Parallel Kalman Filter Based Time-Domain Harmonic State Estimation

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***Abstract*—A time domain methodology is proposed for the harmonic state estimation (HSE) in power systems based on parallel Kalman filter (PKF) algorithm implemented on a graphical processing unit (GPU). The output variable measurements to be used by the HSE are taken from the simulation of the harmonics propagation in the power system. The time domain HSE solution process is based on the application of the PKF algorithm to estimate the waveforms for nodal voltages and line currents with various sources of harmonics, time-varying harmonics and inter-harmonics. The results obtained with the PKF to solve the HSE are validated against the transient program PSCAD/EMTDC. The PKF algorithm is implemented using the Compute Unified Device Architecture (CUDA) platform and the CUDA Basic Linear Algebra Subprograms (CUBLAS) library on a NVIDIA GPU card. Case studies show the effectiveness of the PKF to solve the HSE on the GPU, the speed-up is dependent of the size and complexity of the network model.**

I. Introduction

The main objective of the Power Quality State Estimation (PQSE) is to determine the power quality indices during the network operation using models and measurements from the system. The PQSE particularly evaluates the power system dynamic operation through the HSE and the transient state estimation (TSE) [1]. Traditionally, the HSE has been evaluated in the frequency-domain [2]. In this contribution the HSE is solved in the time domain by means of a PKF [3], [4]. The PKF algorithm is implemented using the CUDA platform [5], [6] and the CUBLAS [7] library on a GPU and is applied to a power system with time varying harmonic and inter-harmonics sources, making the analysis and the state estimation more convenient in the time-domain as highly distorted waveforms may appear. The network model can be replicated to examine larger systems and to verify the execution time using parallel processing techniques.

The objective of the HSE is to estimate the harmonic levels in the network using a system model and noisy measurements from the system [8]. The model, parameters, measurement points and quantities to be measured are important aspects to be taken into account. The analog signals are sampled and converted to digital form. The sampling frequency is related to the number of points per cycle in the discrete-time solution, thus the measurements have to be synchronized. This HSE requirement can be solved with the measurement time stamping using the global positioning system (GPS) [1]. A determined number of cycles are measured and the synchronized measurements are sent to the control center to be concentrated to estimate. The discrete waveforms of voltages and currents are the input data to the HSE [9].

The HSE follows variations in the waveforms and harmonics by means of the PKF approach using the criterion of minimizing the error between the measured and estimated values [10]. The time varying nonlinear components in the power system originate the injection of harmonics and generate voltage and current waveform distortion [11]. It is important to locate and mitigate the harmonic sources to have better power quality indices. This can be achieved with the HSE assessment estimating the voltage and current harmonics in the nodes and elements of the system [12].

Power system state estimation requires reliable measurement instruments, safe and high-speed communication and computers with enough memory and adequate numerical processing speed to obtain the state estimation results. These same requirements are being implemented practically for the smart grid where the devices for measurement, control, protection and operation generate signals that should be processed quickly to operate the system with safety and efficiency. Thus, the state estimation can be seen as an important component of the smart grid infrastructure [13].

In this contribution, the PKF is implemented on a GPU using parallel programming, as a proposal to evaluate the harmonic state estimation numerical process in a fast and convenient way for the operation of the power systems.

One of the actual programming tendencies is the heterogeneous programming to execute the sequential parts of the algorithms in the CPU cores and the steps that are possible to be executed in parallel are programmed for execution on GPUs; mainly when the dimensions of the related algorithms

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and matrices are significantly large and the associated matrix operations are possible to be evaluated in parallel form to reduce the execution time. The GPU acts as a coprocessor of the CPU for the parallel parts of the algorithms [14].

