

13527
NATIONAL LIBRARY
OTTAWA



BIBLIOTHÈQUE NATIONALE
OTTAWA

NAME OF AUTHOR... PLAIN.....
TITLE OF THESIS... *Inter-regional & inter-industry
Differences in the Elasticity
of Factor Substitutⁿ & the Offsetting of Labor
in Canada*.....
UNIVERSITY... ALTA.....
DEGREE FOR WHICH THESIS WAS PRESENTED... P.H.D.....
YEAR THIS DEGREE GRANTED... 72.....

Permission is hereby granted to THE NATIONAL LIBRARY
OF CANADA to microfilm this thesis and to lend or sell copies
of the film.

The author reserves other publication rights, and
neither the thesis nor extensive extracts from it may be
printed or otherwise reproduced without the author's
written permission.

(Signed) *R. H. McPlain*

PERMANENT ADDRESS:

*39 Bellevue Crescent
St. Albert, Alberta*

DATED... *Oct 6*.....1972

NL-91 (10-68)

THE UNIVERSITY OF ALBERTA

INTER-REGIONAL AND INTER-INDUSTRY DIFFERENCES IN
THE ELASTICITY OF FACTOR SUBSTITUTION AND
THE EFFICIENCY OF LABOUR IN CANADA

by



RICHARD HAYWARD MCVICAR PLAIN

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE
OF DOCTOR OF PHILOSOPHY

DEPARTMENT OF ECONOMICS

EDMONTON, ALBERTA

FALL 1972

UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled INTER-REGIONAL AND INTER-INDUSTRY DIFFERENCES IN THE ELASTICITY OF FACTOR SUBSTITUTION AND THE EFFICIENCY OF LABOUR IN CANADA submitted by Richard H. M. Plain in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

..... *Le. Dy. Sr.*

Supervisor

..... *C. G. Hoskie*

..... *Stephen D. Lewis*

..... *E. J. Hamm*

..... *Jack C. Stables*

External Examiner

Date..... *Sept. 26 / 72*

ABSTRACT

One of the central tenets of economic theory is that every commodity has a common production function. An examination of two digit manufacturing industries located in various regions of Canada suggests that the aggregative counterpart of this micro-economic assumption does not hold for all cases. The evidence for this conclusion rests on findings which indicate there are significant inter-regional differences in the elasticity of substitution and the efficiency of labour use in two digit manufacturing industries. The evidence suggests that Western and Central Canada have similar manufacturing technologies.

An examination of the factor-price equalization theorem suggests that its conclusions do not provide a satisfactory basis for explaining inter-regional movements in factors and commodities. It is demonstrated that the possibility of factor reversals holds true for a large number of manufacturing industries.

Manufacturing industries which do not exhibit significant regional differences in production functions have elasticities of substitution that are approximately equal to one. This suggests that the Cobb-Douglas production function has a wide degree of general applicability for cross-sectional purposes.

Finally, it is noted that the elasticity of substitution and the efficiency of labour use exhibit a high degree of stability over short

periods of time. This conclusion rests on an analysis of seventeen, two digit manufacturing industries for the years 1964 and 1965.

ACKNOWLEDGEMENTS

I am indebted to Professors S. E. Drugge, A. Buse and S. D. Lewis for the time they have spent with me discussing various portions of the thesis.

My deepest thanks are extended to my wife and family who have displayed exceptional fortitude both during the stage when the thesis was being written and finally in carrying out the laborious tasks involved in typing and editing the manuscript.

TABLE OF CONTENTS

	Page
ABSTRACT.	iii
TABLE OF CONTENTS	vi
LIST OF TABLES.	viii
LIST OF FIGURES	x
CHAPTER	
I INTRODUCTION	1
II THE THEORETICAL FRAMEWORK AND RELATED EMPIRICAL RESULTS.	3
III INTER-REGIONAL DIFFERENCES IN THE ELASTICITY OF FACTOR SUBSTITUTION AND THE EFFICIENCY OF LABOUR USE.	30
IV INTER-INDUSTRY DIFFERENCES IN THE ELASTICITY OF FACTOR SUBSTITUTION	85
CONCLUSIONS AND OBSERVATIONS	102
BIBLIOGRAPHY	105

LIST OF TABLES

Table	Page	
II-2-1	Summary of Time Series and Cross-Section Estimates of the Elasticity of Substitution Between Capital and Labour for Two-Digit Manufacturing Industries.....	15
II-2-2	Cross-Section Estimates of the Elasticity of Substitution Between Capital and Labour in Manufacturing Industries.....	17
II-2-3	Elasticities of Substitution on Two-Digit Manufacturing Industries, United States, 1957.....	21
II-2-4	Intercountry Estimates of the Elasticity of Substitution for Two-Digit Manufacturing Industry, Murata-Arrow (1965).....	22
II-2-5	Summary of Liu-Hildebrand (1965) Regressions for Two-Digit Industries.....	26
III-1	Classification of Regional Groups.....	34
III-2	Classification of Industry Groups.....	35
III-1-1	Joint Tests of Significance Pertaining to the Pooling of Data for 1964 and 1965 in Seventeen, Two Digit Manufacturing Industries.....	39
III-1-2	Results of the Full Regression Analyses Pertaining to the Pooling of Data in Seventeen Manufacturing Industries in Canada for the Years 1964 and 1965.....	40
III-2-1	Joint Tests of Significance (differential intercepts and slopes) for Regional Influences in Eight Two Digit Manufacturing Industries Located in Western Canada, Central Canada and the Atlantic Provinces.....	44
III-2-2	Tests of Significance for Inter-Regional Differences in the Elasticity of Factor Substitution in Five, Two Digit Manufacturing Industries.....	47
III-2-3	Tests of Significance for Inter-Regional Differences in the Efficiency of Labour in Five, Two Digit Manufacturing Industries in Canada.....	48
III-2-4	Joint Tests of Significance (intercepts and slopes) for Regional Influences in Nine Two Digit Manufacturing Industries Located in Western and Central Canada..	53

Table	Page
III-3-1 Point Estimates of the Substitution (ρ) and Coefficient of Labour Parameters (α) for the Three Region, Eight Industry Breakdown of Manufacturing Activity.....	59
III-3-2 Point Estimates of the Substitution (ρ) and the Coefficient of Labour (α) Parameters for the Two Region, Nine Industry Breakdown of Manufacturing Activity in Canada.....	61
III-3-3 Tests of Significance for the Magnitude of the Elasticity of Substitution in Eight, Two Digit Manufacturing Industries Located in Canada and Western and Central Canada.....	78
III-3-4 Preliminary Estimates of the Elasticity of Substitution for Two-Digit Industries, Standard Errors, and Some Test Statistics.....	80
III-3-5 Estimates of the Elasticity of Substitution for Three-Digit Industries and Standard Errors--1963.....	83
IV-1 Joint Tests of Significance (Intercepts and Slopes) for Inter-Industry Effects in Four Regions of Canada.....	89
IV-2 Joint Tests of Significance (Differential Slopes) for Inter-Industry Effects in Four Regions of Canada.....	90
IV-3 The Full Regression for the Secondary Manufacturing Industries in Canada with Industry Dummies for the Intercept and Slope.....	92
IV-4 Tests of Significance for Inter-Industry Differences in Elasticities of Substitution in Canada.....	93
IV-5 The Full Regression for the Secondary Manufacturing Industries in the Atlantic Provinces with Industry Dummies for the Intercept and Wage Rate (X).....	95
IV-6 Tests of Significance for Inter-Industry Differences in the Elasticity of Substitution in the Atlantic Provinces.....	96
IV-7 The Full Regression for the Secondary Manufacturing Industries in Central Canada With Industry Dummies for the Intercept and Wage Rate (X).....	97
IV-8 Test of Significance for Inter-Industry Differences in Elasticities of Substitution in Central Canada.....	98

Table		Page
IV-9	The Full Regression for the Secondary Manufacturing Industries in Western Canada With Industry Dummies for the Intercept and Wage Rate (X).....	99
IV-10	Tests of Significance for Inter-Industry Differences in Elasticities of Substitution in Western Canada.....	100

CHAPTER I

INTRODUCTION

A cursory examination of the locational pattern of secondary manufacturing industries in Canada indicates there are substantial differences in the level and type of economic activities carried out in various regions of the country. There are a number of factors responsible for the existing pattern of industrial location ranging from federal and provincial policies, the proximity to large consumer markets, historical accident and the least cost locational decisions made by various corporate enterprises. It seems clear that if any attempt is to be made to successfully shape the pattern of industrial development within various regions of Canada that an attempt must be made to explicitly delineate the form of the underlying economic relationships.

This thesis attempts to explicate one facet of these complex sets of economic relationships by examining some of the technical characteristics of various two digit manufacturing industries located in certain regions of the country. In particular an attempt is made to determine whether there are inter-regional and inter-industry differences in the elasticity of substitution of capital for labour and the efficiency of labour use in various manufacturing industries. Both of these parameters are important in determining the relative share of income received by various factors and the pattern of inter-regional trade.

Format of the Thesis

Chapter II consists of a brief survey of the theory of production and an overview of the econometric studies which have dealt with the elasticity of substitution between capital and labour.

Chapter II is divided into two parts. Part 1 is concerned with the temporal stability of the elasticity of substitution and the efficiency of labour use in Canadian manufacturing industries. Part 2 deals with the effect of regional influences on these two parameters.

Chapter IV is concerned with the effect of inter-industry differences in the elasticity of substitution on the returns received by factors of production located in various regions of Canada.

Chapter V consists of the conclusions and observations.

CHAPTER II

THE THEORETICAL FRAMEWORK AND RELATED EMPIRICAL RESULTS

Part 1 of the following discussion provides a brief overview of the salient aspects of production theory which are required for the econometric analyses provided in Chapters III and IV. Part 2 consists of a brief outline of some of the empirical results which have been obtained by a selected group of investigators.

Part 1: The Theoretical Framework

The primary objective is to briefly outline the theoretical relationship that exists between a C.E.S. (Constant Elasticity of Substitution) production function homogeneous of degree '1' expressed by:

$$V = [\beta K^{-\rho} + \alpha L^{-\rho}]^{-1/\rho} \quad (\text{II-1-1})$$

where V is output, K is capital, L is labour, α , β , ρ are parameters-- and the following relationship given by:

$$\log y = \log a + b \log w \quad (\text{II-1-2})$$

where $y = \frac{V}{L}$ $w = \frac{W}{L}$. (W is the total wage bill and 'a' and 'b' are parameters.)

The relationship between (II-1-1) and (II-1-2) is developed in two stages. Stage I briefly outlines the general neo-classical properties

of the production function and the definition of the standard terms and concepts. Attention is paid to the establishment of the conditions under which estimates of the elasticity of factor substitution can be obtained without reliance being placed on capital stock data.

Stage II deals with the way in which C.E.S. production functions can be generated from (II-1-2). The relationship between the various parameters are carefully delineated. Finally, a brief overview is presented of some of the specific economic results that are obtained by using a C.E.S. production function.¹

Stage I: The Production Function and the Elasticity
of Factor Substitution

The conventional way of expressing the general form of the production function is as follows: (II-1-3). $V = F(K, L)$; where V is output, K is capital stock and L is the labour force. The following restrictions are imposed:²

¹It should be noted that the discussion follows well worn paths in economic theory. In the majority of cases the treatment or discussion of standard topics is considered to fall within the "province of common knowledge" though it should be carefully noted that the bulk of the discussion relating to the C.E.S. production function has been adopted or adapted from the article by K. J. Arrow, H. B. Chenery, B. S. Minhas and R. M. Solow, "Capital-Labour Substitution and Economic Efficiency", The Review of Economics and Statistics, Volume XLII, (August 1961), pp. 225-250.

²The import of these standard restrictions have more to do with mathematical and statistical tractability than with general production theory. Bodkin and Klein note: "The textbook form of the production function, in two dimensions, is an S shaped curve passing through the stages of increasing marginal productivity, an inflection point, and decreasing marginal productivity. Eventually, production function specification must deal with multi-variate parametric forms of this type. In most cases, investigators have confined themselves, in empirical research, to a single branch of this neo-classical production function--namely the branch of diminishing marginal productivity."
R.G. Bodkin and L.R. Klein, "A Non-Linear Estimation of Aggregate Production Function", Review of Economics and Statistics, Volume XLIX, (February 1967), p. 29.

(1) $K \geq 0, L \geq 0$; (2) $\frac{\partial V}{\partial K} = F_K > 0, \frac{\partial V}{\partial L} = F_L > 0$; (3) $\frac{\partial^2 V}{\partial L^2} = F_{LL} < 0$;
 and (4) $\frac{\partial^2 V}{\partial K^2} = F_{KK} < 0$. (F_K and F_L are the marginal physical products of capital and labour.)

Since it is assumed that the production function is homogeneous of degree '1' the above may be written in per capita form.

$$y = f(x), \quad x = \frac{K}{L}, \quad y = \frac{V}{L} \quad (\text{II-1-4})$$

and the marginal products of capital and labour may be expressed as:

$$\frac{\partial V}{\partial K} = \frac{\partial y}{\partial x} = f^1(x) \quad (\text{II-1-5})$$

and $\frac{\partial V}{\partial L} = f(x) - xf^1(x) \quad (\text{II-1-6})$

The movement along a given isoquant--the marginal rate of substitution of capital for labour (R)--is given by:

$$R(K, L) = \frac{F_L}{F_K} = \frac{f(x) - xf^1(x)}{f^1(x)} \quad (\text{II-1-7})$$

Elasticity of Factor Substitution

The responsiveness of changes in the capital-labour ratio due to changes in the marginal rate of substitution is given by the elasticity of substitution of capital for labour (σ) which is expressed by:

$\sigma = \frac{d \log x}{d \log R}$. After the appropriate substitutions are carried out:

$$\sigma = \frac{-f^1(x) [f(x) - xf^1(x)]}{xf(x) f^{11}(x)} \quad (\text{II-1-8})$$

Under perfect competition (factor and product markets) and profit maximizing behaviour every factor receives the value of its marginal product. More explicitly, if 'w' and 'r' are the real wage rate and the

rate of return to capital, then

$$w = f(x) - f^1(x) \quad (\text{II-1-9})$$

and $r = f^1(x) \quad (\text{II-1-10})$

The price of the product serves as the numeraire.

Given the above equilibrium conditions, Arrow, Chenery, Minhas and Solow¹ (hereinafter referred to as A.C.M.S.) demonstrated that (II-1-8) can be expressed as:

$$\sigma = \frac{d \log w}{d \log y} = \frac{w}{y} \cdot \frac{dy}{dw} \quad (\text{II-1-11})$$

This result ensures that ordinary least squares regression analysis can be used to provide an estimate of the elasticity of factor substitution by regressing:

$$\log y = \log a + b \log w$$

where 'b' is to be interpreted as an estimate of the elasticity of substitution and log a is the intercept term.

Stage II: Derivation of the C.E.S. Production Function

A.C.M.S. demonstrated the manner in which a production function could be generated from (II-1-2). Given (II-1-4) and the equilibrium condition set out in (II-1-9) it follows that the functional relationship between 'w' and 'x' can be inverted to give a monotonic increasing relationship between 'y' and 'w'; namely, $y = \phi(y - x \frac{dy}{dx})$ which has a

¹ op. cit., p. 229

solution $y = F(x,A)$.

More specifically, if $w = y - x \frac{dy}{dx}$, then (II-1-2) can be re-written as

$$\log y = \log a + b \log (y - x \frac{dy}{dx}) \quad (\text{II-1-12})$$

Taking anti-logarithms and solving for $\frac{dy}{dx}$,

$$\frac{dy}{dx} = \frac{a^{1/b}y - y^{1/b}}{a^{1/b}x} = \frac{y(1 - ay^\rho)}{x} \quad (\text{II-1-13})$$

where $a = a^{-1/b}$ and $\rho = 1/b - 1$.

After re-arranging and integrating the resultant production function is expressed by:

$$y = (\beta x^{-\rho} + \alpha)^{-1/\rho} \quad (\text{II-1-14})$$

Rewritten in its full form:

$$V = [\beta K^{-\rho} + \alpha L^{-\rho}]^{-1/\rho} \quad (\text{II-1-15})$$

The net result is that given the appropriate assumptions the simple regression $\log y = \log a + b \log w$, can be used to estimate two of the parameters of a C.E.S. production function. The slope coefficient 'b'--the estimate of the elasticity of factor substitution--is used to determine the magnitude of the substitution parameter 'ρ'.

$$\rho = \frac{1}{b} - 1$$

The intercept term 'a' yields some interesting information. In the log linear expression 'a' determines the height of the line or the value added

per man hour per dollar expended on labour in a certain industry. In this sense this term is a measure of the efficiency of labour. (In its original form $y = aw^b$ which implies $a = \frac{y}{w^b}$.) In (II-1-15) the coefficient of labour 'a' determines the portion of the product which is divided between capital and labour--'ceteris paribus' the other parameters. Since $a = a^{-1/b}$ it is evident that the portion of the product accruing to labour depends on the "efficiency" of labour and the elasticity of factor substitution.

It is useful to expand this point further by noting that if the C.E.S. production function is written in the form¹

$$y = \gamma[\delta x^{-\rho} + (1 - \delta)]^{-1/\rho} \quad (\text{II-1-16})$$

where $\alpha + \beta = \gamma^{-\rho}$ and $\beta\gamma^{\rho} = \delta$, that (II-1-2) can be expressed as

$$\log y = - \frac{1}{1+\rho} \log \gamma^{-\rho}(1-\delta) + \frac{1}{1+\rho} \log w \quad (\text{II-1-17})$$

If γ is interpreted as the neutral efficiency parameter, δ as the distribution parameter and ρ as noted previously, the constant intercept term noted as 'a' in (II-1-2) is a composite of a number of parameters. One useful result obtained from this type of a breakdown is that given all other parameters significant differences in intercept terms between industries would be based on differences in the "neutral efficiency" term set out in (II-1-17).

Economic Results and Properties of the C.E.S.

Production Function

The C.E.S. production function (constant returns to scale) has

¹ibid., p. 230.

all of the "well behaved" properties required of the general production relationship.¹ The marginal products of capital and labour are given as follows:

$$\frac{\partial V}{\partial K} = \beta \left(\frac{V}{K}\right)^{1+\rho} = \beta \left(\frac{V}{K}\right)^{1/\sigma} \qquad \frac{\partial V}{\partial L} = \alpha \left(\frac{V}{L}\right)^{1+\rho} = \alpha \left(\frac{V}{L}\right)^{1/\sigma}$$

The marginal rate of technical substitution is expressed by

$$R(K, L) = \frac{\alpha}{\beta} \left(\frac{K}{L}\right)^{1/\sigma} = \frac{\alpha}{\beta} (x)^{1/\sigma} \qquad \text{(II-1-18)}$$

Under competition and profit maximization $R(K, L) = \frac{w}{r}$ or

$$\frac{\alpha}{\beta} x^{1/\sigma} = \frac{w}{r} \qquad \text{(II-1-19)}$$

The portion of the product accruing to capital and labour is given by:

$$\frac{\beta}{\alpha} = \left(\frac{r}{w}\right) \left(\frac{K}{L}\right)^{1+\rho} \qquad \text{(II-1-20)}$$

The Economic Implications of Different Elasticities of Factor Substitution

$\sigma > 1$:

Since $\sigma = \frac{1}{1+\rho}$, $\sigma > 1$ implies $-1 < \rho < 0$. If $\rho = -1$, then $\sigma = \infty$

and the resultant isoquants are simply straight lines; however, in the case where $-1 < \rho < 0$ the isoquants are convex to the origin and cut both axes. It is clear from inspection of the following $y = (\beta x^{-\rho} + \alpha)^{-1/\rho}$, that output per unit of labour becomes infinitely large as the capital labour ratio

¹Refer to R. G. D. Allen, Macro-Economic Theory, (New York: Macmillan, 1967). pp. 52-55.

increases without limit; however, if the capital labour ratio tends towards zero the average product of labour is maintained at some positive minimum, namely, $a^{-1/\rho}$.

$\sigma=1$

If $\rho=0$ then $\sigma=1$ and the resultant production function is of the Cobb-Douglas form.¹

$\sigma<1$

If $0<\rho<\infty$, then $\sigma<1$. Let $V = \bar{V}$ and express the equation for an isoquant at $\bar{V} = \left(\frac{\beta}{K^\rho} + \frac{\alpha}{L^\rho}\right)^{-1/\rho}$. The isoquant does not approach both axes asymptotically since in the limit $K = V \cdot \beta^{1/\rho}$ and $L = V \cdot \alpha^{1/\rho}$.

$\sigma=0$

As ρ approaches ∞ , σ approaches zero. The resulting isoquants are right angled and the production function is of the Leontief form.

Relative Factor Intensities

As the previous discussion has indicated one of the most interesting features of the C.E.S. production function is centered around the fact that estimates of the elasticity of substitution may range from zero to infinity. This implies that it is virtually impossible to characterize one industry as being relatively more capital or labour intensive than another in the absence of factor price information since differences in the elasticity of substitution in the industries guarantee that there

¹Refer to A.C.M.S. p. 231 for a discussion of this point since it is not obvious from inspection of any of the above results.

always exists a critical wage profit ratio at which factor intensity ratios will be switched.

