University of Alberta
Department of Civil Engineering

Structural Engineering Report No. 2

The Torsional Strength of Rectangular Reinforced Concrete Beams Subjected to Combined Loading

by

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and

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July, 1967

THE TORSIONAL STRENGTH OF RECTANGULAR REINFORCED CONCRETE BEAMS SUBJECTED TO COMBINED LOADING

An Investigation Conducted by
the Department of Civil Engineering at
the University of Alberta, in Cooperation
with the National Research Council
of Canada

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Preliminary Draft
Not for Publication

Report No. 2
July 1967

ABSTRACT

The primary objective of this investigation was to study the behavior of reinforced concrete beams subjected to combined loading with particular regard to their behavior at failure.

The experimental phase of this investigation consisted of testing 34 reinforced concrete beams. Twenty two beams were subjected to various combinations of bending and torsion and twelve were subjected to various combinations of bending, torsion and shear. All beams had a nominal cross section of 6" x 12" and a nominal concrete strength of 5000 psi. Both longitudinal and transverse reinforcement in various combinations was provided in all beams.

The testing equipment that was designed and fabricated for this investigation permitted independent application of the twisting and transverse loads. The ratio between the twisting moment and the bending moment could be changed at any time during a test.

All beams were tested to failure by applying the load in a series of increments. Each increment consisted of increasing to a predetermined level the transverse load or the twisting load or both, depending on the type of test. In the cases where both types of load were applied in the same increment, the transverse load was applied first. Twenty nine beams were subjected to loads such that for any one test the ratio of twisting moment to bending moment at the end of each increment

was a constant. For the other five beams this ratio was different at the end of each increment and four of these beams were subjected to various sequences of load.

Based on the observations made in the experimental phase of this investigation three idealized failure surfaces have been presented, two of which were first suggested by Lessig. Equations for the ultimate torsional strength of a beam based on each of the three failure surfaces have been developed and their method of solution has been presented. These equations have also been simplified and an interaction diagram consisting of three straight lines has been presented along with the applicable equations.

The correlation between the experimental results of 109 beams and the theoretical results obtained from both the simplified analysis and the more comprehensive analysis has been given. Of these beams, 34 are the beams tested in this investigation and 75 are beams that have been tested by other investigators.

The observations and test results indicate that reinforced concrete beams that are subjected to bending, torsion and moderate amounts of transverse shear can fail by three different modes. These modes of failure are characterized by the formation of a hinge adjacent to one face of the beam and yielding of the reinforcement adjacent to the face opposite to the hinge. The modes of failure predicted by the analysis agree with the observed modes of failure.

ACKNOWLEDGEMENTS

This investigation was made possible through the financial assistance provided by the National Research Council of Canada. The testing facilities at the Structural Laboratory together with technical assistance were provided by the University of Alberta.

This investigation was directed by Dr. J. Warwaruk, Associate Professor in the Department of Civil Engineering and is based on a doctoral dissertation written by A.E. McMullen.

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LIST OF SYMBOLS

<u>Dimensions and Areas</u>

b = beam width

h = beam depth

L_1 = length of compression zone for a mode 1 failure

L_2 = length of compression zone for a mode 2 failure

L_3 = length of compression zone for a mode 3 failure

c_1 = length of mode 1 failure surface projected on the longitudinal axis of the beam

c_2 = length of mode 2 failure surface projected on the longitudinal axis of the beam

c_3 = length of mode 3 failure surface projected on the longitudinal axis of the beam

c = c_1 , c_2 or c_3 for a predicted mode 1, 2 or 3 failure respectively

c_{m1} = maximum length of mode 1 failure surface projected on the longitudinal axis of the beam

c_{m2} = maximum length of mode 2 failure surface projected on the longitudinal axis of the beam

c_{m3} = maximum length of mode 3 failure surface projected on the longitudinal axis of the beam

c_m = c_{m1} , c_{m2} or c_{m3} for a predicted mode 1, 2 or 3 failure respectively

S = spacing of ties

LIST OF SYMBOLS (continued)

a_1 = distance from bottom face of beam to centroid of A_{s1}

a_2 = distance from vertical face of beam to centroid of A_{s2}

a_3 = distance from top face of the beam to centroid of A_{s3}

a_{1t} = distance from bottom face of beam to centroid of bottom leg of tie

a_{2t} = distance from vertical face of beam to centroid of vertical leg of tie

a_{3t} = distance from top face of beam to centroid of top leg of tie

x_1 = distance from top face of beam to the neutral axis

x_2 = distance from vertical face of beam on which the compression zone is located to the neutral axis

x_3 = distance from bottom face of beam to the neutral axis

x = x_1 , x_2 or x_3 for a predicted mode 1, 2 or 3 failure respectively

z_1 = $h_{01} - k_1x_1/2$

z_2 = $b_{02} - k_1x_2/2$

z_3 = $h_{03} - k_1x_3/2$

h_{01} = $h - a_1$

h_{03} = $h - a_3$

b_{02} = $b - a_2$

y_1 = $k_{01}(h - a_{1t} - k_1x_1/2) + (1 - k_{01})\ (1 - k_{01} - 4a_{2t}/b)\ b/4$

y_2 = $k_{02}(b - a_{2t} - k_1x_2/2) + (1 - k_{02})\ (1 - k_{02} - 4a_{1t}/h)h/4$

y_3 = $k_{03}(h - a_{3t} - k_1x_3/2) + (1 - k_{03})\ (1 - k_{03} - 4a_{2t}/b)\ b/4$

α_1 = $A_{s1}\ S/a_v$

LIST OF SYMBOLS (continued)

α_2 = $A_{s2}\ S/a_v$

α_3 = $A_{s3}\ S/a_v$

β_1 = $2h + b$

β_2 = $2b + h$

a_v = area of one leg of a tie

A_{s1} = area of longitudinal reinforcement at the bottom face of the beam

A_{s2} = area of longitudinal reinforcement at the vertical face of the beam opposite to the face on which the compression zone is located

A_{s3} = area of longitudinal reinforcement at the top face of the beam

Moments and Forces

M_{t1} = theoretical ultimate twisting moment of a beam for a mode 1 failure

M_{t2} = theoretical ultimate twisting moment of a beam for a mode 2 failure

M_{t3} = theoretical ultimate twisting moment of a beam for a mode 3 failure

M_t = theoretical ultimate twisting moment of a beam

M_{b1} = theoretical ultimate bending moment of a beam for a mode 1 failure

M_{b2} = theoretical ultimate bending moment of a beam for a mode 2 failure

M_{b3} = theoretical ultimate bending moment of a beam for a mode 3 failure

M_{tt} = test value of ultimate twisting moment of a beam

M_{bt} = test value of ultimate bending moment of a beam

M_{tr} = theoretical ultimate twisting moment of an over reinforced beam

M_{t1a} = theoretical ultimate twisting moment of a beam for a mode 1 failure at ϕ equal to ∞

LIST OF SYMBOLS (continued)

M_{t2a} = theoretical ultimate twisting moment of a beam for a mode 2 failure at $\emptyset$ equal to ∞

M_{t3a} = theoretical ultimate twisting moment of a beam for a mode 3 failure at $\emptyset$ equal to ∞

M_{ta} = the smallest of the values M_{t1a} , M_{t2a} and M_{t3a}

M_{t1b} = theoretical ultimate twisting moment of a beam for a mode 1 failure at $\emptyset$ equal to 1.0

M_{t2b} = theoretical ultimate twisting moment of a beam for a mode 2 failure at $\emptyset$ equal to 1.0

M_{t3b} = theoretical ultimate twisting moment of a beam for a mode 3 failure at $\emptyset$ equal to 1.0

M_{tb} = the smallest of the values M_{t1b} , M_{t2b} and M_{t3b}

M_{bb} = theoretical ultimate bending moment of a beam at $\emptyset$ equal to 1.0

M_{t1c} = theoretical ultimate twisting moment of a beam for a mode 1 failure at $\emptyset$ equal to 0.25

M_{t2c} = theoretical ultimate twisting moment of a beam for a mode 2 failure at $\emptyset$ equal to 0.25

M_{t3c} = theoretical ultimate twisting moment of a beam for a mode 3 failure at $\emptyset$ equal to 0.25

M_{tc} = the smallest of the values M_{t1c} , M_{t2c} and M_{t3c}

M_{bc} = theoretical ultimate bending moment of a beam at $\emptyset$ equal to 0.25

M_{bu} = theoretical ultimate bending moment of a beam subjected to pure bending

V = theoretical ultimate transverse shear force of a beam

V_t = test value of the ultimate transverse shear force of a beam

LIST OF SYMBOLS (continued)

V_u = ultimate transverse shear force of a beam subjected only to bending and transverse shear as defined in ACI Standard 318-63

Stresses

f_{yt} = yield stress of ties

f_{y1} = yield stress of the longitudinal reinforcement at the bottom face of the beam

f_{y2} = yield stress of the longitudinal reinforcement at the vertical face of the beam opposite to the face on which the compression zone is located

f_{y3} = yield stress of the longitudinal reinforcement at the top face of the beam

f'_c = compressive strength of concrete determined by tests on 6 x 12 inch concrete cylinders

Dimensionless Factors

k_1 = a coefficient used to determine the depth of the equivalent compression zone

k_e = a coefficient related to the effective length of the compression zone and the strength of the concrete in the compression zone

k_{01} = $b/(2h + b)$

k_{02} = $h/(2b + h)$

k_{03} = k_{01}

k_{y1} = f_{y1}/f_{yt}

LIST OF SYMBOLS (continued)

k_{y2} = f_{y2}/f_{yt}

k_{y3} = f_{y3}/f_{yt}

p_1 = $f_{yt}\ a_v\ h/f_{y1}\ A_{s1}\ S$

p_2 = $f_{yt}\ a_v\ b/f_{y2}\ A_{s2}\ S$

p_3 = $f_{yt}\ a_v\ h/f_{y3}\ A_{s3}\ S$

ϕ = ratio of torsional moment to bending moment

δ = $\frac{b}{2}\left[\frac{\text{transverse shear force}}{\text{twisting moment}}\right]$

m_1 = $p_1\ b/h$

m_2 = $p_2\ h/b$

m_3 = $p_3\ b/h$

m_{01} = optimum value of m_1

m_{02} = optimum value of m_2

m_{03} = optimum value of m_3

m = m_1 , m_2 or m_3 for a predicted mode 1, 2 or 3 failure respectively

m_o = m_{01} , m_{02} or m_{03} for a predicted mode 1, 2 or 3 failure respectively

Miscellaneous

ψ = ratio of transverse shear force to bending moment

CHAPTER I

INTRODUCTION

1-1 General Remarks

The members of a reinforced concrete structure are subjected to transverse shear forces, axial forces, bending moments and torsional moments in various combinations. The behavior and ultimate strength of reinforced concrete members subjected to various combinations of transverse shear force, axial force and bending moment have been investigated rather extensively. However, the behavior and ultimate strength of reinforced concrete members subjected to a torsional moment combined with transverse shear, axial load or bending moment have received less attention. The reasons for this are:

(a) Torsion is usually a secondary effect in concrete structures

(b) It is usually possible to arrange the members in a structure so that they are subjected to only very small twisting moments

(c) Torsion tests require special testing equipment which is not readily available.

Torsion is seldom taken into consideration in the design of concrete structures. Current practice assumes that if the members are arranged so that they are subjected to only very small twisting moments the factor of safety will adequately provide for the effects of torsion.

However, since knowledge of the behavior of reinforced concrete has increased considerably in recent years and since design procedures are continually being refined it is essential that the effects of torsion be understood so that provision can be made for them in design. In addition, having to arrange the members so that they are subjected to only small twisting moments imposes an undesirable restriction on the designer.

Nylander (1945) tested beams that were reinforced with longitudinal reinforcement only since he considered this to be the type of beam most often encountered in actual structures. He found that for low stress in the reinforcement, the bending moment exerted a favorable influence on the torsional strength of a beam. However, since current practice is to provide most reinforced concrete beams with at least nominal transverse reinforcement, it is doubtful if Nylander's beams can be considered typical.

Cowan (1953) found that subjecting a beam to a small bending moment increases its torsional strength. He suggested that the torsional strength of a beam was equal to the sum of the torsional strength of the plain concrete section and the contribution of the reinforcement. However, since his equations are based on elastic theory they can not be used to determine the ultimate strength of a beam.

Lessig (1959) suggested two possible modes of failure and derived equations for the torsional strength of a beam. These modes of failure involve the formation of an inclined hinge adjacent to one of the faces of the beam. In the equations the torsional strength of the plain concrete section is not considered. Furthermore, the equations indicate that the addition of a bending moment can only reduce the torsional strength of a beam.

Yudin (1962) and Gesund, Schuette, Buchanan and Gray (1964) developed equations for the torsional strength of a beam that are based on the formation of a hinge parallel to the axis of the beam. These equations indicate that addition of a bending moment can only reduce the torsional strength of a beam.

Pandit (1965), similarly to Cowan (1953), assumed that the torsional strength of a beam was equal to the sum of the torsional strength of the plain concrete section and the contribution of the reinforcement. He stated that the beams he tested showed no evidence of the formation of hinges. The beams tested by Pandit were first subjected to bending and were then twisted to failure.

Goode and Helmy (1965) performed tests in which the beams were first twisted and then bent to failure. They stated that these beams sustained an ultimate bending moment at least equal to the ultimate bending moment that would be expected in pure flexure.

It can easily be seen that there is disagreement with regard to the behavior of beams subjected to combined loading. Before a design procedure can be formulated it is essential to obtain a better understanding of this behavior. It is hoped that this investigation will make a contribution to that end.

1-2 Object

The objectives of this investigation are:

(1) To observe the behavior of reinforced concrete beams subjected to combined loading with particular regard to their behavior at failure.

(2) To study the effect of bending moment and transverse shear on the torsional strength of beams.

(3) To determine the effect of sequence of loading on the ultimate strength of beams.

(4) To study the effect of various combinations of longitudinal and transverse reinforcement on the behavior and strength of beams.

(5) To develop a method for predicting the strength of rectangular, reinforced concrete beams subjected to combined loading.

1-3 Scope

In this investigation a total of 34 beams were subjected to various combinations of load and sequences of load. All beams had a nominal cross section of 6 inches by 12 inches, a nominal concrete strength of 5000 psi and were reinforced with both longitudinal and transverse reinforcement.

Three beams were subjected to pure torsion and nineteen beams were subjected to various combinations of bending and torsion. These beams were divided into four groups. Twelve beams divided into three groups were subjected to various combinations of bending, torsion and shear.

All beams were tested to failure by applying the load in a series of increments. Each increment consisted of increasing to a predetermined level the transverse load or the twisting load or both, depending on the type of test. In those cases where both types of load were applied in the same increment, the transverse load was applied first. For 29 beams the ratio of twisting moment to bending moment at the end of each increment was a constant in any one test. For the other five beams this ratio was different at the end of each increment and four of these beams were subjected to various sequences of load.

An analysis is presented which is based on observations made during testing. This analysis was performed with the aid of an IBM 7040 computer and the results obtained are compared with the experimental results. In addition, the experimental results of 75 beams tested by other investigators are compared with the theoretical results obtained from the analysis. The analysis is then simplified and the results from the simplified analysis are compared to the experimental results.

CHAPTER II

REVIEW OF PREVIOUS RESEARCH

2-1 Introduction

Although torsion in reinforced concrete beams has been investigated for many years, most investigations dealing with combined bending and torsion have been performed within the last decade. In general, the behavior and ultimate strength of beams subjected to pure bending is well understood but the behavior and ultimate strength of beams subjected to either pure torsion or combined bending and torsion is not. Since both the limiting cases of pure bending and pure torsion have been adequately reviewed previously, they will not be reviewed here. This chapter presents a review of previous research in which the specimens were subjected to various combinations of bending, torsion and transverse shear.

2-2 Previous Research

Nylander (1945) tested forty four beams of square, rectangular, and T shaped cross section. Thirty four were tested in combined bending and torsion and ten were tested in combined bending, torsion and shear. None of the beams was provided with transverse reinforcement. He stated that if the stress in the reinforcement due to bending was low, the bending moment exerted a favorable effect on the torsional strength. In addition, he concluded that the ultimate capacity of a member subjected to bending, torsion and shear could be obtained by equating the sum of the torsional and transverse shearing stresses to the ultimate tensile strength of the concrete.

Cowan (1953) proposed equations to predict the ultimate strength of reinforced concrete beams under combined loading and classified all failures as either primary torsion or primary bending failures. He stated that primary bending failure was governed by crushing of the compressed concrete whereas primary torsion failure was governed by the development of 45° helical cracks. The failure theory that he used is a combination of Rankine's maximum principal stress theory and Coulomb's internal friction theory. His equations indicate that the addition of bending increases the torsional strength whereas the addition of torsion reduces the bending capacity only slightly. Hence, he recommended designing a reinforced concrete section independently for bending and torsion without reduction of the maximum permissible stresses. This reduces the shape of the bending-torsion interaction diagram to a rectangle. Furthermore, he recommended adding the torsional resistance due to the reinforcement to that of a plain concrete section in order to obtain the torsional resisting moment of the reinforced concrete section.

Cowan and Armstrong (1955) tested seven rectangular reinforced concrete beams. The results of these tests were used to substantiate the theory outlined above.

Lessig (1959) defined two possible modes of failure for rectangular reinforced concrete beams subjected to combined loading. These failure modes are based on the assumption that at failure a plastic hinge that is inclined to the longitudinal axis of the beam forms on either the top or the side of the beam. By making a number of assumptions and using two equations of equilibrium, one for moments and the other for forces, an equation for the ultimate torsional strength of a beam subjected to combined loading is derived for each of the modes of failure. These equations indicate that a bending moment can only reduce the ultimate torsional moment of a beam. A significant assumption that is made is that at failure the tensile strength of the concrete does not contribute to the torsional moment. This is in direct contrast with Cowan's assumption that the torsional resisting moment of a reinforced concrete beam is composed of the torsional resistance of the plain concrete section plus the contribution of the reinforcement.

Chinenkov (1959) tested a total of 36 beams in combined bending and torsion. The results of these tests were used to substantiate the theory proposed by Lessig. Twenty three of these beams were reinforced with two plane welded skeletons of reinforcement placed adjacent to the vertical faces of the beam. Failure of these beams was generally by splitting of the concrete on the unreinforced sides of the beam. The remaining 13 beams were reinforced with both longitudinal bars and closed ties. Chinenkov stated that at failure these beams developed a plastic hinge adjacent to their top face. It was noted that the appearance of cracks on the flexural compression face did not lead to failure; failure occurred only when the flexural tension reinforcement yielded. The ratio between the experimental and the theoretical ultimate twisting moments for these beams was generally greater than 1.0. This was interpreted as evidence that the uncracked portion of concrete was resisting tension. However, the theoretical ultimate twisting moments were computed by simplified versions of Lessig's original equations. The effect of the simplifications on the theoretical ultimate twisting moments was not stated.

All thirteen of the beams with closed ties were tested at a low ratio of twisting moment to bending moment. This ratio varied between 0.1 and 0.4. Consequently, no information was obtained regarding the effect of small bending moments on the torsional strength of the beams.

Lyalin (1959) tested thirty six beams in combined bending, torsion and shear and compared the experimental failure moments to the theoretical failure moments computed using Lessig's simplified equations. The ratio of twisting moment to bending moment at which thirty of these beams were tested was between 0.1 and 0.5. Of the remaining six, two were tested at a ratio of 1.0 and four were tested in pure torsion.

Lyalin concluded from the test results that full redistribution of stress between longitudinal and transverse reinforcement is obtained over a large range of the ratio between these two types of reinforcement. He also concluded that the amount of reinforcement necessary to cause a compression failure in a beam subjected to bending and torsion is less than the amount required in a beam that is subjected to pure flexure and that this amount varies with the ratio of twisting moment to bending moment.

Lessig (1961) reported tests on forty two beams. Six of these were tested in pure torsion, six were tested in combined bending and torsion, and the remainder were tested in combined bending, torsion and shear. Except for the beams tested in pure torsion, they were all tested at a ratio of twisting moment to bending moment of 0.8 or less. On the basis of the test results, equations were proposed for determining the minimum amount of reinforcement necessary to cause a compression failure. In addition, limits on the permissible variation of the ratio between the longitudinal and the transverse reinforcement were proposed.

Yudin (1962) stated that the equations proposed by Lessig indicated that a deficiency in longitudinal reinforcement could be compensated for by an excess of transverse reinforcement and vice versa. He claimed that the most rational procedure was not to take moments about an inclined axis but to use equilibrium of internal and external moments about two axes to establish two equations. One of these axes should be parallel to the longitudinal axis of the beam and the other should be perpendicular to it. Furthermore, he claimed that three equilibrium equations instead of two should be used for each possible mode of failure, that the failure surface could be defined with sufficient accuracy by assuming that on three faces of the beam it makes an angle of 45° with the longitudinal axis, and that since the depth of the compression zone was small it could be neglected. On the basis of these premises he derived equations for determining the ultimate strength of a reinforced concrete beam subjected to combined loading.

Yudin (1964) reported the results of tests on twenty two beams. He claimed that the results substantiated the validity of his equations. These beams were tested at a ratio of twisting moment to bending moment that was between zero and 0.2.

Gesund and Boston (1964) reported results of tests on ten beams. Eight of these beams were square and two were rectangular. All beams were reinforced with longitudinal reinforcement only. Two beams were subjected to pure torsion whereas the remainder were subjected to combined bending and torsion. It was assumed that after cracking had defined a failure surface the torsional moment was resisted by the combined action of the uncracked portion of the concrete and the dowel action of the reinforcement. It was not found possible to write an expression for the first of these two quantities but an expression was derived for the torsional resistance due to the dowel action. The ultimate torsional moment computed using this expression was lower than the experimental moment in all cases. They concluded that beams subjected to bending and torsion may fail in one of several different torsional modes, or in bending and that if there is no transverse reinforcement, the dowel action of the longitudinal reinforcement is of paramount importance in resisting the torsion. They also stated that the bending resistance of a beam may be severely reduced by the torsion if the failure hinge forms on one side and thus greatly reduces the lever arm.

Gesund, Schuette, Buchanan and Gray (1964) reported tests on twelve beams subjected to combined bending and torsion. Eight of these beams were square, four were rectangular and all were provided with both longitudinal and transverse reinforcement. A theoretical failure surface was established utilizing a hinge that was located on the top surface of the beam and was parallel to the longitudinal axis of the beam. On the basis of this failure surface and some further assumptions, equations were derived for the ultimate torsional moment and the ultimate flexural moment of a reinforced concrete beam. Several of these equations are similar to Yudin's equations and indicate that a bending moment of any magnitude will not increase the torsional strength of a beam. Two equations for torsional resistance were proposed. The first was based on the assumption that the entire torsional resistance was provided by the tensile forces in the ties and the second on the assumption that the total torsional resistance was provided by the dowel action of the ties plus the dowel action of the longitudinal reinforcement. The ultimate torque was taken as the greater of the two values given by these two equations.

The investigators classed all failures as either predominantly bending or predominantly torsion. They concluded that even a small amount of transverse reinforcement was sufficient to produce a bending rather than a torsional failure for a ratio of twisting moment to bending moment equal to or less than 0.5. Other conclusions stated were that the transverse reinforcement transforms torque on a reinforced concrete beam into additional bending moment, that if sufficient transverse reinforcement is provided to prevent torsional failure, the stresses in this reinforcement have no effect on the strength of the beam, and that the dowel action of the longitudinal reinforcement frequently provides greater torsional resistance than the transverse reinforcement.

Collins, Walsh and Hall (1965) stated that the results of a series of beams that they tested indicated that a bending moment could increase the torsional strength of some beams. They reported that one beam which exhibited this increased torsional strength failed by rotating about a hinge that was inclined to the axis of the beam and was located on the bottom face.

Goode and Helmy (1965) stated that in their opinion the contribution of the concrete to the torsional resistance could not be ignored. They also stated that beams which they tested by first applying a twisting moment and then applying a bending moment until failure occurred exhibited an ultimate bending moment that was at least as great as the ultimate bending moment of the beam in pure flexure.

