NAME OF AUTHOR/*NOM DE L'AUTEUR* Meiun Shintani

TITLE OF THESIS/*TITRE DE LA THÈSE* Abelian Gauge Field And Goldstone Theorem

UNIVERSITY/*UNIVERSITÉ*

DEGREE FOR WHICH THESIS WAS PRESENTED/ *GRADE POUR LEQUEL CETTE THÈSE FUT PRÉSENTÉE* Master of Science

YEAR THIS DEGREE CONFERRED/*ANNÉE D'OBTENTION DE CE GRADE* 1977

NAME OF SUPERVISOR/*NOM DU DIRECTEUR DE THÈSE*

DATED/*DATÉ* October 21 SIGNED/*SIGNÉ* Meiun Shintani

PERMANENT ADDRESS/*RÉSIDENCE FIXÉ* 7222 - 112 St. Edmonton
Alberta. T6G 1J3

THE UNIVERSITY OF ALBERTA

ABELIAN GAUGE FIELDS AND GOLDSTONE THEOREM

BY

© MEIUN SHINTANI

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

OF MASTER OF SCIENCE

IN

THEORETICAL PHYSICS

DEPARTMENT OF PHYSICS

EDMONTON, ALBERTA

FALL 1977

THE UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled ABELIAN GAUGE FIELDS AND GOLDSTONE THEOREM submitted by MEIUN SHINTANI in partial fulfilment of the requirements for the degree of Master of Science.

Y. Takahashi
Supervisor

[signature]

Garry Ludwig

[signature]

DATE October 21 1977

ABSTRACT

A covariant quantization of the electromagnetic field in the Hilbert space with indefinite metric is studied. There are two cases which enable us to take the view-point of the particle picture, for example, to show the existence of the three-dimensional Fourier transforms for $A_\mu^{in}(x)$.

Each case is investigated in connection with the Goldstone theorem and the Higgs phenomenon using the Higgs Lagrangian. A universal form of the generators for the gauge transformations is found. Moreover, it is shown that spontaneous breakdown of the global gauge transformation occurs, regardless of the charged fields which interact with the electromagnetic field through the minimal coupling.

It is shown that certain conditions on the c-number functions appearing in conserved currents J_μ restrict the spectral representation for $<[\phi_i(x),J_\mu(y)]>_o$, where $\phi_i(x)$ represents a spinless field. It is shown that the Goldstone theorem does not always apply when conserved currents involve c-number functions.

ACKNOWLEDGEMENTS

Without the patient assistance of Professor Y. Takahashi, my adviser, this thesis would have never been possible.

Thanks are due to T. Goto, H. Matsumoto, H. Umezawa, R. Kubo and K. Yokoyama for helpful discussions.

I am indebt to N. Nakanishi for his kind correspondence.

Finally, I wish to express my gratitude to Mrs. Lee Cech for her fast and beautiful typing of the manuscript.

TABLE OF CONTENTS

LIST OF TABLES

NOTATION

The following conventions and notation are used in this article.

The Lorentz sufficies are indicated by Greek letters which run from 0 to 3, and the Latin letters which run from 1 to 3 stand for the space components. In the covariant calculation, the metric

$$g_{oo} = 1, \quad g_{ii} = -1 \qquad \text{for} \qquad i=1,2,3$$

$$g_{\mu\nu} = 0 \qquad \text{for} \qquad \mu \neq \nu$$

will be used. We designate

$$x^{\mu} \equiv (x^{o}, \underset{\sim}{x}) \equiv (t, \underset{\sim}{x})$$

and

$$x_{\mu} = g_{\mu\nu} x^{\nu} = (t, -\underset{\sim}{x})$$

The unit time-like vector n_{μ} is defined by

$$n_{\mu} n^{\mu} = 1, \quad n_{o} > 0 \quad .$$

The notation ∂_{μ} implies

$$\partial_{\mu} \equiv \frac{\partial}{\partial x^{\mu}} = g_{\mu\nu} \partial^{\nu}$$

and the space-like derivative $\partial_{\mu,s}$ is defined by

$$\partial_{\mu,s} \equiv \partial_{\mu} - n_{\mu}(n\partial)$$

where

$$(n\partial) = n^{\mu}\partial_{\mu} \quad .$$

The symbol $\overleftrightarrow{\partial}_{\mu}$ appears in the combination

$$f\overleftrightarrow{\partial}_{\mu}g \equiv \partial_{\mu}f\cdot g - f\partial_{\mu}g \quad .$$

We define a four-vector differential surface area as

$$d\sigma^{\mu} = (dx^1dx^2dx^3, dx^2dx^3dx^o, dx^3dx^odx^1, dx^odx^1dx^2) \quad .$$

The symbol $\langle\ \rangle_o$ stands for the vacuum expectation value. We use † for hermitean conjugate and * for complex conjugation.

The notation

$$\dot{f} \equiv \frac{df}{dt} \equiv \frac{df}{dx^o}$$

will sometimes be used. The abbreviation h.c. and c.c. are used, respectively, for hermitean and complex conjugation.

Finally, we use natural units throughout this article, in which

$$\hbar = c = 1 \quad .$$

CHAPTER I

INTRODUCTION

This chapter is devoted to (1) a statement of the problems, and (2) an outline of the following chapters.

(1) The Statement of the Problems

We shall state the main problems to be dealt with in the following chapters and explain them separately.

(i) Is the covariant quantization of the electromagnetic field possible?

Mathews et al. (1974) proved that a covariant quantization of the electromagnetic field must be formulated in an indefinite-metric Hilbert space. There appears the negative norm in the quantization in the Hilbert space with an indefinite metric. This fact makes the probability interpretation difficult. However, by introducing a certain covariant subsidiary condition, we may avoid this difficulty. Strocchi (1964) proved that the subsidiary condition was indispensable for the covariant quantization in the Hilbert space with an indefinite metric.

Nakanishi (1966) proposed a method for the covariant quantization for the electromagnetic field by introducing the auxiliary scalar field B(x). His formalism consists of the following elements:

(a) Hilbert space with an indefinite metric,

(b) the auxiliary field B(x),

(c) the covariant subsidiary condition which defines the physical states |phys>;

$$B^{(+)}_{(x)}|phys\rangle = 0 \quad ,$$

where the symbol (+) stands for the positive frequency part of the field. Here, the physical state is subject to the probability interpretation.

His theory is satisfactory in the sense that it removes most of the well-known difficulties (for example, the difficulties of the positive definiteness for the Hamiltonian, normalization of the state vectors, unitarity of the S-matrix, the manifest covariance of the theory). However, due to the existence of the pathological function E(x) in his theory, the particle picture does not apply in the usual sense. Is there any method to avoid this difficulty? We shall accomplish this as follows.

We shall adopt (a), (b) and (c) and choose the gauge parameter α in such a fashion that the pathological function E(x) disappears from some of the covariant commutators. In this way, we can take the view-point of the particle picture.

(ii) Is there a universal form of the generators associated with the gauge transformations?

When we introduce the electromagnetic interaction with the minimal coupling, the generators for the gauge transformations can be written in a universal form regardless of the charged fields employed. This implies that the discussion associated with the gauge transformations has universality independently of the detailed structures for the charged fields.

(iii) What is the applicability of the Goldstone theorem?

It is well-known that the Goldstone theorem has been proved only for the case of conserved q-number currents (Goldstone, Salam, Weinberg, 1962). Is it applicable when the conserved currents involve c-number functions? As an example of such cases, if we apply the theorem to the local gauge transformation, the spontaneous breakdown of the local gauge transformation must occur and there exists a Goldstone boson which yields the degeneracy of the vacuum. However, as will be seen in Chapter III, the state $G^{II}[\Lambda]|o\rangle$ is not the vacuum state. In spite of the existence of the massless particle, the discussion of the local gauge transformation cannot say anything about the degeneracy of the vacuum. Moreover, we will arrive at the result that when conserved current J_μ involving c-number functions has a specific form, there can exist a massive particle in the system. These results imply that the Goldstone theorem must be extended to more general cases in which conserved currents involve c-number functions.

(2) Outline of the Following Chapters

In Chapter II, we introduce the electromagnetic interaction with the minimal coupling. The total Lagrangian is shown to be invariant under the local guage transformations. It is shown that we can take the stand-point of the particle picture only when either

(d) $\alpha = Z_3$, if $m=0$, or

(e) $\alpha = 0$, if $m\neq 0$.

Here, m and Z_3 are, respectively, the physical mass and renormalization constant. In particular, case (e) implies that if the physical mass for A_μ is non-zero, we must choose the Landau gauge ($\alpha=0$) (Takahashi, Goto, 1976).

In Chapter III, we shall investigate the limit in which the bare charge is put equal to zero in the discussion of Chapter II. In this limit, it is shown that the case (e) will not happen but only the case (d) will occur.

The latter case is nothing but the Gupta-Bleuler formalism in the Fermi gauge ($\alpha=1$) (Gupta 1950, Bleuler 1950).

In Chapter IV, we take the Higgs Lagrangian to investigate, from the view-point of the particle picture, under what situation A_μ becomes massless or massive.

In Chapter V, with the help of the equation of motion for the electromagnetic field, we shall rewrite the generators for the gauge transformation in a universal form. Using this universal form, we shall discuss the group properties for the gauge transformation. Furthermore, we derive the commutator of the total Hamiltonian and the generator which will be used in Chapter VII for the discussion of the Goldstone theorem concerning the gauge transformation.

In Chapter VI, by applying the Goldstone theorem to the global gauge transformation, its spontaneous breakdown occurs regardless of the charged fields employed. Furthermore, we shall investigate a role of the subsidiary condition in case of the Higgs Lagrangian and see explicitly the way in which the Goldstone bosons appear.

In Chapter VII, it is shown that certain conditions on the c-number functions appearing in conserved currents J_μ restrict the spectral representation for $<[\phi_i(x),J_\mu(y)]>_o$ where $\phi_i(x)$ represents a spinless field. Moreover, the question as to whether or not the Goldstone theorem applies when conserved currents involve c-number functions is investigated.

CHAPTER II

ELECTROMAGNETIC INTERACTION

(1) The Lagrangian with the Minimal Coupling

We write the free Lagrangian as

$$L^{free} = L_{E.M.} + L_{matter} \tag{2.1}$$

where $L_{E.M.}$ is given by

$$L_{E.M.} = -\tfrac{1}{4} F^{\mu\nu}F_{\mu\nu} + \tfrac{1}{2}\alpha B^2 + B\partial^\mu A_\mu \quad , \tag{2.2}$$

and L_{matter} is the free Lagrangian for the charged matter field.

In what follows, we assume that the interaction is introduced by the following replacement in L_{matter}:

for the scalar field

$$\begin{aligned} \partial_\mu\phi &\rightarrow (\partial_\mu - ieA_\mu)\phi \\ \partial_\mu\phi^\dagger &\rightarrow (\partial_\mu + ieA_\mu)\phi^\dagger \end{aligned} \tag{2.3}$$

for the spinor field

$$\begin{aligned} \partial_\mu\psi &\rightarrow (\partial_\mu - ieA_\mu)\psi \\ \bar{\psi}\cdot\overleftarrow{\partial}_\mu &\rightarrow \bar{\psi}(\overleftarrow{\partial}_\mu + ieA_\mu) \quad , \end{aligned} \tag{2.4}$$

for the spinor field with the pseudovector coupling

$$\partial_\mu \psi \rightarrow (\partial_\mu - ie\gamma_5 A_\mu)\psi$$
$$\bar{\psi}\overleftarrow{\partial}_\mu \rightarrow \bar{\psi}(\overleftarrow{\partial}_\mu - ie\gamma_5 A_\mu) \quad . \tag{2.5}$$

The interactions induced by the above replacement will be called "minimal coupling".†

The total Lagrangian can then be written as

$$L = -\tfrac{1}{4} F^{\mu\nu}F_{\mu\nu} + B\partial^\mu A_\mu + \frac{\alpha}{2} B^2 + L_{matter} + L_{int} \quad . \tag{2.6}$$

In this case, the charged current can be written as

$$j^\mu_{(x)} = - \frac{\partial L_{int}}{\partial A_\mu(x)} \quad , \tag{2.7}$$

and it is conserved, i.e.

$$\partial^\mu j_\mu(x) = 0 \quad . \tag{2.8}$$

(2) Gauge Transformations

The Lagrangian (2.6) is invariant under the local gauge transformations

$$B(x) \rightarrow B'(x) = B(x) \tag{2.9}$$

$$A_\mu(x) \rightarrow A'_\mu(x) = A_\mu(x) + \partial_\mu \Lambda(x) \tag{2.10}$$

---

† Originally, the minimal coupling guarantees that the charged particles move under the Lorentz force. However, here, the terminology "minimal coupling" is used in the broad sense that it includes the replacement (2.5) as well as (2.3) and (2.4).

and for the scalar field

$$\phi(x) \rightarrow \phi'(x) = e^{ie\Lambda(x)}\phi(x)$$
$$\phi^{+}(x) \rightarrow \phi^{+'}(x) = e^{-ie\Lambda(x)}\phi^{+}(x) \quad , \qquad (2.11)$$

for the spinor field

$$\psi(x) \rightarrow \psi'(x) = e^{ie\Lambda(x)}\psi(x)$$
$$\bar{\psi}(x) \rightarrow \bar{\psi}'(x) = e^{-ie\Lambda(x)}\bar{\psi}(x) \quad , \qquad (2.12)$$

for the spinor field with the pseudovector coupling

$$\psi(x) \rightarrow \psi'(x) = e^{ie\gamma_5\Lambda(x)}\psi(x)$$
$$\bar{\psi}(x) \rightarrow \bar{\psi}'(x) = \bar{\psi}(x)e^{ie\gamma_5\Lambda(x)} \quad , \qquad (2.13)$$

provided that the c-number gauge function $\Lambda(x)$ satisfies the massless Klein-Gordon equation. In particular, the transformation (2.9), (2.10) and (2.13) are called the chiral gauge transformation. When $\Lambda=\theta=$constant, the above transformations are termed the global gauge transformations.

(3) <u>Canonical Quantization</u>

From (2.6) and (2.7) the electromagnetic field equations are

$$\Box A_\mu - \partial_\mu \partial^\nu A_\nu - \partial_\mu B = j_\mu \qquad (2.14)$$

and

$$\partial^\mu A_\mu + \alpha B = 0 \quad , \qquad (2.15)$$

It follows from these equations that

$$\Box A_\mu - (1-\alpha)\partial_\mu B = j_\mu \quad . \tag{2.16}$$

Equation (2.14) with (2.8) implies that

$$\Box B = 0 \quad . \tag{2.17}$$

The canonical momenta conjugate to A_μ are

$$\Pi^o \equiv \frac{\partial L}{\partial \dot{A}_o} = B \quad , \tag{2.18}$$

$$\Pi^k \equiv \frac{\partial L}{\partial \dot{A}_k} = F_{ok} \quad , \tag{2.19}$$

and the canonical momenta conjugate to the charged fields ϕ_i, $\phi_i^\dagger$ are respectively,

$$\pi^i \equiv \frac{\partial L}{\partial \dot{\phi}_i} \quad , \tag{2.20}$$

$$\pi^{i\dagger} \equiv \frac{\partial L}{\partial \dot{\phi}_i^\dagger} \quad , \tag{2.21}$$

where ϕ_i represents the charged field regardless of its statistics. They satisfy the equal time commutators,

$$[A_\mu([illegible]) \; [illegible]]\delta(x_o-y_o) = i\delta_{\mu\nu}\delta^{(4)}(x-y) \quad , \tag{2.22}$$

$$[\phi_i([illegible], \pi [illegible]]\delta(x_o-y_o) = [\phi_i^\dagger(x), \pi_j^\dagger(y)]\delta(x_o-y_o) = [illegible]\,\delta^{(4)}(x-y) \tag{2.23}$$

and all [illegible] [illegible]ro.

The relation (2.22) can also be written as

$$[A_\mu(x),A_\mu(y)]\delta(x_o-y_o) = 0 \quad , \tag{2.24}$$

$$[A_\mu(x),\dot{A}_\ell(y)]\delta(x_o-y_o) = i\delta_{\mu\ell}\delta^{(4)}(x-y) \quad , \tag{2.25}$$

$$[\dot{A}_k(x),\dot{A}_\ell(y)]\delta(x_o-y_o) = 0 \quad , \tag{2.26}$$

$$[A_\mu(x),B(y)]\delta(x_o-y_o) = i\delta_{\mu o}\delta^{(4)}(x-y), \tag{2.27}$$

$$[\dot{A}_k(x),B(y)]\delta(x_o-y_o) = i\partial_k^x\delta^{(4)}(x-y) \quad , \tag{2.28}$$

$$[B(x),B(y)]\delta(x_o-y_o) = 0 \quad . \tag{2.29}$$

Due to the fact that $j_o(x)$ can be written only in terms of canonical variables ϕ_i, $\phi_j^\dagger$, π_i and $\pi_j^\dagger$, we have

$$[B(x),j_o(y)]\delta(x_o-y_o) = 0 \quad . \tag{2.30}$$

Since, from the field equations (2.14) and (2.15), we have

$$\dot{A}_o = \sum_{k=1}^{3} \partial_k A_k - \alpha B \tag{2.31}$$

and

$$\dot{B} = \sum_{k=1}^{3} \partial_k \dot{A}_k - \Delta A_o - j_o \quad , \tag{2.32}$$

it follows that

$$[B(x),\dot{B}(y)]\delta(x_o-y_o) = 0 \quad . \tag{2.33}$$

It is also possible to derive at $x_o=y_o$

$$[\dot{B}(x),\phi_j(y)] = [-j_o(x),\phi_j(y)] = -e\phi_j(y)\delta(\underset{\sim}{x}-\underset{\sim}{y})$$ for scalar and spinor fields,

$$= -e\gamma_5\psi(x)\delta(\underset{\sim}{x}-\underset{\sim}{y})$$ for the spinor with pseudovector coupling. (2.34)

On using the field equations and the equal time commutators obtained above, some of the commutation relations can be written in the covariant form (Nakanishi 1967, 1973, 1975):

$$[B(x),B(y)] = 0 \quad , \tag{2.35}$$

$$[A_\mu(x),B(y)] = -[B(x),A_\mu(y)] = -i\partial_\mu^x D(x-y) \quad , \tag{2.36}$$

$$[B(x),j_\mu(y)] = 0 \quad , \tag{2.37}$$

$$\left.\begin{aligned}[B(x),\phi(y)] &= e\phi(y)D(x-y)\\ [B(x),\phi^+(y)] &= -e\phi(y)D(x-y)\end{aligned}\right. \tag{2.38}$$

for the scalar fields,

$$\left.\begin{aligned}[B(x),\psi(y)] &= e\psi(y)D(x-y)\\ [B(x),\bar{\psi}(y)] &= -e\bar{\psi}(y)D(x-y)\end{aligned}\right. \tag{2.39}$$

for the spinor fields,

$$\left.\begin{aligned}[B(x),\psi(y)] &= -e\gamma_5\psi(y)D(x-y)\\ [B(x),\bar{\psi}(y)] &= -e\bar{\psi}(y)\gamma_5 D(x-y)\end{aligned}\right. \tag{2.40}$$

for the spinor fields with the pseudovector coupling.