The theoretical basis for this conclusion is set out in the following discussion. Given (II-1-15) and (II-1-19) the capital labour ratio in a particular industry is expressed by:

$$x = \frac{K}{L} = \left[\frac{\beta}{\alpha} \cdot \frac{w}{r} \right]^\sigma \quad (\text{II-1-21})$$

If we consider a case where there are two industries each with a different C.E.S. production function incurring equal costs per unit for their factors of production it follows that the relative factor intensities of the two industries are given by:

$$\frac{x_1}{x_2} = \left(\frac{\beta_1}{\alpha_1} \right)^{\sigma_1} \cdot \left(\frac{\beta_2}{\alpha_2} \right)^{-\sigma_2} \cdot \left(\frac{w}{r} \right)^{\sigma_1 - \sigma_2} \quad (\text{II-1-22})$$

It is evident the relative factor intensities of the two industries are only invariant with respect to w/r if $\sigma_1 = \sigma_2$. For example, if the entire set of industries under consideration happened to fall within either the Cobb-Douglas case ($\sigma=1$) or the Leontief case ($\sigma=0$) then it would be possible to classify industries as capital or labour intensive irrespective of the factor price ratios. It is most unlikely that this fortuitous state of affairs is likely to occur in all instances.

The problem of determining the critical factor price ratio at which the factor intensities of a particular set of industries will be re-

versed is largely an empirical one. Minhas¹ spells out the procedure in some detail. The general point to note is that given a production function which allows any values for the elasticity of factor substitution between 0 and ∞ one must clearly allow for the fact that given different elasticity estimates changes in the ratio of factor prices can lead to quite different mixes in the quantity of labour and capital used in a particular industry.²

Part 2: A Survey of the Empirical Results

The path breaking article written by A.C.M.S. in 1961 'touched off' a veritable explosion in terms of the number of articles and studies which have been carried out in the segment of the literature dealing with the economics of production. At a very early stage it was realized that the special A.C.M.S. case which assumed that the C.E.S. production function was homogeneous of degree '1' could be easily extended to a more general formulation of degree 'h'.³ The next step in the sequence was centered around the developments of more generalized production relationships which included among other things a Variable Elasticity of Substitution (V.E.S.) production function which allowed the elasticity of substitution to vary

¹B.S. Minhas, An International Comparison of Factors Costs and Factor Use, North-Holland Publishing Co., Amsterdam, 1963, pp. 32-42.

²Reference should be made to J. S. Chipman, "A Survey of International Trade: Part 3, The Modern Theory", Econometrica, Volume 34, No.1, (January 1966), pp. 57-70, for an excellent discussion of the C.E.S. production function and the potential impact the A.C.M.S. results have had on conventional international trade theory which is predicated on the invariance of factor intensities.

³J. Paroush, "The h-Homogeneous Production Function with Constant Elasticity of Substitution: A Note", Econometrica, Volume 34, No. 1, (January 1966), pp. 225-227.

freely with the capital-labour ratio.¹ The obvious non-linearity of the C.E.S. function and its more complex successors led to the development of a small body of highly specialized literature dealing with the success obtained in trying out a number of non-linear estimation techniques.² All of these developments were further augmented by investigators who were interested in applying these results in the areas of economic growth, international trade and regional economics. The net result is that at the present time there are a large number of empirical and theoretical articles dealing with the estimation of aggregation production relationships in various temporal and spatial settings.³

It is not particularly edifying to spend time in attempting to survey all of these studies given the fact that a number of the major theoretical issues and empirical results have been set out in publications

¹Refer to the Articles by Y. L. Lu and L. B. Fletcher, "A Generalization of the C.E.S. Production Function" and R. Suto and R. Hoffman, "Production Functions with Variable Elasticity of Factor Substitution: Some Analysis and Testing", contained in "A Symposium on C.E.S. Production Function", Review of Economics and Statistics, Volume L, Number 4, 1968, pp. 443-481.

²Refer to R. G. Bodkin and L. R. Klein, "Non-Linear Estimation of Aggregate Production Functions", Review of Economics and Statistics, Volume XLIX, (February 1967), pp. 28-41 and Hiroki Tsurumi, "Non-Linear Two Stage Least Squares Estimation of C.E.S. Production Functions Applied to the Canadian Manufacturing Industries, 1926-1939, 1946-1967", The Review of Economics and Statistics, Volume LII, Number 2, (May 1970), pp. 200-206.

³It should be noted in passing that the article by F. M. Fisher, "The Existence of Aggregate Production Functions", Econometrica, (October 1969), pp. 553-558 has highlighted the old question dealing with the existence of aggregate production functions. Developments and argumentation on this theme could eventually lead to the use of more meaningful production relationships and/or the reformulation of much of the basic econometric work in this area.

produced by the National Bureau of Economics Research¹ and the Review of Economics and Statistics.² In view of the above, the following procedure has been followed. Nerlove's table summarizing the set of time series and cross-sectional estimates of the elasticity of substitution (Table II-2-1) has been reproduced to give an overview of the wide variation in the results which have been obtained. It is realized that these results in and of themselves are not completely meaningful unless the underlying theoretical models, estimation procedures and sources of data are noted; however, it is equally clear that this end could only be achieved by duplicating Nerlove's work. The following brief overview of the empirical studies will concentrate on the cross-sectional work. The time series studies will be dealt with in a few general comments. The sole justification for treating the time series studies in what may be judged to be a cavalier manner simply rests on the fact that the primary thrust of this thesis lies in the adaptation of the A.C.M.S. procedure to cross-sectional estimates of the elasticity of substitution in Canadian manufacturing industries. The time series work is considerably more complicated both in terms of the theoretical questions that are being pursued--economic growth, technical advance etc.--and the

¹"The Theory and Empirical Analysis of Production", Murray Brown, Editor, National Bureau of Economics Research Studies in Income and Wealth, (Columbia University Press, 1967), The article written by Marc Nerlove, "Recent Empirical Studies of the C.E.S. and Related Production Functions", pp. 55-122, is extremely useful for obtaining an overview of the applied econometric aspects of various production function developments.

²"A Symposium on C.E.S. Production Function Extensions", Review of Economics and Statistics, Volume L, Number 4, (November 1968).

TABLE II-2-1.--Summary of Time Series and Cross-Section Estimates of the Elasticity of Substitution Between Capital and Labour for Two-Digit Manufacturing Industries¹

Industry	Time Series Estimates										Cross-Section Estimate									
	McKinnon (1982)		McKinnon (1983a)		Kendrick (1984)		Ferguson (1985b)		Maddala (1985)		Lucas (1983)	Arrow et al. (1981)	Minnis (1981)	Solo (1984)	All Employees (1965)	Hildebrand (1965)	Liu-Murata-Arrow (1985), Intercountry Data	1953-56	1957-59	Dunne (1985)
Food and kindred products	0.37	n.o.	0.25	0.24	.03- 0.14	.40	0.93	0.58	0.69	2.15	1.29	.72	.73	.56-0.97						
Tobacco manufactures	0.92	n.o.	0.88	1.18	.09- 0.46	.15	n.o.	3.46	1.98	n.o.	n.o.	.83	.83	n.o.						
Textile mill products	0.16	0.44	0.59	1.10	.06- 0.10	.13	0.80	1.58	1.27	1.65	2.08	.79	.83	.69-1.03						
Apparel, etc.	0.69	1.44	0.09	1.08	-.05--0.13	.48	n.o.	n.o.	1.01	1.43	2.38	.66	.80	.54-1.03						
Lumber and timber	0.80	0.56	0.40	0.91	.17- 0.26	.51	0.84	0.94	0.99	0.99	0.91	.82	.92	.78-1.1						
Furniture and fixtures	1.02	0.91	1.86	1.12	.11- 0.21	.49	n.o.	1.09	1.12	0.92	0.96	.90	.79	.70-1.39						
Paper, etc.	0.09	0.94	0.55	1.02	.17- 0.23	.51	1.14	1.60	1.77	1.06	0.71	.90	.79	.20-0.64						
Printing and publishing	0.84	0.94	0.18	1.15	-.04--0.10	.49	1.21	n.o.	1.02	n.o.	n.o.	.84	.93	.68-1.11						
Chemicals, etc.	-1.11	1.12	0.65	1.25	.10- 0.22	.69	0.90	n.o.	0.14	1.25	.88	.84	.83	.31-1.03						
Petroleum and coal	n.o.	n.o.	0.51	1.30	.27- 0.37	.38	n.o.	-0.54	1.45	n.o.	n.o.	.84	.77	.11-1.31						
Rubber and plastics	0.35	n.o.	0.35	0.76	.19- 0.34	.32	0.98	0.82	1.48	1.45	1.39	.83	.70	.40-1.04						
Leather, etc.	0.25	0.52	0.47	0.87	-.01--1.32	.41	0.72	0.86	0.89	0.79	0.93	.71	.70	.51-1.13						
Stone, clay, glass	-1.12	1.08	0.89	0.67	.27- 0.40	-.21	1.08	0.59	0.32	1.28	1.44	.85	.86	.49-0.89						
Primary metals	0.03	n.o.	0.81	1.20	.22- 0.27	.64	n.o.	0.92	1.87	0.99	1.00	.86	.87	.10-0.97						
Fabricated metal products	0.33	n.o.	0.78	0.93	.04- 0.41		n.o.	n.o.	0.80	0.70	0.45	.92	.92	.40-0.95						
Nonelectric machinery	0.75	n.o.	0.50	1.04	.15- 0.25	.48	0.97	1.28	0.37	0.78	1.10	n.o.	n.o.	.12-0.25						
Electrical machinery	0.43	0.64	0.80	0.64	.11- 0.22							n.o.	n.o.	.19-0.62						
Transportation equipment	0.18	n.o.	0.65	0.24	.05- 0.46	.73 ^b	1.04	2.04	0.06	2.01	1.91	n.o.	n.o.	n.o.						
Instruments	0.38	n.o.	-0.14	0.76	.42- 0.58	n.o.	n.o.	n.o.	1.59	1.24	1.65	n.o.	n.o.	n.o.						

n.o. = not obtained.

^aBased on a comparison of the United States and Japan only.

^bAutomobiles only.

¹Merlove, *Op. cit.*, pp. 102-103.

type of econometric techniques which are used.¹ Furthermore, in virtually all of the time series work capital stock data of various sorts and quality are utilized. This data cannot be obtained at a provincial level in Canada.²

Cross-sectional Estimates of Factor Substitution

The results of the cross-sectional studies noted in Table II-2-1 are set out in more detail in Tables II-2-2, 3 and 4. The following points should be noted with respect to these studies. First, the A.C.M.S. study ($\log \left(\frac{V}{L}\right) = \log a + b \log w$) utilized sets of observations on value added per unit of labour and the wage rate in twenty-four, three digit industries located in nineteen countries for a time period spanning the years 1950 to 1955. The general conclusion was that the elasticity of substitution (σ) was greater than zero and less than one for fourteen of the twenty-four industries.

Secondly, Fuchs (Table II-2-2) used the same data and modified the specification to allow for a possible shift in the intercept term. The effect of this type of specification was to allow for differences in the efficiency of labour between two classifications based on a grouping of countries into a 'developing' vis-à-vis a 'developed' category. Unfor-

¹A brief comparison of the econometric models specified in Chapter III and the sophisticated treatment carried out by Yehuda Kotowitz, "Capital-Labour Substitution in Canadian Manufacturing 1926-39 and 1946-61", The Canadian Journal of Economics, (1968), pp. 619-631 clearly illustrates the point. This point is further re-enforced by examining the article written by H. Tsurumi, op. cit., pp. 200-206.

²It was impossible to obtain capital stock data from the Dominion Bureau of Statistics (D.B.S.) for various industries at a provincial level. A letter received from D. Bruce Petrie, Chief, National Wealth and Capital Stock Section, Business Finance Division of D.B.S., July 19, 1969 stated, "Unfortunately, we are unable to provide any capital stock estimates on a regional basis.--and given present priorities and resources it will be some time before we will be able to consider developing regional estimates.-- I am not aware of any comprehensive or reliable set of estimates which would be of use to you."

TABLE II-2-2.--Cross-Section Estimates of the Elasticity of Substitution Between Capital and Labour in Manufacturing Industries

Industry	Arrow, et al. (1961)	Fuchs ^a (1963)	Minsian (1961)	Solow (1964)	Darynes (1965) ^c			
					Liu-Hildebrand (1965) ^b All Employees	Elasticity of Substitution from Regression I	Elasticity of Substitution from Regression II	
Food and kindred products			0.58 (.16)	0.69 (.22)	2.15 ^d	.560 (.122)	0.972 (.132)	
Dairy products	0.72 (.07)	0.90 (.08)						
Fruit and vegetable canning	0.86 (.08)	1.09 (.10)						
Grain and mill products	0.91 (.10)	1.32 (.17)						
Bakery products	0.90 (.07)	1.07 (.11)						
Sugar	0.78 (.12)	0.90 (.18)						
Tobacco	0.75 (.15)	1.22 (.21)	2.48 (.52)	1.96 (.30)				
Textile mill products			1.58 (.35)	1.27 (.15)	1.65	2.08	.976 (.115)	1.033 (.163)
Spinning and weaving	0.61 (.07)	0.98 (.10)						

/continued...

TABLE II-2-2.---continued

Industry	Arrow, et al. (1961)	Fuchs ^a (1963)	Minasian (1961)	Solow (1964)	Lise-Hildebrand (1963) ^b		Dwyane (1963) ^c	
					All Employees	Production Workers	Elasticity of Substitution from Regression I	Elasticity of Substitution from Regression II
Krafting mills	0.79 (.06)	0.95 (.08)		1.01 (.13)	1.43	2.39 ^d	.538 (.134)	1.029 (.181)
Apparel and related products								
Lumber and wood products	0.86 (.07)	1.08 (.14)	0.94 (.11)	0.99 (.09)	1.00	0.91	.779 (.076)	1.101 (.111)
Furniture and fixtures	0.89 (.04)	1.04 (.09)	1.09 (.23)	1.12 (.11)	0.92 ^d	0.96 ^e	.696 (.079)	1.394 (.060)
Pulp, paper, and products	0.97 (.10)	0.91 (.18)	1.60 (.35)	1.77 (1.01)	1.06 ^d	0.72 ^d	.203 (.062)	0.638 (.078)
Printing and publishing	0.87 (.06)	1.02 (.09)		1.02 (.21)			.681 (.125)	1.106 (.061)
Chemicals and products								
Basic chemicals	0.83 (.07)	1.11 (.10)		0.14 (.95)	1.24	0.88	.309 (.096)	1.030 (.063)
Misc. chemicals	0.90 (.06)	1.06 (.09)						
Fats and Oils	0.84 (.09)	1.06 (.18)						

Continued...../

TABLE II-2-2. ---continued

Industry	Arrow, et al. (1961)	Fucho ^e (1963)	Minsian (1961)	Solow (1964)	Liu-Hildebrand (1965) ^b		Dirynee (1965) ^c	
					All Employees	Production Workers	Elasticity of Substitution from Regression I	Elasticity of Substitution from Regression II
Petroleum and coal products			-0.54 (1.06)	1.45 (.71)	Not calculated	.113 (.111)	1.311 (.083)	
Rubber products			0.62 (.29)	1.46 (.88)	1.44	.403 (.088)	1.037 (.144)	
Leather and leather goods	0.66 (.06)	0.98 (.10)	0.93 (.29)	0.89 (.27)	0.79	.508 (.149)	1.126 (1.117)	
Stone, clay, and glass products			0.59 (.25)	0.32 (.46)	1.28 ^d	.491 (.110)	0.887 (.077)	
Clay products	0.92 (.10)	0.66 (.20)						
Glass	1.00 (.08)	1.27 (.10)						
Ceramics	0.90 (.04)	1.08 (.13)						
Cement	0.92 (.15)	1.31 (.22)						
Primary metal products			0.92 (.24)	1.67 (1.25)	0.99 ^e	1.00 ^e	0.969 (.136)	
Iron and steel	0.61 (.05)	0.76 (.11)				.065 (.061)		

/continued.....

TABLE II-2-2.--continued¹

Industry	Arrow, et al. (1961)	Fuchs ^a (1963)	Minesian (1961)	Solow (1964)	Lie-Hildebrand (1965) ^b		Dhrymes (1965) ^c	
					All Employees	Production Workers	Elasticity of Substitution from Regression I	Elasticity of Substitution from Regression I
Nonferrous metals	1.01 (.12)	0.84 (.20)						
Fabricated metal products	0.90 (.09)	1.01 (.17)		0.80 (.29)	0.70 ^d	0.45 ^e	.401 (.135)	0.950 (.149)
Nonelectrical machinery			0.31 (.21)	0.64 (.45)	0.60 ^d	0.44 ^e	.121 (.071)	0.245 (.702)
Electrical machinery	0.67 (.12)	1.03 (.21)	1.26 (.33)	0.37 (.54)	0.79 ^d	1.10 ^d	.194 (.109)	0.620 (.350)
Transportation equipment			2.04 (.49)	0.06 (.82)	2.01 ^d	1.91 ^d		
Instruments and related products				1.59 (.15)	1.24 ^e	1.65 ^d		

^aCountries broken into two groups; shift variable - 1 for Group I, 0 for Group II introduced. Group I: United States, Canada, New Zealand, Australia, Denmark, Norway, United Kingdom, Ireland, Puerto Rico. Group II: Colombia, Brazil, Mexico, Argentina, El Salvador, Southern Rhodesia, Iraq, Ceylon, Japan, India.

^bComputed at 1957 share of capital in value added. See Tables 4-6 for the method and results.

^cRegression I is logarithmic regression of value added per unit of labor on the wage rate; regression II is logarithmic regression of value added per unit of capital on the rate of return to capital; state data from 1957 Census of Manufactures.

^dCapital-labor ratio coefficient in regression is more than twice its standard error.

^eCoefficient of capital-labor ratio nearly twice its standard error.

¹Nerlove, *op. cit.*, pp.61-63.

TABLE II-2-3.--Elasticities of Substitution on Two-Digit Manufacturing Industries, United States, 1957¹

$$Q = B(a_1K^\alpha + a_2L^\beta)^{1/\sigma}, \quad \sigma = \frac{1}{1-\delta}$$

$$\frac{Q}{L} = A^*W^\alpha \quad \frac{Q}{K} = A^{**}P^\alpha$$

Industrial Classification	Industry	σ_1	R^2	σ_2	R^2	Number of Observations
20	Food and Kindred Products	0.2222 (0.1636) ^a	0.3966	0.7683 (0.0690)	0.7655	40
22	Textile Mill Products	0.9675 (0.1540)	0.6752	0.8128 (0.0953)	0.7930	21
23	Apparel and Related Products	1.2140 (0.1965)	0.6345	0.7681 (0.0763)	0.8216	24
24	Lumber and Wood Products	0.8750 (0.0694)	0.8833	0.6352 (0.0774)	0.7624	23
25	Furniture and Fixtures	1.1730 (0.1250)	0.8150	0.7427 (0.0309)	0.9665	22
26	Pulp, Paper and Products	1.4321 (0.4541)	0.2626	1.1558 (0.1088)	0.8012	30
27	Printing and Publishing	0.9980 (0.2061)	0.6436	0.8255 (0.0586)	0.9385	15
28	Chemicals and Products	0.8697 (0.2715)	0.2554	0.9059 (0.0481)	0.9219	32
29	Petroleum and Coal Products	0.8915 (0.5491)	0.1500	0.7702 (0.0370)	0.9666	17
30	Rubber Products	1.5625 (0.3491)	0.5888	0.8082 (0.0970)	0.8322	16
31	Leather and Leather Goods	0.8573 (0.2612)	0.4348	0.8170 (0.0825)	0.8751	16
32	Stone, Clay and Glass Products	1.0273 (0.1920)	0.5341	0.9682 (0.0646)	0.9000	27
33	Primary Metal Industries	0.7654 (0.3965)	0.1255	0.8360 (0.0963)	0.7436	28
34	Fabricated Metal Products	0.5570 (0.1982)	0.2043	0.7172 (0.0955)	0.6451	33
35	Machinery, Except Electrical	0.7468 (0.4662)	0.0843	0.3644 (0.1218)	0.2422	30
36	Electrical Machinery	0.5915 (0.3625)	0.1041	0.7294 (0.0596)	0.8667	25
37	Transportation Equipment	1.2428 (0.6367)	0.1281	0.6854 (0.1310)	0.5129	28

^aNumber in parenthesis is the standard error of the estimate.

¹P. J. Dhrymes and P. Zarembka, "Elasticities of Substitution for Two Digit Manufacturing Industries: A Correction", Research and Statistics, Volume LII, No. 1, (February 1970), pp. 115-117.

