-0.8470-05	-0.6160-05	-0.4590-05	-0.3450-05	-0.2590-05	-0.1930-05	-0.1420-05
	-0.1030-05	-0.7320-06	-0.4110-06	-0.3000-06	-0.2360-06	-0.1550-06	-0.1010-06	-0.6400-07	-0.4000-07	-0.2460-07
	-0.1490-07	-0.8920-08	-0.5270-08	-0.3080-08	-0.1780-08	-0.1020-08	-0.5750-09	-0.3230-09	-0.1800-09	-0.9970-10
0.278E 03	-0.5480-10	-0.2990-10	-0.1630-10	-0.8800-11	-0.4730-11	-0.2540-11	-0.1380-11	-0.7190-12	-0.3800-12	-0.2010-12
	-0.2460-04	-0.2020-04	-0.1190-04	-0.8220-05	-0.6040-05	-0.4600-05	-0.3540-05	-0.2730-05	-0.2100-05	-0.1610-05
	-0.1220-05	-0.9070-06	-0.6680-06	-0.4850-06	-0.3460-06	-0.2440-06	-0.1680-06	-0.1150-06	-0.7690-07	-0.5070-07
	-0.3300-07	-0.2120-07	-0.1340-07	-0.8390-08	-0.5190-08	-0.3170-08	-0.1920-08	-0.1150-08	-0.6850-09	-0.4040-09
0.338E 03	-0.2370-09	-0.1380-09	-0.7540-10	-0.4560-10	-0.2600-10	-0.1480-10	-0.8330-11	-0.4680-11	-0.2620-11	-0.1460-11
	-0.2370-04	-0.1940-04	-0.1150-04	-0.7990-05	-0.5940-05	-0.4580-05	-0.3590-05	-0.2830-05	-0.2230-05	-0.1760-05
	-0.1370-05	-0.1060-05	-0.8160-06	-0.6190-06	-0.4640-06	-0.3430-06	-0.2510-06	-0.1810-06	-0.1280-06	-0.9010-07
	-0.6240-07	-0.4270-07	-0.2880-07	-0.1920-07	-0.1270-07	-0.8290-08	-0.5360-08	-0.3430-08	-0.2170-08	-0.1370-08
0.409E 03	-0.8530-09	-0.5240-09	-0.3240-09	-0.1980-09	-0.1260-09	-0.7240-10	-0.4340-10	-0.2590-10	-0.1540-10	-0.9060-11
	-0.2280-04	-0.1870-04	-0.1110-04	-0.7750-05	-0.5820-05	-0.4530-05	-0.3400-05	-0.2890-05	-0.2330-05	-0.1870-05
	-0.1500-05	-0.1200-05	-0.9510-06	-0.7470-06	-0.5820-06	-0.4490-06	-0.3430-06	-0.2690-06	-0.1930-06	-0.1430-06
	-0.1040-06	-0.7530-07	-0.5380-07	-0.3610-07	-0.2660-07	-0.1840-07	-0.1260-07	-0.6980-08	-0.45780-08	-0.3050-08
0.489E 03	-0.2530-08	-0.1670-08	-0.1090-08	-0.7060-09	-0.4340-09	-0.3090-09	-0.1840-09	-0.1160-09	-0.7310-10	-0.4570-10
	-0.2200-04	-0.1810-04	-0.1080-04	-0.7540-05	-0.5690-05	-0.4470-05	-0.3590-05	-0.2920-05	-0.2390-05	-0.1460-05
	-0.1610-05	-0.1310-05	-0.1670-05	-0.8630-06	-0.6530-06	-0.5530-06	-0.4370-06	-0.3430-06	-0.2660-06	-0.2050-06
	-0.1570-06	-0.1180-06	-0.8860-07	-0.6570-07	-0.4830-07	-0.3520-07	-0.2540-07	-0.1820-07	-0.1290-07	-0.9070-08
	-0.6330-08	-0.4340-08	-0.3010-08	-0.2060-08	-0.1400-08	-0.9400-09	-0.6290-09	-0.4190-09	-0.2770-09	-0.1820-09

FRACTION OF GAS PRODUCED AT LAST FIVE TIME LEVELS

FRACTION OF GAS PRODUCED= 0.842E-05 0.973U-05 0.112E-04 0.129E-04 0.148E-04







0.108E 03	-0.3150-04	-0.2550-04	-0.1450-04	-0.9300-08	-0.6110-05	-0.3970-05	-0.2510-05	-0.1520-05	-0.8680-06	-0.4970-06
	-0.2670-06	-0.1390-06	-0.7030-07	-0.3460-07	-0.1670-07	-0.7910-08	-0.3680-08	-0.1690-08	-0.7690-09	-0.3460-09
	-0.1540-09	-0.6820-10	-0.3000-10	-0.1310-10	-0.8590-11	-0.2460-11	-0.1060-11	-0.4850-12	-0.1950-12	-0.8310-13
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.143E 03	-0.2930-04	-0.2380-04	-0.1380-04	-0.9080-08	-0.6250-05	-0.4320-05	-0.2950-05	-0.1970-05	-0.1280-05	-0.6040-06
	-0.4900-06	-0.2890-06	-0.1650-06	-0.8220-07	-0.5010-07	-0.2670-07	-0.1390-08	-0.7150-08	-0.3620-08	-0.1810-08
	-0.6910-09	-0.4350-09	-0.2110-09	-0.1010-09	-0.4820-10	-0.2280-10	-0.1070-10	-0.5020-11	-0.2340-11	-0.1090-11
	-0.5020-12	-0.2310-12	-0.1050-12	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.183E 03	-0.2750-04	-0.2250-04	-0.1310-04	-0.8020-08	-0.6250-05	-0.4390-05	-0.3240-05	-0.2300-05	-0.1610-05	-0.1100-05
	-0.7310-06	-0.4740-06	-0.3000-06	-0.1850-06	-0.1120-06	-0.6580-07	-0.3800-07	-0.2150-07	-0.1200-07	-0.6580-08
	-0.3360-08	-0.1900-08	-0.1010-08	-0.5240-09	-0.2720-09	-0.1400-09	-0.7140-10	-0.3620-10	-0.1820-10	-0.9110-11
	-0.4530-11	-0.2250-11	-0.1110-11	-0.5440-12	-0.2660-12	-0.1300-12	0.0	0.0	0.0	0.0
0.228E 03	-0.2610-04	-0.2140-04	-0.1260-04	-0.8860-05	-0.6190-05	-0.4590-05	-0.3410-05	-0.2530-05	-0.1660-05	-0.1340-05
	-0.9550-06	-0.6660-06	-0.4550-06	-0.3040-06	-0.1940-06	-0.1290-06	-0.3040-07	-0.4960-07	-0.3010-07	-0.1800-07
	-0.1060-07	-0.6120-08	-0.3510-08	-0.1980-08	-0.1110-08	-0.6150-09	-0.3380-09	-0.1840-09	-0.9940-10	-0.5330-10
	-0.2840-10	-0.1510-10	-0.7940-11	-0.4160-11	-0.2170-11	-0.5640-12	-0.3010-12	-0.1550-12	0.0	0.0
0.278E 03	-0.2500-04	-0.2050-04	-0.1210-04	-0.8320-05	-0.6100-05	-0.4610-05	-0.3520-05	-0.2690-05	-0.2050-05	-0.1540-05
	-0.1130-05	-0.8440-06	-0.6100-06	-0.4330-06	-0.3030-06	-0.2080-06	-0.1400-06	-0.9300-07	-0.6070-07	-0.3900-07
	-0.2460-07	-0.1940-07	-0.9450-08	-0.5740-08	-0.3450-08	-0.2080-08	-0.1200-08	-0.7010-09	-0.4040-09	-0.2310-09
	-0.1320-09	-0.7420-10	-0.4160-10	-0.2320-10	-0.1280-10	-0.7070-11	-0.3880-11	-0.2120-11	-0.1150-11	-0.6220-12
0.338E 03	-0.2400-04	-0.1970-04	-0.1170-04	-0.8080-05	-0.5990-05	-0.4600-05	-0.3580-05	-0.2800-05	-0.2190-05	-0.1700-05
	-0.1320-05	-0.1000-05	-0.7690-06	-0.5560-06	-0.4160-06	-0.3020-06	-0.2160-06	-0.1520-06	-0.1060-06	-0.7250-07
	-0.4890-07	-0.3260-07	-0.2150-07	-0.1390-07	-0.8960-08	-0.5570-08	-0.3580-08	-0.2230-08	-0.1370-08	-0.6390-09
	-0.5090-09	-0.3060-09	-0.1830-09	-0.1090-09	-0.6410-10	-0.3760-10	-0.2190-10	-0.1270-10	-0.7330-11	-0.4210-11
0.409E 03	-0.2310-04	-0.1900-04	-0.1130-04	-0.7850-05	-0.5870-05	-0.4560-05	-0.3600-05	-0.2870-05	-0.2300-05	-0.1830-05
	-0.1400-05	-0.1150-05	-0.8990-06	-0.6970-06	-0.5340-06	-0.4050-06	-0.3040-06	-0.2250-06	-0.1650-06	-0.1190-06
	-0.6540-07	-0.6030-07	-0.4210-07	-0.2910-07	-0.1990-07	-0.1340-07	-0.8980-08	-0.5440-08	-0.3690-08	-0.2530-08
	-0.1630-08	-0.1440-08	-0.6620-09	-0.4170-09	-0.2610-09	-0.1630-09	-0.1010-09	-0.6180-10	-0.3760-10	-0.2300-10
0.485E 03	-0.2230-04	-0.1830-04	-0.1090-04	-0.7630-05	-0.5750-05	-0.4500-05	-0.3600-05	-0.2910-05	-0.2370-05	-0.1930-05
	-0.1570-05	-0.1270-05	-0.1020-05	-0.8160-06	-0.6470-06	-0.5090-06	-0.3970-06	-0.3060-06	-0.2340-06	-0.1770-06
	-0.1330-06	-0.9840-07	-0.7220-07	-0.5280-07	-0.3700-07	-0.2690-07	-0.1900-07	-0.1330-07	-0.9200-08	-0.6320-08
	-0.4310-08	-0.2910-08	-0.1550-08	-0.1300-08	-0.8610-09	-0.5660-09	-0.3700-09	-0.2400-09	-0.1550-09	-0.9940-10

FRACTION OF GAS PRODUCED AT LAST FIVE TIME LEVELS

FRACTION OF GAS PRODUCED= 0.856E-05 0.991E-05 0.114E-04 0.131E-04 0.150E-04





0.108E 03	-0.320D-04	-0.258D-04	-0.147D-04	-0.954D-05	-0.637D-05	-0.824D-05	-0.276D-05	-0.174D-05	-0.105D-05	-0.615D-06
	-0.347D-06	-0.190D-06	-0.160D-06	-0.510D-07	-0.262D-07	-0.130D-07	-0.635D-08	-0.368D-08	-0.140D-08	-0.607D-09
	-0.321D-06	-0.144D-06	-0.664D-10	-0.313D-10	-0.142D-10	-0.644D-11	-0.291D-11	-0.131D-11	-0.584D-12	-0.261D-12
	-0.116D-12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.143E 03	-0.297D-04	-0.241D-04	-0.146D-04	-0.646D-05	-0.495D-05	-0.317D-05	-0.213D-05	-0.146D-05	-0.947D-06	-0.329D-08
	-0.400D-06	-0.769D-06	-0.220D-06	-0.128D-06	-0.720D-07	-0.404D-07	-0.220D-07	-0.119D-07	-0.629D-08	-0.369D-11
	-0.170D-08	-0.064D-09	-0.430D-09	-0.220D-09	-0.109D-09	-0.541D-10	-0.266D-10	-0.130D-10	-0.632D-11	0.0
	-0.149D-11	-0.710D-12	-0.340D-12	-0.162D-12	0.0	0.0	0.0	0.0	0.0	0.0
0.183E 03	-0.260D-04	-0.227D-04	-0.130D-04	-0.644D-05	-0.495D-05	-0.317D-05	-0.213D-05	-0.146D-05	-0.947D-06	-0.329D-08
	-0.857D-06	-0.574D-06	-0.378D-06	-0.128D-06	-0.720D-07	-0.404D-07	-0.220D-07	-0.119D-07	-0.629D-08	-0.369D-11
	-0.423D-08	-0.747D-08	-0.191D-08	-0.104D-08	-0.545D-09	-0.266D-09	-0.130D-09	-0.632D-10	0.0	0.0
	-0.121D-10	-0.423D-11	-0.320D-11	-0.162D-12	0.0	0.0	0.0	0.0	0.0	0.0
0.228E 03	-0.260D-04	-0.217D-04	-0.127D-04	-0.644D-05	-0.495D-05	-0.317D-05	-0.213D-05	-0.146D-05	-0.947D-06	-0.329D-08
	-0.171D-07	-0.103D-07	-0.616D-08	-0.302D-08	-0.144D-08	-0.711D-08	-0.311D-08	-0.174D-08	-0.850D-07	-0.240D-07
	-0.689D-10	-0.380D-10	-0.268D-10	-0.114D-10	-0.617D-11	-0.334D-11	-0.180D-11	-0.562D-12	-0.514D-12	-0.274E-12
0.276E 03	-0.254D-04	-0.208D-04	-0.123D-04	-0.644D-05	-0.495D-05	-0.317D-05	-0.213D-05	-0.146D-05	-0.947D-06	-0.329D-08
	-0.124D-05	-0.960D-05	-0.711D-05	-0.519D-06	-0.375D-06	-0.254D-06	-0.164D-06	-0.126D-06	-0.853D-07	-0.517E-07
	-0.372D-07	-0.241D-07	-0.154D-07	-0.971D-08	-0.635D-08	-0.374D-08	-0.228D-08	-0.134D-08	-0.820D-07	-0.473E-07
	-0.249D-10	-0.171D-09	-0.948D-10	-0.576D-10	-0.413D-10	-0.270D-10	-0.169D-10	-0.126D-10	-0.853E-09	-0.144D-11
0.338E 03	-0.244D-04	-0.200D-04	-0.118D-04	-0.644D-05	-0.495D-05	-0.317D-05	-0.213D-05	-0.146D-05	-0.947D-06	-0.329D-08
	-0.144D-05	-0.112D-05	-0.863D-06	-0.630D-06	-0.498D-06	-0.319D-06	-0.271D-06	-0.181D-06	-0.394E-07	-0.161E-06
	-0.004E-07	-0.478E-07	-0.324D-07	-0.219D-07	-0.144D-07	-0.948D-08	-0.625D-08	-0.443D-08	-0.294D-06	-0.161E-06
	-0.103D-08	-0.641D-09	-0.457D-09	-0.243D-09	-0.150D-09	-0.948D-10	-0.650D-10	-0.443D-10	-0.194D-08	-0.115D-10
0.409E 03	-0.335D-04	-0.192D-04	-0.114D-04	-0.794D-05	-0.600D-05	-0.406D-05	-0.372D-05	-0.249D-05	-0.242D-05	-0.147D-05
	-0.157D-05	-0.107D-05	-0.180D-05	-0.700D-06	-0.610D-06	-0.470D-06	-0.370D-06	-0.270D-06	-0.210D-06	-0.150D-06
	-0.115D-06	-0.833D-07	-0.595D-07	-0.420D-07	-0.300D-07	-0.200D-07	-0.145D-07	-0.970D-08	-0.671E-08	-0.442D-06
	-0.701D-08	-0.169D-08	-0.131D-08	-0.854D-09	-0.593D-09	-0.400D-09	-0.280D-09	-0.143D-09	-0.910D-10	-0.579D-08
0.489E 03	-0.327D-04	-0.184D-04	-0.111D-04	-0.770D-05	-0.587D-05	-0.401D-05	-0.371D-05	-0.249D-05	-0.242D-05	-0.147D-05
	-0.167D-05	-0.137D-05	-0.112D-05	-0.900D-06	-0.732D-06	-0.556D-06	-0.465D-06	-0.367D-06	-0.280D-06	-0.222D-06
	-0.117D-06	-0.129D-06	-0.574D-07	-0.427D-07	-0.330D-07	-0.240D-07	-0.170D-07	-0.200D-07	-0.147D-07	-0.100D-07
	-0.274D-08	-0.512D-08	-0.394D-08	-0.244D-08	-0.166D-08	-0.113D-08	-0.762D-09	-0.511D-09	-0.340D-09	-0.226D-09

