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by

ADONIS FEDNANT LABOR

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled Functional Homology Between Attitudes Toward and Performance on Mathematics Problems, submitted by Adonis Fednant Labor in partial fulfilment of the requirements for the Degree of Doctor of Philosophy.

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ABSTRACT

The purpose of this study was twofold. The first was to determine which dimensions, if any, of junior high school student attitudes toward each of four types of mathematics problems relate to performance on the corresponding type of problem. A type of mathematics problem was considered as a cell in a two-way classification, cognitive objectives of mathematics by content areas in the mathematics program. The types of mathematics problems considered in this study were those that belong to one content area, algebra, and which were hierarchically structured with respect to the cognitive complexities associated with the problems. The second purpose was to investigate whether student attitudes toward, and performances on, the four types of mathematics problems reflected the hierarchy of cognitive complexities associated with the problems.

Before the objectives of the study could be met, however, a preliminary study employing Q-technique (inverse factor analytic technique) was undertaken to identify the structure of students' attitudes toward each of the four types of mathematics problems. To this end, the description of a particular type of mathematics problem followed by examples of that type was each time presented to a sample of twenty-four carefully selected grade 9 students. On each presentation, the responses of the students were to Q-sort the same list of 60 attitude statements according to a specified distribution.

The results of a Q-analysis on each of the four sets of 24 sorts indicated that junior high school student attitudes toward

mathematics problems vary depending on the kind of thinking called for by the problem. A subsequent confirmatory study on the experimental group, using R-technique, reproduced fairly well the attitude dimensions built in from the Q-technique. From the measurement point of view, this latter result suggests that mathematics attitude factor scales can be economically constructed using a small sample of carefully selected students via Q-technique rather than using a large sample of students via R-technique.

The four attitude scales developed from a combination of the Q- and R-techniques, and an algebra test having four subtests whose items exemplified the four types of mathematics problems, were administered to an experimental group of 350 grade 9 students from four Separate Junior High Schools in Edmonton. From an analysis of the data based on 312 students, the following major conclusions were drawn:

1. For both males and females, evasive attitudes toward algebra problems requiring lower level cognitive behaviors correlated with performances on these problems; such attitudes toward algebra problems that require higher level cognitive behaviors were uncorrelated with performances on those types of problem.
2. For males only, their evasive attitudes toward, and performances on, the four types of algebra problems reflected the hierarchy of cognitive complexities associated with the problems. This result is suggestive that such attitudes may be reflections of the complexities associated with the problems. In the case of the females, their performance on the problems reflected the hierarchy

of cognitive complexities associated with the problems, but not their attitudes toward the problems. This result seems to suggest sex difference in the structuring of students' attitudes toward algebra problems of varying cognitive complexities.

Some suggestions for further research are noted in the dissertation.

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To my son Adrian

As an incentive to achieve more than Daddy

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CHAPTER 1

INTRODUCTION TO THE PROBLEM

1. INTRODUCTION

Research studies on student attitudes toward mathematics have been many and diverse. A partial list of reports includes those concerned with measures and methods of improving measures of attitudes toward mathematics (Ellingson, 1962; Milliken and Spilka, 1962; Brown and Abell, 1965; Nealeigh, 1967; Kane, 1968; Capps and Cox, 1969; McClure, 1970; Roberts, 1970; Evans, 1971; McCallon and Brown, 1971); grade distribution and stability of attitudes toward mathematics (Stright, 1960; Herman, 1963; Morrisett and Vinsonhaler, 1965; Osborn, 1965; Dutton and Blum, 1968; Anttonen, 1969); comparative evaluation of students' attitudes toward different mathematics programs (Phelps, 1963; Remai, 1965; Woodall, 1966; Hungerman, 1967; Yasui, 1967; Ryan, 1967; Ryan, 1970; Frase, 1971; Ronshausen, 1971); the effect of mathematical games, discovery methods, specific instructions, homework, and supplementary materials on attitudes toward mathematics (Lerch, 1961; Maertens, 1968; Jones, 1968; Duncan, 1970; Studer, 1971; Urwiller, 1971; Wilkinson, 1971); the relationship of attitudes toward mathematics to ability, personality and social factors (Shapiro, 1961; Aiken, 1963; Lindgren, Silva, Faraco, and Da Rocha, 1964; Karas, 1964), to parental attitudes and expectations (Aiken and Dreger, 1961; Alpert, Stellwagon, and Becker, 1963; Weston, 1966; Hill, 1967), to teacher characteristics, attitudes, and behaviors (Garner, 1963; Peskin, 1964; Torrance, 1966; Keane, 1968; Caezza, 1969; Starkey, 1970), and to achievement in mathematics (Stephens, 1960; Degnan, 1967; Devine, 1967; Farley, 1968; Neale, 1969; Moore, 1971).

Implicit in this growing concern is an awareness that development of favorable attitudes is included among the objectives of a mathematics program, and that there is possibly a relationship between attitudes and performance in mathematics.

Despite the many investigations in this area, it seems likely that these studies have not been maximally useful because of the manner in which attitudes toward mathematics have been analyzed. Most studies have been concerned with attitudes toward mathematics in general, and this fact raises various questions. Some of these are: (a) Are there different attitudes toward the different apsects of mathematics, such as the concepts, the generalizations, and the sets of relationships which are central to mathematics and serve to integrate the many parts into a unified body? (b) What is the nature of these different attitudes? (c) Are the attitudes toward, say, the concepts undimensional or multidimensional? (d) If multidimensional, what are these dimensions. These and many similar questions seemingly remain unanswered despite the numerous investigations in this area.

Romberg (1969) stated part of the problem when he said that:

> Most investigators use a single, global measure of attitudes toward mathematics. This is certainly not realistic, since there is probably a set of dispositions or feelings that vary from computation to problem solving, etc. (p. 481-482)

Aiken (1970a), reviewing recent researches in mathematics education, added to this when he said:

> although the majority of investigations have dealt with attitudes toward mathematics in general, attitudes toward specific courses or types of mathematics problem can also be assessed. (p. 552)

It is reasonable to suggest that a student's attitudes toward solving routine problems need not be the same across the different content areas of a mathematics program. This may be one of the reasons why studies have been concerned with attitudes specific to arithmetic (Bassham, Murphy, and Murphy, 1964; Dutton, 1968; Dutton and Blum, 1968; Capps and Cox, 1969; Evans, 1971), or specific to algebra (Garner, 1963). Furthermore, a student's attitudes may vary across types of mathematics problems to be solved in a particular content area. Thus, accepting this premise, two valid general mathematics attitude inventories, depending on whether most of the statements in each inventory pertain to routine or non-routine problems in one or more content areas of the mathematics program, may reveal contrasting results.

Moreover, there exists the possibility that students, particularly at the junior high school level, may, unless under carefully-guided instructions, interpret the word 'mathematics' to mean specific content areas or cognitive objectives in mathematics, or such related aspects of mathematics learning (homework in mathematics, length of mathematics class period) rather than the sum-total of all mathematics activities.

These are some of the criticisms of previous studies in the area of attitudes toward mathematics in general. Such shortcomings may, in part, account for some of the contrasting findings that have been reported regarding attitudes toward mathematics, particularly at the junior high school level.

With a view to eliminating the shortcomings of previous studies, some of the suggestions put forth by Aiken (1970a) have been employed in this study. Specifically, the study focusses on student attitudes toward each of four types of mathematics problems.* These problems are those that belong to one content area in the junior high school mathematics program and which are hierarchically structured with respect to the levels of thinking inherent in the cognitive objectives associated with the problems.

Such an approach, when extended to include all cognitive objectives in mathematics across all possible content areas in the school mathematics program, will provide a more in-depth method of evaluation and measuring students' attitudes toward the cognitive aspects of mathematics than a global measure of attitudes toward mathematics with a single over-all score.

2. THE PROBLEM AND ITS SIGNIFICANCE

The study has two objectives. First, it seeks to determine which dimension of student attitudes toward each of four types of mathematics problems relate to performance on the corresponding type of mathematics problem. The four types of mathematics problems are

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*A type of mathematics problem is to be considered as a cell in a two-way classification, cognitive objectives of mathematics by content areas in the school mathematics program. A cognitive objective of mathematics refers to the level of thinking required of the student, and a content area in mathematics refers to a subdivision of the subject-matter.

(i) Recalling Factual Knowledge in Algebra (RFKA), (ii) Performing Algebraic Manipulations (PAM), (iii) Solving Routine Algebra Problems (SRAP), and (iv) Solving Non-routine Algebra Problems (SNrAP).** Second, the study investigates whether both attitudes toward, and performance on, the four types of mathematics problems reflect the hierarchy of cognitive complexity associated with the problems.

A recurring theme in mathematics education is the adverse attitudes toward mathematics held by many students. Neale (1969a), in presenting a paper at the National Council of Teachers in Mathematics (NCTM) Annual Meeting, expressed this theme when he said in his opening remarks:

> Mathematics educators are troubled because many students have mistaken impression about mathematics and dislike mathematics activities. More than that, many students seem to fear, even hate, mathematics.

Although researchers are aware that generalized attitudes or interests in mathematics are major factors in determining success or failure in the subject, they are less aware of specific attitudes that may be associated with patterns of success or failure in mathematics. For example, the question may be asked: To what extent do students' attitudes toward mathematics problems reflect the differences in the complexity among the problems?

While the content areas of mathematics programs may be diverse, it is thought that only a limited number of thought processes exist in

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**The cognitive objectives associated with these four types of mathematics problems are levels 1, 2, 3, and 5 of the Educational Testing Service (ETS) taxonomy. Algebra is to be understood as a content area in the Junior High School mathematics program of the Edmonton Separate School Board.

the school mathematics program, and these are those that the students are called upon to perform frequently. It is, therefore, felt that a study of students' attitudes toward the cognitive objectives in mathematics may help focus attention on the role of specific attitudes on over-all mathematics learning.

In the preface to the twenty-fourth year book of the National Council of Teachers of Mathematics (1959), it is stated:

> A major objective of mathematics learning is the training of mathematical modes of thought [cognitive objectives in mathematics]. These modes are not quite mathematical concepts themselves but are rather understanding and procedures which are implicit in the study of all mathematics topics.

If, while learning these "modes of thought", the student acquires a dislike for the process, further learning may be inhibited and part of the purpose of instruction may be lost. "The greatest novelty in the new mathematics curriculum is not the content, not the instructional methods, not the grade placement of topics [but] . . . the objectives from which all else stems" (Cronbach, 1965, p. 121). The essential aim of the new curriculum, as Romberg (1969) pointed out, is "to have mathematics taught as a discipline - a system of thought as the specialist knows it, which includes the systematic contents of the discipline and its procedures" (p. 474), and "an anticipated outcome . . . that students' attitudes toward mathematics would be greatly improved" (p. 481). If so, then the question can rightly be asked: To what extent has the new mathematics program fostered favorable attitudes toward theses "modes of thought" or "objectives" or "procedures" of mathematics?

Part of this study attempts to answer this question in one content objective of the school mathematics problem. Evidence (Remmers, 1963; Ehnam, 1970) indicates that student reports can provide an accurate picture of classroom activities and that the reports can serve as a technique for evaluating by "less formal procedures" (Grossnickle, Brueckner, and Reckzeh, 1968) an outcome of educational instructions. Thus, the outcome of this study may not only provide an opportunity for the school to view itself through the eyes of its students as to whether it meets these objectives, but it may also offer a framework within which the mathematics teacher can amend, if need be, his teaching strategy.

The study by Carey (1958) represents one of few attempts that have been made to relate attitudes specific to mathematics problems to performances on these problems. Using college sophomores as her subjects, Carey (1958) found "some" relationship between scores on a questionnaire of attitudes toward problem-solving activities and success in problem-solving. She reported correlation coefficients of .57 (N=24) for males and .29 (N=24) for females, using one form of an attitude scale and a set of problems administered to an experimental group, and .24 (N=24) for males and -.08 (N=24) for females using an alternate form of the attitude scale and a second comparable set of problems administered to another group.

Despite a substantial difference between the correlation coefficients for the two experimental groups, which might be due to differences in the instruments and the samples, the results of this study (Carey, 1958) suggest a possibility that certain dimension within an attitude (toward a type of mathematics problem) structure may be related to performance on the mathematics problem. It is possible that the dimensions within an attitude structure may have different and possibly opposing correlates with performance in the mathematics problem which are minimized or lost when the dimensions combine into a variable of attitudes toward a type of mathematics problem. Such might have been the case in Carey's study.

In sum, the present study seeks to determine which dimensions of student attitudes toward each of four types of mathematics problems, RFKA, PAM, SRAP, and SNrAP, relate to performance on the corresponding type of mathematics problem. It also investigates whether both attitudes towards, and performance on, the mathematics problems reflect the hierarchy of cognitive complexity associated with the mathematics problems.

3. CLARIFICATION OF TERMS

In the interest of clarity, certain terms need to be defined here.

Cognitive objectives in mathematics. Cognitive objectives in mathematics will mean educational objectives listed in the Educational Testing Service (ETS) taxonomy (See Epstein, 1968). In this study, cognitive objectives in mathematics will specifically refer to those listed under levels 1, 2, 3, and 5. They are: (i) Recalling factual knowledge, (ii) Performing mathematics manipulations, (iii) Solving routine problems, (iv) Solving non-routine problems requiring insight or ingenuity.

Content areas in mathematics. Content areas in mathematics will mean all mathematics (subject-matter) currently taught at the junior high school level. In this study, the content area in mathematics will refer specifically to that of 'algebra', as listed in the mathematics course outlined for the ninth grade of all schools within the Edmonton Separate School Board. The first-order sub-categories are given in Table 38.

A Type of mathematics problem. A type of mathematics problem is to be considered as a cell in a two-way classification, cognitive objectives of mathematics by content areas in the mathematics program.

Attitudes toward a type of mathematics problem. In this study attitudes toward a type of mathematics problem will mean response to statements used in the investigation. These represent a sample of statements from a defined universe of statements that have to do with (i) school involvement, (ii) out of school involvement, (iii) general liking to solve the mathematics problems, (iv) involvement with parents, mathematics teacher and friends, and (v) concern for tests and grades in the mathematics problems. A student's score on a dimension within an attitude structure will be taken as his score on the subscale designed to measure that dimension.

Performance on a type of mathematics problem. A student's performance on a type of mathematics problem will mean his test score on the examples of that type of mathematics problem.

4. LIMITATIONS OF THE STUDY

The study was limited in scope by (a) the sample of students, (b) the attitude statements employed, (c) the stimulus-response (S-R) format used in collecting the data, (d) the types of mathematics problems employed, (e) the Q-technique (inverse factor analytic technique) which served as the basic research tool for identifying the structure of student attitudes toward each of the four types of mathematics problems and which also provided the basis for statement selection in developing short attitude subscales, and (f) the measures of students' performances on the four types of mathematics problems.

A further limitation of this investigation lies in using the novel approach to the broad area of attitudes toward mathematics. To the investigator's knowledge, attitudes toward mathematics have never before been analyzed in this detailed manner. Thus, this study constituted an attempt at laying the foundations for further work on attitudes toward mathematics learning using a more diagnostic procedure.

The instruments used in this study were of the author's own making. As is well known, reliability is a function of test length. But, given the testing time approved by the Edmonton Separate School Board and the exploratory nature of the study, the strategy was to use short scales and tests. Consequently there were limitations on their reliabilities.

CHAPTER 2

REVIEW OF SOME RELATED LITERATURE

1. INTRODUCTION

In this chapter, some of the many and diverse definitions which have been proposed for the concept of 'attitude' are discussed. Next, the structure of attitude is reviewed from two viewpoints, namely, "instrumentality-value analyses" and "cognitive-affective-conative analyses."

Following this section on the nature of attitude, a review is made of various methods which have been employed in measuring attitudes toward mathematics. The third section deals with a review of studies relating attitudes toward mathematics to achievement in the subject. A summary is included in the final section of the chapter.

2. ATTITUDE

Definition of Attitude

The numerous definitions given to the term 'attitude' reflect the many theoretical orientations which exist among psychologists and sociologists. Krech and Crutchfield (1948), for example, viewed attitude as "an enduring organization of motivational, emotional, perceptual, and cognitive processes with respect to some aspect of the individual's world" (p. 152). For Campbell (1950), an individual's attitude was a syndrome of response consistency with respect to the object.

Allport (1935), after reviewing about fifteen earlier definitions, defined attitude as "a mental and neural state of readiness exerting a directive influence upon the individual's response to all objects and situations with which it is related" (p. 810).

Doob (1947) stated that an attitude was an implicit drive-producing response that was both anticipatory and mediating in reference to patterns of overt responses, and which was considered significant in the individual's society.

A summary of the different ways in which writers in the 1930's viewed the concept of attitude was drawn up by Nelson (1939). He listed twenty-three such definitions -- "organic drives, purposes, motives. . . , an integration of specific responses into a general set" (see p. 380) -- before he proposed his own as the twenty-fourth. For Nelson, "an attitude may be considered a felt disposition arising from the integration of experience and innate tendencies which disposition modifies in a general way the responses to psychological objects" (p. 381)

Campbell (1947) and DeFleur and Westie (1963), among others, reviewed many more. In particular, DeFleur and Westie (1936) attempted to reduce "the chaos of conflicting and diverse definitions" by fitting most of them into one or the other of two categories, the "probability conceptions" and the "latent process conceptions." Although both conceptions employ the stimulus-response framework, they differ with respect to the inferences which are to be drawn regarding the observable attitudinal responses.

The probability conceptions emphasize the consistency of an individual's behavior toward the attitudinal object. The latent process conceptions imply that such response consistencies are manifestations of underlying variables which also mediate the form of the attitudinal behavior. The definitions of Fuson (1942), Krech and Crutchfield (1948), and Campbell (1950) would probably be examples of the probability conceptions, while that of Allport (1935) and Doob (1947) would likely belong to the latent process conceptions camp.

Although, as DeFleur and Westie (1963) observed, the probability conceptions of attitude have the virtue of simplicity and can be easily translated into behavioral terms, such conceptions seem too general. They do not specify the exact nature of the behavior whose probabilities are to be observed, nor do they indicate the observable operations which constitute the operational definition of the concept.

The latent process conceptions, on the other hand, suffer from "the fallacy of expected correspondence." That is, the latent process conceptions presuppose that an individual will show consistency between his verbal attitude and other forms of his actions toward the attitudinal object. But, attitude researchers (LaPiere, 1934; Saegner and Gilbert, 1950; Kunter, Wilkins and Yarrow, 1952; Mann, 1959; Festinger, 1964) have indicated that verbal attitude has a low correlation with actual behavior toward the object of the attitude. Thus, accepting the latent process conceptions point of view, the findings of the studies cited above raise the question as to which of the behaviors, verbal or overt, represent the manifestations of true attitude.

Realizing some of the limitations that are associated with either point of view, DeFleur and Westie advocated that one of the steps "for refinement of our conceptual apparatus in this area would be to link our definitions more firmly to the methods we employ in measurement."*

Very recently, McGuire (1968) has summarized five ways in which the concept of attitude is currently used in psychology. Using Allport's (1935) definition as a basis for reviewing the conceptual differences that have arisen out of attempts to define attitude, McGuire (1968) has distinguished five approaches "positivistic, paradigmatic, mediationalist, class-inclusionist, interactionist" (see p. 143), in which attitude is used to describe the sum-total of the relationship between a set of stimulus conditions and a set of response conditions.

These five approaches represent five gradations on a continuum ranging from these theorists (for example, Bain, 1928; Horowitz, 1944) who conceive of attitude as "response" -- the positivistic approach -- to those (for example, Allport, 1935; Doob, 1947; Chein, 1948) who regard attitude as "readiness to respond" -- the interactionist approach.

On the basis of this lengthy review, McGuire concluded that:

> It is unlikely that any one approach to defining attitudes will be superior to the others in all regards. There are numerous desiderata for such definitions -- testability, parsimony, ... -- and it is unlikely that one choice of definition will optimize all of them. Since the relative importance of these criteria will vary with different aspects of the scientific enterprise, it may be convenient to allow somewhat different definitional tactics for different purpose (p. 149).

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*Quoted in Melvin L. DeFleur and Frank Westie, "Attitude as a Scientific Concept," in G.F.N. Fearn (ed.), Conceptual Fundamentals in Social Psychology (New York: Simon and Schuster, 1970), page 2794-14.

Agreeing with DeFleur and Westie (1963), McGuire said that the term "can very readily be given an operational definition in terms of observable and scorable responses" (p. 149).

It is apparent from the above review that the concept of attitude is not uniformly defined by contemporary writers. In fact, it is possible that there may be as many definitions of attitude as there are writers on the subject. The diversity of definitions, as Campbell (1963) pointed out, poses an unnecessary impediment to investigation and fuller understanding in the area of attitude, a point of view to which the present investigator associates himself.

With the increased awareness for operationalism, it is suggested that a strict operational definition suffices just as well as any complete, logical definition. Therefore, this study will adopt a "definitional tactic" regarding attitudes toward the various types of mathematics problems, namely, an operational definition in terms of observable and scorable responses.

In this study, attitudes toward a type of mathematics problem will mean response to the Q-sort statements used in the investigation? These statements represent a sample from a defined universe of statements that have to do with (a) school involvement, (b) out of school involvement, (c) general liking to solve the problems, (d) involvement with parents, mathematics teacher and friends, and (e) concern for tests and grades in the problems. A student's score on a particular dimension within an attitude (factor) structure will be taken as his score on the subscale designed to measure that dimension.

The Structure of Attitude

Current theories of attitude structure can be divided into two categories, the "perceived instrumentality-value importance" approach, and the "cognitive-affective-conative" approach. The earlier versions of the first approach, put forward by Di Vesta and Mervin (1960), Rosenberg (1956), Smith (1949), Smith, Bruner, and White (1956), and by Woodruff and Di Vesta (1948), assume that favorable attitudes toward an object result from perceptions or beliefs that the attitude object facilitates need or value satisfaction, and unfavorable attitudes result from perceptions that the attitude object hinders need satisfaction.

As an example of this approach, Rosenberg (1956) asked his subjects to rate 35 diverse values as to how much satisfaction each gave them. He also asked them to rate the attitude object as to whether it facilitated their reaching these values. A subject's attitude score was then calculated as the algebraic sum of the product of the valence of each goal and the object's perceived instrumentality to that goal. It was found that the attitude scores obtained in this manner related fairly well to an independent measure of favorableness toward the object of the attitude.

In particular, some of Rosenberg's data were interpreted as suggesting that 'value satisfaction' and 'perceived instrumentality' are separate and manipulable dimensions of attitude structure. It has also been shown that the attitude changes could be induced experimentally either by changes in the relationship perceived between the attitude object and the goal (Carlson, 1956), or by changes in the perceived valence of the goal (Rosenberg, 1956).

Fishbein, amongst others, has attempted to quantify this notion (Fishbein, 1963; Fishbein and Hunter, 1964; Fishbein, 1965). For him, an individual's attitude toward an object is the pooling of his beliefs about the object weighted by the evaluative aspects of those beliefs.

The other approach to the structuring of attitudes, the cognitive-affective-conative approach, sprung largely from Campbell (1947), Krech and Crutchfield (1948), Krech, Crutchfield, and Ballachey (1962), and has been adopted by Katz and Stotland (1959), Lambert and Lambert (1964), and by Newcomb, Turner, and Converse (1964). In this approach attitudes are viewed as having three components, cognitive or perceptual component, affective or emotional component, and the conative or behavioral component.

The cognitive component of attitudes refers to the manner in which the attitude object is perceived and thought about. Katz and Stotland (1959) suggest a further breakdown of this component in terms of (a) the number of cognitive elements (i.e., the number of beliefs), (b) the organization of these elements into a hierarchical pattern, and (c) the generality or specificity of the beliefs. A typical measure of this component, according to McGuire (1968), has been the adjectival check list (see Gilbert, 1951) which has been used for revealing stereotypes about ethnic groups.

The affective component refers to the feelings or emotions of likes and dislikes about the object of the attitude. G.S.R., heart beat, and pupil dilation provide measures of this component.

The third component within this camp, the conative component, refers to overt behavior elicited in the presence of the object of the attitude. The social distance scale of Bogardus (1925) has been used to provide an index of this component. Chein (1951) suggests that in addition to the cognitive, affective, and conative components, attitudes can be broken down further into various other dimensions which cut across the three components.

Granting that this latter approach to the structure of attitudes can be applied to mathematics problems, the question arises; How closely related are the cognitive, affective, and conative components of attitudes toward mathematics problems? If these different components do not give identical results, then which component or weighted composite of the components is the most valid measure of attitudes toward mathematics problems? Certain researchers (Katz, 1960; Katz and Stotland, 1959) have suggested that some attitudes are primarily cognitive, others affective, and still others conative, while others have a strong mixture of all three and cannot, therefore, be studied in isolation. McGuire (1968) said, "the three components have proven to be so highly intercorrelated that theorists who insist on distinguishing them should bear the burden of proving that the distinction is worthwhile" (p. 157). It is believed that in the realm of mathematics problems, it may be difficult to distinguish among the three components of a person's <u>actual</u> (not his expressed) attitudes because of the not too ideal measuring procedures. And, unless such distinction can be made with precision, it does not seem "worthwhile" to attempt to measure the different components separately.

Furthermore, realizing the difficulty of answering the questions posed above, this study will not attempt to adopt the cognitive-affective-conative point of view in its conceptualization of attitudes toward mathematics problems, nor will the study adopt the perceived instrumentality-value importance approach. Rather, the study will adopt an operational point of view, that the response of a subject to the list of statements used in this study is an "indicant" of the construct 'attitudes toward mathematics problems.' It is also realized that the measurement of this construct may be difficult and that expressed attitudes need not be the same as actual attitudes. As Torgerson (1958) pointed out, "at best, the two are presumed to be monotonically related to each other. At worst, merely a positive low correlation of unknown magnitude is persumed to exist" (p. 7).

3. MEASURES OF ATTITUDES TOWARD MATHEMATICS

Introduction

Many techniques have been devised to measure attitudes toward mathematics. Several of these techniques include: (a) observation and interviews (Shapiro, 1961; Ellingson, 1962; Brown and Abell, 1965), (b) attitude scales--Thurstone's method of successive intervals (Dutton, 1954; Dutton, 1962), Likert-type scales (Gladstone, Deal, and Drevdahl, 1960; Aiken, 1963; Remai, 1965; Dutton and Blum, 1968), Semantic Differentials (Anttonen, 1969; Neale and Proshek, 1967; Johnson, 1970; McCallon and Brown, 1971), and Guttman-type scales (Anttonen, 1969), (c) projective measures (Fedon, 1958; Nealeigh, 1967; Jones, 1968), and (d) neuro-physiological correlates, e.g. breathing rate, blood presure, heart beat and changes in electrical skin resistance (Dreger and Aiken, 1957; Milliken and Spilka, 1962).

The popular devices for measuring attitudes toward mathematics have been the Thurstone's method of successive intervals (Thurstone, 1928) and the Likert's method of summated rating (Likert, 1932). The Guttman-type scale (Guttman, 1944) a deterministic model for determining from the response patterns of subjects to a set of statements whether or not the statements form a scale, has rarely been employed in measuring attitudes toward mathematics (only one published study by Anttonen (1969) was found to have employed this technique). This may be due to the criticisms that center around this technique (see, for example, Festinger, 1947; Clark and Kriedt, 1948; Edwards, 1948; Edwards and Kilpatrick, 1948; Eysensk and Crown, 1949): (a) that the criterion of a perfect scale, that is, if the subject endorses one statement he will endorse all statements having a lower scale value, is rarely obtained in practice, (b) that placing subjects whose response patterns do not fit the ideal model presents a great difficulty, and (c) that the coefficient of reproducibility is a function of the response popularity and, therefore, cannot be lower than the proportion of subjects in the most popular category (Torgerson, 1958).

Naturally, the question arises as to whether the Semantic Differential measures the same construct as the Thurstone-type or Likert-type scales. Evans (1971) compared the test-retest reliability coefficients for four arithmetic attitude scales, Dutton-Thurston, Dutton-Likert, Hoyt, as revised by Anttonen, and Semantic Differential. He correlated the scores from two administrations of the instruments about a year apart, and found that the test-retest reliability coefficients ranged from 0.35 to 0.61. The intercorrelations among the scales ranged from 0.59 to 0.83. On the basis of these results, he concluded that the four attitude scales sampled a common construct.

Some of the varied methods which have been employed in measuring attitudes toward mathematics are reviewed under four headings, (a) observations and interviews, (b) attitude scales, (c) projective measures and (d) breathing rate, blood pressure and changes in electrical skin resistance.

