University of Alberta

Three Essays on Seigniorage

by

Ebenezer Asem 

A thesis submitted to the **Faculty of Graduate Studies and Research** in partial fulfillment of the requirements for the degree of **Doctor of Philosophy**

Department of Economics

Edmonton, Alberta

Spring, 1997

University of Alberta

Library Release Form

Name of Author: Ebenezer Asem

Title of Thesis: Three Essays on Seigniorage

Degree: Doctor of Philosophy

Year this Degree Granted: 1997

P. O. Box 702

Accra, Ghana.

April 7, 1997.

University of Alberta

Faculty of Graduate Studies and Research

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled Three Essays on Seigniorage submitted by Ebenezer Asem in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

K. L. Gupta, Supervisor

B. G. Reid

L. S. Wilson

B. Scholnick

K. Phylaktis
(External Examiner)

April 7, 1997
Date

To my mother and father, Vicentia Duse and the late S. K. Asem.

Abstract

Essay 1.

One of the main objectives of imposing reserve requirements is a desire for seigniorage revenue. This essay examines how this objective is influenced by two things; the particular asset banks reduce to accommodate increases in cash reserve requirements and the type of reserve requirement used. On the former issue, the study suggests that the objective would be better achieved if the banks reduce assets that are less "taxed" and less "efficient" in production. On the latter, cash reserve and liquid reserve requirements are compared as instruments for boosting seigniorage revenue. It is demonstrated that cash reserve requirement should be used the higher the government's deficit, the lower the cash reserve, and the less the government earns on treasury bills.

Essay 2.

This essay explores the effect of interest rate deregulation on seigniorage. It is shown that in a financially repressed economy, the effect on seigniorage of interest rate deregulation depends on how economic agents adjust their asset portfolios to changes in controlled interest rates. It is demonstrated that deregulation of interest rates would increase seigniorage the higher the interest elasticity of the demand for informal deposits, the lower the interest elasticity of the demand for money, and the higher the reserve requirement. It is also argued that there are three possible dynamics of seigniorage during gradual interest rate adjustment, and that the interactions between the formal and the informal rates are also important in determining the effect on seigniorage.

Essay 3.

This essay examines the welfare effects of different uses of the revenue from seigniorage on the welfare cost of inflation. In most models of the welfare cost of inflation, the revenue from the inflation tax has been modelled as lump-sum transfer payments to economic agents. Since there are many different types of government expenditure, modelling the use of seigniorage this way will only be appropriate if the benefit from the various types of government expenditures are the same. Calibrations using U.S. data indicates that public capital expenditure is more desirable from the welfare perspective and hence the transfer payment modelling strategy overestimates the welfare cost of inflation for the US.

ACKNOWLEDGEMENT

I would like to express my sincere thanks to my dissertation committee, Dr. Kanhaya L. Gupta, Dr. Bradford Reid, and Dr. Leonard S. Wilson, and to the external committee member, Dr. Barry Scholnick, and to the external examiner, Dr. Kate Phylaktis for the time and effort they have expended on my behalf.

In particular, I am indebted to my supervisor, Dr. K. L. Gupta, who not only directed and supervised this thesis but had a profound impact my life throughout my graduate studies in economics. I would also like to thank Dr. S.K. Landon and Dr. S. Sharir for their encouragement and advice at various stage of my studies.

I am very grateful for the financial assistance provided by the Department of Economics and the Faculty of Graduate Studies and Research at the University of Alberta.

A special thanks to Edna Djokoto for being there.

LIST OF TABLES

Introduction

The dissertation contains three essays on seigniorage. The first essay investigates the effects of different bank asset portfolios and the use of different types of reserve requirements on governments' abilities to raise seigniorage. As pointed out in the literature, one of the major objectives of imposing reserve requirements is to boost governments' abilities to raise seigniorage [see, for example, Espinosa Vega (1995)]. The studies on this issue have concentrated on how to set cash reserve requirement and inflation to maximize seigniorage. Although, it is clear in the literature that seigniorage is influenced, among other things, by the behaviour of the banking system no attempt has been made to investigate how specific behaviours of banks affect the government's ability to raise seigniorage. One such behaviour is the type of asset that the banks change in order to comply an increase in binding reserve requirement. Banks have different types of assets on their balance sheets and it is likely that reducing different types of assets to finance an increase in a binding cash reserve requirement would have different implications for seigniorage. This is one of the issues explored in Essay 1. The second issue examined in this essay is how the government's ability to raise seigniorage is influenced by the type of reserve instrument the government uses (cash versus liquid reserve). Since cash and other liquid assets, such as treasury bills, have different characteristics an increase in these different types of assets would have different impact on the government's ability to raise seigniorage.

In Essay 1 a three-asset model is constructed which allows banks to hold three types of assets: cash, treasury bills, and private loans. Thus an increase in a binding cash

reserve requirement can be satisfied by reducing treasury bills or private loan or both by the banks. It is shown that the type of asset (treasury bills versus private loans) that the banks reduce in order to comply with an increase in a binding cash reserve requirement influences the effect of an increase in cash reserve requirement on seigniorage. In particular, it is shown that if the banks reduce assets that are less "productive" and less "taxed" by the government, the government's objective of increasing it's seigniorage capacity would be better enhanced. Also the three assets held by banks in the model allows an investigation of using cash reserve requirement as an instrument of boosting seigniorage capacity versus liquid reserve requirement (treasury bills). The results suggest that the type of reserve instrument used by the government affects it's objective of enhancing seigniorage. The use cash reserve requirement would be better at enhancing seigniorage than liquid reserve requirement the higher the government budget deficit, the less the "tax" on the liquid asset, and the less the level of cash reserve requirement.

In Essay 2, the effect on seigniorage of interest rate deregulation is investigated. This issue is important because of the importance of seigniorage to many developing countries implementing adjustment programs [see Cukierman, Edwards, and Tabellini (1992)]. Although the importance of seigniorage for many developing countries need no emphasis here, the issue raised has not been explored in the literature with the exception of Kapur (1992). He, however, did not focus primarily on interest rate deregulation and seigniorage, and therefore it is not surprising that he did not consider one of the important base on which seigniorage is levied; that is cash demand by households. In Essay 2, a much more focused analyzes of interest rate deregulation on seigniorage is carried out in

an economic environment with a thriving informal sector. Also, the dynamics of seigniorage during gradual interest rate deregulation is investigated since gradual adjustments are essential features of any financial liberalization program.

The economic environment used is one in which formal sector institutions are required to hold cash reserves but the informal sector institutions do not hold such reserves. Thus for a given deposit, the demand for cash by the formal sector institutions would be higher than the informal sector institutions. An increase in controlled interest rate increases the attractiveness of formal sector deposits and hence induces economic agents to shift out of both cash and informal deposits holdings. Thus whereas a reduction in the demand for cash by economic agents reduces the base for seigniorage, a reduction in the demand for informal deposits increases the base of seigniorage. The effect of interest rate deregulation on seigniorage, therefore, depends on the sensitivities of the demand for cash and the demand for informal deposit to changes in the controlled formal interest rate as well as the level of reserve requirement. Interest rate deregulation is likely to have a positive effect on seigniorage the higher the level of cash reserve requirement, the higher the interest sensitivity of the demand for informal deposit, and the lower the interest sensitivity of cash demand. If cash demand is very sensitive to formal interest rates, then a large proportion of the increase in formal sector deposits consequent on an increase in controlled interest rates will come from cash holdings, and this would reduce the base of seigniorage. On the other hand, if informal deposits are very sensitive to controlled rates then a large fraction of the increase in commercial banks deposit will come from the informal sector and this would increase the base of seigniorage. Also the

higher the level of reserve requirement the more the increase in deposits with the commercial banks that will be held as cash reserve requirement. This result is in sharp contrast to Kapur's (1992) results that seigniorage will increase if interest rates are deregulated.

Another issue explored in Essay 2 is the time path for seigniorage during interest rate deregulation. It is shown that there are three possible dynamics of seigniorage. If cash reserve requirement is not "sufficiently" high, seigniorage would decrease throughout. If cash reserve requirement is high seigniorage can increase throughout or increase and decrease at some later stage. The conditions under which seigniorage would follow a particular time path is established in the Essay.

In Essay 3, the effect of the use of seigniorage on the welfare cost of inflation is investigated. As pointed out by Fischer (1981) inflation, especially in the long-run, is an endogenous variable and hence the welfare cost of inflation must take into account the cost and benefits of inflation. In the literature, attempts to provide a better assessment of the welfare cost of inflation has focused essentially on "properly" capturing the cost of inflation [for examples are distortions of labour supply through the CIA model by Cooley and Hanson (1989), costly avoidance of the use of cash by the introduction of costly credits Gillman (1993), and distortions of productive labour by Dotey and Ireland (1996)]. However, little attention has devoted to the benefits from inflation. The benefits from inflation is, in general, given by the revenue from inflation tax by the welfare triangle measures [see Fischer (1981) and Lucas (1981)]. Most others researchers [for examples Gillman (1993), Gomme (1993), Dotsey and Ireland (1996)] have gone further

and modelled the use of seigniorage as lump-sum transfer payments. However, the government can put seigniorage into many different types of uses and there is no reason to limit the use of seigniorage to transfer payments. In particular, the use of seigniorage will be important if the marginal benefit from the use of government revenue is not equal across the different possible uses. Also if a large amount of seigniorage revenue is involved, its use will be important for the welfare cost of inflation whether the marginal benefits from the different government spendings are the same or not.

It is argued, in this essay, that the standard way of modelling seigniorage as transfer payments can potentially underestimate or overestimate the 'true' cost of inflation. Using a framework where seigniorage can be transferred to economic agents or can be used to fund public capital, it is demonstrated with US data that modelling seigniorage as transfer payment overestimates the welfare cost of inflation. This is because the calibrations showed that public capital expenditure is more desirable than lump-sum transfer payments from the welfare perspective, and hence modelling seigniorage as transfer payments would underestimate the benefits of inflation unless one can argue that seigniorage is, indeed, transferred to economic agents. If, however, the benefits from the use of marginal government revenue is the same, then the use of marginal seigniorage would be irrelevant. In the absence of empirical evidence, however, there is no reason to assume that in practice the marginal benefits from the various types of government expenditures are equal. Even if they are, the use of seigniorage would still be important where welfare cost evaluation experiments entail large seigniorage revenue.

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ESSAY 1

SEIGNIORAGE AND BANK ASSET PORTFOLIOS

1. Introduction

Most economists would agree with Tobin's (1986) definition of seigniorage as the ability of governments to finance expenditures by issuing money. This ability is associated with a government's sovereign monopoly in issuing money. However, there is some disagreement over the concept of "inflation" tax. Consequently, there are different measures of the "proceeds" from taxing money. These include the rate of inflation multiplied by the real value of outside money [Friedman (1953), and Bailey (1956)], the rate of monetary growth multiplied by real outside money [Cagan (1956), and Marty (1967)], and the nominal interest rate multiplied by real outside money [Phelps (1971, 1972), and Marty (1978)]. Drazen (1985, 1989) clarifies the problems of defining seigniorage in the budget identity. He presents a general measure of inflation tax revenue which includes all previous measures as special cases[1].

The base for levying taxes on real money balances includes the cash reserve holdings of financial institutions [see Siegel (1981), and Calvo and Fernandez (1983)]. Various studies have been carried out to examine the effects of changes in cash reserve requirements on seigniorage. These studies can, broadly, be classified into two groups. On the one hand authors like Freeman (1987), Drazen (1989), Romer (1985), and Mourmouras and Russell (1992) have focused on the welfare effects of changes in cash reserve

requirements and the rate of inflation[2]. Others, such as Siegel (1981), Calvo and Fernandez (1983), McClure (1986) and Brock (1989) have explored the seigniorage maximizing rate of inflation and cash reserve requirements. This paper follows this line of research and abstracts from the optimality question.

Although seigniorage maximization has received extensive attention in the literature, the seigniorage effects of the specific asset that the banks change[3] to comply with the changes in cash reserve requirement has not been studied. It is well documented in the literature that seigniorage depends, among other things, on reserve requirements and the behaviour of the banking institutions. One such behaviour is the specific asset that the banks change in order to satisfy the required changes in their reserve. Fama (1985) hinted at this when he demonstrated that the issue of reserve requirement operating as a tax is rather complex in a world where banks have many assets and liabilities. Despite this hint, formal analysis of multiple reserve requirement schemes has not been carried out with the exception of Espinosa-Vega (1995). He examined the inflationary and the welfare consequences of adopting a single reserve requirement as against adopting multiple reserve requirements. Like Espinosa-Vega, this analysis is conducted within a framework of multiple assets which allows for the analysis of different reserve requirement schemes.

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1 See Drazen (1989) for how the various measures are derived from the general measure of seigniorage.

2 Freeman (1987) argued that the optimal monetary policy when reserve requirement is in place is to minimize reserve requirement and to inflate the stock of money without limit. This suggests that reserve requirement is not part of an optimal finance plan. The result follows from the argument that reserve requirement is a tax on the earning assets of a bank and an increase in inflation is equivalent to raising that tax [Siegel (1981)]. Mourmouras and Russel (1992), on the other hand, used a model that generalized Freeman's by introducing uncertainty about the returns on physical assets and showed that a binding reserve ratio can form part of an "optimal" financing scheme.

3 This, of course, assumes a fixed asset position of the banks. For example, if the cash reserve requirement is increased, the banks will necessarily have to reduce at least one of their assets. Thus the issue of banks mobilizing deposits or engaging in other activities such as borrowing to satisfy increased reserve requirement is ignored.

However, the focus here is on how the government's ability to raise seigniorage is affected by the way the banks respond to a change in cash reserve requirement and by the kind of reserve requirement adopted. These analyzes are important because the desire for seigniorage revenue is often a major objective behind the imposition of reserve requirement [see, for example, Brock (1989)]. If governments impose reserve requirements to boost seigniorage, then they should be worried about how this objective is influenced by banks reactions to increased reserve requirements and by the kind of reserve requirement they use. These are the issues explored in this essay.

The study would be relevant to countries that have persistent deficits and/or debt problems, and are considering reserve requirements as a means of boosting seigniorage revenue. It would also be of some relevance to many developing countries implementing the World Bank Structural Adjustment Program, since these countries often increase their cash and/or liquidity ratio(s) in an attempt to mop up excess liquidity in the economy[4] and set the stage for financial liberalization. Since seigniorage is a major source of revenue[5] for many of these countries, it is important that the consequences of these actions for seigniorage be widely understood. Apart from these many developed countries have switched from the use of one ratio in favour of the other at some stage in their development, and this analysis would throw some light on why a particular reserve requirement would be desirable.

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[4]For example, in Ghana the government increased the cash ratio requirement from 8.3% to 22.6% in 1987 to check excess liquidity, and in 1993 it increased liquidity reserves from 24% to 52% for the same purpose [Bank of Ghana Annual Report (1987) and Bank of Ghana Quarterly Economic Bulletin (March 1994) respectively].

[5] For example, the average seigniorage as a proportion of government revenue was 28%, 24.8% and 23.9% for Ghana, Uganda, and Mexico respectively for the period 1971 to 1982 [Cukierman, Edwards, and Tabellini (1992)].

Seigniorage maximization studies, typically, deal with maximizing the tax revenue on money balances by setting inflation and cash reserve requirement. Calvo and Fernandez (1983), Brock (1989), and Drazen (1989) are among some of the authors who examined this subject. Using a partial equilibrium model, Calvo and Fernandez (1983) showed that the revenue maximizing inflation rate is not unique under fractional reserve banking system. Thus various combinations of inflation rate and reserve requirement can yield the maximum revenue. In particular, the revenue maximizing inflation rate increases without bound as the reserve ratio is lowered towards zero. Their conclusion also carries over to a growing economy. Brock (1989) explored the same issue in a general equilibrium framework. He showed that the revenue-maximizing cash reserve ratio is obtained when elasticity of demand for the monetary base with respect to the reserve ratio is set equal to zero[6]. If there are no output effects of the inflation tax, then the usual partial equilibrium result that the monetary authority would set the elasticity of demand for currency or for currency plus demand deposits with respect to the nominal interest rate equal to one[7] is obtained. Drazen (1989) argued in his short-run dynamic analysis that the use of reserve requirement in seigniorage maximization policy depends on the extent to which monetary policy is ready to accommodate an increase in reserves by offsetting its effects on the supply of loans and hence growth.

In this study, however, the issue is not to find the seigniorage maximizing rate of inflation and reserve requirement, but rather to study how the government's ability to raise

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6 In this case, Brock (1989) argues that the magnitudes of the elasticities of demand for currency and currency plus demand deposits with respect to the interest rate will depend on the relative magnitudes of consumption with respect to interest rate and reserve ratio.

7 Siegel (1981) arrived at the same condition in his partial equilibrium setting. These studies adopt the definition

seigniorage is affected by banks' asset portfolios and the different reserve requirements that the government can use. Using the measure of seigniorage advocated by Friedman (1953) and Bailey (1956)[8], the government's ability to raise seigniorage would be enhanced by an increase in the demand for real money and/or a decrease in the rate of inflation. Since the demand for real balances is the base on which seigniorage is raised, an increase in the demand for money is equivalent to increasing the tax base which will strengthen the government's ability to extract seigniorage. The rate of inflation is the tax rate on real balances, and hence a decrease in this rate which is not due to a contraction in the money supply will boost the seigniorage capacity of the economy[9].

A cash reserve requirement is a legal restriction which is measured as the ratio of cash to deposits. Other legal restrictions, such as minimum liquidity ratio, have featured in the literature. These have taken the form of "storage" which yields some returns [as in Diamond and Dybvig (1983) and Freeman (1988)] or investment in some liquid asset [as in Bencivenga and Smith (1991)] or holdings of some government securities [as in Espinosa-Vega (1995)]. If the government changes the liquidity ratio, seigniorage will be sensitive to the type of assets that qualify as 'liquid' (for example call-in loans and treasury bills), among other things. To simplify the analysis, banks' liquid assets will be defined as treasury (government) bills, and hence the liquidity ratio will be measured by the ratio of treasury

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of seigniorage suggested by Phelps (1971).

[8] Since the model used here abstracts from growth, this measure is equivalent to the one suggested by Cagan (1956). The measure recommended by Phelps (1971, 1972) would only be appropriate if money is fully backed by assets.

[9] In a zero growth model, the rate of inflation is equal to the rate of monetary expansion, *ceteris paribus*. Thus governments can increase seigniorage revenue by increasing the rate of inflation (printing more money). Other factors, such as banks credit creation, which can result in inflation weakens the government's ability to collect seigniorage in the sense that it increases the tax rate on real balance without generating any revenue for the government. This, of course, assumes that the maximum seigniorage is unaffected by the factors which cause the

bills banks hold to total deposits.

An increase in the cash reserve requirement will increase the cash holdings of financial institutions and hence has the effect of increasing the base of seigniorage. If the banks increase cash reserves by reducing the demand for the liquid asset (treasury bills), then the government budget will be adversely affected[10]. To accommodate the adverse budgetary shock, the government can cut spending or increase income tax or increase inflationary finance or engage any combination of these. If the government cuts spending and/or increase income taxes, this will reduce incomes and the demand for real money which is the base of seigniorage. On the other hand, if the government increases inflation tax, this will also reduce the demand for real cash balance which will, again, unfavourably affect the base of seigniorage. Thus whether the government responds to the fall in demand for it's bills by reducing expenditure or by increasing income taxes or by printing more money or by applying any combination of these, increases in the cash reserve requirement which is complied with by decreasing liquid assets will have both positive and negative effects on seigniorage capacity.

On the other hand, if banks reduce their loans to meet increases in cash reserve requirement, private investment will be negatively affected[11] and this will reduce output and the demand for real money which will adversely affect the government's ability to collect seigniorage. Thus a reduction in loans to accommodate an increase in cash reserve

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inflation.

[10] A decrease in the demand for treasury bills will reduce government's revenue directly if the government issues less bills to accommodate the reduction in demand. On the other hand, the government can fix its issues of treasury bills and increase the rate of return on them to absorb the adverse demand shock. In either case or a combination of them, the government's budget will be adversely affected.

[11] Loans are assumed to be used for private sector productive activities.

requirement also has both positive and negative effects on the government's ability to tax real balances. However, the two strategies[12] of complying with an increase in the cash reserve requirement elicit different responses in the economy and hence their seigniorage implications would be different. In particular, the strategy of reducing treasury bills holdings directly affects the government's budget while reducing private sector loans does not directly affect the government's budget. In the model examined here, when banks reduce their treasury bill asset, it will be assumed that the government will accommodate the fall in demand by reducing the issue of treasury bills. Thus, to the extent that a reduction in the demand for treasury bills affects the government's revenue directly and the use of the government's funds from treasury bills and private investment affect the economy differently, the particular asset that the banks change to comply with changes in cash reserve requirement will be relevant to the economy and the government's ability to raise seigniorage. If, for example, the government invests the proceeds from treasury bills but private investment is more productive, then output and income will be hurt more in the case where private loans[13] are reduced.

One would also expect some difference in the government's ability to raise seigniorage if the government increases liquid rather than cash reserve requirement. An increase in liquid reserve requirement would not be as destructive to production as an increase in the cash reserve requirement if part or all the funds raised from the liquid assets are used for productive purposes. This will favour using liquid reserve requirement to

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12 The issue of which strategy will be optimal for the banks is not pursued here.

