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REINFORCED CONCRETE BUILDINGS

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Reinforced Concrete Buildings

LECTURE 1

INTRODUCTION TO THE 1971 ACI CODE

by

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INTRODUCTION TO THE 1971 A.C.I. CODE

R. N. McManus, P. Eng.

One would think, after more than 60 years of reinforced concrete design by code that very little scope would remain for change.

Changes in design practice however, as reflected in building codes, have been continuous since the first joint committee report.

Throughout this period there has been a regular, if at times unsteady, transition from elastic analysis and permissible stresses to ultimate strength concepts and acceptable factors of safety.

The flat slab enjoyed an advantage over other types of construction for years because the code recognized, from tests, its capability of mobilizing capacity beyond elastic limits at any one point.

Similarly load tests on columns proved their ultimate capacity went beyond transformed elastic limits, particularly eccentrically loaded compression members.

The most recent A.C.I. Code, - 318 - 63 continued this trend to the point where working stress design and ultimate strength design were, to all intents and purposes, given equal billing.

A.C.I. 318 - 71 has carried this transition to the point where it is basically an ultimate strength design code which pays small lip service to working stress design.

There are other basic changes in philosophy and technical approach as well. They will be dealt with briefly at this time to highlight the concepts but most of them will be covered in more detail by others.

First of all the provisions of the code are all based on normal reinforcement having a yield point of 60,000 p.s.i. Other grades of reinforcement may be used, with modification to the code requirements.

Although there is a separate chapter covering prestressed concrete, the requirements for prestressed concrete have been pretty well integrated throughout the code. This is approaching the European practice of treating prestressed concrete as reinforced concrete under special initial conditions.

The general trend of reducing the factors of safety has continued but not without the imposition of additional safeguards. Chapter 4 "Concrete Quality" requires that the history of strength and standard deviation of the proposed mix be known or else the mix must be designed for much higher strength than required. Where durability and water tightness are factors, minimum values of f'c are set for both normal weight and lightweight aggregate concretes.

Chapter 8 - "Analysis and Design - General Considerations" appears to be working stress design. Close examination will show, however, that this is really only true with regard to flexure. Other sections of this chapter all refer to corresponding chapters in the code which themselves are based on ultimate strength design.

Chapter 9 - "Strength and Serviceability Requirements". Based on the increased quality control requirements of Chapter 4, the load factors of Chapter 9 are reduced an average of about 6 per cent compared to 318 - 63.

Also in this chapter deflection control is spelled out with lightweight aggregate concrete covered for the first time.

Chapter 12 - "Development of Reinforcement" represents a departure from the time-honoured procedure of relating unit bond stress to shear. All reinforcement is now simply required to have sufficient length of embedment, or end anchorage, to develop the necessary tension or compression. The critical points are defined and formulae for development length of deformed bars, deformed wire, welded wire mesh and prestressing strand are given. These formulae for normal reinforcing are stated in terms of the size of the bar, the yield point of the steel and the strength of the concrete.

Lightweight aggregate concrete is covered by factors related to the type of mix or the strength of concrete and its tensile splitting strength when this is known. A.C.I. - 318 - 63 did not differentiate between normal and lightweight aggregate concrete so far as bond stress was concerned.

Chapter 15 - "Footings" has been changed notably with regard to the transfer of stress at the base of a column or pedestal. Dowels are only required to carry the excess load over that permitted on the concrete itself. The dowel area is not necessarily related to the area of reinforcement in the compression member at all.

This change is somewhat odd in that the bearing stress permitted on concrete is quite a bit higher than permissible axial stresses under service load conditions on a column. This section would appear to be satisfactory so far as the footing is concerned where the total area is considerably greater than the loaded area. It would appear, however, that the lower end of the column concrete, at least over the development length of the reinforcement, could be highly stressed.

Chapter 19 - "Shells and Folded Plate Members" is a completely new section in this code. The provisions are quite general and elastic analysis is specified.

Chapter 20 - "Strength Evaluation of Existing Structures" is considerably expanded, over A.C.I. - 318 - 63 - "Load Tests of Structures". The general criterion for a structure under test load is that it shall not show "visible evidence of failure". The deflection and recovery criterion include prestressed concrete and the total test load set up is a constant percentage of theoretical design strength.

In summary the new code:

1. Leans heavily towards U.S.D.
2. Has a completely new approach to bond in terms of development length.
3. Integrates prestressed concrete to a large degree.
4. Continues the trend of lower factors of safety combined with higher standards of quality control.
5. Recognizes the increased use of structural lightweight concrete.

A word on the C.S.A. - N.B.C. Joint Committee on Reinforced Concrete Design. The Committee has met twice in preparation of a new code. The 1970 edition is not really new since it is simply an edited version of the 1965 code.

The A.C.I. - 1971 is being used as a working model and committee members have now prepared the first draft of the proposed chapters of our code based largely on the A.C.I. - 318 - 71.

These drafts will be considered at a meeting in June 1971 and re-worked for a fall meeting, after which the first draft of the new code will go out for public comment.

A final meeting in 1972 should deal with the public comments and C.S.A. - A 23-3 should be ready for publication in late 1972 or early 1973.

Reinforced Concrete Buildings

LECTURE 2

SHEAR DESIGN PROVISIONS FOR DEEP BEAMS BRACKETS AND SLABS

by

J.G. MacGregor

CHAPTER 11

SHEAR STRENGTH

Chapter 11 deals with the shear and torsional strength of prestressed and reinforced concrete beams, and the shear strength of deep beams, brackets and corbels, slabs and walls. Dr. Warwaruk will deal with design for torsion, this paper will consider the rest of the Chapter.

Basic Design Relationships for Shear Strength--11.2, 11.4, 11.5

The basic design procedure and the basic equations for v_c for reinforced or prestressed concrete beams are essentially unchanged from the 1963 Code. The reader should refer to References 1, 2, or 3 or any standard textbook on reinforced or prestressed concrete for an explanation of the use of these equations.

Throughout the chapter the capacity reduction factor, ϕ, has been included in the equations for computing the ultimate shear stresses, v_u, rather than in the equations for the shear carried by the concrete or the web reinforcement.

Minimum Web Reinforcement--11.1 and 11.2

Whenever the nominal ultimate shear stress exceeds 1/2 v_c, the 1971 Code requires a minimum amount of web reinforcement. This is desirable because web reinforcement restrains the growth of inclined cracking increasing the ductility and provides a warning in situations where inclined cracking may form suddenly due to an unexpected loading. The required minimum amount of web reinforcement is given by:

$$A_v = 50\ b's/f_y \qquad (11\text{-}1)$$

or: $$v_u - v_c = \frac{A_u f_y}{b's} = 50 \text{ psi}$$

Thus, whenever $v_u \leq v_c + 50$ psi it is not necessary to design web reinforcement beyond providing the minimum amount.

Shear Strength of Lightweight Concrete Beams--11.3

This section gives two ways in which the shear strength equations can be modified when lightweight aggregate concrete is used. The concept is essentially the same as in the 1963 ACI Code except that the values have been somewhat liberalized and simplified.

Combined Shear and Axial Load--11.43 and 11.4.4

The 1963 Code required that members subjected to combined axial load and bending should be designed using:

$$v_c = 1.9\sqrt{f'_c} + 2500 p_w \frac{V_u d}{M'} \qquad (11\text{-}4)$$

where: $$M' = M_u - N_u \frac{(4t-d)}{8} \qquad (11\text{-}5)$$

Because this leads to a very tedious design procedure, the 1971 Code allows the use of a simple alternate procedure based on Equations (11-6) and (11-7):

Shear plus Axial Compression:

$$v_c = 2(1 + 0.0005\ N_u/A_g)\sqrt{f'_c} \qquad (11\text{-}7)$$

Shear plus Axial Tension:

$$v_c = 2(1 + 0.002\ N_u/A_g)\ \sqrt{f'_c} \qquad (11\text{-}8)$$

where N_u is negative in tension and N_u/A_g is expressed in psi. These four equations are discussed in Reference 3 and are compared in Figure 1.

<u>Shear Strength of Prestressed Concrete Building Members--11.5.1</u>

The equations for the shear in the concrete in prestressed beams, v_{ci} and v_{cw} (11-11 and 11-12), are essentially the same as the corresponding equations in the 1963 Code. These equations were developed for use in the design of composite prestressed concrete bridge members and become very tedious when used to check the shear strength of slender uniformly loaded building members. Equation (11-10) has been added to provide a simple means of computing v_c in such cases. It becomes excessively conservative for composite I-section bridge girders but is reasonable in the range of parameters involved in buildings.[3]

$$v_c = 0.6\sqrt{f'_c} + 700\,\frac{V_u d}{M_u} \qquad (11\text{-}10)$$

where: $2\sqrt{f'_c} \leq v_c \leq 5\sqrt{f'_c}$

In applying this equation to simply supported uniformly loaded beams $V_u d/M_u$ can be expressed as:

$$\frac{V_u d}{M_u} = \frac{d}{x}\frac{(L-2x)}{(L-x)} \qquad (A)$$

These three relationships can be plotted in design charts of the form shown in Figure 2. The use of equation (11-10) is illustrated in Reference 3.

Shear Strength of Deep Beams--11.9

In the ACI Code, the design of web reinforcement is based on the equation:

$$v_u = v_c + v'_v \qquad (B)$$

where v'_v is the shear carried by the web reinforcement. For beams having a span to depth ratio, ℓ_c/d of 5 or more and for deep beams ($\ell_c/d < 5$) which are not loaded at the top of the compression face, v_c is taken as the inclined cracking shear for concrete as given in Sections 11.4.1 and 2. For the special case of deep beams loaded on their top surface, the shear carried by the concrete, v_c is greater than the inclined cracking shear as shown in Figures 3 and 4. This is caused by arch action in the concrete between the load and the support. Equation (11-22) takes this into account by multiplying the cracking shear, given by the second term of the equation, by a linear increase for small M/Vd ratios, given by the first term:

$$v_c = [3.5 - (2.5)(M_u/V_ud)] \times [1.9\sqrt{f'_c} + 2500p_w \frac{V_ud}{M_u}] \qquad (11\text{-}22)$$

where $2\sqrt{f'_c} \leq v_c \leq 6\sqrt{f'_c}$ and the first term shall not exceed 2.5. M_u and V_u are the shear and moment at the critical sections described in Section 11.9.3 and shown in Figures 3 and 4. Values of v_c from this equation are plotted in Figure 5. It should be noted, however, that unsightly inclined cracks may be present at working loads if the working load shear stress is significantly higher than the dashed line in Figure 5.

The upper limit on shear stress at ultimate is given by Section 11.9.4 and Equation (11-23). The upper and lower limits are illustrated in Figure 6.

If the ultimate shear stress exceeds v_c, web reinforcement must be provided for the excess. Based on the classical truss analogy for web reinforcement it can be shown that the shear carried by stirrups in a beam is equal to:

$$(v_u - v_c) = \frac{A_v f_y}{b's} [\sin \alpha (\sin \alpha \cot \theta + \cos \alpha)] \qquad (C)$$

where θ = inclination of crack

α = inclination of web reinforcement

The ACI equations for the design of vertical and inclined stirrups (11-3 and 11-4) are derived from this by assuming $\theta = 45^{\circ}$. In a deep beam, however, the diagonal cracks will be considerably steeper than 45° and Equations (11-3 and 11-4) do not apply to this case. Due to the steep cracks the efficiency of vertical web reinforcement is greatly reduced and as a result, both vertical and horizontal web reinforcement are required. For the special case of combined vertical reinforcement (area = A_v, $\alpha = 90^0$) and horizontal reinforcement (area = A_{vh}, $\alpha = 0^0$), Equation C becomes:

$$(v_u - v_c) = \frac{A_v f_y}{b's} \cos^2 \theta + \frac{A_{vh} f_y}{b's_h} \sin^2 \theta \qquad \text{(D)}$$

Tests have shown that θ is a function of M/Vd or ℓ_c/d and this has been taken into account in deriving the code equation 11-24 from Equation D.

$$\left(\frac{A_v}{s}\right)\left(\frac{1-\ell_c/d}{12}\right) + \left(\frac{A_{vh}}{s_h}\right)\left(\frac{11-\ell_c/d}{12}\right) = \frac{(v_u - v_c)\ b'}{f_y} \qquad \text{(11-24)}$$

Example 1 in the Appendix illustrates the shear design procedure for deep beams.

Shear Friction--11.15

In very short members such as corbels, brackets or the end regions of precast beams the concrete is stressed essentially in pure shear. If a crack forms due to any cause in an unfavorable location as shown in Figure 7 shear stresses along the crack may be resisted by friction, provided there is a normal force across the crack to develop the frictional force.[4] This normal force may be obtained by placing reinforcement across the crack. As slip occurs along the crack the irregularities of the crack cause the opposing faces to separate, stressing the reinforcement in tension. A balancing compressive stress will then exist in the concrete and friction will be developed.

The successful application of the shear-friction concept depends on the proper choice of the assumed crack. Typical examples are shown in Figure 7.

The required area, A_{vf}, of shear friction reinforcement is computed by:

$$A_{vf} = \frac{V_u}{\phi f_y \mu} \tag{11-30}$$

where ϕ is the capacity reduction factor given in Section 9.2 which is equal to 0.85 for shear, and μ is a coefficient of friction ranging from 1.4 for monolithically cast concrete to 0.7 for concrete placed against clean, unpainted, as-rolled structural steel. The upper limit of the shear stress on the failure surface is given as the smaller of $0.2f'_c$ or 800 psi.

If tensile stresses are present across the assumed crack, reinforcement for the tension must be provided in addition to that provided for shear friction.

The designer must also ensure that the reinforcement is all adequately anchored and this frequently requires special details.

Example 2 illustrates the use of the Shear-Friction concept in the design of a corbel.

Special Provisions for Brackets and Corbels--11.14

Although brackets and corbels can be designed satisfactorily using the shear-friction concept, additional design equations and requirements are given in Section 11.14. In general these are considerably more complicated than the shear-friction concept and should only be used when shear-friction is not applicable, e.g., if a/d exceeds 0.5. The equations in 11.14 are based on tests reported in Reference 5.

The corbel designed in Example 2 is redesigned in Example 3 using Section 11.14 for comparison of the two methods.

Shear Strength of Slabs and Footings--11.10 and 11.12

The basic requirements for punching shear and one-way shear in slabs and footings are unchanged in this Code. Similarly, the effect of holes on the perimeter effective for shear remains essentially unchanged.

Shear Reinforcement in Slabs and Footings--11.11

The three types of web reinforcement shown in Figure 8 are considered in this section. The major problem involved in the type shown in Figure 8(a) is the anchorage of the bars at the top and bottom of each of the legs. Tests have shown that type (b) provides a positive anchorage and leads to a very ductile failure.[6] The steel shearhead shown in (c) will be discussed more fully in the next section of this paper.

In a slab which is not reinforced for shear the maximum allowable shear stress at ultimate is $4\sqrt{f'_c}$ on a section at d/2 from the column. Because of the triaxial compressions under and around the column this value is twice that allowed in a beam. As the section is moved away from the column, the confining compressive stress disappears rapidly and the shear that can be carried by the concrete drops to $2\sqrt{f'_c}$. For this reason, shear reinforcement in slabs must be designed for all shear in excess of $2\sqrt{f'_c}$ and must continue until the shear stresses drop below this value.

Structural Steel Shearheads--11.11.2

Based on extensive tests reported in Reference 7 structural steel shearheads are permitted at interior supports of slabs. The design of a shear head must satisfy three criteria:

1. The flexural capacity of the shearhead must be large enough that the shear capacity of the slab is reached before the shearhead yields. The moments in the shearheads are a function of the ratio, K, of the stiffnesses of a shearhead arm and a specified strip of slab, and the projection of the shearhead from the face of the column. Based on an assumed shear distribution along the arm of the shearhead the plastic moment capacity of each arm must equal or exceed:

$$M_p = \frac{V_u}{\phi 8} [h + K(L_s - \frac{c_1}{2})] \qquad (11\text{-}26)$$

2. The ultimate shear stress v_u shall not exceed $4\sqrt{f'_c}$ on a critical section located as shown in Figure 9.

3. Once items 1 and 2 are satisfied, the negative moment reinforcement in the slab can be reduced within limits to account for the moment in the shearhead. The amount of this reduction is given in Section 11.11.2.5.

Reference 7 gives an example of the design of a shearhead.

Transfer of Moments to Columns--11.13

Section 11.13.1 requires lateral reinforcement equal to the minimum amount given by Equation (11-1) within all exterior beam-column or slab-column connections. Such reinforcement restrains the diagonal cracks which occur in the exterior joints of a building.[8] In seismic regions ties are also required in interior joints. The forces leading to these cracks are illustrated in Figure 10.

The computation of the shear stresses resulting from the transfer of moments from a slab to a column is made more explicit in the 1971 Code than it was in the 1963 Code. The critical section for transferring shears due to unbalanced moments is assumed to be the same as for vertical shears. About 40 per cent of the unbalanced moments are transferred by shear stresses and about 60 per cent by flexure. The shear stresses on this section are assumed to be distributed as shown in Figure 11 and are calculated using Equations E and F:

$$v_{AB} = \frac{V_u}{A_c} + \frac{KMC_{AB}}{J_c} \qquad \text{(E)}$$

and

$$v_{CD} = \frac{V_u}{A_c} - \frac{KMC_{CD}}{J_c} \qquad \text{(F)}$$

where K is the fraction of moment transferred by shear on this perimeter as given by section 11.13.2. For a square column, K = 0.4. J_c is the moment of inertia used in computing the shear stresses due to flexure. It is equal to the sum of polar moments of inertia of sides AD and BC plus $A\bar{y}^2$ terms for AB and CD. For an interior column J_c is given by:

$$J_c = 2\left\{\underbrace{\frac{d(c_1+d)^3}{12} + \frac{(c_1+d)d^3}{12}}_{\text{Polar I of AD and BC}}\right\} + 2\left\{\underbrace{d(c_2+d)\left(\frac{c_1+d}{2}\right)^2}_{A\bar{y}^2 \text{ for AB and CD}}\right\} \qquad (G)$$

The area A_c is the perimeter A-B-C-D multiplied by the effective depth, d.

A similar derivation can be followed for an exterior column using the critical section shown in Figure (b). Example 4 illustrates the calculation of the shearing stresses due to moment-transfer in a flat plate.

Special Provisions for Shear Walls--11.16

The designer of a shear wall is allowed to use the regular equations for combined shear and axial load given in Section 11.4.4, or he can use Equations (11-31) and (11-32). These latter two equations, respectively, are similar to Equations (11-12) and (11-11) for prestressed concrete beams. Special limits on the shear and longitudinal reinforcement are given for various ratios of v_u to v_c and various wall heights.

Summary

Many of the gaps in Chapters 12 and 17 of the 1963 ACI Code have been filled in the new Code. Since the various relationships are essentially empirical they tend to be complex and limited in application. A major effort will be made in the next decade to simplify this Chapter of the ACI Code.

APPENDIX

DESIGN EXAMPLES

Example 1--Design a Deep Beam for Two Concentrated Loads

Design a beam to span 21 feet and support its own dead load plus two concentrated loads located 7 feet from each support.

$P_u = 1.4D + 1.7L = 350K$

$f'_c = 3000$ psi

f_y - main reinforcement = 60,000 psi

web reinforcement = 40,000 psi

Choose the basic section on the basis of flexure assuming normal assumptions hold.

Require: $d = 72$ in.

$b = 12$ in.

$\ell_c/d = 21/6 = 3.5$

From Section 10.7 it is all right to use normal flexural equations.

Main reinforcement $A_s = 9.36$ in.^{2}

Ultimate DL of beam $W_D = 1.5\ (6.5 \times 1 \times 0.15)$

$= 1.46$ K/ft.

Reaction $R = 350 + 1.46 \times 21/2 = 365.2K$

Check Shear at Critical Section (Section 11.9.3)

Critical section is $0.5a = 0.5 \times 7$ ft. $= 3.5$ ft. from support.

$V_u = R - 3.5w_D = 365.3 - 5.1 = 360.2K$

$v_u = \dfrac{360200}{12x72x0.85} = 491$ psi (Eqn. 11-3)

Max. allowable $v_u = \frac{2}{3}(10+3.5)\sqrt{f'_c} = 9\sqrt{f'_c} = 493$ psi

therefore all right.

Compute v_c at Critical Section

$$M_u = (0.5a)(\frac{R+V_u}{2}) = 3.5\ (\frac{365.3 + 360.2}{2}) = 1270 \text{ ft.K}$$

$M_u/V_ud = 1270/(360.2 \times 6) = 0.586$

$V_ud/M_u = 1.71$

$p = 9.36/(12 \times 72) = 0.0108$

From Eqn.(11-22)

$[3.5 - 2.5(M_u/V_ud)] = 2.03 < 2.5$

and $v_c = 2.03\ (1.9\sqrt{3000} + 2500 \times 0.0108 \times 1.71)$

$= 2.03 \times 150 = 305$ psi

Max $v_c = 6\sqrt{f'_c} = 329$ psi

Use $v_c = 305$ psi

Since $v_c = 305$ psi $< v_u = 491$ psi Require Web Reinforcement

Design Web Reinforcement

Equation 11-24:

$$(\frac{A_v}{s})\ (\frac{1 + \ell_c/d}{12}) + (\frac{A_{vh}}{s_h})\ (\frac{11 - \ell_c/d}{12}) = \frac{(v_u - v_c)b'}{f_y}$$

where: $\frac{(v_u - v_c)b'}{f_y} = \frac{(491 - 305) \times 12}{40000} = 0.056$

$$\left(\frac{1 + \ell_c/d}{12}\right) = \frac{1 + 3.5}{12} = 0.375$$

$$\left(\frac{11 - \ell_c/d}{12}\right) = 0.625$$

Therefore:

$$0.375 \left(\frac{A_v}{s}\right) + 0.625 \left(\frac{A_{vh}}{s_h}\right) = 0.056$$

From this equation it can be seen that A_{vh} will be more effective than A_v. Therefore assume:

$$A_v/s = \frac{1}{2} (A_{vh}/s_h)$$

$$0.056 = 0.375 \times \frac{1}{2} (A_{vh}/s_h) + 0.625 (A_{vh}/s_h)$$

and $A_{vh}/s_h = 0.069 \text{ in}^2/\text{in} = 0.82 \text{ in}^2/\text{ft}$

Minimum $A_{vh} = 0.0025 \, b_{s_h} = 0.35 \text{ in}^2/\text{ft}$ (11.9.6)

Maximum s_h = d/3 = 24 in or 18 in maximum

Horizontal Stirrups--Use #6 @ 6 in. on centers.

$$A_v/s = 0.035 \text{ in}^2/\text{in} = 0.41 \text{ in}^2/\text{ft.}$$

Minimum A_v = 0.0015 bs = 0.22 in^2/ft.

Maximum s_h = d/5 = 14.4 in.

Vertical Stirrups--Use #5 at 9 in. on center.

Note that the inclined cracking shear stress is about $1.9 \sqrt{f'_c} + 2500p \frac{Vd}{M} = 150$ psi. This is only 30% of the shear stress at ultimate and as a result there probably will be visible inclined cracks at working loads.

Example 2--Design a Corbel using Shear-Friction for Vertical and Horizontal Loads

Given--Corbels on two sides of a 14 inch square column support beam reactions which act 4 inches from the face of the column. Due to restraint of shrinkage and time dependent camber a horizontal force acts on this connection.

Reactions--Dead Load = 25.0K

Live Load = 40.0K

Horizontal Load = 40.0K

From Section 9.3: $V_u = 1.4D + 1.7L = 103K$

From Section 11.14.2 (as revised): $T_u = 1.7H = 68K$

Use $f'_c = 5000$ psi

$f_y = 40{,}000$ psi

Assume the corbels have the shape shown. Assume that the critical crack intersects the sloping side about half-way along the side as shown in Figure 12(a). Length of crack = 12 inches.

$V_u = 103K$

$v_u = \frac{103000}{14 \times 12} = 614$ psi

Maximum allowable $v_a = 0.2f'_c = 1000$ psi or 800 psi

therefore all right.

Design Reinforcement for Shear Friction

Eqn. (11-30): $A_{vf} = \dfrac{V_u}{\phi f_y \mu}$

where μ = 1.4.

$$A_{vf} = \frac{103}{0.85 \times 40 \times 1.4} = 2.17 \text{ in}^2$$

Design Reinforcement for Horizontal Tension

$$A_{st} = \frac{68}{0.85 \times 40} = 2.00 \text{ in}^2$$

Total as required = 2.17 + 2.00 = 4.17 in^2

It would appear to be good practice to satisfy Section 11.14.4 regarding the amount and distribution of the reinforcement:

Top steel = $A_s \simeq \frac{2}{3} \times 4.17 = 2.78 \text{ in}^2$

Stirrups = $A_{s_h} \simeq \frac{1}{3} \times 4.17 = 1.39 \text{ in}^2$

Use 4 #8 Bars as top reinforcement, A_s = 3.16 in^2

Use 4 #4 Closed Ties, A_{s_h} = 1.60 in^2

Place reinforcement as shown in the sketch. According to 11.14.2 :

$$p = \frac{A_s}{bd} \leq 0.13 \frac{f'_c}{f_y} = 0.0163$$

where d is at face of column.

$$p = \frac{3.16}{14(16 - 2.0)} = 0.0161$$

therefore all right.

Use Corbel as shown in Fig. 12(a).

Example 3--Redesign Corbel from Example 2 Using 11.14

Loading, material strengths, etc. as given for Example 2.

Compute Allowable Ultimate Shear Stress:

$$v_a = [6.5 - 5.1\sqrt{68/103}][1 - 0.5a/d] \times \left\{1 + [64 + 160\sqrt{(68/103)^3}]p\right\}$$

$$\sqrt{5000} = 167\ (1 - 0.5a/d)(1 + 148p)$$

Assume: Overall depth = 16 inches therefore d = 16 - 2 = 14 in.

$$a/d = 4/14 = 0.286$$

$$p = p_{max} = 0.13f'_c/f_y = 0.0167 \text{ (Section 11.14.2)}$$

Therefore $v_u = 167(1 - 0.5 \times 0.286)(1 + 148 \times 0.0167)$

$= 500$ psi

Compute Depth Required

$$d = \frac{V_u}{bv_u} = \frac{103000}{14x500} = 14.75 \text{ in.}$$

Height required = 14.75 + 2.0 in. = 16.75 in.

Say 17 in. d = 15 in.

Choose Reinforcement

As required = 0.0167 x 14 x 15 = 3.50 in.^{2}

Use 2 #9 and 2 #8 Top Reinforcement $A_s = 3.58$ in^2

Compute Stirrups

$$A_{s_h} = 0.5A_s \quad \text{(Section 11.14.4)}$$
$$= 1.75 \text{ in}^2$$

Use 3 double leg #5 ties $A_{s_h} = 1.86\ in^2$

Use Corbel as shown in Figure 12(b).

Example 4--Compute the Shear Stresses at an Interior Column Due to Shear and Moment

Given a 7 inch flat plate supported by 14 inch by 16 inch columns. At a typical interior column the shear force on the critical perimeter for two-way shear (Section 11.10.2) is V_u = 95 kips. The unbalanced moment computed according to Section 13.3.5 is 13.5 ft. K about an axis parallel to the 14 inch side of the column.

$f'_c = 3000$ psi

Compute the maximum shear stresses in the slab at this joint.

Average Effective depth d = 5.75 inches

From Equation E:

$$v = \frac{V_u}{A_c} + \frac{KMc_{AB}}{J_c}$$

where: $A_c = 2[(14+5.75)+(16+5.75)] \times 5.75$

$= 478\ in^2$

K = fraction of moment transferred by shear

$$= \left(1 - \frac{1}{1 + \frac{2}{3}\sqrt{\frac{c_1+d}{c_2+d}}}\right) \qquad \text{(Section 11.13.2)}$$

$= 0.41$

J_c = "moment of inertia" of shear surface. Eqn. G

$$= 2\left\{\frac{5.75(16+5.75)^3}{12} + \frac{(16+5.75)5.75^3}{12}\right\} + 2\left\{5.75(14+5.75)\left(\frac{16+5.75}{2}\right)^2\right\}$$

$= 37411\ in.^4$

$$v = \frac{94000}{478} + \frac{0.41x13.5x12x21.25/2}{37411}$$

$$= 196 + 18.8 \text{ psi}$$

$$= 214.8 \text{ psi}$$

V_u allow = $f\sqrt{f'_c}$ = 219 psi

The shearing stresses are ok at an interior column.

Note that in this case the shears due to moment were 12 percent of the total shear at this point.

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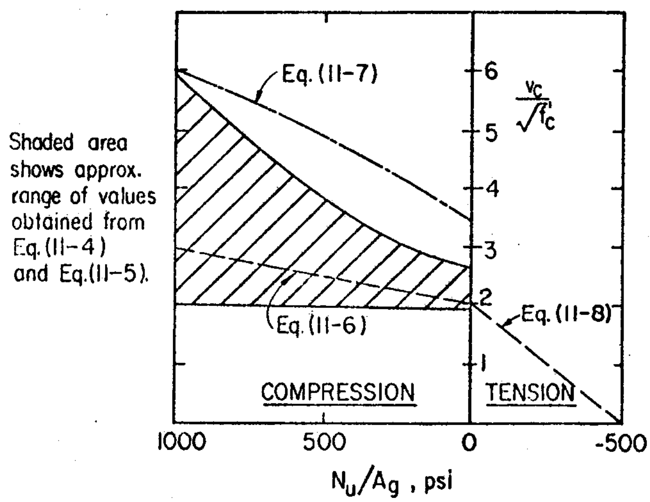


FIG. 1: COMPARISON OF DESIGN EQUATIONS FOR SHEAR AND AXIAL LOAD

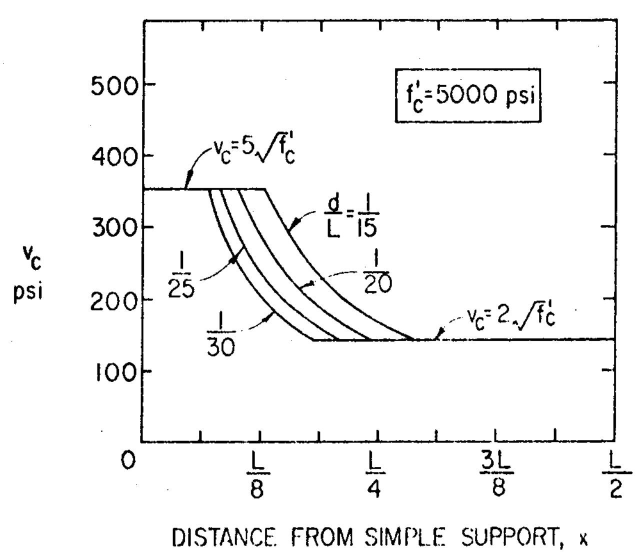


FIG. 2: APPLICATION OF EQUATION (11-10) TO UNIFORMLY LOADED PRESTRESSED CONCRETE MEMBERS

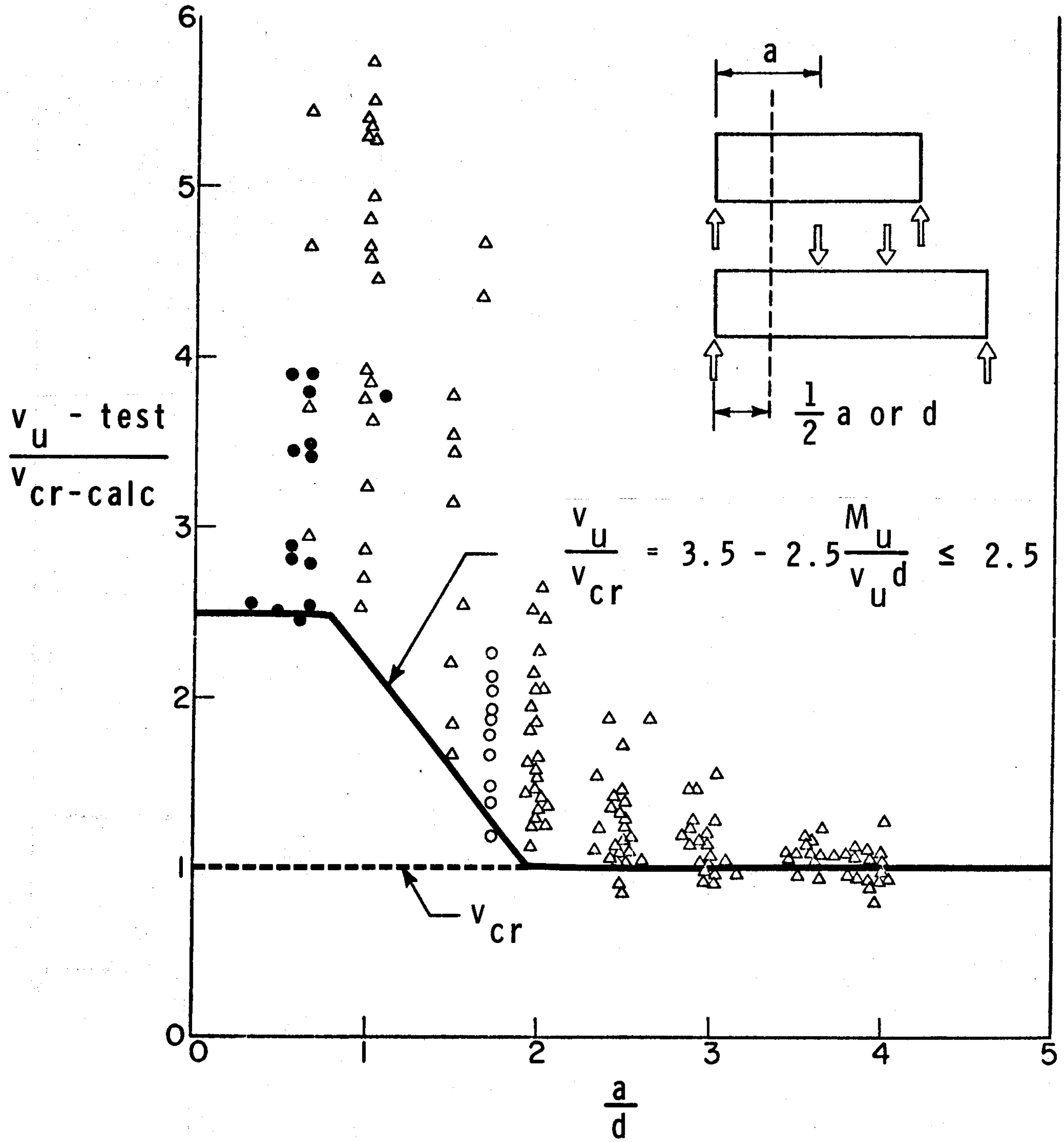


FIG. 3: ULTIMATE STRENGTH OF DEEP BEAMS - CONCENTRATED LOADS

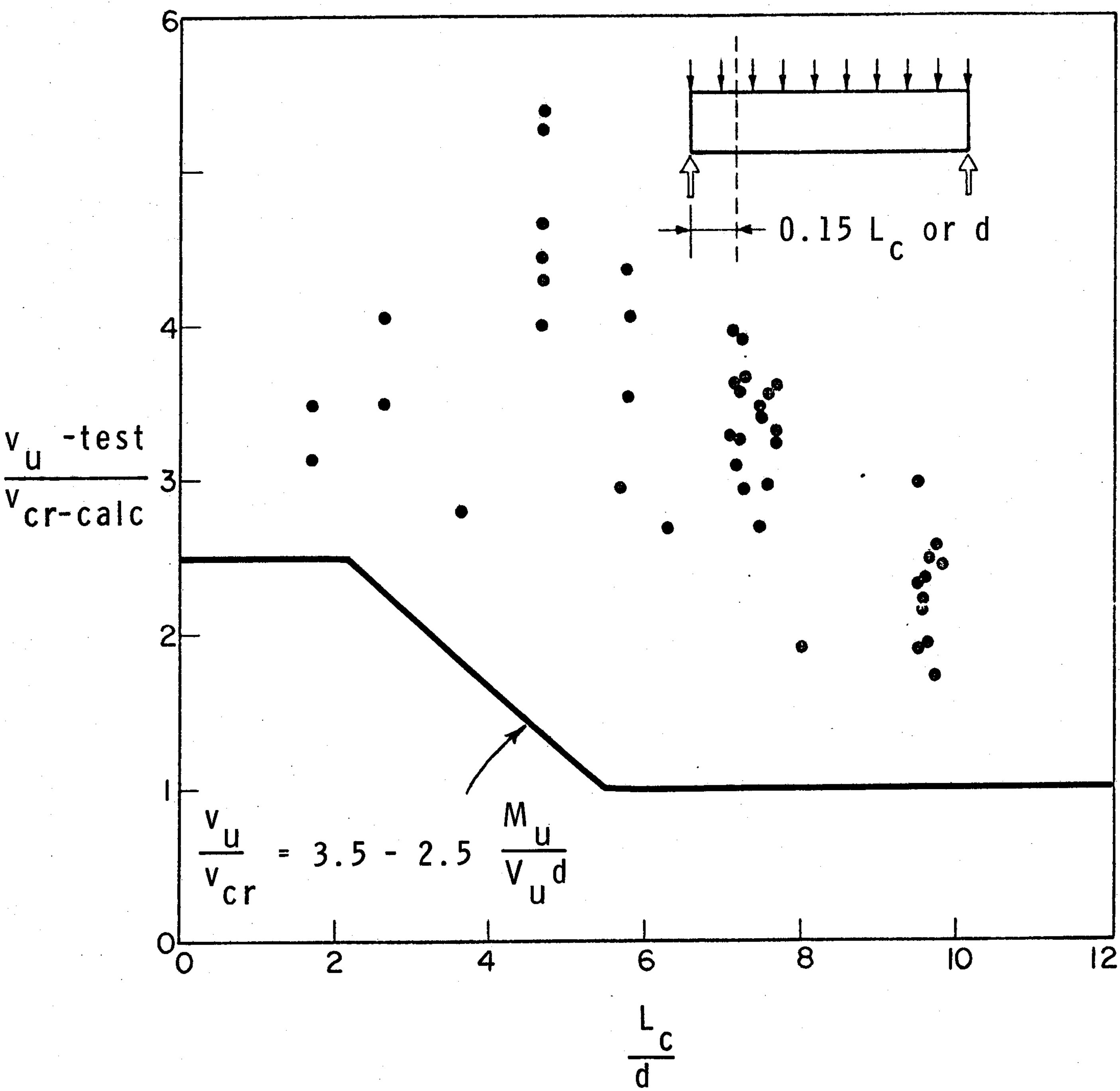


FIG. 4: ULTIMATE STRENGTH OF DEEP BEAMS - UNIFORM LOADS

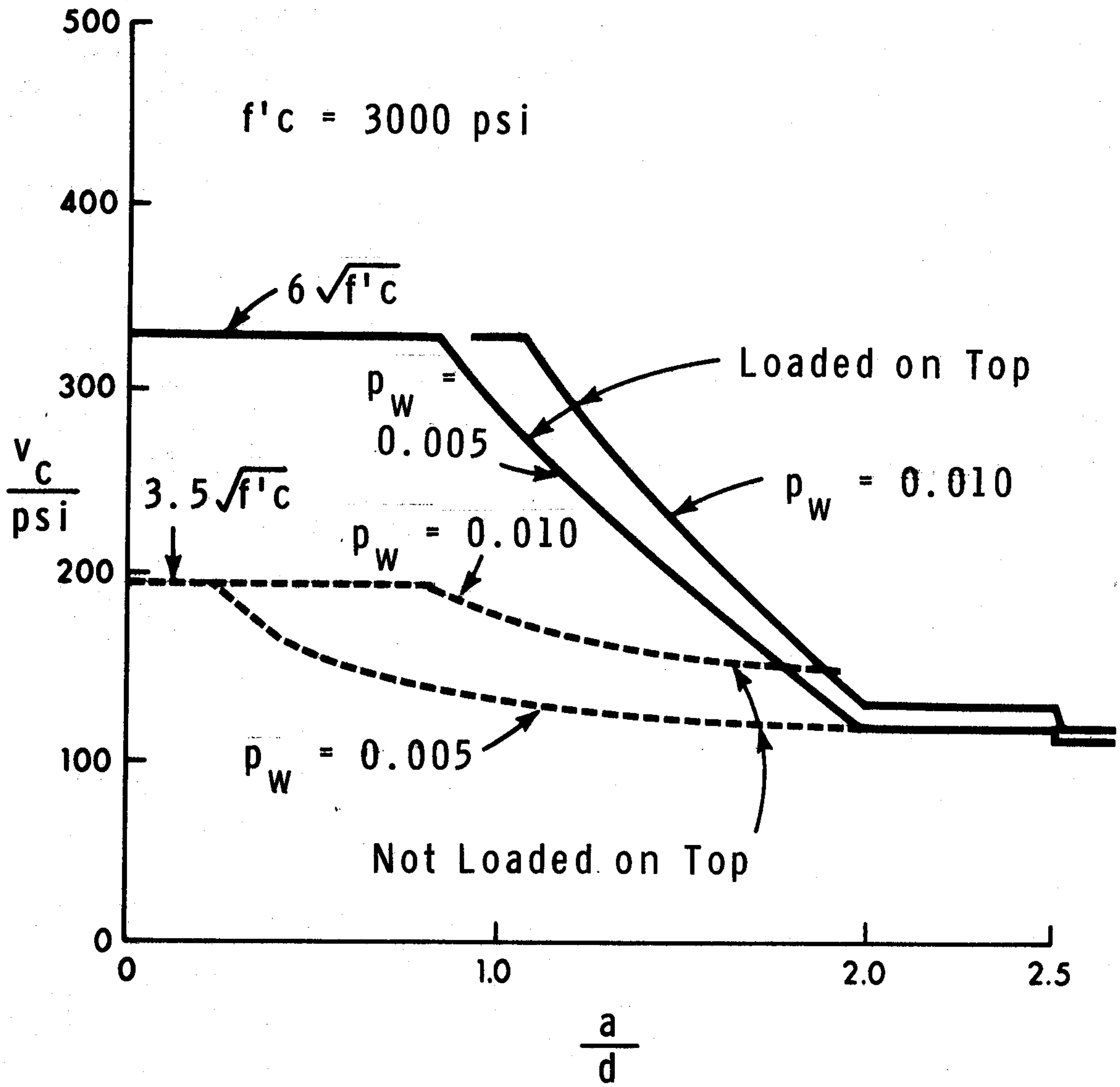


FIG. 5: ALLOWABLE v_c FOR DEEP BEAMS

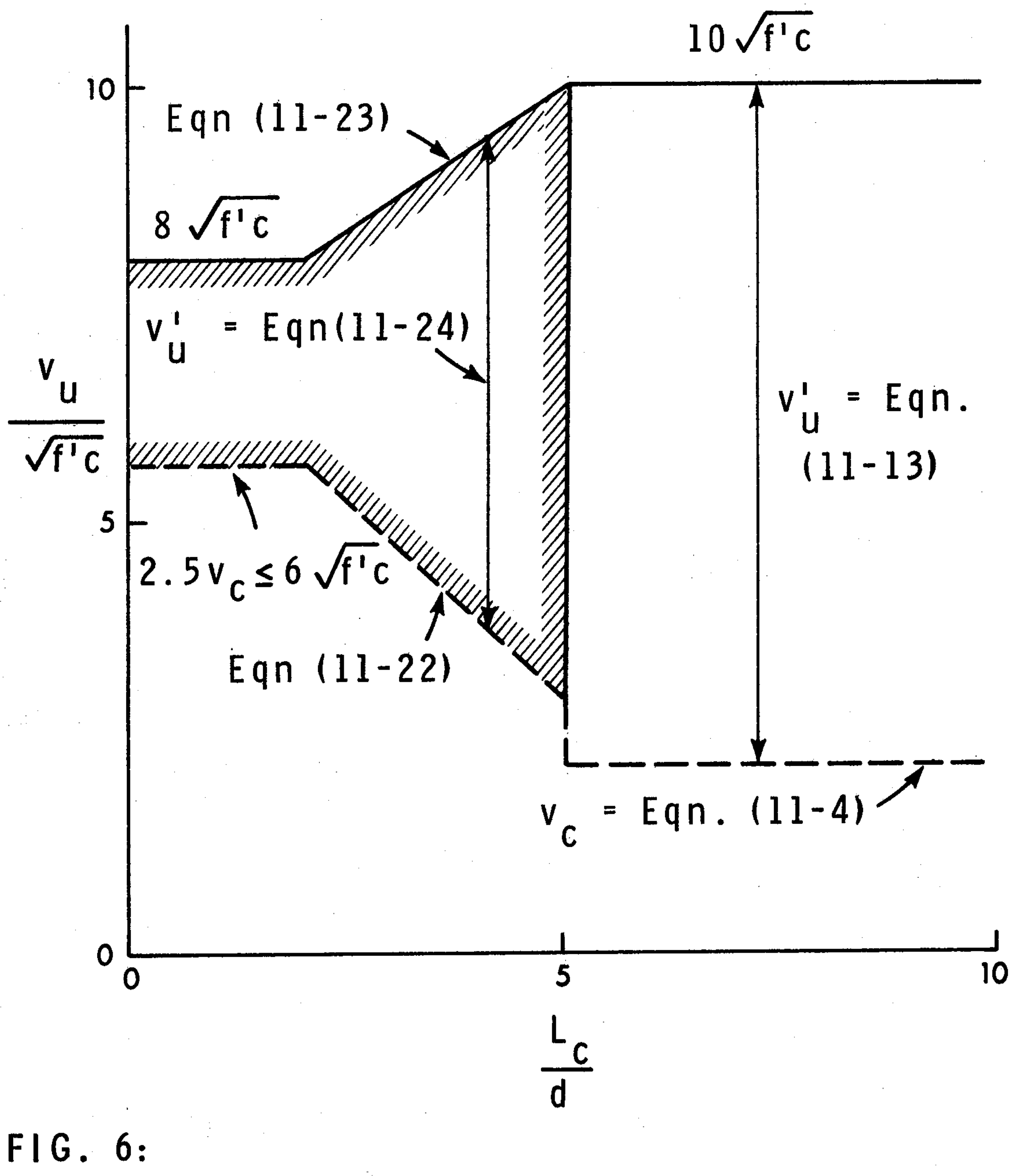


FIG. 6:

