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Controlled synchronization of nonlinear teleoperation in task-space with time-varying delays

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***Abstract*—This paper introduces a novel control framework for bilateral teleoperation system with the redundant remote robot to ensure the end-effectors' position tracking while satisfying a sub-task control such as obstacle avoidance in the presence of the nonlinear dynamics for the manipulators and bounded time-varying delays in the communication channels. The asymptotic stability of the closed-loop dynamics is studied using a Lyapunov-Krasovskii functional under conditions on the controller parameters and the maximum values of time-varying delays. Simulation and experimental results are provided to validate the theoretical findings.**

***Index Terms*—Asymptotic stability, redundant robot, semi-autonomous system, synchronization, time-varying delay**

I. INTRODUCTION

The principle idea of synchronization is to design control algorithms such that a group of agents can reach certain coordination. In literature, synchronization has been studied in various disciplines. In [1], using memristive neural networks, an exponential synchronization of coupled stochastic memristor-based neural networks with probabilistic time-varying delay coupling and time-varying impulsive delay is discussed. Synchronization of complex dynamical networks with impulsive coupling is presented in [2] in which a unified synchronization criterion is derived for directed impulsive dynamical networks. In [3], using a fixed communication topology, pinning synchronization of a class of complex dynamical networks has been investigated. Since Euler-Lagrange models can be used to model a large class of physical systems of practical interests, the synchronization of networked Euler-Lagrange systems is of paramount importance, and synchronization problem of teleoperation systems is an example of this area where local and remote robots are modeled as Euler-Lagrange systems that is the work of this study.

A bilateral teleoperation system is composed of local and remote robots where various signals are exchanged between them via a communication channel. A human operator manipulates the local robot, and the controlled coupling between robots makes it possible for carrying out tasks through the remote robot. The capability of remotely doing tasks is a privilege for the teleoperation system and provides a stable interaction with risky environments. Teleoperation systems have broad application domain in areas like outer-space manipulation, navigating a nuclear reactor station, defusing a bomb, undersea exploration, remote medical operation, telerehabilitation, haptics-assisted training, etc. [4]. The physical distance between robots poses inevitable time delays in their communications which can destabilize the telerobotic system [5]. In practice, the communication delay can be time-varying and asymmetric in the forward and backward paths between the operator and the remote environment [6], [7]. There are a number of control schemes for time-varying delay compensation in the literature [8]–[11]. In literature, there have been significant developments to control the bilateral teleoperated systems [4], [12].

In practice, most of the bilateral teleoperation systems are geared up such as to make it possible for the local operators to control the remote robot's end-effector, and therefore the study of teleoperation in task-space is of great practical importance. In [13], scaled synchronization has been proposed for bilateral teleoperators with different configurations, but the local and remote robots were assumed to be kinematically identical and non-redundant manipulators. In [14], a nonlinear robust adaptive bilateral impedance controller is proposed to provide the absolute stability of multi-DoF teleoperation systems with communication delays. An adaptive switching-based control framework with asymmetric time-varying delays is investigated for task-space performance in teleoperation system [15]. A kinematically redundant remote robot generally provides greater manipulability for the human operator to perform complex tasks. Teleoperation of redundant manipulators was studied in [16] where the robots are assumed to track the desired trajectory in task-space. However, the teleoperation system was developed without considering communication delays, and the local and remote robots were required to have the same degrees of freedom. Synchronization of heterogeneous robotic manipulators following a desired trajectory in the task-space has been presented in [17]. Even though heterogeneity of the robotic manipulators and communication delays were considered, the controller required all agents to have knowledge of a common trajectory, which may not be practical for all teleoperation systems. In [18], task-space teleoperation with a redundant remote robot has been studied in which control theoretic framework was used to guarantee the position and velocity tracking between the local and remote robots in the presence of constant delays. In [19], the system utilizes dual local robots to control different frames assigned to the remote robot. Even though the local and remote robots

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were considered to be non-identical, the remote robot required complete control from the human operator. Moreover, the issue of communication delays was not considered.

As far as the remote robot's surroundings are concerned, overlooking unforeseen limitations of the remote site can restrict the performance of the teleoperation system [20]. To cope with these circumstances, the semi-autonomous teleoperation framework with a redundant remote robot is proposed in [21] in which the redundancy of the remote robot is utilized for achieving an autonomous sub-task control, such as singularity or obstacle avoidance. In [22], a control algorithm is proposed to guarantee the stability and task-space position tracking when teleoperation systems are subject to time-varying delay. In [23], adaptive control of semi-autonomous teleoperation is developed to address the task-space bilateral teleoperation system with asymmetric time-varying delays for heterogeneous local and remote robots to guarantee stability and tracking performance.

In this paper, a novel nP+D like controller that incorporates gravity compensation is proposed for the control problem of task-space nonlinear teleoperation involving a redundant remote robot. The nP+D [24] controller is similar to the proportional plus damping (P+D) controller [8], with the difference that it alters the proportional term by passing it through a nonlinear function. The main contributions of this paper are summarized as follows.