13 Empirical estimates of public and private productivities are quite sensitivity to the estimation techniques. Aschauer (1989) estimated output elasticity of U.S. government capital to be 0.39 for the period 1949-1985. Finn (1993) estimated this to be 0.16 even though she did not consider some unproductive government capital. Lucas

increase seigniorage capacity. On the other hand, an increase in cash reserve requirement would increase the inflationary tax base while an increase in liquid reserve requirement may not. This lends some support to increasing cash reserve requirement as a means of augmenting the government's ability to raise seigniorage. Since an increase in cash reserve requirement and an increase in liquid reserve requirement elicit different economic responses, one cannot conclude *a priori* which instrument will be "better" if the government's aim is to raise seigniorage. This is the second investigation carried out in this paper.

In most seigniorage models the menu of assets is restricted and hence they cannot address the issues raised here. In order to handle them, the model must be a three-asset model and structured in a way that will allow the holding of cash and liquid reserves simultaneously. In this regard, the three period overlapping generations model used by Bencivenga and Smith (1992) will be used with some alterations that will allow financial intermediaries to hold both cash and liquid reserves. One way to do this is to add one period to their setting, say period zero. In this case, it will be assumed that agents work in period zero, and have certain probabilities of consuming in periods one, two, and three. These probabilities are realised at the beginning of period one after decisions about portfolio holdings have been made.

(1990) estimated that private capital output elasticity is 0.3.

2. The Model

2.1 Economic agents

The model used here is an extension of the Bencivenga and Smith (1992) model by one period. This section studies the economy where banks are free of binding cash and liquid reserve requirements. The economy consists of an infinite sequence of four-period-lived overlapping generations. Time is discrete, and indexed t=0,1,... Since only steady states will be analyzed, a description of initial conditions is omitted.

In this economy, a non-storable consumption good is produced using capital and labour. Capital is owned by the old agents and the government. Output is produced using private capital acquired by investing loans from the banks plus some amount of government capital[14]. It is assumed that government capital is shared equally among the firms and there are no rental capital markets. Let k_t, L_t, and y_t denote the capital stock, employment, and output of a representative firm at time t. Then a constant returns to scale Cobb-Douglas production function[15] implies that $y_t = k_t^{\theta} L_t^{1-\theta}$; $\theta \in (0,1)$. A simplifying assumption is made that capital depreciates completely in one period.

Capital is produced using an investment technology according to which one unit of the consumption good invested at time t yields R units of capital at time t+3. This delay represents the "slow cycle of production" emphasized by Cameron (1967). Young agents, intermediaries and the government have access to the investment technology. Capital that accrues at time t+3 can only be received by the originating investor and the government in

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[14] As in Finn (1993) this can be construed as government-owned privately operated capital or that the government is in joint venture with the private sector.

[15] This specification of a standard production function simplifies the analysis without any loss of generality.

proportion to capital invested.

At each date t a young generation is born. Young agents are identical and each generation contains a continuum of (ex ante) identical agents. They are also endowed with a single unit of labour, which is supplied inelastically. Agents can work only when young and there is no endowment of the consumption or capital good at any date. It is assumed that all young agents have the following utility function;

$$u(c_0, c_1, c_2, c_3) = \ln(c_1 + \phi_1 c_2 + \phi_2 c_3), \qquad (1)$$

where c_i denotes consumption at age i (i=0,1,2,3), ϕ_j(j=1,2) is an individual-specific random variable realized at the beginning of age 1. Consequently there are three types of agents ex-post:

type 1 occur with probability π_1 and experience $\phi_1 = \phi_2 = 0$,

type 2 occur with probability π_2 and experience $\phi_1 = 1$, $\phi_2 = 0$,

type 3 occur with probability π_3 and experience $\phi_1 = \phi_2 = 1$.

This formulation is similar to Bencivenga and Smith (1992) and Diamond and Dybvig (1983). It implies a 'desire for liquidity' on the part of savers that leads to the formation of financial intermediaries. Only type 3 individuals can operate firms, since they are the only ones that can benefit from consuming in period 3. Both intragenerational and intergenerational loans are precluded.

Each type 3 agent operates a firm with capital stock, k_t, which is made up of capital financed from the bank loans and the government capital. Taking this capital stock and the real wage as given, each firm chooses an employment level L_t to maximize $k_t^{\theta} L_t^{1-\theta} - w_t L_t$. The solution to this problem is to set

$$L_t = k_t[(1-\theta)/w_t]^{1/\theta}, \tag{2}$$

and this yields a per firm profit of

$$\theta k_t^{\theta} L_t^{1-\theta} = \theta k_t[(1-\theta)/w_t]^{\alpha}, \tag{3}$$

where $\alpha = (1-\theta)/\theta$. Since each young agent supplies one unit of labour, the labour supply per firm is $1/\pi_3$. Using this and the firm's labour demand gives equilibrium wage

$$w_t = (1-\theta)\pi_3^{\theta} k_t^{\theta}. \tag{4}$$

2.2 The Government

There are three primary assets in the economy; capital, treasury bills, and fiat money. The government issues both treasury bills and fiat money and it runs a per capita expenditure of $g > 0$ at each date. The government generates revenue by levying a constant proportional income tax on young agents' earnings at the rate τ, paying less on treasury bills than it earns by investing the proceeds in capital[16], and printing money. Thus the government can increase its revenue by increasing the tax rate or the liquid reserve requirement or printing money. Let w_t denote the time t real wage rate. Then the per capita deficit is given by $q_2(1-\tau)w_t R_2 + g - \tau w_t - q_2(1-\tau)w_t R\theta[(1-\theta)/w_{t+3}]^{\alpha}$, where R_2 is the real interest that government pays per unit of the consumption good invested in its bills, $R\theta[(1-\theta)/w_{t+3}]^{\alpha}$ is the return to government investment per unit of the consumption good, and q_2 is the proportion of deposits held in treasury bills. R is the return in units of the capital good per unit of the consumption good invested. The situation of interest is when

16 This ensures that capital is not dominated by treasury bills as an asset in the model to economic agents.

government expenditure exceeds its revenue and seigniorage has to be used to finance the deficit. In this case the government budget constraint per capita will be given by $q_2(1-\tau)w_t R_2 + g = \tau w_t + q_2(1-\tau)w_t R\theta[(1-\theta)/w_{t+3}]^{\alpha} + (M_t - M_{t-1})/p_t$, where M_t denotes the per capita stock of fiat money in circulation at time t, and p_t the time t price level.

2.3 Portfolio Decisions

Young agents allocate their savings between money, bank deposits and capital. Banks promise to pay gross returns of r_1 and r_2 units of the consumption good if withdrawal occurs in period one and two respectively, and if withdrawal occurs in period three r_3 units of the capital good will be paid.

If $r_3\theta[(1-\theta)/w_{t+3}]^{\alpha} > r_2 > r_1$, then agents will withdraw in periods 1, 2, and 3 only if they are types 1, 2, and 3, respectively. This together with three other facts permit the portfolio problem of young agents to be derived. The first is that when the government is running a deficit, the real return on cash balances is less than 1. In this case, all real balances will be liquidated after one period no matter the type of agent. Secondly, direct purchase of treasury bills has value only if the individual is type 2 or 3, and the gross return is R_2. Thirdly, individual investment in capital has value only if the agent is type 3, and it will return $R\theta[(1-\theta)/w_{t+3}]^{\alpha}$. Let ω_1, ω_2, ω_3, and ω_4 denote the proportions of bank deposit, real balance, treasury bill and capital in young agents' asset portfolios, then the young agents' problem is to choose ω_1, ω_2, ω_3, and ω_4 to maximise their expected utilities. Thus young agents will choose ω_1, ω_2, ω_3, and ω_4 to solve equation (5);

$$\max \ln\{(1-\tau)w_t\} + \pi_1 \ln\{\omega_1 r_1 + \omega_2 p_t / p_{t+1}\} + \pi_2 \ln\{\omega_1 r_2 + \omega_2 (p_t / p_{t+1}) + \omega_3 R_2\}$$
$$+ \pi_3 \ln\{\omega_1 r_3 \theta[(1-\theta)/w_{t+3}]^{\alpha} + \omega_2 p_t / p_{t+1} + \omega_3 R_2 + (1-\omega_1-\omega_2-\omega_3)R\theta[(1-\theta)/w_{t+3}]^{\alpha}\}$$
$$\text{for } 0 \le \omega_i \le 1;\ i = 1,2,3,4. \tag{5}$$

At time t=1 real balances are liquidated so $\omega_2(p_t / P_{t+1})(1-\tau)w_t$ is consumed, and if type 1 an additional $\omega_1 r_1(1-\tau)w_t$ is consumed. Type 2 and type 3 agents consume an additional $(\omega_1 r_2 + \omega_3 R_2)(1-\tau)w_t$ and $\omega_3 R_2(1-\tau)w_t$ respectively at time t=2. At time t=3, type 3 agents again consume $\omega_1 r_3 \theta[(1-\theta)/w_{t+3}]^{\alpha} + (1-\omega_1-\omega_2)R\theta[(1-\theta)/w_{t+3}]^{\alpha}$.

If $p_t / p_{t+1} \le r_1$ and $(\pi_2+\pi_3)R_2 \le r_2$, that is if deposits dominate real balances and treasury bills as an asset to young savers, then $\omega_2 = \omega_3 = 0$. Using this, together with the conditions that $r_3\theta[(1-\theta)/w_{t+3}]^{\alpha} > r_2 > r_1$ and $R > r_3$, maximization of (5) will yield

$$\omega_1 = \min\left[\frac{(\pi_1+\pi_2)R}{R-r_3},\ 1\right] \tag{6}$$

From (6), if the expected return from individual capital investment is less than the return on deposits withdrawn after three periods, then individuals will not invest in capital. If $\frac{(\pi_1+\pi_2)R}{R-r_3} < 1$, then the expected return on the use of the investment technology is greater than the return on deposits withdrawn after three periods, and $\frac{\pi_3 R - r_3}{R-r_3}$ of income will be placed in the investment technology by the young agents.

2.4 Intermediary Behaviour

Financial intermediaries accept deposits from young savers at time t, which they hold in three types of assets; cash, treasury bills, and loans. The banks, thus, choose the

proportions of their assets that would be held in cash (q_1), treasury bills (q_2), and loans (q_3). In addition to this, the banks choose gross interest payments r_1, r_2 and r_3 which are paid per unit of deposit to agents who withdraw in periods one, two and three, respectively.

As in Diamond and Dybvig (1983) banks are co-operative entities consisting of coalitions[17] of young agents at t. These coalitions choose r_1, r_2, r_3, q_1, and q_2 to maximize the expected utility of a representative depositor evaluated at date t. In doing so they take the time paths of $\{w_t\}$ and $\{p_t\}$ as given or they behave competitively. Their choices must, of course, satisfy a set of resource constraints. Assuming $r_3\theta[(1-\theta)/w_{t+3}]^{\alpha} > r_2 > r_1$, then agents would withdraw one and two periods after making a deposit iff they are type 1 and type 2, respectively, and the relevant resource constraints would be given by;

$$\left.\begin{array}{rcl} \pi_1 r_1 & = & q_1(p_t / p_{t+1}) \\ \pi_2 r_2 & = & q_2 R_2 \\ \pi_3 r_3 & = & q_3 R \end{array}\right\} \qquad (7)$$

As long as the real return on capital dominates that of cash, ω_1 will be equal to 1 under laissez-faire[18]. Substituting these into (5) yields the objective of banks; that is to solve

$$\max_{0 \le q_1+q_2 \le 1} \ln\{(1-\pi)w_t\} + \pi_1 \ln\{r_1\} + \pi_2 \ln\{r_2\} + \pi_3 \ln\{r_3\theta[(1-\theta)/w_{t+3}]^{\alpha}\} \qquad (8)$$

Using the resource constraints in (7) and $q_1+q_2+q_3=1$, the solution set yields $q_1=\pi_1$, $q_2=\pi_2$, and $q_3=\pi_3$.

Plugging these values in (7) we obtain

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17 These deposit coalitions are responses to a crucial feature of this economy. That is while each individual faces significant uncertainty about his type, it is assumed there is no uncertainty about the aggregate distribution of agent types.

18 This is because deposits will dominate individual investments in treasury bills and capital by avoiding the risk associated with them.

$$r_1 = p_t/p_{t+1}, \tag{9}$$

$$r_2 = R_2, \tag{10}$$

$$r_3 = R. \tag{11}$$

Equation (9) asserts that depositors who withdraw after one period obtain the real return on cash reserves, (10) states that those who withdraw after two periods obtain the return on treasury bills, and (11) says type 3 agents receive the return on capital.

2.5 Steady State Equilibrium

In the absence of government intervention, ω_1=1, so all capital formation is intermediated. The use of investment technology has value for only type 3 agents and one unit of savings placed in the investment technology returns R units of capital at t+3, and hence the per firm capital stock at time t+3 is given by $k_{t+3} = (q_2 + q_3)(1-\tau) w_t R / \pi_3$.

This is because $(1-\tau) w_t$ is time t savings of which $q_3(1-\tau) w_t$ is invested in capital formation by the banks and government invests $q_2(1-\tau) w_t$. The resulting per capita capital at time 3 is $Rq_3(1-\tau) w_t + Rq_2(1-\tau) w_t$ which is divided among the fraction of old type three agents for production. Substituting for w_t by using (4) and the fact that $q_3=\pi_3$ gives the equilibrium law of motion for the per firm capital stock:

$$k_{t+3} = R(1-\tau)(q_2 + q_3)(1-\theta)\pi_3^{\theta-1} k_t^{\theta}. \tag{12}$$

In steady state $k_{t+3} = k_{t+2} = k_{t+1} = k^*$ which gives steady state capital stock as

$$k^* = 1/\pi_3 [R(1-\theta)(1-\tau)(q_2 + q_3)]^{1/(1-\theta)}. \tag{13}$$

Substituting this into (4) yields steady state equilibrium wage,

$$w^* = (1-\theta)[R(1-\tau)(q_2+q_3)(1-\theta)]^{1/\alpha}, \tag{14}$$

and steady state real balance

$$M_t / P_t = \omega_1 q_1 (1-\tau) w^* = q_1 (1-\tau) w^* = \pi_1 (1-\tau) w^*. \tag{15}$$

From the government budget constraint the steady state real return on cash balances is:

$$\frac{p_t}{p_{t+1}} = \frac{q_1(1-\tau)w^* - (g-\tau w^*) - \{q_2(1-\tau)w^* R_2 - q_2(1-\tau)w^* R\theta[(1-\theta)/w^*]^{\alpha}\}}{q_1(1-\theta)w^*} \tag{16}$$

If the government intervenes by imposing binding minimum cash and liquid reserve requirements, then the banks will be constrained to hold cash and liquid reserves such that $q_1 \geq \bar{q}_1 \geq \pi_1$ and $q_2 \geq \bar{q}_2 \geq \pi_2$ where $\bar{q}_1$ and $\bar{q}_2$ are cash and liquid reserve requirements set by the government. In this case ω_1=1 can still hold, but may not if reserve requirements are set "too" high. In either case

$$r_1 = \bar{q}_1 (p_t / p_{t+1}) / \pi_1 \geq p_t / p_{t+1} \tag{17}$$

$$r_2 = \bar{q}_2 R_2 / \pi_2 \geq R_2 \tag{18}$$

$$r_3 = \bar{q}_3 R / \pi_3 \tag{19}$$

Since reserve requirements are binding (17), (18), and (19) can fail to hold if banks alter their optimal strategy of asset liquidations. The only way they can do so is by either attempting to induce agents to withdraw some of their deposits before they actually need them or by not liquidating all reserve holdings when they are due. But since $r_3\theta[(1-\theta)/w_t]^{1/\alpha} > r_2 > r_1$ agents will not voluntarily withdraw their deposits before they need them, and banks cannot improve the expected utility of the representative consumer by not liquidating all reserves when due.

From (6), capital is intermediated if $r_3 \geq \pi_3 R$ and using (19) this implies that $\bar{q}_3 \geq \pi_3^2$ [$\bar{q}_3$ is the capital holdings of the banks when the government interferes with their cash and/or liquid asset(s) holdings]. If $\bar{q}_3 \in [\pi_3, \pi_3^2]$, then the per firm capital stock evolves according to

$$k_{t+3} = R(1-\tau)(1-\theta)(\bar{q}_2 + \bar{q}_3)\pi_3^{\theta-1} k_t^{\theta}. \tag{20}$$

In steady state the capital stock is given by

$$\bar{k}^* = \frac{1}{\pi_3}[R(1-\theta)(1-\tau)(\bar{q}_2 + \bar{q}_3)]^{1/(1-\theta)}, \tag{21}$$

and the steady state real wage rate is

$$\bar{w}^* = (1-\theta)[R(1-\tau)(1-\theta)(\bar{q}_2 + \bar{q}_3]^{1/\alpha}. \tag{22}$$

Since ω_1=1 the steady state equilibrium level of real balance is given by

$$M_t / p_t = \bar{q}_1(1-\tau)\bar{w}^* = \bar{q}_1(1-\tau)(1-\theta)[R(1-\tau)(1-\theta)(\bar{q}_2 + \bar{q}_3)]^{1/\alpha}. \tag{23}$$

Then using the government budget constraint, the steady state inflation rate is given by

$$\frac{p_t}{p_{t+1}} = \frac{\bar{q}_1(1-\tau)\bar{w}^* - (g - \tau\bar{w}^*) - \bar{q}_2(1-\tau)\bar{w}^*(R_2 - R\theta[(1-\theta)/\bar{w}^*]^{\alpha})}{\bar{q}_1(1-\tau)\bar{w}^*}. \tag{24}$$

Some features of this equilibrium equations merit comments. From (21), an increase in the cash reserve requirement will reduce steady state capital whether banks reduce their treasury bills or loans. An increase in liquid reserve requirement, on the other hand, will increase the capital stock if part or all of the increase is financed by a decrease in cash reserves. If the whole increase comes from bank loans, then there will be no effect on steady state capital stock. This suggests that an increase in cash holdings will reduce capital and growth more than an increase in liquid reserves. From (22), an increase in liquid reserve

requirements and/or bank loans financed by reducing cash reserves would increase steady state real wage by increasing the capital stock. However, an increase in the liquid reserve requirement financed by a reduction in loans does not affect the steady state real wage.

Equation (23) suggests that an increase in the liquid reserve requirement achieved by a reduction in bank loans has no effect on real balances. If, however, the increase is achieved by a reduction in cash reserves then both positive and negative effects could be traced. Real balances will be reduced directly by the reduced cash reserve, but will be increased as a result of increased wages and deposits.

From (24), the equilibrium inflation rate depends on the size of the fiscal deficit. An increase in the deficit will increase inflation and vice versa. Deficits will increase if there is an increase in the interest government pays on treasury bills or if there is a reduction in liquid reserve requirements or the tax rate. These changes reduce government revenue and hence increase the deficit for any given amount of government spending. The higher deficit will require more seigniorage resulting in higher inflation. The effect of changes in the cash reserve requirement on inflation is not readily apparent and is examined in section 4.

3. Real Balances

3.1 Real Balances and reserve requirements

This section investigates the effect of changes in cash and liquid reserve requirements on real balances. Differentiating (23) with respect to the proportions of the various assets in the banks' portfolios gives

$$d(M_t/p_t) = (1-\tau)(1-\theta)[R(1-\theta)(1-\tau)(\bar{q}_2+\bar{q}_3)]d\bar{q}_1 + \frac{1}{\alpha}\bar{q}_1R(1-\tau)^2(1-\theta)^2[R(1-\theta)(1-\tau)(\bar{q}_2+\bar{q}_3)]^{\frac{1}{\alpha}-1}(d\bar{q}_2+d\bar{q}_3). \quad (25)$$

Equation (25) suggests that maximization of real balances by setting cash reserve requirement is independent of whether banks change their treasury bills or loans to comply with the required changes in the cash reserves. This is because the public and the private sectors are equally efficient in production, and hence changing treasury bills or bank loans by the same amount has the same effect on output, real wage, and real balance. Real balance holding is maximised by setting $\bar{q}_1 = \frac{\alpha}{1+\alpha}$, and it can easily be verified that the second order condition holds. If $\bar{q}_1 < \frac{\alpha}{1+\alpha}$, then the direct effect of changes in cash reserve requirements outweighs the indirect effect via incomes, and a positive relationship between cash reserve requirements and real balances is obtained.

In the real world situation, however, there may be differences in the productivities of public and private capital. If productivities differ, the type of asset the banks change to comply with the required change will be relevant. If, for example, public investment is less productive[19], then an increase in cash reserve requirement which is implemented by reducing bank loans will reduce output more than if liquid reserves are reduced. Lower output will result in less real income for labour, and hence less deposit and real balances. This suggests that increases in cash reserve requirement should come from assets that are less efficient in production if seigniorage is important as a source of revenue for the

[19] Lucas (1990) estimated private investment productivity to be 0.3 which is greater than Finn (1993) estimate of 0.16 for public investment for the U.S. economy. Public sector productivity is also likely to be less than the private sector's in many developing countries where the government participates actively in the provision of private goods.

government.

Maximization of (25) with respect to liquid reserves gives $\bar{q}_2 = \frac{\bar{q}_1}{\alpha} - \bar{q}_3$[20]. This does not pin down the proportion of liquid reserve that maximizes real balances. Thus the maximum real balance only depends on the maximum cash reserve ratio, and the composition of treasury bills and loans is irrelevant.

3.2 Real Balances and Utility

In this model, bank profit maximization is equivalent to maximizing the expected utilities[21] of economic agents. Without government intervention, profit maximization leads banks to choose the proportion of their cash, treasury bills, and loans to equal the probabilities of type 1, 2, and 3 agents, respectively. Maximization of real balances requires that q_1 be set equal to labour's share in output. Thus if the goal is to maximize real balances, then there would be a need for intervention only when banks' laissez-fair choice of the proportion of cash holdings is different from the proportion of labour's share of output. In particular, if banks choose cash reserve holdings which are less than the proportion of labour's share in output, then the government will impose a minimum cash reserve requirement which is equal to labour's share of output. In this case, expected utilities will be lowered by the government's action.