SUMMARY OF DESIGN EQUATIONS FOR DEEP BEAMS

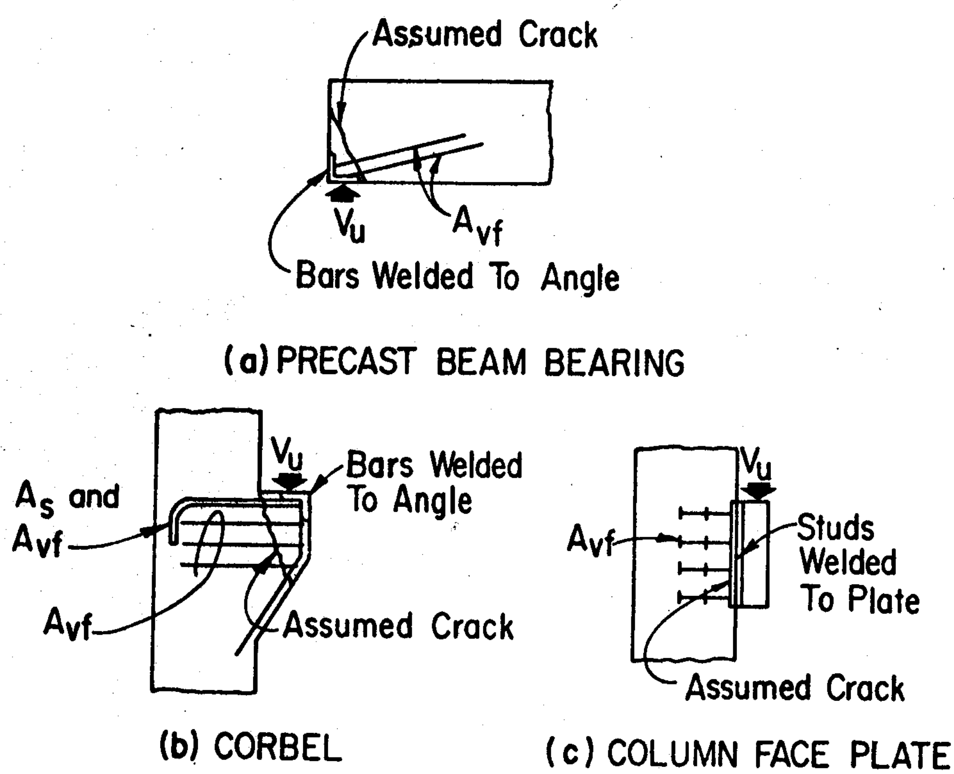
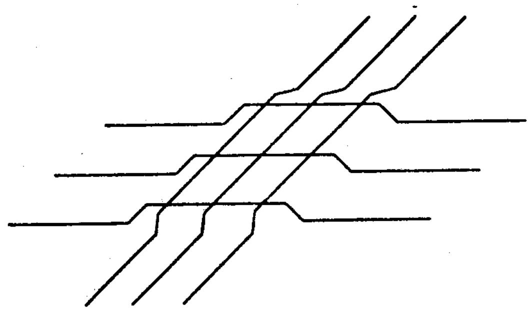
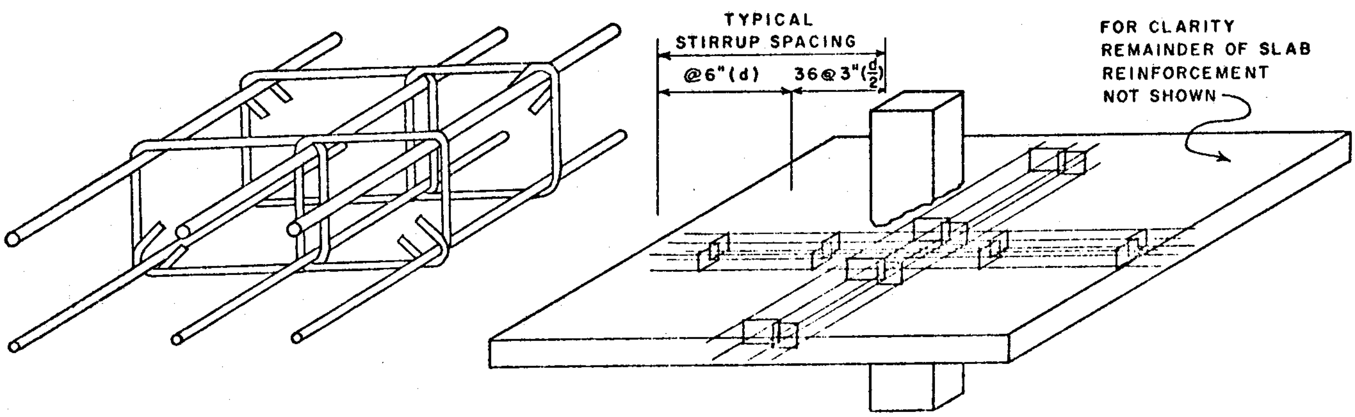


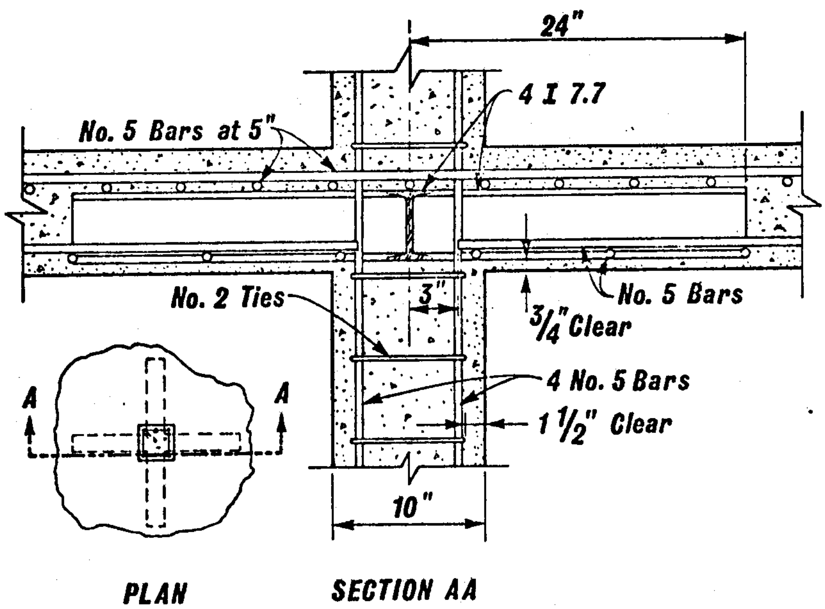
FIG. 7: CRITICAL LOCATIONS FOR SHEAR-FRICTION.



(a) Bent Bars



(b) Slab Stirrups



(c) Shearheads

FIG. 8: SHEAR REINFORCEMENT IN SLABS

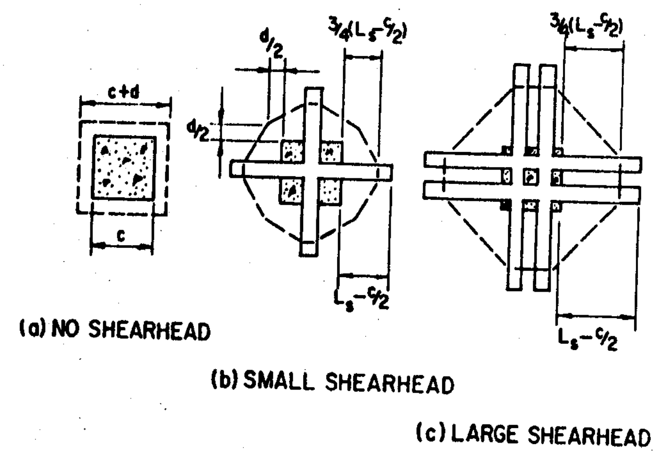


FIG. 9: LOCATION OF CRITICAL SECTIONS AND SHEARHEADS

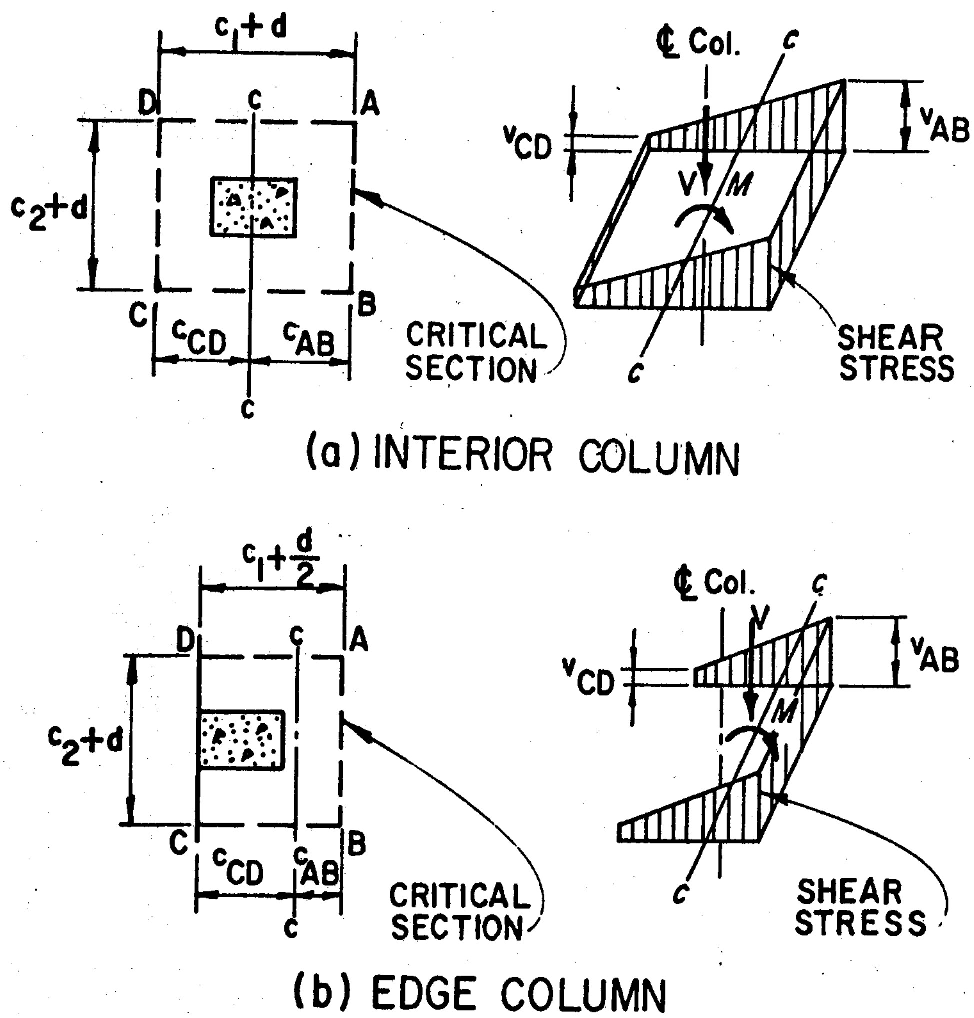
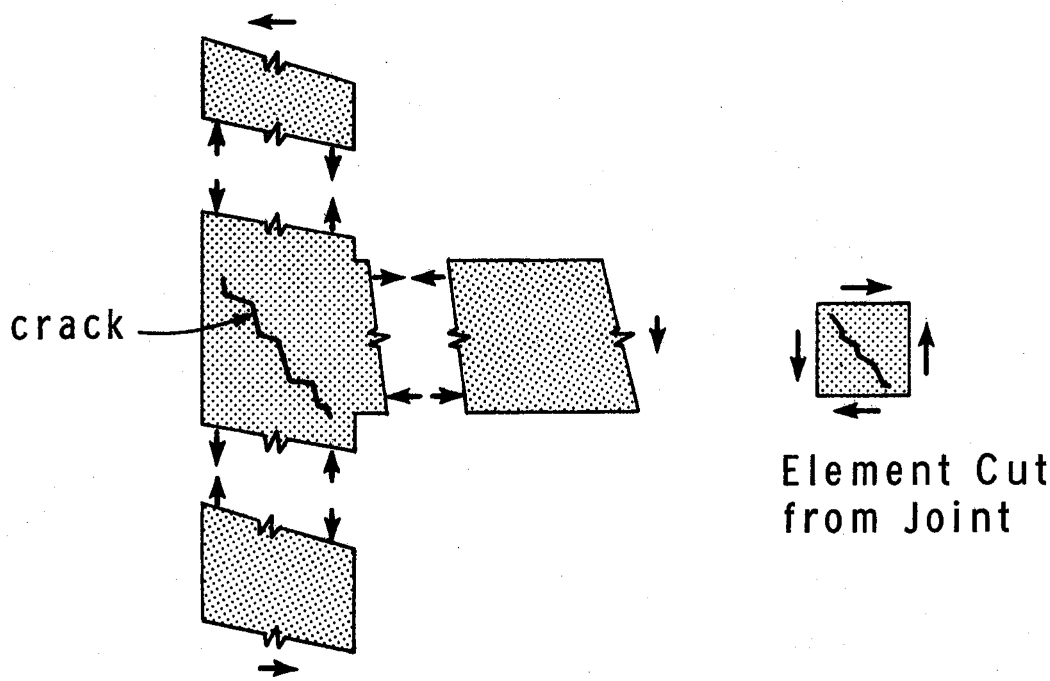
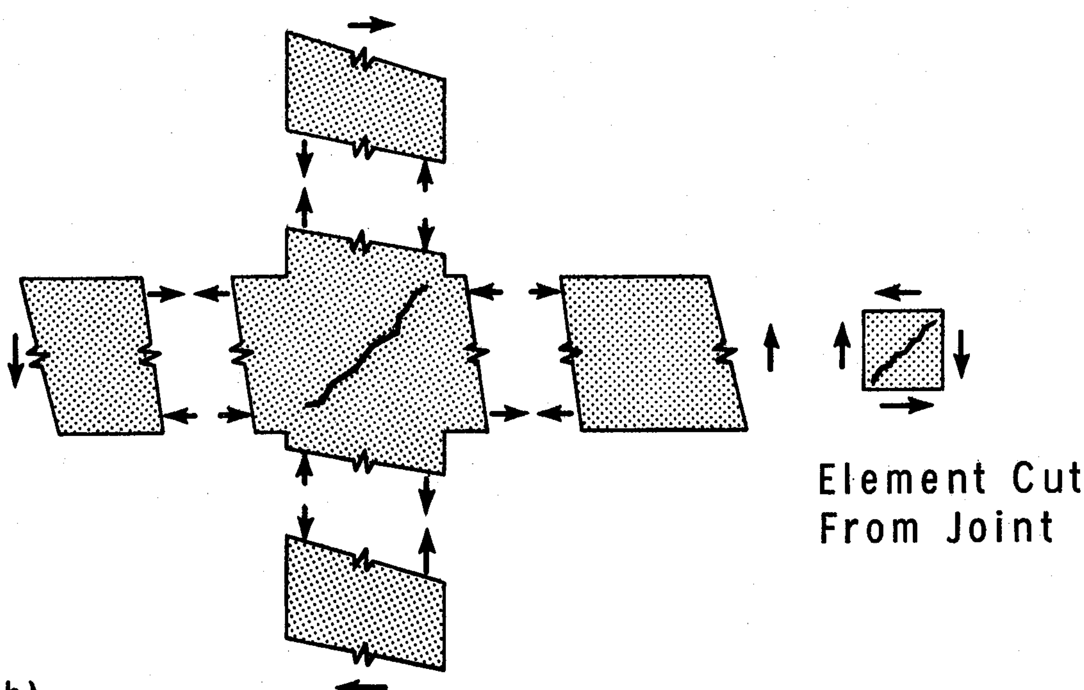


FIG. 11: SHEARING STRESSES AROUND A COLUMN

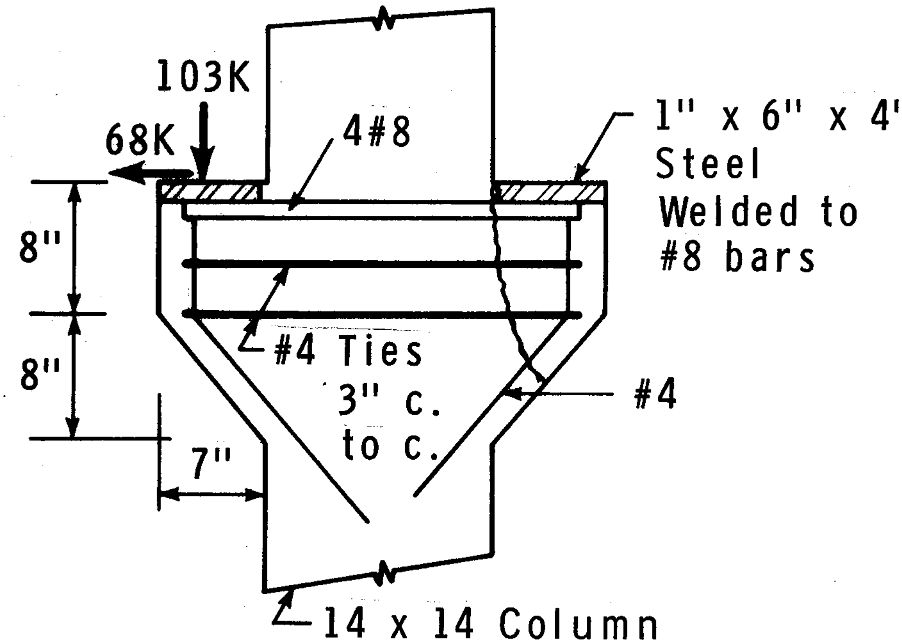


(a) EXTERIOR BEAM - COLUMN JOINT

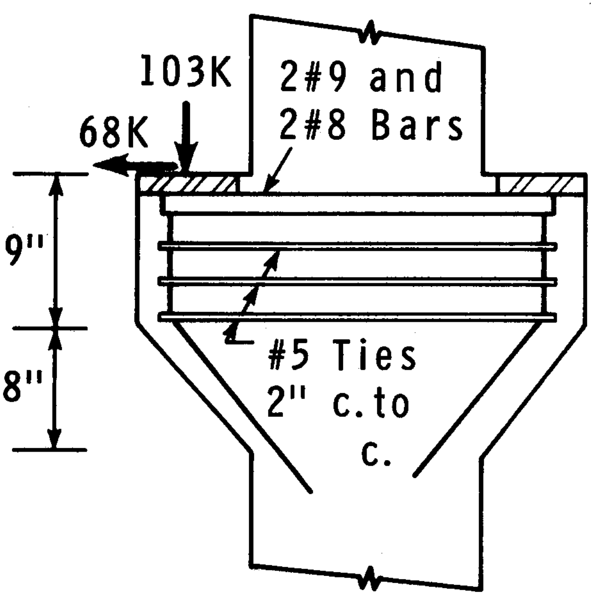


(b)
BEAM-COLUMN JOINT IN LATERALLY LOADED FRAME

FIG. 10: CRACKING IN BEAM-COLUMN JOINTS



(a) Corbel - Example 2



(b) Corbel - Example 3

FIG. 12: DETAILS OF CORBELS

Reinforced Concrete Buildings

LECTURE 3

SLAB DESIGN

by

S.H. Simmonds

REINFORCED CONCRETE SLABS

Introduction

The provisions for reinforced concrete slabs in the new building code are substantially revised from the previous requirements. In fact, the word "revised" may be inadequate and it would be more accurate to state that the old provisions have been deleted and a new set inserted in their place. These new provisions reflect a new philosophy in the design of reinforced concrete slabs. It is this new philosophy that is examined in this paper.

For better understanding of this new approach let us first review the philosophy or basis of the previous code requirments for slab design. Immediately one is confronted with a decision, whether to examine the basis for "two-way" slabs or the basis for "flat" slabs, since each is treated as a separate structural component even to the extent of placing the requirements in different chapters of the code. The reason for the very different requirements is that each system has a unique genesis.

Patents were issued for reinforced concrete slabs as early as 1854 and 1867 but the analyses were based on the reinforcing acting as either the tie to an arch or as a catenary, the concrete being used primarily as a filler. The structures built were restricted to small spans on a grid-work of closely spaced beams and were not economical. Lacking a means of analysing such highly redundant structures, it was not until the end of the nineteenth century that designers showed an interest in reinforced concrete slabs.

Not surprisingly, early designers turned to classical plate theory for means of analysis, the governing differential equation for plates having been presented by Lagrange as early as 1807. However, it was not until 1899 that Levy, using an infinite series of hyperbolic functions, solved this equation for single rectangular panels supporting a distributed load. In 1909 similar problems were solved by Ritz using energy methods. Since these solutions assumed simply-supported or clamped edges only, the panel boundaries were non-deflecting and the slab bending moments based on these solutions are valid only when stiff beams are present on all four sides of each panel. This type of solution forms the basis for the present methods of "two-way" slab design wherein the slab moments are obtained from a set of coefficients based on an elastic analysis, and the beams are proportioned for the remainder of the static moment. Consistent with the original assumptions of non-deflecting panel boundaries the required beams are generally quite stiff.

In 1903, C.A.P. Turner received a patent for a "mushroom" slab, that is, a slab supported directly on the column with no beams spanning between columns. His first design was rejected by the local building authority but two years later he built the first such slab in Minneapolis. Since there was no rigorous mathematical basis for the design, a performance load test was required. Without becoming involved with the details of the long standing debate as to the validity of these load tests, suffice it to say that his flat slab and the slabs of his many competitors were deemed satisfactory. By 1913 over 1,000 such slabs had been built. However, in 1914, Nichols established a simple criteria for the minimum moment that must exist across critical sections to satisfy basic equilibrium which seemed contrary to the widely accepted understanding of the behaviour of flat slabs. This difference between Nichols' theoretical requirements and the supposedly measured moments in the load tests was partially reconciled by Westergaard and Slater in their now famous paper. Based on this paper the provisions for flat slabs were introduced into the 1921 edition of the ACI code and except for the introduction of the frame method in the 1941 edition, there have been no major changes in the code requirements.

Although the two design procedures have existed side by side for half a century, it has long been realized that neither procedure was entirely satisfactory. To begin with the very names, "two-way and flat" are meaningless, (See Fig. 1), since both systems carry load by two-way action, both have flat top surfaces and except for the special case of the flat plates, both have projections below the bottom surface. These terms have been discarded in the new code and have been replaced by the simpler, more descriptive terms of"slabs with beams" and "slabs without beams", respectively. In the past there has been no provision for designing slab systems that fall between these two defined systems, ie: a slab supported on shallow beams of specified depth. However, the most unsatisfactory aspect of the two systems was the great difference in factors of safety based on ultimate load carrying capacity. A series of test slabs were initiated at the University of Illinois in 1958 to obtain reliable data at failure and to serve as the basis of evaluating several analytical solutions which were being obtained concurrently. These test slabs which were chosen to be typical of their respective prototypes yielded the following results.

Type of Slab	Design Live Load	F of S = $\frac{F.L. - 1.5\ DL}{L.L.}$
Two-way	50 psf	6.4
Flat Plate	40 psf	3.3
Flat Slab	100 psf	2.1

Obviously such discrepancies in ultimate behaviour could not be permitted to continue to exist for essentially the same structural element, especially since other sections of the code are based on load factors applied to an ultimate strength design procedure. A new philosphy of design was needed which could incorporate much of the vast accumulated new knowledge of slab behaviour. It had been observed that reinforced concrete slabs were capable of a great deal of redistribution of the stress resultants and that the slab could carry any load if capacity corresponding to any statically admissable moment distribution was provided. However, it was also observed that the redistribution of the stress resultants was accompanied by cracking and deflections which may not be tolerable. Thus, some effort must be made to distribute moments such that serviceability requirements are assured. It is on such observations that the new code provisions are based.

Slab Thickness

No discussion of reinforced concrete slabs would be complete without examining the requirements for slab thickness. Previous codes have specified minimum slab thicknesses as a means of controlling deflections to acceptable limits and, in keeping with the concept of designing slabs according to their origin, two sets of minimum thicknesses were required. To be consistent with the new philosophy of a common design procedure for all slabs, new criteria were required. A logical beginning to develop this criteria is to list the major factors which influence slab deflections. Briefly these are:

- Magnitude of Live Load
- Slab Stiffness
- Presence of stiffening elements
- Time - dependent effects

The magnitude of live load has an obvious influence on deflection but since its distribution and occurrence is frequently of a random nature and since its effects on deflection are considered indirectly when proportioning for the required moment capacity, the new code provisions, like the previous provisions, do not consider live load as a primary variable.

Previously in determining slab thickness no distinction was made between edge and interior panels, panel shape or the size of the supporting columns and/or capitals, if any, all of which have a substantial influence and are considered in the new provisions. In addition to considering the presence of drop panels and the use of higher strength steels, the two factors presently considered for flat slabs,the new provisions must also be able to consider the effects of any stiffening beams.

The expression for minimum slab thickness, which is applicable to all slabs reinforced in two directions, is given in the code as follows:

$$t = \frac{\ell_c\,(800 + 0.005\,f_y)}{36{,}000 + 5000\,S\left[H_{av} - 0.5\,(1-R_a)(1+1/S)\right]}$$

By specifying the minimum thickness in terms of clear span, ℓ_c, instead of the center to center span used previously, advantage is taken of the beneficial effect of large column sections or column capitals in reducing slab deflections.

The term in the numerator in parentheses provides the greater slab thickness required to compensate for the descrease in stiffness caused by the greater degree of cracking associated with the higher allowable steel stresses. In ACI 318-63 this increase in thickness was 10% for each 10,000 psi increase retained in the proposed revisions for one-way slabs. However, this increase for two-way slabs is reduced to 5% for each 10,000 psi increase in f_y above 40,000 psi, since under service load conditions this type of slab is less likely to be cracked and thus is less sensitive to changes in steel stresses.

The shape and position of the panel is considered by terms S and R_a, respectively. S is defined as the ratio of long to short clear spans and R_a as the ratio of length of continuous edges to total perimeter of a slab panel. The effect of a stiffening beam of any depth is given by H_{av} which is defined as the average value of the ratios of beam to slab flexural stiffnesses on all sides of the panel. The effects of these variables is clearly seen in Fig. 2 where plots are presented for corner and interior panels.

For many slabs commonly used in practice, for example slabs with c/ℓ ratios of 0.1, the thickness required by the above equation is greater than that required by the present code. For this reason it is specified that the thickness need not be greater than

$$t = \frac{\ell_c\,(800 + 0.005\,f_y)}{36{,}000}$$

which for f_y = 40,000 psi, reduces to $\ell_c/36$.

On the other hand, for very stiff beams, the required thickness can become less than desireable, so a limiting minimum thickness of

$$t = \frac{\ell_c\,(800 + 0.005\,f_y)}{36{,}000 + 5000\,(1+R_a)}$$

is specified. For square panels, this corresponds to beams having an average flexural stiffness reatio of 2.0.

In addition, for panels having discontinuous edges, either an edge beam having a minimum stiffness of H = 0.8 shall be provided or the minimum thicknesses shall be increased by 10%. However, if a drop panel having dimensions at least one-third of the clear spans and a projection below the slab of at least t/4, the required minimum thickness may be reduced by 10%.

Notwithstanding the computed thicknesses from the above expression, the thickness shall not be less than the following values.

For slabs without beams or drop panels. 5 in.

For slabs without beams but with drop panels. 4 in.

For slabs with beams on all four edges with a value of H_{av} at least equal to 2.0 3 1/2 in.

With the development of such numerical techniques as finite element and finite difference methods,it is possible to compute slab deflections taking into account the size and shape of the panel, the condition of support, the nature of restraint at the panel edges, the modulus of elasticity and variations in the effective moment of inertia of the concrete sections. When such factors are considered, the designer has the option of using any slab section, providing the computed deflections are less than the tabulated maximum allowable computed deflections listed in the code. Expressions for the modulus of elasticity for the concrete and the effective moment of inertia of the critical sections to be used in the deflection calculations are also specified in the code but, since they are not restricted to slab design, will not be discussed further.

To compute the additional long time reflections the computed immediate deflections should be multiplied by the new factor $\left[2 - 1.2\ (A_s' / A_s)\right]$ but not less than 0.6. This factor is identical with the previous factor when the area of compression steel is zero or equal to the area of tension steel but slightly greater for intermediate values.

Methods of Analysis

Why should a building code detail the procedure for the analysis of slab systems when it does not do so for any other structural element? Why not specify "A slab system may be designed by any procedure satisfying the conditions of equilibrium and geometrical compatability, provided it is shown that the strength furnished is at least that of the specified load factors and all serviceability conditions, including the specified limits on deflections, are met"? The portion in quotations is essentially the first paragraph of the section entitled Design Procedures in the new code. However, since analyses meeting these requirements are not readily available, the code outlines two procedures which may be followed.

The first procedure is known as the Direct Design Method (DDM) and may be used for all slabs consisting of more or less rectangular panels and supported on reasonably regularly spaced supports. Where these conditions are not met a more general method, known as the Equivalent Frame Method (EFM), may be used which assumes that the slab-beam column system may be analysed as a series of plane frames. Both procedures are described in detail in the following sections.

Direct Design Method

Most concrete floor systems consist of rectangular panels supported on columns of more or less regular spacing. Stiffening elements such as beams, drop panels or column capitals may also be provided. For such cases a simple straightforward method of analysis referred to as the Direct Design Method, is permitted. In summary this procedure consists of the following steps:

(a) Determination of total static design moment for each span.

(b) Proportioning this total moment to positive and negative design moments at the critical sections.

(c) Proportioning these design moments between column and middle strips.

(d) Checking support stiffnesses for pattern loading effects.

(e) Design for shear.

Although the method resembles the former empirical method for flat slabs there are many important differences. However, like the empirical method, there must be some limitations of geometry for its use. These limitations are compared below.

Empirical Method ACI318-63	Direct Design Method ACI318-71
3 spans continuous	no change
maximum column offset 10%	no change
$\frac{\text{panel length}}{\text{panel width}} \leq 1.33$	2.0
$\frac{\text{span}}{\text{adjacent span}} \leq 1.20$	1.33
$\frac{\text{live load}}{\text{dead load}} \leq 3.0$	no change

In general the direct Design Method may be used for a greater range of slab geometry that the previous empirical method. An additional restriction is placed on the use of the Direct Design Method in that where a panel is supported by beams on all sides, the relative stiffness of the beams in the two perpendicular directions given by $H_1 \ell_2{}^2/H_2\ell_1{}^2$ shall not be less than 0.2 and 5.0. During the compilation of the numerical solutions for slabs in the Illinois slab program on which the new code provisions are based, it became apparent that it would be impossible to include all possible variables. It was decided to define two dimensionless ratios, namely $H_1\ell_2{}^2/H_2\ell_1{}^2$ and $H_1\ell_{2}/\ell_1$, which would be used to proportion slab moments. The first ratio is equal to unity when the beam stiffness in each direction is proportional to the span length, a condition that was assumed throughout the slab program and also exists in most slabs in practice. The range of variation of this ratio was rather arbitrarily set. Subsequent analyses at the University of Alberta show that even within the limit set by the code it is possible to have localized conditions of one-way behaviour which results in substantially different patterns of moments. The second ratio $H_1 \ell_2/\ell_1$ is the parameter used to define column strip stiffness.

The total static design moment is computed for each span in a strip bounded laterally by the centerline of the panel on each side of the supports and is defined as the absolute sum of the positive and average negative moments. By equilibrium this moment is

$$M_o = \frac{w\ell_2 \ell_c^2}{8}$$

The critical section for negative moment is the face of the column, capital or wall. The clear spans ℓ_c is also measured from this position and must be greater than 0.65 times the corresponding center to center span. This value of M_o is compared with the corresponding value in the previous code in Fig. 3.

This total moment, M_o, must now be proportioned between the negative and positive critical sections. This split is made on the basis of the stiffnesses of the supporting columns as shown in Fig. 4. It is assumed that interior columns do not rotate, hence the split is similar to that of a continuously, uniformly loaded beam, in this case for interior panels, 0.65 M_o for negative moments and 0.35 M_o for positive moments. Studies have shown that the moments in the exterior panel are sensitive to the degree of restraint provided at the discontinuous edge. For example when the exterior column has essentially no flexural stiffness the exterior negative design moment is approximately zero. However, even if the exterior column has infinite flexural stiffness the moments will still not be distributed as for an interior panel unless an extremely stiff beam in torsion is also provided to prevent rotation of the discontinuous edge. For this reason the exterior panel moments are expressed in terms of an equivalent column stiffness ratio K_e', which is a function of both the exterior column stiffness and torsional stiffness of the edge slab and beam strip. Equivalent column stiffnesses are discussed in detail in the next section for the Equivalent Frame Method.

Once the negative and positive design moments across critical sections have been determined for each panel they must be distributed laterally. This is simplified, as in previous codes, to assuming uniform distribution across column and middle strips. However, a column strip is now defined as a design strip with a width of 0.25 ℓ_2, but not greater than 0.25 ℓ_1, on each side of the column centerline and includes beams, if any. In short, the width of the column strip is one-half the shorter panel span. The middle strip is the design strip bounded by the column strips. At interior sections the portion assigned to the column strip is specified as a function of the effective column strip stiffness and the panel aspect ratio as shown in Fig. 5. Across the exterior section the proportion assigned to the column strip is reduced as the torsional stiffness of the edge members is increased, reflecting the more uniform distribution when a large edge member is provided.

Where beams are part of the column strip, the beam shall be proportioned to resist 85 percent of the column strip moment if $H_1\ell_2/\ell_1$ is equal or greater than 1.0. For values of this ratio between 1.0 and zero, the proportion resisted by the beam shall be obtained by linear interpolation between 85 and zero percent. The beam must also be designed to carry any loading that is applied directly to the beam.

Middle strips are designed for the remainder of the design moment not specified for column strips. The code also permits modifying any moment value up to 10% providing the total moment in any panel remains unaltered.

Since the above distributions are based on no rotation of interior supports, the supporting elements above and below the slab shall be designed to resist in direct proportion to their stiffnesses, a minimum moment equal to

$$M = \frac{0.08[(w_D+0.5w_L)\ell_2\ell_c{}^2 - w_D'\ell_2'(\ell_c')^2]}{1 + \frac{1}{K_e'}}$$

where w_D', ℓ_2' and ℓ_c' refer to the shorter span.

In addition, where the ratio of live to dead load exceeds 0.5 the designer must consider the effects of pattern loading by either:

(a) provide a sum of flexural stiffnesses of the columns above and below the slab, K', greater than a tabulated minimum stiffness K_r' given in the code, or

(b) multiply the positive bending moments in the panels supported by those columns by the factor

$$F = 1 + \frac{2-w_D/w_L}{4+w_D/w_L}(1 - \frac{K'}{K'_r})$$

It is seen that this ratio will always be greater than 1.0 when its use is required.

The total shear on each panel must be accounted for. Where beams are present with a ratio, $H_1 \ell_2/\ell_1$, equal to or greater than 1.0, the beam shall resist the shear caused by loads in tributary areas bounded by 45 degree lines drawn from the panel corners and the panel centerline parallel to the long sides. For values of $H_1 \ell_2'/\ell_1$ less than 1.0 the shear is obtained by linear interpolation assuming for $H_1 = 0$ the beam carries no load.

<u>Equivalent Frame Method</u>

In the Equivalent Frame Method the structure is considered as a series of frames taken in both the longitudinal and transverse directions. Each frame consists of a row of equivalent columns or supports and slab-beam strips bounded laterally by the center line of the panel on each side of the center line of the columns or supports. For vertical loading, each floor together with its column above and below may be analyzed separately and the far ends of the columns may be assumed fixed. However, the stiffnesses assigned to the various members of these frames have been changed completely from previous procedures to permit a more accurate representation of the actual three-dimensional slab-beam column system by the equivalent two-dimensional elastic frame. The factors required for the Cross moment distribution, namely the fixed end moment coefficient, the stiffness factor and the carry-over factor, must be evaluated for each element of the equivalent frame.

A typical slab-beam element for a portion of an equivalent frame is shown in Fig. 6. The moment of inertia for this element between the faces of the columns, brackets or columns is based on the uncracked cross-section of the concrete including beam or drop panel, if any. The moment of inertia between the center of the column and the face of the column, bracket or capital is to be considered finite and is dependent on the transverse dimensions of the panel and support.

