University of Alberta

Scaling the Heights in Unity

by

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Introduction

While many mountain climbers aspire to climb Mount Everest "because it's there," it would be more accurate to say they can manage this feat, albeit with considerable difficulty, "because we're here."

This paper examines an anthropic argument that suggests human observers are likely to exist only on planets where the scale height of the atmosphere is roughly equal to the maximum height of the mountains on that planet. This one-to-one correlation means there is just enough oxygen in the atmosphere at the top of the highest possible mountain so that mountain climbers can reach the apex without the assistance of breathing gear (with some tragic exceptions). Mount Everest is the tallest mountain on the planet at 8,850 metres, and the scale height (h) of the atmosphere is 8,430 metres. A number of physicists have played with universal constants to come up with an approximation of maximum mountain height (H). With some refinement, as will be shown in this paper, these approximations give that H is of the order of 10^4 metres. Mount Everest is close to being about as high as a mountain can be on this planet, and humans *can* climb to the top of the mountain because the temperature, atmospheric gases and conditions that affect bonding energy are the values they are. Indeed, it would appear that, from arguments using the Weak Anthropic Principle, humans exist on Earth where $H/h \approx 1$, and humans do not exist on planets where H/h varies significantly from unity.

Universal constants have a credible track record of being used to tease information out of apparent numerical coincidences to see if they can lead to some useful insights. The limitations set by the anthropic principle — the idea that the conditions in this inhabited patch of the Universe had to be suitable for the evolution of planets with human inhabitants — has offered some insights into why the fundamental constants have the observed values, and why physical conditions here on Earth are as they are. It has been known for some time that if the fundamental constants such as the mass of an electron or the gravitational constant varied only a little from the known values, life as we know it would not exist [Barrow and Tipler, 1986].

This paper will examine the coincidence that human observers exist on planets where $H/h \approx 1$ to see whether this is significant and more than a random coincidence. Are there reasons why planets with complex observers should be limited to $H \approx h$?

1. BUILDING A FOUNDATION

One of the challenges of physics is to understand the world at its most fundamental level, and to that end, much has been learned from being able to build concepts from just a handful of constants[i]:

$c = 2.99 \times 10^{8}$ m s^{-2}, velocity of light,

$\hbar = 1.054 \times 10^{-34}$ J s, (reduced) Planck's constant,

$G = 6.672 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, gravitational constant,

$k = 1.380 \times 10^{-23}$ J K^{-1}, Boltzmann's constant,

$e = 1.602 \times 10^{-19}$ C, charge of an electron,

$m_e = 9.109 \times 10^{-31}$ kg, mass of an electron,

$m_p = 1.672 \times 10^{-27}$ kg, mass of a proton,

$k_e = 1/(4\pi\varepsilon_0) = 8.987 \times 10^9$ N m^2 C^{-2}, Coulomb constant.

From these constants, the following three dimensionless universal constants have been derived (for purposes of simplification, Planck units will be used, where $c = \hbar = G = k = 4\pi\varepsilon_0 = 1$):

$$\alpha \equiv e^2(4\pi\varepsilon_0)/\hbar c = e^2 \approx 7.3 \times 10^{-3},$$

the fine structure constant,

$$\beta \equiv m_e/m_p \approx 5.4 \times 10^{-4},$$

the ratio of the electron and proton masses, and

$$\alpha_G \equiv Gm_p^2 / \hbar c = m_p^2 \approx 5.9 \times 10^{-39},$$

the gravitational fine structure constant.

There has, since the time of Galileo, been a debate about whether the "constants" are really constant or whether they are subject to change over time. A variety of hypotheses were floated from the 1930s to the '70s (Dirac, Brans-Dicke, Gamow and Teller), and these were reviewed by Freeman Dyson [Dyson, 1972]. He concluded that only two were viable: the conventional view that α, β and α_G are indeed constant, and the

[i] Unless otherwise indicated, the values given are provided by the National Institute of Standards and Technology (NIST) and are available at www.nist.gov.

Brans-Dicke hypothesis that α and β are constant, but α_G is subject to a very slight decrease in the order of a fractional amount of 10^{-12} to 10^{-11} per year. But Dyson warned that neither was provable based on observational capacity at that time.

Observational capacity in cosmology has improved significantly in the thirty years since Dyson's review, but there is still no consensus on which, if any, of the universal constants is truly constant. Indeed, there has been considerable interest of late in variations of the gravitational constant, particularly as it might relate to the cosmological constant. There is also a significant number of papers being produced that focus on variations in the fine structure constant. (Indeed, one of John Barrow's students completed a PhD thesis in early 2004 giving a detailed investigation of the cosmological evolution of the fine structure constant [Mota, 2004]).

