Maximizing Transmission Rates for Pairwise Multiway Relay Channel

by

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Abstract

In this dissertation, we study the effect of users' transmission ordering on the common rate and sum rate of pairwise multiway relay channels (MWRCs). As an extension for two-way relay channel (TWRC), MWRC has been proposed to improve the spectral efficiency in wireless networks. In a pairwise scheme, a set of pairs, known as ordering, is defined that represents the users' transmission schedule. Each pair of users form a TWRC and simultaneously send their data to the relay in an uplink phase. There are different strategies for the relay to form the downlink message. We consider decode-and-forward and functional-decode-forward relaying strategies for our study. We find the ordering that achieves the maximum efficiency of the pairwise MWRC. To find transmission orderings that maximize the common rate and sum rate of the system, we first develop a graphical model for the data transmission in a pairwise MWRC. Using the proposed graphical model, we, then, find the necessary and sufficient conditions for an ordering to be feasible (i.e., allows for successful decoding). Using this model, we finally find the optimal orderings that achieve the maximum common rate and sum rate of the system, respectively. Closed form expressions for the maximum achievable common rate and sum rate are also found. Computer simulations are presented for better illustration and comparison between the rate metrics of the proposed optimal orderings and random orderings.

To Hossein

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List of Abbreviations

List of commonly used abbreviations

AF	Amplify-and-Forward
AWGN	Additive White Gaussian Noise
CF	Compress-and-Forward
DF	Decode-and-Forward
FDF	Functional-Decode-Forward
MIMO	Multiple-Input Multiple-Output
MWRC	Multiway Relay Channel
TWRC	Two-Way Relay Channel

Chapter 1

Introduction

Keeping records of real events and sharing them has been a matter of interest since a long time ago. Lots of effort has been put into this matter so that one can record a beautiful scene as a digital image, an aesthetically pleasing melody as a music file or a more sophisticated event as a movie. These are all examples of what is called *data* today. The demand for data sharing includes not only sharing of recorded events, but also real time sharing applications. Data sharing is not possible without having proper data communication protocols and equipment. As an engineering problem, data communication should be first modeled mathematically. Communication theory pursues the modelling of different aspects of data communication [1, 2].

1.1 Communications Theory

The central problem in communication theory is to find an efficient way for a single transmitter to transmit its message to a receiver [1, 2, 3]. This problem, by itself, brings up many other questions including but not limited to the following:

- How should one describe the message?

- What resources are available to the transmitter?
- How fast does the data transmission process take place?
- How reliable is the received message at the receiver?

The classical theory of communication can successfully formulate the above questions in the language of mathematics. In fact, this is done by the father of information theory, Claude Shannon, in his paper *"A Mathematical Theory of Communication"* [4]. In this 60 years old paper, Shannon provides a mathematical framework which is still being used widely. More specifically, in this paper he provides:

- The basics of information theory as a tool to describe the messages in terms of information,
- The available bandwidth, time and signal power as the available resources,
- The concept of data transmission rate as a measure of transmission speed,
- Probability of error as a measure of reliability.

As an example, one realization of the traditional question mentioned at the beginning, and perhaps the most widely used communication scenario, is communication between mobile phones. A mobile handset (transmitter) wants to transmit its owner's voice (the massage) to another mobile phone (the receiver). However, a one-to-one communication model is too simple to be useful in this case. For example, in reality, even when two mobile phone users are speaking to each other on their phone a lot of middle *nodes* (communication devices existing in a network) are facilitating this communication (e.g., the base station, or network switches between base stations) [5]. Therefore, more sophisticated models are proposed and used. *Cooperative communication* is one such case [6].

1.2 Cooperative Communications

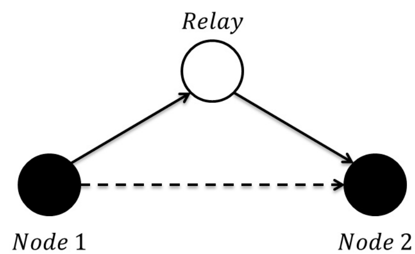

Figure 1.1: The fundamental relay channel.

As the name suggests, in cooperative communications some of the nodes can help other nodes to improve efficiency [7]. There might also be some *dedicated relays* that are placed only to facilitate communications for other nodes. These relays increase the degrees of freedom in the system and thus increase the *spectral efficiency* defined as data transmission rate per bandwidth unit. For instance, a *relay channel* [8, 9] consists of a transmitter, a receiver and at least one relay node, as shown in Figure 1.1. Recently, relay channels have attracted lots of interest in the literature [10, 11, 12, 13, 14]. For example, *two-way relay channel* (TWRC), in which two users exchange their data by means of a common relay terminal, has been studied in detail [15, 16, 17, 18]. Introducing the concept of *multiway relay channel* (MWRC) [19, 20], cooperative communications goes even further by describing a system wherein more than two users receive service from a common relay. MWRC is a multi-user communication system in which users share their data with the help of a relay. Similar to TWRC, the network structure makes it possible for the relay to benefit from *network coding* [16, 21, 22, 23]. Thus, MWRC can potentially improve the spectral efficiency in data sharing applications.

1.3 Summary of Contributions and Dissertation Organization

1.3.1 Motivation

In a special case of multiway relaying, known as *pairwise* multiway relaying, users transmit their data to the relay in pairs. Thus, analysis of pairwise MWRC is based on TWRC. Not only does pairwise relaying have a low decoding complexity, but it also offers interesting capacity achieving properties in various MWRC setups [24, 25]. For instance, it has been shown that pairwise multiway relaying along with rate splitting and joint source-channel decoding achieves the capacity region of MWRC over finite fields [25]. However, one open problem is that which users should be paired together in order to achieve the maximum efficiency of the pairwise MWRC. Considering common rate and sum rate of the system as measures of efficiency, we address this problem in this thesis for different practical setups.

1.3.2 Summary of Contributions

In this dissertation, we address the effect of ordering for pairwise MWRC scenarios. More precisely, we consider pairwise *decode-and-forward* (DF) and *functional-decode-forward* (FDF) scenarios where there is no restriction on the number of uplink transmissions by the users. In this case, we first discuss that there exist N^{N-2} distinct orderings. Thus, finding the optimal ordering through brute-force search becomes expensive for large N. Then, under a reasonable assumption on user's SNR, we analytically find the optimal orderings to maximize the common rate and the sum rate. Also, closed form expressions for the maximum achievable common rate and sum rate in both DF and FDF schemes are presented.

In an MWRC network, there are cases where the available transmit power of users may be limited. This limit imposes upper bounds on the number of uplink transmissions that a user can participate in. For this situation, a more general case is considered in which an upper bound on the number of pairs that a user can belong to is specified. Optimal orderings that achieve the maximum common rate under these restrictions are found for both DF and FDF. When the limit is two, the solutions simplify to those of [26] and [27]. A part of this work has been published in [28].

1.3.3 Organization of the Thesis

In Chapter 2, we describe the system model and provide the needed background on TWRCs. Then, we describe the pairwise MWRC. Furthermore, a graphical representation for pairwise MWRC and the necessary and sufficient conditions for an ordering to be feasible are described. In Chapter 3, we formulate the common rate problem for pairwise MWRC. In addition, the orderings that maximize the common rate for DF and FDF relaying are found. Moreover, the performance of the optimal ordering is compared to those of random orderings using computer simulations. In Chapter 4, we define the sum rate maximization problem and solve it for both DF and FDF relaying in a pairwise MWRC. Again, we compare the performance of our proposed orderings with those of random orderings via simulations. Chapter 5 concludes this work and provides some directions for future research.

Chapter 2

Background

In this chapter, we describe the so called *two-way relay channel* (TWRC) [15, 18, 29]. Further, we explain existing relaying strategies for TWRC. The achievable rates of these relaying strategies are also presented. Then, we present a general system model for MWRC. Finally, pairwise relaying for MWRC and its relation to TWRC is described.

2.1 Two-Way Relay Channel

Consider a relay channel as shown in Figure 2.1. Two users, say Node 1 and Node 2, want to communicate with each other and share their data. It is assumed that users cannot communicate directly, that is, they can only communicate via the relay.

2.1.1 System Parameters

We denote the channel input at Node i by x_i. The data at Node i, shown by X_i, is a vector with elements chosen from a field $\mathbb{F}$. Assume that the channel from Node i to the relay has *additive white Gaussian noise* (AWGN) with noise variance σ^2 and gain g_i. Channels are assumed to be reciprocal. In other words, channel from the relay to Node i has the same characteristics as channel from

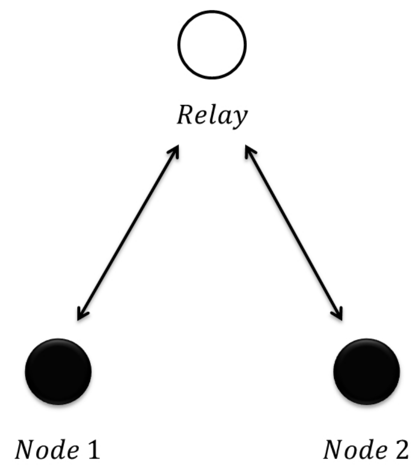

Figure 2.1: A typical two-way relay channel

Node i to the relay. The transmit power of the relay is assumed to be P_r. Also, the transmit power of Node i during each transmission period is assumed to be P_i. Then, we define a *signal to noise ratio* (SNR) for Node i as $\gamma_i \triangleq \frac{P_i|g_r|^2}{\sigma^2}$. The received signal at the relay will be

$$y_r = g_1x_1 + g_2x_2 + n_r \tag{2.1}$$

where n_r is a zero-mean Gaussian noise with variance σ^2. The relay input to the channel is denoted by x_r and the received signal at Node i is shown by y_i. We denote the channel noise at Node i by n_i which is assumed to be a zero-mean Gaussian noise with variance σ^2. n_r, n_1 and n_2 are assumed to be independent. We say that R_1 is an achievable rate for Node 1 if there exists a communication scheme such that Node 1 transmit its data with rate R_1 bits per second and the maximal probability of error in decoding X_1 at Node 2 tends to zero. Similarly, we define the achievable rate for Node 2 as R_2.

2.1.2 Relaying Strategies

There are several relaying protocols to facilitate the communications via the relay. The first approach is the so called *one way relaying* (OWR) [7, 30]. In

OWR, the users share their data in four time slots. In the first time slot X_1 is sent to the relay and $x_2 = 0$. The relay then forwards X_1 to Node 2 in the second time slot. This is then followed by transmission of X_2 to the relay during the third time slot, while $x_1 = 0$. In the fourth time slot, the relay transmits X_2 to Node 1. Another relaying approach that is based on the idea of *network coding* is as follows: in the first time slot, X_1 is transmitted to the relay and $x_2 = 0$. In the second time slot, Node 2 transmits X_2 to the relay and Node 1 transmit nothing. In the third time slot, the relay transmits $X_1 \oplus X_2$ to both Node 1 and Node 2. This means that if X_1 and X_2 are vectors with elements chosen from a field $\mathbb{F}$, then the relay transmits $X_1 \oplus X_2$, where $\oplus$ denotes element-wise summation of X_1 and X_2 over $\mathbb{F}$. Thus, this approach needs only three time slots and have a better spectral efficiency than the OWR approach. The spectral efficiency can be further improved as in what we know as TWRC [31, 32, 33]. For TWRC, in the first time slot, Node 1 and Node 2 simultaneously send their data to the relay. Then, in the second time slot the relay broadcasts the summation of the two users' messages to both of them. Thus, TWRC only needs two time slots to perform the whole transmission. Figure 2.2 illustrates these three strategies.