(4) Subsidiary Condition and Singularities in the Spectral Representation

We demand that all physical states must satisfy the subsidiary condition

$$B^{(+)}(x)|\text{phys}> = 0 \tag{2.41}$$

This ensures that the expectation value of (2.14) with respect to the physical states reduces to the Maxwell equation. Since the vacuum is a physical state, we have

$$B^{(+)}(x)|0\rangle = 0 \tag{2.42}$$

Now let us consider the spectral representation of the vacuum expectation value for the commutator $[A_\mu(x),A_\nu(y)]$. The spectral representation for $\langle[A_\mu(x),A_\nu(y)]\rangle_o$ is written as

$$\langle[A_\mu(x),A_\nu(y)]\rangle_o = -i\int_0^\infty d\kappa^2\rho_1(\kappa^2)g_{\mu\nu}\Delta(x-y:\kappa^2)$$
$$-i\int_0^\infty d\kappa^2\rho_2(\kappa^2)\partial_\mu^x\partial_\nu^x\Delta(x-y:\kappa^2) \tag{2.43}$$

Differentiating both sides of (2.43) with respect to x_μ and using (2.15) and (2.36), we have

$$-i\alpha\partial_\nu^x D(x-y) = -i\int_0^\infty d\kappa^2(\rho_1(\kappa^2) - \kappa^2\rho_2(\kappa^2))\partial_\nu^x\Delta(x-y:\kappa^2) .$$

Since this relation holds for any x and y, we obtain

$$\rho_1(\kappa^2) - \kappa^2\rho_2(\kappa^2) = \alpha\delta(\kappa^2) \tag{2.44}$$

On the other hand, if we set $\mu=k$, $\nu=\ell$ in (2.43) and differentiate with respect to y_o and put $y_o=x_o$, we obtain

$$\int_0^\infty d\kappa^2\rho_1(\kappa^2) = 1 \tag{2.45}$$

and
$$\int_0^\infty d\kappa^2 \; \rho_2(\kappa^2) = 0 \quad . \tag{2.46}$$

If we assume
$$\rho_1(\kappa^2) \equiv Z_3\delta(\kappa^2-m^2) + \sigma(\kappa^2) \quad , \tag{2.47}$$

where
$$\sigma(\kappa^2) = \begin{cases} 0, \text{ for } \kappa^2 \leq m^2 \\ \text{positive for } \kappa^2 > m^2 \end{cases} \quad , \tag{2.48}$$

we have, from equations (2.45) and (2.47),
$$Z_3 = 1 - \int_b^\infty d\kappa^2 \; \sigma(\kappa^2), \tag{2.49}$$
$$b > m^2 \quad .$$

Moreover, from (2.44) and (2.47) we obtain
$$\rho_2(\kappa^2) = -K\delta(\kappa^2) - \frac{\alpha}{\kappa^2}\delta(\kappa^2) + \frac{Z_3}{\kappa^2}\delta(\kappa^2-m^2) + \frac{\sigma(\kappa^2)}{\kappa^2} \quad , \tag{2.50}$$

where K is a real constant to be determined from (2.46). We notice that there are strong singularities in the second and third terms with m=0 which destroy the particle picture. We can, however, remove these singularities by setting

$$\begin{aligned} &(i)\ \alpha = Z_3 \quad \text{if} \quad m=0 \\ &(ii)\ \alpha = 0 \quad \text{if} \quad m\neq 0 \quad . \end{aligned} \tag{2.51}$$

Now let us discuss each case separately.

(i) $\alpha = Z_3$, m=0, whence ρ_1 and ρ_2 become

$$\rho_1^{(o)}(\kappa^2) = Z_3\delta(\kappa^2) + \sigma(\kappa^2) \quad \text{and} \tag{2.52}$$

$$\rho_2^{(o)}(\kappa^2) = -K_o\delta(\kappa^2) + \frac{\sigma(\kappa^2)}{\kappa^2} \quad . \tag{2.53}$$

Due to equation (2.46), K_o becomes

$$K_o = \int_b^\infty d\kappa^2 \frac{\sigma(\kappa^2)}{\kappa^2} \quad , \; (b>0) \quad , \tag{2.54}$$

where the symbol "o" of K_o, $\rho_1^{(o)}$ and $\rho_2^{(o)}$ imply K, ρ_1 and ρ_2 for m=0.

Thus we arrive at the spectral representation

$$<[A_\mu(x),A_\nu(y)]>_o = -i\, g_{\mu\nu}Z_3D(x-y) + i(\Box_x g_{\mu\nu}-\partial_\mu^x\partial_\nu^x) \int_b^\infty d\kappa^2$$

$$\frac{\sigma(\kappa^2)}{\kappa^2}(\Delta(x-y:\kappa^2) - D(x-y)) \quad , \; (b>0) \quad . \tag{2.55}$$

(ii) $\alpha=0$, $m\neq 0$, whence ρ_1 and ρ_2 become

$$\rho_1^{(m)}(\kappa^2) = Z_3\; \delta(\kappa^2-m^2) + \sigma(\kappa^2) \tag{2.56}$$

and

$$\rho_2^{(m)}(\kappa^2) = -K_m\delta(\kappa^2) + \frac{Z_3}{\kappa^2}\delta(\kappa^2-m^2) + \frac{\sigma(\kappa^2)}{\kappa^2} \; . \tag{2.57}$$

where the symbol "m" of K_m, $\rho_1^{(m)}$, $\rho_2^{(m)}$ is associated with the quantities with $m\neq 0$. From equation (2.46), we obtain

$$K_m = \frac{Z_3}{m^2} + \int_{b'}^\infty d\kappa^2 \frac{\sigma(\kappa^2)}{\kappa^2} \quad , \quad (b'>m^2) \quad . \tag{2.58}$$

Thus, the spectral representation is

$$\langle[A_\mu(x),A_\nu(y)]\rangle_0 = -iZ_3(g_{\mu\nu}+m^{-2}\partial^x_\mu\partial^x_\nu)\Delta(x-y:m^2)$$
$$+ i\frac{Z_3}{m^2}\partial^x_\mu\partial^x_\nu D(x-y) + i(g_{\mu\nu}\Box_x-\partial^x_\mu\partial^x_\nu)\int_{b'}^{\infty} d\kappa^2\,\frac{\sigma(\kappa^2)}{\kappa^2}$$
$$\times\ (\Delta(x-y:\kappa^2) - D(x-y)) \quad . \tag{2.59}$$

It should be emphasized that the above argument does not imply the existence of a massive vector field, but restricts the form of the commutation relation when $m \neq 0$.

It is very interesting to see how the massive mode is created in quantum electrodynamics. We investigate this problem in connection with a Higgs phenomenon in a later chapter.

CHAPTER III

FREE ELECTROMAGNETIC FIELD

In order to discuss the quantization of the free electromagnetic field, we consider the case of $j_\mu=0$ in the argument given in Chapter II and see what mode of A_μ will exist.

(1) Spectral Functions in the Limit of e=0 †

In the limit e=0, the field equation (2.16) becomes

$$\Box A_\mu(x) - (1-\alpha)\partial_\mu B(x) = 0 \quad . \tag{3.1}$$

On operating $\Box_x$ and $\Box_y$ on the commutator (2.43) and using (3.1) and (2.35), we obtain a further restriction on ρ_1 and ρ_2;

$$0 = \int_0^\infty d\kappa^2 \cdot \kappa^4 \rho_1(\kappa^2) g_{\mu\nu}\Delta(x-y:\kappa^2)$$
$$+ \int_0^\infty d\kappa^2 \cdot \kappa^4 \rho_2(\kappa^2)\partial_\mu^x\partial_\nu^x\Delta(x-y:\kappa^2) \quad .$$

Hence, we have

$$\kappa^4 \rho_1(\kappa^2) = 0 \tag{3.2}$$

and

$$\kappa^4 \rho_2(\kappa^2) = 0 \tag{3.3}$$

From equation (3.3), we have

$$\kappa^2 \rho_2(\kappa^2) = (1-\alpha)\delta(\kappa^2) \quad , \tag{3.4}$$

† This procedure does apply when the perturbative solutions exist.

hence $\rho_2(\kappa^2)$ has a strong singularity. To remove this, we put $\alpha=1$ which implies that

$$\alpha = Z_3 = 1 \quad ,$$
$$m = 0 \quad ,$$
$$\sigma(\kappa^2) = 0 \quad . \tag{3.5}$$

Then, from equations (2.44), (2.45), (2.46), (3.4) and (3.5), we obtain

$$\rho_1(\kappa^2) = \delta(\kappa^2)$$
$$\rho_2(\kappa^2) = 0 \tag{3.6}$$

Thus, we have

$$[A_\mu(x), A_\nu(y)] = -ig_{\mu\nu}D(x-y) \quad , \tag{3.7}$$

and

$$[B(x), B(y)] = 0 \tag{3.8}$$

$$[A_\mu(x), B(y)] = -[B(x), A_\mu(y)] = -i\partial_\mu^x D(x-y) \tag{3.9}$$

Field equations become

$$\Box A_\mu(x) = 0 \tag{3.10}$$
$$\partial^\mu A_\mu(x) + B = 0 \tag{3.11}$$

and

$$\Box B = 0 \quad . \tag{3.12}$$

Here we have seen that in the limit of $e=0$, the system can be characterized by $\alpha=Z_3=1$ and $m=0$ which is a special case of (i), and the massive mode never appears. This is nothing but the free electromagnetic field in the Fermi (or Feynman) gauge.

(2) Subsidiary Condition

The subsidiary condition (2.41) becomes

$$(\partial^\mu A_\mu)^{(+)}|\text{phys}\rangle = 0 \quad , \tag{3.13}$$

where (+) is the positive frequency part. Therefore, from the above argument, the theory in the Fermi gauge reduces itself to the Gupta-Bleuler formalism. Since A_μ satisfies the massless Klein-Gordon equation (3.10), A_μ can be expressed by 3-dimensional momentum space representation a usual manner.

(3) Momentum Representation

We shall express the commutators (3.7), (3.8) and (3.9) in momentum space.

From equation (3.10) we may write $A_\mu(x)$ as

$$A_\mu(x) \equiv \frac{1}{(2\pi)^{3/2}} \sum_{\lambda=0}^{3} \int \frac{d^3\underset{\sim}{k}}{\sqrt{2|\underset{\sim}{k}|}} e_\mu^{(\lambda)}(\underset{\sim}{k})[a_\lambda(\underset{\sim}{k})e^{-ikx} + \text{h.c.}], \tag{3.14}$$

where $e_\mu^{(\lambda)}$ are the polarization vectors.†

Similarly B(x) can be written as

$$B(x) \equiv \frac{1}{(2\pi)^{3/2}} \int \frac{d^3\underset{\sim}{p}}{\sqrt{2|\underset{\sim}{p}|}} (b(\underset{\sim}{p})e^{-ipx} + \text{h.c} \quad , \tag{3.15}$$

whence equation (3.11) becomes in momentum space

$$b(\underset{\sim}{k}) = i|\underset{\sim}{k}|(a_o(\underset{\sim}{k}) - a_3(\underset{\sim}{k})) \quad , \tag{3.16}$$

† The definition of $e_\mu^{(\lambda)}$ is given in Appendix (A-ii).

and commutators (3.7), (3.8) and (3.9) become

$$[b(\underset{\sim}{k}),b^{\dagger}(\underset{\sim}{k}')] = [b(\underset{\sim}{k}),b(\underset{\sim}{k}')] = [b^{\dagger}(\underset{\sim}{k}),b^{\dagger}(\underset{\sim}{k}')] = 0 \quad , \tag{3.17}$$

$$[a_{\lambda}(\underset{\sim}{k}),a^{\dagger}_{\lambda'}(\underset{\sim}{k}')] = -g_{\lambda\lambda'}\delta^{(3)}(\underset{\sim}{k}-\underset{\sim}{k}') \quad , \tag{3.18}$$

and $$[a_{\lambda}(\underset{\sim}{k}),b^{\dagger}(\underset{\sim}{k}')] = i|\underset{\sim}{k}|\delta(\underset{\sim}{k}-\underset{\sim}{k}')(g_{o\lambda}-g_{3\lambda}) \quad . \tag{3.19}$$

The subsidiary condition (2.41) becomes

$$b(\underset{\sim}{p})|phys\rangle = 0 \quad \text{for any } \underset{\sim}{p} \quad . \tag{3.20}$$

CHAPTER IV

HIGGS LAGRANGIAN

In this Chapter we take the Higgs Lagrangian to investigate, from the view-point of the particle picture, under what situation A_μ becomes massive. Moreover, a covariant treatment of the Higgs phenomenon is given.

(1) Canonical Quantization and Commutation Relations

We employ the Lagrangian

$$L = L_{E.M.} + L_\phi \quad , \tag{4.1}$$

where

$$L_\phi = (\partial^\mu + ieA^\mu)\phi^\dagger \cdot (\partial_\mu - ieA_\mu)\phi - u\phi^\dagger \cdot \phi - \frac{\lambda}{4}(\phi^\dagger \phi)^2 \quad . \tag{4.2}$$

We assume that the constant u is real and λ satisfies

$$\lambda > 0 \tag{4.3}$$

Field equations are

$$\partial^\mu A_\mu + \alpha B = 0 \quad , \tag{4.4}$$

$$\Box A_\mu - (1-\alpha)\partial_\mu B = j_\mu \quad , \tag{4.5}$$

$$\Box B = 0 \quad , \tag{4.6}$$

$$\Box \phi - ie\partial_\mu A^\mu \cdot \phi - 2ieA^\mu \cdot \partial_\mu \phi - e^2 A_\mu A^\mu \cdot \phi + u\phi + \frac{\lambda}{2}\phi(\phi^\dagger \phi) = 0 \quad , \tag{4.7}$$

where

$$j_\mu = -\frac{\partial L_\phi}{\partial A^\mu} = -ie[\phi^\dagger \partial_\mu \phi - \partial_\mu \phi^\dagger \cdot \phi] - 2e^2 \phi^\dagger \phi \cdot A_\mu \quad . \qquad (4.8)$$

Obviously the current is conserved. Following the general argument given in Chapter II, we obtain

$$\pi \equiv \frac{\partial L_\phi}{\partial \dot{\phi}} = \dot{\phi}^\dagger + ieA_o \cdot \phi^\dagger \quad , \qquad (4.9)$$

$$\pi^\dagger \equiv \frac{\partial L_\phi}{\partial \dot{\phi}^\dagger} = \dot{\phi} - ieA_o \cdot \phi \quad , \qquad (4.10)$$

and the equal time commutators for scalar fields are

$$[\phi(x),\pi(y)] = [\phi^\dagger(x),\pi^\dagger(y)] = i\delta^3(\underset{\sim}{x}-\underset{\sim}{y}) \quad \text{for} \quad x_o=y_o \quad , \qquad (4.11)$$

and all others are zero.

For further discussion, it is convenient to set

$$\sqrt{2}\phi(x) \equiv v + \psi(x) + i\chi(x) \quad , \qquad (4.12)$$

where

$$v = \begin{cases} 0 & \text{for } u \geq 0 \\ \sqrt{-\frac{4u}{\lambda}} & \text{for } u<0 \end{cases} \quad , \qquad (4.13)$$

and ψ and χ are assumed to be Hermitian with the vanishing vacuum expectation value, i.e.,

$$\langle\psi(x)\rangle_o = \langle\chi(x)\rangle_o = 0 \quad \text{for arbitrary } x \quad . \qquad (4.14)$$

By substituting (4.12) into (2.38), we have

$$[B(x),\psi(y)] = ie\chi(y)D(x-y) \quad , \tag{4.15}$$

$$[B(x),\chi(y)] = -i(M + e\psi(y))D(x-y) \quad , \tag{4.16}$$

where
$$M = ev \quad . \tag{4.17}$$

With the aid of (4.14), the vacuum expectation values of (4.15) and (4.16) are given by

$$<[B(x),\psi(y)]>_o = 0 \quad , \tag{4.18}$$

$$<[B(x),\chi(y)]>_o = -iMD(x-y) \quad . \tag{4.19}$$

Furthermore, as will be derived in Appendix (C-1), we have

$$<[A_\mu(x),\phi(y)]>_o = -\alpha M\partial^x_\mu E(x-y) \quad , \tag{4.20}$$

which can be written in terms of χ and ψ as

$$<[A_\mu(x),\chi(y)]>_o = i\alpha M\partial^x_\mu E(x-y) \quad , \tag{4.21}$$

$$<[A_\mu(x),\psi(y)]>_o = 0 \quad , \tag{4.22}$$

where use has been made of the property of the function $E(x)$, i.e., the reality of $E(x)$.[†]

(2) Asymptotic Limit and Particle Picture

When we take the limit $x_o \to -\infty$, the Heisenberg fields behave asymptotically as

† The derivation of equation (4.20) is seen in Appendix (C-1). The definition of the function E(x) is given in Appendix (A-1).

$$A_\mu(x) \to A_\mu^{(o)}(x) \quad , \quad B(x) \to B^{(o)}(x) = B(x)$$
$$\chi(x) \to \chi^{(o)}(x) \quad , \quad \psi(x) \to \psi^{(o)}(x) \tag{4.23}$$

Since (o)-fields contribute only to the discrete spectra, the following commutation relations are concluded from (2.55), (2.59), (2.37), (2.36), (4.21), (4.22), (4.19) and (4.18);

$$[A_\mu^{(o)}(x), A_\nu^{(o)}(y)] = -ig_{\mu\nu}Z_3 D(x-y) \quad \text{for} \quad \alpha = Z_3, m=0 \tag{4.24}$$

$$= -iZ_3(g_{\mu\nu} + m^{-2}\partial_\mu^x\partial_\nu^x)\Delta(x-y:m^2)$$
$$+iZ_3 m^{-2}\partial_\mu^x\partial_\nu^x D(x-y) \quad \text{for } \alpha=0, m\neq 0 \tag{4.25}$$

$$[A_\mu^{(o)}(x), B^{(o)}(y)] = -i\partial_\mu^x D(x-y) \tag{4.26}$$

$$[A_\mu^{(o)}(x), \chi^{(o)}(y)] = i\alpha M\partial_\mu^x E(x-y) \tag{4.27}$$

$$[A_\mu^{(o)}(x), \psi^{(o)}(y)] = 0 \tag{4.28}$$

$$[B^{(o)}(x), B^{(o)}(y)] = 0 \tag{4.29}$$

$$[B^{(o)}(x), \chi^{(o)}(y)] = -iMD(x-y) \tag{4.30}$$

$$[B^{(o)}(x), \psi^{(o)}(y)] = 0 \quad . \tag{4.31}$$

We assume that the following commutators hold;

$$[\psi^{(o)}(x), \chi^{(o)}(y)] = 0 \tag{4.32}$$

$$[\psi^{(o)}(x), \psi^{(o)}(y)] \neq 0 \tag{4.33}$$

$$[\chi^{(o)}(x), \chi^{(o)}(y)] \neq 0 \quad . \tag{4.34}$$

If the conditions

$$(A)\ M=0 \quad \text{and} \quad \alpha\neq0$$
$$(B)\ M=0 \quad \text{and} \quad \alpha=0 \qquad (4.35)$$
$$(C)\ M\neq0 \quad \text{and} \quad \alpha=0$$

are satisfied, the pathological function $E(x-y)$ in the commutators disappears.† In case of (C), the spontaneous breakdown of the global transformation necessarily occurs due to $M\neq0$ (or $v\neq0$). We point out that as will be shown in Chapter V, the condition $M=0$ does not imply that the spontaneous breakdown of the symmetry never takes place.