The term "constant", at this point, is just a useful approximation. At a 1973 conference on Confrontation of Cosmological Theories with Observational Data, Brandon Carter gave a talk on the anthropic principle and possible models for various universes [Carter, 1974]. Another physicist asked him "why anything in nature has to be constant at all." Carter's reply:

> It's true of course that once one had admitted the possibility that parameters such as the fine structure 'constant' e^2 or the gravitational coupling constant m_p^2 might vary from one universe to another, one could conceive that they might vary within our own Universe. However (like most other physicists) I prefer to work with the *simplest* hypothesis compatible with the observational evidence, which is that these particular quantities are indeed constant in space and time. (There is strong evidence against even very small variations in the ratio m_e/m_p or in the electromagnetic coupling constant e^2. For the coupling constant m_p^2 the evidence is less conclusive — the possibility of a small variation as postulated by the Brans Dicke theory cannot be absolutely ruled out.)

For the purposes of this discussion, it would be prudent to follow Carter's advice and work with the simplest hypothesis available. The magnitude and time scale under study — planetary mountains and atmospheres — can be easily accommodated by assuming that the constants *are* constant.

Applying the anthropic principle

For about the past fifty years, physicists have been using the limiting conditions of the anthropic principle to understand better the fundamental nature of the universe we live in. The anthropic principle, in its simplest form, states only that the basic features of this universe *must* be suitable for the existence of observers. Such a claim might very well be considered as stating the obvious and of limited value. But as Barrow and Tipler argue in *The Anthropic Cosmological Principle* [Barrow and Tipler, 1986], "the Copernican Revolution was initiated by the application of the Weak Anthropic Principle." Of course, Copernicus would not have been familiar with the Weak Anthropic Principle, since that label was introduced by Robert Dicke in 1957 [Dicke, 1957]. It's the idea that while observers such as humans do not occupy a privileged central place in this universe, the universe still had to evolve in such a manner as to produce observers such as humans in order that the universe be observed.

Up until the time of Copernicus and Galileo, humans held a central role in the explanation of the meaning and function of the universe. The Aristotelian cosmological model located humans at the centre of the universe, and that model held for nearly two millennia.[ii]

Since the Copernican Revolution, there has been a trend toward marginalizing humans, at least with respect to cosmology. The Darwinian Revolution had the effect of returning humans to the top of the heap, so to speak, by characterizing humans as the most sophisticated and advanced species — the apex of evolution. But in cosmological terms, humans are not really much of a factor at all. In his 2004 book, *The Fabric of the Cosmos*, physicist Brian Greene relegated humans to a passing reference as a chemistry accident, echoing what appears to be the prevalent viewpoint in cosmology [Greene, 2004]. This may be a continuation of the need to dissociate modern, post-Darwinian science from pre-modern science and its integration of human-centred science, mysticism and religion.

However, physicists are cautioned against adopting an "exaggerated subservience" to the Copernican Principle [Carter, 1974] or an unwarranted "cosmic modesty" [Rees, 1997] by assuming that we (human observers) cannot be privileged in any sense.

John Barrow and Frank Tipler have neatly combined the work of Dicke and Carter in defining the weak and strong anthopric principles.

> **The Weak Anthropic Principle (WAP)**: The observed values of all physical and cosmological quantitites are not equally probable but take on values restricted by the requirement that there exist sites where carbon-based life can evolve and by the requirement that the Universe be old enough for it to have already done so [Barrow and Tipler, 1986].

[ii] There are some indications that the creation mythology of the ancient Babylonians (c. 1800 BCE) gives humans a less central role. According to the ancient poem or hymn *Enuma Elish*, the gods of the heavens were in need of a rest and humans were created to serve as their assistants and take over some of the workload. See *The Fabric of the Heavens*, S. Toulmin and J. Goodfield, Harper & Row, 1961.

On the other hand, the Strong Anthropic Principle states that the laws governing the universe *must* be such that observers can exist. This may stem from a teleological interpretation whereby an intelligent designer is deemed necessary to create a universe with the very specific conditions necessary for the existence of observers. Although predating modern discussions of the anthropic principle by about 200 years, philosopher William Paley argued that the complexity of nature was evidence of design and purpose, and hence, there must exist a designer with a purpose [W. Paley, *Natural Theology*, 1802]. The Strong Anthropic Principle is anthropocentric, placing humans at the centre of the universe in terms of its design and purpose.