Observations and Interviews

Shapiro (1961) employed a semi-structured interview consisting of 19 questions designed to reveal each student's feelings toward arithmetic. A student's attitude score was then determined by the composite rating of the interviews made by three judges using a scoring scheme which had been set up to quantify the responses.

Ellingson's (1962) study employed two measures of attitudes toward mathematics. One was a Thurstone-type inventory constructed from statements and opinions about mathematics, collected from mathematics students and teachers in selected Oregon high schools. The other was teachers' ratings of students' attitudes on a nine-point scale identical to the one used by the inventory. A significant positive correlation ($r = .48$, $N = 755$) was reported between the two measures. On the other hand, Brown and Abell (1965) reported that teachers' observations appeared to be inadequate as a method of appraising students' attitudes toward mathematics.

Kane (1968) devised a "neutral" instrument for assessing attitudes of prospective elementary school teachers by asking them to respond to items which seemed to have no connection with mathematics. Each teacher was asked to rank-order the subject areas of English, Mathematics, Science, and Social Studies in response to 6 statements having to do with (a) how well he enjoyed working in that area, (b) whether that area of study was worthwhile for him during his high school days, (c) whether he enjoyed most of the courses in that area during college days, (d) how well he learned the courses in that area during college days, (e) whether he would enjoy teaching the subject the most, and (f) whether he would probably be the most competent teacher in that subject area. An attitude score was taken as the extent to which mathematics was ranked higher than the other three subjects.

Attitude Scales

The Thurston-type attitude scale which has been widely employed in studies on mathematics is the Dutton's scale (Dutton, 1954; Dutton, 1962). It consists of statements expressing negative and positive feelings toward arithmetic. It was originally a 22-item scale designed to assess prospective elementary teachers' attitudes toward arithmetic, but it has since been modified into a 15-item scale and has been administered to junior high school students (Dutton, 1968). Several studies (Cleveland and Bosworth, 1967; Neale, 1969; Frase, 1971; Studer, 1971) have adapted Dutton's scale to meet the purposes of their investigations.

Studies (Edwards and Kenny, 1946; Barclay and Weaver, 1962) have shown that the Likert type scale is easier to construct and more reliable than the Thurstone-type scale. This probably explains why a majority of studies on attitudes toward mathematics have employed Likert-type scales of the researchers' own making. Aiken (1963) reported a revised version of Aiken and Deeger's (1961) scale. Dutton and Blum (1968) reworded certain statements of the original Dutton's scale into a Likert-type form.

The Semantic Differential (SD), invented by Osgood and illustraded in Osgood, Suci, and Tannenbaum (1957) for measuring the connotative meanings of concepts as points in a semantic space, has been explored by Anttonen (1969) for use in measuring attitudes toward mathematics. In Anttonen's study, the students were required to respond to the stimulus "Mathematics and Me" on a series of 18 bipolar adjective scales. The responses on the evaluative dimension were summed to obtain a single score representing a student's attitude toward mathematics.

Using the students' responses on the eight adjectival pairs which loaded on the evaluative dimension, Anttonen estimated the reliability by Hoyt's (1941) formula, to be 0.91. He also reported a correlation coefficient of 0.7 with a Likert-type scale. Neale and Proshek (1970), and McCallon and Brown (1971) have also employed the SD to measure attitudes toward mathematics.

Projective Measures

Despite the lack of objectivity characteristic of projective techniques, few studies have explored its use for measuring attitudes toward mathematics. In Fedon's (1958) study, the intensity of the students' feelings toward arithmetic was determined by a color scheme, ranging from red for most favorable attitudes through yellow for neutral attitudes and to black for very unfavorable attitudes.

Nealeigh (1967) explored the possibility of measuring students' attitudes toward mathematics using a picture preference test. The 310 test items consisted of pairs of pictures. The members of a given pair differed from one another in that one of four concepts -- symmetry, similarity, order, and pattern -- was introduced into one of the pair and was omitted in the other. Attitudes toward mathematics were also assessed using another method, and the results obtained were compared with those from the picture preference test. The results indicated that it was possible to construct a picture preference test that discriminated among students on the basis of their attitudes toward the study of mathematics. It was also found that the same pictures did not discriminate at all grade levels.

Jones (1968) employed a structured sentence completion survey to assess students' attitudes toward mathematics. The students were required to complete each of twenty-five stem sentences using any responses they felt appropriate. Four mathematics instructors then rated each of the completed sentences as favorable, unfavorable, and neutral responses.

Neuro-physiological Correlates, e.g. Breathing Rate, Blood Pressure, Heart Beat, and Changes in Electrical Skin Resistance

Dreger and Aiken (1957) employed Nichols and Daroge's (1955) psychogalvanometer to measure changes in electrical skin resistance (GSR) of 40 university students at the same time as the administration of the Wechsler-Bellevue Intelligence (WB-I) test. Besides recording the GSR deflection for each subtest, the deflection was also noted during the administration of the arithmetic instructions which read "I want to see how good you are in arithmetic." The only significant changes in the GSR occurred for those students who previously had been classified as "number anxious."

By means of a polygraph, Milliken and Silkaa (1962) recorded breathing rate, blood pressure, heart rate, and psychogalvanic skin response of students during the first 30 seconds and last 30 seconds of the testing time of each subtest of the ACE. The results indicated that those who previously had scored low on the quantitative section of the Scholastic Aptitude Test (SAT) and high on the verbal section of the SAT made greater physiological responses during the administration of the mathematics subtest of the ACE. Moreover, males showed greater physiological responses than female during the administration of the ACE mathematics subtest.

4. ATTITUDES TOWARD MATHEMATICS AND ACHIEVEMENT IN MATHEMATICS

Harrington (1960) explored the relationship between attitudes toward mathematics, as measured by three different approaches to attitude measurement, and mathematics achievement in college students. Information was also sought from persons thought influential in the development of such attitudes among the students. The results indicated no significant relationship between attitudes and performance in the mathematics course. However, the selection against non-selection of a mathematics course was significantly related to attitudes.

Stephen (1960) compared the attitudes of accelerated and remedial classes. She administered Dutton's attitude scale to six seventh-grade and six eighth-grade classes. The mean attitude score for the accelerated group was found to be significantly higher than that for the remedial group. On the basis of this result, she recommended that attitude scores might be used together with achievement test scores for placement in special classes.

Aiken and Dreger (1961) employed correlational and multiple regression analyses to determine the contribution of past performance, mathematics attitude, and ability measures in predicting mathematics achievement. Tests of significance on the partial regression coefficients in the regression equation indicated that, for the males, all variables except mathematics attitude contributed significantly. For females, only mathematics attitude and Differential Aptitude Tests (DAT) Numerical Ability made a significant contribution.

Shapiro (1961), using a semi-structured interview method to obtain measures of the attitudes toward arithmetic of fourth, fifth, and sixth graders, found that those who reported liking arithmetic had higher IQ scores, higher grade placement scores on the Wide Range Achievement Test, all parts of the arithmetic section of the California Achievement Test, and on arithmetic school marks than those who reported disliking arithmetic.

Cristantiello (1962) investigated the relationship between quantitative aptitude and achievement at various mathematics attitude levels. College sophomores were classified according to their major field of study -- business administration, social science, and natural science. Within each discipline, the students were divided into high, medium, and low attitude groups on the basis of their scores on a mathematics attitude inventory. For each of the nine resulting cells, a correlation coefficient was calculated between the students ACE-Q scores and their mathematics grade.

It was found that the correlation coefficient was positive and significantly higher for the middle attitude group than for the high or low attitude group. This suggests the possibility that only very positive or very negative attitudes affect achievement, whereas moderate attitudes do not affect performance.

A study aimed at determining the effects on fourth graders' attitudes of twenty-one periods of planned arithmetic instructions was carried out by Lyda and Morse (1963). Using Dutton's attitude scale as a measure of the students' attitudes, and the Stanford Arithmetic Achievement Test, Elementary Form L as a measure of students achievement in arithmetic, they found a marked trend toward positive attitudes toward arithmetic. Associated with these changes in attitudes were significant gains in achievement.

Bassham, Murphy, and Murphy (1964) investigated the relationship between attitudes toward arithmetic and achievement in arithmetic while controlling for individual differences in mental ability and reading comprehension. A difference in mean scores of mastery in fundamental concept of arithmetic was found to exist between those students classified as in the upper two-fifths and those classified as in the lower two-fifths on the distribution of attitude scale scores. An obvious criticism of this investigation, as the authors pointed out, was that "the design of the study did not allow the authors to establish a level of confidence at which this difference could be accepted as not occurring by chance" (p. 71).

In a study involving 108 fourth year Brazilian elementary school children, Lindgren, Silva, Faraco, and DaRocha (1964) investigated the relationship between attitudes toward problem-solving activities and success in solving problems. A small significantly positive correlation was found between problem-solving activities and arithmetic achievement. The results of this study, which seemed consistent with those of Carey (1958), also showed a positive, but not significant, correlation between the attitudes and school marks in arithmetic.

In the academic year of 1962/63, the subcommittee of the Department of Education in the Province of Alberta initiated an experimental program in modern mathematics at the junior high school level. Remai's (1965) study was aimed at comparing the attitudes toward mathematics of students in the modern-mathematics group and those in the traditional-mathematics group. A second objective was to investigate the relationship of sex, scholastic ability, and problem solving to these attitudes.

No significant difference was found between the attitude scores of students in the modern program and those of students in the traditional mathematics program. Also, no significant difference was found to exist between the attitudes of boys and girls in the modern mathematics program. Significant positive relationships were found to exist between attitudes toward mathematics and scholastic ability, and between attitudes toward mathematics and problem solving skill. These results seem consistent with those of a similar study by Yasui (1967).

Cleveland and Bosworth (1967) investigated whether there were significant differences between the top-quarter arithmetic achievers and the bottom-quarter arithmetic achievers at sixth-grade level in certain psychological and sociological characteristics. Included in the battery of tests was an adapted form of Dutton's attitude scale which was to provide a measure of the students' attitudes toward arithmetic.

The students were first categorized into three ability groups, (a) low, IQ's 75-89, (b) medium, IQ's 90-110, and (c) high, IQ's 111-125. Within each ability group, the students were then classified into high achievers and low achievers by taking the top and bottom 25 percent of achievers in arithmetic problem solving. The results indicated that positive attitudes toward arithmetic were correlated with achievement in fundamentals among students in the two lower IQ ranges.

An international study (Husen, 1967) was designed to compare the mathematics abilities of secondary school students in twelve countries. Three of the five attitude scales which were employed pertained to mathematics. The first of these measured the extent to which mathematics was viewed as a fixed system. Low scores on this scale indicated a view that mathematics is a fixed system which is learned by mechanical application of set rules. High scores indicated a view that mathematics is viewed as a developing and flexible system.

The second scale assessed the extent to which mathematics is perceived as a difficult subject. Low socres on this scale indicated that mathematics is viewed as a difficult subject which is reserved for the intellectually gifted. High scores indicated that mathematics is viewed as within the reach of every student.

The third scale pertained to the role of mathematics in the present society. Low scores indicated a view that mathematics is of little value, whereas high scores indicated mathematics is viewed as playing a vital role in the society.

The countries were ranked with respect to the mean score on each of the attitude scale and on the total score in mathematics. Significantly negative rank-order correlations were reported across countries. In summarizing the results, Husen concluded that "in those countries in which achievement is high pupils have a greater tendency to perceive mathematics as a fixed and closed system, as difficult to learn and for an intellectual elite, and as important to the future of human society" (p. 45-46).

In a study of the effects of the discovery method on students' attitudes and achievement, Price (1967) compared the pretest and post-test scores on attitude and achievement measures for three groups of students taught in three different ways, discovery, conventional, and transfer methods. Both the discovery and transfer groups showed greater increases in mathematics reasoning than did the conventional group. These successful groups also showed positive attitude changes toward mathematics while the conventional group showed a negative change. Price suggested that the results indicated that success was closely related to attitude and attitude change.

Kane (1968), using a sample of elementary school children from an urban district, found no relationship between their mathematics attitude scores and achievement in arithmetic. A similar result was also reported by Anttonen (1969).

Anttonen (1969) examined through a longitudinal study the stability of mathematics attitude over a 6-year period extending from the fifth and sixth grade levels to the eleventh and twelfth grade levels. Attempts were also made to relate mathematics attitude to mathematics achievement.

Anttonen suggested that the low correlations found between elementary mathematics attitude scores and both elementary mathematics achievement and secondary mathematics achievement indicated that the prediction of both elementary and secondary achievement from fifth and sixth grade mathematics attitude instruments alone would be risky.

Moderate correlations were reported between mathematics attitude scores and both grade-point averages and standardized test scores in grades eleven and twelve. This, according to Anttonen, seemed to indicate that better predictions of mathematics achievement from mathematics attitude alone could be obtained at the high school level.

Neale (1969) investigated the relative contributions of IQ, prior mathematics achievement, and attitude in predicting mathematics achievement. His subjects were 105 sixth grade boys in a suburban elementary school. They were measured at the beginning and end of a school year.

Multiple regression analyses were undertaken to predict later achievement from prior measures. Wisler's (1968) partition analysis was employed to partition the explained variation in the regression analysis into a number of components. The results indicated that much of the explained variation was attributed to independent variables acting jointly. On the basis of this result, no definite conclusion was reached as to whether attitudes cause learning. Neale suggested that a properly controlled experimental study would be necessary to demonstrate whether or not favorable attitudes lead to mathematics achievement.

Moore's (1971) study was one of few studies aimed at relating attitudes toward mathematics to specific components of mathematics achievement. Moore investigated the relationship between attitudes toward mathematics and achievement in arithmetical computation, concepts, and application. Using a Likert-type attitude scale administered six months after the achievement measure, Moore reported significant correlations between attitudes toward mathematics and achievements in computation, concepts, and application.

The relationships between attitudes and any two of the mathematics achievement components were also found to be significant. This study seems to bring to focus the role which specific attainments in mathematics play in overall attitudes toward mathematics.

One final study (Studer, 1971) of interest was a correlational study designed to determine the relationship of the use of discovery methods in mathematics to creative thinking and positive attitudes toward mathematics. Ninety-seven fourth and sixth grade mathematics classes, comprising about three thousand students, were assigned an expository-discovery value by averaging the class mean and the teacher score on two check lists.

Of these ninety-seven classes, twenty-two were classified as expository-oriented and twenty-one as discovery-oriented. These two groups of classes also represented two grade levels and two different socio-economic groups (inner city and non-inner city groups). Torrance Tests of Creative Thinking and Dutton's attitude scale were administered to a twenty-five percent random sample of each of these forty-three classes. The results indicated that inner city classes had more positive attitudes toward mathematics than students in non-inner city classes, regardless of the teaching method. Furthermore, inner city classes using an expository approach had the most positive attitudes toward mathematics of all the groups.

Summary

In general, results from studies relating attitudes toward mathematics to achievement in mathematics indicate a positive relationship. The size of the correlation coefficient seems to vary, depending on the population sampled and the instruments used. The concern of most studies seems to be directed to a global attitude toward mathematics. Two criticisms seem to be in order. These were outlined in Chapter 1 in providing a rationale for the present investigation.

The first concerns the word 'mathematics.' Do junior high school students interprete the word 'mathematics' appearing in mathematics attitude scales to mean the sum-total of all mathematics activities? Secondly, do the attitude scales used in the different studies sample the same attitude dimensions? Aiken (1970A) has emphasized the point -- and the author is in full agreement with him -- that the concept of attitudes toward mathematics be supplemented with that of, say, attitudes toward computation or attitudes toward problem solving.

Also, most researchers seem to consider attitude towards mathematics in the global sense only. Needless to say, the correlation coefficient between attitudes toward mathematics and achievement in mathematics depends also on the dimensions which the attitude scale samples.

One possible approach to overcome these limitations would be to design a multivariate mathematics attitude inventory using a stimulus-response (S-R) format like that adopted in the present study. Such a model is by no means new. Endler, Hunt, and Fosenstein (1962) employed a stimulus-response model for the concept of anxiety. But, to the author's knowledge, such a model has never been used before for the concept of attitudes toward mathematics. Such instruments should provide a more penetrating study of students' attitudes toward mathematics than a global measure of their attitudes toward mathematics.

CHAPTER 3

DESIGN OF THE STUDY

1. INTRODUCTION

Main Study

The primary purpose of this study was to provide answers to three interrelated questions, as follows:

(1) Which dimensions of student attitudes toward each of the following four types of mathematics problems,

- (i) Recalling Factual Knowledge in Algebra (RFKA),
- (ii) Performing Algebraic Manipulations (PAM),
- (iii) Solving Routine Algebra Problems (SRAP), and
- (iv) Solving Non-routine Algebra Problems (SNrAP),

relate to performance on the corresponding type of mathematics problem?

(2) Do student attitudes toward the four types of mathematics problems reflect the hierarchy of cognitive complexity associated with the problems?

(3) Do student performances on the four types of mathematics problems reflect the hierarchy of cognitive complexity associated with the problems?

Preliminary Study

Before the main study could be carried out, however, it was first necessary to conduct a preliminary study to identify the dimensions of student attitudes toward each of the four types of mathematics problems. Q-technique (inverse factor analysis) was chosen for that purpose. A case for employing Q-technique and not the usual R-technique is outlined in the next section.

2. A CASE FOR Q-TECHNIQUE

Q-technique is a factor analytic technique for the study of types (Stephenson, 1953), and is a variation of correlational analysis (Cattell, 1952; Mowrer, 1953). The Q-technique has been widely used in the field of personality and psychotherapy (Friedman, 1955; Turner and Vanderlippe, 1958; Whiting, 1959; Engel, 1959; Fiske and Van Buskirk, 1959; Kelly, 1963; Phillips, Raiford, and El-Batrawi, 1965). It has also been employed in the field of general attitudes.

Kerlinger and Kaya (1959), in a Q study of educational attitudes followed by a "confirmatory" study using the usual R-technique concluded that if the evidence from their study was representative of other attitudinal studies, scale construction could be considerably improved. They added:

> One, Q methodology evidently can, in some cases, be used in place of, or rather, prior to, the usual type of factor analysis (so-called R methodology) and the usual item analysis as a potent logical validity tool. Two, Q methodology is a much quicker, simpler, more economical, and perhaps better procedure than the usual inter correlations of a number of tests using a large number of subjects--at least in situations similar to the one reported here (p. 27).

However, they cautioned that "whether the procedure . . . will hold up in other fields and other types of scales is, of course, not known" (p. 27).

Recently, Gooding and Wilbur (1971) advocated the use of Q-technique as an effective measure of teacher attitudes. Its two major advantages, according to them, are that it is cast in a personalistic frame of reference and it is an idiographic rather than a nomothetic technique. To the author's knowledge, such an approach has never before been employed in the area of mathematics attitudes.

The preliminary study was undertaken with a view to determine if Q-technique would yield meaningful dimensions of student attitudes toward mathematics problems. That is, it sought to find out if the attitude dimensions which emerged using Q-technique with a relatively small sample would be interpretable, and whether these dimensions would generalize to a much larger sample. It is felt that such an approach, if successful, would have implications for selection of statements in constructing mathematics attitude scales. Statements for factor scales can be selected via a Q-sort using a small sample of carefully selected subjects rather than via the usual intercorrelations of statements, using a large number of subjects (R-technique).

A few remarks seem to be in order over the issue of whether Q-technique yields results identical to, or different from, R-technique, Broverman (1961) agreed with Stephenson (1936, 1952, 1953) who maintained that Q-technique gives results that are different from those of R-technique. Eysenck (1953, 1954) and Burt (1937), on the other hand, argued that each technique is a transposition of the other. Eysenck (1954) stated:

> There appears to be no doubt that, statistically, factors derived from the intercorrelations between persons (Q-technique) are transposable from factors derived from intercorrelations between tests (R-technique) (p. 340).

Cattell (1952), remaining definitely uncommitted about the question of transposability of the Q- and R-techniques, pointed out certain differences which make the R-technique preferable to Q-technique, but reported that Burt (1937) claimed to have demonstrated the transposability of the two techniques and that some statisticians agree with this demonstration (p. 503). However, he said "Q-technique has its chief use as a classificatory device for finding the sub-populations in a non-homogeneous population" (p. 502). Cronbach (1953) commented on a possible difference between the Q- and R-techniques.

Lorr, Jenkins and Medland (1955), in a comparative two-way factor analytic study utilizing physical models (cones, cylinders, pyramids and triangular prisms) so as to incorporate known subgroups of objects of known physical dimensions into the study, concluded that "it is inferred that in some instances a Q-analysis may be more powerful and enlightening than an R-analysis (p. 448)." Block (1955) believed that the controversy over the R- and Q-techniques stemmed from different initial assumptions about the nature of personality, and was not an issue in "the realm of matrix algebra." Specifically the conditions under which the R-technique and the Q-technique will or will not give equivalent results, he (Block, 1955) says:

> For the results of Q-technique are not convertible into the results of R where the subject population providing the data for analysis is heterogeneous with respect to the modes of interaction between variables (i.e. the variables intercorrelate differently in one subgroup of subjects than they do in a second subgroup). Where homogeneity of interaction between variables exists for all individuals within the subject sample, results from Q and R are transposable (p. 356).

It appears that definite evidence is lacking over the issue of whether or not the Q-technique will always give results identical to the R-technique; therefore, the ultimate decision to use one or the other must rest on the investigator. The choice to employ the Q-technique in this investigation is to be regarded as purely exploratory.

3. SEQUENCE OF STEPS IN THE STUDY

The sequence of investigations employed in this study is provided below:

A. Pilot Studies.

(i) Selection of four types, and examples of each type, of mathematics problems. These types of mathematics problems were (a) Recalling Factual Knowledge in Algebra (FRKA), (b) Performing Algebraic Manipulations (PAM), (C) Solving Routine Algebra Problems (SRAP), and (d) Solving Non-routine Algebra Problems (SNrAP). This selection was made so that the distinction among the problems could be realized by ninth graders for whom the problems were intended. A detailed report is provided in Appendix B1.

(ii) Selection of 24 students which constituted the sample for the Q-sort. Three criteria, sex, quantitative aptitude, and motivation toward school, were used in selecting these students. Complete details are given in Appendix B3.

The quantitative score on the School and College Ability Tests (SCAT) was used as a measure of students' quantitative aptitude. The Junior Index of Motivation (JIM), whose validity was assessed prior to this pilot study, was used as an index of students' motivation toward school. The validity study of the JIM is reported in Appendix B2.

(iii) Selection of 60 attitude statements to be Q-sorted by the 24 students. This is reported in Appendix B4.

B. Preliminary Study.

Determination of the dimensions that underlie student attitudes toward each of the four types of mathematics problems, RFKA, PAM, SRAP, AND SNrAP, using Q-technique as the factor analytic technique. The design is detailed in Chapter 3, Section 4, and the results are discussed in Chapter 4, Section 2, of this dissertation.

C. Development and Validation of Four Attitude Scales.

The dimensions which emerged from the Q-analysis provided the basis for statement selection in developing these scales. This is discussed in detail in Chapter 4, Section 3.

D. Main Study

(i) Determination of the relationship between the dimensions of student attitudes toward each of the four types of mathematics problems and performance on the corresponding type of problem.

(ii) Investigation as to whether both attitudes toward, and performance on, the four types of mathematics problems reflected the hierarchy of cognitive complexity associated with the problems. The design is reported in Chapter 3, Section 5, and the results are discussed in Chapter 4, Section 4.

4. DESIGN, INSTRUMENTS AND PROCEDURE: PRELIMINARY STUDY

Design

The technique which was used to collect the data employed a stimulus-response (S-R) format. That is, the description of a particular type of mathematics problem followed by examples of that type constituted a stimulus. The responses of each subject in the sample were to Q-sort a list of 60 statements into seven groups on an agree-disagree continuum. The arrangement of the groups and the number of statements in each group were as follows:

STRONGLY AGREE						STRONGLY DISAGREE
2	7	12	18	12	7	2
7	6	5	4	3	2	1

The figures above the line indicate the number of cards in each group and those below the line indicate the values given to the cards in the groups for statistical purposes. The S-R format was repeated for each of the four stimuli. That is, on each occasion, a stimulus consisting of the description of that particular type of mathematics problem, together with examples of that type, was presented to the subjects. The latter were required to Q-sort the same 60 statements in the manner described in Appendix A. A detailed account of the method by which these four types of mathematics problems and their examples were selected is outlined in a pilot study.(See Appendix B1). Four sets of 24 sorts resulted from the above procedure, one for each type of mathematics problem presented to the subjects.

Intercorrelations (product moment) among individual subjects were computed for each set of 24 sorts. The resulting (persons-by-person) matrix was then factored by principal component analysis. Dimensionality was examined by graphing the size of the latent roots (Cattell's Scree criterion). The axes were then rotated orthogonally to the varimax criterion of Kaiser (1958).

Strictly speaking, persons do not define a factor, statements do. Consequently, it was necessary to compute a factor-array for each 'pure person factor' type;* that is, an arrangement of the 60 statements which best typified that pure person factor type extracted from the factor analysis. Following Stephenson's (1953) suggestion and the adaptations by Kerlinger and Kaya (1959), Kerlinger (1966, 1967), and Howell (1968), the factor-array was calculated using Spearman's differential weighting procedure of the factor loading of those persons whose loading on only one factor was significant.

More precisely, persons whose loading were 0.4 and higher on one factor and not on the others were first identified. Each statement-value in the Q-sort responses of those persons was then weighted, using Spearman's formula:

$$W_a = \frac{g_a}{1 - g_a^2}$$

where g_a is the factor loading for person 'a'.

The total value for a statement was then given by the sum, across all 'pure persons', of the weighted scores. The final factor-

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*Persons that load significantly on only one factor and not on the other are said to belong to that pure person factor type.

array was then the arrangement of these 60 statements in descending order of their total scores. Obviously, there were as many factor-arrays uncorrelated with each other as there were person factors. These factor-arrays were identified on the basis of the statements which had high saturations on any given factor. The entire procedure was done separately for the four sets of 24 sorts.

Sample

The subjects consisted of 24 grade 9 students. These students were both males and females of low- and high motivation toward school, as well as of low- and high quantitative aptitude.

A factorial design was employed in selecting these students so that the three variables - sex, motivation toward school, and quantitative aptitude-- had a fair representation in the resulting sample. Table 1 presents the profile of the subjects. The procedure by which these students were selected is reported in detail in Appendix B3.

Q-sort Statements

The Q-sort statements, sixty in all, were considered to be representative of junior high school student attitudes toward mathematics problems. Table 2 presents a list of these 60 statements. Pilot Study B4 was carried out for the purpose of selecting these statements. Additional details are reported in Appendix B4.

5. DESIGN, INSTRUMENTS AND PROCEDURES: MAIN STUDY

Design

Question 1, regarding the relationship between the dimensions

Table 1

Subject Profile Data

Student ID	Sex	Motivational Level	Quantitative Aptitude Level
1	Male	High	High
2	Male	High	High
3	Male	High	High
4	Female	High	High
5	Female	High	High
6	Female	High	High
7	Male	High	Low
8	Male	High	Low
9	Male	High	Low
10	Female	High	Low
11	Female	High	Low
12	Female	High	Low
13	Male	Low	High
14	Male	Low	High
15	Male	Low	High
16	Female	Low	High
17	Female	Low	High
18	Female	Low	High
19	Male	Low	Low
20	Male	Low	Low
21	Male	Low	Low
22	Female	Low	Low
23	Female	Low	Low
24	Female	Low	Low

Table 2

List of Q Sort Attitude Statements

Statement Number	Statement
1.	I am quite 'at home' when this type of problem is discussed in class.
2.	I wish a longer time is spent in class discussing this type of problem.
3.	I feel sorry when I miss a class period in which this type of problem is discussed.
4.	I prefer to do other things in class than to listen to the discussion about this type of problem.
5.	It's thrilling when this type of problem is discussed in class.
6.	I wish students could work this type of problem in groups and check it out among themselves.
7.	I like having the mathematics teacher ask me questions about this type of problem.
8.	I like to talk to the mathematics teacher about this type of problem.
9.	I like to ask the mathematics teacher questions that have to do with this type of problem.
10.	I like to answer questions that have to do with this type of problem.
11.	I prefer the mathematics teacher to work more examples of this type of problem than of other types.
12.	I wish the mathematics teacher could explain this type of problem more and make sure the students understand.
13.	I am happy on days when I do not have to work this type of problem.