The NVIDIA CUDA (Compute Unified Device Architecture) is a hardware-software platform that can be used to execute parallel algorithms with program code in C++ which is augmented with special commands to use the GPU hardware [5]. The sequential parts of an algorithm are executed in the CPU or host while the parts that are convenient to be executed in parallel are run in the device using kernels, the parallel functions for the GPU. When a kernel is executed, blocks with equal number of threads are created to execute the parallel function. Blocks of threads form a grid [6], [15].

The CUBLAS library is a parallel equivalent of the Basic Linear Algebra Subprograms (BLAS) library to evaluate vector and matrix operations of linear algebra on a GPU using the CUDA platform and parallel programming [7].

An important fact in parallel programming using a GPU is the necessary exchange of data between the CPU and the GPU, as indicated in Fig. 1. This flow of data takes some time and is part of the program execution time. To decrease the execution time, the flow of data between the CPU and the GPU is minimized allocating first the necessary data to execute the Kalman filter (KF) algorithm, then these data are numerically processed and the results are saved back in the CPU each time step, when the simulation is completed, the GPU memory is cleared [7].

This paper proposes a KF methodology to assess the HSE using a CPU-GPU parallel program, which is presented in Section II; Section III presents the case studies for harmonics, variable harmonics and inter-harmonics, Section IV draws the main conclusions of this work.

II. Methodology

The PKF evaluates the HSE by means of the following steps:

1) Data allocation in GPU memory.
2) Recursive execution of PKF on GPU.
3) The HSE result is saved each time-step.
4) The GPU memory is free when the case study ends.

The linear state-space model for a power system can be represented by means of a first-order ODE set, as,

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}, \quad (1)$$

$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{u}. \quad (2)$$

An iterative process can be defined for the discrete-time case, by converting of the state-space model from continuous-time to discrete-time, approximating the differential equations to difference equations [16], i.e.,

$$\boldsymbol{x}_{k+1} = \boldsymbol{F}_k \boldsymbol{x}_k + \boldsymbol{B}_k \boldsymbol{u}_k + \boldsymbol{v}_k, \quad (3)$$

$$\boldsymbol{y}_k = \boldsymbol{C}_k \boldsymbol{x}_k + \boldsymbol{w}_k, \quad (4)$$

where $\boldsymbol{F}$ is the state transition matrix, $\boldsymbol{B}$ the control-input matrix, $\boldsymbol{u}$ the input vector, $\boldsymbol{C}$ the output matrix, $\boldsymbol{y}$ the output vector, $\boldsymbol{v}$ is the process noise and $\boldsymbol{w}$ is the measurement noise. These noises regularly are considered stationary, Gaussian, zero averaged, and uncorrelated:

$$\boldsymbol{w}_k \approx N(0, \boldsymbol{R}_k), \quad (5)$$

$$\boldsymbol{v}_k \approx N(0, \boldsymbol{Q}_k). \quad (6)$$

$\boldsymbol{Q}$ and $\boldsymbol{R}$ are the process and measurement noise covariance matrices, respectively.

The expressions (1) and (3) are related to the system dynamics.

The linear measurement state estimation equation is,

$$\boldsymbol{z} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{e} \quad (7)$$

where $\boldsymbol{z}$ is the measurements vector, $\boldsymbol{x}$ the state variables vector, $\boldsymbol{H}$ the measurement state estimation matrix relating the measurements to state variables and $\boldsymbol{e}$ is the error state estimation vector, which is the difference between the estimated and actual or real measurement values [17].

A. The Parallel Kalman Filter using CUBLAS

The KF estimator is based on the concept that it evaluates the estimates for the state variables by filtering the process and measurement noises.

The KF uses the matrices of the state space formulation (3, 4) and the measurement equation (7), to follow the dynamics of the system in the time-domain and to evaluate the state and output variables in the time interval under consideration, taking into account the measurements from the system, which can be contaminated with noise. Fig. 1 indicates the KF numerical process and Table I indicates the order of the associated matrices, with n states, i inputs and m measurements [18].

Sources, harmonic and inter-harmonic levels in a power system vary with time and can be followed with the KF algorithm.

