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Topics in Charm Meson Decays

By

Farhad Ghoddoussi ©

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment of the requirements for the degree of

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In The Name of God

The Most Merciful, The Most Compassionate

And of everything created We two Kinds;

haply you will remember.

Koran 51:49

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FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled "Topics in Charm Meson Decays" submitted by Farhad Ghoddoussi in partial fulfilment of the requirements for the degree of Master of Science in Theoretical Physics

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To my late grandmothers

Hosniyeh and Nosrat.

Abstract

Through a specific example of two-body color-favored charm decay, $D_s^+ \to \phi\pi^+$, we illustrate how an effective and complex (unitarized) a_1, denoted by $a_1^{U,eff}$, may be defined such that it includes nonfactorized, annihilation and inelastic final state interaction (fsi) effects. The procedure can be generalized to color-suppressed processes to define an effective, and complex $a_2^{U,eff}$. We determine $|a_1^{U,eff}|$ and, where relevant, $|a_2^{U,eff}|$ for $D \to \bar{K}\pi, \bar{K}\rho, \bar{K}^*\pi, D_s^+ \to \eta\pi^+$, $\eta'\pi^+$, $\eta\rho^+, \eta'\rho^+$, and for $B^0 \to D^-\pi^+$ and $D^-\rho^+$ from the hadronic and semileptonic decay data.

After discussing how our view of the phenomenological constants a_1 and a_2 have evolved, we have concluded that , treating the phenomenological parameters a_1 and a_2 as process independent is untenable. It does not explain the experimental data. For the processes which involve only one Lorentz structure, it is always possible to define an effective and complex (unitarized) $a_1^{U,eff}$ and $a_2^{U,eff}$ that include nonfactorized, annihilation and inelastic final state interaction effects. A corollary of our point of view is that the purported test of factorization that compares the hadronic rate to the semileptonic should be used instead as a tool to determine the modulus of these effective parameters.

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Chapter 1

Introduction

Historically weak decays have always been a rich source of information about the form and symmetry of the basic interactions as well as about the structure of the constituents of matter. Weak decays are essential for testing the Standard Model, specially its $SU(2) \otimes U(1)$ electroweak sector, theory of electromagnetic and weak interactions. This model has been very successful for the last thirty years, although all features of this theory have not been verified. The heavy quark sector, for example, is still far from being well established since many important parameters are not or only poorly known, like the charged current couplings of heavy quarks.

Therefore the physics of heavy quarks, charm and beauty (they are heavy relative to the QCD mass scale Λ_{QCD}), presents a great environment for testing and studying the Standard Model. The study of heavy quarks involves examining the weak interaction under the influence of strong interaction, as the heavy quarks are not free but are necessarily confined inside hadronic bound states. As a result of this, obtaining any new information about the standard model, from the weak decays of heavy quarks is not straight forward.

Strong interaction which is responsible for the binding of quarks and gluons in hadronic states is described by Quantum Chromodynamics (QCD). The confinement mechanism is still not understood completely; weak decays of heavy quarks thus also offer a chance for achieving a deeper insight into strong interaction dynamics.

It is assumed that the short distance nature of weak decays allows one to separate the possible corrections from strong interactions into short and long distance contributions. The asymptotic-freedom property of QCD allows a perturbative calculation of the short distance corrections. They arise from the exchange of hard gluons and modify the structure of the weak interaction hamiltonian. The light quarks produced in the decay of a charm or beauty quark necessarily have to combine with the spectator quark to form color singlets i.e. hadrons. This is a non-perturbative process and, so far cannot be calculated from first principles . We therefore have to rely on phenomenological approaches. In other words,we assume
(1) the short distance corrections are associated with the effective weak Hamiltonian H^{W}_{eff}, and (2) all long distance effects are absorbed in the initial and final states.

The interplay between weak and strong interactions at short and long distances has been studied for a long time in strange quark decays. Many features of strange quark decays, e.g. the $\Delta I = \frac{1}{2}$ rule, have never been fully understood and the disagreement between theoretical predictions and experimental findings has been attributed to the unknown long distance corrections. The situation is more encouraging in charm and beauty decays where the mass of the decaying quark may be large enough that long distance corrections become unimportant for the understanding of the decay dynamics. This has turned out to be essentially the case although hadronization still plays a more important role in charm decays than anticipated. The best known example of it is the life time difference between the various charmed mesons, namely D^0, D^+ and D_s^+. Beauty decays seem to be dominated by short distance dynamics and they are therefore a very important laboratory for testing the standard model and looking for new physics.

In the next chapter we briefly review the standard model, including the electroweak and strong interactions and, also detail what are the reasons and motivations for studying heavy flavors.

In chapter 3 the effective low energy weak Hamiltonian, with the inclusion of short distance QCD effects (Hard gluon effects), is examined. Using this effective Hamiltonian in the context of "Valence Quark Approximation" model, we study the inclusive, semileptonic and nonleptonic weak decays of charmed mesons. As a first approximation inclusive weak decays allow us, to ignore non-perturbative effects such as the hadronization of the produced quarks and final state interactions between produced hadrons. By treating the light quark in the initial meson as inert quark, that is it does not participate in the decay, the calculations can be done at essentially free quark level.

In chapter 4, by using the factorization model and by introducing the phenomenological form factors and constants a_1 and a_2 we study the exclusive weak decays of charmed mesons. Comparison with experiments shows that overall, the factorization model works well and its predictions for the decay rates of many channels are in good agreement with experiment, but there are some channels where it's predictions fail badly . The possible remedies are to consider the final state interactions or to consider non-factorized effects.

In chapter 5, we introduce the non-factorized phenomenological form factors in close analogy with the introduction of their counterparts in the factorization model. We also introduce the effective phenomenological constants a_1^{eff} and a_2^{eff}, and show how non-factorized and annihilation effects can be absorbed in the phenomenological factors a_1^{eff} and a_2^{eff} . Through a specific example, $D_s^+ \to \phi\pi$, we examine the effect of final state interactions on the effective constants a_1^{eff} and a_2^{eff}. We conclude that by a moderate change in the phenomenological constants a_1 and a_2, which can be due to the effects of annihilation, non-factorization and final state interactions, we can restore the consistency between the theoretical predictions and experiments in the exclusive decays where the factorization model fails to predict results in agreement with experiment. In chapter 6 we give summary and conclusion.

Chapter 2

A Brief Introduction to the Standard Model

2.1 Introduction

The Standard Model ties the theory of electroweak interactions of Glashow-Weinberg-Salam [1, 2] and Quantum chromodynamics (QCD), the theory of strong interactions [3] in a unified non-abelian gauge theory based on $SU(3) \otimes SU(2) \otimes U(1)$ group. (For a detailed review see [4]).

The basic constituents of matter are quarks and leptons (both fermions)and the fundamental interactions are mediated by the gauge particles (bosons). See Tables (2.1) and (2.2) (There are also corresponding antiparticles for each quark and lepton which are not shown in Table (2.1)).

In the Standard Model three of the four known interactions, namely, the strong, electromagnetic and weak are described in a common framework. They have yet to be pulled together into a grand unified theory incorporating the gravitational interaction. The quarks interact via all the four interactions, the charged leptons feel all except the strong interactions while the neutrinos, which by construction are massless in the standard model, are only sensitive to the weak interactions.

Table 2.1: Gauge Particles

Interaction	Mediator	Mass	Spin
Strong	gluons, g	0	1
Electromagnetic	photon γ	0	1
Weak	$W^{\pm}, Z^0$	$\sim 80, 91$ GeV	1
Gravitational	graviton, G	0	2

Table 2.2: Quarks and Leptons

	Generation				
Particle	1	2	3	Charge	Spin
Quarks:	$\begin{pmatrix} u \\ d \end{pmatrix}$	$\begin{pmatrix} c \\ s \end{pmatrix}$	$\begin{pmatrix} t \\ b \end{pmatrix}$	$+\frac{2}{3}e$ $-\frac{1}{3}e$	$\frac{1}{2}$ $\frac{1}{2}$
Leptons:	$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$	$\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}$	$\begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$	0 -1	$\frac{1}{2}$ $\frac{1}{2}$

2.2 Electromagnetic and Weak interactions

The electromagnetic and weak interactions are described by a unified gauge theory [1] based on the group $SU(2) \otimes U(1)$. The left handed fermion fields form doublets with respect to the $SU(2)$ of weak isospin, whereas the right handed components are kept as singlets. Left and right-handed field components are defined by $\psi_{\substack{L \\ R}} = \frac{1}{2}(1 \mp \gamma^5)\psi$, where ψ is the four component Dirac spinor of the fermion.

Corresponding to the four generators of the group $SU(2) \otimes U(1)$ four gauge bosons exist, which are the quanta of electroweak interaction. Renormalizability of the theory requires the vector bosons to be massless [5], a theoretical necessity which is at variance with the observation of only one long-range force in nature (disregarding gravity) namely electromagnetism. Thus the problem is to give mass to the gauge fields without destroying the renormalizability of the theory. The only mechanism known is spontaneous symmetry breaking. Without going into detail we remark that in the standard model spontaneous symmetry breaking is achieved via the Higgs mechanism [2, 5]. One adds a complex scalar doublet $\varphi = \begin{bmatrix} \varphi^+ \\ \varphi^0 \end{bmatrix}$, to the theory with gauge invariant couplings to the vector bosons and fermions, and a potential of the form $V = \mu^2\varphi^\dagger\varphi + \lambda(\varphi^\dagger\varphi)^2$. For $\mu^2 < 0$, V acquires its minimum for a nonzero vacuum expectation value of φ. In order not to violate charge conservation one must choose $< 0|\varphi|0 >= \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ V \end{bmatrix}$. Thus the $SU(2) \otimes U(1)$ symmetry is spontaneously broken by the vacuum, however in such away that a (new) $U(1)$ symmetry remains intact. [1] The latter is identified with the gauge symmetry of electromagnetism. As a consequence three of the four gauge fields, the $W^\pm$ and Z^0 bosons, became massive through their couplings to the scalar field, whereas one gauge boson, the photon stays massless.

[1]The generator of this new $U(1)$ symmetry is a mixture of the two neutral generators of the original $SU(2) \otimes U(1)$. The parameter which describes this mixing is the Weinberg angle θ_W.

The masses of $W^{\pm}$ and Z^0 bosons in terms of θ_W (Weinberg angle) are :

$$m_W = \sqrt{\frac{\pi\alpha}{\sqrt{2}G_F \sin\theta_W{}^2}}$$

$$m_Z = \frac{m_W}{\cos\theta_W}$$

where α is the fine structure constant [6]

$$\frac{1}{\alpha} = 137.035963(15) \quad (2.1)$$

G_F is the Fermi constant, which is known to a high precision through measurements of the μ lifetime [6]

$$G_F = 1.16639920 \times 10^{-5} GeV^{-2} \quad (2.2)$$

M_Z and M_W are experimentally measured to be [6]:

$$M_W = 80.22 \pm 0.26 GeV$$

$$M_Z = 91.187 \pm 0.007 GeV$$

which are in good agreement with the theoretical predictions.

The Feynman rules for tree-level vertices of the $SU(2) \otimes U(1)$ gauge structure are depicted in Fig. (2.1). The electromagnetic interaction occurs through vector currents. The weak charged current interaction has a (V - A) (V for vector and A for axial vector) structure. The weak neutral current is part vector, part axial vector, where the couplings C_V^f and C_A^f are analogous to a 'weak-charge' and are predicted within the context of the standard model in terms of the charge of the fermion Q_f and the third component of weak isospin, T_3^f as specified in Fig. (2.1).

In this work we are concerned with the flavor changing weak decays which proceed via charged current interactions, that is to say via exchange or radiation of $W^{\pm}$ bosons. The relevant term of the fundamental Lagrangian is [2]:

$$L_{CC} = \frac{g_W}{2\sqrt{2}} \left[(J_{CC}^{-})^{\mu} W_{\mu}^{+} + (J_{CC}^{+})^{\mu} W_{\mu}^{-} \right] \quad (2.3)$$

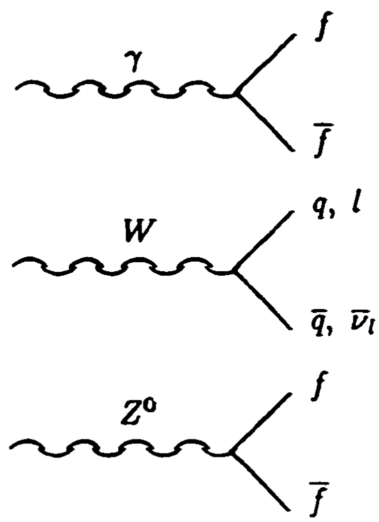
QED		$-ie\, Q_f\, \gamma^\mu$	V
EW - CC		$\frac{-ig}{\sqrt{2}}\, \gamma^\mu\, \frac{1}{2}(1-\gamma^5)$	V−A
EW - NC		$\frac{-ig}{\cos\theta_W}\, \gamma^\mu\, \frac{1}{2}\left(c_V^f - c_A^f\gamma^5\right)$	part V part A

Figure 2.1: Feynman rules for tree-level vertices in electroweak interactions where $g = \frac{e}{\sin\theta_W}$ and c_V and c_A are the vector and axial-vector couplings, $c_V^f = T_3^f - 2Q_f \sin^2\theta_W$ and $c_A^f = T_3^f$, where T_3^f = weak isospin, $\frac{1}{2}$ or $-\frac{1}{2}$.

where the charged currents $(J_{CC}^{\pm})^\mu$ are given by

$$(J_{CC}^{-})^\mu = (\bar{u},\bar{c},\bar{t})\gamma^\mu(1-\gamma^5)V_{CKM}\begin{pmatrix} d \\ s \\ b \end{pmatrix} + (\bar{\nu}_e,\bar{\nu}_\mu,\bar{\nu}_\tau)\gamma^\mu(1-\gamma^5)\begin{pmatrix} e^- \\ \mu^- \\ \tau^- \end{pmatrix} \tag{2.4}$$

and

$$(J_{CC}^{+}) = (J_{CC}^{-})^\dagger \tag{2.5}$$

where V_{CKM} is the Cabibbo-Kobayashi-Maskawa quark-mixing matrix:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{2.6}$$

$W_\mu^{\pm}$'s are vector fields (similar to A^μ the vector field for photon) representing the vector bosons $W^{\pm}$.

V_{CKM} is inserted in the charged current, because the weak eigenstates and flavor (mass) eigenstates are not the same. The quark weak eigenstates are related to the mass eigenstates by a generalized rotation, specified by the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix [7], which in the standard model is necessarily unitary. The elements of this matrix act as coupling constants for the relevant pair of quarks.

By convention, the $Q = +\frac{2}{3}$ quarks are unmixed i.e. the weak and flavor eigenstates are the same, while the mixing in the $Q = -\frac{1}{3}$ sector is specified by the matrix V_{CKM} that is :

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{2.7}$$

where (d', s', b') are the weak eigenstates and (d, s, b) are the flavor eigenstates. In other words weak isospin doublets are:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \begin{pmatrix} u \\ d' \end{pmatrix}_L ; \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \begin{pmatrix} c \\ s' \end{pmatrix}_L ; \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \begin{pmatrix} t \\ b' \end{pmatrix}_L \tag{2.8}$$

The unitary V_{CKM} matrix is usually parametrised by three real angles and one complex phase (CP-violating phase) [6, 8].

Unfortunately no theory exists to predict the values of the CKM matrix elements. They may however be measured and constrained experimentally. At 90% confidence limit the magnitude of the CKM matrix elements are [6]:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.9747\,to\,0.9759 & 0.218\,to\,0.224 & 0.002\,to\,0.005 \\ 0.218\,to\,0.224 & 0.9738\,to\,0.9752 & 0.032\,t0\,0.048 \\ 0.004\,to\,0.015 & 0.030\,to\,0.048 & 0.9988\,to\,0.9995 \end{pmatrix} \tag{2.9}$$

At present the CKM matrix elements are as fundamental as quark masses in the standard model. A better understanding of the matrix elements is essential and could give us a sneak preview to new physics beyond the standard model. For example, the measurements suggest a pattern as noted by Wolfenstein [9]:

$$\begin{pmatrix} 1 & \lambda & \lambda^3 \\ -\lambda & 1 & \lambda^2 \\ -\lambda^3 & -\lambda^2 & 1 \end{pmatrix} \tag{2.10}$$

(ignoring CP violation) with $\lambda \sim \sin\theta_c \sim 0.22$. This striking pattern hints at perhaps a hierarchy of the couplings between quark generations, perhaps a new symmetry beyond standard model.

2.3 Strong Interactions

Quantum Chromodynamics (QCD), the gauge theory of strong interactions [10], is based on the group $SU(3)$. Each quark flavor exists in three colors which form a $SU(3)$ triplet, $q = (q_1, q_2, q_3)$. The leptons, of course, are color-neutral. Corresponding to the eight generators of $SU(3)$ one has a color-octet of gauge bosons, the gluons G^a_μ $(a = 1, 2, \cdots 8)$. The latter are flavor-neutral. In contrast to the spontaneously broken electroweak gauge symmetry, the $SU(3)$-color symmetry is assumed to be unbroken. This means that the gluons are massless [11]. Apart from its non-abelian nature, the QCD Lagrangian resembles the familiar Lagrangian of QED. Explicitly,

$$L_{QCD} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \sum_q \bar{q}_i \left[i\gamma_\mu D^\mu_{ij} - m - q\delta_{ij}\right] q_j \,, \tag{2.11}$$

where the field strength $F^a_{\mu\nu}$ is defined by

$$F^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s f^{abc} G^b_\mu G^c_\nu \tag{2.12}$$

and the covariant derivative D^μ_{ij} is given by

$$D^\mu_{ij} = \partial^\mu \delta_{ij} + i g_s \frac{\lambda^a_{ij}}{2} G^{a\mu} \,. \tag{2.13}$$

In the above λ^a $(a = 1, 2, \cdots 8)$ are the usual $SU(3)$ matrices (generators) with the commutation relations

$$[\lambda^a, \lambda^b] = 2i f^{abc} \lambda^c \tag{2.14}$$

and the normalization

$$Tr(\lambda^a \lambda^b) = 2\delta^{ab} \,. \tag{2.15}$$

f^{abc} denote the $SU(3)$ structure constants. The essential new feature is the self-interaction of the gluons which reflects the non-abelian structure of QCD. m is a diagonal 3×3 mass matrix.

QCD is believed to show completely different behavior at short and long distances. It is shown by using renormalization group analysis [12] that QCD is an asymptotically free theory. This means at small distances or equivalently, at large momentum transfers, quarks and gluons interact relatively weakly. The effective strong coupling $\bar{g}_s\left(\frac{Q}{\mu}, g_s\right)$ (g_s is the renormalized coupling constant of QCD defined at the scale μ, that is $\bar{g}_s(1, g_s) = g_s$) or equivalently $\alpha_s(Q^2)$ defined as

$$\alpha_s(Q^2) \equiv \frac{\bar{g}_s^2\left(\frac{Q}{\mu}, g_s\right)}{4\pi} \tag{2.16}$$

changes with the scale Q at which it is probed according to the following equation

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \alpha_s(\mu^2)\frac{b}{4\pi}\ln\frac{Q^2}{\mu^2}} \tag{2.17}$$

or equivalently

$$\alpha_s(Q^2) = \frac{4\pi}{b \ln \frac{Q^2}{\Lambda_{QCD}^2}} \tag{2.18}$$

where $b = 11 - \frac{2}{3}N_F$ and N_F is the number of the 'open" flavors. Both of the above expressions for the running coupling constant are commonly used. The value of $\alpha_s(\mu^2)$ or, equivalently, the scale Λ_{QCD} must be determined from experiment. One typically finds [3]

$$\Lambda_{QCD} \approx (100 - 500)MeV \tag{2.19}$$

The essential point of Eqs. (2.17) and (2.18), is the logarithmic decrease of the effective strong coupling constant as one goes to higher momenta. This is simply the statement of asymptotic freedom. An important technical consequence is the applicability of perturbation theory to strong interaction phenomena at sufficiently short distances.

Conversely, as the momentum scale is lowered the effective strong coupling increases. It is conjectured that at large separations the interactions between colored fields eventually become so strong that it would require infinite energy to separate color charges. If this is true, quarks and gluons are permanently confined within color neutral bound states, the hadrons. Since confinement [11] is an entirely non-perturbative phenomenon (large coupling region), no real quantitative technique to calculate confinement aspects of QCD exists at present apart from the lattice approach [13]. Hence, for phenomenological applications of QCD it is important to know roughly at which scale the transition from confinement to the asymptotically free regime takes place. From the early onset of approximate scaling in deep inelastic scattering, a typical consequence of asymptotic freedom [3], one expects the transition to occur at $Q^2 \sim few\ GeV^2$. On the other hand characterizing the confinement region by the distance $R = \frac{1}{Q}$, at which $\alpha_s(Q^2) \approx 1$, one finds from Eqs. (2.17) and (2.19), $R \approx (0.2 - 1)$fermi, that is the typical size of hadrons.

2.4 Why Study Heavy Flavors?

The fundamental parameters of the standard model are the electromagnetic coupling constant or the fine structure constant α, the fermi coupling constant G_F, the strong coupling constant of QCD, α_S. Two parameters are necessary to describe the weak coupling constant and now that m_Z has been measured to better precision than $\sin^2\theta_W, m_Z$ and G_F are usually chosen as the weak coupling fundamental parameters of the standard model.

There are also at least 16 more parameters of the standard model which hopefully are not fundamental. These include the mass of 6 quarks and 3 leptons (neutrinos are considered massless), the Higgs mass, the quarks mixing (specified by three angles plus a phase), the strong CP phase angle and a classical gravitational constant.

In the study of heavy quarks, many of these parameters are experimentally accessible in particular: $\alpha_s, V_{cd}, V_{cs}, V_{cb}, V_{ub}, V_{td}, V_{ts}, m_c, m_b, m_t$ and δ which in order are strong interaction coupling constant, CKM matrix elements , quark mass and CP-phase angle.

Returning to the question: Why Study Heavy Flavors? we list the following reasons and and motivations for studying the flavor changing weak decays of heavy quarks:

• Through these decays the V-A structure of the charged current $(J^{\pm}_{CC})^{\mu}$ may be tested.

• Heavy quarks have large mass, therefore a perturbative expansion in $\frac{\Lambda_{QCD}}{m_q}$ makes possible some QCD calculations.

• $B^0 - \bar{B}^0$ mixings are observed to be large, suggesting that CP-violation and CP asymmetries in the B system may likewise be relatively large.

• Measurements of heavy quark decays will help improve errors on many of the CKM matrix elements. See Table (2.3).

Table 2.3: The relation between some measurable decay parameters of heavy quark decays and CKM matrix elements. Where τ is used for lifetime and Br for branching ratio.

Measurement	CKM Matrix Element
τ_c, τ_D	V_{cd}, V_{cs}
τ_b, τ_B	V_{ub}, V_{cb}
$Br(B \to X_c(X_u)l\nu)$	V_{cb}, V_{ub}
$Br(D \to X_s(X_d)l\nu)$	V_{cs}, V_{cd}
$B^0 - \bar{B}^0$ Mixing	V_{td}, V_{ts}
Penguin B decays	V_{td}, V_{ts}

Chapter 3

Effective Weak Hamiltonian and Inclusive Charm Meson Decays

3.1 Introduction

We saw in the previous chapter that flavor changing weak decays proceed via charged current interactions, i.e. through exchange (emission or absorbtion) of $W^{\pm}$ bosons where the corresponding term of the fundamental Lagrangian is :

$$L_{CC} = \frac{g_W}{2\sqrt{2}}\left[(J^-_{CC})^\mu W^+_\mu + (J^+_{CC})^\mu W^-_\mu\right] \tag{3.1}$$

where the charged currents are,

$$(J^-_{CC})^\mu = (\bar{u},\bar{c},\bar{t})\,\gamma^\mu(1-\gamma^5)V_{CKM}\begin{pmatrix} d \\ s \\ b \end{pmatrix} + (\bar{\nu}_e,\bar{\nu}_\mu,\bar{\nu}_\tau)\gamma^\mu(1-\gamma^5)\begin{pmatrix} e^- \\ \mu^- \\ \tau^- \end{pmatrix} \tag{3.2}$$

and $(J^+_{CC})^\mu = (J^-_{CC})^{\mu\dagger}$ and V_{CKM} is the Cabibbo-Kobayashi-Maskawa matrix Eq. (2.6).

Since all the quarks (except the top quark) and leptons are much lighter than the W boson, the momentum transfer involved in the weak decays of strange, charm and beauty hadrons is considerably smaller than the W mass. Hence we can neglect the q^2 dependence of the W propagator $-i\frac{g^{\mu\nu}-q^\mu q^\nu/M_W^2}{q^2-M_W^2}$. Therefore, in the absence of QCD effects the effective low energy Hamiltonian for the charged weak interaction of quarks and leptons has the current×current form:

$$H^0_{eff} = \frac{G_F}{\sqrt{2}} \left\{ J^+_{CC\mu}(0) J^{-\ \mu}_{CC}(0) + h.c. \right\} \tag{3.3}$$

where $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$ and $G_F = (1.16639 \pm 0.0002) \times 10^{-5} GEV^{-2}$ is the Fermi coupling constant[6]. The charm lowering part of the Hamiltonian is obtained from Eqs. (3.2) and (3.3) as

$$H^0_{eff}(\Delta c = -1) = \frac{G_F}{\sqrt{2}} \left[V_{cs}(\bar{s}c)_L + V_{cd}(\bar{d}c)_L \right] \left[V^*_{ud}(\bar{u}d)_L + V^*_{us}(\bar{u}s)_L + (\bar{\nu}_e e)_L + (\bar{\nu}_\mu \mu)_L \right] \tag{3.4}$$

where in the above we have used the following notation,

$$(\bar{q}_2 q_1)_L = \sum_i \bar{q}^i_2 \gamma^\mu (1 - \gamma^5) q^i_1, \tag{3.5}$$

where i is the color index and summation over i implies that the charged currents $(\bar{q}_2 q_1)_L$ in H^0_{eff} are color singlets. The nonleptonic part may be explicitly written as:

$$\begin{aligned} H^0_{NL}(\Delta c = -1) = \frac{G_F}{\sqrt{2}} [& V_{cs}V^*_{ud}(\bar{s}c)_L(\bar{u}d)_L + V_{cs}V^*_{us}(\bar{s}c)_L(\bar{u}s)_L \\ & + V_{cd}V^*_{ud}(\bar{d}c)_L(\bar{u}d)_L + V_{cd}V^*_{us}(\bar{d}c)_L(\bar{u}s)_L] \,. \end{aligned} \tag{3.6}$$

The term with $V_{cs}V^*_{ud}$, which is the product of two diagonal elements of V_{CKM} matrix and is of order unity, is called Cabibbo-favored; the terms $V_{cs}V^*_{us}$ and $V_{cd}V^*_{ud}$ which are products of a diagonal and off-diagonal elements of V_{CKM} matrix and the term $V_{cd}V^*_{us}$ which is the product of two off diagonal matrix elements are called Cabibbo-suppressed and doubly-Cabibbo-suppressed terms, respectively.

For the semileptonic part we have,

$$\begin{aligned} H^0_{SL}(\Delta c = -1) = \frac{G_F}{\sqrt{2}} [& V_{cs}(\bar{s}c)_L(\bar{\nu}_e e)_L + V_{cs}(\bar{s}c)_L(\bar{\nu}_\mu \mu)_L \\ & + V_{cd}(\bar{d}c)_L(\bar{\nu}_e e)_L + V_{cd}(\bar{d}c)_L(\bar{\nu}_\mu \mu)_L] \end{aligned} \tag{3.7}$$

The corresponding nonleptonic and semileptonic effective Hamiltonian for beauty decays ($\Delta b = -1$) can be derived similarly [8]. We will concentrate on charm lowering decays, but the generalization to b decays is clear and straightforward.

