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Solution techniques for transient stability-constrained optimal power flow – Part II

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Abstract: Transient stability-constrained optimal power flow is an important emerging problem with power systems pushed to the limits for economic benefits, dense and larger interconnected systems, and reduced inertia due to expected proliferation of renewable energy resources. In this study, two more approaches: single machine equivalent and computational intelligence are presented. Also discussed are various application areas, and future directions in this research area. A comprehensive resource for the available literature, publicly available test systems, and relevant numerical libraries is also provided.

1 Introduction

The aim of this series of papers is to present the incorporation of the transient stability constraints in a traditional optimal power flow (OPF) formulation. Assessing transient stability is an important analysis in power engineering to decide preventive control actions or corrective remedial action schemes. Interfacing it with an OPF, the defacto steady-state optimisation tool, is both mathematically challenging and computationally rigorous. Research in this area, also termed as the transient stability constrained OPF (TSC-OPF) problem, has resulted in the investigation of various approaches to solve this problem. Part I of this series of papers provided an in-depth overview of dynamic optimisation-based methods for solving TSC-OPF. Several variants of the interior point method (IPM), which have been used for solving the resultant non-linear optimisation problem, were also discussed.

This paper continues the discussion on techniques for solving TSC-OPF by presenting two other categories of approaches; single machine equivalent (SIME), and computational intelligence (CI), which have demonstrated unique advantages and have been widely investigated in the literature. The basic motivations of these two categories of approaches are to focus the two following extreme variants of TSC-OPF problem, compared with conventional dynamic optimisation-based approaches reviewed in the Part I.

SIME-based approaches are developed for the situations that only rotor-angle stability is concerned, therefore it is possible to utilise equivalent single machine model to simplify the formulation. In contrast, CI-based approaches are commonly adopted when the formulation becomes too complicated or too large to be handled by existing approaches. A derivative-free, self-adaptable, black-box approach is preferred in such situations, where CI algorithms exactly fit the requirement.

As a bridge connecting steady-state operation decision making, and transient-state performance, TSC-OPF is capable of being extended to various applications in different fields of power system planning, operation and control, especially when power system dynamic performance and its criteria are observed as an emerging limiting factor. Many examples available in the literature are classified in this survey paper to demonstrate the extensive applicability of TSC-OPF problems.

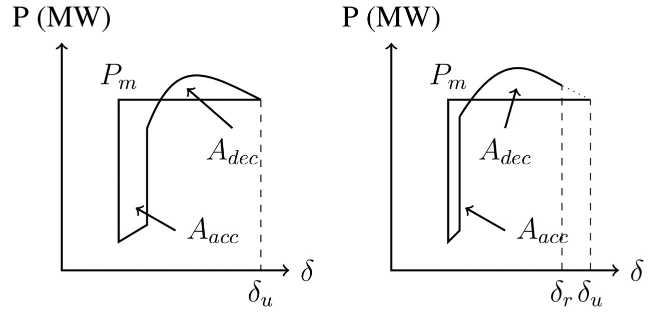
This paper is organised as follows: Section 2 summarises the SIME approach for solving TSC-OPF and details the steps involved in obtaining the single machine equivalent and its stability assessment. Section 3 provides an overview of two computationally intelligent approaches: metaheuristics optimisation and artificial neural networks (ANNs), and provides a literature survey of these approaches for solving TSC-OPF. Various applications of TSC-OPF formulation in its standard form as described in part I, or its variants, are discussed in Section 4. Some of the future research directions in this area are postulated in Section 5. In addition, the Appendix provides a comprehensive resource for the literature available in this area, publicly available test systems, and numerical libraries.

2 SIME approach

SIME is a hybrid temporal-direct method: temporal, since it relies on multi-machine system evolution with time, and direct, like the extended equal area criterion from which it originates [1]. The underlying idea of SIME is to reduce a multi-machine system into an equivalent one-machine-infinite-bus (OMIB) representation at each time step of the time-domain simulation and to calculate its stability margin. The advantage of the SIME approach, as compared with time-domain simulations, is that it provides a deterministic measure of the stability of the post-disturbance trajectory. Thus, when the instability criterion is met the time-domain simulation can be terminated. The SIME approach comprises the following three main steps.

i. At each step of the post-fault trajectory, SIME first group machines into 'critical machines' (CMs), which are responsible for loss of synchronism, and 'non-CMs' (NMs). This is based on the proposition that the mechanism of loss of synchronism in a power system originates from the irrevocable separation of the CM and NM [1]. In order to identify the CMs and NMs, the rotor angles of all machines at each time step of the post-disturbance trajectory are sorted in decreasing order. The largest rotor angular deviations between any two machines are used as a demarcation to create the two groups of machines.

ii. Next, an OMIB equivalent is created from the centre of inertia (COI) equivalent of the two machine groups. With subscript C to denote the critical group and N for the non-critical group, the rotor angles for the COI equivalent of the two groups are as follows:



a *b*

Fig. 1 *OMIB $P - \delta$ trajectory for*
(a) Unstable case, decelerating area A_{dec} < accelerating area A_{acc}, ***(b)*** Stable case $A_{dec} > A_{acc}$

$$\delta_C(t) = M_C^{-1} \sum_{k \in C} M_k \delta_k(t),$$
$$\delta_N(t) = M_N^{-1} \sum_{j \in N} M_j \delta_j(t) \tag{1}$$

The speeds $\omega_C(t)$ and $\omega_N(t)$ can be computed similarly. In the above formulae, one has the following inertia equivalents:

$$M_C = \sum_{k \in C} M_k, \quad M_N = \sum_{j \in N} M_j$$

With the two equivalents thus defined, the rotor angle and speed of the equivalent OMIB are given as follows:

$$\delta(t) = \delta_C(t) - \delta_N(t)$$
$$\omega(t) = \omega_C(t) - \omega_N(t) \tag{2}$$