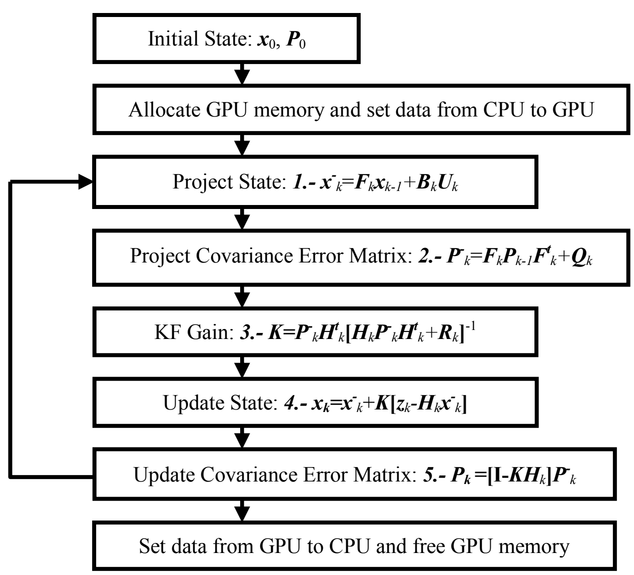

Figure 1. Kalman filter algorithm

TABLE I. MATRIX ORDER KF ALGORITHM

X	*n*	***B***	*n×i*	***K***	*n×m*
P	*n×n*	***U***	*i*	***H***	*m×n*
F	*n×n*	***Q***	*n×n*	***R***	*m×m*

The PKF is designed to execute the matrix operations as inversion, multiplication, addition and subtraction in the algorithm each time step in parallel form. The matrices and vectors indicated in Table I are allocated and set with initial values, then the recursive KF algorithm is executed on the GPU using the CUBLAS library, the result of the state estimation is sent from the GPU to CPU.

The CUBLAS functions to evaluate the steps of the KF algorithm in Fig. 1 are indicated in Table II. They assess vector-vector sum (Daxpy), matrix-vector multiplication (Dgemv), matrix-matrix sum (Dgeam), matrix-matrix multiplication (Dgemm), these functions are implemented in the CUBLAS library to be executed in parallel form on the GPU [7].

The inverse matrix in Step 3 of the KF algorithm is evaluated with the LU decomposition and forward-backward substitutions, the CUBLAS functions used to implement this decomposition are Dscal and Dger. These functions evaluate the Crout's reduction algorithm and the Dtrsm function implements the forward and backward substitutions [7]. These functions are executed in parallel form on the GPU. This step consumes most of the time execution of the KF algorithm and is calculated each time-step of the state estimation [19].

III. TIME-DOMAIN SIMULATION CASE STUDIES

Fig. 2 illustrates the modified IEEE 14 bus test system for the case studies; a base power of 100 MVA and a base voltage of 132 kV are assumed.

The lines 1-2, 1-5, 2-3, 2-4, 2-5, 3-4 and 4-5 are represented by the *pi* model, and the rest of the lines by a series impedance; the transformers 4-9, 5-6, 4-8-9, are represented by an inductive reactance, according to the IEEE 14 bus test system [20]. The generators are represented by sinusoidal voltage sources at constant fundamental frequency, inductive loads are connected to the indicated loads according to the test system specification.

The test system is modified including harmonic sources connected to nodes 5 and 13 to represent nonlinear loads that inject harmonic currents to the system; under this condition the HSE can be evaluated using the PKF algorithm [11].

A set of 43 first-order differential equations representing (1) is solved to obtain the time-domain harmonic propagation using the fourth-order Runge-Kutta (RK4) method with a time-step of 512 points per cyle or 32.55 µs. Table III shows the state variables and the output variables to be measured.