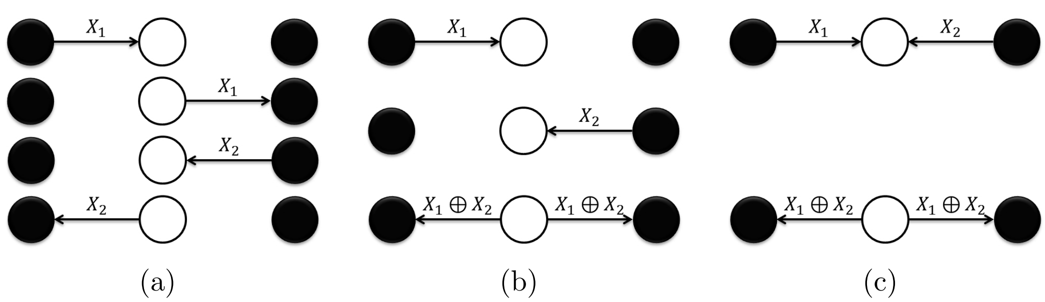

Figure 2.2: Three different relaying approaches for a relay channel.

In TWRC, depending on relay's strategy for forming $X_1 \oplus X_2$, several forwarding strategies are developed, namely *amplify-and-forward* (AF), *decode-and-forward*, *compress-and-forward* (CF) and *functional-decode-forward* [15, 16, 19, 34]. In the next few sections, we describe these relaying strategies in more detail. However, in the rest of this thesis, we deal with DF and FDF strategies only.

2.1.3 Amplify-and-Forward

For a TWRC with AF relaying strategy, both users send their data to the relay simultaneously. Then, the relay amplifies the received signal and broadcasts it back to both users. The amplification factor, α, is chosen such that the transmit power of the relay does not exceed P_r. Thus,

$$x_r = \alpha y_r = \alpha(g_1 x_1 + g_2 x_2 + n_r) \tag{2.2}$$

in which

$$\alpha = \sqrt{\frac{P_r}{|g_1|^2 P_1 + |g_2|^2 P_2 + \sigma^2}} \tag{2.3}$$

After subtracting its own signal, the received signal at at Node 1 is

$$\begin{aligned} y_1 &= \alpha g_1(g_2 x_2 + n_r) + n_1 \\ &= \alpha g_1 g_2 x_2 + (\alpha g_1 n_r + n_1) \end{aligned} \tag{2.4}$$

which is similar to the signal model for an AWGN channel with noise $\alpha g_1 n_r + n_1$. In a TWRC with a pair of users, a rate tuple (R_1, R_2) is said to be achievable if R_i, for both $i = 1, 2$, is an achievable rate for Node i. Thus, for a TWRC with AF relaying R_1 and R_2 are limited by the following achievable bounds [35]:

$$R_i \leq \frac{1}{2} \log_2 (1 + \Gamma_i) \tag{2.5}$$

$$R_1 + R_2 \leq \frac{1}{2} \log_2 (1 + \Gamma_1 + \Gamma_2) \,. \tag{2.6}$$

in which $\Gamma_i = \frac{|\alpha g_1 g_2|^2 P_j}{|(\alpha g_i + 1)|^2 \sigma^2}$ such that $\{i, j\} = \{1, 2\}$.

2.1.4 Decode-and-Forward

For a TWRC with DF relaying strategy, the relay first decodes the received messages completely and then broadcasts the summation of them to the users. In other words, it can be assumed that in the *uplink* phase Node 1 and Node 2 transmit their data to the relay. The relay first decodes both X_1 and X_2 and, in the consecutive *downlink* phase, the relay transmits $X_1 \oplus X_2$ to both users.

It is assumed that the data rates are limited by the uplink phase, not by the downlink phase. This is usually the case in practical wireless systems where the users are low-power small transmitters and the relay is a powerful station. Also, in various practical configurations that the transmit power of the relay increases with the number of users, it has been shown that data rates are limited by the uplink phase [35].

For a TWRC with DF relaying, R_1 and R_2 are limited by the following achievable bounds [8, 36]:

$$R_i \leq \frac{1}{2}\log_2\left(1+\gamma_i\right) \tag{2.7}$$

$$R_1 + R_2 \leq \frac{1}{2}\log_2\left(1+\gamma_1+\gamma_2\right). \tag{2.8}$$

2.1.5 Compress-and-Forward

CF was first proposed by Cover and El Gamal in [37]. For CF, relay quantizes its received signal in the uplink phase and then compresses it. Denoting the quantized version of y_r by $\hat{y}_r$, we have

$$\hat{y}_r = g_1 x_1 + g_2 x_2 + n_r + n_q \tag{2.9}$$

where, n_q is the quantization error. In the next step, the relay applies a source coding scheme on $\hat{y}_r$ and forms x_r. Several approaches have been proposed for forming the relay output message. For details and achievable rates of these approaches, the interested reader is referred to [18, 29, 37, 38] and references therein.

2.1.6 Functional-Decode-Forward

FDF is first proposed by Nam *et al.* [17]. For a TWRC with FDF relaying, the relay decodes the sum of the received signals and sends it back to the users [16, 34]. In other words, relay tries to find the sum of the two received signals directly without decoding each of them. Nam *et al.* showed that a realization of FDF with nested lattice codes can achieve transmission rates upto within 1/2 bit of the capacity of TWRC [17, 39]. Considering AWGN links from users to the relay and with the same assumptions as the previous section, R_i in an FDF TWRC is limited by the following achievable upper bounds:

$$R_1 \leq \max\left\{0, \frac{1}{2}\log_2\left(\frac{\gamma_1}{\gamma_1+\gamma_2}+\gamma_1\right)\right\}, \tag{2.10}$$

$$R_2 \leq \max\left\{0, \frac{1}{2}\log_2\left(\frac{\gamma_2}{\gamma_2+\gamma_1}+\gamma_2\right)\right\}. \tag{2.11}$$

To the best of our knowledge, these are the largest achievable bounds existing for R_i with FDF relaying.

2.2 Multiway Relay Channel

We consider an MWRC in which N users, namely $U_1, U_2, \ldots, U_N$, perform full data exchange. This means that each user wants to decode all other users data, denoted by X_is. It is assumed that users cannot communicate directly, meaning that they can communicate only via the relay $\mathcal{R}$. The channels from U_i to $\mathcal{R}$ are denoted by $C_{i\mathcal{R}}$. We denote the channel gain for $C_{i\mathcal{R}}$ by $g_{i\mathcal{R}}$ and assume that all channels are AWGN with equal noise variance σ^2. Figure 2.3 illustrates an MWRC with N users.

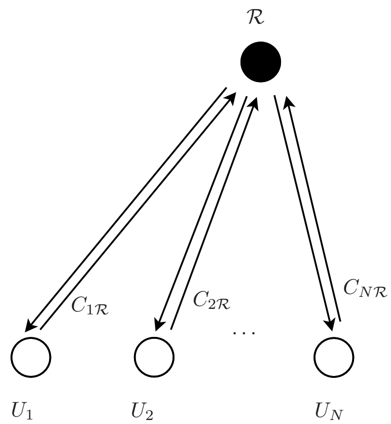

Figure 2.3: A typical multiway relay channel.

2.2.1 Pairwise Relaying

In a pairwise transmission scheme, users are grouped in M pairs. These pairs are not necessarily disjoint meaning that a specific user can appear in more than one pair. A division of the users to subsets of pairs is called an *ordering* of the users and is denoted by $O = \{\{u_{11}, u_{12}\}, \ldots, \{u_{M1}, u_{M2}\}\}$ where $u_{\ell 1}$ and $u_{\ell 2} \in \{U_1, U_2, \ldots, U_N\}$. We presume a half-duplex communication in which a full data exchange consists of M uplink phases and each uplink phase is followed by a downlink phase. Each of the M pairs transmit in one and only one uplink phase. In each downlink phase, the relay transmits the sum of the two messages received in the last uplink phase to all of the users. This means that if X_i and X_j are vectors with elements chosen from a field $\mathbb{F}$, then the relay decodes $X_i \oplus X_j$ where $\oplus$ means element-wise summation of X_i and X_j over $\mathbb{F}$. These pairwise transmissions continue until the last pair of the ordering. We assume that all users know which pair is associated with the received signals. Having its own data, each user is able to decode the data of others at the end of one round of full data exchange. In the rest of this work, we assume that users have an unlimited amount of data to transmit such that none of them will reach the end point before the others.

The transmit power of U_i during an uplink phase is assumed to be P_i.

Furthermore, an uplink signal to noise ratio for user U_i, namely γ_i, is defined as $\gamma_i \triangleq \frac{P_i|g_{i\mathcal{R}}|^2}{\sigma^2}$. Without loss of generality, we assume that

$$\gamma_N \geq \gamma_{N-1} \geq \cdots \geq \gamma_1 > 0.$$

It is also assumed that the data rates are limited by the uplink phase, not by the downlink phase.

For a wireless MWRC with DF relaying strategy, the relay first decodes the received signals completely and then broadcasts the sum of them to the users. In other words, assume that at the ℓth uplink phase $u_{\ell 1} = U_i$ and $u_{\ell 2} = U_j$ are paired and transmit their data to the relay. Denoting the data of $u_{\ell 1}$ and $u_{\ell 2}$ by X_i and X_j, the relay decodes both X_i and X_j and in the consecutive downlink phase, the relay transmits $X_i \oplus X_j$ to all of the users. For FDF relaying, the relay decodes the sum of the received signals and sends it back to the users. In other words, relay tries to find the sum of the two received signals directly without decoding each of them. We consider both FDF and DF strategies in the rest of this work.

Assume a pairwise relaying with $N = 3$ users and

$$O = \{\{U_1, U_2\}, \{U_2, U_3\}, \{U_1, U_3\}\} \tag{2.12}$$

as shown in Figure 2.4. After one round of communication, assuming ideal channels, each user has the following set of equations:

$$\begin{aligned} X_1 \oplus X_2 &= X_r^{(1)} \\ X_2 \oplus X3 &= X_r^{(2)} \\ X_3 \oplus X_1 &= X_r^{(3)} \end{aligned} \tag{2.13}$$

where $X_r^{(1)}$, $X_r^{(2)}$ and $X_r^{(3)}$ are the messages transmitted by the relay to the users. As a result, $X_r^{(1)}$, $X_r^{(2)}$ and $X_r^{(3)}$ are known at each user. Considering that

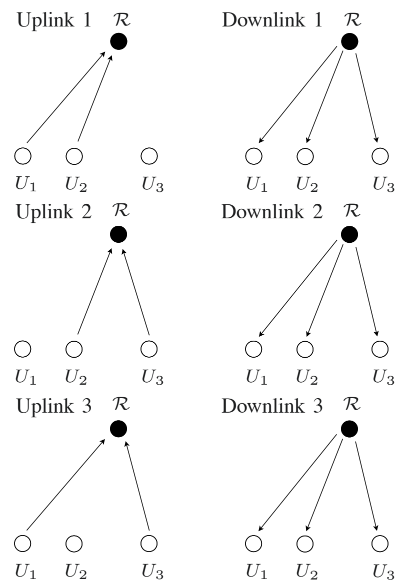

Figure 2.4: A pairwise ordering with $M = N = 3$.

each user knows its own data, one can easily see that the system of equations at each user is solvable. In a general N-MWRC, if the system of equations at each user is solvable, we say that the corresponding ordering is *feasible*. Feasibility implies that M should not be less than $N-1$ because each user needs to find $N-1$ other users' messages. As an example of an ordering with $M = N-1$, we could arbitrarily remove one of the pairs in the previous example and the system of equations at each user is still feasible.