Although the decisions by banks to hold assets in the form of treasury bills or loans does not affect steady state real balances, they are relevant for utility. If the government sets

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20 The real cash balance maximizing cash reserve requirement is obtained by using $q_1+q_2+q_3=1$.

21 This is because banks are cooperative entities consisting of coalitions of young agents.

a binding cash reserve requirement in order to maximize real balances then bank profit maximization will be subject to this restriction, and the expected utility maximization problem in (8) will becomes

$$\max_{0\le\bar{q}_2\le 1} \ln[(1-\tau)w_t] + \pi_1 \ln(r_1) + \pi_2 \ln(r_2) + \pi_3 \ln(r_3\theta[(1-\theta)/w_{t+3}]^{\alpha}). \qquad (26)$$

Using the government's restriction and the resource constraints in (7), the solution to (26) is $\bar{q}_2 = \frac{\pi_2\theta}{\pi_2+\pi_3}$. This suggests that the imposition of a binding minimum cash reserve requirement will force banks to reduce both liquid reserve and loans in response[22]. In particular, they will reduce liquid reserves and loans in proportion to the probabilities of type 2 and 3 agents, respectively, given that agents are type 2 or 3. Thus for real balance maximization, the model suggests that the government should only set a cash reserve requirement and allow banks to choose the composition of the remaining assets. Government intervention with the choice of liquid reserves may adversely affect utility without any positive effect on real balances.

Maximization of real balances which has been tackled so far may not be of interest *per se*. Nonetheless, it is important in the studies of seigniorage as an increase in real balance will increase the base of seigniorage and vice versa. Another important factor in the studies of seigniorage is the dynamics of the price levels and this is taken up in the next section.

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22 The optimal settings with the imposition of $\bar{q}_1 = 1-\theta$ requires banks to set $\bar{q}_2 = \frac{\pi_2\theta}{\pi_2+\pi_3}$ and $\bar{q}_3 = \frac{\pi_3\theta}{\pi_2+\pi_3}$.

4. Inflation

4.1 Inflation and changes in cash reserve requirements

This section analyzes the effect of changes in cash and liquid reserves on steady state inflation. Substituting for real wages in (24) and differentiating it with respect to the proportions of assets in the banks' portfolios gives (27);

$$d\left(\frac{p_t}{p_{t+1}}\right)=\left(\frac{g}{\bar{q}_1^2(1-\tau)(1-\theta)[R(1-\theta)(1-\tau)(\bar{q}_2+\bar{q}_3)]^{1/\alpha}}-\frac{\theta\bar{q}_2}{\bar{q}_1^2(1-\theta)(1-\tau)(\bar{q}_2+\bar{q}_3)}-\frac{\tau-(1-\tau)\bar{q}_2R_2}{\bar{q}_1^2(1-\tau)}\right)d\bar{q}_1$$

$$+\left(\frac{R\theta g}{\bar{q}_1(1-\theta)[R(1-\theta)(1-\tau)(\bar{q}_2+\bar{q}_3)]^{1/(1-\theta)}}-\frac{\theta\bar{q}_2}{\bar{q}_1(1-\theta)(1-\tau)(\bar{q}_2+\bar{q}_3)^2}-\frac{R_2}{\bar{q}_1}+\frac{\theta}{\bar{q}_1(1-\theta)(1-\tau)(\bar{q}_2+\bar{q}_3)}\right)d\bar{q}_2$$

$$+\left(\frac{R\theta g}{\bar{q}_1(1-\theta)[R(1-\theta)(1-\tau)(\bar{q}_2+\bar{q}_3)]^{1/(1-\theta)}}-\frac{\theta\bar{q}_2}{\bar{q}_1(1-\theta)(1-\tau)(\bar{q}_2+\bar{q}_3)}\right)d\bar{q}_3 \qquad (27)$$

Unlike the effect on real balances, the effect on inflation of a change in cash reserve requirement is sensitive to whether the change is accomplished through changing treasury bills or private loans. The effect on inflation of a change in the cash reserve requirement achieved solely by changing liquid reserves is given by

$$\frac{d(p_t/p_{t+1})}{d\bar{q}_1}=\frac{gR[\bar{q}_2+\bar{q}_3-\theta]}{\bar{q}_1^2(1-\theta)[R(1-\theta)(1-\tau)(\bar{q}_2+\bar{q}_3)]^{1/(1-\theta)}}-\frac{\tau}{\bar{q}_1^2(1-\tau)}$$
$$+\frac{R_2(\bar{q}_2+\bar{q}_3)}{\bar{q}_1^2}-\frac{\theta[\bar{q}_2\bar{q}_3+\bar{q}_2^2+\bar{q}_1\bar{q}_3]}{\bar{q}_1^2(1-\theta)(1-\tau)(\bar{q}_2+\bar{q}_3)^2}. \qquad (28)$$

Equation (28) indicates that there are both negative and positive effects on inflation. There are basically two reasons for this. First, an increase in the cash reserve requirement can

increase or decrease real balances. A reduction in real balances will have inflationary consequences whilst an increase will have deflationary effects. Second, a fall in liquid reserves reduces government revenue[23], investment, and steady state output, which has inflationary effects on the economy. A trivial solution of an increase in inflation is obtained when real balances fall[24], and hence attention is restricted to the situation where real balances increase.

In this model, steady state inflation is basically due to government deficit spending. Thus, the consequence for inflation is captured by the influence of the changes in real balance and output on government spendings and revenues. The government's budget in the model is composed of

(a) direct spending [g] minus income taxes (direct government balance), and

(b) treasury bill expenses minus its revenues (indirect government balance).

The effects through direct government spending and income taxes are captured by the first two terms of (28). Consistent with the literature, direct government spending is assumed fixed[25] and unaffected by changes in output and real balance. An increase in real cash balances increases the inflationary tax base and thus reduces the inflation required to finance any given spending. This effect is captured by the positive coefficient of g. With a given g, a reduction in output (due to decreases in treasury bills) will compel agents to compete for less output and hence has inflationary repercussions. This effect is represented

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23 Romer (1985) recognizes that reserve requirements affect the economy-wide composition of assets as well as government revenue. However, he asserts that steady state inflation is unaffected. It appears that this reflects his apparent adjustment of government spending to match the changes in revenue.

24 In the model by Bencivenga and Smith (1992) where the government does not experience any adverse revenue effect through a decrease in treasury bills, an increase in reserve requirements which reduces real balances is still sufficient to raise inflation.

by the negative coefficient of g. The net effect of these on inflation depends on the magnitudes of the liquid plus loan asset ratios and capital's share in output. If the liquid plus loan ratio is greater than the share of capital in output, then the real balance effect will dominate[26], and the effect of an increase in cash reserve requirement will be deflationary. Intuitively, since labour supply is perfectly inelastic, net resource to the economy is maximized when the proportion of the liquid plus the loan ratio is set equal to the share of capital in output. Thus, if the liquid plus the loan ratio is higher than θ, then a reduction in liquid reserve will increase net resource and hence results in deflation. However, if the proportion of private and public investment is less than θ, a reduction in liquid reserve would reduce net resources and will result in inflation.

The effect through the government's income tax revenue is unambiguously negative. This is because an increase in real balances reduces the rate of deflation[27] for any given wage rate, and also the reduction in the demand for treasury bills reduces government revenue and investment which adversely affects income tax. These two are effects are inflationary. Thus, if the adverse output effect dominates in direct government spending [g], the direct government balance will produce inflation. In particular, inflationary results will be obtained if the direct government budget balances or is in a surplus[28]. The result, however, is not conclusive if there is a deficit. If the deficit is large enough, that is if

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25 See Mourmouras and Russell (1992).

26 In the extreme, where the contribution of capital to output is zero changes in liquid reserve does not affect output.

27 Income taxes from the economy deflates the economy by reducing the amount of money available to the economy.

28 In this cases, any deflationary direct government spending effect is too weak to dominate the inflationary direct government revenue effect.

$\frac{g}{w\tau} > \frac{(1-\theta)(\bar{q}_2+\bar{q}_3)}{\bar{q}_2+\bar{q}_3-\theta}$[29], then deflationary consequences will be obtained. Thus for a given deficit, a decrease in capital's share and/or an increase in investment will make inflation more likely through the direct government balance effect.

There are both positive and negative effects from government's issuing of treasury bills. The effect coming through payments of gross returns on treasury bills is unambiguously deflationary. Both the increase in real balance and the reduction in the demand for treasury bills will reduce the inflationary impact of government spending on gross interest payments. On the other hand, the reduction in the demand for treasury bills and the increase in real balances reduce the rate of deflation of withdrawing money by issuing treasury bills (government's revenue from investing Treasury Bill funds)[30]. However, this inflationary effect is mitigated by the increase in returns to capital[31]. The total effect from the issue of treasury bills is thus ambiguous. This is contrary to the expectation that a reduction in treasury bills would reduce government revenue, and hence result in more inflationary financing and inflation. The reason for this is the deflationary effect of the increase in returns on treasury bills to the government. However, the higher the mark-up on treasury bills, the more likely the inflationary effect[32] because the revenue loss from the

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[29] It should be noted that if the deficit is "too" large, it may not be feasible to inflationary finance it. In particular, if real balances are decreasing in inflation, then feasible monetization of the deficit requires that the deficit should not exceed real balance. However, Bencivenga and Smith (1992) argued that if the deficit is sufficiently close to the real balance, then inflationary financing it still possible if real balance can be increased by other means, such as increasing cash requirements.

[30] Note that in steady state the principal amounts involved in the treasury bills deals cancel out and are not relevant for inflation

[31] Since a constant returns to scale production function is used, a reduction in capital input with fixed labour input will increase the returns to capital by increasing its marginal product.

[32] In particular, if the mark-up on treasury bills is greater than $\frac{\bar{q}_1\bar{q}_2}{\bar{q}_2+\bar{q}_3(1-\bar{q}_3)}$ inflation will occur. Thus

reduction in treasury bills is greater. If the government does not make profit on the treasury bills, then the effect will be deflationary. This real world possibility[33] cannot be handled in this model, as paying more than is earned on treasury bills will make them dominate private investment as an asset. The total effect on the price dynamics of an increase in cash reserve requirements financed by an equal reduction in liquid assets is, therefore, ambiguous. The possibility of an inflationary outcome is enhanced the smaller the deficit and/or the higher the mark-up on treasury bills.

If banks comply with an increase in cash reserve requirement by reducing their private sector loans, the following equation will give the relevant relationship between changes in inflation and changes in cash reserves;

$$\frac{d(p_t / p_{t+1})}{d\bar{q}_1} = \frac{gR[\bar{q}_2 + \bar{q}_3 - \theta]}{\bar{q}_1(1-\theta)[R(1-\theta)(1-\tau)(\bar{q}_2 + \bar{q}_3)]^{1/(1-\theta)}} - \frac{\tau}{\bar{q}_1^2(1-\tau)} - \frac{\theta\bar{q}_2(\bar{q}_2 + \bar{q}_3 - \bar{q}_1)}{\bar{q}_1^2(1-\theta)(1-\tau)(\bar{q}_2 + \bar{q}_3)^2} + \frac{R_2\bar{q}_2}{\bar{q}_1^2}. \tag{29}$$

The effects through direct government spending and taxes are the same as in (28). This hinges critically on the assumption that both government and private investments are equally efficient in production. If for example, private investment is more productive, output will be hurt more in this scenario, and the inflationary consequences will be stronger.

A comparison of the magnitudes of the effects due to treasury bills in (28) and (29)

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inflation is likely to occur the less the product of the liquid and cash reserves. If there is no private investment, then inflation will result if the mark-up is greater than the cash reserve ratio.

33 This situation can arise, if for example, the government uses revenues from treasury bills to finance recurrent expenditure or projects that have rates of return less than the returns on the bills.

shows that the inflationary impact from (28) is stronger than from (29)[34]. This suggests that an increase in cash reserves achieved by a reduction in liquid reserves results in more inflationary pressures than if banks reduce their loans. The difference hinges on the assumption that the government pays less on treasury bills than it earns from investing the proceeds (in essence the government taxes treasury bills). In this model, government income tax revenue falls by the same amount whether the increase in cash reserve requirement results in a reduction in liquid or loan asset of the banks (because the output effect is the same). However, in the case of a reduction in treasury bills, the government's net revenue from the treasury bills is also adversely affected. This results in a larger public budget deficit and hence requires more inflationary financing. It should be mentioned that the argument is reversed if government pays more on treasury bills than it earns on it. Thus under the assumptions of this model, the room for seigniorage is better enhanced if banks reduce their loans rather than their liquid assets to comply with increases in their cash reserve requirement. This indicates that seigniorage is sensitive to the particular asset banks change to meet changes in cash reserve[35] requirement. This sensitivity depends on the productivity and the taxation of the asset by the government. The analysis suggests that for any increase in cash reserve requirement, seigniorage capacity will be better enhanced if the banks reduce assets that are less productive and less taxed by the government.

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[34] Subtraction of (28) from (29) gives $\frac{\theta}{\bar{q}_1(1-\theta)(1-\tau)(\bar{q}_2+\bar{q}_3)} - \frac{R_2}{\bar{q}_1}$ which is positive under the assumption that government pays less on treasury bills than it earns from investing the revenues.

[35] Siegel (1981) could not point this out because his asset menu was not rich enough. The same applies to Calvo and Fernandez (1983) and McClure (1986). Fama (1985) shows that the taxation of deposits by reserve requirements is a complex issue in the world of several assets and liabilities.

4.2 Inflation and changes in the liquid reserve requirement

An increase in the liquid reserve requirement which results in an equal decrease in cash reserves is the reverse of an increase cash reserve requirement which is complied with by decreasing liquid reserves. The effects can thus be analyzed in a similar fashion as has been done in section 4.1. The focus here will, therefore, be on an increase in liquid reserve requirement which banks implement by reducing their loans. In this case, the relevant equation is

$$\frac{d(p_t / p_{t+1})}{d\bar{q}_2} = \frac{\theta}{\bar{q}_1(1-\theta)(1-\tau)(\bar{q}_2+\bar{q}_3)} - \frac{R_2}{\bar{q}_1} \qquad (30)$$

Steady state returns from government investment in capital per unit of the consumption good is given by $\theta/(1-\theta)(1-\tau)(\bar{q}_1+\bar{q}_2)$ which is assumed to be greater than the returns on treasury bills, R_2, and hence inflation is decreasing in liquid reserve requirement. In this scenario, output and real balance holdings are not affected and hence a reduction in the rate of inflation enhances the ability to raise seigniorage. If the government pays all the returns on investment as interest on the bills, then changes in the liquid reserve requirement will not affect inflation and seigniorage potential.

5. Cash versus liquid reserve requirement

The effect of increasing cash or liquid reserve requirements on the government's ability to collect seigniorage can be properly compared if the banks reduce a particular asset (loans in this model) to satisfy the required increase in reserve. The effect of a change in cash reserve requirement and liquid reserve requirement on real balances can be analyzed

by examining equation (25). From (25),

$$\frac{d(M_t/P_t)}{d\bar{q}_1} - \frac{d(M_t/P_t)}{d\bar{q}_2} = (1-\tau)(1-\theta)[R(1-\theta)(1-\tau)(\bar{q}_2+\bar{q}_3)]^{1/\alpha}\left\{\frac{\theta-\bar{q}_1}{\theta(1-\bar{q}_1)}\right\}. \quad (32)$$

Equation (32) will be positive if $\bar{q}_1$ is less than θ, and negative if $\bar{q}_1$ is greater than θ. It should be noted that real balance is maximized when $\bar{q}_1$ equals θ[36]. Thus, when real balances are below their maximum, an increase in the cash reserve requirement will have more favourable effects on real money balances, and when real balances are above the maximum, liquid reserves will have more favourable effects. The reason is that when the cash reserve requirement is at the real balance maximizing level, a further increase in the cash reserve requirement will reduce real cash balances whilst an increases in the liquid reserve requirement does not affect them.

The effect of changes in cash reserve requirement and liquid reserve requirement on inflation can be analyzed by examining equations (29) and (30). From these equations

$$\frac{d(p_t/p_{t+1})}{d\bar{q}_1} - \frac{d(p_t/p_{t+1})}{d\bar{q}_2} = \frac{gR[\bar{q}_2+\bar{q}_3-\theta]}{\bar{q}_1^2(1-\theta)[R(1-\theta)(1-\tau)(\bar{q}_2+\bar{q}_3)]^{1/(1-\theta)}} - \frac{\tau}{\bar{q}_1^2(1-\tau)}$$
$$- \frac{\theta}{\bar{q}_1(1-\theta)(1-\tau)(\bar{q}_2+\bar{q}_3)}\left[\frac{\bar{q}_2(\bar{q}_2+\bar{q}_3-\bar{q}_1)}{\bar{q}_1(\bar{q}_2+\bar{q}_3)}+1\right] + \frac{R_2}{\bar{q}_1}[\bar{q}_2+1] \quad (33)$$

The first two terms are the same as the first two terms in equations (28) and (29). This means that the sum of these two terms will be negative if direct government budget

[36] In Brock (1989), when the revenue maximizing reserve ratio is chosen the elasticity of demand for base

balances or is in a surplus. In this case, if the sum of the last two terms is negative, that is if the "profit"[37] on treasury bills is sufficiently low such that $\frac{\theta}{R_2(1-\theta)(1-\tau)(\bar{q}_2+\bar{q}_3)} < \frac{\bar{q}_1(\bar{q}_2+1)(1-\bar{q}_1)}{\bar{q}_2(1-2\bar{q}_1)-\bar{q}_1(1-\bar{q}_1)}$, an increase in liquid reserve requirement will increase inflation more than an increase in the cash reserve requirement.

Combining the real balance and the inflationary effects, cash reserve requirement will be more successful in augmenting the government's ability to raise seigniorage than liquid reserve requirement, if (i) the government budget is in a surplus or is balanced, (ii) the cash reserve requirement is below it's real balance maximizing level, and (iii) the government's return on treasury bills is low, that is $\frac{\theta}{R_2(1-\theta)(1-\tau)(\bar{q}_2+\bar{q}_3)} < \frac{\bar{q}_1(\bar{q}_2+1)(1-\bar{q}_1)}{\bar{q}_2(1-2\bar{q}_1)-\bar{q}_1(1-\bar{q}_1)}$. On the other hand, a sufficient condition for liquid reserve requirement to outperform cash reserve requirement in boosting the government's ability to collect seigniorage is if (i) the primary budget deficit is sufficiently large, that is when $\frac{g}{\tau w} > \frac{(1-\theta)(\bar{q}_2+\bar{q}_3)}{\bar{q}_2+\bar{q}_3-\theta}$, (ii) the cash reserve requirement is greater or equal to the real balance maximizing requirement, and (iii) the government's return on treasury bills is high, that is $\frac{\theta}{R_2(1-\theta)(1-\tau)(\bar{q}_2+\bar{q}_3)} > \frac{\bar{q}_1(\bar{q}_2+1)(1-\bar{q}_1)}{\bar{q}_2(1-2\bar{q}_1)-\bar{q}_1(1-\bar{q}_1)}$.

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money with respect to the reserve ratio is zero. This means that the seigniorage maximizing reserve ratio also maximizes the real cash balance for any given inflation rate.

[37] The profit on treasury bills to the government is the difference between the returns on investment of funds from the bills and the amount the government pays on the bills $\left[\frac{\theta}{(1-\theta)(1-\tau)(\bar{q}_2+\bar{q}_3)} - R_2\right]$.

6. Conclusion

This essay, basically, examines two issues. The first is whether an increase in cash reserve requirements will produce different seigniorage results if banks reduce different assets to comply with the required changes in cash reserves. It was shown that seigniorage is, indeed, sensitive to the particular asset that the banks change in their portfolio to satisfy the required changes in their cash reserves. This sensitivity depends on the rate of tax on and the productivity of the asset. If in response to an increase in cash reserve requirement, the banks decrease assets that are less taxed by the government and/or less productive, the government's ability to raise seigniorage will be more enhanced. Intuitively, if banks reduce assets that are more taxed by the government, the government's revenue will be adversely affected. It is the government's attempt to deal with the adverse budgetary shock either through cutting spending or increasing income taxes or increasing inflationary finance or engaging in any combination of these that adversely affects it's ability to raise seigniorage. Also if the bank reduce assets that are very productive, output will be hurt more and this will reduce the base of seigniorage by reducing the demand for real balances.

The second issue examined is the effect of an increase in cash and liquid reserve requirements on the governments ability to raise seigniorage. It was demonstrated that if seigniorage is an important objective in imposing reserve requirements, then targeting a specific reserve requirement will be beneficial. More specifically, the analysis suggests that increases in the cash reserve requirement will be more effective in enhancing the government's ability to collect seigniorage the smaller the budget deficit, the lower the cash

reserve and the smaller the return on treasury bills to the government.

The analysis indicates that one cannot prescribe on *a priori* basis which assets the banks should change in order to increase the room for seigniorage or which reserve requirement the governments should target if seigniorage is a major objective behind the imposition of reserve requirements. The questions can only be settled empirically.

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ESSAY 2

SEIGNIORAGE AND INTEREST RATE DEREGULATION

1. Introduction

Financial liberalization, among other things, involves the removal of interest rate controls. Studies of interest rate deregulation have focused on its effects on savings, inflation, and growth, and little attention has been paid to its consequences for seigniorage. This is probably because of the less obvious relationship between interest rate deregulation and seigniorage. Upwards adjustment of controlled interest rates will influence the financial asset portfolios of economic agents and change the economy's demand for money. Since the demand for money is the base on which seigniorage is levied, changes in money demand will influence seigniorage.

