*Revision, changes marked

Dynamic Fragmentation of Planetary Materials: Sub-Hypervelocity Ejecta Measurements and Velocity Scaling

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Abstract

Ejecta velocities derived during impacts into planetary materials has been examined for sub-sonic impact speeds and weak-shock conditions. Ejecta velocity fields were able to be collapsed when normalized by v_{max}, $v_{50\%\ mass}$, and $v_{50\%\ KE}$. These correspond to the maximum velocity and median values of mass and kinetic energy among ejecta velocities. Semi-empirical models were developed to provide predictive capabilities of 10^{th}, 50^{th}, and 90^{th} percentiles of the distributions of mass, momentum and kinetic energy with respect to ejecta velocity. Lastly, a functional form describing the probability density distribution of mass, momentum and kinetic energy among ejecta velocities was derived. Data and predictive models are valuable in the development and validation of numerical models, where comparison between experiments and simulations rely on well characterized measurements.

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Keywords: dynamic fragmentation, ejecta measurements, non-dimensional analysis, brittle failure, particle tracking velocimetery, debris field quantification, planetary materials

1. Introduction

Well characterized experiments are needed in the development and validation of constitutive modelling (Graham-Brady, 2010, Paliwal and Ramesh, 2008) of planetary materials and in simulating impact events (Jutzi et al., 2010). Numerical outputs may be compared with experimental results to improve fragmentation schemes (e.g., what constitutes a fragment, how do they interact), or may facilitate simulation refinement or simplification (e.g., ejecta contact algorithms). Improvements to numerical codes will enable a better understanding of material ejection during impacts into planetary materials (Housen and Holsapple, 2012, Osinski et al., 2011).

A better understanding of material ejection requires an improved assessment of ejecta field shape and velocity (Hermalyn et al., 2012, Hogan et al., 2013a, Housen and Holsapple, 2011, Jutzi et al., 2010, Piekutowski et al., 1977). Ejecta velocity measurement data has been obtained in numerical simulations (Artemieva et al., 2009, Jutzi et al., 2010). However, despite vast improvements in numerical modelling schemes, current numerical simulations are unable to fully reproduce ejecta cloud formation and deposit characteristics (Artemieva et al., 2009).

Experimental measurements of ejecta velocity have been previously investigated by many authors (Cintala et al., 1999, Gault and Heitowit, 1963, Hartmann, 1985, Hermalyn et al., 2012, Housen and Holsapple, 2011, Jutzi et al., 2010, Michikami et al., 2007, Piekutowski et al., 1977, Schultz, 2006, Shuvalov and

Trubetskaya, 2008, Stoffler et al., 1975, Yamamoto et al., 2005, Yamamoto and Nakamura, 1997). A compilation of past experiments can be found in the recent review of Housen and Holsapple (2011). Past studies have primarily deployed vertical impacts to simulate impact cratering processes into granular (Braslau, 1970, Hermalyn and Schultz, 2010) and analog lunar and asteroid materials (Hartmann, 1985). Ejecta velocity is commonly plotted against radial distance from the impact point and cumulative distributions of mass among velocity are derived for discrete time intervals (Hermalyn and Schultz, 2010). Bridging results for all times and obtaining a more exhaustive set of measurements will enable better interpretations of key results and provide a more complete understanding of excavation of these materials.

There have been numerous methods used to measure ejecta velocity. Piekutowski (1977) and Cintala et al. (1999) determined ejecta trajectories using a laser sheet to illuminate fragments captured by a high-speed camera. Once fragments were identified, ballistic equations were used to back calculate velocities. Vector fields of discernible ejecta have also been traced onto photographs (Fujiwara and Tsukamoto, 1980). Restricted interrogation area and image resolution, coupled with triggering issues, reduce the total number of fragments that can be measured using these methods. Particle image velocimetery (Anderson et al., 2003) and particle tracking velocimetry (Hermalyn and Schultz, 2010) have also been used to track ejecta fields. Such experiments are difficult to perform and the total number of fragments recorded is limited due to the cluttered nature of the debris field at impact speeds >1 km/s. This renders achieving a complete data set challenging.

Non-dimensional scaling models have been previously developed in order to extrapolate laboratory results to planetary scales (Hermalyn et al., 2012, Housen and Holsapple, 2011, Schultz, 2006, Shuvalov and Trubetskaya, 2008). Models generally employ non-dimensional ratios involving projectile size, impact speed and target strength. These models lead to characterization of early-time crater formation (Hermalyn and Schultz, 2010), later-stage ejecta velocity relations Jennifer L. B. Anderson and Heineck (2003), and ejection velocity and field profile predictions (Housen et al., 1983). Further development of models predicting mechanisms of ejecta cloud formation is critical for the improvement and validation of high level computation models (Jutzi et al., 2010).

This paper examines material ejection during impact testing of planetary materials. Ejecta velocity and field shapes are considered in detail. This investigation is a part a broader study by Hogan et al. (2013a,b, 2012, 2011) on the dynamic fragmentation of planetary materials during impact. Two important stages of the impact events are quantified: (1) fragmentation and (2) material ejection. To date, this work has been primarily focused on quantifying fragmentation (Hogan et al., 2013b, 2012) and investigating micro-scale thermal and fracture effects (Hogan et al., 2013b, 2012, 2011). Fragmentation results have been shown to have good agreement with theoretical models of fragment sizes (e.g., Grady (2009), Zhou et al. (2006)). In a recent study by Hogan et al. (2013a), ejecta velocity, size, mass, momentum and kinetic energy distributions during dynamic fragmentation of gabbro were quantified using particle tracking methods. In this work, image enhancement and post-processing improvements have been made to the tracking algorithm and tests have been performed for an additional three types of granitoid. The results of a total of 76 experiments for six target thicknesses (7 mm to 55 mm) and impact velocities of 20 to 550 m/s are compiled. Ejecta field shapes and the contributions of the mass, momentum, and kinetic energy among ejecta velocities are examined. Non-dimensional scaling laws are developed from the extensive set of experimental results and implications discussed.