In a pairwise MWRC with M pairs, a rate tuple $(R_1, R_2, \ldots, R_N)$ is achievable if any U_i can reliably (with arbitrarily small probability of error) transmit its data to all other users with rate R_i after each round's M uplink and downlink phases. The achievable rate tuple depends on the transmit power of the users and the relay as well as the channel gains, the noise power and relaying strategy.

When U_i participates in a pairwise transmission, say with U_j, during an uplink phase, for DF relaying R_i and R_j are limited by the following achievable bounds [8]

$$R_i \leq \frac{1}{2M}\log_2\left(1+\gamma_i\right) \tag{2.14}$$

$$R_j \leq \frac{1}{2M}\log_2\left(1+\gamma_j\right) \tag{2.15}$$

$$R_i + R_j \leq \frac{1}{2M}\log_2\left(1+\gamma_i+\gamma_j\right) \tag{2.16}$$

For an FDF MWRC, R_i in an FDF MWRC is limited by the following achievable upper bounds according to [17, 39, 28]

$$R_i \leq \max\left\{0, \frac{1}{2M}\log_2\left(\frac{\gamma_i}{\gamma_i+\gamma_j}+\gamma_i\right)\right\}. \tag{2.17}$$

The maximum achievable upper bound on R_i can be found by calculating upper bounds, given by (2.14)-(2.17), for R_i over all pairs that U_i is part of and then taking the minimum of these bounds. Instead of studying individual rates, we study the common rate and sum rate of the system.

2.2.2 Common Rate and Sum Rate of Pairwise MWRC

Common rate and sum rate of an MWRC are two of the most famous metrics that are used in the literature to measure the performance of different relaying scenarios. Here, we define them as follows:

Definition 1. *For a pairwise MWRC communicating with the rate tuple* $\boldsymbol{R} = (R_1, \ldots, R_N)$*, we define the common rate,* C_R*, and the sum rate,* S_R*, as follows*

$$C_R \triangleq \min_i R_i, \tag{2.18}$$

$$S_R \triangleq \sum_{i=1}^{N} R_i. \tag{2.19}$$

As pointed out in [28] and according to (2.14)-(2.17), one can verify that the ordering of the users affects these upper bounds and consequently C_R and S_R. Using Definition 1, we find the orderings that attains the maximum possible

C_R and S_R in Chapters 3 and 4, respectively, for both DF and FDF pairwise multiway relaying.

2.2.3 Client Graph

Here, we introduce the concept of *client graph* which provides a convenient representation of an ordering for pairwise MWRC. As we discuss in the following, the client graph is a useful mathematical tool facilitating the comparison of common rate and sum rate of different orderings.

An undirected graph G is an ordered pair $G = (V, E)$ comprising a set $V = \{v_1, v_2, \dots, v_K\}$ of vertices together with a set E of edges. Each edge is a 2-element subset of V. For simplicity, if $\{v_i, v_j\} \in E$, we say $v_i v_j \in E$. If $v_i v_j \in E$, we say v_j is *adjacent* to v_i. The set of adjacent vertices of v_i, denoted by A_i^G, is called the set of neighbors of v_i. Also the *degree* of node v_i is $deg(v_i) = |A_i^G|$. The adjacency matrix of G, denoted by $\mathcal{A} = (a_{ij})$, is a $K \times K$ matrix in which $a_{ij} = 1$ iff $v_i v_j \in E$; otherwise, a_{ij} is 0. A *path* in G is a sequence of consecutive edges that connects a sequence of vertices. G is called connected if there is at least one path between every pair of its vertices. A non-empty path with the same endpoints is called a *cycle*.

For a given pairwise ordering O, we define a *client graph* $G_O(V, E)$ where $V = \{v_1, v_2, \dots, v_N\}$ is the set of vertices. There is a vertex v_i in V corresponding to each user U_i. There exists an edge $e = v_i v_j \in E$ iff $\{U_i, U_j\} \in O$. Note that there is a one-to-one correspondence between all possible client graphs and all possible pairwise orderings.

The overall time/frequency resources consumed in a communication round is directly affected by the number of pairs. As a result, we are interested in identifying feasible orderings with minimum number of pairs which, as we mentioned, is $M = N - 1$. To this end, we state the following theorem. As we mention later in Chapter 3 and 4, no pairwise scheme with the aforementioned configuration can have a better performance in terms of metrics that we consider in this work.

Theorem 1. *An ordering with $M = N-1$ pairs is feasible iff the corresponding client graph is a tree.*

Proof. For the forward direction, note that if the client graph is not a tree, then it has an isolated vertex (a vertex with no neighbors). This means that there is a user that do not participate in any of the pairs. Thus, no one receives any data from this isolated user. This contradicts the feasibility of the ordering. For the backward direction, we use the fact that if the client graph is a tree then there is exactly one path $P_{i,j}$ between any pair of nodes v_i and v_j. Then we show that U_i can decode X_j for any j. Assume that $P_{i,j} = \{v_i v_{i_1}, v_{i_1} v_{i_2}, \ldots, v_{i_n} v_j\}$. The equations corresponding to the edges in this path are:

$$\begin{aligned} X_i \oplus X_{i_1} &= X_r^{(m_1)} \\ X_{i_1} \oplus X_{i_2} &= X_r^{(m_2)} \\ &\vdots \\ X_{i_n} \oplus X_j &= X_r^{(m_n)} \end{aligned} \tag{2.20}$$

in which $X_r^{(m_k)}$ represent the relay message at the corresponding downlink. Manipulating this system of equations, we wind up with

$$X_i \oplus (-1)^{m_n - 1} X_j = \bigoplus_{k=1}^{n} (-1)^{m_k - 1} X_r^{(m_k)}. \tag{2.21}$$

Knowing its own data, U_i can decode X_j for all $j \neq i$. Thus, if the client graph is a tree the corresponding ordering is feasible. □

In the rest of this thesis, we assume $M = N - 1$ and use the terms *client tree* and client graph, interchangeably. We denote the maximum achievable common rate and sum rate for a client graph G_O by $C_R(G_O)$ and $S_R(G_O)$, respectively.

Chapter 3

Common Rate Maximization for Pairwise Multiway Relay Channels

In this chapter, first we state the common rate maximization problem. Then we present the optimal orderings that achieve the maximum common rate for a pairwise MWRC with AWGN channels. Additionally, a more general case is considered in which an upper bound on the number of pairs that a user can belong to is specified. Optimal orderings that achieve the maximum common rate under these restrictions are found for both DF and FDF. Finally, simulation results are presented for better illustration of the results.

3.1 Problem Statement

By common rate maximization problem, we mean finding the ordering with maximum $C_R(G_O)$. We focus on this problem under two setups, namely *unconstrained common rate maximization* and *constrained common rate maximization problem.* Denoting the set of all feasible orderings whit $\mathcal{O}$, we formulate an unconstrained common rate maximization problem as:

$$O_{\text{UCR}} = \underset{O \in \mathcal{O}}{\operatorname{argmax}}\, C_R(G_O) \tag{3.1}$$

In a pairwise MWRC, with $G_O(V, E)$ as its client tree, the average transmit power of U_i over a complete full-data exchange round may be limited. This could potentially force a bound on the number of transmissions by each user. Thus, in the second common rate maximization scenario, we set a bound on the number of pairs a user participate in. We denote this bound for U_i by B_i and call it *transmission bound* of U_i. We assume that $B_i \geq 2$ for all $i = 1, 2, \ldots, N$. Thus, a constrained common rate maximization problem is defined as:

$$\begin{aligned} O_{\text{CCR}} = & \underset{O \in \mathcal{O}}{\operatorname{argmax}}\, C_R(G_O) \\ & \textit{subject to } \textit{deg}(V_i) \leq B_i. \end{aligned} \tag{3.2}$$

This means that, given B_i's, we want to find an ordering that achieves the maximum C_R in an MWRC.

In order to solve a common rate maximization problem, we need to find a client graph G_O with greatest $C_R(G_O)$ among all client trees. One way is to search over all of the possible client trees and find the one that maximizes $C_R(G_O)$. According to Cayley's formula [40], this necessitates us searching over N^{N-2} client trees which is impractical even if the number of users is not very large. This motivates us to develop efficient solutions for finding the optimal client trees without searching all possible client trees.

3.2 Problem Solution

In this section, we provide solutions to the unconstrained common rate maximization and constrained common rate maximization. We solve these for both DF and FDF relaying.

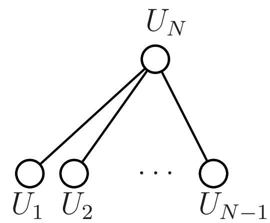

Figure 3.1: Client tree that maximizes $C_R(G_O)$ for a pairwise MWRC with DF relaying.

3.2.1 Unconstrained Common Rate Maximization for DF Relaying

Considering (2.14), (2.15) and (2.16), our goal is to find the ordering that achieves the maximum $C_R(G_O)$. Theorem 2 provides the solution to this maximization problem.

Theorem 2. *For an unconstrained common rate problem, the optimal ordering is*

$$O_{UCR} = \{\{U_1,U_N\},\{U_2,U_N\},\{U_3,U_N\},\ldots,\{U_{N-1},U_N\}\}$$

and the maximum achievable common rate is

$$C_R(G_O) = \frac{1}{4(N-1)} min \left\{\log_2\left(1+\gamma_1+\gamma_N\right), 2\log_2\left(1+\gamma_1\right)\right\}. \tag{3.3}$$

Proof. We first define an $N \times N$ weight matrix $W = (w_{i,j})$ as

$$w_{i,j} = \frac{1}{2(N-1)} \min\left\{\log_2\left(1+\gamma_j\right), \frac{1}{2}\log_2\left(1+\gamma_i+\gamma_j\right)\right\} \tag{3.4}$$

for all pairs of i and j with $i \neq j$ and $w_{i,i} = 0$. According to this definition and our assumption that

$$\gamma_N \geq \gamma_{N-1} \geq \cdots \geq \gamma_1 > 0 \tag{3.5}$$

$w_{i,j}$ is an increasing function of both j and i, for $j \neq i$. For a given client graph G_O with adjacency matrix $\mathcal{A} = (a_{i,j})$, we have

$$C_R(G_O) = \min \{w_{i,j} | a_{i,j} \neq 0\}. \tag{3.6}$$

The client graph G_O is connected and $\mathcal{A}$ is symmetric. As a result, for every $i = 1, 2, \ldots, N-1$, there exist at least one j such that $a_{i,j} = 1$. If we define the minimum value of row i in W, with respect to G_O, as $r_i = \min\{w_{i,j} | a_{i,j} \neq 0; j = 1, 2, \ldots, N\}$ for $i = 1, 2, \ldots, N$, and also define the minimum value of column k with respect to G_O as $c_k = \min\{w_{j,k} | a_{j,k} \neq 0; j = 1, 2, \ldots, N\}$ for every $k = 1, 2, \ldots, N$, we can rewrite equation (3.6) as:

$$C_R(G_O) = \min \{c_1, c_2, \ldots, c_N\} \tag{3.7}$$

and

$$C_R(G_O) = \min \{r_1, r_2, \ldots, r_N\}. \tag{3.8}$$

The ordering which achieves the maximum C_R is the one which maximizes the right hand side of equations (3.7) and (3.8). However, we know that $c_k \leq w_{N,k}$ and $r_i \leq w_{i,N}$. The ordering $O = \{\{U_1, U_N\}, \{U_2, U_N\}, \ldots, \{U_{N-1}, U_N\}\}$ maximizes both c_ks and r_is by picking the most significant entries in each column and each row, simultaneously. Consequently, it achieves the maximum C_R in a DF MWRC. As a result, the common rate is given by

$$C_R(G_O) = \min\{w_{1,N}, w_{N,1}\} \tag{3.9}$$

which is equivalent to (3.3). $\square$

The intuition behind this theorem is that the transmission rate of each user may increase if we increase the SNR of its pair. In the optimal ordering, all users (except the best user in terms of SNR) get their highest possible rate, while the best user gets its worst possible rate. Yet, our theorem shows that this ordering is the optimal one in terms of common rate. Figure 3.1 illustrates the optimal ordering for a DF MWRC that achieves the maximum C_R. It shows that for having the maximum possible common rate, the user with the highest SNR should be paired with all of the other users.