Repression of the financial sector of developing countries usually results in the development of a thriving informal financial sector[38]. The financial sectors of these economies are therefore characterised by a formal financial sector, which is dominated by commercial banks and regulated by the government, and an informal one which is "free" of such regulations. The informal sector is not, for instance, subject to legal reserve requirements and its interest rates are not controlled by the government. The size of households' deposits with the informal sector can be substantial. For example, in Taiwan a large-scale survey of households' portfolio holdings by the Chung-Hua Institution for Economic Research in 1984 shows that households' deposits with the informal sector is

[38] Informal credit markets can take various forms which includes the village moneylenders making loans to local farmers, family members making loans to less fortunate kins, established lenders making loans in the cities, and cooperative arrangements to raise funds and share credit among members.

about 60% of their deposits with the formal banking institutions [see Ming-Yih Liang (1988)]. In Ghana, Aryeetey and Gockel (1989) contends that the proportion of urban savings with the informal sector is more than 60%.

Various reasons can be advanced for why utility maximizing agents would hold deposits in the formal sector despite the fact that its rate of return is dominated by the informal deposit rate. One such reason is that deposits with the formal sector are more liquid than deposits with the informal sector [see Kapur (1992)]. This is because the formal financial institutions hold reserve requirements which enable them to meet unexpected cash withdrawals which the informal institutions cannot meet. Thus, if individuals face some unforeseen contingencies they will hold some deposits with the formal sector as an insurance against these unexpected expenditures. Another reason why individuals would hold deposits with the formal sector is the higher risk that is often associated with the holding of deposits with the informal sector. Although loan default rates are often higher[39] in the formal sector than the informal sector, the formal sector deposits are usually riskless since the central government ensures that they receive significant injections of capital to enable them to continue functioning.

In a financial sector characterised by a formal and an informal sectors, an increase in formal interest rates will reduce the relative attractiveness of informal deposits and hence result in a shift of deposits from the informal to the formal sector. This will increase the demand for money since the formal sector institutions hold required reserves against their

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39 Since the formal sector institutions are controlled in various ways, they are often poorly run with little capitalization and large proportion of non-performing loan portfolios. In Ghana, for example, the banks were required to put more than 40% of the value of their loans into loan loss reserves during its financial liberalization program [see Younger (1991)].

deposits whilst the informal sector institutions do not. Thus switching of deposits from the informal sector to the formal one will potentially boost seigniorage. An increase in controlled deposit rates can also lead to a reduction in the demand for high powered money by economic agents in favour of formal sector deposits. This will reduce the base of the inflation tax and hence reduce seigniorage. Thus in these economies, one can easily identify both positive and negative effects of changes in controlled interest rates on seigniorage. The purpose of this paper is to explore these seigniorage consequences.

Major work on the impact of increasing formal controlled interest rates on savings and growth began with McKinnon (1973) and Shaw (1973). They argued that deregulating interest rates would improve the allocation of resources in repressed economies, increasing their productive capabilities and growth. In particular, they used non-optimizing models to show that if controlled deposits rates are adjusted upwards it would increase output and lower inflation in the short run, and in the medium term increase growth by raising the savings rate.

This finding, however, came under some attack by what has been termed the "neo-structuralist" critique [see, for example, van Wijnbergen (1982, 1983), and Buffie (1984)]. This critique hinges on the argument that the adjustment of controlled interest rate upwards will induce a shift of deposits from the informal sector to the formal sector, and this will reduce the supply of credit since the formal institutions will hold part of the shifted deposit in required reserves. The reduction in productive lending will have a negative effect on output and lead to more inflation. Whilst the arguments put forward by McKinnon and Shaw have favourable implications for seigniorage (increase in output and decrease in

inflation), the neo-structuralist argument suggests that the scope for seigniorage would be adversely affected (reduction in growth and increase in inflation).

The issue of the effect of interest rate deregulation on seigniorage was drawn into the debate by Kapur (1992). He attacked the neo-structuralist critique by arguing that it ignored seigniorage which may result from a policy of financial liberalization. His argument centers on the micro-theoretic examination of cash reserve requirements held by the commercial banks, and on the failure of previous portfolio-theoretic models of the financial sectors of LDCs to capture the liability side of the consumer's balance sheet. He argued that an increase in the interest rates of commercial banks would induce a shift of deposits from the informal financial market to the formal one. Consequently, the economy's demand for high-powered money increases as reserve holdings of commercial banks increase. As a result of the increase in the demand for money, the government receives a "windfall" gain in seigniorage which it can use for productive lending. This can be done by channelling the newly-created high-powered money to 'development banks' or to the commercial banks themselves. He argued that if the government does exercise this option, the neo-structuralist critique would be completely neutralised.

Although, Kapur's analysis is not centered on exploring the seigniorage effects of interest rate deregulation, it made an important stride in this area. However, the importance of seigniorage for many liberalizing economies[40] calls for a much more focused work on the issue. In particular, Kapur's analysis abstracts from any effects changes in the commercial bank interest rates would have on the cash holdings of economic agents, and it ignores the

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[40] For example, the average percentages of seigniorage to governments' revenues were 21.2, 28, and 24.8 for

effect of any interaction between the official and the informal interest rates. In Kapur's model, deposits dominate fiat currency in rates of return and since currency has no other function that would increase its effective rate of return, agents will not hold fiat currency in equilibrium, except to satisfy reserve requirements. Thus fiat money derives its value from the legal restrictions of reserve requirements. Although, Wallace (1980, 1983) argues that the demand for money can be properly understood only in the context of the various legal restrictions that inhibit financial intermediation, it is also true that an extreme version of this approach in which fiat currency is valueless in the absence of legal restrictions can produce results[41] very different from moderating this view which provides a more plausible economic environment.

As pointed out in the literature [see, for example, Espinosa-Vega (1995)] a key characteristic of any economy and especially of developing economies is that the non-bank public holds fiat money. In developing economies, the proportion of money holdings relative to other financial assets is likely to be high because of their peculiar characteristics. In particular, there are restrictions on currency substitution, institutional factors limit the domestic financial assets available to the private sector[42], and the technology of financial intermediation is underdeveloped [Adam, Ndulu and Sowa (1996)]. These features limit the substitution opportunities of private economic agents and imposes a high cost of portfolio adjustment on economic agents which the government can exploit to extract seigniorage.

Mexico, Ghana, and Uganda's respectively for the period 1971 to 1982 [Cukierman, Edwards and Tabellini (1992)].

41 For example, Mourmouras and Russell (1992) showed that Freeman's (1987) result that pure deposit taxation dominates reserve requirement as a tax is due to his (Freeman's) model in which fiat currency derives its value entirely from external restrictions (reserve requirements). Using a generalized version where currency has value without any legal restriction, Mourmouras and Russell demonstrated that Freeman's results do not necessarily go through.

This suggests that studies that ignore the cash holdings of the non-bank sector in analyzing the seigniorage consequences of changes in policies in developing countries can only be partial in nature and potentially lead to misleading results. If account is taken of the demand for money by the economic agents, then both positive and negative effects on seigniorage can be identified when controlled interest rates are adjusted upwards, and Kupur's (1992) result will be a special case where demand for money by economic agents is zero.

The other weakness of Kapur's analysis which will be addressed here is the effect of interactions between the controlled interest rates and the informal rates. If, as theory and evidence suggest, there is an interaction between official and informal interest rates, the seigniorage implications of changes in controlled interest rates would be affected. Most economists agree that an increase in the official rates would lead to a fall in the informal interest rates[43]. A negative relationship between the formal and informal rates will have a positive effect on the economy's demand for money. This is because with financial liberalization, formal deposits will not only become more attractive as a result of an increase in the formal interest rate, but also as a result of a decrease in the informal rates. The portfolio shift out of the informal sector will thus be greater than is suggested before, and hence the increase in the demand for money by the economy as a whole will be greater.

Against the traditional suggestion that an increase in formal interest rates will cause informal rates to fall, Liang-Yih (1988) argued both theoretically and empirically (evidence

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42 Typically cheques are not widely used, credit cards and convenience cards are non-existent or used in a limited fashion in most of these economies.

43 For example, Tsiang (1979) recommended gradual deregulation of the controlled interest rate which would (i) decrease "the gap between the interest rates of the banking system and the outside money market" and (ii) reduce "the scale of operations of the curb money market" (pp. 590).

from Taiwan) that this relationship is positive[44]. A positive relationship will adversely affect money demand and seigniorage when formal controlled rates are adjusted upwards. This is because an increase in formal rates would also increase the informal rates thus offsetting part of the relative increase in the attractiveness of deposits with the commercial banks. This will result in a lesser switch of deposits from the informal sector and a lower demand for fiat money. Also, the increase in informal rates consequent on an increase in controlled rates will induce a switch from cash holdings by households to informal deposits, which will reduce the base of seigniorage further.

Thus if account is taken of the effect of an increase in controlled interest rates on the non-bank cash holdings and of the interaction between the formal and the informal interest rates, it is doubtful whether the seigniorage effect will be unambiguously positive as asserted by Kapur (1992). This paper is aimed at exploring the effects on seigniorage of changes in controlled interest rates in a more plausible economic environment in order to throw more light on the seigniorage consequences of interest rates deregulation, an aspect of a financial liberalization program.

It must be mentioned that the focus of the paper is not to directly enter the debate on the thesis advanced by Mckinnon (1973) and Shaw (1973) and the subsequent critique by the neo-structuralist, but rather to analyze the effect of interest rate deregulation on seigniorage. To this end, the paper will abstract from any inflation, savings, and growth

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44 Liang (1988) demonstrates analytically that there is a positive relation between formal and informal interest rates. He argued that if interest rates are controlled, funds from the formal sector can be used to finance low return projects. Thus, with deregulation, part of the deposit switched from the informal sector would be allocated to inefficient projects which would reduce the funds allocated to the efficient projects. This would put an upwards pressure on informal interest rates as they are determined by the demand and supply of funds for the efficient projects. This argument is likely to hold for many developing countries where the government owns the major

effects of financial liberalization which are the main issues in the debate.

2. The Model

The model retains most of the features of Kapur's (1992) model. Modifications of the model are made, essentially, to allow fiat money to have some value in the absence of legal restrictions on financial intermediation. A variation of the two-period-lived overlapping-generations model introduced by Samuelson (1957) is used to model economic agents. Various authors have made use of a variant of this model [for example, Wallace (1980) and Espinosa-Vega (1995)]. However, unlike in the other overlapping generation models, the structure of missing links among economic agents is not used to inhibit the operations of loans and insurance markets in order to create a potential role for unbacked currency [for example, see Sargent (1987)]. Instead, it is used to provide some stationarity in the structure of commercial banks' assets and liabilities, and cash goods are used to provide a motivation for the demand for cash.

The financial sector is made up of a formal sector and an informal sector. Commercial banks form the formal sector financial institutions, and they hold bank reserves which enable them to meet immediate cash demand that are in excess of any concurrent cash inflows. As such, the holding of reserves allows banks to offer a liquidity facility to their depositors, in that ready withdrawals are permitted. This is the bank deposit risk referred to by Melitz and Bordes (1991). This risk translates into costs for the banks by imposing the need to hold some non-interest-bearing reserves and/or low-interest-bearing

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commercial banks and loan default rates are very high due to poor allocation.

assets. The informal sector enterprises, on the other hand, avoid this risk and the consequential cost by holding zero or minimal reserves. For this reason, informal deposits are not as liquid as the formal sector ones. In fact, Luckett (1984) argues that informal financial institutions are not in a position to provide the same degree of 'short-run liquidity' as commercial banks do unless they maintain at least comparable reserve-deposit ratios[45], in which case they would not be different from the commercial banks. However, the informal sector enterprises offer higher-yielding deposit instruments, where the higher yield stems from their holding zero or minimal reserves and unregulated interest rates.

It is assumed that each of the formal and the informal financial institutions offers a single type of deposit. A deposit with an informal institution pays a higher interest rate than the one with the formal sector, but it would not permit any "premature" withdrawal of deposits since it does not hold any reserve.

2.1 Individual Optimization Problem

The economy is assumed to consist of an infinite sequence of two-period-lived overlapping generations. Agents are assumed to maximize a utility function that is strictly quasi-concave, strictly increasing in its arguments and twice differentiable. For analytical simplicity, it is assumed that agents are endowed with a fixed amount of money (w) in each period of their lives and have no bequest motives. The utility function of each agent is assumed to be given by $U=U(C_s)$; where C_s denotes the consumption of good s [for s=1

[45] This is because informal sector enterprises are usually small and localized. Consequently, the degree of synchronization between withdrawals and deposits is likely to be less than that of the banking system in the course of normal economic activities. Also they are unlikely to be linked by the interbank market which permits banks that unexpectedly experience reserve 'deficits' to borrow from those that unexpectedly experience surpluses.

(cash good) and s=2 (credit good)] at period t (for t=1, 2).

The use of cash and credit goods in the literature is the result of modification of the basic cash in advance model by Lucas and Stokey (1983). Their model and its variants [for example Hartley (1988), Englund and Svensson (1988)] do not, however, explain why some purchases have to be financed with cash while others can be financed with trade credits or check account balances. They have appealed to a trading scenario in which a shopper buys on credit from stores at which he is known, but must use cash otherwise. Subsequently, efforts have been made to endogenize the mix of cash and credit used in exchange through the introduction of explicit transaction costs associated with processing trade credit [see Schreft (1992) and Gillman (1993)]. This essay abstracts from endogenizing the cash and credit mix, and appeals to external restrictions to make some goods purchasable with cash and some with credit as in the earlier models.

In this model the credit good is also purchasable by cash, but since consumers benefit from using credit[46], they will not use cash for the purchases of the credit good in equilibrium. In order to ensure an interior solution to agent j's problem, it is assumed that $U_{1t}/U_{2t} \to \infty$ as $C_{1t}/C_{2t} \to 0$ and $U_{1t}/U_{2t} \to 0$ as $C_{1t}/C_{2t} \to \infty$ (where U_{st} denotes the derivative of U with respect to the s-th good at period t); it is also assumed that the income effect is positive for the two goods. With these assumptions, it is straight forward to determine the first- and second- order conditions of the optimal solutions of any intertemporal utility function. However, any further characterization of the solution is rather problematic [for example, see Levhari, Mirmam and Zilcha (1980)]. For the purpose of the analysis here,

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46 Since costs associated with processing trade credits are assumed away, the consumers will gain if they buy

however, further characterization is essential and hence the commonly used simple-additively-separable utility function will be used. Specifically, it is assumed that

$$U = \sum_{s=1}^{2}\sum_{t=1}^{2} InC_{st} \quad [47] \qquad (1)$$

For the maximization problem to be non-trivial it is assumed that $p_t / p_{t+1} - 1 < r < i < h$, where r is the net real returns on deposits with the commercial banks, i is the net real returns on deposits with the informal institutions, h is the real rate of interest on commercial banks' loans, and p_t is the price level in period t. Following Kapur (1992), agents are assumed to encounter a random uninsurable expense mid-way in each period of their lives. Consequently, agents maximize their utilities at the beginning of each period of their lives by choosing cash and credit goods as well as making some provisions for the random expense that they would encounter mid-period of their lives. The random expenditure can best be defrayed by holding deposits with the commercial banks. This is because holding illiquid deposits with the informal sector imposes a higher cost of financing these unforeseen expenditures (the cost of loans is higher than the return on informal deposits)[48]. Also the holding of money against unexpected expenses is inferior to holding commercial banks' deposits. The reason is straight forward. If the random expense

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goods on credit and pay later with monies upon which they have earned interest.

[47] Kapur (1992), for example, used $U=\min[E(C_1),E(C_2),...E(C_T)]$, where $E(Ci)$ is the expected consumption of an agent in period i. This is a more tractable function, but it will result in an increase in the consumption of all goods in response to an increase in income. Thus if income increases due to an increase in the rate of interest, consumption of all goods will increase including that of the cash good, and hence there will be a positive relationship between interest rates and money demand. This problem is because the utility function does not capture the substitution effects of price changes.

[48] It is not unreasonable to assume that a half-period borrowing rate exceeds r and i (one period returns) when one considers cost of borrowing in these economies which include substantial transactions costs such as costs of negotiating a loan, preparing documents and some times the bribes that the bank officials take which are borne by the borrower [see Ahmed (1989)].

is less than the cash holding provided for it, the consumer will forgo the commercial bank interest income that would have been earned on the portion of the "surplus" cash that will be allocated to credit good consumption. Therefore, agents will hold all three types of assets in equilibrium. They will hold cash against the consumption of cash goods, commercial banks deposits against unexpected expenses, and deposits with the informal sector against the consumption of the credit good.

To introduce the model formally, let D_j denote agent j's deposit with a commercial bank, m_j be the agent's cash holding, $(w-D_j-m_j)$ be the amount placed with the informal sector, and X_j be the random expense that the agent encounters mid-way in the first period of life. X_j is assumed to be distributed independently across time and individuals with continuous probability density function $f(X_j)$, and $0 \leq X_j \leq \overline{X} < w$. That is, the random expense is not allowed to exceed some limit, $\overline{X}$, which is less than the wealth of the consumer. This is to rule out equilibria in which the optimal deposit with the commercial banks is greater than the consumer's wealth. Lastly, it is also assumed that interest is only paid on a full period's deposit, and hence the consumer receives no interest on deposit withdrawn to finance unexpected expense at the time it occurs.

Consumers have to pay for the random expense when it occurs in order to forestall unacceptable declines in utility that will otherwise occur. This means *ex post* the consumer maximizes utility by allocating 'net' wealth (endowment minus random expense) to the consumption of cash and credit goods. Consequently, the consumer's resource allocation between cash holdings and informal deposits can be determined as a function of 'net' wealth. Assuming, for simplicity, that the prices of the two goods are equal and normalizing

it to one, the first order conditions for the optimal allocation of resources between cash and credit goods can be obtained by maximizing equation (1) with respect to consumption and subject to the consumer's budget constraint (2);

$$C_{11} + \frac{C_{21}}{1+i} + \frac{C_{12}}{1+i} + \frac{C_{22}}{(1+i)^2} \leq \frac{(2+i)}{1+i}(w-D). \qquad (2)$$

Equation (2) also assumes that the consumer's provision for the random expense is the same for the two periods. The following optimal values of consumption are obtained:

$$C_{11} = \frac{2+i}{4(1+i)}(w-D), \qquad (3)$$

$$C_{12} = C_{21} = \frac{2+i}{4}(w-D), \qquad (4)$$

$$C_{22} = \frac{(1+i)(2+i)}{4}(w-D). \qquad (5)$$

Thus in the first period of the agent's life, the consumer will hold cash worth C_{11} and deposit the remainder of his wealth ($w-D-C_{11}$) with the informal sector. From equations (3), (4), and (5) the consumption of the various categories of goods depend on wealth, formal sector deposit, and the interest rate offered by the informal sector.

To determine the optimal provision for the random expense, D^*, one has to characterise the behaviour of the consumer when the random expense is realized. It is assumed that the consumer allocates the difference in the provision for the random expense and it's realized value by maximizing (1) subject to the relevant budget constraint. The relevant budget constraint depends on whether the realized random expense is greater or

less than the provision for it[49]. The maximum utility from this strategy[50] would not, in general, be equal to the maximum that would be obtained if account is taken of the utility before the random expense occurred. If the consumer's provision for the random expense is more than the actual value, the consumer will allocate the "excess" deposit among the four different consumption categories by following the initial maximization principles except that in this case the relevant opportunity cost changes. In particular, the consumption of the cash good in period one will increase by (X-D)/4, C_{12} and C_{21} will increase by (1+r)(D-X)/4, and C_{22} will increase by (1+r)(1+i)(D-X)/4[51]. However, it is only the increase in the consumption of the cash good in the first period that entails withdrawal of cash from the commercial banks. This is because transferring deposits from the commercial banks to the informal sector will involve the loss of interest that could have been earned in the formal sector. The consumer will therefore increase his cash holding by a quarter of the "excess"

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49 When the random expense is less than the provision for it, the budget constraint is $C_{11} + \frac{C_{12}}{1+r} + \frac{C_{21}}{1+r} + \frac{C_{22}}{(1+r)(1+i)} = D - X_1$, and when the random expense is greater than the provision for it, the budget constraint is $C_{11} + \frac{C_{12}(1+h)}{1+i} + \frac{C_{21}(1+h)}{1+i} + \frac{C_{22}(1+h)}{(1+i)^2} = D - X_1$ (where X_1 is the realized random expense in period 1).

50 This strategy is used to avoid some potential technical problems that might result if account is taken of the utility derive before the random event occurs. For example, it can result in an increase in the consumption of the credit good at the expense of the cash good when 'net' income is reduced by a higher-than-provided-for random expense. This can be illustrated with a one period problem. If realized random expense is greater than the provision for it then utility maximization will require that the cash good be decreased by $\frac{X_1 - D}{2} + \frac{(w-D)(h-i)}{4(1+h)}$ (part paid by cash) and the credit good by $\frac{X_1 - D}{2} - \frac{(w-D)(h-i)}{4(1+h)}$ (part paid by borrowing). This means that if w-D=100, X_1-D=2, h=12%, and i=5%, then cash good consumption will be reduced by 2.59 and credit good by -0.59. Thus the consumer will increase the consumption of the credit good by .59 at the expense of the cash good. This suggests that the initial allocation is not optimal in the sense that the consumer has to increase the consumption of the credit good whilst reducing the consumption of the cash good following a "decline" in income. If the initial allocation is optimal, then credit good must be inferior which is inconsistent with the assumption of normal goods. This apparent problem is because the opportunity cost of a unit of the cash good at the beginning of the period is the gross rate of return in the informal sector, but when trying to mobilize funds to pay an excess random expense the opportunity cost is the gross cost of loans.

