A Granular Multicriteria Group Decision Making for Renewable Energy Planning Problems

Ye Cui [1], Hanyu E [1,*], Witold Pedrycz [1,2], and Aminah Robinson Fayek [3]

[1] Department of Electrical and Computer Engineering, Faculty of Engineering, University of Alberta, 9211 116 Street NW, Edmonton, Alberta, Canada, T6G 1H9.
[2] Systems Research Institute, Polish Academy of Sciences Warsaw, Warsaw, Poland, PL 01-447.
[3] Department of Civil and Environmental Engineering, Faculty of Engineering, University of Alberta, 11328 89 Ave NW, Edmonton AB, Canada, T6G 2J7.
[*] Corresponding author. Email address: hanyu6@ualberta.ca

Abstract In this study, we introduce and develop a novel decision-making model that provides an original solution to problems of renewable energy site planning. The main idea is to construct a comprehensive and systematic methodology of ranking alternatives encountered in the environment of multicriteria group decision making. The proposed framework is systematically structured with the aid of information granules, in particular, intervals and fuzzy sets. An overall architecture is developed in a comprehensive manner. The inherent facet of uncertainty, it is formalized and processed with the aid of information granules. The two main design phases involve the determination of preference degrees of alternatives with respect to the set of criteria and the weights of the corresponding criteria. The underlying estimation process is realized with the use of the pairwise comparison method (analytical hierarchy process-AHP) resulting in information granules (fuzzy sets) quantifying degrees of preference and relevance of the weights. In light of the group nature of the decision process and diversity of views and opinions conveyed by the individual decision-makers, the results provided by them are aggregated and the diversity (variability) in the individual assessments is captured through information granules of type-2. Finally, a variety of ranking procedures is analyzed and carefully assessed. A case study of selection of solar site is provided to demonstrate the usefulness of the developed approach. Compared with the existing decision-making scenarios, we show that the new model exhibits a significant level of reliability and is characterized by better interpretability.

Keywords: group decision-making, renewable energy, multicriteria problem, ranking information granule, solar site selection.

1. Introduction

Decision-making is a process of determining and ranking alternatives for multiple options (alternatives). For individuals, when being faced with simple and low-dimensional multi-criteria problems, it is relatively straightforward to determine the best solution by engaging human intuition and understanding the underlying problem. But in numerous real-world problems, intuition is usually not sufficient. Thus, how to choose the pros and cons for individual alternatives in a systematic and efficient manner, identifying sound trade-offs and finding optimal solutions has been a challenging task. Usually in decision-making, what we have to consider is not only the individual's evaluation of all aspects of the problem (criteria), at the same time, but also weigh each person's preferences (weights) considered for different criteria. At present, decision-making is widely used in urban planning [1]-[4], company operations [5]-[7] and many other fields [8]-[14]. Sound decision-making plans are aimed at maximization of benefits and reducing unavoidable risks and losses.

Nowadays, with the development of technology and the progress of the society, the public's awareness of environmental protection has gradually increased. How to develop sustainable renewable energy to replace non-renewable fossil energy has become the focus of various countries. With the reduction of fossil energy, it is foreseeable that reliance on renewable energy will inevitably become the key development direction in the future. The commonly considered renewable energy sources include solar energy, wind energy, tidal energy, etc. Meanwhile, it is anticipated that at the decision-making level, we are facing with issues which kind of renewable energy to develop and how to choose a site for the facility development.

The objective of this study is to develop an efficient granular decision-making model for the site selection of renewable energy sources. The AHP algorithm is commonly used in decision-making model while the granular decision-making model needs more attention. The principle of justifiable granularity and the concept of fuzzy sets established by engaging the AHP algorithm generates the granular (interval-valued) results for the alternative location sites. Then the quality of granular results has to be evaluated by engaging a method of ranking fuzzy sets. Compared to the previous studies, the result of our model exhibits high consistency with the results of the existing models. At the same time, due to the concept of information granules introduced to the commonly used AHP, the interpretability of the model has been significantly improved in the sense the credibility of the ranking results is quantified.

The originality in this study involves the following main aspects:

development of alternative numerical scales. In the AHP process, the consistency of a pairwise comparison matrix is essential. We discuss ways on the selection of the optimal numerical scale for the matrix to meet the consistency.

generation of weighted results. The consistency index from AHP algorithm is used to generate weight for each matrix and its corresponding eigenvector, which balances decision aspects to obtain reliable results.

introduction of the concept of information granularity. With the use of the principle of justifiable granularity, the combination of eigenvectors produced by the AHP method is represented in terms of interval-valued fuzzy sets, resulting in the improvement of credibility of the final results.

efficient ranking of fuzzy sets. Since the interval-valued fuzzy sets are generated through the involvement of information granularity leading to information granules of type-2, the commonly used ranking methods are not suitable and are carefully revisited and augmented.

The study is organized in the following way. In Section 2, we elaborate on recent developments in the field of decision-making, especially multi-criteria group decision making used in the realm of problems of renewable energy. Then we present problem formulation in Section 3. Subsequently, in Section 4, we briefly complete a pertinent literature review of some underlying concepts and methods used in this study, especially AHP and ranking information granules. The overall algorithm is discussed in Section5, while the experimental results are shown in Section 6. In the last section, we summarize the main findings and identify future directions.

2. Literature review

To position the formulated problem in a broader context and highlight the multi-criteria and group decision nature of decision problems in the area of renewable energy, in what follows, we complete a concise and focused literature review.

A selection of a suitable location for renewable energy facilities, especially a solar one, gives rise to a challenging decision-making problem, which involves a series of usually conflicting criteria and the decision process is realized by a group of experts (group decision-making). Solar energy only needs sunlight as a source, but its acquisition costs are high and it is directly impacted by the weather [15]-[17]. Wind energy requires open terrain and has a great impact on wildlife, especially birds [18]-[21]. Although tidal energy contains huge energy, how to effectively use and exploit it becomes a huge problem [22]. Apart from the choice of energy type, there are also a lot of issues that need to be considered for a certain type of energy. Take solar energy as an example. It is a widely distributed, low-density, and intermittent energy source in nature, so choosing the location for solar energy development is particularly important. At the same time, the storage, transmission, and subsequent maintenance of electricity should also be considered. In the recent decades, there have been some studies on these issues. Yuan proposed a new multi-criteria decision-making method based on linguistic hesitant fuzzy set (LHFS), This approach helps better express the hesitancy, inconsistency and uncertainty present in decision makers' preferences. The result shows that the preferred renewable energy for Jilin is biomass energy, followed by wind energy, hydro energy and solar energy [23]. A hybrid multi-criteria decision-making model was proposed based on interval type-2 (IT2) fuzzy sets and alpha cut levels for the renewable energy investments. Within this context, hesitant IT2 fuzzy DEMATEL-Based Analytic Network Process (DANP) with alpha cut levels was applied for weighting the customer requirements [24]. Ilbahar proposed multi-attribute decision making and fuzzy set theory to evaluate multiple sustainable energy sources. The main aim of the study was to determine the reasons and factors explaining why geothermal, hydro and secondary waste have been employed [25]. In 2015, Sunil *et al*. proposed a fuzzy Decision-Making Trial and Evaluation Laboratory-based methodology to evaluate the solar power development key enables. Through the analysis of 16 factors in India, it was found that the key was state level and power sector reforms [26]. In terms of the optimal solar site selection in Isfahan-Iran, Mahmood *et al*. combined fuzzy logic, weighted linear combination (WLC) and Multiple Criteria Decision Making (MCDM) Process. It is finally determined that some area in Isfahan, Borkhar etc. have the potential and suitability for the construction of solar power plants see [27]. Similarly, Seda *et al*. used the combination of both pyranometer and Photovoltaic Geographical Information System (PVGIS) and AHP to analyze the ideal location for installing PV power plant, the experiment showed that the best location is Kulluk [28]. Lindberg *et al*. used multi-criteria analysis with a Boolean approach and power flow simulations to do the geographical assessment and the impact analysis on the grid, respectively, of three different sizes of PV parks to find the optimal one for utility-scale solar guides [29]. Davoudabadi *et al*. presented an approach based on data envelopment analysis (DEA) and fuzzy simulation of interval-valued intuitionistic fuzzy sets (IVIFSs) to evaluate renewable energy project. Among multiple candidate energy options, wind energy and fossil energy are determined to be the better options. With further analysis, the advantages of wind energy have gradually expanded and become the most suitable choice [30]. Hirushie *et al*. proposed a framework that use system dynamics for rating renewable energy project deployment scenarios and the fuzzy logic-based optimization algorithm introduced to optimize system capacities and energy mix [31].

In the above problems, the decision-making process is essentially a process of comparison and selection among a limited number of alternatives. Through a comprehensive consideration of various factors and at the same time assigning appropriate weights to each factor, an optimal solution is finally constructed.

3. Problem formulation – an overall of design process

In this study, we consider a decision-making scenario in which there we have p criteria, c decision-makers (DMs) and n alternatives $\{a_1, a_2, \ldots, a_n\}$. The problem is structured as illustrated in Figure

1; the same figure shows also the main phases of processing with emphasis focused on the visualization of criteria and individual decision-makers by forming a criteria-participant matrix $R^{[i,j]}$ and information granule $E^{[i]}$. Furthermore, we highlight the aspect of information granularity which is inherently associated with the diversity of the group nature of the decision process and its quantification in Figure 2. It shows that the evaluation result of the preference of alternatives and criteria by j-th decision maker (DM_j) is type-1 set. However, through the introduction of information granularity, the aggregation of c evaluation results will result in type-2 fuzzy sets. It is worth emphasizing here that the use of type-2 granular constructs is beneficial to capture and quantify and reconcile the diversity of preferences expressed by the individual decision-maker.

Fig. 1. Criteria-decision-makers (DM_j) array; see a detailed description in the text.

