

## **INFORMATION TO USERS**

**This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.**

**The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.**

**In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.**

**Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps.**

**Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.**

**ProQuest Information and Learning  
300 North Zeeb Road, Ann Arbor, MI 48106-1346 USA  
800-521-0600**

**UMI<sup>®</sup>**



**University of Alberta**

**Behaviour of Steel Columns Reinforced with Welded Steel Plates**

by

Ziqi Wu



**A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment  
of the requirements for the degree of Master of Science**

in

**Structural Engineering**

**Department of Civil and Environmental Engineering**

**Edmonton, Alberta**

**Spring, 2002**



**National Library  
of Canada**

**Acquisitions and  
Bibliographic Services**

**395 Wellington Street  
Ottawa ON K1A 0N4  
Canada**

**Bibliothèque nationale  
du Canada**

**Acquisitions et  
services bibliographiques**

**395, rue Wellington  
Ottawa ON K1A 0N4  
Canada**

*Your file Votre référence*

*Our file Notre référence*

**The author has granted a non-exclusive licence allowing the National Library of Canada to reproduce, loan, distribute or sell copies of this thesis in microform, paper or electronic formats.**

**The author retains ownership of the copyright in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author's permission.**

**L'auteur a accordé une licence non exclusive permettant à la Bibliothèque nationale du Canada de reproduire, prêter, distribuer ou vendre des copies de cette thèse sous la forme de microfiche/film, de reproduction sur papier ou sur format électronique.**

**L'auteur conserve la propriété du droit d'auteur qui protège cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.**

0-612-69778-9

**Canada**



**University of Alberta**

**Library Release Form**

**Name of Author:** Ziqi Wu

**Title of Thesis:** Behaviour of Steel Columns Reinforced with Welded Steel Plates

**Degree:** Master of Science

**Year this Degree Granted:** 2002

Permission is hereby granted to the University of Alberta Library to reproduce single copies of this thesis and to lend or sell such copies for private, scholarly or scientific research purposes only.

The author reserves all other publication and other rights in association with the copyright in the thesis, and except as herein before provided, neither the thesis nor any substantial portion thereof may be printed or otherwise reproduced in any material form whatever without the author's prior written permission.



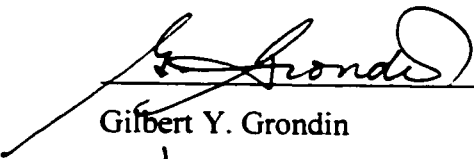
601D, Michener Park  
Edmonton, Alberta  
T6H 5A1

Jan. 3, 2007  
(Date)

**University of Alberta**

**Faculty of Graduate Studies and Research**

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled Behaviour of Steel Columns Reinforced with Welded Steel Plates submitted by Ziqi Wu in partial fulfillment of the requirements for the degree of Master of Science in Structural Engineering.



---

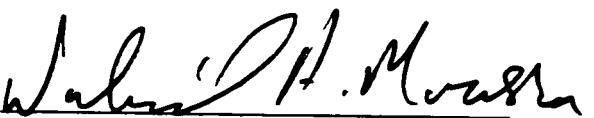
Gilbert Y. Grondin

Supervisor



---

Alaa E. Elwi



---

Walied A. Moussa

Dec. 21, 2001  
(Date)

**To My Family**

## **Abstract**

Current design criteria for steel columns reinforced with welded steel plates, usually based on the SSRC column curve 2, have not been verified. Considering the complexity of different influencing factors of the reinforced columns, the use of current design criteria may not be appropriate. A better understanding of the parameters associated with reinforced columns is therefore required.

A parametric study, using 317 finite element models of reinforced steel columns with varying parameters, was conducted. The study showed that column slenderness and initial out-of-straightness remain the important factors for reinforced columns. The interactions of the orientation of the reinforcing plates and the buckling direction were observed to affect the strength of reinforced columns. These observations require further experimental confirmation. A detailed statistical analysis was then conducted to determine the factored resistance of reinforced columns and to evaluate their performances. The results showed that the current design approach of using the SSRC column curve 2 is appropriate for use with a resistance factor of 0.9.

## **Acknowledgements**

The author would like to express his sincere gratitude to his supervisor, Dr. G.Y. Grondin, for his great help, guidance, suggestions and continuous encouragement throughout the preparation of this thesis.

Thanks are due to Dave Clyburn for his great help in proofreading sections of this thesis.

## **Table of Contents**

<b>1. Introduction</b> .....	<b>1</b>
<b>1.1 General Background</b> .....	<b>1</b>
<b>1.2 Statement of the Problem</b> .....	<b>2</b>
<b>1.3 Objectives and Scope</b> .....	<b>2</b>
<b>1.4 Organization of the Thesis</b> .....	<b>4</b>
<b>2. Literature Review</b> .....	<b>5</b>
<b>2.1 Factors Influencing Column Strength</b> .....	<b>5</b>
2.1.1 Introduction .....	5
2.1.2 Initial Out-of-straightness .....	8
2.1.3 Residual Stresses .....	10
<b>2.2 Research on Reinforced Columns</b> .....	<b>11</b>
2.2.1 Work of Tall and Co-workers .....	12
2.2.2 Work of Brown (1988) .....	15
<b>2.3 Summary</b> .....	<b>15</b>
<b>3. Finite Element Modelling of Reinforced Steel Column</b> .....	<b>18</b>
<b>3.1 General</b> .....	<b>18</b>
<b>3.2 Description of the Model</b> .....	<b>18</b>
3.2.1 Finite Element Mesh .....	19
3.2.2 Material Properties .....	20
3.2.3 Boundary Conditions .....	21
3.2.4 Initial Conditions .....	22
3.2.5 Loading Process .....	23
<b>3.3 Validation of the Finite Element Model</b> .....	<b>24</b>
3.3.1 General .....	24
3.3.2 Description of the Tests .....	24
3.3.3 Initial Conditions of the Numerical Analyses .....	25
3.3.4 Behaviour of the Column Reinforced under Load .....	26
3.3.5 Behaviour of the Column Reinforced under no Load .....	27

3.3.6	Validation of the Finite Element Models in the Intermediate Length Range .....	28
4.	Parametric Study .....	41
4.1	General .....	41
4.2	Buckling Behaviour .....	42
4.3	Selection of Parameters .....	42
4.4	Analysis Results .....	44
4.5	Effect of Column Slenderness .....	45
4.6	Effect of Residual Stresses .....	45
4.6.1	Residual Stresses before Welding .....	46
4.6.2	Effect of the Magnitude of the Welding Residual Stresses .....	48
4.7	Effect of the Initial Out-of-straightness .....	49
4.8	Effect of the Pre-load .....	50
4.9	Effect of Steel Grade .....	51
4.10	Effect of Reinforcing Plate Orientation .....	52
4.11	Effect of Buckling Axis .....	52
4.12	Effect of W-Shape to Plate Area Ratio .....	53
4.13	Summary .....	54
5.	Limit States Design .....	80
5.1	Background .....	80
5.1.1	Column Resistance Based on CAN/CSA-S16.1-94 .....	80
5.1.2	Principles of the Limit States Design .....	81
5.1.3	Determination of the Resistance of a Steel Column .....	85
5.1.4	The Material Factor .....	86
5.1.5	Summary .....	89
5.2	Statistical Parameters .....	92
5.2.1	Geometrical Variations .....	93
5.2.2	Material Variations .....	96
5.2.3	Professional Factors .....	98
5.2.3.1	The Effect of Out-of-straightness .....	99

5.2.3.2	Miscellaneous Factors	100
5.2.3.3	Experimental Factors	101
5.2.3.4	Summary	102
5.3	Evaluation of the Performance Factors	103
6.	Summary, Conclusions, and Recommendations	141
6.1	Summary	141
6.2	Conclusions	143
6.3	Recommendations for Future Research	144
	List of References	146
Appendix A.	Analysed Reinforced Column Description	150
Appendix B.	Analysis Result Description	172
Appendix C.	Statistical Analysis Data for the Professional Factors for the Columns from Group 2	195



## **List of Tables**

<b>Table 3.1</b>	<b>Models Used for the Validation of the Finite Element Method for Reinforced Columns</b>	<b>30</b>
<b>Table 3.2</b>	<b>Finite Element Models for the Unreinforced Columns</b>	<b>31</b>
<b>Table 4.1</b>	<b>Fractional Factorial Design and Designation of the Numerical Models</b>	<b>56</b>
<b>Table 4.2</b>	<b>Models Used to Study the Effect of Initial Residual Stresses In the I-section before Welding</b>	<b>58</b>
<b>Table 4.3</b>	<b>Models Used to Study the Effect of the Initial Residual Stresses In the Cover Plates before Welding</b>	<b>59</b>
<b>Table 4.4</b>	<b>Models Used to Study the Effect of Varying Welding Residual Stress Magnitude</b>	<b>59</b>
<b>Table 4.5</b>	<b>Models Used to Study the Effect of the Initial Out-of-straightness</b>	<b>60</b>
<b>Table 4.6</b>	<b>Models Used to Study the Effect of the Preload</b>	<b>61</b>
<b>Table 4.7</b>	<b>Models Used to Study the Effect of Steel Grades</b>	<b>62</b>
<b>Table 4.8</b>	<b>Models Used to Study the Effect of Reinforced Plate Orientations</b>	<b>63</b>
<b>Table 4.9</b>	<b>Models Used to Study the Effect of the Buckling Axis</b>	<b>64</b>
<b>Table 4.10</b>	<b>Models Used to Study the Effect of Cover Plate Size</b>	<b>65</b>
<b>Table 4.11</b>	<b>Models Used to Study the Effect of the Size of the I-section</b>	<b>65</b>
<b>Table 5.1.a</b>	<b>Statistical Parameters for Rolled W Sections (from Kennedy and Gad Aly, 1979)</b>	<b>104</b>
<b>Table 5.1.b</b>	<b>Statistical Parameters for Cover Plates (from Chernenko and Kennedy, 1988)</b>	<b>104</b>
<b>Table 5.2.a</b>	<b>Statistical Parameters for the Geometric Properties of the Reinforced Columns</b>	<b>105</b>
<b>Table 5.2.b</b>	<b>Statistical Parameters for the Geometric Properties of the Reinforced Columns</b>	<b>111</b>
<b>Table 5.3</b>	<b>Statistical Quantities, <math>\rho_G</math> and <math>V_G</math>, for Geometric Variations</b>	<b>117</b>
<b>Table 5.4</b>	<b>Statistical Parameters for the Material Properties</b>	<b>118</b>

<b>Table 5.5 Simulated Professional Factors for Columns</b>	
<b>from Group 1 (<math>\lambda = 0.4</math>)</b>	119
<b>Table 5.6 Simulated Professional Factors for Columns</b>	
<b>from Group 1 (<math>\lambda = 1.1</math>)</b>	120
<b>Table 5.7 Simulated Professional Factors for Columns</b>	
<b>from Group 1 (<math>\lambda = 1.5</math>)</b>	123
<b>Table 5.8.a Best Fit Lines for the Professional Factors</b>	
<b>for Columns from Group 1</b>	125
<b>Table 5.8.b Best Fit Lines for the Professional Factors</b>	
<b>for Columns from Group 2</b>	125
<b>Table 5.9 Normalized Professional Factors for Columns</b>	
<b>from Group 1 (<math>\lambda = 0.4</math>)</b>	126
<b>Table 5.10 Normalized Professional Factors for Columns</b>	
<b>from Group 1 (<math>\lambda = 1.1</math>)</b>	127
<b>Table 5.11 Normalized Professional Factors for Columns</b>	
<b>from Group 1 (<math>\lambda = 1.5</math>)</b>	130
<b>Table 5.12.a Statistical Parameters for the Professional Factors</b>	
<b>for Columns from Group 1</b>	132
<b>Table 5.12.b Statistical Parameters for the Professional Factors</b>	
<b>for Columns from Group 2</b>	132
<b>Table 5.13 Professional Factors for Unreinforced Columns</b>	133
<b>Table 5.14.a Resistance Factors for Reinforced Columns from Group 1</b>	134
<b>Table 5.14.b Resistance Factors for Reinforced Columns from Group 2</b>	135
<b>Table A.1 Analysed Reinforced Column Description</b>	153
<b>Table B.1 Analysis Result Description</b>	175
<b>Table C.1 Simulated Professional Factors for Columns</b>	
<b>from Group 2 (<math>\lambda = 0.4</math>)</b>	197
<b>Table C.2 Simulated Professional Factors for Columns</b>	
<b>from Group 2 (<math>\lambda = 1.1</math>)</b>	198

<b>Table C.3 Simulated Professional Factors for Columns</b>	
<b>from Group 2 (<math>\lambda = 1.5</math>)</b> .....	<b>201</b>
<b>Table C.4 Normalized Professional Factors for Columns</b>	
<b>from Group 2 (<math>\lambda = 0.4</math>)</b> .....	<b>203</b>
<b>Table C.5 Normalized Professional Factors for Columns</b>	
<b>from Group 2 (<math>\lambda = 1.1</math>)</b> .....	<b>204</b>
<b>Table C.6 Normalized Professional Factors for Columns</b>	
<b>from Group 2 (<math>\lambda = 1.5</math>)</b> .....	<b>207</b>

## **List of Figures**

<b>Figure 2.1</b>	<b>Formation of the Analysis Models</b> .....	<b>17</b>
<b>Figure 2.2</b>	<b>Welding Sequences Investigated by Nagaraja Rao and Tall (1963)</b> ..	<b>17</b>
<b>Figure 3.1</b>	<b>Position of the Tie Connection between Column and Cover plates</b> ..	<b>32</b>
<b>Figure 3.2</b>	<b>Finite Element Mesh of Column Reinforced with the Plates Parallel to the Flanges</b> .....	<b>33</b>
<b>Figure 3.3</b>	<b>Finite Element Mesh of Column Reinforced with the Plates Parallel to the Web</b> .....	<b>34</b>
<b>Figure 3.4</b>	<b>Initial Residual Stress Pattern Used in the Numerical Models</b> .....	<b>35</b>
<b>Figure 3.5</b>	<b>Initial Residual Stress Distribution in I-Section and Cover Plates of Finite Element Models</b> .....	<b>36</b>
<b>Figure 3.6</b>	<b>Welding Residual Stresses in the Column Reinforced under Load</b> ..	<b>37</b>
<b>Figure 3.7</b>	<b>Axial Load versus Lateral Deflection Curves for the Column Reinforced under Load</b> .....	<b>38</b>
<b>Figure 3.8</b>	<b>Welding Residual Stresses in the Column Reinforced under no Load</b> .....	<b>39</b>
<b>Figure 3.9</b>	<b>Axial Load versus Lateral Deflection Curves for the Column Reinforced under no Load</b> .....	<b>40</b>
<b>Figure 4.1</b>	<b>Deformed Shape of Columns Reinforced with Plates Parallel to the Flanges</b> .....	<b>66</b>
<b>Figure 4.2</b>	<b>Deformed Shape of Columns Reinforced with Plates Parallel to the Web</b> .....	<b>67</b>
<b>Figure 4.3</b>	<b>Local Buckled Shape of the Column Reinforced with Plates Parallel to the Web</b> .....	<b>68</b>
<b>Figure 4.4</b>	<b>Load versus Lateral Deflection Curves for the Columns with Overall Buckling and Local Buckling</b> .....	<b>69</b>
<b>Figure 4.5</b>	<b>Strength of All Reinforced Column Samples</b> .....	<b>69</b>
<b>Figure 4.6</b>	<b>Initial Residual Stress Patterns before Welding</b> .....	<b>70</b>
<b>Figure 4.7</b>	<b>Residual Stress Distributions after Welding for Maximum Initial Residual Stress of <math>0.3F_y</math></b> .....	<b>73</b>

<b>Figure 4.8</b>	<b>Residual Stress Distributions after Welding for Maximum Initial Residual Stress of <math>0.1F_y</math></b>	<b>74</b>
<b>Figure 4.9</b>	<b>Effect of Initial Residual Stress Patterns for Maximum Magnitude of <math>0.3F_y</math></b>	<b>75</b>
<b>Figure 4.10</b>	<b>Effect of Initial Residual Stress Patterns for Maximum Magnitude of <math>0.1F_y</math></b>	<b>75</b>
<b>Figure 4.11</b>	<b>Residual Stress Patterns after Welding</b>	<b>76</b>
<b>Figure 4.12</b>	<b>Effect of Varying Initial Residual Stress Patterns before Welding in the Cover Plates</b>	<b>77</b>
<b>Figure 4.13</b>	<b>Effect of Varying Welding Residual Stresses with Different Slenderness Ratios</b>	<b>77</b>
<b>Figure 4.14</b>	<b>Effect of the Initial Out-of-straightness (<math>\lambda = 1.1</math>)</b>	<b>78</b>
<b>Figure 4.15</b>	<b>Effect of the Preload Magnitudes (<math>\lambda = 1.1</math>)</b>	<b>78</b>
<b>Figure 4.16</b>	<b>Effect of the Reinforcing Plate Orientation (<math>\lambda = 1.1</math>)</b>	<b>79</b>
<b>Figure 5.1</b>	<b>Frequency Distributions for Load Effect, S, and Resistance, R</b>	<b>136</b>
<b>Figure 5.2</b>	<b>Risk Frequency Distribution for <math>\ln(R/S)</math></b>	<b>136</b>
<b>Figure 5.3</b>	<b>Geometric Dimensions for Reinforced Columns</b>	<b>137</b>
<b>Figure 5.4</b>	<b>Simulated Professional Ratio vs. Value of Out-of-straightness for Columns from Group 1 (<math>\lambda = 0.4</math>)</b>	<b>138</b>
<b>Figure 5.5</b>	<b>Simulated Professional Ratio vs. Value of Out-of-straightness for Columns from Group 1 (<math>\lambda = 1.1</math>)</b>	<b>138</b>
<b>Figure 5.6</b>	<b>Simulated Professional Ratio vs. Value of Out-of-straightness for Columns from Group 1 (<math>\lambda = 1.5</math>)</b>	<b>139</b>
<b>Figure 5.7</b>	<b>Normalized Professional Ratio vs. Value of Out-of-straightness for Columns from Group 1 (<math>\lambda = 0.4</math>)</b>	<b>139</b>
<b>Figure 5.8</b>	<b>Normalized Professional Ratio vs. Value of Out-of-straightness for Columns from Group 1 (<math>\lambda = 1.1</math>)</b>	<b>140</b>
<b>Figure 5.9</b>	<b>Normalized Professional Ratio vs. Value of Out-of-straightness for Columns from Group 1 (<math>\lambda = 1.5</math>)</b>	<b>140</b>

<b>Figure C.1</b>	<b>Simulated Professional Ratio vs. Value of Out-of-straightness</b> for Columns from Group 2 ( $\lambda = 0.4$ )	209
<b>Figure C.2</b>	<b>Simulated Professional Ratio vs. Value of Out-of-straightness</b> for Columns from Group 2 ( $\lambda = 1.1$ )	209
<b>Figure C.3</b>	<b>Simulated Professional Ratio vs. Value of Out-of-straightness</b> for Columns from Group 2 ( $\lambda = 1.5$ )	210
<b>Figure C.4</b>	<b>Normalized Professional Ratio vs. Value of Out-of-straightness</b> for Columns from Group 2 ( $\lambda = 0.4$ )	210
<b>Figure C.5</b>	<b>Normalized Professional Ratio vs. Value of Out-of-straightness</b> for Columns from Group 2 ( $\lambda = 1.1$ )	211
<b>Figure C.6</b>	<b>Normalized Professional Ratio vs. Value of Out-of-straightness</b> for Columns from Group 2 ( $\lambda = 1.5$ )	211

## List of Symbols

Note: Symbols appearing in the text with and without a bar, e.g.,  $\bar{A}$  and  $A$ , denote the mean and nominal values, respectively.

<b>A</b>	Gross area of the cross section
<b>A<sub>C</sub></b>	Area of the rolled section
<b>A<sub>P</sub></b>	Area of the cover plates
<b>B</b>	Buckling axis of the reinforced column
<b>b<sub>f</sub></b>	Flange width of the rolled section
<b>b<sub>p</sub></b>	Plate width
<b>CF<sub>y</sub></b>	Yield strength of the rolled section columns
<b>C<sub>r</sub></b>	Factored compressive resistance of steel columns
<b>D</b>	Direction of reinforcing plates
<b>d</b>	Depth of the rolled section
<b>E</b>	Modulus of elasticity of steel
<b>E<sub>t</sub></b>	Tangent modulus of steel
<b>F</b>	The stress defined by an expression of $F_y$ and $\lambda$
<b>F<sub>e</sub></b>	Elastic buckling stress
<b>F<sub>y</sub></b>	Yield strength of steel
<b>f(<math>\lambda</math>)</b>	A function of the non-dimension slenderness ratio, $\lambda$
<b>G</b>	Geometric properties
<b>g</b>	Side of the fillet welds connecting the plates to the rolled section
<b>IRS</b>	Initial residual stresses in the rolled section and the cover plates before reinforcing
<b>I<sub>x</sub>, I<sub>y</sub></b>	Moment of inertia of cross section about principal x and y axes
<b>K</b>	Effective length factor
<b>k</b>	Side of the fillet connecting the flanges to the web in the rolled section
<b>L</b>	The column length
<b>MF</b>	Maximum magnitude of the residual stress at the flange tips
<b>MP</b>	Maximum magnitude of the residual stress at the reinforced plate edges
<b>n</b>	Coefficient in the equation for $C_r$ in the Clause 13.3.1 of CSA/CAN-S16.1-94
<b>P</b>	External axial load of the column

$P_0$	Preload before reinforcing
$P_i$	Functions of the non-dimension slenderness ratio, $\lambda$ ( $i = 1, 2, 3$ )
$P_{cr}$	Column strength
$P_{exp}$	Load carrying capacity of the reinforced column obtained from experiment
$P_{fea}$	Load carrying capacity of the reinforced column obtained from finite element analysis
$P_{F_y}$	Yield strength of the cover plates
$P_{r1}$	Load carrying capacity of the reinforced column predicted using SSRC curve 1
$P_{r2}$	Load carrying capacity of the reinforced column predicted using SSRC curve 2
$P_{rc1}$	Load carrying capacity of the reinforced column predicted using CSA curve 1
$P_{rc2}$	Load carrying capacity of the reinforced column predicted using CSA curve 2
$P_{r_y}$	Yield strength of the reinforced column
PS	Designation of the initial residual stress pattern before reinforcing
$P_{u2}$	Load carrying capacity of the unreinforced column predicted using the SSRC column curve 2
$P_{u_y}$	Yield strength of the unreinforced rolled section column
R	The resistance
r	Radius of gyration of the cross section
$r_x, r_y$	Radius of gyration of the cross section about principal x and y axes
S	The load effect
$t_f$	Thickness of the flanges in the rolled section
$t_p$	Thickness of the cover plates
$V_A$	Coefficient of variation of the area
$V_{CF_y}$	Coefficient of variation of the yield strength of the rolled section columns
$V_{Cr}$	Coefficient of variation of the factored compressive resistance of steel columns
$V_E$	Coefficient of variation of the modulus of elasticity
$V_F$	Coefficient of variation of the stress defined as a function of $F_y$ and $\lambda$
$V_{F_y}$	Coefficient of variation of the yield stress
$V_G$	Coefficient of variation of the relevant geometric property
$V_I$	Coefficient of variation of the moment of inertia
$V_M$	Coefficient of variation of the relevant material property



$V_P$	Coefficient of variation of the ratio of the strength of a column obtained from test to the strength predicted using the design equation
$V_{PF_y}$	Coefficient of variation of the yield strength of the cover plates
$V_R$	Coefficient of variation of the resistance
$V_r$	Coefficient of variation of the radius of gyration
$V_S$	Coefficient of variation of the load effect
$w$	Thickness of the web
$\alpha$	Separation factor
$\alpha'$	The load factor
$\alpha_R$	Separation factor for the resistance
$\alpha_S$	Separation factor for the load effect
$\alpha_x$	Coefficient of thermal expansion in the longitudinal direction of the steel column
$\beta$	Safety index
$\Delta\tau$	Temperature change in the steel column
$\delta_0$	Initial imperfection of the unreinforced column
$\delta_i$	Initial out-of-straightness of the reinforced column
$\lambda$	Non-dimensional slenderness parameter
$\rho_A$	Measured-to-nominal ratio of the area
$\rho_{CF_y}$	Measured-to-nominal ratio of the yield strength of rolled section columns
$\rho_{C_r}$	Measured-to-nominal ratio of the factored compressive resistance of steel columns
$\rho_E$	Measured-to-nominal ratio of the modulus of elasticity
$\rho_{ex}$	Measured-to-nominal experimental ratio
$\rho_F$	Measured-to-nominal ratio of the stress defined as a function of $F_y$ and $\lambda$
$\rho_{F_y}$	Measured-to-nominal ratio of the yield strength
$\rho_{f(\lambda)}$	Measured-to-nominal ratio of $f(\lambda)$
$\rho_G$	Measure-to-nominal ratio of the relevant geometric property
$\rho_I$	Measured-to-nominal ratio of the moment of inertia
$\rho_M$	Measure-to-nominal ratio of the relevant material property
$\rho_n$	Normalized professional ratio

$\rho_p$	Ratio of the strength of a column obtained from test to the strength predicted by the design equation
$\rho_{PFy}$	Measured-to-nominal ratio of the yield strength of cover plates
$\rho_R$	Measured-to-nominal ratio of the resistance
$\rho_r$	Measured-to-nominal ratio of the radius of gyration
$\rho_S$	Measured-to-nominal ratio of the load effect
$\rho_s$	Simulated professional ratio
$\rho_{seq}$	Simulated professional ratio predicted by the best-fit equation
$\rho_\lambda$	Measured-to-nominal ratio of the non-dimensional slenderness ratio
$\sigma_{b_f}$	Standard deviation of the width of the flange of the rolled section
$\sigma_{b_p}$	Standard deviation of the width of the cover plate
$\sigma_d$	Standard deviation of the depth of the rolled section
$\sigma_E$	Standard deviation of the modulus of elasticity
$\sigma_F$	Standard deviation of the stress defined by an expression of $F_y$ and $\lambda$
$\sigma_{Fy}$	Standard deviation of the yield strength
$\sigma_g$	Standard deviation of side of fillet welds connecting the plates to the rolled section
$\sigma_k$	Standard deviation of the side of the fillet connecting the flanges to the web of the rolled section
$\sigma_R$	Standard deviation of the resistance
$\sigma_r$	Standard deviation of the radius of gyration
$\sigma_S$	Standard deviation of the load effect
$\sigma_{t_f}$	Standard deviation of the thickness of the flange of the rolled section
$\sigma_{t_p}$	Standard deviation of the thickness of the cover plate
$\sigma_w$	Standard deviation of the thickness of the web of the rolled section
$\sigma_{wr}$	Residual stress resulting from welding
$\sigma_x$	Longitudinal thermal stress in the steel column
$\phi$	Resistance factor

# **Chapter 1**

## **Introduction**

### **1.1 General Background**

It might be necessary to strengthen steel columns many years after construction. Most columns in existing structures carry some load at the time of reinforcing. Common reinforcement of a steel column is welding or bolting cover plates on the column. In bridges, for example, the cover plates would preferably be bolted since welding would introduce potential fatigue problems, while columns in other structures have been reinforced by welding cover plates.

There are no specific design criteria for reinforced columns in Canada up to now. In practice, many engineers would use the same design criteria for reinforced columns as those for rolled W section columns.

Since the limit states design method was employed in Canada in 1974 (Canadian Standards Association, 1974), the design criteria for steel columns have been used based on the SSRC multiple curves. CSA standard CAN3-S16.1-M84 – “Steel Structures for Building – Limit States Design” (Canadian Standards Association, 1984) adopted the SSRC curve 1 and the SSRC curve 2 for the design of steel columns. Each curve presents the behaviour and strength of different kinds of steel columns. The lower curve of clause 13.3.1 (based on the SSRC curve 2) is used for hot-rolled W section columns. Kennedy and Gad Aly (1980) suggested that the higher curve of clause 13.3.2 (based on the SSRC curve 1) was appropriate for class H hollow structural sections. Based on the study of Chernenko and Kennedy (1989), clause 13.3.2 is also used for Canadian WWF columns.

In the current Canadian standard the five-part equation developed by the SSRC was replaced by a double exponential representation with a single parameter, which was proposed by Loov (1996). The column curve described by the expression corresponding to the SSRC curve 1 is CSA curve 1, and the column curve described by the expression corresponding to the SSRC curve 2 is CSA curve 2. The CSA curves were demonstrated to accurately approximate the corresponding SSRC curves.

## **1.2 Statement of the Problem**

In the columns reinforced with welding cover plates, welding can introduce tensile residual stresses at the flange tips of the rolled section and the edges of the reinforcing plates. Since yielding begins at the tips and progresses inwardly, these tensile residual stresses may be beneficial to the column strength by delaying the deterioration in minor axis stiffness.

Out-of-straightness, more specifically called camber or sweep depending about which axis the out-of-straightness occurs, is generally understood to be an important influencing factor for any column. The current S16.1 column curve (CSA curve 2) for rolled W sections is based on a maximum allowable out-of-straightness of  $L/1000$  for both axes (Bjorhovde, 1972). It is more acceptable for S16.1 column curve to be based on statistical quantities, that is, on the mean values and associated coefficients of variations.

Furthermore, more influencing factors exist in reinforced columns, such as orientation of reinforcing plates, welding residual stress, geometric and material properties of the rolled section and plates, comparing to rolled W sections. The addition of reinforcing plates may affect the behaviour and strength of reinforced columns a lot. These parameters may have individual and combined effects on the prediction of reinforced column strength.

The differences between reinforced columns with welded cover plates and rolled W sections affect column strengths over the full range of column lengths, and suggest that reinforced columns with welded cover plates may be unnecessarily penalized with the CSA curve 2 (or SSRC curve 2) along with rolled W sections. Different column curves should be used for the two types of sections.

## **1.3 Objectives and Scope**

In order to understand the uncertainty problems in reinforced columns with welded cover plates, the research was designed with the following objectives:

1. To review the existing literature on reinforced columns.
2. To develop a finite element model for reinforced columns under load.
3. To select the parameters influencing the behaviour and the strength of reinforced columns and to study the effects of parameters on reinforced columns.
4. To investigate statistically the resistance of reinforced columns produced in Canada by evaluating resistance factors appropriate for use with existing column curves for reinforced columns with welded cover plates under load.
5. To assess existing design criterion for reinforced columns with welded cover plates under load.

In the research, the analyzed rolled W section columns were only reinforced with welding cover plates because there is a potential for the welding residual stresses to improve the strength of welded reinforced columns. The beneficial effect of the welding residual stresses is not present when the reinforcing plates are bolted on the column.

A finite element program, ABAQUS (Hibbitt *et al*, 1997), was used to assess the effects of variations in parameters on the behaviour and the strength of reinforced columns. Out-of-straightness was restricted to a superposition of four buckling modes of the column. The study was limited to centrally loaded, pin-ended columns, buckling about the major or minor centroidal axis, and laterally supported about the other axis when required. Local buckling, buckling about both axes simultaneously, and lateral torsional buckling were not considered. Resistance factors were evaluated for values of the slenderness parameter,  $\lambda$ , of 0.4, 1.1 and 1.5 in two categories, in respect of the design criteria of CAN3-S16.1-M84 and CAN/CSA-S16.1-94, respectively. The first category includes columns reinforced with plates parallel to the flanges with buckling about the strong axis of the W shape and columns reinforced with plates parallel to the web with buckling about the weak axis of the W shape. The second category includes columns reinforced with plates parallel to the flanges with buckling about the weak axis of the W shape, and columns reinforced with plates parallel to the web with buckling about the strong axis of the W shape.

## **1.4 Organization of the Thesis**

A literature review is presented in Chapter 2. This outlines the research done for parameters influencing the behaviour and strength of reinforced columns. A design method for reinforced columns is also discussed. Chapter 3 presents a description of a finite element model setup and analytical procedure for reinforced columns with welding cover plates. Parametric studies to assess the effect of the parameters on the reinforced columns are presented in Chapter 4. Based on a review of the principle of limit states philosophy associated with the column design process, the statistical analysis to give the resistance factor for the design of a reinforced column and to verify which design curve given in the code is appropriate to the reinforced column design is treated in Chapter 5. Finally, a summary, conclusion and recommendations for further research are presented in Chapter 6.

Initial geometrical, material and load conditions of all the finite element analytical models are tabulated in Appendix A. Appendix B presents the results of the analytical models. The statistical analysis data for the columns in category 2 (columns reinforced with plates parallel to the flanges and buckling about the weak axis of the rolled section and columns reinforced with plates parallel to the web and buckling about the strong axis of the rolled section) are presented in Appendix C.

## **Chapter 2**

### **Literature Review**

#### **2.1 Factors Influencing Column Strength**

##### **2.1.1 Introduction**

A seemingly simple structural column in fact functions as a complex individual structural member because of the effects of various parameters such as the interaction between the responses and characteristics of the material, the cross-section, the method of fabrication, the imperfections and other geometric factors, and the end conditions. Geschwindner *et al.* (1994) suggested that the following parameters affect column strength:

1. **Material properties**
  - (a) **Stress-strain relationship**
  - (b) **Yield strength**
2. **Shape of cross-section**
  - (a) **Area of steel**
  - (b) **Shape of the cross-section (W, C, WT, etc.)**
  - (c) **Buckling axis**
3. **Length**
4. **End support conditions**
  - (a) **Without sway, pinned or otherwise**
  - (b) **With sway, pinned or otherwise**
5. **Residual stress magnitude and distribution**
6. **Initial imperfections**
  - (a) **Magnitude**
  - (b) **Distribution along column length**

It is generally accepted that the yield strength and the modulus of elasticity are the most important material properties. For very short columns, the load carrying capacity may reflect a strength increase due to strain-hardening, but for hot-rolled structural

shapes, other factors such as local buckling may limit this strength increase. Therefore, the yield strength represents the practical limit of capacity of a very short column. For long columns, the capacity is influenced more by stiffness, which is a function of the magnitude of the tangent modulus and the cross-section moment of inertia (Galambos, 1998; Geschwindner *et al.*, 1994).

The shape of a cross-section is obviously important. For a given stress level, the load-carrying capacity will be larger for a column of larger area. The distribution of the area in the cross-section is expressed as the moment of inertia, which affects the capacity of columns that fail by buckling. The buckling axis is another factor influencing the behaviour of columns. The buckling capacity of columns about different axes is governed by different moments of inertia and corresponding slenderness ratios, which are defined as ratio of the effective column length to the radius of gyration. The geometry of the cross-section also influences the residual stress distribution.

The effective length concept has been introduced to account for the effect of column length and boundary conditions on the capacity of columns (Galambos, 1968). Euler first developed an analytical model based on the assumption that both ends of the column were completely free to rotate as the column reached its buckling strength. This situation will sometimes arise. For other end restraint conditions, the actual length of the column,  $L$ , is replaced by its effective length,  $KL$ , that is, the length of a pin-ended column of the same capacity as the column with other end restraint conditions. This effective length corresponds to the distance between points of inflection (points of zero bending moment) on the buckled shape. As a parameter of the effective length, the end support condition also has been generally understood as a column strength parameter.

The influence of column effective length is explicit in design calculations, whereas the influence of other factors such as initial imperfections and residual stresses may be hidden in the design approach, although they may also be important. In design practice, a non-dimensional slenderness parameter,  $\lambda$ , has been found to be the most important factor (Chen and Lui, 1987). This parameter is taken as the square root of the ratio of the yield stress to the elastic buckling stress, which is expressed as:



$$\lambda = \sqrt{\frac{F_y}{F_e}} = \frac{KL}{r} \sqrt{\frac{F_y}{\pi^2 E}} \quad [2.1]$$

where  $F_e$  is the elastic buckling stress,  $E$  is the modulus of elasticity,  $r$  is the radius of gyration of the cross-section, and  $F_y$  is the yield strength of the column. With respect to their slenderness, columns are generally referred to as short columns, intermediate columns or slender columns. Slender columns buckle when the cross-section is still elastic. Intermediate columns buckle when part of the cross-section has yielded under the combined action of the applied axial load and the residual stresses. Short columns usually fail by local buckling after yielding of the full cross-section.

Residual stresses, formed during the cooling process after hot rolling, are influenced by the distribution of material in the cross-section. It is commonly accepted that the flange tips of a rolled I-shape section are subjected to residual compressive stresses because these areas, which possess less material and more exposed surface area, cool down faster than the flange to web junctions, which possess a larger volume of material to surface area ratio. This differential cooling gives rise to compressive residual stresses at the flange tips and tensile residual stresses at the flange to web junctions. Residual stresses have a major impact on the load-carrying capacity of a steel column. The investigation conducted by Huber (1956) demonstrated that the strength of a column could be reduced by the presence of residual stresses. Residual stresses also cause non-linearity of the stress versus strain relationship as soon as any part of the cross-section starts to yield due to the combination of the applied stresses and the residual stresses. The tangent and reduced modulus theories developed by Engesser (Galambos, 1968) are commonly applied for these inelastic columns. The critical loads computed by these theories correspond closely to experimental results.

Initial out-of-straightness has long been recognized as a significant column strength parameter. It has been found that the effect of initial out-of-straightness is different for columns of different lengths (Galambos, 1998). Short and slender columns are not affected much by initial out-of-straightness, but intermediate columns are significantly affected (Galambos, 1998). The magnitude of initial out-of-straightness in any member is limited by manufacturing tolerance limits set by CSA standard G40.20-92 (Canadian

Standards Association, 1992), which reflects standard mill practice. Thus, the maximum value of out-of-straightness for rolled wide-flange shapes is set at  $L/1000$ , where  $L$  is the member length, with some minor modifications for longer sections and certain geometries (Canadian Standards Association, 1992).

The studies show that the influence of above parameters is more severe for intermediate length columns than for short or slender columns (Kulak and Gilmor, 1998; Galambos, 1998; Kennedy *et al.*, 1976). The effect on the reinforced column also has to be studied to verify their importance.

Tall (1989) suggested that other factors influence the strength of reinforced steel columns as follows:

1. The orientation of the cover plates welded to the column;
2. Different grades of steel for cover plates and rolled section.

Reinforcement is usually understood to be the welding or bolting of cover plates to the flanges or the web of the cross section (Tall, 1989). The orientation of cover plates welded to the flanges can be either parallel to the flanges or to the web, as shown in Figure 2.1. Different reinforcing plate orientations for the same size of cross-section will introduce different moments of inertia and slenderness ratios for the reinforced column, both of which can affect the strength of the reinforced column significantly.

### **2.1.2 Initial Out-of-straightness**

Although not perfectly straight, structural shapes are expected to satisfy the straightness requirements of the applicable materials delivery standard, CSA standard G40.20-92. Before the limit states design philosophy was developed, the effects of any initial imperfections were covered through the factor of safety (Geschwindner *et al.*, 1994). The strength of the straight member was used as the actual criterion. With the limit states design philosophy, all of the major parameters have to be accounted for. Therefore, the influence of initial out-of-straightness must be reflected in the strength equations.

Initial imperfections are the result of the cooling process for the shape once the column has been rolled to its final dimensions. When the column is left on the cooling bed to cool in air for a period of time, other members are also placed on the bed, and the heat dissipation from the member is not uniform neither throughout the cross section nor along the length or around its sides. As a result, heat is usually retained longer in the midlength portion of the column and in the parts of the cross section exposed most directly to the heat of the adjacent members on the cooling bed.

The resulting non-uniform cooling leads not only to residual stresses but also a steel member in a curved configuration along its length. The amount of curvature or initial out-of-straightness is difficult to predict because of a number of uncontrollable factors. The maximum out-of-straightness is limited typically based on the length of the member by CAN/CSA-G40.20-92. The maximum values are set as  $\delta_i = L/1000$  for column length less than 14 m and  $\delta_i = 10 + (L - 14000)/1000$  for column lengths larger than 14 m. The value is different for different cross sections. A column that does not meet this straightness requirement is rotorized or gag-straightened to bring it into compliance with the code. Bjorhovde (1972) found that the mean value of initial out-of-straightness of rolled W sections, is  $L/1500$ , which is less than the code limit.

It was found that the shape of the initial imperfections, i.e., their variation along the length of the column, differs from the commonly assumed half sine wave (Bjorhovde, 1972). However, if it is assumed that the initial shape of the axis of the pinned-ended column is sinusoidal, the resulting average stress at which the maximum stress in the column equals the yield stress can be obtained by the Perry-Robertson equation (Johnston, 1976). This average stress gives a conservative estimate of the strength of an initially curved column.

Geschwindner *et al.* (1994) suggested that the combined effect of residual stresses and initial out-of-straightness could not be obtained merely by combining the two terms. In some cases, and for certain slenderness ratio ranges, the strength of a column with residual stresses and initial out-of-straightness is less than what would be found if the

effects of both were added. In other cases, it is not as critical as the sum would seem to indicate.

### 2.1.3 Residual Stresses

The manufacturing method is one of the primary factors influencing the distribution and magnitude of initial residual stresses in the cross-section. The research in this paper is limited to hot-rolled sections with welded reinforcement. Therefore, the review of the literature was limited to factors related to these specific manufacturing processes.

Under the combined action of residual stresses and applied axial loads, yielding of a column will start when the sum of the applied stress and the maximum compressive residual stress reaches the yield strength of the material. Beyond this point, the column becomes inelastic. Considering the inelastic behaviour of columns, Engesser presented the tangent modulus theory (Galambos, 1998). According to this theory, once the column has become inelastic, its behaviour is dictated by the tangent modulus. The tangent modulus column strength equation can be expressed as:

$$P_{cr} = \frac{\pi^2 E_t}{(KL/r)^2} A \quad [2.2]$$

where  $P_{cr}$  is the column strength,  $E_t$  is the tangent modulus, and  $A$  is the gross area of the cross section.

Huber (1956) measured residual stresses in a series of hot-rolled W-shapes ranging from a light section: 4WF13 (W100x19) to a heavy section: 36WF150 (W920x223). The results showed that there are significantly different patterns for different sections. In rolled sections the residual stresses at the flange tips and the middle of the web are compressive stresses, while the residual stresses in the flange to web junctions are tensile. It was also observed that in most hot-rolled W sections the maximum compressive residual stress is approximately 30% of the yield strength (Chen and Atsuta, 1976).

Residual stress patterns and magnitude also vary widely in steel plates. According to CSA standard G40.20-92, plates are classified as *universal mill* (UM) plates, *sheared* plates, or *flame cut* plates. The initial residual stresses are significantly different in the

three types of plates. The edges of a UM plate or a sheared plate are in compression and the edges of a flame cut plate are in tension (Geschwindner *et al.*, 1994). Tall (1961) and Nagaraja Rao and Tall (1963) investigated residual stress patterns for UM plates ranging from 150 x 6 mm to 300 x 18 mm. All the plates investigated had almost the same initial residual stress pattern with different maximum magnitudes at the edges of the plates. The measured maximum compressive residual stresses were about 30% of the yield strength.

The welding of built-up shapes is an even greater contributor to residual stresses than the differential cooling of hot-rolled shapes (Nagaraja Rao *et al.*, 1964). Nonuniform cooling and restrained shrinkage of welds cause high residual stresses. Masubuchi (1980) observed that the maximum magnitude of tensile residual stress at the weld center is as high as the yield strength of the weld metal. Tall (1961) presented residual stress patterns measured in plates after welding of the plates to wide flange sections. Plate sizes ranged from 150x6 mm to 300x18 mm. The zone of tensile residual stresses resulting from welding along the plate edges was observed to extend from 18 mm to 37.5 mm. The thicker plates have wider zones of tensile residual stresses. The welding residual stresses were observed to reach the yield point value only at and in the vicinity of the weld.

## **2.2 Research on Reinforced Columns**

Very little research has been conducted on reinforced steel columns, although many structures have been strengthened. The processes used to reinforce columns were simply presumed to be safe. Nagaraja Rao and Tall (1962) reported on the work of Wilson and Brown who conducted tests on the strengthening of columns of a viaduct in 1935. Cover plates were welded to the existing sections. It was observed that in some cases the residual stresses in reinforced columns after welding might reach the yield point.

Sparagen and Grapnel (1946) presented an early review of all the literature on structures reinforced under load. They concluded that residual stresses, although high, did not seriously affect the ultimate strength of a column.

### **2.2.1 Work of Tall and Co-workers**

Nagaraja Rao and Tall (1963) conducted an experimental investigation of the effect of welding cover plates to a wide flange column under load. The program consisted of residual stress distribution determination, tension coupon testing, stub column tests, and column tests.

Three pin-ended columns were tested: an unreinforced column, a column reinforced under load, and a column reinforced under no load. Ancillary tests were conducted to investigate the residual stress distribution and the yield strength of the columns.

In the investigation, a 8WF31 (W200x46) shape was selected as the core column and 180x9.5 mm plates were used for the cover plates. The rolled shape and the cover plates were of ASTM A7 steel. Because the selected rolled shape has one of the lowest shape factors (the area, the moment of inertia and so on) and b/t ratio in available rolled sections, the results of tests would be conservative for other sections. The preload at the time of welding was fixed to 405 kN (the stress on the section was 69 MPa).

For convenience in testing and in comparing results, all the three pin-ended columns were tested with boundary conditions that allowed buckling about the weak axis. The column specimens were 2440 mm long, resulting in a slenderness ratio of about 48. The reinforced columns had an out-of-straightness of 0.51 mm ( $L/4880$ ) for the column reinforced under load and 0.77 mm ( $L/3190$ ) for the column reinforced under no load, whereas the unreinforced column had an out-of-straightness of 4.335 mm ( $L/560$ ).

Residual stresses were measured both in the unreinforced and in the reinforced column using the method of sectioning. It was observed that the compressive residual stresses at the flange tips of the unreinforced section were changed to high tensile residual stresses as a result of welding of the reinforcing plates.

The effect of welding sequence on residual stresses in a reinforced column was also investigated. In the experiments, two welding sequences were selected. The welding methods conformed to ASCE-AWS standards then. The two welding sequences were:

- 1) Welding each flange one after another, stage by stage, as shown in Figure 2.2 (a).
- 2) Welding two diagonally opposite flanges simultaneously, as shown in Figure 2.2 (b).

The residual stress distributions with different welding sequences were also measured by the method of sectioning. Welding residual stresses in the flanges and plates were almost the same under different welding sequences, whereas welding residual stresses in the web were found to be different. The maximum magnitudes of the residual stresses in the flanges and plates were close to each other under different welding sequences. Masubuchi (1980) suggested that as far as residual stresses along the weld are concerned, the effect of welding sequence is minor.

The test results on the reinforced columns indicated that the stress level in the reinforced columns at buckling is greater than the buckling stress for the unreinforced section. A theoretical tangent modulus column curve for the reinforced section was developed from the stub-column test results (Nagaraja Rao and Tall, 1963). The resulting column curve for a reinforced W200x46 fell above the Column Research Council (CRC) curve, indicating a higher strength from the reinforced section. The CRC curve represents an average curve for bending about either the strong or the weak axis (Huber, 1958).

The following observations were made from the test results:

1. The pin-ended column showed that the reinforced sections had a higher capacity. One of the pin-ended columns tested reached 98% of its yield strength. The stub column tests and the pin-ended column tests showed that welding for shorter reinforced columns did not reduce the buckling stresses.
2. The influence of welding is confined to a very small area in the vicinity of the weld. The properties of the material in the major portion of the section are not affected enough to change the strength of the reinforced section.

A later paper presented by Tall (1989) extended the discussion of the welded reinforcement based on the previous investigation. Further discussion of the residual stress magnitude and distribution was presented.

The initial compressive residual stresses in W-shapes would contribute to a reduction of compressive strength. To minimize the loss of compressive strength, it is desirable to change the compressive residual stresses to tensile stresses at the critical portion of the rolled section, i.e. the tips of the flanges, to delay yielding of this portion under a compressive load. Welding of cover plates to the flanges or simply laying a weld bead on the flange tips are two ways to achieve this. The change of residual stress distribution resulting from welding alone was demonstrated to result in a marked improvement in column strength (Fujita, 1960). Tall (1989) proposed that the increase in strength resulting from welded cover plates on a rolled section is substantially greater than for welding alone because of the combined effect of the additional material and welding residual stresses.

The test results presented by Nagaraja Rao and Tall (1963) showed a 10% increase in strength after reinforcement. Because the non-dimensionalized strength is defined with respect to the yield strength of the total cross section, which differs before and after reinforcement, the actual absolute increase in strength would be considerably higher. Therefore, the reinforcement of a column may result in the column being assigned a higher column curve if the concept of multiple column curves is considered.

It has been shown (Alpsten and Tall, 1970; Brozzetti *et al.*, 1970; Bjorhovde *et al.*, 1972; Kishima *et al.*, 1969) that welding has a greater influence on the overall distribution of residual stresses in small and medium-size shapes, than in the case of heavy shapes. Tall (1989) therefore proposed that welding alone on the flange tips would improve the strength of rolled sections of light and medium size more than for heavy rolled shapes.

Tall (1989) suggested that the width of the cover plate should not be smaller than the width of the flange less the size of two fillet welds -- this ensures that the weld is as close as possible to the flange tips so as to be effective in changing the residual stresses at the flange tips from compression to tension. The maximum effect of reinforcement is obtained when the reinforcing weld is as close as possible to the edge of the flange of the rolled section.



Comparing test results for columns reinforced under load and reinforced under no load, Tall (1989) suggested that preload would not affect the strength of short reinforced columns.

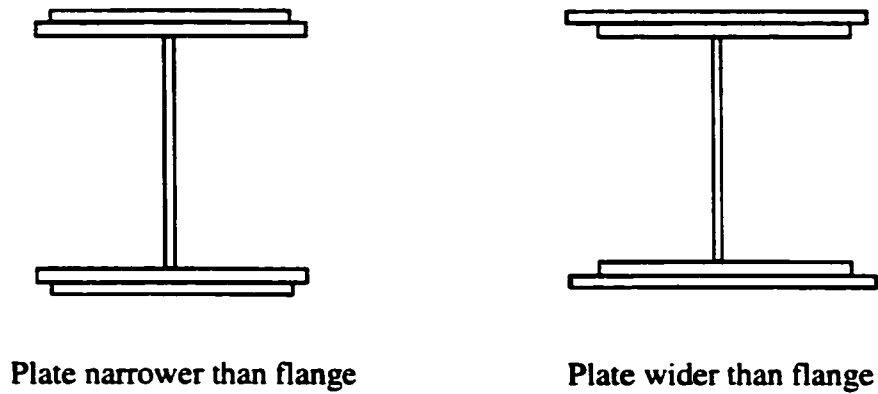
### **2.2.2 Work of Brown (1988)**

Brown (1988) proposed a simplified model of a reinforced steel column to evaluate its strength when reinforced under load. The model consists of two flexible columns, one representing the unreinforced column (or column core) and the other representing the reinforcing plates, tied together with rigid links to enforce compatibility of displacements between the column core and the reinforcing plates. Depending on the slenderness of the column, three ranges of column response were identified: 1) the core column forms a plastic hinge and the reinforcement provides the additional capacity until it reaches the maximum load capacity (i.e. the capacity of the column reinforced under load is the same as a column reinforced under no load); 2) after the core column fails, the reinforcing plates provide additional capacity, but the reinforced column does not reach the full capacity of a column reinforced under no load; 3) failure of the reinforced column takes place when the stress in both the column core and the reinforcing plates is below the buckling stress of the same column reinforced under no load. The model proposed by Brown does not account for any of the residual stresses and depends on SSRC column design curves to account for the presence of the residual stresses.

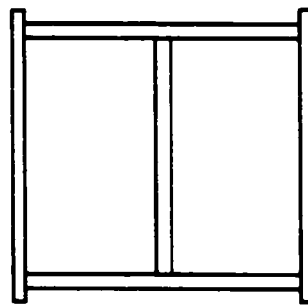
### **2.3 Summary**

Residual stresses, initial geometric imperfections, and material and geometric properties were identified as the most influential factors for the capacity of unreinforced columns. A review of the literature has indicated that additional factors affect the strength and behaviour of welded reinforced columns. These factors, however, have received little attention. The direction of cover plates, the grade of steel used for the columns and the cover plates, the magnitude of the load carried by the unreinforced column when the reinforcing plates are welded to the column, and the buckling axes were identified as

**potentially important parameters for reinforced columns. The effect of these parameters on the strength and behaviour of reinforced steel columns needs to be investigated.**

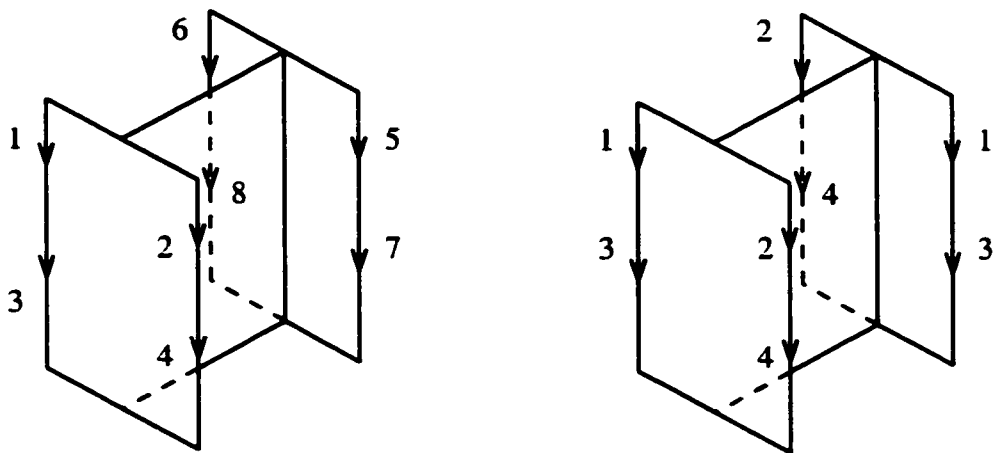


(a) Reinforcing Plates Parallel to Flange



(b) Reinforcing Plates Parallel to Web

**Figure 2.1 Formation of the Analysis Models**



(a) Sequence No. 1

(b) Sequence No. 2

**Figure 2.2 Welding Sequences Investigated by Nagaraja Rao and Tall (1963)**

## **Chapter 3**

### **Finite Element Modelling of Reinforced Steel Column**

#### **3.1 General**

A review of the literature indicated that the number of tests conducted on steel columns reinforced under load with welded steel plates is very limited. To fully understand the behaviour of reinforced steel columns, a large number of tests are required in order to incorporate a wide range of parameters that affect the behaviour of reinforced steel columns. However, it is uneconomical to conduct a large experimentally based investigation. In order to extend the database of test results, a numerical model was used to investigate the full range of parameters not covered by the test. The performance of this model was first verified by comparing the predicted strength and behaviour with test results.

The objective of this chapter is to develop and validate a finite element model of a reinforced steel column. The finite element model was developed using the commercial software ABAQUS, version 5.7 (Hibbitt *et al.*, 1997). ABAQUS was used because of its ability to perform non-linear large displacement and finite strain post-buckling analysis.

This chapter consists of two parts. In the first part, the geometry and the boundary conditions of the finite element model are described. The initial conditions and load process of the numerical analysis are also discussed. In the second part, the results of the numerical analysis are compared with the results of physical tests to validate the numerical models.

#### **3.2 Description of the Model**

To investigate the full range of possible cross sections for reinforced columns, two groups of finite element models were studied in the following investigation:

- 1) Columns reinforced with plates parallel to the flanges as shown in Figure 2.1(a) and,

- 2) Columns reinforced with cover plates parallel to the web as shown in Figure 2.1(b).

The finite element model of a reinforced column is composed of three parts: the rolled section, the reinforcing plates, and the welds connecting the reinforcing plates to the rolled section. The rolled section and reinforcing plates were discretized using element S4R from the ABAQUS finite element library. Element S4R is a four-node, doubly curved, general purpose shell element with finite strain capability and six degrees of freedom per node (Hibbitt *et al.*, 1997). The welded joint between the reinforcing plates and the rolled section was modelled using the two-node linear beam element B31. Element B31 has six degrees of freedom per node and transverse shear deformation capability.

When the beam elements were introduced into the model to simulate the welds, the minimum stiffness required to prevent relative displacement between two corresponding nodes on the rolled section and the reinforcing plate was determined by gradually increasing the beam stiffness until the relative displacements were considered negligible. This process was necessary to avoid potential convergence problems with excessively stiff beam elements.

The beam elements were added when the finite element model was built. In order to prevent interaction between the rolled shape and the reinforcing plates when residual stresses and column preload were added, the beam elements had to be deactivated in the first load step. To prevent rigid body motions of the reinforcing plates before their re-attachment to the rolled section, the plates were connected to the rolled section using a tie connection at the column mid-height (see Figure 3.1). The single tie connection prevented rigid body motion of the reinforcing plates while allowing independent straining of the plates and rolled section during the application of the residual stresses.

### **3.2.1 Finite Element Mesh**

The mesh size used for modelling the reinforced columns was based on two principal considerations: 1) a sufficient number of elements had to be used to form the cross-

section so that the residual stress pattern could be modelled accurately, and 2) the aspect ratio of the elements was below 3.0 to avoid potential numerical problems (Gaylord *et al.*, 1997).

Figures 3.2 and 3.3 show the finite element mesh of the column reinforced with plates parallel to the flanges and that of the column reinforced with plates parallel to the web, respectively. To allow for welding of the cover plates, the cover plates were narrower and shorter than the W-shape column by about 20 mm.

The width of the elements adjacent to the tips of the flanges and webs were determined to accurately simulate the welding residual stresses in the cross-section. Welding residual stress distributions in steel plates were investigated both experimentally and analytically by Tall (1961). The thickness of the plates ranged from 1/4 inch (6 mm) to 3/4 inch (19 mm). The distance from the welded edge of the plate to the point where a residual stress reversal occurs was found to vary from 18 mm to 38 mm. Nagaraja Rao and Tall (1963) also investigated the welding residual stress distribution in a 8WF31 (W200x46) section with 180x9.5 mm plates welded parallel to the flanges. The residual stresses changed from positive to negative about 25 mm from the welded edge in the flanges and the cover plates. Therefore, the width of the elements adjacent to the edges in the flanges and the cover plates was chosen as 25 mm. This was found to provide welding residual stress patterns similar to the residual stress patterns obtained experimentally. This will be discussed further in the following.

### **3.2.2 Material Properties**

An isotropic, elastic-perfectly plastic material model was used for all the elements of the reinforced columns. The elastic range for all the elements was defined with an elastic modulus of 200 000 MPa and Poisson's ratio of 0.3. The yield stress level was varied for the rolled section and the reinforcing plates as described in the following chapter. The material properties for the beam elements, used to model the welds, were the same as for the cover plates.

The residual stresses in the plates and rolled section were introduced by imposing a temperature gradient in the cross-section. In order to introduce longitudinal residual stresses only, orthotropic thermal expansion properties were used. The value of the coefficient of thermal expansion was taken as  $1.17 \times 10^{-6}/^{\circ}\text{C}$  in the longitudinal direction and zero in the transverse and through-thickness directions. The stress caused by constrained thermal expansion is expressed as

$$\sigma_x = -E \alpha_x \Delta\tau \quad [3.1]$$

where  $\sigma_x$  is the longitudinal stress,  $E$  is the modulus of elasticity,  $\alpha_x$  is the coefficient of thermal expansion in the longitudinal direction and  $\Delta\tau$  is the temperature change.

### 3.2.3 Boundary Conditions

The parametric study presented in the following chapter was limited to centrally loaded, pin-ended columns. The pinned ends were modelled using constraint equations between the centroid of the end cross-sections and each node on the cross-sections to force the ends of the column to remain plane and create a hinge about the centroid at each end of the column.

A restraint about the weak axis or the strong axis was added at the column end cross-section to make the corresponding slenderness ratio less than the slenderness ratio about the other axis. These end restraints were added either to promote buckling about the strong axis in some of the specimens included in the parametric study, or to prevent simultaneous buckling about two axes in very few of the specimens investigated with nearly equal stiffness about the strong and the weak axes.

In some of the reinforced column models with reinforcing plates parallel to the web and  $\lambda = 0.4$ , unexpected local buckling occurred at the ends of the column. This failure mode will be discussed further in section 4.2. However, because an investigation of local buckling of reinforced steel columns is beyond the scope of this research project, the thickness of the first three rows of elements near the ends was increased by a factor of three to prevent local buckling.

### **3.2.4 Initial Conditions**

The response of the column in the post-buckling range is of interest to assess the stability, or lack thereof, of the column after the peak load has been reached. To analyse the post-buckling behaviour, the bifurcation problem that exists when no initial imperfections are present in the model must be transformed into a problem with continuous response. This can be accomplished by introducing a geometric imperfection. Since the exact shape of the initial imperfections was not known a priori, initial imperfections, consisting of a superposition of multiple buckling modes, were introduced in the unreinforced column model. In ABAQUS this is accomplished in two analysis runs: 1) in the first run an eigenvalue buckling analysis is performed on the "perfect" geometry to determine the possible buckling modes; 2) in the second analysis run the initial imperfections are introduced by adding the buckling modes to the "perfect" geometry. The first four buckling modes were selected in the second analysis run. Research demonstrated that the first eigen mode provides the most critical imperfections (Galambos, 1968; Chen and Atsuta, 1976). In this research project the first four modes were used with different scaling factors to form the initial imperfections. The largest scaling factor was used for the first mode and the scaling factors of the 2nd mode, 3rd mode, and 4th mode were taken as 1/5, 1/10 and 1/20 of the first mode's scaling factor, respectively. Since the position of the maximum perturbation differs for all four modes, a trial and error method was adopted in the analysis to make the maximum magnitude of the superimposed eigenmodes equal to the desired magnitude of initial imperfection.

The second, third, and fourth buckling modes are not necessarily in the same plane as the first, and predominant, mode. Therefore, the superposition of the first four buckling modes introduces initial imperfections in the weak and in the strong axis directions. The magnitude of the initial imperfections in the strong direction was approximately 20%, or less, of the magnitude in the weak direction.

Initial temperatures at all the nodes in the model were defined as zero when the initial condition was defined. Based on this initial condition, temperature changes could be



introduced to simulate the initial residual strains, and stresses, and welding residual stresses in the load step described in the following section.

### **3.2.5 Loading Process**

Because reinforcement of steel columns is usually performed while the column is carrying some load, loading of the reinforced steel column models had to be performed in several steps. The following steps were adopted in the analysis:

- 1) The finite element model initially consisted of the rolled section and the reinforcing plates attached to the rolled section with beam elements. It was therefore necessary in the first load step to de-activate the beam elements and perform an equilibrium iteration on the structure.
- 2) Initial residual stresses were introduced in the rolled section and in the reinforcing plates in accordance to the temperature versus stress relationship presented in Equation [3.1].
- 3) An axial load representing the dead load and partial live load on the unreinforced column was introduced. The preload was varied from 40% to 60% of the unreinforced column strength.
- 4) The beam elements used to simulate the weld attachment between the reinforcing plates and the rolled section were re-activated. This was performed within a load step to ensure that equilibrium is maintained in the process of attaching the reinforcing plates to the wide flange section.
- 5) The welding residual stresses were introduced by increasing the temperature at the flange tips to create a strain at the flange tips of 70% or 100% of the yield strength of the wide flange section. It is generally accepted that the residual stress due to welding (at the tips of the flange and plates) can reach the yield stress (Tall, 1961).
- 6) The next step consisted of removing the initial preload applied in load step 3. Both the residual stresses and initial imperfections in the reinforced column were determined at the end of this step.
- 7) Riks' method was used to load the reinforced steel column into the pre- and post-buckling ranges.

Nonlinear static stress analysis was used for stable problem analyses such as removing and adding elements, imposing initial residual stress, pre-loading, welding effects, and removing pre-loading. As a consequence, large-displacement effects were included in all the steps of the loading process. The first five steps were performed using a load control Newton-Raphson procedure. In order to trace the post-buckling response of the reinforced columns the modified Riks method (Riks,1979) was used in the last load step.

### **3.3 Validation of the Finite Element Model**

#### **3.3.1 General**

Because of the limited number of test results on reinforced steel columns, it is difficult to collect enough test data to fully validate the numerical model. Nagaraja Rao and Tall (1963) provided a set of test results for columns reinforced under load and under no load. To validate the numerical model as much as possible, both cases were compared with the analysis results.

#### **3.3.2 Description of the Tests**

The experimental investigation presented by Nagaraja Rao and Tall (1962) used W200x47 (8WF31) columns with 178x9.5 mm reinforcing plates, both of ASTM A7 structural steel. The reinforcing plates were placed parallel to the flanges. The weighted mean yield stress, determined from a stub-column test, was 256.5 MPa. The length of the column was 2440 mm, giving a non-dimensional slenderness ratio,  $\lambda$ , of about 0.5.

The column reinforced under load had an out-of-straightness of 0.5 mm ( $L/4900$ ) after reinforcing. The out-of-straightness of the column reinforced under no load was 0.762 mm ( $L/3200$ ). It should be noted that the reported initial out-of-straightness for both unreinforced columns was  $L/565$ , which is significantly greater than the maximum allowable initial out-of-straightness for wide flange sections.

An axial load was applied through end fixtures that allowed the columns to buckle freely about their weak axes. For the column reinforced under load, the pre-load applied before reinforcing was 30 percent of the capacity of the rolled section, namely 405 kN (Nagaraja Rao and Tall, 1963).

### **3.3.3 Initial Conditions of the Numerical Analyses**

Based on the investigation by Nagaraja Rao and Tall (1963), two numerical models were developed to model column reinforcement under load and reinforcement under no load. The geometrical details and the numerical analysis results of the two models are summarized in Table 3.1. The first model consists of the column reinforced under load and the second model represents the column reinforced under no load.

The same initial residual stresses were used for the two numerical models as illustrated in Figure 3.4. Columns (8) and (9) of Table 3.1 present the initial residual stress distributions in the sections. Column (8) shows the maximum magnitude at the tips of the flanges and Column (9) shows the maximum magnitude at the reinforcing plate edges.

Figure 3.5 shows a comparison between the initial residual stress distributions in the cross-section of the numerical models and the test specimen. The figure presents both the input values of residual stresses in the finite element model and the output value, obtained at the end of an equilibrium step in the loading process described above. The numerical model replicates successfully the measured residual stresses.

The magnitude of the initial imperfections reported by Nagaraja Rao and Tall (1962) was also replicated in the finite element models. Since the residual stresses introduce deformations in the model, the initial geometry had to be adjusted so that the magnitude of initial imperfections at the end of the residual stress load step was equal to the measured value. A trial and error procedure was used for this purpose. Columns (11) and (12) of Table 3.1 present the magnitude of the initial out-of-straightness before reinforcing and the ratio of this initial out-of-straightness to the column length, respectively. Columns (13) and (14) present the value of the out-of-straightness after

reinforcing and the ratio of this out-of-straightness to the column length, respectively. Column (17) presents the magnitude of the preload applied on the rolled section before welding of the reinforcing plates. A comparison of the initial out-of-straightness of the unreinforced columns in the numerical models with the measured initial out-of-straightness ( $L/565$ ) indicates that the model cannot predict accurately the effect of the reinforcing plates addition on the initial imperfections. The assumed initial imperfections in the two models before reinforcement of the columns were significantly smaller than the measured values, although the final initial imperfection magnitude in the numerical model is almost identical to the measured value.

Nagaraja Rao and Tall (1963) suggested that average stress at the flange tips after reinforcing was 70% of the measured yield strength. This magnitude was used also for the finite element models.

### **3.3.4 Behaviour of the Column Reinforced under Load**

The welding residual stress patterns in the test specimens were investigated for different welding sequences (Nagaraja Rao and Tall, 1963). Figure 3.6 shows the residual stresses at mid-thickness of the plates after welding. The figure shows residual stresses obtained from the finite element analysis and the residual stresses measured on test specimens fabricated using two different welding sequences as described in Section 2.3.1. Since the experimental data represent surface residual stresses, interpolation between the two surfaces was used to obtain the mid-thickness residual stresses for the flanges and the reinforcing plates. Measured pattern I was obtained for the first welding sequence and measured pattern II was obtained for the second welding sequence. Figure 3.6 shows that the predicted residual stress patterns in the flange and cover plates are similar to the measured patterns. Although the measured residual stresses showed a significant gradient through the thickness near the flange tips, no attempt was made to incorporate this phenomenon in the numerical model. The model therefore used an average stress through the thickness.

The next step in the validation process is to compare the predicted load response of the reinforced column with the reported test results. Figure 3.7 compares the axial load ratio,  $P/P_{ry}$ , versus the mid-height lateral deflection response for the numerical model with the test result. The out-of-straightness value after welding for both cases was 0.5 mm. From the figure, we can make the following observations:

- 1) The shapes of the curves are similar.
- 2) The post-buckling range is accurately predicted by the finite element method.
- 3) The predicted and measured peak strengths are almost the same. Column (19) from Table 3.1 presents the ratio of the predicted to measured peak load. The difference between the predicted capacity and measured capacity is only 0.1%.
- 4) The slopes of the elastic portion of the response curves are identical.

It can therefore be concluded that the strength and behaviour of steel columns reinforced under load can be predicted very well with the proposed finite element model for the slenderness tested. It should be noted that the capacity of the column was very close to its yield strength, indicating that the reinforced column fell into the short column range. The model still remains to be validated in the intermediate length range.

### **3.3.5 Behaviour of the Column Reinforced under no Load**

To further validate the numerical model, the column reinforced under no load was also modelled and analysed. The geometrical and material properties of the experimental model were the same as those of the first numerical model except for the magnitude of the pre-load, as presented in Table 3.1. The initial conditions used in the numerical model were discussed in section 3.3.3.

A comparison between measured and predicted residual stresses in the cross-section for the column reinforced under no load is presented in Figure 3.8. Again, a good agreement between the measured and predicted residual stresses is observed. Although the discrepancy between measured predicted values is more significant in the web the residual stresses in the web are not as influential on the column behaviour and capacity as those encountered at the flange tips.

The measured and predicted axial load versus mid-height lateral deflection response for the column reinforced under no load are shown in Figure 3.9. It can be observed that the curves are similar. The predicted peak strength from the numerical model is 97.7% of the yield strength. This is only slightly higher than the measured strength of 96% of the yield strength. It can therefore be concluded once more that the finite element model predicts the test results accurately.

### **3.3.6 Validation of the Finite Element Models in the Intermediate Length Range**

Because of insufficient experimental data to validate the finite element models over the full range of material response, experimental data for unreinforced columns of intermediate length were used to validate the finite element models in the elastic-to-plastic range.