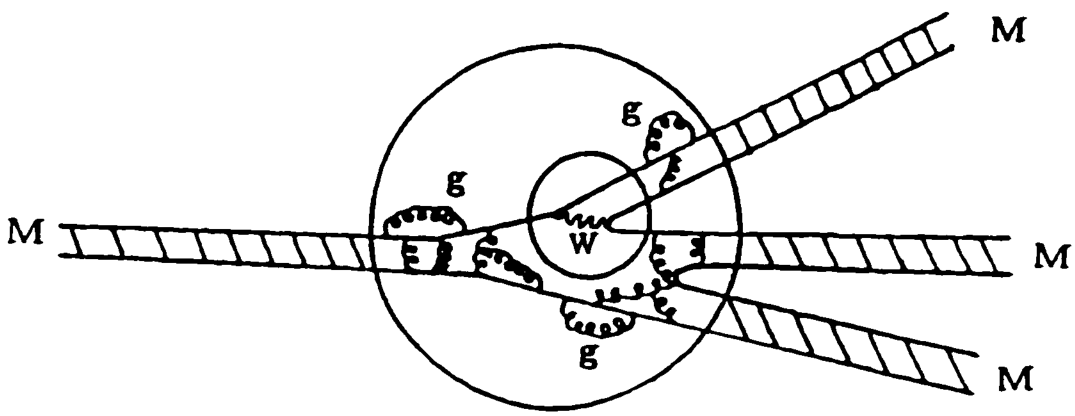


Figure 3.1: The Weak decays involve both electroweak and QCD effects. The short range weak interaction occurs in the small inner circle, with the W intermediate vector boson, while the QCD confinement region is the larger outer circle. Intial and final state mesons are denoted by M , and some of the soft gluons are denoted by g.

3.2 Strong Interaction Effects

Because, the decaying heavy quarks are necessarily confined inside hadronic bound states, the interaction responsible for binding (confining) the quarks and gluons in the hadronic states, namely the strong interaction will necessarily be involved in the weak decays of hadrons. In other words the theory of weak processes is inextricably linked to the theory of strong interactions. Fig. (3.1) shows, qualitatively, how weak and strong processes might come into play in the decay of a heavy meson.

The dynamics can qualitatively be described as follows:

(a) Hadrons are complicated superposition of an infinite number of short lived quark and gluon configurations. The simplest of such fluctuations for a meson M is the valence quark state $(Q\bar{q})$ [1] depicted in Fig. (3.1).

[1] Q will be used for the heavy quark, q and $\bar{q}$ for light quark and antiquark respectively.

(b) The weak interaction time-scale, given by $\tau \approx \frac{1}{M_W}$, is considerably shorter than the typical lifetime of the above fluctuations, because of the large mass of the W. Therefore in one of the configurations mentioned above, the heavy quark will transform into lighter quarks and leptons via a charged current interaction and thus initiate the decay of the hadron.

(c) The left over (spectator quarks, sea quarks and gluons) or newly created quarks and gluons move relatively freely within the confinement radius. However, after the much longer time-scale $\tau = \frac{1}{\Lambda}$ ($\Lambda \simeq 250$MeV is the scale parameter of QCD [14]), the confining forces become important and cause the stored energy to materialize as the final state hadrons.

Now the crucial assumption [15] usually made is that, due to the differing time scale one can separate long and short distance contributions of the strong interaction in spite of complexity of their interplay illustrated in Fig. (3.1). The following picture then emerges

(1) All the long distance effects including bound state wave functions, soft gluon radiation and final state interactions are absorbed into the initial and final state hadronic wave functions.

(2) The short distance effects originating in hard gluon interactions are calculated perturbatively (due to the asymptotic freedom property of strong interactions) and included in the effective weak Hamiltonian.

(3)The weak amplitudes are given by matrix elements of H_{eff} between asymptotic initial and final states (leptons and hadrons):

$$A(a \rightarrow b + c + ...) = \langle bc...|H_{eff}|a\rangle \,, \tag{3.8}$$

where H_{eff} include the hard gluon (short range strong interaction effects) corrections. For example the amplitude of $D^0 \rightarrow K^-\pi^+$ decay is

$$A(D^0 \rightarrow K^-\pi^+) = \left\langle K^-\pi^+|H_{eff}|D^0\right\rangle . \tag{3.9}$$

It should be realized that in the presence of strong interactions Eq. (3.8) is a highly non-trivial approximation.

It is then, natural to attack the problem of weak decays in two steps. First, one derives the effective weak Hamiltonian including all hard gluon corrections. This can basically be done in perturbation theory because of the asymptotic freedom property of QCD. The second, and a much more difficult step, is to evaluate the matrix element of H_{eff}. Since the long distance aspects of QCD cannot yet be calculated from first principles, one must rely on various approximations and physically motivated models to obtain reasonable estimates. This is the weakest point of the whole procedure and therefore will be the subject of our later attention, but for now we consider the effective weak Hamiltonian which includes the hard gluon corrections.

3.3 Hard Gluon Effects

In the absence of strong interaction effects, the hadronic part of the weak Hamiltonian has the following form (we only consider the leading term, the Cabibbo-favored part),

$$H^0_{eff} = \frac{G_F}{\sqrt{2}} V_{q_1 q_2} V^*_{q_4 q_3} \left(\bar{q}_2 q_1\right) \left(\bar{q}_4 q_3\right) \ . \tag{3.10}$$

Diagrammatically, the operator H^0_{eff}, Eq. (3.10), can be represented as shown in Fig. (3.2).

To $O(\alpha_s(\mu))$, where α_s is the strong coupling constant and μ is the probing scale, the correction to the bare 4-quark operator given in Eq. (3.10) arises from the one loop diagrams depicted in Fig. (3.3).

The diagrams of Fig. (3.3a) contribute to the vertex and self-energy corrections which are absorbed in the physical coupling constant, G_F. The gluon exchange

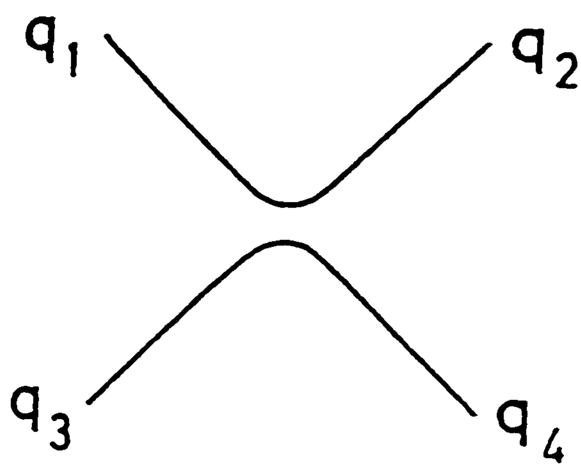


Figure 3.2: Diagram representing the 4-quark operator $(\bar{q}_2 q_1)_L (\bar{q}_4 q_3)_L$.

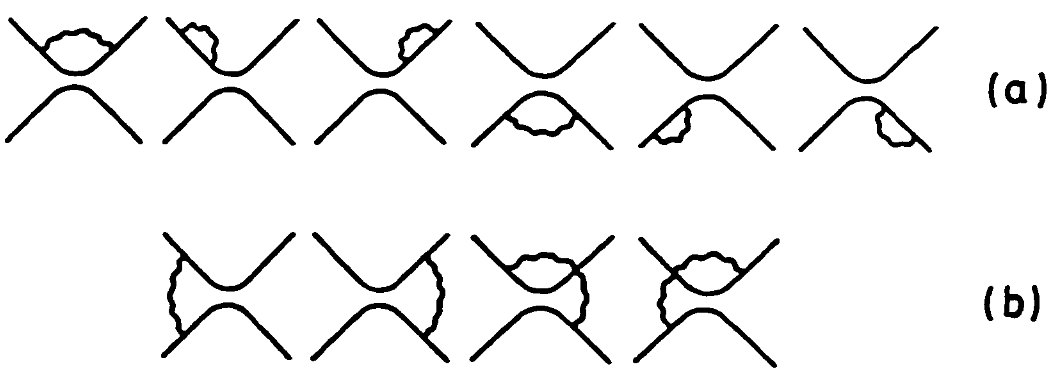


Figure 3.3: One loop gluon corrections to the 4-quark operator.

diagrams in Fig. (3.3b) result in the following corrected Hamiltonian at scale μ [16]:

$$H^1_{eff} = H^0_{eff} - \frac{G_F}{\sqrt{2}} V_{q_1 q_2} V^*_{q_4 q_3} \frac{3\alpha_s}{8\pi} \ln \frac{M_W^2}{\mu^2} (\bar{q}_2 \lambda^a q_1)_L (\bar{q}_4 \lambda^a q_3)_L \tag{3.11}$$

where λ_a, $a = 1, ..., 8$ are $SU(3)$ color group generators and,

$$(\bar{q}_2 \lambda^a q_1)_L = \sum_{ij} \bar{q}_2^i \gamma_\mu (1 - \gamma^5) \lambda^a_{ij} q_1^j. \tag{3.12}$$

We see that hard gluon exchange induces an additional local 4-quark operator which has the same chiral and flavor structure as the original operator H^0_{eff}, but instead of being composed of product of color-singlet currents, the induced operator involves product of color-octet currents. Qualitatively, this result could have been anticipated from the fact that gluon exchange conserves helicity and flavor, but, of course, transfers color and therefore changes the color structure. It is also clear that similar corrections do not occur for semi-leptonic operators since leptons do not interact with gluons.

Combining the Fierz-identity [17],

$$\left[\gamma_\mu \left(1 - \gamma^5\right)\right]_{\alpha\beta} \left[\gamma^\mu \left(1 - \gamma^5\right)\right]_{\delta\epsilon} = - \left[\gamma_\mu \left(1 - \gamma^5\right)\right]_{\alpha\epsilon} \left[\gamma^\mu \left(1 - \gamma^5\right)\right]_{\delta\beta} \tag{3.13}$$

and the SU(3) algebra relation,

$$\sum_{a=1}^{8} \lambda^a_{ij} \lambda^a_{kl} = -\frac{2}{3} \delta_{ij} \delta_{kl} + 2 \delta_{il} \delta_{kj} \,, \tag{3.14}$$

one has the following important relation between product of color-octet current and product of color-singlet currents:

$$(\bar{q}_2 \lambda^a q_1)_L (\bar{q}_4 \lambda^a q_3)_L = -\frac{2}{3} (\bar{q}_2 q_1)_L (\bar{q}_4 q_3)_L + 2 (\bar{q}_2 q_3)_L (\bar{q}_4 q_1)_L \,. \tag{3.15}$$

Using Eq. (3.14) we can rewrite the Hamiltonian H^1_{eff} in the following form:

$$\begin{aligned} H^1_{eff} = \frac{G_F}{\sqrt{2}} V_{q_1 q_2} V^*_{q_4 q_3} \Bigg\{ & \left[1 + \frac{\alpha_s}{4\pi} \ln \frac{M_W^2}{\mu^2} \right] (\bar{q}_2 q_1)_L (\bar{q}_4 q_3)_L \\ & - \frac{3\alpha_s}{4\pi} \ln \frac{M_W^2}{\mu^2} (\bar{q}_2 q_3)_L (\bar{q}_4 q_1)_L \Bigg\} \end{aligned} \tag{3.16}$$

This way of writing the H^1_{eff} shows that hard gluon exchanges renormalize the original charged current interactions and, as a new feature, induce an additional effective neutral current interaction.

The four quark interaction has the color structure $3 \times 3 = \bar{3} + 6$. The operators corresponding to the sextet (symmetric) and antitriplet (antisymmetric) are :

$$O_{\pm} = \frac{1}{2}\{(\bar{q}_2 q_1)_L(\bar{q}_4 q_3)_L \pm (\bar{q}_2 q_3)_L(\bar{q}_4 q_1)_L\} \ . \tag{3.17}$$

The Hamiltonian without QCD corrections, Eq. (3.10), can then be written in the form,

$$H^0_{eff} = \frac{G_F}{\sqrt{2}} V_{q_1 q_2} V^*_{q_4 q_3}\left[C^0_+ O_+ + C^0_- O_-\right] \tag{3.18}$$

where $C^0_+ = C^0_- = 1$. The first order corrected Hamiltonian may be similarly decomposed,

$$H^1_{eff} = \frac{G_F}{\sqrt{2}} V_{q_1 q_2} V^*_{q_4 q_3}\left[C^1_+ O_+ + C^1_- O_-\right] \tag{3.19}$$

where,

$$C^1_+ = 1 - \frac{\alpha_s}{2\pi}\ln\frac{M^2_W}{\mu^2} \quad and \quad C^1_- = 1 + \frac{\alpha_s}{\pi}\ln\frac{M^2_w}{\mu^2} \ . \tag{3.20}$$

The strong interaction effects modify the coefficients C_+ and C_-, but do not mix the operators O_+ and O_-, which are even and odd under interchange of color indices respectively. Note that QCD corrections, increase C_- but decrease C_+.

The hard gluons can be summed up to all orders in α_s using the techniques of operator product expansion [16, 18] and renormalization group [16, 19]. The result is:

$$C_+ = \left(\frac{\alpha_s(M^2_W)}{\alpha_s(\mu^2)}\right)^{\frac{6}{33-2N_F}} \ , \ C_- = \left(\frac{\alpha_s(M^2_W)}{\alpha_s(\mu^2)}\right)^{\frac{-12}{33-2N_F}} \tag{3.21}$$

where N_F is the number of the so called 'open' flavor and

$$\alpha_s(\mu^2) = \frac{4\pi}{\left(\frac{33-2N_F}{3}\right)\ln\frac{\mu^2}{\Lambda^2_{QCD}}} \tag{3.22}$$

Notice, $C_+^2 = C_-^{-1}$.We define

$$C_1 = \frac{1}{2}(C_+ + C_-), \qquad C_2 = \frac{1}{2}(C_+ - C_-). \tag{3.23}$$

Choosing $\Lambda_{QCD} \simeq 250MeV$ typical numerical values for the coefficients C_1 and C_2 are :

$$\begin{aligned} &At\ \ \mu = m_c = 1.5GeV \quad C_1 \approx 1.21 \quad C_2 = -0.42 \\ &At\ \ \mu = m_b = 5.0GeV \quad C_1 \approx 1.10 \quad C_2 = -0.24 \end{aligned} \tag{3.24}$$

In terms of the coefficients C_1 and C_2 our effective Hamiltonian becomes:

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{q_1 q_2} V^*_{q_4 q_3} \left[C_1(\mu)(\bar{q}_2 q_1)(\bar{q}_4 q_3) + C_2(\mu)(\bar{q}_2 q_3)(\bar{q}_4 q_1)\right] \tag{3.25}$$

Note that , the case with no QCD corrections is simply $C_1 = 1$ and $C_2 = 0$. Specifically the corrected Cabibbo-favored charm lowering Hamiltonian may now be written as

$$H_{NL}(\Delta C = -1) = \frac{G_F}{\sqrt{2}} V_{cs} V^*_{ud} \left[C_1(\bar{s}c)_L(\bar{u}d)_L + C_2(\bar{u}c)_L(\bar{s}d)_L\right] \tag{3.26}$$

3.4 Valence Quark Approximation

In the previous section the effective weak Hamiltonian for weak interactions in which hard gluon effects (short range QCD corrections) were included, was found. Now to determine the amplitude for any weak decay one needs to calculate the matrix element of this effective weak Hamiltonian. But, as we have said, we have to deal with non-perturbative phenomena like bound state effects. However because there are no known methods for calculating these non-perturbative effects we have to resort to approximate models. for the following discussion we distinguish between two types of decays: 1) Inclusive weak decays, and 2) exclusive weak decays. In inclusive decays we are not interested in a specific decay channel (final state), but we are interested in all (or a collection of) channels together. For example when we calculate the time life of $D^0(c\bar{u})$, that is when we are considering the following amplitude $A(D^0 \to anything)$, we are interested in all decay channels together and the above amplitude is equal to the sum of the amplitudes of all decay channels. In the total rate each channel contributes incoherently. Another type of inclusive decays of interest is the total hadronic decay modes of a charmed meson that is, for example:

$$D^0 \to X \tag{3.27}$$

where X stands for hadrons. The corresponding amplitude $A(D^0 \to X)$ is the sum of the amplitude of all non-leptonic decay modes. Again, each channel contributes incoherently to the rate. Also of interest are the semi-inclusive decays with a specific lepton in the final state, for example,

$$D^0 \to Xe^+\nu \tag{3.28}$$

where the corresponding amplitude is the sum of the decay amplitudes of the channels where $e^+\nu$ occur in the final state. The rate is the incoherent sum of the individual rates. In exclusive decays, we examine a specific decay channel, for example,

$$D^0 \to K^-\pi^+ . \tag{3.29}$$

Here we have to deal with the bound state structure of final state particles , K^- and π^+, and the process of their hadronization. Inclusive decays, both experimentally and theoretically, are simpler to study than the exclusive decays. In the rest of this chapter we will consider the inclusive decays (time life, total nonleptonic and total specific semileptonic branching ratios) of the charmed mesons D^+, D^0 and D_s^+. As we will see later in calculating inclusive decay rates, with some approximations, we need worry about the details of the process of hadronization or bound state effects of the final state particles.

The simplest approach [16] to the inclusive decays is that of the " Valence Quark Approximation" (VQA). In this approximation,

(a) Calculations are done at quark level .

(b) The initial hadron is represented by its valence configuration; more complex bound state fluctuations, often addressed as the sea of quarks and gluons are disregarded.

(c) Soft gluon interactions accompanying the weak process are neglected.

(d) The inclusive sum of hadronic final states is replaced by the final state of free quarks emitted in the decay. [2]

The above approximations reduce heavy flavor inclusive decays to a few simple processes. In the Fig. (3.4) the hadronic decay modes for the case of a meson P ($Q\bar{q}$) are shown.. The diagram in Fig. (3.4a) represents flavor decay while Figs. (3.4b) and (3.4c) are flavor annihilation via W-exchange and W-annihilation respectively [3].

In flavor decay mechanism (also referred to as the spectator diagram), the spectator quark remains passive. The Feynman diagram of both semileptonic $Q \to q_1 \nu_l \bar{l}$ and nonleptonic $Q \to q_1(q_2\bar{q}_3)$ decays of the heavy (free) quark Q is similar to the corresponding one of $\mu^- \to \nu_\mu e^- \bar{\nu}_e$ as shown in Fig. (3.5).

[2]This means that at the Cabibbo-favored level we effectively take $A(D \to hadronic) \approx A(c \to su\bar{d})$. Because at the Cabibbo-favored level c can only decay to s and the resulting W gauge boson can decay to a ($u\bar{d}$) pair in hadronic decays or to a ($l\bar{\nu}_l$) pair in semi-leptonic decays. The produced quarks, hadronize with the light antiquark of the heavy meson ($\bar{u}$ in $D^0(c\bar{u})$, $\bar{d}$ in $D^+(c\bar{d})$ and $\bar{s}$ in $D_s^+(c\bar{s})$), into hadrons with 100% efficiency. That is the probability of that the charmed meson decay to a hadronic mode is equal to the probability of the charm quark decay to $su\bar{d}$. Similarly for the semileptonic decays, for example, $A(D \to Xe^+\nu) \approx A(D(c\bar{q}) \to \bar{q}se^+\nu)$.

[3]To be more accurate, corresponding to each of the two 4-quark operator terms of the corrected effective Hamiltonian

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{Qq_1} V^*_{q_3q_2} [C_1(\mu)(\bar{q}_1Q)_L(\bar{q}_3q_2)_L + C_2(\mu)(\bar{q}_3Q)_L(\bar{q}_1q_2)_L] \qquad (3.30)$$

a set of diagrams in Fig. (3.4) exist. The diagrams in the Fig. (3.4) correspond to the 4-quark operator term $(\bar{q}_1Q)_L(\bar{q}_3q_2)_L$. The corresponding diagrams for the 4-quark operator $(\bar{q}_3Q)_L(\bar{q}_1q_2)_L$ can be drived from diagrams of Fig. (3.4) by changing $\bar{q}_1 \leftrightarrow \bar{q}_3$ in the diagrams of Fig. (3.4) .

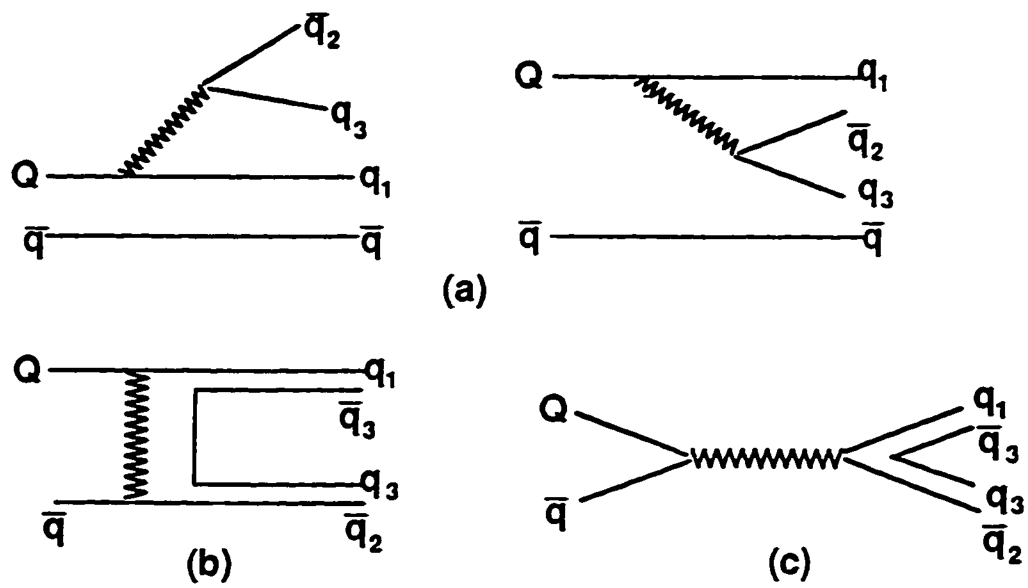


Figure 3.4: Decay mechanisms in valence quark approximation: (a) flavor decay, (b) flavor annihilation via W-exchange and (c) flavor annihilation via W-annihilation mechanism .

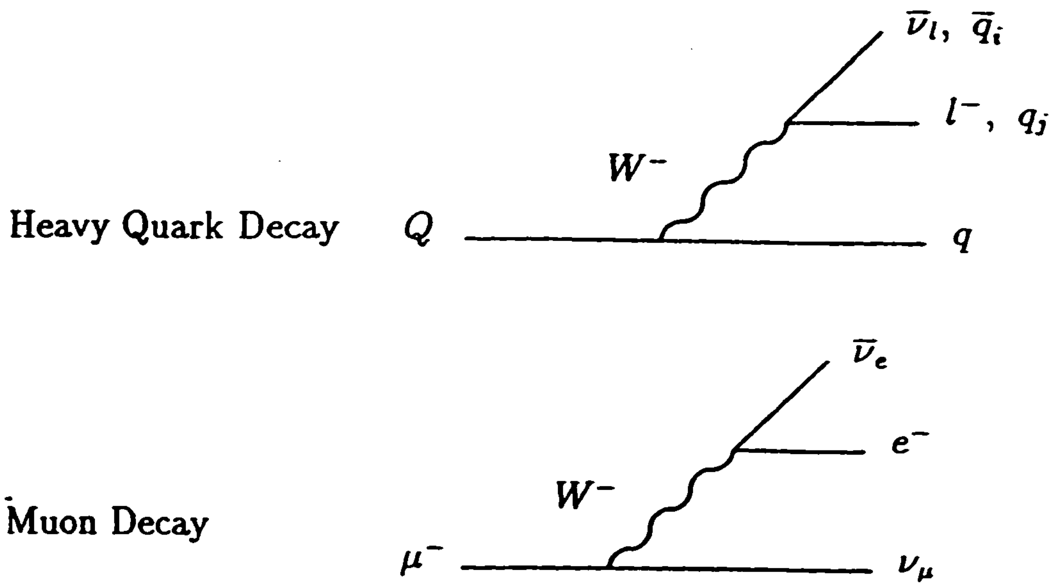


Figure 3.5: Feynman diagrams for μ decay and for free quark decay.

The decay rate for $\mu^- \to \nu_\mu e^- \bar{\nu}_e$ (ignoring electron mass) is [20]:

$$\Gamma(\mu^- \to \nu_\mu e^- \bar{\nu}_e) = \frac{G_F^2 m_\mu^5}{192\pi^3}. \tag{3.31}$$

Now ignoring the correction from the hard gluon interactions and using H^0_{eff},

$$H^0_{eff} = \frac{G_F}{\sqrt{2}} V_{Qq_1} \left[V^*_{q_2q_3} (\bar{q}_1 Q)_L (\bar{q}_3 q_2)_L + (\bar{q}_1 Q)_L (\bar{l}\nu_l)_L \right], \tag{3.32}$$

the semileptonic Γ_{SL} and nonleptonic decay rates in the valence quark approximation for meson $P(Q\bar{q})$ are:

$$\Gamma_{SL}\left(P \to \bar{l}\nu_l X\right) \simeq \Gamma\left(Q \to q_1 \bar{l}\nu_l\right) = \frac{G_F^2 m_Q^2}{192\pi^3} |V_{Qq_1}|^2 F \tag{3.33}$$

and

$$\Gamma_{NL}(P \to X_H) \simeq \Gamma(Q \to q_1 \bar{q}_2 q_3) = 3\frac{G_F^2 m_Q^2}{192\pi^3} |V_{Qq_1}|^2 |V_{q_2q_3}|^2 F \tag{3.34}$$

where F is the phase space factor which is unity for massless final particles [21], and the color factor 3 in Eq. (3.29) accounts for the fact that the quark pair $\bar{q}_2 q_3$ can be produced in three different color states [4].

Although it is clear from the above results, but we emphasize that the flavor decay mechanism (spectator diagram) does not refer to the bound state nature of the initial hadron at all, in particular, the light constituents have no influence on the decay rates.

Just the opposite is the case for the flavor annihilation mechanism shown in Figs. (3.4b) and (3.4c), where the light quark plays an active role. Since weak

[4]More accurately, consider the corresponding amplitude

$$\begin{aligned} \langle X_H | (\bar{q}_3 q_2)_L (\bar{q}_1 Q)_L | P(Q\bar{q}) \rangle &\cong \langle \bar{q} q_1 \bar{q}_2 q_3 | (\bar{q}_3 q_2)_L (\bar{q}_1 Q)_L | P(Q\bar{q}) \rangle \\ &\cong \frac{1}{\sqrt{3}} \langle q_1 \bar{q}_2 q_3 | (\bar{q}_3 q_2)_L (\bar{q}_1 Q)_L | Q \rangle \\ &= \sqrt{3} \left(\bar{U}_{q_3} \gamma^\mu \left(1 - \gamma^5\right) V_{q_2} \right) \left(\bar{U}_{q_1} \gamma_\mu \left(1 - \gamma^5\right) U_Q \right) , \end{aligned}$$

where U and V are the usual Dirac spinors. Apart from the color factor $\sqrt{3}$ the last expression is formally identical to the μ decay amplitude [20].

interactions are point-like the heavy quark and the light antiquark state functions have to overlap for the annihilation process to occur. Therefore the decay probability depends on the bound state wave function. An explicit calculation omitting mixing angles and hard gluon corrections, yields [16]:

$$\Gamma_{Ann}(P \to X_H) = C\frac{G_F^2}{8\pi} f_P^2 M_P \left[m_1^2 + m_2^2\right] I_{Ann}\left(\frac{m_1^2}{m_P^2}, \frac{m_2^2}{m_P^2}\right) \tag{3.35}$$

where M_P and $m_i\,(i = 1, 2)$ denote the mass of the initial meson and the mass of the final state quarks respectivbely. The function $I_{Ann}(x, y)$ in Eq. (3.30) describes the mass corrections to the two body phase space [16]:

$$I_{Ann}(x, y) = \left[1 - \frac{(x-y)^2}{x+y}\right] \lambda^{\frac{1}{2}}(1, x, y) \tag{3.36}$$

with

$$\lambda(z, x, y) = (z - x - y)^2 - 4xy. \tag{3.37}$$

The parameter f_P is the meson decay constant, which characterizes the overlap of the constituent quarks in the intial meson $P(Q\bar{q})$ and is defined by

$$\langle 0|\,(\bar{q}Q)_L\,|P\rangle = i f_P P^{\mu}, \tag{3.38}$$

where P^{μ} is the four momentum of P. f_P can be measured experimentally from purely leptonic meson decays, $P \to l\bar{\nu}_l$, shown in Fig. (3.5). C is the color factor where for W-exchange $C = \frac{1}{3}$ and for W-annihilation $C = 3$ [16].

