Probabilistic and Transform Analyses of Amplify-And-Forward Wireless Relaying

by

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Abstract

Wireless cooperative networks have recently attracted the attention of many researchers as well as industry because such networks promise large diversity gains and increased capacity compared to other wireless communication systems. Accurate performance analysis of wireless cooperative systems is now important, since it enables the design of wireless communication systems with enhanced performance.

The thesis focuses on two main categories of systems: dual-hop amplify-and-forward (AF) relaying systems and multihop AF relaying systems. For dual-hop AF systems, general probabilistic analysis is developed to obtain novel, exact analytical expressions for the probability density function (PDF) and the cumulative distribution function (CDF) of the instantaneous end-to-end signal-to-noise ratio (SNR) of variable gain AF relaying systems operating over Rayleigh, Nakagami-m and Rician fading channels, as well as asymmetric systems operating over mixed Nakagami-m and Rician fading links. Performance metrics, such as the average symbol error probability, outage probability and ergodic capacity, are calculated using the derived PDF and CDF expressions. Dual-hop AF systems under adaptive power transmission are also studied and closed-form expressions for the ergodic capacity of such systems are obtained. Dual-hop AF systems with relay selection are also analyzed and various techniques of relay selection are considered. This probabilistic analysis permits the comparison of dual-hop AF systems with different relay selection criteria, and leads to the factors that should be considered in designing such systems.

For multihop AF systems, the generalized transformed characteristic function (GTCF) methods are proposed. The GTCF methods are new transform methods that constitute a general framework for exact analysis of generic multihop cooperative relaying systems. This

framework is valid for any modulation scheme, any fading channel distribution and any number of relays. The GTCF method is used in the thesis to obtain exact solutions for the ergodic capacity, outage probability and the average symbol error probability of multihop AF relaying systems. A strength of the GTCF approach is that it can be used with tractable computational effort. The thesis shows the cases where the strength of the GTCF method is paramount, and identifies as well the cases where the use of the GTCF method is not recommended. The thesis also studies the effects of the numbers of hops, as well as the parameters of the fading channels on the system performance in multihop AF relaying. The GTCF methods are also used to analyze multi-branch multihop AF systems, obtain exact performance metrics of such systems and study the effects of the numbers of branches as well as the numbers of relays per branch on performance metrics of these systems.

Finally, dual-hop AF systems with relay selection are compared to multihop AF systems, through studying the dependence of the performance metrics on the numbers of relays and the links' fading parameters. The purpose of this comparison is to identify the strengths and weaknesses of multihop AF configurations as opposed to relay selection in dual-hop AF configurations. As a result of this comparison, system design criteria are proposed.

Preface

The research included in this thesis was conducted under the supervision of, and in collaboration with Professor Norman C. Beaulieu. The findings of the research are published as co-authored articles in The Institute of Electrical and Electronics Engineers (IEEE) journals and conference proceedings.

Chapter 3 of this thesis is published as:

S. S. Soliman and N. C. Beaulieu, "The bottleneck effect of Rician fading in dissimilar dual-hop AF relaying systems," *IEEE Trans. Veh. Technol.*, vol. 63, no. 4, pp. 1957–1965, May 2014.

Chapter 4 of this thesis is published as:

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Chapter 7 of this thesis is published as:

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Chapter 8 of this thesis is published as:

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Chapter 9 of this thesis is published as:

S. S. Soliman and N. C. Beaulieu, "Dual-hop Vs multihop AF relaying systems," in *IEEE Global Telecommun. Conf.*, Dec. 2013, pp. 4299–4305.

To my wife, *Martina*

my Kids, *Jessica* and *Stephen*

my brother, *Fady*

And my parents, *Soliman* and *Mervat*

Acknowledgments

"*The Lord has done great things for us, And we are glad.*"

Psalms 126:3

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Table of Contents

List of Tables

List of Figures

Chapter 1

Introduction

Today's demand for wireless communication services is rapidly increasing, and customers require high quality services with high data rates, continuous connectivity, enhanced security, high levels of mobility and low costs. Meanwhile, wireless communication channels suffer from many factors that degrade the performance of the communication system in terms of the quality and the capacity[(1)] of transmission. Impairments in wireless channels may include signal multi-path fading, co-channel interference, adjacent channel interference, noise, signal shadowing and propagation loss. Faced with an ever increasing demand for user services and a finite spectrum resource, new and improved technologies are being developed to permit future wireless networks to sustain the demands and quality of these services.

The cooperative network concept represents a technique that is not only a method to provide extended coverage of the wireless communication systems, but also can significantly increase the system capacity and diversity gain[(2)] in wireless networks. Therefore, cooper-

[(1)] Capacity in general can refer to the number of users of a communication system or the amount of data transmitted over a communication channel. In the context of this thesis, channel capacity refers to the Shannon capacity, defined as the theoretical upper bound for the maximum error-free data transmission rate that a communication channel can support [1]. In this thesis, the term "ergodic capacity" is also used referring to the ensemble average of the maximum achievable information rate over the distribution of the communication channel [2].

[(2)] The term diversity gain or diversity order is used to express the increase in the limiting slope of the average error probability versus average signal-to-noise ratio (SNR) curve [1]. Mathematically, diversity gain can be defined in general as;

$$G_d = -\lim_{SNR\to\infty} \frac{\log P_e(SNR)}{\log SNR}$$

where $P_e(SNR)$ is the average error probability and G_d is the diversity gain [3].

ative systems are under intense study for current and future generation wireless standards [4]–[17]. For example, relaying is a key new functionality introduced in LTE-Advanced(3). In LTE-Advanced systems, the relay nodes are low power base stations that provide enhanced coverage and capacity, especially at cell edges [20].

1.1 Cooperative Systems

Cooperative communication systems are different from the conventional direct source-to-destination communication systems. In cooperative systems, idle nodes, whether stationary or mobile, cooperate with nearby active sources to send data signals to an intended receiver. The source node sends the data signal to one or more of the intermediate nodes, called relays. These relays in turn perform some distributed processing on the received signals before retransmitting the signal to the next node. The next node may be the destination, in which case the system is referred to as a dual-hop relaying system. When the next node is another relay, the system is referred to as a multihop relaying system. Cooperation can be employed when reliable direct communication(4) is not possible, because the source and destination are too far away from each other, or the connecting channel is severely faded. It can be used as well in the existence of direct access to enhance the overall system performance. Through cooperative communication, a virtual multi antenna system(5) is created, resulting in enhanced reliability, throughput, and broader coverage due to the effectiveness of such cooperative communication in mitigating pass loss and fading [21].

1.2 Thesis Motivation and Contributions

Cooperative communication systems have the potential for a wide range of practical applications such as local/home networking, environmental data collection and control, security

(3) LTE-Advanced is the mobile communication standard approved as the 4G system, and it is a major enhancement of the Long Term Evolution (LTE) standard [18]–[20].

(4) Note that direct communication does not necessarily mean the presence of line-of-sight between the source and the destination.

(5) Note that multi antenna systems were proposed to mitigate channel impairments and to increase data rates. However, cost, size and hardware limitations can prevent these systems from practical implementation, such as in wireless sensor networks.

monitoring, and crisis management applications. The common requirements of these applications are wide coverage area, large number of cheap nodes, availability of alternative communication paths in cases of link failures, low outage and error probabilities as well as high reliability. These requirements can be achieved by relays with group cooperation. For this reason much research has been devoted to the study of cooperative systems, performance analysis, developing relaying protocols, designing efficient receivers, and optimizing power allocation among nodes. An important percentage of the state-of-the-art research is devoted to amplify-and-forward (AF) relaying systems. Compared to other relay processing techniques, AF relaying is advantageous in terms of power consumption and complexity of implementation. Moreover, for applications in which the security of information is paramount, AF relaying provides a higher level of security than other processing techniques. Details about AF relaying and the relay processing techniques in general are discussed in Chapter 2. In spite of the fact that hundreds of research contributions are concerned with cooperative communication systems, a number of challenging problems are still unsolved, and need to be addressed before the standardization and implementation of cooperative systems can be in effect.

For these reasons, this thesis focuses on AF wireless cooperative communication systems. Throughout this thesis, novel probabilistic and transform analyses are proposed to address interesting open challenges. For dual-hop AF systems, theoretical probabilistic and statistical analysis is used as a mathematical tool to address some of the open challenges. For multihop AF systems, new transform methods are proposed and used with the standard characteristic function approaches to solve problems in relaying that previously were unsolved. Numerical calculations, where deemed practical, are also used in-part to evaluate integral expressions for which a closed-form solution is not tractable. Note that the trapezoidal rule [22,23] in calculating integrals is followed in evaluating such integral expressions. Monte Carlo simulations are also used, wherever needed, to verify the results obtained by the analytical and numerical methods. In the following, open challenges and the thesis contributions are outlined, followed by the significance of these contributions and their impact in theory and practice.

- **Performance analysis of dual-hop relaying systems**

  Relaying systems offer a technique that can be used in ad-hoc or cellular networks for performance enhancement and coverage extension with low complexity. Dual-hop AF relaying systems are the simplest of relaying systems, yet they can provide performance enhancement comparable to conventional direct communication systems. Performance metrics of dual-hop AF systems were evaluated in [5, 6, 24, 25] and many other publications. However, performance of these systems was evaluated in terms of bounds and/or approximations. Moreover, performance metrics were obtained for systems operating over Rayleigh or Nakagami-m fading links, and none of these publications considered the case of systems operating over Rician fading links.

  Although both the Nakagami-m and the Rician fading distributions converge to the Rayleigh fading distribution, they represent models with different physical origins and each of them describes the fading process from a different perspective. While the Nakagami-m model gives the best fit to land-mobile and indoor-mobile multipath propagation, the Rician model is used to model propagation paths consisting of one strong direct line-of-sight (LOS) component in addition to random weaker scattered components [1]. The study of systems operating over Rician fading links is as important as the study of systems operating over Nakagami-m fading links, because in many practical situations, the propagation link includes a strong LOS component and is more accurately fitted to the Rician model. For example, in an IEEE802.16j relay network, a link between a WiMAX base station, located at the top of a mountain, and a relay station is a Rician fading link because of the LOS component, while a link between a relay station and a mobile station, in an urban area, is best modeled by Nakagami-m fading(6). Although the authors in [1] proposed a one-to-one mapping between the m parameter of the Nakagami-m distribution and the K parameter of the Rician distribution, the differences in the performance metrics, of systems operating over Rician and Nakagami-m fading links, will not diminish for values of m and K chosen according to the "equivalence" suggested in [1, eq. (2.26)]. This is shown

(6) For this reason, relay stations in WiMAX systems are supposed to work in LOS as well as non-LOS propagation conditions [26].

by examples presented in Chapter 3 and Chapter 7 for dual-hop and multihop AF systems, respectively.

Relaying systems operating over Rician fading links were addressed in [27]–[29]. However, results reported therein are either limited to performance bounds, or approximations. Results on the ergodic capacity of similar systems were also obtained in [30] in forms of infinite summations that are mathematically involved due to the mathematical forms of their summands. However, the results in [30] can not be extended to obtain other performance metrics of these systems, such as outage probability and average error probability. In this thesis, Chapter 3 studies dual-hop AF systems operating over Rician fading links as well as dissimilar dual-hop AF systems operating over mixed Nakagami-m/Rician fading links. Solutions for the statistics as well as the performance metrics[(7)] of these systems are evaluated and compared to performance bounds in the literature. Solutions presented in Chapter 3 are precise exact results rather than bounds, and although some of those solutions are expressed in the form of infinite summations, the summands are basic functions that can be readily found in software packages and a finite number of terms of the summations are sufficient to get highly accurate results and achieve the required precision. Using these performance metrics, the LOS component present in Rician fading links is studied, and its effect on the overall system performance is considered.

Another challenge is transmission rate and power in relaying systems. Adaptive transmission[(8)] can be employed in relaying systems to provide enhanced utilization of the channel [32]. In such adaptive systems, the source adapts its rate and/or power based on the channel variations using feedback from the destination to the source. Multiple source-adaptive transmission techniques were studied in [33]–[36]. However, results in the literature are based either on the harmonic mean bound or the minimum SNR bound of the end-to-end SNR. Although performance metrics based on these bounds

[(7)] Statistics obtained in this thesis are the probability density function (PDF) and the cumulative distribution function (CDF). Other statistics such as the moment generating function (MGF) can be evaluated in terms of the PDF or the CDF. In this thesis also, the performance metrics considered are the outage probability, the average symbol error probability as well as the ergodic capacity.

[(8)] In this thesis, as well as in many publications, adaptive transmission refers to the rate of transmission and the power of transmission. Other techniques of adaptation can be considered, such as adaptive modulation at the source [31]. However, this is not the scope of this thesis.

may be acceptable for certain system parameters and operating conditions, they can also be highly unacceptable for other practical system parameters and operating conditions. In Chapter 4 of this thesis, exact expressions are obtained in closed-form for the capacity of dual-hop AF systems with adaptive channel inversion. The outage probability of these systems is also obtained. In addition, the effects of unbalanced links, where the average SNRs of the two hops are not necessarily similar, are studied. This study is motivated by the fact that power efficiency is an important factor for any communication system. Consequently, achieving the optimal power allocation is an important design criterion.

- **Relay selection and diversity systems**

  Diversity techniques have been used for decades to enhance the performance of communication systems. In relaying systems, cooperative diversity is shown to provide spatial diversity even when multiple antennas are not available at each node. Relay selection has been among the diversity techniques that attracted many scholars due to its low implementation complexity and comparable performance to more complicated techniques like maximal ratio combining (MRC) [37]–[44]. In systems employing relay selection techniques, only the best relay is selected for cooperation, and the selection algorithm is implemented at the destination, and a feedback channel is used to activate the selected relay. Different selection techniques can be considered and the pool of available paths varies according to the selection technique. Although many results of the performance metrics of relay selection systems are presented in the literature, these results are approximate and are not tight for some ranges of SNR of practical interest.

  There are two major drawbacks of the insufficient tightness of such results. The first of them is the magnitude of the inaccuracy of the performance bound relative to the exact performance. The second drawback is that those performance bounds overestimate the actual system performance. Such drawbacks disqualify those performance bounds as tools for conservative system design. An example in Chapter 5 shows that the exact average error probability of relay selection dual-hop AF systems, with practical system and channel parameters, is around an order of magnitude higher than that estimated by the performance bounds at low-to-medium SNR levels. The gap increases to two orders of magnitude at higher SNR levels.

In Chapter 5 and Chapter 6, the thesis investigates relay selection in AF relaying systems to obtain accurate results on performance metrics of these systems. Systems studied include those operating over Nakagami-m as well as Rician fading links. It is shown in Chapters 5 and 6 that performance bounds in the literature can be unacceptably loose[(9)], especially for less severe fading links and for high numbers of available relays. While in Chapter 5, only dual-hop links are considered for the selection pool, Chapter 6 considers also the direct link between the source and the destination for selection. Selection criteria based on local as well as global channel estimations are considered and results of both cases are put in comparison to investigate the practical benefit of each criterion.

- **Performance analysis of multihop and multihop diversity relaying systems**

  Performance metrics of multihop AF relaying systems have been studied in [7]–[14], as well as in other publications. However, performance of these relaying systems has been evaluated in terms of bounds or in terms of approximations. Unfortunately, the performance bounds can be loose for moderate-to-large values of SNR and for larger numbers of hops [8]. It is then important to find methods to obtain accurate solutions of the statistics and the performance metrics of multihop AF relaying systems.

  In Chapter 7, a transform technique is proposed to solve a family of problems whose exact solutions were intractable using the common PDF approaches or the known MGF approaches. This technique is referred to as the generalized transform characteristic function (GTCF) method. This method can be applied for systems operating over general fading links to transform the analysis into a new domain, so that the characteristic function of the newly transformed parameters is used to obtain exact statistics of the end-to-end SNR. Such statistics can be used to obtain the performance metrics of multihop AF systems.

(9) In some of the examples shown in the thesis, the performance gap between exact results obtained in this thesis and approximate results in the literature exceeds an order of magnitude.

Another open problem that was not thoroughly studied in the literature is the evaluation of performance metrics of multi-branch multihop AF systems. Although those systems were addressed in [45, 46], solutions therein are valid only for systems operating over lognormal fading channels [45] or for systems operating over Rayleigh fading links [46]. To-date, no exact results for performance metrics have been reported for such systems operating over Nakagami-m fading links nor for systems operating over Rician fading links. Chapter 8 solves this challenging problem using a modified version of the GTCF method. The modified GTCF (M-GTCF) method is developed in that chapter to reduce the computational complexity that can arise from using the GTCF method, as described in Chapter 7, to evaluate the performance of multihop diversity systems. Chapter 8 also studies the effects of the channel parameters and the system parameters on the performance metrics of the whole system.

- **Dual-hop or multihop?**

  Although the literature is rich in results on the analysis of multihop AF relaying systems, yet there are very few results comparing different variations of systems to each other. Such comparisons are necessary to discover the areas of application, and operating conditions under which each system is superior in performance and achieves the least costs in terms of complexity, resources and overhead. Chapter 9 of this thesis addresses this issue by comparing dual-hop AF systems with relay selection to single-branch multihop AF systems. This comparison is important because while it is known that dual-hop AF systems operate better in terms of the maximum achievable throughput, multihop AF systems experience enhanced performance as the number of intermediate relays increases. The comparison in Chapter 9 takes into consideration the possibility that individual links are independent but non-identically distributed to explore the actual dependence of performance enhancements on the channel fading parameters.

1.3 Impact and Significance of Thesis Contributions

The impact of the contributions of this thesis, detailed in Chapter 2, is multi-fold. From the theoretical engineering perspective, the thesis influences the theory of analysis and design of AF cooperative relaying systems through presenting novel analytical solutions, some of them in closed-form, for the statistics and performance metrics of dual-hop AF systems. Those proposed expressions do not only allow the accurate analysis of AF relaying systems, which was previously done through bounds and/or approximations that were not sufficiently accurate in many cases, but also they allow the investigation of the effects of system parameters on the performance. This in turns leads to an enhanced design and optimization of such systems.

Other significantly important contributions of this thesis are the GTCF methods. The GTCF methods proposed in this thesis represent new theoretical approaches which use a new transform domain to find solutions for a problem which was extremely difficult to solve. The new transform domain can be used not only for the problems under consideration in this thesis, but also for other similar problems of similar difficulty, and hence the GTCF methods can be considered a new engineering science transform method for the field of cooperative wireless communication.

From the practical perspective, the findings of this thesis have a significant impact on the implementation of AF relaying systems. As will be shown in the subsequent chapters, although AF relaying systems in general are promising systems with many potential applications, they can be of marginal benefits in some cases depending on the operating conditions. It is globally known that the enhancement of the communications system performance and achieving high-level application-based requirements occurs at the expense of added costs to the system implementation. In this thesis, system configurations with diminishing benefits are distinguished from those with practically valuable benefits to provide design engineers with a helpful tool in the cost-benefit analysis of AF relaying systems. Such an important tool is not only useful in cellular communications systems, but also in many application-driven cooperative systems.

1.4 Thesis Structure

This thesis is written in a paper-based format, with the exception of Chapter 2, which describes the basics of cooperative communications and presents a background on the related works. The rest of the chapters is a collection of peer reviewed research contributions(10) from my investigations on general dual-hop and multihop AF relaying systems. Consequently, each chapter has its own introduction, literature review and list of references. The chapters of this thesis can be grouped in two major parts; dual-hop AF systems and multihop AF systems. Contributions to the probabilistic analysis and design of dual-hop AF systems are presented in Chapters 3 to 6. Chapter 3(11) studies conventional dual-hop AF relaying systems where only one relay node is available to relay the radio signal to the destination. Dual-hop AF systems operating over Rician fading links are analyzed in Section 3.3 and dissimilar dual-hop AF systems operating over mixed Nakagami-m/Rician fading links are analyzed in Section 3.4. In Chapter 4(12), dual-hop AF systems under adaptive transmission are studied. One of the main source-adaptive transmission techniques is considered, namely, truncated channel inversion with fixed rate. Section 4.2 explains the model of the system under study in this chapter, and Section 4.3 includes analysis and derivation of the capacity expressions.

In Chapter 5(13), dual-hop AF relaying systems with relay selection are studied. Two relay selection criteria are considered in this chapter. In Section 5.3, systems adopting maximum end-to-end SNR relay selection technique are studied. In Section 5.4, dual-hop AF systems with partial relay selection (maximum relay-to-destination SNR relay selection is taken as example) are investigated. Section 5.5 uses numerical examples to verify the analytical results, compare the novel results to those in the literature, and to study the

(10) Note that the paper contributions used to structure this thesis are published in top international journals in the field of communications, such as the IEEE Transactions on Communications and the IEEE Transactions on Vehicular Technology. The included conference papers are also published in the proceedings and presented in top, flagship IEEE conferences, such as IEEE Global Communications Conference (GLOBECOM) and IEEE International Conference on Communications (ICC). These conferences usually attract a large number of scientific papers, from which only a small fraction (around 30%) are accepted for publication after rigorous peer review.

(11) The results in Chapter 3 are presented as a version of [47]. However, more related results were presented at the IEEE Global Communications Conference (GLOBECOM) 2012, and can be found in [48].

(12) The results in Chapter 4 are presented as a version of [49].

(13) The results in Chapter 5 are presented as a version of [50]. However, more related results were presented at the IEEE Vehicular Technology Conference (VTC) Fall 2012 [51], and at the IEEE International Conference on Computing, Networking and Communications (ICNC) 2013 [52].

effects of the number of relays in the selection pool size on system performance. At the end of this part of the thesis, dual-hop AF relaying systems with full selection diversity are investigated in Chapter 6(14).

The second part of the thesis focuses on multihop AF systems. Chapter 7(15) presents the GTCF method, and uses it to obtain solutions for multihop single-branch AF relaying systems. Section 7.2 explains the system model of multihop AF relaying networks. The GTCF method is explained in detail in Section 7.3 and Section 7.4. A study on the computational complexity of the GTCF method is presented in Section 7.5. In Section 7.6, simulation results and discussions are presented, and the impact of system and channel parameters on system performance is explored. The existence of a limiting distribution for the instantaneous end-to-end SNR of multihop AF systems is explored in Section 7.7. The rate of convergence and accuracy of this limiting distribution are discussed in the same section. Section 7.8 concludes the main results on multihop AF relaying systems.

In Chapter 8(16) the M-GTCF method is presented and is used to analyze multi-branch multihop AF systems. Section 8.2 explains the steps of the M-GTCF method and shows its advantage over other methods. Multi-branch multihop AF relaying systems are studied in Section 8.3, and numerical examples are shown in Section 8.4 to study the effects of the link fading parameters as well as the system parameters on the performance metrics. Finally, Chapter 9(17) compares dual-hop AF systems to multihop AF systems. More insights into dual-hop AF systems with relay selection are discussed in Section 9.2 to show the dependence of the diversity orders of such systems on the fading links' parameters. Similar insights are obtained for multihop AF systems in Section 9.3, and Section 9.4 compares dual-hop AF systems with relay selection to multihop AF systems in order to derive practical design criteria for AF relaying systems.

Finally, Chapter 10 summarizes the concluding remarks and discusses the possible future work using results presented in this thesis.

(14)The results in Chapter 6 are presented as a version of [53].
(15)The results in Chapter 7 are presented as a version of [54]. Related results were presented at the IEEE Global Communications Conference (GLOBECOM) 2011 [55], and at the IEEE Vehicular Technology Conference (VTC) Fall 2012 [56].
(16)The results in Chapter 8 are presented as a version of [57].
(17)The results in Chapter 9 are presented as a version of [58].

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Chapter 2

Background and Related Works

In this chapter, the main concepts of cooperative networks are described. In addition, a literature review of the related work and its limitations are presented, followed by a detailed description of the major thesis contributions.

2.1 Concepts of Cooperative Communications

2.1.1 Processing Techniques at Relays

The most popular and studied relaying techniques [1] are amplify-and-forward (AF), or equivalently non-regenerative relaying, and decode-and-forward (DF), called also regenerative relaying. In AF relaying systems, the relays amplify the received signals before forwarding to the next node, while in DF relaying systems, the relay nodes completely decode the received signals, regenerate them and forward the reconstructed signals to the next node. DF relaying requires more processing at the relay nodes than AF relaying. Complexity and power consumption are disadvantages of the DF scheme. AF relaying systems can be further classified based on the amplification factor used at the relaying nodes. If the amplification factor is adapted so that to provide constant power at the relay's transmitter output, the technique is referred to as variable-gain AF relaying. In order to implement variable-gain

AF, channel state information (CSI) should be known at the relays(1), that is why this technique is referred to as CSI-assisted AF relaying [3,4]. On the other hand, if the amplification factor at the relays is fixed, the relaying technique is referred to as fixed-gain AF relaying.

2.1.2 Cooperative Relaying Methods

Cooperative relaying methods are classified into fixed relaying, selective relaying and incremental relaying. In dual-hop cooperative systems using fixed relaying, the relay receives the data signal transmitted by the source, processes it according to the underlying processing technique and retransmits the processed signal to the destination. In multihop cooperative system, the procedure is performed at each relay in turn. Although increasing the number of hops helps improving the system performance, that improvement is limited to SNR gain. On the other hand, selective relaying methods can be used to enhance the system performance through diversity gain. Many selective relaying methods are proposed in literature. In the most common selective relaying methods, one relay is selected to relay the data signal to the destination. Selection methods usually require centralized implementation and the availability of CSI at the selection center. The most common selection criteria are:

a) Partial relay selection, where the relay selection is based on CSI about either the source-to-relay link or the relay-to-destination link.

b) Maximum end-to-end SNR relay selection, where the relay selection is based on CSI about the source-to-destination, or equivalently the end-to-end, link.

c) Full selection diversity, where the single-hop path from the source to the destination is considered in the selection process and end-to-end SNR is taken as the selection metric.

Incremental relaying is another form of selective relaying techniques, in which the single-hop link from the source to the destination is triggered for transmission if the received SNR over this link is above a certain SNR threshold, otherwise the relaying link is preferred for transmission. Although incremental relaying results in enhanced system performance, that enhancement occurs at the expense of added system complexity and overhead.

(1)Note that the required CSI at the relay can be obtained by employing conventional channel estimation techniques [2]

2.1.3 Modes of Operation of Relays

Modes of operation of relays, as of all communication nodes, can be classified as full-duplex and half-duplex operation. In full-duplex operation, relays can receive and transmit simultaneously, while in half-duplex operation, relays can either receive or transmit in a given time slot over a given frequency band. Due to the broadcasting nature and the practical difficulty of electrically isolating transmit and receive circuitry, full-duplex operation is not yet practically implemented. Instead, half-duplex operation is commonly used in relaying systems. Moreover, time division multiple access is employed to achieve orthogonality and reduce interference.

2.2 General System Configurations and Models

In this section, cooperative AF systems studied in this thesis are explained. Fig. 2.1 shows the simplest dual-hop cooperative system, where a source node, S, and a destination node, D, communicate through one intermediate relay node, R. The source node, S, transmits the data signal to the relay node, R, over the first (source-to-relay) link, $S - R$. Assuming that the source node transmits a signal $s(t)$, with average source transmitter power, P_0, and that the source-to-relay link experiences channel fading of amplitude α_1, the signal, $r(t)$, received at the relay node is expressed as

$$r(t) = \alpha_1 \times \sqrt{P_0} \times s(t) + n_1(t) \tag{2.1}$$

where $n_1(t)$ is additive white Gaussian noise (AWGN), with single-sided power spectral density (PSD) N_{0_1}, at the relay node. The relay node, in turn, amplifies the received signal and transmits the amplified signal to the destination node D over the second (relay-to-destination) link, $R - D$. The signal, $d(t)$, received at the destination node can be then expressed as

$$d(t) = \alpha_2 \times \left(A \left[\alpha_1 \times \sqrt{P_0} \times s(t) + n_1(t) \right] \right) + n_2(t) \tag{2.2}$$

where A is the amplification factor at the relay node, α_2 is the fading amplitude of the relay-to-destination link, and where $n_2(t)$ is AWGN with single-sided PSD N_{0_2},at the destination node. The instantaneous end-to-end SNR at the destination node can be then written as

$$\begin{aligned}\gamma_t &= \frac{P_0|\alpha_2 A \alpha_1|^2}{|\alpha_2 A|^2 \times N_{0_1} + N_{0_2}} \\ &= \frac{\dfrac{P_0|\alpha_1|^2}{N_{0_1}} \times \dfrac{P_1|\alpha_2|^2}{N_{0_2}}}{\dfrac{P_1|\alpha_2|^2}{N_{0_2}} + \dfrac{P_1}{A^2 N_{0_1}}}.\end{aligned} \tag{2.3}$$

In the case of variable-gain AF relaying systems, the amplification factor at the relay node is determined so as to provide a fixed output at the relay node and is given as

$$A = \sqrt{\frac{P_1}{P_0\left|\alpha_1\right|^2 + N_{0_1}}} \tag{2.4}$$

where P_1 is the average transmitter power at the relay node. Substituting the amplification factor (2.4) into (2.3) results in

$$\gamma_t = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + 1} \tag{2.5a}$$

where $\gamma_i,\ i = 1, 2$ represents the instantaneous received SNR on the i^{th} link defined as

$$\gamma_i = \frac{P_{i-1}}{N_{0_i}}\left|\alpha_i\right|^2. \tag{2.5b}$$

For multihop AF relaying systems, shown in Fig. 2.2. The source node, R_0, and the destination node, R_N, communicate through $N-1$ intermediate relay nodes, $R_1, R_2, \cdots, R_{N-1}$. The source node, R_0, transmits the data signal to the first relay, R_1, over the first link. The first relay, in turn, amplifies the received signal and transmits the amplified signal to the next node, and so on to the destination node. Similar to (2.4), the amplification factor, A_i, at the i^{th} relay node, R_i, is given as

$$A_i = \sqrt{\frac{P_i}{P_{i-1}\left|\alpha_i\right|^2 + N_{0i}}} \tag{2.6}$$

where $P_i,\ i = 0, 1, \cdots, N-1$, is the transmitter power at the i^{th} node, $\alpha_i,\ i = 1, 2, \cdots, N$, is the amplitude fading gain of the i^{th} link, between node R_{i-1} and node R_i, and where $N_{0i},\ i = 1, 2, \cdots, N$, is the single-sided PSD of AWGN at the i^{th} node. Extending the input/output signal model in (2.1)-(2.3) to the case of multihop AF systems, and using the variable-gain amplification factor (2.6), the instantaneous end-to-end received SNR at the destination, γ_t, is given as

$$\gamma_t = \left[\prod_{i=1}^{N} \left(1 + \frac{1}{\gamma_i} \right) - 1 \right]^{-1} \tag{2.7}$$

where $\gamma_i,\ i = 1, 2, \cdots, N$ represents the instantaneous received SNR over the i^{th} link and is defined in (2.5b).

2.3 Related Work and Limitations

2.3.1 Conventional AF Relaying Systems

- **Performance Bounds for Multihop AF Systems**

Much research and publication has focused on the performance of AF relaying systems. The average probability of error and outage probability of dual-hop transmission over Rayleigh fading channels were studied in [5, 6]. In [5], the authors assumed semi-blind relays, which do not have access to the instantaneous CSI of the first hop, but have only statistical CSI about the first hop. In [6], the authors presented an approximation for the end-to-end received signal-to-noise ratio (SNR) using the harmonic mean of two independent exponential random variables and in [7], the harmonic mean approximation was used to obtain an approximate expression for the moment generating function (MGF) of the reciprocal of the end-to-end SNR for multihop transmission over Nakagami-m fading channels. An approximation to outage probability was then obtained via numerical inversion of the Laplace transform. Performance bounds on outages and the average error probabilities of AF transmission over Nakagami-m fading channels were also obtained in [8]. In [9], single-integral expressions for the average error probability of general wireless communication systems were obtained in terms of the MGF of the reciprocal of the instantaneous end-to-end SNR. This general framework was applied to AF relaying systems. The authors in [10] used the same MGF approach to find expressions for the average error probabilities and outage probabilities for some special cases of Nakagami-m fading channels, where the fading index of each hop is an odd multiple of one-half. The MGF approach was also adopted in [4] and [11]. However, all the results in [4,8]–[11] are based on an approximate harmonic mean expression for the instantaneous end-to-end SNR. It was found that the bounds based on the direct harmonic mean approximation of the individual per hop instantaneous SNRs, are not tight for small-to-moderate values of SNR, and for Nakagami-m fading channels for large values of m [8, 9]. Moreover, increasing the number of hops may decrease the tightness of those bounds.

- **Performance Approximations for Multihop AF Systems**

In [12], a new approximation was proposed, in which the harmonic mean of the minimum of the SNRs of the first P hops and the minimum SNRs of the remaining hops was used to obtain new bounds. The new bounds were obtained for independent and non-identically distributed Rayleigh fading channels and for independent and identically distributed Nakagami-m fading channels for integer values of m. It was concluded in [12] that the proposed bounds outperform the existing bounds, particularly for severe fading environments and for medium-to-large SNRs. In [13] and [14] another new approximation was proposed that has the same computational complexity as the previous bounds in the literature, while it is more accurate, especially for small-to-medium values of SNR. The key idea was to use a scaling factor to reduce the approximation errors at small-to-medium values of SNR. Compared to previous bounds, the new approximation results in more accurate prediction of the performance. All of the previously mentioned work reports approximations because no exact solution for the end-to-end received SNR of multihop AF relaying was known at that time.

- **Performance of Fixed-Gain Dual-Hop AF Systems**

Reference [15] presented a closed-form expression for the average bit error probability of AF fixed gain dual-hop relaying systems operating over Nakagami-m links for integer values of m. The expression presented in [15] involves a double-summation of complicated operands. The asymptotic average bit error probability, at large SNR, for arbitrary m was also evaluated in [15]. In [16] also, closed-form expressions for the probability density function (PDF) and the cumulative distribution function (CDF) of dual-hop AF relaying systems operating over Nakagami-m fading channels with integer values of m were derived. However, those expressions were not used to evaluate performance metrics of AF systems with variable gain amplification factor, such as the average symbol error probability and the ergodic capacity.

- **Performance of Dual-Hop AF Systems With Selection Diversity**

The authors in [17] studied the performance of beamforming in dual-hop AF relay networks operating over Rayleigh fading links. In the analysis conducted in [17], the authors obtained closed-form expressions for the CDF and the PDF of dual-hop AF networks. However, those expressions are valid only for Rayleigh fading links, and the PDF expression is valid only in the case of the harmonic mean bound. In [18], the authors obtained closed-form expressions for the average bit error rate and outage probability for differential equal gain combining in cooperative diversity networks. Those expressions are valid only for the dual-hop case and for Nakagami-m fading links with integer values of the fading parameter m. However, the results in [17, 18] were not extended to the case of multihop AF relaying systems. In [19], dual-hop AF systems with selection diversity were studied and an exact closed-form expression for the outage probability was derived for the case of Nakagami-m fading links with integer values of m, yet that expression was not used to obtain other performance metrics.

- **Ergodic Capacity of Multihop AF Systems**

Performance metrics, especially the ergodic capacity, of dual-hop and multihop AF relaying systems were also studied in [20]–[23]. In [20], the authors used an infinite series for the logarithmic function to compute the ergodic capacity of multihop AF relaying systems operating over generalized fading links. The expression for the ergodic capacity in the case of Rician fading links involves *(N+1)*-fold nested infinite summations of arguments which involve Whittaker or Meijer-G functions for N-hop AF relaying systems. The authors in [20] also presented an alternative framework for the special case of dual-hop AF relaying systems operating over Nakagami-m and generalized-K fading links. However, no special results were reported for the case of Rician fading links. In [21], the authors used a similar approach to that used in [20] to obtain a closed-form expression for the ergodic capacity of dual-hop AF relaying systems with relay selection. Although the result in [21] can be used for the case of dual-hop AF relaying without relay selection, it is limited to systems operating over Rayleigh fading links, and generalizing the results to the case of Rician fading links seems intractable. The authors in [22] have proposed an error analysis for the average error probability in multihop AF relaying systems. However the results in [22] are based on the harmonic mean approximation of the end-to-end SNR and are only valid for systems operating over Nakagami-m links. In [23], the authors applied the harmonic mean bound and used numerical inversion of the Laplace transform to obtain an upper bound to the outage probability of dual-hop AF systems with maximum-ratio combining in Rician fading environments.

2.3.2 Asymmetric Dual-Hop AF Relaying Systems

Asymmetric dual-hop AF relaying systems are dual-hop AF systems in which the individual hops' fadings follow different fading distribution families. In this thesis, the particular case of interest is the case of asymmetric (sometimes referred to as dissimilar or mixed) dual-hop systems where one of the fading hops follows a Nakagami-m distribution, while the other follows Rician fading links. These systems are of practical significance, due to the difference in nature of the underlying models for Nakagami-m and Rician fading distributions. Such AF relaying systems were partially studied in [24]–[26]. The authors in [24] obtained an exact expression for the outage probability of dual-hop AF systems operating over mixed Rayleigh/Rician fading channels; however, that exact expression was not used to obtain other performance metrics. Instead, the authors used an approximation of the end-to-end SNR to obtain the average error probability of the system. The authors also presented a lower bound on the average error probability based on approximating the end-to-end SNR by the minimum value of the SNRs on the individual fading links. In [25] also, the authors studied the case of mixed Rayleigh/Rician fading channels for dual-hop, fixed gain relaying systems, and obtained infinite-series representations for the outage probability and the average bit error probability of these systems. The case of dual-hop AF systems with mixed Nakagami-m and Rician fading links was studied in [26]. The authors in [26] adopted the approximation used in [24] to approximate the outage probability and the average error probability of the proposed system. Since the results presented in [24] and [26] are based on approximations of the instantaneous end-to-end SNR, they are not exact.

2.3.3 AF Relaying Systems With Adaptive Transmission

All the previous work considered fixed rate and fixed power transmission. Adaptive transmission can be employed in cooperative wireless systems to provide enhanced utilization of the channel [27]. In [28]–[31], the authors used adaptive source transmission with AF relaying. The authors in [28] derived upper bounds to adaptive transmission for dual-hop AF systems with MRC at the destination in the case of Rayleigh fading links. In [29], the authors developed a framework based on the MGF approach to find bounds to the ergodic capacity of adaptive transmission AF systems operating over Nakagami-m fading links. The performance bounds in [28] and [29] were based on the minimum SNR bound of the end-to-end SNR. The authors in [30] used the harmonic mean bound to obtain theoretical approximations for the capacity, as well as the outage probability, of multihop AF systems under adaptive transmission. In [31], the authors studied the effect of outdated channel information on the capacity of dual-hop AF selection relaying systems under adaptive transmission in the case of Rayleigh fading links. However, the authors in [31] used the minimum SNR bound for the end-to-end SNR.

2.3.4 AF Relaying Systems With Relay Selection

This subsection presents literature review on relaying systems with relay selection. Relay selection was proposed to enhance the performance of AF relaying systems and was studied in [32]–[46]. In [32], the authors developed a distributed relay selection method based on measurements of the instantaneous channel conditions. That selection method requires no topology information. The authors in [33] and [34] approximated the end-to-end SNR, for each possible relay connection, by the minimum value of the SNRs of the source-to-relay and relay-to-destination links. This selection criterion can be referred to as "Best Worse Channel Selection" [46]. The approximate SNR was then used to obtain closed-form approximations for the average symbol error probability of AF selection systems operating over Nakagami-m fading channels and over generalized gamma fading channels. In [35], the authors examined the problem of opportunistic communications in dual-hop AF relaying systems, and they considered the presence of the direct path link between the source and the destination. The authors in [35], used MRC at the receiver for combining signals from the direct path and the best dual-hop path. However, the authors used the harmonic mean approximation for the end-to-end SNR, and therefore did not obtain exact solutions for the performance metrics of such systems. This selection criterion can be referred to as "Best Harmonic Mean Selection" [46].

The authors in [36] analyzed the same system as in [35], but they used the minimum SNR bound for their analysis to obtain bounds to the performance metrics rather than exact results. Reference [37] also used the minimum SNR bound to get bounds on the performance metrics of dual-hop systems with relay selection, but they proposed a K^{th} opportunistic relaying scheme, where the K^{th} best relay is selected to relay the radio signal, rather than the best relay. Note that, while the authors in [35] examined systems operating over Nakagami-m fading channels, the analysis in [36] and [37] was applied only to systems operating over Rayleigh fading channels. In [38], the authors used the minimum SNR approximation to obtain lower bounds and asymptotic expressions for the average error probability of the AF relaying systems with full selection diversity, where the direct single-hop path between the source and the destination is considered in the path selection set. In other work, such as [21], the authors used a partial relay selection method, which is a version of the "Nearest Neighbor Selection" criterion [46], based on measurements of the relay-to-destination channel conditions only. This method of relay selection requires fewer channel measurements as it depends on the estimated conditions of the relay-to-destination links only. While the authors in [32]–[45] studied only single relay selection, single as well as multiple relay selection were rigorously studied in [46].

2.4 Detailed Thesis Contributions

The work presented in this thesis contributes to the exact statistical and performance analysis of dual-hop AF systems, as a special case, and multihop AF systems, as a general case.

2.4.1 Contributions to Dual-Hop AF Systems

Precise analytical[(2)] expressions, for the exact PDF and the exact CDF of the end-to-end SNR of dual-hop AF relaying systems operating over Rician fading links, or asymmetric dual-hop AF systems operating over mixed Nakagami-m/Rician fading links have not been previously obtained. Consequently, exact performance metrics of such systems were not available. In this thesis, dual-hop AF relaying systems operating over Rician/Rician fading channels and those operating over mixed Nakagami-m/Rician are studied. The main contributions in this aspect are:

1. A precise analytical expression for the PDF of the exact end-to-end SNR of dual-hop AF relaying systems operating over Rician fading links is obtained. This is different from the expression obtained in [23], because it is based on the exact expression of the end-to-end SNR rather than the harmonic mean bound used in [23].

2. An exact expression for the outage probability is also derived. This is also different from the expression obtained in [23] because it is exact, and does not require numerical calculations of inverse Laplace transform. Although the authors in [16] derived closed-form expressions for the PDF and outage probability of dual-hop AF systems operating over Rayleigh fading links and Nakagami-m fading links, no closed-form expressions for these statistics were reported in literature for the case of Rician fading links.

[(2)] An analytical expression is a mathematical expression, which is constructed using basic well-known operations and functions. In general, the concept of well-known functions varies according to the context. In engineering problems, well-known functions are more numerous than in other contexts. However, it is still generally accepted to consider expressions including Bessel functions, gamma functions and often infinite series as analytical expressions. This definition of analytical expressions is used in literature for infinite summations of readily available expressions such as in [3]. As a special case, a closed-form expression is expressed in terms of finite number of well-known functions. However, many publications in the field of wireless communications consider that expressions involving infinite summations of well-known functions are also closed-form expressions, e.g. [20, Table I].

3. A solution for the average symbol error probability is obtained, taking into account the exact end-to-end SNR. Exact solutions for the outage probability and the average symbol error probability have not been previously reported.

4. A solution for the ergodic capacity is obtained. This solution is obtained in terms of the modified Bessel function of the second kind and has the same formal computational complexity as the solution presented in [20].

5. Precise expressions for the PDF and the CDF of the end-to-end SNR of dissimilar dual-hop AF systems. Note that outage probability derived in [24] represents only a special case of the expression presented in this thesis.

6. Exact performance analysis of dissimilar dual-hop AF systems. Solutions derived for the average symbol error probability and outage probability represent the first results that are based on the exact expression of the end-to-end SNR. Solution for the ergodic capacity represents the first result for the ergodic capacity performance of dual-hop AF systems operating over mixed fading channels. Note that this thesis studies dual-hop, variable-gain AF systems, rather than the fixed-gain AF systems studied in [25].

The thesis also studies dual-hop AF relaying systems under adaptive transmission. In here, an adaptive scheme consisting of channel inversion with fixed rate is adopted because it has the lowest implementation complexity among other adaptive schemes, as the source only adjusts its transmission power. The thesis contributions to these systems are:

1. Derivation of the first exact closed-form expression of the capacity of dual-hop AF relaying systems with adaptive channel inversion in the case of Nakagami-m fading links. An exact closed-form expression for the outage probability of these systems is also derived. These expressions are the first exact expressions in the literature that are based on the exact end-to-end SNR.

2. The effects of unbalanced links, where the average SNRs of the first and the second links are not necessarily similar, are studied. System performance is compared to that of systems with balanced links.

Another contribution of the thesis is the study of dual-hop AF systems with relay selection. Relay selection in dual-hop AF systems can be based on various selection criteria as explained in the previous sections. In this thesis, three main selection criteria are considered, and the thesis contributes the first exact statistical analysis and performance results for dual-hop AF systems with relay selection as follows:

1. Exact closed-form expressions for the PDF and the CDF of the end-to-end SNR of dual-hop AF relaying systems with partial relay selection, operating over Nakagami-m fading channels. Also for dual-hop AF systems with maximum end-to-end SNR relay selection, and dual-hop AF systems with full selection diversity, precise analytical results for the statistics of the end-to-end SNR are obtained for systems operating over Nakagami-m and Rician fading links as well as for asymmetric systems operating over mixed Nakagami-m/Rician fading links. Both cases of independent and identically distributed (*i.i.d.*) links as well as independent and non-identically distributed (*i.ni.d.*) links are considered.

2. The derived expressions are used to obtain precise results for outage probability, the average symbol error probability and ergodic capacity, of dual-hop AF systems with relay selection.

3. Studying the effects of increasing the numbers of relays in the relay selection pool, as well as the effects of the parameters of the fading links on the performance metrics in relaying systems with relay selection.

4. Comparing the exact results obtained in the thesis to performance bounds presented previously in the literature such as those presented in [32]–[34, 38].

5. Comparisons between dual-hop AF systems without relay selection and those with different relay selection criteria.

6. Recommendations to designers of practical systems with relay selection are presented. These recommendations are to distinguish between operating conditions that can result in worthwhile system improvements, and other conditions which result in diminishing benefits at the expense of excess costs of resources.

2.4.2 Contributions to Multihop AF Systems

For the more general case of multihop AF systems, a novel theoretical approach is developed for the analysis and design of multihop AF relaying systems. A generalized transformed characteristic function (GTCF) approach is proposed to obtain an exact integral solution for the PDF of the instantaneous end-to-end SNR. This solution leads to exact integral solutions for the average symbol error probability, outage probability and ergodic capacity. These solutions are valid for any fading distribution. It is important to confirm that the GTCF is different from the conventional MGF approaches. It uses a new transform domain to solve a family of problems whose exact solutions were intractable using the common probability density approaches or the MGF approaches. It is shown in this thesis that performance bounds based on the harmonic mean approximation for the end-to-end SNR are only acceptable for large values of SNR, and can become dramatically inaccurate for small-to-medium values of SNR. This emphasizes the value of exact analysis of multihop AF systems. Note that the use of transform methods to solve difficult problems in a transform domain is a key approach in wireless communication theory (e.g. Fourier and Laplace transforms) and other recent work [47, 48] has contributed new transform methods. The method proposed represents a novel transform technique that can be used to solve heretofore difficult problems in cooperative wireless networks.

In this thesis, The GTCF method is proposed and is used to obtain exact performance metrics of multihop AF relaying networks. A strength of the GTCF approach is that it can be used for any fading channel distribution with tractable computational effort. A computational complexity analysis is presented to show such strength. Examples consider Rayleigh, Rician and Nakagami-m fading distributions. Both cases of *i.i.d.* fading links and *i.ni.d.* fading links are examined. The GTCF is also used for the cases of asymmetric multihop systems, where the individual links follow different fading distribution families. This thesis also studies the effects of the numbers of hops on the system performance of multihop AF relaying systems. This study shows that there is a trade-off for the improvement of the average symbol error probability and outages against the achievable system throughput.

In this thesis also, a modification to the GTCF method is proposed to reduce the computational complexity of the GTCF method. For multihop AF relaying systems operating over arbitrary Nakagami-m or Rician fading links, the GTCF method is used to obtain two-fold integral expressions for outage probability, average symbol error probability and the ergodic capacity. For systems involving selection diversity, the complexity of the GTCF method increases to three-fold integral calculation. The modified GTCF (M-GTCF) proposed in this thesis reduces the computational complexity of outage probability to a single-fold semi-infinite integral for multihop AF systems, as well as for AF systems involving selection diversity such as multi-branch multihop AF systems with full selection diversity. The M-GTCF method also reduces the computational complexity in the calculation of the average symbol error probability of systems with diversity selection into two-fold integration rather than three-fold integration.

The M-GTCF method is used to obtain exact solutions for the performance metrics of multi-branch multihop AF relaying systems, operating over Nakagami-m and Rician fading links. Multi-branch multihop AF systems were studied in [49, 50]. However the results obtained in [49] are valid only for systems operating over lognormal fading channels, and require computational complexity higher than that required for the GTCF and the M-GTCF methods. Also the results in [50] are valid only for systems operating over Rayleigh fading channels, and are based on the harmonic mean approximation of the end-to-end SNR. It will be shown throughout the thesis that performance bounds based on the harmonic mean approximation for the end-to-end SNR is only acceptable for large values of SNR, and can become dramatically inaccurate for small-to-medium values of SNR. To the best of the author's knowledge, no exact results for performance metrics have been reported in the literature for such systems operating over Nakagami-m fading channels with arbitrary fading parameters, nor for systems operating over Rician fading channels. This emphasizes the value of exact analysis of multihop AF systems using the GTCF and the M-GTCF transform methods. With the help of the results obtained using the M-GTCF method, the effects of the fading parameters, the numbers of hops per branch, and the numbers of branches on the performance metrics of multi-branch multihop AF systems are studied. General cases where the different branches have different numbers of hops, and where the different hops are non-identically distributed fading links are considered.

Finally, the thesis compares dual-hop AF relaying systems with relay selection to multihop AF systems. The purpose of this comparison is to provide design recommendations for the practical planning of wireless cooperative networks. Note that the general topology setup of a group of intermediate relay nodes is a multi-branch multihop topology. The thesis compares only the two extreme cases of multi-branch dual-hop systems with relay selection[3] and single-branch multihop systems.

[3] Note that relay selection in this case is equivalent to branch selection, since each branch includes only one intermediate relay node.

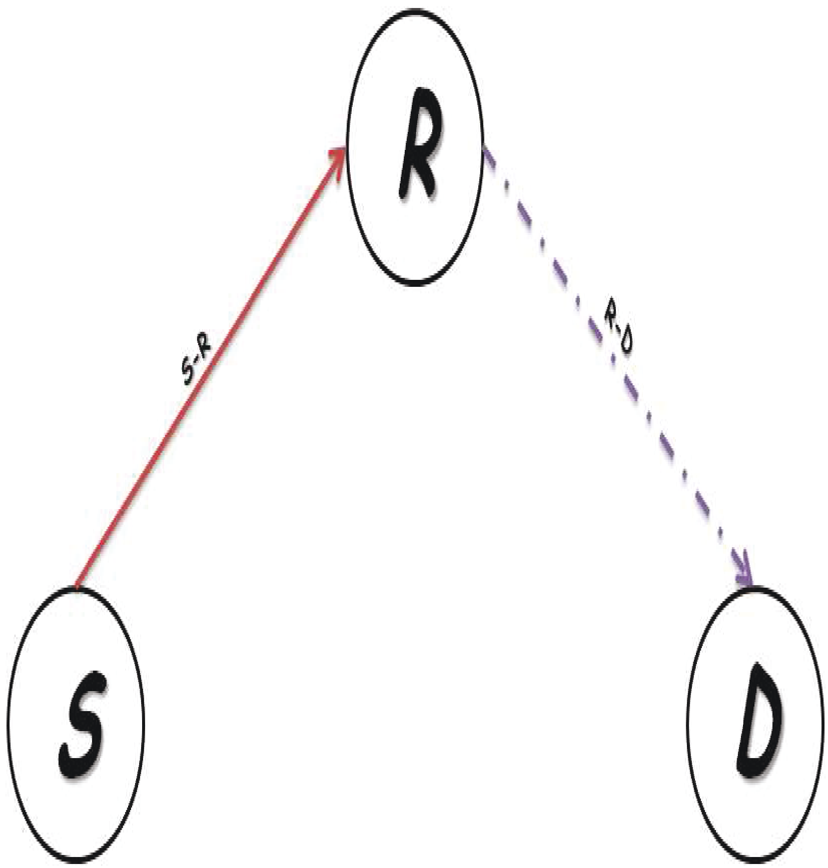

Figure 2.1: Dual-hop AF relaying systems. The source node, S, and the destination node, D, communicate through the intermediate relay node, R.

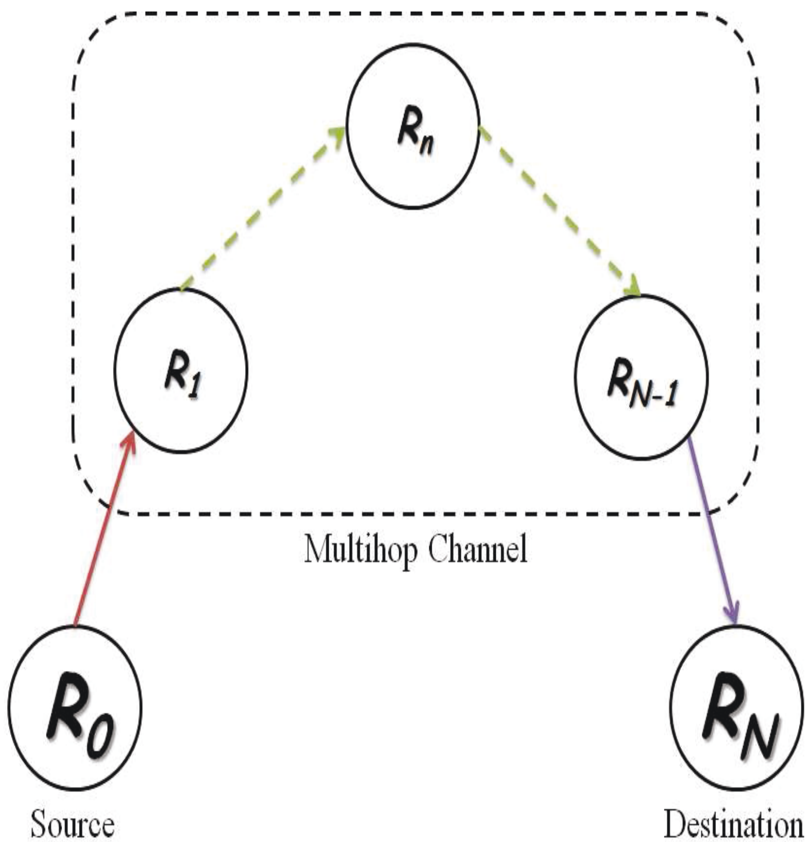

Figure 2.2: Multihop amplify-and-forward relaying network. The source, R_0, and the destination, R_N, communicate through $N - 1$ intermediate relay nodes, R_1, R_2, ..., R_{N-1}.

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Chapter 3

The Effect of Rician Fading in Dual-Hop AF Relaying Systems

In this chapter[(1)], dual-hop amplify-and-forward (AF) relaying systems without relay selection are studied. New exact expressions of the probability density function (PDF) and the cumulative distribution function (CDF) of the instantaneous end-to-end signal-to-noise ratio (SNR) are obtained for dual-hop AF relaying systems operating over Rician fading channels as well as for dissimilar dual-hop AF systems operating over mixed Nakagami-m/Rician fading channels. Exact solutions for the outage probability, and exact single-integral solutions for the ergodic capacity and the average symbol error probability are derived. For the case of dissimilar dual-hop AF systems, the exact performance metrics are compared to performance bounds in the literature. It is shown that the existing performance bounds are not tight for medium ranges of SNR. The effects of the Rician and Nakagami-m fading parameters on the performance metrics of the different systems are also studied.

[(1)] A version of this chapter has been published in the IEEE Transactions on Vehicular Technology: S. S. Soliman and N. C. Beaulieu, "The bottleneck effect of Rician fading in dissimilar dual-hop AF relaying systems," *IEEE Trans. Veh. Technol.*, vol. 63, no. 4, pp. 1957–1965, May 2014.