Huber and Beedle (1954) presented the results of a series of tests on 8WF31 (W200x46) steel columns of different lengths. The steel was ASTM A7 structural steel, with a weighted average yield strength of 260 MPa. A residual stress pattern similar to the pattern illustrated in Figure 3.4 was used in the finite element models. The value of the peak residual stress was measured using the sectioning method (Huber and Beedle, 1954) and was reported to be 84 MPa. The details for two of the test specimens from Huber and Beedle (1954) are presented in Table 3.2. Other test specimens used to validate the finite element models were obtained from Beedle and Tall (1960) who reported tests on W-shape columns performed by other investigators. The material was also reported to be ASTM A7 structural steel. Since material properties were not specifically reported for these specimens, the same value as reported by Huber and Beedle (1954), namely, 260 MPa, was used for these test specimens. Table 3.2 also presents a summary of the properties used for these columns. It should be noted that since the magnitude of initial imperfections was not reported for these columns, values were assumed. In order to attempt to bracket the actual magnitude of initial imperfections two values were assumed, namely,  $L/1500$  and  $L/10\,000$ . The larger of the two values is significantly larger than those reported for the columns tested by Huber and Beedle

(1954) whereas the smaller of the two values is significantly smaller than those reported by Huber and Beedle.

Finite element analyses for the test specimens presented in Table 3.2 were conducted and the results are reported in column (8). The test results are reported in column (9) and the predicted-to-test ratios are reported in column (10). As can be seen, the test to predicted ratio for the first two test specimens is very close to 1.0, indicating an excellent correlation between the finite element models and the test results. The other predicted column capacities are not in such good agreement with the test results, however. The lack of agreement is attributed to the uncertainty in some of the important parameters of the finite element model that had to be assumed. It can be seen from Table 3.2 that a reduction of initial imperfection improves considerably the prediction of the test results. It is also expected that the assumption made about the actual yield strength of the test specimens would have an effect on the test-to-predicted ratio. Considering these later uncertainties, it is considered that the finite element models are able to predict accurately the strength of columns in the intermediate length range.

**Table 3.1 Models Used for the Validation of the Finite Element Method for Reinforced Columns**

FEA model I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	L <sup>c</sup>	$\lambda^d$	RSBR <sup>e</sup>		MP <sup>g</sup>	Welding Residual		$\delta_0^h$	$\delta_1^i$	Yield Strength		Preload				
						MF <sup>f</sup>	MP <sup>g</sup>		Stress	M'(mm)			M'(mm)	M'(mm)	M'(mm)	I-section plate	P <sub>0</sub> <sup>j</sup>	P <sub>u2</sub> <sup>m</sup>	P <sub>feff</sub> <sup>n</sup> /P <sub>ry</sub> <sup>n</sup>
No	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
1	W200x46	180x9.52	F	W	2440	0.5	0.3F <sub>y</sub>	0.15F <sub>y</sub>	0.7F <sub>y</sub>	0.47	L/5200	0.50	L/4900	260	260	405	0.3	0.97	
2	W200x46	180x9.52	F	W	2440	0.5	0.3F <sub>y</sub>	0.15F <sub>y</sub>	0.7F <sub>y</sub>	0.49	L/5000	0.50	L/4900	260	260	0	0.0	0.98	

a) D - Direction of reinforcing plates

b) B - Buckling axis of the reinforced column

c) L - Column length

d)  $\lambda$  - Non-dimensional slenderness parameter of the reinforced column

e) RSBR - Residual stress before reinforcing

f) MF - Maximum magnitude of the residual stress at the flange tips

g) MP - Maximum magnitude of the residual stress at the reinforcing plate edges

h)  $\delta_0$  - Initial imperfection before reinforcing.

i)  $\delta_1$  - Out-of-straightness after reinforcing, no load.

j) M - Out-of-straightness in the weak direction.

k) ratio - The ratio of the out-of-straightness to the column length, L.

l) P<sub>0</sub> - Preload before reinforcing

m) P<sub>u2</sub> - Load carrying capacity of the unreinforced column predicted using SSRC curve 2

n) P<sub>feff</sub> - Load carrying capacity obtained from the finite element analysis

P<sub>ry</sub> - Yield strength of the reinforced column

F - Parallel to the flanges

W - Weak axis of the I-section

F<sub>y</sub> - Yield stress of the unreinforced column



**Table 3.2 Finite Element Models for the Unreinforced Columns**

I-section	Column	RSBR <sup>b</sup>		Initial Imperfection		$F_y$	$P_{fea}/P_{uy}$ <sup>f</sup>	$P_{exp}/P_{uy}$ <sup>g</sup>	$P_{fea}/P_{exp}$	
	Length	$\lambda^a$	MF <sup>c</sup>	before reinforcing						
	L (mm)			ratio <sup>d</sup>	M <sup>e</sup> (mm)	(MPa)				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
W200x46*	4166	0.9	0.3F <sub>y</sub>	L/6300	0.66	260	0.73	0.75	0.97	
W200x46*	2946	1.2	0.3F <sub>y</sub>	L/2400	1.22	260	0.83	0.82	1.01	
W310x74**	4034	0.9	0.4F <sub>y</sub>	L/1500	2.69	260	0.67	0.76	0.88	
W310x74**	4034	0.9	0.4F <sub>y</sub>	L/10000	0.40	260	0.75	0.76	0.98	
W200x36**	3436	1.0	0.25F <sub>y</sub>	L/1500	2.29	260	0.65	0.73	0.89	
W200x36**	3436	1.0	0.25F <sub>y</sub>	L/10000	0.34	260	0.72	0.73	0.98	
W150x22**	3210	1.0	0.25F <sub>y</sub>	L/1500	2.14	260	0.70	0.73	0.96	
W150x22**	3210	1.0	0.25F <sub>y</sub>	L/10000	0.32	260	0.73	0.73	1.00	

Note: \* Test results reported by Huber and Beedle (1954)

\*\* Test results reported by Beedle and Tall (1960)

a)  $\lambda$  - Slenderness parameter of the reinforced column

b) RSBR - Residual stress before reinforcing

c) MF -Maximum magnitude of the residual stress in the flange.

$F_y$  - Yield stress of the unreinforced column

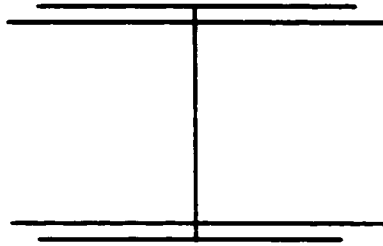
d) ratio - The ratio of the out-of-straightness to the column length, L.

e) M - Out-of-straightness in the weak direction.

f)  $P_{fea}$  - Load carrying capacity obtained from the finite element analysis

$P_{uy}$  - Yield strength of the rolled section column

g)  $P_{exp}$  - Experimental strength of the rolled section column

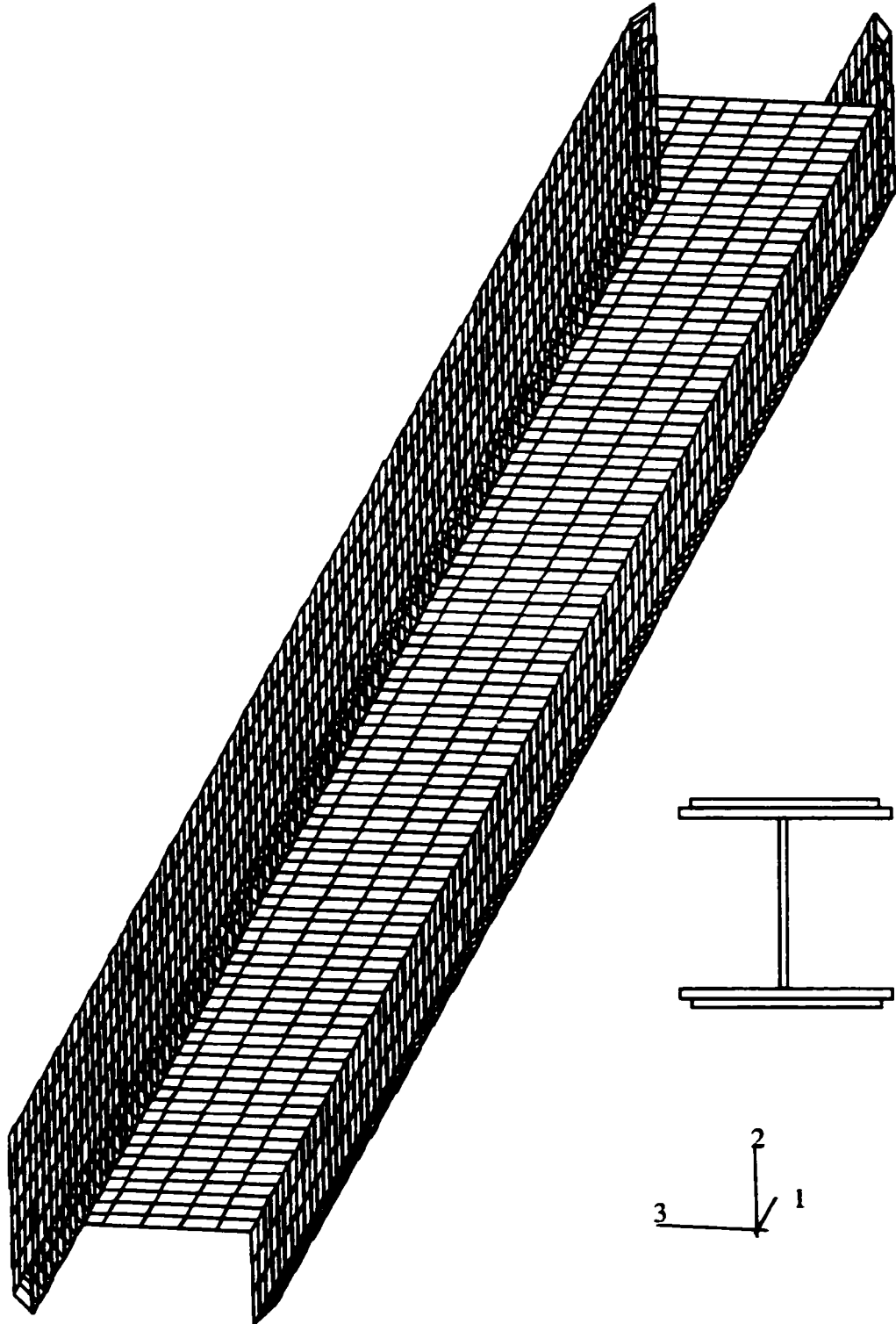


a) Cross-section

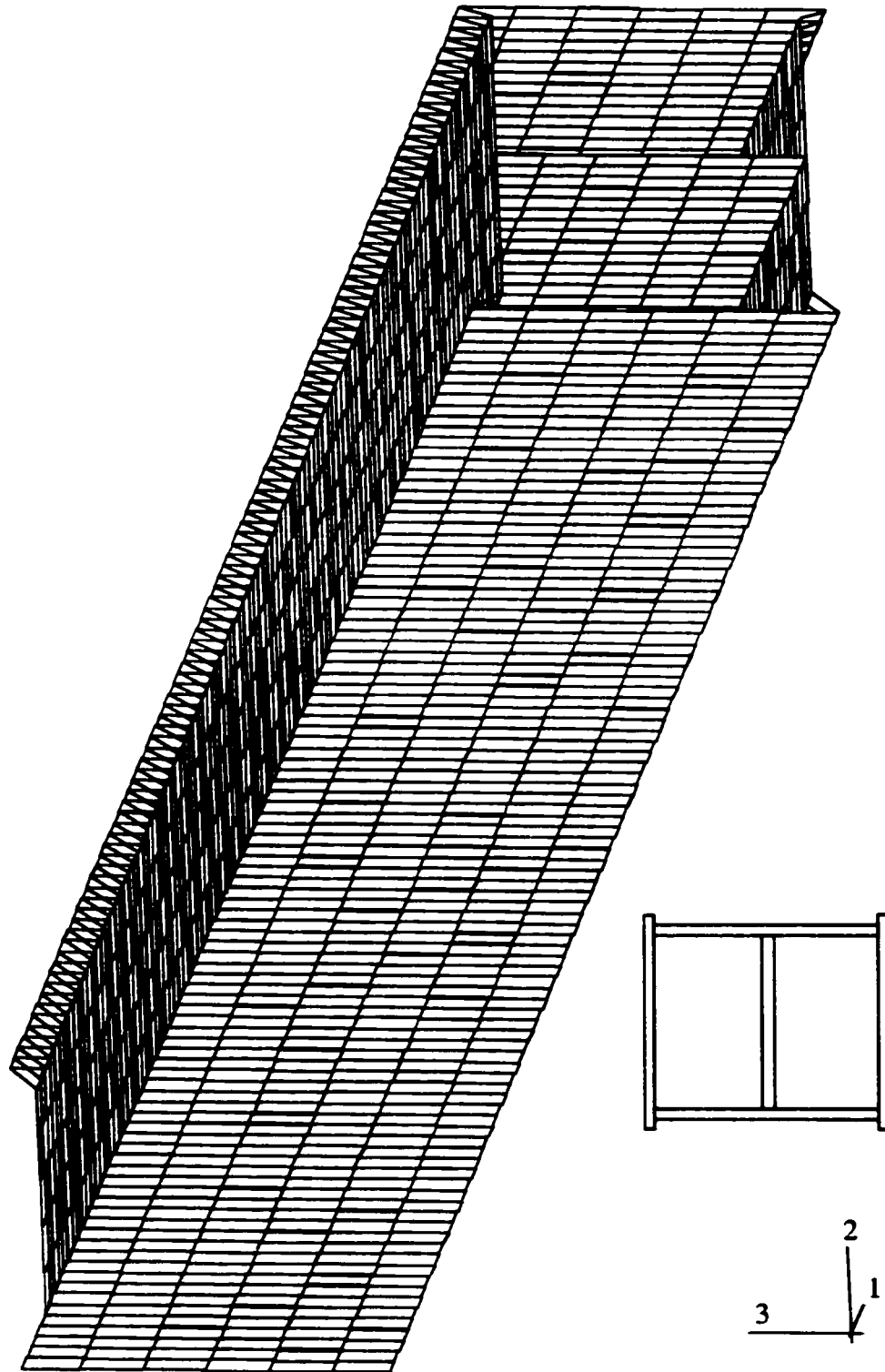


b) The View from the Direction Parallel to the Web

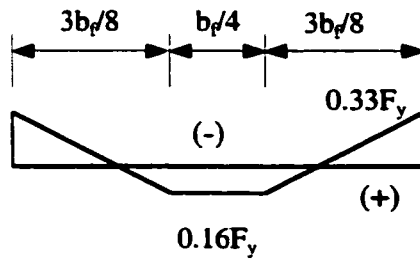
**Figure 3.1 Position of the Tie Connection between Column and Cover Plates**



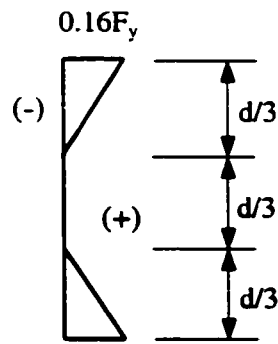
**Figure 3.2 Finite Element Mesh of Column Reinforced with the Plates Parallel to the Flanges**



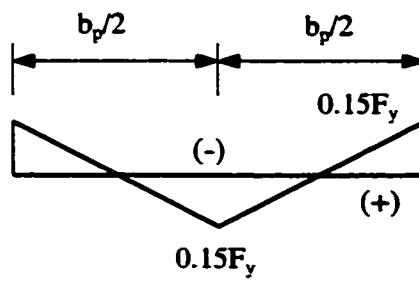
**Figure 3.3 Finite Element Mesh of Column Reinforced with the Plates Parallel to the Web**



a) In the Flanges



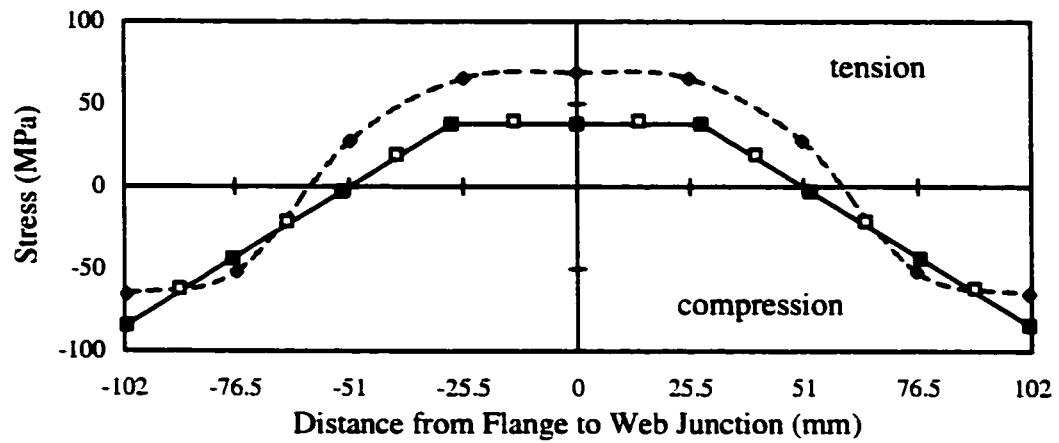
b) In the Web



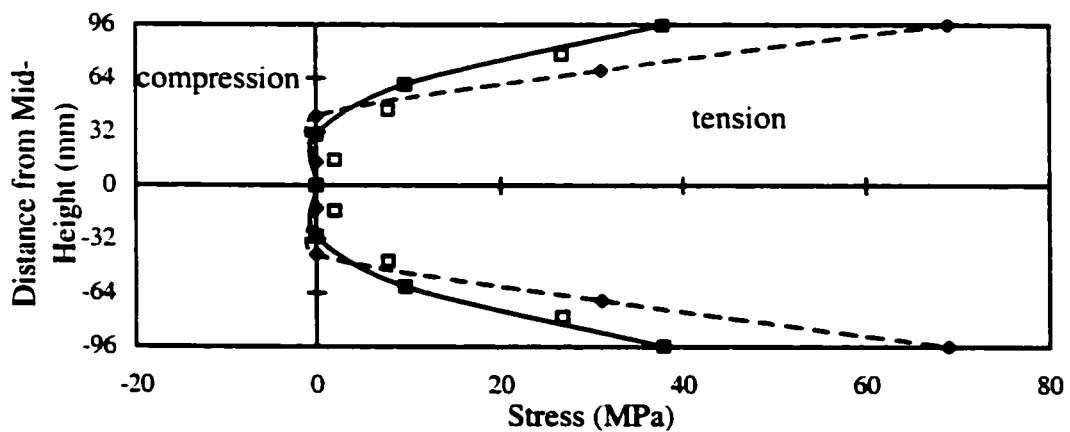
a) In the Plates

$b_f$  - the width of the flanges       $w$  - the width of the web  
 $b_p$  - the width of the plates

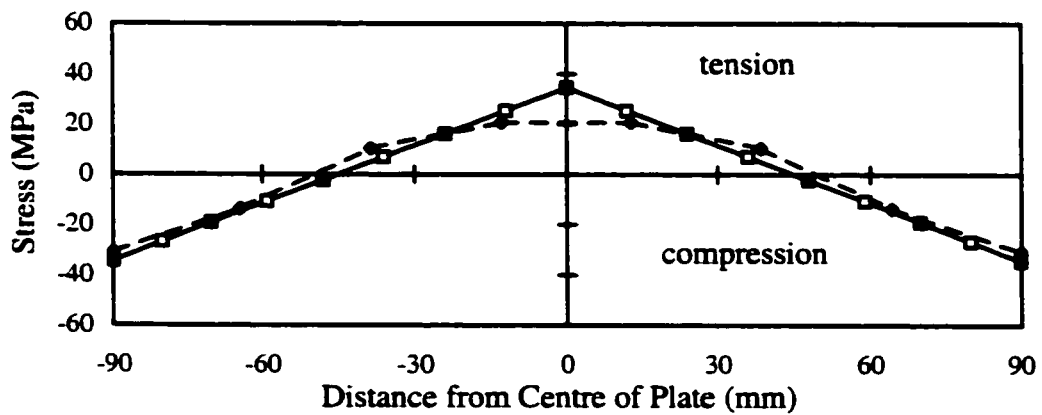
**Figure 3.4 Initial Residual Stress Pattern Used in the Numerical Models**



a) In the Flanges



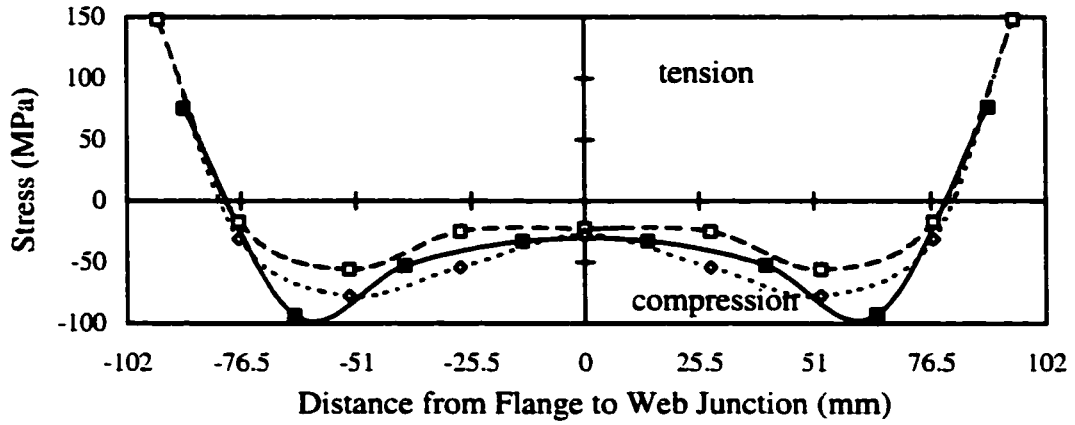
b) In the Web



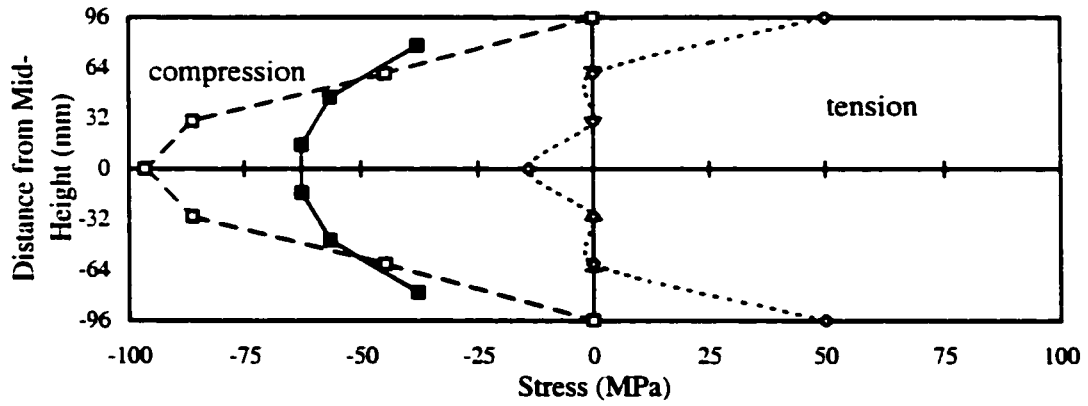
c) In the Cover Plates

—■— Input Value    □— Output Value    -●- Experimental Measurement  
 Section: W200x46; Reinforcing Plate: 180x9.52 mm;  $F_y = 260$  MPa

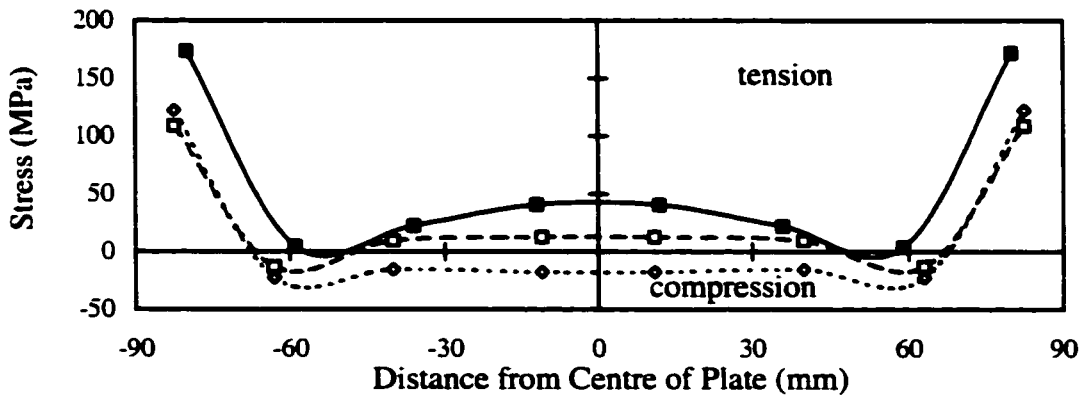
**Figure 3.5 Initial Residual Stress Distribution in I-Section and Cover Plates of Finite Element Models**



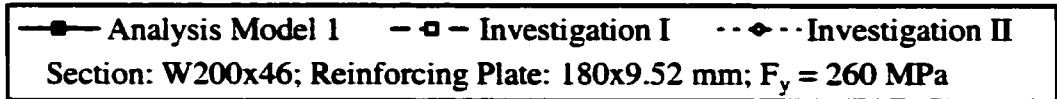
a) In the Flanges



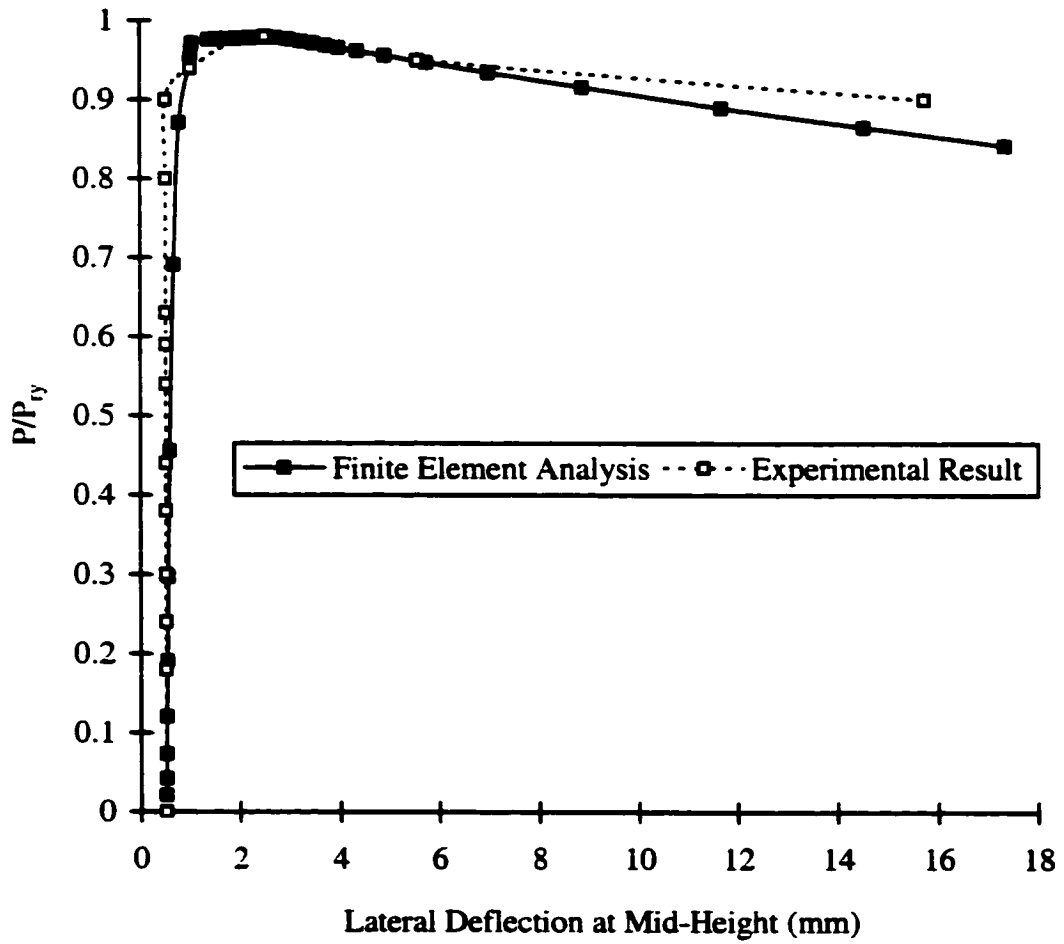
b) In the Web



c) In the Cover Plates

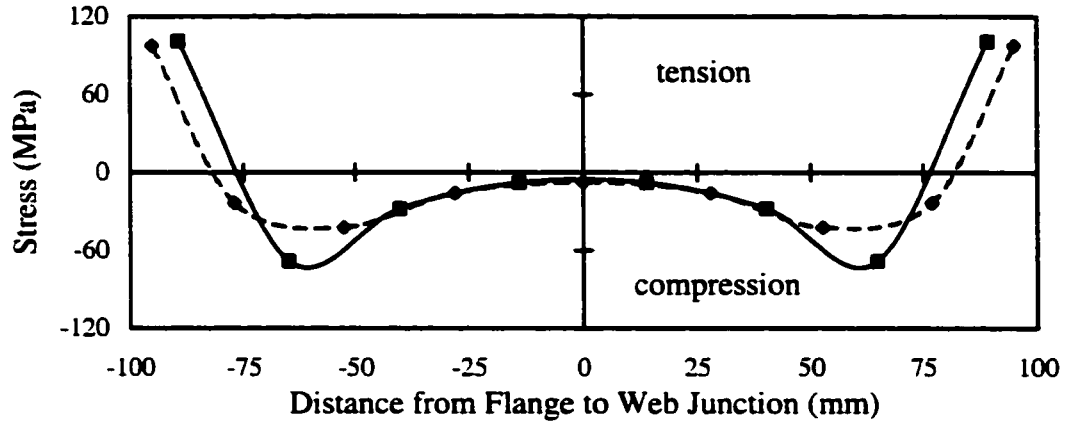


**Figure 3.6 Welding Residual Stresses in the Column Reinforced under Load**

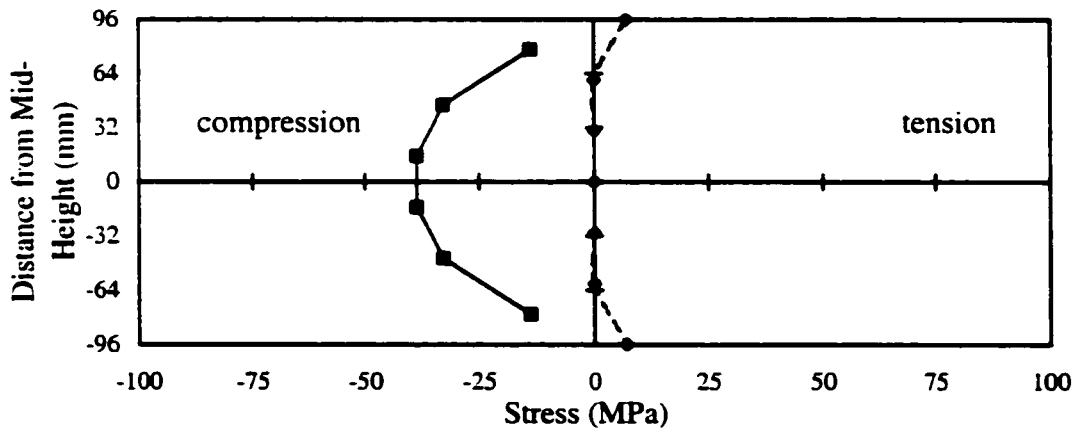


**Figure 3.7 Axial Load versus Lateral Deflection Curves for the Column Reinforced under Load**

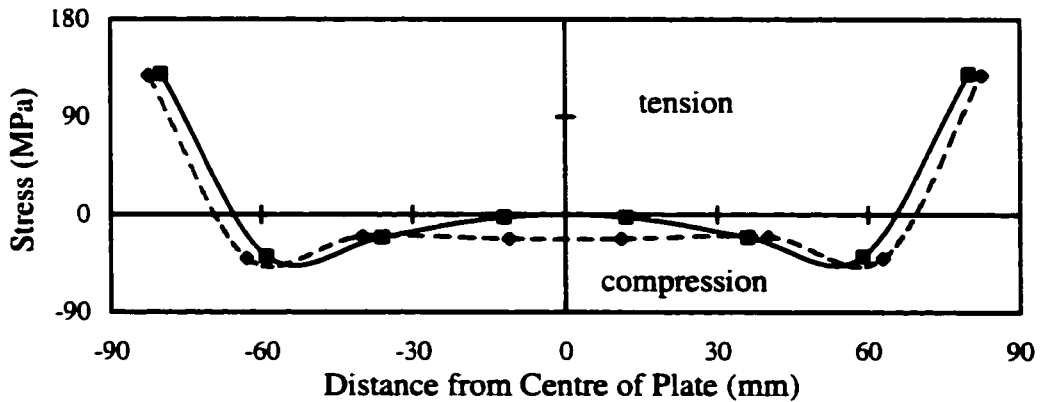




a) In the Flanges



b) In the Web



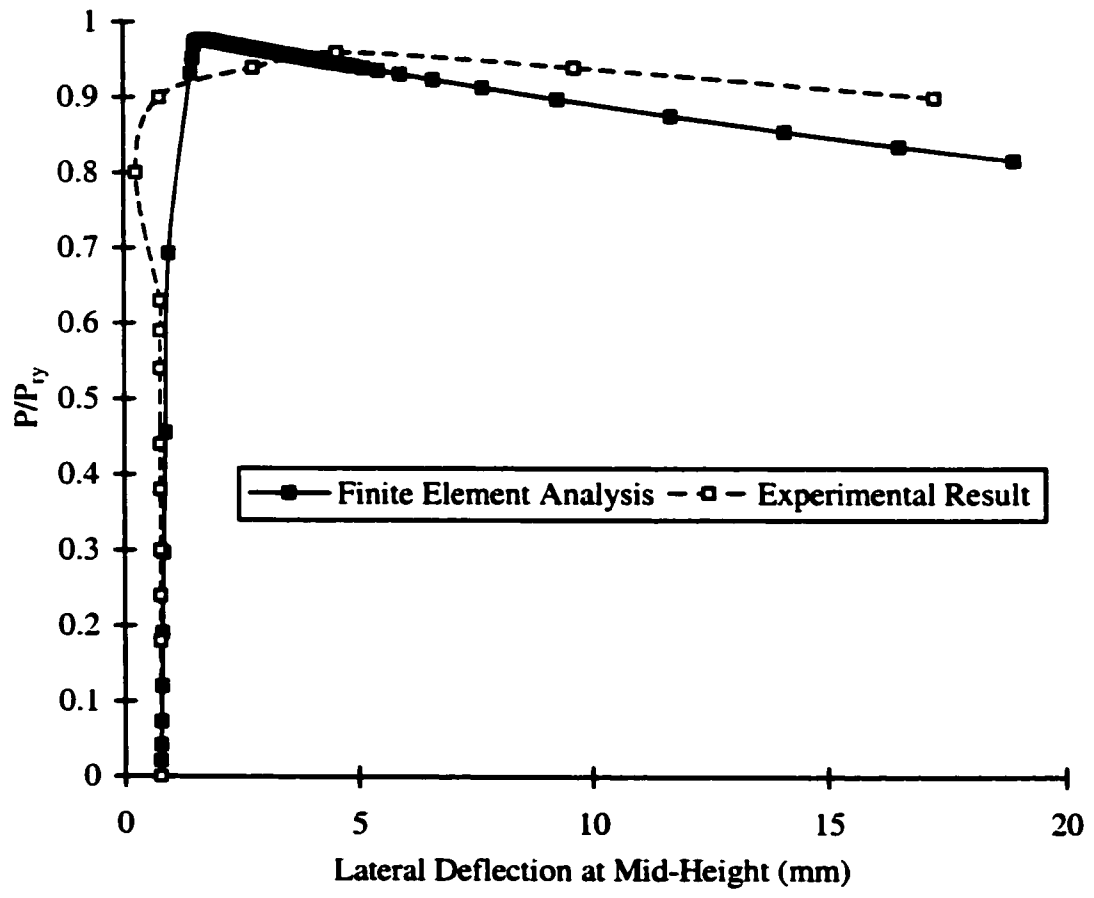
c) In the Cover Plates

Finite Element Analysis
 

 Experimental Measurement

Section: W200x46; Reinforcing Plate: 180x9.52;  $F_y = 260$  MPa

**Figure 3.8 Welding Residual Stresses in the Column Reinforced under no Load**



**Figure 3.9 Axial Load versus Lateral Deflection Curves for the Column Reinforced under no Load**

## **Chapter 4**

### **Parametric Study**

#### **4.1 General**

Testing of full-scale columns is the most direct and reliable approach to examine the strength and behaviour of reinforced steel columns. However, because of the lack of previous test results and the impossibility of testing a large number of specimens to examine all the parameters that may affect the strength and behaviour of reinforced steel columns an alternative approach is desirable. A practical and expedient approach is to use the finite element analysis model presented and validated in Chapter 3 to expand the limited database of test results.

The first stage of this investigation will consist of identifying the parameters that affect the strength and behaviour of reinforced columns. A parametric study using a range of values for each parameter was then conducted. The database obtained from this parametric study provided sufficient information to perform a statistical analysis, which will be discussed in the next chapter. This chapter presents a description of buckling behaviour, the selection of the parameters affecting the strength and behaviour of reinforced columns, and the results of the parametric study.

#### **4.2 Buckling Behaviour**

A study of buckling modes was conducted using the finite element models presented in Chapter 3. Figure 4.1 shows the buckled shape of columns reinforced with plates parallel to the flanges. The figure illustrates buckling about the weak axis (Figure 4.1 (a)) and buckling about the strong axis (Figure 4.1 (b)). Figure 4.2 shows the buckled shape of columns reinforced with plates parallel to the web. Buckling about the weak axis is illustrated in Figure 4.2 (a) and buckling about the strong axis is illustrated in Figure 4.2 (b). For the specimens under investigation, the expected buckled shape always occurs as overall buckling about the weak axis of the reinforced column. For columns reinforced with plates parallel to the flanges the weak axis of the reinforced section always coincides with the weak axis of the rolled shape. When a column is reinforced

with plates parallel to the web, however, the weak axis of the reinforced section may coincide with the strong axis of the rolled shape. In order to trigger buckling about the strong axis of the reinforced section, some column models were restrained at both ends to provide rotational fixity in the weak direction of the reinforced section.

Unexpected local buckling of the reinforcing plates near the supports was found to govern the capacity of some columns reinforced with plates parallel to the web. Figure 4.3 shows the buckled shape of a W310x179 column reinforced with 350x25 mm plates parallel to the web and  $\lambda = 0.4$ . In order to prevent local buckling of the reinforcing plates, the thickness was increased three times near the ends. Figure 4.4 shows the resulting load versus mid-height lateral deflection curves for the above local buckling model and the corresponding overall buckling model. It can be observed that, for the example shown in Figure 4.4, the overall buckling capacity is about 10 percent higher than the local buckling capacity. Although local buckling failure is beyond the scope of this investigation, it is a possible failure mode that should be investigated experimentally.

### **4.3 Selection of Parameters**

Many parameters were found to influence the strength and behaviour of steel columns, with or without reinforcement (Geschwindner *et al.*, 1994; Feder and Lee, 1959; Tall, 1961; Nagaraja Rao and Tall, 1963). The following parameters were selected for this investigation:

1. Reinforced column slenderness.
2. Residual stress pattern and magnitude before welding.
3. Magnitude of residual stresses after welding of reinforcing plates.
4. Initial out-of-straightness of the reinforced columns.
5. Magnitude of the load applied on the column during the reinforcing process.
6. Steel grades of the I-section and cover plates.
7. Orientation of the reinforcing plates.
8. Direction of the buckling of the reinforced columns.
9. I-section to cover plates area ratio.

In order to investigate the effect of these parameters on reinforced columns, short columns, intermediate, and long columns were analysed. The slenderness parameter,  $\lambda$ , of the reinforced columns was taken as 0.4, 1.0, 1.1, and 1.5 for the parametric study.

The effect of the magnitude and pattern of residual stresses in the rolled section and the reinforcing plates before welding was investigated for 12 cases: six residual stress patterns with two different magnitudes for each pattern. The magnitude of the peak residual stresses was taken as 0.3 and 0.1 times the yield strength of the material for the rolled section, and 0.3 and 0.15 times the yield strength of the material for the reinforcing plates. All 12 cases were expected to cover the full range of initial residual stresses before welding (Huber, 1956; Tall, 1961; Nagaraja Rao and Tall, 1963). In addition, two cases were used to investigate the effect of welding residual stresses. The peak residual stresses at the flange tip were taken as 70% of the yield strength as suggested by Nagaraja Rao and Tall [1963], or 100% of the yield strength as suggested by the work of Tall [1961] and Huber [1956].

After the cover plates are added on to the rolled section under preload, it is difficult to control the out-of-straightness of the reinforced columns. In the following work three values for the initial imperfections before reinforcing were mainly used to investigate the effect of the initial out-of-straightness of reinforced columns, namely,  $L/8000$ ,  $L/2000$ , and  $L/1000$ .  $L/8000$  was used to simulate very small initial imperfections.  $L/2000$  is close to the mean value reported for columns of hot rolled wide flange shapes (Bjorhovde, 1988), and  $L/1000$  is the maximum initial imperfection allowed by CSA/CAN-G40.20-92. For columns longer than 10 m, CSA standard G40.20-92 suggests a different allowable initial imperfection as follows:

1. The allowable initial imperfection is 10 mm when  $10 \text{ m} < L \leq 14 \text{ m}$ ;
2. The allowable initial imperfection is  $[10+(L-14000)/1000]$  mm, when  $L > 14 \text{ m}$ .

The pre-load on columns before reinforcing consists of dead loads and a portion of the design live loads. The magnitudes of the dead and live loads vary depending on the type of structure. In the parametric study, two pre-load magnitudes were selected,

namely, 40% of the load-carrying capacity of the unreinforced column predicted using the SSRC column curve 2 and 60% of load carrying capacity of the unreinforced column.

Three combinations of steel grades of I-sections and cover plates were investigated: both the rolled section and the reinforcing plates have a yield strength of 260 MPa or 300 MPa, and the rolled shape has a yield strength of 230 MPa and the reinforcing plates have a yield strength of 350 MPa. The third combination is believed to represent a condition where the difference in grades is maximised.

The effect of reinforcing plate orientation was investigated for two cases: reinforcing plates parallel to the flanges and reinforcing plates parallel to the web. The effect of the buckling direction for reinforced columns was also investigated for two cases: column buckling about the weak axis of the rolled section and column buckling about the strong axis of the rolled section.

Nine different combinations of rolled section sizes and reinforcing plate sizes were selected in the analysis: W200x46 with 180x9.52 mm cover plates, similar to the test specimens used by Nagaraja and Tall (1962); W310x179 with four different cover plate sizes, namely 290x25 mm, 290x16 mm, 350x25 mm and 350x16 mm; and W150x30 with four different cover plate sizes, namely 130x5 mm, 130x8 mm, 175x5 mm and 175x8 mm. All the different cases investigated cover a range of W-shape to reinforcing plate area ratio from 2.71 to 5.83.

#### **4.4 Analysis Results**

With a total of 39 variables selected for nine parameters, the corresponding number of combinations for a full factorial design would equal  $4 \times 12 \times 2 \times 3 \times 2 \times 3 \times 2 \times 2 \times 9 = 62208$ , which is too many. In order to reduce the total number of samples considerably, a fractional factorial design (Hines and Montgomery, 1972) was adopted for the parametric study. Table 4.1 summarizes the various combinations of parameters investigated in the parametric study. A total of 317 numerical models with different variables were analysed. Table A.1 of Appendix A presents the details concerning the geometric and material properties for each model investigated. Table B.1 of Appendix B summarises the

magnitude of the pre-loads used in each case and the analysis results compared with the load carrying capacity predicted using different SSRC column curves and CSA column curves for each case. The load carrying capacity is presented as a ratio of the peak load determined from the finite element analysis to the yield strength.

The following sections present a detailed discussion of the parametric study. The effects of the various parameters investigated on the strength and behaviour of reinforced steel columns are discussed.

#### **4.5 Effect of Column Slenderness**

It is commonly understood that the slenderness parameter is the most important factor affecting the strength of columns. Based on the definition of the slenderness parameter given in Section 2.1, different geometric and material properties result in different non-dimensional slenderness ratios for columns. The effect of slenderness parameters on the strength of 315 reinforced columns is illustrated in Figure 4.5 where the load carrying capacity is plotted against the slenderness parameter. Although the results show a fairly large scatter, the relationship between the slenderness parameter and the strength of the columns is obvious. The large scatter results from the large range of parameters investigated in the parametric study.

#### **4.6 Effect of Residual Stresses**

The residual stresses in reinforced steel columns were modelled in two different stages, namely, the residual stresses in the rolled section and reinforcing plates before welding and the residual stresses after welding the reinforcing plates to the rolled section. Both stages must be considered to obtain a representative residual stress distribution in the reinforced column.

#### **4.6.1 Residual Stresses before Welding**

The residual stresses in the rolled section and in the reinforcing plates before welding were investigated separately. Their effect on the strength and behaviour of the reinforced column are presented in the following.

##### *Residual Stresses in the Rolled Section*

The patterns and magnitudes of the initial residual stresses in the rolled section depend on factors related to the manufacturing process, as discussed in Chapter 2. The initial residual stress distributions vary from section to section. Based on an investigation presented by Huber [1956], four different initial residual stress patterns were selected in this research for the wide flange section. The peak compressive residual stresses selected for this investigation were taken as 30% and 10% of the yield strength of the rolled section. The 30% level is representative of rolled sections (Chen and Atsuta, 1976) and the 10% level represents a lower bound value.

Six different patterns of residual stresses in the rolled section and in the reinforcing plates were considered and illustrated in Figure 4.6. For each residual stress pattern, two magnitudes of the peak residual stresses in the wide flange sections were investigated. The first four and the sixth residual stress patterns presented in Figure 4.6 are studied in this section. The fifth pattern is discussed in the next section.

Two columns configurations were used to investigate the effect of initial residual stresses. The first configuration consisted of a W200x46 section reinforced with 180x9.5 mm plates parallel to the flanges and buckling about the weak axis of the rolled section. The second configuration consisted of a W310x179 section reinforced with 350x25 mm plates parallel to the web and buckling about the strong axis of the rolled section. Table 4.2 summarises the finite element analysis models used in this part of the investigation. A description of the initial residual stress pattern and magnitudes is presented in columns (2) to (4). The other parameters are kept constant and are summarised in Table A.1.



Figures 4.7 and 4.8 depict the residual stress distributions after welding in the cross-section of a W200x46 column reinforced with 180x9.5 mm plates. The maximum magnitude of residual stresses resulting from welding was taken as  $1.0 F_y$  at the flange tips for each model. Figures 4.7 and 4.8 show that the residual stresses after welding in the reinforced section are very similar despite the significant difference in initial residual stress patterns and magnitudes.

Figures 4.9 and 4.10 show the axial load response for the eight reinforced columns described in Table 4.2. Except for residual stress pattern 4-1, all other initial residual stress patterns investigated resulted in the same behaviour and strength of the reinforced columns. Residual stress pattern 4-1 resulted in about a 7% reduction in strength compared to the other specimens investigated. A summary of the peak to yield strength ratio for each case investigated is presented in column (5) of Table 4.2. An examination of the analysis results for two W310x179 columns reinforced with 350x25 mm plates parallel to the web and buckling about the strong axis of the rolled section, as shown in Table 4.2, also indicates that initial residual stresses have little effect on the strength of reinforced steel columns.

#### *Residual Stresses in the Cover Plates*

In order to investigate the effect of initial residual stresses in the reinforcing plates, two peak magnitudes ( $0.3F_y$  and  $0.15F_y$ ) were chosen for the initial residual stresses in the reinforcing plates based on investigations by Tall [1961] and Nagaraja Rao and Tall [1963]. Four W200x46 columns reinforced with 180x9.5 mm plates parallel to the flanges and buckling about the weak axis of the rolled section were used to investigate the effect of this parameter on the strength and behaviour of reinforced steel columns. The models used for this study are described in detail in Table 4.3.

Despite differences in initial residual stresses, the residual stress patterns and magnitudes after welding the reinforcing plates were essentially all the same. All the welding residual stress patterns for these four models are similar to the pattern shown in

Figure 4.11 (a). The axial load response of columns with different residual stress magnitude is presented in Figure 4.12. As expected from an examination of the residual stresses after welding, all the specimens display the same strength and behaviour. The ratios of peak load to yield load for these cases are summarized in Column (5) of Table 4.3. Since this parameter was found to have little effect, a typical initial stress pattern can therefore be used for the remaining part of this study. Pattern 1-3, illustrated in Figure 4.6, was selected for all the following numerical models.

#### **4.6.2 Effect of the Magnitude of the Welding Residual Stresses**

Nagaraja Rao and Tall (1963) have shown that high tensile residual stresses are developed at the flange tips as a result of welding reinforcing plates to a rolled W-shape. The distributions of the welding residual stresses were found to be very similar in the research, as shown in Figure 4.11. The magnitudes of these welding residual stresses were reported to be in the order of 70 percent of the yield strength of the material. Welding residual stresses equal to the yield strength of the material have also been reported elsewhere (Masubuchi, 1980). In order to cover the full range of possible welding residual stresses, a residual stress pattern was investigated with two residual stress magnitudes, namely 70 percent and 100 percent of the yield strength at the flange tips, as illustrated in Figure 4.11. The control parameter in the analysis is the magnitude of the residual stresses at the flange tips. The residual stresses in the remaining portions of the cross-section are governed by the size of the reinforced cross-section and the initial residual stresses in the reinforcing plates and the wide flange section.

This section presents the procedure used for four W310x179 columns reinforced with 290x16 mm plates parallel to the flanges and buckling about the weak axis of the rolled section (all samples used the same procedure). Two different values of the slenderness parameter,  $\lambda$ , were investigated, namely 1.1 and 1.5. The residual stress distribution in the reinforcing plates and the wide flange section before welding is pattern 1-3 of Figure 4-6. A description of the models used for this part of the investigation is given in Table 4.4 where column (4) lists the magnitude of the peak welding residual stress. A comparison of the load carrying capacity listed in column (5) shows that, for a given

slenderness parameter, the difference in the strength of columns with different welding residual stresses is negligibly small.

The effect of welding residual stress magnitude on the strength and the behaviour of reinforced columns is illustrated in Figure 4.13. As for the peak strength, the effect of welding residual stress magnitude on the strength and the behaviour of the reinforced steel columns is negligible. The following investigation therefore uses a representative peak welding residual stress of  $1.0 F_y$  at the flange tips.

#### **4.7 Effect of the Initial Out-of-straightness**

The shape and magnitude of initial out-of-straightness in a reinforced column result from a combination of deformations. These deformations are the initial imperfection of the unreinforced rolled section resulting from the rolling process, the deformation resulting from the preload on the unreinforced column, and the deformation resulting from the welding process during reinforcement of the columns. Although the magnitude of initial out-of-straightness must be controlled in rolled shapes and other fabricated columns, current Canadian standards do not provide any specific requirement for the initial out-of-straightness in a reinforced column. However, CAN/CSA G40.20-92 specifies some limitations for the initial imperfection in unreinforced rolled sections, as described in Section 4.3. The effect of initial out-of-straightness in reinforced columns was therefore investigated in light of the limitations set for unreinforced columns.

The effect of initial out-of-straightness on the strength and behaviour of reinforced steel columns is illustrated using nine W310x179 columns reinforced with 290x25 mm plates parallel to the flanges. All the reinforced columns have their weak axes in the same direction as the weak axes of the rolled sections. Table 4.5 presents a description of the models used for this investigation. Three different values of slenderness were used in the columns, as shown in Column (3) of Table 4.5. In order to obtain different magnitudes of initial out-of-straightness in the reinforced columns, the magnitudes of the initial imperfections in the unreinforced columns were varied as shown in Column (4). Column (5) of Table 4.5 presents the initial out-of-straightness of the reinforced column for each

model. This out-of-straightness value was obtained following the removal of the axial load on the column after welding the plates to the column. It can be observed that the magnitude of the out-of-straightness increases after strengthening the column and this effect is more significant with slender columns than with short ones.

The predicted load carrying capacity of the reinforced columns is presented in column (6) of Table 4.5. It can be seen that the initial out-of-straightness for intermediate and long columns significantly affects column strength. The column strength decreases with increasing initial out-of-straightness magnitude. For example, for  $\lambda = 1.1$ , a change in the initial out-of-straightness from  $L/1790$  to  $L/820$  results in a decrease in load carrying capacity of 8.5%, as shown in the table. An increase in the initial out-of-straightness from  $L/7190$  to  $L/820$  results in a reduction in strength of 15%. A similar trend is observed in columns with  $\lambda = 1.5$ , but the strength of short columns ( $\lambda = 0.4$ ) is not significantly affected by the magnitude of the initial out-of-straightness.

Figure 4.14 shows the axial load versus lateral deflection at mid-height for columns with different initial out-of-straightness after reinforcing for  $\lambda = 1.1$ . It can be observed that with increasing initial out-of-straightness of the reinforced columns, the lateral deflections at the peak load increase, and the load carrying capacity decreases.

#### **4.8 Effect of the Pre-load**

Six W310x179 columns reinforced with 290x25 mm plates parallel to the flanges were used to present the effect of pre-load on the strength and behaviour of reinforced columns. Their buckling axis was the weak axis of the rolled section. A description of the reinforced columns is presented in Table 4.6. Columns with two different preloads, namely 0.4 and 0.6 times the load carrying capacity of the unreinforced column predicted using the SSRC column curve 2, and three slenderness values ( $\lambda = 0.4, 1.1, \text{ and } 1.5$ ) were investigated. Columns (3) and (4) present the pre-load magnitude and the ratio of the pre-load to the load carrying capacity of the unreinforced column predicted using SSRC column curve 2, respectively. An examination of the predicted capacity presented in

column (5) of Table 4.6 indicates that the magnitude of the preload does not significantly affect the strength of reinforced columns.

Plots of the axial load versus axial deformation for different pre-load magnitudes are presented in Figure 4.15 for columns with  $\lambda$  of 1.1. It can be observed that the shapes of the curves are identical. The same observation can also be made for columns with  $\lambda$  of 0.4 and 1.5. The pre-load magnitude does not significantly affect the pre- and post-buckling behaviour of reinforced columns within the range of preload investigated. The following investigation was therefore carried out with a preload of 0.6 times the load carrying capacity of the unreinforced column predicted using the SSRC column curve 2.

#### **4.9 Effect of Steel Grade**

Columns in many older structures are either of grade A9 or A36 steel, a relatively low nominal yield strength compared to more modern structural steels that would typically be used for reinforcing plates. Reinforced steel columns may therefore be composite columns with different steel grades. In order to cover a broad range of these composite columns, columns with two different combinations of steel grades were investigated: 1) columns with the same steel grade for the plate and rolled section ( $F_y = 300$  MPa), which will serve as a reference, and; 2) reinforced columns with  $F_y = 230$  MPa for the rolled section and  $F_y = 350$  MPa for the plates.

Table 4.7 gives a description of the numerical models used to illustrate this investigation. The effect of material yield strength was studied for two different reinforcing plate orientations, buckling about the weak axis and buckling about the strong axis of the rolled section, and three different values for the slenderness parameter,  $\lambda$ . A comparison of the predicted load carrying capacities presented in column (5) of Table 4.7 indicates that varying the steel grades of the reinforced column does not significantly affect the strength of reinforced columns when the capacity is expressed as a ratio of the yield capacity of the cross-section.

#### **4.10 Effect of Reinforcing Plate Orientation**

Steel columns can be reinforced with steel plates either welded parallel to the flanges or parallel to the web in Figure 2.1. In order to investigate the effect of plate orientation on the strength and behaviour of reinforced steel columns, columns of different slenderness and different slenderness were modelled for buckling about either the strong or the weak axis. A summary of these models is presented in Table 4.8.

An examination of Table 4.8 reveals that short columns ( $\lambda = 0.4$ ) are not affected by the orientation of the reinforcing plates. This is expected since short columns fail by yielding, rather than by buckling. For buckling about the weak axis of the W-shape section and  $\lambda = 1.1$  and  $1.5$ , it seems that columns are weaker when the reinforcing plates are parallel to the flanges. A reduction of strength of 7.5% to 10% is observed in the sample columns presented in Table 4.8. When the buckling axis is the strong axis of the strong W-shape section, columns with reinforcing plates parallel to the web are weaker than the columns with plates parallel to the flanges. A reduction in strength-to-yield ratio of about 7% is observed for the selected sample columns.

#### **4.11 Effect of Buckling Axis**

The investigation has so far focused on columns buckling about the weak axis of the reinforced column. When wide flange sections are reinforced with plates parallel to the flanges, the weak axis of the reinforced section coincides with the weak axis of the wide flange section. However, when the reinforcing plates are parallel to the web of the wide flange section, the weak axis of the reinforced section may be at right angle to the weak axis of the unreinforced section. On the other hand, buckling of a column may take place about the strong axis of the cross-section if the braced length in the weak axis direction is shorter than the braced length in the strong direction. In some cases presented in this section, additional bracing in the weak axis direction was provided to force the column to buckle about its strong axis.

Table 4.9 presents a summary of the columns used to illustrate the effect of the buckling axis. Columns reinforced with plates parallel to the flanges and with plates

parallel to the web were investigated. The direction of the buckling axis orientation relative to the unreinforced and reinforced sections are presented in columns (2) and (3), respectively. It is seen that the effect of plate orientation on the buckling capacity of columns varies according to column slenderness and the buckling direction relative to the unreinforced section major axis. For intermediate and long columns ( $\lambda = 1.1, 1.5$ ) with reinforcing plates parallel to the flanges, buckling about the strong axis of the rolled section was observed to result in a larger strength-to-yield ratio than in the case of buckling about the weak axis. For intermediate and long columns with reinforcing plates parallel to the web, buckling about the strong axis of the rolled section was observed to result in a lower strength-to-yield ratio than buckling about the weak axis of the rolled section. This observation holds whether or not the strong axis of the reinforced section is in the same direction as the strong axis of the unreinforced section.

#### **4.12 Effect of W-Shape to Plate Area Ratio**

The effect of the ratio of the wide flange section area to the reinforcing plate area on the strength and behaviour of the reinforced columns was investigated for three non-dimensional slenderness ratios. For each slenderness ratio, the area ratio was varied either by: 1) changing the plate area while keeping the wide flange section constant; and, 2) by changing both the I-section and the reinforcing plate dimensions. The results of this investigation are presented in Table 4.10 and Table 4.11.

The results presented in Table 4.10 show that, despite a variation in the area ratio from 1.57 to 2.46, the predicted strength-to-yield ratio remained essentially the same for all three non-dimensional slenderness ratios. Table 4.11 shows a variation in area ratio from 1.57 to 2.92 obtained by changing both the size of the rolled section and the size of the reinforcing plates. Except for the columns with a slenderness ratio,  $\lambda$ , of 1.1, the change in capacity is insignificant. Although a significant change in capacity is observed for the column with  $\lambda = 1.1$ , considering the large change in area ratio used for these analyses, the strength of the columns is considered to be insensitive to the ratio of the rolled section area to the cover plate area.

### **4.13 Summary**

This chapter presents some of the results of a parametric study that includes a total of 315 reinforced steel columns. The parameters investigated were column slenderness, residual stresses, initial out-of-straightness, preload magnitude, yield strength of the I-section and reinforcing plates material, plate orientation, buckling axis, and the I-section area to the cover plate area ratio. The following conclusions were drawn from this parametric study.

1. The non-dimensional column slenderness, expressed as the slenderness parameter,  $\lambda$ , is the most important parameter affecting column strength.
2. While the initial residual stress is an important factor for the load carrying capacity of an unreinforced column, the investigation demonstrated that variations in the initial residual stresses, before welding the reinforcing plates, do not affect significantly the load carrying capacity of a reinforced column. The investigation also demonstrated that varying the maximum welding residual stress from 70% to 100% of the yield strength of the materials does not affect significantly the predicted strength of reinforced columns.
3. Initial out-of-straightness affects the behaviour and strength of intermediate and long reinforced columns significantly. The strength of reinforced columns decreases as the initial out-of-straightness increases.
4. A change in the preload from 40% to 60% of the load carrying capacity of the unreinforced column does not affect the behaviour and the predicted strength-to-yield strength ratio of reinforced columns significantly.
5. The use of different grades in reinforced columns was found to have a negligible effect on the strength-to-yield ratio of reinforced columns.
6. In the numerical model results, it was observed that the interaction of the plate orientation and the buckling axis affects the behaviour and the strength of



intermediate and long columns significantly. For columns of same slenderness and reinforced with plates parallel to the web the capacity of the column is larger when buckling occurs about the weak axis of the unreinforced section. The converse was observed when columns are reinforced with plates parallel to the flanges. Intermediate and long columns buckling about the strong axis of the rolled section, have a higher strength-to-yield ratio when the reinforcing plates are parallel to the flanges compared to columns reinforced with plates parallel to the web. The converse was observed for intermediate and long columns with reinforcing plates parallel to the web.

7. The effect of I-section to reinforcing plate area ratio on the predicted strength-to-yield ratio was found to be insignificant.

**Table 4.1 Fractional Factorial Design and Designations of the Numerical Models\***

		I-section				W310x179				W150x30																						
		Orientation of Reinforcing Plates		// Flange		// Flange		// Web		// Flange		// Web																				
		Size of Reinforcing Plates		180x9.52		290x25		290x16		350x16		350x25		350x16		130x5		130x8		175x5		175x8										
		Buckling Axis		W		S		W		S		W		S		W		S		W		S										
$\lambda^b$	$\delta_0^c$	IRS <sup>e</sup>	WRS <sup>f</sup>	$P_0^g$	$F_y^h$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)							
I	L/1000	1-1	CF <sub>y</sub>	0.4P <sub>u2</sub>	A	3	13																									
		1-2				5	14																									
		1-3				4																										
		1-4				6																										
		2-1				7																										
		2-2				8																										
		3-1				9	15																									
		3-2				10	16																									
		4-1				11																										
		4-2				12																										
		0.4				L/1000	1-3	CF <sub>y</sub>	0.6P <sub>u2</sub>	B	17																					
											18																					
19	36		84	100	115						130																					
21	38		86	102	117						132																					
20	37		85	101	116						131																					
22	39		87	103	118						133																					
23	40		88	104	119						134																					
24	41		89	105	120						135																					
25	42		90	106	121						136																					
28	45		93	109	124						139																					
27	44		92	108	123						138																					
29	46		94	110	125						140																					
1.1	L/1000 <sup>d</sup>	1-3	CF <sub>y</sub>	0.4P <sub>u2</sub>	C	26	43	91	107	122	137																					
						190																										
						191																										
						175	192	247	262																							
						177	194	249	264																							
						176	193	248	263																							
						178	195	250	265																							
						179	196	251	266																							
						180	197	252	267																							
						181	198	253	268																							
						184	201	256	271																							
						183	200	255	270																							
185	202	257	272																													
182	199	254	269																													

**Table 4.1 (cont'd)**

		I-section		W200x46		W310x179		W150x30																		
		Orientation of Reinforcing Plates		// Flange		// Flange		// Flange																		
		Size of Reinforcing Plates		180x9.52		290x25 290x16 350x16		130x5 130x8																		
		Buckling Axis		W S		W S W S W S		W S W S																		
$\lambda^b$	$\delta_0^c$	IRS <sup>e</sup>	WRS <sup>f</sup>	$P_0^g$	F <sup>h</sup>	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	
1.1	L/1000 <sup>d</sup>	1-4			B																					
		3-3	CF <sub>y</sub>	0.6P <sub>u2</sub>	B																					
		3-4																								
1.5	L/8000	1-3	CF <sub>y</sub>	0.6P <sub>u2</sub>	B			30										203								
		1-3	CF <sub>y</sub>	0.6P <sub>u2</sub>	B			31											204							
	L/2000			0.6P <sub>u2</sub>	B			32	47	95	111	126	141	156	171	186	205	258	273	224	243	288	303			
				CF <sub>y</sub>	C			34	49	98	113	128	143	158	173	188	207	260	275	226	245	290	305			
					B			33	48	97	112	127	142	157	172	187	206	259	274	225	244	289	304			
L/1000 <sup>d</sup>			0.4P <sub>u2</sub>	C			35	50	99	114	129	144	159	174	189	208	261	276	227	246	291	306				
			0.7CF <sub>y</sub>	B								96														

a) The number in the table refers to the finite element analysis model number described in detail in Appendix A and Appendix B.

b)  $\lambda$  - Slenderness parameter of the reinforced column

c)  $\delta_0$  - Initial imperfection of the unreinforced column presented by a ratio of the imperfection to the column length, L.

d) The allowable maximum initial imperfection varies for the column longer than 10 m.

e) IRS - Initial residual stress before welding

f) WRS - Residual stress after welding

g) P<sub>0</sub> - Preload.

h) F<sub>y</sub> - Yield strength of the reinforced column

P<sub>u2</sub> - Load carrying capacity of the unreinforced column predicted using the SSRC column curve 2

A - Yield strength of the rolled section and the cover plates = 260 MPa.

B - Yield strength of the rolled section and the cover plates = 300 MPa.