Evans and Sarkar (1965) reported tests on eighteen hollow reinforced concrete beams tested in combined bending and torsion. The failure surface that they used is defined by cracks on the bottom and the two vertical faces of the beam. The angle that the crack on the bottom face of the beam makes with the longitudinal axis is determined by the direction of the maximum principal stress. This crack is assumed to continue up the vertical faces of the beam at the same angle to the longitudinal axis for a vertical distance equal to 0.6 times the beam depth. From this level to the top surface of the beam the cracks are assumed to be at 45° to the longitudinal axis of the beam. The failure surface is defined on the top face of the beam by a rectangular compression zone making an angle of 45° with the longitudinal axis. The failure surface is discontinuous at the top corners of the beam. An equation for the ultimate torsional moment is derived by using equilibrium of internal and external moments about the "compression fulcrum" which is located at mid depth of the compression zone.

Pandit (1965) reported tests on thirty six plain and reinforced rectangular concrete beams. In these tests the bending load was applied first and then twisting moment was applied until failure occurred. Nine of the beams were plain concrete and twenty seven were reinforced with both longitudinal and transverse reinforcement. One of the plain concrete beams was tested in pure bending, two in pure torsion, three in bending and torsion and three in bending, torsion and shear. Seven of the reinforced concrete beams were tested in pure torsion, one in pure bending, ten in combined bending and torsion and nine in combined bending, torsion and shear.

All failures were classified as either cleavage or hybrid and it is stated that there was no evidence of the formation of plastic hinges on the top or the sides of the beams. The test results indicated that a small bending moment could in some cases increase the torsional strength of the member.

An analysis for the ultimate strength of reinforced concrete beams subjected to combined loading was proposed. In this analysis it was assumed that the torsional strength of a reinforced concrete beam was the sum of the torsional strength of the plain concrete beam and the contribution due to the reinforcement. The torsional strength of the plain concrete beam was calculated by using a modification of the sand heap analogy which considered the beam to be composed of a region of compressed concrete and another region of uncompressed concrete. The unit torsional strength of the compressed concrete was defined by Cowan's theory of failure and the unit torsional strength of the uncompressed concrete was established empirically from the test results. It was assumed that the flexural compression was uniformly distributed over an area equal to one-fourth of the area of the cross section of the beam. It was also assumed that the splitting of the concrete on the top face of the beam caused by the twisting moment reduced the lever arm of the internal flexural moment. The lever arm was assumed to vary linearly with respect to bending moment over the entire range from pure flexure to pure torsion.

The number of ties participating in resisting the torsional moment was taken as the smallest number of ties intersected by a potential crack making an angle of 45^{o} with the axis of the beam. The ties intersected by this potential failure crack were assumed to yield at failure provided there was adequate top and bottom longitudinal reinforcement present to provide for the longitudinal component of the diagonal tension. The contribution of the reinforcement to the torsional strength was assumed to be governed by the yield of top, bottom or transverse reinforcement, whichever occurred first. When the beam was subjected to a bending moment in addition to a torsional moment the longitudinal reinforcement was required to be sufficient to resist both the bending moment and the longitudinal component of the diagonal tension. As a further contribution of the reinforcement to the torsional strength, the longitudinal bars were considered to exhibit dowel strength proportional to the plastic moments they could develop.

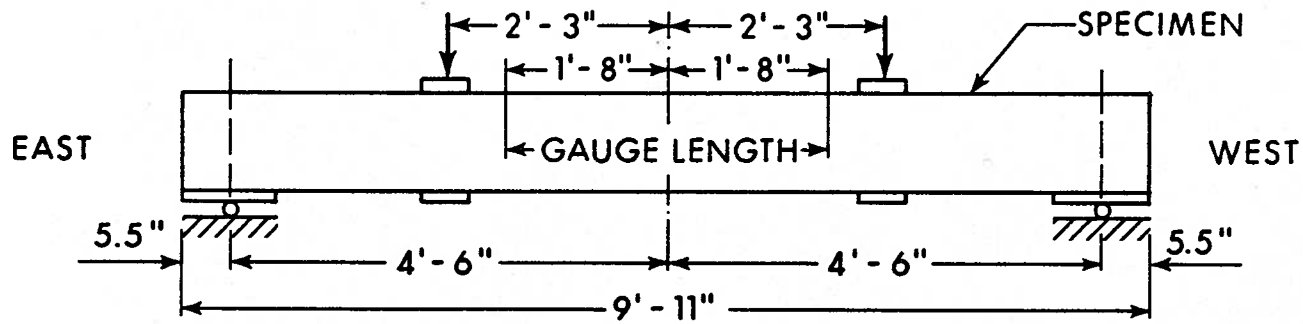
CHAPTER III

TEST SPECIMENS, EQUIPMENT AND PROCEDURE

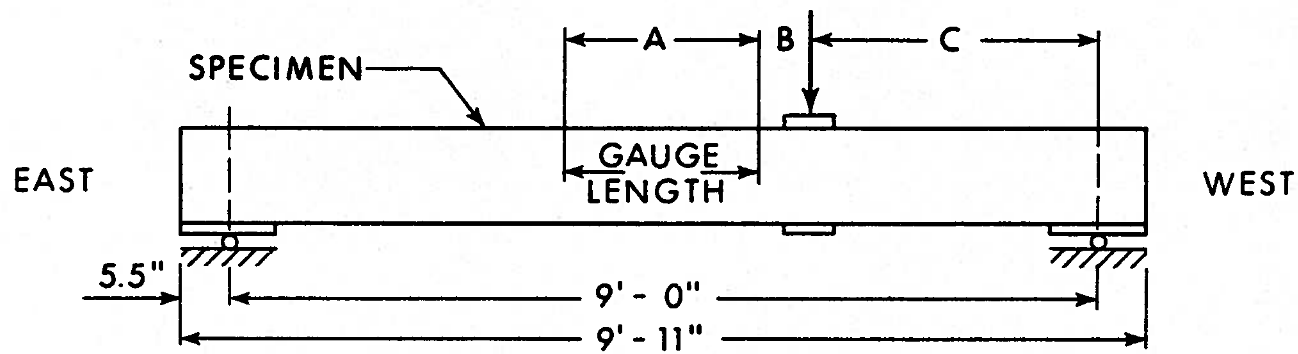
3-1 Test Specimens

The thirty-four beams reported in this investigation were all provided with both longitudinal and transverse reinforcement. They were divided into seven groups as outlined in TABLE 3-1. The beams of Groups 1 to 4 inclusive were designed to be tested in combined bending and torsion, whereas those of Groups 5 to 7 were designed to be tested in combined bending, torsion and shear. All beams had a nominal cross section of 6 x 12 inches and their overall length was 9'-11". The nominal compressive strength of the concrete was 5000 psi, the concrete mix being the same for all beams. The compressive and tensile strengths of the concrete were determined by means of compression and splitting tensile tests respectively on 6 x 12 inch control cylinders. These are given in TABLE 4-1 of CHAPTER IV. The method of fabrication of the specimens and the properties of the sand, coarse aggregate and reinforcing steel are given in Appendix A. The transverse reinforcement provided in these beams was obtained from the manufacturer in two batches which were designated as lot number one and two. All beams of Group 3 and Beams 1-1 to 1-5 of Group 1 were reinforced with transverse reinforcement from lot number 1, whereas all other beams were fabricated with transverse reinforcement from lot number two. Extra reinforcement, both longitudinal and transverse, was provided outside of the gauge length to ensure that failure occurred within the gauge length. The gauge lengths of the beams and their position in the beams are shown in FIGURE 3-1.

The details of the reinforcing steel are shown in FIGURE 3-2 and TABLE 3-1. The reinforcement for the beams of Group 1 was designed such that the top longitudinal reinforcement was less than the bottom. The reinforcement for the beams of Group 2 was identical to that of Group 1 except that the top longitudinal reinforcement was equal to the bottom. The beams of Group 3 were reinforced identically to those of Group C tested by Pandit (1965). Group 4 beams were reinforced such that for Beams 4-1 to 4-4 inclusive the longitudinal reinforcement remained constant whereas the spacing of the transverse reinforcement was progressively increased. The reinforcement was designed such that the test results of Beam 3-4 could be utilized in conjunction with these beams. Beams 4-5 and 4-6 were designed so they could be utilized in conjunction with Beam 1-5. Both the bottom longitudinal and the transverse reinforcement were progressively reduced from that of Beam 1-5. The reinforcement for all beams of Groups 5 and 6 was identical to that of Group 1. The beams of Group 7 were provided with larger bottom reinforcing bars than the beams of Groups 1, 5 and 6, and the spacing of the transverse reinforcement was increased. This was done to obtain beams that would be stronger in flexure but weaker in transverse shear.



(a) COMBINED BENDING AND TORSION



GROUP	BEAM NO.	A	B	C
5	5 - 1	2' - 2"	7"	2' - 10"
5	5 - 2 TO 5 - 4	2' - 2"	6"	2' - 10"
6	6 - 1 TO 6 - 4	2' - 2"	6"	5' - 8"
7	7 - 1 TO 7 - 4	2' - 2"	6"	5' - 8"

(b) COMBINED BENDING TORSION AND SHEAR

FIGURE 3 - 1 TEST ARRANGEMENT FOR COMBINED LOADING

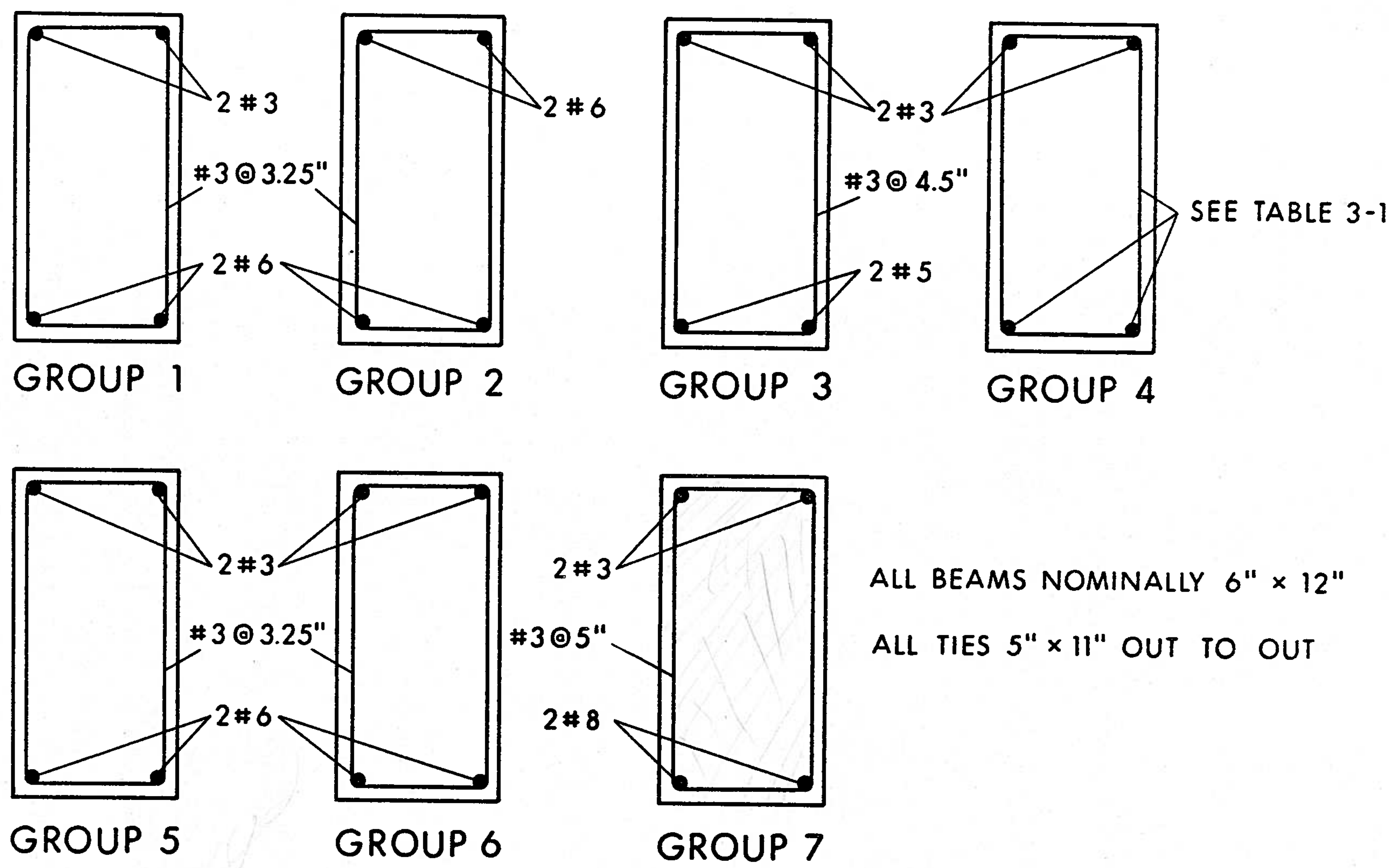


FIGURE 3 - 2 REINFORCING STEEL DETAILS

TABLE 3-1

REINFORCING STEEL DETAILS

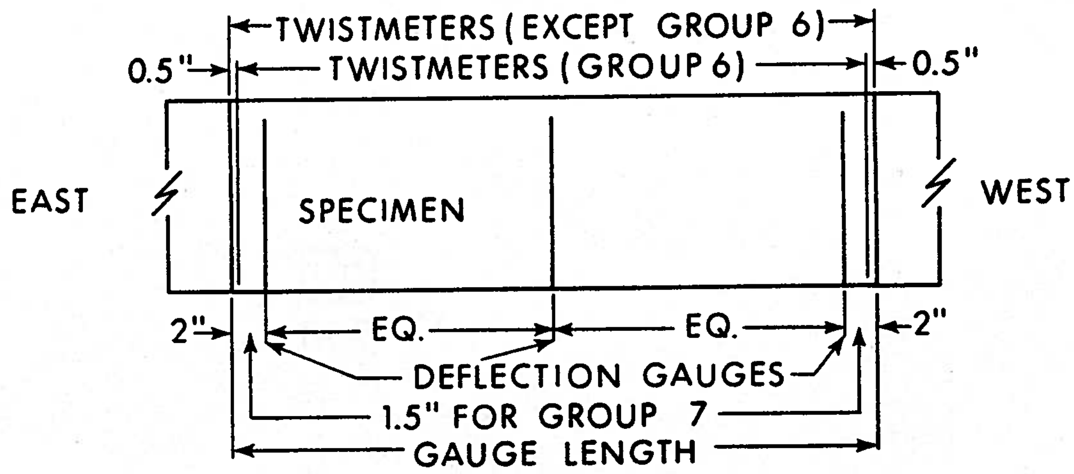
Group	Beam No.	Longitudinal Bars Top	Longitudinal Bars Bottom	Ties Size	Ties Spacing inches	Lot No. of Ties
1	1-1 to 1-5	2#3	2#6	#3	$3\frac{1}{4}$	1
	1-6	2#3	2#6	#3	$3\frac{1}{4}$	2
2	2-1 to 2-5	2#6	2#6	#3	$3\frac{1}{4}$	2
3	3-1 to 3-5	2#3	2#5	#3	$4\frac{1}{2}$	1
4	4-1	2#3	2#5	#3	3	2
	4-2	2#3	2#5	#3	6	2
	4-3	2#3	2#5	#3	$7\frac{1}{2}$	2
	4-4	2#3	2#5	#3	9	2
	4-5	2#3	2#5	#3	$4\frac{3}{8}$	2
	4-6	2#3	2#4	#3	7	2
5	5-1 to 5-4	2#3	2#6	#3	$3\frac{1}{4}$	2
6	6-1 to 6-4	2#3	2#6	#3	$3\frac{1}{4}$	2
7	7-1 to 7-4	2#3	2#8	#3	5	2

3-2 Instrumentation of Specimens

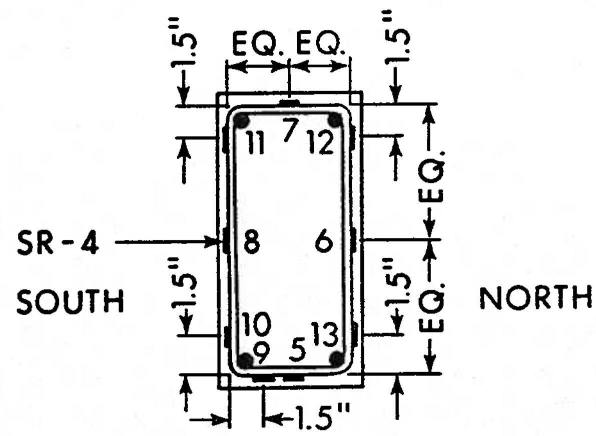
The specimens were instrumented to enable measurement of angle of twist, deflections and reinforcement strains.

(a) Angle of Twist

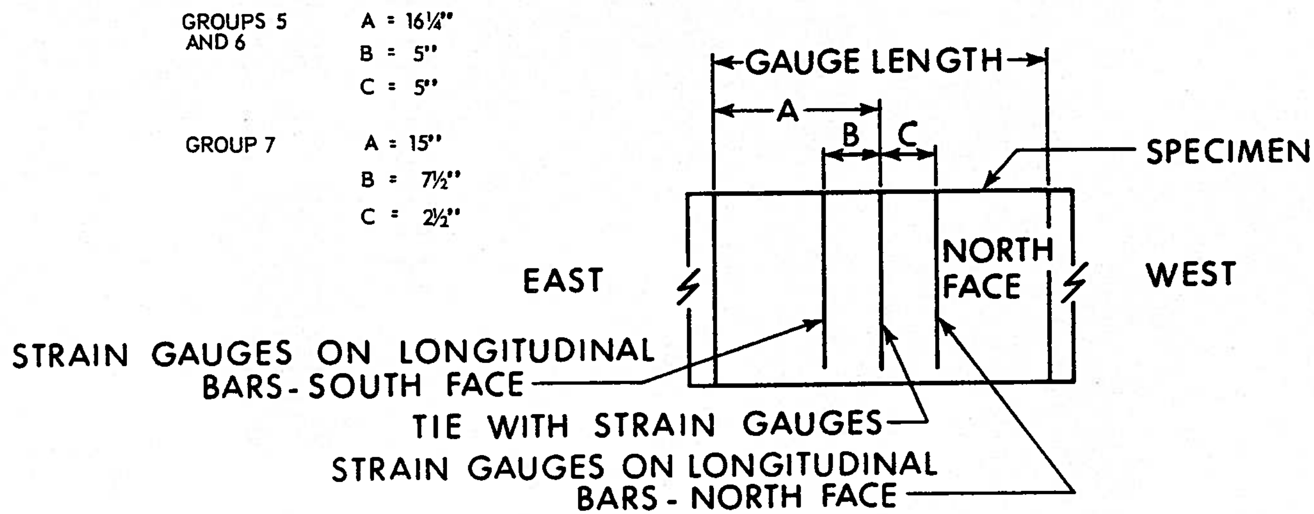
The twistmeters were designed to measure total angle of twist over the gauge length and were mounted at each end of the gauge length as shown in FIGURE 3-3(a). These twistmeters consisted of a level bubble mounted on a length of 1 1/2" x 1" channel which was fixed to a base at



(a) TWISTMETERS AND DEFLECTION GAUGES



(b) BEAM 3 - 2



(c) LOCATION OF STRAIN GAUGES

FIGURE 3 - 3 INSTRUMENTATION OF SPECIMENS

one end by a pin joint. The other end of the channel was supported by the needle of a micrometer screw that was mounted on the base. Close contact between the channel and the micrometer screw needle was maintained by springs. This assembly was then attached to the top of the beam by means of a clamping bracket. The distance between the pin joint and the micrometer screw needle was 15" and the smallest division on the micrometer was 0.001". The angle of twist through which each twistmeter rotated was computed from the difference in the readings of the micrometer screw. The total angle of twist over the gauge length was computed by taking the difference between the angles of twist obtained from each twistmeter.

(b) Deflections

The deflections of the beam were measured at three locations which are shown in FIGURE 3-3(a). The deflection gauges consisted of a clamping bracket mounted on the beam, from which a rod projected horizontally on each side of the beam and at mid depth. From these rods, scales were suspended at equal distances from the face of the beam. The smallest division on the scales was 0.01". The scales were read by using two precise levels located to each side of the beam. The deflection of the beam was obtained by averaging the readings obtained from each side of the gauge. The deflection obtained in this manner is the vertical deflection of the center of the cross section.

(c) Reinforcement Strains

SR-4 electrical resistance strain gauges were used to measure strains in the reinforcement. In general, one gauge was mounted on each of the longitudinal bars and four gauges were mounted on one of the ties. In order to facilitate reference to these gauges, the following numbering system was established.

<u>Longitudinal Bars</u>

<u>Location of Bar</u>	<u>No. of Gauge</u>
Top face, north side	1
Top face, south side	2
Bottom face, north side	3
Bottom face, south side	4

<u>Tie</u>

<u>Location of Gauge</u>	<u>No. of Gauge</u>
Bottom horizontal leg of tie	5
North vertical leg of tie	6
Top horizontal leg of tie	7
South vertical leg of tie	8

The locations of gauges 5 to 8 are illustrated in FIGURE 3-3(b).

Beams 3-2 and 3-5 were provided with additional gauges as discussed below.

(i) Beam 3-2: Five additional gauges were mounted on the tie. These gauges were numbered 9 through 13. The locations are detailed in FIGURE 3-3(b).

(ii) Beam 3-5: Twelve additional gauges were mounted on the longitudinal bars and on other ties. These gauges were numbered as follows:

Longitudinal Bars

Location of Bar	No. of Gauge
Top face, north side	9
Top face, south side	10,17
Bottom face, north side	11,18
Bottom face, south side	12

Tie

Location of Gauge	No. of Gauge
Bottom horizontal leg of tie	13,19
North vertical leg of tie	14,20
Top horizontal leg of tie	15
South vertical leg of tie	16

Gauges 13 to 16 were all mounted on the same tie, whereas gauges 19 and 20 were mounted on another tie.

For the beams of Groups 5, 6 and 7, the location of the gauges on the longitudinal reinforcement and the location of the instrumented tie are shown in FIGURE 3-3(c).

3-3 Test Equipment

The arrangement of the equipment used for testing the beams is illustrated by FIGURES 3-4, 3-5 and 3-6.