3.1 Introduction

In this chapter, we study dual-hop AF relaying systems operating over Rician fading channels and dissimilar dual-hop AF systems operating over Rician/Nakagami-m fading channels. To the best of the authors' knowledge, closed-form expressions for the exact PDF and the exact CDF of the end-to-end SNR of dual-hop AF relaying systems operating over Rician fading links, have not been obtained previously. The main contributions of the chapter are:

1) A precise analytical expression for the PDF of the exact end-to-end SNR of dual-hop AF relaying systems operating over Rician fading links is obtained. This is different from an expression obtained in [1], because it is based on the exact expression of the end-to-end SNR rather than the harmonic mean bound used in [1].

2) A precise analytical expression for the outage probability is also derived. This is also different from the expression obtained in [1] because it is exact, and does not require numerical calculations of inverse Laplace transforms. Although the authors in [2] and [3] derived closed-form expressions for the PDF and the outage probability of dual-hop AF relaying systems operating over Rayleigh fading links and Nakagami-m fading links, respectively, no precise expression for these statistics were reported in literature for the Rician fading case.

3) An analytical solution for the average symbol error probability is obtained, taking into account the exact end-to-end SNR. Exact solutions for the outage probability and the average symbol error probability have not been previously reported.

4) An analytical solution for the ergodic capacity is obtained. This solution has the same formal computational complexity as the solution presented in [4], with the advantage of involving the modified Bessel function of the second kind instead of the Meijer-G function. The Bessel functions are available as built-in functions in more software packages than the Meijer-G function.

5) A comparison of the performance metrics of dual-hop AF relaying systems operating over Rician fading links with those operating over Rayleigh and Nakagami-m fading links.

In this chapter, we also study dissimilar dual-hop AF relaying systems operating over mixed Nakagami-m/Rician fading links. The case of mixed, or asymmetric, fading links in

AF relaying systems was partially studied in [5]–[7]. The authors in [5] obtained an expression for the outage probability of dual-hop AF systems operating over mixed Rayleigh/Rician fading channels; however, that expression was not used to obtain other performance metrics. Instead, the authors used an approximation of the end-to-end SNR to obtain the average error probability of the system. The authors also presented a lower bound to the average error probability based on approximating the end-to-end SNR by the minimum value of the fading SNRs on the individual links. In [6] also, the authors studied the case of mixed Rayleigh/Rician fading channels for dual-hop, fixed gain relaying systems, and obtained infinite-series representations for the outage probability and the average bit error probability of these systems. The case of dual-hop AF systems with mixed Nakagami-m and Rician fading links was studied in [7]. The authors in [7] adopted the approximation used in [5] to approximate the outage probability and the average error probability of the proposed system. Since the results presented in [5] and [7] are based on approximations for the end-to-end SNR, they are not exact. This chapter presents the first precise statistical analysis of dissimilar dual-hop, variable gain AF systems operating over mixed Nakagami-m/Rician fading channels. Also the first precise analytical results for the average symbol error probability, outage probability and ergodic capacity of such dual-hop AF systems are obtained.

3.2 System And Channel Models

The system under study is a dual-hop AF relaying system, shown in [8, Fig. 1]. A source node, S, and a destination node, D, communicate through one intermediate relay node, R. The source node, S, transmits the data signal to the relay node, R, over the first link, $S-R$. The relay node, in turn, amplifies the received signal and transmits the amplified signal to the destination node D over the second link, $R-D$. The amplification factor, A, at the relay node, R, is given by [9] as $A = \sqrt{\frac{P_1}{P_0|\alpha_1|^2+N_{0_1}}}$, where $P_i,\ i = 0, 1$, is the transmitter power at the source node and the relay node, respectively, and where $\alpha_i,\ i = 1, 2$, and $N_{0i},\ i = 1, 2$, are respectively the fading gain of the i^{th} link, and the noise power at the i^{th} node. Time division multiple access is assumed for transmission over both links and orthogonal, half-duplex operation is implemented to avoid inter-signal interference. Note that the direct source-to-destination link is considered absent. This does not only mean the absence of a line-of-sight from the source to the destination, but also that a multipath link from the source to the destination cannot be established. This assumption is realistic in many cases, for example when the distance between the source and the destination is larger than the radio coverage of the source transmitter. The exact instantaneous end-to-end received SNR at the destination, γ_t, is given by [8, eq. (5)]

$$\gamma_t = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + 1} \tag{3.1}$$

where $\gamma_i = \frac{P_{i-1}}{N_{0_i}} |\alpha_i|^2$, $i = 1, 2$ represents the instantaneous received SNR on the i^{th} link. The PDF and CDF expressions for the common fading channel distributions can be found in [10]. When the PDF of the instantaneous end-to-end SNR, γ_t, is known, different system performance metrics can be evaluated. The ergodic capacity, the outage probability and the average symbol error probability can be obtained using expressions in [10, eq. (15.21)], [10, eq. (15.6)] and [10, eq. (5.1)], respectively.

3.3 Dual-Hop AF Relaying Over Rician Fading Links

In order to study the performance of dual-hop AF systems, the expression (3.1) can be used for direct and exact integral evaluation of the performance metrics. Although such a "brute-force" method results in exact performance metrics, more elegant analytical solution is possible. It can be shown that the PDF and the CDF of the end-to-end SNR, γ_t, can be found respectively as

$$f_{\gamma_t}(r) = \int_r^\infty \frac{\gamma_2(\gamma_2 + 1)}{(\gamma_2 - r)^2} f_{\gamma_1} \left(\frac{(\gamma_2 + 1)r}{\gamma_2 - r} \right) f_{\gamma_2}(\gamma_2)\, d\gamma_2 \tag{3.2}$$

and

$$\begin{aligned} P_{out} &= F_{\gamma_t}(\gamma_{th}) \\ &= 1 - \int_{\gamma_{th}}^\infty \left[1 - F_{\gamma_1} \left(\frac{(\gamma_2 + 1)\gamma_{th}}{\gamma_2 - \gamma_{th}} \right) \right] f_{\gamma_2}(\gamma_2)\, d\gamma_2 . \end{aligned} \tag{3.3}$$

Some different versions of the integral expression in (3.3) do exist in literature, as in [11, eq. (9)] and [12, eq. (24)]. However, in all of these publications, the expression (3.3) was used either for Rayleigh or Nakagami-m fading links, and in none of them was the expression used to determine the outage probability for dual-hop AF systems operating over Rician links or over mixed Nakagami-m/Rician fading links with arbitrary fading parameters.

3.3.1 The PDF of γ_t

The PDF of γ_i for Rician fading links [10, eq. (2.16)] is substituted into (3.2) resulting in

$$f_{\gamma_t}(r) = \int_{x=r}^{\infty} \frac{x(x+1)}{(x-r)^2} \left(\frac{1+K_2}{\overline{\gamma}_2} e^{-K_2} \exp\left[-\frac{1+K_2}{\overline{\gamma}_2} x \right] I_0\left(2\sqrt{\frac{K_2(1+K_2)}{\overline{\gamma}_2} x} \right) \right)$$
$$\times \left(\frac{1+K_1}{\overline{\gamma}_1} e^{-K_1} \exp\left[-\frac{1+K_1}{\overline{\gamma}_1} \left(\frac{(x+1)r}{x-r} \right) \right] I_0\left(2\sqrt{\frac{K_1(1+K_1)}{\overline{\gamma}_1} \left(\frac{(x+1)r}{x-r} \right)} \right) \right) dx. \tag{3.4}$$

To proceed, we use the infinite series representation [13, eq. (9.6.10)] for the Bessel function $I_0(\cdot)$, namely,

$$I_0\left(2\sqrt{\frac{K_i(1+K_i)}{\overline{\gamma}_i} x} \right) = \sum_{n=0}^{\infty} a_i(n) x^n \tag{3.5a}$$

where

$$a_i(n) = \frac{1}{(n!)^2} \left(\frac{K_i(1+K_i)}{\overline{\gamma}_i} \right)^n. \tag{3.5b}$$

Note that the expansion of the Bessel function in the form of an infinite series is widely known. However, it is used here to get a closed-form solution to the integral in (3.4). Substituting the Bessel function expansion into (3.4) results in

$$f_{\gamma_t}(r) = \int_{x=r}^{\infty} \frac{x(x+1)}{(x-r)^2} \left(\frac{1+K_2}{\overline{\gamma}_2} e^{-K_2} \exp\left[-\frac{1+K_2}{\overline{\gamma}_2} x \right] \sum_{m=0}^{\infty} a_2(m) x^m \right)$$
$$\times \left(\frac{1+K_1}{\overline{\gamma}_1} e^{-K_1} \exp\left[-\frac{1+K_1}{\overline{\gamma}_1} \left(\frac{(x+1)r}{x-r} \right) \right] \sum_{n=0}^{\infty} a_1(n) \left(\frac{(x+1)r}{x-r} \right)^n \right) dx. \tag{3.6}$$

Interchanging the order of summation and integration, we obtain

$$f_{\gamma_t}(r) = \left(\frac{1+K_1}{\overline{\gamma}_1}\right)\left(\frac{1+K_2}{\overline{\gamma}_2}\right) e^{-(K_1+K_2)} \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} a_1(n)a_2(m) \int_{x=r}^{\infty} \frac{x(x+1)}{(x-r)^2}$$
$$\times \left(\frac{(x+1)r}{x-r}\right)^n \exp\left[-\frac{1+K_1}{\overline{\gamma}_1}\left(\frac{(x+1)r}{x-r}\right)\right] x^m \exp\left[-\frac{1+K_2}{\overline{\gamma}_2}x\right] dx. \quad (3.7)$$

The interchange of order is justified by a theorem on the integration of an infinite series over an infinite interval in [14, pp. 452 − 453]; The conditions of the theorem in [14] are satisfied by the operands in (3.6). The integral in (3.7) can be solved, and an infinite-series expression [(2)] for the PDF of the instantaneous end-to-end SNR for dual-hop AF relaying systems operating over Rician fading links is obtained as

$$f_{\gamma_t}(r) = C \times \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} a_1(n)a_2(m) \times I_1\left(r; n, m, \frac{1+K_1}{\overline{\gamma}_1}, \frac{1+K_2}{\overline{\gamma}_2}\right) \quad (3.8)$$

where $C = \left(\frac{1+K_1}{\overline{\gamma}_1}\right)\left(\frac{1+K_2}{\overline{\gamma}_2}\right) e^{-(K_1+K_2)}$ and $I_1(r; \alpha, \beta, \theta, \phi)$ is defined and evaluated in closed-form in Appendix A (3.28), in terms of the modified Bessel function of the second kind [13, eq. (9.6.2)]. Recall that two-fold infinite summations, such as that in (3.8), are widely accepted in the literature, such as in [15, eq. (14)-(20)], due to the increased computational capabilities of software packages on the current computing platforms.

Note also that although the expression in (3.8) involves two-fold infinite summations, the summands decay (slightly faster than) exponentially [(3)] with increase of n and m, because of the $\frac{1}{(n!)^2}$ and $\frac{1}{(m!)^2}$ factors in the Bessel function expansion, and hence, a truncated summation with a finite number of terms will achieve a required accuracy. A precise analytical finite-series expression of the PDF can be then obtained as

$$f_{\gamma_t}(r) = C \times \sum_{n=0}^{N}\sum_{m=0}^{M} a_1(n)a_2(m) \times I_1\left(r; n, m, \frac{1+K_1}{\overline{\gamma}_1}, \frac{1+K_2}{\overline{\gamma}_2}\right). \quad (3.9)$$

Table 3.1 shows the results of evaluating the PDF in (3.9) at upper limits N and M. The case of $K_1 = 1$, $K_2 = 1$ is taken as an example to construct the table for several combinations of the input parameters $\overline{\gamma}_1$, $\overline{\gamma}_2$ and r. The table shows the decimal places that did not change,

(2) Note that the expression in (3.8) is different from the expression in [1, eq. (12)] because the later is based on the harmonic mean bound of the end-to-end SNR.

(3) Stirling's approximation specifies that $n!$ grows as $e^{n \ln n}$, and hence $\frac{1}{(n!)^2}$ decays as $e^{-2n \ln n}$.

Table 3.1: $f_{\gamma_t}(r)$ in (3.9) for different levels of truncation at N and M ($K_1 = 1$, $K_2 = 1$)

N M	$\overline{\gamma}_1 = \overline{\gamma}_2 = 1$, $r = 1$	$\overline{\gamma}_1 = \overline{\gamma}_2 = 10$, $r = 1$	$\overline{\gamma}_1 = \overline{\gamma}_2 = 10$, $r = 10$	$RTE_{f_{\gamma_t}(r)}$ ($\overline{\gamma}_1 = \overline{\gamma}_2 = r = 1$)
10	**0.0658024**49485052	**0.1910575**26388050	**0.01344058**2557497	1.4761×10^{-7}
15	**0.0658024591982**37	**0.19105752793406**2	**0.01344058358768**3	2.7024×10^{-13}
20	**0.065802459198255**	**0.191057527934065**	**0.013440583587685**	1.0704×10^{-19}
25	**0.065802459198255**	**0.191057527934065**	**0.013440583587685**	1.3242×10^{-26}

by adding more terms to the summations, in bold. Table 3.1 indicates that a finite number of terms can be used for the summation in (3.8) with a negligible truncation error.

To show that truncation error arising in the evaluation of (3.9) is negligible, an analysis of the truncation errors is presented. The truncation error resulting from truncating the summations in (3.9) at upper limits N and M can be obtained as

$$TE\left(f_{\gamma_t}(r)\right) = C \times \left[\sum_{n=0}^{\infty} \sum_{m=M+1}^{\infty} a_1(n)a_2(m) \times I_1\left(r; n, m, \frac{1+K_1}{\overline{\gamma}_1}, \frac{1+K_2}{\overline{\gamma}_2}\right) + \sum_{n=N+1}^{\infty} \sum_{m=0}^{M} a_1(n)a_2(m) \times I_1\left(r; n, m, \frac{1+K_1}{\overline{\gamma}_1}, \frac{1+K_2}{\overline{\gamma}_2}\right)\right]. \quad (3.10)$$

The relative truncation error in the PDF calculation, $RTE_{f_{\gamma_t}(r)}$, can be consequently obtained as

$$RTE_{f_{\gamma_t}(r)} = \left| \frac{TE\left(f_{\gamma_t}(r)\right)}{C \times \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} a_1(n)a_2(m) I_1\left(r; n, m, \frac{1+K_1}{\overline{\gamma}_1}, \frac{1+K_2}{\overline{\gamma}_2}\right)} \right| \quad (3.11)$$

where $|A|$ is the absolute value of A. The last column of Table 3.1 shows the values of the relative truncation error in (3.11) for different values of the upper limits N and M. It can be seen that the truncation error is negligible for a moderate number of terms, and the truncation error decreases dramatically as the number of terms increases. It is concluded then that the finite-series expression (3.9) can be used to obtain precise values of the PDF of the end-to-end SNR in the case of dual-hop systems operating over Rician fading links.

3.3.2 The CDF of γ_t

In the following, we use the single-fold integral (3.3) to obtain the CDF of the instantaneous end-to-end SNR of dual-hop AF relaying systems operating over Rician fading links. Substituting the CDF of Rician distribution into (3.3) results in

$$F_{\gamma_t}(\gamma_{th}) = 1 - \int_{x=\gamma_{th}}^{\infty} Q_1\left(\sqrt{2K_1}\,,\sqrt{2\left(\frac{1+K_1}{\overline{\gamma}_1}\right)\left(\frac{(x+1)\gamma_{th}}{x-\gamma_{th}}\right)}\right) f_{\gamma_2}(x)\,dx. \tag{3.12}$$

Using the infinite series representation of the first-order Marcum Q-function in [10, eq.(4.35)], in addition to the infinite series representation of the modified Bessel function of the first kind in [13, eq.(9.6.10)], the expression in (3.12) can be rewritten, after mathematical simplification, as

$$\begin{aligned} F_{\gamma_t}(\gamma_{th}) = 1 - \exp[-(K_1+K_2)] \int_{x=\gamma_{th}}^{\infty} \sum_{l=0}^{\infty}\sum_{n=0}^{\infty} \frac{1}{(l+n)!\,n!} {K_1}^{l+n} \left(\frac{1+K_1}{\overline{\gamma}_1}\right)^n \\ \times \exp\left[-\frac{1+K_1}{\overline{\gamma}_1}\left(\frac{(x+1)\gamma_{th}}{x-\gamma_{th}}\right)\right]\left(\frac{(x+1)\gamma_{th}}{x-\gamma_{th}}\right)^n \\ \times \left(\frac{1+K_2}{\overline{\gamma}_2}\right)\exp\left[-\frac{1+K_2}{\overline{\gamma}_2}x\right]\sum_{m=0}^{\infty}\left(\frac{1}{(m!)^2}\left(\frac{K_2(1+K_2)}{\overline{\gamma}_2}\right)^m\right) x^m\,dx. \end{aligned} \tag{3.13}$$

Interchanging the order of summation and integration in (3.13), and after further mathematical manipulation, the CDF of the end-to-end SNR in the case of mixed fading links can be expressed as

$$F_{\gamma_t}(\gamma_{th}) = 1 - \sum_{l=0}^{\infty}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} D(n,m) \times I_2\left(\gamma_{th}, n, m, \frac{1+K_1}{\overline{\gamma}_1}, \frac{1+K_2}{\overline{\gamma}_2}\right) \tag{3.14a}$$

where

$$D(n,m) = \frac{{K_1}^{l+n}\,{K_2}^m}{(l+n)!\,n!\,(m!)^2}\left(\frac{1+K_1}{\overline{\gamma}_1}\right)^n\left(\frac{1+K_2}{\overline{\gamma}_2}\right)^{m+1}\exp\left[-(K_1+K_2)\right]. \tag{3.14b}$$

$I_2(r;\alpha,\beta,\theta,\phi)$ is defined and evaluated in Appendix B (3.31). Although the expression in (3.14) involves nested infinite summations, the summands decay rapidly with increase of l, n and m, because of the $\frac{1}{(l+n)!\,n!\,(m!)^2}$ factor. The rate of decay is $e^{-(l+n)\ln(l+n)-n\ln n-2m\ln m}$

and hence a truncated summation with a finite number of terms will achieve required accuracy. A precise analytical finite-series expression of the CDF can be then obtained as

$$F_{\gamma_t}(\gamma_{th}) = 1 - \sum_{l=0}^{L}\sum_{n=0}^{N}\sum_{m=0}^{M} D(n,m) \times I_2\left(\gamma_{th}, n, m, \frac{1+K_1}{\overline{\gamma}_1}, \frac{1+K_2}{\overline{\gamma}_2}\right). \quad (3.15)$$

For Rayleigh fading links, putting $K_1 = K_2 = 0$ into (3.8) and (3.14) results in the expressions [2, eq. (3)] and [2, eq. (2)], respectively, as a special case of the Rician expressions.

3.4 Dual-Hop AF Relaying Over Mixed Fading Links

In this section, the case when the source-to-relay link and the relay-to-destination link experience different fading conditions is studied. The Rician fading distribution is used to characterize fading channels consisting of a strong direct line-of-sight (LOS) component as well as other randomly scattered components. On the other hand, in the absence of a LOS component, the Nakagami-m fading channel model becomes more suitable to represent the wireless channel propagation. Moreover, there are many practical situations where the Rician links will be cascaded with Nakagami-m links or vice verse. For example, an up-link from downtown to a mountain repeater represents a scatter channel environment, appropriately modeled by the Nakagami-m model, while a down-link from the mountain repeater to a destination will include a LOS component, and hence be properly represented by the Rician fading model. Therefore, it is important to study dual-hop AF transmission with mixed fading links [5]–[7].

The PDF of γ_t is obtained by substituting the PDF of the Nakagami-m fading distribution for $f_{\gamma_1}(x)$ and the PDF of the Rician fading distribution for $f_{\gamma_2}(x)$ into (3.2) resulting in

$$f_{\gamma_t}(r) = \left(\frac{m_1}{\overline{\gamma}_1}\right)^{m_1} \frac{1}{\Gamma(m_1)} \frac{1+K_2}{\overline{\gamma}_2} \exp\left[-K_2\right] \int_{x=r}^{\infty} \frac{x(x+1)}{(x-r)^2} \left(\frac{(x+1)r}{x-r}\right)^{m_1-1}$$
$$\times \exp\left[-\frac{m_1}{\overline{\gamma}_1}\left(\frac{(x+1)r}{x-r}\right)\right] \exp\left[-\frac{1+K_2}{\overline{\gamma}_2}x\right] I_0\left(2\sqrt{\frac{K_2(1+K_2)}{\overline{\gamma}_2}x}\right) dx. \quad (3.16)$$

Following similar procedure as in the previous section, a closed-form expression for the PDF of the end-to-end SNR in the case of mixed fading links can be then obtained as

$$f_{\gamma_t}(r) = \left(\frac{m_1}{\overline{\gamma}_1}\right)^{m_1} \frac{1}{\Gamma(m_1)} \frac{1+K_2}{\overline{\gamma}_2} e^{-K_2} \sum_{n=0}^{N} a_2(n) I_1\left(r; m_1-1, n, \frac{m_1}{\overline{\gamma}_1}, \frac{1+K_2}{\overline{\gamma}_2}\right). \quad (3.17)$$

The CDF of the instantaneous end-to-end SNR of dual-hop AF relaying systems operating over mixed fading channels is obtained by substituting the PDF of the Rician fading distribution for $f_{\gamma_2}(x)$, and the CDF of Nakagami-m fading channels for $F_{\gamma_1}(r)$, into (3.3). Using the finite series representation of the lower incomplete gamma function in the CDF of the Nakagami-m fading distribution [16, eq.(8.352.1)], and the series expansion of the Bessel function we get

$$F_{\gamma_t}(r) = 1 - \sum_{j=0}^{m_1-1} \sum_{n=0}^{\infty} \left(\frac{m_1}{\overline{\gamma}_1}\right)^{j} \frac{1}{j!} \frac{1+K_2}{\overline{\gamma}_2} e^{-K_2} a_2(n) \int_{x=r}^{\infty} \left(\frac{(x+1)r}{x-r}\right)^{j} \times \exp\left[-\frac{m_1}{\overline{\gamma}_1}\left(\frac{(x+1)r}{x-r}\right)\right] x^n \exp\left[-\frac{1+K_2}{\overline{\gamma}_2} x\right] dx. \quad (3.18)$$

The CDF of the end-to-end SNR of dual-hop AF systems operating over mixed Nakagami-m/Rician fading links is then given as

$$F_{\gamma_t}(r) = 1 - \sum_{j=0}^{m_1-1} \sum_{n=0}^{N} \left(\frac{m_1}{\overline{\gamma}_1}\right)^{j} \frac{1}{j!} \frac{1+K_2}{\overline{\gamma}_2} e^{-K_2} a_2(n) \times I_2\left(r; j, n, \frac{m_1}{\overline{\gamma}_1}, \frac{1+K_2}{\overline{\gamma}_2}\right). \quad (3.19)$$

Note that for the special case when the first link follows a Rayleigh fading distribution, the expression in (3.19) reduces to [5, eq. (8)], as required.

3.4.1 Diversity Order

The diversity gain for dual-hop AF systems operating over Rician fading links and dissimilar dual-hop AF systems operating over Nakagami-m/Rician fading links can be obtained from the PDF or CDF expressions using asymptotic analysis and the method described in [17]. The average symbol error probability of the system can be obtained by substituting the PDF, in (3.17) for example, into [10, eq. (5.1)], $P_s = E\left\{b\, Q\left(\sqrt{a\,\gamma_t}\right)\right\}$, and it can be expressed as

$$P_s \approx (G_a \overline{\gamma})^{-G_d} + o\left(\overline{\gamma}^{-(G_d)}\right) \tag{3.20}$$

where G_d is the system's diversity order. To obtain the diversity gain, it is assumed that the ratio between $\overline{\gamma}_2$ and $\overline{\gamma}_1$ is fixed, i.e. $\overline{\gamma}_2 \propto \overline{\gamma}_1 \propto \overline{\gamma}$. We substitute $r = \beta\overline{\gamma}$, and use mathematical analysis to express the PDF of the end-to-end SNR, $f_{\gamma_t}(r)$, in the form, $f_{\gamma_t}(\beta) = \alpha(\beta)^t + o(\beta^t)$. This results in $G_d = t + 1$ as shown in [17]. To get to the end result, we use the limiting form of the modified Bessel function of the second kind [13, eq. (9.6.9)], namely,

$$K_\nu(z) \sim \frac{1}{2}\Gamma(\nu)\left(\frac{1}{2}z\right)^{-\nu}. \tag{3.21}$$

The form in (3.21) is valid for fixed ν and for $z \to 0$. Recognizing that $a_2(n) \propto \overline{\gamma}^{-n}$ in (3.17), it can be found that $f_{\gamma_t}(\beta) \propto (\beta)^0 + o\left(\beta^0\right)$, and hence the diversity order of dissimilar dual-hop AF systems operating over mixed Nakagami-m/Rician fading links is limited to 1 irrespective the value of the fading parameter m_1. This will be shown using numerical examples in the next section. Note that the approximation $e^{-x} \sim 1 - x$, when $x \to 0$, was used to obtain the previous result. Similar analysis can be applied to the case of dual-hop AF systems operating over Rician fading links.

3.5 Numerical Examples

In this section, different performance metrics are used to study the behaviours of dual-hop variable gain AF relaying systems. Outage probabilities are directly evaluated using the CDF expressions of the end-to-end SNR. The average symbol error probability and the ergodic capacity can be obtained by substituting the PDF expressions of the end-to-end SNR into [10, eq. (5.1)] and [10, eq. (15.21)], respectively. Alternate methods use the CDF expressions to obtain these performance metrics. For example, the ergodic capacity can be obtained by substituting the CDF expressions into [18, eq. (4)].

Figs. 3.1 and 3.2 show, respectively, the average symbol error probability versus the link average SNR, $\overline{\gamma}$, and the outage probability versus the threshold SNR, γ_{th}, for dual-hop AF relaying systems. Identical Rayleigh fading links, Nakagami-m fading links with $m = 2, 3,$

and Rician fading links with $K = 1, 2$, as well as mixed Nakagami-m/Rician fading links with $m_1 = 3, K_2 = 2$, are assumed. M-QAM modulation with $M = 8$, is assumed, so the values of the parameters (a, b) of the average symbol error probability, $P_s = E\left\{b\, Q\left(\sqrt{a\, \gamma_t}\right)\right\}$, are set to $\left(\frac{3}{M-1}, 4 - \frac{4}{\sqrt{M}}\right)$ [10]. We assume also an uniform power allocation policy, that is the total available power, P, is evenly allocated to the source and the relay. Without loss of generality, we assume equal noise powers, N_0, at all the nodes. We assume the links are balanced and $\overline{\gamma}_1 = \overline{\gamma}_2 = \overline{\gamma}$. Note that we make these simplifying assumptions in the examples for convenience; they are not required by the analysis. The figures show, as expected, precise agreement between the results obtained from the theoretical analysis and simulation results at all values of SNR, for all the different performance measures.

Many important observations can be made from Figs. 3.1 and 3.2. Note first that the system performance improves, as expected, for the less severe Rician and Nakagami-m fading channels. It is observed also that the limiting slopes of the average symbol error probability curves in Fig. 3.1 and the limiting slopes of the outage probability curves in Fig. 3.2 are proportional to the value of the fading parameter m in the case of Nakagami-m fading links. On the other hand, the limiting slopes of the average symbol error probability and outage probability curves in the case of Rician fadings and mixed Nakagami-m/Rician fadings, are the same as the limiting slopes for Rayleigh fading links, irrespective of the values of the fading parameters. This results in a considerable SNR gain for systems subject to Nakagami-m fading links over those subject to Rician or Rayleigh fading links for sufficiently large SNR. It can be observed also that the performance of a dual-hop AF relaying system operating over mixed Nakagami-m/Rician fading links with $m_1 = 3$ and $K_2 = 2$ is better than for a system operating over Rician fading links with $K_1 = K_2 = 2$ and worse than one operating over Nakagami-m fading links with $m_1 = m_2 = 3$. For example, the outage probability at threshold SNR $\gamma_{th} = -6$ dB is 1.3×10^{-2} for the mixed links system, while it is 2.7×10^{-2} for the Rician links system and 2.5×10^{-4} for the Nakagami-m links system.

In the next example, we compare the exact average symbol error probability of dual-hop AF relaying systems operating over mixed fading channels to lower bounds reported in the literature. Fig. 3.3 shows the average symbol error probability versus the link average SNR, $\overline{\gamma}$, of dual-hop AF systems operating over different groups of mixed fading channels.

For this figure, binary phase shift keying (BPSK) is assumed, so that the values of the parameters (a, b) in $P_s = E\left\{b\, Q\left(\sqrt{a\, \gamma_t}\right)\right\}$ are set to $(2, 1)$ [10]. Lower bounds, based on the approximation $\gamma_{t_1} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2}$ in [5] and [7] are shown, and lower bounds, based on the approximation $\gamma_{t_2} = \min\{\gamma_1, \gamma_2\}$ in [5] are also shown for the cases when the first link fading is Rayleigh distributed. The figure shows precise agreement between the results obtained from the analytical method presented in this chapter, and the simulation results. Here, the summation in (3.17) was truncated at the 35^{th} term. It was found that adding more terms does not affect the result in the 15^{th} decimal place.

We next develop two sets of examples, for the case of mixed fading links, to explore the effects of changing the Nakagami-m parameter, m_1, while having the Rician parameter fixed, as well as the effects of changing the Rician parameter, K_2, while having the Nakagami-m parameter unchanged. Figs. 3.4 and 3.5 show respectively the outage probability, at $\overline{\gamma} = 10$ dB, versus the threshold SNR, γ_{th}, and the ergodic capacity versus the average link SNR, $\overline{\gamma}$. Dual-hop AF systems with Rician parameter $K_2 = 5$ and different values of the Nakagami-m parameter, $m_1 = 1, 3, 5$ and 11, are considered. The performance metrics for a dual-hop AF system operating over Rayleigh fading links are shown also for comparison. Many worthwhile observations can be drawn from these figures. It can be seen from Fig. 3.5 that the ergodic capacity is improved, as expected, with less severe fading links. For example, as the fading parameter m_1 increases, the ergodic capacity increases at all values of $\overline{\gamma}$, however the improvement, which can be achieved in the ergodic capacity, is modest. Improvements can be also observed in the outage probability curves in Fig. 3.4. However, it is observed that the limiting slopes of the outage probability curves for the different cases of m_1 are the same as the limiting slopes of the curves for the case of Rayleigh fading links, $m_1 = 1$ and $K_2 = 0$. This shows that the presence of a Rician fading link represents a bottleneck to the performance improvement, and can be intuitively explained for the system model considered. It is well known that the overall limiting slope of the performance curves is proportional to the minimum of the limiting slope caused by the first link and that caused by the second link [3]. Since the Rician fading link results in a limiting slope, of performance curves, similar to that of Rayleigh fading links, it represents a bottleneck preventing the increase of the overall limiting slope, even with increase of Nakagami-m parameter, m_1. Moreover, it can be observed that as the fading parameter, m_1, increases, the system improvement can become negligible. For example, an outage probability of 2×10^{-3} occurs at $\gamma_{th} = -17.87$ dB when $m_1 = 1$ and $K_2 = 5$, and it occurs at $\gamma_{th} = -5.72$ dB when $m_1 = 3$ and $K_2 = 5$, which is a large SNR gain of 12.15 dB. On the other hand, the same outage occurs at $\gamma_{th} = -5.32$ dB when $m_1 = 5$ and $K_2 = 5$, and at $\gamma_{th} = -5.24$ dB when $m_1 = 11$ and $K_2 = 5$, which is a negligible improvement from the case where $m_1 = 3$. It can be concluded, then, that increasing m_1 at fixed values of K_2 has diminishing returns.

Fig. 3.6 shows the average symbol error probability of dual-hop AF systems with Nakagami-m parameter $m_1 = 4$ and different values of the Rician parameter, $K_2 = 0, 3, 5$ and 7. As observed in the previous example set, the limiting slopes of the average symbol error probability for the different cases of the K_2 parameter, are the same as the limiting slopes of the average symbol error probability for the case of Rayleigh fading links. However, in contrast to the results of the previous example set, there exists a substantial SNR gain resulting from increasing the fading parameter, K_2. For example, an average symbol error probability of 10^{-3} occurs at $\overline{\gamma} = 24$ dB when $m_1 = 4$ and $K_2 = 0$, and it occurs at $\overline{\gamma} = 17.71$ dB when $m_1 = 4$ and $K_2 = 3$, at $\overline{\gamma} = 14.27$ dB when $m_1 = 4$ and $K_2 = 5$ and at $\overline{\gamma} = 12.99$ dB when $m_1 = 4$ and $K_2 = 7$. It can be concluded that the Rician fading parameter has a great impact on the performance metrics of dual-hop AF relaying systems operating over mixed Nakagami-m/Rician fading links.

3.6 Conclusion

New theoretical, precise expressions for the probability density function and the cumulative distribution function of the instantaneous end-to-end SNR of dual-hop AF relaying systems have been derived. The expressions were obtained for dual-hop systems operating over Rician fading channels, as well as for dissimilar dual-hop systems operating over mixed Nakagami-m/Rician fading channels. The derived expressions were used to obtain the outage probability, average symbol error probability and ergodic capacity of different AF systems. It was shown that the effects of the channel fading parameters on the performance metrics vary according to the nature of the communication links. It was shown also that for the case of dissimilar dual-hop systems with mixed Nakagami-m/Rician fading links, an increase in the Nakagami-m parameter has diminishing returns on the performance metrics as long as the Rician parameter is unchanged, while an increase in the Rician parameter results in a notable SNR gain even when the Nakagami-m parameter is fixed.

3.A Evaluation of the Integral I_1

The integral $I_1\,(r;\alpha,\beta,\theta,\phi)$ is defined as

$$I_1\,(r;\alpha,\beta,\theta,\phi) = \int_{x=r}^{\infty} \frac{x(x+1)}{(x-r)^2}\left[\frac{(x+1)\,r}{x-r}\right]^{\alpha} x^{\beta} \times \exp\left(-\theta\left[\frac{(x+1)\,r}{x-r}\right]\right)\exp\left(-\phi\, x\right) dx. \quad (3.22)$$

Using a change of variables, $y = \frac{x}{r} - 1$, the integral I_1 can be written as

$$I_1\,(r;\alpha,\beta,\theta,\phi) = \int_{y=0}^{\infty} \left[r + (2r+1)\,y^{-1} + (r+1)\,y^{-2}\right]\left[r + \frac{r+1}{y}\right]^{\alpha} \left[r\,(y+1)\right]^{\beta} \times \exp\left(-\theta\left[r + \frac{r+1}{y}\right]\right)\exp\left(-\phi\left[r\,y + r\right]\right) dy \quad (3.23)$$

which can be rewritten as

$$I_1\,(r;\alpha,\beta,\theta,\phi) = \exp\left[-\left(\theta+\phi\right)r\right]\; r^{(\alpha+\beta)} \int_{y=0}^{\infty} \left[r + (2r+1)\,y^{-1} + (r+1)\,y^{-2}\right] \times \left[1 + \frac{r+1}{r}y^{-1}\right]^{\alpha} \left[1+y\right]^{\beta} \exp\left[-\theta\;(r+1)\,y^{-1} - \phi\, r\, y\right] dy. \quad (3.24)$$

Assuming integer values for both α and β, the binomial expansion in [16, eq. (1.111)] can be applied to the two terms, $\left[1 + \frac{r+1}{r}y^{-1}\right]^{\alpha}$ and $\left[1+y\right]^{\beta}$, resulting in

$$I_1\,(r;\alpha,\beta,\theta,\phi) = \exp\left[-\left(\theta+\phi\right)r\right]\; r^{(\alpha+\beta)} \int_{y=0}^{\infty} \left[r + (2r+1)\,y^{-1} + (r+1)\,y^{-2}\right] \times \left[\sum_{k=0}^{\alpha}\binom{\alpha}{k}\left(\frac{r+1}{r}\right)^{k} y^{-k}\right]\left[\sum_{j=0}^{\beta}\binom{\beta}{j} y^{j}\right] \exp\left[-\theta\;(r+1)\,y^{-1} - \phi\, r\, y\right] dy. \quad (3.25)$$

Exchanging the order of summation and integration results in

$$I_1\left(r;\alpha,\beta,\theta,\phi\right) = \exp\left[-\left(\theta+\phi\right)r\right]\; r^{(\alpha+\beta)} \sum_{k=0}^{\alpha}\sum_{j=0}^{\beta} \binom{\alpha}{k}\binom{\beta}{j}\left(\frac{r+1}{r}\right)^{k}$$
$$\times \int_{y=0}^{\infty} \left[r + \left(2r+1\right)y^{-1} + \left(r+1\right)y^{-2}\right]\; y^{j-k}\; \exp\left[-\theta\;\left(r+1\right)y^{-1} - \phi\, r\, y\right] dy. \quad (3.26)$$

The expression in (3.26) can be written as

$$I_1\left(r;\alpha,\beta,\theta,\phi\right) = \exp\left[-\left(\theta+\phi\right)r\right]\; r^{(\alpha+\beta)} \sum_{k=0}^{\alpha}\sum_{j=0}^{\beta} \binom{\alpha}{k}\binom{\beta}{j}\left(\frac{r+1}{r}\right)^{k}$$
$$\times \left[r\, I(j-k) + (2r+1)\, I(j-k-1) + (r+1)\, I(j-k-2)\right] \quad (3.27a)$$

where the integral $I(\nu)$ is defined as

$$I(\nu) = \int_{y=0}^{\infty} y^{\nu}\; \exp\left[-\theta\;\left(r+1\right)y^{-1} - \phi\, r\, y\right] dy \quad (3.27b)$$

and can be solved using [16, eq. (3.471.9)] as

$$I(\nu) = 2\left(\frac{\theta\,(r+1)}{\phi\, r}\right)^{\left(\frac{\nu+1}{2}\right)} K_{\nu+1}\left(2\sqrt{\theta\,\phi\, r(r+1)}\right). \quad (3.27c)$$

After some mathematical manipulation, the integral I_1 can be evaluated as

$$I_1\left(r;\alpha,\beta,\theta,\phi\right) = 2\;\exp\left[-\left(\theta+\phi\right)r\right]\; r^{(\alpha+\beta)} \sum_{k=0}^{\alpha}\sum_{j=0}^{\beta} \binom{\alpha}{k}\binom{\beta}{j}\left(\frac{\theta}{\phi}\right)^{\frac{j-k}{2}}\left(\frac{r+1}{r}\right)^{\frac{j+k}{2}}$$
$$\times \left[\sqrt{\frac{\theta}{\phi}\, r(r+1)}\, K_{j-k+1}\left(2\sqrt{\theta\,\phi\, r(r+1)}\right) + (2r+1)\, K_{j-k}\left(2\sqrt{\theta\,\phi\, r(r+1)}\right)\right.$$
$$\left. + \sqrt{\frac{\phi}{\theta}\, r(r+1)}\, K_{j-k-1}\left(2\sqrt{\theta\,\phi\, r(r+1)}\right)\right]. \quad (3.28)$$

3.B Evaluation of the Integral I_2

The integral $I_2\left(r;\alpha,\beta,\theta,\phi\right)$ is defined as

$$I_2\left(r;\alpha,\beta,\theta,\phi\right) = \int_{x=r}^{\infty} \left[\frac{(x+1)\,r}{x-r}\right]^{\alpha} x^{\beta} \exp\left(-\theta\left[\frac{(x+1)\,r}{x-r}\right]\right) \exp\left(-\phi\,x\right) dx. \tag{3.29}$$

Using a change of variables $y = \dfrac{x}{r} - 1$, the integral I_2 can be written as

$$\begin{aligned} I_2\left(r;\alpha,\beta,\theta,\phi\right) = \int_{y=0}^{\infty} r^{\alpha} \left[1 + \frac{r+1}{r} y^{-1}\right]^{\alpha} r^{\beta} \left[y+1\right]^{\beta} \\ \times \exp\left(-\theta\left[r + \frac{r+1}{y}\right]\right) \exp\left(-\phi\left[r\,y + r\right]\right)\, r\,dy. \end{aligned} \tag{3.30}$$

Assuming integer values for both α and β, the binomial expansion can be used as previously, and exchanging the order of summation and integration, I_2 can be obtained as

$$\begin{aligned} I_2\left(r;\alpha,\beta,\theta,\phi\right) = 2\,\exp\left[-\left(\theta+\phi\right)r\right]\, r^{(\alpha+\beta+1)} \sum_{k=0}^{\alpha}\sum_{j=0}^{\beta} \binom{\alpha}{k}\binom{\beta}{j} \\ \times \left(\frac{\theta}{\phi}\right)^{\frac{j-k+1}{2}} \left(\frac{r+1}{r}\right)^{\frac{j+k+1}{2}} K_{j-k+1}\left(2\sqrt{\theta\,\phi\,r(r+1)}\right). \end{aligned} \tag{3.31}$$

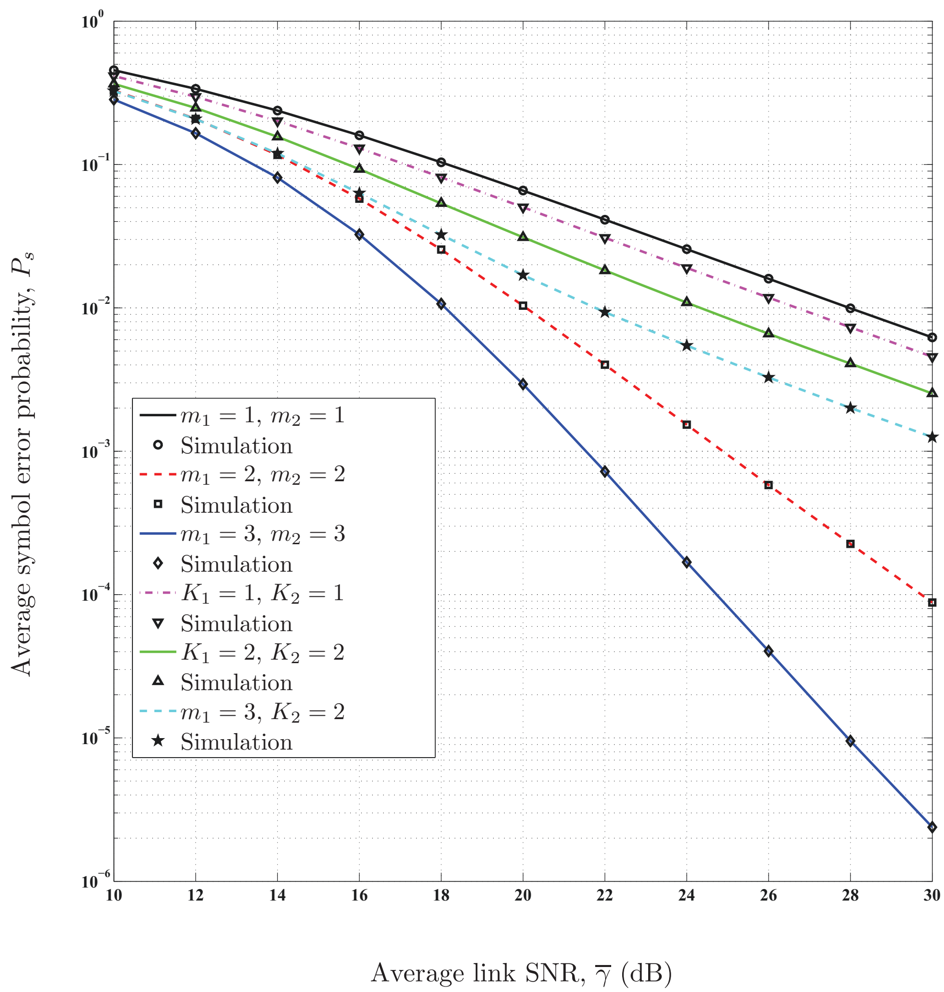

Figure 3.1: The average symbol error probability for dual-hop AF relaying systems with 8-QAM modulation. Identically distributed Rayleigh, Nakagami-m ($m = 2, 3$) and Rician ($K = 1, 2$) fading links, as well as mixed Nakagami-m/Rician links ($m_1 = 3, K_2 = 2$), are assumed.

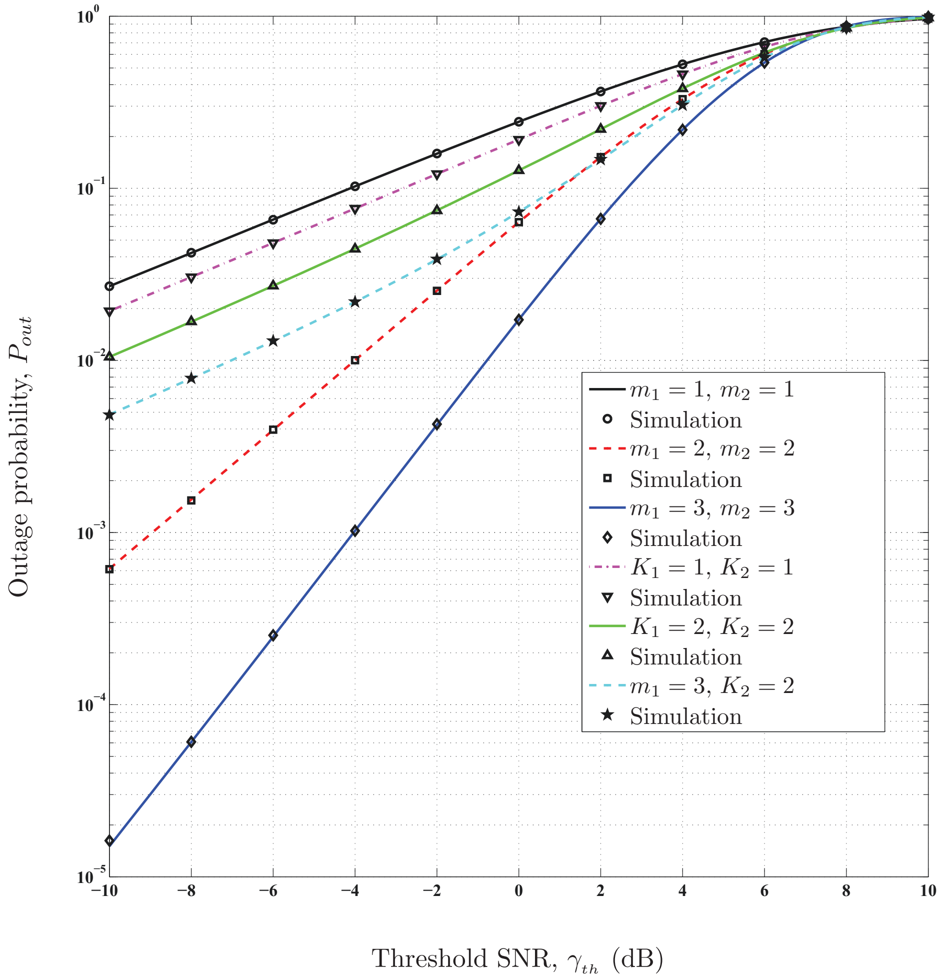

Figure 3.2: Outage probability for dual-hop AF relaying systems for $\overline{\gamma} = 10$ dB. Identically distributed Rayleigh, Nakagami-m ($m = 2, 3$) and Rician ($K = 1, 2$) fading links, as well as mixed Nakagami-m/Rician links ($m_1 = 3, K_2 = 2$), are assumed.

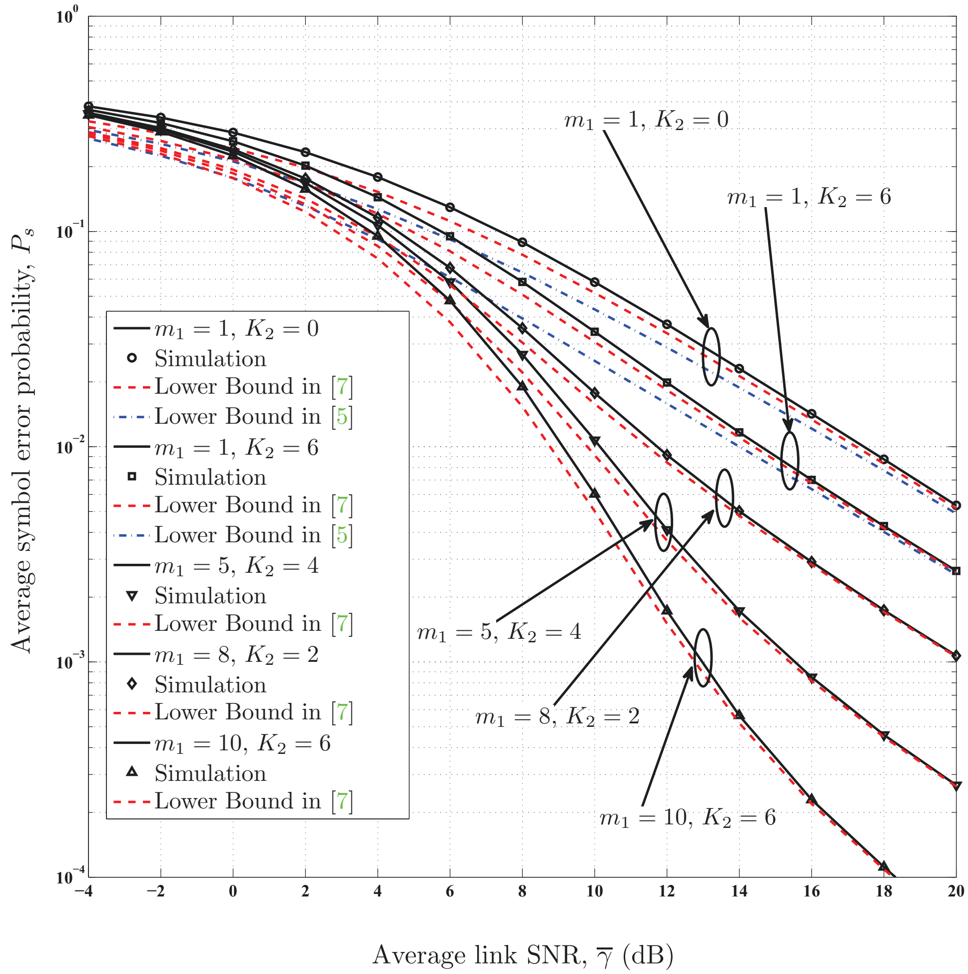

Figure 3.3: The average symbol error probability for dual-hop AF relaying systems with mixed Nakagami-m/Rician fading links and with BPSK modulation. Lower bounds in [5], shown in dashed-doted lines, and lower bounds in [7], shown in dashed lines, are plotted for comparison.

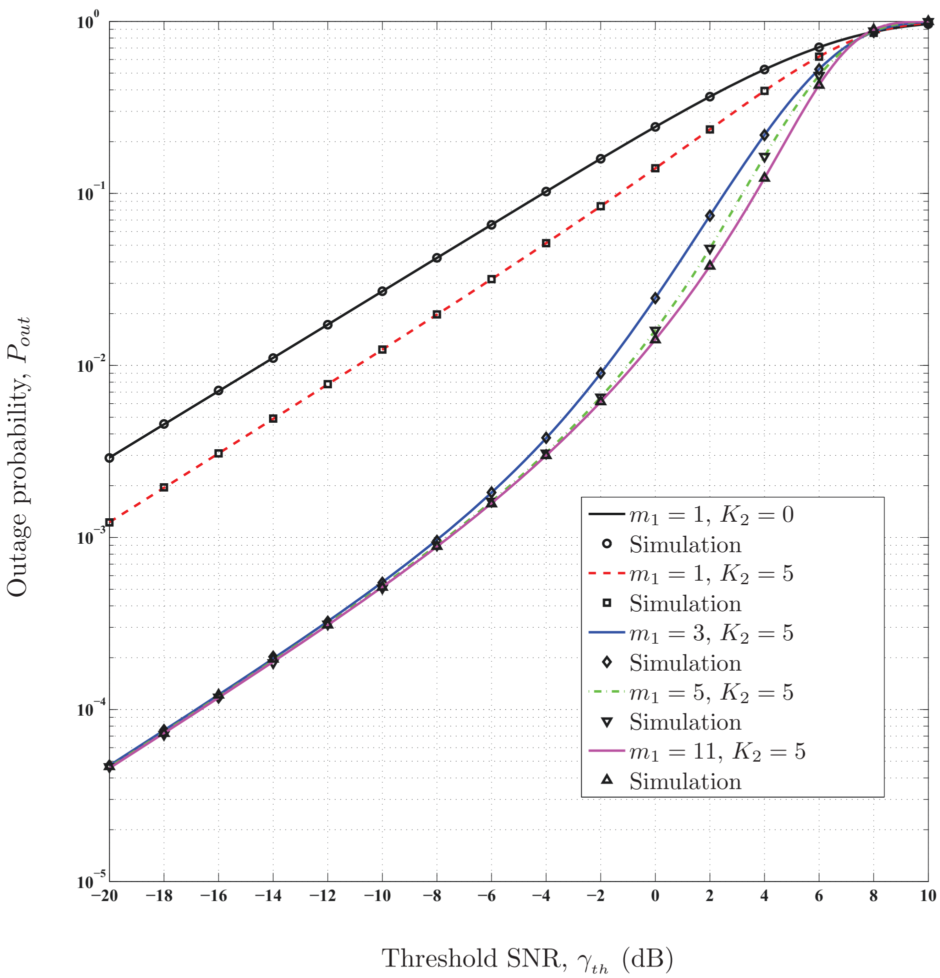

Figure 3.4: Outage probability for dual-hop AF relaying systems with mixed Nakagami-m/Rician fading links for $\overline{\gamma} = 10$ dB. Different cases of the Nakagami-m fading parameter, m_1, are shown.

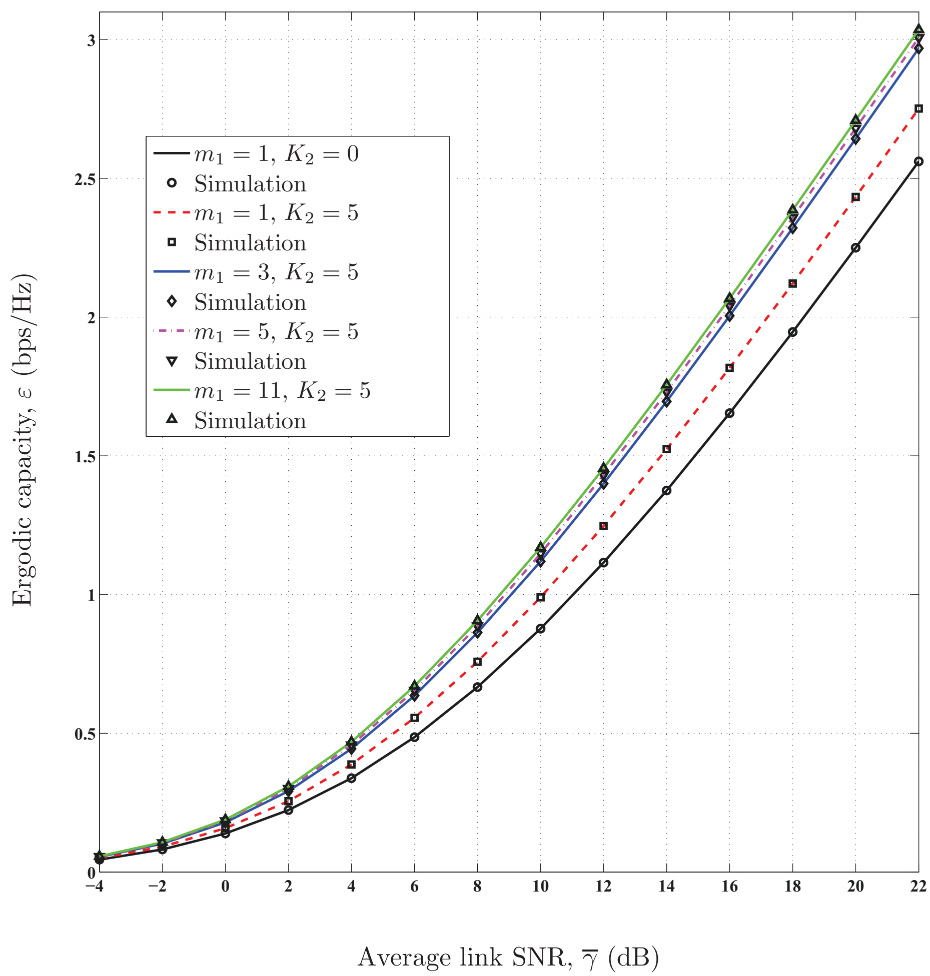

Figure 3.5: The ergodic capacity for dual-hop AF relaying systems with mixed Nakagami-m/Rician fading links. Different cases of the Nakagami-m fading parameter, m_1, are shown.