Table 2 (continued)

Statement Number	Statement
14.	I like to do extra work in this type of problem whenever I have time.
15.	I like to work this type of problem at school, but not for homework.
16.	I enjoy working this type of problem for homework.
17.	I would like to read other books about this type of problem in addition to the one used in class.
18.	I never use the ideas in this type of problem outside school.
19.	When I grow up, I would like a job that makes use of the knowledge about this type of problem.
20.	I like people who know how to work this type of problem.
21.	I like to study the section in the mathematics textbook that deals with this type of problem.
22.	I prefer to take notes about this type of problem from the mathematics teacher than to read the mathematics textbook.
23.	Out of school, I forget much about how to work this type of problem.
24.	I like to begin my homework by working this type of problem.
25.	I would like to work harder problems than this type of problem.
26.	If I do not get this type of problem right the first time, I like to keep working until I get the right answer.
27.	I would rather be given the right answer to this type of problem than work it out myself.
28.	I would never work this type of problem if I did not have to.

Table 2 (continued)

Statement Number	Statement
29.	I like to work this type of problem.
30.	I would rather read books than spend my time working this type of problem.
31.	I never know how to start working this type of problem.
32.	I feel confident about myself when working this type of problem.
33.	This type of problem is easy to work.
34.	Working this type of problem requires too much thinking.
35.	No matter how hard I try, I never get this type of problem right.
36.	I always say to myself 'I can't do it' whenever I have to work this type of problem.
37.	I could work this type of problem with a little help from the mathematics teacher.
38.	I often forget how to work this type of problem after I have worked on other types.
39.	Working this type of problem takes too much time.
40.	I have always liked working this type of problem.
41.	I never do my best in this type of problem.
42.	I rely on memory to work this type of problem.
43.	I like to chat with my friends about this type of problem.
44.	I like to work this type of problem with my friends.
45.	My friends are good at working this type of problem.
46.	I like to chat with my parents about this type of problem.

Table 2 (continued)

Statement Number	Statement
47.	I like to work this type of problem by myself, but check the working out with friends who know how to work it.
48.	I would rather figure out this type of problem by myself than request help from my parents.
49.	I work hard to get good marks in this type of problem.
50.	I worry about my marks in this type of problem.
51.	I would rather have good marks in this type of problem than in other types of problem.
52.	I always have good marks in this type of problem.
53.	I wish mathematics tests are made up only of this type of problem.
54.	Tests in this type of problem are easy.
55.	I like taking tests in this type of problem.
56.	I like taking mathematics tests that involve harder problems than this type.
57.	I always wish I have fewer problems of this type to do for homework.
58.	I usually get this type of problem right in class, but not in a test.
59.	This type of problem takes too long to work in a test or in an exam.
60.	I like to work this type of problem first in a test or in an exam.

of student attitudes toward each of the four types of mathematics problems, RFKA, PAM, SRAP, and SNrAP, and their performances on the corresponding type of problem (see the questions that are listed under the main study at the start of this Chapter), was answered using correlational analysis. Question 2, enquiring as to whether student attitudes toward the four types of mathematics problems reflected the hierarchy of cognitive complexity associated with the problems, was answered by means of correlational analysis and Guttman's simplex technique. Question 3, enquiring as to whether student performances on the four types of mathematics problems reflected the hierarchy of cognitive complexity, was also answered using correlational analysis and Guttman's simplex technique.

Sample

All grade 9 classes in four Junior High Schools of the Edmonton Separate School System were tested. However, complete data were available for only 312 students. The analysis was therefore restricted to this subsample. The distribution of subjects by sex and school is given in Table 3.

Instruments

The algebra test and attitude scales employed in the main study were constructed by the author himself. The 52 examples of the four types of mathematics problems, 15 examples of RFKA, 14 examples of PAM, 14 examples of SRAP, and 9 examples of SNrAP, were the test items in algebra. These test items are reported in Appendix B1. The attitude scales employed were Attitude Scale RFKA (AS-RFKA), Attitude Scale PAM

Table 3

Number of Subjects by Sex and School in The Main Study

School	Sex		
	Male	Female	Total
St. Kevin Separate School	48	39	87
St. Gabriel Separate School	39	31	60
Cartier-McGee Separate School	60	69	129
Academie Assomption Separate School	--	26	26
Total	147	165	312

(AS-PAM), Attitude Scale SRAP (AS-SRAP), and Attitude Scale SNrAP (AS-SNrAP). The dimensions which emerged from the Q-technique provided the basis for statement selection in developing these scales. The scales are reported in Appendix C. Their development and validation are outlined in Chapter 4, Section 3, of this dissertation.

CHAPTER 4

ANALYSIS AND RESULTS

1. INTRODUCTION

Following the pattern adopted in Chapter 3, the analyses and and results of the preliminary and main studies are discussed separately in this Chapter.

2. ANALYSIS AND RESULTS: PRELIMINARY STUDY

It will be recalled that the purpose of the preliminary study was to identify the dimensions of junior high school student attitudes toward each of four types of mathematics problems, RFKA, PAM, SRAP, and SNrAP. More specifically, it sought to determine if the attitude dimensions which emerged, employing Q-technique with only 24 grade 9 students, would be interpretable, and whether these dimensions would generalize to a much larger sample of grade 9 students.

Attempts will now be made to see what conclusions can be drawn concerning the dimensions which have emerged. The generalizability of the attitude dimensions to a much larger sample of grade 9 students is discussed in Section 3 of this Chapter.

Tables 4, 7, 10, and 13 respectively, present the intercorrelations among the twenty-four persons for each of the four stimulus presentations, RFKA, PAM, SRAP, and SNrAP. The method by which these four (person-by-person) correlation matrices were obtained has already been outlined in Chapter 3 of the dissertation. Each correlation matrix was factored by principal component analysis.

A graph of the size of the latent roots (Cattell's Scree criterion) indicated that three factors could explain each of the observed correlation matrices. Besides, this decision agreed with the criterion of retaining all the eigenvalues which accounted for almost all of the total communality. It could, therefore, be argued that in each case three factors could adequately explain the observed intercorrelations. Thus, three factors were retained for each correlation matrix.

The varimax rotating by Kaiser was used to obtain the rotated factors. Tables 5, 8, 11, and 14 respectively, contain the varimax rotated factors. The corresponding unrotated factors are presented in Appendices D1, D2, D3, and D4.

For each rotated matrix, it was desired to identify persons which belonged to a pure person factor type, from which, using Spearman's differential weighting procedure (see Chapter 3), the arrangement of the 60 statements which typified that pure person factor type could be calculated.

Following the suggestion by Stephenson (1953) and the adaptation by Kerlinger and Kaya (1959), Kerlinger (1966, 1967), and Howell (1968), persons with loading of 0.4 and higher on only one factor were assigned to that factor. They were said to belong to that pure person factor type. Persons with loading of 0.4 and higher on more than one factor were not assigned to any of the factors, and thus did not belong to any pure person factor type. Persons with negative loadings were omitted.

From each arrangement of the 60 statements, about 6 statements with high total score after applying Spearman's weighting procedure* were selected to provide the operational definition of the attitude dimension which typified that pure person factor type.

It will be recalled from Chapter 3 of the dissertation that four stimuli, RFKA, PAM, SRAP, and SNrAP, were presented to the subjects. Each presentation gave rise to a (persons-by-person) correlation matrix from which a varimax rotated matrix (person loading on a factor) was derived factor-analytically. Consequently, four sets of attitude dimensions resulted from applying Spearman's differential procedure, one set for each stimulus presented. The four sets of attitude dimensions are reported in Tables 6, 9, 12, and 15 respectively. The pure person factor types and the attitude dimensions which typified them are discussed below in greater detail.

Three graduate students in the Department of Educational Psychology, working in collaboration, named the attitude dimensions. The naming should therefore be regarded as tentative. Each dimension was identified by two sets of symbols. The first was used to indicate approximate similarity between dimensions, and the second, in parenthesis, indicated the type of mathematics problem presented to the subjects that gave rise to that dimension.

As an example, attitude dimension A(RFKA) will mean a dimension which emerged when the stimulus presented to the subjects was a

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*The term 'saturation' will, hereafter, be used instead of the expression 'total score assigned to a statement after applying Spearman's differential procedure'.

description and examples of RFKA. Likewise, attitude dimension A(SRAP) will mean a dimension which emerged when the stimulus presented to the subjects was a description and examples of SRAP, and which is approximately similar to the dimension A(FRKA).

Interpretation of Pure Person Factor Types and the Attitude Dimensions which Typified each Pure Person Factor Type when the Stimulus Presented was RFKA

The person factor types are presented in Table 5 and the corresponding attitude dimensions are presented in Table 6.

Pure Person Factor Type 1. Persons belonging to this factor type were Numbers 3, 4, 8, 15, 20, 23, and 25 (see Table 5). The attitude dimension which typified this pure person factor type was obtained through Spearman's differential weighting procedure as detailed in Chapter 3 of this dissertation. An inspection of the attitude dimension (see Table 6) reveals that they (the persons) seem to possess evasive attitudes toward mathematics problem of this kind. Specifically, they tend to avoid working the problem; would not work it if they did not have to; would not do it for homework; and would rather be given the right answer to the problem than work it themselves. The dimension corresponding to this factor is named 'evasive' attitudes and is labelled A(RFKA).

Pure Person Factor Type 2. Persons of this factor type were Numbers 2, 10, 11, 17, 21, and 22 (see Table 5). They appear to be the class-participating type (see the attitude dimension in Table 6 which typified this pure person factor type). They tend to welcome longer class

Table 4

Q Correlations for Twenty-four Persons as Variables (n=60 Attitude Statements). Stimulus Presented was RFKA.

Person ID	Correlation Coefficient*																						
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1																								
2	-18																							
3	-55	38																						
4	03	-03	34																					
5	54	-26	-40	-12																				
6	57	-35	-47	13	66																			
7	17	04	11	22	-11	19																		
8	-29	15	29	20	-40	-33	15																	
9	-02	25	09	06	10	04	28	11																
10	-16	63	32	07	-25	-38	-01	15	15															
11	-09	61	41	08	-20	-42	-05	17	09	63														
12	-14	12	31	26	-11	-20	-16	10	-14	14	12													
13	54	-20	-40	-21	60	41	10	-33	10	-25	-30	-18												
14	24	-10	-21	11	29	38	14	-18	21	-01	-09	00	08											
15	-42	21	52	35	-62	-43	07	35	-10	30	18	09	-43	-24										
16	42	-14	-19	07	30	39	35	-24	14	-13	-16	-08	40	12	-20									
17	03	24	06	-04	-02	01	10	-06	-03	26	20	-11	07	09	06	12								
18	-08	37	29	08	-18	-24	13	05	-04	31	18	15	-13	-18	14	-02	08							
19	-14	03	24	07	-27	-11	11	22	09	10	15	10	-35	-02	20	-04	02	18						
20	-19	09	33	27	-15	-09	-07	34	07	20	10	46	-28	20	11	-16	-03	05	20					
21	22	22	19	21	02	-02	28	15	08	26	20	04	-10	-09	23	06	05	08	19	-03				
22	-01	40	32	-08	-05	-23	12	14	14	39	28	07	-03	-15	12	-06	28	06	17	10	38			
23	-27	12	52	57	-39	-24	04	41	-09	29	16	32	-39	-10	60	-02	-01	27	14	36	21	05		
24	-21	04	49	26	-32	-21	13	28	06	09	12	11	-34	-03	48	00	04	26	24	28	13	06	53	

*Decimal points have been omitted

Table 5

Varimax Rotated Factor Loadings* for Twenty-Four Persons as Variables (n=60 Attitude Statements). Stimulus Presented was RFKA.

Person ID	1	2	3	h^2
1	-384	-017	<u>689</u>	622
2	072	<u>811</u>	-175	693
3	<u>657</u>	375	-221	621
4	<u>662</u>	-078	358	572
5	-539	-143	<u>539</u>	602
6	-275	-364	<u>713</u>	717
7	220	159	<u>557</u>	384
8	<u>542</u>	109	-168	334
9	024	297	322	193
10	186	<u>769</u>	-132	643
11	137	<u>715</u>	-189	566
12	393	006	-090	163
13	-569	-087	<u>461</u>	543
14	-032	-112	<u>464</u>	228
15	<u>678</u>	169	-263	557
16	-099	-016	<u>656</u>	441
17	-093	<u>448</u>	129	226
18	206	325	143	169
19	376	145	-003	162
20	<u>531</u>	000	010	283
21	241	<u>442</u>	335	366
22	037	<u>669</u>	038	451
23	<u>822</u>	065	-003	680
24	<u>683</u>	045	031	469

*Decimal points have been omitted. Persons with loading of 0.4 and higher on only one factor are said to belong to that pure person factor type. These loadings have been underlined.

Table 6

Dimensions Derived from Q-Technique
when Stimulus Presented was RFKA*

Statement Number	Statement
Dimension A(RFKA) typifying Pure Person Factor Type 1: Evasive attitudes.	
28	I would never work this type of problem if I did not have to.
57	I always wish I have fewer problems of this type to do for homework.
13	I am happy on days when I do not have to work this type of problem.
27	I would rather be given the right answer to this type of problem than work it out myself.
18	I never use the ideas in this type of problem outside school.
Dimension B(RFKA) typifying Pure Person Factor Type 2. Attitudes toward teacher instruction and preference for group work.	
2	I wish a longer time is spent in class discussing this type of problem.
12	I wish the mathematics teacher could explain this type of problem more and make sure the students understand.
6	I wish students could work this type of problem in groups and check it out among themselves.
42	I rely on memory to work this type of problem.
22	I prefer to take notes about this type of problem from the mathematics teacher than to read the mathematics textbook.
47	I like to work this type of problem by myself, but check the working out with friends who know how to work it.
Dimension C(RFKA) typifying Pure Person Factor Type 3. General liking attitudes.	
33	This type of problem is easy to work

Table 6 (continued)

Statement Number	Statement
1	I am quite 'at home' when this type of problem is discussed in class.
10	I like to answer questions that have to do with this type of problem.
60	I like to work this type of problem first in a test or in an exam.
40	I have always liked working this type of problem.
54	Tests in this type of problem are easy.

*Only those statements with high saturations are given in Table 6. The complete arrangement of the 60 statements in descending order of their total scores is given in Appendix D5.

periods for discussion, and seem to like the idea of checking the working of the mathematics problem in groups. The dimension is named attitudes toward 'teacher instruction' and 'preference for group work', and is labelled B(RFKA).

Pure Person Factor Type 3. Persons numbered 1, 5, 6, 7, 13, 14, and 16 belong to this factor type (see Table 5). They tend to have a general liking to work this type of problem (see the attitude dimension in Table 6 which typified this pure person factor type). The emergence of statement Numbers 40 and 45 - 'I have always liked working this type of problem', and 'Tests in this type of problem are easy' - suggests that this general liking may have been the result of past experience. This attitude dimension is, therefore, named 'general liking'. It is labelled C(RFKA).

Interpretation of Pure Person Factor Types and the Attitude Dimensions which Typified each Pure Person Factor Type when the Stimulus Presented was PAM

The person factor types are presented in Table 8 and the corresponding attitude dimensions are presented in Table 9.

Pure Person Factor Type 1. Persons numbered 1, 2, 3, 4, 6, 13, and 17 belong to this factor type (see Table 8). They appear to be persons possessing high quantitative aptitude. They tend to have a general liking to work this type of mathematics problem at school and at home (see the attitude dimension in Table 9 which typified this pure person factor type). To a first approximation, the attitude dimension corresponds to C(RFKA). It is, therefore, labelled C(PAM) and named 'general

Table 7

Q Correlations for Twenty-four Persons as Variables (n=60 Attitude Statements). Stimulus Presented was PAM.

Person ID	Correlation Coefficient*																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1																								
2	53																							
3	36	30																						
4	52	33	34																					
5	34	23	18	22																				
6	47	24	34	39	47																			
7	14	10	-06	09	-11	-08																		
8	-24	-28	-22	-15	-17	-08	-02																	
9	-16	-13	-22	-22	07	10	-03	15																
10	-03	-25	-20	07	14	28	-09	17	22															
11	-01	01	-14	14	20	09	-12	02	47	36														
12	19	04	04	-05	33	21	01	-09	17	26	26													
13	39	33	35	18	24	17	01	-22	-16	-05	-16	09												
14	00	-13	-30	-01	10	-05	00	11	27	28	26	11	-31											
15	-44	-40	-41	-30	-17	-22	09	39	19	09	-03	-05	-41	20										
16	02	-10	-15	07	28	11	06	10	35	26	41	16	-08	41	20									
17	52	28	28	52	34	18	03	-04	-37	-08	07	10	32	-06	-19	07								
18	-09	-09	-16	-06	17	07	-04	17	36	28	35	13	-28	48	45	53	-03							
19	-32	-30	-24	-23	-10	-16	-03	27	16	10	07	-13	-32	41	47	21	-26	44						
20	-50	-40	-34	-41	-27	-34	-22	39	33	14	10	-05	-41	25	41	16	-31	33	54					
21	-22	-23	-11	-12	-10	-25	-09	30	01	05	11	-02	-35	10	31	05	07	26	35	28				
22	-22	-32	-12	-04	-04	-03	-23	49	10	20	03	-06	-18	34	22	18	-10	22	48	35	35			
23	-28	35	16	07	16	15	-04	-18	17	04	02	05	30	-09	-09	07	13	08	00	-06	02	-22		
24	-34	-48	-27	-11	08	-01	-02	29	39	22	38	14	-35	25	48	47	-03	35	22	35	39	22	-08	

*Decimal points have been omitted

Table 8

Varimax Rotated Factor Loadings* for Twenty-Four Persons as Variables (n=60 Attitude Statements). Stimulus Presented was PAM.

Person ID	1	2	3	h^2
1	762	-284	089	669
2	494	-440	-029	438
3	554	-201	-191	384
4	753	018	023	568
5	507	-110	464	484
6	542	-166	356	448
7	001	-127	-052	019
8	-127	629	-012	412
9	-379	-055	683	614
10	003	151	544	319
11	008	040	693	483
12	102	-194	495	293
13	432	-437	-103	388
14	-055	387	484	387
15	-421	537	144	486
16	066	246	682	530
17	768	119	-051	606
18	-049	423	607	550
19	-271	625	144	485
20	-551	482	159	561
21	-045	649	-018	424
22	-001	730	041	535
23	196	-260	225	156
24	-228	455	491	500

*Decimal points have been omitted. Person with loading of 0.4 and higher on only one factor are said to belong to that pure person factor type. These loadings have been underlined.

Table 9

Dimensions Derived from Q-technique when Stimulus Presented was PAM*

Statement Number	Statement
Dimension C(PAM) typifying Pure Person Factor Type 1: General liking attitudes.	
16	I enjoy working this type of homework.
40	I have always liked working this type of problem.
29	I like to work this type of problem.
10	I like to answer questions that have to do with this type of problem.
48	I would rather figure out the type of problem by myself than request help from my parent.
24	I like to begin my homework by working this type of problem.
Dimension A(PAM) typifying Pure Person Factor Type 2: Avoidance attitudes.	
4.	I prefer to do other things in class than to listen to the discussion about this type of problem.
39	Working this type of problem takes too much time.
36	I always say to myself 'I can't do it' whenever I have to work this type of problem.
13	I am happy on days when I do not have to work this type of problem.
34	Working this type of problem requires too much thinking.
Dimension B(PAM) typifying Pure Person Factor Type 3: Attitudes toward teacher instruction.	
2	I wish a longer time is spent in class discussing this type of problem.
11	I prefer the mathematics teacher to work more examples of this type of problem than of other types.

Table 9 (continued)

Statement Number	Statement
8	I like to talk to the mathematics teacher about this type of problem.
12	I wish the mathematics teacher could explain this type of problem more and make sure the students understand.
9	I like to ask the mathematics teacher questions that have to do with this type of problem.
22	I prefer to take notes about this type of problem from the mathematics teacher than to read the mathematics textbook.

*Only those statements with high saturations are given in Table 9. The complete arrangement of the 60 statements in descending order of their total score is given in Appendix D6.

liking'.

Pure Person Factor Type 2. Persons of this factor type were Numbers 8, 15, 19, 20, 21, and 22 (see Table 8). They appear to be self-defeated persons (see the attitude dimension in Table 9 which typified this pure person factor type). They avoid working this problem because, for them, the problem requires too much thinking and takes up much time. This dimension is named 'avoidance' attitudes and is labelled A(PAM) since it compares favorably with A(RFKA).

Pure Person Factor Type 3. Persons belonging to this factor type were Numbers 9, 10, 11, 12, 14, and 16 (see Table 8). They may be termed 'interested' students (see the attitude dimension in Table 9 which typified this pure person factor type). They tend to welcome longer discussion in class about this type of mathematics problem; like to chat with the mathematics teacher and ask him questions that have to do with this type of problem.

They prefer to take notes from the mathematics teacher than to read the mathematics textbook, presumably because it is easier to understand the teacher's notes than it is to comprehend the mathematics textbook. The dimension corresponding to this factor is named attitudes toward 'teacher instruction'. It compared roughly to B(RFKA), and is labelled B(PAM).

Interpretation of Pure Person Factor Types and the Attitude Dimensions which Typified each Pure Person Factor Type when the Stimulus Presented was SRAP

The person factor types are presented in Table 11 and the corresponding attitude dimensions are presented in Table 12.

Pure Person Factor Type 1. Persons belonging to this factor type were Numbers 3, 8, 9, 12, 14, 17, 20, and 23 (see Table 11). They seem to possess evasive attitudes to mathematics problem of this kind (see the attitude dimension in Table 12 which typified this pure person factor type). They would not work the problem if they did not have to. But since they have to, presumably because of school requirements, they are prepared to go as far as memory can tolerate.

The principles involved in working the problem are not internalized and, as a result, they tend to forget how to work the problem after they have worked on other kinds. This dimension corresponds roughly to A(RFKA) or A(PAM). It has been named "Memory-cum-Evasive' attitudes and labelled A(SRAP).

Pure Person Factor Type 2. Persons numbered 1, 4, 10, 11, 18, 21, and 22 belong to this factor type (see Table 11). They appear to be persons who are self-initiated as well as group-oriented (see the attitude dimension in Table 12 which typied this pure person factor type). That is, they like to work the mathematics problem by themselves or in group, but check it out with the teacher or friends who know how to work it. The dimension has been named 'self-initiated within a group context' attitudes. It is labelled D(SRAP).

Table 10

Q Correlations for Twenty-Four Persons as Variables (n=60 Attitude Statements). Stimulus Presented was SRAP.

Person ID	Correlation Coefficient*																						
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1																								
2	06																							
3	23	-42																						
4	25	26	-22																					
5	31	-44	53	-16																				
6	37	-10	32	11	53																			
7	-15	14	03	16	-18	-16																		
8	05	-31	45	-21	22	05	16																	
9	12	-30	25	-02	24	19	-02	40																
10	33	-24	22	11	28	17	03	20	09															
11	50	04	36	21	38	45	-08	01	12	38														
12	20	-35	44	-24	50	24	-02	29	27	14	20													
13	-13	38	-45	12	-24	-05	-03	-46	-22	-20	-06	-28												
14	20	-32	43	-03	35	20	-03	49	44	39	15	35	-34											
15	-24	45	-25	27	-47	-33	36	-04	-27	-09	-09	-22	33	-19										
16	01	02	07	11	26	30	12	-01	-03	20	14	14	-11	01	03									
17	19	-55	54	-10	47	04	06	53	19	54	24	47	-46	56	-10	02								
18	42	-22	43	-02	43	41	-09	43	16	25	29	41	-33	33	-25	25	44							
19	35	-10	21	02	29	18	-10	26	38	34	39	35	-35	48	-06	15	36	23						
20	09	-22	10	05	16	03	00	20	31	14	16	20	-40	32	-12	10	27	04	39					
21	17	-03	25	29	01	13	04	15	22	24	33	11	-20	35	17	16	31	29	24	24				
22	21	-08	16	03	42	25	09	19	11	52	45	22	-21	23	-03	26	34	16	42	21	16			
23	-08	-40	30	-04	11	-01	-12	35	35	09	08	35	-26	33	02	18	37	13	33	24	28	-02		
24	17	-50	71	-09	53	37	-03	35	35	23	28	43	-43	43	-38	16	47	35	35	16	22	13	33	

*Decimal points have been omitted

Table 11

Varimax Rotated Factor Loadings* for Twenty-Four Persons as Stimulus (n=60 Attitude Statements). Stimulus Presented was SRAP.

Person ID	1	2	3	h^2
1	-012	675	229	508
2	-620	162	-435	599
3	598	227	387	558
4	-222	460	-445	458
5	334	424	646	709
6	-023	591	496	596
7	106	-002	-482	243
8	730	-013	-009	533
9	525	091	089	292
10	314	554	-044	407
11	053	781	116	626
12	517	186	343	420
13	-642	-068	-128	433
14	688	255	034	539
15	-146	-018	-786	640
16	049	389	-015	154
17	761	239	071	641
18	355	432	329	421
19	476	477	-055	457
20	465	165	-146	265
21	362	436	-357	449
22	212	593	-023	397
23	637	-050	-085	415
24	589	227	424	578

*Decimal points have been omitted. Person with loading of 0.4 and higher on only one factor are said to belong to that pure person factor type. These loadings have been underlined.

Table 12

Dimensions Derived from Q-technique when Stimulus Presented was SRAP*

Statement Number	Statement
Dimension A(SRAP) typifying Pure Person Factor Type 1: Memory-cum-evasive attitudes.	
42	I rely on memory to work this type of problem.
38	I often forget how to work this type of problem after I have worked on other kinds.
18	I never use the ideas in this type of problem outside school.
28	I would never work this type of problem if I did not have to.
4	I prefer to do other things in class than to listen to the discussion about this type of problem.
13	I am happy on days when I do not have to work this type of problem.
Dimension D(SRAP) typifying Pure Person Factor 2: Self-initiated within a group context attitudes.	
47	I like to work this type of problem by myself, but check it out with friends who know how to work it.
8	I like to talk to the mathematics teacher about this type of problem.
6	I wish students could work this type of problems in groups and check it out among themselves.
44	I like to work this type of problem with my friends.

*Only those statements with high saturations are given in Table 12. The complete arrangement of the 60 statements in ascending order of their total scores is given in Appendix D7.

Pure Person Factor Type 3. It will be recalled that a factor loading of 0.4 and above on only one factor was used as the criterion for assigning a person to a particular factor. No such person exists for this factor.

Howell (1968), in her Q study of Medical Interns, assigned persons with loadings of 0.4 or higher on more than one factor to the factor on which the highest loading occurred, providing this loading was .05 higher than the next loading. Such a strategy was considered by the author to be inappropriate since it artificially forces the person to belong to one or the other of the factors. Attempts to interpret the corresponding dimension using statements with low negative saturations failed. For these reasons, the attitude dimension corresponding to this factor was omitted from further consideration.

Interpretation of Pure Person Factor Types and the Attitude Dimensions which Typified each Pure Person Factor Type when the Stimulus Presented was SNrAP

The person factor types are presented in Table 14 and the corresponding attitude dimensions are presented in Table 15.

Pure Person Factor Type 1. Persons of this factor type were Numbers 3, 4, 5, 8, 10, 15, 16, 17, 20, 22, and 23 (see Table 14). They are persons who tend to avoid working this type of mathematics problem (see the attitude dimension in Table 15 which typified this pure person factor type).

The profile of these persons include both sexes, low- and high-motivated persons, as well as persons low and high in quantitative

Table 13

Q Correlations for Twenty-four Persons as Variables (n=60 Attitude Statements). Stimulus Presented was SNrAP.

Person ID	Correlation Coefficient*																						
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1																								
2	35																							
3	12	-47																						
4	18	-06	45																					
5	19	-42	60	46																				
6	36	01	28	17	22																			
7	01	-14	21	14	-08	02																		
8	-15	-19	34	36	21	19	12																	
9	19	29	-37	-22	-42	02	02	-26																
10	07	-28	30	32	36	22	-06	44	-17															
11	41	01	20	13	15	51	03	-01	16	15														
12	24	17	-24	11	-05	03	06	-21	15	11	20													
13	36	28	-04	-08	-10	22	-02	-14	10	-12	13	10												
14	12	26	-08	17	02	10	23	-05	12	-11	03	03	11											
15	-05	-50	49	42	54	-02	02	46	-39	38	04	-16	-41	-16										
16	06	-10	33	41	17	21	20	47	-16	31	37	-05	14	05	22									
17	05	-25	32	34	23	21	26	30	-18	31	26	04	-07	04	37	47								
18	25	16	23	17	17	41	-35	19	-11	28	21	-08	-11	-20	11	01	06							
19	28	09	23	30	35	23	14	06	03	10	26	17	-16	21	08	10	-01	11						
20	-09	-32	42	45	36	08	14	38	-36	30	12	06	-32	00	46	23	34	08	22					
21	47	38	-03	32	07	23	13	-03	-01	10	22	21	12	13	-09	25	16	16	17	-02				
22	29	-16	30	25	42	28	-08	29	-04	56	34	06	-08	-07	38	12	26	28	24	36	-03			
23	-10	-34	43	26	35	-04	20	41	-32	22	11	-14	-35	-05	52	22	34	08	09	36	00	16		
24	-03	17	-11	16	-14	-16	29	-06	23	-17	03	15	07	12	-05	00	01	-17	07	-11	22	-19	-03	

*Decimal points have been omitted

Table 14

Varimax Rotated Factor Loadings* for Twenty-Four Persons as Stimulus (n=60 Attitude Statements). Stimulus Presented was SNrAP.