The equipment for applying the twisting moment was completely independent of the equipment used to apply the bending moment. The transverse load was applied by a 100 kip Amsler jack. This load was first transferred to a distributing beam that was supported at one end by a ball and at the other end by a roller. This ball and roller each rested on a roller assembly which in turn rested on a pipe collar. The specimen

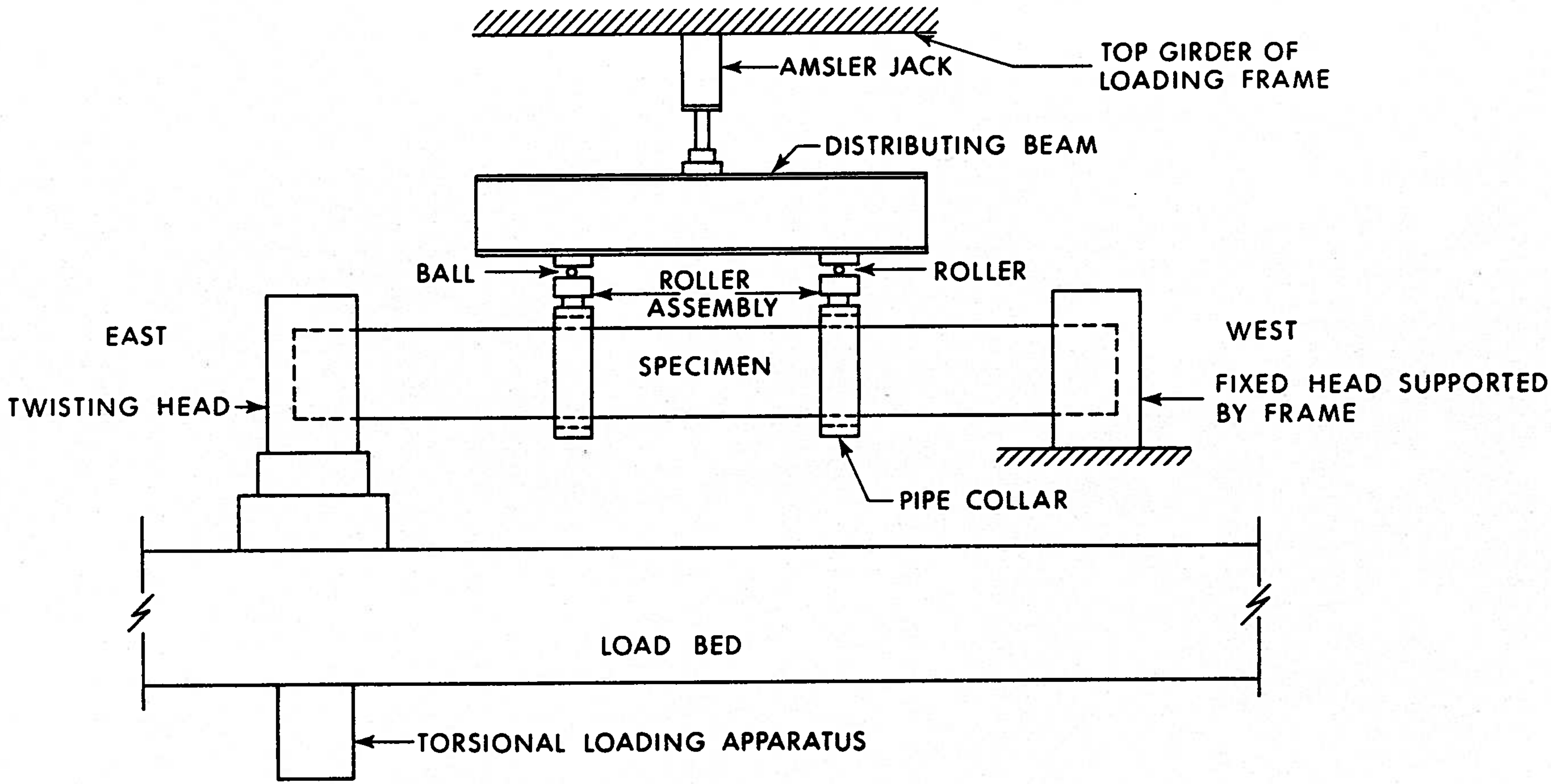


FIGURE 3 - 4 EQUIPMENT ARRANGEMENT FOR COMBINED BENDING AND TORSION

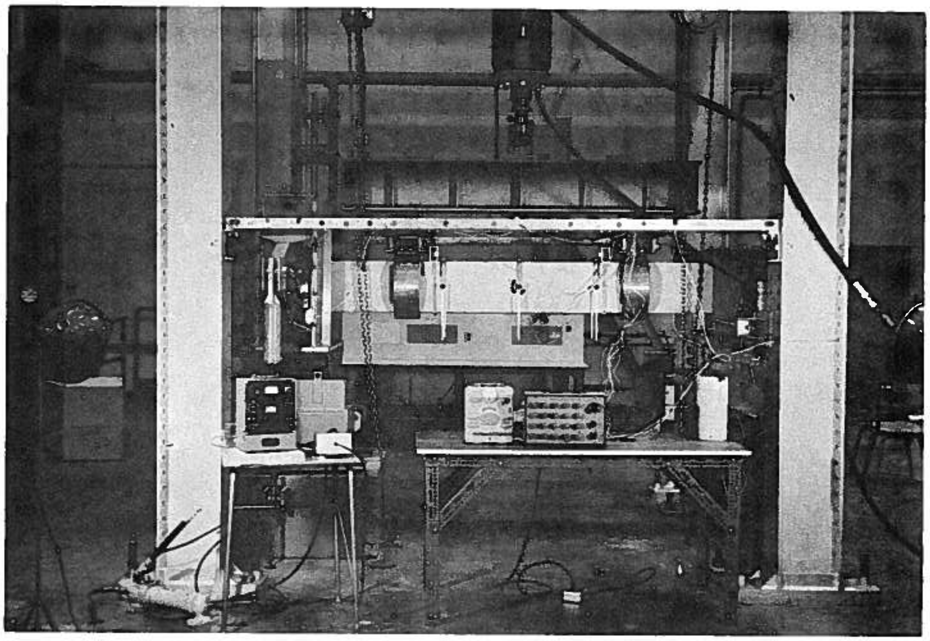


FIGURE 3 - 5 TEST SETUP FOR COMBINED BENDING AND TORSION

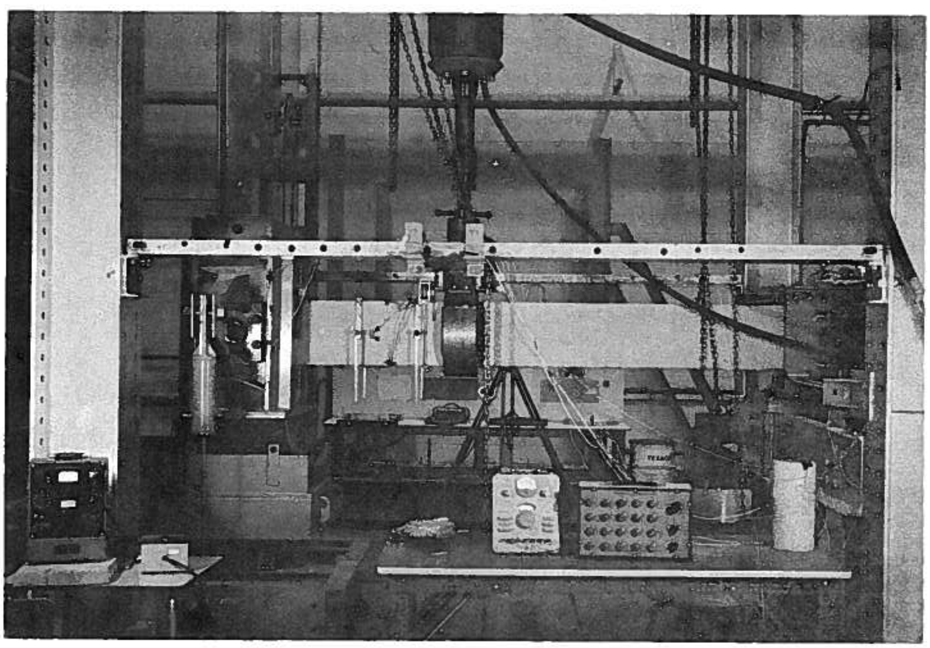


FIGURE 3 - 6 TEST SETUP FOR COMBINED BENDING, TORSION AND SHEAR

was clamped inside the pipe collars. They permitted the twisting moment to be transmitted freely from one end of the specimen to the other. For those specimens subjected to a single transverse load, the distributing beam was not used; the transverse load was transferred directly to the roller assembly. The position of the single transverse load is shown in FIGURE 3-1.

The east end of the specimen was supported by the twisting head through which the torsional moment was applied. The twisting head is illustrated by FIGURES 3-7, 3-8 and 3-9. It permitted the specimen to rotate about its longitudinal axis and about both a horizontal and vertical axis perpendicular to the axis of the specimen. The forces in the cables which were attached to the arms of the twisting head were measured by load cells and an indicator. These had been calibrated using a 200 kip universal testing machine.

The forces in the cables were produced by the torsional loading system illustrated in FIGURES 3-10 and 3-11. The cables attached to the torsion arms of the twisting head were connected to the ends of the cross head by adjustable connecting devices. Each end of the cross head was provided with a roller system to ensure that binding did not occur as the cross head moved within the guides provided. The cross head was loaded at its center by two 20 kip hydraulic jacks coupled in series. The hydraulic pressure was generated by a manual pump. The purpose of coupling the jacks in series was to increase the stroke to 16", which corresponds to a rotation of the twisting head of approximately 26°. This is slightly

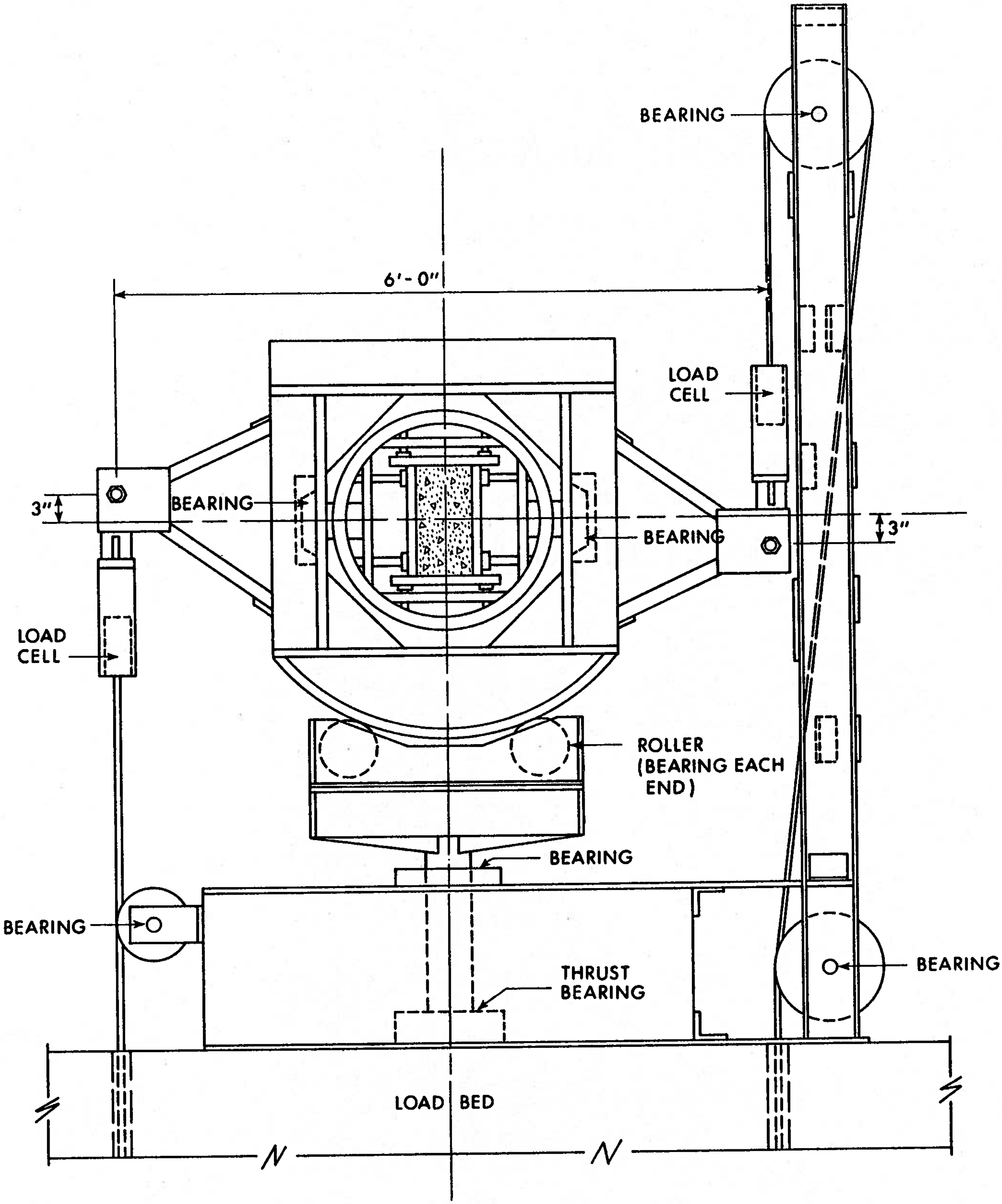


FIGURE 3-7 TWISTING HEAD

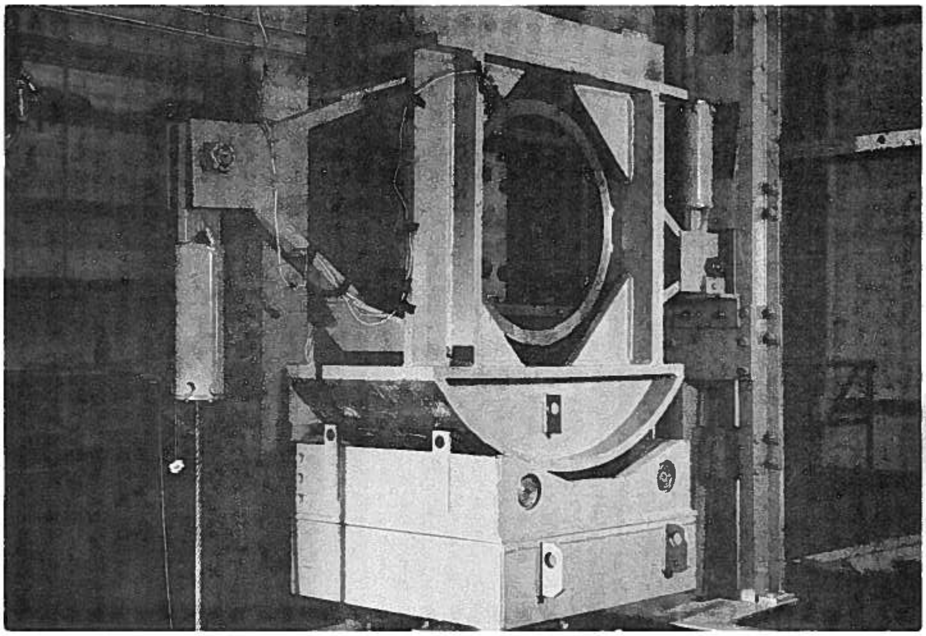


FIGURE 3 - 8 GENERAL VIEW OF TWISTING HEAD

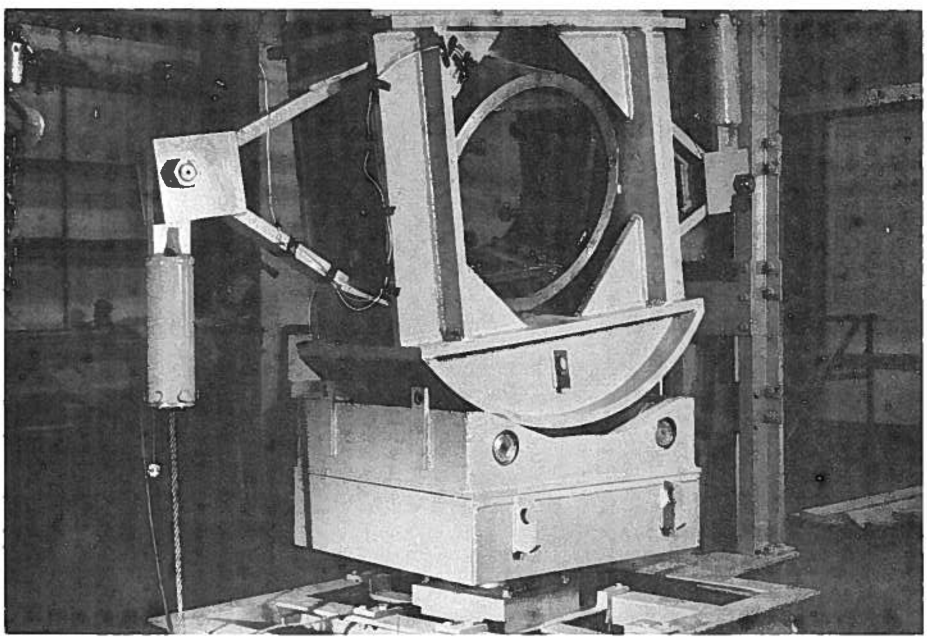


FIGURE 3 - 9 VIEW OF TWISTING HEAD SHOWING ROTATION ABOUT THREE AXES

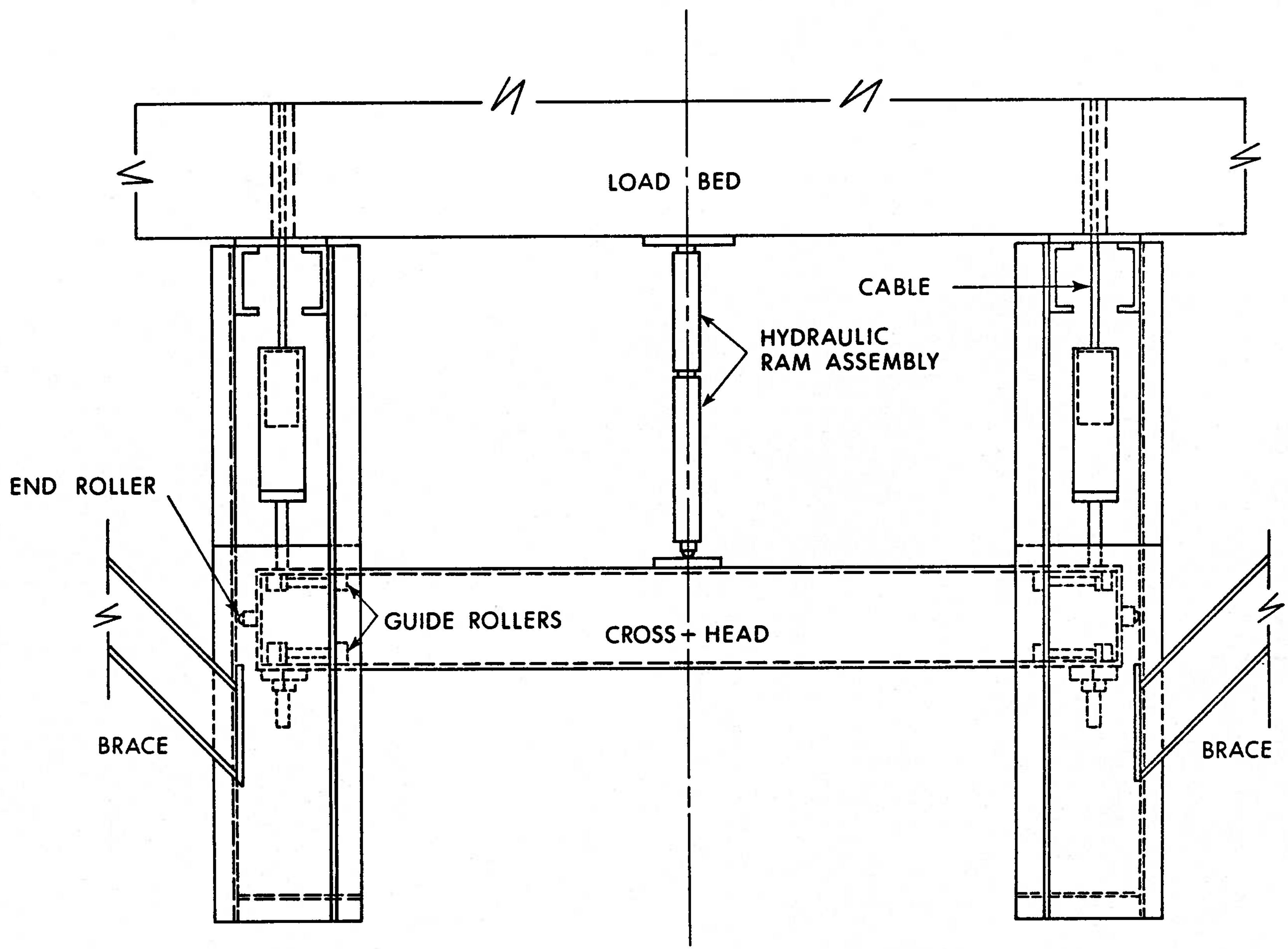


FIGURE 3 - 10 TORSIONAL LOADING EQUIPMENT

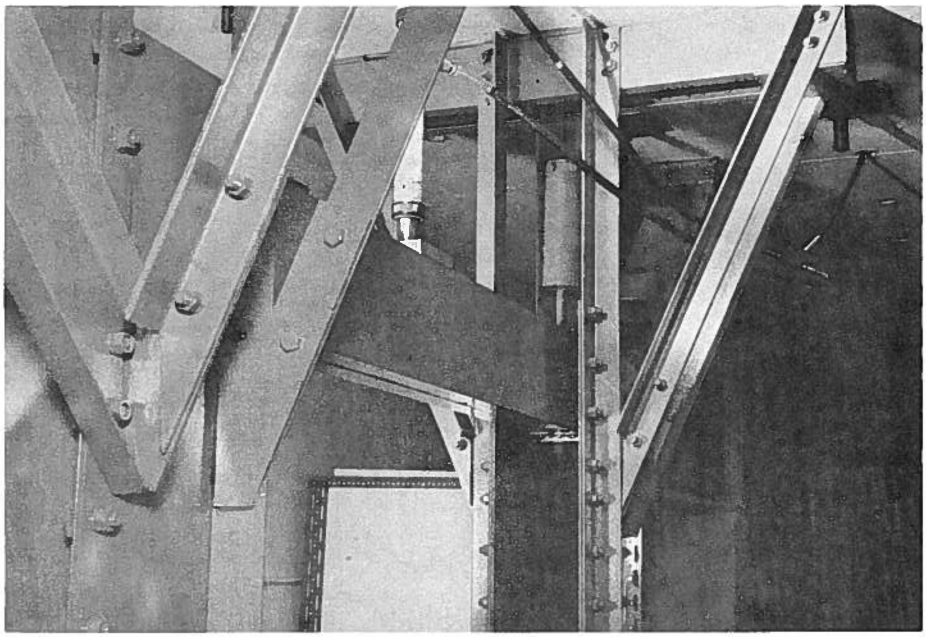


FIGURE 3 - 11 GENERAL VIEW OF TORSIONAL LOADING EQUIPMENT

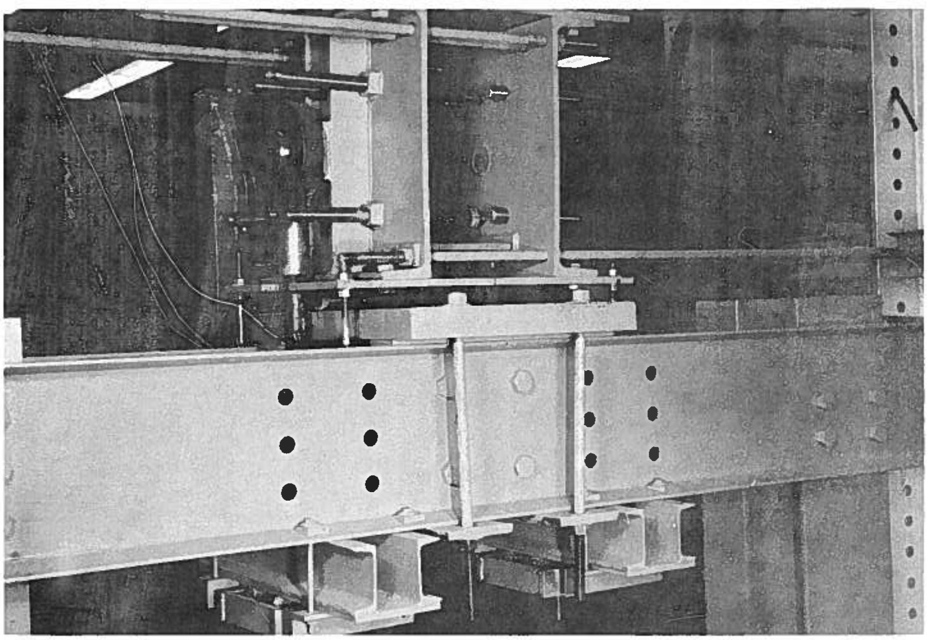


FIGURE 3 - 12 GENERAL VIEW OF FIXED HEAD

greater than the maximum rotation of which the twisting head is capable.

The west end of the specimen was supported by the fixed head which is illustrated by FIGURES 3-12 and 3-13. This fixed head permitted the specimen to translate in a direction parallel to its longitudinal axis, and to rotate about a horizontal axis perpendicular to its longitudinal axis.