- The time-varying delays and the redundancy of the remote robot are incorporated into the controller design so that the sub-task control such as obstacle avoidance can be achieved against a backdrop of the main-task accomplishment (Theorems 1 and 2, and Section V).
- In contrast to [22], using the proposed controller, there is no need for measuring the real-time rates of change of the delays. Instead, it only requires the maximum of the upper bound of the time-varying delays. Also, one can expand the systems tolerance to larger upper bound of the time-varying delays by simply increasing one of the controller parameters (see (17)) without any consequences. For instance, in [25], tolerance to the larger upper bound of the time-varying delays comes at the cost of increased computational complexities.
- It is assumed that the Jacobian matrix of the local robot is full rank. Nevertheless, in contrast to [21]–[23] the local robots controller is still well-defined at a singularity (since it does not contain the inverse of the Jacobian matrix). Also, with an appropriate vector function assigned for the sub-task control, the achievement of both the sub-task control and the task-space position synchronization are guaranteed. In other words, the vector function is a blessing in disguise for the main-task achievement, when the local robot faces a singularity.
- The proposed controller can fulfill the function of both the main-task control and the sub-task control, not only in free motion but also when the human operator exerts a bounded force (Remark 1). Depending on how large the applied force is in comparison with the amplitude of the nonlinear function used in the controller, a relation between the operators applied force and the reflected force from the environment is established. Also, it is shown that the signals of the system are ultimately bounded in contact motion. Moreover, it is pointed out that when the operator exerts a constant force and the remote robot is in free motion, the achievement of both the main-task and sub-task will come at the cost of a singularity for the local robot.

This paper is organized in sections as follows. Section 2 gives problem formulation and the proposed controller and its stability analysis are studied in Sections 3 and 4, respectively. Also, sub-task control, simulation and experimental results, conclusion, and appendix are presented in Sections 5, 6, 7 and 8, respectively.

II. Problem Formulation

Suppose that the manipulators in the teleoperation system are modeled by Lagrangian systems and driven by actuated revolute joints. Let the dynamics of the local (l) and the remote (r) robots be given by

$$M_k(q_k)\ddot{q}_k+C_k(q_k,\dot{q}_k)\dot{q}_k+G_k(q_k)=\tau_{e_k}+\tau_k, \tag{1}$$

where for $k\in\{l,r\}$, $q_k,\dot{q}_k,\ddot{q}_k\in\mathbb{R}^{\beta_k\times 1}$ are the vectors of the joint positions, velocities and accelerations of the robots such that $\beta_l=n$ and $\beta_r=m$, and n and m are the number of joints in the local and remote robots, respectively. $M_k(q_k)\in\mathbb{R}^{\beta_k\times\beta_k}$, $C_k(q_k,\dot{q}_k)\in\mathbb{R}^{\beta_k\times\beta_k}$ and $G_k(q_k)\in\mathbb{R}^{\beta_k\times 1}$ are the inertia matrix, the Coriolis/centrifugal matrix and the gravitational vector, respectively. Moreover, $\tau_{e_k}\in\mathbb{R}^{\beta_k\times 1}$ denotes the exerted torques and $\tau_k\in\mathbb{R}^{\beta_k\times 1}$ denotes the control signals.

Let $X_k\in\mathbb{R}^{z\times 1}$ represent the end-effectors' positions in the task-space. The relation between the task-space positions and the joint-space positions of the robots are as

$$X_k=h_k(q_k);\qquad \dot{X}_k=J_k(q_k)\dot{q}_k, \tag{2}$$

where $h_k(q_k){:}\mathbb{R}^{\beta_k\times 1}\to\mathbb{R}^{z\times 1}$ denotes the position mapping between the joint-space and the task-space, and $J_k(q_k)\in\mathbb{R}^{z\times\beta_k}$ signifies the Jacobian matrices such that $J_k(q_k)=\frac{\partial h_k(q_k)}{\partial q_k}$. The task-space position errors are defined as

$$\begin{aligned} e_l&\triangleq X_l(t)-X_r(t-d_r(t)); \quad & e_l^0&\triangleq X_l(t)-X_r(t),\\ e_r&\triangleq X_r(t)-X_l(t-d_l(t)); \quad & e_r^0&\triangleq X_r(t)-X_l(t),\end{aligned} \tag{3}$$

where $d_r(t)$ and $d_l(t)$ are backward and forward time-varying delays between the robots. In the rest of the paper, notations M_k, M_k^{-1}, C_k, C_k^T, J_k, J_k^T and J_r^+ are used instead of $M_k(q_k)$, $M_k^{-1}(q_k)$, $C_k(q_k,\dot{q}_k)$, $C_k^T(q_k,\dot{q}_k)$, $J_k(q_k)$, $J_k^T(q_k)$ and $J_r^+(q_r)$ (being the pseudo-inverse of J_r defined later), respectively. Inspired by the works in [21], [26], the modified form of the local and remote robots' dynamics are obtained to incorporate the sub-task control into the controller development. To this end, let us define the signals $\zeta_k\in\mathbb{R}^{\beta_k\times 1}$ and $s_k\in\mathbb{R}^{\beta_k\times 1}$ as

$$s_k\triangleq\dot{q}_k-\zeta_k;\qquad \zeta_k\triangleq\begin{cases}0 & \text{if}\quad k=l\\ [\mathbb{I}_m-J_r^+J_r]g_r & \text{if}\quad k=r\end{cases}, \tag{4}$$

where $\mathbb{I}_m$ is the identity matrix of size m, $g_r\in\mathbb{R}^{m\times 1}$ is the negative gradient of an appropriately defined function for the sub-task control, and $J_r^+ \in \mathbb{R}^{m\times z}$ is the pseudo-inverse of J_r which is defined by $J_r^+ \triangleq J_r^T(J_rJ_r^T)^{-1}$ and satisfies $J_rJ_r^+ = \mathbb{I}_m$. Taking time derivative of both sides of the equation $s_k = \dot{q}_k - \zeta_k$, premultiplying by the inertia matrix M_k and substituting $M_k\ddot{q}_k$ from (1), the modified form of the local and the remote robots' dynamics can be found as

$$M_k\dot{s}_k + C_ks_k = \Theta_k + \tau_{e_k} + \tau_k; \quad \Theta_k \triangleq -M_k\dot{\zeta}_k - C_k\zeta_k - G_k. \quad (5)$$