3.2.2 Constrained Common Rate Maximization for DF Relaying

Given the upper bounds B_i's, the optimal ordering for constrained common rate maximization problem is found through the following theorem.

Theorem 3. *In an MWRC with DF relaying and transmission bounds $B_i \geq 2$, for $i = 1, 2, \ldots, N$, the ordering that achieves the maximum C_R is given by*

$$O_{CCR} = \bigcup_{i=0}^{m-1} \{\{U_{b(N-i)+j}, U_{N-i}\} |\ j = 1, 2, \ldots, B_{N-i}\} \cup \\ \{\{U_{b(N-m)+1}, U_{N-m}\}, \{U_{b(N-m)+2}, U_{N-m}\}, \ldots, \{U_{N-m+1}, U_{N-m}\}\} \tag{3.10}$$

where,

$$b(N-i) = \begin{cases} \sum_{k=0}^{i-1} (B_{N-k} - 1) & i \neq 0 \\ 0 & i = 0 \end{cases}$$

and m is the smallest integer such that $b(N-m) + B_{N-m} \geq N - m - 1$.

Proof. We prove the theorem by contradiction. Assume that the ordering O given by (3.10) is not optimal and there is another ordering O' that has a higher C_R, i.e., $C_R(O') > C_R(O)$. Assume that

$$C_R(O) = \frac{1}{4(N-1)} \log_2 \left(1 + \gamma_{b(N-i)+1} + \gamma_{N-i}\right) \tag{3.11}$$

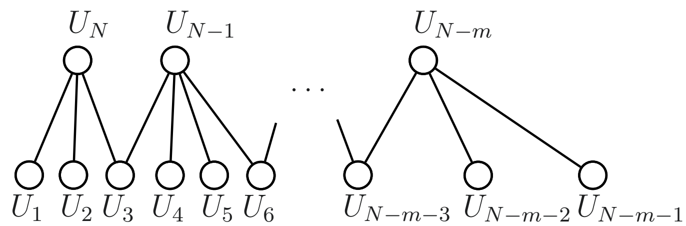

Figure 3.2: Client tree that maximizes $C_R(G_O)$ for a pairwise MWRC with DF relaying.

for an i such that $m \geq i \geq 1$. Then, according to the monotonicity of columns of W in (3.4), $U_{b(N-i)+1}$ can not be paired with any of $\{U_j | j < N - i\}$ in O'. Monotonicity of rows and columns of W also implies that none of the U_j's for $j \in \{1, 2, \ldots, b(N-i)\}$, can be paired with U_k when $k \leq N - i$ in O'. As a result, $G_{O'}$ should be disconnected which contradicts Theorem 1. $\square$

According to Theorem 3, in order to construct the optimal client tree, we should keep pairing the unpaired user with smallest SNR with U_N till $deg(V_N) = B_N$. Then we do the same thing for users with smallest SNRs that are not paired yet, but this time we pair them with U_{N-1} until $deg(V_{N-1}) = B_{N-1} - 1$. To keep the graph connected, we also pair U_{B_N} and U_{N-1}. We continue this process, until the graph is connected. Figure 3.2 illustrates the optimal ordering for a DF MWRC with $B_N = 3$, $B_{N-1} = 4$ and $B_{N-m} \geq 3$.

3.2.3 Unconstrained and Constrained Common Rate Maximization for FDF Relaying

Considering (2.17), we find the ordering that achieves the maximum $C_R(G_O)$ for FDF relaying. Theorem 4 gives the optimal ordering for this scenario. Interestingly, the optimal ordering in this case satisfies $deg(v_i) \leq 2$ for all $i \in \{1, 2, \ldots, N\}$. Subsequently, it is also the optimal ordering for the energy-limited case, i.e. the constrained common rate maximization.

Theorem 4. *The ordering given by*

$$O_{UCR} = O_{CCR} = \{\{U_1, U_2\}, \{U_2, U_3\}, \{U_3, U_4\}, \ldots, \{U_{N-1}, U_N\}\} \tag{3.12}$$

achieves the maximum common rate in an MWRC with FDF relaying and the maximum achievable common rate is

$$C_R(G_O) = \max\left\{\min_{i\in\{1,\ldots,N\}}\left\{\frac{1}{2(N-1)}\log_2\left(\gamma_i + \frac{\gamma_i}{\gamma_i+\gamma_{i+1}}\right)\right\}, 0\right\}. \tag{3.13}$$

Proof. Here, by an optimal tree, we mean a client tree that achieves the maximum C_R with respect to (2.17). There are two statements regarding (2.17) which we use to prove the theorem:

1. The function $f(x) = x\left(1 + \frac{1}{x+\alpha}\right)$ is an increasing function of x for $\alpha > 0$.
2. The function $g(x) = \left(1 + \frac{1}{\alpha+x}\right)$ is a decreasing function of x.

In this proof, without loss of generality we assume that $\gamma_1 > 1$. Discussion for the general case that some of the pairs transmit with rate $R = 0$ is straightforward. Given a client tree, $G_O(V, E)$, with an FDF MWRC, we have

$$C_R(G_O) = \min_{i,j}\left\{\frac{1}{2(N-1)}\log_2\left(\gamma_i + \frac{\gamma_i}{\gamma_i+\gamma_j}\right)\right\}. \tag{3.14}$$

where $\gamma_i \le \gamma_j$ and $v_iv_j \in E$. Using (3.14), we prove the following lemma.

Lemma 1. *There exists an optimal tree, $G_O(V, E)$, in which $A_1^{G_O} = \{v_2\}$.*

Proof. We adapt $G_{O'}(V, E')$ from G_O such that we disconnect all of the neighbors of v_1 from v_1 and connect them to v_2. We also make v_1 and v_2 neighbors. More precisely,

$$E' = (E - \{v_1v_i | v_i \in A_1^{G_O}\}) \cup \{v_2v_i | v_i \in A_1^{G_O}; i \neq 2\} \cup \{v_1v_2\} \tag{3.15}$$

Because of monotonicity of $f(x)$ and $g(x)$, to verify that $C_R(G_O) \leq C_R(G_{O'})$, we just need to show

$$\gamma_1 \left(1 + \frac{1}{\gamma_1 + \gamma_{\min}}\right) \leq \gamma_2 \left(1 + \frac{1}{\gamma_2 + \gamma 1}\right) \tag{3.16}$$

where, $\gamma_{min} = \min\{\gamma_i | v_i \in A_1^{G_O}\}$. After some manipulation, we find that (3.16) is equivalent to

$$0 \leq (\gamma_2 - \gamma_1)(\gamma_1 + \gamma_{\min})(\gamma_2 + \gamma_1) + \gamma_2\gamma_{\min} - \gamma_1^2 \tag{3.17}$$

which, according to the fact that $\gamma_1 \leq \gamma_{\min}$, is true. □

We prove the theorem by induction. If $N = 2$ the theorem obviously holds. Now, assume that the statement of the theorem holds for every FDF MWRC with $N = k$. We show that it also holds for any FDF MWRC with $N = k + 1$. For $N = k + 1$, according to Lemma 1, there exists an optimal tree $G_O(V, E)$ in which $A_1^{G_O} = \{v_2\}$. From equation (3.14), we also have:

$$\begin{aligned} C_R(G_O) = &\min_{i,j} \left\{ \frac{1}{2(N-1)} \log_2 \left(\gamma_i + \frac{\gamma_i}{\gamma_i + \gamma_j}\right) | 1 < i \leq j; v_i v_j \in E \right\} \\ &\cup \left\{ \frac{1}{2(N-1)} \log_2 \left(\gamma_1 + \frac{\gamma_1}{\gamma_1 + \gamma_2}\right) \right\} \end{aligned} \tag{3.18}$$

If the second term in (3.18) is the limiting term in all of the possible client trees with $A_1^{G_O} = \{v_2\}$, the proposed ordering is optimal. Otherwise, maximizing $C_R(G_O)$ is equivalent to maximizing

$$\min \left\{ \gamma_i \left(1 + \frac{1}{\gamma_i + \gamma_j}\right) | 1 < i \leq j; v_i v_j \in E \right\}. \tag{3.19}$$

It is equivalent to maximizing the C_R for $G_{O'}(V', E')$, in which $V' = V - \{v_1\}$ and $E' = E - \{v_1 v_m | v_m \in A_1^{G_O}\}$. According to the induction hypothesis, it

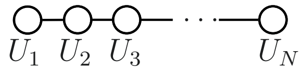

Figure 3.3: Client tree that maximizes $C_R(G_O)$ for a pairwise MWRC with FDF relaying.

happens when

$$O' = \{\{v_2v_3\}, \{v_3v_4\}, \ldots, \{v_{N-1}v_N\}\} \tag{3.20}$$

and as a reslut

$$O = \{\{v_1v_2\}, \{v_2v_3\}, \ldots, \{v_{N-1}v_N\}\} \tag{3.21}$$

□

Figure 3.3 illustrates the optimal ordering for an FDF MWRC that achieves the maximum C_R.

Remark: The proofs given in this chapter are based on two main assumptions. First, we assumed that the transmit power for each user during each uplink phase is fixed to a constant. Second, is the fact that during all uplink phases that U_i participates in them, for one round of full data exchange, U_i always transmit the same packet of information X_i. Based on these assumptions, we can show that there is no feasible ordering with more than $N-1$ pairs with a higher common rate than that of the optimal tree.

To show that, first notice that from the proof of Theorem 1 we can conclude that a general ordering with any arbitrary number of pairs is feasible iff the corresponding client graph is connected. From this, we can conclude that the client graph of each feasible ordering has at least one spanning tree (i.e., a subgraph that is a tree and includes all nodes.) We want to show that the common rate of a general client graph is not higher than the maximum common rate of its spanning trees.

According to proof of Theorem 2, we can see that for DF relaying the transmission rate of user U_i, when it is paired with U_j, depends on the SNR of U_j in such a way that if we increase γ_j, R_i may increases. As a result, the transmission rate of U_i is *only* affected by the pair with the lowest SNR. In other words, the transmission rate of a user is dominantly affected only by its neighbor with the lowest SNR. As a result, adding more edges to a spanning tree will not increase the common rate in this case. A similar statement can be made for FDF relaying. From the proof of Theorem 4, we can see that the transmission rate of a user is dominantly affected this time *only* by its neighbor with the highest SNR. So, the common rate of one connected client graph can not be higher than the maximum common rate of its spanning trees.

3.3 Simulation Results

In this section, we investigate the performance of the optimal ordering in comparison with random orderings. We use Monte Carlo simulation to average over common rate for the optimal ordering and a randomly selected ordering, for both DF and FDF relaying. For each simulation round, random ordering is selected uniformly at random from all of the feasible client trees. We again, assume that the data rates are limited by the uplink phase. Similar to [27], it is assumed that the channels between the users and the relay are Rayleigh fading. The number of users is set to $N = 4$ and 8. Figure 3.4 and 3.5 depict the comparison between the common rate of the optimal ordering and random ordering for DF and FDF relaying, respectively, in low to high SNR regimes. The upper bounds are given by max-flow min-cut theorem [8]. In order to illustrate the difference between optimal ordering and random orderings, we define the *common rate gap* of random ordering and optimal ordering as $G_C = \frac{C_R(G_O) - C_R(G_{O'})}{C_R(G_O)}$ where, by abuse of notation, we denote the average of common rate over all of the simulation rounds by $C_R(\cdot)$. The subscripts O and O' denote optimal ordering and randomly chosen orderings, respectively. Figure 3.6 illustrates the aforementioned gap and feature the effect of optimal ordering on common rate of the system. This figure also shows that for high SNR regime, the performance of a randomly chosen ordering for FDF coincides with that of optimal ordering. However, for DF relaying, the effect of ordering becomes more important as SNR increases.