2. Experimental Setup and Analysis Methods

The impact tests were performed at the French-German Research Institute of Saint-Louis (ISL), France. Target materials, target thicknesses, and impact velocities and energies are displayed in Table 1. Target materials, target configuration and projectiles are shown in Figure 1. The order of materials based on increasing SiO_2 content are gabbro, coarser grained monzonitic granitoid (monzonite), finer grained syenite granitoid (syenite), and finer grained tonalitic granitoid (tonalite). Glass-fibre reinforced composite projectiles (45 g) were used for the fine grained syenite and aluminum projectiles (65 g) were used for the others. The finer-grained block experiments with the composite projectile were performed in a series of experiments earlier than the others and before aluminum projectiles were developed. The effect of projectile density is not considered in this paper, but it is worth noting that projectile density and strength will have an effect on the early-time energy coupling of the impact. Composite projectiles can explode upon impact, thereby coupling relatively less kinetic energy to the target. The flat projectile face configuration yields flyer-plate conditions at impact, where the propagated shock-wave induces fragmentation and ejection of the material. Fragmentation through crushing also occurs at the projectile-target interface. Targets were sandwiched in fitted windowed metal plates that were allowed to expand laterally.

3. Particle Tracking Methods

A Photron APX Ultima video camera filming at a 8 kHz frame rate captured images of material ejected at the rear of the targets. Two high-powered lamps were used to back-illuminate the particles against a black background (Figure 2). Proper lighting and contrast between fragments and background was found to be critical for image enhancement. A tracking algorithm written in Matlab (2013) was implemented to track ejecta larger than 1 mm (determined by resolution of the camera as three pixels) over multiple high-speed camera images. Here it is assumed that two-dimensional projection of the field onto the image is suitable for reliable results.

Pre-processing involves background subtraction and image enhancement within an interrogation window to make the ejecta more distinguishable (Figure 2b). The size of the window is determined by the expansion of the debris cloud, where a greater expansion results in a larger initial window size. This is done in order to maximize the highest number of possible fragments to be tracked during the early stages of the debris field formation. Shown in Figure 2a and b are examples of video and enhanced images for tonalite at 20 m/s and a target thickness of 10 mm. In this case, and in many low-speed cases, there are relatively few, but easily distinguishable fragments.

For highly cluttered debris fields, image enhancement is performed in two stages. The first stage involves identifying and enhancing fragments $>$3 mm, as was done for less cluttered fields. Connected larger fragments are isolated, identified fragments are removed, and the second stage is applied. The second stage involves discretizing the remaining window and performing sub-enhancement of cluttered regions. Fragments are identified as brighter areas in these sub-regions.

Once the images are enhanced, fragment size and area centers are determined. An example of a enhanced highly cluttered debris field is shown in Figure 2d and e.

Probable matches between consecutive enhanced high-speed video frames, in conjunction with penalty functions of fragment sizes, shape and circularity are used to identify fragments. The displacement of the particle over time yields velocity. In order to improve algorithm computation times, fragments were assumed to travel in the positive x-direction and remain ordered in space and time.

Post-processing of the velocity fields involves removing erroneous vectors in highly cluttered areas by using a weighted spatial average of larger well-determined larger fragment velocities. Examples of velocity vectors are shown Figure 2c. θ is defined as arctan(vy/vx) and is referred to here as the ejection angle. The ejecta angles are taken as the projections in the image plane and referred to the normal of the target surface. A measurement of v_z is needed to determine the true ejection angle. Estimates of fragment masses are obtained by multiplying the projected area with the minor axis dimension[1] and density. For larger plate-like fragments (e.g., Figure 2a), a maximum in-plane length of 4 mm of the target thickness is assumed based on post-impact analysis of these fragments. Individual ejecta momentum and translational kinetic energy are estimated and the distribution of mass, momentum, and kinetic energy among ejecta velocity is calculated. Rotational energy has been shown to be two orders of magnitude smaller than translational kinetic energy (Fujiwara and Tsukamoto, 1980) and is not considered here.

The particle tracking methods applied have been used in a recent paper by Hogan et al. (2013a). Other methods to estimate ejecta velocity have been previously outlined (e.g., hand-tracing vector fields, penetrating foils, mass bins downstream).

[1]Taken as the perpendicular axis to the largest spanning dimension.

Previous measurements have been limited in the total number of experiments due to the associated cost and difficulty in performing high-speed image experiments, limited in the total number of ejecta when velocities are recorded and, when performed, in combined ejecta size (or mass) and velocity measurements. Challenges in obtaining these measurements are associated with cluttered debris fields, lack of computational power, camera resolution and triggering (e.g., laser sheet with proper field expansion). No data exists for solid targets at the velocities reported in this paper (i.e., sub-hypersonic).

The total number of fragments, especially the sub-mm fragments, is limited by camera resolution and the two-dimensional projection of the field in the high-speed video image. Fragments may be hidden in the image. These limitations are expected when performing ejecta measurements in highly cluttered debris fields. It is believed that particle tracking methods employed here provide the greatest possibility to achieve an almost complete set among all other techniques and experimental configuration. Ejecta measurements at discrete time intervals are combined into one extensive data set.

Recent particle size measurements of collected ejecta from these experiments by Hogan et al. (2013b) have shown that, while fragments smaller than 1 mm represent $> 99\ \%$ of the total number of fragments, they contain less than $1\ \%$ of the volume (or mass). The majority of these smaller fragments are formed ahead of the projectile and, therefore, have low ejection velocities (and do not contribute much to the total kinetic energy transfer). From this, it is assumed that they can be neglected when developing semi-empirical models predicting distributions of mass and kinetic energy among ejecta velocities.

The ability to track a representative amount of mass with the algorithm is also briefly considered. For all cases, over 93 $\%$ of the mass (i.e., the collected mass of ejecta after experimentation) is tracked by the algorithm. This provides further justification that smaller and, potential, hidden fragments can be neglected.

4. Experimental Results

4.1. Ejecta Field Scaling: Maximum, Mass and Kinetic Energy

Velocity components (v_x and v_y), and the ejection angle, θ (arctan(v_y/v_x)), plotted against the resultant velocity, v, are shown in Figures 9 to 6. Each point corresponds to a measured fragment and colors are used to denote different experiments. It is common to plot, for example, cumulative or average plots of statistical values. However, some features are better observed when the entire debris field is examined. Ejecta velocity fields are normalized three ways: the maximum resultant ejecta velocity, v_{max}; the velocity above which 50 $\%$ of the total mass is distributed, $v_{50\%\ mass}$; or the velocity above which 50 $\%$ of the total kinetic energy is distributed, $v_{50\%\ KE}$. Note that these normalizing values are different for each test case. Kinetic energy is estimated assuming that the out-of-plane velocity component, v_z, is equal to v_y. Normalizing by v_{max} provides information on how the material is fragmented within the thickness of the body; normalizing by $v_{50\%\ mass}$ indicates the proportion of fragmented mass through the body; and normalizing by $v_{50\%\ KE}$ reflects the expulsion of material from the body. Normalization of the velocity fields allows for common features to be extracted and expulsion mechanisms to be explored. As a note, the ejecta fields for the 7 mm and 10 mm tonalite targets are included in the Appendix.

