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THE UNIVERSITY OF ALBERTA

NON-INERTIAL QUANTUM FIELD THEORY IN FLAT SPACE-TIME

BY

KWOK PAN NG

A THESIS

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Date : APRIL 8 1988

DEDICATION

TO

my FAMILY, my BRETHRENS and above all the LORD

whom I treasure above all else on this earth, and who has given meaning to my life

Abstract

The earlier development of non–inertial quantum field theory is traced back to the by–product of formulating a quantum field theory in curved space time. The well known 'Fulling-Davies-Unruh' effect revealed that a thermal character which obeys the Bose-Einstein statistics is inherent in the quantum field as viewed from a uniformly accelerated frame of reference. Then by formulating the theory in a large class of coordinate systems or accelerated frames, the observed spectrum was found to exhibit a complicated form in general. Moreover, the limited applicability of the usual concepts in Minkowskian quantum field theory is recognised when applied in non-trivial space-time. Also discussed is the closely related topic of quantum detection process in non-inertial frames by constructing a simple model particle detector (the De-Witt monopole detector) in probing the 'particle content' or 'vacuum noise' along its motion in flat space-time.The phenomenon of the 'apparent inversion of statistics' for a uniformly accelerated detector in flat space-times of arbitrary dimensions is discussed as well as some experimental facts about the detection of the 'Fulling-Davies-Unruh' effect.

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TABLE OF CONTENTS

LIST OF FIGURES

Conventions and abbreviations

Our notation for quantum field theory mainly follows that of Bjorken & Drell (1965). The metric signature is (+,−,−,−) in usage. Throughout the thesis, natural units are used i.e., $\hbar = c = G = 1$ unless otherwise stated. Bold characters indicate vector quantities in the thesis.

The following special symbols and abbreviations are used throughout :

$*$	complex conjugate
$+$ or h.c.	Hermitian conjugate
∇_μ	covariant derivative
$\partial/\partial x^\mu$ or ∂_μ	partial derivative
Re(Im)	real (imaginary) part
Tr	trace
ln	natural logarithm
k_B	Boltzmann constant
$[A,B]$	$AB - BA$
$[A,B]_+$	$AB + BA$
$\sim$	order of magnitude estimate
$\approx$	asymptotically approximate to
$\equiv$	defined to be equal to

Chapter 1 Introduction

The discovery of the Hawking effect : the quantum thermal radiance from a black hole, is one of the most spectacular results of the construction of quantum field theory in curved space-time (Hawking 1975). It provides a theoretical foundation for further development of the thermodynamics of a black hole originally conceived phenomenologically by Bekenstein (1973).

The fact that a thermal effect arises in a situation which seems to have no relation to ordinary statistical thermodynamics is a surprising one, and has therefore attracted a great deal of theoretical attention concerning its foundations. In particular, it has been suggested that a similar effect could occur even in flat space-time (Davies 1975). Indeed, it was subsequently shown that "a uniformly accelerated 'particle detector' will perceive a thermal bath of particles" (Unruh 1976). This effect, which occurs in flat space-time, hinges on one of the fundamental concepts in physics : the principle of equivalence, becomes important as a byproduct of the efforts to quantize gravity (Unruh 1986b).

These developments revealed that the essential feature of the Hawking effect was embedded in a simpler situation involving the uniform acceleration in flat space-time. It was also realized that a relevant mathematical framework for the latter had already been developed in the work which was concerned with various possible definitions of vacuums and associated particles in the quantum field theory (Fulling 1973). Actually, this investigation was also motivated by the search for a quantum theory of gravity. The structure of this mathematical framework is similar to that underlying the Hawking effect apart from the complications due to the curvature of the space-time with a black-hole. The significance of this framework became appreciated after the discovery of the Hawking effect, and so for the analogous effect in flat space-time, it was called the 'Fulling-Davies-Unruh' effect.

It is an interesting historical fact that some people were developing a general axiomatic theory of quantum fields which found its natural application to wedge shaped manifolds (Bisognano and Wichmann 1975, 1976). Its relevance to the above mentioned effects was recognised much later and further developed into a general 'thermalization theorem' which contained the mathematical structure devised by Fulling as a special case of a free field (Sewell 1982, Kay 1985).

This thermalization theorem states that the pure state which is the vacuum state for an inertial observer is a canonical ensemble from the point of view of a uniformly accelerated observer. The temperature characterizing the ensemble is proportional to the magnitude of the acceleration of the observer. If the acceleration is replaced by the surface gravity of a black hole, it is precisely equal to the Hawking temperature of the black hole.

The discovery of the Fulling-Davies-Unruh effect prompted a flood of researches on this subject and on quantum field theory in curved space-time in general. The word 'Fulling Davies-Unruh' effect is used with various meanings in the literature and the discoverers are assigned different credits for this effect. Here, we have adopted the term which includes two distinct physical aspects: firstly, the thermalization theorem holds and secondly, from an operational viewpoint, a uniformly accelerated detector observes the Planckian spectrum. It should be pointed out that these two features are not equivalent to one another.

The aim of the present thesis is to review the investigations done by people and present their results in a coherent way with the main theme centering about the formulation of quantum field theory in non-inertial frames in flat space-time. The starting point is the phenomenon of the 'Fulling-Davies-Unruh' effect for it provides a suitable candidate for the canonical quantization of a matter field in a uniformly accelerated frame: the 'Rindler' frame. Then, the scope of the formulation will be widened and extended, in order to assimilate the changes in conditions of the problem. These change in conditions include the motion of a non-uniformly accelerated observer; the coordinate systems are no longer the

Rindler frame but indeed a variety of systems represent all possible kinds of motions. A brief account of quantum field theory in analytic accelerated frames is given because of its relevant relation to our interest. In the course of the presentation, the results of the investigations are highlighted and their physical interpretations are emphasized. Their unusual features are pointed out with regard to those in ordinary quantum field theory. Furthermore, we focus our attention on one of the conceptual aspects of the subject, namely the quantum measurement process associated with the motion of the observer. Two main problems are addressed, concerning the method and the interpretation of the result of the measurement process. An account of the working mechanism of a model particle detector and what it measures when coupled to a matter field is given. The phenomenon of 'apparent statistics inversion' comes out unexpectedly from the coupling of a model particle detector to a matter field in arbitrary dimension. This opens a new vista for the detection process. The problem of verifying the effect is also a point of interest but still in the primitive stage of doing actual experiments.

This review follows a pedagogical approach to the subject, presenting the conceptual development pertinent to the formulation of the theory, looking at it from various angles, so as to give a coherent picture about development of the theory. The materials is organized in an orderly manner with emphasis on the general framework of conceptual investigation, instead of concentrating on the details of the derivations or techniques employed by people. It is hoped that this will make the main theme explicit. At least some of the technical aids to carry out calculations are presented but we do not dwell on tedious calculations and arguments. Excellent reviews have been published (De-Witt 1979; Sciama, Candelas and Deutsch 1981; Birrell and Davies 1982; Wald 1984; Takagi 1986) and the reader can refer to them for a more detail treatment of individual subjects.

The plan of this review is as follows.

In chapter 2, an historical background of the work of those 'pioneers' is given so that reader can have a basic idea of the early stages of development of field theory in curved space time and its subsequent simplications and redevelopments in flat space time. The term 'Fulling Davies Unruh' effect with its associated temperature is introduced.

Chapter 3 deals with some of the heuristic features in the field theory in Minkowski space time and in particular, the problems of adapting those features to a generic space time without otherwise modifying and reconsidering the usual field theoretic construction and definition.

Then we proceed to chapter 4, which forms the basic background of the subject and so is treated rather extensively. The Rindler coordinates are introduced followed by the exposition of the quantization of the field. Here, the approach of quantizing the field over the whole Minkowski space time is adopted as opposed to the familiar Fulling quantization or other methods. The global properties of quantum field theory are spelled out by using PCT symmtry on the wavefunctions in the Rindler manifolds. The Bogoliubov cofficients relating the creation annihilation operators that define the two vacuum states are calculated. Finally, the thermalization theorem is stated as a relation between the two vacuum states $|0_M>$ and $|0_R>$ for both the scalar and Dirac fields. This result is generalized to interacting fields in association with the KMS (Kubo-Martin-Schwinger) condition as was first done by Sewell (1982).

Having the relevant concepts thus introduced, chapter 5 embarks upon the canonical quantization of a matter field in other coordinate systems in flat space-time. The case of a rotating observer is discussed in detail due to its interesting physical implications. It is done in the general situation by considering a time-dependent angular velocity in the formulation so that the results from both uniform and non-uniform circular motion can be compared. On the other hand, the main results from the quantization of the field in other stationary coordinate systems are presented that yield a surprising conclusion: only two kinds of vacuum states exist in the formulation of the theory in flat space-time and those vacuum states are distinguished from one another by the existence of an event horizon.

Chapter 6 is devoted solely to the formulation of quantum field theory in analytic accelerated frames. This approach was first considered by Sanchez in flat space-time but is generalized to curved space-time later. However, we will concentrate on the ideas brought into the formulation in flat space-time. The merits of this approach are not only that all the quantum, classical and thermal aspects of the theory can be explicitly expressed in terms of the nature of the analytic mappings; but also the theory of the detection process is incorporated into it since all the measured physical magnitutes are to be interpreted in terms of the measurements carried out by a comoving measuring device.

In chapter 7, the ideas of quantum detection process are introduced. This is done by first considering the conceptually simplest model of a detector coupled to the matter field, called the 'De-Witt monopole detector'. The response of this detector is given by the product of a factor depending on the structure of the detector and another factor depending only on the intrinsic nature of the field. The latter factor is expressed in terms of the 'Wightman two-point ' function and is interpreted as the vacuum noise of the power spectrum by some authors. Then, a calculation is carried out to obtain the power spectrum as seen by the detector in both inertial and uniformly accelerated motion when coupled to a scalar field. The related topic of the measurement of the vacuum fluctuation of the 'zero-point field' in accelerated frames by a detector is also discussed. A comparison is made between the results from canonical quantization of a scalar field and those measured by a De-Witt monopole detector so that the usage of a model particle detector may be justified. The interesting phenomenon of the 'apparent statistics inversion' governing the power spectrum of the 'Rindler noise' is discussed and the expression of it is compared to that from a ordinary thermal noise for both scalar and Dirac cases. The massive matter field complicates the expressions of the vacuum noise but information can still be obtained. Finally, the problems that plague the theory of quantum detection processes are again brought into discussion with an eye for further clarification.

In chapter 8, the experimental facts and plausibility of actually performing an experiment to test the 'Fulling-Davies-Unruh' effect are conveyed to the reader. It is hoped that what we discussed in the preceding chapters can be further understood by considering an experimental verification with an expectation for further improvements and refinement of the existing model.

Lastly, chapter 9 summarizes the main ideas developed in the review and two appendices are given to provide information about the Bogoliubov transformation and the KMS condition.

Chapter 2 Historical Background

The goal of quantizing the gravitational field and hence unifying quantum theory with the general theory of relativity remains unattainable since the early constructions of various formalisms. In the past decades, a semi-classical approach has been used in which a matter field that satisfies the linear field equations ('a linear matter field') is quantized on a given fixed and unquantized curved background. This so called 'background field' method was started by De-Witt as a possible approach to a complete theory of quantization of gravity (De-Witt 1967a,b). According to this method, the new metric is then given by

$$g_{\mu\nu} = g^{c}_{\mu\nu} + \bar{g}_{\mu\nu} \tag{2.1}$$

where $g^{c}_{\mu\nu}$: the classical metric of some background space-time

$\bar{g}_{\mu\nu}$: a quantum field propagating in the background space-time

However, some of the aspects of such a theory have not been explicitly solved such as the regularization and renormalization difficulties of the quantum stress tensor $<T_{\mu\nu}>$. Different techniques are needed for getting around those plagues that exist in the theory. On the other hand, the remarkable discovery by Hawking in 1975 brought new excitement in the formulation of quantum field theory in a curved space-time. The so called 'black-hole evaporation' or 'Hawking effect' states that if a linear matter field is quantized in the presence of a black-hole, then the black-hole produces quanta of this field and radiates them exactly as if it were a black body at a temperature inversely proportional to the mass of the black-hole.

$$k_B T = \frac{hc}{2\pi\,(8GM)} = \frac{\hbar c}{8GM} \tag{2.2}$$

In natural units : $\hbar = c = G = 1$, we have

$$k_B T = \frac{1}{8M} \tag{2.3}$$

where k_B : Boltzmann constant

T : Temperature (K)

M : Mass of the black hole

This result is obtained for a four-dimensional simplified model in which a spherically symmetric ball or shell of matter collapses inwards under the gravitational force. This temperature can also be expressed in geometric units in the form of:

$$k_B\, T = \frac{\hbar\, \kappa}{2\pi c} \tag{2.4}$$

where κ is the surface gravity of the black-hole. An inertial observer in the vicinity of the event horizons of the black-hole would then detect a thermal flux of outgoing particles. We may then also describe the black-hole as a thermal mixed state.

The Hawking effect connects the physics of black-holes with other branches of physics such as thermodynamics etc., and also has various implications for quantum gravity itself. Moreover, it has potentially interesting implications for quantum theory itself, namely the association of the radiation with an event horizon. For some isotropic cosmological models such as the de-Sitter space, an inertial observer finds himself bounded by an event horizon and detects thermal radiation (Parker 1976). Since these horizons could be different for different observers, an idea of observer dependence is introduced into the quantum theory. Actually, we already have this physical implication for flat space-time. This was first discussed by Fulling (1973) in his thesis about the non-uniqueness of canonical quantization of a scalar field in Riemannan space-time. He showed that the 'vacuum' or 'no particle' state in the rest frame coordinate adapted to a uniformly accelerated observer is not unitarily equivalent to the ordinary Minkowski-space vacuum. Working in the Heisenberg picture, the Bogoliubov transformation coefficients between the accelerated and Minkowski frame 'vacuum states' were calculated by Fulling and by Davies (1975) in a different context by using an accelerating moving mirror. They found that the Minkowski 'vacuum' would appear as an infinite sum of multi-particle states with a thermal distribution in the accelerated frame. The associated local temperature is given by

$$k_B T = \frac{a\hbar}{2\pi c} \tag{2.5}$$

$$\text{or} \quad k_B T = \frac{a}{2\pi} \quad \text{in natural units} \tag{2.6}$$

where 'a' is the magnitude of the proper acceleration of the accelerated frame. Similarly, we can use a thermal density matrix to describe the Minkowski vacuum restricted to the space of interest under the Fulling quantization.

The next major step was taken by Unruh (1976), who first studied the operational significance of this result. He considered a 'Gedanken' experiment of how a simple quantum mechancial particle detector such as a model atom with various energy levels put inside a box, when coupled to a massless scalar field would behave if it were moved along a uniformly accelerated world-line through the Minkowski vacuum. The calculated result showed that the detector became excited with a thermal spectrum as if it were immersed in a heat bath with a temperature given by eqn. (2.6). In other words, the response of the detector was the same as that of the detector at rest in a thermal bath of radiation at the temperature given above. In this case, 'a' is the magnitude of the acceleration of the detector.

It should be pointed out that we were addressing two different physical processes while the two temperatures obtained coincided but with totally different interpretations. We then have the 'Fulling-Davies-Unruh' effect for which a uniformly accelerated observer or detector detects a thermal spectrum of radiation when coupled to a 'linear matter field' in Minkowski space-time with the 'Fulling-Davies-Unruh' temperature given by eqn. (2.6).

In summary, the scenario has been changed from curved space-time manifold to static flat space-time but we still obtained similar results. The 'Hawking effect' can then be interpreted as acceleration radiation in a simple physical sense. Actually, the theory of the 'Hawking effect' has a close mathematical relation to that of the 'Fulling-Davies-Unruh' effect since under a conformal transformation, the space-time manifolds are identical to one another. More than this, the 'Fulling-Davies-Unruh' effect paved the way for investigating the physical phenomenon that arose from a uniformly accelerated observer or particle detector and subsequently to non-uniformly accelerated motions when coupled to a 'linear matter free field'.

Chapter 3 Heuristic quantum field theory features

Before embarking on a discussion of the methods and results of field quantization in accelerated frames, we make some heuristic remarks concerning the quantum field theoretic features of the problem. Isham (1977) stated that a linear quantum field is determined by the following features:

1). Linear field equations for operator-valued distributions.

2). Canonical equal-time or covariant commutator relations.

3). Boundary conditions for fields.

4). The mathematical definition of states and observables.

5). The association of physical states and observables with their mathematical correlates.

It should be emphasized that the above five features are intimately connected and cannot truly be separated. Some of the problems inherent in those features manifested themselves in non-inertial frames or a generic curved space-time will be discussed briefly.

On a Minkowski space-time or even a curved space-time, there is no problem in rigorously constructing quantum fields that satisfy the first two conditions (with an appropriate generalization of 'equal time'). The space-time manifold is required to be globally hyperbolic, thus ensuring the existence of a global Cauchy hypersurface on which the classical Cauchy problem is well posed. Thus, there exist unique advanced and retarded Green's functions for the evolution of initial data in the hypersurface. Then the quantum field automatically satisfies the required covariant field equations and commutation relations. However, the main problem is that there exist an infinite number of representations of either the canonical or covariant commutation relations, and we need to have the 'physically relevant' one. Moreover, the suitably smeared field operators are assumed to exist on a fixed Hilbert space and obey the covariant equations. But some people maintain the use of different Hilbert spaces at each times, each with its own representation of the canonical commutation relations. The motivation is that for the particle creation and annihilation operators at each time, the Hamiltonian can be written in the symbolic form:

$$H(t) = \sum_n \omega_n(t)\, a_n^+(t)\, a_n(t) \tag{3.1}$$

This is the so called 'Hamiltonian diagonalization' method.

The effects of boundary conditions for fields are prevalent in Minkowski space-time. The Casimir effect is the result of the difference in vacuum energy between a field in a box and fields in flat space. The moving mirror effect stems from having the moving boundaries coupled to a matter field in a background space-time. In general, the boundary conditions reflect the global topological structure of the space-time that have already been incorporated in the first and second features.

The construction of mathematical states and observables has a problem that requires to be tackled, namely the association of a specific quantum operator with a classical observable which is a non-linear function of the field (e.g. the stress tensor $T_{\mu\nu}$) that is formally divergent in the quantum theory.

The last feature really causes many ambiguities when we have a non-inertial observer or a generic curved space-time. In Minkowski space-time we usually use a Fock representation with its associated particle labels for states and observables. However, these definitions involve global concepts such as positive frequency classical solutions (for particle states), or the Poincaré group of inertial observers, which are not generally applicable in a generic curved space-time. Without the Poincaré invariance of the vacuum and the required positivity (i.e., positive norm) of the associated Hamiltonian, we do not have the exact technical tools that provide the unique conventional free quantum field theory. Quantum fields may be readily constructed using various Fock spaces, but there is no reason to say that field quanta in these spaces are actual 'physical particles' and there is no general scheme of choosing the physically correct representation. Indeed, in the case of background space-times, the notion of frequency which is intimately connected with the idea of a particle is only applicable for wavelengths less than the local radius of curvature. Thus, it is unclear what precisely is meant by a 'vacuum' or a 'particle state'. It is this loosely defined notion that affects the physical interpretation of the results in field quantization and especially in the measurement process.

There are other quantum field theoretic problems that need to be faced up to in addition to those mentioned above. The general covariance and group invariance of the quantum theory are found to be violated by the divergent field quantities (such as $T_{\mu\nu}$) and the non-preservation of the conformal invariance at the quantum level. The phenomenon of infrared divergences of a massless theory could radically change in non-Minkowskian topologies. The final problem of incorporating genuine quantum gravity effects into the theory will be certainly important at dimensions of the Planck length ($L_p \sim 10^{-33}$ cm.) , the Planck time ($t_p \sim 10^{-43}$ sec.) , the Planck mass ($M_p \sim 10^{-5}$gm. $\sim 10^{-28}$ eV.), and the Planck temperature ($T_p \sim 10^{32}$ K).

Chapter 4 Quantum field theory in Rindler frame

The implicit utilization of non-inertial frames in Minkowski space-time has been briefly discussed in Chapter 2 .Basically, we employed two kinds of techniques : one is the use of conventional "in / out " quantum field theory in Fock space. The main tool is the Bogoliubov transformation and the other is the construction of model particle detectors for investigating the 'particle content' of the theory in some physically motivated way. In the following sections, we are going to review the phenomenon of a uniformly accelerated observer in a matter field with emphasis on the methods and the physical interpretation of the obtained result.

4.1 Rindler coordinate system and uniform acceleration

Let 'a' be the proper acceleration of an observer (i.e., the acceleration of an observer relative to his instanteous rest frame) in Minkowski space-time that satisfies the following :

$$a_{\mu}\, a^{\mu} = -a^2 \tag{4.1}$$

with the metric given as

$$\eta^{\mu\nu} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

for two-dimensional space-time.The observer is said to be uniformly accelerated when 'a' is a constant.