> **The Strong Anthropic Principle (SAP)**: The Universe must have those properties which allow life to develop within it at some stage in its history. This principle has several variations, including:
>
> - **Design Argument**: There exists one possible Universe "designed" with the goal of generating and sustaining "observers."
> - **Participatory Anthropic Principle (PAP)**: Observers are necessary to bring the Universe into being.
> - **Many-Worlds or Sum-Over-Histories Argument**: An ensemble of other different universes is necessary for the existence of our universe.
> - **Final Anthropic Principle (FAP)**: Intelligent information-processing must come into existence in the Universe, and once it comes into existence, it will never die out [Barrow and Tipler, 1986].

The Design Argument has more or less fallen out of favour since the advent of Darwinian evolution, but still has some adherents. Astronomer Fred Hoyle was advocating SAP in 1959 [Hoyle, *Religion and Scientists*, 1959, quoted in Barrow and Tipler]. More recently, the development of consciousness theories has blurred the line between the designer and participatory arguments. Physicist Amit Goswami, for instance, states quite unambiguously that, "We are the centre of the universe because we are its meaning" [Goswami,1993].

The last two variations of SAP are still providing food for thought, mainly because they have some application in the fields of quantum theory and information theory. For instance, in the discussion of the Everett Many Worlds Interpretation, the wave function that, in theory, describes the universe would be different for the SAP and WAP universes. The wave function for an ensemble of worlds limited by SAP would not contain any worlds where observers did not exist, according to Barrow and Tipler's use of SAP. On the other hand, if WAP were the only limitation on the wave function, the world ensemble could contain all conceivable combinations of initial conditions and fundamental constants, of which at least one would allow for the existence of observers [Barrow and Tipler, 1986; Carter, 1974]. A SAP multiverse would be distinctly different from a WAP multiverse.

Combining universal constants and the anthropic principle

Utilizing the universal constants and the anthropic principle in the search of meaningful coincidences has something of a playful/dismissive element to it, as if it should not be taken too seriously. It's been characterized as "diversionary physics" [Press and Lightman, 1983] that produces a number of "amusing relationships" [Carr and Rees, 1979], or as something to be ashamed of being associated with [Kolb and Turner, quoted in Linde, 2003].

In a 2002 contribution to the celebration of John Wheeler's 90th birthday, Andre Linde says of the anthropic principle [Linde, 2003], "...This principle can help us understand that some of the most complicated and fundamental problems may become nearly trivial if one looks at them from a different perspective. Instead of denying the anthropic principle or uncritically embracing it, one should take a more patient approach and check whether it is really helpful or not in each particular case."

This paper looks at the numerical coincidence that suggests humans are likely to exist only on planets where $H/h \sim 1$ to see if there is something helpful to be learned here.

2. SCALING THE HEIGHTS

Maximum Mountain Height (*H*)

The question of maximum mountain heights was dealt with initially by Victor Weisskopf [Weisskopf, 1975] who arrived at his conclusions based on the fundamental constants. His work was expanded upon by Bernard Carr and Martin Rees [Carr and Rees, 1979] and by William Press and Alan Lightman [Press and Lightman, 1983].

Weisskopf derives the maximum mountain height using the fundamental constants and gravitational acceleration and arrives at the equation:

$$H = \xi\gamma \frac{Ry}{Am_p g}$$

- ξ is a factor by which the binding energy of molecules that make up the mountain material is reduced, since the binding energy is not broken; only its directional stiffness is removed. Weisskopf sets $\xi = 0.05$ for minerals and metals;

- γ is a "fudge factor" which reduces the binding energy from the Rydberg energy for simple molecules in solids. Weisskopf sets $0.3 > \gamma > 0.1$;
- A is the atomic or molecular weight of mountain material:
- Ry is the Rydberg energy, $Ry = ½\, m_e\, c^2\, \alpha^2$;
- m_p is the mass of a proton; and
- g is the acceleration of gravity on the surface of the planet where the mountain is. (Note that acceleration of gravity is not a fundamental constant, but g cancels when we calculate H/h.)

Weisskopf's hypothesis is fairly straightforward. The gravitational energy gained by the sinking of a mountain of height H is equal to the energy needed to plasticize or melt the same amount of mountain material at its base, such that $E_{melt} = Am_pHg$ is the energy per molecule needed for melting. Weisskopf sets the melting energy as a small fraction of the Rydberg energy, with $E_{melt} = \xi\gamma Ry$, The deforming of the mountain base due to gravity, molecular structure and the bonding energy limits how high a mountain can be.

Of course, it's not as simple as it looks because real mountains are not the nice, neat, rectangular obelisks used in Weisskopf's approximation. Mountains are subject to fractures and impurities, and these are difficult to quantify. However, mountains can generally be considered to be made of "crud" [Press and Lightman, 1983], a substance that is neither metallic nor single-crystalline.