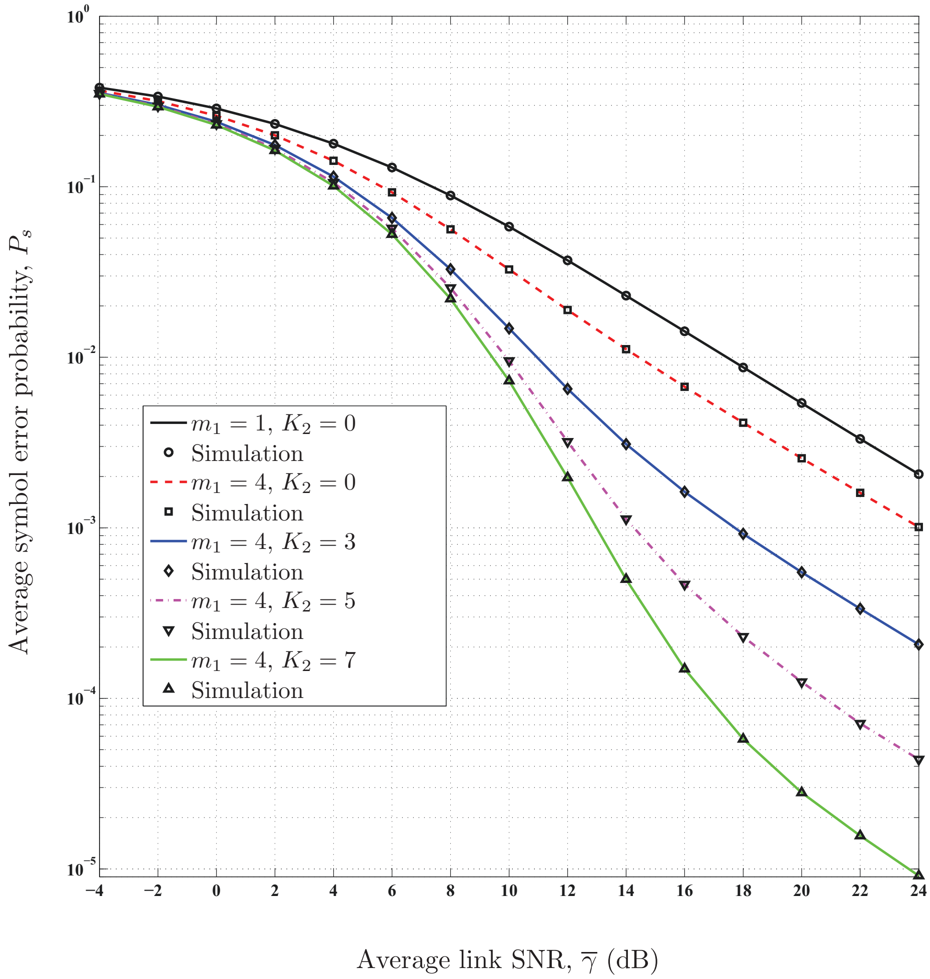

Figure 3.6: The average symbol error probability for dual-hop AF relaying systems with mixed Nakagami-m/Rician fading links and with BPSK modulation. Different cases of the Rician fading parameter, K_2, are shown.

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Chapter 4

Dual-Hop AF Systems With Adaptive Power Transmission

In this chapter[(1)], dual-hop AF relaying systems with adaptive power transmission are studied. A new exact closed-form expression is derived for the capacity of dual-hop AF relaying systems with adaptive channel inversion operating over Nakagami-m fading channels. Truncated channel inversion, at a certain cutoff signal-to-noise ratio (SNR), is assumed to avoid capacity loss resulting in the case of non-truncated channel inversion. The cutoff SNR is selected so as to maximize the system capacity. The results represent the first exact solutions for dual-hop AF systems under adaptive power transmission. The capacity of the systems under study is derived in terms of modified Bessel functions of the second kind, commonly available in mathematical software. The accuracies of the analytical results presented in this chapter are verified by Monte Carlo simulations.

[(1)] A version of this chapter has been presented in the IEEE Global Communications Conference (GLOBECOM) 2012:
S. S. Soliman and N. C. Beaulieu, "On the exact capacity of dual-hop AF relaying with adaptive channel inversion," in *GLOBECOM Workshops (GC Wkshps), 2012 IEEE*, Dec. 2012, pp. 441–446.

4.1 Introduction

Cooperative networks have received much interest in the literature recently because of their capability of providing extended coverage for wireless networks in addition to offering significant increases in the capacity and diversity gain of the networks [1]–[5]. The authors in [1] studied the outage probability and the average error probability of dual-hop relaying over Rayleigh fading channels using a harmonic mean bound for the end-to-end signal-to-noise ratio (SNR). In [2], the authors also used the harmonic mean bound to derive single-integral expressions for the average error probability of general wireless communication systems in terms of the moment generating function (MGF) of the reciprocal of the instantaneous end-to-end SNR and applied this approach to AF relaying systems. In [3], the authors studied dual-hop amplify-and-forward (AF) systems with selection diversity and derived closed-form expressions for the outage probability for systems operating over Nakagami-m fading links with integer values of m. The authors in [4, 5] presented the generalized transformed characteristic function (GTCF) approach to obtain exact analytical solutions for the performance metrics of multihop AF relaying systems operating over general fading links. Results derived in [4, 5] are based on the exact end-to-end SNR of the system, and it was shown that the harmonic mean bound can be loose for small-to-moderate values of SNR and for certain fading conditions. It was shown also in [5] that the ergodic capacity of multihop AF systems decreases as the number of hops increases.

The capacity of AF relaying systems has been considered in some works. The authors in [6] computed the ergodic capacity of multihop AF systems using an infinite series of the logarithmic function. The authors also presented an alternative framework for the special case of dual-hop AF relaying systems operating over Nakagami-m and generalized-K fading links. In [7], the authors used a similar approach to that used in [6] to obtain a closed-form expression for the ergodic capacity of dual-hop AF relaying systems with relay selection for the case of Rayleigh fading links.

However, all the previous work considered fixed rate and fixed power transmission. Adaptive transmission can be employed in cooperative wireless systems to provide enhanced utilization of the channel [8]. In [9]–[12], the authors used adaptive source transmission with

AF relaying. The authors in [9] derived upper bounds to adaptive transmission for dual-hop AF systems with maximum ratio combining (MRC) at the destination in the case of Rayleigh fading links. In [10], the authors developed a framework based on the MGF approach to find bounds for the ergodic capacity of adaptive transmission AF systems operating over Nakagami-m fading links. The performance bounds in [9] and [10] were based on the minimum SNR bound of the end-to-end SNR. The authors in [11] used the harmonic mean bound to obtain theoretical approximations for the capacity, as well as the outage probability, of multihop AF systems under adaptive transmission. Recently, the authors in [12] studied the effect of outdated channel information on the capacity of dual-hop AF selection relaying systems under adaptive transmission in the case of Rayleigh fading links. However, the authors in [12] used the minimum SNR bound for the end-to-end SNR.

In this chapter, we derive the first exact closed-form expression for the capacity of dual-hop AF relaying systems with adaptive channel inversion in the case of Nakagami-m fading links. An exact closed-form expression for the outage probability of these systems is also derived. To the best of the authors' knowledge, these expressions are the first exact expressions in the literature that are based on the exact end-to-end SNR. We adopt the adaptive scheme consisting of channel inversion with fixed rate because it has the lowest implementation complexity among adaptive schemes, as the source only adjusts its transmission power. We study the effect of unbalanced links, where the average SNRs of the first and the second links are not necessarily similar, and compare the system performance to that of systems with balanced links.

The remainder of the chapter is organized as follows. In Section 4.2, the system and channel models are presented. A closed-form expression for the capacity of dual-hop AF relaying systems under adaptive truncated channel inversion is derived in Section 4.3. In Section 4.4, numerical examples and discussions are presented. Finally, the chapter is concluded in Section 4.5.

4.2 System And Channel Models

The system under study is a dual-hop AF relaying system with adaptive source transmission, shown in Fig. 4.1. The source node and the destination node communicate through one intermediate relay node. The source node transmits the data signal to the relay node over the first link, $S-R$. The relay node, in turn, amplifies the received signal and transmits the amplified signal to the destination node over the second link, $R-D$. The amplification factor, A, at the relay node, is given by [11, 13] as

$$A = \sqrt{\frac{P_1}{P_0 \left|\alpha_1\right|^2 + N_{0_1}}} \tag{4.1}$$

where P_0 is the transmitter power at the source node, P_1 is the transmitter power at the relay node, and where α_1, and N_{01}, are respectively the fading gain of the first link, and the noise power at the relay node. Time division multiple access is assumed for transmission over both links and orthogonal, half-duplex operation is implemented to avoid inter-signal interference. The exact instantaneous end-to-end received SNR at the destination, γ_t, is given by [1, eq. (5)]

$$\gamma_t = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + 1} \tag{4.2a}$$

where $\gamma_i,\ i = 1, 2$ represents the instantaneous received SNR over the i^{th} link, defined as

$$\gamma_i = \frac{P_{i-1}}{N_{0_i}} \left|\alpha_i\right|^2. \tag{4.2b}$$

It was found in [14] that the probability density function (PDF) of the instantaneous end-to-end SNR, γ_t, as defined in (4.2a), can be obtained in a single integral closed-form as

$$f_{\gamma_t}(r) = \int_{x=r}^{\infty} \frac{x(x+1)}{(x-r)^2} f_{\gamma_1}\left(\frac{(x+1)r}{x-r}\right) f_{\gamma_2}(x)dx. \tag{4.3}$$

The expression in (4.3) can be used with any arbitrary fading distributions, $f_{\gamma_1}(\gamma_1)$ and $f_{\gamma_2}(\gamma_2)$, to obtain the PDF of the instantaneous end-to-end SNR.

For the adaptive systems considered in this chapter, the source adjusts its transmission power based on variations in the channel measurements. This necessitates estimation of channel conditions at the receiver and channel state information (CSI) feedback to the power control module at the source. In the following section, an analysis of the channel capacity of dual-hop AF systems with adaptive channel inversion is presented.

4.3 Capacity Analysis

In the original adaptive dual-hop AF systems, the source only adapts its power through channel inversion techniques, to maintain a constant SNR level at the destination, while the transmission rate is kept fixed. The channel capacity per unit bandwidth of a dual-hop AF relaying system employing channel inversion (CI) is given by [8, eq. (46)]

$$C_{CI} = \frac{1}{2}\log_2\left(1 + \frac{1}{\int_0^{\infty} \frac{1}{\gamma} f_{\gamma_t}(\gamma) d\gamma}\right) \tag{4.4}$$

where the $\frac{1}{2}$ factor accounts for the bandwidth used by orthogonal transmission over both the $S-R$ and the $R-D$ links. A drawback of this direct channel inversion technique is that the transmitter power has to compensate for deep fades of the channel, which results in large capacity loss [8]. A better approach is to use a truncated inversion technique. This technique inverts only the channel fadings above a certain SNR cutoff level, γ_0. Data transmission is stopped whenever the received SNR is below than the cutoff level, resulting in an outage of $P_{out}(\gamma_0) = Pr(\gamma_t \leq \gamma_0)$. The channel capacity per unit bandwidth of a dual-hop AF relaying system with truncated channel inversion (TCI) is given by [8, eq. (47)]

$$C_{TCI} = \frac{1}{2}\log_2\left(1 + \frac{1}{I(\gamma_0)}\right) Pr(\gamma_t \geq \gamma_0) \tag{4.5a}$$

where $Pr(\gamma_t \geq \gamma_0)$ is recognized as $1 - P_{out}(\gamma_0)$, and where

$$I(\gamma_0) = \int_{\gamma_0}^{\infty} \frac{1}{\gamma} f_{\gamma_t}(\gamma) d\gamma. \tag{4.5b}$$

In the following, we derive a closed-form expression for the channel capacity in (4.5) for the case of Nakagami-m fading links.

Substituting by the PDF expression (4.3) into (4.5b), the integral in (4.5b) can be written as

$$\begin{aligned} I(\gamma_0) &= \int_{\gamma_0}^{\infty} \frac{1}{r} f_{\gamma_t}(r) dr \\ &= \int_{\gamma_0}^{\infty} \frac{1}{r} \left[\int_{x=r}^{\infty} \frac{x(x+1)}{(x-r)^2} f_{\gamma_1}\left(\frac{(x+1)r}{x-r} \right) f_{\gamma_2}(x) dx \right] dr \\ &= \int_{x=\gamma_0}^{\infty} f_{\gamma_2}(x) \int_{r=\gamma_0}^{\infty} \frac{1}{r} \frac{x(x+1)}{(x-r)^2} f_{\gamma_1}\left(\frac{(x+1)r}{x-r} \right) dr dx \\ &= \int_{x=\gamma_0}^{\infty} f_{\gamma_2}(x) \zeta(x;\gamma_0) dx \end{aligned} \tag{4.6a}$$

where

$$\zeta(x;\gamma_0) = \int_{r=\gamma_0}^{\infty} \frac{1}{r} \frac{x(x+1)}{(x-r)^2} f_{\gamma_1}\left(\frac{(x+1)r}{x-r} \right) dr. \tag{4.6b}$$

Using a change of variables, $y = \dfrac{(x+1)r}{x-r}$, the integral expression in (4.6b) can be rewritten as

$$\begin{aligned} \zeta(x;\gamma_0) &= \int_{y=y_0}^{\infty} \left(\frac{1}{x} + \frac{1}{y} + \frac{1}{xy} \right) f_{\gamma_1}(y)\, dy \\ &= \left(1 + \frac{1}{x} \right) \xi_1(x;y_0) + \frac{1}{x} \xi_0(x;y_0) \end{aligned} \tag{4.7a}$$

where

$$\xi_i(x;y_0) = \int_{y=y_0}^{\infty} \frac{1}{y^i} f_{\gamma_1}(y)\, dy \tag{4.7b}$$

and

$$y_0 = \frac{(x+1)\gamma_0}{x-\gamma_0}. \tag{4.7c}$$

To get a closed-form expression for the integral in (4.7b) for the case of Nakagami-m fading links, the PDF $f_{\gamma_1}(y)$ can be replaced by the PDF expression for the Nakagami-m distribution found at [15, eq. (2.21)] resulting in

$$\xi_i(x;y_0) = \left(\frac{m_1}{\overline{\gamma}_1} \right)^i \frac{1}{\Gamma(m_1)} \Gamma_{upp}\left(m_1 - i\,, \frac{m_1}{\overline{\gamma}_1} y_0 \right) \tag{4.8}$$

where $\overline{\gamma}_j$ is the average SNR of the j^{th} link, and where $\Gamma_{upp}(\alpha, y)$ is the upper incomplete gamma function [16, eq. (8.350.2)]. Substituting (4.8) into (4.7a), we obtain a closed-form expression for (4.6b).

To find a closed-form expression for the integral in (4.6a), we use the finite series expansion of the upper incomplete gamma function in [16, eq. (8.352.2)], resulting in

$$\xi_i(x; y_{_0}) = \left(\frac{m_1}{\overline{\gamma}_1}\right)^i \frac{\Gamma(m_1 - i)}{\Gamma(m_1)} \exp\left[-\frac{m_1}{\overline{\gamma}_1} y_{_0}\right] \times \sum_{n=0}^{m_1-1-i} \frac{1}{n!} \left(\frac{m_1}{\overline{\gamma}_1}\right)^n y_{_0}^n. \tag{4.9}$$

Note that the expression in (4.9) is valid for integer values of $m_1 > i$. The expression (4.6a) can be then written as

$$\begin{aligned} I(\gamma_{_0}) = &\int_{x=\gamma_{_0}}^{\infty} f_{\gamma_2}(x) \left(1 + \frac{1}{x}\right) \frac{m_1}{\overline{\gamma}_1} \frac{1}{m_1 - 1} \exp\left[-\frac{m_1}{\overline{\gamma}_1} y_{_0}\right] \sum_{n=0}^{m_1-2} \frac{1}{n!} \left(\frac{m_1}{\overline{\gamma}_1}\right)^n y_{_0}^n dx \\ &+ \int_{x=\gamma_{_0}}^{\infty} f_{\gamma_2}(x) \frac{1}{x} \exp\left[-\frac{m_1}{\overline{\gamma}_1} y_{_0}\right] \sum_{n=0}^{m_1-1} \frac{1}{n!} \left(\frac{m_1}{\overline{\gamma}_1}\right)^n y_{_0}^n dx. \end{aligned} \tag{4.10}$$

Substituting for $f_{\gamma_2}(x)$ from [15, eq. (2.21)] into (4.10), and exchanging the order of summation and integration, we obtain

$$\begin{aligned} I(\gamma_{_0}) = C_2\, C_1 &\sum_{n=0}^{m_1-2} \frac{1}{n!} \left(\frac{m_1}{\overline{\gamma}_1}\right)^n \times (\Psi_{m_2-1} + \Psi_{m_2-2}) \\ &+ C_2 \sum_{n=0}^{m_1-1} \frac{1}{n!} \left(\frac{m_1}{\overline{\gamma}_1}\right)^n \times \Psi_{m_2-2} \end{aligned} \tag{4.11a}$$

where

$$C_2 = \left(\frac{m_2}{\overline{\gamma}_2}\right)^{m_2} \frac{1}{\Gamma(m_2)} \tag{4.11b}$$

$$C_1 = \frac{m_1}{\overline{\gamma}_1} \frac{1}{m_1 - 1} \tag{4.11c}$$

and

$$\Psi_\alpha = \int_{x=\gamma_{_0}}^{\infty} x^\alpha \exp\left[-\frac{m_2}{\overline{\gamma}_2} x\right] y_{_0}^n \exp\left[-\frac{m_1}{\overline{\gamma}_1} y_{_0}\right] dx \tag{4.11d}$$

where $y_{_0}$ was defined in (4.7c). Note that we can interchange the order of summation and integration in (4.10) based on a theorem on the infinite interval integration of a finite interval series found in [17, p. 453]. The theorem in [17] can be applied because its conditions are satisfied by the operands in (4.10).

Substituting (4.7c) into (4.11d), and using a change of variables, namely, $z = \frac{x}{\gamma_0} - 1$, the integral Ψ_α can be written as

$$\Psi_\alpha = \int_{z=0}^{\infty} \gamma_0^\alpha \left[1+z\right]^\alpha \exp\left[-\frac{m_2}{\overline{\gamma}_2}\left[\gamma_0 z + \gamma_0\right]\right] \gamma_0^n \times \left[1 + \frac{\gamma_0+1}{\gamma_0} z^{-1}\right]^n \exp\left[-\frac{m_1}{\overline{\gamma}_1}\left(\gamma_0 + \frac{\gamma_0+1}{z}\right)\right] \gamma_0 \, dz. \quad (4.12)$$

Applying the binomial expansion in [16, eq. (1.111)] to the two power terms in (4.12), namely, $[1+z]^\alpha$ and $\left[1 + \frac{\gamma_0+1}{\gamma_0} z^{-1}\right]^n$, and exchanging the order of summation and integration results

$$\Psi_\alpha = 2 \exp\left[-\left(\frac{m_1}{\overline{\gamma}_1} + \frac{m_2}{\overline{\gamma}_2}\right)\gamma_0\right] \gamma_0^{(n+\alpha+1)} \sum_{k=0}^{n}\sum_{j=0}^{\alpha} \binom{n}{k}\binom{\alpha}{j} \times \left(\frac{m_1/\overline{\gamma}_1}{m_2/\overline{\gamma}_2}\right)^{\frac{j-k+1}{2}} \left(\frac{\gamma_0+1}{\gamma_0}\right)^{\frac{j+k+1}{2}} K_{j-k+1}\left(2\sqrt{\frac{m_1}{\overline{\gamma}_1}\frac{m_2}{\overline{\gamma}_2}\gamma_0(\gamma_0+1)}\right) \quad (4.13)$$

where $K_\nu(\cdot)$ is the ν^{th}- order modified Bessel function of the second kind [18, eq. (9.6.2)], and $\binom{P}{Q}$ is the binomial coefficient. Substituting (4.13) into (4.11a), a closed-form expression is obtained for $I(\gamma_0)$.

An expression for $P_{out}(\gamma_0)$ can be obtained following similar procedure, resulting in

$$P_{out}(\gamma_0) = 1 - C_2 \sum_{n=0}^{m_1-1} \frac{1}{n!}\left(\frac{m_1}{\overline{\gamma}_1}\right)^n \Psi_{m_2-1}. \quad (4.14)$$

A similar expression was used in [3, eq. (12)] to obtain the outage probability of dual-hop AF systems with path selection diversity. However, this expression was not used to obtain other system performance metrics in [3]. This expression is reproduced here for easy reference. Substituting (4.14) and (4.11a) into (4.5a), a closed-form expression for the capacity of dual-hop AF relaying systems with adaptive truncated channel inversion is obtained.

In this chapter, we chose the cutoff SNR level which maximizes the system capacity as defined in (4.5a). This cutoff SNR level can be obtained by solving the equation $\frac{\partial C_{TCI}}{\partial \gamma_0} = 0$.

That is

$$\frac{\partial}{\partial\gamma_{_0}}\left[\frac{1}{2}\log_2\left(1+\frac{1}{I(\gamma_{_0})}\right)\left(1-P_{out}(\gamma_{_0})\right)\right]=0 \tag{4.15}$$

which can be solved resulting in the equation

$$\ln\left(1+\frac{1}{I(\gamma_{_0})}\right)=\frac{1-P_{out}(\gamma_{_0})}{\gamma_{_0}\,I(\gamma_{_0})\left(1+I(\gamma_{_0})\right)}\,. \tag{4.16}$$

Substituting (4.11) and (4.14) for $I(\gamma_{_0})$ and $P_{out}(\gamma_{_0})$, respectively, into (4.16), the cutoff SNR level can be numerically obtained.

Finally, the adaptive power transmission $P_{_{TCI}}(\gamma_{_t})$ at a source employing truncated channel inversion can vary subject to an average power constraint P_{av} as [19, eq. (4.19)]

$$\frac{P_{_{TCI}}(\gamma_{_t})}{P_{av}}=\begin{cases}0, & \gamma_{_t}<\gamma_{_0}\\ \dfrac{1}{\gamma_{_t}\,I(\gamma_{_0})}, & \gamma_{_t}\geq\gamma_{_0}\end{cases}\,. \tag{4.17}$$

4.4 Numerical Examples

In this section, we use some examples to show the behaviour of dual-hop AF relaying systems operating over Nakagami-m fading channels employing the truncated channel inversion adaptation technique. We assume equal noise powers, N_0, at the relay and the destination. We assume also that time division multiple access is considered for transmission over both links and orthogonal, half-duplex operation is implemented to avoid inter-signal interference. Finally, we assume and study both the cases of balanced links, where $\overline{\gamma}_1=\overline{\gamma}_2=\overline{\gamma}$, and unbalanced links, where $\overline{\gamma}_2=\alpha\,\overline{\gamma}_1$.

In the first example, Fig. 4.2 shows the channel capacity per unit bandwidth, of dual-hop AF relaying systems employing truncated channel inversion adaptive transmission, versus the cutoff threshold SNR level $\gamma_{_0}$, for different values of $\overline{\gamma}_1$, and for the case of $m_1=2$ and $m_2=3$. Different cases of balanced links, $\overline{\gamma}_2=\overline{\gamma}_1$, and unbalanced links $\overline{\gamma}_2=0.5\,\overline{\gamma}_1$, $\overline{\gamma}_2=2\,\overline{\gamma}_1$, are plotted for comparison. The figure shows that there is a cutoff SNR level for which the capacity per unit bandwidth is maximized. For example, the capacity per unit bandwidth is maximized for the balanced links case at $\gamma_{_0}=-1.0259$ dB for $\overline{\gamma}_1=5$

dB, while it is maximized at $\gamma_{_0} = 2.5324$ dB for $\overline{\gamma}_1 = 10$ dB. It can be seen also that, at any cutoff SNR level, the channel capacity per unit bandwidth for the case of $\overline{\gamma}_2 = 2\,\overline{\gamma}_1$ is higher than that for the case of $\overline{\gamma}_2 = \overline{\gamma}_1$, which in turn is higher than that for the case of $\overline{\gamma}_2 = 0.5\,\overline{\gamma}_1$. This is expected because in the case of $\overline{\gamma}_2 = 2\,\overline{\gamma}_1$, more power is allocated to the system than in the other cases. It can be shown also that the optimum cutoff SNR value is affected by the imbalance factor α. In order to have fair comparison between different systems employing balanced and unbalanced links, the total system power should be kept fixed for all systems.

Fig. 4.3 shows the channel capacity per unit bandwidth for two different sets of systems, each having the same total system power-to-noise power ratio, P/N_0 (this preserves the total system power). The systems of the first set are operating over non-identically distributed Nakagami-m fading links with $m_1 = 2$ and $m_2 = 3$, and the value of P/N_0 is set to 10 dB. The average link SNRs are related to each other and to P/N_0 as $\overline{\gamma}_2 = \alpha\,\overline{\gamma}_1 = \dfrac{\alpha}{\alpha+1}P/N_0$, where α is the imbalance factor. Fig. 4.3 shows that systems with balanced links exhibit higher channel capacity than systems with unbalanced links, and that the optimum cutoff SNR which maximizes the channel capacity, is also higher for systems with balanced links. It can be observed also that as the imbalance factor, α, increases in the range $0 < \alpha \leq 1$, the channel capacity increases, while as it increases in the range $\alpha \geq 1$, the channel capacity decreases, for all values of the cutoff SNR $\gamma_{_0}$. Similar observations can be seen in the curves of the second set of systems in Fig. 4.3. The second set of systems are operating over identically distributed Nakagami-m fading links with $m_1 = m_2 = 5$, and the value of P/N_0 is set to 5 dB. In addition to the previous observations, a symmetry in performance can be seen for both unbalanced systems with $\overline{\gamma}_2 = \alpha\,\overline{\gamma}_1$, and unbalanced systems with $\overline{\gamma}_2 = \dfrac{1}{\alpha}\,\overline{\gamma}_1$. This symmetry is achieved because the instantaneous end-to-end SNR, $\gamma_{_t}$, is symmetric with respect to $\gamma_{_1}$ and $\gamma_{_2}$ as evident in (4.2a), in addition to the assumption that both the first and the second fading links are identically distributed. This symmetry does not exist if the fading links are non-identically distributed as shown by the curves for the first set of systems. It can be concluded then that systems with balanced links achieve higher channel capacity per unit bandwidth, whether the fading links are identically or non-identically distributed. This represents a design criteria for system designers.

Table 4.1: Values of the optimum cutoff threshold level, $\gamma_{_{opt}}$, for different Nakagami-m fading links with $\overline{\gamma}_2 = 0.5\,\overline{\gamma}_1$

$\overline{\gamma}_1$ (dB)	Optimum $\gamma_{_0}$ ($\gamma_{_{opt}}$) $m_1 = 2, m_2 = 3$	Optimum $\gamma_{_0}$ ($\gamma_{_{opt}}$) $m_1 = 5, m_2 = 5$
0	0.2110	0.2034
5	0.5835	0.6034
10	1.3695	1.4764
15	2.9972	3.3474
20	6.6782	7.7332

In the next example, we study the performance of dual-hop AF relaying systems under adaptive truncated channel inversion and cutoff SNR optimization. Figs. 4.4 and 4.5 show respectively the channel capacity per unit bandwidth, ε, and the outage probability, $P_{out}(\gamma_{_{opt}})$, versus the first link's average SNR, $\overline{\gamma}_1$, for different Nakagami-m fading links with $\overline{\gamma}_2 = 0.5\,\overline{\gamma}_1$. Table 4.1 shows the values of the optimum cutoff SNR, $\gamma_{_{opt}}$, for two different cases of Nakagami-m fading links. The results in Table 4.1 emphasize a previous conclusion that the optimum cutoff SNR increases unbounded with increase in the average link SNRs. These results are also used to generate the curves in Figs. 4.4 and 4.5. Precise agreement between Monte Carlo simulation results and the analytical results can be seen in both figures. Fig. 4.4 shows that the channel capacity increases with increase of the channel fading parameters m_1 and m_2, however, the increase is negligible. For example, the channel capacity per unit bandwidth at $\overline{\gamma}_1 = 20$ dB is 2.223 bps/Hz for systems with $m_1 = 2, m_2 = 3$, while it is 2.449 pbs/Hz for systems with $m_1 = 10, m_2 = 7$. On the other hand, the outage probability of the system is significantly improved by larger values of the channel fading parameters. For example, the outage probability at $\overline{\gamma}_1 = 20$ dB is 1.61×10^{-2} for systems with $m_1 = 2, m_2 = 5$, while it is 5.76×10^{-4} for systems with $m_1 = 5, m_2 = 7$. The reason for such remarkable improvement is that the limiting slopes of the outage probability curves are proportional to the m parameter of Nakagami-m fading links.

4.5 Conclusion

New, exact closed-form expression for the channel capacity of dual-hop AF relaying systems with adaptive truncated channel inversion was derived for systems operating over Nakagami-m fading links. The derived expression is the first exact solution for the channel capacity of dual-hop systems with truncated channel inversion power adaptation. A maximum channel capacity criteria was considered to find the optimum cutoff SNR truncation levels. The results obtained analytically were verified by Monte Carlo simulations. Different cases of systems with balanced and unbalanced links were considered. It was shown that systems with balanced links achieve higher channel capacity for all cutoff SNR values and exhibit higher values of the optimum cutoff SNR.

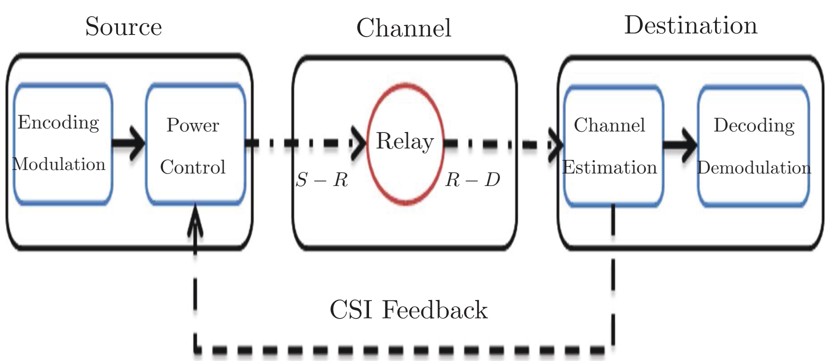

Figure 4.1: Dual-hop AF relaying system with adaptive source power transmission / channel inversion.

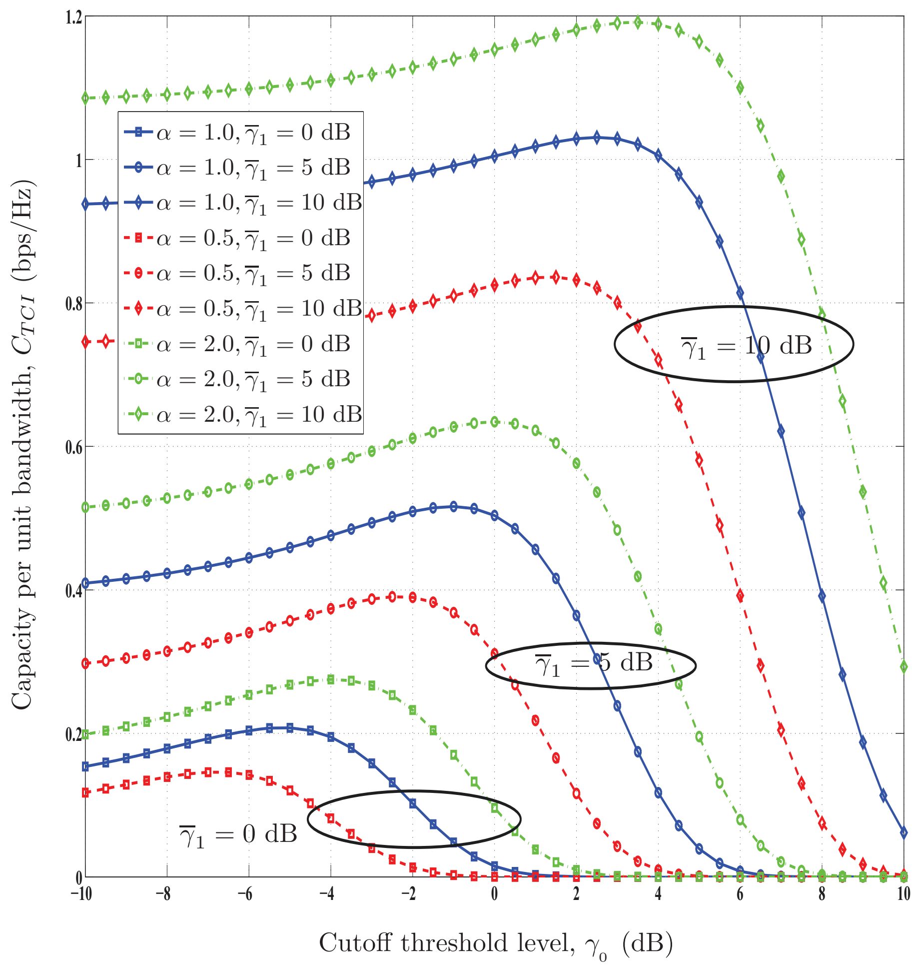

Figure 4.2: The channel capacity per unit bandwidth versus the cutoff threshold level, γ_0. Dual-hop AF relaying systems operating over Nakagami-m fading links with $m_1 = 2$ and $m_2 = 3$ are assumed. A truncated channel inversion adaptation scheme is employed. Different cases of balanced and unbalanced SNR are assumed, $\overline{\gamma}_2 = \alpha\, \overline{\gamma}_1$.

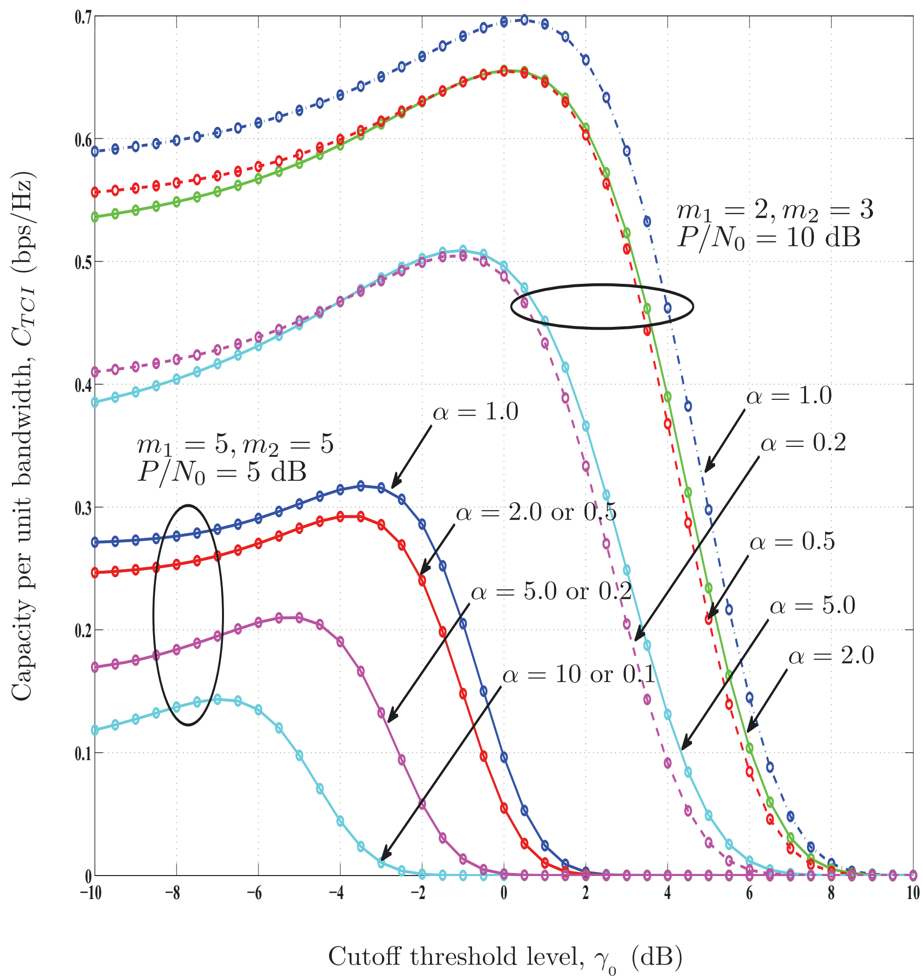

Figure 4.3: The channel capacity per unit bandwidth versus the cutoff threshold level, γ_0. Dual-hop AF systems operating over Nakagami-m fading links are assumed. A truncated channel inversion adaptation scheme is employed. Different cases of balanced and unbalanced SNR are assumed, $\overline{\gamma}_2 = \alpha\, \overline{\gamma}_1$, where the system's total available power is assumed fixed.

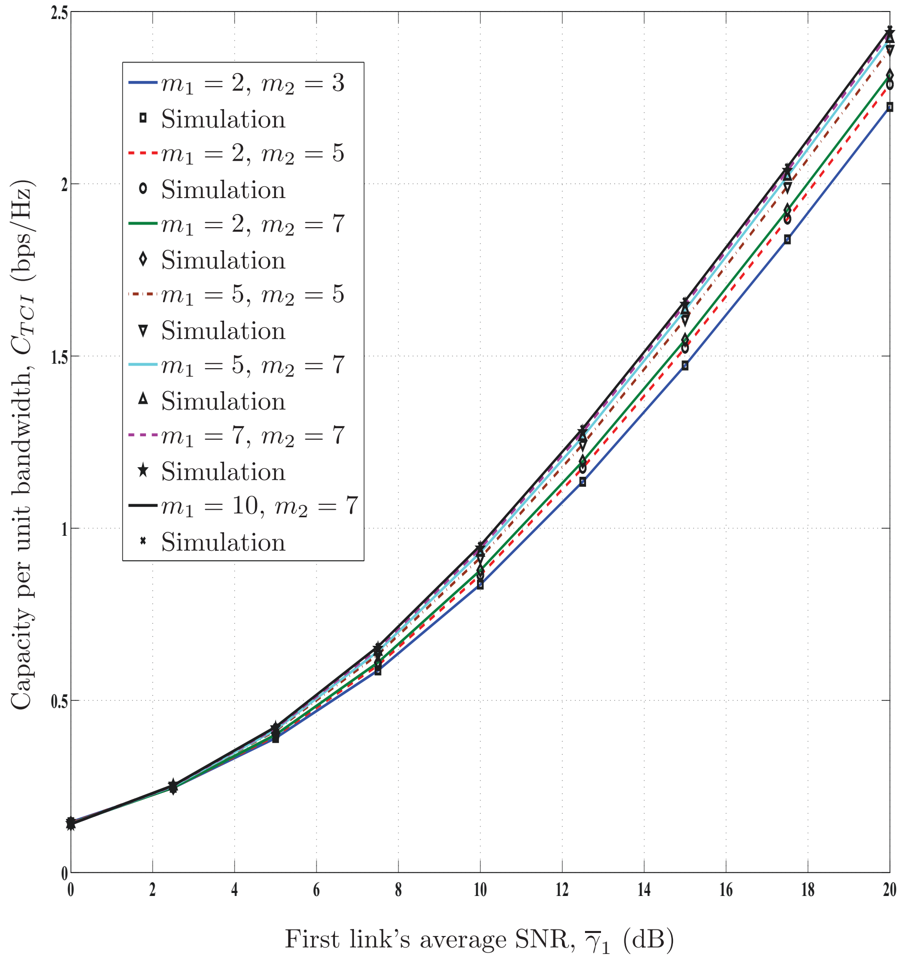

Figure 4.4: The channel capacity per unit bandwidth of dual-hop AF relaying systems operating over Nakagami-m fading links, employing truncated channel inversion adaptation.

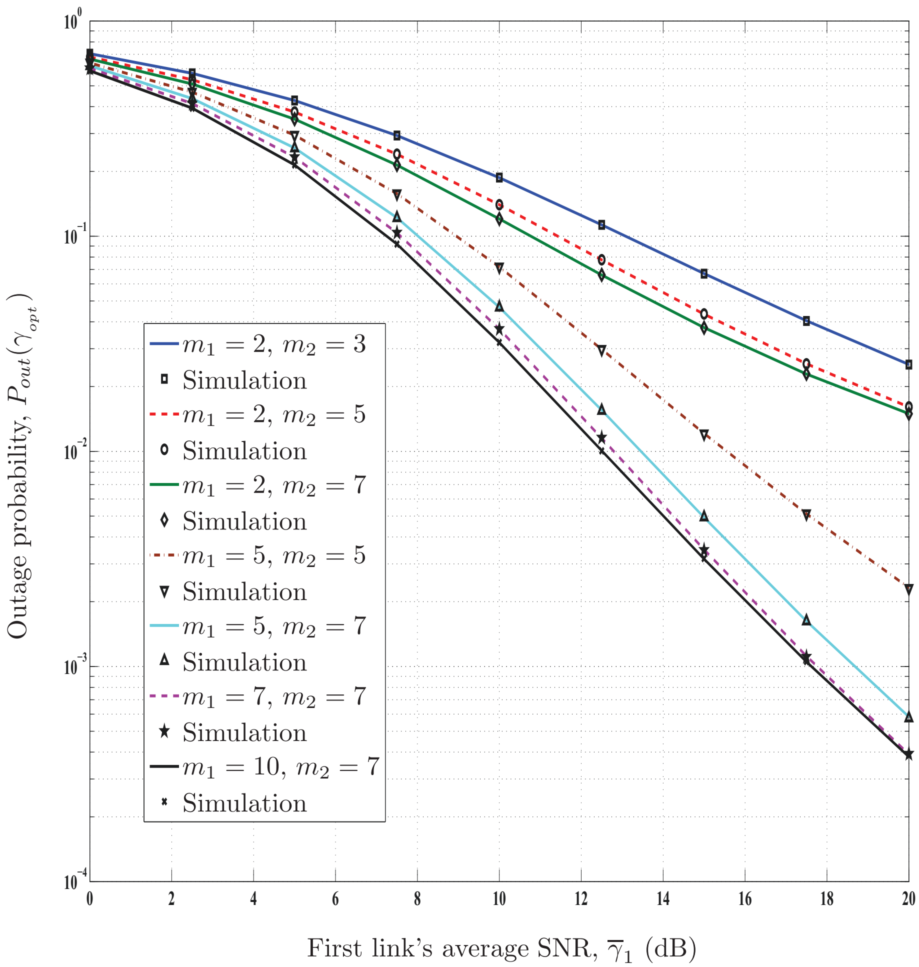

Figure 4.5: Outage probability of dual-hop AF relaying systems operating over Nakagami-m fading links, employing truncated channel inversion adaptation.

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Chapter 5

Dual-Hop AF Systems with Maximum End-to-End SNR Relay Selection

In this chapter[(1)], dual-hop amplify-and-forward (AF) relaying systems with maximum end-to-end SNR relay selection are studied. In these systems, a relaying node is selected from N available relays based on a maximum end-to-end SNR policy. New, precise expressions are derived for the PDF and the CDF of the end-to-end SNR of such systems. The results are precise for any number of candidate relays and Rayleigh, Nakagami-m or Rician fading distributions. The effects of different channel fading parameters and the number of relays in the relay selection pool are studied. The system performance is compared to that of dual-hop AF systems without relay selection and to dual-hop AF relaying systems with partial relay selection. This selection method provides diversity gain over dual-hop AF relaying systems without relay selection and over systems with partial relay selection. The diversity gain is proportional to the relay selection pool size, N.

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5.1 Introduction

Cooperative schemes have been recently proposed as a promising technique for high quality wireless communication services. The concept of cooperative networking represents a technique that can significantly increase the system capacity and diversity gain in wireless networks [1]–[9]. In cooperative networks, the source sends the data signal to one or more intermediate nodes called relays. These relays perform some distributed processing on the received signals and then retransmit the processed signal to the next node. The next node may be the destination, in the case of dual-hop relaying systems, or it may be another relay node, in the case of multihop relaying systems. The most popular and studied relaying protocols are amplify-and-forward (AF) and decode-and-forward (DF). In AF relaying systems, the relays amplify the received signal before forwarding it to the next node while in DF relaying systems, the relay nodes completely decode the received signal, regenerate it and forward the reconstructed signal to the next node. This requires more processing power and complexity at the relay nodes than the AF technique and represents the main disadvantages of the DF scheme.

Much research has focused on predicting the performance of AF relaying systems. The outage probability and the average symbol probability of error of dual-hop transmission over Rayleigh fading channels were studied in [2]. In [2], the authors presented an approximation for the end-to-end received signal-to-noise ratio (SNR) using the harmonic mean of two independent exponential random variables and in [3], the harmonic mean approximation was again used to obtain an approximate expression for the moment generating function (MGF) of the reciprocal of the end-to-end SNR for multihop transmission over Nakagami-m fading channels. The approximate outage probability was then obtained via numerical inversion of the Laplace transform. It has since been found, that the bounds based on the direct harmonic mean approximation of the individual per hop instantaneous SNRs are not tight for small-to-moderate values of SNR, and for Nakagami-m fading channels for large values of m [4,5]. Moreover, increasing the number of hops may decrease the tightness of those bounds. In [6] and [7] an approximation was proposed that has the same computational complexity as the previous bounds in the literature, while being more accurate, especially for small-to-medium values of SNR. Compared to other bounds, the new approximation gives more accurate prediction of the exact performance. In [8] and [9], a novel theoretical technique, the generalized transformed characteristic function (GTCF) approach, was proposed and was used to obtain the first exact analytical results for the ergodic capacity, the average error probability and outage probability of multihop AF systems operating over general fading channels. In [10], the dual-hop AF system with selection diversity was studied and an exact closed-form expression for the outage probability was derived for the case of Nakagami-m fading links with integer values of m; yet, that expression was not used to obtain other important performance metrics. In [11], dual-hop AF systems were studied, and exact closed-form expressions for the PDF and the CDF of the end-to-end SNR were derived for the general cases of Nakagami-m, Rician fading links and mixed Nakagami-m/Rician fading links. Exact closed-form expressions for the PDF and the CDF of the end-to-end SNR of opportunistic dual-hop AF systems were derived in [12]. The expressions derived in both [11] and [12] were then used to obtain exact solutions for the ergodic capacity, outage probability and the average error probability of dual-hop AF systems without relay selection, and opportunistic dual-hop AF relaying systems with relay selection based on the relay-to-destination link transmission statistics, respectively.

In this chapter, we use the closed-form expressions for the PDF and the CDF of the end-to-end SNR of dual-hop AF systems derived in [11](2) to obtain the first exact closed-form expressions for the PDF and the CDF of the end-to-end SNR of a dual-hop opportunistic AF relaying system with maximum end-to-end SNR relay selection. The maximum end-to-end SNR relay selection scheme was proposed in [13]–[15]. The work in [11] and [12] did not consider the maximum end-to-end SNR relay selection criterion. Relay selection in AF relaying systems was previously proposed in [13]–[19] to enhance the performance of AF relaying systems. In [13], the authors developed a distributed relay selection method that requires no topology information and it was based on measurements of the instantaneous channel conditions. In [14], the authors adopted that relay selection method in AF relaying systems, while also combining both the direct path and the dual-hop path signals with MRC receivers at the destination. They obtained closed-form expressions for the average sym-

(2) These PDF and CDF expressions can be found in Chapter 3 of this thesis.

bol error probability for such systems operating over Nakagami-m fading channels. In [15], the authors also adopted the relay selection method in [13] and derived closed-form expressions for performance bounds on dual-hop AF relaying systems operating over generalized gamma fading channels. In [13]–[15], the authors approximated the end-to-end SNR, for each possible relay connection, using the minimum value of the SNRs of the source-to-relay and relay-to-destination links. This approximation is not tight and results obtained using it are not tight for some ranges of SNR of practical interest. This limitation will be shown in the examples of the present chapter. In [16, 17], the authors examine the same problem considered in [14], but they used the harmonic mean approximation of the end-to-end SNR, and therefore did not obtain exact solutions. In other works, [18, 19], the authors used another relay selection method that was based on measurements of the relay-to-destination channel conditions, but exact results for the performance metrics of such systems operating in Nakagami-m fading channels were not derived. Exact results for this selection method were obtained in [12] and it was found that increasing the number of relays in the relay selection pool has diminishing returns because the relay selection is based only on the relay-to-destination link statistics(3).

The contributions of the chapter are 1) obtaining the first exact results for the performance metrics of dual-hop opportunistic AF systems with maximum end-to-end SNR relay selection, 2) obtaining these performance metrics for systems operating over Rician fading links as well as over Nakagami-m fading links, 3) determining how loose the performance bounds presented previously in the literature are, 4) comparing the results obtained for the relay selection system to the results when no relay selection is adopted, 5) showing the large benefits of using a maximum end-to-end SNR relay selection criterion over a maximum relay-to-destination SNR relay selection criterion through numerical examples, and 6) studying the effect of increasing the number of relays in the relay selection pool on the performance metrics.

The remainder of the chapter is organized as follows. In Section 5.2, the system and channel models under study are presented along with the system assumptions. The maximum end-to-end SNR relay selection method is explained in detail, discussed and analyzed

(3) Detailed analysis and general results for this selection method can be found in [20]

in Section 5.3. In Section 5.4, some particulars of the maximum relay-to-destination SNR relay selection method are recalled briefly for the purpose of comparison with the maximum end-to-end SNR relay selection method. In Section 5.5, the new solutions are used to obtain numerical results for the ergodic capacity, outage probability and average error probability of dual-hop opportunistic AF relaying with maximum end-to-end SNR relay selection for the cases of Rayleigh, Nakagami-m and Rician fading channels. Finally, the chapter is concluded in Section 5.6.

5.2 System And Channel Models

The system under study is a dual-hop opportunistic AF relaying system, shown in Fig. 5.1. The source node, S, and the destination node, D, communicate through one intermediate relay node, R, selected from N candidate relay nodes, $R_1, R_2, \cdots, R_N$. The source node, S, transmits the data signal to the selected (*best*) relay, R, over the first link, $S-R$. The relay node, in turn, amplifies the received signal and transmits the amplified signal to the destination node over the second link, $R-D$. The amplification factor, A, at the relay node, R, is given in [1] as $A = \sqrt{\frac{P_1}{P_0 \left|\alpha_1\right|^2 + N_{0_1}}}$, where P_0 is the transmitter power at the source node, P_1 is the transmitter power at the relay node, $\alpha_i,\ i = 1, 2$ is the fading gain of the i^{th} link and where N_{0_1} and N_{0_2} are respectively, the noise power at the relay node and the destination node. Time division multiple access is assumed for transmission over both links and orthogonal, half-duplex operation is implemented to avoid inter-signal interference. The exact instantaneous end-to-end received SNR at the destination, γ_t, is given by [2, eq. (2)]

$$\gamma_t = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + 1} \tag{5.1a}$$

where $\gamma_i,\ i = 1, 2$ represents the instantaneous received SNR over the i^{th} link defined as

$$\gamma_i = \frac{P_{i-1}}{N_{0_i}} \left|\alpha_i\right|^2. \tag{5.1b}$$

Note that γ_1 and γ_2 are the SNRs of the fading links associated with the selected relaying node.

Assuming that all the link fadings are statistically independent Nakagami-m fading links, the probability density function (PDF) of the SNR, γ_n, for any of the source-to-relay links, $S - R_n$, or any of the relay-to-destination links, $R_n - D$, is given by [21],

$$f_{\gamma_n}(r) = \left(\frac{m_n}{\overline{\gamma}_n}\right)^{m_n} \frac{1}{\Gamma(m_n)} r^{m_n - 1} \exp\left[-\frac{m_n}{\overline{\gamma}_n} r\right], \quad r \geqslant 0 \tag{5.2}$$

where $\Gamma(\cdot)$ is the gamma function and $\overline{\gamma}_n = E\{\gamma_n\}$ is the link's average SNR, where $E\{\cdot\}$ is the expectation operator. For Rician fading links, the PDF of γ_n is given by [21],

$$f_{\gamma_n}(r) = \left(\frac{1+K_n}{\overline{\gamma}_n}\right) \exp\left[-K_n\right] \exp\left[-\frac{1+K_n}{\overline{\gamma}_n} r\right] \times I_0\left(2\sqrt{K_n\left(\frac{1+K_n}{\overline{\gamma}_n}\right) r}\right), \quad r \geqslant 0 \tag{5.3}$$

where K_n is the ratio of the power of the line-of-sight (LOS) component to the average power of the scattered component and $I_\alpha(\cdot)$ is the α^{th}- order modified Bessel function of the first kind [22, eq. (9.6.10)]. Note that for Rayleigh fading links, the PDF of γ_n is obtained by substituting $m = 1$ into (5.2), or by substituting $K = 0$ into (5.3). The cumulative distribution function (CDF) of γ_n for Nakagami-m fading links is

$$F_{\gamma_n}(r) = \frac{\Gamma_{inc}\left(m_n\,, \frac{m_n}{\overline{\gamma}_n} r\right)}{\Gamma(m_n)}, \quad r \geqslant 0 \tag{5.4}$$

where $\Gamma_{inc}(\alpha, x)$ is the lower incomplete gamma function [23, eq. (8.350.1)]. For Rician fading channels, the CDF of γ_n is,

$$F_{\gamma_n}(r) = 1 - Q_1\left(\sqrt{2K_n}\,, \sqrt{2\left(\frac{1+K_n}{\overline{\gamma}_n}\right) r}\right), \quad r \geqslant 0 \tag{5.5}$$

where $Q_j(\alpha, \beta)$ is the j^{th}- order Marcum Q-function [21, eq. (4.60)]. For the special case of Rayleigh fading, the CDF of γ_n is obtained by substituting $m = 1$ into (5.4), or by substituting $K = 0$ into (5.5), resulting in

$$F_{\gamma_n}(r) = 1 - \exp\left[-\frac{1}{\overline{\gamma}_n} r\right], \quad r \geqslant 0. \tag{5.6}$$

5.3 Maximum End-to-End SNR Relay Selection

In this section, we adopt a distributed method to select the best relay. The selection method is independent of the network topology and is based on local measurements of the instantaneous channel transmission conditions. The selection method follows a SNR policy in the sense that the selected best relay is the relay which, when used for relaying, achieves a maximum instantaneous SNR.

The work presented here is different from that presented in [12]. In [12], the instantaneous SNRs of only the relay-to-destination links were taken into consideration in the selection process. This selection is optimum, from the SNR perspective, if the fading of the source-to-relay link is the same for all of the N candidate relays. However, the more general and more practical case is when the different source-to-relay links exhibit different fadings with possibly the same fading statistics. In this practical case, it makes more sense that the selection process should be based on both the source-to-relay link SNR and the relay-to-destination link SNR.

The system presented here adopts the latter selection method, in which the relaying node is selected such that the instantaneous end-to-end SNR is maximized [13]–[15], i.e.

$$\max_{R_n \in \{R_1,\cdots,R_N\}} \gamma_{S-R_n-D} \Rightarrow \gamma_t \tag{5.7a}$$

where γ_{S-R_n-D} is the instantaneous end-to-end SNR of the path through the n^{th} relay, R_n, defined as

$$\gamma_{S-R_n-D} = \frac{\gamma_{S-R_n}\gamma_{R_n-D}}{\gamma_{S-R_n} + \gamma_{R_n-D} + 1} \tag{5.7b}$$

where γ_{S-R_n} is the instantaneous SNR of the $S - R_n$ link between the source and the n^{th} relay, and γ_{R_n-D} is the instantaneous SNR of the $R_n - D$ link between the n^{th} relay and the destination.