III. Proposed Controller

Consider the dynamical system (1) and let the control signal be given by

$$\tau_k = -\Theta_k - J_k^T\Omega P(e_k) - \Lambda_k s_k, \quad (6)$$

where $\Lambda_k \in \mathbb{R}^{\beta_k\times\beta_k}$ and $\Omega \in \mathbb{R}^{z\times z}$ are positive-definite diagonal matrices with elements $\Lambda_{k_i} \in \mathbb{R}_{>0}$ and $\Omega_j \in \mathbb{R}_{>0}$, respectively, such that $\Lambda_{k_{min}} \triangleq \min_i\{\Lambda_{k_i}\}$ and $\Omega_{max} \triangleq \max_j\{\Omega_j\}$. Also, $P(e_k): \mathbb{R}^{z\times 1} \to \mathbb{R}^{z\times 1}$ is a nonlinear vector function with elements $p_j(e_{k_j}): \mathbb{R} \to \mathbb{R}$ which are required to be strictly increasing, bounded, continuous, passing through the origin, concave for positive e_{k_j} and convex for negative e_{k_j} with continuous first derivative around the origin such that $|p_j(e_{k_j})| \le |e_{k_j}|$ and $p_j(-e_{k_j}) = -p_j(e_{k_j})$ [24]. For instance, by choosing $p_j(e_{k_j}) = b_j\tan^{-1}(e_{k_j})$; $0 < b_j \le 1$, all the mentioned properties are satisfied, $\frac{\partial p_j(e_{k_j})}{\partial e_{k_j}}$ is bounded and $N_j \triangleq \sup p_j(e_{k_j}) = b_j\pi/2$. Recalling that the manipulators in the teleoperation system (1) are revolute joint robots, important properties of the nonlinear dynamic models are revisited here [27], [28]:

Property 1. *The inertia matrix $M_k \in \mathbb{R}^{\beta_k\times\beta_k}$ is symmetric positive-definite and has the upper and lower bounds as $0 < \lambda_{min}(M_k)\mathbb{I}_{\beta_k} \le M_k \le \lambda_{max}(M_k)\mathbb{I}_{\beta_k} < \infty$ where $\mathbb{I}_{\beta_k}$ is the identity matrix of size β_k.*

Property 2. *$\dot{M}_k - 2C_k$ is a skew symmetric matrix.*

Property 3. *The time derivative of C_k is bounded if $\ddot{q}_k$ and $\dot{q}_k$ are bounded.*

Property 4. *For a manipulator with revolute joints, there exists a positive σ bounding the Coriolis/centrifugal term as $\|C_k(q_k,x)y\|_2 \le \sigma\|x\|_2\|y\|_2$.*

Also, some assumptions are made as follows.

Assumption 1. *The operator and the environment are passive, i.e., there exist positive constants $\varphi_k < \infty$ such that $\varphi_k + \int_0^t -\dot{q}_k^T(\mu)\tau_{e_k}(\mu)d\mu > 0$.*

Assumption 2. *The time derivative of the forward and backward time-varying delays are bounded.*

IV. Stability Analysis

In this section, the stability and asymptotic performance of the system (1) with the proposed controller (6) is analyzed. Applying the controller to the modified dynamics (5), the following closed-loop dynamics can be found:

$$M_k\dot{s}_k + C_ks_k = \tau_{e_k} - J_k^T\Omega P(e_k) - \Lambda_k s_k. \quad (7)$$

Theorem 1. Assume that the Jacobian matrix of the local manipulator is full rank, and the operator and the environment are passive. Given teleoperation system (1) with the controller (6), the end-effector velocities $\dot{X}_k$ and task-space position errors e_k and e_k^0 are bounded for any bounded forward and backward time delays provided that

$$\Lambda_{k_{min}} \ge 2T\Omega_{max}\beta_k J^{(2)}_{k_{max}}, \quad (8)$$

where T is the maximum round-trip delay defined as $T \triangleq (d_{l_{max}} + d_{r_{max}})$ such that $d_{l_{max}} \triangleq \max\{d_l(t)\}$ and $d_{r_{max}} \triangleq \max\{d_r(t)\}$.