Person ID	1	2	3	h^2
1	-105	777	-098	625
2	-593	417	133	543
3	724	143	-109	557
4	594	376	222	544
5	652	200	-237	522
6	150	640	-255	497
7	238	036	707	558
8	633	007	-019	402
9	-518	213	146	335
10	538	235	-305	437
11	151	646	-059	444
12	-152	362	175	185
13	-362	382	066	282
14	-065	249	461	279
15	787	-134	-128	653
16	481	324	253	401
17	566	206	201	404
18	136	375	-599	517
19	218	442	090	251
20	684	020	018	468
21	-010	609	273	446
22	444	369	-410	501
23	666	-132	086	468
24	-097	094	613	394

*Decimal points have been omitted. Person with loading of 0.4 and higher on only one factor are said to belong to that pure person factor type.

Table 15

Dimensions Derived from Q-technique when Stimulus Presented was SNrAP*

Statement Number	Statement
Dimension A(SNrAP) typifying Pure Person Factor Type 1: Evasive attitudes.	
39	Working this type of problem takes too much time.
4	I prefer to do other things in class then to listen to the discussion about this type of problem
57	I always wish I have fewer problems of this type to do for homework.
13	I am happy on days when I do not have to work this type of problem.
59	This type of problem takes too long to work in a test or in an exam.
Dimension B(SNrAP) typifying Pure Person Factor Type 2: Attitudes toward Teaching instruction and preference for group work.	
9	I like to ask the mathematics teacher questions that have to do with this type of problem.
6	I wish students could work this type of problem in groups and check it out among themselves.
26	If I do not get this type of problem right the first time, I like to keep working until I get the right answer.
2	I wish a longer time is spent in class discussing this type of problem.
12	I wish the mathematics teacher could explain this type of problem more and make students understand.
15	I like to work this type of problem at school, but not for homework.
Dimension corresponding to Factor 3: Unnamed	
29	I like to work this type of problem.

Table 15 (continued)

Statement Number	Statement
28	I would never work this type of problem if I did not have to.
26	If I do not get this type of problem right the first time, I like to keep working until I get the right answer.
41	I never do my best in this type of problem.
18	I never use the ideas in this type of problem outside school.

*Only those statements with high positive saturation are given in Table 15. The complete arrangement of the 60 statements in descending order of their total scores is given in Appendix D8.

aptitude. This does not seem surprising since, for most students, working a non-routine mathematics problem is a frightening experience. The attitude dimension compared favorably with A(RFKA), A(PAM), or A(SRAP). It has been named appropriately 'evasive' attitudes and labelled A(SNrAP).

Pure Person Factor Type 2. Persons belonging to this factor type were Numbers 1, 2, 6, 11, 19, and 21 (see Table 14). They appear to be persons who are group-oriented (see the attitude dimension in Table 15 which typified this pure person factor type). In particular, they prefer working the problem at school where, if the need arises, assistance can be readily obtained from either the mathematics teacher or from friends. The attitude dimension corresponds roughly to B(RFKA) as attitudes toward 'teaching instruction' and 'preference for group work.' It is therefore labelled B(SNrAP).

Pure Person Factor Type 3. Person Numbers 7, 14, and 24 belong to this factor type. The statements with high saturations that provided the operational definition of the attitude dimension which typified this factor type are shown at the bottom of Table 15. The judges agreed that this dimension was not readily interpretable and was therefore unnamed. The attitude dimension was therefore omitted from further analysis.

Summary

The preliminary study was undertaken to determine if Q-technique would reveal meaningful attitude dimensions to four types of mathematics problems. These problems were (a) Recalling Factual

Knowledge in Algebra(RFKA), (b) Performing Algebraic Manipulations(PAM), (c) Solving Routine Algebra Problems(SRAP), and (d) Solving Non-routine Algebra Problems(SNrAP).

A sample of 24 grade 9 students Q-sorted a list of 60 statements according to a specified distribution for each of four stimulus presentations, RFKA, PAM, SRAP, and SNrAP. For each set of 24 sorts, product moment intercorrelations were calculated among individual subjects. The resulting (person-by-person) correlation matrix was factored by principal component analysis. Three factors were retained. The axes were rotated to a varimax criterion. For each rotated matrix (persons loading on factors), persons with loadings of 0.4 and higher on only one factor were selected as defining that pure person factor type.

An arrangement of the 60 statements which typified that pure person factor type was calculated using the adapted form of Spearman's differential weighting procedure. About 6 statements with high saturations* were selected to provide the operational definition of the attitude dimension which typified that pure person factor type. The entire procedure was done separately for the four stimulus presentations.

Three interpretable dimensions each were extracted in the case of RFKA and PAM, and two dimensions each in the case of SRAP and SNrAP. Table 16 provides a summary of the attitude dimensions associated with

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*A saturation is to be understood as the total score assigned to a statement after applying Spearman's differential weighting procedure.

Table 16

Summary of Attitude Dimensions Associated with the Four Types of Mathematics Problem.

Type of Mathematics Problem	Attitude Dimension
RFKA	Evasive attitudes (A(RFKA)), Attitudes toward teacher instruction and preference for group work (B(RFKA)), and General liking attitudes (C(RFKA)).
PAM	General liking attitudes (C(PAM)), Avoidance attitudes (A(PAM)), and Attitudes toward teacher instruction (B(PAM).
SRAP	Memory-cum-Evasive attitudes (A(SRAP)) and Self-Initiated within a Group Context attitudes (D(SRAP)).
SNrAP	Evasive attitudes (A(SNrAP)) and Attitudes toward teacher instruction and preference for group work (B(SNrAP)).

each type of problem. It must be emphasized again that the statements provided the operational definition of an attitude dimension. Since the naming of the dimensions necessarily involves subjectivity, this was by no means a straightforward task. Therefore, the names given to some of the dimensions may be open to question. The generalizability of these dimensions to a much larger sample of grade 9 students is discussed in the next section.

3. DEVELOPMENT AND VALIDATION OF ATTITUDE SCALES

Introduction

As remarked before, the interpretation of an attitude dimension was based on the statements, arising from the Q-technique which had high saturations on that dimension. This also formed the basis for statement selection in developing short subscales to measure the dimensions. In some cases, this yielded four or five statements instead of the required six. Therefore, more statements were taken from each pure person sort* until there was a total of six statements for each dimension. Statements lower in an attitude dimension were added provided they did not have high saturations on other dimensions. Four attitude scales resulted from such procedure; each scale consisting of subscales designed to measure the attitude dimensions associated with one type of mathematics problem.

An important consideration in Q-technique is whether or not the dimension are "real". As Kerlinger and Kaya (1959) expressed it,

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*A pure person sort is to be understood as the arrangement of the 60 statements which typified a pure person factor type.

"Naturally, any hypotheses as to the nature of the factors are subject to further empirical inquiry before acceptance of the true nature and reality of the factor" (p. 15). In essence, two questions arose: Do the attitude dimensions from the Q-technique really exist for the experimental group? Are the scales valid for the experimental group?

For each scale, it was felt necessary, therefore, to obtain confirmatory evidence of the existence of the dimensions for the experimental group. To the extent that the dimensions could be reproduced using an entirely different procedure, this would seem to establish the existence of the dimension and, in part, validate the scale. That is to say, if the same dimensions built in from the Q-technique were reproduced on the experimental group using an R-technique, this would provide much stronger evidence of the existence of the dimensions than a conclusion based on just one procedure. In addition, it would indicate that an attitude scale actually consisted of one or more unidimensional subscales. This result, together with reliability estimates of the subscales, provided the basis for judging the validity of an attitude scale. It is worth pointing out that the usual method of establishing validity by checking against a criterion proved impossible. No criterion was found to exist. Each scale was therefore judged in terms of content and factorial validities. This section reports in detail an attempt to follow this line of reasoning to establish the existence of the dimensions as well as to validate the scales.

Additional Notation

At this point, it is necessary to introduce some additional notations which will be used throughout the remainder of the thesis.

Attitude Scale RFKA (AS-RFKA) will mean an attitude scale with subscales to measure the dimensions A(RFKA), B(RFKA), C(RFKA).

Attitude Scale PAM (AS-PAM) will mean an attitude scale with subscales to measure the dimensions C(PAM), A(PAM), and B(PAM).

Attitude Scale SRAP (AS-SRAP) will mean an attitude scale with subscales to measure the dimensions A(SRAP) and D(SRAP).

Attitude Scale SNrAP (AS-SNrAP), will mean an attitude scale with subscales to measure the dimensions A(SNrAP) and B(SNrAP).

The four attitude scales are referred to in this thesis as AS-RFKA, AS-PAM, AS-SRAP, and AS-SNrAP.

Concurrent measures of a subject's performances on the four types of mathematics problems were required for the main study. Consequently, an algebra test composed of items which earlier had formed parts of the four stimuli in the S-R format used in obtaining the students' attitude responses to the four types of mathematics problem was administered about the same time. The entire data were collected over a period of four weeks. Special precautions were taken to exclude all teachers while collecting data so that the students could feel free to express their opinions. Four graduate students helped with the administration.

Intercorrelations of all the statements of AS-RFKA, those of AS-PAM, those of AS-SRAP, and those of AS-SNrAP were computed separately. These yielded four correlation matrices. All four matrices were factor analyzed by the principal component method. As before, dimensionality was determined by two criteria - Cattell's Scree criterion and 'eigenvalues accounting for almost the total communality'

- and the axes rotated to the varimax criterion. The factors were compared with the dimensions emerging from the Q technique. A more detailed discussion is provided below for each scale.

Since the emphasis at this stage of the study was on the rotated solution, only the varimax solutions are reported in the main body of the thesis. The intercorrelation matrices and the unrotated factor matrices are presented in Appendices D9 through D16.

Attitude Scale RFKA (AS-RFKA)

Statement Numbers 28, 57, 13, 27, 18, and 4 (statement Number 4 was added to bring the number of statements to six) designed to measure A(FRKA), 2, 12, 6, 42, 22 and 47 to measure B(RFKA), and 33, 1, 10, 60, 40, and 54 to measure C(RFKA) were incorporated randomly as AS-RFKA. The factor analysis of AS-RFKA yielded four factors, one of which, the fourth factor, seemed uninterpretable and not pertinent to the present discussion. The varimax rotated solution, shown in Table 17, brings out fairly well the three dimensions which were originally built in from the Q-technique.

All C(RFKA) statements load substantially on the first factor. This factor clearly represents what has been labelled C(RFKA). In the case of the second factor which seems to correspond to A(RFKA), two of the A(RFKA) statements, Numbers 18 and 4, load as well on other factors. Number 18, which is about the usefulness of the ideas in this type of problem outside school, load negatively on the third factor. This suggests that it measures the positive end of A(RFKA) and the negative end of B(RFKA). Number 4 seems to be measuring part A(RFKA) and part of the fourth factor (unnamed).

Table 17

Varimax Rotated Factor Loadings
of 18 Statements, AS-RFKA. N=340

Statement Number	1	2	3	4	h^2
A(RFKA)					
28	090	711	-002	026	514
57	128	735	-057	-170	589
13	119	679	-002	-222	524
27	057	587	163	258	441
18	099	413	-452	122	400
4	081	489	-055	428	431
B(RFKA)					
2	-094	-084	-146	761	616
12	-260	-046	351	616	572
6	000	-085	625	-093	406
42	175	-067	563	028	353
22	106	175	446	311	337
47	-052	189	735	052	581
C(RFKA)					
33	790	133	052	-239	701
1	726	122	034	-069	551
10	747	226	024	090	617
60	490	-008	131	-051	260
40	766	067	-097	175	632
54	798	047	-017	-179	671
Σ =	3.35	2.440	1.86	1.546	9.196

*Decimal points have been omitted. Loadings of 0.4 and higher in absolute value were significant.

Table 18

Reliability Coefficients (α) for Factor Subscales A(RFKA), B(RFKA), and C(RFKA). N=340

A(RFKA)		B(RFKA)		C(RFKA)	
Male	Female	Male	Female	Male	Female
N=162	N=186	N=162	N=186	N=162	N=186
.66	.70	.54	.48	.83	.82
		.56*	.52*		

*Reliability values after statement Number 2 has been omitted

The third factor seems to represent B(RFKA). One of the B(RFKA) statements, Number 2, did not show up as measuring that factor. It seems to be measuring the fourth factor rather than the third factor. The decision was made therefore to drop the statement and so increase the internal consistency measure of the subscale by 0.02 in the case of males and 0.04 in the case of females (see Table 18). Statement Number 12 had a factor loading of 0.35 on the third factor. This was just below the cutting point of 0.4. The statement was therefore retained.

The internal consistency of each subscale of AS-RFKA expressed in terms of Cronbach's α is given in Table 18.

Attitude Scale PAM (AS-PAM)

Statement Numbers 16, 40, 29, 10, 48, and 24 as a measure of C(PAM), 4, 39, 36, 13, 34, and 35 (statement Number 35 was added to bring the number of statements to six) as a measure of A(PAM), and 2, 11, 8, 12, 9, and 22 as a measure of B(PAM) were incorporated randomly as AS-PAM. Principal component analysis yielded three factors. The varimax rotation presented in Table 19 shows clearly that all expectations are fulfilled. This is, all C(PAM) statements load on the first factor, all A(PAM) statements load on the second factor, and all B(PAM) statements load on the third factor. This confirms fairly well that all three dimensions from the Q-technique exist for the experimental group.

The internal consistency of each subscale of AS-PAM expressed in terms of Cronbach's α is given in Table 20.

Table 19

Varimax Rotated Factor Loadings
of 18 Statements, AS-PAM. N=340

Statement Number	1	2	.3	h^2
C(PAM)				
16	817	254	010	373
40	804	205	-098	698
29	784	366	-060	751
10	806	158	016	676
48	498	322	-012	367
24	760	110	-054	593
A(PAM)				
4	224	421	391	380
39	272	643	029	488
36	068	718	-052	523
13	221	604	041	415
34	251	755	041	634
35	179	717	022	547
B(PAM)				
2	-169	-017	730	561
11	-105	-146	697	518
8	040	028	717	516
12	-260	-049	605	436
9	012	060	701	495
22	108	181	370	182
$\Sigma =$	3.844	2.950	2.717	9.511

*Decimal points have been omitted. Loadings of 0.4 and higher in absolute value were considered significant.

Table 20

Reliability Coefficients (α) for Factor Subscales C(PAM), A(PAM), and B(PAM). N=340

C(PAM)		A(PAM)		B(PAM)	
Male N=161	Female N=189	Male N=161	Female N=189	Male N=161	Female N=189
.87	.89	.72	.81	.68	.74

Attitude Scale SRAP (AS-SRAP)

Statement Number 42, 38, 18, 28, 4, and 13 as a measure of A(SRAP) and 47, 8, 6, 44, 11, and 2, (Numbers 11 and 2 were added to increase the number of statements to six) as a measure of D(SRAP) made up AS-SRAP. Two factors were extracted using principal component analysis.

A look at the varimax rotated matrix in Table 21 reveals that all but one of the A(SRAP) statements load on the first factor. The loading of statement Number 42 shows that, at least for this group, it measures neither the first nor the second factor. It was therefore dropped from the A(SRAP) subscale.

All the D(SRAP) statements except Number 8 load significantly on the second factor. Statement Number 8 seems to be measuring more of the first factor than of the second factor. However, it was retained since its loading of 0.37 was barely short of the cutting point of 0.4, and omitting the statement resulted in a reduction of the reliability of the subscale.

The internal consistency of each subscale of AS-SRAP expressed in terms of Cronbach's α is presented in Table 22.

Attitude Scale SNrAP (AS-SNrAP)

Statement Number 39, 57, 13, 59, and 34(Number 34 was added to bring the number to six) as a measure of A(SNrap), and 9, 6, 26, 2, 12, and 15 as a measure of B(SNrAP) were incorporated randomly as AS-SNrAP. The solution derived from principal component analysis yielded two factors.

The varimax solution is shown in Table 23. All the A(SNrAP) Statements load on the first factor. One of the A(SNrAP) Statements,

Table 21

Varimax Rotated Factor Loadings
of 12 Statements, AS-SRAP. N=344

Statement Number	1	2	h^2
A(SRAP)			
42	-034	-164	028
38	456	-258	274
18	618	-046	383
28	732	144	556
4	685	180	502
13	563	-165	344
D(SRAP)			
47	192	591	386
8	454	368	341
6	-081	597	363
44	050	735	543
11	-142	416	193
2	-320	452	307
Σ =	2.287	1.935	4.222

*Decimal points have been omitted. Loadings of 0.4 and higher in absolute value were considered significant.

Table 22

Reliability Coefficients (α) for Factor Subscales A(SRAP) and D(SRAP). N=344

A(SRAP)		D(SRAP)	
Male N=157	Female N=187	Male N=157	Female N=187
.49	.58	.65	.64
.53*	.70*		

*Reliability values after statement Number 42 has been omitted.

Table 23

Varimax Rotated Factor Loadings
of 12 Statements, AS-SNrAP. N=344

Statement Number	1	2	h^2
A(SNrAP)			
39	720	000	518
4	471	461	434
57	751	150	587
13	724	022	524
59	730	-128	549
34	717	025	515
B(SNrAP)			
9	166	650	450
6	-019	363	132
26	332	480	341
2	-105	778	616
12	-190	659	470
15	-381	316	245
Σ =	3.206	2.176	5.381

*Decimal points have been omitted. Loadings of 0.4 and higher in absolute value were considered significant.

Table 24

Reliability Coefficients (α) for Factor Scales A(SNrAP) and B(SNrAP). N=344

A(SNrAP)		B(SNrAP)	
Male	Female	Male	Female
N=157	N=187	N=157	N=187
.77	.81	.53	.59

Statement Number 4, seems to be measuring part of the second factor as well. This statement was retained.

Four of the six B(SNrAP) statements load on the second factor. The remaining two statements Number 6 and 15, did not load significantly on this factor. One of them, statement Number 6, has a loading of 0.36 which was just below the cutting point. The other, statement Number 15, seems to be measuring more of the negative pole of dimension A(SNrAP) than of the positive pole of B(SNrAP). Both statements, however, were retained since their loadings were just short of the 0.4 cutting point, and omitting them resulted in a reduction of the reliability of the subscale.

The internal consistency of the subscales of AS-SNrAP expressed in terms of Cronbach's α is given in Table 24.

Summary

Four attitude scales, AS-RFKA, AS-PAM, AS-SRAP, and AS-SNrAP, developed from the Q-technique employed in the preliminary study, were subjected to a validation study on the experimental group.

The view held here is that if the attitude dimensions resulting from the use of Q-technique on a sample of 24 grade 9 students can be reproduced with another sample of 344 grade 9 students and employing R-technique, this provides much stronger evidence of the existence of the dimensions than a conclusion based on either procedure. For this reason, it was decided to validate the attitude dimensions of each scale on the experimental group using an R-technique.

A value of 0.4 was chosen as the cutting point of significance for a factor loading.

A study of the Tables presented in Section 3 of this Chapter indicates that all expectations seem to be fairly well met. That is, after the orthogonal varimax rotation, the statements comprising one subscale seem to load on one factor, and those comprising another subscale within the same attitude structure seem to load on another factor. Thus the problem of factorial validities for each scale seems fairly well solved.

In a very few cases, certain statements selected from the Q-technique did not load appropriately in the R-technique. This might be due to inadequate sampling of persons and statements. Some of them loaded on a different factor from what was anticipated. These were dropped from the respective subscales. Others loaded on more than one factor, thereby suggesting that the statement belongs to more than one attitude dimension. These were retained for reasons of reliability.

The Cronbach's α estimates of reliability of the subscales on the experimental group were considered satisfactory for this exploratory study. Out of the twenty estimates (the reliabilities were reported for males and females separately), five were in the 0.5 to 0.59 range, four were in the 0.6 to 0.69 range, five were in the 0.7 to 0.79 range, and six were in the 0.8 to 0.89 range.

4. ANALYSIS AND RESULTS: MAIN STUDY

Introduction

The objectives of the main study were twofold. First, it sought to determine which dimensions of student attitudes toward each of four types of mathematics problems, RFKA, PAM, SRAP, SNrAP, related significantly to performance on the corresponding mathematics problems. Secondly, it sought to determine whether both attitudes toward, and performance on, the four types of mathematics problems reflected the differences in the cognitive complexity among the mathematics problems.

Since the results of previous studies on attitudes toward mathematics have indicated sex differences, the analysis of the main study was done for males and females separately.

Relationship Between Attitude Dimensions and Performance on Corresponding Mathematics Problems

This study was limited to four types of mathematics problems and hence four attitude structures, one for each type of mathematics problem. Question 1 (see the questions that are listed under the design of the main study in Chapter 3), therefore, consisted of four sub-questions. These are dealt with separately.

Question 1A

Which dimensions within attitude structure RFKA relate significantly to performance on RFKA?

Tables 25 and 26 respectively provide the correlations between AS-RFKA subscale scores and RFKA test scores for males and females. For the males, as Table 25 shows, A(RFKA) subscale scores correlate with RFKA test scores ($r=18$, $p<.05$), and C(RFKA) subscale scores correlate with RFKA test scores ($r=.23$, $p< .01$). These results suggest that for the males, both evasive and general liking attitudes toward recalling factual knowledge in algebra correlate at low levels with performance on this type of mathematics problem.

Table 25

Correlations Between AS-RFKA Subscale Scores
and RFKA Test Scores for Males. N=147

	A(RFKA)	B(RFKA)	C(RFKA)
RFKA	.18*	.07	.23**

*$P < .05$
**$P < .01$

Table 26

Correlations Between AS-RFKA Subscale Scores
and RFKA Test Scores for Female. N=165

	A(RFKA)	B(RFKA)	C(RFKA)
RFKA	.20*	.08	.13

*$P < .05$

In the case of the females (see Table 26), a positive relationship ($r=.20$, $p<.05$) exists only between evasive attitudes (A(RFKA)) toward, and performance on, recalling factual knowledge in algebra.

Question 1B

Which dimensions within attitude structure PAM relate significantly to performance on PAM?

Tables 27 and 28 respectively present the correlations between AS-PAM subscale scores and PAM test scores for males and females. Table 27 shows that in the case of males the correlations between C(PAM) subscale scores and PAM test scores is $r=.23$ ($p<.01$), between A(PAM) subscale scores and PAM test scores is $r=.19$ ($p<.05$), and between B(PAM) subscale scores and PAM test scores is $r=-.18$ ($p<.05$). For females, the correlation between C(PAM) subscale scores and PAM test scores is $r=.31$ ($p<.01$), and between A(PAM) subscale scores and PAM test scores is $r=.19$ ($p<.05$).

These results suggest that for both males and females general liking and avoidance attitudes toward algebraic manipulations correlate at low levels with performance on this type of mathematics problem. Attitudes toward teacher instruction and preference for group work in this type of mathematics problem seem to relate inversely to the corresponding performance for the males only.

Question 1C

Which dimensions within attitude structure SRAP relate significantly to performance on SRAP?

Tables 29 and 30 respectively contain the correlations between

Table 27

Correlations Between AS-PAM Subscale Scores and PAM Test Scores for Males. N=147

	C(PAM)	A(PAM)	B(PAM)
PAM	.23**	.19*	-.18*

*P < .05
**P < .01

Table 28

Correlations Between AS-PAM Subscale Scores and PAM Test Scores for Females. N=165

	C(PAM)	A(PAM)	B(PAM)
PAM	.31**	.19*	-.05

*P < .05
**P < .01

Table 29

Correlations Between AS-SRAP Subscale Scores and SRAP Test Scores for Males. N=147

	A(SRAP)	D(SRAP)
SRAP	.23**	-.02

**P <.01

Table 30

Correlations Between AS-SRAP Subscale Scores and SRAP Test Scores for Females. N=165

	A(SRAP)	D(SRAP)
SRAP	.23**	-.04

**P <.01

Table 31

Correlations Between AS-SNrAP Subscale Scores and SNrAP Test Scores for Males. N=147

	A(SNrAP)	B(SNrAP)
SNrAP	.00	-.08

Table 32

Correlations Between AS-SNrAP Subscale Scores and SNrAP Test Scores for Females. N=165

	A(SNrAP)	B(SNrAP)
SNrAP	.05	-.06

AS-SRAP subscale scores and SRAP test scores for males and females. The tables show that for both males and females there exist a low positive correlation ($r=.23$, $p<.01$ in both cases) between memory-cum-evasive attitudes (A(SRAP)) toward, and performance on, routine algebra problems.

Question 1D

Which dimensions within attitude structure SNrAP relate significantly to performance of SNrAP?

In this case of non-routine algebra problems (SNrAP), Tables 31 and 32 show that for both males and females none of the SNrAP attitude subscale scores correlate significantly with performance on this type of mathematics problem.

Hierarchical Nature of Attitudes

Needless to say, an investigation into the hierarchical nature of the attitudes toward the four types of mathematics problems lacks meaning if the attitudes are not of the same kind. Consequently, the analysis in this section was limited only to attitude dimensions which were invariant across the four types of mathematics problems.

It will be recalled from the results of the preliminary study, discussed in Chapter 4, that a visual similarity was established among the four attitude dimensions, A(RFKA), A(PAM), A(SRAP), and A(SNrAP). The hierarchical investigation was therefore performed only on the four attitude dimensions, A(RFKA), A(PAM), A(SRAP) and A(SNrAP). For convenience, each dimension has been termed 'evasive' attitudes.

Preliminary Consideration

Guttman (1954) has developed a notion of complexity among tests, such that a test t_2 is more complex than a test t_1 if "it requires everything that t_1 does and more." His formulation led him to define intermediacy among variables as "a statistical variable z will be said to be intermediate to x and y if the following partial correlation vanishes: $r_{xy.z}=0$" (Guttman, 1954, p. 273). From this he deduced that if we are given a set of variables t_1, t_2,.....t_n, this set will be said to form a perfect simplex in this order if $r_{ik.j}=0$ $(i<j<k)$.

The investigator has adapted the above consideration in defining "betweenness" among similar attitude dimensions.*

A dimension D_z will be said to be between dimensions D_x and D_y if

$$r_{xy.z} = 0 \text{ whenever } D_x \subset D_z \subset D_y.$$

Using the above definition, it can be seen that a set of similar attitude dimensions D_1, D_2, D_3,.....D_n, will exhibit hierarchical cumulative property in this rank order if

$$r_{ik.j} = 0 \text{ whenever } D_i \subset D_j \subset D_k.$$

The cumulative property of similar attitude dimensions can, therefore, be investigated using Guttman's simplex technique.

For a perfect simplex, according to the theory, the relationship

$$r_{ik} = r_{ij}r_{jk} \text{ where } i<j<k \text{ should hold for all i, j, and k.}$$

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*Similar attitude dimensions are to be understood as dimensions which are invariant over all four types of mathematics problem. Thus, they are of the same kind but may differ in degree.

If this relationship is satisfied, the correlation matrix should be such that its highest correlations are along the main diagonal, and decrease gradually the farther they are from the diagonal.

When the correlation matrix approaches the simplex form but does not quite meet the rigorous requirement of a perfect simplex, the matrix is classified as pseudo-simplex.

The sequence of the hierarchical investigation was as follows:

1. An investigation into the existence of a pseudo-simplex among similar attitude dimensions.
2. Dependent on the positive outcome of (1), an investigation into the measure of goodness of fit to a perfect simplex as judged by the size of the discrepancy between r_{ik} and the product $r_{ij}r_{jk}$ (Humphreys, 1960).

<u>Question 2</u>

Do the evasive attitudes toward the mathematics problems, RFKA, PAM, SRAP, and SNrAP reflect the differences in the cognitive complexity among the problems?

Table 33 presents the matrix of intercorrelations among the dimensions A(RFKA), A(PAM), A(SRAP), and A(SNrAP) for the males. The pattern of the correlations indicates that the matrix satisfies the pseudo-simplex requirement; the highest correlations appear along the diagonal and decrease gradually away from the diagonal.

The absolute mean discrepancy was found to be 0.075. This value is slightly higher than the traditionally accepted value of 0.05. However, this seems to be a fairly good fit for the data.

