Stability Analysis of Teleoperation Systems under Strictly Passive and Non-passive Operator

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Abstract

A bilateral teleoperation system includes a human operator and an environment, which make the system stability analysis complicated due to their unknown, time-varying and nonlinear nature. Unable to have exact models for the human operator and the environment, it is typically assumed that they are passive but otherwise arbitrary. In this paper, through a set of experiments, first we show that a human operator's relaxed arm is strictly passive while voluntary motions of the human operator's arm involve non-passive characteristics. Then, we adjust the passivity assumption of the human operator's arm (by tightening it for an input-strictly-passive arm and relaxing it for a non-passive arm) in order to enable a more precise stability analysis of the teleoperation system. Inspired by Llewellyn's absolute stability criterion, a powerful stability analysis approach is developed to investigate the stability of a two-port network when it is coupled to an input-strictly-passive or a non-passive termination. Although this new stability criterion is applicable to any two-port network system, we apply it to a position-error-based bilateral teleoperation system as a case study.

Index Terms: H.5.2 [Information Interfaces and Presentation]: User Interfaces—Haptic I/O; I.2.9 [Artificial Intelligence]: Problem Solving, Control Methods, and Search—Control theory

1 Introduction

A bilateral teleoperation system comprises a human operator manipulating a master robot, a communication channel, and a slave robot working on an environment [1]. We define a teleoperator to be the combination of the master and the slave, their controllers, and the communication channel. Stability of a teleoperation system is investigated in passivity-based frameworks in order to accommodate the fact the teleoperator's terminations (i.e., the human operator and the environment) are uncertain, nonlinear and/or time-varying. In the passivity-based stability analysis, the teleoperator is modelled as a two-port network and made passive through appropriate control. The human operator and the environment as the two terminations for this two-port network are assumed to be passive. The interconnection of a passive two-port network with two passive terminations is passive and therefore stable [1], thus the passivity of the two-port network teleoperator is sufficient for stability of the teleoperation system.

In a less conservative approach to stability analysis of teleoperation systems, a well-known absolute stability criterion has been proposed by Llewellyn [2]. Absolute stability of a two-port network is equivalent to passivity of the driving-point impedance seen from one of the ports when the other port is terminated to a passive one-port network [3] (Fig. 1). Equivalently, a two-port network is absolutely stable if it results in a stable teleoperation system when coupled to two passive termination. By not requiring the teleoperator to be passive, absolute stability reduces the conservatism in the stability analysis and makes it possible to design controllers that lead to improved teleoperation transparency.

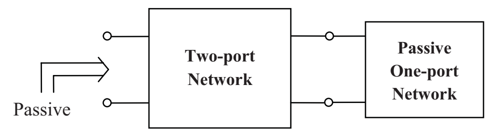

Figure 1: Absolute stability of a two-port network is equivalent to passivity of the driving-point impedance at a port when the other port is terminated to a passive one-port network.

Investigating stability of teleoperation systems through both passivity-based and absolute stability approaches involve the common assumption that the teleoperator's terminations are passive. We claim that in many practical systems, the human operator (or the environment) may be either input-strictly-passive (i.e., overly passive) or non-passive. While coupling an input-strictly-passive termination to an absolutely stable two-port network does not impose more stability issues than when the termination is passive, using the Llewellyn's absolute stability criterion for the teleoperator will result it overly conservative stability conditions. In fact, the excess of passivity (EOP) of the input-strictly passive termination may allow the two-port network to be non-passive, which provides an opportunity to improve the transparency of the teleoperation system. On the other hand, a non-passive termination coupled to a two-port network that satisfies the Llewellyn's absolute stability criterion may lead to an unstable coupled system.

When minimum and maximum bounds on the impedance of a termination in a teleoperation system are known, the termination can be modelled by series and shunt impedances combined with an arbitrary impedance [4, 5]. In another work, the stability condition has been found as a function of the impedance parameter of the termination after using reflective coefficient parameters in the scattering domain [6]. In a recent work, Llewellyn's criterion has been revisited to accommodate investigating stability of a two-port network coupled with a non-passive termination [7]. Specifically, the largest region in the complex impedance plane was found such that any termination with an impedance in that region – regardless of being passive or non-passive – when coupled to a two-port network would result in a passive driving-point impedance at the other port of the two-port network. In this paper, this work has been extended to when the termination is overly passive which allows the relaxation of stability conditions on the two-port network and improved transparency of the overall teleoperation system.

Llewellyn's absolute stability criterion determines bounds on the teleoperator assuming the human operator and the environment are passive [4, 8]. When a human operator is holding a robot through a relaxed grasp, he/she is passive [9]. However, in general, the human operator may be a source of energy in a teleoperation system. In this paper, coinciding with Hogan's results, a relaxed human arm is identified as a passive system. On the other hand, during operator's voluntary motion, the human operator is found to act as an active system. Instead of labeling a termination as passive or non-

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passive, how much a termination is passive or non-passive – excess of passivity (EOP) and shortage of passivity (SOP) – is considered for stability of the teleoperation systems.