Proof of Theorem 1. Let $x_t = x(t+\varrho)$ [29], [30] be the state of the system where $x(t) \triangleq [s_l\ s_r\ X_l\ \dot{X}_l\ X_r\ \dot{X}_r]$, $-d_{max} \le \varrho \le 0$ and $d_{max} \triangleq \max\{d_{l_{max}}, d_{r_{max}}\}$ is the maximum delay. Consider the Lyapunov-Krasovskii functional $V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t)$ where

$$V_1(x_t) = \sum_k \left(\frac{1}{2}s_k^TM_ks_k + \int_0^t -\dot{q}_k^T(\mu)\tau_{e_k}(\mu)d\mu + \varphi_k\right), \quad (9)$$

$$V_2(x_t) = \sum_{j=1}^{z}\int_0^{X_{l_j}-X_{r_j}} \Omega_j p_j(\gamma_j)d\gamma_j, \quad (10)$$

$$V_3(x_t) = 2\sum_k \int_{-d_{k_{max}}}^{0}\int_{t+\gamma}^{t} \dot{X}_k^T(\eta)\Omega P(\dot{X}_k(\eta))d\eta d\gamma. \quad (11)$$

Defining $D_k \triangleq \dot{X}_k^T\Omega P(e_k)$ and $D_k^{'} \triangleq -\dot{X}_k^T\Omega P(e_k)$ such that $D_k + D_k^{'} = 0$, considering Property 2, given the facts that $\tau_{e_k} = J_k^TF_{e_k}$, $\zeta_k^TJ_k^T = 0$, $s_k^T\tau_{e_k} = \dot{q}_k^T\tau_{e_k}$ and $s_k^TJ_k^T = \dot{X}_k^T$, and regarding the closed-loop dynamics (7), we get

$$\dot{V}_1(x_t) + \dot{V}_2(x_t) + \sum_k D_k = \sum_k\left(\dot{X}_k^T\Omega P(e_k^0) - s_k^T\Lambda_k s_k\right), \quad (12)$$

$$\dot{V}_3(x_t) \le 2\sum_k\left(d_{k_{max}}\dot{X}_k^T\Omega P(\dot{X}_k) - \int_{t-d_k(t)}^{t}\dot{X}_k(\tau)^T\Omega P(\dot{X}_k(\tau))d\tau\right). \quad (13)$$

Now, using Lemma 3.2 [31] and adding $\sum_k D_k^{'}$ to (12) yields

$$(12) + \sum_k D_k^{'} \le 2|\dot{X}_l|^T\Omega\int_{t-d_r(t)}^{t}P(|\dot{X}_r(\tau)|)d\tau + 2|\dot{X}_r|^T\Omega\int_{t-d_l(t)}^{t}P(|\dot{X}_l(\tau)|)d\tau - \sum_k s_k^T\Lambda_k s_k \quad (14)$$

and adding (13) to (14) and using Lemma 3.3 [31], we get

$$\dot{V}_3(x_t) + (14) \le \sum_k\left(2T\Omega_{max}\dot{X}_k^TP(\dot{X}_k) - \Lambda_{k_{min}}s_k^Ts_k\right). \quad (15)$$

Defining $\psi_k \triangleq \frac{\dot{X}_k^TP(\dot{X}_k)}{\|s_k\|_2^2}$ and substituting it into (15) leads to

$$\dot{V}(x_t) \le -\sum_k[\Lambda_{k_{min}} - 2T\Omega_{max}\psi_k]s_k^Ts_k. \quad (16)$$

Therefore, a sufficient condition for $\dot{V}(x_t) \le 0$ is $\Lambda_{k_{min}} \ge 2T\Omega_{max}\psi_k$. Using Lemma 3.1 [31] and the properties that for any $x_j \in \mathbb{R}$, $x_jp_j(x_j) \ge 0$ and $|p_j(x_j)| \le |x_j|$ [24], it can be concluded that $\psi_k = \frac{1}{\|s_k\|_2^2}\dot{X}_k^TP(\dot{X}_k) \le \frac{1}{\|s_k\|_2^2}\dot{X}_k^T\dot{X}_k \le \beta_k J^{(2)}_{k_{max}}$. Therefore, the sufficient condition for $\dot{V}(x_t) \le 0$ can be rewritten as

$$\Lambda_{k_{min}} \geq 2T\Omega_{max}\beta_k J^{(2)}_{k_{max}}. \tag{17}$$

In conclusion, if (17) is satisfied, then $\dot{V}(x_t) \leq 0$ which means all terms in $V(x_t)$ are bounded, i.e., $\dot{X}_k, e^0_k, s_k \in L_\infty$. Noting that $e_l = e^0_l + \int_{t-d_r(t)}^{t} \dot{X}_r(\tau)d\tau$ and given $\int_{t-d_r(t)}^{t} \dot{X}_r(\tau)d\tau \in L_\infty$, it can be concluded that $e_l \in L_\infty$. Similarly, $e_r \in L_\infty$ and the proof of Theorem 1 has been completed.

Theorem 2. Given Assumption 2, for the closed-loop dynamics described by (7), in free motion ($\tau_{e_k}=0$) the task-space position errors (3) asymptotically converge to the origin if all conditions in Theorem 1 are satisfied.

Proof of Theorem 2. Integrating both sides of (16), we get $s_k \in L_2$. Based on the results of Theorem 1 we have $\dot{X}_k, s_k, e_k \in L_\infty$ and given Assumption 2 it can be concluded that $\dot{e_k} \in L_\infty$. Considering (4), $s_k \in L_\infty$ results in $\dot{q}_l \in L_\infty$ and by the assumption that the remote manipulator is able to avoid the singularities, we get $\dot{q}_r \in L_\infty$. Also, $\dot{q}_k \in L_\infty$ leads to $\dot{M}_k \in L_\infty$. Using Properties 1 and 4, and since the term $J_k^T \Omega P(e_k)$ is bounded, it can be seen from (7) that $\dot{s}_k \in L_\infty$. Because $s_k \in L_2$ and $\dot{s}_k \in L_\infty$, using the Barbalat's lemma we have $s_k \to 0$. Given (4) and using the fact that $\dot{X}_k = J_k s_k$, we have $\dot{X}_k, \dot{q}_l \to 0$. If the remote manipulator is able to avoid the singularities, then $\dot{q}_r \to 0$. Applying $\tau_{e_k}=0$ into (7) and differentiating both sides with respect to time and given $\frac{d}{dt}(M_k^{-1}) = -M_k^{-1}(C_k + C_k^T)M_k$, we get