51 The consumer increases the consumption of each category of the consumption goods by a quarter of the "excess" cash multiplied by their gross rates of return.

deposit.

On the other hand, if the random expense is greater than the provision for it, the consumer will have to reduce consumption. A dollar withdrawn from the consumption of C_{11} or C_{12} or C_{22} will cost the consumer $1/p_t$ or $(1+i)/p_t$ or $(1+i)^2/p_t$ units of goods, respectively (both cash and credit goods cost p_t per unit). Since agents cannot draw on their informal deposits, they have to borrow funds at an interest rate of h and they pay for these loans by using deposits on which interest i is earned. Thus the cost of C_{12} and C_{21} in terms of C_{11} is $(1+i)/(1+h)$, and the cost of C_{22} relative to C_{11} is $(1+i)^2/(1+h)$. Therefore, given the utility function, the consumer will reduce the consumption of C_{11}, C_{12}, C_{21}, and C_{22} in the ratio of $(1+h):(1+i):(1+i):(1+i)^2$. Thus the consumption of C_{11} will be reduced by $\frac{(1+h)(X-D)}{4+h+i^2+4i}$, C_{12} and C_{21} by $\frac{(1+i)(X-D)}{4+h+i^2+4i}$, and C_{22} by $\frac{(1+i)^2(X-D)}{4+h+i^2+4i}$. This implies that $(1+h)/(4+h+i^2+4i)$ of the excess is financed by reducing cash and $(3+i^2+4i)/(4+h+i^2+4i)$ is financed by borrowing. Since the return on informal sector deposits is less than the loan rate, it would be more expensive to reduce C_{12} and C_{21} than C_{11}.

An agent's expected 'net' wealth at the beginning of a period is described by equation (6);

$$E(w-D) = (1+r)D + (1+i)(w-D-m) + m - (1+r)\int_0^D Xf(X)dX - (1+r)D\int_D^{\bar{X}} f(X)dX - \frac{(3+i^2+4i)(1+h)}{4+i^2+h+4i}\int_D^{\bar{X}}(X-D)f(X)dX - \frac{1+h}{4+h+i^2+4i}\int_D^{\bar{X}}(X-D)f(X)dX \quad . \quad (6)$$

Since this is a one period problem and all agents are identical in terms of endowments, the time and agent specific subscripts are suppressed. The first three terms on the RHS of (6)

constitutes the wealth of an agent for a period if there were no random expense. The fourth term on the RHS is the expected value of deductions from this amount occasioned by the set of values of X in the interval (0, D) -such values being paid for by the consumer drawing on his commercial bank deposit-, while any alternative value of X between D and $\bar{X}$ will be paid for by withdrawing his bank deposit completely (the fifth term), and by borrowing at rate h as well as reducing his cash holdings (the last two terms respectively).

The first order condition for the maximization of equation (6) with respect to the consumer's choice of commercial bank deposit is

$$\int_{D^*}^{\bar{X}} f(X)dX = \frac{(i-r)(4+h+i^2+4i)}{h(3+i^2+4i)-r(4+i^2+h+4i)}. \qquad (7)$$

From equation (7), the solution of D^* will be positive if $\frac{i(4+i^2+4i)}{3+i^2+3i} < h$, and D^* will be equal to the maximum unforeseen expense if i=r. The intuition for the latter result is quite straight-forward. If there is no difference between the formal and the informal deposit interest rates, then D^* will be, at least, equal to the maximum unforeseen expense since deposits with the formal sector are more liquid and are not dominated by informal returns. The condition $\frac{i(4+i^2+4i)}{3+i^2+3i} < h$ [52], suggests that if the cost of borrowing is not high relative to the return on the informal deposits, then agents would not hold deposits with the formal sector. They would prefer to pay for the random expenses by using cash and borrowing than to forego the difference between the informal and the formal rates. Since the task here is to

52 This condition is stronger than the one in Kapur's (1992) model. In his case, h>i ensures that agents will hold deposits with the formal sector. This is because the cost of financing a random expense that is higher than its provision is lesser here since part of it is paid for by cash.

ensure that all three types of assets are held, it will be assumed that this condition holds. The condition also ensures that the denominator of (7) cannot be negative and that the second order conditions are satisfied provided that f(X) is positive for all $X \in (0, \overline{X})$. From equation (7), the optimal D does not depend on the level of income and consumption. It is influenced by the distribution of the random expense, deposit rates in the formal and the informal sectors, and the loan rate.

To find the response in commercial banks deposits to their rates of return, equation (7) is differentiated with respect to r holding h and i constant. This gives equation (8);

$$\frac{dD^*}{dr} = \frac{(4+h+i^2+4i)[h(3+i^2+3i) - i(4+i^2+4i)]}{f(D^*)[h(3+i^2+4i) - r(4+i^2+h+4i)]^2}. \qquad (8)$$

From (8), provided that it is worthwhile to hold some deposits with the commercial banks, these deposits will be an increasing function of their own rate of return as expected.

The solutions to the demand for money and deposits with the informal sector are simplified by the fact that D^* is independent of the levels of both the demand for money and informal deposits. As mentioned earlier, the consumer's demand for money will be given by the money necessary to finance C_{II}. Thus from equation (3)

$$M^d = \frac{2+i}{4(1+i)}(w - D^*). \qquad (9)$$

From (9), an increase in the controlled formal interest rate will decrease money demand since D^* is positively related to the controlled official rate, r. Although, the reduction in the demand for money is achieved through a negative 'net' income effect, this 'net' wealth effect is due to the higher demand for commercial banks deposits resulting from the change in relative interest rates. An increase in the informal rates, on the other hand, has a positive

'net' income effect and a negative substitution effect. The income effect is due to the fall in D^* when i increases. It is assumed that the substitution effect is stronger[53] so that cash good in the first period and the other categories of goods will be gross substitutes [see Varian (1978)]. This will ensure that the demand for money is negatively related to the informal deposit rate, which is consistent with theory. An increase in the endowment will increase the demand for money.

Deposits with the informal sector will be given by

$$IFD = \frac{2+3i}{4(1+i)}(W - D^*). \qquad (10)$$

An increase in informal interest rates will increase informal sector deposits both through the 'net' income and the substitution effects. An increase in commercial bank interest rate, on the other hand, will decrease informal deposits. Thus an upwards adjustment of the controlled commercial bank deposit rate will decrease the demand for money and deposits with the informal sector.

2.2 The Banking Sector

It is assumed for the purpose of stationarity in the banking system that there are two types of economic agents; types A and B. These two types of agents are identical except for differences in the time they give birth and the type of agent to whom they give birth. It is assumed that type A agents give birth to type B agents, and type Bs gives birth to type As. Further, type As give birth in the middle of the first period of their lives while type Bs give

birth in the middle of the second period of their lives. This will ensure that there are both type A and B agents of different generations at each point in time, except for the initial period. This initial period problem can be overcome by assuming that $-\infty < t < \infty$ (where t is time) or that the economy starts with a generation of type A and B agents.

An important feature of this formulation is that at every half period when one generation experiences its random shock, the other receives its endowment. Since type As give birth in the middle of the first period of their lives, the new borns receive their endowments whilst their "parents" experience their random shocks. At the end of As' first period, they receive their endowment while the Bs experience their shocks. In the middle of the second period of the As' lives (when they are experiencing their shocks), the Bs will be receiving their second endowments. When the Bs are one and half periods old, they will experience their random shocks while the As pass away. But this is the time that the Bs give birth to new type As who come into being with their endowments. In the middle of the new As' first period the Bs (their parents) pass away but then this is the time that new Bs are born. This formulation, which is a slight modification of Kapur's (1992) group demarcation style, ensures stationarity of the structure of the asset and liability position of the commercial banking system.

It is assumed that the size of a generation at time t, N(t), is large enough to allow the law of large numbers to apply, and that N(t)=N(t-1) for all t. Since the real growth rate in this economy is given by the population growth rate, this condition amounts to assuming a zero real growth rate. From the law of large numbers, the average amount of deposits

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[53] This condition will be satisfied if $\frac{(w - D^*)}{1+i} \phi (2+i)\frac{dD^*}{di}$.

withdrawn and loans incurred at each mid-period to meet the random expenses can be made arbitrarily close to their respective expected values $\int_0^{D^*} Xf(X)dX + D^* \int_{D^*}^{\bar{X}} f(X)dX$ and $\frac{3+i^2+4i}{4+h+i^2+4i} \int_{D^*}^{\bar{X}} (X - D^*)f(X)dX$. Thus for large numbers we can approximate these averages by the corresponding expected values[54]. At each half period therefore the average inflow of new deposits, D^*, will be equal to the average outflow, consisting of $[\int_0^{D^*} Xf(X)dX + D^* \int_{D^*}^{\bar{X}} f(X)dX]$ which is withdrawn by the generation experiencing their expenditure shock and $[D^* - \int_0^{D^*} Xf(X)dX - D^* \int_{D^*}^{\bar{X}} f(X)dX]$ by those who have completed their transaction period. In addition, the average amount of loans issued to finance expenditures beyond the provision for them by some agents, $\frac{3+i^2+4i}{4+h+i^2+4i} \int_{D^*}^{\bar{X}} (X - D^*)f(X)dX$, will be equal to the average repayments of such loans contracted a half period earlier by the generation that has just completed its transaction period. Agents pay for such loans by using part of their matured deposits with the informal sector and part of their income if they are a period old. Thus at each half period, after transactions have been completed, the average size of new deposits by the new borns or those one period old is D^*, and of those who have just experienced their random shock will be $[D^* - \int_0^{D^*} Xf(X)dX - D^* \int_{D^*}^{\bar{X}} f(X)dX]$, so that the

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54 One way to deal with this is to assume that the number of agents is infinite [see, for example, Williamson (1988)].

there are both type A and B agents of different generations at each point in time, except for the initial period. This initial period problem can be overcome by assuming that $-\infty < t < \infty$ (where t is time) or that the economy starts with a generation of type A and B agents.

An important feature of this formulation is that at every half period when one generation experiences its random shock, the other receives its endowment. Since type As give birth in the middle of the first period of their lives, the new borns receive their endowments whilst their "parents" experience their random shocks. At the end of As' first period, they receive their endowment while the Bs experience their shocks. In the middle of the second period of the As' lives (when they are experiencing their shocks), the Bs will be receiving their second endowments. When the Bs are one and half periods old, they will experience their random shocks while the As pass away. But this is the time that the Bs give birth to new type As who come into being with their endowments. In the middle of the new As' first period the Bs (their parents) pass away but then this is the time that new Bs are born. This formulation, which is a slight modification of Kapur's (1992) group demarcation style, ensures stationarity of the structure of the asset and liability position of the commercial banking system.

It is assumed that the size of a generation at time t, N(t), is large enough to allow the law of large numbers to apply, and that N(t)=N(t-1) for all t. Since the real growth rate in this economy is given by the population growth rate, this condition amounts to assuming a zero real growth rate. From the law of large numbers, the average amount of

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[53] This condition will be satisfied if $\frac{(w-D^*)}{1+i} \phi\ (2+i)\frac{dD^*}{di}$.

deposits withdrawn and loans incurred at each mid-period to meet the random expenses can be made arbitrarily close to their respective expected values $\int_0^{D^*} Xf(X)dX + D^* \int_{D^*}^{\bar{X}} f(X)dX$ and $\frac{3+i^2+4i}{4+h+i^2+4i} \int_{D^*}^{\bar{X}} (X-D^*)f(X)dX$. Thus for large numbers we can approximate these averages by the corresponding expected values[54]. At each half period therefore the average inflow of new deposits, D^*, will be equal to the average outflow, consisting of $[\int_0^{D^*} Xf(X)dX + D^* \int_{D^*}^{\bar{X}} f(X)dX]$ which is withdrawn by the generation experiencing their expenditure shock and $[D^* - \int_0^{D^*} Xf(X)dX - D^* \int_{D^*}^{\bar{X}} f(X)dX]$ by those who have completed their transaction period. In addition, the average amount of loans issued to finance expenditures beyond the provision for them by some agents, $\frac{3+i^2+4i}{4+h+i^2+4i} \int_{D^*}^{\bar{X}} (X-D^*)f(X)dX$, will be equal to the average repayments of such loans contracted a half period earlier by the generation that has just completed its transaction period. Agents pay for such loans by using part of their matured deposits with the informal sector and part of their income if they are a period old. Thus at each half period, after transactions have been completed, the average size of new deposits by the new borns or those one period old is D^*, and of those who have just experienced their random shock will be $[D^* - \int_0^{D^*} Xf(X)dX - D^* \int_{D^*}^{\bar{X}} f(X)dX]$, so that the average deposit across both

54 One way to deal with this is to assume that the number of agents is infinite [see, for example, Williamson (1988)].

generations denoted by $\bar{D}$ will be

$$\bar{D} = \frac{1}{2}[2D^{*} - \int_{0}^{D^{*}} Xf(X)dX - D^{*}\int_{D^{*}}^{\bar{X}} f(X)dX].$$

(11)

Also, the average volume of outstanding loans issued for the purpose of financing the random expenditures of the half-period old agents is given by equation (12);

$$\bar{R} = \frac{3 + i^{2} + 4i}{2(4 + h + i^{2} + 4i)} \int_{D^{*}}^{\bar{X}} (X - D^{*})f(X)dX.$$

(12)

Following Kapur (1992), it will be assumed that the values of r, i, and h are such that D^{*} is sufficiently close to $\bar{X}$ and that $\bar{D}$ exceeds $\bar{R}$ by more than the average size of bank reserves held. This assumption allows commercial banks to have some resources for lending to the productive sector after meeting their reserve requirements and the loan requirements of their depositors.

3. Effects of Interest Rate Deregulation

Having characterized the economic environment which gives rise to the demand for money, formal deposits, and informal deposits, the remainder of the essay is devoted to analyzing the effect of interest rate deregulation or upwards adjustment of the formal sector interest rates on seigniorage. Since the model abstracts from growth and inflationary effects of interest rate deregulation, the seigniorage effect is captured by the changes in the demand for money in the economy.

From equation (11) $\frac{d\bar{D}}{dr}=\left(1-\frac{1}{2}\int_{D^*}^{\bar{X}} f(X)dX\right)\frac{dD^*}{dr}$, and since dD^*/dr is positive [from equation (8)], it follows that the average commercial bank deposit ($\bar{D}$) is an increasing function of its rate of return (r). Also from equation (10), $\frac{d\bar{R}}{dr}=\frac{-(3+i^2+4i)}{2(4+h+i^2+4i)}\int_{D^*}^{\bar{X}} f(X)dX\frac{dD^*}{dr}$, thus the per capita loans for unforeseen contingencies, $\bar{R}$, is a decreasing function of r. Commercial banks lending to firms for productive purposes can be described by (13);

$$\bar{L}=(1-k)\bar{D}-\bar{R}, \qquad (13)$$

where k is the reserve requirement of the commercial banks and is assumed for simplicity to be fixed. Financial repression can take the form of pegging the interest rate on loans below its equilibrium market rate or pegging both the interest on loans and the deposit rate, r, below their equilibrium values [see Fry (1988)]. Firms which cannot obtain "adequate" bank financing turn to the informal sector for their remaining credit requirements. Interest rate deregulation will involve increasing formal sector loan rate which would allow the banks to increase r, or in the case where both the loan rate and r are pegged, it would involve increasing both rates. The resulting change in per capita deposits and lendings of the commercial banks are given by $d\bar{D}/dr$ and $d\bar{L}/dr$, respectively. Also the demand for money changes by dM/dr and the change in average informal deposit holdings is given by dIFD/dr.

In the above, the reduction in the demand for money and informal deposits is greater than the increase in average deposits with the commercial banks. This is because

as more funds are deposited with the commercial banks, more of it is used to pay for the random expense and hence the average deposit does not increase by as much. As asset-holders have more commercial bank deposits to draw upon, they need to borrow less, and the resulting funds are "freed" for lending to productive enterprises. The "freed" funds can be derived from equation (12) by differentiating it with respect to D^*. Thus, as pointed out by Kapur (1992) the adding up constraint need not bind when both the asset and liability sides of the balance sheet are considered.

In the model examined here, change in total productive lending as a result of an increase in the controlled interest rate is given by

$$(1-k)\frac{d\bar{D}}{dr}-\frac{d\bar{R}}{dr}+\frac{dIFD}{dr}=(1-k)\left[1-\frac{(i-r)(4+h+i^2+4i)}{2[(4+i^2+4i)(h-r)-(1+r)h]}\right]\frac{dD^*}{dr}$$
$$+\left[\frac{(3+i^2+4i)(i-r)}{2[(4+i^2+4i)(h-r)-(1+r)h]}\right]\frac{dD^*}{dr}-\left[\frac{2+3i}{4(1+i)}\right]\frac{dD^*}{dr} \qquad (14)$$

The LHS of (14) is made up of changes in commercial banks lending (first two terms) and changes in informal sector lending (last term). Unlike Kapur's model, the RHS of equation (14) *cannot* be shown to be negative (that is total productive lending cannot be shown to be decreasing in controlled interest rate). In fact, equation (14) can be negative, zero or positive depending on the size of the required reserve. In particular, if z is given by equation (15);

$$z=\frac{2(2+i)[(2+i)^2(h-r)-(1+r)h]-(i-r)[2+3i-h(4+i)]}{4(1+i)[(2+i)^2(2h-r-i)-h(2+i+r)]}, \qquad (15)$$

then (14) will be positive if $k < z$, equal to zero if $k = z$, and less than zero if $k > z$. Thus k is very important in determining whether seigniorage will increase or fall when

controlled interest rates are adjusted upwards. Intuitively, an increase in controlled interest rate will induce a portfolio shift from cash holdings to deposit with the commercial banks which will increase productive lending by $(1-k)\Delta M^d$. Also, the reduction in borrowing by depositors to meet their unexpected expenses provides an additional source through which productive lending increases. These have to be compared with the disintermediation result of a shift in deposits from the informal to the formal sector to assess the total effect on productive lending. Informal deposits provide full intermediation (zero reserve requirement) and hence lending will fall by $k\Delta IFD$. Thus, whereas the shift of deposit from the informal to the formal sector increases the base of seigniorage, the shift from cash holdings to commercial bank deposit reduces it. The overall effect depends on which of these two effects is stronger.

For $k < z$, total productive lending will actually increase, and contrary to Kapur's results seigniorage will decrease if controlled interest rates are adjusted upwards. This means that the effect of the switch from cash holding dominates that of the switch from informal deposit. Kapur's result is, therefore, due to the fact that economic agents do not hold cash in his model and hence an increase in controlled rates only draws money from the informal sector where financial intermediation is greater. Although, borrowing to finance unexpected contingencies decrease (this frees resources for productive lending) when deposits with commercial banks increase, this is not enough to counter the decrease in lending occasioned by the switch of deposits from the informal to the formal sector. Consequently, total productive lending will fall in Kapur's model unless reserve

the requirement is zero[55], in which case productive lending will not change. Thus the fact that fiat currency has value even with zero reserves opens up the possibility that seigniorage can fall when interest rates are deregulated.

In the model constructed here, the effect of interest rate deregulation on seigniorage depends on the level of reserve requirement, and the sensitivities of the demand for fiat money and informal deposits to the controlled rate. Given that the cost of borrowing is high enough for agents to hold deposits with the commercial banks, then from equation (15) $0 < z < 1$[56]. This makes it difficult to establish plausible conditions under which an increase in controlled rates will unambiguously lead to a fall or an increase in seigniorage. Conditions that can lead to unambiguous effects on seigniorage cannot be supported in equilibriums where all three assets are held. For example, $k = 0$ ensures that $k < z$, and hence rules out an increase in seigniorage if controlled interest rates are increased [Kapur's (1992) result]. But $k = 0$ also implies that there is no difference between the liquidity of commercial banks and that of the informal financial institutions, and hence agents will not hold commercial banks deposits in equilibrium as their rate of return is dominated by the informal rate. On the other hand if $k = 1$, which ensures that $k > z$ and hence delivers Kapur's result, the banks cannot undertake any financial intermediation activity and they cannot pay any positive nominal interest. This makes commercial bank deposits and cash perfect substitutes, and there is no reason for agents to hold both assets. Thus for a plausible economic environment, the value of k has

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55 A zero reserve holdings by the commercial banks will result in no demand for their deposits since their rates of return is dominated by that of the informal sector and they cannot provide "short-term" liquidity just like the informal institutions.

56 If the condition for holding the three assets is satisfied then the numerator of (15) will be positive and

to be between zero and one, making it important in determining the effect on seigniorage of interest rate deregulation. Thus for $0 < k < 1$, there is no *a priori* reason to expect that seigniorage will increase when controlled interest rates are deregulated. The issue is an empirical one that involves determining whether k is greater than z or not in a particular repressed economy.

To get some idea about the values of z, it's maximum and minimum values were calculated for real rates of return between -10% and 20% subject to the constraints that economic agents hold all three types of assets and $r < i < h$. The values were located by imposing a one percentage point grid on the range of the real returns and searching on the grid points. The maximum value of z located is 52.1% and the minimum value is 36.7%. Within the specified range of values of real returns, the value of z increases when r and/or h increases, and/or i decreases. Thus the minimum value was obtained when r=-10%, i=15% and h is 20%, and the maximum value was obtained when r = -10%, i=-9%, and h=20%[57]. In economies where the real interest rates are in the regions considered, interest rate deregulation will reduce seigniorage if their cash reserve requirement is less than 36.7%, and will increase it if the cash reserve requirement is greater than 52.1%. If the cash reserve ratio is between these two values, then the economy's z will have to be calculated and compared with k in order to access the effect of interest rate deregulation on seigniorage. The maximum and the minimum values do not change much when the

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less than the denominator.