Combining (2.51) and (4.35), we consider three cases:

$$(A)'\ \alpha=Z_3,\ M=0 \quad \text{and} \quad m=0$$
$$(B)'\ \alpha=0,\ M=0 \quad \text{and} \quad m\neq0 \qquad (4.36)$$
$$(C)'\ \alpha=0,\ M\neq0 \quad \text{and} \quad m\neq0$$

(3) Detailed Discussion of the Three Cases

The (o)-fields are related to the in-fields which diagonalize the total Hamiltonian and satisfy free equations of motion and commutators. We consider three cases separately.

(A)' $\alpha=Z_3$, $M=0$ and $m=0$

Following the general argument in the preceding sections commutation relations are given by

† See, for its pathological properties, Nakanishi (1973, 1975).

$$[A_\mu^{(o)}(x), A_\nu^{(o)}(y)] = -iZ_3 g_{\mu\nu} D(x-y) \quad (4.37\text{-}1)$$

$$[A_\mu^{(o)}(x), B^{(o)}(y)] = -i\partial_\mu^x D(x-y) \quad (4.37\text{-}2)$$

$$[A_\mu^{(o)}(x), \chi^{(o)}(y)] = 0 \quad (4.37\text{-}3)$$

$$[A_\mu^{(o)}(x), \psi^{(o)}(y)] = 0 \quad (4.37\text{-}4)$$

$$[B^{(o)}(x), B^{(o)}(y)] = 0 \quad (4.37\text{-}5)$$

$$[B^{(o)}(x), \chi^{(o)}(y)] = 0 \quad (4.37\text{-}6)$$

$$[B^{(o)}(x), \psi^{(o)}(y)] = 0 \quad (4.37\text{-}7)$$

$$[\psi^{(o)}(x), \chi^{(o)}(y)] = 0 \quad (4.37\text{-}8)$$

$$[\psi^{(o)}(x), \psi^{(o)}(y)] \neq 0 \quad (4.37\text{-}9)$$

$$[\chi^{(o)}(x), \chi^{(o)}(y)] \neq 0 \quad (\ .37\text{-}10)$$

The field equations for $A_\mu^{(o)}(x)$ and $B^{(o)}(x)$ consistent with the above commutators are

$$\begin{aligned} \Box A_\mu^{(o)}(x) &= 0 \\ \Box B^{(o)}(x) &= 0 \\ \partial^\mu A_\mu^{(o)}(x) + Z_3 B^{(o)}(x) &= 0 \ . \end{aligned} \quad (4.38)$$

These relations can be reproduced if we put

$$\begin{aligned} A_\mu^{(o)}(x) &= Z_3^{\frac{1}{2}} A_\mu^{in}(x) \\ B^{(o)}(x) &= Z_3^{-\frac{1}{2}} B^{in}(x) \\ \chi^{(o)}(x) &= Z_\chi^{\frac{1}{2}} \chi^{in}(x) \\ \psi^{(o)}(x) &= Z_\psi^{\frac{1}{2}} \psi^{in}(x) \end{aligned} \quad (4.39)$$

in which the commutators are

$$[A_\mu^{in}(x), A_\nu^{in}(y)] = -ig_{\mu\nu}D(x-y)$$

$$[A_\mu^{in}(x), B^{in}(y)] = -i\partial_\mu^x D(x-y)$$

$$[B^{in}(x), B^{in}(y)] = 0$$

$$[A_\mu^{in}(x), \chi^{in}(y)] = [A_\mu^{in}(x), \psi^{in}(y)] = [B^{in}(x), \chi^{in}(y)] = [B^{in}(x), \psi^{in}(y)] = [\psi^{in}(x), \chi^{in}(y)] = 0$$

$$[\psi^{in}(x), \psi^{in}(y)] \neq 0$$

$$[\chi^{in}(x), \chi^{in}(y)] \neq 0$$

(4.40)

and the field equations for $A_\mu^{in}(x)$ and $B^{in}(x)$ are

$$\Box A_\mu^{in}(x) = 0$$

$$\Box B^{in}(x) = 0 \qquad (4.41)$$

$$\partial^\mu A_\mu^{in}(x) + B^{in}(x) = 0 \quad ,$$

the field equations and commutators of $A_\mu^{in}(x)$ and $B^{in}(x)$ are the same as ones obtained in Chapter III.

(B)' $\alpha=0$, $M=0$ and $m\neq 0$

This case leads to a contradiction. To see this, we proceed as follows.

The commutation relations are the same as those of (A)' except the commutator

$$[A_\mu^{(o)}(x), A_\nu^{(o)}(y)] = -iZ_3(g_{\mu\nu} + m^{-2}\partial_\mu^x\partial_\nu^x)\Delta(x-y:m^2)$$
$$+ i\frac{Z_3}{m^2}\partial_\mu^x\partial_\nu^x D(x-y) \quad . \tag{4.42}$$

The field equations consistent with the commutation relations are

$$\Box B^{(o)}(x) = 0 \tag{4.43}$$

$$\partial^\mu A_\mu^{(o)}(x) = 0 \tag{4.44}$$

In order to obtain the field equations for $A_\mu^{(o)}(x)$, we set

$$(\Box+m^2)A_\mu^{(o)}(x) - C_1''\partial_\mu B^{(o)}(x) - C_2''\partial_\mu \chi^{(o)}(x) - C_3''\partial_\mu \psi^{(o)}(x) = 0 \tag{4.45}$$

and calculate each coefficient.

By operating $(\Box_x+m^2)$ to (4.37-3) and using (4.45), we obtain

$$C_2'' = 0 \quad ,$$

similarly from (4.42) and (4.45) we have

$$C_1'' = Z_3 \quad ,$$

and from (4.37-4) and (4.45) we obtain

$$C_3'' = 0 \quad .$$

Thus (4.45) becomes

$$(\Box+m^2)A_\mu^{(o)}(x) - Z_3\partial_\mu B^{(o)}(x) = 0 \quad . \tag{4.46}$$

Operating $(\Box_x+m^2)(\Box_y+m^2)$ to (4.42) and using (4.46), we obtain

$$0 = iZ_3m^2\partial_\mu^x\partial_\nu^x D(x-y) \qquad (4.47)$$

which is unacceptable as long as $Z_3m^2 \neq 0$.

(C)' $\alpha=0$, $M\neq0$ and $m\neq0$

In this case we have the following commutation relations for (o)-fields:

$$[A_\mu^{(o)}(x),A_\nu^{(o)}(y)] = -iZ_3(g_{\mu\nu}+m^{-2}\partial_\mu^x\partial_\nu^x)\Delta(x-y:m^2)$$

$$+ iZ_3m^{-2}\partial_\mu^x\partial_\nu^x D(x-y) \qquad (4.48\text{-}1)$$

$$[A_\mu^{(o)}(x),B^{(o)}(y)] = -i\partial_\mu^x D(x-y) \qquad (4.48\text{-}2)$$

$$[A_\mu^{(o)}(x),\chi^{(o)}(y)] = 0 \qquad (4.48\text{-}3)$$

$$[A_\mu^{(o)}(x), \psi^{(o)}(y)] = 0 \qquad (4.48\text{-}4)$$

$$[B^{(o)}(x),B^{(o)}(y)] = 0 \qquad (4.48\text{-}5)$$

$$[B^{(o)}(x),\chi^{(o)}(y)] = -iMD(x-y) \qquad (4.48\text{-}6)$$

$$[B^{(o)}(x),\psi^{(o)}(y)] = 0 \qquad (4.48\text{-}7)$$

$$[\psi^{(o)}(x),\chi^{(o)}(y)] = 0 \qquad (4.48\text{-}8)$$

$$[\psi^{(o)}(x),\psi^{(o)}(y)] \neq 0 \qquad (4.48\text{-}9)$$

from (4.48-6) we have

$$[\chi^{(o)}(x),\chi^{(o)}(y)] = iZ_\chi D(x-y) \qquad (4.48\text{-}10)$$

Field equations consistent with the above commutators are

$$\Box B^{(o)}(x) = 0 \quad , \tag{4.49}$$

and with the use of the commutators (4.48-2), (4.48-3), (4.48-5), (4.48-6), (4.48-7) and (4.48-9), we obtain

$$\Box \chi^{(o)}(x) = 0 \quad . \tag{4.50}$$

By setting

$$\partial^\mu A_\mu^{(o)}(x) + R_1 B^{(o)}(x) + R_2 \dot{\chi}^{(o)}(x) + R_3 \psi^{(o)}(x) = 0 \tag{4.51}$$

and using the commutators (4.48-2) to (4.48-9), we obtain

$$\partial^\mu A_\mu^{(o)}(x) = 0 \quad . \tag{4.52}$$

The form

$$(\Box + m^2) A_\mu^{(o)}(x) - R_1' \partial_\mu B^{(o)}(x) - R_2' \partial_\mu \chi^{(o)}(x) - R_3' \partial_\mu \psi^{(o)}(x) = 0 \tag{4.53}$$

with $R_1' = Z_3$, $R_2' = Z_\chi^{-1} Z_3 M$ and $R_3' = 0$ is consistent with (4.48) to (4.52). Hence,

$$(\Box + m^2) A_\mu^{(o)}(x) - Z_3 \partial_\mu B^{(o)}(x) - Z_\chi^{-\frac{1}{2}} Z_3 M \partial_\mu \chi^{(o)}(x) = 0 \quad . \tag{4.54}$$

The operation of $(\Box_x + m^2)(\Box_y + m^2)$ on (4.48-1) with the aid of (4.54) and commutators (4.48-5), (4.48-6) and (4.48-10) gives

$$(Z_3^2 Z_\chi^{-1} M^2 - Z_3 m^2) \partial_\mu^x \partial_\nu^x D(x-y) = 0 \quad .$$

Since $Z_3 \neq 0$, the inter sting relation between M and m

$$\frac{Z_3}{Z_\chi} = \left(\frac{m}{M}\right)^2 \tag{4.55}$$

is obtained. Since from (4.54) $A_\mu^{(o)}$ contains the massive mode, $A_\mu^{(o)}$ is written in the form

$$A_\mu^{(o)}(x) = U_\mu^{(o)}(x) + a\partial_\mu\chi^{(o)}(x) + b\partial_\mu B^{(o)}(x) \quad , \tag{4.56}$$

where $U_\mu^{(o)}$ is a massive vector field, and a and b are real constants. From (4.49), (4.50), (4.52) and (4.56), we have

$$\partial^\mu U_\mu^{(o)}(x) = 0 \quad . \tag{4.57}$$

To determine a and b, we assume

$$[U_\mu^{(o)}(x), B^{(o)}(y)] = 0 \tag{4.58}$$

$$[U_\mu^{(o)}(x), \chi^{(o)}(y)] = 0 \quad . \tag{4.59}$$

From (4.48-2), (4.48-5), (4.48-6), (4.56) and (4.57), we have

$$a = M^{-1} \quad ,$$

similarly from (4.48-3), (4.48-6), (4.48-10), (4.56) and (4.59), we have

$$b = M^{-2}Z_\chi = m^{-2}Z_3 \quad .$$

Hence, (4.56) becomes

$$A_\mu^{(o)}(x) = U_\mu^{(o)}(x) + M^{-1}\partial_\mu\chi^{(o)}(x) + m^{-2}Z_3\partial_\mu B^{(o)}(x) \quad . \tag{4.60}$$

From (4.54) and (4.60) we obtain

$$(\Box+m^2)U_\mu^{(o)}(x) = 0 \quad . \tag{4.61}$$

It is clear from (4.57) and (4.61) that $U_\mu^{(o)}(x)$ is the Proca field. The substitution of (4.60) into (4.48-1) with the aid of the commutators (4.48-5), (4.48-6), (4.48-10), (4.58) and (4.59), gives

$$[U_\mu^{(o)}(x),U_\nu^{(o)}(y)] = -iZ_3(g_{\mu\nu}+m^{-2}\partial_\mu^x\partial_\nu^x)\Delta(x-y:m^2) \quad . \tag{4.62}$$

As before, the (o)-fields are related to the in-fields as

$$\begin{aligned} U_\mu^{(o)}(x) &= Z_3^{\frac{1}{2}}u_\mu^{in}(x) \\ \psi^{(o)}(x) &= Z_\psi^{\frac{1}{2}}\psi^{in}(x) \\ \chi^{(o)}(x) &= Z_\chi^{\frac{1}{2}}\chi^{in}(x) \\ B^{(o)}(x) &= Z_3^{-\frac{1}{2}}B^{in}(x) \\ A_\mu^{(o)}(x) &= Z_3^{\frac{1}{2}}A_\mu^{in}(x) \quad . \end{aligned} \tag{4.63}$$

$B^{in}(x)$ can be written in the subtraction form of two fields as

$$B^{in}(x) = m(\Phi_1^{in}(x) - \chi^{in}(x)) \quad . \tag{4.64}$$

where $\Phi_1^{in}(x)$ and $\chi^{in}(x)$ have the following properties;

$$\Box\Phi_1^{in}(x) = 0 \quad , \tag{4.65}$$

$$\Box\chi^{in}(x) = 0 \quad , \tag{4.66}$$

$$\begin{aligned} &[\Phi_1^{in}(x),\Phi_1^{in}(y)] = -[\chi^{in}(x),\chi^{in}(y)] = -iD(x-y) \\ \text{and} \quad &[\Phi_1^{in}(x),\chi^{in}(y)] = 0 \quad . \end{aligned} \tag{4.67}$$

$A_\mu^{in}(x)$ can be written as

$$A_\mu^{in}(x) = u_\mu^{in}(x) + \frac{1}{m}\partial_\mu \phi_1^{in}(x) \quad , \tag{4.68}$$

and other commutation relations and field equations are listed in Appendix (A-iii).

In the present case, since $M \neq 0 (v \neq o)$, the spontaneous breakdown of the symmetry necessarily occurs, and $\phi_1^{in}(x)$ and $\chi^{in}(x)$ are regarded as Goldstone bosons. $\chi^{in}(x)$ and $\phi_1^{in}(x)$, however, never appear in reality due to the subsidiary condition

$$B^{in(+)}(x)|phys\rangle = 0 \quad \text{for any } x \tag{4.69}$$

and commutation relations

$$[B^{in}(x), \phi_1^{in}(y)] = [B^{in}(x), \chi^{in}(y)] = -imD(x-y) \neq 0 \quad \text{for any } x, y \tag{4.70}$$

In this way it is shown that ϕ_1^{in} and χ^{in} are unphysical particles and the gauge field A_μ acquires the mass. This is called the Higgs mechanism (Higgs 1964, 1966; Nakanishi 1973, 1975; Nishijima 1973). As was seen in the above analysis, it should be noted that the Goldstone bosons never disappear from the theory, though they do not play any physical role.

A general treatment of the spontaneous breakdown of symmetry will be given in Chapter VI.

CHAPTER V

GAUGE TRANSFORMATION AND ITS GENERATOR

We shall obtain a universal form of the generator for the gauge transformation. Using its form we shall discuss the group properties of the gauge transformation and show that the successive operations of the infinitesimal will give the finite gauge transformation. We show that the local gauge transformation changes the Hamiltonian by $-G[\dot{\Lambda}]$, but does not change the physical energy spectra.

(1) The Generator of an Infinitesimal Gauge Transformation

Let us consider an infinitesimal gauge transformation*

$$\phi_i(x) \to \phi_i'(x) = \phi_i(x) + \delta\phi_i(x)$$

for any charged field,

$$A_\mu(x) \to A_\mu'(x) = A_\mu(x) + \partial_\mu(\delta\Lambda(x)) \tag{5.1}$$

$$B(x) \to B'(x) = B(x) \quad ,$$

where $\delta\phi_i(x)$ are given by

$$\delta\phi(x) = ie\delta\Lambda(x)\cdot\phi(x)$$
$$\delta\phi^+(x) = -ie\delta\Lambda(x)\cdot\phi^\dagger(x) \quad \text{for scalar field} \tag{5.2}$$

$$\delta\psi(x) = ie\delta\Lambda(x)\cdot\psi(x)$$
$$\delta\bar{\psi}(x) = -ie\delta\Lambda(x)\cdot\bar{\psi}(x) \quad \text{for spinor field} \tag{5.3}$$

$$\delta\psi(x) = ie\gamma_5\delta\Lambda(x)\cdot\psi(x) \tag{5.4}$$

$$\delta\bar{\psi}(x) = \bar{\psi}(x)ie\gamma_5\delta\Lambda(x) \quad \text{for spinor field with p-}\nu\text{ coupling} \quad ,$$

and $\delta\Lambda(x)$ is an infinitesimal c-number function satisfying

$$\Box\delta\Lambda(x) = 0 \quad . \tag{5.5}$$

The change of the action integral due to the transformation (5.1) is given by the Noether Identity

$$\delta I = \delta \int_{\sigma_1}^{\sigma_2} d^4x L(x)$$

$$= - \int_{\sigma_1}^{\sigma_2} d^4x \, \partial^\mu J_\mu(x) = G[\sigma_1] - G[\sigma_2] \quad , \tag{5.6}$$

where

$$J^\mu(x) = - \frac{\partial L}{\partial(\partial_\mu A_\nu(x))} \delta A_\nu(x)$$

$$= \sum_i \left(\frac{\partial L}{\partial(\partial_\mu \phi_i(x))} \delta\phi_i(x) + \text{h.c.}\right)$$

$$= (F^{\mu\nu}(x) - Bg^{\mu\nu})\partial_\nu\delta\Lambda(x)$$

$$- \sum_i \left(\frac{\partial L}{\partial(\partial_\mu \phi_i(x))} \delta\phi_i(x) + \text{h.c.}\right) \tag{5.7}$$

As will be shown in Appendix (C-ii), the current $j_\mu(x)$ given by (2.7) is written as

$$j^{\mu}(x) = \sum_i \left(\frac{\partial L}{\partial(\partial_\mu \phi_i(x))} \delta\phi_i(x) + h.c.\right)/\delta\Lambda(x) \quad , \qquad (5.8)$$

where $J^{\mu}(x)$ becomes

$$J^{\mu}(x) = (F^{\mu\nu}(x) - B(x)g^{\mu\nu})\partial_\nu(\delta\Lambda(x)) - \delta\Lambda(x)\cdot j^{\mu}(x) \quad . \qquad (5.9)$$

Hence the generator of the transformation (5.1) is

$$\begin{aligned} G[\sigma] &= \int_\sigma d\sigma_\mu(x) J^{\mu}(x) \\ &= \int_\sigma d\sigma_\mu(x)\{(F^{\mu\nu}(x) - B(x)g^{\mu\nu})\partial_\nu(\delta\Lambda(x)) \\ &\qquad - \delta\Lambda(x)\cdot j^{\mu}(x)\} \quad . \end{aligned} \qquad (5.10)$$

Since, as can be easily demonstrated, $J^{\mu}(x)$ is conserved, i.e.

$$\partial^{\mu} J_{\mu}(x) = 0 \quad , \qquad (5.11)$$

it follows from (5.6) that the Lagrangian is invariant under the transformation (5.1), and $G[\sigma]$ is independent of the choice of the spacelike hypersurface σ, i.e.

$$\frac{\delta}{\delta\sigma(x)} G[\sigma] = 0 \quad . \qquad (5.12)$$

Now we shall obtain a universal form of the generator of this transformation with the aid of the field equation (2.14) as follows;

$$\begin{aligned} G &= \int d\sigma_\mu(x)\{(F^{\mu\nu}(x) - B(x)g^{\mu\nu})\partial_\nu(\delta\Lambda(x)) \\ &\qquad - \delta\Lambda(x)(\partial_\nu F^{\nu\mu}(x) - \partial^{\mu}B(x))\} \\ &= \int d\sigma_\mu(x) B(x)\overleftrightarrow{\partial^{\mu}}(\delta\Lambda(x)) + \int d\sigma_\mu(x)\{F^{\mu\nu}(x)\partial_\nu(\delta\Lambda(x)) \\ &\qquad + \delta\Lambda(x)\partial_\nu F^{\mu\nu}(x)\} \quad . \end{aligned} \qquad (5.13)$$

On using the formula (B-iii), it is shown that the second term in equation (5.13) vanishes. Indeed,

$$\int d\sigma_\mu \{F^{\mu\nu}\partial_\nu(\delta\Lambda) + \delta\Lambda\partial_\nu F^{\mu\nu}\} = \int d\sigma_\mu \partial_\nu (F^{\mu\nu}\delta\Lambda)$$

$$= \tfrac{1}{2}\{\int d\sigma_\mu \partial_\nu (F^{\mu\nu}\delta\Lambda) + \int d\sigma_\nu \partial_\mu (F^{\nu\mu}\delta\Lambda)\}$$

$$= \tfrac{1}{2}\{\int d\sigma_\mu \partial_\nu (F^{\mu\nu}\delta\Lambda) + \int d\sigma_\mu \partial_\nu (F^{\nu\mu}\delta\Lambda)\}$$

$$= \tfrac{1}{2}\int d\sigma_\mu \partial_\nu \{(F^{\mu\nu} + F^{\nu\mu})\delta\Lambda\} = 0 \quad .$$

Thus we arrive at

$$G = \int d\sigma_\mu(x) B(x) \overleftrightarrow{\partial}{}^\mu_x (\delta\Lambda(x)) \equiv G[\delta\Lambda] \quad . \qquad (5.14)$$

It should be noted that the generator (5.14) is expressed only in terms of the B field and an infinitesimal c-number function $\delta\Lambda(x)$ whatever the matter field may be. As will be seen in the next section, (5.14) is valid for any $\delta\Lambda(x)$ satisfying equation (5.5). In this sense equation (5.14) is a universal form of the generator for the gauge transformation.†

(2) The Gauge Transformation and its Group Properties

We shall study the group properties of the gauge transformation and show that it is possible to extend the infinitesimal to the finite gauge transformation.