FRACTION OF GAS PRODUCED AT LAST FIVE TIME LEVELS

FRACTION OF GAS PRODUCED= 0.071E-05 0.101E-04 0.116E-04 0.134E-04 0.153E-04





0.108E 03	-0.3250-04	-0.2620-04	-0.1490-04	-0.9600-05	-0.6340-05	-0.4140-05	-0.2640-05	-0.1620-05	-0.9520-06	-0.5380-06
	-0.2930-06	-0.1940-06	-0.7870-07	-0.3920-07	-0.1910-07	-0.9150-08	-0.4310-08	-0.2000-08	-0.9210-09	-0.4190-09
	-0.1890-09	-0.8440-10	-0.3750-10	-0.1560-10	-0.8720-11	-0.3180-11	-0.1380-11	-0.6010-12	-0.2600-12	-0.1120-12
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.145E 03	-0.3020-04	-0.2450-04	-0.1420-04	-0.9360-05	-0.6460-05	-0.4490-05	-0.3080-05	-0.2070-05	-0.1360-05	-0.8610-06
	-0.5290-09	-0.3150-06	-0.1820-06	-0.1030-06	-0.5640-07	-0.3030-07	-0.1600-07	-0.8310-08	-0.4250-08	-0.2140-08
	-0.1070-08	-0.5470-09	-0.2580-09	-0.1250-09	-0.6020-10	-0.2880-10	-0.1370-10	-0.6470-11	-0.3050-11	0.0
	-0.6670-12	-0.3100-12	-0.1440-12	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.183E 03	-0.2840-04	-0.2310-04	-0.1350-04	-0.9090-05	-0.6460-05	-0.4670-05	-0.3370-05	-0.2410-05	-0.1690-05	-0.1160-05
	-0.7810-06	-0.5110-06	-0.3260-06	-0.2030-06	-0.1240-06	-0.7360-07	-0.4290-07	-0.2460-07	-0.1380-07	-0.7660-08
	-0.4190-08	-0.2260-08	-0.1210-08	-0.6370-09	-0.3330-09	-0.1750-09	-0.8910-10	-0.4560-10	-0.2320-10	-0.1170-10
	-0.5890-11	-0.2940-11	-0.1470-11	-0.7270-12	-0.3600-12	-0.1770-12	-0.8700-13	0.0	0.0	0.0
0.228E 03	-0.2700-04	-0.2230-04	-0.1290-04	-0.8820-05	-0.6390-05	-0.4750-05	-0.3550-05	-0.2640-05	-0.1950-05	-0.1420-05
	-0.1010-05	-0.7110-06	-0.4890-06	-0.3300-06	-0.2180-06	-0.1410-06	-0.8090-07	-0.5570-07	-0.3410-07	-0.2050-07
	-0.1220-07	-0.7140-08	-0.4120-08	-0.2360-08	-0.1330-08	-0.7480-09	-0.4130-09	-0.2270-09	-0.1240-09	-0.6710-10
	-0.3610-10	-0.1930-10	-0.1030-10	-0.5440-11	-0.2860-11	-0.1500-11	-0.7850-12	-0.4080-12	-0.2120-12	-0.1090-12
0.278E 03	-0.2580-04	-0.2110-04	-0.1240-04	-0.8560-05	-0.6300-05	-0.4770-05	-0.3650-05	-0.2800-05	-0.2140-05	-0.1620-05
	-0.1210-05	-0.8950-06	-0.6510-06	-0.4680-06	-0.3270-06	-0.2260-06	-0.1540-06	-0.1030-06	-0.6770-07	-0.4390-07
	-0.2600-07	-0.1760-07	-0.1090-07	-0.6700-08	-0.4060-08	-0.2430-08	-0.1440-08	-0.8480-09	-0.4940-09	-0.2850-09
	-0.1640-09	-0.9310-10	-0.5270-10	-0.2960-10	-0.1660-10	-0.9200-11	-0.5090-11	-0.2800-11	-0.1540-11	-0.6390-12
0.338E 03	-0.2480-04	-0.2030-04	-0.1200-04	-0.8320-05	-0.6180-05	-0.4750-05	-0.3700-05	-0.2910-05	-0.2280-05	-0.1780-05
	-0.1380-05	-0.1060-05	-0.8040-06	-0.6030-06	-0.4460-06	-0.3260-06	-0.2350-06	-0.1670-06	-0.1170-06	-0.8040-07
	-0.9470-07	-0.3560-07	-0.2440-07	-0.1600-07	-0.1040-07	-0.6630-08	-0.4200-08	-0.2640-08	-0.1640-08	-0.1010-08
	-0.8190-09	-0.3760-09	-0.2270-09	-0.1360-09	-0.0070-10	-0.4770-10	-0.2810-10	-0.1640-10	-0.9350-11	-0.5040-11
0.409E 03	-0.2390-04	-0.1950-04	-0.1160-04	-0.8000-05	-0.6050-05	-0.4700-05	-0.3730-05	-0.2980-05	-0.2390-05	-0.1910-05
	-0.1520-05	-0.1210-05	-0.9480-06	-0.7380-06	-0.5690-06	-0.4340-06	-0.3270-06	-0.2440-06	-0.1800-06	-0.1310-06
	-0.9420-07	-0.6700-07	-0.4710-07	-0.3280-07	-0.2290-07	-0.1530-07	-0.1030-07	-0.8900-08	-0.4500-08	-0.2990-08
	-0.1940-08	-0.1250-08	-0.8620-09	-0.5090-09	-0.3220-09	-0.2020-09	-0.1260-09	-0.7800-10	-0.4810-10	-0.2950-10
0.469E 03	-0.2300-04	-0.1880-04	-0.1120-04	-0.7850-05	-0.5920-05	-0.4560-05	-0.3720-05	-0.3020-05	-0.2460-05	-0.2010-05
	-0.1440-05	-0.1330-05	-0.1070-05	-0.8610-06	-0.6850-06	-0.5410-06	-0.4240-06	-0.3290-06	-0.2530-06	-0.1920-06
	-0.1430-06	-0.1080-06	-0.7980-07	-0.5840-07	-0.4430-07	-0.3030-07	-0.2160-07	-0.1520-07	-0.1060-07	-0.7340-08
	-0.5040-08	-0.3410-08	-0.2320-08	-0.1560-08	-0.1040-08	-0.6870-09	-0.4620-09	-0.2960-09	-0.1920-09	-0.1240-09

FRACTION OF GAS PRODUCED AT LAST FIVE TIME LEVELS

FRACTION U/F GAS PRODUCED= 0.887E-05 0.102E-04 0.118E-04 0.136E-04 0.155E-04





















0.108E 03	-0.220D-04	-0.184D-04	-0.995D-05	-0.606D-05	-0.382D-05	-0.241D-05	-0.151D-05	-0.919D-06	-0.544D-06	-0.312D-06
	-0.174D-06	-0.938D-07	-0.493D-07	-0.253D-07	-0.128D-07	-0.631D-08	-0.307D-08	-0.148D-08	-0.701D-09	-0.330D-09
	-0.154D-09	-0.711D-10	-0.327D-10	-0.149D-10	-0.679D-11	-0.307D-11	-0.138D-11	-0.621D-12	-0.278D-12	-0.124D-12
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.143E 03	-0.209D-04	-0.175D-04	-0.976D-05	-0.620D-05	-0.412D-05	-0.277D-05	-0.166D-05	-0.123D-05	-0.802D-06	-0.510D-06
	-0.317D-06	-0.192D-06	-0.113D-06	-0.652D-07	-0.368D-07	-0.203D-07	-0.110D-07	-0.590D-08	-0.311D-08	-0.162D-08
	-0.632D-09	-0.424D-09	-0.214D-09	-0.107D-09	-0.532D-10	-0.263D-10	-0.129D-10	-0.629D-11	-0.306D-11	-0.148D-11
	-0.713D-12	-0.343D-12	-0.164D-12	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.183E 03	-0.194D-04	-0.165D-04	-0.954D-05	-0.623D-05	-0.430D-05	-0.302D-05	-0.213D-05	-0.150D-05	-0.104D-05	-0.712D-06
	-0.479D-06	-0.316D-06	-0.204D-06	-0.129D-06	-0.800D-07	-0.486D-07	-0.290D-07	-0.178D-07	-0.983D-08	-0.560D-08
	-0.315D-09	-0.175D-09	-0.959D-09	-0.522D-09	-0.281D-09	-0.150D-09	-0.798D-10	-0.420D-10	-0.220D-10	-0.115D-10
	-0.594D-11	-0.306D-11	-0.157D-11	-0.809D-12	-0.409D-12	-0.208D-12	-0.105D-12	0.0	0.0	0.0
0.220E 03	-0.192D-04	-0.162D-04	-0.931D-05	-0.621D-05	-0.439D-05	-0.319D-05	-0.233D-05	-0.171D-05	-0.124D-05	-0.490D-06
	-0.643D-06	-0.449D-06	-0.311D-06	-0.211D-06	-0.141D-06	-0.927D-07	-0.599D-07	-0.341D-07	-0.230D-07	-0.147D-07
	-0.892D-08	-0.535D-08	-0.317D-08	-0.168D-08	-0.108D-08	-0.621D-09	-0.354D-09	-0.200D-09	-0.112D-09	-0.625D-10
	-0.346D-10	-0.191D-10	-0.104D-10	-0.568D-11	-0.308D-11	-0.166D-11	-0.893D-12	-0.474D-12	-0.255D-12	-0.136D-12
0.278E 03	-0.186D-04	-0.157D-04	-0.910D-05	-0.616D-05	-0.444D-05	-0.329D-05	-0.240D-05	-0.187D-05	-0.141D-05	-0.106D-05
	-0.788D-06	-0.530D-06	-0.422D-06	-0.303D-06	-0.215D-06	-0.150D-06	-0.103D-06	-0.701D-07	-0.469D-07	-0.309D-07
	-0.201D-07	-0.129D-07	-0.620D-08	-0.314D-08	-0.319D-08	-0.164D-08	-0.119D-08	-0.718D-09	-0.424D-09	-0.295D-09
	-0.150D-09	-0.876D-10	-0.509D-10	-0.294D-10	-0.169D-10	-0.965D-11	-0.547D-11	-0.310D-11	-0.175D-11	-0.982D-12
0.338E 03	-0.180D-04	-0.152D-04	-0.890D-05	-0.608D-05	-0.445D-05	-0.336D-05	-0.258D-05	-0.200D-05	-0.155D-05	-0.120D-05
	-0.923D-06	-0.706D-06	-0.535D-06	-0.401D-06	-0.298D-06	-0.219D-06	-0.159D-06	-0.114D-06	-0.806D-07	-0.564D-07
	-0.390D-07	-0.266D-07	-0.180D-07	-0.120D-07	-0.749D-08	-0.519D-08	-0.337D-08	-0.216D-08	-0.138D-08	-0.868D-09
	-0.544D-09	-0.310D-09	-0.209D-09	-0.128D-09	-0.782D-10	-0.474D-10	-0.286D-10	-0.172D-10	-0.102D-10	-0.649D-11
0.409E 03	-0.175D-04	-0.143D-04	-0.870D-05	-0.600D-05	-0.444D-05	-0.340D-05	-0.264D-05	-0.210D-05	-0.167D-05	-0.132D-05
	-0.105D-05	-0.824D-06	-0.646D-06	-0.507D-06	-0.387D-06	-0.298D-06	-0.244D-06	-0.188D-06	-0.125D-06	-0.920D-07
	-0.670D-07	-0.442D-07	-0.344D-07	-0.243D-07	-0.170D-07	-0.110D-07	-0.808D-08	-0.550D-08	-0.371D-08	-0.248D-08
	-0.165D-08	-0.108D-08	-0.709D-09	-0.461D-09	-0.298D-09	-0.191D-09	-0.122D-09	-0.774D-10	-0.468D-10	-0.307D-10
0.489E 03	-0.170D-04	-0.144D-04	-0.851D-05	-0.590D-05	-0.441D-05	-0.342D-05	-0.271D-05	-0.218D-05	-0.176D-05	-0.142D-05
	-0.115D-05	-0.929D-06	-0.747D-06	-0.598D-06	-0.476D-06	-0.376D-06	-0.295D-06	-0.230D-06	-0.178D-06	-0.136D-06
	-0.103D-06	-0.779D-07	-0.582D-07	-0.430D-07	-0.314D-07	-0.230D-07	-0.168D-07	-0.119D-07	-0.843D-08	-0.593D-08
	-0.415D-08	-0.288D-08	-0.198D-08	-0.136D-08	-0.924D-09	-0.624D-09	-0.419D-09	-0.280D-09	-0.186D-09	-0.123D-09
FRACTION OF GAS PRODUCED AT LAST FIVE TIME LEVELS										
FRACTION OF GAS PRODUCED	0.554E-05	0.652E-05	0.766E-05	0.895E-05	0.104E-04					





0.168E 03	-0.223E-04	-0.186E-04	-0.100D-04	-0.602D-05	-0.373D-05	-0.232D-05	-0.141D-05	-0.839D-06	-0.483D-06	-0.268D-06
	-0.104E-06	-0.752E-07	-0.382D-07	-0.189E-07	-0.919E-08	-0.438E-08	-0.206E-08	-0.954D-09	-0.437D-09	-0.197D-09
	-0.693E-10	-0.359E-10	-0.177D-10	-0.781E-11	-0.343D-11	-0.150D-11	-0.652D-12	-0.283E-12	-0.122D-12	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.142E 03	-0.211E-04	-0.177E-04	-0.982E-05	-0.619E-05	-0.408E-05	-0.265D-05	-0.178D-05	-0.115D-05	-0.733D-06	-0.454E-06
	-0.427E-06	-0.252E-06	-0.124E-06	-0.601D-06	-0.390E-06	-0.249E-06	-0.162D-06	-0.104D-06	-0.629D-06	-0.382D-06
	-0.517E-09	-0.255E-09	-0.124E-09	-0.601D-09	-0.390E-09	-0.249E-09	-0.162D-09	-0.104D-09	-0.629D-09	-0.382D-09
	-0.318E-12	-0.143E-12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.183E 03	-0.202E-04	-0.170E-04	-0.961D-05	-0.634E-05	-0.426D-05	-0.276D-05	-0.181D-05	-0.117D-05	-0.711D-06	-0.451E-06
	-0.427E-06	-0.252E-06	-0.124E-06	-0.601D-06	-0.390E-06	-0.249E-06	-0.162D-06	-0.104D-06	-0.629D-06	-0.382D-06
	-0.517E-09	-0.255E-09	-0.124E-09	-0.601D-09	-0.390E-09	-0.249E-09	-0.162D-09	-0.104D-09	-0.629D-09	-0.382D-09
	-0.266E-11	-0.143D-11	-0.710D-12	-0.352E-12	-0.174D-12	-0.855D-13	0.0	0.0	0.0	0.0
0.228E 03	-0.194E-04	-0.163E-04	-0.939E-05	-0.623D-05	-0.437D-05	-0.314D-05	-0.228D-05	-0.164D-05	-0.118D-05	-0.836E-06
	-0.565E-06	-0.422E-06	-0.222D-06	-0.140E-06	-0.892E-07	-0.472D-07	-0.292E-07	-0.177D-07	-0.106E-07	-0.477E-07
	-0.625E-08	-0.364E-08	-0.209E-08	-0.119E-08	-0.670E-09	-0.374E-09	-0.207D-09	-0.113E-09	-0.616D-10	-0.337D-10
	-0.175E-10	-0.955E-11	-0.507D-11	-0.266D-11	-0.141D-11	-0.733D-12	-0.385D-12	-0.200D-12	-0.104D-12	0.0
0.278E 03	-0.188E-04	-0.158E-04	-0.918E-05	-0.619E-05	-0.443D-05	-0.326D-05	-0.243D-05	-0.182D-05	-0.135D-05	-0.100D-05
	-0.733E-06	-0.531E-06	-0.379D-06	-0.267E-06	-0.185E-06	-0.126E-06	-0.848D-07	-0.561D-07	-0.365D-07	-0.235E-07
	-0.148E-07	-0.938E-08	-0.572E-08	-0.345E-08	-0.210E-08	-0.125E-08	-0.741D-09	-0.430D-09	-0.252D-09	-0.145E-09
	-0.629E-10	-0.471E-10	-0.266D-10	-0.149D-10	-0.831E-11	-0.461D-11	-0.255D-11	-0.140D-11	-0.766D-12	-0.410E-12
0.338E 03	-0.182E-04	-0.154E-04	-0.898E-05	-0.612E-05	-0.445D-05	-0.334D-05	-0.255D-05	-0.196D-05	-0.150D-05	-0.115E-05
	-0.672E-06	-0.457E-06	-0.490D-06	-0.361E-06	-0.263D-06	-0.190D-06	-0.135E-06	-0.945D-07	-0.655D-07	-0.447E-07
	-0.302E-07	-0.201E-07	-0.132E-07	-0.861E-08	-0.555E-08	-0.353D-08	-0.223D-08	-0.139D-08	-0.862D-09	-0.529D-09
	-0.323E-09	-0.155E-09	-0.117D-09	-0.700D-10	-0.415E-10	-0.245D-10	-0.144D-10	-0.839D-11	-0.488D-11	-0.282E-11
0.409E 03	-0.177E-04	-0.142E-04	-0.878E-05	-0.604E-05	-0.445D-05	-0.339D-05	-0.264D-05	-0.207D-05	-0.162D-05	-0.121E-05
	-0.959E-06	-0.748E-06	-0.610E-06	-0.446E-06	-0.350E-06	-0.263E-06	-0.196E-06	-0.144D-06	-0.105D-06	-0.754D-07
	-0.541E-07	-0.381E-07	-0.266D-07	-0.183E-07	-0.125E-07	-0.847D-08	-0.567E-08	-0.376D-08	-0.247E-08	-0.161D-08
	-0.104E-08	-0.679E-09	-0.427D-09	-0.271D-09	-0.170E-09	-0.106D-09	-0.662D-10	-0.409D-10	-0.252D-10	-0.154E-10
0.489E 03	-0.172E-04	-0.146E-04	-0.859E-05	-0.595E-05	-0.442D-05	-0.342D-05	-0.270D-05	-0.215D-05	-0.172D-05	-0.138D-05
	-0.111E-05	-0.887E-06	-0.7C5D-06	-0.556E-06	-0.438E-06	-0.342D-06	-0.264D-06	-0.203D-06	-0.154D-06	-0.116E-06
	-0.865E-07	-0.659E-07	-0.468E-07	-0.339E-07	-0.244E-07	-0.174D-07	-0.123E-07	-0.858E-08	-0.595D-08	-0.410D-08
	-0.260E-08	-0.190E-08	-0.128D-08	-0.852D-09	-0.566E-09	-0.373D-09	-0.245E-09	-0.160D-09	-0.104D-09	-0.668E-10

FACTION OF GAS PRODUCED AT LAST FIVE TIME LEVELS

FRACTION OF GAS PRODUCED= C.560E-05 0.659E-05 0.775E-05 0.905E-05 0.105E-04

APPENDIX K

COPIES OF THE COMPUTER PROGRAMS FOR CASES  
I, II, III AND IV

K2

K.1

CONSTANT TERMINAL RATE CASE AND CONSTANT  
EXTERNAL PRESSURE: CASE I

## MAIN LINE

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TRANSIENT ISOTHERMAL RADIAL GAS FLOW IN POROUS MEDIA

CASE I

CONSTANT TERMINAL RATE CASE AND CONSTANT  
EXTERNAL PRESSURE

THIS PROGRAM SOLVES A SECOND ORDER NON-LINEAR  
PARABOLIC-TYPE PARTIAL DIFFERENTIAL EQUATION FOR  
TRANSIENT ISOTHERMAL RADIAL GAS FLOW THROUGH POROUS  
MEDIA. THE MATHEMATICAL MODEL IS GENERAL IN THE SENSE  
THAT NOT ONLY INCLUDES SLIPPAGE AND INERTIAL EFFECTS,  
BUT ALSO TAKES INTO ACCOUNT CHANGES OF GAS PROPERTIES  
AND POROSITY WITH PRESSURE. THE SOLUTION FOR THE  
MODEL IS OBTAINED BY USING THE FINITE DIFFERENCES  
TECHNIQUE. THE SYSTEM OF NON-LINEAR ALGEBRAIC  
EQUATIONS OBTAINED AFTER DISCRETIZATION GIVES A  
COEFFICIENT MATRIX WHICH IS TRIDIAGONAL AND THEREFORE  
THOMAS ALGORITHM WAS USED TO THE SOLUTION OF THE  
SYSTEM.