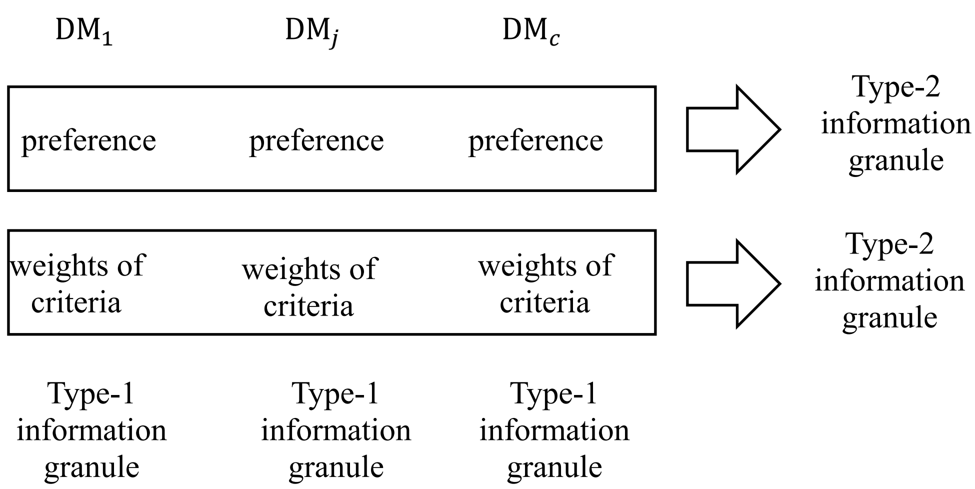


Fig. 2. Elevation of type of information granule being reflective of the diversity of views expressed by individual decision makers: a movement from individual DMs to a group decision-making.

Considering the i-th criterion and the j-th decision maker, the decision process is realized with respect to n alternatives and their weight. Both of them involve a process of pairwise comparison (described later in detail). In a nutshell, an AHP method [32] considered here is used and n alternatives are compared pairwise resulting in an n x n dimensional matrix of pairwise comparisons $R^{[i,j]} = [r^{[i,j]}]$. The same process of pairwise comparison is completed for the weights of the criteria resulting in the p x p matrix in the form $W^{[i,j]} = [w^{[i,j]}]$. Once the matrices have been constructed experimentally, the n-dimensional vector of preferences of the alternatives associated with $R^{[i,j]}$ is formed as $\boldsymbol{e}^{[i,j]}$. The entries of the vector are regarded as degrees of preference assuming values in the [0,1] interval, thus $\boldsymbol{e}^{[i,j]}$ is a discrete fuzzy set defined over a set of alternatives. The method of pairwise comparison also quantifies a level of consistency of the pairwise comparison which is described by the corresponding inconsistency index $\lambda^{[i,j]}$. The same process is carried out for the weights, so for the matrix $W^{[i,j]}$ of dimensionality p by p resulting in weight vector $\boldsymbol{w}^{[i,j]}$ of dimensionality p and the corresponding inconsistency index $\theta^{[i,j]}$. By analyzing a certain row, see Figure 1, it is noticeable that the alternatives evaluated in light of the same criterion by different decision-makers produce a family of fuzzy sets with different membership functions. To aggregate these results and develop a global view conveyed by the group, the individually obtained fuzzy sets have to be aggregated to arrive at the global view. With the intent of describing the diversity of view, the result is formed as an information granule of type-2, namely $E^{[i]}$ (note that $\boldsymbol{e}^{[i,j]}$s are type-1 information granules). This aggregation is obtained with the use of the principle of justifiable granularity [33]; the principle realizes the process of elevation of type of aggregated information granules. In brief, the transformation is expressed as $E^{[i]} = G(E^{[i,1],} E^{[i,2]},.. E^{[i,c]})$ where G stands for the process of granulation guided by the principle of justifiable granularity. As a result, $E^{[i]}$ is a type-2 information granule defined over the space of n alternatives, $E^{[i]} = [E^{[i]}{}_1, E^{[i]}{}_2 ... E^{[i]}{}_n]$. $E^{[i]}{}_j$ denotes a degree of preference (information granule defined over [0,1]) of the j-th alternative expressed with respect to the i-th criterion. In the construction of $E^{[i]}$, the principle involves also the values of the inconsistency index associated with the corresponding fuzzy sets. Likewise we proceed with the fuzzy sets of the weights of the criteria obtained by processing $W^{[i,j]}$, which results in the fuzzy sets of weights of the criteria $\boldsymbol{w}^{[i,j]}$. As in case of preference levels of alternatives, the aggregation of the results delivered by the individual decision-makers is carried out with the aid of the principle of justifiable granularity resulting in the weight expressed as information granule (e.g., a fuzzy set) of type-2, so $W^{[i]}$ is an information granule defined over the space of decision-makers. Finally, $E^{[i]}$ and $W^{[i]}$ are aggregated by taking the following weighted average

$$E = \sum_{i=1 \oplus}^{p} (W^{[i]} \otimes E^{[i]}) \qquad (1)$$

In the above expression, the symbols shown in circles underline that the operations are completed on information granules rather than numeric entities. In sum, E is an information granule of type-2 defined over the space of n alternatives. In other words, the i-th coordinate of E, E_i captures an overall level of preference of the i-th alternative where the level is expressed as an information granule of type-1 defined over the unit interval. Finally, these information granules of preference E_1, E_2, ..., E_n are ranked with the use of ranking methods so that a linear order is established, the expression $E_i < E_j$ denotes that E_j is preferred over E_i.

4. Prerequisites

In this section, we briefly recall the key ideas supporting the development of the model.

4.1 The analytic hierarchy process (*AHP*).

The basic idea of the Analytic Hierarchy Process [32], [34]-[35] refers to a decision-making method that helps determine preference degrees among a set of alternatives by carrying out a series of pairwise comparisons of preferences of two alternatives at a time. The experimentally determined entries of the pairwise comparison matrix satisfies the two essential properties, namely reflexivity and reciprocity:

-*reflexivity*: the elements on the main diagonal are equal to 1 (they represent situations where a preference of alternative is expressed vis-à-vis itself)

-*reciprocity*: the elements located symmetrically with respect to the main diagonal are inverse of each other.

The entries of the matrix are quantified by using a certain scale; usually those values are selected from the set $\{1, 2....9\}$. If $alternative_1$ is strongly preferred over $alternative_2$ then the corresponding entry of the matrix is selected from the upper end of the scale, say 9 or 8. If some moderate preference is considered, the entry assumes values in the range 6-7, etc. At the same time the preference of $alternative_2$ over $alternative_1$ is taken as a reciprocal value of the estimate already completed, say 1/9.

Having the matrix completed, one determines the maximum eigenvalue (ξ) of the matrix and the corresponding eigenvector. The eigenvector (after normalization) is the estimate of the fuzzy set describing degrees of preference assigned to the individual alternatives. The method comes with a flagging mechanism: the maximum eigenvalue quantifies the consistency in terms of the inconsistency index coming in the following form

$$\lambda = (\xi - n)/(n - 1) \tag{2}$$

where n is the number of alternatives. If the inconsistency index λ is greater than some threshold value (say, 0.1), the result lacks consistency and the acquisition of the pairwise matrix has to be repeated.

The method can be augmented by a robustness analysis, which helps express a level of robustness of the results when some changes of the pairwise comparison matrix are encountered. In doing this, we analyze on how the possible perturbations (due to subjective way of determining entries from the scale). To quantify the robustness, the following process is considered:

given some reciprocal matrix R, we perturb their entries in additive way by adding random integers coming from the uniform distribution [-ε, ε] to the integer entries of R. The modified entries of R are clipped to the range of the assumed scale using which R has been estimated. Note that once some entry has been modified, the entry positioned symmetrically with respect to the diagonal also need to be changed. The modified R, denoted as R_ε, produces some fuzzy set of preferences $\boldsymbol{e}_\varepsilon$. It is compared with the original fuzzy set $\boldsymbol{e}$ produced by R. Similarly, one compares the original ranking provided by $\boldsymbol{e}$ to the one produced by $\boldsymbol{e}_\varepsilon$. For given ε, the experiment is repeated a number of times, say 1,000 and the resulting Euclidean distance between $\boldsymbol{e}$ and $\boldsymbol{e}_\varepsilon$ is computed as well as the Hamming distance between the rankings. These two distances obtained for selected values of ε quantify the robustness of the AHP method.

As an example, we consider 5 by 5 matrix R with some entries. We run the calculations 1,000 times with several values of ε, namely 1, 2, 3, and 4. The histograms of inconsistency values and distances between $\boldsymbol{e}$ and $\boldsymbol{e}_\varepsilon$ for different values of ε are shown in Figure 3. Especially when ε is 1 or 2, most values of the inconsistency index are less than or around 0.1 and the corresponding distances are 0.

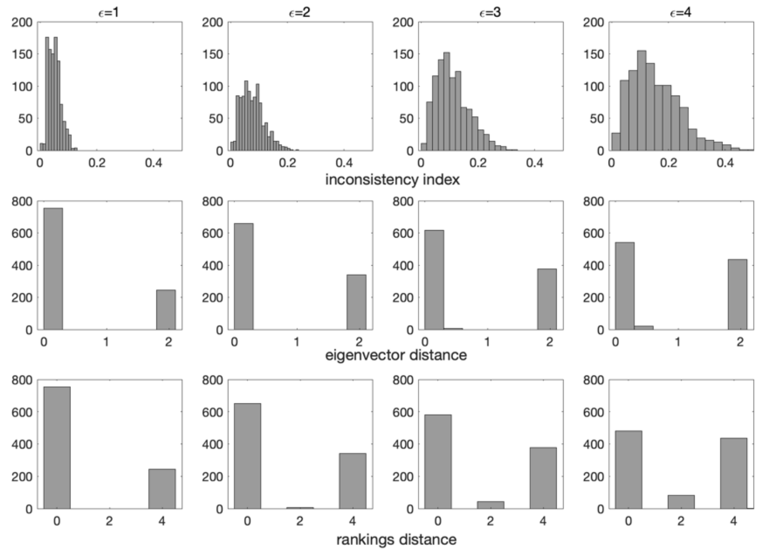


Fig. 3. Difference of results (consistency, distance between fuzzy sets and rankings) between original and modified *R* as a function of ε.