† Matsumoto et al. (1976) obtained the generator of the gauge transformation in the form (5.14) in the case of the quantum electrodynamics with both dimensions of 2 and 4.

Consider a set $\mathcal{U}$ of canonical transformations such that

$$\mathcal{U} = \{U[\delta\Lambda] | U[\delta\Lambda] = e^{iG[\delta\Lambda]} , \Box\delta\Lambda = 0\} \quad . \tag{5.15}$$

On account of the form of equation (5.14), we derive important properties for a set $\mathcal{U}$. For $\forall \delta\Lambda_1, \delta\Lambda_2 \in A$, where A is a set given by

$$A = \{\delta\Lambda | \Box\delta\Lambda = 0\} \quad ,$$

(i) linear dependence on $\delta\Lambda$ of $G[\delta\Lambda]$ leads to the additivity of G, i.e.

$$G[\delta\Lambda_1] + G[\delta\Lambda_2] = G[\delta\Lambda_1 + \delta\Lambda_2] \quad , \tag{5.16}$$

(ii) the fact that the only operator appearing in (5.14) is the B field leads to the commutativity of G, i.e.

$$[G[\delta\Lambda_1], G[\delta\Lambda_2]] = 0 \quad , \tag{5.17}$$

due to the commutator

$$[B(x), B(y)] = 0 \quad .$$

From the properties (i) and (ii), it follows that

(1) a set $\mathcal{U}$ obeys a law of combination, i.e.,

$$U[\delta\Lambda_1]U[\delta\Lambda_2] = e^{iG[\delta\Lambda_1]} e^{iG[\delta\Lambda_2]} = e^{iG[\delta\Lambda_1 + \delta\Lambda_2]}$$

$$= U[\delta\Lambda_1 + \delta\Lambda_2] \in \mathcal{U}$$

$$\text{for } \forall U[\delta\Lambda_1], U[\delta\Lambda_2] \in \mathcal{U} \quad , \tag{5.18}$$

(2) the law of combination is associative, i.e.

$$(U[\delta\Lambda_1]U[\delta\Lambda_2])U[\delta\Lambda_3] = U[\delta\Lambda_1](U[\delta\Lambda_2]U[\delta\Lambda_3])$$

$$\text{for } \forall U[\delta\Lambda_1], U[\delta\Lambda_2], U[\delta\Lambda_3] \in \mathcal{U} , \tag{5.19}$$

(3) the identity element $I = U[o] = 1$ exists satisfying the property

$$U[\delta\Lambda]U[o] = U[o]U[\delta\Lambda] = U[\delta\Lambda]$$

$$\text{for } \forall U[\delta\Lambda] \in \mathcal{U} ,$$

(4) to every element $U[\delta\Lambda]$ in $\mathcal{U}$, the inverse element $U^{-1}[\delta\Lambda] = U[-\delta\Lambda]$ exists such that

$$U^{-1}[\delta\Lambda]\cdot U[\delta\Lambda] = U[\delta\Lambda]U^{-1}[\delta\Lambda] = I .$$

Hence, the set $\mathcal{U}$ forms a "continuous" group. Besides, we obtain from (5.18)

$$[U[\delta\Lambda_1], U[\delta\Lambda_2]] = 0$$

$$\text{for } \forall U[\delta\Lambda_1], U[\delta\Lambda_2] \in \mathcal{U} , \tag{5.20}$$

from which follows that the group $\mathcal{U}$ is an "Abelian" group. Thus the group properties obtained above enable us to extend the infinitesimal operator $U[\delta\Lambda]$ to the finite operator $U[\Lambda] = e^{iG[\Lambda]}$. We can verify explicityly that $U[\Lambda] = e^{iG[\Lambda]}$ generates a finite gauge transformation as follows.

Consider the transformation for the fields such that

$$A(x) \rightarrow A'(x) = U^{\dagger}[\Lambda]A(x)U[\Lambda] \quad , \tag{5.21}$$

where A(x) stands for $A_\mu(x)$, B(x) and charged fields. Equation (5.21) can be computed by the help of Appendix (B-1). Since

$$[G, A_\nu(y)] = i\partial^y_\nu \Lambda(y) \tag{5.22}$$

$$[G, B(y)] = 0 \tag{5.23}$$

$$\begin{aligned} &[G, \phi(y)] = -e\phi(y)\Lambda(y) \\ &[G, \phi^{\dagger}(y)] = e\phi^{\dagger}(y)\Lambda(y) \quad \text{for the scalar fields,} \end{aligned} \tag{5.24}$$

$$\begin{aligned} &[G, \psi(y)] = -e\psi(y)\Lambda(y) \\ &[G, \bar{\psi}(y)] = e\bar{\psi}(y)\Lambda(y) \quad \text{for the spinor fields,} \end{aligned} \tag{5.25}$$

$$\begin{aligned} &[G, \psi(y)] = -e\gamma_5\psi(y)\Lambda(y) \\ &[G, \bar{\psi}(y)] = e\bar{\psi}(y)\gamma_5\Lambda(y) \quad \text{for the spinor fields with p-v coupling,} \end{aligned} \tag{5.26}$$

we obtain

$$A_\mu(x) \rightarrow A'_\mu(x) = A_\mu(x) + \partial_\mu \Lambda(x) \tag{5.27}$$

$$B(x) \rightarrow B'(x) = B(x) \tag{5.28}$$

$$\begin{aligned} &\phi(x) \rightarrow \phi'(x) = e^{ie\Lambda(x)}\phi(x) \\ &\phi^{\dagger}(x) \rightarrow \phi^{\dagger\prime}(x) = e^{-ie\Lambda(x)}\phi^{\dagger}(x) \quad \text{for the scalar fields,} \end{aligned} \tag{5.29}$$

$$\begin{aligned} &\psi(x) \rightarrow \psi'(x) = e^{ie\Lambda(x)}\psi(x) \\ &\bar{\psi}(x) \rightarrow \bar{\psi}'(x) = e^{-ie\Lambda(x)}\bar{\psi}(x) \quad \text{for the spinor fields,} \end{aligned} \tag{5.30}$$

$$\begin{aligned} &\psi(x) \rightarrow \psi'(x) = e^{ie\gamma_5\Lambda(x)}\psi(x) \\ &\bar{\psi}(x) \rightarrow \bar{\psi}'(x) = \bar{\psi}(x)e^{ie\gamma_5\Lambda(x)} \quad \text{for the spinor fields with p-v coupling,} \end{aligned} \tag{5.31}$$

Thus the group $\{U[\Lambda] | U[\Lambda] = e^{iG[\Lambda]}, \Box\Lambda = 0\}$ is a set of operators which induce finite gauge transformations. By the help of equation (5.12), $G[\Lambda]$ may be written as

$$G[\Lambda] = \int d^3x B(x) \overleftrightarrow{\partial}{}_o^x \Lambda(x) \quad , \tag{5.32}$$

defining

$$G[\Lambda] = \begin{cases} G^I[\Lambda] = \int d^3x \dot{B}(x) \cdot \Lambda & \text{for } \partial_\mu \Lambda = 0 \quad (5.33) \\ G^{II}[\Lambda] = \int d^3x (\dot{B}(x)\Lambda(x) - B(x)\dot{\Lambda}(x)) & \text{for } \partial_\mu \Lambda(x) \neq 0 . \quad (5.34) \end{cases}$$

(3) Local Gauge Transformation.

We shall consider the significance of the local gauge transformation in Heisenberg representation and see how the Hamiltonian transforms under the transformation.

Consider a local gauge transformation

$$|\psi_H\rangle \rightarrow |\psi_G\rangle = e^{iG^{II}[\Lambda]} |\psi_H\rangle \quad . \tag{5.35}$$

for the physical Heisenberg state vector satisfying

$$B^{(+)}(x) |\psi_H\rangle = 0 \quad . \tag{5.36}$$

Now, from (3.18) we have

$$[G^{II}[\Lambda], B^{(+)}(x)] = 0 \quad , \tag{5.37}$$

$$\frac{d}{dt} G^{II}[\Lambda] = 0 \quad , \tag{5.38}$$

whence we know that the transformed state $|\psi_G\rangle$ is also a physical state. The expectation value of a Heisenberg operator A_H satisfying

$$[H, A_H(x)] = -i\dot{A}_H(x) \quad , \tag{5.39}$$

is transformed under the transformation (5.35) as

$$\langle\psi_H|A_H(x)|\psi_H\rangle \rightarrow \langle\psi_H|e^{-iG^{II}[\Lambda]}A_H(x)e^{iG^{II}[\Lambda]}|\psi_H\rangle \quad , \tag{5.40}$$

which implies that A_H is transformed as

$$A_H(x) \rightarrow A_G(x) = e^{-iG^{II}[\Lambda]}A_H(x)e^{iG^{II}[\Lambda]} \quad . \tag{5.41}$$

On using the equations (5.38) and (5.41), the Heisenberg equation of motion for A_G becomes

$$[H_G, A_G(x)] = -i\dot{A}_G(x) \quad : \tag{5.42}$$

where $$H_G = e^{-iG^{II}[\Lambda]}He^{iG^{II}[\Lambda]} \quad . \tag{5.43}$$

With the help of Heisenberg equations for $B(x)$ and $\dot{B}(x)$, and (5.32), we obtain

$$[H, G^{II}[\Lambda]] = iG^{II}[\dot{\Lambda}] \quad , \tag{5.44}$$

whence, from (5.43) with a formula (B-1), we have

$$H_G = H + [H, iG^{II}[\Lambda]] = H - G^{II}[\dot{\Lambda}] \quad . \tag{5.45}$$

Hence, Hamiltonian is shifted by $(-G^{II}[\dot{\Lambda}])$ under the local gauge transformation, but the expectation value of Hamiltonian is unchanged, i.e.

$$\langle\psi_H|H_G|\psi_H\rangle = \langle\psi_H|(H-G^{II}[\dot{\Lambda}])|\psi_H\rangle$$

$$= \langle\psi_H|H|\psi_H\rangle \quad , \qquad (5.46)$$

since from the subsidiary condition (5.36) we have

$$\langle\psi_H|G^{II}[\dot{\Lambda}]|\psi_H\rangle = 0 \quad , \qquad (5.47)$$

where $|\psi_H\rangle$ is a physical state. Therefore, the local gauge transformation does not have any influence on the physical energy spectra.

CHAPTER VI

SPONTANEOUS BREAKDOWN OF SYMMETRY AND A ROLE OF SUBSIDIARY CONDITION

We illustrate now the spontaneous breakdown of the global gauge transformation. A role of the subsidiary condition is studied, and we shall see the way in which the Goldstone bosons disappear from the physical states.

(1) Goldstone Theorem

Goldstone theorem can be stated as follows.

Theorem:

If the Lagrangian is invariant under a continuous transformation characterized by the generator Q, either the vacuum state (or the ground state) $|o\rangle$ is invariant under the transformation or there exist elementary excitations the energy of which vanishes as the wave number goes to zero. (Such elementary excitations are called Goldstone bosons.)

The proof of this theorem was first given by Goldstone, Salam and Weinberg (1962), which we shall not repeat here.

Invariance of the Lagrangian under a continuous transformation characterized by Q yields

$$\frac{dQ}{dt} = 0 \quad , \tag{6.1}$$

whence the Heisenberg equation of motion for Q becomes

$$[H,Q] = 0 \quad , \tag{6.2}$$

if Q does not depend explicitly on time. From (6.2) we see, therefore, that the state $Q|o\rangle$ is the vacuum state, i.e.

$$HQ|o\rangle = QH|o\rangle = 0 \quad . \tag{6.3}$$

When the condition (6.3) is satisfied, investigation of the existence of Goldstone bosons becomes equivalent to finding a field which satisfies the relation

$$\langle[\phi(x),Q]\rangle_o \neq 0 \quad . \tag{6.4}$$

This is because equation (6.4) implies that the vacuum state $|o\rangle$ is not an eigenstate of Q, i.e.

$$Q|o\rangle \neq q|o\rangle \quad , \tag{6.5}$$

where q is an arbitrary c-number constant. In other words, the vacuum state has a degeneracy.

On the other hand, from equation (6.4), the spectral representation for $\langle[\phi(x),J_\mu(y)]\rangle_o$ shows the existence of elementary excitations whose energy vanishes as the wave number goes to zero. Therefore, the degeneracy of the vacuum is due to the existence of Goldstone bosons. It is noted that in the relativistic case, the Goldstone boson must be a massless particle.

The condition that equations (6.1) and (6.5) are satisfied, is called spontaneous breakdown of symmetry generated by Q.

(2) Spontaneous Breakdown of the Global Gauge Transformation

As was seen in Chapter II, the Lagrangian (2.6) is invariant under the global gauge transformation whatever the matter field may be. On using this fact and the Goldstone theorem, we now show that spontaneous breakdown of the global gauge transformation takes place regardless of the model employed. To show this, we proceed by using the in-field representation as follows.

As was obtained in Chapter IV,

$$B(x) = Z_3^{-\frac{1}{2}} B^{in}(x) \quad , \tag{6.6}$$

$$[B^{in}(x), B^{in}(y)] = 0 \quad , \tag{6.7}$$

$$\Box B^{in}(x) = 0 \quad , \tag{6.8}$$

whence $B^{in}(x)$ can be written in the subtraction form as

$$B^{in}(x) = c(\phi_1^{in}(x) - \phi_2^{in}(x)) \quad , \tag{6.9}$$

In the above, c is a non-zero real constant and $\phi_i^{in}(x)$ (i=1,2) are real scalar fields satisfying

$$[\phi_i^{in}(x), \phi_j^{in}(y)] = ig'_{ij}D(x-y)$$

with

$$g'_{11} = -g'_{22} = -1, \; g'_{12} = g'_{21} = 0 \quad , \tag{6.10}$$

and

$$\Box\Phi_i^{in}(x) = 0 \quad \text{for} \quad i=1,2 \quad . \tag{6.11}$$

In what follows, we use $\Phi_1^{in}(x)$ instead of $B^{in}(x)$. A substitution of equation (6.9) into the generator (5.33) of the global gauge transformation leads to

$$G^I[\theta] = c\theta Z_3^{-\frac{1}{2}} \int d^3x(\dot{\Phi}_1^{in}(x) - \dot{\Phi}_2^{in}(x))$$

$$\text{for any real constant } \theta \; . \tag{6.12}$$

With the help of the commutator (6.10) and equation (6.12), we obtain

$$[G^I[\theta], \Phi_i^{in}(x)] = ic\theta Z_3^{-\frac{1}{2}} \quad \text{for} \quad i=1,2 \quad , \tag{6.13}$$

whence the fields $\Phi_i^{in}(x)$ are transformed under the global transformation as

$$\Phi_i^{in}(x) \rightarrow \Phi_i^{in\prime}(x) = \Phi_i^{in}(x) + c\theta Z_3^{-\frac{1}{2}}$$

$$\text{for } i=1,2 \quad . \tag{6.14}$$

Furthermore, the vacuum expectation value of (6.13) gives

$$<[G^I[\theta], \Phi_i^{in}(x)]>_o = ic\theta Z_3^{-\frac{1}{2}}(\neq 0) \quad \text{for} \quad i=1,2 \quad , \tag{6.15}$$

which implies

$$G^I[\theta]|o> \neq g|o> \quad , \tag{6.16}$$

where g is an arbitrary c-number constant. Besides equation (6.16), we have, from the argument in Chapter V,

$$[G^I[\theta],H] = 0 \quad , \qquad (6.17)$$

which implies that the state $G^I[\theta]|o\rangle$ is also a vacuum state. From equations (6.16) and (6.17), it follows that the vacuum state is not unique but rather degenerate, and in view of the properties of Φ_1^{in}, i.e. (6.10) ~ (6.17), $\Phi_1^{in}(x)$ and $\Phi_2^{in}(x)$ are Goldstone bosons. Thus the spontaneous breakdown of the global gauge transformation takes place regardless of the model employed.

(3) A Role of the Subsidiary Condition

Let us now investigate a role of the subsidiary condition

$$B^{in(+)}(x)|phys\rangle = 0 \quad \text{at any space-time point,} \qquad (6.18)$$

and consider this in momentum space. As a consequence of equations (6.8) and (6.11), $B^{in}(x)$ and $\Phi_1^{in}(x)$ can be written as

$$B^{in}(x) = 1/(2\pi)^{3/2} \int \frac{d^3\underset{\sim}{p}}{\sqrt{2|\underset{\sim}{p}|}} (b^{in}(\underset{\sim}{p})e^{-ipx} + h.c.) \quad , \qquad (6.19)$$

and

$$\Phi_1^{in}(x) = 1/(2\pi)^{3/2} \int \frac{d^3\underset{\sim}{p}}{\sqrt{2|\underset{\sim}{p}|}} (\varphi_1^{in}(\underset{\sim}{p})e^{-ipx} + h.c.) \quad , \qquad (6.20)$$

where $p_o = |\underset{\sim}{p}|$.