The metric line element in the two-dimensional Minkowski space-time is given by

$$ds^2 = dt^2 - dx^2 \quad \text{with } \eta^{\mu\nu} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{4.2}$$

With the following coordinate transformation

$$t = \alpha^{-1} e^{\alpha\xi} \sinh(\alpha\eta) \quad (4.3)$$

$$x = \alpha^{-1} e^{\alpha\xi} \cosh(\alpha\eta) \quad (4.4)$$

with α= constant > 0 and $-\infty < \eta, \xi < \infty$.

Then eqn. (4.2) becomes

$$ds^2 = e^{2\alpha\xi} (d\eta^2 - d\xi^2) \quad (4.5)$$

The coordinates (η, ξ) cover only a quadrant of the Minkowski space-time, namely the wedge $|x| > t$ as shown in Fig.4.1 . Lines of constant η are straight while lines of constant ξ are hyperbolae.

From eqns.(4.3) and (4.4), we get :

$$x^2 - t^2 = \alpha^{-2} e^{2\alpha\xi} = \text{constant}$$
$$= (\alpha^{-1} e^{\alpha\xi})^2 = \text{constant} \quad (4.6)$$

The world-lines of the uniformly accelerated observers are then represented by the hyperbolae for a particular value ξ. The proper acceleration 'a' can be identified as the following relation:

$$\alpha e^{-\alpha\xi} = \beta^{-1} = \text{'a'} = \text{proper acceleration} \quad (4.7)$$

All the hyperbolae are asymptotic to the null rays $t - x = 0$, $t + x = 0$ which means that the accelerated observers approached the speed of light as $\eta \rightarrow \pm\infty$. The observers' proper time τ is related to ξ and η by

$$\tau = e^{\alpha\xi}\eta \quad (4.8)$$

For each uniformly accelerated observer, we have constructed a non-inertial coordinate system in the Minkowski space time. This system (η, ξ) is known as the 'Rindler coordinate system' with the portion $x > |t|$ of the Minkowski space time known as the 'Rindler space' or 'Rindler wedge'. An observer at rest in Rindler space is equivalent to a uniformly accelerated observer in Minkowski space-time, and experiences an inertial force that by virtue of the equivalence principle,cannot be distinguished from a static gravitational field.

A second Rindler wedge $x < |t|$ can be obtained by the time reflection, followed by the space reflection. The left - ($x < |t|$) and right - ($x > |t|$) hand wedges are labelled as L and R respectively. The null rays $t - x = 0$ and $t + x = 0$ act as event horizons. Then the two Rindler wedges L and R are two causally disconnected segments in the Minkowski space time. The remaining regions are labelled as 'Future (F)' and 'Past (P)' as shown in the Figure. Events in both P and F can be connected by null rays to both L and R.

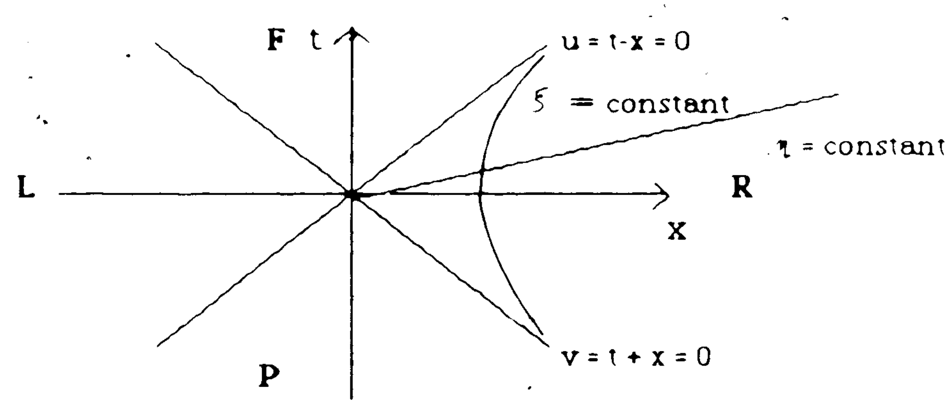

Fig. 4.1. Rindler coordinatization of Minkowski space. The four regions R,L,F and P must be covered by separate coordinate patches. Rindler coordinates are non-analytic across the event horizons.

The four regions can be represented as :

$$R = \{ (t,x) : x > | t | \} \quad (4.9)$$

$$L = \{ (t,x) : x < | t | \} \quad (4.10)$$

$$F = \{ (t,x) : t > | x | \} \quad (4.11)$$

$$P = \{ (t,x) : t < | x | \} \quad (4.12)$$

The event horizons u = 0 and v = 0 are the future and past event horizons respectively. It means that events in F cannot be witnessed in R and similarly for events in R, they cannot be witnessed in P. The equations for u and v are given by:

$$u = \{ (t,x) : t = x \} \quad (4.13)$$

$$v = \{ (t,x) : t = -x \} \quad (4.14)$$

The geometric properties of Rindler space are preserved when extended to four dimensional Minkowski space-time. The coordinate transformation eqns. become

$$t = \alpha^{-1} e^{\alpha\xi} \sinh(\alpha\eta) \quad (4.15)$$

$$x = \alpha^{-1} e^{\alpha\xi} \cosh(\alpha\eta) \quad (4.16)$$

$$y = y, \; z = z \quad (4.17)$$

The metric line element is still defined as :

$$ds^2 = e^{2\alpha\xi}(d\eta^2 - d\xi^2) + dy^2 + dz^2 \quad (4.18)$$

where $-\infty < \eta; \xi < \infty$.

World-lines of constant ξ are still the world-lines of observers with a constant uniform proper acceleration $a = \alpha\, e^{-\alpha\xi}$. Spatial hypersurfaces η = constant are Cauchy hypersurfaces for the evolution of initial data in Rindler space. This hypersurface can be extended through the origin into the left Rindler wedge to form a Cauchy hypersurface for all of Minkowski space-time.

The metric given by eqn. (4.18) is static. Invariance under Rindler time (η) translations corresponds to Lorentz invariance for boosts in the x- direction. Therefore, the velocity of the accelerated observer is being used as the relevant measure of time. Corresponding to this invariance there exist a time-like Killing vector field $\partial/\partial\eta$ which is tangent to the hyperbolic world-lines. The norm of this Killing vector is 1 along the hyperbola $\xi=0$, where Killing time and the observer's proper time agree (see eqn. 4.8). In the right Rindler wedge this Killing field is future directed while in the left wedge it is past directed. In addition, $\partial/\partial y$ and $\partial/\partial z$ are space-like Killing vectors with associated conserved quantities P_y and P_z. In two-dimensions, the space-like vector is a conformal Killing vector but for space dimension $n > 2$, this conformal invariance is lost (Stephens 1986).

4.2 Field quantization in Rindler coordinates

Having discussed some of the geometrical properties of Rindler space, we are now ready to discuss the properties of quantum field therein. In the past decades, the problem of field quantization has been studied extensively for non-interacting field theories, starting with the massless scalar field (Fulling 1973; Davies 1975), Dirac fields (Candelas and Deutsch 1978; Iyer and Kumar 1980; Soffel et.al., 1982) and subsequently other complicated fields. Different methods have also been used by people. For a more detailed discussion of the methods and results, the reader can refer to the reviews mentioned in the introduction.

Although different viewpoints are emphasised in the discussion of field quantization, a similar conclusion is reached : the Rindler representation is inequivalent to that of the Minkowskian. Specifically, the vacuum state of the Minkowski representation is a mixed state of finite temperature of the Rindler representation. However, some authors criticize the usual 'Rindler quantization' of the matter field. The matter field is quantized in the Rindler wedge first and subsequently connected to the Minkowski space-time through a process of continuation such as the Bogoliubov transformation. They stress that this method does not consider the global properties of quantum field theory and a more fruitful way is to do field quantization in the whole of Minkowski space-time (Rumpf 1983; Hughes 1985). The main purpose of Rindler coordinates is to keep track of the segment of Minkowski space-time which is space-like to the uniformly accelerated observer. Then when the field is quantized in Minkowski space-time, it is automatically quantized on the Rindler space too (Hughes 1985).

Rumpf quantized a massive scalar field over the complete two-dimensional Minkowski space-time by the method of mass-analytic quantization. We will follow the treatment of the quantization of a massless scalar field ϕ in four-dimensional Minkowski space-time by Bernard (1984/1985) in which the global properties of the PCT symmetry is spelled out (Hughes 1985).

The Rindler coordinate system is the Fermi-Walker rest frame coordinates of an observer which is moving co-linearly through flat space with a constant proper acceleration of magnitude 'a'. This 'rest frame' coordinate system consists of coordinates carried by an orthonormal tetrad $(e^{\mu})_{\nu}$ such that the following conditions are obeyed :

(a). The timelike tetrad vector $(e^{\mu})_0$ is the four-velocity of the observer u^{μ}.

(b). The tetrad is Fermi-Walker transported (Misner et.al., 1973) along the world-line of the observer. Given a world-line $X^{\mu}(t)$, parametrized by the proper time τ, yields the equations :

$$u^{\mu} = \frac{dX^{\mu}}{d\tau} \; ; \quad a^{\mu} = \frac{du^{\mu}}{d\tau} \qquad (4.19)$$

$$(e^{\mu})_0 = u^{\mu} \qquad (4.20)$$

$$\left[\frac{de^{\mu}}{d\tau}\right]_i = (e^{\mu})_0 \, [(e^{\nu})_i \, a_{\nu}] \quad , \; i = 1,2,3 \qquad (4.21)$$

$$(e^{\mu})_{\sigma}(e_{\mu})_{\nu} = \eta_{\sigma\nu} = \text{diag}\,(1,-1,-1,-1) \qquad (4.22)$$

Now, the Fermi-Walker transport equation can be written as :

$$\frac{d(e^{\mu})}{d\tau} + (e^{\mu})_{\sigma}\,\theta^{\sigma}_{\ v}(\tau) = 0 \qquad (4.23)$$

where θ^{σ}_{v} is the generator of this transport.

It is also a generator of the Lorentz transformation and can be written as:

$$\theta = \frac{1}{2}\varepsilon^{\alpha\beta} L_{\alpha\beta} \qquad (4.24)$$

with

$$\varepsilon^{\alpha\beta} + \varepsilon^{\beta\alpha} = 0 \qquad (4.25)$$

where $L_{\alpha\beta}$ are the generators of the Lorentz transformations.

Let us introduce the acceleration **a** and the rotation Ω as the 'electric' and 'magnetic' part of the antisymmetric tensor

$$\frac{1}{2}\varepsilon^{\alpha\beta}\varepsilon_{\alpha\beta} = \mathbf{a}^2 - \Omega^2 \qquad (4.26)$$

$$\frac{1}{4}(\varepsilon^{\alpha\beta})^{*}\varepsilon_{\alpha\beta} = \mathbf{a}\,.\,\Omega \qquad (4.27)$$

We are looking for accelerating trajectories for which $(\varepsilon^{\alpha\beta})^{*}\,\varepsilon_{\alpha\beta} > 0$ so that the operator θ is a 'good' generator of temporal evolution:

$$[\,\theta,\ \xi^{\mu}\hat{P}_{\mu}\,] \neq 0 \qquad (4.28)$$

for any time-like vector ξ^{μ} ($\hat{P}_{\mu}$ are the translation generator).

The non-inertial coordinates (τ, Z^k)are connected to the inertial ones x^{μ} = (x^0 , x^1, x^2, x^3) (with $x^0 = t$) by the following:

$$x^{\mu} = \Lambda^{\mu}_{k}(\tau)\,Z^{k} \quad , \ k = 1,2,3 \qquad (4.29)$$

with

$$\frac{d}{d\tau} \Lambda^{\mu}_{\ \nu} (\tau) = \theta^{\mu}_{\ \sigma} (\tau) \, \Lambda^{\sigma}_{\ \nu} \tag{4.30}$$

The metric line element can then be written as :

$$ds^2 = -(\theta_{0k} Z^k \, d\tau^2)^2 + \delta_{ij} (dZ^i + \theta^i_{\ k} Z^k d\tau) (dZ^j + \theta^j_{\ k} Z^k \, d\tau) \tag{4.31}$$

Then the Hamiltonian

$$H = \frac{1}{2} \varepsilon_{\alpha\beta} \, L^{\alpha\beta} \tag{4.32}$$

becomes the τ -time evolution operator

$$H = -i \frac{\partial}{\partial \tau} \tag{4.33}$$

The observer's Hamiltonian is represented by either (4.32) or (4.33). For simplicity, we choose θ and τ independent of one another as it is generally valid for an infinitestimal amount of time $\delta\tau$. For a and Ω parallel to each other, the Hamiltonian becomes

$$H = a \, (\hat{K})_1 + \Omega \, (\hat{J}\,)_1 \tag{4.34}$$

where $(\hat{K})_1$: boost generator in the direction 1.

$(\hat{J})_1$: angular momentum operator in the direction 1.

Then the non-inertial coordinates become the rotating Rindler coordinates

$$t = Z^1 \sinh (a\tau) \tag{4.35}$$

$$x^1 = Z^1 \cosh(a\tau) \tag{4.36}$$

$$\theta = \theta' + \Omega\tau \tag{4.37}$$

$$\rho = \rho' \tag{4.38}$$

in cylindrical coordinates. The accelerated coordinates (τ, Z^1, θ', ρ') covers the Rindler space of the Minkowski space-time.

The quantum particle states each with a definite energy for the accelerated observer are chosen to be eigenstates or eigenfunctions of the Hamiltonian H

$$H\,\phi_{\varepsilon,0,0}(t,\mathbf{x}) = \varepsilon\,\phi_{\varepsilon,0,0}(t,\mathbf{x}) \tag{4.39}$$

where $\phi_{\varepsilon,0,0}(t,\mathbf{x})$ vanishes on L or R. It also satisfies the transformation law.

$$\phi_{\varepsilon,0,0}(\Lambda^{\mu}_{\ \nu}(\tau)x^{\nu}) = e^{-i\varepsilon\tau}\phi_{\varepsilon,0,0}(x^{\mu}) \tag{4.40}$$

The wave function is represented by the plane-wave decomposition.

$$\phi_{\varepsilon,0,0}(t,\mathbf{x}) = \int d^3\mathbf{k}\,\frac{e^{i(\mathbf{k}\cdot\mathbf{x}-\omega_k t)}}{(2\pi)^3(2\omega_k)}F_{\varepsilon,0,0}(\mathbf{k}) \tag{4.41}$$

with

$$\omega_k = \sqrt{j^2 + k^2} \tag{4.42}$$

where j is the angular momentum;

Using eqn. (4.40), we get a differential eqn. for $F_{\varepsilon,0,0}(\mathbf{k})$:

$$\left[a\omega_k\frac{\partial}{\partial k^1} + \Omega\frac{\partial}{\partial\theta_k}\right]F_{\varepsilon,0,0}(\mathbf{k}) = -i\varepsilon F_{\varepsilon,0,0}(\mathbf{k}) \tag{4.43}$$

where θ_k is the azimuthal angle of $\mathbf{k}$. The solutions to the above eqns. are

$$\left[\frac{\omega_k + k^1}{\sqrt{j^2+q^2}}\right]^{-\pi\left(\frac{\varepsilon+j\Omega}{a}\right)} e^{i(j\theta_k)} \tag{4.44}$$

$$\omega_k = \sqrt{(k^1)^2 + q^2 + j^2} \tag{4.45}$$

We can use $\phi_{\varepsilon,q,j}(t,\mathbf{x})$ and $\phi^{*}_{\varepsilon,q,j}(t,\mathbf{x})$ as a basis for constructing the Fock-space of the quantum field :

$$\psi(x) = \sum_{\varepsilon,q,j} (\phi_{\varepsilon,q,j}(x)\, a(\varepsilon,q,j) + \phi^{*}_{\varepsilon,q,j}(x)\, a^{\dagger}(\varepsilon,q,j)) \tag{4.46}$$

(in discrete notation). The creation and annihilation operators $a^{\dagger}(\varepsilon,q,j)$ and $a(\varepsilon,q,j)$ define the vacuum state $|0_M>$:

$$a(\varepsilon,q,j)\,|0_M> = 0 \tag{4.47}$$

$|0_M>$ is the Minkowski vacuum as we can notice that the wave-function $\phi_{\varepsilon,q,j}(x)$ have positive energy. We know that the two Rindler wedges L and R are causally disconnected, so L is outside the field of communciation of the uniformly accelerated observer inside R. We want to diagonalize the Hamiltonian separately inside the regions L and R. By the PCT symmetry of quantum field theory, we can link the wavefunction inside R to that inside L. From eqns. (4.41) and (4.44), we get:

$$\phi_{\varepsilon,q,j}(-t,\mathbf{x}) = e^{-\pi\left[\frac{\varepsilon+j\Omega}{a}\right]} \phi_{\varepsilon,q,-j}(t,\mathbf{x}) \quad \text{for } x^1 > 0 \tag{4.48}$$

Since

$$\phi_{\varepsilon,q,j}(t,\mathbf{x}) = \phi_{-\varepsilon,q,-j}(t,\mathbf{x}) \tag{4.49}$$

we have

$$\phi^{*}_{-\varepsilon,q,j}(t,\mathbf{x}) = e^{-\pi\left[\frac{\varepsilon+j\Omega}{a}\right]} \phi_{\varepsilon,q,-j}(t,\mathbf{x}) \quad \text{for } x^1 > 0 \tag{4.50}$$

and a similar relation for $x^1 < 0$.

Therefore, the states

$$\phi_{R,(\varepsilon,q,j)}(x) = \left|2\sinh\left[\frac{(\varepsilon+j\Omega)\pi}{a}\right]\right|^{\frac{1}{2}} \left\{\exp\left[\frac{(\varepsilon+j\Omega)\pi}{2a}\right]\phi_{\varepsilon,q,j}(x) - \exp\left[\frac{-(\varepsilon+j\Omega)\pi}{2a}\right]\phi^{*}_{-\varepsilon,q,-j}(x)\right\} \tag{4.51}$$

vanish in the region L and are eigenfunctions of H .

Similarly, we define

$$\phi_{L,(\varepsilon,q,j)}(x) = \phi^{*}_{R,(\varepsilon,q,j)}(-x) \tag{4.52}$$

which is the PCT symmetric image of $\phi_{R,(\varepsilon,q,j)}(x)$. The $\phi_{L,(\varepsilon,q,j)}(x)$ vanish in the region R and are eigenfunctions of H, too. The normalized wave functions $\phi_R(x)$ and $\phi_L(x)$ and their complex conjugates form a basis which defines the Rindler mode. The quantum field ϕ reads :

$$\phi(x) = \phi_R(x) + \phi_L(x) \quad \text{with} \tag{4.53}$$

$$\phi_R(x) = \sum_{\varepsilon,q,j}\left\{ b_R(\varepsilon,q,j)\,\phi_{\varepsilon,q,j}(x) + b_R^{+}(\varepsilon,q,j)\,\phi^{*}_{\varepsilon,q,j}(x)\right\} \tag{4.54}$$

As $\phi_L(x)$ is related to $\phi_R(x)$ by

$$\phi_L(x) = \Theta^{-1}\,\phi_R\,\Theta \tag{4.55}$$

where Θ is the antiunitary PCT operator. The creation-annihilation operators $b_R(\varepsilon,q,j)$ and $b_L(\varepsilon,q,j) = \Theta^{-1} b_R(\varepsilon,q,j)\Theta$ define the Rindler vacuum : $|0_R>$;

$$b_R|0_R> = b_L|0_R> = 0 \tag{4.56}$$

We see that the field operator $\phi_R(x)$ mixes up positive and negative frequencies and hence the Rindler vacuum is not equivalent to the Minkowskian one. The different creation-annihilation operators are related by the Bogoliubov transformation

$$b_R(\varepsilon,q,j) = \left|2\sinh\left[\frac{(\varepsilon+j\Omega)}{a}\pi\right]\right|^{-\frac{1}{2}}\left\{\exp\left[\frac{(\varepsilon+j\Omega)\pi}{2a}\right]a(\varepsilon,q,j)\quad\exp\left[\frac{-(\varepsilon+j\Omega)\pi}{2a}\right]a^{+}(\ \varepsilon,q,\ j)\right\} \tag{4.57}$$

and similarly for $b_L(\varepsilon,q,j)$. Thus, the Minkowski vacuum $|0_M>$ contains Rindler modes with its density given by :

$$n = <0_M|\, b_R(\varepsilon,q,j)\, b_R^{+}(\varepsilon',q',j')\, |0_M> \tag{4.58}$$

$$= \left[\exp\left[\frac{2\pi(\varepsilon+j\Omega)}{a}\right] - 1\right]^{-1} \delta_{\varepsilon\varepsilon'}\, \delta_{qq'}\, \delta_{jj'} \tag{4.59}$$

which appears as a Planckian spectrum with the acceleration playing the role of the temperature

$$T_o = \frac{a}{2\pi k_B} \tag{4.60}$$

and the rotation velocity appearing as a chemical potential.