For a linear time-invariant (LTI) system, passivity is equivalent to positive realness of its transfer function. In other words, a passive one-port network's complex impedance will be in the right half of the complex plane. For a non-passive one-port network, the complex impedance has a real part greater than a negative number. In contrast, for an input-strictly-passive (ISP) system, the real part of the complex impedance is greater than a positive number. The shift of the right half plane to the right (or the left) corresponds to the, EOP (or SOP).

This paper applies conformal mapping or Mobius transformation to propose a new powerful stability analysis tool for two-port networks when the terminations have excess of passivity or shortage of passivity. Llewellyn's absolute stability criterion for passive terminations has been extended to non-passive and ISP terminations. The resulting absolute stability conditions have a closed form.

The rest of this paper is organized as follow. In Section 2, the fundamental notions such as excess of passivity and shortage of passivity are introduced. In Section 3, experiments involving a human operator are performed to identify when the operator is passive or non-passive. The absolute stability condition for passive termination is derived in Section 4 and its extension to non-passive and input-strictly-passive terminations are developed in Sections 5 and Section 6, respectively. Although the developed stability criteria are valid for any two-port network, they are applied to a position-error-based teleoperation system in Section 7.

2 MATHEMATICAL PRELIMINARIES

This section includes the definitions and prerequisite theorems that are used in the subsequent sections. Notions of passivity, absolute stability, positive realness, and strict passivity are defined.

Definition 1. *A system with input $u(t)$ and output $y(t)$ is passive if there is a constant β such that*

$$\int_0^t y(\tau)u(\tau)d\tau \geq \beta \tag{1}$$

for all $t \geq 0$.

The constant β is the energy storage at time $t = 0$. If the input and output of a mechanical system are chosen to be the force and the velocity, the physical meaning of passivity will be that the system dissipates energy.

Definition 2. *[10] A two-port network is called absolutely stable if for any passive but otherwise arbitrary terminations, the overall two-port network is stable. A two-port network is called potentially unstable if it is not absolutely stable.*

Theorem 1. *[3] A two-port network is absolutely stable if and only if for any passive but otherwise arbitrary termination at each of the ports, the driving-point impedance from the other port is passive.*

The absolute stability according to Definition 2 requires that the terminations are assumed passive. Our paper will consider non-passive and strictly passive terminations defined in the following.

Definition 3. *Consider a system with input $u(t)$ and output $y(t)$. If there is constants β such that for all $t \geq 0$,*

$$\int_0^t y(\tau)u(\tau)d\tau \geq \beta + \delta \int_0^t u(\tau)u(\tau)d\tau \tag{2}$$

for $\delta > 0$, the system is ISP with EOP of δ [11, 12]. For $\delta < 0$, the system is non-passive with SOP of $-\delta$.

For an LTI system, passivity is equivalent to having the system's Nyquist diagram entirely in the right half plane (Fig. 2-a). The Nyquist diagram of an ISP system with transfer function $G(s)$ and EOP of $\delta > 0$ is in the right hand side of the vertical line at δ, i.e. $\mathrm{Re}\{G(s)\} \geq \delta$ (Fig. 2-b). Similarly, for a non-passive transfer function $G(s)$ with SOP of $\lambda > 0$ the Nyquist diagram is in $\mathrm{Re}\{G(s)\} \geq -\lambda$ (Fig. 2-c). Theorems 2 and 3 make the connection between the passivity of LTI system and their corresponding regions in the Nyquist plane.

Theorem 2. *[11, 13, 14] Consider a system with transfer function $G(s)$, input $U(s)$ and output $Y(s)$. Assume that all poles of $G(s)$ have negative real parts. The system is passive if and only if $\mathrm{Re}G(j\omega) \geq 0$ for all frequencies ω [1].*

Theorem 3. *[11] Consider a system with transfer function $G(s)$, input $U(s)$ and output $Y(s)$. Assume that all poles of $G(s)$ have negative real parts. The system is ISP with EOP of δ if and only if $\mathrm{Re}G(j\omega) \geq \delta$ for all frequencies ω. Also, the system is non-passive with SOP of λ if and only if $\mathrm{Re}G(j\omega) \geq -\lambda$ for all frequencies ω.*

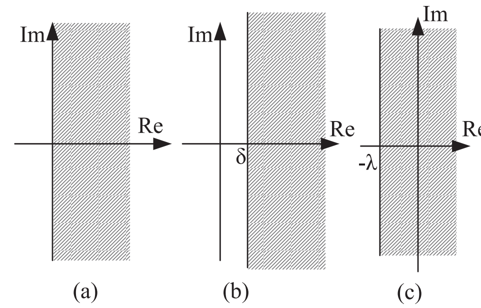

Figure 2: Nyquist diagrams of (a) a passive system, (b) an ISP system with excess of passivity of δ, (c) a non-passive system with shortage of passivity of λ.