TABLE II-2-4.--Intercountry Estimates of the Elasticity of Substitution for Two-Digit Manufacturing Industry, Murata-Arrow (1965)^a

Industry	Data for 1953-56:				Data for 1957-59:			
	Estimate Based on:		Estimate Based on:		Estimate Based on:		Estimate Based on:	
	Log Regression of Value Added per Employee on Wage Rate	Log Regression of Value Added per Employee on Wage Rate	Degrees of Freedom	R ²	Log Regression of Value Added per Employee on Wage Rate	Log Regression of Value Added per Employee on Wage Rate	Degrees of Freedom	R ²
Food, beverages and tobacco	.722 (.054)	.799 (.075)	19	.903	.725 (.054)	0.801 (.074)	19	.906
Textiles	.793 (.049)	.851 (.082)	19	.932	.827 (.069)	0.931 (.084)	18	.888
Clothing, footwear, and made-up textiles	.600 (.067)	.775 (.102)	17	.851	.804 (.043)	0.841 (.054)	16	.950
Wood products and furniture	.818 (.039)	.920 (.083)	18	.880	.919 (.074)	1.025 (.080)	18	.896
Paper and paper products	.904 (.050)	.955 (.055)	18	.947	.788 (.061)	0.874 (.078)	18	.901
Printing and publishing	.836 (.075)	.951 (.090)	17	.879	.926 (.063)	0.999 (.068)	17	.927
Leather and leather goods	.711 (.059)	.801 (.083)	18	.888	.699 (.050)	0.761 (.072)	17	.919
Rubber products	.829 (.058)	.889 (.069)	15	.933	.768 (.106)	1.000 (.137)	16	.768
Chemicals, petroleum, and coal	.838 (.050)	.887 (.060)	16	.946	.834 (.087)	0.988 (.104)	17	.844
Stone, clay, glass	.847 (.046)	.898 (.054)	20	.945	.859 (.051)	0.920 (.060)	20	.934
Primary metals	.856 (.063)	.943 (.077)	17	.909	.873 (.063)	0.946 (.072)	16	.923
Metal products	.917 (.052)	.970 (.053)	18	.945	.922 (.069)	1.011 (.076)	17	.912

Sources: Exchange rate from Year Book of Labor Statistics, 1963, International Labor Organization, Geneva, 1963. Value added (local currency), wage and salary payments (local currency), number of employees from The Growth of World Industry, 1938-1961, United Nations, 1963.

^aCountries used were selected from the following list in each case: Australia, Belgium, Canada, Denmark, El Salvador, Finland, India, Iraq, Ireland, Japan, Luxembourg, Mexico, New Zealand, Norway, Pakistan, Philippines, Portugal, Puerto Rico, Singapore, Sweden, United Arab Republic, United Kingdom, and United States.

^bApproximate standard errors based on Taylor's series expansion; see L. R. Klein, A Textbook of Econometrics, Evanston, Ill. 1953, p. 258.

unately, Fuchs included Japan in the group of developing countries as well as imposing the constraint that the estimates of the elasticity of substitution (b) would be identical for both groups. The results obtained from this specification suggest that twenty-two of the twenty-four industries have an elasticity of substitution which is not significantly different from one.

Thirdly, Minasian and Solow used the original A.C.M.S. specification to obtain estimates of the elasticities for various manufacturing industries in the United States for the years 1956 and 1957. The regional breakdowns used were at the state level (Minasian) and sets of states (Solow). Comparison of the two results indicates that there are substantial differences in certain industries. Nerlove outlined a number of possible factors which might tend to explain these differences ranging from the observation that 1957 was a recessionary year as compared to 1956 to suggesting that the heterogeneity of the product mix at the state level would be more extreme than at a higher level of aggregation. He concludes with the statement that "when there is a lot of noise in the system apparently small changes can produce substantial variation in the system".¹

Fourthly, Dhrymes used the A.C.M.S. specification for the same year and the same type of regional breakdown as employed by Minasian. Inspection of both sets of results indicates that there are substantial differences. Nerlove's remark with respect to this inexplicable state of affairs is as follows: "The only explanation for these gross differences appears to be a slight difference in the basic series employed."² In effect, the

¹Marc Nerlove, op. cit., p. 70.

²ibid., p. 74.

estimates of the elasticities of substitution obtained from cross-sectional analysis seem to be inordinately sensitive to very small changes in the data.¹

Fifthly, Murata and Arrow adopted the A.C.M.S. technique and carried out a cross-sectional study for twelve, two digit industries for the years 1958-1956 and 1957-1959 on an international basis. (Twenty-three countries including Canada.) The results set out in Table II-2-4 indicate a mixture of unitary and inelastic values of the various elasticities. Clearly, there is nothing to support any contention that a two digit industry classification of industrial activity results in significantly different values from a three digit breakdown.

Lastly, the Liu-Hildebrand results (Table II-2-5) are based on estimates of the elasticity derived from a general production function which includes the C.E.S. production homogeneous of degree '1' as a special case. This function can be expressed in per capita terms by:

$$y = [\beta x^{-\rho} + \alpha x^{-m\rho}]^{-1/\rho} \quad (\text{II-2-1})$$

$$\text{where } \rho = \frac{1-b}{b}, \quad m = \frac{g}{1-b}, \quad \alpha = \frac{1-b}{(1-b-g)a^{1/b}}, \quad \beta = \frac{-c(1-b)}{a^{1/b \cdot b}}$$

If $g = 0$ then $m = 0$ which implies that this production function is reduced to the C.E.S. constant returns to scale case.

It should be noted that this function is generated by extending the basic A.C.M.S. specification to include K/L as well as w . In effect,

$$\log y = \log a + b \log w + g \log x \quad (\text{II-2-2})$$

(where $x = K/L$)

¹Reference should be made to Table II-2-3 for a revised version of Dhrymes' results since it was discovered that the results reported in the last two columns of Table II-2-2 were obtained by regressing the wage rate and the return to capital on the value added per unit of labour and capital, respectively.

Substitution of the equilibrium condition

$$w = y - x \frac{dy}{dx}$$

into the above yields a differential equation which upon solution has the form depicted in (II-2-1).

The elasticity of substitution is expressed by

$$\sigma = \frac{1}{1 + \rho - \frac{\rho m}{S_k}} \quad (\text{II-2-3})$$

where (S_k) capital's share in the product is expressed by

$$S_k = \frac{f(x)}{xf'(x)}$$

This indicates that the elasticity of substitution is a function of $x = \frac{K}{L}$ which implies that under the Liu-Hildebrand specification the elasticity of factor substitution varies with changes in the K/L ratio brought about by changes in the factor price ratio.^{1,2}

Liu and Hildebrand used the 1956 "Survey of Manufactures" data of two digit industries partitioned on a state basis to carry out their estimates. Results are reported for two different categories of labour breakdown provided in the manufacturing sector. It is evident from inspection of Table II-2-5 that 'g' is statistically significant and greater than zero in a number of industries. In effect, if $\rho \geq 0$ and $m \geq 0$ given

¹ ibid., p. 75-82.

² The basic Liu-Hildebrand formulations outlined here were also used by Y. L. Lu and L. B. Fletcher, op. cit., pp. 449-450, and by Y. Kotowitz, op. cit., pp. 624-625 and 628. It is interesting to note that Kotowitz found that the Liu-Hildebrand form of the production function did not provide as good a fit as other competing specifications.

TABLE II-2-5.--Summary of Liu-Hildebrand (1965) Regressions for Two-Digit Industries¹

Industry	All Employees				Production Workers Only			
	b	g	R ²	N	b	g	R ²	N
Food and kindred products	0.407 (.177)	.446 (.139)	.548	35	0.282 (.144)	.430 (.148)	.464	35
Textile mill products	0.975 (.175)	.160 (.109)	.695	18	1.427 (.299)	.122 (.156)	.641	18
Apparel and related products	1.071 (.263)	.097 (.086)	.669	18	1.094 (.374)	.211 (.102)	.617	18
Lumber and wood products	0.990 (.135)	.002 (.070)	.943	14	0.989 (.165)	-.073 (.090)	.920	14
Furniture and fixtures	1.258 (.128)	-.154 (.072)	.859	19	1.402 (.177)	-.191 (.102)	.807	19
Pulp, paper, and products	0.386 (.322)	.331 (.050)	.730	28	0.298 (.340)	.304 (.069)	.657	28
Chemicals and products	0.866 (.231)	.201 (.085)	.424	31	0.780 (.254)	.076 (.109)	.309	31
Petroleum and coal products	0.180 (.716)	.282 (.224)	.152	18	-0.027 (.951)	.309 (.283)	.213	18
Rubber products	1.278 (.553)	.018 (.217)	.523	16	1.231 (.286)	-.052 (.132)	.738	16
Leather and leather goods	0.890 (.457)	-.050 (.113)	.368	15	0.926 (.528)	.0003 (.118)	.434	15
Stone, clay, and glass products	0.539 (.177)	.295 (.065)	.611	25	0.568 (.175)	.309 (.069)	.627	25
Primary metal products	0.298 (.704)	.321 (.141)	.234	28	0.187 (.683)	.374 (.154)	.250	28
Fabricated metal products	0.401 (.207)	.178 (.068)	.336	32	0.189 (.208)	.243 (.080)	.298	32
Machinery except electrical	0.222 (.263)	.258 (.100)	.343	25	0.222 (.226)	.204 (.104)	.262	25
Electrical machinery	0.300 (.210)	.278 (.071)	.483	22	0.606 (.233)	.202 (.089)	.494	22
Transportation equipment	1.008 (.448)	.214 (.060)	.504	26	0.998 (.545)	.205 (.073)	.441	26
Instruments and related products	0.601 (.294)	.217 (.116)	.681	12	0.874 (.264)	.196 (.098)	.805	12

N = number of observations.

^aSummary of estimates of regressions of the form

$$\log V/L = a + b \log w + g \log K/L$$

for two-digit industries from *Census of Manufactures, 1957*, in Liu-Hildebrand (1965, pp. 35-39).

¹Nerlove, *op. cit.*, p. 78.

$g \geq 0$ then the estimate of the elasticity obtained--if only 'b', the coefficient of $\log w$ in (II-2-2) was used--would be less than the estimate given by (II-2-3). If $\rho < 0$ and $m < 0$ and $g < 0$ then the Liu-Hildebrand estimate of σ will be less than in the A.C.M.S. case. The point to note is that if the above specification is accepted then the A.C.M.S. estimates of σ are biased in an upward or downward direction.

Time Series and Cross-Sectional Studies

A cursory examination of Table II-2-1 indicates that the cross-sectional estimates of the elasticities of substitution appear to be greater than their time series counterparts. This is not a fleeting impression as is evidenced by the series of comments made by Kotowitz at the close of his rather extensive investigation of capital and labour substitution in Canadian manufacturing between 1926-39 and 1946-61.

The conclusions of the study are:

- (1) the elasticity of factor substitution $0 < \sigma < 1$ for aggregate manufacturing and for various two digit manufacturing industries;
- (2) the elasticity of substitution has decreased from 0.7 in the pre-war period to 0.3 in the post war period; and
- (3) the two digit estimates of σ are similar to the estimates obtained for the United States.^{1,2}

¹Yehuda Kotowitz, "Technical Progress, Factor Substitution and Income Distribution in Canadian Manufacturing 1926-39 and 1946-61", The Canadian Economic Journal, (February 1969), pp. 106-114.

²Kotowitz states that "A comparison with Lucas shows fairly close correspondence for all those industries where the values of σ are significant in both studies...McKinnon's results are generally much poorer. However, for these industries where σ is significant they too correspond fairly closely to our results." (Refer to Table II-2-1 for both McKinnon's and Lucas' results.) Y. Kotowitz, "Capital-Labour Substitution in Canadian Manufacturing 1926-39 and 1946-61", op. cit., p.629.

Given these results he notes:

It must be pointed out that cross-section results suggest that the elasticity of substitution is close to one. Attempts at reconciliation between cross-section and time series analysis have not been successful. Thus, although I believe that the time series results offered here are better for analysis of aggregate movements over time than cross-section results, it must be acknowledged that this unexplained contradiction in the evidence exists.¹

In effect, there are substantial differences within and between various cross-sectional and time series estimates of the elasticity of substitution.

One point that has emerged from the foregoing discussion is that the majority of investigators have assumed that the observations obtained from various industries across different regions (states, countries etc.) are obtained from a common production relationship.² This maintained hypothesis should be subjected to empirical testing since it is not at all clear that one can assume on 'a priori' grounds that the production relationship obtained for some two, three or four digit aggregations of industries--given observable differences in the industrial mix in a country, province or state--can be said to be identical. One of the major reasons for the apparent sensitivity and diversity of the various estimates of σ may simply stem from the fact that various investigators have used different production function mixes to obtain their estimates of the elasticity of substitution in a given industry. One of the chief objectives of the analysis set out in the following chapters is to determine whether there are significant inter-regional differences in the

¹Kotowitz, "Technical Progress, Factor Substitution and Income Distribution in Canadian Manufacturing 1926-39 and 1946-61", op. cit., p. 113.

²Arrow, et. al., in "Capital-Labour Substitution and Economic Efficiency", op. cit., p. 237, noted that their assumption of a common production function across countries was a "naive" hypothesis.

elasticity of substitution in various manufacturing industries located
in Canada.

CHAPTER III

INTER-REGIONAL DIFFERENCES IN THE ELASTICITY OF FACTOR SUBSTITUTION AND THE EFFICIENCY OF LABOUR USE

The brief overview of the various studies surveyed in Chapter II suggests it is highly advisable to determine whether there are significant inter-regional and inter-industry differences in the estimates of the parameters of various aggregate production functions. The discussion and empirical results set out in this Chapter attempt to deal with the first issue; however, the scope of the inquiry is restricted to the examination of the elasticity of substitution and the efficiency of labour parameters. The reason that the investigation is limited in this fashion stems from the fact that there are no reliable breakdowns of capital stock data at the provincial level in Canada.¹ Given this constraint the set of possible theoretical specifications is reduced to one element--the basic A.C.M.S. specification noted in Chapter II.

$$\log \frac{V}{L} = \log a + b \log W \quad (\text{III-1})$$

Given (III-1) an attempt was made to obtain a detailed breakdown of observations on value added per unit of labour and the average wage rate

¹Refer to footnote '2', p. 16, Chapter II.

in each of the twenty, two digit manufacturing industries located in the ten provinces of Canada. The basic objective was to determine whether there were any significant inter-regional differences in the two production coefficients set out in (III-1). The problems and eventual "working compromises" which were reached with respect to the breakdown of the data used in the study are summarized in the following discussion.

The Data

The data used in the study were obtained from the Dominion Bureau of Statistics publications "Manufacturing Industries of Canada" for the years 1964 and 1965.¹ This data consisted of observations on the average wage rate and value added in manufacturing (production) categorized by province and by a twenty, two digit standard classification of industrial activity for each of the respective years. D.B.S. breaks these twenty major (two digit) industry groups into 140 three digit industries. Eighteen of these industries are broken down into fifty-six four digit industries. The confidentiality constraint imposed by the Statistics Act severely restricts the amount of information that is provided at the regional level for certain industries. The net result is that the observations found under any particular two digit heading will vary from a single two digit entry to a set of three and four digit observations.

All of the observations that were published for each two digit industry in each province were utilized--double counting was avoided. This procedure differs from that of a number of other investigators who have employed two or three or four digit observations exclusively;

¹D.B.S. Manufacturing Industries of Canada, sections B, C, D, E and F (1964 and 1965 respectively). (Queen's Printer, Ottawa, July 1968 and 1969).

however, it is argued that the procedure is perfectly legitimate as long as one accepts the maintained hypothesis that two digit industries have a common technology. In effect, investigators who use a single two digit observation for a given region are simply recording a consolidation of three and four digit activity without obtaining any reflection of the inter-industry variation within the region.

It is suggested that even if this argument is not accepted the worst possible interpretation is that the observations reflect the activity of some "average" composite industry which is neither two, three or four digit in nature. This would not be any more or less meaningful than the standard artifact which is an "average" two digit industry. Cross-sectional observations in Canada are obtained from a ten province partitioning of economic activity. One either has to live with this size of sample and in many cases not carry out a study or opt for larger samples and run the risk of losing the purity of a particular industrial classification.

A number of constraints were imposed on the scope of the regression analysis due to the paucity of observations in a number of the twenty industry, ten region cells set out in these publications. The following steps were taken to obtain samples with a 'viable' number of observations. First, observations were pooled for the years 1964 and 1965.¹ The results of the pooling were not favorable in the sense that a large number of the

¹This pooling of the data rests on the assumption of temporal stability. This point is examined in some detail in the following section.

cells still did not contain samples of a 'reasonable' size. This situation provided the impetus for the second step which resulted in a reduction in the number of regions from ten to a group of three: Western Canada, Central Canada and the Atlantic Provinces.¹ As a third measure three of the twenty, two digit industries had to be eliminated because they were either concentrated in one or at most two of the regions or regulations under the Statistics Act prevented D.B.S. from publishing the data.²

The Econometric Analysis

Format of the Analysis

The discussion dealing with the industrial and regional breakdown of manufacturing data indicates that the analysis should be divided into three parts. Part 1 deals with the question of whether any bias is introduced into the analysis by pooling 1964 and 1965 data. Given these results Part 2 deals with the effect regional influences exert on the elasticity of factor substitution and the efficiency of labour in various two digit manufacturing industries located in the Atlantic Provinces and Western and Central Canada. Part 3 deals with economic interpretation and overall significance of the results.

The analyses contained in each of these parts are organized in the following manner. Firstly, the general specification of the appropriate regression models is given. Secondly, a joint test of all the

¹ Refer to Table III-1 for a listing of the individual provinces and a definition of the three major sub-groups.

² The list of two digit manufacturing industries included in the study is set out in Table III-2. The industries excluded due to their limited representation in various regions of the country are the Tobacco Products, Rubber and Knitting Mills Industries.

TABLE III-1.--Classification of Regional Groups.

I. PROVINCIAL GROUPING

<u>Number</u>	<u>Region</u>
0	British Columbia
1	Alberta
2	Saskatchewan
3	Manitoba
4	Ontario
5	Quebec
6	Newfoundland
7	Prince Edward Island
8	Nova Scotia
9	New Brunswick

II. WESTERN, CENTRAL AND ATLANTIC GROUPING

10	Western Canada (British Columbia, Alberta, Saskatchewan, Manitoba)
11	Central Canada (Ontario and Quebec)
12	Atlantic Provinces (Nova Scotia, New Brunswick, Prince Edward Island and Newfoundland)

TABLE III-2.--Classification of Industry Groups

S. I. C. No.	INDUSTRY
1	Food and Beverage
4	Leather Industries
5	Textile
7	Clothing
8	Wood
9	Furniture and Fixture
10	Paper and Allied
11	Printing, Publishing and Allied
12	Primary Metals
13	Metal Fabricating (except Machinery and Transportation Equipment Industries
14	Machinery Industries (except Electrical Machinery)
15	Transportation Equipment
16	Electrical Products
17	Non-Metallic Mineral Products
18	Petroleum and Coal Products
19	Chemical and Chemical Products
20	Miscellaneous Manufacturing

differential intercept and slope coefficients is carried out to determine whether these coefficients differ significantly from zero. If this null hypothesis is rejected individual tests of various dummy coefficients are carried out.

Part 1: Inter-Temporal Effects

The answer to the question of whether the pooling of data for the years 1964 and 1965 introduces any bias into the estimates of the elasticity of factor substitution and the efficiency of labour is obtained by determining whether there are any significant differences in the coefficients of two linear regressions in each of the seventeen, two digit manufacturing industries. This can be shown by the following.

The Specification

Rewrite (III-1) as:

$$Y_i = \alpha + \beta X_i + \epsilon_i \quad (\text{III-1-1})$$

where $Y = \log \frac{V}{L}$, $X = \log W$ (W - wage rate, ϵ - stochastic term and α and β are the intercept and slope coefficients with the same interpretation as noted previously.

The two regression models for the j^{th} industry are:

$$Y_{1i} = \alpha_1 + \beta_1 X_{1i} + \epsilon_{1i}, \quad i = 1 \dots n_1 \quad (\text{III-1-2})$$

$$Y_{2i} = \alpha_2 + \beta_2 X_{2i} + \epsilon_{2i}, \quad i = 1 \dots n_2$$

where the subset of observations labelled '1' refers to 1964 and '2' refers

to 1965. If inter-temporal effects are unimportant $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$ and the specification set out in (III-1-1) can be used for all $i = [1, 2, \dots, N]$ observations where $N = N_1 + N_2$. Clearly, other 'less desirable' results can occur.

The procedures adopted for testing the coefficients are as follows:

$$\text{Let } D_1 = \begin{cases} 1 & \text{for all observations in 1965} \\ 0 & \text{for all observations in 1964} \end{cases}$$

The dummy variable form of the ordinary least squares regression model can be written as:

$$Y_i = \gamma_0 + \gamma_1 D_1 + \gamma_2 X_i + \gamma_3 (D_1 X_i) + U_i \quad (\text{III-1-3})$$

where γ_0 and γ_2 are the intercept and slope coefficient for the base year 1964 and γ_1 and γ_3 are the differential intercept and slope coefficients. γ_0 and γ_2 are equal to α_1 and β_1 respectively while $(\gamma_0 + \gamma_1) = \alpha_2$ and $(\gamma_2 + \gamma_3) = \beta_2$. (U_i is the stochastic term.) Ordinary regression procedures applied to the single equation set out in (III-1-3) provide the basis for determining whether significant differences exist between the coefficients in (III-1-2). In effect, $H_0: \gamma_1=0$ and $H_0^1: \gamma_3=0$ are tested by the standard 't' test for individual regression coefficients.

The above tests of individual dummy coefficients are extremely useful; however, the first point that must be clarified is whether the joint contribution made by all dummy variables (intercept and slopes taken together) are significantly different from zero. This may be achieved by recognizing that (III-1-1) incorporates a hypothesis which assumes that temporal factors do not influence either the efficiency of labour or the elasticity of factor substitution (the intercept or slope terms) while

(III-1-3) allows for temporal influences by allowing both the intercepts and slopes to vary freely. The basis for testing whether temporal effects are important simply amounts to determining whether the increment in the residual sums of squares resulting from the removal of the dummy variables from (III-1-3) is significant. If "S" is the residual sum of squares obtained from the analysis of variance of the completely restricted regression (III-1-1) and "S₁" is the residual sum of squares obtained from (III-1-3)--the completely unrestricted model--then the appropriate joint test of the null hypothesis H₀:γ₁=γ₃=0 is given by:¹

$$F = \frac{(S - S_1)/K(P - 1)}{S_1/N - pk}$$

K refers to the number of explanatory variables; 'p' is the number of classes and 'N' is the total number of observations.