The unitary transformation that links the Rindler modes to the Minkowski ones can be written as :

$$|0_M> = U\,|0_R> \tag{4.61}$$

where

$$U = Z^{-1}\exp\left\{\sum_{\varepsilon,q,j}\left[\exp\left[\frac{-(\varepsilon+j\Omega)\pi}{a}\right] b_R^{+}(\varepsilon,q,j)\, b_L^{+}(\varepsilon,q,j) - h.c.\right]\right\} \tag{4.62}$$

(h.c. means Hermitian conjugate).

The pure Minkowski vacuum state contains pairs of Rindler modes. Each pair contains one 'particle' created in the region R and another created outside the horizons u and v. If we consider an observable A_R in the Rindler space R, we can introduce the density matrix by

$$\hat{\rho} = (\mathrm{Tr})_L \mid 0_M > < 0_M \mid \tag{4.63}$$

i.e., by taking the trace over the states built from $\psi_L(x)$. Then, the expectation value of the observable A_R , in the Minkowski vacuum is given by :

$$< 0_M \mid A_R \mid 0_M > = \mathrm{Tr}\,(\hat{\rho}\, A_R) \tag{4.64}$$

The density matrix $\hat{\rho}$ describes a thermal mixed state and is given by :

$$\hat{\rho} = Z^{-1} \sum e^{-2\pi(\varepsilon + j\Omega)/a} \mid n;\, \varepsilon,q,j > < n;\, \varepsilon,q,j \mid \tag{4.65}$$

where

$$\mid n;\, \varepsilon,q,j > = (n!)^{-\frac{1}{2}} \, [\, b_R(\varepsilon,q,j)\,]^n \mid 0_R > \tag{4.66}$$

are the n-Rindler mode states. The representation of the density matrix as a vector in some larger Hilbert space is precisely the way in which quantum statistical mechanics has been formulated by Takahashi and Umezawa (1975). This connection with thermo-field dynamics was first pointed out by Israel (1976) and further elucidated by examining the thermodynamical properties in two-dimensional space-time taking into account of self-interactions in perturbation theory (Horibe et.al., 1985).

The investigation of the quantization of both massless and massive Dirac fields has been done in two-dimensional Minkowski space-time (Soffel et.al., 1980; Hughes 1985). They found the density of the observed particles (antiparticles) :

$$n = < 0_M \mid b_R^{+}(\varepsilon,q,j)\; b_R(\varepsilon',q',j') \mid 0_M > \tag{4.67}$$

$$= \left[\, \exp \frac{2\pi\varepsilon}{a} + 1 \,\right]^{-1} \delta_{\varepsilon\varepsilon'}\, \delta_{qq'}\, \delta_{jj'} \tag{4.68}$$

(using the same notation as before). Thus, the observer measures a thermal flux of Dirac particles (antiparticles) with a Fermi-Dirac distribution at the temperature :

$$T_o = \frac{a}{2\pi k_B} \quad (4.69)$$

The fermionic mass does not enter into the above result as the temperature is essentially determined at the event horizons where the mass plays no role. Moreover, the energy gap between particle and antiparticle modes breaks down on the event horizon independent of m. Consequently, the spectrum of the 'energy' of the accelerated observer does not exhibit an 'energy' gap because the observer experiences a gravitational potential in his motion. This is also valid for a massive scalar field in quantization (Soffel et.al., 1980; Hughes 1985).

The accelerated observer detects either a Bose-Einstein spectrum or a Fermi Dirac spectrum depending upon whether the quantized field is bosonic or fermionic at the same local temperature which is the Fulling-Davies-Unruh temperature. We get different kinds of distribution for the scalar and Dirac fields because of the nature of the scalar products. The scalar product for the scalar field is not positive definite while the Dirac scalar product has a positive definite nature.

The nature of the radiation will be discussed in connection with the detection process in later chapters. The thermal nature will also be examined therein. The radiation that is detected by the observer at rest with respect to the Rindler frame is itself accelerated (Unruh and Wald 1984). In other words, an isolated observer sees around himself not an homogeneous thermal bath with the temperature given above but instead a thermal bath at equilibrium in a constant gravitational field with the metric coefficient g_{00} given from:

$$ds^2 = e^{2\alpha\xi} (d\eta^2 - d\xi^2) \quad (4.70)$$

i.e., the value of g_{00} can be obtained as $g_{00} = e^{2\alpha\xi}$. The temperature of the radiation as seen by the accelerated observer has the spatial dependence required by the Tolman relation given by:

$$T = (g_{00})^{-\frac{1}{2}} T_o \tag{4.71}$$

(See Tolman 1934; Landau and Lifshitz 1958). It relates the temperature at different points of a system at thermal equilibrium with the gravitational potential at these points. Using eqns. (4.7) and (4.71), we obtain

$$T = \frac{1}{2\pi\beta k_B}, \tag{4.72}$$

4.3 Remarks

The general principle underlying the occurence of the 'Fulling-Davies-Unruh' effect has received much attention in the past decades. In considering the global properties of the field theory, the PCT symmetry seems to be a possible candidate in explaining the known effect (Hughes 1985). Actually, the role of the PCT symmetry of quantum field theory has been highlighted in the axiomatic approach used by Sewell(1982). He examined the thermodynamical properties of field theory in curved space-time from a rigorous axiomatic field theoretic viewpoint. He found that in arbitrary dimensions independent of whether or not the particles are interacting, using the PCT invariance of the theory and the theorem on the Kubo-Martin-Schwinger (KMS) condition proved by Bisogonano and Wichmann (1975; 1976), the equivalence of the accelerated system with thermodynamics can be shown. In particular, the Rindler vacuum $|0_R>$ is neither translationally invariant nor PCT invariant and so the uniformly accelerated observer finds the Minkowski vacuum appearing as if it were in the (KMS) state described by a 'canonical ensemble of states'.

The quantization of a matter field in Rindler space discloses new problems in the formulation of a complete quantum theory applicable in a generic space-time. The theory of measurements seems to play an important part of the formulation as we discuss later

(Unruh 1986b; Sanchez 1985). The basic definitions of some physical quantities such as the 'vacuum' or 'particle states' are under review (Davies 1984) while various new formulations of quantum field theory are devised for handling the conceptual pitfalls embedded in the original formalism (Rumpf 1982; Freese et.al., 1985; Lee 1986).

Chapter 5 Quantized matter field in other coordinate systems of flat space-time

In the preceding chapter, we reviewed the quantization of a matter field in the Rindler frame by deriving the Fulling-Davies-Unruh effect in the case where PCT symmetry holds. However, field quantization has been extended to other coordinate systems as on flat space-time as we discuss in the following sections.

5.1 Quantized field theory in rotating coordinates

The quantization of a matter field in rotating coordinates has received much attention because of its physical implication in measuring the Fulling-Davies-Unruh temperature experimentally (Bell et al., 1983,1987) as well as because it provides a clue to the origin of the 'particles' indicated by the spectrum of radiation. Those inquiries which are closely related to the response of a particle detector in circular motion along with its consequences will be discussed in later chapters.

Let us consider the quantization of a massive scalar field in rotating coordinates first. This has been done by several authors (Letaw and Pfautsch 1980; Denardo and Percacci 1978) using canonical quantization.

Following the treatment by Denardo and Percacci in the quantization of a massive scalar field, we start from cylindrical coordinates (t,z,r,θ) and perform the transformation

$$\varphi = \theta - \int^{t} \Omega(t')\ dt' \qquad (5.1)$$

which allows a more general case of a time dependent angular velocity. In the rotating frame, the metric reads

$$ds^2 = g_{\mu\nu}\, dx^{\mu}\, dx^{\nu} = (1-\Omega^2(t)\, r^2)\, dt^2 - dz^2 - dr^2 - r^2 d\varphi^2 - 2\Omega(t)\, r^2 d\varphi\, dt \qquad (5.2)$$

The rotating coordinates cover all space but the metric is meaningful only for $r < \Omega^{-1}$; the surface $r = \Omega^{-1}$ is an infinite red shift surface and unlike the situation of a uniformly accelerated observer, there is no event horizon here. We have the following Killing vectors (in coordinates t, z, r, φ):

(i) $N^{\mu} \equiv (1,0,0,0)$

N^{μ} is the usual time translation Killing vector of Minkowski space time, [redacted] globally timelike ($N^{\mu} N_{\mu} = 1$) and orthogonal to the hypersurfaces t = constant

(ii). $R^{\mu} \equiv (0,0,0, \Omega)$

R^{μ} is the generator of rotations around the z axis and is everywhere spacelike ($R^{\mu} R_{\mu}$ Ω^2)

(iii) $T^{\mu} \equiv (1,0,0, \Omega r)$

T^{μ} is the time translation Killing vector in the rotating frame and is timelike only for $r < \Omega^{-1}$ (since $T^{\mu} T_{\mu} = (1 \quad \Omega^2 r^2)$).

The Klein-Gordon equation for a scalar field ϕ with mass μ is given by:

$$\frac{\partial^2 \phi}{\partial t^2} \quad \frac{\partial^2 \phi}{\partial Z^2} \leq \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial \phi}{\partial r}) \quad \left[\frac{1}{r^2} \quad \Omega^2(t)\right]\frac{\partial^2 \phi}{\partial \varphi^2} \quad \Omega(t)\frac{\partial \phi}{\partial \varphi} \quad 2\Omega(t)\frac{\partial}{\partial t}\frac{\partial \phi}{\partial \varphi} + \mu^2 \phi = 0 \qquad (5.3)$$

where $\Omega(t)$ is the derivative of Ω with respect to t.

Introducing the trial solution

$$\phi(t,z,r,\varphi) \propto \exp\left[\mp i \int^{t} dt' \, E(t')\right] \exp\,[im\varphi + ikz]\, R(r) \qquad (5.4)$$

we obtain the following radial equation:

$$\left[\frac{1}{r}\frac{d}{dr}(r\frac{dR}{dr})\right] + \left\{ [E(t) \pm m\Omega(t)]^2 - k^2 - \mu^2 + im\dot{\Omega}(t) - \frac{m^2}{r^2} \right\} R = 0 \qquad (5.5)$$

Since R must depend on r only, we impose that

$$\left[E(t) \pm m\Omega(t) \right]^2 - k^2 - \mu^2 + im\dot{\Omega}(t) = q^2 = \text{constant} \qquad (5.6)$$

Solving for E(t) , we obtain the following dispersion relation:

$$E(t) = \sqrt{\omega^2 - m\Omega(t)} + m\Omega(t) \qquad (5.7)$$

where $\omega^2 = \mu^2 + k^2 + q^2$, $m = 0, \pm 1, \pm 2, \ldots\ldots$

and $-\infty < k < +\infty$, $0 < q < \infty$ have continuous spectra. Note that when Ω is a constant, $E = \omega + m\Omega$ is also a constant, if Ω is zero (nonrotating frame), $E = \omega$ and there is an infinite degeneracy in m.

A complete set of normal modes is

$$\Phi_{k,q,m}(t,z,r,\varphi) = \frac{1}{\sqrt{2\omega}} \exp\left[-i\int^{t} E(t')\, dt' \right] \frac{1}{2\pi} \exp[\, ikz + im\varphi\,]\, J_{|m|}(qr) \qquad (5.8)$$

For Ω = constant, we have the mode function $\bar{\phi}_{k,q,m}(t,z,r,\varphi)$ and $\phi_{k,q,m}(t,z,r,\theta)$the mode functions for $\Omega = 0$. Then $\bar{\phi}$ represents a particle mode and $\bar{\phi}^*$an antiparticle (negative energy) mode.

In the general case, the field can be expanded as

$$\psi(t,\hat{x}) = \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} dk \int_{0}^{\infty} dq\, q \left[\, \hat{a}\,(k,q,m)\, \Phi_{k,q,m}(t,\hat{x}) + \text{h.c.} \,\right] \qquad (5.9)$$

For eqn. (5.1), we have the following relation :

$$\bar{\phi}_{k,q,m}(t,z,r,\varphi) = \frac{1}{\sqrt{2E}} \exp[\, -i(\omega - m\Omega)t\,]\, \frac{1}{2\pi} \exp[\, im\varphi + ikz\,]\, J_{|m|}(qr)$$

$$= \frac{1}{\sqrt{2E}} \exp[\, -iEt\,]\, \frac{1}{2\pi} \exp[\, im\theta + ikz\,]\, J_{|m|}(qr)$$

$$= \phi_{k,q,m}(t,z,r,\theta) \qquad (5.10)$$

The energy momentum tensor of the scalar field is:

$$T_{\mu\nu} = \partial_\mu \psi \partial_\nu \psi - g_{\mu\nu} L \qquad (5.11)$$

The total energy as seen by the nonrotating observer is:

$$H = \int d\Sigma^{\mu}\, T_{\mu\nu}\, N^{\nu} = \int d\theta\, dz\, dr\, r\, T_{00} \qquad (5.12)$$

$$= \int d\theta\, dz\, dr\, r\, \frac{1}{2} \left\{ (\partial_0\psi)^2 + (\partial_z\psi)^2 + (\partial_r\psi)^2 + \frac{1}{r^2}(\partial_\theta\psi)^2 + \mu^2\psi^2 \right\} \qquad (5.13)$$

Using eqns. (5.9) and (5.10), we have

$$H = \frac{1}{2} \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} dk \int_{0}^{\infty} dq\, q\omega\, (a^{+}(k,q,m)\, a(k,q,m) + h.c.) \qquad (5.14)$$

On the other hand, the energy measured by the rotating observer is:

$$K = \int d\Sigma^{\mu}\, T_{\mu\nu}\, T^{\nu} = \int d\varphi\, dz\, dr\, r\, (T_{00} - \Omega T_{0\varphi}) \qquad (5.15)$$

$$= \int d\varphi\, dz\, dr\, r\, \frac{1}{2} \left\{ (\partial_0\psi)^2 + (\partial_z\psi)^2 + (\partial_r\psi)^2 + \left(\frac{1}{r^2} - \Omega^2\right)(\partial_\varphi\psi)^2 + \mu^2\psi^2 \right\} \qquad (5.16)$$

$$= \frac{1}{2} \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} dk \int_{0}^{\infty} dq\, q(\omega - m\Omega)\, (a^{+}(k,q,m)\, a(k,q,m) + h.c.) \qquad (5.17)$$

Equations (5.14) and (5.17) show that the energy of the mode (k,q,m) is ω and ω−mΩ for the rotating one. The physical interpretation is that the modes for which m and Ω have the same sign (corresponding to particles co-rotating with respect to the observers) are seen red-shifted, while those for which m and Ω have opposite sign (counter-rotating particles) are seen blue-shifted by the rotating observer.

The energy spectrum of the rotating observer extends from −∞ to +∞ with infinite multiplicity. For sufficiently large values of mΩ , the energy levels of the particle states can be pushed arbitrarily downwards and hence they are unbounded from below. We seem to have an unstable vacuum but because of the conservation of angular momentum, the particle interpretation continues to hold.

For the general case when Ω is time dependent, the relation between nonrotating and rotating mode functions is:

$$\Phi_{k,q,m} = \alpha(k,q,m)\,\phi_{k,q,m} + \beta(k,q,m)\,\phi^{*}_{k,q,m}$$

where the Bogoliubov coefficients are

$$\alpha(k,q,m) = \frac{\omega + \sqrt{\omega^2 - im\Omega(t)}}{2\omega}\exp\left[i\omega t - i\int_0^t dt' \sqrt{\omega^2 - im\Omega(t')}\right] \qquad ([illegible])$$

$$\beta(k,q,m) = \frac{\omega - \sqrt{\omega^2 - im\Omega(t)}}{2\omega}\exp\left[i\omega t + i\int_0^t dt' \sqrt{\omega^2 - im\Omega(t')}\right] \qquad (5.20)$$

The normalization condition is

$$\left|\alpha(k,q,m)\right|^2 - \left|\beta(k,q,m)\right|^2 = \left[\mathrm{Re}\sqrt{1 - \frac{im\Omega(t)}{\omega^2}}\right]\exp\left[2\,\mathrm{Im}\int_0^t dt' \sqrt{\omega^2 - im\Omega(t')}\right] \qquad (5.21)$$

Note that when Ω is always constant, then $\alpha(k,q,m) = 1$ and $\beta(k,q,m) = 0$, which is the trivial case. Moreover, β vanishes also in the stationary regions in which $\Omega = 0$. The physical interpretation of the above results is that if we choose the state of the system to be a Minkowski vacuum defined by the annihilation operators: $a\,|\,0_M> = 0$, there are no real particles present at any time; an observer in the rotating frame agrees with the inertial one whenever he is rotating uniformly, but he will see pairs of particles with spectrum $|\beta(k,q,m)|^2$ whenever Ω increases or decreases. In other words, the number operator as defined by the uniformly rotating observer is:

$$N = \sum_{m=-\infty}^{\infty} \int_{k,q,\omega<0} \bar{a}^{+} a \; q\,dq\,dk \qquad (5.22)$$

which is the same as the inertial observer's number operator, so

$$N\,|\,0_M> = 0 \qquad (5.23)$$

and there are no particles detected by the uniformly rotating observer just as if he were in the Minkowski vacuum. Furthermore, the creation (annihilation) operators in the uniformly rotating frame can be identified with creation (annihilation) operators in the Minkowski frame with the same appearance and hence the vacuum expectation value of the stress tensor $<T_{\mu\nu}>$ is identically the same, namely zero in each frame.

The analogy of a uniformly rotating observer to a uniformly accelerated observer seems to hold as each of them experiences a constant acceleration in the Minkowski vacuum and with the same vacuum expectation value of the stress tensor. However, this analogy breaks down when we consider the particle interpretation, since the uniformly rotating observer detects no particles at all while the uniformly accelerated observer detects a thermal spectrum of particles. This fact can be connected to the global structure of the model used: in the case of the uniformly accelerated observer, there is a boostlike Killing vector, which is timelike in some regions, spacelike in others and null on an horizon. In the case of a uniformly rotating observer, the Killing vector T^{μ} has all these properties but the surface $r = \Omega^{-1}$ is not an event horizon. Furthermore, from eqn. (5.10), we see that waves which have positive frequency with respect to N^{μ} also have positive frequency with respect to T^{μ} as long as Ω is a constant, and hence the vacuums coincide. For $d\Omega/dt \neq 0$, there is a distinct vacuum associated with the vector T^{μ} , annihilated by the operators a(k,q,m) appearing in eqn.(5.9). This new vacuum state consists of pairs of Minkowski particles with opposite angular momentum along the axis of rotation.

The quantization of a massive scalar field by a uniformly rotating observer has also been examined by Letaw and Pfautsch in the context of using the two parameters: curvature and torsion of a non-null curve in a flat three-dimensional space. Their conclusion agrees with that of Denardo and Percacci but they point out that several criteria have not been satisfied in their (Letaw and Pfautsch) procedures, namely: they do not give an exact definition of positive and negative frequency modes relative to the Killing vector ξ^{μ} ; nor

nor do they employ a strictly canonical procedure and regard a and a^+ as strictly annihilation and creation operators respectively (i.e. satisfy the commutation relation $[a, a^+] = 1$).

Following the work of Letaw and Pfautsch, Iyer (1982) examined the result of quantization of a Dirac field, both massless and massive in rotating coordinates. He found that the natural procedure of defining particles via the Killing vector of the rotating observer yields a canonical quantization scheme. This scheme is inequivalent to the usual Minkowski quantization and furthermore, the rotating observer detects, in the Minkowski vacuum, a nonthermal spectrum of particles and antiparticles.