To approximate actual slab behavior under unequal loading on adjacent panels, or unequal adjacent panel lengths, the equivalent column is considered to consist of the actual column plus an attached torsional member running transverse to the direction in which the moments are being determined as shown in Fig. 7. The transverse slab-beam may rotate even though the column is infinitely stiff, thus permitting consideration of the moment redistribution between adjacent panels. This equivalent column element is also used for the exterior columns in the Direct Design Method.

The stiffness of the equivalent column, K_{ec}, is determined from the expression

$$\frac{1}{K_{ec}} = \frac{1}{\Sigma K_c} + \frac{1}{K_t}$$

where $K_c = \frac{k_c E_{cc} I_c}{L}$

= stiffness of attached column

$$K_t = \frac{\Sigma\ 9\ E_{cs} C}{\ell_2 (1 - C_2/\ell_2)^3}$$

= torsional stiffness of attached transverse slab-beam elements.

In computing the stiffness of the column, K_c , the moment of inertia is to be assumed infinite from the top to the bottom of the slab-beam or capital as indicated in Fig. 6. The attached torsional members for monolithic construction are considered to include, in addition to the beam, that portion of the slab on each side of the beam extending a distance equal to the beam projection either above or below the slab, whichever is greater, but not more than four times the slab thickness. The torsional constant, C, can be computed for this transverse member by dividing the cross section into separate rectangular parts and carrying out the summation

$$C = \Sigma(1 - 0.63\ \frac{x}{y})\frac{x^3 y}{3}$$

After evaluating the necessary factors from the slab-beam and column elements the equivalent frame is analyzed by elastic analysis. Where live load does not exceed three-quarters of the dead load or nature of live load is such that all panels must be loaded simultaneously, maximum moments are obtained assuming full live load over entire system. For other conditions maximum moments may be obtained by considering pattern loading using three-quarters of full design live load on appropriate panels.

Since the analysis of the frame is based on center to center lengths the negative moments may be reduced to those occurring at a critical section taken at the face of rectilinear supports. At exterior supports provided with brackets or capitals the critical section for negative moment in the direction perpendicular to the edge shall be taken at a distance not greater than one-half the projection of the bracket or capital beyond the face of the column. It should be noted that this is different than the location of the critical section by the Direct Design Method. Circular or regular polygonal supports shall be treated as square supports having the same area. These design moments at critical sections are distributed between column and middle strips in the same manner as for the Direct Design Method. Columns, however, are designed to resist the moments obtained from the frame analysis.

Where the slab-system is within the limitations of geometry required for the Direct Design Method and analysis has been performed by the Equivalent Frame Method, the absolute sum of the positive and average negative design moments in any panel need not exceed the total static moment required by the former method.

Conclusion

Initially the new provisions may appear more complex than former procedures but this is due to lack of familiarity with the new and familiarity with the old. In addition to a rational theoretical background, the new procedures enjoy a logic of design which, once grasped, permits the design to flow smoothly from one step to another.

The new provisions were neither divinely inspired nor do they cover all possible slab configurations, deficiencies which also existed with the previous code. While the new provisions have less restrictions for their use than the proliferation of methods that existed previously they were designed to apply only to slab systems consisting of reasonably regular rectangular panels on isolated column supports and so carry load in two directions. Thus any discontinuities of support such as randomly placed stiff beams, columns elongated in plan or other factors which produce localized one-way behavior should be carefully examined by the designer and appropriate modifications made.

Notwithstanding the limitations of the new provisions, they do provide for the first time, a method of design that provides a consistent factor of safety to all types of slab systems. The elimination from the code of the artificial differences between slabs with or slabs without beams must be considered an achievement.

Appendix - Design Example

To illustrate the use of the design procedures outlined in the code consider the strip of a slab system shown in Fig. 8. A variety of support conditions are used to demonstrate the calculations for stiffness. The ratio of adjacent span lengths, numbers of spans etc. were chosen so that the Direct Design Method may be used but since the geometry is at the extreme limits permitted for this method it is expected that some differences may be expected from the Equivalent Frame Method solution.

For computing the stiffnesses of column and slab-beam elements for the Equivalent Frame Method the new code gives specific procedures for determining the moments of inertia to be used at the various sections. No such definitions are included with the Direct Design Method so it is permissable to consider the column and slab-beam elements as prismatic and compute stiffnesses by the well known expression 4 EI/ℓ for each member where ℓ is the center to center length. For most cases the difference between this assumption and the assumption of stiffnesses for the EFM are not large. For this design example the design moments were obtained assuming prismatic sections for the DDM. The corresponding moments using the stiffnesses as obtained from the EFM are shown at the bottom in parenthesis.

The stiffnesses required for the column and slab-beam elements in the EFM (see Fig. 8) may be obtained using column analogy and tabulated.

Since both c_1/ℓ_1 and c_2/ℓ_2 can vary independently at each end a comprehensive table of the required factors for all practical condition would be voluminous. However studies by the writer have indicated that by assuming the ratios c_1/ℓ_1 and c_2/ℓ_2 at the far end are equal to the respective ratios at the near end, relatively simple tables can be produced in terms of the remaining two variables, namely c_1/ℓ_1 and c_2/ℓ_2 at the near end. Examples of these tables taken from a paper by the writer which has been accepted for publication in the Journal of the American Concrete Institute, are included to assist in evaluating the necessary factors. Values in the calculations obtained from the tables are indicated by square parenthesis.

Since the live load is only three-quarters of the dead load the frame need only be analyzed for full load on each span. After balancing the moments in the frame the negative moments may be reduced to those at the critical sections located at the face of the columns or capitals except for line D where the critical section is at one-half the projection of the capital from the column face (see Section 13.4.2). Also for span CD the final moments may be reduced so that the sum of the positive and average negative moments does not exceed M_o (see Section 13.4.5).

Only the design moments for the columns and critical slab sections are shown. A complete design would of course include design for shear and choice of reinforcing.

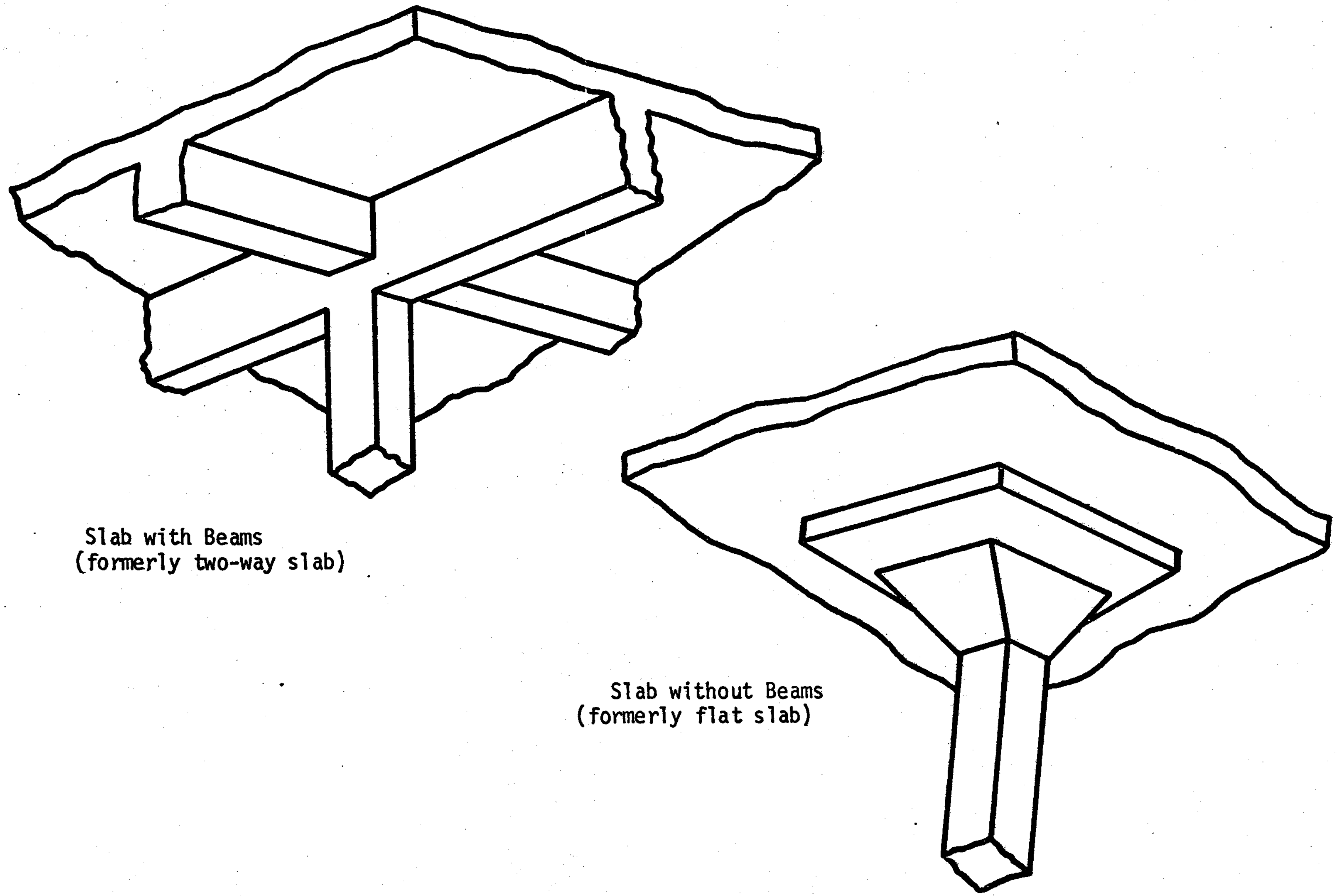


FIG. 1: TYPES OF SLAB SYSTEMS

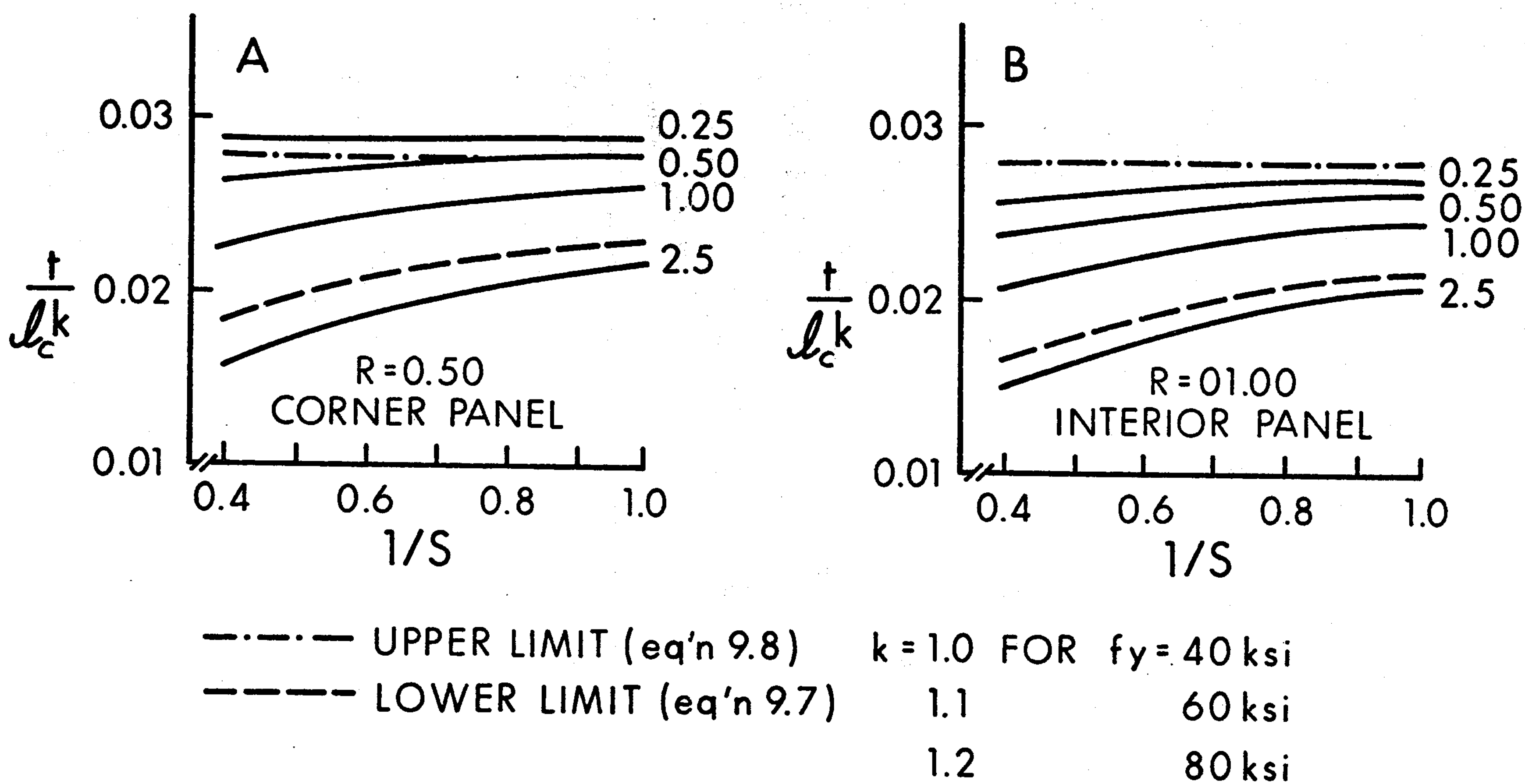


FIG. 2: MINIMUM THICKNESS TO CLEAR SPAN RATIOS

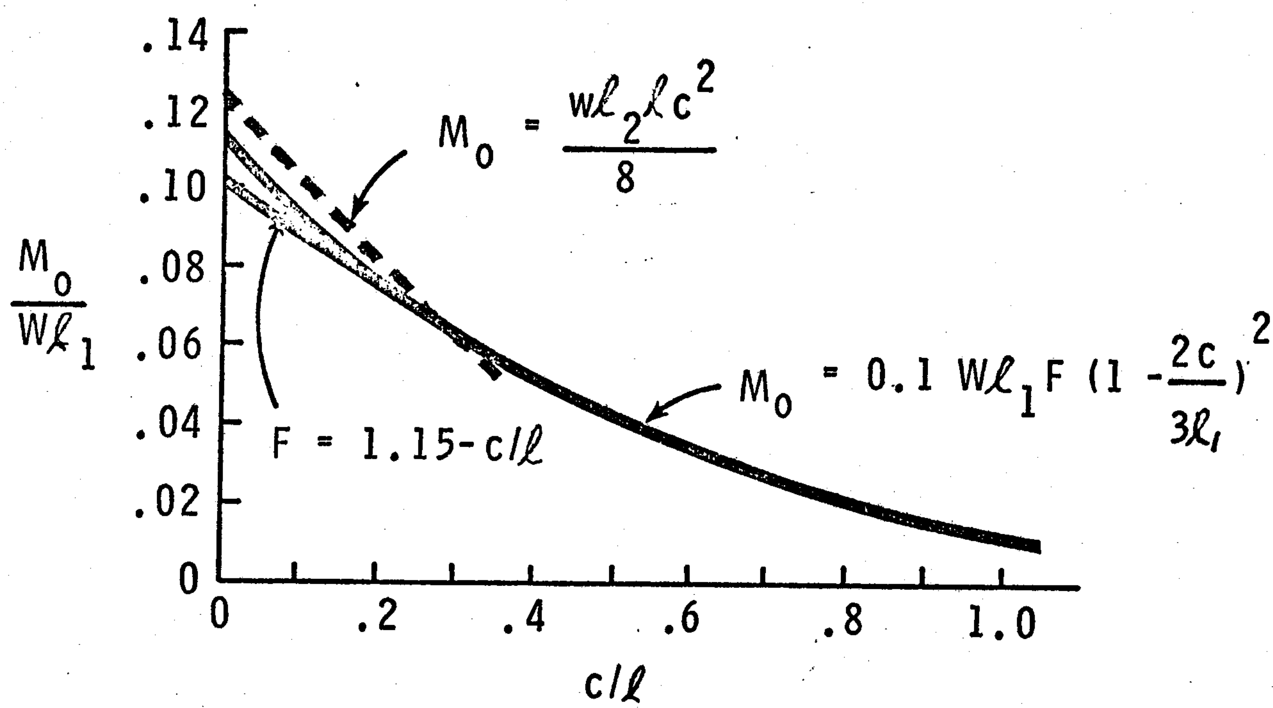


FIG. 3: COMPARISON OF TOTAL STATIC MOMENT M_o

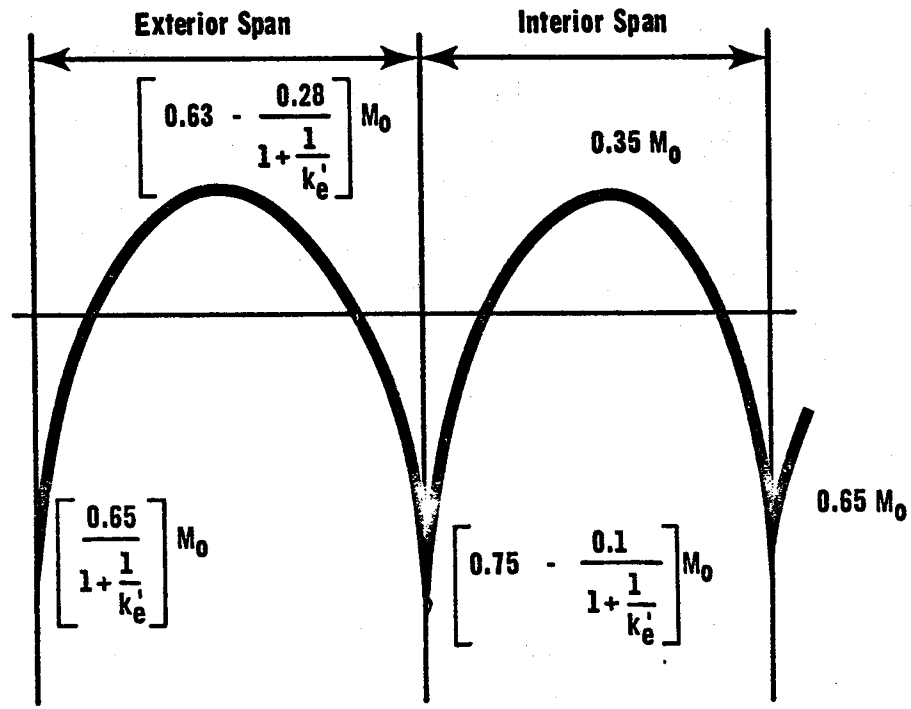


FIG. 4: SPLIT OF M_o INTO NEGATIVE AND POSITIVE MOMENTS

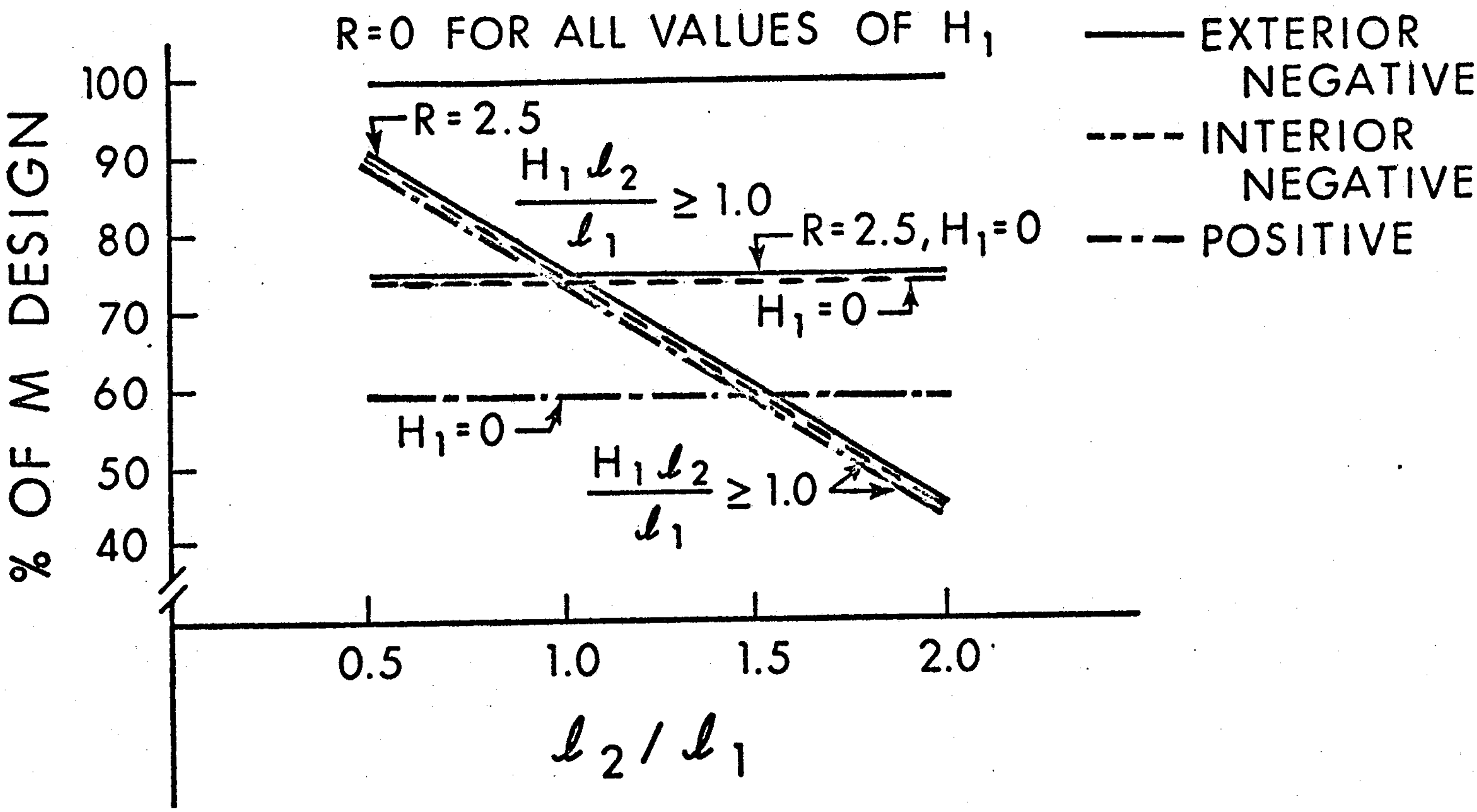


FIG. 5: PERCENTAGE OF M DESIGN
ASSIGNED TO COLUMN STRIP

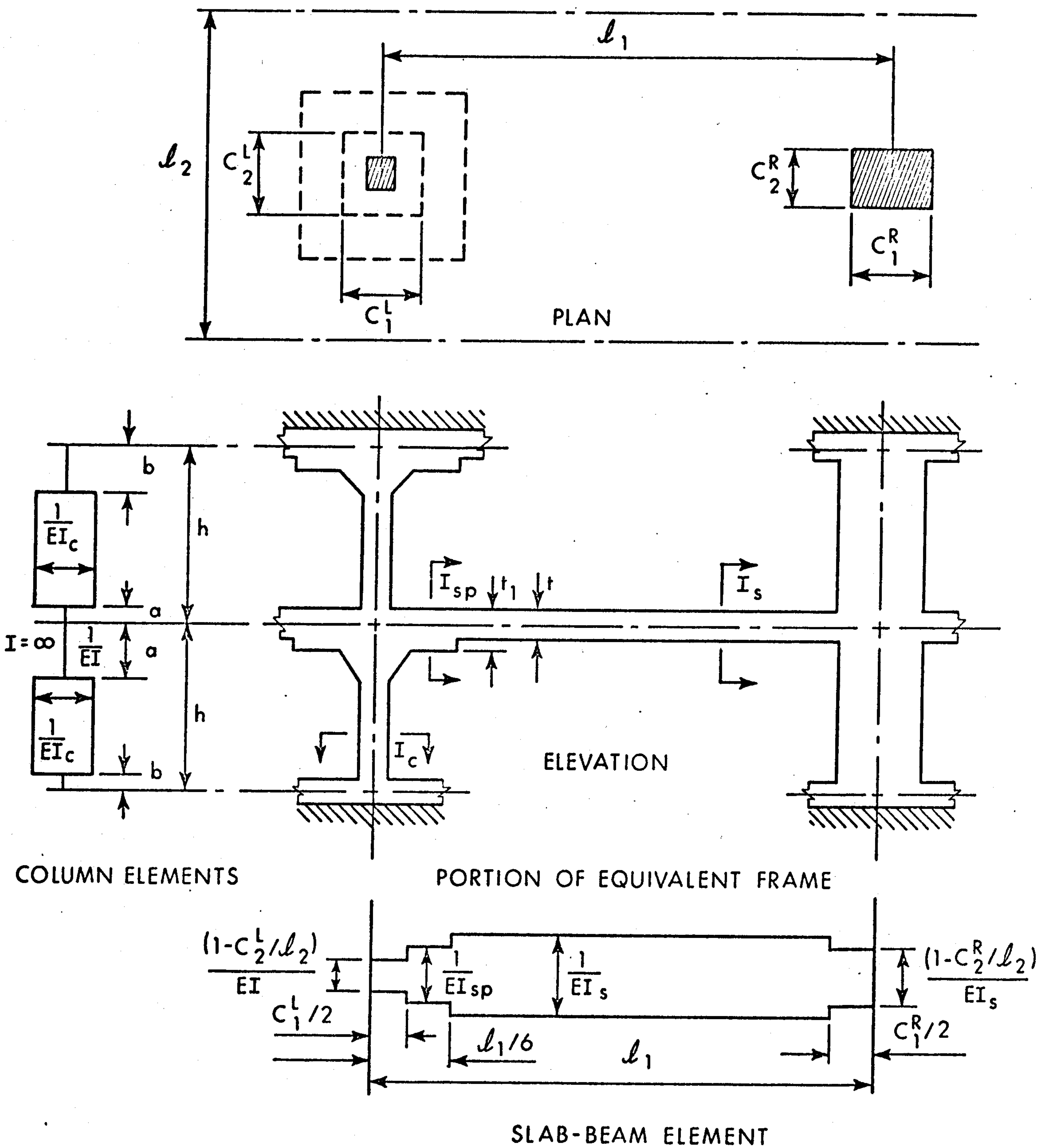


FIG. 6: COLUMN AND SLAB-BEAM ELEMENTS FOR THE EQUIVALENT FRAME METHOD

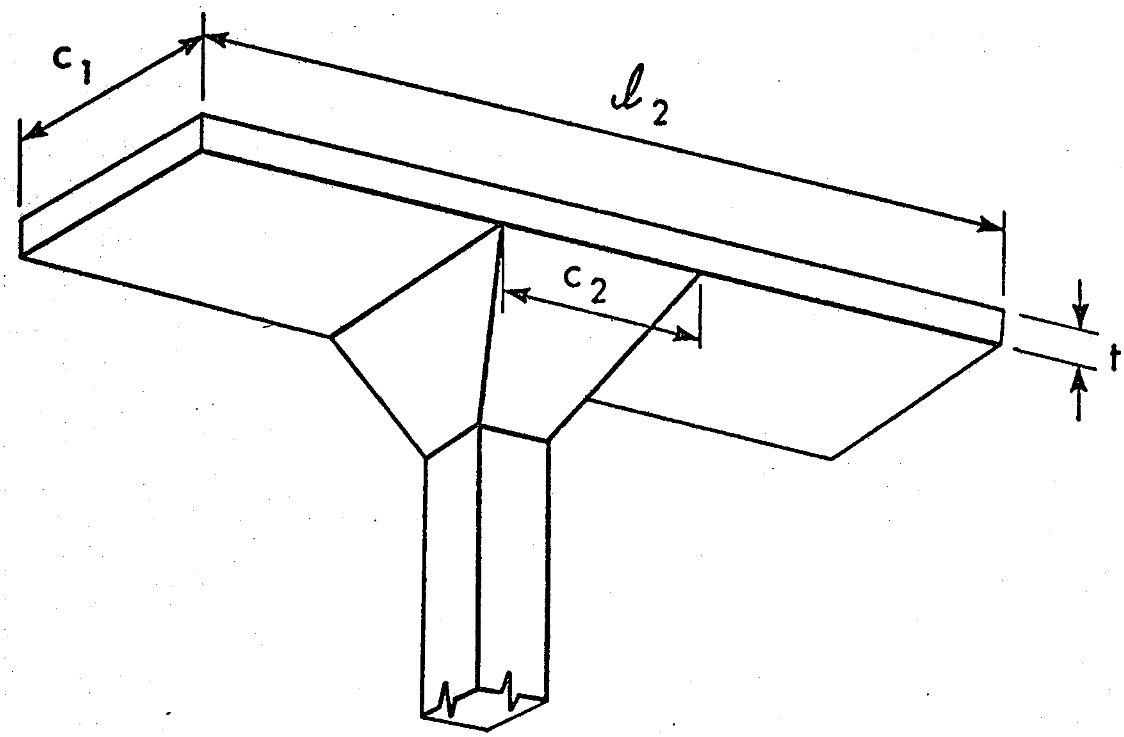


FIG. 7: EQUIVALENT COLUMN

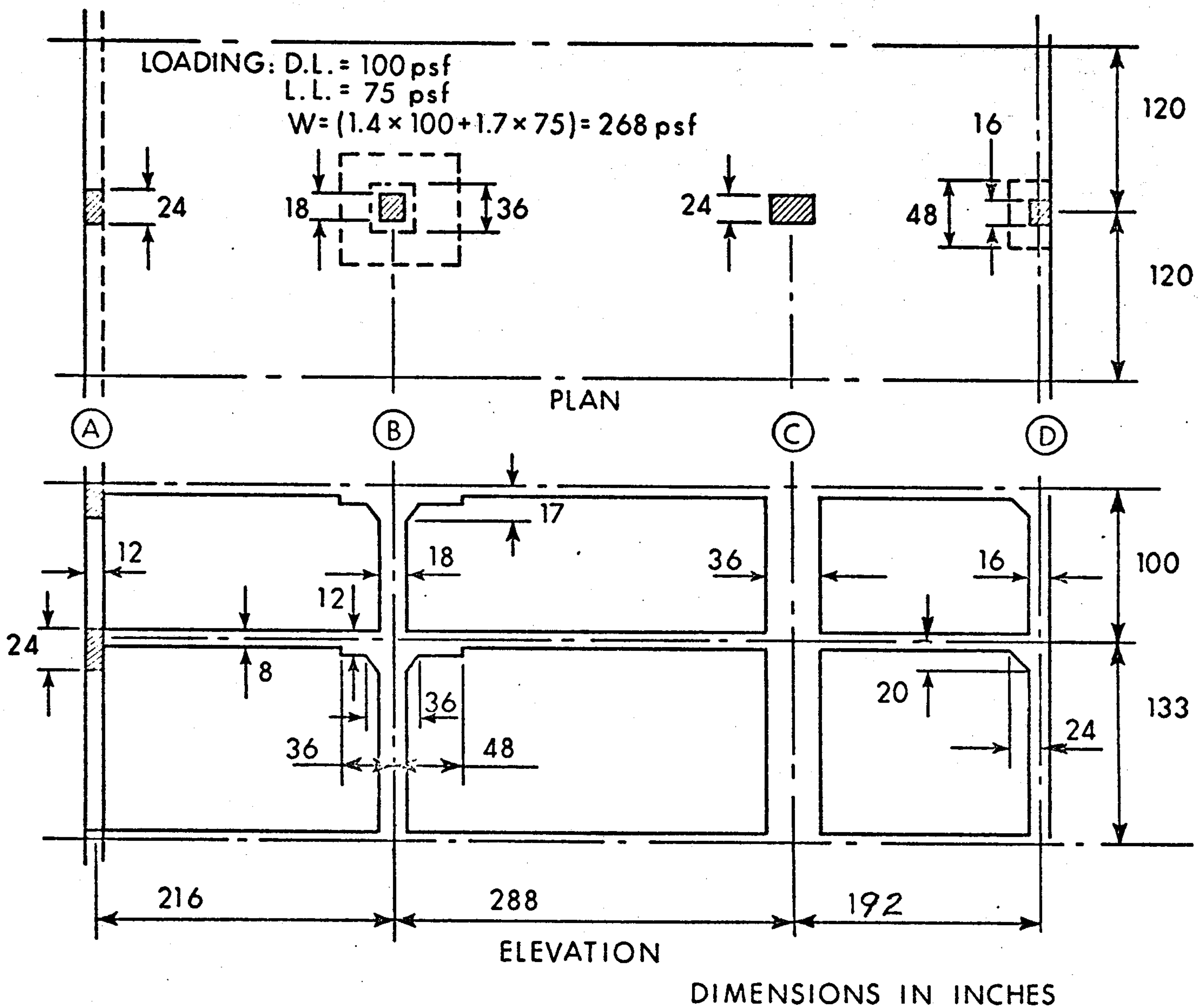


FIG. 8: SLAB FOR DESIGN EXAMPLE

TABLE 2

MOMENT DISTRIBUTION FACTORS FOR SLAB-BEAM ELEMENTS

(FLAT PLATE)

c_1/ℓ_1 \ c_2/ℓ_2		0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
0.00	M	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083
	k	4.000	4.000	4.000	4.000	4.000	4.000	4.000	4.000	4.000	4.000	4.000
	C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
0.05	M	0.083	0.084	0.084	0.084	0.085	0.085	0.085	0.086	0.086	0.086	0.086
	k	4.000	4.047	4.093	4.138	4.181	4.222	4.261	4.299	4.334	4.368	4.398
	C	0.500	0.503	0.507	0.510	0.513	0.516	0.518	0.521	0.523	0.526	0.528
0.10	M	0.083	0.084	0.085	0.085	0.086	0.087	0.087	0.088	0.088	0.089	0.089
	k	4.000	4.091	4.182	4.272	4.362	4.449	4.535	4.618	4.698	4.774	4.846
	C	0.500	0.506	0.513	0.519	0.524	0.530	0.535	0.540	0.545	0.550	0.554
0.15	M	0.083	0.084	0.085	0.086	0.087	0.088	0.089	0.090	0.090	0.091	0.092
	k	4.000	4.132	4.267	4.403	4.541	4.680	4.818	4.955	5.090	5.222	5.349
	C	0.500	0.509	0.517	0.526	0.534	0.543	0.550	0.558	0.565	0.572	0.579
0.20	M	0.083	0.085	0.086	0.087	0.088	0.089	0.090	0.091	0.092	0.093	0.094
	k	4.000	4.170	4.346	4.529	4.717	4.910	5.108	5.308	5.509	5.710	5.908
	C	0.500	0.511	0.522	0.532	0.543	0.554	0.564	0.574	0.584	0.593	0.602
0.25	M	0.083	0.085	0.086	0.087	0.089	0.090	0.091	0.093	0.094	0.095	0.096
	k	4.000	4.204	4.420	4.648	4.887	5.138	5.401	5.672	5.952	6.238	6.527
	C	0.500	0.512	0.525	0.538	0.550	0.563	0.576	0.588	0.600	0.612	0.623
0.30	M	0.083	0.085	0.086	0.088	0.089	0.091	0.092	0.094	0.095	0.096	0.098
	k	4.000	4.235	4.488	4.760	5.050	5.361	5.692	6.044	6.414	6.802	7.205
	C	0.500	0.514	0.527	0.542	0.556	0.571	0.585	0.600	0.614	0.628	0.642
0.35	M	0.083	0.085	0.087	0.088	0.090	0.091	0.093	0.095	0.096	0.098	0.099
	k	4.000	4.264	4.551	4.864	5.204	5.575	5.979	6.416	6.888	7.395	7.935
	C	0.500	0.514	0.529	0.545	0.560	0.576	0.593	0.609	0.626	0.642	0.658
0.40	M	0.083	0.085	0.087	0.088	0.090	0.092	0.094	0.095	0.097	0.099	0.100
	k	4.000	4.289	4.607	4.959	5.348	5.778	6.255	6.782	7.365	8.007	8.710
	C	0.500	0.515	0.530	0.546	0.563	0.580	0.598	0.617	0.635	0.654	0.672
0.45	M	0.083	0.085	0.087	0.088	0.090	0.092	0.094	0.096	0.098	0.100	0.101
	k	4.000	4.311	4.658	5.046	5.480	5.967	6.517	7.136	7.836	8.625	9.514
	C	0.500	0.515	0.530	0.547	0.564	0.583	0.602	0.621	0.642	0.662	0.683
0.50	M	0.083	0.085	0.087	0.088	0.090	0.092	0.094	0.096	0.098	0.100	0.102
	k	4.000	4.331	4.703	5.123	5.599	6.141	6.760	7.470	8.239	9.234	10.329
	C	0.500	0.515	0.530	0.547	0.564	0.583	0.603	0.624	0.645	0.667	0.690
$X=(1-c_2/\ell_2)^3$		1.000	0.556	0.729	0.613	0.512	0.421	0.343	0.274	0.216	0.166	0.125

FEM (uniform load w) $= Mw\ell_2\ell_1^2$

K (stiffness) $= kE\ell_2 t^3/12\ell_1$

Carry Over Factor $= C$

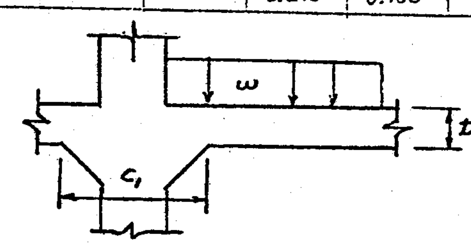


TABLE 4

MOMENT DISTRIBUTION FACTORS FOR SLAB-BEAM ELEMENTS
($t_1 = 1.5t$)

c_1/ℓ_1 \ c_2/ℓ_2		0.00	0.05	0.10	0.15	0.20	0.25	0.30
0.00	M	0.093	0.093	0.093	0.093	0.093	0.093	0.093
	k	5.837	5.837	5.837	5.837	5.837	5.837	5.837
	C	0.589	0.589	0.589	0.589	0.589	0.589	0.589
0.05	M	0.093	0.093	0.093	0.093	0.094	0.094	0.094
	k	5.837	5.890	5.942	5.993	6.041	6.087	6.131
	C	0.589	0.591	0.594	0.596	0.598	0.600	0.602
0.10	M	0.093	0.093	0.094	0.094	0.094	0.095	0.095
	k	5.837	5.940	6.042	6.142	6.240	6.335	6.427
	C	0.589	0.593	0.598	0.602	0.607	0.611	0.615
0.15	M	0.093	0.093	0.094	0.095	0.095	0.096	0.096
	k	5.837	5.986	6.135	6.284	6.432	6.579	6.723
	C	0.589	0.595	0.602	0.608	0.614	0.620	0.626
0.20	M	0.093	0.093	0.094	0.095	0.096	0.096	0.097
	k	5.837	6.027	6.221	6.418	6.616	6.816	7.015
	C	0.589	0.597	0.605	0.613	0.621	0.628	0.635
0.25	M	0.093	0.094	0.094	0.095	0.096	0.097	0.098
	k	5.837	6.065	6.300	6.543	6.790	7.043	7.298
	C	0.589	0.598	0.608	0.617	0.626	0.635	0.644
0.30	M	0.093	0.094	0.095	0.096	0.097	0.098	0.099
	k	5.837	6.099	6.372	6.657	6.953	7.258	7.571
	C	0.589	0.599	0.610	0.620	0.631	0.641	0.651