In the case of females (see Table 34), the matrix does not

Table 33

Intercorrelations of Scores on A(RFKA), A(PAM), A(SRAP), and A(SNrAP) for Males.* N=147

	A(RFKA)	A(PAM)	A(SRAP)	A(SNrAP)
A(RFKA)				
A(PAM)	37			
A(SRAP)	22	23		
A(SNrAP)	07	16	18	

*Decimal points have been omitted

Table 34

Intercorrelations of Scores on A(RFKA), A(PAM), A(SRAP), and A(SNrAP) for Females.* N=165

	A(RFKA)	A(PAM)	A(SRAP)	A(SNrAP)
A(RFKA)				
A(PAM)	25			
A(SRAP)	49	28		
A(SNrAP)	22	12	36	

*Decimal points have been omitted

satisfy the pseudo-simplex criterion. With the exception of the first variable, the pseudo-simplex form appears. The result suggests that, for the females, their evasive attitudes toward the mathematics problems do not reflect the differences in the complexity among the problems. It therefore seems to indicate a sex difference in the structuring of student evasive attitudes toward mathematics problems of varying cognitive complexity.

Question 3. Hierarchical Nature of Performance on RFKA, PAM, SRAP, and SNrAP

Do students' performance on the mathematics problems, RFKA, PAM, SRAP, and SNrAP reflect the differences in the cognitive complexity among the problems?

Tables 35 and 36 respectively present the intercorrelations among the test scores on the four types of mathematics problems for males and females. Both matrices satisfy the pseudo-simplex form.

The goodness of fit was found to be 0.17 for males and 0.18 for females. These values are higher than the traditionally accepted cutting point of 0.05. Few writers have commented on the possibility of discovering real data which will satisfy the rigorous definition of a perfect simplex. Dubois (1960) observed that it is "unlikely that a rigorously perfect simplex will ever be discovered with real data" (p. 179). Kropp, Stoker, and Bashaw (1966), after an extensive validation study on Bloom's taxonomy concluded that "if data are gathered to measure the six cognitive levels and if these data do yield a quasi-simplical structure, then there is evidence of the hypothesized hierarchy of complexity" (p. 74). Extrapolating from the above

Table 35

Intercorrelations of Test Scores on RFKA, PAM, SRAP, and SNrAP for Males*. N=147

	RFKA	PAM	SRAP	SNrAP
RFKA				
PAM	48			
SRAP	42	65		
SNrAP	39	25	32	

*Decimal points have been omitted

Table 36

Intercorrelations of Test Scores on RFKA, PAM, SRAP, and SNrAP for Females*. N=165

	RFKA	PAM	SRAP	SNrAP
RFKA				
PAM	53			
SRAP	51	57		
SNrAP	39	27	36	

*Decimal points have been omitted

comments, the results of this study suggest, though not strongly, that the empirical data largely support the hierarchical nature of the mathematics problems.

Summary

This section summarizes the results of the main study. The objective was two fold. First, it sought to determine which dimensions of student attitude toward each of four types of mathematics problems, Recalling Factual Knowledge in Algebra(RFKA), Performing Algebraic Manipulations(PAM), Solving Routine Algebra Problems(SRAP), and Solving Non-routine Algebra Problems(SNrAP), related significantly to performance on the corresponding type of mathematics problem. Secondly, it sought to determine whether student evasive attitudes toward, and their performances on, the four types of mathematics problems reflected the hierarchy of cognitive complexity associated with the problems.

The relationships between the attitude dimensions and performance on the corresponding mathematics problem have been very revealing. The trend of the correlation coefficients suggests that evasive attitudes toward algebra problems that call for lower level cognitive behaviors correlate to a lesser extent with performance on those types of problem. Attitudes toward teacher instruction and preference for group work in any of the mathematics problems seem unrelated to the corresponding performance. One exception seems to exist. In the case of males, such attitudes toward algebra problems that require basic computations correlate inversely with performance on this type of problem.

In the case of 'general liking' attitudes toward mathematics problems, the trend of the correlation coefficients is not readily interpretable. For both males and females, general liking attitudes toward, and performance on, algebra problems that require basic computation seem to be related. However, such attitudes toward recalling factual knowledge in algebra seem to be related in the case of males, but not in the case of females.

The results of the investigation into the hierarchical nature of the evasive attitudes toward the four types of mathematics problems suggest that, for males only, evasive attitudes seem to reflect the differences in the complexity among the mathematics problems. This result is interpreted as reflecting sex difference in the structuring of student evasive attitudes toward the problems.

The empirical data for both males and females seem to support, though not strongly, the hierarchical nature of the four types of mathematics problems.

CHAPTER 5

SUMMARY, CONCLUSIONS, IMPLICATIONS AND SUGGESTIONS FOR FUTURE RESEARCH

1. SUMMARY

This exploratory study had two objectives. First, it sought to determine which dimensions of student attitudes toward each of four types of mathematics problems related significantly to performance on the corresponding mathematics problem. Secondly, it investigated whether both attitudes toward, and performance on, the four types of mathematics problems reflected the hierarchy of cognitive complexity associated with the mathematics problems.

An operational definition of 'a type of mathematics problem' was first adopted for the study, namely, a cell in a two-way classification: 'cognitive objectives in mathematics' by 'content areas in the mathematics program.' The taxonomy of Educational Testing Service (ETS) furnished the cognitive objectives, and the subject-matter of 'algebra' was selected as the content area.

A pilot study revealed that two of the six levels of the ETS taxonomy could not be applied to junior high school students. Consequently, only four levels, levels 1, 2, 3, and 5, were employed. The investigation was limited to four types of mathematics problems. These were those that belong to one content area, algebra, and which were hierarchically structured with respect to the levels of thinking inherent in the cognitive objectives that are associated with the mathematics problems. Specificially, the mathematics problems were (a) Recalling Factual Knowledge in Algebra (RFKA), (b) Performing Algebraic Manipulations (PAM), (c) Solving Routine Algebra Pooblem (SRAP), and (d) Solving Non-routine Algebra Problem (SNrAP). Before the study could be carried out, it was first necessary to determine the attitude structure toward each of the four types of mathematics problems. Q-technique was the factor analytic technique that was chosen to identify the attitude dimensions. Recognizing that the generalizability of the results from Q-technique depended on the selection of the sample as well as on the coverage of the attitude area, two additional pilot studies were undertaken.

One of these pilot studies was for selecting 60 statements representative of junior high school student attitudes toward mathematics problems. The other was for selecting a sample of 24 students for the Q-sort. In the latter pilot study, three variables, sex, quantitative aptitude, and motivation toward school, were taken into account. This required the use of a 2x2x2 factorial layout with 3 students per cell.

For the identification of the attitude dimensions, a stimulus-response (S-R) format was employed in collecting the data. A stimulus consisted of the description of a particular type of mathematics problem followed by examples of that type. The responses of the students were to Q-sort the list of 60 attitude statements.

The stimuli were presented to the 24 students, one at a time, and on each occasion the students were to Q-sort the same list of 60 statements according to a specified (approximately normal) distribution.

Each of the 24 sorts resulting from a stimulus presentation was correlated (product moment correlation) with each of the other sorts and the (person) correlation matrix was factored by principal component analysis. Dimensionality was determined by two criteria-Cattell's Scree criterion and 'eigen-values accounting for almost total communality'. The axes were then rotated to the varimax criterion.

A person's loading of 0.4 and above on only one factor was used as the criterion for judging persons that belong to a 'pure person factor' type. Following Stephenson's (1953) suggestion and the adaptations by Kerlinger and Kaya (1959), Kerlinger (1966, 1967), and by Howell (1968), the arrangement of the 60 statements which best typified a pure person factor type was found by means of Spearman's differential weighting procedure. Thus, there were as many arrangements of the 60 statements as there were pure person factor types, each arrangement uncorrelated with any other.

About 6 statements most highly saturated with a factor were selected arbitrarily to provide an operational definition of an attitude dimension. This also provided the basis for statement selection in developing a short subscale to measure the dimension. The entire procedure was done separately for each set of 24 sorts resulting from a stimulus presentation. Through visual inspection, a rough similarity was found among dimensions resulting from different stimulus presentations.

Four Likert-type attitude scales were developed using the above procedure. Each scale consisted of subscales designed to measure the attitude dimensions associated with one type of mathematics problem.

Because of the unavailability of criteria against which to validate the scales, the validities of the scales were judged on the basis of content and factorial purity. All the four attitude scales were factorially validated on the experimental group using an R-technique.

The attitude scales were administered to an experimental group consisting of about 350 grade 9 students from four Separate Junior High Schools in Edmonton. Concurrent measures of the students' performances on the four types of mathematics problems were also required to meet part of the second objective. Thus, an algebra test comprising of items which were examples of the mathematics problems were also administered about the same time to the experimental group.

Product moment intercorrelations of all the statements of each attitude scale were computed separately. This yielded four matrices. As in the case of Q-technique, each matrix was factored by principal component analysis, dimensionality was determined by Cattell's Scree criterion and the eigenvalue requirements, and the axes were rotated to varimax criterion.

The factors which emerged were then compared with the dimensions which emerged from the Q-study. In a very few cases, statements were found to load inappropriately, possibly because of inadequate sampling of persons and statements. Some of the statements loaded on factors different from what was anticipated. These were dropped. Others loaded on more than one factor, indicating that the statement belonged to more than one factor. These were retained. The reliability coefficients of the subscales were found satisfactory for the study.

Attitude scores required for the main study were based on the attitude scales constructed through Q-technique and revised as a result of the R-technique. Performance scores came from the algebra test devised specially for this study.

For the main study, the relationship between the dimensions within each attitude structure and performance on the corresponding mathematics problem was investigated, using Pearson product moment correlational analysis; the hierarchical nature of the attitudes toward, and performance on, the mathematics problems were investigated using Pearson product moment correlational analysis and Guttman's simplex technique.

2. CONCLUSIONS AND IMPLICATIONS

The relationships between the dimensions within an attitude structure and performance on the corresponding mathematics problem have been very revealing. The results of this study seem to indicate that the relationship between attitudes toward mathematics and perfomance in mathematics is of a complex nature. It varies depending on the attitude dimension as well as on the type of mathematics problem. Thus the result is suggestive that the variation in the size of the correlation coefficients obtained in previous studies could, among other things, be due to differences in attitude dimensions which the different scales measured as well as the types of mathematics problem which made up the mathematics test in these investigations.

The trend of the correlation coefficients between attitudes toward mathematics problems and performance on the mathematics problems is worth noticing.

For both males and females, the correlation coefficient between evasive attitudes toward algebra problems that require lower level cognitive behaviors and performance on these types of problems is positive, although not high. It is speculated that this result suggests that high-ability students prefer solving non-routine algebra problems which do not provide a challenge for them. Evasive attitudes toward algebra problems that require higher level cognitive behaviors seem to be unrelated to performance on these types of problems.

Attitudes toward 'teacher instruction' and 'preference for group work' in algebra problems seem not to be directly related to performances on these problems. In the case of males, however, such attitudes toward algebraic computation seem to be inversely related to the corresponding performance. It can therefore be speculated that males tend not to welcome group work on mathematics problems that require basic computation and which presumably they can solve.

In the case of 'general liking' attitude toward mathematics problems, the trend of the correlation coefficients does not lend itself to an easy interpretation. For both sexes, general liking attitudes toward, and performance on, algebra problems that require basic computations seem to be related. However, such attitudes toward algebra problems that require the recall of factual knowledge seem to be related to performance on these problems for males only.

Particularly noteworthy is the fact that none of the attitude dimensions associated with non-routine algebra problems seems to be related to performance in this type of problem. Carey's (1958) study, which represents one of the few studies relating attitudes toward problem-solving to performance in problem-solving, reported that "attitude scores were found to have some positive relationship to performance scores" (p. 260). Accepting Carey's findings, the result of this investigation seems to indicate that while overall attitudes toward problem solving or non-routine problem may have "some" relationship to performance on this type of problem, the two dimensions within the attitude structure that were extracted in this study seem to be unrelated to performance.

It is noted by this author that the over-all results of the correlational analyses between attitudes toward mathematics problems and performance on the problems suggest that performance on a particular type of mathematics problem can be better predicted by differential weighting of scores on the various dimensions within the corresponding attitude structure than a global measure of attitudes toward the mathematics problem.

It is interesting also to note the result of the investigation into the hierarchical nature of both the attitudes toward, and performance on, the mathematics problems. The findings seem to indicate that, for males, both the evasive attitudes toward the mathematics problems as well as their performance on the problems reflect the differences in the complexity among the problems. That is, the more complex the mathematics problem becomes, the more evasive the attitudes toward the problems become. No such trend was found in the case of females. Their performance on the mathematics problems reflected the differences in the complexity among the problems, but not their attitudes. The result therefore seems to indicate a sex difference in the manner in which evasive attitudes of students toward different types of algebra problems are structured.

Remarkable as this result may seem, it empirically supports a claim by Johnson (1957) that, while teaching the rudiments of mathematics, teachers unconsciously teach attitudes. As he put it, "while teaching how to solve equation . . . our students may be learning . . . to dislike mathematics . . . . Attitudes will inevitably be learned by the poor achievers as well as the superior students" (p. 113).

For the males, the result is suggestive that evasive attitudes toward the four types of mathematics problem may be a reflection of the differences in the complexity among the problems. Further work, it is hoped, will be undertaken to determine whether or not an isomorphism (one-to-one correspondence) exists between the evasive attitudes of males toward, and their performance on, the mathematics problems.

A secondary objective of this investigation was to explore the possibility of employing Q-technique (inverse factor analytic technique) as a factor analytic tool for identifying the dimensions of student attitudes toward mathematics problems. The concern in this regard was to find out whether or not Q-technique would yield interpretable dimensions of junior high school student attitudes toward <u>four</u> types of mathematics problems, and whether these dimensions would generalize to a much larger sample of students.

From the Q-analysis, three attitude dimensions were extracted from RFKA and PAM presentations, and two dimensions from each of the

SRAP and SNrAP presentations.

Generally speaking, the results suggest that evasive attitudes, attitudes toward teacher instruction, and preference for group (including the mathematics teacher) work were associated with all four types of mathematics problem, whereas general liking attitudes were associated only with mathematics problems that requires basic recall of factual knowledge in algebra and that which requires algebraic computations.

It is realized that these findings, based on the analysis of the results of Q sort by 24 students, representing both males and females of low and high quantitative aptitude, as well as of low and high motivation, cannot be regarded as conclusive. It may be that these and other attitude dimensions not extracted in this study will be replicated in other investigation. Also, whether or not these attitude dimensions will remain invariant over a different content area in mathematics is unknown. The study, however, claims to provide a foundation, an indication of the varying attitude dimensions associated with different types of mathematics problems, for a more thorough investigation from which it is hoped to establish conclusive results. It provides empirical evidence of a suggestion that "there is probably a set of dispositions or feelings that vary from computation to problem-solving" (Romberg, 1969, pp. 481-482.).

Aiken (1969, 1970a, 1970b) advocated a novel and "diagnostic" approach to measuring and evaluating students' attitudes toward mathematics. He suggested that the notion of attitudes toward mathematics be replaced by attitudes toward different types of mathematics problems.

According to Brock (1965), the concern of mathematics programs today is understanding the process rather than mere computation. The ability to comprehend, apply, and analyze, he emphasized, has become the major concern in school mathematics programs. It seems necessary, therefore, that in order to guide the student and evaluate the program thoroughly, we need to pay more attention to the students' attitudes toward these more abstract objectives of mathematics learning.

It will be recalled that the attitude scales developed from Q methodology using a small sample size held up fairly well with a much larger experimental group. This finding was not at all surprising. For, in choosing the sample for the Q sort, special consideration was given to selecting a sample such that three variables, sex, quantitative aptitude, and motivation toward school, were accounted for. It is speculated that if students of only one particular profile, say, high quantitative aptitude, had been selected for the Q-sort, the results from the Q-analysis would have failed to hold up as it did with the larger experimental group. It is also felt that if two more variables, past performance and attitude toward the mathematics teacher, were taken into account in selecting the students for the Q-sort, the "confirmatory" results would have held up even better than they did in the study.

From the measurement point of view, the implication of this finding seems to be that mathematics attitude scale construction may be facilitated by the use of a small sample of carefully selected students via Q-technique than the intercorrelation of a large number of statements using a large number of subjects. In other situations, perhaps similar to that of this study, a combination of Q- and R-techniques may be found to be an effective item selection procedure.

3. SUGGESTIONS FOR FUTURE RESEARCH

During the course of this investigation, certain ideas have come to mind. These should provide useful avenues for further research.

(1) An investigation to determine whether the attitude dimensions extracted in this study are invariant over a different content area in mathematics.

(2) A Q study of students' attitudes toward different types of mathematics problems, and a subsequent "confirmation" of the existence of the attitude dimensions on a larger group using an R-technique. This time, two more variables, past performance in mathematics and attitudes toward the mathematics teacher, could be accounted for in selecting the students for the Q-sort.

(3) A more detailed study of the relationship between students' attitudes toward one particular type of mathematics problem and performance in that type.

(4) An investigation into whether or not there exists an isomorphism (one-to-one correspondence) between the evasive attitudes of males toward, and performance on, different types of mathematics problem. These types being those that belong to one content area, and which are hierarchically structured with respect to the levels of thinking associated with the mathematics problems.

(5) The usual practice of computing a factor-array in a Q-study is based on the adaptation by Kerlinger (1966, 1967) of Spearman's differential weighting procedure. Research is needed to ascertain whether or not there is loss of information in not using the original procedure.

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APPENDIX A

Q-SORT EXERCISE AND INSTRUCTIONS FOR ADMINISTRATION

Q-SORT EXERCISE

The purpose of this exercise is to find out the extent to which you agree or disagree with 60 statements which junior high school students like yourselves have made concerning ALGEBRA PROBLEMS or QUESTIONS. For the purpose of this exercise, ALGEBRA PROBLEMS are classified into FOUR types, namely:-

TYPE 1. Those that require the student just to recall some fact in algebra.

TYPE 2. Those that require algebraic manipulations or calculations.

TYPE 3. Those that are routine algebra problems.

TYPE 4. Those that are non-routine algebra problems and which require the student to develop his own technique to work them.

On the following pages you will find a detailed description of each type of algebra problem together with examples so as to help you understand the description.

You will be required to read the description of each type of algebra problem very carefully using the examples that follow each description to aid your understanding. After making sure that you understand the description, you are required to sort a deck of 60 cards into seven piles such that the number in each pile is as given below:

Agree	Pile 1	Pile 2	Pile 3	Pile 4	Pile 5	Pile 6	Pile 7	Disagree
(end)	2	7	12	18	12	7	2	(end)

On each card in the deck is written a statement that has to do with that particular type of algebra problem. The entire sorting procedure will be repeated each time for the four types of algebra problem.

To begin the sorting, select from the 60 statements 2 statements which you agree with the most and put them in pile 1; next, select from the remaining 58 statements 2 statements which you disagree with the most and put them in pile 7; next, select from the remaining 56 statements 7 statements which you agree with the most and put them in pile 2; next, select from the remaining 49 statements 7 statements which you disagree with the most and put them in pile 6, and so on, approaching the centre of the distribution always from the two extremes. You will eventually be left with 18 statements which will fall in pile 4.

TYPE 1 ALGEBRA PROBLEM

RECALLING FACTS IN ALGEBRA

DESCRIPTION: In problem of this type, the student is only required to recall (or remember) definitions, facts, or theorems in algebra. This type of problem is useful to find out whether a student knows basic facts in algebra.

Examples are

1. What is the reciprocal of -a?

   (a) a (c) $\frac{1}{a}$

   (b) $\frac{-1}{a}$ (d) -1

2. Which of these is a monomial?

   (a) y^2-5 (c) y^2+3x-5

   (b) 3x (d) 3x-5

3. Any monomial is a

(a) binomial | (c) trinomial

(b) polynomial | (d) none of these

4. Which of these polynomials is a binomial?

(a) $x^3 + 3x^2 + 2x + 1$ | (c) $5x$

(b) $x + 2$ | (d) $2x^2 + 5x + 2$

5. The additive inverse of $5x^2 - 4x - 3$ is

(a) $5x^2 - 4x - 3$ | (c) $-5x^2 - 4x + 3$

(b) $-5x^2 + 4x + 3$ | (d) $-5x^2 + 4x - 3$

6. $(-6)^\circ =$

(a) -6 | (c) -1

(b) 0 | (d) 1

7. Which of these polynomials is a trinomial?

(a) $3x^2 + x - 4$ | (c) $3x^3 + 3x^2 + 2x + 7$

(b) $x + 4$ | (d) $3x^2$

8. The product of $-.2$ and 0 is

(a) $^{-}.2$ | (c) $^{-}.20$

(b) $^{+}.2$ | (d) 0

9. The statement below that is <u>TRUE</u> is

(a) $x^3 + x^3 = x^6$ | (c) $x^3 + x^3 = 2x^3$

(b) $x^3 + x^3 = x^9$ | (d) $x^3 + x^3 = (x^3)^2$

10. An example of a number raised to a negative integral power is

(a) $(-6)^4$ | (c) $3^{-1/2}$

(b) $\frac{1}{3^{-2}}$ | (d) 6^{-4}

11. The additive inverse of 25 is

(a) 25

(b) -25

(c) $\frac{1}{25}$

(d) $-\frac{1}{25}$

12. What is the constant term in the polynomial $5x^2 - 2x + 3$?

(a) 5

(b) 2

(c) 3

(d) -2

13. The statement below that is FALSE is

(a) $x^n \cdot x^m = x^{n+m}$

(b) $(x^n) = x^{nm}$

(c) $x^n + x^m = x^{n/m}$

(d) $(xy)^m = x^m y^m$

14. What is the coefficient of y in the expansion $2y^5+6y^4-4y^2-5y+1$?

(a) -5

(b) 5

(c) -1

(d) +1

15. What is the reciprocal of $-\frac{b}{4}$?

(a) $\frac{b}{4}$

(b) $\frac{-4}{b}$

(c) $\frac{4}{b}$

(d) 4

TYPE 2 ALGEBRA PROBLEM

PERFORMING CALCULATIONS IN ALGEBRA

DESCRIPTION: In problem of this type, the student is required to perform particular operations in algebra such as addition, subtraction, multiplication, etc. etc. These questions will vary in level of difficulty or complexity, but the problem tells the student what to do to solve the problem.

Examples are

1. $(4x+3y) - (x+y) - (3x-4y) =$

   (a) $x-y$ (c) $8x$

   (b) $6y$ (d) $8x-8y$

2. $\frac{x^7}{x^3} =$

   (a) $x^{3.5}$ (c) x^{10}

   (b) x^4 (d) x^{21}

3. $\frac{m^3n^5}{m^5n^3} =$

   (a) 1 (c) $(m^2)(n^{-2})$

   (b) $\frac{m^2}{n^2}$ (d) $(m^{-2})(n^2)$

4. $(3a^3)(1/3a^3b)(3b^4) =$

   (a) $3a^6b^5$ (b) $3a^9b^5$

   (b) $3a^9b^4$ (d) $6\ 1/3a^6b^5$

5. $(3x^2 + 2y^2) + (2xy-x^2) =$

   (a) $2x^2 + 4xy^3$ (c) $4x^2 + 4xy^3$

   (b) $2x^2 + 2xy^+ 2y^2$ (d) $4x^2 + 2xy + 2y^3$

6. $(-5x-3) - (5x+3) =$

   (a) 0 (c) $-10x$

   (b) $-10x - 6$ (d) $10x + 6$

7. $(x+y)\ (x+2y) =$

(a) $x^2 + 2y^2$

(c) $x^2 + 2xy + 4y^2$

(b) $2x + 3y$

(d) $x^2 + 3xy + 2y^2$

8. $(3x^2-5x+6) + (10x-4x^3+x^2) =$

(a) $4x^3 + 4x^2 - 5x + 6$

(c) $-4x^3 + 2x^2 + 5x + 6$

(b) $-4x^3 + 4x^2 + 5x + 6$

(d) $4x^3 + 2x^2 + 5x + 6$

9. $(y^3 + 2y^2 + y) + (y^2 + 2y + 2) =$

(a) $y^5 + 4_y{}^3 + 2y$

(c) $y^3 + 3y^2 + 3y + 2$

(b) $2y^3 + 4y^2 + 2y$

(d) $2y^2 + 2y + 2$

10. $(x + 2)\ (4x - 7) =$

(a) $4x^2 - 14$

(c) $4x^2 - 15x - 14$

9b) $4x^2 + 15x - 14$

(d) $4x^2 + x - 14$

11. $(5abc)\ (2a^2b^4) =$

(a) $7a^2b^4c$

(c) $10a^2b^4c$

(b) $10a^3b^5c$

(d) $10a^{10}b^{10}c^{10}$

12. The simplest form of $3x-6-2x-7-3x+4$ is

(a) $-2x+17$

(c) $2x+3$

(b) $-2x-9$

(d) $8x+9$

13. $(3y)^3 =$

(a) $3y^3$

(c) $9y^3$

(b) $27y^3$

(d) $9y$

14. $(2a^2)(1/4b^2)(6a^3b) =$

(a) $6a^5b^2$ (c) $3a^6b^2$

(d) $4a^5b^3$ (d) $3a^5b^3$

TYPE 3 ALGEBRA PROBLEM

SOLVING ROUTINE ALGEBRA PROBLEM

DESCRIPTION: This type of problem includes problem in which the method to be used to find the solution is not directly given. Problem in this category requires the student to apply a definition, fact, or theorem in algebra in order to arrive at the solution.

Also, the student has to decide which operations in algebra to use in order to work the problem, but usually there is one particular method which has already been taught in class.

Examples are

1. An automobile is moving at r miles per hour, and an airplane is moving three times as fast. How many hours will the plane require for a 500-mile flight?

(a) $\frac{500}{3+r}$ (c) $\frac{500}{r^3}$

(b) $\frac{500}{3r}$ (d) $\frac{3(500)}{r}$

2. What is the square root of $16b^8$?

(a) $2b^2$ (c) $4b^4$

(b) $4b^2$ (d) $8b^4$

3. Factor $3x^2-4x-4$

(a) (3x-2) (x+2)   (c) (3x+1) (x-4)

(b) (3x+2) (x-2)   (d) (3x-4) (x+1)

4. If $\frac{N}{34} = 22$, then $\frac{N}{17} = ?$

(a) 11   (c) 22

(b) 44   (d) 748

5. If n is an even number, what is the next larger even number

(a) n-2   (c) n+2

(b) n+1   (d) 2n

6. b^2-81 can be factored into

(a) b(b-9)   (c) $(b+9)^2$

(b) $(b-9)^2$ (d) (b+9) (b-9)

7. If 3x-1 = 8, then 2x-5

(a) -7   (c) 2 1/2

(b) 1   (d) 3

8. The factors of $a^2 + 6ab + 5b^2$ are

(a) (a+5b) (a+b)   (c) (a-5b) (a-b)

(b) (a-2b) (a-3b)   (d) (a+5b) (a-b)

9. Which of the following equations expresses the condition that "the product of two numbers R and S is one less than twice their sum?"

(a) 2(RxS)-1 = R+S   (c) RxS = 2(R+S)+1

(b) RxS = 2 (RxS)   (d) RxS-1 = 2(RxS)

10. The middle term of the trinomial that is the product of $(3a-4)^2$ is

(a) 12a   (c) -12a

(b) 24a   (d) -24a

11. The statement "A certain number f increased by twice another number n is equal to 30" can be written as

(a) $f + 2n = 30$
(c) $2f + n = 30$
(b) $2fn = 30$
(d) $f + 2f = 30$

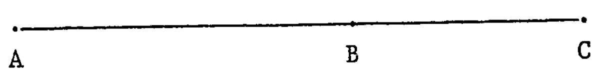
12. A rectangle has length 2x+1 inches and width x-7 inches, what is its area?

(a) $(2x+1)^2$
(c) $(2x+1) + (x-7)$
(b) $1/2(2x+1)\ (x-7)$
(d) $2x^2-13x-7$

13. The equation $x^2+1 = 10$ may be interpreted as

(a) a number when doubled and increased by 1 is 10
(b) a number whose square increased by 1 is 10
(c) a number increased by 1 and squared is 10
(d) a number squared is one more than 10

14.



The distance AB is given by 3x+1 inches and the distance AC is 5x+2 inches. How far is the distance BC?

(a) $8x+3$
(c) $\frac{5x+2}{2}$
(b) $2x+1$
(d) $-2x-1$

TYPE 4 ALGEBRA PROBLEM

SOLVING NON-ROUTINE ALGEBRA PROBLEM

DESCRIPTION: This type of algebra problem requires the student to use insight to work them. The student has probably not met this type of problem in the mathematics textbook. He will have to develop his own method for working the problem. Sometimes a series of logical steps will be necessary to arrive at the correct solution, and it will be up to the student to figure these steps. Some will be simple and others will be hard, but in order to arrive at the answer the student will have to demonstrate a degree of mathematical capability.