Next, the passivity of a human operator is experimentally tested for two tasks involving relaxed arm and voluntary motions.

3 PASSIVITY OF HUMAN ARM

The passivity of a human operator can be analyzed from records of the force and velocity obtained while the operator interacts with a haptic device. For this purpose, experiments were performed with a two degree-of-freedom (DOF) robot manufactured by Quanser, Inc. (Markham, ON, Canada). The robot, which operates in a horizontal plane, is readily backdrivable and features a two-motor capstan drive mechanism capable of exerting forces in excess of 50 N throughout its workspace. Optical encoders on the motor shafts provide a Cartesian resolution of better than 0.002 mm at the end effector. End effector velocities are calculated through differentiation and low-pass filtering of position signals. A 2-DOF strain gauge force sensor measures the forces the operator exerts on the device. The robot is controlled by a closed-loop feedback linearized position control algorithm. All data logging and robot control activities occur at a 1 kHz sampling frequency.

Data was collected from a 29-year-old right-handed male with no history of motor impairment. In each trial, the subject sat in front of the robot and rested his right hand on the robot's end-effector (handle). The subject's elbow was supported against gravity by a sling attached to a rope connected to the ceiling. The subject's hand was centered at a home position 52 cm anterior to the body in the sagittal plane intersecting the shoulder joint.

[1] In fact, for an LTI system, passivity is equivalent to positive realness. Positive realness is (a) having no pole in the right half plane, (b) poles on the $j\omega$-axis are not repeated and have positive residues, (c) $\mathrm{Re}G(j\omega) \geq 0$. Conditions (a) and (b) are for stability and condition (c) ensures passivity.

The subject was instructed to perform a relaxed arm task and a voluntary motion task. In the first task, the subject relaxed his arm while the robot applied a series of 5 mm step-like underdamped position perturbations to the subject's hand. Following each perturbation, the robot's position controller gently repositioned the hand at the home position such that each subsequent perturbation was delivered from the same location. To prevent the subject from voluntarily intervening as the measurements were performed, each perturbation had a random direction (selected from 8 possible directions at 45° intervals around the home position), duration(1.5–2.0 s), and onset time (2.5–7.0 s after the conclusion of the previous perturbation). A total of 16 perturbations were delivered in each of the five trials.

In the second task, the robot's position controller was turned off and the subject was instructed to move his arm. Two dots were displayed on a computer screen, representing the robot's end-effector position and a target position for the subject to track as closely and quickly as possible. In each of the four trials, the target moved 5 mm away from the home position a total of 16 times, returning to the home position after each motion. As in the relaxed arm task, the direction, duration, and onset of each movement were selected randomly.

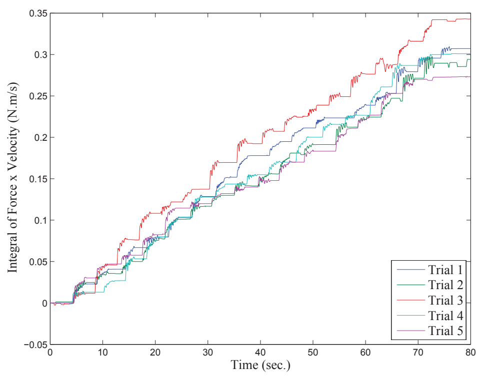

Figure 3: The integral of force times velocity is an indication of passivity. Staying non-negative for all times corresponds to passivity of the system. The operator is asked to have his arm on the robot but does not apply any deliberate force to the robot.

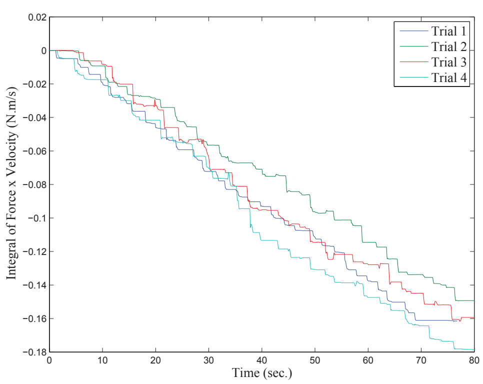

Figure 4: The integral of force times velocity is an indication of passivity. Becoming negative implies that the system is non-passive. The operator is asked to voluntarily apply perturbations to the robot.