The mechanical power $P_m(t)$ and electrical power $P_e(t)$ of the equivalent OMIB are

$$P_m(t) = M\left(M_C^{-1} \sum_{k \in C} P_{mk}(t) - M_N^{-1} \sum_{j \in N} P_{mj}(t)\right)$$
$$P_e(t) = M\left(M_C^{-1} \sum_{k \in C} P_{ek}(t) - M_N^{-1} \sum_{j \in N} P_{ej}(t)\right) \tag{3}$$

with the accelerating power given by the following:

$$P_a(t) = P_m(t) - P_e(t) \tag{4}$$

In the above expressions, M denotes the equivalent OMIB inertia coefficient:

$$M = \frac{M_C M_N}{M_C + M_N} \tag{5}$$

3) Fig. 1*a* shows the variation of OMIB electrical power P_e with respect to the rotor angle δ for the fault-on trajectory for two stability scenarios. If the instability criterion (6) is satisfied at some time t_u, the candidate OMIB is declared as the critical OMIB equivalent. This time t_u also corresponds to the time at which the electrical power $P_e(t)$ and mechanical power $P_m(t)$ intersect each other at angle δ_u as shown in Fig. 1*b*. The unstable margin is computed by using (7), where $\omega(t_u)$ is the rotor speed at t_u

$$P_a(t_u) = P_m(t_u) - P_e(t_u) = 0, \quad \dot{P}_a(t_u) > 0 \tag{6}$$

$$\eta_u = -M(\omega(t_u))^2/2 \tag{7}$$

For marginally stable cases, the stable margin is computed by (8), where δ_r is the rotor angle at time t_r at which δ starts decreasing and $P_a < 0$. The angle δ_r is also referred to as the return angle

$$\eta_{st} = \int_{\delta_u}^{\delta_r} |P_a(t_r)| \, d\delta \tag{8}$$

Equation (8) can be approximated by a triangular or a curve-fitting method as detailed in [1].

SIME has been employed in a variety of TSC-OPF applications of which optimal preventive generator rescheduling [2–5] and maximising available transfer capacity [1, 6] have been the most researched. A detailed discussion of the SIME concepts and the associated applications – preventive and corrective control, open-loop and closed-loop emergency control, and others – is given in [7].

Compared with dynamic optimisation-based approaches, the SIME approach of providing a direct measure of system instability (unstable margin). Thus, the instability margin can be implicitly incorporated as a transient stability constraint. For instance, transient stability constraints provided by SIME are directly expressed in terms of the power limits of the CMs. Since these are constraints already considered in the conventional optimal power flow model [(1)–(4) of the companion paper], this approach does not need to consider the sets of dynamic and stability constraints. In this way, this approach has never been limited by the size of the system and by the modelling detail, and has been successfully used to optimise large realistic systems reported in [1–3, 6, 7].

Another advantage of SIME is that it provides the time at which the instability occurs thus yielding a termination time for the TSC-OPF time-domain integration. SIME also implicitly provides the limit on the rotor angle of the OMIB, δ^+, that one may use in the transient stability constraint. Pizano-Martinez *et al.* [4] use a simultaneous discretisation approach along with SIME to solve the TSC-OPF problem. In their approach, the simulation end time is set to t_u calculated by using the SIME approach when an unstable margin is detected. Further, they use only a transient stability constraint for time t_u instead of a constraint for each time step. This transient stability constraint is given by the following:

$$\delta_{UT}(t_u) - \delta_{CT}(t_u) - T_h \le 0 \tag{9}$$

Here δ_{UT} and δ_{CT} are the OMIB rotor angles for the unstable case and the marginally stable case, respectively, and T_h is a 'desired' deviation threshold. This desired deviation threshold is an additional cushion for maintaining stability so that the system is operated well within the marginally stable angle $\delta_{CT}(t_u)$. $\delta_{CT}(t_u)$, being dependent on the prevailing operating practices, differs from one system to another. The 'desired' threshold used in [4] is 1×10^{-4}. In [5], their approach is improved by constraining the rotor angle deviation only at the initial time t_0 as follows:

$$\delta_{UT}(t_0) - \delta_{sh}(t_0) - T_h \le 0 \tag{10}$$

Here $\delta_{sh}(t_0)$ is the OMIB rotor angle that results in a stable case. This rotor angle is computed by using the SIME sensitivity analysis described in detail in [1]. Thus, this approach results in an optimisation problem involving only the steady-state OPF constraints and a single constraint on the initial condition of dynamic state variables. We note that in both approaches, Pizano-Martinez *et al.* solve the optimisation separately to compute the initial operating point and then perform a time-domain simulation complemented by SIME. In [8], it was shown that this approach could consider detailed modelling of the power system.

Zárate-Minano *et al.* [9] use the SIME approach in conjunction with the simultaneous approach as their solution, similar to [4]. A difference is that their formulation has transient stability constraints at each time-step. They also use a reduced admittance matrix, Y_{bus}, to reduce the dimensions of the variables to be solved. Xia *et al.* [10] propose the use of minimum kinetic energy for normal unstable cases or the minimum accelerating power distance for extreme unstable cases as a stability performance index, obtained from SIME simulation. This index is then integrated as a stability constraint in an OPF formulation.

However, SIME may face convergence issues due to identifying different OMIB equivalents during the TSC-OPF iterations. The reason is the OMIB equivalent system structure (composition of the CMs and NMs groups) may change during the stabilisation procedure. These discontinuities may invalidate the TSC-OPF procedure with uncertain renewable energy generation output. Since the transient stability constraint (9) is set with the initial operation point, during restarting the stabilisation procedure, as detailed in [4], it is difficult to get a converge result with the changed OMIB structure.

3 CI approaches

Adaptation and *self-organisation* are two main features that make an algorithm computationally intelligent. Adaptation is the ability of an algorithm to change or evolve its parameters to better meet its objectives, while self-organisation is a system's attempt to organise itself into different complex structures [11].

For the sake of brevity, consider a simplified transient stability constraint as (11), where x represents steady-state operating condition to be optimised by TSC-OPF. The function $g(x)$ indicates the transient stability assessment conclusion based on the operating condition x given a certain contingency. Different approaches may use slightly different forms of such function, but the concept is similar

$$g(x) = \begin{cases} 1 & \text{(unstable)} \\ 0 & \text{(stable)} \end{cases} \tag{11}$$

Therefore, the formulation of TSC-OPF can be re-written as (12), where $\mathbb{S}_C$ is the set of contingencies. $f(x)$ is the objective function similar to the ones in other approaches surveyed in this paper. x may be also subjected to other steady-state constraints, e.g. line flow or bus voltage constraints, which are not explicitly shown

$$\begin{aligned} \min_{x} \quad & f(x) \\ \text{s.t.} \quad & g_k(x) = 0, \quad \forall k \in \mathbb{S}_C \end{aligned} \tag{12}$$

Due to the complexity of transient stability, it is commonly difficult, if not possible, to establish the analytic expression of $g(x)$. Its sensitivity w.r.t. x is also sometimes difficult to obtain, due to various aspects like large dimension, ill-condition, discontinuity and so on. The idea of CI-based approaches is to treat $g(x)$ as a black-box and investigate its input/output behaviour, instead of looking at its detailed mathematical model.