## INPUT DATA

PI -INITIAL PRESSURE=PF, PSIA  
R -UNIVERSAL GAS CONSTANT=10.72  
MW -NITROGEN MOLECULAR WEIGHT=28.0  
RW -WELLBORE RADIUS, INCHES. LATER ON IS  
NORMALIZED WITH RESPECT TO RE  
RE -EXTERNAL RESERVOIR RADIUS, FEET  
CR -ROCK COMPRESSIBILITY, 1/PSIA  
PHI -ROCK POROSITY AT SURFACE CONDITIONS,  
FRACTION  
DR -DIMENSIONLESS SPACE INCREMENT IN THE  
RADIAL DIRECTION  
NSETS-AN INTEGER TO INDICATE NUMBER OF SETS  
TO BE RUN  
MM -AN INTEGER TO INDICATE THE NUMBER OF  
TIME STEPS PLUS ONE. ACTUAL NUMBER OF  
TIME STEPS IS (MM-1)  
LL -AN INTEGER TO INDICATE TOTAL NUMBER OF  
GRID POINTS TO BE USED SUCH THAT THE LAST  
POINT CORRESPONDS TO A NORMALIZED DISTANCE  
EQUALS TO 1.0  
T -TEMPERATURE, DEG. F  
EPS -AN ERROR CRITERION FOR CONVERGENCE OF  
SOLUTION  
DTIME-SET OF VALUES FOR DIMENSIONLESS TIME

## MAIN LINE ... (CONT'D)

C           STEPS  
 C   MT   -REFERENCES POINTS FOR TIME INCREMENT  
 C           (SAY THE 3RD., 5TH., ETC.) AT WHICH  
 C           FRACTION OF GAS PRODUCED WILL BE CALCULATED  
 C   ICONT-AN INTEGER TO INDICATE WHETHER ONE OR  
 C           TWO VALUES OF DELTA R WILL BE USED  
 C           ICONT=0 TO INDICATE THAT ONLY ONE VALUE  
 C                    OF DELTA R WILL BE USED THROUGHOUT  
 C                    THE REGION  
 C           ICONT=1 TO INDICATE THE USE OF TWO  
 C                    DIFFERENT VALUES OF DELTA R  
 C   IFG   -AN INTEGER TO INDICATE THE SWITCHING  
 C           POINT TO A DIFFERENT VALUE OF DELTA R  
 C   IPOR   -AN INTEGER HAVING VALUES 0 OR 1  
 C           IPOR=0 RATE OF CHANGE OF POROSITY WITH  
 C           PRESSURE IS A CONSTANT  
 C           IPOR=1 RATE OF CHANGE OF POROSITY WITH  
 C           PRESSURE IS A FUNCTION OF PRESSURE  
 C   FLOW   -CONSTANT DIMENSIONLESS FLUX PRODUCING  
 C           AT THE WELLBORE  
 C   PERM   -ROCK PERMEABILITY, MILLIDARCIES  
 C   B       -SLIPPAGE COEFFICIENT, PSIA  
 C   BB      -INERTIAL COEFFICIENT, 1/FT  
 C  
 C           REMARKS  
 C  
 C   1. SINCE THE VALUE OF THE NON-LINEAR COEFFICIENT  
 C       K, IN THE PARABOLIC DIFFERENTIAL EQUATION  
 C       CANNOT BE NEGATIVE, EXECUTION WILL BE  
 C       SUPPRESSED IF  $0.5 \cdot \Delta R$  EXCEEDS THE VALUE OF THE  
 C       NORMALIZED  $r_w$ . IT WILL PRINT A MESSAGE  
 C   2. ONE ASTERISK WILL BE PRINTED TO INDICATE  
 C       THAT THE SOLUTION AT A GIVEN TIME STEP DID  
 C       NOT CONVERGE TO THE DESIRED ACCURACY AFTER  
 C       TEN ITERATIONS  
 C   3. THE PROGRAM CAN ACCEPT A CONSTANT DELTA R  
 C       THROUGHOUT THE REGION OR TWO DIFFERENT  
 C       VALUES THE SWITCHING POINT BEING SPECIFIED  
 C       BY THE PARAMETER IFG  
 C  
 C           SUBROUTINES REQUIRED  
 C  
 C   BCOND  
 C   FPRESS  
 C

REAL#8 K1(196,36),K2(196,36),G1(196,36),ERR(196),  
 IPO(36),UO(36),BA(196),K1AVG(196,36),K2AVG(196,36),  
 2WX(196,36),WWX(196,36),P(196,36),PP(196,36),U(196,36)  
 3,UU(196,36),G1AVG(196,36),ACD(196),BCD(196),DCD(196),



## MAIN LINE ... (CONT'D)

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4CC(196),DTIME(36),DSQRT,RHO,DEXP,DABS,WX1,WX2
  REAL#8 DER(196,36),DERAVG(196,36),E(196),G(196),D,BC,
1RW,RE,B,BB,FL,DR,PAVG,PAVG1,PAVG2,CG,CR,Z,VIS,PHI,PF,
2FLOW,CONS,DT,PRE,R,MW,PERM,KEY,ALPHA,PI,VISI,CGI,ZI,
3RHO1,RHO2,DR2,PSTD,PHIO,PHIRI
  DIMENSION GP2(20),MT(20)
  COMMON WX,WWX,P,PP,U,UU,Z,CG,VIS,FLOW,LL,MM,ITN
  INTEGER L,LL,LLL,LLLL,M,MM,MMM,J,MB,ITN
  INTEGER IFG,ICONT

```

```

C   READ IN INITIAL PRESSURE, PI AND OTHER RELEVANT DATA
C

```

```

  READ(5,520) PI
  READ(5,520) R,MW
520  FORMAT(5D13.6)
  READ(5,520) RW,RE,CR,PHI
  READ(5,520) PF,DR,DR2
  READ(5,521) NSETS,MM,LL,T,EPS
521  FORMAT(3I4,2E13.6)
  MMM=MM-1
  READ(5,520) (DTIME(I),I=1,MMM)

```

```

C

```

```

  II=5

```

```

C   READ IN REFERENCES OF TIME INCREMENTS MT(I) AT WHICH
C   FRACTION OF GAS PRODUCED WILL BE CALCULATED
C   USING AVERAGE FLUXES
C

```

```

  READ(5,293) (MT(I),I=1,II)
293  FORMAT(6I6)
  READ(5,293) ICONT,IFG,IPOR
  WRITE(6,3015) ICONT,IFG,DR,DR2
3015  FORMAT(2I6,2F12.7)
  ALPHA=MW/(R*(T+460.))
  CONS=DSQRT(0.72D 02#0.32174D 02/ALPHA)
  LLL=LL-1
  RW=RW/(0.12D 02*RE)
  PSTD=0.14697D 02
  PHIO=PHI
  JK=0

```

```

C   SETTING DIMENSIONLESS FLUX
C

```

```

  READ(5,520) FLOW
  CALL FPRESS(PI,T)

```

## MAIN LINE ... (CONT'D)

VIS=VIS/(0.1D 03\*0.6891D 05)

C CALCULATING POROSITY AT INITIAL RESERVOIR PRESSURE  
C

PHIRI=0.1D 01-(0.1D 01-PHI0)\*DEXP(-CR\*(PI-PSTD))

C STORING INITIAL VALUES  
C

VISI=VIS

CGI=CG

ZI=Z

999 READ(5,520) PERM,B,BB

JK=JK+1

CRIT=0.5\*DR-RW

IF(CRIT) 875,875,876

876 WRITE(6,295)

295 FORMAT(//,8X,'VALUE OF K2AVG AT POINT 1 WILL BE  
1NEGATIVE BECAUSE 0.5\*DR EXCEEDS THE VALUE OF RW.  
2THIS ORIGINATES ANOMALOUS SOLUTIONS AND THEREFORE  
3EXECUTION WAS SUPPRESSED')

GO TO 613

875 CONTINUE

KEY=PERM\*0.1062D-13

TIME=0.

WRITE(6,231)

231 FORMAT('1')

WRITE(6,230)

230 FORMAT('-')

WRITE(6,1059)

1059 FORMAT(36X,'TRANSIENT RADIAL GAS FLOW')

C WRITE(6,8667) IPOR

C8667 FORMAT(/,44X,'IPOR=',I2,/) )

WRITE(6,230)

WRITE(6,205) B,BB,PERM

205 FORMAT(1H .,13X,'SLIP COEFF(B)=' ,F7.3, ' INERTIAL  
\* COEFF', '(BB)=' ,

1D10.3, ' PERMEABILITY(K)=' ,F8.3,1X, 'MD')

WRITE(6,206) PHI,CR,PI

206 FORMAT(/,14X,'POROSITY=' ,F7.3,8X, 'ROCK  
\* COMPRESSIBILITY=' ,

1D10.3,2X, 'INITIAL PRESSURE=' ,D10.3)

WRITE(6,207)

207 FORMAT(1H0,35X,'PRESSURE SQUARE DISTRIBUTION')

WRITE(6,220)



## MAIN LINE ... (CONT'D)

```

CALL FPRESS(PRE,T)
VIS=VIS/(0.1D 03*0.6891D 05)
K2(L,M)=-(0.5D 00*DR-RW)*KEY*PF*(PAVG2+B)/(0.2D 01*RE
**VIS*Z*
1PAVG2*CONS+RE*BB*KEY*PF*(PAVG2+B)*Z*DABS(WX(L,M)))
GO TO 5
9994 FL=IFG-1
KK=KK+1
F1=KK
FLL=F1-0.5
K2(L,M)=(FL*DR+RW+FLL*DR2)*KEY*PF*(PAVG2+B)/(0.2D 01
**RE*VIS*Z*
1PAVG2*CONS+RE*BB*KEY*PF*(PAVG2+B)*Z*DABS(WX(L,M)))
5 CONTINUE

```

```

C CALCULATING K1(L+1/2) AT (M)
C

```

```

KK=0
DO 4 L=1,LLL
PAVG1=0.5*(P(L+1,M)+P(L,M))
PRE=PAVG1*PF
CALL FPRESS(PRE,T)
VIS=VIS/(0.1D 03*0.6891D 05)
IF(ICONT.EQ.0) GO TO 9999
IF(L.GE.IFG) GO TO 9991
9999 FLL=L
FL=FLL-0.5
K1(L,M)=(FL*DR+RW)*KEY*PF*(PAVG1+B)/(0.2D 01*RE*VIS*Z
**PAVG1*
1CONS+RE*BB*KEY*PF*(PAVG1+B)*Z*DABS(WX(L+1,M)))
GO TO 4
9991 FL=IFG-1
KK=KK+1
F1=KK
FLL=F1-0.5
K1(L,M)=(FL*DR+RW+FLL*DR2)*KEY*PF*(PAVG1+B)/(0.2D 01
**RE*VIS*Z*
1PAVG1*CONS+RE*BB*KEY*PF*(PAVG1+B)*Z*DABS(WX(L+1,M)))
4 CONTINUE

```

```

C CALCULATING G1 AT (M)
C

```

```

KK=0
DO 5000 L=1,LLL
PAVG3=P(L,M)
PRE=PAVG3*PF
CALL FPRESS(PRE,T)

```

## MAIN LINE ... (CONT'D)

```

C   THE ALTERNATIVE TO DROP THE ASSUMPTION OF RATE OF
C   CHANGE OF POROSITY WITH PRESSURE BEING A CONSTANT,
C   CAN BE OBTAINED BY SETTING IPOR=1 AND USING THE
C   STATEMENTS INDICATED WITH A C FORTRAN COMMENT.
C
C   IF(IPOR.EQ.0) GO TO 8666
      IF(ICONT.EQ.0) GO TO 9995
C
C   PHI=0.1D 01-(0.1D 01-PHI0)*DEXP(-CR*(PRE-PSTD))
C8666 IF(ICONT.EQ.0) GO TO 9995
      IF(L.GE.IFG+1) GO TO 9996
C9995 FL=L-1
      IF(L-1) 536,536,537
C536 G1(L,M)=PHI*RW*PF/Z*(CG+CR*(0.1D 01/PHI-0.1D 01))
      536 G1(L,M)=RW*PF/Z*(CG+CR*(0.1D 01/PHI-0.1D 01))
      GO TO 5000
C537 G1(L,M)=PHI*(FL*DR+RW)*PF/Z*(CG+CR*(0.1D 01/PHI-0.1D
C537 01))
      537 G1(L,M)=(FL*DR+RW)*PF/Z*(CG+CR*(0.1D 01/PHI-0.1D 01))
      GO TO 5000
C9996 FL=IFG-1
      KK=KK+1
      F1=KK
C
C   G1(L,M)=PHI*(FL*DR+RW+F1*DR2)*PF/Z*(CG+CR*(0.1D
C   01/PHI-0.1D 01))
      G1(L,M)=(FL*DR+RW+F1*DR2)*PF/Z*(CG+CR*(0.1D 01/PHI
*-0.1D 01))
C5000 CONTINUE
C
C   STORING FIRST GUESSES OF P AND U AT (M+1)
C
      DO 454 L=1,LL
      PP(L,M+1)=P(L,M)
      UU(L,M+1)=U(L,M)
C454 WWX(L,M+1)=WX(L,M)
      WWX(1,M+1)=FLOW
C
C   CALCULATING NON-LINEAR COEFFICIENTS AT TIME
C   STEP (M+1) TO OBTAIN AVERAGE VALUES OF THESE
C   COEFFICIENTS
C

```

## MAIN LINE ... (CONT'D)

C CALCULATING K2AVG  
C

```

24  CONTINUE
    KK=0
    DO 1051 L=1,LLL
    IF(L.EQ.1) GO TO 5004
    PAVG2=0.5*(PP(L,M+1)+PP(L-1,M+1))
    PRE=PAVG2*PF
    CALL FPRESS(PRE,T)
    VIS=VIS/(0.1D 03*0.6891D 05)
    IF(ICONT.EQ.0) GO TO 3001
    IF(L.GE.IFG+1) GO TO 3002
3001 FLL=L
    FL=FLL-1.5
    K2(L,M+1)=(FL*DR+RW)*KEY*PF*(PAVG2+B)/(0.2D 01*RE*VIS
**Z*PAVG2*
1CONS+RE*BB*KEY*PF*(PAVG2+B)*Z*DABS(WWX(L,M+1)))
    K2AVG(L,M)=(K2(L,M)+K2(L,M+1))/2.
    GO TO 1051
5004 PAVG2=PP(1,M+1)
    PRE=PAVG2*PF
    CALL FPRESS(PRE,T)
    VIS=VIS/(0.1D 03*0.6891D 05)
    U0(M+1)=0.4D 01*DR*RE*VIS*Z*PP(1,M+1)*CONS*WWX(1,M+1)
*/(KEY*PF*
1 (PP(1,M+1)+B)) + 0.2D 01*DR*RE*BB*Z*WWX(1,M+1)
**DABS(WWX(1,M+1))
2+UU(2,M+1)
    IF(U0(M+1)) 955,957,957
955  P0(M+1)=0.1D-09
    GO TO 961
957  P0(M+1)=DSQRT(U0(M+1))
961  PAVG2=0.5*(P0(M+1)+PP(1,M+1))
    PRE=PAVG2*PF
    CALL FPRESS(PRE,T)
    VIS=VIS/(0.1D 03*0.6891D 05)
    K2(L,M+1)=-(0.5D 00*DR-RW)*KEY*PF*(PAVG2+B)/(0.2D 01
**RE*VIS*Z*
1PAVG2*CONS+RE*BB*KEY*PF*(PAVG2+B)*Z*DABS(WWX(L,M+1)))
    K2AVG(L,M)=0.5*(K2(L,M)+K2(L,M+1))
    GO TO 1051
3002 FL=IFG-1
    KK=KK+1
    F1=KK
    FLL=F1-0.5
    K2(L,M+1)=(FL*DR+RW+FLL*DR2)*KEY*PF*(PAVG2+B)/(0.2D 01
**RE*VIS
1*Z*PAVG2*CONS+RE*BB*KEY*PF*(PAVG2+B)*Z*DABS(WWX(L,M
*+1)))

```

## MAIN LINE ... (CONT'D)

```

K2AVG(L,M)=0.5*(K2(L,M)+K2(L,M+1))
1051 CONTINUE

C   CALCULATING K1AVG
C

KK=0
DO 1050 L=1,LLL
PAVG1=0.5*(PP(L+1,M+1)+PP(L,M+1))
PRE=PAVG1*PF
CALL FPRESS(PRE,T)
VIS=VIS/(0.1D 03*0.6891D 05)
IF(ICONT.EQ.0) GO TO 3003
IF(L.GE.IFG) GO TO 3004
3003 FLL=L
FL=FLL-0.5
K1(L,M+1)=(FL*DR+RW)*KEY*PF*(PAVG1+B)/(0.2D 01*RE*VIS
**Z*PAVG1*
1CONS+RE*BB*KEY*PF*(PAVG1+B)*Z*DABS(WWX(L+1,M+1)))
K1AVG(L,M)=0.5*(K1(L,M)+K1(L,M+1))
GO TO 1050
3004 FL=IFG-1
KK=KK+1
F1=KK
FLL=F1-0.5
K1(L,M+1)=(FL*DR+RW+FLL*DR2)*KEY*PF*(PAVG1+B)/(0.2D 01
**RE*VIS
1*Z*PAVG1*CONS+RE*BB*KEY*PF*(PAVG1+B)*Z*DABS(WWX(L+1,M
**+1)))
K1AVG(L,M)=0.5*(K1(L,M)+K1(L,M+1))
1050 CONTINUE

C   CALCULATING G1AVG(L,M)
C

KK=0
DO 5006 L=1,LLL
PAVG3=PP(L,M+1)
PRE=PAVG3*PF
CALL FPRESS(PRE,T)

C   TO DROP THE ASSUMPTION OF CONSTANT RATE OF CHANGE
C   OF POROSITY WITH PRESSURE, FOLLOW THE SAME PROCEDURE
C   INDICATED ABOVE WHEN EVALUATING G1 AT M.
C
C   IF(IPOR.EQ.0) GO TO 7836
C   PHI=0.1D 01-(0.1D 01-PHI0)*DEXP(-CR*(PRE-PSTD))
C7836 IF(ICONT.EQ.0) GO TO 3005

IF(ICONT.EQ.0) GO TO 3005

```

## MAIN LINE ... (CONT'D)

```

IF(L.GE.IFG+1) GO TO 3006
3005 FL=L-1
IF(L-1) 694,694,695

C694 G1(L,M+1)=PHI#RW#PF/Z*(CG+CR*(0.1D 01/PHI-0.1D 01))

694 G1(L,M+1)=RW#PF/Z*(CG+CR*(0.1D 01/PHI-0.1D 01))
GO TO 5005

C695 G1(L,M+1)=PHI*(FL#DR+RW)#PF/Z*(CG+CR*(0.1D 01/PHI-0.1D
C695 01))

695 G1(L,M+1)=(FL#DR+RW)#PF/Z*(CG+CR*(0.1D 01/PHI-0.1D
* 01))
5005 G1AVG(L,M)=0.5*(G1(L,M)+G1(L,M+1))
GO TO 5006
3006 FL=IFG-1
KK=KK+1
F1=KK

C G1(L,M+1)=PHI*(FL#DR+RW+F1#DR2)#PF/Z*(CG+CR*(0.1D
C 01/PHI-0.1D 01))

G1(L,M+1)=(FL#DR+RW+F1#DR2)#PF/Z*(CG+CR*(0.1D 01/PHI
*-0.1D 01))
G1AVG(L,M)=0.5*(G1(L,M)+G1(L,M+1))
5006 CONTINUE

C SETTING THE COEFFICIENTS AND GETTING THE SOLUTION
C AT (M+1) USING THE THOMAS ALGORITHM
C

RHO=2.#DR**2/DT
RHO1=DR2#DR*(DR2+DR)/DT
RHO2=0.2D 01#DR2**2/DT

C FOR L=1
C

BC=-RHO#G1AVG(1,M)-K1AVG(1,M)-K2AVG(1,M)
E(1)=BC
G(1)=(-K2AVG(1,M)*(UO(M+1)-UU(2,M+1))-K2AVG(1,M)*UO(M)
#+
1(K1AVG(1,M)+K2AVG(1,M)-RHO#G1AVG(1,M))*U(1,M)-K1AVG(1
*,M)
2*U(2,M))/BC

C FOR INTERMEDIATE GRID POINTS
C

```



## MAIN LINE ... (CONT'D)

```

JF=LLL-1
DO 3 L=2,JF
IF(ICONT.EQ.0.OR.L.LT.IFG) GO TO 3007
IF(L.EQ.IFG) GO TO 3008
RHO=RHO2
3007 BC=-RHO*G1AVG(L,M)-K1AVG(L,M)-K2AVG(L,M)
IF(L.EQ.IFG+1) GO TO 3016
IF(L.GT.2) GO TO 666
E(L)=BC-K2AVG(L,M)*(K1AVG(L-1,M)+K2AVG(L-1,M))/E(L-1)
GO TO 444
666 E(L)=BC-K2AVG(L,M)*K1AVG(L-1,M)/E(L-1)
GO TO 444
3016 E(L)=BC-K2AVG(L,M)*K1AVG(L-1,M)*DR/E(L-1)
444 D=-K2AVG(L,M)*U(L-1,M)+(K1AVG(L,M)+K2AVG(L,M)-RHO
**G1AVG(L,M))*
1U(L,M)-K1AVG(L,M)*U(L+1,M)
G(L)=(D-K2AVG(L,M)*G(L-1))/E(L)
GO TO 3
3008 BC=-RHO1*G1AVG(L,M)-K1AVG(L,M)*DR-K2AVG(L,M)*DR2
E(L)=BC-K2AVG(L,M)*DR2*K1AVG(L-1,M)/E(L-1)
D=-K2AVG(L,M)*DR2*U(L-1,M)+(K1AVG(L,M)*DR+K2AVG(L,M)
**DR2-RHO1*
1G1AVG(L,M)*U(L,M)-K1AVG(L,M)*DR*U(L+1,M)
G(L)=(D-K2AVG(L,M)*DR2*G(L-1))/E(L)
3 CONTINUE