The Results:

The appropriate 'F' values for the seventeen, two digit manufacturing industries are set out in Table (III-1-1). Casual inspection is sufficient to indicate that the null hypothesis cannot be rejected in any of the industries. Reference to the regression results set out in Table (III-1-2) indicates that the 't' values for all of the individual dummy coefficients are highly insignificant. On the basis of these two sets of results it is reasonable to conclude that there are no intertemporal differences in the estimates of the efficiency of labour and the elasticity of factor substitution in Canadian manufacturing between 1964 and 1965.

¹Refer to J. Stewart and A. J. Rayner, "Qualitative Factors in Linear Regression Analysis", The Manchester School (April 1971), p. 350 for a definition of this test and an extensive discussion of the dummy variable and related techniques. A text book exposition is given by J. Johnston, Econometric Methods, 2nd. edition, (McGraw Hill, 1972), pp. 192-207.

TABLE III - 1 - 1: Joint Tests of Significance Pertaining to the Pooling of Data for 1964 and 1965 in Seventeen, Two digit Manufacturing Industries

S.I.C. Name	"F" Value	Degrees of Freedom	
		Numer-ator	Denom-inator
Food and Beverage	0.29	2	211
Wood	0.55	2	89
Paper and Allied	0.81	2	54
Printing, Publishing and Allied	0.75	2	50
Metal Fabricating	0.13	2	106
Transportation	0.83	2	58
Non-Metallic Minerals	0.47	2	100
Miscellaneous Manufacturing	0.45	2	163
Leather	0.03	2	22
Textile	0.55	2	69
Clothing	0.03	2	56
Furniture and Fixture	1.11	2	32
Primary Metal	0.22	2	37
Machinery Industries	0.58	2	27
Electrical Products	0.11	2	37
Petroleum and Coal Products	0.37	2	16
Chemical and Chemical Products	0.05	2	63

TABLE III-1-2.--Results of the Full Regression Analyses Pertaining to the Pooling of Data in Seventeen Manufacturing Industries in Canada for the Years 1964 and 1965.

Industry Name	Intercept for Base year 1964	Differential Intercept	Slope Coefficient for Base year 1964	Differential slope coefficient	Sample size	R ²
Food and Beverage	1.065 (0.082) t=12.935	-0.057 (0.123) t=-0.458	1.663 (0.144) t=11.521	0.029 (-0.209) t=0.137	215	0.550
Wood	0.865 (0.067) t=12.982	-0.084 (0.096) t=-0.867	0.765 (0.150) t=5.098	-0.218 (0.208) t=1.050	93	0.452
Paper and Allied	1.055 (0.205) t=5.148	-0.217 (0.303) t=-0.717	1.184 (0.287) t=4.119	0.181 (0.412) t=0.438	58	0.416
Printing, Publishing and Allied	0.650 (0.355) t=1.832	0.458 (0.477) t=0.960	1.513 (0.423) t=3.581	-0.616 (0.571) t=-1.079	54	0.270
Metal Fabricating	1.039 (0.105) t=9.936	-0.070 (0.154) t=-0.453	0.824 (0.141) t=5.842	0.100 (0.204) t=0.492	110	0.412
Transportation Equipment	0.616 (0.142) t=4.334	0.179 (0.205) t=0.872	1.576 (0.194) t=5.962	-0.155 (0.273) t=-0.567	62	0.534
Non-Metallic Mineral Products	0.843 (0.195) t=4.331	0.222 (0.297) t=0.749	1.441 (0.285) t=5.051	-0.249 (0.418) t=-0.595	104	0.307
Miscellaneous Manufacturing	1.152 (0.075) t=15.309	0.061 (0.111) t=0.546	0.601 (0.152) t=3.957	-0.052 (0.216) t=-0.242	157	0.158

/continued....

TABLE III-1-2 continued.

Industry Name	Intercept for Base year 1964	Differential Intercept	Slope coef- ficient for Base year 1964	Differential slope coef- ficient	Sample size	R ²
Leather	0.680 (0.083) t=8.230	0.011 (0.132) t=0.085	1.072 (0.235) t=4.567	-0.053 (0.347) t=-0.154	26	0.635
Textile	1.007 (0.084) t=11.954	-0.076 (0.119) t=-0.638	0.910 (0.209) t=4.352	0.080 (0.273) t=0.294	73	0.427
Clothing	0.721 (0.071) t=10.122	-0.025 (0.115) t=-0.221	1.095 (0.208) t=5.275	0.045 (0.305) t=0.148	60	0.498
Furniture and Fixture	0.954 (0.113) t=8.457	0.132 (0.165) t=0.797	0.714 (0.223) t=3.198	-0.138 (0.319) t=-0.433	36	0.384
Primary Metals	1.164 (0.533) t=2.182	-0.067 (0.792) t=-0.084	0.730 (0.615) t=1.187	0.155 (0.894) t=0.174	41	0.101
Machinery Industries	1.391 (0.246) t=5.657	0.195 (0.361) t=0.540	0.567 (0.327) t=1.734	-0.148 (0.466) t=-0.317	31	0.190
Electrical	1.476 (0.218) t=6.765	0.049 (0.288) t=0.169	0.599 (0.328) t=1.827	-0.031 (0.425) t=-0.073	41	0.183
Petroleum and Coal Products	2.236 (0.561) t=3.988	-0.517 (0.699) t=-0.739	0.638 (0.572) t=1.115	0.464 (0.716) t=0.649	20	0.341
Chemical and Chemical Products	2.054 (0.255) t=8.064	0.045 (0.341) t=0.131	0.611 (0.351) t=1.739	-0.091 (0.463) t=-0.197	67	0.087

Part 2: Regional Influences on the "Efficiency" of
Labour and the Elasticity of Factor Substitution

The seventeen, two digit set of manufacturing industries are divided into an eight industry, three region classification covering the Dominion and a nine industry two region grouping which is confined to Central and Western Canada. The same procedure as outlined in Part 1 is employed; however, it is useful to set out the general format of the models and the requisite null hypotheses.

Given a situation in which there are $j = 1, 2, \dots, J$ industries, $r = 1, 2, \dots, R$ regions and $i = 1, 2, \dots, N_j$ observations on value added per unit of labour (Y) and the wage rate (X) in each industry--one can express the completely unrestricted model for the j^{th} industry in the following manner. Let $r = 1$ be the base region and $D = [D_1 D_2 \dots D_R]$ be a $1 \times R$ vector of dummy variables, where $D_1 \equiv 1$ and $D_k = 1$ for all observations in the k^{th} region and '0' for all observations located in any other region.

$$B_1 = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_R \end{bmatrix}$$

Rx1

is a column vector of intercept coefficients where β_1 is the intercept for the base region and $\beta_2 \dots \beta_R$ are differential intercepts for the remaining regions.

$$B_2 = \begin{bmatrix} \beta_{R+1} \\ \beta_{R+2} \\ \vdots \\ \beta_{2R} \end{bmatrix}$$

Rx1

where β_{R+1} is the slope coefficient for the base region and the remaining elements are differential slope coefficients.

The dummy variable form of the ordinary least squares model for the j^{th}

industry is expressed as:

$$Y_i = D B_1 + X_i D B_2 + U_i \quad (\text{III-2-1})$$

where $i = (1, 2, \dots, N)$ $N = \sum_{r=1}^R N_r$, and U is the stochastic variable.

Alternatively one can express (III-2-1) as:

$$Y_i = \beta_1 + \sum_{r=2}^R \beta_r D_r + \beta_{R+1} X_i + \sum_{r=2}^R \beta_{R+r} (D_r X_i) + U_i$$

In the above model all of the intercept and slope coefficients are allowed to vary freely. The appropriate null hypothesis is:

$$H_0: \beta_2 = \dots = \beta_R = \beta_{R+2} = \dots = \beta_{2R} = 0$$

The totally restricted model was set out in (III-1-1) as:

$$Y_i = \alpha + \beta X_i + \epsilon_i \quad i = 1, 2, \dots, N$$

The Eight Industry, Three Region Results

The results of the tests for the overall homogeneity of the differential intercept and slope coefficients in eight, two digit manufacturing industries located in the Atlantic Provinces, Central and Western Canada are set out in Table (III-2-1). Five industries exhibit significant regional differences at the five percent and ten percent levels vis-à-vis four industries at the one percent level.

The regression results obtained for each of the eight industries are set out below. It should be noted that only five of these industries-- Food and Beverages, Printing, Publishing and Allied, Metal Fabricating, Transportation and Miscellaneous Manufacturing--qualify for a detailed examination of individual dummy coefficients. The appropriate results

TABLE III-2-1.--Joint Tests of Significance (differential Intercepts and Slopes) for Regional Influences in Eight Two Digit Manufacturing Industries Located in Western Canada, Central Canada and the Atlantic Provinces

S.I.C. Name	"F" Value	Degrees of Freedom		Significance Levels	
		Numer-ator	Demon-inator	10%	5%
Food and Beverages	6.561*	4	209	R	R
Wood	1.538	4	87	A	A
Paper and Allied	2.036	4	52	A	A
Printing, Publishing and Allied	6.763*	4	48	R	R
Metal Fabricating	2.600	4	104	R	R
Transportation Equipment	5.820*	4	56	R	R
Non-Metallic Minerals	1.010	4	98	A	A
Miscellaneous Manufacturing	3.552*	4	161	R	R

*reject at the 1% level

A - accept or R - reject the null hypothesis

for the Wood, Paper and Allied and Non-Metallic Minerals industries are obtained from specification (III-1-1); however, both sets of results are presented for the sake of completeness.

The Significant Industries

Food and Beverage Industries¹

$$\hat{Y} = 0.976 - 0.240D_1 + 0.150D_2 + 1.598X + 0.702(D_1X) + 0.249(D_2X) \quad (\text{III-2-2})$$

(8.182) (-1.420) (1.006) (8.853) (2.666) (0.732)

$N = 215, R^2 = 0.599$

Printing, Publishing and Allied Industries

$$\hat{Y} = 0.895 - 1.061D_1 - 0.744D_2 + 1.096X + 1.033(D_1X) + 1.706(D_2X) \quad (\text{III-2-3})$$

(1.667) (-1.109) (-1.234) (1.810) (1.008) (2.299)

$N = 54, R^2 = 0.523$

Metal Fabricating Industries

$$\hat{Y} = 0.986 + 0.194D_1 + 0.301D_2 + 0.904X - 0.223(D_1X) - 0.650(D_2X) \quad (\text{III-2-4})$$

(9.997) (0.878) (1.484) (7.128) (-0.777) (-2.031)

$N = 110, R^2 = 0.462$

$$D_1 = \begin{cases} 1 & \text{for observations in Central Canada and} \\ 0 & \text{otherwise} \end{cases}$$
$$D_2 = \begin{cases} 1 & \text{for observations in the Atlantic Provinces} \\ 0 & \text{otherwise} \end{cases}$$

The figures in brackets are the 't' results.

Transportation Equipment Industries

$$\hat{Y} = 0.758 + 0.024D_1 + 0.216D_2 + 0.971X - 0.136(D_1X) - 0.721(D_2X) \quad (\text{III-2-5})$$

(5.096) (0.110) (0.922) (5.085) (0.485) (-1.955)

$N = 62, R^2 = 0.662$

Miscellaneous Industries

$$\hat{Y} = 1.303 - 0.192D_1 - 0.163D_2 + 0.272X + 0.553(D_1X) + 0.101D_2X \quad (\text{III-2-6})$$

(13.043) (-1.551) (-0.995) (1.490) (2.365) (0.290)

$N = 167, R^2 = 0.222$

The tests of significance for the individual dummy slope and intercept coefficients are set out in Tables III-2-2 and 3. An examination of the first table indicates that in three out of the five industries there is a significant difference between the estimates of the elasticity of substitution in Western and Central Canada vis-a-vis the Atlantic Provinces. This difference exists at the five percent level or better in the Printing, Publishing and Allied and Metal Fabricating industries while relatively weaker support is found in the Transportation Equipment Industries at the ten percent level. In the Miscellaneous Manufacturing and Food and Beverage industries the dummy slope coefficients are significant at the five and one percent levels for Central Canada vis-a-vis Western Canada and the Atlantic Provinces.

Table III-2-3 indicates that there are no significant inter-regional differences in the intercept terms in the three regions. It should be noted 'en passant' that the existence of significant differences in the elasticities ensures that there are differences in the efficiency of labour even though a common intercept terms holds across all of the various regions.

TABLE III-2-2.---Tests of Significance for Inter-Regional Differences in the Elasticity of Factor Substitution in Five, Two Digit Manufacturing Industries

Industry	Estimates of the Differential Slope Coefficients		Degrees of Freedom	Tests of Significance ¹		
	Central	Atlantic		Central	Atlantic	5%
Food and Beverage	0.702 (0.264) t=2.666*	0.249 (0.341) t=0.732	209	R	A	A
Printing, Publishing and Allied	1.033 (1.025) t=1.008	1.706 (0.742) t=2.299	48	A	R	R
Metal Fabricating	-0.223 (0.287) t=-0.777	-0.650 (0.320) t=-2.031	104	A	R	R
Transportation Equipment	-0.136 (0.281) t=0.485	-0.721 (0.369) t=-1.955	56	A	R	A
Miscellaneous Manufacturing	0.553 (0.234) t=2.365	0.101 (0.349) t=0.290	161	R	A	A

¹All tests are two tailed.

- () - standard error of coefficient
- A - accept the null hypothesis $H_0: \beta_T=0$
- R - reject the null hypothesis
- * - reject at the one percent level

TABLE III-2-3.--Tests of Significance for Inter-Regional Differences in the Intercept Terms in Five, Two Digit Manufacturing Industries in Canada.

Industry	Estimates of the Differential Intercept Coefficients		Degrees of Freedom	Tests of Significance ¹		
				10%		5%
				Central	Atlantic	Central Atlantic
Food and Beverage	-0.240 (0.169) t=-1.420	0.150 (0.149) t=1.006	209	A	A	A
Printing, Publishing and Allied	-1.061 (0.957) t=-1.109	-0.744 (0.603) t=-1.234	48	A	A	A
Metal Fabricating	0.194 (0.221) t=0.878	0.301 (0.203) t=1.484	104	A	A	A
Transportation Equipment	0.024 (0.221) t=0.110	0.216 (0.234) t=0.922	56	A	A	A
Miscellaneous Manufacturing	-0.192 (0.124) t=-1.551	-0.163 (0.163) t=-0.995	161	A	A	A

A - accept the null hypothesis
R - reject the null hypothesis

¹All tests are two-tailed.

The regression results set out in (III-2-2) to (III-2-6) inclusive indicate relatively low R^2 values compared to those which are typically encountered in time series studies or in inter-national cross-sectional studies. A comparison of Table II-2-3, page 21 and Table II-2-4 exemplifies the fact that inter-regional cross-sectional studies within a given nation typically produce lower R^2 . The explanation is centered around the relative lack of variation in the observations in a given year within a single country as compared to the wide extremes encountered in cross-country comparisons. It should be noted that the estimate of the elasticity of substitution for the base region is not significant for the Miscellaneous Manufacturing industry at the ten percent level. Given the nebulous nature of this classification it is felt that the statistical results reflect the lack of meaningful economic interpretation that can be given to this classification.

The "Non-Significant Industries"

The null hypothesis which states that the sets of differential slope and intercept coefficients in a given industry are equal to zero could not be rejected--even at a generous ten percent level of significance--for the Wood, Paper and Allied and Non-metallic Mineral Products Industries. An examination of the regression results set out below indicates that this result is generally supported by the insignificant "t" values obtained for each of the dummy coefficients. An exception to this statement is found in (III-2-9) in the "t" values obtained for the differential intercept and slope coefficients for Central Canada which are significant at the ten percent and five percent levels respectively.

Wood Industries

$$\hat{Y} = 0.941 - 0.214D_1 - 0.030D_2 + 0.746X + 0.323(D_1X) - 0.554(D_2X) \quad (\text{III-2-7})$$

(9.659) (-1.157) (-0.235) (4.749) (0.728) (-1.326)

N = 93, R² = 0.482

Paper and Allied Industries

$$\hat{Y} = 0.950 - 0.116D_1 + 0.257D_2 + 1.403X - 0.042(D_1X) - 0.656(D_2X) \quad (\text{III-2-8})$$

(4.290) (-0.323) (0.745) (4.795) (-0.085) (-1.417)

N = 58, R² = 0.480

Non-Metallic Mineral Products Industries

$$\hat{Y} = 1.405 - 0.798D_1 - 0.506D_2 + 0.667X + 1.109(D_1X) + 0.687(D_2X) \quad (\text{III-2-9})$$

(4.801) (-1.943) (-1.178) (1.655) (1.994) (0.925)

N = 104, R² = 0.329

It is useful to briefly comment on the seemingly paradoxical situation which exists in the Non-Metallic Mineral Products industry. The "t" values indicate that β_2 and β_5 are significant at the ten percent level (two-tailed test) while $H_0: \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$ is rejected. Some writers have loosely stated that if any one of the estimated regression coefficients are significantly different from zero then the resultant "F" value--for the joint test--will also be significant.¹ It is evident that if this statement is correct then the above set of results could not occur other than through some type of computational error.

A note by Geary and Leser in the American Statistician,² demonstrates that it is possible to have a situation in which all the coefficients

¹Jan Kmenta, Elements of Econometrics, (New York: MacMillan, 1971), p. 367. It should be noted in all fairness that Kmenta does qualify this statement.

²R. C. Geary and C. E. V. Leser, "Significance Tests in Multiple Regression", The American Statistician, Vol. 22, (February 1968), pp.20-21.

of a multiple regression have significant "t" values and still accept the null hypothesis that $\beta_2 = \beta_3 \dots \beta_k = 0$. It is suggested that this result ensures that the case under consideration is not unique.

Reference to Wonnacott and Wonnacott's¹ discussion and portrayal of joint confidence intervals--two dimensional case--offers some intuitive basis for explaining the above results since it certainly appears possible to have a situation where two of four possible regression estimates are significantly different from zero and yet have a joint confidence interval which includes the point (0,0;0,0) in four space. One cannot proceed very far with four dimensional problems in two space. The procedure outlined earlier will be maintained--that is, attention will not be paid to individual dummy coefficients unless the "F" test for the overall homogeneity of the differential intercept and slope coefficient is significant.

Given the foregoing discussion and argumentation the regression results for the 'non-significant' industries can be obtained by regressing $Y = F(X)$. (In effect, it is argued that regional influences do not significantly affect the estimates.)²

Wood Industries

$$\hat{Y} = 0.824 + 0.879X$$

(2.54) (8.55)

(III-2-10)

¹R. J. Wonnacott and L. H. Wonnacott, Econometrics, (New York: J. Wiley and Sons Inc., 1970), pp. 248-258.

²The 't' values for the intercept coefficient are accurate only to three figures. One of the regression programs used in the study did not produce standard errors for the intercept; thus a number of laborious calculations were required before an estimate could be obtained and only slide rule accuracy could be maintained.

Paper and Allied Industries

$$\hat{Y} = 0.972 + 1.239X \quad (III-2-11)$$

(3.09) (6.09)

Non-Metallic Mineral Industries

$$\hat{Y} = 0.927 + 1.353X \quad (III-2-12)$$

(2.60) (6.62)

It is evident that the slope and intercept coefficients are all significant at the one percent level.

The Nine Industry Two Region Results

The results of the joint tests for the existence of significant regional influences in nine, two digit manufacturing industries located in Western and Central Canada are set out in Table III-2-4. None of the "F" values are significant at the one percent level; however, significant differences are found in the Leather and Primary Metal industries at the ten percent level and the Electrical Products and Chemical and Chemical Products industries at the five percent level. The individual regression results results for the significant and 'non-significant' industries are presented below.

The Significant Industries¹

Leather Industry

$$\hat{Y} = 0.500 + 0.256D_1 + 1.694X - 0.858D_1X \quad (III-2-13)$$

(4.989) (2.109) (5.711) (-2.492)

N = 26, R² = 0.717

¹ $D_1 = \begin{cases} 1 & \text{for observations located in Central Canada} \\ 0 & \text{otherwise} \end{cases}$

TABLE (III-2-4).--Joint Tests of Significance (Intercepts and Slopes) for Regional Influences in Nine Two Digit Manufacturing Industries Located in Western and Central Canada.

S.I.C. NAME	"F" Value	Degrees of Freedom		Significance Levels ¹	
		Numerator	Denominator	10%	5%
Leather	3.288	2	22	R	A
Textile	0.334	2	69	A	A
Clothing	0.578	2	56	A	A
Furniture and Fixture	0.247	2	32	A	A
Primary Metal	2.654	2	37	R	A
Machinery Industries	2.193	2	27	A	A
Electrical Products	4.358	2	37	R	R
Petroleum and Coal Products	1.058	2	16	A	A
Chemical & Chemical Products	3.883	2	63	R	R

R - reject, A - accept the null hypothesis

¹Not significant at the one percent level.