$$\begin{aligned}\ddot{s}_k = &\frac{d}{dt}\left(M_k^{-1}\right)\left(-C_k s_k - J_k^T \Omega P(e_k) - \Lambda_k s_k\right) \\ &+ M_k^{-1}\frac{d}{dt}\left(-C_k s_k - J_k^T \Omega P(e_k) - \Lambda_k s_k\right)\end{aligned} \tag{18}$$

and based on Properties 1 and 4, and given $\dot{q}_k \in L_\infty$, it is easy to see that $\frac{d}{dt}(M_k^{-1})$ is bounded. Also, from

$$\frac{d}{dt}\left(J_k^T \Omega P(e_k)\right) = \left(\frac{\partial}{\partial q_k} J_k^T\right)\dot{q}_k \Omega P(e_k) + J_k^T \Omega \left(\frac{\partial}{\partial e_k} P(e_k)\right)\dot{e}_k, \tag{19}$$

it can be concluded that $\frac{d}{dt}(J_k^T \Omega P(e_k)) \in L_\infty$. Also, given (1) it is possible to see that $\ddot{q}_k \in L_\infty$ and based on Property 3 the time derivative of C_k is bounded. Therefore, $\ddot{s}_k \in L_\infty$ and given that $s_k \to 0$, using the Barbalat's lemma yields $\dot{s}_k \to 0$. Considering the closed-loop dynamics (7) in free motion, having shown that $s_k, \dot{s}_k \to 0$, we get $J_k^T \Omega P(e_k) \to 0$. When $\dot{q}_k \to 0$ we have $P(e_l) = P(e^0_l) = -P(e^0_r)$ and so $J_k^T \Omega P(e^0_k) \to 0$. Therefore, if the remote manipulator is able to avoid the singularities ($g_r \neq 0$), then it can be concluded that $P(e^0_r) \to 0$. Noting that $P(e^0_r)$ passes through the origin, we get $e^0_r, e^0_l \to 0$. Now assume that in (4) we have $g_r = 0$, then $\dot{q}_l, \ddot{q}_l, J_l^T \Omega P(e_l) \to 0$ and so $e_l \to 0$. Given $\dot{q}_l, \ddot{q}_l \to 0$ and $e_l \to 0$ it can be concluded that $\dot{q}_r \to 0$ and so $e^0_l \to 0$ and so $e^0_r \to 0$. In other words, $e_k, e^0_k \to 0$ are valid for both $g_r = 0$ and $g_r \neq 0$. Thus, the proof has been completed.

Remark 1. Suppose that bounded forces are exerted on the end-effectors of the robots, and the remote robot is able to avoid the singularities. It can still be concluded that $s_k, \dot{s}_k \to 0$. Therefore,

$$\begin{bmatrix} J_l^T \\ -J_r^T \end{bmatrix} \Omega P(e^0_l) \to \begin{bmatrix} \tau_{e_l} \\ \tau_{e_r} \end{bmatrix} = \begin{bmatrix} J_l^T F_{e_l} \\ J_r^T F_{e_r} \end{bmatrix} \tag{20}$$

and supposing that $\Omega_j N_j \geq F_{e_{l_j}}$, it is possible to see that $\Omega P(e^0_l) \to F_{e_l}$ and so $F_{e_l} + F_{e_r} \to 0$. If $\Omega_j N_j < F_{e_{l_j}}$ then $F_{e_r} \to -\Omega P(e^0_l) = \Omega P(e^0_r)$. When the applied force on the end-effector of the local robot dwindles, the remote robot will move away to vanish the reflected force from the environment and the task-space position errors will converge to the origin asymptotically. Also, when the operator exerts a constant force and the remote robot is in free motion, the achievement of both the main-task and sub-task will come at the cost of F_{e_l} being in the null space of J_l^T matrix. In other words, the local robot will approach a singularity.

V. Sub-Task Control

Kinematic redundancy in the task-space of a manipulator can be exploited either to provide increased manipulability [32] or to achieve a sub-task control [21]. Also, as demonstrated in [33], the redundant manipulators as haptic interfaces for teleoperated surgical systems can provide better and more realistic force feedback to the user than the non-redundant haptic interfaces. The joint velocity of the redundant remote robot in the null space of J_r can be regulated in such a way as to not affect the position and velocity of its end-effector. This regulation leads to the so-called self-motion movement [34] since the manipulator's movement in joint space is not observed at the end-effector. Besides the main task and for various applications, the self-motion can be controlled by designing an appropriate auxiliary function g_r to achieve a sub-task control. To resolve the redundancy and achieve a controlled self-motion movement, we use the gradient projection method [35] which involves defining any differentiable cost function (expressed in terms of the joint angles or end-effector position) that has a minimum or maximum value at a desirable configuration. Using the gradient (or its negative) of this function to control the joint velocity in the redundant directions, the manipulator will tend to seek the optimal configuration.