[57] The maximum value is perhaps exaggerated for financially repressed economies because in such economies the freely determined interest on informal deposits is likely to be more than one percentage point above the controlled interest rates offered by the commercial banks.

domain of real returns is changed[58].

The behaviour of z with respect to gradual changes in controlled interest rates, r, deserves some attention since gradual adjustment of controlled interest rates is a characteristic of many financial liberalization programs. This behaviour will give us some idea about the dynamics of the effect on seigniorage of interest rate deregulation during a liberalization phase. From equation (15),

$$\frac{dz}{dr} = \frac{(1+i)[(2+i)^2(h-i)-h(1+i)][2(1+h)(1+i)-i(3+i^2+4i)]}{2(1+i)^2[(4+i^2+4i)(2h-i-r)-h(2+i+r)]^2}. \qquad (16)$$

It can easily be shown that equation (16) is positive provided agents hold all three types of assets[59]. This means that an increase in controlled interest rate will increase z, making it more likely that z>k (for a fixed k), which makes seigniorage gains less likely. This gives three possible dynamic effects on seigniorage during gradual upwards adjustment of the controlled interest rate. If k<z at the outset of interest rate deregulation, then this will hold during the liberalization period and seigniorage will fall throughout. If k>z at the inception of deregulation, then two possible scenarios will result. One is that this relation holds throughout the adjustment period, which implies that seigniorage will increase throughout. The second scenario is that z will become greater than k at some stage. This suggests that it is possible for seigniorage to increase at the initial stages of interest rate deregulation and to fall at some later stage, which implies an inverted U-

58 For example, when the domain of real returns is restricted to [0%, 10%], the maximum value decreased to 48.8% [obtained when r=0%, i=1%, and h=10%] and the minimum value increased to 38.4% [obtained when r=0, i=7%, and h=10%], respectively.

59 Thus the relation between z and r is not only true for the range of real interest rates considered [-10%, 20%], it holds for all possible values of real interest rate provided agents hold all three types of assets.

shape curve for seigniorage as a function of commercial banks deposit rate, r[60]. In this case, the maximum seigniorage is attained when k=z. Thus, the effect of interest rate deregulation on seigniorage is ambiguous and so are the dynamics of seigniorage during gradual liberalization of controlled interest rates.

4. Interaction between formal and informal interest rates

This section deals with the implications for seigniorage if there is an interaction between formal and informal interest rates. It is well documented both theoretically and empirically that there is a relationship between formal and informal interest rates [see Fry (1988), and Ming-Yih Liang (1988)]. However, as mentioned earlier, there is no clear consensus on the direction of this relation. It will therefore be appropriate to explore the seigniorage implications of both positive and negative relationships. If an increase in formal interest rates reduces informal rates then this will make the formal rates more attractive than is hitherto suggested. Thus the decrease in the informal deposits will be larger and the reduction in productive lending through this channel will be more than before. To formally see the effect of an interaction between the informal and the formal rates on seigniorage equation (7) is differentiated again allowing for this interaction. This gives

60 Various studies of the effects of financial liberalization on seigniorage suggest a uni-directional relationship between liberalization and seigniorage. For example, Kapur (1992) suggests an increase in seigniorage. Sowa, Ndulu and Adam (1996) suggest that seigniorage will fall with liberalization due to increased degree of substitutability between domestic currency and other assets.

$$\frac{dD^*}{dr} = (8) - \frac{\partial i}{\partial r}\left[(8) + \frac{(i-r)[(4+h+i^2+4i)^2 - h(h+1)(2i+4)]}{f(D^*)[h(3+i^2+4i) - r(4+h+i^2+4i)]^2}\right]. \qquad (17)$$

The term multiplying $\partial i/\partial r$ is unambiguously positive, and hence if r and i are negatively related then equation (17) will be greater than equation (8). As discussed above, this shows that the increase in commercial banks deposits in this case will be more than the situation where $\partial i/\partial r=0$. This will tend to have a favourable effect on seigniorage as more deposits are switched from the informal to the formal sector.

Liang (1988), on the other hand, suggested a positive relationship. In fact, he asserts that

> "...our analysis reveals, that while it is true that the size of the black market may diminish as the official rate rises, the gap between the official rate and the black market rate does not necessarily narrow in the process" pp. 549.

This suggests that the black market rate can even increase more than the official rate. There are, therefore, two possibilities in this case. One is where the return on informal deposit increases slower and the gap narrows, and the other is where the informal rate increases faster and the gap widens. In the first scenario deposits with the informal sector will still fall but by less than the fall if there were no relationship. In the second scenario deposits with the informal sector will actually increase. In this scenario therefore there is no way productive lending can fall and hence Kapur's (1992) result is entirely ruled out.

A positive relationship between the formal and the informal rates implies that equation (17) is less than (8). Thus the increase in commercial banks' deposits resulting from an increase in their rates of return will be less than the zero relationship case. If the gap between the formal and the informal rates widens when official rates are adjusted

upwards, then $\partial i/\partial r>1$. In this case, equation (17) will be negative. This is because the term multiplying $\partial i/\partial r$ is greater than equation (8). Consequently, as mentioned earlier commercial bank deposits will be negatively related to their own rate of return, and hence the economy's demand for money will fall when controlled interest rates are adjusted upwards leading to a reduction in the base of seigniorage. This suggests that seigniorage will fall. In fact, this results can go through even with $\partial i/\partial r<1$ (that is, it is not necessary for the gap to widen). In particular, if equation (18) holds, seigniorage will fall when controlled interest rates are increased no matter the level of reserve requirement.

$$1>\partial i/\partial r>\frac{(4+i^2+h+4i)[h(3+i^2+4i)-i(4+i^2+h+4i)]}{(4+i^2+h+4i)[h(3+i^2+4i)-i(4+i^2+h+4i)]+(i-r)[(4+i^2+h+4i)^2-h(1+i)(2i+4)]} \tag{18}$$

It can be argued that the model is flexible enough to accommodate the argument that the relationship between the formal and the informal rates can be positive or negative. If cash reserve requirement is large enough to ensure that the supply of productive lending falls with an increase in controlled rates, then for a given demand for credits the informal rate will increase. On the other hand, if the reserve requirement is small then the total supply of productive lending will increase, and for a given demand for these funds, the informal rate will decrease. In Kapur's model where fiat money has no value without external restriction only positive relationship can be supported.

5. Conclusion

The analysis reveals that the effect on seigniorage of interest rate deregulation is not straight forward in economies with active informal financial markets. The effect on seigniorage can be radically different depending on whether fiat money has value in the absence of restrictions on financial intermediation or not. This underpins the general results obtained here compared to the one obtained by Kapur (1992).

The results suggest that the change in seigniorage in response to an increase in controlled interest rates depends on the level of reserve requirements and the responses of the demand for money and informal deposit to the change in the controlled rate. Specifically, seigniorage is more likely to increase in response to deregulation of interest rates the higher the level of cash reserve requirement, the higher the interest sensitivity of informal deposit demand, and the lower the interest elasticity of money demand. Intuitively, an increase in controlled interest rates will increase deposit with commercial banks, and the effect on seigniorage depends in part on the assets that are used to finance the increase in commercial banks deposits. If cash demand is very sensitive to controlled interest rate then a large proportion of the increase in deposit with the formal sector will come from cash holding, and this will reduce the base of seigniorage since only part of the shifted money will be held by banks as reserve. On the other hand, if informal sector deposits are very sensitive to controlled rates then a large fraction of the increase in commercial bank deposit will come from the informal sector and this will increase the base of seigniorage since the informal institutions do not hold reserves. Not only are the particular assets involved in the repackaging of the consumers' portfolios important, the

amount required to be held as cash reserve is also important in determining the seigniorage effect. A high level of reserve requirement means that a high proportion of any increase in commercial banks deposits will be held as reserves, which favours an increase in seigniorage.

Another interesting result is the dynamics of seigniorage when controlled interest rates are gradually deregulated. The results indicate that seigniorage can increase or decrease or take the form of an inverted U in controlled interest rate during a liberalization phase. The inverted U-shape contrasts sharply with the uni-directional paths suggested by the other studies of financial deregulation, and it suggests that the initial pains of liberalization can be reduced by the positive effect on seigniorage of interest rate deregulation, but the pains can be exacerbated at later stages by dwindling seigniorage.

It was also shown that in financially repressed economies with thriving informal markets, the relationship between formal and the informal sector interest rates is very important for seigniorage. A negative relationship will have positive effects on seigniorage whilst a negative relationship will have an adverse effect on seigniorage.

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ESSAY 3

THE USE OF SEIGNIORAGE AND THE WELFARE COST OF INFLATION

1. Introduction

As pointed out by Dotsey and Ireland (1996), among others, the welfare cost of inflation has been a topic of active research because of its potential in influencing monetary policy. Serious work on the issue began with Bailey (1956), who modelled money as a consumption good and considered inflation as a tax on real balances. This technique measures the welfare cost of inflation as the appropriate area under the money demand curve. Fischer (1981) and Lucas (1981) updated Bailey's estimates and calculated the welfare cost of a 10% inflation relative to an optimal monetary policy rule to be 0.8% and 0.5%[61] of income, respectively, in steady state. Reactions to these estimates have been mixed and over the years a number of papers have appeared questioning the accuracy of these estimates. Departures from these estimates have not gone consistently in any one direction. Whilst some economists argue that these figures underestimate the 'true' cost, others argue that they exaggerate it. The disagreements basically hinge on different motives for modelling the demand for money and other 'subsidiary' costs of inflation that were ignored in the calculations.

Imrohoroglu (1992), Gillman (1993) and Dotsey and Ireland (1996) are some of the authors who argue that these figures are relatively small and that they provide little support for price stability as an essential goal for monetary policy. Imrohoroglu (1992),

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61 The difference in Lucas and Fischer's estimates can be traced to the different definitions of money used (Lucas used M1 and Fischer used high-powered money) and the different values they used for the interest

using a model in which optimizing agents hold money to insure against unemployment, reports that the welfare triangle calculations of the cost of inflation underestimate it by a factor of three for a 5% inflation and four for a 10% inflation. Gillman (1993), making provision for the Bailey-type real resource cost of avoiding inflation by introducing costly credits, puts the figure of the welfare cost of 10% inflation at 2.19% of output. Dotsey and Ireland (1996) argue that the partial equilibrium approaches used in assessing the welfare cost of inflation underestimate it by ignoring other distortions that inflation causes. Using a model in which inflation causes inefficient allocation of productive labour resources, they estimate the welfare cost of 10% inflation to be 1.7% of output under an M1 specification (the corresponding estimate under a currency specification is 0.92% of output). In contrast, Gomme (1993) argues that the cost of inflation is lower than suggested by Fischer (1981) and Lucas (1981). He calculates the welfare cost of an 8.5% inflation (10% money growth rate) to be 0.0273% of output. The lower welfare cost is due to the introduction of human capital, a new choice variable which enables the households to avoid some of the cost of inflation[62].

In the literature, differences in the cost of inflation could be attributed to different modelling of the demand for money, different distortions of marginal decisions, and different losses of real resources caused by inflation. This paper contributes to this literature by exploring how different uses of the proceeds from inflation (inflation tax)[63]

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elasticity of money demand (Lucas chose 3 and Fisher used 5).

62 Essentially, within Gomme's framework, the steady state capital stock is unaffected by the rate of money growth. This means that a reduction in the real growth rate (due to an increase in inflation) allows reallocation of output from capital to consumption which mitigates the welfare cost of inflation.

63 In the models of welfare cost of inflation, inflation has been generated by increases in the money supply and this constitutes the revenue from inflation. In the zero growth versions of these models, the growth rate of

could impact on the welfare cost of inflation. As pointed out by Fischer (1981), inflation is basically an endogenous variable to the economy. He, therefore, argues that attention should be focused on the costs and benefits of alternative policies rather than on the cost of inflation *per se*. In his assessment, as in Lucas (1981), he measures the revenue from the inflation tax, but did not explore the effect of the uses of this revenue.

For any given inefficient allocation of resources and real losses caused by inflation, different benefits from the revenue of monetary expansion, which causes the inflation[64], can result in different welfare costs of inflation. The benefits derived from seigniorage depend on what the government or the monetary authorities use the seigniorage for. Smith (1994), for example, points out that there is a difference in the economic performance between the American colonies that printed money and lent it to private agents and the ones that printed money for deficit financing. This indicates that the use of seigniorage is not trivial and different ways of modelling it can potentially influence the cost of inflation[65]. Also, in the public finance literature, different types of government spending (for example, current versus capital expenditure) have yielded different results for any given government revenue [see, for example, Baxter and King (1993)].

It is difficult in practice to pinpoint the expenditures that absorb monies printed

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money is equal to the rate of inflation (assuming velocity is constant). In the growth models, the growth rate of money supply is approximately equal to the growth rate of output plus the rate of inflation. In these models, therefore, it is possible for zero seigniorage to be obtained and inflation caused by negative growth. This type of inflation is, however, not sustainable in the long-run as it implies driving output to zero.

64 It is perhaps this strong relationship between money and inflation which led to the use of the "money-cum-inflation" term in Gomme (1993).

65 It is implicitly assumed here that the distortions caused by inflation are not sensitive to the use of seigniorage. If this assumption does not hold, the distortions caused by the different uses of seigniorage will be additional source of difference.

by the central bank. If this were possible, the benefits derived from seigniorage can be properly accounted for and a better assessment of the cost of inflation can be made. Since one cannot easily identify the uses of seigniorage in practice, it is not surprising that seigniorage has been modelled as a lump sum transfer payment to households in most models[66] of the welfare cost of inflation. This modelling technique has not been rationalized in the literature apart from, of course, simplifying the analytical work and relieving researchers of the trouble of "tracking" seigniorage revenue. There are numerous categories of government expenditure in any government budget and there is no reason to restrict the use of seigniorage to transfer payments[67] in the analysis of the welfare cost of inflation. Thus, it will be equally plausible to model seigniorage to be used to finance other expenditure items in the government budget. Since it is impossible to examine every government expenditure item in this paper, I will focus on the current expenditure (transfer payments) and capital expenditure dichotomy and investigate the effects of modelling seigniorage to finance these types of public expenditures.

There are basically four broad ways of modelling public expenditure. Public spending can be modelled as transfer payments to economic agents or as an argument in the production function or as an argument in the utility function or as a pure waste. Since seigniorage in most models of the welfare cost of inflation is transferred to households, I will ignore the discussion of transfer payments here and concentrate on the remaining three. Examples of public spending which affect production can be found in Barro

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66 This is especially true in the Cash-In-Advance models [examples are Cooley and Hansen (1989), Gomme (1993), Dotsey and Ireland (1996)].

67 This, in particular, limits fiscal policy to expenditure on transfer payments (recurrent spending) and does not permit any form of capital expenditure.

(1990), Garcia-Milla and McGuire (1992), and Finn (1993), among others. In modelling government spending as an argument in the production function, Barro (1990) contends that it is this productive role of public spending which potentially creates the positive relationship between government spending and growth[68]. As in Aschauer (1989), Barro argues that government infrastructural services are important complementary inputs to private sector inputs and hence must be treated as a separate distinct input from private input in the production function. Another reason for treating government input separately is that private activity would not readily replace public activity if user charges are difficult to implement, as in the case of nonexcludable services such as national defense and the maintenance of law and order. This, conceptually, means that the government buys a flow of output (highways, sewers, etc.) from the private sector and makes it available to economic agents. It is the availability of this infrastructure to the households that matters for private sector production. Finn (1993), however, argues that government-owned-privately-operated capital (for example, research and development facilities, shipyards, etc.) and government enterprise capital (for example, gas and electricity, post office, etc.) can enter the production function through the same channel as private capital. To capture this effect, the standard production function is maintained but capital is made up of both private and government capital. Another way that government spending can influence production can be found in Christiano (1991) and Andofatto and Gomme (1996). In this case, the government transfers seigniorage to financial intermediaries that

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68 Aschauser (1989), for example, estimated the output elasticity of government capital to be 0.39 for the period 1945-1985 for the US economy. This figure is higher than the estimate of the output elasticity of private capital [0.3] calculated by Lucas (1990). Finn (1993) criticized Aushauser's total government method and suggested that only government-owned privately operated capital, government enterprise capital, and

lend it to firms to finance production.

Barro (1990) also captured the use of government revenue by assuming that it can directly enter the utility function [also see Baxter and King (1993)]. Although Barro did not consider seigniorage, placing money directly in the utility function is not new in monetary economics. This technique, in fact, was used by Lucas (1981) and Fischer (1981) to model the demand for money in their analysis of the welfare cost of inflation. This method of modelling money demand, of course, is *ad hoc* and has been criticized by many economists. However, other methods developed to introduce money into economic models, such as the Cash-In-Advance (CIA) model, are no less *ad hoc*[69] although they have been "rationalized" in much more sophisticated ways. Under some transactions technology specifications, however, the Cash-In-Advance model can be formally equivalent to the money-in-utility function specification[70].

The fourth way of modelling government expenditure in the literature is to treat it as a complete waste [for example as in Braun (1994)]. Using this technique, the government's spending, whether financed by taxes or by seigniorage, does not affect production, utility or the consumers' budgets constraints.

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government highways capital directly contribute to private production.

69 These models are all ad hoc, because the competitive equilibrium without cash is often Pareto efficient as is the case of the money-in-utility-function models, when cash does not enter the utility function. This suggests that cash actually makes agents worse off which conflicts with the ideas involved in the so called Hahn problem that the mere presence of money reflects inherent market failure, frictions, absent of markets and so on in the real world.

70 An example of this can be found in Dotsey and Ireland (1996). In their model, the transaction technology allows agents to buy goods either in cash (which involves inflation cost via CIA) or by the use of credits (which involves real resource cost -time). Writing the labour supply to intermediaries as a function of the real money supply and substituting it into the time constraint and the utility function yields the money in utility specification. They, however, contend that the households' trading opportunities they described provides a precise argument of how and why money enters into the utility function.

Putting money into different uses by the government would likely affect the economy in different ways and could potentially influence the welfare cost of inflation. In the Cash-In-Advance models, inflation reduces the real returns to labour and this reduces labour supply which results in loss of output and welfare. It can also induce agents to use other costly ways to execute transactions [for example, using costly credits in Dotsey and Ireland (1996)]. The benefit from the revenue of the money which causes the inflation, however, depends on how the use of seigniorage is modelled. Where seigniorage is modelled as transfer payments to households, the benefit is captured by a relaxation of the household's budget constraint. The effect of this on household welfare depends on how the households' maximum value function responds to changes in real balances. If seigniorage is not modelled as transfer payments, but as waste [as in Braun (1994)], real balances would be lower and this would lower utility levels provided real balances are not superneutral[71]. If, instead, the revenue is used to provide infrastructure (such as highways) or invested in government-owned-privately-operated capital or in government enterprise capital, the economy would be affected in a different way. In particular, it would increase productivity and output which would improve welfare, but it would also crowd out private capital and consumption due to the resource constraint, which would adversely affect welfare. This suggests that benefits from seigniorage accrue in different ways depending on the use of seigniorage, and this would potentially affect the welfare cost of inflation. This is the main theme of this paper. That is, to investigate whether the welfare cost of inflation is sensitive to the use of seigniorage. In

particular, I will compare the welfare cost of inflation when the government transfers seigniorage to households and when it uses it for production enhancing government capital spending.

2. The Model

2.1 The Economic Environment

The model used here retains all the essential features of the model used by Gomme (1993), except that the production function has been modified. Specifically, the production of human capital is not considered here as a labour augmenting argument, rather public capital is included as one of the arguments of the production function. The economy is assumed to be inhabited by infinitely-lived economic agents with preferences defined over consumption and leisure. It is also assumed that households are identical, and that the representative household's optimization decision involves the maximization of the expected value of a discounted stream of utilities given by;

$$\max_{C_t, l_t} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, l_t), \qquad (1)$$

where C_t and l_t are consumption and leisure respectively at time t and b is the rate of time discount. E_0 is the expectation conditional on information at time t=0. The utility function is assumed to be well-behaved, being concave and increasing in both consumption and leisure.

The representative household enters any period t with physical capital, k_t, and

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71 See Wang and Yip (1993) for discussion of the real effects of money supply and the welfare cost of inflation.

nominal cash balance, m_t. The government's only role in the economy is to provide households with non-interest-bearing fiat money and/or to finance capital projects. In any period t, the government makes a lump-sum transfer of V_t to each economic agent as well as finances the purchases of public capital. Thus, the gross growth rate of money supply at any period t, m_t, can be described by;

$$\mu_t = \frac{V_t + P_t I_{gt}}{M_t} + 1, \tag{2}$$

where $P_t I_{gt}$ is the per capita nominal public investment (that is I_{gt} is real government per capita investment measured in consumption units), and M_t is the per capita money balance. The resulting law of motion for money supply is

$$M_{t+1} = \mu_t M_t . \tag{3}$$

It is assumed that the government is credible in its announcement of money growth rates at the beginning of each period, and hence the model abstracts from any unanticipated money growth or unexpected inflation effect.