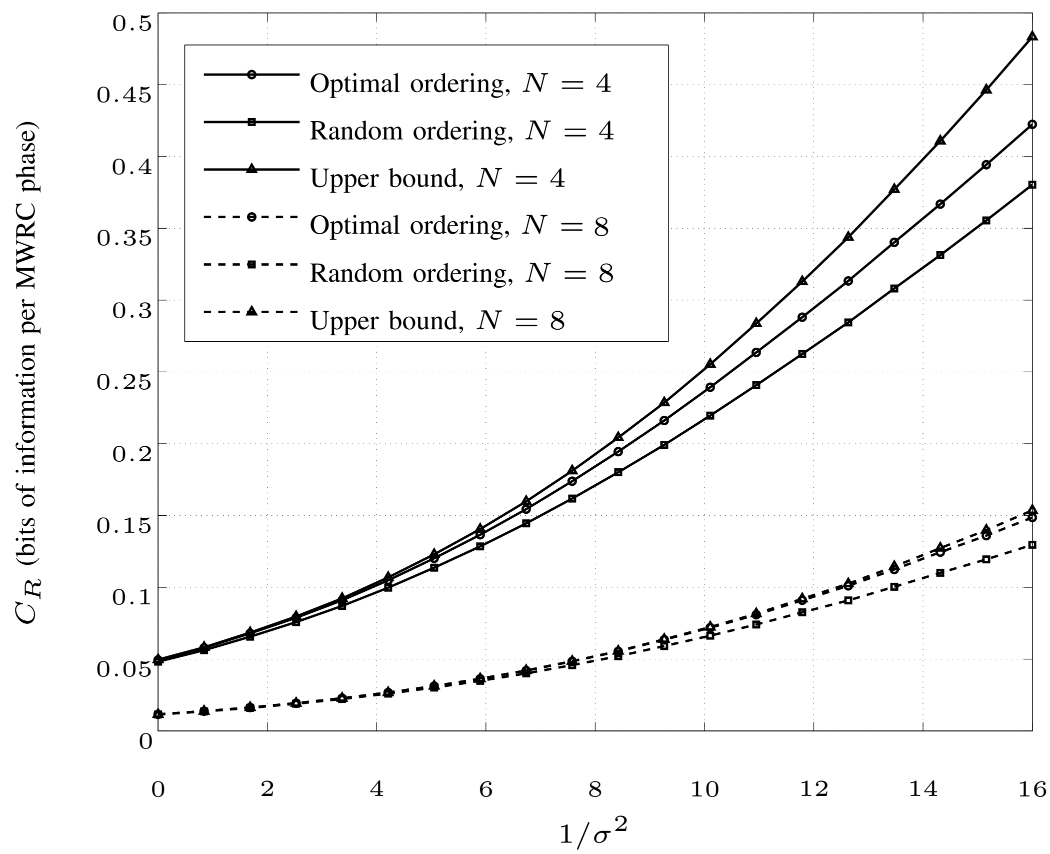

Figure 3.4: Comparison between the common rate of the optimal ordering and random ordering in MWRC with DF relaying for $N = 4$ and 8.

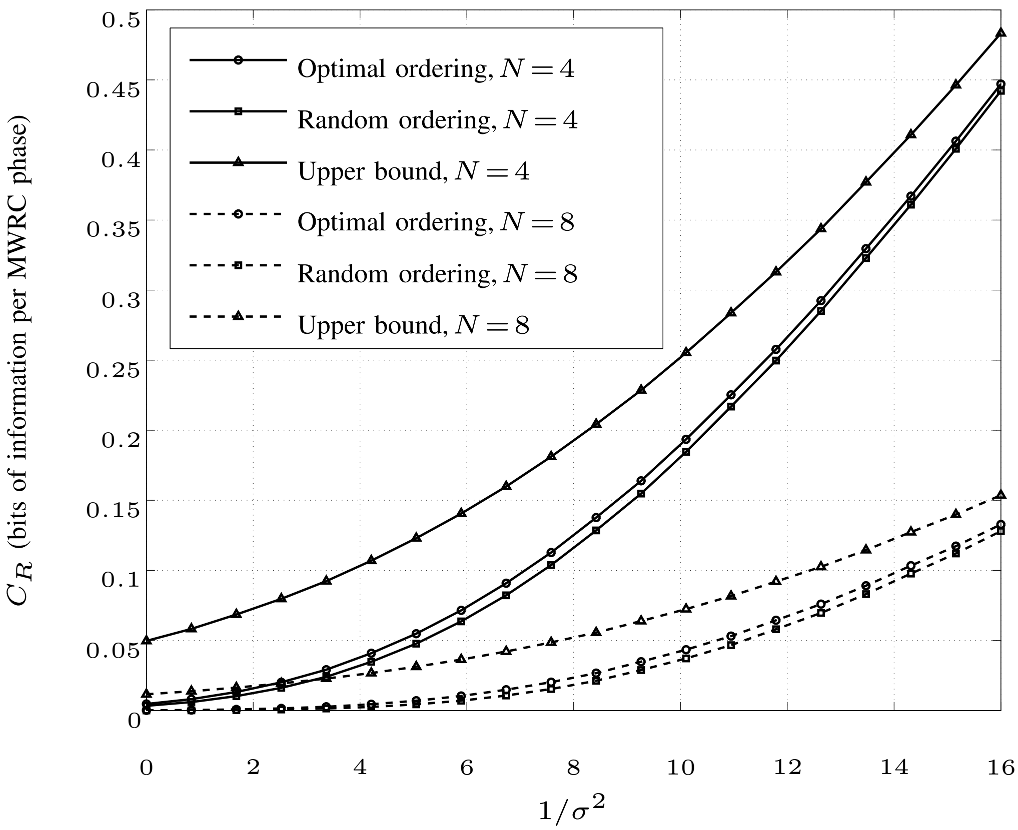

Figure 3.5: Comparison between the common rate of the optimal ordering and random ordering in MWRC with FDF relaying for $N = 4$ and 8.

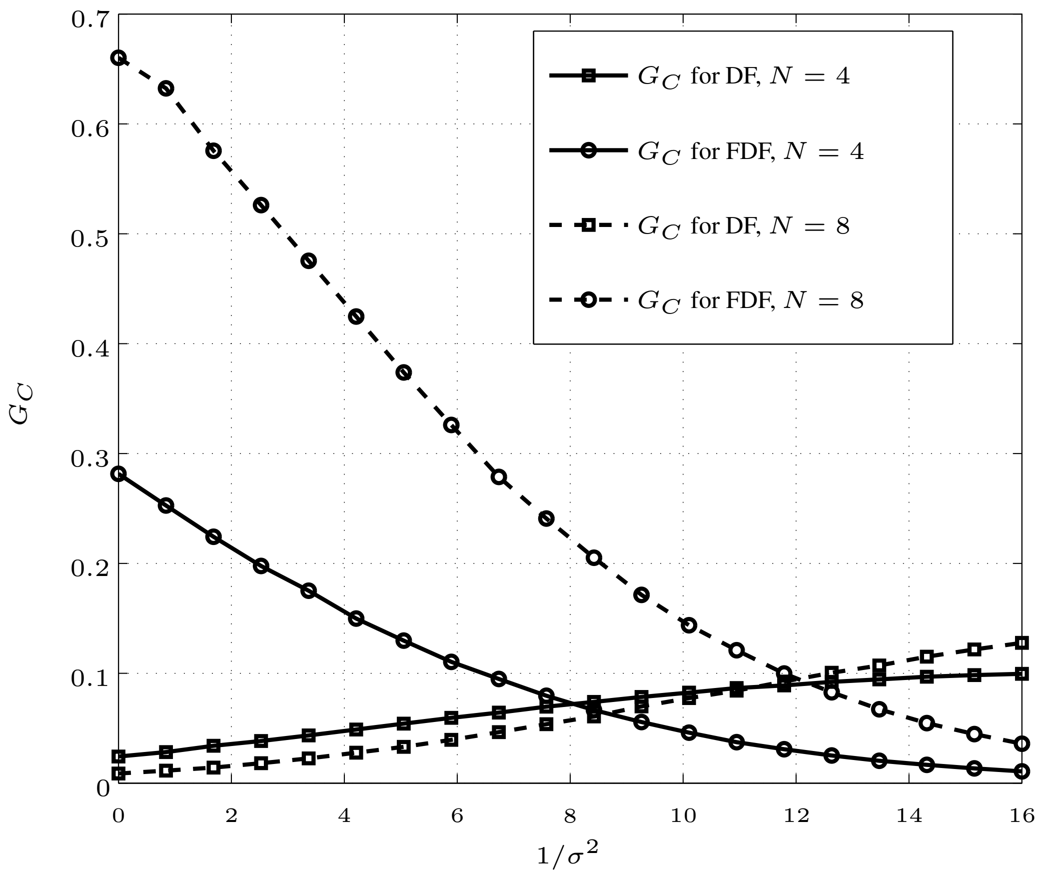

Figure 3.6: Common rate gap between optimal ordering and random ordering for DF and FDF relaying with $N = 4$ and 8.

Chapter 4

Sum Rate Maximization for Pairwise Multiway Relay Channels

In this chapter, first we state the sum rate maximization problem. Then we present the optimal orderings that achieve the maximum sum rate for a pairwise MWRC with AWGN channels. Simulation results are presented for better illustration of the results.

4.1 Problem Statement

In an *unconstrained sum rate maximization*, we want to find the ordering that maximizes the sum rate of the considered MWRC network. More precisely, an unconstrained sum rate maximization problem is formulated as:

$$O_{\text{USR}} = \arg\max_{O \in \mathcal{O}} S_R(G_O). \tag{4.1}$$

In this case, as we discussed earlier, there exist N^{N-2} distinct feasible orderings. Thus, finding the optimal ordering through brute-force search becomes expensive for large N. Then, under a reasonable assumption on user's SNR, we analytically find the optimal orderings to maximize the sum rate for both DF and FDF schemes.

4.2 Problem Solution

In this section, we provide solution to the unconstrained sum rate maximization. We solve this problem for both DF and FDF relaying.

4.2.1 Unconstrained Sum Rate Maximization for DF Relaying

The ordering given by Theorem 5 achieves the maximum sum rate in an unconstrained MWRC with DF relaying.