Exact closed-form expressions for the PDF and the CDF of the SNR, γ_{S-R_n-D}, of dual-hop AF systems were obtained in [11] for the common link fading distributions. For Nakagami-m fading links, the PDF of γ_{S-R_n-D} is given by

$$f_{\gamma_{S-R_n-D}}(r) = 2\left(\frac{m_1}{\overline{\gamma}_1}\right)^{m_1} \frac{1}{\Gamma(m_1)} \left(\frac{m_2}{\overline{\gamma}_2}\right)^{m_2} \frac{1}{\Gamma(m_2)} \exp\left[-\left(\frac{m_1}{\overline{\gamma}_1}+\frac{m_2}{\overline{\gamma}_2}\right) r\right] r^{m_1+m_2-2}$$
$$\times \sum_{k=0}^{m_1-1} \sum_{j=0}^{m_2-1} \binom{m_1-1}{k} \binom{m_2-1}{j} \left(\frac{m_1\overline{\gamma}_2}{m_2\overline{\gamma}_1}\right)^{\frac{j-k}{2}} \left(\frac{r+1}{r}\right)^{\frac{j+k}{2}}$$
$$\times \left[\sqrt{\frac{m_1\overline{\gamma}_2}{m_2\overline{\gamma}_1}\, r(r+1)}\, K_{j-k+1}\left(2\sqrt{\frac{m_1 m_2}{\overline{\gamma}_1\overline{\gamma}_2} r(r+1)}\right) + (2r+1)\, K_{j-k}\left(2\sqrt{\frac{m_1 m_2}{\overline{\gamma}_1\overline{\gamma}_2} r(r+1)}\right)\right.$$
$$\left. + \sqrt{\frac{m_2\overline{\gamma}_1}{m_1\overline{\gamma}_2}\, r(r+1)}\, K_{j-k-1}\left(2\sqrt{\frac{m_1 m_2}{\overline{\gamma}_1\overline{\gamma}_2} r(r+1)}\right)\right] \quad (5.8)$$

where $K_\nu(\cdot)$ is the ν^{th}- order modified Bessel function of the second kind [22, eq. (9.6.2)] and where $\binom{P}{Q}$ is the binomial coefficient. The CDF of γ_{S-R_n-D} is given by

$$F_{\gamma_{S-R_n-D}}(r) = 1 - 2\left(\frac{m_2}{\overline{\gamma}_2}\right)^{m_2} \frac{1}{\Gamma(m_2)} \exp\left[-\left(\frac{m_1}{\overline{\gamma}_1}+\frac{m_2}{\overline{\gamma}_2}\right) r\right] \sum_{n=0}^{m_1-1} \frac{1}{n!} \left(\frac{m_1}{\overline{\gamma}_1}\right)^{n} r^{n+m_2}$$
$$\times \sum_{k=0}^{n} \sum_{j=0}^{m_2-1} \binom{n}{k} \binom{m_2-1}{j} \times \left(\frac{m_1\overline{\gamma}_2}{m_2\overline{\gamma}_1}\right)^{\frac{j-k+1}{2}} \left(\frac{r+1}{r}\right)^{\frac{j+k+1}{2}} K_{j-k+1}\left(2\sqrt{\frac{m_1 m_2}{\overline{\gamma}_1\overline{\gamma}_2} r(r+1)}\right). \quad (5.9)$$

It is noted that a similar expression to (5.9) has been previously presented in [10, eq. (12)], however the system performance metrics for the variable-gain AF relaying system in [10] were not obtained. The PDF in (5.8) can be derived by taking the derivative of (5.9) but we present both for easy reference. Note also that the expressions in (5.8) and (5.9) are valid for integer values of m_1 and m_2 [11]. For Nakagami-m fading links with non-integer values of m, interpolation can be used to obtain extended results. For Rician fading links, the PDF, $f_{\gamma_{S-R_n-D}}(r)$, and the CDF, $F_{\gamma_{S-R_n-D}}(r)$, are given in closed-forms as

$$f_{\gamma_{S-R_n-D}}(r) = 2\, A_1\, A_2\, e^{-(K_1+K_2)} \exp\left[-\left(A_1+A_2\right) r\right] \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} a_1(n) a_2(m)\, r^{n+m}$$
$$\times \sum_{k=0}^{n} \sum_{j=0}^{m} \binom{n}{k} \binom{m}{j} \left(\frac{A_1}{A_2}\right)^{\frac{j-k}{2}} \left(\frac{r+1}{r}\right)^{\frac{j+k}{2}}$$
$$\times \left[\sqrt{\frac{A_1}{A_2}\, r(r+1)}\, K_{j-k+1}\left(2\sqrt{A_1\, A_2\, r(r+1)}\right) + (2r+1)\, K_{j-k}\left(2\sqrt{A_1\, A_2\, r(r+1)}\right)\right.$$
$$\left. + \sqrt{\frac{A_2}{A_1}\, r(r+1)}\, K_{j-k-1}\left(2\sqrt{A_1\, A_2\, r(r+1)}\right)\right] \quad (5.10)$$

and

$$F_{\gamma_{S-R_n-D}}(r) = 1 - 2\, e^{-(K_1+K_2)} \exp\left[-\,(A_1 + A_2)\, r\right] \sum_{l=0}^{\infty} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{{K_1}^{l+n}\, {K_2}^{m}}{(l+n)!\; n!\; (m!)^2}$$
$$\times\, (A_1)^n\, (A_2)^{m+1}\, r^{n+m+1} \sum_{k=0}^{n} \sum_{j=0}^{m} \binom{n}{k} \binom{m}{j} \times \left(\frac{A_1}{A_2}\right)^{\frac{j-k+1}{2}} \left(\frac{r+1}{r}\right)^{\frac{j+k+1}{2}}$$
$$\times\, K_{j-k+1}\left(2\sqrt{A_1\, A_2\, r(r+1)}\right) \quad (5.11)$$

respectively, where

$$A_i = \frac{(1+K_i)}{\overline{\gamma}_i} \quad (5.12a)$$

$$a_i(n) = \frac{1}{(n!)^2}\, (K_i \times A_i)^n\, . \quad (5.12b)$$

The analytical derivations of (5.10) and (5.11) have been presented in detail in [11]. For the special case of Rayleigh fading links, the PDF and the CDF expressions reduce to

$$f_{\gamma_{S-R_n-D}}(r) = \frac{2}{\overline{\gamma}_1 \overline{\gamma}_2} \exp\left[-\left(\frac{1}{\overline{\gamma}_1} + \frac{1}{\overline{\gamma}_2}\right) r\right]$$
$$\times \left[\left(\sqrt{\frac{\overline{\gamma}_2}{\overline{\gamma}_1}} + \sqrt{\frac{\overline{\gamma}_1}{\overline{\gamma}_2}}\right) \sqrt{r(r+1)} K_1\left(2\sqrt{\frac{r(r+1)}{\overline{\gamma}_1 \overline{\gamma}_2}}\right) + (2r+1)\, K_0\left(2\sqrt{\frac{r(r+1)}{\overline{\gamma}_1 \overline{\gamma}_2}}\right)\right] \quad (5.13)$$

and

$$F_{\gamma_{S-R_n-D}}(r) = 1 - 2 \exp\left[-\left(\frac{1}{\overline{\gamma}_1} + \frac{1}{\overline{\gamma}_2}\right) r\right] \sqrt{\frac{r(r+1)}{\overline{\gamma}_1 \overline{\gamma}_2}}\, K_1\left(2\sqrt{\frac{r(r+1)}{\overline{\gamma}_1 \overline{\gamma}_2}}\right). \quad (5.14)$$

With the knowledge of the PDF, $f_{\gamma_{S-R_n-D}}(r)$, and the CDF, $F_{\gamma_{S-R_n-D}}(r)$, of γ_{S-R_n-D}, order statistics can be used to obtain the PDF and the CDF of the end-to-end SNR, γ_t. The PDF of the k^{th} order statistic is given by [24, p. 198]

$$f_{y_k}(y_k) = \frac{N!}{(k-1)!(N-k)!} f(y_k)\, [F(y_k)]^{k-1}\, [1 - F(y_k)]^{N-k} \quad (5.15)$$

where y_k is the k^{th} order statistic, defined as the k^{th} minimum random variable of total N random variables [25, p. 192], and $f(\cdot)$ and $F(\cdot)$ are the PDF and the CDF, respectively, of the N independent and identically distributed ($i.i.d.$) ordered random variables. Since γ_t is selected as the maximum (N^{th} order statistic) of the N source-to-destination link SNRs, γ_{S-R_n-D}, the PDF of γ_t can be written as,

$$f_{\gamma_t}(r) = N f_{\gamma_{S-R_n-D}}(r) \left[F_{\gamma_{S-R_n-D}}(r)\right]^{N-1} \tag{5.16}$$

and the CDF of γ_t can be written as,

$$F_{\gamma_t}(r) = \left[F_{\gamma_{S-R_n-D}}(r)\right]^{N}. \tag{5.17}$$

To get the end result, we substitute (5.8) and (5.9) for Nakagami-m fading links, or (5.10) and (5.11) for Rician fading links or (5.13) and (5.14) for Rayleigh fading links into (5.16) and (5.17) to obtain closed-form expressions for the PDF and the CDF of the instantaneous end-to-end SNR of opportunistic dual-hop AF relaying system with maximum end-to-end SNR relay selection. These expressions represent the first exact, closed-form expressions obtained for γ_t defined in (5.7). Note that although the expression in (5.10) and the expression in (5.11) involve nested infinite summations, the summands decay (slightly faster than) exponentially [(4)] for the high order terms, because of the "*factorial*" factors, and hence, a truncated summation with a finite number of terms will achieve a required accuracy. Owing to this rapid rate of convergence, a careful empirical test for convergence is allowable and also for truncation error estimation. Estimation of the truncation errors will be shown through the numerical examples.

When the PDF and the CDF of the instantaneous end-to-end SNR, γ_t, are known, different system performance metrics can be evaluated. The outage probability, P_{out}, defined as the probability that the instantaneous end-to-end SNR falls below a certain threshold SNR, γ_{th}, is given by [21, eq. (15.6)]

$$P_{out} = Pr\{\gamma_t \leqslant \gamma_{th}\} = \int_0^{\gamma_{th}} f_{\gamma_t}(r)dr = F_{\gamma_t}(\gamma_{th}). \tag{5.18}$$

[(4)] Stirling's approximation specifies that $n!$ grows as $e^{n\ln n}$, and hence $\frac{1}{(n!)^2}$ decays as $e^{-2n\ln n}$, and $\frac{1}{(l+n)!n!(m!)^2}$ decays as $e^{-(l+n)\ln(l+n)-n\ln n-2m\ln m}$.

The ergodic capacity, ε, can be evaluated using [21, eq. (15.21)]

$$\begin{aligned}\varepsilon &= \frac{1}{2} E\left\{\log_2(1+\gamma_t)\right\} \\ &= \frac{1}{2}\int_0^{\infty} \log_2(1+r) f_{\gamma_t}(r) dr.\end{aligned} \tag{5.19}$$

where the $\frac{1}{2}$ factor accounts for orthogonal transmission over the two links, and the average symbol error probability, P_s, is defined as [21, eq. (5.1)]

$$P_s = E\left\{b\, Q\left(\sqrt{a\,\gamma_t}\right)\right\} = \int_0^{\infty} b\, Q\left(\sqrt{a\,r}\right) f_{\gamma_t}(r) dr \tag{5.20}$$

where $Q(\cdot)$ is the Gaussian Q-function [21] and the parameters (a, b) depend on the modulation scheme. Note that the outage probability, P_{out}, can be obtained by direct substitution into the closed-form CDF expression (5.17), while the calculation of the ergodic capacity, ε, and the average error probability, P_s, involves a single-fold integral that is evaluated numerically using common software packages such as MATLAB. Numerical evaluation of single-fold integrals is widely practiced in the literature [21].

5.4 Partial Relay Selection

In this section, we recall briefly the mechanics of dual-hop AF relaying systems with maximum relay-to-destination SNR relay selection [18, 19][(5)]. To the best of the authors' knowledge, a comparison between the performance of dual-hop AF relaying systems employing maximum end-to-end SNR relay selection method and those employing maximum relay-to-destination relay selection method does not exist in the literature. In this chapter, we undertake such comparisons between both relay selection methods and we will see the large performance benefits of systems adopting the maximum end-to-end SNR relay selection method over those adopting the maximum relay-to-destination relay selection method. For the maximum relay-to-destination selection method, the relaying node is selected such

[(5)]Note that maximum relay-to-destination SNR relay selection and maximum source-to-relay SNR relay selection are possible versions of partial relay selection. Detained analysis and results on partial relay selection can be found in [20] that the instantaneous SNR of the relay-to-destination link is maximized [12, 18, 19], i.e.

$$\max_{R_n \in \{R_1, \cdots, R_N\}} \gamma_{R_n - D} \Rightarrow \gamma_2 \tag{5.21}$$

where γ_{R_n-D} is the instantaneous SNR of the $R_n - D$ link between the n^{th} relay and the destination. A detailed derivation for the exact PDF and the exact CDF of the instantaneous end-to-end SNR of dual-hop AF relaying systems following this selection method was reported in [12]. It was found that the PDF of γ_t, in the case of Nakagami-m fading links for integer values of m_1 and m_2, is

$$\begin{aligned} f_{\gamma_t}(r) = N \left(\frac{m_1}{\overline{\gamma}_1}\right)^{m_1} \left(\frac{m_2}{\overline{\gamma}_2}\right)^{m_2} \frac{1}{\Gamma(m_1)\Gamma(m_2)} \sum_{l=0}^{N-1} \binom{N-1}{l} (-1)^l \sum_{n_1=0}^{m_2-1} \cdots \sum_{n_l=0}^{m_2-1} \left(\frac{m_2}{\overline{\gamma}_2}\right)^{\sum_{i=1}^{l} n_i} \\ \times \frac{1}{\prod_{i=1}^{l} n_i!} I_1 \left(r; m_1 - 1,\, m_2 - 1 + \sum_{i=1}^{l} n_i,\, \frac{m_1}{\overline{\gamma}_1},\, \frac{m_2}{\overline{\gamma}_2}(1+l)\right) \end{aligned} \tag{5.22a}$$

where $I_1(r; \alpha, \beta, \theta, \phi)$ is defined as

$$\begin{aligned} I_1\left(r; \alpha, \beta, \theta, \phi\right) = 2 \, \exp\left[-\left(\theta + \phi\right) r\right] \, r^{(\alpha+\beta)} \sum_{k=0}^{\alpha} \sum_{j=0}^{\beta} \binom{\alpha}{k} \binom{\beta}{j} \left(\frac{\theta}{\phi}\right)^{\frac{j-k}{2}} \left(\frac{r+1}{r}\right)^{\frac{j+k}{2}} \\ \times \left[\sqrt{\frac{\theta}{\phi}\, r(r+1)}\, K_{j-k+1}\left(2\sqrt{\theta\,\phi\, r(r+1)}\right) + (2r+1)\, K_{j-k}\left(2\sqrt{\theta\,\phi\, r(r+1)}\right) \right. \\ \left. + \sqrt{\frac{\phi}{\theta}\, r(r+1)}\, K_{j-k-1}\left(2\sqrt{\theta\,\phi\, r(r+1)}\right)\right]. \end{aligned} \tag{5.22b}$$

The CDF of γ_t was also obtained as

$$\begin{aligned} F_{\gamma_t}(r) = 1 - N \left(\frac{m_2}{\overline{\gamma}_2}\right)^{m_2} \frac{1}{\Gamma(m_2)} \sum_{n=0}^{m_1-1} \frac{1}{n!} \left(\frac{m_1}{\overline{\gamma}_1}\right)^{n} \sum_{l=0}^{N-1} \binom{N-1}{l} (-1)^l \\ \times \sum_{n_1=0}^{m_2-1} \cdots \sum_{n_l=0}^{m_2-1} \left(\frac{m_2}{\overline{\gamma}_2}\right)^{\sum_{i=1}^{l} n_i} \frac{1}{\prod_{i=1}^{l} n_i!} \\ \times I_2 \left(r; n,\, m_2 - 1 + \sum_{i=1}^{l} n_i,\, \frac{m_1}{\overline{\gamma}_1},\, \frac{m_2}{\overline{\gamma}_2}(1+l)\right) \end{aligned} \tag{5.23a}$$

where $I_2(r;\alpha,\beta,\theta,\phi)$ is defined as

$$I_2\left(r;\alpha,\beta,\theta,\phi\right) = 2\,\exp\left[-\left(\theta+\phi\right)r\right]\,r^{(\alpha+\beta+1)}\sum_{k=0}^{\alpha}\sum_{j=0}^{\beta}\binom{\alpha}{k}\binom{\beta}{j} \times \left(\frac{\theta}{\phi}\right)^{\frac{j-k+1}{2}}\left(\frac{r+1}{r}\right)^{\frac{j+k+1}{2}} K_{j-k+1}\left(2\sqrt{\theta\,\phi\,r(r+1)}\right). \quad (5.23b)$$

Note that for the special case of Rayleigh fading links, $m_1 = m_2 = 1$, and the PDF in (5.22) and the CDF in (5.23) reduce to

$$f_{\gamma_t}(r) = \frac{2N}{\overline{\gamma}_1\overline{\gamma}_2}\sum_{l=0}^{N-1}\binom{N-1}{l}(-1)^l\,\exp\left[-\left(\frac{1}{\overline{\gamma}_1}+\frac{1+l}{\overline{\gamma}_2}\right)r\right] \times\left[\left(\sqrt{\frac{\overline{\gamma}_2}{(1+l)\overline{\gamma}_1}}+\sqrt{\frac{(1+l)\overline{\gamma}_1}{\overline{\gamma}_2}}\right)\sqrt{r(r+1)}K_1\left(2\sqrt{\frac{(1+l)r(r+1)}{\overline{\gamma}_1\overline{\gamma}_2}}\right) +(2r+1)\,K_0\left(2\sqrt{\frac{(1+l)r(r+1)}{\overline{\gamma}_1\overline{\gamma}_2}}\right)\right] \quad (5.24)$$

and

$$F_{\gamma_t}(r) = 1 - 2N\sum_{l=0}^{N-1}\binom{N-1}{l}(-1)^l\,\exp\left[-\left(\frac{1}{\overline{\gamma}_1}+\frac{1+l}{\overline{\gamma}_2}\right)r\right] \times\sqrt{\frac{r(r+1)}{\overline{\gamma}_1\overline{\gamma}_2(1+l)}}\,K_1\left(2\sqrt{\frac{(1+l)r(r+1)}{\overline{\gamma}_1\overline{\gamma}_2}}\right) \quad (5.25)$$

respectively. Note that systems with relay selection based on maximum source-to-relay SNR policy as in [26] will have the same analysis as that presented for systems with relay selection based on maximum relay-to-destination SNR policy, because the instantaneous end-to-end SNR in (5.1a) is symmetric with respect to γ_1 and γ_2.

5.5 Numerical Examples

In this section, results obtained using the exact analysis of dual-hop opportunistic AF relaying with maximum end-to-end SNR selection are presented. Recall that no exact solutions for such systems, in the case of general fading environments, are known and only performance bounds have been reported in the literature. Many of these bounds, as in [13]–[15], are based on the approximation $\gamma_{S-R_n-D} = \min\left(\gamma_{S-R_n}, \gamma_{R_n-D}\right)$. In notable recent work, performance bounds were derived in [15] for the case of statistically independent distributed generalized gamma fading channels. However, it will be shown that those bounds are loose for the Nakagami-m fading channels, in spite of the fact that Nakagami-m distribution is a special case of the generalized gamma distribution.

Different performance metrics are used to study the behaviour of opportunistic dual-hop AF relaying system with maximum end-to-end SNR relay selection. Figs. 5.2 and 5.3 show, respectively, the average symbol error probability versus the link average SNR, $\overline{\gamma}$, and outage probability, at $\overline{\gamma} = 10$ dB, versus the threshold SNR, γ_{th} for opportunistic dual-hop AF systems. Identical Rayleigh, Nakagami-m fading links with $m = 2, 3$, and Rician fading links with $K = 1, 2$ are assumed for the case of $N = 3$ available relays. Binary phase shift keying (BPSK) is assumed, so the values of the parameters (a, b) in (5.20) are set to $(2, 1)$. The figures show precise agreement between the results obtained from the analytical solution and simulation results at all values of SNR, for all the link fading distributions. Note that for the cases of Rician fading links, each of the infinite summations in (5.10) and (5.11) was truncated at the 20^{th} term. It was found that adding more terms does not affect the result in the 10^{th} decimal figure. Table 5.1 shows the results of evaluating the PDF in (5.10) using summations truncated at upper limits N and M, respectively. The case of $K_1 = K_2 = 2$ is taken as an example to construct the table for several combinations of the input parameters $\overline{\gamma}_1$, $\overline{\gamma}_2$ and r. The table shows the decimal places that did not change, by adding more terms to the summations, in bold. Table 5.1 indicates that a finite number of terms can be used for the summations in (5.10) with a negligible truncation error.

Many important observations can be drawn from these figures. As expected, the performance is improved for the less severe Nakagami-m and Rician fading channels compared to the performance of Rayleigh fading channels. For example, we see in Fig. 5.2 that an average error probability of 10^{-3} occurs at $\overline{\gamma} = 13.32$ dB for Rayleigh fading links, while it occurs at $\overline{\gamma} = 12.43$ dB and $\overline{\gamma} = 11.39$ dB for Rician fading links with $K = 1$ and $K = 2$, respectively and it occurs at $\overline{\gamma} = 10.85$ dB and $\overline{\gamma} = 10.26$ dB for Nakagami-m fading links with $m = 2$ and $m = 3$, respectively. However, there is a dramatic difference in the system

Table 5.1: Truncated values of $f_{\gamma_{S-R_n-D}}(r)$ in (5.10) for different values of truncated summations at N and M. Values are calculated for $K_1 = K_2 = 2$

$N = M$	$\overline{\gamma}_1 = \overline{\gamma}_2 = 1$ and $r = 1$	$\overline{\gamma}_1 = \overline{\gamma}_2 = 10$ and $r = 1$	$\overline{\gamma}_1 = \overline{\gamma}_2 = 10$ and $r = 10$
10	**0.0472**65324863485	**0.1502**49941032347	**0.01212**5231960593
15	**0.04727387**4549155	**0.150250869**199197	**0.0121262863**03458
20	**0.0472738750894**65	**0.15025086925133**3	**0.0121262863621**09
25	**0.047273875089472**	**0.150250869251334**	**0.012126286362110**
30	**0.047273875089472**	**0.150250869251334**	**0.012126286362110**

performance in the case of Nakagami-m fading links compared to the system performance in the case of Rician fading links. It can be noticed that the limiting slopes of the average error probability curves in Fig. 5.2 and the limiting slopes of the outage probability curves in Fig. 5.3 in the case of Rician fading links are the same as the limiting slopes for the average error probability and outage probability curves in the case of Rayleigh fading links, irrespective of the value of the line of sight factor K. On the other hand, the limiting slopes of the average error probability curves and the outage probability curves in the case of Nakagami-m fading links are affected by the value of the fading parameter m. For example, in the case of Nakagami-m fading links with $m = 2$ and $m = 3$, the limiting slopes of the average error probability and outage probability curves are 2 and 3 times the limiting slopes for the case of Rayleigh fading links ($m = 1$). This implies that the high SNR system performance is strongly influenced by the Nakagami fading parameter m in contrast with the Rician fading parameter K. For example reading from Fig. 5.3, at a threshold SNR $\gamma_{th} = 2$ dB, the outage probability is 4.85×10^{-2} in the case of Rayleigh fading links, while it is 2.75×10^{-2} and 1.07×10^{-2} for Rician fading links with $K = 1$ and $K = 2$, respectively and it is 3.50×10^{-3} and 2.87×10^{-4} for Nakagami-m fading links with $m = 2$ and $m = 3$ respectively.

It is worth mentioning that although both the Nakagami-m and the Rician fading distributions converge to the Rayleigh fading distribution, they represent models with different physical origins and each of them describes the fading process from a different perspective. The differences in the "limiting slopes" of the average error probability curves and the outage probability curves will not diminish for values of m and K chosen according to the "equivalence" suggested in [21, eq. (2.26)]. The authors in [21] proposed a one-to-one mapping between the m parameter of the Nakagami-m distribution and the K parameter of the Rician distribution. This mapping was originally proposed by Nakagami in [27, eqs. (55) and (56)]. The mapping in [27] was based on equating statistical quantifiers, namely the mean and the variance, of both distributions, and in [21], the authors reproduced the mapping relations through equating the amount of fading associated with the Nakagami-m fading distribution to that associated with the Rician distribution, resulting in the mapping equations [21, eq. (2.26)]. However, the author in [28] questioned the practical utility of mapping techniques based on fitting envelope statistics around the mean or median, rather than in the tails of the distributions. It can be shown that although the mapping equations in [21, eq. (2.26)] can result in Nakagami-m and Rician fading distributions with similar mean and variance, the tails of the distributions will be dramatically different, which in turn affects the slopes of the average error probability curves and the outage probability curves in different manners as shown and explained in our numerical examples.

We next compare the results obtained by the exact method to the recent performance bounds proposed in [15]. It was claimed in [15] that the proposed performance bounds are tight in the case of generalized gamma fading channels. Rayleigh and Nakagami-m fadings are special cases of the generalized gamma fading, yet the authors did not show in [15] any examples concerning the performance of the system for these common fading distributions. In Figs. 5.4 and 5.5 we show the ergodic capacity and the average error probability obtained by the exact integral solution for the dual-hop opportunistic AF system with maximum end-to-end SNR relay selection for $N = 3$ and 8. We show also the upper bound on the ergodic capacity and the lower bound on the average error probability proposed in [15]. Nakagami-m fading links with $m = 3$ are assumed. The figures show that the bounds proposed in [15] are loose for all the SNR ranges of interest. For example, the lower bound for $N = 3$ estimates the value of $\overline{\gamma}$, at which an average error probability of 10^{-4} occurs, 1.7 dB lower than its precise value. The bound becomes more loose for $N = 8$. For example, an average error probability of 10^{-5} occurs at $\overline{\gamma} = 12.1$ dB, while the lower bound estimates that the same average error probability occurs at $\overline{\gamma} = 9.7$ dB. Note that the lower bound *underestimates* the required SNR by 2.4 dB. This emphasizes two reasons for the importance of the exact analysis presented in this chapter. The magnitude of the inaccuracy of the bound is unacceptable for some design prediction applications, and the fact that the bound underestimates, rather than overestimates, a required SNR disqualifies it as a tool for conservative system design.

In the next example set, we study the effect of increasing the number of relays, N, in the relay selection pool on the system performance. Figs. 5.6 and 5.7 show respectively the average error probability and the outage probability, at $\overline{\gamma} = 10$, for dual-hop AF maximum end-to-end SNR relay selection with identical Rayleigh, Nakagami-m, with $m = 3$, and Rician, with $K = 2$, fading links. Different numbers of the available candidate relays are assumed, $N = 4, 8$. The case of dual-hop AF systems without relay selection, $N = 1$, is shown as well for comparison. The figures show, as expected, that the performance is improved with increasing the number of available relays for the different channel fading distributions. It can be shown also that the limiting slopes of the average error probability and the outage probability curves are affected by the number of relays in the relay selection pool. This represents a major advantage of maximum end-to-end SNR relay selection; it achieves a diversity gain proportional to the number of relays in the selection pool. This behaviour is different from the behaviour of opportunistic dual-hop AF systems with relay selection methods that rely on the relay-to-destination SNR only [12]. In the case of maximum relay-to-destination SNR relay selection, discussed in [12], it was found that the limiting slopes of the average error probability and outage probability curves are not affected by the increase of the number of the available relays, and that the performance enhancement returns are diminishing for larger numbers of relays, N. This will be shown thoroughly in the next example set. On contrary, the performance of dual-hop AF maximum end-to-end SNR relay selection is well improved by increasing the number of relays. For example, an average error probability of 10^{-3} occurs at $\overline{\gamma} = 13.65$ dB, $\overline{\gamma} = 9.77$ dB and $\overline{\gamma} = 8.79$ dB for $N = 1$, $N = 4$ and $N = 8$, respectively, in the case of Nakagami-m fading links with $m = 3$. Also an outage probability of 10^{-3} occurs at $\gamma_{th} = 1.235$ dB and $\gamma_{th} = 4.421$ dB for $N = 4$ and $N = 8$, respectively, in the case of Rician fading links with $K = 2$.

In the last example set, we show some important aspects of the system performance and how strongly it is influenced by the method of relay selection adopted by the system. Figs. 5.8, 5.9 and 5.10 show the ergodic capacity, the average error probability and outage probability for dual-hop AF systems with maximum end-to-end SNR relay selection, denoted $E-E$, and dual-hop AF systems with maximum relay-to-destination SNR relay selection, denoted $R{-}D$. Performance metrics of dual-hop AF relaying systems without relay selection, $N=1$, are also shown for comparison. Identical Nakagami-m fading links, with $m=2$, are assumed. Fig. 5.8 shows that the ergodic capacity is improved, when any of the two relay selection methods is used, over the case of dual-hop AF relaying without relay selection. The results in the figure also show that the ergodic capacity of the dual-hop AF relaying systems adopting the maximum end-to-end SNR relay selection method is higher than that of dual-hop AF relaying systems adopting the maximum relay-to-destination SNR relay selection method. For example, the ergodic capacity at $\overline{\gamma} = 10$ dB is 1.053 for the case of dual-hop AF systems without relay selection, while it is 1.236 and 1.299 for $N=3$ and $N=6$, respectively, in the case of dual-hop AF systems with maximum relay-to-destination SNR relay selection, and it is 1.344 and 1.479 for $N=3$ and $N=6$, respectively, in the case of dual-hop AF systems with maximum end-to-end SNR relay selection. This behaviour is expected because relay selection in the latter method is based on more link state information. Even more dramatic differences in performance arising from the method of relay selection can be observed from the average error probability curves in Fig. 5.9 and the outage probability curves in Fig. 5.10. It is observed that for the maximum relay-to-destination SNR selection method, increasing the number of relays in the selection pool to $N=3$ and to $N=6$, does not change the limiting slope of the average error probability curves in Fig. 5.9 and the limiting slopes of the outage probability curves in Fig. 5.10; they are the same as the limiting slopes of the average error probability curves and the outage probability curves for the case of no relay selection ($N=1$). Moreover, the improvement in performance when N increases from 3 to 6 is modest. For example it can be observed from Fig. 5.9 that at $\overline{\gamma} = 12$ dB, the average error probability of the dual-hop AF system with maximum relay-to-destination SNR relay selection is 3.38×10^{-3} for the case of $N=3$, while it is reduced only to 3.04×10^{-3} for the case of $N=6$. This improvement is not worthwhile and may well be vitiated by implementation losses or component variation in practical systems. On the other hand, when the maximum end-to-end SNR relay selection method is used, the limiting slopes of the average error probability curves and the outage probability curves are proportional to the number of relays in the selection pool, and hence the improvement in performance is huge. This emphasizes the diversity gain advantage achieved by maximum end-to-end SNR relay selection systems. For example, the average error probability at $\overline{\gamma} = 12$ dB is 3.45×10^{-4} for $N = 3$ and 3.75×10^{-5} for $N = 6$, when the maximum end-to-end SNR relay selection method is used, an order of magnitude improvement. These observations lead to the conclusion that increasing the size of the relay selection pool in the case of the maximum relay-to-destination SNR relay selection method may be a waste of power and computational resources because the returns are only modest improvements in the system performance, while using more relays in the selection pool has significant benefits in enhancing the system performance in the case of the maximum end-to-end SNR relay selection method. For example, reading from Fig. 5.10 for average SNR, $\overline{\gamma} = 10$ dB, and at a threshold SNR $\gamma_{th} = 2$ dB, the outage probability for maximum relay-to-destination SNR relay selection method with $N = 6$ is 5.39×10^{-2}, while the outage probability for maximum end-to-end SNR relay selection method with only $N = 3$ is 3.49×10^{-3}.

5.6 Conclusion

New analytical, exact closed-form expressions for the PDF and the CDF of a dual-hop opportunistic AF relaying system with maximum end-to-end SNR relay selection have been derived. The derived expressions were used to obtain the first exact results for the ergodic capacity, outage probability and the average symbol error probability of opportunistic dual-hop AF relaying wireless networks with relay selection over Rayleigh, Nakagami-m and Rician fading channels. The exact results will be useful in design applications improving upon some of the existing performance bounds that underestimate system performance. The effect of increasing the number of relays in the relay selection pool was investigated and it was observed that the ergodic capacity, outage probability and the average error probability are improved with increasing the size of the relay selection pool, as expected. Moreover, it was observed that at high SNR values, the limiting slopes of the average error probability curves and the outage probability curves are strongly influenced by the selection pool size in contrast to the behaviour of dual-hop AF systems following a maximum relay-to-destination SNR selection method[6]. It was shown that the dual-hop AF relaying systems, adopting maximum end-to-end SNR relay selection, exhibit a diversity gain over dual-hop AF systems without relay selection and over those using maximum relay-to-destination relay selection, for the same relay selection pool size.

[6] Note that this conclusion is valid for systems operating over *i.i.d.* fading links. General conclusions for the cases of systems operating over *i.ni.d.* can be found in Chapter 9 of this thesis.

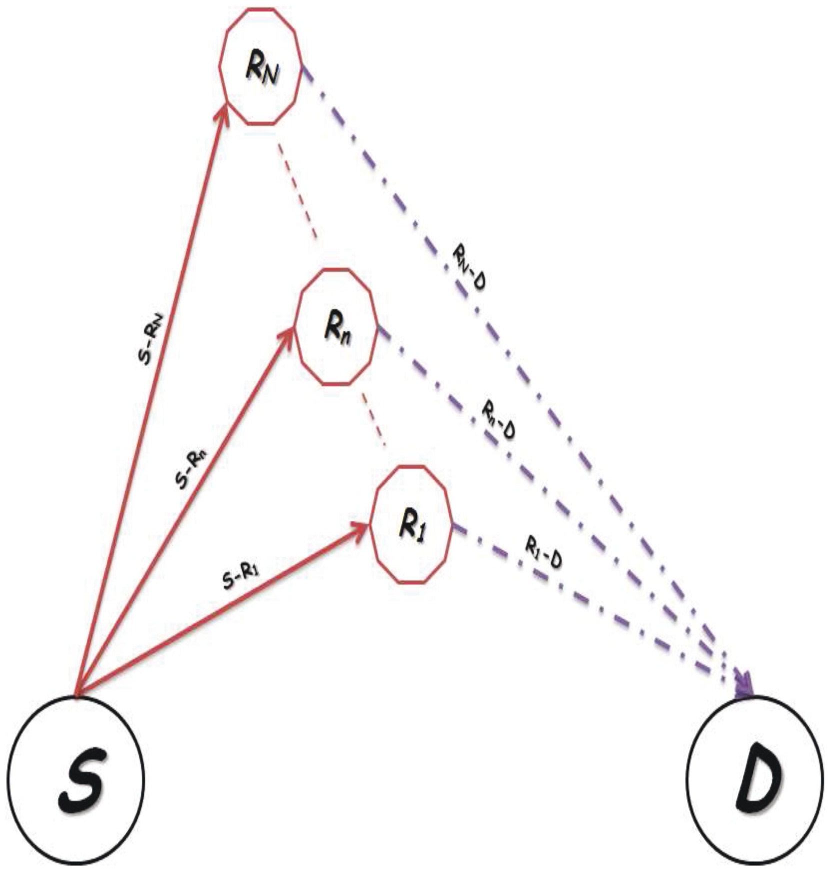

Figure 5.1: Dual-hop amplify-and-forward relaying network. One of the N relays, R_1, R_2, $\cdots$, R_N is selected to relay the data signal between the source, S, and the destination, D.

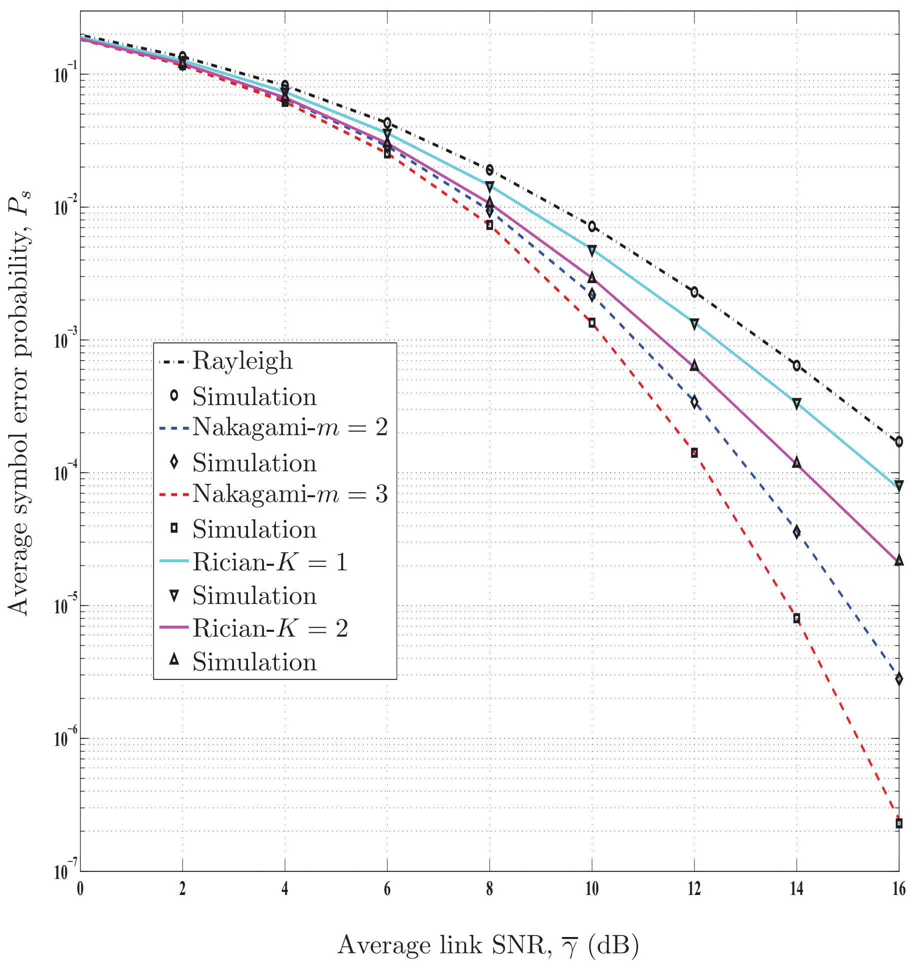

Figure 5.2: The average symbol error probability of dual-hop opportunistic AF relaying systems with maximum end-to-end SNR relay selection and BPSK modulation. Identically distributed Rayleigh, Nakagami-m ($m = 2, 3$) and Rician ($K = 1, 2$) fading links are assumed. Relay selection pool of size $N = 3$ is assumed.

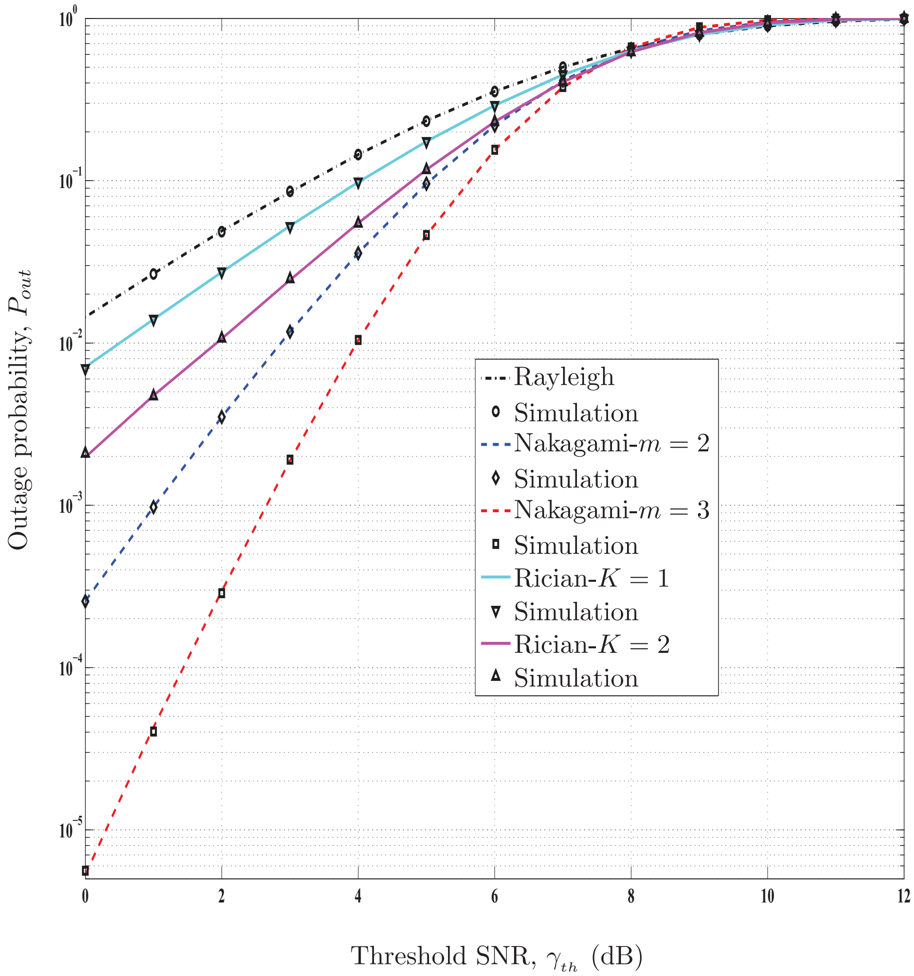

Figure 5.3: Outage probability of dual-hop opportunistic AF relaying systems with maximum end-to-end SNR relay selection. Identically distributed Nakagami-m ($m = 1, 2, 3$) and Rician ($K = 1, 2$) fading links, with average link SNR $\overline{\gamma} = 10$ dB, are assumed. Relay selection pool of size $N = 3$ is assumed.

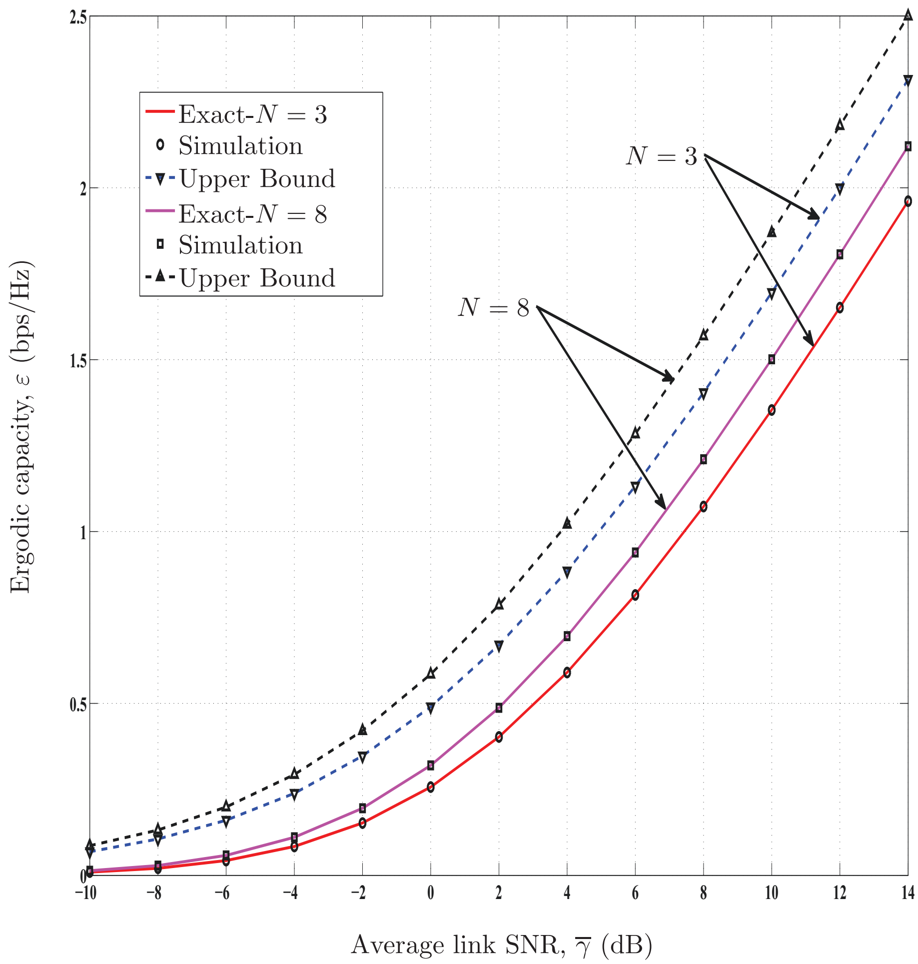

Figure 5.4: The ergodic capacity of dual-hop opportunistic AF relaying systems with maximum end-to-end SNR relay selection. Identically distributed Nakagami-m ($m = 3$) fading links are assumed. Different numbers of relays are assumed, $N = 3, 8$. The upper bounds of [15] are shown with dashed lines.

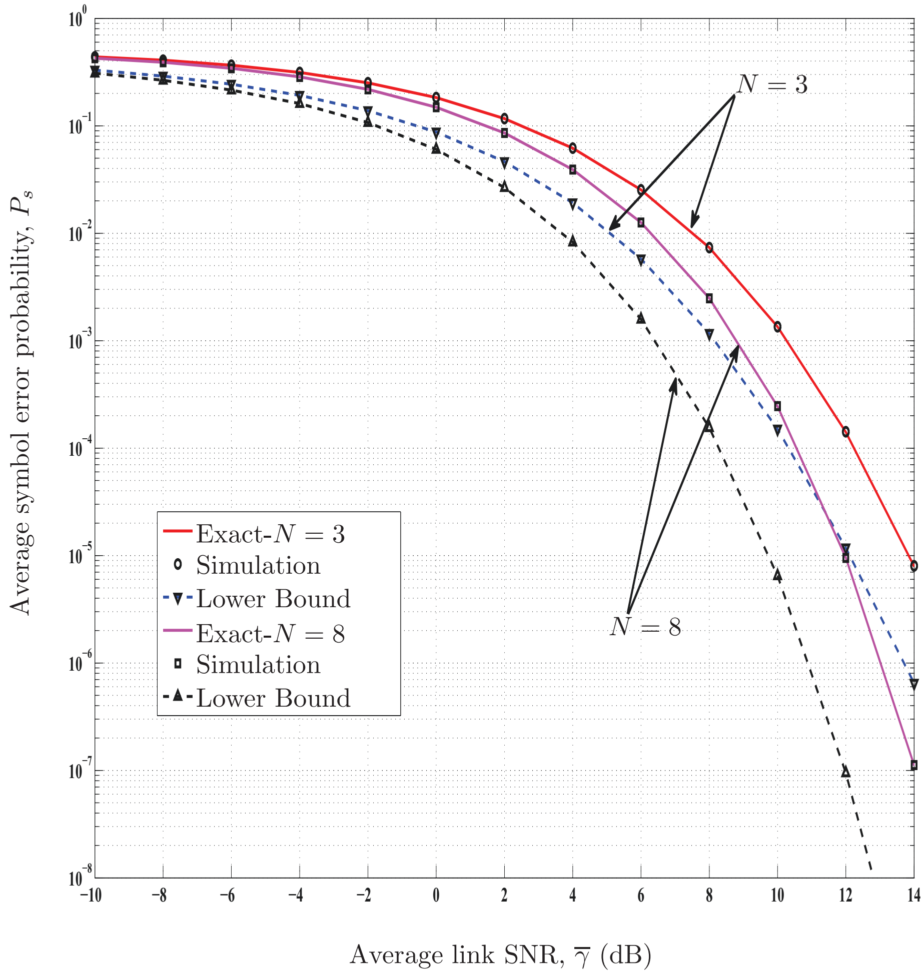

Figure 5.5: The average symbol error probability of dual-hop opportunistic AF relaying systems with maximum end-to-end SNR relay selection and BPSK modulation. Identically distributed Nakagami-m ($m = 3$) fading links are assumed. Different numbers of relays, $N = 3, 8$, are assumed. The lower bounds of [15] are shown with dashed lines.

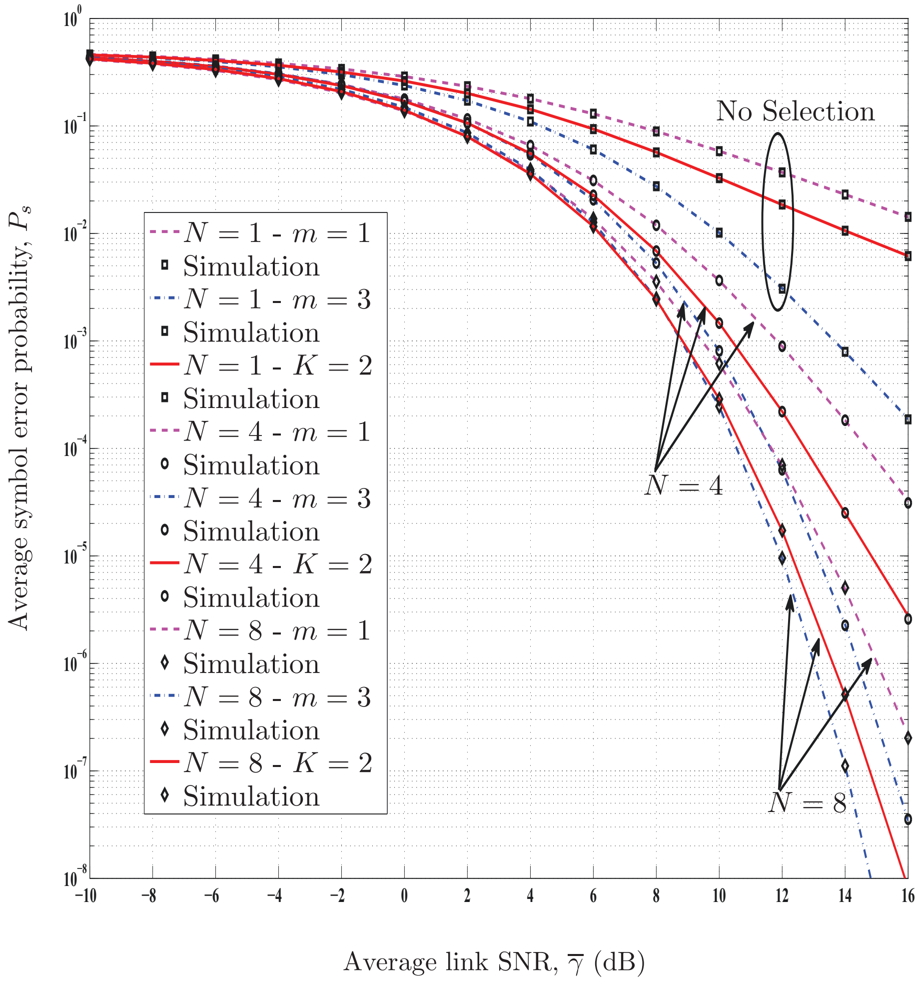

Figure 5.6: The average symbol error probability of dual-hop opportunistic AF relaying systems with maximum end-to-end SNR relay selection and BPSK modulation. Identically distributed Rayleigh, Nakagami-m ($m = 3$) and Rician ($K = 2$) fading links are assumed. Different numbers of relays, $N = 1, 4, 8$, are assumed.

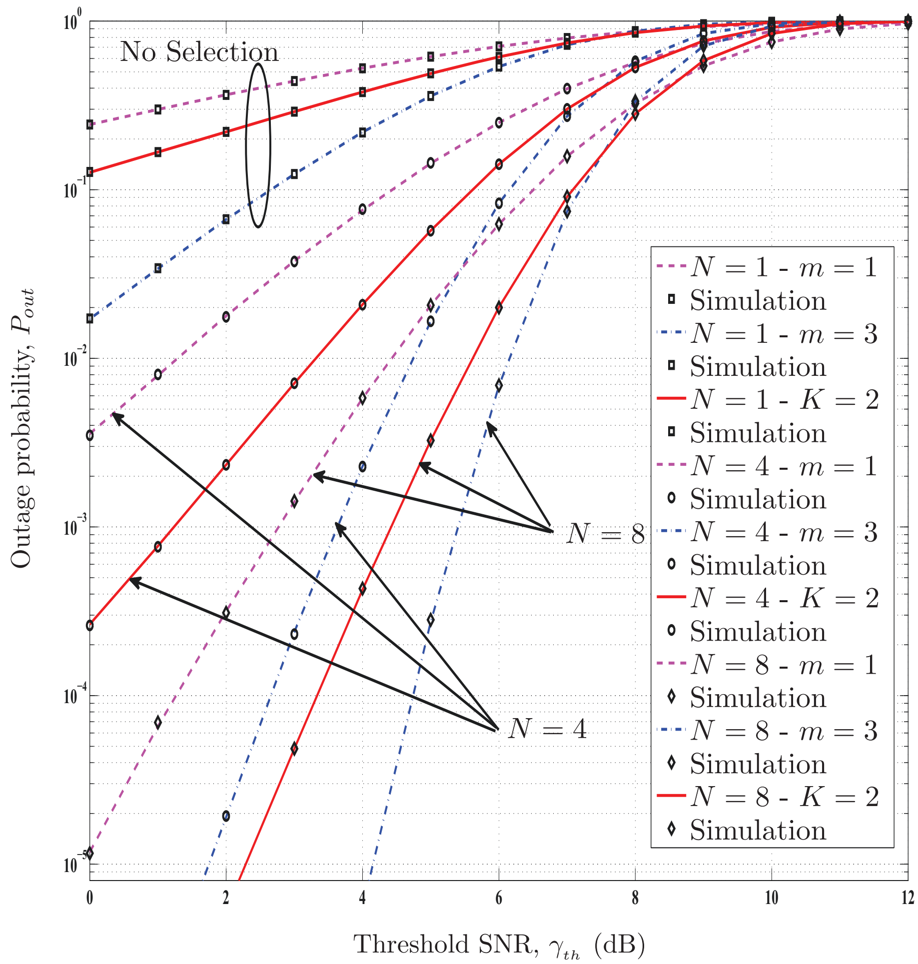

Figure 5.7: Outage probability of dual-hop AF relaying systems with maximum end-to-end SNR relay selection. Identically distributed Rayleigh, Nakagami-m ($m = 3$) and Rician ($K = 2$) fading links with $\overline{\gamma} = 10$ dB are assumed. Different numbers of relays, $N = 1, 4, 8$, are assumed.

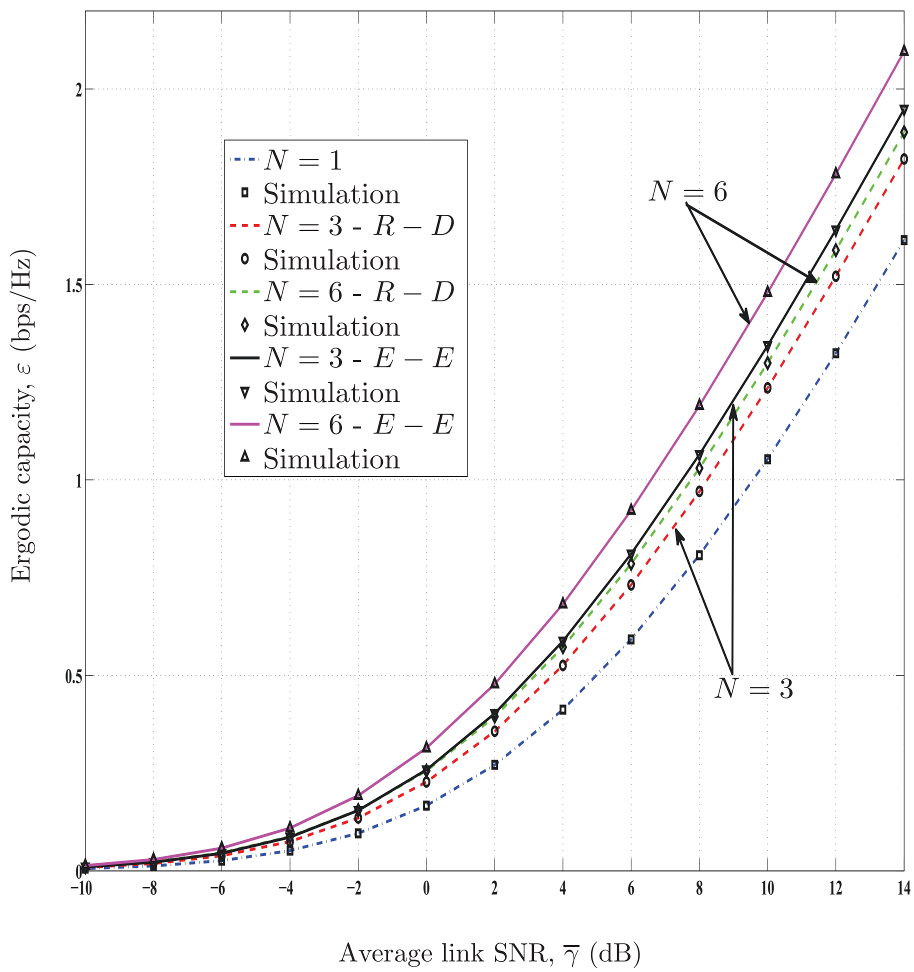

Figure 5.8: The ergodic capacity of dual-hop AF relaying systems with relay selection. Identically distributed Nakagami-m ($m = 2$) fading links are assumed. Different numbers of relays, $N = 1, 3, 6$, are assumed. Results for both the partial relay selection method, denoted $R-D$, and the maximum end-to-end SNR relay selection method, denoted $E - E$, are shown.