Examples are

1. There are 33 students in a class, and the ratio of boys to girls is 7:4. One day a boy was absent. What was the ratio of boys to girls on that day?

   (a) 3:2 (c) 7:8

   (b) 5:3 (d) 21:12

2. If the number b is between a and c, then

   (a) b + 2 is between c + 2 and a + 2
   (b) c + 2 is between a + 2 and b + 2
   (c) a + 2 is between b + 2 and c + 2
   (d) all of the above is true

3. If a and b are different numbers, which of the following is always between a and b?

   (a) a+1 (c) $\frac{a+b}{2}$

   (b) $\frac{b-a}{2}$ (d) $a + \frac{b}{2}$

4. Two of a student's test marks are 64 and 84. A third mark is at least 40. What is the lowest possible average for the three tests?

   (a) 40 (c) 64

   (b) 58 (d) 76

5. The mailman delivers 9 letters to 4 houses. If each house gets at least one letter, which of the following is necessarily true?

   (a) each gets more than 1
   (b) none gets more than 3
   (c) none got more than 6
   (d) each got at least 3

6. Which value of x satisfies every condition of the form $ax^2 + bx = 0$, where a and b are any real numbers?

   (a) 1 (c) 2

   (b) 0 (d) 10

7. If x is greater than 3, which of the following is the smallest?

(a) $\frac{3}{x}$

(b) $\frac{3}{x+1}$

(c) $\frac{3}{x-1}$

(d) $\frac{x}{3}$

8. How many cents are there in x dollars and y quarters?

(a) x + 4y

(b) 4x + y

(c) 100x + 4y

(d) 100x + 25y

9. If a + b = 0, then

(a) a must be negative
(b) b must be negative
(c) Both a and b can be negative
(d) either a or b can be negative

APPENDIX B

PILOT STUDIES

Introduction

Part of the main objective of the study was to explore the relationship between attitudes toward, and performance on, certain types of mathematics problems.* The types of mathematics problems to be considered were those that belong to the content area of algebra** and which were hierarchically structured with respect to the levels of thinking inherent in the cognitive objectives associated with the mathematics problems.

The classification system developed by the Educational Testing Services (ETS) (Epstein, 1968) was to serve as the taxonomy of cognitive objectives. However, it was felt that grade 9 students might not be able to conceptualize the difference between, say, a mathematics problem which "demonstrates comprehension of ideas and concepts" (level 4 of the ETS taxonomy) and one which is nonroutine and "requires insight and ingenuity" (level 5 of the ETS taxonomy).

For this reason, a pilot study, referred to in this dissertation as Pilot Study B1, was undertaken to choose certain types of mathematics problems such that the distinctions among them could be clearly understood by grade 9 students. The second objective of

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*A type of mathematics problem is to be considered as a cell in a two-way classification, cognitive objectives of mathematics by content in the school mathematics program. A cognitive objective in mathematics refers to the level of thinking required of the student, and a content area in mathematics refers to a subdivision of the subject matter.

**Algebra is to be understood as a content area within the Junior high school mathematics program of the Separate School Board in Edmonton.

Pilot Study B1 was to select examples of each type of mathematics problem which would serve as part of a stimulus in a stimulus-response (S-R) format to be used in collecting the data as well as test items.

Q-technique (inverse factor analytic technique) was chosen as the research design for identifying the dimensions of student attitudes toward the mathematics problems. Recognizing that the generalizability of the inverse factor analytic solution depends largely on the selection of the subjects as well as on the coverage of the attitude area, three additional pilot studies were undertaken.

The first of these, Pilot Study B2, was aimed at estimating the possibility of the Junior Index of Motivation (JIM) for use in Pilot Study B3 as an index of a student motivation toward school. The second, Pilot Study B3, was designed to select the 24 students for the Q-sort. The third, Pilot Study B4, was to select 60 Q-sort statements representative of grade 9 student attitudes toward mathematics problems. This section reports the results of these four pilot studies.

APPENDIX B1

THE SELECTION OF TYPES, AND EXAMPLES OF EACH TYPE, OF MATHEMATICS PROBLEMS

Introduction

Prior to meeting the aims of this pilot study, an operational definition for 'a type of mathematics problem' was first adopted, namely, that 'a type of mathematics problem' would refer to one cell in a two-way classification of cognitive objectives of mathematics by content areas in the mathematics program.

This cognitive-by-content matrix scheme was patterned after that used by the School Mathematics Study Group (SMSG) Research and Test Development staff in preparing test batteries for use in the National Longitudinal Study of Mathematical Abilities (NLSMA). Such a system of classification, as Wilson, Cahen, and Begle (1969) pointed out, increases the content validity of mathematics tests, identifies specific components in mathematics achievement, and provides a comprehensive picture of the domain of behaviors associated with mathematics.

The objectives listed under the Educational Testing Service (ETS) taxonomy (see Epstein, 1968) were selected as the cognitive objectives in mathematics. The content area of ALGEBRA as outlined in the Edmonton Separate School Board syllabus for ninth graders was selected as the content objective for this investigation. Thus a structure was established whereby subject-matter knowledge was cross-coded with the behavior required to successfully solve the mathematics problem.

Table 37 presents the cognitive objectives of the ETS taxonomy, and Table 38 presents the first-order subcategories of the content area of algebra.

Table 37

Cognitive Objectives of the ETS Taxonomy*

1. Recall of factual knowledge

This category is restricted to questions which requires only the recall of a definition, fact or theorem without doing anything to it. Questions in this category are useful in the quick quiz to check on whether a group has acquired important basic information.

2. Performing mathematical manipulations

This category is used for questions, regardless of complexity or level of mathematics, that call for the application of a technique that has been learned - where no decision is required on how to approach the solution.

3. Solving routine problems

This category includes questions in which a choice of the technique to be used is necessary or a definition or theorem recalled and applied, but where there is a straight forward technique available which is commonly taught.

4. Demonstrating comprehension of mathematical ideas and concepts

This category is used for questions which require some understanding of the underlying concepts. They may still be familiar or textbook-like, but in them the student must decide not only what to do but, how to do it. Also included in this category would be questions on theory and those calling for translation, comparison, approximation, extrapolation, or the use of new symbolism.

5. Solving non-routine problems requiring insight or ingenuity

Questions are classified in this category if they require a student to develop his own technique for solving a problem he probably has not met in a textbook. The solution may be straightforward or simple, but some insight should be needed to find it. This category is also used for questions in which the student is required to marshall a variety of techniques or carry out a self-determined sequence of logical steps.

Table 37 (continued)

6. Apply "higher" mental processes to mathematics

In this category are classified questions testing generalization, evaluation, the nature of proof, induction, logical inference and decisions about sufficiency of data.

*Reproduced from Epstein (1968)

Table 38

First-Order Subcategories of the Content Area of Algebra*

1. Introduction to Polynomial
2. Addition, Subtraction and Multiplication of Polynomial
3. Factor of squares.
    (a) Common factor
    (b) Difference of squares
    (c) Trinomials that are perfect squares
    (d) Trinomials that are the products of binomials
4. Division of Polynomials, Extension of Rational Expressions
5. Addition, Subtraction, Multiplication and Division of Relation Expressions
6. Solution of problems involving linear conditions

*Obtained from the Mathematics Educator, Edmonton Separate School Board.

Aim of Pilot Study B1

(1) To select types of mathematics problems so that the distinctions among them could be realized by ninth graders.

(2) To choose examples of each type which would constitute part of a stimulus in a stimulus-response (S-R) format to be used in collecting the data and also serve as test items for the main investigation.

Method

Under the model developed above, an algebra test item appropriate for ninth graders within the Edmonton Separate School System could, theoretically at least, be categorized into one or the other of six types of mathematics problems. But extensive discussions with five grade 9 mathematics teachers* conversant with the ETS taxonomy, and try-out sessions with ninth graders using a language compatible with their understanding, revealed that such a system of categorization was unworkable. A decision, based on the recommendations of the five mathematics teachers, was taken to omit levels 4 and 6 of the ETS taxonomy.

This revision resulted in fewer cognitive objectives and hence a smaller set of types of mathematics problems than was originally anticipated. However, there was unanimous agreement among the mathematics teachers that the revised system of categorizing algebra test items could be realized by ninth graders. Consequently, FOUR types of mathematics problems were selected for the main investigation. These were (a) Recalling Factual Knowledge in Algebra, (b) Performing

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*The author expresses gratitude to Messers M. Hrynew, K. Schlender, and P. Sorochuk of Holy Cross Separate Junior High School, and Messers R. Forest and B. Cartnell of Sir John Thompson Separate Junior High School for their valuable time and suggestions which helped to formulate the main investigation.

Algebraic Manipulations, (c) Solving Routine Algebra Problem, and (d) Solving Non-routine Algebra Problem, hereafter denoted as RFKA, PAM, SRAP, and SNrAP respectively.

90 multiple-choice test items in algebra, felt appropriate for ninth graders, were assembled. Some of these items were especially built for this investigation and were based on the prescribed mathematics textbook.

Others were selected or adapted from the Cooperative Mathematics Tests, Form A, Algebra 1, developed by the Cooperative Test Division of the ETS. In certain cases, attempts were made to select items that span the content objective of algebra rather than to focus on each cell within the cognitive-by-content matrix scheme. Also, since the content subcategory of 'Rational Expression' had not yet been covered at the time of testing, there were no test items in that area. Table 39 presents the list of 90 test items in Algebra.

Four other grade 9 mathematics teachers* familiar with the ETS taxonomy independently judged the appropriateness of the 90 test items as belonging to the content area of algebra, their suitability and clarity for ninth graders, and classified them according to four types of mathematics problem. Because of the cumulative hierarchy of the ETS taxonomy, each test item could be categorized by the number of different cognitive objectives associated with the test item, but the teachers were instructed to categorize the test item by the highest cognitive objective associated with the test item.

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*The author wishes to thank Messers G. McNally and S. Damarais of St. Alphonsus Separate Junior High School, and Messers A.B. Wacowich and L. Mackie of Sir John Thompson Separate Junior High School for serving as judges.

Table 39

List of Algebra Test Items

1. If Mary has (3x+2) dimes and Jane has (x+5) nickles, the difference between the amounts of money is

   (a) (25x−5) cents (c) (35x−5) cents

   (b) (35x + 45) cents (d) (25x+5) cents

2. There are 33 students in a class, and the ratio of boys to girls is 7:4. One day a boy was absent. What was the ratio of boys to girls on that day?

   (a) 3:2 (c) 7:8

   (b) 5:3 (d) 21:12

3. What is the reciprocal of −a?

   (a) a (c) $\frac{1}{a}$

   (b) $\frac{-1}{a}$ (d) −1

4. Each of five children, P, Q, R, S and T chooses his favorite color. P and Q do not choose the same color, nor do Q and R, nor do R and S, nor do S and T. What is the smallest number of different colors that could have been chosen?

   (a) 2 (c) 4

   (b) 3 (d) 5

5. (4x+3y) − (x+y) − (3x−4y) =

   (a) x − y (c) 8x

   (b) 6y (d) 8x−8y

6. What number must be added to X^2−6x+4 in order to make it a perfect square?

   (a) −4 (c) 2

   (b) 0 (d) 5

Table 39 (continued)

7. An automobile is moving at 4 miles per hour, and an airplane is moving three times as fast. How many hours will the plane require for a 500-mile flight?

(a) $\frac{500}{3+r}$ (c) $\frac{500}{r^3}$

(b) $\frac{500}{3r}$ (d) $3\frac{(500)}{r}$

8. A boy who has x quarters and y dimes buys p pencils at 5 cents each. How many cents does he have left?

(a) $x + y = 5p$ (c) $25x + 10y - p$

(b) $25x + 2\ (y-p)$ (d) $25x + 10y - 5p$

9. If Susan is younger than Mary, and if Mary is older than Jane, then we know that

(a) Susan is younger than Jane
(b) Jane is younger than Susan
(c) Susan and Jane are the same age
(d) Jane is more than half as old as Mary

10. Which of these is a monomial?

(a) $y^2 - 5$ (c) $y^2 + 3x - 5$

(b) $3x$ (d) $3x - 5$

11. If the number $\underline{b}$ is between $\underline{a}$ and $\underline{c}$, then

(a) $\underline{b} + 2$ is between $\underline{c} + 2$ and $\underline{a} + 2$
(b) $\underline{c} + 2$ is between $\underline{a} + 2$ and $\underline{b} + 2$
(c) $\underline{a} + 2$ is between $\underline{b} + 2$ and $\underline{c} + 2$
(d) all of the above is true

12. Any nominal is a

(a) bionomial
(b) polynomial
(c) trinomial
(d) none of these

Table 39 (continued)

13. $\frac{x^7}{x^3}$ =

(a) $x^{3.5}$ (c) x^{10}

(b) x^4 (d) x^{21}

14. What is the square root of $16b^8$?

(a) $2b^2$ (c) $4b^4$

(b) $4b^2$ (d) $8b^4$

15. (7y-5) - (2y+3) =

(a) 5y-8 (c) 9y-2

(b) 5y-2 (d) 9y-8

16. Jane is twice as old as Mary. In 12 years times the sum of their ages will be 48. How old is Mary?

(a) 8 (c) 12

(b) 4 (d) 16

17. How many elements do (all odd integers from 1 to 10) and (all integral multiples of 3) have in common?

(a) 1 (c) 3

(b) 2 (d) 5

18. The product of (a+b+c) and (x+y+z) contains _?_ terms.

(a) 3 (c) 6

(b) 9 (d) 12

Table 39 (continued)

19. $\frac{m^3n^5}{m^5n^3} =$

(a) 1

(b) $\frac{m^2}{n^2}$

(c) $(m^2)(n^{-2})$

(d) $(m^{-2})(n^2)$

20. What is the reciprocal of $\frac{2}{a}$?

(a) a

(b) $\frac{a}{2}$

(c) 2

(d) $\frac{2}{a}$

21. If x is a real number, what are <u>all</u> values of x for which x^2-16 is a negative number?

(a) all x less than zero
(b) all x less than 4
(c) all x between -4 and 4
(d) all x less than -4

22. If a and b are different numbers, which of the following is always between a and b

(a) a + 1

(b) $\frac{b - a}{2}$

(c) $\frac{a + b}{2}$

(d) $\frac{a + b}{2}$

23. A quadratic polynomial is always a

(a) monomial
(b) binomial
(c) trinomial
(d) second degree polynomial

24. $(3a^3)(1/3\ a^3b)(3b^4) =$

(a) ea^6b^5

(b) $3a^9b^4$

(c) $3a^9b^5$

(d) $6\ 1/3\ a^6b^5$

Table 39 (continued)

25. Factor $3x^2-4x-4$

(a) (3x-2) (x+2)
(b) (3x+2) (x-2)
(c) (3x+1) (x-4)
(d) (3x-4) (x+1)

26. If 7a-2a=5a, then 2a is called

(a) difference
(b) minued
(c) subtrahend
(d) sum

27. If $\frac{N}{34} = 22$, the $\frac{N}{17} = ?$

(a) 11
(b) 44
(c) 22
(d) 748

28. The factors of $a^2+6ab+5b^2$ are

(a) (a+5b) (a+b)
(b) (a-2b) (a-3b)
(c) (a-5b) (a-b)
(d) (a-5b) (a-b)

29. An example of a number raised to a positive integral power is

(a) $\frac{1}{3^2}$
(b) 2^{-2}
(c) $(-\sqrt{3})^4$
(d) $(\frac{2}{3})^{\circ}$

30. Two of a student's test marks are 68 and 84. A third mark is at least 40. What is the lowest possible average for the three tests?

(a) 40
(b) 58
(c) 64
(d) 76

31. Which of these polynomial is a binomial?

(a) $x^4 + 3x^2 + 2x + 1$
(b) $x + 2$
(c) $5x$
(d) $2x^2 + 5x + 2$

Table 39 (continued)

32. The multiplicative inverse of 5 is

(a) -5 (c) $\frac{1}{5}$

(b) $-\frac{1}{5}$ (d) 1

33. The additive inverse of $5x^2 - 4x - 3$ is

(a) $5x^2 - 4x - 3$ (c) $-5x^2 - 4x + 3$

(b) $-5x^2 + 4x + 3$ (c) $-5x^2 + 4x - 3$

34. The greatest common factor of the terms of the polynomial $24a^2b^4-36a^3b^6$ is

(a) 2ab (c) $12a^2b^4$

(b) $24a^2b^4$ (d) 12ab

35. The simple form of 3x+4-2y-3-5x-2y+1 is

(a) $-2x+4y$ (c) $-8x+4y$

(b) $-2x+2$ (d) $-2x-4y+2$

36. If m=5 and n>5, then the value of $4m^2n$ is greater than

(a) 200 (c) 800

(b) 600 (d) 1600

37. If $m^2 + \underline{?} + \frac{1}{25}$ is a trinomial square, then the missing term is

(a) $\frac{1}{625}$ m (b) $\frac{2}{25}$ m

(b) $\frac{1}{25}$ m (d) $\frac{2}{5}$ m

Table 39 (continued)

38. If $m^2 + \underline{?} + \frac{1}{25}$ is a trinomial square, then the missing term is

(a) $\frac{1}{625}$ m

(b) $\frac{1}{25}$ m

(c) $\frac{2}{25}$ m

(d) $\frac{2}{5}$ m

39. $(-6)^{\circ}$ =

(a) -6

(b) 0

(c) -1

(d) 1

40. Which one of the following equations expresses the condition that "the product of two numbers R and S is one less than twice their sum"?

(a) 2(RxS)-1 = R + S

(b) RxS = 2(R+S) - 1

(c) RxS = 2(RxS) + 1

(d) RxS-1 = 2(RxS)

41. Jim was elected President of the Students' Union. The ratio of voters was 5/4. If there are 900 votes cast, how many votes did Pete, Jim's opponent receive?

(a) 225

(b) 400

(c) 500

(d) 720

42. (-5x-3) - (5x+3) =

(a) 0

(b) -10x-6

(c) -10x

(d) 10x+6

43. Which of these polynomial is a trinomial?

(a) $3x^2+x+4$

(b) x + 4

(c) $3x^3+3x^2+2x+7$

(d) $3x^2$

Table 39 (continued)

44. If a represents a certain odd integer, then three times the next larger odd integer is represented by

(a) $3(a + 1)$ (c) $3a$

(b) $3(a + 2)$ (d) $3a + 2$

45. (?) is a second-degree polynomial in one variable

(a) $x + 1$ (c) $0x^2 + x + 1$

(b) $3x^2$ (d) $3x^3 + 5x + 1$

46. The product of −.2 and 0 is

(a) −.2 (c) −.20

(b) +.2 (d) 0

47. The statement below that is TRUE is

(a) $x^3 + x^3 = x^6$ (c) $x^3 + x^3 = 2x^3$

(b) $x^3 + x^3 = x^9$ (d) $x^3 + x^3 = (x^3)^2$

48. An example of a number raised to a negative integral power is

(a) $(-6)^4$ (c) $3^{-1/2}$

(b) $\frac{1}{3^{-2}}$ (d) 6^{-4}

49. If $(5x+a)$ is a factor of $10x^2-13x-3$, the value of a is

(a) −13 (c) 1

(b) −3 (d) 10

Table 39 (continued)

50. $\frac{3x^7y^2}{3x^2y^7}$ =

(a) $3(\frac{x}{y})^5$

(b) $(\frac{x}{y})^5$

(c) $\frac{3x^5}{y^5}$

(d) $\frac{x}{y}$

51. When the ploynomials $2x^2+3$ and $x-3$ are added together, the resulting expression is a

(a) monomial
(b) trinomial
(c) polynomial with four terms
(d) binomial

52. If xy=1 and x is greater than 0, which of the following statements is true

(a) when x is greater than 1, y is greater than 1
(b) when x is less than 1, y is less than 1
(c) as x increases, y increases
(d) as x increases, y decreases

53. The mailman delivers 9 letters to 4 houses. If each house gets at least one letter, which of the following is necessarily true

(a) each gets more than 1
(b) none gets more than 3
(c) none got more than 6
(d) each got at least 3

54. A class consists of 14 girls and a number of boys. There are fewer than 20 boys, but more than 30 children altogether, the number of boys is

(a) (1, 2, 3, ..., 19)

(b) (17)

(c) (17, 18, ..., 20)

(d) (17, 18, 19)

Table 39 (continued)

55. The middle term of the trinomial that is the product of $(3a-4)^2$ is

(a) $12a$
(b) $24a$
(c) $-12a$
(d) $-24a$

56. The statement "A certain number is increased by twice another number n is equal to 30" can be written as

(a) $f + 2n = 30$
(b) $2fn = 30$
(c) $2f + n = 30$
(d) $f + 2f = 30$

57. $(x + y)\ (x + 2y) =$

(a) $x^2 + 2y^2$
(b) $2x + 3y$
(c) $x^2 + 2xy + 4y^2$
(d) $x^2 + 3xy + 2y^2$

58. $(3x^2 - 5x + 6) + (10x - 4x^3 + x^2) =$

(a) $4x^3 + 4x^2 - 5x + 6$
(b) $-4x^3 + 4x^2 + 5x + 6$
(c) $-4x^3 + 2x^2 + 5x + 6$
(d) $4x^3 + 2x^2 + 5x + 6$

59. The additive inverse of -5 is

(a) $\frac{-1}{5}$
(b) $\frac{1}{5}$
(c) 0
(d) 5

60. $(y^3 + 2y^2 + y) + (y^2 + 2y + 2) =$

(a) $y^5 + 4y^3 + 2y$
(b) $2y^3 + 4y^2 + 2y$
(c) $y^3 + 3y^2 + 3y + 2$
(d) $2y^2 + 2y + 2$

Table 39 (continued)

61. $(x + 2)^2 =$

(a) $2x + 4$

(b) $x^2 + 4x + 4$

(c) $x^2 + 4$

(d) $x^2 + 4x + 2$

62. The additive inverse of 25 is

(a) 25

(b) -25

(c) $\frac{1}{25}$

(d) $\frac{-1}{25}$

63. What is the constant term in the polynomial $5x^2 - 2x + 3$

(a) 5

(b) 2

(c) 3

(d) -2

64. The statement below that is <u>FALSE</u> is

(a) $x^n . x^m = x^{n+m}$

(b) $(x^n)^m = x^{nm}$

(c) $x^n \div x^m = x^{n/m}$

(d) $(xy)^m = x^m y^m$

65. $(x + 2)(4x - 7) =$

(a) $4x^2 - 14$

(b) $4x^2 + 15x - 14$

(c) $4x^2 - 15x - 14$

(d) $4x^2 + x - 14$

66. What value of x, when substituted in $\frac{1}{x-2}$, will make this fraction meaningless?

(a) -2

(b) 0

(c) 2

(d) any number between 0 and 2

Table 39 (continued)

67. $(5abc)\ (2a^2b^4) =$

(a) $7a^2b^4c$

(b) $10a^3b^5c$

(c) $10a^2b^4c$

(d) $10a^{10}b^{10}c^{10}$

68. A rectangle has length 2x+1 inches and width x-7 inches, what is its area.

(a) $(2x+1)^2$

(b) 1/2 (2x+1) (x-7)

(c) (2x+1) + (x-7)

(d) $2x^2-13x-7$

69. The greatest common factor of the terms of the polynomial $8x^4 - 14x^3 - 6x^2$ is

(a) 2

(b) 2x

(c) x^2

(d) $2x^2$

70. The multiplicative inverse of -25 is

(a) +25

(b) 1/25

(c) -1/4

(d) -1/25

71. The equation $x^2 + 1 = 10$ may be interpreted as

(a) a number when doubled and increased by 1 is 10.
(b) a number whose square increased by 1 is 10.
(c) a number increased by 1 and squared is 10.
(d) a number squared is one more tnan 10.

72. If a=3 and b=2 then $\frac{ab^3}{(a-b)^2}$ =

(a) $\frac{18}{5}$

(b) $\frac{24}{5}$

(c) 18

(d) 24

Table 39 (continued)

73. Which value of x satisfies every condition of the form $ax^2+bx=0$, where a and b are any real numbers

(a) 1 (c) 2

(b) 0 (d) 10

74. Suppose that an operation * on any numbers a and b is defined by a * b = a x (a+b) then 2 * 3 equals

(a) 6 (c) 10

(b) 7 (d) 12

75. This line represents a piece of string

Suppose it is cut a few times. Since each cut changes the total number of pieces, how many pieces will you have if you make 17 cuts

(a) 16 (c) 18

(b) 17 (d) 19

76. $(\frac{3ab^2}{4})\ (\frac{10a}{9b}) =$

(a) $6a^5b^2$ (c) $3a^6b^2$

(b) $4a^5b^3$ (d) $3a^5b^3$

77. What is the coefficient of y in the expression $2y^5+6y^4-4y^2=5y+1$

(a) -5 (c) -1

(b) 5 (d) +1

Table 39 (continued)

78. What is the reciprocal of $-\frac{b}{4}$

(a) $\frac{b}{4}$

(b) $\frac{-4}{5}$

(c) $\frac{4}{b}$

(d) 4

79. The simplest form of 3x-6-2x-7-3x+4 is

(a) -2x+17

(b) -2x-9

(c) 2x+3

(d) 8x+9

80. $(3y)^3 =$

(a) $3y^3$

(b) $27y^3$

(c) $9y^3$

(d) 9y

81. When x is replaced by -3 in $x^2+9x+20$, the number obtained is

(a) -16

(b) 2

(c) 38

(d) 56

82. $(2a^2)\ (1/4\ b^2)\ (6a^3b) =$

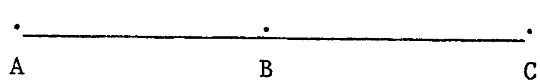
(a) $6a^5b^2$

(b) $4a^5b^3$

(c) $3a^6b^2$

(d) $3a^5b^3$

83.



The distance AB is given by 3x+1 inches and the distance AB is 5x+2 inches. How far is the distance BC?

(a) 8x + 3

(b) 2x + 1

(c) $\frac{5x+2}{2}$

(d) -2x-1

Table 39 (continued)

84. How many pieces of ribbon each exactly x ins. long can be cut from a ribbon which is 3x ins. long?

(a) 4x
(b) 2x
(c) 3
(d) $3x^2$

85. If x is greater than 3, which of the following is the smallest

(a) $\frac{3}{x}$
(b) $\frac{3}{x+1}$
(c) $\frac{3}{x-1}$
(d) $\frac{x}{3}$

86. How many cents are there in x dollars and y quarters

(a) x + 4y
(b) 4x + y
(c) 100x + 4y
(d) 100x + 25y

87. If a + b = 0, then

(a) a must be negative
(b) b must be negative
(c) both a and b can be negative
(d) either a or b can be negative

88. If n is an even number, what is the next larger even number

(a) n-2
(b) n+1
(c) N+2
(d) 2n

89. b^2-81 can be factored into

(a) b(b-9)
(b) $(b-9)^2$
(c) $(b+9)^2$
(d) (b+9)(b-9)

90. If 3x-1 = 8, then 2x-5 =

(a) -7
(b) 1
(c) 2 1/2
(d) 3

For each type of mathematics problem, all the test items on which the teachers' agreement was unanimous were grouped together to form a pilot subtest designed to measure performance in that type of mathematics problem. Four pilot subtests resulted. The remaining items were deleted from further consideration.

Given the testing time approved by the Separate School Board, a decision was made to use two schools for the pilot testing. All efforts were made to have the students of the two schools comparable in ability and socioeconomic status. Pilot subtests RFKA and PAM, each consisting of 21 items, were administered at two different sessions the same day to 78 ninth graders at St. Alphonsus Seperate Junior High School. About the same time, pilot subtests SRAP and SNrAP consisting of 20 items and 16 items respectively were administered in one session to 90 ninth graders at Sir John Thompson Separate Junior High School.

The results of this pilot testing were then subjected to item analysis. The decision to retain an item in the final form of each subtest was based on the item statistics, namely, the difficulty index, the biserial correlation, and the item reliability resulting from the item analysis. The goal was to obtain four valid and discriminatory subtests measuring the four components of mathematics achievement while still maintaining the highest possible reliability.