The quantization of a Dirac field is done in the following manner: the Dirac equation in general coordinates may be written as

$$\gamma^a \nabla_a \psi + i \mu \psi = 0 \tag{5.24}$$

where γ^a are the 4 by 4 flat space-time Dirac matrices satisfying

$$[\gamma^a, \gamma^b]_+ = 2\eta^{ab} \quad \text{and} \tag{5.25}$$

$$\nabla_a \psi = e_a^\mu (\partial_\mu - \Gamma_\mu)\psi \tag{5.26}$$

Here $e_a{}^\mu$ are the chosen tetrad fields and Γ_μ the spinor affine connections given by

$$\Gamma_\mu = -\frac{1}{4} \gamma^a \gamma^b e_a^\nu e_{b\nu;\mu} \tag{5.27}$$

Using cylindrical Minkowski coordinates, the Dirac equation in rotating coordinates is given by:

$$\left\{ \gamma^0(\partial_t - \Omega\partial_\phi) + \gamma^1[\partial_r + \frac{1}{2r}] + \frac{1}{r}\gamma^2\partial_\phi + \gamma^3\partial_z + i\mu \right\} \psi = 0 \tag{5.28}$$

In the case of a massless Dirac field (e.g. neutrino), the normal modes are written as:

$$\psi'(\omega, m, k; x) = \exp[-i(\varpi - m\phi - kz)] \left[QJ_{m-1/2}, i(\omega-k)J_{m+1/2}, QJ_{m-1/2}, i(\omega-k)J_{m+1/2} \right]^T \tag{5.29}$$

where $Q = +(\omega^2 - k^2)$, $\varpi = \omega - m\Omega$, $J_{m\pm 1/2} \equiv J_{m\pm 1/2}(Qr)$

These solutions are the Minkowski modes transformed to the rotating coordinates since the Dirac field is a scalar under coordinate transformation. The normal modes given by eqn. (5.29) are orthogonal and yield

$$(\psi'(\omega,m,k),\ \psi'(\omega',m',k')) = 8\pi^2\ |\omega - k|\ \delta(\omega - \omega')\ \delta(k - k')\ \delta_{mm'} \quad (5.30)$$

Thus,

$$\psi(\omega,m,k;\ x) \equiv [\ 2\pi(\ 2|\omega - k|\ )^{1/2}]^{-1}\ \psi'(\omega,m,k;\ x)$$

are a convenient set of orthonormal modes for the massless Dirac field in rotating coordinates. In terms of these normal modes, an arbitrary Dirac field may be expanded as

$$\Psi = \sum_{m=-\infty}^{\infty} \int_{\varpi>0} d\omega \int_{-|\omega|}^{+|\omega|} dk\ \{a(\omega,m,k)\psi(\omega,m,k) + b^{+}(\omega,m,k)\ \psi(-\omega,\ -m,k;\ x)\} \quad (5.31)$$

Note that the rotating observer defines positive frequency via his Killing vector $\partial/\partial t$ so that modes with $\varpi > 0$ are his particles. On the other hand, the inertial observer considers the $\omega > 0$ modes as the Minkowski particles. Eqn. (5.31) may be inverted to yield

$$a(\omega,m,k) = (\ \psi(\omega,m,k;\ x),\ \Psi\ ),\quad \varpi > 0 \quad (5.32)$$

$$b^{+}(\omega,m,k) = (\ \psi(-\omega,-m,k;\ x),\ \Psi\ ),\quad \varpi > 0 \quad (5.33)$$

It follows that

$$[\ a(\omega,m,k),\ a^{+}(\omega',m',k')\ ]_{+} = \delta(\omega-\omega')\ \delta(k-k')\ \delta_{mm'} \quad (5.34)$$

$$[\ b(\omega,m,k),\ b^{+}(\omega',m',k')\ ]_{+} = \delta(\omega-\omega')\ \delta(k-k')\ \delta_{mm'} \quad (5.35)$$

All other anticommutators vanish. The rotating observer defines his vacuum by

$$a(\omega,m,k)\ |\ 0_{\Omega} > = b(\omega,m,k)\ |\ 0_{\Omega} > = 0,\quad \varpi > 0 \quad (5.36)$$

It means that the natural vacuum $|0_\Omega>$ has no particles, i.e., $\varpi > 0$ states and all modes with $\varpi < 0$ (holes) are filled. For the inertial observer, the Dirac field is expanded in the following way:

$$\Psi = \sum_{m=-\infty}^{\infty} \int_0^{\infty} d\omega \int_{-\omega}^{+\omega} dk \left[\hat{a}(\omega,m,k)\, \psi(\omega,m,k;x) + \hat{b}^{+}(\omega,m,k)\, \psi(-\omega,-m,k;x) \right] \tag{5.37}$$

with $\hat{a}(\omega,m,k)$ and $\hat{b}(\omega,m,k)$ satisfying the canonical anticommutation relations as before.

His vacuum is defined by

$$\hat{a}(\omega,m,k)\,|0_M> = \hat{b}(\omega,m,k)\,|0_M> = 0 \tag{5.38}$$

so that all $\omega > 0$ states are empty and all 'holes' $\omega < 0$ are filled. It can be shown that

$$a(\omega,m,k) = \hat{a}(\omega,m,k) \quad , \omega > 0 \tag{5.39}$$

$$a(\omega,m,k) = \hat{b}^{+}(-\omega,-m,k), \ \omega < 0 \tag{5.40}$$

Similarly,

$$b(\omega,m,k) = \hat{b}(\omega,m,k) \quad , \omega > 0 \tag{5.41}$$

$$b(\omega,m,k) = \hat{a}^{+}(-\omega,-m,k) \quad , \omega < 0 \tag{5.42}$$

The number of 'rotating' particles in the Minkowski vacuum is now given by:

$$N = <0_M|\, N^{+}_{\Omega}(\omega,m,k)\, |0_M>$$

$$\equiv <0_M|\, a^{+}(\omega,m,k)\, a(\omega,m,k)\, |0_M> \tag{5.43}$$

$$= 0 \quad , \omega > 0$$

$$= 1 \quad , \omega < 0 \tag{5.44}$$

Similarly, it follows that

$$N = \langle 0_M \mid N_\Omega^- (\omega,m,k) \mid 0_M \rangle$$

$$\equiv \langle 0_M \mid b^+(\omega,m,k)\, b(\omega,m,k) \mid 0_M \rangle \tag{5.45}$$

$$= 0 \quad , \omega > 0$$

$$\tag{5.46}$$

$$= 1 \quad , \omega < 0$$

The above results can be illustrated by the following figure:

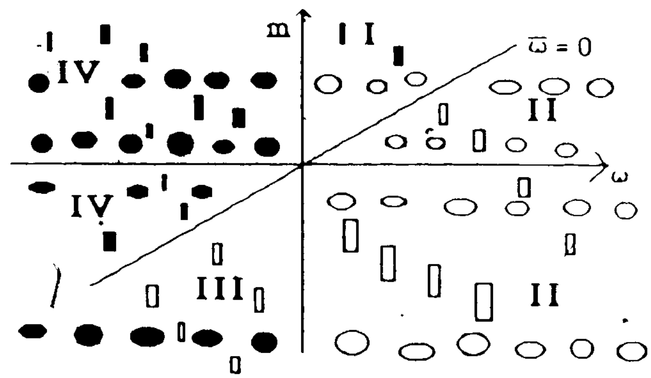

Fig. 5.1. The figure shows the vacuum states of the inertial and rotating observers. The open ovals(rectangles) indicate that the Minkowski(rotating) vacuum has no 'particles' The dark ovals(rectangles) indicate the filled negative-energy states as defined by the Minkowski(rotating) observer.

From the figure, we see that in the (ω, m) plane for the state $\mid 0_M \rangle$, all the states to the right of the $\varpi = 0$ lines are empty while all holes to the left are filled. However, to the rotating observer all states below the $\varpi = 0$ line are empty while those above are filled. Thus, if the rotating observer sees a Minkowski vacuum, then he finds states in sector III filled (particles) and those in sector I empty (antiparticles). The Minkowski vacuum consequently contains for the rotating observer a nonthermal spectrum of particles and antiparticles when ϖ is positive but ω is negative. However, the rotating observer does not see any charges in the Minkowski vacuum.

For a massive Dirac field, the normal modes may be written as

$$\psi(\omega,m,k;x) = (4\pi|\omega-\mu|^{1/2})^{-1}\exp[-i(\varpi t - m\phi - kz)]\ X$$

$$\left[(Q+ik)J_{m-1/2}(qr),\ (Q+ik)J_{m+1/2}(qr),\ i(\omega-\mu)J_{m-1/2}(qr),\ i(\omega-\mu)J_{m-1/2}(qr)\right]^{T} \tag{5.47}$$

with $q = +(\omega^2 - \mu^2 - k^2)^{1/2} \equiv (\omega_o^2 - k^2)^{1/2}$

The massive Dirac field can be expressed as:

$$\Psi = \sum_{m}^{\infty} \int_{\varpi>0,\ |\omega|>\mu} d[illegible]\ dk\ [a(\omega,m,k)\ \psi(\omega,m,k;x) + b^{+}(\omega,m,k)\ \psi(-\omega,-m,k;x)] \tag{5.48}$$

The constructions are the same as before and similar conclusions can be obtained for the massive case.

5.2 Quantized scalar field in the stationary coordinate systems

Letaw and Pfautsch extended their work from rotating coordinates to other classes of stationary coordinate systems in Minkowski space-time (Letaw and Pfautsch 1981;1982). They concluded that there were only two stationary vacuums in flat space-time : the Minkowski vacuum $|0_M>$ and the Rindler vacuum $|0_R>$. The Minkowski vacuum is found in those coordinate systems without event horizons while the Rindler vacuum is found in those with identical event horizons. The vacuum state is not in general the lowest energy state but is stable because of the presence of additional symmetries with no ambiguity of a particle interpretation. The stationary coordinate systems are those in which the particle states are defined to be 'time-independent'. There are six classes of systems corresponding to the six different types of timelike Killing vector fields in flat space-time, which have been classified on the basis of the six types of stationary world-lines in Minkowski space (Letaw and Pfautsch 1982). The stationary coordinate systems correspond to the Minkowski vacuum are classified as class A, C, and D while the Rindler vacuum is found in class B, E and F systems. Canonical quantization of a massive scalar field is done in those coordinate systems. For diagrammic representations of the coordinate systems, the reader can refer to Letaw and Pfautsch (1982).

(i). Class A coordinates

The Killing vector field is:

$$\xi^{\mu}(x) = (1,0,0,0) \tag{5.49}$$

a). Coordinate system: Rectangular Minkowski coordinates

$$x^0 = t,\ \ x^1 = x,\ \ x^2 = y,\ \ x^3 = z$$

$$\text{Metric: } ds^2 = dt^2 - dx^2 - dy^2 - dz^2 \tag{5.50}$$

The Klein Gordon equation is

$$\left[\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} + \mu^2\right]\phi = 0 \tag{5.51}$$

Positive-norm modes in these coordinates are

$$\phi = \frac{1}{(2\pi)^{3/2}(2\omega)^{1/2}}\, e^{-i\omega t}\, e^{i\mathbf{k}.\mathbf{x}} \tag{5.52}$$

where $\omega^2 = |\mathbf{k}|^2 + \mu^2$ and $\omega > 0$

The modes are also of 'positive frequency' and so the vacuum state in this system is the lowest energy state. The field operators is expanded in terms of these modes as

$$\psi = \int d^3k\, [a(\mathbf{k})\, \phi(\mathbf{k}) + a^{+}(\mathbf{k})\, \phi^{*}(\mathbf{k})] \tag{5.53}$$

and the Minkowski vacuum is defined by

$$a(\mathbf{k}) \mid 0_M> = 0 \tag{5.54}$$

b). Coordinate system: cylindrical Minkowski coordinates

$x^0 = t,\ x^1 = r,\ x^2 = \theta,\ x^3 = z$ with $r = (x^2 + y^2)^{1/2},\ \theta = \tan^{-1}(y/x)$

Metric: $ds^2 = dt^2 - dr^2 - r^2 d\theta^2 - dz^2$ (5.55)

The Klein Gordon equation is

$$\left[\frac{\partial^2}{\partial t^2} - \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} - \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} - \frac{\partial^2}{\partial z^2} + \mu^2\right]\phi = 0 \tag{5.56}$$

Positive norm modes in these coordinates, chosen to be well behaved at r=0, are

$$\Phi = \frac{1}{(2\pi)\,(2\omega)^{1/2}}\, e^{-i\omega t} e^{im\theta} e^{ik_z z} J_m(qr) \tag{5.57}$$

where $\omega^2 = (k_z)^2 + q^2 + \mu^2$, $\omega > 0$ and the modes are also of positive frequency.

The field operator is then:

$$\psi = \sum_m \int q\,dq\,dk_z [\hat{a}(q,m,k_z)\,\Phi(q,m,k_z) + \hat{a}^{+}(q,m,k_z)\,\Phi^{*}(q,m,k_z)] \tag{5.58}$$

Relating the modes ϕ and Φ by a Bogoliubov transformation, we have

$$\alpha(q.m.k_z; \mathbf{k}) = \frac{1}{(2\pi)^{1/2}}\left[\frac{k_x - ik_y}{q}\right]^m \frac{\delta(q - (k_x^2 + k_y^2)^{1/2})}{q}\,\delta(k_z - k_z) \tag{5.59}$$

$$\beta(q.m,k_z; \mathbf{k}) = 0 \tag{5.60}$$

Thus, the vacuum states in rectangular and cylindrical Minkowski coordinates are the same.

(ii). Class-C-coordinates

The Killing vector field:

$$\xi^{\mu}(x) = (1+\kappa x,\ \kappa t - \tau y,\ \tau x,\ 0)\ ,\ \tau > \kappa \tag{5.61}$$

Coordinate system : rotating coordinates

$$x^0 = t,\ x^1 = r,\ x^2 = \bar{\theta},\ x^3 = z \qquad \text{where } \bar{\theta} = \theta - \Omega t$$

$$\text{Metric: } ds^2 = dt^2 - dr^2 - r^2[\,d(\bar{\theta})^2 + \Omega dt\,]^2 - dz^2 \qquad (5.62)$$

The Klein Gordon equation is

$$\left[\left(\frac{\partial}{\partial t} - \Omega\frac{\partial}{\partial \bar{\theta}}\right)^2 - \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} - \frac{1}{r^2}\frac{\partial^2}{\partial(\bar{\theta})^2} - \frac{\partial^2}{\partial z^2} + \mu^2 \right] \phi = 0 \qquad (5.63)$$

where Ω is a constant.

Positive norm modes in those coordinates, chosen to be well behaved at $r=0$, are

$$\bar{\phi} = \frac{1}{(2\pi)\,[\,2(\varpi + \bar{m}\,\Omega)\,]^{1/2}}\, e^{-i\varpi t}\, e^{i\bar{m}\bar{\theta}}\, e^{ik_z z}\, J_{\bar{m}}(qr) \qquad (5.64)$$

with $(\varpi + \bar{m}\,\Omega)^2 = k_z^2 + q^2 + \mu^2$ and $\varpi + \bar{m}\,\Omega > 0$

These modes are not generally of positive frequency and the vacuum will thus not to be the lowest energy state (see the preceding section). The field operator is expressed as:

$$\psi = \sum_{\bar{m}} \int q dq\, dk_z\, [\bar{a}\,(q,\bar{m},k_z)\,\bar{\phi}(q,\bar{m},k_z) + \bar{a}^+(q,\bar{m},k_z)\,\bar{\phi}^*(q,\bar{m},k_z)] \qquad (5.65)$$

The Bogoliubov transformation between the rotating modes $\bar{\phi}$ and the cylindrical modes Φ is :

$$\alpha(q',\bar{m},k_z';\, q,m,k_z) = \frac{\delta(q-q')}{q}\,\delta_{m\bar{m}}\,\delta(k_z - k_z') \qquad (5.66)$$

$$\beta(q',\bar{m},k_z';\, q,m,k_z) = 0 \qquad (5.67)$$

Thus, the vacuum defined in rotating coordinates is just the Minkowski vacuum.

(iii) Class D-coordinates

The Killing vector field

$$\xi^{\mu}(x) = (1+\kappa x,\ \kappa t - \kappa y,\ \kappa x,\ 0) \tag{5.68}$$

Coordinate system: null parabolic coordinates

$$x^0 = \frac{1}{6}\kappa^2 t^3 + (\kappa x + \frac{1}{2})t + y$$

$$x^1 = \frac{1}{2}\kappa t^2 + x - \frac{1}{2\kappa}$$

$$x^2 = \frac{1}{6}\kappa^2 t^3 + (\kappa x - \frac{1}{2})t + y$$

$$x^3 = z$$

$$\text{Metric: } ds^2 = 2\kappa x(dt)^2 + 2dy\,dt - (dx)^2 - (dz)^2 \tag{5.69}$$

The Klein Gordon equation is

$$\left[2\frac{\partial^2}{\partial t\,\partial y} - \frac{\partial^2}{\partial x} - 2\kappa x\frac{\partial^2}{(\partial y)^2} - \frac{\partial^2}{\partial z^2} + \mu^2\right]\phi = 0 \tag{5.70}$$

Positive norm modes in these coordinates, chosen to be well behaved as $x \to \infty$, are

$$\tilde{\phi} = \frac{1}{(4\pi)}[4/\kappa l^2]^{1/6} e^{-i\tilde{\omega}t} e^{-ily} e^{ik_z z} \mathrm{Ai}(\lambda + \alpha\tilde{x}) \tag{5.71}$$

for $l > 0$. Here $\alpha = (2\kappa l^2)^{1/2}$, $\lambda = \alpha^{-2}(k_z^2 - l\tilde{\omega} + \mu^2)$ and Ai is an Airy function.

We see that the positive norm modes need not be positive-frequency modes and the positive-norm modes are characterized solely by l. The field operator is expanded in terms of the modes as:

$$\psi = \int dl\, dk_z\, d\lambda\, [\tilde{a}(l,k_z,\lambda)\,\tilde{\phi}(l,k_z,\lambda) + \tilde{a}^{+}(l,k_z,\lambda)\,\tilde{\phi}^{*}(l,k_z,\lambda)] \tag{5.72}$$

The Bogoliubov transformation between the modes $\bar{\phi}$ and the modes ϕ yields

$$\alpha(l,k_z,\lambda,\mathbf{k}) = \frac{1}{2}\left[\frac{\omega}{\pi\kappa l}\right]^{1/2} \exp\left[i\frac{k_x}{2\kappa l^2}(k_x^2 + l^2 + \lambda\omega^2)\right]\delta(k_z - k_z')\,\delta(l - (\omega - k_x)) \qquad (5.73)$$

$$\beta(l,k_z,\lambda;\mathbf{k}) = 0 \qquad (5.74)$$

Thus, the vacuum is again the Minkowski vacuum.

(iv). Class B coordinates

The Killing vector field is:

$$\xi^\mu(x) = (1+\kappa x,\ \kappa t,\ 0,\ 0) \qquad (5.75)$$

Coordinate system: pseudocylindrical coordinates

$x^0 = \tau,\ x^1 = \xi,\ x^2 = y,\ x^3 = z$ where $\xi = (x^2 - t^2)$ and $\tau = \tanh^{-1}(t/x)$

Metric: $ds^2 = \xi^2\, d\tau^2 - d\xi^2 - dy^2 - dz^2$ (5.76)

The Klein Gordon equation is:

$$\left[\frac{1}{\xi^2}\frac{\partial^2}{\partial\tau^2} - \frac{1}{\xi}\frac{\partial}{\partial\xi}\xi\frac{\partial}{\partial\xi} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} + \mu^2\right]\phi = 0 \qquad (5.77)$$

The positive norm modes in these coordinates is

$$\phi_R = \frac{(\sinh(\pi E))^{1/2}}{2\pi^2}\, e^{-iE\tau}\, e^{ik_y y}\, e^{ik_z z}\, K_{iE}(Q\xi) \qquad (5.78)$$

where $Q^2 = (k_y)^2 + (k_z)^2 + \mu^2$ and $E > 0$. K_{iE} is a Macdonald function, a Bessel function of pure imaginary order and argument. As $\xi = (x^2 - t^2)^{1/2}$ is defined only for $x > |t|$, these functions are only defined in the right Rindler wedge. Similarly, we have mode functions defined in the left Rindler wedge

$$\phi_L = \frac{(\sinh(\pi E))^{1/2}}{2\pi^2} e^{iE\tau} e^{ik_y y} e^{ik_z z} K_{iE}(Q\xi) \qquad (5.79)$$

It is notable that both ϕ_R and ϕ_L are positive frequency mode functions. The field operator is expanded in terms of the modes ϕ_R and ϕ_L as

$$\psi = \int dE\, dk_y\, dk_z\, [b_R(E,k_y,k_z)\,\phi_R(E,k_y,k_z) + b_R^{*}(E,k_y,k_z)\,\phi_R^{*}(E,k_y,k_z)$$
$$+ b_L(E,k_y,k_z)\,\phi_L(E,k_y,k_z) + b_L^{*}(E,k_y,k_z)\,\phi_L^{*}(E,k_y,k_z)] \qquad (5.80)$$

The modes ϕ_R and ϕ_L are related to the rectangular Minkowski modes by the Bogolubov coefficients:

$$\alpha_R(E,k_y,k_z;\mathbf{k}) = \left[2\pi\omega(1-e^{-2\pi E})\right]^{-1/2} [(\omega+k_x)/Q]^{iE}\,\delta(k_y-k_y)\,\delta(k_z-k_z) \qquad (5.81)$$

$$\alpha_L(E,k_y,k_z;\mathbf{k}) = \alpha_R^{*}(E,k_y,k_z;\mathbf{k}) = \left[2\pi\omega(1-e^{-2\pi E})\right]^{-1/2} [(\omega+k_x)/Q]^{iE}\,\delta(k_y-k_y)\,\delta(k_z-k_z)$$

(5.82)

$$\beta_R(E,k_y,k_z;\mathbf{k}) = -e^{-\pi E}\alpha_R(E,k_y,k_z;\mathbf{k}) \qquad (5.83)$$

$$\beta_L(E,k_y,k_z;\mathbf{k}) = -e^{-\pi E}\alpha_L(E,k_y,k_z;\mathbf{k}) \qquad (5.84)$$

The Rindler vacuum is defined by:

$$b_{R(L)}(E,k_y,k_z)\,|0_R\rangle = 0 \qquad (5.85)$$

which is not equivalent to the Minkowski vacuum. $|0_M\rangle$ is related to $|0_R\rangle$ by

$$|0_M\rangle = \frac{1}{c}\exp\left[\int dE\, dk_y\, dk_z\, e^{-\pi E}\, b_R^{+}(E,k_y,k_z)\, b_L^{+}(E,k_y,k_z)\right]|0_R\rangle \qquad (5.86)$$

Thus,the Minkowski vacuum consists of a mixture of multiparticle states composed of particle pairs, one each in the right and left Rindler wedges, distributed in a thermal fashion.