The proportionality of the weak annihilation to the final state fermion masses is known as helicity suppression and reflects the fact that a pseudoscalar meson can not decay into a massless fermion-antifermion pair if only left handed fermions participate in weak interactions. The classical example is the suppression of $\pi, K \to e^-\bar{\nu}_e$ versus $\pi, K \to \mu\bar{\nu}_\mu$.

Both the dependence on the decay constant f_P and helicity suppression make the annihilation rates of heavy mesons into two mesons less favorable than the flavor decay mechanism rates.

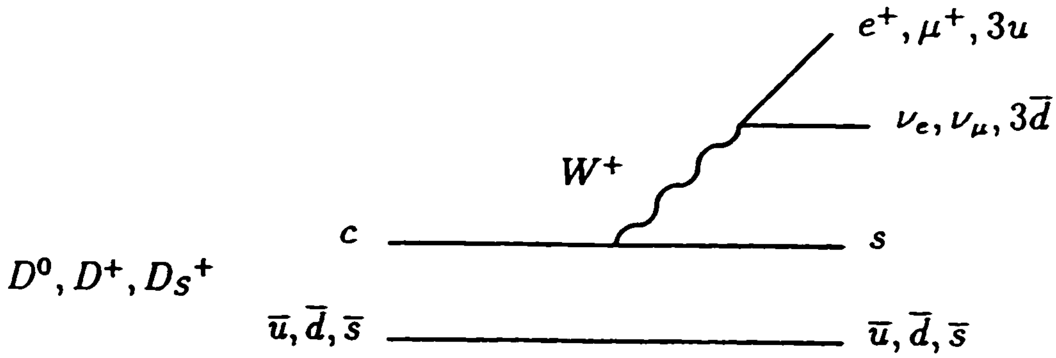


Figure 3.6: Spectator decay of charmed mesons.

3.5 Spectator Model

In the so called spectator model [22], the light constituents of the decaying heavy meson are assumed to be inert. The annihilation process is neglected, as implied above, and the decay is assumed to occur only via the spectator diagram. Thus in this model the lifetimes of $D^+(c\bar{d})$, $D^0(c\bar{u})$ and $D_S^+(c\bar{s})$ mesons should all be the same and essentially determined by the lifetime τ_c of the charm quark:

$$\tau\left(D^+\right) \simeq \tau\left(D^0\right) \simeq \tau\left(D_S^+\right) = \tau_C. \tag{3.39}$$

In the charm quark decay, at the Cabibbo-allowed level, only two leptonic modes ($c \rightarrow se^+\nu_e$ and $c \rightarrow s\mu^+\nu_\mu$) and one hadronic mode ($c \rightarrow su\bar{d}$) is possible (See Fig. (3.6)).

If we ignore quark mixing, namely if we take $|V_{ud}| = |V_{us}| \simeq 1$ and if we also neglect the mass of final state particles or equivalently if we take $F \approx 1$, then using

Eqs. (3.26),(3.28) and (3.29) we have :

$$\tau_c = \frac{1}{\Gamma_c} \simeq \frac{1}{5}\left(\frac{m_\mu}{m_c}\right)^5 \tau_\mu \approx 7 \times 10^{-13} s, \tag{3.40}$$

for $m_c \approx 1.5$GeV.

The experimental values of the life times of D^0, D^+ and D_s^+ are [6]:

$$\begin{aligned} \tau\left(D^0\right) &= (4.15 \pm 0.04) \times 10^{-13} s, \\ \tau\left(D^+\right) &= (10.57 \pm 0.15) \times 10^{-13} s, \\ \tau\left(D_s^+\right) &= (4.67 \pm 0.17) \times 10^{-13} s\,. \end{aligned} \tag{3.41}$$

Notice,

$$\tau\left(D^+\right) > \tau\left(D_s^+\right) > \tau\left(D^0\right). \tag{3.42}$$

The naive prediction of the spectator model is quite close to the experimental values, specifically its prediction is very close to the average of the three D mesons life times, namely $\tau_D = (6.46 \pm 0.07) \times 10^{-13} s$. Thus the spectator model does provide a very useful qualitative basis for calculating the weak decay of heavy quarks. However, one notes that the lifetimes of D^+ and D^0 are significantly different, $\tau(D^+) \approx 2.5\tau(D^0)$. The simple spectator model, thus, fails to describe the lifetimes of the charmed mesons.

In the spectator model, the semileptonic branching ratios of all weakly decaying states with the same heavy flavor, will be the same (considering the leptons are massless). For charm quark decays, one out of five decays lead to a particular lepton in the final state, therefore

$$B(c \to \bar{l}\nu_l X) = \frac{1}{5} = 20\%\,. \tag{3.43}$$

But this result is at variance with the observed semileptonic branching ratios [6]:

$$B(D^+ \to e^+\nu_e X^0) = (17.2 \pm 1.9)\% \tag{3.44}$$

and

$$B(D^0 \to e^+\nu_e X^-) = (7.7 \pm 1.2)\% \tag{3.45}$$

Since the electroweak sector is well understood, it may be expected that the reason for the lifetime difference must be hidden in the purely hadronic sector. By definition we have

$$\frac{B(D^+ \to e^+\nu_e X^0)}{B(D^0 \to e^+\nu_e X^-)} = \frac{\Gamma(D^+ \to e^+\nu_e X^0)}{\Gamma(D^0 \to e^+\nu_e X^-)}\frac{\tau(D^+)}{\tau(D^0)} \tag{3.46}$$

As we saw the spectator model states that

$$\Gamma(D^+ \to e^+\nu_e X^0) = \Gamma(D^0 \to e^+\nu_e X^-)\ . \tag{3.47}$$

These semi-leptonic widths should be nearly the same, because dominant (i.e. Cabibbo-favored) semi-leptonic Hamiltonian, $H_{SL} \sim (\bar{\nu}_l l)(\bar{s}c)$ is an isoscalar and is invariant under an arbitrary rotation in the isospin space. Therefore the ratio of the semileptonic branching ratios should be equal to the lifetime ratios, which experimentally indeed is the case. That is

$$\frac{B(D^+ \to e^+\nu_e X^0)}{B(D^0 \to e^+\nu_e X^-)} = 2.23 \pm 0.42, \ \ and \ \ \frac{\tau(D^+)}{\tau(D^0)} = 2.54 \pm 0.04 \tag{3.48}$$

Note that the short distance QCD corrections were ignored in the above. However, within the framework of spectator model, their inclusion still results in equal lifetimes and equal semileptonic branching ratios for three D^+, D^0 and D_s^+, charmed mesons. The nonleptonic rate which includes the hard gluon corrections is [16, 23]:

$$\Gamma_{NL}(c \to s\bar{d}u) = (2C_+^2 + C_-^2)\frac{G_F^2 m_c^5}{192\pi^3}, \tag{3.49}$$

where we have neglected quark mixing and have considered the final-state particles as massless. Using $C_1 \simeq 1.21$ and $C_2 \simeq -0.42$ at $\mu = m_c \simeq 1.5$GeV, we have $2C_+^2 + C_-^2 \approx 3.9$. Therefore the hard gluon corrections enhance the nonleptonic rate by a factor of $\frac{1}{3}(2C_+^2 + C_-^2) = \frac{3.9}{3} = 1.3$ with respect to the rate obtained using the uncorrected Hamiltonian. The nonleptonic QCD enhancement would lead to the following branching ratio for each mode of the semileptonic part

$$B\left(c \to s\bar{l}\nu_l\right) = \frac{1}{2 + 2C_+^2 + C_-^2} \approx 16\% \tag{3.50}$$

instead of 20% using the uncorrected Hamiltonian.

In the preceding estimates we neglected the quarks mixing and we also considered that all the final state particles are massless, which effectively means we took the phase space factor $F \simeq 1$ for all the different decay modes. Although the consideration of quark mixing and the final state particle masses will not improve our basic understading of the problem, that is the equal life time prediction for the three charmed D^0, D^+ and D_s^+ mesons, but for a better understanding and appreciation of the standard model and also for the sake of completeness we will give the decay rates that includes these effects. The results can be summarized as follows :

$$\Gamma_Q = \Gamma_{SL} + \Gamma_{NL} \tag{3.51}$$

where

$$\Gamma_{SL} = \sum_{l,q} |V_{Qq}|^2 I\left(\frac{m_q^2}{m_Q^2}, \frac{m_l^2}{m_Q^2}, 0\right) \frac{G_F^2 m_Q^5}{192\pi^3} \tag{3.52}$$

and

$$\Gamma_{NL} = \sum_{q_1,q_2,q_3} |V_{Qq_1}|^2 |V_{q_2q_3}|^2 I\left(\frac{m_{q_1}^2}{m_Q^2}, \frac{m_{q_2}^2}{m_Q^2}, \frac{m_{q_3}^2}{m_Q^2}\right) \left(2C_+^2 + C_-^2\right) \frac{G_F^2 m_Q^5}{192\pi^3} \tag{3.53}$$

where $I(x, y, z)$ is the three-body phase space integral [24]:

$$I(x,y,z) = 12 \int_{(x+y)^2}^{(1-z)^2} \frac{d\xi}{\xi}(\xi - x^2 - y^2)(1 + z^2 - \xi) W\left(\xi, x^2, y^2\right) W(1, z^2, \xi) \tag{3.54}$$

and

$$W(a,b,c) = \left(a - \left(\sqrt{b} + \sqrt{c}\right)^2\right)^{\frac{1}{2}} \left(a - \left(\sqrt{b} - \sqrt{c}\right)^2\right)^{\frac{1}{2}} \tag{3.55}$$

Note that in the Eqs. (3.48) and (3.49) the Cabibbo-suppressed and Cabibbo-doubly-suppressed decay modes of the heavy quark Q have also been included.

The effect of including quark mixings and the final state particles, masses is not more than 20% and as a general trend the semi-leptonic branching ratios increase if one includes the quark masses [16].

Thus the spectator model, which takes the light quark (of the heavy meson) as inert , and only incorporates the short -distance structure of heavy flavor decays, is not able to reproduce the detailed pattern of charmed mesons inclusive decays. This indicates that charmed meson decays are not solely determined by the decay properties of the charmed quark , but are also influenced by the light constituents (light quark and gluons) present in the bound state of the decaying charmed meson. To improve the theoretical predictions of charmed meson inclusive decays in the context of the valence quark approximation , some non-spectator effects such as quark interference (final state interaction and Pauli interference) and flavor annihilation have been considered . Reference [16] discusses in detail. But consideration of these effects doesn't change the overall picture very much,namely, that the reason for the lifetime difference must be hidden in the non-leptonic sector. In the next chapter, by taking a phenomenological approach and by introducing suitable form factors, we will examine the exclusive decays of charmed mesons.

Chapter 4

Factorization Model

4.1 Introduction

In the previous chapter we considered the inclusive charmed mesons (D^0, D^+ and D_s^+) decays and we did a free quark calculation for the decay rates (3.33) and (3.34). But, although the free quark calculation is very simple and straightforward it suffers from various weak features. The total rates as well as the shapes of the lepton spectra (in the semileptonic decays) depend strongly on unknown quark masses which occur in the amplitudes and, thus determine the phase space. In Table (4.1) the phase space factor F of the various charm quark decay channels is given for current and constituent quark masses. These corrections are quite sizable and - especially for charm quark, c, decay - very uncertain .

An alternative approach is to study specific exclusive channels (both in semileptonic and nonleptonic decays) and doing the calculations on the hadronic level, by introducing suitable phenomenological formfactors, which enables us to avoid uncertainties connected with quark masses, quark phase space and various confinement effects inherent in a pure quark decay picture and also let us to consider the bound state structure of the intial(decaying) and final (produced) mesons.

Since the discovery of charmed mesons, many exclusive decay channels of these particles have been observed and their branching ratios measured [25, 26] by various

Table 4.1: Phase space factor, F, of various charm quark decay channels for current (first row) and constituent (second row) quark masses.

Masses [GeV]	$c \to se\nu_e$	$c \to s\mu\nu_e$	$c \to de\nu_e$	$c \to d\mu\nu_\mu$	$c \to su\bar{d}$	$c \to su\bar{s}$	$c \to du\bar{d}$	$c \to du\bar{s}$
$m_c = 1.35$ $m_s = 0.15$ $m_{u,d} = 0.006$	0.91	0.86	1.00	0.95	0.91	0.82	1.00	0.91
$m_c = 1.70$ $m_s = 0.51$ $m_{u,d} = 0.34$	0.52	0.50	0.74	0.72	0.19	0.09	0.33	0.19

collaborations. Two-body and quasi two-body modes seem to dominate the non-leptonic D decays [25]. Due to a larger phase space, two-body decays of B mesons are not expected to be as dominant as in the case of D decays. In this chapter we will mainly consider different exclusive two-body Cabibbo-favored decay modes of charmed mesons in the context of the factorization model.

4.2 The valence quark approximation and the factorization ansatz

Adapting the inclusive Valence quark approximation (VQA) -Spectator picture to the exclusive non-leptonic two-body decays, one obtains the prototype of a phenomenological model [27, 28] which can be characterized as follows:

(a) The weak amplitude is given by the matrix element of the short distance corrected weak Hamiltonian, H_{NL}^{eff}, between the initial and final state hadrons, which requires separability of long and short distance processes as discussed in Section (3.2).

(b) Hadronic bound states are represented by their simplest valence quark

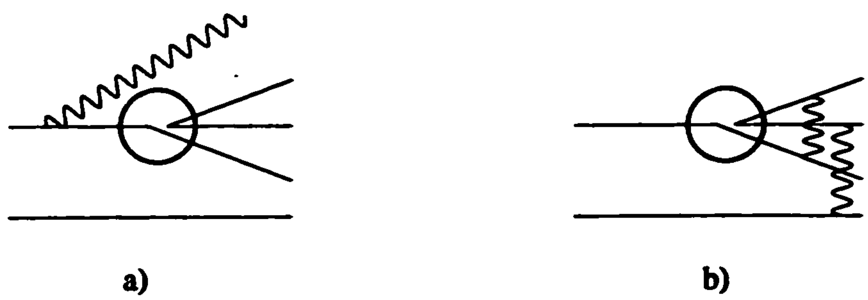


Figure 4.1: Soft gluon interactions in exclusive decays. Circles represent hard gluon corrections to H_{NL}^{eff}.

configuration. The gluon content and the quark sea are disregarded.

(c) Soft gluon interactions such as those illustrated in Fig. (4.1) are neglected.
(d) For two-body final states consisting of different combinations of pseudoscalar (P), vector (V) and axial vector(A) mesons one employs the factorization ansatz :

$$< f|\tilde{J} \cdot J|D > \simeq < P, V or A|\tilde{J}|0 >< P, V or A|J|D > \qquad (4.1)$$

where J and $\tilde{J}$ are $(\bar{q}_2 q_1)_L$ color-singlet currents. Therefore the matrix element of the H_{NL}^{eff} can be written in terms of simpler matrix elements of single weak currents with the vacuum as an intermediate state and thus are determined in terms of meson decay constants and hadron current matrix elements. The amplitude of the semileptonic decays factorizes exactly, hadronic current can only lead to hadrons and the leptonic current to leptons:

$$< X l \bar{\nu}_l|(\bar{l}\nu_l)(\bar{q}Q)|D > = < X l \bar{\nu}_l|(\bar{l}\nu_l)|0 >< X|(\bar{q}Q)|D > \ . \qquad (4.2)$$

In one sense the factorization ansatz in nonleptonic processes is a natural extension of the factorization of semileptonic decay amplitudes, asserting that similar to semileptonic decays that the matrix element is a product of the matrix elements of one hadronic and one leptonic current, in the nonleptonic decays the amplitudes are also the product of the matrix elements of two hadronic currents.

4.3 Phenomenological constants a_1 and a_2.

We saw in the chapter 3 that the QCD improved effective weak Hamiltonian for Cabibbo-favored nonleptonic charm decays is :

$$H_{NL}^{eff}(\Delta c = -1) = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* \left[C_1 (\bar{u}d)_L (\bar{s}c)_L + C_2 (\bar{s}d)_L (\bar{u}c)_L \right], \tag{4.3}$$

where C_1 and C_2 are the Wilson coefficients for which we adopt the values

$$C_1 = 1.26 \pm 0.04, \qquad C_2 = -0.51 \pm 0.05. \tag{4.4}$$

The central values of C_1 and C_2 are taken from [29] and the errors are from [30].

According to the above effective Hamiltonian there are two possible decay mechanisms for D (Charmed- mesons) decay: 1) Flavor (quark) decay Fig. (4.2) and 2) Flavor (quark) annihilation Fig. (4.3). The indices i and j in the Figs. (4.2) and (4.3) are color indices. As we saw before in the flavor decay mechanism the light quark doesn't participate in the decay process and remains a spectator; hence the name spectator diagrams for the corresponding diagrams of flavor decay mechanism. However in the flavor annihilation process the light quark is an active participant - both the heavy and light quarks annihilate and two new quarks are created. In the flavor decay process the left-over quarks recombine directly to the final state hadrons, whereas in the annihilation process an additional quark pair $(u\bar{u}, d\bar{d}, s\bar{s})$ must be created from the vacuum.

As shown in Fig. (4.2), the final state quarks and anitquarks produced as a result of flavor-decay can recombine in two different ways to produce color-singlet hadrons. Therefore, corresponding to each term of the effective Hamiltonian there exist two flavor-decay diagrams. Diagrams (4.2a) and (4.2c) correspond to the charged-current $(\bar{u}d)_L(\bar{s}c)_L$ (C_1-term) and the diagrams (4.2b) and (4.2d) correspond to the effective neutral current $(\bar{s}d)_L(\bar{u}c)_L$ (C_2-term). Diagrams (4.2a), (4.2b) and (4.2c),

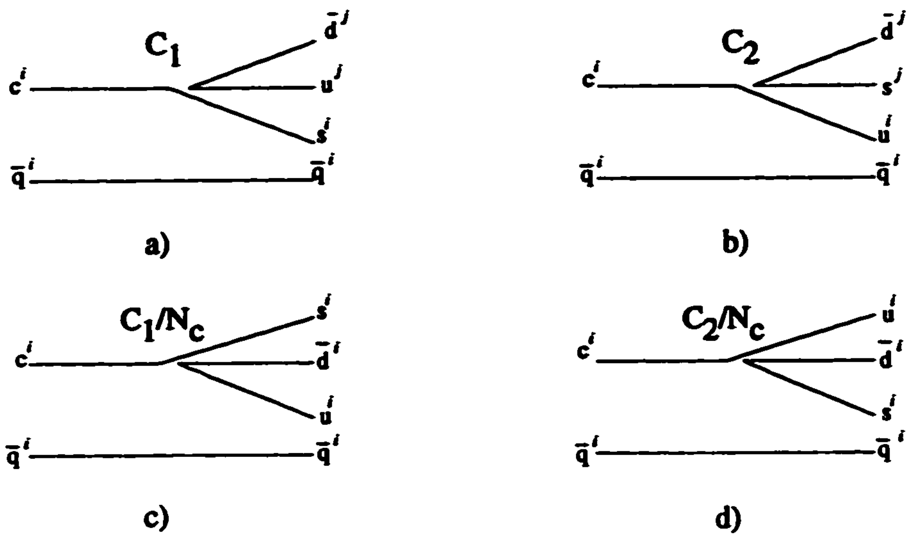


Figure 4.2: D mesons flavor decay diagrams.

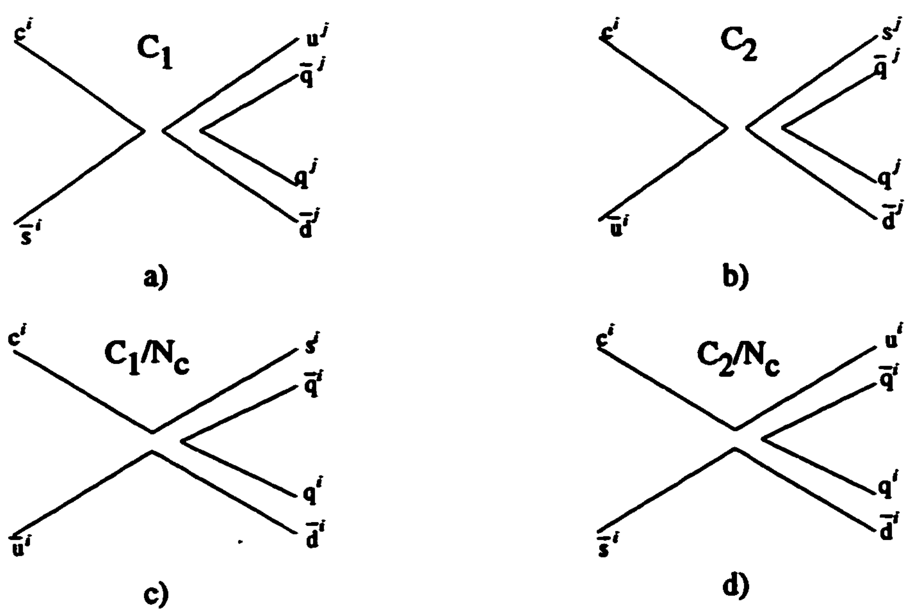


Figure 4.3: D mesons flavor annihilation diagrams.

(4.2d) are also called external and internal spectator diagrams respectively. The internal spectator diagrams (4.2c) and (4.2d) are, suppressed due to the color mismatch. In other words, both charged and neutral current receive contributions from a term where the color indices are automatically matched to form color singlet hadrons and a second term where the color indices are mismatched. ξ is the color mismatch factor and we expect $\xi = \frac{1}{N_c}$ for N_c colors. We know that there are three colors, $N_c = 3$, but the value $\xi = \frac{1}{3}$ cannot be trusted because of the uncertain contributions of color-octet current products obtained by the Fierz transformation [31], therefore it is instructive to keep ξ arbitrary for a phenomenological treatment.

Similarly we see that the two flavor annihilation diagrams Fig. (4.3c) and Fig. (4.3d) which respectively correspond to the C_1-term and C_2-term of the effective Hamiltonian are suppressed, due to the color mismatch, relative to the flavor annihilation diagrams Fig. (4.3a) and Fig. (4.3b).

Consider now the following three Cabibbo-favored two-body charmed mesons decays: 1) $D^0 \rightarrow K^-\pi^+$ 2) $D^0 \rightarrow \bar{K}^0\pi^0$ and 3) $D^+ \rightarrow \bar{K}^0\pi^+$. The $D^0 \rightarrow K^-\pi^+$ decay proceeds via the two flavor decay diagrams Fig. (4.2a) and Fig. (4.2d) and also via the two flavor annihilation diagrams Fig. (4.3b) and Fig. (4.3c). Similarly $D^0 \rightarrow \bar{K}^0\pi^0$ decay proceeds via the two flavor-decay diagrams Fig. (4.2b) and Fig. (4.2c) and also again via the two flavor-annihilation diagrams Fig. (4.3b) and Fig. (4.3c). $D^+ \rightarrow \bar{K}^0\pi^+$ decay can proceed via all the four flavor decay diagrams of Fig. (4.2). An examination of Fig. (4.3) shows that flavor-annihilation mechanism is not viable for Cabibbo-favored D^+ decays including $D^+ \rightarrow \bar{K}^0\pi^+$. Therefore none of the four diagrams of Fig. (4.3) participate in $D^+ \rightarrow \bar{K}^0\pi^+$ decay. The two diagrams Fig. (4.3a) and Fig. (4.3b) correspond to the flavor-annihilation of D_s^+.

Let us consider the amplitude of one of the above decays, for example $D^+ \rightarrow \bar{K}^0\pi^+$ in more detail. Starting from Eq. (4.3) according to VQA we have,

$$A(D^+ \to \bar{K}^0\pi^+) =< \bar{K}^0\pi^0|H_w^{eff}|D^+ >= \tilde{G}_F \Big\{C_1 < \bar{K}^0\pi^+|(\bar{u}d)_L(\bar{s}c)_L|D^+ >$$
$$+ C_2 < \bar{K}^0\pi^+|(\bar{s}d)_L(\bar{u}c)_L|D^+ >\Big\} \quad (4.5)$$

where $\tilde{G}_F = \frac{G_F}{\sqrt{2}}V_{cs}V_{ud}^*$. Operator d in $(\bar{u}d)_L$ and $(\bar{s}d)_L$ can operate on the 'd ' antiquark of both $|\pi^+>$ and $|\bar{K}^0>$ states, therefore we have

$$\begin{aligned} A(D^+ \to \bar{K}^0\pi^+) = \frac{G_F}{\sqrt{2}}V_{cs}V_{ud}^* \Big\{ & C_1 < \bar{K}^0\,\underbrace{\pi^+|(\bar{u}}\,d)_L(\bar{s}c)_L|D^+ > \\ & +C_1 < \underbrace{\bar{K}^0\pi^+|(\bar{u}}\,d)_L(\bar{s}c)_L|D^+ > \\ & +C_2 < \bar{K}^0\,\underbrace{\pi^+|(\bar{s}}\,d)_L(\bar{u}c)_L|D^+ > \\ & + C_2 < \underbrace{\bar{K}^0\pi^+|(\bar{s}}\,d)_L(\bar{u}c)_L|D^+ >\Big\} \end{aligned} \quad (4.6)$$

We will use the following Fierz identities

$$\begin{aligned} (\bar{s}c)_L(\bar{u}d)_L &= \frac{1}{N_c}(\bar{s}d)_L(\bar{u}c)_L + \frac{1}{2}\sum_{a=1}^{8}(\bar{s}\lambda^a d)_L(\bar{u}\lambda^a c)_L \\ (\bar{s}d)_L(\bar{u}c)_L &= \frac{1}{N_c}(\bar{u}d)_L(\bar{s}c)_L + \frac{1}{2}\sum_{a=1}^{8}(\bar{u}\lambda^a d)_L(\bar{s}\lambda^a c)_L \end{aligned} \quad (4.7)$$

to transform the second and fourth terms of Eq. (4.6). Here λ^a are the Gell-Mann matrices.That is

$$\begin{aligned} A(D^+ \to \bar{K}^0\pi^+) = \frac{G_F}{\sqrt{2}}V_{cs}V_{ud}^* \Big\{ & C_1 < \bar{K}^0\,\underbrace{\pi^+|(\bar{u}}\,d)_L(\bar{s}c)_L|D^+ > \\ +C_1 < \underbrace{\bar{K}^0\pi^+|\frac{1}{N_c}(\bar{s}}\,d)_L(\bar{u}c)_L & + \frac{1}{2}\sum_{a=1}^{8}(\bar{s}\lambda^a d)_L(\bar{u}\lambda^a c)_L|D^+ > \\ & +C_2 < \bar{K}^0\,\underbrace{\pi^+|(\bar{s}}\,d)_L(\bar{u}c)_L|D^+ > \\ + C_2 < \underbrace{\bar{K}^0\pi^+|\frac{1}{N_c}(\bar{u}}\,d)_L(\bar{s}c)_L & + \frac{1}{2}\sum_{a=1}^{8}(\bar{u}\lambda^a d)_L(\bar{s}\lambda^a c)_L|D^+ >\Big\}. \end{aligned} \quad (4.8)$$

Rearranging the terms we get

$$A(D^+ \to \bar{K}^0\pi^+) = \frac{G_F}{\sqrt{2}}V_{cs}V_{ud}^* \Big\{(C_1 + \frac{C_2}{N_c}) < \bar{K}^0\,\underbrace{\pi^+|(\bar{u}}\,d)_L(\bar{s}c)_L|D^+ >$$

$$+C_1 < \bar{K^0}\pi^+|\frac{1}{2}\sum_{a=1}^{8}(\bar{s}\lambda^a d)_L(\bar{u}\lambda^a c)_L|D^+ >$$

$$+(C_2 + \frac{C_1}{N_c}) < \underbrace{\bar{K^0}\pi^+|(\bar{s}}d)_L(\bar{u}c)_L|D^+ >$$

$$+ C_2 < \bar{K^0}\pi^+|\frac{1}{2}\sum_{a=1}^{8}(\bar{u}\lambda^a d)_L(\bar{s}\lambda^a c)_L|D^+ >\Big\}. \quad (4.9)$$

We define η_1 and η_2 as:

$$\eta_1 = \frac{< \bar{K^0}\pi^+|\frac{1}{2}\sum_{a=1}^{8}(\bar{s}\lambda^a d)_L(\bar{u}\lambda^a c)_L|D^+ >}{< \underbrace{\bar{K^0}\pi^+|(\bar{s}}d)_L(\bar{u}c)_L|D^+ >}$$

$$\eta_2 = \frac{< \bar{K^0}\pi^+|\frac{1}{2}\sum_{a=1}^{8}(\bar{u}\lambda^a d)_L(\bar{s}\lambda^a c)_L|D^+ >}{< \bar{K^0}\,\underbrace{\pi^+|(\bar{u}}d)_L(\bar{s}c)_L|D^+ >} \quad (4.10)$$

In terms of η_1 and η_2 we have

$$A(D^+ \to \bar{K^0}\pi^+) = \tilde{G}_F\Big\{(C_1 + (\frac{1}{N_c} + \eta_2)C_2) < \bar{K^0}\,\underbrace{\pi^+|(\bar{u}}d)_L(\bar{s}c)_L|D^+ >$$

$$+ (C_2 + (\frac{1}{N_c} + \eta_1)C_1) < \underbrace{\bar{K^0}\pi^+|(\bar{s}}d)_L(\bar{u}c)_L|D^+ >\Big\}\,. \quad (4.11)$$

We define the phenomenological, color mismatch factors ξ and ξ' as

$$\xi = \frac{1}{\tilde{N}_c} = (\frac{1}{N_c} + \eta_2)$$

$$\xi' = \frac{1}{\tilde{N}_c'} = (\frac{1}{N_c} + \eta_1) \quad (4.12)$$

where $\tilde{N}_c$ and $\tilde{N}_c'$ are "effective" color numbers.The coefficients a_1 and a_2 are defined in terms of the Wilson coefficients C_1, C_2 and the color mismatch factors ξ, ξ' as:

$$a_1 = C_1 + \xi C_2, \qquad a_2 = C_2 + \xi' C_1. \quad (4.13)$$

In terms of a_1 and a_2 we have

$$A(D^+ \to \bar{K^0}\pi^+) = \frac{G_F}{\sqrt{2}}V_{cs}V_{ud}^*\big\{a_1 < \bar{K^0}\pi^+|(\bar{u}d)_{L,H}(\bar{s}c)_{L,H}|D^+ >$$

$$+ a_2 < \bar{K^0}\pi^+|(\bar{s}d)_{L,H}(\bar{u}c)_{L,H}|D^+ >\big\} \quad (4.14)$$

The subscript H instructs us to treat the Dirac bilinear $(\bar{q}_2 q_1)_{L,H}$ as an interpolating hadron field of a hadron carrying the quantum numbers of the quark currents, that is, no further Fierz reordering in flavor and color need be done.