4.1.1. Tonalite: 20 mm and impact energies of 38 J to 1,265 J

The normalized v_{max} ejecta field for 11 tonalite cases with target thickness of 20 mm for impact energies of 38 J to 1,265 J is shown in Figure 3a. Although not explicitly visible, the fields for 38 J to 292 J collapse (i.e., overlie on each other) for $|v_y/v_{max}|$ <0.3 and the fields for the 633 J to 1,265 J cases collapse for $|v_y/v_{max}|$ <0.2. The corresponding angle and resultant velocity plot are shown in Figure 3b. All fields are bounded by $|\theta|\leq 30°$ for $|v/v_{max}|\geq 0.5$.

The $v_{50\%\ mass}$ scaled ejecta field is shown in Figure 3c. Fields collapse well for $v_x/v_{50\%\ mass}\leq 2.1$ for impact energies of 38 J to 292 J, and for $v_x/v_{50\%\ mass}\leq 3.3$ for impact energies of 633 J to 1,265 J. All fields overlay well for $|v_y/v_{50\%\ mass}|<0.5$. Corresponding ejecta angles and resultant velocities are shown Figure 3d. The transition to $|\theta|\leq 30°$ begins at $|v/v_{max}|\geq 1.5$.

The normalized $v_{50\%\ KE}$ ejecta field is shown in Figure 3e. The fields for 38 J to 292 J collapse for $|v_y/v_{50\%\ KE}|\leq 0.4$ and the fields for the 633 J to 1,265 J cases collapse to $|v_y/v_{50\%\ KE}|\leq 0.2$. All fields are bounded by $v_x/v_{50\%\ KE}\leq 1.5$. Fields are bounded by $|\theta|\leq 30°$ for $v/v_{50\%\ KE}\geq 0.5$ (Figure 3f).

4.1.2. Tonalite: 30 mm (286 J to 2,500 J) and 40 mm (906 J to 2,243 J)

Shown in Figure 4a is the ejecta field normalized by v_{max} 13 cases of tonalite with target thicknesses of 30 mm and 40 mm. The fields collapse well, and have maximum expansions of $|v_y/v_{max}|\leq 0.3$ to 0.4. Angles gradually become bounded $|\theta|\leq 30°$ for $v/v_{max}\geq 0.5$ (Figure 4b). The maximum angle bounds is $|\theta|\leq 60°$ for the lowest v/v_{max}. The wave-like structures observed in the angle figures are artifacts of the algorithm. Time steps are chosen so as to not record the same ejecta twice. This is done by allowing the ejecta with the minimum x-velocity to travel a defined distance, thereby allowing the field to expand further.

Ejecta velocities greater than the minimum x-velocity in previous time steps are not recorded. The majority of KE and mass is contained above when these are first observed. Improvements to the algorithm are ongoing to mix two time steps and allow better overlapping.

Shown in Figure 4a is the ejecta field normalized by $v_{50\%\ mass}$. The fields collapse well for $|v_y/v_{50\%\ mass}| \leq 0.6$ and $|v_x/v_{50\%\ mass}| \leq 3.3$. The initial cone portion of the field expands for $v_x/v_{50\%\ mass} \leq 0.5$. This indicates that 25% of the mass is contained in this region. Recall, that $v_x/v_{50\%\ mass} = 1$ corresponds to 50 % of the mass. Angles become bounded by $|\theta| \leq 30°$ for $v/v_{50\%\ mass} \geq 1.5$ (Figure 4d).

The ejecta fields scaled by $v_{50\%\ KE}$ for the 30 mm and 40 mm tonalite experiments are shown in Figure 4e. Like most cases, the fields collapse well for $|v_y/v_{50\%\ KE}| \leq 0.3$ to 0.4 and $v_x/v_{50\%\ KE} \leq 1.3$. Angles become bounded by $|\theta| \leq 30°$ for $v/v_{50\%\ KE} \geq 0.5$ (Figure 4f).

4.1.3. Gabbro: 10 mm and impact energies of 21 J to 305 J

The normalized ejecta fields for the 19 gabbro tests (10 mm thick and impact energies of 21 J to 305 J) are shown in Figure 5. Ejecta fields collapse well for $|v_y/v_{max}| \leq 0.3$ for $v_x/v_{max} \geq 0.2$. The ejecta field is bounded by a cone for $v_x/v_{max} \leq 0.2$. Angles become bounded by $|\theta| \leq 30°$ and $v/v_{max} \geq 0.5$, with angles for $v/v_{max} \leq 0.5$ steadily increasing to 50° (Figure 5b).

Shown in Figure 5c is the ejecta field normalized by $v_{50\%\ mass}$. Ejecta fields do not collapse and are bounded by $v_x/v_{50\%\ mass} \leq 5$ and $|v_y/v_{50\%\ mass}| \leq 2$. Ejecta angles are bounded by $|\theta| \leq 30°$ for $v_x/v_{50\%\ mass} > 1$ (Figure 5d). Ejecta angles steadily decrease to a maximum of 50° for $v_x/v_{50\%\ mass} < 1$.

Shown in Figure 5e is the normalized $v_{50\%\ KE}$ ejecta field. The field is bounded by $v_x/v_{50\%\ KE} \leq 1.4$ and $|v_y/v_{50\%\ KE}| \leq 0.3$. The cone portion of the field is bounded by $v_x/v_{50\%\ KE} \leq 0.5$. This corresponds to 25% of the total kinetic energy. Ejecta angles are bounded by $|\theta| \leq 30°$ for $v_x/v_{50\%\ KE} \geq 0.6$ (Figure 5f).