In calculating H for Earth, Weisskopf assumes mountain crud to be primarily SiO_2, with $A \approx 50$, and $\gamma = 0.2$. Plugging in the appropriate values (with the observed value $g = 9.81$ m/s^2) gives $H \sim 26$ km. Weisskopf admits this is higher than it should be because the calculation does not account for other factors on planets that might reduce *H*, such as tectonic plates, a liquid core and high volcanic activity. As he points out [Weisskopf, 1975], "the energy necessary to produce plastic flow should be less than the liquefaction energy."

Press and Lightman approach the problem of determining maximum mountain heights (H_{PL}) by equating the pressure at the base of a mountain as it deforms with tensile or shear strength of the "crud" the mountain is made of:

$$H_{PL}\, g\, \rho_0 \sim [Ry / (2\, a_0)^3]\, \beta^{1/2}$$

- ρ_0 is the density, very roughly, for all solid matter, given by the mass of a proton per cubic Bohr diameter, so $\rho_0 \equiv m_p\,(2a_0)^{-3}$;
- $Ry / (2\, a_0)^3$ is the binding energy per cubic Bohr diameter of a solid; and
- $\beta^{1/2}$, in this case, is a fudge factor used to accommodate the weaker characteristic bond energy of "crud".

Press and Lightman initially reduce the molecular binding energy based on the idea that the "vibrational frequencies of molecular bonds are smaller than the electronic transitions by a factor of $(m_e/m_p)^{1/2}$" [Press and Lightman, 1983], then apply the same factor to reduce the Rydberg energy. However, in discussions at the end of the same paper, Press clarifies that $(m_e/m_p)^{1/2}$ should not be considered a physical description but rather a *mnemonic*. He adds, "Numerically, however, it does also give the correct (rough) factor by which molecular bindings are smaller than typical atomic ones." As such, $\beta^{1/2} = (m_e/m_p)^{1/2}$ is really a numerical fudge factor.

According to Press and Lightman, when the pressure at the base of the mountain overcomes the reduced binding energy, the base "melts" and the mountain sinks until the pressure and tensile strength of the crud at the mountain base are in equilibrium. The maximum mountain height is given by

$$H_{PL} \sim Ry\,(2\,a_0)^{-3}\,\beta^{1/2}\,\rho_0^{-1}\,g^{-1}$$

$$\sim Ry\,\beta^{1/2}\,m_p^{-1}\,g^{-1}$$

Using Earth's observed value of $g = 9.81$ m/s^2, $H_{PL} \sim 3{,}100$ km which is just slightly less than the planet's radius of 6,370 km. Press and Lightman admit that Weisskopf has done a more careful job in determining *H*. When they add Weisskopf's fudge factors after the fact, their approximation is closer to the estimated maximum mountain height for Earth of about 10 km.

Press and Lightman also point out that matter appears to be significantly softer than indicated by the fundamental constants "by a significant factor," but do not address what that factor might be. They do, however, approximate the binding energy of organic materials as less than the binding energy of crud by a fudge factor of $\varepsilon = 0.1$, which takes care of "all the abhorrent details of chemistry" that the authors do not want to deal with [Press and Lightman, 1983].

In both cases, the authors are able to derive approximate values for the maximum mountain height on Earth by approximating the fudge factors. Dropping the fudge factors (including $\beta^{1/2}$),

$$H \sim H_W \sim H_{PL} \sim \frac{Ry}{Am_pg} \sim \frac{kT_{melt}}{Am_pg}. \qquad (1)$$

$\underline{T_{melt}}$ is the temperature sufficient to plasticize crud at the base of a mountain. But what is the value of $\underline{T_{melt}}$? Both Weisskopf and Press and Lightman state there are other factors not considered in their approximations. Mountains are also complex structures, albeit not as delicately structured as organic material. If we suppose that the binding energy for mountain material is twice that proposed by Press and Lightman for organic material, we can approximate the binding energy for complex mountain material to be $2\varepsilon Ry\ \beta^{1/2}$, so that $\underline{T_{melt}} \sim$ 700K. Rock melts at about 1000K, so this is not a bad approximation of the temperature needed to soften mountain material sufficiently for it to deform.