Fig. 3 and Fig. 4 display the result of the integral (1) for the relaxed arm and the voluntary motions, respectively. The measured forces were filtered by a 10th-order Butterworth low-pass filter with a cutoff frequency of 20 Hz. Staying non-negative at all times for the relaxed arm, in Fig. 3, indicates that the relaxed arm is passive. In fact, the relaxed arm experiment resulted in an ISP system which provides some level of flexibility in the control design. According to (2), the EOP is found to be 10.81 ± 0.78 for Fig. 3. On the other hand, the negative result of the integral in Fig. 4 demonstrates that voluntary motion implies non-passivity of the human operator with SOP of 27.19 ± 3.46. We have shown passivity and non-passivity of human arm in a related work using impedance identification [15]. Before addressing stability conditions of the non-passive and ISP terminations, we will consider absolute stability for a passive termination in the following.

4 Absolute Stability Criterion under Passive Terminations

A two-port network (Fig. 5-a) is modelled by its impedance (Z) parameters as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{3}$$

Theorem 4. *[3] A two-port network is absolutely stable if and only if*

(i) Z_{11} and Z_{22} have no poles in the right half of the complex plane (RHP),

(ii) Pure imaginary poles of Z_{11} and Z_{22} are simple and have positive residues, and

(iii) For all real positive frequencies ω,

$$\begin{aligned} \operatorname{Re} Z_{11}(j\omega) &\geq 0 \\ \operatorname{Re} Z_{22}(j\omega) &\geq 0 \\ 2 \operatorname{Re} Z_{11}(j\omega) \operatorname{Re} Z_{22}(j\omega) - \operatorname{Re}\{Z_{12}(j\omega)Z_{21}(j\omega)\} & \\ -|Z_{12}(j\omega)Z_{21}(j\omega)| &\geq 0 \end{aligned} \tag{4}$$

The two-port network impedance parameters may be replaced by any immitance parameters.

Proof. [7, 16] Conditions (i) and (ii) are necessary for ensuring positive realness of Z_{11} and Z_{22} in zero-impedance conditions for ports 2 and 1, respectively [3]. Let us consider Condition (iii). As shown in Fig. 5-b, the two-port network is connected to a passive impedance z_2 and the driving-point impedance is called Z_{a1}. The two-port network will be absolutely stable if Z_{a1} is passive as well. It is easy to show that Z_{a1} can be expressed based on the two-port network impedance parameters Z_{ij}'s and the termination impedance z_2 as

$$Z_{a1} = Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22} + z_2} \tag{5}$$

This relationship between the impedances z_2 and Z_{a1} can be expressed as a Mobius transformation as

$$Z_{a1} = \frac{z_2(Z_{11}) + (Z_{11}Z_{22} - Z_{12}Z_{21})}{z_2 + (Z_{22})} \tag{6}$$

Mobius transformation maps circles and lines from one complex plane to lines and circles in another complex plane [17, 18]. The borderline of passivity in the z_2 complex plane is a vertical line coincident with the $j\omega$-axis; any impedance to the right of this line (i.e., with a positive real value) is passive. If $\operatorname{Re} Z_{22} \geq 0$, then the Mobius transformation of the $j\omega$-axis is a circle with a radius r_o and a centre located at ω_o where

$$r_o = \frac{|Z_{12}Z_{21}|}{2R_{22}} \tag{7}$$

$$\omega_o = Z_{11} - \frac{Z_{12}Z_{21}}{2R_{22}} \tag{8}$$

where $R_{22} = \operatorname{Re} Z_{22}$. This means that the right half plane in the z_2-plane (i.e., class of positive-real impedances) is mapped to a disc as depicted in Fig. 6. Now, the condition for passivity of the resulting mapped impedance, i.e. Z_{a1}, is that it entirely lies in the right half plane. In other words,

$$\mathrm{Re}\,\omega_o - r_o \geq 0 \tag{9}$$

Substituting (7) and (8) in the above leads to

$$\frac{2\,\mathrm{Re}\,Z_{11}\,\mathrm{Re}\,Z_{22} - \mathrm{Re}(Z_{12}Z_{21}) - |Z_{12}Z_{21}|}{2R_{22}} \geq 0 \tag{10}$$

Here, $R_{ij} = \mathrm{Re}Z_{ij}$. This completes the proof. □

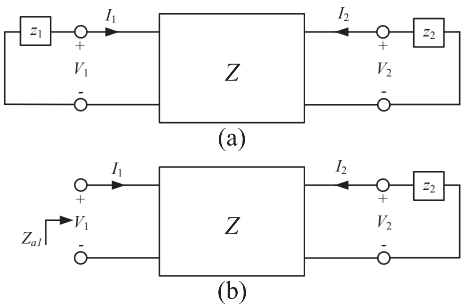

Figure 5: (a) Two-port network and (b) driving-point impedance $Z_{a1} = V_1/I_1$ when port 2 is terminated to a passive impedance z_2.