Following this idea, the following two categories of CI-based approaches are surveyed: meta-heuristics-based approach and ANNs-based approach.

ANNs are another paradigm of CI, inspired by the massively parallel structure of the mammalian brain. Classification, clustering and pattern recognition, are some applications of ANNs.

3.1 Metaheuristic optimisation algorithms

Metaheuristics is a problem-independent algorithm framework that provides a higher-level set of strategies, compared with heuristic algorithms, to solve optimisation problems [12]. In other words, instead of using simple strategies such as trial and error to find the solution to a problem, metaheuristics provides higher-level global optimisers that organise and guide the search process to reach to the global solution. As it does not depend any property of the optimisation problem, it can be easily utilised to solve TSC-OPF problem like (12).

Metaheuristics-based approaches commonly address the following unconstrained optimisation formulation, after introducing a sufficient large barrier coefficient μ

$$F(x) = f(x) + \mu \sum_{k \in \mathbb{S}_C} g_k(x) \tag{13}$$

The idea of metaheuristic methods is to perform direct search in the space of x, it evaluates the values of $F(x)$ using a series of attempts on x and then determines which new x to try next time. Such iteration continues until a sufficiently good solution is found. Such direct search process only depends on the evaluation of $F(x)$ and does not require any knowledge of the convexity, non-linearity, discontinuity, uncertainty of the function studied. This is root reason behind why it has been widely used to solve TSC-OPF.

Generally, a meta-heuristic method includes the following three steps:

i. Find an initial guess of x_0.
ii. Evaluate the value of y_k given x_k in kth iteration, where y_k is the function value as

$$y_k = F(x_k) \tag{14}$$

iii. If y_k is sufficiently good, then return it as the optimal solution, otherwise determine x_{k+1} to be evaluated in next iteration

$$x_{k+1} = h(x_k, y_k) \tag{15}$$

iv. where the function $h(x, y)$ denotes to the strategy used to update x.

It is observed that many variants can be developed following the framework shown above. First, one is able to evaluate and update a single x one-by-one or multiple x in a batch, which can be parallelised in nature. Second, the strategy implemented in function $h(x, y)$ can be inspired by the advancement in other disciplines, such as the progress of evolution or the behaviour of a swarm and so on.

Evolutionary algorithms are designed based on the principles of Darwinian evolution and take advantage of genetic mechanisms such as mutation, crossover, recombination and selection of the best individuals. Genetic algorithms, evolutionary programming, and differential evolution are examples of evolutionary optimisation algorithms. With a genetic algorithm an initial population of chromosomes (individuals) is evolved at each iteration of the algorithm, employing genetic mechanisms. At each iteration, the fitness of each individual is assessed, the most promising individuals are selected to build the new population, and crossover and mutation are applied to the population. The algorithm terminates once a criterion (e.g. maximum number of iterations) is fulfilled. Evolutionary programming is similar to genetic algorithms, but some differences between the two methods do exist. For example, the crossover operator is not used in evolutionary programming. Differential evolution is another member of the evolutionary optimisation family in which the mutation and recombination operators are utilised to evolve the candidate solution. However, as a parallel direct search method, differential evolution employs *np* *d*-dimensional parameter vectors $x_{i,G}, i \in 0, 1, \ldots, np-1$ as the population at each iteration. The algorithm generates a new parameter vector by adding a scaled difference between two population vectors to a third vector. If the resulting vector offers a better objective value than a selected vector in the population, it will replace that selected vector in the next iteration.

Swarm-based optimisation algorithms are inspired by the intelligent and organised behaviour of different types of animals, such as insects and birds in search for food. Particle swarm optimisation (PSO) is an important member of this family. PSO works based on exchanging data between different particles of a swarm that are exploring the search space. Each particle is represented by using a velocity and a position vector. The velocity and position vectors of each particle are updated at each iteration with respect to the particle's best visited position, p_{best}, and the best solution the swarm has ever encountered, g_{best}. The final solution to the problem is the value of g_{best} after the last iteration.

Problem independence is an important characteristic of metaheuristic algorithms. This feature makes them good candidates for solving the non-linear TSC-OPF problem. The application of metaheuristic algorithms for solving TSC-OPF has been studied in

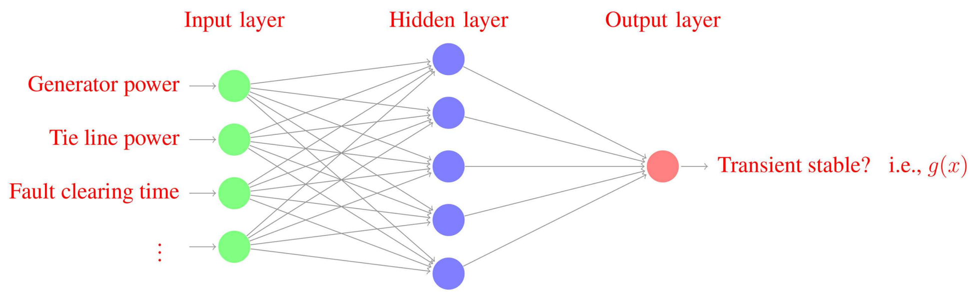


Fig. 2 *Illustration of using ANN to emulate $g(x)$ for a TSC-OPF problem*

several articles. This includes applying genetic algorithms [13] differential evolution [14] and PSO [15] to TSC-OPF problem. In these studies the objective function typically has been to minimise generation cost. Some of the recent developments on the application of CI for solving TSC-OPF include (a) population based methods: micro-PSO method [16], NSGA-II [17], self-adaptive differential evolution [18], krill herd algorithm [19], (b) direct search methods: improved group search optimisation [20], pattern search [21], and (c) support vector machine: support vector machines (SVMs) are used to classify whether an operating condition satisfies predefined transient contingencies in [22].

In order to handle the power flow and transient stability constraints, a penalty function approach is used [i.e. μ in the illustrative example of (13)]. Based on this method, a penalty term is defined for each constraint. If a constraint is satisfied, its penalty term will be zero, otherwise the penalty term will be a positive value proportional to the amount of constraint violation. The penalty term of each constraint is multiplied by a large enough positive (scaling) value and added to the objective function. Therefore, in order to minimise the penalty terms, the optimiser is forced to search inside the feasible region for the optimal solution, otherwise the added penalty terms to the objective function will produce large constraint violation values. We note here that, although, the solution of TSC-OPF, and in general SCOPF, by metaheuristics is a promising research area, its practical applicability and scalability still needs to be proven.