4.2 Arithmetic of information granules

Given two information granules *A* and *B* defined in [0,1], we determine their product and sum. We consider both intervals, namely *A* = [*a, b*] and *B*= [*c, d*] and fuzzy set with triangular membership functions. The information granules are defined in the [0,1] interval; *m* and *n* are the modal values of *A* and *B* while *a* and *c* are the lower bounds and *b* and *d* are upper bounds.

Let us proceed with the addition of *A* and *B*, *A*⨁*B*:

-intervals. We obtain the following result

A⨁*B*= [*a*+*c*, *b*+*d*]

-triangular fuzzy sets

To calculate the resulting fuzzy set *C*, we use the extension principle [36]. Given are the triangular fuzzy sets *A* and *B* as follows

$$A(x) = \begin{cases} (x-a)/(m-a)\,, \text{if } x \in [a,m] \\ (b-x)/(b-m)\,, \text{if } x \in [m,b] \\ 0, \qquad \text{otherwise} \end{cases};\quad B(y) = \begin{cases} (y-c)/(n-c)\,, \text{if } y \in [c,n] \\ (d-y)/(d-n)\,, \text{if } y \in [n,d] \\ 0, \qquad \text{otherwise} \end{cases} \quad (3)$$

The sum of them is $C(z) = A(x) \oplus B(y)$, where modal value ($C(z) = 1$) is attained at $z = m + n$.

If $x < m$ and $y < n$, one has $z = x + y$ and we involve the increasing parts of fuzzy sets *A* and *B*. $z = x + y = (m-a)\alpha + a + (n-c)\alpha + c = (a+c) + (m+n-(a+c))\alpha$, thus the membership function is

$$C(z) = \alpha = (z-(a+c))/((m+n)-(a+c)) \quad (4)$$

If $x > m$ and $y > n$, $z = x + y = b - (b-m)\alpha + d - (d-n)\alpha = (b+d) - (b+d-(m+n))\alpha$, the membership function of the decreasing part of the resulting fuzzy set is

$$C(z) = \alpha = ((b+d) - z)/((b+d) - (m+n)) \quad (5)$$

Multiplication of A and B, $A \otimes B$

-intervals. The resulting interval is

$A \otimes B$= [min(ac, ad, bc, bd), max(ac, ad, bc, bd)]

-triangular fuzzy sets

Similarly, we have the triangular fuzzy sets A and B as mentioned above, the product of them is $D(z) = A(x) \otimes B(y)$, where modal value is obtained at $z = mn$.

If $x < m$ and $y < n$, one has $z = xy = [(m-a)\alpha + a][(n-c)\alpha + c] = ac + [a(n-c) + c(m-a)]\alpha + (m-a)(n-c)\alpha^2 = f(\alpha)$, thus the membership function of the increasing part of D is

$$\begin{aligned} D(z) &= \frac{-[a(n-c) + c(m-a)] + \sqrt{[a(n-c) + c(m-a)]^2 - 4(m-a)(n-c)(ac-z)}}{2(m-a)(n-c)} \\ &= \frac{-[a(n-c) + c(m-a)] + \sqrt{[a(n-c) - c(m-a)]^2 + 4(m-a)(n-c)z}}{2(m-a)(n-c)} \end{aligned} \quad (6)$$

If $x > m$ and $y > n$, $z = xy = [b - (b-m)\alpha][d - (d-n)\alpha] = bd - [b(d-n) + d(b-m)]\alpha + (b-m)(d-n)\alpha^2 = g(\alpha)$, the membership function of the decreasing part is

$$\begin{aligned} D(z) &= \frac{[b(d-n) + d(b-m)] - \sqrt{[b(d-n) + d(b-m)]^2 - 4(b-m)(d-n)(bd-z)}}{2(b-m)(d-n)} \\ &= \frac{[b(d-n) + d(b-m)] - \sqrt{[b(d-n) - d(b-m)]^2 + 4(b-m)(d-n)z}}{2(b-m)(d-n)} \end{aligned} \quad (7)$$

As seen from (6) and (7), the membership function of D is not a linear function. An illustrative example for the sum and product is shown in Figure 4; C is the addition of fuzzy sets A and B, and D is the product of these two fuzzy sets. It is evident that the sum result of triangular fuzzy sets is still triangular however the product is not.

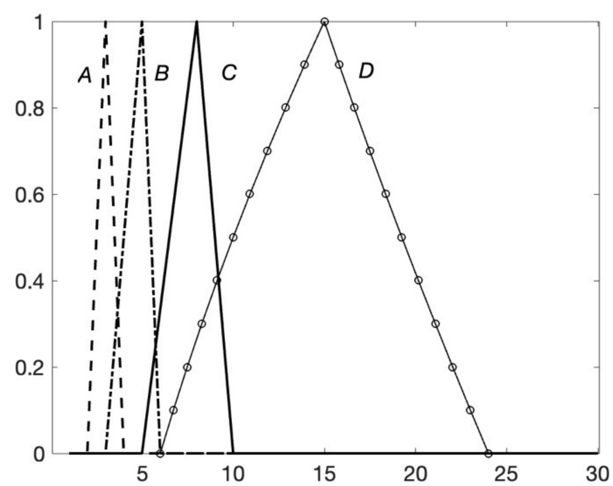


Fig. 4. Addition and multiplication of triangular fuzzy numbers.

4.3 Ranking information granules

Ranking alternatives described by information granules is far more challenging than ranking numbers (which reduces to a simple comparison of numeric values and ordering them in an increasing order). Let us consider information granules A_1, A_2, ..., A_n (intervals or fuzzy sets) defined in the [0,1] interval. In what follows, we briefly describe some selected methods by highlighting the underlying rationale behind the ranking technique.

Method #1: The method was proposed in [37]. Assume we have fuzzy sets A_1, A_2, ..., A_n needed to be ranked. $A_{max}(x)$ is the maximizing set based on the largest value in the support of all subsets as below.

$$A_{max}(x) = \begin{cases} x/5, x \in [0,5] \\ 0, \text{ otherwise} \end{cases} \tag{8}$$

Then we rank the fuzzy sets by the intersection point between them and $A_{max}(x)$. The larger the value of the intersection point, the higher the preference of the corresponding fuzzy set is.

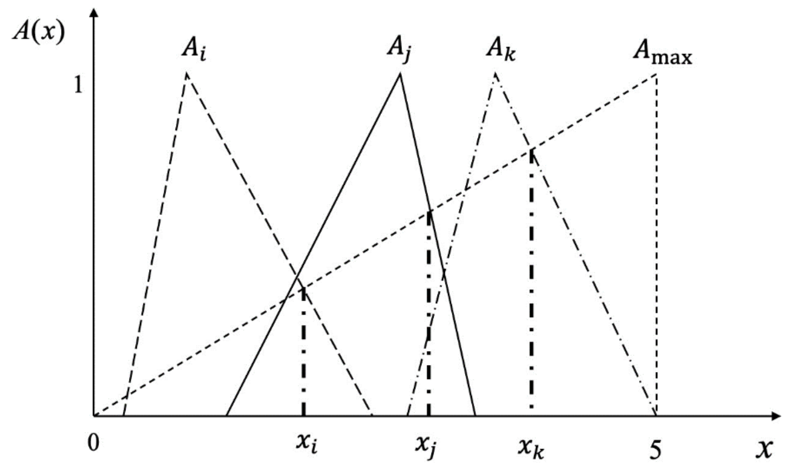


Fig. 5. Example of ranking.

Method #2: This is a center of gravity method [38]. The fuzzy set A_i is represented by the center of gravity of the membership function, $CoG(A_i)$ computed as

$$CoG(A_i) = \int_R xA_i(x)dx \Big/ \int_R A_i(x)dx \tag{9}$$

The larger the value of $CoG(A_i)$, the higher preference of A_i.

Method #3: The relative anteriority index in [39] is expressed in the form

$$I(A_i, A_j) = \begin{cases} K(A_i)/(K(A_i) + K(A_j)), & if\ K(A_i) + K(A_j) > 0 \\ 1, & if\ K(A_i) + K(A_j) = 0. \end{cases} \tag{10}$$

where $K(A_i)$ is computed out by comparing the Hamming distances D_H between fuzzy sets and the maximum (union) of them.

$$K(A_i) = D_H(A_i, \max(A_1, A_2, .., A_n)) = \int |A_i(x) - \max(A_1, A_2, .., A_n)(x)| dx \tag{11}$$

Given the anteriority index, we compare the two fuzzy numbers A_i, $A_j, i \neq j$, producing the corresponding preference relationship as follows:

$$\begin{cases} A_i \geq A_j, if\ 0 \leq I(A_i, A_j) \leq 0.5\ or\ 0.5 \leq I(A_j, A_i) \leq 1. \\ A_i \leq A_j,\ \ 0.5 \leq I(A_i, A_j) \leq 1\ or\ 0 \leq I(A_j, A_i) \leq 0.5. \end{cases} \tag{12}$$

Method #4: R. In this approach [40], the ranking is based on the magnitude of fuzzy sets defined as

$$Mag(A_i) = \left(\int_0^1 (A_i{}^L(\alpha) + A_i{}^R(\alpha) + A_i{}^L(1) + A_i{}^R(1)) f(\alpha) d\alpha\right)/2 \tag{13}$$

Where f is a nonnegative weight function and increasing on [0,1], α is the α-cut value, $\alpha \in [0,1]$. $A_i{}^L(\alpha)$, $A_i{}^R(\alpha)$ are the bounded left-continuous non-decreasing function and right-continuous non-increasing function, respectively, over [0,1].

The larger the value of Mag, the higher the preference of the fuzzy number becomes.

Method #5: Median of fuzzy numbers [41] is computed in the following form

$$\int_{-\infty}^{Med} A_i(x)dx = \int_{Med}^{\infty} A_i(x)dx \tag{14}$$

The larger Med is, the larger the preference of the fuzzy number is.