As in Lucas and Stokey (1983), the household is assumed to consist of two members, a worker and a shopper. The worker rents out the household's capital stock, k_t, at real interest rate r_t, and also supplies N_t units of labour at wage rate w_t in any period t to the firms. The worker's labour supply is subject to the time constraint (4), where time is normalized to one.

$$1 \geq l_t + N_t \tag{4}$$

The shopper, on the other hand, goes to the trading market to purchase consumption goods for the household with previously acquired money. Unlike Lucas

and Stokey, however, there are no credit goods in this economy. The shopper thus faces a cash-in-advance constraint given by (5);

$$P_t C_t \leq M_t + V_t . \tag{5}$$

In any period t, the household receives $r_t K_{p,t}$ as payment for hiring out its capital, $w_t N_t$ as wage earning, and V_t as lump sum transfer payment from the government. These, in addition to the household's undepreciated capital and it's unspent money balances constitute the funds available to the households in any period t. The household's resources are used to finance consumption as well as capital and money balances that will be carried into period t+1. Thus the typical household's budget constraint can be described by equation (6);

$$C_t + K_{p,t+1} + \frac{M_{t+1}}{P_t} \leq r_t K_{p,t} + w_t N_t + (1-\delta_K) K_{p,t} + \frac{M_t + V_t}{P_t} . \tag{6}$$

The representative firm hires K_t units of capital and n_t units of labour, and combines these with available government capital to produce output Y_t. Output can be consumed or invested by either the private or the public sector; that is $Y_t = C_t + I_{pt} + I_{gt}$, where I_{pt} is gross per capita private investment in period t, and Y_t is per capita output. The production function is assumed to be described by equation (7);

$$Y_t = F(K_{p,t}, n_t, K_{g,t}; z_t) , \tag{7}$$

where z_t is the productivity shock, $K_{g,t}$ and $K_{p,t}$ are per capita[72] government and private capital respectively in period t. Public and private capital evolve analogously and they are described by equation (8);

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72 Other ways of modelling government capital spending can be found in Barro and Salai-i-Martin (1992) where total government capital spending entering the production function is divided by the aggregate private

$$K_{a,t+1} = (1-\delta_K) K_{a,t} + I_{a,t}, \qquad a = g, p \tag{8}$$

where d_K is the rate of depreciation which is assumed to be the same for both private and public capital. Unlike household's gross investment, however, public gross investment is exogenously determined by the government's fiscal policy. Thus from (2) and (3); $I_{g,t}=(M_{t+1}-M_t-V_t)/P_t$[73].

2.2. Equilibrium Conditions

The representative household's problem is to choose consumption, labour supply, stock of capital, and money balances such that equation (1) is maximized subject to the constraints in (4), (5), and (6). The solution to the household's problem can be obtained by solving the following Bellman's equation;

$$V(K_{p,t}, M_t, K_{g,t}; S_t) = \max_{C_t, I_t, K_{p,t+1}, M_{t+1}} [U(C_t, l_t) + \beta E_t V(K_{p,t+1}, M_{t+1}, K_{g,t+1}; S_{t+1})], \tag{9}$$

where $S_t=(K_{p,t}, M_t, K_{g,t}; z_t)$ is the state of the economy at period t.

The firm's optimization problem involves choosing $K_{p,t}$ and n_t to maximize profits. That is, they solve equation (10);

$$\max_{K_{p,t}, n_t} [F(K_{p,t}, n_t, K_{g,t}; z_t) - r_t K_{p,t} - w_t n_t]. \tag{10}$$

By definition, competitive equilibrium in this economy consists of the following allocations;

(a) the households choose sequences of consumption (C_0, C_1, C_2, ...), capital holding

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capital, and Glomm and Ravikummar (1992) where government capital stock is divided by the economy's private output.

73 This formulation allows the government to finance investment by lump-sum taxes (that is V_t can be negative).

($K_{p,0}$, $K_{p,1}$, $K_{p,2}$, ...), money holding (M_0, M_1, M_2,...), and labour supply (N_0, N_1, N_2, ...) which solves equation (9) for given sequences of price (P_0, P_1, P_2, ...), wage (w_0, w_1, w_2, ...), rent (r_0, r_1, r_2, ...), transfer payment (V_0, V_1, V_2, ...), government capital ($K_{g,0}$, $K_{g,1}$, $K_{g,2}$, ...), and productivity shock (z_0, z_1, z_2, ...)

(b) firms hire sequences of labour (n_0, n_1, n_2, ...), and capital ($K_{p,0}$, $K_{p,1}$, $K_{p,2}$, ...) such that equation (10) is solved given wage rate (w_0, w_1, w_2, ...), government capital ($K_{g,0}$, $K_{g,1}$, $K_{g,2}$, ...), interest rate (r_0, r_1, r_2, ...), and productivity shock (z_0, z_1, z_2, ...) and

(c) the goods, the labour, and the money markets clear. That is;

$$C_t + K_{p,t+1} + K_{g,t+1} = F(K_{p,t}, n_t, K_{g,t}; z_t) + (1-\delta_K)K_{p,t} + (1-\delta_K)K_{g,t}, \tag{11}$$

$$N_t = n_t, \tag{12}$$

and $$M_{t+1} = M_t + V_t + P_t I_{gt}. \tag{13}$$

For sufficiently high inflation, the cash-in-advance constraint will hold with equality and the budget constraint will also hold with equality due to nonsatiation. From the definition of competitive equilibrium, the allocation rules for the households and the firms are defined by the market clearing conditions in (11)-(13), and equations (14)-(16);

$$U_2(C_t, l_t) = \beta E_t\left[\frac{U_1(C_{t+1}, l_{t+1})}{P_{t+1}/P_t}\right] F_2(K_{p,t}, n_t, K_{g,t}; z_t), \tag{14}$$

$$\frac{U_2(C_t, l_t)}{F_2(K_{p,t}, n_t, K_{g,t}; z_t)} = \beta E_t\left[\frac{U_2(C_{t+1}, l_{t+1})}{F_2(K_{p,t+1}, n_{t+1}, K_{g,t+1}; z_{t+1})}\right][F_1(K_{p,t+1}, n_{t+1}, K_{g,t+1}; z_{t+1}) + 1 - \delta_K], \tag{15}$$

$$P_t C_t = M_t + V_t, \tag{16}$$

where U_i and F_i are the differentials of the respective function with respect to the i-th argument and E_t is the expectation operator at time t.

Equation (14) guides the consumer's labour supply decision. From this equation, the consumer's equilibrium will be characterized by equating the marginal cost of reducing leisure to the marginal benefit of labour supply in terms of utility. The LHS of (14) can be interpreted as measuring the cost of reducing leisure, and the RHS interpreted as measuring the benefit from the last unit of labour supply. The benefit from the last unit of labour supply is equal to the time t expected discounted marginal benefit of the real returns from labour. The presence of inflation in the allocation rule is due to the cash-in-advance constraint which ensures that income earned in any period can only be spent the next period. Thus, perfectly anticipated inflation distorts labour supply and hence influences real variables in the economy. This, potentially, affects the economy in two ways. Firstly, in multiple goods environment with credit goods, this can lead to a deviation of the marginal rate of substitution between cash and credit goods from their marginal rate of transformation [see, for example, Braun (1994)]. Secondly, the reduction in labour supply will reduce overall output. These effects of inflation underpin most of the welfare cost of inflation studied within the cash-in-advance models [see Cooley and Hansen (1989, 1991)]. Since only the second inflationary distortion is captured in the setup here, the model abstracts from a more general economic environment where inflation causes additional distortions and rather focuses on the effect of alternative uses of seigniorage revenue on the welfare cost of inflation.

Equation (14) governs the households capital demand. The equation indicates that the consumer will equate the cost of acquiring capital to it's benefits in equilibrium. The cost of acquiring the marginal unit of capital is given by the LHS of equation (14).

The benefit is the return earned by holding the marginal unit of capital for one period which is given by the next period's marginal product of capital plus undepreciated capital valued in that period's utility and discounted to period t.

The allocation decision rule in equation (13) emphasizes the distortions caused by inflation within the cash-in-advance model and the subsequent real loss due to it. The benefits, on the other hand, can only be explicitly analyzed if the maximum value function is derived[74] and the effect of changes in the money supply on utility analyzed. The benefits will then be the absolute value of the changes in the maximum value function resulting from the decline in real balances as a result of modelling money as a pure waste rather than transfer payments to economic agents. Alternatively, the values of the nonnegative multipliers on the cash-in-advance constraints which are the shadow prices of money in the various periods give some indication of the value of money.

The gains from seigniorage will potentially be different if the use of seigniorage is modelled differently, as for example, modelled to finance government capital. Again, an increase in inflation will reduce the effective return on labour which will induce a fall in the labour supply[75] and output, but output may not be hurt as much in this scenario. The reason is that in this situation money is introduced in a productive way and this cushions the reduction in output via the reduction in labour supply. As in Baxter and King (1993), output will increase directly due to the increase in public capital as well as

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74 Unfortunately, there is no known formal way of deriving the maximum value function for these set of problems [see Cooley and Hansen (1989) and Sargent (1987)]. Various techniques such as linearization and quadratic approximations are, however, used to approximate these value functions.

75 It is possible, in this model, for labour supply to increase if the effect of the increase in marginal product of labour, as a result of increased government capital spending, outweighs the inflationary effect on labour supply. In this case, the standard results in public policy analysis, where an increase in government purchases reduces

indirectly due to increases in both the demand for labour and private capital (these result from increases in their marginal products). This, however, does not guarantee an improvement in welfare. The reason is that accumulation of higher stocks of public capital and provision of higher depreciation to maintain the higher levels of public and private capital will divert output from private consumption[76].

The gains from investing seigniorage in public capital would have to be compared to the gains that accrue when money is directly transferred to consumers[77] in order to access any welfare differences. Since the cost of inflation *per se*, however, would remain the same in the sense that inflation will reduce effective wages and hence reduce labour supply and output, differences in the benefits from printing money would imply different welfare cost of inflation.

3. Model Parameterization and Calibration

In order to solve for the competitive equilibrium, specific forms of the utility and the production functions are assumed and values assigned to the various parameters in the model. After this, the model's results are analyzed and compared to other models that studied the welfare cost of inflation.

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private income and leads to an increase in labour supply (because leisure is considered normal), is obtained [see, for example, Baxter and King (1993) and Aschauser (1989)].

76 In Baxter and King (1993), consumption increases when government increases capital spending for a wide range of public capital productivity (consumption falls when public capital is not productive). This result is due to the financing of public capital by lump-sum taxes which did not have any adverse inflationary effect.

77 The gains in the lump-sum transfer scenario will be zero if money is superneutral. Although superneutrality of money has been discussed and debated for a long time there is no definite conclusion on whether money is superneutral or not [see Danthine, Donaldson, and Smith (1987)].

3.1 Steady State

For steady state to be feasible, permanent technical change must be expressible in labour augmenting form [see Swan (1963) and Phelps (1966)]. Consistent with this and adopting a production function which is similar in spirit to the one used by Baxter and King (1993) resulted in the specification of the technical production constraint (17);

$$Y_t = (K_{g,t}/z_t)^{\theta_3} K_{p,t}^{\theta_1} (z_t n_t)^{\theta_2}, \qquad \theta_1+\theta_2=1^{78}, \tag{17}$$

where z_t is permanent productivity shock. Temporary productivity shocks are ignored here and attention is focused on steady state welfare. Welfare costs can be appropriately measured by comparing steady states since the cyclical characteristics of the model, like most other models of the welfare cost of inflation, are unaffected by the average growth rate of money [see Cooley and Hansen (1989)]. Also, as pointed out by King, Plosser, and Rebelo (1988), steady state is a characteristic of many developed countries. The production function, as specified by (17), implies that the steady state growth rate of output, consumption, public capital, and private capital per capita are all equal to the growth rate of the labour augmenting technical progress, and the growth rate of labour is zero. That is;

$$m_{K_p} = m_Y = m_{K_g} = m_z = m_c \text{ and } m_n=0 \tag{18}$$

It is worth noting that equation (17) exhibits constant returns to scale only in labour and private capital[79]. This specification, as in Baxter and King (1993), allows for increases in government capital to be captured in public capital productivity. Since the

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78 Compared to Baxter and King (1993), public capital stock is divided by permanent productivity shock. This is to allow real variables to be transformed to stationary in steady state.

79 This specification of the production function has also been used by various authors to generate increasing-

paper explores the influences of financing government capital through printing money, it is essential that both the growth of public capital and it's productivity are not constrained by technical requirements, and hence Baxter and King's (1993) modelling strategy is appropriate here[80].

Government spending can, basically, be modelled to affect production in two ways. One of these is to assume that the government uses seigniorage to finance government capital which is a perfect substitute for private capital. This corresponds to government financing of government-owned-privately-operated capital and government enterprise capital in Finn's (1993) study. In this case, the standard production function is maintained, but capital comprise both government and private capital. Modelling government capital this way would require some assumptions about the capital accumulation process. One could assume that capital is "residually" acquired by private agents. This will mean that equation (15) will still guide the capital accumulation process and as a result it will not make any difference whether the government transfers seigniorage directly to individuals or invest it in productive capital [complete crowding out occurs]. Alternatively, one can assume that the government accumulates capital residually without paying any attention to equation (15) which guides equilibrium capital accumulation. There is, however, a strong implicit assumption underlying this; that is either private agents do not care about the government's capital expenditure or that they cannot observe it.

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returns-to-scale in endogenous growth models [for example, Dotsey and Ireland (1996)]. The government capital stock argument is, however, interpreted as the aggregate capital stock or some variant of it.

80 Baxter and King (1993) captured increases in government investment by increasing the output elasticity of government capital. This technique cannot be used if the production function exhibits constant-returns-to-scale

The second way that one can examine the effect of government capital spending, which is much more satisfactory in this context, is to model it as a separate distinct capital from private capital[81]. Such capital spending includes the construction of highways, among other things [Finn (1993)]. This is the situation described by the production function in (17), which still retains constant-returns-to-scale features in labour and private capital in order to ensure that the function is consistent with steady-state or balanced growth in these two arguments.

The feasible steady state growth rate would be compatible with an optimal competitive equilibrium if restrictions are imposed on the utility function [see King et al (1988)]. These restrictions include; (i) the intertemporal elasticity of substitution in consumption must be invariant to the scale of consumption and (ii) the income and substitution effects associated with sustained growth in labour productivity must not alter labour supply. These conditions imply the following class of utility functions;

$$U(C,l) = \frac{C^{1-\sigma}}{1-\sigma} V(l), \tag{19}$$

where 1/s is the constant intertemporal elasticity of substitution in consumption for these functions[82]. To fully parameterize the utility function, the following constant relative risk aversion functional form is adopted;

$$U(C_t, l_t) = \frac{[C_t^{\alpha} l_t^{1-\alpha}]^{1-\gamma}}{1-\gamma}, \qquad 0 < \gamma < 1 < \gamma, \tag{20}$$

(CRS) in all three factors because of the restriction it imposes on the productivity coefficients.

81 This is the modelling technique used in most studies [see, for example, Barro (1989), Baxter and King (1993), and Dotsey and Mao (1993)].

82 The assumption of concavity of U(C,l) implies that V(l) is increasing and concave for s less than or equal to 1 and decreasing and convex for s greater than 1.

where g is the coefficient of relative risk aversion and w determines the relative importance of consumption and leisure.

Given the current parameterization for taste and technology, the steady-state versions of the market clearing condition (11), and the efficiency equations (14) and (15), can be written as (21), (22), and (23) when real growth is zero.

$$C = K_g^{\theta_3} K_p^{\theta_1} n^{\theta_2} - \delta_K (K_g + K_p). \tag{21}$$

$$(1-\omega)(1-n)^{-\omega} = \frac{\beta\omega C^{\omega-1}\theta_2 K_g^{\theta_3} K_p^{\theta_1} n^{\theta_2-1}}{P_{t+1}/P_t}. \tag{22}$$

$$1 = \beta[\theta_1 K_g^{\theta_3} K_p^{\theta_1-1} n^{\theta_2} + 1 - \delta_K]. \tag{23}$$

Equation (21), (22) and (23) can be used to solve for C, n, and K_p for any given level of government capital stock and growth rate of money (which is also equal to the rate of inflation in this case). However, the price level and the money stock are indeterminate resulting in the classical dichotomous economy since the cash-in-advance constraint in equation (5) can be used to determine the price level for any given nominal balance.

3.2 Steady State Growth

To analyze the model with steady state growth, the economy is transformed into a stationary one where the dynamics facilitate computation. This balance growth transformation involves dividing all real variables in the system by the growth component z_t, so that $c_t=C_t/z_t$, $y_t=Y_t/z_t$, $k_{p,t}=K_{p,t}/z_t$, $k_{g,t}=K_{g,t}/z_t$, $i_{p,t}=I_{p,t}/z_t$, and $p_t=P_t z_t/M_t$ are stationary. Since the government's expenditure on capital is exogenous, public capital

growth may be different from the exogenous technical growth. In order to address this problem, increases in public investments would be captured in increases in public capital productivity [*à la* Baxter and King (1993)] rather than increases in the stock of public capital[83]. This means that an increase in government's investment which is in excess of the amount required to maintain public capital stock influences the economy by increasing public capital productivity.

Given the homogeneity properties of the utility and the production functions, the efficiency equations (14) and (15), and market clearing conditions (11), (13), and (16) can be transformed to give the following equations which facilitate computation of equilibrium;

$$c_t + \left(\frac{z_{t+1}}{z_t}\right)[k_{p,t+1} + k_{g,t+1}] = F(K_{g,t}, k_{p,t}, n_t) + (1-\delta_K)[k_{p,t} + k_{g,t}], \tag{25}$$

$$\mu U_2(c_t, l_t) = \beta\left(\frac{z_{t+1}}{z_t}\right)^{1-\sigma} E_t\left[\frac{U_1(c_{t+1}, l_{t+1})}{p_{t+1}/p_t}\right] F_2(K_{g,t}, k_{p,t}, n_t), \tag{26}$$

$$\frac{U_2(c_t, l_t)}{F_2(K_{g,t}, k_{p,t}, n_t)} = \beta\left(\frac{z_{t+1}}{z_t}\right)^{-\sigma} E_t\left[\frac{U_2(c_{t+1}, l_{t+1})}{F_2(K_{g,t+1}, k_{p,t+1}, n_{t+1})}\right][F_1(K_{g,t+1}, k_{p,t+1}, n_{t+1}) + 1 - \delta_K], \tag{27}$$

$$m_{t+1} = m, \tag{28}$$

$$p_t c_t = m - I_{gt} p_t / z_t \tag{29}$$

The US economy is calibrated using the procedure advocated by Kyland and Prescott (1982), and subsequently used by various authors. They recommend that as

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83 Baxter and King (1993) argue that increases in government capital works like increases in the productivity of capital from the standpoint of the demand for labour and private capital. Thus increases in government capital can be captured by increases in its productivity. Economically, one can attempt to justify this by arguing that government investments are targeted at research and development which results in the production of more efficient capital. This is, however, not the motivation for adopting this technique here. Rather, it is to facilitate computation.

many parameter values as possible be set on *a prior* information about their magnitudes or be set so that the model's deterministic steady-state values for the various endogenous variable match their averages for the postwar U.S. economy.

In line with these recommendations capital's share of income is set equal to 0.34, its average quarterly share for the sampling period 1959Q1-1994Q4. This implies a value of .66 for q_2[84]. The average share of public investment in output of .05 is used as public capital productivity in the benchmark economy. That is q_3=.05 [also see King et al (1988), and Baxter and King (1993)]. Depreciation rate of 0.025 per quarter is used for both public and private capital. This corresponds to the value used by Kyland and Prescott (1982) and subsequently by other authors in business cycle studies [examples are Hansen (1985), Cooley and Hansen (1989), and King et al (1988)]. Evidence from the study by Mehra and Prescott (1985) suggests that the coefficient of relative risk aversion lies between 1 and 2. This guides the setting of this value, and like in most business cycle models, it is set equal to 1.5. Evidence also indicate that the per capita time spent on work by US working population is about 24%[85]. The parameter w, which governs the importance of consumption relative to leisure is set such that households, on average, spend this proportion of their time on work in the benchmark economy. This implies a value of 0.2472 for w. The value of w together with the value of s implies that g is equal to 3.0226. The time discount factor, b, is chosen such that the real interest rate of 4% per annum is obtained as in most business cycle models. This yielded a value of

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84 Kyland and Prescot (1982) calculated labour's share of income as 0.64 for the US in 1976. This share is, however, higher than in many studies with public investment as an input [see, for example Finn (1993)].

85 This figure has been used by many authors, for example, Gomme (1993). Other authors have used 0.2 [for example, Dotsey and Ireland (1996)].

0.996 by equation (15).