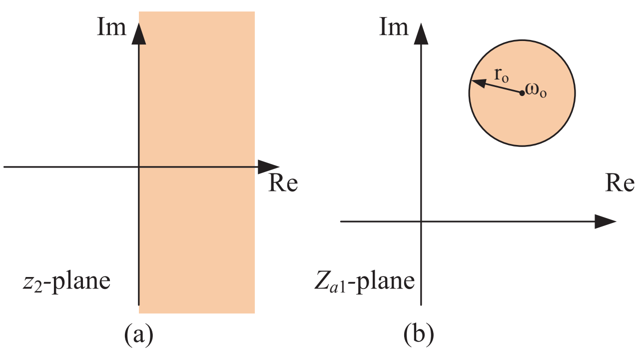

Figure 6: Mobius transformation maps the right half of the z_2-plane (a) to a disc in the Z_{a1}-plane (b).

Having Llewellyn's absolute stability criterion derived in the above, our two extensions for stability analysis of a two-port network systems with non-passive and strictly passive terminations are elaborated in Section 5 and Section 6.

5 Stability of Two-port Networks Comprising a Non-passive Rectangular Impedance Termination

Assume that impedance z_2 that terminates port 2 of a two-port network has a negative real part with magnitude of SOP equal to a, i.e. $\mathrm{Re}z_2 \geq -a$. Also, to cover a more general case of impedance shapes in the z_2-plane, assume that there is an upper bound for the real part of the impedance, i.e., $\mathrm{Re}z_2 \leq b, b > 0$, and that the imaginary part of the impedance in the complex plane is bounded between $c > 0$ and $-d < 0$. The impedance is boxed in the rectangle $-a \leq \mathrm{Re}z_2 \leq b, -d \leq \mathrm{Im}z_2 \leq c$ as shown in Fig. 7-a. By setting $a = 0, b = \infty, c = \infty$ and $d = \infty$ the impedance will span the right half plane (passive).

Theorem 5. *Consider a two-port network system (3). Assume that port 2 of the two-port network is terminated to an impedance z_2 and the driving-point impedance seen from port 1 is Z_{a1}. Assume that z_2 satisfies $-a \leq \mathrm{Re}z_2 \leq b$ and $-d \leq \mathrm{Im}z_2 \leq c$, where $a, b, c, d > 0$. Also, assume port 1 of the two-port network is terminated to a passive impedance. Then, the necessary and sufficient condition for stability of the coupled system is*

(i) Z_{11} and Z_{22} have no poles in the right half of the complex plane,

(ii) Pure imaginary poles of Z_{11} and Z_{22} are simple and have positive residues, and

(iii) For all real positive frequencies ω,

$$\begin{aligned} R_{11} &\geq 0 \\ R_{22} &\geq a \\ 2R_{11}R_{22} - \mathrm{Re}\{Z_{12}Z_{21}\} - |Z_{12}Z_{21}| - 2R_{11}a &\geq 0 \end{aligned} \tag{11}$$

In (11), the $j\omega$ arguments of the impedances are not displayed for the sake of brevity.

Proof. Similar to the proof of Theorem 4, Conditions (i) and (ii) are necessary condition for stability of the two-port network. Condition (iii) is derived below.

- Step 1: When z_2 is connected to port 2 of the two-port network, for stability of the overall system we require that Z_{a1} is passive. The transformation from z_2 into Z_{a1} is

$$Z_{a1} = Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22} + z_2} \tag{12}$$

  which can be expressed as a Mobius transformation from z_2 into Z_{a1} consistent with (6).

- Step 2: The rectangle $-a \leq \mathrm{Re}\,z_2 \leq b, -d \leq \mathrm{Im}z_2 \leq c$ is mapped to a portion of a crescent in Z_{a1}-plane (hatched region in Fig. 7-b). Proof is omitted for brevity.

- Step 3: The portion of the crescent found in Step 2 needs to be entirely in the right half the complex Z_{a1}-plane for passivity of the driving-point impedance at port 1. Because the outer edge of the crescent (mapping of $\mathrm{Re}z_2 = -a$) contains the inner edge of the crescent (mapping of $\mathrm{Re}\,z_2 = b$) and the two circles overlap each other in a region farthest possible from the origin, the necessary and sufficient condition for passivity of the driving-point impedance at port 1 is

$$\mathrm{Re}\,{}^{a}\omega_o - {}^{a}r_o \geq 0 \tag{13}$$

  The transformation of the line $\mathrm{Re} = -a$ into the Z_{a1}-plane is a circle with radius and centre of

$${}^{a}r_o = \frac{|Z_{12}Z_{21}|}{2(R_{22} - a)} \tag{14}$$

$${}^{a}\omega_o = Z_{11} - \frac{Z_{12}Z_{21}}{2(R_{22} - a)} \tag{15}$$

  After substituting for ${}^{a}r_o$ and ${}^{a}\omega_o$ from (14) and (15), (13) becomes

$$\mathrm{Re}\{Z_{11} - \frac{Z_{12}Z_{21}}{2(R_{22} - a)}\} - \frac{|Z_{12}Z_{21}|}{2(R_{22} - a)} \geq 0 \tag{16}$$

  which can be rearranged as (11). This completes the proof.