C - Yield strength of I-section = 230 MPa and yield strength of the cover plates = 350 MPa.

i) W - The weak axis of the rolled section

S - The strong axis of the rolled section

**Table 4.2 Models Used to Study the Effect of Initial Residual Stresses  
in the I-Section before Welding**

FEA model	Initial Residual Stresses before Welding			$P_{fea}/P_{ry}^c$
Number <sup>a</sup>	PS <sup>b</sup>	MF <sup>c</sup>	MP <sup>d</sup>	
(1)	(2)	(3)	(4)	(5)
<b>W200x46 column with 180x9.52 mm plates parallel to the flanges</b>				
<b>Buckling about the weak axis of the rolled section</b>				
3	1-1	0.3F <sub>y</sub>	0.15F <sub>y</sub>	0.65
5	1-2	0.1F <sub>y</sub>	0.15F <sub>y</sub>	0.65
7	2-1	0.3F <sub>y</sub>	0.15F <sub>y</sub>	0.63
8	2-2	0.1F <sub>y</sub>	0.15F <sub>y</sub>	0.66
9	3-1	0.3F <sub>y</sub>	0.15F <sub>y</sub>	0.65
10	3-2	0.1F <sub>y</sub>	0.15F <sub>y</sub>	0.66
11	4-1	0.3F <sub>y</sub>	0.15F <sub>y</sub>	0.60
12	4-2	0.1F <sub>y</sub>	0.15F <sub>y</sub>	0.65
<b>W310x179 column with 350x25 mm plates parallel to the web</b>				
<b>Buckling about the strong axis of the rolled section</b>				
82	3-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	0.54
83	3-4	0.1F <sub>y</sub>	0.3F <sub>y</sub>	0.54

- a) The FEA model number refers to the finite element analysis model number described in detail in Appendix A and Appendix B.
- b) PS - Designation of the residual stress pattern
- c) MF - Magnitude of the initial residual stresses before welding at the flange tips
- d) MP - Magnitude of the initial residual stresses before welding at the plate edges
- F<sub>y</sub> - Yield stress of the rolled section column
- e) P<sub>fea</sub> - load carrying capacity of the reinforced column obtained from the finite element analysis
- P<sub>ry</sub> - Yield strength of the reinforced column

**Table 4.3 Models Used to Study the Effect of Initial Residual Stresses in the Cover Plates before Welding**

FEA model Number <sup>a</sup>	Initial Residual Stresses before Welding			$P_{fea}/P_{ry}^c$
	PS <sup>b</sup>	MF <sup>c</sup>	MP <sup>d</sup>	
(1)	(2)	(3)	(4)	(5)
<b>W200x46 column with 180x9.52 mm plates parallel to the flanges</b>				
<b>Buckling about the weak axis of the rolled section</b>				
3	1-1	0.3F <sub>y</sub>	0.15F <sub>y</sub>	0.65
4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	0.65
5	1-2	0.1F <sub>y</sub>	0.15F <sub>y</sub>	0.65
6	1-4	0.1F <sub>y</sub>	0.3F <sub>y</sub>	0.65

- a) The FEA model number refers to the finite element analysis model number described in detail in Appendix A and Appendix B.
- b) PS - Designation of the initial residual stress pattern
- c) MF - Magnitude of the initial residual stresses before welding at the flange tips
- d) MP - Magnitude of the initial residual stresses before welding at the plate edges  
F<sub>y</sub> - Yield stress of the unreinforced column
- e) P<sub>fea</sub> - load carrying capacity of the reinforced column obtained from the finite element analysis  
P<sub>ry</sub> - Yield strength of the reinforced column

**Table 4.4 Models Used to Study the Effect of Varying Welding Residual Stress Magnitude**

FEA model Number <sup>a</sup>	Column Length L (mm)	Slenderness Parameter $\lambda$	Welding Residual Stress <sup>b</sup>	$P_{fea}/P_{ry}^c$
<b>W310x179 column with reinforcing plates 290x16 parallel to the flanges</b>				
<b>Buckling about the weak axis of the rolled section</b>				
90	7197	1.1	F <sub>y</sub>	0.56
91	7197	1.1	0.7F <sub>y</sub>	0.57
95	9813	1.5	F <sub>y</sub>	0.36
96	9813	1.5	0.7F <sub>y</sub>	0.36

- a) The FEA model number refers to the finite element analysis model number described in detail in Appendix A and Appendix B.
- b) F<sub>y</sub> - Yield stress of the steel of the rolled section
- c) P<sub>fea</sub> - load carrying capacity of the reinforced column obtained from the finite element analysis  
P<sub>ry</sub> - Yield strength of the reinforced column

**Table 4.5 Models Used to Study the Effect of the Initial Out-of-straightness**

FEA model Number <sup>a</sup>	Column Length L (mm)	Slenderness Parameter $\lambda$	$\delta_0^b$	$\delta_i^c$	$P_{fca}/P_{ry}^d$
(1)	(2)	(3)	(4)	(5)	(6)
W310x179 column with 290x25 mm plates parallel to the flangs					
Buckling about the weak axis of the rolled section					
17	2631	0.4	L/8000	L/7850	1.00
18	2631	0.4	L/2000	L/1930	0.99
19	2631	0.4	L/1000	L/970	0.97
23	7235	1.1	L/8000	L/7190	0.64
24	7235	1.1	L/2000	L/1790	0.60
25	7235	1.1	L/1000	L/820	0.56
30	9866	1.5	L/8000	L/6270	0.42
31	9866	1.5	L/2000	L/1570	0.38
32	9866	1.5	L/1000	L/790	0.35

- a) The FEA model number refers to the finite element analysis model number described in detail in Appendix A and Appendix B.
- b)  $\delta_0$  - Initial imperfection of the unreinforced rolled section column  
L - The column length
- c)  $\delta_i$  - Initial out-of-straightness of the reinforced column
- d)  $P_{fca}$  - load carrying capacity of the reinforced column obtained from the finite element analysis  
 $P_{ry}$  - Yield strength of the reinforced column

**Table 4.6 Models Used to Study the Effect of the Preload**

FEA model Number <sup>a</sup>	Slenderness Parameter $\lambda$	Preload, $P_0$		$P_{fea}/P_{ry}$ <sup>c</sup>
		$P_0$ (kN)	$P_0/P_{u2}$ <sup>b</sup>	
(1)	(2)	(3)	(4)	(5)
W310x179 column with 290x25 mm plates parallel to the flanges				
Buckling about the weak axis of the rolled section				
19	0.4	3760	0.6	0.97
20	0.4	2507	0.4	0.98
25	1.1	2152	0.6	0.56
27	1.1	1435	0.4	0.57
32	1.5	1401	0.6	0.35
33	1.5	934	0.4	0.36

- a) The FEA model number refers to the finite element analysis model number described in detail in Appendix A and Appendix B.
- b)  $P_{u2}$  - Load carrying capacity of the rolled section (predicted using SSRC curve 2);
- c)  $P_{fea}$  - load carrying capacity of the reinforced column obtained from the finite element analysis
- $P_{ry}$  - Yield strength of the reinforced column

**Table 4.7 Models Used to Study the Effect of the Steel Grades**

FEA model Number <sup>a</sup>	Slenderness Parameter $\lambda$	Yield Strength, $F_y$ (MPa)		$P_{fea}/P_{ry}^b$
		Rolled Section	Plates	
(1)	(2)	(3)	(4)	(5)
<b>W310x179 column with 290x25 mm plates parallel to the flanges</b>				
<b>Buckling about the weak axis of the rolled section</b>				
19	0.4	300	300	0.97
21	0.4	230	350	0.96
25	1.1	300	300	0.56
28	1.1	230	350	0.51
32	1.5	300	300	0.35
34	1.5	230	350	0.34
<b>W310x179 column with 290x25 mm plates parallel to the flanges</b>				
<b>Buckling about the strong axis of the rolled section</b>				
36	0.4	300	300	0.93
38	0.4	230	350	0.94
42	1.1	300	300	0.61
45	1.1	230	350	0.57
47	1.5	300	300	0.40
49	1.5	230	350	0.37
<b>W310x179 column with 350x16 mm plates parallel to the web</b>				
<b>Buckling about the weak axis of the rolled section</b>				
145	0.4	300	300	0.95
147	0.4	230	350	0.97
151	1.1	300	300	0.60
154	1.1	230	350	0.62
156	1.5	300	300	0.40
158	1.5	230	350	0.40
<b>W310x179 column with 350x16 mm plates parallel to the web</b>				
<b>Buckling about the strong axis of the rolled section</b>				
160	0.4	300	300	0.93
162	0.4	230	350	0.89
166	1.1	300	300	0.56
169	1.1	230	350	0.51
171	1.5	300	300	0.38
173	1.5	230	350	0.36

a) The FEA model number refers to the finite element analysis model number described in detail in Appendix A and Appendix B.

b)  $P_{fea}$  - load carrying capacity of the reinforced column obtained from the finite element analysis

$P_{ry}$  - Yield strength of the reinforced column



**Table 4.8 Models Used to Study the Effect of Reinforced Plate Orientations**

FEA model Number <sup>a</sup>	Cover Plates Orientation <sup>b</sup>	Column Length L (mm)	Slenderness Parameter $\lambda$	$\delta_i^c$	$P_{fea}/P_{ry}^d$
(1)	(2)	(3)	(4)	(5)	(6)
<b>W310x179 column with 350x16 mm plates and buckling about the weak axis of the I-section</b>					
115	F	2827	0.4	L/974	0.98
307	W	3720	0.4	L/979	0.97
121	F	7772	1.1	L/786	0.56
308	W	10229	1.1	L/789	0.62
126	F	10598	1.5	L/788	0.37
156	W	13948	1.5	L/791	0.40
<b>W310x179 column with 350x16 mm plates and buckling about the strong axis of the I-section</b>					
130	F	4928	0.4	L/974	0.94
160	W	4155	0.4	L/995	0.93
135	F	13551	1.1	L/1582	0.63
309	W	11425	1.1	L/1574	0.59
310	F	18479	1.5	L/1249	0.41
171	W	15579	1.5	L/1245	0.38

- a) The FEA model number refers to the finite element analysis model number described in detail in Appendix A and Appendix B.
- b) F - Cover plates parallel to the flanges  
W - Cover plates parallel to the web
- c)  $\delta_i$  - Initial out-of-Straightness of the reinforced column  
L - The column length
- d)  $P_{fea}$  - load carrying capacity of the reinforced column obtained from the finite element analysis  
 $P_{ry}$  - Yield strength of the reinforced column

**Table 4.9 Models Used to Study the Effect of the Buckling Axis**

FEA model Number <sup>a</sup>	Buckling Axis		Column Length (mm)	Slenderness Parameter $\lambda$	$\delta_i^b$	$P_{fea}/P_{ry}^c$
	Unreinforced Section	Reinforced Section				
(1)	(2)	(3)	(4)	(5)	(6)	(7)
<b>W150x30 column with 130x5 mm plates parallel to the flanges</b>						
175	Weak axis	Weak axis	1236	0.4	L/973	0.97
192	Strong axis	Strong axis	2300	0.4	L/964	0.95
181	Weak axis	Weak axis	3399	1.1	L/885	0.60
311	Strong axis	Strong axis	6326	1.1	L/858	0.63
186	Weak axis	Weak axis	4635	1.5	L/848	0.37
312	Strong axis	Strong axis	8626	1.5	L/808	0.44
<b>W310x179 column with 350x25 mm plates parallel to the web</b>						
313	Weak axis	Weak axis	4103	0.4	L/983	0.96
66	Strong axis	Strong axis	4030	0.4	L/984	0.93
314	Weak axis	Weak axis	11281	1.1	L/985	0.63
72	Strong axis	Strong axis	11083	1.1	L/1000	0.54
315	Weak axis	Weak axis	15383	1.5	L/1194	0.41
77	Strong axis	Strong axis	15113	1.5	L/1200	0.37
<b>W310x179 column with 350x16 mm plates parallel to the web</b>						
307	Weak axis	Weak axis	3720	0.4	L/974	0.97
160	Strong axis	Strong axis	4155	0.4	L/995	0.93
316	Weak axis	Weak axis	10229	1.1	L/1862	0.68
165	Strong axis	Strong axis	11425	1.1	L/1870	0.60
317	Weak axis	Weak axis	13948	1.5	L/1227	0.42
171	Strong axis	Strong axis	15579	1.5	L/1235	0.38

- a) The FEA model number refers to the finite element analysis model number described in detail in Appendix A and Appendix B.
- b)  $\delta_i$  - Initial out-of-Straightness of the reinforced column  
L - The column length
- c)  $P_{fea}$  - load carrying capacity of the reinforced column obtained from the finite element analysis  
 $P_{ry}$  - Yield strength of the reinforced column

**Table 4.10 Models Used to Study the Effect of Cover Plate Size**

FEA model number <sup>a</sup>	I-Section	Plate	I-section Area Plate Area	Column Length (mm)	Slenderness parameter $\lambda$	$P_{fea}/P_{ry}$ <sup>b</sup>
(1)	(2)	(3)	(4)	(5)	(6)	(7)
19	W310x179	290x25	1.57	2631	0.4	0.97
84	W310x179	290x16	2.46	2617	0.4	0.98
115	W310x179	350x16	2.04	2827	0.4	0.98
25	W310x179	290x25	1.57	7235	1.1	0.56
90	W310x179	290x16	2.46	7197	1.1	0.56
121	W310x179	350x16	2.04	7772	1.1	0.56
32	W310x179	290x25	1.57	9866	1.5	0.35
95	W310x179	290x16	2.46	9813	1.5	0.36
126	W310x179	350x16	2.04	10598	1.5	0.36

a) The FEA model number refers to the finite element analysis model number described in detail in Appendix A and Appendix B.

b)  $P_{fea}$  - load carrying capacity of the reinforced column obtained from the finite element analysis

$P_{ry}$  - Yield strength of the reinforced column

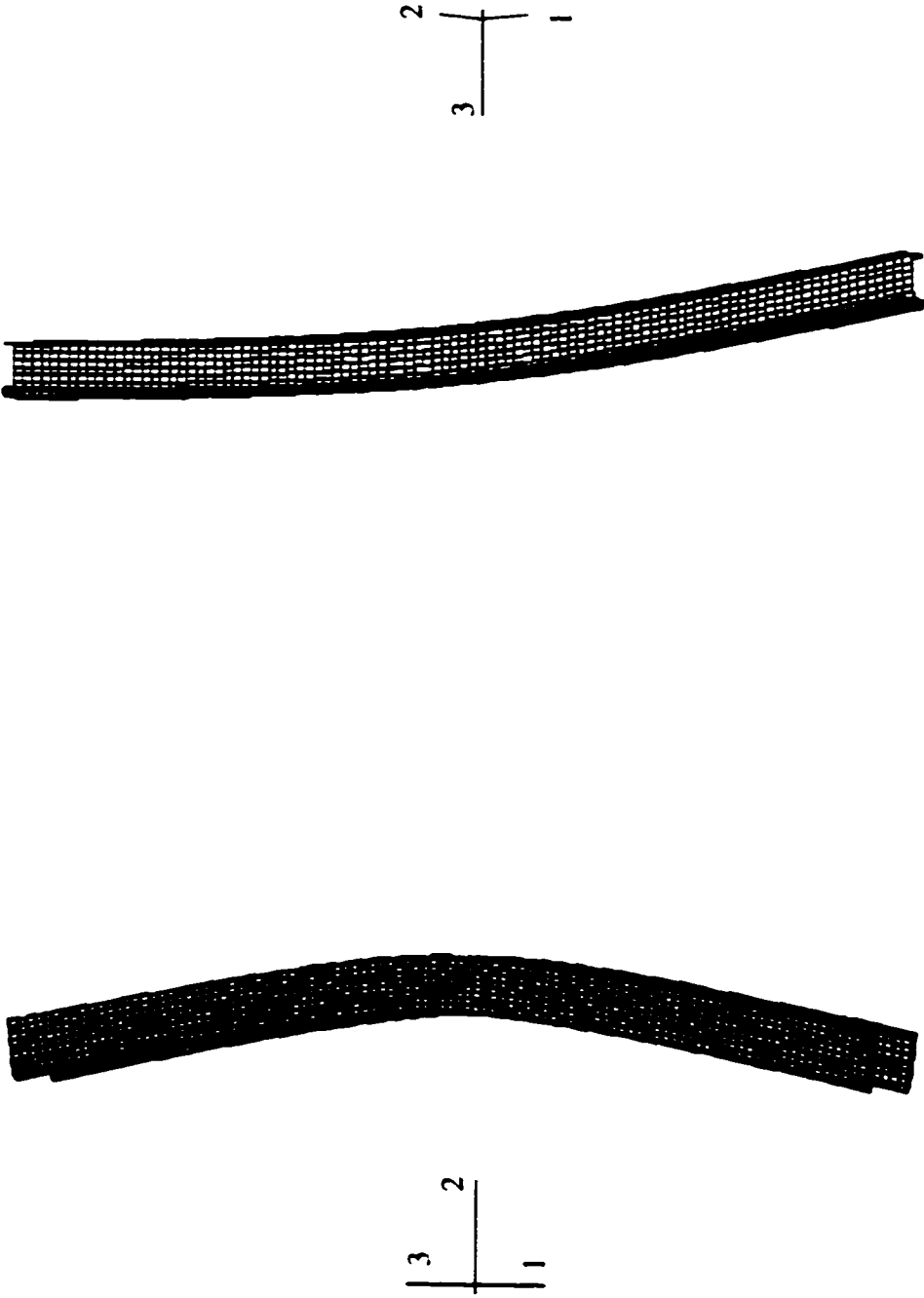
**Table 4.11 Models Used to Study the Effect of the Size of the I-section**

FEA model Number <sup>a</sup>	I-Section	Plate	I-section Area Plate Area	Column Length L (mm)	Slenderness parameter $\lambda$	$P_{fea}/P_{ry}$ <sup>b</sup>
(1)	(2)	(3)	(4)	(5)	(6)	(7)
19	W310x179	290x25	1.57	2631	0.4	0.97
175	W150x30	130x5	2.92	1236	0.4	0.97
25	W310x179	290x25	1.57	7235	1.1	0.56
181	W150x30	130x5	2.92	3399	1.1	0.60
32	W310x179	290x25	1.57	9866	1.5	0.35
186	W150x30	130x5	2.92	4635	1.5	0.37

a) The FEA model number refers to the finite element analysis model number described in detail in Appendix A and Appendix B.

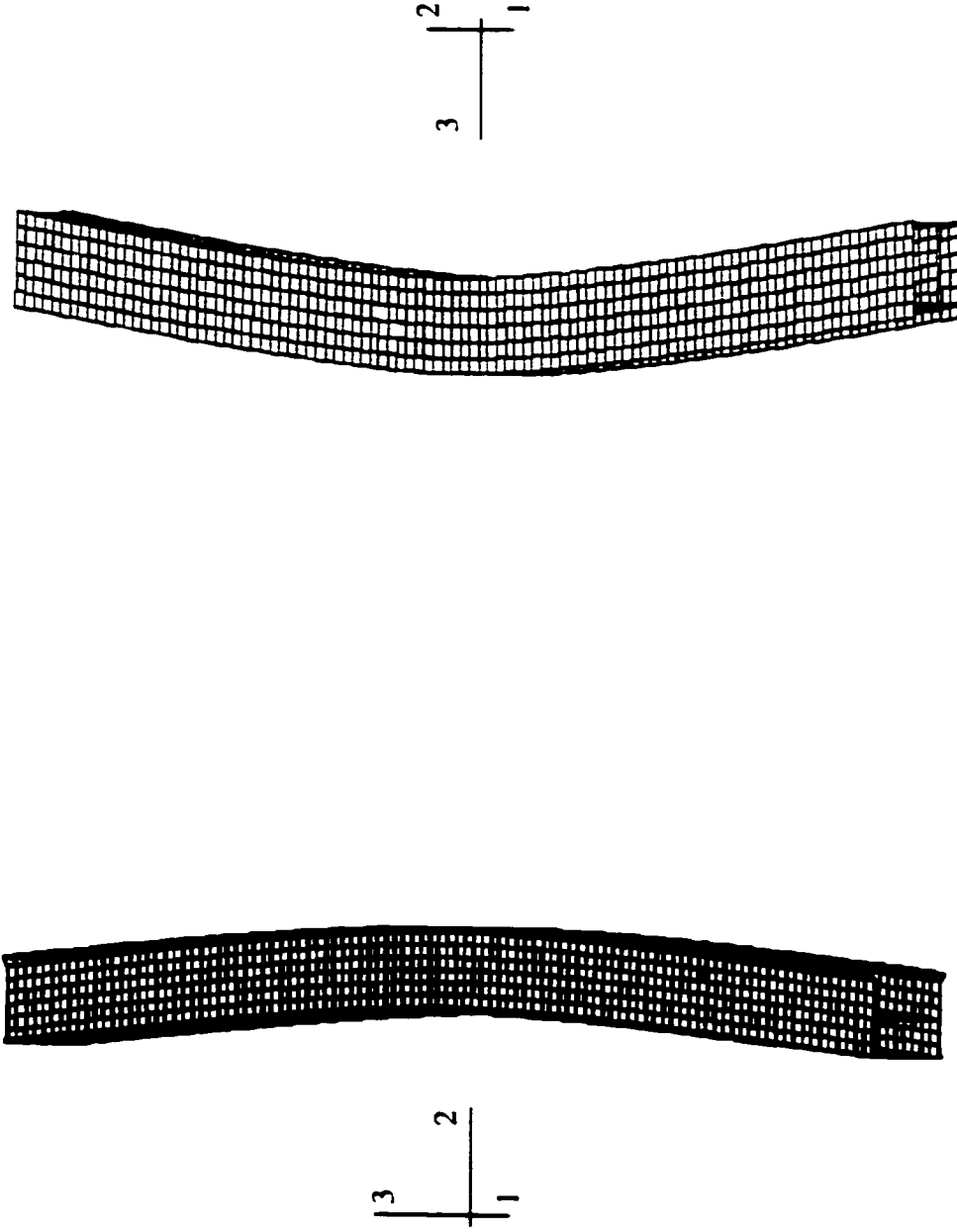
b)  $P_{fea}$  - load carrying capacity of the reinforced column obtained from the finite element analysis

$P_{ry}$  - Yield strength of the reinforced column



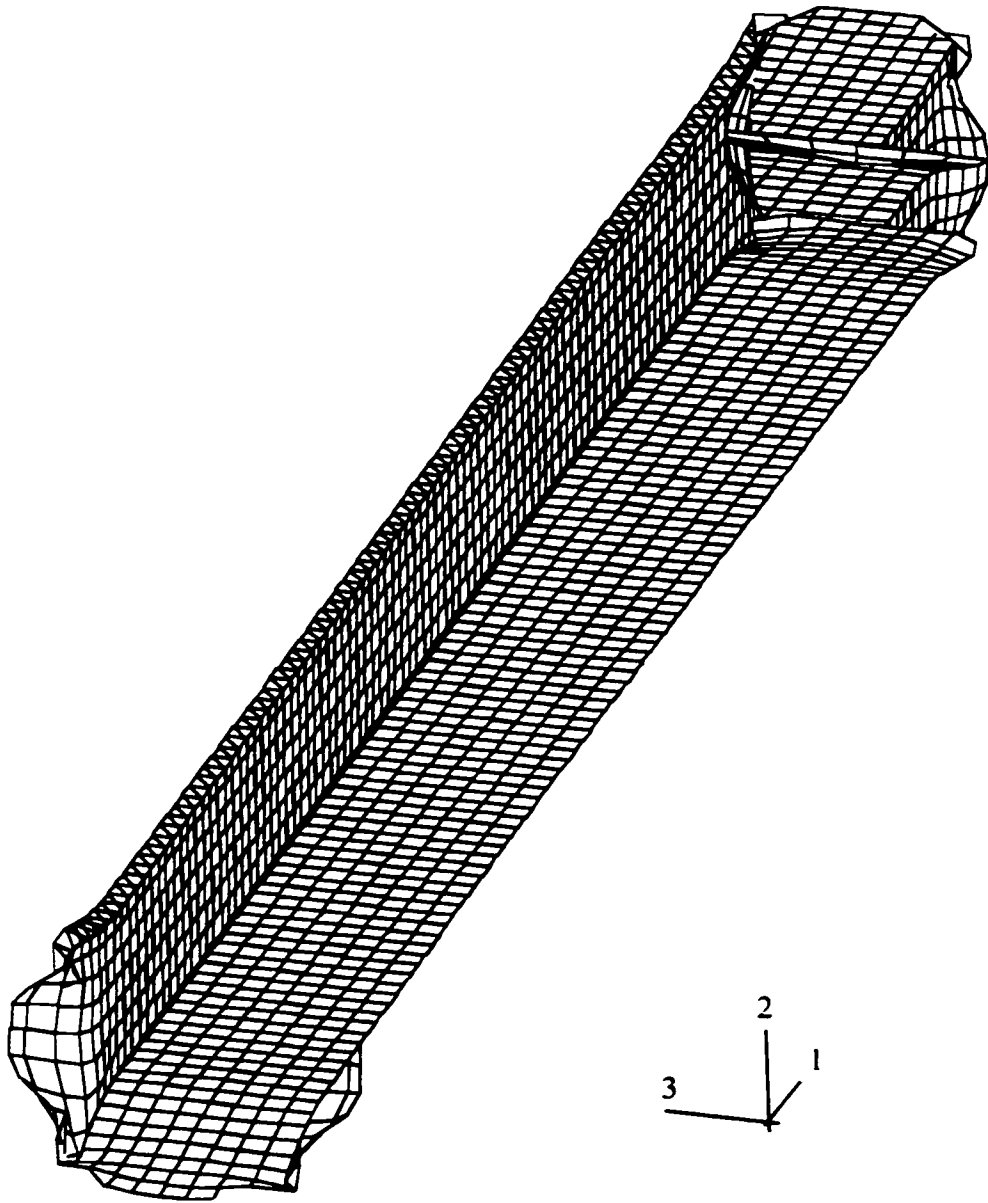
a) Buckling about the Weak Axis of the Rolled Section      b) Buckling about the Strong Axis of the Rolled Section

**Figure 4.1 Deformed Shape of Columns Reinforced with Plates Parallel to the Flanges**



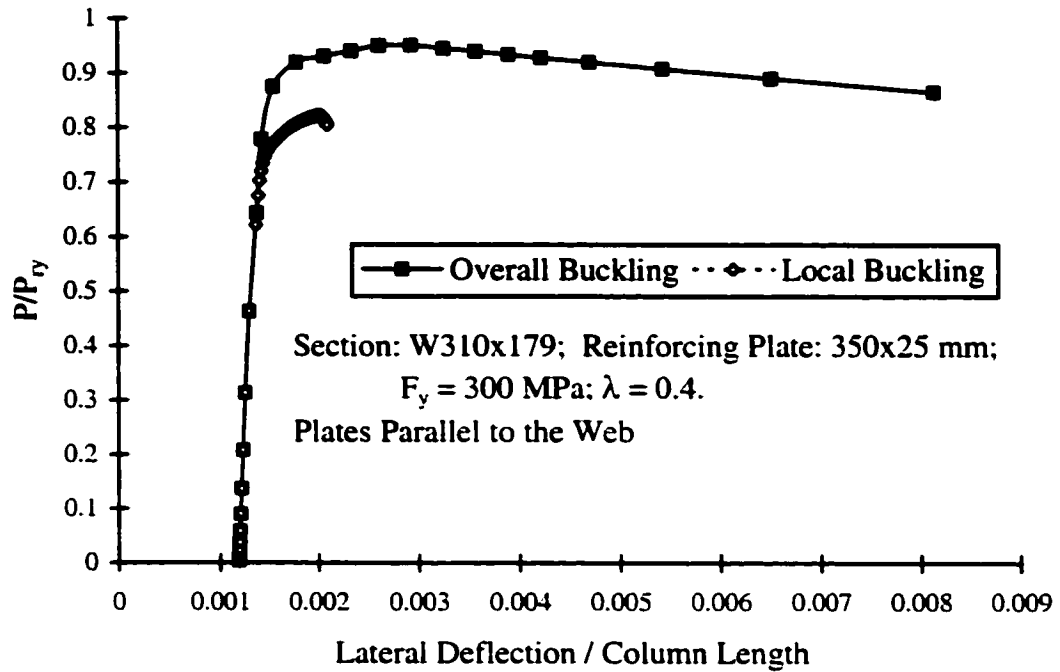
a) Buckling about the Weak Axis of the Rolled Section      b) Buckling about the Strong Axis of the Rolled Section

**Figure 4.2 Deformed Shape of Columns Reinforced with Plates Parallel to the Web**

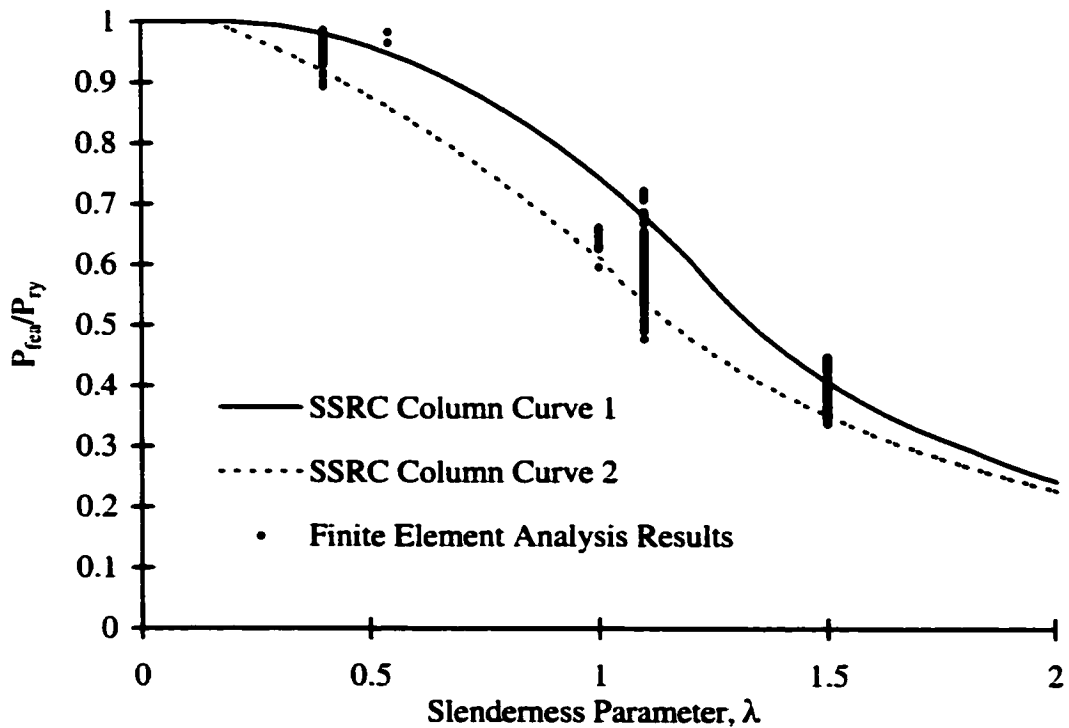


Section: W310x179; Reinforced Plate: 350x25 mm;  
 $F_y = 300 \text{ MPa}$ ;  $\lambda = 0.4$

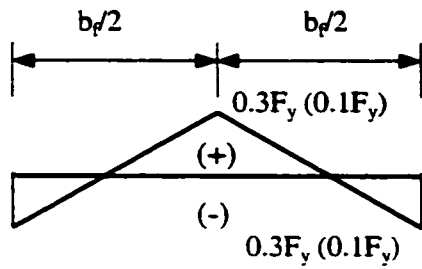
**Figure 4.3 Local Buckled Shape of the Column Reinforced with Plates Parall to the Web**



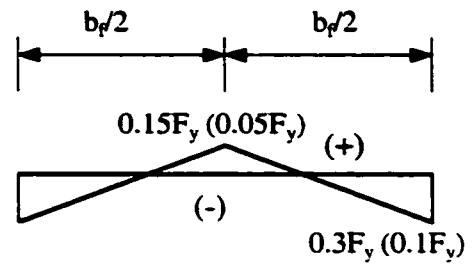
**Figure 4.4 Load versus Lateral Deflection Curves for the Columns with Overall Buckling and Local Buckling**



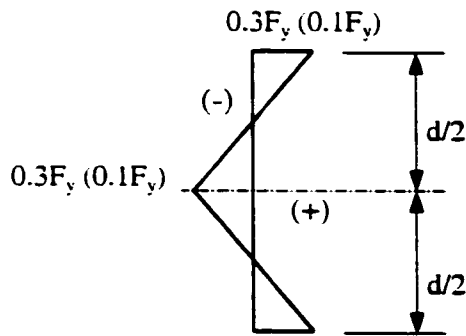
**Figure 4.5 Strength of All Reinforced Column Samples**



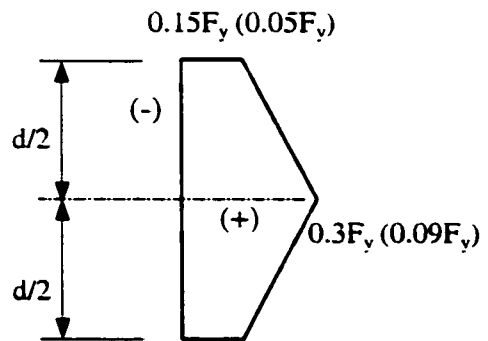
(a) In the Flanges



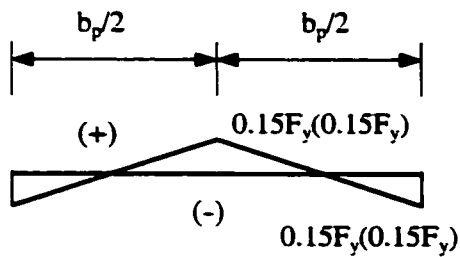
(a) In the Flanges



(b) In the Web

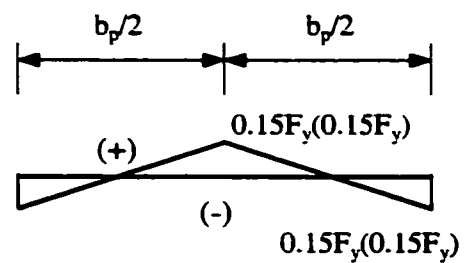


(b) In the Web



(c) In the Plates

Pattern 1-1 (1-2)

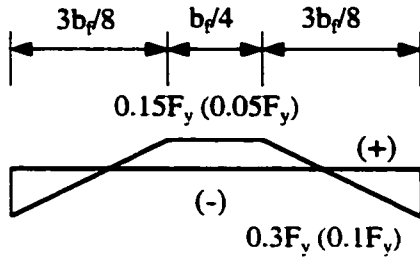


(c) In the Plates

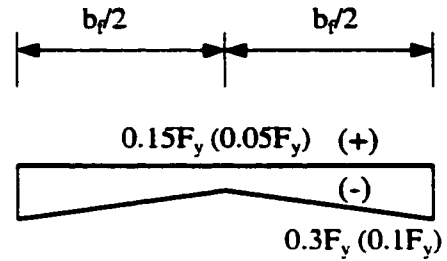
Pattern 2-1 (2-2)

**Figure 4.6 Initial Residual Stress Patterns before Welding**

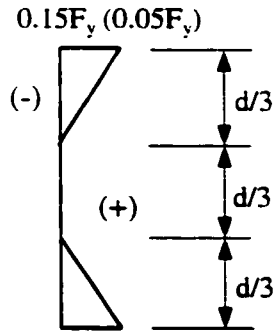




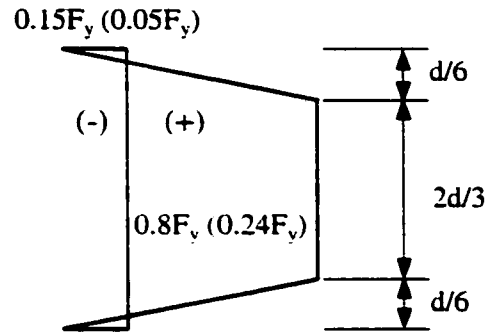
(a) In the Flanges



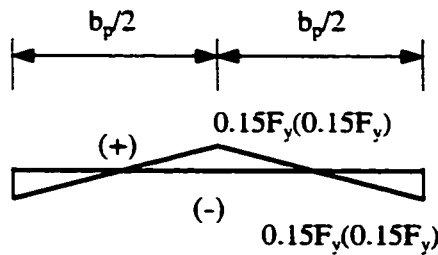
(a) In the Flanges



(b) In the Web

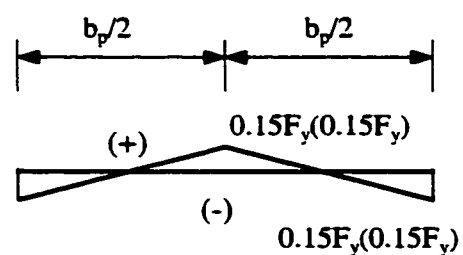


(b) In the Web



(c) In the Plates

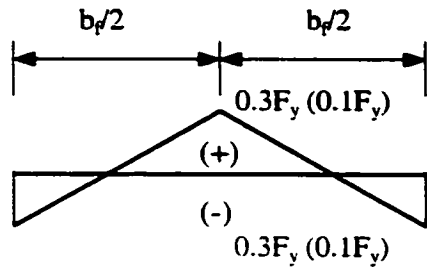
Pattern 3-1 (3-2)



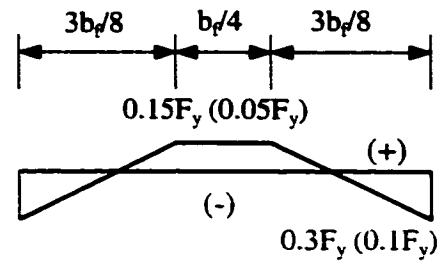
(c) In the Plates

Pattern 4-1 (4-2)

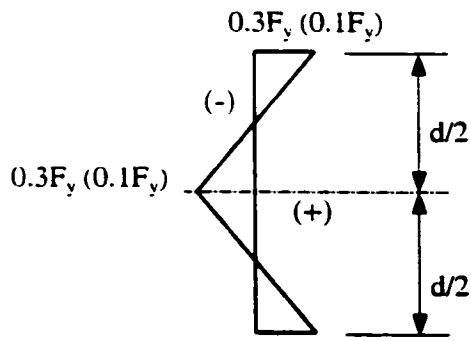
Figure 4.6 (Cont'd)



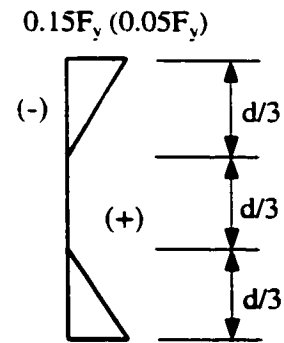
(a) In the Flanges



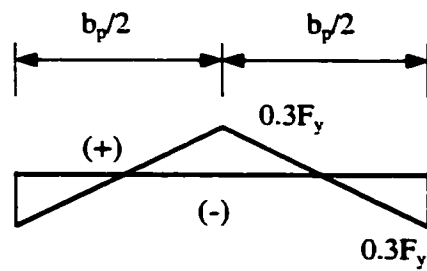
(a) In the Flanges



(b) In the Web

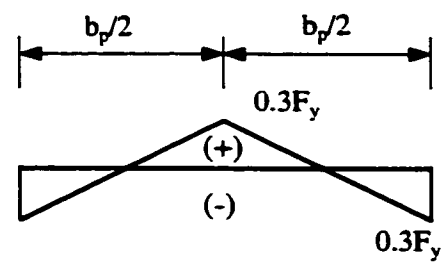


(b) In the Web



(c) In the Plates

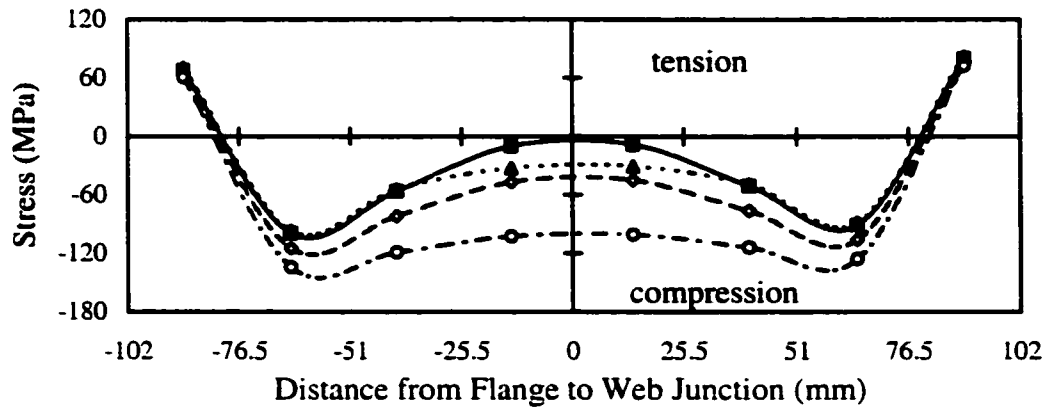
Pattern 1-3 (1-4)



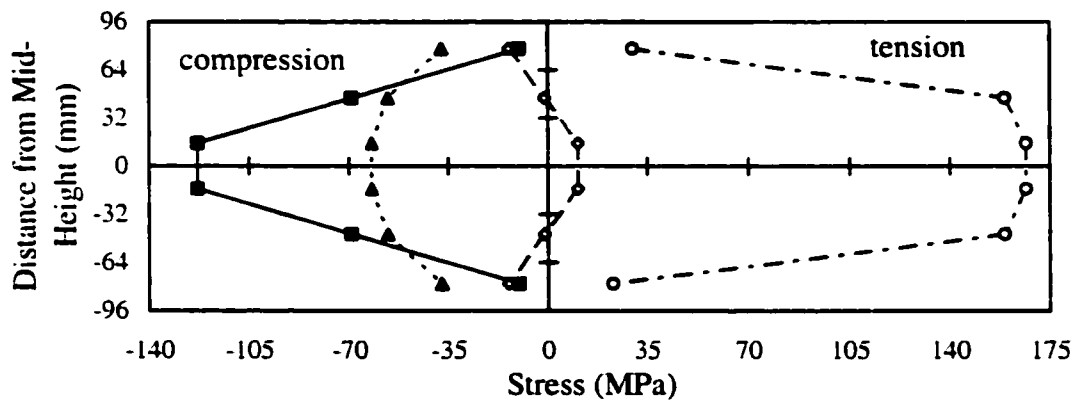
(c) In the Plates

Pattern 3-3 (3-4)

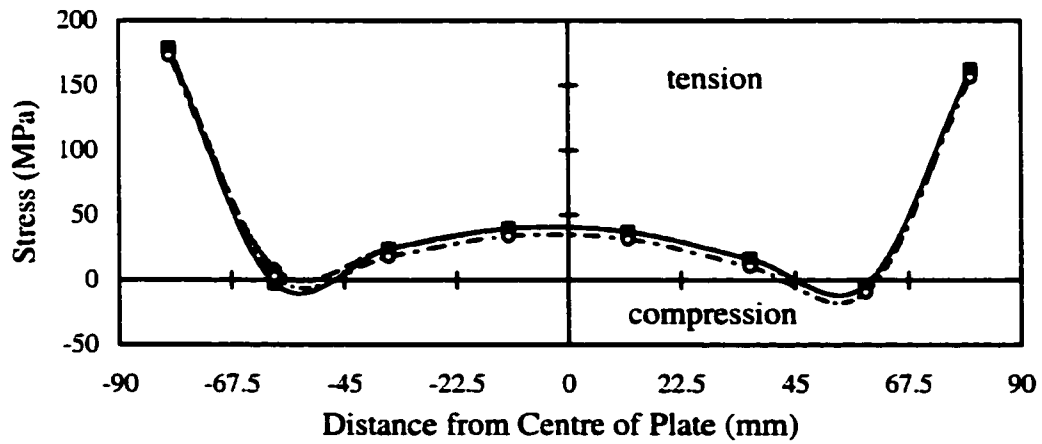
**Figure 4.6 (Cont'd)**



a) In the Flanges



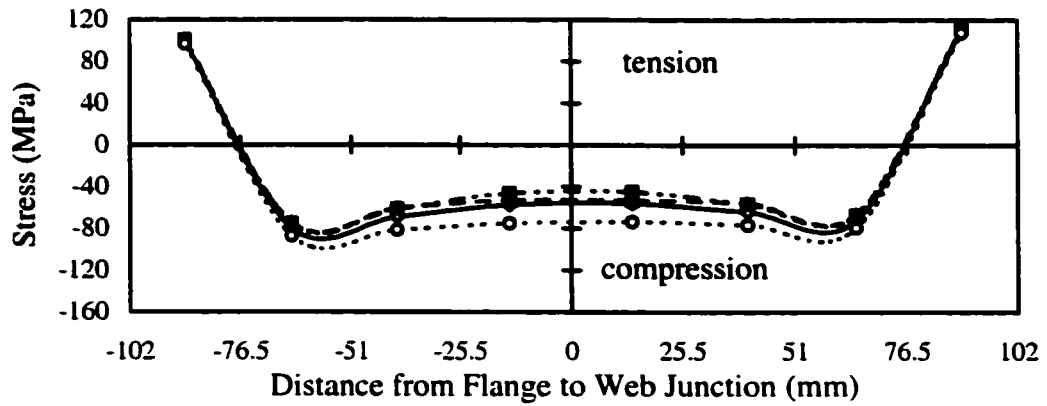
b) In the Web



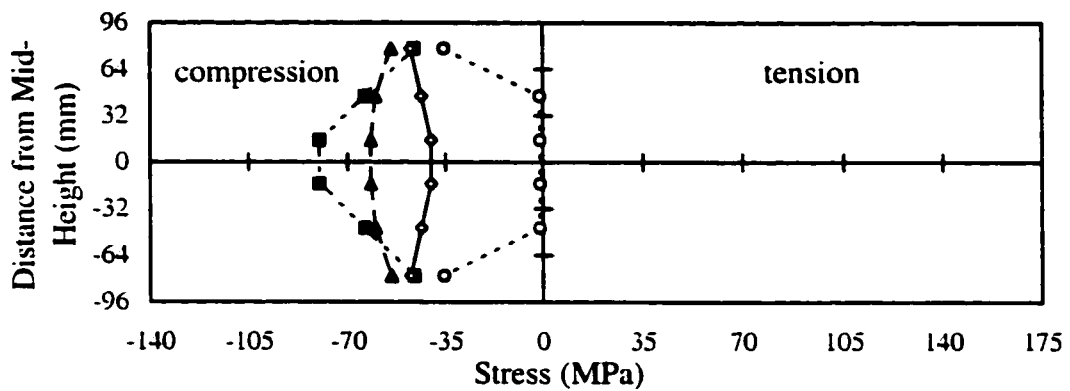
c) In the Reinforcing Plates

—■— Pattern 1-1    -◇- Pattern 2-1    ···▲··· Pattern 3-1    -○- Pattern 4-1  
 Section: W200x46; Reinforcing Plate: 180x9.52 mm;  $F_y = 260$  MPa

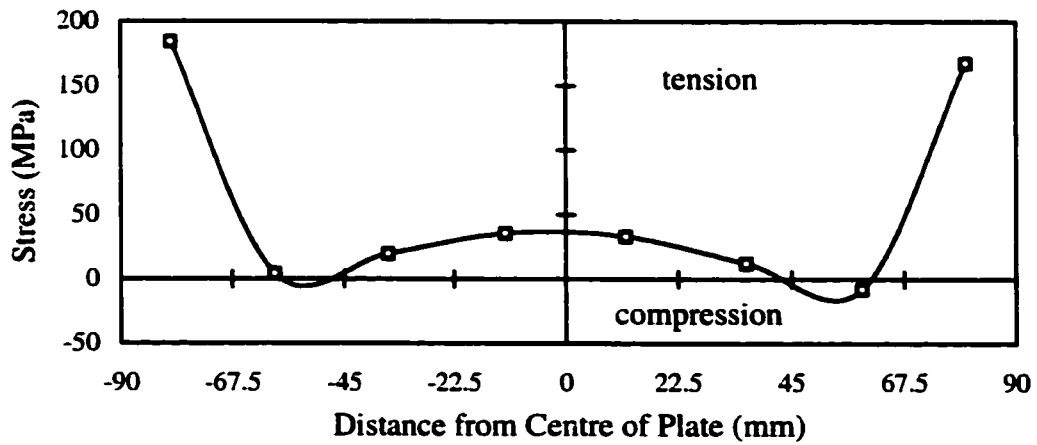
**Figure 4.7 Residual Stress Distributions after Welding for Maximum Initial Residual Stress of  $0.3 F_y$**



a) In the Flanges



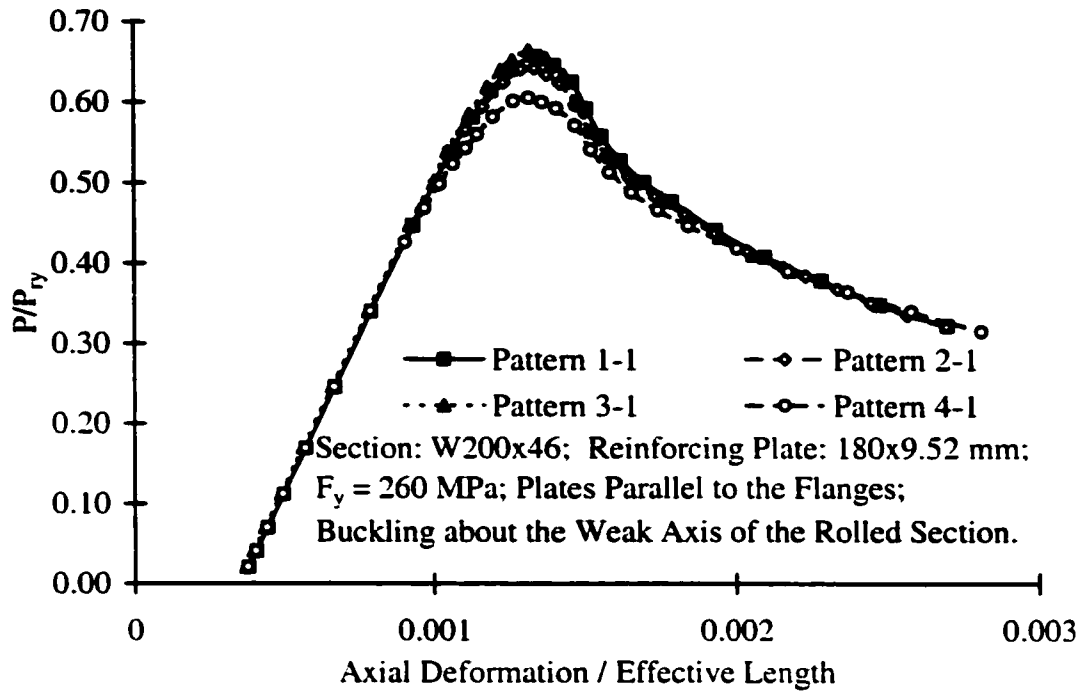
b) In the Web



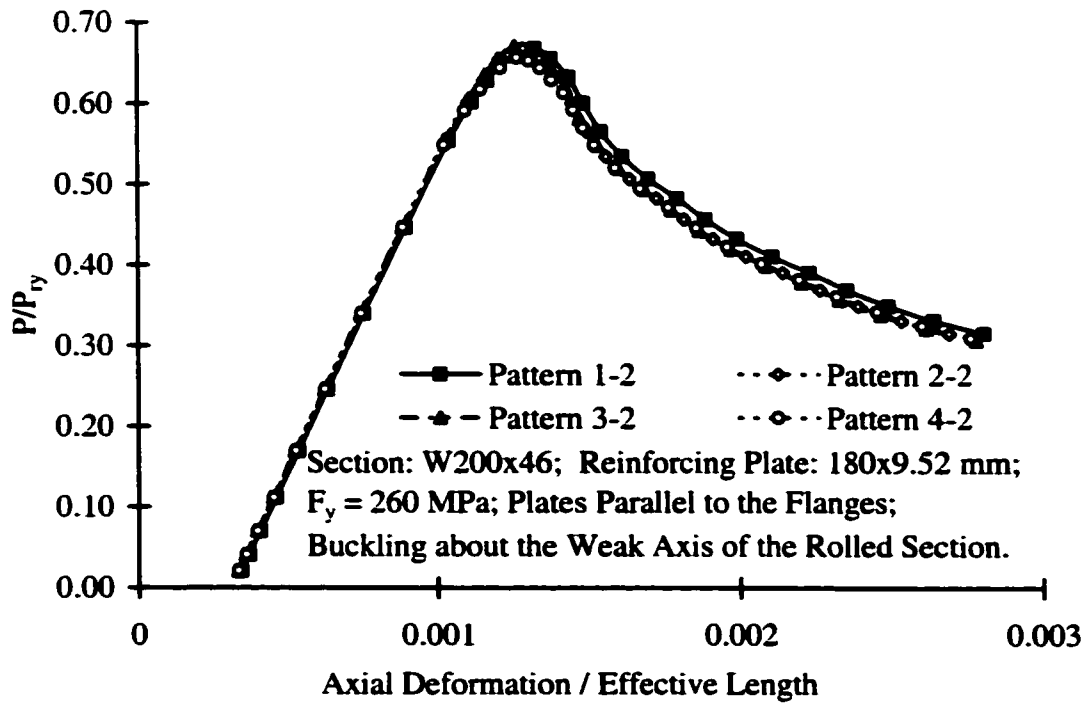
c) In the Reinforcing Plates

-■- Pattern 1-2    -○- Pattern 2-2    -▲- Pattern 3-2    ··◇·· Pattern 4-2  
 Section: W200x46; Reinforcing Plate: 180x9.52 mm;  $F_y = 260$  MPa

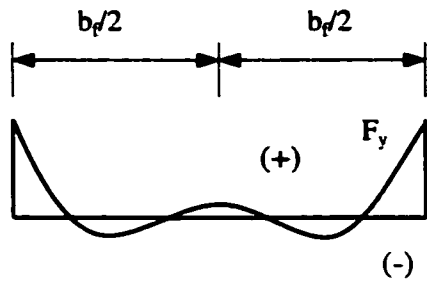
**Figure 4.8 Residual Stress Distributions after Welding for Maximum Initial Residual Stress of  $0.1F_y$**



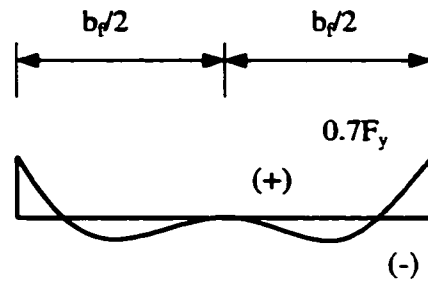
**Figure 4.9 Effect of Initial Residual Stress Patterns for Maximum Magnitude of  $0.3F_y$**



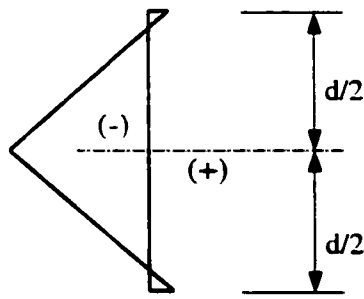
**Figure 4.10 Effect of Initial Residual Stress Patterns for Maximum Magnitude of  $0.1F_y$**



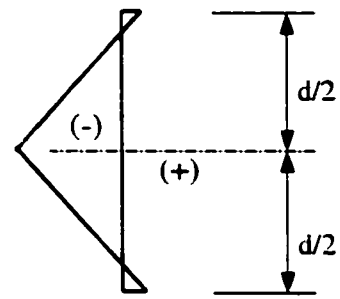
In the Flanges



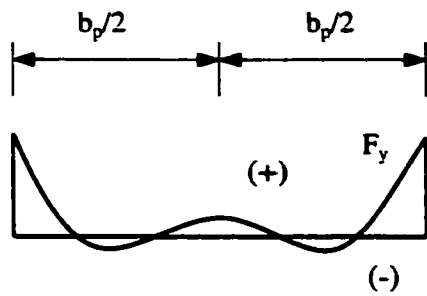
In the Flanges



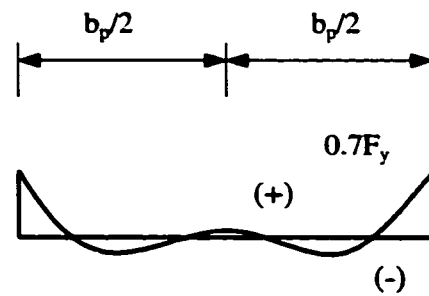
In the Web



In the Web



In the Plates

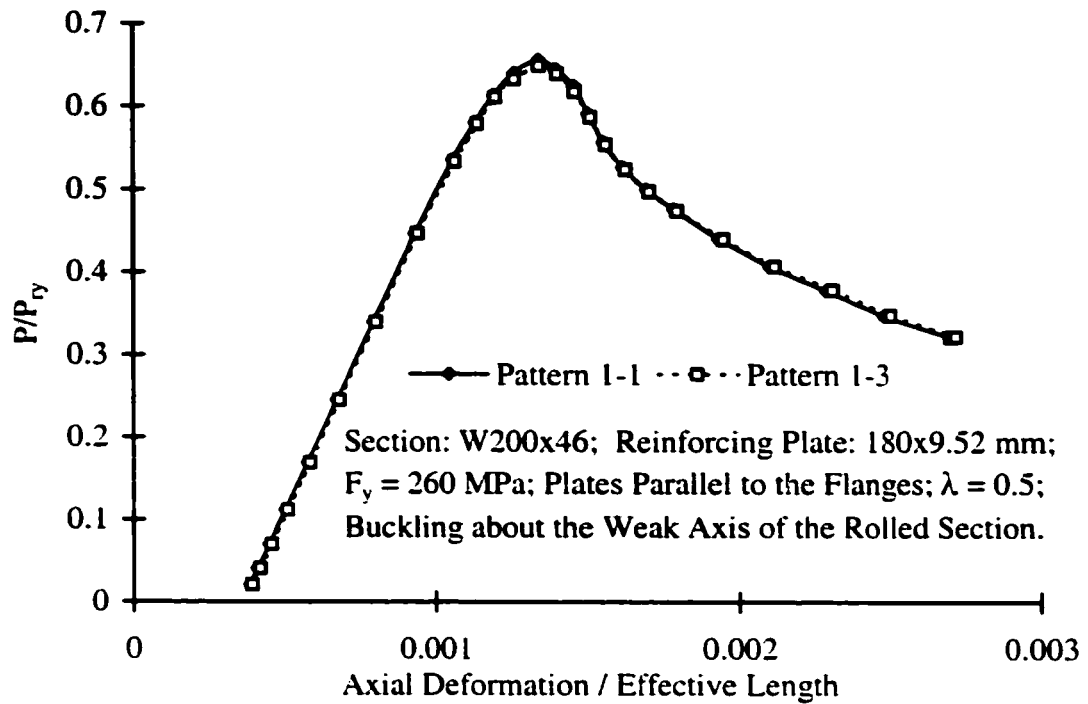


In the Plates

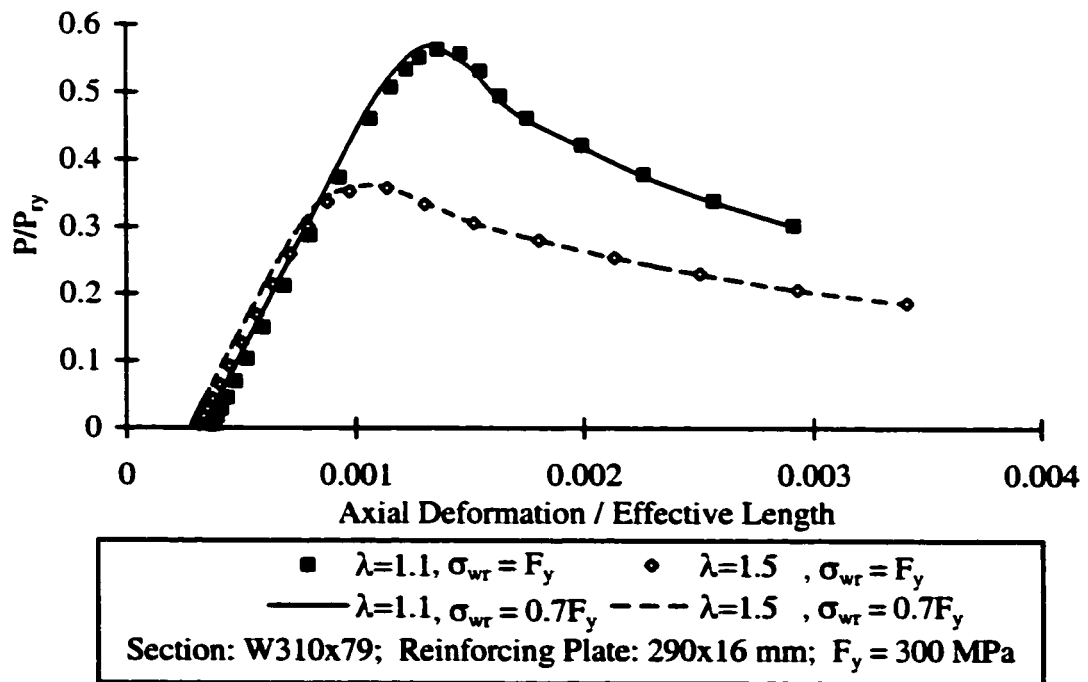
(a) Maximum Magnitude =  $1.0F_y$

(b) Maximum Magnitude =  $0.7F_y$

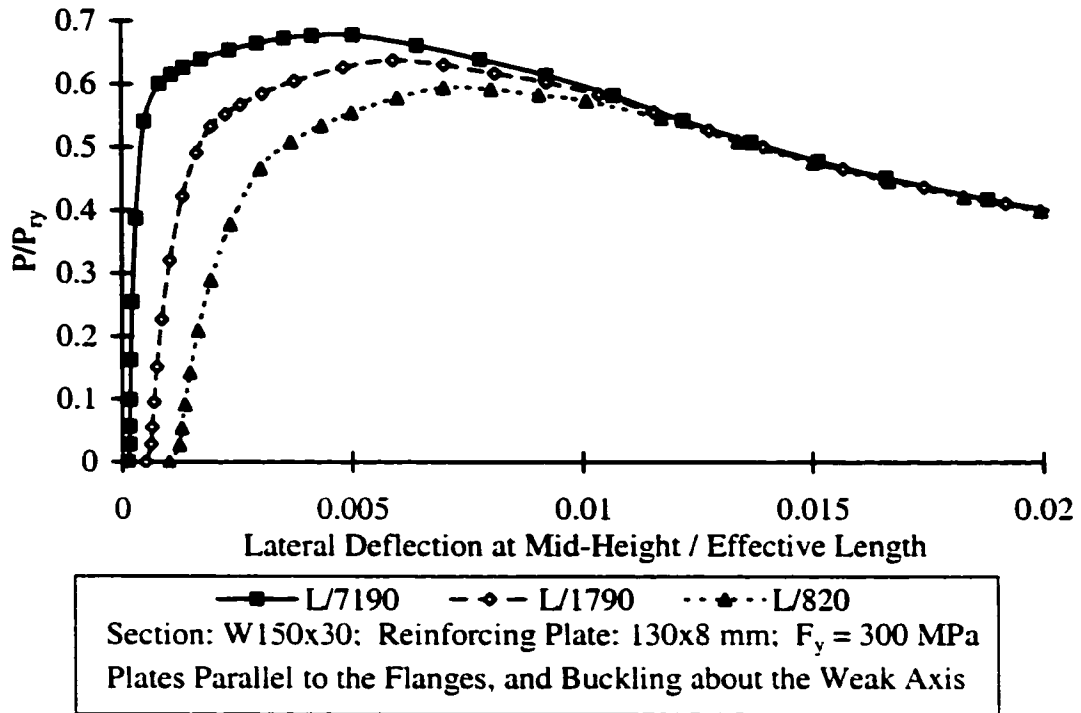
**Figure 4.11 Residual Stress Patterns after Welding**



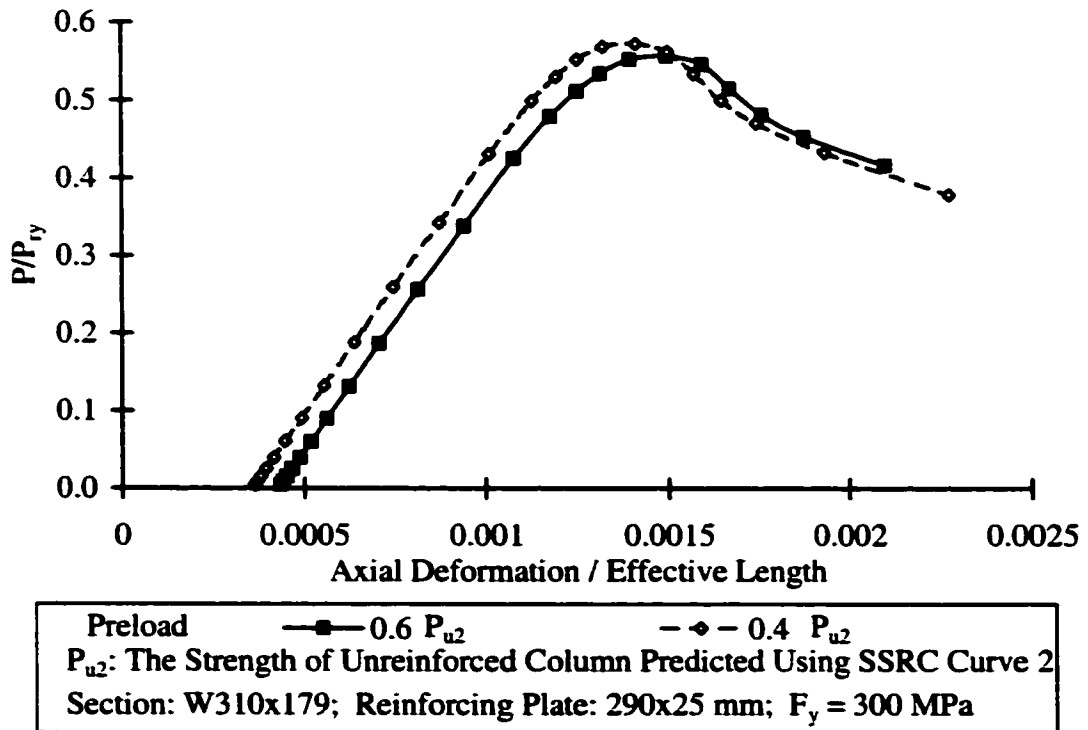
**Figure 4.12 Effect of Varying Initial Residual Stress Patterns before Welding in the Cover Plates**



**Figure 4.13 Effect of Varying Welding Residual Stresses with Different Slenderness Ratios**

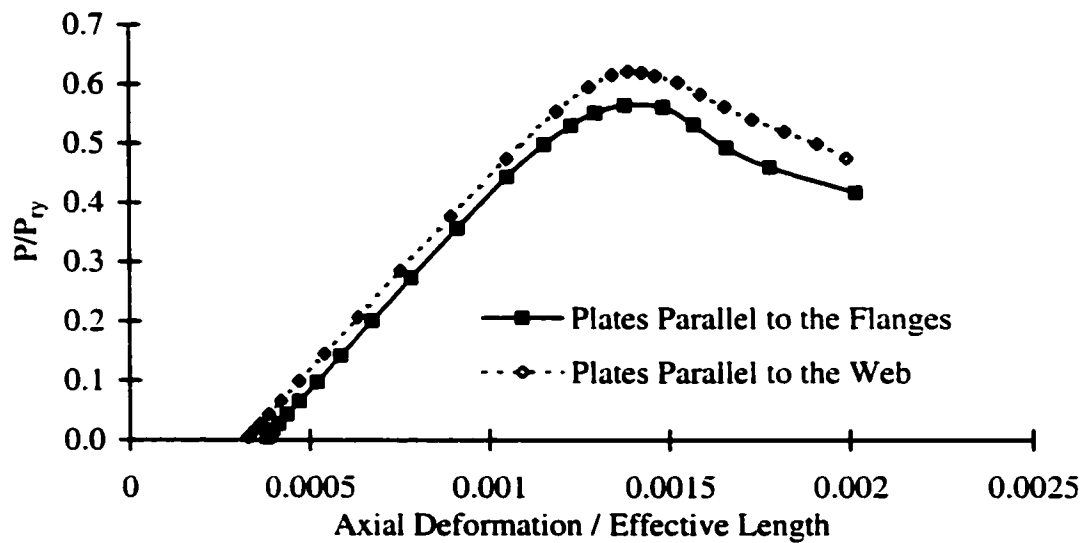


**Figure 4.14 Effect of the Initial Out-of-straightness**  
( $\lambda = 1.1$ )



**Figure 4.15 Effect of the Preload Magnitudes** ( $\lambda=1.1$ )





Section: W310x179; Reinforcing Plate: 350x16 mm;  $F_y = 300$  MPa  
 Columns Buckle about the Weak Axis of the Rolled Section.

**Figure 4.16 Effect of the Reinforcing Plate Orientation**  
 ( $\lambda = 1.1$ )

## Chapter 5

### Limit States Design

#### 5.1 Background

A statistical-based design philosophy, which provides a uniform level of safety for various structural components, is used for the design of steel structures in Canada. Considering the variation in the resistance and load effects, corresponding resistance and load factors are determined using statistical analysis. The resistance factor for steel columns reinforced with welded steel plates is the prime concern in this research.

##### 5.1.1 Column Resistance Based on CAN/CSA-S16.1-94

Based on the investigation conducted by Bjorhovde (1972), three strength curves have been developed to predict the strength of steel columns of different shapes and types (Johnston, 1976). The Structural Stability Research Council proposed the equations used to describe these column curves, and these three curves are therefore called SSRC column curves. CSA standard CAN3-S16.1-M84 - "Steel Structures for Building – Limit States Design" (Canadian Standards Association, 1984) adopted the first two SSRC curves for the design of steel columns. The equations for the first two SSRC curves are in five parts and are expressed as follows:

###### SSRC column curve 1

- |     |                                  |  |       |
|-----|----------------------------------|--|-------|
| (1) | For $0 \leq \lambda \leq 0.15$   | $C_r = \phi A F_y$ (stub column)                           |       |
| (2) | For $0.15 \leq \lambda \leq 1.2$ | $C_r = \phi A F_y (0.990 + 0.122\lambda - 0.367\lambda^2)$ |       |
| (3) | For $1.2 \leq \lambda \leq 1.8$  | $C_r = \phi A F_y (0.051 + 0.801\lambda^{-2})$             | [5.1] |
| (4) | For $1.8 \leq \lambda \leq 2.8$  | $C_r = \phi A F_y (0.008 + 0.942\lambda^{-2})$             |       |
| (5) | For $2.8 \leq \lambda$           | $C_r = \phi A F_y \lambda^{-2}$                            |       |

###### SSRC column curve 2

- |     |                                  |  |  |
|-----|----------------------------------|--|--|
| (1) | For $0 \leq \lambda \leq 0.15$   | $C_r = \phi A F_y$ (stub column)                           |  |
| (2) | For $0.15 \leq \lambda \leq 1.0$ | $C_r = \phi A F_y (1.035 - 0.202\lambda - 0.222\lambda^2)$ |  |

$$(3) \quad \text{For } 1.0 \leq \lambda \leq 2.0 \quad C_r = \phi A F_y (-0.111 + 0.636\lambda^{-1} + 0.087\lambda^{-2}) \quad [5.2]$$

$$(4) \quad \text{For } 2.0 \leq \lambda \leq 3.6 \quad C_r = \phi A F_y (0.009 + 0.877\lambda^{-2})$$

$$(5) \quad \text{For } 3.6 \leq \lambda \quad C_r = \phi A F_y \lambda^{-2}$$

where Part (5) of equations [5.1] and [5.2] corresponds to the Euler's elastic buckling resistance. SSRC column curve 1 represents a higher strength than SSRC column curve 2. In the Canadian standard, SSRC columns curve 1 is adopted for the hollow structural shape of Class H, which are sections that are hot formed or cold formed followed by stress relieving, and for welded wide flange sections with flanges made of flame cut plates. These particular sections possess higher strength in compression because of the more favourable residual stress pattern present in these sections. All other sections are designed based on SSRC column curve 2. It should be noted that Equations [5.1] and [5.2] provide a factored resistance, with the resistance factor,  $\phi$ , taken as 0.9 for columns.

In 1995, Loov proposed a double exponential equation using a single parameter,  $n$ , to replace the five-part equations proposed by SSRC. This expression, which was adopted by the CSA standard CAN/CSA-S16.1-94 (Canadian Standards Association, 1994), takes the following form:

$$C_r = \phi A F_y (1 + \lambda^{2n})^{-1/n} \quad [5.3]$$

where  $n = 2.24$  for CSA column curve 1, corresponding to SSRC curve 1, and  $n = 1.34$  for CSA column curve 2, corresponding to SSRC curve 2. It was demonstrated that this expression never deviates by more than approximately 3% from the corresponding values given by Equations [5.1] and [5.2] (Loov, 1996).

### **5.1.2 Principles of Limit States Design**

Limit states design is a design method that requires the structure not to exceed the limit states that govern its strength and behaviour for any realistic load or load combinations. There are basically two categories of limit states that are pertinent to the structural design process: ultimate limit states (ULS) and serviceability limit states (SLS). Ultimate limit states deal with strength conditions for the structure. Exceeding an ULS

implies a local or overall structural failure. On the other hand, exceeding a serviceability limit state means that a structure is not behaving or serving in the way it was intended to. The SLS therefore considers the performance under normal operating conditions. In the design of columns, serviceability limit states are seldom a concern. The following, therefore, focuses on ultimate limit states.

The general criterion for an ultimate limit state can be expressed in the following form:

$$\phi R \geq \alpha' S \quad [5.4]$$

where  $R$  is the nominal resistance,  $\phi$  is the resistance factor,  $S$  is the nominal value of the load effect, and  $\alpha'$  is the load factor.

It is clearly understood that there is always a possibility that failure will occur. In order to ensure that the probability of failure is acceptably small, the factors  $\alpha'$  and  $\phi$  have to be set at a suitable value by applying the principles of probability theory to the statistical analysis of the load effects and the resistance.

Figure 5.1 shows possible distribution curves for the load effect,  $S$ , and the resistance,  $R$ . The variables are assumed to be statistically independent. Galambos and Ravindra (1973a) combined the two curves to produce a risk frequency distribution curve, as illustrated in Figure 5.2. The probability of failure is equivalent to the probability of the ratio  $R/S$  being less than 1.0 (the load effect exceeding the resistance), or the natural log of  $(R/S)$  being less than 0. This probability of failure is therefore a function of the distance  $\beta\sigma_{\ln(R/S)}$  shown in Figure 5.2, which provides the margin of safety. The factor  $\beta$  is called the safety index and  $\sigma_{\ln(R/S)}$  is the standard deviation of the natural log of  $(R/S)$ . The probability of failure can be set at any desired level by selecting an appropriate value of  $\beta$ . The safety index is, therefore, a measure of the safety or reliability of the structure. Galambos and Ravindra (1973a) proposed a first order simplification method to express the safety index,  $\beta$ , in algebraic form. In Figure 5.2,

$$\sigma_{\ln(R/S)}^2 \cong \left(\frac{\partial \ln(R/S)}{\partial R}\right)^2 \cdot \sigma_R^2 + \left(\frac{\partial \ln(R/S)}{\partial S}\right)^2 \cdot \sigma_S^2 = \frac{\sigma_R^2}{R^2} + \frac{\sigma_S^2}{S^2} = V_R^2 + V_S^2 \quad [5.5]$$

Thus,

$$\beta(V_R^2 + V_S^2)^{1/2} = \ln \overline{R/S} \quad [5.6]$$

from which,

$$\overline{R/S} = e^{\beta(V_R^2 + V_S^2)^{1/2}} \quad [5.7]$$

Then,

$$\beta = \frac{\ln \frac{\overline{R}}{\overline{S}}}{(V_R^2 + V_S^2)^{1/2}} \quad [5.8]$$

Allen (1975) proposed a more accurate expression for the safety index as follows:

$$\beta = \frac{\ln \left[ \frac{\overline{R}}{\overline{S}} \left( \frac{1 + V_S^2}{1 + V_R^2} \right)^{1/2} \right]}{\ln \left[ (1 + V_S^2)(1 + V_R^2) \right]^{1/2}} \quad [5.9]$$

Based on the investigation of Allen (1975) and the work done by Galambos and Ravindra (1973b), a value of 3.0 was adopted for  $\beta$  for most members of building structures in Canada.