C FOR L=LLL
C

BC=-RHO*G1AVG(LLL,M)-K1AVG(LLL,M)-K2AVG(LLL,M)
D=-0.2D 01*K1AVG(LLL,M)-K2AVG(LLL,M)*U(LLL-1,M)
**+(K1AVG(LLL,M)+
1K2AVG(LLL,M)-RHO*G1AVG(LLL,M))*U(LLL,M)
E(LLL)=BC-K2AVG(LLL,M)*K1AVG(LLL-1,M)/E(LLL-1)
G(LLL)=(D-K2AVG(LLL,M)*G(LLL-1))/E(LLL)
U(LLL,M+1)=G(LLL)

C GETTING THE SOLUTIONS FOR THE REST OF THE GRID POINTS
C

DO 8 J=1,JF
I=LLL-J
IF(I.EQ.1) GO TO 333
IF(I.EQ.IFG) GO TO 3017
U(I,M+1)=G(I)-K1AVG(I,M)*U(I+1,M+1)/E(I)
GO TO 8
333 U(I,M+1)=G(I)-(K1AVG(I,M)+K2AVG(I,M))*U(I+1,M+1)/E(I)
GO TO 8
3017 U(I,M+1)=G(I)-K1AVG(I,M)*DR*U(I+1,M+1)/E(I)
8 CONTINUE

```

## MAIN LINE ... (CONT'D)

```

C   SETTING PRESSURES AT (M+1) FOR L=1,LL
C
      DO 6 L=1,LLL
      IF(U(1,M+1).GT. 0.005) GO TO 6

C   OTHERWISE TERMINATE PRODUCTION
C
      DO 69 K=1,LL
      WX(K,M+1)=0.1D-16
      U(K,M+1)=0.005
69   CONTINUE
      MB=M
      GO TO 678
6   P(L,M+1)=DSQRT(U(L,M+1))

C   CALCULATING THE FLUX AT TIME STEP (M+1) USING SOLUTION
C   OBTAINED AT (M+1) LEVEL
C
      DO 7 L=2,LL
      IF(L.GT. IFG) GO TO 3009
      DER(L,M+1)=(U(L,M+1)-U(L-1,M+1))/DR
      DER(L,M)=(U(L,M)-U(L-1,M))/DR
      GO TO 3010
3009 DER(L,M+1)=(U(L,M+1)-U(L-1,M+1))/DR2
      DER(L,M)=(U(L,M)-U(L-1,M))/DR2
3010 DERA VG(L,M)=0.5*(DER(L,M+1)+DER(L,M))
      PAVG1=0.5*(P(L,M+1)+P(L-1,M+1))
      PAVG2=0.5*(P(L,M)+P(L-1,M))
      PAVG=0.5*(PAVG1+PAVG2)
      PRE=PAVG*PF
      CALL FPRESS(PRE,T)
      VIS=VIS/(0.1D 03*0.6891D 05)
      BCO(L)=-0.2D 01*RE*VIS*Z*PAVG*CONS/(KEY*PF*(PAVG+B))
      ACO(L)=-RE*BB*Z
      DCO(L)=DSQRT(BCO(L)*BCO(L)-4.*ACO(L)*DERAVG(L,M))
      IF(DERA VG(L,M).LE.0.1E-05) GO TO 17

C   ALTERNATIVE FOR QUADRATIC CONSTANT = 0.
C
      IF(DABS(ACO(L)).GT.0.10E-03)GO TO 15
      WX(L,M+1)=-1.*(-DERAVG(L,M)/BCO(L))
      GO TO 7
15   WX1=-(-BCO(L)-DCO(L))/(2.0*ACO(L))
      WX2=-(-BCO(L)+DCO(L))/(2.0*ACO(L))
      IF((WX1+DABS(WX1)).LT.0.10E-04) GO TO 18

```

## MAIN LINE ... (CONT'D)

```

18 IF((WX2+DABS(WX2)).LT.0.10E-04) GO TO 9
   WX(L,M+1)=WX1
   IF((WX2+DABS(WX2)).LT.0.10E-04) GO TO 7
   GO TO 7
17 WX(L,M+1)=0.
   GO TO 7
9   WX(L,M+1)=WX2
7   CONTINUE

C   CHECKING CONVERGENCE OF THE SOLUTION
C
   DO 10 L=1,LLL
   ERR(L)=DABS(PP(L,M+1)-P(L,M+1))
   IF(ERR(L).LE.EPS) GO TO 10
   GO TO 429
10  CONTINUE
   GO TO 11
429 DO 13 L=1,LL
   PP(L,M+1)=P(L,M+1)
   UU(L,M+1)=U(L,M+1)
13  WWX(L,M+1)=WX(L,M+1)
   ITN=ITN+1
   IF(ITN.GT.10) GO TO 12
   GO TO 24
11  CONTINUE

C   PROBLEM HAS CONVERGED FOR TIME STEP (M+1)
C
   GO TO 678
12  WRITE(6,713)
713 FORMAT(2X,'*')
678 TIME=TIME+DTIME(M)
   IF(M.EQ.11.OR.M.EQ.21) GO TO 476
   GO TO 477
476 WRITE(6,231)
   WRITE(6,230)
   WRITE(6,230)
477 WRITE(6,208) TIME
208 FORMAT(6X,E10.3)
78  WRITE(6,224) (U(L,M+1),L=1,LL,3)
224 FORMAT(16X,F7.4,2X,F7.4,2X,F7.4,2X,F7.4,2X,F7.4,2X
*,F7.4,2X,
1F7.4,2X,F7.4,2X,F7.4,2X,F7.4)
   IF(U(1,M+1).LE.0.005) GO TO 679
99  CONTINUE
   MB=M
679 MC=MB-4

```

## MAIN LINE ... (CONT'D)

```

C      CALCULATING FRACTION OF GAS PRODUCED USING AVERAGE
C      FLUXES
C
      DO 291 I=1,II
      MK=MT(I)
      IF(U(1,M+1).LE.0.005) MK=MC
      FLUX1=0.
      FLUXLL=0.
      DO 395 K=1,MK
      FAVG1=DABS(WX(1,K)+WX(1,K+1))/2.
      FAVGLL=DABS(WX(LL,K)+WX(LL,K+1))/2.
      OUT=FAVG1*DTIME(K)
      IN=FAVGLL*DTIME(K)
      FLUX1=FLUX1+OUT
      FLUXLL=FLUXLL+IN
395    CONTINUE

C      IF IPOR=1 USE THE STATEMENT WITH A C FORTRAN
C      COMMENT TO CALCULATE FRACTION OF GAS PRODUCED
C
      GP2(I)=ZI/(PHIRI*(1.-RW**2))*(RW*FLUX1-FLUXLL)

      GP2(I)=ZI*PHIO/(PHIRI*(1.-RW**2))*(RW*FLUX1-FLUXLL)
      MC=MC+1
291    CONTINUE
      WRITE(6,231)
      WRITE(6,230)

C      WRITE(6,8667) IPOR

      WRITE(6,230)
      B=B*PF
      WRITE(6,205) B,BB,PERM.
      WRITE(6,210)
210    FORMAT(1H0,33X,'FLUX DISTRIBUTION FOR THE FIRST 40
      * GRID POINTS')
      WRITE(6,230)
      TIME=0.
      DO 75 M=1,MB
      TIME=TIME+DTIME(M)
      IF(M.EQ.9.OR.M.EQ.17) GO TO 494
      GO TO 445
494    WRITE(6,231)
      WRITE(6,230)
      WRITE(6,230)
445    WRITE(6,208) TIME
      WRITE(6,515) (WX(L,M+1),L=1,40)
515    FORMAT(16X,10D11.3)
75    CONTINUE

```

K17

MAIN LINE ... (CONT'D)

```
WRITE(6,9000)
9000 FORMAT(1H0,30X,'FRACTION OF GAS PRODUCED AT LAST FIVE
* TIME LEVELS'
1)
WRITE(6,9003) (GP2(I),I=1,II)
9003 FORMAT(/,7X,'FRACTION OF GAS PRODUCED=',5(3X,E10.3))
IF(JK.EQ.NSETS) GO TO 613
GO TO 999
613 WRITE(6,231)
STOP
END
```

## SUBROUTINE BCOND

```

C *****
C
C THIS SUBROUTINE SETS UP THE INITIAL AND BOUNDARY
C CONDITIONS FOR EACH SET OF THE BOUNDARY CONDITIONS
C STUDIED.
C
C PARAMETERS
C
C INDEX -CODE FOR EACH SET OF BOUNDARY CONDITIONS
C INDEX=1 CASE I
C INDEX=2 CASE II
C INDEX=3 CASE III
C INDEX=4 CASE IV
C *****
C
C SUBROUTINE BCOND(INDEX)
C
C REAL*8 WX(196,36),WWX(196,36),P(196,36),PP(196,36)
C 1,U(196,36),UU(196,36),FLOW,Z,CG,VIS
C COMMON WX,WWX,P,PP,U,UU,Z,CG,VIS,FLOW,LL,MM,ITN
C INTEGER L,LL,LLL,LLLL,M,MM,MMM,J,MB,ITN
C
C INITIAL CONDITION
C
C DO 10 L=1,LL
C P(L,1)=1.
C WX(L,1)=0.0
C 10 U(L,1)=P(L,1)*P(L,1)
C
C BOUNDARY CONDITIONS
C
C GO TO (300,400,500,600), INDEX
C 300 DO 13 J=2,MM
C P(LL,J)=P(LL,1)
C U(LL,J)=U(LL,1)
C WX(1,J)=FLOW
C 13 CONTINUE
C GO TO 700
C
C BOUNDARY CONDITIONS
C
C 400 P(LL+1,1)=P(LL-1,1)
C U(LL+1,1)=U(LL-1,1)
C DO 14 J=2,MM

```

## SUBROUTINE BCOND ... (CONT'D)

```
WX(LL+1,J)=0.
WX(1,J)=FLOW
14  CONTINUE
    GO TO 700

C   BOUNDARY CONDITIONS
C
500 DO 15 J=2,MM
    P(LL,J)=P(LL,1)
    U(LL,J)=U(LL,1)
    P(1,J)=0.5
    U(1,J)=0.25
15  CONTINUE
    GO TO 700

C   BOUNDARY CONDITIONS
C
600 P(LL+1,1)=P(LL-1,1)
    U(LL+1,1)=U(LL-1,1)
    DO 16 J=2,MM
    P(LL,J)=P(LL,1)
    U(LL,J)=U(LL,1)
    WX(LL+1,J)=0.
    P(1,J)=0.5
    U(1,J)=0.25
16  CONTINUE
700 RETURN
    END
```

## SUBROUTINE FPRESS

```

C *****
C
C THIS SUBROUTINE CALCULATES DEVIATION FACTOR Z, GAS
C COMPRESSIBILITY CG, AND VISCOSITY VIS, FOR NITROGEN
C AS A FUNCTION OF PRESSURE. VALUES OF Z AND CG ARE
C CALCULATED USING B-W-R EQUATION AND VALUES FOR VIS
C USING THE POLYNOMIAL FIT GIVEN BY J. KESTING AND E.
C WANG (SEE REFERENCES).
C
C PARAMETERS
C
C Y -PRESSURE, PSIA
C T -TEMPERATURE, DEG. F
C *****

```

```

SUBROUTINE FPRESS(Y,T)

```

```

REAL*8 WX(196,36),WWX(196,36),P(196,36),PP(196,36)
I,U(196,36),UU(196,36),FLOW,Z,CG,VIS,Y
COMMON WX,WWX,P,PP,U,UU,Z,CG,VIS,FLOW,LL,MM,ITN
INTEGER L,LL,LLL,LLLL,M,MM,MMM,J,MB,ITN

```

```

C B-W-R COEFFICIENTS FOR NITROGEN
C

```

```

AO=1.053642
BO=0.0407426
CO=8059.00
W=0.025102
B=0.0023277
C=728.41
ALPH=0.0001272
GAM=0.0053

```

```

C DETERMINATION OF GAS DENSITY, D
C

```

```

AOM=SQRT(AO)
BOM=BO
COM=SQRT(CO)
Q=1./3.
AM=W**Q
BM=B**Q
CM=C**Q
ALPHM=ALPH**Q
GAMM=SQRT(GAM)
AOM=AOM*AOM

```



## SUBROUTINE FPRESS ... (CONT'D)

```

COM=COM*COM
AM=AM*AM*AM
BM=BM*BM*BM
CM=CM*CM*CM
ALPHM=ALPHM*ALPHM*ALPHM
GAMM=GAMM*GAMM
R=0.0825
TV=(T+459.6)/1.8
Z1=BOM*R*TV-AOM-COM/(TV*TV)
Z2=BM*R*TV-AM
Z3=AM*ALPHM
D=0.
COR=0.1
DIFFE=10.
ZZ=0.0001
101 IF(DIFFE-ZZ) 112,112,102
102 DO 106 J=1,200
    D=D+COR
    PCALC=R*TV*D+Z1*D*D+Z2*D*D*D+Z3*D**6+(CM*D*D*D/(TV
**TV))*(1.+GAMM
1*D*D)*EXP(-GAMM*D*D)
    DIFE=Y/14.696 - PCALC
    DIFFE=ABS(DIFE)
103 IF(DIFE) 104,104,106
104 GO TO 107
106 CONTINUE
107 IF(COR-0.00001) 112,112,109
109 D=D-COR
    COR=0.1*COR
    GO TO 101
112 CONTINUE
    ROD=D

C      DETERMINATION OF GAS DEVIATION FACTOR, Z
C
    CONS=R*TV
    A1=(BO-AO/CONS-CO/(CONS*TV**2))*CONS
    B1=(B-W/CONS)*CONS
    C1=W*ALPH
    D1=C/TV**2
    PAT=Y/14.696
    Z=PAT/(ROD*R*TV)

C      DETERMINATION OF GAS COMPRESSIBILITY, CG
C
    DENOM=CONS+2.*A1*ROD+3.*B1*ROD**2+6.*C1*ROD**5+2.*D1
**GAM*ROD**4*
1EXP(-GAM*ROD**2)*(1.-(1.+GAM*ROD**2))+3.*D1*ROD**2*(1.

```

K22

SUBROUTINE FPRESS ... (CONT'D)

```
#+GAM#ROO**2
1)*EXP(-GAM#ROO**2)
DRHO=1./DENOM
DZ=-PAT/(CONS#ROO**2)*DRHO+1./(CONS#ROO)
CGATM=1./PAT-1./Z*DZ
CG=CGATM/14.696
```

C DETERMINATION OF GAS VISCOSITY, VIS  
C

```
VIS=1778.0E-05*(1.+8.958E-04*(PAT-1.))+6.120E-07*(PAT
*-1.)**2+
13.997E-08*(PAT-1.)**3)+4.55E-05*(TV-298.)
RETURN
END
```

K.2

CONSTANT TERMINAL RATE CASE AND SEALED  
EXTERNAL BOUNDARY: CASE II

## MAIN LINE

```

C *****
C
C TRANSIENT ISOTHERMAL RADIAL GAS FLOW IN POROUS MEDIA
C
C CASE II
C
C CONSTANT TERMINAL RATE CASE AND SEALED
C EXTERNAL BOUNDARY
C *****
C
C REAL#8 K1(196,36),K2(196,36),G1(196,36),ERR(196),
C IPO(36),UO(36),BA(196),K1AVG(196,36),K2AVG(196,36),
C 2WX(196,36),WWX(196,36),P(196,36),PP(196,36),U(196,36)
C 3,UU(196,36),G1AVG(196,36),ACO(196),BCO(196),DCO(196),
C 4CC(196),DTIME(36),DSQRT,RHO,DEXP,DABS,WX1,WX2
C REAL#8 DER(196,36),DERAVG(196,36),E(196),G(196),D,BC,
C 1RW,RE,B,BB,FL,DR,PAVG,PAVG1,PAVG2,CG,CR,Z,VIS,PHI,PF,
C 2FLOW,CONS,DT,PRE,R,MW,PERM,KEY,ALPHA,PI,VISI,CGI,ZI,
C 3RH01,RHO2,DR2,PSTD,PHIO,PHIRI
C DIMENSION GP2(20),MT(20)
C COMMON WX,WWX,P,PP,U,UU,Z,CG,VIS,FLOW,LL,MM,ITN
C INTEGER L,LL,LLL,LLLL,M,MM,MMM,J,MB,ITN
C INTEGER IFG,ICONT
C
C READ INPUT DATA
C
C READ(5,520) PI
C READ(5,520) R,MW
520 FORMAT(5D13.6)
C READ(5,520) RW,RE,CR,PHI
C READ(5,520) PF,DR,DR2
C READ(5,521) NSETS,MM,LL,T,EPS
521 FORMAT(3I4,2E13.6)
C MMM=MM-1
C READ(5,520) (DTIME(I),I=1,MMM)
C II=5
C
C READ IN REFERENCES OF TIME INCREMENTS MT(I) AT WHICH
C FRACTION OF GAS PRODUCED WILL BE CALCULATED
C USING AVERAGE FLUXES
C
C READ(5,293) (MT(I),I=1,II)
293 FORMAT(6I6)
C READ(5,293) ICONT,IFG,IPOR
C WRITE(6,3015) ICONT,IFG,DR,DR2
3015 FORMAT(2I6,2F12.7)

```

## MAIN LINE ... (CONT'D)

```

ALPHA=MW/(R*(T+460.))
CONS=DSQRT(0.72D 02#0.32174D 02/ALPHA)
LLL=LL-1
RW=RW/(0.12D 02#RE)
PSTD=0.14697D 02
PHIO=PHI
JK=0

C   SETTING DIMENSIONLESS FLUX
C

READ(5,520) FLOW
CALL FPRESS(PI,T)
ZI=Z

C   CALCULATING POROSITY AT INITIAL RESERVOIR PRESSURE
C

PHIRI=0.1D 01-(0.1D 01-PHIO)*DEXP(-CR*(PI-PSTD))
999 READ(5,520) PERM,B,BB
    JK=JK+1
    CRIT=0.5#DR-RW
    IF(CRIT) 875,875,876
876  WRITE(6,295)
295  FORMAT(/,8X,'VALUE OF K2AVG AT POINT 1 WILL BE
1NEGATIVE BECAUSE 0.5#DR EXCEEDS THE VALUE OF RW.
2THIS ORIGINATES ANOMALOUS SOLUTIONS AND THEREFORE
3EXECUTION WAS SUPPRESSED')
    GO TO 613
875  CONTINUE
    KEY=PERM#0.1062D-13
    TIME=0.
    WRITE(6,231)
231  FORMAT('1')
    WRITE(6,230)
230  FORMAT('-')
    WRITE(6,1059)
1059 FORMAT(36X,'TRANSIENT RADIAL GAS FLOW')