3.2 Artificial neural networks

ANNs simulate the massively interconnected parallel structure of the brain using simple interconnected nervous cells called *neurons*. This simulation is presented by two main structural components in a typical ANN: connection weights and processing elements [11]. An ANN can contain several layers of processing elements in which each processing element in each layer calculates its output value based on an activating function and the weighted outputs of the previous layer. If the weighted summation of the input values to a processing element exceeds a threshold, its output is activated. Training (adaptation) plays the major role in the behaviour of ANNs. Once an ANN has been designed, it must be trained. Based on the supervised training strategy [11], a set of input–output data is provided for training the ANN. The ANN uses the dataset to adjust the weights of its internal connections based on a method such as back-propagation of the error between the desired and computed output through the system. The trained ANN is then tested for accuracy by using another set of input–output data. If the testing results are satisfactory, the ANN is ready to use.

For the problem of TSC-OPF (12), ANN can be used to describe the transient stability constraint $g(x)$. ANN utilises one or more hidden layers to emulate the relationship from the inputs, including generator power, tie line power, or fault clearing time and so on, to the output, i.e. the value of $g(x)$. After training using sufficient amount of samples obtained by transient stability assessment computation, an ANN shown in Fig. 2 can be established to quickly estimate transient stability performance given any values or combinations of inputs. Therefore, one is able to significantly speed up the modelling and solving of TSC-OPF problems based on the trained ANN.

This idea was originally developed to provide an online estimation of the stability condition of the system for TSC-OPF study, so as to reduce the computationally expensive numerical stability analysis. ANNs are used in [23] to evaluate the sensitivity of the transient energy function with respect to the generators' output power, emfs, and machine inertia constants. The estimation is then included in the economic dispatch problem and solved by using a gradient-based method. In [24] ANNs alongside evolutionary programming have been employed to solve the TSC-OPF problem. In both approaches, evolutionary programming is used for optimising the objective function while the ANN estimates the degree of stability of each individual of the population.

4 Applications of TSC-OPF

TSC-OPF can be used either for preventive control or for corrective/remedial action schemes. We highlight six TSC-OPF applications that have been extensively investigated within the power system research community.

4.1 Optimal generation scheduling

One important application widely investigated in the literature is of preventive control through optimal generation rescheduling for maintaining certain transient stability criteria. This task is traditionally achieved by a conventional OPF or a steady-state security constrained OPF formulation. Stability assessment is commonly performed in offline study and ignored in online operation, which increases the risk of instability or even blackouts. In order to find an OPF solution that satisfies stability constraints, a trial-and-error method that is not only computationally cumbersome but also can miss 'optimal solutions' [25] and produce discrimination among market players in stressed power systems [26]. The TSC-OPF formulation with minimisation of generation cost as the objective function is used for optimal generation rescheduling.

4.2 Dynamic available transfer capability (ACT)

As steady-state OPF is one of the approaches to determine the ATC of a transmission line or a corridor. Similarly, when dynamic constraints are considered in ATC calculation, it comes to the concept of dynamic ATC, which can be solved by TSC-OPF. In this application, the transfer capability of the studied assets is maximised while ensuring that the transient response after a large disturbance remains stable. Dynamic ATC addresses post-contingency transient stability and leads to a more secure ATC estimation for system operation. Hiskens *et al.* [27] and Zhang *et al.* [28] explain the concept of dynamic ATC and present supporting results.

4.3 Dynamic reactive power dispatch

Dynamic VAR devices, such as SVCs and Flexible Alternating Current Transmission Systems (FACTS) devices, supply reactive power locally to ensure acceptable transient voltage performance and short-term voltage stability following a severe disturbance. These devices can be used to mitigate short-term voltage stability issues, such as fault-induced delayed voltage recovery (FIDVR), near inductive load centres. Moreover, their capacity can be used for steady-state reactive power compensation to lower line losses. With appropriate transient voltage stability constraints incorporated in TSC-OPF formulation, for example as done in [29], the short-term voltage stability performance can be improved by appropriately sizing and dispatching dynamic VAR devices. Geng *et al.* [30] provide an example of dynamic reactive power reserve dispatch using a TSC-OPF formulation.

4.4 Dynamic VAR allocation

As stated in Section 4.3, TSC-OPF formulation is able to dispatch dynamic VAR devices in order to achieve better short-term voltage stability performance. Similarly, it is able to assist system planning engineers to select candidate sites for installing dynamic VAR devices, which is addressed in [25], Tiwari and Ajjarapu [32], and Paramsivam *et al.* [33]. During the allocation of these devices, one has to consider system dynamics after dynamic reactive power compensation devices are installed on candidate sites. To this end, an extended TSC-OPF formulation is constructed with binary variables deciding dynamic VAR location.

4.5 Location marginal price (LMP) calculations

LMP calculations, dual variables of an OPF solution, form the fundamental pricing mechanism for electricity markets. Similar concept can be established in TSC-OPF solution, where the price of transient stability is investigated. Nguyen *et al.* [34] extend the OPF formulation by including transient stability constraints to calculate nodal price and their components, taking into account the cost for maintaining transient stability. In their formulation, they also consider the contribution of FACTS devices to LMP calculation.

4.6 Emergency control

TSC-OPF operates power system in a preventive manner; that is, the steady-state operating condition is adjusted in order to keep system transient response stable after a large disturbance in the predefined contingency set. If an unpredictable fault occurs, however, corrective actions, such as load shedding and generator tripping, must be taken in order to ensure post-fault rotor angle stability. In this decision-making process, a transient stability constrained emergency control (TSCEC) problem has to be solved. The formulation of TSCEC is also similar to TSC-OPF: they share the same dynamic constraint incorporation technique and optimisation algorithm.

Jiang *et al.* [35] utilise an orthogonal collocation discretisation-based reduced-space IPM algorithm to solve an emergency control problem with a first-swing stability consideration. Wang *et al.* [36] propose a risk-based coordinating framework for preventive and corrective control with transient stability constraints. High-risk and low-risk contingencies are considered in generation rescheduling-based preventive control and load shedding corrective control. Transient stability performance is shown to be enhanced by this coordination strategy.

5 Future directions

TSC-OPF is an extremely challenging problem, both mathematically and computationally, coupling different power system time-scales. It has been explored by power system researchers over the last two decades with a variety of solution approaches and applications of TSC-OPF been devised. Yet, there are still many advances needed to be made on TSC-OPF to make it practically viable for industry use. We highlight a few future directions for TSC-OPF to improve its robustness and computational efficiency, expand its use in new applications, and address practical limitations. In addition, we also provide the advancements made by researchers in this space.