Applying the factorization assumption, that is, matrix element of the product of two hadronic currents is equal to the product of matrix elements of the individual currents we get:

$$A(D^+ \to \bar{K^0}\pi^+) = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* \{ a_1 < \pi^+|(\bar{u}d)_{L,H}|0 >< \bar{K^0}|(\bar{s}c)_{L,H}|D^+ > \\ + a_2 < \bar{K^0}|(\bar{s}d)_{L,H}|0 >< \pi^+|(\bar{u}c)_{L,H}|D^+ > \}. \quad (4.15)$$

Similarly the factorized amplitudes for $D^0 \to K^-\pi^+$ and $D^0 \to \bar{K^0}\pi^0$, respectively, are

$$A(D^0 \to K^-\pi^+) = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* \Big\{ a_1 < \pi^+|(\bar{u}d)_{L,H}|0 >< K^-|(\bar{s}c)_{L,H}|D^0 > \\ + a_2 < K^-\pi^+|(\bar{s}d)_{L,H}|0 >< 0|(\bar{u}c)_{L,H}|D^0 > \Big\} , \quad (4.16)$$

and

$$A(D^0 \to \bar{K^0}\pi^0) = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* \Big\{ a_2 < \bar{K^0}|(\bar{s}d)_{L,H}|0 >< \pi^0|(\bar{u}c)_{L,H}|D^0 > \\ + a_2 < \bar{K^0}\pi^0|(\bar{s}d)_{L,H}|0 >< 0|(\bar{u}c)_{L,H}|D^0 > \Big\}. \quad (4.17)$$

The second term in both of the decay amplitudes (4.16) and (4.17) is the contribution of annihilation to the corresponding decay. But in the factorization approximation the annihilation amplitudes are small (suppressed) since they involve the divergence of vector currents formed by the lighter quarks and thus go to zero in the limit of vanishing light quark masses [1] [33, 34] .Furthermore the current divergence

[1]

$$< K\pi|(\bar{s}d)_L|0 >< 0|(\bar{u}c)_L|D^0 > \sim f_D < K\pi|\partial_\mu(\bar{s}\gamma^\mu d)|0 > \\ \sim f_D(m_d - m_s) < K\pi|(\bar{s}d|0 >,$$

where f_D is the meson decay constant defined by $< 0|(\bar{u}c)|D^0 >= -if_D p_D^\mu$

is to be taken at $q^2 = m_D^2$ or $m_{D_s}^2$ i.e. values which are large compared to the quark masses in the decay products. Since all currents are presumably asymptotically (for large q^2) conserved the corresponding contribution is expected to be very small [28]. Therefore we will neglect the annihilation amplitudes. But we notice that due to final state interactions amplitudes which come out small in factorization approximation may recieve large contributions via mixing with other channels [35]. This possibility we will consider later.

After ignoring the annihilation effects we have,

$$A(D^0 \to K^-\pi^+) \simeq \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* a_1 < \pi^+ |(\bar{u}d)_{L,H}| 0 >< K^- |(\bar{s}d)_{L,H}| D^0 > \qquad (4.18)$$

and

$$A(D^0 \to \bar{K^0}\pi^0) \simeq \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* a_2 < \bar{K^0} |(\bar{s}d)_{L,H}| 0 >< \pi^0 |(\bar{u}c)_{L,H}| D^0 > \qquad (4.19)$$

We can distinguish three types of decays: 1) Class I transitions, determined by a_1 only, 2) Class II transitions, determined by a_2, and 3) Class III transitions, where the amplitudes are proportional to a superposition of a_1 and a_2, $(a_1 + xa_2)$. (Only D^+ decays occur in class III.)

By examining the amplitudes (4.15), (4.18) and (4.19) we see that the non-leptonic Hamiltonian can be written effectively, for all the three above type of decays as:

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* \left\{ a_1 (\bar{u}d)_{L,H} (\bar{s}c)_{L,H} + a_2 (\bar{s}d)_{L,H} (\bar{u}c)_{L,H} \right\}. \qquad (4.20)$$

Class III transitions, where a_1 and a_2 amplitudes interfere, can be used for determining the relative sign of a_1 and a_2. In $D \to PP$ transitions (P: pseudoscalar nonet $\pi, K, ...$) the relevant combination is of the form $a_1 + xa_2$ with $x = +1$ in the $SU(3)$ symmetry limit [35]. D^+-decay rates immediately show that the amplitudes interfere destructively with $x = +1$ this gives $a_2/a_1 < 0$ in agreement with what is expected from Eqs. (4.4) and (4.13) that $\frac{C_2}{C_1} < 0$ and if ξ and ξ' are near $\frac{1}{3}$ then $\frac{a_2}{a_1} < 0$ too.

Till now we have treated a_1 and a_2 as free parameters and we haven't yet given any numerical value for them. Bauer, Stech and Wirbel (BSW) [35]used the $D \to \bar{K}\pi$ decay amplitudes namely $D^0 \to K^-\pi^+, D^0 \to \bar{K^0}\pi^0$ and $D^+ \to \bar{K^0}\pi^+$ to fix the parameters a_1 and a_2 (or correspondingly ξ and ξ'). Note, however, that final state interactions (FSI) can seriously affect the decay rates [36]. An isospin analysis gives some information about the effect of final state interaction (elastic scattering). The three $D \to \bar{K}\pi$ amplitudes, $D^0 \to K^-\pi^+, D^0 \to \bar{K^0}\pi^0$ and $D^+ \to \bar{K^0}\pi^+$ can be expressed in terms of two isospin amplitudes with isospin $\frac{1}{2}$ and $\frac{3}{2}$ in the final state as

$$A(D^0 \to K^-\pi^+) = \frac{1}{\sqrt{3}}(\sqrt{2}A_{\frac{1}{2}} + A_{\frac{3}{2}}) \tag{4.21}$$

$$A(D^0 \to \bar{K^0}\pi^0) = \frac{1}{\sqrt{3}}(-A_{\frac{1}{2}} + \sqrt{2}A_{\frac{3}{2}}) \tag{4.22}$$

$$A(D^+ \to \bar{K^0}\pi^+) = \sqrt{3}A_{\frac{3}{2}} \tag{4.23}$$

where $A_{\frac{1}{2}} = |A_{\frac{1}{2}}|e^{i\delta_{\frac{1}{2}}}$ and $A_{\frac{3}{2}} = |A_{\frac{3}{2}}|e^{i\delta_{\frac{3}{2}}}$and the third relation is written by using the following isospin relation [37] :

$$A(D^0 \to K^-\pi^+) + \sqrt{2}A(D^0 \to \bar{K^0}\pi^0) = A(D^+ \to \bar{K^0}\pi^+) \tag{4.24}$$

Without final state interactions the amplitudes $A_{\frac{1}{2}}$ and $A_{\frac{3}{2}}$ would be real. However, Mark III [38] measurements indicate that they need to be complex, and the phase shift between the two isospin amplitude is $\delta_{\frac{1}{2}} - \delta_{\frac{3}{2}} = 77^0 \pm 11^0$.The individual rates, especially those of Class II transitions, are modified by final state interactions. For instance $\Gamma(D^0 \to \bar{K^0}\pi^0)$ would decrease by a factor of 3 if we set $\delta_{\frac{1}{2}} - \delta_{\frac{3}{2}} = 0$ keeping $|A_{\frac{1}{2}}|$ and $|A_{\frac{3}{2}}|$ fixed. The isospin analysis for $D \to \bar{K}\pi$ decays can be used to obtain values for the coefficients $a_1$and a_2. Assuming factorization for the bare amplitude and neglecting scattering into other channels (inelastic effects) one finds [32]:

$$a_1 \simeq 1.2 \pm 0.1, \qquad a_2 \simeq -0.5 \pm 0.1\,. \tag{4.25}$$

Using Eqs. (4.13) and (4.25) and the Wilson coefficients $C_1 = 1.26 \pm 0.04$ and $C_2 = -0.51 \pm 0.05$ we find the following equation for ξ :

$$1.2 \pm 0.1 = (1.26 \pm 0.04) + \xi(-0.51 \pm 0.05) \tag{4.26}$$

and for ξ':

$$(-0.5 \pm 0.1) = (-0.51 \pm 0.05) + \xi'(1.26 \pm 0.04) \tag{4.27}$$

The solutions for Eqs. (4.26) and (4.27) are, $\xi \approx 0$ and $\xi' \approx 0$ or equivalently we have :

$$a_1 \approx C_1(m_c), \qquad a_2 \approx C_2(m_c). \tag{4.28}$$

This is different from what we anticipated from the naive color counting that $\xi \simeq \frac{1}{N_c} = \frac{1}{3}$ and also $\xi' \simeq \frac{1}{N_c} = \frac{1}{3}$ with the corresponding

$$a_1^0 = C_1 + \frac{C_2}{3} = 1.1, \qquad a_2^0 = C_2 + \frac{C_1}{3} = -0.1. \tag{4.29}$$

a_1^0 and a_2^0 can be looked at as the first order approximation of the phenomenological constants a_1 and a_2, that is they are the numerical values of a_1 and a_2 in a pure factorized model with $N_c = 3$. The result $\xi = 0$ and $\xi' = 0$ indicates an effective total color mismatch and in physical terms this means that quarks associated with different color singlet currents do not easily combine to form a single meson [39] . The case $\xi = 1$ corresponds to no color suppression which is ruled out by Bauer and Stech [40]. Although in the naive quark model the parameter ξ has the value $\frac{1}{3}$, in presence of soft gluons i.e. nonperturbative effects, this may not be justified. By retaining the matrix element of the color-octet current products in Eq. (4.9) and introducing η_1 and η_2 we have implicitly considered such nonperturbative effects. Thus by retaining these matrix elements we have done the first level of improvement in our factorization model by considering a nonfactorized effect. In a purely factorized model these matrix elements have to be considered negligible [16], that is $\eta_1 \approx 0$ and $\eta_2 \approx 0$. In other words, a_1 and a_2 in Eq. (4.13) consist of two parts, one of which has the factorized effects in it and the other which has nonfactorized effects implicitly in it, that is

$$a_1 = a_1^0 + a_1^{nf}, \qquad a_2 = a_2^0 + a_2^{nf} \tag{4.30}$$

where "nf" stands for "non-factorized".

According to Eq. (4.12), $\xi \approx 0$ and $\xi' \approx 0$ implies that $\tilde{N}_c \to \infty$ and $\tilde{N}'_c \to \infty$. Considering the result $\xi \approx 0$ and $\xi' \approx 0$, Buras et al. [41] have emphasized that factorization together with $\xi \approx 0$ corresponds to the lowest order in $\frac{1}{N_c}$ expansion. This effectively means that we take $\xi \approx 0$ and $\xi' \approx 0$ for all charm meson decays,

$$a_1 \approx C_1(m_c), \qquad a_2 \approx C_2(m_c)\,. \tag{4.31}$$

Therefore in this model a_1 and a_2 become process independent phenomenological coefficients. In the rest of this chapter we will consider the theoretical predictions of factorization model and compare these predictions with the experimental results.

4.4 Form Factors

We saw in Sec. (4.2)that for two-body final states consisting of pseudoscalar (P), vector (V), and axial vector (A) mesons, the factorization assumption allows the matrix element of $H_{NL}^{eff}$to be written in terms of simpler matrix elements of single currents and are thus determined in terms of meson decay constants and hadron current matrix elements. Meson decay constants are defined[29, 35, 42] in terms of the matrix elements of weak, vector $j_\mu^V = (\bar{q}_2\gamma^\mu q_1)$ and axial vector $j_\mu^A = (\bar{q}_2\gamma_5\gamma^\mu q_1)$ currents, between vacuum and pseudoscalar (P), vector(V) and axial vector (A) states as

$$\begin{aligned}
\left\langle V(p,\varepsilon)|j_\mu^V|0\right\rangle &= \varepsilon_\mu^* m_V f_V\,, & (4.32)\\
\left\langle A(p,\varepsilon)|j_\mu^A|0\right\rangle &= \varepsilon_\mu^* m_A f_A\,, & (4.33)\\
\left\langle P(p)|j_\mu^A|0\right\rangle &= -if_P p_\mu\,, & (4.34)
\end{aligned}$$

where m, p, ϵ and f are in order mass, four-momentum, polarization and the decay constant of the corresponding pseudoscalar, vector and axial vector meson. The other three possible matrix elements are zero, namely

$$\begin{aligned}
\left\langle V(p,\varepsilon)|j_\mu^A|0\right\rangle &= 0\,,\\
\left\langle A(p,\varepsilon)|j_\mu^V|0\right\rangle &= 0\,,\\
\left\langle P(p)|j_\mu^V|0\right\rangle &= 0\,, \qquad (4.35)
\end{aligned}$$

From Lorentz invariance one finds the most general form factor decompositions for the transition of a pseudoscalar meson (M) to pseudoscalar(P), vector(V) and axial vector mesons to be [29, 35, 42]

$$\begin{aligned}
\left\langle P(p')|j_\mu^V|M(p)\right\rangle = \left\{(p+p')_\mu - \frac{m_M^2-m_P^2}{q^2}q_\mu\right\} F_1^{MP}\left(q^2\right)\\
+\frac{m_M^2-m_P^2}{q^2}q_\mu F_0^{MP}\left(q^2\right)\,, \qquad (4.36)
\end{aligned}$$

$$\begin{aligned}
\left\langle V(p',\varepsilon)|j_\mu^V - j_\mu^A|M(p)\right\rangle &= i\Big\{(m_M+m_V)\varepsilon_\mu^* A_1^{MV}(q^2)\\
&- \frac{\varepsilon^*.q}{m_M+m_V}(p+p')_\mu A_2^{MV}(q^2)\\
&- 2m_V\frac{\varepsilon^*.q}{q^2}q_\mu\left(A_3^{MV}(q^2)-A_0^{MV}(q^2)\right)\Big\}\\
&+ \frac{2}{m_M+m_V}\varepsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}p^\rho p'^\sigma V^{MV}(q^2)\,, \qquad (4.37)
\end{aligned}$$

and

$$\begin{aligned}
\left\langle A(p',\varepsilon)|j^V_\mu - j^A_\mu|M(p)\right\rangle &= -i\Big\{(m_M+m_A)\varepsilon^*_\mu V_1^{MA}(q^2) \\
&- \frac{\varepsilon^*.q}{m_M+m_A}(p+p')_\mu V_2^{MA}(q^2) \\
&- 2m_A\frac{\varepsilon^*.q}{q^2}q_\mu\left(V_3^{MA}(q^2)-V_0^{MA}(q^2)\right)\Big\} \\
&- \frac{2}{m_M+m_A}\varepsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}p^\rho p'^\sigma A^{MA}(q^2)\,, \qquad (4.38)
\end{aligned}$$

where $q_\mu = (p-p')_\mu$. In addition, the following constraint applies at all q^2,

$$2m_V A_3^{MV}(q^2) = (m_M+m_V)A_1^{MV}(q^2) - (m_M-m_V)A_2^{MV}(q^2)\,, \qquad (4.39)$$

$$2m_A V_3^{MA}(q^2) = (m_M+m_A)V_1^{MA}(q^2) - (m_M-m_A)V_2^{MA}(q^2) \qquad (4.40)$$

The following relations are also needed to cancel the poles at $q^2=0$,

$$F_0^{MP}(0) = F_1^{MP}(0)\,, \qquad (4.41)$$

$$A_0^{MV}(0) = A_3^{MV}(0)\,, \qquad (4.42)$$

$$V_0^{MA}(0) = V_3^{MA}(0)\,. \qquad (4.43)$$

The task is now to estimate the invariant form factors appearing in Eqs. (4.36), (4.37) and (4.38). Once they are known the calculation of decay rates and spectra is straightforward. Different models for the determination of form factors have been proposed [29, 42, 43, 44, 45]. In the (original) model of Bauer, Stech and Wirbel (called BSWI here)[42], the values of the form factors are calculated at $q^2=0$ and extrapolated to finite q^2 using a monopole form for all the form factors , that is all the form factors are considered to have the following general form

$$\frac{h_i}{1-\frac{q^2}{m_i^2}}. \qquad (4.44)$$

This ansatz assures asymptotic ($q^2 \to \infty$) current conservation. The form factor decomposition in Eqs. (4.36 -4.38) has been done in such away that each form factor

Table 4.2: Values of pole masses (in GeV)

Current	$m(0^-)$	$m(1^-)$	$m(0^+)$	$m(1^+)$
$\bar{d}c$	1.87	2.01	2.47	2.42
$\bar{s}c$	1.97	2.11	2.60	2.53

is dominated by one and the nearest pole only: the vector form factors F_0 and F_1, V by the 0^+ and 1^- poles and the axial vector form factors A_0 and A_1, A_2 by the 0^- and 1^+ poles, respectively. Precise positions of most poles are not known, however, approximate values are sufficient in most cases. For numerical calculations the mass values displayed in Table (4.2) have been used. In the BSW approach the problem is reduced to an estimation of the form factors at $q^2 = 0$, i.e. the constants h_i. In the limit of exact SU(3) symmetry h_i's are unity, therefore h_i's in a sense a measure of $SU(3)$ breaking. h_i's can be expressed by the overlap of initial and final meson wave functions. The mesons are described as relativistic bound states of a quark Q_1 and antiquark $\bar{q}_2$ which are modelled as a relativistic harmonic oscillator in BSW model [42].For numerical calculation we will use the values of h_i's given in Table (4.3).

4.5 $M \to PP, PV, PA, VP$ and VA decay amplitudes

We saw that the factorized decay amplitude for a pseudoscalar meson M decaying to two mesons (pseudoscalar(P), vector(V) or axial vector (A)) can be written in the following general form

$$\left\langle P, V or A|\tilde{J}^\mu|0\right\rangle \langle P, V or A|J^\mu|M\rangle \qquad (4.45)$$

In this section we will examine the factorized decay amplitude for $M \to PP, PV, PA, VP$ and AP decays in more detail.

Table 4.3: Formfactors at zero momentum transfer for $P \to P$ and $P \to V$ transitions

Decay	$h_{F_1} = h_{F_0}$	h_V	h_{A_1}	h_{A_2}	$h_{A_3} = h_{A_0}$
$D \to K$	0.762				
$D \to \pi$	0.692				
$D \to \eta$	0.681				
$D \to \eta'$	0.655				
$D \to K^*$		1.226	0.880	1.147	0.733
$D \to \rho$		1.225	0.775	0.923	0.669
$D \to \omega$		1.236	0.772	0.920	0.669
$D_s \to \eta$	0.723				
$D_s \to \eta'$	0.704				
$D_s \to K$	0.643				
$D_s \to K^*$		1.250	0.717	0.853	0.634
$D_s \to \phi$		1.319	0.820	1.076	0.700

4.5.1 $M \to PP$

The spectator part of the PP decay amplitudes involve the following type of terms

$$A_{PP}(M \to P_1P_2) \equiv \left\langle P_2(q)|\tilde{J}_\mu|0\right\rangle \langle P_1(p')|J^\mu|M(p)\rangle . \tag{4.46}$$

Using Eqs. (4.34) and (4.36) we get

$$A_{PP}(M \to P_1P_2) = -if_{P_1}q^\mu \left\{ \left\{ (p+p')_\mu - \frac{m_M^2 - m_{P_1}^2}{q^2} q_\mu \right\} F_1^{MP_1}\left(q^2\right) + \frac{m_M^2 - m_{P_1}^2}{q^2} q_\mu F_0^{MP_1}\left(q^2\right) \right\} . \tag{4.47}$$

Since $q^\mu = P^\mu - P'^\mu$ and $q^2 = m_{P_2}^2$ we have

$$A_{PP}(M \to P_1P_2) = -if_{P_2}(m_M^2 - m_{P_1}^2)F_0^{MP_1}(m_{P_2}^2) \tag{4.48}$$

where

$$F_0^{MP_1}(q^2) = \frac{h_{F_0}}{1 - \frac{q^2}{m_{0+}^2}} \tag{4.49}$$

and m_{0+} is the mass of the scalar particle giving the nearest pole singularity for the vector part of current J^μ.

4.5.2 $M \to PV$

The spectator part of the PV decay amplitude consist of the following type of term :

$$A_{PV}(M \to PV) \equiv < V(\varepsilon, q)|\tilde{J}_\mu|0 >< P(p')|J^\mu|M(p) > . \tag{4.50}$$

Using Eqs. (4.32) and (4.36), we get

$$A_{PV}(M \to PV) = m_V f_V \varepsilon^{*\mu} \left\{ \left\{ (p+p')_\mu - \frac{m_M^2 - m_P^2}{q^2} q_\mu \right\} F_1^{MP}\left(q^2\right) + \frac{m_M^2 - m_P^2}{q^2} q_\mu F_0^{MP}\left(q^2\right) \right\} . \tag{4.51}$$

Since $q^\mu = p^\mu - p'^\mu$ is the four momentum of V we have $q \cdot \varepsilon = 0$ according to definition of ε^μ. Hence we have

$$\begin{aligned} A_{PV}(M \to PV) &= (p+p') \cdot \varepsilon m_V f_V F_1^{MP}(m_V^2) \\ &= 2p \cdot \varepsilon m_V f_V F_1^{MP}(m_V^2) , \end{aligned} \tag{4.52}$$

where

$$F_1^{MP}(q^2) = \frac{h_{F_1}}{1 - \frac{q^2}{m_{1^-}^2}} \tag{4.53}$$

and m_{1^-} is the vector pole mass for the appropriate current J^μ.

4.5.3 $M \to PA$

The spectator part of the PA decay amplitude involve the following type of term

$$A_{PA}(M \to PA) \equiv < A(\varepsilon, q)|\tilde{J}_\mu|0 >< P(p')|J^\mu|M(p) > . \tag{4.54}$$

Using Eqs. (4.33) and (4.36), we have

$$A_{PA}(M \to PA) = m_A f_A \varepsilon^{*\mu} \left\{ \left\{ (p+p')_\mu - \frac{m_M^2 - m_P^2}{q^2} q_\mu \right\} F_1^{MP}\left(q^2\right) + \frac{m_M^2 - m_P^2}{q^2} q_\mu F_0^{MP}\left(q^2\right) \right\} . \tag{4.55}$$

After simplification, similar to PV decays we get

$$A_{PA}(M \to PA) = 2p \cdot \varepsilon m_A f_A F_1^{MP}(m_A^2) \tag{4.56}$$

where $F_1^{MP}(q^2)$ is defined in Eq. (4.53) and it must be evaluated at $q^2 = m_A^2$.

4.5.4 $M \to VP$

The spectator part of the $M \to VP$ decay amplitudes involves the following type of term:

$$A_{VP}(M \to VP) \equiv < P(q)|\tilde{J}_\mu|0 >< V(\varepsilon, p')|J^\mu|M(p) > . \tag{4.57}$$

Eqs. (4.34) and (4.37) imply that

$$\begin{aligned} A_{VP}(M \to VP) &= -if_P q^\mu \Big\{ i \Big\{ (m_M + m_V)\varepsilon^*_\mu A_1^{MV}(q^2) \\ &- \frac{\varepsilon^*.q}{m_M + m_V}(p+p')_\mu A_2^{MV}(q^2) \\ &- 2m_V \frac{\varepsilon^*.q}{q^2} q_\mu \left(A_3^{MV}(q^2) - A_0^{MV}(q^2) \right) \Big\} \\ &+ \frac{2}{m_M + m_V} \varepsilon_{\mu\nu\rho\sigma} \varepsilon^{*\nu} p^\rho p'^\sigma V^{MV}(q^2) \Big\} . \end{aligned} \tag{4.58}$$

Noticing, $q^\mu = p^\mu - p'^\mu$, therefore $(p+p') \cdot q = m_M^2 - m_V^2$ and also by definition $\varepsilon^* \cdot p = 0$. Using Eq. (4.39) and after simplification we have

$$A_{VP}(M \to VP) = 2m_V f_P (\varepsilon^* \cdot p) A_0^{MV}(m_P^2) \tag{4.59}$$

where

$$A_0^{MV}(q^2) = \frac{h_{A_0}}{1 - \frac{q^2}{m_{0^-}^2}} \tag{4.60}$$

m_{0^-} is the pseudoscalar particle mass that corresponds to the nearest pole.