Scale height (*h*)

Scale heights are the purview of atmospheric sciences and are used as a means of calculating atmospheric densities at varying heights and for modelling atmospheric turbulence. Pilots of small planes risk blacking out if they fly too high without the appropriate breathing gear, and commercial airlines pressurize their cabins because the atmosphere is so "thin" at the altitudes large planes normally fly that passengers would quickly die of oxygen deprivation at those elevations. Mountain climbers face a similar difficulty. The higher they climb, the thinner the atmosphere gets, and at a certain point, they cannot continue without the aid of breathing gear or they'll die. The scale height describes how quickly atmospheric pressure changes as altitude (z) increases, and that's a pretty important thing for some people to know.

The scale height (h) is essentially the characteristic vertical dimension of atmospheric gas distributions and is given at the surface of the planet (say, at sea level for Earth) by the formula

$$h = -\left(\frac{d\ln P}{dz}\right)^{-1}$$

$$= \frac{k\,T_{planet}}{mg},$$

where P = atmospheric pressure,

z = altitude,

k = Boltzmann's constant,

T_{planet} = mean atmospheric surface temperature of planet,

g = acceleration of gravity, and

$m = \mu\, m_p$ (atomic weight of gas x proton mass).

(See Appendix B for the derivation of h.) The scale height depends on the surface temperature of the planet, the kind of gas in the atmosphere, and planet size. A large planet with a low temperature will have a compact atmosphere (h will be small) while a small, hot planet will have a very extended atmosphere (h will be large).

The scale height can thus be written as

$$h \sim \frac{kT_{planet}}{\mu\, m_p\, g} \tag{2}$$

Approximating *H/h*

A first, crude approximation of H/h can be made using Eqs. (1) and (2).

$$\frac{H}{h} \sim \frac{kT_{melt}}{A\, m_p\, g}\;\frac{\mu\, m_p g}{kT_{planet}} \sim \frac{\mu\, T_{melt}}{A\, T_{planet}} \tag{3}$$

If we assume molecular weights μ and A to be similar, then $\mu \sim A$, and the ratio H/h becomes, in its simplest form, just the ratio of the temperature needed to "melt" rock and the surface temperature of the planet. The quotes on "melt" are a reminder that it's not necessary to melt the mountain material, only to heat it sufficiently so that it deforms.

$$\frac{H}{h} \sim \frac{T_{melt}}{T_{planet}} \quad (4)$$

The numerical coincidence of $H \sim h$ arises from the fact that $T_{melt} \sim T_{planet}$, within an order of magnitude.

The Weak Anthropic Principle states that physical values are restricted by the requirement that there exist sites in the universe where carbonaceous life forms can evolve. But such life forms are restricted to sites where conditions are neither so cold as to inhibit chemical reactions nor so hot as to render the environment chemically unstable and therefore inhibit the evolution of complex life forms. The temperature range suitable for the existence of carbonaceous life forms is about $270K < T_{bio} < 400K$[iii]; therefore on a planet with complex life forms, $T_{bio} \sim T_{planet}$. Press and Lightman approximate the bonding energy of organic matter as $\varepsilon Ry(m_e/m_p)^{1/2}$, resulting in $T_{organic} \sim 350K \approx T_{bio}$. If $T_{organic} \sim T_{bio} \sim T_{planet}$, then T_{planet} must be about 270K - 400K as well. It is not surprising that on a planet that we know with certainty contains complex carbonaceous life forms, the temperature would be ideally suited to the evolution of complex life forms at $T_{Earth} \sim 300K$.

[iii] Carbon-based life forms are restricted to temperature ranges of about -5°C to 110°C. Outside this range, organisms would either be frozen into a state of stasis or prevented from forming the chemical chains necessary for the creation of proteins that are crucial to the evolution of complex organisms. See *Biology of Microorganisms*, 9th ed., M.T. Madigan, J.M. Martinko and J.Parker, eds., Prentice Hall, 2000

And when the ratio of the molecular weights of atmospheric gas and rock (μ/A) is included, the idea that $H/h \approx$ unity on Earth is really the numerical coincidence that $(\mu/A)T_{melt} \approx T_{bio} \approx T_{Earth}$.

H/h for Earth, Mars and Venus

We have pretty good data for the terrestrial planets in our solar neighbourhood. Using Eq. (3), we can get a quick and dirty approximation of whether complex life forms are likely to exist knowing only the temperature and the atmospheric gases of Earth ($T_E \approx$ 300K, $\mu_E \approx 30$), Mars ($T_M \approx 225$K, $\mu_M \approx 44$) and Venus ($T_V \approx 730$K, $\mu_V \approx 44$) [Carroll and Ostie, 1996]. Since all three planets evolved from the same debris of ancient stars, it is safe to assume $T_{melt} \sim 700$K and the atomic mass for SiO_2 ($A \approx 60$) holds for all terrestrial planets in this solar system.