5.1 TSC-OPF coupled unit commitment

While TSC-OPF has been hitherto used for coupling transient constraints in optimal power flow, its expansion to a unit commitment is an interesting research topic, particularly to assess the impacts of switching quick start units. The application of TSC-OPF for unit commitment explodes the mathematical complexity due to the combination of a mixed-integer (for committing units and their integer constraints such as start-up/shut-down) and dynamic optimisation (TSC-OPF) formulation. Jiang *et al.* [37] have attempted to solve the TSCUC problem by formulating transient stability constraints for each period in unit commitment. Their solution approach is based on an augmented Lagrangian relaxation algorithm that decomposes the problem into a master dynamic programming problem to handle mixed-integer variables and a set of TSC-OPF problems for different time periods. Reduced-space IPM-based simultaneous discretisation is utilised to solve each subproblem. Also, since these TSC-OPF subproblems are independent of each other, they are processed in parallel processing units.

5.2 Practical application in electricity markets

Currently, ISOs uses a DC-based economic dispatch for market operations due to its robustness and, more importantly, the confidence operators have in its approach. Usage of an ACOPF for market operations is still being debated in the industry with a few ISOs warming up to the idea of piloting an ACOPF solution (though there are concerns expressed for cases where ACOPF diverges and does not provide a solution). Not only does TSC-OPF carry the same adoption issues of ACOPF but further expands it due to the inclusion of transient constraints. Before even surmising the adoption of TSC-OPF in a market environment, important questions such as (a) how to quantify the risk of instability under low-probability high-impact contingencies and allocated additional operational cost caused by stability constraints, and (b) how to decide locational marginal prices for TSC-OPF, need to be addressed. The authors of [34, 38] have sketched out interesting ideas in this direction. Other issues for TSC-OPF adoption with respect to its algorithm robustness and computational complexity must be tackled to make it a practical solution.

5.3 Dynamic parameter estimation

Application of TSC-OPF for estimation of system and model parameters is a very timely topic with the increasing penetration of sensors, particularly phasor measurement units, and distributed energy resources. The parameter estimation with TSC-OPF problem inherits a similar formulation [39] and solution technique [29]. A best fit for the model parameters can be estimated by minimising the difference between the observed measurements and model response. Hiskens [31] uses a trajectory sensitivity analysis approach for dynamic parameter estimation from system wide measurements. Choi *et al.* [40] focus on identifying the parameters for dynamic load models, among multiple chosen candidate models, akin to a grey box estimation approach.

5.4 Uncertainty quantification

Power system is witnessing a great paradigm shift due to the increasing penetration of variable and intermittent renewable energy resources and price-responsive loads resulting in the transition from deterministic to stochastic approaches for decision making. Understanding the impact of stochasticity on stability and its application to economic dispatch (e.g. preventive TSC-OPF with high penetration of renewables) is an unexplored territory, though researchers are exploring methods to address uncertainty in steady-state voltage stability and small-signal stability-constrained OPF, using boundary-based methods [41, 42].

5.5 Parallel computing

Since TSC-OPF is a computationally intensive problem, especially with the consideration of multiple contingencies, parallel computing techniques to accelerate the solution process is a natural fit. Several efforts to accelerate the TSC-OPF solution have been undertaken by the power system community in the recent past.

Cai *et al.* [14] use differential evolution as optimisation algorithm and divide overall population into several sub-populations evenly among processing units on a Beowulf cluster. Since the computations of subpopulations (i.e. power flow calculation and transient stability assessment) are done simultaneously, the overall computation time is reduced by this simple but effective parallelisation.

Based on a similar multi-core CPU based Beowulf cluster, Geng and Jiang [43] present a two-level parallel decomposition approach for a simultaneous discretisation-based multi-contingency TSC-OPF problem. Their approach involves dividing the contingency list in the first level followed by a multi-threaded solution approach for each contingency. The effectiveness of the proposed parallel algorithm is benchmarked with 16 nodes with 128 CPU cores using test cases up to 2746 buses and 16 contingencies. In [30], Geng *et al.* parallelise the multiple-shooting method for TSC-OPF with each computing task comprised of a time-domain simulation and trajectory sensitivity analysis on different shooting intervals.

Parallel and distributed computing approaches for solving the security constrained optimal power problem [44] (without transients) are also applicable for the TSC-OPF problem (with contingencies) as it has a similar problem structure.

5.6 Modelling complexity and extreme events

Transient stability analysis is becoming more challenging because of the need for simulating various practical behaviours of power system dynamics, including response of high-order dynamic device models (e.g. FACTS, HVDC – high-voltage, direct current and renewable generations) and complicated system-level responses (e.g. cascading failures and protection schemes). Calle *et al.* [45] address an HVDC–line-commutated converters (LCC) link in a TSCOPF formulation for a practical transmission system. Different recovery patterns of the HVDC link after a severe fault are evaluated. This highlights one of the extended applications of TSC-OPF. Modelling these behaviours in a TSC-OPF formulation is a difficult task, and the majority of the work done in this area has used simplified dynamic models. Interfacing with industry-proven simulation tools is a practical and efficient way to tackle this problem, as shown in [1–3, 6, 7]. However, most existing power system time-domain simulation tools lack the feature of trajectory sensitivity analysis, which is an essential algorithmic component in incorporating dynamic constraints for TSC-OPF problems. Further efforts must be taken to develop this feature in an efficient and seamless way for existing simulation tools.

6 Conclusions

This paper presented a survey of the SIME and computationally intelligent approaches for solving the TSC-OPF problem. Extensive applications of TSC-OPF in various fields of power system analysis were discussed. We highlighted potential future research areas in this domain and provided an extensive list of references. Information on publicly available data for test systems and relevant numerical libraries are listed.

In summary, transient stability-constrained optimal power flow problem is a challenging problem that is both computationally intensive and mathematically rigorous. TSC-OPF offers an effective and efficient solution for power system decision-making when dynamic performance becomes a major concern. Intensive research efforts to address different aspects of TSC-OPF solution techniques are needed in order to make TSC-OPF a practical operational tool in the industry.