Primary Metal Industry

$$\hat{Y} = 2.219 - 1.576D_1 - 0.311X + 1.617D_1X$$

(3.387) (-1.987) (-0.428) (1.820) (III-2-14)

$N = 41, R^2 = 0.204$

Electrical Products Industry

$$\hat{Y} = 1.133 + 0.481 D_1 + 1.329X - 0.953D_1X$$

(5.313) (1.785) (3.885) (-2.298) (III-2-15)

$N = 41, R^2 = 0.494$

Chemical and Chemical Products Industry

$$\hat{Y} = 1.795 + 0.866D_1 + 0.879X - 1.010D_1X$$

(9.101) (2.575) (3.212) (-2.235) (III-2-16)

$N = 67, R^2 = 0.186$

Reference to the individual regression coefficients in the above four regressions indicates that the differential intercept terms are significant at the five percent level in the Leather, Primary Metal and Chemical and Chemical Products industries as compared to a significance level of ten percent in the Electrical Products industry. The differential slope coefficients are significant at the five percent level in all of the industries with the exception of the Primary Metal Industry (significant at the ten percent level).

The Non-Significant Industries

The results of the full dummy variable and the restricted regressions are presented below. As noted previously, the restricted regressions are based on the prior information obtained from the joint tests.

Textile Industry

Full Regression

$$\hat{Y} = 0.928 + 0.088D_1 + 1.011X - 0.149(D_1X)$$

(9.771) (0.729) (4.328) (-0.525)

(III-2-17)

$N = 73, R^2 = 0.424$

Restricted

$$\hat{Y} = 0.980 + 0.925X$$

(5.34) (7.136)

(III-2-18)

$R^2 = 0.425$

Clothing Industry

Full Regression

$$\hat{Y} = 0.767 - 0.098D_1 + 1.043X + 0.130D_1X$$

(8.660) (-0.860) (3.960) (0.407)

(III-2-19)

$N = 60, R^2 = 0.507$

Restricted

$$\hat{Y} = 0.711 + 1.111X$$

(3.23) (7.572)

(III-2-20)

$R^2 = 0.497$

Furniture and Fixture Industry

Full Regression

$$\hat{Y} = 0.994 + 0.005D_1 + 0.668X + 0.053D_1X$$

(7.329) (0.031) (2.677) (0.158)

(III-2-21)

$N = 36, R^2 = 0.352$

Restricted

$$\hat{Y} = 1.007 + 0.669X$$

(7.66) (4.199)

(III-2-22)

$R^2 = 0.341$

Machinery Industry

Full Regression

$$\hat{Y} = 1.799 - 0.654D_1 + 0.066X + 0.904D_1X$$

$$(7.417) \quad (-1.929) \quad (0.212) \quad (2.063)$$

(III-2-23)

$$N = 31, R^2 = 0.305$$

Restricted

$$\hat{Y} = 1.464 + 0.525X$$
$$(6.54) \quad (2.035)$$

(III-2-24)

$$R^2 = 0.148$$

Petroleum and Coal Products Industry

Full Regression

$$\hat{Y} = 1.795 + 0.089D_1 + 0.965X + 0.090D_1X$$

$$(3.910) \quad (0.137) \quad (2.105) \quad (0.135)$$

(III-2-25)

$$N = 20, R^2 = 0.236$$

Restricted

$$\hat{Y} = 1.904 + 0.941X$$
$$(4.65) \quad (2.843)$$

(III-2-26)

$$R^2 = 0.321$$

The "t" results obtained for individual dummy coefficients support the overall findings indicated by the "F" values in Table III-2-4 with the exception of differential intercept and slope coefficients for the Machinery industry which are significant at the five percent level. This result is not entirely unexpected given the fact that $F = 2.193$ for this industry while $F_{2,27}^{.10} = 2.51$. The intercept and slope coefficients in the restricted regressions are all significant at the one percent level.

In summary, it has been shown that there are reasonable grounds for believing that regional influences must be accounted for if the A.C.M.S. specification is to be applied to the manufacturing sector of the Canadian economy. Five of the eight industries located in Western Canada, Central

Canada and the Atlantic Provinces have significantly different elasticities of substitution. In three out of five of these industries Western and Central Canada shared common elasticities of substitution which were significantly different from the estimates obtained for the Atlantic region. The efficiency of labour use in each of these three industries is accounted for by the differences in elasticities.

Four of the industries included in the nine industry partitioning of manufacturing activity in Western and Central Canada exhibited significant differences in both the intercept and the elasticity of substitution. Significant regional regional influences were not detected in the remaining five industries.

Part 3: The Over-all Significance and Economic

Interpretation of the Empirical Results

The objective of this portion of the study is three fold. Firstly, the estimates of the two parameters obtained from the A.C.M.S. specification are used to obtain point measures of the substitution and coefficient parameters contained in a C.E.S. production function homogeneous of degree one. Secondly, each of the industries exhibiting significant regional influences are examined in an attempt to determine whether there are any readily discernible characteristics of these industries which would account for the significant differences in the efficiency of labour use and the elasticity of substitution. Finally, a detailed comparison is made between the estimates of the elasticity of substitution obtained for the non-significant industries and the results obtained from a study of two digit manufacturing industries in the United States at approximately the same period of time.

The Existence of Non-Identical Production Functions

The results of the analyses carried out in Part 2 of Chapter III indicates that there are significant inter-regional differences in the efficiency of labour use and the elasticity of factor substitution in a number of two digit manufacturing industries in Canada. It is suggested that differences in these two estimates provide sufficient grounds for arguing that there are different C.E.S. production functions for each of the regional partitions of the given industries. In effect, if

$$V = [\beta K^{-\rho} + \alpha L^{-\rho}]^{-1/\rho}$$

where $\rho = \frac{1}{b} - 1$, $\alpha = a^{-1/b}$ and $\log \frac{V}{L} = \log a + b \log W$

then it follows that significant inter-regional differences in the slope and intercept terms are reflected in turn by different values for the production parameters. Estimates of these production parameters are set out in Tables III-3-1 and III-3-2. The following points should be noted. First, the intercept term in the A.C.M.S. regression is defined as $a = \frac{(\frac{V}{L})}{W^b}$. The efficiency of labour is defined as $a^{1/b} = \frac{(\frac{V}{L})^{1/b}}{W}$, which implies that $\alpha^{-1} = a^{1/b}$. In effect, the reciprocal of the proportion of the product distributed to labour-- $\alpha = a^{-1/b} = W \left(\frac{L}{V}\right)^{1/b}$ --is a measure of efficiency of labour use. An examination of the last column in Tables III-3-1 and 2 provides the information required to provide an ordinal ranking of relative labour efficiencies within and between various industries. Secondly, reference to Table III-3-1 indicates that differences in the elasticity of substitution lead to marked differences in the efficiency of labour and the portion of the product accruing to labour even when the intercept terms are identical. Thirdly, a limited number of the 't' values for estimates of the elasticity of factor substitution and the intercept terms--in particular

TABLE III-3-1.--Point Estimates of the Substitution (ρ) and Coefficient of Labour Parameters (α) for the Three Region, Eight Industry Breakdown of Manufacturing Activity¹

Industry Name	Region ²	Estimates of Regression Coefficients		Estimates of Production Parameters	
		Intercept log a	Elasticity b	$\rho = \frac{1}{b} - 1$	$\alpha = a^{-1/b}$
Food and Beverage	Western & Atlantic	0.976 (8.182)	1.598 (8.853)	-0.374	0.543
	Central		2.300 (11.95)	-0.565	0.654
Printing, Publishing and Allied	Western & Central	0.895 (1.667)**	1.096 (1.810)*	-0.088	0.442
	Atlantic		2.447		
Metal Fabricating	Western & Central	0.986 (9.997)	0.904 (7.128)	0.106	0.336
	Atlantic		2.680	-0.643	0.727
Transportation Equipment	Western & Central	0.758 (5.096)	0.254 (0.867)	2.937	0.021
	Atlantic		0.971 (5.085)	0.030	0.458
Miscellaneous Industries	Western & Atlantic	1.303 (13.043)	0.249 (0.790)	3.016	0.048
	Central		0.272 (1.490)**	2.676	0.061
Wood	Western & Central	0.824 (2.54)	1.325 (4.460)	-0.245	0.374
	Atlantic		2.280	0.138	0.392

/continued....

TABLE III-3-1.--continued

Industry Name	Region	Estimates of Regression Coefficients		Estimates of Production Parameters	
		Intercept $\log a$	Elasticity $\frac{a}{b}$	$\rho = \frac{1}{b} - 1$	$\alpha = a^{-1/b}$
Paper and Allied	Western, Central & Atlantic	0.972 (3.09)	2.643 1.239 (6.09)	-0.193	0.456
Non-Metallic Mineral	Western, Central & Atlantic	0.927 (2.60)	2.527 1.353 (6.62)	-0.261	0.504

* significant at 5% level

** significant at 10% level

¹ Each of the figures in brackets are 't' values (one-tail) $H_0: \beta_1 = 0$

² Each industry has a common intercept across all three regions.

³ It should be noted that the standard error for the estimate of the elasticities provided in industries exhibiting significant regional differences is obtained by:

$$\text{S.e. } (\beta_{R+1} + \beta_{R+2}) = [\text{Var } (\beta_{R+1}) + \text{Var } (\beta_{R+2}) + 2\text{Cov } (\beta_{R+1}, \beta_{R+2})]^{1/2}$$

TABLE III-3-2.--Point Estimates of the Substitution (ρ) and the Coefficient of Labour (α) Parameters for the Two Region, Nine Industry Breakdown of Manufacturing Activity in Canada.

Industry Name	Region	Estimates of Regression Coefficients ¹		Estimates of Production Parameters		
		Intercept	Elasticity	$\rho = \frac{1}{b} - 1$	$\alpha = a^{-1/b}$	
Leather	Western	0.500 (4.989)	1.649	1.694 (5.711)	-0.410	0.744
	Central	0.756 (3.53)	2.130	0.736 (4.19)	0.359	0.358
Primary Metal	Western	2.219 (3.387)	9.198	-0.311 (-0.428)	-4.215	1255.2
	Central	0.643 (1.47)	1.902	1.306 (2.60)	-0.234	0.611
Electrical Products	Western	1.133 (5.313)	3.105	1.329 (3.885)	-0.248	0.426
	Central	1.614 (9.91)	5.023	0.376 (1.61)**	1.660	0.014
Chemical and Chemical Products	Western	1.795 (9.101)	6.019	0.879 (3.212)	0.138	0.130
	Central	2.661 (9.77)	14.311	-0.131 (0.956)	-8.634	6.636 ²
Textile	Western & Central	0.980 (5.34)	2.664	0.925 (7.136)	0.081	0.347
	Western & Central	0.711 (3.23)	2.036	1.111 (7.572)	-0.099	0.527
Furniture and Fixture	Western & Central	1.007 (7.66)	2.757	0.669 (4.199)	0.495	0.222
	Western & Central	1.464 (6.54)	4.323	0.525 (2.035)	0.905	0.062
Petroleum & Coal Products	Western & Central	1.904 (4.65)	6.713	0.941 (2.843)	0.063	0.132

¹The numbers in brackets are 't' values.

²6.636 x 10⁸

**Significant at 10% level

estimates obtained in industries exhibiting regional differences--are not significantly different from zero. These results account for the seemingly ridiculous point estimates of ρ and α' . Reference to Table III-3-1 indicates that the 't' values are insignificant at the ten percent and lower levels for estimates of the elasticity of substitution obtained in the Metal Fabricating industry (Atlantic region) and the Transportation Equipment industry (Atlantic region). The remainder of the intercept and elasticity terms are significant at the one percent level with the exception of the three estimates which are significant at the five and ten percent levels. Similar observations can be made with respect to Table III-3-2. Insignificant 't' values are obtained for the estimate of the elasticity of substitution in the Primary Metal (Western Canada) and the Chemical and Chemical Products industries (Central Canada). The intercept and the elasticity of substitution estimates are significant at the ten percent level in the Primary Metal and Electrical Products industries, respectively, while the remainder of the values are significant at the one percent level.

It should be noted that the acceptance of the null hypothesis that the elasticity of substitution in a particular industry is equal to zero implies that the Leontief type production function--elasticity of substitution equal to zero--is supported rather than the Cobb-Douglas or C.E.S. form. This situation differs considerably from the normal case in which the null hypothesis is simply a 'straw man'.

Finally, it is worth re-emphasizing the point that the lack of regional capital stock data has proven to be rather costly in the sense that it is impossible to obtain estimates of β , the coefficient of capital in the C.E.S. production function or to try various types of alternative specifications.

An Appraisal of the Economic Results Obtained for the
Significant Industries

The previous discussion has established the fact that regional differences in the efficiency of labour use and the elasticity of substitution (the intercept and slope terms in the regressions) are reflected in terms of different production parameters. Clearly the important questions are why do these differences exist and how can they be accounted for. The standard micro-economic theory of production has always posited the assumption that there is a one to one correspondence between a given production function and a given commodity or sets of commodities regardless of the spatial location of the production processes. This assumption was adopted in aggregative analysis under the 'reasoning by analogy' rule which has been widely used in formulating theories in the macro area; however, as noted the results of the study suggest that in certain industries the assumption is of dubious merit.

A.C.M.S. attempted to answer this question by arguing that inter-regional differences in production functions were based on neutral differences in factor use. This simply amounts to stating that the shape of the isoquants in various regions are identical except for a pure scale effect. Their findings indicated that the constancy of the capital and labour coefficients held up but the neutral efficiency parameter experienced wide inter-national variations.

In effect, by rewriting the C.E.S. function as

$$V_i = \gamma_i [\delta_i K^{-\rho_i} + (1 - \delta_i) L^{-\rho_i}]^{-1/\rho_i}$$

where i refers to the i^{th} industry, δ the distribution parameter and γ the

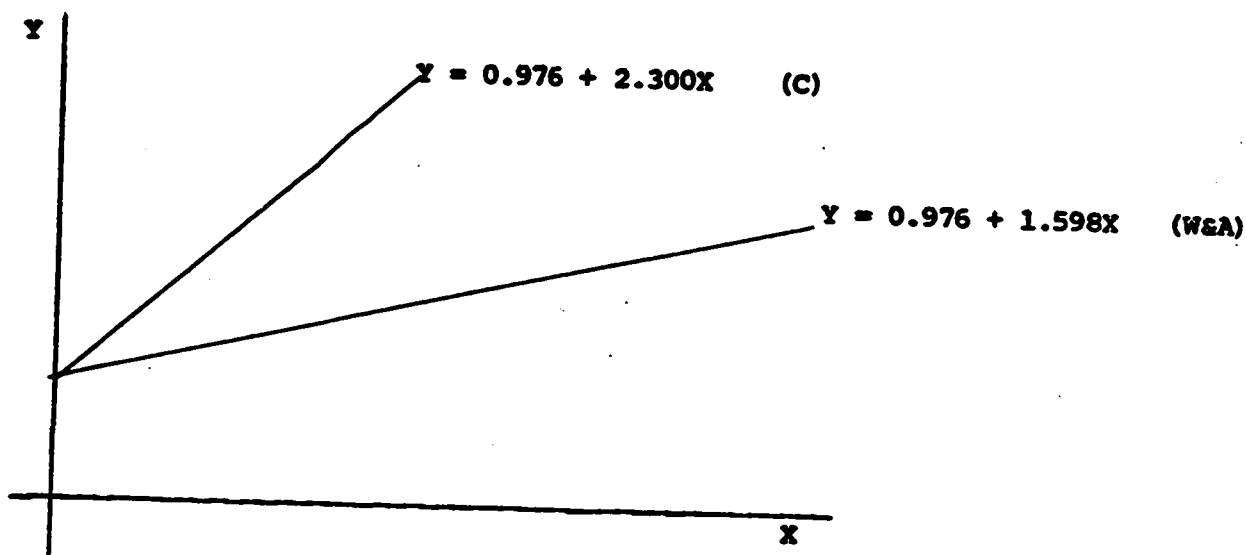
neutral efficiency parameter $\delta = \beta\gamma^p$ and $\alpha + \beta = \gamma^{-p}$. A.C.M.S. found that $\frac{\delta}{1-\delta}$ remained constant while γ varied widely.¹ Clearly, data limitations ensure that estimates of δ and γ cannot be provided in this study. In addition, the statement that production functions are identical with the exception of differences in the neutral efficiency parameter seems to be rather facile in the light of seemingly large disparities in the quantity and quality of human capital stock located in various regions in Canada. In effect, it would appear that differences in efficiency could be biased towards capital or labour as well as being neutral.

The net result is that the data constraints prevent one from pursuing various avenues which could produce some fruitful results. The objective of the remainder of this section is to explicitly depict the results obtained for the various industries exhibiting regional influences and to speculate on some of the factors which may account for the differences. No pretense is made to provide a definitive answer to the question as to why regional differences in production functions exist. This answer will only emerge after a substantial amount of empirical and theoretical research has been carried out; however, it is suggested the exercise will provide some insight into the matter.

Food and Beverage Industry

The estimates of the elasticity of substitution and the intercept term for the regions exhibiting significant differences in the Food and Beverage industry are set out in Table III-3-1 and represented graphically in Figure III-3-1.

1x. J. Arrow et. al. op. cit., p. 233.



C - Central Canada, W & A - Western Canada and the Atlantic Provinces
 FIGURE III-3-1.--Regional Influences in the Food and Beverage Industry

Tests of the null hypothesis that the elasticity of substitution is equal to one indicates that $t = 3.3$ for Central Canada and 2.1 for Western Canada and the Atlantic Provinces. This implies that the Cobb-Douglas form of the production function is rejected at the one percent level. The estimates of the elasticities indicate that a ten percent change in the factor price ratio will lead to a twenty-three percent increase in Central Canada vis-à-vis a sixteen percent increase in Western Canada.¹ This high degree of substitution of capital for labour in response to relatively nominal increases in the price of labour relative to

¹This result follows from:
$$\sigma = \frac{d \log \frac{K}{L}}{d \log R} = \frac{d \log \frac{V}{L}}{d \log W}$$

and the equilibrium condition:

$$\frac{P_L}{P_K} = \frac{\frac{\partial V}{\partial L}}{\frac{\partial V}{\partial K}} = R(K, L)$$

capital could prove to be troublesome in terms of the employment tradeoffs that would be faced if labour negotiated relatively favourable contracts or provincial and federal governments made substantial changes in minimum wage laws.

The existence of the significant differences in elasticities between Central Canada vis-à-vis Western Canada and the Maritimes offers some grounds for the following speculation. Casual empiricism suggests that the output of the Food and Beverage industries in the Western and Atlantic provinces is characterized by a substantial amount of primary processing as compared to the finished commodity mix produced by Central Canadian manufacturers for consumption in the major metropolitan areas of Ontario and Quebec. If one accepts the premise that the primary processing carried on by the export oriented industries in the West : are relatively more capital intensive and thus have a relatively lower elasticity of factor substitution the empirical results seem to make a great deal of sense. For example, a quick comparison of the output of the large meat packing establishments in Alberta and Manitoba suggests that large volumes of beef and pork carcasses and animal by-products are shipped to Central Canadian markets. These products are produced by dressing the animals on large scale beef and hog conveyor systems. The processing of table-ready meats (sliced meat products etc.) on the other hand is relatively more labour intensive. It is suggested that the output mix in the Toronto and Montreal meat packing plants is weighted much more heavily towards the production of the relatively more labour intensive items than is the case in the export oriented Western provinces.

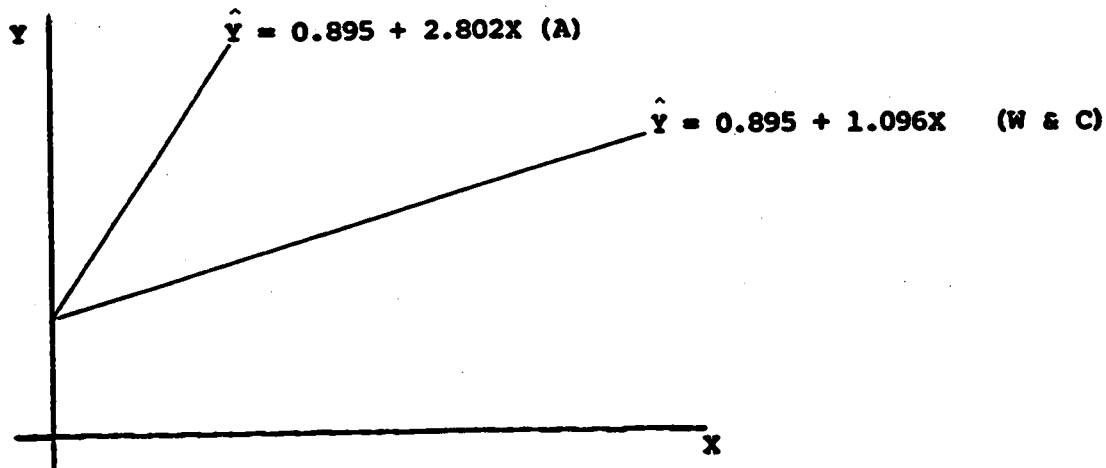
It should be noted that the argument made in the above example implicitly assumes that the production function is identical for similar

commodities in various regions. It is assumed that differences in the mix of commodities (mix of production functions) account for the regional differences in the aggregate production function.