TABLE II. CUBLAS FUNCTIONS FOR THE PKF ALGORITHM

Step	CUBLAS functions
1	Dgemv, Daxpy
2	Dgemm, Dgeam
3	Dgemm, Dgeam, Dger, Dscal, Dtrsm
4	Dgemv, Daxpy
5	Dgemm, Dgeam

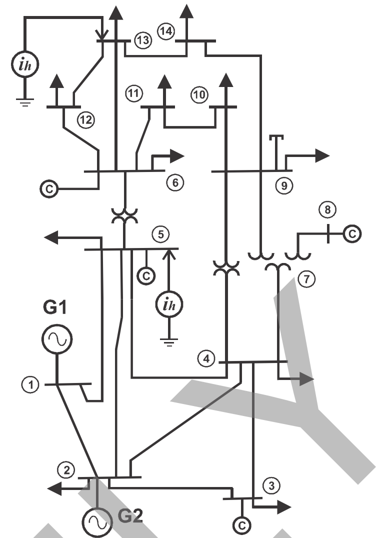

Figure 2. Modified IEEE 14 bus test system with harmonic sources connected to nodes 5 and 13.

The HSE is evaluated taking the observation or measurement equation $z=Hx$ into the PKF algorithm as shown in Fig. 1. The measurement vector z takes the 38 measurements shown of Table III, 15 line currents, 11 load currents and 12 nodal voltages. With this set of measurements the observation equation is under-determined, 38 states are observable while 5 are partially observable. The PKF estimates the state variables in a minimum mean squared error sense between the real and estimated values.

A. *Harmonic state estimation with harmonic sources at nodes 5 and 13*

Table IV shows the harmonic injections, which can be present during the normal operation of the power system when nonlinear loads are connected, e.g., a AC/DC converter, a HVDC link or an uninterruptible power supply [21].

The PKF is able to estimate the state variables using the first-order ODE model of the system and the available measurements, which are added with a stationary and Gaussian noise having a 2% signal-to-noise ratio (SNR).

A close agreement is obtained between the true and proposed PKF responses; the maximum difference is approximately of 2% due to the noisy measurements used in the PKF algorithm. Fig. 3 shows the true and estimated state variables and their difference for the line currents. The true values were obtained from a harmonic propagation simulation

TABLE III. STATE AND MEASURED VARIABLES

Description	State Variables	Measured Variables
Line current	1-20	1-15
Nodal voltage	21-32	21-32
Load current	33-43	33-43

TABLE IV. HARMONIC INJECTIONS NODES 5 AND 13

Node	5				13			
Harmonic	5	7	11	13	5	7	11	13
Peak Value A	6	3	1.5	0.75	3	1.5	0.75	0.37

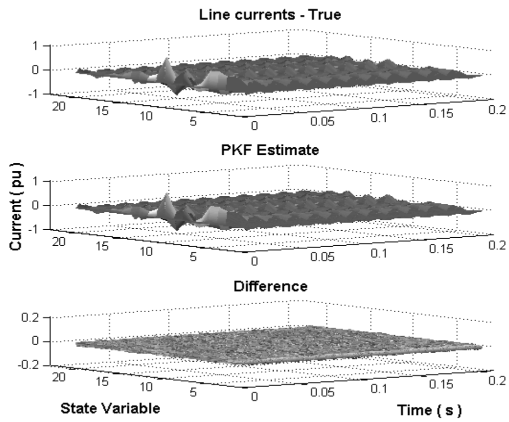

Figure 3. Actual, PKF estimate, and difference for line currents.

using the state-space model and the RK4 method with a time step of 32.55 μs.

The generator currents at nodes 1 and 2 are calculated using the estimates for the line currents and applying the KCL to evaluate the total harmonic distortion (THD). Fig. 4 shows these currents, true and estimate waveforms are represented and validated by direct comparison against the PSCAD/EMTDC response. A close agreement is obtained between the waveforms for the proposed PKF methodology.