4. Welfare Analysis

This section deals with the welfare measures of different monetary and fiscal policies for the economic framework described earlier. This enables us to compute the welfare losses that result from the different monetary and fiscal policies, and in particular to examine the effect of different fiscal policies on welfare. To do this, let the sum of discounted utilities of the representative agent be V^{al}, then V^{al} is given by equation (30);

$$V^{a1}(m_0, k_0, h_0; \lambda) = E_0 \sum_{t=0}^{\infty} \beta^t U(c_t^{a1} + \lambda y_t^{a1}, l_t^{a1}), \tag{30}$$

where the superscripts *a* and *1* denote equilibrium values of the relevant variable for monetary policy *a* and fiscal policy *1*, respectively[86]. ly_t is a lump-sum equivalent variation payment made to households to attain a certain utility level. Using these definitions, the welfare cost of operating monetary policy *a* and fiscal policy *1* rather than monetary policy *b* and fiscal policy *2* is measured by the value of l satisfying equation (31);

$$V^{a1}(k_0, m_0, g_0; s, \lambda) = V^{b2}(k_0, m_0, g_0; s, \lambda = 0). \tag{31}$$

The value function in (30) can be rewritten as;

$$V^{a1}(k_0, m_0, g_0; s, \lambda) = z_0^{1-\sigma} E_0 \sum_{t=0}^{\infty} \beta^t U(c_t^{a1} + \lambda y_t^{a1}, l_t^{a1}) \left[\prod_{\tau=0}^{t} \frac{Z_{\tau+1}^{a1}}{Z_\tau^{a1}} \right]^{\sigma}, \tag{32}$$

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86 The first superscript describes monetary policy (uses alphabets), and the second denotes fiscal policy (uses numbers).

and in steady state equation (32) can be written as

$$V^{a1}(k_0, m_0, g_0; s, \lambda) = \frac{z_0^{1-\sigma} U(c^{a1} + \lambda y^{a1}, l^{a1})}{1 - \beta\left[\left(\frac{z_{t+1}}{z_t}\right)^{a1}\right]^{1-\sigma}}. \tag{33}$$

If the optimal monetary policy is denoted by an asterisks superscript, then the cost of operating monetary policy *a* relative to the optimal plan in steady state will be given by (34);

$$\frac{U(c^{*\Omega}, l^{*\Omega})}{1 - \beta\left[\left(\frac{z_{t+1}}{z_t}\right)^{*\Omega}\right]^{1-\sigma}} = \frac{U(c^{a\Omega} + \lambda y^{a\Omega}, l^{a\Omega})}{1 - \beta\left[\left(\frac{z_{t+1}}{z_t}\right)^{a\Omega}\right]^{1-\sigma}}, \qquad \text{for } \Omega = 1,2,... \tag{34}$$

Similarly the cost of operating fiscal policy *2* relative to *1* is;

$$\frac{U(c^{\phi 1}, l^{\phi 1})}{1 - \beta\left[\left(\frac{z_{t+1}}{z_t}\right)^{\phi 1}\right]^{1-\sigma}} = \frac{U(c^{\phi 2} + \lambda y^{\phi 2}, l^{\phi 2})}{1 - \beta\left[\left(\frac{z_{t+1}}{z_t}\right)^{\phi 2}\right]^{1-\sigma}}, \qquad \text{for } \phi = *, a, b,... \tag{35}$$

4.1 Empirical Results

The steady state results obtained from experimentations with various monetary and fiscal policies are reported in Table 1. Experiments were conducted for the optimal, zero, 5%, 10%, 20%, and 100% inflation for a fixed exogenous gross growth rate of 1.0041 per quarter, the average quarterly gross growth rate for the sample period (1959Q1-1994Q4). For each inflation rate considered, two fiscal policies were examined. The first is where the government's investment is just enough to maintain the stock of government capital, and seigniorage revenue is transferred to economic agents in a lump-sum fashion. This

corresponds to the standard way of evaluating the welfare cost of inflation [see Cooley and Hansen (1989), and Dotsey and Ireland (1996), among others]. The second fiscal policy experiment describes a situation where the government invests the revenue from seigniorage in capital. Experiments can be conducted with various compositions of current and capital spendings, but the study here is limited to the extreme cases to highlight the potential influences of capital expenditure on the welfare cost of inflation.

A 10% inflation in this model costs about .1775% of income, when seigniorage revenue is transferred to economic agents. This estimate is higher than the figure reported by Gomme (1993), where 10% money growth (8.7% inflation) cost .0238% of income, but is lower than the estimates reported by many other researchers. Cooley and Hansen (1989), Dotsey and Ireland (1996), for example, calculate the welfare cost of 10% inflation to be .4%, and .92% of income, respectively.

On the other hand, when the government invests the revenue from seigniorage in capital, the cost of a 10% inflation reduces to .0197% of income. This recovers about 88.9% of the welfare loss in the lump-sum transfer experiment. The reason is that current and capital expenditures elicit different responses in the economy. In particular, a 10% inflation allows the government to print money at a rate of 11.8% per annum[87]. This "extra" revenue[88] permits the government to increase it's capital expenditure which increases the productivity of both labour and private capital. Consequently, labour supply increases from 23.605% of total time endowment to 23.64%, an increase of .148%, and private capital

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87 This assumes that velocity is constant. As noted in Cooley and Hansen (1989), allowing velocity to vary or output growth to vary would not alter the results much.

88 This represents an extra revenue of 13.4% of fiat money compared to the optimal scenario where government

increases from 6.7842 to about 6.8136, a gain of about .433%. These increases in factor inputs mitigate the loss in output occasioned by the 10% increase in inflation, and hence reduces the welfare cost.

These results suggest that the way the inflation tax is used is not trivial in evaluating the welfare cost of inflation. The welfare loss under the different inflation experiments are smaller when the government invests seigniorage in capital. However, the results indicate that the percentage gain in the welfare cost of inflation when seigniorage is switched from transfer payments to capital investment falls as the level of inflation increases. This is evidenced by the 96%, 89%, 73%, and 31% gains in the welfare cost of 5%, 10%, 20%, and 100% inflation, respectively, when the government diverts seigniorage revenue from transfer payment to capital investment. Thus the standard way of calculating the welfare cost of inflation (where seigniorage is transferred to economic agents) is more suitable for high inflation economies than low inflation ones[89]. This, however, does not provide a justification for ignoring the influence of the use of inflation tax in evaluating the welfare cost of inflation in high inflation economies. The gain in the welfare cost of inflation in the current versus capital expenditure scenarios are .001101, .001578, .00271, and .008076 of income for 5%, 10%, 20%, and 100% inflation, respectively. These increasing absolute differences reflect the fact that larger amounts are switched from transfer payments to finance capital at higher levels of inflation and this results in larger improvements in welfare (absolute). The reason for this, is that the government capital stock from the data is

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has to contract money at a rate of about .8% per.

89 The estimates show that the margin of error arising from modelling seigniorage as transfer payments rather than capital investment at 100% inflation is about a third the margin of error at 5% inflation.

below the optimal level[90], and a shift of seigniorage from transfer payments to capital expenditure is desirable from a welfare perspective. In the cash-in-advance model used here, money is neither neutral nor superneutral because effective returns on factor incomes are influenced by inflation. Thus the improvement in welfare, consequent on switching revenue from transfer payments to capital investment, means the reduction in welfare due to withdrawing seigniorage from transfer payment is less than the gain in welfare due to investing it in public capital. This implies that the results hinge on the welfare cost of withdrawing the transfer payments and the welfare gains of investing the withdrawn funds in public capital. If the welfare cost of withdrawing seigniorage from transfer payments is higher than the gain in welfare when it is invested in public capital, then welfare will decline when revenue is switched from transfer payments to capital investment. This can be the case if there is overaccumulation of public capital[91].

Although the gain in welfare increases as higher seigniorage revenues are diverted from transfer payments to capital expenditure, the cost per unit of seigniorage (gain in welfare divided by seigniorage revenue) does not increase monotonically in inflation. The gain of switching, for example, 1% of income of seigniorage from transfer payment to investment in public capital is .1338%, .1155%, .1154%, and .1186% of income for 5%, 10%, 20%, and 100% inflation, respectively. This suggests a parabolic relationship between

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90 In calibrating the model the ratio of public capital to income (gross national product) is set to the ratio observed in the US data, which is about 50%. See Aschauer (1989) for estimation of public capital stock. Note that model can be calibrated to equate the marginal benefits of transfer payments and capital expenditure. In this case, the *marginal* use of seigniorage will be irrelevant. This, however, allows an extra degree of freedom in the calibration process which is contrary to the recommendations by Kyland and Prescott (1982).

91 It is possible for capital, at least private capital, to be overaccumulated especially in the so called "low-interest-rate economies" which can occur when the economy is dynamically inefficient. See Cass (1972) on some of the ways to tackle this problem.

gains in welfare and the per unit inflation tax switched from transfer payment to investment in public capital as inflation levels increase.

It would also be observed that whereas the welfare cost of inflation is monotonically increasing in inflation under the transfer payment scenario (for the range of inflation considered), the welfare cost under the capital expenditure scenario does not exhibit the same relationship. In the capital expenditure scenario, the welfare cost of inflation attains a minimum value of .0000296 at 3.55% inflation for positive inflation.

5. Seigniorage and distortionary labour income tax

Another distortionary tax (labour income tax)[92] is introduced in order to compare the responses of the welfare cost of these distortionary taxes (seigniorage and labour income tax) to the composition of public expenditure. To appropriately compare these distortionary taxes, the tax revenue of the government is fixed at 10% of income by adjusting the labour income tax rate as the inflation tax rate changes. This experiment was conducted for both transfer payments and capital spending [transfer payment results are reported in Table 2(a) and capital expenditure results are reported in Table 2(b)]. The results indicate that welfare is sensitive to the composition of both government revenue and expenditure. As the government's tax revenue is fixed at 10% of income, the inflation levels differ a bit for the capital and current expenditure scenarios. In particular, public capital is higher in the capital expenditure experiment for any given positive seigniorage leading to higher incomes and consumption. However, consumption increases faster than income and thus increases the

[92] Because the model is calibrated using a real capital return of 4% per annum, introducing capital taxes will

demand for money faster than the increase in income. The higher relative (relative to income) base of seigniorage means that less inflation will be required to raise a given proportion of income tax by seigniorage.

From the results, seigniorage is a superior form of distortionary tax in the economy at low levels of inflation and inferior at high ones. When government expenditure is composed entirely of transfer payments, the welfare cost of collecting 10% of income as taxes decreases from about 2.31% of income when no money is printed to about 2.02% of income when seigniorage constitutes about 42.2% of government revenue (optimal tax mix), and it increases to about 2.13% of income when seigniorage constitutes about 69% of the tax revenue (100% inflation). In the scenario where government invests its revenue, welfare loss decreases from about 1.59% of income to about 1.31% when seigniorage increases from zero to about 42.4% of government's revenue, and it increases to about 1.47% of income when seigniorage is about 69% of the government's revenue.

In collecting 10% of income by distortionary tax, the optimal taxes mix is obtained at the same level of inflation (48.8%) for both transfer payment and capital expenditure experiments. At this rate of inflation about 42% of the government's revenue is collected through the inflation tax and the remainder through labour income tax. Since the optimal inflation rate is the same for both kinds of public expenditure, the optimal mix of seigniorage and labour income tax is not sensitive to the composition of government expenditure in this model. This suggests that a positive inflation tax is optimal in a second-best economy. The question of whether a positive inflation tax is optimal in an economy

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distort the returns to capital which will change the rate of time preference in the model.

where distortionary taxes have to be raised has attracted considerable attention from monetary economists. Drazen (1979) argues that there is no *a priori* reason to expect that the optimal rate of inflation would be positive. He contends that the sign and magnitude of the optimal rate of inflation are questions which must be settled by empirical estimations of demand functions. The results of the theoretical debates depend to a large extent on the motive for holding money in the model and other specification issues[93]. The optimality of the inflation tax has been studied within the cash-in-advance model by authors such as Lucas and Stokey (1983), Chari, Christiano, and Kehoe (1991), Cooley and Hanson (1992), and Braun (1994), among others. Lucas and Stokey (1983), and Chari, Christiano, and Kehoe (1991) argue that the Friedman rule is optimal in the cash-in-advance environment. Cooley and Hanson (1992) argue that some inflation tax may be required if distortionary taxes have to raised in the cash-in-advance model. This assertion is supported by Braun (1994) study. Using a cash-in-advance model, he argues that if the income elasticity of money demand is greater than or equal to one then Friedman's rule is optimal. If not, some inflation tax would be optimal. He estimates that the optimal inflation rate for the U.S. economy ranges between 29% to over 200% per year, using monthly post-war U.S. data. Using annual data dating back to the beginning of this century, he estimates the optimal inflation rate to be between 1% and 6% per annum. Given that the data used here are post-war quarterly data, the estimates of the optimal inflation appear reasonable.

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93 For example, Phelps (1973), by modelling money as consumption good, argues that money should be taxed like any other good in Ramsey-like fashion. Other modelling techniques used such as the 'shopping time' transactions technology [see, for example, Kimborough (1986)] and consumption smoothening [for example, Prescott and Imrohoroglu (1991)] suggests results that money should not be taxed. The transactions technology result is consistent with the results in the public finance literature that intermediary goods should not be taxed in a second-best world. However, Guidotti and Vegh (1993) pointed out that these findings are sensitive to timing

Table 3 summarizes the changes in the welfare cost as the composition of revenue is shifted away from labour income tax to seigniorage. As mention earlier, seigniorage is superior to labour income tax at low levels of inflation, but inferior to it at high levels of inflation. Put differently, the welfare gain of replacing, for example, 1% of revenue collected by labour income tax with seigniorage is higher at 100% rate of inflation than at no inflation. The average welfare gain of replacing 1% of government revenue collected by labour income tax with seigniorage is about .013% of income, when inflation is between 0-5% and the government transfers seigniorage revenue. The corresponding cost when government invests seigniorage in capital is about .012%. For inflation rates between 20% and 100%, welfare on average deteriorates when a labour income tax is substituted by an inflation tax. In this case, the average cost of substituting 1% of revenue collected by a labour income tax with seigniorage is .0006% of income in the transfer payment scenario, and .002% in the capital expenditure scenario. This suggests that if seigniorage revenue is invested in capital, the gains of switching from labour income tax to seigniorage will be less than if the revenue is transferred to economic agents at low levels of inflation. At high levels of inflation, investing seigniorage will result in higher welfare gains than in the transfer payment scenario when seigniorage is replaced with labour income tax. However, these expenditure effects are not large. There are only small deteriorations in welfare costs (in terms of the difference in welfare when the government transfers seigniorage to economic agents and when the government invests the revenue in capital) as the mix of government revenue is changed. The difference in welfare when government transfers

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and homogeneity assumptions on the transaction cost technology.

revenue and when it invests in capital reduces from .7219% to .6351% when inflation is increased from 5% to 20%, and welfare cost increases to .662% when inflation is 100%. The implication of this is that adjustments on the revenue side of the government's budget cannot be used to eliminate the welfare consequences of the composition of government expenditure. This stresses the importance of the use of inflation tax in the calculation of the welfare cost of inflation. This result is consistent with the robustness of the optimal inflation tax to the composition of government expenditure.

6. Conclusion

The main lesson from this study is that fiscal policy, in terms of the use of the revenue from printing money which generates inflation, has implications for the welfare cost of inflation. In particular, the welfare cost could be mitigated if the revenue from seigniorage allows the government to increase its capital expenditure in economies where public capital is below its optimal level. This suggests that to measure the welfare cost of inflation more appropriately, the benefits derived from the revenue of the inflation tax must also be considered. The welfare triangles measure of the cost of inflation [for example, Lucas (1981) and Fischer (1981)] takes account of the revenue from the inflation tax. This study goes a step further and examines the effect of the use of this revenue on the welfare cost of inflation. As demonstrated, different uses of the inflation tax elicit different responses in the economy which have different implications for the welfare cost of inflation. In particular, it was shown that if government invests seigniorage rather than transfers it, the 'destructive' effects of inflation could be reduced resulting in lower welfare cost of

inflation. Thus better estimates of the welfare cost of inflation can be obtained by "tracking" the use of seigniorage and modelling it accordingly. In practice, however, it is difficult to trace the uses of particular government revenue, but this difficulty should not prevent further research in the area. This is because just as it is erroneous to allocate inflationary tax revenue entirely to capital expenditure, it is also erroneous to allocate it entirely to recurrent expenditure. The marginal use of seigniorage will be irrelevant if the marginal benefits derived from the various government expenditure items are equal. However, the use of the entire revenue from most of the seigniorage experiments would be relevant.

It was found that failure to take account of the use of the revenue from seigniorage can lead to exaggeration of the welfare cost of inflation from about 96% at 5% inflation to about 31% at 100% inflation for the US economy. These extreme errors will result when government invests all seigniorage revenue and this is ignored. It was also found that the direction of the error depends on whether there is overaccumulation of public capital or not. Where there is overaccumulation, modelling seigniorage as transfer payment may well underestimate the welfare cost of inflation.

It was also found that the inflation tax is superior to labour income tax at low rates of inflation and inferior at high ones. This suggests a positive optimal inflation rate in a second-best world, which is in tune with many studies in this area. This optimal inflation rate is not sensitive to the composition of government expenditure. However, the welfare effects of switching from the use of inflation tax to labour income tax is sensitive to the composition of government expenditure.

Although attempting to capture both the benefits and costs of inflation for the

purposes of calculating the welfare cost of inflation can be analytically challenging, it is worth investigating this area further in view of the fact these results can potentially drive monetary policy.

Table 1(a): Welfare Cost of Inflation (Transfer Payment versus Capital Expenditure)

Variable	I N F L A T I O N					
	Optimal	z e r o		4.38%*	5%	
		Transfer Pymt.	Capital Expt.	Capital Expt	Transfer Pymt.	Capital Expt
n	0.245139	.242799	.242858	.240479	0.239319	0.239539
k	7.045418	6.978174	6.982951	6.920831	6.87817	6.896214
c	0.513936	0.508931	.509226	.504477	0.501489	0.502595
y	0.729346	0.722385	0.722879	.716449	0.712032	0.7139
u	-1.062125	-1.062372	-1.06219	-1.062147	-1.062863	-1.062154
v	-175.978302	-176.019287	-175.989075	-175.982056	-176.100708	-175.983139
wel cost	0	.000328	.000086	.0000296	.001139	.000038

Variable	I N F L A T I O N					
	1 0 %		2 0 %		1 0 0 %	
	Transfer Pymt.	Capital Expt.	Transfer Pymt.	Capital Expt.	Transfer Pymt	Capital Expt.
n	0.23605	0.236399	0.230026	0.230629	0.197439	0.199009
k	6.7842	6.8136	6.611073	6.660457	5.674505	5.795766
c	0.494496	0.496286	0.481611	0.484603	0.411911	0.418954
y	0.702304	0.705348	0.684382	0.689494	0.587428	0.599981
u	-1.063463	-1.062274	-1.064925	-1.062885	-1.081833	-1.075868
v	-176.19997	-176.003052	-176.442322	-176.104294	-179.243637	-178.255371
wel cost	.001775	.000197	.003716	.001006	.026263	.018187

Note: The * denotes the inflation rate at which welfare loss is minimized for positive inflation. The variables n, k, c, y, u, and v denote equilibrium labour supply, capital, consumption, income, utility and the value function respectively.

Table 2(a): Seigniorage versus Labour Income Tax (Transfer Payments).

	INFLATION					
	-1.65%	5%	10%	20%	48.8%*	100%
Wel. Cost	.023112	.022237	.021728	.020988	.020225	.021276
c	.418605	.420554	.421709	.423417	.425213	.422748
n	.200569	.20148	.20202	.202818	.203658	.202506
k	5.76445	5.79064	5.80616	5.82911	5.85325	5.82012
y	.596739	.59945	.601057	.603433	.605932	.602503
Gov't Revenue						
Income tax /output	0.1	.0932	.0884	.0790	.0578	.0313
Seigniorage/output	0	.0068	.0116	.0210	.0422	.0687

Table 2(b): Seigniorage versus Labour Income Tax (Capital Expenditure).

	INFLATION					
	-1.65%	5%	10%	20%	48.8%*	100%
Wel cost	.015893	.015098	.014637	.013712	.013087	.014656
c	.425054	.426863	.427931	.430111	.431616	.427888
n	.20199	.20282	.20331	.204309	.204999	.20329
k	5.87489	5.8993	5.91371	5.94312	5.96341	5.91312
y	.608177	.610699	.61219	.615235	.617336	.61213
Gov't Revenue						
Income Tax/Output	0.1	.0932	.0884	.0790	.0576	.0313
Seigniorage/Output	0	.0068	.0116	.0210	.0424	.0687

Table 3: Welfare Comparison (Transfer Payments versus Capital Expenditure)

	INFLATION			
	5%	10%	20%	100%
Δ in welfare/Δ in rev. shift(TP)	.12877	.105711	.078908	-.006035
Δ in welfare/Δ in rev. shift(CE)	.116998	.095742	.098635	-.019782
Δ in Welfare (TP - CE) —(1)	.007219	.007139	.006351	.00662
(1)/Δ in rev. Shift	1.0624	1.48265	.67722	.138723

Note: TP denotes transfer payment, and CE denotes capital expenditure.

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Concluding Remarks

The thesis consists of three essays analyzing various aspects of seigniorage. Essay 1 explores the effect of different bank asset portfolios and the use of different kinds of reserve requirement on the government's ability to collect seigniorage. In Essay 2, the effect of deregulating controlled interest rate on seigniorage is investigated and in Essay 3 the effect of different uses of seigniorage on the welfare cost of inflation is explored. Each essay contains a detailed analysis of some aspect of seigniorage and the conclusions from the studies. However, brief concluding remarks on the three essays are warranted here.

In Essay 1, it is shown that the type of asset that the banks reduce in order to comply with an increase in a binding cash reserve requirement is important in achieving a government's objective of boosting its seigniorage capacity by increasing cash reserve requirement. In particular, the analysis reveals that if banks reduce assets that are less "productive" and less "taxed" by the government, the government's objective of enhancing seigniorage by increasing cash reserve requirement will be better achieved. Also, the analysis show that the type of reserve instrument, that cash versus liquid reserve requirement, is important in achieving a government's goal of increasing seigniorage by increasing reserve requirements. The results suggest that the use of cash reserve requirement would be better at enhancing seigniorage than the use of liquid reserve requirement the higher the government's budget deficit, the less the "tax" on the liquid asset, and the less the level of cash reserve requirement.

In Essay 2, the analysis indicates that the effect of interest rate deregulation on seigniorage depends on the sensitivities of the demand for cash and informal deposit to changes in the controlled rates, and the level of cash reserve requirement. Interest rate deregulation is likely to have a positive effect on seigniorage the higher the level of cash reserve requirement, the higher the formal interest elasticity of the demand for informal deposits, and the lower the formal interest elasticity of the demand for cash. The study also indicates that seigniorage can decrease throughout during gradual interest rate deregulation program or increase throughout or increase and decrease at some later stages of interest rate deregulation.

In Essay 3 the effect of different uses of seigniorage on the welfare cost of inflation is explored. It is shown that the welfare cost of inflation is sensitive to the use of seigniorage. Using US data, it is demonstrated that the standard way of modelling seigniorage as transfer payment overestimates the welfare cost of inflation. This is because the calibrations show that public capital expenditure is more desirable than transfer payments from the welfare perspective.