4.4 The principle of justifiable granularity

The principle of justifiable granularity [42] is one of the methods to construct an information granule on a basis of some experimental evidence. In the setting of this study (as we are provided with fuzzy sets), for x_i, using this principle aggregates the numeric membership grades and forms a single information granule of higher type, namely an interval or a fuzzy set. Recall that fuzzy sets are information granules of type-1, viz. their degrees of membership are numeric. The result of aggregation of several information granules is an information granule of type-2, say an interval-valued fuzzy set (where the degrees of membership are intervals) or more generally a fuzzy set whose membership grades are fuzzy sets themselves. An illustration of the resulting information granules is presented in Figure 6.

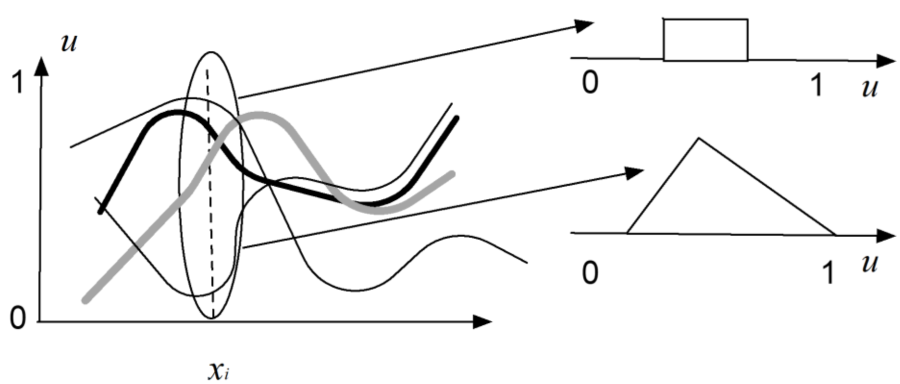


Fig. 6. Example of resulting information granules (interval-valued or triangular fuzzy set).

To proceed with the detailed algorithm, let us assume that for some alternatives, the degrees of membership form a set $Z = \{z_1, z_2, \dots, z_c\}$ (recall that we have c decision-makers). They give rise to an information granule defined in the [0,1] interval being a result of maximization of the product of coverage and specificity. If we consider an interval information granule $A = [a, b]$, the coverage (*cov*) and specificity (*sp*) are defined as follows

$$cov(A) = card(z_k | z_k \in [a, b])/c$$
$$sp(A) = 1 - (|b - a| / |\max(z_k) - \min(z_k)|) \tag{15}$$

where $k = 1,2, \dots, c$.

The convenient construction of the information granule is realized as a two-step procedure:

(i)specification of a numeric representative of Z; say by taking the average or modal value, denote the result as z^0.

(ii)maximizing the product of coverage and specificity by choosing the upper bound of the interval b so that this product has been maximized. Maximize the product of coverage and specificity by selecting an optimal value of the lower bound a.

For a triangular fuzzy set A, the calculations of coverage and specificity are modified because of the use of the membership degrees rather than 0-1 values only. The detailed calculations are completed as follows

$$cov(A) = \sum_{\substack{k=1; \\ z_k : a < z_k < b}}^{c} A(z_k)$$
$$sp(A) = \int_0^1 1 - (|b_\alpha - a_\alpha| / |\max(z_k) - \min(z_k)|) d\alpha \tag{16}$$

5. An overall method

In this section, we elaborate on overall decision-making process. It consists of five main phases.

Step 1: Evaluate the alternatives based on the AHP algorithm, assuming a certain scale, say 1-9, the reciprocal matrices are elicited producing $R^{[i,j]}$, where $i = 1,2, \dots, p;\ j = 1,2, \dots, c$.

Step 2: Consider the *i*-th criterion. For each $R^{[i,j]}$, we solve the eigenvalue problem,

$$R^{[i,j]} \boldsymbol{e}^{[i,j]} = \xi^{[i,j]} \boldsymbol{e}^{[i,j]} \tag{17}$$

and determine $\boldsymbol{e}^{[i,j]}$ which is the eigenvector corresponding to the largest eigenvalue $\xi^{[i,j]}$. The entries of $\boldsymbol{e}^{[i,j]}$ stand for the degrees of preference of the alternatives, where the values of these degrees are in the [0,1] interval. The consistency is quantified by computing $\lambda^{[i,j]}$ using (2). The result is used to compute the weight (*weight*) of the corresponding $\boldsymbol{e}^{[i,j]}$

$$weight(\boldsymbol{e}^{[i,j]}) = 1 - ((\lambda^{[i,j]} - \lambda_{\min}) / (\lambda_{\max} - \lambda_{\min})), j = 1,2, \dots, c \tag{18}$$

where $\lambda_{\min}, \lambda_{\max}$ are the minimal and maximal value of $\lambda^{[i,j]}$, $j = 1,2, \dots, c$ for the *i*-th criterion.

Step 3: Again considering the i-th criterion, the principle of justifiable granularity is used to determine the interval-valued fuzzy set $E^{[i]}$.

Analyze the eigenvector $\boldsymbol{e}^{[i,j]}, j = 1,2, \dots, c$ with its weight. That is, for each alternative a_s, the corresponding values in $\boldsymbol{e}^{[i,j]}$ compose one-dimensional weighted data. Based on these data, we invoke the principle of justifiable granularity to generate an optimal interval information granule to represent $\boldsymbol{e}^{[i,j]}$, that is, to find the optimal lower bound (a) and upper bound (b) with the equation (15). Since what we input to the information granularity is weighted data, form (15) is replaced by

$$cov = \sum_{\substack{j=1; \\ \boldsymbol{e}^{[i,j]}: a < \boldsymbol{e}^{[i,j]} < b}}^{c} weight(\boldsymbol{e}^{[i,j]})/c$$
$$sp = 1 - \left(|b - a| / \left| \max\left(\boldsymbol{e}^{[i,j]}\right) - \min\left(\boldsymbol{e}^{[i,j]}\right) \right| \right) \tag{19}$$

In the same way, using the principle of justifiable granularity, one can construct $\tilde{E}^{[i]}$ in the form of triangular fuzzy set.

Step 4: The AHP method is used with regard to pairwise matrices of weights of criteria $W^{[i,j]}, j = 1,2, \dots, c$.

$$W^{[i,j]} \boldsymbol{w}^{[i,j]} = \eta^{[i,j]} \boldsymbol{w}^{[i,j]} \tag{20}$$

where $\eta^{[i,j]}$ is the eigenvalue and $\boldsymbol{w}^{[i,j]}$ is the eigenvector. Then for matrix $W^{[i,j]}$, we generate importance vectors of criteria $\boldsymbol{w}^{[i,j]}$ and these weight vectors are aggregated with the principle of justifiable granularity producing interval-valued fuzzy set $W^{[i]}$ or triangular fuzzy set $\widetilde{W}^{[i]}$ for i-th criterion (the same as before, determine each lower and upper bound by maximizing the product of coverage and specificity).

Step 5: Finally, the vector of preferences of alternatives is constructed by combining interval-valued fuzzy sets $E^{[i]}$ with weights $W^{[i]}$ as described by (1). In addition, for triangular fuzzy sets, the weighted sum is computed by the transformation format of (1) as follows, then we obtain the result $\tilde{E}$ as below.

$$\tilde{E} = \sum_{i=1 \oplus}^{p} (\widetilde{W}^{[i]} \otimes \tilde{E}^{[i]}) \tag{21}$$

Then the results are determined by ranking methods to find the optimal alternative. In addition, to evaluate the reliability of the results, a process described in Section 4.

6. Experimental study

In this section, we use the developed methodology to the decision problem. The objective is to determine a set-up of a solar PV power plant by considering five criteria, namely

Potential energy production,
Environment factors,
Safety,
Distance from existing transmission line, and
Topographical properties.

Three alternative locations are considered. Following the presented approach, the alternative locations and the weights of criteria are estimated by decision-makers by invoking the AHP method. The elicited matrices of pairwise comparison are shown in Appendix. Twelve decision-makers are involved in the evaluation process.

For $R^{[i,j]}$s, the corresponding eigenvectors $\boldsymbol{e}^{[i,j]}$ along with the values of the inconsistency index λ are reported in Table 1. According to Saaty's criteria [35], the value of λ less than 0.1 implies that the results are consistent. As seen in Table 1, the values of λ are different. The smaller value of λ stands for higher consistency of matrix, which means that the corresponding decision-maker has superb logic. Especially considering criterion 4 (distance from existing transmission line), most values of λ are relatively small. In other words, most decision-makers have the same perception about the three locations in terms of this criterion and the resulted pairwise comparison matrices (decision results) are consistent.

Table 1. Eigenvectors and values of inconsistency index of pairwise comparison matrices.