Fig. 5 illustrates the harmonic content for the generator currents obtained by applying the discrete Fourier transform (DFT). The THD in the current is of 7.33% and 3.57% for generators nodes 1 and 2 respectively. This harmonic distortion is originated due to the harmonic currents injected in nodes 5 and 13, they flow through the lines and reach the generators and loads, affecting adversely their operation.

B. Harmonic state estimation with time varying harmonics

Time varying harmonic currents are present during the operation of an electrical system, e.g., a transient condition of a nonlinear component [22]. The injected harmonics of the previous case are increased during the last six cycles, to simulate and estimate this time varying harmonic condition.

The harmonic magnitude is increased two times of their initial values in nodes 5 and 13 as shown in Table V. This condition verifies the use of the PKF to estimate the variant harmonics using the state-space model and noisy measurements from the system. The signal-to-noise ratio SNR remains constant and equal to 2%.

Fig. 6 presents the waveforms for currents in lines 2-5 and 5-6; these lines are connected to node 5 where the variant harmonic current is present. The harmonic distortion changes according to the new injection of harmonics. The fluctuation starts at cycle 6 or 0.1 s, and the PKF estimates the harmonic variable condition; the estimated currents agree with the actual values.

Fig. 7 shows the waveforms for the generator currents under the time-varying harmonic condition which are evaluated using the estimated line currents; the true and estimate values closely agree. The harmonic content is obtained applying the DFT as shown in Fig. 8; the generator node 1 THD changes from 7.95% to 14.73% and for generator node 2 the THD changes from 4.26% to 7.22%.

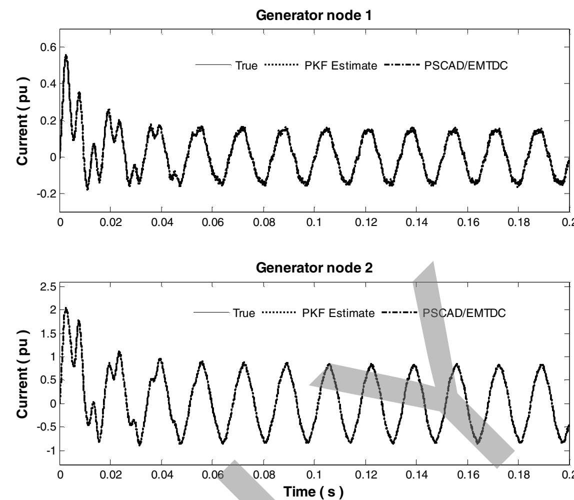

Figure 4. Actual, PKF estimate, and PSCAD/EMTDC response for generator currents at nodes 1-2.

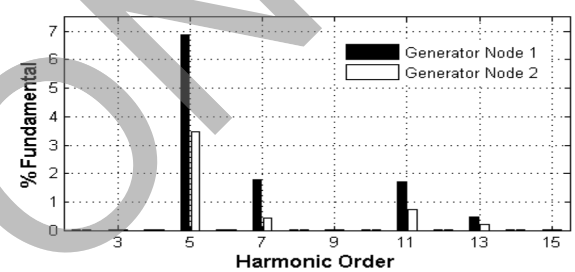

Figure 5. Harmonic spectrum of generator currents at nodes 1-2.

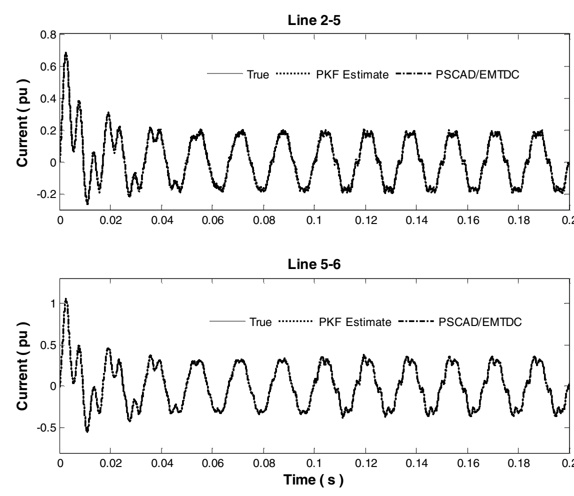

Figure 6. Actual, PKF estimate, and PSCAD/EMTDC response for time varying harmonics in line currents 2-5 and 5-6.