It is interesting to note that by accepting the industry mix argument one is really implicitly assuming that there are significant inter-industry differences in the three digit production function parameters; however, this assumption is contrary to the maintained hypothesis that the industries included in a two digit classification can be said to have a common technology. The only way to resolve this dilemma would be to carry out a study which determined whether there were any significant inter-industry differences in production parameters at the three digit level though it is not at all clear that this procedure would not have to be continued down to at least the four digit level. Given the constraints imposed on the availability of data such a series of studies could not be carried out by individuals outside of the government civil service. This implies that for all practical purposes investigators have had to live with the maintained hypothesis and assume that there are real differences in the production functions within a region at two and three digit levels. This is a dangerous practise since one can confuse apparent differences in aggregate production functions with real differences in the underlying micro relationships. It is suggested that it is intuitively and logically appealing to start at the four digit level and demonstrate that various sets of production relationships are statistically and economically equivalent before investigating the two digit relationships. As matters now stand there is no way of knowing whether aggregate and micro differences in production relationships are related. This is of little consequence for certain broad policy purposes; however, it can be disastrous if due caution is not exercised.

Printing, Publishing and Allied Industries

Reference to Table III-3-1 indicates that the elasticity of substitution in the Printing and Publishing industry ranges from approximately 1.0 for Western and Central Canada vis-a-vis 2.8 for the Atlantic Provinces. The efficiency of labour use is different in both regions.¹ The test of ($H_0: \beta_1=1$) produced a $t = 0.16$ for Western and Central Canada vis-a-vis $t = 4.5$ for the Atlantic region which suggests that a Cobb-Douglas production function is appropriate for the former as compared to the C.E.S. form which is required in the latter region. Figure III-3-2 illustrates the difference in the two regions.



(A) - Atlantic Provinces (W & C) Western and Central Canada

FIGURE III-3-2.--Regional Influences in the Printing, Publishing and Allied Industries

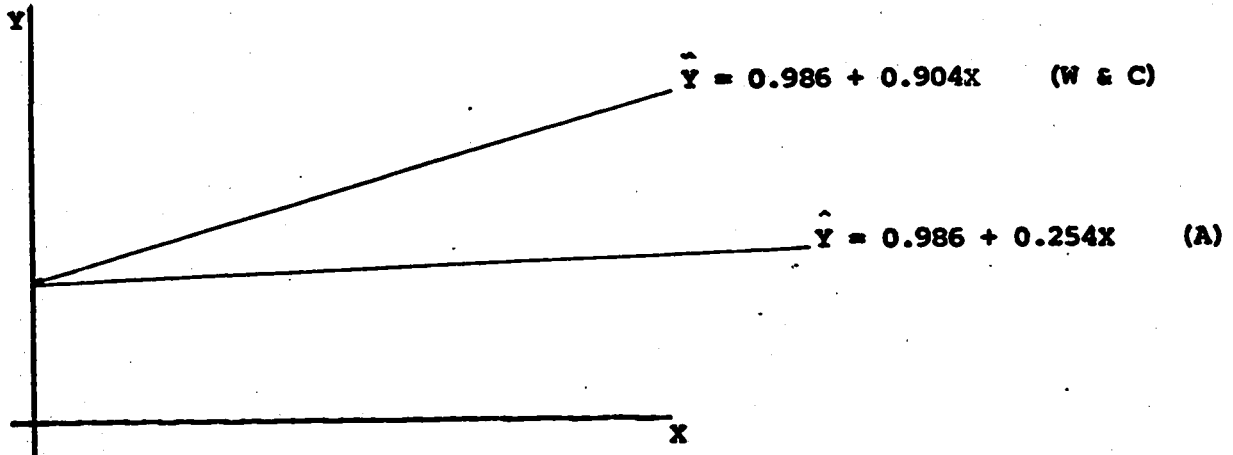
No attempt will be made to speculate on the reasons which account for the major differences in the elasticities. It is suggested that close attention should be paid to highly dis-aggregated (four digit) data.

The Metal Fabricating and Transportation Equipment Industries

Reference to the results set out in Table III-3-1 indicates that quite similar results are obtained for the Metal Fabricating and

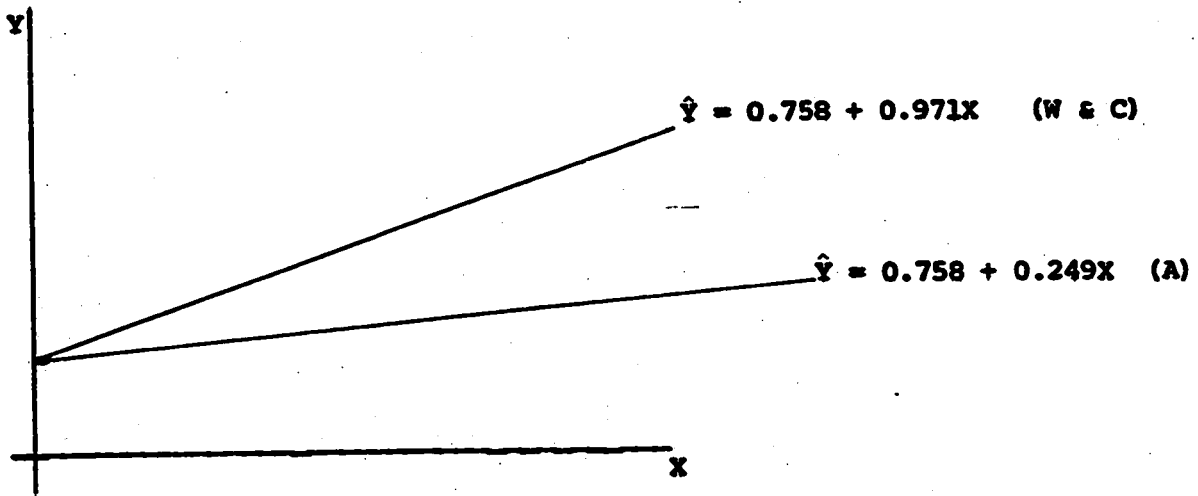
¹Refer to Table III-3-1.

Transportation Equipment industries in the sense that the elasticities of substitution are significantly lower in the Atlantic vis-à-vis the Central and Western regions of Canada. The results are depicted graphically in Figures III-3-3 and III-3-4.



(W&C) - Western and Central Canada (A) - Atlantic Provinces

FIGURE III-3-3.--Regional Effects in the Metal Fabricating Industry.



W & C - Western and Central Canada (A) - Atlantic Provinces

FIGURE III-3-4.--Regional Effects in the Transportation Industry.

Tests to determine whether the elasticities of substitution are different from one indicate that the Cobb-Douglas production function is an acceptable form in both industries for Western and Central Canada. The low 't' values for estimates in each of the two regions suggest that

the Leontief form would be a suitable type of production relationship for the Atlantic region.

The marked regional differences in the elasticities in each industry--unitary vis-a-vis highly inelastic values--are rather interesting in the sense that substantial increases in wages in the Maritimes would not have the employment trade-offs that would be experienced in the rest of Canada. This result could have some useful policy implications if a reasonable explanation could be formulated which would account for the regional effects.

The following comments offer some broad guidance in this direction. An examination of the industrial composition of the Metal Fabricating industries--Boiler and Plate Works, Fabricated Structural Metal, Ornamental and Architectural Metal, Metal Stamping, Pressing and Coating, Wire and Wire Products, Hardware, Tool and Cutlery, Heating Equipment, Machine Shops and Miscellaneous Fabricating--indicates that there are marked differences in the industry mix between the Maritimes and Central Canada. This difference is not surprising; however, recognition of the fact that the Shipbuilding and Repair and Boatbuilding and Repair industries are included under Transportation Equipment highlights the fact that a major section of the Atlantic Provinces' "metal fabricating like activities" are not included in this industry. Western Canadian provinces have enough strength in various three digit Metal Fabricating industries to present an industrial mix which is not significantly different from Ontario and Quebec at least in terms of the estimates of the substitution parameters. This is seemingly lacking in the Atlantic Provinces.

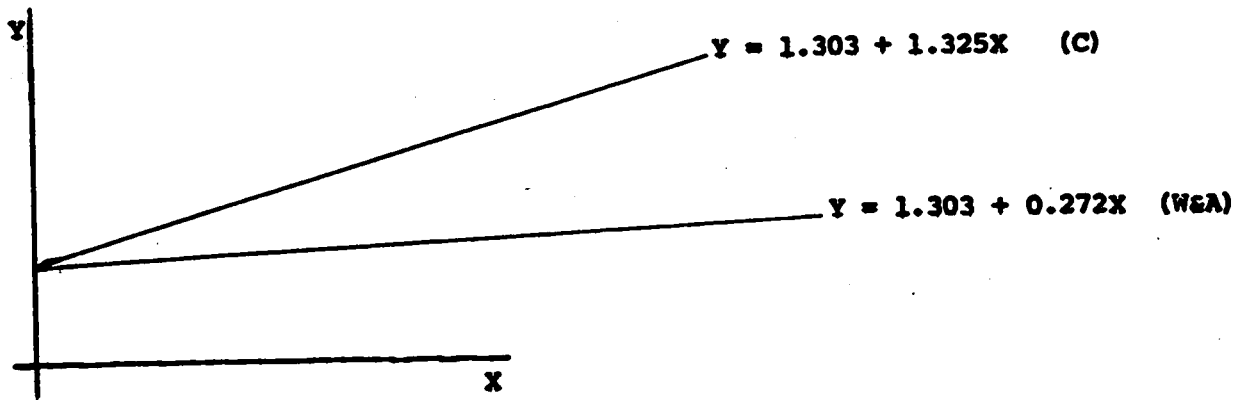
Similar remarks can be made with respect to the Transportation Industry. The Ontario-Quebec mix is heavily weighted by the Motor Car, Aircraft and Parts, Shipbuilding and Motor Vehicles Parts and Accessories

industries while the Atlantic provinces--principally Nova Scotia--are more heavily dominated by Shipbuilding.

In effect, it is suggested that the argument advanced with respect to the Food and Beverage industry applies in the Metal Fabricating and Transportation Equipment industries. Certain regional partitions of the set of economic activities carried out in a two digit industry may only encompass a subset of the entire range of activities carried out in the industry. The aggregate production function for these regions will be atypical in the sense that the investigator has only obtained data for a few of the total number of different production processes.

Miscellaneous Manufacturing Industries

The results obtained for the Miscellaneous Manufacturing industry---refer to Table III-3-1--indicate that all three regions have different efficiencies of factor use; however, the estimate of the elasticity of substitution in the Western and Atlantic regions are equal to 0.272 vis-a-vis a value of 1.325 for Central Canada. The 't' value for the elasticity of substitution ($H_0: \beta_1=1$) indicates that the elasticity of substitution in Western Canada and the Atlantic Provinces is greater than zero and less than one. The null hypothesis could not be rejected for Central Canada.



(C) Central Canada (W&A) Western Canada and Atlantic Provinces

FIGURE III-3-5.--Regional Influences in the Miscellaneous Manufacturing Industry.

No attempt will be made to provide an insight into this 'pot pourri' of manufacturing industries other than to note that regional concentrations are important.

The remaining four industries which exhibit regional influences are found in the nine industry, Western and Central Canada partitioning of manufacturing activity. It is worth noting as a preliminary observation that significant regional differences are found in both the efficiency of labour and elasticity of factor substitution. A brief examination of the point estimates for the substitution and the efficiency of labour parameters in Table III-3-2 indicates that this leads to extremely wide variations compared to the range exhibited by the estimates set out in Table III-3-1.

It is useful to examine the four significant industries as a group. The estimates noted in Table III-3-2 are illustrated by the following figures.

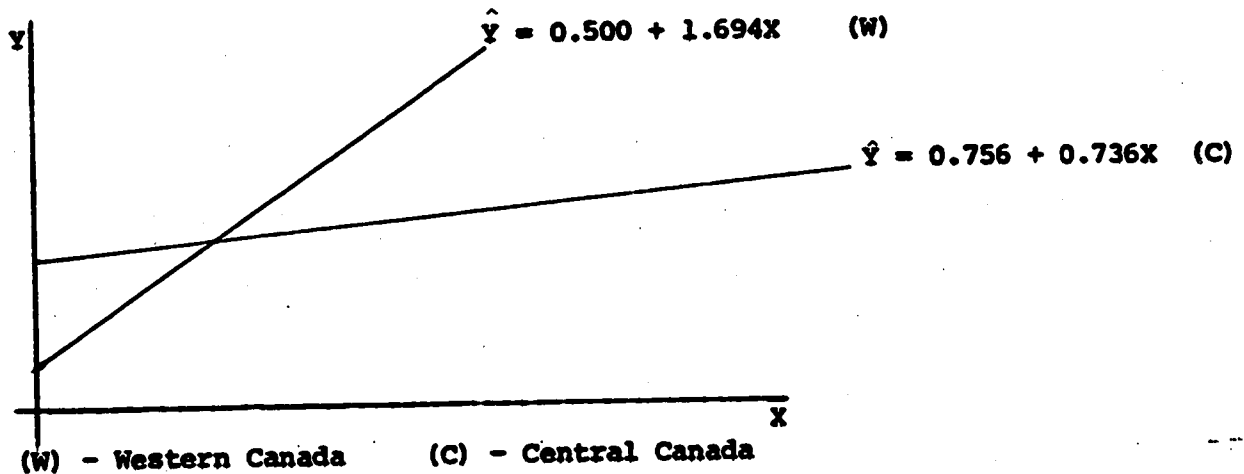


FIGURE III-3-6.--Regional Effects in the Leather Industry.

Tests of $(H_0: \beta_1=1)$ indicate that the hypothesis is rejected at the five percent level for Western Canada ($t = 2.34$). The estimate for Central Canada is not significantly different from one at the ten percent level ($t = 1.47$).

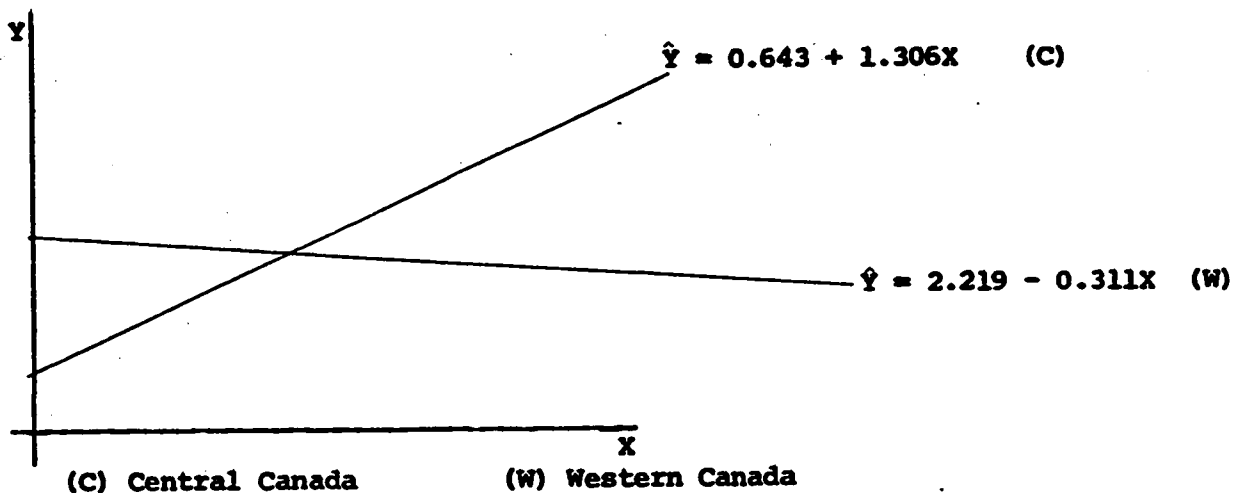


FIGURE III-3-7.--Regional Effects in the Primary Metal Industry

The estimate of the elasticity of substitution has a negative slope which would be meaningless from a theoretical viewpoint if the 't' value did not indicate that the estimate was highly insignificant and not different from zero. (Refer to Table III-3-2). The elasticity of substitution in Central Canada is not significantly different from one at the ten percent level.

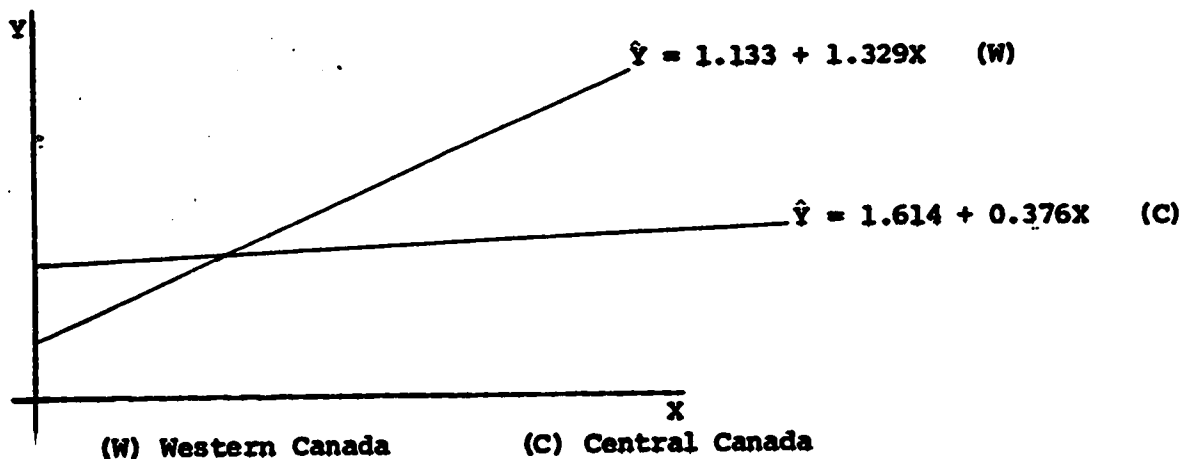


FIGURE III-3-8.--Regional Effects in the Electrical Products Industry.

The elasticity of substitution in Western Canada is not significantly different from one while the estimate for Central Canada is significantly

different from one at the one percent level and significantly different from zero at the ten percent level.

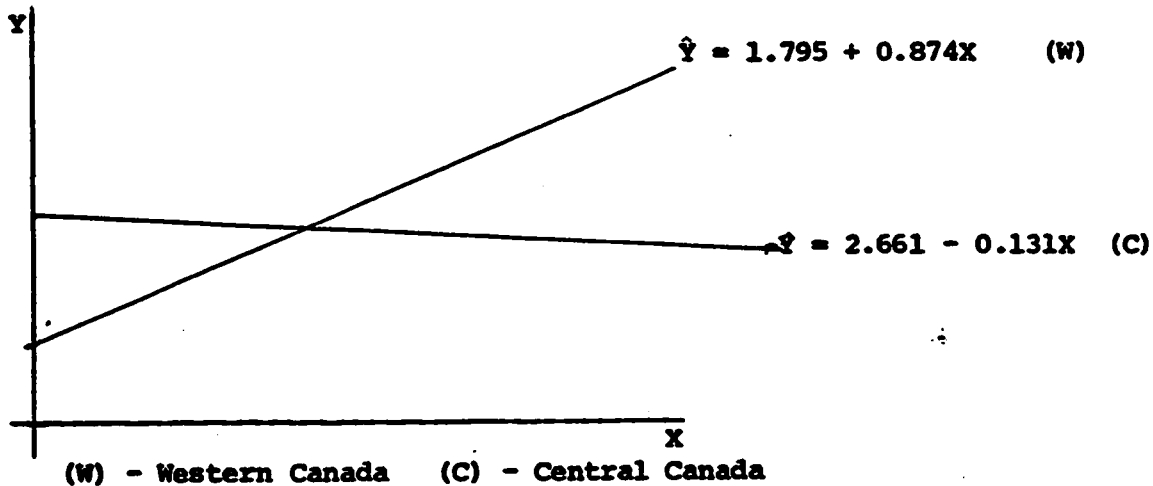


FIGURE III-3-9.--Regional Effects in the Chemical and Chemical Products Industry

The elasticity of substitution in Central Canada has a negative sign; however, it is not different from zero at any acceptable significance level. The estimate for Western Canada is not significantly different from one at the ten percent level ($t = 0.44$).

An examination of Figures III-3-6 to 9 and Table III-3-2 indicates that relatively low values of the elasticity of substitution are associated with relatively high values for the efficiency of labour use term. This trend holds across all of the regions with the exception of the unsupportable negative elasticity estimates which should be ignored. It should be noted that in two of the industries (Leather and Electrical Products) the lower elasticities and higher efficiency of labour use values are associated with the Central Canada rather than Western Canada. If the assumption is made that the negative elasticities are also indicative of a low elasticity high efficiency pattern then this observation can be extended to the Chemical and Chemical Products industry. It should be noted that the regional

positions would be juxtaposed in the Primary industries.

One possible explanation of this result is centered around the assumption that highly capital intensive production processes have relatively low elasticities of substitution and generally speaking a larger value added per man hour of labour employed. In certain instances it can be argued that the capital intensive technology is associated with high volume mass production techniques which can employ highly automated procedures; however, on the other hand a relative shortage of labour which is reflected in higher labour costs can lead to the adopting of capital intensive procedures at relatively lower volumes of output. An examination of the data¹--value added, man hours of production, wage rates in manufacturing industries for various three digit breakdowns of industrial activity--indicates that the bulk of the mass produced items are seemingly concentrated in Central Canada. For example in the Leather industry the major shoe factories are located in Central Canada--in the Electrical Products industry virtually all of the Canadian electronics industry is located in Central Canada. Similar observations can be made with respect to the pharmaceutical and medical products, toilet preparations and the major portion of the industrial chemicals produced in the Chemical and Chemical Products industry. An apparent contradiction of this general trend arises in the Primary Metal industry since it is difficult to reconcile the low elasticity, high efficiency of labour use argument in Western Canada with the higher estimates obtained for Central Canada which contains among other things the Algoma and Stelco steel operations. It is suggested, however, that the differences in the industry mix account for the different configuration of production parameters in the two regions.