	Criterion #1	Criterion #2	Criterion #3	Criterion #4	Criterion #5
DM 1	λ: 0.091; $\boldsymbol{e}^{[1,1]}$: [0.07 0.98 0.18]	λ: 0.099; $\boldsymbol{e}^{[2,1]}$: [0.25 0.97 0.08]	λ: 0.001; $\boldsymbol{e}^{[3,1]}$: [0.72 0.68 0.11]	λ: 0.062; $\boldsymbol{e}^{[4,1]}$: [0.33 0.09 0.94]	λ: 0.072; $\boldsymbol{e}^{[5,1]}$: [0.07 0.94 0.34]
DM 2	λ: 0.003; $\boldsymbol{e}^{[1,2]}$: [0.44 0.86 0.27]	λ: 0.016; $\boldsymbol{e}^{[2,2]}$: [0.31 0.92 0.22]	λ: 0.017; $\boldsymbol{e}^{[3,2]}$: [0.25 0.93 0.26]	λ: 0.004; $\boldsymbol{e}^{[4,2]}$: [0.34 0.93 0.11]	λ: 0.025; $\boldsymbol{e}^{[5,2]}$: [0.45 0.89 0.08]
DM 3	λ: 0.029; $\boldsymbol{e}^{[1,3]}$: [0.33 0.53 0.78]	λ: 0.011; $\boldsymbol{e}^{[2,3]}$: [0.91 0.34 0.23]	λ: 0.023; $\boldsymbol{e}^{[3,3]}$: [0.89 0.41 0.20]	λ: 0.007; $\boldsymbol{e}^{[4,3]}$: [0.94 0.26 0.24]	λ: 0.002; $\boldsymbol{e}^{[5,3]}$: [0.89 0.36 0.27]
DM 4	λ: 0.005; $\boldsymbol{e}^{[1,4]}$: [0.44 0.60 0.67]	λ: 0.016; $\boldsymbol{e}^{[2,4]}$: [0.54 0.72 0.43]	λ: 0.071; $\boldsymbol{e}^{[3,4]}$: [0.54 0.66 0.52]	λ: 0.006; $\boldsymbol{e}^{[4,4]}$: [0.66 0.64 0.40]	λ: 0.001; $\boldsymbol{e}^{[5,4]}$: [0.60 0.69 0.41]
DM 5	λ: 0.047; $\boldsymbol{e}^{[1,5]}$: [0.56 0.82 0.15]	λ: 0.016; $\boldsymbol{e}^{[2,5]}$: [0.37 0.92 0.11]	λ: 0.062; $\boldsymbol{e}^{[3,5]}$: [0.09 0.94 0.33]	λ: 0.005; $\boldsymbol{e}^{[4,5]}$: [0.14 0.95 0.26]	λ: 0.091; $\boldsymbol{e}^{[5,5]}$: [0.29 0.95 0.09]
DM 6	λ: 0.005; $\boldsymbol{e}^{[1,6]}$: [0.90 0.41 0.12]	λ: 0.005; $\boldsymbol{e}^{[2,6]}$: [0.97 0.22 0.12]	λ: 0.015; $\boldsymbol{e}^{[3,6]}$: [0.97 0.23 0.09]	λ: 0.040; $\boldsymbol{e}^{[4,6]}$: [0.98 0.08 0.19]	λ: 0.001; $\boldsymbol{e}^{[5,6]}$: [0.90 0.43 0.13]
DM 7	λ: 0.002; $\boldsymbol{e}^{[1,7]}$: [0.62 0.77 0.15]	λ: 0.076; $\boldsymbol{e}^{[2,7]}$: [0.58 0.79 0.21]	λ: 0.000; $\boldsymbol{e}^{[3,7]}$: [0.34 0.92 0.19]	λ: 0.000; $\boldsymbol{e}^{[4,7]}$: [0.60 0.78 0.15]	λ: 0.000; $\boldsymbol{e}^{[5,7]}$: [0.58 0.80 0.16]
DM 8	θ: 0.009; $\boldsymbol{e}^{[1,8]}$: [0.12 0.54 0.83]	λ: 0.000; $\boldsymbol{e}^{[2,8]}$: [0.36 0.35 0.86]	λ: 0.091; $\boldsymbol{e}^{[3,8]}$: [0.36 0.93 0.07]	λ: 0.009; $\boldsymbol{e}^{[4,8]}$: [0.08 0.65 0.76]	λ: 0.037; $\boldsymbol{e}^{[5,8]}$: [0.19 0.49 0.85]
DM 9	λ: 0.093; $\boldsymbol{e}^{[1,9]}$: [0.30 0.58 0.76]	λ: 0.012; $\boldsymbol{e}^{[2,9]}$: [0.19 0.17 0.97]	λ: 0.043; $\boldsymbol{e}^{[3,9]}$: [0.12 0.82 0.55]	λ: 0.002; $\boldsymbol{e}^{[4,9]}$: [0.93 0.17 0.33]	λ: 0.027; $\boldsymbol{e}^{[5,9]}$: [0.30 0.95 0.13]
DM 10	λ: 0.005; $\boldsymbol{e}^{[1,10]}$: [0.16 0.51 0.85]	λ: 0.052; $\boldsymbol{e}^{[2,10]}$: [0.86 0.40 0.31]	λ: 0.009; $\boldsymbol{e}^{[3,10]}$: [0.20 0.35 0.92]	λ: 0.005; $\boldsymbol{e}^{[4,10]}$: [0.95 0.14 0.26]	λ: 0.003; $\boldsymbol{e}^{[5,10]}$: [0.93 0.33 0.14]
DM 11	λ: 0.052; $\boldsymbol{e}^{[1,11]}$: [0.19 0.18 0.97]	λ: 0.040; $\boldsymbol{e}^{[2,11]}$: [0.40 0.91 0.08]	λ: 0.000; $\boldsymbol{e}^{[3,11]}$: [0.51 0.82 0.26]	λ: 0.003; $\boldsymbol{e}^{[4,11]}$: [0.56 0.81 0.17]	λ: 0.000; $\boldsymbol{e}^{[5,11]}$: [0.11 0.56 0.82]
DM 12	λ: 0.006; $\boldsymbol{e}^{[1,12]}$: [0.12 0.55 0.82]	λ: 0.009; $\boldsymbol{e}^{[2,12]}$: [0.38 0.33 0.87]	λ: 0.052; $\boldsymbol{e}^{[3,12]}$: [0.42 0.91 0.08]	λ: 0.005; $\boldsymbol{e}^{[4,12]}$: [0.14 0.88 0.46]	λ: 0.009; $\boldsymbol{e}^{[5,12]}$: [0.19 0.49 0.85]

Vectors $\boldsymbol{e}^{[i,j]}$ are displayed in the form of the bar chart; refer to Figure 7.

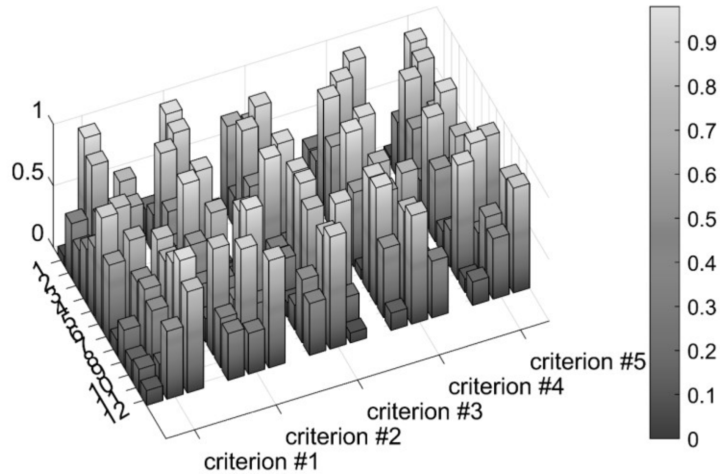


Fig. 7. Visualization of $\boldsymbol{e}^{[i;j]}$.

The principle of justifiable granularity generates the corresponding summarization expressed as interval-valued fuzzy sets $E^{[1]}, E^{[2]}, \dots, E^{[5]}$ or triangular fuzzy sets $\tilde{E}^{[1]}, \tilde{E}^{[2]}, \dots, \tilde{E}^{[5]}$. For the three alternatives under discussion, one obtains the results shown below

$$E^{[1]} = [(0.12,0.44)\ (0.41,0.60)\ (0.55,0.85)]$$
$$E^{[2]} = [(0.31,0.54)\ (0.17,0.52)\ (0.08,0.44)]$$
$$E^{[3]} = [(0.20,0.54)\ (0.66,0.94)\ (0.08,0.30)]$$
$$E^{[4]} = [(0.55,0.98)\ (0.60,0.95)\ (0.11,0.40)]$$
$$E^{[5]} = [(0.53,0.93)\ (0.33,0.59)\ (0.08,0.41)]$$

Likewise, the triangular fuzzy numbers are determined yielding the following membership functions

$$\tilde{E}^{[1]} = [T(0.11,0.12,0.72), T(0.51,0.51,0.98), T(0.57,0.85,0.85)]$$
$$\tilde{E}^{[2]} = [T(0.31,0.31,0.66), T(0.17,0.35,0.44), T(0.07,0.12,0.53)]$$
$$\tilde{E}^{[3]} = [T(0.09,0.15,0.64), T(0.59,0.93,0.94), T(0.07,0.10,0.61)]$$
$$\tilde{E}^{[4]} = [T(0.30,0.95,0.98), T(0.41,0.93,0.93), T(0.15,0.15,0.61)]$$
$$\tilde{E}^{[5]} = [T(0.07,0.11,0.74), T(0.36,0.36,0.76), T(0.11,0.13,0.48)]$$

Proceeding with the evaluation of weights (criteria), we arrive at matrices shown in Appendix. From twelve pairwise comparison matrices, we conclude that the 5th criterion (topographical properties) is considered as the most essential factor identified by the majority of decision-makers.

Subsequently, the corresponding results are displayed in Table 2; refer also to the bar plot, Figure 8.

Table 2. Maximal eigenvectors and inconsistency index of pairwise comparison matrix *F*.