$$\frac{H}{h} \sim \frac{\mu}{60}\frac{700\text{K}}{T_{planet}} \qquad (5)$$

This formula gives for Earth (E), Mars (M) and Venus (V) the values

$$\frac{H_E}{h_E} \sim 1.16, \qquad \frac{H_M}{h_M} \sim 2.28, \qquad \frac{H_V}{h_V} \sim 0.70.$$

Even without including the various fudge factors for H and h, Eq. (5) does show that $T_{melt} \sim T_{bio} \sim T_{planet}$ is a strong indicator of the likelihood of complex life forms being found only on those planets where $H/h \approx 1$. We know that at the present time, Mars and Venus do not support complex life forms, although according to Eq. (5) they might have been amenable to life if, at some time, Mars was hotter and Venus was cooler, putting the surface temperatures within range of T_{bio}.

What happens when we look at the ratio of real-world mountain heights on Earth (Mount Everest), Mars (Olympus Mons) and Venus (Maxwell Montes) to the atmospheric scale height ($h = kT/\mu\, m_p\, g$)? (See Appendix B for the calculation of h_E.)

$$\frac{H_{Everest}}{h_E} \approx \frac{8{,}850 \text{ m}}{8{,}430 \text{ m}} \approx 1.05$$

$$\frac{H_{Olympus}}{h_M} \approx \frac{24{,}000 \text{ m}}{11{,}400 \text{ m}} \approx 2.10$$

$$\frac{H_{Maxwell}}{h_V} \approx \frac{10{,}800 \text{ m}}{15{,}400 \text{ m}} \approx 0.70$$

These results show a satisfying correlation to the results obtained using the very crude approximation of Eq. (5).

3. APPROXIMATING *H/h* IN UNIVERSAL CONSTANTS

It is also possible to arrive at *H/h* using the anthropic principle and universal constants, up to the fudge factors. The translation of H_W (Eq. (1A)) and H_{PL} (Eq. (3A)) into universal constants is covered in Appendix A, and the scale height h (Eq. (1B)) is translated in Appendix B. Taking the ratio of Eq. (1A) to Eq. (1B) gives

$$\frac{H_W}{h} = \frac{F_W}{f}\, \alpha^{-1/2+1/2}\, \beta^{-5/4+5/4}\, \alpha_G^{\,-1+1} = \frac{F_W}{f} \qquad (6)$$

The ratio H/h is not dependent on the universal constants, regardless of whether they are really constant or not; it depends only on the fudge factors, F_W and f. From Eq. (2A),

$$F_W = \varepsilon^{-1/2}\, \xi\gamma A^{-1} = 6 \times 10^{-4}$$

where ε, γ and A are Weisskopf's fudge factors. (See earlier discussion.) From Eq. (1B),

$$f = \varepsilon^{1/2}\, \beta^{1/2}\mu^{-1} = 2 \times 10^{-4}$$

where μ is the atomic weight of atmospheric gases and $\varepsilon\,\beta^{1/2}$ reduces the Rydberg energy. See Appendix B for discussion.

Using the numerical results for Eq. (2A) and (1B), the ratio of $F_W/f = 3$. Because $H/h \sim F/f$, we would expect the ratio of the numerical values from Eq. (1A) $H_W \sim 200$ km and Eq. (1B) $h \sim 100$ km to produce a result similar to F_W/f. It does, with $H_W/h \sim 2$. The difference is due to rounding off the estimates to one significant figure.

Both Weisskopf and Press and Lightman state that the various fudge factors do not allow for all the complexities of the materials involved and the variations in planetary activity that might affect the energy needed to plasticize rock. Approximations using universal constants produce reasonably accurate results within an order of magnitude, but some fine tuning of the fudge factors is still needed for more accurate results.

Conclusion

The Weak Anthropic Principle (WAP) requires that the universe be old enough for the evolution of complex biological life forms to have already occurred, the evidence being that is has, in fact, occurred. But the evolution of carbonaceous life forms restricts evolution to planets that aren't so cold as to severely inhibit chemical activity nor so hot as to render chemical activity highly unstable. These conditions are met when 270K < T_{bio} < 400K and when $T_{bio} \sim T_{planet}$. This implies that the ratio of the maximum mountain height and the scale height (H/h) goes as unity if $T_{bio} \sim T_{melt}$, with T_{melt} being the melting temperature at the mountain base. But that doesn't mean we have only to search for exoplanets with $T_{bio} \sim T_{planet} \sim T_{melt}$ to find other complex life forms. Indeed, there is some consideration that life forms of the evolutionary complexity of humans might be very rare [Rees, 1997; Barrow and Tipler, 1986], even when conditions are optimal.