Theorem 5. *In an MWRC with DF relaying the optimal ordering that achieves the maximum S_R is given by*

$$O_{USR} = \{\{U_2, U_1\}, \{U_3, U_1\}, \dots, \{U_N, U_1\}\} \tag{4.2}$$

and the maximum achievable sum rate is given by:

$$S_R(G_O) = \frac{1}{2(N-1)} \log_2 \left(\frac{\prod_{i=1}^{N} (1+\gamma_i)}{1 + \frac{\gamma_1 \gamma_N}{1+\gamma_1+\gamma_N}} \right). \tag{4.3}$$

Proof. We prove Theorem 5 by induction. For $N = 2$, $S_R = \frac{1}{2} \log_2 (1 + \gamma_1 + \gamma_2)$ which is true for a two way relay channel with half-duplex DF relaying. Now, assume that the statement of Theorem 5 is true for all $N \leq k$. We show that it also holds for $N = k + 1$. If $deg(v_1) = N - 1$, the statement holds. Otherwise, $deg(v_1) < N - 1$. Let us define V' the set of all neighbors of v_1 that have a degree greater than 1, i.e.,

$$V' = \{v_j \in A_1^{G_O} | deg(v_j) > 1\}. \tag{4.4}$$

Note that $deg(v_1) < N - 1$ and as a result, V' is nonempty. Since G_O is a tree, there exists one unique path from v_1 to any other node v_i, $i \neq 1$. We define the set of edges that form a path from v_1 to v_i by $\mathcal{P}_i$. For any $v_j \in V'$ and considering v_1 as the root of G_O, we define the set of descendants of v_j as $D_j = \{v_i \in V | v_1 v_j \in \mathcal{P}_i; i \neq j\}$. Figure 4.1 illustrates the set of descendants of v_j. Since $deg(v_1) < N - 1$, there exists at least one j such that $D_j \neq \{\}$. For

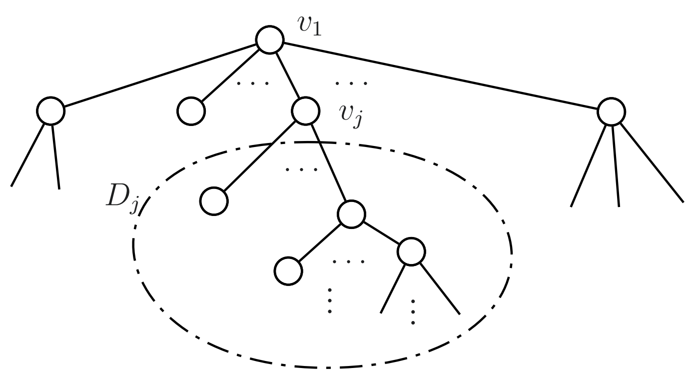

Figure 4.1: Set of descendants of v_j.

these j's, we define the highest SNR among the descendant of v_j and v_j itself, as

$$h^*(v_j) = \max\{\gamma_i | v_i \in D_j \cup v_j\}. \tag{4.5}$$

Now, assume that

$$v^* = \underset{v_j \in A_1^{G_O}, D_j \neq \{\}}{\arg\max} \; h^*(v_j). \tag{4.6}$$

We denote the set of descendants of v^* by D^*. According to the induction hypothesis, by removing v^* and all of its descendants from the client tree G_O, we have

$$R_1 + \sum_{v_i \in A_1^{G_O} - \{v^*\}} R_i \leq \frac{1}{2(N-1)} \log_2 \left(\frac{(1+\gamma_1) \times \prod_{v_i \in A_1^{G_O} - \{v^*\}} (1+\gamma_i)}{1 + \frac{\gamma_1 \gamma_1^*}{1+\gamma_1+\gamma_1^*}} \right) \tag{4.7}$$

where, $\gamma_1^* = \max\{\gamma_i | v_i \in A_1^{G_O} - \{v^*\}\}$. Similarly, by removing v_1 and all of its neighbors, except v^*, we have

$$\sum_{v_i \in D^* \cup v^*} R_i \leq \frac{1}{2(N-1)} \log_2 \left(\frac{(1+h^*(v^*)) \times \prod_{v_i \in D^*} (1+\gamma_i)}{1 + \frac{\gamma_{\min} \times h^*(v^*)}{1+\gamma_{\min}+h^*(v^*)}} \right) \tag{4.8}$$

where, $\gamma_{\min} = \min\{\gamma_i | v_i \in D^* \cup \{v^*\}\}$. Now, we have $\gamma_N = \max\{h^*(v^*), \gamma_1^*\}$ and $\gamma_1 \leq \gamma_{\min}$. Then, as a result of (2.14),(4.7) and (4.8), we have

$$\sum_{i=1}^{N} R_i \leq \frac{1}{2(N-1)} \log_2 \left(\frac{\prod_{i=1}^{N} (1+\gamma_i)}{\left(1 + \frac{\gamma_1 \times \gamma_1^*}{1+\gamma_1+\gamma_1^*}\right)\left(1 + \frac{\gamma_{\min} \times h^*(v^*)}{1+\gamma_{\min}+h^*(v^*)}\right)} \right). \tag{4.9}$$

This configuration achieves a lower sum rate than (4.3) iff

$$\left(1 + \frac{\gamma_1 \times \gamma_1^*}{1+\gamma_1+\gamma_1^*}\right)\left(1 + \frac{\gamma_{\min} \times h^*(v^*)}{1+\gamma_{\min}+h^*(v^*)}\right) \geq \left(1 + \frac{\gamma_1 \gamma_N}{1+\gamma_1+\gamma_N}\right) \tag{4.10}$$

which, according to the fact that $\gamma_N = \max\{h^*(v^*), \gamma_1^*\}$, is true. □

In the optimal ordering, according to the proof of Theorem 5, all of the users except U_1 and U_N, can transmit with their highest transmission rates given by (2.14). Additionally, U_1 and U_N transmit with rates satisfying (2.16). Figure 4.2 shows the optimal ordering for a DF MWRC that achieves the maximum S_R.

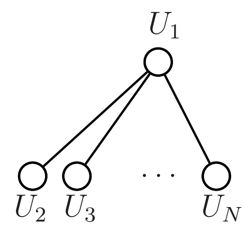

Figure 4.2: Client tree that maximizes $S_R(G_O)$ for a pairwise MWRC with DF relaying and also maximizes $S_R(G_O)$ for a pairwise MWRC with FDF relaying subject to the weakened upper bound given by (4.11).

4.2.2 Unconstrained Sum Rate Maximization for FDF Relaying

In order to find an ordering with maximum sum rate in this case, we consider a weaker condition than (2.17). We assume that the upper bound on achievable rate for U_i, when it is paired with U_j, is given by:

$$R_i \leq \frac{1}{2(N-1)} \log_2 \left(\frac{\gamma_i}{\gamma_i + \gamma_j} + \gamma_i \right). \tag{4.11}$$

This weaker upper bound is still interesting in many practical settings in which the most significant term at the right hand side of (2.17) is the one mentioned in (4.11). For instance, one can easily verify that if $\gamma_1 + \frac{\gamma_1}{\gamma_1+\gamma_N} \geq 1$, (4.11) and (2.17) are equivalent. The optimal ordering in this case is given by the following theorem:

Theorem 6. *The ordering*

$$O_{USR} = \{\{U_2, U_1\}, \{U_3, U_1\}, \ldots, \{U_N, U_1\}\} \tag{4.12}$$

is the optimal ordering for an unconstrained FDF MWRC subject to (4.11). Moreover, the maximum sum rate for this ordering is:

$$S_R(G_O) = \frac{1}{2(N-1)}\log_2\left(\max\left\{1, \left(\gamma_1 + \frac{\gamma_1}{\gamma_1+\gamma_N}\right)\right\} \times \prod_{i=2}^{N}\max\left\{1, \frac{\gamma_i}{\gamma_i+\gamma_1} + \gamma_i\right\}\right). \tag{4.13}$$

Proof. To prove the theorem, we first show that there is an optimal tree with $deg(v_N) = 1$ (Lemma 2). Then we prove that in the optimal tree each node needs to have only one neighbor among nodes with a lower SNR (Lemma 3). We then show that there exist an optimal tree with $deg(v_N) = deg(v_{N-1}) = 1$ (Lemma 4). In the next step, we prove that in an optimal tree for two nodes of degree one, say v_i and v_j, if v_i has a higher SNR than v_j then the neighbor of v_i has a higher SNR than the neighbor of v_j (Lemma 5). Then we prove the theorem by induction (Lemma 6).

We use the following convention for the rest of this proof:

$$d_i \triangleq 2^{2(N-1)R_i}. \tag{4.14}$$

As a result, the bound given by (4.11) is equivalent to

$$d_i \leq \gamma_i\left(1 + \frac{1}{\gamma_i+\gamma_j}\right). \tag{4.15}$$

We also define $D_s(G_O) = \max \prod_{i=1}^{N} d_i = 2^{2(N-1)S_R(G_O)}$. Assume that $G(V,E)$ is a tree such that $\{v_i, v_j, v_k\} \subseteq V$ and$\{v_iv_j, v_iv_k\} \subseteq E$. We define a $\mathcal{V}$-transform on G in such a way that $\mathcal{V}(G, v_i, v_j, v_k) = G'(V, E')$ and $E' = (E - \{v_iv_k\}) \cup \{v_jv_k\}$. Figure 4.3 shows the operation of a $\mathcal{V}$-transform.

Lemma 2. *There exists an optimal tree in which $deg(v_N) = 1$.*

Proof. Assume G_O is an optimal tree in which $deg(v_N) > 1$ and v_i and v_j are two neighbors of v_N and γ_j is the minimum SNR value of the neighbors

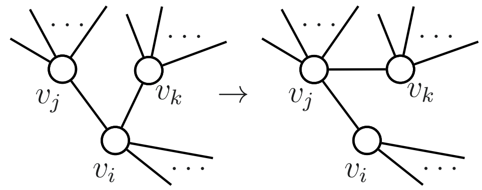

Figure 4.3: Operation of $\mathcal{V}$-transform, $\mathcal{V}(G, v_i, v_j, v_k)$.

of V_n. Consequently, we have $\gamma_i \geq \gamma_j$. It is straightforward to show that by performing a $\mathcal{V}$-transform on G_O and transform it to $G_{O'} = \mathcal{V}(G_O, v_N, v_i, v_j)$, we have $\frac{D_s(G_{O'})}{D_s(G_O)} \geq 1$:

$$\frac{D_s(G_{O'})}{D_s(G_O)} \geq \frac{\left(1+\frac{1}{\gamma_N+\gamma_i}\right)\left(1+\frac{1}{\gamma_i+h^{G_{O'}}(v_i)}\right)\left(1+\frac{1}{\gamma_j+h^{G_{O'}}(v_j)}\right)}{\left(1+\frac{1}{\gamma_i+\gamma_N}\right)\left(1+\frac{1}{\gamma_i+\gamma_N}\right)\left(1+\frac{1}{\gamma_j+\gamma_N}\right)} \geq 1. \tag{4.16}$$

Here, $h^{G_{O'}}(v_m)$ is the highest SNR of neighbors of v_m in $G_{O'}$. It means that the sum rate of $G_{O'}$ is not less than sum rate of G_O. Note that, after applying this $\mathcal{V}$-transform, we have reduced degree of v_N by one. After applying $deg(v_N) - 2$ more $\mathcal{V}$-transforms, we end up with an optimal tree with $deg(v_N) = 1$. Figure 4.4 illustrates a hypothetical optimal tree with $deg(v_N) = 4$. It shows how we apply 3 $\mathcal{V}$-transforms to get an optimal tree with $deg(v_N) = 1$. $\square$

Lemma 3. *There exists an optimal tree, $G_O(V, E)$, such that for any $0 < i < N-1$, $deg(v_{N-i}) \leq i+1$. Furthermore, the number of neighbors of v_{N-i} with a lower SNR than γ_{N-i} is at most one and consequently, the number of neighbors of v_{N-i} which have higher SNR than γ_{N-i} is at least $deg(v_{N-i}) - 1$.*

Proof. If the number of those neighbors of v_{N-i} that have a lower SNR value than γ_{N-i} is a, after applying $a-1$ $\mathcal{V}$-transforms, we end up with an optimal tree in which $deg(v_{N-i}) \leq i+1$. These $a-1$ $\mathcal{V}$-transforms have the form $\mathcal{V}(G, V_{N-i}, v_i, v_k)$ and v_k has the highest SNR value among all of the neighbors of v_{N-i}.