□

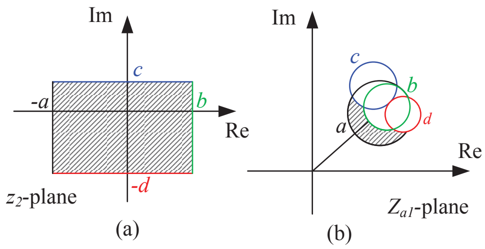

Figure 7: A rectangle in the z_2-plane is transformed into the hatched region in the Z_{a1}-plane.

Remarks:

- The value of the upper limit of the z_2 impedance b does not appear in the stability condition (11) due to the fact that the inner circle of the crescent in Fig. 7-b is not the source of any constraint when ensuring the passivity of Z_{a1}. Also, c and d do not enter (11). In other words, stability only is affected by the lower limit of the real part of the impedance connected to port 2 of the two-port network.

- The difference between the new stability condition (11) for rectangular termination impedances and Llewellyn's absolute stability criterion is in the last term of (11).

- Unlike Llewellyn's absolute stability criterion, the new stability conditions in Theorem 5 are not symmetric with respect to the network parameters. In other words, swapping the terminations at ports 1 and 2 of a two-port network does not change Llewellyn's absolute stability conditions but it may affect the new conditions in Theorem 5. This is due to the fact that in this paper we assume that a port of the two-port network is connected to a potentially non-passive terminations and then require the driving-point impedance seen from the other port to be passive in order to ensure the stability of the overall system once the open port is terminated to a passive impedance.

6 Stability of Two-port Networks Comprising an Input Strictly Passive Impedance Termination

Theorem 6. *For a two-port network, assume that the termination impedance of z_2 is ISP with $\delta \geq 0$. Also, assume that $R_{22} \geq -\delta$. Then, the necessary and sufficient condition for stability of the coupled system is*

(i) Z_{11} and Z_{22} have no poles in the right half of the complex plane,

(ii) Pure imaginary poles of Z_{11} and Z_{22} are simple and have positive residues, and

(iii) For all real positive frequencies ω,

$$R_{11} \geq 0$$

$$R_{22} \geq -\delta$$

$$2R_{11}R_{22} - \text{Re}\{Z_{12}Z_{21}\} - |Z_{12}Z_{21}| + 2R_{11}\delta \geq 0 \quad (17)$$

Proof. As depicted in Fig. 5, port 2 of a two-port network is terminated to an impedance of z_2 which has the Nyquist plot of Fig. 2-b and the objective of the rest of the proof is to find the transformed region in the Z_{a1}-plane. Then, this region in the Z_{a1}-plane requires to be entirely placed in the right half plane.

With the assumption of $R_{22} \geq -\delta$, the region of Fig. 2-b in the z_2-plane is transformed to a disc in the Z_{a1}-plane with a radius of

$$r_o = \frac{|Z_{12}Z_{21}|}{2(R_{22}+\delta)} \quad (18)$$

and a centre at

$$\omega_o = Z_{11} - \frac{Z_{12}Z_{21}}{2(R_{22}+\delta)} \quad (19)$$

This region in the Z_{a1}-plane should be entirely in the right half plane, leading to (17). □

Remark:

- The last condition in (17) for stability under an ISP termination should be compared to its non-passive counterpart in (11). Given that $a, \delta > 0$, it is clear that the latter is more restrictive. Indeed, a two-port network with a non-passive termination should be overly passive to compensate for non-passivity of the termination. On the other hand, a passive termination requires less energy absorption by the two-port network for ensuring stability of the coupled system.

7 Application to Bilateral Teleoperation

While the stability conditions in Section 5 and Section 6 are valid for any two-port network, an important application is in stability analysis of a bilateral teleoperation system. Assume the LTI dynamic models of the master and the slave are

$$sX_m = Z_m^{-1}(F_h + F_m) \quad (20)$$

$$sX_s = Z_s^{-1}(F_e + F_s) \quad (21)$$

where subscript m and s correspond to the master and the slave, X denotes the position, Z is the robot impedance, F is the robot controller output, F_h and F_e are the operator's hand and the environment forces, and s is the Laplace variable. Position-error-based controllers for the master and the slave are

$$F_m = C_m(s)(X_s - X_m) \quad (22)$$

$$F_s = C_s(s)(X_m - X_s) \quad (23)$$

The impedance matrix representing the teleoperator is

$$Z = \begin{bmatrix} Z_{tm} & C_m \\ C_s & Z_{ts} \end{bmatrix} \quad (24)$$

Here, $Z_{ts} = Z_s + C_s$ and $Z_{tm} = Z_m + C_m$, where the master and the slave robots are modelled as $Z_m = sM_m + B_m$ and $Z_s = sM_s + B_s$ and the local position controllers are proportional derivative controllers: $C_m = K_{v_m} + K_{p_m}/s$ and $C_s = K_{v_s} + K_{p_s}/s$.