Lind (1971) proposed an approximate equation as follows:

$$(V_R^2 + V_S^2)^{1/2} = \alpha(V_R + V_S) \quad [5.10]$$

where  $\alpha$  is called a separation variable. Galambos and Ravindra (1973b) extended this concept further by introducing two separation variables,  $\alpha_R$  and  $\alpha_S$ , such that:

$$(V_R^2 + V_S^2)^{1/2} = \alpha_R V_R + \alpha_S V_S \quad [5.11]$$

Galambos and Ravindra (1973b, 1977) also used an error minimization process to demonstrate that a single value of  $\alpha = 0.55$  could be used for the conservative approximate equation [5.10], leading to an acceptably small error. Substituting Equation [5.10] into Equation [5.8] results in the following expression for the safety index:

$$\beta = \frac{\ln \frac{\bar{R}}{\bar{S}}}{\alpha(V_R + V_S)} \quad [5.12]$$

Solving for the mean value of resistance,  $\bar{R}$ , we obtain

$$\bar{R} = \bar{S} e^{\beta\alpha(V_R + V_S)} \quad [5.13]$$

which can be rewritten as follows

$$\bar{R} e^{-\beta\alpha V_R} = \bar{S} e^{\beta\alpha V_S} \quad [5.14]$$

Equation [5.14] relates to the mean values of the resistance and load effect. If the ratio of the mean to nominal value of the resistance (also called the bias coefficient for the resistance) is expressed as

$$\rho_R = \frac{\bar{R}}{R} \quad [5.15]$$

and the ratio of the mean to nominal value of the load effect (the bias coefficient of the load effect) is expressed as

$$\rho_S = \frac{\bar{S}}{S} \quad [5.16]$$

Equation [5.14] can be rewritten as follows

$$\rho_R \cdot e^{(-\beta\alpha V_R)} \cdot R = \rho_S \cdot e^{(\beta\alpha V_S)} \cdot S \quad [5.17]$$

from which, compared with [5.4], the resistance factor,  $\phi$ , can be defined as

$$\phi = \rho_R \cdot e^{(-\beta\alpha V_R)} \quad [5.18]$$

and the load effect factor,  $\alpha'$ , can be defined as

$$\alpha' = \rho_S \cdot e^{(\beta\alpha V_S)} \quad [5.19]$$

The ratio of the mean to the nominal resistance,  $\rho_R$ , of a member consists of three parts: the ratio of the mean to the nominal cross-sectional properties,  $\rho_G$ ; the ratio of the mean to the nominal material properties,  $\rho_M$ ; and the professional ratio,  $\rho_P$  (i.e., the ratio of the actual load carrying capacity of a column to that predicted by the design equation). The professional ratio indicates how well the design equation fits the test results. Then,

$$\rho_R = \rho_G \cdot \rho_M \cdot \rho_P \quad [5.20]$$

The above ratios are assumed to be independent random variables. Therefore, the coefficient of variation for the resistance,  $V_R$ , of a member is given by:

$$V_R^2 = V_G^2 + V_M^2 + V_P^2 \quad [5.21]$$

where  $V_G$ ,  $V_M$ , and  $V_P$  are the coefficient of variation associated with  $\rho_G$ ,  $\rho_M$ , and  $\rho_P$  respectively.

### 5.1.3 Determination of the Resistance of a Steel Column

For steel columns, CSA standards CAN3-S16.1-M84 and CAN/CSA-S16.1-94 use different equations based on different approximations, i.e., the SSRC column curves and the CSA curves as shown in Equation [5.1] through [5.2] and [5.3] respectively. In general, the factored resistance of a steel column can be expressed as:

$$C_r = \phi \cdot A \cdot F_y \cdot f(\lambda) \quad [5.22]$$

where  $f(\lambda)$  is a function of the non-dimensional slenderness ratio,  $\lambda$ , defined in Equation [4.1]. Therefore, the mean-to-nominal ratio of the resistance,  $\rho_R$  for intermediate columns becomes

$$\rho_R = \rho_A \cdot \rho_{F_y} \cdot \rho_{f(\lambda)} \cdot \rho_P \quad [5.23]$$

Because the slenderness parameter,  $\lambda$ , is a function of the yield strength,  $F_y$ , the two terms  $F_y$  and  $f(\lambda)$  can be grouped as

$$F = F_y \cdot f(\lambda) \quad [5.24]$$

where  $F$  is a function of the terms  $F_y$  and  $f(\lambda)$ , and

$$\bar{F} = \bar{F}_y \cdot f(\bar{\lambda}) \quad [5.25]$$

Thus,

$$\rho_F = \rho_{F_y} \cdot \rho_{f(\lambda)} \quad [5.26]$$

As discussed above, the properties governing the strength of intermediate columns are the yield strength,  $F_y$ , the slenderness parameter,  $\lambda$ . As a result, Equation [5.23] becomes

$$\rho_R = \rho_A \cdot \rho_F \cdot \rho_P \quad [5.27]$$

and the coefficient of variation,  $V_R$ , for intermediate columns follows

$$V_R = (V_A^2 + V_F^2 + V_P^2)^{1/2} \quad [5.28]$$

However, the resistance of stub columns depends on the area of the column,  $A$ , and yield strength,  $F_y$ , as shown in the first part of equations [5.1] and [5.2]. Therefore, the mean-to-nominal ratio of the resistance,  $\rho_R$ , for short columns becomes

$$\rho_R = \rho_A \cdot \rho_{F_y} \cdot \rho_P \quad [5.29]$$

and the coefficient of variation,  $V_R$ , for short columns follows

$$V_R = (V_A^2 + V_{F_y}^2 + V_P^2)^{1/2} \quad [5.30]$$

Furthermore, the resistance of slender columns depends on the moment of inertia of the cross-section,  $I$ , and elastic modulus of the column,  $E$ , as seen by comparing Equation [4.1] with the fifth part of equations [5.1] and [5.2]. Therefore, the mean-to-nominal ratio of the resistance,  $\rho_R$ , for slender columns becomes

$$\rho_R = \rho_I \cdot \rho_E \cdot \rho_P \quad [5.31]$$

and the coefficient of variation,  $V_R$ , for slender columns follows

$$V_R = (V_I^2 + V_E^2 + V_P^2)^{1/2} \quad [5.32]$$

The values for the mean-to-nominal ratios and the coefficients of variation for the cross-sectional properties are investigated by using simple statistical analyses. The method of probability study for the professional factor is presented in section 5.2.3. The following derivation of  $\rho_F$  and  $V_F$  follow the work of Kennedy and Gad Aly (1980).

#### 5.1.4 The Material Factor

The bias coefficient and the coefficients of variation for the material factor vary with the different design criteria for the resistance of a steel column. The column equations of clause 13.3.1 from CAN3-S16.1-M84 (identical to SSRC column curve 2) are taken in this sub-section to illustrate the procedure for determining the bias coefficients and the coefficients of variation for the material factor in the research. From a comparison of Equation [5.22] with Equation [5.2], the expressions for  $f(\lambda)$  are obtained as follows:



- (1)  $f(\lambda) = 1.0$  , for  $0 \leq \lambda \leq 0.15$
- (2)  $f(\lambda) = 1.035 - 0.202\lambda - 0.222\lambda^2$  , for  $0.15 \leq \lambda \leq 1.0$
- (3)  $f(\lambda) = -0.111 + 0.636\lambda^{-1} + 0.087\lambda^{-2}$  , for  $1.0 \leq \lambda \leq 2.0$  [5.33]
- (4)  $f(\lambda) = 0.009 + 0.877\lambda^{-2}$  , for  $2.0 \leq \lambda \leq 3.6$
- (5)  $f(\lambda) = \lambda^{-2}$  , for  $3.6 \leq \lambda$

Thus, the mean of  $f(\lambda)$  is:

- (1)  $f(\bar{\lambda}) = 1.0$  , for  $0 \leq \bar{\lambda} \leq 0.15$
- (2)  $f(\bar{\lambda}) = 1.035 - 0.202\bar{\lambda} - 0.222\bar{\lambda}^2$  , for  $0.15 \leq \bar{\lambda} \leq 1.0$
- (3)  $f(\bar{\lambda}) = -0.111 + 0.636\bar{\lambda}^{-1} + 0.087\bar{\lambda}^{-2}$  , for  $1.0 \leq \bar{\lambda} \leq 2.0$  [5.34]
- (4)  $f(\bar{\lambda}) = 0.009 + 0.877\bar{\lambda}^{-2}$  , for  $2.0 \leq \bar{\lambda} \leq 3.6$
- (5)  $f(\bar{\lambda}) = \bar{\lambda}^{-2}$  , for  $3.6 \leq \bar{\lambda}$

The mean-to-nominal ratio of  $\lambda$  is defined as

$$\rho_\lambda = \frac{\bar{\lambda}}{\lambda} \quad [5.35]$$

Based on the definition of the slenderness parameter,  $\lambda$ , given by Equation [4.1], the mean-to-nominal ratio of the slenderness parameter can be obtained as

$$\rho_\lambda = \left( \frac{\rho_{F_y}}{\rho_r^2 \cdot \rho_E} \right)^{1/2} \quad [5.36]$$

Combining [5.25], [5.34], and [5.35] gives

- (1)  $\bar{F} = \bar{F}_y$  , for  $0 \leq \lambda \leq 0.15$
- (2)  $\bar{F} = \bar{F}_y (1.035 - 0.202\lambda\rho_\lambda - 0.222\lambda^2\rho_\lambda^2)$  , for  $0.15 \leq \lambda \leq 1.0$
- (3)  $\bar{F} = \bar{F}_y (-0.111 + 0.636\lambda^{-1}\rho_\lambda^{-1} + 0.087\lambda^{-2}\rho_\lambda^{-2})$  , for  $1.0 \leq \lambda \leq 2.0$  [5.37]
- (4)  $\bar{F} = \bar{F}_y (0.009 + 0.877\lambda^{-2}\rho_\lambda^{-2})$  , for  $2.0 \leq \lambda \leq 3.6$

$$(5) \quad \bar{F} = \bar{F}_y (\lambda^{-2} \rho_\lambda^{-2}) \quad , \text{ for } 3.6 \leq \lambda$$

Thus, the mean-to-nominal ratio for the material factor with SSRC column curve 2 can be deduced as follows:

$$\begin{aligned} (1) \quad \rho_F &= \rho_{F_y} \quad , \text{ for } 0 \leq \lambda \leq 0.15 \\ (2) \quad \rho_F &= \rho_{F_y} \frac{(1.035 - 0.202\lambda\rho_\lambda - 0.222\lambda^2\rho_\lambda^2)}{(1.035 - 0.202\lambda - 0.222\lambda^2)} \quad , \text{ for } 0.15 \leq \lambda \leq 1.0 \\ (3) \quad \rho_F &= \rho_{F_y} \frac{(-0.111 + 0.636\lambda^{-1}\rho_\lambda^{-1} + 0.087\lambda^{-2}\rho_\lambda^{-2})}{(-0.111 + 0.636\lambda^{-1} + 0.087\lambda^{-2})} \quad , \text{ for } 1.0 \leq \lambda \leq 2.0 \quad [5.38] \\ (4) \quad \rho_F &= \rho_{F_y} \frac{(0.009 + 0.877\lambda^{-2}\rho_\lambda^{-2})}{(0.009 + 0.877\lambda^{-2})} \quad , \text{ for } 2.0 \leq \lambda \leq 3.6 \\ (5) \quad \rho_F &= \rho_E \rho_r^2 \quad , \text{ for } 3.6 \leq \lambda \end{aligned}$$

The mean-to-nominal ratios for the material factors derived from the other criteria can be obtained using the same procedure.

Applying the definition of the associated coefficient of variation,  $V_F$ , gives

$$V_F = \frac{\sigma_F}{\bar{F}} \quad [5.39]$$

Having assumed that the variables affecting  $F$ , that is,  $F_y$ ,  $r$ , and  $E$ , are independent, fundamental statistical equations for the standard deviation (Kennedy and Neville, 1976) are adopted to calculate the value of  $V_F$  as follows:

$$\sigma_F = \left[ \left( \frac{\partial \bar{F}}{\partial \bar{F}_y} \right)^2 \cdot \sigma_{F_y}^2 + \left( \frac{\partial \bar{F}}{\partial r} \right)^2 \cdot \sigma_r^2 + \left( \frac{\partial \bar{F}}{\partial E} \right)^2 \cdot \sigma_E^2 \right]^{1/2} \quad [5.40]$$

The components in Equation [5.40] can be obtained respectively. Using the second part of SSRC curve 2 as an example, the terms in [5.40] can be obtained as follows:

$$\begin{aligned} (1) \quad \left( \frac{\partial \bar{F}}{\partial \bar{F}_y} \right)^2 \cdot \sigma_{F_y}^2 &= (1.035 - 0.303\lambda - 0.444\lambda^2)^2 \cdot \frac{\bar{F}_y^2}{\bar{F}_y^2} \cdot \sigma_{F_y}^2 \\ &= (1.035 - 0.303\lambda - 0.444\lambda^2)^2 \cdot \bar{F}_y^2 \cdot V_{F_y}^2 \\ &= P_1^2 \cdot \bar{F}_y^2 \cdot V_{F_y}^2 \end{aligned}$$

$$(2) \left( \frac{\partial \bar{F}}{\partial r} \right)^2 \cdot \sigma_r^2 = (0.202\bar{\lambda} + 0.444\bar{\lambda}^2)^2 \cdot \frac{\bar{F}_y^2}{r^2} \cdot \sigma_r^2 \quad [5.41]$$

$$= (0.202\lambda + 0.444\lambda^2)^2 \cdot \bar{F}_y \cdot V_r^2$$

$$= P_2^2 \cdot \bar{F}_y \cdot V_r^2$$

$$(3) \left( \frac{\partial \bar{F}}{\partial E} \right)^2 \cdot \sigma_E^2 = (0.101\lambda + 0.222\lambda^2)^2 \cdot \frac{\bar{F}_y^2}{E^2} \cdot \sigma_E^2$$

$$= (0.101\lambda + 0.222\lambda^2)^2 \cdot \bar{F}_y \cdot V_E^2$$

$$= P_3^2 \cdot \bar{F}_y \cdot V_E^2$$

where  $P_1$ ,  $P_2$ , and  $P_3$  present the portions of Eq. [5.41] that are functions of  $\lambda$ . Therefore, Eq. [5.40] can be expressed as

$$\sigma_F = \bar{F}_y \cdot (P_1^2 \cdot V_{F_y}^2 + P_2^2 \cdot V_r^2 + P_3^2 \cdot V_E^2)^{1/2} \quad [5.42]$$

and thus [5.39], in general form, becomes

$$V_F = \frac{\sigma_F}{\bar{F}} = \frac{\bar{F}_y \cdot (P_1^2 \cdot V_{F_y}^2 + P_2^2 \cdot V_r^2 + P_3^2 \cdot V_E^2)^{1/2}}{\bar{F}_y \cdot f(\bar{\lambda})} \quad [5.43]$$

$$= \frac{(P_1^2 \cdot V_{F_y}^2 + P_2^2 \cdot V_r^2 + P_3^2 \cdot V_E^2)^{1/2}}{f(\bar{\lambda})}$$

### 5.1.5 Summary

In general, the statistical quantities  $\rho_R$  and  $V_R$  for the SSRC curves are:

In accordance to SSRC column curve2:

[5.44]

I. For short columns:  $0 \leq \lambda \leq 0.15$

$$\rho_R = \rho_A \cdot \rho_{F_y} \cdot \rho_P$$

$$V_R = (V_A^2 + V_{F_y}^2 + V_P^2)^{1/2}$$

II. For intermediate columns:  $0.15 \leq \lambda \leq 3.6$

$$\rho_R = \rho_A \cdot \rho_F \cdot \rho_P$$

$$\text{where } \rho_F = \rho_{F_y} \frac{(1.035 - 0.202 \lambda \rho_\lambda - 0.222 \lambda^2 \rho_\lambda^2)}{(1.035 - 0.202 \lambda - 0.222 \lambda^2)}, \text{ for } 0.15 \leq \lambda \leq 1.0$$

$$\rho_F = \rho_{F_y} \frac{(-0.111 + 0.636 \lambda^{-1} \rho_\lambda^{-1} + 0.087 \lambda^{-2} \rho_\lambda^{-2})}{(-0.111 + 0.636 \lambda^{-1} + 0.087 \lambda^{-2})}, \text{ for } 1.0 \leq \lambda \leq 2.0$$

$$\rho_F = \rho_{F_y} \frac{(0.009 + 0.877 \lambda^{-2} \rho_\lambda^{-2})}{(0.009 + 0.877 \lambda^{-2})}, \text{ for } 2.0 \leq \lambda \leq 3.6$$

$$V_R = (V_A^2 + V_F^2 + V_P^2)^{1/2}$$

$$\text{where } V_F = \frac{(P_1^2 \cdot V_{F_y}^2 + P_2^2 \cdot V_r^2 + P_3^2 \cdot V_E^2)^{1/2}}{f(\bar{\lambda})}$$

$$\text{where } f(\bar{\lambda}) = 1.035 - 0.202 \bar{\lambda} - 0.222 \bar{\lambda}^2, \text{ for } 0.15 \leq \lambda \leq 1.0$$

$$\text{and } P_1 = 1.035 - 0.303 \bar{\lambda} - 0.444 \bar{\lambda}^2$$

$$P_2 = 0.202 \bar{\lambda} + 0.444 \bar{\lambda}^2$$

$$P_3 = 0.101 \bar{\lambda} + 0.222 \bar{\lambda}^2$$

$$\text{where } f(\bar{\lambda}) = -0.111 + 0.636 \bar{\lambda}^{-1} + 0.087 \bar{\lambda}^{-2}, \text{ for } 1.0 \leq \lambda \leq 2.0$$

$$\text{and } P_1 = -0.111 + 0.318 \bar{\lambda}^{-1}$$

$$P_2 = 0.636 \bar{\lambda}^{-1} + 0.174 \bar{\lambda}^{-2}$$

$$P_3 = 0.318 \bar{\lambda}^{-1} + 0.087 \bar{\lambda}^{-2}$$

$$\text{where } f(\bar{\lambda}) = 0.009 + 0.877 \bar{\lambda}^{-2}, \text{ for } 2.0 \leq \lambda \leq 3.6$$

$$\text{and } P_1 = 0.009$$

$$P_2 = 1.754 \bar{\lambda}^{-2}$$

$$P_3 = 0.877 \bar{\lambda}^{-2}$$

III. For long columns:  $\lambda \leq 3.6$

$$\rho_R = \rho_I \cdot \rho_E \cdot \rho_P$$

$$V_R = (V_I^2 + V_E^2 + V_P^2)^{1/2}$$

In accordance to SSRC column curve 1:

[5.45]

I. For short columns:  $0 \leq \lambda \leq 0.15$

$$\rho_R = \rho_A \cdot \rho_{F_y} \cdot \rho_P$$

$$V_R = (V_A^2 + V_{F_y}^2 + V_P^2)^{1/2}$$

II. For intermediate columns:  $0.15 \leq \lambda \leq 3.6$

$$\rho_R = \rho_A \cdot \rho_F \cdot \rho_P$$

$$\text{where } \rho_F = \rho_{F_y} \frac{(0.99 + 0.122\lambda\rho_\lambda - 0.367\lambda^2\rho_\lambda^2)}{(0.99 + 0.122\lambda - 0.367\lambda^2)} \quad , \text{ for } 0.15 \leq \lambda \leq 1.2$$

$$\rho_F = \rho_{F_y} \frac{(0.051 + 0.801\lambda^{-2}\rho_\lambda^{-2})}{(0.051 + 0.801\lambda^{-2})} \quad , \text{ for } 1.2 \leq \lambda \leq 1.8$$

$$\rho_F = \rho_{F_y} \frac{(0.008 + 0.942\lambda^{-2}\rho_\lambda^{-2})}{(0.008 + 0.942\lambda^{-2})} \quad , \text{ for } 1.8 \leq \lambda \leq 2.8$$

$$V_R = (V_A^2 + V_F^2 + V_P^2)^{1/2}$$

$$\text{where } V_F = \frac{(P_1^2 \cdot V_{F_y}^2 + P_2^2 \cdot V_r^2 + P_3^2 \cdot V_E^2)^{1/2}}{f(\bar{\lambda})}$$

$$\text{where } f(\bar{\lambda}) = 0.990 + 0.122\bar{\lambda} - 0.367\bar{\lambda}^2 \quad , \text{ for } 0.15 \leq \lambda \leq 1.2$$

$$\text{and } P_1 = 0.990 + 0.183\bar{\lambda} - 0.734\bar{\lambda}^2$$

$$P_2 = -0.122\bar{\lambda} + 0.734\bar{\lambda}^2$$

$$P_3 = -0.061\bar{\lambda} + 0.367\bar{\lambda}^2$$

$$\text{where } f(\bar{\lambda}) = 0.051 + 0.801\bar{\lambda}^{-2} \quad , \text{ for } 1.2 \leq \lambda \leq 1.8$$

$$\text{and } P_1 = 0.051$$

$$P_2 = 1.602\bar{\lambda}^{-2}$$

$$P_3 = 0.801\bar{\lambda}^{-2}$$

$$\text{where } f(\bar{\lambda}) = 0.008 + 0.942\bar{\lambda}^{-2} \quad , \text{ for } 1.8 \leq \lambda \leq 2.8$$

$$\text{and } P_1 = 0.008$$

$$P_2 = 1.884\bar{\lambda}^{-2}$$

$$P_3 = 0.942\bar{\lambda}^{-2}$$

III. For long columns:  $\lambda \geq 3.6$

$$\rho_R = \rho_I \cdot \rho_E \cdot \rho_P$$

$$V_R = (V_I^2 + V_E^2 + V_P^2)^{1/2}$$

For the column curve from Clause 13.3.1 of CAN/CSA-S16.1-94, the statistical quantities  $\rho_R$  and  $V_R$  are:

$$\rho_R = \rho_A \cdot \rho_F \cdot \rho_P \quad [5.46]$$

$$\text{where } \rho_F = \rho_{F_y} \frac{(1 + \lambda^{2n} \rho_{\lambda}^{2n})^{-1/n}}{(1 + \lambda^{2n})^{-1/n}}$$

$$V_R = (V_A^2 + V_F^2 + V_P^2)^{1/2}$$

$$\text{where } V_F = \frac{(P_1^2 \cdot V_{F_y}^2 + P_2^2 \cdot V_r^2 + V_E^2)^{1/2}}{f(\bar{\lambda})}$$

$$\text{where } f(\bar{\lambda}) = (1 + \bar{\lambda}^{2n})^{-1/n}$$

$$\text{and } P_1 = (1 + \bar{\lambda}^{2n})^{\frac{(n+1)}{n}}$$

$$P_2 = 2\bar{\lambda}^{2n} (1 + \bar{\lambda}^{2n})^{\frac{(n+1)}{n}}$$

$$P_3 = \bar{\lambda}^{2n} (1 + \bar{\lambda}^{2n})^{\frac{(n+1)}{n}}$$

## 5.2 Statistical Parameters

The parametric study presented in Chapter 4 indicated that the orientation of the plates and buckling direction may affect the behaviour and strength of reinforced steel columns. Therefore, different performance factors may be applicable to columns with different reinforcing plate orientations and different buckling directions. It was therefore decided to conduct a statistical analysis on four separate groups, namely, two different reinforcing plate orientations and two different buckling directions. However, a statistical

analysis of these four groups of columns indicated that the above four groups could be merged into two groups since their performance factors were very similar. The first group includes columns reinforced with plates parallel to the flanges and buckling about the strong axis of the rolled section and columns reinforced with plates parallel to the web and buckling about the weak axis of the rolled section. The second group includes columns reinforced with plates parallel to the flanges and buckling about the weak axis of the rolled section, and columns reinforced with plates parallel to the web and buckling about the strong axis of the rolled section. The following sections present the statistical analysis results for these two groups.

The basic data related to cross-sectional and material properties required for the following analysis were obtained from Kennedy and Gad Aly (1980) and Chernenko and Kennedy (1988). The ratios of the mean value to the nominal value and the coefficients of variation for the geometrical and material properties are tabulated in Table 5.1. Since no statistical data were available for the sizes of the welds and fillets between the flanges and web of the rolled section, these values were taken as nominal values.

### 5.2.1 Geometrical Variations

The cross-sectional properties used to determine the capacity of a column are the area,  $A$ , the moment of inertia about principal centroidal axes,  $I_x$  and  $I_y$ , and the corresponding radii of gyration,  $r_x$  and  $r_y$ . Variations in these geometrical properties can be derived from statistical data concerning the flange and web thickness,  $t_f$  and  $w$ , the flange width,  $b_f$ , and the depth,  $d$ , of the rolled section, the reinforcing plate width and thickness,  $b_p$  and  $t_p$ , the size of the fillet at the junction of the web and the flanges,  $k$ , and the size of the fillet weld joining the reinforcing plates to the flanges,  $g$ . These cross-sectional dimensions are summarized in Figure 5.3.

For any geometrical property, the ratio of the mean to the nominal value is defined as

$$\rho_G = \frac{\bar{G}}{G} \quad [5.47]$$

The mean value for the area of a given reinforced column section is given as

$$\bar{A} = 2\bar{b}_f \bar{t}_f + (\bar{d} - 2\bar{t}_f) \bar{w} + 2\bar{b}_p \bar{t}_p + 0.858 \bar{k}^2 + 2\bar{g}^2 \quad [5.48]$$

Since the actual shape of the fillet between the web and the flanges of the rolled shape is not perfectly circular, the actual horizontal and vertical sides of the fillet are not the same. In order to simplify the calculations without sacrificing accuracy significantly, the fillet shape in the numerical model was assumed to be the complement of a quarter circle with identical side lengths,  $k$ . The size of the fillet,  $k$ , was taken as the average of the two sides given in the section properties tables (CISC Handbook, 1995). In addition, the weld face was assumed to be a straight line with equal leg size,  $g$ .

The mean values of the moments of inertia about the major and minor centroidal axes,  $\bar{I}_x$  and  $\bar{I}_y$ , for a column reinforced with plates parallel to the flanges are

$$\begin{aligned} \bar{I}_x = & \frac{1}{6} \bar{b}_f \bar{t}_f^3 + \frac{1}{2} \bar{b}_f \bar{t}_f (\bar{d} - \bar{t}_f)^2 + \frac{1}{12} \bar{w} (\bar{d} - 2\bar{t}_f)^3 + \frac{1}{6} \bar{b}_p \bar{t}_p^3 + \frac{1}{2} \bar{b}_p \bar{t}_p (\bar{d} + \bar{t}_p)^2 \\ & + 0.03\bar{k}^4 + 0.858\bar{k}^2 \left( \frac{1}{2} \bar{d} - \bar{t}_f - 0.223\bar{k} \right)^2 + \frac{1}{9} \bar{g}^4 + 2\bar{g}^2 \left( \frac{1}{2} \bar{d} + \frac{1}{3} \bar{g} \right)^2 \end{aligned} \quad [5.49]$$

and

$$\begin{aligned} \bar{I}_y = & \frac{1}{6} \bar{t}_f \bar{b}_f^3 + \frac{1}{12} (\bar{d} - 2\bar{t}_f) \bar{w}^3 + \frac{1}{6} \bar{t}_p \bar{b}_p^3 + \frac{1}{9} \bar{g}^4 + 2\bar{g}^2 \left( \frac{1}{2} \bar{b}_p + \frac{1}{3} \bar{g} \right)^2 \\ & + 0.03\bar{k}^4 + 0.858\bar{k}^2 \left( \frac{1}{2} \bar{w} + 0.223\bar{k} \right)^2 \end{aligned} \quad [5.50]$$

The mean values for the moment of inertia about the major and minor centroidal axes,  $\bar{I}_x$  and  $\bar{I}_y$ , for a column reinforced with plates parallel to the web are

$$\begin{aligned} \bar{I}_x = & \frac{1}{6} \bar{b}_f \bar{t}_f^3 + \frac{1}{2} \bar{b}_f \bar{t}_f (\bar{d} - \bar{t}_f)^2 + \frac{1}{12} \bar{w} (\bar{d} - 2\bar{t}_f)^3 + \frac{1}{6} \bar{b}_p^3 \bar{t}_p + 0.03\bar{k}^4 \\ & + 0.858\bar{k}^2 \left( \frac{1}{2} \bar{d} - \bar{t}_f - 0.223\bar{k} \right)^2 + \frac{1}{9} \bar{g}^4 + 2\bar{g}^2 \left( \frac{1}{2} \bar{d} + \frac{1}{3} \bar{g} \right)^2 \end{aligned} \quad [5.51]$$

and

$$\begin{aligned} \bar{I}_y = & \frac{1}{6} \bar{t}_f \bar{b}_f^3 + \frac{1}{12} (\bar{d} - 2\bar{t}_f) \bar{w}^3 + \frac{1}{6} \bar{b}_p \bar{t}_p^3 + \frac{1}{2} \bar{b}_p \bar{t}_p (\bar{b}_f + \bar{t}_p)^2 + 0.03\bar{k}^4 \\ & + 0.858\bar{k}^2 \left( \frac{1}{2} \bar{w} + 0.223\bar{k} \right)^2 + \frac{1}{9} \bar{g}^4 + 2\bar{g}^2 \left( \frac{1}{2} \bar{b}_f - \frac{1}{3} \bar{g} \right)^2 \end{aligned} \quad [5.52]$$

The associated radii of gyration for reinforced columns are



$$\bar{r}_x = \sqrt{\left(\frac{\bar{I}_x}{A}\right)} \quad [5.53]$$

and

$$\bar{r}_y = \sqrt{\left(\frac{\bar{I}_y}{A}\right)} \quad [5.54]$$

The coefficient of variation for the cross-sectional properties can be obtained from the assumption that the dimensions are independent variables. It is given as

$$V_G = \frac{1}{\bar{G}} \left[ \left( \frac{\partial \bar{G}}{\partial \bar{b}_f} \right)^2 \sigma_{b_f}^2 + \left( \frac{\partial \bar{G}}{\partial \bar{t}_f} \right)^2 \sigma_{t_f}^2 + \left( \frac{\partial \bar{G}}{\partial \bar{w}} \right)^2 \sigma_w^2 + \left( \frac{\partial \bar{G}}{\partial \bar{b}_p} \right)^2 \sigma_{b_p}^2 + \left( \frac{\partial \bar{G}}{\partial \bar{t}_p} \right)^2 \sigma_{t_p}^2 + \left( \frac{\partial \bar{G}}{\partial \bar{d}} \right)^2 \sigma_d^2 + \left( \frac{\partial \bar{G}}{\partial \bar{k}} \right)^2 \sigma_k^2 + \left( \frac{\partial \bar{G}}{\partial \bar{g}} \right)^2 \sigma_g^2 \right]^{1/2} \quad [5.55]$$

where the partial derivatives are evaluated at the mean. The variation in weld and fillet sizes was not considered in this work because actual measurements were not available. This assumption is justified given their small area compared to the total area of the reinforced cross-section.

Table 5.2 presents a summary of the mean, the measured-to-nominal ratio, and the associated coefficient of variation for the relevant geometric properties of reinforced columns consisting of the most widely used wide flange shapes reinforced with various plate thicknesses. The nominal values for the cross-sectional dimensions were obtained from the handbook of steel construction (CISC Handbook, 1995). With known statistical quantities,  $\rho_G$  and  $V_G$ , the mean values of the geometric properties  $t_f$ ,  $w$ ,  $b_f$ ,  $d$ ,  $k$ , and  $g$  can be obtained. The mean values of the geometric properties  $A$ ,  $I_x$ ,  $I_y$ ,  $r_x$  and  $r_y$  are shown in Table 5.2. The statistical quantities,  $\rho_G$  and  $V_G$ , for the geometric variations  $A$ ,  $I_x$ ,  $I_y$ ,  $r_x$  and  $r_y$  can also be obtained, as shown in the table, using equations [5.47] to [5.55].

In order to consider all the sections as a whole, the mean value of the mean-to-nominal ratios is calculated as follows

$$\bar{\rho}_G = \frac{1}{n} \sum_{i=1}^n \rho_{G_i} = \frac{1}{n} (\rho_{G_1} + \rho_{G_2} + \dots + \rho_{G_n}) \quad [5.56]$$

and the associated coefficient of variation can be obtained using the moment algebra for the distribution of different parameter variables as given by Benjamin and Cornell (1970) as follows:

$$V_G = \frac{1}{\bar{\rho}_G} \cdot \frac{1}{(n-1)^{1/2}} \left[ \sum_{i=1}^n (V_{G_i} \cdot \rho_{G_i})^2 + \sum_{i=1}^n (\rho_G - \rho_{G_i})^2 \right]^{1/2} \quad [5.57]$$

The mean-to-nominal ratios and the coefficient of variation for the geometrical properties are summarized in Table 5.3. The statistical quantities,  $\rho_G$  and  $V_G$ , for the geometric variations  $A$ ,  $I_x$ ,  $I_y$ ,  $r_x$  and  $r_y$ , are obtained using equations [5.56] and [5.57] based on the statistical data shown in Table 5.2. The statistical quantities,  $\rho_G$  and  $V_G$ , for the radius of gyration,  $r$ , and the moment of inertia,  $I$ , were obtained by pooling the data for the  $x$ -axis and  $y$ -axis properties together in equations [5.56] and [5.57]. These quantities are presented in Table 5.3.

### 5.2.2 Material Variations

The important material properties for the calculation of column capacity are the yield strength,  $F_y$ , and the modulus of elasticity,  $E$ . The statistical parameters for the material properties for the rolled section were obtained from Kennedy and Gad Aly (1980). The statistical parameters for the reinforcing plates were obtained from Chernenko and Kennedy (1988). These statistical parameters are summarized in Tables 5.1 (a) and 5.1 (b), respectively.

Possible rolled sections for columns reinforced with corresponding reinforcing plates were chosen from the CISC handbook (1995) to take the ratios of the components' area to the total area into account. The descriptions of the cross-sections are given in Table A.1. With known geometric properties for a section, the mean value of the rolled section area,  $A_C$ , and the plate area,  $A_P$ , can be obtained for each sample. Therefore, the ratio of the rolled section area to the total area,  $\frac{A_C}{A_C + A_P}$ , can be obtained for each sample. An

analysis of the numerical models presented in Table A.1 gives a mean ratio of the rolled

section area to the total area,  $\frac{A_C}{A_C + A_P}$ , of 0.645. Because the sum of the ratio of the reinforcing plate area ( $A_P$ ) to total area and the ratio of the W shape area ( $A_C$ ) to total area should be 1.0, the mean ratio of the plates area to the total area,  $\frac{A_P}{A_C + A_P}$ , for the numerical models is 0.355.

With a known mean ratio of the component area to the total area, the weighted yield strength of the cross-section can be obtained as:

$$F_y = CF_y \cdot \frac{A_C}{A_C + A_P} + PF_y \cdot \frac{A_P}{A_C + A_P} \quad [5.58]$$

where  $CF_y$  is the mean yield strength of the rolled section and  $PF_y$  is the mean yield strength of the reinforcing plates. Furthermore, the mean value of the weighted mean-to-nominal ratios for the yield strength was approximated as follows:

$$\rho_{F_y} = \rho_{CF_y} \cdot \frac{A_C}{A_C + A_P} + \rho_{PF_y} \cdot \frac{A_P}{A_C + A_P} \quad [5.59]$$

The yield strengths of the rolled columns and reinforcing plates are considered as independent random variables. The coefficient of variation for the yield strength thus becomes (Kennedy and Neville 1976),

$$V_{F_y} = \left[ V_{CF_y}^2 + V_{PF_y}^2 \right]^{1/2} \quad [5.60]$$

Following the procedure presented in Section 5.1.4, the statistical parameters for the material properties can be obtained. Table 5.4 presents the statistical parameters for the material properties with different SSRC and CSA curves. The mean-to-nominal ratio of the radius of gyration,  $\rho_r$ , and its associated coefficient of variation,  $V_r$ , are obtained from Table 5.1 for the samples. The mean-to-nominal ratio of the modulus of elasticity,  $\rho_E$ , and its associated coefficient of variation,  $V_E$ , were obtained from the investigation presented by Chernenko and Kennedy (1988) as 1.013 and 0.015, respectively.

### 5.2.3 Professional Factors

As a factor indicating how well the design equation fits the experimental results, the professional factor accounts for variations in column capacity other than those considered as cross-sectional and material properties. The professional factor shows the relationship between the measured strength (or strength predicted using the finite element model) of a reinforced column and that predicted by the design equation. The column design equations evaluated in this work are those in CAN3-S16.1-M84 (equations [5.1] and [5.2]) and CAN/CSA-S16.1-94 (equation [5.3]). These equations were derived based on test results and analysis results that included the effect of initial out-of-straightness and residual stresses. The equations show that the slenderness parameter,  $\lambda$ , is the prime factor determining the load carrying capacity of a column. Furthermore, effects of cross-sectional property, orientation of reinforcing plates, axes of buckling, preload magnitude, and non-linear interaction of the above parameters on the strength of the reinforced column have to be considered.

Chernenko and Kennedy (1988) proposed that the effect of the statistical variation of out-of-straightness and residual stresses on column strength could be assessed independently. The parametric study presented in Chapter 4 indicated that the effect of initial residual stresses on reinforced column strength is negligibly small. Since the residual stresses in the reinforced section do not vary much, only the effect of the statistical variations in out-of-straightness will be assessed independently in the following. The effects of variation in residual stresses, steel grade, buckling axis, preload magnitude, and the geometrical properties of the rolled section and reinforcing plates will be considered as a whole. The professional ratio can be therefore expressed as

$$\rho_P = \rho_s \cdot \rho_n \cdot \rho_{ex} \quad [5.61]$$

where  $\rho_s$  is the simulated professional ratio, that is, the ratio of the strength determined by the computer simulation to that predicted by the design equations for the mean value of out-of-straightness for a given value of  $\lambda$ .  $\rho_n$  is the normalized simulated professional ratio, which accounts for the other parameters. The third term,  $\rho_{ex}$ , is the mean value of

the ratio of the experimental strength to the strength predicted by computer simulations. Consequently, the professional ratio,  $\rho_p$ , is the experimental to the predicted ratio.

### **5.2.3.1 The Effect of Out-of-straightness**

As discussed in Chapter 3, the effect of initial out-of-straightness of the rolled section before reinforcement and the effect of the initial out-of-straightness of the reinforced column can contribute to the scatter in the test results. The following therefore looks at the effect of initial out-of-straightness in the unreinforced wide flange section and the initial out-of-straightness in the reinforced column.

Figures 5.4, 5.5 and 5.6 present analysis data for the columns from Group 1 (columns reinforced with plates parallel to the flanges and buckling about the strong axis of the rolled section and columns reinforced with plates parallel to the web and buckling about the weak axis of the rolled section). Plots of the simulated professional ratio,  $\rho_s$ , versus out-of-straightness for non-dimensional slenderness ratio are presented for values of the slenderness ratio,  $\lambda$ , of 0.4, 1.1, and 1.5. The magnitude of the initial out-of-straightness was varied from 0 to  $L/1000$  (the maximum initial imperfection permissible by CSA standard G40.20). The simulated professional ratio,  $\rho_s$ , is taken as the ratio of the strength obtained from the finite element analysis to the strength predicted using SSRC curve 2. All the data used to plot figures 5.4, 5.5 and 5.6 are presented in Column (7) of Tables 5.5, 5.6 and 5.7, respectively. Columns (6), (8) and (9) from these tables present the simulated professional ratios based on SSRC curve 1, CSA curve 1 and CSA curve 2, respectively. For the second group of columns (columns reinforced with plates parallel to the web and buckling about the weak axis of the rolled section and columns reinforced with plates parallel to the web and buckling about the strong axis of the rolled section) the same procedure was used to obtain the professional ratios. This data is presented in Appendix C.

Figures 5.4, 5.5, and 5.6 show an average line obtained using the method of least squares (Kennedy and Neville, 1976). The equation of the regression line for each value of the slenderness parameter is presented in Table 5.8. Figures 5.4 to 5.6 show that the

ratio of the professional factor decreases with increasing initial imperfection for a given value of  $\lambda$ . The slopes of the average lines (i.e., the rate of decrease of strength with out-of-straightness) for  $\lambda = 1.1$  and  $1.5$  are much greater than those for  $\lambda = 0.4$ . As expected, initial imperfections have a greater effect on reducing the column strength for  $\lambda = 1.1$  and  $1.5$  than for  $\lambda = 0.4$ . In fact, the slope of the regression line is close to zero for  $\lambda = 0.4$ .

To investigate the effect of the variation in initial imperfection on reinforced column strength, the mean value and coefficient of variation are used as a basis of comparison. In this research, a mean initial imperfection of  $L/1500$ , a standard deviation of  $L/15000$ , and a coefficient of variation of  $0.1$  were used based on the work of Bjorhovde (1988) on rolled, unreinforced columns. The limited work on reinforced columns presented by Nagaraja Rao and Tall (1963) indicates that initial out-of-straightness in reinforced columns can be smaller than for unreinforced columns.

A vertical line, representing a mean value of out-of-straightness of  $\delta_0/L = 1/1500 = 0.000667$  is shown in Figures 5.4, 5.5, and 5.6. Two additional vertical lines are drawn at one standard deviation from the mean value in each figure. The intersection point of the mean line with the regression line gives the simulated professional ratio for the mean out-of-straightness,  $\rho_s$ . The vertical distance between the intercepts of the right and left standard deviation lines gives the values of two standard deviations of  $\rho_s$  associated with out-of-straightness. Therefore, the standard deviation for the simulated professional ratio is calculated by multiplying the slope of the equation by one standard deviation for out-of-straightness, that is, slope  $\times 0.000667$ . From this, the coefficient of variation is calculated directly. The mean values of the professional ratios and the corresponding coefficients of variation for the mean out-of-straightness are also given in Table 5.8.

### **5.2.3.2 Miscellaneous Factors**

Figures 5.4, 5.5 and 5.6 show significant scatter of the simulated professional ratios about the regression lines. This scatter is attributed to influencing factors such as initial residual stresses, welding residual stresses, buckling axis, preload magnitude, cross-sectional geometry, and the fundamental non-linearity of the relation between the

strength of the columns and the governing parameters. The effect of these miscellaneous factors can be assessed by normalizing the plotted professional ratio for any slenderness parameter by dividing by the value obtained from the linear expression given in Table 5.8 for that specific slenderness parameter. The normalized professional ratios,  $\rho_n$ , are presented in Tables 5.9, 5.10, and 5.11 for the  $\lambda$  values of 0.4, 1.1, and 1.5, respectively.

The average value of the normalized professional ratio,  $\rho_n$ , should equal 1.00 for any slenderness parameter for which a best-fit straight line has been used. The normalized professional ratio for  $\lambda = 0.4, 1.1, \text{ and } 1.5$  are plotted in Figures 5.7, 5.8, and 5.9 respectively. The mean values in each case are nearly equal to 1.00. The scatter of the distribution is large and, as a result, the coefficient of variation is large. The mean values and coefficients of variations for all different slenderness parameters are given in Table 5.12 as  $\rho_n$  and  $V_n$ , respectively.

### 5.2.3.3 Experimental Factor

The experimental ratio,  $\rho_{ex}$ , is defined as

$$\rho_{ex} = \frac{P_{ex}}{P_{fea}} \quad [5.62]$$

where,  $P_{ex}$  is the strength obtained from the experiment and  $P_{fea}$  is the strength obtained from finite element analysis. Because of the very small number of test results, the statistical value of the experimental ratio cannot be evaluated with any degree of confidence. In order to provide more support for the statistical analysis for the experimental factor for reinforced columns, the experimental ratios for unreinforced columns were used in the research. The unreinforced columns used for the partial validation of the finite element models were used for this purpose. A description of these columns is presented in Table 3.1. The geometric properties and the initial imperfections of the rolled section, as well as the initial residual stresses are the same for both unreinforced and reinforced columns. The loading procedures for both reinforced and unreinforced columns are similar, except for the introduction of the reinforcing plates and the welds in the reinforced columns. Therefore, the experimental ratios for unreinforced columns are expected to be similar to those for reinforced columns.

Table 5.13 presents the results of the analysis of the unreinforced columns described in Table 3.2. Columns (4) and (5) present the predicted and measured capacities, respectively. The results are normalized in terms of the yield strength. Column (6) presents the experimental ratio calculated using Equation [5.62]. The first two test specimens were obtained from the work of Huber and Beedle (1954). The experimental ratio for these two specimens is close to 1.0. The lack of information about initial imperfections for the other three test specimens makes it difficult to obtain an accurate prediction of the test results. In order to verify the effect of initial imperfections on the finite element analysis results, two values of initial imperfection were assumed for each test specimen, as shown in Column (3) of Table 5.13. The results show that the initial imperfection has a significant effect on strength. Within the range of initial imperfections presented in Table 5.13, the experimental results can be accurately simulated.

In general, it can be expected that  $\rho_{ex}$  would be closer to 1.0 with a smaller coefficient of variation as long as the geometric and material properties of the reinforced columns are accurately determined. Therefore, the mean value of  $\rho_{ex}$  was taken as 1.0 and the coefficient of variance was taken as 0.0 to reflect the high accuracy of the finite element analysis when the physical parameters of the column are accurately defined. Further testing of reinforced steel column should be conducted to verify these values.

#### **5.2.3.4 Summary**

The professional factors are given in Table 4.12 for the two groups of columns defined in section 5.2, for the three values of slenderness parameter used in this study, and for the reference strength calculated using four different column curves from two standards. These four column curves are SSRC column curve 1, SSRC column curve 2, and the equivalent curves adopted by CAN/CSA-S16.1-94. The strength predicted using SSRC column curve 2 is lower than that predicted using SSRC column curve 1, which leads to higher values of  $\rho_s$  for SSRC column curve 2 than for SSRC column curve 1.

Since the column curves used in CAN/CSA-S16.1-94 are close approximations of the original SSRC column curves (Loov, 1996), the column strength predicted using



SSRC curves are in very good agreement with that predicted using the CSA curves. The corresponding values of  $\rho_s$  are also similar.

### **5.3 Evaluation of the Performance Factors**

Resistance factors for reinforced columns were calculated using Equation [5.18]. A coefficient of separation,  $\alpha$ , of 0.55, and a reliability index,  $\beta$ , of 3.0, consistent with the limit state design for building in Canada, were used. Table 5.14 presents the assembled data and calculated resistance factors for three values of the slenderness parameter, the two groups of columns, and four columns curves. Examination of the resistance factors for reinforced columns within one group indicates that the CSA curves can approximate the SSRC curves very well. The maximum difference in the values of the performance factors between SSRC column curve 2 and the corresponding CSA column curve is about 2.8% for  $\lambda = 1.1$ .

As expected, SSRC column curve 1 yields lower resistance factors than SSRC column curve 2. Within the range studied, the strength of reinforced steel columns in Group 1 (columns reinforced with plates parallel to the flanges and buckling about the strong axis of the rolled section, and columns with reinforcing plates parallel to the web and buckling about the weak axis of the rolled section) can be predicted conservatively with SSRC curve 1 or CSA column curve with  $n=2.24$ .

On the other hand, resistance factors calculated for the columns from Group 2 for SSRC curve 1 vary from 0.82 to 0.99. It is therefore unconservative to use SSRC column curve 1 to predict the capacity of the columns from Group 2. The resistance factors obtained for SSRC curve 2 vary from 1.02 to 1.06. They are from 1.13 to 1.18 times the current value of  $\phi$ . SSRC column curve 2 and the CSA column curve with  $n=1.34$  can therefore be used conservatively to predict the capacity of reinforced steel columns from Group 2 (columns reinforced with plates parallel to the web and buckling about the strong axis of the rolled section, and columns with reinforcing plates parallel to the flanges and buckling about the weak axis of the rolled section).

**Table 5.1.a Statistical Parameters for Rolled W Sections  
(from Kennedy and Gad Aly, 1979)**

Property	Designation	Statistical Parameters	
		Mean/Nominal, $\rho$	Coefficient of Variation, V
$b_f$	The width of the flanges	1.005	0.014
$t_f$	The thickness of the flanges	0.976	0.042
w	The thickness of the web	1.017	0.038
d	The depth of the section	1.000	0.00195*
k	The side of the fillet	1.000	0.000
$F_y$	The yield strength	1.070	0.065
E	The modulus of elasticity	1.000	0.019

Note: The average side of the fillet was assumed without measurement available.

\* Mean value of the distribution, which depends on range in depths of rolled sections.

**Table 5.1.b Statistical Parameters for Cover Plates  
(from Chernenko and Kennedy, 1988)**

Property	Designation	Statistical Parameters	
		Mean/Nominal, $\rho$	Coefficient of Variation, V
$b_p$	The width of the plates	0.999	0.003
$t_p$	The thickness of the plates	1.010	0.008
g	The side of the weld	1.000	0.000
$F_y$	The yield strength	1.133	0.059
E	The modulus of elasticity	1.038	0.026

Note: The average side of the weld was assumed without measurement available.

**Table 5.2.a Statistical Parameters for the Geometric Properties of the Reinforced Columns**

No.	W-Section	Flange			Web			Fillet		Plate			Weld		$\rho_A$	A (mm <sup>2</sup> )	A (mm <sup>2</sup> )	V <sub>A</sub>	I <sub>x</sub> (10 <sup>6</sup> mm <sup>4</sup> )
		b <sub>f</sub> (mm)	t <sub>f</sub> (mm)	d (mm)	w (mm)	k (mm)	Plate	b <sub>p</sub> (mm)	t <sub>p</sub> (mm)	g (mm)	g (mm)	g (mm)							
1	W310x179	313	28.1	333	18.0	16.3	290x25	290	25	22	38268	38174	0.998	0.0003	940.5				
2	W310x179	313	28.1	333	18.0	16.3	290x20	290	20	18	35048	34927	0.997	0.0004	827.0				
3	W310x179	313	28.1	333	18.0	16.3	290x16	290	16	14	32472	32330	0.996	0.0004	740.2				
4	W310x158	310	25.1	327	15.5	17.0	290x25	290	25	22	35568	35496	0.998	0.0004	865.5				
5	W310x158	310	25.1	327	15.5	17.0	290x20	290	20	18	32348	32249	0.997	0.0004	755.5				
6	W310x158	310	25.1	327	15.5	17.0	290x16	290	16	14	29772	29651	0.996	0.0004	671.5				
7	W310x158	310	25.1	327	15.5	17.0	290x12	290	12	10	27260	27118	0.995	0.0005	592.9				
8	W310x143	309	22.9	323	14.0	13.9	290x20	290	20	18	30448	30366	0.997	0.0004	706.9				
9	W310x143	309	22.9	323	14.0	13.9	290x16	290	16	14	27872	27768	0.996	0.0005	624.8				
10	W310x143	309	22.9	323	14.0	13.9	290x12	290	12	10	25360	25235	0.995	0.0005	548.0				
11	W310x143	309	22.9	323	14.0	13.9	290x10	290	10	8	24128	23992	0.994	0.0005	511.5				
12	W310x129	308	20.6	318	13.1	14.7	290x20	290	20	18	28748	28687	0.998	0.0004	656.9				
13	W310x129	308	20.6	318	13.1	14.7	290x16	290	16	14	26172	26089	0.997	0.0005	577.1				
14	W310x129	308	20.6	318	13.1	14.7	290x12	290	12	10	23660	23556	0.996	0.0005	502.4				
15	W310x129	308	20.6	318	13.1	14.7	290x10	290	10	8	22428	22313	0.995	0.0006	466.9				
16	W310x118	307	18.7	314	11.9	16.2	290x16	290	16	14	24672	24605	0.997	0.0005	538.8				
17	W310x118	307	18.7	314	11.9	16.2	290x12	290	12	10	22160	22071	0.996	0.0006	465.9				
18	W310x118	307	18.7	314	11.9	16.2	290x10	290	10	8	20928	20828	0.995	0.0006	431.3				
19	W310x118	307	18.7	314	11.9	16.2	290x8	290	8	6	19712	19601	0.994	0.0007	397.9				
20	W310x107	306	17.0	311	10.9	14.3	290x16	290	16	14	23272	23219	0.998	0.0006	505.9				
21	W310x107	306	17.0	311	10.9	14.3	290x12	290	12	10	20760	20685	0.996	0.0006	434.3				
22	W310x107	306	17.0	311	10.9	14.3	290x10	290	10	8	19528	19442	0.996	0.0007	400.3				
23	W310x107	306	17.0	311	10.9	14.3	290x8	290	8	6	18312	18216	0.995	0.0007	367.5				
24	W310x86	254	16.3	310	9.1	15.1	230x16	230	16	14	18752	18711	0.998	0.0006	404.2				
25	W310x86	254	16.3	310	9.1	15.1	230x12	230	12	10	16720	16662	0.997	0.0007	346.7				

**Table 5.2.a (Cont'd)**

No.	W-Section	Flange		Web		Fillet		Plate		Weld		A	A	$P_A$	$V_A$	$I_x$ ( $10^6 \text{mm}^4$ )
		$b_f$ (mm)	$t_f$ (mm)	$d$ (mm)	$w$ (mm)	$k$ (mm)	Plate	$b_p$ (mm)	$t_p$ (mm)	$g$ (mm)	$A$ ( $\text{mm}^2$ )					
26	W310x86	254	16.3	310	9.1	15.1	230x10	230	10	8	15728	15662	0.996	0.0008	319.5	
27	W310x86	254	16.3	310	9.1	15.1	230x8	230	8	6	14752	14677	0.995	0.0008	293.3	
28	W310x79	254	14.6	306	8.8	17.0	230x12	230	12	10	15820	15776	0.997	0.0008	322.1	
29	W310x79	254	14.6	306	8.8	17.0	230x10	230	10	8	14828	14776	0.996	0.0008	295.6	
30	W310x79	254	14.6	306	8.8	17.0	230x8	230	8	6	13852	13791	0.996	0.0009	270.1	
31	W310x67	204	14.6	306	8.5	15.3	180x12	180	12	10	13030	13002	0.998	0.0009	259.4	
32	W310x67	204	14.6	306	8.5	15.3	180x10	180	10	8	12238	12203	0.997	0.0009	238.2	
33	W310x67	204	14.6	306	8.5	15.3	180x8	180	8	6	11462	11420	0.996	0.0010	218.0	
34	W310x60	203	13.1	303	7.5	15.1	180x12	180	12	10	12110	12088	0.998	0.0010	240.7	
35	W310x60	203	13.1	303	7.5	15.1	180x10	180	10	8	11318	11289	0.997	0.0010	219.9	
36	W310x60	203	13.1	303	7.5	15.1	180x8	180	8	6	10542	10507	0.997	0.0011	200.0	
37	W250x73	254	14.2	253	8.6	12.5	230x12	230	12	10	15000	14952	0.997	0.0007	213.0	
38	W250x73	254	14.2	253	8.6	12.5	230x10	230	10	8	14008	13951	0.996	0.0007	194.4	
39	W250x73	254	14.2	253	8.6	12.5	230x8	230	8	6	13032	12966	0.995	0.0008	176.6	
40	W250x73	254	14.2	253	8.6	12.5	230x6	230	6	6	12112	12038	0.994	0.0009	160.2	
41	W250x58	203	13.5	252	8.0	12.7	180x12	180	12	10	11940	11911	0.998	0.0008	166.0	
42	W250x58	203	13.5	252	8.0	12.7	180x10	180	10	8	11148	11112	0.997	0.0009	151.2	
43	W250x58	203	13.5	252	8.0	12.7	180x8	180	8	6	10372	10329	0.996	0.0009	137.2	
44	W250x58	203	13.5	252	8.0	12.7	180x6	180	6	6	9652	9603	0.995	0.0010	124.4	
45	W250x49	202	11.0	247	7.4	12.8	180x10	180	10	8	9978	9958	0.998	0.0010	132.2	
46	W250x49	202	11.0	247	7.4	12.8	180x8	180	8	6	9202	9176	0.997	0.0011	118.6	
47	W250x49	202	11.0	247	7.4	12.8	180x6	180	6	6	8482	8449	0.996	0.0012	106.3	
48	W200x59	205	14.2	210	9.1	8.0	180x12	180	12	10	12050	12013	0.997	0.0007	116.5	
49	W200x59	205	14.2	210	9.1	8.0	180x10	180	10	8	11258	11214	0.996	0.0007	106.0	
50	W200x59	205	14.2	210	9.1	8.0	180x8	180	8	6	10482	10431	0.995	0.0008	95.9	

Table S.2.a (Cont'd)

No.	W-Section	Flange		Web		Fillet		Plate		Weld		A	A	P <sub>A</sub>	V <sub>A</sub>	I <sub>x</sub> (10 <sup>6</sup> mm <sup>4</sup> )
		b <sub>f</sub> (mm)	t <sub>f</sub> (mm)	d (mm)	w (mm)	k (mm)	Plate	b <sub>p</sub> (mm)	t <sub>p</sub> (mm)	g (mm)	(mm <sup>2</sup> )					
51	W200x59	205	14.2	210	9.1	8.0	180x6	180	6	6	9762	9705	0.994	0.0009	86.9	
52	W200x52	204	12.6	206	7.9	7.7	180x12	180	12	10	11140	11111	0.997	0.0007	106.1	
53	W200x52	204	12.6	206	7.9	7.7	180x10	180	10	8	10348	10312	0.997	0.0008	95.9	
54	W200x52	204	12.6	206	7.9	7.7	180x8	180	8	6	9572	9529	0.996	0.0009	86.2	
55	W200x52	204	12.6	206	7.9	7.7	180x6	180	6	6	8852	8802	0.994	0.0009	77.5	
56	W200x46	203	11.0	203	7.2	7.7	180x10	180	10	8	9548	9522	0.997	0.0009	87.4	
57	W200x46	203	11.0	203	7.2	7.7	180x8	180	8	6	8772	8739	0.996	0.0010	78.0	
58	W200x46	203	11.0	203	7.2	7.7	180x6	180	6	6	8052	8012	0.995	0.0010	69.5	
59	W200x42	166	11.8	205	7.2	8.1	140x10	140	10	8	8208	8185	0.997	0.0010	74.4	
60	W200x42	166	11.8	205	7.2	8.1	140x8	140	8	6	7592	7564	0.996	0.0010	66.8	
61	W200x42	166	11.8	205	7.2	8.1	140x6	140	6	6	7032	6999	0.995	0.0011	60.1	
62	W200x36	165	10.2	201	6.2	8.0	140x10	140	10	8	7468	7451	0.998	0.0011	66.7	
63	W200x36	165	10.2	201	6.2	8.0	140x8	140	8	6	6852	6830	0.997	0.0011	59.4	
64	W200x36	165	10.2	201	6.2	8.0	140x6	140	6	6	6292	6265	0.996	0.0013	52.9	
65	W200x31	134	10.2	210	6.4	7.9	110x10	110	10	8	6328	6320	0.999	0.0012	59.5	
66	W200x31	134	10.2	210	6.4	7.9	110x8	110	8	6	5832	5820	0.998	0.0013	53.1	
67	W200x31	134	10.2	210	6.4	7.9	110x6	110	6	6	5392	5375	0.997	0.0015	47.6	
68	W200x27	133	8.4	207	5.8	7.8	110x8	110	8	6	5222	5216	0.999	0.0015	47.0	
69	W200x27	133	8.4	207	5.8	7.8	110x7	110	7	6	5002	4994	0.998	0.0016	44.3	
70	W200x27	133	8.4	207	5.8	7.8	110x6	110	6	6	4782	4772	0.998	0.0016	41.6	
71	W150x30	153	9.3	157	6.6	6.0	130x8	130	8	6	5942	5925	0.997	0.0011	31.8	
72	W150x30	153	9.3	157	6.6	6.0	130x7	130	7	6	5682	5663	0.997	0.0011	29.9	
73	W150x30	153	9.3	157	6.6	6.0	130x6	130	6	6	5422	5400	0.996	0.0012	28.0	
74	W310x179	313	28.1	333	18.0	16.3	350x25	350	25	22	41268	41202	0.998	0.0003	653.8	
75	W310x179	313	28.1	333	18.0	16.3	350x20	350	20	18	37448	37350	0.997	0.0003	608.1	

**Table 5.2.a (Cont'd)**

No.	W-Section	Flange		Web		Fillet		Plate		Weld		A	A	$\rho_A$	$V_A$	$I_x$ ( $10^6 \text{ mm}^4$ )
		$b_f$ (mm)	$t_f$ (mm)	d (mm)	w (mm)	k (mm)	Plate	$b_p$ (mm)	$t_p$ (mm)	g (mm)	A ( $\text{mm}^2$ )					
76	W310x179	313	28.1	333	18.0	16.3	350x16	350	16	14	34392	34268	0.996	0.0004	571.7	
77	W310x158	310	25.1	327	15.5	17.0	350x25	350	25	23	38658	38614	0.999	0.0003	597.0	
78	W310x158	310	25.1	327	15.5	17.0	350x20	350	20	18	34748	34671	0.998	0.0004	548.8	
79	W310x158	310	25.1	327	15.5	17.0	350x16	350	16	14	31692	31589	0.997	0.0004	512.7	
80	W310x158	310	25.1	327	15.5	17.0	350x12	350	12	10	28700	28571	0.996	0.0005	478.6	
81	W310x143	309	22.9	323	14.0	13.9	350x20	350	20	18	32848	32788	0.998	0.0004	508.3	
82	W310x143	309	22.9	323	14.0	13.9	350x16	350	16	14	29792	29706	0.997	0.0004	472.3	
83	W310x143	309	22.9	323	14.0	13.9	350x12	350	12	10	26800	26688	0.996	0.0005	438.4	
84	W310x143	309	22.9	323	14.0	13.9	350x10	350	10	8	25328	25203	0.995	0.0005	422.1	
85	W310x129	308	20.6	318	13.1	14.7	350x20	350	20	18	31148	31109	0.999	0.0004	468.1	
86	W310x129	308	20.6	318	13.1	14.7	350x16	350	16	14	28092	28027	0.998	0.0005	432.4	
87	W310x129	308	20.6	318	13.1	14.7	350x12	350	12	10	25100	25009	0.996	0.0005	398.6	
88	W310x129	308	20.6	318	13.1	14.7	350x10	350	10	8	23628	23524	0.996	0.0006	382.4	
89	W310x118	307	18.7	314	11.9	16.2	350x16	350	16	14	26592	26542	0.998	0.0005	400.3	
90	W310x118	307	18.7	314	11.9	16.2	350x12	350	12	10	23600	23524	0.997	0.0006	366.6	
91	W310x118	307	18.7	314	11.9	16.2	350x10	350	10	8	22128	22039	0.996	0.0006	350.5	
92	W310x118	307	18.7	314	11.9	16.2	350x8	350	8	6	20672	20570	0.995	0.0006	334.7	
93	W310x107	306	17.0	311	10.9	14.3	350x16	350	16	14	25192	25157	0.999	0.0005	372.0	
94	W310x107	306	17.0	311	10.9	14.3	350x12	350	12	10	22200	22139	0.997	0.0006	338.4	
95	W310x107	306	17.0	311	10.9	14.3	350x10	350	10	8	20728	20654	0.996	0.0006	322.3	
96	W310x107	306	17.0	311	10.9	14.3	350x8	350	8	6	19272	19185	0.995	0.0007	306.6	
97	W310x86	254	16.3	310	9.1	15.1	330x16	330	16	14	21952	21941	1.000	0.0006	304.3	
98	W310x86	254	16.3	310	9.1	15.1	330x12	330	12	10	19120	19085	0.998	0.0007	275.4	
99	W310x86	254	16.3	310	9.1	15.1	330x10	330	10	8	17728	17680	0.997	0.0007	261.6	
100	W310x86	254	16.3	310	9.1	15.1	330x8	330	8	6	16352	16292	0.996	0.0008	248.2	

Table 5.2.a (Cont'd)

No.	W-Section	Flange		Web		Fillet		Plate			Weld		A	$\rho_A$	$V_A$	$I_x$ ( $10^6 \text{mm}^4$ )
		$b_f$ (mm)	$t_f$ (mm)	$d$ (mm)	$w$ (mm)	$k$ (mm)	Plate	$b_p$ (mm)	$t_p$ (mm)	$g$ (mm)	A	A				
101	W310x79	254	14.6	306	8.8	17.0	330x12	330	12	10	18220	18199	0.999	0.0007	254.4	
102	W310x79	254	14.6	306	8.8	17.0	330x10	330	10	8	16828	16794	0.998	0.0008	240.6	
103	W310x79	254	14.6	306	8.8	17.0	330x8	330	8	6	15452	15406	0.997	0.0008	227.3	
104	W310x67	204	14.6	306	8.5	15.3	330x12	330	12	10	16630	16635	1.000	0.0007	222.0	
105	W310x67	204	14.6	306	8.5	15.3	330x10	330	10	8	15238	15231	1.000	0.0008	208.2	
106	W310x67	204	14.6	306	8.5	15.3	330x8	330	8	6	13862	13843	0.999	0.0009	194.9	
107	W310x60	203	13.1	303	7.5	15.1	330x12	330	12	10	15710	15721	1.001	0.0008	205.3	
108	W310x60	203	13.1	303	7.5	15.1	330x10	330	10	8	14318	14317	1.000	0.0009	191.6	
109	W310x60	203	13.1	303	7.5	15.1	330x8	330	8	6	12942	12929	0.999	0.0010	178.3	
110	W250x73	254	14.2	253	8.6	12.5	280x12	280	12	10	16200	16163	0.998	0.0007	160.0	
111	W250x73	254	14.2	253	8.6	12.5	280x10	280	10	8	15008	14960	0.997	0.0007	151.4	
112	W250x73	254	14.2	253	8.6	12.5	280x8	280	8	6	13832	13774	0.996	0.0008	143.2	
113	W250x73	254	14.2	253	8.6	12.5	280x6	280	6	6	12712	12644	0.995	0.0008	135.8	
114	W250x58	203	13.5	252	8.0	12.7	280x12	280	12	10	14340	14333	1.000	0.0007	134.5	
115	W250x58	203	13.5	252	8.0	12.7	280x10	280	10	8	13148	13131	0.999	0.0008	126.0	
116	W250x58	203	13.5	252	8.0	12.7	280x8	280	8	6	11972	11944	0.998	0.0009	117.7	
117	W250x58	203	13.5	252	8.0	12.7	280x6	280	6	6	10852	10814	0.996	0.0010	110.4	
118	W250x49	202	11.0	247	7.4	12.8	280x10	280	10	8	11978	11977	1.000	0.0009	109.3	
119	W250x49	202	11.0	247	7.4	12.8	280x8	280	8	6	10802	10790	0.999	0.0010	101.0	
120	W250x49	202	11.0	247	7.4	12.8	280x6	280	6	6	9682	9660	0.998	0.0011	93.7	
121	W200x59	205	14.2	210	9.1	8.0	230x12	230	12	10	13250	13224	0.998	0.0007	87.6	
122	W200x59	205	14.2	210	9.1	8.0	230x10	230	10	8	12258	12223	0.997	0.0007	82.6	
123	W200x59	205	14.2	210	9.1	8.0	230x8	230	8	6	11282	11239	0.996	0.0008	77.9	
124	W200x59	205	14.2	210	9.1	8.0	230x6	230	6	6	10362	10310	0.995	0.0008	73.9	
125	W200x52	204	12.6	206	7.9	7.7	230x12	230	12	10	12340	12322	0.999	0.0007	79.0	

**Table 5.2.a (Cont'd)**

No.	W-Section	Flange		Web		Fillet		Plate		Weld		A	A	$\rho_A$	$V_A$	$I_k$ ( $10^6 \text{mm}^4$ )
		$b_f$ (mm)	$t_f$ (mm)	d (mm)	w (mm)	k (mm)	Plate	$b_p$ (mm)	$t_p$ (mm)	g (mm)	A ( $\text{mm}^2$ )					
126	W200x52	204	12.6	206	7.9	7.7	230x10	230	10	8	11348	11321	0.998	0.0008	74.1	
127	W200x52	204	12.6	206	7.9	7.7	230x8	230	8	6	10372	10337	0.997	0.0008	69.4	
128	W200x52	204	12.6	206	7.9	7.7	230x6	230	6	6	9452	9408	0.995	0.0009	65.4	
129	W200x46	203	11.0	203	7.2	7.7	230x10	230	10	8	10548	10531	0.998	0.0008	66.8	
130	W200x46	203	11.0	203	7.2	7.7	230x8	230	8	6	9572	9546	0.997	0.0009	62.2	
131	W200x46	203	11.0	203	7.2	7.7	230x6	230	6	6	8652	8618	0.996	0.0010	58.1	
132	W200x42	166	11.8	205	7.2	8.1	230x10	230	10	8	10008	10002	0.999	0.0008	62.3	
133	W200x42	166	11.8	205	7.2	8.1	230x8	230	8	6	9032	9017	0.998	0.0009	57.6	
134	W200x42	166	11.8	205	7.2	8.1	230x6	230	6	6	8112	8089	0.997	0.0010	53.6	
135	W200x36	165	10.2	201	6.2	8.0	230x10	230	10	8	9268	9268	1.000	0.0009	55.8	
136	W200x36	165	10.2	201	6.2	8.0	230x8	230	8	6	8292	8284	0.999	0.0010	51.1	
137	W200x36	165	10.2	201	6.2	8.0	230x6	230	6	6	7372	7355	0.998	0.0011	47.1	
138	W200x31	134	10.2	210	6.4	7.9	230x10	230	10	8	8728	8742	1.002	0.0010	53.2	
139	W200x31	134	10.2	210	6.4	7.9	230x8	230	8	6	7752	7757	1.001	0.0011	48.4	
140	W200x31	134	10.2	210	6.4	7.9	230x6	230	6	6	6832	6829	1.000	0.0012	44.4	
141	W200x27	133	8.4	207	5.8	7.8	230x8	230	8	6	7142	7154	1.002	0.0012	42.9	
142	W200x27	133	8.4	207	5.8	7.8	230x7	230	7	6	6682	6690	1.001	0.0013	40.8	
143	W200x27	133	8.4	207	5.8	7.8	230x6	230	6	6	6222	6226	1.001	0.0014	38.8	
144	W150x30	153	9.3	157	6.6	6.0	180x8	180	8	6	6742	6733	0.999	0.0010	25.4	
145	W150x30	153	9.3	157	6.6	6.0	180x7	180	7	6	6382	6369	0.998	0.0010	24.4	
146	W150x30	153	9.3	157	6.6	6.0	180x6	180	6	6	6022	6006	0.997	0.0011	23.4	