In the anthropic sense, humans exist on Earth because the conditions are suitable for the development of life. Even if it is finally established that the universal constants are indeed changing, this does not affect the ratio H/h, since α, β, and α_G cancel in Eq. (6). Only the fudge factors remain. (Recall that $\beta^{1/2}$ in Eq. (4A) is treated as a fudge factor.) It might be interesting to speculate about how changes in the universal constants might affect the fudge factors. However, the significance of the numerical coincidence of $H \sim h$ arises from the fact that $T_{melt} \sim T_{planet} \sim T_{bio}$, based on the Weak Anthropic Principle.

This does indeed appear to limit the presence of most humans or complex biological life forms to planets where

$$\frac{H}{h} = e^{x} \tag{9}$$

where $|x| < O(1)$.

Therefore, it is possible to conclude that indeed mountain climbers *can* (barely) scale Mount Everest "because we are here."

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Appendix A

It helps to know some simple substitutions for putting commonly used terms into universal constants, using Planck units where $\hbar = c = G = k = 4\pi\varepsilon_0 = 1$.

$$a_0 \equiv \alpha^{-1}\, m_e^{-1} \qquad Ry \equiv e^2 / 2a_0 \sim \alpha^2\, m_e \qquad \alpha_G \equiv m_p^{\,2}\, G = m_p^{\,2}$$

$$\rho_0 \equiv m_p\,(2a_0)^{-3} \sim m_p\, \alpha^3\, m_e^{\,3}$$

- Translating H_W (Weisskopf) into universal constants begins with

$$H = \xi\gamma \frac{Ry}{Am_p g} \sim 26 \text{ km}$$

where $Ry \sim \alpha^2\, m_e$

$g = GN_E\, m_p / R_E^{\;2}$

where N_E is number of nucleons on Earth,

$R_E \sim N_E^{\;1/3}\, A^{-1/3} f\, a_0$ [iv], the radius of Earth,

and $f\, a_0$ is the radius of a simple molecule.

Hence, $g \sim G\, N_E^{\;1/3}\, m_p\, A^{2/3}\, (f\, a_0)^{-2}$. But Weisskopf does not give a value for $N_E^{\;1/3}$, so it's not possible to translate g into universal constants. However Press and Lightman set $g = GM/R^2$, and they use the anthropic principle, with $\varepsilon = 0.1$, to estimate the mass and radius of a habitable planet as

$$M \sim \varepsilon^{3/2}\, m_p\, \beta^{3/4}\, (\alpha/\alpha_G)^{3/2}$$

$$R \sim \varepsilon^{1/2}\, (2a_0)\, \beta^{1/4}\, (\alpha/\alpha_G)^{1/2}$$

which translates into $g \sim \varepsilon^{1/2}\, \alpha^{5/2}\, \beta^{9/4}\, \alpha_G$. This is a fairly good estimate of gravitational acceleration for an Earth-sized planet, giving $g \sim 2$ m/s^2.

[iv] Note that there is a typographical error in Weisskopf's paper that gives the atomic mass as A^{-1} instead of $A^{-1/3}$.

Using Press and Lightman's anthropic estimate of g results in

$$H_W = F_W\,\alpha^{-1/2}\,\beta^{-5/4}\,\alpha_G^{\ -1} \sim 200\text{ km} \tag{1A}$$

$$\text{where } F_w = \varepsilon^{-1/2}\,\xi\,\gamma A^{-1} = 6 \times 10^{-4} \tag{2A}$$

Note that the resulting H_W is a dimensionless number in Planck lengths, with L_P = 1.616 x 10^{-35} m. Eq. (1A) gives $H_W \sim 200$ km which is about an order of magnitude larger than $H \sim 10$ km.

- Translating H_{PL} (Press and Lightman) into universal constants begins with equating the pressure at the base of a mountain (the product of the mountain height, acceleration of gravity and its density) with the shear strength of "crud",

  $$H_{PL}\,g\,\rho_0 \sim Ry\,\beta^{1/2} / (2\,a_0)^3,$$

  giving the maximum mountain height as

  $$H_{PL} \sim Ry\,\beta^{1/2}\,(2\,a_0)^{-3}\,\rho_0^{-1}\,g^{-1} \sim Ry\,\beta^{1/2}\,m_p^{-1}\,g^{-1}$$