C. Harmonic state estimation including interharmonics

The interharmonics are components of a waveform between the harmonics of the fundamental frequency, which can be discrete frequencies or have a wide spectrum. They have an adverse effect on the power quality originating heating, flicker, interference and saturation [23].

TABLE V. TIME VARYING HARMONIC INJECTIONS NODES 5 AND 13

Node	Cycles	Harmonic	Peak Value (Amps)
5	1-6	5	6
		7	3
		11	1.5
		13	0.75
5	7-12	5	12
		7	6
		11	3
		13	1.5
13	1-6	5	3
		7	1.5
		11	0.75
		13	0.375
13	7-12	5	6
		7	3
		11	1.5
		13	0.75

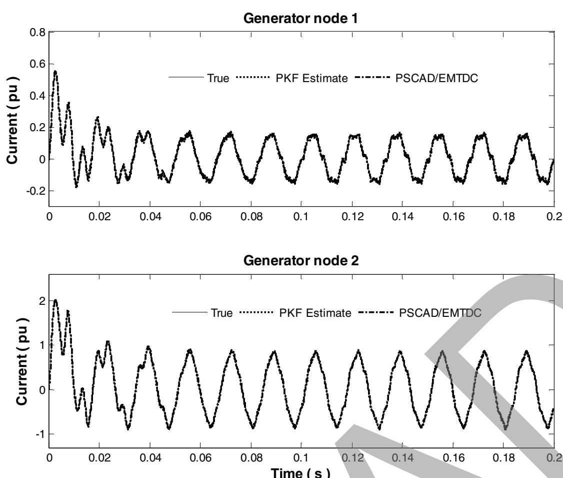

Figure 7. Actual, PKF estimate, and PSCAD/EMTDC response for time varying harmonics in generator current nodes 1-2.

The interharmonics are generated by nonlinear loads such as frequency converters, cycloconverters, speed drives and arc furnaces [24].

The injected harmonics in nodes 5 and 13 are modified to include a subharmonic of 30 Hz with a magnitude of 50 A. Also an interharmonic of 90 Hz and 25 A of magnitude is injected to analyze the effect on the line currents using the PKF-HSE methodology. Fig. 9 shows the actual, PKF estimate and PSCAD/EMTDC waveforms for generator current in nodes 1-2.

The harmonic distortion is high due to the interharmonics injection. A close agreement between the actual and estimated waveforms is obtained. The DFT is applied again to obtain the harmonic content. Generator node 1 presents a THD of 24.98%, and the generator node 2 of 11.47%. In Fig. 10 the magnitude of the fundamental harmonic is 1.0 pu, and the

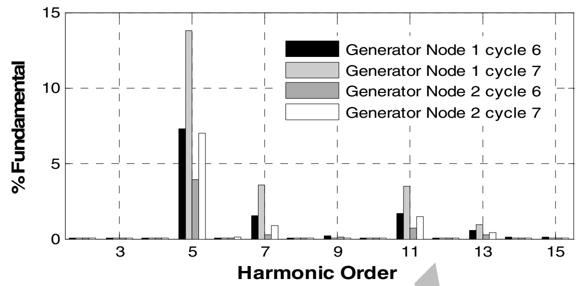

Figure 8. Harmonic spectrum for time varying harmonics in generator current nodes 1-2.