**Table 5.2.b Statistical Parameters of the Geometric Properties of the Reinforced Columns**

No.	$\bar{I}_x$ ( $10^6 \text{mm}^4$ )	$\rho_{I_x}$	$V_{I_x}$	$\bar{I}_y$ ( $10^6 \text{mm}^4$ )	$\rho_{I_y}$	$V_{I_y}$	$\bar{r}_x$ (mm)	$\rho_{r_x}$	$V_{r_x}$	$\bar{r}_y$ (mm)	$\rho_{r_y}$	$V_{r_y}$			
1	940.5	1.000	0.0006	267.9	267.3	0.998	0.0008	156.8	157.0	1.001	0.0003	83.7	83.7	1.000	0.0004
2	825.7	0.998	0.0006	239.9	239.2	0.997	0.0009	153.6	153.8	1.001	0.0004	82.7	82.7	1.000	0.0005
3	738.0	0.997	0.0007	217.6	216.8	0.996	0.0010	151.0	151.1	1.001	0.0004	81.9	81.9	1.000	0.0005
4	865.7	1.000	0.0006	248.9	248.5	0.998	0.0009	156.0	156.2	1.001	0.0003	83.6	83.7	1.000	0.0005
5	754.6	0.999	0.0007	220.8	220.3	0.998	0.0010	152.8	153.0	1.001	0.0004	82.6	82.7	1.000	0.0005
6	669.7	0.997	0.0008	198.6	197.9	0.997	0.0011	150.2	150.3	1.001	0.0004	81.7	81.7	1.000	0.0006
7	590.3	0.996	0.0009	177.9	177.2	0.996	0.0012	147.5	147.5	1.000	0.0005	80.8	80.8	1.000	0.0006
8	706.3	0.999	0.0007	208.8	208.3	0.998	0.0010	152.4	152.5	1.001	0.0004	82.8	82.8	1.000	0.0005
9	623.3	0.998	0.0008	186.5	186.0	0.997	0.0011	149.7	149.8	1.001	0.0005	81.8	81.8	1.000	0.0006
10	545.7	0.996	0.0009	165.9	165.2	0.996	0.0013	147.0	147.1	1.000	0.0005	80.9	80.9	1.000	0.0007
11	508.8	0.995	0.0010	156.1	155.4	0.995	0.0014	145.6	145.6	1.000	0.0006	80.4	80.5	1.001	0.0007
12	656.6	1.000	0.0008	196.5	196.2	0.998	0.0011	151.2	151.3	1.001	0.0004	82.7	82.7	1.000	0.0006
13	575.9	0.998	0.0009	174.2	173.8	0.998	0.0012	148.5	148.6	1.001	0.0005	81.6	81.6	1.000	0.0006
14	500.5	0.996	0.0010	153.6	153.0	0.996	0.0014	145.7	145.8	1.000	0.0006	80.6	80.6	1.000	0.0007
15	464.7	0.995	0.0011	143.8	143.2	0.996	0.0015	144.3	144.3	1.000	0.0006	80.1	80.1	1.000	0.0008
16	538.0	0.998	0.0009	164.1	163.7	0.998	0.0013	147.8	147.9	1.001	0.0005	81.5	81.6	1.000	0.0007
17	464.3	0.997	0.0011	143.4	143.0	0.997	0.0014	145.0	145.0	1.000	0.0006	80.4	80.5	1.000	0.0008
18	429.3	0.995	0.0012	133.7	133.2	0.996	0.0016	143.6	143.6	1.000	0.0007	79.9	80.0	1.000	0.0008
19	395.6	0.994	0.0013	124.3	123.7	0.995	0.0017	142.1	142.1	1.000	0.0007	79.4	79.5	1.000	0.0009
20	505.3	0.999	0.0010	155.0	154.8	0.998	0.0013	147.4	147.5	1.001	0.0006	81.6	81.7	1.000	0.0007
21	432.9	0.997	0.0012	134.4	134.0	0.997	0.0015	144.6	144.7	1.000	0.0007	80.5	80.5	1.000	0.0008
22	398.6	0.996	0.0013	124.7	124.2	0.997	0.0016	143.2	143.2	1.000	0.0007	79.9	79.9	1.000	0.0009
23	365.4	0.994	0.0014	115.3	114.8	0.996	0.0018	141.7	141.6	1.000	0.0008	79.4	79.4	1.000	0.0010
24	403.7	0.999	0.0010	82.6	82.4	0.998	0.0014	146.8	146.9	1.000	0.0006	66.4	66.4	1.000	0.0008
25	345.6	0.997	0.0012	71.7	71.5	0.997	0.0016	144.0	144.0	1.000	0.0007	65.5	65.5	1.000	0.0009

**Table 5.2.b (Cont'd)**

No.	$\bar{I}_x$ ( $10^6 \text{mm}^4$ )	$\rho_{I_k}$	$V_{I_k}$	$I_y$ ( $10^6 \text{mm}^4$ )	$\bar{I}_y$ ( $10^6 \text{mm}^4$ )	$\rho_{I_y}$	$V_{I_y}$	$\bar{I}_x$ (mm)	$\bar{I}_y$ (mm)	$\rho_{I_x}$	$V_{I_x}$	$I_x$ (mm)	$\bar{I}_y$ (mm)	$\rho_{I_y}$	$V_{I_y}$
26	318.1	0.996	0.0013	66.6	66.3	0.996	0.0018	142.5	142.5	1.000	0.0008	65.1	65.1	1.000	0.0010
27	291.7	0.994	0.0014	61.8	61.5	0.995	0.0019	141.0	141.0	1.000	0.0008	64.7	64.7	1.000	0.0010
28	321.3	0.997	0.0013	67.0	66.9	0.997	0.0017	142.7	142.7	1.000	0.0007	65.1	65.1	1.000	0.0010
29	294.5	0.996	0.0014	62.0	61.7	0.997	0.0019	141.2	141.2	1.000	0.0008	64.6	64.6	1.000	0.0010
30	268.7	0.995	0.0015	57.1	56.9	0.996	0.0021	139.6	139.6	1.000	0.0009	64.2	64.2	1.000	0.0011
31	258.7	0.998	0.0013	34.1	34.0	0.997	0.0018	141.1	141.1	1.000	0.0008	51.2	51.1	1.000	0.0010
32	237.4	0.996	0.0014	31.5	31.4	0.996	0.0019	139.5	139.5	1.000	0.0008	50.7	50.7	1.000	0.0011
33	216.9	0.995	0.0015	29.1	28.9	0.996	0.0021	137.9	137.8	0.999	0.0009	50.4	50.3	1.000	0.0012
34	240.2	0.998	0.0014	31.7	31.6	0.997	0.0019	141.0	141.0	1.000	0.0008	51.2	51.1	1.000	0.0011
35	219.2	0.997	0.0015	29.1	29.0	0.997	0.0021	139.4	139.3	1.000	0.0009	50.7	50.7	1.000	0.0011
36	199.1	0.996	0.0017	26.7	26.6	0.996	0.0022	137.7	137.7	0.999	0.0010	50.3	50.3	1.000	0.0013
37	212.6	0.998	0.0013	65.9	65.8	0.997	0.0018	119.2	119.2	1.001	0.0007	66.3	66.3	1.000	0.0010
38	193.8	0.997	0.0014	60.9	60.7	0.997	0.0019	117.8	117.8	1.000	0.0008	65.9	65.9	1.000	0.0010
39	175.7	0.995	0.0016	56.0	55.8	0.996	0.0021	116.4	116.4	1.000	0.0009	65.6	65.6	1.000	0.0011
40	159.2	0.994	0.0017	52.0	51.7	0.995	0.0023	115.0	115.0	1.000	0.0010	65.5	65.5	1.001	0.0012
41	165.6	0.998	0.0013	32.2	32.2	0.997	0.0019	117.9	117.9	1.000	0.0008	52.0	52.0	1.000	0.0010
42	150.8	0.997	0.0014	29.7	29.6	0.997	0.0020	116.5	116.5	1.000	0.0008	51.6	51.6	1.000	0.0011
43	136.6	0.996	0.0016	27.2	27.1	0.996	0.0022	115.0	115.0	1.000	0.0009	51.2	51.2	1.000	0.0012
44	123.7	0.994	0.0018	25.3	25.2	0.995	0.0024	113.5	113.5	1.000	0.0010	51.2	51.2	1.000	0.0013
45	131.9	0.998	0.0016	25.9	25.9	0.998	0.0023	115.1	115.1	1.000	0.0010	51.0	51.0	1.000	0.0012
46	118.2	0.997	0.0018	23.5	23.4	0.997	0.0025	113.5	113.5	1.000	0.0011	50.5	50.5	1.000	0.0014
47	105.8	0.995	0.0020	21.6	21.5	0.996	0.0027	112.0	111.9	0.999	0.0012	50.4	50.4	1.000	0.0015
48	116.3	0.998	0.0012	33.8	33.7	0.997	0.0018	98.3	98.4	1.001	0.0007	53.0	53.0	1.000	0.0010
49	105.6	0.997	0.0014	31.2	31.1	0.996	0.0020	97.0	97.1	1.001	0.0008	52.7	52.7	1.000	0.0011
50	95.5	0.996	0.0015	28.8	28.7	0.996	0.0021	95.7	95.7	1.000	0.0009	52.4	52.4	1.000	0.0011

**Table 5.2.b (Cont'd)**

No.	$\bar{I}_x$ ( $10^6 \text{mm}^4$ )	$\rho_{I_x}$	$V_{I_x}$	$\bar{I}_y$ ( $10^6 \text{mm}^4$ )	$\rho_{I_y}$	$V_{I_y}$	$\bar{r}_x$ (mm)	$\rho_{r_x}$	$V_{r_x}$	$\bar{r}_y$ (mm)	$\rho_{r_y}$	$V_{r_y}$
51	86.4	0.994	0.0017	26.8	0.995	0.0023	94.4	1.000	0.0009	52.4	1.000	0.0012
52	105.9	0.999	0.0014	31.2	0.998	0.0019	97.6	1.001	0.0008	53.0	1.000	0.0010
53	95.6	0.998	0.0015	28.7	0.997	0.0021	96.3	1.001	0.0008	52.6	1.000	0.0011
54	85.9	0.996	0.0017	26.2	0.996	0.0023	94.9	1.000	0.0009	52.3	1.000	0.0012
55	77.1	0.994	0.0018	24.3	0.995	0.0025	93.6	1.000	0.0010	52.4	1.000	0.0013
56	87.2	0.998	0.0016	26.2	0.997	0.0023	95.7	1.000	0.0009	52.3	1.000	0.0012
57	77.7	0.997	0.0018	23.7	0.997	0.0025	94.3	1.000	0.0010	52.0	1.000	0.0013
58	69.2	0.995	0.0020	21.8	0.996	0.0027	92.9	1.000	0.0011	52.0	1.000	0.0015
59	74.3	0.998	0.0016	14.3	0.997	0.0023	95.2	1.000	0.0009	41.7	1.000	0.0012
60	66.6	0.996	0.0017	13.0	0.996	0.0025	93.8	1.000	0.0010	41.4	1.000	0.0014
61	59.8	0.995	0.0019	12.1	0.995	0.0027	92.5	1.000	0.0011	41.5	1.000	0.0015
62	66.6	0.998	0.0017	12.9	0.997	0.0025	94.5	1.000	0.0010	41.5	1.000	0.0013
63	59.2	0.997	0.0019	11.7	0.996	0.0027	93.1	1.000	0.0011	41.3	1.000	0.0015
64	52.6	0.995	0.0022	10.8	0.995	0.0030	91.7	1.000	0.0013	41.4	1.000	0.0016
65	59.4	0.998	0.0017	6.7	0.997	0.0025	97.0	1.000	0.0011	32.6	0.999	0.0014
66	53.0	0.997	0.0019	6.1	0.996	0.0028	95.5	1.000	0.0012	32.4	0.999	0.0016
67	47.4	0.996	0.0021	5.7	0.995	0.0030	94.0	0.999	0.0013	32.4	0.999	0.0017
68	46.9	0.998	0.0022	5.3	0.997	0.0032	94.8	1.000	0.0013	31.9	0.999	0.0018
69	44.1	0.997	0.0023	5.1	0.996	0.0033	94.1	0.999	0.0014	31.9	0.999	0.0018
70	41.5	0.996	0.0024	4.9	0.996	0.0035	93.3	0.999	0.0015	31.9	0.999	0.0019
71	31.7	0.998	0.0020	8.8	0.997	0.0029	73.1	1.000	0.0011	38.5	1.000	0.0015
72	29.8	0.997	0.0021	8.4	0.996	0.0030	72.5	1.000	0.0012	38.5	1.000	0.0016
73	27.9	0.996	0.0022	8.1	0.996	0.0032	71.8	1.000	0.0013	38.6	1.000	0.0017
74	650.2	0.994	0.0007	666.1	1.014	0.0004	125.9	0.998	0.0004	127.0	1.008	0.0003
75	604.2	0.994	0.0008	547.1	1.012	0.0005	127.4	0.998	0.0004	120.9	1.007	0.0003

**Table 5.2.b (Cont'd)**

No.	$\bar{I}_k$ ( $10^6\text{mm}^4$ )	$\rho_{I_k}$	$V_{I_k}$	$\bar{I}_y$ ( $10^6\text{mm}^4$ )	$\rho_{I_y}$	$V_{I_y}$	$\bar{r}_x$ (mm)	$\rho_{r_x}$	$V_{r_x}$	$\bar{r}_y$ (mm)	$\rho_{r_y}$	$V_{r_y}$
76	567.6	0.993	0.0009	456.1	1.011	0.0006	128.9	0.998	0.0005	115.2	1.007	0.0004
77	593.8	0.995	0.0008	639.6	1.014	0.0004	124.3	0.998	0.0005	128.6	1.008	0.0003
78	545.4	0.994	0.0009	520.8	1.013	0.0005	125.7	0.998	0.0005	122.4	1.007	0.0003
79	509.0	0.993	0.0010	431.4	1.011	0.0006	127.2	0.998	0.0005	116.7	1.007	0.0004
80	474.7	0.992	0.0010	347.2	1.009	0.0007	129.1	0.998	0.0006	110.0	1.007	0.0004
81	505.2	0.994	0.0010	506.3	1.013	0.0005	124.4	0.998	0.0005	124.2	1.008	0.0003
82	469.0	0.993	0.0011	417.5	1.012	0.0006	125.9	0.998	0.0006	118.4	1.007	0.0004
83	434.8	0.992	0.0011	333.7	1.010	0.0008	127.9	0.998	0.0006	111.6	1.007	0.0005
84	418.4	0.991	0.0012	293.8	1.009	0.0009	129.1	0.998	0.0006	107.7	1.007	0.0005
85	465.4	0.994	0.0011	491.6	1.014	0.0005	122.6	0.998	0.0006	125.6	1.007	0.0003
86	429.5	0.993	0.0011	403.3	1.012	0.0006	124.1	0.998	0.0006	119.8	1.007	0.0004
87	395.4	0.992	0.0012	320.1	1.011	0.0008	126.0	0.998	0.0007	112.9	1.007	0.0005
88	379.1	0.991	0.0013	280.3	1.009	0.0009	127.2	0.998	0.0007	108.9	1.007	0.0005
89	397.8	0.994	0.0012	391.3	1.013	0.0007	122.7	0.998	0.0007	121.3	1.007	0.0004
90	363.8	0.992	0.0013	308.6	1.011	0.0008	124.6	0.998	0.0007	114.3	1.007	0.0005
91	347.6	0.992	0.0014	269.1	1.010	0.0009	125.8	0.998	0.0008	110.3	1.007	0.0006
92	331.7	0.991	0.0015	230.8	1.008	0.0011	127.2	0.998	0.0008	105.7	1.007	0.0006
93	369.7	0.994	0.0013	380.4	1.013	0.0007	121.5	0.998	0.0007	122.9	1.007	0.0004
94	335.9	0.993	0.0015	298.2	1.012	0.0008	123.5	0.998	0.0008	115.9	1.007	0.0005
95	319.6	0.992	0.0015	258.9	1.011	0.0010	124.7	0.998	0.0008	111.8	1.007	0.0006
96	303.8	0.991	0.0016	220.9	1.009	0.0011	126.1	0.998	0.0009	107.1	1.007	0.0007
97	302.6	0.994	0.0014	243.1	1.014	0.0006	117.7	0.997	0.0007	105.2	1.007	0.0004
98	273.4	0.993	0.0015	187.8	1.013	0.0008	120.0	0.997	0.0008	99.1	1.007	0.0005
99	259.5	0.992	0.0016	161.6	1.012	0.0009	121.5	0.997	0.0009	95.5	1.007	0.0006
100	246.1	0.991	0.0017	136.3	1.010	0.0011	123.2	0.997	0.0009	91.3	1.007	0.0007

**Table 5.2.b (Cont'd)**

No.	$\bar{I}_x$ ( $10^6 \text{mm}^4$ )	$\rho_{I_x}$	$V_{I_x}$	$\bar{I}_y$ ( $10^6 \text{mm}^4$ )	$\rho_{I_y}$	$V_{I_y}$	$\bar{r}_x$ (mm)	$\rho_{r_x}$	$V_{r_x}$	$\bar{r}_y$ (mm)	$\rho_{r_y}$	$V_{r_y}$		
101	252.7	0.993	0.0016	183.2	1.013	0.0008	118.2	117.8	0.997	0.0009	100.3	101.0	1.007	0.0005
102	238.8	0.993	0.0017	156.9	1.012	0.0010	119.6	119.3	0.997	0.0009	96.6	97.3	1.007	0.0006
103	225.4	0.992	0.0018	131.7	1.011	0.0011	121.3	121.0	0.997	0.0010	92.3	92.9	1.007	0.0007
104	220.8	0.995	0.0015	115.1	1.015	0.0008	115.5	115.2	0.997	0.0008	83.2	83.8	1.007	0.0005
105	207.0	0.994	0.0016	97.6	1.014	0.0009	116.9	116.6	0.997	0.0009	80.0	80.6	1.007	0.0006
106	193.5	0.993	0.0017	80.8	1.012	0.0011	118.6	118.2	0.997	0.0010	76.3	76.8	1.007	0.0007
107	204.3	0.995 <sup>1</sup>	0.0016	111.8	1.015	0.0008	114.3	114.0	0.997	0.0009	84.4	85.0	1.007	0.0006
108	190.5	0.994	0.0017	94.5	1.014	0.0009	115.7	115.3	0.997	0.0010	81.2	81.8	1.007	0.0006
109	177.1	0.993	0.0018	77.8	1.013	0.0011	117.4	117.0	0.997	0.0010	77.5	78.1	1.007	0.0007
110	158.9	0.993	0.0017	160.8	1.013	0.0009	99.4	99.1	0.998	0.0009	99.6	100.4	1.007	0.0006
111	150.2	0.992	0.0018	138.4	1.012	0.0010	100.4	100.2	0.998	0.0010	96.0	96.7	1.007	0.0006
112	141.9	0.991	0.0019	116.8	1.010	0.0012	101.7	101.5	0.998	0.0010	91.9	92.6	1.007	0.0007
113	134.5	0.990	0.0020	96.7	1.008	0.0014	103.4	103.2	0.998	0.0011	87.2	87.8	1.007	0.0008
114	133.8	0.995	0.0016	98.5	1.014	0.0008	96.9	96.6	0.997	0.0009	82.9	83.5	1.007	0.0005
115	125.2	0.994	0.0017	83.7	1.013	0.0010	97.9	97.6	0.997	0.0009	79.8	80.3	1.007	0.0006
116	116.9	0.993	0.0018	69.4	1.012	0.0011	99.2	98.9	0.998	0.0010	76.2	76.7	1.007	0.0007
117	109.5	0.992	0.0020	56.3	1.010	0.0014	100.9	100.6	0.998	0.0011	72.0	72.5	1.007	0.0008
118	108.7	0.995	0.0020	79.3	1.014	0.0010	95.5	95.3	0.997	0.0011	81.4	82.0	1.007	0.0007
119	100.4	0.994	0.0021	65.2	1.013	0.0012	96.7	96.5	0.997	0.0012	77.7	78.3	1.007	0.0008
120	93.0	0.993	0.0023	52.2	1.011	0.0015	98.4	98.1	0.997	0.0013	73.4	73.9	1.007	0.0009
121	87.0	0.994	0.0016	87.4	1.013	0.0009	81.3	81.1	0.998	0.0009	81.2	81.8	1.007	0.0005
122	82.1	0.993	0.0017	74.9	1.012	0.0010	82.1	81.9	0.998	0.0009	78.2	78.7	1.007	0.0006
123	77.3	0.992	0.0018	62.9	1.010	0.0012	83.1	82.9	0.998	0.0010	74.7	75.2	1.007	0.0007
124	73.2	0.991	0.0019	51.9	1.008	0.0014	84.4	84.3	0.998	0.0010	70.7	71.2	1.007	0.0008
125	78.5	0.994	0.0018	84.2	1.014	0.0009	80.0	79.8	0.998	0.0010	82.6	83.2	1.008	0.0006

**Table 5.2.b (Cont'd)**

No.	$\bar{I}_x$ ( $10^6 \text{mm}^4$ )	$\rho_{Ix}$	$V_{Ix}$	$\bar{I}_y$ ( $10^6 \text{mm}^4$ )	$\rho_{Iy}$	$V_{Iy}$	$\bar{r}_x$ (mm)	$\bar{r}_y$ (mm)	$\rho_{rx}$	$V_{rx}$	$\bar{r}_x$ (mm)	$\bar{r}_y$ (mm)	$\rho_{ry}$	$V_{ry}$
126	73.6	0.993	0.0019	71.8	1.012	0.0010	80.8	80.6	0.998	0.0010	79.5	80.1	1.007	0.0006
127	68.9	0.992	0.0020	59.9	1.011	0.0012	81.8	81.6	0.998	0.0011	76.0	76.6	1.007	0.0007
128	64.8	0.991	0.0021	49.0	1.009	0.0015	83.2	83.0	0.998	0.0012	72.0	72.5	1.007	0.0009
129	66.4	0.994	0.0021	68.8	1.013	0.0011	79.6	79.4	0.998	0.0011	80.8	81.4	1.007	0.0007
130	61.7	0.993	0.0023	57.0	1.012	0.0013	80.6	80.4	0.998	0.0012	77.2	77.8	1.007	0.0008
131	57.6	0.992	0.0024	46.2	1.010	0.0016	81.9	81.8	0.998	0.0013	73.1	73.6	1.007	0.0009
132	62.0	0.995	0.0018	45.5	1.014	0.0010	78.9	78.7	0.998	0.0010	67.4	67.9	1.007	0.0006
133	57.3	0.994	0.0020	37.3	1.013	0.0012	79.9	79.7	0.998	0.0011	64.3	64.8	1.007	0.0007
134	53.2	0.992	0.0021	29.9	1.011	0.0014	81.3	81.1	0.998	0.0012	60.7	61.1	1.007	0.0009
135	55.5	0.995	0.0020	43.7	1.015	0.0010	77.6	77.4	0.998	0.0011	68.7	69.2	1.007	0.0007
136	50.8	0.994	0.0022	35.7	1.014	0.0012	78.5	78.3	0.998	0.0012	65.6	66.1	1.007	0.0008
137	46.7	0.993	0.0024	28.3	1.012	0.0015	79.9	79.7	0.998	0.0013	62.0	62.4	1.007	0.0009
138	53.0	0.996	0.0019	28.5	1.016	0.0010	78.0	77.8	0.997	0.0011	57.2	57.6	1.007	0.0007
139	48.2	0.995	0.0021	23.0	1.015	0.0012	79.1	78.8	0.997	0.0012	54.4	54.8	1.007	0.0008
140	44.1	0.994	0.0023	17.9	1.013	0.0014	80.6	80.4	0.997	0.0013	51.2	51.6	1.007	0.0009
141	42.7	0.996	0.0024	21.9	1.015	0.0012	77.5	77.2	0.997	0.0013	55.4	55.8	1.007	0.0008
142	40.6	0.995	0.0025	19.4	1.015	0.0013	78.2	77.9	0.997	0.0014	53.9	54.2	1.007	0.0009
143	38.6	0.995	0.0026	16.9	1.014	0.0015	79.0	78.7	0.997	0.0015	52.2	52.5	1.007	0.0010
144	25.2	0.994	0.0024	24.6	1.013	0.0013	61.4	61.2	0.998	0.0013	60.4	60.9	1.007	0.0008
145	24.3	0.993	0.0025	22.1	1.012	0.0014	61.8	61.7	0.998	0.0014	58.8	59.3	1.007	0.0009
146	23.3	0.993	0.0026	19.6	1.011	0.0016	62.4	62.2	0.998	0.0014	57.1	57.5	1.007	0.0010

**Table 5.3 Statistical Quantities,  $\rho_G$  and  $V_G$ , for Geometric Variations**

Geometric Variation	$\rho_G$	$V_G$
G		
A	0.997	0.002
$I_x$	0.995	0.003
$I_y$	1.004	0.008
$r_x$	0.999	0.002
$r_y$	1.004	0.004
I	1.000	0.008
r	1.001	0.004

Note: A - area of the column reinforced with welded steel plates  
 $I_x$  - moment of inertia of cross section about principal x axis  
 $I_y$  - moment of inertia of cross section about principal y axis  
 $r_x$  - radius of gyration of the cross section about principal x axis  
 $r_y$  - radius of gyration of the cross section about principal y axis  
I - moment of inertia of cross section  
r - radius of gyration of the cross section

**Table 5.4 Statistical Parameters for the Material Properties**

Reference Criteria	Slenderness parameter $\lambda$	$\rho_r$	$V_r$	$\rho_{F_y}$	$V_{F_y}$	$\rho_E$	$V_E$	$\rho_\lambda$	$\rho_F$	$V_F$
SSRC Curve 1	0.4	1.001	0.004	1.092	0.088	1.038	0.026	1.025	1.090	0.085
	1.1	1.001	0.004	1.092	0.088	1.038	0.026	1.025	1.062	0.039
	1.5	1.001	0.004	1.092	0.088	1.038	0.026	1.025	1.047	0.026
SSRC Curve 2	0.4	1.001	0.004	1.092	0.088	1.038	0.026	1.025	1.088	0.080
	1.1	1.001	0.004	1.092	0.088	1.038	0.026	1.025	1.057	0.034
	1.5	1.001	0.004	1.092	0.088	1.038	0.026	1.025	1.055	0.031
CSA Curve 1	0.4	1.001	0.004	1.092	0.088	1.038	0.026	1.025	1.091	0.086
	1.1	1.001	0.004	1.092	0.088	1.038	0.026	1.025	1.060	0.036
	1.5	1.001	0.004	1.092	0.088	1.038	0.026	1.025	1.047	0.026
CSA Curve 2	0.4	1.001	0.004	1.092	0.088	1.038	0.026	1.025	1.088	0.080
	1.1	1.001	0.004	1.092	0.088	1.038	0.026	1.025	1.062	0.040
	1.5	1.001	0.004	1.092	0.088	1.038	0.026	1.025	1.053	0.029



**Table 5.5 Simulated Professional Factors for Columns from Group 1 ( $\lambda = 0.4$ )**

FEA model No.	D	B	Out-of-Straightness $\delta_0$	$P_{fea}/P_{ry}$	SSRC 1 $\rho_s$ ( $P_{fea}/P_{r1}$ )	SSRC 2 $\rho_s$ ( $P_{fea}/P_{r2}$ )	CSA 1 $\rho_s$ ( $P_{fea}/P_{rc1}$ )	CSA 2 $\rho_s$ ( $P_{fea}/P_{rc2}$ )
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
36	F	S	L/1000	0.933	0.952	1.016	0.940	0.992
37	F	S	L/1000	0.934	0.953	1.017	0.941	0.994
38	F	S	L/1000	0.936	0.955	1.019	0.943	0.995
39	F	S	L/1000	0.942	0.961	1.025	0.949	1.001
51	G	W	L/1000	0.951	0.970	1.035	0.958	1.011
52	G	W	L/1000	0.957	0.976	1.041	0.964	1.017
53	G	W	L/1000	0.958	0.977	1.043	0.965	1.018
54	G	W	L/1000	0.962	0.982	1.048	0.970	1.023
100	F	S	L/1000	0.939	0.958	1.022	0.946	0.999
101	F	S	L/1000	0.941	0.960	1.024	0.948	1.000
102	F	S	L/1000	0.933	0.952	1.015	0.940	0.992
103	F	S	L/1000	0.939	0.958	1.022	0.946	0.999
130	F	S	L/1000	0.938	0.957	1.021	0.944	0.997
131	F	S	L/1000	0.940	0.959	1.023	0.946	0.999
132	F	S	L/1000	0.934	0.953	1.017	0.941	0.994
133	F	S	L/1000	0.939	0.958	1.022	0.946	0.998
145	G	W	L/1000	0.952	0.971	1.036	0.959	1.012
146	G	W	L/1000	0.969	0.989	1.055	0.976	1.030
147	G	W	L/1000	0.965	0.985	1.051	0.972	1.026
148	G	W	L/1000	0.968	0.987	1.053	0.975	1.029
190	F	S	L/8000	0.977	0.997	1.063	0.984	1.039
191	F	S	L/2000	0.963	0.982	1.048	0.970	1.024
192	F	S	L/1000	0.947	0.966	1.031	0.954	1.007
193	F	S	L/1000	0.954	0.973	1.038	0.961	1.014
194	F	S	L/1000	0.940	0.959	1.023	0.947	1.000
195	F	S	L/1000	0.947	0.966	1.031	0.954	1.007
209	G	W	L/8000	1.000	1.021	1.089	1.008	1.064
210	G	W	L/2000	0.981	1.001	1.067	0.988	1.043
211	G	W	L/1000	0.963	0.982	1.048	0.970	1.024
212	G	W	L/1000	0.968	0.987	1.053	0.975	1.029
213	G	W	L/1000	0.963	0.982	1.048	0.970	1.024
214	G	W	L/1000	0.968	0.987	1.053	0.975	1.029

Note:  $\delta_0$  - Initial imperfection

$P_{ry}$  - Yield strength of reinforced column

$P_{fea}$  - Finite element analysis after reinforcing

$P_{r1}$  - Capacity after reinforcing (SSRC1)

$P_{r2}$  - Capacity after reinforcing (SSRC2)

$P_{rc1}$  - Capacity after reinforcing (CSA1)

$P_{rc2}$  - Capacity after reinforcing (CSA2)

L - Column length

D - Orientation of reinforcing plates

F - Parallel to the flanges

G - Parallel to the web

B - Buckling axis

W - Weak axis of the rolled section

S - Strong axis of the rolled section

**Table 5.5 (Cont'd)**

FEA model No.	D	B	Out-of-Straightness $\delta_0$	$P_{fea}/P_{ry}$	SSRC 1 $\rho_s$ ( $P_{fea}/P_{r1}$ )	SSRC 2 $\rho_s$ ( $P_{fea}/P_{r2}$ )	CSA 1 $\rho_s$ ( $P_{fea}/P_{rc1}$ )	CSA 2 $\rho_s$ ( $P_{fea}/P_{rc2}$ )
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
262	F	S	L/1000	0.944	0.963	1.027	0.951	1.004
263	F	S	L/1000	0.952	0.971	1.036	0.959	1.012
264	F	S	L/1000	0.946	0.966	1.030	0.953	1.006
265	F	S	L/1000	0.952	0.971	1.036	0.959	1.012
277	G	W	L/1000	0.956	0.975	1.041	0.963	1.017
278	G	W	L/1000	0.961	0.980	1.046	0.968	1.021
279	G	W	L/1000	0.961	0.980	1.046	0.968	1.022
280	G	W	L/1000	0.964	0.984	1.049	0.971	1.025

Note:  $\delta_0$  - Initial imperfection

L - Column length

 $P_{ry}$  - Yield strength of reinforced column

D - Orientation of reinforcing plates

 $P_{fea}$  - Finite element analysis after reinforcing

F - Parallel to the flanges

 $P_{r1}$  - Capacity after reinforcing (SSRC1)

G - Parallel to the web

 $P_{r2}$  - Capacity after reinforcing (SSRC2)

B - Buckling axis

 $P_{rc1}$  - Capacity after reinforcing (CSA1)

W - Weak axis of the rolled section

 $P_{rc2}$  - Capacity after reinforcing (CSA2)

S - Strong axis of the rolled section

**Table 5.6 Simulated Professional Factors for Columns from Group 1 ( $\lambda = 1.1$ )**

FEA model No.	D	B	Out-of-Straightness $\delta_0$	$P_{fea}/P_{ry}$	SSRC 1 $\rho_s$ ( $P_{fea}/P_{r1}$ )	SSRC 2 $\rho_s$ ( $P_{fea}/P_{r2}$ )	CSA 1 $\rho_s$ ( $P_{fea}/P_{rc1}$ )	CSA 2 $\rho_s$ ( $P_{fea}/P_{rc2}$ )
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
40	F	S	L/8000	0.682	1.003	1.265	1.033	1.266
41	F	S	L/2000	0.630	0.926	1.168	0.954	1.169
42	F	S	L/1400	0.609	0.895	1.129	0.922	1.130
43	F	S	L/1400	0.618	0.908	1.146	0.935	1.147
44	F	S	L/1400	0.626	0.920	1.161	0.948	1.162
45	F	S	L/1400	0.566	0.832	1.049	0.856	1.050
46	F	S	L/1400	0.585	0.860	1.086	0.886	1.086
55	G	W	L/8000	0.707	1.040	1.312	1.071	1.313

Note:  $\delta_0$  - Initial imperfection

L - Column length

 $P_{ry}$  - Yield strength of reinforced column

D - Orientation of reinforcing plates

 $P_{fea}$  - Finite element analysis after reinforcing

F - Parallel to the flanges

 $P_{r1}$  - Capacity after reinforcing (SSRC1)

G - Parallel to the web

 $P_{r2}$  - Capacity after reinforcing (SSRC2)

B - Buckling axis

 $P_{rc1}$  - Capacity after reinforcing (CSA1)

W - Weak axis of the rolled section

 $P_{rc2}$  - Capacity after reinforcing (CSA2)

S - Strong axis of the rolled section

**Table 5.6 (Cont'd)**

FEA model No.	D	B	Out-of-Straightness $\delta_0$	$P_{fea}/P_{ry}$	SSRC 1 $\rho_s$ ( $P_{fea}/P_{r1}$ )	SSRC 2 $\rho_s$ ( $P_{fea}/P_{r2}$ )	CSA 1 $\rho_s$ ( $P_{fea}/P_{rc1}$ )	CSA 2 $\rho_s$ ( $P_{fea}/P_{rc2}$ )
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
56	G	W	L/2000	0.636	0.935	1.180	0.963	1.181
57	G	W	L/1100	0.588	0.865	1.091	0.890	1.092
58	G	W	L/1100	0.599	0.880	1.111	0.907	1.112
59	G	W	L/1100	0.611	0.898	1.133	0.925	1.134
60	G	W	L/1150	0.609	0.896	1.130	0.922	1.131
61	G	W	L/1150	0.631	0.927	1.170	0.955	1.171
104	F	S	L/8000	0.687	1.011	1.275	1.041	1.276
105	F	S	L/2000	0.639	0.939	1.185	0.967	1.185
106	F	S	L/1350	0.614	0.902	1.138	0.929	1.139
107	F	S	L/1350	0.624	0.917	1.157	0.944	1.158
108	F	S	L/1350	0.630	0.926	1.169	0.954	1.170
109	F	S	L/1400	0.566	0.833	1.051	0.857	1.051
110	F	S	L/1400	0.589	0.865	1.092	0.891	1.093
134	F	S	L/8000	0.683	1.004	1.266	1.034	1.267
135	F	S	L/2000	0.633	0.931	1.174	0.958	1.175
136	F	S	L/1350	0.609	0.896	1.130	0.922	1.131
137	F	S	L/1350	0.619	0.910	1.148	0.937	1.149
138	F	S	L/1350	0.627	0.922	1.163	0.950	1.164
139	F	S	L/1400	0.559	0.822	1.037	0.847	1.038
140	F	S	L/1400	0.582	0.856	1.080	0.882	1.081
149	G	W	L/8000	0.723	1.062	1.340	1.094	1.341
150	G	W	L/2000	0.656	0.965	1.217	0.993	1.218
151	G	W	L/1000	0.601	0.884	1.115	0.910	1.116
152	G	W	L/1000	0.612	0.899	1.135	0.926	1.136
153	G	W	L/1000	0.621	0.913	1.152	0.940	1.152
154	G	W	L/1100	0.620	0.912	1.151	0.939	1.151
155	G	W	L/1100	0.641	0.942	1.188	0.970	1.189
196	F	S	L/8000	0.713	1.049	1.323	1.080	1.324
197	F	S	L/2000	0.670	0.985	1.243	1.014	1.244
198	F	S	L/1000	0.629	0.924	1.166	0.952	1.167
199	F	S	L/1000	0.646	0.950	1.199	0.979	1.200
200	F	S	L/1000	0.643	0.946	1.193	0.974	1.194

Note:  $\delta_0$  - Initial imperfection

$P_{ry}$  - Yield strength of reinforced column

$P_{fea}$  - Finite element analysis after reinforcing

$P_{r1}$  - Capacity after reinforcing (SSRC1)

$P_{r2}$  - Capacity after reinforcing (SSRC2)

$P_{rc1}$  - Capacity after reinforcing (CSA1)

$P_{rc2}$  - Capacity after reinforcing (CSA2)

L - Column length

D - Orientation of reinforcing plates

F - Parallel to the flanges

G - Parallel to the web

B - Buckling axis

W - Weak axis of the rolled section

S - Strong axis of the rolled section

**Table 5.6 (Cont'd)**

FEA model No.	D	B	Out-of-Straightness $\delta_0$	$P_{fea}/P_{ry}$	SSRC 1 $\rho_s$ ( $P_{fea}/P_{r1}$ )	SSRC 2 $\rho_s$ ( $P_{fea}/P_{r2}$ )	CSA 1 $\rho_s$ ( $P_{fea}/P_{rc1}$ )	CSA 2 $\rho_s$ ( $P_{fea}/P_{rc2}$ )
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
201	F	S	L/1000	0.590	0.868	1.095	0.894	1.096
202	F	S	L/1000	0.611	0.899	1.134	0.926	1.135
215	G	W	L/8000	0.723	1.063	1.341	1.094	1.342
216	G	W	L/2000	0.656	0.965	1.218	0.994	1.219
217	G	W	L/1000	0.603	0.886	1.118	0.913	1.119
218	G	W	L/1000	0.615	0.904	1.141	0.931	1.142
219	G	W	L/1000	0.616	0.906	1.143	0.933	1.144
220	G	W	L/1000	0.627	0.922	1.164	0.950	1.165
221	G	W	L/1000	0.650	0.956	1.206	0.985	1.207
266	F	S	L/8000	0.718	1.055	1.332	1.087	1.333
267	F	S	L/2000	0.668	0.983	1.240	1.012	1.241
268	F	S	L/1000	0.627	0.922	1.163	0.949	1.164
269	F	S	L/1000	0.647	0.951	1.200	0.979	1.201
270	F	S	L/1000	0.648	0.953	1.202	0.981	1.203
271	F	S	L/1000	0.603	0.886	1.118	0.912	1.119
272	F	S	L/1000	0.618	0.909	1.147	0.936	1.148
281	G	W	L/8000	0.718	1.056	1.332	1.088	1.333
282	G	W	L/2000	0.646	0.951	1.199	0.979	1.200
283	G	W	L/1000	0.588	0.865	1.091	0.891	1.092
284	G	W	L/1000	0.601	0.884	1.116	0.911	1.117
285	G	W	L/1000	0.608	0.894	1.128	0.921	1.129
286	G	W	L/1000	0.619	0.911	1.149	0.938	1.150
287	G	W	L/1000	0.644	0.947	1.194	0.975	1.195

Note:  $\delta_0$  - Initial imperfection

$P_{ry}$  - Yield strength of reinforced column  
 $P_{fea}$  - Finite element analysis after reinforcing  
 $P_{r1}$  - Capacity after reinforcing (SSRC1)  
 $P_{r2}$  - Capacity after reinforcing (SSRC2)  
 $P_{rc1}$  - Capacity after reinforcing (CSA1)  
 $P_{rc2}$  - Capacity after reinforcing (CSA2)

L - Column length

D - Orientation of reinforcing plates  
F - Parallel to the flanges  
G - Parallel to the web  
B - Buckling axis  
W - Weak axis of the rolled section  
S - Strong axis of the rolled section

**Table 5.7 Simulated Professional Factors for Columns from Group 1 ( $\lambda = 1.5$ )**

FEA model No.	D	B	Out-of-Straightness $\delta_0$	$P_{fea}/P_{ry}$	SSRC 1 $\rho_s$ ( $P_{fea}/P_{r1}$ )	SSRC 2 $\rho_s$ ( $P_{fea}/P_{r2}$ )	CSA 1 $\rho_s$ ( $P_{fea}/P_{rc1}$ )	CSA 2 $\rho_s$ ( $P_{fea}/P_{rc2}$ )
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
47	F	S	L/1900	0.403	0.989	1.145	0.969	1.125
48	F	S	L/1900	0.411	1.011	1.170	0.990	1.150
49	F	S	L/1250	0.375	0.921	1.066	0.902	1.048
50	F	S	L/1250	0.385	0.946	1.095	0.927	1.076
62	G	W	L/1350	0.384	0.944	1.092	0.924	1.074
63	G	W	L/1350	0.396	0.973	1.126	0.953	1.107
64	G	W	L/1350	0.390	0.958	1.109	0.939	1.090
65	G	W	L/1350	0.404	0.992	1.149	0.972	1.129
111	F	S	L/1300	0.394	0.967	1.119	0.947	1.100
112	F	S	L/1300	0.400	0.983	1.138	0.963	1.118
113	F	S	L/1250	0.378	0.928	1.074	0.909	1.056
114	F	S	L/1250	0.388	0.955	1.105	0.935	1.086
141	F	S	L/1300	0.391	0.961	1.113	0.942	1.094
142	F	S	L/1300	0.398	0.977	1.131	0.957	1.112
143	F	S	L/1250	0.375	0.922	1.067	0.903	1.049
144	F	S	L/1250	0.386	0.949	1.098	0.930	1.080
156	G	W	L/1400	0.398	0.978	1.131	0.958	1.112
157	G	W	L/1400	0.412	1.012	1.172	0.991	1.152
158	G	W	L/1350	0.403	0.990	1.146	0.970	1.126
159	G	W	L/1350	0.411	1.011	1.170	0.990	1.150
203	F	S	L/8000	0.490	1.203	1.392	1.178	1.369
204	F	S	L/2000	0.459	1.127	1.304	1.104	1.282
205	F	S	L/1000	0.436	1.072	1.240	1.050	1.219
206	F	S	L/1000	0.441	1.083	1.253	1.061	1.232
207	F	S	L/1000	0.424	1.043	1.207	1.021	1.186
208	F	S	L/1000	0.432	1.061	1.228	1.039	1.207
222	G	W	L/8000	0.456	1.120	1.296	1.097	1.274
223	G	W	L/2000	0.414	1.017	1.177	0.996	1.157
224	G	W	L/1000	0.380	0.933	1.080	0.914	1.061
225	G	W	L/1000	0.393	0.965	1.117	0.946	1.098
226	G	W	L/1000	0.396	0.973	1.127	0.953	1.107
227	G	W	L/1000	0.409	1.005	1.163	0.984	1.143

Note:  $\delta_0$  - Initial imperfection  
 $P_{ry}$  - Yield strength of reinforced column  
 $P_{fea}$  - Finite element analysis after reinforcing  
 $P_{r1}$  - Capacity after reinforcing (SSRC1)  
 $P_{r2}$  - Capacity after reinforcing (SSRC2)  
 $P_{rc1}$  - Capacity after reinforcing (CSA1)  
 $P_{rc2}$  - Capacity after reinforcing (CSA2)  
L - Column length  
D - Orientation of reinforcing plates  
F - Parallel to the flanges  
G - Parallel to the web  
B - Buckling axis  
W - Weak axis of the rolled section  
S - Strong axis of the rolled section

**Table 5.7 (Cont'd)**

FEA model No.	D	B	Out-of-Straightness $\delta_0$	$P_{fea}/P_{ry}$	SSRC 1 $\rho_s$ ( $P_{fea}/P_{r1}$ )	SSRC 2 $\rho_s$ ( $P_{fea}/P_{r2}$ )	CSA 1 $\rho_s$ ( $P_{fea}/P_{rc1}$ )	CSA 2 $\rho_s$ ( $P_{fea}/P_{rc2}$ )
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
273	F	S	L/1000	0.444	1.091	1.263	1.069	1.241
274	F	S	L/1000	0.449	1.102	1.276	1.080	1.254
275	F	S	L/1000	0.427	1.049	1.214	1.028	1.193
276	F	S	L/1000	0.436	1.072	1.241	1.050	1.220
288	G	W	L/1000	0.378	0.930	1.076	0.911	1.058
289	G	W	L/1000	0.395	0.969	1.122	0.950	1.103
290	G	W	L/1000	0.395	0.971	1.124	0.951	1.105
291	G	W	L/1000	0.411	1.010	1.169	0.989	1.149

Note:  $\delta_0$  - Initial imperfection

$P_{ry}$  - Yield strength of reinforced column

$P_{fea}$  - Finite element analysis after reinforcing

$P_{r1}$  - Capacity after reinforcing (SSRC1)

$P_{r2}$  - Capacity after reinforcing (SSRC2)

$P_{rc1}$  - Capacity after reinforcing (CSA1)

$P_{rc2}$  - Capacity after reinforcing (CSA2)

L - Column length

D - Orientation of reinforcing plates

F - Parallel to the flanges

G - Parallel to the web

B - Buckling axis

W - Weak axis of the rolled section

S - Strong axis of the rolled section

**Table 5.8.a Best Fit Lines for the Professional Factors for Columns from Group 1**

Reference Criteria	Equation No.	$\lambda$	Equation for Best Fit Curve $\rho_s = m \delta_o/L + b$	$\rho_s$ $\delta_o/L = 0.00067$	$V_s$
SSRC curve 1	1	0.4	$\rho_s = -44.341x\delta_o/L + 1.014$	0.984	0.003
	2	1.1	$\rho_s = -137.03x\delta_o/L + 1.0277$	0.936	0.010
	3	1.5	$\rho_s = -95.487x\delta_o/L + 1.0807$	1.017	0.006
SSRC curve 2	1	0.4	$\rho_s = -47.304x\delta_o/L + 1.0818$	1.050	0.003
	2	1.1	$\rho_s = -172.88x\delta_o/L + 1.2966$	1.181	0.010
	3	1.5	$\rho_s = -110.51x\delta_o/L + 1.2507$	1.177	0.006
CSA curve 1	1	0.4	$\rho_s = -43.776x\delta_o/L + 1.0011$	0.972	0.003
	2	1.1	$\rho_s = -141.12x\delta_o/L + 1.0583$	0.964	0.010
	3	1.5	$\rho_s = -93.526x\delta_o/L + 1.0585$	0.996	0.006
CSA curve 2	1	0.4	$\rho_s = -46.211x\delta_o/L + 1.0568$	1.026	0.003
	2	1.1	$\rho_s = -173.02x\delta_o/L + 1.2976$	1.182	0.010
	3	1.5	$\rho_s = -108.63x\delta_o/L + 1.2294$	1.157	0.006

**Table 5.8.b Best Fit Lines for the Professional Factors for Columns from Group 2**

Reference Criteria	Equation Number	$\lambda$	Equation for Best Fit Curve $\rho_s = m \delta_o/L + b$	$\rho_s$ $\delta_o/L = 0.00067$	$V_s$
SSRC curve 1	1	0.4	$\rho_s = -45.968x\delta_o/L + 1.0217$	0.991	0.003
	2	1.1	$\rho_s = -133.23x\delta_o/L + 0.9505$	0.861	0.010
	3	1.5	$\rho_s = -164.5x\delta_o/L + 1.0558$	0.946	0.012
SSRC curve 2	1	0.4	$\rho_s = -49.041x\delta_o/L + 1.09$	1.057	0.003
	2	1.1	$\rho_s = -168.09x\delta_o/L + 1.1992$	1.087	0.010
	3	1.5	$\rho_s = -190.38x\delta_o/L + 1.2219$	1.094	0.012
CSA curve 1	1	0.4	$\rho_s = -45.383x\delta_o/L + 1.0087$	0.978	0.003
	2	1.1	$\rho_s = -137.2x\delta_o/L + 0.9788$	0.887	0.010
	3	1.5	$\rho_s = -161.12x\delta_o/L + 1.0341$	0.926	0.012
CSA curve 2	1	0.4	$\rho_s = -47.907x\delta_o/L + 1.0648$	1.033	0.003
	2	1.1	$\rho_s = -168.21x\delta_o/L + 1.2001$	1.087	0.010
	3	1.5	$\rho_s = -187.13x\delta_o/L + 1.2011$	1.076	0.012

**Table 5.9 Normalized Professional Factors for Columns from Group 1 ( $\lambda = 0.4$ )**

FEA model No.	D	B	Out-of-Straightness $\delta_0$	SSRC 2 $\rho_s$ ( $P_{fea}/P_{r2}$ )	$\rho_s = m \delta_0/L + b$ SSRC 2	$\rho_{seq}$	SSRC 2 $\rho_n$ ( $\rho_s/\rho_{seq}$ )
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
36	F	S	L/1000	1.016	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	0.982
37	F	S	L/1000	1.017	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	0.983
38	F	S	L/1000	1.019	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	0.985
39	F	S	L/1000	1.025	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	0.991
51	G	W	L/1000	1.035	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	1.000
52	G	W	L/1000	1.041	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	1.006
53	G	W	L/1000	1.043	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	1.008
54	G	W	L/1000	1.048	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	1.013
100	F	S	L/1000	1.022	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	0.988
101	F	S	L/1000	1.024	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	0.990
102	F	S	L/1000	1.015	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	0.982
103	F	S	L/1000	1.022	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	0.988
130	F	S	L/1000	1.021	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	0.986
131	F	S	L/1000	1.023	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	0.989
132	F	S	L/1000	1.017	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	0.983
133	F	S	L/1000	1.022	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	0.988
145	G	W	L/1000	1.036	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	1.002
146	G	W	L/1000	1.055	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	1.020
147	G	W	L/1000	1.051	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	1.016
148	G	W	L/1000	1.053	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	1.018
190	F	S	L/8000	1.063	$\rho_s = -47.3x\delta_0/L + 1.082$	1.076	0.988
191	F	S	L/2000	1.048	$\rho_s = -47.3x\delta_0/L + 1.082$	1.058	0.990
192	F	S	L/1000	1.031	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	0.996
193	F	S	L/1000	1.038	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	1.003
194	F	S	L/1000	1.023	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	0.989
195	F	S	L/1000	1.031	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	0.996
209	G	W	L/8000	1.089	$\rho_s = -47.3x\delta_0/L + 1.082$	1.076	1.012
210	G	W	L/2000	1.067	$\rho_s = -47.3x\delta_0/L + 1.082$	1.058	1.009
211	G	W	L/1000	1.048	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	1.013
212	G	W	L/1000	1.053	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	1.018
213	G	W	L/1000	1.048	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	1.013
214	G	W	L/1000	1.053	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	1.018
277	G	W	L/1000	1.041	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	1.006

Note: L - Column length  
D - Orientation of reinforcing plate  
F - Parallel to the flanges  
G - Parallel to the web  
B - Buckling axis  
W - Weak axis of the rolled section  
S - Strong axis of the rolled section  
 $P_{r2}$  - Capacity after reinforcing (SSRC2)  
 $P_{fea}$  - Finite element analysis after reinforcing  
 $\rho_{seq}$  - Professional ratio predicted by the best-fit equation



**Table 5.9 (Cont'd)**

FEA model No.	D	B	Out-of-Straightness $\delta_0$	SSRC 2 $\rho_s$ ( $P_{fea}/P_{r2}$ )	$\rho_s = m \delta_0/L + b$ SSRC 2	$\rho_{seq}$	SSRC 2 $\rho_n$ $\rho_s/\rho_{seq}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
278	G	W	L/1000	1.046	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	1.011
279	G	W	L/1000	1.046	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	1.011
280	G	W	L/1000	1.049	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	1.014
262	F	S	L/1000	1.027	$\rho_s = -47.3x\delta_0/L + 1.082$	1.035	0.993
263	F	S	L/1000	1.036	$\rho_s = -47.3x\delta_0/L + 1.082$	1.035	1.001
264	F	S	L/1000	1.030	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	0.996
265	F	S	L/1000	1.036	$\rho_s = -47.3x\delta_0/L + 1.082$	1.034	1.002

Note: L - Column length  
 D - Orientation of reinforcing plate  
 F - Parallel to the flanges  
 G - Parallel to the web  
 $P_{fea}$  - Finite element analysis after reinforcing  
 $\rho_{seq}$  - Professional ratio predicted by the best-fit equation  
 B - Buckling axis  
 W - Weak axis of the rolled section  
 S - Strong axis of the rolled section  
 $P_{r2}$  - Capacity after reinforcing (SSRC2)

**Table 5.10 Normalized Professional Factors for Columns from Group 1 ( $\lambda = 1.1$ )**

FEA model No.	D	B	Out-of-Straightness $\delta_0$	SSRC 2 $\rho_s$ ( $P_{fea}/P_{r2}$ )	$\rho_s = m \delta_0/L + b$ SSRC 2	$\rho_{seq}$	SSRC 2 $\rho_n$ $\rho_s/\rho_{seq}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
40	F	S	L/8000	1.265	$\rho_s = -172.88x\delta_0/L + 1.30$	1.275	0.992
41	F	S	L/2000	1.168	$\rho_s = -172.88x\delta_0/L + 1.30$	1.210	0.965
42	F	S	L/1400	1.129	$\rho_s = -172.88x\delta_0/L + 1.30$	1.172	0.963
43	F	S	L/1400	1.146	$\rho_s = -172.88x\delta_0/L + 1.30$	1.172	0.977
44	F	S	L/1400	1.161	$\rho_s = -172.88x\delta_0/L + 1.30$	1.172	0.990
45	F	S	L/1450	1.049	$\rho_s = -172.88x\delta_0/L + 1.30$	1.177	0.891
46	F	S	L/1400	1.086	$\rho_s = -172.88x\delta_0/L + 1.30$	1.172	0.927
55	G	W	L/8000	1.312	$\rho_s = -172.88x\delta_0/L + 1.30$	1.275	1.029
56	G	W	L/2000	1.180	$\rho_s = -172.88x\delta_0/L + 1.30$	1.210	0.975
57	G	W	L/1100	1.091	$\rho_s = -172.88x\delta_0/L + 1.30$	1.143	0.954
58	G	W	L/1100	1.111	$\rho_s = -172.88x\delta_0/L + 1.30$	1.143	0.971
59	G	W	L/1100	1.133	$\rho_s = -172.88x\delta_0/L + 1.30$	1.143	0.991

Note: L - Column length  
 D - Orientation of reinforcing plate  
 F - Parallel to the flanges  
 G - Parallel to the web  
 $P_{fea}$  - Finite element analysis after reinforcing  
 $\rho_{seq}$  - Professional ratio predicted by the best-fit equation  
 B - Buckling axis  
 W - Weak axis of the rolled section  
 S - Strong axis of the rolled section  
 $P_{r2}$  - Capacity after reinforcing (SSRC2)

**Table 5.10 (Cont'd)**

FEA model No.	D	B	Out-of-Straightness $\delta_0$	SSRC 2 $\rho_s$ ( $P_{fea}/P_{r2}$ )	$\rho_s = m \delta_0/L + b$ SSRC 2	$\rho_{seq}$	SSRC 2 $\rho_n$ ( $\rho_s/\rho_{seq}$ )
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
60	G	W	L/1150	1.130	$\rho_s = -172.88x\delta_0/L + 1.30$	1.148	0.984
61	G	W	L/1150	1.170	$\rho_s = -172.88x\delta_0/L + 1.30$	1.148	1.019
104	F	S	L/8000	1.275	$\rho_s = -172.88x\delta_0/L + 1.30$	1.275	1.000
105	F	S	L/2000	1.185	$\rho_s = -172.88x\delta_0/L + 1.30$	1.210	0.979
106	F	S	L/1350	1.138	$\rho_s = -172.88x\delta_0/L + 1.30$	1.168	0.975
107	F	S	L/1350	1.157	$\rho_s = -172.88x\delta_0/L + 1.30$	1.168	0.990
108	F	S	L/1350	1.169	$\rho_s = -172.88x\delta_0/L + 1.30$	1.168	1.001
109	F	S	L/1400	1.051	$\rho_s = -172.88x\delta_0/L + 1.30$	1.172	0.896
110	F	S	L/1400	1.092	$\rho_s = -172.88x\delta_0/L + 1.30$	1.172	0.932
134	F	S	L/8000	1.266	$\rho_s = -172.88x\delta_0/L + 1.30$	1.275	0.993
135	F	S	L/2000	1.174	$\rho_s = -172.88x\delta_0/L + 1.30$	1.210	0.970
136	F	S	L/1350	1.130	$\rho_s = -172.88x\delta_0/L + 1.30$	1.167	0.968
137	F	S	L/1350	1.148	$\rho_s = -172.88x\delta_0/L + 1.30$	1.169	0.982
138	F	S	L/1350	1.163	$\rho_s = -172.88x\delta_0/L + 1.30$	1.169	0.995
139	F	S	L/1400	1.037	$\rho_s = -172.88x\delta_0/L + 1.30$	1.172	0.885
140	F	S	L/1400	1.080	$\rho_s = -172.88x\delta_0/L + 1.30$	1.172	0.922
149	G	W	L/8000	1.340	$\rho_s = -172.88x\delta_0/L + 1.30$	1.275	1.051
150	G	W	L/2000	1.217	$\rho_s = -172.88x\delta_0/L + 1.30$	1.210	1.006
151	G	W	L/1000	1.115	$\rho_s = -172.88x\delta_0/L + 1.30$	1.128	0.989
152	G	W	L/1000	1.135	$\rho_s = -172.88x\delta_0/L + 1.30$	1.128	1.006
153	G	W	L/1000	1.152	$\rho_s = -172.88x\delta_0/L + 1.30$	1.128	1.021
154	G	W	L/1100	1.151	$\rho_s = -172.88x\delta_0/L + 1.30$	1.136	1.012
155	G	W	L/1100	1.188	$\rho_s = -172.88x\delta_0/L + 1.30$	1.136	1.046
196	F	S	L/8000	1.323	$\rho_s = -172.88x\delta_0/L + 1.30$	1.275	1.038
197	F	S	L/2000	1.243	$\rho_s = -172.88x\delta_0/L + 1.30$	1.210	1.027
198	F	S	L/1000	1.166	$\rho_s = -172.88x\delta_0/L + 1.30$	1.124	1.038
199	F	S	L/1000	1.199	$\rho_s = -172.88x\delta_0/L + 1.30$	1.124	1.067
200	F	S	L/1000	1.193	$\rho_s = -172.88x\delta_0/L + 1.30$	1.124	1.062
201	F	S	L/1000	1.095	$\rho_s = -172.88x\delta_0/L + 1.30$	1.124	0.974
202	F	S	L/1000	1.134	$\rho_s = -172.88x\delta_0/L + 1.30$	1.124	1.009
215	G	W	L/8000	1.341	$\rho_s = -172.88x\delta_0/L + 1.30$	1.275	1.051
216	G	W	L/2000	1.218	$\rho_s = -172.88x\delta_0/L + 1.30$	1.210	1.006
217	G	W	L/1000	1.118	$\rho_s = -172.88x\delta_0/L + 1.30$	1.124	0.995

Note: L - Column length  
D - Orientation of reinforcing plate  
F - Parallel to the flanges  
G - Parallel to the web  
P<sub>fea</sub> - Finite element analysis after reinforcing  
ρ<sub>seq</sub> - Professional ratio predicted by the best-fit equation  
B - Buckling axis  
W - Weak axis of the rolled section  
S - Strong axis of the rolled section  
P<sub>r2</sub> - Capacity after reinforcing (SSRC2)

**Table 5.10 (Cont'd)**

FEA model No.	D	B	Out-of-Straightness $\delta_0$	SSRC 2 $\rho_s$ ( $P_{fea}/P_{r2}$ )	$\rho_s = m \delta_0/L + b$ SSRC 2	$\rho_{seq}$	SSRC 2 $\rho_n$ ( $\rho_s/\rho_{seq}$ )
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
218	G	W	L/1000	1.141	$\rho_s = -172.88x\delta_0/L + 1.30$	1.124	1.015
219	G	W	L/1000	1.143	$\rho_s = -172.88x\delta_0/L + 1.30$	1.124	1.017
220	G	W	L/1000	1.164	$\rho_s = -172.88x\delta_0/L + 1.30$	1.124	1.036
221	G	W	L/1000	1.206	$\rho_s = -172.88x\delta_0/L + 1.30$	1.124	1.073
281	G	W	L/8000	1.332	$\rho_s = -172.88x\delta_0/L + 1.30$	1.275	1.045
282	G	W	L/2000	1.199	$\rho_s = -172.88x\delta_0/L + 1.30$	1.210	0.991
283	G	W	L/1000	1.091	$\rho_s = -172.88x\delta_0/L + 1.30$	1.124	0.971
284	G	W	L/1000	1.116	$\rho_s = -172.88x\delta_0/L + 1.30$	1.124	0.993
285	G	W	L/1000	1.128	$\rho_s = -172.88x\delta_0/L + 1.30$	1.124	1.004
286	G	W	L/1000	1.149	$\rho_s = -172.88x\delta_0/L + 1.30$	1.124	1.023
287	G	W	L/1000	1.194	$\rho_s = -172.88x\delta_0/L + 1.30$	1.124	1.063
266	F	S	L/8000	1.332	$\rho_s = -172.88x\delta_0/L + 1.30$	1.276	1.044
267	F	S	L/2000	1.240	$\rho_s = -172.88x\delta_0/L + 1.30$	1.214	1.022
268	F	S	L/1000	1.163	$\rho_s = -172.88x\delta_0/L + 1.30$	1.124	1.035
269	F	S	L/1000	1.200	$\rho_s = -172.88x\delta_0/L + 1.30$	1.124	1.068
270	F	S	L/1000	1.202	$\rho_s = -172.88x\delta_0/L + 1.30$	1.124	1.070
271	F	S	L/1000	1.118	$\rho_s = -172.88x\delta_0/L + 1.30$	1.124	0.995
272	F	S	L/1000	1.147	$\rho_s = -172.88x\delta_0/L + 1.30$	1.124	1.021

Note: L - Column length  
D - Orientation of reinforcing plate  
F - Parallel to the flanges  
G - Parallel to the web  
B - Buckling axis  
W - Weak axis of the rolled section  
S - Strong axis of the rolled section  
P<sub>r2</sub> - Capacity after reinforcing (SSRC2)  
P<sub>fea</sub> - Finite element analysis after reinforcing  
ρ<sub>seq</sub> - Professional ratio predicted by the best-fit equation

**Table 5.11 Normalized Professional Factors for Columns from Group 1 ( $\lambda = 1.5$ )**

FEA model No.	D	B	Out-of-Straightness $\delta_0$	SSRC 2 $\rho_s$ ( $P_{fea}/P_{r2}$ )	$\rho_s = m \delta_0/L + b$ SSRC 2	$\rho_{seq}$	SSRC 2 $\rho_n$ ( $\rho_s/\rho_{seq}$ )
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
47	F	S	L/1900	1.145	$\rho_s = -110.51x\delta_0/L + 1.25$	1.193	0.960
48	F	S	L/1900	1.170	$\rho_s = -110.51x\delta_0/L + 1.25$	1.193	0.981
49	F	S	L/1250	1.066	$\rho_s = -110.51x\delta_0/L + 1.25$	1.163	0.917
50	F	S	L/1250	1.095	$\rho_s = -110.51x\delta_0/L + 1.25$	1.163	0.942
62	G	W	L/1350	1.092	$\rho_s = -110.51x\delta_0/L + 1.25$	1.169	0.934
63	G	W	L/1350	1.126	$\rho_s = -110.51x\delta_0/L + 1.25$	1.169	0.964
64	G	W	L/1350	1.109	$\rho_s = -110.51x\delta_0/L + 1.25$	1.168	0.950
65	G	W	L/1350	1.149	$\rho_s = -110.51x\delta_0/L + 1.25$	1.168	0.983
111	F	S	L/1300	1.119	$\rho_s = -110.51x\delta_0/L + 1.25$	1.164	0.961
112	F	S	L/1300	1.138	$\rho_s = -110.51x\delta_0/L + 1.25$	1.164	0.977
113	F	S	L/1300	1.074	$\rho_s = -110.51x\delta_0/L + 1.25$	1.163	0.923
114	F	S	L/1300	1.105	$\rho_s = -110.51x\delta_0/L + 1.25$	1.163	0.950
141	F	S	L/1300	1.113	$\rho_s = -110.51x\delta_0/L + 1.25$	1.164	0.956
142	F	S	L/1300	1.131	$\rho_s = -110.51x\delta_0/L + 1.25$	1.164	0.972
143	F	S	L/1300	1.067	$\rho_s = -110.51x\delta_0/L + 1.25$	1.163	0.918
144	F	S	L/1300	1.098	$\rho_s = -110.51x\delta_0/L + 1.25$	1.163	0.945
156	G	W	L/1400	1.131	$\rho_s = -110.51x\delta_0/L + 1.25$	1.171	0.966
157	G	W	L/1400	1.172	$\rho_s = -110.51x\delta_0/L + 1.25$	1.171	1.000
158	G	W	L/1350	1.146	$\rho_s = -110.51x\delta_0/L + 1.25$	1.170	0.979
159	G	W	L/1350	1.170	$\rho_s = -110.51x\delta_0/L + 1.25$	1.170	1.000
203	F	S	L/8000	1.392	$\rho_s = -110.51x\delta_0/L + 1.25$	1.237	1.126
204	F	S	L/2000	1.304	$\rho_s = -110.51x\delta_0/L + 1.25$	1.195	1.091
205	F	S	L/1000	1.240	$\rho_s = -110.51x\delta_0/L + 1.25$	1.140	1.088
206	F	S	L/1000	1.253	$\rho_s = -110.51x\delta_0/L + 1.25$	1.140	1.099
207	F	S	L/1000	1.207	$\rho_s = -110.51x\delta_0/L + 1.25$	1.140	1.058
208	F	S	L/1000	1.228	$\rho_s = -110.51x\delta_0/L + 1.25$	1.140	1.077
222	G	W	L/8000	1.296	$\rho_s = -110.51x\delta_0/L + 1.25$	1.237	1.048
223	G	W	L/2000	1.177	$\rho_s = -110.51x\delta_0/L + 1.25$	1.195	0.985
224	G	W	L/1000	1.080	$\rho_s = -110.51x\delta_0/L + 1.25$	1.140	0.947
225	G	W	L/1000	1.117	$\rho_s = -110.51x\delta_0/L + 1.25$	1.140	0.980
226	G	W	L/1000	1.127	$\rho_s = -110.51x\delta_0/L + 1.25$	1.140	0.988
227	G	W	L/1000	1.163	$\rho_s = -110.51x\delta_0/L + 1.25$	1.140	1.020
288	G	W	L/1000	1.076	$\rho_s = -110.51x\delta_0/L + 1.25$	1.140	0.944

Note: L - Column length  
 D - Orientation of reinforcing plate  
 F - Parallel to the flanges  
 G - Parallel to the web  
 B - Buckling axis  
 W - Weak axis of the rolled section  
 S - Strong axis of the rolled section  
 $P_{r2}$  - Capacity after reinforcing (SSRC2)  
 $P_{fea}$  - Finite elemetn analysis after reinforcing  
 $\rho_{seq}$  - Professional ratio predicted by the best-fit equation

**Table 5.11 (Cont'd)**

FEA model No.	D	B	Out-of- Straightness $\delta_0$	SSRC 2 $\rho_s$ ( $P_{fea}/P_{r2}$ )	$\rho_s = m \delta_0/L + b$ SSRC 2	$\rho_{seq}$	SSRC 2 $\rho_n$ $\rho_s/\rho_{seq}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
289	G	W	L/1000	1.122	$\rho_s = -110.51x\delta_0/L + 1.25$	1.140	0.984
290	G	W	L/1000	1.124	$\rho_s = -110.51x\delta_0/L + 1.25$	1.140	0.986
291	G	W	L/1000	1.169	$\rho_s = -110.51x\delta_0/L + 1.25$	1.140	1.025
273	F	S	L/1000	1.263	$\rho_s = -110.51x\delta_0/L + 1.25$	1.140	1.108
274	F	S	L/1000	1.276	$\rho_s = -110.51x\delta_0/L + 1.25$	1.140	1.119
275	F	S	L/1000	1.214	$\rho_s = -110.51x\delta_0/L + 1.25$	1.140	1.065
276	F	S	L/1000	1.241	$\rho_s = -110.51x\delta_0/L + 1.25$	1.140	1.088

Note: L - Column length

B - Buckling axis

D - Orientation of reinforcing plate

W - Weak axis of the rolled section

F - Parallel to the flanges

S - Strong axis of the rolled section

G - Parallel to the web

 $P_{r2}$  - Capacity after reinforcing (SSRC2) $P_{fea}$  - Finite element analysis after reinforcing $\rho_{seq}$  - Professional ratio predicted by the best-fit equation

**Table 5.12.a Statistical Parameters for the Professional Factors  
for Columns from Group 1**

Reference criteria	$\lambda$	$\rho_s$	$V_s$	$\rho_n$	$V_n$	$\rho_{ex}$	$V_{ex}$	$\rho_p$	$V_p$
SSRC curve 1	0.4	0.984	0.003	1.000	0.012	1.000	0.000	0.984	0.013
	1.1	0.936	0.010	1.000	0.043	1.000	0.000	0.936	0.044
	1.5	1.017	0.006	1.000	0.061	1.000	0.000	1.017	0.061
SSRC curve 2	0.4	1.050	0.003	1.000	0.012	1.000	0.000	1.050	0.013
	1.1	1.181	0.010	1.000	0.043	1.000	0.000	1.181	0.044
	1.5	1.177	0.006	1.000	0.061	1.000	0.000	1.177	0.061
CSA curve 1	0.4	0.972	0.003	1.000	0.012	1.000	0.000	0.972	0.013
	1.1	0.964	0.010	1.000	0.043	1.000	0.000	0.964	0.044
	1.5	0.996	0.006	1.000	0.061	1.000	0.000	0.996	0.061
CSA curve 2	0.4	1.026	0.003	1.000	0.012	1.000	0.000	1.026	0.013
	1.1	1.182	0.010	1.000	0.043	1.000	0.000	1.182	0.044
	1.5	1.157	0.006	1.000	0.061	1.000	0.000	1.157	0.061

**Table 5.12.b Statistical Parameters for the Professional Factors  
for Columns from Group 2**

Reference criteria	$\lambda$	$\rho_s$	$V_s$	$\rho_n$	$V_n$	$\rho_{ex}$	$V_{ex}$	$\rho_p$	$V_p$
SSRC curve 1	0.4	0.991	0.003	1.000	0.023	1.000	0.000	0.991	0.023
	1.1	0.861	0.010	1.000	0.057	1.000	0.000	0.861	0.057
	1.5	0.946	0.012	0.997	0.043	1.000	0.000	0.942	0.045
SSRC curve 2	0.4	1.057	0.003	1.000	0.023	1.000	0.000	1.057	0.023
	1.1	1.087	0.010	1.000	0.057	1.000	0.000	1.087	0.057
	1.5	1.094	0.012	1.000	0.043	1.000	0.000	1.094	0.045
CSA curve 1	0.4	0.978	0.003	1.000	0.023	1.000	0.000	0.978	0.023
	1.1	0.887	0.010	1.000	0.057	1.000	0.000	0.887	0.057
	1.5	0.926	0.012	1.000	0.043	1.000	0.000	0.926	0.045
CSA curve 2	0.4	1.033	0.003	1.000	0.023	1.000	0.000	1.033	0.023
	1.1	1.087	0.010	1.000	0.057	1.000	0.000	1.087	0.057
	1.5	1.076	0.012	1.000	0.043	1.000	0.000	1.076	0.045