(v). Class E-coordinates

The Killing vector field:

$$\xi^\mu = (1+\kappa x,\ \kappa t - \tau y,\ \tau x,\ 0)\ ,\quad \kappa > \tau \tag{5.87}$$

Coordinate system: pseudorotating coordinates

$$x^0 = \tau,\ x^1 = \xi,\ x^2 = \bar{y},\ x^3 = z \quad \text{where } \bar{y} = y - \bar{\Omega}\tau$$

$$\text{Metric: } ds^2 = \xi^2 d\tau^2 - d\xi^2 - (d\bar{y} + \Omega d\tau)^2 - dz^2 \tag{5.88}$$

The Klein Gordon equation is

$$\left[\frac{1}{\xi^2}\left(\frac{\partial}{\partial\tau} - \bar{\Omega}\frac{\partial}{\partial\bar{y}}\right)^2 - \frac{1}{\xi}\frac{\partial}{\partial\xi}\xi\frac{\partial}{\partial\xi} - \frac{\partial^2}{\partial\bar{y}^2} - \frac{\partial^2}{\partial z^2} + \mu^2\right]\phi = 0 \tag{5.89}$$

where $\bar{\Omega}$ is a constant. Positive norm modes in these coordinates are

$$\bar{\phi}_R = \frac{[\sinh(\bar{E} + \bar{k}_y\bar{\Omega})\pi]^{1/2}}{2\pi^2}\, e^{-i\bar{E}\tau} e^{i\bar{k}_y\bar{y}} e^{ik_z z} K_{i(\bar{E}+\bar{k}_y\bar{\Omega})}(Q\xi) \tag{5.90}$$

where $Q^2 = (\bar{k}_y)^2 + k_z^2 + \mu^2$ and $\bar{E} + \bar{k}_y\bar{\Omega} > 0$ defined on the right Rindler wedge. We can define $\bar{\phi}_L$ on the left Rindler wedge in a similar way.

Note that these mode functions are not in general of positive frequency. The field operator can then be expanded as:

$$\begin{aligned}\psi = \int dE\, d\bar{k}_y\, dk_z\, [\bar{b}_R(\bar{E},\bar{k}_y,\bar{k}_z)\, \bar{\phi}_R(\bar{E},\bar{k}_y,\bar{k}_z) + \bar{b}_R^{+}(\bar{E},\bar{k}_y,\bar{k}_z)\, \bar{\phi}_R^{*}(\bar{E},\bar{k}_y,\bar{k}_z) \\ + \bar{b}_L(\bar{E},\bar{k}_y,\bar{k}_z)\, \bar{\phi}_L(\bar{E},\bar{k}_y,\bar{k}_z) + \bar{b}_L^{+}(\bar{E},\bar{k}_y,\bar{k}_z)\, \bar{\phi}_L^{*}(\bar{E},\bar{k}_y,\bar{k}_z)]\end{aligned} \tag{5.91}$$

The Bogoliubov coefficients between the set of modes $\phi_{R(L)}$ and $\bar{\phi}_{R(L)}$ are given by :

$$\alpha_{R(L)}(\bar{E},\bar{k}_y,k_z;\ E,k_y,k_z) = \delta[\ E - (\bar{E}+\bar{k}_y\bar{\Omega})\ ]\ \delta(k_y - \bar{k}_y)\ \delta(k_z - k_z) \tag{5.92}$$

$$\beta_{R(L)}(\bar{E},\bar{k}_y,k_z;\ E,k_y,k_z) = 0 \tag{5.93}$$

Thus, the vacuum state in pseudorotating coordinates is the Rindler vacuum.

(vi). Class F coordinates

The timelike Killing vector field:

$$\xi^{\mu} = (1 + \kappa x,\ \kappa t - \tau y,\ \tau x - \nu z,\ \nu y) \tag{5.94}$$

Coordinate system: rotating pseudocylindrical coordinates

$$x^0 = \tau,\ x^1 = \xi,\ x^2 = r,\ x^3 = \Theta \quad \text{where } \Theta = \theta - \tilde{\Omega}\tau$$

$$\text{Metric: } ds^2 = \xi^2 d\tau^2 - d\xi^2 - dr^2 - r^2(d\Theta + \tilde{\Omega}\, d\tau) \tag{5.95}$$

The Klein Gordon equation is:

$$\left[\frac{1}{\xi^2}\left(\frac{\partial}{\partial\tau} - \tilde{\Omega}\frac{\partial}{\partial\Theta}\right)^2 - \frac{1}{\xi}\frac{\partial}{\partial\xi}\xi\frac{\partial}{\partial\xi} - \frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r} - \frac{1}{r^2}\frac{\partial^2}{\partial\Theta^2} + \mu^2\right]\phi = 0 \tag{5.96}$$

where $\tilde{\Omega}$ is a constant.

Positive norm modes in these coordinates are given by:

$$\tilde{\phi}_R = \sqrt{\frac{\sinh(\tilde{E} + \tilde{m}\tilde{\Omega})\pi}{2\pi^3}}\ e^{-i\tilde{E}\tau} e^{i\tilde{m}\theta} J_{\tilde{m}}(qr)\, K_{i(\tilde{E}+\tilde{m}\tilde{\Omega})}(Q\xi) \tag{5.97}$$

where $Q^2 = q^2 + \mu^2$ and $\tilde{E} + \tilde{m}\tilde{\Omega} > 0$ on the right Rindler wedge.

We can define ϕ_L on the left Rindler wedge in a similar way. They are not generally of positive frequency.

The field operator is expanded in the following:

$$\psi = \sum_{\tilde{m}} \int q\,dq\, d\tilde{E}\ [\ \tilde{b}_R(\tilde{E},q,\tilde{m})\,\tilde{\phi}_R(\tilde{E},q,\tilde{m}) + \tilde{b}^{+}_R(\tilde{E},q,\tilde{m})\,\tilde{\phi}^{*}_R(\tilde{E},q,\tilde{m})$$
$$+ \tilde{b}_L(\tilde{E},q,\tilde{m})\,\tilde{\phi}_L(\tilde{E},q,\tilde{m}) + \tilde{b}^{+}_L(\tilde{E},q,\tilde{m})\,\tilde{\phi}^{*}_L(\tilde{E},q,\tilde{m})\ ] \tag{5.98}$$

The Bogoliubov coefficients between the two set of modes $\tilde{\phi}_{R(L)}$ and $\Phi_{R(L)}$ are :

$$\alpha_{R(L)}(\tilde{E},q',\tilde{m};E,q,m) = \frac{\delta(E-(\tilde{E}+\tilde{m}\tilde{\Omega}))\,\delta_{m\tilde{m}}\,\delta(q-q')}{q} \tag{5.99}$$

$$\beta_{R(L)}(\tilde{E},q',\tilde{m};E,q,m) = 0 \tag{5.100}$$

Thus, the vacuum state is again the Rindler vacuum.

The significance of Letaw and Pfautsch's work is that it encompasses all possible coordinate systems in Minkowski space-time in the quantization of a scalar field. Furthermore, the role of the event horizon in defining a vacuum state is spelled out. The event horizon reduces the vacuum states in all of the chosen classes of stationary coordinate system in flat space-time to only two possibilities, namely, the Minkowski vacuum and the Rindler vacuum. The interpretation of a particle state remains valid since the vacuum is stable because of the presence of a conserved quantity associated with the definition of positive-norm mode functions. The positive-norm modes ensure the existence of a stable vacuum. However, the benefits of using the positive-norm modes may not appear in a generic space-time. Actually, the distinction between the positive frequency and positive-norm modes vanishes when there is not an initial value hypersurface on which the energy-defining Killing vector is everywhere timelike.

In the context of quantum field theory, we considered only two inequivalent Fock representations for field quantization in all the coordinate systems of Minkowski space-time, one based on the Minkowski vacuum $|0_M\rangle$ and the other on the Rindler vacuum $|0_R\rangle$. However, it was shown that despite the fact that two stationary observers agree on the notion of 'particles', they may still disagree about the spectrum of vacuum fluctuations as described by the Wightman function along the observer's trajectory (Letaw and Pfautsch 1981; Takagi 1984).

5.3 Other types of motion

A general type of motion is examined by Gerlach in considering the Fourier analysis of the spectrum of the Minkowski normal modes (Gerlach 1983a). This motion consists of either (a). uniform drift or (b). uniform rotating motion, in addition to the uniform linear acceleration in a specific direction. An interesting aspect of his investigation is that he concludes that the blackbody radiation (thermal ambience) surrounding a linearly and uniformly accelerated observer is totally isotropic although the observer has a preferred direction (namely, his acceleration). A totally isotropic radiation means the following conditions are satisfied:

(i). its power spectrum is isotropic;

(ii). its fluctuation spectrum is isotropic and gives the power spectrum a thermal signature;

(iii). its stress energy tensor is isotropic, and

(iv). classical radiation reaction forces are absent from a point detector carried along by the observer.

However, the anisotropy of the thermal radiation has been suggested by people in their work of detection process (Israel 1983; Hinton et al., 1983). When the observer undergoes the above mentioned types of motion, this isotropy is broken. This breaking of the spatial isotropy manifests itself as a non-zero chemical potential of the thermal radiation surrounding the observers. This chemical potential is proportional to the drift velocity or the angular velocity of the respective accelerated observers.

For an observer having a uniform drift velocity 'v' (say in the y direction), the radiation spectrum is given by:

$$\{ \exp [\hbar (\omega \pm k_y v) /kT] - 1 \}^{-1}$$

where 'v' is the drift velocity in the y-direction. When the observer is orbiting uniformly in addition to his linearly uniformly accelerated motion, the radiation spectrum has the form of:

$$\{ \exp [\hbar (\omega \pm m\Omega) / kT] - 1 \}^{-1}$$

where Ω is the angular velocity and m is the azimuthal wave number (see section 4.3).

The chemical potential in either cases is given by : (i) $\pm \hbar k_y v$ for uniform drift motion and (ii) $\pm\hbar m\Omega$ for uniform orbiting motion behaving in such a way that the intensity of wave modes propagating against (i) the drift or (ii) the rotation, exceeds that of wave field modes propagating in the same sense. Thus, the thermal background tends to slow down the drift or angular velocity of an observer until his surrounding is totally isotropic.

Chapter 6 Quantum Field theory in analytic accelerated frame

In the preceding chapters, we reviewed the formulation of quantum theory in various non-inertial coordinate systems which predicted the appearance of the characteristics of thermal effects even in flat space-time. However, two facts were addressed by Sanchez (1981), namely the lack of a general description of quantum fields in accelerated coordinates involving non-uniform accelerations (or non-Planckian spectra) and secondly, no explicit relation between the thermal aspects and the structure of space-time has been obtained. It was accepted that the temperature in this context is due to the presence of event horizons.

A new approach was used in the formulation of the theory initially in two dimensional flat space-time (Sanchez 1981b). By the method of analytic mappings, quantum field theory has been defined in a wide class of accelerated space-times preserving the light cone structure of flat Minkowski space-time. The coordinates in which quantum field theory can be consistently formulated are defined by holomorphic (or anti-holomorphic) mappings in Euclidean space (imaginary time). Each analytic function defines an accelerated frame and a quantum production rate associated with it. Classical, quantum and thermal aspects of the theory are explicitly expressed in terms of the mappings. In particular, the accelerated frames, their associated particle production rates and temperature can be classified in terms of the nature of the singular and critical points of the mappings. Furthermore, the physical magnitutes predicted by the theory such as the vacuum spectra could be interpreted in terms of the measurements carried out by accelerated detectors.

6.1 Contextual background

The 'principle of covariance', highlighting the independence of coordinates, is the underlying principle in the theory of gravitation. Sanchez emphasized that this principle is also prevalent in all descriptions of physical laws. Thus, the seeming difference in results from the treatment of a quantum field in a variety of coordinate systems in flat space-time is not due to the choice of the coordinate systems that were used, but is a consequence of the fact that physically different quantum states are correctly described by the quantum theory as being physically distinct. In other words, the 'canonical' states for different coordinate systems are physically different for each time-like vector field. The correct approach is to consider the formulation of quantum field theory in accelerated frames instead of relative to an (accelerated) observer following a particular world-line. Then the boundary conditions of the quantum theory can be described physically in terms of the asymptotic behaviour of those accelerated frames. This automatically specifies the quantum state to be examined. Moreover, the physical consequences of the quantum theory for the chosen state can be examined by the appropriate coordinates. These consequences are completely independent of the coordinates used to evaluate them. The'principle of covariance' seems to be manifested in the above approach.

6.2 Analytic mappings and accelerated frames

We consider the two-dimensional space-time case because of its simplicity. In such a space-time, a formal analytic continuation of the time variable t ($t \rightarrow i\tau$) is made, making it imaginary and then we look for real analytic functions

$$u = f(u') \tag{6.1}$$

as establishing a conformal mapping between the points $u = x+i\tau$ of the Euclidean plane and the points $u' = x'+i\tau'$ of a transformed one. Points where $f'(u') = 0$ are critical points and the transformation is not conformal there.

In the primed coordinates, the metric takes the form

$$ds^2 = | f'(u') |^2 (dx'^2 + d\tau'^2) \tag{6.2}$$

The real critical points of the conformal transformation determine event horizons in Minkowski space time. In Minkowski space-time, the mapping

$$x \pm t = f(x' \pm t') \tag{6.3}$$

represents a transformation from an inertial frame (x,t) to an accelerated one (x',t'). It defines x' and t' as even and odd function of t. Constant values of x' define the world-lines of the accelerated observers in the (x,t) plane. The velocity of these observers is given by

$$v = \frac{f'(x'+t') - f'(x'-t')}{f'(x'+t') + f'(x'-t')} \tag{6.4}$$

The proper acceleration is given by

$$a = \frac{1}{[\Lambda(x',t')]^{1/2}} \partial_x [\ln \Lambda(x',t')] \tag{6.5}$$

where $\Lambda(x', t') = f'(x'+t') f'(x'-t')$

This involves a very large class of accelerated motions. In particular, the Rindler frame corresponds to the analytic mapping

$$f(u') = \beta e^{u'/\alpha} \tag{6.6}$$

and describes uniform acceleration. The bilinear transformations

$$f(u') = \frac{\alpha u' + \beta}{\gamma u' + \delta} \qquad (\alpha\delta - \beta\gamma \neq 0) \tag{6.7}$$

where $\alpha, \beta, \gamma, \delta$ are real parameters also describe uniform acceleration. However, the two mappings are not the same and additional assumptions must be imposed on the wave functions to guarantee the self-adjointness of propagation equations, completeness and orthogonality of their solutions.

It is necessary to carry out a field quantization procedure in those accelerated frames defined by analytic functions to ensure a one-to-one mapping from an interval $[u_-, u_+] \in$

Re u into the whole real u' axis. That is, we require monotonic functions f(u') such that

$$f(\pm\infty) = u_{\pm} \tag{6.8}$$

where one of the $u_{\pm}$ or both u_+ and u_- can be infinite. For finite $u_{\pm}$, conditions (6.8) imply that

$$[\, f'(u')]|_{u'=\pm\infty} = 0 \tag{6.9}$$

i.e., critical points of f lie at the ends of the real u' axis.

An accelerated frame defined from eqns. (6.1) and (6.8) covers a bounded region (a rhombus)

$$u_- < | x \pm t | < u_+ \tag{6.10}$$

of Minkowski space-time; $x \pm t = u_-$ and $x \pm t = u_+$ represent two event horizons. These are the boundaries of the space-time domain over which the (x',t') coordinate is defined. In particular, for $u_- = 0$ and $u_+ = +\infty$, the accelerated frame covers the right-hand wedge of Minkowski space-time. When $u_{\pm} = +\infty$, there are no event horizons, the accelerated frame covers the whole Minkowski space-time.

In terms of the inverse functions

$$u' = F(u) \tag{6.11}$$

condition (6.8) reads

$$F(u_{\pm}) = \pm\infty \tag{6.12}$$

i.e., F(u) has real singularities at $u = u_{\pm}$.

In accelerated frames defined by eqns. (6.1) and (6.8), self-adjointness of the propagation equations and orthogonality and completeness of their solutions are guaranteed.

Physically, condition (6.8) mean that event horizons move at the speed of light (light rays take an infinite time t' to reach them). For $t'\to\pm\infty$, world-lines of accelerated observers tend asymptotically to the event horizons $x \pm t = u_-$ and $x \pm t = u_+$ where their velocity reaches the values $\pm c$.

6.3 Quantum states and vacuum spectra in accelerated frames

A free massless scalar field may be quantized in the accelerated frames described in the preceding sections. a complete set of solutions of the conformally invariant wave function $(g^{\mu\nu}\partial_\mu\partial_\nu)\phi = 0$ is given by

$$\phi_\lambda(u') = \frac{1}{2(\pi\lambda)^{1/2}} e^{i\lambda u'} \qquad (6.13)$$

$$\phi^*_\lambda(v') = \frac{1}{2(\pi\lambda)^{1/2}} e^{i\lambda v'} \qquad (6.14)$$

where $u' = x' - t'$, $v' = x' + t'$, and $\lambda > 0$. Because of eqn.(6.12), they are orthogonal with the scalar product

$$(\phi_1, \phi_2) = i \int \left[\phi^*_1 [\sqrt{g}\, g^{\mu\nu} \partial_\nu \phi_2] - [\partial_\nu \phi^*_1\, g^{\mu\nu} \sqrt{g}\,] \phi_2 \right] d\Sigma_\mu \qquad (6.15)$$

A Bogoliubov transformation

$$C_\lambda = \int_0^\infty dk\, [A_\lambda(k)\, a_k + B_\lambda(k)\, a^+_k] \qquad (6.16)$$

with coefficients

$$B_\lambda(k) = (\phi_\lambda, \varphi^*_k) \quad , \quad A_\lambda(k) = (\phi_\lambda, \varphi_k) \qquad (6.17)$$

relates the annihilation opereators C_λ of the modes ϕ_λ (positive-frequency modes with respect to the time t') to the annihilation and creation operators a_k , $(a_k)^+$ of the modes φ_k (positive-frequency modes with respect to the time t). We have

$$\varphi_k(u') = \frac{1}{2(\pi\,|k|\,)^{1/2}}\, e^{iku'} \tag{6.18}$$

$$\varphi_k^*(v') = \frac{1}{2(\pi\,|k|\,)^{1/2}}\, e^{-ikv'} \tag{6.19}$$

The condition $C_\lambda\,|\,0'>=0$ for all λ does not define the vacuum state of the theory; this is defined by

$$a_k\,|\,0>=0 \qquad \forall k \tag{6.20}$$

The production function is defined to be

$$N(\lambda,\lambda') = <0\,|\,C^+_{\lambda'}C_\lambda\,|\,0> = \int_0^\infty dk\; B^*_\lambda(k)\; B_{\lambda'}(k) \tag{6.21}$$

For $\lambda=\lambda'$, it gives the number $N(\lambda)$ of quanta of frequency λ in the inertial vacuum state on the total volume. The number $N_v(\lambda)$ of λ quanta per unit volume is obtained by introducing wave packets; i.e.,

$$N_v(\lambda) = \lim_{y\to\infty} \int_0^\infty\!\!\int_0^\infty d\lambda'\, d\lambda''\; g_y(\lambda,\lambda')\; g^*_y(\lambda',\lambda'')\; N(\lambda,\lambda') \tag{6.22}$$

where $g_y(\lambda,\lambda')$ is such that

$$\int_0^\infty d\lambda\,|\,g_y(\lambda,\lambda')\,|^2 = 1 \tag{6.23}$$

From eqns.(6.13,6.14) and (6.21) and by using

$$\psi = \int_0^\infty d\lambda\,(C_\lambda\phi_\lambda + C^+_\lambda\phi^*_\lambda\,)\,. \tag{6.24}$$

the vacuum energy H and momentum densities are given by:

$$H(x',t') = \frac{1}{2}[\, H(x'-t',0) + H(x'+t',0) \,] \qquad (6.25)$$

$$P(x',t') = \frac{1}{2}[\, H(x'-t',0) - H(x'+t',0) \,] \qquad (6.26)$$

where $H(x',0) = \dfrac{Re}{\pi}\displaystyle\int_0^\infty\int_0^\infty d\lambda\, d\lambda' \, (\lambda\lambda')^{1/2} \, [e^{i(\lambda-\lambda')x'} N(\lambda,\lambda') - e^{i(\lambda+\lambda')x'} R(\lambda,\lambda')] \qquad (6.27)$

with $R(\lambda,\lambda') = \langle 0 | C_\lambda C_{\lambda'} | 0 \rangle = -\displaystyle\int_0^\infty A_\lambda(k)\, B_\lambda(k) \qquad (6.28)$

H and P satisfy

$$\partial_t H(x',t') + \partial_x P(x',t') = 0 \qquad (6.29)$$

$N(\lambda,\lambda')$ and $R(\lambda,\lambda')$ describe interferences between the created modes with different frequencies λ,λ'. These interferences cancel over the whole volume as can be seen from the relation

$$E = \int_{-\infty}^{+\infty} H(x')\, dx' = \int_0^\infty \lambda N(\lambda)\, d\lambda \qquad (6.30)$$

for the total energy E. The total momentum of created modes over the whole space is zero. Given $N(\lambda,\lambda')$, we reconstruct the mapping, i.e.

$$f(u') = f(u_0)\exp\left\{ -4\pi i \, Re \int_0^\infty \frac{d\lambda}{\lambda} \exp[-i\lambda u'] \, (\lambda\lambda')^{1/2} \, N(\lambda,\lambda') \,\Big|_{\lambda'=0} \right\} \qquad (6.31)$$

where $f(u_0)$ is an integration constant (scale factor of the transformation).