4.5.5 $M \to AP$

The spectator part of the $M \to AP$ decay amplitude involve the following type of term

$$A_{AP}(M \to AP) \equiv < P(q)|\tilde{J}_\mu|0 >< A(\varepsilon, p')|J^\mu|M(p) > . \qquad (4.61)$$

Eqs. (4.34) and (4.38) imply that

$$\begin{aligned} A_{AP}(M \to AP) &= -if_P q^\mu \Big\{ i \Big\{ (m_M + m_A)\varepsilon^*_\mu V_1^{MA}(q^2) \\ &- \frac{\varepsilon^*.q}{m_M + m_A}(p+p')_\mu V_2^{MA}(q^2) \\ &- 2m_A \frac{\varepsilon^*.q}{q^2} q_\mu \left(V_3^{MA}(q^2) - V_0^{MA}(q^2) \right) \Big\} \\ &+ \frac{2}{m_M + m_A} \varepsilon_{\mu\nu\rho\sigma} \varepsilon^{*\nu} p^\rho p'^\sigma A^{MA}(q^2) \Big\} . \end{aligned} \qquad (4.62)$$

$q^\mu = p^\mu - p'^\mu$ therefore $(p+p')\cdot q = m_M^2 - m_A^2$ and also by definition $\varepsilon^* \cdot p = 0$. Using Eq. (4.40) after simplification we have

$$A_{AP}(M \to AP) = 2m_A f_P (\varepsilon^* \cdot p) V_0^{MA}(m_P^2) . \qquad (4.63)$$

4.6 Branching Ratios

The branching ratio for for $M \to P_1 P_2$ decay where P_1 and P_2 are pseudoscalar mesons is given by

$$B(M \to P_1 P_2) = \tau_M \Gamma(M \to P_1 P_2) = \tau_M \frac{|\vec{p}|}{8\pi m_M^2} |A(M \to P_1 P_2)|^2 , \qquad (4.64)$$

where τ_M is the lifetime of M and $|\vec{p}|$ is the magnitude of the final state mesons three-momentum in the M rest frame.

For the decays where one or both of final state particles are vector or axial vector mesons we have a summation over the polarizations. For example for a $M \to$

PV decay we have

$$B(M \to PV) = \tau_M \frac{|\vec{p}|}{8\pi m_M^2} \sum_\lambda |A(M \to PV)_\lambda|^2 \,. \tag{4.65}$$

In the decays of our interest summation over polarization usually means calculation of terms like

$$\sum_{\lambda=1}^{3} |\varepsilon \cdot p|^2 \tag{4.66}$$

where ε is the polarization vector of the vector or axial vector meson and p is the four-momentum of the pseudoscalar meson M. The calculation is straightforward (See [46]), leading to

$$\sum_{\lambda=1}^{3} |\varepsilon \cdot p|^2 = |\vec{p}_{V,A}|^2 \frac{m_M^2}{m_{V,A}^2} \,, \tag{4.67}$$

where $\vec{p}_{V,A}$ is the three-momentum of the vector or axial-vector meson in the rest frame of M and $m_{V,A}$ its mass. This term is the source of $|\vec{p}|^3$ dependence of the branching ratios of two-body decays with vector or axial vector mesons in the final state.

4.7 Two-body Non-leptonic Charm meson Decays in Factorization Model

After fixing the values of parameters a_1 and a_2 from $D \to \bar{K}\pi$ decays, Bauer, Stech and Wirbel [35] proceeded to calculate many Cabibbo-favored and also some Cabibbo-suppressed, exclusive two-body D^0, D^+ and D_s^+ decays. The comparison of the theoretical branching ratios predicted by the factorization model with experimental branching ratios shows overall agreement between theoretical and experimental numbers which is quite remarkable considering the simplicity of the model and the fact that D decays occur in a resonance region. It can be concluded from the experience with D decays that factorization together with $a_1 \approx C_1(m_c)$ and $a_2 \approx C_2(m_c)$ gives generally good results at least for energetic two-body decays.

Table 4.4: Two-body non-leptonic decay widths summed up and compared with the measured total nonleptonic widths for D^0, D^+ and D_s^+ transitions. The Units are $10^{10} s^{-1}$

Meson	$\Gamma^{th}(D \to XY)$	$\Gamma^{exp}(D \to nonleptonic)$
D^0	153 ± 22	194 ± 48
D^+	56 ± 15	71 ± 14
D_s^+	126 ± 15	—

Encouraged by their results BSW computed [35] many more two-body decays and summed them up in order to estimate their contribution to the nonleptonic decays widths. In their calculation they included all particles which belong to $SU(3)$ pseudoscalar or vector nonets. They also consider $a_1(J^P = 1^+)$ and $K_1(J^P = 1^+)$ particles but only as far as they can be produced directly from the corresponding currents.

Using for the parameters a_1, a_2 the values 1.3 ± 0.1 and -0.55 ± 0.1 respectively, it turns out that about 70-80 % of the total nonleptonic transition rates D^0 and D^+ are accounted for by two-body decays (Table (4.4)). More important, the calculated ratio $\Gamma(D^0 \to XY)/\Gamma(D^+ \to XY)$ for these nonleptonic two-body decays turned out to be $\simeq 2.7$ (Table (4.4)). Thus a sizable part of the lifetime difference between D^0 and D^+ arises from two-body decays due to the destructive interference ($a_2/a_1 < 0$) in several important D^+ decays [39]. They didn't consider any annihilation contribution in their calculations.

4.8 Test of Factorization by Comparison of Semileptonic to Nonleptonic Decay Rates

As we saw in Sec. (4.4), to calculate the theoretical branching ratios with the factorized amplitude we have to use different form factors such as $F_0(q^2), F_1(q^2), \ldots$.As we have remarked earlier different models have been proposed for the evaluation of the form factors at $q^2 = 0$, h_i's. Further, a q^2 dependence of the form factors has to be assumed. Thus the theoretical branching ratios, and as a result the tests of factorization, become much dependent on the theoretical model for the form factors at $q^2 = 0$ and their assumed q^2 dependence. the determination of h_i's and q^2 dependence of the form factors.

One way of avoiding this problem [47] is to test the factorization assumption by comparing two-body hadronic decay rates , for example $\Gamma(D^0 \to K^-\pi^+)$ with the differential rate of a suitable semileptonic decay for which in this case the suitable choice is $\frac{d}{dq^2}\Gamma(D^0 \to K^-e^+\nu)$. The reason is that they both involve one and the same form factor which drops out by taking the ratio of the hadronic decay rate to the differential semileptonic decaty rate evaluated at a suitable fixed value of q^2.

But, unfortunately, this needs very good data of $\frac{d\Gamma}{dq^2}$ at the required q^2 which in most cases of interest is lacking. The alternative is to compare the nonleptonic two-body decay rates with the semileptonic decay rate, that is, comparing $\Gamma(D^0 \to K^-\pi^+)$ with $\Gamma(D^0 \to K^-e^+\nu)$ instead of $\frac{d}{dq^2}\Gamma(D^0 \to K^-e^+\nu)$. In this way h_i's cancel in the ratio and the uncertainty comes from the assumed q^2 behavior of the form factor, in this case, $F_1(q^2)$, in performing the q^2 integration in the derivation of $\Gamma(D^0 \to K^-e^+\nu)$ from $\frac{d}{dq^2}\Gamma(D^0 \to K^-e^+\nu)$.

In the following we will consider the test of factorization by comparing

1) $B(D^0 \to K^-\pi^+), B(D^0 \to K^-\rho^+)$ and $B(D^0 \to K^-a_1^+)$ with B ($D^0 \to K^-e^+\nu$).

2)$B(D^0 \to \pi^-\pi^+)$ with $B(D^0 \to \pi^- e^+\nu)$.

3) $B(D_s^+ \to \eta\pi^+), B(D_s^+ \to \eta\rho^+)$ with $B(D_s^+ \to \eta e^+\nu)$ and $B(D_s^+ \to \eta'\pi^+)$, $B(D_s^+ \to \eta'\rho^+)$ with $B(D_s^+ \to \eta' e^+\nu)$.

4.8.1 Comparison of $B(D^0 \to K^-\pi^+), B(D^0 \to K^-\rho^+)$ and $B(D^0 \to K^-a_1^+)$ with $B(D^0 \to K^-e^+\nu)$.

The mesons π^+, ρ^+ and a_1^+ are pseudoscalar, vector and axial vector respectively. Therefore, the decays $D^0 \to K^-\pi^+, D^0 \to K^-\rho^+$ and $D^0 \to K^-a_1^+$ are of type $D \to PP, D \to PV$ and $D \to PA$ respectively . Also all the three decays belong to Class I type .

Using Eqs. (4.20) and (4.48) we have

$$A(D^0 \to K^-\pi^+) = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* a_1 < \pi^+|(\bar{u}d)_{L,H}|0 >< K^-|(\bar{s}c)_{L,H}|D^0 >$$
$$= i\frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* a_1 f_\pi (m_D^2 - m_K^2) F_0^{DK}(m_\pi^2) \,. \quad (4.68)$$

Therefore using Eqs. (4.64) and (4.68) and $\tilde{G}_F = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^*$ we get

$$B(D^0 \to K^-\pi^+) = \tau_{D^0} \frac{\tilde{G}_F^2}{8\pi^2} |a_1|^2 f_\pi^2 \frac{(m_D^2 - m_K^2)^2}{m_D^2} [F_0^{DK}(m_\pi^2)]^2 |\vec{p}| \quad (4.69)$$

For two-body decays ($a \to bc$), $|\vec{p}|$ is:

$$|\vec{p}| = \frac{\lambda(m_a^2, m_b^2, m_c^2)}{2m_a} \quad (4.70)$$

where $\lambda(x, y, z) = (x^2 + y^2 + z^2 - 2xy - 2xz - 2yz)^{1/2}$. By substituting for $|\vec{p}|$ in Eq. (4.69) from Eq. (4.70) we get

$$B(D^0 \to K^-\pi^+) = \tau_{D^0} \frac{\tilde{G}_F^2}{8\pi^2} |a_1|^2 f_\pi^2 \frac{(m_D^2 - m_K^2)^2}{m_D^3} [F_0^{DK}(m_\pi^2)]^2 \lambda(m_D^2, m_K^2, m_\pi^2) \quad (4.71)$$

Similarly, using Eqs. (4.20), (4.52) and (4.56) we have

$$A(D^0 \to K^-\rho^+) = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* a_1 (2m_\rho) f_\rho F_1^{DK}(m_\rho^2) \varepsilon^* \cdot p \quad (4.72)$$

$$A(D^0 \to K^- a_1^+) = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* a_1 (2m_{a_1}) f_{a_1} F_1^{DK}(m_{a_1}^2) \varepsilon^* \cdot p \tag{4.73}$$

where p is the four-momentum of D^0. Using Eqs. (4.72), (4.73), (4.65), (4.67) and (4.70) we get

$$B(D^0 \to K^- \rho^+) = \tau_{D^0} \frac{G_F^2}{32\pi} \frac{|V_{cs}|^2 |V_{ud}|^2}{m_D^3} |a_1|^2 f_\rho^2 |F_1^{DK}(m_\rho^2)|^2 \lambda^3(m_D^2, m_K^2, m_\rho^2) \tag{4.74}$$

$$B(D^0 \to K^- a_1^+) = \tau_{D^0} \frac{G_F^2}{32\pi} \frac{|V_{cs}|^2 |V_{ud}|^2}{m_D^3} |a_1|^2 f_{a_1}^2 |F_1^{DK}(m_{a_1}^2)|^2 \lambda^3(m_D^2, m_K^2, m_{a_1}^2) \tag{4.75}$$

The differential decay rate for the semileptonic decay $D^0 \to K^- e^+ \nu$ is given by [48]

$$\frac{d}{dq^2}\Gamma(D^0 \to K^- e^+ \nu) = \frac{G_F^2}{192\pi^3} |V_{cs}|^2 \frac{\lambda^3(m_D^2, m_K^2, q^2)}{m_D^3} |F_1^{DK}(q^2)|^2 \tag{4.76}$$

Comparing $B(D^0 \to K^- \pi^+)$ with $\frac{d}{dq^2}\Gamma(D^0 \to K^- e^+ \nu)$ at $q^2 = m_\pi^2$ and using $F_0^{DK}(m_\pi^2) \approx F_0^{DK}(0) = F_1^{DK}(0)$, one finds that in the absence of FSI's the factorization model implies the following local relation:

$$\Gamma(D^0 \to K^- \pi^+)_{no\ FSI} \approx 6\pi^2 a_1^2 f_\pi^2 |V_{ud}|^2 \frac{(m_D^2 - m_K^2)^2}{\lambda^2(m_D^2, m_K^2, m_\pi^2)} \times \frac{d}{dq^2}\Gamma(D^0 \to K^- e^+ \nu)|_{q^2 = m_\pi^2} \tag{4.77}$$

Similarly comparing $B(D^0 \to K^- \pi^+)$ with $\frac{d}{dq^2}\Gamma(D^0 \to K^- e^+ \nu)$ at $q^2 = m_\rho^2$ and also comparing $B(D^0 \to K^- a_1^+)$ with $\frac{d}{dq^2}\Gamma(D^0 \to K^- e^+ \nu)$ at $q^2 = m_{a_1}^2$ one finds that in the absence of final state interactions (FSI) the factorization model requires the following local relations:

$$\Gamma(D^0 \to K^- \rho^+)_{no\ FSI} \approx 6\pi^2 a_1^2 f_\rho^2 |V_{ud}|^2 \frac{d}{dq^2}\Gamma(D^0 \to K^- e^+ \nu)|_{q^2 = m_\rho^2} \tag{4.78}$$

$$\Gamma(D^0 \to K^- a_1^+)_{no\ FSI} \approx 6\pi^2 a_1^2 f_{a_1}^2 |V_{ud}|^2 \frac{d}{dq^2}\Gamma(D^0 \to K^- e^+ \nu)|_{q^2 = m_{a_1}^2} \tag{4.79}$$

As we have said before, testing the local relations (4.77), (4.78) and (4.79) requires very good data of $\frac{d\Gamma}{dq^2}$ at the required q^2. In the absence of high precision data an alternative test of factorization assumption is to compare the calculated ratios $\frac{\Gamma(D^0\to K^-\pi^+)}{\Gamma(D^0\to K^-e^+\nu)}$, $\frac{\Gamma(D^0\to K^-\rho^+)}{\Gamma(D^0\to K^-e^+\nu)}$, and $\frac{\Gamma(D^0\to K^-a_1^+)}{\Gamma(D^0\to K^-e^+\nu)}$ with the experimental ones.

For calculating $\Gamma(D^0 \to K^-e^+\nu)$ we have to assume a specific q^2 dependence for $F_1^{DK}(q^2)$. We use the one suggested in the BSWI model namely

$$F_1^{DK}(q^2) = \frac{F_1^{DK}(0)}{1-\frac{q^2}{\Lambda_1^2}} \tag{4.80}$$

with $\Lambda_1 = 2.11$ GeV (D_s^* mass). The semileptonic rate is evaluated to be

$$\Gamma(D^0 \to K^-e^+\nu) = \frac{G_F^2}{192}|V_{cs}|^2\frac{|F_1^{DK}(0)|^2\Lambda_1^4}{m_D^3}I(m_D, m_K, \Lambda_1)\,, \tag{4.81}$$

where

$$I(m_D, m_K, \Lambda_1) \equiv \int_0^{(m_D-m_K)^2} \frac{\lambda^3(m_D^2, m_K^2, q^2)}{(q^2-\Lambda_1^2)^2}dq^2\,. \tag{4.82}$$

which can be determined both numerically and analytically [47].

Comparing Eqs. (4.69), (4.74) and (4.75) with Eq. (4.81) we find that the factorization hypotheses requires:

$$\begin{aligned}\frac{\Gamma(D^0 \to K^-\pi^+)_{no\ FSI}}{\Gamma(D^0 \to K^-e^+\nu)} &\simeq \frac{6\pi^2 a_1^2 f_\pi^2 (m_D^2 - m_K^2)^2 |V_{ud}|^2 \lambda(m_D^2, m_K^2, m_\pi^2)}{\Lambda_1^4 I(m_D, m_K, \Lambda_1)} \\ &= 1.63 \pm 0.33\end{aligned} \tag{4.83}$$

$$\frac{\Gamma(D^0 \to K^-\rho^+)_{no\ FSI}}{\Gamma(D^0 \to K^-e^+\nu)} \simeq \frac{6\pi^2 a_1^2 f_\rho^2 |V_{ud}|^2 \lambda^3(m_D^2, m_K^2, m_\rho^2)}{(\Lambda_1^2 - m_\rho^2)^2 I(m_D, m_K, \Lambda_1)} = 3.0 \pm 0.6 \tag{4.84}$$

$$\frac{\Gamma(D^0 \to K^-a_1^+)_{no\ FSI}}{\Gamma(D^0 \to K^-e^+\nu)} \simeq \frac{6\pi^2 a_1^2 f_{a_1}^2 |V_{ud}|^2 \lambda^3(m_D^2, m_K^2, m_{a_1}^2)}{(\Lambda_1^2 - m_{a_1}^2)^2 I(m_D, m_K, \Lambda_1)} = 0.43 \pm 0.05 \tag{4.85}$$

where we have used $f_\pi = 131$ MeV, $f_\rho = f_{a_1} = 220$ MeV and $a_1 = 1.2$.The errors come from assigning 10% uncertainty to a_1 .

The experimental ratios are :

$$\left.\frac{\Gamma(D^0 \to K^-\pi^+)}{\Gamma(D^0 \to K^-e^+\nu)}\right|_{expt} = 1.10 \pm 0.11 \qquad [6]\,, \tag{4.86}$$

$$\left.\frac{\Gamma(D^0 \to K^-\rho^+)}{\Gamma(D^0 \to K^-e^+\nu)}\right|_{expt} = 2.20 \pm 0.38 \qquad [6]\,, \tag{4.87}$$

$$\left.\frac{\Gamma(D^0 \to K^-a_1^+)}{\Gamma(D^0 \to K^-e^+\nu)}\right|_{expt} = 2.24 \pm 0.44 \qquad [6]\,, \tag{4.88}$$

Although Eqs. (4.83) and (4.86) appear to be in disagreement, but after the theoretical prediction of factorization model is corrected for FSI's, a very good agreement between theory and experiment is found [47].

Within errors factorization model prediction for the ratio in Eq. (4.84) agreess with the experimental ratio Eq. (4.87), therefore it seems the "raw" test (uncorrected for FSI effects) of factorization for $D^0 \to K^-\rho^+$ is satisfactory. If the effects of FSI and mass smearing over the ρ width are taken into consideration, the agreement between theory and experiment improves [47]. The theoretical prediction of factorization model in Eq. (4.85) is in strong disagreement with the experimental ratio Eq. (4.88). The ratio in Eq. (4.85) is not very sensitive to mass smearing [47] and FSI does not seem to solve the problem. Perhaps factorization ought not to be expected to work well for decays involving small energy release such as $D^0 \to K^-a_1^+$.

4.8.2 Comparison of $B(D^0 \to \pi^-\pi^+)$ to $B(D^0 \to \pi^-e^+\nu)$.

Contrary to the cases discussed so far $D^0 \to \pi^-\pi^+$ and $D^0 \to \pi^-e^+\nu$ decays are Cabibbo-suppressed processes, which their decay amplitude involve the CKM matrix element V_{cd} appears instead of V_{cs}. However, V_{cd} cancels in the ratio. Exactly similar to the $D^0 \to K^-\pi^+$ case one can derive the following predicted ratio of the factorization model:

$$\frac{\Gamma(D^0 \to \pi^-\pi^+)_{no\ FSI}}{\Gamma(D^0 \to \pi^-e^+\nu)} \simeq \frac{6\pi^2 a_1^2 f_\pi^2 (m_D^2 - m_\pi^2)^2 |V_{ud}|^2 \lambda(m_D^2, m_\pi^2, m_\pi^2)}{\Lambda_2^4 I(m_D, m_\pi, \Lambda_2)} \tag{4.89}$$
$$= 1.02 \pm 0.20$$

where $\Lambda_2 = 2.01GeV(D^*$ mass) has been used and the errors in Eq. (4.89) result from a 10%uncertainty assigned to a_1. The experimental ratio is

$$\left.\frac{\Gamma(D^0 \to \pi^-\pi^+)}{\Gamma(D^0 \to \pi^- e^+ \nu)}\right|_{expt} = 0.42^{+0.25}_{-0.14} \quad [6]\,, \tag{4.90}$$

The disagreement between theory, Eq. (4.89), and experiment, Eq. (4.90), is largely due to the FSI's. It is shown in Ref. [49] that the BSWI model for the form factors does a fair job of reproducing $D \to \pi\pi$ data with $\delta_0 - \delta_2 \approx 90^0$ [50], where δ_0 and δ_2 are the phase of $I = 0$ and 2 decay amplitudes. The suppression of $D^0 \to \pi^-\pi^+$ rate in the BSW model with $\delta_0 - \delta_2 \approx 90^0$ is by a factor of 0.68. Thus the effect of FSI's would be to lower the ratio in Eq. (4.89) to 0.69 ± 0.14 which would be in agreement with experiment Eq. (4.90).

4.8.3 Comparison of $B(D_s^+ \to \eta\pi^+), B(D_s^+ \to \eta\rho^+)$ with $B(D_s^+ \to \eta e^+\nu)$ and $B(D_s^+ \to \eta'\pi^+), B(D_s^+ \to \eta'\rho^+)$ with $B(D_s^+ \to \eta' e^+\nu)$.

In some ways two-body Cabibbo-favored decays of D^0 and D_s^+ involving η or η' in the final state are cleaner systems as the final state only involves one isospin (η and η' are isospinless states), yet these decays have proven to be problematic for the theory[49, 51].

Factorization model relates the nonleptonic processes $D_s^+ \to \eta\pi^+, D_s^+ \to \eta\rho^+$ and $D_s^+ \to \eta'\pi^+, D_s^+ \to \eta'\rho^+$ to the semileptonic $D_s^+ \to \eta e^+\nu$ and $D_s^+ \to \eta' e^+\nu$ respectively. A comparison between the theoretical predictions and experimental results of the ratio of the above nonleptonic and semileptonic decays would further test the validity or otherwise of the factorization assumption.

In describing the η and η' system we use the conventional mixing

$$|\eta >= |8 > cos\theta_p - |0 > sin\theta_p\,,$$

$$|\eta' >= |8 > sin\theta_p + |0 > cos\theta_p \ , \qquad (4.91)$$

where the flavor-singlet and -octet are defined as

$$|0 >= \frac{1}{\sqrt{3}}|u\bar{u} + d\bar{d} + s\bar{s} > \ ,$$
$$|8 >= \frac{1}{\sqrt{6}}|u\bar{u} + d\bar{d} - 2s\bar{s} > \ , \qquad (4.92)$$

and the mixing angle θ_p is taken to be $\simeq -19^0$ [6].

$D_s^+ \to \eta\pi^+$ and $D_s^+ \to \eta\rho^+$ decays are repectively $M \to PP$ and $M \to PV$, therefore using Eqs. (4.32), (4.34) and (4.36) the factorized decay amplitudes are

$$A(D_s^+ \to \eta\pi^+) = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* C_\eta a_1 f_\pi (m_{D_s}^2 - m_\eta^2) F_0^{D_s\eta}(m_\pi^2) \ , \qquad (4.93)$$
$$A(D_s^+ \to \eta\rho^+) = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* C_\eta a_1 (2m_\rho f_\rho) \varepsilon^* \cdot p_{D_s} F_1^{D_s\eta}(m_\rho^2) \ , \qquad (4.94)$$

where

$$C_\eta = \sqrt{\frac{2}{3}} \left(\cos\theta_P + \frac{1}{\sqrt{2}} \sin\theta_P \right) \ . \qquad (4.95)$$

The resulting decay rates, using Eqs. (4.64) and (4.65) are

$$\Gamma(D_s^+ \to \eta\pi^+) = \frac{\tilde{G}_F^2}{16\pi} C_\eta^2 f_\pi^2 \frac{(m_{D_s}^2 - m_\pi^2)^2}{m_{D_s}^2} \lambda(m_{D_s}^2, m_{\eta^2}, m_\pi^2) |F_1^{D_s\eta}(0)|^2 \ , \qquad (4.96)$$

$$\Gamma(D_s^+ \to \eta\rho^+) = \frac{\tilde{G}_F^2}{16\pi m_{D_s}^3} C_\eta^2 f_\rho^2 \lambda^3(m_{D_s}^2, m_\eta^2, m_\rho^2) \frac{|F_1^{D_s\eta}(0)|^2 \Lambda_1^4}{(\Lambda_1^2 - m_\rho^2)^2} \ , \qquad (4.97)$$

where in Eq. (4.96) we have used $F_0^{D_s\eta}(m_\pi^2) \approx F_0^{D_s\eta}(0) = F_1^{D_s\eta}(0)$ and in Eq. (4.97) Λ_1 is taken to be 2.11 GeV, the D_s^* mass.

The differential and the total rates for the semileptonic decay $D_s^+ \to \eta e^+ \nu$ are given by

$$\frac{d}{dq^2}\Gamma(D_s^+ \to \eta e^+ \nu) = \frac{G_F^2}{192\pi^3} \frac{|V_{cs}|^2 C_\eta^2}{M_{D_s}^3} \lambda^3(m_{D_s}^3, m_\eta^2, q^2) |F_1^{D_s\eta}(q^2)|^2 \qquad (4.98)$$

and

$$\Gamma(D_s^+ \to \eta e^+ \nu) = \frac{G_F^2}{192\pi^3}\frac{|V_{cs}|^2 C_\eta^2}{M_{D_s}^3}|F_1^{D_s\eta}(0)|^2 \Lambda_1^4 I(m_{D_s}, m_\eta, \Lambda_1)\,. \tag{4.99}$$

where $I(m_{D_s}, m_\eta, \Lambda_1)$ is defined as in Eq. (4.82) .