3-4 Testing Procedure

Each specimen was tested to failure by applying the load in a series of increments. Each increment consisted of increasing to a predetermined level the transverse load or the twisting load or both, depending on the type of test. In those cases where both types of load were applied in the same increment, the transverse load was applied first. In the advanced stages of a test, the loads were generally applied in fractions of an increment. If failure did not occur during or after the application of a fraction of an increment, a further fraction was applied until failure occurred or a full increment was reached. After each full increment of load was reached, the transverse and twisting loads were held constant and recorded. Readings of the electrical resistance strain gauges, the twistmeters and the deflection gauges were also recorded and the crack patterns were marked. In all instances load application was continued well beyond the stage at which peak values were observed.

The first transverse load applied to a specimen was the load due to the testing equipment, i.e. the distributing beam and roller assemblies. Included in this load for the computation of the bending moment

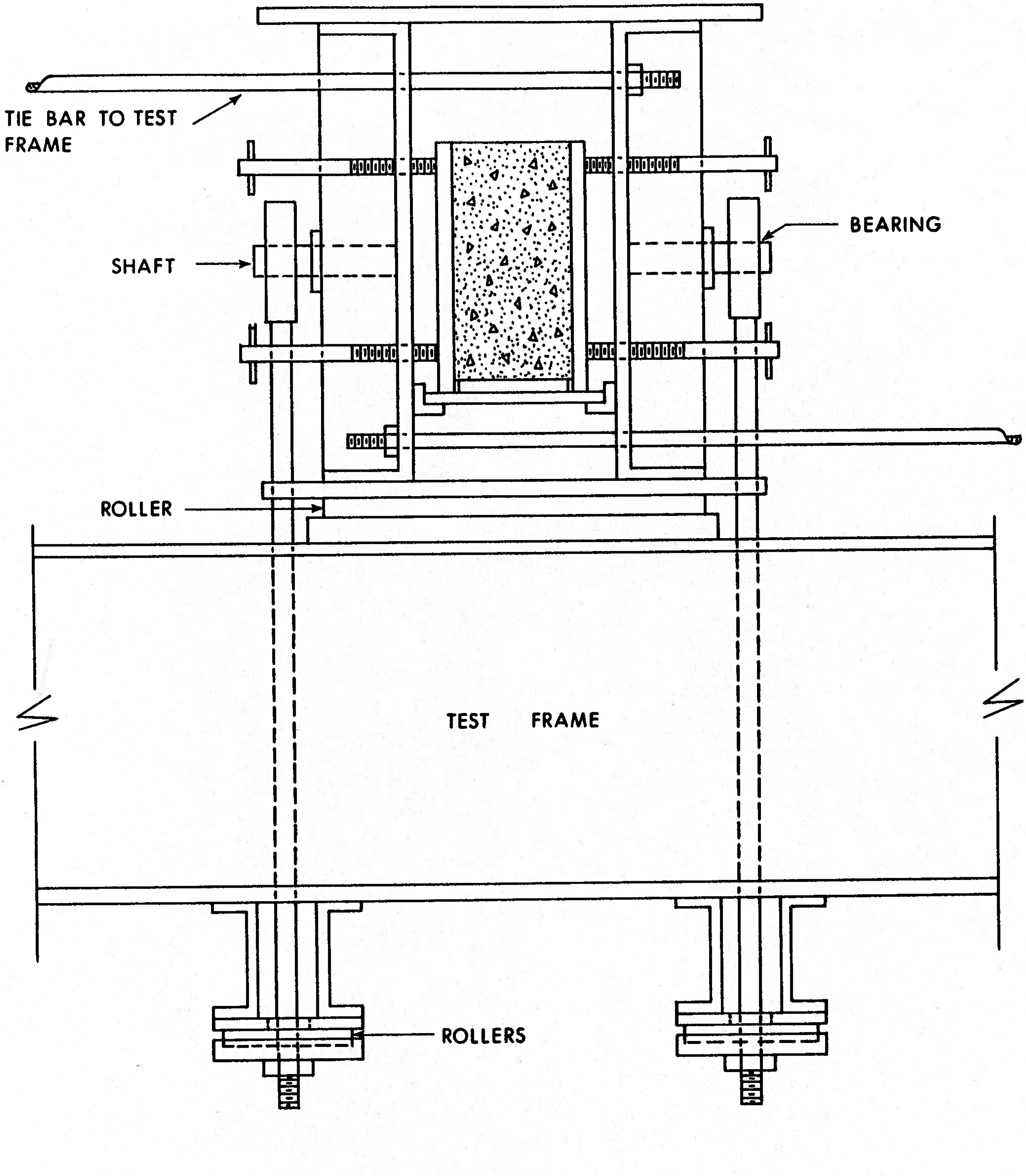


FIGURE 3-13 FIXED HEAD

acting on the beam was the load due to the pipe collars and the dead weight of the beam. The first torsional load applied to a specimen was the weight of the cross head.

All beams except those of Group 3 and Beam 1-2 were subjected to loadings such that for any one test the ratio of twisting moment to bending moment at the end of each increment was a constant. For those beams which had a bending moment gradient within the gauge length, this ratio was computed using the bending moment at the west end of the gauge length. This bending moment was the largest within the gauge length.

In the test of Beam 1-2 the ratio of twisting moment to bending moment after the first load increment was 2.0. Due to an error, subsequent increments were applied such that the increase in twisting moment was four times as large as the increase in bending moment.

Group 3 beams were tested in the following manner:

(a) Beam 3-1: This beam was tested in pure torsion.

(b) Beam 3-2: A small bending moment due to the weight of the testing equipment was first applied to this beam. Next, a twisting moment of approximately 75 percent of the ultimate twisting moment exhibited by Beam 3-1 was applied. Bending moment was then applied until a predetermined level was reached. Finally, the specimen was twisted to failure.

(c) Beam 3-3: This beam was first subjected to transverse load until the bending moment was equal to the maximum bending moment sustained by Beam 3-2. It was then twisted to failure.

(d) Beam 3-4: The bending moment due to the weight of the testing equipment was first applied. Next, a twisting moment of approximately 75 percent of the ultimate twisting moment exhibited by Beam 3-1 was applied. Then transverse load was applied until the bending moment was equal to approximately one-half of the ultimate bending moment of Beams 3-2 and 3-3. The twisting moment was then increased to the ultimate value of twisting moment exhibited by Beam 3-1. Finally, the specimen was bent to failure.

(e) Beam 3-5: The bending moment due to the weight of the testing equipment was first applied. The beam was then twisted to a predetermined level. Finally, the specimen was bent to failure.

CHAPTER IV

TEST RESULTS

4-1 Introduction

The principal test results and the general behavior of the beams are presented in this chapter. More detailed results, which include readings at the end of each load increment and plots of twisting moment versus angle of twist, deflection, and reinforcement stress for each beam, are presented in Appendix B.

4-2 Principal Test Results

The principal test results, which include the material and geometrical properties of the beams and the maximum moments and forces sustained by them, are presented in TABLE 4-1. The maximum recorded twist is generally the angle of twist recorded at the end of the load increment immediately preceeding failure. For those beams tested in combined bending, torsion and shear (Groups 5, 6, and 7) the bending moment listed is the maximum bending moment within the gauge length and occurred at the west end of the gauge length. The ratio of twisting moment to bending moment at which the tests were conducted is denoted as ϕ , and is discussed in SECTION 4-3(d). The value of ϕ listed in TABLE 4-1 for any beam is the value which existed at failure. For those beams which failed during application of an increment of load, the value of ϕ listed in TABLE 4-1

TABLE 4-1

PRINCIPAL TEST RESULTS

Beam	Concrete Strength Compressive* p.s.i.	Concrete Strength Tensile* p.s.i.	Beam Dimensions Width in.	Beam Dimensions Depth in.	Max Bending Moment in.kips	Max Twisting Moment in.kips	Max Recorded Twist Radians/in. x 10^6	$\phi = \frac{\text{Max.Tw.Mom.}}{\text{Max.B. Mom.}}$	Max Shear Force kips
1-1	5190	495	6.19	12.13	0	122	1233	∞	--
1-2	4440	398	6.38	12.13	47	138	1435	2.94	--
1-3	5060	416	6.00	12.13	70	140	1730	2.00	--
1-4	4970	440	6.38	12.13	159	159	1258	1.00	--
1-5	4820	433	6.00	12.13	267	131	1210	0.49	--
1-6	5540	522	6.00	12.13	362	90	315	0.25	--
2-1	5750	460	6.38	12.13	0	181	1483	∞	--
2-2	5020	327	6.38	12.13	88	172	1593	1.96	--
2-3	5500	480	6.00	12.13	166	166	1850	1.00	--
2-4	5220	495	6.25	12.13	267	134	875	0.50	--
2-5	5310	433	6.38	12.13	362	90	335	0.25	--
3-1	5330	462	6.25	12.13	0	115	1440	∞	--
3-2	5330	515	6.38	12.13	111	117	1733	1.05	--
3-3	5460**	-	6.38	12.13	111	120	1430	1.08	--
3-4	5460**	-	6.25	12.13	188	115	1253	0.61	--
3-5	5820	497	6.38	12.13	265	73	738	0.28	--
4-1	5380	504	6.25	12.13	200	120	818	0.60	--
4-2	5700	442	6.38	12.13	166	101	935	0.61	--
4-3	5850	513	6.00	12.13	153	93	1094	0.61	--
4-4	5670	475	6.25	12.13	146	85	1019	0.58	--
4-5	5570	433	6.38	12.13	212	103	717	0.49	--
4-6	6280	477	6.00	12.13	132	66	567	0.50	--
5-1	5710	446	6.25	12.13	65	128	1528	1.97	0.76
5-2	6370	428	6.38	12.13	143	141	1746	0.99	1.89
5-3	6060	418	6.00	12.13	278	130	1390	0.47	3.87
5-4	5800	409	6.25	12.13	389	99	1815	0.25	5.51
6-1	5860	459	6.25	12.13	65	129	1397	1.98	1.81
6-2	5930	424	5.94	12.13	149	145	1821	0.97	4.27
6-3	5700	413	6.25	12.13	264	132	1459	0.50	7.67
6-4	5720	504	6.38	12.13	427	107	1006	0.25	12.45
7-1	6080	482	5.94	12.13	56	112	1592	2.00	1.56
7-2	5210	438	6.25	12.13	115	115	955	1.00	3.32
7-3	5700	382	6.38	12.13	275	132	1504	0.48	8.11
7-4	5340	447	5.94	12.13	505	125	870	0.25	14.97

*Average value from two 6 x 12 inch cylinders
**Compressive strength of Beams 3-3 and 3-4 is based on one 6 x 12 inch cylinder

is lower than that discussed in SECTION 4-3(d).

4-3 Behavior

(a) General Behavior Prior to Failure

The twisting moment versus angle of twist relationship for each beam is presented in Appendix B. For those beams tested in pure torsion, the initial stage of this relationship was linear. However, after the level of twisting moment at which cracks in the concrete were observed was reached, this relationship became curved. The length of the linear portion of this relationship decreased for those beams tested in combined bending and torsion and the complete relationship was curved for those beams tested with a value of $\emptyset = 0.25$.

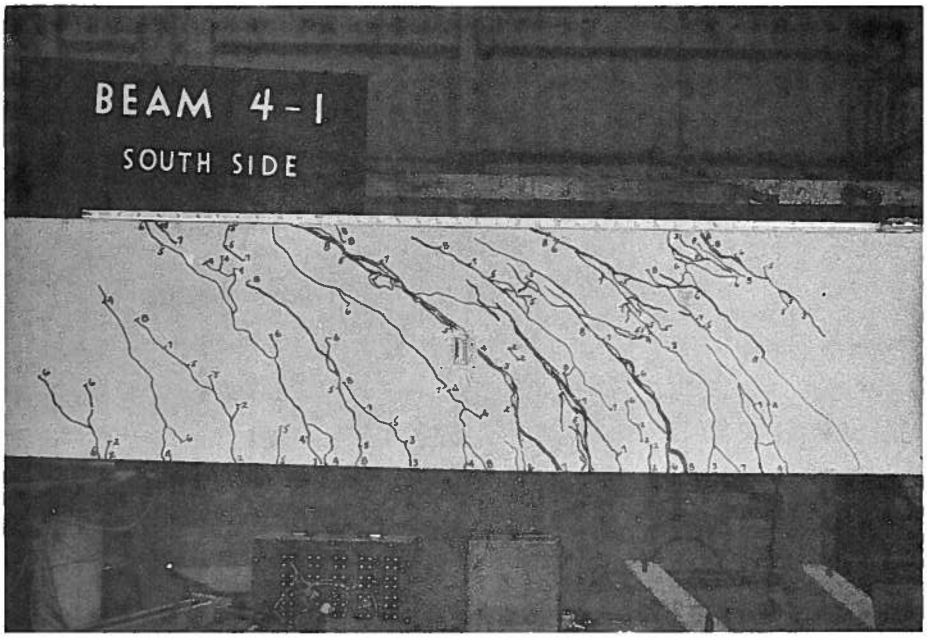
The twisting moment-deflection relationship for each beam is presented in Appendix B. This relationship varied considerably and was found to be dependent on the value of $\emptyset$ and on the amount of top longitudinal reinforcement relative to the amount of bottom longitudinal reinforcement. Those beams which had equal top and bottom reinforcement and which were tested with a high value of $\emptyset$ exhibited small deflections. For purposes of this discussion a high value of $\emptyset$ is regarded as 2.0 or greater and a low value as 0.5 or less. As the value of $\emptyset$ at which these beams were tested was decreased, the deflection of the beams at any particular stage of the test became progressively larger. The beams that were tested at a high value of $\emptyset$ and provided with less top longitudinal reinforcement than bottom exhibited negative or upward deflections in the latter stages of the test. As the value of $\emptyset$ at which these beams were tested was decreased the deflection at any particular stage of the test became less negative and subsequently became positive. At ϕ equal to 0.25 this type of beam exhibited a positive deflection of the same order of magnitude as exhibited by the type of beam which had equal top and bottom reinforcement and which was tested at ϕ equal to 0.25. The beams that exhibited negative deflection during the latter stages of the test usually had zero or a small positive deflection up to the level of twisting moment at which cracking occurred. After cracking occurred the deflection became negative.

Neither the quantity nor the distribution of the reinforcement affected the appearance and initial development of cracks. However, the value of ϕ at which the test was conducted affected them considerably. For those beams tested at a high value of ϕ , the first cracks appeared at mid depth of the vertical faces and were inclined at approximately 45° to the axis of the beam. Further loading extended these cracks to the top and bottom faces. These cracks, where they first appeared on the top and bottom faces, made an angle of approximately 90° with the axis of the beam. However, as they progressed they became inclined to the axis of the beam at approximately 45°. All faces exhibited numerous cracks at twisting moments considerably less than the ultimate values. The effect of reducing the value of ϕ was to delay the appearance of cracks on the top face and to cause the cracks on the vertical faces to progress downward more rapidly than upward. Nevertheless, cracks generally appeared on the top face within one or two increments of first cracking and at twisting moments considerably less than the ultimate.

First cracking of the beams tested at a low value of ϕ generally occurred on the bottom face and the lower portion of the vertical faces. On the bottom face these cracks were nearly perpendicular to the axis of the beam and on the vertical faces they were nearly vertical. As further loading was applied to the beam, these cracks at first progressed vertically up the vertical faces but subsequently the extensions became inclined. At the same time, inclined cracks formed on the vertical faces. They were either new cracks or cracks which started at and progressed downward from the vertical cracks. Inclined cracks were also observed on the bottom face at this time. All faces of the beams were cracked at twisting moments less than the ultimate values.

FIGURE 4-1 illustrates the effect of the amount of reinforcement in a beam on the crack pattern. Beams with large amounts of reinforcement exhibited more numerous but narrower cracks than those with small amounts of reinforcement.

The sequence of development of cracks on several beams tested with a single transverse load and at a low value of ϕ was somewhat different than that described above and is illustrated by FIGURES 4-2 and 4-3. The first cracks observed on these beams were inclined cracks that occurred on the south face, i.e. the face on which the diagonal tension stresses due to transverse shear and torsional shear were additive. Further loading caused new inclined cracks to appear on the south face and the existing ones to extend toward the top and bottom faces. Inclined cracks on the top and bottom faces also appeared at this time. The north face was either



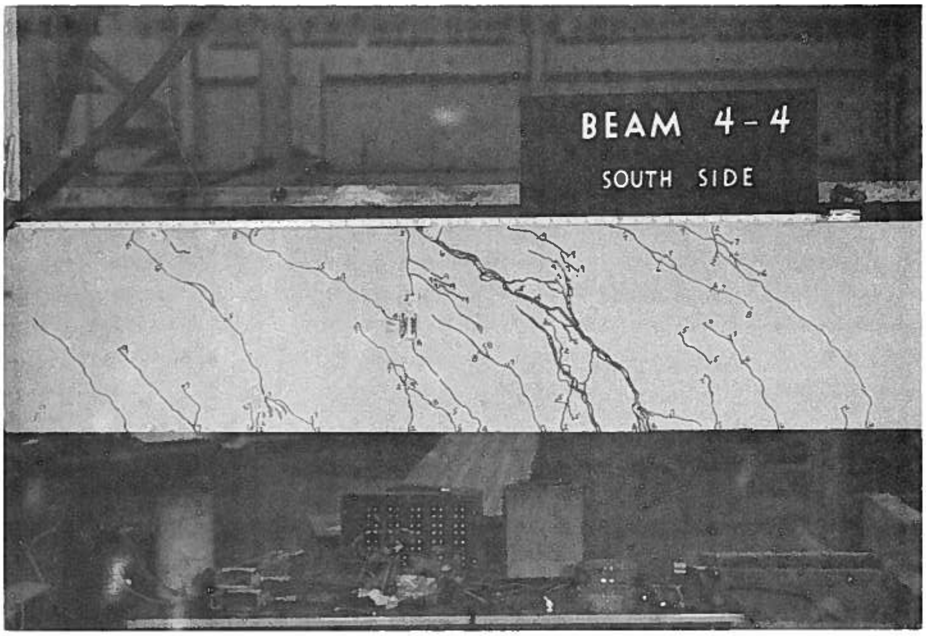
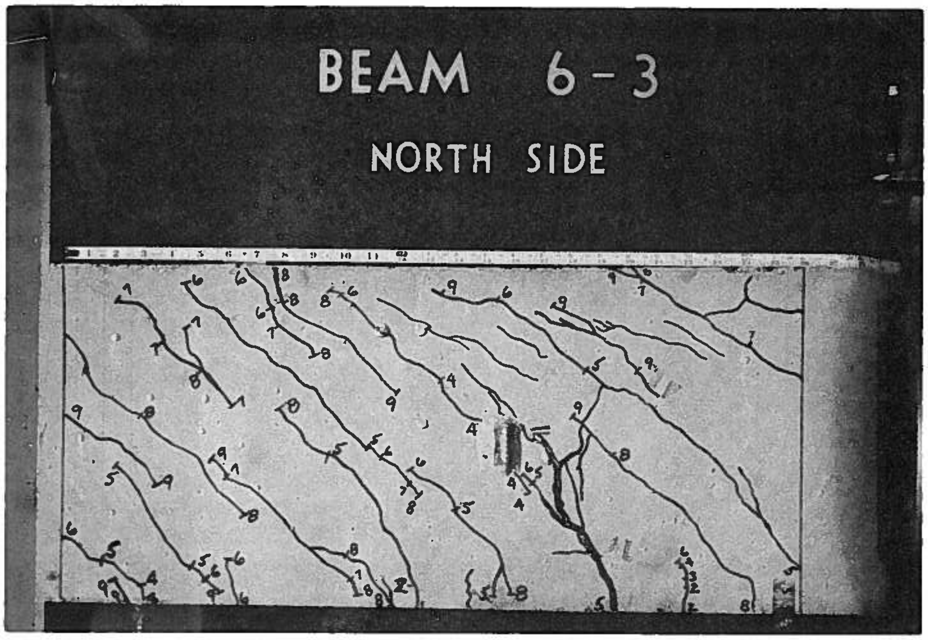


FIGURE 4 - 1 CRACK PATTERN OF BEAMS 4 - 1 AND 4 - 4



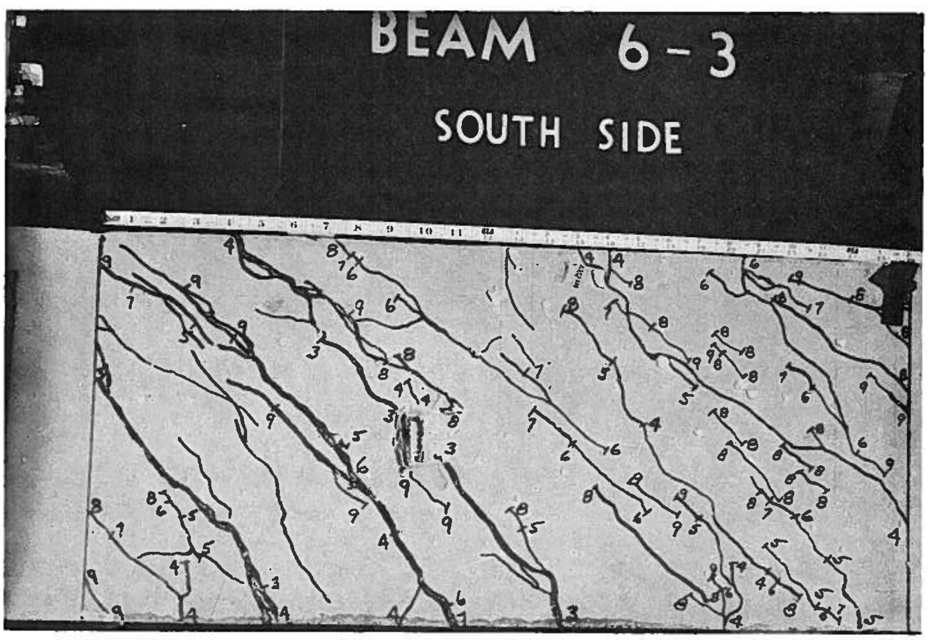
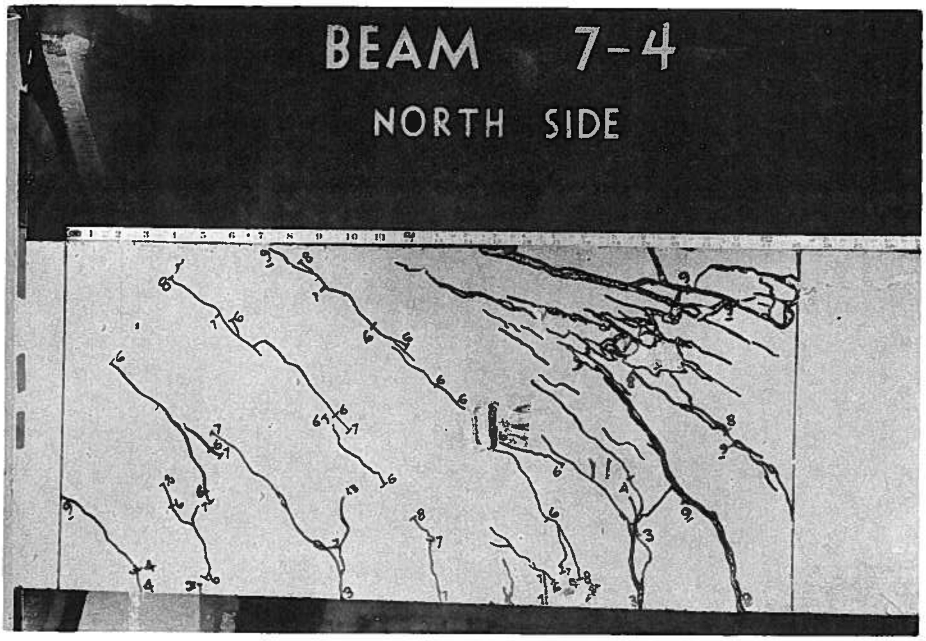


FIGURE 4 - 2 CRACK PATTERN OF BEAM 6 - 3



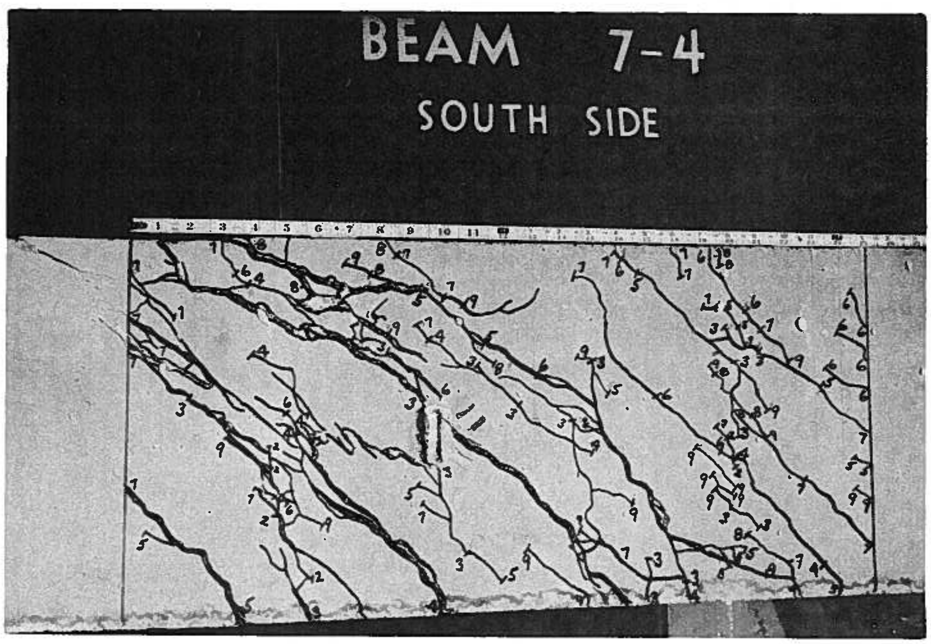


FIGURE 4 - 3 CRACK PATTERN OF BEAM 7 - 4

uncracked or had a few small vertical or inclined cracks. Subsequent increments of load produced inclined cracks on the north face as well as the other faces.