4.1.4. *Syenite (2,710 J to 6,810 J) and coarser grained monzonite (1,940 J to 3,040 J): 55 mm*

Normalized ejecta fields for the syenite (five cases from 2,710 J to 6,810 J) and monzonite (four cases from 1,940 J to 3,040 J) 55 mm thick blocks are shown in Figure 6. The normalized v_{max} field is bounded by $|v_y/v_{max}| \leq 0.4$. Ejecta angles are bounded by $\pm 30°$ for all v/v_{max} (Figure 6b).

Shown in Figure 6c is the normalized $v_{50\%\ mass}$ ejecta field. The field collapses well for $v_x/v_{50\%\ mass} \leq 3$ and $|v_y/v_{50\%\ mass}| \leq 0.8$. The two highest energy cases have a few scattered points for $v_x/v_{50\%\ mass} > 3$. 50% of the mass, corresponding to $v_x/v_{50\%\ mass} = 1$, is contained in the dense region of field. Ejecta angles are bounded by $\pm 30°$ for all $v/v_{50\%\ mass}$ (Figure 6d).

Shown in Figure 6e is the normalized $v_{50\%\ KE}$ ejecta field. The field collapses well when normalized by $v_{50\%\ KE}$ and is bounded by $|v_y/v_{50\%\ KE}| \leq 0.4$ and $v_x/v_{50\%\ KE} \leq 1.5$. The dense region (the region bounded by $v_x/v_{max} < 0.3$ and $v_x/v_{50\%\ mass} < 1$) is bounded by $v_y/v_{50\%\ KE} \leq 0.5$, which contains 25% of the total kinetic energy. Ejecta angles are bounded by $\pm 30°$ for all $v/v_{50\%\ KE}$ (Figure 6f).

4.2. *Velocity Scales: Distributions of Mass, Momentum and Kinetic Energy*

Semi-empirical models relating experimental results with non-dimensional groups are developed in this section. Experimental data sets include 19 experiments involving gabbro tiles (10 mm thick), 9 experiments involving monzonite and syenite blocks, and 48 additional experiments with a tonalite (7 mm to 40 mm thick). The total number of experiments is 76. Dimensionless groups are formed using Buckingham Pi theorem (Munson et al., 1990). An attempt is made here to consider parameters that, when altered, affect the prediction of experimental results. Target thickness, t, is varied during experimentation and is considered influential during the fragmentation and ejection of the target material. Projectile length is taken as the characteristic length parameter in Housen and Holsapple (2011) to normalize the radial distance, R, from the impact point in vertical impacts. Projectile length is not considered here as it does not vary.

The input energy (KE) is also important in fragmentation and ejection processes during impact. Kinetic energy incorporates projectile dimensions (e.g., length), density, and velocity and is used here as a simplification to reduce the total number of non-dimensional groups. At hypervelocities, projectile density may be considered more important as target-projectile density mismatches affect shock wave generation at impact. Shown in Table 1 is, among other things, a summary of target thicknesses and impact energies for all experimental trials.

Target properties such as density (ρ: kg/m^3), yield strength (Y: MPa) and fracture toughness (K_c:MPa $\sqrt{m}$) are important during the dynamic fracture of planetary materials. These are also common material properties in theoretical predictions of dominant size during fragmentation (e.g., Grady (2009), Zhou et al. (2006)). Under the experimental conditions used here, target density represents a quantifier of compactness (e.g., similar feldspar-rich materials will have different densities if more flaws are present). Yield strength characterizes the ability of the target to deform before failure and fracture toughness is an indicator of post-failure (i.e., fracture) behaviour.

Housen and Holsapple (2011) discuss the challenges associated with choosing an appropriate strength parameter (e.g., yield strength or shear strength). Further complications arise when assigning values for these inputs as limited data exists in the literature for material properties of planetary materials. Static tensile yield strength values are chosen here because brittle materials typically fail in tension. The effect of loading rate (units: 1/s), which can be estimated as the ratio of impact velocity and target thickness, on material properties is not considered.

Housen and Holsapple (2011) discuss the challenges associated with choosing an appropriate strength parameter (e.g., yield strength or shear strength). Further complications arise when assigning values for these inputs as limited data exists in the literature for material properties of planetary materials. Tensile yield strength values are chosen here because brittle materials typically fail in tension. The effect of loading rate on material properties is not considered in the current paper, but can vary upwards of 5 to 10 times over a few orders of magnitude increase in strain rate (Housen, 2009). A value of 148 MPa is taken from Ai (1 February 2004) for the granitoid material. This seems reasonable based on work by Housen (2009), where strain rates in the current study are estimated as the ratio of impact velocity and target thickness with values between 10^3 to 10^4 s^{-1}.

Shown in Table 2 is a summary of target density, yield strength and fracture toughness. Material properties are similar for all materials; a result of choosing solid rock targets. Regardless, these properties vary slightly among the considered materials and are believed to be important in the dynamic fragmentation of planetary materials. A study involving a broader range of planetary materials (e.g., porous materials) or other brittle materials (e.g., boron carbide) would provide a greater insight into the effect of yield strength. There are many choices for input parameters and the selection of impact energy, target thickness, density, yield strength and fracture toughness does not represent a unique solution.

According to the Buckingham Pi Theorem (Munson et al., 1990), two non-dimensional groups can be formed with five independent variables (impact energy, target thickness, density, yield strength and fracture toughness) and three units (length, mass and time). Target thickness t is taken as the characteristic length ($L^\star$) term, ρt^3 is taken as the characteristic mass ($M^\star$) term and $\rho^{1/2} t\ Y^{-1/2}$ is taken as the characteristic time ($T^\star$) term. The resulting non-dimensional groups are:

$$KE^\star = \left(\frac{KE}{Y\ t^3}\right) \tag{1}$$

and

$$K_c^\star = \left(\frac{K_c}{Y\ t^{1/2}}\right) \tag{2}$$

The resulting form of the non-dimensional fit is thus:

$$aKE^{\star b} K_c^{\star c} \tag{3}$$

where a, b, and c are fitted coefficients obtained using a least-squares process. Fits yield semi-empirical models of experimental results. Values for ρ, Y and K_c are displayed in Table 2. The variation in target thickness (7 mm to 55 mm: 690 $\%$ difference) and kinetic energy (10 to 6,810 J: 68,100 $\%$ difference) will have a greater effect on a, b, and c than density (23 $\%$ variation), yield strength (3 $\%$) and fracture toughness (22 $\%$). Material property selections, regardless of similarity, are considered justifiable and enable normalization of the experimental results.