**Table 5.13 Professional Factors for Unreinforced Columns**

Section	Slenderness parameter $\lambda$	Initial Imperfection $\delta_0$	$P_{fea}/P_{uy}$	$P_{ex}/P_{uy}$	$\rho_{ex} = P_{fea}/P_{ex}$
(1)	(2)	(3)	(4)	(5)	(6)
W200x46*	0.92	L/6308	0.73	0.75	0.973
W200x46*	1.22	L/2417	0.83	0.82	1.011
W310x74**	0.925	L/1500	0.67	0.76	0.879
W310x74**	0.925	L/8000	0.75	0.76	0.980
W200x36**	0.955	L/1500	0.65	0.73	0.892
W200x36**	0.955	L/6000	0.72	0.73	0.979
W150x22**	0.992	L/1500	0.70	0.73	0.956
W150x22**	0.992	L/5000	0.73	0.73	1.000

Note: \* Test results reported by Huber and Beedle (1954)

\*\* Test results reported by Beedle and Tall (1960)

$\delta_0$  - Initial imperfection of the rolled section columns

L - Column length

$P_{fea}$  - Finite element analyzed critical load capacity of the rolled section column

$P_{uy}$  - Yielding strength of the rolled section column

$P_{ex}$  - Experimental strength of the rolled section column

$\rho_{ex}$  - Experimental ratio

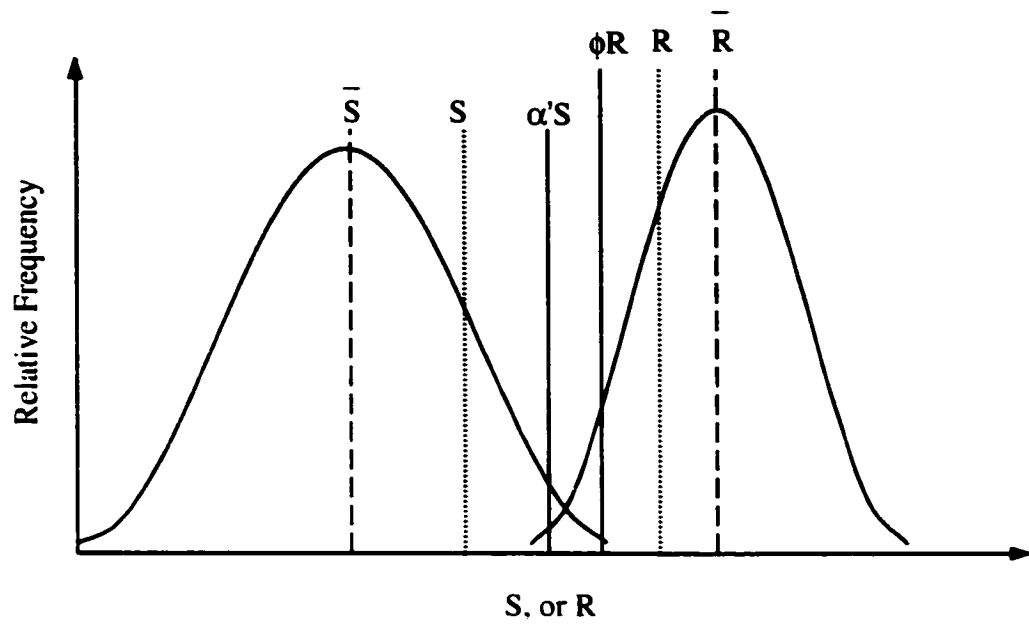
**Table 5.14.a Resistance Factors for Reinforced Columns from Group 1**

Reference criteria	$\lambda$	Mean/Nominal Ratio			Coefficient of Variation			$\rho_{Cr}$	$V_{Cr}$	$\phi$
		$\rho_G = \rho_A$	$\rho_F$	$\rho_P$	$V_G$	$V_F$	$V_P$			
SSRC curve 1	0.4	0.997	1.090	0.984	0.002	0.085	0.013	1.070	$[V_G^2 + V_F^2 + V_P^2]^{1/2}$	$\rho_{Cr} e^{-0.4\lambda V_{Cr}}$
	1.1	0.997	1.062	0.936	0.002	0.039	0.044	0.991	0.058	0.90
	1.5	0.997	1.047	1.017	0.002	0.026	0.061	1.062	0.066	0.95
SSRC curve 2	0.4	0.997	1.088	1.050	0.002	0.080	0.013	1.139	0.081	1.00
	1.1	0.997	1.057	1.181	0.002	0.034	0.044	1.245	0.055	1.14
	1.5	0.997	1.055	1.177	0.002	0.031	0.061	1.238	0.069	1.11
CSA curve 1	0.4	0.997	1.091	0.972	0.002	0.086	0.013	1.058	0.087	0.92
	1.1	0.997	1.060	0.964	0.002	0.036	0.044	1.019	0.057	0.93
	1.5	0.997	1.047	0.996	0.002	0.026	0.061	1.040	0.066	0.93
CSA curve 2	0.4	0.997	1.088	1.026	0.002	0.080	0.013	1.113	0.081	0.97
	1.1	0.997	1.062	1.182	0.002	0.040	0.044	1.252	0.059	1.14
	1.5	0.997	1.053	1.157	0.002	0.029	0.061	1.215	0.068	1.09

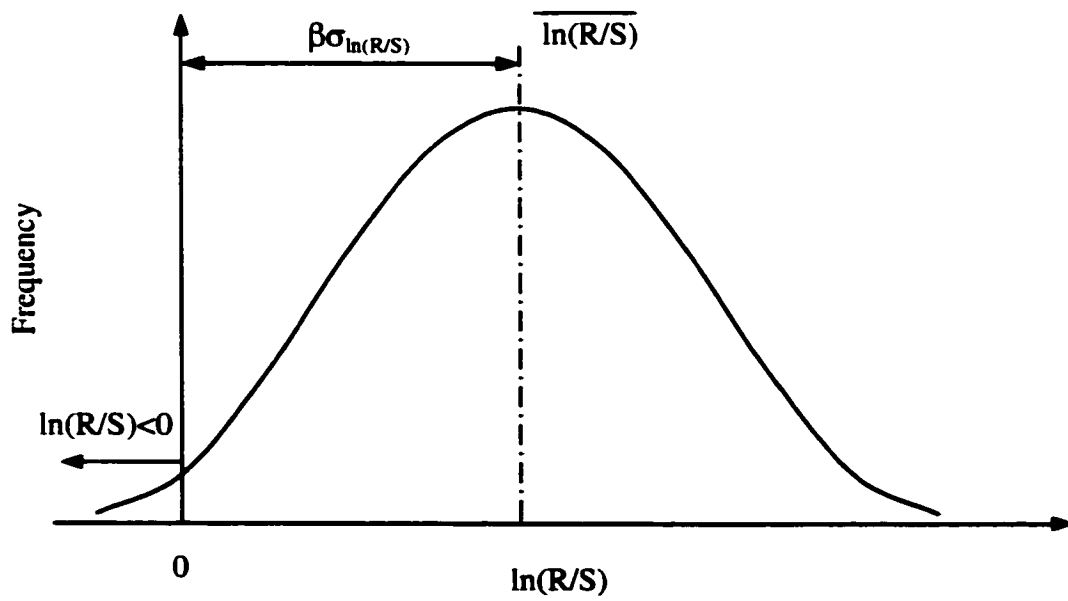


**Table 5.14.b Resistance Factors for Reinforced Columns from Group 2**

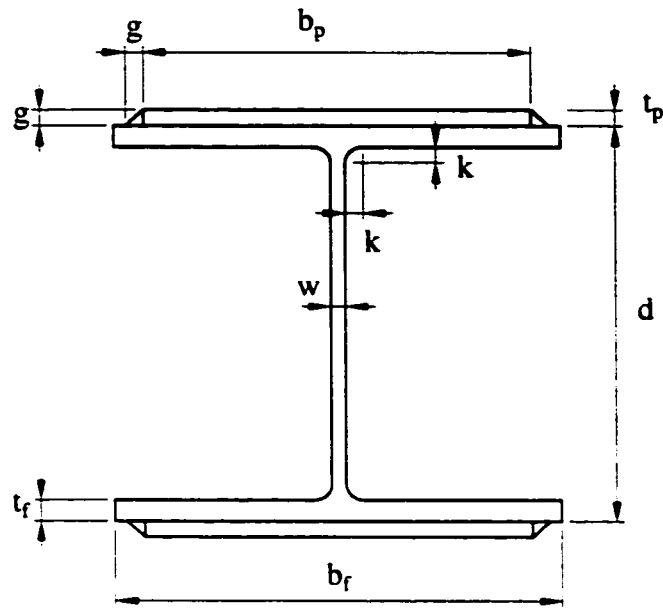
Reference criteria	$\lambda$	Mean/Nominal Ratio			Coefficient of Variation			$\rho_{cr}$	$V_{c_r}$	$\phi$
		$\rho_G = \rho_A$	$\rho_F$	$\rho_P$	$V_G$	$V_F$	$V_P$			
SSRC curve 1	0.4	0.997	1.090	0.991	0.002	0.085	0.023	1.077	0.088	0.93
	1.1	0.997	1.062	0.861	0.002	0.039	0.057	0.912	0.069	0.81
	1.5	0.997	1.047	0.942	0.002	0.026	0.045	0.984	0.052	0.90
SSRC curve 2	0.4	0.997	1.088	1.057	0.002	0.080	0.023	1.147	0.084	1.00
	1.1	0.997	1.057	1.087	0.002	0.034	0.057	1.146	0.067	1.03
	1.5	0.997	1.055	1.094	0.002	0.031	0.045	1.151	0.055	1.05
CSA curve 1	0.4	0.997	1.091	0.978	0.002	0.086	0.023	1.065	0.089	0.92
	1.1	0.997	1.060	0.887	0.002	0.036	0.057	0.937	0.068	0.84
	1.5	0.997	1.047	0.926	0.002	0.026	0.045	0.967	0.052	0.89
CSA curve 2	0.4	0.997	1.088	1.033	0.002	0.080	0.023	1.120	0.084	0.98
	1.1	0.997	1.062	1.087	0.002	0.040	0.057	1.152	0.070	1.03
	1.5	0.997	1.053	1.076	0.002	0.029	0.045	1.130	0.053	1.03



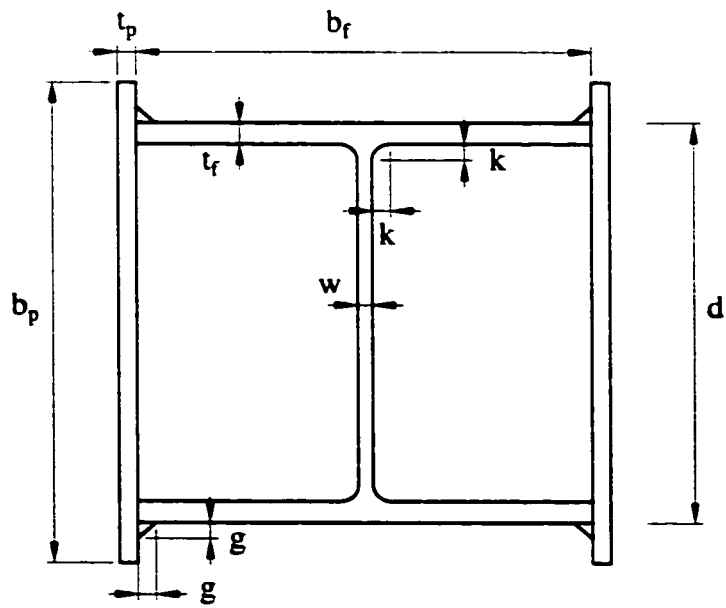
**Figure 5.1 Frequency Distributions for Load Effect,  $S$ , and Resistance,  $R$**



**Figure 5.2 Frequency Distribution for  $\ln(R/S)$**

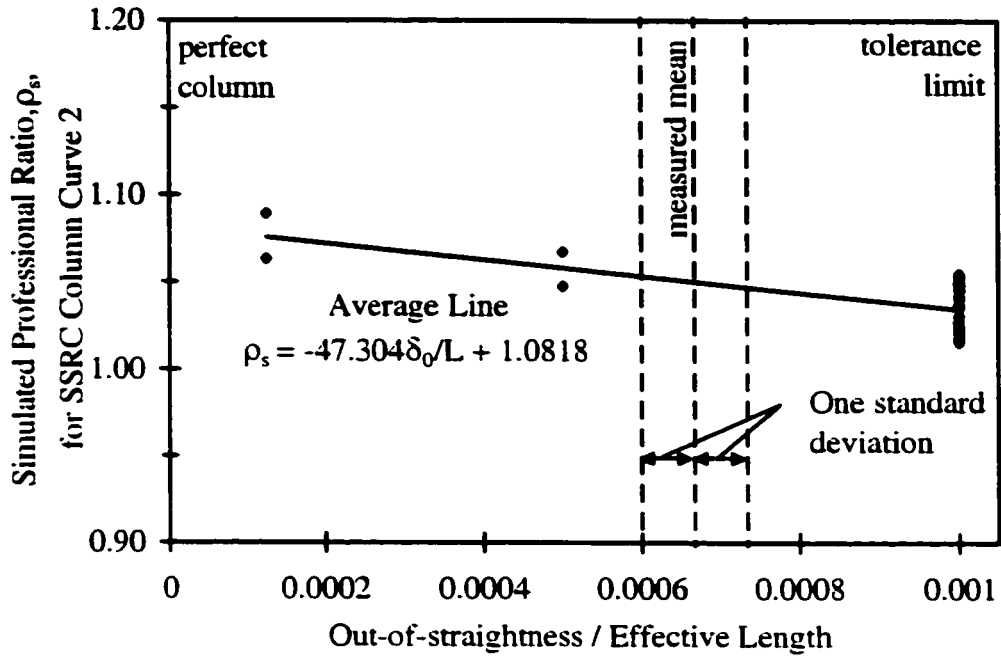


a) Column Reinforced with Plates Parallel to the Flanges

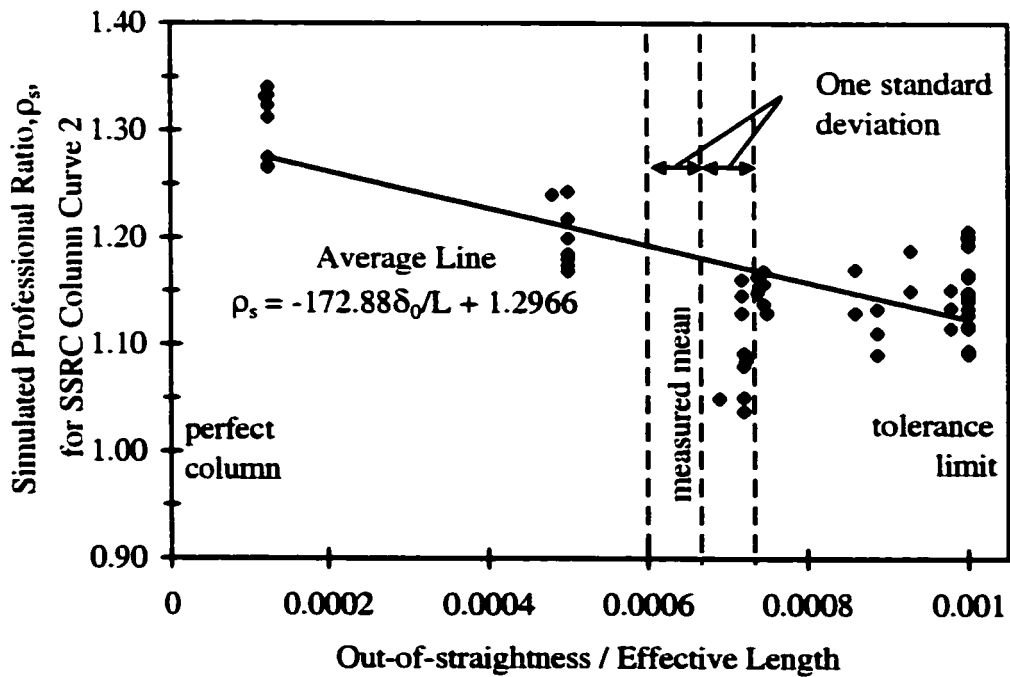


b) Column Reinforced with Plates Parallel to the Web

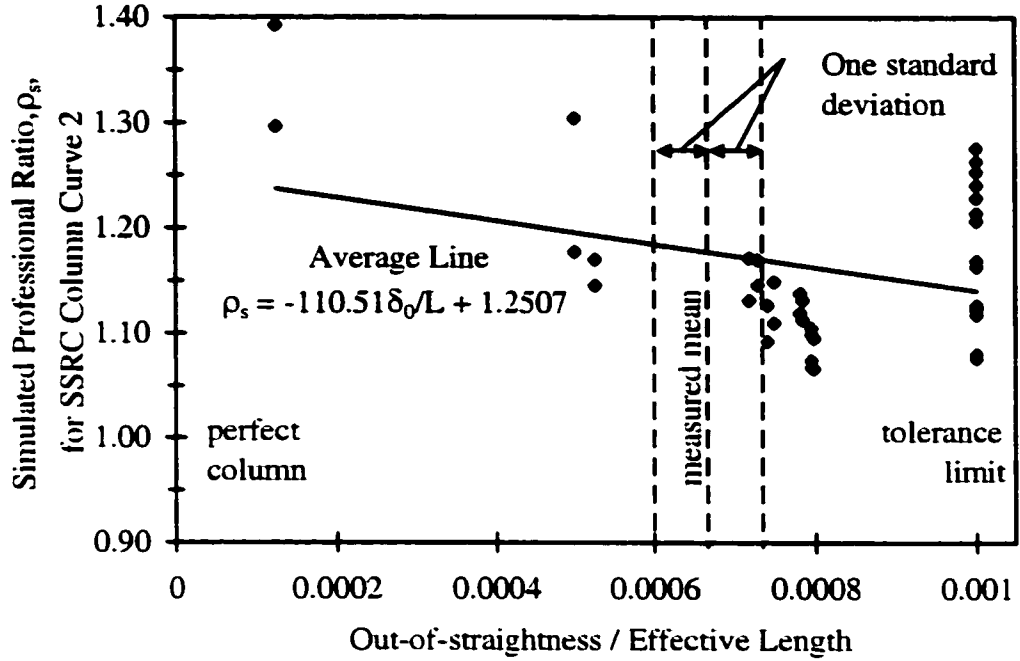
**Figure 5.3 Geometric Dimensions for Reinforced Columns**



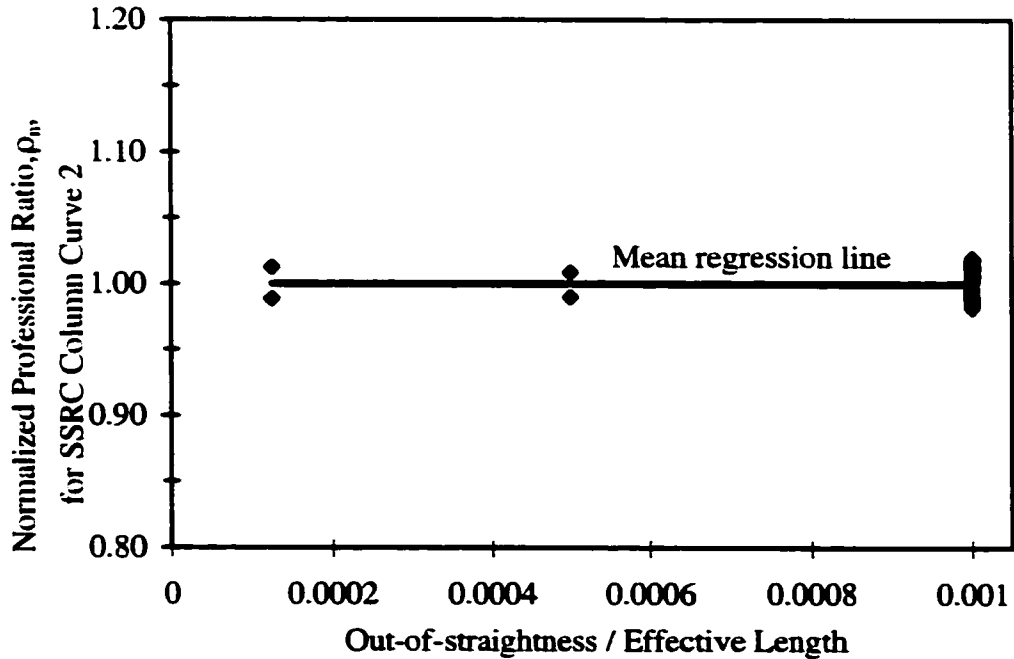
**Figure 5.4 Simulated Professional Ratio vs. Value of Out-of-straightness for Columns from Group 1 ( $\lambda = 0.4$ )**



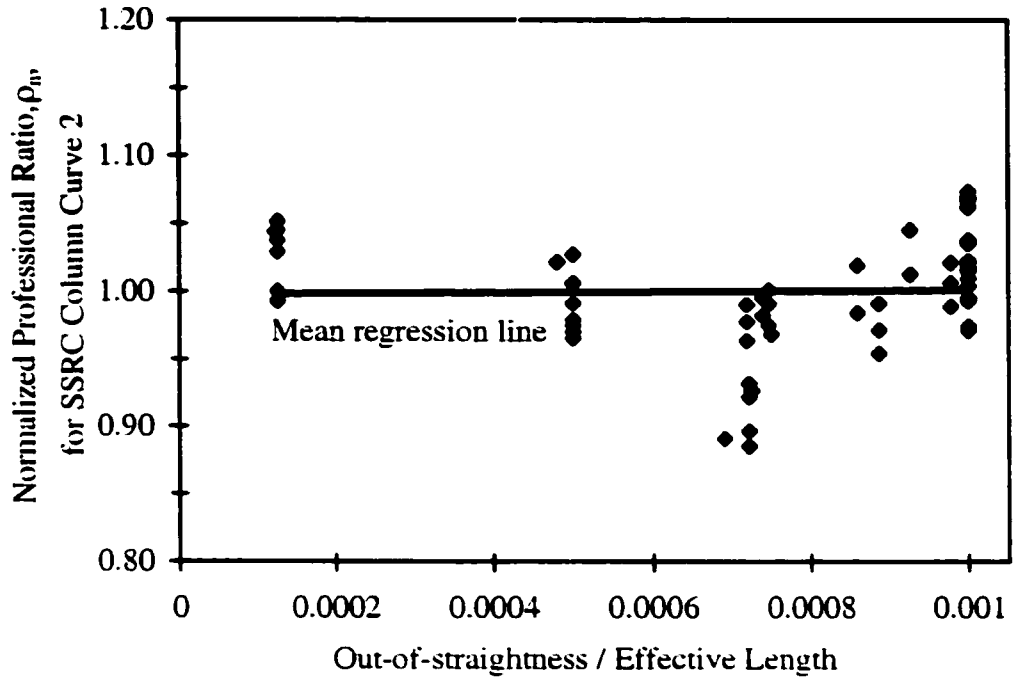
**Figure 5.5 Simulated Professional Ratio vs. Value of Out-of-straightness for Columns from Group 1 ( $\lambda = 1.1$ )**



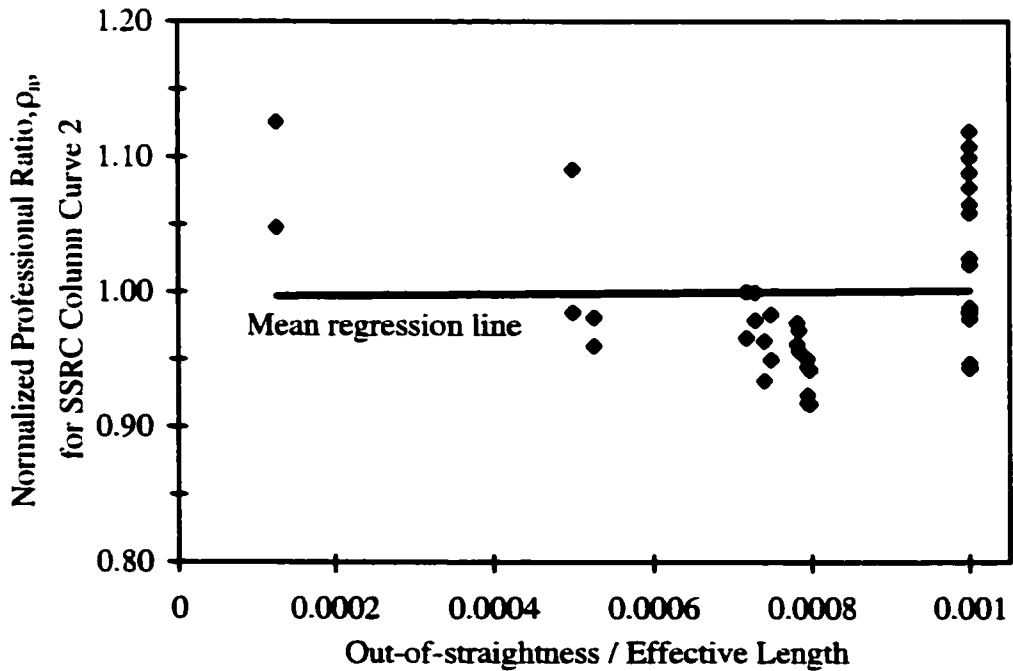
**Figure 5.6 Simulated Professional Ratio vs. Value of Out-of-straightness for Columns from Group 1 ( $\lambda = 1.5$ )**



**Figure 5.7 Normalized Professional Ratio vs. Value of Out-of-straightness for Columns from Group 1 ( $\lambda = 0.4$ )**



**Figure 5.8 Normalized Professional Ratio vs. Value of Out-of-straightness for Columns from Group 1 ( $\lambda = 1.1$ )**



**Figure 5.9 Normalized Professional Ratio vs. Value of Out-of-straightness for Columns from Group 1 ( $\lambda = 1.5$ )**

## **Chapter 6**

### **Summary, Conclusions and Recommendations**

#### **6.1 Summary**

A study of loaded steel wide flange columns reinforced with welded steel plates has been presented in this thesis. A review of the literature has indicated that there is no specific guideline to assess the strength of steel columns reinforced with welded cover plates, although many steel columns have been reinforced in this way. With little knowledge about the effect of parameters that may affect the strength and behaviour of reinforced steel columns, reinforced columns are commonly designed using the lower column curve in S16.1. Although this column curve gives the lowest predicted column capacity, the level of safety obtained from such a design procedure is not known. It is also possible that the residual stresses introduced by the welding process may improve the capacity of reinforced column beyond that predict by the commonly used column curve. Research on the effect of parameters and a design guideline for steel columns reinforced with welded cover plates is therefore necessary to obtain an appropriate column curve for steel columns reinforced under load with welded cover plates.

Numerous parameters may affect the strength of steel columns reinforced with welded cover plates that would not affect the strength of rolled W section columns. Welding of reinforcing plates to a wide flange section changes the compressive residual stresses normally present at the tip of the flanges to high tensile residual stresses, which may be beneficial to the strength of the reinforced column. Interaction of influencing factors, such as residual stresses, preload magnitude, orientation of reinforcing plates, buckling direction, steel grades and geometric properties, may also significantly affect the behaviour and strength of reinforced columns. To understand the behaviour and strength of the columns reinforced with welded cover plates, the effect of these influencing parameters was studied numerically.

A finite element model was developed to study the effect of varying parameters on reinforced columns. The model and analysis procedure were validated by comparing the

strength and behaviour of reinforced and unreinforced columns with the results of tests on ten columns failing in various ranges of material response. A total of 317 finite element models of wide flange steel columns reinforced under load were developed to study specifically the effect of: 1) residual stress pattern and magnitude in the wide flange; 2) residual stress magnitude in the reinforcing plates; 3) the magnitude of the load on the unreinforced section at the time of the reinforcing procedure; 4) steel grade both in the wide flange section and the reinforcing plates; 5) the orientation of the reinforcing plates; 6) the buckling axis; and 7) the relative area of reinforcing plates and wide flange section. The finite element software ABAQUS was used to perform the analysis.

The load versus deflection response for the reinforced columns investigated was obtained for values of the non-dimensional slenderness parameters,  $\lambda$ , of 0.4, 1.1 and 1.5, which cover the range from short to close to the limit between intermediate and slender columns. Residual stresses before welding varied from 10 to 30 percent of the yield strength of the wide flange section and from 15 to 30 percent of the yield strength of the reinforcing plates. The peak welding residual stress was varied from 70 to 100 percent of the yield strength of the rolled section. The initial imperfections of the unreinforced column investigated ranged from near zero to the maximum limit of  $L/1000$  permitted by industry standards. The preload magnitudes investigated were taken as 40 percent and 60 percent of the load carrying capacity of the reinforced columns predicted using SSRC curve 2. The orientation of reinforcing plates was parallel to the flanges or parallel to the web. Buckling about the weak axis of the wide flange section or its strong axis were both investigated. All most commonly used wide flange sections varying from W310x179 to W150x30 were investigated.

A statistical analysis of the analysis results was performed to evaluate the performance of reinforced steel columns based on the limit states philosophy. Statistical data on geometric properties, material properties, and initial imperfections for both rolled sections and plates were collected from the literature. A statistical analysis of the data was performed to obtain the appropriate magnitude of resistance factor to use with each of the two column curves used in the Canadian standard. The design criteria for steel



columns reinforced with welded cover plates were recommended based on this statistical analysis.

## **6.2 Conclusions**

The following conclusions can be drawn based on the results of the work described above.

1. The slenderness parameter and out-of-straightness have the most significant effect on the strength and behaviour of reinforced steel columns.
2. Varying the initial residual stress pattern does not significantly affect the behaviour and strength of reinforced columns. The investigation also demonstrated that varying the maximum welding residual stress pattern from 70% to 100% of the yield strength of the materials does not significantly affect the predicted strength of reinforced columns either.
3. There is an interactive effect between the orientation of the reinforcing plates and the buckling direction on intermediate and slender reinforced columns. For intermediate and slender reinforced columns buckling about the weak axis of the rolled section, columns reinforced with plates parallel to the web show a higher strength than columns reinforced with plates parallel to the flanges. For intermediate reinforced columns buckling about the strong axis of the rolled section, columns reinforced with plates parallel to the flanges show a higher strength than columns reinforced with plates parallel to the web. On the other hand, for intermediate and long columns with reinforcing plates parallel to the flanges, buckling about the strong axis of the rolled section may introduce a higher strength-to-yield ratio of the reinforced column. For intermediate columns with reinforcing plates parallel to the web, buckling about the weak axis of the rolled section may introduce a higher strength-to-yield ratio of the reinforced column.

4. Difference in steel grades between the wide flange section and the reinforcing plates was found to have a negligible effect on the behaviour and strength of reinforced columns.
5. The effect of the rolled section area to the reinforcing plate area ratio on the predicted strength-to-yield ratio of reinforced columns was found to be insignificant.
6. The effect of the preload magnitude varying from 40% to 60% of the load carrying capacity of the unreinforced column was found to be negligible.
7. SSRC curve 2 and corresponding CSA curve 2 used with a resistance factor of 0.9 are appropriate for reinforced steel columns.

### **6.3 Recommendations for Future Research**

The work presented herein is only based on the numerical analyses. Since the lack of experimental data in the intermediate to slender range, the finite element model used for this study had only been partly validated by comparison with existing test results in the short range. Consequently, there is a need for more tests in the intermediate to slender range. The following issues should be investigated based on the test results.

1. Fully validating the finite element model used for this study by comparison with the test results in the intermediate to slender range.
2. The ratio of test strength to the computer simulation developed in this thesis.
3. The effect of reinforcing process on the out-of-straightness of the column, i.e., if the welding process increases the out-of-straightness of the intermediate or long column, as shown in the numerical analysis.
4. The local buckling shapes observed in the short columns described in Section 4.2.

5. **The interactive effect between the buckling axis and the direction of the reinforced plates on the strength and behaviour of the columns reinforced with welded cover plates.**

## List of References

- Allen, D.E. (1975). **Limit States Design – A Probabilistic Study**. Canadian Journal of Civil Engineers, Vol. 2, No. 1, pp. 36-49.
- Alpsten, G.A., and Tall, L. (1970). **Residual Stresses in Heavy Welded Shapes**. Supplement to the Welding Journal, Welding Research Council of the Engineering Foundation, Vol. 49, March, pp. 93s – 105s.
- American Institute of Steel Construction, Inc. (1980). **Manual of Steel Construction**. 8th Ed, Chicago, III.
- Beedle, L.S., and Tall, L. (1960). **Basic Column Strength**. Journal of the Structural Division, American Society of Civil Engineers, Vol. 86, No. ST7, pp. 139 –173.
- Benjamin, J.R., and Cornell, C.A. (1970). **Probability, Statistics and Decision for Civil Engineers**. McGraw-Hill book Company, New York, NY.
- Bjorhovde, R. (1972). **Deterministic and Probabilistic Approaches to the Strength of Steel Columns**. PhD. Thesis, Lehigh University, Bethlehem, PA, May.
- Bjorhovde, R. (1988). **Columns: From Theory to Practice**. Engineering Journal, American Institute of Steel Construction, Inc., Vol. 25, 1st Quarter, pp. 21 – 34.
- Bjorhovde, R., Brozzetti, J., Alpsten, G.A., and Tall, L. (1972). **Residual Stress in Thick Welded Plates**. Supplement to the Welding Journal, Welding Research Council of the Engineering Foundation, Vol. 51, August, pp. 392s – 405s.
- Brozzetti, J., Alpsten, G.A., and Tall, L. (1970). **Welding Parameters, Thick Plates, and Column Strength**. Fritz Engineering Laboratory Report No. 337.21, Lehigh University, Bethlehem, PA, February.
- Brown, J.H. (1988). **Reinforcing Loaded Steel Compression Members**. Engineering Journal, American Institution of Steel Construction, Inc., Vol. 25, 4th Quarter, pp. 161-168.
- Canadian Standards Association (1992). **General Requirements for Rolled and Welded Structural Quality Steel**. CSA Standard G40.20-92, Canadian Standards Association, Rexdale, Ontario.
- Canadian Institute of Steel Construction (1995). **Handbook of Steel Construction**. 6th Edition, Canadian Institute of Steel Construction, Willowdale, Ontario.
- Canadian Standards Association (1974). **Steel Structures for Building (Limit States Design)**. CSA Standard S16.1-1974, Canadian Standards Association, Rexdale, Ontario.

- Canadian Standards Association (1984). **Steel Structures for Building - Limit States Design**. CSA Standard CAN3-S16.1-M84, Canadian Standards Association, Rexdale, Ontario.
- Canadian Standards Association (1992). **Structural Quality Steel**. CSA Standard G40.21-92, Canadian Standards Association, Rexdale, Ontario.
- Canadian Standards Association (1994). **Limit State Design of Steel Structures**. CSA Standard CAN/CSA-S16.1-94, Canadian Standards Association, Rexdale, Ontario.
- Chen, W.F. and Atsuta, T. (1976). **Theory of Beam-Columns (Volume 1): In-Plane Behavior and Design**. McGraw-Hill, New York, NY.
- Chen, W.F. and Lui, E.M. (1987). **Structural Stability: Theory & Implementation**. Elsevier, New York, NY.
- Chernenko, D.E., and Kennedy, D.J.L. (1988). **An Analysis of the Performance of Welded Wide Flange Columns**. Structural Engineering Report 163, Dept. of Civil Engineering, University of Alberta, Edmonton, Alberta, December.
- Feder, D.K. and Lee, G.C. (1959). **Residual Stresses in High Strength Steel**. Fritz Engineering Laboratory Report No. 269.2, Lehigh University, Bethlehem, PA, April.
- Fujita, Y. (1960). **Ultimate Strength of Columns with Residual Stresses**. Journal of the Society of Naval Architects, Japan, January.
- Galambos, T.V. (1968). **Structural Members and Frames**. Prentice-Hall International Inc., Englewood Cliffs, New Jersey.
- Galambos, T.V. (1998). **Guide to Stability Design Criteria for Metal Structures**. 5th Ed., Structural Stability Research Council, John Wiley & Sons, Inc., New York.
- Galambos, T.V., and Ravindra, M.K. (1973a). **Load Factor Design for Combinations of load**. National Structural Engineer Meeting, American Society of Civil Engineers, San Francisco, California.
- Galambos, T.V., and Ravindra, M.K. (1973b). **Tentative Load and Resistance Factor Design Criteria for Steel Buildings**. Research Report 18, Civil Engineering Department, Washington University, Saint Louis, Missouri.
- Galambos, T.V., and Ravindra, M.K. (1977). **The Basis for Load and Resistance Factor Design Criteria of Steel Building Structures**. Canadian Journal of Civil Engineers, Vol. 4, No. 1, pp. 178-189.

- Gaylord, Jr., E. H., Gaylord, C. N., and Stallmeyer, J. E. (1997). **Structural Engineering Handbook**. 4th Ed., McGraw-Hill Inc., New York, NY.
- Geschwindner, L.F., Disque, R.O., and Bjorhovde, R. (1994). **Load and Resistance Factor Design of Steel Structures**. Prentice-Hall, Inc, Englewood Cliffs, New Jersey.
- Hibbitt, H.D., Karlsson, and Sorensen. *et al.* (1997). **ABAQUS/Standard User's Manual**. Hibbitt, Karlsson & Sorensen, Inc.
- Hines, W.W., and Montgomery, D.C. (1972). **Probability and Statistics in Engineering and Management Science**. The Ronald Press Company, New York, NY.
- Huber, A.W. (1956). **The Influence of Residual Stresses on the Instability of Columns**. Ph.D. Dissertation, Lehigh University, Bethlehem, PA, May.
- Huber, A.W. (1958). **Fixture for Testing Pin-End Columns**. ASTM Bulletin No. 234, December.
- Huber, A.W. (1959). **Residual Stresses in Wide-Flange Beams and Columns**. Fritz Engineering Laboratory Report No. 220A.25, Lehigh University, Bethlehem, Pennsylvania, July.
- Huber, A.W., and Beedle, L.S. (1954). **Residual Stress and the Compressive Strength of Steel**. Supplement to the Welding Journal, Welding Research Council of the Engineering Foundation, Vol. 33, December, pp. 589s – 614s.
- Johnston, B.G. (1976). **Guide to Stability Design Criteria for Metal Structures**. 3rd Ed., Structural Stability Research Council, John Wiley & Sons, Inc., New York.
- Kennedy, D.J.L., Allen, D.E., Adams, P.F., Kulak, G.L., Tarlton, D.L., and Turner, D.K. (1976). **Limit States Design**. Proceeding of the Canadian Structural Engineering Conference, Canadian Institute of Steel Construction, February.
- Kennedy, D.J.L., and Gad Aly, M. (1980). **Limit States Design of Steel Structures – Performance Factors**. Canadian Journal of Civil Engineering, Vol. 7, No. 1, pp. 45 – 77.
- Kennedy, J.B., and Neville, A.M. (1976). **Basic Statistical Methods for Engineers and Scientists**. 3rd Ed., Harper and Row Publishers Inc., New York, NY.
- Kishima, Y., Alpsten, G.A., and Tall, L. (1969). **Residual Stresses in Welded Shapes of Flame-Cut Plates in ASTM A572(50) Steel**. Fritz Engineering Laboratory Report No. 321.2, Lehigh University, Bethlehem, PA, June.

- Kulak, G.L. and Gilmor, M.I. (1998). **Limit States Design in Structural Steel**. 6th Ed., Canadian Institute of Steel Construction, Willowdale, Ontario.
- Lind, N.C. (1971). **Consistent Partial Safety Factors**. Journal of the Structural Division, American Society of Civil Engineers, Vol. 97, No. ST5, pp. 1651 – 1670.
- Loov, R. (1996). **A Simple Equation for Axially Loaded Steel Column Design Curves**. Canadian Journal of Civil Engineers, Vol. 23, No. 1, pp. 272-276.
- Masubuchi, K. (1980). **Analysis of Welded Structures**. 1st Ed., Pergamon Press, Oxford, NY.
- Nagaraja Rao, N.R., Estuar, F.R., and Tall, L. (1964). **Residual Stresses in Welded Plates**. Supplement to the Welding Journal, Welding Research Council of the Engineering Foundation, Vol. 43, July, pp. 295s – 306s.
- Nagaraja Rao, N.R. and Tall, L. (1962). **Columns Reinforced under Load**. Fritz Laboratory Report No. 286.1, Lehigh University, Bethlehem, PA, April.
- Nagaraja Rao, N.R. and Tall, L. (1963). **Columns Reinforced under Load**. Supplement to the Welding Journal, Welding Research Council of the Engineering Foundation, Vol. 39, April, pp. 177s - 185s
- Riks, E. (1979). **An Incremental Approach to the Solution of Snapping and Buckling Problems**. International Journal of Solids Structures, Vol. 15, pp. 529-551.
- Sparagen, W., and Grapnel, L. (1944). **Reinforcing Structures under Load**. Supplement to the Welding Journal, Welding Research Council of the Engineering Foundation, Vol. 23, February, pp. 65s – 92s.
- Tall, L. (1961). **Residual Stresses in Welded Plates – A Theoretical Study**. Fritz Laboratory Report No. 249.11, Lehigh University, Bethlehem, PA, July.
- Tall, L. (1989). **The Reinforcement of Steel Columns**. Engineering Journal, American Institute of Steel Construction, Vol. 26, 1st Quarter, pp. 33 – 37.

## **Appendix A**

### **Analysed Reinforced Columns Description**



## **Analysed Reinforced Columns Description**

All 317 model analyses were performed to verify the behaviour of the reinforced steel columns with welded cover plates in the analysis. The initial geometrical conditions and the material conditions of the analysed specimen are presented in Table A.1.

Column (1) of Table A.1 presents the number of the finite element analysed models. Columns (2) and (3) present the designations of rolled sections and reinforcing plates respectively.

Column (4) "D" is the direction of reinforcing plates. In this column, F represents that the cover plates are reinforced on the column along the flanges, as shown in Figure 2.1 (a). G represents that the cover plates are attached to the column at the flange tips of the column and parallel to the web, as shown in Figure 2.1 (b).

Column (5) "B" is the buckling axis of the reinforced column. W represents the same axis as the weak axis of the I-section before reinforcing, and S represents the same axis as the strong axis of the I-section before reinforcing. Column (6) presents the length of the reinforced column. This length is the same as that of the I-section. The length of the cover plates is 20 mm shorter than this length. Column (7) " $\lambda$ " is the slenderness parameter of the reinforced column on the buckling direction.

Columns (8), (9) and (10) present the initial residual stress before welding in the cross section. Column (8) "PS" is the pattern of the initial residual stress before welding. The patterns are shown in the Figures 4.6. Column (9) "MF" is the maximum magnitude of the initial residual stress before welding at the flange tips in the rolled section. The values are presented by the ratio of the maximum residual stress to the yield strength of the rolled section. Column (10) "MP" is the maximum magnitude of the initial residual stress before welding at the edges of the cover plates. The values are presented by the ratio of the maximum residual stress to the yield strength of the rolled section. Column (11) presents the maximum magnitude residual stress after welding at the flange tips of the

rolled section. The values are presented by the ratio of the maximum residual stress to the yield strength of the rolled section.

Columns (12), (13) and (14) present the initial imperfection of the rolled section columns before reinforcing. The column (12) presents the ratio of the out-of-straightness to the column length,  $L$ . The out-of-straightness is on the expected buckling direction of the reinforced column. The maximum allowable initial out-of-straightness varies for columns longer than 10 m in accordance of CAN/CSA-S16.1-94. The column (13) “W” is the magnitude of the initial imperfection on the weak axis of the unreinforced column. The column (14) “S” is the initial imperfection on the strong axis of the unreinforced column.

Columns (15), (16) and (17) present the out-of-straightness of the reinforced columns after reinforcing the cover plates to the rolled section without any pre-load. The column (15) presents the ratio of the out-of-straightness to the column length,  $L$ . The out-of-straightness is on the expected buckling direction of the reinforced column. The column (16) “W” is the magnitude of the out-of-straightness on the weak axis of the unreinforced column. The column (14) “S” is the out-of-straightness on the strong axis of the unreinforced column.

Columns (18) and (19) present the yield strength of the I-section and the cover plates respectively.

**Table A.1 Analysed Reinforced Column Description**

FEA model I-section No.	I-section Plate	D <sup>a</sup>	B <sup>b</sup>	Length λ <sup>c</sup>	L (mm)	Column			IRS <sup>d</sup>		Welding Residual		Initial Imperfection before reinforcing		Out-of-straightness after reinforcing, no load		Yield Strength I-section plate (MPa) (18)	Yield Strength (MPa) (19)
						(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)		
1	W200x46	180x9.52	F	W	2440	0.5	2-1	0.3F <sub>y</sub>	0.15F <sub>y</sub>	0.7F <sub>y</sub>	L/5200	0.47	0.00	L/4900	0.50	260	260	
2	W200x46	180x9.52	F	W	2440	0.5	2-1	0.3F <sub>y</sub>	0.15F <sub>y</sub>	0.7F <sub>y</sub>	L/5000	0.49	0.00	L/4900	0.50	260	260	
3	W200x46	180x9.52	F	W	4504	1.0	1-1	0.3F <sub>y</sub>	0.15F <sub>y</sub>	F <sub>y</sub>	L/1000	4.51	0.00	L/890	5.04	260	260	
4	W200x46	180x9.52	F	W	4504	1.0	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	4.51	0.00	L/890	5.04	260	260	
5	W200x46	180x9.52	F	W	4504	1.0	1-2	0.1F <sub>y</sub>	0.15F <sub>y</sub>	F <sub>y</sub>	L/1000	4.50	0.00	L/890	5.04	260	260	
6	W200x46	180x9.52	F	W	4504	1.0	1-4	0.1F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	4.50	0.00	L/890	5.04	260	260	
7	W200x46	180x9.52	F	W	4504	1.0	2-1	0.3F <sub>y</sub>	0.15F <sub>y</sub>	F <sub>y</sub>	L/1000	4.50	0.00	L/890	5.04	260	260	
8	W200x46	180x9.52	F	W	4504	1.0	2-2	0.1F <sub>y</sub>	0.15F <sub>y</sub>	F <sub>y</sub>	L/1000	4.50	0.00	L/890	5.04	260	260	
9	W200x46	180x9.52	F	W	4504	1.0	3-1	0.3F <sub>y</sub>	0.15F <sub>y</sub>	F <sub>y</sub>	L/1000	4.50	0.00	L/890	5.04	260	260	
10	W200x46	180x9.52	F	W	4504	1.0	3-2	0.1F <sub>y</sub>	0.15F <sub>y</sub>	F <sub>y</sub>	L/1000	4.50	0.00	L/890	5.04	260	260	
11	W200x46	180x9.52	F	W	4504	1.0	4-1	0.3F <sub>y</sub>	0.15F <sub>y</sub>	F <sub>y</sub>	L/1000	4.50	0.00	L/890	5.04	260	260	
12	W200x46	180x9.52	F	W	4504	1.0	4-2	0.1F <sub>y</sub>	0.15F <sub>y</sub>	F <sub>y</sub>	L/1000	4.50	0.00	L/890	5.04	260	260	
13	W200x46	180x9.52	F	S	8316	1.0	1-1	0.3F <sub>y</sub>	0.15F <sub>y</sub>	F <sub>y</sub>	L/1000	1.66	8.32	L/850	9.83	260	260	
14	W200x46	180x9.52	F	S	8316	1.0	1-2	0.1F <sub>y</sub>	0.15F <sub>y</sub>	F <sub>y</sub>	L/1000	1.66	8.32	L/850	9.83	260	260	
15	W200x46	180x9.52	F	S	8316	1.0	3-1	0.3F <sub>y</sub>	0.15F <sub>y</sub>	F <sub>y</sub>	L/1000	1.66	8.32	L/850	9.83	260	260	
16	W200x46	180x9.52	F	S	8316	1.0	3-2	0.1F <sub>y</sub>	0.15F <sub>y</sub>	F <sub>y</sub>	L/1000	1.66	8.32	L/850	9.83	260	260	
17	W310x179	290x25	F	W	2631	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/8000	0.33	0.00	L/7850	0.34	300	300	

- a) D - Direction of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web  
b) B - Buckling axis of the reinforced column      W - Weak axis of the I-section      S - Strong axis of the I-section  
c) λ - Slenderness parameter of the reinforced column      d) IRS - Initial residual stress before reinforcing  
e) PS - Residual stress pattern, as illustrated in Figure 4-1.      f) MF - Maximum magnitude of the residual stress in the flange.  
g) MP - Maximum magnitude of the residual stress in the reinforcing plate.      F<sub>y</sub> - Yield stress of the unreinforced column  
h) ratio - The ratio of the out-of-straightness to the column length, L.  
i) W - Out-of-straightness in the weak direction.      j) S - Out-of-straightness in the strong direction.

**Table A.1 (cont'd)**

FEA model I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	Length λ <sup>c</sup>	PS <sup>e</sup>	MP <sup>f</sup>	MP <sup>g</sup>	Welding Residual	Initial Imperfection before reinforcing	Out-of-straightness after reinforcing, no load	Yield Strength I-section plate								
No.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
						L (mm)			Stress ratio	W <sup>h</sup> (mm)	S <sup>i</sup> (mm)	ratio	W <sup>h</sup> (mm)	S <sup>i</sup> (mm)	ratio	W <sup>h</sup> (mm)	S <sup>i</sup> (mm)	(MPa)	(MPa)
18	W310x179	290x25	F	W	2631	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/2000	1.32	0.00	0.00	L/1930	1.36	0.00	300	300
19	W310x179	290x25	F	W	2631	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	2.64	0.00	0.00	L/970	2.72	0.00	300	300
20	W310x179	290x25	F	W	2631	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	2.63	0.00	0.00	L/980	2.69	0.00	300	300
21	W310x179	290x25	F	W	2740	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	2.75	0.00	0.00	L/970	2.83	0.00	230	350
22	W310x179	290x25	F	W	2740	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	2.75	0.00	0.00	L/980	2.80	0.00	230	350
23	W310x179	290x25	F	W	7235	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/8000	0.83	0.00	0.00	L/7190	1.01	0.00	300	300
24	W310x179	290x25	F	W	7235	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/2000	3.34	0.00	0.00	L/1790	4.06	0.00	300	300
25	W310x179	290x25	F	W	7235	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	7.23	0.00	0.00	L/820	8.82	0.00	300	300
26	W310x179	290x25	F	W	7235	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	0.7F <sub>y</sub>	L/1000	7.25	0.00	0.00	L/820	8.86	0.00	300	300
27	W310x179	290x25	F	W	7235	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	7.25	0.00	0.00	L/900	8.09	0.00	300	300
28	W310x179	290x25	F	W	7535	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	7.54	0.00	0.00	L/830	9.03	0.00	230	350
29	W310x179	290x25	F	W	7535	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	7.54	0.00	0.00	L/900	8.34	0.00	230	350
30	W310x179	290x25	F	W	9866	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/8000	1.24	0.00	0.00	L/6270	1.57	0.00	300	300
31	W310x179	290x25	F	W	9866	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/2000	4.94	0.00	0.00	L/1570	6.28	0.00	300	300
32	W310x179	290x25	F	W	9866	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	9.88	0.00	0.00	L/790	12.56	0.00	300	300
33	W310x179	290x25	F	W	9866	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	9.88	0.00	0.00	L/890	11.13	0.00	300	300
34	W310x179	290x25	F	W	10275	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	10.00	0.00	0.00	L/830	12.46	0.00	230	350

a) D - Direction of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web  
b) B - Buckling axis      W - Weak axis of the I-section      S - Strong axis of the I-section  
c) λ - Slenderness parameter of the reinforced column      d) IRS - Initial residual stress before reinforcing  
e) PS - Residual stress pattern, as illustrated in Figure 4-1.      f) MF - Maximum magnitude of the residual stress in the flange.  
g) MP - Maximum magnitude of the residual stress in the reinforcing plate.      F<sub>y</sub> - Yield stress of the unreinforced column  
h) ratio - The ratio of the out-of-straightness to the column length, L.      j) S - Out-of-straightness in the strong direction.  
i) W - Out-of-straightness in the weak direction.

Table A.1 (cont'd)

FEA model	I-section	Plate	Column			Welding			Initial Imperfection			Out-of-straightness			Yield Strength			
			D <sup>a</sup>	B <sup>b</sup>	L (mm)	λ <sup>c</sup>	PS <sup>e</sup>	MP <sup>f</sup>	IRS <sup>d</sup>	MP <sup>g</sup>	Residual	before reinforcing	W <sup>h</sup> (mm)	S <sup>i</sup> (mm)		ratio	W <sup>h</sup> (mm)	S <sup>i</sup> (mm)
No.	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
35	W310x179	290x25	F	W	10275	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	10.00	0.00	L/920	11.18		230	350
36	W310x179	290x25	F	S	5064	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.02	5.06	L/950		5.33	300	300
37	W310x179	290x25	F	S	5064	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.02	5.06	L/960		5.25	300	300
38	W310x179	290x25	F	S	5273	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.08	5.27	L/960		5.52	230	350
39	W310x179	290x25	F	S	5273	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.08	5.27	L/970		5.43	230	350
40	W310x179	290x25	F	S	13923	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/8000	0.35	1.74	L/6050		2.30	300	300
41	W310x179	290x25	F	S	13923	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/2000	1.39	6.96	L/1510		9.22	300	300
42	W310x179	290x25	F	S	13923	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1400	1.99	10.00	L/1050		13.25	300	300
43	W310x179	290x25	F	S	13923	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	0.7F <sub>y</sub>	L/1400	1.99	10.00	L/1051		13.27	300	300
44	W310x179	290x25	F	S	13923	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1400	1.99	10.00	L/1190		11.71	300	300
45	W310x179	290x25	F	S	14500	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1450	1.96	10.00	L/1070		13.57	230	350
46	W310x179	290x25	F	S	14500	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1400	2.06	10.49	L/1200		12.15	230	350
47	W310x179	290x25	F	S	18986	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1900	1.99	9.99	L/1320		14.42	300	300
48	W310x179	290x25	F	S	18986	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1900	1.99	9.99	L/1560		12.14	300	300
49	W310x179	290x25	F	S	19772	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1250	3.06	15.77	L/900		22.09	230	350
50	W310x179	290x25	F	S	19772	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1250	3.06	15.77	L/1050		18.92	230	350
51	W310x179	350x25	G	W	4103	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	4.11	0.00	L/940	4.89		300	300

- a) D - Direction of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web
- b) B - Buckling axis      W - Weak axis of the I-section      S - Strong axis of the I-section
- c) λ - Slenderness parameter of the reinforced column      d) IRS - Initial residual stress before reinforcing
- e) PS - Residual stress pattern, as illustrated in Figure 4-1.      f) MF - Maximum magnitude of the residual stress in the flange.
- g) MP - Maximum magnitude of the residual stress in the reinforcing plate.      F<sub>y</sub> - Yield stress of the unreinforced column
- h) ratio - The ratio of the out-of-straightness to the column length, L.      j) S - Out-of-straightness in the strong direction.
- i) W - Out-of-straightness in the weak direction.

**Table A.1 (cont'd)**

FEA		Column		Welding		Initial Imperfection		Out-of-straightness		Yield Strength								
model I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	Length λ <sup>c</sup>	PS <sup>c</sup>	MP <sup>d</sup>	MP <sup>d</sup>	Residual	before reinforcing	after reinforcing, no load	I-section	plate						
No.	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
					L (mm)				Stress ratio	W <sup>h</sup> (mm)	S <sup>i</sup> (mm)	W <sup>h</sup> (mm)	S <sup>i</sup> (mm)	ratio	W <sup>h</sup> (mm)	S <sup>i</sup> (mm)	(MPa)	(MPa)
52	W310x179	350x25	G	W	4103	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	4.11	0.00	L/900	4.58	300	300	300
53	W310x179	350x25	G	W	4231	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	4.23	0.00	L/870	4.89	230	230	350
54	W310x179	350x25	G	W	4231	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	4.23	0.00	L/910	4.64	230	230	350
55	W310x179	350x25	G	W	11281	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/8000	1.41	0.00	L/4530	2.49	300	300	300
56	W310x179	350x25	G	W	11281	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/2000	5.64	0.00	L/1140	9.91	300	300	300
57	W310x179	350x25	G	W	11281	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1150	10.00	0.00	L/640	17.57	300	300	300
58	W310x179	350x25	G	W	11281	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	0.7F <sub>y</sub>	L/1150	10.00	0.00	L/640	17.61	300	300	300
59	W310x179	350x25	G	W	11281	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1150	10.00	0.00	L/820	13.65	300	300	300
60	W310x179	350x25	G	W	11634	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1150	10.00	0.00	L/700	16.79	230	230	350
61	W310x179	350x25	G	W	11634	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1150	10.00	0.00	L/880	13.36	230	230	350
62	W310x179	350x25	G	W	15383	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1350	11.38	0.20	L/720	21.36	300	300	300
63	W310x179	350x25	G	W	15383	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1350	11.38	0.20	L/970	15.92	300	300	300
64	W310x179	350x25	G	W	15864	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1350	11.86	0.21	L/720	22.03	230	230	350
65	W310x179	350x25	G	W	15864	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1350	11.86	0.21	L/970	16.53	230	230	350
66	W310x179	350x25	G	S	4030	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	0.90	4.03	L/980	4.09	300	300	300
67	W310x179	350x25	G	S	4030	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	0.90	4.03	L/990	4.08	300	300	300
68	W310x179	350x25	G	S	4156	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	0.80	4.15	L/991	4.22	230	230	350

a) D - Direction of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web  
b) B - Buckling axis      W - Weak axis of the I-section      S - Strong axis of the I-section  
c) λ - Slenderness parameter of the reinforced column      d) IRS - Initial residual stress before reinforcing  
e) PS - Residual stress pattern, as illustrated in Figure 4-1.      f) MF - Maximum magnitude of the residual stress in the flange.  
g) MP - Maximum magnitude of the residual stress in the reinforcing plate.      F<sub>y</sub> - Yield stress of the unreinforced column  
h) ratio - The ratio of the out-of-straightness to the column length, L.      j) S - Out-of-straightness in the strong direction.  
i) W - Out-of-straightness in the weak direction.

**Table A.1 (cont'd)**

FEA model I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	Column		Welding		Initial Imperfection		Out-of-straightness		Yield Strength							
				Length	$\lambda^c$	Residual	before reinforcing	after reinforcing	ratio	ratio	W <sup>b</sup> (mm)	S'(mm)	S'(mm)	(MPa)	(MPa)				
No.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
69	W310x179	350x25	G	S	4156	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	0.80	4.15	L/992	4.19	230	350	350	
70	W310x179	350x25	G	S	11083	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/8000	0.30	1.39	L/7240	1.53	300	300	300	
71	W310x179	350x25	G	S	11083	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/2000	1.08	5.54	L/1810	6.12	300	300	300	
72	W310x179	350x25	G	S	11083	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1100	1.94	9.99	L/1000	11.04	300	300	300	
73	W310x179	350x25	G	S	11083	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	0.7F <sub>y</sub>	L/1100	1.94	9.99	L/1001	11.06	300	300	300	
74	W310x179	350x25	G	S	11083	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1100	1.94	9.99	L/1050	10.55	300	300	300	
75	W310x179	350x25	G	S	11429	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1150	1.85	10.00	L/1050	10.87	230	350	350	
76	W310x179	350x25	G	S	11429	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1150	1.85	10.00	L/1050	10.47	230	350	350	
77	W310x179	350x25	G	S	15113	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1350	2.20	11.11	L/1200	12.64	300	300	300	
78	W310x179	350x25	G	S	15113	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1350	2.20	11.11	L/1280	11.85	300	300	300	
79	W310x179	350x25	G	S	15585	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1350	2.30	11.58	L/1200	12.99	230	350	350	
80	W310x179	350x25	G	S	15585	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1350	2.30	11.58	L/1270	12.27	230	350	350	
81	W310x179	350x25	G	S	11083	1.1	I-4	0.1F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1100	1.94	9.98	L/1000	11.01	300	300	300	
82	W310x179	350x25	G	S	11083	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1100	1.94	9.99	L/1000	11.04	300	300	300	
83	W310x179	350x25	G	S	11083	1.1	I-3	0.1F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1100	1.94	9.99	L/1000	11.04	300	300	300	
84	W310x179	290x16	F	W	2617	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	2.62	0.00	L/980	2.66	300	300	300	
85	W310x179	290x16	F	W	2617	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	2.62	0.00	L/981	2.68	300	300	300	

a) D - Direction of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web  
b) B - Buckling axis      W - Weak axis of the I-section      S - Strong axis of the I-section  
c)  $\lambda$  - Slenderness parameter of the reinforced column      d) IRS - Initial residual stress before reinforcing  
e) PS - Residual stress pattern, as illustrated in Figure 4-1.      f) MF - Maximum magnitude of the residual stress in the flange.  
g) MP - Maximum magnitude of the residual stress in the reinforcing plate.      F<sub>y</sub> - Yield stress of the unreinforced column  
h) ratio - The ratio of the out-of-straightness to the column length, L.  
i) W - Out-of-straightness in the weak direction.      j) S - Out-of-straightness in the strong direction.

**Table A.1 (cont'd)**

FEA model I-section		Column		Welding		Initial Imperfection		Out-of-straightness		Yield Strength								
Plate	D <sup>a</sup>	B <sup>b</sup>	Length λ <sup>c</sup>	PS <sup>e</sup>	MP <sup>f</sup>	MP <sup>g</sup>	Residual	before reinforcing	after reinforcing, no load	I-section	plate							
No.	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
					L (mm)				Stress ratio	W <sup>h</sup> (mm)	S <sup>i</sup> (mm)	W <sup>h</sup> (mm)	S <sup>i</sup> (mm)	ratio	W <sup>h</sup> (mm)	S <sup>i</sup> (mm)	(MPa)	(MPa)
86	W310x179	290x16	F	W	2786	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	2.79	0.00	L/982	2.86		230	350
87	W310x179	290x16	F	W	2786	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	2.79	0.00	L/983	2.83		230	350
88	W310x179	290x16	F	W	7197	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/8000	0.90	0.00	L/6900	1.04		300	300
89	W310x179	290x16	F	W	7197	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/2000	3.60	0.00	L/1730	4.17		300	300
90	W310x179	290x16	F	W	7197	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	7.20	0.00	L/860	8.34		300	300
91	W310x179	290x16	F	W	7197	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	0.7F <sub>y</sub>	L/1000	7.20	0.00	L/861	8.36		300	300
92	W310x179	290x16	F	W	7197	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	7.20	0.00	L/920	7.80		300	300
93	W310x179	290x16	F	W	7662	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	7.66	0.00	L/870	8.78		230	350
94	W310x179	290x16	F	W	7662	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	7.66	0.00	L/930	8.25		230	350
95	W310x179	290x16	F	W	9813	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	9.81	0.00	L/840	11.71		300	300
96	W310x179	290x16	F	W	9813	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	0.7F <sub>y</sub>	L/1000	9.81	0.00	L/840	11.76		300	300
97	W310x179	290x16	F	W	9813	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	9.81	0.00	L/920	10.69		300	300
98	W310x179	290x16	F	W	10447	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1050	10.00	0.00	L/890	11.77		230	350
99	W310x179	290x16	F	W	10447	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1050	10.00	0.00	L/970	10.82		230	350
100	W310x179	290x16	F	S	4880	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	0.97	4.88	L/970		5.04	300	300
101	W310x179	290x16	F	S	4880	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	0.97	4.88	L/980		5.00	300	300
102	W310x179	290x16	F	S	5196	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.06	5.20	L/970		5.37	230	350

a) D - Direction of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web  
 b) B - Buckling axis      W - Weak axis of the I-section      S - Strong axis of the I-section  
 c) λ - Slenderness parameter of the reinforced column      d) IRS - Initial residual stress before reinforcing  
 e) PS - Residual stress pattern, as illustrated in Figure 4-1.      f) MF - Maximum magnitude of the residual stress in the flange.  
 g) MP - Maximum magnitude of the residual stress in the reinforcing plate.      F<sub>y</sub> - Yield stress of the unreinforced column  
 h) ratio - The ratio of the out-of-straightness to the column length, L.      j) S - Out-of-straightness in the strong direction.  
 i) W - Out-of-straightness in the weak direction.



**Table A.1 (cont'd)**

FEA		Column		Welding		Initial Imperfection		Out-of-straightness		Yield Strength								
model	I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	Length λ <sup>c</sup>	PS <sup>e</sup>	MP <sup>f</sup>	MP <sup>g</sup>	Residual	before reinforcing	after reinforcing, no load	I-section plate						
No.	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
					L (mm)				Stress	ratio	ratio	W <sup>h</sup> (mm)	S <sup>i</sup> (mm)	ratio	W <sup>h</sup> (mm)	S <sup>i</sup> (mm)	(MPa)	(MPa)
103	W310x179	290x16	F	S	5196	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.06	5.20	L/980	5.31	230	350	
104	W310x179	290x16	F	S	13420	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/8000	0.33	1.68	L/6500	2.07	300	300	
105	W310x179	290x16	F	S	13420	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/2000	1.32	6.71	L/1620	8.27	300	300	
106	W310x179	290x16	F	S	13420	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1350	1.96	10.00	L/1090	12.32	300	300	
107	W310x179	290x16	F	S	13420	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	0.7F <sub>y</sub>	L/1350	1.96	10.00	L/1091	12.34	300	300	
108	W310x179	290x16	F	S	13420	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1350	1.96	10.00	L/1200	11.23	300	300	
109	W310x179	290x16	F	S	14287	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1400	2.02	10.29	L/1150	12.49	230	350	
110	W310x179	290x16	F	S	14287	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1400	2.02	10.29	L/1250	11.47	230	350	
111	W310x179	290x16	F	S	18300	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1300	2.78	14.29	L/970	18.81	300	300	
112	W310x179	290x16	F	S	18300	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1300	2.78	14.29	L/1110	16.50	300	300	
113	W310x179	290x16	F	S	19483	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1250	3.01	15.48	L/970	19.99	230	350	
114	W310x179	290x16	F	S	19483	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1250	3.01	15.48	L/1100	17.72	230	350	
115	W310x179	350x16	F	W	2827	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	2.83	0.00	L/970	2.90	300	300	
116	W310x179	350x16	F	W	2827	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	2.83	0.00	L/970	2.92	300	300	
117	W310x179	350x16	F	W	2982	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	2.99	0.00	L/970	3.10	230	350	
118	W310x179	350x16	F	W	2982	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	2.99	0.00	L/970	3.06	230	350	
119	W310x179	350x16	F	W	7772	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/8000	0.97	0.00	L/6300	1.23	300	300	

a) D - Direction of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web  
 b) B - Buckling axis      W - Weak axis of the I-section      S - Strong axis of the I-section  
 c) λ - Slenderness parameter of the reinforced column      d) IRS - Initial residual stress before reinforcing  
 e) PS - Residual stress pattern, as illustrated in Figure 4-1.      f) MF - Maximum magnitude of the residual stress in the flange.  
 g) MP - Maximum magnitude of the residual stress in the reinforcing plate.      F<sub>y</sub> - Yield stress of the unreinforced column  
 h) ratio - The ratio of the out-of-straightness to the column length, L.      j) S - Out-of-straightness in the strong direction.  
 i) W - Out-of-straightness in the weak direction.

**Table A.1 (cont'd)**

FEA model I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	Column		λ <sup>c</sup>	PS <sup>e</sup>	IRS <sup>d</sup>		MP <sup>f</sup>	MP <sup>g</sup>	Welding Residual		Initial Imperfection before reinforcing		Out-of-straightness after reinforcing, no load		Yield Strength I-section plate (MPa)	
				Length	L (mm)			λ <sup>c</sup>	PS <sup>e</sup>			Stress	ratio	W <sup>h</sup> (mm)	S <sup>i</sup> (mm)	ratio	W <sup>h</sup> (mm)		S <sup>i</sup> (mm)
No.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
120	W310x179	350x16	F	W	7772	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/2000	3.88	0.00	L/1570	4.94			300	300
121	W310x179	350x16	F	W	7772	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	7.77	0.00	L/790	9.88			300	300
122	W310x179	350x16	F	W	7772	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	0.7F <sub>y</sub>	L/1000	7.77	0.00	L/790	9.90			300	300
123	W310x179	350x16	F	W	7772	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	7.77	0.00	L/880	8.88			300	300
124	W310x179	350x16	F	W	8200	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	8.20	0.00	L/800	10.23			230	350
125	W310x179	350x16	F	W	8200	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	8.20	0.00	L/890	9.28			230	350
126	W310x179	350x16	F	W	10598	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1400	7.57	0.00	L/790	13.44			300	300
127	W310x179	350x16	F	W	10598	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1050	10.00	0.00	L/910	11.63			300	300
128	W310x179	350x16	F	W	11182	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1100	10.00	0.00	L/850	13.13			230	350
129	W310x179	350x16	F	W	11182	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1100	10.00	0.00	L/970	11.51			230	350
130	W310x179	350x16	F	S	4928	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	0.97	4.93	L/960		5.11		300	300
131	W310x179	350x16	F	S	4928	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	0.97	4.93	L/970		5.07		300	300
132	W310x179	350x16	F	S	5199	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.04	5.20	L/960		5.40		230	350
133	W310x179	350x16	F	S	5199	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.04	5.20	L/980		5.33		230	350
134	W310x179	350x16	F	S	13551	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/8000	0.33	1.69	L/6330		2.14		300	300
135	W310x179	350x16	F	S	13551	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/2000	1.33	6.77	L/1590		8.56		300	300
136	W310x179	350x16	F	S	13551	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1350	1.96	10.00	L/1070		12.64		300	300

a) D - Direction of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web  
b) B - Buckling axis      W - Weak axis of the I-section      S - Strong axis of the I-section  
c) λ - Slenderness parameter of the reinforced column      d) IRS - Initial residual stress before reinforcing  
e) PS - Residual stress pattern, as illustrated in Figure 4-1.      f) MF - Maximum magnitude of the residual stress in the flange.  
g) MP - Maximum magnitude of the residual stress in the reinforcing plate.      F<sub>y</sub> - Yield stress of the unreinforced column  
h) ratio - The ratio of the out-of-straightness to the column length, L.      j) S - Out-of-straightness in the strong direction.  
i) W - Out-of-straightness in the weak direction.