¹Manufacturing Industries of Canada, Sections A, B, C, D, E, and F, Dominion Bureau of Statistics, (Ottawa: Queen's Printer) 1964 and 1965.

In summary, it is suggested that four of the nine industries contained in the nine industry breakdown of manufacturing activity exhibit significant regional differences in both the efficiency of labour use and the elasticity of substitution. There appears to be a systematic relationship between low elasticity and high efficiency of labour use values across the regional breakdown contained in each industry. The suggestion was made that the regional concentration of this difference in three of the four industries simply reflects the difference in technology underlying the capital intensive mass production techniques employed in Central Canada. A cursory examination of the industry mix in the various regions indicated that this argument must be qualified by the observation that Central Canada is the sole region in which certain goods and services are produced. This implies that regional partitions of two digit manufacturing industries may differ partially because of the differences in the technology which can be utilized in producing similar goods and services and partially because of the differences in the product mix.

The point to note is that once regional differences in aggregate production functions are encountered one must revert to less aggregate analysis which provides relatively fine commodity and industrial breakdowns. It is impossible to achieve this objective within the confines of this study; however, it is felt that a sufficient number of problems have been raised to discourage the naive use of aggregative results.

An Appraisal of the Economic Results Obtained for the Non-Significant Industries

The results of the discussion and econometric analyses dealing with the question of inter-regional differences in production functions within various industries in Canada indicates that there are no significant differences in the elasticity of substitution and the efficiency of factor

use in three of the eight industry three region and five of the nine industry two region partitions of industrial activity. The objective of the following discussion is to: (1) determine whether the special cases of the C.E.S. production function--the Leontief or Cobb-Douglas forms--can be used in place of the more complex function; and (2) compare the estimates of the elasticity of substitution obtained for these two digit industries with similar results obtained in the United States.

The results of the 't' tests are set out in Table III-3-3.

The Leontief type production function is excluded from consideration at the one percent level in all industries except the Machinery industry (the null hypothesis is rejected at the 2.5% level). The tests of significance for the Cobb-Douglas case indicates that the hypothesis $\beta_j = 1$ cannot be rejected at the one percent level though significant differences in the Machinery and Furniture and Fixture industries offer some support for $0 < \beta < 1$ at the 2.5% level. It is suggested that the Cobb-Douglas production function is suitable for at least six of the eight industries.

One interesting speculation should be noted in connection with industries noted in Table III-3-3. The first two industries could be called ubiquitous industries since it can be argued that their basic function is to process a basic Canadian staple (wood) which is widely scattered throughout the Dominion. The basic reason for this natural resource orientated dispersion rests on the weight losing characteristics of the raw material inputs. In effect, it is suggested that it is the rather limited processing of relatively homogeneous products which has ensured that their aggregate production functions are virtually identical across Western and Central Canada and the Atlantic Provinces. A similar comment could be made with respect to the Non-Metallic Mineral Industry though it should be noted that it is the weight gaining characteristics of its output which has led to a wide dispersal throughout Canada.

TABLE III-3-3.---Tests of Significance for the Magnitude of the Elasticity of Substitution in Eight, Two Digit Manufacturing Industries Located in Canada and Western and Central Canada.¹

Industry No.	Name	Estimate of 'b'	R ²	d.F.	Test of Hypothesis	
					H ₀ :β _i =0 1%	H ₀ :β _i =1 1%
1	Wood	0.879 (0.103) t=8.552	0.445	91	R	A
2	Paper and Allied	1.239 (0.203) t=6.092	0.398	56	R	A
3	Non-Metallic Mineral	1.353 (0.203) t=6.619	0.300	102	R	A
4	Textile	0.925 (0.130) t=7.136	0.417	71	R	A
5	Clothing	1.110 (0.147) t=7.572	0.497	58	R	A
6	Furniture & Fixture	0.669 (0.159) t=4.199	0.341	34	R	A*
7	Machinery Industries	0.525 (0.228) t=2.305	0.154	29	A*	A*
8	Petroleum & Coal Products	0.941 (0.331) t=2.843	0.310	18	R	A
					*Significant at 2.5% level	
R - Reject Null Hypothesis						

¹It should be carefully noted that the estimates of the elasticity of substitution that are set out for each of the first three industries were obtained from the three region partitioning of manufacturing activity. The remainder are restricted to the Western and Central Canada breakdown.

It should be noted that the marked similarity in the elasticity of substitution and the efficiency of labour use between Western and Central Canada extends over eleven of the seventeen manufacturing industries. (The eight industries set out in Table III-3-3 and the Printing, Publishing and Allied, Metal Fabricating and Transportation Equipment industries noted in Table III-2-2). This suggests that common national policies which perturb these parameters will have a common effect on both these regions.

Estimates of the Elasticities of Substitution in Two and
Three Digit Industries

It is useful to examine a study carried out by Zarembka and Chernicoff for two and three digit manufacturing industries in the United States.¹ A dummy variable form of the A.C.M.S. specification was used in this study. Regional dummy variables (intercept dummies) were added to determine whether product prices and wage rates were correlated among regions and a slope dummy was added to determine whether regional differences in product mixes led to significantly different estimates of the elasticity of substitution. In effect:

$$Y_i = \alpha_0 + \alpha_1 D_1 + \alpha_2 D_2 + \alpha_3 D_3 + \alpha_4 D_4 + \alpha_5 X_i + \alpha_6 (D_5 X_i) + U_i \quad (\text{III-3-1})$$

D_1 to D_4 are intercept dummies for various regions, D_5 is a slope dummy for a partitioning of the regions in two groups, Y and X are the same as noted previously and U is the stochastic term.

The resultant estimates of the elasticity of substitution and the 't' test for the slope dummy are set out in Table III-3-4. The first point

¹p. Zarembka and H. B. Chernicoff, "Further Results on the Empirical Relevance of the C.E.S. Production Function", Review of Economics and Statistics, Volume LIII, Number 1, (February 1971), pp. 106-107.

TABLE III-3-4.--Preliminary Estimates of the Elasticity of Substitution for Two-Digit Industries (United States 1963)¹

Two-digit Industry	1963			
	All Observations			t-test for Regional Shift
	δ_1	R ²	d.f.	
20. Food	0.692 (.104)	.516	97	1.638
22. Textile	1.236 (.157)	.571	62	0.384
23. Apparel	1.371 (.184)	.507	90	1.025
24. Lumber and Wood	0.926 (.087)	.734	95	2.638
25. Furniture	1.046 (.121)	.638	87	0.264
26. Paper	1.592 (.140)	.679	82	0.702
27. Printing and Publishing	0.818 (.087)	.585	97	0.816
28. Chemicals	1.230 (.177)	.427	91	0.858
29. Petroleum and Coal	1.361 (.424)	.234	52	0.065
30. Rubber and Plastics	1.041 (.135)	.537	67	0.156
31. Leather	1.015 (.149)	.598	40	0.277
32. Stone, Clay, and Glass	1.204 (.122)	.604	97	0.040
33. Primary Metals	1.306 (.139)	.574	80	0.214
34. Fabricated Metals	0.887 (.110)	.576	96	0.716
35. Machinery, except Electrical	0.681 (.146)	.261	94	1.975
36. Electrical Machinery	0.563 (.280)	.085	81	1.593
37. Transportation Equipment	0.242 (.105)	.251	74	2.853
38. Instruments	1.143 (.213)	.395	54	1.927
39. Miscellaneous Manufacturing	1.082 (.134)	.492	92	0.202

¹Data Sources: For 1963, see text; for 1957 and 1958, see Zarembka [7, table 1].

²Test of $\sigma_{12} = \sigma_{21}$ rejected at 5 percent, see Zarembka [7, table 1].

³Test not undertaken since the 1963 t-test for regional shift is significant at 5 percent.

to consider is that significant regional differences in the elasticity of substitution at the ten percent level or better are found in the United States: Food, Lumber and Wood, Machinery except Electrical, Electrical, Transportation and Instrument industries. Reference to Tables III-3-1 and III-3-2 indicates that regional differences were found in the Food and Beverage, Printing, Publishing and Allied, Transportation Equipment, Metal Fabricating, Miscellaneous, Leather, Primary Metal, Electrical Products and Chemical and Chemical Products industries. Regional influences seem to be more prevalent in Canada; however, it should be noted that Zarembka and Chernicoff used regions for their slope dummy that were larger than the regions that were used for the intercept dummies. This implies, other things being equal, that there is less chance of obtaining regional variations the larger the size of the region. In any event regional influences were evident in roughly thirty-one percent of the two digit United States industries in 1963 and approximately fifty-three percent of Canadian manufacturing in 1964 and 1965.

A pairwise comparison of the industries in each country--industries exhibiting regional influences were excluded--indicates there is a high degree of mutual compatibility between the various elasticity estimates. In the Canadian case the estimates are not significantly different from one at the one percent level. This holds true for the United States as well though a relaxation to the five percent level provides grounds for arguing that the estimate of the elasticity in the Apparel industry in the United States would become greater than one whereas the estimate for the Furniture and Fixture industry in Canada would become inelastic. In general Chernicoff and Zarembka argue that the elasticity of substitution in United States manufacturing is not significantly different from one. The results of the eight industry breakdown for Canada would support this contention;

however, the full seventeen industry breakdown has a number of regional estimates of the elasticity which are not significantly different from zero.

Table III-3-5 sets out various estimates of the elasticity of substitution that were obtained for the set of three digit industries contained in each two digit United States manufacturing industry. These results are extremely interesting since it has been suggested that one of the basic reasons for inter-regional differences in two digit production function parameters may simply stem from the fact that certain three digit industries are not located in one or more of the regions under consideration. If there are no significant differences among these industries then this problem is of little consequence; however, if the converse is true then significant differences in elasticities at a two digit level could not be claimed until the investigator had demonstrated that the industry mixes in various regions were comparable. An examination of Table III-3-5 indicates there are significant differences in the elasticity of substitution ($H_0: \sigma=1$) in the three digit industries contained in the following two digit industries: Food, Apparel, Paper, Chemicals, Fabricated Metals, Transportation Equipment, Instruments and Miscellaneous Manufacturing. Reference to Table III-3-4 indicates that the majority of the two digit industries with regional differences in their elasticities of substitution do not have significant differences in elasticities among their three digit industries. (Compare the results set out for the Lumber industry in both Tables.) This result is seemingly explained by the fact that Zarembka and Chernicoff used a different size of region to estimate their differential elasticities and their estimates of the elasticities set out in Table III-3-5. The net result is that the existence of wide variations among the three digit industries contained in a two digit industry has been

TABLE III-3-5.--Estimates of the Elasticity of Substitution for Three-Digit Industries and Standard Errors--1963¹

Two-Digit Industry	Third-Digit									Ho: $\sigma = 1$ (Accept/Reject)	
	1	2	3	4	5	6	7	8	9	all Estimates	when $\sigma \geq 1.5$
20	0.558* (.113)	0.714* (.106)	1.259 (.340)	0.867 (.231)	0.949 (.135)		1.081 (.259)	1.309* (.139)	1.779* (.244)	4/4	1/3
22					0.927 (.387)			0.082 (.714)	1.386 (.429)	3/0	0/0
23	1.540 (.363)	1.959* (.431)	1.476* (.219)	2.363* (.457)		1.869 (.497)		1.207 (.231)	1.068 (.102)	4/3	1/0
24	0.990 (.178)	0.469 (.281)	0.835 (.123)	0.964 (.138)					0.959 (.168)	5/0	2/0
25	1.134 (.150)		0.747 (.206)	0.953 (.176)					0.562 (.263)	4/0	1/0
26		2.308* (.521)	0.883 (.629)	1.172 (.458)	0.727 (.225)					3/1	0/0
27	0.868 (.101)	1.108 (.102)	1.059 (.188)	0.998 (.171)	0.818 (.095)	0.884 (.437)		0.832 (.197)	1.066 (.100)	8/0	4/0
28	0.939 (.311)		1.796 (.497)	2.231* (.414)	2.053* (.291)		1.406 (.258)		1.170 (.306)	4/2	0/0
29	1.053 (.437)				0.879 (.459)					2/0	0/0
30						0.649 (.377)	0.643 (.278)			2/0	0/0
31				0.991 (.407)						1/0	0/0
32			1.235 (.344)	0.366 (.699)	1.424 (.311)		1.207 (.120)	0.912 (.183)	1.107 (.318)	6/0	1/0
33	0.999 (.317)	1.151 (.154)			1.297 (.337)	1.164 (.149)			0.680 (.286)	5/0	1/0
34		1.248 (.209)	1.323 (.291)	0.692* (.120)	0.811 (.191)	0.786 (.158)	0.659 (.209)	0.907 (.363)	0.680* (.156)	6/2	0/1
35		0.717 (.205)	0.786 (.317)	0.882 (.122)	1.119 (.149)	0.853 (.138)		0.907 (.314)	0.820 (.125)	7/0	4/0
36	1.015 (.230)	0.923 (.293)	1.098 (.600)	0.816 (.482)		0.763 (.292)	0.875 (.225)		1.027 (.249)	7/0	0/0
37	1.706 (.362)		1.556* (.263)						0.854 (.281)	2/1	0/0
38	0.543* (.203)	1.006 (.272)		1.213 (.250)						2/1	0/0
39	-0.253* (.512)			1.259 (.226)	0.805 (.312)	0.787 (.353)			1.215 (.132)	4/1	1/0

Data Sources: See text.
* Significantly different from one at 5 per cent.

¹ Ibid., p. 109, 110.

established. In the case under question the finer three digit classification did not vitiate the conclusion that an elasticity of substitution equal to one was appropriate for most estimation purposes; however, it is clear that variations in the industry mix and inter-industry differences at the three digit level can easily combine to produce substantial regional differences in two digit estimates of various production parameters. This implies that more care must be taken in carrying out inter-regional cross-sectional studies to ensure that industries under examination have reasonably comparable industry mixes rather than similar names.

CHAPTER IV

INTER-INDUSTRY DIFFERENCES IN THE ELASTICITY OF FACTOR SUBSTITUTION

The Heckscher-Ohlin model has been widely used to provide an explanation for the movement of goods and services between various regions. Given the following assumptions:

- (1) Identical production functions for all countries;
 - (2) Constant returns to scale;
 - (3) Perfect mobility of factors within a country and complete immobility between countries;
 - (4) Inelastic supply of factors; and
 - (5) Perfect competition in all markets and an absence of transport costs;
- it is relatively easy to demonstrate that differences in the comparative costs of producing various commodities are based on differences in relative factor endowments among various countries. The basis for trade rests on the emergence of comparative advantages predicated on differences in the factor intensities of various commodities. Under perfectly competitive conditions countries will export commodities which use their relative factor endowments most intensively. The eventual equalization of commodity prices eventually leads to the equalization of factor prices.¹

¹Refer to Clement, Pfister and Rothwell, Theoretical Issues in International Economics, (Boston, Massachusetts: Houghton Mifflin Company), 1967, pp. 81-122, for a detailed treatment of the Heckscher-Ohlin model and a review of the empirical studies and issues which challenge its validity. Attention should also be paid to the discussion dealing with the factor price equalization theorem and factor reversals on pages 18 to 60.

The purpose of this Chapter is to briefly test whether the pre-conditions for factor price equalization exist within the domestic economy. This may be achieved fairly easily by following the procedure adopted by Minhas¹ which simply amounts to determining whether the strong factor intensity assumption--the optimal ratio of capital to labour in any given industry is always greater or always less than in any other industry regardless of the ratio of factor prices--holds in a C.E.S. world. In effect, if an ordering of industries according to their capital-labour ratio is affected by changes in the factor price ratio little faith can be placed in the assumption that inter-regional trade in commodities will result in factor price equalization within the economy.

The basis for testing the strong intensity hypothesis has been set out in Chapter II, pages ten to twelve. Reference to equation (II-1-22) indicates that if the elasticities of substitution in a given set of industries are identical then the strong intensity assumption remains valid. If the elasticities differ the factor price equalization theorem is refuted.

The following econometric analysis examines the question of whether or not there are significant differences in elasticities of substitution within a given region. Four regions are examined--Canada, the Atlantic provinces, Western Canada and Central Canada. The latter three regions are included since inter-industry differences in elasticities of substitution for the nation will be affected by the regional differences noted in Chapter III. Reduction of the region to its basic sub-component should eliminate this objection.

¹B. S. Minhas, op. cit., Chapter IV.

The Econometric Analysis

The Completely Unrestricted Model

Given a situation in which there are $j = 1, 2, \dots, J$ industries located in $r = 1, 2, \dots, R$ regions with $i = 1, 2, \dots, N_j$ observations on log value added per man hour (Y) and the log wage rate (X)--one can express the requisite dummy variable form of the ordinary least squares regression model for the r^{th} region as

$$Y_i = DA_1 + X_i DA_2 + \epsilon_i \quad (IV-1)$$

where $i = 1, 2, \dots, N$, $N = \sum_{j=1}^J N_j$. D is a $(1 \times J)$ vector of dummy variable

$D = [D_1 \dots D_k \dots D_J]$ with $D \equiv 1$ and $D_k = 1$ for all observations in the k^{th} industry and $D_k = 0$ for observations in other industries.

$$A_1 = \begin{bmatrix} a_1 \\ a_2 \\ \cdot \\ \cdot \\ a_J \end{bmatrix} \quad J \times 1$$

is a column vector of intercept coefficients where a_1 is the intercept for the base industry and $a_2 \dots a_J$ are differential intercepts for the remaining industries.

$$A_2 = \begin{bmatrix} a_{J+1} \\ a_{J+2} \\ \cdot \\ \cdot \\ a_{2J} \end{bmatrix} \quad J \times 1$$

where a_{J+1} is the slope coefficient for the base region and $a_{J+2} \dots a_{2J}$ are differential slope coefficients for the remaining industries.

There are R models of this type.

The Fully Restricted Model

The fully restricted model is expressed by:

$$Y_i = \alpha + \beta X_i + U_i \quad (IV-2)$$

where $i = 1 \text{---} N$

THE EMPIRICAL RESULTS

Inter-Industry Results for Four Regions of Canada

The results of the tests for the homogeneity of the differential slope and intercept coefficients in four regions of Canada are set out in Table IV-1. Each region exhibits highly significant inter-industry differences at the one percent level.

Given these results it is necessary to determine whether the subset of differential slope (elasticity) coefficients are significantly different from zero. The tests¹ of the requisite null hypothesis for each region are set out in Table IV-2. It is evident that there are significant inter-industry differences in the elasticity of factor substitution in all regions though the evidence is weaker for Western Canada compared to the other three regions (five percent vis-a-vis a one percent level of significance).

¹The "F" test is given by J. Johnson.

$$F = \frac{S_3 / (pk - p - k + 1)}{S_1 / (N - pk)}$$

where p - is the number of industries;
 k - is the number of explanatory variables (excluding dummy);
 N - is the total number of observations;
 S_3 - is the increment in the residual sum of squares obtained by subtracting
 S - the residual sum of squares for the full dummy variable regression from
 S_2 - the residual sum of squares for the partially restricted regression
J. Johnston, Econometric Methods, 2nd edition, (New York: McGraw-Hill, 1972), pp. 198-199.

TABLE IV-1.--Joint Tests of Significance (Intercepts and Slopes) for Inter-Industry Effects in Four Regions of Canada.

Region	"F" Value	Degrees of Freedom		Significance Levels
		Numerator	Denominator	
Atlantic Provinces ¹	9.10	14	141	R
Central Canada	26.07	32	537	R
Western Canada	12.11	32	496	R
Canada	32.2	32	1224	R

R - Reject the null Hypothesis

A - Accept the null hypothesis

¹Only eight industries are included in the Atlantic Provinces. The remaining three regions contain seventeen of the twenty two digit manufacturing industries.

TABLE IV-2.--Joint Tests of Significance (Differential Slopes) for Inter-Industry Effects in Four Regions of Canada

Region	*F* Value	Degrees of Freedom		Significance Level 1%
		Numerator	Denominator	
Atlantic Provinces	5.04	7	141	R
Central Canada	7.57	16	537	R
Western Canada	1.67	16	496	A*
Canada	4.41	16	1224	R

*Reject the null hypothesis at the five percent level.

It is an interesting exercise to attempt to identify the various industries in each region which exhibit significant differences in the elasticity of substitution. This implies that attention must be paid to the full (completely unrestricted) regression results and the "t" values for the individual dummy slope coefficients. (Refer to the specification set out in (IV-1).

Tables IV-3 and IV-4 provide the requisite information for Canada as a whole. Nine industries are significant at the one percent level, eleven industries at the five percent level and twelve industries at the ten percent level. Five industries--Leather, Paper and Allied, Primary Metals, Non-Metallic Mineral Products and the Petroleum and Coal Products--have elasticities of substitution which are not significantly different from the Food and Beverage industry (approximately 1.6). This result is interesting since it suggests that there is an important sub-group of industries in the Canadian economy in which commodity price equalization could potentially lead to factor price equalization though it should be noted that this comment is highly speculative since it is not at all clear that one can meaningfully separate portions of the economy on this basis.

It should also be noted that generally speaking the output of these six industries has a high raw material content. It would be useful to estimate a specific form of the production function for each industry which included natural resources as well as capital and labour as factors of production.