(b) General Behavior at Failure

When the beam could no longer sustain the loads to which it was subjected or could not sustain an increase in these loads, it was considered to have failed. The forces acting on it at this time were considered to be the failure forces.

In all tests, application of load was carried well beyond observance of peak values of twisting load and transverse load. Most beams failed while the twisting moment portion of the load increment was being applied. However, several failed during application of the bending portion and several failed after application of a full load increment and while readings were being taken.

At failure there was a large increase in the angle of twist and, if the level of transverse load was high, in the vertical deflection. At this time the transverse load and the twisting load decreased noticeably. Due to the load maintaining feature of the transverse loading equipment, the previous level of transverse load was quickly regained, but further application of twist in an effort to reach the previous level of twisting load resulted only in further rotation of the beam with little or no corresponding increase in load. However, when twisting was stopped and the beam was allowed to stabilize, the level of twisting load was found to be 80 to 90 percent of the peak value, with the transverse load at its peak value.

However, the magnitude of the twisting load slowly decreased with time. Further application of twist in an effort to increase the twisting load resulted in a temporary increase in twisting load accompanied by a large rotation of the beam. All beams tested exhibited appreciable ductility.

The large rotation which was observed at failure and also upon unloading of the specimen took place about an axis located in different regions of the beam cross section in different beams. The beams that were subjected to a low value of $\emptyset$ at failure rotated about an axis in the vicinity of the top reinforcement. The beams that were subjected to high values of $\emptyset$ and were provided with less top longitudinal reinforcement than bottom rotated about an axis located in the vicinity of the bottom reinforcement. The location of the axis of rotation in beams tested with an intermediate value of $\emptyset$ was less obvious but could be deduced from the shape and width of the failure cracks. Several specimens appeared to have two failure planes, each involving an axis of rotation on a different face of the beam. This is discussed further in SECTION 4-3(d).

Failure of a beam was accompanied by a large rotation and a widening of cracks on the faces of the beam other than the one adjacent to the axis of rotation. In most cases the cracks which widened at failure and defined the failure plane were cracks which had first appeared at moments considerably lower than the ultimate moments. These cracks were essentially a continuous crack which spiralled around the three faces of the beam. However, failure of some specimens was accompanied by the formation of new cracks that joined two sets of previously existing spiral cracks. In addition, some beams appeared to have a failure plane defined by parts of two sets of spiral cracks that, on the surface of the beam, were not joined. FIGURE 4-4 shows the crack pattern of Beam 5-3 and illustrates this type of failure plane. It appeared that the failure plane in this beam was defined on the north face of the beam by a crack from one set of spiral cracks, on the south face of the beam by a crack from another set, and on the bottom face by a combination of two cracks. This type of failure surface appeared most often in those beams tested with a bending moment gradient within the gauge length. The failure surface appeared to be crowded to one end of the gauge length.

Many of the beams that were tested with low or high values of ϕ and had an axis of rotation adjacent to the top or bottom surface of the beam developed a "Z" shaped crack at failure. This crack occurred on the face adjacent to the axis of rotation and connected the failure cracks appearing on the vertical faces. It generally did not form on those specimens tested with an intermediate value of ϕ , or on those specimens in which the axis of rotation appeared to be located adjacent to one of the vertical faces.

(c) <u>Reinforcement Strains</u>

Strains in the reinforcement were small until cracking of the concrete occurred. After cracking occurred, the strains increased at a greater rate and were observed to be dependent on the amount of top longitudinal reinforcement and on the value of ϕ . The beam that was tested in pure torsion and was provided with equal top and bottom reinforcement exhibited tensile strains in the longitudinal reinforcement that

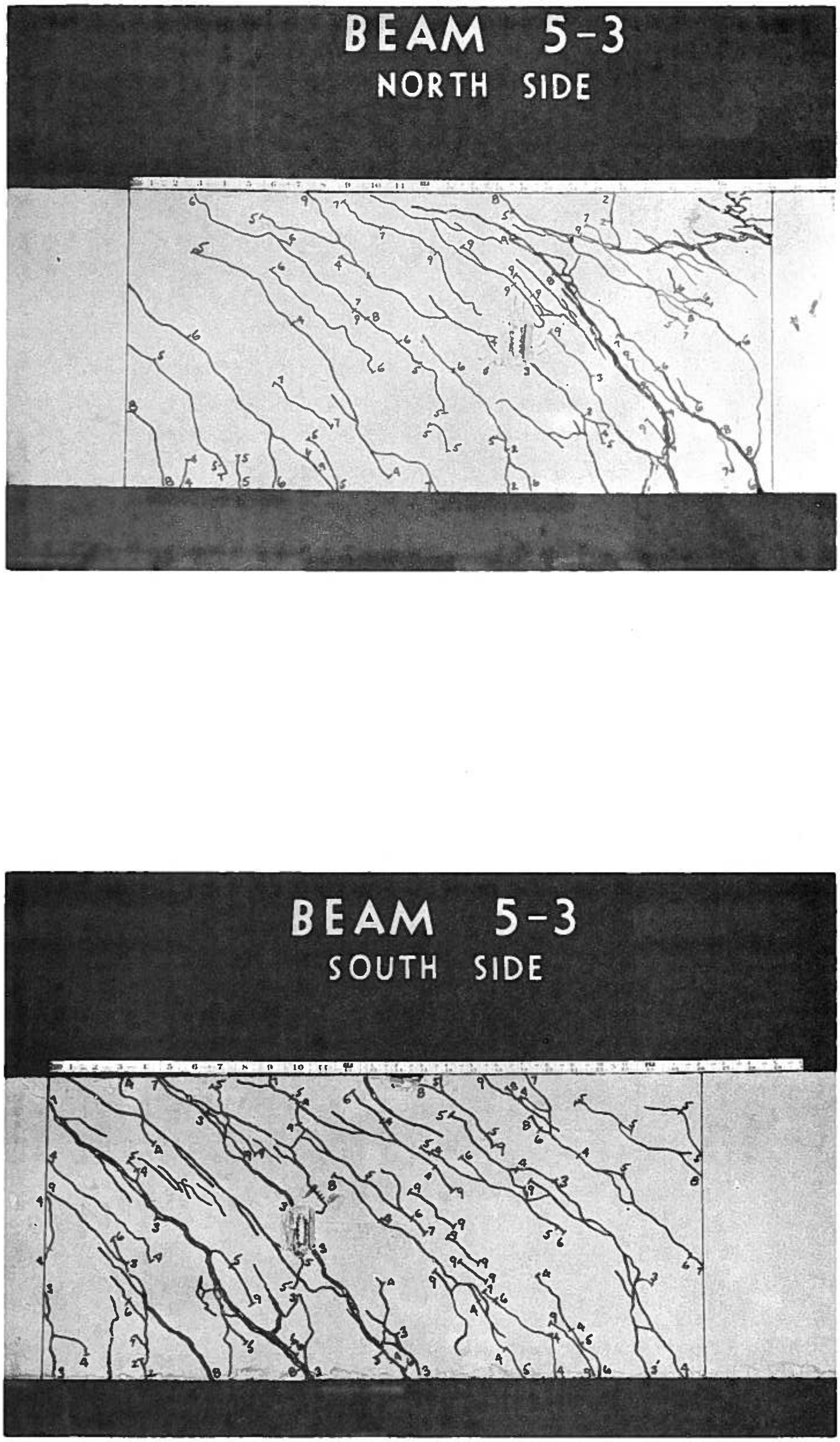


FIGURE 4 - 4 CRACK PATTERN OF BEAM 5 - 3

were nearly the same in all bars. The effect of decreasing the value of Ø for beams of this type was to decrease the strain in the top bars and to increase the strain in the bottom bars. For those beams that were tested with a high value of Ø and were provided with less top reinforcement than bottom, the strain in the top reinforcement at failure was tensile and exceeded the yield strain. The strain in the bottom reinforcement was tensile but was less than the yield strain. The effect of decreasing the value of Ø was to increase the tensile strain that existed in the bottom reinforcement when the beam failed. When beams of this type were tested at low values of Ø , the strain in the top reinforcement at failure was compressive whereas the strain in the bottom reinforcement was tensile and exceeded the yield strain.

In many cases the failure plane did not cross the tie which was instrumented and when it did cross this tie, it intersected only one leg of it. Consequently, it was difficult to establish if the ties intersecting the failure plane yielded at failure. One further difficulty was that the equipment used for reading the strain was not of the continuous recording type. As a result, no strain readings could be obtained when failure occurred during application of the load. This was not a serious disadvantage with regard to the longitudinal reinforcement since either the top or bottom reinforcement usually yielded several load increments before failure occurred. However, this was not so for the ties, particularly with regard to the top and bottom legs. In order to obtain further information, one gauge was constantly observed during load application in the latter stages of the test. It was noted that if the failure plane crossed the leg of the tie which was being observed, this gauge indicated a strain larger than the yield strain at the time the moments were at their peak values. In these cases the strain in this particular leg of the tie at the end of the load increment immediately preceeding failure was often considerably less than the yield strain but immediately before failure occurred, the strain began to increase rapidly. In an attempt to obtain additional information, readings were taken in many of the tests after failure had occurred and while the beam was still supporting loads less than the ultimate values. The strains in the ties are discussed further in SECTION 4-3(d).

(d) Detailed Observations

(i) Group 1

The beams of this group had less longitudinal reinforcement at the top than at the bottom. Except for Beam 1-2 they were tested such that for each beam the value of ϕ at the end of each full increment of the load was a constant. Within the gauge length the twisting moment and the bending moment were constant.

Beam 1-1: This beam was tested in pure torsion ($\phi = \infty$). At failure, rotation occurred about an axis near the bottom face. The failure plane, which was well defined, did not cross the tie which was instrumented. The negative deflection of the beam at failure was pronounced.

Beam 1-2: At the end of the first increment of load, the twisting moment was twice the bending moment. For all subsequent load increments the increase in twisting moment applied to the beam was four times as large as the increase in bending moment. Consequently, the value of Ø varied from 2.0 at the end of the first increment to 2.9 at failure. The first cracks on this specimen, which were inclined cracks on both vertical faces, were observed after the first load increment. At failure, which occurred while the twistmeters were being read, the beam rotated about an axis located near the bottom face of the beam. The failure plane did not intersect the tie which was instrumented.

Beam 1-3: The value of Ø at which this specimen was tested was 2.0. Failure occurred while the strain gauges were being read. The hinge, or axis of rotation at failure, was located near the bottom of the beam.

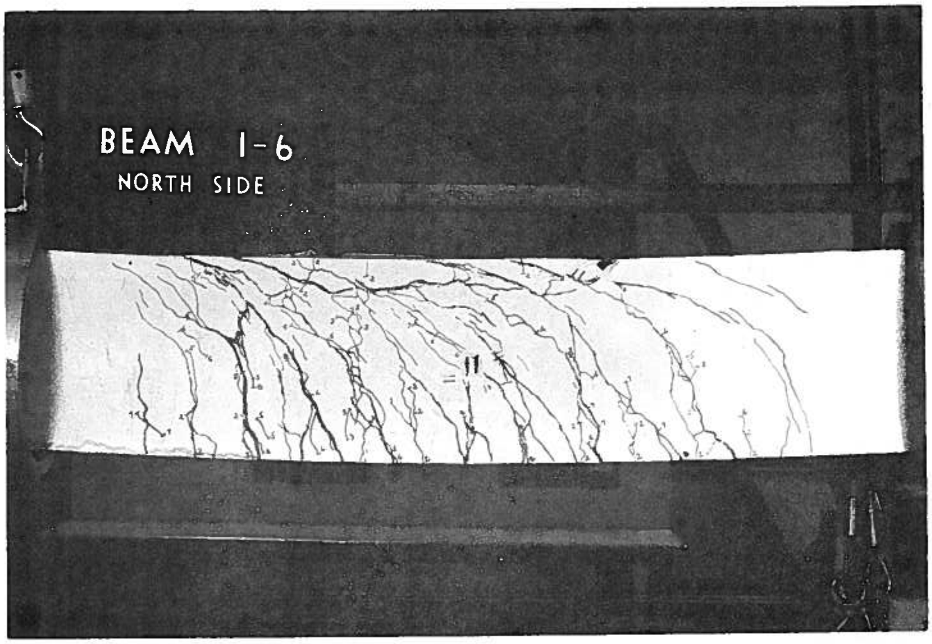
Beam 1-4: This beam was tested at Ø equal to 1.0 and failed immediately after the twisting portion of the last increment was applied. The development of the failure plane indicated that the hinge was located adjacent to the north face. However, the beam had a second failure plane which was fairly well developed. This second failure plane indicated that a hinge had begun to form in the vicinity of the top face. On the top face there was a fairly well defined "Z" shaped crack. Strain readings taken after failure but while the beam was still sustaining load indicated strains larger than yield strains at gauges 2, 3, 4, 5, 7 and 8. The locations of these gauges have been detailed in CHAPTER III.

Beam 1-5: Testing of this beam was carried out with Ø equal to 0.5. The axis of rotation at failure, which occurred while torsion was being applied, was located near the top surface of the beam. Considerable spalling of the concrete occurred in the region of the top north corner. During application of the last increment of load, the strain indicated by gauge 5 was observed continually. While the last increment of twisting was being applied the strain steadily increased and when failure occurred the strain was increasing very rapidly. Readings taken subsequent to failure indicated strains in excess of yield strains at gauges 3, 4, 5, 6, and 8.

Beam 1-6: For this beam, which was tested at ϕ equal to 0.25, the axis of rotation at failure was near the top face of the beam. Failure occurred during reading of the strain gauges and considerable spalling of the concrete on the top surface of the beam was observed. The peak values of moments were held throughout a considerable rotation of the beam but eventually could not be maintained as the rate of rotation and deflection began to increase rapidly. During this rotation, the strain indicated by gauge 5 was observed and was found to be increasing very rapidly. Subsequent to failure, the readings of gauges 3, 4, 5 and 6 indicated tensile strains larger than the yield strains; gauge 1 indicated compressive strain larger than the yield strain. At failure, numerous cracks that had not been evident previously appeared and joined cracks that had existed previously. The failure plane was defined by a combination of these new cracks and cracks that had existed before failure. The crack pattern of this beam is shown in FIGURE 4-5.

(ii) Group 2

This group of beams was subjected to uniform bending and twisting moments and for each beam ϕ was a constant at the end of any increment of load. The beams were provided with top longitudinal reinforcement that



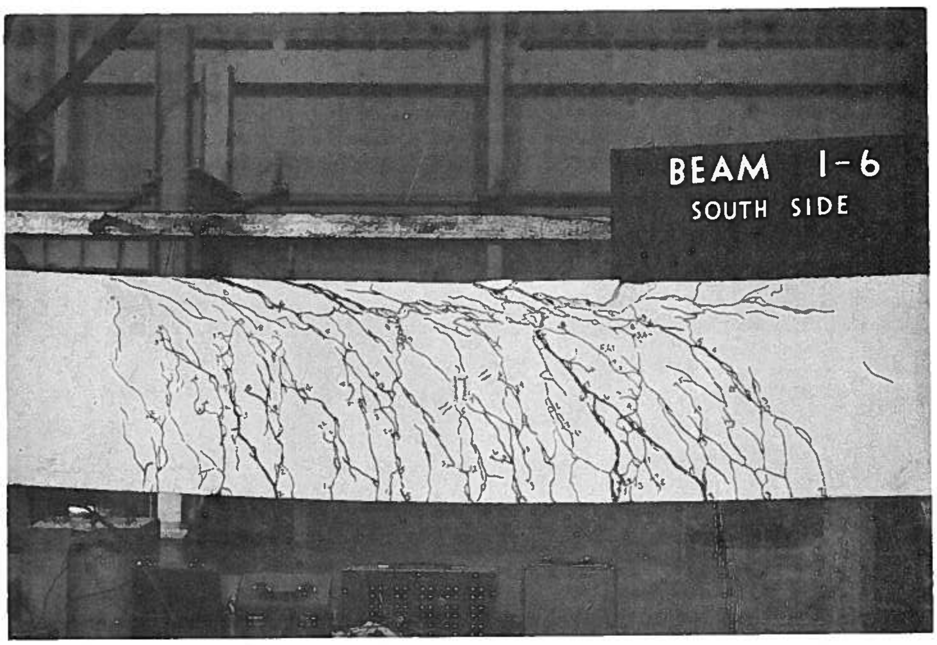


FIGURE 4 - 5 CRACK PATTERN OF BEAM 1 - 6

was equal to the bottom reinforcement.

Beam 2-1: This beam, tested in pure torsion, failed during reading of the strain gauges. Since wide cracks appeared on all faces at failure it was difficult to define the failure plane. However, the widest crack was an inclined crack on the south face that also extended across the top and bottom faces. Thus, the hinge was likely located adjacent to the north face. On the top face a "Z" shaped crack had begun to form indicating that a hinge was beginning to form on this face. Readings taken after failure indicated strains in excess of yield strains at all gauges except number 4. The crack pattern of this beam is shown in FIGURE 4-6.

Beam 2-2: Failure of this beam occurred during application of the twisting portion of the last load increment. The test was conducted at Ø equal to 2.0. The failure plane was difficult to define and appeared to be composed of portions of two sets of spiral cracks. However, the axis of rotation at failure was located in the vicinity of the top face of the beam. Readings taken after failure indicated strains larger than yield strains at gauges 3, 5, 6 and 8.

Beam 2-3: The value of Ø at which this beam was tested was 1.0. Failure, with the hinge located near the top surface, occurred immediately after application of the twisting portion of the last load increment. Strain readings taken after failure indicated strains larger than yield strains at gauges 3, 4, 5, 6 and 8.

Beam 2-4: This beam failed soon after application of the last load increment and developed a hinge in the vicinity of the top surface. The test

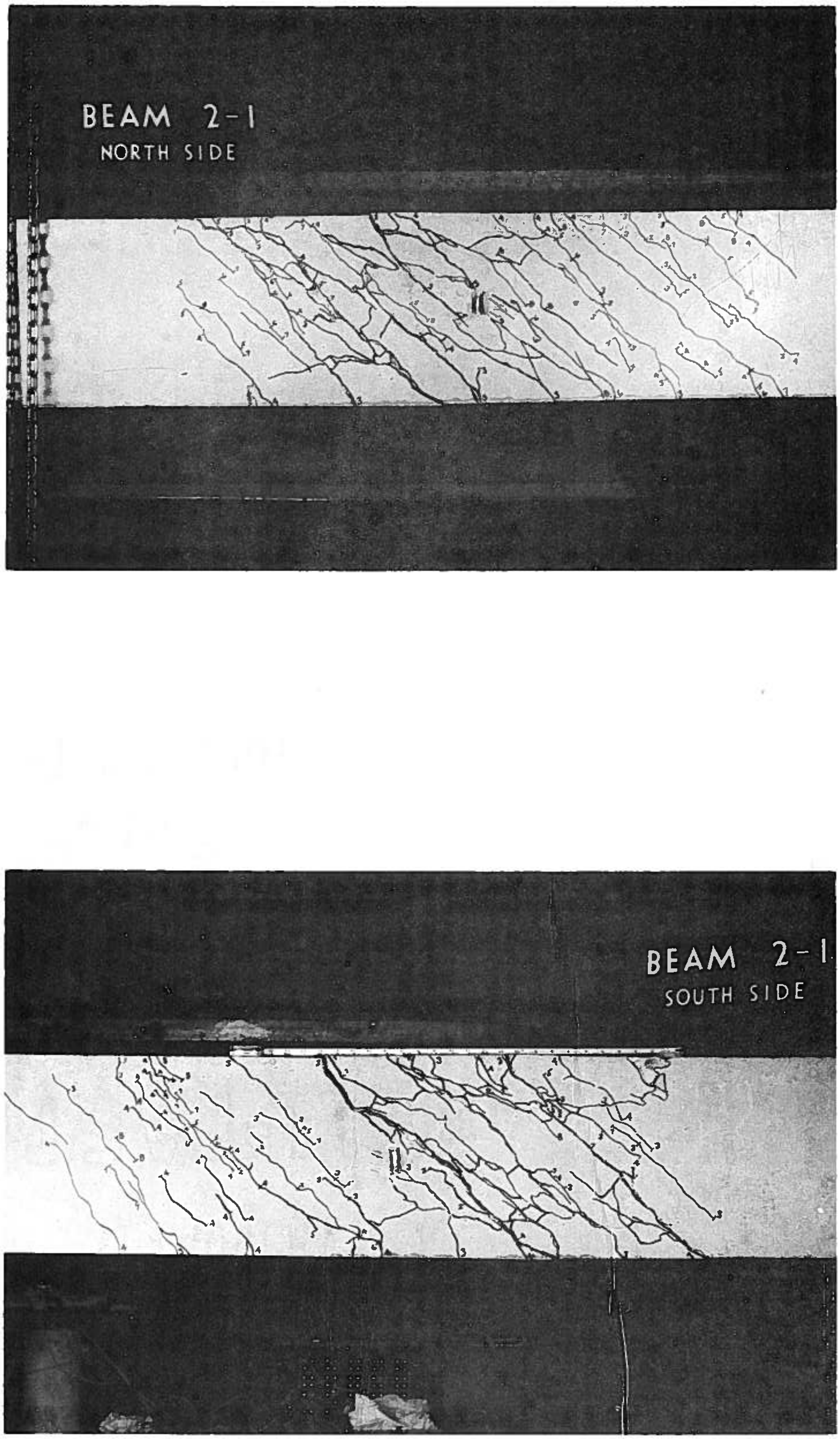


FIGURE 4 - 6 CRACK PATTERN OF BEAM 2 - 1

was performed at ϕ equal to 0.5. Readings obtained after failure from strain gauges 3, 4 and 5 indicated strains larger than yield strains.

Beam 2-5: The testing of this specimen was conducted at ϕ equal to 0.25. Failure, accompanied by rotation about a hinge near the top surface of the beam, occurred after application of the last load increment but peak values of moments were maintained throughout considerable rotation of the beam. Readings taken subsequent to failure indicated strains larger than yield strains at gauges 3, 4, 5 and 8.

(iii) Group 3

The sequence of loading for each beam of this group is described in CHAPTER III. These beams had less top longitudinal reinforcement than bottom and were similar to Group C of Pandit (1965).