Premultiplying s_r by $[\mathbb{I}_m - J_r^+ J_r]$ and using the property that $[\mathbb{I}_m - J_r^+ J_r][\mathbb{I}_m - J_r^+ J_r] = \mathbb{I}_m - J_r^+ J_r$, the relation between the Sub-task Tracking Error [34] (let it be STE) and s_r is obtained as [21]

$$STE \triangleq [\mathbb{I}_m - J_r^+ J_r]s_r = [\mathbb{I}_m - J_r^+ J_r](\dot{q}_r - g_r). \tag{21}$$

Therefore, if $s_r \to 0$ (Theorem 2), then the sub-task tracking error would approach the origin. Given (21) and the property that $J_r[\mathbb{I}_m - J_r^+ J_r] = 0$, the function $[\mathbb{I}_m - J_r^+ J_r]g_r$ can be considered as the desired velocity in the null space of J_r. As mentioned earlier, we can define a differentiable function $f(q_r): R^{m\times 1} \to R$ for which a lower value corresponds to more desirable configurations. Then, the auxiliary function $g_r = -\frac{\partial}{\partial q_r} f(q_r)$ is utilized for achieving the sub-task control of the remote robot. It is worth noting that as mentioned in Remark 1, not only in free motion but also when the operator exerts a bounded force on the local robot, $s_k \to 0$ and $\dot{s}_k \to 0$

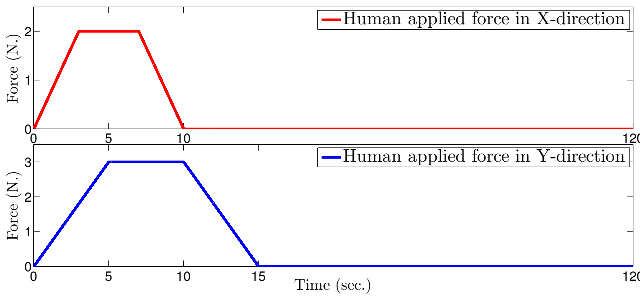


Fig. 1: Human-applied forces in X and Y directions.

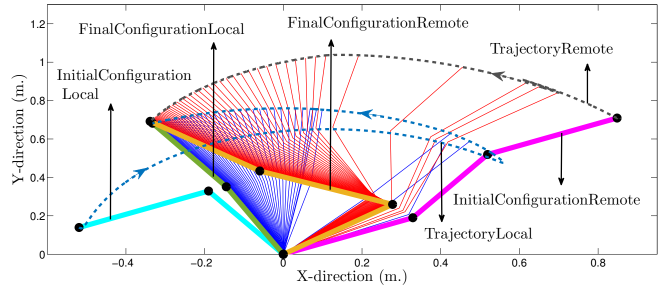
are still valid, the sub-task tracking error will still converge to zero, and thus the sub-task control also will still be achieved.

VI. Simulation and Experimental Results

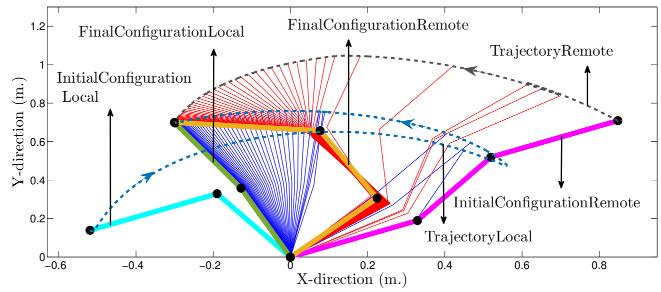
In this section, simulation and experimental results are presented to verify the theoretical findings. In simulations, the local and remote manipulators are considered to be 2-DOF and 3-DOF planar robots, respectively with revolute joints. Link mass parameters are $m_{1_l}{=}m_{2_l}{=}m_{1_r}{=}m_{2_r}{=}m_{3_r}{=}0.3kg$ and link length parameters are $L_{1_l}{=}L_{2_l}{=}L_{1_r}{=}L_{2_r}{=}L_{3_r}{=}0.38m$. The forward and backward time delays are considered to be identical and equal to $d_r(t){=}d_l(t){=}0.1{+}0.1\sin(t)$ (i.e., $T{=}0.4s$). Initial conditions $q_l(0){=}[2\pi/3,\ \pi/2]^T\ rad$ and $q_r(0){=}[\pi/6,\ \pi/6,\ -\pi/6]^T\ rad$ are chosen for the local and remote robots, respectively. Also, it is assumed that $\dot{q}_k(0){=}\ddot{q}_k(0){=}0$. The nonlinear function $p_j(e_{kj}){=}\tan^{-1}(e_{kj})$ is chosen (i.e., $N_j{=}N_{max}{=}\pi/2$) to be used in the robots' controllers. The gains $\Omega_1{=}1$, $\Omega_2{=}1$, $\Lambda_{l_1}{=}0.94$, $\Lambda_{l_2}{=}0.94$, $\Lambda_{r_1}{=}3.19$, $\Lambda_{r_2}{=}3.20$ and $\Lambda_{r_3}{=}3.21$ are chosen according to the stability condition. Therefore, the controllers become

$$\tau_l{=}G_l{-}J_l^T\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\tan^{-1}(e_l){-}\begin{bmatrix}0.94 & 0\\ 0 & 0.94\end{bmatrix}\dot{q}_l,$$
$$\tau_r{=}{-}\Theta_r{-}J_r^T\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\tan^{-1}(e_r){-}\begin{bmatrix}3.19 & 0 & 0\\ 0 & 3.20 & 0\\ 0 & 0 & 3.21\end{bmatrix}s_r. \tag{22}$$