FEM (uniform load w) $= Mw\ell_2\ell_1^2$

K (stiffness) $= kE\ell_2 t^3/12\ell_1$

Carry Over Factor $= C$

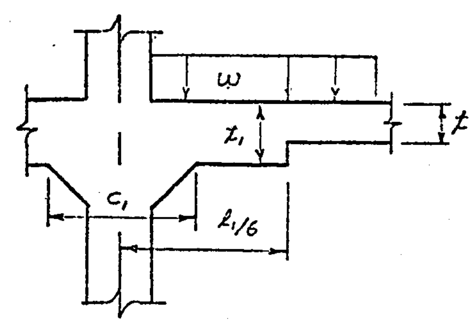


TABLE 5

COLUMN STIFFNESS COEFFICIENTS, k_c

a/h \ b/h	.00	.02	.04	.06	.08	.10	.12	.14	.16	.18	.20	.22	.24
.00	4.000	4.082	4.167	4.255	4.348	4.444	4.545	4.651	4.762	4.878	5.000	4.128	5.263
.02	4.337	4.433	4.533	4.638	4.747	4.862	4.983	5.110	5.244	5.384	5.533	5.690	5.856
.04	4.709	4.882	4.940	5.063	5.193	5.330	5.475	5.627	5.787	5.958	6.138	6.329	6.533
.06	5.122	5.252	5.393	5.539	5.693	5.855	6.027	6.209	6.403	6.608	6.827	7.060	7.310
.08	5.581	5.735	5.898	6.070	6.252	6.445	6.650	6.868	7.100	7.348	7.613	7.897	8.203
.10	6.091	6.271	6.462	6.665	6.880	7.109	7.353	7.614	7.893	8.192	8.513	8.859	9.233
.12	6.659	6.870	7.094	7.333	7.587	7.859	8.150	8.461	8.796	9.157	9.546	9.967	10.430
.14	7.292	7.540	7.803	8.084	8.385	8.708	9.054	9.426	9.829	10.260	10.740	11.250	11.810
.16	8.001	8.291	8.600	8.931	9.287	9.670	10.080	10.530	11.010	11.540	12.110	12.740	13.420
.18	8.796	9.134	9.498	9.888	10.310	10.760	11.260	11.790	12.370	13.010	13.700	14.470	15.310
.20	9.687	10.080	10.510	10.970	11.470	12.010	12.600	13.240	13.940	14.710	15.560	16.490	17.530
.22	10.690	11.160	11.660	12.200	12.800	13.440	14.140	14.910	15.760	16.690	17.210	18.870	20.150
.24	11.820	12.370	12.960	13.610	14.310	15.080	15.920	16.840	17.870	19.000	20.260	21.650	23.260

$$K_c = \frac{k_c E_c I_c}{h}$$

Note: a is length of rigid column section at near end

b is length at rigid column section at far end

TABLE 6

VALUES OF TORSION CONSTANT, C*

y \ x	4	5	6	7	8	9	10	12	14	16
12	202	369	592	868	1188	1538	1900	2557		
14	245	452	736	1096	1529	2024	2566	3709	4738	
16	388	534	880	1325	1871	2510	3233	4861	6567	8083
18	330	619	1024	1554	2212	2996	3900	6013	8397	10813
20	373	702	1167	1782	2553	3482	4567	7165	10226	13544
22	416	785	1312	2011	2895	3968	5233	8317	12055	16275
24	548	869	1456	2240	3236	4454	5900	9469	13885	19005
27	522	994	1672	2583	3748	5183	6900	11197	16628	23101
30	586	1119	1888	2926	4260	5912	7900	12925	19373	27197
33	650	1243	2104	3269	4772	6641	8900	14653	22117	31293
36	714	1369	2320	3612	5284	7370	9900	16381	24860	35389
42	842	1619	2752	4298	6308	8828	11900	19837	30349	43581
48	970	1869	3184	4984	7332	10286	13900	23293	35836	51773
54	1098	2119	3616	5670	8356	11744	15900	26749	41325	59965
60	1226	2369	4048	6356	9380	13202	17900	30205	46813	68157

$$*C = (1 - 0.63x/y)\frac{x^3 y}{3}$$

x is smaller dimension of rectangular cross section

Analysis by Direct Design Method

	Ⓐ	A–B	Ⓑ	B–C	Ⓒ	C–D	Ⓓ	
ℓ_1 (center to center span)		18		24		16		ft.
ℓ_c (clear span)		16		21		12.5		ft.
ℓ_2/ℓ_1 (panel aspect ratio		1.11		0.83		1.25		
$M_o = \frac{w \ell_2 \ell_c^2}{8}$		172		296		105		ft. Kips
$K_c = \Sigma \frac{4I_c}{h}$ ($k_c = 4$)	242		613		6500		383	in⁴/in
K_t (Eqn. 13.6 see cal. EFM)	1167		646		544		674	in⁴/in
K_{ec} (Eqn. 13.5)	200		315		502		245	in⁴/in
$K_s = \Sigma \frac{4I_s}{\ell_1}$ ($k_c = 4$)		190		142		213		in⁴/in
K_e'	1.05		0.95		1.42		1.15	
% of M_o (Sect. 13.3.3.3)	33	49	70 65	35	65 70	48	35	
M design	-57	+84	-120 -192	+104	-192 -73	+50	-37	ft. kips
% of M_{DES} to Col. Strip	94.5	60	75	60	75	60	97.5	
DESIGN MOMENTS { Column Strip	-54	+50	-144	+62	-144	+30	-36	ft. kips
DESIGN MOMENTS { Middle Strip	-3	+34	-48	+42	-48	+20	-1	ft. Kips.
Column M_{DES} (Eqn 13-3)	57		27.5		42.5		37	ft. Kips.
K_e' (Stiffnesses from EFM)	(1.57)		(0.75)		(1.33)		(1.36)	
M design	(-68)	(+79)	(-118) (-192)	(+104)	(-192) (-73)	(+49)	(-40)	ft. Kips.
DESIGN MOMENTS { Column Strip	(-64)	(+47)	(-144)	(+62)	(-144)	(+29)	(-37)	ft. Kips
DESIGN MOMENTS { Middle Strip	(-4)	(+32)	(-48)	(+42)	(-48)	(+20)	(-1)	ft. Kips
Column M_{DESIGN}	(-68)		(24)		(41)		(40)	ft. Kips

Analysis by Equivalent Frame Method

GEOMETRY

		A	B	C	D
COLUMN ABOVE	b/h	20/100 = 0.20	17/100 = 0.17	4/100 = 0.04	20/100 = 0.20
	a/h	4/100 = 0.04	4/100 = 0.04	4/100 = 0.04	4/100 = 0.04
SLAB	c_1/ℓ_1 (LEFT)	—	36/216 = 0.17	36/288 = 0.13	32/192 = 0.17
	c_2/ℓ_2	24/240 = 0.10	36/240 = 0.15	24/240 = 0.10	48/240 = 0.20
	c_1/ℓ_1 (RIGHT)	12/216 = 0.06	36/288 = 0.13	36/192 = 0.19	—
COLUMN BELOW	a/h	20/133 = 0.15	17/133 = 0.13	4/133 = 0.03	20/133 = 0.15
	b/h	4/133 = 0.03	4/133 = 0.03	4/133 = 0.03	4/133 = 0.03

STIFFNESSES

	A	B	C	D
$K_c^{ABOVE} = [k_c]\, Ic/h$	[6.138] 3456/100 = 212	[5.872] 8748/100 = 514	[4.940] 93312/100 = 4609	[6.138] 5461/100 = 335
$K_c^{BELOW} = [k_c]\, Ic/h$	[8.045] 3456/133 = 209	[7.327] 8748/133 = 482	[4.676] 93312/133 = 3281	[8.045] 5461/133 = 330
ΣK_c	421	996	7890	665
$\Sigma [C]$	[9469] + [1871] = 11340	[5284]	[5284]	[4601]
$[X]$	[0.729]	[0.613]	[0.729]	[0.512]
$K_T = \Sigma (9/\ell_2)\, C/X$	1167	646	544	674
$K_{ec} = 1/(1/\Sigma K_c + 1/K_T)$	310	392	509	335
$K_s^{LEFT} = [k_s]\, I_s/\ell_1$	—	[6.338] 10240/216 = 300	[4.233] 10240/288 = 151	[4.611] 10240/192 = 246
$K_s^{RIGHT} = [k_s]\, I_s/\ell_1$	[4.174] 10240/216 = 198	[6.227] 10240/288 = 221	[4.330] 10240/192 = 231	—
ΣK	508	913	891	581

FACTORS

	Col. A	Span A–B (A end)	Span A–B (B end)	Col. B	Span B–C (B end)	Span B–C (C end)	Col. C	Span C–D (C end)	Span C–D (D end)	Col. D
MOMENT COEFFICIENT [M]		[0.084]	[0.095]		[0.094]	[0.085]		[0.086]	[0.087]	
CARRY-OVER FACTOR		[0.509] →	← [0.610]		[0.606] →	← [0.515]		[0.521] →	← [0.539]	
SLAB DISTRIBUTION FACTOR	—	0.39	0.33		0.24	0.17		0.23	0.42	—
COLUMN DISTRIBUTION FACTOR	0.61			0.43			0.57			0.58

DISTRIBUTION

	Col. A	Span A–B (A end)	Span A–B (mid)	Span A–B (B end)	Col. B	Span B–C (B end)	Span B–C (mid)	Span B–C (C end)	Col. C	Span C–D (C end)	Span C–D (mid)	Span C–D (D end)	Col. D	
FEM = $[M]\, W \ell_2 \ell_1^2$		-146		+165		-290		+262		-118		+119		
bal	+89	+57		+41	+54	+30		-25	-82	-37		-53	-69	
co		+25		+29		-13		+18		-27		-19		
bal	-15	-10		-5	-7	-4		+2	+5	+2		+8	+11	
co		-3		-5		+1		-2		+5		+1		
bal	+2	+1		+1	+2	+1			-1	-1			-1	
MOMENTS AT COLUMN ℄	+76	-76		+226	+49	-275		+253	-78	-167		+59	-59	ft-kips
DESIGN MOMENTS		-57	+67	-149		-183	+121	-166		-90	+54	-14		ft-kips

Reinforced Concrete Buildings

LECTURE 4

DESIGN OF TALL REINFORCED CONCRETE BUILDINGS

by

M. Fintel

DESIGN OF TALL REINFORCED CONCRETE BUILDINGS

M. Fintel

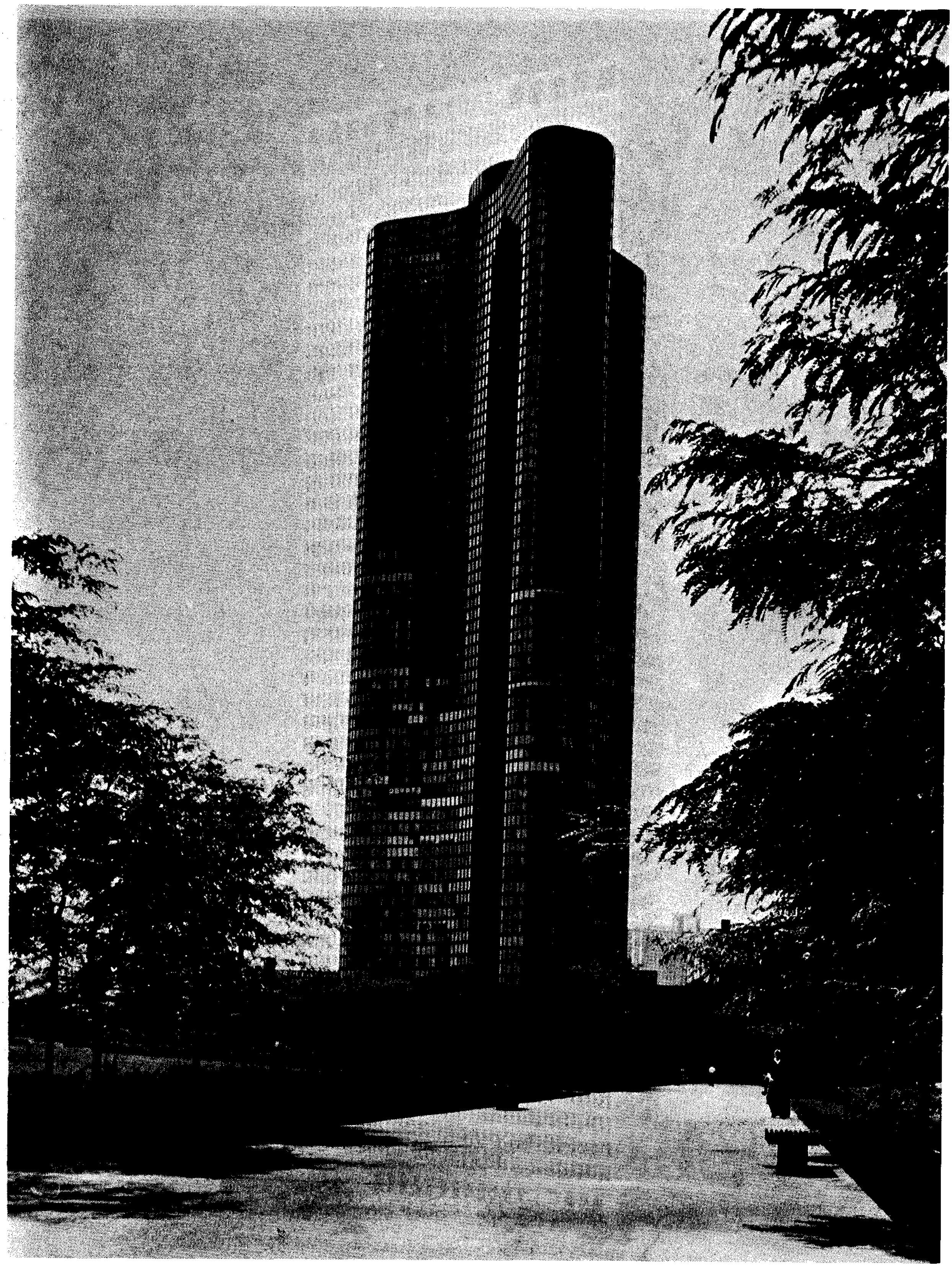
Taller and taller buildings are now being built of reinforced concrete. Figure 1 shows the 70 story, 645 foot high Lake Point Tower in Chicago. This building has three wings 65 feet wide, each of which extends 117 feet from the triangular shear wall assembly in the center of the building. The floors are 8 inch lightweight concrete flat plates. The columns are normal weight concrete. Weather permitting, the building was cast at the rate of one floor every three working days.

Figure 2 shows a model of the 713 foot high 52 story - One Shell Plaza in Houston. This is an office building built entirely of lightweight concrete. The building is a "tube-in-tube" structure with a shear wall core and exterior columns and spandrel beams resisting lateral loads. The floors consist of one way concrete joists spanning from the outside walls to the core with two way joists in the corners.

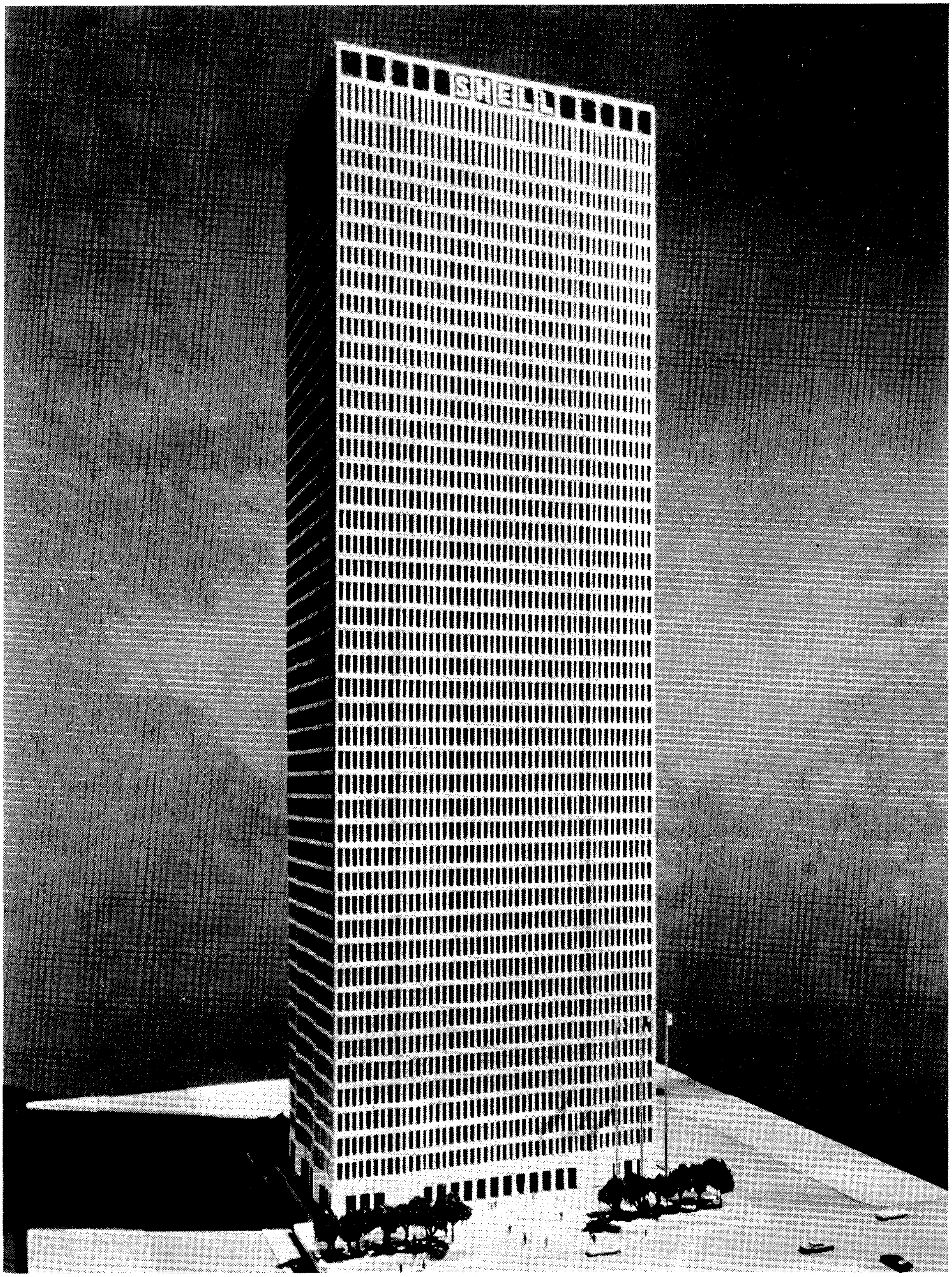
This paper presents a review of the various structural systems currently employed in the construction of tall concrete buildings. The choice of building type, the problems in the design of such buildings and the reinforcement required for seismic conditions are discussed in this paper.

Reference:

ACI Committee 442, "Response of Buildings to Lateral Forces", ACI Journal, February 1971, pp. 81-107.



Lecture 4 Figure 1 Lake Point Tower, Chicago



Lecture 4 Figure 2 Model of One Shell Plaza, Houston

Reinforced Concrete Buildings

LECTURE 5

DESIGN FOR TORSION

by

J. Warwaruk

TORSION

by
J. Warwaruk
Professor of Civil Engineering
The University of Alberta

INTRODUCTION

Torsion was generally not taken into account in the design of reinforced concrete structures in the past as it was assumed that the torsional stresses were minor and could be taken care of by large factors of safety used in design. More recently as design procedures have been refined and factors of safety reduced it becomes necessary to take torsion explicitly into design. This is particularly important for those cases where torsion moments may be of the same order of magnitude as flexural moments as for example, in beams curved in plan, beams framing into other beams, cantilever slabs, spiral staircases, etc.

This paper presents the basis for the current torsion design procedures for reinforced concrete as presented in ACI 318-71 and the NBC/CSA building codes. These design requirements are based on the work of ACI Committee 438 - Torsion and over the last half decade have gone through several modifications to reach the form in which they exist today. Although these requirements are empirical in nature, they represent a reasonably conservation design procedure for members subjected to torsion combined with other forces and moments.

BROAD PHILOSOPHY OF DESIGN

The approach to design of a reinforced concrete member subjected to torsion alone is analogous to that for design in flexural shear. Figure 1 illustrates that the resistance of a member subjected either to flexural shear or to torsion is made up of two parts:

(i) the contribution of the strength of the concrete compression zone;

(ii) the contribution of the web reinforcement.

It is difficult to assess precisely the contribution of the compression zone because of the very complex stress situation existing there but results of tests are useful for establishing such contribution. For flexural shear it has been shown (1) that the contribution of the concrete compression zone to shear strength is approximately equal to the shear which causes diagonal tension to occur. For torsion alone the contribution of the concrete compression zone to the ultimate torsional strength may be taken as about one-half the torsional moment which causes diagonal tension cracking to occur (2).

The action of web reinforcement in resisting torsion alone is similar to its action in resisting flexural shear. In both cases, as shown in Figure 2 the web reinforcement may be considered to act as part of a truss in combination with inclined compression struts of concrete which are formed by diagonal tension cracking, and with longitudinal reinforcement.

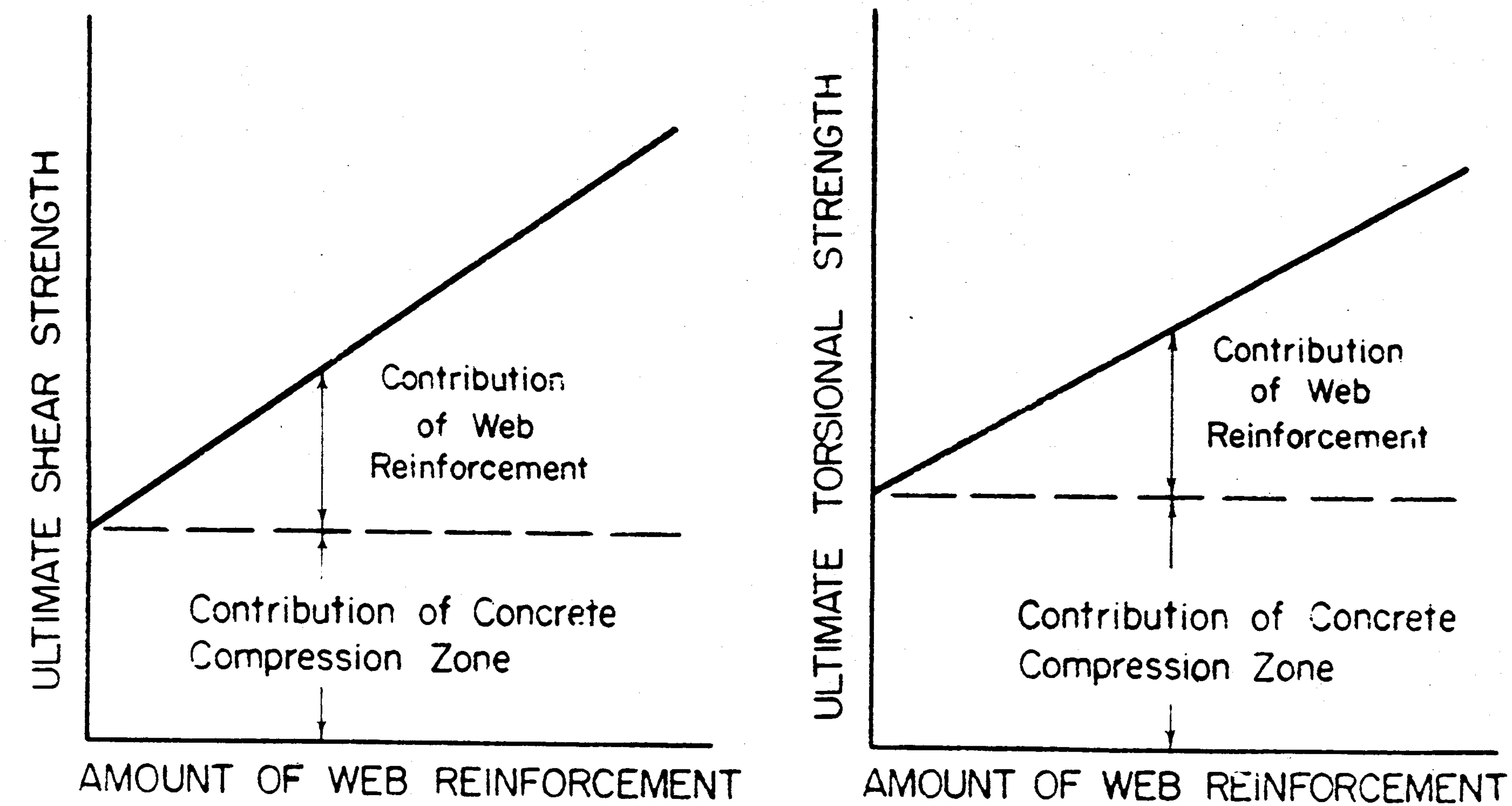


FIG. 1: ULTIMATE STRENGTH OF REINFORCED CONCRETE BEAMS IN SHEAR AND IN TORSION.

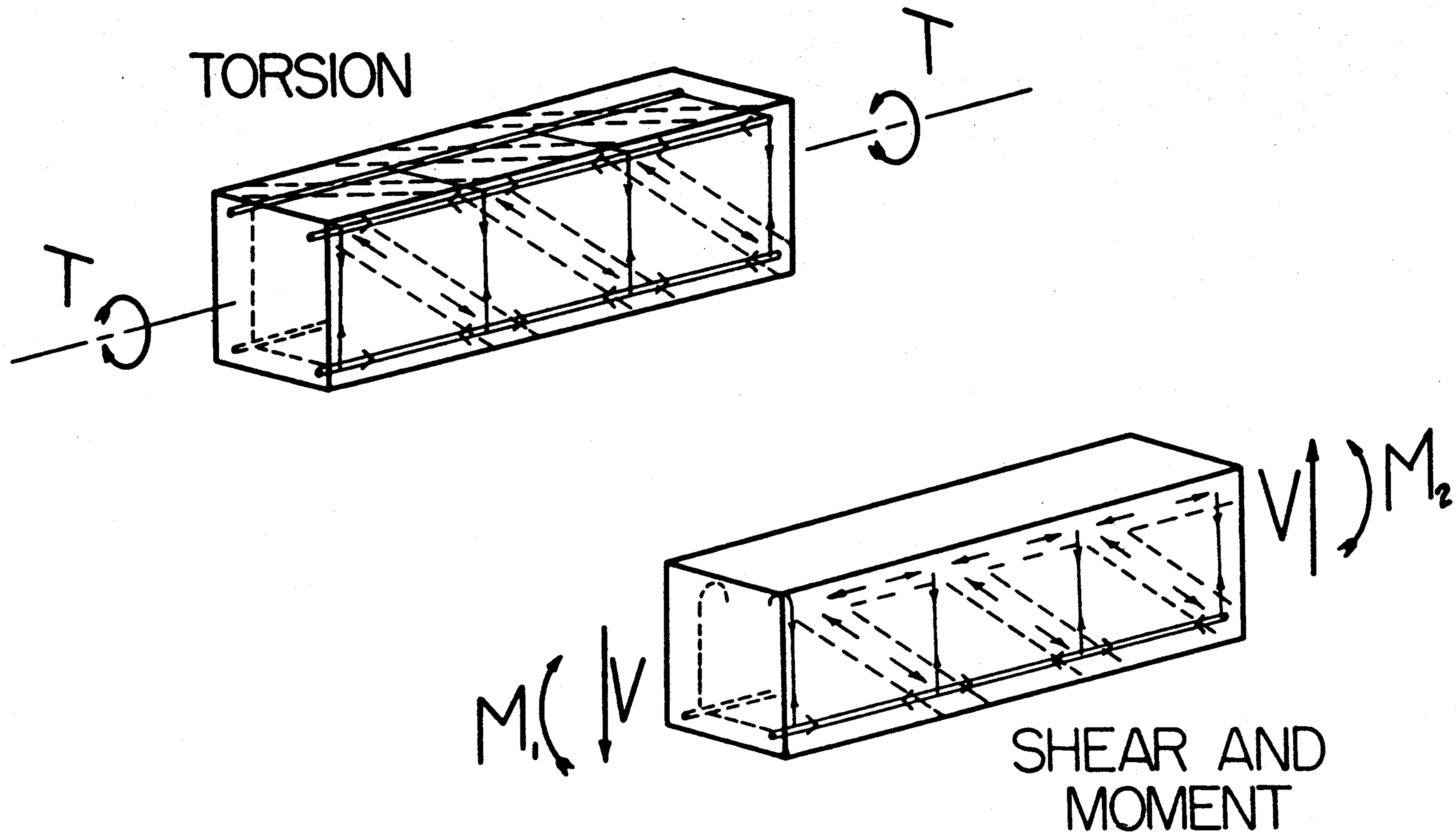


FIG. 2: THE TRUSS ANALOGY AS APPLIED TO TORSION AND TO SHEAR

Torsion causes principal tension stresses on all faces of a beam and web reinforcement therefore must be provided in each face. For this closed loop stirrups must be used; U shaped stirrups commonly used for flexural shear reinforcement are not suitable for torsion.

To this point the discussion has been restricted to design philosophy for torsion alone or flexural shear alone. In practice members must usually be designed for combinations of torsion, shear and bending moment. When both flexural shear and torsion are present in a member they interact with one another and the shear and the torsion which a member can resist simultaneously is less than the flexural shear or the torsion which the member can resist when either shear or torsion acts alone. This is illustrated by results of tests (2) summarized in Figure 3 for different cross-sections of beams having no web reinforcement, and in the case of the rectangular beams having M/Vd values ranging from 3 to 6. The value of M/Vd does not significantly affect the shape of the interaction curve although it does affect the value of V_o . A circular arc interaction curve was therefore used to calculate the contribution of concrete to strength in flexural shear and torsion when shear and torsion act simultaneously in beams with no web reinforcement.

The interaction of torsion with flexural shear in beams having web reinforcement is not well understood. The design procedure adopted for beam thus loaded is as follows. For the part

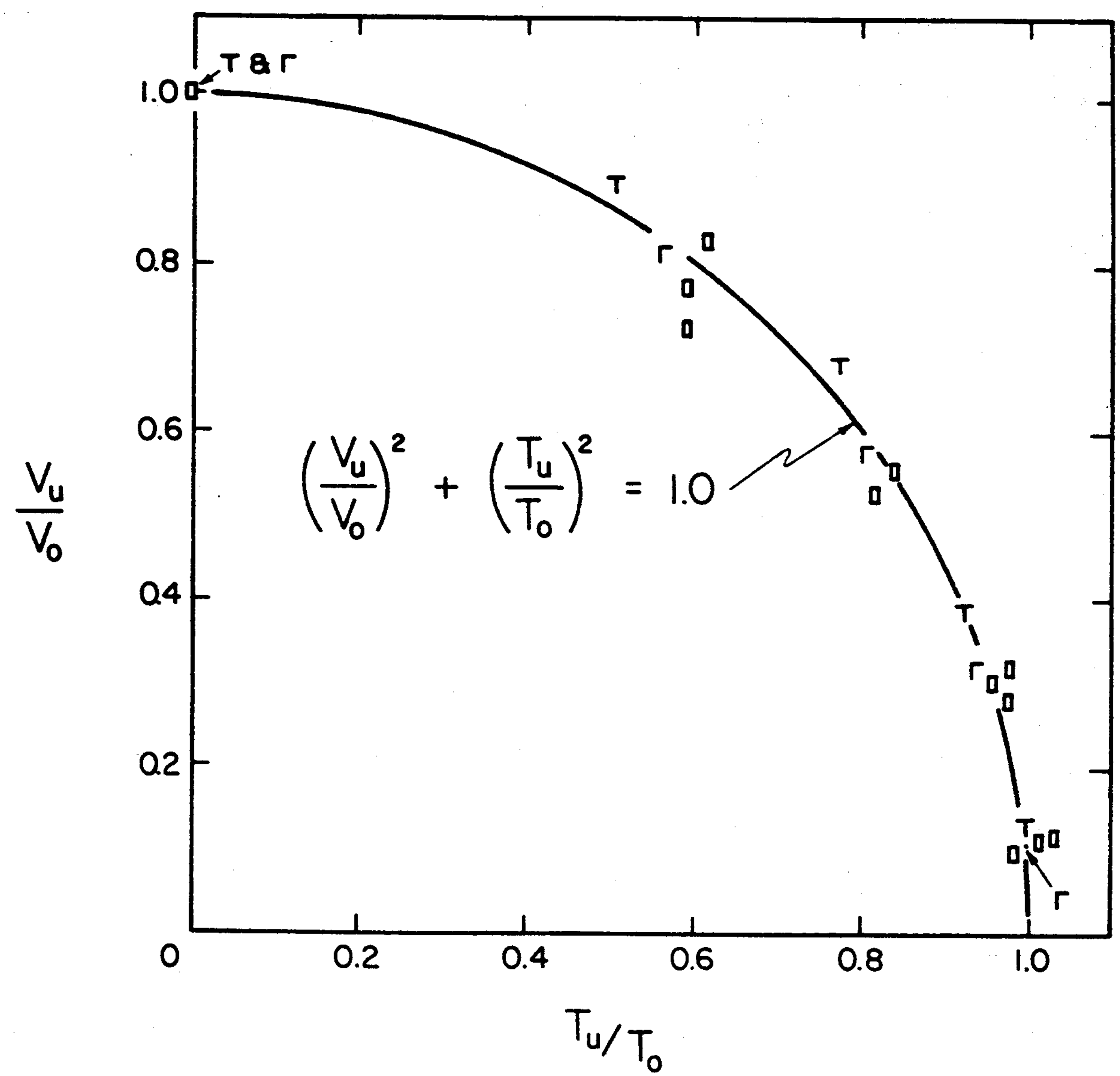


FIG. 3: INTERACTION OF TORSION AND SHEAR

of the strength contributed by the concrete a circular interaction curve is used, as discussed in the preceding paragraph. For the other part of the torsional and flexural shear strengths contributed by the reinforcement the interaction curve between torsion and shear lies close to a straight line. Hence a straight line interaction curve was adopted. This is the same as saying that the reinforcement for torsion and flexural shear should be added.

In summary, the procedure for design for torsion and other forces acting on a reinforced concrete section is that listed below.

(i) Compute the nominal shear stresses due to torsion and flexural shear.

(ii) Determine the contribution of the concrete to the resistance of nominal shear stresses.

(iii) Provide web and longitudinal reinforcement to take the difference between (i) and (ii) above.

BACKGROUND OF DESIGN RELATIONSHIPS

The following sections present relationship for the design of reinforced concrete cross-sections when subjected to torsion and other forces.

(a) ACI 318-71: Equation 11-16

The strength of beams with no web reinforcement subjected to pure torsion can be expressed as

$$M_{tu} = \alpha x^2 y f_t \tag{1}$$

for both elastic and plastic theories. Based on elastic theory α is the St. Venant's coefficient and varies from 1/5 to 1/3. Based on the plastic theory α varies from 1/3 to 1/2. Recently a new theory based on an ultimate strength approach (3) has been formulated; it uses a constant value for $\alpha = 1/3$. The term f_t is the diagonal tension stress and is numerically equal to the shear stress due to torsion, τ_u. Thus for a rectangular beam at ultimate strength

$$M_{tu} = 1/3 \; x^2 y \tau_u$$

and rearranging

$$\tau_u = \frac{3 M_{tu}}{x^2 y} \tag{2}$$

If the member has flanges they contribute to the strength in torsion. Details of such flange contributions are presented in Appendix 1. Thus, taking the contribution of the flanges and incorporating the capacity reduction factor ϕ , Equation 2 becomes

$$\tau_u = \frac{3\ M_{tu}}{\phi\ \Sigma\ x^2\ y} \qquad (11\text{-}16)$$

Limit on τ : For pure torsion the shear stress due to torsion τ_c shall not exceed $2.4\sqrt{f_c'}$. This is based on the contribution of concrete to the ultimate strength of a beam with web reinforcement (4). This torsional shear stress corresponds to a torque about 40 percent of the cracking torque of a beam without web reinforcement (5), and is conservative for predicting torsional cracking.

When the nominal ultimate shear stress due to torsion only is less than $1.5\sqrt{f_c'}$ torsion effects are neglected. This stress corresponds to about 25 percent of the pure torsional strength of a member without web reinforcement and torsional stresses of such magnitude will not cause a significant reduction in ultimate strength either in flexure or shear.

(b) ACI 318-71: Equation 11-3

$$v_u = \frac{V_u}{\phi\ b'\ d} \qquad (11\text{-}3)$$

This has been discussed in shear design.

Limit on v : The shear stress carried by the concrete shall not exceed $2\sqrt{f_c'}$.

(c) ACI 318-71: Equation 11-17

In the case of combined torsion, flexural shear and flexure the interaction of torsion and shear is taken into account explicitly (2) using a circular interaction curve between torsion and shear. The detailed derivation of this relationship is given in Appendix 2.

Thus the contribution to the total capacity of a beam by concrete resisting torsion when subjected to combined loading is

$$\tau_c = \frac{2.4\sqrt{f_c'}}{\sqrt{1 + (1.2\ v_u/\tau_u)^2}} \qquad (11\text{-}17)$$

Limit on τ : The maximum value of the torsion stress that a member may carry is restricted to

$$\tau_u = \frac{12\sqrt{f_c'}}{\sqrt{1 + (1.2\ v_u/\tau_u)^2}} \qquad (11\text{-}18)$$

and is based on Equation 11-17 but limiting the numerator to $12\sqrt{f_c'}$ which is the maximum torsional stress observed in tests of beams subjected to pure torsion in which the reinforcement still yielded before concrete crushed.

(d) ACI 318-71: Equation 11-9

Equation 11-9 can be derived in a manner similar to that used in deriving Equation 11-17. It represents the nominal permissible shear stress carried by the concrete for combined torsion, flexural shear and flexural loading.

$$v_c = \frac{2\sqrt{f_c'}}{\sqrt{1 + (\tau_u/1.2\ v_u)^2}} \qquad (11\text{-}9)$$

(e) ACI 318-71: Equation 11-19

When the ultimate torsional shear stress τ_u , exceeds the permissible torsional shear stress, τ_c , the excessive shear stress $(\tau_u - \tau_c)$ must be resisted by torsional reinforcement (5).

For beams containing longitudinal and transverse web reinforcement and subjected to torsion only it is shown (6) that the ultimate torque can be calculated by

$$T_u = T_o + \Omega\ x_1\ y_1\ \frac{A_t\ f_y}{s} \qquad (3)$$

Where x_1 and y_1 are the smaller and larger legs of a closed stirrup A_t is the cross-sectional area of one leg is a closed stirrup, f_y is the yield strength of the reinforcement, and s is the stirrup spacing. The parameter $x_1\ y_1\ A_t\ f_y/s$ is referred to as the reinforcement factor. Figure 4 presents typical results of tests on beams at PCA (6,7). T_o is the vertical intercept, Ω is the slope of the straight line. The value of Ω was found to be mainly a function of m , the ratio of the volume of longitudinal reinforcement to volume of stirrups, and y_1/x_1 . These test data indicate that Ω is equal to $(0.66m + 0.33\ y_1/x_1)$. The following equation was presented (7) as representing the strength in torsion of beams reinforced with longitudinal steel and stirrups. Committee 438 (5) assumed for

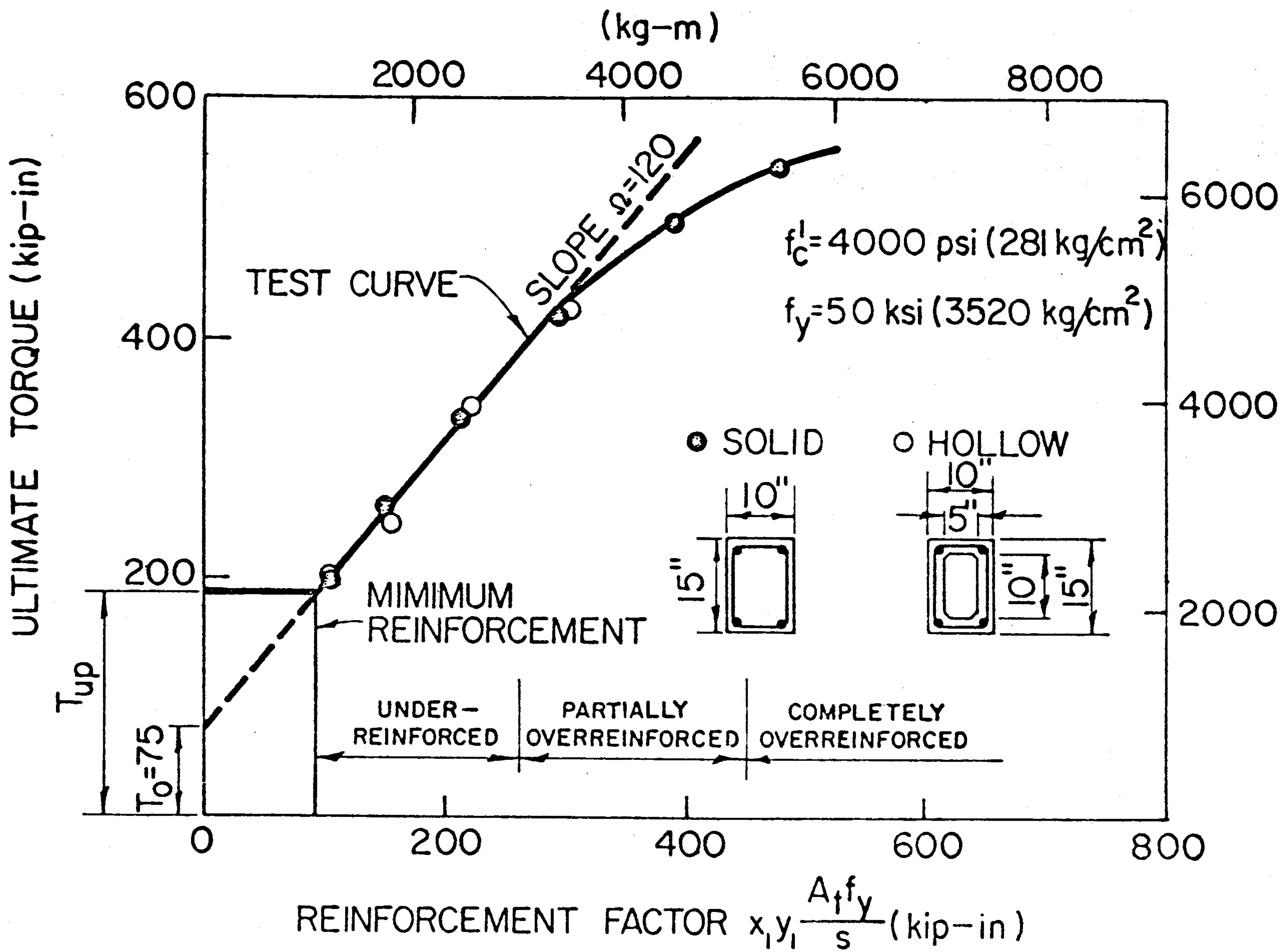


FIG. 4: ULTIMATE TORQUE VERSUS REINFORCEMENT FACTOR FOR BEAM TESTS

practical purposes that the amount of longitudinal steel would be equal to the amount of stirrups and hence $m = 1$.

$$T_u = \frac{x^2 y}{3} (2.4\sqrt{f_c'}) + (0.66 + 0.33 \frac{y_1}{x_1}) x_1 y_1 \frac{A_t f_y}{s} \quad (4)$$

The first term of Equation 4 is the strength of concrete beams without web reinforcement where $2.4\sqrt{f_c'}$ was the recommended maximum shear stress due to torsion only carried by an unreinforced concrete cross-section.