Thus, in momentum space (6.9) and (6.10) become

$$b^{in}(\underset{\sim}{p}) = c(\varphi_1^{in}(\underset{\sim}{p}) - \varphi_2^{in}(\underset{\sim}{p})) \quad \text{for any } \underset{\sim}{p} \ , \tag{6.21}$$

and

$$\begin{aligned} [\varphi_i^{in}(\underset{\sim}{p}), \varphi_j^{in^\dagger}(\underset{\sim}{q})] &= g'_{ij}\delta^3(\underset{\sim}{p}-\underset{\sim}{q}) \quad , \\ [\varphi_i^{in}(\underset{\sim}{p}), \varphi_j^{in}(\underset{\sim}{q})] &= [\varphi_i^{in^\dagger}(\underset{\sim}{p}), \varphi_j^{in^\dagger}(\underset{\sim}{q})] = 0 \end{aligned} \tag{6.22}$$

where $\varphi_i^{in}(\underset{\sim}{p}), \varphi_i^{in^\dagger}(\underset{\sim}{p})$ are, respectively, annihilation and creation operators. The subsidiary condition in momentum space is written as

$$b^{in}(\underset{\sim}{p})|phys\rangle = 0 \tag{6.23}$$

or $$(\varphi_1^{in}(\underset{\sim}{p}) - \varphi_2^{in}(\underset{\sim}{p}))|phys\rangle = 0 \text{ for any } \underset{\sim}{p} \quad . \tag{6.24}$$

Now consider a state $\varphi_1^{in^\dagger}(\underset{\sim}{p})|phys\rangle$. The use of the subsidiary condition (6.24) and the commutator (6.22) yields

$$(\varphi_1^{in}(\underset{\sim}{q}) - \varphi_2^{in}(\underset{\sim}{q}))\varphi_1^{in^\dagger}(\underset{\sim}{p})|phys\rangle = -\delta^3(\underset{\sim}{p}-\underset{\sim}{q})|phys\rangle \neq 0 \quad . \quad \text{for any } \underset{\sim}{p} \text{ and } \underset{\sim}{q} \quad . \tag{6.25}$$

Hence, the state $\varphi_1^{in^\dagger}(\underset{\sim}{p})|phys\rangle$ is unphysical. It is concluded, therefore, that the subsidiary condition prohibits the Goldstone bosons from being observed in reality. It is noted that the above argument is true independently of the Higgs Phenomenon.

<u>Example</u>

We now apply the above argument to the case of the Higgs Lagrangian studied in Chapter IV, and see explicitly the way in which the Goldstone bosons appear.

Case (A)': $\alpha=Z_3$, $M=0$, $m=0$

In this case we have

$$\Box A_\mu^{in}(x) = 0 \quad , \tag{6.26}$$

whence $A_\mu^{in}(x)$ is written as

$$A_\mu^{in}(x) = 1/(2\pi)^{3/2} \int \frac{d^3\tilde{p}}{\sqrt{2|\tilde{p}|}} \sum_{\lambda=0}^{3} e_\mu^{(\lambda)}(\tilde{p})[a_\lambda^{in}(\tilde{p})e^{-ipx} + h.c.]$$

$$\text{with } p_o = |\tilde{p}| \quad . \tag{6.27}$$

The field equation

$$\partial^\mu A_\mu^{in}(x) + B^{in}(x) = 0 \tag{6.28}$$

becomes, in momentum space,

$$b^{in}(\tilde{p}) = i|\tilde{p}|(a_o^{in}(\tilde{p}) - a_3^{in}(\tilde{p})) \quad , \tag{6.29}$$

and the commutators become

$$[a_\lambda^{in}(\tilde{p}),a_{\lambda'}^{in\dagger}(\tilde{q})] = -g_{\lambda\lambda'}\delta^3(\tilde{p}-\tilde{q}) \quad ,$$
$$[a_\lambda^{in}(\tilde{p}),b^{in\dagger}(\tilde{q})] = i|\tilde{q}|\delta^3(\tilde{p}-\tilde{q})(g_{o\lambda}-g_{3\lambda}) \quad , \tag{6.30}$$
$$[b^{in}(\tilde{p}),b^{in\dagger}(\tilde{q})] = [b^{in}(\tilde{p}),b^{in}(\tilde{q})] = [b^{in\dagger}(\tilde{p}),b^{in\dagger}(\tilde{q})] = 0 \quad .$$

Comparison of (6.21) and (6.22) with (6.29) and (6.30) gives

$$C\cdot\mathcal{P}_1^{in}(\underset{\sim}{p}) = i|\underset{\sim}{p}|a_o^{in}(\underset{\sim}{p}) \quad ,$$
$$C\cdot\mathcal{P}_2^{in}(\underset{\sim}{p}) = i|\underset{\sim}{p}|a_3^{in}(\underset{\sim}{p}) \qquad (6.31)$$

from which it follows that both scalar and longitudinal photons are interpreted as Goldstone bosons and would never be observed in reality due to the subsidiary condition

$$(a_o^{in}(\underset{\sim}{p}) - a_3^{in}(\underset{\sim}{p}))|phys\rangle = 0 \quad \text{for any } \underset{\sim}{p} \quad . \qquad (6.32)$$

Case (C)': $\alpha=0$, $M\neq 0$, $m\neq 0$

This is the case for which the Higgs Phenomenon occurs. As was studied before, we have

$$B^{in}(x) = m(\phi_1^{in}(x) - \chi^{in}(x)) \quad ,$$

whence

$$c = m \quad , \qquad (6.33)$$

$$\phi_1^{in}(x) = \phi_1^{in}(x)$$
$$\phi_2^{in}(x) = \chi^{in}(x) \qquad (6.34)$$

Thus, we again see that both $\phi_1^{in}(x)$ and $\chi^{in}(x)$ are interpreted as Goldstone bosons which would never be observed in reality due to the subsidiary condition.

CHAPTER VII

CONSERVED CURRENTS INVOLVING C-NUMBER FUNCTIONS AND GOLDSTONE THEOREM

Up to the present, Goldstone's conjecture (1961) has been proved only for the case of conserved q-number current (Goldstone, Salam and Weinberg, 1962). However the question as to whether or not the theorem applies when conserved currents or generators involve c-number functions has not been discussed.

We shall show that certain conditions on the c-number functions appearing in conserved currents restrict the properties of $<[\phi_i(x), J_\mu(y)]>_o$ in the spectral representation, where $\phi_i(x)$ represents a spinless field.† Having assumed various special forms for J_μ, we investigate the conditions under which a massless or massive mode emerge.

(1) Basic Assumptions

We restrict ourselves to J_μ of the form

$$J_\mu(x) = Q_\mu(x)C(x) + Q_{\mu\nu}(x)C^\nu(x) \quad , \tag{7.1}$$

where $Q_\mu(x)$ and $Q_{\mu\nu}(x)$ are Heisenberg operators, and $C(x)$ and $C^\nu(x)$ are c-number functions.

† The index i of $\phi_i(x)$ does not imply the Lorentz suffix.

The law of current conservation

$$\partial^\mu J_\mu(x) = 0 \quad , \tag{7.2}$$

implies that

$$\partial^\mu Q_\mu(x)\cdot C(x) + Q_\mu(x)\partial^\mu C(x) + \partial^\mu Q_{\mu\nu}(x)\cdot C^\nu(x) + Q_{\mu\nu}(x)\partial^\mu C^\nu(x) = 0 \quad . \tag{7.3}$$

Now we begin the analysis by adopting the following assumptions:

(i) At least one of the two c-number functions $C(x)$, $C_\nu(x)$ is not constant,

(ii) both of the c-number functions are independent of any choice of spacelike hypersurface, and

(iii) the theory is relativistically invariant.

Assumption (i) is needed in order that $J_\mu(x)$ involves at least one (not constant) c-number function.

(2) Spectral Representation

Let us consider a physical system with a conserved current (1), and denote a set of spinless fields by

$$\mathscr{S} = \{\phi_1(x), \phi_2(x), \ldots, \phi_i(x), \ldots\} \quad .^{\dagger} \tag{7.4}$$

The vacuum expectation value of the commutator of $J_\mu(x)$ and $\forall \phi_i(x) \in \mathscr{S}$ can be written as

---

† The fields ϕ_i need not be "fundamental" here; all remarks will apply equally well if the ϕ_i are synthetic objects.

$$\langle[\phi_i(x), J_\mu(y)]\rangle_o$$

$$= \langle[\phi_i(x), Q_\mu(y)]\rangle_o C(y) + \langle[\phi_i(x), Q_{\mu\nu}(y)]\rangle_o C^\nu(y)$$

$$= \int d\kappa^2 \{\partial_\mu^x \Delta(x-y:\kappa^2)\rho_i^{(o)}(\kappa^2)C(y)$$

$$+ g_{\mu\nu}\Delta(x-y:\kappa^2)\rho_i^{(1)}(\kappa^2)C^\nu(y)$$

$$+ \partial_\mu^x\partial_\nu^x\Delta(x-y:\kappa^2)\rho_i^{(2)}(\kappa^2)C^\nu(y)\} \quad , \qquad (7.5)$$

where $\rho_i^{(o)}$, $\rho_i^{(1)}$ and $\rho_i^{(2)}$ are spectral functions satisfying

$$\rho_i^{(o)}, \ \rho_i^{(1)} \text{ and } \rho_i^{(2)} \geq 0 \quad . \qquad (7.6)$$

It is noted here that the antisymmetric part of $Q_{\mu\nu}$ cannot contribute to the spectral representation in equation (7.5). Differentiation of equation (7.5) with respect to y_μ gives

$$0 = \langle[\phi_i(x), \partial^\mu J_\mu(y)]\rangle_o$$

$$= \int d\kappa^2\{\kappa^2\Delta(x-y:\kappa^2)\rho_i^{(o)}(\kappa^2)C(y) + \partial_\mu^x\Delta(x-y:\kappa^2)\rho_i^{(o)}(\kappa^2)\partial_y^\mu C(y)$$

$$- \partial_\nu^x\Delta(x-y:\kappa^2)\rho_i^{(1)}(\kappa^2)C^\nu(y) + \Delta(x-y:\kappa^2)\rho_i^{(1)}(\kappa^2)\partial_\nu C^\nu(y)$$

$$+ \kappa^2\partial_\nu^x\Delta(x-y:\kappa^2)\rho_i^{(2)}(\kappa^2)C^\nu(y) + \partial_\mu^x\partial_\nu^x\Delta(x-y:\kappa^2)\rho_i^{(2)}(\kappa^2)$$

$$\partial^\mu C^\nu(y)\} \quad . \qquad (7.7)$$

If we integrate (7.7) with respect to y along σ indicated by

$$\sigma = \{y | n_\mu y^\mu + \tau = 0\} \quad ,$$

and put x on σ, where τ is a real constant and n_μ is a timelike vector with $n_\mu n^\mu = 1$, then we obtain, with the help of formulae (B.17), (B.18) and (B.21),

$$0 = \int_\sigma d\sigma_\lambda(y)\langle[\phi_i(x),\partial^\mu J_\mu(y)]\rangle_o$$
$$= \int d\kappa^2 n_\lambda \frac{d}{d\tau}\int d^4y\delta(ny+\tau)\Delta(x-y:\kappa^2)\{\rho_i^{(o)}(\kappa^2)(n\partial)C(y) - \rho_i^{(1)}(\kappa^2)n_\nu C^\nu(y) + \kappa^2\rho_i^{(2)}(\kappa^2)n_\nu C^\nu(y) + \rho_i^{(2)}(\kappa^2)$$
$$(n_\mu\partial_{\nu\cdot s} + n_\nu\partial_{\mu\cdot s})\partial^\mu C^\nu(y)\} \quad . \tag{7.8}$$

Here we have used the shorthand notation

$$\partial_{\mu\cdot s} \equiv \partial_\mu - n_\mu(n\partial) \quad ,$$
$$n\partial \equiv n_\mu\partial^\mu \quad . \tag{7.9}$$

In view of the fact that

$$\left[\frac{d}{d\tau}\int d^4y\delta(ny+\tau)\Delta(x-y:\kappa^2)\right]_{x/\sigma} = \left[\int d^4y(n\partial^y)\delta(ny+\tau)\Delta(x-y:\kappa^2)\right]_{x/\sigma}$$

is independent of the choice of n_μ, we may choose $n_\mu = (1.0,0.0)$, whence the above quantity is equal to (-1) and is independent of κ^2. Thus, from (7.8) we obtain

$$\int d\kappa^2\{\rho_i^{(o)}(\kappa^2)(n\partial)C(y) - \rho_i^{(o)}(\kappa^2)n_\nu C^\nu(y) + \kappa^2\rho_i^{(2)}(\kappa^2)n_\nu C^\nu(y)$$
$$+ \rho_i^{(2)}(\kappa^2)(n_\mu\partial_{\nu\cdot s} + n_\nu\partial_{\mu\cdot s})\partial^\mu C^\nu(y)\} = 0 \quad . \tag{7.10}$$

In a similar manner, if we integrate (7.7) along the surface σ with respect to x and put y on σ, we have

$$0 = \int_\sigma d\sigma_\lambda(x)\langle[\phi_i(x),\partial^\mu J_\mu(y)]\rangle_o$$

$$= \int d\kappa \text{ [illegible] } d^4x\delta(nx+\tau)\Delta(x-y:\kappa^2)\{\rho_i^{(o)}(\kappa^2)(n\partial)C(y)$$

$$\text{[illegible]} C^\nu(y) + \kappa^2\rho_i^{(2)}(\kappa^2)n_\nu C^\nu(y)\} \text{ for } y/\sigma \quad . \qquad (7.11)$$

Hence we have

$$\int d\kappa^2\{\rho_i^{(o)}(\kappa^2)(n\partial)C(y)-\rho_i^{(1)}(\kappa^2)n_\nu C^\nu(y)+\kappa^2\rho_i^{(2)}(\kappa^2)n_\nu C^\nu(y)\}=0 \quad . \qquad (7.12)$$

From (7.10) and (7.12) we have

$$\int d\kappa^2\rho_i^{(2)}(\kappa^2)(n_\mu\partial_{\nu\cdot s}+n_\nu\partial_{\mu\cdot s})\partial^\mu C^\nu(y) = 0 \quad . \qquad (7.13)$$

If we perform the differential operation $(n\partial^x)$ on both sides of equation (7.7), integrate it along the surface σ with respect to y and put x on σ, then we have

$$0 = \int_\sigma d\sigma_\lambda(y)\langle[n\partial^x\phi_i(x),\partial^\mu J_\mu(y)]\rangle_o$$

$$= n_\lambda \int d\kappa^2 \frac{d}{d\tau} \int d^4y\delta(ny+\tau)\Delta(x-y:\kappa^2)[-\rho_i^{(o)}(\kappa^2)\partial_{\mu\cdot s}\partial^\mu{}_{\cdot s}C(y)$$

$$+ \rho_i^{(1)}(\kappa^2)\partial_{\nu\cdot s}C^\nu(y)-\kappa^2\rho_i^{(2)}(\kappa^2)\partial_{\mu\cdot s}C^\mu(y)+\rho_i^{(2)}(\kappa^2)\partial_{\mu\cdot s}\partial^\mu{}_{\cdot s}\partial_{\nu\cdot s}C^\nu(y)$$

$$- \rho_i^{(2)}(\kappa^2)\partial_{\mu\cdot s}\partial^\nu{}_{\cdot s}(n\partial)n_\nu C^\nu(y)$$

$$+ \{\kappa^2\rho_i^{(o)}(\kappa^2)C(y)+\partial_\nu C^\nu(y)\rho_i^{(1)}(\kappa^2)-\kappa^2\rho_i^{(2)}(\kappa^2)(n\partial)n_\nu C^\nu(y)\}]$$

$$\text{for } x/\sigma \quad ,$$

where use has been made of formulae (B.20), (B.21) and (B.26). Hence we obtain

$$\int d\kappa^2[-\rho_i^{(o)}(\kappa^2)\partial_{\mu\cdot s}\partial^{\mu}_{\cdot s}C(y) + \rho_i^{(1)}(\kappa^2)\partial_{\mu\cdot s}C^{\mu}(y)-\kappa^2\rho_i^{(2)}(\kappa^2)\partial_{\mu\cdot s}C^{\mu}(y)$$

$$+ \rho_i^{(2)}(\kappa^2)\partial^{\mu}_{\cdot s}\partial_{\mu\cdot s}\partial_{\nu\cdot s}C^{\nu}(y)-\rho_i^{(2)}(\kappa^2)\partial_{\mu\cdot s}\partial^{\mu}_{\cdot s}(n\partial)n_{\nu}C^{\nu}(y)$$

$$+\{\kappa^2\rho_i^{(o)}(\kappa^2)C(y)+\partial_{\nu}C^{\nu}(y)\rho_i^{(1)}(\kappa^2)-\kappa^2\rho_i^{(2)}(\kappa^2)(n\partial)n_{\nu}C^{\nu}(y)\}]=0. \tag{7.14}$$

Similarly if we perform the operation $(n\partial^x)$ on equation (7.7) and integrate it along the surface σ with respect to x, moreover, put y on σ, then we obtain

$$0 = \int_{\sigma} d\sigma_{\lambda}(x)\langle[n\partial^x\phi_i(x),\partial^{\mu}J_{\mu}(y)]\rangle_o$$

$$= \int d\kappa^2 \frac{d}{d\tau}\int d^4x\delta(nx+\tau)\Delta(x-y:\kappa^2)(-n_{\lambda}\kappa^2\rho_i^{(o)}(\kappa^2)C(y)$$

$$- n_{\lambda}\rho_i^{(1)}(\kappa^2)\partial_{\nu}C^{\nu}(y) + n_{\lambda}\kappa^2\rho_i^{(2)}(\kappa^2)(n\partial)n_{\nu}C^{\nu}(y))$$

for y/σ ,

whence we have

$$\int d\kappa^2(\kappa^2\rho_i^{(o)}(\kappa^2)C(y)+\rho_i^{(1)}(\kappa^2)\partial_{\nu}C^{\nu}(y)-\kappa^2\rho_i^{(2)}(\kappa^2)(n\partial)n_{\nu}C^{\nu}(y))=0. \tag{7.15}$$

From (7.14) and (7.15) we obtain

$$\int d\kappa^2[-\rho_i^{(o)}(\kappa^2)\partial_{\mu\cdot s}\partial^{\mu}_{\cdot s}C(y)+\rho_i^{(1)}(\kappa^2)\partial_{\mu\cdot s}C^{\mu}(y)-\kappa^2\rho_i^{(2)}(\kappa^2)\partial_{\mu\cdot s}C^{\mu}(y)$$

$$+\rho_i^{(2)}(\kappa^2)\partial_{\mu\cdot s}\partial^{\mu}_{\cdot s}\partial_{\nu\cdot s}C^{\nu}(y)-\rho_i^{(2)}(\kappa^2)\partial_{\mu\cdot s}\partial^{\mu}_{\cdot s}(n\partial)n_{\nu}C^{\nu}(y)]=0 \tag{7.16}$$

The operation $(n\partial^y)$ on (7.12) gives

$$\int d\kappa^2[-\rho_i^{(o)}(\kappa^2)(n\partial)^2C(y)+\rho_i^{(1)}(\kappa^2)(n\partial)n_\mu C^\mu(y)-\kappa^2\rho_i^{(2)}(\kappa^2)(n\partial)n_\mu C^\mu(y)] = 0 \qquad (7.17)$$

{(7.15)-(7.16)+(7.17)} with the help of (7.13) gives

$$\int d\kappa^2[\rho_i^{(o)}(\kappa^2)(\Box+\kappa^2)C(x)+\rho_i^{(2)}(\kappa^2)(\Box+\kappa^2)\partial_{\mu\cdot s}C^\mu(x)] = 0. \qquad (7.18)$$

The argument in the following section is based on the above five relations (7.12), (7.13), (7.15), (7.16) and (7.18).