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8 References

[1] Pavella, M., Ernst, D., Ruiz-Vega, D.: '*Transient stability of power systems: a unified approach to assessment and control*' (Kluwer Academic Publishers, 2000)

[2] Ruiz-Vega, D., Pavella, M.: 'A comprehensive approach to transient stability control. Part i: near optimal preventive control', *IEEE Trans. Power Syst.*, 2003, **18**, (4), pp. 1446–1453

[3] Ruiz-Vega, D., Salinas, O., Pavella, M.: 'Simultaneous optimization of transient stability-constrained transfer limits of multi-area power systems'. Proc. Med Power 2002 Conf., 2002

[4] Pizano-Martianez, A., Fuerte-Esquivel, C.R., Ruiz-Vega, D.: 'Global transient stability-constrained optimal power flow using an omib reference trajectory', *IEEE Trans. Power Syst.*, 2010, **25**, (1), pp. 392–403

[5] Pizano-Martinez, A., Fuerte-Esquivel, C.R., Ruiz-Vega, D.: 'A new practical approach to transient stability-constrained optimal power flow', *IEEE Trans. Power Syst.*, 2011, **26**, (3), pp. 1686–1696

[6] Bettiol, A.L., Wehenkel, L., Pavella, M.: 'Transient stability constrained maximum allowable transfer', *IEEE Trans. Power Syst.*, 1999, **14**, (2), pp. 654–659

[7] Pavella, M., Murthy, P.G.: '*Transient stability of power systems: theory and practice*' (Wiley, 1994)

[8] Tu, X., Dessaint, L., Kamwa, I.: 'A global approach to transient stability constrained optimal power flow using a machine detailed model', *Can. J. Electr. Comput. Eng.*, 2013, **36**, (1), pp. 32–41

[9] Zárate-Minano, R., Van Cutsem, T., Milano, F., *et al.*: 'Securing transient stability using time-domain simulations within an optimal power flow', *IEEE Trans. Power Syst.*, 2010, **25**, (1), pp. 243–253

[10] Xia, S., Chan, K.W., Guo, Z.: 'A novel margin sensitivity based method for transient stability constrained optimal power flow', *Electr. Power Syst. Res.*, 2014, **108**, pp. 93–102

[11] Eberhart, R.C., Shi, Y.: '*Computational intelligence: concepts to implementations*' (Elsevier Science, 2011)

[12] Sorensen, K., Glover, F.: 'Metaheuristics', in Gass, S.I., Fu, M.C. (Eds.): '*Encyclopedia of operations research and management science*' (Wiley, 2013)

[13] Zhang, X., Dunn, R.W., Li, F.: 'Stability constrained optimal power flow for the balancing-market using genetic algorithms'. Power Engineering Society General Meeting, 2003, July 2003, vol. **2**, pp. 932–937

[14] Cai, H.R., Chung, C.Y., Wong, K.P.: 'Application of differential evolution algorithm for transient stability constrained optimal power flow', *IEEE Trans. Power Syst.*, 2008, **23**, (2), pp. 719–728

[15] Mo, N., Zou, Z.Y., Chan, K.W., *et al.*: 'Transient stability constrained optimal power flow using particle swarm optimisation', *IET Gener. Transm. Distrib.*, 2007, **1**, (3), pp. 476–483

[16] Wu, D., Gan, D., Jiang, J.N.: 'An improved micro-particle swarm optimization algorithm and its application in transient stability constrained optimal power flow', *Int. Trans. Electr. Energy Syst.*, 2014, **24**, (3), pp. 395–411

[17] Ye, C.J., Huang, M.X.: 'Multi-objective optimal power flow considering transient stability based on parallel NSGA-ii', *IEEE Trans. Power Syst.*, 2015, **30**, (2), pp. 857–866

[18] Chen, Y., Luo, F., Xu, Y., *et al.*: 'Self-adaptive differential approach for transient stability constrained optimal power flow', *IET Gener. Transm. Distrib.*, 2016, **10**, (15), pp. 3717–3726

[19] Mukherjee, A., Roy, P.K., Mukherjee, V.: 'Transient stability constrained optimal power flow using oppositional krill herd algorithm', *Int. J. Electr. Power Energy Syst.*, 2016, **83**, pp. 283–297

[20] Xia, S.W., Zhou, B., Chan, K.W., *et al.*: 'An improved fGSOg method for discontinuous non-convex transient stability constrained optimal power flow with complex system model', *Int. J. Electr. Power Energy Syst.*, 2015, **64**, pp. 483–492

[21] Li, Z., Geng, G., Jiang, Q.: 'Transient stability emergency control using asynchronous parallel mixed-integer pattern search', *IEEE Trans. Smart Grid*, 2016, **PP**, (99), pp. 1–1

[22] Nguyen-Duc, H., Tran-Hoai, L., Cao-Duc, H.: 'An approach to solve transient stability-constrained optimal power flow problem using support vector machines', *Electr. Power Compon. Syst.*, 2017, **45**, (6), pp. 624–632

[23] Miranda, V., Fidalgo, J.N., Lopes, J.A.P., *et al.*: 'Real time preventive actions for transient stability enhancement with a hybrid neural network-optimization approach', *IEEE Trans. Power Syst.*, 1995, **10**, (2), pp. 1029–1035

[24] Tangpatiphan, K., Yokoyama, A.: 'Evolutionary programming incorporating neural network for transient stability constrained optimal power flow'. Joint Int. Conf. Power System Technology and IEEE Power India Conf. 2008, POWERCON 2008, October 2008, pp. 1–8

[25] Xin, H., Gan, D., Huang, Z., *et al.*: 'Applications of stability-constrained optimal power flow in the east china system', *IEEE Trans. Power Syst.*, 2010, **25**, (3), pp. 1423–1433

[26] Gan, D., Thomas, R.J., Zimmerman, R.D.: 'Stability-constrained optimal power flow', *IEEE Trans. Power Syst.*, 2000, **15**, (2), pp. 535–540

[27] Hiskens, I.A., Pai, M.A., Sauer, P.W.: 'Dynamic atc'. Power Engineering Society Winter Meeting, 2000, January 2000, vol. **3**, p. 1629

[28] Zhang, X., Song, Y.H., Qiang, L., *et al.*: 'Dynamic available transfer capability (ATC) evaluation by dynamic constrained optimization', *IEEE Trans. Power Syst.*, 2004, **19**, (2), pp. 1240–1242

[29] Zavala, V.M., Laird, C.D., Biegler, L.T.: 'Interior-point decomposition approaches for parallel solution of large-scale nonlinear parameter estimation problems', *Chem. Eng. Sci.*, 2008, **63**, (19), pp. 4834–4845

[30] Geng, G., Ajjarapu, V., Jiang, Q.: 'A hybrid dynamic optimization approach for stability constrained optimal power flow', *IEEE Trans. Power Syst.*, 2014, **29**, (5), pp. 2138–2149