C   WRITE(6,8667) IPOR
C8667 FORMAT(/,44X,'IPOR=',I2,/)

    WRITE(6,230)
    WRITE(6,205) B,BB,PERM
205  FORMAT(1H ,13X,'SLIP COEFF(B)=' ,F7.3,' INERTIAL
* COEFF', '(BB)=' ,
1D10.3,' PERMEABILITY(K)=' ,F8.3,1X,'MD')
    WRITE(6,206) PHI,CR,PI
206  FORMAT(/,14X,'POROSITY=' ,F7.3,8X,'ROCK
* COMPRESSIBILITY=' ,

```

## MAIN LINE ... (CONT'D)

```

1D10.3,2X,'INITIAL PRESSURE=',D10.3)
WRITE(6,207)
207 FORMAT(1H0,35X,'PRESSURE SQUARE DISTRIBUTION')
WRITE(6,220)
220 FORMAT(8X,'TIME',26X,'DIMENSIONLESS DISTANCE')
WRITE(6,221)
221 FORMAT(8X,'(TD)          RW          DELTA R1=0.0010
* DELTA R2=0.
1032724')
WRITE(6,223)
223 FORMAT(1X,' ')

C   SETTING THE DIMENSIONLESS SLIPPAGE
C

B=B/PF
INDEX=2
CALL BCOND(INDEX)
DO 99 M=1,MMM
DT=DTIME(M)
ITN=0

C
C   CALCULATING K2(L-1/2) AT (M)
C

KK=0
DO 5 L=1,LL
IF(L.EQ.1) GO TO 777
PAVG2=0.5*(P(L,M)+P(L-1,M))
PRE=PAVG2*PF
CALL FPRESS(PRE,T)
VIS=VIS/(0.1D 03*0.6891D 05)
IF(ICONT.EQ.0) GO TO 9993
IF(L.GE.IFG+1) GO TO 9994
9993 FLL=L
FL=FLL-1.5
K2(L,M)=(FL*DR+RW)*KEY*PF*(PAVG2+B)/(0.2D 01*RE*VIS*Z
**PAVG2*
1CONS+RE*BB*KEY*PF*(PAVG2+B)*Z*DABS(WX(L,M)))
GO TO 5
777 PAVG2=P(1,M)
PRE=PAVG2*PF
CALL FPRESS(PRE,T)
VIS=VIS/(0.1D 03*0.6891D 05)
UO(M)=0.4D 01*DR*RE*VIS*Z*P(1,M)*CONS*WX(1,M)/(KEY*PF
** (PAVG2+B))
1+0.2D 01*DR*RE*BB*Z*WX(1,M)*DABS(WX(1,M)) + U(2,M)
IF(UO(M)) 95,97,97
95 PO(M)=0.1D-09

```

## MAIN LINE ... (CONT'D)

```

GO TO 962
97  PO(M)=DSQRT(UG(M))
962  PAVG2=0.5*(PO(M)+P(1,M))
     PRE=PAVG2*PF
     CALL FPRESS(PRE,T)
     VIS=VIS/(0.1D 03*0.6891D 05)
     K2(L,M)=-((0.5D 00*DR-RW)*KEY*PF*(PAVG2+B))/(0.2D 01*RE
**VIS*Z*
1PAVG2*CONS+RE*BB*KEY*PF*(PAVG2+B)*Z*DABS(WX(L,M)))
     GO TO 5
9994 FL=IFG-1
     KK=KK+1
     F1=KK
     FLL=F1-0.5
     K2(L,M)=(FL*DR+RW+FLL*DR2)*KEY*PF*(PAVG2+B)/(0.2D 01
**RE*VIS*Z*
1PAVG2*CONS+RE*BB*KEY*PF*(PAVG2+B)*Z*DABS(WX(L,M)))
5   CONTINUE

C   CALCULATING K1(L+1/2) AT (M)
C

     KK=0
     DO 4 L=1,LL
     PAVG1=0.5*(P(L+1,M)+P(L,M))
     PRE=PAVG1*PF
     CALL FPRESS(PRE,T)
     VIS=VIS/(0.1D 03*0.6891D 05)
     IF(ICONT.EQ.0) GO TO 9999
     IF(L.GE.IFG) GO TO 9991
9999 FLL=L
     FL=FLL-0.5
     K1(L,M)=(FL*DR+RW)*KEY*PF*(PAVG1+B)/(0.2D 01*RE*VIS*Z
**PAVG1*
1CONS+RE*BB*KEY*PF*(PAVG1+B)*Z*DABS(WX(L+1,M)))
     GO TO 4
9991 FL=IFG-1
     KK=KK+1
     F1=KK
     FLL=F1-0.5
     K1(L,M)=(FL*DR+RW+FLL*DR2)*KEY*PF*(PAVG1+B)/(0.2D 01
**RE*VIS*Z*
1PAVG1*CONS+RE*BB*KEY*PF*(PAVG1+B)*Z*DABS(WX(L+1,M)))
4   CONTINUE

C   CALCULATING G1 AT (M)
C

     KK=0
     DO 5000 L=1,LL

```

## MAIN LINE ... (CONT'D)

```

PAVG3=P(L,M)
PRE=PAVG3*PF
CALL FPRESS(PRE,T)

C   IF(IPOR.EQ.0) GO TO 8666
      IF(ICONT.EQ.0) GO TO 9995

C   PHI=0.1D 01-(0.1D 01-PHI0)*DEXP(-CR*(PRE-PSTD))
C8666 IF(ICONT.EQ.0) GO TO 9995
      IF(L.GE.IFG+1) GO TO 9996
9995  FL=L-1
      IF(L-1) 536,536,537

C536  G1(L,M)=PHI*RW*PF/Z*(CG+CR*(0.1D 01/PHI-0.1D 01))
      536  G1(L,M)=RW*PF/Z*(CG+CR*(0.1D 01/PHI-0.1D 01))
      GO TO 5000

C537  G1(L,M)=PHI*(FL*DR+RW)*PF/Z*(CG+CR*(0.1D 01/PHI-0.1D
C537  01))
      537  G1(L,M)=(FL*DR+RW)*PF/Z*(CG+CR*(0.1D 01/PHI-0.1D 01))
      GO TO 5000
9996  FL=IFG-1
      KK=KK+1
      F1=KK

C   G1(L,M)=PHI*(FL*DR+RW+F1*DR2)*PF/Z*(CG+CR*(0.1D
C   01/PHI-0.1D 01))
      G1(L,M)=(FL*DR+RW+F1*DR2)*PF/Z*(CG+CR*(0.1D 01/PHI
      *-0.1D 01))
5000  CONTINUE

C   STORING FIRST GUESSES OF P AND U AT (M+1)
C
      DO 454 L=1,LL
      PP(L,M+1)=P(L,M)
      UU(L,M+1)=U(L,M)
454  WWX(L,M+1)=WX(L,M)
      PP(LL+1,M+1)=PP(LL-1,M+1)
      UU(LL+1,M+1)=UU(LL-1,M+1)
      WWX(1,M+1)=FLOW

C   CALCULATING K2AVG
C

```



## MAIN LINE ...(CONT'D)

```

24  CONTINUE
    KK=0
    DO 1051 L=1,LL
    IF(L.EQ.1) GO TO 5004
    PAVG2=0.5*(PP(L,M+1)+PP(L-1,M+1))
    PRE=PAVG2*PF
    CALL FPRESS(PRE,T)
    VIS=VIS/(0.1D 03*0.6891D 05)
    IF(ICONT.EQ.0) GO TO 3001
    IF(L.GE.IFG+1) GO TO 3002
3001 FLL=L
    FL=FLL-1.5
    K2(L,M+1)=(FL*DR+RW)*KEY*PF*(PAVG2+B)/(0.2D 01*RE*VIS
**Z*PAVG2*
1CONS+RE*BB*KEY*PF*(PAVG2+B)*Z*DABS(WWX(L,M+1)))
    K2AVG(L,M)=(K2(L,M)+K2(L,M+1))/2.
    GO TO 1051
5004 PAVG2=PP(1,M+1)
    PRE=PAVG2*PF
    CALL FPRESS(PRE,T)
    VIS=VIS/(0.1D 03*0.6891D 05)
    U0(M+1)=0.4D 01*DR*RE*VIS*Z*PP(1,M+1)*CONS*WWX(1,M+1)
*/(KEY*PF*
1 (PP(1,M+1)+B) + 0.2D 01*DR*RE*BB*Z*WWX(1,M+1)
**DABS(WWX(1,M+1))
2+UU(2,M+1)
    IF(U0(M+1)) 955,957,957
955  P0(M+1)=0.1D-09
    GO TO 961
957  P0(M+1)=DSQRT(U0(M+1))
961  PAVG2=0.5*(P0(M+1)+PP(1,M+1))
    PRE=PAVG2*PF
    CALL FPRESS(PRE,T)
    VIS=VIS/(0.1D 03*0.6891D 05)
    K2(L,M+1)=-((0.5D 00*DR-RW)*KEY*PF*(PAVG2+B)/(0.2D 01
**RE*VIS*Z*
1PAVG2*CONS+RE*BB*KEY*PF*(PAVG2+B)*Z*DABS(WWX(L,M+1)))
    K2AVG(L,M)=0.5*(K2(L,M)+K2(L,M+1))
    GO TO 1051
3002 FL=IFG-1
    KK=KK+1
    F1=KK
    FLL=F1-0.5
    K2(L,M+1)=(FL*DR+RW+FLL*DR2)*KEY*PF*(PAVG2+B)/(0.2D 01
**RE*VIS
1*Z*PAVG2*CONS+RE*BB*KEY*PF*(PAVG2+B)*Z*DABS(WWX(L,M
**+1)))
    K2AVG(L,M)=0.5*(K2(L,M)+K2(L,M+1))
1051 CONTINUE

```

## MAIN LINE ... (CONT'D)

C CALCULATING K1AVG  
C

```

KK=0
DO 1050 L=1,LL
PAVG1=0.5*(PP(L+1,M+1)+PP(L,M+1))
PRE=PAVG1*PF
CALL FPRESS(PRE,T)
VIS=VIS/(0.1D 03*0.6891D 05)
IF(ICONT.EQ.0) GO TO 3003
IF(L.GE.IFG) GO TO 3004
3003 FLL=L
FL=FLL-0.5
K1(L,M+1)=(FL*DR+RW)*KEY*PF*(PAVG1+B)/(0.2D 01*RE*VIS
**Z*PAVG1*
1CONS+RE*BB*KEY*PF*(PAVG1+B)*Z*DABS(WWX(L+1,M+1)))
K1AVG(L,M)=0.5*(K1(L,M)+K1(L,M+1))
GO TO 1050
3004 FL=IFG-1
KK=KK+1
F1=KK
FLL=F1-0.5
K1(L,M+1)=(FL*DR+RW+FLL*DR2)*KEY*PF*(PAVG1+B)/(0.2D 01
**RE*VIS
1*Z*PAVG1*CONS+RE*BB*KEY*PF*(PAVG1+B)*Z*DABS(WWX(L+1,M
**1)))
K1AVG(L,M)=0.5*(K1(L,M)+K1(L,M+1))
1050 CONTINUE

```

C CALCULATING G1AVG(L,M)  
C

```

KK=0
DO 5006 L=1,LL
PAVG3=PP(L,M+1)
PRE=PAVG3*PF
CALL FPRESS(PRE,T)

C IF(IPDR.EQ.0) GO TO 7836
C PHI=0.1D 01-(0.1D 01-PHI0)*DEXP(-CR*(PRE-PSTD))
C7836 IF(ICONT.EQ.0) GO TO 3005

IF(ICONT.EQ.0) GO TO 3005
IF(L.GE.IFG+1) GO TO 3006
3005 FL=L-1
IF(L-1) 694,694,695

C694 G1(L,M+1)=PHI*RW*PF/Z*(CG+CR*(0.1D 01/PHI-0.1D 01))

694 G1(L,M+1)=RW*PF/Z*(CG+CR*(0.1D 01/PHI-0.1D 01))

```

## MAIN LINE ... (CONT'D)

```

GO TO 5005

C695 G1(L,M+1)=PHI*(FL*DR+RW)*PF/Z*(CG+CR*(0.1D 01/PHI-0.1D
C695 01))

695 G1(L,M+1)=(FL*DR+RW)*PF/Z*(CG+CR*(0.1D 01/PHI-0.1D
* 01))
5005 G1AVG(L,M)=0.5*(G1(L,M)+G1(L,M+1))
GO TO 5006
3006 FL=IFG-1
KK=KK+1
F1=KK

C G1(L,M+1)=PHI*(FL*DR+RW+F1*DR2)*PF/Z*(CG+CR*(0.1D
C 01/PHI-0.1D 01))

G1(L,M+1)=(FL*DR+RW+F1*DR2)*PF/Z*(CG+CR*(0.1D 01/PHI
*-0.1D 01))
G1AVG(L,M)=0.5*(G1(L,M)+G1(L,M+1))
5006 CONTINUE

C SETTING THE COEFFICIENTS AND GETTING THE SOLUTION
C AT (M+1) USING THE THOMAS ALGORITHM
C

RHO=2.*DR**2/DT
RHO1=DR2*DR*(DR2+DR)/DT
RHO2=0.2D 01*DR2**2/DT

C FOR L=1

BC=-RHO *G1AVG(1,M)-K1AVG(1,M)-K2AVG(1,M)
E(1)=BC
G(1)=(-K2AVG(1,M)*(UO(M+1)-UU(2,M+1))-K2AVG(1,M)*UO(M)
*+
1(K1AVG(1,M)+K2AVG(1,M)-RHO*G1AVG(1,M))*U(1,M)-K1AVG(1
*,M)
2*U(2,M))/BC

C FOR INTERMEDIATE GRID POINTS
C

DO 3 L=2,LLL
IF(ICONT.FQ.0.OR.L.LT.IFG) GO TO 3007
IF(L.EQ.IFG) GO TO 3008
RHO=RHO2
3007 BC=-RHO*G1AVG(L,M)-K1AVG(L,M)-K2AVG(L,M)
IF(L.EQ.IFG+1) GO TO 3016
IF(L.GT.2) GO TO 666
E(L)=BC-K2AVG(L,M)*(K1AVG(L-1,M)+K2AVG(L-1,M))/E(L-1)

```

## MAIN LINE ... (CONT'D)

```

      GO TO 444
666  E(L)=BC-K2AVG(L,M)*K1AVG(L-1,M)/E(L-1)
      GO TO 444
3016 E(L)=BC-K2AVG(L,M)*K1AVG(L-1,M)*DR/E(L-1)
444  D=-K2AVG(L,M)*U(L-1,M)+(K1AVG(L,M)+K2AVG(L,M)-RHO
      **G1AVG(L,M))*
      1U(L,M)-K1AVG(L,M)*U(L+1,M)
      G(L)=(D-K2AVG(L,M)*G(L-1))/E(L)
      GO TO 3
3008 BC=-RHO1*G1AVG(L,M)-K1AVG(L,M)*DR-K2AVG(L,M)*DR2
      E(L)=BC-K2AVG(L,M)*DR2*K1AVG(L-1,M)/E(L-1)
      D=-K2AVG(L,M)*DR2*U(L-1,M)+(K1AVG(L,M)*DR+K2AVG(L,M)
      **DR2-RHO1*
      1G1AVG(L,M)*U(L,M)-K1AVG(L,M)*DR*U(L+1,M)
      G(L)=(D-K2AVG(L,M)*DR2*G(L-1))/E(L)
3    CONTINUE

C    FOR L=LL
C

      BC=-RHO*G1AVG(LL,M)-K1AVG(LL,M)-K2AVG(LL,M)
      D=(K1AVG(LL,M)+K2AVG(LL,M)-RHO*G1AVG(LL,M))*U(LL,M)+(
      *-K1AVG(LL,M)
      1-K2AVG(LL,M))*U(LL-1,M)
      E(LL)=BC-(K1AVG(LL,M)+K2AVG(LL,M))*K1AVG(LL-1,M)/E(LL
      *-1)
      G(LL)=(D-(K1AVG(LL,M)+K2AVG(LL,M))*G(LL-1))/E(LL)
      U(LL,M+1)=G(LL)

C    GETTING THE SOLUTIONS FOR THE REST OF THE GRID POINTS
C

      DO 8J=1,LLL
      I=LL-J
      IF(I.EQ.1) GO TO 333
      IF(I.EQ.1FG) GO TO 3017
      U(I,M+1)=G(I)-K1AVG(I,M)*U(I+1,M+1)/E(I)
      GO TO 8
333  U(I,M+1)=G(I)-(K1AVG(I,M)+K2AVG(I,M))*U(I+1,M+1)/E(I)
      GO TO 8
3017 U(I,M+1)=G(I)-K1AVG(I,M)*DR*U(I+1,M+1)/E(I)
      8    CONTINUE

C    SETTING PRESSURES AT (M+1) FOR L=1,LL
C

      DO 6 L=1,LL
      IF(U(1,M+1).GT. 0.005) GO TO 6

C    OTHERWISE TERMINATE PRODUCTION

```

## MAIN LINE ... (CONT'D)

```

C
DO 69 K=1,LL
WX(K,M+1)=0.1D-16
U(K,M+1)=0.005
69 CONTINUE
MB=M
GO TO 678
6 P(L,M+1)=DSQRT(U(L,M+1))

C CALCULATING THE MASS FLUX AT TIME STEP (M+1) USING
C SOLUTION OBTAINED AT (M+1) LEVEL
C

DO 7 L=2,LL
IF(L.GT.1FG) GO TO 3009
DER(L,M+1)=(U(L,M+1)-U(L-1,M+1))/DR
DER(L,M)=(U(L,M)-U(L-1,M))/DR
GO TO 3010
3009 DER(L,M+1)=(U(L,M+1)-U(L-1,M+1))/DR2
DER(L,M)=(U(L,M)-U(L-1,M))/DR2
3010 DERAvg(L,M)=0.5*(DER(L,M+1)+DER(L,M))
PAVG1=0.5*(P(L,M+1)+P(L-1,M+1))
PAVG2=0.5*(P(L,M)+P(L-1,M))
PAVG=0.5*(PAVG1+PAVG2)
PRE=PAVG*PF
CALL FPRESS(PRE,T)
VIS=VIS/(0.1D 03*0.6891D 05)
BCO(L)=-0.2D 01*RE*VIS*Z*PAVG*CONS/(KEY*PF*(PAVG+B))
ACO(L)=-RE*BB*Z
DCO(L)=DSQRT(BCO(L)*BCO(L)-4.*ACO(L)*DERAVG(L,M))
IF(DERAvg(L,M).LE.0.1E-05) GO TO 17

C ALTERNATIVE FOR QUADRATIC CONSTANT = 0.
C

IF(DABS(ACO(L)).GT.0.10E-03)GO TO 15
WX(L,M+1)=-1.*(-DERAVG(L,M)/BCO(L))
GO TO 7
15 WX1=-(-BCO(L)-DCO(L))/(2.0*ACO(L))
WX2=-(-BCO(L)+DCO(L))/(2.0*ACO(L))
IF((WX1+DABS(WX1)).LT.0.10E-04) GO TO 18
IF((WX2+DABS(WX2)).LT.0.10E-04) GO TO 9
18 WX(L,M+1)=WX1
IF((WX2+DABS(WX2)).LT.0.10E-04) GO TO 7
GO TO 7
17 WX(L,M+1)=0.
GO TO 7
9 WX(L,M+1)=WX2
7 CONTINUE

```

## MAIN LINE ... (CONT'D)

C CHECKING CONVERGENCE OF THE SOLUTION  
C