7.1 Passive termination

If the teleoperator terminations are passive and controller gains are non-negative, i.e., $K_{v_m}, K_{v_s}, K_{p_m}, K_{p_s} \geq 0$, the teleoperation system is stable if and only if

$$B_{ms} \geq \frac{(K_{v_m}K_{p_s} - K_{p_m}K_{v_s})^2}{4K_{p_m}K_{p_s}} \quad (25)$$

where $B_{ms} = B_m B_s + B_m K_{v_s} + B_s K_{v_m}$. This is found by applying the Llewellyn's criterion (4), which results in the inequality condition

$$4B_{ts}B_{tm}B_{ms}\omega^2 + 4K_{p_m}K_{p_s}B_{ms} - (K_{v_m}K_{p_s} - K_{p_m}K_{v_s})^2 \geq 0 \quad (26)$$

where $B_{ts} = B_s + K_{v_s}$ and $B_{tm} = B_m + K_{v_m}$, and then setting $\omega = 0$ as the worst case. Condition (25) implies that stability is guaranteed for any value of robot damping if the controller gains are selected to satisfy

$$\frac{K_{p_s}}{K_{v_s}} = \frac{K_{p_m}}{K_{v_m}} \quad (27)$$

This is consistent with the result reported in [8]. When (27) does not hold, a lower bound on B_{ms} is found, which resulting in lower bounds on the master and the slave damping terms B_m and B_s and bounds on their derivative control gains K_{v_m} and K_{v_s}.

7.2 Non-passive termination

If the terminating one-port network has an impedance of a rectangular shape as in Fig. 7-a, i.e., $-a \le \mathrm{Re}z_2 \le b$ and $-d \le \mathrm{Im}z_2 \le c$ where $a, b, c, d \ge 0$ and $K_{v_m}, K_{v_s}, K_{p_m}, K_{p_s} \ge 0$, the stability condition (11) yields

$$4(B_{ts} - a)B_{tm}B'_{ms}\omega^2 + 4K_{p_m}K_{p_s}B'_{ms} - (K_{v_m}K_{p_s} - K_{p_m}K_{v_s})^2 \ge 0 \quad (28)$$

$$B_{ts} \ge a \quad (29)$$

where $B'_{ms} = (K_{v_m} + B_m)(B_s - a) + B_m K_{v_s} = B_{ms} - (K_{v_m} + B_m)a$ and results in a similar condition as (26) where B_s is replaced by $B_s - a$. With $\omega = 0$, the worst case for (28) is found as

$$B'_{ms} \ge \frac{(K_{v_m}K_{p_s} - K_{p_m}K_{v_s})^2}{4K_{p_m}K_{p_s}} \quad (30)$$

which is similar to (25) with the difference that B_{ms} has been replaced by B'_{ms} in the left-hand side. Noting that $B'_{ms} = B_{ms} - (K_{v_m} + B_m)a$, we arrive at the conclusion that (30) is more restrictive than (25), which is expected because of the nature of the termination for each case. Note that (29) has to be fulfilled, too.

If the master and slave damping terms B_m and B_s are zero, the stability conditions (29) and (30) are satisfied only if the controller gains are proportional as in (27) and also $a = 0$, which means that there is no non-passive termination for which stability is ensured.

The condition (29) implies a lower bound on the termination non-passivity. In other words, to be able to accommodate more non-passive terminations, the damping terms of the master and the slave robots should be higher or otherwise the derivative term of the controller should be selected high enough to overcome non-passivity of the termination $B_{ts} = B_s + K_{v_s}$.

7.3 Input-strictly-passive termination

Assume that the termination is input-strictly-passive with EOP of δ. The impedance of the termination is a half plane shifted to the right expressed as $\mathrm{Re}z_2 \ge \delta > 0$. The stability condition (17) becomes

$$4(B_{ts} + \delta)B_{tm}B''_{ms}\omega^2 + 4K_{p_m}K_{p_s}B''_{ms} - (K_{v_m}K_{p_s} - K_{p_m}K_{v_s})^2 \ge 0 \quad (31)$$

where $B''_{ms} = (K_{v_m} + B_m)(B_s + \delta) + B_m K_{v_s} = B_{ms} + (K_{v_m} + B_m)\delta$. Since (31) must hold for all frequencies $0 \le \omega < \infty$, the stability condition under the ISP termination becomes

$$B''_{ms} \ge \frac{(K_{v_m}K_{p_s} - K_{p_m}K_{v_s})^2}{4K_{p_m}K_{p_s}} \quad (32)$$

which is similar to the first condition for a non-passive termination (30) with an exception that $-a$ is now replaced by δ. In fact, the stability condition for an ISP termination (32) is less restrictive than the stability conditions for passive and non-passive terminations, i.e., (25) and (29)-(30), respectively. For instance, when (27) is violated, and the robot damping terms B_m and B_s are zero, the excess of passivity δ is able to make the stability condition (32) satisfied; this was not the case for the stability condition (25) for a passive termination. This means that flexibility in controller design may result from the any excess of passivity in a teleoperator's termination. The more passive the termination is (larger EOP), the more flexibility we will have in the design of a stabilizing controller.