From Eqs. (4.96), (4.97) and (4.98) we obtain the following local relations

$$\Gamma(D_s^+ \to \eta\pi^+) \approx 6\pi^2 a_1^2 f_\pi^2 |V_{ud}|^2 \frac{(m_{D_s}^2 - m_\eta^2)^2}{\lambda^2(m_{D_s}^2, m_\eta^2, m_\pi^2)} \times \frac{d}{dq^2}\Gamma(D_s^+ \to \eta e^+\nu)\Big|_{q^2=m_\pi^2} \tag{4.100}$$

$$\Gamma(D_s^+ \to \eta\rho^+) \approx 6\pi^2 a_1^2 f_\rho^2 |V_{ud}|^2 \frac{d}{dq^2}\Gamma(D_s^+ \to \eta e^+\nu)\Big|_{q^2=m_\rho^2} \tag{4.101}$$

and from Eqs. (4.96), (4.97) and (4.99) we get

$$\frac{\Gamma(D_s^+ \to \eta\pi^+)}{\Gamma(D_s^+ \to \eta e^+\nu)} \simeq \frac{6\pi^2 a_1^2 f_\pi^2 (m_D^2 - m_\eta^2)^2 |V_{ud}|^2 \lambda(m_D^2, m_\eta^2, m_\pi^2)}{\Lambda_1^4 I(m_D, m_\eta, \Lambda_1)} = 1.47 \pm 0.14 \tag{4.102}$$

$$\frac{\Gamma(D_s^+ \to \eta\rho^+)}{\Gamma(D_s^+ \to \eta e^+\nu)} \simeq \frac{6\pi^2 a_1^2 f_\rho^2 |V_{ud}|^2 \lambda^3(m_D^2, m_\eta^2, m_\rho^2)}{(\Lambda_1^2 - m_\rho^2)^2 I(m_D, m_\eta, \Lambda_1)} = 2.85 \pm 0.28 \tag{4.103}$$

An analogous treatment of the decays involving η' with using

$$C_{\eta'} = \sqrt{\frac{2}{3}}\left(-\sin\theta_P + \frac{1}{\sqrt{2}}\cos\theta_P\right) \tag{4.104}$$

instead of C_η, results in the following predictions of the factorization model

$$\Gamma(D_s^+ \to \eta'\pi^+) \approx 6\pi^2 a_1^2 f_\pi^2 |V_{ud}|^2 \frac{(m_{D_s}^2 - m_{\eta'}^2)^2}{\lambda^2(m_{D_s}^2, m_{\eta'}^2, m_\pi^2)} \times \frac{d}{dq^2}\Gamma(D_s^+ \to \eta' e^+\nu)\Big|_{q^2=m_\pi^2}\,, \tag{4.105}$$

$$\Gamma(D_s^+ \to \eta'\rho^+) \approx 6\pi^2 a_1^2 f_\rho^2 |V_{ud}|^2 \frac{d}{dq^2}\Gamma(D_s^+ \to \eta' e^+\nu)\Big|_{q^2=m_\rho^2}\,, \tag{4.106}$$

and

$$\frac{\Gamma(D_s^+ \to \eta' \pi^+)}{\Gamma(D_s^+ \to \eta' e^+ \nu)} \simeq \frac{6\pi^2 a_1^2 f_\pi^2 (m_D^2 - m_{\eta'}^2)^2 |V_{ud}|^2 \lambda(m_D^2, m_{\eta'}^2, m_\pi^2)}{\Lambda_1^4 I(m_D, m_{\eta'}, \Lambda_1)} = 3.07 \pm 0.30 \tag{4.107}$$

$$\frac{\Gamma(D_s^+ \to \eta' \rho^+)}{\Gamma(D_s^+ \to \eta' e^+ \nu)} \simeq \frac{6\pi^2 a_1^2 f_\rho^2 |V_{ud}|^2 \lambda^3(m_D^2, m_{\eta'}^2, m_\rho^2)}{(\Lambda_1^2 - m_\rho^2)^2 I(m_D, m_{\eta'}, \Lambda_1)} = 2.88 \pm 0.28 \tag{4.108}$$

CLEO collaboration has measured [52] the following ratios,

$$\frac{\Gamma(D_s^+ \to \eta e^+ \nu)}{\Gamma(D_s^+ \to \phi e^+ \nu)} = 1.24 \pm 0.12 \pm 0.15 \,,$$

$$\frac{\Gamma(D_s^+ \to \eta' e^+ \nu)}{\Gamma(D_s^+ \to \phi e^+ \nu)} = 0.43 \pm 0.11 \pm 0.07 \,. \tag{4.109}$$

If we combine this with the following measured ratios,

$$\frac{\Gamma(D_s^+ \to \phi \pi^+)}{\Gamma(D_s^+ \to \phi e^+ \nu)} = 0.54 \pm 0.05 \pm 0.04 \qquad [6] \,, \tag{4.110}$$

$$\frac{\Gamma(D_s^+ \to \eta \pi^+)}{\Gamma(D_s^+ \to \phi \pi^+)} = 0.54 \pm 0.09 \pm 0.06 \qquad [53] \,, \tag{4.111}$$

$$\frac{\Gamma(D_s^+ \to \eta' \pi^+)}{\Gamma(D_s^+ \to \phi \pi^+)} = 1.2 \pm 0.15 \pm 0.11 \qquad [53] \,, \tag{4.112}$$

$$\frac{\Gamma(D_s^+ \to \eta \rho^+)}{\Gamma(D_s^+ \to \phi \pi^+)} = 2.86 \pm 0.38^{+0.36}_{-0.38} \qquad [54] \,, \tag{4.113}$$

and

$$\frac{\Gamma(D_s^+ \to \eta' \rho^+)}{\Gamma(D_s^+ \to \phi \pi^+)} = 3.44 \pm 0.62^{+0.44}_{-0.46} \qquad [54] \,. \tag{4.114}$$

We obtain the following experimental ratios:

$$\frac{\Gamma(D_s^+ \to \eta \pi^+)}{\Gamma(D_s^+ \to \eta e^+ \nu)} = 0.81 \pm 0.23 \,, \tag{4.115}$$

$$\frac{\Gamma(D_s^+ \to \eta\rho^+)}{\Gamma(D_s^+ \to \eta e^+\nu)} = 4.27 \pm 1.13 \,, \tag{4.116}$$

$$\frac{\Gamma(D_s^+ \to \eta'\pi^+)}{\Gamma(D_s^+ \to \eta' e^+\nu)} = 5.17 \pm 1.86 \,, \tag{4.117}$$

and

$$\frac{\Gamma(D_s^+ \to \eta'\rho^+)}{\Gamma(D_s^+ \to \eta' e^+\nu)} = 14.81 \pm 5.81 \,, \tag{4.118}$$

The errors here are probably overestimated as we propagated all errors as if they were independent while some systematic errors in the products of ratios would cancel. Comparing the theoretical predictions with the experimental resuls (Table (4.5), for the decays involving η and η', we see the theoretical and experimental results hardly match. Factorization model predictions for $\frac{\Gamma(D_s^+ \to \eta\pi^+)}{\Gamma(D_s^+ \to \eta e^+\nu)}$, $\frac{\Gamma(D_s^+ \to \eta\rho^+)}{\Gamma(D_s^+ \to \eta e^+\nu)}$ and $\frac{\Gamma(D_s^+ \to \eta'\pi^+)}{\Gamma(D_s^+ \to \eta' e^+\nu)}$ ratios, although not exactly inside the range allowed by the experiments, but are close to the experimental results. However factorizartion fails badly in the prediction of $\frac{\Gamma(D_s^+ \to \eta'\rho^+)}{\Gamma(D_s^+ \to \eta' e^+\nu)}$ ratio.

It seems that although the factorization model is successful in predicting the decay rates of some exclusive charm meson decay channels, it ought not to be expected to work well for decays involving small energy release such as $D^0 \to K^- a_1^+$ and $D_s^+ \to \eta'\rho^+$ (rows 4 and 8 in Table (4.5)).

In the next chapter we proceed to show that by inclusion of nonfactorized amplitudes, it is possible to restore the mutual agreement between experiment and theoretical prediction.

Table 4.5: The theoretical predictions of the factorization model and the experimental results for the ratio of hadronic decay rates and the corresponding semtileptonic decay rates

Row	Ratio	Theoretical	Experiment
1	$\frac{\Gamma(D^0 \to K^- \pi^+)}{\Gamma(D^0 \to K^- e^+ \nu)}$	1.63 ± 0.33	1.10 ± 0.11
2	$\frac{\Gamma(D^0 \to K^- \rho^+)}{\Gamma(D^0 \to K^- e^+ \nu)}$	3.0 ± 0.6	2.20 ± 0.38
3	$\frac{\Gamma(D^0 \to K^- a_1^+)}{\Gamma(D^0 \to K^- e^+ \nu)}$	0.43 ± 0.05	2.24 ± 0.44
4	$\frac{\Gamma(D^0 \to \pi^- \pi^+)}{\Gamma(D^0 \to \pi^- e^+ \nu)}$	1.02 ± 0.20	$0.42^{+0.25}_{-0.14}$
5	$\frac{\Gamma(D_s^+ \to \eta \pi^+)}{\Gamma(D_s^+ \to \eta e^+ \nu)}$	1.47 ± 0.14	0.81 ± 0.23
6	$\frac{\Gamma(D_s^+ \to \eta \rho^+)}{\Gamma(D_s^+ \to \eta e^+ \nu)}$	2.85 ± 0.28	4.27 ± 1.13
7	$\frac{\Gamma(D_s^+ \to \eta' \pi^+)}{\Gamma(D_s^+ \to \eta' e^+ \nu)}$	3.07 ± 0.30	5.17 ± 1.86
8	$\frac{\Gamma(D_s^+ \to \eta' \rho^+)}{\Gamma(D_s^+ \to \eta' e^+ \nu)}$	2.88 ± 0.28	14.81 ± 5.81

Chapter 5

Nonfactorization

5.1 Introduction

As we saw in the previous chapter factorization scheme with the choice $a_{1,2} = C_{1,2}$ worked as a reasonable approximation for charmed decays. The most successful example of this was the decay $D \to \bar{K}\pi$ [35, 65]. This led to the belief that $N_c \to \infty$ was a good approximation in charmed decays. This idea, when carried over to hadronic B decays failed as theory wanted a_2 to be negative [29] while experiments [74] left no doubt that in B decays a_2 was positive. In other words, the observed destructive interference pattern in $D^+ \to \bar{K}^0\pi^+$, $\bar{K}^0\rho^+$, $\bar{K}^{*0}\pi^+$ cannot be extrapolated to B decays.

Also, recently, it was shown [56] that in the factorization hypothesis all commonly used models of hadronic form factors had difficulty in explaining the polarization data [74, 57, 58] in $B \to \psi K^*$ decay. It was subsequently shown in [59] that even a modest amount ($\sim$ 10%) of nonfactorized contribution made all form factor models consistent with the polarization data.

As a result of the above, the view that the phenomenological parameters a_1 and a_2 are effective and process dependant, was proposed and pursued in Refs. [59] - [63]. In these studies it is suggested that a_1 and a_2, be evaluated with $N_c = 3$, however the nonfactorization contributions be included.

Motivated by the works in B decays in considering nonfactorization effects, and having in mind that factorization model fails badly in predicting the decay rates of some of the exclusive charm decays, such as $D^0 \to K^- a_1^+$ and $D_s^+ \to \eta' \rho^+$ which involve small energy release, in this chapter we will investigate nonfactorization in hadronic two-body Cabibbo-favored decays of D meson, phenomenologically. This will be done in two steps; in the first step we will ignore the nonfactorized annihilation and inelastic final state interaction effects, in the second step we will introduce these ignored aspects through an illustrative example.

The material presented in this chapter, which contains the contributions of the author of this thesis to the subject is mostly based on the work done in the following two papers [30, 64].

5.2 Nonfactorization in hadronic two-body Cabibbo-favored decays of D^0 and D^+ mesons

We saw in Sec. (4.3) that the effective Hamiltonian for the Cabibbo-favored charm decays is (Eq. (4.20))

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* \left\{ a_1 (\bar{u}d)_{L,H} (\bar{s}c)_{L,H} + a_2 (\bar{s}d)_{L,H} (\bar{u}c)_{L,H} \right\}, \tag{5.1}$$

where in the factorization model the phenomenological constants a_1 and a_2, were considered to be process independent and

$$a_1 = C_1 + \xi C_2 \approx C_1(m_c), \qquad a_2 = C_2 + \xi' C_1 \approx C_2(m_c). \tag{5.2}$$

That is, the color mismatch factors ξ and ξ' defined in Eq. (4.12) were taken to be $\xi \approx 0$ and $\xi' \approx 0$. As we discussed in Sec. (4.3), we anticipate that $\xi = \frac{1}{N_c} = \frac{1}{3}$ and $\xi' = \frac{1}{N_c} = \frac{1}{3}$, $N_c = 3$ is the number of colors. By taking $\xi \approx 0$ and $\xi' \approx 0$ in the factorization model one has implicitly allowed some new effects. Explicitly a_1 and

a_2 consist of two parts, one which has the factorized effects in it and the other the nonfactorized effects , that is

$$a_1 = a_1^0 + a_1^{nf}, \qquad a_2 = a_2^0 + a_2^{nf} \tag{5.3}$$

and

$$a_1^0 = C_1 + \frac{C_2}{3}, \qquad a_2^0 = C_2 + \frac{C_1}{3}\,, \tag{5.4}$$

where "nf" stands for "non-factorized". In this chapter, whose main aim is to consider the non-factorized effects, we prefer to separate factorized and non-factorized parts. Defining

$$H_w^{(8)} \equiv \frac{1}{2}\sum(\bar{s}\lambda^a c)(\bar{u}\lambda^a d)\,, \qquad \tilde{H}_w^{(8)} \equiv \frac{1}{2}\sum(\bar{s}\lambda^a d)(\bar{u}\lambda^a c)\,. \tag{5.5}$$

and using Eqs. (4.10)-(4.13) we rewrite the effective Hamiltonian Eq. (5.1) as

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* \left\{ \left[a_1^0 (\bar{u}d)_{L,H} (\bar{s}c)_{L,H} + C_2 H_w^{(8)} \right] + \left[a_2^0 (\bar{s}d)_{L,H} (\bar{u}c)_{L,H} + C_1 \tilde{H}_w^{(8)} \right] \right\}. \tag{5.6}$$

We drop the superscript "0' from a_1^0 and a_2^0 as a matter of notation such that

$$a_1 = C_1 + \frac{C_2}{3} = 1.09 \pm 0.04\,, \qquad a_2 = C_2 + \frac{C_1}{3} = -0.09 \pm 0.05\,. \tag{5.7}$$

where

$$C_1 = 1.26 \pm 0.04\,, \qquad C_2 = -0.51 \pm 0.05 \tag{5.8}$$

are used in determining the numerical values of a_1 and a_2. The central values of C_1 and C_2 are taken from [29] and the errors are from [30]. The effective Hamiltonian then becomes

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* \left\{ \left[a_1 (\bar{u}d)_{L,H} (\bar{s}c)_{L,H} + C_2 H_w^{(8)} \right] + \left[a_2 (\bar{s}d)_{L,H} (\bar{u}c)_{L,H} + C_1 \tilde{H}_w^{(8)} \right] \right\}. \tag{5.9}$$

We will be concerned with Class I (Color-favored) and Class II (Color-suppressed) decays. The effective Hamiltonians for these decays are

$$H_w^{CF} \;=\; \tilde{G}_F \left\{ a_1 (\bar{u}d)(\bar{s}c) + C_2 H_w^8 \right\}\,, \tag{5.10}$$

and

$$H_w^{CS} = \tilde{G}_F \left\{ a_2 (\bar{u}c)(\bar{s}d) + C_1 \tilde{H}_w^8 \right\} , \qquad (5.11)$$

where (CF) and (CS) refer to "color-favored" and "color-suppressed", respectively. The matrix elements of the first terms in Eqs. (5.10) and (5.11) are expected to be dominated by factorized contributions; any nonfactorized contributions arising from them are parametrized as we will see later . The second terms, $H_w^{(8)}(\equiv \frac{1}{2}\sum(\bar{u}\lambda^a d)(\bar{s}\lambda^a c))$ and $\tilde{H}_w^{(8)}(\equiv \frac{1}{2}\sum(\bar{u}\lambda^a c)(\bar{s}\lambda^a d))$, involving color-octet currents (as we stressed before) generate only nonfactorized contributions. It should be clear from Eqs. (5.10), (5.11) and (5.7) that nonfactorized effects are more likely to manifest themselves in color-suppressed decays than in color-favored decays due to the fact that C_1 is much larger than a_2 in magnitude.

In the following sections we will study some exclusive charm meson decays. Our aim is to extract the size and the sign of the nonfactorized terms in each decay. The introduction and description of nonfactorized terms is purely phenomenological, as it is in the Refs. [59]-[63]. No attempt is made to calculate the nonfactorized terms but, rather, the emphasis is to glean some systematic behavior of these terms.

The decays we study are:
1) $D \to \bar{K}\pi$, that is the three decays $D^0 \to K^-\pi^+$, $D^0 \to \bar{K}^0\pi^0$ and $D^+ \to \bar{K}^0\pi^+$ which are $D \to P_1P_2$ type of decays.
2)$D \to \bar{K}^*\pi$, that is the three decays $D^0 \to K^{*-}\pi^+$, $D^0 \to \bar{K}^{*0}\pi^0$ and $D^+ \to \bar{K}^{*0}\pi^+$ which are $D \to VP$ type of decays.
3)$D \to \bar{K}\rho$, that is the three decays $D^0 \to K^-\rho^+$, $D^0 \to \bar{K}^0\rho^0$ and $D^+ \to \bar{K}^0\rho^+$ which are $D \to PV$ type of decays.

These decays are all two-body Cabibbo-favored decays of D^0 and D^+ mesons that involve two isospins in the final state. We will see, for decays involving a single

Lorentz scalar structure (one form factor, see Sec. (4.4)), such as $D \to \bar{K}\pi$, $\bar{K}^*\pi$ and $\bar{K}\rho$, one can extract an "effective" a_1 and a_2 which it is shown to be process-dependent.

5.3 $D \to \bar{K}\pi$ Decays

Using Eq. (5.10), for the effective Hamiltonian, the decay amplitude of $D^0 \to K^-\pi^+$ is,

$$A(D^0 \to K^-\pi^+) = \tilde{G}_F \left\{ a_1 \left\langle K^-\pi^+ |(\bar{s}c)(\bar{u}d)| D^0 \right\rangle + C_2 \left\langle K^-\pi^+ | H_w^{(8)} | D^0 \right\rangle \right\} . \quad (5.12)$$

We write the first term as a sum of a factorized and a nonfactorized part,

$$\begin{aligned} \left\langle K^-\pi^+ |(\bar{s}c)(\bar{u}d)| D^0 \right\rangle &= \left\langle \pi^+ |(\bar{u}d)| 0 \right\rangle \left\langle K^- |(\bar{s}c)| D^0 \right\rangle + \left\langle \pi^+ K^- |(\bar{s}c)(\bar{u}d)| D^0 \right\rangle^{nf} \\ &= -if_\pi (m_D^2 - m_K^2)(F_0^{DK}(m_\pi^2) + F_0^{(1)nf}) , \end{aligned} \quad (5.13)$$

where the nonfactorized matrix element of the product of the color-singlet currents is defined as $(\bar{s}c)(\bar{u}d)$ as

$$\left\langle K^-\pi^+ |(\bar{s}c)(\bar{u}d)| D^0 \right\rangle^{nf} \equiv -if_\pi (m_D^2 - m_K^2) F_0^{(1)nf} . \quad (5.14)$$

For the second term in Eq. (5.12) we write,

$$\left\langle K^-\pi^+ | H_w^{(8)} | D^0 \right\rangle \equiv -if_\pi (m_D^2 - m_K^2) F_0^{(8)nf} . \quad (5.15)$$

Both $F_0^{(1)nf}$ and $F_0^{(8)nf}$ (as also all nonfactorized contributions to follow) are functions of the Mandelstam variables, $s = m_D^2$, $t = m_K^2$ and $u = m_\pi^2$. We have chosen to suppress these variables in writing the last three and all ensuing equations. The decay amplitude of Eq. (5.12) is then written in the form,

$$A(D^0 \to K^-\pi^+) = -i\tilde{G}_F (a_1^{eff})_{K\pi} f_\pi \left(m_D^2 - m_K^2 \right) F_0^{DK}(m_\pi^2) , \quad (5.16)$$

where,

$$\left(a_1^{eff}\right)_{K\pi} = a_1\left(1 + \frac{F_0^{(1)nf}}{F_0^{DK}(m_\pi^2)} + \frac{C_2}{a_1}\frac{F_0^{(8)nf}}{F_0^{DK}(m_\pi^2)}\right) . \tag{5.17}$$

This defines a process-dependent effective a_1. We shall see that it is possible to do so for all decays involving a single Lorentz structure. We notice also that as the coefficient C_2/a_1 (≈ -0.47) is smaller than unity, the effect of the nonfactorized amplitude arising from $H_w^{(8)}$ is suppressed relative to the factorized amplitude in color-favored decays. For the same reason, the nonfactorized term proportional to $F_0^{(1)nf}$ could compete favorably with $F_0^{(8)nf}$.

The decay amplitude for the color-suppressed decay $D^0 \to \bar{K}^0\pi^0$ by following an analogous procedure is given by,

$$A(D^0 \to \bar{K}^0\pi^0) = -i\frac{\tilde{G}_F}{\sqrt{2}}(a_2^{eff})_{K\pi} f_K \left(m_D^2 - m_\pi^2\right) F_0^{D\pi}(m_K^2) , \tag{5.18}$$

where,

$$\left(a_2^{eff}\right)_{K\pi} = a_2\left(1 + \frac{\tilde{F}_0^{(1)nf}}{F_0^{D\pi}(m_K^2)} + \frac{C_1}{a_2}\frac{\tilde{F}_0^{(8)nf}}{F_0^{D\pi}(m_K^2)}\right) . \tag{5.19}$$

In writing Eq. (5.18) we have used,

$$\begin{aligned} \left\langle \bar{K}^0\pi^0|(\bar{u}c)(\bar{s}d)|D^0\right\rangle &= \left\langle \bar{K}^0|(\bar{s}d)|0\right\rangle \left\langle \pi^0|(\bar{u}c)|D^0\right\rangle + \left\langle \bar{K}^0\pi^0|(\bar{u}c)(\bar{s}d)|D^0\right\rangle^{nf} , \\ &= -i\frac{f_K}{\sqrt{2}}(m_D^2 - m_\pi^2)\left(F_0^{D\pi}(m_K^2) + \tilde{F}_0^{(1)nf}\right) , \end{aligned} \tag{5.20}$$

and

$$\begin{aligned} \left\langle \bar{K}^0\pi^0|(\bar{u}c)(\bar{s}d)|D^0\right\rangle^{nf} &\equiv -i\frac{f_K}{\sqrt{2}}(m_D^2 - m_\pi^2)\tilde{F}_0^{(1)nf} , \\ \left\langle \bar{K}^0\pi^0|\tilde{H}_w^{(8)}|D^0\right\rangle &\equiv -i\frac{f_K}{\sqrt{2}}(m_D^2 - m_\pi^2)\tilde{F}_0^{(8)nf} . \end{aligned} \tag{5.21}$$

Now, as $\frac{C_1}{a_2}$ in Eq. (5.19) is large (≈ -14), the nonfactorized contribution $\tilde{F}_0^{(8)nf}$, arising from $\tilde{H}_w^{(8)}$, is greatly enhanced in comparison to $\tilde{F}_0^{(1)nf}$. Nevertheless, it is not possible to separate the individual contributions from $\tilde{F}_0^{(8)nf}$ and $\tilde{F}_0^{(1)nf}$.

The amplitude for $D^+ \to \bar{K}^0\pi^+$ decay is obtained from Eqs. (5.16) and Eqs. (5.18) via the isospin sum rule

$$A(D^+ \to \bar{K}^0\pi^+) = A(D^0 \to K^-\pi^+) + \sqrt{2}A(D^0 \to \bar{K}^0\pi^0)\ . \tag{5.22}$$

In terms of isospin amplitudes $A_{1/2}$ and $A_{3/2}$ and the final-state interaction (fsi) phases,

$$\begin{aligned} A(D^0 \to K^-\pi^+) &= \frac{1}{\sqrt{3}}\left(A_{3/2}exp(i\delta_{3/2}) + \sqrt{2}A_{1/2}exp(i\delta_{1/2})\right) \\ A(D^0 \to \bar{K}^0\pi^0) &= \frac{1}{\sqrt{3}}\left(\sqrt{2}A_{3/2}exp(i\delta_{3/2}) - A_{1/2}exp(i\delta_{1/2})\right) \\ A(D^+ \to \bar{K}^0\pi^+) &= \sqrt{3}A_{3/2}exp(i\delta_{3/2}) \end{aligned} \tag{5.23}$$

The relative phase is known [65] to be

$$\delta^{\bar{K}\pi} \equiv \delta^{\bar{K}\pi}_{1/2} - \delta^{\bar{K}\pi}_{3/2} = (86 \pm 8)^0\ , \tag{5.24}$$

with the relative sign of $A_{1/2}$ and $A_{3/2}$ positive. There would be another solution where the relative sign of $A_{1/2}$ and $A_{3/2}$ is odd and $\delta \to (\pi - \delta)$ [66].

$A_{1/2}$ and $A_{3/2}$ are determined by equating Eqs. (5.16) and (5.18) to Eq. (5.23) with the phases $\delta_{1/2}$ and $\delta_{3/2}$ set equal to zero; and then reinstating the phases to calculate the branching ratios from Eq. (4.64). This procedure is equivalent to assuming that the effect of fsi in this mode is simply to rotate the isospin amplitudes without effecting their magnitudes. For the form factors the following normalizations at $q^2 = 0$ were used ,

$$\begin{aligned} F_0^{DK}(0) &= 0.77 \pm 0.04\ , \qquad [67] \\ F_0^{D\pi}(0) &= 0.83 \pm 0.08\ . \qquad [65,\ 68,\ 69] \end{aligned} \tag{5.25}$$

$F_0^{DK}(q^2)$ and $F_0^{D\pi}(q^2)$ are extrapolated as monopoles with 0^+ pole masses of 2.01 and 2.47 GeV respectively as in [35, 42] and our examples in the previous chapter.

As these form factors are needed at a relatively small q^2 ($=m_\pi^2$ or m_K^2), the results are not very sensitive to the manner of extrapolation.

$A_{1/2}$ and $A_{3/2}$ are first determined from Eqs. (5.16), (5.18) and (5.23) with $\delta_{1/2}^{K\pi}$ and $\delta_{3/2}^{K\pi}$ set equal to zero. Next the allowed ranges of $(a_1^{eff})_{K\pi}$, $(a_2^{eff})_{K\pi}$ are searched, that fit the branching ratio data [6] as $\delta_{1/2}^{K\pi} - \delta_{3/2}^{K\pi}$ are allowed to vary in the range indicated in Eq. (5.24) for $A_{3/2}/A_{1/2} > 0$. The resulting ranges were,

$$1.11 \leq (a_1^{eff})_{K\pi} \leq 1.17\,, \qquad -0.46 \leq (a_2^{eff})_{K\pi} \leq -0.39 \qquad (5.26)$$

Equivalently, we define the following two parameters (which are measures of nonfactorization effects)

$$\chi_{K\pi} \equiv \frac{F_0^{(8)nf}}{F_0^{DK}(m_\pi^2)} + \frac{a_1}{C_2}\frac{F_0^{(1)nf}}{F_0^{DK}(m_\pi^2)} \qquad \text{and} \qquad \xi_{K\pi} \equiv \frac{\tilde{F}_0^{(8)nf}}{F_0^{D\pi}(m_K^2)} + \frac{a_2}{C_1}\frac{\tilde{F}_0^{(1)nf}}{F_0^{D\pi}(m_K^2)} \qquad (5.27)$$

such that

$$(a_1^{eff})_{K\pi} = a_1(1 + \frac{C_2}{a_1}\chi_{K\pi})\,, \qquad \text{and} \qquad (a_2^{eff})_{K\pi} = a_2(1 + \frac{C_1}{a_2}\xi_{K\pi})\,. \qquad (5.28)$$

The allowed ranges of these two parameters are

$$-0.22 \leq \chi_{K\pi} \leq 0\,, \qquad -0.32 \leq \xi_{K\pi} \leq -0.21 \qquad (5.29)$$

In calculating the ranges of $\chi_{K\pi}$ and $\xi_{K\pi}$ the errors in C_1 and C_2,Eq. (5.8), and also the errors in a_1 and a_2, Eq. (5.7), were considered .

There is another allowed solution where $\frac{A_{3/2}}{A_{1/2}}$ is negative and $\delta^{\bar{K}\pi}$ is replaced by $(\pi - \delta^{\bar{K}\pi})$ [66]. This solution requires,

$$0.76 \leq (a_1^{eff})_{K\pi} \leq 0.84\,, \qquad \text{and} \qquad -0.92 \leq (a_2^{eff})_{K\pi} \leq -0.87 \qquad (5.30)$$

However, this solution requires $\chi_{K\pi}$ and $\xi_{K\pi}$, which are measures of nonfactorization effects, to be larger and of opposite sign. That is,

$$0.41 \leq \chi_{K\pi} \leq 0.76\,, \qquad \text{and} \qquad -0.71 \leq \xi_{K\pi} \leq -0.58 \qquad (5.31)$$

Of the two solutions, the solution shown in Eq. (5.26) and (5.29), which yields $\frac{A_{3/2}}{A_{1/2}} > 0$ has $(a_1^{eff})_{K\pi}$ and $(a_2^{eff})_{K\pi}$ closer to the values that have been in vogue over the past decade and it also requires a smaller nonfactorized contribution. In principle, there is a sign ambiguity in a_1^{eff} and a_2^{eff}: One could reverse the signs of both a_1^{eff} and a_2^{eff} which only serves to change the sign of the decay amplitudes. This is a very unlikely solution as a sign reversal of a_1^{eff} can only be accomplished at the expense of a very large nonfactorized term (> (3 − 4) times the factorized term) in Class I [35] decays.