Modifying Equation 4 and expressing it in stress terms and substituting A_o for A_t

$$\frac{3 T_u}{x^2 y} = 2.4\sqrt{f_c'} + (0.66 + 0.33 \frac{y_1}{x_1}) \frac{x_1 y_1}{x^2 y} \frac{A_o f_y .3}{3}$$

letting: $T_u = M_{tu}$, substituting $\Sigma x^2 y$ for $x^2 y$, and solving for A_o yields

$$A_o = \frac{(\tau_u - \tau_o) s \Sigma x^2 y}{3 \Omega x_1 y_1 f_y} \quad (11\text{-}19)$$

where $\Omega = (0.66 + 0.33 y_1/x_1)$

with a maximum value for Ω of 1.5 (5).

(f) ACI 318-71: Equation 11-20

Torsional reinforcement must consist of longitudinal and web reinforcement because both types of reinforcement are needed to resist diagonal tension stresses. The ultimate torsional strength cannot be increased if either one of the two types of reinforcement is absent. Equation 11-20 which forms part of the requirement for longitudinal bars is derived from the assumption that the volume percentage of longitudinal reinforcement is equal to the volume percentage of stirrups (4).

$$A_\ell = 2A_o \left[\frac{x_1 + y_1}{s}\right] \tag{11-20}$$

(g) ACI 318-71: Equation 11-21

This equation forms the other part of the requirement for longitudinal bars and it was first presented in (8) with its objective being to increase longitudinal torsional reinforcement for small amounts of torsion stirrups.

$$A_\ell = \left[\frac{400 \times s}{f_y} \left(\frac{\tau_u}{\tau_u + v_u}\right) - 2\,A_o\right] \left[\frac{x_1 + y_1}{s}\right] \tag{11-21}$$

(h) Other Reinforcement Details

The reinforcement required to resist torsion is simply added to that required to resist flexural shear and bending (5).

The spacing of closed stirrups shall not exceed

$$\frac{(x_1 + y_1)}{4} \text{ or 12 inches,}$$

which ever is smaller. This requirement is based on test results (8,9).

The minimum area of closed stirrups provided shall be

$$A_v + 2A_o = 50\ b's/f_y$$

This is a recommendation by Committee 438 (9) and represents minimum amount of web reinforcement for combined torsion and shear be made the same as that for shear without torsion and that the accompanying longitudinal reinforcement be increased so that the total volume of minimum torsion reinforcement remains the same as in the original Committee 438 report (5). This requirement complements Equation 11-21.

Distribution of longitudinal reinforcement around the perimeter of the stirrups involves placing at least one longitudinal bar in each corner and spaced no more than 12 inches apart. Longitudinal bars must be #3 or larger.

The provision requiring torsional reinforcement to be provided at least (d + b) beyond the point theoretically required is to account for and intercept torsional diagonal tension cracks which normally develop in a spiral form.

DESIGN EXAMPLE

This example involves the design of the beam shown in elevation in Fig. 5 . The beam carries the loading shown in section on Fig. 5 . Since the analysis of the structure is not the subject of this example, only a summary of the design moment, shear and torque are presented in Fig. 6 . The material strengths used in this design are f'_c = 4000 psi and f_y = 60,000 psi.

The flexural reinforcement requirement at the column face, using a singly reinforced rectangular section is 4.64 sq.in. At midspan, using an L section, 2.88 sq.in are required. These are indicated in Fig. 7 .

The design which follows is mainly that for the section at a distance d from the column face. Only a summary of the design at other sections is presented.

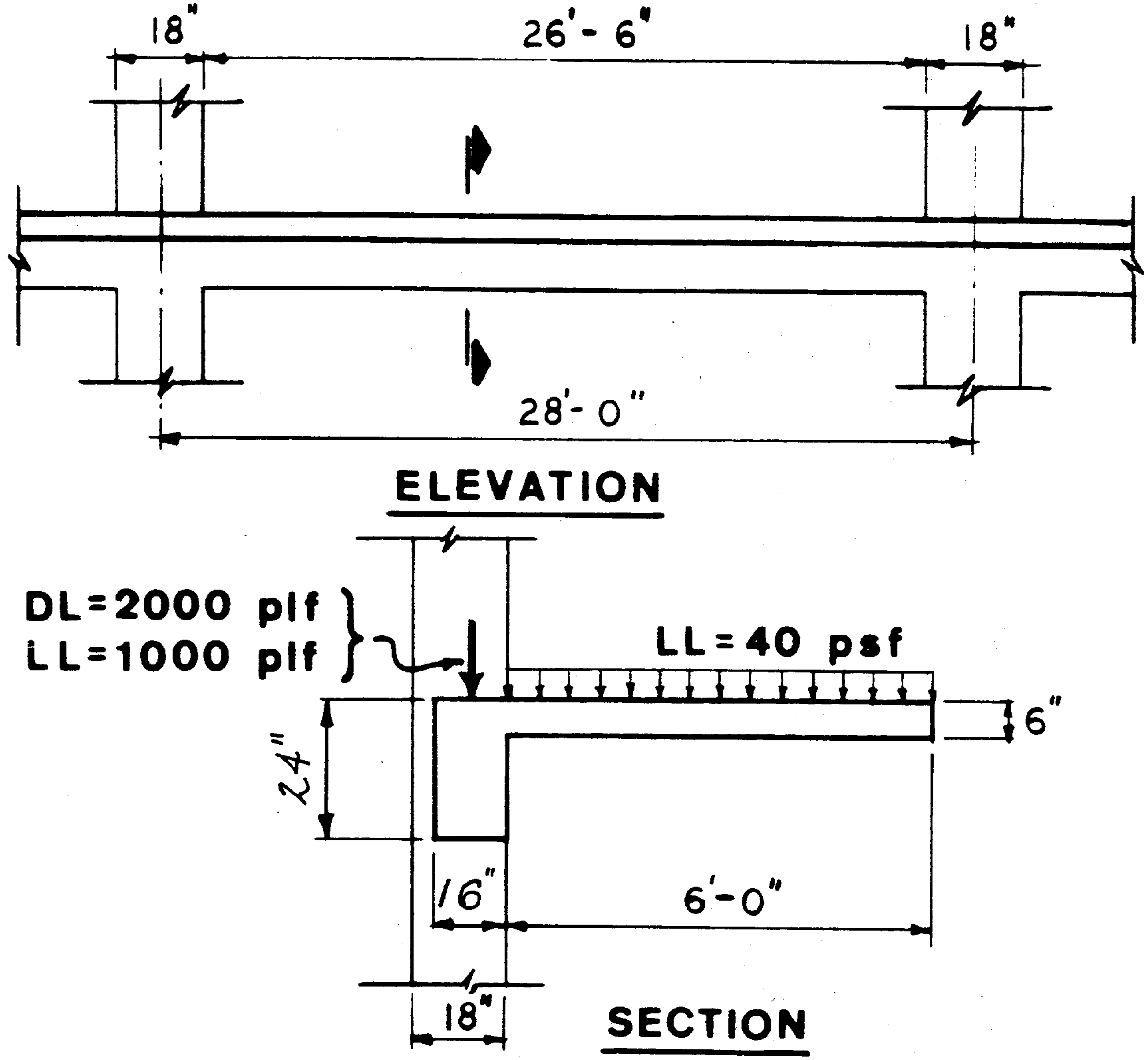


FIG. 5: SAMPLE PROBLEM

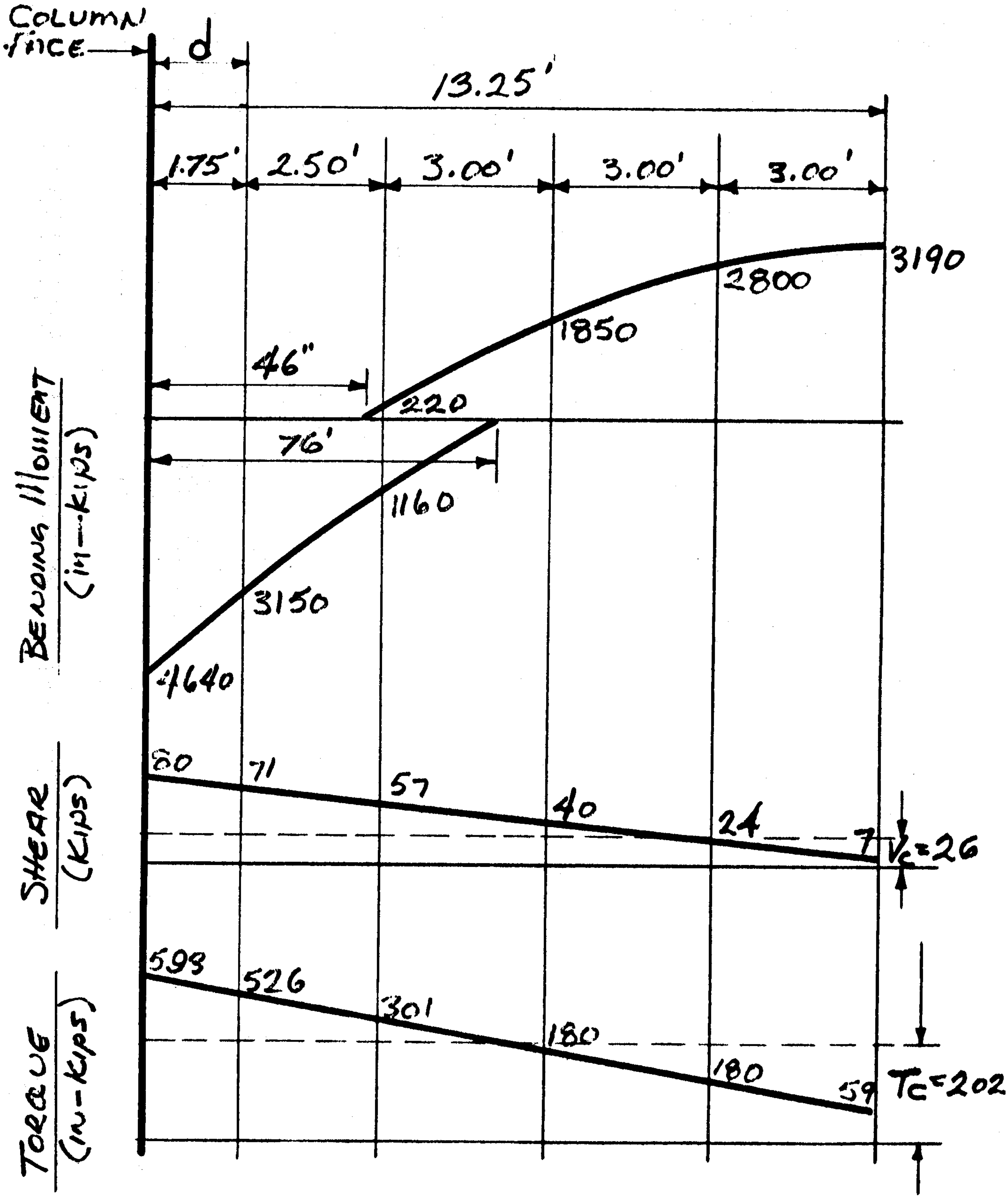
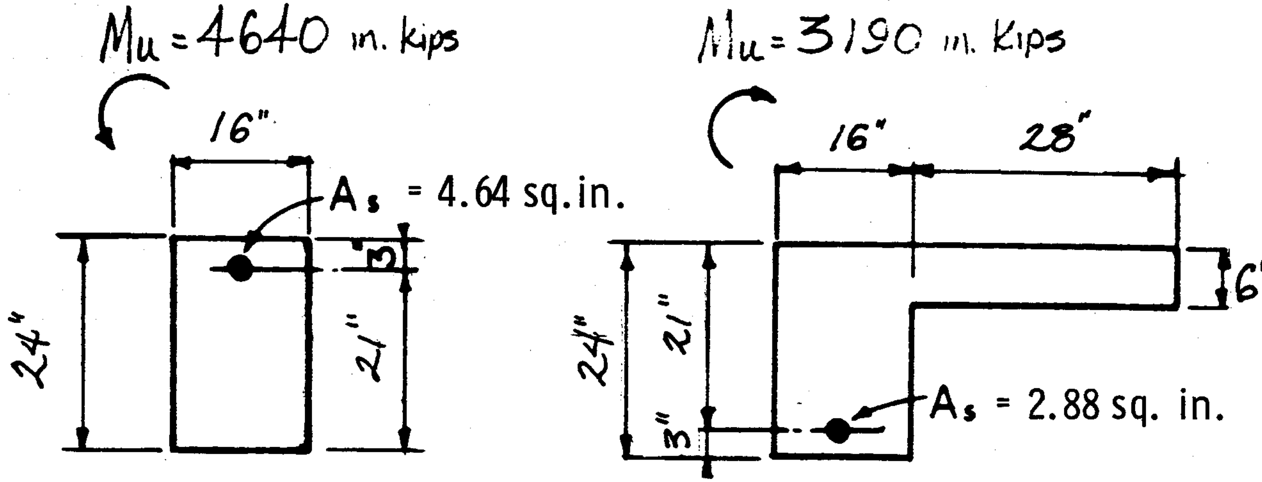


FIG. 6: BENDING MOMENT, SHEAR AND TORQUE ENVELOPES



(a) AT COL. FACE **(b) AT MIDSPAN OF BM.**

FIG. 7: CROSS-SECTIONS FOR FLEXURAL DESIGN

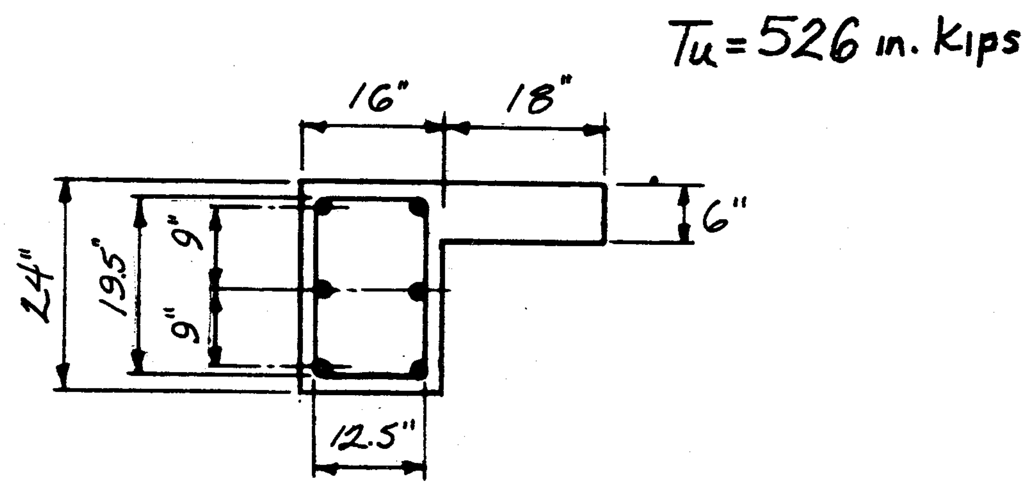


FIG. 8: CROSS-SECTION FOR TORSION DESIGN

Flexural and Torsional Shear Stresses

Effective overhanging width = 3 (Thickness of Slab) = 18 in.

From Fig. 8:

$$\Sigma x^2 y = 16^2 \times 24 + 6^2 \times 18 = 6792 \text{ in}^3$$

At distance d from column face, V_u = 71 kips T_u = 526 in-kips

Eq. (11-3) $$v_u = \frac{V_u}{\phi bd} = \frac{7100}{0.85 \times 16 \times 21} = 249 \text{ psi}$$

Eq. (11-16) $$\tau_u = \frac{3T_u}{\phi \Sigma x^2 y} = \frac{3 \cdot 526000}{0.85 \cdot 6792} = 273 \text{ psi}$$

Sect. 11.7.1 $$\tau_{u,\,min} = 1.5 \sqrt{f'_c} = 1.5 \sqrt{4000} = 95 \text{ psi} < 273 \text{ psi}$$

Eq. (11-18) $$\tau_{u,max} = \frac{12\sqrt{f'_c}}{\sqrt{1 + (1.2v_u/\tau_u)^2}} = \frac{12\sqrt{4000}}{\sqrt{1 + (1.2 \times 249/273)^2}} = 524>273$$

Eq. (11-9) $$v_c = \frac{2\sqrt{f'_c}}{\sqrt{1 + (\tau_u/1.2v_u)^2}} = \frac{2\sqrt{4000}}{\sqrt{1 + (273/1.2 \times 249)^2}} = 91 \text{ psi}$$

Eq. (11-17) $$\tau_c = \frac{2.4\sqrt{f'_c}}{\sqrt{1 + (1.2v_u/\tau_u)^2}} = \frac{2.4\sqrt{4000}}{\sqrt{1 + (1.2 \cdot 249/273)^2}} = 105 \text{ psi}$$

Flexural Shear and Torque Carried by Concrete

$$V_c = \phi v_c \, b \, d = 0.85 \cdot 91 \times 16 \cdot 21 = 26 \text{ kips}$$

$$T_c = \phi \tau_c \frac{\Sigma x^2 y}{3} = 0.85 \times 105 \frac{6792}{3} = 202 \text{ in kips}$$

(These values are entered in Fig. 6)

Shear Web Reinforcement

Eq. (11-13) $\frac{A_v}{s} = \frac{(v_u - v_c)b}{f_y} = \frac{(249 - 91)16}{60000} = 0.0421 \text{ in}^2/\text{in}$

Eq. (11-1) $(\frac{A_v}{s})_{min} = \frac{50b'}{f_y} = \frac{50 \cdot 16}{60000} = 0.0133 \text{ in}^2/\text{in} < 0.0421 \text{ in}^2/\text{in}$

Sect. 11.1.4 $s_{max} = \frac{d}{2} = \frac{21}{2} = 10.5 \text{ in}$

Torsional Web Reinforcement

Concrete cover is 2.0 in. for bottom and top faces and 1.5 for side faces. Assume No. 4 bars for closed stirrups.

$$x_1 = 16 - 2\ (1.75) = 12.5 \text{ in}$$

$$y_1 = 24 - 2\ (2.25) = 19.5 \text{ in}$$

Sect. 11.8.2 $\Omega = 0.66 + 0.33\ \frac{y_1}{x_1} = 0.66 + 0.33\ \frac{19.5}{12.5} = 1.175 < 1.5$

At distance d from column face

Eq. (11-19) $\frac{A_o}{s} = \frac{(\tau_u - \tau_c)\Sigma x^2 y}{3\Omega\ x_1 y_1 f_y} = \frac{(273-105)6792}{3x1.175x12.5x19.5x60000} = 0.0221 \text{ in}^2/\text{in}$

Sect. 11.8.3 $s_{max} = \frac{x_1 + y_1}{4} = \frac{12.5 + 19.5}{4} = 8 \text{ in} \leftarrow$ governs

$$s_{max} = 12 \text{ in}$$

Total Web Reinforcement

At distance d from column face

$$\frac{A_o}{s} + \frac{1}{2}\frac{A_v}{s} = 0.0221 + \frac{1}{2}\,0.0421 = 0.0432 \text{ in}^2/\text{in}$$

$$s = \frac{0.20}{0.0432} = 4.6 \text{ in} \quad \text{use } 4.0 \text{ in} \qquad \text{for No. 4 bars}$$

Torsional Longitudinal Reinforcement

At distance d from column face

Eq. (11-20) $\quad A_\ell = 2\frac{A_o}{s}(x_1 + y_1) = 2 \times (0.02221)(12.5 + 19.5) = 1.41 \text{ in}^2 \leftarrow \text{governs}$

Eq. (11-21) $\quad A_\ell = \left[\frac{400x}{f_y}\left(\frac{\tau_u}{\tau_u + v_u}\right) - 2\frac{A_o}{s}\right](x_1 + y_1)$

$$= \left[\frac{400 \cdot 14}{60000}\left(\frac{273}{273 + 249}\right) - 2\,(0.0221)\right](12.5 + 19.5) = 0.37 \text{ in}^2$$

$$s_{max} = 12 \text{ in}$$

Longitudinal bars are needed at mid-height to satisfy the spacing requirement.

$$s' = \frac{1}{2}(24 - 3.0 - 3.0) = 9.0 < 12.0$$

Assume equal area of steel at the three levels

$$\frac{A_\ell}{3} = \frac{1.41}{3} = 0.47 \text{ in}^2$$

Use 2 No. 5 bars at the middle level.

$$\frac{A_\ell}{3} = 2\,(0.31) = 0.62 \text{ in}^2 > 0.47 \text{ in}^2$$

Then the steel area for the top and bottom levels are

$$A_{\ell,\,top} = A_{\ell,\,bottom} = \frac{1}{2}(1.41 - 0.62) = 0.40 \text{ in}^2$$

Total Longitudinal Reinforcement

Torsional longitudinal bars at column face should be identical to those at a distance and from column face.

Top Bars at Column Face

$A_s + A_\ell$, top = 4.64 + 0.40 = 5.04 in^2

Use 5 No. 9 bars $A_s + A_\ell$, top = 5(1.00) = 5.00 ≈ 5.04

Bottom Bars at Column Face

$A_s' + A_\ell$, bottom = 0 + 0.40 = 0.40 in^2

Sect. 12.2.1 $\frac{1}{4}$ A_s (at mid-span) = $\frac{1}{4}$ (2.88) = 0.72 in^2 ← governs

Summary of Reinforcement

See Table 1.

Table 1. Summary of Reinforcement

Items	Units	Column Face	Distance d from column face	Distance from Center of Span - ft. 9	6	3	0
Web Reinforcement							
V_u-V_c	kips	---	45	31	14	0	0
T_u-T_c	in.-kips	---	320	219	99	0	0
$A_v/2s$	in.^{2}/in.	---	0.0211	0.0145	0.0065	0	0
A_o/s	in.^{2}/in	---	0.0221	0.0151	0.0068	0	0
$(A_v/2s)+(A_o/s)$	in.^{2}/in.	---	0.0432	0.0296	0.0133	0	0
s	in.	---	4.6 use 4 in.	6.7 use 6 in.	15.0 use 8 in.*	∞ use 8 in.*	∞ use 8 in.*
Top Longitudinal Bars							
$-M_u$	in.-kips	4640	3150	1160	0	0	0
T_u-T_c	in.-kips	---	320	219	99	0	0
A_s	in.^{2}	4.64	3.15	1.16	0	0	0
A_ℓ,top	in.^{2}	0.40	0.40	0.27	0.12	0	0
A_s+A_ℓ,top	in.^{2}	5.04	3.45	1.43	0.12	0	0
Bars		5 No. 9					
Bottom Longitudinal Bars							
M_u	in.-kips	0	0	220	1850	2750	3190
T_u-T_c	in.-kips	---	320	219	99	0	0
A_s	in.^{2}	0	0	0.20	1.67	2.48	2.88
A_ℓ,bottom	in.^{2}	0.40	0.40	0.27	0.12	0	0
A_s+A_ℓ,bottom	in.^{2}	0.40	0.40	0.47	1.79	2.48	2.88
Bars		2 No. 7**					5 No. 7

* Governed by Sect. 11.1.3

** Governed by Sect. 12.2.1

Check Minimum Web Width

Use "Ultimate Strength Design Hanbook", Vol. I, ACI SP-17, P.93.

Top bars at Column Face (5 No. 9 Bars)

b_{min} = 14 in. for 5 No. 9 bars

14 < 16

Bottom Bars at Center of Beam (5 No. 7 Bars)

b_{min} = 12.5 < 16

Cut-off Points for Top Bars

Extension beyond the point at which it is no longer required to resist flexure (Section 12.1.4)

d = 21 in ← governs

12 (Bar Diameter) = 12 x 1.13 = 13.5 in

First Cut-off Length for 1 No. 9 Bar

Steel area requirement (Fig. 9):

$A_s = 4 \times 1.00 = 4.00 \text{ in}^2$

$$a = \frac{A_s f_y}{0.85 f'_c\, b} = \frac{4.00 \times 60000}{0.85 \times 4000 \times 16} = 4.41$$

$$M_u = 0.9 \times 4.0 \times 60000 \left(21 - \frac{4.41}{2}\right) = 4059$$

Entering Fig. 6 $\therefore$ $L_1 = 7"$

Development length required for top bars

Sect. 12.5.1 a, and b,

$$L_d = 1.4 \frac{0.04\, a_s f_y}{\sqrt{f_c}} = \frac{1.4 \times 0.04 \times 1.00 \times 60000}{\sqrt{4000}} = 53 \leftarrow \text{governs}$$

Section 12.1.4

$d + L_d = 21 + 53 = 74$ in

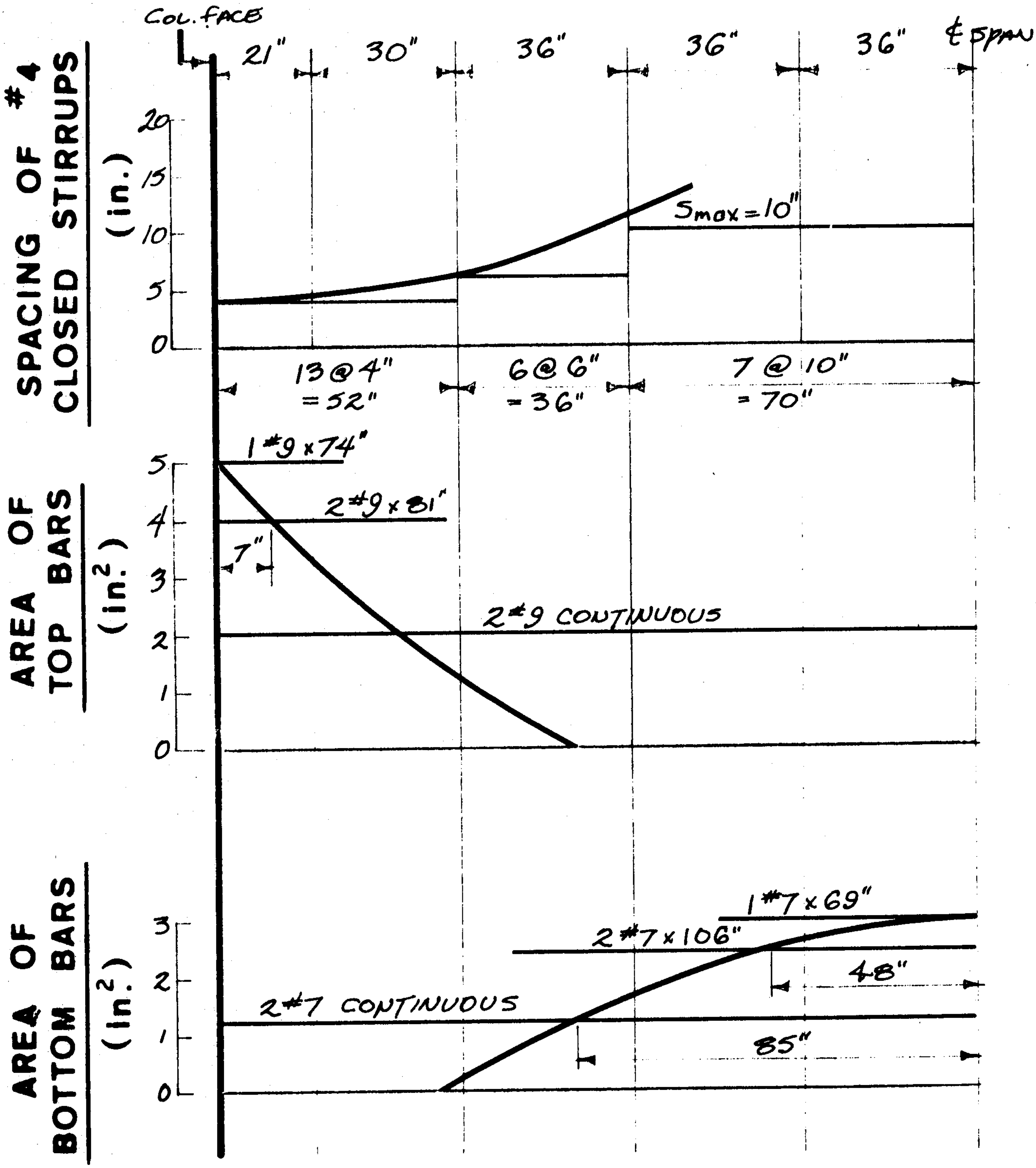


FIG. 9: REINFORCEMENT REQ'D AND PROVIDED

Second Cut-off Length for 2 No. 9 Bars

$$d + L_1 + L_d = 21 + 7 + 53 - 81 \text{ in}$$

The remaining 2 No. 9 bars will be carried throughout the beam to satisfy Sect. 11.8.5

Cut-off Points for Bottom Bars

Extension beyond moment diagram

Sect 12.1.4 $d = 21$ in ← governs

$12 \text{ (Bar Diameter)} = 12 \times 0.875 = 10.5 \text{ in}$

First Cut-off Length for 1 No. 7 Bar

Steel area requirement (from Fig. 9)

$$A_s = 4 \times 0.60 = 2.40$$

$$d = \frac{A_s f_y}{0.85 f'_c b} = \frac{2.40 \times 60000}{0.85 \times 4000 \times 16} = 2.647$$

$$M_u = 0.9 \times 2.40 \times 60000 \left(21 - \frac{2.647}{2}\right) = 2550 \text{ in kips}$$

Entering Fig. 6 $\therefore$ $L_1 = 4' = 48''$ ← governs

Development length requirement for bottom bars

Sect. 12.5.1 a, $L_d = \dfrac{0.04\, a_s f_y}{\sqrt{f'_c}} = \dfrac{0.04 \times 0.60 \times 60000}{\sqrt{4000}} = 23 \text{ in}$

Sect. 12.1.4 $d + L_1 = 21 + 48 = 69 \text{ in}$

Check Cut-off Bars in Tension Zone (59" from Center of Span)

Shear at cut of point = 34 kips

Sect. 12.1.6.1 $\frac{2}{3}$ of permitted shear $= \frac{2}{3}\left(V_c + \phi A_v f_y \frac{d}{s}\right)$

$$= \frac{2}{3}\left[26 + 0.85 \times 0.40 \left(60 \frac{21}{10}\right)\right] = 46 > 34$$

Second Cut-off Length for 2 No. 7 Bars

Steel area requirement (from Fig. 9)

$$A_s = 2 \times 0.60 = 1.20$$

$$a = \frac{1.20 \times 60000}{0.85 \times 4000 \times 16} = 1.324$$

$$M_u = 0.9 \times 1.20 \times 60000\ (21 - \frac{1.324}{2}) = 1318 \text{ in kips}$$

Entering Fig. 6 $\therefore$ $L_2 = 7.1' = 85"$

Sect. 12.1.4 $d + L_2 = 21 + 85 = 106$ in

$d + L_1 + L_d = 21 + 48 + 17 = 86"$ ← governs

Check 2 No. 7 Bars at Inflection Points

$$V_u = \frac{80-7}{159} \times 113 + 7.0 = 59 \text{ kips}$$

$$M_{uo} = A_s f_y (d - \frac{a}{2}) = 1.20 \times 60000\ (21 - \frac{1.324}{2}) = 1.464 \text{ in kips}$$

Sect. 12.2.3 $L_a = d = 21$ in

Sect. 12.2.3 $\frac{M_{uo}}{V_u} + L_a = \frac{1464}{59} + 21 = 46$ in

$L_d = 23 \text{ in} < 46 \text{ in}$

Reinforcement Details

See Fig.10.

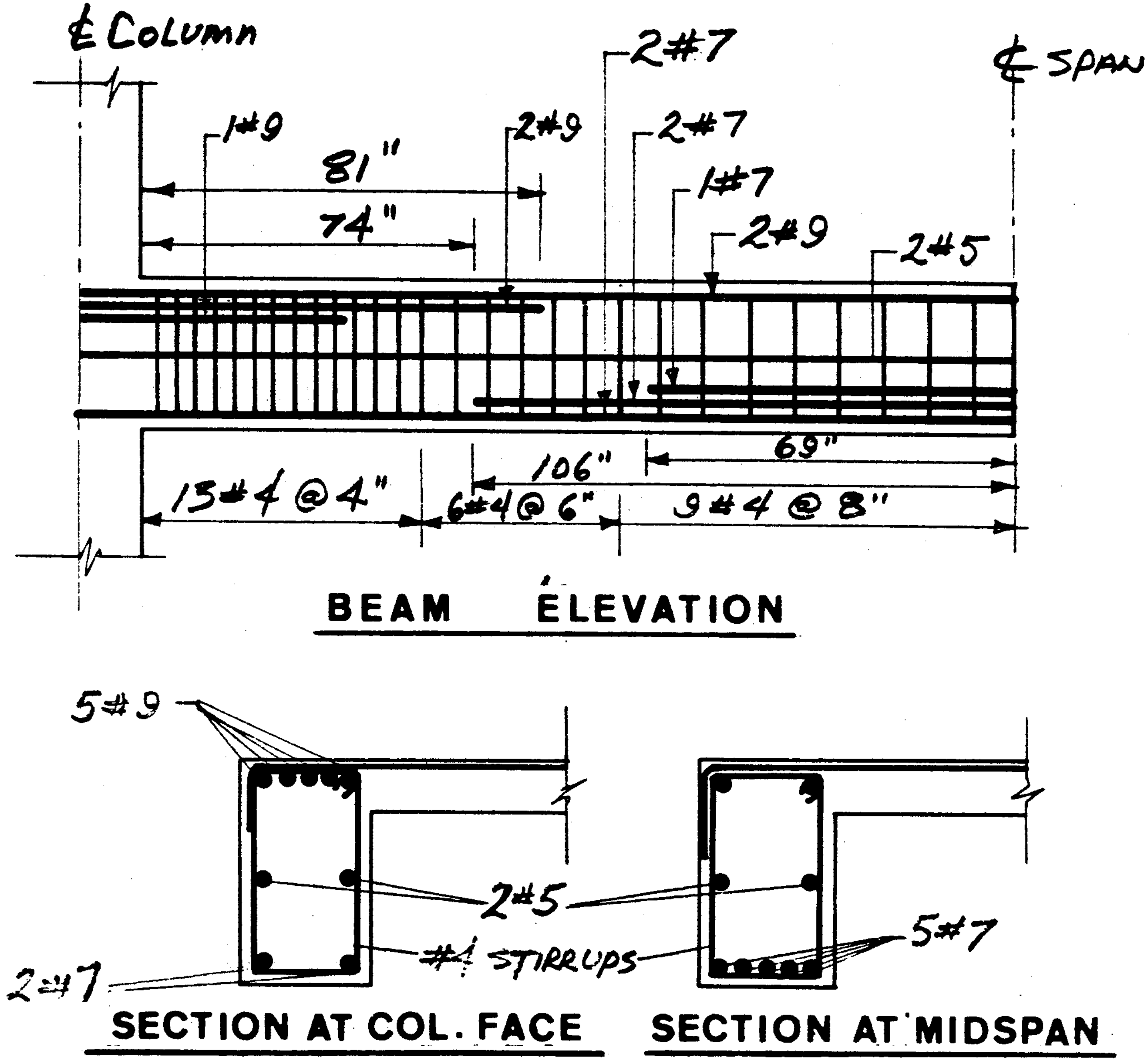


FIG. 10: REINFORCEMENT DETAILS

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APPENDIX I

Flanged Sections

In calculating the nominal total design torsion stress, τ_u, the denominator contains Σx^2y which: "shall be taken as the component rectangles of the section but the overhanging flange width used in design shall not exceed three times the flange thickness". The following paragraph presents more details regarding this calculation.

The calculation of the quantity Σx^2y for flanges sections depends on the selection of component rectangles. These rectangles should not overlap. In the normal case where the closed stirrups are installed in the stem, as shown in Fig. A-1-b, the quantity Σx^2y should be taken as the x^2y values of the web extending through the over-all depth of the section plus the x^2y values of the outstanding flanges. However, in the special case of cross sections such as Fig. A-1-c, it would be more advantageous to install the closed stirrups in the upper wider rectangular portion. In the latter case the Σx^2y value should be taken as the x^2y value of the upper wide component rectangle plus the narrow vertical outstanding stem. In the case of members without web reinforcement, the component rectangles should not overlap and may be taken so as to result in the highest possible Σx^2y.

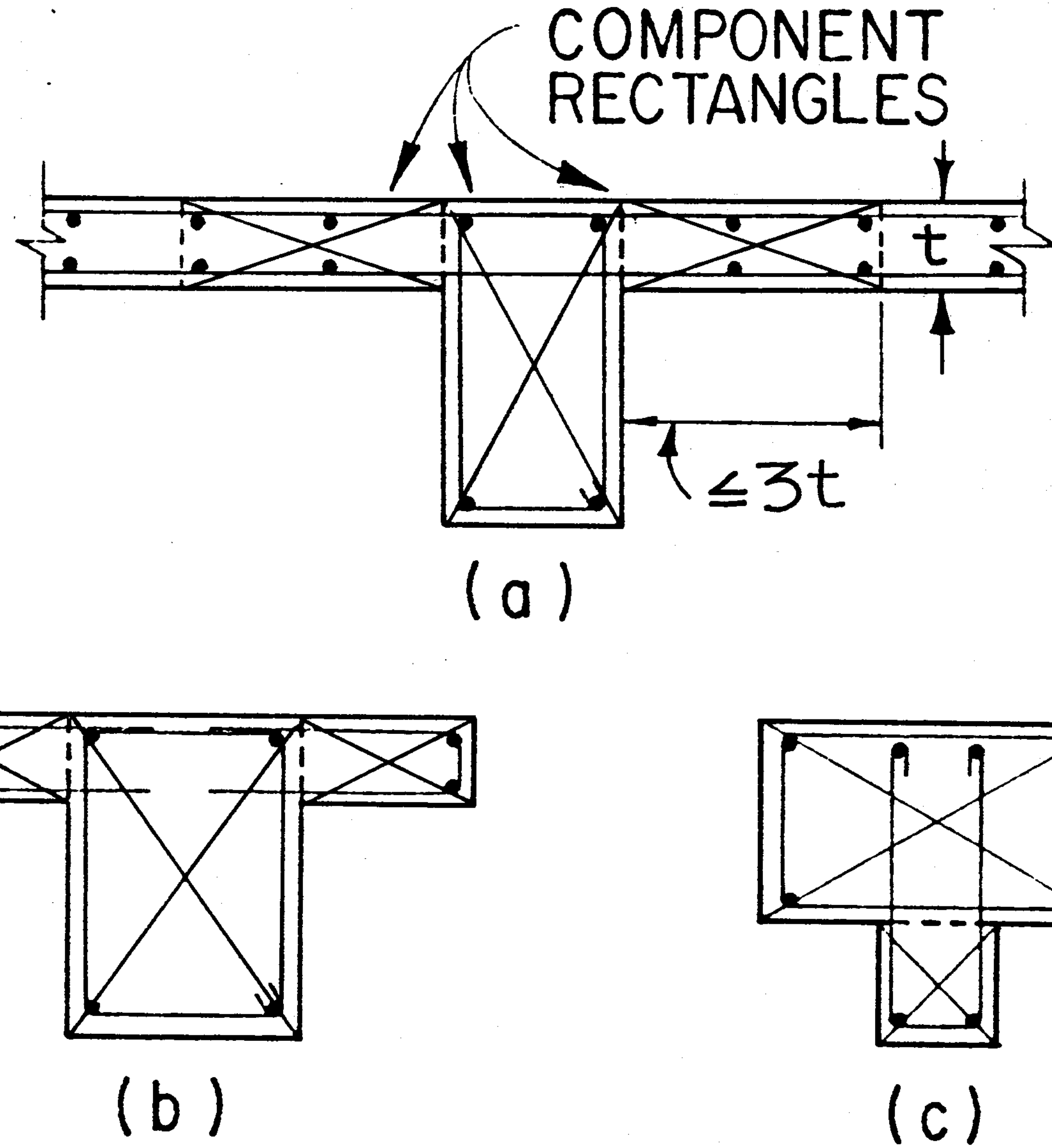

FIG. A-1: COMPONENT RECTANGLES FOR THE CALCULATION OF $\sum x^2y$

APPENDIX II

Derivation of Equations 11-17, 11-9, and 11-18.