Results

Table 40 provides a summary count of the judges' agreement on the 90 test items. The four teachers who categorized the items agreed as follows: 78 items were unanimously classified congruent with the

Table 40

Summary Count of Judges' Agreement
on Algebra Test Items

Item Number	RFKA	PAM	SRAP	SNrAP	Judges' Comments (if any)
1				4	
2				4	
3	4				
4				4	
5		4			
6				4	
7			4		
8				4	
9					No correct answer
10	4				
11				4	
12	4				
13		4			
14			4		
15		4			
16				4	
17					Not belonging to the content objective of algebra
18			4		
19		4			
20	4				
21				4	
22				4	

Table 40 (continued)

Item Number	RFKA	PAM	SRAP	SNrAP	Judges' Comments (if any)
23					The expression 'quadratic polynomial' was not used
24		4			
25			4		
26	4				
27			4		
28			4		
29	4				
30				4	
31	4				
32	4				
33	4				
34		3	1		
35		4			
36		2	2		
37		4			
38			4		
39	4				
40			4		
41			4		
42		4			
43	4				
44			4		
45					The expression 'second degree polynomial' was not used

Table 40 (continued)

Item Number	RFKA	PAM	SRAP	SNrAP	Judges' Comments (if any)
46	4				
47	4				
48	4				
49			3	1	
50		4			
51			4		
52			1	3	
53				4	
54				4	
55			4		
56			4		
57		4			
58		4			
59	4				
60		4			
61		4			
62	4				
63	4				
64	4				
65		4			
66		2	2		
67		4			
68			4		
69		3	1		
70	4				
71			4		
72		4			
73				4	

Table 40 (continued)

Item Number	RFKA	PAM	SRAP	SNrAP	Judges' Comments (if any)
74			1	3	
75					Not belonging to the content objective of algebra
76		4			
77	4				
78	4				
79		4			
80		4			
81		4			
82		4			
83			4		
84			4		
85				4	
86				4	
87				4	
88			4		
89			4		
90			4		

intended categories; for 5 items, item Numbers 34, 49, 52, 69 and 74, three of the judges agreed with each other and with the intended categories; for 2 other items, item Numbers 36 and 66, there was an even split among the four teachers; on 2 items, item Numbers 17 and 75, the teachers agreed that those items did not belong to the content area of algebra; item Number 9 was found not to have a correct answer; for the remaining 2 items, item Numbers 23 and 54, two of the four teachers commented that the particular expressions used in the test items were unfamiliar to their students.

Of the 78 items on which there was unanimous agreement, 21 items were classified as belonging to RFKA, 21 items belonging to PAM, 20 items belonging to SRAP, and 16 items belonging to SNrAP. Each of these sets of items constituted a pilot subtest for measuring the particular component of mathematics achievement. Table 41 presents the item Numbers within each pilot subtest.

On the basis of the item statistic resulting from the item analysis of the pilot data, items which were too easy or too difficult, and hence not discriminating well, were rejected. Ideally, each item should have a difficult index (p) of 0.5 and a corresponding biserial correlation (r_{bis}) of about 1, thus given an item reliability index (i.r.i) of 0.5 (i.r.i. = $r_{bis}\sqrt{pq}$, q = 1 - p). However, for practical purposes, difficulty indices not less than 0.20 and not greater than 0.75 and biserials above 0.3 were considered satisfactory.

Using these criteria, 6 items each were dropped from pilot subtests RFKA and SRAP, and 7 items each from pilot subtests PAM and SNrAP. The rejected items are indicated by asterisks in Table 41.

Table 41

Item Numbers within Pilot Subtests
RFKA, PAM, SRAP, and SNrAP

RFKA			PAM			SRAP			SNrAP		
3,	10,	12,	5,	13,	15*,	7,	14,	18*,	1*,	2,	4*,
20*,	26*,	29*,	19,	24,	35*,	25,	27,	28,	6*,	8*,	11,
31,	32*	33	37,	42,	50*,	38*,	40,	41*,	16*,	21*,	22,
39,	43,	46,	57,	58,	60,	44*,	51*,	55,	30,	53,	54*,
47,	48,	59*,	61*,	65,	67,	56,	68,	71	73,	85,	86,
62,	63,	64,	72*,	76*,	79,	83,	84*,	88,	87,		
70*,	77,	78,	80,	81*,	82,	89,	90,				
	n = 21			n = 21			n = 20			n = 60	

*Items rejected on the basis of item statistics

The KR-20 reliabilities for the subtests after rejected items have been removed were: RFKA, .70; PAM, .77; SRAP, .64; andSNrAP, .36.

Because the KR-20 reliability for the subtest SNrAP turned out to be fairly low even after item selection an attempt was made to select a subtest of these items with the maximum possible reliability. Cluster analysis technique (Loevinger, Gleser, and Dubois, 1953) revealed that subtest SNrAP was composed of three clusters. The first and second clusters, for example, consisted of 3 items each with KR-20 reliabilities of 0.49 and 0.45 respectively.

Thus, given the results of this cluster analysis, a choice had to be made between measuring a factor within SNrAP more accurately or measuring SNrAP less accurately. Sacrificing unifactorial accuracy within the SNrAP component for broader coverage of that component was judged to be acceptable. This was because the objective of the study was to measure the SNrAP component and not a factor within the component. Furthermore, it is well known that a test which has high reliability for individuals need not necessarily measure groups accurately. And, since the outcome of this investigation was to be applicable to groups and not individuals, it was decided to retain all items in the final form of subtest SNrAP. Tables 42 through 45 respectively present the test and item statistics on the experimental group for the subtests RFKA, PAM, SRAP, and SNrAP.

Table 42

Test and Item Statistics on the Experimental Group for Subtest RFKA

RFKA (15 items). This subtest is designed to measure ability to recall factual knowledge in algebra.

Male Sample

Subtest Statistics:

Mean = 9.02

Variance = 7.34

Sample Size = 168

KR-20 Reliability = 0.59

Err. of Measurement = 1.73

Item Statistics:

Item Number	3	10	12	31	33
Difficulty Index	0.37	0.90	0.82	0.77	0.82
Biserial Correlation	0.45	0.73	0.55	0.58	0.62
Item Reliability Index	0.22	0.22	0.21	0.24	0.24

Item Number	39	43	46	47	48
Difficulty Index	0.42	0.74	0.64	0.42	0.59
Biserial Correlation	0.51	0.59	0.41	0.55	0.47
Item Reliability Index	0.25	0.26	0.20	0.27	0.23

Item Number	62	63	64	77	78
Difficulty Index	0.67	0.64	0.57	0.21	0.44
Biserial Correlation	0.44	0.57	0.67	0.43	0.59
Item Reliability Index	0.21	0.27	0.33	0.18	0.29

Table 42 (continued)

Female Sample

Subtest Statistics:

Mean	= 8.93	KR-20 Reliability	= 0.60
Variance	= 7.58	Err. of Measurement	= 1.74
Sample Size	= 191		

Item Statistics:

Item Number	3	10	12	31	33
Difficulty Index	0.42	0.88	0.79	0.77	0.87
Biserial Correlation	0.50	0.55	0.39	0.62	0.67
Item Reliability Index	0.25	0.18	0.16	0.26	0.22
Item Number	39	43	46	47	48
Difficulty Index	0.44	0.75	0.63	0.32	0.50
Biserial Correlation	0.67	0.73	0.55	0.55	0.48
Item Reliability Index	0.33	0.32	0.27	0.25	0.24
Item Number	62	63	64	77	78
Difficulty Index	0.68	0.55	0.60	0.18	0.56
Biserial Correlation	0.56	0.39	0.60	0.51	0.49
Item Reliability Index	0.26	0.19	0.30	0.19	0.24

Table 43

Test and Item Statistics on the Experimental Group for Subtest PAM

PAM (14 items). This subtest is designed to measure ability to recall factual knowledge in algebra.

Male Sample

Subtest Statistics:

Mean = 6.98

Variance = 12.80

Sample Size = 168

KR-20 Reliability = 0.74

Err. of Measurement = 1.83

Item Statistics:

Item Number	5	13	19	24	27
Difficulty Index	0.38	0.68	0.42	0.41	0.45
Biserial Correlation	0.81	0.75	0.43	0.60	0.49
Item Reliability Index	0.39	0.35	0.21	0.29	0.24

Item Number	42	57	58	60	65
Difficulty Index	0.49	0.33	0.49	0.56	0.46
Biserial Correlation	0.80	0.73	0.64	0.74	0.70
Item Reliability Index	0.40	0.34	0.32	0.37	0.35

Item Number	67	79	80	82
Difficulty Index	0.64	0.69	0.52	0.48
Biserial Correlation	0.56	0.78	0.59	0.71
Item Reliability Index	0.27	0.36	0.30	0.35

Table 43 (continued)

Female Sample

Subtest Statistics:

Mean = 7.30

Variance = 13.70

Sample Size = 191

KR-20 Reliability = 0.76

Err. of Measurement = 1.81

Item Statistics:

Item Number	5	13	19	24	27
Difficulty Index	0.43	0.67	0.44	0.45	0.47
Biserial Correlation	0.77	0.73	0.46	0.60	0.55
Item Reliability Index	0.38	0.35	0.23	0.30	0.28
Item Number	42	57	58	60	65
Difficulty Index	0.48	0.33	0.58	0.65	0.44
Biserial Correlation	0.71	0.87	0.69	0.74	0.82
Item Reliability Index	0.35	0.41	0.34	0.35	0.40
Item Number	67	79	80	82	
Difficulty Index	0.72	0.66	0.56	0.46	
Biserial Correlation	0.67	0.74	0.62	0.76	
Item Reliability Index	0.30	0.35	0.31	0.38	

Table 44

Test and Item Statistics on the Experimental Group for Subtest SRAP

SRAP (14 items). This subtest is designed to measure ability to solve routine problems in algebra.

Male Sample

Subtest Statistics:

Mean = 6.52

Variance = 9.48

Sample Size = 168

KR-20 Reliability = 0.66

Err. of Measurement = 1.80

Item Statistics:

Item Number	7	14	25	27	28
Difficulty Index	0.55	0.42	0.41	0.27	0.45
Biserial Correlation	0.55	0.59	0.62	0.40	0.77
Item Reliability Index	0.27	0.29	0.31	0.18	0.39

Item Number	40	55	56	68	71
Difficulty Index	0.35	0.39	0.55	0.27	0.55
Biserial Correlation	0.36	0.59	0.57	0.75	0.50
Item Reliability Index	0.17	0.29	0.29	0.33	0.25

Item Number	83	88	89	90
Difficulty Index	0.56	0.66	0.48	0.62
Biserial Correlation	0.51	0.53	0.77	0.57
Item Reliability Index	0.25	0.25	0.38	0.28

Table 44 (continued)

Female Sample

Subtest Statistics:

Mean	= 6.47	KR-20 Reliability	= 0.61
Variance	= 8.11	Err. of Measurement	= 1.78
Sample Size	= 191		

Item Statistics:

Item Number	7	14	25	27	28
Difficulty Index	0.60	0.43	0.48	0.20	0.51
Biserial Correlation	0.44	0.53	0.59	0.45	0.66
Item Reliability Index	0.22	0.26	0.30	0.18	0.33
Item Number	40	55	56	68	71
Difficulty Index	0.26	0.38	0.55	0.33	0.45
Biserial Correlation	0.40	0.63	0.46	0.59	0.57
Item Reliability Index	0.17	0.31	0.23	0.28	0.28
Item Number	83	88	89	90	
Difficulty Index	0.64	0.71	0.47	0.48	
Biserial Correlation	0.53	0.44	0.65	0.63	
Item Reliability Index	0.25	0.20	0.32	0.31	

Table 45

Test and Item Statistics on the Experimental Group for Subtest SNrAP

SNrAP (9 items). This subtest is designed to measure ability to solve non-routine problems in algebra.

Male Sample

Subtest Statistics:

Mean	= 4.39	KR-20 Reliability	= 0.50
Variance	= 4.18	Err. of Measurement	= 1.44
Sample Size	= 168		

Item Statistics:

Item Number	2	11	22	30	53
Difficulty Index	0.63	0.55	0.48	0.35	0.36
Biserial Correlation	0.49	0.72	0.64	0.26	0.68
Item Reliability Index	0.34	0.36	0.32	0.13	0.32

Item Number	73	85	86	87
Difficulty Index	0.60	0.28	0.55	0.69
Biserial Correlation	0.69	0.49	0.70	0.71
Item Reliability Index	0.34	0.22	0.35	0.33

Table 45 (continued)

Female Sample

Subtest Statistics:

Mean = 4.32

Variance = 2.92

Sample Size = 191

KR-20 Reliability = 0.32

Err. of Measurement = 1.41

Item Statistics:

Item Number	2	11	22	30	53
Difficulty Index	0.51	0.65	0.39	0.31	0.26
Biserial Correlation	0.54	0.47	0.50	0.27	0.64
Item Reliability Index	0.27	0.22	0.24	0.13	0.28

Item Number	73	85	86	87
Difficulty Index	0.67	0.29	0.55	0.72
Biserial Correlation	0.57	0.55	0.60	0.56
Item Reliability Index	0.27	0.25	0.30	0.25

APPENDIX B2

ESTIMATING THE POSSIBILITY OF THE JUNIOR INDEX OF MOTIVATION (JIM) FOR USE IN PILOT STUDY B3 TO ASSESS STUDENTS' MOTIVATION TOWARD SCHOOL

Introduction

The objective of this pilot study was to select a valid, reliable, and relatively short objective measure of motivation toward school which would serve as one of three criteria to be used in selecting 24 students for the Q-sort.

The problem in this area of general motivation (or n achievement) seem to be one of measurement. That is, how can motivation be best measured? Projective measures have been widely used to quantify this construct. These include McClelland's Thematic Apperception Test (TAT: McClelland, Atkinson, Clark, and Lowell, 1953), the French Test of Insight (FTI: French, 1958), and the Iowa Picture Interpretation Test (IPIT: Johnston, 1957).

Such disadvantages of projective measures of n Achievement which result from lack of internal consistency and test-retest reliabilities, and from low intercorrelations among other projective n Achievement measures (Klinger, 1966), have lead researchers to develop objective measures of n Achievement.

Barnette (1961), amongst others, found almost zero correlation (r = .094, N = 175) between two achievement motivation measures. One of these was a multiple-choice version of achievement using TAT cards (IPIT); the other, a personality scale which has since become part of Gough's California Psychological Inventory (1957). This finding suggests that the two scales might have been measuring different component of achievement motive.

Because there are many, possibly uncorrelated, aspects of n (generalized) Achievement, say, the need to achieve academically, the need to achieve in sports etc., which come under the general title 'n Achievement', it is perhaps better to speak about specific n Achievements.

The purpose of this pilot study was not to find a measure of a student's generalized n Achievement, but rather his motivation toward school or what might be termed 'n Academic Achievement'. Recent contribution to the realm of questionnaire measures of n Achievement included Myers' (1965) 12-item Achievement Motivation Scale, Hermans' (1970) 29-items Questionnaire Measure of Achievement Motivation, and Frymier's (1970) 50-item Junior Index of Motivation (JIM).

Myers' and Hermans' scales seemed unsuitable for the present study. Both seemed applicable for measuring generalized n Achievement, and both had been developed for use with college students. Thus a certain amount of rewriting and adaptation would be necessary before either could be used as a measure of motivation toward school or 'n academic achievement' applicable to grade 9 students.

The JIM Scale looked promising. It was specifically designed "for assessing students' motivation toward school", and was developed for use with junior high school students. It was thought advisable to carry out a preliminary study to re-assess its validity and reliability for us with the present sample, Gulliksen (1950) emphasized this point that when a test is in repeated use, its validity must be redetermined at intervals.

Basically, this pilot study was intended to answer two questions. Firstly, is the JIM Scale valid for the present sample? Secondly, is the JIM Scale reliable for the present sample?

Hypotheses: To answer these questions, the following three hypotheses were advanced.

1) Given that the JIM Scale does measure motivation toward school, it should correlate positively and significantly with teachers' estimates of students' motivational level.
2) Given that the JIM Scale does measure motivation toward school, the partial correlation coefficient between the JIM Scale scores and the grade point average (GPA), partialling out the effects of intelligence, should be positively and significantly related. This correlation coefficient would be taken as evidence that students who have higher JIM Scale scores would have higher GPA, since motivation toward school should, to some extent, show itself as accomplishment in school.
3) A relatively high value of Cronbach's Alpha (Cronbach, 1951) would be taken as evidence that the JIM Scale is internally consistent.

Procedure

The sample for this study was 71 ninth-grade pupils, 37 males and 34 females, of the Holy Cross Separate Junior High School within the city of Edmonton. It was understood from the Principal of the school and fellow graduate students that the pupils of the above-named school came from homes of varying socioeconomic status. Thus the sample was considered heterogeneous insofar as socioeconomic status was concerned and, therefore, appropriate for this validation study.

The JIM Scale was administered to all three ninth grade classes at the same time about the middle of the school year. Two education students at the University of Alberta helped with two of the classes. The scale was administered with no interference from staff members so as to minimize the strain which might prevail in a class situation.

Two teachers familiar with the grade 9 students were afterwards asked to rate each student independently on a five-point scale according to their estimates of the student's motivation toward school.

GPA was calculated for each student. This was based on the mean standardized scores on the core subjects--language arts, social studies, mathematics, and science. These scores were their mid-year marks which, by coincidence, were being compiled about the time these data were being collected. The scores with each of these students made on the Canadian Lorge-Thorndike Intelligence Test, administered about the middle of their eighth grade, were collected from their cumulative records.

The data were analyzed by means of a matrix of product moment correlation coefficients. Realizing the relatively small sample size, and the fact that the objective was to assess the validity of the JIM for use with the students as a group, it was considered unadvisable to analyze the results separately for each sex.

Results

Table 46 shows the intercorrelations between the JIM Scale, GPA, teachers' A rating, teachers' B rating, teachers' average rating, and the Canadian Lorge-Thordike Intelligence Test.

Table 46

Intercorrelations of School Motivation Index (JIM), Grade Point Average (GPA), Teachers' Ratings, and Intelligence (IQ) Scores (N=71).

	GPA	Rating A	Rating B	AV. Rating	
JIM	.36**	.17	.26*	.26*	.30*
GPA		.35	.55	.56	.51**
Rating A			.28*	.81	-.02
Rating B				.77	.10
AV. Rating					.05

*Significant beyond the 0.05 level

**Significant beyond the 0.01 level

The point of view is advanced that the need for academic achievement includes, but is a different concept from, the need for generalized achievement. Consequently, the results of this study were compared only with those studies which had employed measures specific to academic or scholastic achievement.

The JIM Scale scores correlated significantly with GPA ($N = 71$, $r = .36$ $p < .01$). This value compared reasonably well with the 0.38 ($N = 175$) reported by Barnette (1961, p. 652) and the 0.32 ($N = 400$) by Bending and Klugh (1965, p. 521), both using Gough (1953) objective measure which had been specifically constructed to predict scholastic achievement.

Frymier (1970), the author of the JIM Scale, reported a correlation coefficient of 0.44 ($N = 69$) in one of his studies aimed at validating the scale. The value found in this pilot study differed slightly from his. This difference, in part, could be explained on the grounds that two different measures of achievement were used. The Iowa Test of Education Development (ITED) employed in Frymier's study might be measuring a different aspect of academic achievement from that measured by grade point average.

The correlation ($N = 71$, $r = .30$) of JIM Scale scores with intelligence scores was slightly higher than a similar correlation ($N = 175$, $r = .26$) found in Barnette's study. Obviously, students who scored higher on the JIM Scale had more intelligence, as reflected by this significant correlation ($N = 71$, $r = .30$, $.01 < p < .02$). But this value of 0.30 was lower than the correlation ($r = .36$) of JIM Scale scores with GPA, which suggested an independent contribution of

JIM scores to grade point average.

Regression analysis, using IQ scores and JIM scores as predictors and GPA as criterion measure, yielded a multiple correlation coefficient (R) of 0.55. Analysis of variance on the significance of R^2 (Cooley and Lohnes, 1966, p. 34) gave an F ratio of 14.63 which was significant beyond the 0.01 level. The fact that $R^2 = .31$ indicated that 31 per cent of the variance in grade point average was predictable from a combination of IQ and JIM Scores. Of this 31 per cent, the JIM scores contributed 8 per cent. This indicated, as one would expect, that the academic motivation measure contributed over and above intelligence measure in predicting grade point average.

It was also considered appropriate to partial out the effect of intelligence so as to get a clearer picture of the relationship between JIM Scale scores and GPA. Even after partialling out the effects of intelligence, the JIM scores were positively correlated with GPA ($N = 71$, $t = 2.17$, $p < .05$). This seems to confirm hypothesis 2 that those who scored high on the JIM Scale also had high GPA even after statistically controlling for differences in intelligence, thereby lending support to the claim that the JIM is a measure of motivation toward school.

Of the two teachers' ratings, only one correlated significantly with JIM Scores ($N = 71$, $r = .26$, $p < .05$). The correlation coefficient between the two ratings was 0.28. This reflected a lack of agreement between the two ratings. The two teachers might have rated the students on different aspects of school work, as became evident in a personal communication from one of them in which he said, "It must be remembered that because of departmentalization, teachers know these students only in a severely limited context."

Although the average rating of the teachers correlated ($N = .71$, $r = .26$, $p < .05$) with the JIM Scale socres, the ratings were considered unreliable to serve as the only criterion for judging the validity of the JIM Scale. The internal consistency of the JIM Scale as measured by Cronbach's alpha (Cronbach, 1951) was 0.82.

Conclusion

On the basis of the outcome of testing hypothesis 2, and the fact that the JIM Scale has shown itself to be internal consistent, the JIM seemed to be discriminating between low-motivated and high-motivated students. The decision was therefore made to use the JIM Scale in Pilot Study B3 as an index of students' motivation toward school.

APPENDIX B3

SELECTION OF 24 STUDENTS FOR THE Q-SORT

Introduction

The purpose of this pilot study was to select a sample of 24 students such that three variables, sex, quantitative aptitude, and motivation toward school, could be accounted for in the selection of the sample.

Any investigation pertaining to attitudes toward mathematics in general, and to mathematics problems in particular, is plagued with many confounding variables. Chief amongst these are sex, quantitative aptitude, attitude toward the mathematics teacher, past performance in mathematics, and to a lesser extent, personality factors. However, bearing in mind the results which have emerged from previous studies that attitudes toward mathematics are related to quantitative aptitude and attitude toward school, and the fact that the generalizability of the results from inverse factor analytic procedures is determined by the adequacy of the subjects, it was decided to devote particular attention to assembling a representative sample so that sex, quantitative aptitude, and motivation toward school could be accounted for in the selection. That is, the 24 students should comprise both males and females of low and high motivation, as well as of low and high quantitative aptitude.

Procedure

The initial sample for this study was 71 ninth graders at Holy Cross Separate Junior High School. This was the same sample that was used in Pilot Study B2 for estimating the validity of the JIM, and from which the 24 students for the Q-sort were to be selected. The selection of the students was based on sex and on their scores on two tests.

One of the tests, the JIM, had already been administered a week before in connection with Pilot Study B2. Since the JIM had already shown signs of being valid for these students, it was considered legitimate to use their scores on the JIM Scale as measures of their motivation toward school. Their scores on the quantitative subtest of the School and College Ability Tests (SCAT, Form 3B), were taken as measures of the students' quantitative aptitude.

Since there were three variables (sex, motivation, quantitative aptitude) involved, and two levels within each variable (male and female for the variable of sex, high and low motivated students for the variable of motivation, and high and low aptitude students for the variable of quantitative aptitude), the schema constituted a 2 x 2 x 2 factorial layout with 8 cells. And, since a total of 24 students were to be selected, that meant that the number of students within each cell should be three.

For purposes of selection, the students' scores on the SCAT were first ranked, and those with high scores were designated as high quantitative aptitude students, and those with low scores were designated as low quantitative aptitude students. A similar strategy was employed in selecting the high motivated and low motivated students. If a student with a high score had already been chosen, the student with the next highest score was selected. A similar method of selection was applied to the low scorers.

Results

Table 47 provides the student IDs, sex, motivational and aptitude levels of the 24 students selected for the Q-sort.

Table 47

Subject Profile Data

Student ID	Sex	Motivational Level	Quantitative Aptitude Level
1	Male	High	High
2	Male	High	High
3	Male	High	High
4	Female	High	High
5	Female	High	High
6	Female	High	High
7	Male	High	Low
8	Male	High	Low
9	Male	High	Low
10	Female	High	Low
11	Female	High	Low
12	Female	High	Low
13	Male	Low	High
14	Male	Low	High
15	Male	Low	High
16	Female	Low	High
17	Female	Low	High
18	Female	Low	High
19	Male	Low	Low
20	Male	Low	Low
21	Male	Low	Low
22	Female	Low	Low
23	Female	Low	Low
24	Female	Low	Low

APPENDIX B4

SELECTION OF THE Q-SORT ATTITUDE STATEMENTS

Aim

To select 60 Q-sort attitude statements which represent a random sample from a defined universe of statements that have to do with the following categories.

1. School involvement.
2. Out of school involvement.
3. General liking to work the mathematics problems.
4. Involvement with parents, mathematics teacher and friends.
5. Concern for tests and grades in the mathematics problems.

Introduction

Theoretically, any sample of statements is acceptable as any other for the same Q-analytic design (Stephenson, 1953). In this study, however, the strategy was to first define a broad classification scheme consisting of areas thought important in any study of mathematics attitude. These categories were not to be understood as mutually exclusive, but rather as a starting point for compiling the initial pool of attitude statements. Wilson, Cahen, and Begle (1969), in the National Longitudinal Study of Mathematics Abilities (NLSMA), Report No. 9, on the development of tests, emphasized this point that

> It is likely that different students avoid or approach engagements in mathematics for quite different reasons, and an attitude instrument must cast an unbelievably wide net in order to tap these many facets (p. 158).

The categories listed above, modelled after those used in developing the Minnesota Pupil Opinion Instrument, were found appropriate for this purpose.

Method

Group sessions were held with students from three Separate Junior High Schools in Edmonton. The objective was to elicit the students' attitudes toward each of the four types of mathematics problems selected in Pilot Study B1. A schedule had been prepared beforehand and this was used as a rough guide. The students were told the objective of the sessions. In particular, they were told that the study had nothing to do with their school work. Their cooperation was solicited. The students seemed thrilled with the idea and promised to cooperate.

For each type of mathematics problem, each student was requested to write down in statement form five opinions, feelings, experiences, or views which they held to that type of mathematics problem. These five statements were to pertain to the five categories listed before, one for each category. The students were encouraged to write whatever they felt like writing.

These preliminary statements were then edited so that the various categories could be fairly represented. It was the original in tention to select all 60 statements by this procedure, but the number fell short of sixty. To boost the number to sixty, additional statements were adapted from statements in the (a) Minnesota Pupil Mathematics Opinions instrument (Form A), which, according to Attonen (1969), was devised by Dr. Cyril J. Hoyt in an unpublished study in 1960 for use with junior high school students, (b) Aiken's (1963) Revised Mathematics Attitude Scale, and from (c) Dutton's Scale (Dutton, 1951; 1962).

The adaptation in these statements consisted mainly of substituting the expression 'this type of problem' for the word 'mathematics' or 'arithmetic', and changing an item from a question format to a statement format. Also, some questions were changed slightly, for example, "Have you always like arithmetic" became 'I have always liked working this type of problem'.

Results

Table 48 presents the final list of the 60 Q-sort attitude statements.

Table 48

List of Q Sort Attitude Statements

Statement Number	Statement
1.	I am quite 'at home' when this type of problem is discussed in class.
2.	I wish a longer time is spent in class discussing this type of problem.
3.	I feel sorry when I miss a class period in which this type of problem is discussed.
4.	I prefer to do other things in class than to listen to the discussion about this type of problem.
5.	It's thrilling when this type of problem is discussed in class.
6.	I wish students could work this type of problem in groups and check it out among themselves.
7.	I like having the mathematics teacher ask me questions about this type of problem.
8.	I like to talk to the mathematics teacher about this type of problem.
9.	I like to ask the mathematics teacher questions that have to do with this type of problem.
10.	I like to answer questions that have to do with this type of problem.
11.	I prefer the mathematics teacher to work more examples of this type of problem than of other types.
12.	I wish the mathematics teacher could explain this type of problem more and make sure the students understand.
13.	I am happy on days when I do not have to work this type of problem.

Table 48 (continued)

Statement Number	Statement
14.	I like to do extra work in this type of problem whenever I have time.
15.	I like to work this type of problem at school, but not for homework.
16.	I enjoy working this type of problem for homework.
17.	I would like to read other books about this type of problem in addition to the one used in class.
18.	I never use the ideas in this type of problem outside school.
19.	When I grow up, I would like a job that makes use of the knowledge about this type of problem.
20.	I like people who know how to work this type of problem.
21.	I like to study the section in the mathematics textbook that deals with this type of problem.
22.	I prefer to take notes about this type of problem from the mathematics teacher than to read the mathematics textbook.
23.	Out of school, I forget much about how to work this type of problem.
24.	I like to begin my homework by working this type of problem.
25.	I would like to work harder problems than this type of problem.
26.	If I do not get this type of problem right the first time, I like to keep working until I get the right answer.
27.	I would rather be given the right answer to this type of problem than work it out myself.