Beam 3-1: This beam was tested in pure torsion and at failure developed a hinge in the vicinity of the bottom reinforcement. Deflection of this beam was not measured.

Beam 3-2: The value of ϕ at failure of this beam was 1.05. Although the failure plane was not well defined and large cracks existed on all faces, the failure hinge appeared to be located in the top portion of the beam. The relationship between the stresses in the tie that was instrumented and the sum of the twisting and bending moment is shown in FIGURE 4-7. The sum of twisting and bending moment is used simply for convenience.

Beam 3-3: At failure, the value of ϕ was 1.08 and the hinge was located adjacent to the top surface of the beam.

Beam 3-4: The value of ϕ at failure of this beam was 0.61. The hinge was

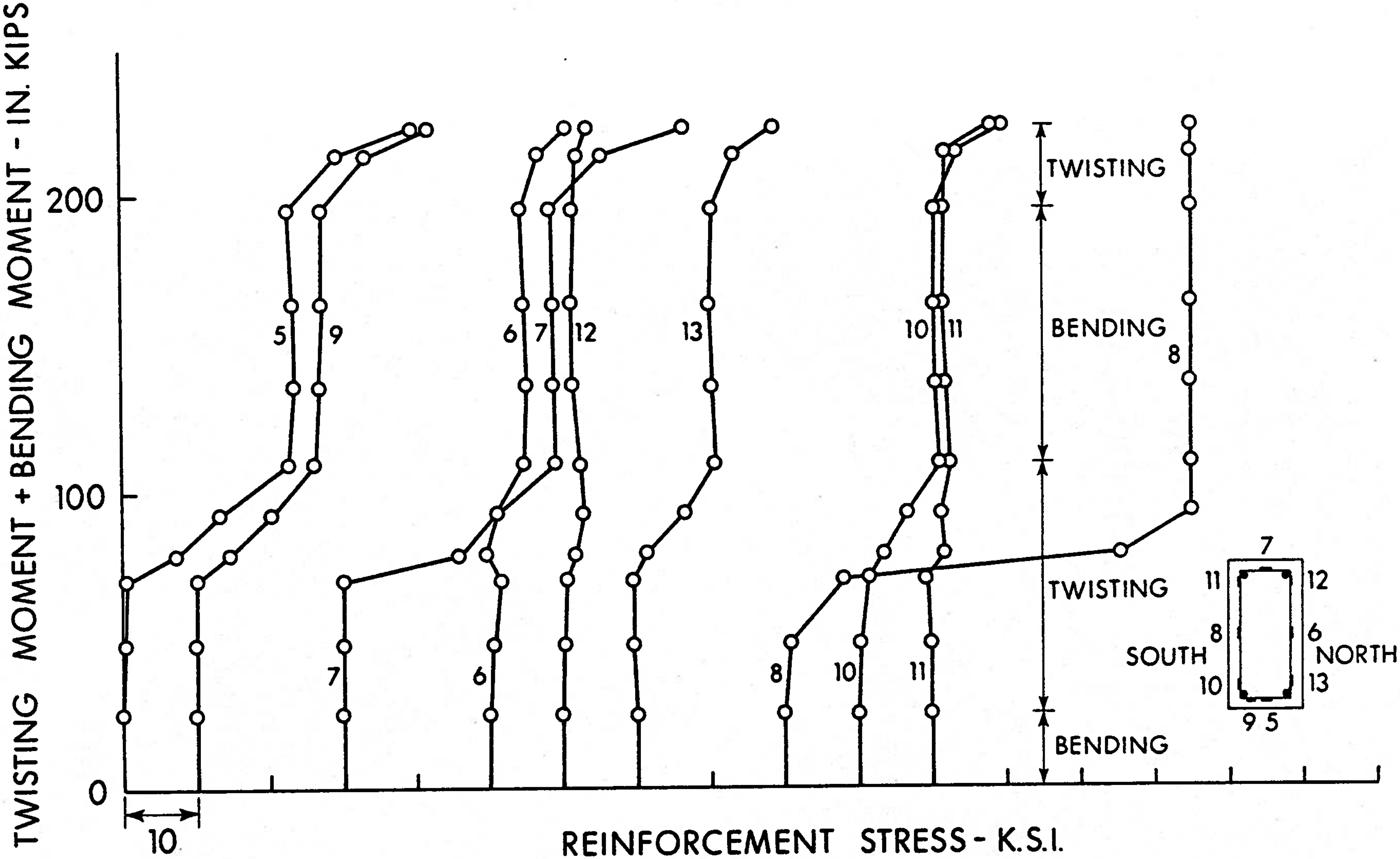


FIGURE 4-7 TIE STRESSES IN BEAM 3 - 2

located in the vicinity of the top face and some spalling of the concrete was observed on this face. The effect of the bending load, which was applied after the twisting load, was to widen the cracks that had first appeared during the twisting stage of the test. No deflections were measured during this test.

<u>Beam 3-5:</u> The hinge that developed in this beam at failure was located near the top surface of the beam. The failure value of ϕ was 0.28. At failure, pieces of concrete on the top face of the beam, particularly the top, south corner, broke away from the remainder of the beam. Since gauges 1 and 18 were damaged early in the test, no useful strain readings were obtained from them. After failure, gauges 2 and 10 indicated compressive strains larger than yield strains and gauges 3, 4, 5, 6, 7, 8, 11, 12, 16, 19 and 20 indicated tensile strains larger than yield strains. The relationship between the reinforcement stresses and the sum of the twisting and bending moment is shown in FIGURES 4-8 and 4-9. The crack pattern of this beam is shown in FIGURE 4-10.

(iv) <u>Group 4</u>

The details of the reinforcement of these beams are given in CHAPTER III. A plot of twisting moment versus unit angle of twist for these beams and Beams 3-4 and 1-5 is presented in FIGURE 4-11. The irregular shape of this relationship for Beam 3-4 is due to the different sequence of loading to which this beam was subjected.

<u>Beam 4-1:</u> This beam was tested at ϕ equal to 0.61 and at failure developed a hinge adjacent to the top face. Failure occurred during appli-

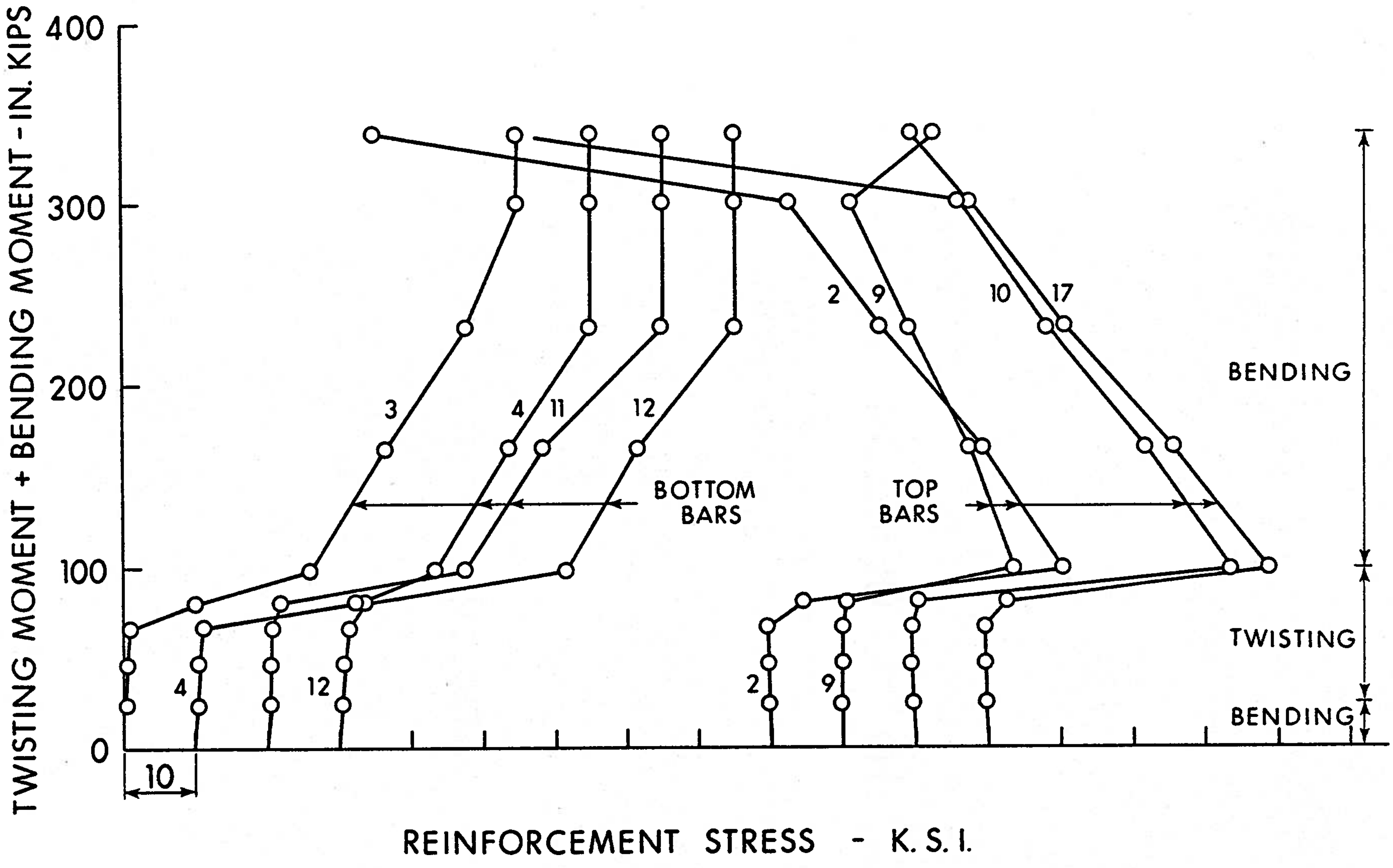


FIGURE 4 - 8 LONGITUDINAL BAR STRESSES IN BEAM 3 - 5

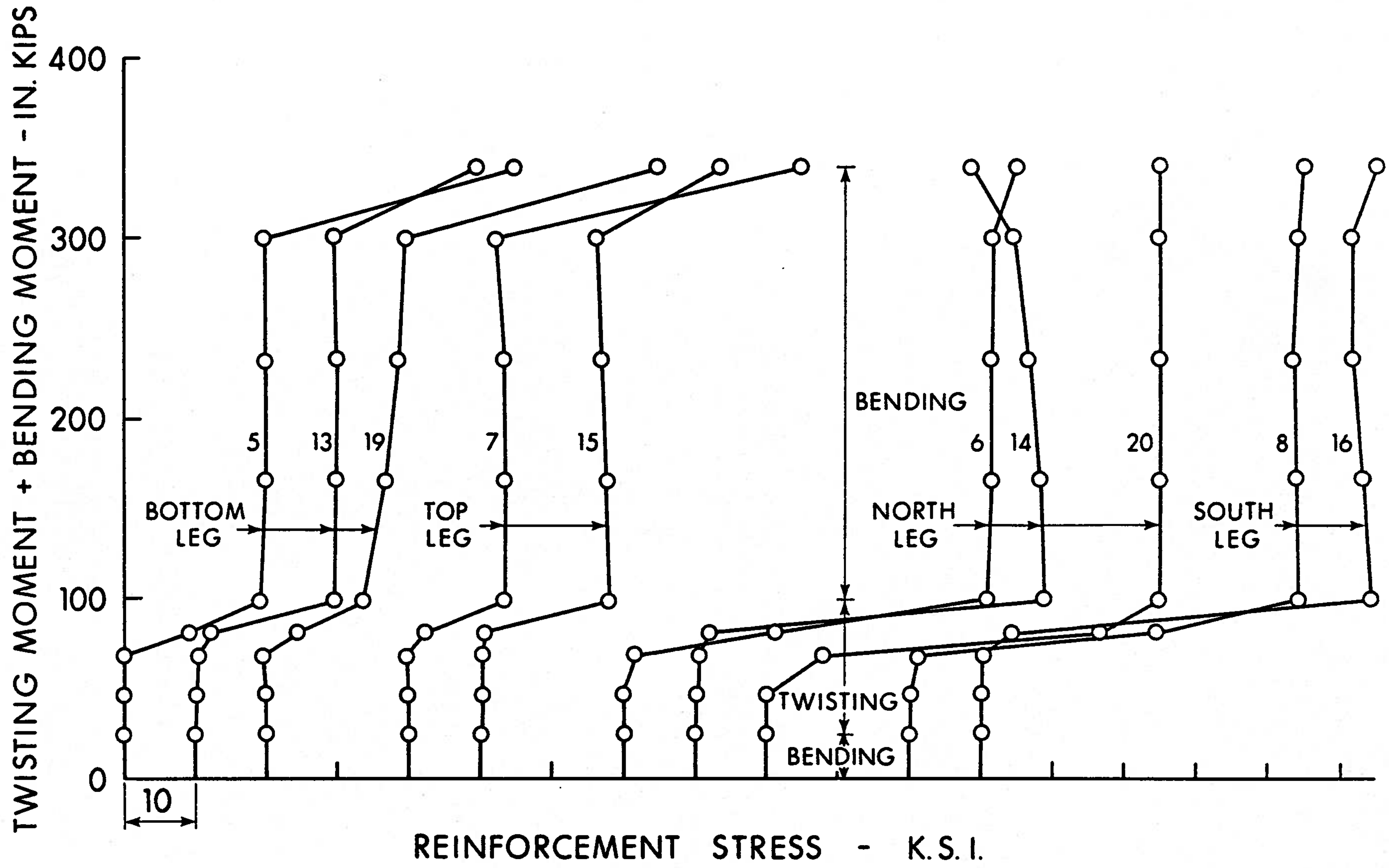
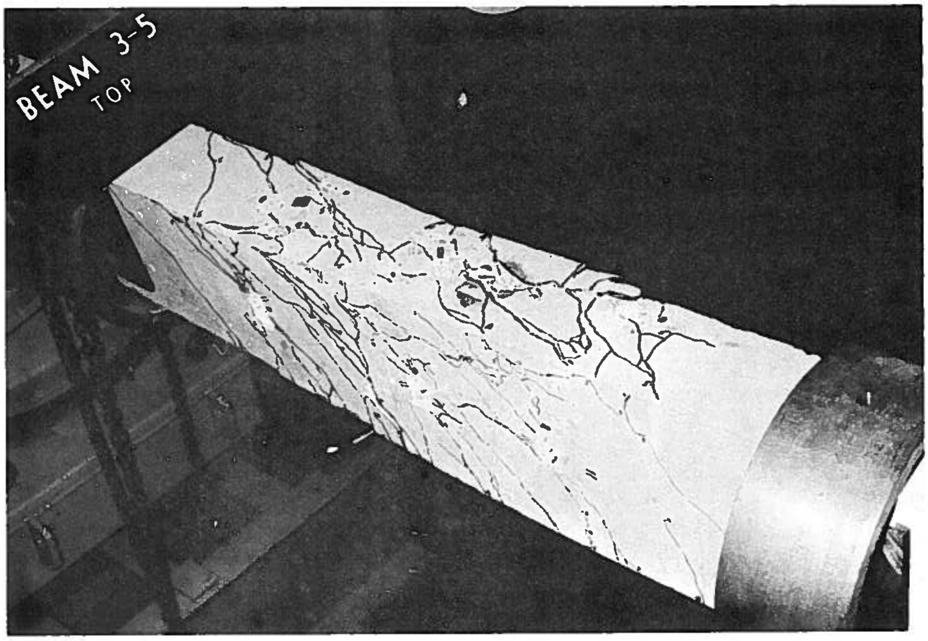


FIGURE 4 - 9 TIE STRESSES IN BEAM 3 - 5



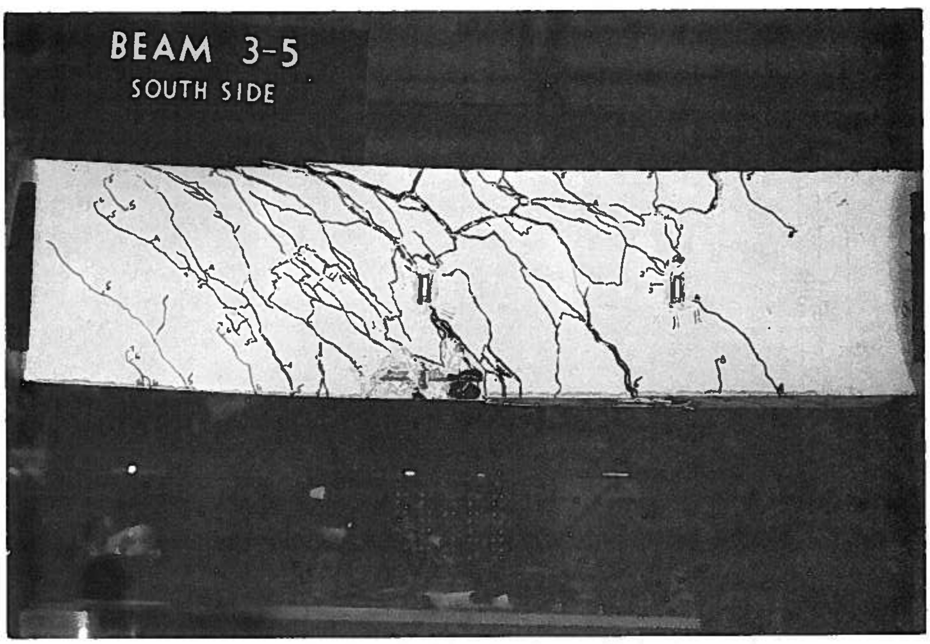


FIGURE 4 - 10 CRACK PATTERN OF BEAM 3 - 5

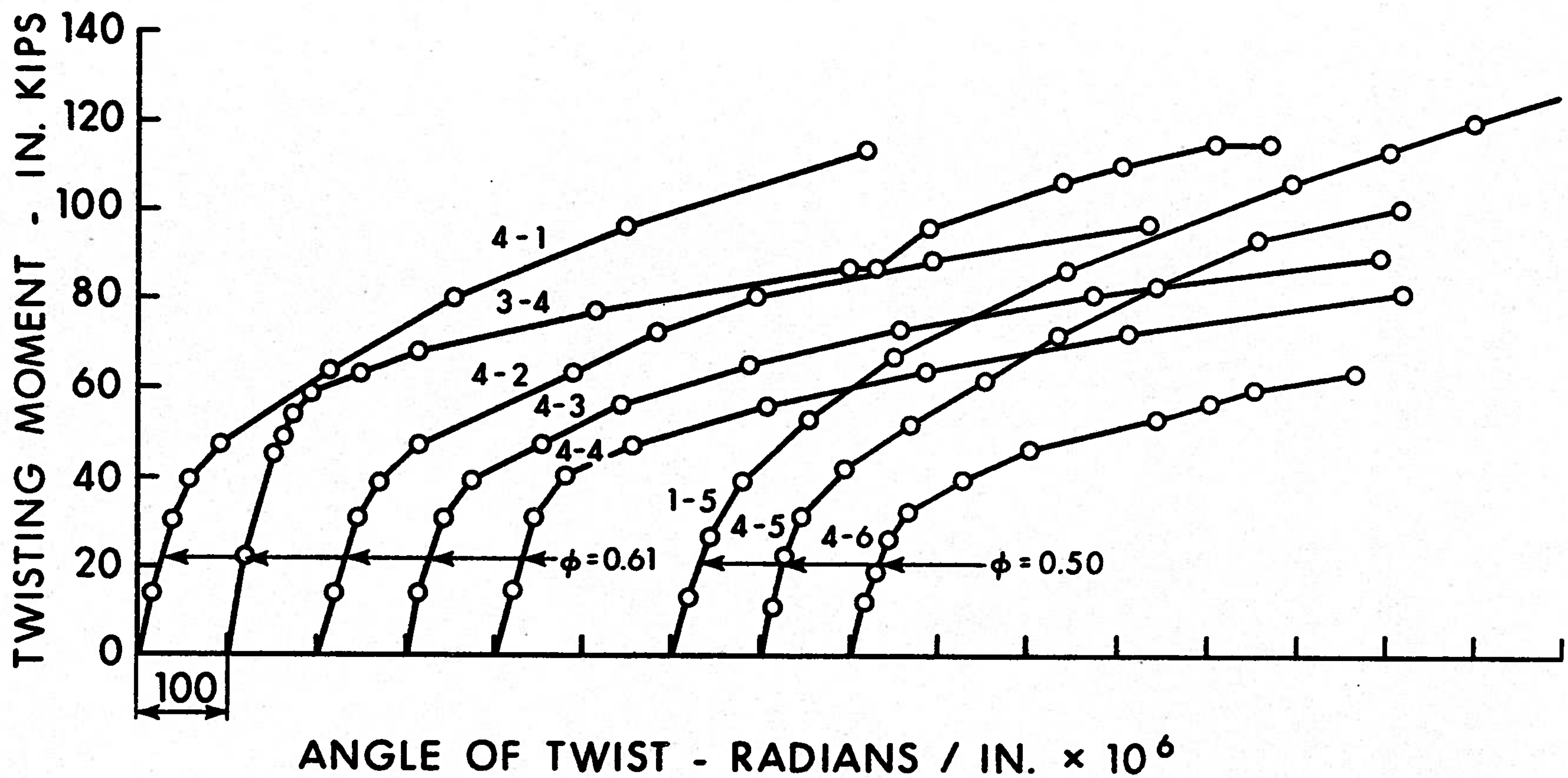


FIGURE 4 - 11 TORQUE TWIST CURVES FOR BEAMS WITH DIFFERENT REINFORCEMENT

cation of the twisting portion of the load increment. Strain readings taken subsequent to failure indicated strains larger than yield strains at gauges 3 and 4.

Beam 4-2: The failure hinge in this beam, which was also tested at Ø equal to 0.61, was located near the top face of the beam. Failure occurred immediately after the twisting portion of the load increment was applied. Strain readings taken subsequent to failure indicated strains larger than yield strains at gauges 3, 4, 7 and 8.

Beam 4-3: This beam was tested at Ø equal to 0.61. Failure occurred immediately after application of the twisting portion of the load increment. The axis of rotation at failure was located near the top face of the beam but the width of cracks suggested that a hinge was about to form on the north face. Readings taken subsequent to failure indicated strains larger than yield strains at gauges 3, 4, 6, 7 and 8.

Beam 4-4: This test was conducted at Ø equal to 0.61. At failure, which occurred during application of the twisting portion of the load increment, a hinge formed adjacent to the top face of the beam but the width of cracks suggested that formation of a hinge on the north face was imminent. Strain gauges 3, 5 and 8, when read after failure, indicated strains larger than yield strains. Gauge 4 was damaged during testing.

Beam 4-5: The hinge occurring in this beam at failure was located near the top face of the beam. The value of Ø at which this beam was tested was 0.50. Failure occurred during application of the bending portion of the load increment.

Beam 4-6: This beam also was tested at Ø equal to 0.50 and at failure a hinge formed in the vicinity of the top face. Failure occurred while the strain gauges were being read. The peak moments were maintained throughout a large rotation.

(v) Group 5

The beams of this group, which were reinforced identically to the beams of Group 1, were loaded using a single concentrated transverse load in addition to a twisting moment. Consequently, the gauge length was subjected to a constant twisting moment, a constant shear force, and a linearly varying bending moment. The maximum bending moment in the gauge length, which occurred at the west end of the gauge length, was used in establishing the value of Ø .

Beam 5-1: Failure of this beam occurred during application of twisting moment. The failure hinge was near the bottom face of the beam and the failure plane was in the east portion of the gauge length. The value of Ø at which this test was conducted was 2.0.

Beam 5-2: The value of Ø at which this test was conducted was 1.0. At failure, which occurred during application of twisting moment, a hinge formed near the north face of the beam. However, the width of cracks suggested that the formation of a hinge near the bottom face was imminent. Strain readings taken subsequent to failure indicated strains larger than yield strains at gauges 2, 3, 4, 5, 7 and 8.