Important velocity scales during the dynamic fragmentation of brittle materials during impact testing are shown in Figure 7. In each case, velocity is normalized with the characteristic time and length scales. Corresponding non-dimensional groups are plotted with fitted coefficients using a least squares fit of the 50th percentile (or median). The maximum fragment velocity plotted against fitted non-dimensional groups is shown in Figure 7a. The maximum velocity increases for increasing $KE^{\star}$ (b=0.51) and $K_c{}^{\star}$ (c=0.18). Coefficient fits for the normalized v_{max} are reasonable to collapse the data (R^2=0.81).

Normalized 10^{th}, 50^{th}, and 90^{th} percentiles of $v_{mass}T^{\star}/L^{\star}$ are plotted against coefficient-fitted non-dimensional groups in Figure 7b. The v_{mass} notation is used to represent the distribution of mass among velocity. Coefficients are obtained through a least squares fit of equation (3) of the median value. As an example, 10% of the total mass lies below the 10^{th} percentile of v_{mass}. 50^{th} percentile values scale linearly with KE^{*} ($b\approx$1) and inversely linearly with $K_c{}^{*}$ ($c\approx$-1). The other percentile values ($P(x)$) are fitted with power-law curves in the form of:

$$P(x) = Kx^n \tag{4}$$

using a least squares fit to obtain coefficients K and n, where x is the fitted non-dimensional groups for the 50^{th} percentile (i.e., the values on the x-axis). By definition, the fit for the 50^{th} is $1x^1$. 50^{th} (R^2=0.89) and 90^{th} (R^2=0.70) percentile values are fitted well, with the 90^{th} percentile velocities increasing at a slower rate ($n_{90^{th}}$=0.6 vs. $n_{50^{th}}$=1). 10^{th} percentiles of the distribution of mass among velocities do not collapse well (R^2=0.40) and decrease at a slower rate ($n_{10^{th}}$=0.24).

Normalized 10^{th}, 50^{th}, and 90^{th} percentiles of $v_{mom}T^{\star}/L^{\star}$ are plotted against coefficient-fitted non-dimensional groups in Figure 7c. The v_{mom} notation is used to represent the distribution of momentum among ejecta velocity. Again, power coefficients on the x-axis are fitted for 50^{th} percentiles. The percentile values collapse well for the fitted coefficients (${R_{10^{th}}}^2$=0.88, ${R_{50^{th}}}^2$=0.89, and ${R_{90^{th}}}^2$=0.84). The power law exponents are similar in value and range from $n_{10^{th}}$=1.16 to $n_{50^{th}}$=1 and $n_{90^{th}}$=1.11.

Lastly, normalized 10^{th}, 50^{th}, and 90^{th} percentiles of $v_{KE}T^{\star}/L^{\star}$ are plotted against coefficient-fitted non-dimensional groups in Figure 7d. The v_{KE} notation is used to represent the distribution of kinetic energy among velocities. Percentiles values collapse well for the fitted coefficients (${R_{10^{th}}}^2$=0.94, ${R_{50^{th}}}^2$=0.93, and ${R_{90^{th}}}^2$=0.94). Power law exponents of equation (4) for the fitted percentile values decrease slightly, and are $n_{10^{th}}$=1.16, $n_{50^{th}}$=1, and $n_{90^{th}}$=0.94.

4.3. Cumulative Distributions: The Contribution of Mass, Momentum and Kinetic Energy Among Ejecta Velocities

Cumulative distributions of the percentage contribution of mass among velocities are shown in Figure 8a. Velocities are normalized by $v_{50\%\ mass}$. Commonly, distributions (or data sets) are non-dimensionalized using more complex forms (e.g., non-dimensional groups raised to exponents (Hermalyn and Schultz, 2010, Housen and Holsapple, 2011)). An example of a common type of data set is the cumulative distribution of mass versus velocity. In previous studies (e.g., Hermalyn and Schultz (2010), Housen and Holsapple (2011)),velocity is normalized by incoming projectile velocity multiplied by the ratio of target and projectile density raised to a power. Mass is normalized by the projectile mass. This does not necessarily produce universal collapse of all data sets over a range of impact velocities.

Another approach to collapse the curves is to normalize them by a single descriptor (e.g., average or maximum fragment size (Melosh, 1984)). In fragmentation studies by Zhou et al. 2005, the average fragment size is used to normalize the cumulative distributions of fragments for one-dimensional fragmentation of a bar. Average, or percentile values (e.g., median), based on number distributions are not advisable in impact experiments because their values depend on the total number of fragments measured. For example, a greater number of smaller fragments results in a much smaller median value, or a few large fragments results in a higher average value. Measuring all fragments, which may span from <1μm to 10 mm (Hogan et al., 2013b), during the dynamic fragmentation of brittle materials is not practical. Instead, median values of the distributions of mass, momentum and kinetic energy for ejecta velocities are considered in the current study. Their values do not depend on the total number of fragments, provided the more massive and faster ones are measured. This normalization provides reasonable collapse for all of the cumulative distribution curves.

By definition, curves pass through the point (1, 50) (Figure 8a). Most plots collapse reasonably well for v/v$_{mass}\geq$1, with the exception of the 7 mm tonalite data set and the 10 mm tonalite and gabbro data sets (noted in the figure). Maximums of v/v$_{mass}$ range from 1.3 to 4.1 for curves that collapse and 10 to 26 for 7 mm tonalite and 10 mm tonalite and gabbro sets. All data does not collapse well for v/v$_{mass}$<1, with no discernible trend in the order or shapes of the curves.

Also shown in Figure 8 is a fitted function of the form:

$$F\left(\frac{v}{v_i}\right) = C_1 e^{-C_2 * v/v_i} * v/v_i^{C_3} + 100 \tag{5}$$

where C$_i$ are constants and v$_i$ correspond to either v/v$_{mass}$, v/v$_{mom}$, or v/v$_{KE}$. The constants are obtained using a least squares fit and are also shown in the figure. Single power-law curves are often used for grouped data (Grady, 2009, Miljkovic et al., 2011). For example, the total number of ejecta between 10 mm and 100 mm in size is plotted at the midpoint, 55 mm. Power-law functions are also plotted for discrete time intervals and individual ejecta measurements (e.g., velocity plotted against launch position (Housen and Holsapple, 2012)). These are number dependent. The compilation of data for all time, and the use of the mass distribution instead of number, provides a more universal function form of the cumulative distributions of mass among velocity.