5.4 $D \to \bar{K}^*\pi$ decays

Using the definitions introduced in Sections (4.4) and (4.5) and the method of calculation detailed for $D \to \bar{K}\pi$ decays in the previous section , the amplitudes for the decays $D^0 \to K^*\pi$ are given by

$$\begin{aligned} A(D^0 \to K^{*-}\pi^+) &= 2\tilde{G}_F f_\pi m_{K^*} A_0^{DK^*}(m_\pi^2)(\varepsilon^*.p_D)(a_1^{eff})_{K^*\pi}\,, \\ A(D^0 \to \bar{K}^{*0}\pi^0) &= \sqrt{2}\tilde{G}_F f_{K^*} m_{K^*} F_1^{D\pi}(m_{K^*}^2)(\varepsilon^*.p_D)(a_2^{eff})_{K^*\pi}\,, \\ A(D^+ \to \bar{K}^{*0}\pi^+) &= A(D^0 \to K^{*-}\pi^+) + \sqrt{2}A(D^0 \to \bar{K}^{*0}\pi^0)\,. \end{aligned} \quad (5.32)$$

where

$$\begin{aligned} \left(a_1^{eff}\right)_{K^*\pi} &= a_1\left(1 + \frac{A_0^{(1)nf}}{A_0^{DK^*}(m_\pi^2)} + \frac{C_2}{a_1}\frac{A_0^{(8)nf}}{A_0^{DK^*}(m_\pi^2)}\right)\,, \\ \left(a_2^{eff}\right)_{K^*\pi} &= a_2\left(1 + \frac{\tilde{F}_1^{(1)nf}}{F_1^{D\pi}(m_{K^*}^2)} + \frac{C_1}{a_2}\frac{\tilde{F}_1^{(8)nf}}{F_1^{D\pi}(m_{K^*}^2)}\right)\,. \end{aligned} \quad (5.33)$$

In Eqs. (5.32) and (5.33), in addition to Eqs. (4.36) and (4.37) the following definitions have been used

$$\left\langle K^{*-}\pi^+|(\bar{s}c)(\bar{u}d)|D^0\right\rangle^{nf} \equiv 2\tilde{G}_F f_\pi m_{K^*} A_0^{(1)nf}(\varepsilon^*.p_D)\,,$$

$$\begin{aligned}
\left\langle K^{*-}\pi^+|H_w^{(8)}|D^0\right\rangle &\equiv 2\tilde{G}_F f_\pi m_{K^*} A_0^{(8)nf}(\varepsilon^*.p_D)\,,\\
\left\langle \bar{K}^{*0}\pi^0|(\bar{u}c)(\bar{s}d)|D^0\right\rangle^{nf} &\equiv \sqrt{2}\tilde{G}_F f_{K^*} m_{K^*} \tilde{F}_1^{(1)nf}(\varepsilon^*.p_D)\,,\\
\text{and}\quad \left\langle \bar{K}^{*0}\pi^0|\tilde{H}_w^{(8)}|D^0\right\rangle &\equiv \sqrt{2}\tilde{G}_F f_{K^*} m_{K^*} \tilde{F}_1^{(8)nf}(\varepsilon^*.p_D)\,. \qquad (5.34)
\end{aligned}$$

It is known [65] from an analysis of 1994 listed data [6] that fsi phases in this decay are large, $\delta^{K^*\pi} \equiv \delta_{1/2}^{K^*\pi} - \delta_{3/2}^{K^*\pi} = (103 \pm 17)^0$ for $\frac{A_{3/2}}{A_{1/2}} > 0$. To take the fsi phases into account a procedure similar to that for $D \rightarrow K\pi$ decays has been used ; the isospin amplitudes are calculated by equating the amplitudes in Eq. (5.32) to those in Eq. (5.23) with phases set equal to zero. Determining $A_{1/2}$ and $A_{3/2}$ in this way, then the phases are reinstated. For the form factors the following normalizations at $q^2 = 0$ are used,

$$\begin{aligned}
A_0^{DK^*}(0) &= 0.70 \pm 0.09\,, \qquad [65,\ 67]\\
F_1^{D\pi}(0) &= 0.83 \pm 0.08\,. \qquad [65,\ 68] \qquad (5.35)
\end{aligned}$$

Besides using the normalization of the form factors given in Eq. (5.35), monopole (Bauer-Stech-Wirbel I (BSWI) model) forms for the q^2 extrapolation of the form factors $A_0^{DK^*}(q^2)$ and $F_1^{D\pi}(q^2)$ with pole masses 2.11 and 1.87 GeV, respectively, have been assumed . Allowing $\delta^{K^*\pi}$ to vary in the range $(103 \pm 17)^0$, the allowed ranges of $(a_1^{eff})_{K^*\pi}$ and $(a_2^{eff})_{K^*\pi}$ for $\frac{A_{3/2}}{A_{1/2}} > 0$ and $\frac{A_{3/2}}{A_{1/2}} < 0$, which fit the branching ratio data [6], were determined. The results were,

$\frac{A_{3/2}}{A_{1/2}} > 0$:

$$1.74 \leq (a_1^{eff})_{K^*\pi} \leq 1.96\,, \qquad -0.53 \leq (a_2^{eff})_{K^*\pi} \leq -0.43\,, \qquad (5.36)$$

$\frac{A_{3/2}}{A_{1/2}} < 0$:

$$1.29 \leq (a_1^{eff})_{K^*\pi} \leq 1.51\,, \qquad -0.90 \leq (a_2^{eff})_{K^*\pi} \leq -0.81\,, \qquad (5.37)$$

From Eq. (5.33) the parameters $\chi_{K^*\pi}$ and $\xi_{K^*\pi}$, analogous to $\chi_{K\pi}$ and $\xi_{K\pi}$ defined through Eqs. (5.27) and (5.28), are estimated to be

$\frac{A_{3/2}}{A_{1/2}} > 0$:

$$-1.91 \leq \chi_{K^*\pi} \leq -1.14\,, \qquad -0.38 \leq \xi_{K^*\pi} \leq -0.25\,, \tag{5.38}$$

$\frac{A_{3/2}}{A_{1/2}} < 0$:

$$-0.93 \leq \chi_{K^*\pi} \leq -0.34\,, \qquad -0.68 \leq \xi_{K^*\pi} \leq -0.53\,, \tag{5.39}$$

Simply by fitting the three branching ratios for $(D^0, D^+) \to \bar{K}^*\pi$, it is not possible to favor one solution or the other. Here, the solutions obtained with the constraint $\frac{A_{3/2}}{A_{1/2}} > 0$ require large nonfactorization contributions to $(a_1^{eff})_{K^*\pi}$ as evidenced by $\chi_{K^*\pi}$ in Eq. (5.38). On the contrary, the solutions corresponding to $\frac{A_{3/2}}{A_{1/2}} < 0$ (with $\delta^{K^*\pi} \to (\pi - \delta^{K^*\pi})$) require larger nonfactorization contributions to $(a_2^{eff})_{K^*\pi}$ as seen by comparing $\xi_{K^*\pi}$'s of Eq. (5.39) with those of Eq. (5.38).

5.5 $D \to \bar{K}\rho$

Using the definitions given in Sections (4.4) and (4.5), the amplitudes for the decays $D^0 \to K\rho$ are given by

$$\begin{aligned} A(D^0 \to K^-\rho^+) &= 2\tilde{G}_F f_\rho m_\rho (\varepsilon^*.p_D) F_1^{DK}(m_\rho^2)(a_1^{eff})_{K\rho}\,, \\ A(D^0 \to \bar{K}^0\rho^0) &= \sqrt{2}\tilde{G}_F f_K m_\rho (\varepsilon^*.p_D) A_0^{D\rho}(m_K^2)(a_2^{eff})_{K\rho}\,, \\ \text{and} \quad A(D^+ \to \bar{K}^0\rho^+) &= A(D^0 \to K^-\rho^+) + \sqrt{2}A(D^0 \to \bar{K}^0\rho^0)\,, \end{aligned} \tag{5.40}$$

where

$$\begin{aligned} \left(a_1^{eff}\right)_{K\rho} &= a_1 \left(1 + \frac{F_1^{(1)nf}}{F_1^{DK}(m_\rho^2)} + \frac{C_2}{a_1}\frac{F_1^{(8)nf}}{F_1^{DK}(m_\rho^2)}\right)\,, \\ \left(a_2^{eff}\right)_{K\rho} &= a_2 \left(1 + \frac{\tilde{A}_0^{(1)nf}}{A_0^{D\rho}(m_K^2)} + \frac{C_1}{a_2}\frac{\tilde{A}_0^{(8)nf}}{A_0^{D\rho}(m_K^2)}\right)\,. \end{aligned} \tag{5.41}$$

In addition to Eqs. (4.36) and (4.37), the following definitions have been used

$$\begin{aligned}
\left\langle K^-\rho^+|(\bar{s}c)(\bar{u}d)|D^0\right\rangle^{nf} &\equiv 2\tilde{G}_F f_\rho m_\rho F_1^{(1)nf}(\varepsilon^*.p_D)\,, \\
\left\langle K^-\rho^+|H_w^{(8)}|D^0\right\rangle &\equiv 2\tilde{G}_F f_\rho m_\rho F_1^{(8)nf}(\varepsilon^*.p_D)\,, \\
\left\langle \bar{K}^0\rho^0|(\bar{u}c)(\bar{s}d)|D^0\right\rangle^{nf} &\equiv \sqrt{2}\tilde{G}_F f_K m_\rho \tilde{A}_0^{(1)nf}(\varepsilon^*.p_D)\,, \\
\left\langle \bar{K}^0\rho^0|\tilde{H}_w^{(8)}|D^0\right\rangle &\equiv \sqrt{2}\tilde{G}_F f_K m_\rho \tilde{A}_0^{(8)nf}(\varepsilon^*.p_D)\,.
\end{aligned} \tag{5.42}$$

Fits [65, 70] to $D \to \bar{K}\rho$ data admit a solution with $\frac{A_{3/2}}{A_{1/2}} > 0$ and a relative fsi phase $\delta^{K\rho} \equiv \delta_{1/2}^{K\rho} - \delta_{3/2}^{K\rho} = (0 \pm 30)^0$. $F_1^{DK}(0)$ is given in Eq. (5.25) and the value of $A_0^{D\rho}(0) = 0.67$ is taken form the BSW [35, 42] . In this decay too, monopole (BSWI) extrapolations of the form factors $F_1^{DK}(q^2)$ and $A_0^{D\rho}(q^2)$ with 1^- pole at 2.11 GeV and 0^- pole at 1.87 GeV, respectively[35, 42], have been assumed. To search for the allowed ranges of $(a_1^{eff})_{K\rho}$ and $(a_2^{eff})_{K\rho}$, the same procedure which was outlined in the analysis of $D \to \bar{K}\pi$ and $\bar{K}^*\pi$ decays was followed, that is by varying $\delta^{K\rho}$ in the domain $(0 \pm 30)^0$ and searching for allowed values of a_1^{eff} and a_2^{eff} that fit the data [6]. The following allowed ranges for $(a_1^{eff})_{K\rho}$ and $(a_2^{eff})_{K\rho}$ were found:

$\frac{A_{3/2}}{A_{1/2}} > 0$:

$$1.17 \leq (a_1^{eff})_{K\rho} \leq 1.32\,, \qquad -1.00 \leq (a_2^{eff})_{K\rho} \leq -0.75\,,\,. \tag{5.43}$$

$\frac{A_{3/2}}{A_{1/2}} < 0$:

$$0.71 \leq (a_1^{eff})_{K\rho} \leq 0.85\,, \qquad -2.53 \leq (a_2^{eff})_{K\rho} \leq -2.19\,,\,. \tag{5.44}$$

These ranges translate into the following limits on $\chi_{K\rho}$ and $\xi_{K\rho}$ defined in analogy with $\chi_{K\pi}$ and $\xi_{K\pi}$ of Eq. (5.27),

$\frac{A_{3/2}}{A_{1/2}} > 0$:

$$-0.52 \leq \chi_{K\rho} \leq -0.11\,, \qquad -0.75 \leq \xi_{K\rho} \leq -0.50\,,\,. \tag{5.45}$$

$\frac{A_{3/2}}{A_{1/2}} < 0$:

$$-0.41 \leq \chi_{K\rho} \leq -0.85\ , \qquad -2.00 \leq \xi_{K\rho} \leq -1.60\ ,\ . \tag{5.46}$$

Note that the solutions with $\frac{A_{3/2}}{A_{1/2}} < 0$ require much larger nonfactorized contributions.

The results for a_1^{eff} , a_2^{eff}, χ and ξ, for $D \to \bar{K}\pi$, $D \to \bar{K}^*\pi$ and $D \to \bar{K}\rho$ decays are tabulated in Table (5.1).

5.6 Summary of the consideration of the nonfactorized effects in $D \to \bar{K}\pi$, $D \to \bar{K}^*\pi$ and $D \to \bar{K}\rho$ decays.

We saw that an analysis of those Cabibbo-favored two-body decays of D^0 and D^+ which involve two isospins in the final state in a formalism that uses $N_c = 3$ and includes nonfactorized amplitudes, was carried out. The decays considered were: $D \to \bar{K}\pi$, $D \to \bar{K}^*\pi$ and $D \to \bar{K}\rho$ decays. The measured phases of the (elastic) final state interactions (FSI) were also included, but only in so far as they rotate the isospin amplitudes without affecting their magnitudes. The annihilation terms and inelastic final state interactions were ignored.

The rationale for ignoring the annihilation terms in $D \to \bar{K}\pi$ decays is that these terms are proportional to $a_2(m_K^2 - m_\pi^2)$ while the terms that are kept are proportional to $a_1(m_D^2 - m_K^2)$ or $a_2(m_D^2 - m_\pi^2)$.

Justifying the neglect of annihilation terms in $D \to K^*\pi$ or $\bar{K}\rho$ is harder as they involve the divergence of the axial current. If, however, the annihilation form factors $A_0^{K^*\pi}(m_D^2)$ and $A_0^{\rho\bar{K}}(m_D^2)$ would be much smaller than the form factors $A_0^{DK^*}(m_\pi^2)$ or $A_0^{D\rho}(m_K^2)$, the annihilation term would be smaller than the terms retained. The

Table 1: a_1^{eff}, a_2^{eff}, χ and ξ for the two solutuions $\frac{A_{3/2}}{A_{1/2}} > 0$ and $\frac{A_{3/2}}{A_{1/2}} < 0$ for $D \to \bar{K}\pi$, $D \to \bar{K}^*\pi$ and $D \to \bar{K}\rho$ decays.

Solutions	Quantities	Decays		
		$D \to K\pi$	$D \to K^*\pi$	$D \to K\rho$
$\frac{A_{3/2}}{A_{1/2}} > 0$	a_1^{eff}	$1.11 \leq (a_1^{eff})_{K\pi} \leq 1.17$	$1.74 \leq (a_1^{eff})_{K^*\pi} \leq 1.96$	$1.17 \leq (a_1^{eff})_{K\rho} \leq 1.32$
	a_2^{eff}	$-0.46 \leq (a_2^{eff})_{K\pi} \leq -0.39$	$-0.53 \leq (a_2^{eff})_{K^*\pi} \leq -0.43$	$-1.00 \leq (a_2^{eff})_{K\rho} \leq -0.75$
	χ	$-0.22 \leq \chi_{K\pi} \leq 0$	$-1.91 \leq \chi_{K^*\pi} \leq -1.14$	$-0.52 \leq \chi_{K\rho} \leq -0.11$
	ξ	$-0.32 \leq \xi_{K\pi} \leq -0.21$	$-0.38 \leq \xi_{K^*\pi} \leq -0.25$	$-0.75 \leq \xi_{K\rho} \leq -0.50$
$\frac{A_{3/2}}{A_{1/2}} < 0$	a_1^{eff}	$0.76 \leq (a_1^{eff})_{K\pi} \leq 0.84$	$1.29 \leq (a_1^{eff})_{K^*\pi} \leq 1.51$	$40.71 \leq (a_1^{eff})_{K\rho} \leq 0.85$
	a_2^{eff}	$-0.92 \leq (a_2^{eff})_{K\pi} \leq -0.87$	$-0.90 \leq (a_2^{eff})_{K^*\pi} \leq -0.81$	$-2.53 \leq (a_2^{eff})_{K\rho} \leq -2.19$
	χ	$0.41 \leq \chi_{K\pi} \leq 0.76$	$-0.93 \leq \chi_{K^*\pi} \leq -0.34$	$-0.41 \leq \chi_{K\rho} \leq -0.85$
	ξ	$-0.71 \leq \xi_{K\pi} \leq -0.58$	$-0.68 \leq \xi_{K^*\pi} \leq -0.53$	$-2.00 \leq \xi_{K\rho} \leq -1.60$

neglect of the inelastic FSI is largely due to ignorance of the parameters to be used in implementing a believable calculation.

Despite the statement above regarding the size of the annihilation terms, perhaps it is fair to say that at our present situation we do not understand the role of the nonfactorization effects fully.

From the data, one only determines $(a_1)^{eff}$ and $(a_2)^{eff}$ which, as it is shown here and in Refs. [30] and [63] are process-dependent. The next question is: What effects contribute to $(a_1)^{eff}$ and $(a_2)^{eff}$ in a scheme that uses $N_c = 3$? It is tacitly assumed in Refs. [59] - [63] that these effects arise from the following sources: the nonfactorized matrix elements of $H_w^{(8)} = \frac{1}{2}\sum_a (\bar{s}\lambda^a c)(\bar{u}\lambda^a d)$, $\tilde{H}_w^{(8)} = \frac{1}{2}\sum_a (\bar{s}\lambda^a d)(\bar{u}\lambda^a c)$ and parts of the effective Hamiltonian made up of color-singlet currents $(\bar{s}c)(\bar{u}d)$ and $(\bar{u}c)(\bar{s}d)$. With these assumptions, the relative size of the nonfactorized contribution in each specific channel was extracted.

We saw that in the decays that involve a single Lorentz structure, we can absorb the factorized and nonfactorized effects in a_1^{eff} and a_2^{eff}. We left the phases of the final state interactions (elastic) out of a_1^{eff} and a_2^{eff}, that is by definition we take them to be real. That is after the elastic final state interactions (rotation of the isospin amplitudes) we have:

$$\begin{aligned} A(D^0 \to K^-\pi^+) &\equiv -i\tilde{G}_F(a_1^{eff})_{K\pi} f_\pi \left(m_D^2 - m_K^2\right) F_0^{DK}(m_\pi^2) \exp i\phi_{+-} \\ &= \frac{1}{\sqrt{3}}\left(A_{3/2} exp(i\delta_{3/2}) + \sqrt{2}A_{1/2} exp(i\delta_{1/2})\right) , \end{aligned} \tag{5.47}$$

$$\begin{aligned} A(D^0 \to \bar{K}^0\pi^0) &\equiv -i\frac{\tilde{G}_F}{\sqrt{2}}(a_2^{eff})_{K\pi} f_K \left(m_D^2 - m_\pi^2\right) F_0^{D\pi}(m_K^2) \exp i\phi_{00} \\ &= \frac{1}{\sqrt{3}}\left(\sqrt{2}A_{3/2} exp(i\delta_{3/2}) - A_{1/2} exp(i\delta_{1/2})\right) , \end{aligned} \tag{5.48}$$

$$\begin{aligned} A(D^+ \to \bar{K}^0\pi^+) &= A(D^0 \to K^-\pi^+) + \sqrt{2}A(D^0 \to \bar{K}^0\pi^0) \\ &= \sqrt{3}A_{3/2} exp(i\delta_{3/2}) . \end{aligned} \tag{5.49}$$

The phases ϕ_{+-} and ϕ_{00} are a result of the final state interactions (here elastic rotation of the isospin amplitudes under the influence of strong interaction).

In the next section by doing a specific example we attempt to include the effects of inelastic final state interactions and annihilation terms. We will slightly generalize our definitions of a_1^{eff} and a_2^{eff} to include the phases produced by the final state interactions, that is, by definition we allow them to be complex.

5.7 Annihilation and inelastic Final State Interactions Effects

In this section we will examine the effect of annihilation terms and final state interactions (inelastic) on the phenomenological constants a_1^{eff} and a_2^{eff} through an illustrative example. We will consider the charm decay: $D_s^+ \to \phi\pi^+$. The reason for choosing this Cabibbo-favored decay is that it has only one isospin which makes the fsi calculation somewhat simpler ($\phi(s\bar{s})$ is an isoscalar). $D_s^+ \to \phi\pi^+$ is a $D \to VP$ and Class I type of decay (see Sec. (4.5.4)) . Using Eq. (5.10) , the decay amplitude (before fsi effects are brought into play) for $D_s^+ \to \phi\pi^+$ is given by,

$$A(D_s^+ \to \phi\pi^+) = \tilde{G}_F \left\{ a_1 \left\langle \phi\pi^+ | (\bar{s}c)(\bar{u}d) | D_s^+ \right\rangle + C_2 \left\langle \phi\pi^+ | H_w^{(8)} | D_s^+ \right\rangle \right\} . \qquad (5.50)$$

While the matrix element of $H_w^{(8)}$ is completely nonfactorized, the first term in Eq. (5.50) includes a (i) factorized (spectator) term, (ii) any nonfactorized contributions in addition to the factorized (spectator) term and (iii) a W-annihilation term which in turn has a factorized and a nonfactorized part. These individual contributions to the decay amplitude are parametrized as follows:

$$\left\langle \phi\pi^+ | (\bar{s}c)(\bar{u}d) | D_s^+ \right\rangle^{fact} = f_\pi (2m_\phi) \varepsilon . p_{D_s} A_0^{D_s \phi}(m_\pi^2) , \qquad (5.51)$$

$$\left\langle \phi\pi^+ | (\bar{s}c)(\bar{u}d) | D_s^+ \right\rangle^{nf} \equiv f_\pi (2m_\phi) \varepsilon . p_{D_s} A_0^{(1)nf} , \qquad (5.52)$$

$$\left\langle \phi\pi^+ | (\bar{s}c)(\bar{u}d) | D_s^+ \right\rangle^{ann} \equiv f_{D_s}(2m_\phi)\varepsilon.p_{D_s} A_0^{ann} , \tag{5.53}$$

$$\left\langle \phi\pi^+ | H_w^{(8)} | D_s^+ \right\rangle \equiv f_\pi(2m_\phi)\varepsilon.p_{D_s} A_0^{(8)nf} . \tag{5.54}$$

The superscript 'ann' stands for annihilation and ε is the polarization four-vector for the ϕ. the Eqs. (5.51), (5.52) and (5.53), respectively, are the factorized (spectator), non-factorized and annihilation contributions of the 4-quark operator $(\bar{s}c)(\bar{u}d)$ to the decay amplitude (Eq. (5.50)) , and Eq. (5.54) is the contribution of $H_w^{(8)}$. Putting it all together, one can define an effective a_1 as follows,

$$A(D_s^+ \to \phi\pi^+) = \tilde{G}_F a_1^{eff} f_\pi(2m_\phi)\varepsilon.p_{D_s} A_0^{D_s\phi}(m_\pi^2) , \tag{5.55}$$

where

$$a_1^{eff} = a_1 \left\{ 1 + \frac{A_0^{(1)nf}}{A_0^{D_s\phi}(m_\pi^2)} + \frac{C_2}{a_1}\frac{A_0^{(8)nf}}{A_0^{D_s\phi}(m_\pi^2)} + \frac{f_{D_s}}{f_\pi}\frac{A_0^{ann}}{A_0^{D_s\phi}(m_\pi^2)} \right\} . \tag{5.56}$$

Up to this stage, all quantities are taken to be real, including A_0^{ann}. Complex amplitudes will emerge as the result of fsi at the hadronic level.

Consider now the final state interactions. For illustrative purposes we consider a two-channel model which is adequate to illustrate our ideas. Consider an inelastic coupling of $\phi\pi$ channel with G-parity even, to the G-parity even eigenstate of $\bar{K}^0 K^{*+}$ and $\bar{K}^{*0} K^+$ (See Fig. (5.1)). Channel $\phi\pi$ will couple, among others, to $\eta\rho$ and $\eta'\rho$ channels also. Our intention is not to calculate numerically the effect of these channels but to illustrate how fsi enter the description. Both of these decays, $D_s^+ \to \bar{K}^0 K^{*+}$ and $D_s^+ \to \bar{K}^{*0} K^+$, are color-suppressed. Following an analogous procedure to the one that led us to Eq. (5.55), we find,

$$A(D_s^+ \to \bar{K}^0 K^{*+}) = \tilde{G}_F a_2^{eff} f_K(2m_{K^*})\varepsilon.p_{D_s} A_0^{D_s K^*}(m_K^2) , \tag{5.57}$$

where

$$a_2^{eff} = a_2 \left\{ 1 + \frac{B_0^{(1)nf}}{A_0^{D_s K^*}(m_K^2)} + \frac{C_1}{a_2}\frac{B_0^{(8)nf}}{A_0^{D_s K^*}(m_K^2)} + \frac{a_1}{a_2}\frac{f_{D_s}}{f_K}\frac{B_0^{ann}}{A_0^{D_s K^*}(m_K^2)} \right\} . \tag{5.58}$$

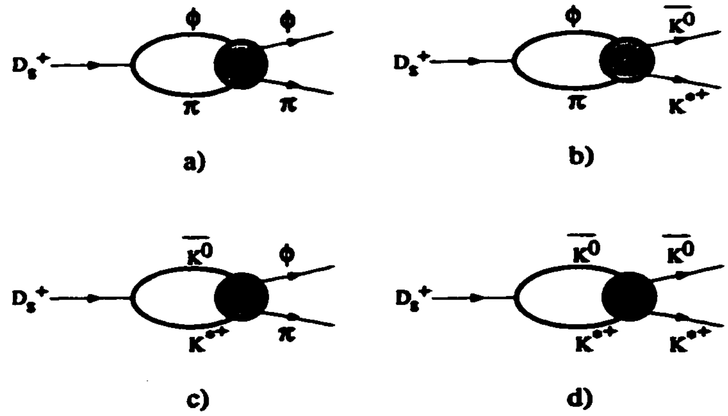


Figure 5.1: Inelastic coupling of $\phi\pi$ channel with G-parity even, to the G-parity even eigenstate of $\bar{K}^0 K^{*+}$ and $\bar{K}^{*0} K^+$

Here $B_0^{(1)nf}$, $B_0^{(8)nf}$ and B_0^{ann} are the analogues of $A_0^{(1)nf}$, $A_0^{(8)nf}$ and A_0^{ann} of Eqs. (5.51) to (5.54) . Similarly,

$$A(D_s^+ \to \bar{K}^{*0} K^+) = \tilde{G}_F \hat{a}_2^{eff} f_{K^*} (2m_{K^*}) \varepsilon . p_{D_s} F_1^{D_s K}(m_{K^*}^2) \tag{5.59}$$

where

$$\hat{a}_2^{eff} = a_2 \left\{ 1 + \frac{\hat{B}_0^{(1)nf}}{F_1^{D_s K}(m_{K^*}^2)} + \frac{C_1}{a_2} \frac{\hat{B}_0^{(8)nf}}{F_1^{D_s K}(m_{K^*}^2)} + \frac{a_1}{a_2} \frac{f_{D_s}}{f_{K^*}} \frac{\hat{B}_0^{ann}}{F_1^{D_s K}(m_{K^*}^2)} \right\} . \tag{5.60}$$

The hatted quantities here refer to the decay channel $\bar{K}^{*0} K^+$.

Now the eigenstates of G-parity are [71],

$$|K^* K\rangle_{S,A} = \frac{1}{\sqrt{2}} \left\{ |K^{*+} \bar{K}^0\rangle \pm |K^+ \bar{K}^{*0}\rangle \right\} , \tag{5.61}$$

where the symmetric (antisymmetric) combination has G-parity even (odd). Thus it is only $|K^* K\rangle_S$ that couples to $\phi\pi$.

Next we set up a coupled channel fsi between the decays $D_s^+ \to \phi\pi^+$ and $D_s^+ \to (K^* K)_S$ following the formalism described in [72]. Though the method of unitarization, the K-matrix method, which amounts to retaining only the on-shell contribution from re-scattering loops, is not unique, it serves adequately to describe the effects of final state interactions.