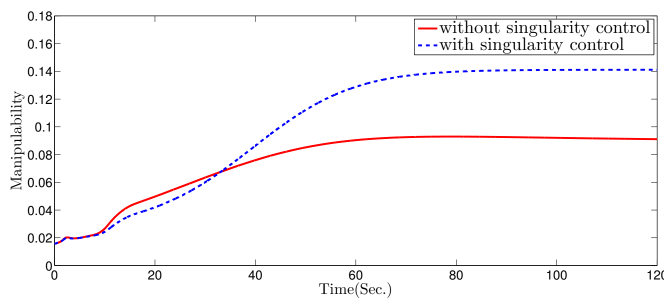
Singularity avoidance is an important objective in choosing the function g_r for the control law. As mentioned earlier a common method is to define a function; $f(q_r)$, for which a lower value is associated with a more desirable configuration. Motivated by [26], [34], first $g_r{\triangleq}{-}0.01(q_{1_r}{-}2q_{2_r}{+}q_{3_r})[1,{-}2,1]^T$ is chosen for the manipulability improvement and singularity avoidance. Note that the selected g_r is the negative of the gradient of the cost function $f(q_r){=}0.005(q_{3_r}{-}2q_{2_r}{+}q_{1_r})^2$ and the control law will try to minimize the cost function. Now, suppose that the operator applies its force as shown in Fig. 1. The simulation results without (i.e., $g_r{=}0$) and with the singularity avoidance control are presented in Fig. 2. As noticeable in Fig. 2(b), the defined function g_r has increased the flexibility of the redundant robot, and the manipulability index (defined as the determinant of the matrix $J_rJ_r^T$) is increased (see Fig. 2(c)). Therefore, both the end-effectors



(a) Without singularity avoidance control



(b) With singularity avoidance control

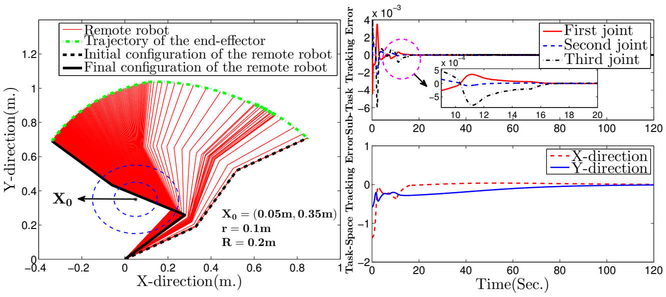


(c) Manipulability index

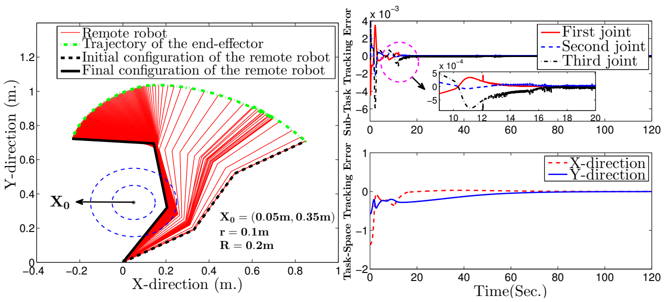
Fig. 2: Simulation results for the singularity control.

convergence task and the singularity avoidance control are accomplished.

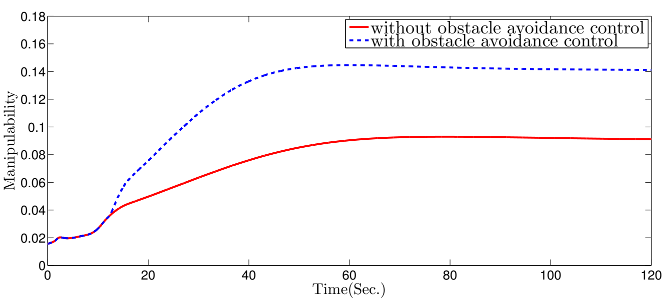
The extra degrees of the redundant manipulator can also be used for avoiding an obstacle while performing the desired end-effector task. In this case, the strategy for choosing the function g_r is adopted from [21]. Again, suppose that the operator applies its force as shown in Fig. 1. The simulation results without (i.e., $g_r{=}0$) and with the obstacle avoidance control are presented in Fig. 3. The obstacle that the remote robot needs to avoid is located at $X_0{=}[0.05m{,}0.35m]$ and the collision distance and the safe distance are given as $R{=}0.2m$ and $r{=}0.1m$, which are shown as the dashed blue circles in the simulations (Figs. 3(a) and 3(b)). The middles of the three links of the remote robot are chosen as the collision-free points to avoid the obstacle. Note that the Task-Space Tracking Error is defined as $TTE{\triangleq}X_l{-}X_r$. As we see in Fig. 3(b), by incorporating the obstacle avoidance control into the proposed controller, the redundant manipulator changes its configuration to avoid the obstacle while achieves the



(a) Without obstacle avoidance control.



(b) With obstacle avoidance control.



(c) Manipulability index.

Fig. 3: The simulation results for the obstacle control.