  Translating this into universal constants, using $g \sim \varepsilon^{1/2}\,\alpha^{5/2}\,\beta^{9/4}\,\alpha_G$, results in

  $$H_{PL} = F_{PL}\,\alpha^{-1/2}\,\beta^{-3/4}\,\alpha_G^{\ -1} \tag{3A}$$

  $$\text{where } F_{PL} = \varepsilon^{-1/2}$$

Eq. (1A) differs from (2A) by a factor of $\beta^{1/2}$, but that's because Press and Lightman dispense with the fudge factors used by Weisskopf, and instead reduce binding energy by $\beta^{1/2} = 2.33 \times 10^{-2} \approx 0.02$ rather than Weisskopf's $\xi\gamma = 0.01$. By including $\beta^{1/2}$ in the fudge factor, Eq. (2A) can be rewritten as

$$H_{PL} = F_{PL'}\,\alpha^{-1/2}\,\beta^{-5/4}\,\alpha_G^{-1} \sim 30{,}000 \text{ km} \tag{4A}$$

$$\text{where } F_{PL'} = \varepsilon^{-1/2}\,\beta^{1/2} = 7 \times 10^{-2} \tag{5A}$$

Note that H_{PL} is also a dimensionless number in Planck lengths, and $\varepsilon^{-1/2}$ arises from Press and Lightman's use of ε as a fudge factor. Eq. (4A) gives $H_{PL} \sim 30{,}000$ km, which is significantly different from $H \sim 10$ km.

Press and Lightman admit Weisskopf has done a more careful job in calculating *H*, so preference is given to Eq. (1A) and fudge factor Eq. (2A).

Appendix B

Scale Height

The scale height is essentially the characteristic vertical dimension of atmospheric gas distribution. Atmospheric pressure (P) drops as the altitude (z) increases. This relationship is given [Chamberlain and Hunten, 1987] as

$$\Delta P = e^{-\Delta z/h},$$

where h is the scale height.

The scale height (h) is determined by

$$h = \frac{-dz}{dlnP}$$

$$= -\left(\frac{dlnP}{dz}\right)^{-1}$$

$$= -\left(\frac{1}{P}\frac{dP}{dz}\right)^{-1}$$

$$= \frac{P}{\rho g},$$

using the equation of hydrostatic equilibrium $\frac{dP}{dz} = -\rho g$.

The equation of state for an ideal gas is $PV = nRT$ and hence, $P = nRT/V$. Substituting $n = N/N_0$ (the number of molecules per mole), $\rho = N/V$ (the number of molecules per volume) and $k = R/N_0$ (the universal gas constant divided by Avogadro's number) results in $P = \rho kT/m$. Substituting for P,

$$h = \frac{kT}{mg}, \text{ where } k = \text{Boltzmann's constant,}$$

T = mean surface temperature,

g = acceleration of gravity,

$m = \mu\, m_p$ (atomic weight of gas x mass of proton).

Scale height: Earth[v]

To determine the scale height for Earth's atmosphere, it is first necessary to determine the value for μ. The chemical composition of the atmosphere is approximately

N_2 (80%)
O_2 (20%)

with some trace amounts of CO_2, H_2O, Ar, Ne and O_3 that are not significant for this calculation. The atomic weights are then

$$\mu = (14)(2)(80\%) + (16)(2)(20\%) = 28.8.$$

The other factors for Earth's scale height are

$T = 288$ K

$k = 1.38 \times 10^{-23}$ J/K

$g = 9.81$ m/s^2

$m_p = 1.67 \times 10^{-27}$ kg

giving

$$h_E = 8{,}432 \text{ m } (8.4 \text{ km})$$

This compares favourably with the value of the U.S. Standard Atmosphere, $h_\oplus = 8{,}434.5$ m[vi].

- Translating h into universal constants, using Press and Lightman's anthropic approximation of g as in Appendix A,

$$h = \frac{kT}{\mu m_p g} \sim \frac{\varepsilon \beta^{1/2}}{\mu} \frac{Ry}{m_p g} \sim f \alpha^{-1/2} \beta^{-5/4} \alpha_G^{-1} \sim 100 \text{ km}, \qquad (1B)$$

$$\text{where } f = \varepsilon^{1/2} \beta^{1/2} \mu^{-1} = 2 \times 10^{-4}. \qquad (2B)$$

The fudge factors $\varepsilon \beta^{1/2}$ reduce the Rydberg energy so that the temperature of a habitable planet is about $T \sim 350$K [Press and Lightman, 1983] to allow for an atmosphere which is neither a vacuum nor primordial hydrogen and helium [Press, 1980].

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[v] Unless otherwise indicated, the planetary data used here are from NASA, and are available at \\pds.jpl.nasa.gov/planets.

[vi] CRC Handbook of Chemistry and Physics, 1984, R.C. Weast, M.J. Astle and W.H. Beyer, eds, 65th Ed, CRC Press, Inc.