From eqn.(6.31), we get the relation

$$\frac{d}{du'}\,[\ln f(u')] = 4\pi \,\mathrm{Re} \int_0^{\infty} d\lambda \, e^{i\lambda u'} [\, (\lambda\lambda')^{1/2} \, N(\lambda,\lambda') \,]_{\lambda' = 0} \qquad (6.32)$$

From the above results, we have the following theorem: each one of the following statements implies the other two.

(i). The production function has the form

$$N(\lambda,\lambda') = N_v(\lambda)\, \delta\, (\lambda-\lambda') \qquad (6.33)$$

(ii). The Bogoliubov transformation can be expressed as a two term one, i.e.,

$$C_\lambda = [\, 1 + N_v(\lambda) \,]^{1/2}\, \overline{C}_{\lambda(+)} - [\, N_v(\lambda) \,]^{1/2}\, \overline{C}_{\lambda(-)} \qquad (6.34)$$

(iii). The analytic mapping is

$$F(u) = \frac{1}{2\pi T} \ln \beta u \qquad (6.35)$$

$$\text{where} \quad T = [\, \lambda \, N_v(\lambda) \,]_{\lambda=0} \qquad (6.36)$$

and β is an integration constant.

In proving the theorem, we used the relation

$$A_\lambda(k) = \left[\frac{1 + N_v(\lambda)}{N_v(\lambda)} \right]^{1/2} B_\lambda(k) \qquad (6.37)$$

which is the necessary and sufficient condition for the Bogoliubov transformation to be simplified. This condition allows a basis to be defined by:

$$\bar{C}_{\lambda(+)} = \int_{-\infty}^{+\infty} d^3k \, \frac{A_\lambda(k)}{[1+N_v(\lambda)]^{1/2}} \, a_k \tag{6.38}$$

$$\bar{C}_{\lambda(-)} = \int_{-\infty}^{+\infty} d^3k \, \frac{B^*_\lambda(k)}{[\, N_v(\lambda)\,]^{1/2}} \, a^*_k \tag{6.39}$$

such that

$$[\, \bar{C}_{\lambda(+)}, \bar{C}^+_{\lambda'(+)} \,] = \int_0^{\infty} A^*_\lambda(k)\, A_{\lambda'}(k)\, dk$$

$$= [1+N_v(\lambda)]^{1/2}\, \delta(\lambda - \lambda') \tag{6.40}$$

$$[\, \bar{C}_{\lambda(-)}, \bar{C}^+_{\lambda'(-)} \,] = \int_0^{\infty} B^*_\lambda(k)\, B_{\lambda'}(k)\, dk$$

$$= [\, N_v(\lambda)]^{1/2}\, \delta(\lambda - \lambda') \tag{6.41}$$

$$[\, \bar{C}_{\lambda(+)}, \bar{C}^+_{\lambda'(-)} \,] = [\, \bar{C}^+_{\lambda(+)}, \bar{C}_{\lambda'(-)} \,] = 0 \tag{6.42}$$

A corollary of the theorem is the following : If $N(\lambda,\lambda')$ satisfies the statement (i), then $N_v(\lambda)$ is given by

$$N_v(\lambda) = \frac{1}{(e^{\lambda/T} - 1)} \tag{6.43}$$

but the converse is not true.

The paremater T as defined by eqn.(6.43) plays the role of a temperature. For any of the statements (i), (ii), and (iii), the left hand side of eqn.(6.32) evaluated at u' =0 is equal to a constant of value $2\pi T$. The theorem characterize a situation of global thermal equilibrium over the whole accelerated space. This situation implies the presence of event horizons. For example, the Rindler frame has either one event horizon or two event horizons at the same temperature. The important point is that the presence of event horizons is a necessary (but not a sufficient) condition for global thermal equilibrium. The temperature associated with each boundary $u' = \pm\infty$ is defined by:

$$T_{\pm} = \frac{1}{2\pi} \frac{d}{du'} [\, \ln f(u') \,]_{u' = \pm\infty} \qquad (6.44)$$

Thus, each asymptotic region $u' = -\infty$ ($+\infty$), has a temperature associated with it. If $u' = +\infty$ ($-\infty$) is a critical point of $f(u')$, T_+ (T_-) is the temperature of that horizon. Otherwise, T_+ (T_-) is the temperature at infinity.

All the above results enable us to study the quantum spectra themselves in terms of the analytic properties of the mappings. There are basically three different types of vacuum spectra; each of the following spectra $N_v(\lambda)$ characterizes a class of accelerated frames having the same asymptotic properties and temperature. $N_v(\lambda)$ reflects the asymptotic properties of the acceleration but not their detailed behaviour.

$$\text{(i)} \quad N_v(\lambda) = \frac{1}{2}\left[\frac{1}{e^{\lambda/T_-} - 1} + \frac{1}{e^{\lambda/T_+} - 1} \right]$$

It corresponds to accelerated frames with logarithmic singularities for both asymptotic regions.

$$\text{(ii)} \quad N_v(\lambda) = 0$$

In this case, $N(\lambda)$ is finite and non-zero and it is non-thermal and both singularities are of the power or essential type.

$$\text{(iii)} \quad N_v(\lambda) = \infty$$

It corresponds to log-log singularities.

Each of the above classes involves nonuniform acceleration and one, two or no event horizons.

The physical interpretation of the vacuum spectra has to be done in terms of the measurments of accelerated detectors and this brings the detection process naturally into the theory.

Chapter 7 Particle detectors and quantum detection process

We have reviewed the results of field quantization in a variety of coordinate systems and were led to the 'thermalization' theorem (Takagi 1986), i.e., the pure state which is the vacuum as seen by an inertial observer is a canonical ensemble from the view point of a uniformly accelerated observer. The temperature characterizing the ensemble is proportional to the magnitude of the acceleration of the observer. Although this result is obvious mathematically, its physical significance is not obvious immediately. One of the questions that follows naturally concerns the actual detection of such particles predicted by the theory. In other words, can an observer make measurement of his motion and if so, then how are the measurements to be interpreted. We stated in Chapter 1 that a uniformly accelerated detector such as the Unruh box records a Planckian spectrum when coupled to a massless scalar field. However, in Chapter 2 we pointed out that the concept of a 'particle' is ill-defined and ambiguous in a generic space-time. We will not delve into the problem of finding a real definition of a 'particle' but rather we state that the concept of a particle remains purely mathematical until a method of observation is specified (Birrell and Davies 1982, section 3.3; Takagi 1986; Unruh 1986b). Therefore, we consider a concrete, albeit highly idealized model of a particle detector and in particular we discuss how the detector responds when it is uniformly accelerated. The response of the detector then provides us with an observable which in principle is a physical effect that reflects a certain aspect of the thermalization theorem. However, whether the "particle detector' can be reasonably said to detect a particle as it is purported to or not is an important matter to be examined (Grove and Ottewill 1983; Davies 1984; Grove 1986a).

Let us stress that our consideration of an accelerated detector (uniform or non-uniform) is a 'Gedanken' experiment, although some closely related physical phenomena might be observed experimentally as will be discussed in Ch. 8. Our next step will be to focus on certain aspects of the detection process that emerged. Firstly, we have the concept of 'quantum noise' which is to be regarded as the fundamental physical quantity actually observed by the particle detector (Sciama et al., 1981; Takagi 1986). Secondly, when the 'effective particle content' is considered, then the discussion ranges over various possibilities depending on the type of the detector (Unruh 1976; Grove and Ottewill 1983; Hinton 1983,1984). The problems concerning the mechanism of the excitations of a detector have also been investigated extensively (Unruh and Wald 1984; Boyer 1984; Padmanabhan 1985; Grove 1986b; Frolov and Ginzburg 1986) as well as the seeming anisotropy in the detection of acceleration radiation (Israel and Nester 1983; Hinton et al., 1983; Grove 1985).

In this chapter, we give a survey of the construction of a model particle detector and its response in various coordinate systems. The possible mechanism of the quantum detection process and its inherent difficulties will be addressed.

7.1 The De-Witt particle detector

The idea of a particle detector originated with Unruh, who carried out an approximate analysis using the so called 'Unruh box' coupled to a massive scalar field in four-dimensional space-time, taking into account the line width of the detector qualitatively (Unruh 1974). De-Witt simplied the detection process further by considering a point-like object endowed with an internal structure characterized by energy levels and coupled linearly to a free complex scalar field via a 'monopole moment' (De-Witt 1979). This object is called the 'De-Witt point-like detector of monopole type' or the'De-Witt detector'. Specifically, let H be the Hamiltonian describing the internal structure of the detector with a discrete spectrum $\{ E_i \}$ and corresponding eigenstates $\{ | E_i > \}$,

$$H_D | E_i > = E_i | E_i > \tag{7.1}$$

Let the detector be endowed with a 'monopole moment' M such that

$$M(\tau) = \exp(iH_D\tau)\, M(0)\, \exp(-iH_D\tau) \qquad (7.2)$$

in the Heisenberg picture, where τ is the proper time of the detector. It is assumed that the detector is linearly coupled to the scalar field ϕ via this monopole as described by the interaction Lagrangian.

$$L_{int} = \{ M(\tau)\,\Phi(\tau) + M^{+}(\tau)\,\Phi^{+}(\tau) \}\, e^{-s|\tau|} \qquad (7.3)$$

where

$$\Phi(\tau) = \phi(x(\tau)) \qquad (7.4)$$

and $x(\tau)$ denotes the world line of the detector. In writing down eqn.(7.3), we assumed that the detector interacts with the field at a point and so the detector is an idealized point without size. The adiabatic switching factor has been introduced in order to suppress spurious transient effects; the interaction is kept switched on for the duration of proper time ~ 1/s, where s will be taken less than the spacing of detector's energy levels.

Let the entire system, consisting of the detector and the field, be treated in the 'interaction picture'. Suppose that at time $\tau = -\tau_o$, the detector was in one of its energy eigenstates, $| E_1 >$, and the field was in the Minkowski vacuum, $| 0_M >$, i.e., the entire system was in the state $| E_1, 0_M >$. The probability amplitude for the entire system to be found in a state $| E_2, \psi >$ at time $\tau = \tau_0$ is given by

$$i < E_2, \psi | \int_{-\tau_0}^{+\tau_0} d\tau\, e^{-s|\tau|} \{ M(\tau)\,\Phi(\tau) + M^{+}(\tau)\,\Phi^{*}(\tau) \} | E_1, 0_M > \qquad (7.5)$$

where it has been assumed that the matrix elements of M is small enough for the first order perturbation to be appropriate. Using eqn. (7.2), this can be written as

$$i < E_2 \mid M(0) \mid E_1 > \int_{-\tau_0}^{+\tau_0} d\tau \, e^{i(E_2 - E_1)\tau - s|\tau|} < \psi \mid \Phi(\tau) \mid 0_M >$$

$$+ i < E_2 \mid M^{+}(0) \mid E_1 > \int_{-\tau_0}^{+\tau_0} d\tau \, e^{i(E_2 - E_1)\tau - s|\tau|} < \psi \mid \Phi^{+}(\tau) \mid 0_M > \qquad (7.6)$$

The transition rate, namely the probability per unit proper time, for the detector to make transition from one energy eigenstate to another is obtained by taking the square modulus of the expression (7.6), summing it over all the final states $\mid \psi >$ of the field and finally dividing the result by $2\tau_0$. Moreover, we consider the situation when the detector-field is switched on and off infinitely slowly. This may be achieved by letting τ_0 tend to infinity first, and then letting s appearing in the exponent of the adiabatic factor tend to zero at the end of the calculation. Denoting the transition rate by $R(E_2/E_1)$, we obtain

$$R(E_2/E_1) = \{ \mid < E_2 \mid M(0) \mid E_1 > \mid^2 + \mid < E_2 \mid M^{+}(0) \mid E_1 > \mid^2 \} \, F(E_2 - E_1) \qquad (7.7)$$

where

$$F(\omega) = \lim_{s \to 0} \lim_{\tau_0 \to \infty} \frac{1}{2\tau_0} \int_{-\tau_0}^{+\tau_0} d\tau \int_{-\tau_0}^{+\tau_0} d\tau' \exp\{ -i\omega(\tau - \tau') - s|\tau| - s|\tau'| \} \, g(\tau,\tau') \qquad (7.8)$$

and

$$g(\tau,\tau') = < 0_M \mid \Phi(\tau) \, \Phi^{+}(\tau') \mid 0_M > \qquad (7.9)$$

In deriving eqn. (7.7), we used the relations

$$< 0_M \mid \phi^{+}(x) \, \phi(x') \mid 0_M > = < 0_M \mid \phi(x) \, \phi^{+}(x') \mid 0_M > \qquad (7.10)$$

$$< 0_M \mid \phi(x) \, \phi(x') \mid 0_M > = 0 \qquad (7.11)$$

The expression (7.7), which is the consequence of the standard Fermi's golden rule, shows that the transition rate of the De-Witt detector is proportional to the 'response function':F (ω), which depends only on the field but not on the structure of the detector. The remaining factor in eqn. (7.7) represents the 'selectivity' of the detector to the radiation bath and clearly depends on the internal structure of the detector itself.

When $E_2 > E_1$, then the detector has detected a quantum of energy $E_2 - E_1$, and we interpret this event as the absorption of a particle by the detector. If $E_2 < E_1$, we may interpret that the detector has emitted a particle of energy $E_1 - E_2$.

An important thing we need to reconsider concerns the 'size' of the detector. Actually, we get an infinite value for the vacuum fluctuation of the field at a point from eqn. (7.10) when the two points x and x' coincide. We may remedy this by using the 'iε prescription' for the function in eqn. (7.10), i.e., the function is defined in the lower half of the complex x plane and its boundary value is taken at the real axis. However, this seems to be unjustifiable on physical grounds as we are concerned with an 'observable' quantity. Thus, the point-like detector is simply a much idealized case and any actual detector must be of finite size (Unruh 1976; Grove and Ottewill 1983). A smearing factor for a distribution will be naturally provided if the effect of finite size is taken into account (De-Witt 1979). In other words, we consider a particle detector of infinitesimal size, but not a point one. The result can be put formally in the form of eqn. (7.7), but the infinitesimal size of the detector presents itself as the infinitesimal negative imaginary part added to the Minkowski time coordinate x^0,

$$g(\tau,\tau') = G^+(x(\tau), x(\tau')) \tag{7.12}$$

where

$$G^+(x,x') = \langle 0_M | \phi(x)\, \phi^+(x') | 0_M \rangle \big|_{\mathrm{Im}\, x^0 = -\varepsilon} \tag{7.13}$$

ε is an infinitesimal positive quantity of dimension of length, and all the coordinates except for x^0 are real. Since ε is taken to be infinitesimal, the time coordinate x^0 is independent of the choice of the Lorentz frame. The quantity G^+ is called the positive frequency Wightman function.

The De Witt particle detector is of the omnidirectional and monopole type, which is the simplest kind in a variety of differently designed detectors and/or with different detector field couplings (e.g. linear; quadratic; derivative) (Hinton et al., 1983). A classification scheme for particle detector types has been put forward by Hinton (1984). The use of particle detectors associated with quantum detection processes will be discussed later, while the De Witt detector will be used as the prototype for examining its response in different coordinate systems and in n-dimensions in general (Letaw and Pfautsch 1981; Takagi 1986) when coupled to a matter field.

7.2 Vacuum noise

The considerations of a model of a 'particle detector' in the preceding section led us to focus our attention on the two-point function

$$g(\tau,\tau') = \langle 0_M \mid \Phi(\tau)\, \Phi^+(\tau') \mid 0_M \rangle \tag{7.14}$$

This is a correlation function of the field at two times τ and τ' along a world-line $x(\tau)$. More general correlation functions can be written as

$$\langle 0_M \mid \Phi(\tau_1)\, \Phi^+(\tau_1)\, \Phi(\tau_2)\, \Phi^+(\tau_2) \;......\Phi(\tau_m)\, \Phi^+(\tau_m) \mid 0_M \rangle \tag{7.15}$$

which can be expressed as a sum of products of the two-point correlation functions. Moreover, any correlation function involving an odd number of points vanishes identically. In particular

$$\langle 0_M \mid \Phi(\tau) \mid 0_M \rangle = 0 \tag{7.16}$$

When we consider the correlation function from another perspective, namely in the theory of stochastic processes, where a Brownian particle is coupled linearly to a random force, then the detector corresponds to the Brownian particle and the field to the random force. In replacing the field Φ by a random force and the vacuum average by a stochastic ensemble everywhere in eqns. (7.14-7.16), all the above mentioned properties become the properties of the so called Gaussian random force; the two-point correlation function is the noise generated by the random force. Thus, the quantity defined in (7.14) is a kind of noise, "the quantum noise in the Minkowski vacuum along the world line $x(\tau)$", whose stochastic properties are determined by the nature of the Minkowski vacuum and the world line. This viewpoint is much emphasized by Sciama and his co-workers (Sciama et al., 1982). For free fields, the quantum noise is Gaussian as remarked above. If the world lines belong to a class of stationary world lines (Letaw and Pfautsch 1981) as discussed in section 5.2, of which among others the world lines of uniform acceleration in either linear or circular motions are members, the noise is also stationary, i.e., the right hand side of eqn. (7.14) depends on the difference $\tau-\tau'(\Delta\tau)$. We shall concentrate our discussion on stationary noise and so we write

$$g(\tau-\tau') \equiv g(\tau,\tau')$$

The quantum noise is both Gaussian and stationary but need not be a 'white' one and indeed the response function defined by eqn. (7.8) can be simplified to

$$F(\omega) = \lim_{s\to 0} \int_{-\infty}^{+\infty} d\tau\, e^{-i\omega\tau - s|\tau|}\; g(\tau) \tag{7.17}$$

If g (τ) vanishes as $|\tau|$ tends to infinity, the adiabatic factor can be ignored. Eqn. (7.18) then says that the response function is just the power spectrum of the noise according to the Wiener-Khinchin theorem. The response function depends on ω in general and hence the noise is not white. The 'particle detector' acts as a 'noise detector' or a 'fluctuometer' (Candelas and Sciama 1977) for the probing of the colour of the noise. The detector can be said to detect particles as the spectrum of vacuum fluctuation and it need not be interpreted as meaning that the detected 'particles' are 'real'.