Table A.1 (cont'd)

FEA model I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	Column		λ <sup>c</sup>	PS <sup>e</sup>	IRS <sup>d</sup>		Welding Residual	Initial Imperfection before reinforcing		Out-of-straightness after reinforcing, no load		Yield Strength I-section plate (MPa)				
				L (mm)	Length			MF	MP <sup>f</sup>		Stress ratio	W <sup>h</sup> (mm), S <sup>i</sup> (mm)	ratio	W <sup>h</sup> (mm), S <sup>i</sup> (mm)					
No.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
137	W310x179	350x16	F	S	13551	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	0.7F <sub>y</sub>	L/1350	1.96	10.00	L/1070	12.66	300	300	300	
138	W310x179	350x16	F	S	13551	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1350	1.96	10.00	L/1190	11.40	300	300	300	
139	W310x179	350x16	F	S	14297	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1400	2.02	10.30	L/1120	12.79	230	230	350	
140	W310x179	350x16	F	S	14297	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1400	2.02	10.30	L/1230	11.64	230	230	350	
141	W310x179	350x16	F	S	18479	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1300	2.81	14.48	L/940	19.68	300	300	300	
142	W310x179	350x16	F	S	18479	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1300	2.81	14.48	L/1090	17.03	300	300	300	
143	W310x179	350x16	F	S	19496	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1250	3.01	15.49	L/940	20.59	230	230	350	
144	W310x179	350x16	F	S	19496	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1250	3.01	15.49	L/1090	18.03	230	230	350	
145	W310x179	350x16	G	W	3720	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	3.72	0.00	L/890	4.20	300	300	300	
146	W310x179	350x16	G	W	3720	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	3.72	0.00	L/920	4.04	300	300	300	
147	W310x179	350x16	G	W	3925	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	3.93	0.00	L/900	4.39	230	230	350	
148	W310x179	350x16	G	W	3925	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	3.93	0.00	L/930	4.22	230	230	350	
149	W310x179	350x16	G	W	10229	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/8000	1.28	0.00	L/4990	2.05	300	300	300	
150	W310x179	350x16	G	W	10229	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/2000	5.11	0.00	L/1250	8.21	300	300	300	
151	W310x179	350x16	G	W	10229	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	10.00	0.00	L/640	16.05	300	300	300	
152	W310x179	350x16	G	W	10229	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	0.7F <sub>y</sub>	L/1000	10.00	0.00	L/640	16.08	300	300	300	
153	W310x179	350x16	G	W	10229	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	10.00	0.00	L/790	12.99	300	300	300	

- a) D - Direction of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web
- b) B - Buckling axis      W - Weak axis of the I-section      S - Strong axis of the I-section
- c) λ - Slenderness parameter of the reinforced column      d) IRS - Initial residual stress before reinforcing
- e) PS - Residual stress pattern, as illustrated in Figure 4-1.      f) MF - Maximum magnitude of the residual stress in the flange.
- g) MP - Maximum magnitude of the residual stress in the reinforcing plate.      F<sub>y</sub> - Yield stress of the unreinforced column
- h) ratio - The ratio of the out-of-straightness to the column length, L.      j) S - Out-of-straightness in the strong direction.
- i) W - Out-of-straightness in the weak direction.

**Table A.1 (cont'd)**

FEA model I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	Column Length λ <sup>c</sup>	PS <sup>c</sup>	MP <sup>d</sup>	MP <sup>e</sup>	IRS <sup>d</sup>	Welding Residual	Initial Imperfection before reinforcing	Out-of-straightness after reinforcing, no load	Yield Strength I-section plate							
No.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
						L (mm)				Stress ratio	W <sup>h</sup> (mm)	S <sup>i</sup> (mm)	W <sup>h</sup> (mm)	S <sup>i</sup> (mm)	ratio	W <sup>h</sup> (mm)	S <sup>i</sup> (mm)	(MPa)	(MPa)
154	W310x179	350x16	G	W	10792	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1100	10.00	0.00	0.00	L/700	15.54		230	350
155	W310x179	350x16	G	W	10792	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1100	10.00	0.00	0.00	L/850	12.80		230	350
156	W310x179	350x16	G	W	13948	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1400	10.00	0.18	0.18	L/800	17.63		300	300
157	W310x179	350x16	G	W	13948	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1400	10.00	0.18	0.18	L/1030	13.50		300	300
158	W310x179	350x16	G	W	14716	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1350	10.71	0.19	0.19	L/790	18.77		230	350
159	W310x179	350x16	G	W	14716	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1350	10.71	0.19	0.19	L/1020	14.42		230	350
160	W310x179	350x16	G	S	4155	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	0.85	4.15	4.15	L/1000		4.17	300	300
161	W310x179	350x16	G	S	4155	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	0.85	4.15	4.15	L/1000		4.19	300	300
162	W310x179	350x16	G	S	4383	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	0.99	4.38	4.38	L/1000		4.43	230	350
163	W310x179	350x16	G	S	4383	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	0.99	4.38	4.38	L/1000		4.41	230	350
164	W310x179	350x16	G	S	11425	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/8000	0.28	1.43	1.43	L/7480		1.53	300	300
165	W310x179	350x16	G	S	11425	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/2000	1.11	5.71	5.71	L/1870		6.12	300	300
166	W310x179	350x16	G	S	11425	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1150	1.94	10.00	10.00	L/1070		10.71	300	300
167	W310x179	350x16	G	S	11425	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	0.7F <sub>y</sub>	L/1150	1.94	10.00	10.00	L/1070		10.73	300	300
168	W310x179	350x16	G	S	11425	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1150	1.94	10.00	10.00	L/1120		10.37	300	300
169	W310x179	350x16	G	S	12054	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/2750	0.99	4.38	4.38	L/2420		5.00	230	350
170	W310x179	350x16	G	S	12054	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1200	1.96	10.00	10.00	L/1170		10.32	230	350

- a) D - Direction of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web
- b) B - Buckling axis      W - Weak axis of the I-section      S - Strong axis of the I-section
- c) λ - Slenderness parameter of the reinforced column      d) IRS - Initial residual stress before reinforcing
- e) PS - Residual stress pattern, as illustrated in Figure 4-1.      f) MF - Maximum magnitude of the residual stress in the flange.
- g) MP - Maximum magnitude of the residual stress in the reinforcing plate.      F<sub>y</sub> - Yield stress of the unreinforced column
- h) ratio - The ratio of the out-of-straightness to the column length, L.      j) S - Out-of-straightness in the strong direction.
- i) W - Out-of-straightness in the weak direction.

Table A.1 (cont'd)

FEA model No.	I-section (1)	Plate (2)	D <sup>a</sup> (3)	B <sup>b</sup> (4)	S (5)	Column Length L (mm) (6)	$\lambda^c$ (7)	PS <sup>e</sup> (8)	IRS <sup>d</sup>		Welding Residual Stress ratio (11)	Initial Imperfection before reinforcing (12)		Out-of-straightness after reinforcing, no load (16)	Yield Strength I-section plate (MPa) (18)		
									MF (9)	MP <sup>f</sup> (10)		W <sup>h</sup> (mm) (13)	S'(mm) (14)			ratio (15)	S'(mm) (17)
171	W310x179	350x16	G	S	15579	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1350	2.27	11.58	L/1240	12.61	300	300
172	W310x179	350x16	G	S	15579	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1350	2.27	11.58	L/1290	12.06	300	300
173	W310x179	350x16	G	S	16437	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1300	2.44	12.44	L/1220	13.44	230	350
174	W310x179	350x16	G	S	16437	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1300	2.44	12.44	L/1270	12.90	230	350
175	W150x30	130x5	F	W	1236	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.24	0.00	L/980	1.27	300	300
176	W150x30	130x5	F	W	1236	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.24	0.00	L/980	1.27	300	300
177	W150x30	130x5	F	W	1327	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.32	0.00	L/980	1.35	230	350
178	W150x30	130x5	F	W	1327	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.32	0.00	L/980	1.34	230	350
179	W150x30	130x5	F	W	3399	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/8000	0.43	0.00	L/7100	0.48	300	300
180	W150x30	130x5	F	W	3399	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/2000	1.70	0.00	L/1940	1.75	300	300
181	W150x30	130x5	F	W	3399	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	3.40	0.00	L/890	3.84	300	300
182	W150x30	130x5	F	W	3399	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	0.7F <sub>y</sub>	L/1000	3.40	0.00	L/890	3.84	300	300
183	W150x30	130x5	F	W	3399	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	3.40	0.00	L/930	3.64	300	300
184	W150x30	130x5	F	W	3647	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	3.65	0.00	L/900	4.07	230	350
185	W150x30	130x5	F	W	3647	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	3.65	0.00	L/940	3.88	230	350
186	W150x30	130x5	F	W	4635	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	4.64	0.00	L/850	5.47	300	300
187	W150x30	130x5	F	W	4635	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	4.64	0.00	L/920	5.05	300	300

- a) D - Direction of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web
- b) B - Buckling axis      W - Weak axis of the I-section      S - Strong axis of the I-section
- c)  $\lambda$  - Slenderness parameter of the reinforced column      d) IRS - Initial residual stress before reinforcing
- e) PS - Residual stress pattern, as illustrated in Figure 4-1.      f) MF - Maximum magnitude of the residual stress in the flange.
- g) MP - Maximum magnitude of the residual stress in the reinforcing plate.      F<sub>y</sub> - Yield stress of the unreinforced column
- h) ratio - The ratio of the out-of-straightness to the column length, L.      j) S - Out-of-straightness in the strong direction.
- i) W - Out-of-straightness in the weak direction.

**Table A.1 (cont'd)**

FEA model I-section No.	Plate	D <sup>a</sup>	Column		Welding		Initial Imperfection		Out-of-straightness		Yield Strength							
			B <sup>b</sup>	Length	λ <sup>c</sup>	PS <sup>e</sup>	MP <sup>f</sup>	MP <sup>g</sup>	Residual	before reinforcing		after reinforcing, no load	I-section plate					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
					L (mm)				Stress ratio	W <sup>h</sup> (mm)	S <sup>i</sup> (mm)	W <sup>h</sup> (mm)	S <sup>i</sup> (mm)	ratio	W <sup>h</sup> (mm)	S <sup>i</sup> (mm)	(MPa)	(MPa)
188	W150x30	130x5	F	W	4973	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	4.97	0.00	L/860	5.78		230	350
189	W150x30	130x5	F	W	4973	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	4.97	0.00	L/920	5.38		230	350
190	W150x30	130x5	F	S	2300	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/8000	0.06	0.29	L/7730		0.30	300	300
191	W150x30	130x5	F	S	2300	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/2000	0.23	1.15	L/1940		1.19	300	300
192	W150x30	130x5	F	S	2300	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	0.45	2.30	L/960		2.38	300	300
193	W150x30	130x5	F	S	2300	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	0.45	2.30	L/980		2.36	300	300
194	W150x30	130x5	F	S	2468	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	0.49	2.47	L/960		2.56	230	350
195	W150x30	130x5	F	S	2468	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	0.49	2.47	L/980		2.52	230	350
196	W150x30	130x5	F	S	6326	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/8000	0.16	0.79	L/6600		0.96	300	300
197	W150x30	130x5	F	S	6326	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/2000	0.62	3.16	L/1650		3.85	300	300
198	W150x30	130x5	F	S	6326	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.24	6.33	L/820		7.71	300	300
199	W150x30	130x5	F	S	6326	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	0.7F <sub>y</sub>	L/1000	1.24	6.33	L/820		7.72	300	300
200	W150x30	130x5	F	S	6326	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.24	6.33	L/880		7.17	300	300
201	W150x30	130x5	F	S	6787	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.33	6.79	L/810		8.40	230	350
202	W150x30	130x5	F	S	6787	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.33	6.79	L/890		7.66	230	350
203	W150x30	130x5	F	S	8626	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/8000	0.00	1.08	L/6050		1.43	300	300
204	W150x30	130x5	F	S	8626	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/2000	0.00	4.31	L/1500		5.74	300	300

- a) D - Direction of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web
- b) B - Buckling axis      W - Weak axis of the I-section      S - Strong axis of the I-section
- c) λ - Slenderness parameter of the reinforced column      d) IRS - Initial residual stress before reinforcing
- e) PS - Residual stress pattern, as illustrated in Figure 4-1.      f) MF - Maximum magnitude of the residual stress in the flange.
- g) MP - Maximum magnitude of the residual stress in the reinforcing plate.      F<sub>y</sub> - Yield stress of the unreinforced column
- h) ratio - The ratio of the out-of-straightness to the column length, L.      j) S - Out-of-straightness in the strong direction.
- i) W - Out-of-straightness in the weak direction.

**Table A.1 (cont'd)**

FEA model I-section No.	Plate	D <sup>a</sup>	B <sup>b</sup>	Column Length L (mm)	$\lambda^c$	PS <sup>e</sup>	IRS <sup>d</sup>		Welding Residual Stress	Initial Imperfection before reinforcing	Out-of-straightness after reinforcing, no load		Yield Strength I-section plate (MPa)					
							MP <sup>f</sup>	MF <sup>g</sup>			ratio W <sup>h</sup> (mm)	ratio S <sup>i</sup> (mm)		ratio W <sup>h</sup> (mm)	ratio S <sup>i</sup> (mm)			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
205	W150x30	130x5	F	S	8626	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	0.00	8.63	L/750	11.57	300	300	
206	W150x30	130x5	F	S	8626	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	0.00	8.63	L/860	10.06	300	300	
207	W150x30	130x5	F	S	9254	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.82	9.25	L/750	12.28	230	350	
208	W150x30	130x5	F	S	9254	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.82	9.25	L/860	10.76	230	350	
209	W150x30	175x5	G	W	1770	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/8000	0.22	0.00	L/7050	0.25	300	300	
210	W150x30	175x5	G	W	1770	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/2000	0.88	0.00	L/1770	1.00	300	300	
211	W150x30	175x5	G	W	1770	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.77	0.00	L/880	2.01	300	300	
212	W150x30	175x5	G	W	1770	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.77	0.00	L/920	1.92	300	300	
213	W150x30	175x5	G	W	1873	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.87	0.00	L/900	2.09	230	350	
214	W150x30	175x5	G	W	1873	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.87	0.00	L/930	2.01	230	350	
215	W150x30	175x5	G	W	4867	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/8000	0.61	0.00	L/4900	0.99	300	300	
216	W150x30	175x5	G	W	4867	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/2000	2.43	0.00	L/1220	3.99	300	300	
217	W150x30	175x5	G	W	4867	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	4.86	0.00	L/610	7.97	300	300	
218	W150x30	175x5	G	W	4867	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	0.7F <sub>y</sub>	L/1000	4.86	0.00	L/610	7.98	300	300	
219	W150x30	175x5	G	W	4867	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	4.86	0.00	L/760	6.41	300	300	
220	W150x30	175x5	G	W	5150	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	5.15	0.00	L/630	8.17	230	350	
221	W150x30	175x5	G	W	5150	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	5.15	0.00	L/780	6.68	230	350	

a) D - Direction of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web  
b) B - Buckling axis      W - Weak axis of the I-section      S - Strong axis of the I-section  
c)  $\lambda$  - Slenderness parameter of the reinforced column      d) IRS - Initial residual stress before reinforcing  
e) PS - Residual stress pattern, as illustrated in Figure 4-1.      f) MF - Maximum magnitude of the residual stress in the flange.  
g) MP - Maximum magnitude of the residual stress in the reinforcing plate.      F<sub>y</sub> - Yield stress of the unreinforced column  
h) ratio - The ratio of the out-of-straightness to the column length, L.      j) S - Out-of-straightness in the strong direction.  
i) W - Out-of-straightness in the weak direction.

**Table A.1 (cont'd)**

FEA model I-section No.	(1)	(2)	(3)	Plate	D <sup>a</sup>	B <sup>b</sup>	Length	$\lambda^c$	(7)	(8)	(9)	IRS <sup>d</sup>		Welding Residual Stress	(11)	(12)	Initial Imperfection before reinforcing	W <sup>h</sup> (mm)	S <sup>i</sup> (mm)	(14)	Out-of-straightness after reinforcing	ratio	W <sup>h</sup> (mm)	S <sup>i</sup> (mm)	(17)	Yield Strength I-section plate (MPa)	(18)	(19)
												MP <sup>e</sup>	MF <sup>f</sup>															
222	W150x30	G	W	6637	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	(10)	F <sub>y</sub>	L/8000	0.83	0.00	L/4410	1.50	300	300											
223	W150x30	G	W	6637	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	(10)	F <sub>y</sub>	L/2000	3.32	0.00	L/1100	6.05	300	300											
224	W150x30	G	W	6637	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	(10)	F <sub>y</sub>	L/1000	6.64	0.00	L/550	12.10	300	300											
225	W150x30	G	W	6637	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	(10)	F <sub>y</sub>	L/1000	6.64	0.00	L/730	9.16	300	300											
226	W150x30	G	W	7023	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	(10)	F <sub>y</sub>	L/1000	7.02	0.00	L/570	12.32	230	350											
227	W150x30	G	W	7023	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	(10)	F <sub>y</sub>	L/1000	7.02	0.00	L/740	9.46	230	350											
228	W150x30	G	S	2022	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	(10)	F <sub>y</sub>	L/8000	0.05	0.25	L/7860	0.26	300	300											
229	W150x30	G	S	2022	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	(10)	F <sub>y</sub>	L/2000	0.20	1.01	L/1960	1.03	300	300											
230	W150x30	G	S	2022	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	(10)	F <sub>y</sub>	L/1000	0.40	2.02	L/980	2.06	300	300											
231	W150x30	G	S	2022	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	(10)	F <sub>y</sub>	L/1000	0.40	2.02	L/990	2.05	300	300											
232	W150x30	G	S	2140	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	(10)	F <sub>y</sub>	L/1000	0.42	2.14	L/990	2.17	230	350											
233	W150x30	G	S	2140	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	(10)	F <sub>y</sub>	L/1000	0.42	2.14	L/990	2.16	230	350											
234	W150x30	G	S	5561	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	(10)	F <sub>y</sub>	L/8000	0.14	0.69	L/7200	0.77	300	300											
235	W150x30	G	S	5561	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	(10)	F <sub>y</sub>	L/2000	0.54	2.78	L/1800	3.10	300	300											
236	W150x30	G	S	5561	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	(10)	F <sub>y</sub>	L/1000	1.09	5.56	L/900	6.22	300	300											
237	W150x30	G	S	5561	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	(10)	0.7F <sub>y</sub>	L/1000	1.09	5.56	L/900	6.21	300	300											
238	W150x30	G	S	5561	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	(10)	F <sub>y</sub>	L/1000	1.09	5.56	L/940	5.95	300	300											

a) D - Direction of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web  
b) B - Buckling axis      W - Weak axis of the I-section      S - Strong axis of the I-section  
c)  $\lambda$  - Slenderness parameter of the reinforced column      d) IRS - Initial residual stress before reinforcing  
e) PS - Residual stress pattern, as illustrated in Figure 4-1.      f) MF - Maximum magnitude of the residual stress in the flange.  
g) MP - Maximum magnitude of the residual stress in the reinforcing plate.      F<sub>y</sub> - Yield stress of the unreinforced column  
h) ratio - The ratio of the out-of-straightness to the column length, L.      j) S - Out-of-straightness in the strong direction.  
i) W - Out-of-straightness in the weak direction.



**Table A.1 (cont'd)**

FEA model I-section No.	(1)	(2)	(3)	Plate	D <sup>a</sup>	B <sup>b</sup>	Column		IRS <sup>d</sup>		Welding		Initial Imperfection		Out-of-straightness		Yield Strength	
							Length	$\lambda^c$	PS <sup>e</sup>	MF <sup>f</sup>	MP <sup>g</sup>	Residual	before reinforcing	after reinforcing	ratio	ratio		W <sup>h</sup> (mm)
L (mm)																		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
239	W150x30	175x5	G	S	5885	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.15	5.88	L/900	6.51	230	350	
240	W150x30	175x5	G	S	5885	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.15	5.88	L/940	6.26	230	350	
241	W150x30	175x5	G	S	7583	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/8000	0.19	0.95	L/6890	1.10	300	300	
242	W150x30	175x5	G	S	7583	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/2000	0.74	3.79	L/1700	4.44	300	300	
243	W150x30	175x5	G	S	7583	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.49	7.58	L/850	8.94	300	300	
244	W150x30	175x5	G	S	7583	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.49	7.58	L/910	8.35	300	300	
245	W150x30	175x5	G	S	8024	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.58	8.02	L/860	9.34	230	350	
246	W150x30	175x5	G	S	8024	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.58	8.02	L/910	8.78	230	350	
247	W150x30	130x8	F	W	1234	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.23	0.00	L/970	1.28	300	300	
248	W150x30	130x8	F	W	1234	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.23	0.00	L/980	1.26	300	300	
249	W150x30	130x8	F	W	1295	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.30	0.00	L/970	1.33	230	350	
250	W150x30	130x8	F	W	1295	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.30	0.00	L/980	1.32	230	350	
251	W150x30	130x8	F	W	3392	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/8000	0.42	0.00	L/6770	0.50	300	300	
252	W150x30	130x8	F	W	3392	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/2000	1.70	0.00	L/1700	2.01	300	300	
253	W150x30	130x8	F	W	3392	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	3.39	0.00	L/850	4.01	300	300	
254	W150x30	130x8	F	W	3392	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	0.7F <sub>y</sub>	L/1000	3.39	0.00	L/850	4.02	300	300	
255	W150x30	130x8	F	W	3392	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	3.39	0.00	L/910	3.72	300	300	

a) D - Direction of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web  
b) B - Buckling axis      W - Weak axis of the I-section      S - Strong axis of the I-section  
c)  $\lambda$  - Slenderness parameter of the reinforced column      d) IRS - Initial residual stress before reinforcing      f) MF - Maximum magnitude of the residual stress in the flange.  
e) PS - Residual stress pattern, as illustrated in Figure 4-1.      f) MP - Maximum magnitude of the residual stress in the flange.  
g) MP - Maximum magnitude of the residual stress in the reinforcing plate.      F<sub>y</sub> - Yield stress of the unreinforced column  
h) ratio - The ratio of the out-of-straightness to the column length, L.      j) S - Out-of-straightness in the strong direction.  
i) W - Out-of-straightness in the weak direction.

**Table A.1 (cont'd)**

FEA model I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	Length L (mm)	Column			Welding			Initial Imperfection			Out-of-straightness			Yield Strength I-section plate (MPa)	
					λ <sup>c</sup>	PS <sup>e</sup>	MF <sup>f</sup>	MP <sup>g</sup>	Residual Stress	ratio	W <sup>h</sup> (mm)	S <sup>i</sup> (mm)	ratio	W <sup>h</sup> (mm)	S <sup>i</sup> (mm)	after reinforcing, no load		
No.	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
256	W150x30	130x8	F	W	3560	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	3.56	0.00	L/860	4.13		230	350
257	W150x30	130x8	F	W	3560	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	3.56	0.00	L/920	3.86		230	350
258	W150x30	130x8	F	W	4626	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	4.63	0.00	L/800	5.76		300	300
259	W150x30	130x8	F	W	4626	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	4.63	0.00	L/890	5.18		300	300
260	W150x30	130x8	F	W	4854	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	4.85	0.00	L/820	5.91		230	350
261	W150x30	130x8	F	W	4854	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	4.85	0.00	L/900	5.37		230	350
262	W150x30	130x8	F	S	2366	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	0.46	2.36	L/950		2.50	300	300
263	W150x30	130x8	F	S	2366	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	0.46	2.36	L/970		2.45	300	300
264	W150x30	130x8	F	S	2483	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	0.23	2.48	L/950		2.61	230	350
265	W150x30	130x8	F	S	2483	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	0.23	2.48	L/970		2.56	230	350
266	W150x30	130x8	F	S	6507	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/8200	0.00	0.79	L/6140		1.06	300	300
267	W150x30	130x8	F	S	6507	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/2100	0.00	3.12	L/1560		4.19	300	300
268	W150x30	130x8	F	S	6507	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	0.00	6.51	L/740		8.75	300	300
269	W150x30	130x8	F	S	6507	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	0.7F <sub>y</sub>	L/1000	0.00	6.51	L/740		8.76	300	300
270	W150x30	130x8	F	S	6507	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	0.00	6.51	L/850		7.71	300	300
271	W150x30	130x8	F	S	6828	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	0.00	6.83	L/760		9.00	230	350
272	W150x30	130x8	F	S	6828	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	0.00	6.83	L/850		8.02	230	350

a) D - Direction of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web

b) B - Buckling axis      W - Weak axis of the I-section      S - Strong axis of the I-section

c) λ - Slenderness parameter of the reinforced column      d) IRS - Initial residual stress before reinforcing

e) PS - Residual stress pattern, as illustrated in Figure 4-1.      f) MF - Maximum magnitude of the residual stress in the flange.

g) MP - Maximum magnitude of the residual stress in the reinforcing plate.      F<sub>y</sub> - Yield stress of the unreinforced column

h) ratio - The ratio of the out-of-straightness to the column length, L.

i) W - Out-of-straightness in the weak direction.      j) S - Out-of-straightness in the strong direction.

Table A.1 (cont'd)

FEA model I-section No.	Plate	D <sup>a</sup>	B <sup>b</sup>	Column Length L (mm)	$\lambda^c$	Column		PS <sup>e</sup>	MP <sup>f</sup>	IRS <sup>g</sup>	Welding Residual Stress ratio W <sup>h</sup> (mm)	Initial Imperfection before reinforcing S <sup>i</sup> (mm)	Out-of-straightness after reinforcing, no load S <sup>j</sup> (mm)	Yield Strength I-section plate (MPa)			
						D <sup>a</sup>	B <sup>b</sup>								(10)	(11)	(12)
273	W150x30	130x8	F	S	8873	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	-0.04	8.87	L/680	13.00	300	300
274	W150x30	130x8	F	S	8873	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	-0.04	8.87	L/810	10.93	300	300
275	W150x30	130x8	F	S	9310	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	-0.04	9.31	L/700	13.34	230	350
276	W150x30	130x8	F	S	9310	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	-0.04	9.31	L/820	11.38	230	350
277	W150x30	175x8	G	W	1947	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.95	0.00	L/850	2.31	300	300
278	W150x30	175x8	G	W	1947	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.95	0.00	L/900	2.17	300	300
279	W150x30	175x8	G	W	2012	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	2.01	0.00	L/870	2.31	230	350
280	W150x30	175x8	G	W	2012	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	2.01	0.00	L/920	2.20	230	350
281	W150x30	175x8	G	W	5353	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/8000	0.67	0.00	L/4420	1.21	300	300
282	W150x30	175x8	G	W	5353	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/2000	2.68	0.00	L/1100	4.85	300	300
283	W150x30	175x8	G	W	5353	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	5.35	0.00	L/550	9.71	300	300
284	W150x30	175x8	G	W	5353	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	0.7F <sub>y</sub>	L/1000	5.35	0.00	L/550	9.72	300	300
285	W150x30	175x8	G	W	5353	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	5.35	0.00	L/720	7.46	300	300
286	W150x30	175x8	G	W	5531	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	5.53	0.00	L/580	9.56	230	350
287	W150x30	175x8	G	W	5531	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	5.53	0.00	L/730	7.53	230	350
288	W150x30	175x8	G	W	7300	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	7.30	0.00	L/530	13.92	300	300
289	W150x30	175x8	G	W	7300	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	7.30	0.00	L/710	10.33	300	300

- a) D - Direction of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web
- b) B - Buckling axis      W - Weak axis of the I-section      S - Strong axis of the I-section
- c)  $\lambda$  - Slenderness parameter of the reinforced column      d) IRS - Initial residual stress before reinforcing
- e) PS - Residual stress pattern, as illustrated in Figure 4-1.      f) MF - Maximum magnitude of the residual stress in the flange.
- g) MP - Maximum magnitude of the residual stress in the reinforcing plate.      F<sub>y</sub> - Yield stress of the unreinforced column
- h) ratio - The ratio of the out-of-straightness to the column length, L.      j) S - Out-of-straightness in the strong direction.
- i) W - Out-of-straightness in the weak direction.

**Table A.1 (cont'd)**

FEA model I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	Column Length λ <sup>c</sup>	Column			Welding		Initial Imperfection		Out-of-straightness		Yield Strength I-section plate (MPa)				
					(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)		(12)	(13)	(14)	(15)
No.	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
290	W150x30	175x8	G	W	7543	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	7.54	0.00	L/530	14.19		230	350
291	W150x30	175x8	G	W	7543	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	7.54	0.00	L/710	10.60		230	350
292	W150x30	175x8	G	S	1966	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	0.38	1.96	L/970		2.02	300	300
293	W150x30	175x8	G	S	1966	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	0.38	1.96	L/980		2.00	300	300
294	W150x30	175x8	G	S	2032	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	0.40	2.03	L/980		2.08	230	350
295	W150x30	175x8	G	S	2032	0.4	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	0.40	2.03	L/980		2.07	230	350
296	W150x30	175x8	G	S	5407	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/8000	0.13	0.68	L/6910		0.78	300	300
297	W150x30	175x8	G	S	5407	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/2000	0.53	2.70	L/1730		3.13	300	300
298	W150x30	175x8	G	S	5407	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.06	5.41	L/860		6.28	300	300
299	W150x30	175x8	G	S	5407	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	0.7F <sub>y</sub>	L/1000	1.06	5.41	L/860		6.27	300	300
300	W150x30	175x8	G	S	5407	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.06	5.41	L/910		5.92	300	300
301	W150x30	175x8	G	S	5587	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.09	5.59	L/880		6.35	230	350
302	W150x30	175x8	G	S	5587	1.1	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.09	5.59	L/930		6.04	230	350
303	W150x30	175x8	G	S	7373	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.45	7.37	L/810		9.14	300	300
304	W150x30	175x8	G	S	7373	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.45	7.37	L/880		8.37	300	300
305	W150x30	175x8	G	S	7618	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.50	7.62	L/820		9.26	230	350
306	W150x30	175x8	G	S	7618	1.5	1-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1000	1.50	7.62	L/890		8.55	230	350

- a) D - Direction of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web
- b) B - Buckling axis      W - Weak axis of the I-section      S - Strong axis of the I-section
- c) λ - Slenderness parameter of the reinforced column      d) IRS - Initial residual stress before reinforcing
- e) PS - Residual stress pattern, as illustrated in Figure 4-1.      f) MF - Maximum magnitude of the residual stress in the flange.
- g) MP - Maximum magnitude of the residual stress in the reinforcing plate.      F<sub>y</sub> - Yield stress of the unreinforced column
- h) ratio - The ratio of the out-of-straightness to the column length, L.      j) S - Out-of-straightness in the strong direction.
- i) W - Out-of-straightness in the weak direction.

**Table A.1 (cont'd)**

FEA model I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	Column		$\lambda^c$	PS <sup>e</sup>	IRS <sup>d</sup>		MP <sup>f</sup>	MP <sup>g</sup>	Welding Residual	Initial Imperfection before reinforcing		Out-of-straightness after reinforcing, no load		Yield Strength I-section plate	
				Length	L (mm)			(5)	(6)				(7)	(8)	(9)	(10)		(11)
No.	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
307	W310x179	350x16	G	W	3720	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1105	3.37		L/979	3.80		300	300
308	W310x179	350x16	G	W	10229	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1267	8.07		L/789	12.96		300	300
309	W310x179	350x16	G	S	11425	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1697		6.73	L/1574		7.26	300	300
310	W310x179	350x16	F	S	18479	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/2029		9.11	L/858		7.37	300	300
311	W150x30	130x5	F	S	6326	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1081		5.85	L/808		10.67	300	300
312	W150x30	130x5	F	S	8626	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1133		7.61	L/860		10.03	300	300
313	W310x179	350x25	G	W	4103	0.4	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1171	3.50		L/983	4.17		300	300
314	W310x179	350x25	G	W	11281	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/1741	6.48		L/985	11.45		300	300
315	W310x179	350x25	G	W	15383	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/2263	6.80		L/1194	12.89		300	300
316	W310x179	350x16	G	W	10229	1.1	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/3010	3.40		L/1862	5.49		300	300
317	W310x179	350x16	G	W	13948	1.5	I-3	0.3F <sub>y</sub>	0.3F <sub>y</sub>	F <sub>y</sub>	L/2189	6.37		L/1227	11.36		300	300

a) D - Direction of reinforcing plates

b) B - Buckling axis

c)  $\lambda$  - Slenderness parameter of the reinforced column

e) PS - Residual stress pattern, as illustrated in Figure 4-1.

g) MP - Maximum magnitude of the residual stress in the reinforcing plate.

h) ratio - The ratio of the out-of-straightness to the column length, L.

i) W - Out-of-straightness in the weak direction.

F - Parallel to the flanges

W - Weak axis of the I-section

d) IRS - Initial residual stress before reinforcing

f) MF - Maximum magnitude of the residual stress in the flange.

F<sub>y</sub> - Yield stress of the unreinforced column

j) S - Out-of-straightness in the strong direction.

G - Parallel to the web

S - Strong axis of the I-section

## **Appendix B**

### **Analysis Results Description**

## Analysis Results Description

All 317 model analyses were performed to verify the behavior of the reinforced steel columns with welded cover plates in the analysis. The preload condition and analysis results of the analysed models are presented in Table B.1.

Column (1) of Table B.1 presents the number of the finite element analysed models. Columns (2) and (3) present the designations of I-sections and cover plates respectively.

Column (4) “D” is the direction of reinforcing plates. In this column, F represents that the cover plates are reinforced on the column along the flanges, as shown in Figure 2.1 (a). G represents that the cover plates are attached to the column at the flange tips of the column and parallel to the web, as shown in Figure 2.1 (b).

Column (5) “B” is the buckling axis of the reinforced column. W represents the same axis as the weak axis of the I-section before reinforcing, and S represents the same axis as the strong axis of the I-section before reinforcing. Column (6) “ $\lambda$ ” is the slenderness parameter of the reinforced column on the buckling direction. Column (7) “A” is the area of the reinforced cross-section consisting of the rolled section and cover plates.

Columns (8) and (9) present the yield strength of the I-section and the cover plates respectively. Columns (10) and (11) present the pre-loads on the column before reinforcing. Column (10) “ $P_0$ ” is the magnitude of the pre-load on the column before reinforcing. Column (11) “ $P_0/P_{u2}$ ” presents the ratio of the pre-load to the expected load carrying capacity of the unreinforced column predicted using SSRC curve 2.

Column (12) “ $P_{fca}/P_{ry}$ ” presents the ratio of the load carrying capacity of reinforced column by mathematical model analyses to the yield strength of the reinforced column. Column (13) “ $P_{r1}/P_{ry}$ ” presents the ratio of the load carrying capacity of the reinforced column predicted using SSRC curve 1 to the yield strength of the reinforced column. Column (14) “ $P_{r2}/P_{ry}$ ” presents the ratio of the load carrying capacity of the reinforced column predicted using SSRC curve 2 to the yield strength of the reinforced column. Column (15) “ $P_{rc1}/P_{ry}$ ” presents the ratio of the load carrying capacity of the reinforced

column predicted using CSA curve 1 to the yield strength of the reinforced column. Column (16) " $P_{rc2}/P_{ry}$ " presents the ratio of the load carrying capacity of the reinforced column predicted using CSA curve 2 to the yield strength of the reinforced column.



**Table B.1 Analysis Result Description**

FEA model	I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	$\lambda^c$	Area (mm <sup>2</sup> )	I-section (MPa)	plate (MPa)	Preload		P <sub>0</sub> <sup>d</sup> (kN)	P <sub>u2</sub> <sup>e</sup>	P <sub>rc1</sub> <sup>f</sup>	P <sub>rc2</sub> <sup>g</sup>	P <sub>rc1</sub> <sup>h</sup>	P <sub>rc2</sub> <sup>i</sup>	P <sub>rc1</sub> <sup>j</sup>	P <sub>rc2</sub> <sup>k</sup>
									(8)	(9)								
No.	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
1	W200x46	180x9.52	F	W	0.5	9287	260	260	405	0.3	0.97	0.95	0.86	0.97	0.88	0.97	0.88	
2	W200x46	180x9.52	F	W	0.5	9287	260	260	0	0.0	0.98	0.95	0.86	0.97	0.88	0.97	0.88	
3	W200x46	180x9.52	F	W	1.0	9287	260	260	405	0.4	0.65	0.75	0.61	0.73	0.60	0.73	0.60	
4	W200x47	180x9.53	F	W	1.0	9287	260	260	405	0.4	0.65	0.75	0.61	0.73	0.60	0.73	0.60	
5	W200x46	180x9.52	F	W	1.0	9287	260	260	405	0.4	0.65	0.75	0.61	0.73	0.60	0.73	0.60	
6	W200x47	180x9.53	F	W	1.0	9287	260	260	405	0.4	0.65	0.75	0.61	0.73	0.60	0.73	0.60	
7	W200x46	180x9.52	F	W	1.0	9287	260	260	405	0.4	0.63	0.75	0.61	0.73	0.60	0.73	0.60	
8	W200x46	180x9.52	F	W	1.0	9287	260	260	405	0.4	0.66	0.75	0.61	0.73	0.60	0.73	0.60	
9	W200x46	180x9.52	F	W	1.0	9287	260	260	405	0.4	0.65	0.75	0.61	0.73	0.60	0.73	0.60	
10	W200x46	180x9.52	F	W	1.0	9287	260	260	405	0.4	0.66	0.75	0.61	0.73	0.60	0.73	0.60	
11	W200x46	180x9.52	F	W	1.0	9287	260	260	405	0.4	0.60	0.75	0.61	0.73	0.60	0.73	0.60	
12	W200x46	180x9.52	F	W	1.0	9287	260	260	405	0.4	0.65	0.75	0.61	0.73	0.60	0.73	0.60	
13	W200x46	180x9.52	F	S	1.0	9287	260	260	405	0.5	0.63	0.75	0.61	0.73	0.60	0.73	0.60	
14	W200x46	180x9.52	F	S	1.0	9287	260	260	405	0.5	0.63	0.75	0.61	0.73	0.60	0.73	0.60	
15	W200x46	180x9.52	F	S	1.0	9287	260	260	405	0.5	0.63	0.75	0.61	0.73	0.60	0.73	0.60	
16	W200x46	180x9.52	F	S	1.0	9287	260	260	405	0.5	0.63	0.75	0.61	0.73	0.60	0.73	0.60	

a) D - Orientation of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web

b) B - Buckling axis of the reinforced column      W - Weak axis of the rolled section      S - Strong axis of the rolled section

c)  $\lambda$  - Slenderness parameter.      d) P<sub>0</sub> - Pre-load      e) P<sub>u2</sub> - Load carrying capacity of the I-section (predicted using the SSRC curve 2)

f) P<sub>rc1</sub> - Load carrying capacity obtained from the finite element analysis      P<sub>rc2</sub> - Yield strength of the reinforced column

g) P<sub>r1</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 1)

h) P<sub>r2</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 2)

i) P<sub>rc1</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 1)

j) P<sub>rc2</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 2)

**Table B.1 (cont'd)**

FEA model	I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	λ <sup>c</sup>	Area (mm <sup>2</sup> )	Yield Strength			Preload			P <sub>rc1</sub> /P <sub>ry</sub> <sup>i</sup>	P <sub>rc2</sub> /P <sub>ry</sub> <sup>j</sup>	
							I-section (MPa)	plate (MPa)	P <sub>0</sub> (kN)	P <sub>0</sub> /P <sub>u2</sub> <sup>d</sup>	P <sub>rcd</sub> /P <sub>ry</sub> <sup>e</sup>	P <sub>r1</sub> /P <sub>ry</sub> <sup>f</sup>			P <sub>r2</sub> /P <sub>ry</sub> <sup>g</sup>
No.	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
17	W310x179	290x25	F	W	0.4	37300	300	300	3760	0.6	1.00	0.98	0.92	0.99	0.94
18	W310x179	290x25	F	W	0.4	37300	300	300	3760	0.6	0.99	0.98	0.92	0.99	0.94
19	W310x179	290x25	F	W	0.4	37300	300	300	3760	0.6	0.97	0.98	0.92	0.99	0.94
20	W310x179	290x25	F	W	0.4	37300	300	300	2507	0.4	0.98	0.98	0.92	0.99	0.94
21	W310x179	290x25	F	W	0.4	37300	230	350	2926	0.6	0.96	0.98	0.92	0.99	0.94
22	W310x179	290x25	F	W	0.4	37300	230	350	1950	0.4	0.96	0.98	0.92	0.99	0.94
23	W310x179	290x25	F	W	1.1	37300	300	300	2152	0.6	0.64	0.68	0.54	0.66	0.54
24	W310x179	290x25	F	W	1.1	37300	300	300	2152	0.6	0.60	0.68	0.54	0.66	0.54
25	W310x179	290x25	F	W	1.1	37300	300	300	2152	0.6	0.56	0.68	0.54	0.66	0.54
26	W310x179	290x25	F	W	1.1	37300	300	300	2152	0.6	0.56	0.68	0.54	0.66	0.54
27	W310x179	290x25	F	W	1.1	37300	300	300	1435	0.4	0.57	0.68	0.54	0.66	0.54
28	W310x179	290x25	F	W	1.1	37300	230	350	1866	0.6	0.51	0.68	0.54	0.66	0.54
29	W310x179	290x25	F	W	1.1	37300	230	350	1244	0.4	0.54	0.68	0.54	0.66	0.54
30	W310x179	290x25	F	W	1.5	37300	300	300	1401	0.6	0.42	0.41	0.35	0.42	0.36
31	W310x179	290x25	F	W	1.5	37300	300	300	1401	0.6	0.38	0.41	0.35	0.42	0.36
32	W310x179	290x25	F	W	1.5	37300	300	300	1401	0.6	0.35	0.41	0.35	0.42	0.36

a) D - Orientation of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web

b) B - Buckling axis of the reinforced column      W - Weak axis of the rolled section      S - Strong axis of the rolled section

c) λ - Slenderness parameter.      d) P<sub>0</sub> - Pre-load      e) P<sub>u2</sub> - Load carrying capacity of the I-section (predicted using the SSRC curve 2)

f) P<sub>rc1</sub> - Load carrying capacity obtained from the finite element analysis      P<sub>ry</sub> - Yield strength of the reinforced column

g) P<sub>r1</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 1)

h) P<sub>r2</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 2)

i) P<sub>rc1</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 1)

j) P<sub>rc2</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 2)

**Table B.1 (cont'd)**

FEA model	I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	$\lambda^c$	Area (mm <sup>2</sup> )	Yield Strength		Preload		P <sub>rc1</sub> /P <sub>ry</sub> <sup>i</sup>	P <sub>rc2</sub> /P <sub>ry</sub> <sup>j</sup>			
							I-section (MPa)	plate (MPa)	P <sub>0</sub> <sup>d</sup>	P <sub>u2</sub> <sup>e</sup>			P <sub>rc1</sub> /P <sub>ry</sub> <sup>f</sup>	P <sub>rc2</sub> /P <sub>ry</sub> <sup>h</sup>	P <sub>rc1</sub> /P <sub>ry</sub> <sup>i</sup>
No.	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
33	W310x179	290x25	F	W	1.5	37300	300	300	934	0.4	0.36	0.41	0.35	0.42	0.36
34	W310x179	290x25	F	W	1.5	37300	230	350	1224	0.6	0.34	0.41	0.35	0.42	0.36
35	W310x179	290x25	F	W	1.5	37300	230	350	816	0.4	0.35	0.41	0.35	0.42	0.36
36	W310x179	290x25	F	S	0.4	37300	300	300	3697	0.6	0.93	0.98	0.92	0.99	0.94
37	W310x179	290x25	F	S	0.4	37300	300	300	2465	0.4	0.93	0.98	0.92	0.99	0.94
38	W310x179	290x25	F	S	0.4	37300	230	350	2883	0.6	0.94	0.98	0.92	0.99	0.94
39	W310x179	290x25	F	S	0.4	37300	230	350	1922	0.4	0.94	0.98	0.92	0.99	0.94
40	W310x179	290x25	F	S	1.1	37300	300	300	1901	0.6	0.68	0.68	0.54	0.66	0.54
41	W310x179	290x25	F	S	1.1	37300	300	300	1901	0.6	0.63	0.68	0.54	0.66	0.54
42	W310x179	290x25	F	S	1.1	37300	300	300	1901	0.6	0.61	0.68	0.54	0.66	0.54
43	W310x179	290x25	F	S	1.1	37300	300	300	1901	0.6	0.62	0.68	0.54	0.66	0.54
44	W310x179	290x25	F	S	1.1	37300	300	300	1268	0.4	0.63	0.68	0.54	0.66	0.54
45	W310x179	290x25	F	S	1.1	37300	230	350	1651	0.6	0.57	0.68	0.54	0.66	0.54
46	W310x179	290x25	F	S	1.1	37300	230	350	1101	0.4	0.59	0.68	0.54	0.66	0.54
47	W310x179	290x25	F	S	1.5	37300	300	300	1227	0.6	0.40	0.41	0.35	0.42	0.36
48	W310x179	290x25	F	S	1.5	37300	300	300	818	0.4	0.41	0.41	0.35	0.42	0.36

a) D - Orientation of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web

b) B - Buckling axis of the reinforced column      W - Weak axis of the rolled section      S - Strong axis of the rolled section

c)  $\lambda$  - Slenderness parameter.      d) P<sub>0</sub> - Pre-load      e) P<sub>u2</sub> - Load carrying capacity of the I-section (predicted using the SSRC curve 2)

f) P<sub>fea</sub> - Load carrying capacity obtained from the finite element analysis      P<sub>ry</sub> - Yield strength of the reinforced column

g) P<sub>r1</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 1)

h) P<sub>r2</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 2)

i) P<sub>rc1</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 1)

j) P<sub>rc2</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 2)

**Table B.1 (cont'd)**

FEA model	I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	λ <sup>c</sup>	Area (mm <sup>2</sup> )	Yield Strength		Preload		P <sub>0</sub> <sup>d</sup> (kN)	P <sub>u2</sub> <sup>e</sup> (MPa)	P <sub>u2</sub> <sup>e</sup> (kN)	P <sub>re1</sub> <sup>f</sup> /P <sub>y</sub> <sup>i</sup>	P <sub>re2</sub> <sup>g</sup> /P <sub>y</sub> <sup>j</sup>	P <sub>re1</sub> <sup>h</sup> /P <sub>y</sub> <sup>k</sup>	P <sub>re2</sub> <sup>l</sup> /P <sub>y</sub> <sup>m</sup>
							I-section (MPa)	plate (MPa)	P <sub>0</sub> <sup>d</sup>	P <sub>u2</sub> <sup>e</sup>							
No.	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)		
49	W310x179	290x25	F	S	1.5	37300	230	350	1075	0.6	0.37	0.41	0.35	0.42	0.36		
50	W310x179	290x25	F	S	1.5	37300	230	350	717	0.4	0.39	0.41	0.35	0.42	0.36		
51	W310x179	350x25	G	W	0.4	40300	300	300	3353	0.6	0.95	0.98	0.92	0.99	0.94		
52	W310x179	350x25	G	W	0.4	40300	300	300	2235	0.4	0.96	0.98	0.92	0.99	0.94		
53	W310x179	350x25	G	W	0.4	40300	230	350	2662	0.6	0.96	0.98	0.92	0.99	0.94		
54	W310x179	350x25	G	W	0.4	40300	230	350	1775	0.4	0.96	0.98	0.92	0.99	0.94		
55	W310x179	350x25	G	W	1.1	40300	300	300	1151	0.6	0.71	0.68	0.54	0.66	0.54		
56	W310x179	350x25	G	W	1.1	40300	300	300	1151	0.6	0.64	0.68	0.54	0.66	0.54		
57	W310x179	350x25	G	W	1.1	40300	300	300	1151	0.6	0.59	0.68	0.54	0.66	0.54		
58	W310x179	350x25	G	W	1.1	40300	300	300	1151	0.6	0.60	0.68	0.54	0.66	0.54		
59	W310x179	350x25	G	W	1.1	40300	300	300	768	0.4	0.61	0.68	0.54	0.66	0.54		
60	W310x179	350x25	G	W	1.1	40300	230	350	1026	0.6	0.61	0.68	0.54	0.66	0.54		
61	W310x179	350x25	G	W	1.1	40300	230	350	684	0.4	0.63	0.68	0.54	0.66	0.54		
62	W310x179	350x25	G	W	1.5	40300	300	300	668	0.6	0.38	0.41	0.35	0.42	0.36		
63	W310x179	350x25	G	W	1.5	40300	300	300	445	0.4	0.40	0.41	0.35	0.42	0.36		
64	W310x179	350x25	G	W	1.5	40300	230	350	622	0.6	0.39	0.41	0.35	0.42	0.36		

a) D - Orientation of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web

b) B - Buckling axis of the reinforced column      W - Weak axis of the rolled section      S - Strong axis of the rolled section

c) λ - Slenderness parameter.      d) P<sub>0</sub> - Pre-load      e) P<sub>u2</sub> - Load carrying capacity of the I-section (predicted using the SSRC curve 2)

f) P<sub>fea</sub> - Load carrying capacity obtained from the finite element analysis      P<sub>y</sub> - Yield strength of the reinforced column

g) P<sub>r1</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 1)

h) P<sub>r2</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 2)

i) P<sub>re1</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 1)

j) P<sub>re2</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 2)

**Table B.1 (cont'd)**

FEA model	I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	$\lambda^c$	Area (mm <sup>2</sup> )	Yield Strength		Preload		P <sub>0</sub> <sup>d</sup> (kN)	P <sub>0</sub> /P <sub>u2</sub> <sup>e</sup> (11)	P <sub>rc1</sub> /P <sub>ry</sub> <sup>f</sup> (12)	P <sub>rc1</sub> /P <sub>ry</sub> <sup>g</sup> (13)	P <sub>rc2</sub> /P <sub>ry</sub> <sup>h</sup> (14)	P <sub>rc1</sub> /P <sub>ry</sub> <sup>i</sup> (15)	P <sub>rc2</sub> /P <sub>ry</sub> <sup>j</sup> (16)
							I-section (MPa)	plate (MPa)	P <sub>0</sub> <sup>d</sup> (kN)	P <sub>0</sub> /P <sub>u2</sub> <sup>e</sup> (11)							
No.	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)		
65	W310x179	350x25	G	W	1.5	40300	230	350	414	0.4	0.40	0.41	0.35	0.42	0.36		
66	W310x179	350x25	G	S	0.4	40300	300	300	3840	0.6	0.93	0.98	0.92	0.99	0.94		
67	W310x179	350x25	G	S	0.4	40300	300	300	2560	0.4	0.94	0.98	0.92	0.99	0.94		
68	W310x179	350x25	G	S	0.4	40300	230	350	2982	0.6	0.91	0.98	0.92	0.99	0.94		
69	W310x179	350x25	G	S	0.4	40300	230	350	1988	0.4	0.92	0.98	0.92	0.99	0.94		
70	W310x179	350x25	G	S	1.1	40300	300	300	2562	0.6	0.60	0.68	0.54	0.66	0.54		
71	W310x179	350x25	G	S	1.1	40300	300	300	2562	0.6	0.57	0.68	0.54	0.66	0.54		
72	W310x179	350x25	G	S	1.1	40300	300	300	2562	0.6	0.54	0.68	0.54	0.66	0.54		
73	W310x179	350x25	G	S	1.1	40300	300	300	2562	0.6	0.54	0.68	0.54	0.66	0.54		
74	W310x179	350x25	G	S	1.1	40300	300	300	1708	0.4	0.57	0.68	0.54	0.66	0.54		
75	W310x179	350x25	G	S	1.1	40300	230	350	2149	0.6	0.48	0.68	0.54	0.66	0.54		
76	W310x179	350x25	G	S	1.1	40300	230	350	1433	0.4	0.51	0.68	0.54	0.66	0.54		
77	W310x179	350x25	G	S	1.5	40300	300	300	1699	0.6	0.37	0.41	0.35	0.42	0.36		
78	W310x179	350x25	G	S	1.5	40300	300	300	1132	0.4	0.38	0.41	0.35	0.42	0.36		
79	W310x179	350x25	G	S	1.5	40300	230	350	1498	0.6	0.35	0.41	0.35	0.42	0.36		
80	W310x179	350x25	G	S	1.5	40300	230	350	999	0.4	0.36	0.41	0.35	0.42	0.36		

a) D - Orientation of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web

b) B - Buckling axis of the reinforced column      W - Weak axis of the rolled section      S - Strong axis of the rolled section

c)  $\lambda$  - Slenderness parameter.      d) P<sub>0</sub> - Pre-load      e) P<sub>u2</sub> - Load carrying capacity of the I-section (predicted using the SSRC curve 2)

f) P<sub>rc1</sub> - Load carrying capacity obtained from the finite element analysis      P<sub>ry</sub> - Yield strength of the reinforced column

g) P<sub>r1</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 1)

h) P<sub>r2</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 2)

i) P<sub>rc1</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 1)

j) P<sub>rc2</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 2)

**Table B.1 (cont'd)**

FEA model	I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	λ <sup>c</sup>	Area (mm <sup>2</sup> )	Yield Strength		Preload		P <sub>rc1</sub> /P <sub>ry</sub> <sup>i</sup>	P <sub>rc2</sub> /P <sub>ry</sub> <sup>j</sup>			
							I-section (MPa)	plate (MPa)	P <sub>0</sub> <sup>d</sup> (kN)	P <sub>0</sub> /P <sub>u2</sub> <sup>e</sup> (11)					
No.	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
81	W310x179	350x25	G	S	1.1	40300	300	300	2562	0.6	0.54	0.68	0.54	0.66	0.54
82	W310x179	350x25	G	S	1.1	40300	300	300	2562	0.6	0.54	0.68	0.54	0.66	0.54
83	W310x179	350x25	G	S	1.1	40300	300	300	2562	0.6	0.54	0.68	0.54	0.66	0.54
84	W310x179	290x16	F	W	0.4	32080	300	300	3764	0.6	0.98	0.98	0.92	0.99	0.94
85	W310x179	290x16	F	W	0.4	32080	300	300	2509	0.4	0.98	0.98	0.92	0.99	0.94
86	W310x179	290x16	F	W	0.4	32080	230	350	2918	0.6	0.95	0.98	0.92	0.99	0.94
87	W310x179	290x16	F	W	0.4	32080	230	350	1945	0.4	0.96	0.98	0.92	0.99	0.94
88	W310x179	290x16	F	W	1.1	32080	300	300	2168	0.6	0.64	0.68	0.54	0.66	0.54
89	W310x179	290x16	F	W	1.1	32080	300	300	2168	0.6	0.60	0.68	0.54	0.66	0.54
90	W310x179	290x16	F	W	1.1	32080	300	300	2168	0.6	0.56	0.68	0.54	0.66	0.54
91	W310x179	290x16	F	W	1.1	32080	300	300	2168	0.6	0.57	0.68	0.54	0.66	0.54
92	W310x179	290x16	F	W	1.1	32080	300	300	1445	0.4	0.58	0.68	0.54	0.66	0.54
93	W310x179	290x16	F	W	1.1	32080	230	350	1825	0.6	0.53	0.68	0.54	0.66	0.54
94	W310x179	290x16	F	W	1.1	32080	230	350	1217	0.4	0.54	0.68	0.54	0.66	0.54
95	W310x179	290x16	F	W	1.5	32080	300	300	1412	0.6	0.36	0.41	0.35	0.42	0.36
96	W310x179	290x16	F	W	1.5	32080	300	300	1412	0.6	0.36	0.41	0.35	0.42	0.36

a) D - Orientation of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web

b) B - Buckling axis of the reinforced column      W - Weak axis of the rolled section      S - Strong axis of the rolled section

c) λ - Slenderness parameter.      d) P<sub>0</sub> - Pre-load      e) P<sub>u2</sub> - Load carrying capacity of the I-section (predicted using the SSRC curve 2)

f) P<sub>rc1</sub> - Load carrying capacity obtained from the finite element analysis      P<sub>ry</sub> - Yield strength of the reinforced column

g) P<sub>r1</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 1)

h) P<sub>r2</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 2)

i) P<sub>rc1</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 1)

j) P<sub>rc2</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 2)

**Table B.1 (cont'd)**

FEA model	I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	$\lambda^c$	Area (mm <sup>2</sup> )	Yield Strength		Preload		P <sub>0</sub> <sup>d</sup> (kN)	P <sub>0</sub> /P <sub>u2</sub> <sup>e</sup> (11)	P <sub>fc1</sub> /P <sub>ty</sub> <sup>f</sup> (12)	P <sub>fc2</sub> /P <sub>ty</sub> <sup>g</sup> (13)	P <sub>rc1</sub> /P <sub>ty</sub> <sup>h</sup> (14)	P <sub>rc2</sub> /P <sub>ty</sub> <sup>i</sup> (15)	P <sub>rc2</sub> /P <sub>ty</sub> <sup>j</sup> (16)
							I-section (MPa) (8)	plate (MPa) (9)	P <sub>0</sub> (10)	P <sub>u2</sub> (11)							
97	W310x179	290x16	F	W	1.5	32080	300	300	941	0.4	0.37	0.41	0.35	0.42	0.36		
98	W310x179	290x16	F	W	1.5	32080	230	350	1196	0.6	0.34	0.41	0.35	0.42	0.36		
99	W310x179	290x16	F	W	1.5	32080	230	350	797	0.4	0.35	0.41	0.35	0.42	0.36		
100	W310x179	290x16	F	S	0.4	32080	300	300	3724	0.6	0.94	0.98	0.92	0.99	0.94		
101	W310x179	290x16	F	S	0.4	32080	300	300	2482	0.4	0.94	0.98	0.92	0.99	0.94		
102	W310x179	290x16	F	S	0.4	32080	230	350	2890	0.6	0.93	0.98	0.92	0.99	0.94		
103	W310x179	290x16	F	S	0.4	32080	230	350	1927	0.4	0.94	0.98	0.92	0.99	0.94		
104	W310x179	290x16	F	S	1.1	32080	300	300	1999	0.6	0.69	0.68	0.54	0.66	0.54		
105	W310x179	290x16	F	S	1.1	32080	300	300	1999	0.6	0.64	0.68	0.54	0.66	0.54		
106	W310x179	290x16	F	S	1.1	32080	300	300	1999	0.6	0.61	0.68	0.54	0.66	0.54		
107	W310x179	290x16	F	S	1.1	32080	300	300	1999	0.6	0.62	0.68	0.54	0.66	0.54		
108	W310x179	290x16	F	S	1.1	32080	300	300	1333	0.4	0.63	0.68	0.54	0.66	0.54		
109	W310x179	290x16	F	S	1.1	32080	230	350	1685	0.6	0.57	0.68	0.54	0.66	0.54		
110	W310x179	290x16	F	S	1.1	32080	230	350	1123	0.4	0.59	0.68	0.54	0.66	0.54		
111	W310x179	290x16	F	S	1.5	32080	300	300	1295	0.6	0.39	0.41	0.35	0.42	0.36		
112	W310x179	290x16	F	S	1.5	32080	300	300	863	0.4	0.40	0.41	0.35	0.42	0.36		

a) D - Orientation of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web

b) B - Buckling axis of the reinforced column      W - Weak axis of the rolled section      S - Strong axis of the rolled section

c)  $\lambda$  - Slenderness parameter.      d) P<sub>0</sub> - Pre-load      e) P<sub>u2</sub> - Load carrying capacity of the I-section (predicted using the SSRC curve 2)

f) P<sub>fc1</sub> - Load carrying capacity obtained from the finite element analysis      P<sub>ty</sub> - Yield strength of the reinforced column

g) P<sub>r1</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 1)

h) P<sub>r2</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 2)

i) P<sub>rc1</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 1)

j) P<sub>rc2</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 2)

**Table B.1 (cont'd)**

FEA model	I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	$\lambda^c$	Area (mm <sup>2</sup> )	Yield Strength		Preload		P <sub>0</sub> <sup>d</sup> (kN)	P <sub>0</sub> /P <sub>u2</sub> <sup>e</sup>	P <sub>rc2</sub> /P <sub>ry</sub> <sup>f</sup>	P <sub>rc1</sub> /P <sub>ry</sub> <sup>g</sup>	P <sub>rc2</sub> /P <sub>ry</sub> <sup>h</sup>	P <sub>rc1</sub> /P <sub>ry</sub> <sup>i</sup>	P <sub>rc2</sub> /P <sub>ry</sub> <sup>j</sup>
							I-section (MPa)	plate (MPa)	P <sub>0</sub>	P <sub>0</sub> /P <sub>u2</sub> <sup>e</sup>							
No.	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)		
113	W310x179	290x16	F	S	1.5	32080	230	350	1098	0.6	0.38	0.41	0.35	0.42	0.36		
114	W310x179	290x16	F	S	1.5	32080	230	350	732	0.4	0.39	0.41	0.35	0.42	0.36		
115	W310x179	350x16	F	W	0.4	34000	300	300	3712	0.6	0.98	0.98	0.92	0.99	0.94		
116	W310x179	350x16	F	W	0.4	34000	300	300	2474	0.4	0.98	0.98	0.92	0.99	0.94		
117	W310x179	350x16	F	W	0.4	34000	230	350	2887	0.6	0.96	0.98	0.92	0.99	0.94		
118	W310x179	350x16	F	W	0.4	34000	230	350	1924	0.4	0.97	0.98	0.92	0.99	0.94		
119	W310x179	350x16	F	W	1.1	34000	300	300	1954	0.6	0.65	0.68	0.54	0.66	0.54		
120	W310x179	350x16	F	W	1.1	34000	300	300	1954	0.6	0.61	0.68	0.54	0.66	0.54		
121	W310x179	350x16	F	W	1.1	34000	300	300	1954	0.6	0.56	0.68	0.54	0.66	0.54		
122	W310x179	350x16	F	W	1.1	34000	300	300	1954	0.6	0.57	0.68	0.54	0.66	0.54		
123	W310x179	350x16	F	W	1.1	34000	300	300	1302	0.4	0.58	0.68	0.54	0.66	0.54		
124	W310x179	350x16	F	W	1.1	34000	230	350	1667	0.6	0.54	0.68	0.54	0.66	0.54		
125	W310x179	350x16	F	W	1.1	34000	230	350	1111	0.4	0.55	0.68	0.54	0.66	0.54		
126	W310x179	350x16	F	W	1.5	34000	300	300	1263	0.6	0.37	0.41	0.35	0.42	0.36		
127	W310x179	350x16	F	W	1.5	34000	300	300	842	0.4	0.37	0.41	0.35	0.42	0.36		
128	W310x179	350x16	F	W	1.5	34000	230	350	1086	0.6	0.35	0.41	0.35	0.42	0.36		

a) D - Orientation of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web

b) B - Buckling axis of the reinforced column      W - Weak axis of the rolled section      S - Strong axis of the rolled section

c)  $\lambda$  - Slenderness parameter.      d) P<sub>0</sub> - Pre-load      e) P<sub>u2</sub> - Load carrying capacity of the I-section (predicted using the SSRC curve 2)

f) P<sub>rc2</sub> - Load carrying capacity obtained from the finite element analysis      P<sub>ry</sub> - Yield strength of the reinforced column

g) P<sub>r1</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 1)

h) P<sub>r2</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 2)

i) P<sub>rc1</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 1)

j) P<sub>rc2</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 2)



**Table B.1 (cont'd)**

FEA model	I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	λ <sup>c</sup>	Area (mm <sup>2</sup> )	Yield Strength		Preload		P <sub>rc1</sub> /P <sub>ry</sub> <sup>i</sup>	P <sub>rc2</sub> /P <sub>ry</sub> <sup>j</sup>			
							I-section (MPa)	plate (MPa)	P <sub>0</sub> <sup>d</sup> (kN)	P <sub>0</sub> /P <sub>u2</sub> <sup>e</sup>					
No.	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
129	W310x179	350x16	F	W	1.5	34000	230	350	724	0.4	0.36	0.41	0.35	0.42	0.36
130	W310x179	350x16	F	S	0.4	34000	300	300	3717	0.6	0.94	0.98	0.92	0.99	0.94
131	W310x179	350x16	F	S	0.4	34000	300	300	2478	0.4	0.94	0.98	0.92	0.99	0.94
132	W310x179	350x16	F	S	0.4	34000	230	350	2890	0.6	0.93	0.98	0.92	0.99	0.94
133	W310x179	350x16	F	S	0.4	34000	230	350	1927	0.4	0.94	0.98	0.92	0.99	0.94
134	W310x179	350x16	F	S	1.1	34000	300	300	1973	0.6	0.68	0.68	0.54	0.66	0.54
135	W310x179	350x16	F	S	1.1	34000	300	300	1973	0.6	0.63	0.68	0.54	0.66	0.54
136	W310x179	350x16	F	S	1.1	34000	300	300	1973	0.6	0.61	0.68	0.54	0.66	0.54
137	W310x179	350x16	F	S	1.1	34000	300	300	1973	0.6	0.62	0.68	0.54	0.66	0.54
138	W310x179	350x16	F	S	1.1	34000	300	300	1315	0.4	0.63	0.68	0.54	0.66	0.54
139	W310x179	350x16	F	S	1.1	34000	230	350	1683	0.6	0.56	0.68	0.54	0.66	0.54
140	W310x179	350x16	F	S	1.1	34000	230	350	1122	0.4	0.58	0.68	0.54	0.66	0.54
141	W310x179	350x16	F	S	1.5	34000	300	300	1276	0.6	0.39	0.41	0.35	0.42	0.36
142	W310x179	350x16	F	S	1.5	34000	300	300	851	0.4	0.40	0.41	0.35	0.42	0.36
143	W310x179	350x16	F	S	1.5	34000	230	350	1097	0.6	0.38	0.41	0.35	0.42	0.36
144	W310x179	350x16	F	S	1.5	34000	230	350	732	0.4	0.39	0.41	0.35	0.42	0.36

- a) D - Orientation of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web
- b) B - Buckling axis of the reinforced column      W - Weak axis of the rolled section      S - Strong axis of the rolled section
- c) λ - Slenderness parameter.      d) P<sub>0</sub> - Pre-load      e) P<sub>u2</sub> - Load carrying capacity of the I-section (predicted using the SSRC curve 2)
- f) P<sub>fea</sub> - Load carrying capacity obtained from the finite element analysis      P<sub>ry</sub> - Yield strength of the reinforced column
- g) P<sub>r1</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 1)
- h) P<sub>r2</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 2)
- i) P<sub>re1</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 1)
- j) P<sub>re2</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 2)

**Table B.1 (cont'd)**

FEA model	I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	$\lambda^c$	Area (mm <sup>2</sup> )	Yield Strength			Preload		P <sub>rc1</sub> /P <sub>ry</sub> <sup>i</sup>	P <sub>rc2</sub> /P <sub>ry</sub> <sup>j</sup>			
							I-section (MPa)	plate (MPa)	P <sub>0</sub> <sup>d</sup> (kN)	P <sub>0</sub> /P <sub>u2</sub> <sup>e</sup>	P <sub>rc1</sub> /P <sub>ry</sub> <sup>f</sup>			P <sub>rc2</sub> /P <sub>ry</sub> <sup>g</sup>		
No.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
145	W310x179	350x16	G	W	0.4	34000	300	300	300	3468	0.6	0.95	0.98	0.92	0.99	0.94
146	W310x179	350x16	G	W	0.4	34000	300	300	300	2312	0.4	0.97	0.98	0.92	0.99	0.94
147	W310x179	350x16	G	W	0.4	34000	230	350	350	2721	0.6	0.97	0.98	0.92	0.99	0.94
148	W310x179	350x16	G	W	0.4	34000	230	350	350	1814	0.4	0.97	0.98	0.92	0.99	0.94
149	W310x179	350x16	G	W	1.1	34000	300	300	300	1330	0.6	0.72	0.68	0.54	0.66	0.54
150	W310x179	350x16	G	W	1.1	34000	300	300	300	1330	0.6	0.66	0.68	0.54	0.66	0.54
151	W310x179	350x16	G	W	1.1	34000	300	300	300	1330	0.6	0.60	0.68	0.54	0.66	0.54
152	W310x179	350x16	G	W	1.1	34000	300	300	300	1330	0.6	0.61	0.68	0.54	0.66	0.54
153	W310x179	350x16	G	W	1.1	34000	300	300	300	887	0.4	0.62	0.68	0.54	0.66	0.54
154	W310x179	350x16	G	W	1.1	34000	230	350	350	1142	0.6	0.62	0.68	0.54	0.66	0.54
155	W310x179	350x16	G	W	1.1	34000	230	350	350	762	0.4	0.64	0.68	0.54	0.66	0.54
156	W310x179	350x16	G	W	1.5	34000	300	300	300	805	0.6	0.40	0.41	0.35	0.42	0.36
157	W310x179	350x16	G	W	1.5	34000	300	300	300	536	0.4	0.41	0.41	0.35	0.42	0.36
158	W310x179	350x16	G	W	1.5	34000	230	350	350	718	0.6	0.40	0.41	0.35	0.42	0.36
159	W310x179	350x16	G	W	1.5	34000	230	350	350	479	0.4	0.41	0.41	0.35	0.42	0.36
160	W310x179	350x16	G	S	0.4	34000	300	300	300	3824	0.6	0.93	0.98	0.92	0.99	0.94

a) D - Orientation of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web

b) B - Buckling axis of the reinforced column      W - Weak axis of the rolled section      S - Strong axis of the rolled section

c)  $\lambda$  - Slenderness parameter.      d) P<sub>0</sub> - Pre-load      e) P<sub>u2</sub> - Load carrying capacity of the I-section (predicted using the SSRC curve 2)

f) P<sub>rc1</sub> - Load carrying capacity obtained from the finite element analysis      P<sub>ry</sub> - Yield strength of the reinforced column

g) P<sub>r1</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 1)

h) P<sub>r2</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 2)

i) P<sub>rc1</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 1)

j) P<sub>rc2</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 2)