We simplify our notations further by using the following short-hand notations for the thus far real amplitudes,

$$A(D_s^+ \to \phi\pi^+) \equiv \varepsilon.p_{D_s} A^{\phi\pi} , \tag{5.62}$$

with

$$A^{\phi\pi} = \tilde{G}_F a_1^{eff} f_\pi (2m_\phi) A_0^{D_s\phi}(m_\pi^2) , \tag{5.63}$$

and

$$A(D_s^+ \to (K^*K)_S) \equiv \varepsilon.p_{D_s} A^{K^*K} , \tag{5.64}$$

where

$$A^{K^*K} = \tilde{G}_F \frac{(2m_{K^*})}{\sqrt{2}} \left\{ a_2^{eff} f_K A_0^{D_s K^*}(m_K^2) + \hat{a}_2^{eff} f_{K^*} F_1^{D_s K}(m_{K^*}^2) \right\} . \tag{5.65}$$

The two amplitudes, Eqs. (5.62) and (5.64), couple via fsi and get unitarized. The unitarized decay amplitudes, $\boldsymbol{A}^U$, are given by [72],

$$\boldsymbol{A}^U = \left(\boldsymbol{1} - i\boldsymbol{k}^3\boldsymbol{K}\right)^{-1T} \boldsymbol{A} , \tag{5.66}$$

where $\boldsymbol{A}$ is a column with entries $A^{\phi\pi}$ and A^{K^*K}, $\boldsymbol{k}^3$ is a diagonal matrix with entries k_1^3 and k_2^3, k_1 and k_2 being the center of mass momenta in the channels $\phi\pi$ and K^*K respectively and $\boldsymbol{K}$ is the symmetric, real (2×2) K-matrix,

$$\boldsymbol{K} = \begin{pmatrix} a & b \\ b & c \end{pmatrix} , \tag{5.67}$$

where a, b and c are assumed to be constants with dimensions GeV^{-3}.

The parameters of the K-matrix can be evaluated from the knowledge of the coupled channel scattering problem. In absence of this information, they remain undetermined in our case, which wasn't our aim from the beginning .

Carrying out the unitarization of the decay amplitude as indicated in Eq. (5.66), we obtain a unitarized $A^{U,\phi\pi}$ which is complex and given by,

$$A^U(D_s^+ \to \phi\pi^+) = \tilde{G}_F a_1^{U,eff} f_\pi \varepsilon.p_{D_s}(2m_\phi) A_0^{D_s\phi}(m_\pi^2) , \tag{5.68}$$

where

$$a_1^{U,eff} = \frac{a_1^{eff}}{\Delta}\left\{1 - ik_2^3 c + i\frac{m_{K^*}}{\sqrt{2}m_\phi} k_2^3 b\left(\frac{a_2^{eff}}{a_1^{eff}}\frac{f_K}{f_\pi}\frac{A_0^{D_s K^*}(m_K^2)}{A_0^{D_s\phi}(m_\pi^2)} + \frac{\hat{a}_2^{eff}}{a_1^{eff}}\frac{f_{K^*}}{f_\pi}\frac{F_1^{D_s K}(m_{K^*}^2)}{A_0^{D_s\phi}(m_\pi^2)}\right)\right\}, \tag{5.69}$$

with $\Delta = \det(\mathbf{1} - i\boldsymbol{k}^3\boldsymbol{K})$.

If the fsi were elastic, $b = c = 0$ and $\Delta = 1 - ik_1^3 a$, we would have obtained

$$a_1^{U,eff} = \frac{a_1^{eff}}{\sqrt{1 + k_1^6 a^2}} e^{i\delta} , \tag{5.70}$$

where $\delta = \tan^{-1}(ak_1^3)$ is the elastic P-wave $\phi\pi$ scattering phase.

A similar expression to Eq. (5.69) can be written down for $D_s^+ \to K^{*+}\bar{K}^0$ (and for $D_s^+ \to K^+\bar{K}^{*0}$) which would define $a_2^{U,eff}$ and $\hat{a}_2^{U,eff}$.

One should view Eq. (5.68) as the defining equation for $a_1^{U,eff}$ which includes the important physics, and which is process-dependent and complex. If we view $a_1^{U,eff}$ and $a_2^{U,eff}$ in the manner we are proposing, then the comparison of two body hadronic decays of D and B mesons with semileptonic decays which in past has been claimed [73, 74, 75, 76] to be tests of factorization, becomes merely determinations of $|a_1^{U,eff}|$ and $|a_2^{U,eff}|$. In the next section we will use the defining equations like Eq. (5.68) for determining $|a_1^{U,eff}|$ and $|a_2^{U,eff}|$ for different charm decay processes.

5.8 Estimates of $|a_1^{U,eff}|$ and $|a_2^{U,eff}|$ from charm decay data .

The same procedure that we took for defining $a_1^{U,eff}$ in $D_s^+ \to \phi\pi^+$ decay in the last section can be used in defining $a_1^{U,eff}$ and $a_2^{U,eff}$ in, say, $D^0 \to \bar{K}\pi$ decays. There is an added complication here, that of two isospins in the final state . The fsi unitarization has to be carried out in each of the two isospin states separately. Nevertheless, one can define, following the same procedure as we have used for the simpler case of $D_s^+ \to \phi\pi^+$,

$$\begin{aligned}
A(D^0 \to K^-\pi^+) &= \tilde{G}_F|a_1^{U,eff}|f_\pi(m_D^2 - m_K^2)F_0^{DK}(m_\pi^2)e^{i\phi_{+-}} \\
A(D^0 \to \bar{K}^0\pi^0) &= \frac{\tilde{G}_F}{\sqrt{2}}|a_2^{U,eff}|f_K(m_D^2 - m_\pi^2)F_0^{D\pi}(m_K^2)e^{i\phi_{00}} \\
A(D^+ \to \bar{K}^0\pi^+) &= A(D^0 \to K^-\pi^+) + \sqrt{2}A(D^0 \to \bar{K}^0\pi^0)\ .
\end{aligned} \tag{5.71}$$

$|a_1^{U,eff}|$ can be determined by relating $\Gamma(D^0 \to K^-\pi^+)$ to $\Gamma(D^0 \to K^-l^+\nu)$ and $|a_2^{U,eff}|$ by relating $\Gamma(D^0 \to \bar{K}^0\pi^0)$ to $\Gamma(D^0 \to \pi^-l^+\nu)$ (Similar to the ones we did in Sec. (4.8)) . Finally, $\phi_{+-} - \phi_{00}$ is obtainable from $\Gamma(D^+ \to \bar{K}^0\pi^+)$.We determined the products $|a_1^{U,eff}|F_0^D K(m_\pi^2)$ and $|a_2^{U,eff}|F_0^{D\pi}(m_K^2)$ and the relative phase $(\phi_{+-} - \phi_{00})$ from the branching ratios $B(D^0 \to K^-\pi^+)$, B (D^0 $\to$ $\bar{K}^0$ π^0) and $B(D^+ \to \bar{K}^0\pi^+)$ [6] with the result:

$$\begin{aligned}
D \to \bar{K}\pi : \qquad |a_1^{U,eff}|F_0^{DK}(m_\pi^2) &= \ 0.767 \pm 0.014 \\
|a_2^{U,eff}|F_0^{D\pi}(m_K^2) &= \ 0.593 \pm 0.038 \\
cos(\phi_{+-} - \phi_{00}) &= \ -0.867 \pm 0.089.
\end{aligned} \tag{5.72}$$

If we use the experimental determinations [6] of $F_0^{DK}(0)$ and $F_0^{D\pi}(0)$ from semileptonic decays (assuming monopole extrapolation),

$$F_0^{DK}(0) = \ 0.75 \pm 0.02 \pm 0.02 \qquad [6]$$

$$F_0^{D\pi}(0)/F_0^{DK}(0) = \quad 1.0^{+0.3}_{-0.2} \pm 0.04 \qquad [77] \tag{5.73}$$
$$= \quad 1.3 \pm 0.2 \pm 0.1 \qquad [78]$$

we obtain,

$$|a_1^{U,eff}| = 1.02 \pm 0.04$$
$$|a_2^{U,eff}| = (0.76^{+0.26}_{-0.16}), \quad (0.58 \pm 0.08) \tag{5.74}$$

In $a_2^{U,eff}$ above, the two values correspond to the two values of the form factor ratio $F_0^{D\pi}(0)/F_0^{DK}(0)$ given in Eq. (5.73) respectively. We have used a monopole extrapolation with pole mass 2.47 GeV [35, 42] in calculating $F_0^{D\pi}(m_K^2)$.

We define $a_1^{U,eff}$ and $a_2^{U,eff}$ for $D \to \bar{K}\rho$ and $\bar{K}^*\pi$ decays as

$$\begin{aligned} A(D^0 \to K^-\rho^+) &= 2|a_1^{U,eff}| m_\rho f_\rho F_1^{DK}(m_\rho^2)\epsilon^* \cdot p e^{i\phi_{+-}} , \\ A(D^0 \to \bar{K}^0\rho^0) &= 2|a_2^{U,eff}| \frac{m_\rho}{\sqrt{2}} f_K A_0^{D\rho}(m_K^2)\epsilon^* \cdot p e^{i\phi_{00}} , \\ A(D^+ \to \bar{K}^0\rho^+) &= A(D^0 \to K^-\rho^+) + \sqrt{2}A(D^0 \to \bar{K}^0\rho^0) . \end{aligned} \tag{5.75}$$

and

$$\begin{aligned} A(D^0 \to K^{*-}\pi^+) &= 2|a_1^{U,eff}| m_{K^{*-}} f_\pi A_0^{DK^*}(m_\pi^2)\epsilon^* \cdot p e^{i\phi_{+-}} , \\ A(D^0 \to \bar{K}^{*0}\pi^0) &= 2|a_2^{U,eff}| \frac{m_{K^*}}{\sqrt{2}} f_{K^*} F_1^{D\pi}(m_{K^*}^2)\epsilon^* \cdot p e^{i\phi_{00}} , \\ A(D^+ \to \bar{K}^{*0}\pi^+) &= A(D^0 \to K^{*-}\pi^+) + \sqrt{2}A(D^0 \to \bar{K}^{*0}\pi^0) . \end{aligned} \tag{5.76}$$

A similar analysis, similar to the one we did for $D \to \bar{K}\pi$ decays, of the branching ratios in $D \to \bar{K}\rho$ and $\bar{K}^*\pi$leads to:

$$\begin{aligned} D \to \bar{K}\rho : \qquad |a_1^{U,eff}| F_1^{DK}(m_\rho^2) &= 1.097 \pm 0.069 \\ |a_2^{U,eff}| A_0^{D\rho}(m_K^2) &= 0.672 \pm 0.055 \\ cos(\phi_{+-} - \phi_{00}) &= -1.046 \pm 0.205 \end{aligned} \tag{5.77}$$

and

$$D \to \bar{K}^*\pi: \qquad \begin{aligned} |a_1^{U,eff}| A_0^{DK^*}(m_\pi^2) &= 1.138 \pm 0.070 \\ |a_2^{U,eff}| F_1^{D\pi}(m_{K^*}^2) &= 0.747 \pm 0.061 \\ cos(\phi_{+-} - \phi_{00}) &= -0.926 \pm 0.166 \end{aligned} \tag{5.78}$$

From the form factors at $q^2 = 0$ listed in [6] we can calculate all the form factors needed by using monopole extrapolation for all of them except $A_0^{D\rho}(m_K^2)$ for which we adopt the theoretical value given in [35, 42] . The resulting $a_1^{U,eff}$ and $a_2^{U,eff}$ are :

$$D \to \bar{K}\rho: \qquad \begin{aligned} |a_1^{U,eff}| &= 1.27 \pm 0.09 \\ |a_2^{U,eff}| &= 0.93 \pm 0.08 \end{aligned} \tag{5.79}$$

and

$$D \to \bar{K}^*\pi: \qquad \begin{aligned} |a_1^{U,eff}| &= 1.76 \pm 0.23 \\ |a_2^{U,eff}| &= (0.8^{+0.27}_{-0.17}), \quad (0.61 \pm 0.09) \end{aligned} \tag{5.80}$$

The two values of $a_2^{U,eff}$ correspond to the two values of the ratio $F_0^{D\pi}(0)/F_0^{DK}(0)$ respectively, given in Eq. (5.73).

We end with a determination of the process-dependent $|a_1^{U,eff}|$ in $D_s^+ \to \eta\pi^+$, $\eta'\pi^+$, $\eta\rho^+$ and $\eta'\rho^+$ from hadronic and semileptonic data.

The defining equation for $a_1^{U,eff}$ in $D_s^+ \to \eta\pi^+$ and $\eta\rho^+$ decay amplitudes is obtained by simply replacing a_1 in the expression for the factorized amplitude, Eqs. (4.93) and (4.94), by $a_1^{U,eff}$ as in Eq. (5.68). Thus

$$A(D_s^+ \to \eta\pi^+) = \tilde{G}_F C_\eta (a_1^{U,eff})_{\eta\pi^+} f_\pi (m_{D_s}^2 - m_\eta^2) F_0^{D_s\eta}(m_\pi^2) \ , \tag{5.81}$$

and

$$A(D_s^+ \to \eta\rho^+) = \tilde{G}_F C_\eta (a_1^{U,eff})_{\eta\rho^+} (2m_\rho f_\rho) \varepsilon^* . p_{D_s} F_1^{D_s\eta}(m_\rho^2) \ , \tag{5.82}$$

where C_η is defined in Eq. (4.95).

Now using Eqs. (4.102) and (4.103) the ratios of the nonleptonic to semileptonic decay rates in terms of $|(a_1^{U,eff})|$ are

$$\frac{\Gamma(D_s^+ \to \eta\pi^+)}{\Gamma(D_s^+ \to \eta e^+\nu)} \simeq |(a_1^{U,eff})_{\eta\pi^+}|^2 \frac{6\pi^2 f_\pi^2 (m_D^2 - m_\eta^2)^2 |V_{ud}|^2 \lambda(m_D^2, m_\eta^2, m_\pi^2)}{\Lambda_1^4 I(m_D, m_\eta, \Lambda_1)} \quad (5.83)$$

$$\frac{\Gamma(D_s^+ \to \eta\rho^+)}{\Gamma(D_s^+ \to \eta e^+\nu)} \simeq |(a_1^{U,eff})_{\eta\rho^+}|^2 \frac{6\pi^2 f_\rho^2 |V_{ud}|^2 \lambda^3(m_D^2, m_\eta^2, m_\rho^2)}{(\Lambda_1^2 - m_\rho^2)^2 I(m_D, m_\eta, \Lambda_1)} \quad (5.84)$$

where $I(m_D, m_\eta, \Lambda_1)$ is defined in Eq. (4.82) and Λ_1 is taken to be 2.11 GeV, the D_s^* mass.

A similar treatment for η' using Eqs. (4.107) and (4.108) will result in the following ratios of the nonleptonic and semileptonic decay rates in terms of $|(a_1^{U,eff})|$

$$\frac{\Gamma(D_s^+ \to \eta'\pi^+)}{\Gamma(D_s^+ \to \eta' e^+\nu)} \simeq |(a_1^{U,eff})_{\eta'\pi^+}|^2 \frac{6\pi^2 f_\pi^2 (m_D^2 - m_{\eta'}^2)^2 |V_{ud}|^2 \lambda(m_D^2, m_{\eta'}^2, m_\pi^2)}{\Lambda_1^4 I(m_D, m_{\eta'}, \Lambda_1)} \quad (5.85)$$

$$\frac{\Gamma(D_s^+ \to \eta'\rho^+)}{\Gamma(D_s^+ \to \eta' e^+\nu)} \simeq |(a_1^{U,eff})_{\eta'\rho^+}|^2 \frac{6\pi^2 f_\rho^2 |V_{ud}|^2 \lambda^3(m_D^2, m_{\eta'}^2, m_\rho^2)}{(\Lambda_1^2 - m_\rho^2)^2 I(m_D, m_{\eta'}, \Lambda_1)} \quad (5.86)$$

By equating the theoretical ratios, Eqs. (5.83) - (5.86), to the experimental ones, Eqs. (4.115) - (4.118), we have evaluated the following (we have used $V_{ud} = 0.975$, $f_\pi = 130.7$ MeV and $f_\rho = 216.0$ MeV):

$$|(a_1^{U,eff})_{\eta\pi^+}| = 0.89 \pm 0.13\,, \quad (5.87)$$

$$|(a_1^{U,eff})_{\eta'\pi^+}| = 1.56 \pm 0.28\,, \quad (5.88)$$

$$|(a_1^{U,eff})_{\eta\rho^+}| = 1.49 \pm 0.20\,, \quad (5.89)$$

$$|(a_1^{U,eff})_{\eta'\rho^+}| = 2.77 \pm 0.55\,. \quad (5.90)$$

5.9 Summary

We defined effective, and unitarized a_1 and a_2 by the following prescription: In decays where the decay amplitudes involve only one form factor, the true decay amplitude is given by replacing a_1 and a_2 by $a_1^{U,eff}$ and $a_2^{U,eff}$ repectively in the factorized amplitude. Defined in this manner, we saw how these effective parameters get contributions from annihilation and nonfactorizable processes as well as the final state interactions. As these effective parameters are process-dependent, the purported test of factorization that compares the hadronic rate to the semileptonic should be used instead as a tool to determine the modulus of these effective parameters.

We determined $|a_1^{U,eff}|$ and $|a_2^{U,eff}|$ in $D \to \bar{K}\pi, \bar{K}\rho$ and $\bar{K}^*\pi$ decays using experimental input on formfactors (with monopole extrapolation) as much as possible (See Table (5.2)). The values of these parameters, particularly $|a_2^{U,eff}|$, imply large departures from the factorization expectation when compared with a_1 and a_2 given by Eq. (5.7)) with $N_c = 3$.

For $D_s^+ \to \eta'\pi^+$, $\eta\rho^+$ and $\eta'\rho^+$, $a_1^{U,eff}$ was found to be significantly different from a_1 of Eq. (5.7) ($N_c = 3$ scheme) signifying that the simple factorization prescription would not apply to these cases (see Table (5.2)).

We reemphasize that effective a_1 and a_2 can be defined only for those decays whose amplitudes involve a single Lorentz scalar structure. Thus they can not be defined for decays of D and B mesons involving two vector particles in the final state. Consequently, our analysis applies only to those cases where the decay amplitudes involve a single Lorentz scalar structure.

Table 1: The values of $|(a_1^{U,eff})|$ and $|(a_2^{U,eff})|$ for some exclusive D meson decays.

	$\|(a_1^{U,eff})\|$	$\|(a_2^{U,eff})\|$
Pure Factorization (N_c=3 scheme)	$a_1 = C_1 + \frac{C_2}{3} = 1.09 \pm 0.04$	$a_2 = C_2 + \frac{C_1}{3} = -0.09 \pm 0.05$
Factorization Model ($N_c \to \infty$ scheme)	$a_1 \approx C_1(m_c) = 1.26 \pm 0.04$	$a_2 \approx C_2(m_c) = -0.51 \pm 0.05$
$D \to \bar{K}\pi$	$\|(a_1^{U,eff})_{\bar{K}\pi}\| = 1.02 \pm 0.04$	$\|(a_2^{U,eff})_{\bar{K}\pi}\| = (0.76^{+0.26}_{-0.16})$, (0.58 ± 0.08)
$D \to \bar{K}\rho$	$\|(a_1^{U,eff})_{\bar{K}\rho}\| = 1.27 \pm 0.09$	$\|(a_2^{U,eff})_{\bar{K}\rho}\| = 0.93 \pm 0.08$
$D \to \bar{K}^*\pi$	$\|(a_1^{U,eff})_{\bar{K}^*\pi}\| = 1.76 \pm 0.23$	$\|(a_2^{U,eff})_{\bar{K}^*\pi}\| = (0.8^{+0.27}_{-0.17})$, (0.61 ± 0.09)
$D_s^+ \to \eta\pi^+$	$\|(a_1^{U,eff})_{\eta\pi^+}\| = 0.89 \pm 0.13$	
$D_s^+ \to \eta'\pi^+$	$\|(a_1^{U,eff})_{\eta'\pi^+}\| = 1.56 \pm 0.28$	
$D_s^+ \to \eta\rho^+$	$\|(a_1^{U,eff})_{\eta\rho^+}\| = 1.49 \pm 0.20$	
$D_s^+ \to \eta'\rho^+$	$\|(a_1^{U,eff})_{\eta'\rho^+}\| = 2.77 \pm 0.55$	

Chapter 6

Summary and Conclusion

Due to confinement, the decaying heavy quarks are necessarily bound in hadronic bound states (mesons or baryons). Thus the study of heavy quarks involves examining the weak interaction under the influence of strong interaction . It is assumed that the short distance nature of weak decays allows one to separate possible corrections from strong interactions into short and long distance contributions. The asymptotic-freedom property of QCD allows a perturbative calculation of the short distance corrections. They arise from the exchange of hard gluons and modify the structure of the weak interaction Hamiltonian. The light quarks produced in the decay of a charm or beauty quark necessarily have to combine with the spectator quark to form color singlets i.e. hadrons. This is a non-perturbative process and, so far cannot be calculated from first principles. Therefore a phenomenological approach has to be adopted with model form factors. The overall picture is the following

(1) Some long distance effects are included in the bound state wave functions. Other long distance effects such as final state interactions among the produced hadrons are added separately.

(2) The short distance effects originating in hard gluon interactions are calculated perturbatively (due to the asymptotic freedom property of strong interactions) and included in the effective weak Hamiltonian.

(3) The weak amplitudes are given by matrix elements of H_{eff} between asymptotic initial and final states (leptons and hadrons):

$$A(a \to b + c + ...) = \langle bc...|H_{eff}|a\rangle \,, \tag{6.1}$$

where H_{eff} includes the hard gluon (short range strong interaction effects) corrections.

We started with the charm lowering part of the effective low energy weak Hamiltonian, Eq. (3.4), where the nonleptonic Cabibbo-favored part of it is given by

$$H^0_{NL}(\Delta C = -1) = \frac{G_F}{\sqrt{2}} V_{cs} V^*_{ud} (\bar{s}c)_L (\bar{u}d)_L \,. \tag{6.2}$$

Hard gluonic effects were calculated perturbatively and included in the weak Hamiltonian. The nonleptonic part of the resulting effective weak Hamiltonian once QCD corrections have been included is given by

$$H_{NL}(\Delta C = -1) = \frac{G_F}{\sqrt{2}} V_{cs} V^*_{ud} \left[C_1(m_c)(\bar{s}c)_L(\bar{u}d)_L + C_2(m_c)(\bar{u}c)_L(\bar{s}d)_L\right] \,. \tag{6.3}$$

The Wilson Coefficients, C_1 and C_2, are given by

$$C_1(m_c) = 1.26 \pm 0.04 \,, \qquad C_2(m_c) = -0.51 \pm 0.05 \,. \tag{6.4}$$

That is, hard gluon exchanges renormalize the original charged current interaction (first term) and, as a new feature, induce an additional effective neutral current interaction (second term).

We distinguished between inclusive and exclusive decay of charm mesons. In the context of the valence quark approximation model we examined the inclusive decays of charm mesons, $D^0(c\bar{u})$, $D^+(c\bar{d})$ and $D^+_s(c\bar{s})$. We saw that although the valence quark approximation model gives correct order of magnitude for the lifetimes of charm mesons, it could not explain the observed pattern of these lifetimes, that is,

$$\tau\left(D^+\right) > \tau\left(D^+_s\right) > \tau\left(D^0\right). \tag{6.5}$$

In particular the significant lifetime difference of D^+ and D^0 ($\tau(D^+) \approx 2.5\tau(D^0)$) remains a puzzle for the valence quark approximation model which predicts nearly equal lifetimes for the three D mesons.

In chapter 4 we studied the exclusive decays of charm mesons. We adapted the valence quark approximation model used in inclusive decays for exclusive decays

(Sec. (4.2)) and calculated the decay amplitudes in the factorization ansatz, which states that the matrix element of a product of two currents can be approximated by a product of matrix elements of currents,

$$< M_1, M_2|\tilde{J}\cdot J|D > \approx < M_1|\tilde{J}|0 >< M_2|J|D >, \tag{6.6}$$

where J and $\tilde{J}$ are $(\bar{q}_2 q_1)_L$ color-singlet currents. Therefore the matrix element of the H_{NL}^{eff} can be written in terms of simpler matrix elements of single weak currents with the vacuum as an intermediate state and thus are determined in terms of meson decay constants and hadron form factors. The amplitude of the semileptonic decays factorizes exactly as the hadronic current can only lead to hadrons and the leptonic current to leptons:

$$< Xl\bar{\nu}_l|(\bar{l}\nu_l)(\bar{q}Q)|D > = < l\bar{\nu}_l|(\bar{l}\nu_l)|0 >< X|(\bar{q}Q)|D > \ . \tag{6.7}$$

For calculating the decay rates of the exclusive decay, we have to introduce different phenomenological formfactors which we discussed in chapter 4. We also introduced phenomenological constants a_1 and a_2, and as a result the nonleptonic Cabibbo-favored part of the effective Hamiltonian takes the following form

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^{*} \left\{ a_1 (\bar{u}d)_{L,H} (\bar{s}c)_{L,H} + a_2 (\bar{s}d)_{L,H} (\bar{u}c)_{L,H} \right\}. \tag{6.8}$$

Now the question is what is the nature of the a_1 and a_2 or what effects are included in it? We can summarize the development of our understanding of the phenomenological constants a_1 and a_2 in three steps:

1) In the pure factorized model with $N_c = 3$ and without considering any nonfactorized effects, a_1 and a_2 are considered to be process independent constants which are given by

$$a_1 = C_1 + \frac{C_2}{3} = 1.09 \pm 0.04, \qquad a_2 = C_2 + \frac{C_1}{3} = -0.09 \pm 0.05\,. \tag{6.9}$$

We have not discussed the predictions of this model in this dissertation because from the beginning it has been known that it does not accomodate experimental results due to the smallness of a_2. In particular it fails badly in predicting the observed decay rates of $D \rightarrow \bar{K}\pi$ decays. For details see Refs. [16, 32].

By contrast, in the so called factorization model with $N_c \rightarrow \infty$, (which we discussed in chapter 4) a_1 and a_2 are considered to be process independent and their values are taken to be

$$a_1 \approx C_1(m_c) = 1.26 \pm 0.04\,, \qquad a_2 \approx C_2(m_c) = -0.51 \pm 0.05\,. \tag{6.10}$$

These values of a_1 and a_2 lead to successful predictions of some of the exclusive charm decay rates, most notably $D \rightarrow \bar{K}\pi$ decays. This is a procedure that finds some theoretical justification in $1/N_c$ expansion arguments [41]. Factorization model, however, fails badly in predicting the decay rates of some of the exclusive charm decays, such as $D^0 \rightarrow K^- a_1^+$ and $D_s^+ \rightarrow \eta' \rho^+$ which involve small energy release.

2) In the second step (which we discussed in chapter 5) we took a_1 and a_2 to be process-dependent effective phenomenological constants. By a redefinition some nonfactorized effects were includecd in a_1 and a_2 thereby generating effective a_1 and a_2 (real at this stage) which come close to Eq. (6.10) for $D \rightarrow \bar{K}\pi$ decays. We emphasize that effective a_1 and a_2 can be defined only for those decays whose amplitudes involve a single Lorentz scalar structure. Thus they can not be defined for decays of D and B mesons involving two vector particles in the final state. In this step we ignored the annihilation terms and inelastic final state interactions.

3) We generalized our definition of a_1 and a_2 and replaced them by complex unitarized $a_1^{U,eff}$ and $a_2^{U,eff}$. We defined effective, and unitarized a_1 and a_2 by the following prescription: In decays where the decay amplitudes involve only one form factor, the true decay amplitude is given by replacing a_1 and a_2 by $a_1^{U,eff}$ and $a_2^{U,eff}$ respectively in the factorized amplitude. Defined in this manner, we saw how these effective parameters get contributions from annihilation and nonfactorizable processes as well

as the final state interactions.

In conclusion, we assert that, treating the phenomenological parameters a_1 and a_2 as process independent is untenable. It does not explain the experimental data. For the processes which involve only one Lorentz structure, it is always possible to define an effective and complex (unitarized) $a_1^{U,eff}$ and $a_2^{U,eff}$ that include nonfactorized, annihilation and inelastic final state interaction effects. A corollary of our point of view is that the purported test of factorization that compares the hadronic rate to the semileptonic should be used instead as a tool to determine the modulus of these effective parameters.

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