The basic derivation of these equations involves the use of a circular interaction curve between flexural shear and torsion:

$$\left(\frac{V_u}{V_o}\right)^2 + \left(\frac{T_u}{T_o}\right)^2 = 1.0$$

This has been expressed (2) in terms of stresses

$$\left(\frac{v_{ca}}{v_c}\right)^2 + \left(\frac{\tau_{ca}}{\tau_c}\right)^2 = 1.0$$

Modifying $$v_{ca}^{\ 2}\left[1 + \left(\frac{\tau_{ca}}{v_{ca}}\right)^2 \cdot \left(\frac{v_c}{\tau_c}\right)^2\right] = v_c^{\ 2}$$

at a particular section: τ/v is constant

at diagonal cracking: $\tau/v = 2\ \tau_{ca}/v_{ca}$

at ultimate $\tau/v = \tau_u/v_u$

from above $\tau_{ca}/v_{ca} = \tau/2v = \tau_u/2v_u$

so that $$v_{ca}^2\left[1 + \left(\frac{\tau_u}{2v_u}\right)^2\left(\frac{v_c}{\tau_c}\right)^2\right] = v_c^{\ 2}$$

or $$v_{ca} = \frac{v_c}{\sqrt{1 + \left(\frac{\tau_u}{2v_u}\right)^2\left(\frac{v_c}{\tau_c}\right)^2}}$$

similarly $$\tau_{ca} = \frac{\tau_c}{\sqrt{1 + \left(\frac{\tau_c}{v_c}\right)^2\left(\frac{2v_u}{\tau_u}\right)^2}}$$

Using design consistent with ACI 318-63 - v_c and τ_c become $2\phi \sqrt{f'_c}$ and $2.4\phi \sqrt{f'_c}$ resp., and if $\phi = 0.85$ is used for torsion as for shear then $\tau_c/v_c = 1.2$ and

$$\tau_{ca} = \frac{\tau_c}{\sqrt{1 + \left(\frac{2.4 v_u}{\tau_u}\right)^2}}$$

$$v_{ca} = \frac{v_c}{\sqrt{1 + \left(\frac{\tau_u}{2.4 v_u}\right)^2}}$$

Committee 438 (5) suggested that the coefficient 2.4 in the above equations be replaced by 3. In the Proposed Revision of ACI 318-63, February 1970, Eqs. 11-9 and 11-17 contained the constant 3 in the denominator. Subsequently in discussion of the revisions to ACI 318-63 (9), Committee 438 reduced the constant from 3 to 1.2 stating - "The revised Eq. 11-9 and 11-17 provide a practically more convenient transition from the case of pure torsion to that of shear-flexure."

Equation 11-18 is derived using a circular interaction curve as follows:

$$\left[\frac{\tau_u(\max)}{12 \sqrt{f'_c}}\right]^2 + \left[\frac{v_u(\max)}{10 \sqrt{f'_c}}\right]^2 = 1.0$$

Reinforced Concrete Buildings

LECTURE 6

DESIGN OF SLENDER REINFORCED CONCRETE COLUMNS

and

CHAPTER 10 - FLEXURE AND AXIAL LOADS

by

J.G. MacGregor

DESIGN OF SLENDER REINFORCED CONCRETE COLUMNS

J.G. MacGregor

BEHAVIOR OF SLENDER REINFORCED CONCRETE COLUMNS

In this paper the term "short column" is used to denote a column which has a strength equal to or greater than that computed for the cross section using the forces and moments obtained from a nominal analysis and the normal assumptions for combined bending and axial load. A "slender column" is defined as a column whose strength is reduced by second-order deformations. By these definitions a column with a given slenderness ratio may be a short column under one set of restraints and loadings and a slender column under another combination of restraints.

The effect of slenderness on a slender column is illustrated in Fig. 1. The maximum moment in the column occurs at Section A-A, due to the combination of the initial eccentricity e in the column and the deflection Δ at this point. Two types of failure can occur. First, the column may be stable at the deflection Δ_1, but the axial load P and the moment M at Section A-A may exceed the strength of that cross section. This type of failure, known as a "material failure," is illustrated by Column 1 in Fig. 1 (c) and is the type which will generally occur in practical building columns which are braced against sway. Second, as shown for Column 2 in Fig. 1 (c), if the column is very slender it may reach a deflection Δ_2 due to the axial force P and the end moment Pe, such that the value of $\delta M/\delta P$ is zero or negative. This type of failure is known as a "stability

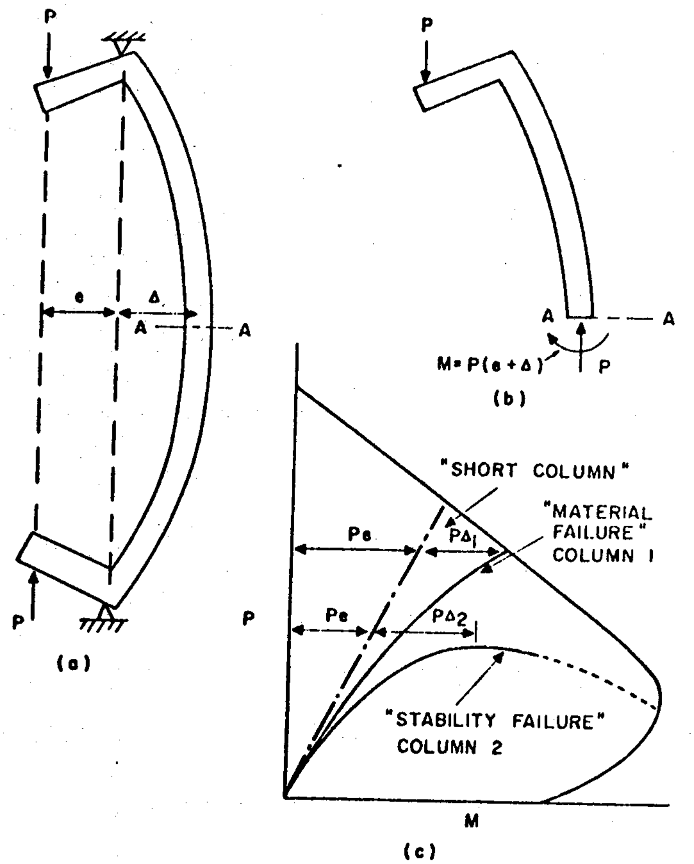

Fig. 1—Load and moment in a slender column

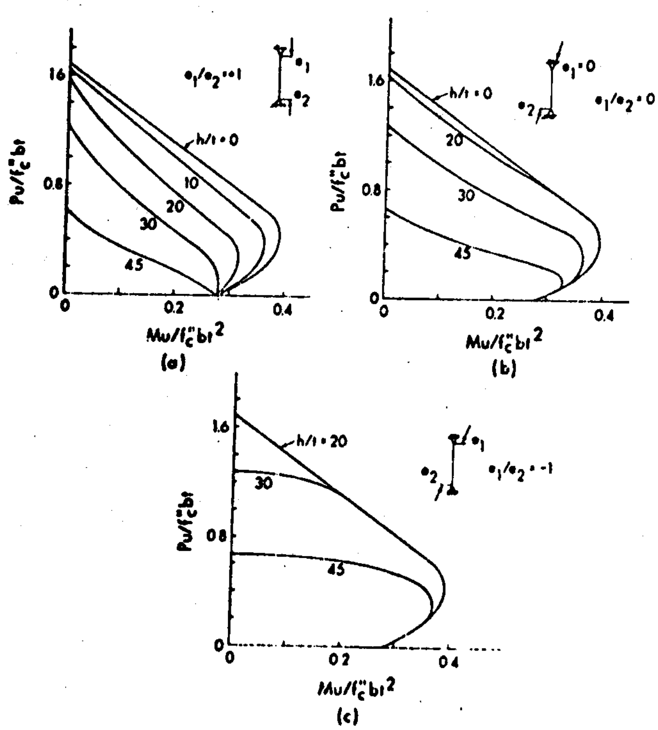

Fig. 2—Effect of deflected shape on interaction diagrams for long hinged columns

failure" and may occur in slender columns in sway frames.

Major Variables Affecting the Strength of Slender Hinged Columns

Broms and Viest[1] and, more recently, Pfrang and Siess[2] have presented comprehensive discussions of the effects of a number of variables on the strength of restrained and unrestrained columns. These two studies have shown that the three most significant variables affecting the strength and behavior of a slender hinged column are: the slenderness ratio h/t; the end eccentricity e/t; and the ratio of end eccentricities e_1/e_2. The effects of these variables are strongly interrelated, as illustrated in Fig. 2. This figure presents three series of load-moment interaction curves for hinged columns with various e_1/e_2 ratios. The interaction diagrams are presented in terms of the maximum loads and moments that can be applied at the ends of columns with various slenderness ratios, as discussed in the previous section.[2]

A hinged column will be weakened if at any section the sum of the moments due to the end eccentricities or imperfections and the column deflections exceeds the maximum moment in the undeflected column. In a column subjected to symmetrical single curvature the column deflections will always increase the column moments. Thus, in Fig. 2 (a) the interaction diagrams for all h/t values greater than zero fall inside the interaction diagram for the cross section (h/t = 0).

In the case of double curvature, however, this will not always be true, since the maximum applied moment occurs at one or both ends of the column while the maximum deflection moments occur between the column ends. This is illustrated by the interaction diagrams for h/t = 30 in Fig. 2 (c). This column is weakened by the column deflections for small eccentricities where the sum of the deflection moments and the applied moments lead to maximum moments greater than the applied moments. For larger eccentricities, however, the maximum moments will always occur at the ends of the column and as a result there is no weakening due to length.

Broms and Viest[1] showed that an increase in the proportion of the load carried by the reinforcement led to a more stable column. Thus, columns with high concrete strengths f'_c and/or low reinforcement percentages p tended to be most strongly affected by length. In other words, as the p/f'_c ratio is increased, the column tends to be more stable.

Sustained loads tend to weaken a hinged slender column by increasing the column deflections. Hinged columns bent in symmetrical single curvature will always be weakened by sustained loads. The effect of sustained loads on the strength of hinged columns bent in double curvature is much less pronounced, especially if the end eccentricities are large.

<u>Behavior of Columns in Braced Frames</u>

Fig. 3 shows a restrained column bent in single curvature. An axial load P and an unbalanced moment M_{ext} are applied to the joint at each end of the column. The external moment M_{ext} is resisted by the restraints M_r and the column M_c as shown in Fig. 3 (b). The maximum moment in the column is the sum of the column end moment M_c and the deflection moment

M_d as shown in Fig. 3 (c) and Eq. (1):

$$M_{max} = M_c + M_d = M_{ext} \left(\frac{K_c}{K_c + K_b} \right) + M_d \tag{1}$$

The column end moment is a direct function of the relative stiffnesses of column and beam. As the column deflects laterally under axial load and moment, the column stiffness is reduced so that more of the external moment at the joint is resisted by the beams and less by the column. Inelastic action in the column tends to hasten this reduction in the column stiffness thus reducing the moment developed at the ends of the column still further. On the other hand, inelastic action in the beam tends to throw moment back into the column.

The deflection moment is affected by the same variables that affect the column stiffness. An increase in the deflection moment tends to weaken the column.

For short restrained columns, the reduction in the column end moment due to axial load may be larger than the increase in moment due to deflections. As a result, the maximum moment in the column is decreased below the value from a first-order analysis and the axial load capacity of the column increases. For a slender restrained column, however, the deflection moments tend to increase more rapidly than the restraint moments and the over-all effect is a weakening of the column because the maximum moment is increased.

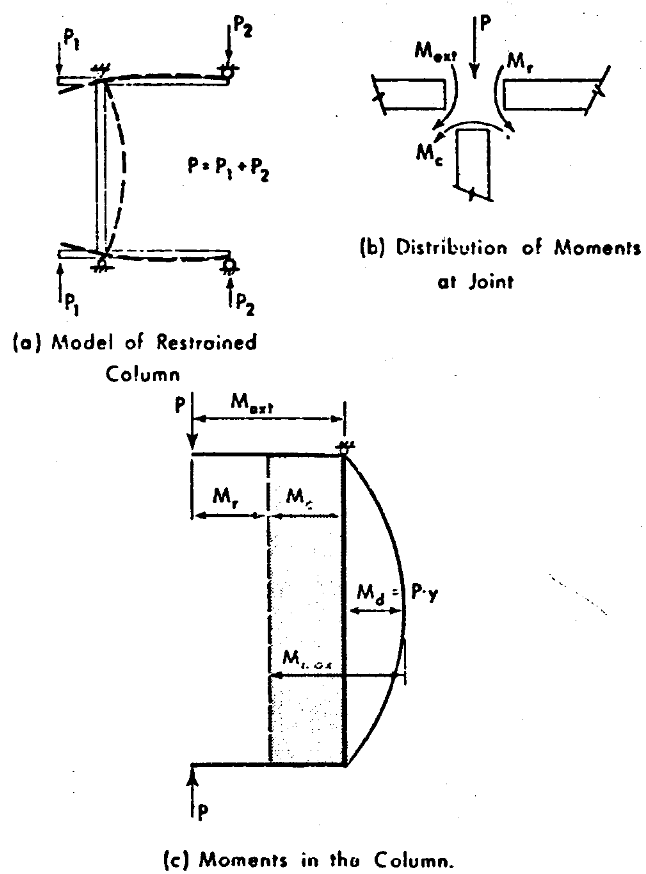

Fig. 3—Moments in a restrained column

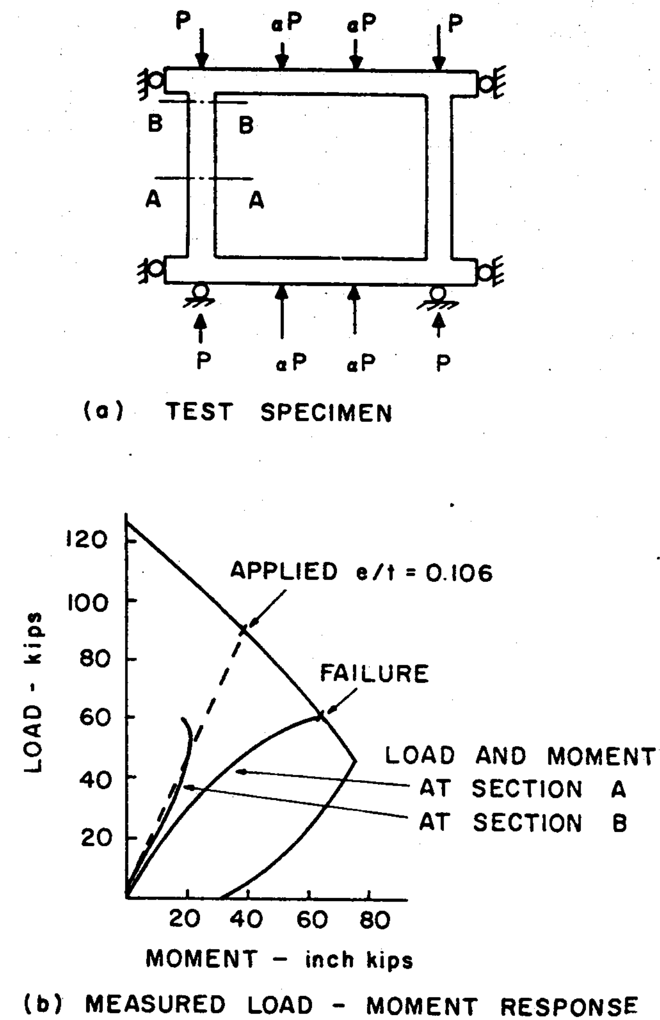

Fig. 4—Behavior of a column in a frame

Fig. 4 (a) shows Frame F2 tested by Furlong and Ferguson[3]. This frame was loaded so that the columns deflected in a single-curvature mode. The columns had h/t = 20 (kh/r = 57) and an initial eccentricity ratio e/t = 0.106. Failure occurred at Section A at midheight of one of the columns. In Fig. 4 (b) the load and moment history at Sections A and B is superimposed on an interaction diagram for that column. The moment at B corresponds to the moment M_c at the end of the column in Fig. 3 (c). Although the loads P and αP were proportionally applied, the variation in moment at Section B is not linear with increasing load because the column stiffness decreased more rapidly than the beam stiffness. The moment at Section A, the failure section, is equal to the sum of the moment at Section B plus the moments due to the column deflection, as indicated by Eq. (1) and Fig. 3 (c). The intersection of the interaction diagram and the line for Section A corresponded to column failure.

Figure 4 also illustrates a common error in frame analysis. The e/t of the applied loads computed using the moments of inertia of the uncracked concrete sections is shown by the dashed line. The actual initial e/t measured in the test would be represented by a line lying between the two solid curves with an e/t of about 0.14. Thus, the column attracted more moment than predicted by the analysis because the column was uncracked through most of the loading history while the beams were cracked. This problem is discussed more fully in Reference 4.

Behavior of Slender Columns in Sway Frames

The behavior of a column free to sway is markedly different from that of a column restrained against sway. As the stiffness of the lateral restraints is decreased, there is a transition from the no-sway to the sway case.

A column in a frame without lateral restraint must depend largely on the beams to prevent sidesway collapse. If the beams are quite flexible, the column will behave essentially as a rigid body and the frame will deflect laterally due primarily to the bending of the beams. If the restraints are stiff, the column will resist some lateral deflection by bending of the column itself. In either case, however, the frame will form an unstable mechanism if the beams yield and can no longer restrain the columns. An increase in the degree of rotational restraint will always increase the strength of a column that is free to sway, unless, of course, the restraints yield.

With the presence of even a small amount of lateral restraint the behavior and capacity of the column changes significantly[5]. Since part of the lateral load is resisted by the lateral restraint, the column no longer deflects as a rigid body and flexural deflections of the column begin to play an important role in the column behavior. However, since a laterally unrestrained column depends entirely on its rotational restraints for stability, the column becomes unstable if these restraints yield. Because of this potentially large reduction in the strength of the frame due to yielding of the restraining beams, it is essential that design of these beams reflect the magnified column end moments which they will be called on to resist.

Summary of the Primary Factors Affecting the Strength of Slender Columns

This brief review of column behavior suggests that the principal variables affecting the strength of slender columns are:

1. The degree of rotational end restraint. An increase in the degree of end restraint will increase the capacity of a column. The effect of the restraints is less marked if they yield under the moments transferred to them by the column.
2. The degree of lateral restraint. A completely unbraced column is significantly weaker than a braced column, but a relatively small amount of bracing is enough to increase the strength almost to that of a completely braced frame. The strength of an unbraced column is strongly dependent on the rotational capacity of the restraining beams.
3. The slenderness ratio h/t, the end eccentricity e/t, and the ratio of end eccentricities e_1/e_2. All have a significant and strongly interrelated effect.
4. The ratio p/f'_c. An increase in this ratio tends to increase the stability of a column.
5. Sustained loads. These loads increase the column deflections and usually decrease the strength of slender columns.

THE DESIGN OF SLENDER COLUMNS

Sections 10.10 and 10.11 of ACI 318-71[6] recommend the use of a second-order structural analysis in the design of slender columns. The term "second-order" analysis is used to describe an analysis that considers the effects of column and frame deflections on moments and the effects of axial loads on the member stiffnesses. Such an analysis should be carried out for the frames in combination with any bracing members or shear walls that exist.

Alternately, design may be based on the axial loads and moments from a conventional structural analysis and the effects of slenderness on column strengths are approximated by multiplying the column moments by a "moment magnifier" F which will be defined later in the paper. This procedure is similar to the beam column design procedure in the recent CSA Standard S16-1969, "Steel Structures for Buildings"[7]. It cannot be emphasized too strongly that, for unbraced frames, the design procedure based on a second-order frame analysis is preferable because it adequately estimates the behavior of the structure. Such analyses are currently available and during the 1970's when the next ACI Code is in use should become readily available to consulting firms.

Approximate Design Method for Slender Concrete Columns

Figure 1 (a) shows an elastic beam-column bent in single curvature. The maximum moment in this column is given by Equation (2):

$$M_{max} = M_o + P\Delta \tag{2}$$

where $M_o = P\,e$

A good approximation of the maximum moment in a beam-column can be found from Equation (3)

$$M_{max} = M_o + \frac{P\delta_o}{1-P/P_c} \tag{3}$$

where δ_o is the deflection caused by M_o if $P = 0$. This equation can be conveniently rewritten as:

$$M_{max} = \frac{M_o\ (1 + \psi P/P_c)}{(1 - P/P_c)} \tag{4}$$

where for a simply supported member with uniform cross-section:

$$\psi = \frac{\pi^2 \delta_o EI}{M_o L^2} - 1 \tag{5}$$

For the column shown in Figure 1 (a) $\psi = 0.273$. Values for other cases are given in Reference 8.

Equation (4) is approximated for design purposes by Equation (6):

$$M_{max} = \frac{M_o}{1 - (P/P_c)} \tag{6}$$

Equation (6) is reasonably accurate for a column bent in uniform single curvature because in this case the maximum moment and maximum deflection occur at the same point. In the more usual case where the end moments are not equal, the maximum moment may be estimated using an "equivalent uniform moment" $C_m M_o$, which would lead to the same long column strength as the actual moment diagram. Thus, Equation (6) becomes:

$$M_{max} = \frac{C_m M_o}{1 - (P/P_c)} \geq M_o \qquad (7)$$

where C_m is the ratio of the equivalent uniform end moment to the numerically larger end moment. The CSA Standard S-16[7] calls for the working stress design of eccentrically loaded steel columns using Equation (8):

$$\frac{P}{P_{allow}} + \frac{M_{max}}{M_{allow}} \leq 1 \qquad (8)$$

where M_{max} is defined using Equation (7).

For reinforced concrete columns, the design can be based on the axial load P from a first-order analysis and the moment M_{max} from Equation (7). This design procedure closely approximates the actual case shown in Figure 1 (b) in which the most highly stressed section, Section A-A, is loaded with an axial load P and a moment $Pe + P\Delta$ equivalent to M_{max}.

The application of this design procedure to concrete columns is discussed briefly in the following section. For more details and a design example the reader is referred to Reference (9).

Application of Approximate Design Method

The 1971 ACI Code[6] states that:

> 10.11.5--Compression members shall be designed using the design axial load from a conventional frame analysis and a magnified moment M defined by Equation (10-4).

$$M = FM_2 \qquad (10\text{-}4)$$

where

$$F = \frac{C_m}{1 - P_u/\phi P_c} > 1.0 \qquad (10\text{-}5)$$

and

$$P_c = \frac{\pi^2 EI}{(kh)^2} \qquad (10\text{-}6)$$

In lieu of a more precise calculation, EI in Equation (10-6) may be taken either as

$$EI = \frac{E_c I_g/5 + E_s I_s}{1 + R_m} \qquad (10\text{-}7)$$

or conservatively

$$EI = \frac{E_c I_g/2.5}{1 + R_m} \qquad (10\text{-}8)$$

In Equation (10-5), for members braced against sidesway and without transverse loads between supports C_m may be taken as

$$C_m = 0.6 + 0.4(M_1/M_2) \qquad (10\text{-}9)$$

but not less than 0.4.

For all other cases C_m shall be taken as 1.0.

Definition of EI

The moment magnifier is computed using Equation (10-5). The main problem in applying the moment magnifier concept to an inelastic, non-homogeneous material such as reinforced concrete is the manner in which the critical load of the column is defined. In particular, it is difficult to choose a value of the stiffness parameter EI which will reasonably approximate the variations in stiffness due to cracking, creep, and the nonlinearity of the concrete stress-strain curve.

The values of EI defined by Equations (10-7) and (10-8) are recommended for use when more precise values are not available. These equations approximate the lower limits to EI for practical cross sections and, hence are conservative for secondary moment calculations. Three procedures were followed to arrive at these two expressions for EI:

1. EI values were estimated from a systematically varied series of theoretical load-moment-curavture diagrams for practical column sections.
2. The effective EI values were computed for each of the University of Texas frame tests.

3. Finally, similar effective EI values were computed for a series of frames simulated by the computer.

Because slender columns generally fail in compression with small e/t values as shown in a survey of actual building cases, and because the effect of axial loads is more pronounced for small e/t values and high P/P_o values, Equations (10-7) and (10-8) were chosen to fit this region.

Equation (10-7) represents a lower limit of the practical range of column EI values especially for columns with large percentages of reinforcement. On the other hand, the EI values from Equation (10-8) are not unreasonable for steel percentages up to about 2.5 per cent but greatly underestimate the effect of the reinforcement in heavily reinforced columns[9]. In many cases where reinforcement percentages are low or slenderness effects are not very substantial the relative simplicity of Equations (10-8) is desirable[9].

Research is currently underway at the University of Alberta to develop improved equations for the EI of practical column sections.

The recently published "ACI Ultimate Strength Design Handbook, Volume 2--Columns"[10] contains tables of EI values computed using Equation (10-8) and presents equations for I_s for three practical reinforcement patterns.

The Effect of Sustained Loads

The creep due to sustained loads increases the column deflections and in many cases this additional deflection results in a reduction in the column capacity. This effect may be taken into account by increasing the deflection term δ_o in Equation (3) or (5), or by increasing the moment multiplier. As an interim solution the latter has been accomplished by reducing the value of EI to an "Effective Modulus." This is done by dividing the EI term by $(1 + R_m)$ where R_m is the ratio of dead load moment to total load moment. This factor has been chosen to give the correct trend when compared to analyses and tests of columns under sustained loads. This procedure is also under study at the present time.

The Equivalent Uniform Moment Term, C_m

The column is designed for the maximum end moment M_2 multiplied by the moment magnifier F and by an equivalent moment correction factor C_m. The derivation of the moment-magnifier term F assumes that the maximum moment is at or near the midheight of the column. If the maximum applied moment occurs at one end of the column, the maximum moment along the length of the column cannot be found by adding the maximum end moment and the maximum deflection moment, since the two occur at different sections in the column. The design of such columns can be based on an equivalent symmetrical single curvature bending moment diagram which would give rise to the same maximum moment as occurs under the actual loading. In the equivalent moment diagram each end of the column is subjected to a moment equal to C_mP times the maximum end eccentricity e_2. However, the maximum moment from such an analysis cannot be less than the maximum end moment Pe_2.

For braced columns C_m is defined by Equation (10-9) in the same manner as in Reference 7.

The Effective Length of Columns

The basic slender compression member design equations in Section 10.11.5 were derived for hinged columns. For restrained columns the basic equations must be modified to account for the effect of the end restraints. This is done by using an "effective length" kh in the computation of slenderness ratios, as is used for beam column design in the CSA Standard S-16[7]. Section 10.11.3 requires the following values of k:

> 10.11.3--For compression members braced against sidesway, the effective length factor k shall be taken as 1.0, unless an analysis shows that a lower value may be used. For compression members not braced against sidesway, the effective length factor k shall be determined with due consideration of cracking and reinforcement on relative stiffness, and shall not be less than 1.0.

The ACI-ASCE Committee on Reinforced Concrete Columns proposed that the effective lengths be computed in a more or less standard way by use of the Jackson and Moreland Alignment Charts[8]. These charts allow graphical determination of the value of k for a typical interior column of constant cross section in a high multi-bay frame.

Because the behavior of braced and unbraced frames is so different, it is difficult to derive a single all-encompassing design rule for columns in both types of frames. For this reason it has been traditional to present one set of effective length factors for "completely braced frames" and another set of factors for "completely unbraced" frames. This distinction has been retained in Paragraph 10.11.3 of the 1971 ACI Code. However, no guidance is given for what constitutes bracing. This problem has recently been clarified by Adams in a discussion of the design of steel columns[11].

Figure 5 shows columns with and without lateral displacements of the ends. If translation is prevented, the buckled shape is as shown in Figure 5 (a). The moments M_1 and M_2 are the applied end moments while M_{R1} and M_{R2} are restraining moments caused by the rotations of the end restraints as the column deflects. Horizontal forces, H, are present if the end moments are unequal. At midheight there are secondary moments equal to the axial load times the deflections shown shaded. To account for the restraining moments M_{R1} and M_{R2} in the design of this "braced" column an effective length less than the real length is used to compute the lateral deflections.

If, however, the column is free to sway laterally as shown in Figure 5 (b), the moments M_1 and M_2 must equilibriate not only any horizontal load, H, but also a moment $P\Delta$. The secondary moments in this column can be divided into two components, one due to the additional horizontal reaction or sway force $P\Delta/h$ necessary to resist the axial force in the deformed position and the second equal to the axial load times the deflections from the chord line, shown shaded. If there is no bracing the sway force $P\Delta/h$ must be provided by increased column moments. These are accounted for in design

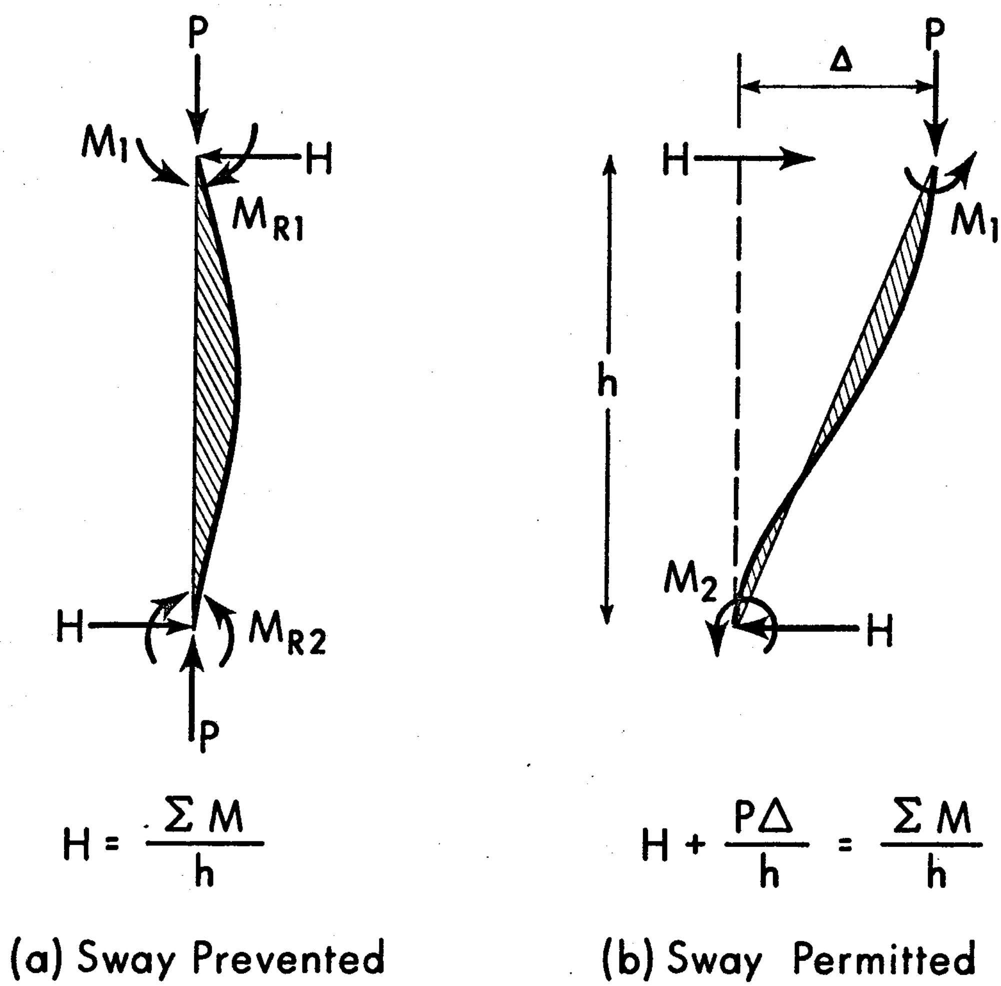

FIGURE 5 Forces in Deflected Columns

if the effective length factors for the unbraced case are used in calculating the moment magnifier.

On the other hand, if a "second-order" structural analysis is carried out including the effects of both the applied loads and the sway forces, the latter have been accounted for in the analysis and need not be considered a second time in proportioning the column. Under these conditions the design would be based on the effective length for a "braced" column to include the effect of the deflections of the column from the chord.

The computation of sway forces for the combined loading case is relatively simple. The lateral and vertical loads are applied to the structure and the relative lateral displacement at each floor level is computed by a first order elastic analysis ignoring PΔ terms. These displacements are denoted as Δ_i in Figure 6, where i denotes the floor level. The additional story shears due to the vertical loads are computed as $\Sigma P_i \left(\frac{\Delta_i}{h_i}\right)$ where P_i represents the axial force in a column of the ith story and the summation is performed for all columns in the story. At a given floor level, the sway force will be the algebraic sum of the story shears from the columns above and below the floor, as shown in Figure 6.

If the sway forces are small relative to the applied lateral loads then the deflections, Δ_i, will be approximately correct and the calculated sway forces will be sufficiently accurate. These are added to the applied lateral loads and the total forces and moments in the structure are computed. Generally one cycle will be enough except in the case of tall slender structures.

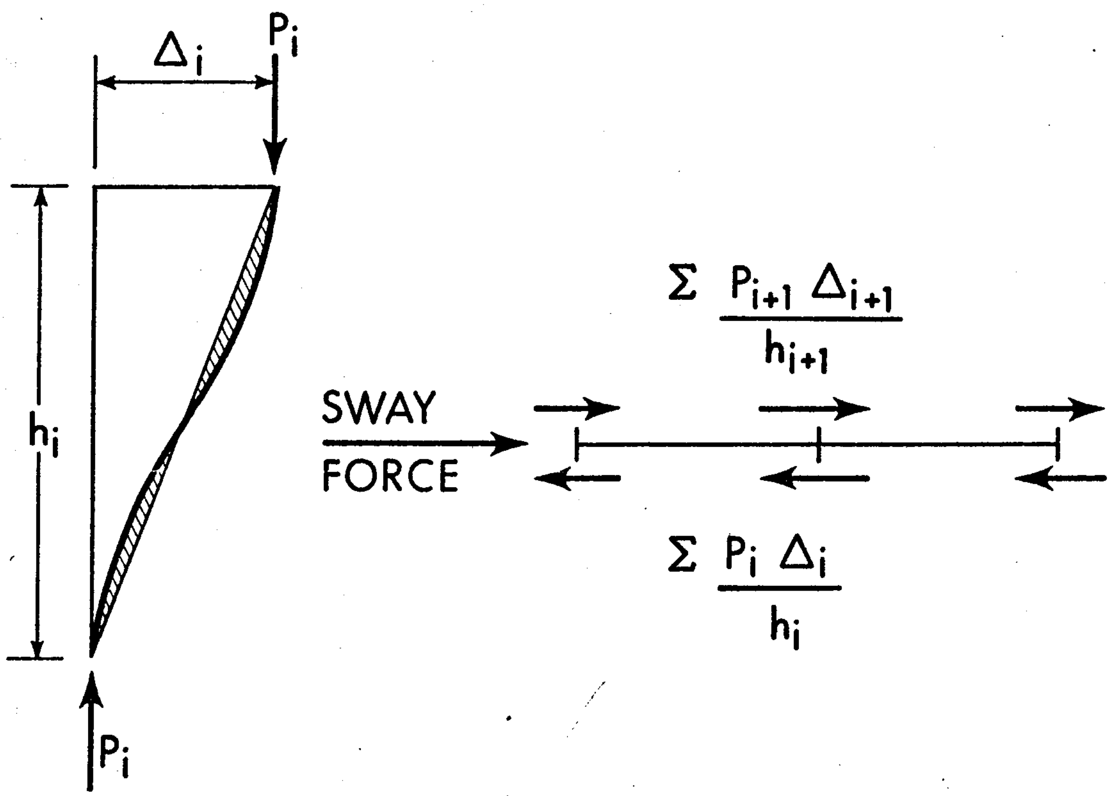

FIGURE 6 Calculation of Sway Force

If the sway forces are large these must be added to the original lateral loads and new deflections computed. New sway forces are computed and the process repeated until convergence is obtained.

Fey[12] and Parme[13] have both shown that the total first and second order deflections in the ith story, $(\Delta + \Delta_2)_i$ can be estimated directly by Equation (9):

$$\frac{(\Delta + \Delta_2)_i}{\Delta_i} = \frac{1}{1 - \delta_T/N \; \Sigma P_i} \qquad (9)$$

where δ_T is the first order lateral deflection at the top of the building due to a unit lateral load applied at each story, and N is the overall height of the building.

Although design procedures based on an elastic second-order analysis and ultimate strength design of cross-sections are a lower-bound solution for the strength of a tall building frame, the true strength of such a frame is not known from such a procedure. For this reason, elastic-plastic, second-order analyses of frames and frame-shear wall structures are highly desirable. One such analysis is described in References 14 and 15.

<u>Moments in Beams in Unbraced Frames</u>

The strength of a laterally unbraced frame is governed by the stability of the columns and by the degree of end restraint provided by the beams in the frame. If plastic hinges form in the restraining beams, the structure approaches a mechanism and its axial load capacity is drastically reduced. In the absence of a comprehensive structural analysis of the type described above the designer is required to make certain that the restraining flexural members have the capacity to resist the amplified column moments and hence can restrain the columns in the manner assumed in the derivation of the slender column design equations.

Limits of Applicability of the Approximate Method

A great many columns are sufficiently stocky or sufficiently well-restrained that they can essentially develop the full cross section strength (assumed here as anything up to a 5 per cent strength loss). The designer's job is considerably simplified if these columns are excluded from the range of slenderness ratios to be considered in design. For compression members braced against sidesway the 1971 ACI Code states that the effects of slenderness may be neglected when kh/r is less than $34-12M_1/M_2$. For compression members not braced against sidesway, the effects of slenderness may be neglected when kh/r is less than 22.

The lower limit, kh/r = 22, applies for columns bent in symmetrical single curvature and columns in unbraced frames. This corresponds to h/t values of about 7.5 for hinged columns, 9 for restrained columns in braced frames, and 4.5 for restrained columns in unbraced frames. For a column in double curvature with one end moment equal to minus one-half the other, the column

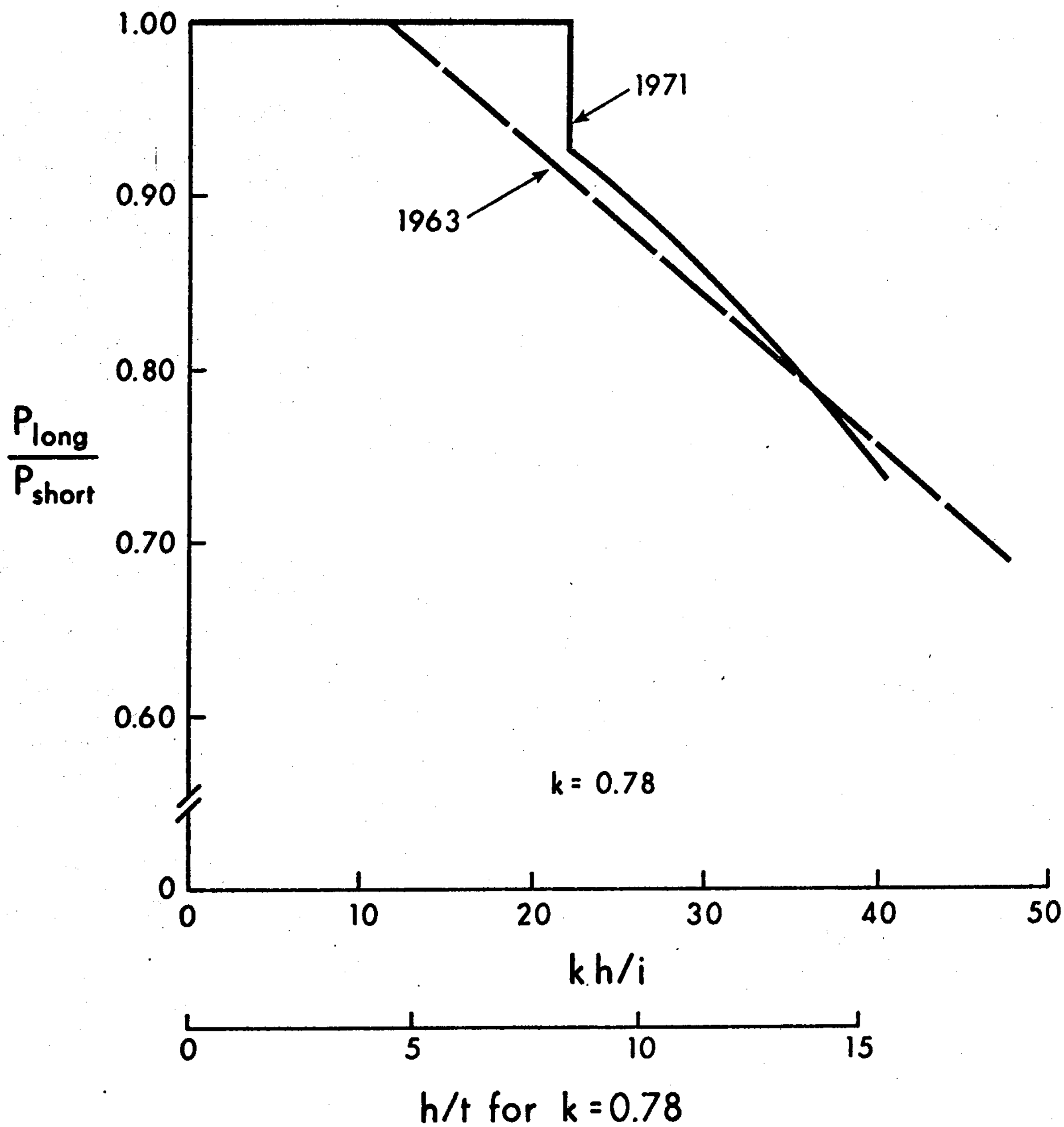

FIGURE 7 Effect of Slenderness for Columns for which Moments have not been Computed

is assumed to be short if kh/r is less than 40. This corresponds to h/t values of about 13 and 16, respectively, for hinged and restrained columns in braced frames. A survey of the normal range of column variables suggests that less than 10 per cent of the columns in braced frames and less than 60 per cent of the columns in sway frames qualify as slender columns.