```

DO 10 L=1,LL
ERR(L)=DABS(PP(L,M+1)-P(L,M+1))
IF(ERR(L).LE.EPS) GO TO 10
GO TO 429
10 CONTINUE
GO TO 11
429 DO 13 L=1,LL
PP(L,M+1)=P(L,M+1)
UU(L,M+1)=U(L,M+1)
13 WWX(L,M+1)=WX(L,M+1)
ITN=ITN+1
IF(ITN.GT.10) GO TO 12
GO TO 24
11 CONTINUE

```

C PROBLEM HAS CONVERGED FOR TIME STEP (M+1)  
C

```

GO TO 678
12 WRITE(6,713)
713 FORMAT(2X,'*')
678 TIME=TIME+DTIME(M)
IF(M.EQ.11.OR.M.EQ.21) GO TO 476
GO TO 477
476 WRITE(6,231)
WRITE(6,230)
WRITE(6,230)
477 WRITE(6,208) TIME
208 FORMAT(6X,E10.3)
78 WRITE(6,224) (U(L,M+1),L=1,LL,3)
224 FORMAT(16X,F7.4,2X,F7.4,2X,F7.4,2X,F7.4,2X,F7.4,2X
*,F7.4,2X,
1F7.4,2X,F7.4,2X,F7.4,2X,F7.4)
IF(U(1,M+1).LE.0.005) GO TO 679
99 CONTINUE
MB=M
679 MC=MB-4

```

C CALCULATING FRACTION OF GAS PRODUCED USING AVERAGE  
C FLUXES  
C

```

DO 291 I=1,II
MK=MT(I)
IF(U(1,M+1).LE.0.005) MK=MC
FLUX1=0.

```

## MAIN LINE ... (CONT'D)

```

FLUXLL=0.
DO 395 K=1,MK
FAVG1=DABS(WX(1,K)+WX(1,K+1))/2.
FAVGLL=DABS(WX(LL,K)+WX(LL,K+1))/2.
OUT=FAVG1*DTIME(K)
IN=FAVGLL*DTIME(K)
FLUX1=FLUX1+OUT
FLUXLL=FLUXLL+IN
395 CONTINUE

C GP2(I)=ZI/(PHIRI*(1.-RW**2))*(RW*FLUX1-FLUXLL)

GP2(I)=ZI*PHIO/(PHIRI*(1.-RW**2))*(RW*FLUX1-FLUXLL)
MC=MC+1
291 CONTINUE
WRITE(6,231)
WRITE(6,230)

C WRITE(6,8667) IPOR

WRITE(6,230)
B=B*PF
WRITE(6,205) B,BB,PERM
WRITE(6,210)
210 FORMAT(1H0,33X,'FLUX DISTRIBUTION FOR THE FIRST 40
* GRID POINTS')
WRITE(6,230)
TIME=0.
DO 75 M=1,MB
TIME=TIME+DTIME(M)
IF(M.EQ.9.OR.M.EQ.17) GO TO 494
GO TO 445
494 WRITE(6,231)
WRITE(6,230)
WRITE(6,230)
445 WRITE(6,208) TIME
WRITE(6,515) (WX(L,M+1),L=1,40)
515 FORMAT(16X,10D11.3)
75 CONTINUE
WRITE(6,9000)
9000 FORMAT(1H0,30X,'FRACTION OF GAS PRODUCED AT LAST FIVE
* TIME LEVELS'
1)
WRITE(6,9003) (GP2(I),I=1,II)
9003 FORMAT(/,7X,'FRACTION OF GAS PRODUCED=',5(3X,E10.3))
IF(JK.EQ.NSETS) GO TO 613
GO TO 999
613 WRITE(6,231)
STOP
END

```

K.3

CONSTANT TERMINAL PRESSURE CASE AND CONSTANT  
EXTERNAL PRESSURE: CASE III



## MAIN LINE

```

C *****
C
C TRANSIENT ISOTHERMAL RADIAL GAS FLOW IN POROUS MEDIA
C
C CASE III
C
C CONSTANT TERMINAL PRESSURE CASE AND CONSTANT
C EXTERNAL PRESSURE
C *****
C

```

```

REAL*8 K1(196,36),K2(196,36),G1(196,36),ERR(196),
1P0(36),U0(36),BA(196),K1AVG(196,36),K2AVG(196,36),
2WX(196,36),WWX(196,36),P(196,36),PP(196,36),U(196,36)
3, UU(196,36),G1AVG(196,36),ACD(196),BCD(196),DCD(196),
4CC(196),DTIME(36),DSQRT,RHO,DEXP,DABS,WX1,WX2
REAL*8 DER(196,36),DERAVG(196,36),E(196),G(196),D,BC,
1RW,RE,B,BB,FL,DR,PAVG,PAVG1,PAVG2,CG,CR,Z,VIS,PHI,PF,
2FLOW,CONS,DT,PRE,R,MW,PERM,KEY,ALPHA,PI,VISI,CGI,ZI,
3RH01,RH02,DR2,PSTD,PHI0,PHIRI
DIMENSION GP2(20),MT(20)
COMMON WX,WWX,P,PP,U,UU,Z,CG,VIS,FLOW,LL,MM,ITN
INTEGER L,LL,LLL,LLLL,M,MM,MMM,J,MB,ITN
INTEGER IFG,ICONT

```

```

C READ INPUT DATA
C

```

```

READ(5,520) PI
READ(5,520) R,MW
520 FORMAT(5D13.6)
READ(5,520) RW,RE,CR,PHI
READ(5,520) PF,DR,DR2
READ(5,521) NSETS,MM,LL,T,EPS
521 FORMAT(3I4,2E13.6)
MMM=MM-1
READ(5,520) (DTIME(I),I=1,MMM)
II=5

```

```

C READ IN REFERENCES OF TIME INCREMENTS MT(I) AT WHICH
C FRACTION OF GAS PRODUCED WILL BE CALCULATED
C

```

```

READ(5,293) (MT(I),I=1,II)
293 FORMAT(6I6)
READ(5,293) ICONT,IFG,IPOR
WRITE(6,3015) ICONT, IFG, DR, DR2
3015 FORMAT(2I6,2F12.7)
ALPHA=MW/(R*(T+460.))

```

## MAIN LINE ... (CONT'D)

```

CONS=DSQRT(0.72D 02*0.32174D 02/ALPHA)
LLL=LL-1
RW=RW/(0.12D 02*RE)
PSTD=0.14697D 02
PHIO=PHI
JK=0
CALL FPRESS(PI,T)
ZI=Z

C   CALCULATING POROSITY AT INITIAL RESERVOIR PRESSURE
C

PHIRI=0.1D 01-(0.1D 01-PHIO)*DEXP(-CR*(PI-PSTD))
999 READ(5,520) PERM,B,BB
    JK=JK+1
    CRIT=0.5*DR-RW
    IF(CRIT) 875,875,876
876 WRITE(6,295)
295 FORMAT(/,8X,'VALUE OF K2AVG AT POINT 1 WILL BE
1NEGATIVE BECAUSE 0.5*DR EXCEEDS THE VALUE OF RW.
2THIS ORIGINATES ANOMALOUS SOLUTIONS AND THEREFORE
3EXECUTION WAS SUPPRESSED')
    GO TO 613
875 CONTINUE
    KEY=PERM*0.1062D-13
    TIME=0.
    WRITE(6,231)
231 FORMAT('1')
    WRITE(6,230)
230 FORMAT('-')
    WRITE(6,1059)
1059 FORMAT(36X,'TRANSIENT RADIAL GAS FLOW')

C   WRITE(6,8667) IPOR
C8667 FORMAT(/,44X,'IPOR=',I2,/)

    WRITE(6,230)
    WRITE(6,205) B,BB,PERM
205 FORMAT(1H ,13X,'SLIP COEFF(B)=' ,F7.3, ' INERTIAL
* COEFF', '(BB)=' ,
1D10.3, ' PERMEABILITY(K)=' ,F8.3,1X, 'MD')
    WRITE(6,206) PHI,CR,PI
206 FORMAT(/,14X,'POROSITY=' ,F7.3,8X, 'ROCK
* COMPRESSIBILITY=' ,
1D10.3,2X, 'INITIAL PRESSURE=' ,D10.3)
    WRITE(6,207)
207 FORMAT(1H0,35X,'PRESSURE SQUARE DISTRIBUTION')
    WRITE(6,220)
220 FORMAT(8X, 'TIME',26X, 'DIMENSIONLESS DISTANCE')
    WRITE(6,221)

```

## MAIN LINE ... (CONT'D)

```

221  FORMAT(8X,'(TD)          RW          DELTA R1=0.0010
      * DELTA R2=0.
      1032724')
      WRITE(6,223)
223  FORMAT(1X,'  ')

C    SETTING THE DIMENSIONLESS SLIPPAGE
C

      B=B/PF
      INDEX=3
      CALL BCOND(INDEX)
      DO 99 M=1,MMM
      . DT=DTIME(M)
      ITN=0

C
C    CALCULATING K2(L-1/2) AT (M)
C

      KK=0
      DO 5 L=2,LLL
      IF(L.EQ.1) GO TO 777
      PAVG2=0.5*(P(L,M)+P(L-1,M))
      PRE=PAVG2*PF
      CALL FPRESS(PRE,T)
      VIS=VIS/(0.1D 03*0.6891D 05)
      IF(ICONT.EQ.0) GO TO 9993
      IF(L.GE.IFG+1) GO TO 9994
9993  FLL=L
      FL=FLL-1.5
      K2(L,M)=(FL*DR+RW)*KEY*PF*(PAVG2+B)/(0.2D 01*RE*VIS*Z
      **PAVG2*
      1CONS+RE*BB*KEY*PF*(PAVG2+B)*Z*DABS(WX(L,M)))
      GO TO 5
777  PAVG2=P(1,M)
      PRE=PAVG2*PF
      CALL FPRESS(PRE,T)
      VIS=VIS/(0.1D 03*0.6891D 05)
      U0(M)=0.4D 01*DR*RE*VIS*Z*P(1,M)*CONS*WX(1,M)/(KEY*PF
      ** (PAVG2+B)
      1+0.2D 01*DR*RE*BB*Z*WX(1,M)*DABS(WX(1,M)) + U(2,M)
      IF(U0(M)) 95,97,97
95   PO(M)=0.1D-09
      GO TO 962
97   PO(M)=DSQRT(U0(M))
962  PAVG2=0.5*(PO(M)+P(1,M))
      PRE=PAVG2*PF
      CALL FPRESS(PRE,T)
      VIS=VIS/(0.1D 03*0.6891D 05)

```

## MAIN LINE ... (CONT'D)

```

      K2(L,M)=-(.05D 00*DR-RW)*KEY*PF*(PAVG2+B)/(0.2D 01*RE
**VIS*Z*
      1PAVG2*CONS+RE*BB*KEY*PF*(PAVG2+B)*Z*DABS(WX(L,M)))
      GO TO 5
9994 FL=IFG-1
      KK=KK+1
      F1=KK
      FLL=F1-0.5
      K2(L,M)=(FL*DR+RW+FLL*DR2)*KEY*PF*(PAVG2+B)/(0.2D 01
**RE*VIS*Z*
      1PAVG2*CONS+RE*BB*KEY*PF*(PAVG2+B)*Z*DABS(WX(L,M)))
5      CONTINUE

C      CALCULATING K1(L+1/2) AT (M)

      KK=0
      DO 4 L=2,LLL
      PAVG1=0.5*(P(L+1,M)+P(L,M))
      PRE=PAVG1*PF
      CALL FPRESS(PRE,T)
      VIS=VIS/(0.1D 03*0.6891D 05)
      IF(ICONT.EQ.0) GO TO 9999
      IF(L.GE.IFG) GO TO 9991
9999 FLL=L
      FL=FLL-0.5
      K1(L,M)=(FL*DR+RW)*KEY*PF*(PAVG1+B)/(0.2D 01*RE*VIS*Z
**PAVG1*
      1CONS+RE*BB*KEY*PF*(PAVG1+B)*Z*DABS(WX(L+1,M)))
      GO TO 4
9991 FL=IFG-1
      KK=KK+1
      F1=KK
      FLL=F1-0.5
      K1(L,M)=(FL*DR+RW+FLL*DR2)*KEY*PF*(PAVG1+B)/(0.2D 01
**RE*VIS*Z*
      1PAVG1*CONS+RE*BB*KEY*PF*(PAVG1+B)*Z*DABS(WX(L+1,M)))
4      CONTINUE

C      CALCULATING G1 AT (M)
C

      KK=0
      DO 5000 L=2,LLL
      PAVG3=P(L,M)
      PRE=PAVG3*PF
      CALL FPRESS(PRE,T)

C      IF(IPOR.EQ.0) GO TO 8666

      IF(ICONT.EQ.0) GO TO 9995

```

## MAIN LINE ... (CONT'D)

```

C      PHI=0.1D 01-(0.1D 01-PHI0)*DEXP(-CR*(PRE-PSTD))
C8666 IF(ICONT.EQ.0) GO TO 9995

      IF(L.GE.IFG+1) GO TO 9996
9995  FL=L-1
      IF(L-1) 536,536,537

C536  G1(L,M)=PHI*RW*PF/Z*(CG+CR*(0.1D 01/PHI-0.1D 01))

      536  G1(L,M)=RW*PF/Z*(CG+CR*(0.1D 01/PHI-0.1D 01))
      GO TO 5000

C537  G1(L,M)=PHI*(FL*DR+RW)*PF/Z*(CG+CR*(0.1D 01/PHI-0.1D
C537  01))

      537  G1(L,M)=(FL*DR+RW)*PF/Z*(CG+CR*(0.1D 01/PHI-0.1D 01))
      GO TO 5000
9996  FL=IFG-1
      KK=KK+1
      F1=KK

C      G1(L,M)=PHI*(FL*DR+RW+F1*DR2)*PF/Z*(CG+CR*(0.1D
C      01/PHI-0.1D 01))

      G1(L,M)=(FL*DR+RW+F1*DR2)*PF/Z*(CG+CR*(0.1D 01/PHI
*-0.1D 01))
5000 CONTINUE

C      STORING FIRST GUESSES OF P AND U AT (M+1)
C

      DO 454 L=1,LL
      PP(L,M+1)=P(L,M)
      UU(L,M+1)=U(L,M)
454   WWX(L,M+1)=WX(L,M)
      PP(1,M+1)=0.5
      UU(1,M+1)=0.25

C      CALCULATING K2AVG
C

24   CONTINUE
      KK=0
      DO 1051 L=2,LLL
      IF(L.EQ.1) GO TO 5004
      PAVG2=0.5*(PP(L,M+1)+PP(L-1,M+1))
      PRE=PAVG2*PF
      CALL FPRESS(PRE,T)
      VIS=VIS/(0.1D 03*0.6891D 05)

```

## MAIN LINE ... (CONT'D)

```

IF(ICONT.EQ.0) GO TO 3001
IF(L.GE.IFG+1) GO TO 3002
3001 FLL=L
    FL=FLL-1.5
    K2(L,M+1)=(FL*DR+RW)*KEY*PF*(PAVG2+B)/(0.2D 01*RE*VIS
**Z*PAVG2*
1CONS+RE*BB*KEY*PF*(PAVG2+B)*Z*DABS(WWX(L,M+1)))
    K2AVG(L,M)=(K2(L,M)+K2(L,M+1))/2.
    GO TO 1051
5004 PAVG2=PP(1,M+1)
    PRE=PAVG2*PF
    CALL FPRESS(PRE,T)
    VIS=VIS/(0.1D 03*0.6891D 05)
    U0(M+1)=0.4D 01*DR*RE*VIS*Z*PP(1,M+1)*CONS*WWX(1,M+1)
*/(KEY*PF*
1 (PP(1,M+1)+B)) + 0.2D 01*DR*RE*BB*Z*WWX(1,M+1)
**DABS(WWX(1,M+1))
2+UU(2,M+1)
    IF(U0(M+1)) 955,957,957
955 P0(M+1)=0.1D-09
    GO TO 961
957 P0(M+1)=DSQRT(U0(M+1))
961 PAVG2=0.5*(P0(M+1)+PP(1,M+1))
    PRE=PAVG2*PF
    CALL FPRESS(PRE,T)
    VIS=VIS/(0.1D 03*0.6891D 05)
    K2(L,M+1)--(0.5D 00*DR-RW)*KEY*PF*(PAVG2+B)/(0.2D 01
**RE*VIS*Z*
1PAVG2*CONS+RE*BB*KEY*PF*(PAVG2+B)*Z*DABS(WWX(L,M+1)))
    K2AVG(L,M)=0.5*(K2(L,M)+K2(L,M+1))
    GO TO 1051
3002 FL=IFG-1
    KK=KK+1
    F1=KK
    FLL=F1-0.5
    K2(L,M+1)=(FL*DR+RW+FLL*DR2)*KEY*PF*(PAVG2+B)/(0.2D 01
**RE*VIS
1*Z*PAVG2*CONS+RE*BB*KEY*PF*(PAVG2+B)*Z*DABS(WWX(L,M
**+1)))
    K2AVG(L,M)=0.5*(K2(L,M)+K2(L,M+1))
1051 CONTINUE

```

```

C    CALCULATING K1AVG
C

```

```

KK=0
DO 1050 L=2,LLL
PAVG1=0.5*(PP(L+1,M+1)+PP(L,M+1))
PRE=PAVG1*PF
CALL FPRESS(PRE,T)

```

## MAIN LINE ... (CONT'D)

```

VIS=VIS/(0.1D 03*0.6891D 05)
IF(ICONT.EQ.0) GO TO 3003
IF(L.GE.IFG) GO TO 3004
3003 FLL=L
    FL=FLL-0.5
    K1(L,M+1)=(FL*DR+RW)*KEY*PF*(PAVG1+B)/(0.2D 01*RE*VIS
**Z*PAVG1*
I CONS+RE*BB*KEY*PF*(PAVG1+B)*Z*DABS(WWX(L+1,M+1)))
    K1AVG(L,M)=0.5*(K1(L,M)+K1(L,M+1))
    GO TO 1050
3004 FL=IFG-1
    KK=KK+1
    F1=KK
    FLL=F1-0.5
    K1(L,M+1)=(FL*DR+RW+FLL*DR2)*KEY*PF*(PAVG1+B)/(0.2D 01
**RE*VIS
1*Z*PAVG1*CONS+RE*BB*KEY*PF*(PAVG1+B)*Z*DABS(WWX(L+1,M
**+1))
    K1AVG(L,M)=0.5*(K1(L,M)+K1(L,M+1))
1050 CONTINUE

C    CALCULATING G1AVG(L,M)
C

    KK=0
    DO 5006 L=2,LLL
    PAVG3=PP(L,M+1)
    PRE=PAVG3*PF
    CALL FPRESS(PRE,T)

C    IF(IPOR.EQ.0) GO TO 7836
C    PHI=0.1D 01-(0.1D 01-PHI0)*DEXP(-CR*(PRE-PSTD))
C7836 IF(ICONT.EQ.0) GO TO 3005

    IF(ICONT.EQ.0) GO TO 3005
    IF(L.GE.IFG+1) GO TO 3006
3005 FL=L-1
    IF(L-1) 694,694,695

C694 G1(L,M+1)=PHI*RW*PF/Z*(CG+CR*(0.1D 01/PHI-0.1D 01))
694 G1(L,M+1)=RW*PF/Z*(CG+CR*(0.1D 01/PHI-0.1D 01))
    GO TO 5005