Differentiation provides the functional form for the probability density distribution, given by:

$$p.d.f.\left(\frac{v}{v_i}\right) = C_1 * C_2 * e^{C_2 * \frac{v}{v_i}} * \left(\frac{v}{v_i}\right)^{C_3} + C_1 * e^{C_2 * \frac{v}{v_i}} * \left(\frac{v}{v_i}\right)^{C_3} * C_3 * \frac{v_i}{v} \quad (6)$$

The function is obtained by curve-fitting the normalized data and provides predictive capabilities to others researchers and those numerically modelling these events.

The cumulative distributions of the percentage of momentum among velocities are shown in Figure 8b. Velocities on the x-axis are normalized by $v_{50\%\ mom}$. The data collapses much better when normalized by $v_{50\%\ mom}$ than previously by $v_{50\%\ mass}$, with maxima ranging from $v/v_{50\%\ mom}$=1.3 to 2.3. Again, data does not collapse as well for $v/v_{50\%\ mom} < 1$. Further improvements to collapsing occurs when the distribution of kinetic energy among velocity is normalized by $v_{50\%\ KE}$ (Figured 8c). Maxima range from $v/v_{50\%\ KE}$=1.3 to 2. Curve fits in the function form of equation (5) are shown in each sub-figure.

5. Discussion

Ejecta fields (i.e., v_y vs. v_x and θ vs. v) have been normalized with v_{max}, $v_{50\%\ mass}$ and $v_{50\%\ KE}$ to better understand fragmentation and material ejection processes at the rear of the target. The maximum velocity was found to be predictable using semi-empirical models of fitted non-dimensional groups. Velocity field shapes were observed to collapse over the range of KE* when normalized by v_{max}. This is important because it indicates self-similarity of the field envelope.

Maximum field expansions of $|v_y/v_{max}|$ $\leq$0.3 to 0.4 were found to be independent of experimental conditions for $KE^\star\leq$0.89 (i.e., excluding the tonalite 10 mm series 2). The field has maximum expansions of $|v_y/v_{max}|$ $\leq$0.5 for $KE^\star\geq$1.59. This suggests a shift towards more uniform expansion, where the length of the lateral field expansion (i.e., -0.5$\leq v_y/v_{max}\leq$0.5, or $||v_y/v_{max}||$) is equal to that of the streamwise (direction of v_x) expansion. That is, the field expands the same amount laterally than it does in the streamwise direction.

Ejecta angles were also examined. The outer curtain of the ejecta angles of the fastest moving fragments is approximately $\pm$20°. These are in agreement with asymptotical (i.e., for later time) angle values by Hoerth et al. 2013 for wet sandstone targets. Of note: angles in Hoerth et al. 2013 are complementary to those here. Angles become more conical-shaped for an increase in target thickness. This angle is governed by the angle at which the shock wave interacts with the back surface. The conical region at the beginning of each velocity field was observed to correspond to $|\theta|\leq$30°, where $|\theta|$ is defined as arctan(v_y/v_x). $|\theta|\leq$30° corresponds to Hertzian fracture zone (Kocer and Collins, 1998) and is commonly associated with the formation of cones in the targets observed in impact tests (Hogan et al., 2011). Trajectories of Hertzian cone cracks are defined by the stress field in the body at impact (Kocer and Collins, 1998). Crack trajectories, defined here as the outer edge of the cone, will follow the direction of maximum energy release (Kocer and Collins, 1998). For thinner targets, $|\theta|\leq$30° for v/v_{max} $\geq$0.2. For the other cases, $|\theta|\leq$30° for v/v_{max} $\geq$0.5. Angles steadily increase for lower v/v_{max} to approximately 50° to 70°. This is likely related to increased through-cracking and fragmentation (i.e., the whole target is fragmented and material ejected).

The normalized $v_{50\%\ mass}$ ejecta fields do not collapse as well as v_{max} for the 7 mm and 10 mm tonalite and gabbro data sets. Bounds range from $|v_y/v_{50\%\ mass}| \leq 0.5$ to 6 and $v_x/v_{50\%\ mass} \leq 3$ to 25. In these cases, it is difficult to obtain global features. The wide range is related to nonuniform fragmentation where larger $v_x/v_{50\%\ mass}$ bounds are associated with higher velocity cases. In higher velocity and thin target cases, a few spalled smaller fragments stretch out the field. Lower $v_x/v_{50\%\ mass}$ bounds are associated with lower impact energies and a more uniform distribution of ejected mass.

The other data sets were observed to collapse well for $v_x/v_{50\%\ mass} \leq 2$ to 3.6, with no clear trend for target thickness, material type or impact energy. Fields for $KE^\star \leq 0.89$ have lateral expansions of $|v_y/v_{50\%\ mass}| \leq 0.5$ to 0.6. Fields for $KE^\star \geq 1.59$ have lateral expansions of $|v_y/v_{50\%\ mass}| \leq 1.2$ to 1.5. In all cases, the total width of the lateral field is approximately 0.80 of the streamwise direction. For the cases that do collapse under $v_{50\%\ mass}$, the Hertzian fracture zone is bounded by $|\theta| \leq 30°$ for $v/v_{50\%\ mass} \geq 1.5$. The ability to collapse the fields using $v_{50\%\ mass}$ has implications for extending this work to greater $KE^\star$ and, in particular, the hypervelocity regime. In particular, more uniform crushing for large $KE^\star$ results in collapsing for the normalized $v_{50\%\ mass}$ ejecta field. The self-similarity of the field envelope (i.e., the field bounds) allows predictive capabilities for the distribution of mass among velocity (two independent parameters).

Ejecta fields normalized by $v_{50\%\ KE}$ produce collapsed fields for all cases, with streamwise bounds of $v_x/v_{50\%\ KE} \leq 1.3$ to 1.4 and $|v_y/v_{50\%\ KE}| \leq 0.3$ to 0.4. Commonly, ejecta angles are bounded by $|\theta| \leq 30°$ for $v/v_{50\%\ KE} > 0.5$. This corresponds to 75 $\%$ of the residual kinetic energy and reflects a universal trend in the energy dissipated in ejecting the material from the target.