(3) Special Cases

We shall discuss the following cases separately with the help of equations (7.12), (7.13), (7.15), (7.16) and (7.18) under the assumption (i), (ii) and (iii).

(A) $J_\mu = Q_\mu(x)\cdot C(x)$

(B) $J_\mu = Q_{\mu\nu}(x)\cdot C_\nu(x)$

(C) $J_\mu = Q_\mu(x)\cdot C(x) + Q(x)\cdot C_\mu(x)$

(A) $J_\mu = Q_\mu(x)\cdot C(x)$

In this case assumption (i) implies

$$C(x) \neq 0 \quad , \qquad (7.19)$$

$$\partial_\mu C(x) \neq 0 \quad , \qquad (7.20)$$

and equation (7.12) becomes

$$\int d\kappa^2\rho_i^{(o)}(\kappa^2)n_\mu\partial^\mu C(x) = 0 \quad . \qquad (7.21)$$

From assumption (i) and condition (7.20), $n_\mu \partial^\mu C(x)$ does not vanish. Hence, equation (7.21) together with (7.6) reduces to

$$\rho_i^{(o)}(\kappa^2) = 0 \quad \text{for any } i \quad . \tag{7.22}$$

It is concluded therefore, that there is no Goldstone boson in this case. However, if we remove the condition (7.20) from the theory, that is, if J_μ is a q-number current, we cannot derive equation (7.22). However, we have, from equation (7.15) with the condition (7.19)

$$\int d\kappa^2 \rho_i^{(o)}(\kappa^2)\cdot\kappa^2 = 0 \quad ,$$

which implies

$$\rho_i^{(o)}(\kappa^2) = \beta_i^{(o)}\delta(\kappa^2) \quad \text{for any } i \quad , \tag{7.23}$$

where $\beta_i^{(o)}$ is a positive constant. Therefore, when $C(x) = \theta$ = constant, there exists a massless mode if $\beta_i^{(o)} \neq 0$. We will not discuss this mode subsequently, for this is nothing but Goldstone, Salam and Weinberg's well-known result.

(B) $J_\mu(x) = Q_{\mu\nu}(x)C^\nu(x)$

In this case the assumption (i) implies

$$C_\nu(x) \neq 0 \quad , \tag{7.24}$$

$$\partial_\mu C_\nu(x) \neq 0 \quad . \tag{7.25}$$

With the help of assumption (ii) and the condition (7.24), basic equations (7.12), (7.15), (7.16) and (7.18) become

$$\int d\kappa^2[\rho_i^{(1)}(\kappa^2)-\kappa^2\rho_i^{(2)}(\kappa^2)] = 0 \quad , \tag{7.26}$$

$$\int d\kappa^2[\rho_i^{(1)}(\kappa^2)\partial_\nu C^\nu(y)-\kappa^2\rho_i^{(2)}(\kappa^2)n_\mu n_\nu\partial^\mu C^\nu(y)] = 0 \quad , \tag{7.27}$$

$$\int d\kappa^2[-\rho_i^{(2)}(\kappa^2)\partial_{\mu\cdot s}\partial^\mu_{\cdot s}n_\nu n_\rho\partial^\nu C^\rho+\rho_i^{(2)}(\kappa^2)\partial_{\mu\cdot s}\partial^\mu_{\cdot s}\partial_{\nu\cdot s}C^\nu] = 0 \tag{7.28}$$

$$\int d\kappa^2\rho_i^{(2)}(\kappa^2)(\Box+\kappa^2)\partial_{\mu\cdot s}C^\mu = 0 \quad \text{for any } i \quad , \tag{7.29}$$

and equation (7.13) is still the same. The second term in equation (7.27) depends on the choice of the spacelike hypersurface. There are, however, the following cases in which this dependence does not appear.

I. $\partial^\mu C^\nu = \partial^\nu C^\mu$ (symmetrical case)

This case will further be divided into two cases

(a) $\partial^\mu C^\nu = g^{\mu\nu}C'(x)$

(b) $\partial^\mu C^\nu = \partial^\mu\partial^\nu C''(x)$

In case (b), we must put

$$\int d\kappa^2\kappa^2\rho_i^{(2)}(\kappa^2) = 0 \quad \text{for any } i \quad ,$$

II. $\partial^\mu C^\nu + \partial^\nu C^\mu = 0$ (antisymmetrical case)

Now let us discuss the above cases separately.

I. $\partial^\mu C^\nu = \partial^\nu C^\mu$

(a) $\partial^\mu C^\nu = g^{\mu\nu} C'(x)$ (7.30)

From the condition (7.25), we have

$$C'(x) \neq 0 \quad , \tag{7.31}$$

and equation (7.27) and (7.26) under the conditions (7.6) and (7.31) give

$$\rho_i^{(1)}(\kappa^2) = 0 \quad , \tag{7.32}$$

$$\rho_i^{(2)}(\kappa^2) = \beta_i^{(2)} \delta(\kappa^2) \quad \text{for any } i \quad , \tag{7.33}$$

where $\beta_i^{(2)}$ is a positive constant. Thus, if $\beta_i^{(2)} \neq 0$, we see that there exists a massless mode. When $\beta_i^{(2)} \neq 0$ for a ϕ_i, equations (7.28) and (7.29) together with (7.33) become

$$(\; -(n\partial)^2) C'(x) = 0 \quad , \tag{7.34}$$

$$\Box C'(x) = 0 \quad , \tag{7.35}$$

respectively. Hence, with assumption (ii) we obtain

$$\partial_\mu \partial_\nu C'(x) = 0 \quad . \tag{7.36}$$

Also, equation (7.5) reduces to

$$\langle [\phi_i(x), J_\mu(y)] \rangle_0 = \beta_i^{(2)} \partial_\mu^x \partial_\nu^x D(x-y) C^\nu(y) \quad , \tag{7.37}$$

and the surface integral of (7.37) with respect to y gives

$$<[\phi_i(x),Q]>_o = -3\beta_i^{(2)}\cdot C'(x) \neq 0 \quad \text{for } \beta_i^{(2)}\neq 0 \quad , \tag{7.38}$$

from which it follows that

$$Q|o> \neq q|o> \quad . \tag{7.39}$$

Here

$$Q = \int d\sigma^\mu(y) J_\mu(y) \quad , \tag{7.40}$$

and q is an arbitrary c-number constant.

As an example, we may take a scale invariant model for a real scalar field (Aurilia et al. 1971, Umezawa 1974), whereby the current J^μ is given by

$$J_\mu = \theta_{\mu\nu}\cdot x^\nu \quad , \tag{7.41}$$

where $\theta_{\mu\nu}$ is the energy-stress tensor satisfying

$$\partial^\mu \theta_{\mu\nu} = 0 \quad , \tag{7.42}$$

$$\theta_\mu{}^\mu = 0 \quad . \tag{7.43}$$

The commutator

$$[H,D] = i\frac{\partial}{\partial t} D(\neq 0) \quad , \tag{7.44}$$

where $\quad D = \int d^3x J_o(x) \quad .$

does not vanish, but since $i\frac{\partial}{\partial t}D$ is equal to iH, we have

$$i\frac{\partial}{\partial t} D|o> = 0 \quad , \tag{7.45}$$

which implies equation (6.3). Therefore, in this system there exists a Goldstone boson for $\beta_i^{(2)} \neq 0$.

(b) $\partial^\mu C^\nu = \partial^\mu \partial^\nu C''(x)$

From condition (7.25), we have

$$C''(x) \neq 0 \quad , \tag{7.47}$$

and from (7.46) we obtain

$$\partial_\mu C^\mu(x) = \Box C''(x) \quad , \tag{7.48}$$

$$n_\mu n_\nu \partial^\mu C^\nu = (n\partial)^2 C''(x) \neq 0 \quad . \tag{7.49}$$

Since $n_\mu n_\nu \partial^\mu C^\nu \neq 0$, in equation (7.27) we must put

$$\int d\kappa^2 \kappa^2 \rho_i^{(2)}(\kappa^2) = 0 \quad , \text{ i.e.,}$$

$$\rho_i^{(2)}(\kappa^2) = \beta_i^{(2)} \delta(\kappa^2) \quad \text{for any } i \quad , \tag{7.50}$$

which implies that there exists a massless mode if $\beta_i^{(2)} \neq 0$. From equations (7.26) and (7.50) with the condition (7.6), we obtain

$$\rho_i^{(1)}(\kappa^2) = 0 \quad \text{for any } i \quad . \tag{7.51}$$

By comparing equations (7.32) and (7.33) with (7.51) and (7.52), respectively, we see that both cases (a) and (b) give the same result concerning the spectral functions. Equation (7.5) also reduces to the same result as (7.37) but the commutator $<[\phi_i(x),Q]>_o$ is given by

$$<[\phi_i(x),Q]>_o = -\beta_i^{(2)} \sum_k \partial_k C_k(x) \neq 0 \quad \text{for } \beta_i^{(2)} \neq 0 \ , \tag{7.52}$$

which implies

$$Q|o> \neq q|o> \quad . \tag{7.53}$$

II. $\partial^\mu C^\nu + \partial^\nu C^\mu = 0$ (7.54)

In this case we have, from (7.54),

$$n_\mu n_\nu \partial^\mu C^\nu = 0 \quad , \tag{7.55}$$

$$\partial_\mu C^\mu = 0 \quad , \tag{7.56}$$

from which it follows that

$$\partial_{\mu \cdot s} C^\mu \doteq 0 \quad . \tag{7.57}$$

Moreover, from equations (7.55), (7.56) and (7.57), we see that equations (7.13), (7.27), (7.28) and (7.29) do not yield information, but equation (7.26)

$$\rho_i^{(1)}(\kappa^2) = \kappa^2 \rho_i^{(2)}(\kappa^2) \quad , \tag{7.58}$$

implies that we cannot conclude that there exists either a massless or massive mode. Equation (7.5) reduces to

$$<[\phi_i(x),J_\mu(y)]>_o = \int d\kappa^2 \rho_i^{(2)}(\kappa^2)(g_{\mu\nu}\kappa^2 + \partial_\mu^x \partial_\nu^x)\Delta(x-y:\kappa^2)C^\nu(y), \tag{7.59}$$

and its surface integration with respect to y gives

$$<[\phi_i(x),Q]>_o = -(\int d\kappa^2 \rho_i^{(2)}(\kappa^2)) \sum_k \partial_k^x C_k(x) = 0$$

$$\text{for } \rho_i^{(2)}(\kappa^2) \neq 0 \quad , \tag{7.60}$$

on account of (7.57). This implies that even if

$$\rho_i^{(2)}(\kappa^2) = \beta_i^{(2)}\delta(\kappa^2) \quad \text{for } \beta_i^{(2)} \neq 0 \quad ,$$

equation (7.59) is always satisfied.

(C) $J_\mu(x) = Q_\mu(x) \cdot C(x) + Q(x) \cdot C_\mu(x)$

We assume

$$C(x) \neq 0 \tag{7.61}$$

$$C_\mu(x) \neq 0 \quad . \tag{7.62}$$

With the help of assumption (ii), the basic equations (7.12), (7.15), (7.16) and (7.18) become, respectively,

$$\int d\kappa^2[\rho_i^{(o)}(\kappa^2)\partial^\mu C(x) - \rho_i^{(1)}(\kappa^2)C^\mu(x)] = 0 \quad , \tag{7.63}$$

$$\int d\kappa^2[\kappa^2\rho_i^{(o)}(\kappa^2) + \rho_i^{(1)}(\kappa^2)\partial_\mu C^\mu(x)] = 0 \quad , \tag{7.64}$$

$$\int d\kappa^2[\rho_i^{(o)}(\kappa^2)\partial_{\mu\cdot s}\partial^\mu_{\cdot s}C(x) - \rho_i^{(o)}(\kappa^2)\partial_{\mu\cdot s}C^\mu(x)]=0, \tag{7.65}$$

$$\int d\kappa^2\rho_i^{(o)}(\kappa^2)(\Box+\kappa^2)C(x) = 0 \quad , \tag{7.66}$$

and equation (7.13) remains unaltered.

Let us consider three cases:

(a) $\partial_\mu C(x) \neq 0$ and $\partial_\mu C_\nu(x) \neq 0$,

(b) $\partial_\mu C(x) = 0$ and $\partial_\mu C_\nu(x) \neq 0$,

(c) $\partial_\mu C(x) \neq 0$ and $\partial_\mu C_\nu(x) = 0$,

separately.

(a) $\partial_\mu C \neq 0$, $\partial_\mu C_\mu \neq 0$

This case is divided into two sub-cases:

(i) $C^\mu(x) \neq \alpha\partial^\mu C(x)$,

(ii) $C^\mu(x) = \alpha\partial^\mu C(x)$,

where α is any non-zero c-number constant.

(i) $C^\mu \neq \alpha\partial^\mu C$

From equation (7.63), together with the condition (7.6), we have

$$\int d\kappa^2 \rho_1^{(o)}(\kappa^2) = 0$$

$$\text{i.e.} \quad \rho_1^{(o)}(\kappa^2) = 0 \qquad (7.67)$$

$$\int d\kappa^2 \rho_1^{(1)}(\kappa^2) = 0$$

$$\text{i.e.} \quad \rho_1^{(1)}(\kappa^2) = 0 \quad , \qquad (7.68)$$

whence there is no Goldstone boson. Equation (7.5) reduces to

$$<[\phi_1(x), J_\mu(y)]>_o = 0 \quad , \qquad (7.69)$$

so that

$$<[\phi_1(x), Q]>_o = 0 \quad , \qquad (7.70)$$

and thus

$$Q|o> = q|o> \quad . \qquad (7.71)$$

Therefore, we see that the vacuum is not degenerate.

(ii) $C^\mu = \alpha\partial^\mu C \ (\alpha\neq 0)$

From equation (7.63), we have

$$\int d\kappa^2[\rho_i^{(o)}(\kappa^2) - \alpha\rho_i^{(1)}(\kappa^2)] = 0 \quad \text{for any } i \quad , \tag{7.72}$$

and using (7.72) in (7.66), we obtain

$$\int d\kappa^2\cdot\rho_i^{(1)}(\kappa^2)(\Box+\kappa^2)C(x) = 0 \quad \text{for any } i \quad . \tag{7.73}$$

Hence, from equation (7.73) and (7.66), we have

$$\rho_i^{(1)}(\kappa^2) = \beta_i^{(1)}\delta(\kappa^2-m^2) \quad , \tag{7.74}$$

$$\rho_i^{(o)}(\kappa^2) = \beta_i^{(o)}\delta(\kappa^2-m^2) \quad , \quad \text{for any } i \quad , \tag{7.75}$$

where $\quad \beta_i^{(o)} = \alpha\beta_i^{(1)} \quad , \quad$ for any i $\quad , \qquad (7.76)$

and m^2 is given by the relation ,

$$\frac{\Box C}{C} = -m^2 \quad \text{or} \quad (\Box+m^2)C = 0 \quad . \tag{7.77}$$

Therefore, it is concluded that there can exist either a massless or a massive mode if $\beta_i^{(o)}\neq 0$. Equation (7.5) reduces to

$$<[\phi_i(x),J_\mu(y)]>_o = \beta_i^{(o)}\{\partial_\mu^x\Delta(x-y:m^2)C(y) + \Delta(x-y:m^2)\partial_\mu C(y)\} \quad , \tag{7.78}$$

and its surface integral with respect to y gives

$$<[\phi_i(x),Q]>_o = -\beta_i^{(o)}C(x) \neq 0 \qquad (7.79)$$

$$\text{for} \quad \beta_i^{(o)} \neq 0 \quad , \qquad (7.80)$$

which implies that

$$Q|o> \neq q|o> \quad . \qquad (7.81)$$

Thus, we see that even if we have equation (7.79) or (7.81), we cannot conclude that there exists a Goldstone boson.

Now we shall present three example models, the conserved currents of which take the form

$$J_\mu(x) = Q_\mu(x)C(x) + \alpha Q(x)\partial_\mu C(x) \quad , \qquad (7.82)$$

but have no massless mode.

(1) Free real scalar field b(x)

The Lagrangian

$$L = -\tfrac{1}{2}\,\partial^\mu b\partial_\mu b + \frac{m^2}{2}\,b^2 \quad (m\neq 0) \quad , \qquad (7.83)$$

is invariant under the transformation

$$b(x) \rightarrow b(x) + m\Lambda(x) \quad , \qquad (7.84)$$

where $\Lambda(x)$ is an arbitrary, real, scalar, c-number function satisfying

$$(\Box+m^2)\Lambda(x) = 0 \quad . \qquad (7.85)$$

The conserved current

$$J_\mu(x) = b(x)\overleftrightarrow{\partial}_\mu \Lambda(x) \tag{7.86}$$

gives the relation

$$<[b(x),Q]>_o = i\Lambda(x) \neq 0 \quad , \tag{7.87}$$

which implies that we have

$$Q|o> \neq q|o> \quad , \tag{7.88}$$

but also

$$\Box_x <[b(x),Q]> = -im^2\Lambda(x) \neq 0 \quad . \tag{7.89}$$

Therefore, though (7.88) holds, from (7.89) it is seen that there exists no massless mode .

(2) Stueckelberg Field (Stueckelberg 1938)

The Lagrangian

$$L = -\tfrac{1}{2}\partial^\mu A^\nu \cdot \partial_\mu A_\nu + \frac{m^2}{2} A^\mu A_\mu - \tfrac{1}{2}\partial^\mu b \partial_\mu b + \frac{m^2}{2} b^2 \quad \text{with } m^2 \neq 0 \tag{7.90}$$

is invariant under the gauge transformation

$$\begin{aligned} A_\mu(x) &\to A_\mu(x) + \partial_\mu \Lambda(x) \\ b(x) &\to b(x) + m\Lambda(x) \end{aligned} \tag{7.91}$$

where $\Lambda(x)$ is the same function as in (1).

The conserved current

$$J_\mu(x) = (\partial^\nu A_\nu(x) + mb(x))\overleftrightarrow{\partial}_\mu \Lambda(x) \quad , \qquad (7.92)$$

gives the relations

$$\langle[b(x),G]\rangle_o = im\Lambda(x) \quad , \qquad (7.93)$$

$$\langle[\partial^\mu A_\mu(x),G]\rangle_o = -im^2\Lambda(x) \quad , \qquad (7.94)$$

which imply

$$G|o\rangle \neq g|o\rangle \quad , \qquad (7.95)$$

where $$G = \int d^3y J_o(y) \quad . \qquad (7.96)$$

However, since we have

$$\Box_x\langle[b(x),G]\rangle_o = -im^3\Lambda(x) \neq 0 \quad , \qquad (7.97)$$

$$\Box_x\ [\partial^\mu A_\mu(x),G]\rangle_o = im^4\Lambda(x) \neq 0 \quad , \qquad (7.98)$$

we conclude that there exists no massless mode. The subsidiary condition

$$(\partial^\mu A_\mu(x) + mb(x))^{(+)}|phys\rangle = 0 \quad \text{for any } x \quad , \qquad (7.99)$$

is used for this system (Gupta 1951).