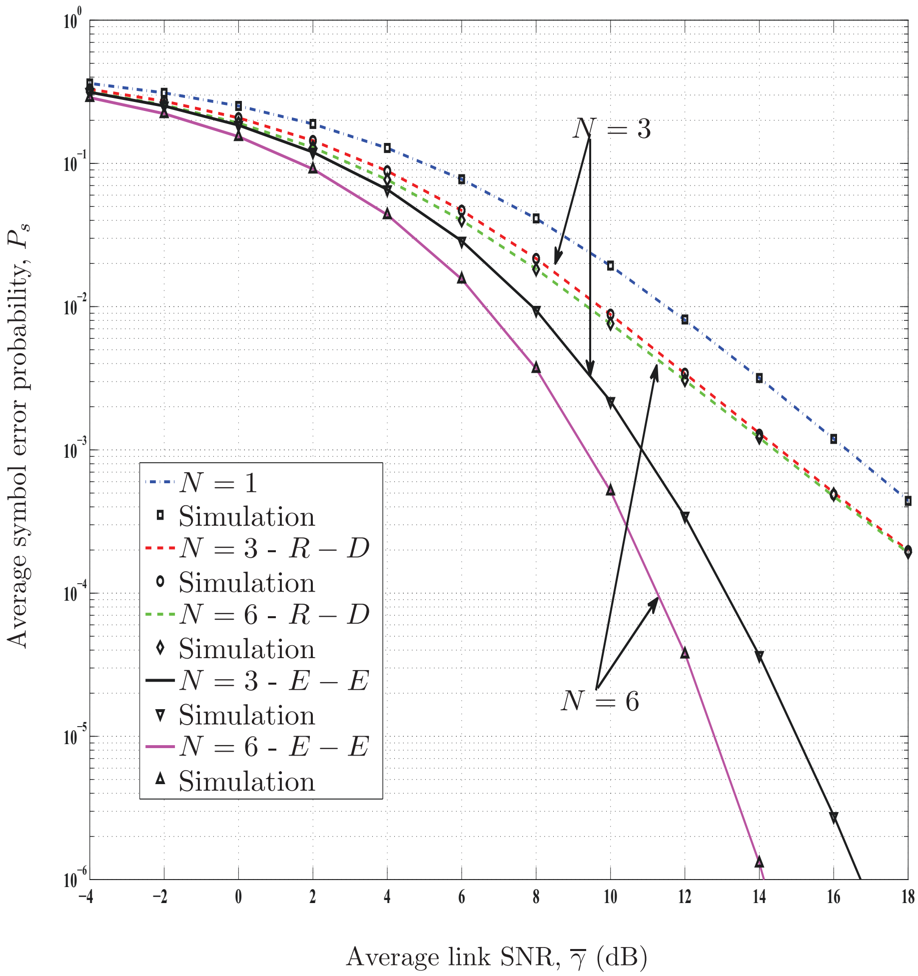

Figure 5.9: The average symbol error probability of dual-hop AF relaying system with relay selection and BPSK modulation. Identically distributed Nakagami-m ($m = 2$) fading links are assumed. Different numbers of relays, $N = 1, 3, 6$, are assumed. Results for both the partial relay selection method, denoted $R - D$, and the maximum end-to-end SNR relay selection method, denoted $E - E$, are shown.

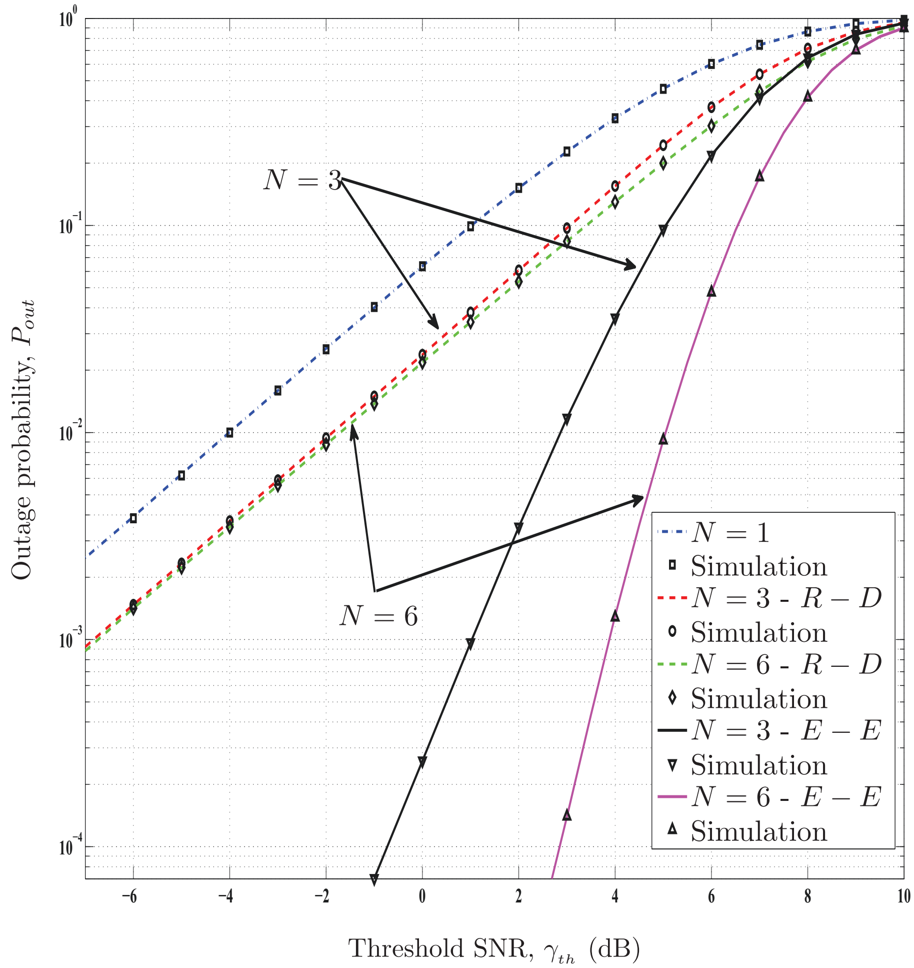

Figure 5.10: Outage probability of dual-hop AF relaying system with relay selection. Identically distributed Nakagami-m ($m = 2$) fading links with, $\overline{\gamma} = 10$ dB, are assumed. Different numbers of relays, $N = 1, 3, 6$, are assumed. Results for both the partial relay selection method, denoted $R - D$, and the maximum end-to-end SNR relay selection method, denoted $E - E$, are shown.

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Chapter 6

Dual-Hop AF Relaying Systems With Full Selection Diversity

In this chapter[1], dual-hop amplify-and-forward (AF) relaying systems with full selection diversity are studied. When the direct path, from the source to the destination is available and considered in the selection set, full selection diversity is achieved. The selection process follows a maximum SNR policy, such that the path of the maximum end-to-end SNR is used as the communication link, whether this path is the direct path or one of the dual-hop paths. Expressions are derived for the PDF and the CDF of the end-to-end SNR. These expressions are used to obtain the first precise results for the average symbol error probability and the outage probability of such AF relaying systems. The system performance is compared to that of conventional wireless systems which use only the direct link between the source and the destination for communication. The system performance is compared also to that of dual-hop AF system with other relay selection criteria. Results show that the performance of full selection diversity systems is superior to those in the comparison, and that the system provides diversity gain, proportional to the selection set size, $N + 1$.

[1] A version of this chapter has been presented in the IEEE International Conference on Communications (ICC) 2012:
S. S. Soliman and N. C. Beaulieu, "'Exact analytical solution for AF relaying systems with full selection diversity," in *IEEE Int. Conf. Commun.*, Jun. 2012, pp. 3995–4000.

6.1 Introduction

The concept of cooperative networks has been recently proposed for wireless communication systems as a solution for the increasing number of users and the increasing demand for high quality wireless services [1]–[7]. Cooperative networks are based on cooperation from idle users to relay the radio signals from the source to the destination. The most popular and studied relaying protocols are amplify-and-forward (AF) and decode-and-forward (DF). In AF relaying systems, the relays amplify the received signal before forwarding it to the next node, while in DF relaying systems, the relay nodes completely decode the received signal, regenerate it and forward the reconstructed signal to the next node. This requires more processing power and complexity at the relay nodes than the AF technique and represents the main disadvantages of the DF scheme. In [1], the authors used a harmonic mean approximation to obtain an approximate expression for the moment generating function (MGF) of the reciprocal of the end-to-end SNR for multihop transmission over Nakagami-m fading channels. The authors then used numerical inversion of the Laplace transform to obtain an approximate expression for the outage probability. It has since been found, that the bounds based on the direct harmonic mean approximation of the individual per hop instantaneous SNRs are not tight for small-to-moderate values of SNR, nor for Nakagami-m fading channels for large values of m [2, 3]. Moreover, increasing the number of hops may decrease the tightness of those bounds. In [4] and [5] an approximation was proposed that has the same computational complexity as the previous bounds in the literature, while being more accurate, especially for small-to-medium values of SNR. In [6], the special case of dual-hop AF systems was studied, and exact closed-form expressions for the PDF and the CDF of the end-to-end SNR were derived for the general cases of Nakagami-m and Rician fading links. The derived expressions were then used to obtain exact solutions for the outage probability, ergodic capacity and the average error probability of dual-hop AF systems. In [7], a novel theoretical technique, the generalized transformed characteristic function (GTCF) approach, was proposed and was used to obtain the first exact analytical results for the outage probability, ergodic capacity and the average error probability of multihop AF systems operating over general fading channels.

Relay selection in cooperative networks has been also a voracious field that attracted many authors. Relay selection in AF relaying systems was proposed in [8]–[12] to enhance the performance of AF relaying systems. In [8], the authors developed a distributed relay selection method, based on measurements of the instantaneous channel conditions. In [9], the authors adopted that relay selection method in AF relaying systems, while also combining both the direct path and the dual-hop path signals with MRC receivers at the destination. They obtained closed-form expressions for bounds on the average symbol error probability for such systems operating over Nakagami-m fading channels. In [8, 9], the authors approximated the end-to-end SNR, for each possible relay connection, using the minimum value of the SNRs of the source-to-relay and relay-to-destination links. This approximation is not tight and results obtained using it are not tight for some ranges of SNR of practical interest. This limitation will be shown in the examples of the present chapter. In other works, [10, 11], the authors used another relay selection method that was based on measurements of the relay-to-destination channel conditions, but exact results for the performance metrics of such systems operating in Nakagami-m fading channels were not derived. Exact results for this selection method were obtained in [6] and it was found that increasing the number of relays in the relay selection pool has diminishing returns because the relay selection is based only on the relay-to-destination link statistics. In a recent work [12], the authors studied AF systems with selection diversity. In [12], the strongest link among the direct path and the N dual-hop AF path is selected for communication between the source and the destination. The authors therein obtained a lower bound and asymptotic expressions for the average error probability of the proposed system.

In this chapter, we use the closed-form expressions for the PDF and the CDF of the end-to-end SNR of dual-hop AF systems derived in [6] to obtain the first exact closed-form expressions for the PDF and the CDF of the end-to-end SNR of the full selection dual-hop opportunistic AF relaying system presented in [12] . We then obtain the first exact analytical results for the performance metrics of such opportunistic AF systems which consider both the direct path and the candidate dual-hop paths for path selection. We compare the results obtained for the full selection system to those of a conventional direct path communication system and to those of an opportunistic dual-hop AF relaying system with maximum end-to-end SNR relay selection, where the direct path does not exist, and we show that the full selection system is superior to both. We also study the effect of increasing the number of candidate paths on the performance metrics.

The remainder of the chapter is organized as follows. In Section 6.2, the system model under study is presented along with the system assumptions. The full selection AF system is analyzed, closed-form expressions are obtained for the PDF and the CDF of the end-to-end SNR, and exact solutions for the performance metrics are obtained in Section 6.3. In Section 6.4, the derived analytical solutions are used to obtain numerical results for the outage probability and the average error probability of full selection dual-hop opportunistic AF relaying systems for different cases of Nakagami-m fading channels. Finally, the chapter is concluded in Section 6.5.

6.2 System Model

The system under study is shown in Fig. 6.1. In a full selection AF relaying system, there exists a set of potential paths for communication between the source node, S, and the destination node, D. The source and the destination can communicate directly through the direct path link, $S - D$, or they can communicate through one of the N dual-hop path links, $S - R_n - D$. Only one path is selected for communication based on a maximum SNR policy. For the direct path, $S - D$, the source node transmits the radio signal directly, in a single-hop, to the destination node. In this case, the end-to-end SNR is given as $\gamma_0 = \frac{P}{N_0}|\alpha_0|^2$, where P is the total available power for transmission, N_0 is the noise power at the destination node, and α_0 is the fading gain of the direct path link. For any of the N dual-hop paths, $S - R_n - D$, the source node transmits the radio signal to the relay node, $R_n, n = 1, 2, \cdots, N$, over the $S - R_n$ link. The relay node, in turn, amplifies the received signal and retransmits the amplified signal to the destination node over the $R_n - D$ link. For variable gain AF relaying, the amplification factor, A, at any relay node, R_n, depends on the channel state information and it is given in [4, 13] as $A = \sqrt{\frac{P_1}{P_0|\alpha_1|^2 + N_{0_1}}}$, where P_0 is the transmitter power at the source node in the case of dual-hop transmission, P_1 is the transmitter power at any relay node, $\alpha_i,\ i = 1, 2$ is the fading gain of the i^{th} link and where

N_{0_1} and N_{0_2} are respectively, the noise power at any relay node and at the destination node. Orthogonal, half-duplex operation is implemented to avoid inter-signal interference and time division multiple access is assumed for transmission over both links. The exact instantaneous received SNR at the destination through any of the dual-hop paths, γ_n, is given by [12, eq. (1)]

$$\gamma_n = \frac{\gamma_{S-R_n}\gamma_{R_n-D}}{\gamma_{S-R_n} + \gamma_{R_n-D} + 1} \tag{6.1}$$

where $\gamma_{S-R_n} = \frac{P_0}{N_{0_1}} |\alpha_1|^2$ represents the instantaneous received SNR over the $S - R_n$ link and $\gamma_{R_n-D} = \frac{P_1}{N_{0_2}} |\alpha_2|^2$ represents the instantaneous received SNR over the $R_n - D$ link.

The channel conditions are estimated at the receiver, and a decision is taken to select one of the potential $N + 1$ paths for the radio signal transmission. The path with the maximum end-to-end SNR is selected. If the direct link path is selected, the signal is transmitted directly from the source to the destination using the total available power per transmission, P. If one of the dual-hop paths is selected, a portion of the total power, P_0, is used at the source to transmit the signal to the selected relay, while the remaining portion, $P_1 = P - P_0$, is used at the relay to amplify the signal and transmit it to the destination. We assume that all the link fadings are statistically independent, and that they follow a Nakagami-m fading distribution. Note that the Rayleigh fading distribution is a special case of the Nakagami-m fading distribution. The probability density functions (PDFs) and the cumulative distribution function (CDFs) of the common fading distributions can be found in [14].

6.3 Full Selection Dual-Hop AF Systems

In this section, we derive exact expressions for the PDF and the CDF of the instantaneous end-to-end SNR of full selection AF relaying systems, and use these expressions to obtain exact solutions for the outage probability and the average symbol error probability of the studied systems. We adopt a SNR based method to select the "best" path. The selection method is independent of the network topology and is based on local measurements of the instantaneous channel transmission conditions. The selection method follows a SNR policy in the sense that the selected best path is the path which achieves a maximum instantaneous end-to-end SNR. In some of the related research work [10, 11], the direct path was excluded from the path selection process. Moreover, the instantaneous SNRs of only the relay-to-destination links were taken into consideration in the selection process. This selection is optimum, from the SNR perspective, if the direct path is not available for communication, for example when the source and the destination are too far from each other, and if the link fading of the source-to-relay link is the same for all of the N candidate relays. However, the more general and more practical case is when the different source-to-relay links exhibit different fadings with possibly the same fading statistics, and when the direct path communication is available. Note that a line of sight (LOS) may not exist between the source and the destination, but still direct path communication is possible via multipath propagation. In this practical case, the selection process should be based on both the direct source-to-destination link SNR and the dual-hop source-relay-destination link SNR. The system presented here adopts the latter selection method, in which the communication path is selected such that the instantaneous end-to-end SNR is maximized, i.e.

$$\max\{\gamma_0, \gamma_m\} \Rightarrow \gamma_t \tag{6.2a}$$

where γ_0 is the end-to-end SNR of the direct path, defined in the previous section, and where

$$\max_{n\in\{1,2,\cdots,N\}} \gamma_n \Rightarrow \gamma_m \tag{6.2b}$$

where $\gamma_n, n = 1, 2, \cdots, N$ is the end-to-end SNR of any dual-hop path, previously defined in (6.1).

With the knowledge of the PDFs, $f_{\gamma_0}(r)$ and $f_{\gamma_m}(r)$, and the CDF, $F_{\gamma_0}(r)$ and $F_{\gamma_m}(r)$, of γ_0 and γ_m, respectively, order statistics can be used to obtain the PDF and the CDF of the end-to-end SNR, γ_t, [15, p. 192] as

$$f_{\gamma_t}(r) = f_{\gamma_0}(r)F_{\gamma_m}(r) + F_{\gamma_0}(r)f_{\gamma_m}(r) \tag{6.3}$$

and

$$F_{\gamma_t}(r) = F_{\gamma_0}(r)F_{\gamma_m}(r). \tag{6.4}$$

Since γ_m is selected as the maximum of the N candidate source-relay-destination link SNRs, γ_n, as in (6.2b), the PDF of γ_m can be written as,

$$f_{\gamma_m}(r) = N f_{\gamma_n}(r) \left[F_{\gamma_n}(r)\right]^{N-1} \tag{6.5}$$

and the CDF of γ_m can be written as,

$$F_{\gamma_m}(r) = \left[F_{\gamma_n}(r)\right]^{N}. \tag{6.6}$$

Exact closed-form expressions for the PDF and the CDF of the SNR, γ_n, of dual-hop AF systems were obtained in [6] for the common link fading distributions. For Nakagami-m fading links, the PDF of γ_n is given by

$$f_{\gamma_n}(r) = C_f \, I_1(r; m_1 - 1, m_2 - 1, \frac{m_1}{\overline{\gamma}_1}, \frac{m_2}{\overline{\gamma}_2}) \tag{6.7a}$$

where $C_f = \left(\frac{m_1}{\overline{\gamma}_1}\right)^{m_1} \frac{1}{\Gamma(m_1)} \left(\frac{m_2}{\overline{\gamma}_2}\right)^{m_2} \frac{1}{\Gamma(m_2)}$, where $\overline{\gamma}_i, i = 1, 2$ is the average SNR of the i^{th} link in the dual-hop path, and where $I_1(r; \alpha, \beta, \theta, \phi)$ is given as [6, eq. (6b)]

$$\begin{aligned} I_1(r; \alpha, \beta, \theta, \phi) = 2 \, \exp\left[-(\theta + \phi) r\right] \, r^{(\alpha+\beta)} \sum_{k=0}^{\alpha} \sum_{j=0}^{\beta} \binom{\alpha}{k} \binom{\beta}{j} \left(\frac{\theta}{\phi}\right)^{\frac{j-k}{2}} \left(\frac{r+1}{r}\right)^{\frac{j+k}{2}} \\ \times \left[\sqrt{\frac{\theta}{\phi} r(r+1)} \, K_{j-k+1}\left(2\sqrt{\theta \, \phi \, r(r+1)}\right) + (2r+1) \, K_{j-k}\left(2\sqrt{\theta \, \phi \, r(r+1)}\right) \right. \\ \left. + \sqrt{\frac{\phi}{\theta} r(r+1)} \, K_{j-k-1}\left(2\sqrt{\theta \, \phi \, r(r+1)}\right)\right]. \end{aligned} \tag{6.7b}$$

where $K_\nu(\cdot)$ is the ν^{th}- order modified Bessel function of the second kind [16, eq. (9.6.2)] and where $\binom{P}{Q}$ is the binomial coefficient. The CDF of γ_n is given by

$$F_{\gamma_n}(r) = 1 - \sum_{n=0}^{m_1-1} C_F(n) I_2(r; n, m_2 - 1, \frac{m_1}{\overline{\gamma}_1}, \frac{m_2}{\overline{\gamma}_2}) \tag{6.8a}$$

where $C_F(n) = \left(\frac{m_1}{\overline{\gamma}_1}\right)^{n} \frac{1}{n!} \left(\frac{m_2}{\overline{\gamma}_2}\right)^{m_2} \frac{1}{\Gamma(m_2)}$, and where $I_2(r; \alpha, \beta, \theta, \phi)$ is given in [6, eq. (8b)] as

$$I_2\left(r;\alpha,\beta,\theta,\phi\right) = 2\,\exp\left[-\left(\theta+\phi\right)r\right]\,r^{(\alpha+\beta+1)}\sum_{k=0}^{\alpha}\sum_{j=0}^{\beta}\binom{\alpha}{k}\binom{\beta}{j} \times\left(\frac{\theta}{\phi}\right)^{\frac{j-k+1}{2}}\left(\frac{r+1}{r}\right)^{\frac{j+k+1}{2}} K_{j-k+1}\left(2\sqrt{\theta\,\phi\,r(r+1)}\right). \quad (6.8b)$$

Note that the expressions in (6.7) and (6.8) are valid for integer values of m_1 and m_2 [6]. For Nakagami-m fading links with non-integer values of m, interpolation can be used to estimate results for non-integer values of m.

To get the end result, we substitute (6.7) and (6.8) into (6.5) and (6.6), and substitute the result into (6.3) and (6.4) to obtain closed-form expressions for the PDF and the CDF of the instantaneous end-to-end SNR of full selection dual-hop AF relaying systems. These expressions represent the first exact, closed-form expressions obtained for γ_t defined in (6.2).

Note that the same analysis can be extended to the case of Rician fading channels, and closed-form expressions for the PDF and the CDF of the SNR, γ_n, of dual-hop AF systems were obtained in [6] for Rician link fadings, but this analysis is omitted here because of space restrictions.

When the PDF and the CDF of the instantaneous end-to-end SNR, γ_t, are known, different system performance metrics can be evaluated. The outage probability, P_{out}, defined as the probability that the instantaneous end-to-end SNR falls below a certain threshold SNR, γ_{th}, is given by [14, eq. (15.6)]

$$P_{out} = Pr\left\{\gamma_t \leqslant \gamma_{th}\right\} = F_{\gamma_t}(\gamma_{th}). \quad (6.9)$$

The average symbol error probability, P_s, is defined as [14, eq. (5.1)]

$$P_s = E\left\{b\,Q\left(\sqrt{a\,\gamma_t}\right)\right\} = \int_0^{\infty} b\,Q\left(\sqrt{a\,r}\right) f_{\gamma_t}(r)dr \quad (6.10)$$

where $Q(\cdot)$ is the Gaussian Q-function [14] and the parameters (a,b) depend on the modulation scheme.

Note that the outage probability, P_{out}, can be obtained by direct substitution into the closed-form CDF expression (6.4), while the calculation of the average error probability,

P_s, involves a single-fold integral that can be evaluated numerically using common software packages such as MATLAB.

6.4 Numerical Examples

In this section, we present results obtained using the exact analysis of full selection dual-hop AF relaying systems. We use the outage probability and the average error probability to study the behaviour of such systems. Binary phase shift keying (BPSK) is assumed and the values of the parameters (a, b) in (6.10) are set to $(2, 1)$. We assume an uniform power allocation policy, in the case of dual-hop transmission, that is the total available power, P, is evenly allocated to the source and the relay. Without loss of generality, we assume equal noise powers, N_0, at all the nodes. The relays are assumed to be located at equal distances from the source and the destination and normalized distances are assumed, i.e. $d_{hop} = \frac{1}{M}$, so according to the Friss propagation model [17], the average SNR over any link is given as $\overline{\gamma}_i = M^{\delta-1}\frac{P}{N_0}$ where $M = 1$ in the case of direct path transmission and $M = 2$ in the case of dual-hop transmission, and where δ is the path loss exponent, taken equal to 3 in the following example sets. According to these assumptions, $\overline{\gamma}_1 = \overline{\gamma}_2 = 2 \times \overline{\gamma}_0 = \overline{\gamma}$, where $\overline{\gamma}_0$ is the average SNR of the direct path link.

In the first example set, Figs. 6.2 and 6.3 show, respectively, the average symbol error probability versus the link average SNR, $\overline{\gamma}$, and outage probability, at $\overline{\gamma} = 10$ dB, versus the threshold SNR, $\gamma_{_{th}}$ for full selection dual-hop AF systems. Identically distributed Rayleigh, Nakagami-m fading links, as well as non-identically distributed Nakagami-m fading links, are assumed for the case of $N = 3$ available relays. The figures show precise agreement between the results obtained from the analytical solution and simulation results at all values of SNR, for all the different cases of link fading distributions. Many important observations can be made from these figures. As expected, the performance is improved with the less severe Nakagami-m fading channels compared to the performance of Rayleigh fading channels. For example, an outage probability of 10^{-2} occurs at $\gamma_{_{th}} = 1.65$ dB in the case of Rayleigh fading links, while it occurs at $\gamma_{_{th}} = 4.84$ dB in the case of Nakagami-m fading links with $m_0 = m_1 = m_2 = 3$.

It is also observed that the limiting slopes of the average error probability curves and the outage probability curves depend on the fading parameters m_0, m_1 and m_3. For example, it can be seen from Fig. 6.2, that an average error probability of 10^{-4} occurs at $\overline{\gamma} = 13.05$ dB for the case of $m_0 = 3, m_1 = m_2 = 1$, while it occurs at $\overline{\gamma} = 11.73$ dB for the case of $m_0 = 1, m_1 = m_2 = 3$. The same observation can be made from Fig. 6.3. For example, for the different cases of $m_0 = 3$, an outage probability of 10^{-3} occurs at $\gamma_{th} = 0.93$ dB in the case of $m_1 = m_2 = 1$, while it occurs at $\gamma_{th} = 1.76$ dB in the case of $m_1 = 1, m_2 = 2$ and at $\gamma_{th} = 3.67$ dB in the case of $m_1 = m_2 = 3$.

We next compare the accuracy obtained by the exact analytical method to that of one of the recent, state-of-the-art performance bounds. Recall that no exact solutions for such systems, in the case of general fading environments, are known and only performance bounds have been reported in the literature. Many of these bounds, as in [9, 12], are based on the approximation $\gamma_n = \min\left\{\gamma_{S-R_n}, \gamma_{R_n-D}\right\}$, for the exact expression in (6.1). In [12], the authors used this approximation to obtain lower bounds and asymptotic expressions for the average error probability of the AF relaying systems under study. In Fig. 6.4, we show the average error probability obtained by the exact analytical method for the full selection dual-hop AF system for $N = 1, 3, 5$ and 15. We show also the lower bound on the average error probability proposed in [12]. Rayleigh fading links are assumed. The figure shows that the bounds proposed in [12] are not precise for some SNR ranges of interest. For example, the lower bound for the case of $N = 3$ estimates the value of $\overline{\gamma}$, at which an average error probability of 10^{-4} occurs, 0.9 dB lower than its precise value. The bound becomes more loose for $N = 15$. For example, an average error probability of 10^{-4} occurs at $\overline{\gamma} = 9.77$ dB, while the lower bound estimates that the same average error probability occurs at $\overline{\gamma} = 7.92$ dB. Note that the lower bound *underestimates* the required SNR by 1.85 dB. This emphasizes two reasons for the value of the exact analysis presented in this chapter. The magnitude of the inaccuracy of the bound may be undesirable for some design prediction applications, and the fact that the bound underestimates, rather than overestimates, a required SNR disqualifies it as a tool for conservative system design.

In the next example set, we study the effect of increasing the number of candidate relays, N, on the system performance. Figs. 6.5 and 6.6 show respectively the average error probability and the outage probability, at $\overline{\gamma} = 10$ dB, for full selection dual-hop AF relaying systems with identical Rayleigh and Nakagami-m, with $m = 3$, fading links. Different numbers of available candidate relays are assumed, $N = 2, 4$ and 8. The figures show that the limiting slopes of the average error probability curves and the outage probability curves are affected by the number of candidate dual-hop paths and that the system performance is greatly improved by increasing the number of candidate relays. For example, an average error probability of 10^{-5} occurs at $\overline{\gamma} = 13.82$ dB, $\overline{\gamma} = 12.57$ dB and $\overline{\gamma} = 11.68$ dB for $N = 2$, $N = 4$ and $N = 8$, respectively, in the case of Nakagami-m fading links with $m = 3$. Also an outage probability of 10^{-3} occurs at $\gamma_{th} = -3.68$ dB for $N = 2$, while it occurs at $\gamma_{th} = 0.44$ dB for $N = 4$ and at $\gamma_{th} = 3.47$ dB for $N = 8$, in the case of Rayleigh fading links. It can be concluded that the system provides diversity gain, proportional to the selection set size, $N + 1$.

In the last example, we compare the system performance to that of the conventional wireless systems which use only the direct link between the source and the destination for communication. The system performance is compared also to that of an opportunistic dual-hop AF system with maximum end-to-end SNR relay selection that excludes the direct path from the selection set. Fig. 6.7 shows the average error probability of the three systems operating over Nakagami-m fading links with $m = 4$. For the systems with path selection, different numbers of relays are assumed, $N = 2, 6$. Results show that the performance of a full selection dual-hop AF relaying system is superior to that of both of the other two systems. For example, an average error probability of 2×10^{-4} occurs at $\overline{\gamma} = 15.27$ dB in the case of the conventional direct path communication system, and it occurs at $\overline{\gamma} = 12.16$ dB for the opportunistic dual-hop AF system with maximum end-to-end SNR relay selection excluding the direct path, while it occurs at $\overline{\gamma} = 11.21$ dB for the full selection dual-hop AF system with $N = 2$. This indicates that both the direct path link, and the candidate dual-hop links should be included in the selection process for maximum system performance.

6.5 Conclusion

Novel exact closed-form expressions for the PDF and the CDF of a full selection dual-hop AF relaying system have been analytically derived. The derived expressions were used to obtain the first exact analytical results for the outage probability and the average error probability of dual-hop AF relaying wireless networks with full selection over Nakagami-m fading channels. It was shown that the derived analytical method provides exact results, in contrast to state-of-the-art works that report only performance bounds. The effect of increasing the number of candidate relays was investigated and it was observed that a diversity gain, proportional to the total number of possible paths, is provided. The system performance was compared to that of a conventional, direct path communication system and to that of opportunistic dual-hop AF relaying systems with maximum end-to-end SNR relay selection, which neglect the direct path. It was found that the full selection system performance is superior(2) to both.

(2)Note that performance metrics considered for comparison in this Chapter are the outage and average symbol error probabilities. However, from a system throughput perspective, transmission over a dual-hop path represents a loss of throughput compared to transmission over direct single-hop path because in the first, end-to-end transmission occurs over two time slots, while in the later, end-to-end transmission occurs over only one time slot.

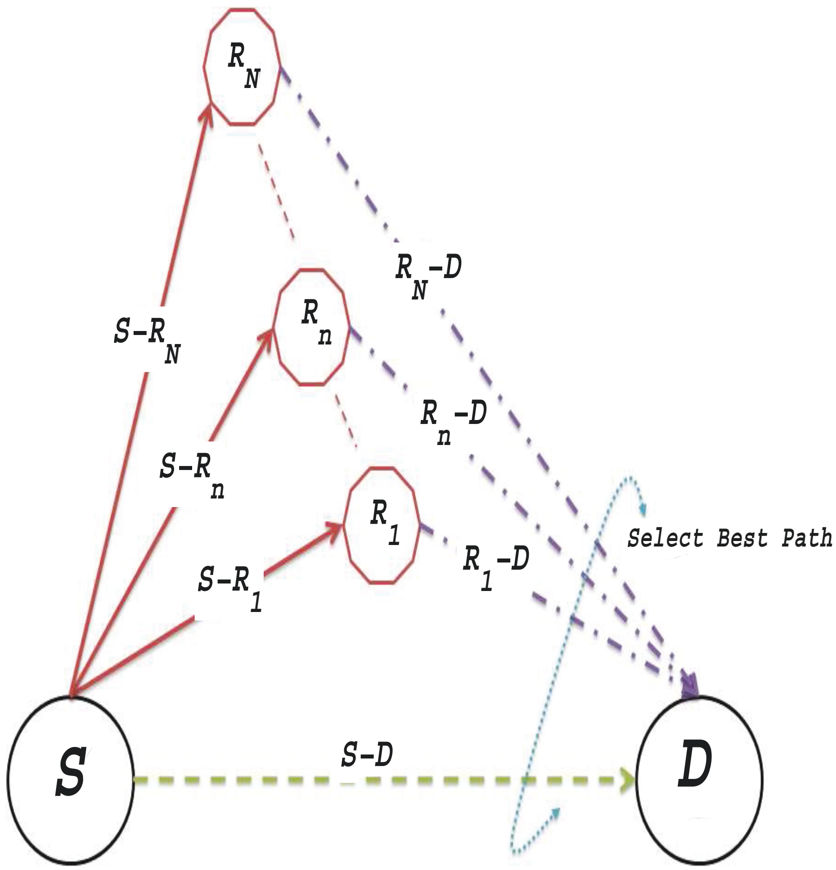

Figure 6.1: Dual-hop full selection amplify-and-forward relaying network. Either the direct path, $S - D$, is selected for direct data signal transmission or one of the N relays, R_1, R_2, $\cdots$, R_N is selected to relay the data signal between the source, S, and the destination, D.

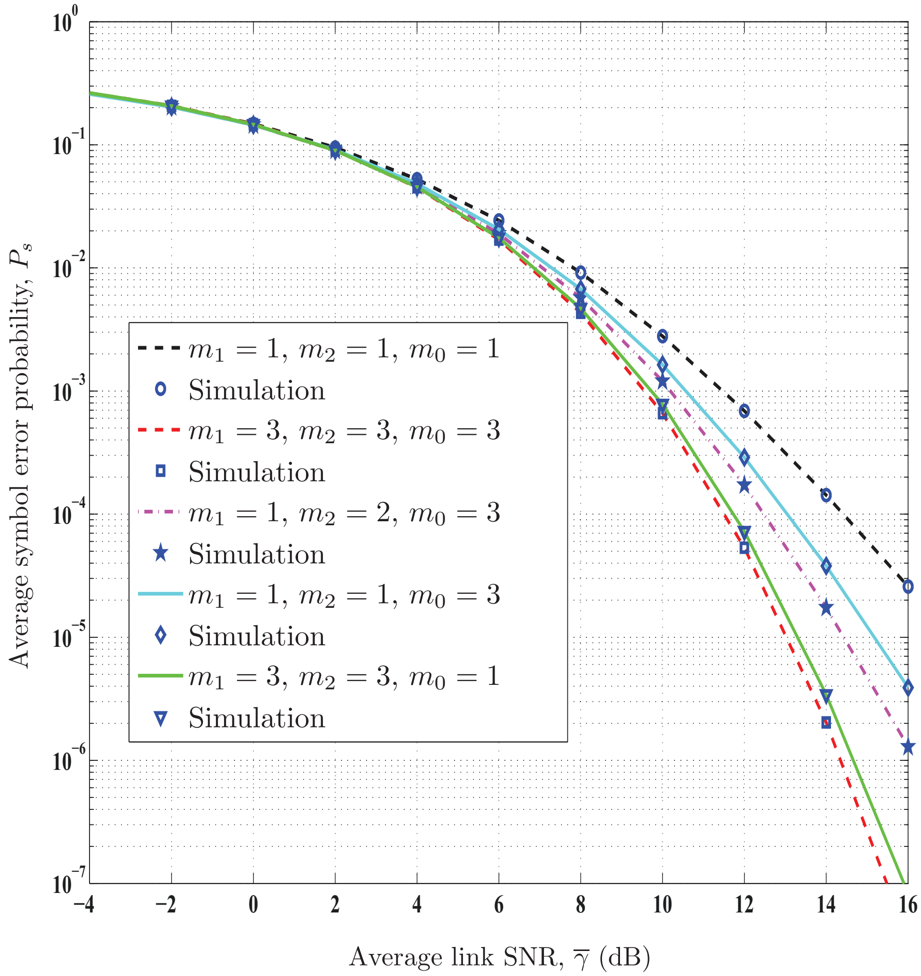

Figure 6.2: The average symbol error probability of dual-hop full selection diversity AF relaying systems with BPSK modulation. The number of candidate relays is assumed to be $N = 3$.

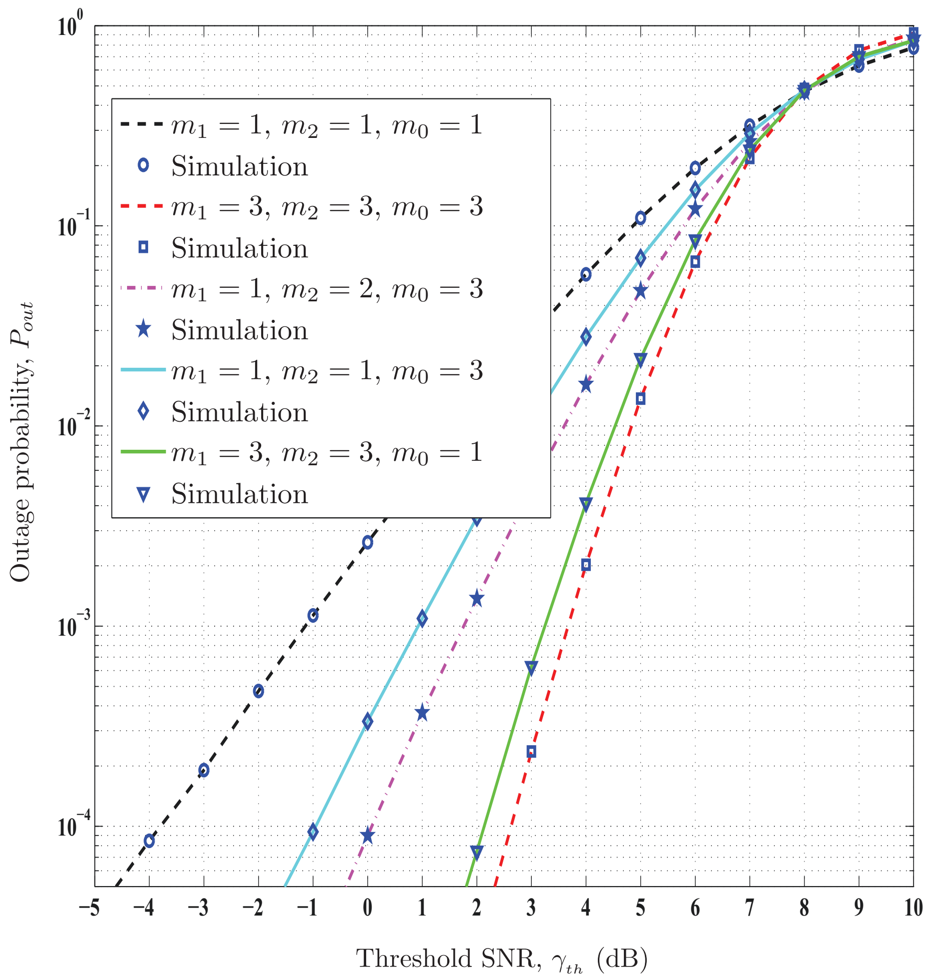

Figure 6.3: Outage probability of dual-hop full selection diversity AF relaying systems at $\overline{\gamma} = 10$ dB. The number of candidate relays is assumed to be $N = 3$.

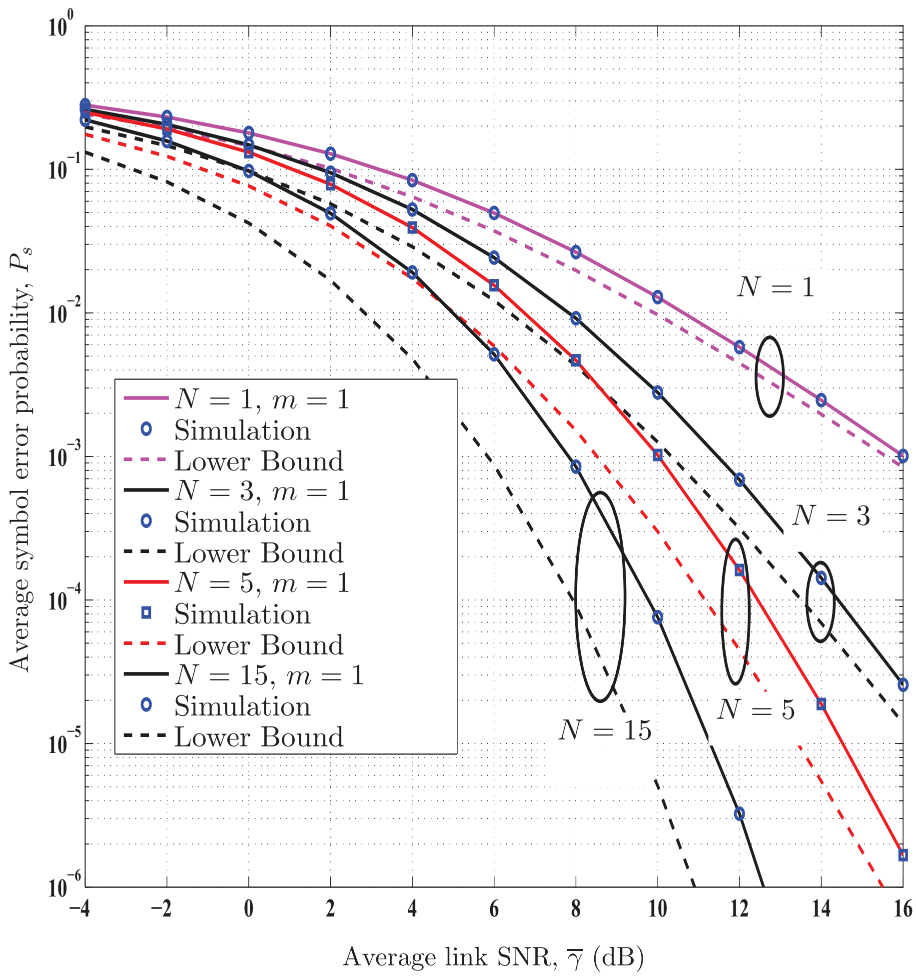

Figure 6.4: The average symbol error probability of dual-hop full selection diversity AF relaying system with BPSK modulation. Identically distributed Rayleigh fading links are assumed. Lower bounds of [12] are shown with dashed lines.

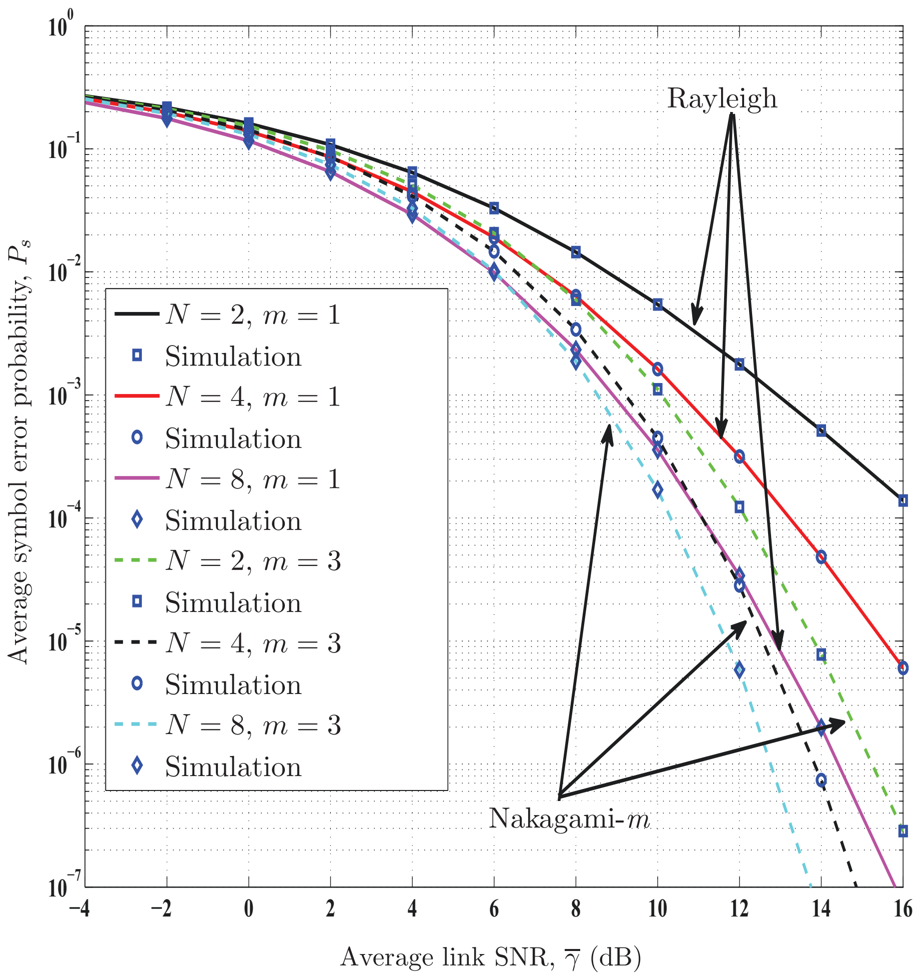

Figure 6.5: The average symbol error probability of full selection diversity dual-hop AF relaying systems with BPSK modulation. Identically distributed Rayleigh and Nakagami-m ($m = 3$) fading links are assumed. Different numbers of relays, $N = 2, 4, 8$, are assumed.

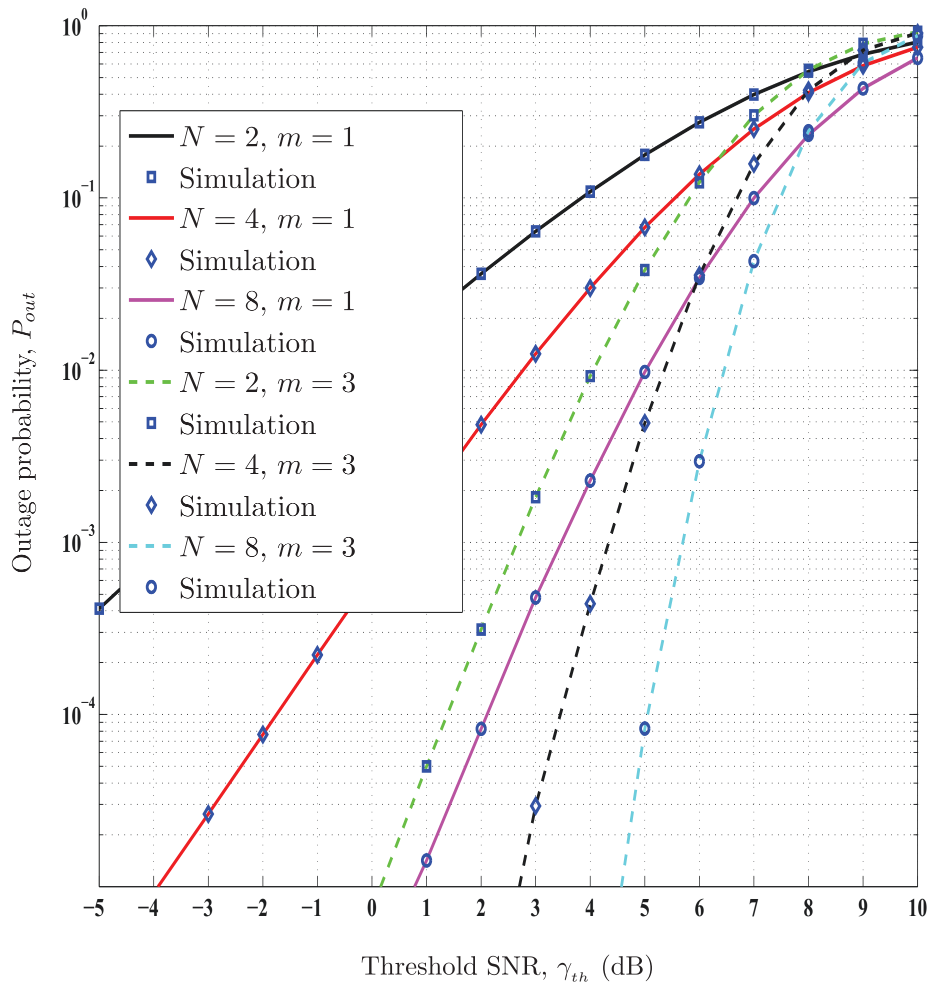

Figure 6.6: Outage probability of dual-hop full selection diversity AF relaying systems at $\overline{\gamma} = 10$ dB. Identically distributed Rayleigh and Nakagami-m ($m = 3$) fading links are assumed. Different numbers of candidate relays, $N = 2, 4, 8$, are assumed.

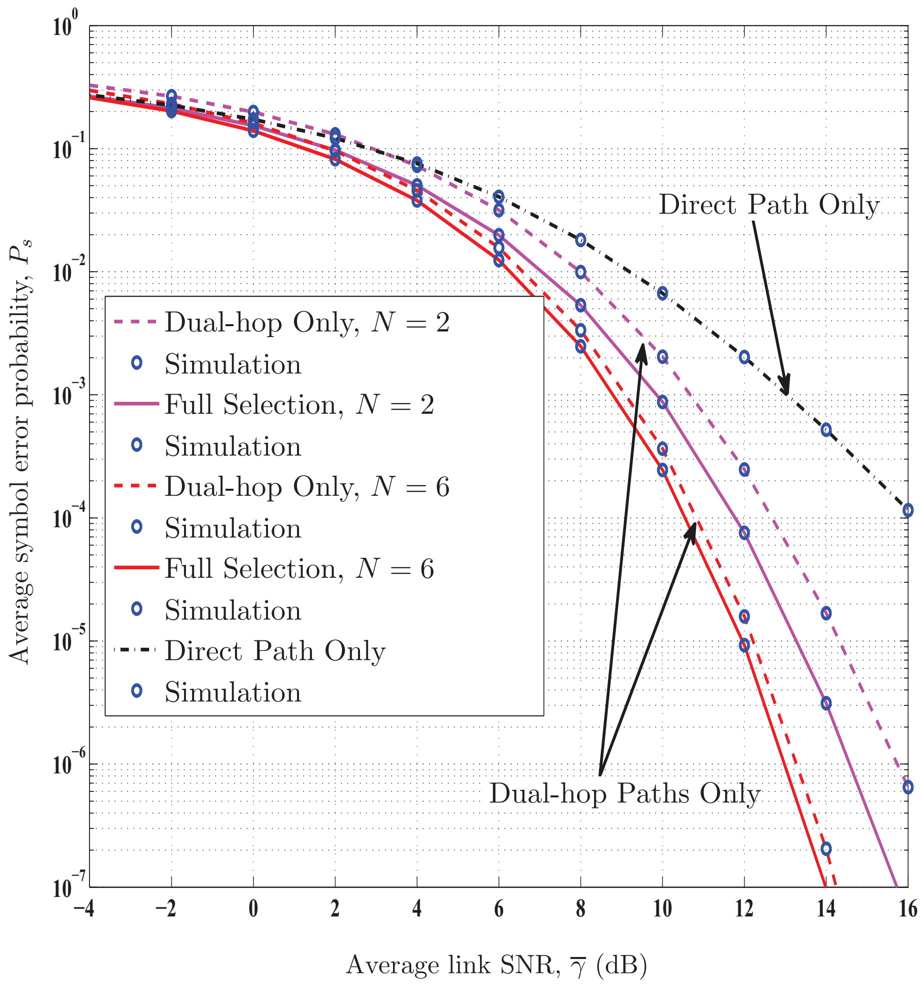

Figure 6.7: The average symbol error probability of dual-hop full selection diversity AF relaying system with BPSK modulation. Systems using only the direct single-hop path, and systems using only dual-hop paths for relay selection are shown for comparison. Identically distributed Nakagami-m ($m = 4$) fading links are assumed. Different numbers of relays, $N = 2, 6$, are assumed.

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Chapter 7

The GTCF Method for Multihop AF Systems

In this chapter[1], a theoretical approach is developed for the analysis and design of multihop AF relaying systems. A generalized transformed characteristic function (GTCF) approach is proposed to obtain an exact integral solution for the probability density function (PDF) of the instantaneous end-to-end signal-to-noise ratio (SNR) of general multihop AF relaying systems. This solution leads to exact integral solutions for the average symbol error probability, outage probability and ergodic capacity that are valid for any fading distribution. The results are precise for any number of hops and can accommodate any channel fading distribution. The theory provides the first exact theoretical solutions for numbers of hops $N > 2$. Cases of identically and non-identically distributed fading links are investigated, as well as the case of dissimilar links, in which the link fadings follow different distributions. The effects of the numbers of hops, N, on the performance metrics are also studied. The presence of a limiting distribution for the end-to-end received SNR, with increasing number of relays, as well as the accuracy and rate of convergence of such limiting distribution are explored.

[1] A version of this chapter has been published in the IEEE Transactions on Communications: N. C. Beaulieu and S. S. Soliman, "Exact analysis of multihop amplify-and-forward relaying systems over general fading links," *IEEE Trans. Commun.*, vol. 60, no. 8, pp. 2123–2134, Aug. 2012.

7.1 Introduction

Wireless channels suffer from many factors that degrade the performance of the communication system in terms of the quality and the capacity of transmission. Capacity may refer to the number of users or the amount of data transmitted over the wireless channel. Impairments in the wireless channel may include signal fading, co-channel interference, adjacent channel interference, noise, signal shadowing and propagation loss. Faced with an ever increasing demand for user services and a finite spectrum resource, new and improved techniques are being developed to permit future wireless networks to sustain the increasing demands for services and the quality of these services. Cooperative networks, recently proposed, are receiving intense study for future generation wireless standards [1]–[15]. The cooperative network concept represents a technique that can significantly increase the system capacity and diversity gain in wireless networks.

In cooperative systems, idle users (nodes) are used as relays between the source and the destination. Accurate performance analysis of cooperative diversity systems is important since it enables the design of networks for maximum utility and performance. The most popular and studied relaying techniques are amplify-and-forward (AF), or equivalently non-regenerative relaying, and decode-and-forward (DF), called also regenerative relaying. DF relaying requires more processing at the relay nodes than AF and the complexity and power consumption are disadvantages of the DF scheme. In cooperative diversity networks, the same signal is transmitted through a direct path, from the source to the destination, as well. Once the destination receives all the different versions of the signal, through the direct path and the relays, they can be combined using any of the well known receiver diversity combining techniques.