Semi-empirical models in the form $aKE^{\star b}K_c^{\star c}$ were developed and fitted with coefficients to provide predictive capabilities of 10th, 50th, and 90th percentiles for the non-dimensionalized distribution of mass, momentum and kinetic energy among ejecta velocities. 10th percentiles characterize material crushed ahead of the projectile and 90th percentiles characterize the material ejected at the rear of the target. For all cases considered, the non-dimensional velocities (i.e., v_{max}, v_{mass}, v_{mom}, and v_{KE}) increase for increasing $KE^\star$. 50th and 90th percentiles of $v_{mass}T^\star/L^\star$ are predictable ($R^2 > 0.70$). Coefficients of $b \approx 1$ and $c \approx -1$ indicate that Y can be removed and the fit reduced. 10th percentiles do not collapse well ($R^2 = 0.40$). This highlights the challenges in predicting the distribution of mass among velocities immediately ahead of the projectile. Suitable collapsing, or fitting, of all percentile values for $v_{mom}T^\star/L^\star$ and $v_{KE}T^\star/L^\star$ is obtained ($R^2 > 0.81$ for the median value).

Curves collapse reasonably well, with the exception of the 7 mm and 10 mm targets, for $v/v_{mass} \geq 1$ and have maxima between $v/v_{mass} = 1.3$ and 4.1. Curves do not collapse well for $v/v_{mass} < 1$. Improved collapsing was achieved for v/v_{mom} and v/v_{KE} curves. Maxima range from 1.2 to 2.3 for v/v_{mom} and 1.3 to 2 for v/v_{KE}. The ability to collapse the curve using a simple normalization highlights the similarities of fragmentation and ejection mechanisms over the broad range of materials, impact energies and target thicknesses tested in this study.

Fits of the cumulative distribution functions (equation (5)) were differentiated to obtain probability density distribution functions for mass, momentum and kinetic energy among ejecta velocities (equation (6)). Functional forms of particle distributions can take many forms (e.g., Rayleigh (Levy and Molinari, 2010), Weibull (Cheong et al., 2004), log-normal distributions (Wang and Ramesh, 2004)). Differentiating the cumulative distribution function here provides a relatively simple functional form for the probability distribution.

5.1. Implications

Methodologies provided in this study facilitate a framework for those performing similar ejecta measurements. These relatively low velocity measurements can be bridged with higher velocity experiments to provide further insight into impacts in planetary materials. This data is also important when developing and validating numerical codes (e.g., Jutzi et al., 2010). For example, material models, fragment determining schemes, and fragment interaction algorithms may be improved through comparison with these experimental results. Velocity, size, momentum, and kinetic energy fields may also be evaluated in these simulations immediately after fragmentation and preceding material ejection. However, it currently remains computationally expensive to track ejecta for time periods that allow the majority of the fragment interactions to occur (i.e., reach steady-state).

6. Concluding Remarks

The dynamic fragmentation of selected planetary materials during impact has been examined. Ejecta velocity field shapes collapse when normalized by v_{max}, $v_{50\%\ mass}$ and $v_{50\%\ KE}$. The total width of the lateral field expansion for the normalized v_{max} field increases from 0.6 of the streamwise expansion to unity as $KE^\star$ increases. Ejecta fields normalized by $v_{50\%\ KE}$ had streamwise bounds of $v_x/v_{50\%\ KE}\leq$1.3 to 1.4 and lateral bounds of $|v_y/v_{50\%\ KE}|\leq$0.3 to 0.4. Ejecta fields normalized by $v_{50\%\ mass}$ had streamwise bounds of $v_x/v_{50\%\ mass}\leq$2 to 3.6 and lateral bounds of $|v_y/v_{50\%\ mass}|\leq$0.5 to 0.6. Combined, this highlights the self-similarity of underlying fragmentation and ejection mechanisms over a wide range of experimental conditions.

Non-dimensional parameters $KE^{\star}$ and $K_c^{\star}$ have been obtained using the Buckingham Pi approach. Semi-empirical models in the form $aKE^{\star b}K_c^{\star c}$ were developed and fitted with coefficients to provide predictive capabilities of 10^{th}, 50^{th}, and 90^{th} percentiles for the non-dimensionalized distribution of mass, momentum and kinetic energy among ejecta velocities. 50^{th} and 90^{th} percentiles non-dimensional $v_{mass}T^{\star}/L^{\star}$ were predictable ($R^2>0.70$). 10^{th} percentiles did not collapse well ($R^2=0.40$). This highlights the challenges in predicting the distribution of mass for small ejection velocities (e.g., those formed ahead of the projectile). Suitable collapsing of all percentile values for $v_{mom}T^{\star}/L^{\star}$ and $v_{KE}T^{\star}/L^{\star}$ was obtained ($R^2>0.81$ for the median values). The development of scaling relations highlights the applicability of these methods to such data, and provides for predictive capabilities.

A function form describing the probability density distributions of mass, momentum and kinetic energy among velocity has been provided. Cumulative mass distributions among ejecta velocity collapse for $v/v_{mass}\geq 1$. Maxima range between $v/v_{mass}=1.3$ and 4.1. Curves do not collapse well for $v/v_{mass}<1$, with no apparent relationship between experimental conditions. Improved collapsing is achieved for v/v_{mom} and v/v_{KE} curves. Maxima range from 1.2 to 2.3 for v/v_{mom} and 1.3 to 2 for v/v_{KE}. The ability to collapse the data highlights the similarities of fragmentation and ejection mechanisms over the broad range of conditions examined in this study.

This work provides the groundwork for future studies. The data and predictive models provided here are valuable for the development and validation of numerical models, where comparisons between high quality experiments and simulations rely on quantitative measurements. This will lead to a better understanding of the dynamic fragmentation of brittle materials and improved interpretation of impact events.

Acknowledgements

This work was supported by a Natural Sciences and Engineering Research Council (NSERC) PGS-D scholarship to JDH and funding from NSERC, the Canada Research Chairs program and the Canada Foundation for Innovation to JGS. The authors would also like to thank the effort and contribution during the experimental phase of the study of Yannick Boehrer, David Bluntzer and Philippe Baumann at ISL. Planetary and Space Science Centre contribution 79.

7. Appendix

Additional ejecta fields for the 7 mm and 10 mm tonalite data sets are provided for further reference.