Beam 5-3: This beam was tested at Ø equal to 0.50 and failed during application of twisting moment. The hinge that developed at failure was located in the vicinity of the top surface. The failure plane was composed of parts of two sets of spiral cracks and was crowded toward the west end of the gauge length. Subsequent to failure, strain gauges 3, 4, 5 and 8 indicated strains larger than yield strains.

Beam 5-4: The value of $\emptyset$ at which this beam was tested was 0.25. The hinge that formed at failure was located near the top face of the beam; the failure surface was in the west portion of the gauge length. Failure of this beam occurred after application of the twisting portion of the load increment and was accompanied by spalling of the concrete at the top face of the beam. The peak values of load were maintained throughout a very large rotation.

(vi) Group 6

These beams were reinforced identically to the beams of Group 5 and were also subjected to bending, torsion and transverse shear. However, the point of application of the transverse load was changed in order to increase the shear in the gauge length.

Beam 6-1: At failure, which occurred during application of twisting moment, the axis of rotation was near the bottom surface of the beam. The test was conducted at $\emptyset$ equal to 2.0. The failure plane was located in the east portion of the gauge length.

Beam 6-2: This beam was tested at $\emptyset$ equal to 1.0 and failed during application of twisting moment. The failure surface was not well defined but was located in the east half of the gauge length. The failure hinge appeared to be located near the bottom face but some aspects of the crack pattern indicated its location to be in the vicinity of the north side. Readings after failure indicated strains larger than yield strains at gauges 1, 2 and 6.

Beam 6-3: Failure of this beam occurred during application of twisting moment. The test was conducted at ϕ equal to 0.50 and at failure the peak values of moments were maintained through a large rotation. The failure surface was located in the west portion of the gauge length; the failure hinge was near the top surface of the beam. Subsequent to failure, strain gauges 3, 4 and 5 indicated strains larger than yield strains.

Beam 6-4: Failure occurred after application of the twisting portion of the last load increment but peak values of the moments were maintained throughout a very large rotation. The value of ϕ for this test was 0.25 and the failure surface was located in the west portion of the gauge length. Considerable spalling of the concrete at the top surface of the beam occurred. The hinge that formed at failure was located near the top face of the beam. Strain readings taken after failure indicated compressive strain larger than the yield strain at gauge 1 and tensile strains larger than the yield strains at gauges 3, 4 and 5.

(vii) Group 7

In comparison to the beams of Groups 5 and 6, the beams of this group had larger bottom reinforcement and larger spacing of ties. The loading arrangement for this group was the same as for Group 6.

Beam 7-1: This beam was tested at ϕ equal to 2.0 and failed during reading of the deflection gauges. The axis of rotation at failure was in the vicinity of the bottom face of the beam. The failure surface, which was composed of portions of two sets of spiral cracks, was crowded toward the east end of the gauge length.

Beam 7-2: The value of $\emptyset$ at which this test was conducted was 1.0 and failure occurred during reading of the strain gauges. The failure hinge formed near the bottom face of the beam and the failure surface was located in the east portion of the gauge length. Strain readings taken after failure indicated strains larger than yield strains at gauges 2 and 8.

Beam 7-3: The failure plane was crowded toward the east end of the gauge length and the hinge was located near the bottom face. However, the width of cracks suggested that another failure plane was about to form in the central portion of the gauge length. This failure plane would have involved a hinge near the north face. This test was carried out at $\emptyset$ equal to 0.50 and failure occurred during application of twisting moment. Readings obtained after failure indicated strains larger than yield strains at gauges 2, 6, 7 and 8.

Beam 7-4: The test of this beam was carried out at $\emptyset$ equal to 0.25 and failure occurred during application of twisting moment. The failure surface was a composite of several sets of spiral cracks and was crowded to the west end of the gauge length. The failure hinge was located near the top face of the beam. However, similarly to Beam 7-3, the width of cracks suggested that another failure plane, which would have involved a hinge near the north face, was about to form in the central portion of the gauge length. Some spalling of the concrete occurred on the top, north corner of the beam. Readings obtained after failure indicated strains larger than yield strains at gauges 3, 4, 7 and 8.

CHAPTER V

ANALYSIS OF ULTIMATE STRENGTH IN COMBINED LOADING

5-1 Introduction

The assumptions and the three failure surfaces on which the analysis is based are presented in this chapter. For each failure surface an equation for the torsional strength of a rectangular reinforced concrete beam is derived. In addition, the method used to solve these equations is presented.

5-2 General Remarks

The results presented in CHAPTER IV and in Appendix B indicate that the specimens exhibited two different stages of behavior during a test. Before the concrete cracked, the relationship between torque and twist was nearly linear. In this stage, the reinforcement strains were low and the deflection was zero or slightly positive. After cracking, the slope of the torque-twist relationship decreased considerably and this relationship often became curved. The strains in the reinforcement increased with load at a greater rate and in many cases the deflection became more pronounced. After cracking occurred the specimen appeared to assume a new configuration of equilibrium which was dependent mainly on the value of ϕ and the reinforcement of the beam.

When failure occurred, cracks on three faces of the beam usually widened to define the failure plane. The cracks on two of these faces decreased in width as they approached the fourth face. Although the fourth face was cracked, these cracks did not widen appreciably at failure. Rotation of the beam at failure occurred about an axis located near this face. Consequently, the failure plane appeared to consist of a tension zone on one side of a neutral axis and a compression zone on the other side, with the neutral axis being the hinge or axis of rotation.

A "Z" shaped crack often developed on the face of the beam adjacent to the hinge, with the central portion of the "Z" being parallel to the longitudinal axis of the beam. However, this is not sufficient justification to assume that the hinge was parallel to the axis of the beam. Furthermore, rotation about a longitudinal hinge could not explain the vertical deflections observed at failure. It is important that any method of analysis used to predict the ultimate strength of beams subjected to combined loading be valid at the limiting cases of pure bending and pure torsion. The neutral axis of a beam subjected to pure bending is perpendicular to the axis of the beam. Adoption of an inclined hinge that is parallel to one face of the beam but which is inclined to the longitudinal axis of the beam was first suggested by Lessig (1959). A smooth transition from pure bending to pure torsion is obtained when this type of hinge is used. The angle this hinge makes with the longitudinal axis is a function of ϕ and the properties of the beam and is discussed in SECTION 5.4. Lessig denoted failure on a warped plane with a hinge near the top face of the beam as mode 1; failure on a warped plane with a hinge located near one of the sides was denoted as mode 2. This designation will be followed here, but in addition, failure on a warped plane with a hinge located near the bottom face will be denoted as mode 3.

5-3 Assumptions of Analysis

The method of analysis is based on the following assumptions:

(1) Failure occurs on a warped plane. The boundaries of the warped plane are defined on three sides of the beam by a spiral crack and on the fourth side by a rectangular compression zone which joins the ends of the spiral crack.

(2) The crack defining the failure plane on three sides of the beam is composed of three straight lines spiralling around the beam at a constant angle. The angle between these cracks and the longitudinal axis of the beam is never less than 45^{o}.

(3) The concrete outside of the rectangular compression zone is cracked and carries no tension.

(4) The reinforcement near the face of the beam on which the compression zone is located is neglected.

(5) No local loads are present within the length of the failure plane.

(6) The reinforcement has a well defined yield plateau. All reinforcement crossing the failure plane outside of the compression zone yields at failure.

(7) The cross sectional area of transverse reinforcement intersected by the failure plane is constant per unit length of the beam.

(8) At failure, the characteristics of the concrete in the compression zone are known and the concrete reaches its strength in flexural compression.

Assumptions 1 and 2 define the failure plane. The three lines composing the spiral crack are assumed to be straight for simplicity. The assumption that these cracks do not make an angle of less than 45° with the longitudinal axis of the beam is strongly supported by the tests of Chinenkov (1959), Pandit (1965) and this investigation.

The third, fourth and fifth assumptions are introduced to simplify the analysis.

The sixth assumption is strongly supported by the tests of Chinenkov (1959), Lyalin (1959), Yudin (1964), Evans and Sarkar (1965) and this investigation. However, Lessig (1961) has suggested that this assumption is true only within a specific range of the ratio of transverse to longitudinal reinforcement.

The seventh assumption is an idealization which is introduced for simplicity.

The eighth assumption is true only for pure bending i.e. $\phi = 0$. As torsion is added and the value of ϕ is increased, the compression zone is subjected to stress conditions which are complex and which are further complicated by inclined cracks crossing the compression zone. This assumption is discussed further in CHAPTER VIII.

5-4 Failure Surfaces

(a) Mode 1

The failure surface for this mode of failure is illustrated in FIGURE 5-1. Three sides of the failure surface are defined by a spiral crack which is located on the bottom and the two vertical faces of the beam. The ends of this spiral crack are joined by a compression zone located at the top face of the beam.

The notation used in the equations which follow is:

b = beam width

h = beam depth

L_1 = length of the compression zone

c_1 = length of the warped failure plane projected on the longitudinal axis of the beam

c_{m1} = the maximum value that c_1 can have

A_{s1} = area of longitudinal reinforcement at the bottom face of the beam

f_{y1} = yield stress of longitudinal reinforcement

a_v = area of one leg of a tie

f_{yt} = yield stress of tie

S = spacing of ties

a_1 = distance from bottom face of beam to centroid of longitudinal reinforcement

a_{1t} = distance from bottom face of beam to centroid of bottom leg of tie

a_{2t} = distance from vertical face of beam to centroid of vertical leg of tie

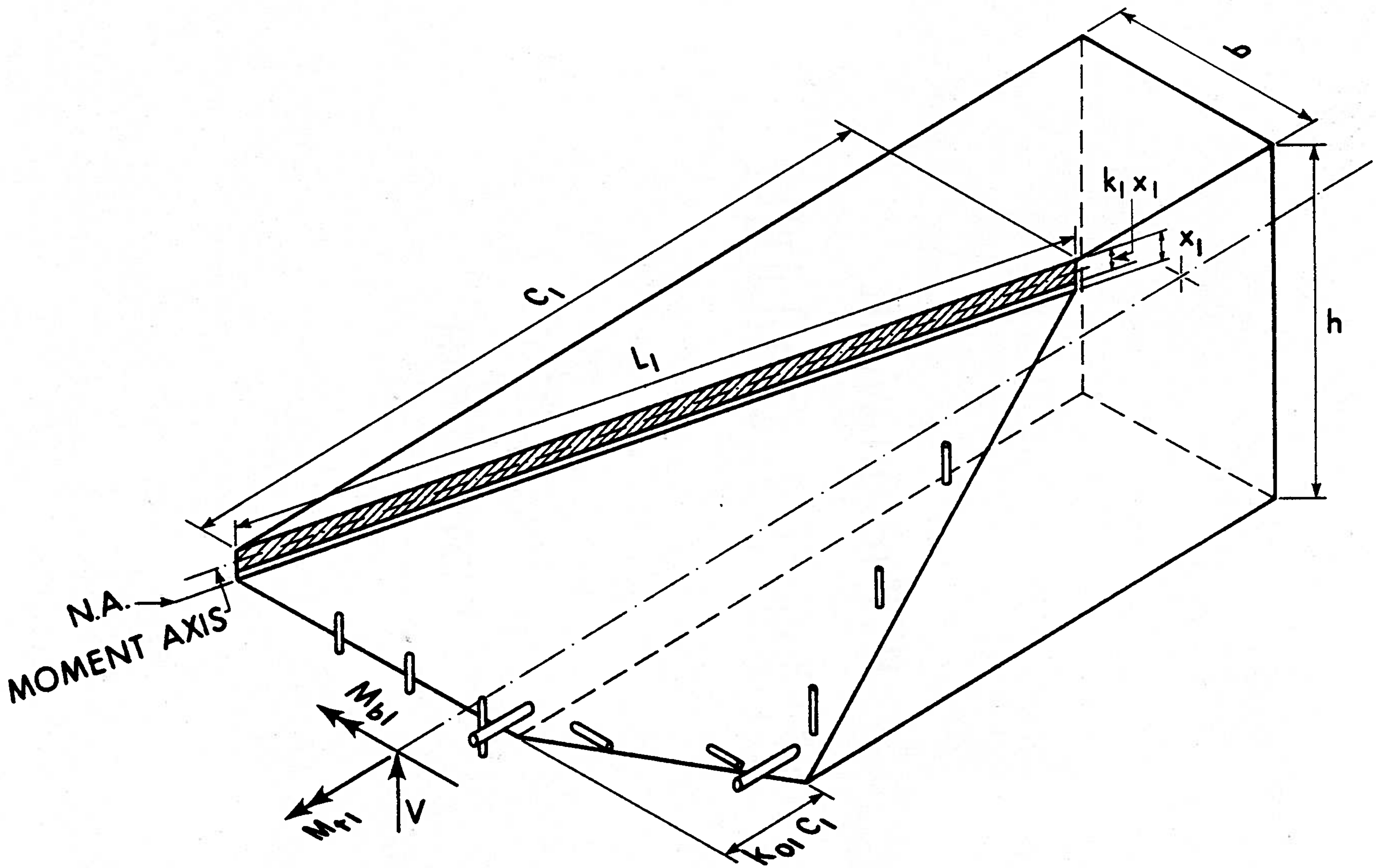


FIGURE 5 - 1 MODE 1 FAILURE SURFACE

x_1 = distance from top face of the beam to the neutral axis

f'_c = compressive strength of concrete determined by tests on 6 x 12 inch concrete cylinders

k_{01} = the ratio between the length of the failure crack on the bottom face when projected on the longitudinal axis of the beam and the total length of the failure plane measured parallel to the longitudinal axis

M_{t1} = ultimate twisting moment of the beam according to this mode of failure

M_{tt} = test value of ultimate twisting moment of the beam

M_{b1} = ultimate bending moment of the beam according to this mode of failure

M_{bt} = test value of ultimate bending moment of the beam

A rectangular stress block in the concrete compression zone is used. A concrete stress intensity of 0.85 f'_c is assumed to be uniformly distributed over an equivalent compression zone. This zone is bounded by the edges of the cross section and a straight line parallel to the neutral axis at a distance equal to $k_1 x_1$ from the top face of the beam. The value of k_1 is defined as follows:

For $f'_c \leq 4000$ psi $\qquad k_1 = 0.85$

For $f'_c > 4000$ psi $\qquad k_1 = 0.85 - 0.05 \dfrac{(f'_c - 4000)}{1000}$

From equilibrium of internal and external moments about an axis parallel to the neutral axis and located at mid-depth of the equivalent compression zone, the following equation is obtained.

$$M_{t1} = A_{s1} f_{y1} \left[\frac{z_1 + p_1 y_1 c_1^2/bh}{\frac{c_1}{b} + \frac{1}{\phi}} \right] \tag{5-1}$$

where
$z_1 = h_{01} - k_1 x_1/2$

$h_{01} = h - a_1$

$\phi = M_{t1}/M_{b1}$

$p_1 = \dfrac{f_{yt} a_v h}{f_{y1} A_{s1} S}$

$y_1 = k_{01} \left(h - a_{1t} - \dfrac{k_1 x_1}{2}\right) + \dfrac{b}{4} (1 - k_{01})(1 - k_{01} - 4a_{2t}/b)$

$k_{01} = b/(2h + b)$

The external moments about this axis are the components of twisting moment and bending moment. The internal moments about this axis are the moments of the forces in the longitudinal reinforcement, the bottom leg of the ties and the vertical legs of the ties. Detailed derivation of this and subsequent equations is presented in Appendix C.

From equilibrium of forces acting perpendicular to the plane which contains the neutral axis and is perpendicular to the top face of the beam, the following equation is obtained.

$$x_1 = \frac{A_{s1} f_{y1} \left(b + \frac{p_1 k_{01} c_1^2}{h}\right)}{0.85\ k_1 f'_c\ L_1^2} \tag{5-2}$$

To determine the value of c_1 corresponding to the minimum value of M_{t1}, Equation (5-1) is differentiated with respect to c_1, equated to zero and solved for c_1. The result is:

$$c_1 = -\frac{b}{\phi} + b\left[\frac{1}{\phi^2} + \frac{z_1 h}{p_1 y_1 b}\right]^{1/2} \tag{5-3}$$

Since negative values of c_1 have no physical significance the minimum value of c_1 is zero. In accordance with the second assumption of SECTION 5-3 and neglecting the depth of the compression zone, the maximum value of c_1 is:

$$c_{m1} = 2h + b \tag{5-4}$$

To solve these equations, the value of ϕ and the necessary properties of the section must be computed. The equations may then be solved by an iterative procedure. The initial value of x_1 is assumed to be zero and initial values of z_1 and y_1 are computed. The value of c_1 can then be computed by Equations (5-3) and (5-4). This value of c_1 is used in Equation (5-2) to calculate a value for x_1. The calculated value of x_1 is then used to compute new values of z_1 and y_1 and the procedure is repeated. With each iteration the difference between the initial and the computed value of x_1 decreases. The process is continued until the desired degree of accuracy is obtained. The ultimate twisting moment is then calculated using Equation (5-1).

(b) Mode 2

The failure surface for this mode of failure is illustrated in FIGURE 5-2. Three sides of the failure surface are defined by a spiral crack which is located on one of the vertical faces of the beam and the two horizontal faces. The ends of this spiral crack are joined by a compression zone located on one side of the beam.

The notation used is the same as for mode 1 with the following exceptions:

L_2 = length of the compression zone

c_2 = length of the warped failure plane projected on the longitudinal axis of the beam

c_{m2} = the maximum value that c_2 can have

A_{s2} = area of longitudinal reinforcement at the vertical face of the beam opposite to the face on which the compression zone is located

f_{y2} = yield stress of longitudinal reinforcement

a_2 = distance from vertical face of beam to centroid of longitudinal reinforcement

x_2 = distance from vertical face of beam on which compression zone is located to the neutral axis

k_{02} = the ratio between the length of the failure crack on the vertical face when projected on the longitudinal axis of the beam and the total length of the failure plane measured parallel to the longitudinal axis

M_{t2} = ultimate twisting moment of the beam according to this mode of failure

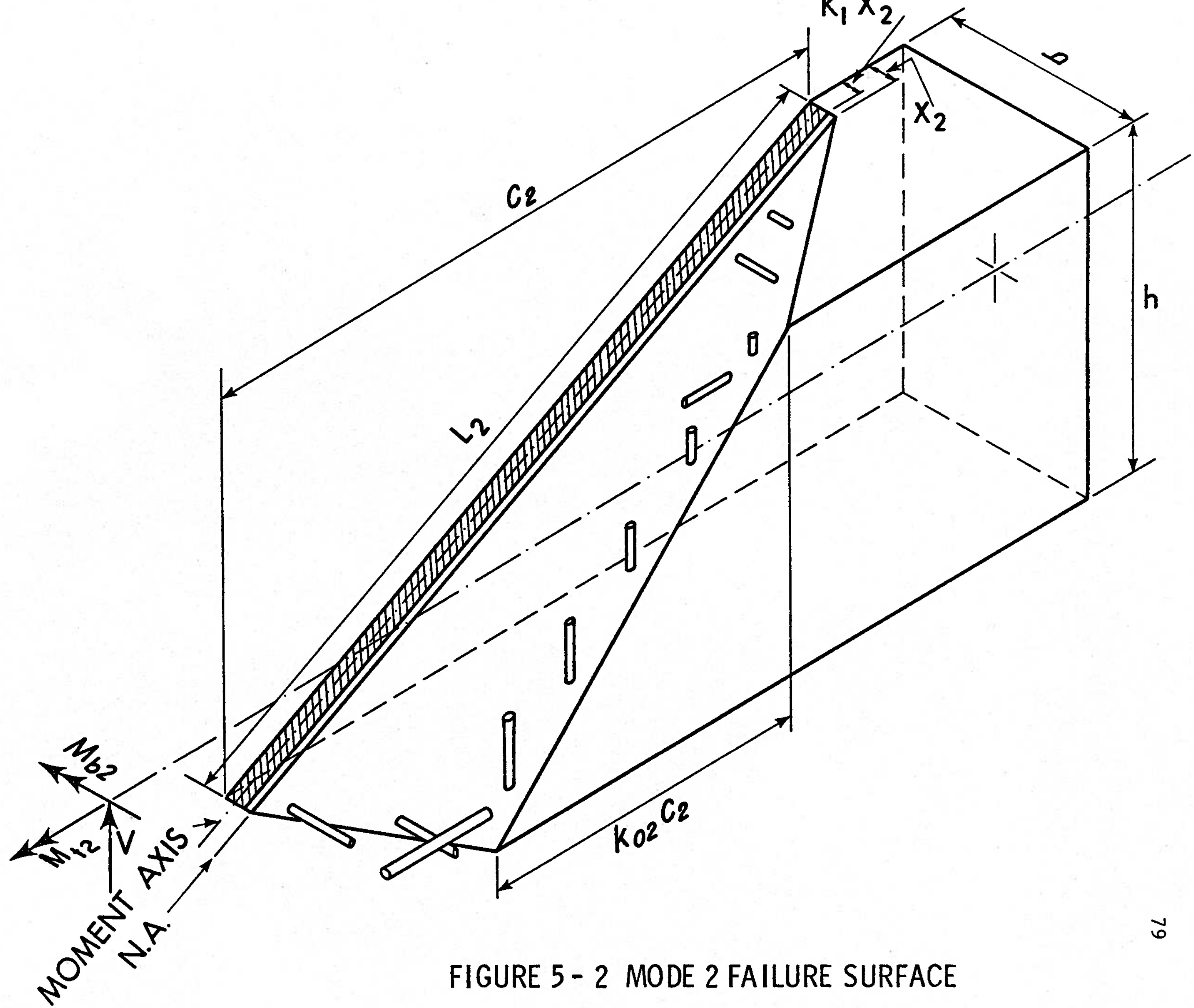


FIGURE 5-2 MODE 2 FAILURE SURFACE

V = the theoretical ultimate transverse shear force

V_t = test value of the ultimate transverse shear force

The stress block in the compression zone is considered to be the same as for mode 1. The transverse shear is assumed to be concentrated at mid width of the beam.

From equilibrium of internal and external moments about an axis parallel to the neutral axis and at a distance $k_1x_2/2$ from the vertical face of the beam, the following equation is obtained.

$$M_{t2} = \frac{A_{s2}f_{y2}h}{c_2}\left[\frac{z_2 + p_2y_2c_2{}^2/bh}{1 + \delta - \dfrac{\delta\, k_1x_2}{b}}\right] \qquad (5\text{-}5)$$

where $z_2 = b_{02} - k_1x_2/2$

$$b_{02} = b - a_2$$

$$y_2 = k_{02}(b - a_{2t} - \frac{k_1x_2}{2}) + (1 - k_{02})\,\frac{h}{4}\,(1 - k_{02} - \frac{4a_{1t}}{h})$$

$$p_2 = \frac{f_{yt}\ a_v\ b}{f_{y2}A_{s2}\ S}$$

$$\delta = \frac{V\ b}{2M_{t2}}$$

$$k_{02} = h/(2b + h)$$

From equilibrium of forces acting perpendicular to the plane that contains the neutral axis and is perpendicular to the vertical face of the beam on which the compression zone is located, the following equation is obtained.

$$x_2 = \frac{A_{s2}\, f_{y2} \left[h + \frac{p_2 k_{02} c_2^{\,2}}{b}\right] - V\, c_2}{0.85\, k_1 f'_c\, L_2^{\,2}} \tag{5-6}$$

An expression for c_2 is determined in the same manner as for mode 1. The resulting equation is:

$$c_2 = \left[\frac{h\, z_2\, b}{p_2 y_2}\right]^{1/2} \tag{5-7}$$

The maximum value that c_2 can have is:

$$c_{m2} = 2b + h \tag{5-8}$$

After the necessary properties of the section and the value of δ have been computed, the equations can be solved by iteration. Assuming that x_2 and V are equal to zero, z_2 and y_2 are computed. Equations (5-7) and (5-8) are then used to compute c_2. Using this value of c_2, Equation (5-6) is used to determine x_2 after which new values of z_2 and y_2 are computed. This procedure is repeated until the difference between the values of x_2 computed from any two consecutive iterations is less than a specified value. The twisting moment is then computed by Equation (5-5) and is used to compute a new value of V by the following equation:

$$V = \frac{2M_{t2}\delta}{b} \tag{5-9}$$

Again assuming x_2 to be zero but using the value of V obtained from Equation (5-9), the above procedure is repeated until a new value of V is obtained. When the difference between the values of V computed from any two consecutive iterations is less than a specified value, the procedure is terminated. The values of M_{t2} and V determined in the last iteration are the theoretical ultimate values of twisting moment and transverse shear.