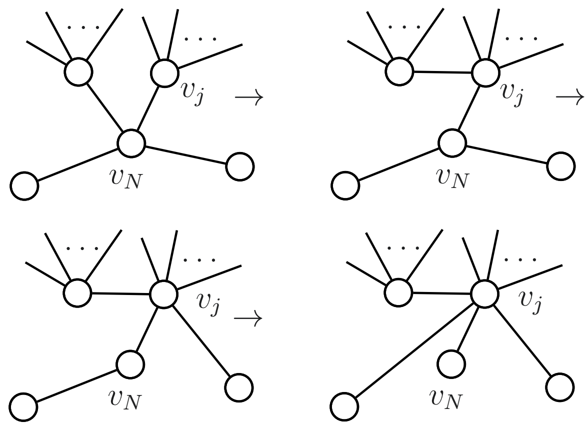

Figure 4.4: Applying 3 $\mathcal{V}$-transform on an optimal tree with $deg(v_N) = 4$.

Now, assume that $deg(v_{N-i}) \leq i + 1$ and v_{N-i} has at most one neighbor v_j such that $j < N - i$. Then, we have that the number of neighbors of v_{N-i} that have a higher SNR than γ_{N-i} is greater than or equal to $|A_{N-i}^{G_O}| - 1 = deg(v_{N-i}) - 1$. □

Lemma 4. *There exists an optimal tree, $G_O(V, E)$, in which $deg(v_N) = 1$ and $deg(v_{N-1}) = 1$. Moreover, if v_j is the only neighbor of v_{N-1} and v_i is the only neighbor of v_N, then $\gamma_i \geq \gamma_j$*

Proof. If $deg(v_{N-1}) = 2$, according to Lemma 2 and 3, there exists an optimal tree $G_O(V, E)$ in which $deg(v_N) = 1$ and $v_N v_{N-1} \in E$. Let the other neighbor of v_{N-1} be v_j. Then, $G_{O'} = \mathcal{V}(G_O, v_N, v_i, v_j)$ is an optimal tree in which $deg(v_{N-1}) = 1$. So, there always exists an optimal tree G_O, with $deg(v_N) = deg(v_{N-1}) = 1$. Now, assume that $deg(v_{N-1}) = 1$ and the only neighbor of v_{N-1} is v_j. If $v_j = v_N$, the graph will be disconnected. Otherwise, if the only neighbor of v_N is v_i, we want to prove that $\gamma_i \geq \gamma_j$. We also assume $\gamma_N \neq \gamma_{N-1}$; otherwise, one can rename the nodes in such a way that theorem holds. Assume that $G_{O''}(V, E'')$ is a client tree in which:

$$E'' = (E - \{v_N v_i, v_{N-1} v_j\}) \cup \{v_N v_j, v_{N-1} v_i\}. \tag{4.17}$$

We show that $D_s(G_{O''}) \leq D_s(G_O)$ iff $\gamma_i \geq \gamma_j$:

$$\frac{D_s(G_{O''})}{D_s(G_O)} = \frac{\left(1+\frac{1}{\gamma_N+\gamma_j}\right)^2\left(1+\frac{1}{\gamma_{N-1}+\gamma_i}\right)^2}{\left(1+\frac{1}{\gamma_N+\gamma_i}\right)^2\left(1+\frac{1}{\gamma_{N-1}+\gamma_j}\right)^2} \tag{4.18}$$

and as a result:

$$\begin{aligned}
& \frac{D_s(G_{O''})}{D_s(G_O)} \leq 1 \\
\Leftrightarrow \quad & \left(1+\frac{1}{\gamma_N+\gamma_j}\right)\left(1+\frac{1}{\gamma_{N-1}+\gamma_i}\right) \leq \left(1+\frac{1}{\gamma_N+\gamma_i}\right)\left(1+\frac{1}{\gamma_{N-1}+\gamma_j}\right) \\
\Leftrightarrow \quad & \gamma_N\gamma_j + \gamma_i\gamma_{N-1} \leq \gamma_N\gamma_i + \gamma_{N-1}\gamma_j \\
\Leftrightarrow \quad & \gamma_j \leq \gamma_i.
\end{aligned}$$

□

Next lemma, is a generalization of Lemma 4 and we prove it in a similar way.

Lemma 5. *Assume that $G_O(V,E)$ is an optimal tree in which $deg(v_N) = deg(v_{N-1}) = \cdots = deg(v_{N-i}) = 1$ and $i < N-1$. Also, assume that $q < p \leq i$ and $\{v_jv_{N-p}, v_kv_{N-q}\} \in E$. Then $\gamma_j \leq \gamma_k$.*

Proof. It is obvious that $j > N-i$ and $k > N-i$, otherwise the graph is disconnected. Now, if $\gamma_k < \gamma_j$, according to Lemma 4, the graph $G_{O'}(V,E')$ with $E' = (E - \{v_jv_{N-p}, v_kv_{N-q}\}) \cup \{v_jv_{N-q}, v_kv_{N-p}\}$ has a greater sum rate which contradicts the fact that G_O is optimal. □

Lemma 6. *Assume $G_O(V,E)$ is an optimal tree and i is the largest integer such that*

$$deg(v_N) = deg(v_{N-1}) = \cdots = deg(v_{N-i}) = 1. \tag{4.19}$$

If $i < N-1$, then there exists an optimal tree $G_{O'}(V,E')$ in which

$$deg(v_N) = deg(v_{N-1}) = \cdots = deg(v_{N-i+1}) = 1. \tag{4.20}$$

Proof. Assume that $A^{G_O}_{N-i+1} \cap \{v_N, v_{N-1}, \dots, v_{N-i}\} = \{v_{m_1}, v_{m_2}, \dots, v_{m_n}\}$ where $m_1 > m_2 > \dots > m_n$. Define

$$B = A^{G_O}_{N-i+1} - \{v_N, v_{N-1}, \dots, v_{N-i}\}. \tag{4.21}$$

According to Lemma 3, we assume that $|B| \leq 1$. If $|B| = 0$, G_O is disconnected. Assume $B = \{v_j\}$. Consider $G_{O'}(V, E')$ such that

$$\begin{aligned} E' =& (E - \{v_{m_1}v_{N-i+1}, v_{m_2}v_{N-i+1}, \dots, v_{m_n}v_{N-i+1}\}) \\ & \cup \{v_{m_1}v_j, v_{m_2}v_j, \dots, v_{m_n}v_j\}. \end{aligned} \tag{4.22}$$

Then, one can conclude that $\frac{D_s(G_O)}{D_s(G_{O'})} \geq 1$ as follows:

$$\frac{D_s(G_O)}{D_s(G_{O'})} \geq \frac{\left(1 + \frac{1}{\gamma_{N-i+1}+\gamma_{m_1}}\right)\left(1 + \frac{1}{\gamma_j + h^{G_O(v_j)}}\right)}{\left(1 + \frac{1}{\gamma_{N-i+1}+\gamma_j}\right)\left(1 + \frac{1}{\gamma_j + h^{G_{O'}(v_j)}}\right)} \tag{4.23}$$

$$\Rightarrow \frac{D_s(G_O)}{D_s(G_{O'})} \geq \frac{\left(1 + \frac{1}{\gamma_{N-i+1}+\gamma_{m_1}}\right)}{\left(1 + \frac{1}{\gamma_{N-i+1}+\gamma_j}\right)} \geq 1. \tag{4.24}$$

□

According to Lemma 6, there exists an optimal tree with respect to (4.11) in which

$$deg(v_N) = deg(v_{N-1}) = \dots = deg(v_2) = 1. \tag{4.25}$$

As a result, O is an optimal solution with respect to (4.11). The muximum achievable sum rate, $S_R(G_O)$, could be found directly from (4.13). □

Remark: From Theorem 6 and considering (4.11), one can show that the maximum achievable sum rate for the optimal ordering is

$$S_R(G_O) = \frac{1}{2(N-1)} \times \log_2 \left(\left(\prod_{i=1}^{N} \gamma_i \right) \left(\prod_{i=2}^{N} 1 + \frac{1}{\gamma_i + \gamma_1} \right) \left(1 + \frac{1}{\gamma_1 + \gamma_N} \right) \right). \tag{4.26}$$

Thus, the maximum sum rate can be upper bounded by

$$S_R(G_O) \leq \frac{1}{2(N-1)} \log_2 \left(\prod_{i=1}^{N} \gamma_i \times \left(1 + \frac{1}{2\gamma_1} \right)^N \right). \tag{4.27}$$

Similarly, we can show that for a random ordering O', the corresponding sum rate is lower bounded by

$$S_R(G_{O'}) \geq \frac{1}{2(N-1)} \log_2 \left(\prod_{i=1}^{N} \gamma_i \times \left(1 + \frac{1}{2\gamma_N} \right)^N \right). \tag{4.28}$$

According to (4.27) and (4.28), we find an upper bound for the difference between the sum rate of a random ordering and the optimal ordering as follows

$$S_R(G_O) - S_R(G_{O'}) \leq \frac{1}{2} \log_2 \left(\left(\frac{\gamma_N(1+2\gamma_1)}{\gamma_1(1+2\gamma_N)} \right)^N \right) \tag{4.29}$$

and as a result

$$\lim_{\gamma_1 \to \infty} \left(S_R(G_O) - S_R(G_{O'}) \right) = 0. \tag{4.30}$$

Interestingly, (4.30) shows that for FDF relaying in high SNR regime, the performance of a randomly chosen ordering approaches the performance of the optimal ordering.

Remark: Similar to the argument we had in Chapter 3, we can show that there is no feasible ordering with more than $N-1$ pairs with a higher sum rate than that of the optimal tree.

4.3 Simulation Results

In this section, we investigate the performance of the optimal ordering in comparison with random orderings. Again, we use Monte Carlo simulation to average over sum rate for the optimal ordering and a randomly selected ordering, for both DF and FDF relaying. For each simulation round, random ordering is selected uniformly at random from all of the feasible client trees. We again, assume that the data rates are limited by the uplink phase. Channels from users to the relay are simulated as slow Rayleigh fading. The number of users is set to $N = 4$ and 8.

Figure 4.5 and 4.6 compare the sum rate of the optimal ordering and average performance of random orderings for DF and FDF relaying, respectively. Again, the upper bounds are given by max-flow min-cut theorem [8]. To have a better illustration of the difference between optimal ordering and random orderings, we define the *sum rate gap* of random ordering and optimal ordering similar to the common rate gap. More precisely, it is defined as $G_S = \frac{S_R(G_O)-S_R(G_{O'})}{S_R(G_O)}$. The subscripts O and O' denote optimal ordering and randomly chosen orderings, respectively. Figure 4.7 depicts the aforementioned sum rate gap. As we showed, it can be seen that for high SNR regime the performance of a randomly chosen ordering for FDF coincides with that of optimal ordering. For DF relaying, the sum rate gap increases and the effect of ordering becomes more important as SNR increases.