7.1. Tonalite: 7 mm and impact energies of 66 J to 262 J

The normalized ejecta fields for the tonalite with a target thickness of 7 mm and six impact energies of 66 J to 262 J are shown in Figure 9. The normalized v$_{max}$ ejecta field is asymmetric, with bounds for the maximum lateral field expansion of $|$v$_y$/v$_{max}|\leq$0.3 to 0.4. The debris field is equally densely populated (i.e., points remain in close proximity in the figure). There is a cone region for 0$\leq$v$_x$/v$_{max}\leq$0.05 and $|$v$_y$/v$_{max}|\leq$0.2. Corresponding ejection angles plotted against resultant velocities are shown in Figure 9b. Ejecta angles are bounded by $|\theta|\leq 70^\circ$ for v/v$_{max}$<0.08. The bounds steadily decrease to $|\theta|\leq 30^\circ$ for v/v$_{max}$>0.2 approximately.

The normalized v$_{50\%\ mass}$ ejecta field is shown in Figure 9c. The fields do not collapse as well and are bounded by $|$v$_y$/v$_{50\%\ mass}|\leq$ 0.5 to 6 and v$_x$/v$_{50\%\ mass}$ vary from 3.5 to 27. Commonly, these are observed to increase in the figure for increasing impact energy. Corresponding ejecta angles plotted against the normalized v$_{50\%\ mass}$ resultant velocity are shown in Figure 9d.

Shown in Figure 9e is the normalized v$_{50\%\ KE}$ ejecta field. The field collapses (i.e., overlie and become bounded) for $|$v$_y$/v$_{50\%\ KE}|\leq$0.3 to 0.4 and v$_x$/v$_{50\%\ KE}\leq$1.3. The corresponding angle and v/v$_{50\%\ KE}$ field are shown in Figure 9f. Ejecta angles are bounded by $|\theta|\leq 30^\circ$ for v/v$_{50\%\ KE}$>0.5. This corresponds to 75 % of the kinetic energy.

7.2. *Tonalite: 10 mm and impact energies of 12 J to 280 J*

The normalized ejecta fields for the 10 mm thick tonalite target and 11 impact energies from 12 J to 280 J are shown in Figure 10. The fields collapse well when normalized by v$_{max}$ (Figure 10a), despite all being asymmetric in shape. The field is more densely populated for v$_x$/v$_{max}$ <0.2 and becomes bounded by $|$v$_y$/v$_{max}|$ $\leq$0.2 to 0.3 for v$_x$/v$_{max}$ $\geq$0.2. This corresponds to $|\theta|\leq 30^\circ$ for v/v$_{max}$ $\geq$0.2 (Figure 10b).

The normalized v$_{50\%\ mass}$ field is shown in Figure 10c. The fields do not collapse well, as shown by the large abscissa axis values, with bounds ranging from $|$v$_y$/v$_{50\%\ mass}|\leq$0.5 to 5 and $|$v$_x$/v$_{50\%\ mass}|\leq$3 to 25. Again, values commonly increase for increased impact energy. Corresponding ejection angles are plotted against v in Figure 10d.

The normalized $v_{50\%\ KE}$ ejecta field is shown in Figure 10e. The field collapses well and is bounded by $v_x/v_{50\%\ KE} \leq 1.2$ to 1.3. The field expands conically from 0 to $v_x/v_{50\%\ KE}$=0.3 to 0.5 and remains bounded by $|v_y/v_{50\%\ KE}|$=0.3 for $v_x/v_{50\%\ KE} > 0.5$. This also corresponds to when the ejection becomes bounded by $|\theta| \leq 30°$ for $v/v_{50\%\ KE} > 0.5$ (Figured 10f).

7.3. *Tonalite: 10 mm and impact energies of 716 J to 1,786 J*

Shown in Figure 11 are the scaled ejecta fields for the 10 mm thick tonalite for seven impact energies of 716 J to 1,786 J. The ejecta fields collapse when normalized by v_{max}. The fields consist of a conical region for $v_x/v_{max} < 0.4$ and an expanded region for $v_x/v_{max} \geq 0.4$. The onset of the expanded region occurs for $|v_y/v_{max}| \leq 0.1$ and reaches a maximum of $|v_y/v_{max}|$=0.5 to 0.6. The foremost region exhibits another distinct characteristic. The 'nose' of the field begins to expand at v_x/v_{max}=0.9 and is approximately bounded by $|v_y/v_{max}| \leq 0.1$. The resultant velocity normalized by v_{max} and ejecta angle fields (Figure 11b) indicate that nearly all ejection angles are bounded by $|\theta| \leq 30°$, with a decreasing $|\theta|$ trend for $v_x/v_{max} \leq 0.2$, and an increasing trend for $0.2 < v_x/v_{max} \leq 0.9$.

The ejecta field normalized by $v_{50\%\ mass}$ is shown in Figure 11c. The initial expanded region is bounded by $v_x/v_{50\%\ mass} \leq 1.5$ and $|v_y/v_{50\%\ mass}| \leq 0.5$, which contains approximately 58% of the mass. The maximum field expansion is bounded for $v_y/v_{50\%\ mass} \leq 1.5$ at $v_x/v_{50\%\ mass}$=2.5. The field is fully collapsed for $v_x/v_{50\%\ mass} \leq 3.6$. The corresponding resultant velocity plotted against ejecta angle is shown in Figure 11d. All ejecta angles are bounded by $|\theta| \leq 30°$, with a constricting then expanding transition behaviour before and after $v/v_{50\%\ mass}$=1.5.

Lastly, shown in Figure 11e is the ejecta field normalized by $v_{50\%\ KE}$. The conical region is bounded by $v_x/v_{50\%\ KE} \leq 0.5$ (i.e., corresponding to containing

25 $\%$ of the total kinetic energy in this case). The maximum lateral field expansion is bounded by $|$v$_y$/v$_{50\%\ KE}|\leq$0.5 to 0.6 and occurs at v$_x$/v$_{50\%\ KE}$=1. The whole field is collapsed for v$_x$/v$_{50\%\ KE}\leq$1.3. The resultant ejecta angle field is shown in Figure 11f. All angles are bounded by $|\theta|\leq$30°. There is a notable contracting and expanding transition at v/v$_{50\%\ mass}$=0.5.

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