	Criterion		Criterion
DM 1	θ: 0.036 $\boldsymbol{w}^{[1]}$: [0.11 0.06 0.51 0.23 0.82]	DM 7	θ: 0.038 $\boldsymbol{w}^{[7]}$: [0.09 0.06 0.87 0.20 0.43]
DM 2	θ: 0.086 $\boldsymbol{w}^{[2]}$: [0.15 0.07 0.32 0.34 0.87]	DM 8	θ: 0.095 $\boldsymbol{w}^{[8]}$: [0.12 0.05 0.81 0.53 0.20]
DM 3	θ: 0.065	DM 9	θ: 0.067

	$\boldsymbol{w}^{[3]}$: [0.21 0.63 0.25 0.06 0.71]		$\boldsymbol{w}^{[9]}$: [0.19 0.25 0.46 0.83 0.05]
DM 4	θ: 0.063 $\boldsymbol{w}^{[4]}$: [0.12 0.84 0.50 0.16 0.06]	DM 10	θ: 0.066 $\boldsymbol{w}^{[10]}$: [0.23 0.15 0.10 0.95 0.12]
DM 5	θ: 0.069 $\boldsymbol{w}^{[5]}$: [0.06 0.44 0.33 0.11 0.86]	DM 11	θ: 0.095 $\boldsymbol{w}^{[11]}$: [0.20 0.64 0.35 0.10 0.64]
DM 6	θ: 0.018 $\boldsymbol{w}^{[6]}$: [0.09 0.08 0.50 0.30 0.81]	DM 12	θ: 0.072 $\boldsymbol{w}^{[12]}$: [0.33 0.21 0.23 0.16 0.88]

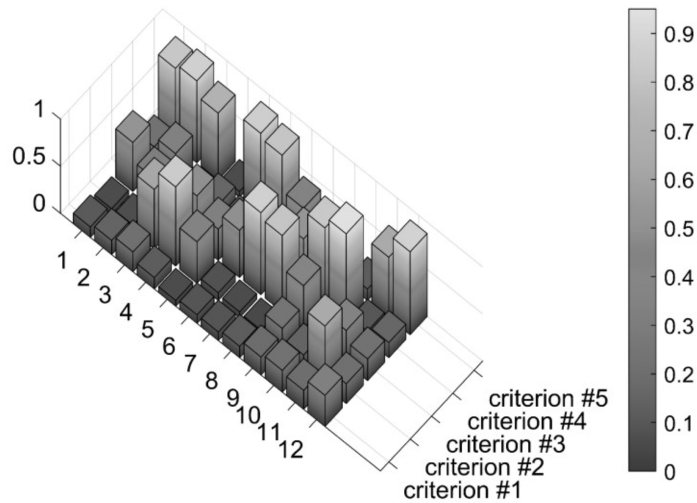


Fig. 8. Pairwise comparison matrices.

Next, by applying the principle of justifiable granularity, we obtain the interval-valued fuzzy set W, say

$$W = \left[W^{[1]}, W^{[2]}, W^{[3]}, W^{[4]}, W^{[5]}\right]$$
$$= [(0.09,0.14)\ (0.06,0.25)\ (0.46,0.51)\ (0.06,0.31)\ (0.58, 0.88)]$$

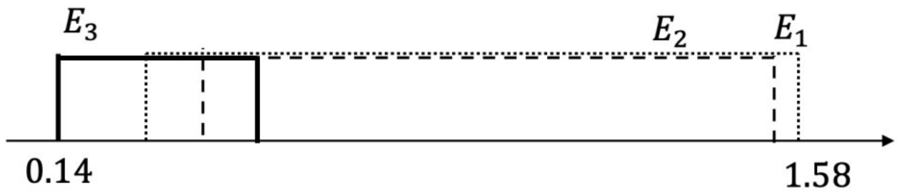
and the triangular fuzzy set $\widetilde{W}$ with the entries

$$\widetilde{W} = \left[\widetilde{W}^{[1]}, \widetilde{W}^{[2]}, \widetilde{W}^{[3]}, \widetilde{W}^{[4]}, \widetilde{W}^{[5]}\right]$$
$$= [T(0.09,0.09,0.25), T(0.05,0.06,0.34), T(0.10,0.50,0.51), T(0.06,0.30,0.35), T(0.64,0.81,0.88)]$$

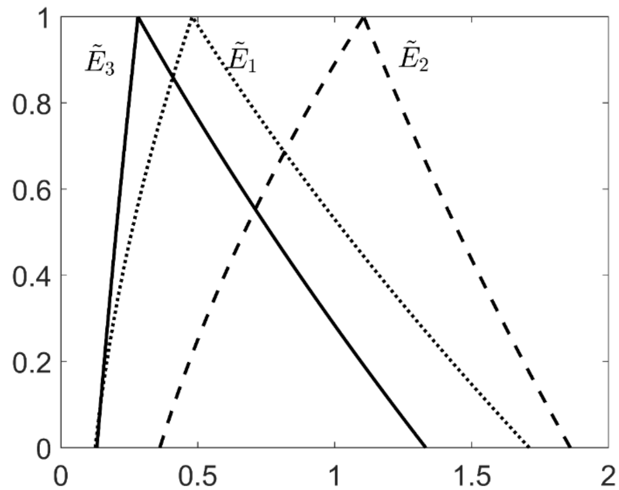
Finally, based on the above result, we calculate the combination of interval-valued fuzzy sets $E^{[i]}$ with weights $W^{[i]}$ following (1). The vector of preferences of alternatives is resulted as follows and shown in Figure 9(*a*).

$$E = [E_1, E_2, E_3] = [(0.41,1.58)\ (0.59,1.51)\ (0.14,0.88)]$$

As for triangular fuzzy sets, the weighted sum $\tilde{E}$ is computed following (21) and shown in Figure 9(*b*), see $\tilde{E}_1, \tilde{E}_2, \tilde{E}_3$.



(*a*)



(*b*)

Fig. 9 Resulting fuzzy sets (*a*) interval-valued; (*b*) weighted sum of triangular fuzzy sets for 3 alternatives.

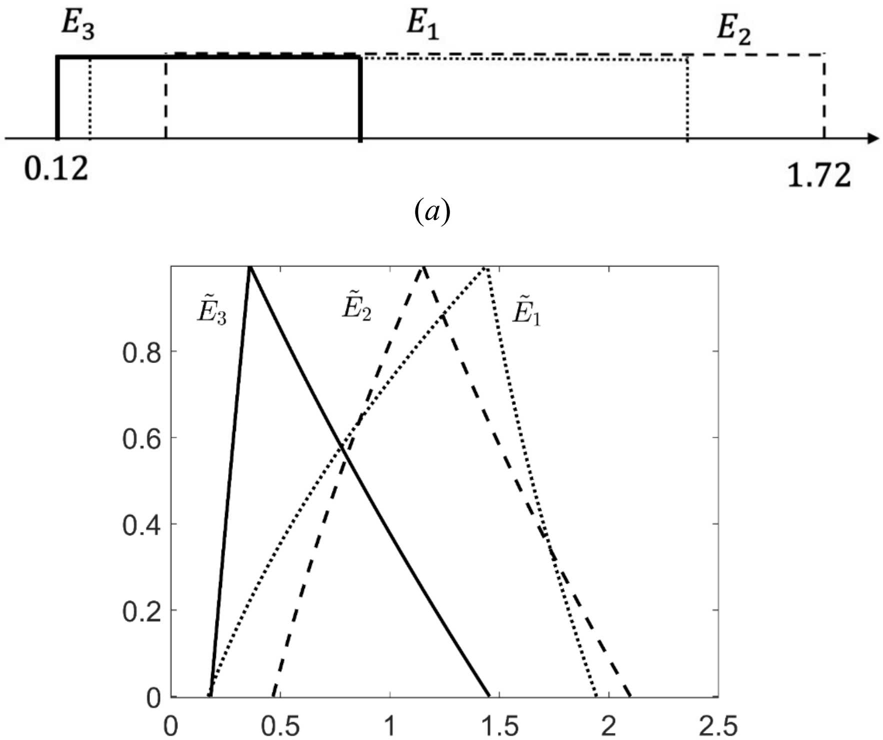
Then, for the interval-valued set E and fuzzy set $\tilde{E}$ in our experiment, according to the ranking methods, the ranking result shown in Table 3.

Table 3. Ranking results obtained using different ranking methods.

	Ranking result for interval-valued fuzzy sets	Ranking result for fuzzy sets
Method #1	$E_1 > E_2 > E_3$	$\tilde{E}_2 > \tilde{E}_1 > \tilde{E}_3$
Method #2	$E_2 > E_1 > E_3$	$\tilde{E}_2 > \tilde{E}_1 > \tilde{E}_3$
Method #3	$E_1 > E_2 > E_3$	$\tilde{E}_2 > \tilde{E}_1 > \tilde{E}_3$
Method #4	$E_2 > E_1 > E_3$	$\tilde{E}_2 > \tilde{E}_1 > \tilde{E}_3$
Method #5	$E_2 > E_1 > E_3$	$\tilde{E}_2 > \tilde{E}_1 > \tilde{E}_3$

The results in Table 3 show that mostly the alternatives are ranked as (2)>(1)>(3).

Then considering the sensitivity analysis, we add the noise in a way mentioned in Section 4, where the noise coming from the random values in the level of [-4, 4] as an example. Saying [4, -3.7, 2.1, 1.3, -3.1, -2.2, 0.5, 2] is added for corresponding integer values [2, 3, 4, 5, 6, 7, 8, 9] in pairwise matrices. Then we do the same experiments for the generated new matrix, the final interval sets E_1, E_2, E_3 and fuzzy sets $\tilde{E}_1, \tilde{E}_2, \tilde{E}_3$ are as following Figure 10 and the ranking results are in Table 4. Compared to the ranking results in Table 3 with the given ranking methods, mostly the Hamming distance between the rankings are 0 except that distance is 2 for Method #1 and #3. It proves the robustness of the results to some extent. Thus, we can conclude that the second location is the best site to install the PV power plant.



(*b*)

Fig. 10. Ranking of fuzzy sets: (*a*) interval-valued; (*b*) weighted sum of triangular fuzzy sets.

The ranking results are reported in Table 4.

Table 4. Ranking results with different methods.