Table 48 (continued)

Statement Number	Statement
28.	I would never work this type of problem if I did not have to.
29.	I like to work this type of problem.
30.	I would rather read books than spend my time working this type of problem.
31.	I never know how to start working this type of problem.
32.	I feel confident about myself when working this type of problem.
33.	This type of problem is easy to work.
34.	Working this type of problem requires too much thinking.
35.	No matter how hard I try, I never get this type of problem right.
36.	I always say to myself 'I can't do it' whenever I have to work this type of problem.
37.	I could work this type of problem with a little help from the mathematics teacher.
38.	I often forget how to work this type of problem after I have worked on other types.
39.	Working this type of problem takes too much time.
40.	I have always liked working this type of problem.
41.	I never do my best in this type of problem.
42.	I rely on memory to work this type of problem.
43.	I like to chat with my friends about this type of problem.
44.	I like to work this type of problem with my friends.

Table 48 (continued)

Statement Number	Statement
45.	My friends are good at working this type of problem.
46.	I like to chat with my parents about this type of problem.
47.	I like to work this type of problem by myself, but check the working out with friends who know how to work it.
48.	I would rather figure out this type of problem by myself than request help from my parents.
49.	I work hard to get good marks in this type of problem.
50.	I worry about my marks in this type of problem.
51.	I would rather have good marks in this type of problem than in other types of problem.
52.	I always have good marks in this type of problem.
53.	I wish mathematics tests are made up only of this type of problem.
54.	Tests in this type of problem are easy.
55.	I like taking tests in this type of problem.
56.	I like taking mathematics tests that involve harder problems than this type.
57.	I always wish I have fewer problems of this type to do for homework.
58.	I usually get this type of problem right in class, but not in a test.
59.	This type of problem takes too long to work in a test or in an exam.
60.	I like to work this type of problem first in a test or in an exam.

APPENDIX C

ATTITUDE SCALES AND INSTRUCTIONS FOR ADMINISTRATION

ATTITUDE SCALES

The purpose of this survey is to find out the extent to which you agree or disagree with statements which junior high school students like yourselves have made concerning ALGEBRA PROBLEMS.

For the purpose of this exercise ALGEBRA PROBLEMS are categorized into FOUR types.

TYPE 1

Those that require the student just to recall some fact in algebra.

TYPE 2

Those that require algebraic calculations.

TYPE 3

Those that are routine algebra problems.

TYPE 4

Those that are non-routine algebra problems which require the student to develop his own technique to work them.

You will find a detailed description of each type of algebra problem together with some examples so as to help you understand the distinctions between the different types.

You will be required to read the description of a particular type of algebra problem very carefully using the examples that follow each description to help your understanding.

After making sure that you understand the description, you are to indicate the extent to which you agree or disagree with those statements which have been made concerning that particular type of algebra problems. Note that the same statements may apply to different types of problem.

DIRECTIONS

Write your name and sex on the appropriate places on the IBM answer sheet. Please indicate your own personal opinion of each statement by marking under the correct space on the answer sheet as follows:

STRONGLY AGREE	AGREE	UNDECIDED	DISAGREE	STRONGLY DISAGREE
A	B	C	D	E

ATTITUDE SCALE 1 (AS-RFKA)

TYPE 1 ALGEBRA PROBLEM: RECALLING FACTS IN ALGEBRA

DESCRIPTION. SAME AS FOR THE Q-SORT

(SEE APPENDIX A)

. . . .
. . . .
. . . .
. . . .
. . . .
. . . .

STATEMENTS PERTAINING TO TYPE 1 ALGEBRA PROBLEM

1. I like to work this type of problem first in a test or in an exam.
2. I like to work this type of problem by myself, but check the working out with friends who know how to work it.
3. I would never work this type of problem if I did not have to.
4. I am happy on days when I do not have to work this type of problem.
5. I prefer to do other things in class than to listen to the discussion about this type of problem.
6. I rely on memory to work this type of problem.
7. I wish students could work this type of problem in groups and check it out among themselves.
8. I would rather be given the right answer to this type of problem than work it out myself.
9. This type of problem is easy to work.
10. I am quite 'at home' when this type of problem is discussed in class.
11. I always wish I have fewer problems of this type to do for homework.
12. I wish the mathematics teacher could explain this type of problem more and make sure the students understand.
13. Tests in this type of problem are easy.
14. I like to work this type of problem at school, but not for homework.
15. I wish a longer time is spent in class discussing this type of problem.

16. I have always liked working this type of problem.

17. I never use the ideas in this type of problem outside school.

18. I like to answer questions that have to do with this type of problem.

ATTITUDE SCALE 2 (AS-PAM)

TYPE 2 ALGEBRA PROBLEM: PERFORMING MANIPULATIONS OR CALCULATIONS IN ALGEBRA

DESCRIPTION. SAME AS FOR THE Q-SORT

(SEE APPENDIX A)

. . .
. . .
. . .
. . .
. . .
. . .

STATEMENTS PERTAINING TO TYPE 2 ALGEBRA PROBLEM

1. I always say to myself 'I can't do it' whenever I have to work this type of problem.

2. Working this type of problem requires too much thinking.

3. I am happy on days when I do not have to work this type of problem.

4. I wish students could work this type of problem in groups and check it out among themselves.

5. I like to ask the mathematics teacher questions that have to do with this type of problem.

6. I like to answer questions that have to do with this type of problem.

7. I have always liked working this type of problem.

8. I enjoy working this type of problem for homework.

9. Working this type of problem takes too much time.

10. I like to work this type of problem.

11. No matter how hard I try, I never get this type of problem right.

12. I would rather figure out this type of problem by myself than request help from my parents.

13. I prefer to do other things in class than to listen to the discussion about this type of problem.

14. I prefer to take notes about this type of problem from the mathematics teacher than to read the mathematics textbook.

15. I wish a longer time is spent in class discussing this type of problem.

16. I like to begin my homework by working this type of problem.

17. I prefer the mathematics teacher to work more examples of this type of problem that of other types.

18. I like to talk to the mathematics teacher about this type of problem.

ATTITUDE SCALE 3 (AS-SRAP)

TYPE 3 ALGEBRA PROBLEM: SOLVING ROUTINE ALGEBRA PROBLEM

DESCRIPTION. SAME AS FOR THE Q-SORT

(SEE APPENDIX A)

. . . .
. . . .
. . . .
. . . .
. . . .
. . . .

STATEMENTS PERTAINING TO TYPE 3 ALGEBRA PROBLEM

1. I am happy on days when I do not have to work this type of problem.
2. I rely on memory to work this type of problem.
3. I like to talk to the mathematics teacher about this type of problem.
4. I wish a longer time is spent in class discussing this type of problem.
5. I would never work this type of problem if I did not have to.
6. I prefer the mathematics teacher to work more examples of this type of problem than of other types.
7. I never use the ideas in this type of problem outside school.
8. I prefer to do other things in class than to listen to the discussion about this type of problem.
9. I like to work this type of problem with my friends.
10. I wish students could work this type of problem in groups and check it out among themselves.
11. I like to work this type of problem by myself, but check the working out with friends who know how to work it.
12. I often forget how to work this type of problem after I have worked on other kinds.

ATTITUDE SCALE 4 (AS-SNrAP)

TYPE 4 ALGEBRA PROBLEM: SOLVING NON-ROUTINE ALGEBRA PROBLEM

DESCRIPTION. SAME AS FOR THE Q-SORT

(SEE APPENDIX A)

. . .
. . .
. . .
. . .
. . .
. . .

STATEMENTS PERTAINING TO TYPE 4 ALGEBRA PROBLEM

1. I am happy on days when I do not have to work this type of problem.
2. I like to work this type of problem at school, but not for homework.
3. I prefer to do other things in class than to listen to the discussion about this type of problem.
4. I always wish I have fewer problems of this type to do for homework.
5. This type of problem takes too long to work in a test or in an exam.
6. Working this type of problem requires too much thinking.
7. I wish the mathematics teacher could explain this type of problem more and make sure the students understand.
8. If I do not get this type of problem right the first time, I like to keep working until I get the right answer.
9. I wish a longer time is spent in class discussing this type of problem.
10. Working this type of problem takes too much time.
11. I wish students could work this type of problem in groups and check it out among themselves.
12. I like to ask the mathematics teacher questions that have to do with this type of problem.

APPENDIX D

TABLES

Appendix D1

Unrotated Factor Loadings* for Twenty-Four Persons
as Variables (n=60 Attitude Statements).
Stimulus Presented was RFKA.

Person ID	1	2	3
1	602	463	-213
2	-513	508	415
3	-777	073	-109
4	-317	084	-682
5	716	290	-073
6	692	197	-446
7	-002	444	-432
8	-541	-086	-185
9	-018	428	-097
10	-562	489	298
11	-524	418	343
12	-344	-098	-186
13	678	288	021
14	279	207	-329
15	-715	-115	-182
16	369	409	-371
17	-083	437	166
18	-373	137	102
19	-358	065	-172
20	-404	-058	-340
21	-247	522	-180
22	-327	536	238
23	-663	-050	-487
24	-533	-028	-430

*Decimal points have been omitted

Appendix D2

Unrotated Factor Loadings* for Twenty-Four Persons
as Variables (n=60 Attitude Statements).
Stimulus Presented was PAM.

Person ID	1	2	3
1	667	434	188
2	634	187	-032
3	564	095	239
4	478	395	428
5	261	645	025
6	356	565	046
7	103	-054	-072
8	-507	-026	393
9	-431	398	-518
10	-275	483	-099
11	-243	607	-235
12	041	464	-276
13	615	093	-036
14	-455	420	064
15	-689	-044	099
16	-342	641	-048
17	443	346	539
18	-514	533	046
19	-650	037	248
20	-742	-100	-020
21	-465	011	456
22	-510	091	516
23	234	272	-166
24	-617	342	014

*Decimal points have been omitted

Appendix D3

Unrotated Factor Loadings* for Twenty-Four Persons as Variables (n=60 Attitude Statements). Stimulus Presented was SRAP.

Person ID	1	2	3
1	408	542	-219
2	-563	513	141
3	723	-156	-109
4	-103	592	311
5	704	081	-455
6	452	418	-466
7	-087	052	483
8	568	-340	308
9	492	-178	134
10	510	347	165
11	472	628	-090
12	623	-145	-101
13	-587	258	-147
14	684	-098	250
15	-402	216	657
16	228	318	031
17	747	-152	246
18	613	144	-157
19	596	209	242
20	399	-037	323
21	379	290	471
22	456	423	104
23	449	-314	339
24	729	-159	-146

*Decimal points have been omitted

Appendix D4

Unrotated Factor Loadings* for Twenty-Four Persons as Variables (n=60 Attitude Statements). Stimulus Presented was SNrAP.

Person ID	1	2	3
1	096	765	-174
2	-491	550	001
3	745	-034	001
4	629	258	287
5	708	023	-142
6	331	559	-275
7	137	062	732
8	613	-135	084
9	-469	337	042
10	615	073	-230
11	306	586	-083
12	-086	404	119
13	-267	458	-024
14	-069	307	424
15	743	-318	013
16	503	236	303
17	564	098	275
18	303	266	-595
19	301	390	090
20	659	-129	129
21	095	623	220
22	572	213	-359
23	596	-265	204
24	-159	182	580

*Decimal points have been omitted

Appendix D5

Arrangements of 60 Attitude Statements in Descending Order of Their Total Scores for Stimulus RFKA

Factor 1		Factor 2		Factor 3	
Statement Number	Total Score	Statement Number	Total Score	Statement Number	Total Score
28*	62.55	2*	50.11	33*	42.67
57*	61.88	12*	48.34	1*	38.52
13*	61.78	6*	46.33	10*	37.26
27*	61.30	42*	45.37	60*	36.84
18*	61.09	22*	45.31	40*	36.58
12	60.02	47*	44.84	54*	36.21
4	59.98	15	42.94	25	34.68
15	59.72	51	42.51	52	34.26
35	58.63	9	41.14	56	34.22
50	57.63	50	40.83	29	34.09
23	56.97	37	40.76	16	33.79
11	55.95	31	40.72	26	33.78
38	54.32	26	40.60	7	32.94
37	53.05	44	39.62	55	32.94
42	52.83	43	38.53	24	32.38
20	51.81	58	38.21	32	31.41
44	51.42	48	37.97	48	31.03
30	50.91	3	37.91	18	30.83
47	49.82	49	37.62	37	30.68
58	48.97	8	37.31	47	29.85
36	48.93	11	37.31	45	29.48
2	48.05	29	36.79	49	29.40
6	47.03	20	35.12	15	28.69
45	46.98	14	34.47	42	27.09
22	45.32	45	34.27	14	26.98
54	44.70	10	32.47	57	26.07
10	43.26	59	32.40	9	25.98
26	41.82	13	32.12	43	25.89
53	40.09	18	31.98	4	25.86
48	39.63	40	31.45	58	25.85
41	37.84	24	30.89	8	25.75
5	37.32	23	30.86	46	25.16
31	35.97	38	30.81	21	25.04
51	34.82	21	30.79	38	24.83
34	33.80	35	30.62	5	24.34

Appendix D5 (continued)

Factor 1		Factor 2		Factor 3	
Statement Number	Total Score	Statement Number	Total Score	Statement Number	Total Score
9	32.76	7	30.25	53	24.04
1	31.28	28	29.71	20	23.72
39	31.06	39	29.04	41	23.60
33	31.02	4	28.73	17	23.41
60	30.19	41	28.72	30	23.40
25	29.78	60	27.67	23	22.99
21	29.63	27	27.26	44	22.94
29	28.58	16	26.95	51	22.74
3	28.35	17	26.67	3	22.65
17	26.79	53	26.43	13	22.62
8	26.70	19	26.29	6	22.12
49	26.53	5	26.29	19	21.48
43	26.21	25	25.98	22	21.36
52	25.95	1	25.49	12	21.31
56	25.87	36	25.08	50	20.31
59	24.20	55	24.52	31	19.81
32	23.63	46	22.92	35	19.78
19	22.99	57	21.88	2	19.43
14	22.07	52	21.76	39	19.28
24	22.02	56	21.11	28	18.70
46	21.66	32	20.44	11	17.06
7	21.64	34	19.65	59	16.95
16	25.50	30	18.41	36	16.82
40	24.98	33	17.10	27	16.68
55	23.56	54	15.94	34	15.92

*Statements considered to have high positive saturations

Appendix D6

Arrangements of 60 Attitude Statements in Descending Order of Their Total Scores for Stimulus PAM

Factor 1		Factor 2		Factor 3	
Statement Number	Total Score	Statement Number	Total Score	Statement Number	Total Score
16*	47.60	4*	36.19	2*	33.40
40*	47.02	39*	35.80	11*	33.17
29*	45.12	36*	34.61	8*	33.06
10*	43.91	13*	34.60	12*	32.42
48*	43.13	34*	34.56	9*	32.24
24*	43.07	12	33.39	22*	31.95
26	42.69	35	33.38	15	30.93
32	42.51	37	33.18	50	30.41
60	41.99	50	32.51	6	30.23
49	40.43	23	31.98	47	29.47
8	40.30	38	31.21	14	28.91
1	40.10	28	31.00	26	28.75
51	39.92	31	30.54	3	28.64
9	39.88	27	30.07	57	28.09
47	39.34	6	29.54	42	27.83
54	39.27	44	29.40	37	27.34
58	38.33	59	29.14	30	26.47
53	38.01	18	28.70	46	26.35
55	37.54	11	28.67	44	26.20
33	37.04	42	28.39	49	26.20
25	36.98	17	27.26	31	25.65
18	36.59	47	27.14	48	25.56
44	35.88	58	26.82	20	25.45
45	35.84	30	26.80	59	24.99
50	35.77	20	26.50	7	24.96
37	35.42	43	26.32	18	24.93
43	34.80	51	26.09	51	24.89
7	33.91	45	25.78	21	24.82
42	32.90	48	25.73	45	24.77
11	32.53	26	25.15	38	24.53
20	32.52	8	25.15	41	24.16
14	32.35	57	23.96	28	24.01
12	32.22	9	23.11	23	23.89
13	32.02	25	22.90	10	23.83
56	31.91	3	22.66	29	23.76
3	31.23	41	22.03	53	23.51

Appendix D6 (continued)

Factor 1		Factor 2		Factor 3	
Statement	Total	Statement	Total	Statement	Total
6	30.40	22	21.80	34	23.41
57	30.35	7	21.46	60	22.77
46	30.25	49	21.30	39	22.73
2	30.01	1	20.92	35	22.40
17	28.96	46	20.23	13	21.60
52	28.88	56	19.75	17	21.09
39	27.89	54	19.62	43	19.87
28	27.65	15	19.44	19	19.18
23	26.66	32	19.28	1	19.13
22	26.40	53	19.25	4	19.02
5	26.31	60	19.08	27	19.02
21	26.21	14	19.03	16	18.71
15	24.55	33	18.81	58	17.92
59	24.30	10	18.53	54	17.65
19	22.82	24	18.46	25	17.47
31	22.66	16	18.23	5	17.04
4	21.79	40	18.23	36	17.01
41	21.29	5	17.70	52	15.64
38	21.19	55	17.53	24	15.06
30	20.81	52	17.23	33	14.26
34	19.94	21	16.67	40	14.09
36	17.80	29	15.16	32	13.69
27	15.74	19	14.65	55	13.12
35	13.07	2	13.03	56	12.44

*Statements considered to have high positive saturations

Appendix D7

Arrangements of 60 Attitude Statements in Descending Order of Their Total Scores for Stimulus SRAP

Factor 1		Factor 2	
Statement Number	Total Score	Statement Number	Total Score
42*	50.10	47*	37.38
38*	48.64	8*	37.13
18*	47.85	6*	37.08
28*	47.60	44*	36.18
4*	46.53	11	33.36
13*	46.17	2	32.93
27	44.97	42	32.90
9	44.83	37	32.41
37	44.70	50	32.01
11	44.36	12	31.92
59	44.13	9	31.74
57	43.62	49	31.27
39	43.01	51	30.63
23	42.90	22	30.32
2	42.75	38	30.31
12	41.35	46	30.11
49	40.94	43	30.03
6	40.23	15	29.92
31	39.82	23	29.54
30	39.74	16	29.28
47	39.10	45	28.82
20	38.76	29	28.80
35	38.75	48	28.50
36	38.51	3	28.50
50	38.18	57	28.45
51	37.68	58	28.14
15	37.63	21	27.32
34	37.18	26	27.29
45	36.85	18	27.20
14	34.43	10	26.95
48	34.13	7	26.02
44	34.03	31	25.71
22	32.87	20	25.57
8	32.86	1	25.50
3	32.16	17	24.53

Appendix D7 (continued)

Factor 1		Factor 2	
Statement Number	Total Score	Statement Number	Total Score
43	31.78	59	24.52
41	31.69	14	24.23
21	31.50	41	24.05
26	31.28	13	24.02
17	31.05	28	23.89
46	30.18	36	23.43
52	28.79	24	23.21
55	28.72	40	22.73
7	28.64	19	22.55
60	28.06	30	21.99
5	28.06	39	21.51
53	27.83	60	21.45
32	27.15	55	20.70
10	27.03	34	19.94
58	26.93	25	19.74
19	25.24	4	19.20
1	25.16	56	19.17
24	24.26	35	18.98
33	23.65	53	18.84
25	22.52	52	17.57
29	22.11	27	17.37
54	20.99	32	17.33
16	20.78	54	17.11
40	20.18	5	14.35
56	22.10	33	14.06

*Statements considered to have high positive saturations

Appendix D8

Arrangments of 60 Attitude Statements in Descending Order of Their Total Scores for Stimulus SNrAP

Factor 1		Factor 2		Factor 3	
Statement Number	Total Score	Statement Number	Total Score	Statement Number	Total Score
39*	49.17	9*	37.57	29*	18.38
4*	47.95	6*	35.42	28*	18.30
57*	46.93	26*	35.16	26*	17.92
13*	46.91	2*	34.46	41*	17.60
59*	46.84	12*	33.75	18*	16.75
34	45.83	15*	33.68	33	15.78
2	45.12	8	32.52	4	15.78
38	43.55	47	32.24	49	15.78
30	42.13	48	32.22	31	15.13
35	42.71	11	32.07	15	14.93
27	42.33	43	31.58	47	14.54
41	41.94	29	31.56	20	14.47
18	41.83	10	31.32	37	14.35
31	41.71	44	31.31	55	13.69
37	41.09	37	30.79	52	13.50
50	40.52	51	28.96	13	13.50
28	40.34	49	28.21	1	13.43
23	40.27	23	27.62	32	13.30
12	40.08	46	27.33	24	13.11
49	40.01	59	27.32	60	13.10
36	39.97	57	27.12	57	12.99
11	39.83	38	27.04	6	12.99
15	39.82	39	26.98	25	12.92
20	39.52	22	26.70	11	12.92
44	39.38	60	26.70	40	12.79
6	39.38	3	26.31	23	12.79
48	38.54	18	25.87	48	12.79
9	38.27	7	25.36	16	12.53
17	37.99	32	25.25	39	12.53
58	37.98	33	24.91	45	12.41
26	37.51	17	24.37	7	12.14
42	36.85	55	24.18	51	12.02
22	36.03	16	23.98	59	11.98
51	34.85	42	23.95	12	11.94
7	34.65	20	23.79	27	11.70

Appendix D8 (continued)

Factor 1		Factor 2		Factor 3	
Statement Number	Total Score	Statement Number	Total Score	Statement Number	Total Score
5	34.10	34	23.32	44	11.56
45	33.17	25	23.27	50	11.43
47	32.80	41	22.88	56	11.36
8	32.50	50	22.68	10	11.29
43	31.87	52	22.63	5	10.25
1	31.87	58	21.96	30	10.12
60	30.56	1	21.17	43	10.00
21	30.23	54	20.84	22	9.93
46	30.19	28	19.74	38	9.93
54	28.55	14	19.66	2	9.54
14	28.24	13	19.63	21	9.54
53	27.80	45	19.63	53	9.35
3	27.76	5	19.16	36	9.22
52	27.48	56	19.12	19	8.84
24	27.43	35	18.90	54	8.81
25	26.66	24	18.87	3	8.50
33	26.50	21	18.69	14	8.38
32	26.26	31	17.60	34	8.38
10	25.91	30	17.53	42	8.30
19	24.74	4	17.24	9	7.60
16	22.08	19	16.78	17	7.60
56	20.83	53	16.49	46	7.53
40	19.08	40	15.58	8	7.07
29	16.22	36	15.04	35	5.97
55	14.12	27	13.05	58	5.97

*Statements considered to have high positive saturations

Appendix D9

Intercorrelations among 18 Statements of AS-RFKA. N=340

Statement Number	60	47	28	13	4	42	6	27	33	1	57	12	54	22	2	40	18	10
60																		
47	-00																	
28	13	07																
13	09	02	34															
4	-00	13	27	17														
42	12	21	-03	-03	-05													
6	08	33	-06	01	-05	17												
27	00	19	33	27	27	05	-07											
33	29	01	16	21	04	13	02	09										
1	25	05	13	18	11	11	-01	13	56									
57	09	06	45	44	23	03	-10	27	24	14								
12	-04	20	06	-13	09	12	13	14	-32	-18	-19							
54	34	-03	13	17	04	09	-01	03	66	50	18	-25						
22	03	27	06	08	11	19	13	16	07	04	06	15	00					
2	-09	-06	-05	-10	14	-01	-05	04	-21	-17	-05	30	-19	07				
40	28	-04	15	10	13	03	-02	13	47	42	15	-17	55	09	01			
18	-01	-19	21	22	23	-16	-14	09	05	13	24	-14	05	02	04	13		
10	26	02	19	26	16	10	02	17	56	53	23	-14	46	15	-05	53	16	

Correlation Coefficient*

*Decimal points have been omitted

Appendix D10

Unrotated Factor Loadings of 18 Statements
of AS-RFKA. N=340*

Statement Number	1	2	3	4
A(RFKA)				
28	422	530	-162	-168
57	490	432	-252	-314
13	461	377	-198	-316
27	305	586	062	005
18	273	259	-486	149
4	253	542	-078	255
B(RFKA)				
2	-213	304	018	691
12	-320	418	415	351
6	-030	003	586	-250
42	115	-002	577	-083
22	140	325	450	100
47	041	324	637	-261
C(RFKA)				
33	776	-295	103	-031
1	701	-195	109	101
10	745	-056	111	217
60	425	-189	198	060
40	674	-168	058	381
54	734	-350	074	070

*Decimal points have been omitted

Appendix D11

Intercorrelations among 18 Statements of AS-PAM. N=340

Statement Number	Correlation Coefficient*																	
	36	34	13	12	9	10	40	16	39	29	35	48	4	22	2	24	11	8
36																		
34	37																	
13	33	31																
12	-03	-02	-16															
9	-06	04	12	31														
10	19	27	28	-19	06													
40	20	30	31	-24	-08	60												
16	22	35	34	-18	02	61	66											
39	36	41	33	-16	08	33	34	38										
29	36	40	36	-17	-05	65	70	72	41									
35	39	55	40	-06	03	33	38	39	52	44								
48	27	36	24	-14	-10	41	38	43	20	50	34							
4	22	25	29	09	19	23	21	28	29	26	36	15						
22	09	17	09	12	25	08	07	11	15	09	12	09	13					
2	-05	-04	-03	44	34	-12	-16	-17	-04	-15	-02	-16	26	15				
24	24	19	22	-25	-01	53	55	57	30	55	27	36	20	09	-16			
11	-17	-08	-06	37	34	-13	-17	-12	-06	-20	-04	-14	14	12	49	-14		
8	03	06	08	30	47	03	-07	05	05	00	04	-01	21	20	37	-01	37	

*Decimal points have been omitted

Appendix D12

Unrotated Factor Loadings of 18 Statements
of AS-PAM. N=340*

Statement Number	1	2	3
C(PAM)			
16	794	-003	-319
40	762	-118	-321
29	843	-040	-198
10	726	-019	-385
48	599	-086	-038
24	665	-094	-376
A(PAM)			
4	403	459	080
39	608	159	304
36	503	117	506
13	544	165	303
34	660	199	398
35	583	178	418
B(PAM)			
2	-203	716	-087
11	-230	648	-214
8	-010	696	-180
12	-283	596	-024
9	-011	690	-134
22	167	392	-022

*Decimal points have been omitted

Appendix D13

Intercorrelations among 12 Statements of AS-SRAP. N=344

Statement Number	Correlation Coefficient*											
	13	42	8	2	28	11	18	4	44	6	47	38
13												
42	03											
8	01	-02										
2	-21	00	-03									
28	27	-02	27	-09								
11	-08	-02	15	26	-05							
18	21	-07	21	-16	36	-07						
4	29	-01	33	03	43	03	28					
44	-01	-05	15	12	10	12	-05	07				
6	-10	-11	03	09	02	04	01	-04	45			
47	06	-05	19	13	16	09	00	12	36	17		
38	20	02	04	-23	19	-13	14	17	-05	-11	02	

*Decimal points have been omitted

Appendix D14

Unrotated Factor Loadings of 12 Statements of AS-SRAP. N=344*

Statement Number	1	2
A(SRAP)		
42	-061	-156
38	407	-329
18	602	-147
28	746	022
4	705	065
13	528	-255
B(SRAP)		
47	286	552
8	508	288
6	018	602
44	170	717
11	-071	434
2	-241	499

*Decimal points have been omitted

Appendix D15

Intercorrelations among 12 Statements of AS-SNrAP. N=344

Statement Number	Correlation Coefficient*											
	13	15	4	57	59	34	12	26	2	39	6	9
13												
15	-27											
4	37	-10										
57	53	-25	41									
59	40	-13	15	36								
34	39	-19	22	41	52							
12	-11	27	11	-02	-12	-07						
26	17	04	34	21	14	29	10					
2	-05	17	23	02	-15	-04	45	26				
39	35	-10	19	50	62	43	-03	16	-03			
6	04	08	04	02	00	-03	14	11	10	-30		
9	09	02	28	20	04	13	22	21	39	08	23	

*Decimal points have been omitted

Appendix D16

Unrotated Factor Loadings of 12 Statements
of AS-SNrAP. N=344*

Statement Number	1	2
A(SNrAP)		
39	707	-135
4	549	365
57	766	007
13	715	-115
59	693	-262
34	709	-110
B(SNrAP)		
9	285	607
6	050	360
26	416	409
2	043	783
12	-063	683
15	-315	382

*Decimal points have been omitted