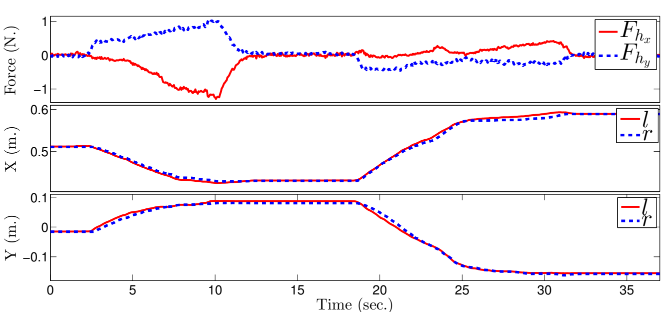
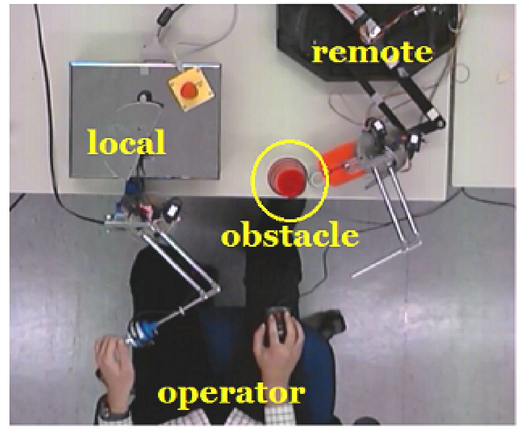


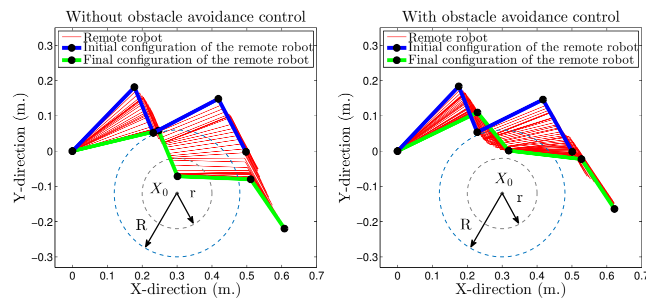
Fig. 4: Experimental results for the task-space position synchronization when g_r=0.

end-effectors synchronization task. Deploying the sub-task control has also increased the manipulability of the redundant manipulator (see Fig. 3(c)). Readers are encouraged to watch downloadable **simulation videos** prepared to shed more light on the simulation results[1].

[1]https://bit.ly/2D2iMRQ



(a) Experiment setup.



(b) With and without obstacle avoidance control.

Fig. 5: The experiment for the obstacle avoidance control.

To show the performance of the proposed controller in practice, we have experimented it on a bilateral teleoperation setup in which a 2-DoF local robot is connected to a 4-DoF remote robot. The local robot is a 2-DoF PHANToM 1.5A (Geomagic Inc., Morrisville, NC, USA) where the base joint of the 3-DoF PHANToM robot has been removed to turn it into a 2-DoF planar robot. The remote robot has four degrees of freedom and the 4-DoF planar RHI is developed by serially connecting two robots, a 2-DoF PHANToM 1.5A (Geomagic Inc., Morrisville, NC, USA) and a 2-DoF planar upper-limb rehabilitation robot 1.0 (Quanser Inc., Markham, ON, Canada). The base joint of the 3-DoF PHANToM robot has been removed to turn it into a 2-DoF planar robot. Also, to measure the applied force on the local robot, a 6-DoF force/torque (f/t) sensor (50M31A3-I25, JR3 Inc., Woodland, CA, USA) is used. The link lengths for the local and remote robots are $[0.21{,}0.2520]\ m$ and $[0.254{,}0.1405{,}0.21{,}0.17]\ m$, respectively.

For g_r=0, the operator's applied force on the local robot and the positions of the robots' end-effectors in X and Y directions are shown in Fig. 4. Also, to show the performance of the proposed controller for concurrently achieving the position synchronization task and the obstacle avoidance control, an experiment has been done in which the remote robot avoids the obstacle located at $[0.3m, -0.12m]$. The experimental results are shown in Fig. 5 in which Fig. 5(a) shows an above view shot of the experiment setup. The collision distance and the safe distance are assumed to be R=0.18m and r=0.1m. The readers are strongly encouraged to watch the downloadable **Experiment videos** prepared to shed more light on the experimental results[2].

VII. Conclusion and Future Work

Given nonlinear dynamics for the manipulators and the bounded time-varying delays, in this paper, a novel (nP+D)-like controller was proposed to ensure the stability and task-space tracking performance of the bilateral teleoperation system. The redundancy of the remote manipulator was exploited to accomplish the end-effectors' convergence task against a backdrop of avoiding an obstacle. It was pointed out that concurrently occurrence of the task-space positions synchronization and sub-task control can take place either in free motion or when the operator exerts a bounded force on the local robot. Moreover, it was indicated that in contact motion the error signals are bounded, the system is stable and there exists a relationship between the interactive forces. Using a Lyapunov-Krasovskii functional, the asymptotic stability and tracking performance of the teleoperation system is established under some conditions on the controller parameters and maximum allowable time delays. The efficiency of the control algorithm was studied using numerical simulations with a 2-DoF planar local robot and a 3-DoF planar redundant remote robot. Also, the experiments were done on a bilateral teleoperation system with a 2-DoF local robot and a 4-DoF remote robot.

As a future work, while incorporated with a speed observer to extricate the system from velocity measurements, it is possible to put forward an enhanced version of the controller to improve the synchronization time. Also, the task-space synchronization problem can be studied in the simultaneous presence of time-varying delays and a problem such as model uncertainties or external disturbances.

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