(3) Massive Neutral Vector Field (Nakanishi 1976)

The Lagrangian

$$L = -\tfrac{1}{4} F^{\mu\nu}F_{\mu\nu} + \frac{m^2}{2} A_\mu A^\mu + B\partial^\mu A_\mu + \frac{B^2}{2} \quad \text{with } m^2 \neq 0$$

$$(7.100)$$

is invariant under the gauge transformation

$$A_\mu(x) \to A_\mu(x) + \partial_\mu \Lambda(x)$$
$$B(x) \to B(x) + m^2 \Lambda(x) \tag{7.101}$$

$$\text{with } (\Box + m^2)\Lambda(x) = 0 \quad . \tag{7.102}$$

The conserved current

$$J_\mu(x) = B(x)\overleftrightarrow{\partial}_\mu \Lambda(x) \tag{7.103}$$

gives the relation

$$<[B(x),G]>_o = -im^2\Lambda(x) \neq 0 \tag{7.104}$$

which implies

$$G|o> \neq g|o> \tag{7.105}$$

where $$G = \int d^3y J_o(y) \quad . \tag{7.106}$$

However, since we have

$$\Box_x <[B(x),G]>_o = im^4\Lambda(x) \neq 0 \quad , \tag{7.107}$$

we conclude there exists no massless mode. The subsidiar condition

$$B^{(+)}(x)|phys> = 0 \quad \text{for any } x \quad , \tag{7.108}$$

is used for this system.

(b) $\partial_\mu C(x)=0$ and $\partial_\mu C_\nu \neq 0$

Equation (7.63) becomes

$$\int d\kappa^2 \rho_i^{(1)}(\kappa^2) = 0 \quad \text{i.e.,}$$

$$\rho_i^{(1)}(\kappa^2) = 0 \quad \text{for any } i \quad , \tag{7.109}$$

and (7.64) with (7.109) gives

$$\int d\kappa^2 \rho_i^{(o)}(\kappa^2)\cdot\kappa^2 = 0 \quad , \text{ i.e.}$$

$$\rho_i^{(o)}(\kappa^2) = \beta_i^{(o)}\delta(\kappa^2) \quad \text{for any } i \quad . \tag{7.110}$$

Equation (7.110) implies that there exists a massless mode if $\beta_i^{(o)} \neq 0$. Equation (7.5) reduces to

$$<[\phi_i(x),J_\mu(y)]>_o = \beta_i^{(o)}\partial_\mu^x\Delta(x-y:m^2)C(y) \quad , \tag{7.111}$$

and its surface integral with respect to y gives

$$<[\phi_i(x),Q]>_o = -\beta_i^{(o)}\cdot C(x)\neq 0 \quad \text{for } \beta_i^{(o)}\neq 0 \tag{7.112}$$

which implies

$$Q|o> \neq q_o|o> \quad . \tag{7.113}$$

(c) $\partial_\mu C \neq 0$ and $\partial_\mu C_\nu = 0$

This case will be divided into two sub-cases.

(i) $C^\mu \neq \alpha\partial^\mu C$

From equation (7.63), we have

$$\rho_i^{(o)}(\kappa^2) = 0 \quad , \tag{7.114}$$

$$\rho_i^{(1)}(\kappa^2) = 0 \quad \text{for any } i \quad . \tag{7.115}$$

Hence, we have

$$<[\phi_i(x),Q]>_o = 0 \quad , \tag{7.116}$$

which implies

$$Q|o> = q|o> \quad . \tag{7.117}$$

(ii) $C^\mu=\alpha\partial^\mu C$

From equation (7.64) we have

$$\rho_i^{(o)}(\kappa^2)=\beta_i^{(o)}\delta(\kappa^2) \text{ for any } i \quad , \tag{7.118}$$

which implies that there exists a massless mode if $\beta_i^{(o)}\neq 0$. From equation (7.63) with (7.118) we have

$$\int d\kappa^2\rho_i^{(1)}(\kappa^2) = \beta_i^{(o)}/\alpha \quad . \tag{7.119}$$

The surface integral of equation (7.5) becomes

$$<[\phi_i(x),Q]>_o = -\beta_i^{(o)}C(x)\neq 0 \text{ for } \beta_i^{(o)}\neq 0 \quad , \tag{7.120}$$

which implies

$$Q|o> \neq q|o> \quad . \tag{7.121}$$

(4) Summary

In the above argument, we have shown that certain conditions on the c-number functions appearing in conserved currents yield relationships among the spectral functions of

$<[\phi_1(x), J_\mu(y)]>_0$. This is summarized in Table 1.

In the case of the local gauge transformation discussed in Chapter V, from equation (5.44) it follows that equations (6.2) and (6.3) are not satisfied. Thus, in spite of the existence of a massless field B(x), we cannot conclude, from the discussion of the local gauge transformation, that the vacuum is degenerate. Moreover, as was shown by three example models in (C-a-ii), the generator Q may involve a massive field. In these cases, equation (6.5) does not always imply the degeneracy of the vacuum (see Table 1).

The above argument implies that the Goldstone theorem does not always apply when conserved currents or generators involve c-number functions, and suggests that the theorem must be extended when conserved currents involve c-number functions.

TABLE 1 List of Structure for Spectral Functions

$J_\mu(x)$	Conditions on c-number functions	Structure of spectral functions	Value of $<[\phi_i(x),Q]>_0$
$Q_\mu(x)C(x)$	I. $\partial_\mu C(x)\neq 0$	$\rho_i^{(0)}(\kappa^2)=0$	0
$C(x)\neq 0$	II. $C(x)=\theta=\text{const.}$	$\rho_i^{(0)}(\kappa^2)=\beta_i^{(0)}\delta(\kappa^2)$	$-\beta_i^{(0)}\cdot\theta$
$Q_{\mu\nu}(x)C^\nu(x)$	I. $\partial^\mu C^\nu=\partial^\nu C^\mu$ (a) $\partial^\mu C^\nu=g^{\mu\nu}C'(x)$	$\rho_i^{(1)}(\kappa^2)=0$ $\rho_i^{(2)}(\kappa^2)=\beta_i^{(2)}\delta(\kappa^2)$	$-3\beta_i^{(2)}C'(x)$
$C_\nu(x)\neq 0$ $\partial_\mu C_\nu(x)\neq 0$	(b) $\partial^\mu C^\nu=\partial^\mu\partial^\nu C''(x)$	$\rho_i^{(1)}(\kappa^2)=0$ $\rho_i^{(2)}(\kappa^2)=\beta_i^{(2)}\delta(\kappa^2)$	$-\beta_i^{(2)}\sum_k\partial_k C_k(x)$
	II. $\partial^\mu C^\nu+\partial^\nu C^\mu=0$	$\rho_i^{(1)}(\kappa^2)=\kappa^2\rho_i^{(2)}(\kappa^2)$	0
$Q_\mu(x)C(x)+Q(x)C_\mu(x)$	(a) $\partial_\mu C\neq 0,\ \partial_\mu C_\nu\neq 0$ (i) $C^\mu\neq\alpha\partial^\mu C$	$\rho_i^{(0)}(\kappa^2)=\rho_i^{(1)}(\kappa^2)=0$	0
$C(x)\neq 0$ $C_\mu(x)\neq 0$	(ii) $C^\mu=\alpha\partial^\mu C$	$\rho_i^{(0)}(\kappa^2)=\beta_i^{(0)}\delta(\kappa^2-m^2)$ $\rho_i^{(1)}(\kappa^2)=\beta_i^{(1)}\delta(\kappa^2-m^2)$	$-\beta_i^{(0)}C(x)$
	(b) $\partial_\mu C=0$ $\partial_\mu C_\nu\neq 0$	$\rho_i^{(0)}(\kappa^2)=\beta_i^{(0)}\delta(\kappa^2)$ $\rho_i^{(1)}(\kappa^2)=0$	$-\beta_i^{(0)}C(x)$
	(c) $\partial_\mu C\neq 0,\ \partial_\mu C_\nu=0$ (i) $C^\mu\neq\alpha\partial^\mu C$	$\rho_i^{(0)}(\kappa^2)=\rho_i^{(1)}(\kappa^2)=0$	0
	(ii) $C^\mu=\alpha\partial^\mu C$	$\rho_i^{(0)}(\kappa^2)=\beta_i^{(0)}\delta(\kappa^2)$ $\rho_i^{(1)}(\kappa^2)=\beta_i^{(0)}/\alpha$	$-\beta_i^{(0)}C(x)$

REFERENCES

A. Aurilia, Y. Takahashi and H. Umezawa: Spontaneous Breakdown of Symmetry and the Gauge Invariance in a Relativistic Field Theory. Progr. Theor. Phys. 48 (1972) 290.

A. Aurilia, Y. Takahashi and H. Umezawa: Dilatation and Dimensional Transformations. Phys. Rev. D5 (1972) 851.

A. Aurilia, Y. Takahashi, N. J. Papastamatiou and H. Umezawa: Spontaneous Violation of Dilatation Invariance in Relativistic Field Theories. Phys. Rev. D5 (1972) 3066.

A. Aurilia and Y. Takahashi: Comment on the Breakdown of Dilatation Invariance. Phys. Rev. D6 (1972) 2294.

K. Bleuler: Eine neue Methode zur Behandlung der longitudinalen und skalaren Photonen. Helv. Phys. Acta 23 (1950) 567.

F. J. Belinfante: Direct Proof of the Covariance of Gupta's Indefinite Metric in Quantum Electrodynamics. Phys. Rev. 96 (1954) 780.

R. A. Brandt and N. W. Chiu: Gauge Invariance and Mass. Phys. Rev. D10 (1974) 4198.

S. Coleman: There are no Goldstone Bosons in Two Dimensions. Commun. Math. Phys. 31 (1973) 259.

P. A. M. Dirac: The Physical Interprepation of Quntum Mechanics. Proc. Roy. Soc. A180 (1942) 1.

K. Dormuth and Y. Takahashi: Hyperquantization of the Electromagnetic and Spin ½ Fields. Progr. Theor. Phys. 43 (1970) 1386.

Y. Fujii and S. Kamefuchi: A Generalization of Stueckelberg Formalism of Vector Meson Fields. Nuovo Cim. 33 (1964) 1639.

Y. Freundlich and D. Lurie: Gauge Fields and Goldstone Bosons in the Nambu Model. Nucl. Phys. B19 (1970) 557.

S. N. Gupta: Theory of Longitudinal Photons in Quantum Electrodynamics. Proc. Phys. Soc. A63 (1950) 681.

S. N. Gupta: On Stueckelberg's Treatment of the Vector Meson Field. Proc. Phys. Soc. A64 (1951) 695.

S. N. Gupta: On the Supplementary Condition in Quantum Electrodynamics.

S. N. Gupta: Quantum Electrodynamics with Auxiliary Fields. Proc. Phys. Soc. [illegible]A (1952) 129.

N. Gupta: Quantum Mechanics with an Indefinite Metric. Canadi. Jour. Phys. 35 (1957) 961.

T. Goto and T. Imamura: Note on the Non-Perturbation Approach to Quantum Field Theory. Progr. Theor. Phys. 14 (1955) 396.

R. J. Glauber: On the Gauge Invariance of the Neutral Vector Meson Theory. Progr. Theor. Phys. 9 (1953) 295.

J. Goldstone: Field Theories with "Superconductor" Solutions. Nuovo. Cim. 19 (1961) 154.

J. Goldstone, A. Salam and S. Weinberg: Broken Symmetries. Phys. Rev. 127 (1962) 965.

P. W. Higgs: Broken Symmetries, Massless Particles and Gauge Fields. Phys. Lett. 12 (1964) 132.

P. W. Higgs: Spontaneous Symmetry Breakdown without Massless Bosons. Phys. Rev. 145 (1966) 1156.

M. Hayakawa and K. Yokoyama: Gauge, Renormalization and G-I-S term in Q.E.D. Progr. Theor. Phys. 44 (1970) 533.

W. Heitler: The Quantum Theory of Radiation. Oxford Univ. Press, Oxford (1949).

E. M. Henley and W. Thirring: Elementary Quantum Field Theory. McGraw-Hill, New York (1962).

K. Johnson: Current-Charge Density Commutation Relations. Nucl. Phys. 25 (1961) 431.

K. Johnson: Relations Between the Bare and Physical Masses of Vector Mesons. Nucl. Phys. 25 (1961) 435.

J. M. Jauch and F. Rohrlich: The Theory of Photons and Electrons. Addison-Wesley Publishing Comp. (1955).

G. Källen: On the Definition of the Renormalization Constants in Quantum Electrodynamics. Helv. Phys. Acta. 25 (1952) 417.

G. Källen: Gradient Terms in Commutators of Currents and Fields. Proceedings of the 14th Conference on Physics at the University of Brussels, Interscience Publisher (1968).

L. D. Landau and I. M. Khalatnikov: The Gauge Transformation of the Green's Function for Charged Particles. Soviet Physics JETP 2 (1956) 69.

B. Lautrap: Canonical Quantum Electrodynamics in Covariant Gauges. K. Danske Vidensk. Selsk. Mat. Fys. Medd 35 (1967) No. 11.

D. Lurie: Particles and Fields. Interscience Publishers, New York (1968).

Y. Miyamoto: On the Interaction of the Meson and Nuclear Field in the Super-Many-Time Theory. Progr. Theor. Phys. 3 (1948) 124.

P. M. Mathews: Indecomposability of Poincaré Group Representations over Massless Fields and the Quantization Problem for Electromagnetic Potentials. Phys. Rev. 9 (1973) 17[illegible]

H. Matsumoto, N. J. Papastamatiou, H. Umezawa and [illegible]tiello: Dynamical Rearrangement in the Anderson-Higgs-[illegible]e Mechanism. Nucl. Phys. B97 (1975) 61.

H. Matsumoto, P. Sodano and H. Umezawa: Dynamical Rearrangement of Symmetry in Quantum Electrodynamics. Phys. Rev. D15 (1977) 2192.

N. Nakanishi: Covariant Quantization of the Electromagnetic Field in the Landau Gauge. Progr. Theor. Phys. 35 (1966) 1111.

N. Nakanishi: Quantum Electrodynamics in the General Covariant Gauge. Progr. Theor. Phys. 38 (1967) 881.

N. Nakanishi: Massive Vector Field and Electromagnetic Field in the Landau Gauge. Phys. Rev. D5 (1972) 1324.

N. Nakanishi: Indefinite-Metric Quantum Field Theory. Progr. Theor. Phys. Supplement No. 51 (1971) 1.

N. Nakanishi: Indefinite-Metric Quantum Theory of Genuine and Higgs-Type Massive Vector Fields. Progr. Theor. Phys. 49 (1973) 640.

N. Nakanishi: Quantum Field Theory with Spontaneous Breakdown of the Chiral Gauge Invariance. Progr. Theor. Phys. 50 (1973) 1388.

N. Nakanishi: A way out of Formal Difficulty Encountered in the Landau Gauge Quantum Electrodynamics. Progr. Theor. Phys. 51 (1973) 952.

N. Nakanishi and M. Konoue: Vector Field with a Bare Mass and the Higgs Mechanism. Preprint, Research Institute for Math. Sciences. Kyoto Univ., Kyoto (1976).

N. Nakanishi: Ba no Ryoshiron. Baihukan, Tokyo (1975).

Y. Nambu and G. Jona-Lasinio: Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity I. Phys. Rev. 122 (1961) 345.

K. Nishijima: Higgs Theorem in the Yang-Feldman Formalism. Progr. Theor. Phys. 50 (1973) 1381.

K. Nishijima: Fields and Particles. Benjamine, New York (1969).

J. Schwinger: Quantum Electrodynamics, I. A Covariant Formulation. Phys. Rev. 74 (1948) 1439.

J. Schwinger: Field Theory Commutators. Phys. Rev. Lett. 3 (1959) 296.

J. Schwinger: Selected Papers on Quantum Electrodynamics. Dover Publ., New York (1958).

F. Strocchi: Gauge Problem in Quantum Field Theory. Phys. Rev. 162 (1967) 1429.

E. C. G. Stueckelberg: Die Wechselwirkungskräfte in der Electrodynamik und in der Feldtheorie der Kernkräfte (Teil I). Helv. Phys. Acta 11 (1938) 299.

S. Sunakawa: Quantum Electrodynamics with the Indefinite Metric. Progr. Theor. Phys. 19 (1958) 221.

Y. Takahashi: On the Gauge Invariance and the Structure of Elementary Particles I, II. Progr. Theor. Phys. 10 (1953) 366, 11 (1953) 251.

Y. Takahashi and S. Kamefuchi: Invariance and the S-matrix, I. Nucl. Phys. 17 (1960) 686.

Y. Takahashi: Some Covariant Identities and the Generators of the Poincaré Group. Proc. Roy. Irish. Acad. 71 (1971) 1.

Y. Takahashi and T. Groto: General Word-like Relations in Canonical Field Theory. Progr. Theor. Phys. 57 (1977) 1732.

Y. Takahashi: An Introduction to Field Quantization. Pergamon Press, Oxford (1968).

Y. Takahashi: Bussei Kenkyusha no tameno Ba no Ryoshiron vol. I, II. Baihukan, Tokyo (1974, 1975).

H. Umezawa: On the Structure of the Interactions of the Elementary Particles, II. Progr. Theor. Phys. 7 (1952) 551.

H. Umezawa: Self-Consistent Quantum Field Theory and Symmetry Breaking, Renormalization and Invariance in Quantum Field Theory, Edited by E. R. Caianiello. Plenum Publishing Company, New York (1974).