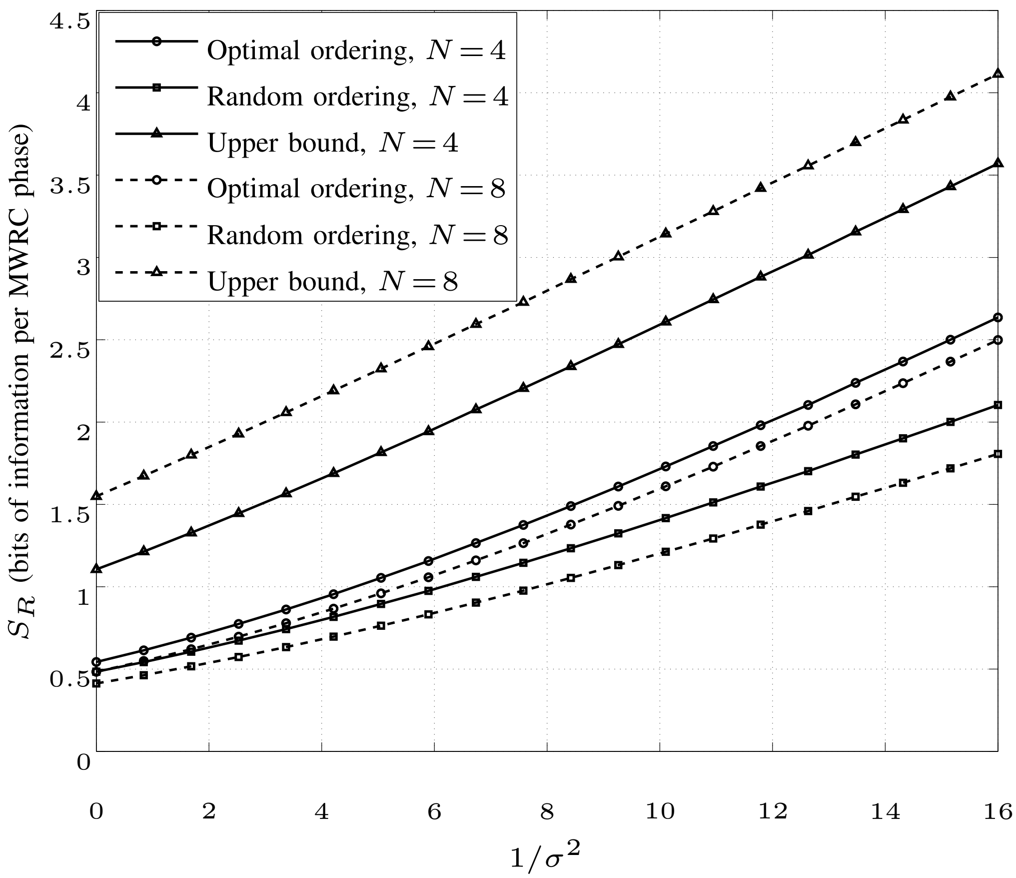

Figure 4.5: Comparison between the sum rate of the optimal ordering and random ordering in MWRC with DF relaying for $N = 4$ and 8.

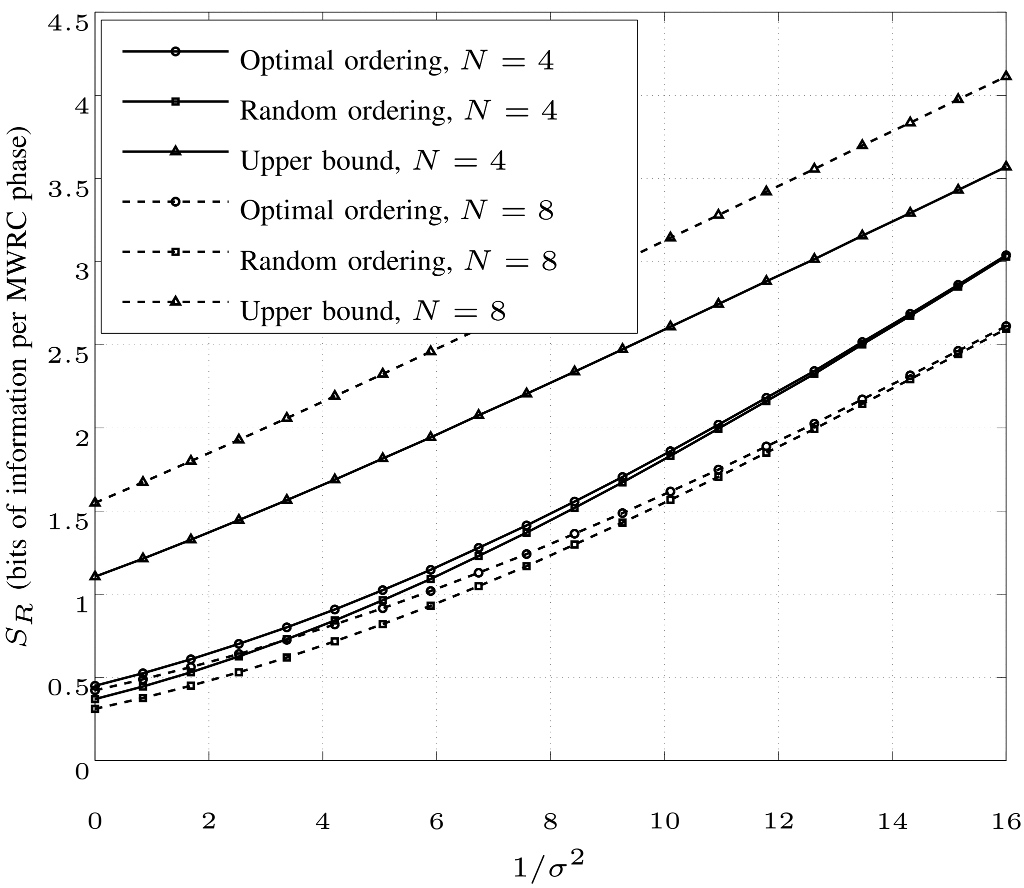

Figure 4.6: Comparison between the sum rate of the optimal ordering and random ordering in MWRC with FDF relaying for $N = 4$ and 8.

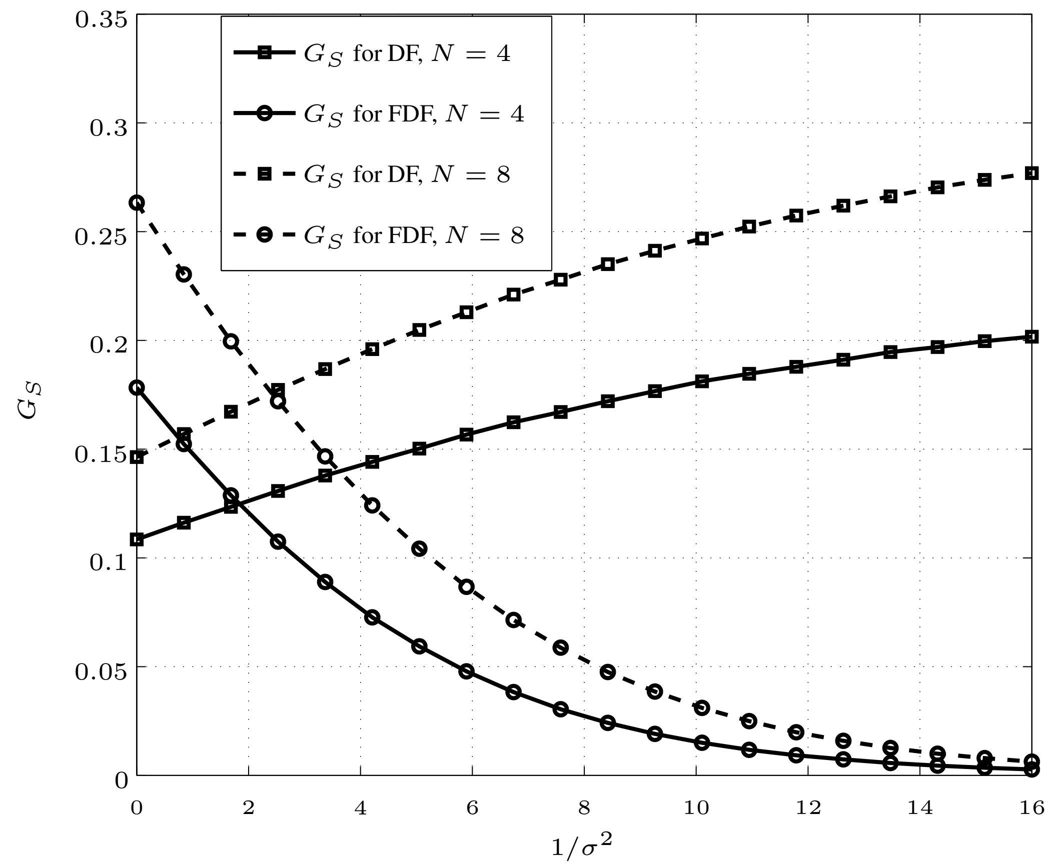

Figure 4.7: Sum rate gap between optimal ordering and random ordering for DF and FDF relaying with $N = 4$ and 8.

Chapter 5

Conclusion

In this work, we studied the effect of users' transmission ordering on the common rate and sum rate of a pairwise MWRC. Both DF and FDF relaying were studied. First, we suggested a graphical model for the data communication between the users. Then, using this model, optimal orderings were found that maximize common rate and (under a mild practical assumption) sum rate in the system. Moreover, we showed that for high SNR regimes, the effect of ordering becomes less important when the relay performs FDF.

Our claims were supported and verified by computer simulations. Based on our simulations, one can compare the maximum performance of pairwise MWRC with cut-set bound and performance of random orderings. Figure 5.1 summarizes the main results of our simulations. In addition, one can investigate the performance of optimal ordering in comparison with random ordering and cut-set bound for different values of N. Based on our simulations, effect of ordering for both common rate and sum rate with both DF and FDF becomes more important. Compared to cut-set bound, the gap between optimal ordering and cut-set bound for DF decreases as the number of users, N, increases. However, the gap between optimal ordering and cut-set bound increases with the increase in N for both common rate and sum rate with FDF.

Problem \ Relaying	DF		FDF	
	Low SNR	High SNR	Low SNR	High SNR
Common Rate	*Optimal* $\approx$ *Random* *Pairwise* $\rightarrow$ *CB*	*Optimal* $\gg$ *Random* *Pairwise* $\rightarrow$ *CB*	*Optimal* $\gg$ *Random* *Pairwise* $\ll$ *CB*	*Optimal* $\approx$ *Random* *Pairwise* $\rightarrow$ *CB*
Sum rate	*Optimal* $\gg$ *Random* *Pairwise* $\ll$ *CB*	*Optimal* $\gg$ *Random* *Pairwise* $\ll$ *CB*	*Optimal* $\gg$ *Random* *Pairwise* $\ll$ *CB*	*Optimal* $\approx$ *Random* *Pairwise* $\ll$ *CB*

Figure 5.1: Summary of simulation results.

5.1 Future Work

In general, for AWGN channels, it is not proven that pairwise scheme can achieve the sum rate capacity. One possible future work is to find other transmission schemes that outperform the pairwise scheme in terms of achievable sum rate. As we showed in Chapter 2, the received system of linear equation at all users should be solvable to have a feasible MWRC. In a pairwise MWRC, each equation is a linear combination of exactly two users' messages. One may think about a more general case that orderings are not limited to pairs. In other words, a group of users that transmit simultaneously in an uplink phase may have more than two users. Finding efficient transmission strategies for such a case is one possible future research direction.

Further, in the optimal ordering for sum rate maximization with FDF relaying, the user with the lowest SNR is sacrificed in terms of transmission rate. Considering the issue of transmission fairness and finding orderings that have a performance near the optimal ordering (e.g., see [41]) is another potential topic for future work.

We did not consider multiple-input multiple-output (MIMO) systems in this work. Recently, research effort has been directed toward TWRCs that users and the relay are equipped with multiple antennas [42, 43, 44, 45]. Different approaches for MIMO TWRC has been proposed and achievable bounds are found [46]. However, the sum rate and common rate maximization problems for MIMO pairwise MWRC is not studied yet. Considering the same problem and finding the optimal transmission schemes for MIMO MWRC systems would be interesting.

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