Table 1 summarizes the stability conditions for different cases of terminations for this PEB teleoperator.

Table 1: Teleoperation system stability condition

Termination	Stability condition
Passive	$B_{ms} \ge \frac{(K_{v_m}K_{p_s} - K_{p_m}K_{v_s})^2}{4K_{p_m}K_{p_s}}$
Non-passive	$B'_{ms} \ge \frac{(K_{v_m}K_{p_s} - K_{p_m}K_{v_s})^2}{4K_{p_m}K_{p_s}}$, $B_{ts} \ge a$
ISP	$B''_{ms} \ge \frac{(K_{v_m}K_{p_s} - K_{p_m}K_{v_s})^2}{4K_{p_m}K_{p_s}}$

8 Conclusions

Llewellyn's absolute stability criterion assumes that the terminations of a teleoperator are passive systems. Through experiments involving a relaxed arm task and a voluntary motion task, we showed that the relaxed arm as a passive system while the arm executing voluntary motion acts as a non-passive system. Using Mobuis transformation, the Llewellyn's absolute stability criterion is modified to accommodate cases where a termination of a teleoperator has excess of passivity (i.e., is strictly passive) or shortage of passivity (is non-passive). The new stability criterion is applied to a position-error-based teleoperator and the resulting stability conditions are compared.

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References

[1] P. F. Hokayem, M. W. Spong, Bilateral teleoperation: An historical survey, Automatica 42 (12) (2006) 2035 – 2057.
[2] F. B. Llewellyn, Some fundamental properties of transmission systems, Proceedings of the IRE 40 (3) (1952) 271 –283.
[3] S. Haykin, Active Network Theory, Addison-Wesley, Reading, MA, 1970.
[4] K. Hashtrudi-Zaad, S. E. Salcudean, Analysis of control architectures for teleoperation systems with impedance/admittance master and slave manipulators, The International Journal of Robotics Research 20 (6) (2001) 419–445.
[5] R. Adams, B. Hannaford, Control law design for haptic interfaces to virtual reality, Control Systems Technology, IEEE Transactions on 10 (1) (2002) 3–13.
[6] A. Haddadi, K. Hashtrudi-Zaad, Bounded-impedance absolute stability of bilateral teleoperation control systems, Haptics, IEEE Transactions on 3 (1) (2010) 15 –27. doi:10.1109/TOH.2009.48.
[7] A. Jazayeri, M. Tavakoli, Revisiting Llewellyn's absolute stability criterion for bilateral teleoperation systems under non-passive operator or environment, in: IEEE/RSJ International Conference on Intelligent Robots and Systems, Algarve, Portugal, 2012, pp. 1624–1630.
[8] M. Tavakoli, A. Aziminejad, R. Patel, M. Moallem, High-fidelity bilateral teleoperation systems and the effect of multimodal haptics, IEEE Transactions on Systems, Man and Cybernetics – Part B 37 (6) (2007) 1512–1528.
[9] N. Hogan, Controlling impedance at the man/machine interface, in: Proceedings of IEEE International Conference on Robotics and Automation, 1989, pp. 1626–1631 vol.3.
[10] R. Adams, B. Hannaford, Stable haptic interaction with virtual environments, IEEE Transactions on Robotics and Automation 15 (3) (1999) 465–474.
[11] R. Lozano, B. Maschke, B. Brogliato, O. Egeland, Dissipative Systems Analysis and Control: Theory and Applications, Springer-Verlag New York, Inc., Secaucus, NJ, USA, 2007.
[12] D. Hill, P. Moylan, Stability results for nonlinear feedback systems, Automatica 13 (4) (1977) 377–382.
[13] J. Slotine, W. Li, Applied nonlinear control, Prentice-Hall Englewood Cliffs, NJ, 1991.
[14] K. J. Astrom, B. Wittenmark, Adaptive Control, 2nd Edition, Addison-Wesley Longman Publishing Co., Inc., Boston, MA, USA, 1994.
[15] M. Dyck, A. Jazayeri, M. Tavakoli, Is the human operator in a teleoperation system passive?, in: IEEE World Haptics Conference, Daejeon, Korea, 2013.
[16] W. H. Ku, Stability of linear active nonreciprocal n-ports, Journal of the Franklin Institute 276 (3) (1963) 207–224.
[17] N. Levinson, R. M. Redheffer, Complex Variables, McGraw-Hill, 1970.
[18] J. Bak, D. J. Newman, Complex Analysis, 3rd Edition, Springer, 2010.