**Table B.1 (cont'd)**

FEA		Yield Strength			Preload											
model	I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	λ <sup>c</sup>	Area (mm <sup>2</sup> )	I-section (MPa)	plate (MPa)	P <sub>0</sub> <sup>d</sup> (kN)	P <sub>y</sub> /P <sub>u2</sub> <sup>e</sup>	P <sub>rc2</sub> /P <sub>ry</sub> <sup>f</sup>	P <sub>r1</sub> /P <sub>ry</sub> <sup>g</sup>	P <sub>r2</sub> /P <sub>ry</sub> <sup>h</sup>	P <sub>rc1</sub> /P <sub>ry</sub> <sup>i</sup>	P <sub>rc2</sub> /P <sub>ry</sub> <sup>j</sup>	
No.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
161	W310x179	350x16	G	S	0.4	34000	300	300	2549	0.4	0.94	0.98	0.92	0.99	0.94	
162	W310x179	350x16	G	S	0.4	34000	230	350	2963	0.6	0.89	0.98	0.92	0.99	0.94	
163	W310x179	350x16	G	S	0.4	34000	230	350	1975	0.4	0.90	0.98	0.92	0.99	0.94	
164	W310x179	350x16	G	S	1.1	34000	300	300	2480	0.6	0.64	0.68	0.54	0.66	0.54	
165	W310x179	350x16	G	S	1.1	34000	300	300	2480	0.6	0.60	0.68	0.54	0.66	0.54	
166	W310x179	350x16	G	S	1.1	34000	300	300	2480	0.6	0.56	0.68	0.54	0.66	0.54	
167	W310x179	350x16	G	S	1.1	34000	300	300	2480	0.6	0.57	0.68	0.54	0.66	0.54	
168	W310x179	350x16	G	S	1.1	34000	300	300	1653	0.4	0.59	0.68	0.54	0.66	0.54	
169	W310x179	350x16	G	S	1.1	34000	230	350	2057	0.6	0.51	0.68	0.54	0.66	0.54	
170	W310x179	350x16	G	S	1.1	34000	230	350	1371	0.4	0.53	0.68	0.54	0.66	0.54	
171	W310x179	350x16	G	S	1.5	34000	300	300	1628	0.6	0.38	0.41	0.35	0.42	0.36	
172	W310x179	350x16	G	S	1.5	34000	300	300	1086	0.4	0.39	0.41	0.35	0.42	0.36	
173	W310x179	350x16	G	S	1.5	34000	230	350	1393	0.6	0.36	0.41	0.35	0.42	0.36	
174	W310x179	350x16	G	S	1.5	34000	230	350	929	0.4	0.37	0.41	0.35	0.42	0.36	
175	W150x30	130x5	F	W	0.4	5090	300	300	627	0.6	0.97	0.98	0.92	0.99	0.94	
176	W150x30	130x5	F	W	0.4	5090	300	300	418	0.4	0.98	0.98	0.92	0.99	0.94	

a) D - Orientation of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web

b) B - Buckling axis of the reinforced column      W - Weak axis of the rolled section      S - Strong axis of the rolled section

c) λ - Slenderness parameter.      d) P<sub>0</sub> - Pre-load      e) P<sub>u2</sub> - Load carrying capacity of the I-section (predicted using the SSRC curve 2)

f) P<sub>rc2</sub> - Load carrying capacity obtained from the finite element analysis      P<sub>ry</sub> - Yield strength of the reinforced column

g) P<sub>r1</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 1)

h) P<sub>r2</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 2)

i) P<sub>rc1</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 1)

j) P<sub>rc2</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 2)

**Table B.1 (cont'd)**

FEA model	I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	$\lambda^c$	Area (mm <sup>2</sup> )	Yield Strength		Preload		P <sub>0</sub> <sup>d</sup>	P <sub>u2</sub> <sup>e</sup>	P <sub>fea</sub> <sup>f</sup>	P <sub>ri</sub> <sup>g</sup>	P <sub>r1</sub> <sup>h</sup>	P <sub>r2</sub> <sup>i</sup>	P <sub>rc1</sub> <sup>j</sup>	P <sub>rc2</sub> <sup>k</sup>
							I-section (MPa)	plate (MPa)	P <sub>0</sub> <sup>d</sup>	P <sub>u2</sub> <sup>e</sup>								
No.	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)			
177	W150x30	130x5	F	W	0.4	5090	230	350	486	0.6	0.95	0.98	0.92	0.99	0.94			
178	W150x30	130x5	F	W	0.4	5090	230	350	324	0.4	0.95	0.98	0.92	0.99	0.94			
179	W150x30	130x5	F	W	1.1	5090	300	300	370	0.6	0.68	0.68	0.54	0.66	0.54			
180	W150x30	130x5	F	W	1.1	5090	300	300	370	0.6	0.64	0.68	0.54	0.66	0.54			
181	W150x30	130x5	F	W	1.1	5090	300	300	370	0.6	0.60	0.68	0.54	0.66	0.54			
182	W150x30	130x5	F	W	1.1	5090	300	300	370	0.6	0.61	0.68	0.54	0.66	0.54			
183	W150x30	130x5	F	W	1.1	5090	300	300	247	0.4	0.62	0.68	0.54	0.66	0.54			
184	W150x30	130x5	F	W	1.1	5090	230	350	308	0.6	0.57	0.68	0.54	0.66	0.54			
185	W150x30	130x5	F	W	1.1	5090	230	350	206	0.4	0.58	0.68	0.54	0.66	0.54			
186	W150x30	130x5	F	W	1.5	5090	300	300	242	0.6	0.37	0.41	0.35	0.42	0.36			
187	W150x30	130x5	F	W	1.5	5090	300	300	161	0.4	0.38	0.41	0.35	0.42	0.36			
188	W150x30	130x5	F	W	1.5	5090	230	350	202	0.6	0.36	0.41	0.35	0.42	0.36			
189	W150x30	130x5	F	W	1.5	5090	230	350	135	0.4	0.37	0.41	0.35	0.42	0.36			
190	W150x30	130x5	F	S	0.4	5090	300	300	621	0.6	0.95	0.98	0.92	0.99	0.94			
191	W150x30	130x5	F	S	0.4	5090	300	300	621	0.6	0.95	0.98	0.92	0.99	0.94			
192	W150x30	130x5	F	S	0.4	5090	300	300	621	0.6	0.95	0.98	0.92	0.99	0.94			

a) D - Orientation of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web

b) B - Buckling axis of the reinforced column      W - Weak axis of the rolled section      S - Strong axis of the rolled section

c)  $\lambda$  - Slenderness parameter.      d) P<sub>0</sub> - Pre-load      e) P<sub>u2</sub> - Load carrying capacity of the I-section (predicted using the SSRC curve 2)

f) P<sub>fea</sub> - Load carrying capacity obtained from the finite element analysis      P<sub>ry</sub> - Yield strength of the reinforced column

g) P<sub>r1</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 1)

h) P<sub>r2</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 2)

i) P<sub>rc1</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 1)

j) P<sub>rc2</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 2)

**Table B.1 (cont'd)**

FEA model	I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	λ <sup>c</sup>	Area (mm <sup>2</sup> )	Yield Strength		Preload		P <sub>rc1</sub> /P <sub>ry</sub> <sup>i</sup>	P <sub>rc2</sub> /P <sub>ry</sub> <sup>j</sup>			
							I-section (MPa)	plate (MPa)	P <sub>0</sub> <sup>d</sup> (kN)	P <sub>u2</sub> <sup>e</sup> (kN)					
No.	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
193	W150x30	130x5	F	S	0.4	5090	300	300	414	0.4	0.95	0.98	0.92	0.99	0.94
194	W150x30	130x5	F	S	0.4	5090	230	350	481	0.6	0.94	0.98	0.92	0.99	0.94
195	W150x30	130x5	F	S	0.4	5090	230	350	321	0.4	0.95	0.98	0.92	0.99	0.94
196	W150x30	130x5	F	S	1.1	5090	300	300	341	0.6	0.71	0.68	0.54	0.66	0.54
197	W150x30	130x5	F	S	1.1	5090	300	300	341	0.6	0.67	0.68	0.54	0.66	0.54
198	W150x30	130x5	F	S	1.1	5090	300	300	341	0.6	0.63	0.68	0.54	0.66	0.54
199	W150x30	130x5	F	S	1.1	5090	300	300	341	0.6	0.65	0.68	0.54	0.66	0.54
200	W150x30	130x5	F	S	1.1	5090	300	300	228	0.4	0.64	0.68	0.54	0.66	0.54
201	W150x30	130x5	F	S	1.1	5090	230	350	285	0.6	0.59	0.68	0.54	0.66	0.54
202	W150x30	130x5	F	S	1.1	5090	230	350	190	0.4	0.61	0.68	0.54	0.66	0.54
203	W150x30	130x5	F	S	1.5	5090	300	300	222	0.6	0.44	0.41	0.35	0.42	0.36
204	W150x30	130x5	F	S	1.5	5090	300	300	222	0.6	0.44	0.41	0.35	0.42	0.36
205	W150x30	130x5	F	S	1.5	5090	300	300	222	0.6	0.44	0.41	0.35	0.42	0.36
206	W150x30	130x5	F	S	1.5	5090	300	300	148	0.4	0.44	0.41	0.35	0.42	0.36
207	W150x30	130x5	F	S	1.5	5090	230	350	186	0.6	0.42	0.41	0.35	0.42	0.36
208	W150x30	130x5	F	S	1.5	5090	230	350	124	0.4	0.43	0.41	0.35	0.42	0.36

- a) D - Orientation of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web
- b) B - Buckling axis of the reinforced column      W - Weak axis of the rolled section      S - Strong axis of the rolled section
- c) λ - Slenderness parameter.      d) P<sub>0</sub> - Pre-load      e) P<sub>u2</sub> - Load carrying capacity of the I-section (predicted using the SSRC curve 2)
- f) P<sub>rc1</sub> - Load carrying capacity obtained from the finite element analysis      P<sub>ry</sub> - Yield strength of the reinforced column
- g) P<sub>r1</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 1)
- h) P<sub>r2</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 2)
- i) P<sub>rc1</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 1)
- j) P<sub>rc2</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 2)

**Table B.1 (cont'd)**

FEA model	I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	$\lambda^c$	Area (mm <sup>2</sup> )	Yield Strength		Preload		P <sub>0</sub> <sup>d</sup> (kN)	P <sub>u2</sub> <sup>e</sup> (kN)	P <sub>rc1</sub> <sup>f</sup> /P <sub>ry</sub> <sup>f</sup>	P <sub>rc2</sub> <sup>g</sup> /P <sub>ry</sub> <sup>g</sup>	P <sub>rc1</sub> <sup>h</sup> /P <sub>ry</sub> <sup>h</sup>	P <sub>rc2</sub> <sup>i</sup> /P <sub>ry</sub> <sup>i</sup>
							I-section (MPa)	plate (MPa)	P <sub>0</sub> <sup>d</sup>	P <sub>u2</sub> <sup>e</sup>						
No.	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	
209	W150x30	175x5	G	W	0.4	5540	300	300	578	0.6	0.96	0.98	0.92	0.99	0.94	
210	W150x30	175x5	G	W	0.4	5540	300	300	578	0.6	0.96	0.98	0.92	0.99	0.94	
211	W150x30	175x5	G	W	0.4	5540	300	300	578	0.6	0.96	0.98	0.92	0.99	0.94	
212	W150x30	175x5	G	W	0.4	5540	300	300	386	0.4	0.97	0.98	0.92	0.99	0.94	
213	W150x30	175x5	G	W	0.4	5540	230	350	453	0.6	0.96	0.98	0.92	0.99	0.94	
214	W150x30	175x5	G	W	0.4	5540	230	350	302	0.4	0.97	0.98	0.92	0.99	0.94	
215	W150x30	175x5	G	W	1.1	5540	300	300	225	0.6	0.72	0.68	0.54	0.66	0.54	
216	W150x30	175x5	G	W	1.1	5540	300	300	225	0.6	0.66	0.68	0.54	0.66	0.54	
217	W150x30	175x5	G	W	1.1	5540	300	300	225	0.6	0.60	0.68	0.54	0.66	0.54	
218	W150x30	175x5	G	W	1.1	5540	300	300	225	0.6	0.61	0.68	0.54	0.66	0.54	
219	W150x30	175x5	G	W	1.1	5540	300	300	150	0.4	0.62	0.68	0.54	0.66	0.54	
220	W150x30	175x5	G	W	1.1	5540	230	350	193	0.6	0.63	0.68	0.54	0.66	0.54	
221	W150x30	175x5	G	W	1.1	5540	230	350	128	0.4	0.65	0.68	0.54	0.66	0.54	
222	W150x30	175x5	G	W	1.5	5540	300	300	137	0.6	0.38	0.41	0.35	0.42	0.36	
223	W150x30	175x5	G	W	1.5	5540	300	300	137	0.6	0.38	0.41	0.35	0.42	0.36	
224	W150x30	175x5	G	W	1.5	5540	300	300	137	0.6	0.38	0.41	0.35	0.42	0.36	

a) D - Orientation of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web

b) B - Buckling axis of the reinforced column      W - Weak axis of the rolled section      S - Strong axis of the rolled section

c)  $\lambda$  - Slenderness parameter.      d) P<sub>0</sub> - Pre-load      e) P<sub>u2</sub> - Load carrying capacity of the I-section (predicted using the SSRC curve 2)

f) P<sub>rc1</sub> - Load carrying capacity obtained from the finite element analysis      P<sub>ry</sub> - Yield strength of the reinforced column

g) P<sub>r1</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 1)

h) P<sub>r2</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 2)

i) P<sub>rc1</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 1)

j) P<sub>rc2</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 2)

**Table B.1 (cont'd)**

FEA model	I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	$\lambda^c$	Area (mm <sup>2</sup> )	Yield Strength		Preload		P <sub>0</sub> <sup>d</sup> (kN)	P <sub>u2</sub> <sup>e</sup> (MPa)	P <sub>u2</sub> <sup>c</sup> (MPa)	P <sub>red</sub> <sup>f</sup> (kN)	P <sub>r1</sub> /P <sub>ry</sub> <sup>g</sup>	P <sub>r2</sub> /P <sub>ry</sub> <sup>h</sup>	P <sub>ret</sub> /P <sub>ry</sub> <sup>i</sup>	P <sub>rec2</sub> /P <sub>ry</sub> <sup>j</sup>
							I-section (MPa)	plate (MPa)	P <sub>0</sub>	P <sub>u2</sub>								
No.	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	
225	W150x30	175x5	G	W	1.5	5540	300	300	91	0.4	0.39	0.41	0.35	0.42	0.36			
226	W150x30	175x5	G	W	1.5	5540	230	350	122	0.6	0.40	0.41	0.35	0.42	0.36			
227	W150x30	175x5	G	W	1.5	5540	230	350	81	0.4	0.41	0.41	0.35	0.42	0.36			
228	W150x30	175x5	G	S	0.4	5540	300	300	634	0.6	0.97	0.98	0.92	0.99	0.94			
229	W150x30	175x5	G	S	0.4	5540	300	300	634	0.6	0.97	0.98	0.92	0.99	0.94			
230	W150x30	175x5	G	S	0.4	5540	300	300	634	0.6	0.97	0.98	0.92	0.99	0.94			
231	W150x30	175x5	G	S	0.4	5540	300	300	423	0.4	0.97	0.98	0.92	0.99	0.94			
232	W150x30	175x5	G	S	0.4	5540	230	350	491	0.6	0.95	0.98	0.92	0.99	0.94			
233	W150x30	175x5	G	S	0.4	5540	230	350	327	0.4	0.95	0.98	0.92	0.99	0.94			
234	W150x30	175x5	G	S	1.1	5540	300	300	406	0.6	0.65	0.68	0.54	0.66	0.54			
235	W150x30	175x5	G	S	1.1	5540	300	300	406	0.6	0.60	0.68	0.54	0.66	0.54			
236	W150x30	175x5	G	S	1.1	5540	300	300	406	0.6	0.56	0.68	0.54	0.66	0.54			
237	W150x30	175x5	G	S	1.1	5540	300	300	406	0.6	0.58	0.68	0.54	0.66	0.54			
238	W150x30	175x5	G	S	1.1	5540	300	300	270	0.4	0.59	0.68	0.54	0.66	0.54			
239	W150x30	175x5	G	S	1.1	5540	230	350	337	0.6	0.51	0.68	0.54	0.66	0.54			
240	W150x30	175x5	G	S	1.1	5540	230	350	225	0.4	0.53	0.68	0.54	0.66	0.54			

a) D - Orientation of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web

b) B - Buckling axis of the reinforced column      W - Weak axis of the rolled section      S - Strong axis of the rolled section

c)  $\lambda$  - Slenderness parameter.      d) P<sub>0</sub> - Pre-load      e) P<sub>u2</sub> - Load carrying capacity of the I-section (predicted using the SSRC curve 2)

f) P<sub>ret</sub> - Load carrying capacity obtained from the finite element analysis      P<sub>ry</sub> - Yield strength of the reinforced column

g) P<sub>r1</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 1)

h) P<sub>r2</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 2)

i) P<sub>ret</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 1)

j) P<sub>rec2</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 2)

**Table B.1 (cont'd)**

FEA model	I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	λ <sup>c</sup>	Area (mm <sup>2</sup> )	I-section (MPa)	plate (MPa)	Preload		P <sub>0</sub> <sup>d</sup> (kN)	P <sub>0</sub> <sup>d</sup> /P <sub>u2</sub> <sup>c</sup> (11)	P <sub>rc2</sub> <sup>f</sup> /P <sub>ry</sub> <sup>i</sup> (12)	P <sub>r1</sub> <sup>g</sup> /P <sub>ry</sub> <sup>h</sup> (13)	P <sub>r2</sub> <sup>g</sup> /P <sub>ry</sub> <sup>h</sup> (14)	P <sub>rc1</sub> <sup>i</sup> /P <sub>ry</sub> <sup>j</sup> (15)	P <sub>rc2</sub> <sup>j</sup> /P <sub>ry</sub> <sup>k</sup> (16)
									(8)	(9)							
241	W150x30	175x5	G	S	1.5	5540	300	300	266	0.6	0.45	0.41	0.35	0.42	0.36		
242	W150x30	175x5	G	S	1.5	5540	300	300	266	0.6	0.42	0.41	0.35	0.42	0.36		
243	W150x30	175x5	G	S	1.5	5540	300	300	266	0.6	0.39	0.41	0.35	0.42	0.36		
244	W150x30	175x5	G	S	1.5	5540	300	300	177	0.4	0.40	0.41	0.35	0.42	0.36		
245	W150x30	175x5	G	S	1.5	5540	230	350	227	0.6	0.37	0.41	0.35	0.42	0.36		
246	W150x30	175x5	G	S	1.5	5540	230	350	151	0.4	0.38	0.41	0.35	0.42	0.36		
247	W150x30	130x8	F	W	0.4	5870	300	300	627	0.6	0.96	0.98	0.92	0.99	0.94		
248	W150x30	130x8	F	W	0.4	5870	300	300	418	0.4	0.97	0.98	0.92	0.99	0.94		
249	W150x30	130x8	F	W	0.4	5870	230	350	487	0.6	0.95	0.98	0.92	0.99	0.94		
250	W150x30	130x8	F	W	0.4	5870	230	350	325	0.4	0.95	0.98	0.92	0.99	0.94		
251	W150x30	130x8	F	W	1.1	5870	300	300	371	0.6	0.68	0.68	0.54	0.66	0.54		
252	W150x30	130x8	F	W	1.1	5870	300	300	371	0.6	0.64	0.68	0.54	0.66	0.54		
253	W150x30	130x8	F	W	1.1	5870	300	300	371	0.6	0.59	0.68	0.54	0.66	0.54		
254	W150x30	130x8	F	W	1.1	5870	300	300	371	0.6	0.59	0.68	0.54	0.66	0.54		
255	W150x30	130x8	F	W	1.1	5870	300	300	247	0.4	0.61	0.68	0.54	0.66	0.54		
256	W150x30	130x8	F	W	1.1	5870	230	350	319	0.6	0.55	0.68	0.54	0.66	0.54		

a) D - Orientation of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web

b) B - Buckling axis of the reinforced column      W - Weak axis of the rolled section      S - Strong axis of the rolled section

c) λ - Slenderness parameter.      d) P<sub>0</sub> - Pre-load      e) P<sub>u2</sub> - Load carrying capacity of the I-section (predicted using the SSRC curve 2)

f) P<sub>rc1</sub> - Load carrying capacity obtained from the finite element analysis      P<sub>ry</sub> - Yield strength of the reinforced column

g) P<sub>r1</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 1)

h) P<sub>r2</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 2)

i) P<sub>rc1</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 1)

j) P<sub>rc2</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 2)



**Table B.1 (cont'd)**

FEA model	I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	λ <sup>c</sup>	Area (mm <sup>2</sup> )	Yield Strength		Preload		P <sub>0</sub> <sup>d</sup> (kN)	P <sub>0</sub> /P <sub>u2</sub> <sup>e</sup> (11)	P <sub>rc1</sub> /P <sub>ry</sub> <sup>f</sup> (12)	P <sub>rc1</sub> /P <sub>ry</sub> <sup>g</sup> (13)	P <sub>rc2</sub> /P <sub>ry</sub> <sup>h</sup> (14)	P <sub>rc2</sub> /P <sub>ry</sub> <sup>i</sup> (15)	P <sub>rc2</sub> /P <sub>ry</sub> <sup>j</sup> (16)
							I-section (MPa)	plate (MPa)	P <sub>u2</sub> <sup>e</sup> (kN)	P <sub>rc1</sub> <sup>f</sup> (kN)							
No.	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)		
257	W150x30	130x8	F	W	1.1	5870	230	350	212	0.4	0.57	0.68	0.54	0.66	0.54		
258	W150x30	130x8	F	W	1.5	5870	300	300	242	0.6	0.37	0.41	0.35	0.42	0.36		
259	W150x30	130x8	F	W	1.5	5870	300	300	162	0.4	0.38	0.41	0.35	0.42	0.36		
260	W150x30	130x8	F	W	1.5	5870	230	350	209	0.6	0.35	0.41	0.35	0.42	0.36		
261	W150x30	130x8	F	W	1.5	5870	230	350	140	0.4	0.36	0.41	0.35	0.42	0.36		
262	W150x30	130x8	F	S	0.4	5870	300	300	617	0.6	0.94	0.98	0.92	0.99	0.94		
263	W150x30	130x8	F	S	0.4	5870	300	300	412	0.4	0.95	0.98	0.92	0.99	0.94		
264	W150x30	130x8	F	S	0.4	5870	230	350	481	0.6	0.95	0.98	0.92	0.99	0.94		
265	W150x30	130x8	F	S	0.4	5870	230	350	320	0.4	0.95	0.98	0.92	0.99	0.94		
266	W150x30	130x8	F	S	1.1	5870	300	300	329	0.6	0.72	0.68	0.54	0.66	0.54		
267	W150x30	130x8	F	S	1.1	5870	300	300	329	0.6	0.67	0.68	0.54	0.66	0.54		
268	W150x30	130x8	F	S	1.1	5870	300	300	329	0.6	0.63	0.68	0.54	0.66	0.54		
269	W150x30	130x8	F	S	1.1	5870	300	300	329	0.6	0.65	0.68	0.54	0.66	0.54		
270	W150x30	130x8	F	S	1.1	5870	300	300	219	0.4	0.65	0.68	0.54	0.66	0.54		
271	W150x30	130x8	F	S	1.1	5870	230	350	282	0.6	0.60	0.68	0.54	0.66	0.54		
272	W150x30	130x8	F	S	1.1	5870	230	350	188	0.4	0.62	0.68	0.54	0.66	0.54		

a) D - Orientation of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web

b) B - Buckling axis of the reinforced column      W - Weak axis of the rolled section      S - Strong axis of the rolled section

c) λ - Slenderness parameter.      d) P<sub>0</sub> - Pre-load      e) P<sub>u2</sub> - Load carrying capacity of the I-section (predicted using the SSRC curve 2)

f) P<sub>rc1</sub> - Load carrying capacity obtained from the finite element analysis      P<sub>ry</sub> - Yield strength of the reinforced column

g) P<sub>r1</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 1)

h) P<sub>r2</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 2)

i) P<sub>rc1</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 1)

j) P<sub>rc2</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 2)

**Table B.1 (cont'd)**

FEA model	I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	λ <sup>c</sup>	Area (mm <sup>2</sup> )	Yield Strength		Preload		P <sub>0</sub> <sup>d</sup> (kN)	P <sub>u2</sub> <sup>e</sup> (MPa)	P <sub>rc1</sub> <sup>f</sup> /P <sub>ry</sub> <sup>g</sup>	P <sub>rc2</sub> <sup>h</sup> /P <sub>ry</sub> <sup>g</sup>	P <sub>rc1</sub> <sup>i</sup> /P <sub>ry</sub> <sup>g</sup>	P <sub>rc2</sub> <sup>j</sup> /P <sub>ry</sub> <sup>g</sup>
							I-section (MPa)	plate (MPa)	P <sub>0</sub> <sup>d</sup> (kN)	P <sub>u2</sub> <sup>e</sup> (MPa)						
No.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
273	W150x30	130x8	F	S	1.5	5870	300	300	300	213	0.6	0.44	0.41	0.35	0.42	0.36
274	W150x30	130x8	F	S	1.5	5870	300	300	300	142	0.4	0.45	0.41	0.35	0.42	0.36
275	W150x30	130x8	F	S	1.5	5870	230	350	350	184	0.6	0.43	0.41	0.35	0.42	0.36
276	W150x30	130x8	F	S	1.5	5870	230	350	350	123	0.4	0.44	0.41	0.35	0.42	0.36
277	W150x30	175x8	G	W	0.4	6590	300	300	300	560	0.6	0.96	0.98	0.92	0.99	0.94
278	W150x30	175x8	G	W	0.4	6590	300	300	300	373	0.4	0.96	0.98	0.92	0.99	0.94
279	W150x30	175x8	G	W	0.4	6590	230	350	350	444	0.6	0.96	0.98	0.92	0.99	0.94
280	W150x30	175x8	G	W	0.4	6590	230	350	350	296	0.4	0.96	0.98	0.92	0.99	0.94
281	W150x30	175x8	G	W	1.1	6590	300	300	300	196	0.6	0.72	0.68	0.54	0.66	0.54
282	W150x30	175x8	G	W	1.1	6590	300	300	300	196	0.6	0.65	0.68	0.54	0.66	0.54
283	W150x30	175x8	G	W	1.1	6590	300	300	300	196	0.6	0.59	0.68	0.54	0.66	0.54
284	W150x30	175x8	G	W	1.1	6590	300	300	300	196	0.6	0.60	0.68	0.54	0.66	0.54
285	W150x30	175x8	G	W	1.1	6590	300	300	300	131	0.4	0.61	0.68	0.54	0.66	0.54
286	W150x30	175x8	G	W	1.1	6590	230	350	350	174	0.6	0.62	0.68	0.54	0.66	0.54
287	W150x30	175x8	G	W	1.1	6590	230	350	350	116	0.4	0.64	0.68	0.54	0.66	0.54
288	W150x30	175x8	G	W	1.5	6590	300	300	300	114	0.6	0.38	0.41	0.35	0.42	0.36

a) D - Orientation of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web

b) B - Buckling axis of the reinforced column      W - Weak axis of the rolled section      S - Strong axis of the rolled section

c) λ - Slenderness parameter.      d) P<sub>0</sub> - Pre-load      e) P<sub>u2</sub> - Load carrying capacity of the I-section (predicted using the SSRC curve 2)

f) P<sub>rc1</sub> - Load carrying capacity obtained from the finite element analysis      P<sub>ry</sub> - Yield strength of the reinforced column

g) P<sub>r1</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 1)

h) P<sub>r2</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 2)

i) P<sub>rc1</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 1)

j) P<sub>rc2</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 2)

**Table B.1 (cont'd)**

FEA model	I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	λ <sup>c</sup>	Area (mm <sup>2</sup> )	Yield Strength			Preload			P <sub>rc1</sub> /P <sub>y</sub> <sup>i</sup>	P <sub>rc2</sub> /P <sub>y</sub> <sup>j</sup>	
							I-section (MPa)	plate (MPa)	P <sub>0</sub> <sup>d</sup> (kN)	P <sub>0</sub> /P <sub>u2</sub> <sup>e</sup>	P <sub>rcd</sub> /P <sub>y</sub> <sup>f</sup>	P <sub>rc1</sub> /P <sub>y</sub> <sup>g</sup>			P <sub>rc2</sub> /P <sub>y</sub> <sup>h</sup>
No.	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
289	W150x30	175x8	G	W	1.5	6590	300	300	76	0.4	0.39	0.41	0.35	0.42	0.36
290	W150x30	175x8	G	W	1.5	6590	230	350	106	0.6	0.40	0.41	0.35	0.42	0.36
291	W150x30	175x8	G	W	1.5	6590	230	350	71	0.4	0.41	0.41	0.35	0.42	0.36
292	W150x30	175x8	G	S	0.4	6590	300	300	636	0.6	0.98	0.98	0.92	0.99	0.94
293	W150x30	175x8	G	S	0.4	6590	300	300	424	0.4	0.98	0.98	0.92	0.99	0.94
294	W150x30	175x8	G	S	0.4	6590	230	350	494	0.6	0.97	0.98	0.92	0.99	0.94
295	W150x30	175x8	G	S	0.4	6590	230	350	330	0.4	0.97	0.98	0.92	0.99	0.94
296	W150x30	175x8	G	S	1.1	6590	300	300	419	0.6	0.62	0.68	0.54	0.66	0.54
297	W150x30	175x8	G	S	1.1	6590	300	300	419	0.6	0.58	0.68	0.54	0.66	0.54
298	W150x30	175x8	G	S	1.1	6590	300	300	419	0.6	0.55	0.68	0.54	0.66	0.54
299	W150x30	175x8	G	S	1.1	6590	300	300	419	0.6	0.57	0.68	0.54	0.66	0.54
300	W150x30	175x8	G	S	1.1	6590	300	300	280	0.4	0.58	0.68	0.54	0.66	0.54
301	W150x30	175x8	G	S	1.1	6590	230	350	352	0.6	0.49	0.68	0.54	0.66	0.54
302	W150x30	175x8	G	S	1.1	6590	230	350	235	0.4	0.52	0.68	0.54	0.66	0.54
303	W150x30	175x8	G	S	1.5	6590	300	300	277	0.6	0.38	0.41	0.35	0.42	0.36
304	W150x30	175x8	G	S	1.5	6590	300	300	184	0.4	0.39	0.41	0.35	0.42	0.36

a) D - Orientation of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web

b) B - Buckling axis of the reinforced column      W - Weak axis of the rolled section      S - Strong axis of the rolled section

c) λ - Slenderness parameter.      d) P<sub>0</sub> - Pre-load      e) P<sub>u2</sub> - Load carrying capacity of the I-section (predicted using the SSRC curve 2)

f) P<sub>rcd</sub> - Load carrying capacity obtained from the finite element analysis      P<sub>y</sub> - Yield strength of the reinforced column

g) P<sub>rc1</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 1)

h) P<sub>rc2</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 2)

i) P<sub>rc1</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 1)

j) P<sub>rc2</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 2)

**Table B.1 (cont'd)**

FEA model	I-section	Plate	D <sup>a</sup>	B <sup>b</sup>	λ <sup>c</sup>	Area (mm <sup>2</sup> )	Yield Strength			Preload			P <sub>rc2</sub> /P <sub>ry</sub> <sup>j</sup>		
							I-section (MPa)	plate (MPa)	P <sub>0</sub> <sup>d</sup> (kN)	P <sub>0</sub> /P <sub>u2</sub> <sup>e</sup>	P <sub>rcd</sub> /P <sub>ry</sub> <sup>f</sup>	P <sub>r1</sub> /P <sub>ry</sub> <sup>g</sup>		P <sub>r2</sub> /P <sub>ry</sub> <sup>h</sup>	P <sub>rc1</sub> /P <sub>ry</sub> <sup>i</sup>
No.	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
305	W150x30	175x8	G	S	1.5	6590	230	350	244	0.6	0.35	0.41	0.35	0.42	0.36
306	W150x30	175x8	G	S	1.5	6590	230	350	162	0.4	0.37	0.41	0.35	0.42	0.36
307	W310x179	350x16	G	W	0.4	34000	300	300	3468	0.6	0.97	0.98	0.92	0.99	0.94
308	W310x179	350x16	G	W	1.1	34000	300	300	1330	0.6	0.62	0.68	0.54	0.66	0.54
309	W310x179	350x16	G	S	1.1	34000	300	300	2480	0.6	0.59	0.68	0.54	0.66	0.54
310	W310x179	350x16	F	S	1.5	34000	300	300	1276	0.6	0.41	0.41	0.35	0.42	0.36
311	W150x30	130x5	F	S	1.1	5090	300	300	341	0.6	0.63	0.68	0.54	0.66	0.54
312	W150x30	130x5	F	S	1.5	5090	300	300	222	0.6	0.44	0.41	0.35	0.42	0.36
313	W310x179	350x25	G	W	0.4	40300	300	300	3353	0.6	0.96	0.98	0.92	0.99	0.94
314	W310x179	350x25	G	W	1.1	40300	300	300	1151	0.6	0.63	0.68	0.54	0.66	0.54
315	W310x179	350x25	G	W	1.5	40300	300	300	668	0.6	0.41	0.41	0.35	0.42	0.36
316	W310x179	350x16	G	W	1.1	34000	300	300	1330	0.6	0.68	0.68	0.54	0.66	0.54
317	W310x179	350x16	G	W	1.5	34000	300	300	805	0.6	0.42	0.41	0.35	0.99	0.36

a) D - Orientation of reinforcing plates      F - Parallel to the flanges      G - Parallel to the web

b) B - Buckling axis of the reinforced column      W - Weak axis of the rolled section      S - Strong axis of the rolled section

c) λ - Slenderness parameter.      d) P<sub>0</sub> - Pre-load      e) P<sub>u2</sub> - Load carrying capacity of the I-section (predicted using the SSRC curve 2)

f) P<sub>rcd</sub> - Load carrying capacity obtained from the finite element analysis      P<sub>ry</sub> - Yield strength of the reinforced column

g) P<sub>r1</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 1)

h) P<sub>r2</sub> - Load carrying capacity of the reinforced column (predicted using the SSRC curve 2)

i) P<sub>rc1</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 1)

j) P<sub>rc2</sub> - Load carrying capacity of the reinforced column (predicted using the CSA curve 2)

## **Appendix C**

### **Statistical Analysis Data for the Professional Factors for the Columns from Group 2**

## **Statistical Analysis Data for the Professional Factors for the Columns from Group 2**

This appendix serves as a supplement to Chapter 5. It presents the statistical analysis data used to obtain the professional factors for the columns from group 2 (columns reinforced with plates parallel to the flanges and buckling about the weak axis of the rolled section and columns reinforced with plates parallel to the web and buckling about the strong axis of the rolled section). The statistical analysis procedures for the columns from group 2 are same as those from group 1 presented in Chapter 5.

Tables C.1 to C.3 present the analysis data for the simulated professional factors for columns from group 2 for values of the slenderness ratio,  $\lambda$ , of 0.4, 1.1 and 1.5 respectively. Tables C.4 to C.6 present the analysis data for the normalized professional factors for columns from group 2 for values of the slenderness ratio,  $\lambda$ , of 0.4, 1.1 and 1.5 respectively.

Plots of the simulated professional ratio,  $\rho_s$ , versus out-of-straightness for columns from group 2 for values of the slenderness ratio,  $\lambda$ , of 0.4, 1.1 and 1.5 are presented in Figures C.1, C.2 and C.3 respectively. The normalized professional ratio for  $l = 0.4, 1.1,$  and 1.5 are plotted in Figures C.4, C.5, and C.6 respectively.

**Table C.1 Simulated Professional Factors for Columns form Group 2 ( $\lambda = 0.4$ )**

FEA model No.	D	B	Out-of-Straightness $\delta_0$	$P_{fea}/P_{ry}$	SSRC 1 $\rho_s (P_{fea}/P_{r1})$	SSRC 2 $\rho_s (P_{fea}/P_{r2})$	CSA 1 $\rho_s (P_{fea}/P_{rc1})$	CSA 2 $\rho_s (P_{fea}/P_{rc2})$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
17	F	W	L/8000	1.005	1.025	1.094	1.012	1.069
18	F	W	L/2000	0.987	1.007	1.074	0.994	1.049
19	F	W	L/1000	0.973	0.993	1.059	0.980	1.035
20	F	W	L/1000	0.978	0.997	1.064	0.985	1.040
21	F	W	L/1000	0.958	0.978	1.043	0.965	1.019
22	F	W	L/1000	0.961	0.981	1.047	0.968	1.022
66	G	S	L/1000	0.934	0.953	1.017	0.941	0.993
67	G	S	L/1000	0.940	0.959	1.023	0.947	1.000
68	G	S	L/1000	0.911	0.930	0.992	0.918	0.969
69	G	S	L/1000	0.917	0.935	0.998	0.923	0.975
84	F	W	L/1000	0.977	0.996	1.063	0.984	1.038
85	F	W	L/1000	0.980	1.000	1.067	0.987	1.042
86	F	W	L/1000	0.952	0.971	1.036	0.959	1.012
87	F	W	L/1000	0.956	0.975	1.040	0.963	1.016
115	F	W	L/1000	0.980	1.000	1.067	0.987	1.042
116	F	W	L/1000	0.983	1.003	1.070	0.990	1.045
117	F	W	L/1000	0.963	0.983	1.049	0.971	1.025
118	F	W	L/1000	0.966	0.986	1.051	0.973	1.027
160	G	S	L/1000	0.929	0.948	1.011	0.936	0.988
161	G	S	L/1000	0.937	0.956	1.020	0.944	0.996
162	G	S	L/1000	0.894	0.912	0.973	0.900	0.950
163	G	S	L/1000	0.902	0.920	0.981	0.908	0.959
175	F	W	L/1000	0.972	0.992	1.058	0.979	1.033
176	F	W	L/1000	0.978	0.998	1.065	0.986	1.040
177	F	W	L/1000	0.947	0.966	1.031	0.954	1.007
178	F	W	L/1000	0.952	0.971	1.036	0.959	1.012
228	G	S	L/8000	0.981	1.001	1.068	0.989	1.044
229	G	S	L/2000	0.981	1.000	1.067	0.988	1.043
230	G	S	L/1000	0.968	0.987	1.053	0.975	1.029
231	G	S	L/1000	0.972	0.992	1.058	0.980	1.034
232	G	S	L/1000	0.951	0.970	1.035	0.958	1.011
233	G	S	L/1000	0.955	0.974	1.039	0.962	1.015

Note:  $\Delta_0$  - Initial imperfection

$P_{ry}$  - Yield strength of reinforced column

$P_{fea}$  - Finite elemetn analysis after reinforcing

$P_{r1}$  - Capacity after reinforcing (SSRC1)

$P_{r2}$  - Capacity after reinforcing (SSRC2)

$P_{rc1}$  - Capacity after reinforcing (CSA1)

$P_{rc2}$  - Capacity after reinforcing (CSA2)

L - Column length

D - Orientation of reinforcing plates

F - Parallel to the flanges

G - Parallel to the web

B - Buckling axis

W - Weak axis of the rolled section

S - Strong axis of the rolled section

**Table C.1 (Cont'd)**

FEA model No.	D	B	Out-of-Straightness $\delta_0$	$P_{fea}/P_{ry}$	SSRC 1 $\rho_s$ ( $P_{fea}/P_{r1}$ )	SSRC 2 $\rho_s$ ( $P_{fea}/P_{r2}$ )	CSA 1 $\rho_s$ ( $P_{fea}/P_{rc1}$ )	CSA 2 $\rho_s$ ( $P_{fea}/P_{rc2}$ )
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
247	F	W	L/1000	0.965	0.984	1.050	0.972	1.026
248	F	W	L/1000	0.972	0.992	1.058	0.979	1.034
249	F	W	L/1000	0.949	0.968	1.033	0.956	1.009
250	F	W	L/1000	0.954	0.973	1.038	0.961	1.014
292	G	S	L/1000	0.977	0.997	1.063	0.984	1.039
293	G	S	L/1000	0.979	0.999	1.066	0.986	1.041
294	G	S	L/1000	0.970	0.990	1.056	0.978	1.032
295	G	S	L/1000	0.970	0.990	1.056	0.977	1.031

Note:  $\Delta_0$  - Initial imperfection

L - Column length

 $P_{ry}$  - Yield strength of reinforced column

D - Orientation of reinforcing plates

 $P_{fea}$  - Finite element analysis after reinforcing

F - Parallel to the flanges

 $P_{r1}$  - Capacity after reinforcing (SSRC1)

G - Parallel to the web

 $P_{r2}$  - Capacity after reinforcing (SSRC2)

B - Buckling axis

 $P_{rc1}$  - Capacity after reinforcing (CSA1)

W - Weak axis of the rolled section

 $P_{rc2}$  - Capacity after reinforcing (CSA2)

S - Strong axis of the rolled section

**Table C.2 Simulated Professional Factors for Columns from Group 2 ( $\lambda = 1.1$ )**

FEA model No.	D	B	Out-of-Straightness $\delta_0$	$P_{fea}/P_{ry}$	SSRC 1 $\rho_s$ ( $P_{fea}/P_{r1}$ )	SSRC 2 $\rho_s$ ( $P_{fea}/P_{r2}$ )	CSA 1 $\rho_s$ ( $P_{fea}/P_{rc1}$ )	CSA 2 $\rho_s$ ( $P_{fea}/P_{rc2}$ )
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
23	F	W	L/8000	0.641	0.942	1.189	0.971	1.190
24	F	W	L/2000	0.601	0.883	1.114	0.909	1.115
25	F	W	L/1000	0.556	0.818	1.032	0.842	1.033
26	F	W	L/1000	0.560	0.823	1.038	0.848	1.039
27	F	W	L/1000	0.572	0.841	1.062	0.867	1.062
28	F	W	L/1000	0.511	0.751	0.947	0.773	0.948
29	F	W	L/1000	0.538	0.791	0.998	0.814	0.999
70	G	S	L/8000	0.603	0.887	1.119	0.914	1.120

Note:  $\Delta_0$  - Initial imperfection

L - Column length

 $P_{ry}$  - Yield strength of reinforced column

D - Orientation of reinforcing plates

 $P_{fea}$  - Finite element analysis after reinforcing

F - Parallel to the flanges

 $P_{r1}$  - Capacity after reinforcing (SSRC1)

G - Parallel to the web

 $P_{r2}$  - Capacity after reinforcing (SSRC2)

B - Buckling axis

 $P_{rc1}$  - Capacity after reinforcing (CSA1)

W - Weak axis of the rolled section

 $P_{rc2}$  - Capacity after reinforcing (CSA2)

S - Strong axis of the rolled section



**Table C.2 (Cont'd)**

FEA model No.	D	B	Out-of-Straightness $\delta_0$	$P_{fea}/P_{ry}$	SSRC 1 $\rho_s (P_{fea}/P_{r1})$	SSRC 2 $\rho_s (P_{fea}/P_{r2})$	CSA 1 $\rho_s (P_{fea}/P_{rc1})$	CSA 2 $\rho_s (P_{fea}/P_{rc2})$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
71	G	S	L/2000	0.575	0.845	1.067	0.871	1.067
72	G	S	L/1100	0.541	0.796	1.004	0.819	1.005
73	G	S	L/1100	0.541	0.795	1.003	0.818	1.004
74	G	S	L/1100	0.573	0.842	1.062	0.867	1.063
75	G	S	L/1150	0.478	0.702	0.886	0.723	0.887
76	G	S	L/1150	0.506	0.743	0.938	0.765	0.938
81	G	S	L/1100	0.544	0.799	1.009	0.823	1.009
82	G	S	L/1100	0.537	0.789	0.996	0.813	0.996
83	G	S	L/1100	0.540	0.794	1.002	0.818	1.003
88	F	W	L/8000	0.643	0.946	1.193	0.974	1.194
89	F	W	L/2000	0.603	0.886	1.118	0.912	1.119
90	F	W	L/1000	0.564	0.829	1.045	0.853	1.046
91	F	W	L/1000	0.569	0.837	1.056	0.862	1.057
92	F	W	L/1000	0.577	0.849	1.071	0.874	1.072
93	F	W	L/1000	0.528	0.776	0.979	0.799	0.980
94	F	W	L/1000	0.544	0.800	1.009	0.823	1.010
119	F	W	L/8000	0.652	0.959	1.210	0.988	1.211
120	F	W	L/2000	0.607	0.893	1.126	0.919	1.127
121	F	W	L/1000	0.564	0.830	1.047	0.855	1.048
122	F	W	L/1000	0.571	0.839	1.058	0.864	1.059
123	F	W	L/1000	0.579	0.852	1.074	0.877	1.075
124	F	W	L/1000	0.538	0.791	0.998	0.815	0.999
125	F	W	L/1000	0.554	0.815	1.028	0.839	1.029
164	G	S	L/8000	0.641	0.942	1.189	0.970	1.190
165	G	S	L/2000	0.598	0.879	1.108	0.905	1.109
166	G	S	L/1150	0.564	0.829	1.045	0.853	1.046
167	G	S	L/1150	0.574	0.845	1.066	0.870	1.066
168	G	S	L/1150	0.589	0.865	1.092	0.891	1.093
169	G	S	L/2750	0.507	0.745	0.941	0.768	0.941
170	G	S	L/1200	0.534	0.785	0.990	0.808	0.991
179	F	W	L/8000	0.681	1.001	1.263	1.031	1.264
180	F	W	L/2000	0.642	0.944	1.191	0.972	1.191

Note:  $\Delta_0$  - Initial imperfection

$P_{ry}$  - Yield strength of reinforced column

$P_{fea}$  - Finite element analysis after reinforcing

$P_{r1}$  - Capacity after reinforcing (SSRC1)

$P_{r2}$  - Capacity after reinforcing (SSRC2)

$P_{rc1}$  - Capacity after reinforcing (CSA1)

$P_{rc2}$  - Capacity after reinforcing (CSA2)

L - Column length

D - Orientation of reinforcing plates

F - Parallel to the flanges

G - Parallel to the web

B - Buckling axis

W - Weak axis of the rolled section

S - Strong axis of the rolled section

**Table 5.6 (Cont'd)**

FEA model No.	D	B	Out-of-Straightness $\delta_0$	$P_{fea}/P_{ry}$	SSRC 1 $\rho_s$ ( $P_{fea}/P_{r1}$ )	SSRC 2 $\rho_s$ ( $P_{fea}/P_{r2}$ )	CSA 1 $\rho_s$ ( $P_{fea}/P_{rc1}$ )	CSA 2 $\rho_s$ ( $P_{fea}/P_{rc2}$ )
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
181	F	W	L/1000	0.605	0.889	1.122	0.916	1.123
182	F	W	L/1000	0.608	0.894	1.128	0.921	1.129
183	F	W	L/1000	0.616	0.906	1.143	0.933	1.143
184	F	W	L/1000	0.572	0.841	1.062	0.867	1.062
185	F	W	L/1000	0.585	0.860	1.085	0.885	1.085
234	G	S	L/8000	0.648	0.953	1.202	0.981	1.203
235	G	S	L/2000	0.604	0.887	1.120	0.914	1.120
236	G	S	L/1000	0.564	0.829	1.046	0.854	1.047
237	G	S	L/1000	0.581	0.854	1.077	0.879	1.078
238	G	S	L/1000	0.590	0.868	1.095	0.894	1.096
239	G	S	L/1000	0.511	0.751	0.947	0.773	0.948
240	G	S	L/1000	0.535	0.787	0.992	0.810	0.993
251	F	W	L/8000	0.677	0.996	1.257	1.026	1.258
252	F	W	L/2000	0.637	0.937	1.182	0.965	1.183
253	F	W	L/1000	0.594	0.873	1.102	0.899	1.103
254	F	W	L/1000	0.593	0.873	1.101	0.899	1.102
255	F	W	L/1000	0.608	0.893	1.127	0.920	1.128
256	F	W	L/1000	0.547	0.804	1.015	0.828	1.015
257	F	W	L/1000	0.566	0.832	1.049	0.857	1.050
296	G	S	L/8000	0.624	0.918	1.158	0.945	1.159
297	G	S	L/2000	0.585	0.860	1.085	0.885	1.086
298	G	S	L/1000	0.551	0.811	1.023	0.835	1.024
299	G	S	L/1000	0.566	0.832	1.049	0.856	1.050
300	G	S	L/1000	0.578	0.849	1.072	0.875	1.072
301	G	S	L/1000	0.494	0.726	0.916	0.747	0.916
302	G	S	L/1000	0.520	0.764	0.964	0.787	0.964

Note:  $\Delta_0$  - Initial imperfection  
 $P_{ry}$  - Yield strength of reinforced column  
 $P_{fea}$  - Finite element analysis after reinforcing  
 $P_{r1}$  - Capacity after reinforcing (SSRC1)  
 $P_{r2}$  - Capacity after reinforcing (SSRC2)  
 $P_{rc1}$  - Capacity after reinforcing (CSA1)  
 $P_{rc2}$  - Capacity after reinforcing (CSA2)  
L - Column length  
D - Orientation of reinforcing plates  
F - Parallel to the flanges  
G - Parallel to the web  
B - Buckling axis  
W - Weak axis of the rolled section  
S - Strong axis of the rolled section

**Table C.3 Simulated Professional Factors for Columns from Group 2 ( $\lambda = 1.5$ )**

FEA model No.	D	B	Out-of-Straightness $\delta_0$	$P_{fea}/P_{ry}$	SSRC 1 $\rho_s$ ( $P_{fea}/P_{r1}$ )	SSRC 2 $\rho_s$ ( $P_{fea}/P_{r2}$ )	CSA 1 $\rho_s$ ( $P_{fea}/P_{rc1}$ )	CSA 2 $\rho_s$ ( $P_{fea}/P_{rc2}$ )
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
30	F	W	L/8000	0.415	1.020	1.181	0.999	1.161
31	F	W	L/2000	0.379	0.930	1.076	0.911	1.058
32	F	W	L/1000	0.355	0.871	1.008	0.853	0.991
33	F	W	L/1000	0.362	0.889	1.029	0.870	1.011
34	F	W	L/1000	0.338	0.830	0.961	0.813	0.945
35	F	W	L/1000	0.351	0.862	0.997	0.844	0.980
77	G	S	L/1350	0.374	0.919	1.064	0.901	1.046
78	G	S	L/1350	0.383	0.942	1.090	0.922	1.071
79	G	S	L/1350	0.349	0.857	0.992	0.839	0.975
80	G	S	L/1350	0.361	0.887	1.027	0.869	1.010
95	F	W	L/1000	0.357	0.878	1.016	0.860	0.998
96	F	W	L/1000	0.360	0.885	1.024	0.866	1.006
97	F	W	L/1000	0.365	0.897	1.038	0.879	1.020
98	F	W	L/1050	0.344	0.846	0.980	0.829	0.963
99	F	W	L/1050	0.351	0.862	0.997	0.844	0.980
126	F	W	L/1400	0.356	0.875	1.013	0.857	0.995
127	F	W	L/1050	0.368	0.903	1.045	0.884	1.027
128	F	W	L/1100	0.349	0.856	0.991	0.839	0.974
129	F	W	L/1100	0.361	0.886	1.026	0.868	1.008
171	G	S	L/1350	0.384	0.943	1.091	0.924	1.073
172	G	S	L/1350	0.389	0.956	1.106	0.936	1.087
173	G	S	L/1300	0.363	0.891	1.031	0.873	1.014
174	G	S	L/1300	0.369	0.908	1.050	0.889	1.032
186	F	W	L/1000	0.374	0.918	1.062	0.899	1.044
187	F	W	L/1000	0.379	0.931	1.077	0.911	1.059
188	F	W	L/1000	0.362	0.890	1.030	0.872	1.012
189	F	W	L/1000	0.371	0.912	1.055	0.893	1.037
241	G	S	L/8000	0.452	1.111	1.286	1.088	1.264
242	G	S	L/2000	0.417	1.024	1.185	1.003	1.164
243	G	S	L/1000	0.391	0.961	1.112	0.941	1.093
244	G	S	L/1000	0.400	0.982	1.136	0.962	1.117
245	G	S	L/1000	0.369	0.907	1.050	0.889	1.032

Note:  $\Delta_0$  - Initial imperfection

L - Column length

 $P_{ry}$  - Yield strength of reinforced column

D - Orientation of reinforcing plates

 $P_{fea}$  - Finite element analysis after reinforcing

F - Parallel to the flanges

 $P_{r1}$  - Capacity after reinforcing (SSRC1)

G - Parallel to the web

 $P_{r2}$  - Capacity after reinforcing (SSRC2)

B - Buckling axis

 $P_{rc1}$  - Capacity after reinforcing (CSA1)

W - Weak axis of the rolled section

 $P_{rc2}$  - Capacity after reinforcing (CSA2)

S - Strong axis of the rolled section

**Table C.3 (Cont'd)**

FEA model No.	D	B	Out-of-Straightness $\delta_0$	$P_{fea}/P_{ry}$	SSRC 1 $\rho_s$ ( $P_{fea}/P_{r1}$ )	SSRC 2 $\rho_s$ ( $P_{fea}/P_{r2}$ )	CSA 1 $\rho_s$ ( $P_{fea}/P_{rc1}$ )	CSA 2 $\rho_s$ ( $P_{fea}/P_{rc2}$ )
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
246	G	S	L/1000	0.379	0.930	1.077	0.911	1.058
258	F	W	L/1000	0.368	0.904	1.046	0.886	1.029
259	F	W	L/1000	0.377	0.925	1.071	0.906	1.052
260	F	W	L/1000	0.353	0.868	1.004	0.850	0.987
261	F	W	L/1000	0.361	0.888	1.027	0.870	1.010
303	G	S	L/1000	0.381	0.937	1.085	0.918	1.066
304	G	S	L/1000	0.393	0.966	1.118	0.946	1.099
305	G	S	L/1000	0.352	0.864	1.000	0.846	0.983
306	G	S	L/1000	0.365	0.897	1.038	0.879	1.020

Note:  $\Delta_0$  - Initial imperfection

$P_{ry}$  - Yield strength of reinforced column

$P_{fea}$  - Finite element analysis after reinforcing

$P_{r1}$  - Capacity after reinforcing (SSRC1)

$P_{r2}$  - Capacity after reinforcing (SSRC2)

$P_{rc1}$  - Capacity after reinforcing (CSA1)

$P_{rc2}$  - Capacity after reinforcing (CSA2)

L - Column length

D - Orientation of reinforcing plates

F - Parallel to the flanges

G - Parallel to the web

B - Buckling axis

W - Weak axis of the rolled section

S - Strong axis of the rolled section



**Table C.4 (Cont'd)**

FEA model No.	D	B	Out-of-Straightness $\delta_0$	SSRC 2 $\rho_s$ ( $P_{fea}/P_{r2}$ )	$\rho_s = m \delta_0/L + b$ SSRC 2	$\rho_{seq}$	SSRC 2 $\rho_n$ $\rho_s/\rho_{seq}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
248	F	W	L/1000	1.06	$\rho_s = -49.041x\delta_0/L + 1.09$	1.041	1.017
249	F	W	L/1000	1.03	$\rho_s = -49.041x\delta_0/L + 1.09$	1.041	0.992
250	F	W	L/1000	1.04	$\rho_s = -49.041x\delta_0/L + 1.09$	1.041	0.998
292	G	S	L/1000	1.06	$\rho_s = -49.041x\delta_0/L + 1.09$	1.041	1.021
293	G	S	L/1000	1.07	$\rho_s = -49.041x\delta_0/L + 1.09$	1.041	1.024
294	G	S	L/1000	1.06	$\rho_s = -49.041x\delta_0/L + 1.09$	1.041	1.015
295	G	S	L/1000	1.06	$\rho_s = -49.041x\delta_0/L + 1.09$	1.041	1.014

Note: L - Column length  
 D - Orientation of reinforcing plate  
 F - Parallel to the flanges  
 G - Parallel to the web  
 $P_{fea}$  - Finite element analysis after reinforcing  
 $\rho_{seq}$  - Professional ratio predicted by the equation  
 B - Buckling axis  
 W - Weak axis of the rolled section  
 S - Strong axis of the rolled section  
 $P_{r2}$  - Capacity after reinforcing (SSRC2)

**Table C.5 Normalized Professional Factors for Columns from Group 2 ( $\lambda = 1.1$ )**

FEA model No.	D	B	Out-of-Straightness $\delta_0$	SSRC 2 $\rho_s$ ( $P_{fea}/P_{r2}$ )	$\rho_s = m \delta_0/L + b$ SSRC 2	$\rho_{seq}$	SSRC 2 $\rho_n$ $\rho_s/\rho_{seq}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
70	G	S	L/8000	1.12	$\rho_s = -168.09x\delta_0/L + 1.1992$	1.18	0.95
71	G	S	L/2000	1.07	$\rho_s = -168.09x\delta_0/L + 1.1992$	1.12	0.96
72	G	S	L/1100	1.00	$\rho_s = -168.09x\delta_0/L + 1.1992$	1.05	0.96
73	G	S	L/1100	1.00	$\rho_s = -168.09x\delta_0/L + 1.1992$	1.05	0.96
74	G	S	L/1100	1.06	$\rho_s = -168.09x\delta_0/L + 1.1992$	1.05	1.01
75	G	S	L/1150	0.89	$\rho_s = -168.09x\delta_0/L + 1.1992$	1.05	0.84
76	G	S	L/1150	0.94	$\rho_s = -168.09x\delta_0/L + 1.1992$	1.05	0.89
81	G	S	L/1100	1.01	$\rho_s = -168.09x\delta_0/L + 1.1992$	1.05	0.96
82	G	S	L/1100	1.00	$\rho_s = -168.09x\delta_0/L + 1.1992$	1.05	0.95
83	G	S	L/1100	1.00	$\rho_s = -168.09x\delta_0/L + 1.1992$	1.05	0.96
23	F	W	L/8000	1.19	$\rho_s = -168.09x\delta_0/L + 1.1992$	1.18	1.01
24	F	W	L/2000	1.11	$\rho_s = -168.09x\delta_0/L + 1.1992$	1.12	0.99

Note: L - Column length  
 D - Orientation of reinforcing plate  
 F - Parallel to the flanges  
 G - Parallel to the web  
 $P_{fea}$  - Finite element analysis after reinforcing  
 $\rho_{seq}$  - Professional ratio predicted by the equation  
 B - Buckling axis  
 W - Weak axis of the rolled section  
 S - Strong axis of the rolled section  
 $P_{r2}$  - Capacity after reinforcing (SSRC2)

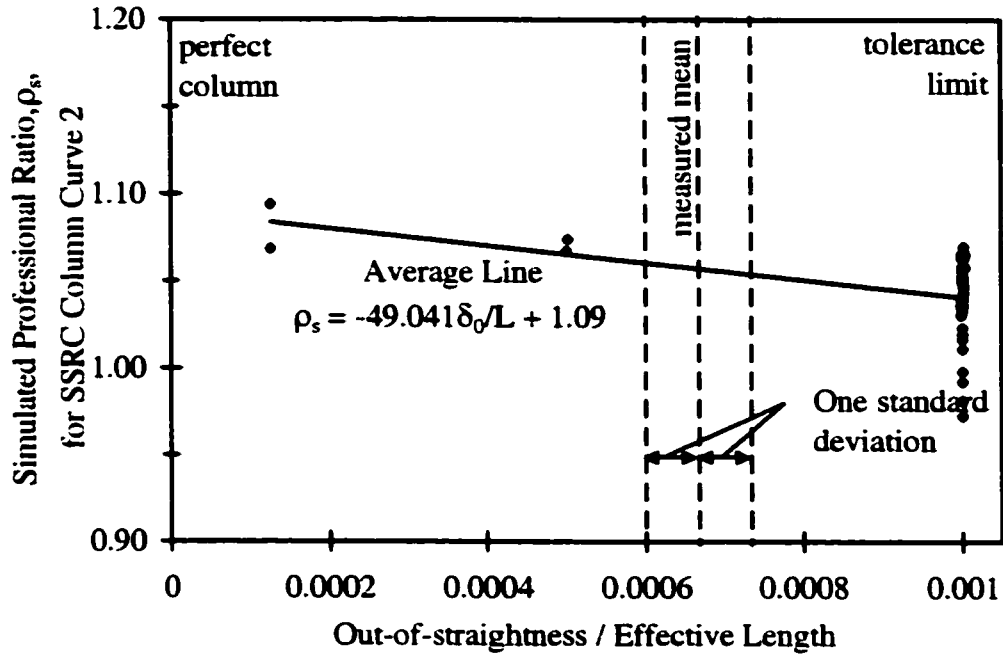




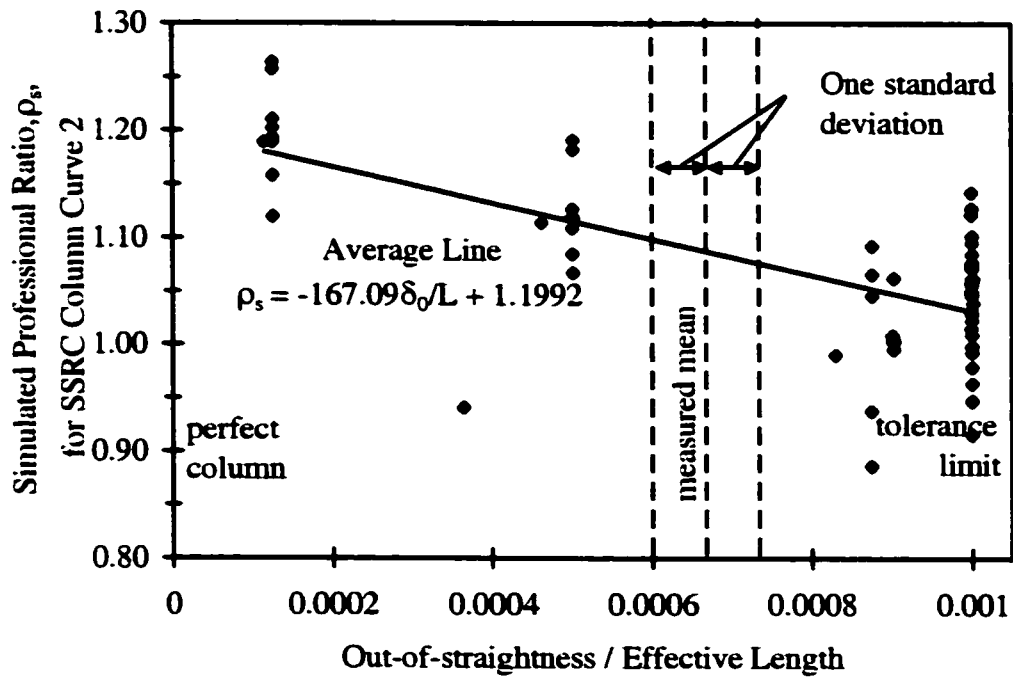




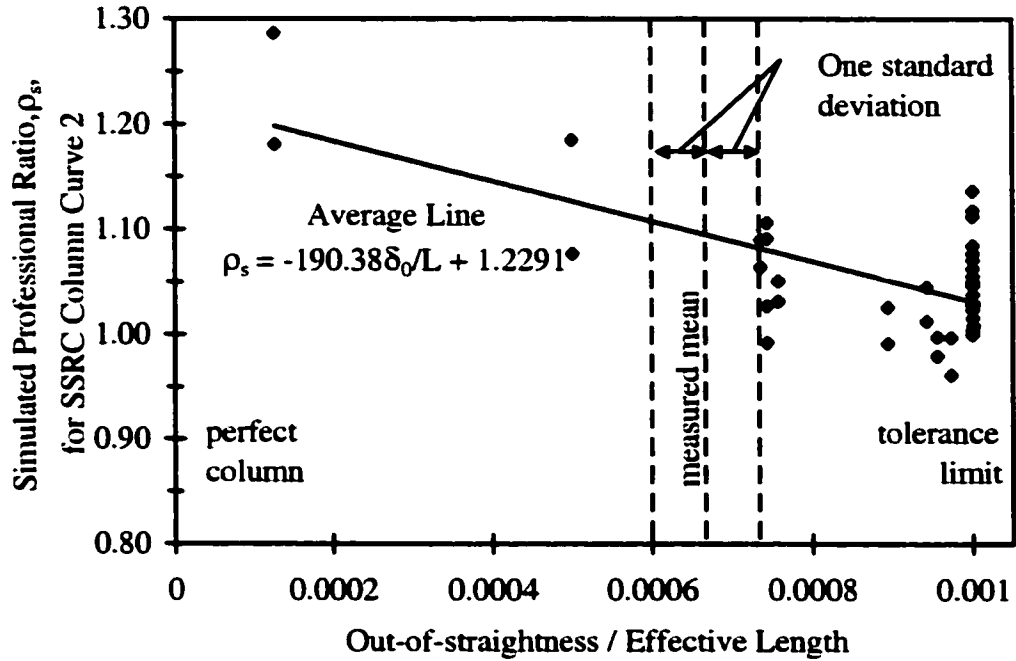




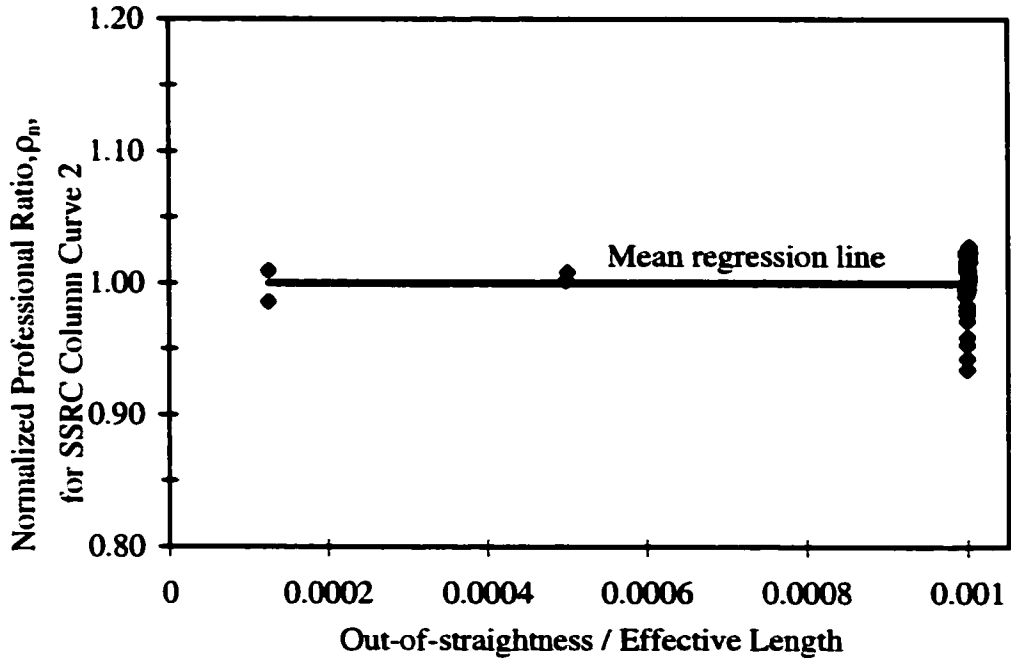
**Figure C.1 Simulated Professional Ratio vs. Value of Out-of-straightness for Columns from Group 2 ( $\lambda = 0.4$ )**



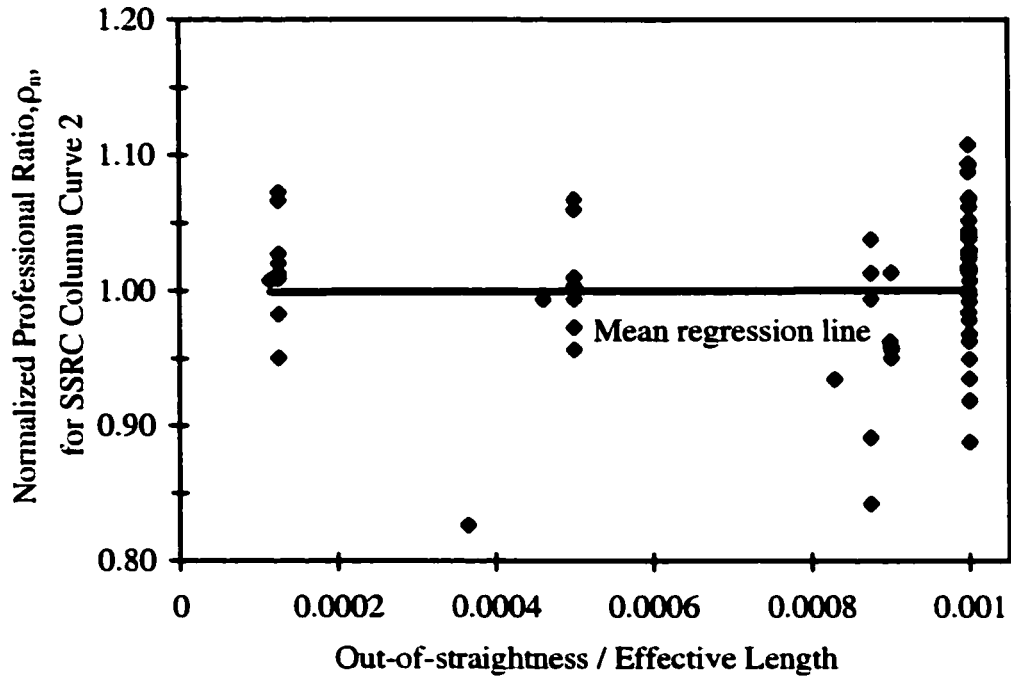
**Figure C.2 Simulated Professional Ratio vs. Value of Out-of-straightness for Columns from Group 2 ( $\lambda = 1.1$ )**



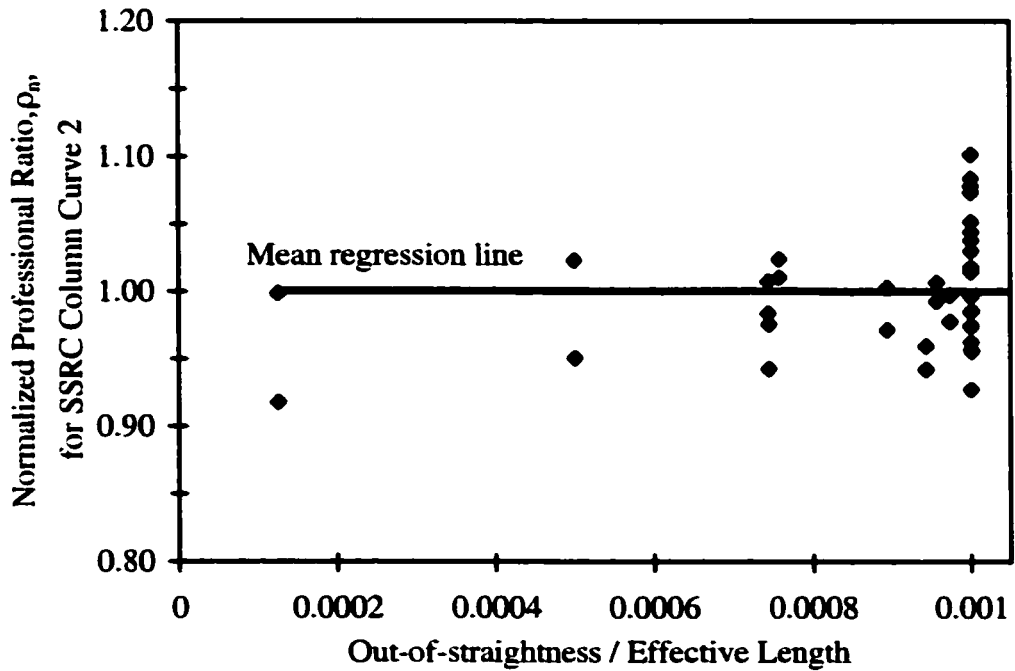
**Figure C.3 Simulated Professional Ratio vs. Value of Out-of-straightness for Columns from Group 2 ( $\lambda = 1.5$ )**



**Figure C.4 Normalized Professional Ratio vs. Value of Out-of-straightness for Columns from Group 2 ( $\lambda = 0.4$ )**



**Figure C.5 Normalized Professional Ratio vs. Value of Out-of-straightness for Columns from Group 2 ( $\lambda = 1.1$ )**



**Figure C.6 Normalized Professional Ratio vs. Value of Out-of-straightness for Columns from Group 2 ( $\lambda = 1.5$ )**