[31] Hiskens, I.A.: 'Nonlinear dynamic model evaluation from disturbance measurements', *IEEE Trans. Power Syst.*, 2001, **16**, (4), pp. 702–710
[32] Tiwari, A., Ajjarapu, V.: 'Optimal allocation of dynamic var support using mixed integer dynamic optimization', *IEEE Trans. Power Syst.*, 2011, **26**, (1), pp. 305–314
[33] Paramsivam, M., Salloum, A., Ajjarapu, V., *et al.*: 'Dynamic optimization based reactive power planning to mitigate slow voltage recovery and short term voltage instability', *IEEE Trans. Power Syst.*, 2013, **28**, (4), pp. 3865–3873
[34] Nguyen, T.T., Nguyen, V.L., Karimishad, A.: 'Transient stability-constrained optimal power flow for online dispatch and nodal price evaluation in power systems with flexible ac transmission system devices', *IET Gener. Transm. Distrib.*, 2011, **5**, (3), pp. 332–346
[35] Jiang, Q., Wang, Y., Geng, G.: 'A parallel reduced-space interior point method with orthogonal collocation for first-swing stability constrained emergency control', *IEEE Trans. Power Syst.*, 2014, **29**, (1), pp. 84–92
[36] Wang, Z., Song, X., Xin, H., *et al.*: 'Risk-based coordination of generation rescheduling and load shedding for transient stability enhancement', *IEEE Trans. Power Syst.*, 2013, **28**, (4), pp. 4674–4682
[37] Jiang, Q., Zhou, B., Zhang, M.: 'Parallel augment Lagrangian relaxation method for transient stability constrained unit commitment', *IEEE Trans. Power Syst.*, 2013, **28**, (2), pp. 1140–1148
[38] Yue, Y., Kubokawa, J., Sasaki, H.: 'Pricing for transient stability'. Proc. 2004 IEEE Int. Conf. Electric Utility Deregulation, Restructuring and Power Technologies, 2004 (DRPT 2004), 2004, vol. **1**, pp. 332–336
[39] Tjoa, I.B., Biegler, L.T.: 'Simultaneous solution and optimization strategies for parameter estimation of differential-algebraic equation systems', *Ind. Eng. Chem. Res.*, 1991, **30**, (2), pp. 376–385
[40] Choi, B.-K., Chiang, H.-D., Li, Y., *et al.*: 'Measurement-based dynamic load models: derivation, comparison, and validation', *IEEE Trans. Power Syst.*, 2006, **21**, (3), pp. 1276–1283
[41] Hamon, C., Perninge, M., Soder, L.: 'A stochastic optimal power flow problem with stability constraints – part i: Approximating the stability boundary', *IEEE Trans. Power Syst.*, 2013, **28**, (2), pp. 1839–1848
[42] Perninge, M., Hamon, C.: 'A stochastic optimal power flow problem with stability constraints, part ii: The optimization problem', *IEEE Trans. Power Syst.*, 2013, **28**, (2), pp. 1849–1857
[43] Geng, G., Jiang, Q.: 'A two-level parallel decomposition approach for transient stability constrained optimal power flow', *IEEE Trans. Power Syst.*, 2012, **27**, (4), pp. 2063–2073
[44] Phan, D., Kalagnanam, J.: 'Some efficient optimization methods for solving the security-constrained optimal power flow problem', *IEEE Trans. Power Syst.*, 2013, **29**, pp. 863–872
[45] Calle, I.A., Ledesma, P., Castronuovo, E.D.: 'Advanced application of transient stability constrained-optimal power flow to a transmission system including an HVDC-LCC link', *IET Gener. Transm. Distrib.*, 2015, **9**, (13), pp. 1765–1772
[46] La Scala, M., Trovato, M., Antonelli, C.: 'On-line dynamic preventive control: an algorithm for transient security dispatch', *IEEE Trans. Power Syst.*, 1998, **13**, (2), pp. 601–610
[47] Yue, Y., Kubokawa, J., Sasaki, H.: 'A solution of optimal power flow with multicontingency transient stability constraints', *IEEE Trans. Power Syst.*, 2003, **18**, (3), pp. 1094–1102
[48] Jiang, Q., Huang, Z.: 'An enhanced numerical discretization method for transient stability constrained optimal power flow', *IEEE Trans. Power Syst.*, 2010, **25**, (4), pp. 1790–1797
[49] Jiang, Q., Geng, G.: 'A reduced-space interior point method for transient stability constrained optimal power flow', *IEEE Trans. Power Syst.*, 2010, **25**, (3), pp. 1232–1240
[50] Chen, L., Taka, Y., Okamoto, H., *et al.*: 'Optimal operation solutions of power systems with transient stability constraints', *IEEE Trans. Circuits Syst.*, 2001, **48**, (3), pp. 327–339
[51] Sun, Y., Xinlin, Y., Wang, H.F.: 'Approach for optimal power flow with transient stability constraints', *IEE Gener. Transm. Distrib.*, 2004, **151**, pp. 8–18
[52] Xia, Y., Chan, K.W., Liu, M.: 'Direct nonlinear primal-dual interior-point method for transient stability constrained optimal power flow', *IEE Proc. Gener. Transm. Distrib.*, 2005, **152**, (1), pp. 11–16
[53] Xia, Y., Chan, K.W.: 'Dynamic constrained optimal power flow using semi-infinite programming', *IEEE Trans. Power Syst.*, 2006, **2**, (3), pp. 1455–1457
[54] Fang, D.Z., Yang, X., Sun, J., *et al.*: 'An optimal generation rescheduling approach for transient stability enhancement', *IEEE Trans. Power Syst.*, 2007, **22**, (1), pp. 386–394
[55] Abhyankar, S., Rao, V., Anitescu, M.: 'Dynamic security constrained optimal power flow using finite difference sensitivities'. Power Engineering Society General Meeting, 2014, 2014
[56] Tu, X., Dessaint, L.-A., Huy, N.-D.: 'Transient stability constrained optimal power flow using independent dynamic simulation', *IEE Gener. Transm. Distrib.*, 2013, **7**, (3), pp. 244–253
[57] Chandrashekhar, K.S., Hill, D.J.: 'Dynamic security dispatch: basic formulation', *IEEE Trans. Power Appar. Syst.*, 1983, **PAS-102**, pp. 2145–2154
[58] Nguyen, T., Pai, M.A.: 'Dynamic security-constrained rescheduling of power systems using trajectory sensitivities', *IEEE Trans. Power Syst.*, 2003, **18**, (2), pp. 848–854
[59] Vittal, V., Zhou, E.Z., Hwang, C., *et al.*: 'Derivation of stability limits using analytical sensitivity of the transient energy margin', *IEEE Trans. Power Syst.*, 1989, **4**, pp. 1363–1372
[60] Sterling, J., Pai, M.A., Sauer, P.W.: 'A method of secure and optimal operation of power system for dynamic contingencies', *Electr. Mach. Power Syst.*, 1991, **19**, pp. 639–655
[61] Fouad, A.A., Jianzhong, T.: 'Stability constrained optimal rescheduling of generation', *IEEE Trans. Power Syst.*, 1993, **8**, (1), pp. 105–112
[62] Kuo, D., Bose, A.: 'A generation rescheduling method to increase the dynamic security of power systems', *IEEE Trans. Power Syst.*, 1995, **10**, pp. 68–76
[63] Singh, S.N., David, A.K.: 'Dynamic security in open power market dispatch', *Electr. Power Compon. Syst.*, 2002, **30**, pp. 315–330
[64] Shubhanga, K.N., Kulkarni, A.M.: 'Stability-constrained generation rescheduling using energy margin sensitivities', *IEEE Trans. Power Syst.*, 2004, **19**, (3), pp. 1402–1413
[65] Sauer, P., Pai, M.A.: '*Power system dynamics and stability*' (Prentice Hall Inc., 1998)
[66] Zimmerman, R., Murillo-Sanchez, C.: 'Matpower 4.0 users manual'. Technical report, Power Systems Engineering Research Center, 2011
[67] Pai, M.A.: '*Energy function analysis for power system stability*' (Kluwer, 1989)
[68] Institute of Electrical Engineers in Japan (IEEJ). Japanese power system models. http://www2.iee.or.jp/ver2/pes/23-st_model/english/index.html, 2014
[69] Andersson, J.: 'A general-purpose software framework for dynamic optimization'. PhD thesis, Arenberg Doctoral School, KU Leuven, Department of Electrical Engineering (ESAT/SCD) and Optimization in Engineering Center, Kasteelpark Arenberg 10, 3001-Heverlee, Belgium, October 2013
[70] Diehl, M., Leineweber, D.B., Schäfer, A.: 'MUSCOD-II Users' Manual', http://www.iwr.uni-heidelberg.de/~agbock/RESEARCH/MUSCOD/usermanual.pdf, 2001
[71] Kirches, C., Leyffer, S.: 'TACO – a Toolkit for AMPL Control Optimization', http://www.iwr.uni-heidelberg.de/groups/agbock/FILES/taco_paper.pdf, 2011
[72] Fourer, R., Gay, D., Kernighan, B.W.: '*The ampl book*' (Cengage Learning, 2002)
[73] Brooke, A., Kendrick, D.A., Meeraus, A., *et al.*: '*GAMS: a user's guide*' (Course Technology, 1988)
[74] Vivlabs LLC: 'Adc: Automatic differentiation software for c/c++', 2014
[75] Bischof, C., Roh, L., Mauer-Oatsy, A.: 'Adic: an extensible automatic differentiation tool for ansi-c', *Urbana*, 1997, **51**, p. 61802
[76] Bell, B.M.: 'Cppad: A package for differentiation of c++ algorithms', 2014
[77] Walther, A., Griewank, A.: 'Getting started with adol-c', in Naumann, U., Schenk, O. (Eds.): '*Comb. Sci. Comput.*, (Chapman & Hall/CRC, 2012), pp. 181–202
[78] Wächter, A., Biegler, L.T.: 'On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming', *Math. Program.*, 2006, **106**, (1), pp. 25–57
[79] Byrd, R.H., Nocedal, J., Waltz, R.A.: 'Knitro: An integrated package for nonlinear optimization', in Di Pillo, G., Roma, M. (Eds.): '*Large-scale nonlinear optimization*' (Springer, 2006), pp. 35–59
[80] Bartlett, R.: 'Moocho: Multi-functional object-oriented architecture for optimization', 2014
[81] Gill, P.E., Murray, W., Saunders, M.A.: 'Snopt: an sqp algorithm for large-scale constrained optimization', *SIAM J. Optim.*, 2002, **12**, (4), pp. 979–1006
[82] Franke, R.: 'Hqp: a solver for sparse nonlinear optimization', 2014
[83] Balay, S., Abhyankar, S., Adams, M.F., *et al.*: 'PETSc Web page', http://www.mcs.anl.gov/petsc, 2014
[84] Hindmarsh, A.C., Brown, P.N., Grant, K.E., *et al.*: 'Sundials: suite of nonlinear and differential/algebraic equation solvers', *ACM Trans. Math. Softw. (TOMS)*, 2005, **31**, (3), pp. 363–396