Let us illustrate the above mentioned materials by the following example. The responses of two detectors, one of which moves inertially while the other is uniformly accelerated will be compared as both coupled to a massless scalar field. The autocorrelation function (positive-frequency Wightman function) as given by eqn. (7.14) for the zero-point fluctuations of a real scalar field $\Phi(x)$ relative to the usual Poincaré invariant vacuum state (Minkowski vacuum) is found to be

$$G^{+}(x, x') = \frac{1}{4\pi^2} \frac{1}{[-(t-t'-i\varepsilon)^2 + (\mathbf{x}-\mathbf{x}')^2]} \tag{7.18}$$

where $x = (t,x,y,z)$ and $\mathbf{x}=(x,y,z)$.

If the detector moves inertially, then the calculation can be done in the rest frame of the detector with the power spectrum of the fluctuation (response function) written as

$$F(\omega) = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty} dt \, \frac{\exp(-i\omega t)}{(t-i\varepsilon)^2}$$

$$= -\frac{\omega}{2\pi}\,\theta(-\omega) \tag{7.19}$$

where θ denotes the step function.

Since F (ω) contains only negative frequencies, it implies that if the detector is prepared in the ground state, then it will never be found in the excited state. On the other hand, when it is prepared in an excited state then it may decay to a lower state as induced by the fluctuation of the field.

The autocorrelation function for a uniformly accelerated detector is given by

$$G^{+} = < 0_M \,|\, \Phi(\tau)\,\Phi(0) \,|\, 0_M >$$

$$= \frac{-a^2}{16\pi^2 \sinh^2(\tau/2 - i\varepsilon)} \tag{7.20}$$

where a is the constant proper acceleration of the detector and $\Phi(\tau)$ denotes $\Phi(x)$ evaluated at the point $x(\tau) = (\tau, \xi, y, z)$

The power spectrum of the fluctuations along the accelerated world line is therefore

$$F(\omega) = -\frac{1}{4\pi^2}\int_{-\infty}^{+\infty} d\tau \frac{a \exp(-i\omega\tau/a)}{4\sinh^2(\tau/2 - i\varepsilon)} \tag{7.21}$$

In order to evaluate the above integral, we first consider the result of integrating the function

$$f(\tau) = -\frac{a}{16\pi^2}\frac{\exp(-i\omega\tau/a)}{\sinh^2(\tau/2)} \tag{7.22}$$

around the contour C as shown in Fig. 7.1.

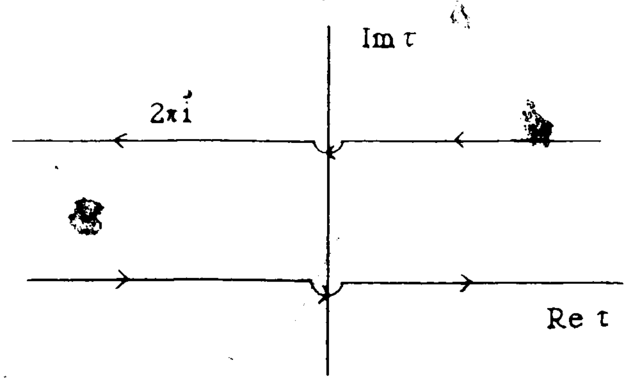

Fig. 7.1 The contour of integration appropriate to the evaluation of F(ω)

The integral over the lower part of the contour yields $F(\omega)$ while that over the upper part yields $-\exp(2\pi\omega/a)F(\omega)$. The sum of these contributions is related to the residue of $f(\tau)$ at $\tau = 0$. Thus, we find

$$F(\omega) = \frac{1}{2\pi}\frac{\omega}{[\exp(2\pi\omega/a) - 1]} \tag{7.23}$$

Hence, the uniformly accelerated detector will be excited to a temperature $T = a/2\pi k_B$, the well known 'Fulling-Davies-Unruh' temperature. Moreover, the uniformly accelerated detector reacts as if it were in a heat bath at temperature T. This follows because the detector is moving through a Gaussian random process and therefore sees a Gaussian random process. This process is completely determined by the second order correlation function which is that of a thermal distribution at temperature T. In this case, the quantum Gaussian fluctuations are promoted to thermal Gaussian Fluctuations, and a truly thermal state is detected.

The concept of the quantum noise was examined further by Takagi in comparing the response of a particle detector uniformly accelerated in Minkowski vacuum (the Rindler detector) to that of the same detector at rest in a thermal bath (the thermal detector) in arbitrary dimensional space time (Takagi 1986).

7.3 Vacuum fluctuation of 'zero-point field' in accelerated frames

As remarked in the preceding section, Sciama and his co-workers considered the thermal effects of acceleration to have their origin in the zero-point fluctuations of the quantum field, without implying that particles are created. Similar physical arguments have been pointed out by Boyer in the context of a classical random field. He considered that the 'vacuum' state is actually formed by a universal random classical electromagnetic field. This random field should exist even at absolute zero temperature; hence it is termed the zero-point field. This zero-point field is considered to be a physically real field interacting with the matter in the universe. Moreover, this zero-point field has a Lorentz-invariant energy spectrum, and therefore cannot be detected in an inertial frame; however, this field manifests itself in the form of a thermal radiation in a uniformly accelerated frame. Thus, the spectrum observed by a uniformly accelerated detector is a distortion of the zero-point field and is not due to the 'creation of particles'(Boyer 1980). It is natural to deduce that the 'zero-point' field will also be distorted by the existence of a gravitational field and manifest itself in some peculiar way. This investigation was carried out in the framework of quantum field theory (Hacyan et al., 1985; Hacyan 1986) in curved space time. We will review the results of the energy spectra of 'zero point' fields observed by an accelerated detector in flat space time.

For a massless scalar field, the standard density current is given by:

$$n_\alpha = i < \phi\,(\partial_\alpha \phi) - (\partial_\alpha \phi)\,\phi > \tag{7.24}$$

Moreover, if ξ^μ is a Killing vector for the background space time, ξ^μ generates a transformation which leaves the action invariant. The conserved Noether current associated with that invariance is

$$J_\alpha = < \phi\,[\,\partial_\alpha(\xi^\beta \partial_\beta \phi)\,] - [\,\partial_\alpha(\xi^\beta \partial_\beta \phi)\,]\,\phi > \tag{7.25}$$

The orbits of the timelike Killing vector can be identified with the world line $x^\alpha = x^\alpha(\tau)$ of an observer whose four velocity is:

$$\frac{dx^\alpha}{d\tau} = u^\alpha = (\xi^\mu \xi_\mu)^{-1/2}\, \xi^\alpha \tag{7.26}$$

Then, for the detector we can define the particle number density (n) and energy density (e) as:

$$n \equiv u^\alpha n_\alpha$$
$$= -i < \phi \frac{d\phi}{d\tau} - \frac{d\phi}{d\tau} \phi > \tag{7.27}$$

$$e \equiv u^\alpha J_\alpha$$
$$= \frac{1}{2} (\xi^\mu \xi_\mu)^{1/2} < -\phi \frac{d^2\phi}{d\tau^2} - \frac{d^2\phi}{d\tau^2} + 2 \frac{d\phi}{d\tau} \frac{d\phi}{d\tau} > \tag{7.28}$$

The Wightman functions $D^{\pm}(x^\mu, x'^\mu)$ are to be evaluated at two points: $x^\mu_+ = x^\mu(\,\tau + (1/2)\sigma\,)$ and $x^\mu_- = x^\mu(\,\tau - (1/2)\sigma\,)$, along a given world line:

$$D^{\pm}(\tau + \frac{1}{2}\sigma, \tau - \frac{1}{2}\sigma) = \langle \phi(\tau \pm \frac{1}{2}\sigma)\, \phi(\tau + \frac{1}{2}\sigma) \rangle \tag{7.29}$$

and by defining the Fourier transforms of the Wightman functions

$$\bar{D}^{\pm}(\omega,\tau) = \int_{-\infty}^{+\infty} d\sigma\, e^{i\omega\sigma}\, D^{\pm}(\tau + \frac{1}{2}\sigma, \tau - \frac{1}{2}\sigma) \tag{7.30}$$

we have for the particle number density

$$n(\omega,\tau) = \frac{1}{\pi}\int_{0}^{\infty} d\omega\, \omega\, [\, \bar{D}^{+}(\omega,\tau) - \bar{D}^{-}(\omega,\tau)\,] \tag{7.31}$$

and similarly for the energy density

$$e(\omega,\tau) = (\xi^{\mu}\xi_{\mu})^{1/2}\int_{0}^{\infty} d\omega\, \frac{\omega^2}{\pi}\, [\, \bar{D}^{+}(\omega,\tau) + \bar{D}^{-}(\omega,\tau)\,] \tag{7.32}$$

Now, ω is the frequency measured by a detector with proper time τ and four velocity u^{α}. From eqn. (7.31), we can define the particle density $f(\omega,\tau)$ as

$$\frac{dn}{d\omega} = f(\omega,\tau) = \frac{1}{(2\pi)^2\omega}\, [\, \bar{D}^{+}(\omega,\tau) - \bar{D}^{-}(\omega,\tau)\,] \tag{7.33}$$

and eqn. (7.32) implies that the energy density per mode is

$$\frac{de}{d\omega} = (\xi^{\mu}\xi_{\mu})^{1/2}\, \frac{\omega^2}{\pi}\, [\, \bar{D}^{+}(\omega,\tau) + \bar{D}^{-}(\omega,\tau)\,] \tag{7.34}$$

For an inertial observer, the Wightman functions for the Minkowski space are

$$D^{\pm}(x,x') = -\frac{1}{4\pi^2}\, [\, (t - t' \mp i\varepsilon)^2 - |\mathbf{x} - \mathbf{x}'|^2\,]^{-1} \tag{7.35}$$

Thus, for this particular observer,

$$D^{\pm}\left(\tau + \frac{1}{2}\sigma, \tau - \frac{1}{2}\sigma\right) = -\frac{1}{4\pi^2}(\sigma \mp i\epsilon)^{-2} \qquad (7.36)$$

and

$$f(\omega,\tau) = (2\pi)^{-3} \qquad [= (2\pi\hbar)^{-3}] \qquad (7.37)$$

$$de = \frac{\omega^3}{2\pi^2}\, d\omega \qquad \left[= \frac{\hbar\,\omega^3}{2\pi^2 c^3}\, d\omega \right] \qquad (7.38)$$

We see that there is one particle in each cell of phase space, which has no significant physical meaning while eqn. (7.37) is the well-known expression for the zero-point energy.

For a uniformly accelerated detector, the Wightman functions are given by:

$$D^{\pm}\left(\tau+\frac{1}{2}\sigma, \tau-\frac{1}{2}\sigma\right) = -\frac{a^2}{16\pi^2}\cosh^{-2}\left[\frac{1}{2}a(\sigma \mp i\epsilon)\right] \qquad (7.39)$$

and

$$f(\omega,\tau) = (2\pi)^{-3} \qquad (7.40)$$

$$de = (\xi^{\mu}\xi_{\mu})^{1/2}\,\frac{\omega^3}{\pi^2}\left[\frac{1}{2} + \frac{1}{e^{2\pi\omega/a} - 1}\right] d\omega \qquad (7.41)$$

Again, there is one particle in each phase-space cell, but now the energy of the zero-point has an additional Planckian term. This implies that an accelerated detector does not count newly created particle; what it actually detects is the zero-point field, which by effect of the acceleration, manifests itself as a Planck spectrum.

For massive matter fields, the expressions for the 'particle density' becomes more complicated while the energy density still retains the familiar form of distribution. In particular, when we have a massive scalar field, the Wightman functions are given by

$$D^{+}(\omega) = \frac{m^2}{4a\pi^2} e^{\pi\omega/a}\left[|K'_{1+i\omega/a}(\frac{m}{a})|^2 - |K_{i\omega/a}(\frac{m}{a})|^2 \right] \qquad (7.42)$$

Thus, the 'particle density' is given by

$$\frac{dn}{d\omega} = f(\omega,\tau) = \frac{m^2\omega}{2a\pi^3} \sinh(\frac{\pi\omega}{a})\left[|K_{1+i\omega/a}(\frac{m}{a})|^2 - |K_{i\omega/a}(\frac{m}{a})|^2 \right] \qquad (7.43)$$

and the energy density can be written in the following form:

$$de = 2\omega\left[\frac{1}{2} + \frac{1}{e^{(2\pi\omega)/a} - 1} \right] dn \qquad (7.44)$$

Here a Bose Einstein distribution function appears superimposed on the zero point energy spectrum but the 'particle density' takes a complicated form.

For a massive Dirac field (e.g. electrons), the energy density can be written in the form

$$de = \left[\omega + \frac{2\omega}{e^{(2\pi\omega)/a} + 1} \right] dn \qquad (7.45)$$

where

$$dn = \frac{m^3}{2a\pi^3} \cosh(\frac{\pi\omega}{a})\left[|K_{(3/2+i/a)}(\frac{m}{a})|^2 - |K_{(1/2+i/a)}(\frac{m}{a})|^2 \right] d\omega \qquad (7.46)$$

The particle density is complicated in its form, but the energy spectrum exhibits a thermal term. According to eqn. (7.47), an accelerated observer detects a non-zero density of 'virtual' particles with energies below mc^2, even though only the states with $h\omega < -mc^2$ are occupied in an inertial frame. This may lie in the fact that the acceleration lifted some electrons above their original energy levels, producing an overall excess of positive energy.

At the same time, the depletion of electrons below the energy level mc^2 is observed as an increase in the 'positrons' energy. Thus, eqn.(7.45) can be interpreted in the sense that the term ω corresponds to the vacuum energy, and the Fermi-Dirac distribution comes from the additional energy detected in the accelerated frame, the factor of 2 being due to the fact that both electrons and positrons contribute to the excess energy.

It should be emphasized that both the positive and negative frequency Wightman functions are used in the formalisms instead of only the positive one we used in the preceding section. As Hacyan pointed out, this omission is justifiable when the negative frequency contributions to the energy can be eliminated which is equivalent to arbitrarily cutting off the zero-point energy.

The distortion of the zero-point field has also been considered in a circular-motion frame when the accelerated detector is coupled to a massless scalar field (Kim et al., 1987). The Wightman functions are found to be:

$$D^{\pm}(\tau+\frac{1}{2}\sigma,\ \tau-\frac{1}{2}\sigma) = -\frac{1}{4\pi^2}\frac{1}{(\gamma\sigma\mp i\varepsilon)^2-4\rho^2\sin^2(\gamma\omega_o\sigma/2)} \qquad (7.47)$$

where ρ is the radius of the circle, ω_o is the angular frequency, $v = \rho\omega_o$ and $\gamma = 1/(1-v^2)^{1/2}$. Thus, the Fourier transforms are

$$\overline{D}^{\pm}(\omega,\tau) = -\frac{1}{4\pi^2}\frac{\omega_o}{2\gamma}\int_{-\infty}^{+\infty} ds\ \frac{\exp(2iWs)}{(s\mp i\varepsilon)^2 - v^2\sin^2(s)} \qquad (7.48)$$

where $s \equiv \gamma\omega_o\sigma/2$, $W \equiv \omega/\gamma\omega_o$

It can be shown that

$$\overline{D}^{+}(\omega) - \overline{D}^{-}(\omega) = \frac{\omega}{2\pi} \qquad (7.49)$$

which means that the particle density is one for each phase-space cell.

The integrand of $\overline{D}^{-}(\omega)$ can be expressed as a power series in v :

$$\bar{D}(\omega) = \frac{1}{4\pi^2}\frac{\omega_o}{2\gamma}\int_{-\infty}^{+\infty} ds\, e^{2iWs} \sum_{n=0}^{\infty} \frac{v^{2n}\sin^{2n}(s)}{(s+i\epsilon)^{2n+2}} \qquad (7.50)$$

which can be integrated term by term. The result is

$$\bar{D}(\omega) = \frac{\omega_o}{2\pi\gamma}\sum_{n=0}^{\infty} \frac{v^{2n}}{2n+1}\sum_{k=0}^{n} (-1)^k \frac{(n-k-W)^{2n+1}}{k!\,(2n-k)!}\,\theta(n-k-W) \qquad (7.51)$$

where θ is the usual step function.

The energy density detected by the detector in circular motion is given by:

$$d\epsilon = \frac{\omega^3}{\pi^2}\left[\frac{1}{2} + \frac{\omega_o}{2\gamma\omega}\sum_{n=0}^{\infty} v^{2n} f_n(W)\right] d\omega \qquad (7.52)$$

The energy spectrum of the distorted zero-point field is not a thermal one and can have dependence on the speed of the detector in addition to its acceleration.

By comparing different kinds of motion, the energy densities were found to depend on the details of motion. The zero-point field seems to be distorted by the motion of the detector and yields the familiar thermal spectrum with the exception of that in circular motion. "This signifies that the similarity between a thermal spectrum and the zero-point spectrum may be only coincidental" (Kim et al., 1987).

7.4 Vacuum spectra in stationary trajectories

In Chapter 5, different classes of coordinate systems were introduced and via canonical quantization of a free scalar field, the vacuum states were found limited to two possibilities: the Minkowski vacuum and the Rindler vacuum. Then in section 7.1, we studied the response function of a De-Witt particle detector coupled to a scalar field in flat space-time. In this section, the results obtained from the 'detector' method are compared to those from canonical quantization in order to check the validity of the statement that the 'model detector' detects particles as defined by the standard canonical quantization. In particular, the nature of the spectra corresponding to the various trajectories will be examined with a discussion of the physical implications of the result of comparison.

The cases for an inertial observer and detector as well as their uniformly accelerated motions have been discussed in previous sections. The results are found to be the same in each class of coordinate systems.

For other coordinate systems such as Class C and Class D, the investigation is as follows:

Class C coordinate system: rotating coordinates

$$\xi^{\mu} = (c, -\alpha y, \alpha x, 0) \tag{7.53}$$

This corresponds to the trajectory:

$$X^{\mu}(\tau) = (C\tau, R\cos\alpha\tau, R\sin\alpha\tau, 0) \quad ; \quad R = \sqrt[\alpha]{c^2 - 1} \tag{7.54}$$

The Wightman function is given by

$$G^{+} = \frac{1}{2\pi^2} \frac{1}{c^2(\tau - i\varepsilon)^2 - 4R^2 \sin^2(\alpha/2)(\tau - i\varepsilon)} \tag{7.55}$$

The above integral cannot be evaluated in terms of simple functions but it is obvious that the particle detector sees a non-Planckian spectrum of radiation. Moreover, the Bogoliubov coefficients are found to be:

$$\alpha(q,m,k_z; \bar{k}) = \frac{1}{(2\pi)^{1/2}} \left[\frac{\bar{k}_x - i\bar{k}_y}{q} \right]^m \frac{\delta[q - (\bar{k}_x^2 + \bar{k}_y^2)^{1/2}]}{q} \, \delta(k_z - \bar{k}_z) \tag{7.56}$$

$$\beta(q,m,k_z; \bar{k}) = 0 \tag{7.57}$$

The barred quantities indicate that Minkowski modes are used in the expressions. Thus, the two vacuum states defined by the two procedures are incompatible with one another.

Lastly, when we consider the Class-D coordinate system, we get that the trajectory of the motion is:

$$X^{\mu}(\tau) = \left(\tau + \frac{1}{6}a^2\tau^3,\ \frac{1}{2}a\tau^2,\ \frac{1}{6}a^2\tau^3,\ 0\right) \tag{7.58}$$

The Wightman function is written as

$$G^{+} = \frac{1}{2\pi^2}\,\frac{1}{(\tau - i\varepsilon)^2 + (a^2/12)(\tau - i\varepsilon)^4} \tag{7.59}$$

and the power spectrum is then defined by

$$F(\omega) = \frac{\omega^2}{8\pi^2\sqrt{3}\,a^2}\left[\exp\left[-3\sqrt{2}\left(\frac{\omega}{a}\right)\right]\right] \tag{7.60}$$

which does not have the form of a Planck spectrum. The Bogoliubov coefficients are then given by:

$$\alpha(k_y, k_z, p; \overline{Q}) = \frac{1}{2}\left|\frac{\omega}{\pi a k_y}\right|^{1/2} \exp\left| i\frac{\overline{Q}_x}{2ak_y^2}\left(\overline{Q}_x^2 + k_y^2 + pq^2\right)\right| \delta(\overline{Q}_z - k_z)\,\delta(k_y - \omega + \overline{Q}_y) \tag{7.61}$$

$$\beta(k_y, k_z, p; \overline{Q}) = 0 \tag{7.62}$$

Again, the detector is not detecting the particles corresponding to the standard canonical quantization.

The conclusions that may be drawn are these: in general, there is no correspondence between the particles detected by the 'model particle' detector and the particles as defined through a canonical quantization procedure. Moreover, the existence of a Planck spectrum for the uniformly accelerated frames is merely a coincidence as stated in the preceding section. Various types of detector motions lead to different kinds of spectra. In particular, no natural definition of temperatures exists for these trajectories (Padmanabhan 1982;

Letaw and Pfautsch 1981).