Much research and publication has focused on the performance of AF relaying systems. The outage probability and the average probability of error of dual-hop transmission over Rayleigh fading channels were studied in [1,2]. In [1], the authors assumed semi-blind relays, which do not have access to the instantaneous channel state information (CSI) of the first hop, but have statistical CSI about the first hop. In [2], the authors presented an approximation for the end-to-end received signal-to-noise ratio (SNR) using the harmonic mean of two independent exponential random variables and in [3], the harmonic mean approximation was again used to obtain an approximate expression for the moment generating function (MGF) of the reciprocal of the end-to-end SNR for multihop transmission over Nakagami-m fading channels. The approximate outage probability was then obtained via numerical inversion of the Laplace transform. Performance bounds on the outages and error probabilities of AF transmission over Nakagami-m fading channels were also obtained in [4]. In [5], single-integral expressions for the average error probability of general wireless communication systems were obtained in terms of the MGF of the reciprocal of the instantaneous end-to-end SNR. This general framework was applied to AF relaying systems. The authors in [6] used the same MGF approach to find expressions for the average error probability and the outage probability for some special cases of Nakagami-m fading channels, where the fading index of each hop is an odd multiple of one-half. The MGF approach was also adopted in [7] and [8]. However, all the results in [4]–[8] are based on the approximate harmonic mean expression for the end-to-end SNR. It was found that the bounds based on the direct harmonic mean approximation of the individual per hop instantaneous SNRs, are not tight for small-to-moderate values of SNR, and for Nakagami-m fading channels for large values of m. Moreover, increasing the number of hops may decrease the tightness of those bounds. In [9], an approximation was proposed in which the harmonic mean of the minimum of the first P hop SNRs and the minimum of the remaining hop SNRs is used to obtain new bounds. The new bounds were obtained for independent and non-identically distributed Rayleigh fading channels and for independent and identically distributed Nakagami-m fading channels for integer values of m. It was concluded in [9] that the proposed bounds outperform the existing bounds, particularly for severe fading environments and for medium-to-large SNRs. In [10] and [11] another new approximation was proposed that has the same computational complexity as the previous bounds in the literature, while it is more accurate, especially for small-to-medium values of SNR. The key idea was to use a scaling factor to reduce the approximation errors at small-to-medium values of SNR. Compared with previous bounds, the new approximation gives more accurate estimate of the performance. All of the previously mentioned work reports approximations because no exact solution for the end-to-end received SNR of multihop AF relaying is known. In [13], the authors found closed-form expressions for the average bit error rate and outage probability for differential equal gain combining in cooperative diversity networks. Those expressions are valid only for the dual-hop case and for Nakagami-m fading links with integer values of the fading parameter m. Reference [14] presented a closed-form expression for the average bit error probability of AF fixed gain dual-hop relaying systems over Nakagami-m links for integer values of m. The asymptotic average bit error probability for arbitrary m was also evaluated. However, the results in [13, 14] were not extended to the case of multihop AF relaying systems.

In this chapter, we develop a novel theoretical approach for the analysis and design of AF relaying systems. A generalized transformed characteristic function (GTCF) approach is proposed to obtain an exact integral solution for the probability density function (PDF) of the end-to-end SNR. This solution leads to exact integral solutions for the average symbol error probability, outage probability and ergodic capacity that are valid for any fading distribution. The GTCF method is used here to obtain the ergodic capacity, outage probability and the average symbol error probability of multihop AF relaying networks. A strength of the GTCF approach is that it can be used for any fading channel distribution with tractable computational effort. Examples will consider Rayleigh, Rician and Nakagami-m fading distributions. Both the independent and identically distributed (*i.i.d.*) case and the independent and non-identically distributed (*i.ni.d.*) case are examined. We show that the GTCF can be also used for the cases of mixed fading, where the links follow different fading distribution families. Note that the GTCF approach presented in this chapter is advantageous over the methods presented in [15]. The authors in [15] presented a framework for performance analysis of multihop multi-branch AF relaying systems. They derived expressions to obtain exact performance metrics for the systems under study. However, the expressions derived therein are valid only for systems operating over lognormal fading channels and can not be extended to other fading channel distributions. Moreover, the computational complexity of those expressions, as studied from [15, Table I], imposes practical limits on their usage. For example, for a N-hop single-branch AF relaying system, the computational complexity of the expressions presented in [15] is exponential in terms of N, while the GTCF method has fixed computational complexity, and this complexity is far less than that in [15] for any number of hops, N. The superiority of the GTCF approach becomes more obvious for large numbers of hops, N. We also study the effect of the number of hops on the system performance in multihop relaying. Finally, we study the existence of a limiting distribution for the end-to-end received SNR with an increasing number of relays, and examine the rate of convergence and accuracy of this limiting distribution for estimating performance in practical applications.

The remainder of the chapter is organized as follows. In Section 7.2, the system model under study is presented along with the system assumptions. The GTCF approach is explained in detail and discussed in Section 7.3. Some closed-form expressions for the transformed characteristic function are obtained in Section 7.4. In Section 7.5, computational considerations of the GTCF method and alternate exact solutions are examined. In Section 7.6, the GTCF method is employed to obtain numerical results for the performance metrics of Rayleigh, Nakagami-m and Rician fading multihop relaying for both *i.i.d.* and *i.ni.d.* cases. A central limit theorem argument is applied in Section 7.7 to find a limiting distribution for the end-to-end received SNR and the Berry-Esseen theorem is used to study the accuracy and the rate of convergence of the limiting distribution for practical applications. Finally, the chapter is concluded in Section 7.8.

7.2 System Model

The system under study is a multihop AF relaying system, shown in [16, Fig. 1]. The source node, R_0, and the destination node, R_N, communicate through (N-1) intermediate relay nodes, $R_1, R_2, \cdots, R_{N-1}$. The source node, R_0, transmits the data signal to the first relay, R_1, over the first link. The first relay, in turn, amplifies the received signal and transmits the amplified signal to the next node, and so on to the destination node. The amplification factor, A_n, at the n^{th} relay node, R_n, is given by [12] as

$$A_n = \sqrt{\frac{P_n}{P_{n-1} \left|\alpha_n\right|^2 + N_{0n}}} \tag{7.1}$$

where $P_n,\ n = 0, 1, \cdots, N-1$, is the transmitter power at the n^{th} node, and where $\alpha_n,\ n = 1, 2, \cdots, N,$ and $N_{0n},\ n = 1, 2, \cdots, N,$ are respectively the fading gain of the n^{th} link, between node R_{n-1} and node R_n, and the noise power at the n^{th} node. Orthogonal time slots are assigned to each of the transmitting nodes in order to avoid inter-signal-interference and to achieve half-duplex operation. The instantaneous end-to-end received SNR at the destination, γ_t, is given by [2, eq. (2)]

$$\gamma_t = \left[\prod_{n=1}^{N} \left(1 + \frac{1}{\gamma_n} \right) - 1 \right]^{-1} \tag{7.2a}$$

where γ_n, $n = 1, 2, \cdots, N$ represents the instantaneous received SNR over the n^{th} link defined as

$$\gamma_n = \frac{P_{n-1}}{N_{0n}} \left| \alpha_n \right|^2. \tag{7.2b}$$

When the PDF of the instantaneous end-to-end SNR, γ_t, is available, different system performance metrics can be evaluated. The ergodic capacity, ε, can be evaluated using [17, eq. (15.21)]

$$\varepsilon = \frac{1}{N} E \left\{ \log_2(1 + \gamma_t) \right\} = \frac{1}{N} \int_0^\infty \log_2(1 + r) f_{\gamma_t}(r) dr. \tag{7.3}$$

where the $\frac{1}{N}$ factor accounts for orthogonal transmission over the N links. Note that the ergodic capacity, or equivalently, the average Shannon capacity per unit bandwidth, is used as a benchmark to compare the spectral efficiency of all practical transmission techniques [17]. The outage probability, P_{out}, defined as the probability that the instantaneous end-to-end SNR falls below a certain threshold SNR, γ_{th}, is given by [17, eq. (15.6)]

$$P_{out} = Pr \left\{ \gamma_t \leqslant \gamma_{th} \right\} = \int_0^{\gamma_{th}} f_{\gamma_t}(r) dr \tag{7.4}$$

and the average symbol error probability, P_s, is defined as [17, eq. (5.1)]

$$P_s = E \left\{ b Q(\sqrt{a \gamma_t}) \right\} = \int_0^\infty b Q(\sqrt{a\, r}) f_{\gamma_t}(r) dr \tag{7.5}$$

where $Q(\cdot)$ is the Gaussian Q-function [17] and the parameters (a, b) depend on the modulation scheme.

7.3 GTCF Method

An exact solution for the instantaneous end-to-end received SNR for multihop AF relaying systems is not known [6,9]–[11]. To find the performance statistics, references [5,6,9,10] use the approximate expression

$$\frac{1}{\gamma_t} = \sum_{n=1}^{N} \left[\frac{1}{\gamma_n} \right] \tag{7.6}$$

which represents an upper bound for the exact end-to-end received SNR. The statistics needed to evaluate the performance in wireless communication systems include the PDF and the moment generating function (MGF) or the characteristic function (CHF) of γ_t.

The generalized transformed characteristic function (GTCF) method is a new approach developed to find the exact statistics of the instantaneous end-to-end SNR, γ_t. This allows finding exact solutions for the performance metrics such as ergodic capacity, outage probability and average symbol error probability. The GTCF can be illustrated by rewriting the expression in (7.2a) as,

$$\left(1 + \frac{1}{\gamma_t}\right) = \prod_{n=1}^{N} \left(1 + \frac{1}{\gamma_n}\right). \tag{7.7}$$

Then, taking the logarithm of both sides of (7.7) results in

$$Z_t = \sum_{n=1}^{N} Z_n \tag{7.8a}$$

where

$$Z_t = \ln\left(1 + \frac{1}{\gamma_t}\right) \tag{7.8b}$$

$$Z_n = \ln\left(1 + \frac{1}{\gamma_n}\right). \tag{7.8c}$$

Since γ_t and γ_n are random variables, then Z_t and Z_n are new transformed random variables. With the assumption that $\gamma_n,\ n = 1, 2, \cdots, N$ are statistically independent, then the transformed random variables, Z_n, are statistically independent as well [18, p. 244], and we can form the GTCF of the new transformed random variable, Z_t, as

$$\Phi_{Z_t}(\omega) = \prod_{n=1}^{N} \Phi_{Z_n}(\omega) \tag{7.9a}$$

where

$$\Phi_{Z_n}(\omega) = \int_0^\infty f_{z_n}(r) e^{j\omega r} dr \tag{7.9b}$$

and

$$f_{z_n}(r) = \frac{e^r}{(e^r - 1)^2} f_{\gamma_n}\left(\frac{1}{e^r - 1}\right) \tag{7.9c}$$

where (7.9c) can be derived according to a theorem on transformation of random variables [18, Ch. 7]

The computation of $\Phi_{Z_n}(\omega)$ by itself involves a single integration. In some cases, (7.9b) will have a closed-form expression and consequently, $\Phi_{Z_t}(\omega)$ will have a closed-form expression. In other cases, when no closed-form expression is known, numerical integration techniques can be used to calculate the single-fold integral (7.9b). The numerical integration will be feasible because the PDF of Z_n, $f_{z_n}(r)$, is a well-behaved function, in the sense that $\lim_{r\to\infty} f_{z_n}(r) = e^{-r} f_{\gamma_n}(0)$. Therefore, $f_{z_n}(r)$, as an integrand, is a rapidly exponentially decaying function. Rice [19, 20] reported the benefits of using the trapezoidal rule in calculating integrals involving such functions. Noteworthy, the trapezoidal rule gives the exact, aliased result when used to calculate integral transforms [19, 20].

As the next step, it is straightforward to find the PDF of the transformed random variable, Z_t, using the inverse transformation,

$$f_{z_t}(r) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi_{Z_t}(\omega) e^{-j\omega r} d\omega. \tag{7.10}$$

To get the end result, the PDF of the instantaneous end-to-end received SNR, γ_t, we need to express the PDF of γ_t in terms of the PDF of Z_t. To do this, we rewrite (7.8b) as,

$$\gamma_t = \frac{1}{e^{Z_t} - 1} \tag{7.11}$$

and then use the standard technique for determining the PDF of a transformation of a random variable [18, Ch. 7] to obtain

$$f_{\gamma_t}(r) = \frac{1}{r(r+1)} f_{z_t}\left(\ln\left(1 + \frac{1}{r}\right)\right). \tag{7.12}$$

Eq. (7.12) will be used with (7.3), (7.4) and (7.5) to calculate the ergodic capacity, outage probability and average symbol error probability, respectively.

7.4 The Characteristic Function of Z_n

As discussed in the previous section, the computation of the transformed characteristic function, $\Phi_{Z_n}(\omega)$, of the transformed random variable, Z_n, involves a single-fold integral as shown in (7.9b). In this section, we show that, for Rayleigh, Nakagami-m and Rician fading links, there exist closed-form expressions for the characteristic functions. To proceed, we substitute (7.9c) into (7.9b) to obtain

$$\Phi_{Z_n}(\omega) = \int_0^\infty \frac{e^r}{(e^r-1)^2} f_{\gamma_n}\left(\frac{1}{e^r-1}\right) e^{j\omega r} dr. \tag{7.13}$$

Then, using the variable transformation, $x = \dfrac{1}{e^r-1}$, and after manipulation, (7.13) is transformed into

$$\Phi_{Z_n}(\omega) = \int_0^\infty f_{\gamma_n}(x)\left(1+\frac{1}{x}\right)^{j\omega} dx. \tag{7.14}$$

For Nakagami-m fading links, $f_{\gamma_n}(x)$ is defined in [17, eq. (2.21)]. Substituting for the Nakagami-m fading distribution into (7.14) results in

$$\Phi_{Z_n}(\omega) = \int_0^\infty C\, x^{m-1} \exp\left[-\frac{m}{\overline{\gamma}_n} x\right]\left(1+\frac{1}{x}\right)^{j\omega} dx \tag{7.15}$$

where $C = \left(\dfrac{m}{\overline{\gamma}_n}\right)^m \dfrac{1}{\Gamma(m)}$ and $\overline{\gamma}_n$ is the channel's average SNR. Eq. (7.15) can be rewritten and solved as

$$\begin{aligned}\Phi_{Z_n}(\omega) &= C\int_0^\infty \exp\left[-\frac{m}{\overline{\gamma}_n}x\right] x^{m-1-j\omega}(1+x)^{j\omega} dx \\ &= C\,\Gamma(m-j\omega)\,U\left(m-j\omega, m+1, \frac{m}{\overline{\gamma}_n}\right)\end{aligned} \tag{7.16}$$

where $U(a,b,z)$ is the Tricomi confluent hypergeometric function defined in [21, eq. (13.2.5)]. Using a Kummer transformation [21, eq. (13.1.29)], namely $U(a,b,z) = z^{1-b}\,U(a-b+1, 2-b, z)$, a closed-form expression for the characteristic function, $\Phi_{Z_n}(\omega)$, in the case of

Nakagami-m fading links, is obtained as

$$\Phi_{Z_n}(\omega) = \frac{1}{\Gamma(m)}\,\Gamma(m - j\omega)\,U\left(-j\omega,\, 1-m,\, \frac{m}{\overline{\gamma}_n}\right). \tag{7.17}$$

For Rician fading links, substituting the infinite series representation for the Bessel function $I_0(\cdot)$ [21, eq. (9.6.10)] into the PDF of the Rician fading distribution [17, eq. (2.16)], and following the same procedure as before, the transformed characteristic function $\Phi_{Z_t}(\omega)$ can be written as

$$\Phi_{Z_n}(\omega) = \int_0^\infty \exp\left[-K\right] \alpha \sum_{i=0}^{\infty} \beta_i \exp\left[-\alpha x\right] x^i \left(1 + \frac{1}{x}\right)^{j\omega} dx \tag{7.18}$$

where $\alpha = \frac{1+K}{\overline{\gamma}}$ and $\beta_i = \frac{(K\alpha)^i}{(i!)^2}$. Eq. (7.18) can be rewritten as

$$\Phi_{Z_n}(\omega) = \exp\left[-K\right] \alpha \sum_{i=0}^{\infty} \beta_i \int_0^\infty \exp\left[-\alpha x\right] x^i \left(1 + \frac{1}{x}\right)^{j\omega} dx \tag{7.19}$$

resulting in

$$\Phi_{Z_n}(\omega) = \exp\left[-K\right] \alpha \sum_{i=0}^{\infty} \beta_i \Gamma(i + 1 - j\omega)\, U\left(i + 1 - j\omega,\, i+2,\, \alpha\right). \tag{7.20}$$

Using the same previous Kummer transformation, a closed-form expression for $\Phi_{Z_n}(\omega)$ in the case of Rician fading links is obtained as

$$\Phi_{Z_n}(\omega) = \sum_{i=0}^{\infty} C_i\, \Gamma(i + 1 - j\omega)\, U\left(-j\omega,\, -i,\, \alpha\right) \tag{7.21}$$

where $C_i = e^{-K} \frac{K^i}{(i!)^2}$ and K is the ratio of the power of the line-of-sight (LOS) component to the average power of the scattered component. Although the expression in (7.21) involves an infinite summation, it is found that the summands decay exponentially, or slightly faster, with the increase of i, because Stirling's approximation specifies that $i!$ grows as $e^{i \ln i}$, and hence $\frac{1}{(i!)^2}$ decays as $e^{-2i \ln i}$, and hence, a truncated summation with a finite number of terms will reliably achieve a required accuracy. Owing to this rapid rate of convergence, a careful empirical test for convergence is allowable and also for truncation error estimation.

7.5 Computational Considerations

A strength of the GTCF method is that in addition to providing exact results, which can be obtained for any general case of multihop AF systems with general fading distributions, it is computationally far less complex than other approaches that will give exact results. Previous exact methods, small in number, apply only for special cases of the AF systems.

In this section, we discuss the computational considerations and the ostensible complexity of the GTCF method. In the best case, (7.10) will have a closed-form solution and then the PDF of γ_t, given by (7.12), will be in closed-form. In other cases, the PDF of γ_t must be computed using (7.12) and numerical integration of (7.10). This is the case for the major fading distributions; Rayleigh, Nakagami-m and Rician link fading.

In the case where no closed-form solutions are available for any of the integrals involved, but the link fadings are statistically identical or statistically identical to a scale factor, the GTCF method will require a 3-fold numerical integration. Note that the links can be statistically identical to a scale factor if the link SNRs follow the same distribution with the same fading parameters but they differ in the value of the average link SNRs, $\overline{\gamma}_n$.

In the worst case where no closed-form solutions are available for any of the integrals involved and the links are not statistically identically distributed, the GTCF method will require a $(N+2)$-fold numerical integration, whether the links have dissimilar fading parameters or have different fading distributions other than the major fading distributions.

To see this, let $G_{\gamma_t}(\gamma_t)$ represent a performance metric; for example, $G_{\gamma_t}(\gamma_t)$ may represent the ergodic capacity for which $G_{\gamma_t}(\gamma_t) = \frac{1}{N}\log_2(1+\gamma_t)$ in (7.3), or it may represent the average symbol error probability for which $G_{\gamma_t}(\gamma_t) = b\,Q\left(\sqrt{a\,\gamma_t}\right)$ in (7.5), or it may represent the outage probability for which $G_{\gamma_t}(\gamma_t) = 1 - U(\gamma_t - \gamma_{th})$ in (7.4), where $U(\cdot)$ is the unit step function. Then an end-to-end performance measure, PM, will have the form

$$PM\left(G_{\gamma_t}\right) = \int_0^\infty G_{\gamma_t}(r)\; f_{\gamma_t}(r)\, dr. \tag{7.22}$$

Table 7.1 shows the composite integral expressions required by the GTCF method, for different cases of complexity, and for exact evaluation using the distributions of the component link SNRs.

The expressions (A), (B) and (D) in Table 7.1, can be derived by substituting (7.12) into (7.22) resulting in

$$\begin{aligned} PM\left(G_{\gamma_t}\right) &= \int_0^\infty G_{\gamma_t}\left(r\right) f_{\gamma_t}(r) dr \\ &= \int_0^\infty G_{\gamma_t}\left(r\right) \frac{1}{r(r+1)} f_{z_t}\left(\ln\left(1+\frac{1}{r}\right)\right) dr. \end{aligned} \tag{7.23}$$

Then using the variable transformation, $x = \ln\left(1+\frac{1}{r}\right)$, the expression in (7.23) becomes

$$PM\left(G_{\gamma_t}\right) = \int_0^\infty G_{\gamma_t}\left(\frac{1}{e^x-1}\right) f_{z_t}(x) dx. \tag{7.24}$$

And finally, using (7.10), the expression in (7.24) becomes

$$PM\left(G_{\gamma_t}\right) = \int_0^\infty G_{\gamma_t}\left(\frac{1}{e^x-1}\right)\left(\frac{1}{2\pi}\int_{-\infty}^\infty \Phi_{Z_t}(\omega) e^{-j\omega x} d\omega\right) dx. \tag{7.25}$$

Eq. (7.25) represents the composite integral expression for the GTCF method in the most general case. Cases (A), (B), (C) and (D) are now explained each in turn.

Case (A) occurs when a closed-form expression for $\Phi_{Z_n}(\omega)$ is known. For example, in the case of Rayleigh, Nakagami-m or Rician fading links, whether the link fadings are statistically identical or not, a closed-form expression for $\Phi_{Z_t}(\omega)$ can be obtained by substituting the suitable expression for $\Phi_{Z_n}(\omega)$, (7.17) for Nakagami-m fading links, or (7.21) for Rician fading links, into (7.9a). Substituting the obtained expression for $\Phi_{Z_t}(\omega)$ into (7.25) results in the two-fold integral expression (A) in Table 7.1. In the worst case, when the double-integral expression (7.25) can not be evaluated in closed-form, it can be numerically evaluated using available software packages such as MATLAB and MATHEMATICA. Note that numerical calculation of a two-fold integral is widely practiced in the literature [17]. Note that in the case of mixed fading links, where the link fadings follow different families of the major fading distributions, the application of the GTCF approach requires only the two-fold integral expression (A).

Case (B) occurs when no closed-form expression for $\Phi_{Z_n}(\omega)$ is known, but the link fadings are statistically identical or statistically identical to a scale factor. The link fadings

Table 7.1: Representations of the integration complexity of the different cases using the GTCF method and exact evaluation using the PDFs of the link SNRs

Case	Representation
(A) *(2)*-fold	$PM\left(G_{\gamma_t}\right) = \int_0^\infty G_{\gamma_t}\left(\frac{1}{e^x - 1}\right)\left(\frac{1}{2\pi}\int_{-\infty}^\infty \prod_{n=1}^N \left[\Phi_{Z_n}(\omega)\right] e^{-j\omega x} d\omega\right) dx$
(B) *(3)*-fold	$PM\left(G_{\gamma_t}\right) = \int_0^\infty G_{\gamma_t}\left(\frac{1}{e^x - 1}\right)\left(\frac{1}{2\pi}\int_{-\infty}^\infty \prod_{n=1}^N \left[\int_0^\infty f_{z_n}(\sigma) e^{j\omega\sigma} d\sigma\right] e^{-j\omega x} d\omega\right) dx$
(C) *(N)*-fold	$PM\left(G_{\gamma_t}\right) = \int_{r_N=0}^\infty \int_{r_{N-1}=0}^\infty \cdots \int_{r_2=0}^\infty \int_{r_1=0}^\infty G_{\gamma_t}\left(\left[\prod_{n=1}^N \left(1 + \frac{1}{r_n}\right) - 1\right]^{-1}\right)$ $\times f_{\gamma_N}(r_N) f_{\gamma_{N-1}}(r_{N-1}) \cdots f_{\gamma_2}(r_2) f_{\gamma_1}(r_1)\ dr_1 dr_2 \cdots dr_{N-1} dr_N$
(D) *(N+2)*-fold	$PM\left(G_{\gamma_t}\right) = \int_0^\infty G_{\gamma_t}\left(\frac{1}{e^x - 1}\right)\left(\frac{1}{2\pi}\int_{-\infty}^\infty \prod_{n=1}^N \left[\int_0^\infty f_{z_n}(\sigma) e^{j\omega\sigma} d\sigma\right] e^{-j\omega x} d\omega\right) dx$

are statistically identical to a factor when the average link SNRs are scaled versions of each other. In this case, the characteristic functions, $\Phi_{Z_n}(\omega)$, are compressed, or expanded, versions of each other. To clarify this, assume that $\overline{\gamma}_n = \alpha_n\, \overline{\gamma}_1, n = 2, 3, \cdots, N$, then

$$f_{z_n}(r) = \frac{1}{\alpha_n} f_{z_1}\left(\frac{r}{\alpha_n}\right) \tag{7.26}$$

resulting in

$$\begin{aligned} \Phi_{Z_n}(\omega) &= \int_0^\infty f_{z_n}(r) e^{j\omega r} dr \\ &= \int_0^\infty \frac{1}{\alpha_n} f_{z_1}\left(\frac{r}{\alpha_n}\right) e^{j\omega r} dr \\ &= \int_0^\infty f_{z_1}(x)\, e^{j\omega\alpha_n x} dx \\ &= \Phi_{Z_1}(\alpha_n\, \omega)\,. \end{aligned} \tag{7.27}$$

In these cases, the integral definition in (7.9b) will be used and the performance metrics are represented by (B) in Table 7.1. Evaluation of this representation involves a 3-fold numerical integral because, either in case of statistically identical links or links statistically identical to a factor, the numerical calculation of the innermost integral in (B) is done only once, resulting in a computational complexity of 3 nested numerical integrals.

Case (C) represents the integration complexity of exact evaluation using the PDFs of the component link SNRs. The representation (C) in Table 7.1 involves a N-fold integral, and it represents the conventional numerical evaluation method, however, to the best of the authors' knowledge, it was not reported before in the literature. This N-fold integral is not simplified even when the links are identically distributed, because of the presence of the complicated, inseparable function, $G_{\gamma_t}(\gamma_t)$, where γ_t is defined in (7.2). Recalling that the computational effort required to compute a N-fold integral grows exponentially with N clarifies the computational advantage of using the GTCF method in cases (A) and (B), especially for large numbers of links, N.

Finally, case (D) occurs when no closed-form expression for $\Phi_{Z_n}(\omega)$ is known and the individual links have different fading distributions or similar fading distributions with different fading parameters, i.e. the individual link PDFs do not follow the relation (7.26). In this case, the calculation of the performance metrics using the GTCF method requires a $(N+2)$-fold numerical integration as shown by (D) in Table 7.1. Using the GTCF method in this case is not indicated because it has higher complexity than the direct method using the PDFs of the component links SNRs.

It should be emphasized that for the different situations where the link fadings follow any of the major fading distributions, the computational complexity of the GTCF will be always as in case (A); a two-fold numerical integration, irrespective of the number of hops.

7.6 Numerical Examples

In this section, the GTCF method is used to examine AF relaying systems operating in Rayleigh, Nakagami-m and Rician fading channels. Multihop systems with *i.i.d.* and *i.ni.d.* links are considered. The links can be non-identically distributed if the fading parameters, m in the case of Nakagami-m fading and K in the case of Rician fading, are different from each other and/or the average link signal-to-noise ratios (SNRs), $\overline{\gamma}$, are not the same for all the links. We assume an uniform power allocation policy, that is the total available power, P, is evenly allocated to the source and the relays. Without loss of generality, we assume equal noise powers, N_0, at all the nodes. The nodes are assumed to be located at equal distances from each other, and fixed and normalized distance between the source and the destination is assumed, so that $d_{hop} = \frac{1}{N}$, and according to the Friss propagation model [22], the average SNR over any link is given by [11]

$$\overline{\gamma} = N^{\delta-1} \frac{P}{N_0} \tag{7.28}$$

where δ is the path loss component, taken equal to 3 in the following examples. Binary phase shift keying (BPSK) modulation is assumed, so the parameter (a, b) in (7.5) is equal to $(2, 1)$ [17].

The first set of examples confirms the accuracy and versatility of the GTCF approach. It shows the average error probability, outage probability and the ergodic capacity for a five-hop ($N = 5$) AF system with *i.i.d.* Rayleigh, Nakagami-m ($m_1 = m_2 = m_3 = m_4 = m_5 = 2.5$) and Rician ($K_1 = K_2 = K_3 = K_4 = K_5 = 2$) fading links. The same system performance metrics are shown for *i.ni.d.* Nakagami-m fading links ($m_1 = 1.5$, $m_2 = 2.0$, $m_3 = 2.5$, $m_4 = 3.0$, $m_5 = 3.5$) and *i.ni.d.* Rician ($K_1 = 1$, $K_2 = 2$, $K_3 = 3$, $K_4 = 4$, $K_5 = 5$) fading links. As expected from the analysis, Figs. 7.1, 7.2 and 7.3 show precise matches between the results obtained from the GTCF method and simulation results at all values of P/N_0 for the different performance measures. Note that for the cases of Rician fading links, the infinite summation of (7.21) was truncated at the 20^{th} term to obtain the characteristic functions of the transformed random variables. It was found that adding more terms does not affect the result in the 10^{th} decimal figure.

It is noteworthy that the GTCF is computationally advantageous over simulations. Due to the limited memory capacity and processing capabilities of software packages, simulations can not be used efficiently to obtain very small values of the average error probability, less than or equal to 10^{-7}, because in order to achieve the required accuracy, a huge number of randomly generated samples, 10^9 samples or more, will be required for the simulations. For most software packages, such as MATLAB, the memory capabilities do not support such sizes of samples. Moreover, the processing time for such numbers of samples will be unacceptably long. For example, in order to obtain the average error probability at $P/N_0 = 20$ dB , assuming 10 iterations with 10^8 samples per iteration, with $N = 5$, the required processing time is 480 seconds on average. On the other hand, using the GTCF method, the problem of memory capacity does not exist, because fewer numbers of samples per characteristic function and probability density function are required to get the results to the required accuracy, even for very small values of the average error probability. The required processing time for the GTCF method is only 51 seconds on average. This emphasizes the computational efficiency of the GTCF method compared to simulations.

The figures show some interesting aspects of the performance measures and their relationship to the type of fading. Fig. 7.1 shows that the ergodic capacity is better for the less severely faded Nakagami-m and Rician fading channels than the Rayleigh channels; however, the differences are modest being less than 15.8% for all values of P/N_0 less than 20 dB. Yet, dramatic differences in performance arising from the type of fading are seen in the average error probability in Fig. 7.3 and the outage probability in Fig. 7.2. It is observed that although the Rician links, for both identical and non-identical cases, outperform the Rayleigh links, the slopes of the Rician average error probability curves in Fig. 7.3 are the same as the slope of the Rayleigh curve for average error probabilities less than 4×10^{-2}. The same behaviour is observed for outage probability in Fig. 7.2. The slopes of the Rician outage probability curves are the same as the slope of the Rayleigh curve for values of outage less than 2×10^{-1}. The behaviour of the Nakagami-m links is markedly different. The limiting slopes of the average error probability curves in Fig. 7.3 and the limiting slope of the outage probability curves in Fig. 7.2 are greater than those for the Rician and Rayleigh curves and the limiting slopes change with variation of the fading parameters, m_1, m_2, m_3, m_4 and m_5. This behaviour is expected [23, Fig. 8]. The limiting slopes of the Rician outage and average error probability curves will be the same as the slope of the Rayleigh curves irrespective of the value of the fading parameter, K [24, Figs. 1–3], [25, Figs. 4–6]. On the contrary, the limiting slopes of the Nakagami-m outage and average error probability curves are functions of the fading parameter m, and increase with increasing m [5, Fig. 2], [10, Fig. 6]. This behaviour of the Nakagami-m model implies that the Nakagami-m model will always exhibit better performance than the Rician model, for sufficiently large SNR for all values of $m > 1$, regardless of the value of the Rician parameter K. For example, assuming $i.i.d$ links, the average error probability for the Nakagami-m links with $m = 2.5$ at $P/N_0 = 15$ dB is not achieved by Rician links unless the fading parameter K is at least equal to 10.2, i.e. the power of the LOS signal component on each of the 5 hops, is 10.2 times the power of the scattered signal component.

The next example set affirms the previous observations and shows further interesting observations. Fig. 7.4 shows the average error probability for 5 different cases of five-hop AF relaying systems over Nakagami-m fading links. The first two cases represent identical Rayleigh ($m_n = 1, n = 1, 2, 3, 4, 5$) links and identical Nakagami-m ($m_n = 2.5, n = 1, 2, 3, 4, 5$) links. The other three cases represent non-identical Nakagami-m links with different values of the fading parameters m_n. In case (3), $\min\{m_n\} = 1.5$ while in cases (4) and (5), $\min\{m_n\} = 1$ and 2.5 respectively. It is observed, as in the previous set of examples, that the limiting slope of the average error probability curves is dependent on the fading parameter m. However, this limiting slope does not depend on the fading parameters of all the links. It is observed that the limiting slope is dependent only on the value of the smallest fading parameter, $\min\{m_n\}$. It is seen in Fig. 7.4 that the limiting slope of case (1), where ($m_n = 1, n = 1, 2, 3, 4, 5$), is the same as the limiting slope of case (4), where $\min\{m_n\} = 1$. The same observation is true for cases (4) and (5). Similar behaviour is seen in [14, Figs. 1, 2] for dual-hop AF systems. Note that the dependence of the limiting slope of the average error probability curve on the value of the smallest fading parameter, $\min\{m_n\}$, is not affected by the placement of the link which attains this smallest value of the fading parameters. This can be seen from (7.2a) which shows that the instantaneous end-to-end SNR, γ_t, is symmetric with respect to all the individual link SNRs, γ_n. On the other hand as expected, the larger the values of the fading parameters, other than the smallest one, the better the performance of the system. For example, at $P/N_0 = 10$ dB, the average error probability for case (2), $m_n = 2.5$, is 1.1×10^{-5}, while it is 2.6×10^{-6} for case (5) where $m_1 = 2.5$, $m_2 = 3.0$, $m_3 = 3.5$, $m_4 = 4.0$, $m_5 = 4.5$.

In the next example, the exact PDF of the instantaneous end-to-end received SNR is obtained by applying the GTCF method to different multihop systems over $i.i.d$ Nakagami-m

and $i.i.d$ Rician links. The value of P/N_0 is taken equal to 0 dB and the PDF is obtained for $N = 4, 5$ and 6. Fig. 7.5 shows the probability density functions for the cases of Nakagami-m links with $m = 2.5$ and Rician links with $K = 2$. From the figure it can be observed that, as the number of intermediate links, N, between the source and the destination increases, the end-to-end SNR will be spread over a wider range of values; the mode of the PDF moves to larger values and its height decreases. Recall that the distance between the source and the destination is assumed fixed for all values of N[(2)]. This behaviour is characteristic of the PDF of a sum of independent random variables, and is interesting given that (7.2a) is a highly nonlinear function of the γ_n, rather than a linear summation. Central limit theorems hold for many linear sums of independent random variables, and it is intriguing to ask whether a limit theorem holds for the nonlinear combination of the γ_n specified by (7.2a). It will be shown in Section 7.7 that a limiting distribution exists for (7.2a) and the limiting distribution will be determined.

The next set of examples study the effect of increasing the number of hops on the different performance metrics. The average error probability and outage probability are shown in Figs. 7.6 and 7.7, respectively, for multihop AF relaying systems with $N = 4, 5$ and 6. Rayleigh, identical Nakagami-m ($m = 2.5$) and identical Rician ($K = 2$) fading channels are considered in the examples. The ergodic capacity is also shown in Fig. 7.8 for the identical Nakagami-m and Rician links. Many observations can be drawn from the figures. The first observation is that the limiting slope of the average error probability curves, for all fading distribution cases, is independent of the number of hops, N. Note that the system model assumes that the data signal is received at the destination through only the multihop relay path. On the other hand, there is an effective SNR gain achieved by using more intermediate relays. For example, to achieve an average error probability of 10^{-5} over the Nakagami-m links, a five-hop system requires 1.50 dB less average SNR than that

[(2)]Note that for the case of systems operating over Nakagami-m fading links, with $m > 1$, as the end-to-end SNR approaches zero, its PDF approaches zero as well, i.e.

$$f_{\gamma_t}(0) = 0.$$

On the other hand, for systems operating over Rayleigh or Rician fading links, as the end-to-end SNR approaches zero, its PDF approaches infinity (which cannot be easily shown in Fig. 7.5), i.e.

$$\lim_{r \to 0} f_{\gamma_t}(r) = \infty.$$

These results can be analytically verified using the GTCF equations.

required by a four-hop system, while a six-hop system requires 2.64 dB less average SNR. Also, the outage probability is improved by increasing the number of hops. For example, in Fig. 7.7, for the Nakagami-m links system, a target value of outage of 10^{-3} occurs at $\gamma_{th} = -6.077$ dB for $N = 4$, while it occurs at $\gamma_{th} = -4.436$ dB and -3.105 dB for $N = 5$ and 6, respectively.

Another observation concerning the ergodic capacity can be drawn from Fig. 7.8. It can be noticed that with increasing the number of hops, N, the ergodic capacity, ϵ, decreases. For example, for the Rician fading case at $P/N_0 = 15$ dB, the ergodic capacities for $N = 4, 5$ and 6 are $1.492, 1.242$ and 1.069 bps/Hz. The decrease in the ergodic capacity is to be expected. Since the relaying is time division multiplexed on orthogonal channels, the time-bandwidth resources consumed to send the message end-to-end increases with the number of relaying transmissions and the capacity per link decreases when fixed bandwidth transmission is employed.

7.7 Limiting Distribution for γ_t

In this section, we investigate the existence of a limiting distribution for the end-to-end received SNR, γ_t. We had earlier noted the similarity of the changes in the end-to-end SNR distribution with an increasing number of relays, to the changes in the distribution of a sum of independent random variables with an increasing number of summands. We start the investigation from the summation of the statistically independent transformed random variables, Z_n, in eq. (7.8a). We assume that all the link fadings are statistically identical, implying that all the random variables Z_n are also identically distributed. In this case, we apply the central limit theorem (CLT) found at [26, Theorem 4.20] to the distribution of the random variable Z_t. Assuming each of the random variables Z_n has mean $E\{Z_n\} = \mu$ and variance $E\left\{(Z_n - \mu)^2\right\} = \sigma^2$, then the distribution of the random variable $Z_t = \sum_{n=1}^{N} Z_n$ approaches a normal distribution with mean $N\mu$ and variance $N\sigma^2$, i.e. $F(Z_t) \simeq \Phi\left(\frac{Z_t - N\mu}{\sqrt{N\sigma^2}}\right)$ as $N \to \infty$ [18, p. 278], or equivalently the distribution of the normalized random variable $\overline{Z}_t = \frac{1}{\sqrt{N\sigma^2}}\left(\sum_{n=1}^{N} Z_n - N\mu\right)$ tends to a standard normal distribution, $\Phi(\overline{Z}_t)$. Since the random variables Z_n are of continuous type, the density function of $\overline{Z}_t$ approaches a normal density $\mathcal{N}(0,1)$. Accordingly, a limiting PDF for the end-to-end received SNR is obtained using (7.12) as

$$f_{\gamma_t}(r) \backsimeq \frac{1}{\sqrt{2\pi N\sigma^2}} \frac{1}{r(r+1)} \times \exp\left(-\frac{1}{2N\sigma^2}\left(\ln\left(1+\frac{1}{r}\right) - N\mu\right)^2\right). \tag{7.29}$$

The accuracy in using the limiting PDF in (7.29) to approximate the true PDF for finite values of N depends on the rate of convergence of the CLT for this case as N increases. Fig. 7.9 shows the CDF of $\overline{Z}_t$ for $N = 20, 200$ and 1000 Nakagami-m fading links with $m = 2.5$ and $\overline{\gamma} = 20$ dB. The CDF of the standard normal distribution is shown as well. It can be seen that the CLT normal approximation is poor even for a number of links as large as 200. We conclude that the limiting distribution cannot be used to approximate the end-to-end SNR distribution for practical numbers of relay links. This reinforces the value of the GTCF method for obtaining accurate results.

The slow rate of convergence evidenced in our examples is surprising, and some affirmation is warranted. The Berry-Esseen theorem [26, Theorem 4.24] gives a bound for the accuracy of the normal approximation and quantifies the rate at which the convergence to normality takes place. The Berry-Esseen theorem states that

$$\sup_{t\in\Re} |F(t) - \Phi(t)| \leq B\frac{\rho}{\sigma^3}\frac{1}{\sqrt{N}} \tag{7.30}$$

where $\Re$ is the set of real numbers, $F(t)$ is the CDF of $\overline{Z}_t = \frac{1}{\sqrt{N\sigma^2}}\left(\sum_{n=1}^{N} Z_n - N\mu\right)$, $\Phi(t)$ is the standard normal CDF, $\rho = E\left\{|Z_n - \mu|^3\right\}$ is the absolute third moment of Z_n and B is a constant that does not depend on N. The exact value of the constant B is not known although there has been much research conducted to find upper and lower bounds for B. It can be seen from this theorem that the rate of convergence of the CLT is proportional to $B\frac{\rho}{\sigma^3}$ and is on the order of $\frac{1}{\sqrt{N}}$.

Table 7.2 shows the mean, μ, the variance, σ^2 and the absolute third moment, ρ, of the random variables Z_n, for Nakagami-m fading links with $m = 2.5$ for the cases when $\overline{\gamma} = 1, 10$ and 20 dB. The ratios $\left(\rho/\sigma^3\right)^2$ are shown as well. Since the value of the constant

Table 7.2: Mean, variance, absolute third moment and minimum acceptable N for CLT approximation

$\overline{\gamma}$	μ	σ^2	ρ	$\left(\rho/\sigma^3\right)^2$	$N\|_{N_u=12}$
0 dB	0.861556	0.180470	0.168039	4.804	35
10 dB	0.144593	0.015523	0.008339	18.589	133
20 dB	0.016331	0.000364	0.0000795	130.825	930

B is crucial in the Berry-Esseen theorem and an exact value for this constant is not known, we will make an assessment based on comparison to the accuracy of the CLT in approximating the distribution of the sum of 12 uniform $\mathcal{U}(0,1)$ random variables. The latter was widely used as an approximation for the Gaussian distribution for decades before the advent of high powered personal computers. In the following, we find the number of links, N, such that the CLT normal approximation of $\overline{Z}_t$, is comparably accurate. Assuming a certain level of accuracy α, such that $\sup_{t\in\Re}|F(t)-\Phi(t)| \leq B\frac{\rho}{\sigma^3}\frac{1}{\sqrt{N}} \leq \alpha$, then the minimum number of *i.i.d.* random variables, N, required to achieve this level of accuracy is $N = \left(B\frac{\rho}{\sigma^3}\frac{1}{\alpha}\right)^2$. Since B and α are constants that do not depend on the distribution of Z_n, then $\frac{N_1}{N_2} = \frac{\left(\rho_1/\sigma_1^3\right)^2}{\left(\rho_2/\sigma_2^3\right)^2}$, where N_i is the minimum number of *i.i.d.* random variables Z_{n_i}, with statistical parameters σ_i and ρ_i, required to achieve the target accuracy. The last column of Table 7.2 shows the number of fading links, N, compared to $N_u = 12$ for the $\mathcal{U}(0,1)$ summation, at which the approximating distribution in (7.29) approaches the exact distribution.

As expected, it can be observed from the values in Table 7.2 that the CLT normal approximation is much more accurate for smaller values of the average SNR $\overline{\gamma}$. Moreover, it can be observed that very large values of N are required to achieve an acceptable accuracy of the CLT approximation in the relaying problem.

7.8 Conclusion

New exact solutions for performance metrics of multihop AF relaying networks have been derived through the application of the generalized transformed characteristic function method. The GTCF approach was used to obtain the first exact theoretical results for the ergodic capacity, outage probability and the average symbol error probability of multihop AF relaying wireless networks with more than two hops. The method can be used for any fading distribution. Closed-form expressions for the transformed characteristic function were obtained for Rayleigh, Nakagami-m and Rician fading links. The computational complexity of the GTCF method was discussed and its computational complexity advantage over other exact methods was quantified. The effect of having non-identical links as well as the effect of increasing the number of relaying nodes were investigated. It was observed that the limiting slope of the average error probability curve is determined only by the smallest value of the fading parameter, m, in the case of Nakagami-m links. It was also found that increasing the number of hops improves the average error probability and outage performance of the AF relaying system under a total network power constraint. On the other hand, increasing the number of hops results in reduction of the ergodic capacity. A limiting distribution for the end-to-end received SNR with increasing number of relays was obtained by applying a central limit theorem. The accuracy and rate of convergence of the limiting distribution were studied using the Berry-Esseen theorem. It was found that for practical numbers of hops, the limiting distribution is not an accurate estimate of the true distribution, especially at high values of the average SNR.

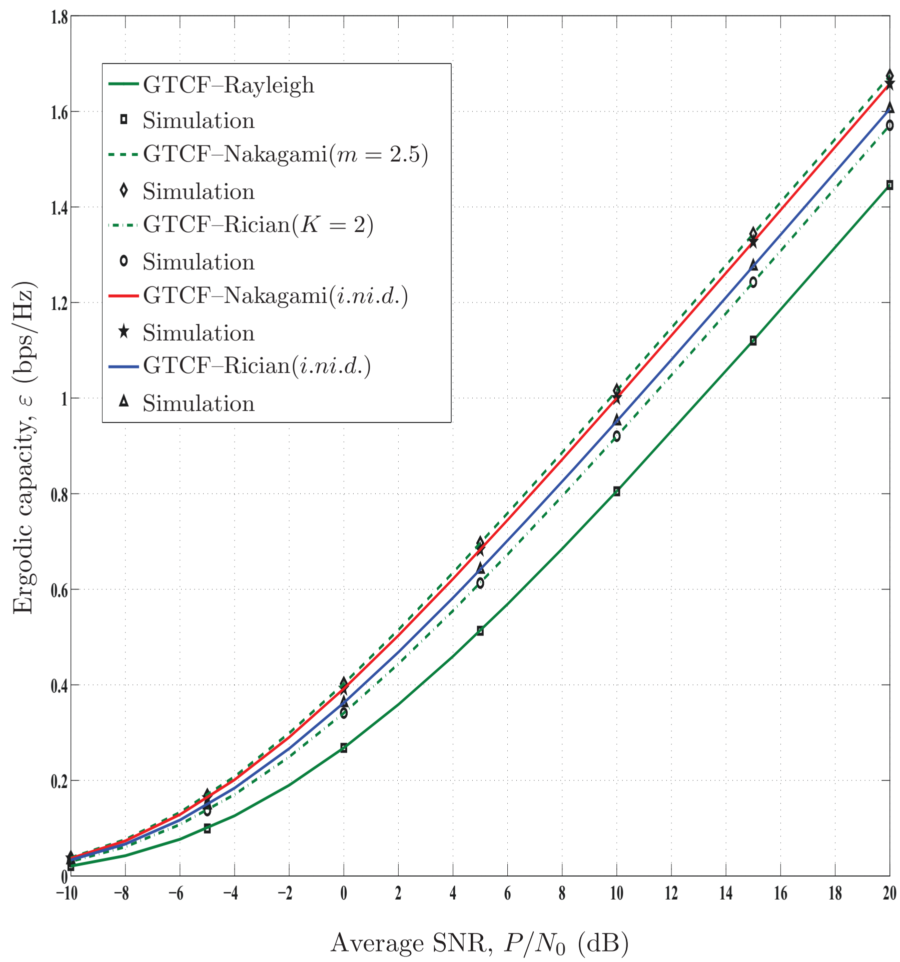

Figure 7.1: The ergodic capacity for five-hop ($N = 5$) AF relaying systems. Rayleigh, *i.i.d.* Nakagami-m ($m = 2.5$) and *i.i.d.* Rician ($K = 2$) fading links are assumed. *i.ni.d.* Nakagami-m ($m_1 = 1.5$, $m_2 = 2.0$, $m_3 = 2.5$, $m_4 = 3.0$, $m_5 = 3.5$) and *i.ni.d.* Rician ($K_1 = 1$, $K_2 = 2$, $K_3 = 3$, $K_4 = 4$, $K_5 = 5$) fading links are also shown.

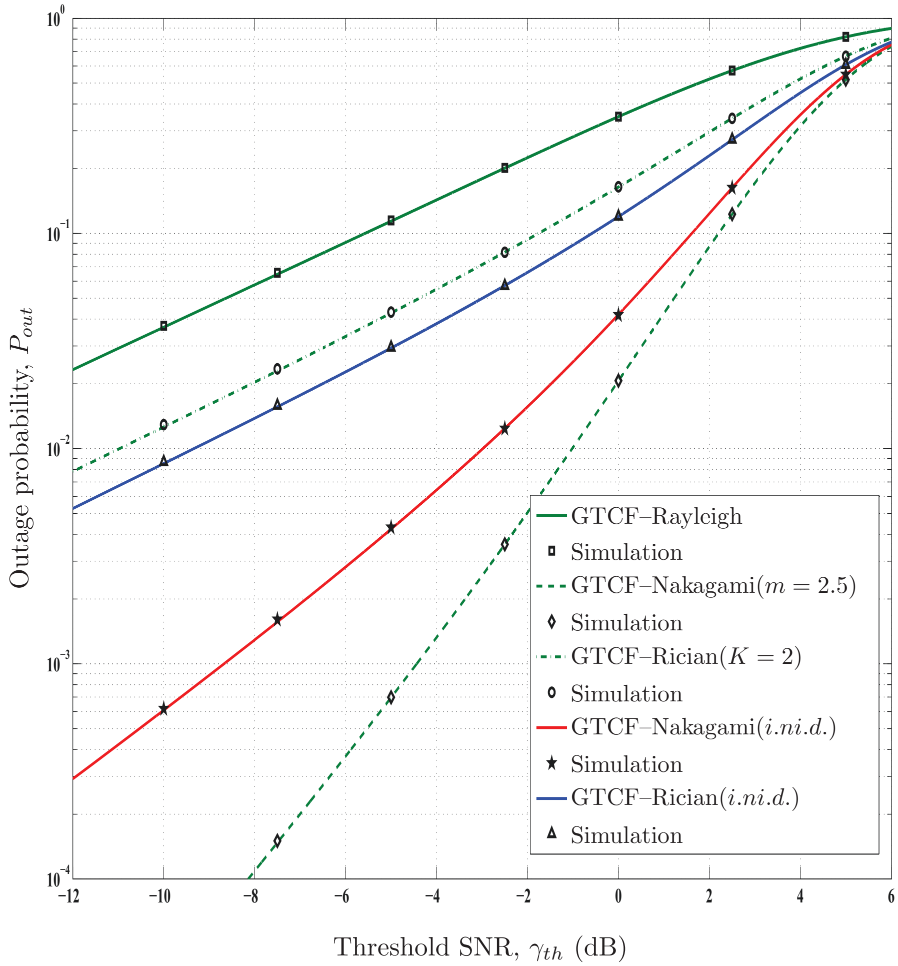

Figure 7.2: The outage probability for five-hop ($N = 5$) AF relaying systems. Rayleigh, *i.i.d.* Nakagami-m ($m = 2.5$) and *i.i.d.* Rician ($K = 2$) fading links are assumed. *i.ni.d.* Nakagami-m ($m_1 = 1.5$, $m_2 = 2.0$, $m_3 = 2.5$, $m_4 = 3.0$, $m_5 = 3.5$) and *i.ni.d.* Rician ($K_1 = 1$, $K_2 = 2$, $K_3 = 3$, $K_4 = 4$, $K_5 = 5$) fading links are also shown.

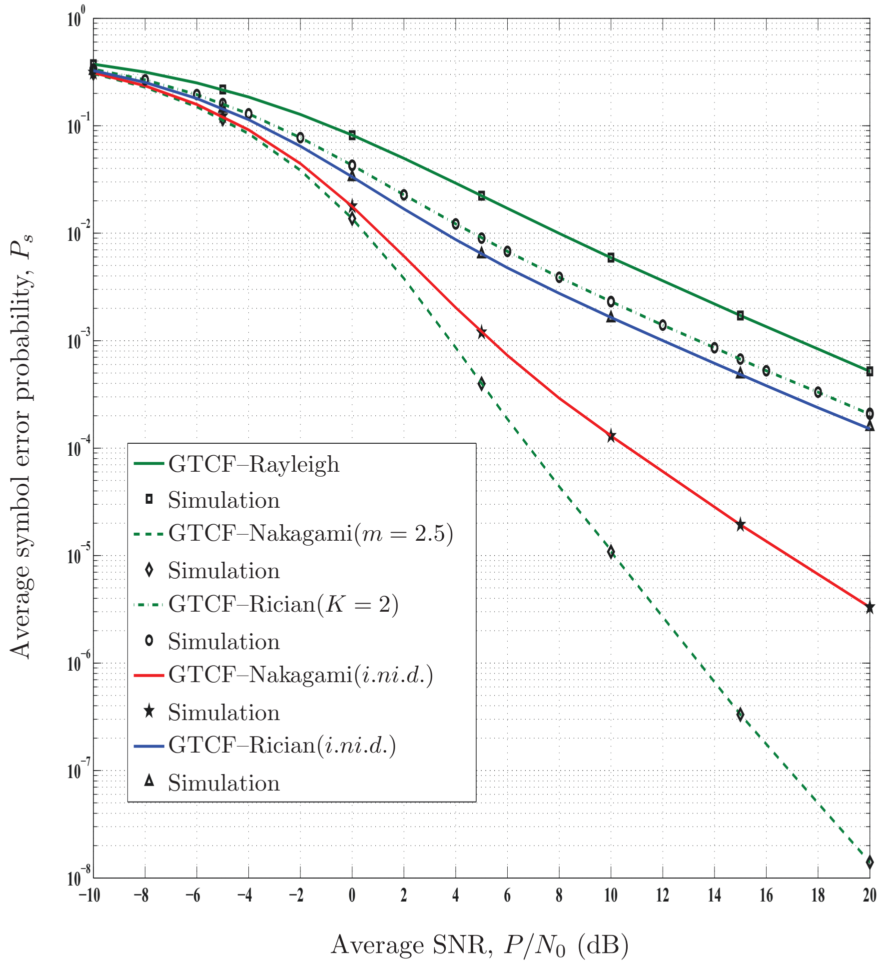

Figure 7.3: The average symbol error probability for five-hop ($N = 5$) AF relaying systems with BPSK modulation. Rayleigh, *i.i.d.* Nakagami-m ($m = 2.5$) and *i.i.d.* Rician ($K = 2$) fading links are assumed. *i.ni.d.* Nakagami-m ($m_1 = 1.5$, $m_2 = 2.0$, $m_3 = 2.5$, $m_4 = 3.0$, $m_5 = 3.5$) and *i.ni.d.* Rician ($K_1 = 1$, $K_2 = 2$, $K_3 = 3$, $K_4 = 4$, $K_5 = 5$) fading links are also shown.

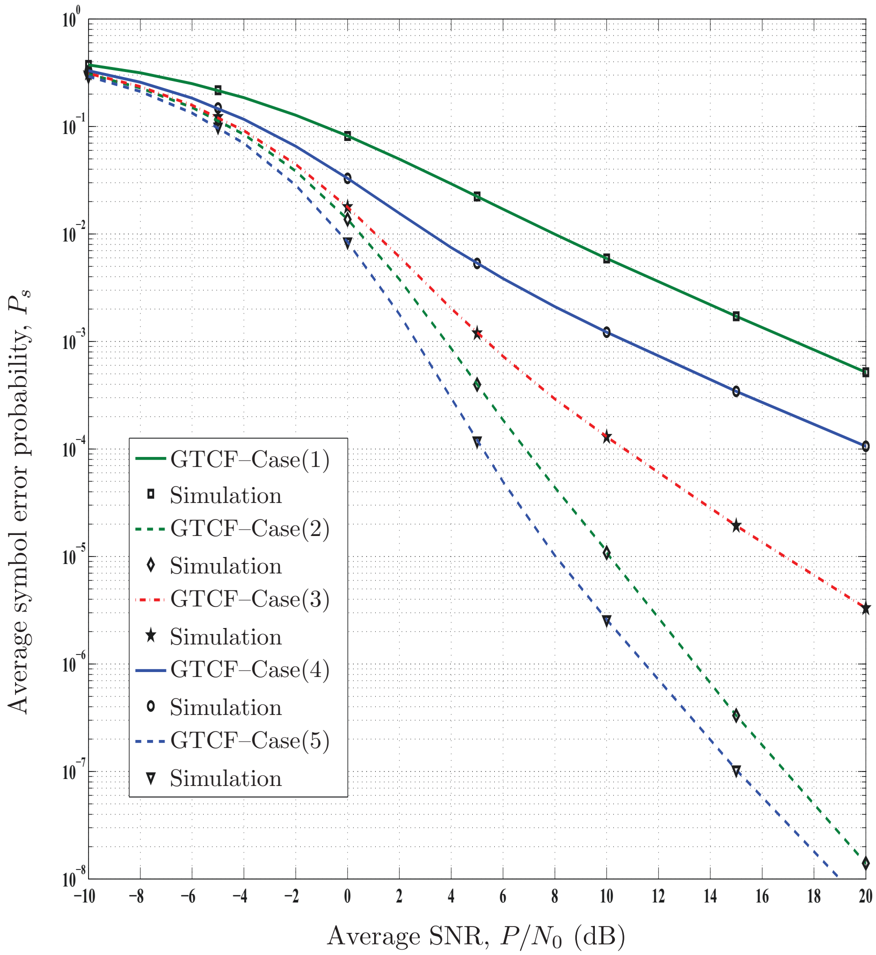

Figure 7.4: The average symbol error probability for five-hop ($N = 5$) AF relaying systems with BPSK modulation. Case (1) represents *i.i.d.* Rayleigh ($m = 1$) links. Case (2) represents *i.i.d.* Nakagami-m ($m = 2.5$) links. Case (3) represents *i.ni.d.* Nakagami-m ($m_1 = 1.5$, $m_2 = 2.0$, $m_3 = 2.5$, $m_4 = 3.0$, $m_5 = 3.5$) links. Case (4) represents *i.ni.d.* Nakagami-m ($m_1 = 1.0$, $m_2 = 1.5$, $m_3 = 2.0$, $m_4 = 2.5$, $m_5 = 3.0$) links. Case (5) represents *i.ni.d.* Nakagami-m ($m_1 = 2.5$, $m_2 = 3.0$, $m_3 = 3.5$, $m_4 = 4.0$, $m_5 = 4.5$) links.