For kh/r greater than 100 the ACI Code requires a second-order analysis.

Design of Columns for which Moments Have Not Been Computed

Both the 1963 and 1971 ACI Codes require all columns to be designed for a minimum eccentricity of 0.1t. In addition, if column moments have not been computed in the analysis, both codes require that the column be considered as bent in single curvature for slenderness computations. The governing relationships from both codes are plotted in Figure 7. The liberalized R factor equation proposed on Page 160 of the February 1970 ACI Journal has not been plotted because this equation ignores sustained loads and in the author's opinion should not be used.

CONCLUSIONS

This paper presents and documents the new procedures for slender column design in the 1971 ACI Code. It proposes use of a rational second-order structural analysis wherever possible or practical. In place of such an analysis an approximate design method is used based on a moment magnifier principal and similar to the procedure used under the CSA Specification S-16.

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CHAPTER 10 - FLEXURE AND AXIAL LOADS

J. G. MacGregor

Portions of Chapters 9, 15, 16 and 19 of the 1963 ACI Code have been collected together in Chapter 10 of the 1971 ACI Code. This chapter deals with the entire range of members from beams through beam-columns to columns.

Perhaps the most noticeable change is the elimination of the detailed equations for M_u and P_u which previously were presented in Chapters 16 and 19. Another major deletion concerns the minimum column sizes in Section 912 (a) of the 1963 code. This has been done to allow the use of very small columns in building systems.

Minimum Eccentricities - Section 10.3.6

The minimum eccentricities for columns, stated in Section 10.3.6, include the old 0.1t and 0.05t rules plus a minimum of 1 inch for cast-in-place columns or 0.6 inches for highly controlled precast columns. This, in part, offsets the fact that smaller columns can be built.

Crack Widths - Section 10.6

Section 10.6 presents limitations on the distribution of flexural reinforcement. These are intended to limit the width of cracks to certain desirable values. Because unforeseen cracks will occur even in the most carefully designed structure, the code provisions are presented in terms of an unidentifiable variable, Z, for the protection of the designer. Crack widths are to be checked whenever the design yield strength exceeds 40,000 psi.

Slender Columns - Section 10.10 and 10.11

A major change in Chapter 10 involves the method of designing of slender columns. Instead of designing for P_u/R and M_u'/R where R was a reduction factor less than 1.0, as in past ACI Codes, the 1971 Code will require that slender columns be designed for the axial load P_u and FM_u where F is a moment multiplier of the form:

$$F = \frac{1}{1 - P_u/P_c}$$

where:

$$P_c = \frac{\pi^2 E\ I}{(kh)^2}$$

This procedure is explained on Pages 6-28 of the January 1970, ACI Journal and on Pages 158 and 159 of the February 1970 ACI Journal.

Bearing - Section 10.14

For the first time, ultimate bearing stresses are presented in the ACI Code. If the bearing area is surrounded by restraining concrete the permissable bearing stress on the loaded area can be increased by up to two times.

Composite Columns - Section 10.15

Composite columns fall in the area between steel and concrete construction. This was considered in preparing the 1969 CSA S-16 and the 1970 CSA A23.3 standards and for this reason these documents appear more satisfactory than Section 10.15 of the ACI Code.

Walls - Section 10.16

Walls can be designed as compression members under Chapter 10 or as walls under Chapter 14. Chapter 14 can only be used if the eccentricity of the load is less than t/6. The wall load equation (14-1) as revised in the September 1970 ACI Journal yields results similar to the slender column provisions in Section 10.11 for e = t/6.

Reinforced Concrete Buildings

APPENDIX

EXERPTS FROM THE 1971 ACI CODE INCLUDING AMENDMENTS AND EDITORIAL REVISIONS

9.5.3 *Nonprestressed two-way construction*

9.5.3.1 Minimum thickness. The minimum thicknesses of slabs or other two-way construction for floors designed in accordance with the provisions of Chapter 13, and having a ratio of long to short span not exceeding two, shall be governed by Eq. (9-6), (9-7), and (9-8), and the other provisions of this section

$$t = \frac{l_n(800 + 0.005f_y)}{36{,}000 + 5000S[H_{av} - 0.5(1 - R_s)(1 + 1/S)]} \quad (9\text{-}6)$$

but not less than

$$t = \frac{l_n(800 + 0.005f_y)}{36{,}000 + 5000S(1 + R_s)} \quad (9\text{-}7)$$

The thickness need not be more than

$$t = \frac{l_n(800 + 0.005f_y)}{36{,}000} \quad (9\text{-}8)$$

However, the thickness shall be not less than the following values:

For slabs without beams or drop panels . . . 5 in.
For slabs without beams, but with drop panels satisfying Section 9.5.3.2 . . . . 4 in.
For slabs having beams on all four edges with a value of H_{av} at least equal to 2.0 . . . . 3½ in.

9.5.3.2 Drop panels. For slabs without beams but with drop panels having a length in each direction equal to at least one-third the clear span length in that direction and a projection below the slab of at least $t/4$, the thickness required by Eq. (9-6) or (9-7) may be reduced by 10 percent.

9.5.3.3 Edge beams. At discontinuous edges, an edge beam shall be provided having a stiffness such that the value of H is at least 0.80, or the minimum thickness required by Eq. (9-6), (9-7), or Section 9.5.3.2 shall be increased by at least 10 percent in the panel having a discontinuous edge.

9.5.3.4 Computation of immediate deflection. Thicknesses less than those required in this section may be used only if it is shown by computation that the deflection will not exceed the limits in Table 9.5(b). Deflections shall be computed taking into account the size and shape of the panel, the conditions of support, and the nature of restraints at the panel edges. For such computations, the modulus of elasticity of the concrete shall be as specified in Section 8.3.1. The effective moment of inertia shall be that given by Eq. (9-4); other values may be used if they result in predictions of deflection in reasonable agreement with the results of comprehensive tests. Long-time deflections shall be computed in accordance with Section 9.5.2.3.

CHAPTER 10—FLEXURE AND AXIAL LOADS

10.0—Notation

a = depth of equivalent rectangular stress block, defined by Section 10.2.7

A = effective tension area of concrete surrounding the main tension reinforcing bars and having the same centroid as that reinforcement, divided by the number of bars, sq in. When the main reinforcement consists of several bar sizes the number of bars shall be computed as the total steel area divided by the area of the largest bar used

A_b = loaded area

A_b' = maximum area of the portion of the supporting surface that is geometrically similar to and concentric with the loaded area"

A_c = area of core of spirally reinforced column measured to the outside diameter of the spiral, sq in.

A_g = gross area of section

A_s = area of tension reinforcement, sq in.

A_s' = area of compression reinforcement, sq in.

A_t = area of structural steel or tubing in a composite section

b = width of compression face of member

c = distance from extreme compression fiber to neutral axis

C_m = a factor relating the actual moment diagram to an equivalent uniform moment diagram

d = distance from extreme compression fiber to centroid of tension reinforcement, in.

D = dead loads, or their related internal moments and forces

e = eccentricity of design load parallel to axis measured from the centroid of the section. It may be calculated by conventional methods of frame analysis

E_c = modulus of elasticity of concrete, psi. See Section 8.3.1

E_s = modulus of elasticity of steel, psi. See Section 8.3.2

EI = stiffness of compression members. See Eq. (10-8) and Eq. (10-9)

f_c' = specified compressive strength of concrete, psi

f_s = calculated stress in reinforcement at service loads, ksi

f_y = specified yield strength of reinforcement, psi

F = moment magnification factor. See Section 10.11.5

h = unsupported length of compression member

I_g = moment of inertia of gross concrete section about the centroidal axis, neglecting the reinforcement

I_s = moment of inertia of reinforcement about the centroidal axis of the member cross section

I_t = moment of inertia of structural steel or tubing in a cross section about the centroidal axis of the member cross section

k = effective length factor for compression members

k_1 = a factor defined in Section 10.2.7

L = live loads, or their related internal moments and forces

M = moment to be used for design of compression member

M_1 = value of smaller end moment on compression member calculated from a conventional elastic frame analysis, positive if member is bent in single curvature, negative if bent in double curvature

M_2 = value of larger end moment on compression member calculated from a conventional elastic frame analysis, always positive

p = A_s/bd

p' = A_s'/bd

p_b = reinforcement ratio producing balanced conditions. See Section 10.3.3

p_s = ratio of volume of spiral reinforcement to total volume of core (out to out of spirals) of a spirally reinforced concrete or composite column

P_c = critical load. See Section 10.11.5

P_u = axial design load in compression member

r = radius of gyration of the cross section of a compression member

R_m = the ratio of maximum design dead load moment to maximum design total load moment, always positive

t = over-all thickness or depth of a section or diameter of a circular section, in.

t_b = thickness of concrete cover measured from the extreme tension fiber to the center of the bar located closest thereto

Z = a quantity limiting distribution of flexural reinforcement. See Section 10.6

ϕ = capacity reduction factor. See Section 9.2

10.1—Scope

10.1.1—This chapter covers the design of members subject to flexure or to axial loads or to both flexure and axial loads.

10.2—Assumptions

10.2.1—The strength design of members for flexure and axial loads shall be based on the assumptions given in this section, and on satisfaction of the applicable conditions of equilibrium and compatibility of strains.

10.2.2—Strain in the reinforcing steel and concrete shall be assumed directly proportional to the distance from the neutral axis.

10.2.3—The maximum usable strain at the extreme concrete compression fiber shall be assumed equal to 0.003.

10.2.4—Stress in reinforcement below the specified yield strength, f_y, for the grade of steel used shall be taken as E_s times the steel strain. For strains greater than that corresponding to f_y, the stress in the reinforcement shall be considered independent of strain and equal to f_y.

10.2.5—Tensile strength of the concrete shall be neglected in flexural calculations of reinforced concrete, except when meeting the requirements of Section 18.4.

10.2.6—The relationship between the concrete compressive stress distribution and the concrete strain may be assumed to be a rectangle, trapezoid, parabola, or any other shape which results in prediction of strength in substantial agreement with the results of comprehensive tests.

10.2.7—The requirements of Section 10.2.6 may be considered satisfied by an equivalent rectangular concrete stress distribution which is defined as follows: A concrete stress of $0.85f_c'$ shall be assumed uniformly distributed over an equivalent compression zone bounded by the edges of the cross section and a straight line located parallel to the neutral axis at a distance $a = k_1c$ from the fiber of maximum compressive strain. The distance c from the fiber of maximum strain to the neutral axis is measured in a direction perpendicular to that axis. The fraction k_1 shall be taken as 0.85 for strengths, f_c', up to 4000 psi and shall be reduced continuously at a rate of 0.05 for each 1000 psi of strength in excess of 4000 psi.

10.3—General principles and requirements

10.3.1 - The design of cross sections subject to flexure or combined flexure and axial load shall be based on stress and strain compatibility using the assumptions in Section 10.2.

10.3.2 - For flexural members, and for members under combined flexure and axial load controlled by Section 9.2.1.2(d), the reinforcement ratio, p, shall not exceed 0.75 of that ratio which would produce balanced conditions for the section under flexure without axial load.

10.3.3—Balanced conditions exist at a cross section when the tension reinforcement reaches its specified yield strength, f_y, just as the concrete in compression reaches its assumed ultimate strain of 0.003.

10.3.4—All cross sections subject to a compression load shall be designed for the applied moments which can accompany this loading condition, including slenderness effects according to the requirements of Sections 10.10 and 10.11.

10.3.5—Compression reinforcement in conjunction with additional tension reinforcement may be used to increase the capacity of a flexural member.

10.3.6—All members subjected to a compression load shall be designed for the eccentricity e corresponding to the maximum moment which can accompany this loading condition, but not less than 1 in., or $0.05t$ for spirally reinforced or composite steel encased compression members, or $0.10t$ for tied compression members, about either principal axis. For precast members, the minimum design eccentricity may be reduced to not less than 0.6 in. provided that the manufacturing and erection tolerances are limited to one-third of the minimum design eccentricity. Include slenderness effects according to the requirements of Sections 10.10 and 10.11

10.4—Distance between lateral supports of flexural members

10.4.1—The spacing of lateral supports for a beam shall not exceed 50 times the least width b of compression flange or face. Effects of lateral eccentricity of load shall be taken into account in determining the spacing of lateral supports.

10.5—Minimum reinforcement of flexural sections

10.5.1—At any section of a flexural member (except slabs of uniform thickness) where positive reinforcement is required by analysis, the ratio p supplied shall not be less than $200/f_y$ unless the area of reinforcement provided at every section, positive or negative, is at least one-third greater than that required by analysis. In T-beams where the stem is in tension, the ratio p shall be computed for this purpose using the width of the stem.

10.5.2—In structural slabs of uniform thickness, the minimum amount of reinforcement in the direction of the span shall not be less than that required for shrinkage and temperature reinforcement (see Section 7.13).

10.6 - Distribution of flexural reinforcement in beams and one-way slabs.

10.6.1 - This section prescribes rules for the distribution of flexural reinforcement in beams and in one-way slabs; that is slabs reinforced to resist flexural reinforcement in beams and in one-way slabs; that is slabs reinforced to resist flexural stresses in only one direction. The distribution of reinforcement in two-way slabs shall be as required in Section 13.5.

10.6.2—Only deformed reinforcement shall be used. Tension reinforcement shall be well distributed in the zones of maximum concrete tension. Where flanges are in tension, a part of the main tension reinforcement shall be distributed over the effective flange width or a width equal to one-tenth of the span, whichever is smaller. If the effective flange width exceeds one-tenth of the span, some longitudinal reinforcement shall be provided in the outer portions of the flange.

10.6.3—When the design yield strength f_y for tension reinforcement exceeds 40,000 psi, the cross sections of maximum positive and negative moment shall be so proportioned that the quantity Z given by

$$Z = f_s \sqrt[3]{t_b A} \qquad (10\text{-}2)$$

does not exceed the values given by Section 10.6.4. The calculated flexural stress in the reinforcement at service loads f_s, in kips per sq in., shall be computed as the bending moment divided by the product of the steel area and the internal moment arm. In lieu of such computations, f_s may be taken as 60 percent of the specified yield strength f_y.

10.6.4—The quantity Z shall not exceed 175 kips per in. for interior exposure and 145 kips per in. for exterior exposure. Eq. (10-2) does not apply to structures subjected to very aggressive exposure or designed to be watertight; special precautions are required and must be investigated for such cases.

10.6.5—If the depth of the web exceeds 3 ft, longitudinal reinforcement having a total area at least equal to 10 percent of the main tension steel area shall be placed near the faces of the web and distributed in the zone of flexural tension with a spacing not more than 12 in. or the width of the web, whichever is less. Such reinforcement may be taken into account in computation of the strength only if a strain compatibility analysis is made to determine stresses in the individual bars.

10.7—Deep flexural members

10.7.1—Flexural members with over-all depth-span ratios greater than 2/5 for continuous spans, or 4/5 for simple spans, shall be designed as deep beams taking account of nonlinear distribution of stress and lateral buckling.

The design of such members for shear effects shall be in accordance with Section 11.9. The minimum horizontal and vertical reinforcement in the faces shall be the greater of the requirements of Section 11.9.6 or Section 14.0.2 The minimum principal tension reinforcement shall conform to Section 10.5.

10.8—Limiting dimensions for compression members

10.8.1 *Isolated compression member with multiple spirals*—If two or more interlocking spirals are used in a compression member, the outer boundary of the compression member shall be taken at a distance outside the extreme limits of the spiral equal to the requirements of Section 7.14.1.

10.9—Limits for reinforcement of compression members

10.9.1—The vertical reinforcement for noncomposite compression members shall be not less than 0.01 nor more than 0.08 times the gross area of the section. The minimum number of bars in compression members shall be six for bars in a circular arrangement and four for bars in a rectangular arrangement.

10.9.2—The ratio of spiral reinforcement p_s shall be not less than the value given by

$$p_s = 0.45\,(A_g/A_c - 1)\,f_c'/f_y \qquad (10\text{-}3)$$

where f_y is the specified yield strength of spiral reinforcement but not more than 60,000 psi.

10.10—Slenderness effects in compression members

10.10.1—The design of compression members shall be based on forces and moments determined from an analysis of the structure. Such an analysis shall take into account the influence of axial loads and variable moment of inertia on member stiffness and fixed-end moments, the effect of deflections on the moments and forces, and the effects of the duration of the loads.

10.10.2—In lieu of the procedure described in Section 10.10.1, the design of compression members may be based on the approximate procedure presented in Section 10.11. The detailed requirements of Section 10.11 do not need to be applied if design is carried out according to Section 10.10.1.

10.11—Approximate evaluation of slenderness effects

10.11.1—The unsupported length h of a compression member shall be taken as the clear distance between floor slabs, girders, or other members capable of providing lateral support for the compression member. Where capitals or haunches are present, the unsupported length shall be measured to the lower extremity of the capital or haunch in the plane considered.

10.11.2—The radius of gyration r may be taken equal to 0.30 times the over-all dimension in the direction in which stability is being considered for rectangular compression members, and 0.25 times the diameter for circular compression members. For other shapes, r may be computed for the gross concrete section.

10.11.3—For compression members braced against sidesway, the effective length factor k shall be taken as 1.0, unless an analysis shows that a lower value may be used. For compression members not braced against sidesway, the effective length factor k shall be determined with due consideration of cracking and reinforcement on relative stiffness, and shall be greater than 1.0.

10.11.4—For compression members braced against sidesway, the effects of slenderness may be neglected when kh/r is less than $34 - 12M_1/M_2$. For compression members not braced against sidesway, the effects of slenderness may be neglected when kh/r is less than 22. For all compression members with kh/r greater than 100, an analysis as defined in Section 10.10.1 shall be made.

10.11.5—Compression members shall be designed using the design axial load from a conventional frame analysis and a magnified moment M defined by Eq. (10-4).

$$M = FM_2 \qquad (10\text{-}4)$$

where

$$F = \frac{C_m}{1 - P_u/\phi P_c} \geq 1.0 \qquad (10\text{-}5)$$

and

$$P_c = \frac{\pi^2 EI}{(kh)^2} \qquad (10\text{-}6)$$

In lieu of a more precise calculation, EI in Eq. (10-6) may be taken either as

$$EI = \frac{E_c I_g/5 + E_s I_s}{1 + R_m} \qquad (10\text{-}7)$$

or conservatively

$$EI = \frac{E_c I_g/2.5}{1 + R_m} \qquad (10\text{-}8)$$

In Eq. (10-5), for members braced against sidesway and without transverse loads between supports C_m may be taken as

$$C_m = 0.6 + 0.4(M_1/M_2) \qquad (10\text{-}9)$$

but not less than 0.4.

For all other cases C_m shall be taken as 1.0.

10.11.5.1 In frames not braced against sidesway, the value of F shall be computed for the entire story assuming all columns to be loaded. In Eq. (10-5), P_u and P_c shall be taken as ΣP_u and ΣP_c for all of the columns in the story. In designing each column within the story, F shall be taken as the larger value computed for the entire story or computed for the individual column assuming its ends to be braced against sidesway.

10.11.5.2 When compression members are subject to bending about both principal axes, the moment about each axis shall be amplified by F, computed from the corresponding conditions of restraint about that axis.

10.11.6 - When design of compression members is governed by the minimum eccentricities specified in Section 10.3.6, M_2 in Equation (10.4) shall be based on the specified minimum eccentricity, with conditions of curvature determined by either of the following:

(a) When the actual computed eccentricities are less than the specified minimum, the computed end moments may be used to evaluate the conditions of curvature.

(b) If computations show that there is no eccentricity at both ends of the member, conditions of curvature shall be based on a ratio of M_1/M_2 equal to one.

10.11.7—In structures which are not braced against sidesway, the flexural members shall be designed for the total magnified end moments of the compression members at the joint.

10.12—Axially loaded members supporting flat slabs

10.12.1—All axially loaded members supporting flat slabs shall be designed as provided in this chapter and in accordance with the additional requirements of Chapter 13.

10.13—Transmission of column loads through floor system

10.13.1—When the specified strength of concrete in columns exceeds that specified for the floor system by more than 40 percent, transmission of load shall be provided by one of the following:

(a) Concrete of the strength specified for the column shall be placed in the floor for an area four times the column areas about the column, well integrated into floor concrete, and placed in accordance with Section 6.4.2.

(b) The capacity of the column through the floor system shall be computed using the weaker concrete strength with vertical dowels and spirals as required.

(c) For columns laterally supported on four sides by beams of approximately equal depth or by slabs, the capacity may be computed by using an assumed concrete strength in the column formulas equal to 75 percent of the column concrete strength plus 35 percent of the floor concrete strength.

10.14—Bearing

10.14.1 — Bearing stresses shall not exceed $0.85\phi f_c'$, except as provided below.

10.14.2 — When the supporting surface is wider than the loaded area on all sides, the permissible bearing stress on the loaded area may be multiplied by $\sqrt{A_b'/A_b}$, but not more than 2.

10.14.3 — When the supporting surface is sloped or stepped, A_b' may be taken as the area of the lower base of the largest frustrum of a right pyramid or cone contained wholly within the support and having for its upper base the loaded area, and having side slopes of 1 vertical to 2 or more horizontal.

10.14.4 — This section does not apply to post-tensioning anchorages.

10.15—Composite compression members

10.15.1—Composite compression members shall include all concrete compression members reinforced longitudinally with structural steel shape, pipe, or tubing with or without longitudinal bars.

10.15.2—The strength of composite compression members shall be computed for the same limiting conditions applicable to ordinary reinforced concrete members. Any direct compression load capacity assigned to the concrete in a member must be transferred to the concrete by members or brackets in direct bearing on the compression member concrete. All compression load capacity not assigned to the concrete shall be developed by direct connection to the structural steel shape, pipe, or tube.

10.15.3—For slenderness calculations, the radius of gyration of the composite section shall be not greater than the value given by

$$r = \sqrt{\frac{\frac{1}{5}E_cI_g + E_sI_t}{\frac{1}{5}E_cA_g + E_sA_t}} \qquad (10\text{-}10)$$

For computing P_c in Eq. (10-6), EI of the composite section shall be not greater than

$$EI = \frac{E_cI_g/5 + E_sI_t}{1 + R_m} \qquad (10\text{-}11)$$

10.15.4—Where the composite compression member consists of a steel encased concrete core, the thickness of the steel encasement shall be greater than

$$b\sqrt{\frac{f_y}{3E_s}}, \text{ for each face of width } b \qquad (10\text{-}12)$$

or

$$t\sqrt{\frac{f_y}{8E_s}}, \text{ for circular sections of diameter } t \qquad (10\text{-}13)$$

Longitudinal bars within the encasement may be considered in computing A_t and I_t.

10.15.5—Where the composite compression member consists of a spiral bound concrete encasement around a structural steel core, f_c' shall be not less than 2500 psi and spiral reinforcement shall conform to Eq. (10-3). The design yield strength of the structural steel core shall be the specified minimum yield strength for the grade of structural steel used but not to exceed 50,000 psi. Longitudinal reinforcing bars within the spiral shall be not less than 0.01 nor more than 0.08 times the net concrete section and may be considered in computing A_t and I_t.

10.15.6—Where the composite compression member consists of laterally tied concrete around a structural steel core, f_c' shall be not less than 2500 psi and the design yield strength of the structural steel core shall be the specified minimum yield strength for the grade of structural steel used but not to exceed 50,000 psi. Lateral ties shall extend completely around the steel core. Lateral ties shall be #5 bars, or smaller bars having a diameter not less than 1/50 the longest side or diameter of the cross section, but not smaller than #3. The vertical spacing of lateral ties shall not exceed one-half the least width of the cross section, or 48 tie bar diameters, or 16 longitudinal bar diameters. Welded wire fabric of equivalent area may be used.

Longitudinal reinforcing bars within the ties, not less than 0.01 nor more than 0.08 times the net concrete section, shall be provided. These shall be spaced not greater than one-half the least width of the cross section. A longitudinal bar shall be placed at each corner of a rectangular cross section. Bars placed within the lateral ties may be considered in computing A_t for strength calculations but not I_t for slenderness calculations.

10.16—Special provisions for walls

10.16.1— Walls may be designed under the provisions of this chapter with the limitations and exceptions of this section or under Chapter 14.

10.16.2

The minimum ratio of vertical reinforcement to gross concrete area shall be:

"(a) 0.0012 for deformed bars not larger than #5 and with a specified yield strength of 60,000 psi or greater, or

"(b) 0.0015 for other deformed bars, or

"(c) 0.0012 for welded wire fabric not larger than ⅝ in. in diameter"

10.16.3—Vertical reinforcement shall be spaced not farther apart than three times the wall thickness nor 18 in.

10.16.4—Vertical reinforcement need not be provided with lateral ties if such reinforcement is 0.01 times the gross concrete area or less, or where such reinforcement is not required as compressive force and if the diameter of reinforcing does not exceet 5/8 in. or one-half the concrete cover, whichever is greater.

10.16.5—The minimum ratio of horizontal reinforcement to gross concrete area shall be:

(a) 0.0020 for deformed bars not larger than #5 and with a specified yield strength of 60,000 psi or greater or

(b) 0.0025 for other deformed bars, or

(c) 0.0020 for welded wire fabric not larger than ⅝ in. in diameter

10.16.6—Horizontal reinforcement shall be spaced not farther apart than one and one-half times the wall thickness nor 18 in.

CHAPTER 11—SHEAR AND TORSION

11.0—Notation

a = shear span, distance between concentrated load and face of support

A_g = gross area of section, sq in.

A_h = area of horizontal wall reinforcement within a distance s_h, sq in.

A_t = area of one leg of a closed stirrup resisting torsion within a distance s, sq in.

A_l = total area of longitudinal reinforcement to resist torsion, sq in.

A_s = area of tension reinforcement, sq in.

A_s^* = area of prestressed reinforcement

A_v = area of shear reinforcement within a distance s, or vertical wall reinforcement within a distance s_v, sq in.

A_{vf} = area of shear-friction reinforcement, sq in.

A_{vh} = area of shear reinforcement parallel to the main tension reinforcement, sq in.

b = width of compression face of member

b' = web width, or diameter of circular section, in.

b_o = periphery of critical section for slabs and footings

c_1 = size of column, capital, or bracket measured in the direction in which moments are being determined

c_2 = size of column, capital, or bracket measured transverse to the direction in which moments are being determined

D = horizontal length of wall

d = the distance from extreme compression fiber to centroid of tension reinforcement, in.

f_c' = specified compressive strength of concrete, psi

$\sqrt{f_c'}$ = square root of specified compressive strength of concrete, psi

f_d = stress due to dead load, at the extreme fiber of a section at which tensile stresses are caused by applied loads, psi

f_{pc} = compressive stress in the concrete, after all prestress losses have occurred, at the centroid of the cross section resisting the applied loads or at the junction of the web and flange when the centroid lies in the flange, psi. (In a composite member, f_{pc} will be the resultant compressive stress at the centroid of the composite section, or at the junction of the web and flange when the centroid lies within the flange, due to both prestress and to the bending moments resisted by the precast member acting alone)

f_{pe} = compressive stress in concrete due to prestress only after all losses, at the extreme fiber of a section at which tensile stresses are caused by applied loads, psi

f_y = specified yield strength of reinforcement, psi

f_s' = ultimate strength of prestressing steel, psi

f_{ct}' = average splitting tensile strength of lightweight aggregate concrete, psi

h = total depth of shearhead cross section

H = total height of wall from its base to its top

I = moment of inertia of section resisting externally applied loads.

K = ratio of stiffness of shearhead arm to surrounding composite slab section. See Section 11.11.2

l_c = clear span, measured face to face of supports

L_v = length of shearhead arm from centroid of concentrated load or reaction.

M_u = applied design load moment at a section, in.-lb

M' = modified bending moment

M_{cr} = cracking moment

M_t = maximum bending moment due to externally applied design loads

M_p = required full plastic moment of shearhead cross section

M_v = moment resistance contributed by shearhead reinforcement

M_{tu} = design torsional moment

N_u = design axial load normal to the cross section occurring simultaneously with V_u to be taken as positive for compression, negative for tension, and to include the effects of tension due to shrinkage and creep

$p = A_s/bd$

P_n = the ratio of vertical reinforcement area to the gross concrete area of a shear wall

P_h = the ratio of horizontal reinforcement area to the gross concrete area of a shear wall

P_v = $(A_s + A_{vh})/bd$

p_w = $A_s/b'd$

s = shear or torsion reinforcement spacing in a direction parallel to the longitudinal reinforcement

s_1 = shear or torsion reinforcement spacing in a direction perpendicular to the longitudinal reinforcement or spacing of horizontal reinforcement in a wall

s_v = spacing of vertical reinforcement in a wall

T = design tensile force on bracket or corbel acting simultaneously with V_u

t = total depth of section or thickness of wall

v_c = nominal permissible shear stress carried by concrete

v_{ci} = shear stress at diagonal cracking due to all design loads, when such cracking is the result of combined shear and moment

v_{cw} = shear stress at diagonal cracking due to all design loads, when such cracking is the result of excessive principal tensile stresses in the web

v_u = nominal total design shear stress

V_d = shear force at section due to dead load

V_i = shear force at section occurring simultaneously with M_i

V_p = vertical component of the effective prestress force at the section considered

V_u = total applied design shear force at section

x = shorter over-all dimension of a rectangular part of a cross section

x_1 = shorter center-to-center dimension of a closed rectangular stirrup

y = longer over-all dimension of a rectangular part of a cross section

y_1 = longer center-to-center dimension of a closed rectangular stirrup

y_t = distance from the centroidal axis of gross section, neglecting the reinforcement, to the extreme fiber in tension

α = angle between inclined web bars and longitudinal axis of member

Ω = a coefficient as a function of y_1/x_1. See Section 11.8.2

μ = coefficient of friction. See Section 11.15

τ_c = nominal permissible torsion stress carried by concrete

τ_u = nominal total design torsion stress

ϕ = capacity reduction factor. See Section 9.2

11.1—General reinforcement requirements

11.1.1—A minimum area of shear reinforcement shall be provided in all reinforced, prestressed, and nonprestressed concrete flexural members except:

(a) Slabs and footings

(b) Concrete joist floor construction defined by Section 8.8

(c) Beams where the total depth does not exceed either 10 in., two and one-half times the thickness of the flange, or one-half the width of the web

(d) Where v_u is less than one-half of v_c

This requirement may be waived if it is shown by test that the required ultimate flexural and shear capacity can be developed when shear reinforcement is omitted.

11.1.2 - Where shear reinforcement is required by Section 11.1.1 or by calculations, and the nominal torsion stress τ_u does not exceed $1.5\sqrt{f_c'}$, the minimum area in square inches shall be

$$A_v = 50b's/f_y \qquad (11\text{-}1)$$

for prestressed and nonprestressed members where b' and s are in inches. Alternatively, a minimum area

$$A_v = \frac{A_s^*}{80}\frac{f_s'}{f_y}\frac{s}{d}\sqrt{\frac{d}{b'}} \qquad (11\text{-}2)$$

may be used for prestressed members having an effective prestress force at least equal to 40 percent of tensile strength of the flexural reinforcement.

Where the nominal torsion stress τ_u is greater than $1.5\sqrt{f_c'}$, and where web reinforcement is required by Section 11.1.1 or by calculations, the minimum area of closed stirrups provided shall be

$$A_v + 2A_o = 50b's/f_y$$

11.1.3—The design yield strength of shear and torsion reinforcement shall not exceed 60,000 psi.

11.1.4—Shear reinforcement may consist of:

(a) Stirrups perpendicular to the axis of the member

(b) Welded wire fabric with wires located perpendicular to the axis of the member

Where shear reinforcement is required and is placed perpendicular to the axis of the member, it shall be spaced not further apart than $0.50d$ in nonprestressed concrete and $0.75t$ in prestressed concrete, but not more than 24 in.

11.1.5—For reinforced concrete members without prestressing, shear reinforcement may also consist of:

(a) Stirrups making an angle of 45 deg or more with the longitudinal tension bars

(b) Longitudinal bars with a bent portion making an angle of 30 deg or more with the longitudinal tensile bars

(c) Combinations of stirrups and bent bars

Inclined stirrups and bent bars shall be so spaced that every 45 deg line, extending toward the reaction from the middepth of the member, $0.50d$, to the longitudinal tensions bars, shall be crossed by at least one line of web reinforcement.

11.1.6—Torsion reinforcement where required by Section 11.7 shall consist of closed stirrups, closed ties, or spirals combined with longitudinal bars.

11.1.7—Stirrups and other bars or wires used as shear or torsion reinforcement shall extend to a distance d from the extreme compression fiber and shall be anchored at both ends according to Sections 7.1 and 12.13 to develop the design yield strength of the reinforcement.

11.2—Shear strength

11.2.1 - The nominal shear stress v_u shall be computed by:

(a) For members of constant depth

$$v_u = \frac{V_u}{\phi b'd} \qquad (11\text{-}3)$$

(b) For members of varying depth

$$v = \frac{V_u \pm \frac{M}{d}\tan\beta}{\phi b'd} \qquad (11\text{-}3a)$$

in which the negative sign applies where the bending moment, M, increases numerically in the same direction as the depth, d, increases and the positive sign where the moment decreases in this direction.

The distance d shall be taken from the extreme compression fiber to the centroid of the longitudinal tension reinforcement, but not less than $0.80t$ for prestressed concrete members.

For circular sections, d need not be taken less than the distance from the extreme compression fiber to the centroid of the longitudinal reinforcement in the opposite half of the member.

11.2.2—When the reaction, in the direction of the applied shear, introduces compression into the end region of the member, sections located less than a distance d from the face of the support may be designed for the same v_u as that computed at a distance d; for prestressed concrete, sections located at a distance less than $t/2$ may be designed for the shear computed at $t/2$.

11.2.3—The shear stress carried by the concrete, v_c, shall be calculated according to Section 11.4 or 11.5. Wherever applicable, the effects of inclined flexural compression in variable-depth members may be included, and effects of axial tension due to restrained shrinkage and creep shall be considered.

11.2.4—When v_u exceeds v_c, shear reinforcement shall be provided according to Section 11.6.

11.2.5—For deep beams, slabs, walls, brackets, and corbels the special provisions of Sections 11.9 through 11.16 shall apply.

11.3—Lightweight concrete shear and torsion stresses

The provisions of this chapter for nominal shear stress v_c and nominal torsion stress τ_c carried by the concrete apply to normal weight concrete. When lightweight aggregate concretes are used, the following modifications shall apply:

11.3.1—The provisions for v_c and τ_c shall be modified by substituting $f'_{sp}/6.7$ for $\sqrt{f'_c}$, but the value of $f'_{sp}/6.7$ used shall not exceed $\sqrt{f'_c}$. The value of f'_{sp} shall be specified and the concrete proportioned in accordance with Section 4.2.

11.3.2 - When f'_{sp} is not specified, all values of $\sqrt{f'_c}$ affecting v_c, τ_c, and M_{cr} shall be multiplied by 0.75 for "all - light weight" concrete, and 0.85 for "sand lightweight" concrete. Linear interpolation may be used when partial sand replacement is used.

11.4—Nominal permissible shear stress for nonprestressed concrete members

11.4.1—The shear stress carried by the concrete, v_c, shall not exceed $2\sqrt{f'_c}$ unless a more detailed analysis is made in accordance with Section 11.4.2 or 11.4.3. For members subjected to axial load or torsion, v_c shall not exceed values given in Sections 11.4.3 through 11.4.5.

11.4.2—The nominal shear stress v_c shall not exceed:

$$v_c = 1.9\sqrt{f_c'} + 2500 p_w \frac{V_u d}{M_u} \quad (11\text{-}4)$$

but v_c shall not be greater than $3.5\sqrt{f_c'}$. M_u is the bending moment occurring simultaneously with V_u at the section considered, but $V_u d/M_u$ shall not be taken greater than 1.0 in computing v_c from Eq. (11-4).

11.4.3—For members subjected to axial compression, Eq. (11-4) may be used, except that M' shall be substituted for M_u, and M' shall be permitted to have values less than $V_u d$

$$M' = M_u - N_u \frac{(4t - d)}{8} \quad (11\text{-}5)$$

Alternatively, v_c may be computed by:

$$v_c = 2(1 + 0.0005\, N_u/A_g)\sqrt{f_c'} \quad (11\text{-}6)$$

However, v_c shall not exceed:

$$v_c = 3.5\sqrt{f_c'}\sqrt{1 + 0.002\, N_u/A_g} \quad (11\text{-}7)$$

The quantity N_u/A_g shall be expressed in psi.

11.4.4—For members subjected to significant axial tension, web reinforcement shall be designed to carry the total shear, unless a more detailed analysis is made using

$$v_c = 2(1 + 0.002\, N_u/A_g)\sqrt{f_c'} \quad (11\text{-}8)$$

where N_u is negative for tension. The quantity N_u/A_g shall be expressed in psi.

11.4.5—At cross sections subjected to a nominal torsion stress, τ_u exceeding $1.5\sqrt{f_c'}$, computed by Eq. (11-16), v_c shall not exceed

$$v_c = \frac{2\sqrt{f_c'}}{\sqrt{1 + (\tau_u/1.2 v_u)^2}} \quad (11\text{-}9)$$

11.5—Nominal permissible shear stress for prestressed concrete members

11.5.1—For members having an effective prestress force at least equal to 40 percent of the tensile strength of the flexural reinforcement, unless a more detailed analysis is made in accordance with Section 11.5.2, the nominal shear stress carried by the concrete, v_c, shall not exceed

$$v_c = 0.6\sqrt{f_c'} + 700 \frac{V_u d}{M_u} \quad (11\text{-}10)$$

but v_c need not be taken less than $2\sqrt{f_c'}$ nor shall v_c be greater than $5\sqrt{f_c'}$. M_u is the bending moment occurring simultaneously with V_u, but $V_u d/M_u$ shall not be taken greater than 1.0. When applying Eq. (11-10), d shall be the distance from the extreme compression fiber to the centroid of the prestressing tendons.

11.5.2—Except as allowed in Section 11.5.1 the shear stress v_c shall be computed as the lesser of v_{ci} or v_{cw}.

$$v_{ci} = 0.6\sqrt{f_c'} + \frac{V_d + (V_i M_{cr}/M_l)}{b'd} \quad (11\text{-}11)$$

but need not be taken less than $1.7\sqrt{f_c'}$, where

$$M_{cr} = (I/y_t)(6\sqrt{f_c'} + f_{pe} - f_d).$$

$$v_{cw} = 3.5\sqrt{f_c'} + 0.3f_{pc} + \frac{V_p}{b'd} \quad (11\text{-}12)$$

Alternately, v_{cw} may be taken as the shear stress corresponding to a multiple of dead load plus live load which results in a computed principal tensile stress of $4\sqrt{f_c'}$ at the centroidal axis of the member, or at the intersection of the flange and the web when the centroidal axis is in the flange. In a composite member, the principal tensile stress shall be computed using the cross section which resists live load.

11.5.2.1 In Eq. (11-11) and (11-12), d shall be the distance from the extreme compression fiber to the centroid of the prestressing tendons or $0.8t$, whichever is greater.

11.5.2 The values of M_l and V_i in Eq. (11-11) shall be computed from the load distribution causing maximum moment to occur at the section.

11.5.3—In a pretensioned member in which the section at a distance $t/2$ from the face of the support is closer to the end of the beam than the development length of the tendons, the reduced prestress shall be considered when calculating v_{cw}. This value of v_{cw} shall also be taken as the maximum limit for Eq. (11-10). The prestress force may be assumed to vary linearly from zero at the end of the tendon to a maximum at a distance from the end of the tendon equal to the transfer length, assumed to be 50 diameters for strand and 100 diameters for single wire.

11.6—Design of shear reinforcement

11.6.1—Shear reinforcement shall conform to the general requirements of Section 11.1. When shear reinforcement perpendicular to the longitudinal axis is used, the required area of shear reinforcement shall be not less than

$$A_v = \frac{(v_u - v_c)b's}{f_y} \quad (11\text{-}13)$$

11.6.2—When inclined stirrups or bent bars are used as shear reinforcement in reinforced concrete members, the following provisions apply:

11.6.2.1 When inclined stirrups are used, the required area shall be not less than

$$A_v = \frac{(v_u - v_c)b's}{f_y(\sin\alpha + \cos\alpha)} \quad (11\text{-}14)$$

11.6.2.2 When shear reinforcement consists of a single bar or a single group of parallel bars, all bent up at the same distance from the support, the required area shall be not less than

$$A_v = \frac{(v_u - v_c)b'd}{f_y \sin\alpha} \quad (11\text{-}15)$$

in which $(v_u - v_c)$ shall not exceed $3\sqrt{f_c'}$.

11.6.2.3 When shear reinforcement consists of a series of parallel bent-up bars or groups of parallel bent-up bars at different distances from the support, the required area shall be not less than that computed by Eq. (11-14).

11.6.2.4 Only the center three-fourths of the inclined portion of any longitudinal bar that is bent shall be considered effective for shear reinforcement.