	Ranking result for interval-valued fuzzy sets	Ranking result for fuzzy sets
Method #1	$E_2 > E_1 > E_3$	$\tilde{E}_1 > \tilde{E}_2 > \tilde{E}_3$
Method #2	$E_2 > E_1 > E_3$	$\tilde{E}_2 > \tilde{E}_1 > \tilde{E}_3$
Method #3	$E_2 > E_1 > E_3$	$\tilde{E}_1 > \tilde{E}_2 > \tilde{E}_3$
Method #4	$E_2 > E_1 > E_3$	$\tilde{E}_2 > \tilde{E}_1 > \tilde{E}_3$
Method #5	$E_2 > E_1 > E_3$	$\tilde{E}_2 > \tilde{E}_1 > \tilde{E}_3$

Through analyzing the above figures and tables, the ranking results of interval-valued fuzzy sets and fuzzy sets demonstrate a visible consistency, namely the alternatives are ranked as (2)> (1)>(3). In addition, compared to the model presented in [28], our model introduces fuzzy logic to further enhance the feasibility and reliability of the method on a basis of retaining the advantages of AHP (establishing the hierarchy of all elements, clearly showing the relationship between each layer, each criterion and each element). It is worth highlighting that the results of our model are in the form of type-2 fuzzy sets. In this way, they also quantify credibility associated with the obtained ranking results induced by the diversity of the opinions expressed by the individual decision-makers involved in the group decision-making.

7. Conclusions

In this study, we have established a model for the planning and site selection of renewable energy. Through the combination of AHP, ranking fuzzy sets and the concept of information granularity, from a number of alternatives, the site selection programs are evaluated and sorted from multiple perspectives, the optimal site is finally selected. The main contribution of this study is the comprehensive development of the fuzzy analytic hierarchy process and the information granule of type-2. The fuzzy analytic hierarchy process adopted in this study maintains the advantages of traditional analytic hierarchy process while the type-2 fuzzy sets are used to estimate the preference of the sites expressed in a linguistic rather than numeric fashion.

There are still some interesting further studies worth pursuing in the next step. The analytic hierarchy process is dependent on the suggestions of experts in related fields. In the next step, people can consider evaluating each participant through some certain indicators and analyzing each decision matrix, assessing appropriate weights to further optimize our decision model.

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Appendix

The pairwise comparison matrix of 3 alternatives coming from 12 decision makers under different criteria.

The 12 comparison matrices of alternatives following criterion #1: Potential energy production

$$R^{[1,1]} = \begin{bmatrix} 1 & \frac{1}{9} & \frac{1}{4} \\ 9 & 1 & 8 \\ 4 & \frac{1}{8} & 1 \end{bmatrix} R^{[1,2]} = \begin{bmatrix} 1 & \frac{5}{9} & \frac{3}{2} \\ \frac{9}{5} & 1 & \frac{7}{2} \\ \frac{2}{3} & \frac{2}{7} & 1 \end{bmatrix} R^{[1,3]} = \begin{bmatrix} 1 & \frac{4}{5} & \frac{1}{3} \\ \frac{5}{4} & 1 & \frac{6}{7} \\ 3 & \frac{7}{6} & 1 \end{bmatrix} R^{[1,4]} = \begin{bmatrix} 1 & \frac{4}{5} & \frac{1}{3} \\ \frac{5}{4} & 1 & 1 \\ \frac{5}{3} & 1 & 1 \end{bmatrix}$$

$$R^{[1,5]}=\begin{bmatrix}1&\frac{1}{2}&5\\2&1&4\\\frac{1}{5}&\frac{1}{4}&1\end{bmatrix}\quad R^{[1,6]}=\begin{bmatrix}1&2&8\\\frac{1}{2}&1&3\\\frac{1}{8}&\frac{1}{3}&1\end{bmatrix}\quad R^{[1,7]}=\begin{bmatrix}1&\frac{3}{4}&\frac{9}{2}\\\frac{4}{3}&1&5\\\frac{2}{9}&\frac{1}{5}&1\end{bmatrix}\quad R^{[1,8]}=\begin{bmatrix}1&\frac{1}{4}&\frac{1}{8}\\4&1&\frac{3}{4}\\8&\frac{4}{3}&1\end{bmatrix}$$

$$R^{[1,9]}=\begin{bmatrix}1&\frac{1}{3}&\frac{3}{5}\\3&1&\frac{1}{2}\\\frac{5}{3}&2&1\end{bmatrix}\quad R^{[1,10]}=\begin{bmatrix}1&\frac{1}{3}&\frac{1}{6}\\3&1&\frac{2}{3}\\6&\frac{3}{2}&1\end{bmatrix}\quad R^{[1,11]}=\begin{bmatrix}1&\frac{3}{2}&\frac{1}{7}\\\frac{2}{3}&1&\frac{1}{4}\\7&4&1\end{bmatrix}\quad R^{[1,12]}=\begin{bmatrix}1&\frac{1}{5}&\frac{1}{6}\\5&1&\frac{3}{5}\\6&\frac{5}{3}&1\end{bmatrix}$$

The comparison matrices following criterion #2: Environment factors

$$R^{[2,1]}=\begin{bmatrix}1&\frac{1}{6}&5\\6&1&8\\\frac{1}{5}&\frac{1}{8}&1\end{bmatrix}\quad R^{[2,2]}=\begin{bmatrix}1&\frac{2}{5}&\frac{7}{6}\\\frac{5}{2}&1&5\\\frac{6}{7}&\frac{1}{5}&1\end{bmatrix}\quad R^{[2,3]}=\begin{bmatrix}1&\frac{7}{3}&\frac{9}{2}\\\frac{3}{7}&1&\frac{5}{4}\\\frac{2}{9}&\frac{4}{5}&1\end{bmatrix}\quad R^{[2,4]}=\begin{bmatrix}1&\frac{5}{8}&\frac{3}{2}\\\frac{8}{5}&1&\frac{7}{5}\\\frac{2}{3}&\frac{5}{7}&1\end{bmatrix}$$

$$R^{[2,5]}=\begin{bmatrix}1&\frac{1}{3}&4\\3&1&7\\\frac{1}{4}&\frac{1}{7}&1\end{bmatrix}\quad R^{[2,6]}=\begin{bmatrix}1&4&9\\\frac{1}{4}&1&\frac{5}{3}\\\frac{1}{9}&\frac{3}{5}&1\end{bmatrix}\quad R^{[2,7]}=\begin{bmatrix}1&\frac{1}{2}&4\\2&1&\frac{5}{2}\\\frac{1}{4}&\frac{2}{5}&1\end{bmatrix}\quad R^{[2,8]}=\begin{bmatrix}1&1&\frac{3}{7}\\1&1&\frac{2}{5}\\\frac{7}{3}&\frac{5}{2}&1\end{bmatrix}$$

$$R^{[2,9]}=\begin{bmatrix}1&\frac{4}{3}&\frac{1}{6}\\\frac{3}{4}&1&\frac{1}{5}\\6&5&1\end{bmatrix}\quad R^{[2,10]}=\begin{bmatrix}1&3&2\\\frac{1}{3}&1&\frac{7}{4}\\\frac{1}{2}&\frac{4}{7}&1\end{bmatrix}\quad R^{[2,11]}=\begin{bmatrix}1&\frac{1}{3}&7\\3&1&9\\\frac{1}{7}&\frac{1}{9}&1\end{bmatrix}\quad R^{[2,12]}=\begin{bmatrix}1&1&\frac{1}{2}\\1&1&\frac{1}{3}\\2&3&1\end{bmatrix}$$

The comparison matrices based on criterion #3: Safety

$$R^{[3,1]}=\begin{bmatrix}1&1&7\\1&1&6\\\frac{1}{7}&\frac{1}{6}&1\end{bmatrix}\quad R^{[3,2]}=\begin{bmatrix}1&\frac{2}{9}&\frac{7}{6}\\\frac{9}{2}&1&3\\\frac{6}{7}&\frac{1}{3}&1\end{bmatrix}\quad R^{[3,3]}=\begin{bmatrix}1&\frac{8}{3}&\frac{7}{2}\\\frac{3}{8}&1&\frac{5}{2}\\\frac{2}{7}&\frac{2}{5}&1\end{bmatrix}\quad R^{[3,4]}=\begin{bmatrix}1&\frac{5}{9}&\frac{3}{2}\\\frac{9}{5}&1&\frac{7}{8}\\\frac{2}{3}&\frac{8}{7}&1\end{bmatrix}$$

$$R^{[3,5]}=\begin{bmatrix}1&\frac{1}{7}&\frac{1}{5}\\7&1&4\\5&\frac{1}{4}&1\end{bmatrix}\quad R^{[3,6]}=\begin{bmatrix}1&5&9\\\frac{1}{5}&1&3\\\frac{1}{9}&\frac{1}{3}&1\end{bmatrix}\quad R^{[3,7]}=\begin{bmatrix}1&\frac{3}{8}&\frac{7}{4}\\\frac{8}{3}&1&5\\\frac{4}{7}&\frac{1}{5}&1\end{bmatrix}\quad R^{[3,8]}=\begin{bmatrix}1&\frac{1}{4}&8\\4&1&9\\\frac{1}{8}&\frac{1}{9}&1\end{bmatrix}$$

$$R^{[3,9]}=\begin{bmatrix}1&\frac{1}{5}&\frac{1}{6}\\5&1&2\\6&\frac{1}{2}&1\end{bmatrix}\quad R^{[3,10]}=\begin{bmatrix}1&\frac{1}{2}&\frac{1}{4}\\2&1&\frac{1}{3}\\4&3&1\end{bmatrix}\quad R^{[3,11]}=\begin{bmatrix}1&\frac{3}{5}&2\\\frac{5}{3}&1&3\\\frac{1}{2}&\frac{1}{3}&1\end{bmatrix}\quad R^{[3,12]}=\begin{bmatrix}1&\frac{1}{3}&7\\3&1&8\\\frac{1}{7}&\frac{1}{8}&1\end{bmatrix}$$