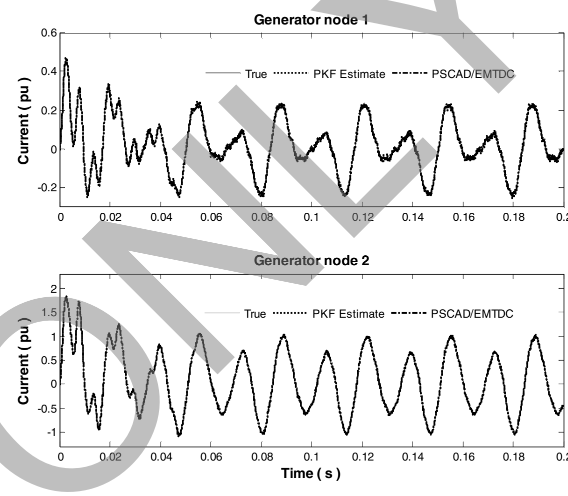

Figure 9. Actual, PKF estimate, and PSCAD/EMTDC response during subharmonics and interharmonics for generator currents at nodes 1-2.

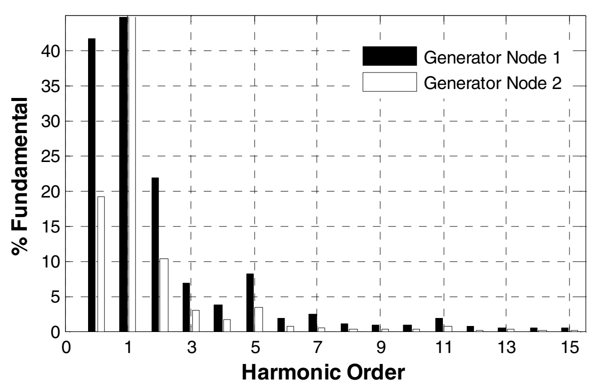

Figure 10. Harmonic spectrum, subharmonics and interharmonics at generator currents at nodes 1-2.

waveforms have a dc component with even harmonics present, which is due to the interharmonic condition.

D. *CPU-GPU Specifications and execution time*

The time-domain HSE case studies were implemented on an Intel Core TM i7-3770 CPU, 3.4 GHz, 16.0 GB RAM with a NVIDIA GeForce GTX 680 GPU. The GPU characteristics are given in Table VI.

TABLE VI. NVIDIA GEFORCE GTX 680 GPU CARD DATA

Processor Cores	1536
GPU Clock rate	1.08 GHz
Memory	2.0 GB
Memory Clock rate	3.0 GHz
Memory Bus Width	256-bit

TABLE VII. EXECUTION TIME (S)

Models	**CPU C++ code**	**CPU-GPU CUBLAS**	**Speed-up**
1	36.29	22.5	1.61
2	434.72	52.24	8.32
3	1965.01	79.13	24.83

The power system model was replicated three times to create larger systems, and the execution time was measured, Table VII presents the results on the execution time. The speed-up increases with the size of power system models as is expected using the parallel processing CPU-GPU and CUBLAS compared against the sequential CPU C++ code.

IV. CONCLUSIONS

A time domain harmonic state estimator based on the application of the PKF implemented using the CUDA platform and the CUBLAS library on a GPU has been proposed. The results obtained with the PKF methodology have been validated against the actual system response and the PSCAD/EMTDC simulation obtaining a close agreement in all cases.

The power system has been represented by a set of first order differential equations modeling the components of the network and the state estimation solution in the time domain has been based on the PKF algorithm. The waveforms of the estimated variables have been obtained with the proposed time domain PKF-HSE methodology and their harmonic content evaluated with the DFT.

The proposed HSE method using the PKF requires a model and a set of synchronized output measurements from the system in order to estimate the state variables; the estimate output variables are in turn calculated to be compared with the measured output variables.

The power system HSE with PKF has been evaluated for computational efficiency on the GPU to demonstrate speed-up in comparison with sequential execution.

The results can be used, for instance, to locate and design the installation of filters and to determine the harmonic flows in the power system.

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