11.6.2.5 Where more than one type of shear reinforcement is used to reinforce the same portion of the web, stirrups shall be provided to carry at least one third of the total shear take by the reinforcement. The required area shall b computed as the sum for the various types separ ately with v_c being included only once.

11.6.3—When $(v_u - v_c)$ exceeds $4\sqrt{f_c'}$, the maximum spacings given in Sections 11.1.4 and 11.1.5 shall be reduced by one-half.

11.6.4—The value of $(v_u - v_c)$ shall not exceed $8\sqrt{f_c'}$.

11.7—Combined torsion and shear for nonprestressed members

11.7.1—Torsion effects shall be included for shear and bending whenever the nominal torsion stress τ_u exceeds $1.5\sqrt{f_c'}$. Otherwise, torsion effects may be neglected.

11.7.2—For members with rectangular or flanged sections, τ_u shall be computed by

$$\tau_u = 3M_{tu}/\phi\Sigma x^2 y \quad (11\text{-}16)$$

The sum $\Sigma x^2 y$ shall be taken for the component rectangles of the section, but the overhanging flange width used in design shall not exceed three times the thickness of the flange.

11.7.3—A rectangular box section may be taken as a solid section, provided that the wall thickness t is at least $x/4$. A box section with a wall thickness less than $x/4$, but greater than $x/10$, may also be taken as a solid section except that $\Sigma x^2 y$ shall be multiplied by $4t/x$. When t is less than $x/10$, the stiffness of the wall shall be considered. Fillets shall be provided at interior corners of all box sections.

11.7.4—Sections located less than a distance d from the face of the support may be designed for the same torsion, τ_u, as that computed at a distance d.

11.7.5—The nominal torsion stress carried by the concrete, τ_c, in reinforced concrete members shall not exceed

$$\tau_c = \frac{2.4\sqrt{f_c'}}{\sqrt{1 + (1.2 v_u/\tau_u)^2}} \quad (11\text{-}17)$$

11.7.6—For members subjected to significant axial tension, torsion reinforcement shall be designed to carry the total torque, unless a more detailed analysis is made in which τ_c given by Eq. (11-17) and v_c given by Eq. (11-9) shall be multiplied by $(1 + 0.002N_u/A_g)$, where N_u is negative for tension.

11.7.7—The torsion stress τ_u shall not exceed

$$\tau_u = \frac{12\sqrt{f_c'}}{\sqrt{1 + (1.2 v_u/\tau_u)^2}} \quad (11\text{-}18)$$

11.8—Design of torsion reinforcement

11.8.1—Torsion reinforcement, where required, shall be provided in addition to reinforcement required to resist shear, flexure, and axial forces. The reinforcement required for torsion may be combined with that required for other forces, provided the area furnished is the sum of the individually required areas and the most restrictive requirements for spacing and placement are met.

11.8.2 The required area of closed stirrups shall be computed by

$$A_t = \frac{(\tau_u - \tau_c)\, s \Sigma x^2 y}{3\Omega x_1 y_1 (f_y)} \quad (11\text{-}19)$$

"where Ω = $[0.66 + 0.33(y_1/x_1)]$, but not more than 1.50."

11.8.3

"The spacing of closed stirrups shall not exceed $(x_1 + y_1)/4$, or 12 in., whichever is the smaller."

"11.8.4 — The required area of longitudinal bars shall be computed by

$$A_l = 2A_t \frac{x_1 + y_1}{s} \quad (11\text{-}20)$$

or by

$$A_l = \left[\frac{400xs}{f_y}\left(\frac{\tau_u}{\tau_u + v_u}\right) - 2A_t\right]\left(\frac{x_1 + y_1}{s}\right) \quad (11\text{-}21)$$

whichever is the greater, where $2A_t$ used in Eq. (11-21) need not be taken as less than $50b's/f_y$."

"11.8.5 — The spacing of longitudinal bars, not less than #3 in size, distributed around the perimeter of the stirrups, shall not exceed 12 in. At least one longitudinal bar shall be placed in each corner of the stirrups."

11.8.6—Torsion reinforcement shall be provided at least a distance $(d + b)$ beyond the point theoretically required.

11.9—Special provisions for deep beams

11.9.1—These provisions apply when l_c/d is less than 5 and the members are loaded at the top or compression face.

11.9.2—The nominal shear stress v_c carried by the concrete shall be determined by

$$v_c = [3.5 - (2.5)(M_u/V_u d)] \times \left[1.9\sqrt{f_c'} + 2500\, p_w \frac{V_u d}{M_u}\right] \quad (11\text{-}22)$$

except that the term $[3.5 - (2.5)(M_u/V_u d)]$ shall not exceed 2.5, and v_c shall not exceed $6\sqrt{f_c'}$. M_u and V_u are the bending moment and shear occurring simultaneously at the critical section defined by Section 11.9.3. In lieu of Eq. (11-22), v_c may be taken as $2\sqrt{f_c'}$.

11.9.3—The critical section for shear measured from the face of the support shall be taken at 0.15 l_c for uniformly loaded beams and $0.50a$ for beams with concentrated loads, but not greater than d. Shear reinforcement required at the critical section shall be used throughout the span.

11.9.4—The shear stress v_u shall not exceed $8\sqrt{f_c'}$ when l_c/d is less than 2. When l_c/d is between 2 and 5, v_u shall not exceed

$$v_u = \frac{2}{3}(10 + l_c/d)\sqrt{f_c'} \quad (11\text{-}23)$$

11.9.5—The area of shear reinforcement shall be computed from

$$\left(\frac{A_v}{s}\right)\left(\frac{1 + l_c/d}{12}\right) + \left(\frac{A_{vh}}{s_h}\right)\left(\frac{11 - l_c/d}{12}\right) = \frac{(v_u - v_c)\,b'}{f_y} \quad (11\text{-}24)$$

11.9.6—The area of shear reinforcement A_v perpendicular to the main reinforcement shall not be less than $0.0015bs$, and s shall not exceed $d/5$ or 18 in. The area of shear reinforcement A_{vh} parallel to the main reinforcement shall not be less than $0.0025bs_h$, and s_h shall not exceed $d/3$ or 18 in.

11.10—Special provisions for slabs and footings

11.10.1—The shear strength of slabs and footings in the vicinity of concentrated loads or reactions is governed by the more severe of two conditions:

(a) The slab or footing acting essentially as a wide beam, with a potential diagonal crack extending in a plane across the entire width. This case shall be considered in accordance with Sections 11.1 through 11.6

(b) Two-way action for the slab or footing, with potential diagonal cracking along the surface of a truncated cone or pyramid around the concentrated load or reaction. In this case, the slab or footing shall be designed as specified in the remainder of this section

11.10-2 - The critical section for two-way action shall be perpendicular to the plane of the slab and located so that its periphery is a minimum and approaches no closer than d/2 to the periphery of the concentrated load or reaction area.

11.10-3 - The nominal shear stress for two-way action shall be computed by

$$v_u = V_u/\phi b_o d \qquad (11\text{-}25)$$

in which V_u and b_o are taken at the critical section specified in Section 11.10.2. The shear stress v_u shall not exceed $v_c = 4\sqrt{f'_c}$ unless shear reinforcement is provided. A maximum increase of 50 percent in v_u is permitted if shear reinforcement is provided in accordance with Section 11.11.1, and a maximum increase of 75 percent is permitted if shearhead reinforcement is provided in accordance with Section 11.11.2.

11.11—Shear reinforcement in slabs and footings

11.11.1 - Shear reinforcement consisting of bars or wires anchored in accordance with Section 12.13 may be provided in slabs. For design of such shear reinforcement, shear stresses shall be investigated at the critical section defined in Section 11.10.2 and at successive sections more distant from the support; and the shear stress, v_c, carried by the concrete at any section shall not exceed $2\sqrt{f'_c}$. Where v_u exceeds v_c, the shear reinforcement shall be provided according to Section 11.6

11.11.2—Shear reinforcement within the slab consisting of steel I or channel shapes shall be designed in accordance with the following provisions, which do not apply where shear is transferred to a column at an edge or a corner of a slab. At exterior columns, special designs are required.

11.11.2.1 Each shearhead shall consist of steel shapes fabricated by welding into four identical arms at right angles and continuous through the column section. The ends of shearheads may be cut at angles not less than 30 deg with the horizontal, provided that the plastic moment capacity of the remaining tapered section is adequate to resist the shear force attributed to that arm of the shearhead. The ratio K between the stiffness for each shearhead arm and that for the surrounding composite cracked slab section of width $(c_2 + d)$ shall not be less than 0.15. All compression flanges of the steel shapes shall be located within 0.3d of the compression surface of the concrete slab. The steel shapes shall not be deeper than 70 times their web thickness.

11.11.2.2 [illegible] full plastic moment of resistance M_p required for each arm of the shearhead shall be computed by

$$M_p = \frac{V_u}{\phi 8}\left[h + K\left(L_v - \frac{c_1}{2}\right)\right] \qquad (11\text{-}26)$$

where ϕ is the capacity reduction factor for flexure and L_v is the minimum length of each shearhead arm required to comply with the requirements of Sections 11.11.2.3 and 11.11.2.4.

11.11.2.3 The critical slab section shall be perpendicular to the plane of the slab. The section shall cross each shearhead arm three-quarters of the distance, $L_v - (c_1/2)$, from the column face to the end of the shearhead, and it shall be so located that its periphery is a minimum. However, the critical section need not approach closer than $d/2$ to the periphery of the column.

11.11.2.4 The shear stress v_u shall not exceed $4\sqrt{f'_c}$ on the critical section.

11.11.2.5 The shearhead may be assumed to contribute a resisting moment M_v to each column strip of the slab computed by

$$M_v = \frac{\phi K V_u}{8}\left(L_v - \frac{c_1}{2}\right) \qquad (11\text{-}27)$$

where ϕ is the capacity reduction factor for flexure, and L_v is the length of each shearhead arm actually provided. However, M_v shall exceed neither 30 percent of the total moment resistance required for each column strip of the slab, nor the change in column strip moment over the length L_v, nor the value of M_p given by Eq. (11-26).

11.12—Openings in slabs

11.12.1—When openings in slabs and footings are located at a distance less than ten times the thickness of the slab from a concentrated load or reaction, or when openings in flat slabs are located within the column strips as defined in Chapter 13, the critical sections specified in Sections 11.10.2 and 11.11.2.3 shall be modified as follows:

(a) For slabs without shearheads, that part of the periphery of the critical section which is enclosed by radial projections of the openings to the centroid of the loaded area shall be considered ineffective

(b) For slabs with shearheads, one-half of that part of the periphery specified in (a) shall be considered ineffective

11.13 - Transfer of moments to columns

11.13.1 - Shear forces exerted by unbalanced loads at connections to columns shall be considered in the design of lateral reinforcement in the column. Lateral reinforcement not less than that required by Eq. (11-1) shall be provided within the connections, except those not part of a primary seismic load-resisting system which are restrained on four sides by beams or slabs of approximately equal depth.

11.13.2—When unbalanced gravity load, wind, earthquake, or other lateral forces cause transfer of bending moment between slab and column, a fraction of the moment given by

$$\left(1 - \frac{1}{1 + \frac{2}{3}\sqrt{\frac{c_1 + d}{c_2 + d}}}\right)$$

shall be considered transferred by eccentricity of the shear about the centroid of the critical section defined in Section 11.10.2. Shear stresses shall be taken as varying linearly about the centroid of the critical section and the shear stress v_u shall not exceed $4\sqrt{f_c'}$.

11.14—Special provisions for brackets and corbels

11.14.1—These provisions apply to brackets and corbels having a shear-span-to-depth ratio, a/d, of unity or less. When the shear-span-to-depth-ratio a/d is one-half or less, the design provisions of Section 11.15 may be used in lieu of Eq. (11-28) and (11-29), except that all limitations on quantity and spacing of reinforcement in Section 11.14 shall apply. The distance d shall be measured at a section adjacent to the face of the support, but shall not be taken greater than twice the depth of the corbel or bracket at the outside edge of the bearing area.

11.14.2—The shear stress shall not exceed

$$v_u = [6.5 - 5.1\sqrt{(T_u/V_u)}]\,[1 - 0.5\,(a/d)]$$
$$\times \{1 + [64 + 160\sqrt{(T_u/V_u)^3}]p\}\sqrt{f_c'} \quad (11\text{-}28)$$

where p shall not exceed $0.13\,f_c'/f_y$ and T_u/V_u shall not be taken less than 0.20. The tensile force T_u shall be regarded as a live load even when it results from creep, shrinkage or temperature change.

11.14.3—When provisions are made to avoid tension due to restrained shrinkage and creep, so that the member is subject to shear and moment only, v_u shall not exceed

$$v_u = 6.5[1 - 0.5\,(a/d)]\,[1 + 64\,p_v]\sqrt{f_c'} \quad (11\text{-}29)$$

where $p_v = (A_s + A_{vh})/bd$, but not greater than $0.20\,f_c'/f_y$, and A_{vh} shall not exceed A_s.

11.14.4—Closed stirrups or ties parallel to the main tension reinforcement having a total cross-sectional area A_{vh} not less than $0.50A_s$ shall be uniformly distributed within two-thirds of the effective depth adjacent to the main tension reinforcement.

11.14.5—The ratio $p = A_s/bd$ shall not be less than $0.04\,(f_c'/f_y)$.

11.15—Shear-friction

11.15.1—These provisions apply where it is inappropriate to consider shear as a measure of diagonal tension, and particularly in design of reinforcing details for precast concrete structures.

11.15.2—A crack shall be assumed to occur along the shear path. Relative displacement shall be considered resisted by friction maintained by shear-friction reinforcement across the crack. This reinforcement shall be approximately perpendicular to the assumed crack.

11.15.3—The shear stress v_u shall not exceed $0.2\,f_c'$, nor 800 psi.

11.15.4—The required area of reinforcement A_{vf} shall be computed by

$$A_{vf} = \frac{V_u}{\phi f_y \mu} \quad (11\text{-}30)$$

The design yield strength f_y shall not exceed 60,-000 psi. The coefficient of friction, μ, shall be 1.4 for concrete cast monolithically, 1.0 for concrete placed against hardened concrete, and 0.7 for concrete placed against as-rolled structural steel.

11.15.5—Direct tension across the assumed crack shall be provided for by additional reinforcement.

11.15.6—The shear-friction reinforcement shall be well distributed across the assumed crack and shall be adequately anchored on both sides by embedment, hooks, or welding to special devices.

11.15.7—When shear is transferred between concrete placed against hardened concrete, the interface shall be rough with a full amplitude of approximately ¼ in. When shear is transferred between as-rolled steel and concrete, the steel shall be clean and without paint.

11.16—Special provisions for walls

"**11.16.1** — Design for horizontal shear forces in the plane of the wall shall be in accordance with Section 11.16. The nominal shear stress, v_u, shall be computed by

$$v_u = \frac{V_u}{\phi\, td} \quad (11\text{-}31)$$

where d shall be taken equal to $0.8D$. A larger value of d, equal to the distance from the extreme compression fiber to the center of force of all reinforcement in tension, may be used when determined by a strain compatibility analysis."

11.16.2—Sections located closer to the base than a distance $D/2$ or one-half of the wall height, whichever is less, may be designed for the same v_u as that computed at a distance $D/2$ or one-half the height.

11.16.3—The shear stress carried by the concrete, v_c, shall not be taken greater than the lesser value computed from

$$v_c = 3.3\sqrt{f_c'} + \frac{N_u}{4Dt} \qquad (11\text{-}32)$$

and

$$v_c = 0.6\sqrt{f_c'} + \frac{D\left(1.25\sqrt{f_c'} + 0.2\,\dfrac{N_u}{Dt}\right)}{\dfrac{M_u}{V_u} - \dfrac{D}{2}} \qquad (11\text{-}33)$$

where N_u is negative for tension.

However, v_c may be taken as $2\sqrt{f_c'}$ if N_u is compression or Section 11.4.4 may be applied if N_u is tension.

"**11.16.4**—When v_u is less than $v_c/2$, reinforcement shall be provided in accordance with the provisions below or in accordance with Chapter 14. When v_u exceeds $v_c/2$, wall reinforcement for resisting shear shall conform to Sections 11.16.4.1 and 11.16.4.2.

"**11.16.4.1** The area of horizontal shear reinforcement shall be not less than than computed by Eq. (11-13). The ratio, p_h, of horizontal reinforcement area to the gross concrete area shall be at least 0.0025. The spacing of horizontal reinforcement shall not exceed $D/5$, $3t$, nor 18 in.

"**11.16.4.2** The ratio of vertical reinforcement area to gross concrete area shall be not less than

$$p_n = 0.0025 + 0.5\left(2.5 - \frac{H}{D}\right)(p_h - 0.0025) \qquad (11\text{-}34)$$

nor 0.0025, but need not be greater than the value of p_h required by Section 11.16.4.1. The spacing of vertical reinforcement shall not exceed $D/3$, $3t$, nor 18 in.

"**11.16.5**—The total design shear stress, v_u, at any section shall not exceed $10\sqrt{f_c'}$."

"**11.16.6**—Design for shear forces perpendicular to the face of the wall shall be in accordance with provisions for slabs in Section 11.10."

PART 5—STRUCTURAL SYSTEMS OR ELEMENTS

CHAPTER 13—SLAB SYSTEMS WITH MULTIPLE SQUARE OR RECTANGULAR PANELS

13.0—Notation

c_2 = Size of rectangular column, capital, wall, or bracket measured transverse to the direction in which moments are being determined

C = cross-sectional constant to define the torsional properties of edge beams and attached torsional members. See Eq. (13-7)

E_{cb} = modulus of elasticity for beam concrete

E_{cc} = modulus of elasticity for column concrete

E_{cs} = modulus of elasticity for slab concrete

h = height of column, center-to-center of floors or roof

H = ratio of flexural stiffness of beam section to the flexural stiffness of a width of slab bounded laterally by the center line of the adjacent panel (if any) on each side of the beam

$= \frac{E_{cb}I_b}{E_{cs}I_s}$

H_1 = H in the direction of l_1

H_2 = H in the direction of l_2

I_b = moment of inertia about centroidal axis of gross section of a beam as defined in Section 13.1.5

I_c = moment of inertia of gross cross section of columns

I_s = moment of inertia about centroidal axis of gross section of slab = $t^3/12 \times$ width of slab specified in definitions H and R

K_b = flexural stiffness of beam; moment per unit rotation

K_c = flexural stiffness of column; moment per unit rotation

K_{ec} = flexural stiffness of an equivalent column; moment per unit rotation (See Eq. 13-5)

K_s = flexural stiffness of slab; moment per unit rotation

K_t = torsional stiffness of torsional member; moment per unit rotation

K' = ratio of flexural stiffness of the columns above and below the slab to the combined flexural stiffness of the slab and beam at a joint taken in the direction moments are being determined

$= \frac{\Sigma K_c}{\Sigma (K_s + K_b)}$

K_e' = ratio of flexural stiffness of the equivalent column to the combined flexural stiffness of the slab and beam at a joint taken in the direction moments are being determined

$= \frac{K_{ec}}{\Sigma (K_s + K_b)}$

K_r' = minimum K' to satisfy Section 13.3.6.1 (a)

l_c = length of clear span, in the direction moments are being determined, measured face-to-face of supports

l_1 = length of span in the direction moments are being determined, measured center-to-center of supports

l_2 = length of span transverse to l_1, measured center-to-center of supports

M_o = total static design moment

R = ratio of torsional stiffness of edge beam section to the flexural stiffness of a width of slab equal to the span length of the beam, center to center of supports

$= \frac{E_{cb}C}{2E_{cs}I_s}$

t = over-all thickness of member, in.

w = design load per unit area

w_D = design dead load per unit area

w_L = design live load per unit area

x = shorter over-all dimension of a rectangular part of a cross section

y = longer over-all dimension of a rectangular part of a cross section

13.1 - Scope and definitions

13.1.1 - The provisions of this chapter govern the design of slab systems reinforced for flexure in more than one direction with or without beams between supports. Solid slabs and slabs with recesses or pockets made by permanent or removable fillers between ribs or joists in two directions are included under this definition. Slabs with paneled ceilings are also included under this definition provided the panel of reduced thickness lies entirely within the middle strips, and is at least two-thirds the thickness of the remainder of the slab, exclusive of the drop panel, and is not less than 4 in. thick. The thicknesses shall satisfy requirements of Sections 9.5.3.

PART 5—STRUCTURAL SYSTEMS OR ELEMENTS

CHAPTER 13—SLAB SYSTEMS WITH MULTIPLE SQUARE OR RECTANGULAR PANELS

13.0—Notation

c_2 = Size of rectangular column, capital, wall, or bracket measured transverse to the direction in which moments are being determined

C = cross-sectional constant to define the torsional properties of edge beams and attached torsional members. See Eq. (13-7)

E_{cb} = modulus of elasticity for beam concrete

E_{cc} = modulus of elasticity for column concrete

E_{cs} = modulus of elasticity for slab concrete

h = height of column, center-to-center of floors or roof

H = ratio of flexural stiffness of beam section to the flexural stiffness of a width of slab bounded laterally by the center line of the adjacent panel (if any) on each side of the beam

$= \dfrac{E_{cb}I_b}{E_{cs}I_s}$

H_1 = H in the direction of l_1

H_2 = H in the direction of l_2

I_b = moment of inertia about centroidal axis of gross section of a beam as defined in Section 13.1.5

I_c = moment of inertia of gross cross section of columns

I_s = moment of inertia about centroidal axis of gross section of slab = $t^3/12$ × width of slab specified in definitions H and R

K_b = flexural stiffness of beam; moment per unit rotation

K_c = flexural stiffness of column; moment per unit rotation

K_{ec} = flexural stiffness of an equivalent column; moment per unit rotation (See Eq. 13-5)

K_s = flexural stiffness of slab; moment per unit rotation

K_t = torsional stiffness of torsional member; moment per unit rotation

K' = ratio of flexural stiffness of the columns above and below the slab to the combined flexural stiffness of the slab and beam at a joint taken in the direction moments are being determined

$= \dfrac{\Sigma K_c}{\Sigma (K_s + K_b)}$

K_e' = ratio of flexural stiffness of the equivalent column to the combined flexural stiffness of the slab and beam at a joint taken in the direction moments are being determined

$= \dfrac{K_{ec}}{\Sigma (K_s + K_b)}$

K_r' = minimum K' to satisfy Section 13.3.6.1 (a)

l_c = length of clear span, in the direction moments are being determined, measured face-to-face of supports

l_1 = length of span in the direction moments are being determined, measured center-to-center of supports

l_2 = length of span transverse to l_1, measured center-to-center of supports

M_o = total static design moment

R = ratio of torsional stiffness of edge beam section to the flexural stiffness of a width of slab equal to the span length of the beam, center to center of supports

$= \dfrac{E_{cb}C}{2E_{cs}I_s}$

t = over-all thickness of member, in.

w = design load per unit area

w_D = design dead load per unit area

w_L = design live load per unit area

x = shorter over-all dimension of a rectangular part of a cross section

y = longer over-all dimension of a rectangular part of a cross section

13.1 - Scope and definitions

13.1.1 - The provisions of this chapter govern the design of slab systems reinforced for flexure in more than one direction with or without beams between supports. Solid slabs and slabs with recesses or pockets made by permanent or removable fillers between ribs or joists in two directions are included under this definition. Slabs with paneled ceilings are also included under this definition provided the panel of reduced thickness lies entirely within the middle strips, and is at least two-thirds the thickness of the remainder of the slab, exclusive of the drop panel, and is not less than 4 in. thick. The thicknesses shall satisfy requirements of Sections 9.5.3.

13.1.2—A column strip is a design strip with a width of $0.25l_2$ but not greater than $0.25l_1$ on each side of the column center line. The strip includes beams, if any.

13.1.3—A middle strip is a design strip bounded by two column strips.

13.1.4—A panel is bounded by column or wall center lines on all sides.

13.1.5—For monolithic or fully composite construction, the beam includes that portion of the slab on each side of the beam extending a distance equal to the projection of the beam above or below the slab whichever is greater, but not greater than four times the slab thickness.

13.1.6—The slab may be supported on walls, columns, or beams. No portion of a column capital shall be considered for structural purposes which lies outside the largest right circular cone or pyramid with a 90 deg vertex which can be included within the outlines of the supporting element.

13.2—Design procedures

13.2.1—A slab system may be designed by any procedure satisfying the conditions of equilibrium and geometrical compatibility provided it is shown that the strength furnished is at least that required considering Sections 9.2 and 9.3, and that all serviceability conditions, including the specified limits on deflections, are met.

13.2.2—A slab system, including the slab and any supporting beams, columns, and walls, may be designed directly by either of the procedures described in this chapter: (A) The Direct-Design Method (Section 13.3) or (B) The Equivalent-Frame Method (Section 13.4).

13.2.3—The slabs and beams shall be proportioned for the design bending moments prevailing at every section.

13.2.4—When unbalanced gravity load, wind, earthquake, or other lateral loads cause transfer of bending moment between slab and column, the flexural stresses on the critical section shall be investigated by a rational analysis, and the cross section proportioned according to the requirements of Section 11.13.2. Concentration of reinforcement over the column head by closer spacing or additional reinforcement may be used to resist the moment on this section. A slab width between lines that are $t/2$ on each side of the column or capital may be considered effective.

13.2.5—Design for the transmission of load from the slab to the supporting walls and columns through shear and torsion shall be in accordance with Chapter 11.

13.3—Direct design method

13.3.1 *Limitations*

13.3.1.1 There shall be a minimum of three continuous spans in each direction.

13.3.1.2 The panels shall be rectangular with the ratio of the longer to shorter spans within a panel not greater than 2.0.

13.3.1.3 The successive span lengths in each direction shall not differ by more than one-third of the longer span.

13.3.1.4 Columns may be offset a maximum of 10 percent of the span, in direction of the offset, from either axis between center lines of successive columns.

13.3.1.5 The live load shall not exceed three times the dead load.

13.3.1.6 If a panel is supported by beams on all sides, the relative stiffness of the beams in the two perpendicular directions

$$\frac{H_1 l_2^2}{H_2 l_1^2} \qquad (13\text{-}1)$$

shall not be less than 0.2 nor greater than 5.0.

13.3.1.7 Variations from the limitations of this section may be considered acceptable if demonstrated by analysis that the requirements of Section 13.2.1 are satisfied.

13.3.2 *Total static design moment for a span*

13.3.2.1 The total static design moment for a span shall be determined in a strip bounded laterally by the center line of the panel on each side of the center line of the supports. The absolute sum of the positive and average negative bending moments in each direction shall be not less than

$$M_o = \frac{w l_2 l_c^2}{8} \qquad (13\text{-}2)$$

Where the transverse span of the panels on either side of the center line of supports varies, l_2 shall be taken as the average of the transverse spans. When the span adjacent and parallel to an edge is being considered, the distance from the edge to the panel center line shall be substituted for l_2 in Eq. (13-2).

13.3.2.2 The clear span l_c shall extend from face to face of columns, capitals, brackets, or walls. The value of l_c used in Eq. (13.2) shall be not less than $0.65l_1$. Circular supports shall be treated as square supports having the same area.

13.3.3 *Negative and positive design moments*

13.3.3.1 The negative design moment shall be located at the face of rectangular supports. Circular supports shall be treated as square supports having the same area.

13.3.3.2 In an interior span, the total static design moment M_o shall be distributed as follows:

Negative design moment 0.65
Positive design moment 0.35

13.3.3.3 In an end span, the total static design moment M_o shall be distributed as follows:

Interior negative design moment .... $[0.75 - 0.10/(1 + 1/K_e')]$

Positive design moment .... $[0.63 - 0.28/(1 + 1/K_e')]$

Exterior negative design moment .... $[0.65/(1 + 1/K_e')]$

where K_e' is computed for the exterior column.

13.3.3.4 The negative moment section shall be designed to resist the larger of the two interior negative design moments determined for the spans framing into a common support unless an analysis is made to distribute the unbalanced moment in accordance with the stiffnesses of the adjoining elements.

13.3.4 *Design moments and shears in column and middle strips and beams*

13.3.4.1 The column strips shall be proportioned to resist the following portions in percent of the interior negative design moment:

l_2/l_1	0.5	1.0	2.0
$H_1 = 0$	75		75
$(H_1l_2/l_1) \geq 1.0$	90		45

Linear interpolations shall be made between the values shown.

13.3.4.2 The column strip shall be proportioned to resist the following portions in percent of the exterior negative design moment:

l_2/l_1		0.5	1.0	2.0
$H_1 = 0$	$R = 0$	100		100
	$R \geq 2.5$	75		75
$(H_1l_2/l_1) \geq 1.0$	$R = 0$	100		100
	$R \geq 2.5$	90		45

Linear interpolations shall be made between the values shown.

Where the exterior support consists of column or wall extending for a distance equal to or greater than three-quarters of the l_2 used to compute M_o, the exterior negative moment shall be considered to be uniformly distributed across l_2.

13.3.4.3 The column strip shall be proportioned to resist the following portions in percent of the positive design moment:

l_2/l_1	0.5	1.0	2.0
$H_1 = 0$	60		60
$(H_1l_2/l_1) \geq 1.0$	90		45

Linear interpolations shall be made between the values shown.

13.3.4.4 The beam shall be proportioned to resist 85 percent of the column strip moment if (H_1l_2/l_1) is equal to or greater than 1.0. For values of (H_1l_2/l_1) between 1.0 and zero, the proportion of moment to be resisted by the beam shall be obtained by linear interpolation between 85 and zero percent. Moments caused by loads applied on the beam and not considered in the slab design, shall be determined directly. The slab in the column strip shall be proportioned to resist that portion of the design moment not resisted by the beam.

13.3.4.5 That portion of the design moment not resisted by the column strip shall be proportionately assigned to the corresponding half middle strips. Each middle strip shall be proportioned to resist the sum of the moments assigned to its two half middle strips. The middle strip adjacent to and parallel with an edge supported by a wall shall be proportioned to resist twice the moment assigned to the half middle strip corresponding to the first row of interior supports.

13.3.4.6 A design moment may be modified by 10 percent provided the total static design moment for the panel in the direction considered is not less than that required by Eq. (13-2).

13.3.4.7 Beams with (H_1l_2/l_1) equal to or greater than 1.0 shall be proportioned to resist the shear caused by loads in tributary areas bounded by 45 deg lines drawn from the corners of the panels and the center line of the panels parallel to the long sides. For values of (H_1l_2/l_1) less than 1.0, the shear on the beam may be obtained by linear interpolation, assuming that for $H = 0$ the beams carry no load. In addition, all beams shall be proportioned to resist the shear caused by directly applied loads.

13.3.4.8 The shear stresses in the slab may be computed on the assumption that the load is distributed to the supporting beams in accordance with Section 13.3.4.7. The total shear occurring on the panel shall be accounted for.

13.3.4.9 The shear stresses shall satisfy the requirements of Chapter 11.

13.3.4.10 Edge beams or the edges of the slab shall be proportioned to resist in torsion their share of the exterior negative design moments.

13.3.5 *Moments in columns and walls*

13.3.5.1 Columns and walls built integrally with the slab system shall resist moments arising from loads on the slab system.

13.3.5.2 At an interior support, the supporting elements above and below the slab shall resist the moment specified by Eq. (13-3) in direct proportion to their stiffnesses unless a general analysis is made.

$$M = 0.08[(w_D + 0.5w_L)l_2l_c^2 - w_D'l_2'(l_c')^2] \Big/ \left(1 + \frac{1}{K_c'}\right) \qquad (13\text{-}3)$$

where w_D', l_2' and l_c' refer to the shorter span.

13.3.6 *Provisions for effects of pattern loadings*

13.3.6.1 Where the ratio of live to dead load exceeds 0.5, one of the following conditions shall be satisfied:

TABLE 13.3.6.1—MINIMUM K_r'

Live load / Dead load	Aspect ratio l_2/l_1	Relative beam stiffness, H				
		0	0.5	1.0	2.0	4.0
0.5	**0.5-2.0**	0	0	0	0	0
1.0	**0.5**	0.6	0	0	0	0
	0.8	0.7	0	0	0	0
	1.0	0.7	0.1	0	0	0
	1.25	0.8	0.4	0	0	0
	2.0	1.2	0.5	0.2	0	0
2.0	**0.5**	1.3	0.3	0	0	0
	0.8	1.5	0.5	0.2	0	0
	1.0	1.6	0.6	0.2	0	0
	1.25	1.9	1.0	0.5	0	0
	2.0	4.9	1.6	0.8	0.3	0
3.0	**0.5**	1.8	0.5	0.1	0	0
	0.8	2.0	0.9	0.3	0	0
	1.0	2.3	0.9	0.4	0	0
	1.25	2.8	1.5	0.8	0.2	0
	2.0	13.0	2.6	1.2	0.5	0.3

(a) The sum of flexural stiffnesses of the columns above and below the slab shall be such that K' is not less than the minimum K_r' specified in Table 13.3.6.1

(b) If the columns do not satisfy (a), the design positive bending moments in the panels supported by those columns shall be multiplied by the coefficient F determined from Eq. (13-4)

$$F = 1 + \frac{2 - w_D/w_L}{4 + w_D/w_L}\left(1 - \frac{K'}{K_r'}\right) \qquad (13\text{-}4)$$

13.4—Equivalent frame method

13.4.1 *Assumptions*—In design by the equivalent frame method the following assumptions shall be used and all sections of slabs and supporting members shall be proportioned for the moments and shears thus obtained.

13.4.1.1 The structure shall be considered to be made up of equivalent frames on column lines taken longitudinally and transversely through the building. Each frame consists of a row of equivalent columns or supports and slab-beam strips, bounded laterally by the center line of the panel on each side of the center line of the columns or supports. Frames adjacent and parallel to an edge shall be bounded by the edge and the center line of the adjacent panel.

13.4.1.2 Each such frame may be analyzed in its entirety, or, for vertical loading, each floor thereof and the roof may be analyzed separately with its columns as they occur above and below, the columns being assumed fixed at their remote ends. Where slab-beams are thus analyzed separately, it may be assumed in determining the bending moment at a given support that the slab-beam is fixed at any support two panels distant therefrom provided the slab continues beyond that point.

13.4.1.3 The moment of inertia of the slab-beam or column at any cross section outside of the joint or column capital may be based on the cross-sectional area of the concrete. Variation in the moments of inertia of the slab-beams and columns along their axes shall be taken into account.

13.4.1.4 The moment of inertia of the slab-beam from the center of the column to the face of the column, bracket, or capital shall be assumed equal to the moment of inertia of the slab-beam at the face of the column, bracket, or capital divided by the quantity $(1 - c_2/l_2)^2$ where c_2 and l_2 are measured transverse to the direction moments are being determined.

13.4.1.5 The equivalent column shall be assumed to consist of the actual columns above and below the slab beam plus an attached torsional member transverse to the direction in which moments are being determined and extending to the bounding lateral panel center lines on each side of the column.

The flexibility (inverse of the stiffness) of the equivalent column shall be taken as the sum of the flexibilities of the columns above and below the slab beam and the flexibility of the torsional members

$$\frac{1}{K_{ec}} = \frac{1}{\Sigma K_c} + \frac{1}{K_t} \qquad (13\text{-}5)$$

In computing the stiffness of the column K_c, the moment of inertia shall be assumed infinite from the top to the bottom of the slab-beam at the joint.

The attached torsional members shall be assumed to have a constant cross section throughout their length consisting of the larger of:

(a) A portion of the slab having a width equal to that of the column, bracket or capital in the direction in which moments are being considered

(b) For monolithic or fully composite construction, the portion of the slab specified in (a) plus that part of the transverse beam above and below the slab

(c) The transverse beam as defined in Section 13.1.5.

The stiffness K_t of the torsional member shall be calculated by the following expression:

$$K_t = \Sigma \frac{9E_{cs}C}{\ell_2(1-c_2/\ell_2)^3} \qquad (13\text{-}6)$$

where c_2 and ℓ_2 are related to the measured transverse spans on each side of the column. The constant C in Eq.(13-6) may be evaluated for the cross section by dividing it into separate rectangular parts and carrying out the following summation:

$$C = \Sigma\left(1 - 0.63\frac{x}{y}\right)\frac{x^3y}{3} \qquad (13\text{-}7)$$

If the values of $1/K_t$ as computed by Eq. (13-6) differ on the two sides of the column, the average of the two values should be used. Where beams frame into the column in the direction moments are being determined, the value of $1/K_t$ as computed by Eq. (13-6) shall be multiplied by the ratio of the moment of inertia of the slab without such beam to the moment of inertia of the slab with such beam.

13.4.1.6 Where metal column capitals are used, account may be taken of their contributions to stiffness and resistance to bending and to shear.

13.4.1.7 The change in length of columns and slabs due to direct stress, and deflections due to shear, may be neglected.

13.4.1.8 When the loading pattern is known, the structure shall be analyzed for that load. When the live load is variable but does not exceed three-quarters of the dead load, or the nature of the live load is such that all panels will be loaded simultaneously, the maximum moments may be assumed to occur at all sections when full design live load is on the entire slab system. For other conditions, maximum positive moment near midspan of a panel may be assumed to occur when three-quarters of the full design live load is on the panel and on alternate panels; and maximum negative moment in the slab at a support may be assumed to occur when three-quarters of the full design live load is on the adjacent panels only. In no case shall the design moments be taken as less than those occurring with full design live load on all panels.

13.4.2 *Negative design moment*—At interior supports, the critical section for negative moment, in both the column and middle strips, shall be taken at the face of rectilinear supports. At exterior supports provided with brackets or capitals, the critical section for negative moment in the direction perpendicular to the edge shall be taken at a distance from the face of the supporting element not greater than one-half the projection of the bracket or capital beyond the face of the supporting element. Circular or regular polygonal supports shall be treated as square supports having the same area.

13.4.3 *Distribution of panel moments*—Bending at critical sections across the slab-beam strip of each frame shall be distributed to the column strips, middle strips, and beams as specified in Section 13.3.4.

13.4.4 *Column moments*—Moments determined for the equivalent columns in the frame analysis shall be used in the design of the columns.

13.4.5 *Sum of positive and average negative moments*—Slabs within the limitations of Section 13.3, when designed by the equivalent frame analysis method, may have the resulting analytical moments reduced in such proportion that the numerical sum of the positive and average negative bending moments used in design need not exceed the value obtained from Eq. (13-2).

13.5—Slab reinforcement

13.5.1—The spacing of the bars at critical sections shall not exceed two times the slab thickness, except for those portions of the slab area which may be of cellular or ribbed construction In the slab over the cellular spaces, reinforcement shall be provided as required by Section

13.5.2—In exterior spans, all positive reinforcement perpendicular to the discontinuous edge shall extend to the edge of the slab and have embedment, straight or hooked, of at least 6 in. in spandrel beams, walls, or columns, where provided. All negative reinforcement perpendicular to the discontinuous edge shall be bent, hooked, or otherwise anchored, in spandrel beams, walls, or columns. Where the slab or columns to be developed at the face of the support according to the provisions of Chapter 12. Where the slab is not supported by a spandrel beam or wall, ow where the slab cantilevers the support, anchorage of reinforcement may be within the slab.

13.5.3—The area of reinforcement shall be determined from the bending moments at the critical sections but shall not be less than required by Section 7.13.

13.5.4—In slabs supported on beams having a value of H greater than 1.0, special reinforcement shall be provided at exterior corners in both the bottom and top of the slab. This reinforcement shall be provided for a distance in each direction from the corner equal to one-fifth the longer span.

The reinforcement in both the top and bottom of the slab shall be sufficient to resist a moment equal to the maximum positive moment per foot of width in the slab. The direction of the moment is parallel to the diagonal from the corner in the top of the slab and perpendicular to the diagonal in the bottom of the slab. In either the top or bottom of the slab, the reinforcement may be placed in a single band in the direction of the moment or in two bands parallel to the sides of the slab.

13.5.5—Where a drop panel is used to reduce the amount of negative reinforcement over the column of a flat slab, such drop shall extend in each direction a distance equal to at least one-third the span length in that direction and the projection below the slab should be at least one-quarter the thickness beyond the drop. For determining reinforcement, the thickness of the drop panel below the slab shall not be assumed to be more than one-fourth the distance from the edge of the drop panel to the edge of the column capital.

13.5.6—In addition to the other requirements of this section, reinforcement shall have the minimum lengths given in Fig. 13.5.6. Where adjacent spans are unequal, the extension of negative reinforcement as prescribed in Fig. 13.5.6 shall be based on the requirements of the longer span.

13.6—Openings in the slab system

13.6.1—Openings of any size may be provided in the slab system if it is shown by analysis that the strength furnished is at least that required with consideration of Sections 9.2 and 9.3, and that all serviceability conditions, including the specified limits on deflections, are met.

13.6.2—Openings conforming to the following requirements may be provided in slab systems not having beams without special analysis as required in Section 13.6.1.

(a) Openings of any size may be placed in the area within the middle half of the span in each direction, provided the total amount of reinforcement required for the panel without the opening is maintained

(b) In the area common to two column strips, not more than one-eighth of the width of strip in either span shall be interrupted by the openings. The equivalent of reinforcement interrupted shall be added on all sides of the openings

(c) In the area common to one column strip and one middle strip, not more than one-quarter of the reinforcement in either strip shall be interrupted by the opening. The equivalent of reinforcement interrupted shall be added on all sides of the openings

(d) The shear requirements of Chapter 11 shall be satisfied