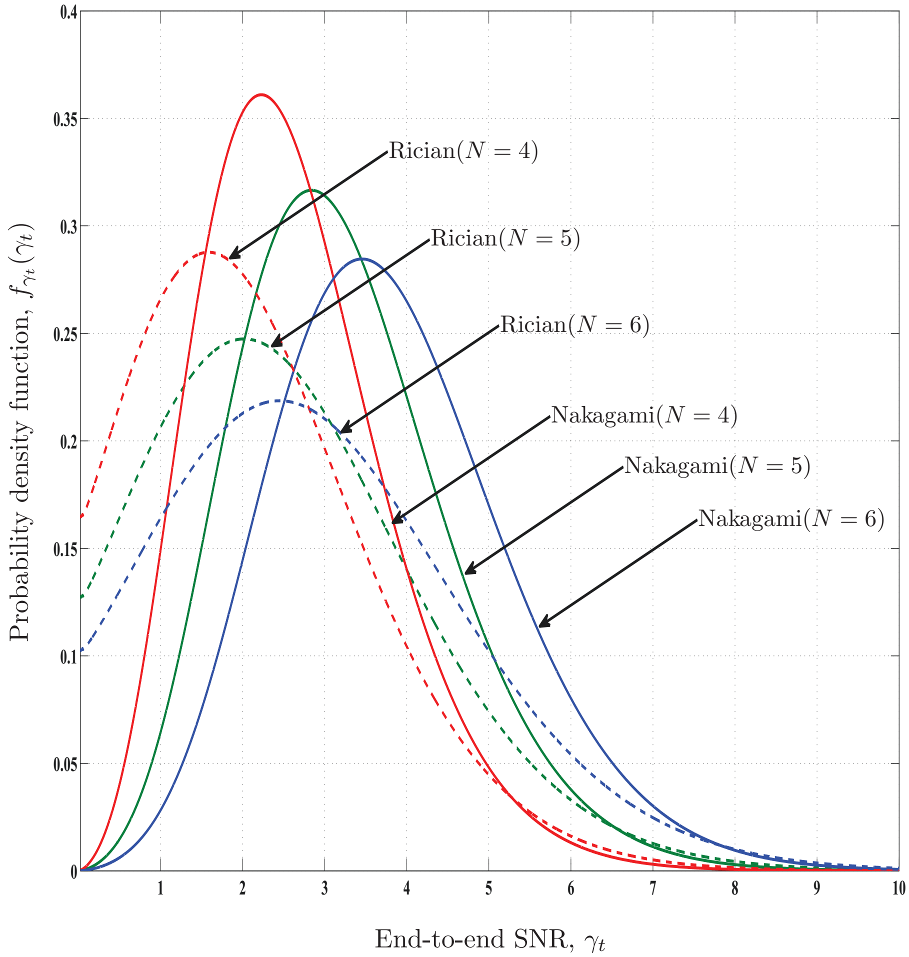

Figure 7.5: The exact probability density function of the instantaneous end-to-end received SNR for different numbers of multihops, $N = 4, 5$ and 6. Identically distributed Nakagami-m ($m = 2.5$) and *i.i.d.* Rician ($K = 2$) fading links are assumed. Fixed total available power, P, is assumed for all cases.

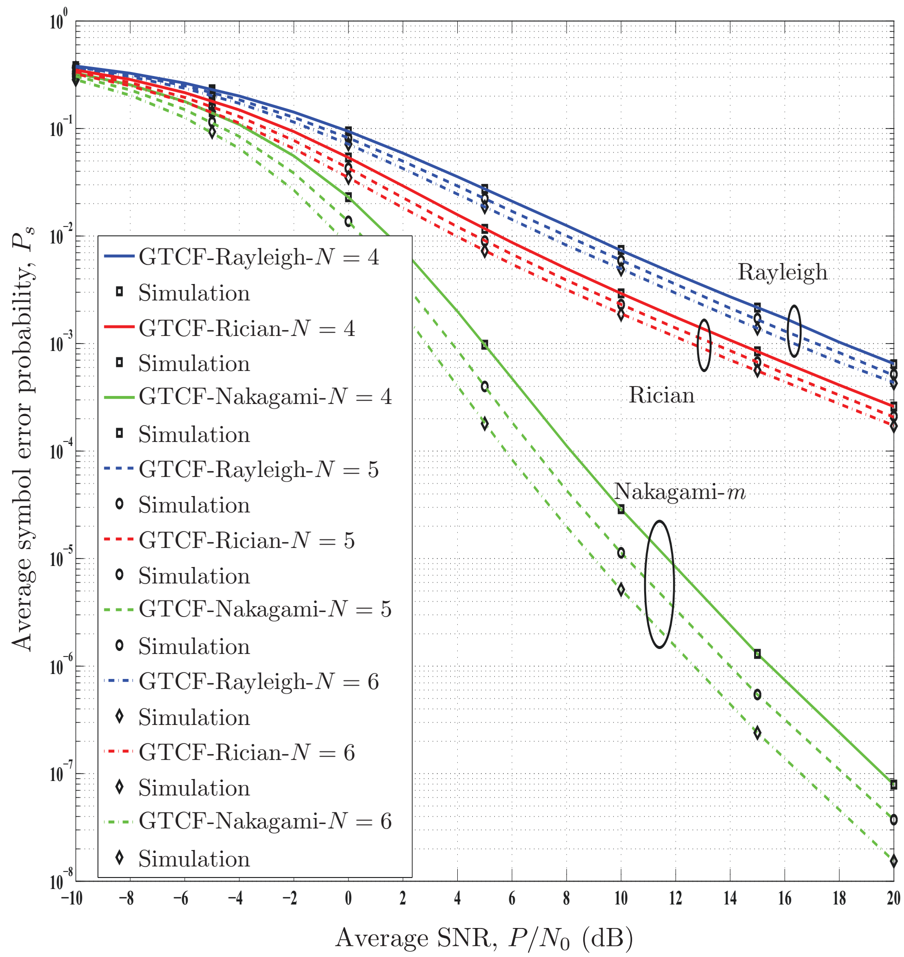

Figure 7.6: The average symbol error probability for multihop ($N = 4, 5$ and 6) AF relaying systems with BPSK modulation. Rayleigh, *i.i.d.* Nakagami-m ($m = 2.5$) and *i.i.d.* Rician ($K = 2$) fading links are assumed.

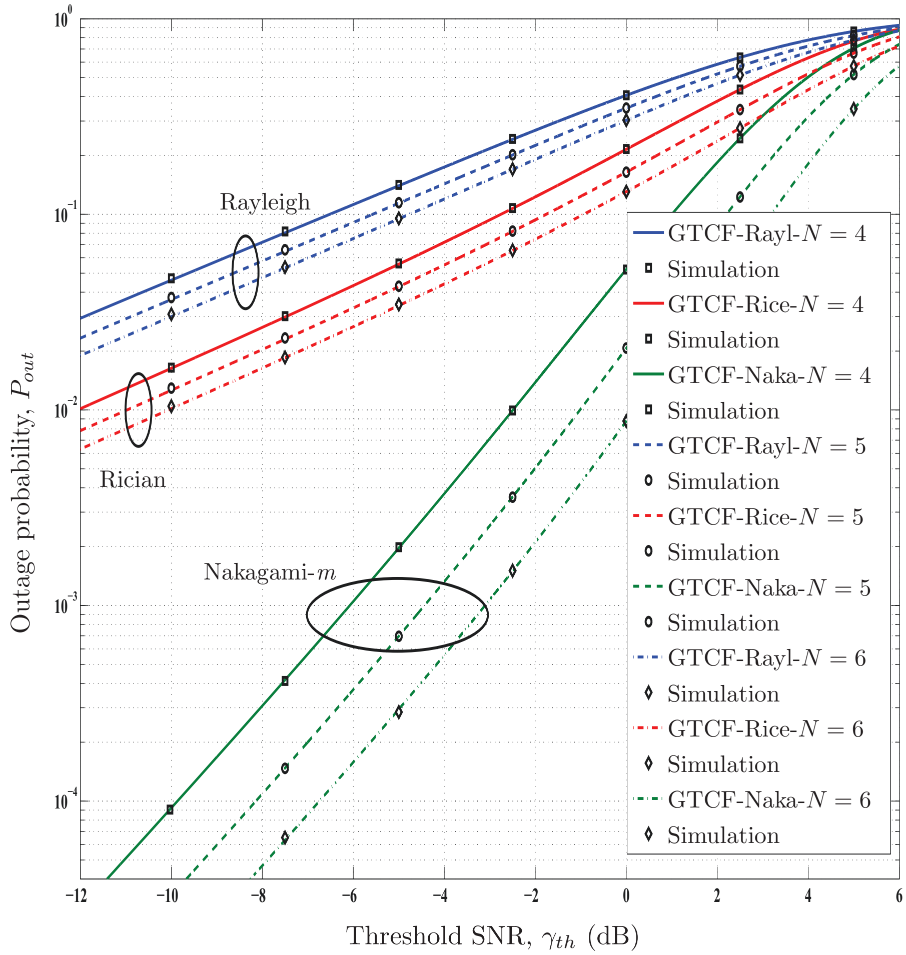

Figure 7.7: The outage probability for multihop ($N = 4, 5$ and 6) AF relaying systems. Rayleigh, *i.i.d.* Nakagami-m ($m = 2.5$) and *i.i.d.* Rician ($K = 2$) fading links are assumed.

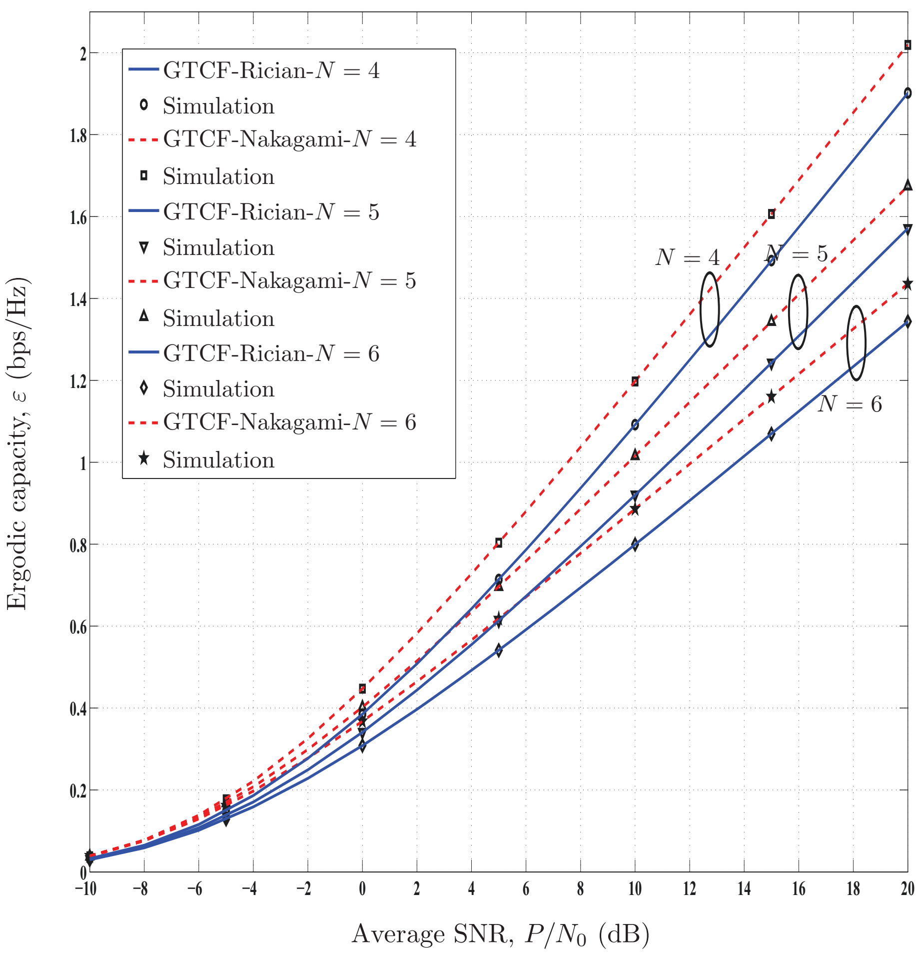

Figure 7.8: The ergodic capacity for multihop ($N = 4, 5$ and 6) AF relaying systems. Identically distributed Nakagami-m ($m = 2.5$) and *i.i.d.* Rician ($K = 2$) fading links are assumed.

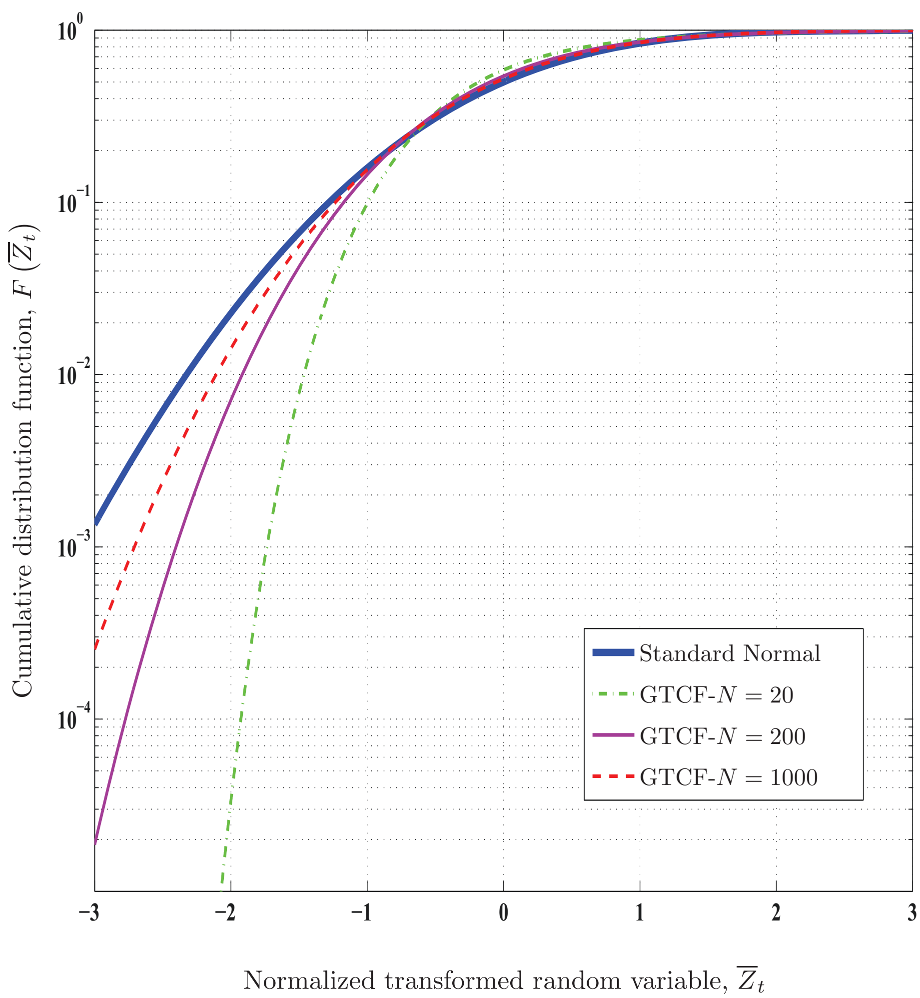

Figure 7.9: The CDF of the random variable $\overline{Z}_t$ obtained using the GTCF method for $N = 20, 200$ and 1000. Nakagami-m fading links with $m = 2.5$ and $\overline{\gamma} = 20$ dB are assumed. The CDF of the standard normal distribution is shown for comparison.

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Chapter 8

The Modified-GTCF Method for Multi-Branch Multihop AF Systems

In this chapter[(1)], a modification to the new transform method, the generalized transformed characteristic function (GTCF), is proposed to reduce its computational complexity. The modified GTCF (M-GTCF) gives exact solutions, in single-integral form, for the probability density function and the cumulative distribution function of the end-to-end received signal-to-noise ratio (SNR) of multihop amplify-and-forward (AF) relaying systems. The M-GTCF method is applied to obtain exact, integral solutions for the average symbol error probability and outage probability of multi-branch, multihop AF relaying systems with full selection diversity. The results are precise for any number of hops, N, any number of branches, L, and can accommodate any channel fading distribution. The GTCF method can provide exact solution for such systems. However, the M-GTCF method provides the first exact theoretical solution for such systems with complexity less than the GTCF method.

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S. S. Soliman and N. C. Beaulieu, "The modified GTCF method and its application to multi-branch multihop relaying systems with full selection diversity," in *6th Joint IFIP Wireless and Mobile Networking Conf. (WMNC), 2013*, Apr. 2013, pp. 1–6.

8.1 Introduction

Cooperative networks have attracted the focus of many researchers as well as industry because such networks promise large diversity gain and increased capacity compared to other wireless systems [1]–[12]. Much research has concentrated on AF cooperative relaying due to its advantages in terms of complexity and power consumption.

The authors in [1] presented an approximation for the end-to-end received signal-to-noise ratio (SNR) using the harmonic mean of two independent exponential random variables in order to study the outage probability and the average error probability of dual-hop systems operating over Rayleigh channels. In [2], the authors used the harmonic mean bound to obtain an approximation of the outage probability of multihop systems, operating over Nakagami-m fading channels, using a numerical inversion of the Laplace transform of the moment generating function (MGF) of the reciprocal of the end-to-end SNR. Meanwhile, it has been found that bounds based on the direct harmonic mean approximation of the individual per hop instantaneous SNRs, are neither tight for small-to-moderate values of SNR, nor for Nakagami-m fading channels for large values of m [3,4]. In [5] the authors used a scaling factor to reduce approximation errors, and presented an approximate solution for the performance metrics of multihop AF systems. That solution has the same computational complexity as some previous results in the literature, while being more accurate, especially for small-to-medium values of SNR.

In [6], the authors examined the problem of opportunistic communications in dual-hop AF relaying systems, where they considered the presence of the direct path link between the source and the destination, and used maximum ratio combining (MRC) at the receiver for combining signals from the direct path and the best dual-hop path. However, the authors used the harmonic mean approximation for the end-to-end SNR, and therefore did not obtain exact solutions for the performance metrics of such systems. The authors in [7] analyzed the same system in [6], but they used the minimum SNR bound for their analysis to obtain bounds to the performance metrics rather than exact results. Reference [8] also used the minimum SNR bound to get bounds to the performance metrics of dual-hop systems with relay selection, but they proposed a K^{th} opportunistic relaying scheme, where the K^{th}

best relay is selected to relay the radio signal, rather than the best relay selection scheme. Note that, while the authors in [6] examined systems operating over Nakagami-m fading channels, the analysis in [7] and [8] was applied only to systems operating over Rayleigh fading channels.

The authors in [1]–[8] presented bounds and/or approximations to the different systems' performance metrics, however, accurate performance analysis of cooperative diversity systems is important, as it enables the design of networks for maximum utility and performance as well as minimum cost. In [9], the authors derived exact closed-form expressions for the PDF and the CDF of the end-to-end SNR of dual-hop AF relaying systems with full selection diversity. Full selection diversity was defined in [9] for relaying systems where a direct path between the source and the destination exists. Exact results on the average symbol error probability, outage probability and the ergodic capacity were obtained in [9] for systems operating over Nakagami-m fading links with integer values of the fading parameter m. In [10]–[12], a new analytical approach, the generalized transformed characteristic function (GTCF), was proposed to obtain the first exact analytical results for the average error probability, outage probability and ergodic capacity for multihop AF relaying systems operating over general fading channels. The GTCF is different from the conventional MGF approaches. It uses a new transform domain to solve a family of problems whose exact solutions were intractable using the common probability density approaches or the MGF approaches. It was shown in [11] that performance bounds based on the harmonic mean approximation for the end-to-end SNR is only acceptable for large values of SNR, and becomes dramatically inaccurate for small-to-medium values of SNR. This emphasizes the value of exact analysis of multihop AF systems. Note that the use of transform methods to solve difficult problems in a transform domain is a key approach in wireless communication theory (e.g. Fourier and Laplace transforms) and other recent work [13,14] has contributed new transform methods. The work herein presents a novel transform technique that can be used to solve heretofore difficult problems in cooperative wireless networks.

In this chapter, we propose a modification to the GTCF method to enhance its computational complexity. For multihop AF relaying systems operating over arbitrary Nakagami-m or Rician fading links, the GTCF method was used to obtain two-fold integral expressions for the outage probability, average symbol error probability and the ergodic capacity [12, Table I]. For systems involving selection diversity, the complexity of the GTCF method will increase to three-fold integral calculation. The modified GTCF (M-GTCF) proposed in this chapter reduces the computational complexity of the outage probability to a single-fold semi-infinite integral for multihop AF systems, as well as AF systems involving selection diversity such as multi-branch multihop AF systems with full selection diversity. The M-GTCF method also reduces the computational complexity of the average error probability of systems with diversity selection to two-fold integration rather than three-fold integration.

The main contributions of this chapter are: 1) Proposing the M-GTCF method with reduced computational complexity compared to the GTCF method and compared to direct evaluation methods. 2) Using the M-GTCF method to obtain exact, single-integral solution for the outage probability of multihop AF systems. 3) Using the M-GTCF method to obtain the first exact, integral solutions for the outage probability and the average symbol error probability of multi-branch multihop AF relaying systems operating over Nakagami-m and Rician fading links. 4) Studying the effects of the fading parameters, the number of hops per branch, and the number of branches on the system performance metrics. To the best of the authors' knowledge, no exact results for performance metrics have been reported in the literature for systems operating over Nakagami-m fading channels with arbitrary fading parameters, nor for systems operating over Rician fading channels. Note that although it is commonly known that Rician fading channels can be modeled by Nakagami-m channels, however, it was shown and proved in [15] that this is not always true.

The remainder of the chapter is organized as follows. In Section 8.2, the modified GTCF method is proposed and its computational complexity is explained. In Section 8.3, an application of the M-GTCF method is discussed. Multi-branch multihop AF relaying systems with full selection diversity are analyzed in Section 8.3 using the M-GTCF method, and the first exact solutions for the outage probability and the average symbol error probability are obtained. Numerical examples are presented in Section 8.4 to study the behaviour of multi-branch multihop AF systems. The effects of the fading parameters, number of hops per branch and number of branches on the system performance are also studied. Finally, the conclusions of the chapter are drawn in Section 8.5.

8.2 The Modified GTCF Method

The GTCF method was proposed in [10]–[12] to obtain the exact statistics of the instantaneous end-to-end SNR of multihop AF relaying systems. It was shown in [11] that for the common channel fading distributions, Rayleigh, Nakagami-m and Rician, the GTCF has a fixed complexity of 2-fold numerical integration to obtain the outage probability and the average symbol error probability of AF systems with number of hops $N > 2$. The complexity of using the GTCF method increases for more complicated systems such as multihop multi-branch AF systems with full selection diversity. In this section, we propose the modified GTCF (M-GTCF) method which employes a modification that reduces the computational complexity of the GTCF method and permits obtaining exact solutions for the outage probability and the average error probability of complicated relaying system with less computational complexity.

The steps of the GTCF method were shown in [12, eq. (2)]. The M-GTCF method starts with the first three steps of the GTCF method [12, eqs. (2a–2c)] to obtain the characteristic function of the end-to-end transformed random variable Z_t in terms of the characteristic functions of the transformed random variables Z_n. The characteristic functions, $\Phi_{Z_n}(\omega)$, of the transformed random variables, Z_n, were obtained in [11] in closed-form expressions for the common channel fading distributions, Nakagami-m [11, eq. (17)] and Rician [11, eq. (21)]. The M-GTCF then continues as follows:

4. Find the CDF of the end-to-end transformed random variable Z_t:

$$F_{z_t}(r) = \frac{1}{2} - \frac{1}{\pi} \int_0^{\infty} \frac{1}{\omega} \Im\left[\Phi_{Z_t}(\omega) e^{-j\omega r}\right] d\omega \tag{8.1a}$$

   where $\Im[h(x)]$ is the imaginary part of $h(x)$.

5. Find the CDF of the end-to-end SNR γ_t:

$$F_{\gamma_t}(r) = 1 - F_{z_t}\left(\ln\left(1 + \frac{1}{r}\right)\right). \tag{8.1b}$$

Note that eq. (8.1a) is obtained using Gil-Pelaez inversion theorem [16]. In the worst case, when the integral in (8.1a) is not solved in closed-form, the CDF of the end-to-end SNR γ_t can be obtained in a single-fold integral form by substituting (8.1a) into (8.1b) resulting in

$$F_{\gamma_t}(r) = \frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} \frac{1}{\omega}\, \Im\left[\Phi_{Z_t}(\omega)\left(1+\frac{1}{r}\right)^{-j\omega}\right] d\omega. \tag{8.2}$$

The PDF of the end-to-end SNR γ_t can be then obtained by differentiating the CDF expression in (8.2). Note that the outage probability, $P_{out}(\gamma_{th}) = F_{\gamma_t}(\gamma_{th})$, of multihop AF systems can be obtained by direct substitution into (8.2). This shows the advantage of the M-GTCF method over the GTCF method, where the outage probability is obtained by a single-fold semi-infinite integral rather than a double-fold integral as provided by the GTCF method.

8.3 Multi-Branch Multihop AF Systems

In this section, the M-GTCF method is used to obtain exact solutions for the outage probability and the average symbol error probability of multi-branch multihop AF systems with full selection diversity. The system under study is shown in Fig. 8.1. The communication between the source node, S, and the destination node, D, can be through a single-hop direct path from S to D, or through one of the available L multihop branches. For the direct path, $S - D$, the source node transmits the radio signal directly, in a single-hop, to the destination node. In this case, the end-to-end SNR is given as $\gamma_0 = \frac{P}{N_0}|\alpha_0|^2$, where P is the total available power for transmission, N_0 is the noise power at the destination node, and α_0 is the fading gain of the direct path link. For any of the L multihop branches, the source node transmits the radio signal to the destination through N AF relays.

For variable gain AF relaying, the exact instantaneous received SNR at the destination through any of the multihop branches, γ_l, is given by [1, eq. (2)]

$$\gamma_l = \left[\prod_{n=1}^{N}\left(1+\frac{1}{\gamma_n}\right) - 1\right]^{-1} \tag{8.3}$$

where $\gamma_n = \frac{P_{n-1}}{N_{0n}}|\alpha_n|^2,\ n = 1, 2, \cdots, N$ represents the instantaneous received SNR over the

n^{th} link, where $P_n,\ n = 1, 2, \cdots, N-1$, is the transmitter power at the n^{th} relay, and where $\alpha_n,\ n = 1, 2, \cdots, N$, and $N_{0n},\ n = 1, 2, \cdots, N$, are respectively the fading gain of the n^{th} link, and the noise power at the n^{th} node.

The channel conditions are estimated at the receiver, and a decision is taken to select one of the potential $L + 1$ branches for the radio signal transmission. The branch with the maximum end-to-end SNR is selected. If the direct link branch is selected, the signal is transmitted directly from the source to the destination using the total available power per transmission, P. If one of the multihop branches is selected, the total power is distributed to be used among the source and the relay nodes to transmit and relay the signal to the destination. We assume that all the link fadings are statistically independent, and that they follow Nakagami-m or Rician fading distributions. Note that the Rayleigh fading distribution is a special case of the Nakagami-m or the Rician fading distribution. Without loss of generality, we assume the preliminary case of independent and identically distributed (*i.i.d.*) branches. The probability density functions (PDFs) and the cumulative distribution function (CDFs) of the common fading distributions can be found in [17].

The communication branch is selected such that the instantaneous end-to-end SNR is maximized, i.e.

$$\max\left\{\gamma_{_0}, \gamma_{_m}\right\} \Rightarrow \gamma_{_t} \tag{8.4a}$$

where γ_0 is the end-to-end SNR of the direct path, and where

$$\max_{l\in\{1,2,\cdots,L\}} \gamma_{_l} \Rightarrow \gamma_{_m} \tag{8.4b}$$

where $\gamma_{_l}, n = 1, 2, \cdots, L$ is the end-to-end SNR of any multihop branch, defined in (8.3). The PDF, $f_{\gamma_{_l}}(r)$, and the CDF, $F_{\gamma_{_l}}(r)$, of the SNR of the multihop branches, $\gamma_{_l}$, can be obtained for using the M-GTCF method by numerical calculation of a single-fold integral, as explained in the previous section.

Since $\gamma_{_m}$ is selected as the maximum of the L *i.i.d.* candidate multihop branch SNRs, $\gamma_{_l}$, as in (8.4b), the PDF of $\gamma_{_m}$ can be written as,

$$f_{\gamma_{_m}}(r) = L f_{\gamma_{_l}}(r)\left[F_{\gamma_{_l}}(r)\right]^{L-1} \tag{8.5}$$

and the CDF of γ_m can be written as,

$$F_{\gamma_m}(r) = \left[F_{\gamma_l}(r)\right]^L. \tag{8.6}$$

With the knowledge of the PDFs, $f_{\gamma_0}(r)$ and $f_{\gamma_m}(r)$, and the CDF, $F_{\gamma_0}(r)$ and $F_{\gamma_m}(r)$, of γ_0 and γ_m, respectively, order statistics can be used to obtain the PDF and the CDF of the end-to-end SNR, γ_t, [18, p. 192] as

$$f_{\gamma_t}(r) = f_{\gamma_0}(r)F_{\gamma_m}(r) + F_{\gamma_0}(r)f_{\gamma_m}(r) \tag{8.7}$$

and

$$F_{\gamma_t}(r) = F_{\gamma_0}(r)F_{\gamma_m}(r). \tag{8.8}$$

The expressions for the PDF in (8.7) and the CDF in (8.8) represent the first exact, single-integral expressions obtained for γ_t defined in (8.4). The outage probability of multi-branch multihop systems with full selection diversity can be then obtained through direct substitution into (8.8), while the average symbol error probability can be obtained by substituting (8.7) into [17, eq. (5.1)].

8.4 Numerical Examples

In this section, the outage probability and the average error probability are used to study the behaviour of multi-branch multihop AF systems with full selection diversity. Binary phase shift keying (BPSK) is assumed as well as an uniform power allocation policy, in the case of multihop transmission, i.e. the total available power, P, is evenly allocated to the source and the relays. We assume also equal noise powers, N_0, at all the nodes. The relays are assumed evenly located between the source and the destination and normalized distances are assumed, i.e. $d_{hop} = \frac{1}{N}$, so according to the Friss propagation model [19], the average SNR over any link is given as $\overline{\gamma}_i = N^{\delta-1}\frac{P}{N_0}$ where $N = 1$ in the case of the direct path, $N \geq 2$ in the case of multihop paths, and δ is the path loss exponent, taken as 3. According to these assumptions, $\overline{\gamma}_l = N^2 \times \overline{\gamma}_0$, where $\overline{\gamma}_0$ is the average SNR of the direct path.

The first example set studies multi-branch multihop AF systems, with $L = 3$ branches, each of $N = 4$ hops, operating over Nakagami-m fading links. Figs. 8.2 and 8.3 show the average symbol error probability, P_s, versus the direct link average SNR, $\overline{\gamma}_0$, and the outage probability, P_{out}, at $\overline{\gamma}_0 = 10$ dB, versus the threshold SNR, γ_{th}, respectively. Without loss of generality, independent and identically distributed (*i.i.d.*) Nakagami-m fading channels, with different values of the fading parameter m, are assumed. Note that the steps of the M-GTCF method do not require the fading links to be *i.i.d.*, and the computational complexity is not affected by this condition. Figs. 8.2 and 8.3 show that results obtained by the M-GTCF method are in precise agreement with the Monte Carlo simulation results, for different values of the Nakagami-m parameter. As expected, it can be seen in the figures that the system performance improves as the fading parameter m increases. It can also be seen that the limiting slopes of the average error probability curves and the outage probability curves are proportional to the fading parameter m. This results in a remarkable improvement in the system performance. For example, the average symbol error probability, at an average SNR of $\overline{\gamma}_0 = 5$ dB, for the case of Rayleigh fading links is 7.47×10^{-4}, while it is 7.37×10^{-5} and 5.022×10^{-6} for Nakagami-m links with $m = 1.7$ and $m = 3.4$, respectively.

The second example set studies multi-branch multihop AF systems operating over Rician fading links. Figs. 8.4 and 8.5 show the average error probability and the outage probability, respectively. Independent and identically distributed Rician fading links, with different values of the line-of-sight (LOS) Rician parameter K are considered. As previously, the results show precise agreement between the results obtained by the M-GTCF method and the simulation results. As expected, the system performance improves as the Rician parameter K increases. For example, an outage probability of 10^{-3} occurs at $\gamma_{th} = 6.42$ dB in the case of Rayleigh fading links, while it occurs at $\gamma_{th} = 8.13$ dB in the case of $K = 1.5$ and at $\gamma_{th} = 11.54$ dB and $\gamma_{th} = 12.86$ dB for $K = 4.2$ and $K = 6.7$, respectively.

The next example studies the effect of the number of hops, N, per branch on the system performance metrics. Fig. 8.6 shows the outage probability at $\overline{\gamma}_0 = 10$ dB, versus the threshold SNR, γ_{th}, for multi-branch systems with $L = 2$ and different values of the number of hops per branch $N = 2, 3$ and 4. Nakagami-m links with $m = 1.7$ and Rician links with $K = 4.2$ are considered. Fig. 8.6 shows that the outage probability of the system improves as the number of hops per branch increases. For example, the outage probability of systems operating over Nakagami-m links, at $\gamma_{th} = 8$ dB, is 1.18×10^{-2} for $N = 2$, while it is 2.63×10^{-3} and 7.02×10^{-4} for $N = 3$ and $N = 4$, respectively. However, it can be seen from the figure that the limiting slopes of the outage probability curve are not affected by the increase in the number of hops per branch. In contrast, an increase in the number of hops per branch results in an SNR gain. For example, an outage probability of 10^{-3} occurs at $\gamma_{th} = 7.25$ dB in the case of Rician fading links and $N = 2$, while it occurs at $\gamma_{th} = 8.62$ dB in the case of $N = 3$ and at $\gamma_{th} = 9.64$ dB in the case of $N = 4$.

The last example studies the effect of the number of potential branches on the system performance. Fig. (8.7) shows the average symbol error probability versus the direct link average SNR, γ_0, for multihop systems with $N = 4$ and different numbers of branches $L = 1, 2$ and 3. Nakagami-m links with $m = 3.4$ and Rician links with $K = 1.5$ are considered. Fig. 8.7 shows that the average error probability of the system improves as the number of branches increases. For example, the average error probability of systems operating over Rician fading channels, at $\overline{\gamma}_0 = 6$ dB is 2.89×10^{-3} for the case of $L = 1$ branch, while it is 3.95×10^{-4} and 8.13×10^{-5} for $L = 2$ and $L = 3$, respectively. Moreover, it can be seen from the figure that the limiting slopes of the average error probability curves are affected by the number of branches. This behaviour is different from the systems' behaviour with respect to the number of hops per branch as shown in the previous example. This behaviour results in a remarkable improvement in the average error probability with an increase in the number of branches. For example, an average error probability of 10^{-4} occurs at $\overline{\gamma}_0 = 7.39$ dB in the case of Nakagami-m fading links and $L = 1$, while it occurs at $\overline{\gamma}_0 = 5.37$ dB and $\overline{\gamma}_0 = 4.62$ dB in the cases of $L = 2$ and $L = 3$, respectively.

8.5 Conclusion

A new theoretical method, the modified generalized transformed characteristic function, is proposed as a modification in the use of the newly proposed transform domain to reduce the computational complexity of the generalized transformed characteristic function. The M-GTCF method provides exact, single-integral solutions for the outage probability of multihop AF systems. The M-GTCF method was used to obtain the first exact results for the average symbol error probability and the outage probability of multi-branch multihop amplify-and-forward relaying systems with full selection diversity. The method can be used for arbitrary link fading distributions. It was shown that the results obtained from the M-GTCF method are in precise agreement with Monte Carlo simulation results, for the common fading distributions with various values of the fading parameters. It was observed that although the limiting slopes of the average symbol error probability and the outage probability curves are not affected by the number of hops per branch, they are directly proportional to the number of potential branches.

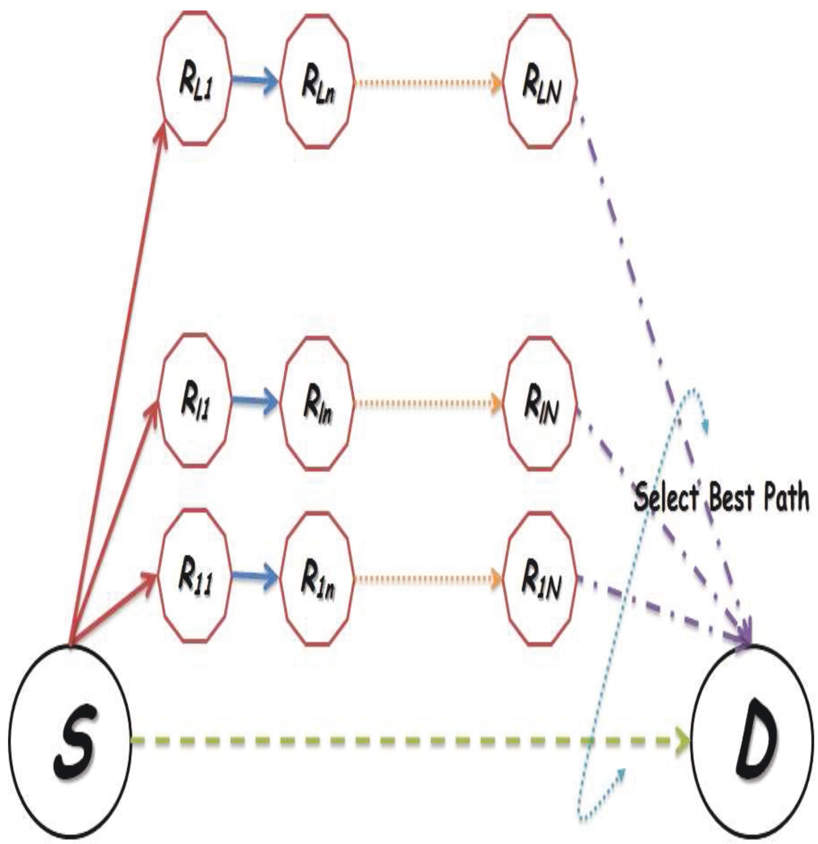

Figure 8.1: Multi-branch, multihop amplify-and-forward relaying network with full route selection diversity. Either the direct path, $S-D$, is selected for direct data signal transmission or one of the L multihop branches is selected to relay the data signal between the source, S, and the destination, D.

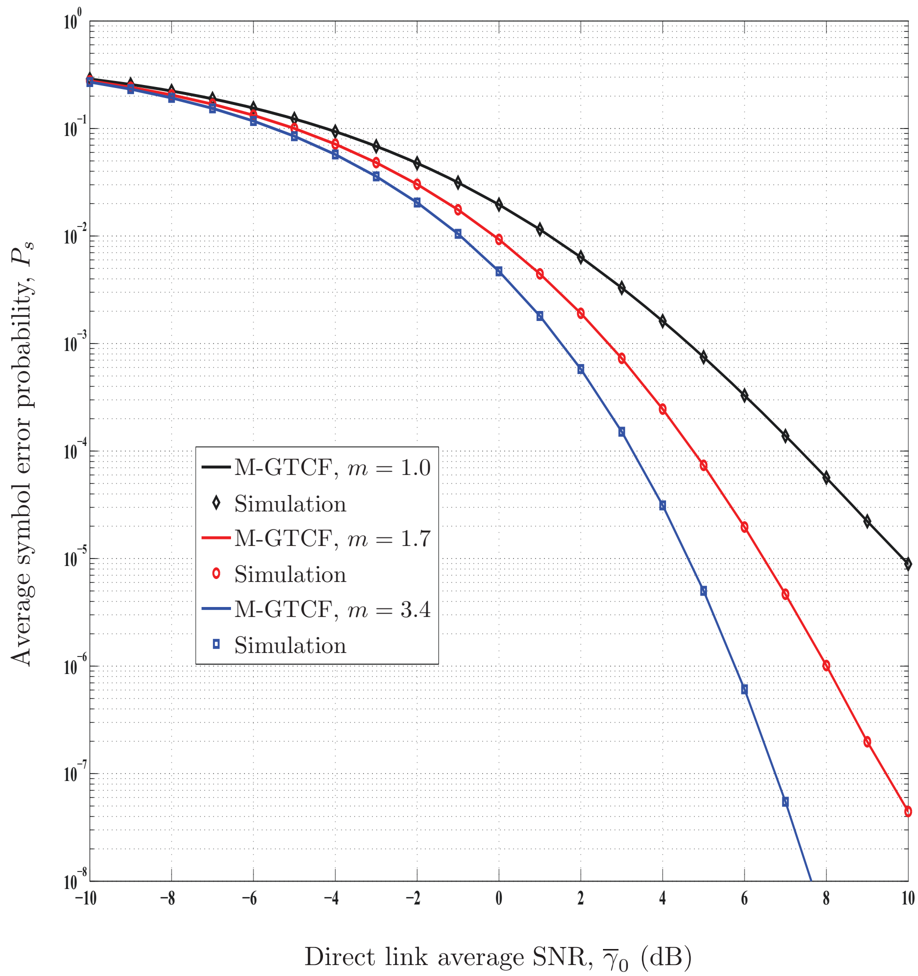

Figure 8.2: The average symbol error probability for multi-branch ($L = 3$), multihop ($N = 4$) AF relaying systems with BPSK modulation. Rayleigh and identically distributed Nakagami-m fading links are assumed.

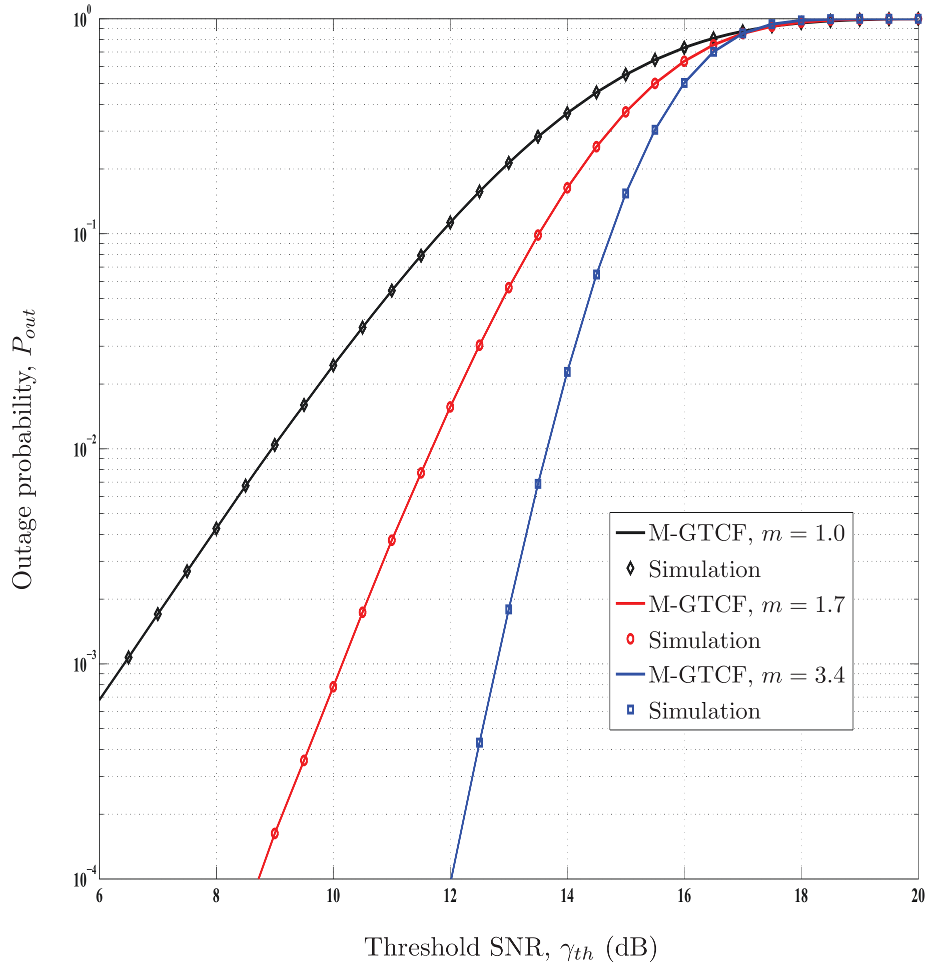

Figure 8.3: The outage probability for multi-branch ($L = 3$), multihop ($N = 4$) AF relaying systems at $\overline{\gamma}_0 = 10$ dB. Rayleigh and identically distributed Nakagami-m fading links are assumed.

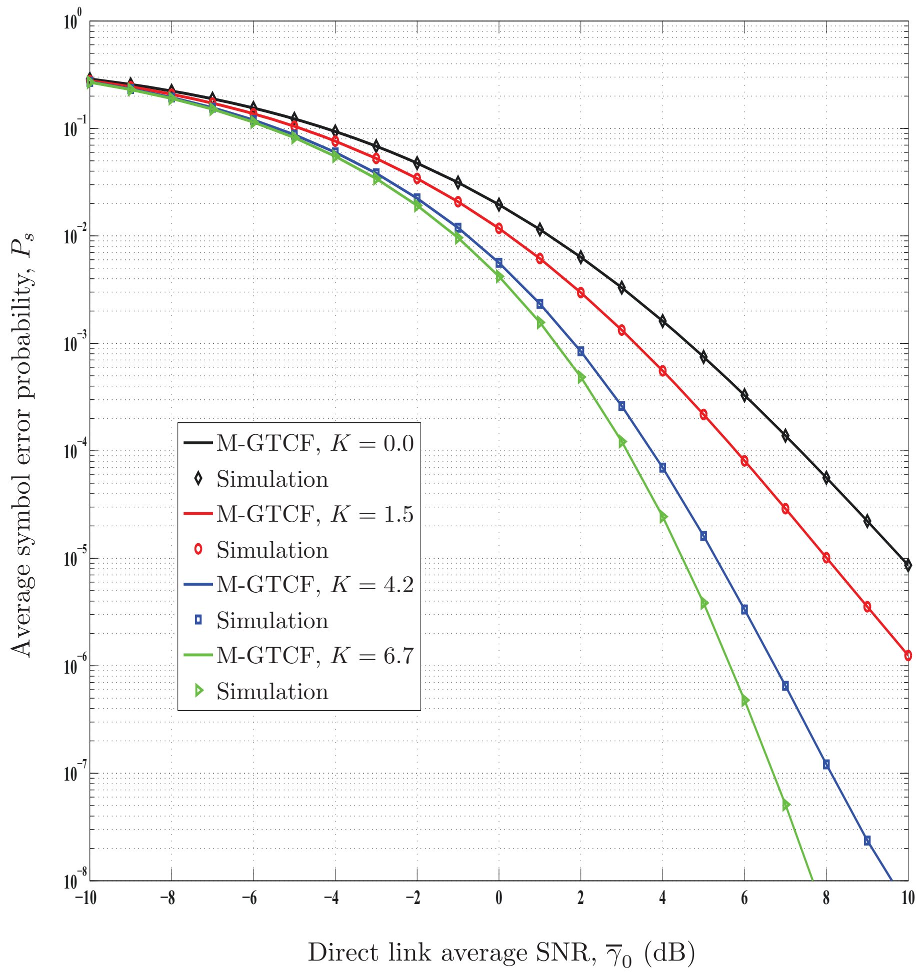

Figure 8.4: The average symbol error probability for multi-branch ($L = 3$), multihop ($N = 4$) AF relaying systems with BPSK modulation. Rayleigh and identically distributed Rician fading links are assumed.

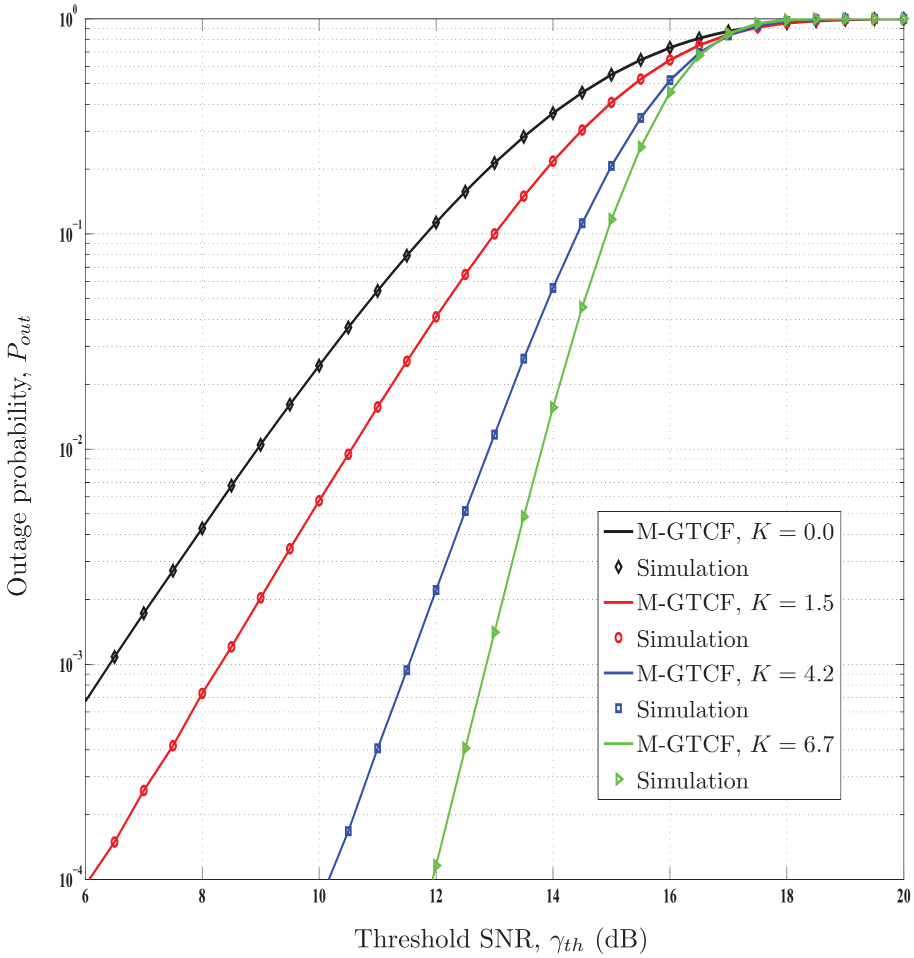

Figure 8.5: The outage probability for multi-branch ($L = 3$), multihop ($N = 4$) AF relaying systems at $\overline{\gamma}_0 = 10$ dB. Rayleigh and identically distributed Rician fading links are assumed.

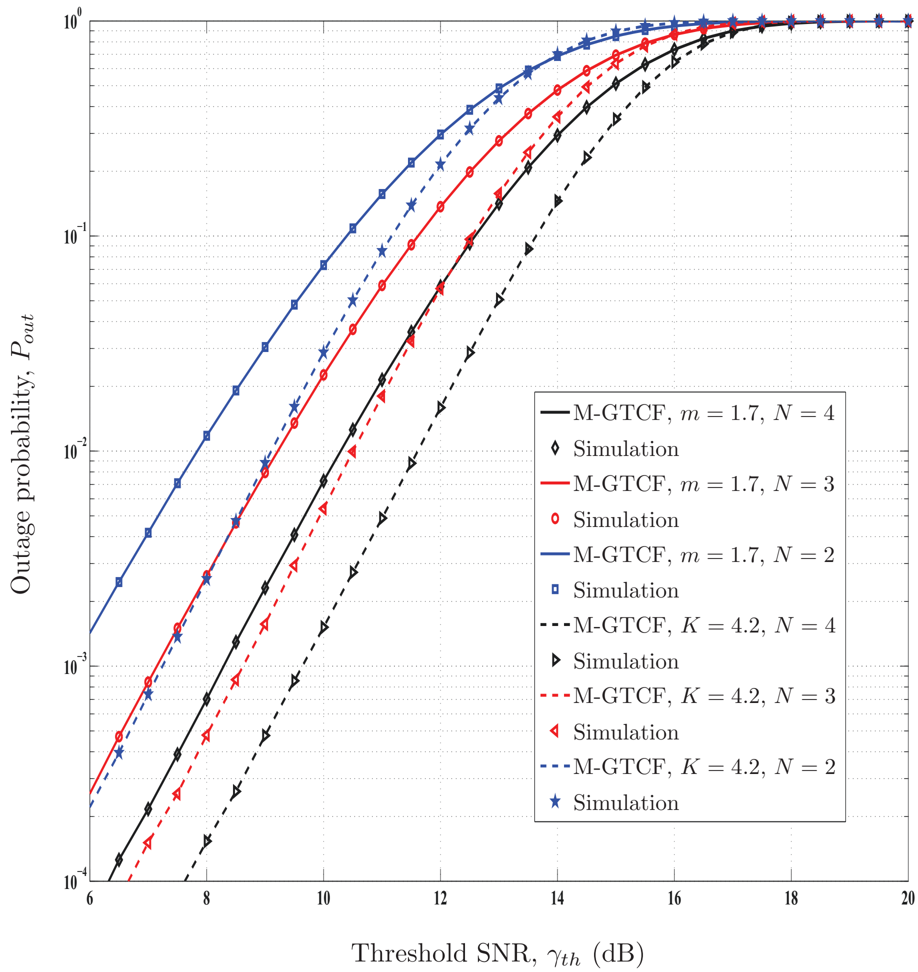

Figure 8.6: The outage probability for multi-branch ($L = 2$), multihop ($N = 2, 3$ and 4) AF relaying systems at $\overline{\gamma}_0 = 10$ dB. Identically distributed Nakagami-m, with $m = 1.7$, and identically distributed Rician, with $K = 4.2$, fading links are assumed.