However, it was pointed out by Myhrvold that the above conclusions were incorrect and he asserted that the canonical quantization and detector methods were guaranteed to give consistent results if they were each applied properly (Myhrvold 1984). The discrepancy between the canonical quantization and the detector methods was due to an incorrect choice of rest frame coordinates. The argument can be stated as the following:

The Wightman function as defined in eqn.(7.14) is a two-point function in the particular state $|0_M>$ and is a completely coordinate independent object. This invariance of the two-point function for a given state under coordinate basis transformations is a general aspect of the formalism of quantum field theory which obtains even in non-static cases. Hence, when the two-point function is evaluated in $|0'>$ in Minkowski coordinates while a Bogoliubov transformation is performed on $|0_M>$ to any other basis, the same result should be obtained regardless of the coordinate system that the vacuum is in. Moreover, the 'rest frame' of the accelerated observer should have the metric written as:

$$ds^2 = -[\,1 + z^i a^d (e_d)_i\,]^2\, d\tau^2 + (dz^1)^2 + (dz^2)^2 + (dz^3)^2 \qquad (7.63)$$

with $i = 1, 2, 3$ and $z^0 = \tau$,

The metric (7.63) is the Fermi-Walker rest frame for any time-like world-line in flat space-time. It is equivalent to the Minkowski metric for observers with $a^b = 0$, otherwise it is a static metric if and only if the world-line has constant co-linear acceleration. The metric used by people in their calculations are incorrect for the accelerated observers.

7.5 Particle detector response in static space-times

In section 7.2, we defined the vacuum noise (i.e., the noise in the Minkowski vacuum due to a complete scalar field seen along a world-line with uniform acceleration 'a') through the power spectrum of the field which is given by:

$$F(\omega) \equiv \int_{-\infty}^{+\infty} e^{i\omega\tau} < 0_M \mid \Phi(x(\tau))\, \Phi(x(0)) \mid 0_M > d\tau \tag{7.64}$$

For a De Witt particle detector moving along a Rindler trajectory with acceleration 'a' through the Minkowski vacuum of a massless scalar field in four-dimensional space-time, it was found that

$$F_4(\omega) = \frac{1}{2\pi} \frac{\omega}{e^{\omega/T} - 1} \tag{7.65}$$

where $T \equiv a / 2\pi$, which is in good intuitive agreement with the thermal description of that state. Indeed eqn.(7.65) can be rewritten as:

$$F(\omega) = \frac{|\omega|}{2\pi} \left[\theta(\omega) \frac{1}{e^{\omega/T} - 1} + \theta(-\omega) \left[1 + \frac{1}{e^{|\omega|/T} - 1} \right] \right] \tag{7.66}$$

This can be interpreted as: The first term determines the rate of absorption by the detector of Rindler particles which obey the Bose-Einstein distribution at the temperature T, and the second term determines the rates of spontaneous and induced emission of Rindler particles.

In previous sections, we noticed that the absence of the Planck spectrum detected by the detector in some coordinate systems. It is then natural to generalize the result given by (7.65) to space-times of arbitrary dimensions and for massive fields as well. This investigation was done by Takagi for both scalar and Dirac fields in n-dimensional space-time (Takagi 1985; 1986).

For a massless scalar field, the power spectrum in n-dimensional space-time is found to be

$$F_n(\omega) = \frac{\pi}{\omega} \frac{D_n^M(\omega)}{e^{\omega/T} - (-1)^n} d_n(\omega) \qquad (7.67)$$

where

$$D_n^M(\omega) = \frac{2^{2-n} \pi^{(1-n)/2}}{\Gamma(\frac{n-1}{2})} \omega^{n-2} \qquad (7.68)$$

is the usual Minkowski density of states per unit volume and the factor $d_n(v)$ is a polynomial in v^{-2} given by the recursion relation

$$d_n(v) = \{ 1 + (\frac{n-4}{2})^2 v^{-2} \} d_{n-2}(v) \quad , \quad n \geq 4 \qquad (7.69)$$

$$d_2(v) = d_3(v) = 1 \qquad (7.70)$$

From eqns. (7.67-7.70), we see that while in even space-time dimensions, the denominator in $F_n(\omega)$ has the form associated with the Planck spectrum for bosons, but in odd space-time dimensions it has the form associated with the Planck spectrum for fermions.

The above strange result was explained by Ooguri using the Huygens' principle (Ooguri 1986) and rederived by Unruh in taking into consideration the details of the Rindler mode function (Unruh 1986b).

It is instructive to compare the 'Rindler noise' with the thermal noise. The thermal noise is defined by :

$$g_{\beta n}(\tau - \tau') = \langle \Phi(\tau) \Phi^+(\tau') \rangle_\beta \qquad (7.71)$$

where

$$\Phi(\tau) = \phi(x(\tau)) \qquad (7.72)$$

with the world-line $x(\tau)$ taken to be at rest with respect to the heat bath at temperature T, i.e.,

$$x^0(\tau) = \tau, \quad x^i(\tau) = \text{constant}, \qquad 1 \le i \le n-1 \tag{7.73}$$

where $\{x^\mu\}$ are the Minkowski coordinates of the rest frame of the heat bath. The bracket $<\ldots>_\beta$ denotes the thermal average

$$<\ldots>_\beta = \mathrm{Tr}[\,\rho_\beta \ldots\,] \tag{7.74}$$

with the canonical density matrix

$$\rho_\beta = \frac{\exp(-H_M/T)}{\mathrm{Tr}\exp(-H_M/T)} \tag{7.75}$$

where H_M is the Minkowski Hamiltonian.

This thermal noise is observable in principle by putting the De-Witt detector in the thermal bath inertially. The power spectrum is found to be:

$$F_{\beta n}(\omega) = \frac{\pi D_n^M(\omega)}{|\omega|}\left[\theta(\omega)\frac{1}{e^{\omega/T}-1} + \theta(-\omega)\left[1 + \frac{1}{e^{|\omega|/T}-1}\right]\right] \tag{7.76}$$

$$= \frac{\pi}{\omega}\frac{D_n^M(\omega)}{e^{\omega/T}-1} \tag{7.77}$$

where $D_n^M(\omega)$ is the usual density of states,

$$D_n^M(\omega) = \frac{2^{2-n}\pi^{(1-n)/2}}{\Gamma(\frac{n-1}{2})}|\omega|(\omega^2-m^2)^{(n-3)/2}\,\theta(|\omega|-m) \tag{7.78}$$

Comparison of eqns. (7.67) and (7.76) for the massless case shows that the Rindler noise is identical with the thermal noise if and only if the dimension of space-time is 2 or 4. In even-dimensional spacetimes with dimension larger than four, the Rindler noise has the extra factor $d_n(\omega)$. In odd-dimensional space-times, the Rindler noise is characterized by the Fermi-Dirac distribution, in contrast to the thermal noise which obeys the Bose-Einstein distribution in any dimensions.

For a massive scalar field, the power spectrum of the Rindler noise is a complicated function. In particular, the asymptotic form of the power spectrum in terms of large m is found to be

$$F_n(\omega) \approx \frac{a}{8\pi}\left(\frac{ma}{4\pi}\right)^{(n/2)-2} e^{-(\omega/2 + m/\pi)/T} \tag{7.79}$$

The power spectrum of the Rindler noise of a massless Dirac field in n dimensions is identical to that of a massless scalar field in n+1 dimensions up to a renormalization factor

$$F_{1/2,n}(\omega) = \alpha_n F_{n+1}(\omega) \tag{7.80}$$

with

$$\alpha_n = 2\pi^{1/2} \frac{\Gamma(\frac{n}{2})}{\Gamma(\frac{n-1}{2})} \tag{7.81}$$

The power spectrum of the Dirac field is characterized by a Fermi dist[illegible] the dimension of spacetime is even and by a Bose-Einstein distribution if the dimension is odd.

When considering the thermal noise in a Dirac field, the power spectrum is found to be

$$F_{\beta,1/2,n}(\omega) = \alpha_n \left| \tanh\left(\frac{\omega}{2T}\right) \right| F_{\beta,n+1}(\omega) \tag{7.82}$$

where α_n is given by (7.81). Thus, the Rindler and the thermal noise are identical in two dimensions but differ for higher dimensions as well as giving the statistics-inversion in odd dimensions.

The work of Takagi pointed out the phenomenon of 'statistics-inversion' in the power spectrum detected by the De-Witt particle detector in n-dimensions. The roles of the Rindler and the thermal detector have also been distinguished from one-another although they exhibit the same spectrum in some particular dimensions.

7.6 Remarks on the quantum detection process

The theory of quantum detection seems to play an important role in the formulation of quantum field theory by non-inertial observers. This is in contrast to the usual formulation and interpretation of quantum mechanics by an inertial observer. The eigenvalues of the operator and the mean values computed in the theory are postulated to be the detectable quantities. The observer and his measurement devices are external classical objects not described by the quantum theory. But, as Sanchez has emphasized the state of motion of the observer appears included in the formulation of quantum field theory by accelerated observers, so in such a theory a description of the observer's detector and of the measurements he carries out should be provided (Sanchez 1985). Then, in order to represent the quantum measurement by an observable, without describing the detection process, we need to answer the following questions (Haag et al., 1984):

a). Which element of the observable algebra represents an ideal detection process?

b). Can we find a transformation law which indicates how this observable is modified when the same detector is forced to move along some other world-lines?

There is no 'quantum covariance principle' to answer these questions. Thus, models of detection processes have been devised. As mentioned at the beginning of this chapter, an idealized particle detector which is the De-Witt monopole detector is constructed to study the results of its motion along some particular trajectories. Then, by studying the response function or power spectrum of the detectors, these models of detectors purport to analyse the 'effective particle content' or 'vacuum noise' of the quantum state seen along its trajectory. However, it is found that the 'effective particle content' is detector-model dependent. Moreover, the incompatibility of the results between the canonical formulation and the models of quantum detection processes by non-inertial observers casts doubts on the plausibility of an ideal model detector (see section 7.4). It is more correct to say the Fulling Davies Unruh effect comprises two physical phenomena: the Planck spectrum that is obtained by a Bogoliubov transformation is not localized but defined over the whole Rindler space, while a uniformly accelerated detector observing this spectrum along its trajectory is a local effect. This special coincidence has its origin in the high degree of symmetry of the Rindler frame. However, in general there is no coordinate system which is locally the Fermi Walker system that is associated with a given flow of trajectories. Hence, the existence of a coordinate system 'naturally' adapted to the flow of trajectories is unlikely.

Some of the problems that were encountered when constructing models of detection processes such as the consideration of the finite size of the detector; the physical interpretation of the observed spectrum etc., remain to be answered. The working mechanism of the detectors also raised different ideas (Unruh and Wald 1984; Grove 1986; Padmanabhan 1982) and for some particular direction-perferred detectors, the detection of anisotropic radiation has been discovered (Israel and Nester 1983; Hinton et al., 1983) but seems to be answered by Grove (1985).

In conclusion, the link between models of detection processes and quantum field theory in accelerated frames has not been established. Concerning the detection process, at present it seems better to formulate a theory of model detectors for measuring a variety of field-related physical quantities such as ϕ^2, $T_{\mu\nu}$ or other vector densities rather than concentrating on the usual concepts (e.g. particle) (Davies 1984).

Chapter 8 Experimental facts about the Fulling-Davies-Unruh effect

After discussing extensively the phenomenon associated with a non-inertial frame in a matter field, one may wonder about the appearence of the Fulling-Davies-Unruh effect in actual experimental set-ups. Indeed, several authors speculated that this effect may have occured and been observed through measurements in high energy physics experiments (Barshay and Troost 1978; Hosoya 1979). In particular, Hosoya stated that the moving (accelerated) mirror effects were quite obvious in the hadronic reactions especially in the $e^+ e^-$ reactions. According to the theory, gluons are thermally produced at the walls of the bag which are pushed by energetic but permanently confined quarks. The walls of the bag are idealized as perfectly reflecting mirrors to the quarks and the gluons. The standard linear potential between quarks implies a constant acceleration of the walls. Then we may consider the appearence of the Fulling-Davies-Unruh temperature due to the acceleration:

$$k_B T = \frac{a}{2\pi} \tag{8.1}$$

Although the temperature corresponding to the acceleration in 'everyday life' is extremely low (10^{-20} K for 9.8 m / sec^2), hadronic reactions give considerably higher temperatures which have the standard values of

$$k_B\, T \sim 130 \text{ MeV}$$

Under some still to be clarified conditions, the measured transverse momentum distribution in the low energy region is of the form

$$P_\perp \exp\left(\frac{-m_\perp}{130 \text{ MeV}}\right) dP_\perp$$

This seems to explain the mysterious thermodynamical features of hadronic reactions (e.g. the transverse momentum distribution) in the confined quark picture coming from the moving mirror effects. This theory may be extended to other hadronic reactions, e.g., pp $\rightarrow \pi X$ but a good model is needed to calculate the distribution function more definitely.

On the other hand, we may consider the actual measurement of the Fulling-Davies-Unruh temperature in an experiment. Indeed, it has been suggested to use an electron in an accelerator ring as a thermometer or detector instead of a linearly accelerated one for which an actual experiment is not feasible (1 K corresponds to a= 2.4x 10^{20} m / sec^2) (Bell and Leinnas 1983). The electron will then be affected in its motion via the linear coupling of its magnetic moment to the magnetic field. The spin of the electron will be flipped by the vacuum fluctuation of the magnetic field. Thus, we have the phenomenon of depolarization of electrons in a magnetic field. It is expected that the rate of spin-flip of the circulating electron is proportional to the power spectrum of a 'noise' and that the population of the spin-up and spin-down levels will approach thermal equilibrium appropriate to the Fulling-Davies-Unruh temperature corresponding to the magnitude of the acceleration. However, this is plausible only in large storage rings where a rotating coordinate system is used in the calculation. Moreover, the depolarization of the electron is found to be complicated by spin-orbit coupling and the fluctuations in the path, rather than the spin polarization of the electrons which are more directly related to the power spectrum (Bell and Leinnas 1987).

The power spectrum of a detector in a rotating frame has been discussed in previous chapters and revealed a difference from that of a Rindler frame. Using the notion of 'circular noise', namely, the noise seen by a detector in a circular motion at a constant speed, we can say that circular noise differs from the Rindler noise in its form and numerically (Letaw and Pfautsch 1980). On the other hand, a notion of the 'effective temperature' of the circular noise has been introduced and it is argued that the effective temperature differs from the value calculated from the Fulling-Davies-Unruh temperature by the factor of $(3)^{-1/2}\pi$ at most (Bell and Leinnas 1983). This notion of effective temperature is objected to by Takagi because of its ambiguity and he tries to relate the two kinds of noises in a precise fashion (Takagi 1986). He finds that the circular noise is related to the 'drifted Rindler noise', i.e., the noise seen by a detector which is uniformly accelerated and at the same time moves at a constant speed in the direction perpendicular to the acceleration, in a high speed circular motion.This drifted motion has also been considered with a different motivation (Letaw 1981; Gerlach 1983a). The question of whether or not the Fulling-Davies-Unruh radiation can be observed experimentally is still open. The numerical calculations are based on simple dimensional arguments and order of magnitude estimates using the Davies and Unruh results which may not be reliable in the actual design of practical experiments.

Chapter 9 Conclusions

The formulation of quantum field theory in non-trivial (curved or flat) space-time has given new fundamental features with respect to the usual understanding of quantum field theory in trivial (Minkowski-flat) space-time. Firstly, the fundamental concepts such as the vacuum or particle states lose their validity and are the source of much confusion. It is possible for a given field theory to have different well-defined vacuum states (non-uniqueness of vacuum) leading to different well-defined Fock spaces. Secondly, the presence of 'intrinsic' statistical features (temperature, entropy) arising from the non-trivial structures (geometry, topology) of the space-time and not from a superimposed statistical description of the quantum matter fields. In particular, these features are seen to be manifested in the formulation of the theory in non-inertial frames and in the stationary coordinate systems in flat space-time. The relevant examples are quantum field theory on the Rindler frame, which yields the well-known Fulling-Davies-Unruh effect and in the rotating frame. Actually, the mathematical structure of non-inertial quantum field theory is identical to that underlying the Hawking effect for a black-hole and even for cosmological (de-Sitter) space-time.

A related problem to the formulation of the theory is the quantum detection process. Keeping in mind the ambiguity of the usual definitions in the canonical quantization of the field in a variety of coordinate systems, it is instructive to relate the results to some operational devices such as a model particle detector. However, the response of the detector cannot be used as a criterion for the existence of a 'particle' due to different physical interpretations and the arbitrary usage of different models of particle detectors coupled to a matter field.

One of the interesting aspects of the Fulling-Davies-Unruh effect is the association of a temperature to the radiation and because of this, the verification of the temperature in actual experiments has been the subject of investigation. This again links the theory to experimental observation and in this interplay experimental data act as a referee to the validity of theoretical prediction. However, a definite conclusion in not forseeable in the near future.

The non-inertial quantum field theory is a first step in describing results from quantum field theory in curved space-time such as the Hawking effect in an analogous way. One can extend the formulation of the theory to a generic space-time by including other non-trivial structures of space-time. In this process, one has to reconsider the applicability of the usual concepts as defined in the trivial case. Moreover, one may have to settle for some field-related quantities (e.g.,the vacuum stress tensor) or other techniques in producing a complete theory which in turn act as a guide to the elusive theory of quantum gravity.

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Appendix 1 Bogoliubov transformation for a scalar field in Minkowski space-time

A scalar field ϕ may be expanded in the complete set of modes as:

$$\phi = \sum_i [\, a_i u_i(x) + a_i^+ u_i^*(x) \,] \qquad (A1.1)$$

We then have the commutation relations

$$[\, a_i, a_j^+ \,] = \delta_{ij} \quad \text{etc.,} \qquad (A1.2)$$

Consider a second complete set of modes $\bar{u}_j(x)$. The field ϕ may be expanded in this set also as:

$$\phi(x) = \sum_i [\, \bar{a}_j \bar{u}_j(x) + \bar{a}_j^+ \bar{u}_j^*(x) \,] \qquad (A1.3)$$

This decomposition of ϕ defines a new vacuum state $|\bar{0}>$:

$$\bar{a}_j |\bar{0}> = 0 \quad , \quad \forall j \qquad (A1.4)$$

and a new Fock space.

As both sets are complete, the new modes $\bar{u}_j$ can be expanded in terms of the old:

$$\bar{u}_j = \sum_i (\, \alpha_{ji} u_i + \beta_{ji} u_i^* \,) \qquad (A1.5)$$

Conversely

$$u_i = \sum_j (\, \alpha_{ji}^* \bar{u}_j - \beta_{ji} \bar{u}_j^* \,) \qquad (A1.6)$$

These relations are known as Bogoliubov transformations. The matrices α_{ij}, β_{ij} are called Bogoliubov coefficients and they can be evaluated as:

$$\alpha_{ij} = <\bar{u}_i \,,\, u_j> \;,\;\; \beta_{ij} = -<\bar{u}_i \,,\, u_j^*>. \qquad (A1.7)$$

with

$$\langle A, B \rangle = i \int [\, A^* (\partial_\mu B) - (\partial_\mu B) A^* \,] \, d\Sigma^\mu \qquad \text{(A1.8)}$$

where $d\Sigma^\mu$ is an appropriate Cauchy surface. Equating the expansion (A1.1) and (A1.3) and making use of (A1.5), (A1.6) and the orthonormally of the modes, one obtains

$$a_i = \sum_j (\alpha_{ji} \bar{a}_j + \beta^*_{ji} \bar{a}^*_j) \qquad \text{(A1.9)}$$

$$\bar{a}_j = \sum_i (\alpha^*_{ji} a_i - \beta^*_{ji} a^*_i) \qquad \text{(A1.10)}$$

The Bogolubov coefficients possess the following properties.

$$\sum_k (\, \alpha_{ik} \alpha^*_{jk} - \beta_{ik} \beta^*_{jk} \,) = \delta_{ij} \qquad \text{(A1.11)}$$

$$\sum_k (\, \alpha_{ik} \beta_{jk} - \beta_{ik} \alpha_{jk} \,) = 0 \qquad \text{(A1.12)}$$

It follows immediately from (A1.9) that the two Fock spaces based on the two choices of modes u_i and $\bar{u}_j$ are different so long as $\beta_{ij} \neq 0$. In fact, the expectation value of the operator $N_i = a_i^* a_i$ for the number of u_i mode particles in the state $|\bar{0}\rangle$ is

$$\langle \bar{0} | N_i | \bar{0} \rangle = \langle \bar{0} | a_i^* a_i | \bar{0} \rangle = \sum_j | \beta_{ji} |^2 \qquad \text{(A1.13)}$$

which is to say the vacuum of the $\bar{u}_j$ modes contains $\sum_j | \beta_{ji} |^2$ particles in the u_i mode.