The comparison matrices coming from criterion #4: Distance from existing transmission line

$$R^{[4,1]}=\begin{bmatrix}1&5&\frac{1}{4}\\\frac{1}{5}&1&\frac{1}{7}\\4&7&1\end{bmatrix}\quad R^{[4,2]}=\begin{bmatrix}1&\frac{1}{3}&\frac{7}{2}\\3&1&8\\\frac{2}{7}&\frac{1}{8}&1\end{bmatrix}\quad R^{[4,3]}=\begin{bmatrix}1&4&\frac{7}{2}\\\frac{1}{4}&1&\frac{5}{4}\\\frac{2}{7}&\frac{4}{5}&1\end{bmatrix}\quad R^{[4,4]}=\begin{bmatrix}1&\frac{7}{6}&\frac{3}{2}\\\frac{6}{7}&1&\frac{9}{5}\\\frac{2}{3}&\frac{5}{9}&1\end{bmatrix}$$

$$R^{[4,5]}=\begin{bmatrix}1&\frac{1}{6}&\frac{1}{2}\\6&1&4\\2&\frac{1}{4}&1\end{bmatrix}\quad R^{[4,6]}=\begin{bmatrix}1&9&7\\\frac{1}{9}&1&\frac{1}{3}\\\frac{1}{7}&3&1\end{bmatrix}\quad R^{[4,7]}=\begin{bmatrix}1&\frac{3}{4}&4\\\frac{4}{3}&1&5\\\frac{1}{4}&\frac{1}{5}&1\end{bmatrix}\quad R^{[4,8]}=\begin{bmatrix}1&\frac{1}{9}&\frac{1}{8}\\9&1&\frac{3}{4}\\8&\frac{4}{3}&1\end{bmatrix}$$

$$R^{[4,9]}=\begin{bmatrix}1&5&3\\\frac{1}{5}&1&\frac{1}{2}\\\frac{1}{3}&2&1\end{bmatrix}\quad R^{[4,10]}=\begin{bmatrix}1&6&4\\\frac{1}{6}&1&\frac{1}{2}\\\frac{1}{4}&2&1\end{bmatrix}\quad R^{[4,11]}=\begin{bmatrix}1&\frac{3}{4}&3\\\frac{4}{3}&1&5\\\frac{1}{3}&\frac{1}{5}&1\end{bmatrix}\quad R^{[4,12]}=\begin{bmatrix}1&\frac{1}{7}&\frac{1}{3}\\7&1&\frac{7}{4}\\3&\frac{4}{7}&1\end{bmatrix}$$

The comparison matrices following criterion #5: Topographical properties

$$R^{[5,1]}=\begin{bmatrix}1&\frac{1}{9}&\frac{1}{7}\\9&1&4\\7&\frac{1}{4}&1\end{bmatrix}\quad R^{[5,2]}=\begin{bmatrix}1&\frac{2}{5}&7\\\frac{5}{2}&1&9\\\frac{1}{7}&\frac{1}{9}&1\end{bmatrix}\quad R^{[5,3]}=\begin{bmatrix}1&\frac{7}{3}&\frac{7}{2}\\\frac{3}{7}&1&\frac{5}{4}\\\frac{2}{7}&\frac{4}{5}&1\end{bmatrix}\quad R^{[5,4]}=\begin{bmatrix}1&\frac{5}{6}&\frac{3}{2}\\\frac{6}{5}&1&\frac{8}{5}\\\frac{2}{3}&\frac{5}{8}&1\end{bmatrix}$$

$$R^{[5,5]}=\begin{bmatrix}1&\frac{1}{5}&5\\5&1&7\\\frac{1}{5}&\frac{1}{7}&1\end{bmatrix}\quad R^{[5,6]}=\begin{bmatrix}1&2&7\\\frac{1}{2}&1&3\\\frac{1}{7}&\frac{1}{3}&1\end{bmatrix}\quad R^{[5,7]}=\begin{bmatrix}1&\frac{3}{4}&\frac{7}{2}\\\frac{4}{3}&1&5\\\frac{2}{7}&\frac{1}{5}&1\end{bmatrix}\quad R^{[5,8]}=\begin{bmatrix}1&\frac{1}{2}&\frac{1}{6}\\2&1&\frac{3}{4}\\6&\frac{4}{3}&1\end{bmatrix}$$

$$R^{[5,9]}=\begin{bmatrix}1&\frac{1}{4}&3\\4&1&6\\\frac{1}{3}&\frac{1}{6}&1\end{bmatrix}\quad R^{[5,10]}=\begin{bmatrix}1&3&6\\\frac{1}{3}&1&\frac{5}{2}\\\frac{1}{6}&\frac{2}{5}&1\end{bmatrix}\quad R^{[5,11]}=\begin{bmatrix}1&\frac{1}{5}&\frac{1}{7}\\5&1&\frac{2}{3}\\7&\frac{3}{2}&1\end{bmatrix}\quad R^{[5,12]}=\begin{bmatrix}1&\frac{1}{3}&\frac{1}{4}\\3&1&\frac{1}{2}\\4&2&1\end{bmatrix}$$

The pairwise comparison matrix of criteria coming from 12 decision makers.

$$W^{[1]} = \begin{bmatrix} 1 & 2 & \frac{1}{5} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{2} & 1 & \frac{1}{7} & \frac{1}{5} & \frac{1}{9} \\ 5 & 7 & 1 & 3 & \frac{1}{2} \\ 3 & 5 & \frac{1}{3} & 1 & \frac{1}{5} \\ 6 & 9 & 2 & 5 & 1 \end{bmatrix} \quad W^{[2]} = \begin{bmatrix} 1 & 3 & \frac{1}{2} & \frac{1}{3} & \frac{1}{5} \\ \frac{1}{3} & 1 & \frac{1}{7} & \frac{1}{3} & \frac{1}{9} \\ 2 & 7 & 1 & \frac{1}{2} & \frac{1}{2} \\ 3 & 3 & 2 & 1 & \frac{1}{5} \\ 5 & 9 & 2 & 5 & 1 \end{bmatrix} \quad W^{[3]} = \begin{bmatrix} 1 & \frac{1}{3} & \frac{4}{5} & 4 & \frac{1}{3} \\ 3 & 1 & 5 & 7 & \frac{1}{2} \\ \frac{5}{4} & \frac{1}{5} & 1 & 6 & \frac{2}{5} \\ \frac{1}{4} & \frac{1}{7} & \frac{1}{6} & 1 & \frac{1}{8} \\ 3 & 2 & \frac{5}{2} & 8 & 1 \end{bmatrix}$$

$$W^{[4]} = \begin{bmatrix} 1 & \frac{1}{8} & \frac{1}{5} & \frac{4}{5} & 3 \\ 8 & 1 & 2 & 7 & 9 \\ 5 & \frac{1}{2} & 1 & 5 & 6 \\ \frac{5}{4} & \frac{1}{7} & \frac{1}{5} & 1 & 5 \\ \frac{1}{3} & \frac{1}{9} & \frac{1}{6} & \frac{1}{5} & 1 \end{bmatrix} \quad W^{[5]} = \begin{bmatrix} 1 & \frac{1}{6} & \frac{1}{5} & \frac{1}{3} & \frac{1}{8} \\ 6 & 1 & 3 & 5 & \frac{1}{3} \\ 5 & \frac{1}{3} & 1 & 3 & \frac{1}{5} \\ 3 & \frac{1}{5} & \frac{1}{3} & 1 & \frac{1}{7} \\ 8 & 3 & 5 & 7 & 1 \end{bmatrix} \quad W^{[6]} = \begin{bmatrix} 1 & 1 & \frac{1}{5} & \frac{1}{3} & \frac{1}{9} \\ 1 & 1 & \frac{1}{7} & \frac{1}{5} & \frac{1}{7} \\ 5 & 7 & 1 & 2 & \frac{1}{2} \\ 3 & 5 & \frac{1}{2} & 1 & \frac{1}{3} \\ 9 & 7 & 2 & 3 & 1 \end{bmatrix}$$

$$W^{[7]} = \begin{bmatrix} 1 & 2 & \frac{1}{8} & \frac{1}{3} & \frac{1}{5} \\ \frac{1}{2} & 1 & \frac{1}{9} & \frac{1}{4} & \frac{1}{7} \\ 8 & 9 & 1 & 5 & 3 \\ 3 & 4 & \frac{1}{5} & 1 & \frac{1}{3} \\ 5 & 7 & \frac{1}{3} & 3 & 1 \end{bmatrix} \quad W^{[8]} = \begin{bmatrix} 1 & 4 & \frac{1}{8} & \frac{1}{6} & \frac{1}{2} \\ \frac{1}{4} & 1 & \frac{1}{8} & \frac{1}{7} & \frac{1}{7} \\ 8 & 8 & 1 & 3 & 5 \\ 6 & 7 & \frac{1}{3} & 1 & 5 \\ 2 & 7 & \frac{1}{5} & \frac{1}{5} & 1 \end{bmatrix} \quad W^{[9]} = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 7 \\ 2 & 1 & \frac{2}{5} & \frac{1}{4} & 6 \\ 3 & \frac{5}{2} & 1 & \frac{1}{2} & 7 \\ 6 & 4 & 2 & 1 & 9 \\ \frac{1}{7} & \frac{1}{6} & \frac{1}{7} & \frac{1}{9} & 1 \end{bmatrix}$$

$$W^{[10]} = \begin{bmatrix} 1 & 3 & 2 & \frac{1}{5} & \frac{7}{6} \\ \frac{1}{3} & 1 & 2 & \frac{1}{9} & 2 \\ \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{7} & 1 \\ 5 & 9 & 7 & 1 & 7 \\ \frac{6}{7} & \frac{1}{2} & 1 & \frac{1}{7} & 1 \end{bmatrix} \quad W^{[11]} = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{5} & 3 & \frac{1}{4} \\ 2 & 1 & 3 & 4 & \frac{3}{2} \\ 5 & \frac{1}{3} & 1 & \frac{5}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{4} & \frac{2}{5} & 1 & \frac{1}{9} \\ 4 & \frac{2}{3} & 2 & 9 & 1 \end{bmatrix} \quad W^{[12]} = \begin{bmatrix} 1 & 2 & 1 & 3 & \frac{1}{3} \\ \frac{1}{2} & 1 & 1 & 2 & \frac{1}{5} \\ 1 & 1 & 1 & \frac{2}{3} & \frac{2}{5} \\ \frac{1}{3} & \frac{1}{2} & \frac{3}{2} & 1 & \frac{1}{7} \\ 3 & 5 & \frac{5}{2} & 7 & 1 \end{bmatrix}$$