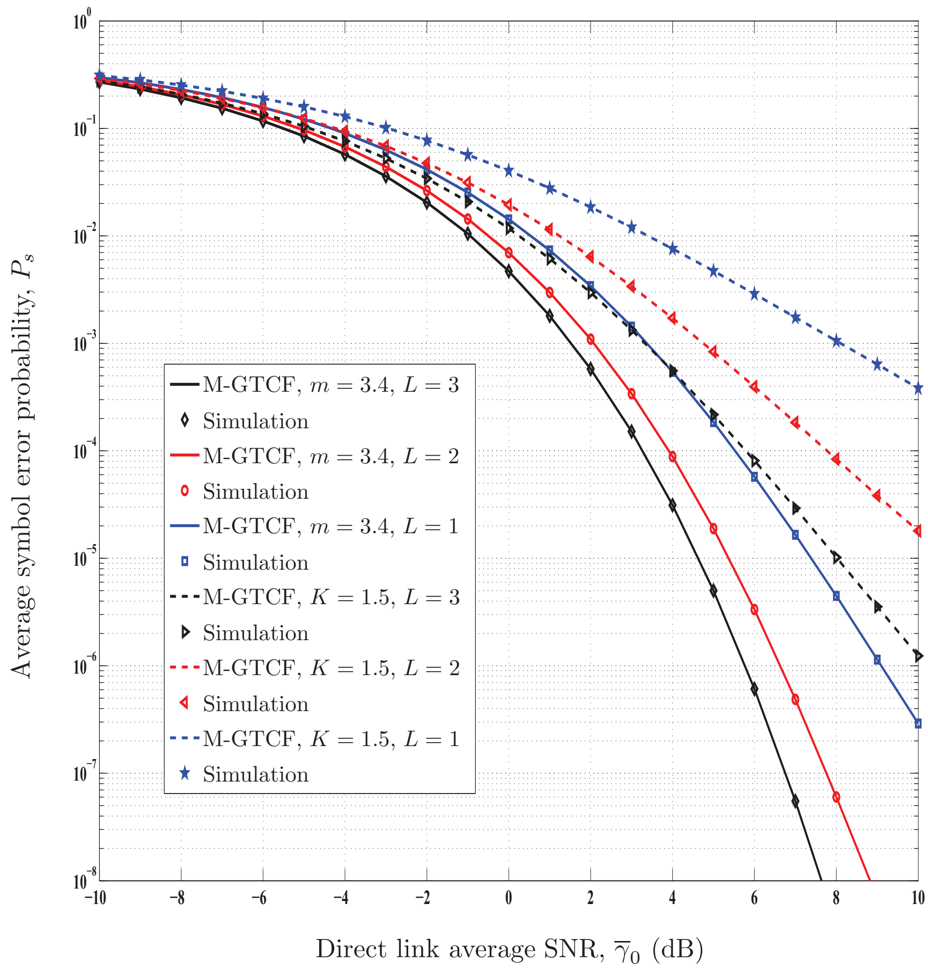

Figure 8.7: The average symbol error probability for multi-branch ($L = 1, 2$ and 3), multihop ($N = 4$) AF relaying systems with BPSK modulation. Identically distributed Nakagami-m, with $m = 3.4$, and identically distributed Rician, with $K = 1.5$, fading links are assumed.

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Chapter 9

Dual-Hop vs Multihop AF Relaying Systems

In this chapter[(1)], dual-hop amplify-and-forward (AF) systems with relay selection are compared to multihop AF systems. Closed-form expression for the outage probability of dual-hop amplify-and-forward (AF) systems with relay selection from N independent and non-identically (*i.ni.d.*) distributed dual-hop branches is obtained. The probability density function of the end-to-end signal-to-noise ratio (SNR) is also obtained and is used to precisely obtain the average symbol error probability for systems operating over *i.ni.d.* Nakagami-m fading links. The modified generalized transformed characteristic function (M-GTCF) is used to analyze multihop AF relaying systems operating over N *i.ni.d.* Nakagami-m fading links. In comparison of systems under study, it is shown that dual-hop AF systems can significantly outperform multihop AF systems. These findings are used to propose criteria for practical design of wireless cooperative systems.

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9.1 Introduction

Wireless cooperative communication systems have been one of the main interests for researchers for a decade [1]–[10]. Amplify-and-forward (AF) relaying systems have gained special interest because of the advantages of AF relaying in terms of power consumption and complexity. The authors in [1] and [2] presented a harmonic mean approximation for the end-to-end received signal-to-noise ratio (SNR) to study bounds to the outage probability and the average error probability of dual-hop systems operating over Rayleigh and Nakagami-m fading channels. However, it has been found that bounds based on the direct harmonic mean approximation of the individual per hop instantaneous SNRs, are neither tight for small-to-moderate values of SNR, nor for Nakagami-m fading channels for large values of m [3].

In [4], the authors used maximum ratio combining (MRC) to combine signals from the direct path, between the source and the destination, and the best dual-hop path. They used the harmonic mean bound to obtain bounds to the performance metrics of such systems. Reference [5] used a different bound, the minimum SNR bound, to get bounds to the performance metrics of dual-hop systems with relay selection, but they proposed a K^{th} opportunistic relaying scheme, where the K^{th} best relay is selected to relay the radio signal, rather than the best relay selection scheme. Note that, while the authors in [4] examined systems operating over Nakagami-m fading channels, the analysis in [5] was applied only to systems operating over Rayleigh fading channels.

The authors in [1]–[5] presented bounds and/or approximations to the different systems' performance metrics, however, accurate performance analysis of cooperative diversity systems is important, as it enables the design of networks for maximum utility and performance as well as minimum cost.

In [6], the authors derived closed-form expressions for the probability density function, cumulative distribution function and the moment generating function (MGF) of dual-hop AF relaying systems operating over Nakagami-m fading links. However, the results in [6] were not used to obtain the exact performance metrics of variable-gain AF relaying systems. In [7], the authors derived exact expressions for the probability density function and the cumulative distribution function of dual-hop AF systems operating over Rician fading links, as well as dual-hop AF systems with partial relay selection. The results in [7] were used to obtain the exact performance metrics of such systems.

In [8]–[10], a new analytical approach, the generalized transformed characteristic function (GTCF), was proposed to obtain the first exact analytical results for the average error probability, outage probability and ergodic capacity for multihop AF relaying systems operating over general fading channels. The GTCF is different from the conventional MGF approaches. It uses a new transform domain to solve a family of problems whose exact solutions were intractable using the common probability density approaches or the moment generating function approaches. Note that the use of transform methods to solve difficult problems in a transform domain is a key approach in wireless communication theory (e.g. Fourier and Laplace transforms) and other recent work [11, 12] has contributed new transform methods. The work in [8]–[10] presents a novel transform technique that can be used to solve heretofore difficult problems in cooperative wireless networks. It was shown in [9] that performance bounds based on the harmonic mean approximation for the end-to-end SNR are only acceptable for large values of SNR, and become dramatically inaccurate for small-to-medium values of SNR. This emphasizes the value of exact analysis of multihop AF systems.

In this chapter, we use the expressions obtained in [6] to obtain the first, exact-closed form results of the outage probability of dual-hop AF systems with relay selection from N available relays operating over Nakagami-m fading links. An exact closed-form expression of the PDF of the end-to-end SNR of such systems is also obtained and is used to obtain the average symbol error probability of these systems. The results in this chapter are obtained for the most general case of independent but non-identically distributed candidate branches, as well as independent but non-identically distributed source-to-relay and relay-to-destination links per dual-hop branch. In [13, 14] the authors studied only the special case of systems operating over independent and identically distributed fading links, and consequently, a full study about the effects of the channel fading parameters was not presented therein. We study the effects of the fading parameters as well as the number of available relays, N, on the systems performance. In this chapter also we use a modified version of the GTCF [8]–[10] to obtain the performance metrics of multihop AF relaying systems operating over independent but non-identically distributed fading links. The modified-GTCF method is used to obtain the performance metrics of multihop AF systems using reduced computational complexity than that needed by the GTCF. The major contribution of this chapter is in presenting the first comparison between multihop AF systems and dual-hop AF systems with relay selection based on fair basis of comparison. The motivation of such comparison is to find the dependence of the performance metrics of both groups of systems on the numbers of relays as well as the links fading parameters, and to use such comparison to propose multiple criteria for the practical design of AF cooperative systems.

9.2 Dual-Hop AF Systems With Relay Selection

Dual-hop AF systems with relay selection from N available relays are shown in Fig. 9.1. The source node, S, and the destination node, D, communicate through a selected relay of the available N relay nodes. The relay node is selected based on local measurements of the instantaneous channel conditions. A maximum end-to-end relay selection policy is adopted, where the relay which achieves the maximum source-to-destination SNR is selected to amplify the signal received from the source, and forward the amplified radio signal to the destination. The amplification factor, A, at the relay node, R, is given in [15] as $A = \sqrt{\frac{P_1}{P_0|\alpha_1|^2+N_{0_1}}}$, where P_0 is the transmitter power at the source node, P_1 is the transmitter power at the relay node, $\alpha_i,\ i=1,2$ is the fading gain of the i^{th} link and where N_{0_1} and N_{0_2} are respectively, the noise power at the relay node and the destination node. Half-duplex operation is implemented to avoid inter-signal interference. For each dual-hop branch, the exact instantaneous end-to-end received SNR at the destination, γ_{S-R_n-D}, is given by [1, eq. (2)]

$$\gamma_{S-R_n-D} = \frac{\gamma_{S-R_n}\gamma_{R_n-D}}{\gamma_{S-R_n}+\gamma_{R_n-D}+1} \tag{9.1a}$$

where γ_{S-R_n} is the instantaneous SNR of the $S-R_n$ link between the source and the n^{th} relay, and γ_{R_n-D} is the instantaneous SNR of the R_n-D link between the n^{th} relay and the destination.

The instantaneous received SNR over any of those links is defined as

$$\gamma_i = \frac{P_{i-1}}{N_{0_i}} |\alpha_i|^2 . \tag{9.1b}$$

Exact closed-form expressions for the PDF and the CDF of the SNR, γ_{S-R_n-D}, of dual-hop AF systems were obtained in [6] for systems operating over Nakagami-m fading links, and in [7] for systems operating over Rician fading links. For the case of Nakagami-m links, the PDF, $f_{\gamma_{S-R_n-D}}(r)$, of γ_{S-R_n-D} can be obtained from [6, eq. (4)], and the CDF, $F_{\gamma_{S-R_n-D}}(r)$, of γ_{S-R_n-D} can be obtained from [6, eq. (2)]. According to the maximum end-to-end relay selection policy taken into consideration, the total end-to-end instantaneous SNR of dual-hop AF systems with relay selection can be obtained as

$$\gamma_t = \max_{R_n \in \{R_1, \cdots, R_N\}} \gamma_{S-R_n-D} \tag{9.2}$$

Assuming the most general case that the available source-to-destination branches are independent but non-identically distributed (*i.ni.d.*), order statistics [16] can be used to obtain the PDF of the total end-to-end SNR, γ_t, as

$$f_{\gamma_t}(r) = \sum_{i=1}^{N} f_i(r) \prod_{\substack{j=1 \\ j \neq i}}^{N} F_j(r) \tag{9.3}$$

where $f_i(r)$ and $F_i(r)$ are, respectively, the PDF and the CDF of the i^{th} source-to-destination branch. The CDF of the total end-to-end SNR, γ_t, can be also obtained as

$$F_{\gamma_t}(r) = \prod_{j=1}^{N} F_j(r). \tag{9.4}$$

With the PDF and the CDF of the end-to-end SNR known, the performance metrics of the system can be evaluated. The average symbol error probability, P_s, is obtained as [17, eq. (5.1)]

$$P_s = E\left\{b\, Q\left(\sqrt{a\,\gamma_t}\right)\right\} = \int_0^\infty b\, Q\left(\sqrt{a\,r}\right) f_{\gamma_t}(r) dr \tag{9.5}$$

where $Q(\cdot)$ is the Gaussian Q-function [17], a and b depend on the modulation scheme.

The outage probability, P_{0ut}, can be also obtained as [17, eq. (15.6)]

$$P_{out} = Pr\left\{\gamma_t \leqslant \gamma_{th}\right\} = \int_0^{\gamma_{th}} f_{\gamma_t}(r)dr = F_{\gamma_t}(\gamma_{th}) \tag{9.6}$$

where γ_{th} is a threshold SNR, below which the system is considered in outage. Note that the result in (9.6) represents the first exact, closed-form solution for the outage probability of dual-hop AF systems with relay selection operating over *i.ni.d.* Nakagami-m fading branches. The exact average symbol error probability of these systems is also obtained by numerically evaluating the single-fold integral in (9.5) using common software packages.

In the following examples, we use the analytical result in (9.6) to study the behaviour of dual-hop AF systems with relay selection, and the dependence of the system performance on the number of available relays, N, in the selection pool as well as on the values of the fading parameters.

The first example studies the effect of the number of the available relays, N, in the relay selection pool on the performance of dual-hop AF systems. Fig. 9.2 shows the outage probability of dual-hop AF systems with relay selection from N available relays, versus the threshold SNR, γ_{th}. It is assumed the N possible dual-hop branches are independent but non-identically distributed (*i.ni.d.*). It is assumed also in this example that for each dual-hop branch, the source-to-relay link and the relay-to-destination link are independent and identically distributed (*i.i.d.*), i.e. $m_{1i} = m_{2i} = m_i$.

The figure shows the curves for the outage probabilities, at $\overline{\gamma} = 10$ dB, of different groups of systems with different numbers of available relays, N, and different fading parameters, m_i. The systems parameters for different systems are shown in Table 9.1. Similar total available power is assumed for all systems to achieve fair basis of comparison.

It can be observed from the figure that there is a precise agreement between the analytical results and the Monte Carlo results. Moreover, It can be observed that the limiting slopes of the outage probability curves are not proportional to the number of relays available in the selection pool as would be expected. Rather, the limiting slopes of the outage probability curves are proportional to the sum of the Nakagami-m parameters of the N available branches, $\sum m_i$. For example, the first group of systems shown in Fig. 9.2 is a group of 3

Table 9.1: Values of the fading parameters for the links of systems in Fig. 9.2

Links Parameters	Group 1	Group 2	Group 3
$N = 2$ (m_1,m_2)	$(4, 2)$	$(3, 2)$	$(3, 1)$
$N = 3$ (m_1,m_2,m_3)	$(2, 1, 3)$	$(2, 2, 1)$	$(2, 1, 1)$
$N = 4$ (m_1,m_2,m_3,m_4)	$(1, 2, 2, 1)$	$(2, 1, 1, 1)$	$(1, 1, 1, 1)$
$\sum m_i$	6	5	4

systems, with different numbers of available relays, N, yet the limiting slopes of their outage probability curves are the same because each of them has $\sum m_i = 6$. Moreover, it can be observed that irrespective the number of available branches, and irrespective the individual values of the fading parameters of the links in each branch, the system performance is dependent only on $\sum m_i$.

The next example emphasizes the previous observation and adds more insight into it. Fig. 9.3 shows the outage probability of dual-hop AF systems with relay selection from $N = 3$ available relays. For this example, the source-to-relay link and the relay-to-destination link are independent and non-identically distributed (*i.ni.d.*), i.e. $m_{1i} \neq m_{2i}$. The values of the Nakagami-m parameter of all $N = 3$ possible pairs of links are shown in the legend of Fig. 9.3. It can be observed that the limiting slopes of the outage probability curve is proportional to the sum of the minimums of the fading parameters of the source-to-relay and relay-to-destination links of each branch, i.e. $\sum_i \min\{m_{1i}, m_{2i}\}$ where $i = 1, 2, \cdots, N$. The values of the fading parameters other than the minimum ones result in an SNR gain. For example, the outage probability curve of dual-hop AF systems with fading parameters $(2, 2)-(1, 3)-(3, 4)$ has a limiting slope equal to that of dual-hop AF systems with $(2, 4)-(2, 3)-(2, 5)$, because both have $\sum_i \min\{m_{1i}, m_{2i}\} = 6$. However, the latter system has an SNR gain advantage because it is operating over less sever fading links.

It can be concluded then that the presence of fewer but less sever available links results in a more significant improvement in the system performance compared to the presence of more available links with smaller values of the fading parameters, m_i. Moreover, for each available dual-hop branch, the weaker link represents a bottleneck for the system performance. These conclusions leads to important design criteria that can be used by designers of practical communication systems.

9.3 The M-GTCF Method For Multihop AF Systems

This section studies the performance metrics of multihop AF systems, shown in Fig. 9.4, operating over independent but non-identically distributed (*i.ni.d.*) Nakagami-m fading links. The exact instantaneous end-to-end SNR of multihop AF systems, with N relays, can be obtained as

$$\gamma_t = \left[\prod_{n=1}^{N+1}\left(1+\frac{1}{\gamma_n}\right)-1\right]^{-1} \tag{9.7}$$

where $\gamma_n,\ n=1,2,\cdots,N+1$ represents the instantaneous received SNR over the n^{th} link.

The generalized transformed characteristic function (GTCF) method was proposed in [8]–[10] to obtain the exact statistics of the instantaneous end-to-end SNR of multihop AF relaying systems. The main idea of the GTCF approach is that the expression in (9.7) can be rewritten as, $\left(1+\frac{1}{\gamma_t}\right)=\prod_{n=1}^{N+1}\left(1+\frac{1}{\gamma_n}\right)$. Then, taking the logarithm of both sides of this equation results in $Z_t=\sum_{n=1}^{N+1} Z_n$, Assuming that $\gamma_n,\ n=1,2,\cdots,N+1$ are statistically independent, then the transformed random variables, Z_n, are statistically independent as well [18, p. 244]. It was shown in [9] that for the common channel fading distributions, Rayleigh, Nakagami-m and Rician, the GTCF has a fixed complexity of 2-fold numerical integration to obtain the outage probability and the average symbol error probability of AF systems with number of hops $N>2$. This complexity can be further reduced to a single-fold numerical integration through introducing a modification to the GTCF method. The steps of the modified GTCF (M-GTCF) can be summarized as follows:

1. Find the PDF of the transformed random variables Z_n:

$$f_{z_n}(r)=\frac{e^r}{(e^r-1)^2}f_{\gamma_n}\left(\frac{1}{e^r-1}\right). \tag{9.8a}$$

2. Find the CHF of the transformed random variables Z_n:

$$\Phi_{Z_n}(\omega) = \int_0^\infty f_{z_n}(r) e^{j\omega r} dr. \tag{9.8b}$$

3. Find the CHF of the end-to-end transformed random variable Z_t:

$$\Phi_{Z_t}(\omega) = \prod_{n=1}^{N+1} \Phi_{Z_n}(\omega). \tag{9.8c}$$

4. Find the CDF of the end-to-end transformed random variable Z_t:

$$F_{z_t}(r) = \frac{1}{2} - \frac{1}{\pi} \int_0^\infty \frac{1}{\omega} \Im\left[\Phi_{Z_t}(\omega) e^{-j\omega r}\right] d\omega \tag{9.8d}$$

where $\Im[h(x)]$ is the imaginary part of $h(x)$, and (9.8d) is obtained using Gil-Pelaez inversion theorem [19].

5. Find the CDF of the end-to-end SNR γ_t:

$$F_{\gamma_t}(r) = 1 - F_{z_t}\left(\ln\left(1 + \frac{1}{r}\right)\right). \tag{9.8e}$$

For Nakagami-m fading links, the CHF of the transformed random variable, Z_n, was obtained as [9]

$$\Phi_{Z_n}(\omega) = \frac{\Gamma(m_n - j\omega)}{\Gamma(m_n)}\, U\left(-j\omega,\, 1 - m_n,\, \frac{m_n}{\overline{\gamma}_n}\right). \tag{9.9}$$

where $U(a, b, z)$ is the Tricomi confluent hypergeometric function defined in [20, eq. (13.2.5)]. The outage probability of multihop AF systems can be then obtained by substituting (9.9) into (9.8c), and then into (9.8d) and (9.8e), respectively, resulting in

$$F_{\gamma_t}(r) = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \frac{1}{\omega} \Im\left[\Phi_{Z_t}(\omega)\left(1 + \frac{1}{r}\right)^{-j\omega}\right] d\omega. \tag{9.10}$$

Differentiating the expression in (9.10), the PDF of the end-to-end SNR of multihop AF systems is obtained and can be used with (9.5) to obtain the exact average symbol error probability of multihop AF relaying systems.

9.4 Dual-Hop With Relay Selection vs Multihop AF Systems

In this section, we use the results obtained in Section 9.2 and in Section 9.3 to compare dual-hop AF systems with relay selection to multihop AF systems, through series of examples. The focus of the following examples will be on the outage probability and the average error probability. It is known that while dual-hop AF systems require only 2 time intervals to fulfill a complete source-to-destination transmission, multihop AF systems with, N intermediate relays, require $N+1$ time intervals. That results in a higher ergodic capacity of dual-hop AF systems than that of multihop AF systems. Figs. 9.5 and 9.6 show respectively the outage probability, at $\frac{P}{N_0} = 10$ dB, versus the threshold SNR, γ_{th}, and the average symbol error probability versus the average SNR, $\frac{P}{N_0}$, for different groups of dual-hop and multihop AF relaying systems.

The first group of systems, denoted "EP" in Figs. 9.5 and 9.6, is a group of multihop AF relaying systems, where the total available power is equally distributed to the source node and the N intermediate relaying nodes. We assume equal noise powers, N_0, at all the nodes. The relays are assumed evenly located between the source and the destination and normalized distances are assumed, i.e. $d_{hop} = \frac{1}{N+1}$. According to the Friss propagation model [21], the average SNR over any link is given as $\overline{\gamma}_i = (N+1)^{\delta-1}\frac{P}{N_0}$ for $i = 1, 2, \cdots, N+1$ where N is the number of available relays in the multihop path, and where δ is the path loss exponent, taken equal to 3 in the following examples.

The second group of systems is a group of dual-hop AF relaying systems with relay selection from N available relays. The total power is equally distributed among the source node and the selected relay. Thus half the total power is used by the source node to broadcast the data signal to the relays, and the selected best relay uses half the total power to forward the radio signal to the destination node. It is assumed also that the N candidate relays are equally located from the source and the destination, so according to the Friss propagation model, the average SNR over both the source-to-relay link and the relay-to-destination link is given as $\overline{\gamma}_i = (2)^{\delta-1}\frac{P}{N_0}$ for $i = 1, 2$. It is assumed that the source-to-relay and the relay-to-destination distances can be covered by signals from the source and the relay.

The third group of systems, denoted "HP" in Figs. 9.5 and 9.6, is a group of multihop AF relaying systems where the total available power is distributed such that the source node uses half of the total available power to broadcast the radio signal, and the remaining power is equally distributed among the N intermediate relays to relay the signal to the destination. With the assumption of equal noise powers at all nodes, and evenly distributed relays between the source and the destination, the average SNR of the first link is given as $\overline{\gamma}_1 = \frac{1}{2}(N+1)^{\delta}\frac{P}{N_0}$, while the average SNR of the subsequent links is given as $\overline{\gamma}_i = \frac{1}{2}(N+1)^{\delta}(N)^{-1}\frac{P}{N_0}$ for $i = 2, 3, \cdots, N+1$. For this example, it is assumed that all the systems are operating over Rayleigh fading links. For the average symbol error probability, binary phase shift keying (BPSK) is assumed. For each of the three groups of systems, the system performance is tested for numbers of relays $N = 2, 3$ and 4. For fair comparison, it is assumed that the total available power, P, is kept fixed for all systems and scenarios. Monte Carlo simulation results for all systems are also shown in Figs. 9.5 and 9.6.

It can be observed that there is precise agreement between the analytical results and the Monte Carlo simulation results for all multihop and dual-hop systems. It can be observed also that for all numbers N of the intermediate relays, multihop AF relaying systems where the power is equally distributed among the source node and the relay nodes outperform multihop AF relaying systems where half the total power is dedicated to the source and the remaining part is equally distributed to the relay nodes. For example, it can be seen in Fig. 9.5 that for $N = 4$, the outage probability of the later system is 3.25×10^{-2} at a threshold SNR of $\gamma_{th} = 0$ dB, while it is 2.32×10^{-2} for multihop systems with equal power distribution. It can be observed also that both the average symbol error probability and the outage probability are improved with larger numbers of hops, $N + 1$. For example, Fig. 9.6 shows that for multihop AF relaying systems with equal power distribution, the average symbol error probability, at $\frac{P}{N_0} = 20$ dB, is 8.61×10^{-4} for $N = 2$, while it is 5.11×10^{-4} for $N = 4$. However, it can be observed that limiting slopes of the outage probability curves and the average symbol error probability curves for both groups of multihop systems are the same, irrespective the number of hops, $N + 1$.

It can be then concluded from the previous observations that multihop AF relaying systems with equal power distribution among the available nodes outperform multihop AF systems with other power distributions. This conclusion is valid with the assumption of equal noise powers at all nodes. It can be concluded also that the performance improvement associated with the increase in the number of hops is limited to an SNR gain.

On the other hand, it can be observed from Figs. 9.5 and 9.6 that dual-hop AF systems with relay selection from N available relays significantly outperform multihop AF systems with N relays. For example, the outage probability at a threshold SNR of $\gamma_{th} = 0$ is 2.32×10^{-2} for multihop AF systems with equal power distribution and $N = 4$ relays, while an outage probability of 1.02×10^{-5} occurs at the same threshold SNR for dual-hop AF systems with relay selection from $N = 4$ available relays. Moreover, it can be observed that, in contrast with multihop AF systems, the limiting slopes of the outage probability curves and the average symbol error probability curves are dependent on the number of available relays in the selection pool, N. This complies with the conclusions drawn from Figs. 9.2 and 9.3, that the limiting slopes of the outage probability curves and the average symbol error probability curves are proportional to $\sum_i \min\{m_{1i}, m_{2i}\}$ where $i = 1, 2, \cdots, N$. In case of Rayleigh fading links, $m_j = 1$ and the limiting slopes are proportional to $\sum_i \min\{m_{1i}, m_{2i}\} = N$. This results in a significantly remarkable improvement in the performance of dual-hop AF systems with the increase in the number of available relays, N, in the selection pool. For example, the average symbol error probability of dual-hop AF systems with relay selection, at $\frac{P}{N_0} = 16$ dB, is 6.71×10^{-5} for $N = 2$, while it is 2.36×10^{-6} and 9.43×10^{-8} for $N = 3$ and $N = 4$, respectively. It can be concluded then that for Rayleigh fading links, for a certain number of available relay nodes and fixed source and destination, planning the system in the form of a dual-hop AF system with best relay selection from the available relays, if the communication radii permit that, results in a better performance than planning it in the form of a multihop AF system. Of course there is an overhead for the selection procedure, but assuming centric selection at the destination with sufficient power resources, the selection overhead becomes negligible compared to the improved system performance. This represents another useful design and planning criteria for wireless cooperative systems. It can be concluded also that adding more relays to the selection pool of dual-hop systems with relay selection remarkably enhances the system performance compared to adding relays to the single multihop branch of multihop systems.

In the last example, we compare the system performance of multihop AF systems with equal power distribution to the source node and the N relay nodes, i.e. $\overline{\gamma}_i = (N+1)^{\delta-1}\frac{P}{N_0}$ for $i = 1, 2, \cdots, N+1$, to the performance of dual-hop AF systems with relay selection from N available relays, when all systems are operating over independent but non-identically distributed Nakagami-m links. Fig. 9.7 shows the average symbol error probability of these systems with $N = 2, 3$ and 4 for different values of the fading parameters, m_i, as shown in the legend of Fig. 9.7. For dual-hop AF systems, it is assumed that for every dual-hop branch, the source-to-relay link and the relay-to-destination link are independent and identically distributed, i,e, $m_{i1} = m_{i2} = m_i$. The results shown in Fig. 9.7 emphasize the previous observations. The first group of curves is for 3 multihop AF systems with fading parameters $m_1 = 1, m_2 = 2, m_3 = 1$, $m_1 = 1, m_2 = 2, m_3 = 1, m_4 = 3$, and $m_1 = 1, m_2 = 2, m_3 = 1, m_4 = 3, m_5 = 6$, respectively. It can be observed that the limiting slopes of the average symbol error probability curves for all three systems are the same and similar to the limiting slopes of the average error probability curves of multihop AF systems operating over Rayleigh fading links in Fig. 9.6. This is because the limiting slope of the average symbol error probability curve in the case of multihop AF relaying systems operating over Nakagami-m fading links is proportional to the minimum of the fading parameters of the individual links, i.e. $\min\{m_i\}$. Thus, for the first group of systems, the limiting slopes of the average error probability curves are proportional to $\min\{m_i\} = 1$. The second group of systems are multihop AF systems with fading parameters $m_1 = 2, m_2 = 3, m_3 = 4$, $m_1 = 2, m_2 = 3, m_3 = 4, m_4 = 5$, and $m_1 = 2, m_2 = 3, m_3 = 4, m_4 = 5, m_5 = 6$, respectively. It can be seen from Fig. 9.7 that the limiting slopes of the average error probability curves, for this group of systems, are proportional to $\min\{m_i\} = 2$.

The third and fourth groups of systems are dual-hop AF systems with relay selection, operating over Nakagami-m fading links of parameter similar to those for the first and second groups of systems, respectively. It can be observed that the limiting slopes for the average error probability curves for these systems is proportional to $\sum\{m_i\}$ rather than $\min\{m_i\}$ in the case of multihop AF systems. This emphasizes the practical benefit of dual-hop AF systems over multihop AF systems. For example, for a multihop AF system with $N = 2$ relays, operating over Nakagami-m fading links with parameters $m_1 = 2, m_2 = 3, m_3 = 4$, the average symbol error probability, at $\frac{P}{N_0} = 14$ dB, is 1.68×10^{-5} and the limiting slope of the average error probability curve is $\min\{m_i\} = 2$. On the other hand, for a dual-hop AF system with $N = 2$ relays, operating over Nakagami-m fading links with parameters $m_1 = 2, m_2 = 3$, the average symbol error probability, at $\frac{P}{N_0} = 14$ dB, is 7.61×10^{-8} and the limiting slope of the average error probability curve is $\sum\{m_i\} = 5$. Similar observations can be drawn for the remaining systems. It can be concluded then that dual-hop AF systems with relay selection have a diversity order advantage over multihop AF systems, resulting from the selection among multiple branches. This advantage exists for systems operating over Rayleigh as well as Nakagami-m fading links, and it results in a significant performance enhancement in favor of dual-hop AF systems with relay selection, compared to multihop AF systems with same number of relays and same link fading parameters. It can be concluded also that while the weakest link, of lowest value of the fading parameter, m, is a bottleneck for the performance improvement in the case of multihop AF systems, because the diversity order is proportional to $\min\{m_i\}$, this bottleneck does not exist for dual-hop AF systems with relay selection because the contribution of the weakest branch is overridden by other branches in the selection pool.

- **Remarks**

Note that the comparisons presented in this Chapter are preliminary investigations about the relative performance of systems under comparison. Further investigations would include the effects of multiple factors that were not addressed in this Chapter. These factors include, but not limited to,

1. The selection overhead added to relay selection systems.
2. The effects of outdated channel estimations.
3. The relative locations of the available relay nodes with respect to the source and destination nodes.

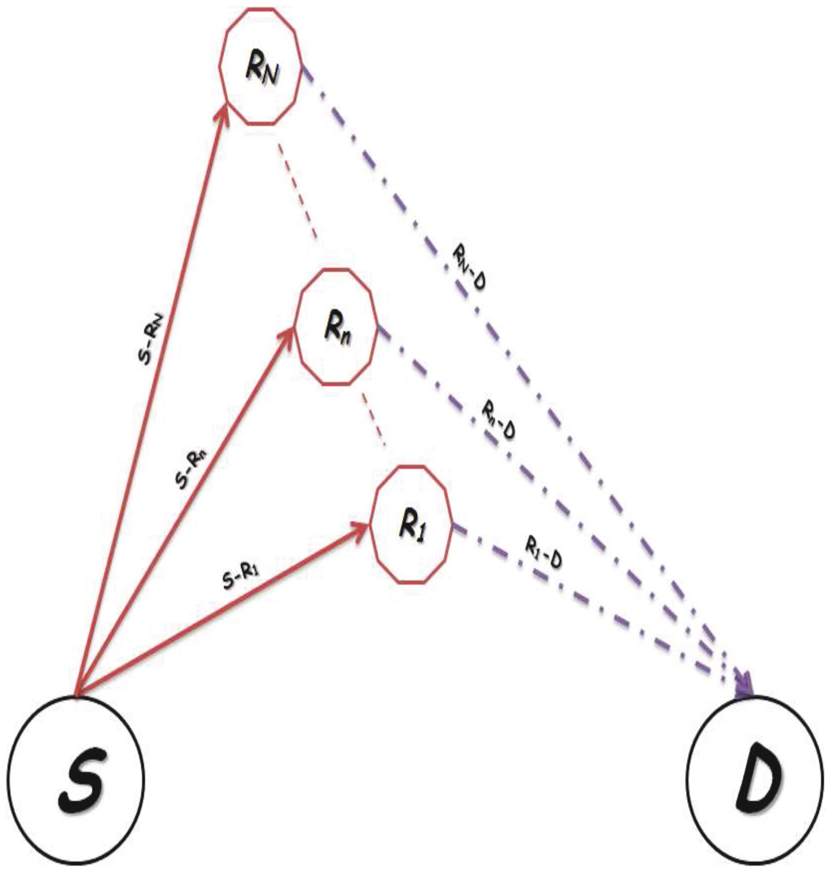

Figure 9.1: Dual-hop amplify-and-forward relaying network. One of the N relays, R_1, R_2, $\cdots$, R_N is selected to relay the data signal between the source, S, and the destination, D.

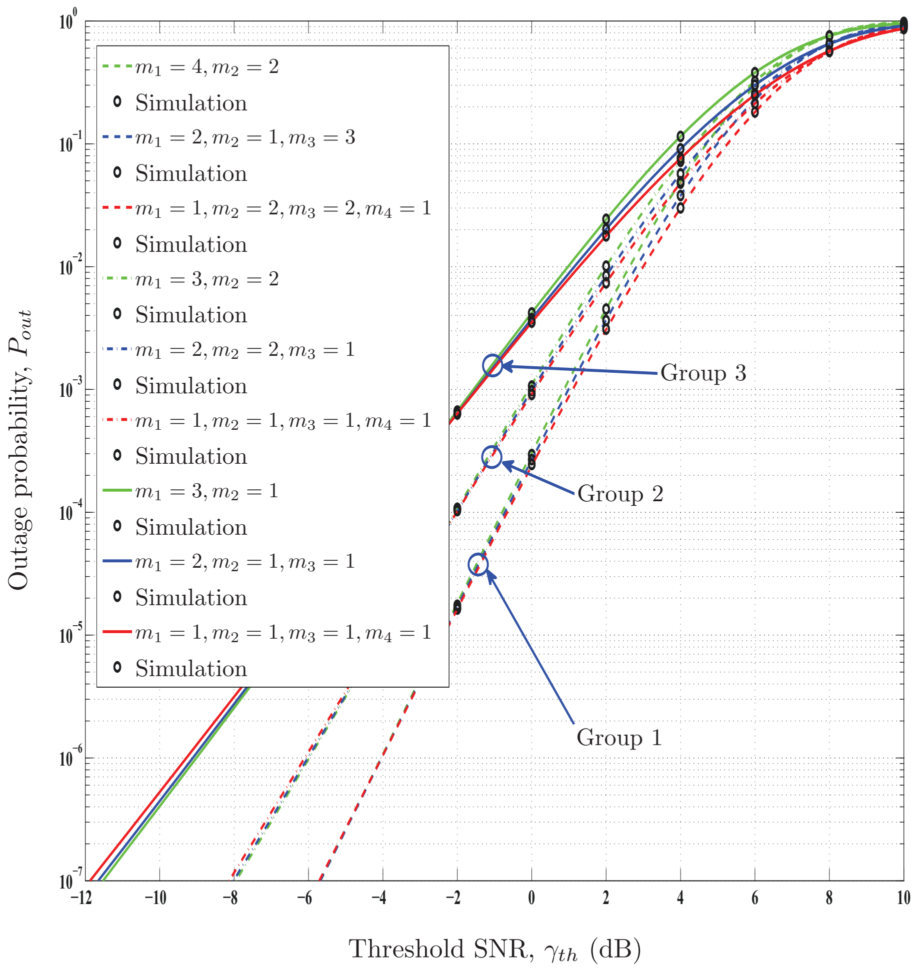

Figure 9.2: The outage probability, at $\overline{\gamma} = 10$ dB, for dual-hop AF relaying systems with relay selection from $N = 2, 3$ and 4 available relays. Identically distributed Nakagami-m links per branch, but non-identically distributed branches are assumed.

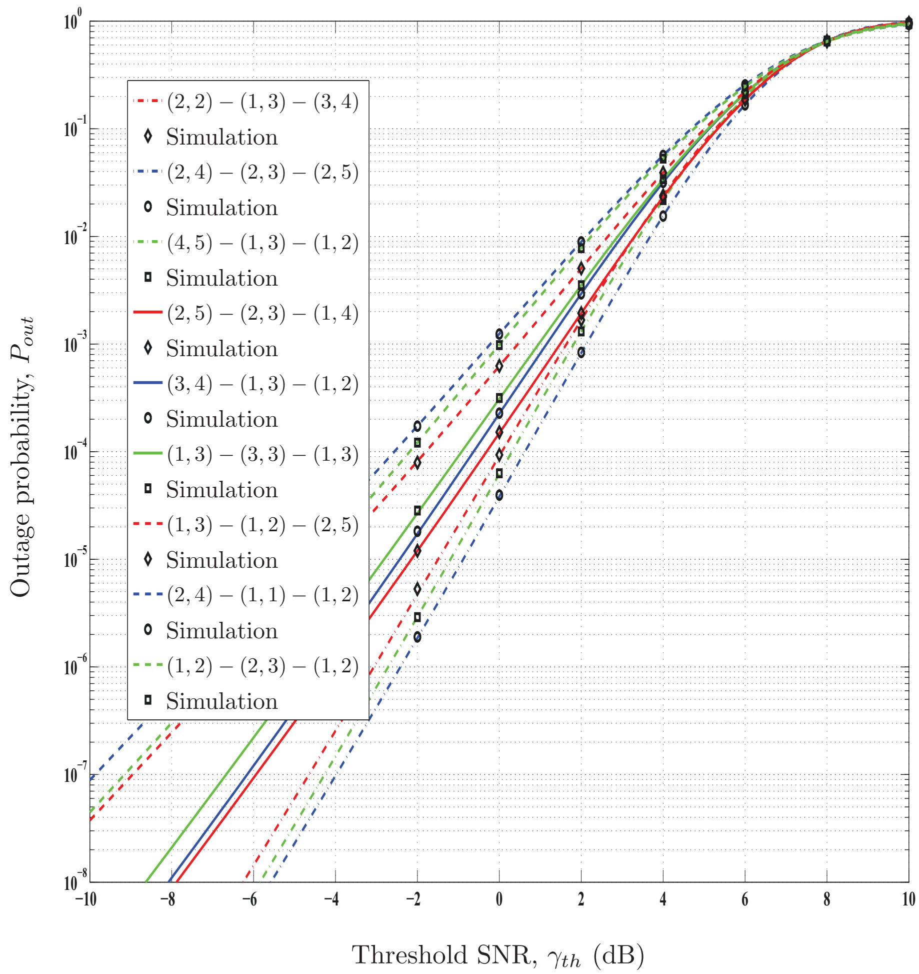

Figure 9.3: The outage probability, at $\overline{\gamma} = 10$ dB, for dual-hop AF relaying systems with relay selection from $N = 3$ available relays. Non-identically distributed Nakagami-m links per branch, and non-identically distributed branches are assumed.

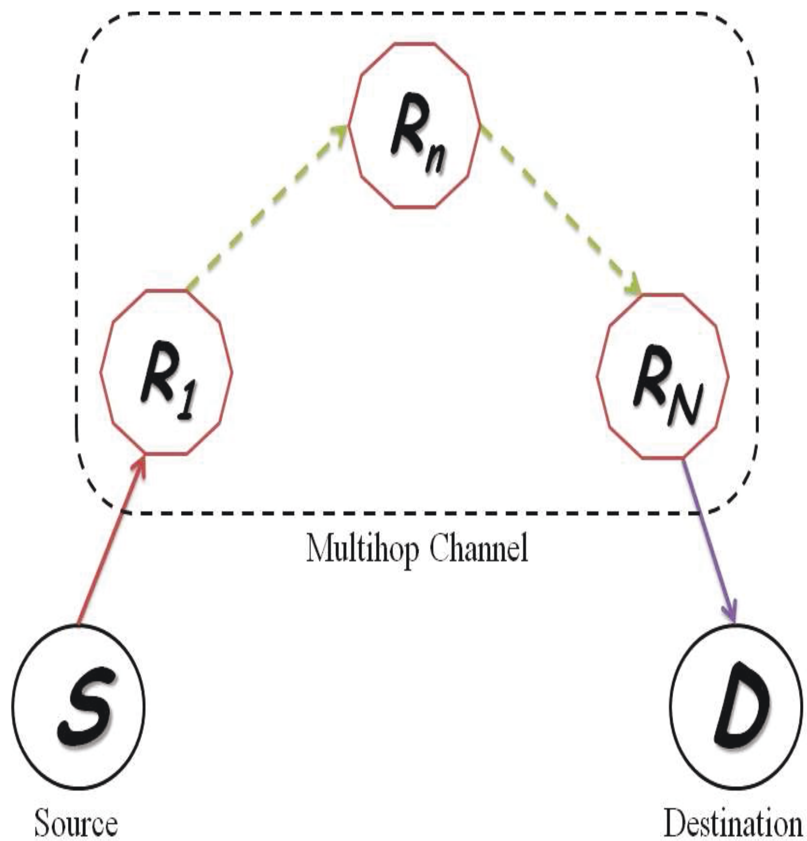

Figure 9.4: Multihop amplify-and-forward relaying network. The source, S, and the destination, D, communicate through N intermediate relay nodes, R_1, R_2, $\cdots$, R_N.

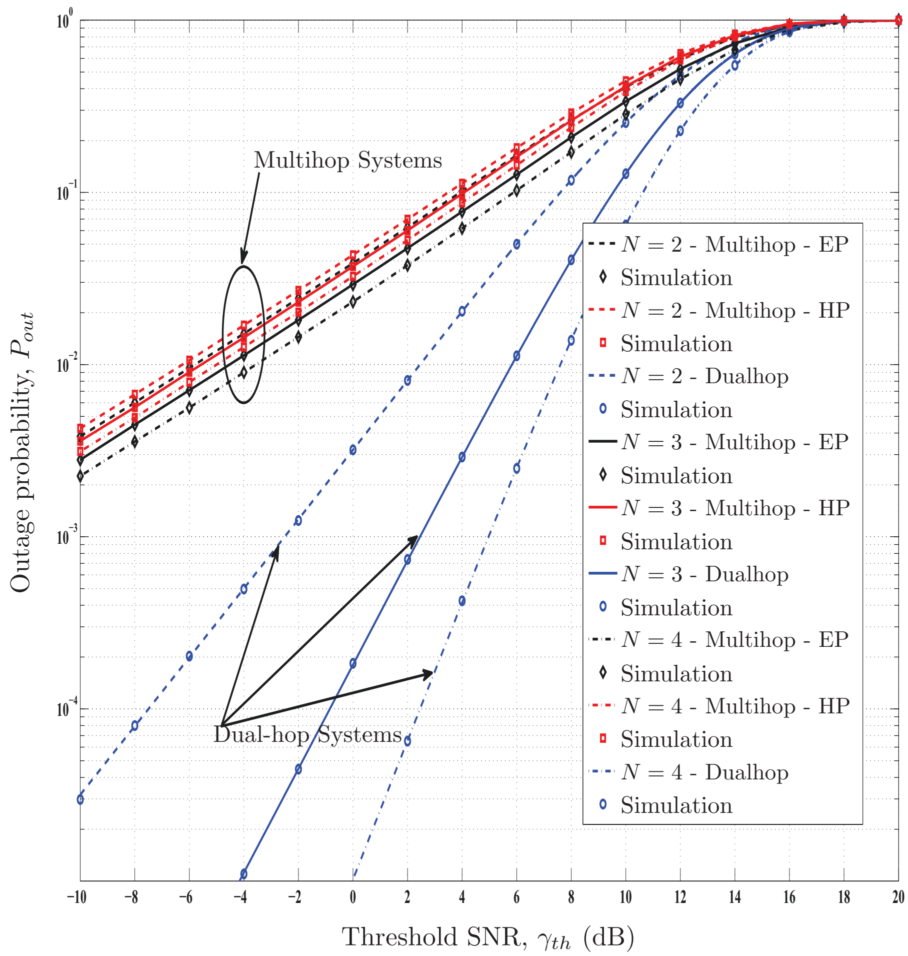

Figure 9.5: The outage probability, at $P/N_0 = 10$ dB, for dual-hop AF relaying systems with relay selection from $N = 2, 3$ and 4 available relays, and multihop AF relaying systems with $N = 2, 3$ and 4 intermediate relays. Multihop AF systems with equal power distribution and other power distributions are considered. The total available system power is fixed for all systems. Rayleigh fading links are assumed.

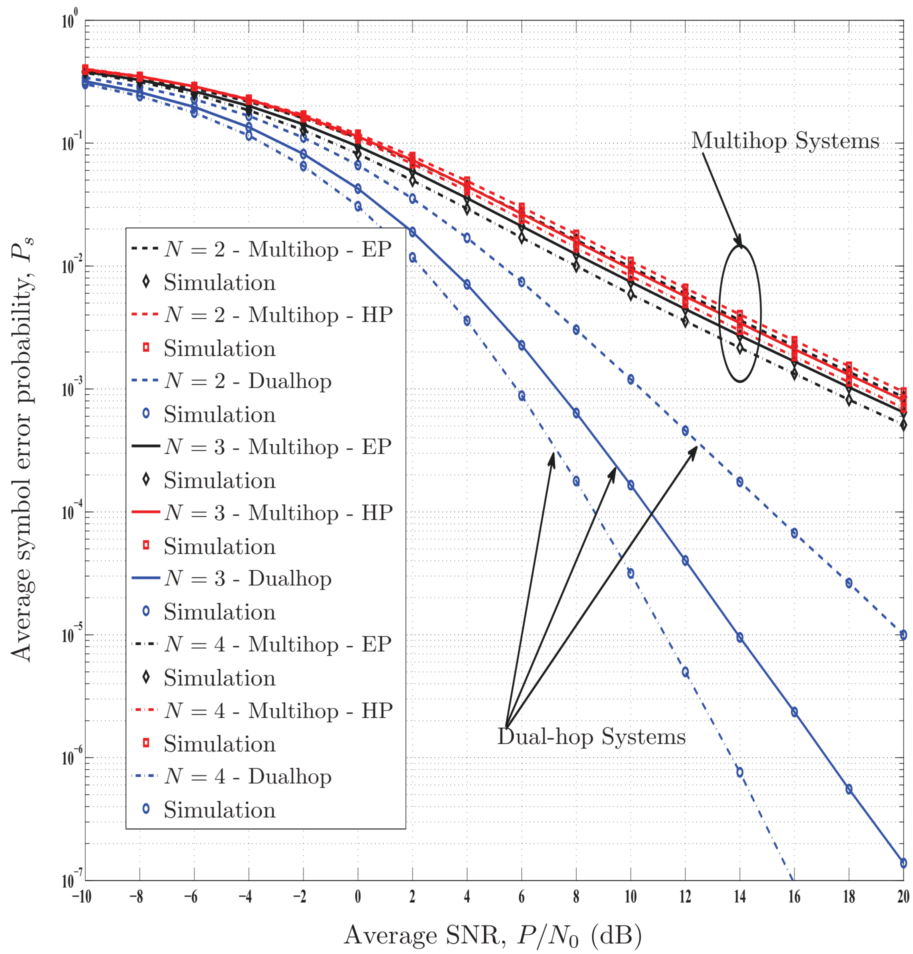

Figure 9.6: The average symbol error probability for dual-hop AF relaying systems with relay selection from $N = 2, 3$ and 4 available relays, and multihop AF relaying systems with $N = 2, 3$ and 4 intermediate relays. Multihop AF systems with equal power distribution and other power distributions are considered. The total available system power is fixed for all systems. Rayleigh fading links and BPSK modulation are assumed.

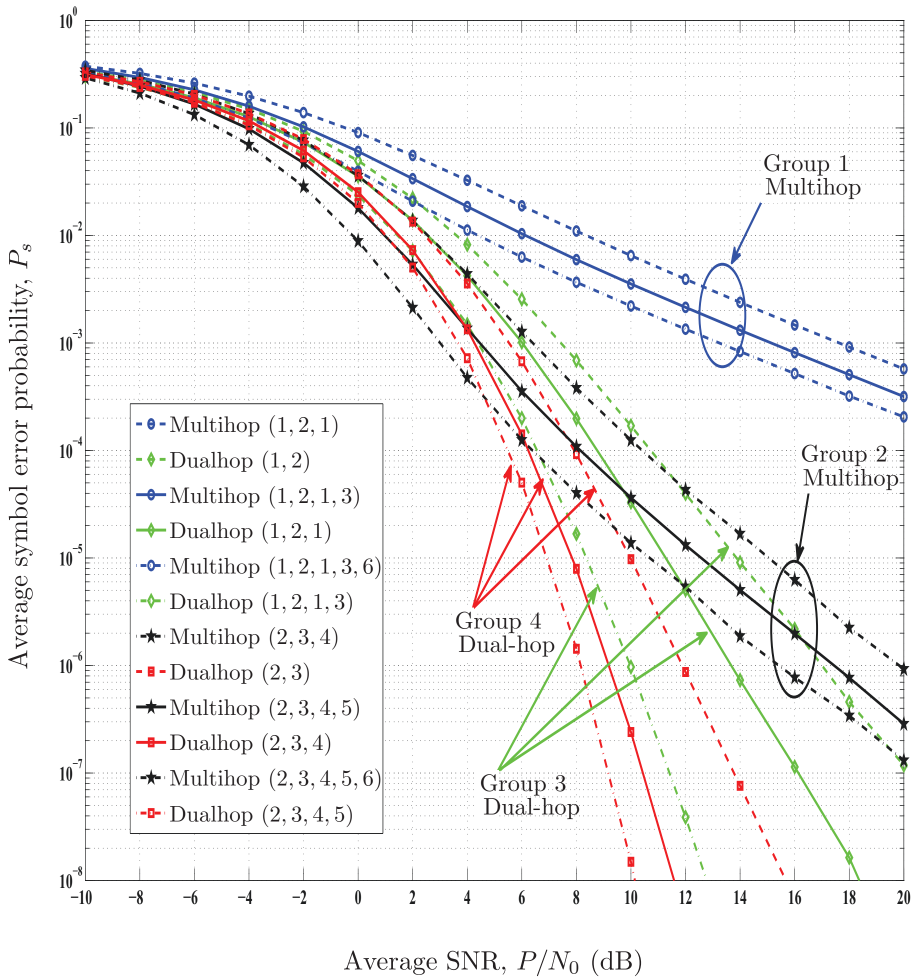

Figure 9.7: The average symbol error probability for dual-hop AF relaying systems with relay selection from $N = 2, 3$ and 4 available relays, and multihop AF relaying systems with $N = 2, 3$ and 4 intermediate relays. Multihop AF systems with equal power distribution are considered. The total available system power is fixed for all systems. Non-identically distributed Nakagami-m fading links and BPSK modulation are assumed.

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Chapter 10

Concluding Remarks and Future Directions

10.1 Concluding Remarks

In this thesis, novel exact performance analysis was presented for different wireless cooperative systems. Systems studied in this thesis are:

1) Dual-hop AF relaying systems without relay selection.

2) Dual-hop AF relaying systems operating under adaptive power transmission.

3) Dual-hop AF relaying systems with partial relay selection.

4) Dual-hop AF relaying systems with maximum end-to-end SNR relay selection.

5) Dual-hop AF relaying systems with full selection diversity.

6) General multihop AF relaying systems.

7) Multi-branch multihop AF relaying systems with full selection diversity.

Dependence of the system performance on the channel fading parameters, system parameters and numbers of relays was explored. It was found that while the Nakagami-m parameter, m, has an effect on the diversity gain, as well as on the SNR gain of the system, the Rician parameter, K, only affects the system's SNR gain. It was shown that when combined together, the Rician parameter results in a bottleneck to the improvement of the system's performance.

It was also found that the dependence of the system's diversity gain on the selection pool size is related to the relay selection technique used in systems with relay selection. For dual-hop AF systems with partial relay selection operating over Nakagami-m fading links, the diversity gain was found proportional to $\min\{m_1, Nm_2\}$, in the case of maximum relay-to-destination SNR selection, or to $\min\{Nm_1, m_2\}$, in the case of source-to-relay SNR selection. For these systems, it was shown that increasing the selection pool size beyond $\frac{m_1}{m_2}$ (or $\frac{m_2}{m_1}$) is of limited practical benefit. On the other hand, for dual-hop AF systems with maximum end-to-end SNR relay selection operating over Nakagami-m fading links, the system's diversity gain is proportional to $\sum_i \min\{m_{1i}, m_{2i}\}$. In the special case of *i.i.d.* links, the diversity gain is proportional to N. This means that the presence of fewer but less severely fading available links results in a more significant improvement in the system performance compared to the presence of more available links with smaller values of the fading parameters. These conclusions lead to important design criteria that can be used by designers of practical communication systems.

For multihop AF systems, the GTCF method was proposed as a general framework that can provide exact solutions for the performance metrics of these systems. The GTCF method presents a transform technique that can be used to solve heretofore difficult problems in cooperative wireless networks. It was found that for multihop AF systems, diversity gain is not affected by numbers of hops. On the other hand, the SNR gain is influenced by numbers of hops. The more hops, the better the system's performance. However, it was found that there is a trade-off for the improvement of system's outage and average symbol error probabilities against the system's ergodic capacity. It was also found that the diversity gain of multihop AF systems operating over Nakagami-m fading links is limited by the fading conditions of the weakest hop, and is proportional to $\min\{m_i\}$. Note that, in addition to systems analyzed by the GTCF method in this thesis, the GTCF method can be used to analyze other systems. The GTCF method was used in [1] to find exact performance metrics of multihop AF systems operating in co-channel interference (CCI). The GTCF method was also used in [2] to investigate multihop cooperative networks implementing CSI-assisted AF relaying in the presence of in-phase and quadrature-phase (I/Q) imbalance. For multi-branch multihop AF relaying systems, it was found that while the system's diversity order is not affected by the number of hops per branch, it is affected by the presence of multiple available branches.

In this thesis also, dual-hop AF systems with maximum end-to-end SNR relay selection were compared to multihop AF systems. It was shown that dual-hop AF systems with relay selection have a diversity order advantage over multihop AF systems, resulting from selection among multiple branches. This advantage results in a significant performance enhancement in favor of dual-hop AF systems with relay selection. It was shown also that while the weakest link is a bottleneck for the performance improvement in the case of multihop AF systems, this bottleneck does not exist for dual-hop AF systems with relay selection because the contribution of the weakest branch is overridden by other branches in the selection pool.

10.2 Future Directions

It is fundamental to study optimized designs of wireless cooperative network. The work presented in this thesis can be considered as basis for other studies in several ways. A number of possible future directions are:

1) Designing multihop AF systems with optimum power allocation among the different nodes of multihop paths. Equal power allocation was considered in the literature and in this thesis. Different power allocation schemes can be promising and may achieve higher system performance if adapted and optimized based on the system parameters.

2) Exact analysis and optimized design of high order multihop AF systems, such as multiple-input multiple-output (MIMO) multihop AF systems. In these systems, the source node, relay nodes and/or destination node can be equipped with multiple transmit/receive antennas. Transmit beamforming and receiver combining techniques proved to enhance the performance of wireless communication systems. Integrating MIMO operation with multihop relaying systems will result in significant improvements to the quality of service provided by wireless systems. Using the methods and frameworks proposed in this thesis permits improved designs of such systems based on different design criteria, such as maximum throughput, minimum outage probability, etc.

3) The results presented in this thesis can be extended to multiuser systems. Multiuser systems provide advances in the system capacity and can provide significant diversity gains. Applying the cooperative concept on multiuser systems will boost the performance of wireless communication systems and promises as a future standard of mobile communications, in particular, and wireless communications, in general.

4) Analysis and design of wireless cooperative systems in dynamic environment. The results in the literature as well as the results presented in this thesis consider static configuration of end nodes and relaying nodes. In a dynamic environment, end nodes as well as relying nodes can be mobile. This imposes new challenges for the analysis and design of cooperative systems.

5) The methods proposed in this thesis can be also used for wireless sensor networks (WSN). Wireless sensor networks have wide range of applications including environmental monitoring, security surveillance, military applications and others. A WSN is composed of large numbers of sensor nodes with relatively limited energy storage, as well as limited processing and communications capabilities. Sensor nodes are responsible for gathering, processing and transmitting data to an end observer. In most cases, direct single-hop transmission is not possible. Using an optimally designed multihop cooperative infrastructure can result in better exploitation of WSN, increases the reliability and enhances fault-tolerance of such systems.

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