H. Umezawa: Quantum Field Theory. North-Holland Publ. Co., Amsterdam (1956).

C. N. Yang and D. Feldman: The Smatrix in the Heisenberg Representation. Phys. Rev. 79 (1950) 972.

APPENDIX A

DEFINITION

(i) Properties of Lorentz Invariant Functions

We define

Δ-function

$$\Delta(x:m^2) \equiv \frac{1}{(2\pi)^3 i} \int_{-\infty}^{+\infty} d^4p\,\varepsilon(p_o)\delta(p^2-m^2)e^{-ipx} \quad , \qquad (A.1)$$

Δ_F-function (causal function)

$$\Delta_F(x:m^2) \equiv \frac{-i}{(2\pi)^4} \int_{-\infty}^{+\infty} d^4p \, \frac{e^{-ipx}}{m^2-p^2-i\varepsilon} \quad , \qquad (A.2)$$

E-function

$$E(x) \equiv - \frac{\partial}{\partial m^2} \Delta(x:m^2) \Big|_{m^2=0}$$

$$\text{or} \quad \equiv \frac{1}{(2\pi)^3 i} \int_{-\infty}^{+\infty} d^4p\,\varepsilon(p_o)\delta'(p^2)e^{-ipx} \quad , \qquad (A.3)$$

E_F-function

$$E_F(x) \equiv - \frac{\partial}{\partial m^2} \Delta_F(x:m^2) \Big|_{m^2=0}$$

$$= - \frac{i}{(2\pi)^4} \int d^4p \, \frac{e^{-ipx}}{(-p^2-i\varepsilon)^2} \quad . \qquad (A.4)$$

These functions have the following properties:

$$(\Box+m^2)\Delta(x:m^2) = 0 \tag{A.5}$$

$$(\Box+m^2)\Delta_F(x:m^2) = -i\delta^4(x) \quad , \tag{A.6}$$

$$\Box E(x) = D(x) \quad , \tag{A.7}$$

$$\Box^2 E(x) = 0 \quad , \tag{A.8}$$

$$\Box E_F(x) = D_F(x) \quad , \tag{A.9}$$

$$\Box^2 E_F(x) = -i\delta^4(x) \quad , \tag{A.10}$$

where

$$D(x) \equiv \Delta(x:m^2=0) = -(2\pi)^{-1}\varepsilon(x_o)\delta(x^2) \quad , \tag{A.11}$$

$$D_F(x) \equiv \Delta_F(x:m^2=0) \quad , \tag{A.12}$$

and

$$\dot{\Delta}(0,\underset{\sim}{x}:m^2) = -\delta^3(\underset{\sim}{x}) \tag{A.13}$$

$$\Delta(0,\underset{\sim}{x}:m^2) = 0 \quad , \tag{A.14}$$

$$E(0,\underset{\sim}{x}) = \dot{E}(0,\underset{\sim}{x}) = \ddot{E}(0,\underset{\sim}{x}) = 0 \quad , \tag{A.15}$$

$$\dddot{E}(0,\underset{\sim}{x}) = -\delta^3(\underset{\sim}{x}) \quad . \tag{A.16}$$

The properties for E(x) are seen in, for example, Lautrap (1967).

(ii) Polarization Vector for $A_\mu(x)$

When we have

$$\Box A_\mu(x) = 0 \quad , \tag{A.17}$$

$A_\mu(x)$ can be expressed in the Fourier transform as

$$A_\mu(x) = 1/(2\pi)^{3/2} \sum_{\lambda=0}^{3} \int \frac{d^3\underset{\sim}{k}}{\sqrt{2|\underset{\sim}{k}|}} e_\mu^{(\lambda)}(\underset{\sim}{k})[a_\lambda(k)e^{-ikx}+h.c.] \tag{A.18}$$

$$\text{with } k_o = |\underset{\sim}{k}| \quad ,$$

where $e_\mu^{(\lambda)}$ is given by

$$e_o^{(\lambda)} = 0 \quad \text{for } \lambda=1,2,3$$

$$e_o^{(o)}(\underset{\sim}{k}) = 1$$

$$\underset{\sim}{e}^{(3)}(\underset{\sim}{k}) = \underset{\sim}{k}/k_o$$

$$\underset{\sim}{e}^{(\lambda)}(\underset{\sim}{k})\cdot\underset{\sim}{k} = 0 \quad \text{for } \lambda=1,2 \quad . \tag{A.19}$$

The photons corresponding to $\lambda=1$ and 2 are called transversely polarized and those corresponding to $\lambda=3$ and $\lambda=0$ are longitudinal and scalar photons, respectively.

The orthogonality condition is given by

$$e_\mu^{(\lambda)}(\underset{\sim}{k})e_\nu^{(\lambda')}(\underset{\sim}{k})g^{\mu\nu} = g^{\lambda\lambda'} \quad , \tag{A.20}$$

and completeness of the polarization states is expressed by

$$\sum_{\lambda=0}^{3} \sum_{\lambda'=0}^{3} e_\mu^\lambda(\underset{\sim}{k})e_\nu^{(\lambda')}(\underset{\sim}{k})g_{\lambda\lambda'} = g_{\mu\nu} \quad . \tag{A.21}$$

(iii) Field Equations and Commutators for In-fields with $\alpha=0$, $M\neq0$, $m\neq0$

We have for this case

Field equations

$$\{(\Box+m^2)g_{\mu\nu}-\partial_\mu\partial_\nu\}u^{\mu\,in}(x) = 0 \quad , \tag{A.22}$$

$$\Box\chi^{in}(x) = \Box\Phi_1^{in}(x) = 0 \quad , \tag{A.23}$$

$$\partial^\mu u_\mu^{in}(x) = 0 \quad , \tag{A.24}$$

where the field $u_\mu^{in}(x)$ is called the Proca field.

The Commutation Relations

$$[u_\mu^{in}(x),u_\nu^{in}(y)] = -i(g_{\mu\nu}+m^{-2}\partial_\mu^x\partial_\nu^x)\Delta(x-y:m^2) \quad , \tag{A.25}$$

$$[u_\mu^{in}(x),\Phi_1^{in}(y)] = 0 \quad , \tag{A.26}$$

$$[u_\mu^{in}(x),\chi^{in}(y)] = 0 \quad , \tag{A.27}$$

$$[\chi^{in}(x),\chi^{in}(y)] = iD(x-y) \quad , \tag{A.28}$$

$$[\Phi_1^{in}(x),\Phi_1^{in}(y)] = -iD(x-y) \quad , \tag{A.29}$$

$$[\Phi_1^{in}(x),\chi^{in}(y)] = 0 \quad . \tag{A.30}$$

APPENDIX B

FORMULAE

(i) Expansion of q-number Function $F' \equiv e^{-iG} F e^{iG}$

$$F' \equiv e^{-iG} F e^{iG} = F + \frac{i}{1!}[F.G] + \frac{i^2}{2!}[[F,G],G] + \ldots$$

or

$$= F + \frac{(-i)}{1!}[G.F] + \frac{(-i)^2}{2!}[G,[G.F]] + \ldots \tag{B.1}$$

(ii)† Integral Representation of the Solutions for Klein-Gordon Equation

When a function g(x) satisfies the Klein-Gordon Equation

$$(\Box + m^2) g(x) = 0 \quad , \tag{B.2}$$

g(x) can be written in the integral representation as

$$g(x) = \int_\sigma d\sigma^\mu(z) \Delta(x-z:m^2) \overleftrightarrow{\partial}^z_\mu g(z)$$

or

$$= \int d^3z \Delta(x-z:m^2) \overleftrightarrow{\partial}^z_0 g(z) \quad , \tag{B.3}$$

where σ is an arbitrary spacelike hypersurface.

(iii)† Identity

$$\int_\sigma d\sigma_\mu(x) \partial_\nu F(x) - \int_\sigma d\sigma_\nu(x) \partial_\mu F(x) = 0 \quad , \tag{B.4}$$

† The proofs for (ii) and (iii) are seen in, for example, Schwinger (1948), Takahashi (1968).

assuming that

$$|\underset{\sim}{x}|^2 F(x) \to 0 \quad \text{as} \quad |\underset{\sim}{x}| \to +\infty \quad . \tag{B.5}$$

(iv) List of Formulae for Integrations in Chapter VII

The integrations in the following are carried out on the flat spacelike hypersurface σ which is determined by the equation

$$n^\mu x_\mu + \tau = 0 \quad , \tag{B.6}$$

where τ is a real constant and n_μ is a timelike unit vector

$$n_\mu n^\mu = 1 \quad . \tag{B.7}$$

Formulae:

$$\int_\sigma d\sigma_\lambda(y) \ldots = \int d^4y n_\lambda \delta(ny + \tau) \ldots , \tag{B.8}$$

$$\partial_\mu^y f(ny+\tau) = n_\mu \frac{d}{d\tau} f(ny+\tau) , \tag{B.9}$$

$$\text{or} \quad (n\partial) f(ny+\tau) = \frac{d}{d\tau} f(ny+\tau) \quad \text{(Takahashi, 1960)} \tag{B.10}$$

$$(\partial_\mu^y - n_\mu(n\partial)) f(ny+\tau) = 0 \quad . \tag{B.11}$$

$$\int_\sigma d\sigma_\lambda(y) \partial_\nu^x \Delta(x-y) f(y) = \int d^4y n_\lambda \delta(ny+\tau) \partial_\nu^x \Delta(x-y) f(y)$$

$$= n_\lambda \{ n_\nu \frac{d}{d\tau} \int d^4y \delta(ny+\tau) \Delta(x-y) f(y)$$

$$+ \int d^4y \delta(ny+\tau) \Delta(x-y) \partial_\nu^y f(y) \} \quad , \tag{B.12}$$

$$\int_\sigma d\sigma_\lambda(y)\partial^x_\mu\partial^x_\nu\Delta(x-y)f(y) = n_\lambda\{n_\mu n_\nu \frac{d^2}{d\tau^2}\int d^4y\delta(ny+\tau)\Delta(x-y)f(y)$$

$$+ \frac{d}{d\tau}\int d^4y\delta(ny+\tau)\Delta(x-y)(n_\mu\partial_\nu+n_\nu\partial_\mu)f(y)$$

$$+ \int d^4y\delta(ny+\tau)\Delta(x-y)\partial_\mu\partial_\nu f(y)\} \quad , \qquad \text{(B.13)}$$

$$\frac{d^2}{d\tau^2}\int d^4y\delta(ny+\tau)\Delta(x-y)g(y)$$

$$= \frac{d}{d\tau}\int d^4y\delta(ny+\tau)\Delta(x-y)(n\partial)g(y) + \int d^4y\delta(ny+\tau)(n\partial^x)^2 \Delta(x-y)g(y)$$

$$- (n\partial^x)\int d^4y\delta(ny+\tau)\Delta(x-y)(n\partial)g(y) \quad , \qquad \text{(B.14)}$$

$$\int_\sigma d\sigma_\lambda(y)\partial^x_\mu\partial^x_\nu\Delta(x-y)\cdot f(y)$$

$$= n_\lambda \frac{d}{d\tau}\int d^4y\delta(ny+\tau)\Delta(x-y)(n_\mu\partial_\nu+n_\nu\partial_\mu-n_\mu n_\nu(n\partial))f(y)$$

$$+ n_\lambda\{n_\mu n_\nu \int d^4y\delta(ny+\tau)(n\partial^x)^2\Delta(x-y)f(y)$$

$$- n_\mu n_\nu(n\partial^x)\int d^4y\delta(ny+\tau)\Delta(x-y)(n\partial)f(y)$$

$$+ \int d^4y\delta(ny+\tau)\Delta(x-y)\partial_\mu\partial_\nu f(y)\} \quad , \qquad \text{(B.15)}$$

$$\frac{d}{d\tau}\int d^4y\delta(ny+\tau)(n\partial^x)^2\Delta(x-y:\kappa^2)g(y)$$

$$= -\frac{d}{d\tau}\int d^4y\delta(ny+\tau)\Delta(x-y:\kappa^2)\{\kappa^2 + \partial_{\mu\cdot s}\partial^\mu_{\cdot s}\}g(y) \qquad \text{(B.16)}$$

These formulae (B.12) to (B.16) hold for any x.

$$\int_\sigma d\sigma_\lambda(y)\partial^x_\nu\Delta(x-y)f(y) = n_\lambda n_\nu \frac{d}{d\tau}\int d^4y\delta(ny+\tau)\Delta(x-y)f(y)$$

$$\text{for } x/\sigma \quad , \qquad \text{(B.17)}$$

$$\int_\sigma d\sigma_\lambda(y)\Delta(x-y)f(y) = n_\lambda \int d^4y\delta(ny+\tau)\Delta(x-y)f(y) = 0$$

for x/σ , (B.18)

$$\frac{d}{d\tau}\int d^4y\delta(ny+\tau)\Delta(x-y)g(y)$$

$$= (n\partial^x)\int d^4y\delta(ny+\tau)\Delta(x-y)g(y) \quad \text{for } x/\sigma \quad , \qquad \text{(B.19)}$$

$$\frac{d^2}{d\tau^2}\int d^4y\delta(ny+\tau)\Delta(x-y)g(y)$$

$$= -2\frac{d}{d\tau}\int d^4y\delta(ny+\tau)\Delta(x-y)(n\partial)g(y) \quad \text{for } x/\sigma \quad , \qquad \text{(B.20)}$$

$$\int_\sigma d\sigma_\lambda(y)\partial^x_\mu\partial^x_\nu\Delta(x-y)f(y)$$

$$= n_\lambda \frac{d}{d\tau}\int d^4y\delta(ny+\tau)\Delta(x-y)(n_\mu\partial_{\nu\cdot s}+n_\nu\partial_{\mu\cdot s})f(y)$$

for x/σ , (B.21)

$$\frac{d}{d\tau}\int d^4y\delta(ny+\tau)(n\partial^x)^2\Delta(x-y)g(y)$$

$$= \int d^4y\delta(ny+\tau)(n\partial^x)^3\Delta(x-y)g(y) \quad \text{for } x/\sigma \quad , \qquad \text{(B.22)}$$

$$\int d^4y\delta(ny+\tau)(n\partial^x)^m\Delta(x-y)f(y) = 0 \quad \text{for } x/\sigma \quad ,$$

and positive and even integer m } (B.23)

$$\frac{d}{d\tau}\int d^4y\delta(ny+\tau)(n\partial^x)^m\Delta(x-y)g(y)$$

$$= \int d^4y\delta(ny+\tau)(n\partial^x)^{m+1}\Delta(x-y)g(y) \quad \text{for } x/\sigma \text{ and,}$$

positive and even integer, (B.24)

$$\sigma = \{y | ny + \tau = 0\} \quad ,$$

and (B.28) holds for any y on the hypersurface σ;

$$\sigma = \{x | nx + \tau = 0\} \quad .$$

APPENDIX C

DERIVATION

(i) Derivation of Equation (4.20)

We set

$$<[A_\mu(x),\phi(y)]>_o = \int_0^\infty d\kappa^2 \rho(\kappa^2) \partial_\mu^x \Delta(x-y:\kappa^2) \quad , \qquad (C.1)$$

and differentiate both sides of (C.1) with respect to x_μ, by the help of equation (4.4) and commutator (2.38). Then we have

$$\alpha MD(x-y) = \int_0^\infty d\kappa^2 \rho(\kappa^2) \kappa^2 \Delta(x-y:\kappa^2) \quad , \qquad (C.2)$$

which implies

$$\kappa^2 \rho(\kappa^2) = \alpha M \delta(\kappa^2) \quad . \qquad (C.3)$$

Therefore, the spectral function $\rho(\kappa^2)$ has neither a massive pole nor a continuous spectra. This fact enables us to rewrite (C.1) as

$$<[A_\mu(x),\phi(y)]>_o = C_1 \partial_\mu E(x-y) + C_2 \partial_\mu D(x-y) \quad , \qquad (C.4)$$

where C_1 and C_2 are constant. In order to determine the coefficients C_1 and C_2, we set $\mu=0$, and $x_o=y_o$, then from the property of equal time commutator we have

$$0 = -C_2 \delta^3(\underset{\sim}{x}-\underset{\sim}{y}) \quad ,$$

which implies $\qquad C_2 = 0 \quad . \qquad (C.5)$

$$\frac{d}{d\tau}\int d^4y\delta(ny+\tau)(n\partial^x)^2\Delta(x-y:\kappa^2)g(y)$$

$$= \int d^4y\delta(ny+\tau)(n\partial^x)^3\Delta(x-y:\kappa^2)g(y)$$

$$= -\frac{d}{d\tau}\int d^4y\delta(ny+\tau)\Delta(x-y:\kappa^2)\{\kappa^2+\partial_{\mu\cdot s}\partial^{\mu}_{\cdot s}\}g(y)$$

$$\text{for } x/\sigma \quad , \qquad (B.25)$$

$$\int_\sigma d\sigma_\lambda(y)(n\partial^x)\partial^x_\mu\partial^x_\nu\Delta(x-y:\kappa^2)f(y)$$

$$= n_\lambda \frac{d}{d\tau}\int d^4y\delta(ny+\tau)\Delta(x-y:\kappa^2)\{-n_\mu(n\partial)\partial_\nu$$

$$- n_\nu(n\partial)\partial_\mu + n_\mu n_\nu(n\partial)^2 + \partial_\mu\partial_\nu$$

$$- n_\mu n_\nu(\kappa^2+\partial_{\rho\cdot s}\partial^{\rho}_{\cdot s})\}f(y) \quad \text{for } x/\sigma \quad , \qquad (B.26)$$

$$\int_\sigma d\sigma_\lambda(x)(n\partial)^x\Delta(x-y)$$

$$= -n_\lambda \frac{d}{d\tau}\int d^4x\delta(nx+\tau)\Delta(x-y) \quad \text{for any } x \quad , \qquad (B.27)$$

$$\int_\sigma d\sigma_\lambda(x)(n\partial^x)\partial^x_\nu\Delta(x-y) = 0 \quad \text{for } y/\sigma \quad , \qquad (B.28)$$

$$\int_\sigma d\sigma_\lambda(x)(n\partial^x)\partial^x_\mu\partial^x_\nu\Delta(x-y:\kappa^2)$$

$$= n_\lambda n_\mu n_\nu \kappa^2 \frac{d}{d\tau}\int d^4x\delta(nx+\tau)\Delta(x-y:\kappa^2) \quad \text{for any } y \quad , \qquad (B.29)$$

the formulae (B.17) to (B.26) hold for any x on the hypersurface σ;

Next on differentiating (C.4) with respect to x_μ, we obtain

$$-\alpha M D(x-y) = C_1 D(x-y) \quad ,$$

from which follows

$$C_1 = -\alpha M \quad . \tag{C.6}$$

Thus, from (C.4), (C.5) and (C.6), we have

$$<[A_\mu(x),\phi(y)]>_0 = -\alpha M \partial^x_\mu E(x-y) \quad . \tag{C.7}$$

(ii) Derivation of Equation (5.8) from Equation (2.7)

We shall show that the current $j_\mu(x)$ given by equation (2.7) is also written in the form of equation (5.8) in case of minimal interactions.

Equation (2.7) can be rewritten step by step as follows:

$$
\begin{aligned}
j^\mu(x) &= -\frac{\partial L_{int}}{\partial A_\mu(x)} \\
&= -\frac{\partial(L_{int}+L_{matter})}{\partial A_\mu(x)} \\
&= -\frac{\partial L_{matter}(\phi_i,\phi_i^\dagger,(\partial_\mu-ieA_\mu)\phi_i,(\partial_\mu+ieA_\mu)\phi_i^\dagger)}{\partial A_\mu(x)} \\
&= -\sum_i\{-ie\frac{\partial L_{matter}}{\partial(\partial_\mu\phi_i)}\phi_i + ie\phi_i^\dagger\frac{\partial L_{matter}}{\partial(\partial_\mu\phi_i^\dagger)}\} \\
&= \sum_i \frac{\partial L}{\partial(\partial_\mu\phi_i)}(ie\delta\Lambda\phi_i) + (-ie\delta\Lambda\phi_i^\dagger(x))\frac{\partial L}{\partial(\partial_\mu\phi_i^\dagger)}\}/\delta\Lambda(x) \\
&= \sum_i\{\frac{\partial L}{\partial(\partial_\mu\phi_i)}\delta\phi_i + \delta\phi_i^\dagger\frac{\partial L}{\partial(\partial_\mu\phi_i^\dagger)}\}/\delta\Lambda(x) \quad .
\end{aligned}
$$

Thus, we have arrived at equation (5.8).