C695 G1(L,M+1)=PHI*(FL*DR+RW)*PF/Z*(CG+CR*(0.1D 01/PHI-0.1D
C695 01))

695 G1(L,M+1)=(FL*DR+RW)*PF/Z*(CG+CR*(0.1D 01/PHI-0.1D
* 01))
5005 G1AVG(L,M)=0.5*(G1(L,M)+G1(L,M+1))

```

## MAIN LINE ... (CONT'D)

```

      GO TO 5006
3006 FL=IFG-1
      KK=KK+1
      F1=KK

C      G1(L,M+1)=PHI*(FL*DR+RW+F1*DR2)*PF/Z*(CG+CR*(0.1D
C      01/PHI-0.1D 01))

      G1(L,M+1)=(FL*DR+RW+F1*DR2)*PF/Z*(CG+CR*(0.1D 01/PHI
      *-0.1D 01))
      G1AVG(L,M)=0.5*(G1(L,M)+G1(L,M+1))
5006 CONTINUE

C      SETTING THE COEFFICIENTS AND GETTING THE SOLUTION
C      AT (M+1) USING THE THOMAS ALGORITHM
C

      RHO=2.*DR**2/DT
      RHO1=DR2*DR*(DR2+DR)/DT
      RHO2=0.2D 01*DR2**2/DT

C      FOR L=2

      BC=-RHO*G1AVG(2,M)-K1AVG(2,M)-K2AVG(2,M)
      E(2)=BC
      G(2)=(-K2AVG(2,M)*U(1,M+1)-K2AVG(2,M)*U(1,M)+(K1AVG(2
      *,M)+
      1K2AVG(2,M)-RHO*G1AVG(2,M))*U(2,M)-K1AVG(2,M)*U(3,M))
      */BC

      JF=LLL-1

C      FOR INTERMEDIATE GRID POINTS
C

      DO 3 L=3,JF
      IF(ICONT.EQ.0.OR.L.LT.IFG) GO TO 3007
      IF(L.EQ.IFG) GO TO 3008
      RHO=RHO2
3007 BC=-RHO*G1AVG(L,M)-K1AVG(L,M)-K2AVG(L,M)
      IF(L.EQ.IFG+1) GO TO 3016
      IF(L.GT.2) GO TO 666
      E(L)=BC-K2AVG(L,M)*(K1AVG(L-1,M)+K2AVG(L-1,M))/E(L-1)
      GO TO 444
666  E(L)=BC-K2AVG(L,M)*K1AVG(L-1,M)/E(L-1)
      GO TO 444
3016 E(L)=BC-K2AVG(L,M)*K1AVG(L-1,M)*DR/E(L-1)
444  D=-K2AVG(L,M)*U(L-1,M)+(K1AVG(L,M)+K2AVG(L,M)-RHO
      **G1AVG(L,M))*

```



## MAIN LINE ... (CONT'D)

```

1U(L,M)-K1AVG(L,M)*U(L+1,M)
  G(L)=(D-K2AVG(L,M)*G(L-1))/E(L)
  GO TO 3
3008 BC=-RHO1*G1AVG(L,M)-K1AVG(L,M)*DR-K2AVG(L,M)*DR2
      E(L)=BC-K2AVG(L,M)*DR2*K1AVG(L-1,M)/E(L-1)
      D=-K2AVG(L,M)*DR2*U(L-1,M)+(K1AVG(L,M)*DR+K2AVG(L,M)
      **DR2-RHO1*
      1G1AVG(L,M))*U(L,M)-K1AVG(L,M)*DR*U(L+1,M)
      G(L)=(D-K2AVG(L,M)*DR2*G(L-1))/E(L)
3  CONTINUE

C  FOR L=LLL
C

      BC=-RHO*G1AVG(LLL,M)-K1AVG(LLL,M)-K2AVG(LLL,M)
      D=-0.2D 01*K1AVG(LLL,M)-K2AVG(LLL,M)*U(LLL-1,M)
      **+(K1AVG(LLL,M)+
      1K2AVG(LLL,M)-RHO*G1AVG(LLL,M))*U(LLL,M)
      E(LLL)=BC-K2AVG(LLL,M)*K1AVG(LLL-1,M)/E(LLL-1)
      G(LLL)=(D-K2AVG(LLL,M)*G(LLL-1))/E(LLL)
      U(LLL,M+1)=G(LLL)
      JF1=JF-1

C  GETTING THE SOLUTIONS FOR THE REST OF THE GRID POINTS
C

      DO 8 J=1,JF1
      I=LLL-J
      IF(I.EQ.1) GO TO 333
      IF(I.EQ.IFG) GO TO 3017
      U(I,M+1)=G(I)-K1AVG(I,M)*U(I+1,M+1)/E(I)
      GO TO 8
333  U(I,M+1)=G(I)-(K1AVG(I,M)+K2AVG(I,M))*U(I+1,M+1)/E(I)
      GO TO 8
3017 U(I,M+1)=G(I)-K1AVG(I,M)*DR*U(I+1,M+1)/E(I)
8  CONTINUE

C  SETTING PRESSURES AT (M+1) FOR L=1,LL
C

      DO 6 L=1,LLL
6  P(L,M+1)=DSQRT(U(L,M+1))

C  CALCULATING THE MASS FLUX AT TIME STEP (M+1) USING
C  SOLUTION OBTAINED AT (M+1) LEVEL
C

      DO 7 L=1,LL
      IF(L.EQ.1) GO TO 3011
      IF(L.GT.IFG) GO TO 3009

```

## MAIN LINE ... (CONT'D)

```

DER(L,M+1)=(U(L,M+1)-U(L-1,M+1))/DR
DER(L,M)=(U(L,M)-U(L-1,M))/DR
GO TO 3010
3009 DER(L,M+1)=(U(L,M+1)-U(L-1,M+1))/DR2
DER(L,M)=(U(L,M)-U(L-1,M))/DR2
GO TO 3010
3011 DER(1,M+1)=(-3.*U(L,M+1)+4.*U(L+1,M+1)-U(L+2,M+1))/(2.
**DR)
DER(1,M)=(-3.*U(L,M)+4.*U(L+1,M)-U(L+2,M))/(2.*DR)
DERAVG(1,M)=0.5*(DER(1,M+1)+DER(1,M))
PAVG=(P(1,M+1)+P(1,M))/2.
GO TO 3012
3010 DERAVG(L,M)=0.5*(DER(L,M+1)+DER(L,M))
PAVG1=0.5*(P(L,M+1)+P(L-1,M+1))
PAVG2=0.5*(P(L,M)+P(L-1,M))
PAVG=0.5*(PAVG1+PAVG2)
3012 PRE=PAVG*PF
CALL FPRESS(PRE,T)
VIS=VIS/(0.1D 03#0.6891D 05)
BCO(L)=-0.2D 01*RE*VIS*Z*PAVG*CONS/(KEY*PF*(PAVG+B))
ACO(L)=-RE*BB*Z
DCO(L)=DSQRT(BCO(L)*BCO(L)-4.*ACO(L)*DERAVG(L,M))
IF(DERAVG(L,M).LE.0.1E-05) GO TO 17

```

C ALTERNATIVE FOR QUADRATIC CONSTANT = 0.  
C

```

IF(DABS(ACO(L)).GT.0.10E-03)GO TO 15
WX(L,M+1)=-1.*(-DERAVG(L,M)/BCO(L))
GO TO 7
15 WX1=(-BCO(L)-DCO(L))/(2.*ACO(L))
WX2=(-BCO(L)+DCO(L))/(2.*ACO(L))
IF((WX1+DABS(WX1)).LT.0.10E-04) GO TO 18
IF((WX2+DABS(WX2)).LT.0.10E-04) GO TO 9
18 WX(L,M+1)=WX1
IF((WX2+DABS(WX2)).LT.0.10E-04) GO TO 7
GO TO 7
17 WX(L,M+1)=0.
GO TO 7
9 WX(L,M+1)=WX2
7 CONTINUE

```

C CHECKING CONVERGENCE OF THE SOLUTION  
C

```

DO 10 L=2,LLL
ERR(L)=DABS(PP(L,M+1)-P(L,M+1))
IF(ERR(L).LE.EPS) GO TO 10
GO TO 429
10 CONTINUE

```

## MAIN LINE ... (CONT'D)

```

GO TO 11
429 DO 13 L=1,LL
    PP(L,M+1)=P(L,M+1)
    UU(L,M+1)=U(L,M+1)
13   WWX(L,M+1)=WX(L,M+1)
    ITN=ITN+1
    IF(ITN.GT.10) GO TO 12
    GO TO 24
11   CONTINUE

C   PROBLEM HAS CONVERGED FOR TIME STEP (M+1)
C

GO TO 678
12   WRITE(6,713)
713  FORMAT(2X,'*')
678  TIME=TIME+DTIME(M)
    IF(M.EQ.11.OR.M.EQ.21) GO TO 476
    GO TO 477
476  WRITE(6,231)
    WRITE(6,230)
    WRITE(6,230)
477  WRITE(6,208) TIME
208  FORMAT(6X,E10.3)
    78  WRITE(6,224) (U(L,M+1),L=1,LL,3)
224  FORMAT(16X,F7.4,2X,F7.4,2X,F7.4,2X,F7.4,2X,F7.4,2X,
*,F7.4,2X,
1F7.4,2X,F7.4,2X,F7.4,2X,F7.4)
99   CONTINUE
    MB=M

C   CALCULATING FRACTION OF GAS PRODUCED USING AVERAGE
C   FLUXES
C

DO 291 I=1,II
    MK=MT(I)
    FLUX1=0.
    FLUXLL=0.
    DO 395 K=1,MK
        FAVG1=DABS(WX(1,K)+WX(1,K+1))/2.
        FAVGLL=DABS(WX(LL,K)+WX(LL,K+1))/2.
        OUT=FAVG1*DTIME(K)
        IN=FAVGLL*DTIME(K)
        FLUX1=FLUX1+OUT
        FLUXLL=FLUXLL+IN
395  CONTINUE

C   GP2(I)=ZI/(PHIRI*(1.0-RW**2))*(RW*FLUX1-FLUXLL)

```

## MAIN LINE ... (CONT'D)

```
GP2(I)=ZI*PHIO/(PHIRI*(1.-RW**2))*(RW*FLUX1-FLUXLL)
291 CONTINUE
    WRITE(6,231)
    WRITE(6,230)

C    WRITE(6,8667) IPOR

    WRITE(6,230)
    B=B*PF
    WRITE(6,205) B,BB,PERM
    WRITE(6,210)
210  FORMAT(1H0,33X,'FLUX DISTRIBUTION FOR THE FIRST 40
    * GRID POINTS')
    WRITE(6,230)
    TIME=0.
    DO 75 M=1,MB
    TIME=TIME+DTIME(M)
    IF(M.EQ.9.OR.M.EQ.17) GO TO 494
    GO TO 445
494  WRITE(6,231)
    WRITE(6,230)
    WRITE(6,230)
445  WRITE(6,208) TIME
    WRITE(6,515) (WX(L,M+1),L=1,40)
515  FORMAT(16X,10D11.3)
    75 CONTINUE
    WRITE(6,9000)
9000 FORMAT(1H0,30X,'FRACTION OF GAS PRODUCED AT LAST FIVE
    * TIME LEVELS'
    1)
    WRITE(6,9003) (GP2(I),I=1,II)
9003 FORMAT(/,7X,'FRACTION OF GAS PRODUCED=',5(3X,E10.3))
    IF(JK.EQ.NSETS) GO TO 613
    GO TO 999
613  WRITE(6,231)
    STOP
    END
```

K.4

CONSTANT TERMINAL PRESSURE CASE AND SEALED  
EXTERNAL BOUNDARY: CASE IV

## MAIN LINE

```

C *****
C
C TRANSIENT ISOTHERMAL RADIAL GAS FLOW IN POROUS MEDIA
C
C CASE IV
C
C CONSTANT TERMINAL PRESSURE CASE AND SEALED
C EXTERNAL BOUNDARY
C *****
C
REAL*8 K1(196,36),K2(196,36),G1(196,36),ERR(196),
1PO(36),UO(36),BA(196),K1AVG(196,36),K2AVG(196,36),
2WX(196,36),WWX(196,36),P(196,36),PP(196,36),U(196,36)
3,UU(196,36),G1AVG(196,36),ACO(196),BCO(196),DCO(196),
4CC(196),DTIME(36),DSQRT,RHO,DEXP,DABS,WX1,WX2
REAL*8 DER(196,36),DERAVG(196,36),E(196),G(196),D,BC,
1RW,RE,B,BB,FL,DR,PAVG,PAVG1,PAVG2,CG,CR,Z,VIS,PHI,PF,
2FLOW,CONS,DT,PRE,R,MW,PERM,KEY,ALPHA,PI,VISI,CGI,ZI,
3RHO1,RHO2,DR2,PSTD,PHIO,PHIRI
DIMENSION GP2(20),MT(20)
COMMON WX,WWX,P,PP,U,UU,Z,CG,VIS,FLOW,LL,MM,ITN
INTEGER L,LL,LLL,LLLL,M,MM,MMM,J,MB,ITN
INTEGER IFG,ICONT

C READ INPUT DATA
C
READ(5,520) PI
READ(5,520) R,MW
520 FORMAT(5D13.6)
READ(5,520) RW,RE,CR,PHI
READ(5,520) PF,DR,DR2
READ(5,521) NSETS,MM,LL,T,EPS
521 FORMAT(3I4,2E13.6)
MMM=MM-1
READ(5,520) (DTIME(I),I=1,MMM)

C READ IN REFERENCES OF TIME INCREMENTS MT(I) AT WHICH
C FRACTION OF GAS PRODUCED WILL BE CALCULATED
C USING AVERAGE FLUXES
C
II=5
READ(5,293) (MT(I),I=1,II)
293 FORMAT(6I6)
READ(5,293) ICONT,IFG,IPOR
WRITE(6,3015) ICONT, IFG, DR, DR2
3015 FORMAT(2I6,2F12.7)

```

## MAIN LINE ... (CONT'D)

```

ALPHA=MW/(R*(T+460.))
CONS=DSQRT(0.72D 02*0.32174D 02/ALPHA)
LLL=LL-1
RW=RW/(0.12D 02*RE)
PSTD=0.14697D 02
PHI0=PHI
JK=0
CALL FPRESS(PI,T)
ZI=Z

C   CALCULATING POROSITY AT INITIAL RESERVOIR PRESSURE
C
    PHIRI=0.1D 01-(0.1D 01-PHI0)*DEXP(-CR*(PI-PSTD))
999 READ(5,520) PERM,B,BB
    JK=JK+1
    CRIT=0.5*DR-RW
    IF(CRIT) 875,875,876
876 WRITE(6,295)
295 FORMAT(/,8X,'VALUE OF K2AVG AT POINT 1 WILL BE
    1NEGATIVE BECAUSE 0.5*DR EXCEEDS THE VALUE OF RW.
    2THIS ORIGINATES ANOMALOUS SOLUTIONS AND THEREFORE
    3EXECUTION WAS SUPPRESSED')
    GO TO 613
875 CONTINUE
    KEY=PERM*0.1062D-13
    TIME=0.
    WRITE(6,231)
231 FORMAT('1')
    WRITE(6,230)
230 FORMAT('-')
    WRITE(6,1059)
1059 FORMAT(36X,'TRANSIENT RADIAL GAS FLOW')

C   WRITE(6,8667) IPOR
C8667 FORMAT(/,44X,'IPOR=',I2,/)

    WRITE(6,230)
    WRITE(6,205) B,BB,PERM
205 FORMAT(1H ,13X,'SLIP COEFF(B)=' ,F7.3,' INERTIAL
    * COEFF', '(BB)=' ,
    1D10.3,' PERMEABILITY(K)=' ,F8.3,1X,'MD')
    WRITE(6,206) PHI,CR,PI
206 FORMAT(/,14X,'POROSITY=' ,F7.3,8X,'ROCK
    * COMPRESSIBILITY=' ,
    1D10.3,2X,'INITIAL PRESSURE=' ,D10.3)
    WRITE(6,207)
207 FORMAT(1H0,35X,'PRESSURE SQUARE DISTRIBUTION')
    WRITE(6,220)
220 FORMAT(8X,'TIME',26X,'DIMENSIONLESS DISTANCE')

```

## MAIN LINE ... (CONT'D)

```

221 WRITE(6,221)
    FORMAT(8X,'(TD)          RW          DELTA R1=0.0010
    * DELTA R2=0.
    1032724')
223 WRITE(6,223)
    FORMAT(1X,' ')

C   SETTING THE DIMENSIONLESS SLIPPAGE

    B=B/PF
    INDEX=4
    CALL BCOND(INDEX)
    DO 99 M=1,MMM
    DT=DTIME(M)
    ITN=0

C
C   CALCULATING K2(L-1/2) AT (M)
C

    KK=0
    DO 5 L=2,LL
    IF(L.EQ.1) GO TO 777
    PAVG2=0.5*(P(L,M)+P(L-1,M))
    PRE=PAVG2*PF
    CALL FPRESS(PRE,T)
    VIS=VIS/(0.1D 03*0.6891D 05)
    IF(ICONT.EQ.0) GO TO 9993
    IF(L.GE.IFG+1) GO TO 9994
9993 FLL=L
    FL=FLL-1.5
    K2(L,M)=(FL*DR+RW)*KEY*PF*(PAVG2+B)/(0.2D 01*RE*VIS*Z
    **PAVG2*
    ICONS+RE*BB*KEY*PF*(PAVG2+B)*Z*DABS(WX(L,M)))
    GO TO 5
777 PAVG2=P(1,M)
    PRE=PAVG2*PF
    CALL FPRESS(PRE,T)
    VIS=VIS/(0.1D 03*0.6891D 05)
    UO(M)=0.4D 01*DR*RE*VIS*Z*P(1,M)*CONS*WX(1,M)/(KEY*PF
    *(PAVG2+B))
    I+0.2D 01*DR*RE*BB*Z*WX(1,M)*DABS(WX(1,M)) + U(2,M)
    IF(UO(M)) 95,97,97
95  PO(M)=0.1D-09
    GO TO 962
97  PO(M)=DSQRT(UO(M))
962 PAVG2=0.5*(PO(M)+P(1,M))
    PRE=PAVG2*PF
    CALL FPRESS(PRE,T)
    VIS=VIS/(0.1D 03*0.6891D 05)

```



## MAIN LINE ... (CONT'D)

```

      K2(L,M)=-(.05D 00*DR-RW)*KEY*PF*(PAVG2+B)/(0.2D 01*RE
**VIS*Z*
      1PAVG2*CONS+RE*BB*KEY*PF*(PAVG2+B)*Z*DABS(WX(L,M))
      GO TO 5
9994 FL=IFG-1
      KK=KK+1
      F1=KK
      FLL=F1-0.5
      K2(L,M)=(FL*DR+RW+FLL*DR2)*KEY*PF*(PAVG2+B)/(0.2D 01
**RE*VIS*Z*
      1PAVG2*CONS+RE*BB*KEY*PF*(PAVG2+B)*Z*DABS(WX(L,M))
5      CONTINUE