9 Appendix

9.1 Summary of different solution approaches

Table 1 (overleaf) provides a comprehensive literature resource for the solution of TSC-OPF problem.

9.2 Available test system data

- *IEEE 9-bus system:* Data for test 9-bus system is available in [65]. This system has three generators, three loads, and nine transmission lines. The generators are modelled as a fourth-order dynamic model describing its mechanical and electrical equations. Each generator is equipped with an IEEE-T1 exciter model.
- *IEEE 39-bus system:* This IEEE 39-bus system consists of n generators, *n* loads, and *n* transmission lines. The steady-state data and generator costs can be obtained from [66]. The dynamic data is available in [67].
- *Reduced regional Chinese system:* The reduced regional Chinese power system available in [51] consists of 36 buses, 7 synchronous generators, 1 synchronous compensator and 26 transmission lines. Synchronous machines are represented by a third-order detailed model, and each of them is equipped with a fourth-order model excitation system.
- *Japanese power system test cases:* Institute of Electrical Engineers in Japan (IEEJ) [68] provides network and dynamic

Table 1 Summary of different solution approaches

Solution approach	References
simultaneous discretisation	[26, 43, 46–49]
constraint transcription	[50–55]
SIME	[2, 4–7, 9, 56]
CI	[13–15, 23, 24]
sensitivity based	[34, 57, 58]
transient energy function	[59–64]

data for four power system test cases comprising of 10 and 30 machines. These models possess the distinctive characteristics of Japanese power systems and have been developed with the objective of providing common system models for engineers and researchers in power system engineering.

9.3 Numerical libraries resources

- *Dynamic optimisation:* CasADi [69], MUSCOD-II [70], TACO [71].
- *Optimisation modelling platform:* AMPL [72], GAMS [73].
- *Automatic differentiation:* ADC [74], ADIC [75], CppAD [76], ADOL-C [77].
- *Non-linear optimisation:* Ipopt [78], KNITRO [79], MOOCHO [80], SNOPT [81], HQP [82].
- *Time-domain simulation:* PETSc [83], SUNDIALS [84].