The results for the three sub-regions of Canada are set out in Tables IV-5 to IV-10 inclusive. In the Atlantic Provinces only the Food

TABLE IV-4.---Tests of Significance for Inter-Industry Differences in Elasticities of Substitution
In Canada

S.I.C. No.	Industry 2 Name	"c" value	Tests of Significance of Differential Slope Coefficients ¹		
			10%	5%	1%
4	Leather	-1.141	A	A	A
5	Textile	-2.956	R	R	R
7	Clothing	-2.232	R	R	R
8	Wood	-4.671	R	R	R
9	Furniture and Fixture	-2.709	R	R	R
10	Paper and Allied	-1.638	A	A	A
11	Printing, Publishing and Allied	-2.966	R	R	R
12	Primary Metals	1.881	R	A	A
13	Metal Fabricating	-3.782	R	R	R
14	Machinery	-3.488	R	R	R
15	Transportation Equipment	-2.597	R	R	A
16	Electrical Products	-2.942	R	R	R
17	Non-Metallic Mineral Products	-1.420	A	A	A
18	Petroleum and Coal Products	-1.573	A	A	A
19	Chemical and Chemical Products	-4.913	R	R	R
20	Miscellaneous Manufacturing	-5.493	R	R	R

¹Two tailed tests with 1224 degrees of freedom

²Base Industry - Food and Beverages

TABLE IV-5.--The Full Regression for the Secondary Manufacturing Industries in the Atlantic Provinces with Industry Dummies for the Intercept and Wage Rate (X)

		Industries ¹									
Base 2 Industry 1		8	10	11	13	15	17	20			
Intercept	1.138	-0.227	0.070	-0.987	0.148	-0.164	-0.242	0.002			
Standard Error	-	0.138	0.335	0.269	0.379	0.310	0.324	0.190			
"t" value	-	-1.643	0.208	-3.673	0.392	-0.530	-0.748	0.011			
X	1.741	-1.549	-0.995	1.060	-1.488	-1.491	-0.348	-1.368			
Standard Error	0.244	0.589	0.504	0.471	0.661	0.577	0.667	0.467			
"t" value	7.127	-2.632	-1.973	2.253	-2.249	-2.583	-0.523	-2.929			
N = 157		R ² = 0.786									

¹The industries are numbered according to the S.I.C. code. (Refer to Table II-2.)

²The food and beverage industry (S.I.C. No. 1) was selected as the base industry.

TABLE IV-6.---Tests of Significance for Inter-Industry Differences in the Elasticity of Substitution in the Atlantic Provinces

S.I.C. No.	Industry Name	"t" value	Tests of Significance of Differential Slope Coefficients ^{1, 2}	
			1%	5%
8	Wood	-2.632	R	R
10	Paper and Allied	-1.973	A	R
11	Printing, Publishing and Allied	2.253	A	R
13	Metal Fabricating	-2.249	A	R
15	Transportation Equipment	-2.583	R	R
17	Non-Metallic Mineral Products	-0.523	A	A
20	Miscellaneous Manufacturing	-2.929	R	R

¹Two tailed tests with degrees of freedom = 141

²The food and Beverage industry (S.I.C. No. 1) was selected as the "base" industry.

R - reject the null hypothesis

A - accept the null hypothesis

TABLE IV-7.--The Full Regression for the Secondary Manufacturing Industries in Central Canada
With Industry Dummies for the Intercept and Wage Rate (X)

	Base Industry 1	Industries									
		4	5	7	8	9	10	11	12		
Intercept	0.735	0.021	0.280	-0.086	-0.008	0.264	0.099	-0.901	-0.092		
Standard Error	-	0.204	0.132	0.116	0.186	0.222	0.281	0.580	0.364		
"t" value	-	0.102	2.124	-0.572	-0.041	1.189	0.352	-1.555	-0.254		
X	2.301	-1.465	-1.439	-1.128	-1.232	-1.580	-0.940	-0.172	-0.994		
Standard Error	0.128	0.500	0.262	0.249	0.461	0.446	0.399	0.613	0.427		
"t" value	17.920	-2.929	-5.502	-4.534	-2.670	-3.538	-2.357	-0.281	-2.328		
	13	14	15	16	17	18	19	20			
Intercept	0.445	0.410	0.047	0.878	-0.129	1.149	1.926	0.376			
Standard Error	0.330	0.295	0.224	0.249	0.236	0.446	0.222	0.110			
"t" value	1.348	1.389	0.211	3.535	-0.546	2.576	8.681	3.408			
X	-1.620	-1.331	-1.193	-1.925	-0.515	-1.246	-2.432	-1.476			
Standard Error	0.435	0.389	0.294	0.359	0.317	0.479	0.302	0.198			
"t" value	-3.721	-3.418	-4.061	-5.365	-1.623	-2.602	-8.060	-7.451			

n = 571 R² = 0.789

TABLE IV-8.--Test of Significance for Inter-Industry Differences in Elasticities of Substitution
In Central Canada

S. I. C. No.	Industry ² Name	"t" value	Tests of Significance of Differential Slope Coefficients ¹	
			5%	10%
4	Leather	-2.929*	R	R
5	Textile	-5.502*	R	R
7	Clothing	-4.534*	R	R
8	Wood	-2.670*	R	R
9	Furniture and Fixture	-3.538*	R	R
10	Paper and Allied	-2.357	R	R
11	Printing, Publishing and Allied	-0.281	A	A
12	Primary Metals	-2.328	R	R
13	Metal Fabricating	-3.721*	R	R
14	Machinery	-3.418*	R	R
15	Transportation Equipment	-4.061*	R	R
16	Electrical Products	-5.365*	R	R
17	Non-Metallic Mineral Products	-1.623	A	A
18	Petroleum and Coal Products	-2.602*	R	R
19	Chemical and Chemical Products	-8.060*	R	R
20	Miscellaneous Manufacturing	-7.451*	R	R

* reject null at the 1% level

¹Two tailed tests with 537 degrees of freedom

²Base Industry - Food and Beverage

TABLE IV-9.--The Full Regression for the Secondary Manufacturing Industries in Western Canada
With Industry Dummies for the Intercept and Wage Rate (X)

		INDUSTRIES											
Base Industry		4	5	7	8	9	10	11	12				
1													
Intercept	0.991	-0.490	-0.057	-0.221	-0.027	0.109	-0.001	0.174	1.228				
Standard Error	-	0.358	0.191	0.163	0.161	0.327	0.279	0.190	0.652				
"t" value	-	-1.371	-0.300	-1.360	-0.165	0.333	-0.002	0.919	1.883				
X	1.495	0.199	-0.492	-0.451	-0.763	-0.982	-0.130	-0.522	-1.805				
Standard Error	0.152	1.026	0.432	0.413	0.255	0.596	0.375	0.255	0.730				
"t" value	9.855	0.194	-1.140	-1.093	-2.993	-1.646	-0.348	-2.044	-2.473				
		13	14	15	16	17	18	19	20				
Intercept	0.043	0.873	-0.233	0.142	0.464	0.781	0.799	0.120					
Standard Error	0.221	0.374	0.257	0.390	0.296	0.550	0.211	0.136					
"t" value	0.196	2.334	-0.906	0.364	1.565	1.419	3.792	0.884					
X	-0.637	-1.487	-0.523	-0.166	-0.873	-0.468	-0.590	-0.642					
Standard Error	0.295	0.489	0.339	0.623	0.418	0.560	0.297	0.230					
"t" value	-2.160	-3.043	-1.545	-0.266	-2.088	-0.837	-1.988	-2.796					
n = 530		R ² = 0.632											

The base industry is the Food and Beverage industry.

TABLE IV-10.--Tests of Significance for Inter-Industry Differences in Elasticities of Substitution
in Western Canada

S.I.C. No.	Industry ² Name	"t" value	Tests of Significance of Differential Slope coefficients ¹	
			10%	5%
4	Leather	0.194	A	A
5	Textile	-1.140	A	A
7	Clothing	-1.093	A	A
8	Wood	-2.993*	R	R
9	Furniture and Fixture	-1.646	R	A
10	Paper and Allied	-0.348	A	A
11	Printing, Publishing and Allied	-2.044	R	R
12	Primary Metals	-2.473	R	R
13	Metal Fabricating	-2.160	R	R
14	Machinery	-3.043*	R	R
15	Transportation Equipment	-1.545	A	A
16	Electrical Products	-0.266	A	A
17	Non-Metallic Mineral Products	-2.088	R	R
18	Petroleum and Coal Products	-0.837	A	A
19	Chemical and Chemical Products	-1.988	R	R
20	Miscellaneous Manufacturing	-2.796*	R	R

*Reject at the one percent level.

¹Two tailed tests with 496 degrees of freedom.

²Base industry - Food and Beverage.

and Beverage industries and the Non-Metallic Mineral Products industries have similar elasticities at the five percent level while in Central Canada the Food and Beverage, Printing, Publishing and Allied and Non-Metallic Mineral Products industries share a common value at the same level of significance. In Western Canada eight industries share a common elasticity at the five percent level with the Food and Beverage industry.

(Refer to Tables IV-9 and IV-10.) The elasticity of substitution for this group is considerably higher than for the remaining nine industries. (It should be noted that this comment holds for the other regions with the exception of the Printing, Publishing and Allied industry in Canada.) The reason for this differential is not clear. The general observation that the high elasticity industries seem to have a high raw material component holds reasonably well for the nation as a whole and is supported somewhat by the results noted in the Maritimes and Central Canada; however, in Western Canada the Textile, Clothing, Electrical Products and Transportation Equipment is included with the Pulp and Paper and Leather industries. It is evident that a detailed analysis must be undertaken before any conclusions can be drawn.

The Over-all Significance of the Findings

The results of the investigation into the question of whether the pre-condition for factor price equalization in inter-regional trade exists within the nation and three of its sub-regions strongly suggests that factor reversals are the rule rather than the exception. This result indicates that differences in elasticities of substitution may lead to quite different capital-labour mixes within industries located in a given region and among various segments of a given industry located in certain regions. In effect, the same relative increase in the price of labour

relative to capital will lead to a greater degree of substitution of capital for labour in various industries which can eventually lead to the switching of a given industry from a relatively labour intensive to a relatively capital intensive industry. As noted in Chapter II (pages ten to twelve) the question of whether this reversal will occur is reduced to determining whether the critical factor price ratio falls within the set of observable prices. This ratio cannot be determined given our data problems.

It is clear that the results in this chapter must be interpreted with a great deal of caution since absolutely nothing has been said about technological change or economic growth. Furthermore, the estimates of the elasticities of substitution have been obtained without direct recourse being made to capital stock data. In addition, it is not at all clear that two or three digit levels of industrial activity are the appropriate vehicles to use in testing a theory which is defined in terms of specific identifiable commodities and factors.

CHAPTER V

CONCLUSIONS AND OBSERVATIONS

The following conclusions may be drawn from this study:

CONCLUSIONS

First, slight variations in time periods do not create any significant differences in estimates of the elasticities of factor substitution and the efficiency of labour use in Canadian manufacturing industries. This conclusion is supported by Zarembka's¹ comparison of elasticities in two digit manufacturing industries in the United States between 1957 and 1958. Both of these results disagree with Nerlove's conclusion that drastically different estimates of cross-sectional elasticities are obtained by slight changes in the time period.²

Secondly, there are significant inter-regional differences in the elasticity of substitution and/or the efficiency of labour use within certain two digit manufacturing industries. This partially confirms the Economic Council's conclusion that regional income disparities in Canada are caused by regional differences in industrial productivity.³ In particular, the Council's observations on the existence of substantial differences in earned income per employed person in aggregate manufacturing in the Atlantic

¹ Zarembka, op. cit.

² ibid.

³ Towards Sustained and Balanced Growth, Second Annual Review, Economic Council of Canada, (Ottawa: Queen's Printer, 1965), p. 124.

Provinces vis-a-vis Central and Western Canada are weakly supported by evidence indicating that there are significant differences in production functions for various two digit manufacturing industries located in these regions. A more limited comparison of a set of Western and Central Canadian manufacturing industries indicated that these two regions tended to have common production functions for the majority of industries that were considered.

The reason for the difference in aggregate production functions was not satisfactorily resolved. This state of affairs partly stems from the lack of capital data; however, a more fundamental problem was encountered--it appears that substantial differences in industry and/or product mixes in various regions are associated with significant inter-regional differences in the aggregate functions. This observation suggests that differences in aggregate production functions simply stem from a different weighting of micro-economic functions. It is evident that this question must be resolved before any attempt is made to formulate rational regional development policies.

Thirdly, a number of two digit industries have common elasticities of substitution. A unitary value for the elasticity cannot be rejected at a one percent level of significance; therefore, the selection of a Cobb-Douglas form of the production function seems to be appropriate for most estimation purposes. This conclusion is supported by Zarembka and Chernicoff's study of United States manufacturing industries in 1963¹ (Table III-3-4) and is within the range of estimates provided in the set of cross-sectional studies noted in Table II-2-1.

¹P. Zarembka and H. B. Chernicoff, op. cit., p. 106-110.

Fourthly, if the assumption is made that the existence of "uncommon" or "non-identical" aggregate production functions reflect similar differences in the underlying micro-economic relationships then it follows that the Ricardian explanation of inter-regional trade--different costs of production attributable to the fact that identical inputs of factors yield different outputs of the same commodity in different regions--has equal standing with the Heckscher-Ohlin theorem in explaining the direction of factor and commodity movements in the manufacturing segment of the Canadian economy.

Fifthly, significant inter-industry differences in the elasticities of substitution ensure that the basic pre-condition for ensuring that free inter-regional trade in commodities will equalize factor prices is vitiated. This result occurred in each of the four regions that were examined.

Observations

It is clear that the lack of regional capital stock data has greatly hampered the scope of this study. This problem should be rectified at the earliest possible opportunity since it is becoming increasingly evident that the sound formulation of regional economic policy is becoming more and more of an issue in the Canadian federation. The study has suggested that certain national economic policies which impact relative factor prices will have a uniform effect on all regions for a limited number of industries; however, in other cases there are sharp divergencies which will lead to diametrically opposite effects in certain regional economies. Clearly, decisions which are related to formulating national and provincial economic priorities cannot be taken in ignorance of the explicit trade-offs that are involved.

The severe limitations imposed on this study have been noted on numerous occasions. It is worth noting that once a number of alternative production function specifications have been tried there is a strong possibility that more tangible economic problems can be examined. Any of the tentative policy suggestions that have been made should be viewed with the 'appropriate' degree of suspicion that is reserved for a preliminary investigation of this nature.

In conclusion this study is only one of a large number of investigations that need to be carried out in the field of regional economics in Canada; however, it is important to stress the fact that the attempt to formulate relatively simple models which can be used to examine regional economic growth problems rests on the determination of the specific form of the underlying aggregate production relationships. It is suggested that the high priority assigned to the promotion of high and balanced rates of regional economic growth in the Canadian federation should result in the allocation of substantial amounts of funds being directed towards the provision of basic economic data and econometric research. This has not occurred in any meaningful way within the Dominion as evidenced by the problems created in this study by the lack of available regional capital stock data. The problem of formulating viable regional economic policies as well as simply trying to obtain some degree of quantitative understanding of basic regional economic relationships is tied to the availability of basic data series. The problems created by the lack of reliable regional data are best summed by Lord Kelvin's comment:

...When you can measure what you are speaking about, and express it in numbers, you know something about it; when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind; it may be the beginning of knowledge; but you have scarcely, in your thoughts, advanced to the stage of science...¹

¹p. A. Samuelson and A. Scott, Economics, Second edition, (Toronto, Ontario: McGraw-Hill, 1968), p. 811.

BIBLIOGRAPHY

Books

- Allen, R. G. D., Macro-Economic Theory, New York: MacMillan, 1967.
- Brown, M., On the Theory and Measurement of Technological Change, Cambridge, England: Cambridge University Press, 1968.
- Brown, M. (editor), The Theory and Empirical Analysis of Production, "Studies in Income and Wealth", Volume XXXI, New York: National Bureau of Economic Research, 1967.
- Clement, M., et. al, Theoretical Issues in International Economics, Boston, Massachusetts: Houghton Mifflin Company, 1967.
- Ferguson, C. E., Microeconomic Theory, Homewood, Illinois: Richard D. Irwin, Inc., 1966.
- Goldberger, Arthur S., Econometric Theory, New York: John Wiley & Sons, Inc., 1964.
- Kmenta, Jan, Elements of Econometrics, New York: The MacMillan Co., 1971.
- Johnston, J., Econometric Methods, second edition, New York: McGraw-Hill Book Co., Inc., 1970.
- Mansfield, Edwin, Microeconomics, New York: W. W. Norton & Co., Inc., 1970.
- McKee, D. L., Dean, R. D., Leahy, W. H., Regional Economics, New York: The Free Press, 1970.
- _____. Spatial Economic Theory, New York: The Free Press, 1970.
- Minhas, B. S., An International Comparison of Factor Costs and Factor Use, Amsterdam: North-Holland Publishing Company, 1962.
- Richardson, H. W., Regional Economics, London, England: Weidenfeld and Nicholson, 1969.
- Samuelson, P. A., Scott, A., Economics, Second Edition, Toronto, Ontario: McGraw-Hill, 1968.
- Shearer, R. A., Young, J. H., Munro, G. R., Trade Liberalization and a Regional Economy: Studies of the Impact of Free Trade on British Columbia, Private Planning Association of Canada, Toronto, Ontario: University of Toronto Press, 1971.

Stiglitz, Joseph E., editor, The Collected Scientific Papers of Paul A. Samuelson, Volume II, Cambridge, Massachusetts, M.I.T. Press, 1966.

Wilkinson, B. W., Canada's International Trade: An Analysis of Recent Trends and Patterns, Private Planning Association of Canada, 1968.

Wonnacott, R. J., and Wonnacott, T. H., Econometrics, New York: John Wiley and Sons, Inc., 1970.

Public Documents

Dominion Bureau of Statistics, Manufacturing Industries of Canada, 1964, Sections A, B, C, D, E, and G, Ottawa: Queen's Printer, 1968.

Dominion Bureau of Statistics, Manufacturing Industries of Canada, 1965, Sections C, D, E and F, Ottawa: Queen's Printer, 1968.

Dominion Bureau of Statistics, Standard Industrial Classification Manual, Ottawa: Information Canada, 1970.

Towards Sustained and Balanced Growth, 2nd Annual Review, Economic Council of Canada, Ottawa: Queen's Printer, 1965.

Articles

Arrow, K. J., Chenery, H. B., Minhas, B.S., and Solow, R. M., "Capital-Labour Substitution and Economic Efficiency", The Review of Economics and Statistics, Volume XLII, (August 1961).

Bell, F. W., "The Role of Capital-Labour Substitution in the Economic Adjustments of an Industry Across Regions", Southern Economic Journal, 31, 1964-65.

Bodkin, R. G., and Klein, L. R., "A Non-Linear Estimation of Aggregate Production Functions", Review of Economics and Statistics, Volume XLIX, (February 1967).

Borts, George, "The Equalization of Returns and Regional Economic Growth", American Economic Review, 50, 1960.

Chipman, J. S., "A Survey of International Trade" Part 3, The Modern Theory", Econometrica, Volume 34, No. 1, (January 1966).

Dhrymes, P. J. and Zarembka, P., "Elasticities of Substitution for Two Digit Manufacturing Industries: A Correction", Research and Statistics, Volume LII, No. 1, (February 1970).

- Ferguson, C. E., "Cross-Section Production Functions and the Elasticity of substitution in American Manufacturing Industry", Review of Economics and Statistics, Volume XLV, (1969).
- Fisher, F. M., "The Existence of Aggregate Production Functions", Econometrica, (October 1969).
- Geary, R. C., and Leser, C. E. V., "Significance Tests in Multiple Regression", The American Statistician, Volume 22, (February 1968).
- Kotowitz, Y., "On the Estimation of a Non-Neutral C.E.S. Production Function", The Canadian Journal of Economics, (May 1968).
- _____. "Capital-Labour Substitution in Canadian Manufacturing 1926-39 and 1946-61", The Canadian Journal of Economics, (August 1968).
- _____. "Technical Progress, Factor Substitution and Income Distribution in Canadian Manufacturing 1926-39 and 1946-61", The Canadian Economic Journal, (February 1969).
- Lu, Y. I. and Fletcher, L. B., "A Generalization of the C.E.S. Production Function", Review of Economics and Statistics, Volume L, Number 4, (1968).
- Minasian, Jora, R., "Elasticities of Substitution and Constant-Output Demand Curves for Labour," Journal of Political Economy, (1961).
- Paroush, "The h'Homogeneous Production Function with Constant Elasticity of Substitution: A Note:", Econometrica, Volume 34, Number 1, (January 1966).
- Stewart, J. and Rayner, A. J., "Qualitative Factors in Linear Regression Analysis", The Manchester School, (April 1971).
- Suto, R. and Hoffman, R., "Production Functions with Variable Elasticity of Factor Substitution: Some Analysis and Testing", Review of Economics and Statistics, Volume L, Number 4, (1968).
- Tsurumi, Hiroki, "Non-Linear Two Stage Least Squares Estimation of C.E.S. Production Functions Applied to the Canadian Manufacturing Industries 1926-1939, 1946-1967", The Review of Economics and Statistics, Volume LII, Number 2, (May 1970).
- Zarembka, P. and Chernicoff, H. B., "Further Results on the Empirical Relevance of the C.E.S. Production Function", Review of Economics and Statistics, Volume LIII, Number 1, (February 1971).

Other Sources

- A letter received from D. Bruce Petrie, Chief, National Wealth and Capital Stock Section, Business Finance Division of the Dominion Bureau of Statistics, July, 19, 1969.