C      CALCULATING K1(L+1/2) AT (M)
C

      KK=0
      DO 4 L=2,LL
      PAVG1=0.5*(P(L+1,M)+P(L,M))
      PRE=PAVG1*PF
      CALL FPRESS(PRE,T)
      VIS=VIS/(0.1D 03*0.6891D 05)
      IF(ICONT.EQ.0) GO TO 9999
      IF(L.GE.IFG) GO TO 9991
9999 FLL=L
      FL=FLL-0.5
      K1(L,M)=(FL*DR+RW)*KEY*PF*(PAVG1+B)/(0.2D 01*RE*VIS*Z
**PAVG1*
      1CONS+RE*BB*KEY*PF*(PAVG1+B)*Z*DABS(WX(L+1,M))
      GO TO 4
9991 FL=IFG-1
      KK=KK+1
      F1=KK
      FLL=F1-0.5
      K1(L,M)=(FL*DR+RW+FLL*DR2)*KEY*PF*(PAVG1+B)/(0.2D 01
**RE*VIS*Z*
      1PAVG1*CONS+RE*BB*KEY*PF*(PAVG1+B)*Z*DABS(WX(L+1,M))
4      CONTINUE

C      CALCULATING G1 AT (M)
C

      KK=0
      DO 5000 L=2,LL
      PAVG3=P(L,M)
      PRE=PAVG3*PF
      CALL FPRESS(PRE,T)

C      IF(IPOR.EQ.0) GO TO 8666

```

## MAIN LINE ... (CONT'D)

```

IF(ICONT.EQ.0) GO TO 9995

C   PHI=0.1D 01-(0.1D 01-PHI0)*DEXP(-CR*(PRE-PSTD))
C8666 IF(ICONT.EQ.0) GO TO 9995

IF(L.GE.IFG+1) GO TO 9996
9995 FL=L-1
IF(L-1) 536,536,537

C536 G1(L,M)=PHI*RW*PF/Z*(CG+CR*(0.1D 01/PHI-0.1D 01))

536 G1(L,M)=RW*PF/Z*(CG+CR*(0.1D 01/PHI-0.1D 01))
GO TO 5000

C537 G1(L,M)=PHI*(FL*DR+RW)*PF/Z*(CG+CR*(0.1D 01/PHI-0.1D
C537 01))

537 G1(L,M)=(FL*DR+RW)*PF/Z*(CG+CR*(0.1D 01/PHI-0.1D 01))
GO TO 5000
9996 FL=IFG-1
KK=KK+1
F1=KK

C   G1(L,M)=PHI*(FL*DR+RW+F1*DR2)*PF/Z*(CG+CR*(0.1D
C   01/PHI-0.1D 01))

G1(L,M)=(FL*DR+RW+F1*DR2)*PF/Z*(CG+CR*(0.1D 01/PHI
*-0.1D 01))
5000 CONTINUE

C   STORING FIRST GUESSES OF P AND U AT (M+1)
C

DO 454 L=1,LL
PP(L,M+1)=P(L,M)
UU(L,M+1)=U(L,M)
454 WXX(L,M+1)=WX(L,M)
PP(LL+1,M+1)=PP(LL-1,M+1)
UU(LL+1,M+1)=UU(LL-1,M+1)
PP(1,M+1)=0.5
UU(1,M+1)=0.25

C   CALCULATING K2AVG
C

24 CONTINUE
KK=0
DO 1051 L=2,LL
IF(L.EQ.1) GO TO 5004
PAVG2=0.5*(PP(L,M+1)+PP(L-1,M+1))

```

## MAIN LINE ...(CONT'D)

```

PRE=PAVG2*PF
CALL FPRESS(PRE,T)
VIS=VIS/(0.1D 03*0.6891D 05)
IF(ICONT.EQ.0) GO TO 3001
IF(L.GE.IFG+1) GO TO 3002
3001 FLL=L
    FL=FLL-1.5
    K2(L,M+1)=(FL*DR+RW)*KEY*PF*(PAVG2+B)/(0.2D 01*RE*VIS
**Z*PAVG2*
I CONS+RE*BB*KEY*PF*(PAVG2+B)*Z*DABS(WWX(L,M+1))
    K2AVG(L,M)=(K2(L,M)+K2(L,M+1))/2.
    GO TO 1051
5004 PAVG2=PP(1,M+1)
    PRE=PAVG2*PF
    CALL FPRESS(PRE,T)
    VIS=VIS/(0.1D 03*0.6891D 05)
    UO(M+1)=0.4D 01*DR*RE*VIS*Z*PP(1,M+1)*CONS*WWX(1,M+1)
    #/(KEY*PF*
    1 (PP(1,M+1)+B)) + 0.2D 01*DR*RE*BB*Z*WWX(1,M+1)
    **DABS(WWX(1,M+1))
    2+UU(2,M+1)
    IF(UO(M+1)) 955,957,957
955  P0(M+1)=0.1D-09
    GO TO 961
957  P0(M+1)=DSQRT(UO(M+1))
961  PAVG2=0.5*(P0(M+1)+PP(1,M+1))
    PRE=PAVG2*PF
    CALL FPRESS(PRE,T)
    VIS=VIS/(0.1D 03*0.6891D 05)
    K2(L,M+1)=-((0.5D 00*DR-RW)*KEY*PF*(PAVG2+B)/(0.2D 01
**RE*VIS*Z*
1PAVG2*CONS+RE*BB*KEY*PF*(PAVG2+B)*Z*DABS(WWX(L,M+1))
    K2AVG(L,M)=0.5*(K2(L,M)+K2(L,M+1))
    GO TO 1051
3002 FL=IFG-1
    KK=KK+1
    F1=KK
    FLL=F1-0.5
    K2(L,M+1)=(FL*DR+RW+FLL*DR2)*KEY*PF*(PAVG2+B)/(0.2D 01
**RE*VIS
1*Z*PAVG2*CONS+RE*BB*KEY*PF*(PAVG2+B)*Z*DABS(WWX(L,M
**+1))
    K2AVG(L,M)=0.5*(K2(L,M)+K2(L,M+1))
1051 CONTINUE

C    CALCULATING K1AVG
C

KK=0
DO 1050 L=2,LL

```

## MAIN LINE ... (CONT'D)

```

PAVG1=0.5*(PP(L+1,M+1)+PP(L,M+1))
PRE=PAVG1*PF
CALL FPRESS(PRE,T)
VIS=VIS/(0.1D 03*0.6891D 05)
IF(ICONT.EQ.0) GO TO 3003
IF(L.GE.IFG) GO TO 3004
3003 FLL=L
FL=FLL-0.5
K1(L,M+1)=(FL*DR+RW)*KEY*PF*(PAVG1+B)/(0.2D 01*RE*VIS
**Z*PAVG1*
ICONS+RE*BB*KEY*PF*(PAVG1+B)*Z*DABS(WWX(L+1,M+1)))
K1AVG(L,M)=0.5*(K1(L,M)+K1(L,M+1))
GO TO 1050
3004 FL=IFG-1
KK=KK+1
F1=KK
FLL=F1-0.5
K1(L,M+1)=(FL*DR+RW+FLL*DR2)*KEY*PF*(PAVG1+B)/(0.2D 01
**RE*VIS
1*Z*PAVG1*ICONS+RE*BB*KEY*PF*(PAVG1+B)*Z*DABS(WWX(L+1,M
**1))
K1AVG(L,M)=0.5*(K1(L,M)+K1(L,M+1))
1050 CONTINUE

C   CALCULATING G1AVG(L,M)
C
KK=0
DO 5006 L=2,LL
PAVG3=PP(L,M+1)
PRE=PAVG3*PF
CALL FPRESS(PRE,T)

C   IF(IPOR.EQ.0) GO TO 7836
C   PHI=0.1D 01-(0.1D 01-PHI0)*DEXP(-CR*(PRE-PSTD))
C7836 IF(ICONT.EQ.0) GO TO 3005

IF(ICONT.EQ.0) GO TO 3005
IF(L.GE.IFG+1) GO TO 3006
3005 FL=L-1
IF(L-1) 694,694,695

C694 G1(L,M+1)=PHI*RW*PF/Z*(CG+CR*(0.1D 01/PHI-0.1D 01))

694 G1(L,M+1)=RW*PF/Z*(CG+CR*(0.1D 01/PHI-0.1D 01))
GO TO 5005

C695 G1(L,M+1)=PHI*(FL*DR+RW)*PF/Z*(CG+CR*(0.1D 01/PHI-0.1D
C695 01))

```

## MAIN LINE ... (CONT'D)

```

695  G1(L,M+1)=(FL*DR+RW)*PF/Z*(CG+CR*(0.1D 01/PHI-0.1D
      * 01))
5005  G1AVG(L,M)=0.5*(G1(L,M)+G1(L,M+1))
      GO TO 5006
3006  FL=IFG-1
      KK=KK+1
      F1=KK

C      G1(L,M+1)=PHI*(FL*DR+RW+F1*DR2)*PF/Z*(CG+CR*(0.1D
C      01/PHI-0.1D 01))

      G1(L,M+1)=(FL*DR+RW+F1*DR2)*PF/Z*(CG+CR*(0.1D 01/PHI
      *-0.1D 01))
      G1AVG(L,M)=0.5*(G1(L,M)+G1(L,M+1))
5006  CONTINUE

C      SETTING THE COEFFICIENTS AND GETTING THE SOLUTION
C      AT (M+1) USING THE THOMAS ALGORITHM
C

      RHO=2.*DR**2/DT
      RHO1=DR2*DR*(DR2+DR)/DT
      RHO2=0.2D 01*DR2**2/DT

C      FOR L=2
C

      BC=-RHO*G1AVG(2,M)-K1AVG(2,M)-K2AVG(2,M)
      E(2)=BC
      G(2)=(-K2AVG(2,M)*U(1,M+1)-K2AVG(2,M)*U(1,M)+(K1AVG(2
      *,M)+
      1K2AVG(2,M)-RHO*G1AVG(2,M))*U(2,M)-K1AVG(2,M)*U(3,M))
      */BC

C      FOR INTERMEDIATE GRID POINTS
C

      JF=LLL-1
      DO 3 L=3,LLL
      IF(ICONT.EQ.0.OR.L.LT.IFG) GO TO 3007
      IF(L.EQ.IFG) GO TO 3008
      RHO=RHO2
3007  BC=-RHO*G1AVG(L,M)-K1AVG(L,M)-K2AVG(L,M)
      IF(L.EQ.IFG+1) GO TO 3016
      IF(L.GT.2) GO TO 666
      E(L)=BC-K2AVG(L,M)*(K1AVG(L-1,M)+K2AVG(L-1,M))/E(L-1)
      GO TO 444
666  E(L)=BC-K2AVG(L,M)*K1AVG(L-1,M)/E(L-1)
      GO TO 444
3016  E(L)=BC-K2AVG(L,M)*K1AVG(L-1,M)*DR/E(L-1)

```

## MAIN LINE ... (CONT'D)

```

444  D=-K2AVG(L,M)*U(L-1,M)+(K1AVG(L,M)+K2AVG(L,M)-RHO
      **G1AVG(L,M))*
      1U(L,M)-K1AVG(L,M)*U(L+1,M)
      G(L)=(D-K2AVG(L,M)*G(L-1))/E(L)
      GO TO 3
3008  BC=-RHO1*G1AVG(L,M)-K1AVG(L,M)*DR-K2AVG(L,M)*DR2
      E(L)=BC-K2AVG(L,M)*DR2*K1AVG(L-1,M)/E(L-1)
      D=-K2AVG(L,M)*DR2*U(L-1,M)+(K1AVG(L,M)*DR+K2AVG(L,M)
      **DR2-RHO1*
      1G1AVG(L,M))*U(L,M)-K1AVG(L,M)*DR*U(L+1,M)
      G(L)=(D-K2AVG(L,M)*DR2*G(L-1))/E(L)
3     CONTINUE

C     FOR L=LL
C

      BC=-RHO*G1AVG(LL,M)-K1AVG(LL,M)-K2AVG(LL,M)
      D=(K1AVG(LL,M)+K2AVG(LL,M)-RHO*G1AVG(LL,M))*U(LL,M)+(
      *-K1AVG(LL,M)
      1-K2AVG(LL,M))*U(LL-1,M)
      E(LL)=BC-(K1AVG(LL,M)+K2AVG(LL,M))*K1AVG(LL-1,M)/E(LL
      *-1)
      G(LL)=(D-(K1AVG(LL,M)+K2AVG(LL,M))*G(LL-1))/E(LL)
      U(LL,M+1)=G(LL)

C     GETTING THE SOLUTIONS FOR THE REST OF THE GRID POINTS
C

      DO 8 J=1,JF
      I=LL-J
      IF(I.EQ.1) GO TO 333
      IF(I.EQ.IFG) GO TO 3017
      U(I,M+1)=G(I)-K1AVG(I,M)*U(I+1,M+1)/E(I)
      GO TO 8
333   U(I,M+1)=G(I)-(K1AVG(I,M)+K2AVG(I,M))*U(I+1,M+1)/E(I)
      GO TO 8
3017  U(I,M+1)=G(I)-K1AVG(I,M)*DR*U(I+1,M+1)/E(I)
8     CONTINUE

C     SETTING PRESSURES AT (M+1) FOR L=1,LL
C

      DO 6 L=1,LL
6     P(L,M+1)=DSQRT(U(L,M+1))

C     CALCULATING THE MASS FLUX AT TIME STEP (M+1) USING
C     SOLUTION OBTAINED AT (M+1) LEVEL
C

      DO 7 L=1,LL

```

## MAIN LINE ... (CONT'D)

```

IF(L.EQ.1) GO TO 3011
IF(L.GT.IFG) GO TO 3009
DER(L,M+1)=(U(L,M+1)-U(L-1,M+1))/DR
DER(L,M)=(U(L,M)-U(L-1,M))/DR
GO TO 3010
3009 DER(L,M+1)=(U(L,M+1)-U(L-1,M+1))/DR2
DER(L,M)=(U(L,M)-U(L-1,M))/DR2
GO TO 3010
3011 DER(L,M+1)=(-3.*U(L,M+1)+4.*U(L+1,M+1)-U(L+2,M+1))/(2.
    *DR)
DER(L,M)=(-3.*U(L,M)+4.*U(L+1,M)-U(L+2,M))/(2.*DR)
DERAVG(L,M)=0.5*(DER(L,M+1)+DER(L,M))
PAVG=(P(L,M+1)+P(L,M))/2.
GO TO 3012
3010 DERAVG(L,M)=0.5*(DER(L,M+1)+DER(L,M))
PAVG1=0.5*(P(L,M+1)+P(L-1,M+1))
PAVG2=0.5*(P(L,M)+P(L-1,M))
PAVG=0.5*(PAVG1+PAVG2)
3012 PRE=PAVG*PF
CALL FPRESS(PRE,T)
VIS=VIS/(0.1D 03*0.6891D 05)
BCO(L)=-0.2D 01*RE*VIS*Z*PAVG*CONS/(KEY*PF*(PAVG+B))
ACO(L)=-RE*BB*Z
DCO(L)=DSQRT(BCO(L)*BCO(L)-4.*ACO(L)*DERAVG(L,M))
IF(DERAVG(L,M).LE.0.1E-05) GO TO 17

C   ALTERNATIVE FOR QUADRATIC CONSTANT = 0.
C
IF(DABS(ACO(L)).GT.0.10E-03)GO TO 15
WX(L,M+1)=-1.*(-DERAVG(L,M)/BCO(L))
GO TO 7
15  WX1=-(-BCO(L)-DCO(L))/(2.0*ACO(L))
    WX2=-(-BCO(L)+DCO(L))/(2.0*ACO(L))
    IF((WX1+DABS(WX1)).LT.0.10E-04) GO TO 18
    IF((WX2+DABS(WX2)).LT.0.10E-04) GO TO 9
18  WX(L,M+1)=WX1
    IF((WX2+DABS(WX2)).LT.0.10E-04) GO TO 7
    GO TO 7
17  WX(L,M+1)=0.
    GO TO 7
9   WX(L,M+1)=WX2
7   CONTINUE

C   CHECKING CONVERGENCE OF THE SOLUTION
C
DO 10 L=2,LL
ERR(L)=DABS(PP(L,M+1)-P(L,M+1))
IF(ERR(L).LE.EPS) GO TO 10

```

## MAIN LINE ... (CONT'D)

```

GO TO 429
10 CONTINUE
GO TO 11
429 DO 13 L=1,LL
    PP(L,M+1)=P(L,M+1)
    UU(L,M+1)=U(L,M+1)
13   WWX(L,M+1)=WX(L,M+1)
    ITN=ITN+1
    IF(ITN.GT.10) GO TO 12
    GO TO 24
11 CONTINUE

C   PROBLEM HAS CONVERGED FOR TIME STEP (M+1)
C

GO TO 678
12 WRITE(6,713)
713 FORMAT(2X,'*')
678 TIME=TIME+DTIME(M)
    IF(M.EQ.11.OR.M.EQ.21) GO TO 476
    GO TO 477
476 WRITE(6,231)
    WRITE(6,230)
    WRITE(6,230)
477 WRITE(6,208) TIME
208 FORMAT(6X,E10.3)
78  WRITE(6,224) (U(L,M+1),L=1,LL,3)
224 FORMAT(16X,F7.4,2X,F7.4,2X,F7.4,2X,F7.4,2X,F7.4,2X,
*,F7.4,2X,
1F7.4,2X,F7.4,2X,F7.4,2X,F7.4)
99  CONTINUE
    MB=M

C   CALCULATING FRACTION OF GAS PRODUCED USING AVERAGE
C   FLUXES
C

DO 291 I=1,II
    MK=MT(I)
    FLUX1=0.
    FLUXLL=0.
    DO 395 K=1,MK
        FAVG1=DABS(WX(1,K)+WX(1,K+1))/2.
        FAVGLL=DABS(WX(LL,K)+WX(LL,K+1))/2.
        OUT=FAVG1*DTIME(K)
        IN=FAVGLL*DTIME(K)
        FLUX1=FLUX1+OUT
        FLUXLL=FLUXLL+IN
395 CONTINUE

```



## MAIN LINE ... (CONT'D)

```
C      GP2(I)=ZI/(PHIRI*(1.-RW**2))*(RW*FLUX1-FLUXLL)

      GP2(I)=ZI*PHIO/(PHIRI*(1.-RW**2))*(RW*FLUX1-FLUXLL)
291    CONTINUE
      WRITE(6,231)
      WRITE(6,230)

C      WRITE(6,8667) IPOR

      WRITE(6,230)
      B=B*PF
      WRITE(6,205) B,BB,PERM
      WRITE(6,210)
210    FORMAT(1H0,33X,'FLUX DISTRIBUTION FOR THE FIRST 40
      * GRID POINTS')
      WRITE(6,230)
      TIME=0.
      DO 75 M=1,MB
      TIME=TIME+DTIME(M)
      IF(M.EQ.9.OR.M.EQ.17) GO TO 494
      GO TO 445
494    WRITE(6,231)
      WRITE(6,230)
      WRITE(6,230)
445    WRITE(6,208) TIME
      WRITE(6,515) (WX(L,M+1),L=1,40)
515    FORMAT(16X,10D11.3)
      75 CONTINUE
      WRITE(6,9000)
9000  FORMAT(1H0,30X,'FRACTION OF GAS PRODUCED AT LAST FIVE
      * TIME LEVELS'
      1)
      WRITE(6,9003) (GP2(I),I=1,II)
9003  FORMAT(/,7X,'FRACTION OF GAS PRODUCED=',5(3X,E10.3))
      IF(JK.EQ.NSETS) GO TO 613
      GO TO 999
613  WRITE(6,231)
      STOP
      END
```