(c) <u>Mode 3</u>

The failure surface for this mode of failure is illustrated in FIGURE 5-3. Three sides of the failure surface are defined by a spiral crack which is located on the top and the two vertical faces of the beam. The ends of this spiral crack are joined by a compression zone located at the bottom face of the beam.

The notation used is the same as for mode 1 with the following exceptions:

L_3 = length of the compression zone

c_3 = length of the warped failure plane projected on the longitudinal axis of the beam

c_{m3} = the maximum value that c_3 can have

A_{s3} = area of longitudinal reinforcement at the top face of the beam

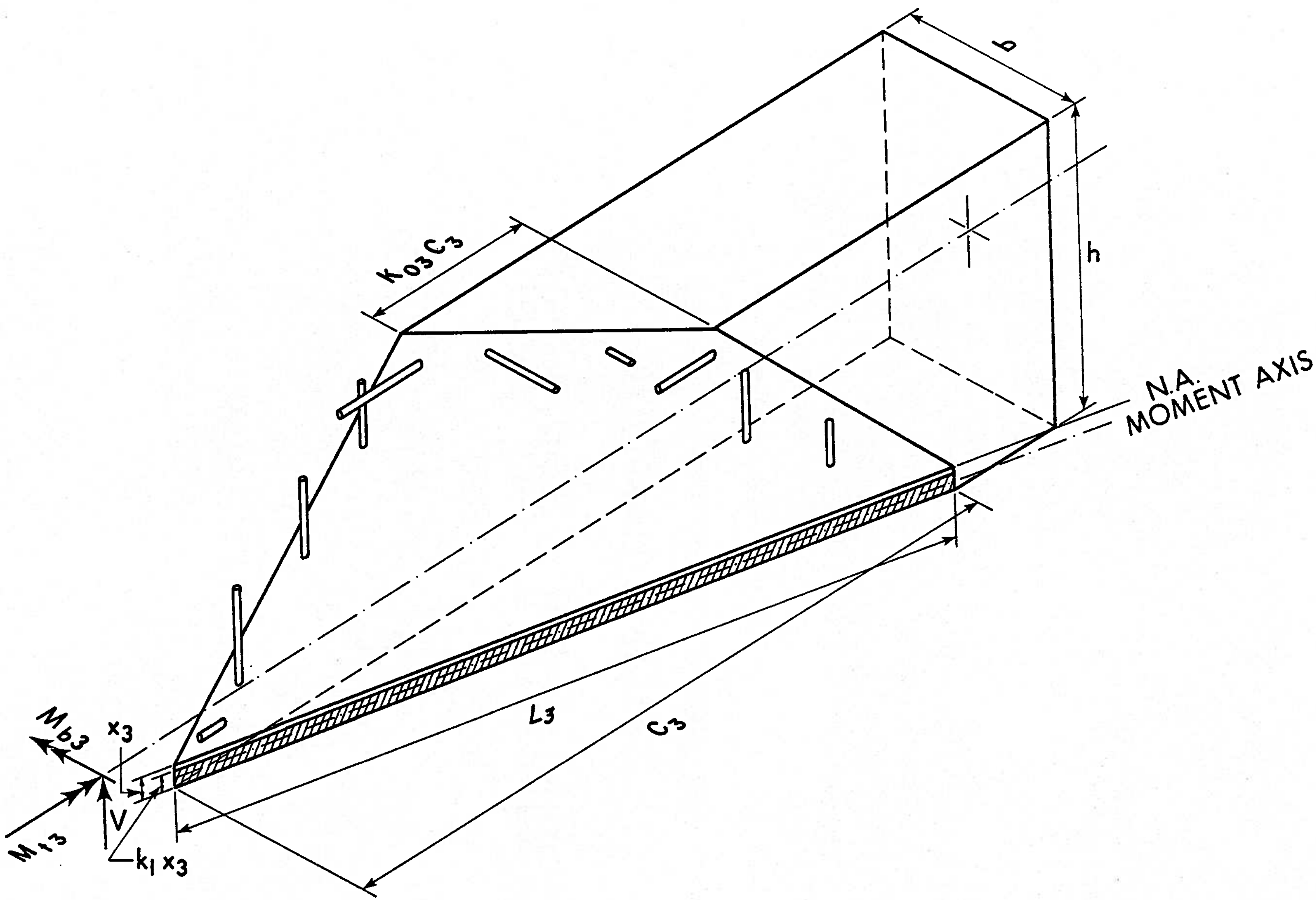


FIGURE 5-3 MODE 3 FAILURE SURFACE

f_{y3} = yield stress of the longitudinal reinforcement

a_3 = distance from top face of the beam to centroid of longitudinal reinforcement

a_{3t} = distance from top face of beam to centroid of top horizontal leg of tie

x_3 = distance from the bottom face of the beam to the neutral axis

k_{03} = the ratio between the length of the failure crack on the top face when projected on the longitudinal axis of the beam and the total length of the failure plane measured parallel to the longitudinal axis

M_{t3} = ultimate twisting moment of the beam according to this mode of failure

M_{b3} = ultimate bending moment of the beam according to this mode of failure

The stress block in the compression zone is considered to be the same as for mode 1.

From equilibrium of internal and external moments about an axis parallel to the neutral axis and at a distance $k_1x_3/2$ from the bottom face of the beam, the following equation is obtained.

$$M_{t3} = A_{s3}f_{y3}\left[\frac{z_3 + p_3y_3c_3^{\,2}/bh}{\frac{c_3}{b} - \frac{1}{\phi}}\right] \tag{5-10}$$

where $z_3 = h_{03} - k_1 x_3/2$

$h_{03} = h - a_3$

$y_3 = k_{03} (h - a_{3t} - \frac{k_1 x_3}{2}) + \frac{b}{4} (1 - k_{03}) (1 - k_{03} - \frac{4a_{2t}}{b})$

$p_3 = \frac{f_{yt} a_v h}{f_{y3} A_{s3} S}$

$\phi = M_{t3}/M_{b3}$

$k_{03} = b/(2h + b)$

From equilibrium of forces acting perpendicular to the plane which contains the neutral axis and is perpendicular to the bottom face of the beam, the following equation is obtained.

$$x_3 = \frac{A_{s3} f_{y3} (b + \frac{p_3 k_{03} c_3^2}{h})}{0.85 k_1 f'_c L_3^2} \tag{5-11}$$

The value of c_3 for which M_{t3} is a minimum is given by:

$$c_3 = \frac{b}{\phi} + b \left[\frac{1}{\phi^2} + \frac{z_3 h}{p_3 y_3 b} \right]^{1/2} \tag{5-12}$$

The maximum value that c_3 can have is:

$$c_{m3} = 2h + b \tag{5-13}$$

After the necessary properties of the section and the value of ϕ have been computed, the equations can be solved by an iterative procedure identical to that used for mode 1.

5-5 Theoretical Ultimate Twisting Moment

The iterative procedure that was used to calculate x_1, x_2, and x_3 was continued until the values calculated in any two consecutive iterations did not differ by an amount greater than 0.001". Similarly, the iterative procedure for mode 2 was continued until the values of V computed in any two successive iterations did not differ by an amount greater than 0.01 kip.

Three theoretical ultimate twisting moments, M_{t1}, M_{t2} and M_{t3}, are obtained for each beam by using Equations (5-1), (5-5) and (5-10). The theoretical ultimate twisting moment is designated as M_t and is equal to the smallest of the three values. The predicted mode of failure is designated as the mode that yields the smallest value of theoretical ultimate twisting moment.

CHAPTER VI

EVALUATION OF TEST RESULTS

6-1 Introduction

This chapter compares the experimental results presented in CHAPTER IV with the theory presented in CHAPTER V. Bending moment versus twisting moment interaction diagrams based on the equations in CHAPTER V are presented. The failure values of twisting moment and bending moment are also shown on the interaction diagrams. Part of an interaction surface for bending moment, twisting moment and shear force is also presented.

The correlation between the experimental results of 109 beams and the theory presented in CHAPTER V is presented in TABLES 6-1 to 6-7. Of these beams, 34 were tested in this investigation and 75 were tested by other investigators.

6-2 Beams Tested in Combined Bending and Torsion

(a) Effect of Top Reinforcement

The beams of Groups 1 and 2 were similar in all respects except for the size of the top longitudinal reinforcement. The top reinforcement for beams of Group 1 consisted of 2#3 bars whereas for beams of Group 2 this reinforcement was 2#6. The bottom longitudinal reinforcement for both groups was 2#6.

The twisting moment versus bending moment interaction diagrams for the beams of Groups 1 and 2 are shown in FIGURE 6-1. The lines labeled mode 1, mode 2 and mode 3 were obtained by solving Equations (5-1), (5-5) and (5-10) respectively for values of ϕ ranging from 0 (pure bending) to ∞ (pure torsion). The material and geometrical properties of the beam used in solution of these equations are the average values for the group. The failure values of twisting moment and bending moment are also shown.

(b) Sequence of Loading

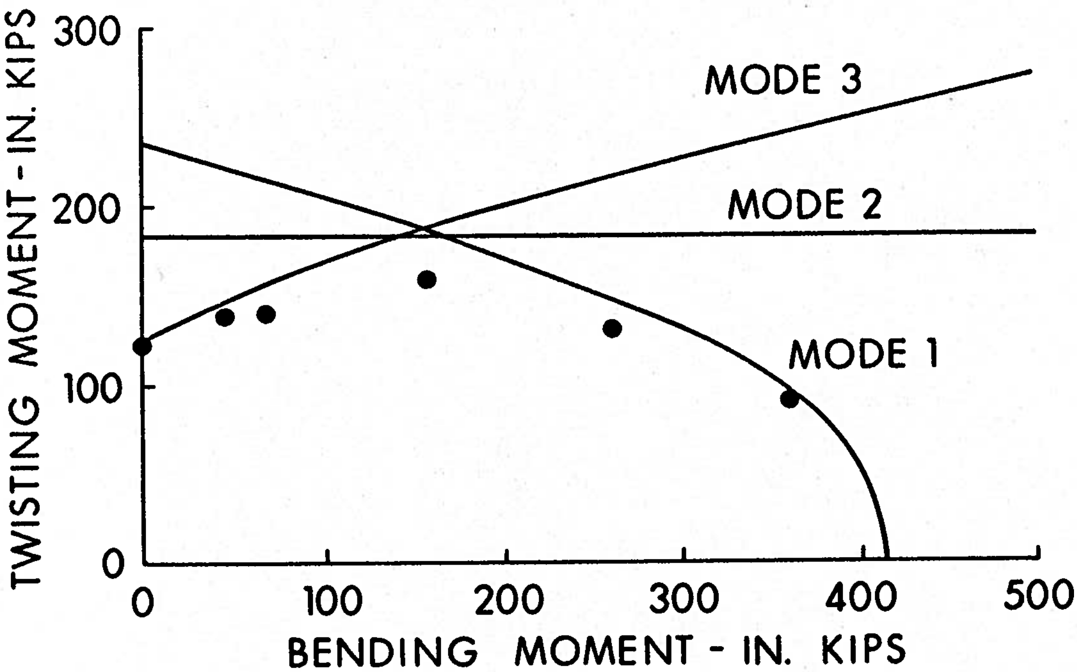
The beams of Group 3 were similar to Group C tested by Pandit (1965). Pandit's beams were tested by first applying the transverse load and then twisting the beam to failure. The beams of Group 3 were tested under various sequences of loading. The loading paths for these beams and the failure values of twisting moment and bending moment are shown in FIGURE 6-2.

The interaction diagram obtained from Equations (5-1), (5-5) and (5-10) is shown in FIGURE 6-2. The properties of the beam used in solution of these equations are the average values for the nine beams considered.

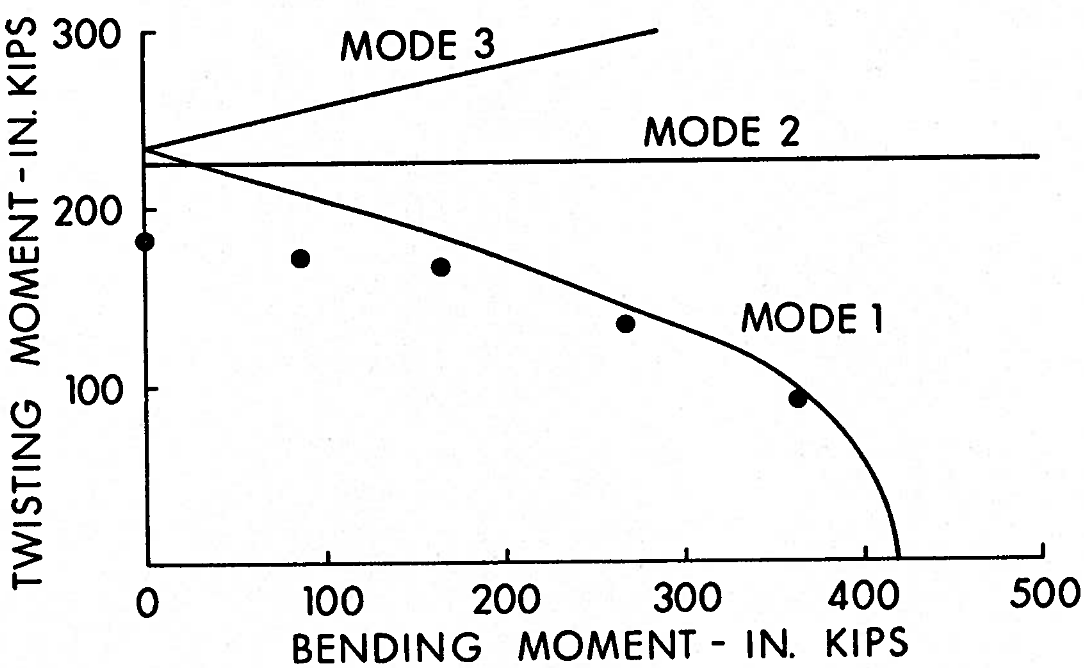
(c) Effect of Stirrup Spacing

(i) General Remarks

Assumption five of SECTION 5-3 states that the cross section of transverse reinforcement per unit length of the beam is constant over any section of its length. Since this assumption is an idealization in that



(a) GROUP 1



(b) GROUP 2

FIGURE 6-1 INTERACTION DIAGRAMS FOR BEAMS OF GROUPS 1 AND 2

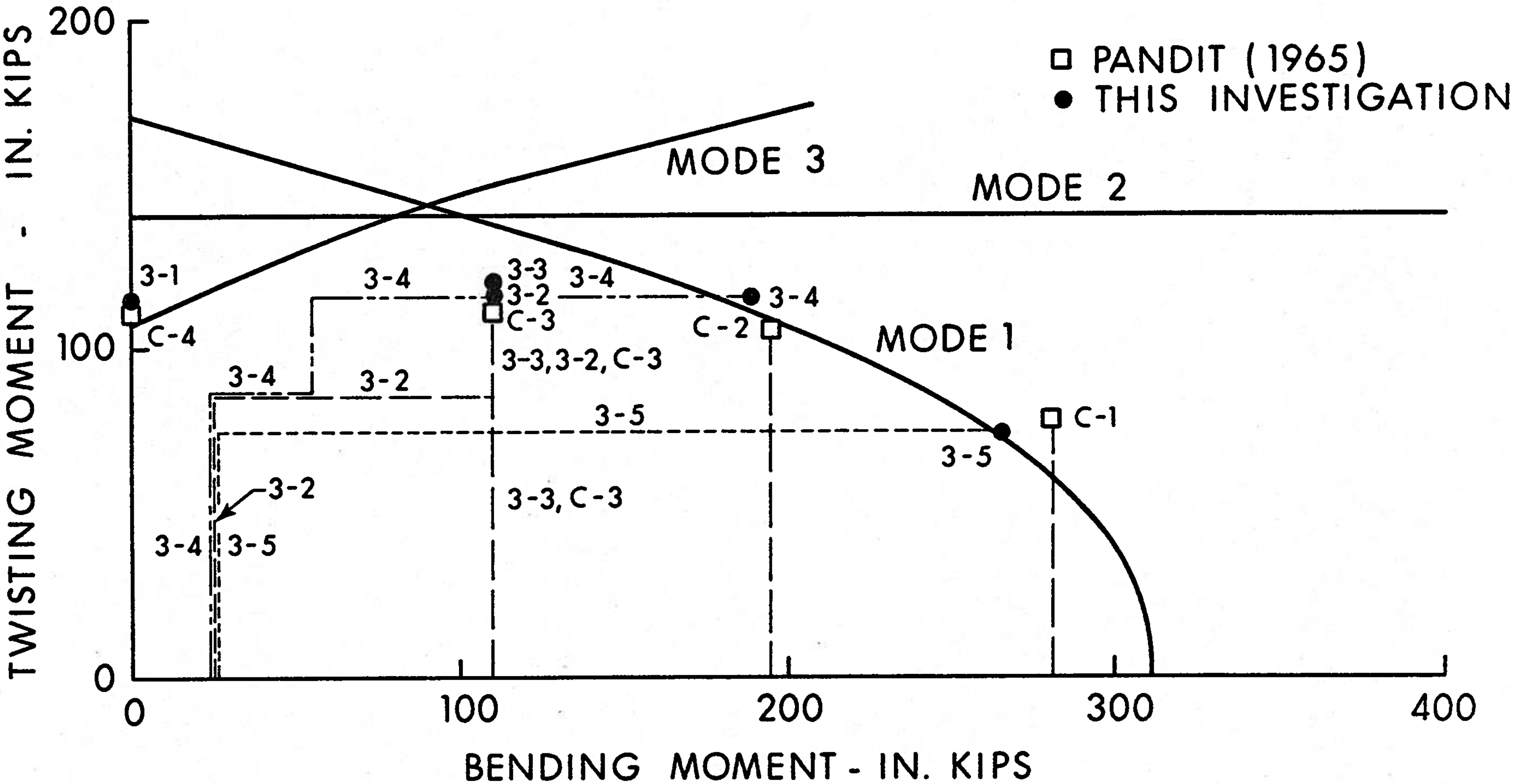


FIGURE 6 - 2 INTERACTION DIAGRAM FOR BEAMS OF GROUP 3

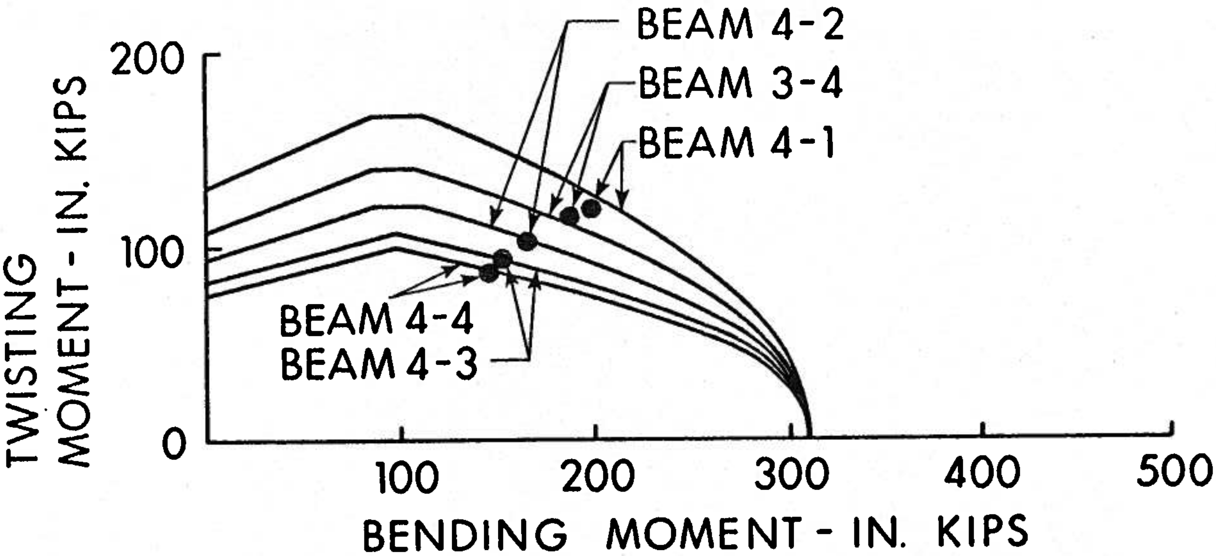
the cross section of transverse reinforcement is concentrated at the tie locations, Lessig (1959) suggested that a correction coefficient be applied to the area of transverse reinforcement. The value suggested for this correction coefficient or efficiency factor was 0.7 or 0.8. To compensate for the transverse reinforcement being concentrated at the tie locations, Pandit (1965) assumed that the ties that are effective in resisting torsion are the least number which are crossed by a potential crack making an angle of 45° with the longitudinal axis of the beam.

(ii) <u>Beams With Similar Longitudinal Reinforcement</u>

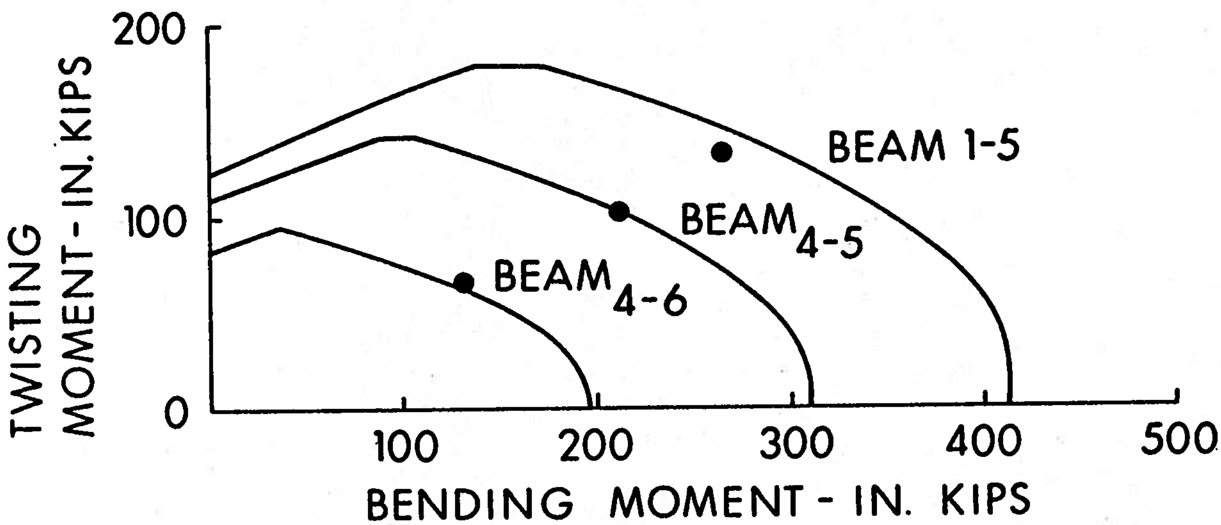
Beams 4-1, 4-2, 4-3, and 4-4 were similar in all respects except the spacing of ties. The tie spacing of Beams 4-1 to 4-4 was 3, 6, 7 1/2 and 9 inches respectively. The tie spacing of Beam 3-4 was 4 1/2 inches. All beams except Beam 3-4 were tested with $\emptyset$ equal to 0.61. The loading sequence for Beam 3-4 is indicated in FIGURE 6-2. The value of $\emptyset$ at failure for this beam was 0.61.

The interaction diagrams obtained from Equations (5-1), (5-5) and (5-10) for each of these beams are shown in FIGURE 6-3(a). In these equations, the total area of transverse reinforcement has been considered effective in resisting the twisting moment. The failure values of twisting moment and bending moment are also shown.

The ratio of the experimental value of twisting moment to the theoretical value of twisting moment for each beam is plotted as a function of tie spacing in FIGURE 6-4(a).

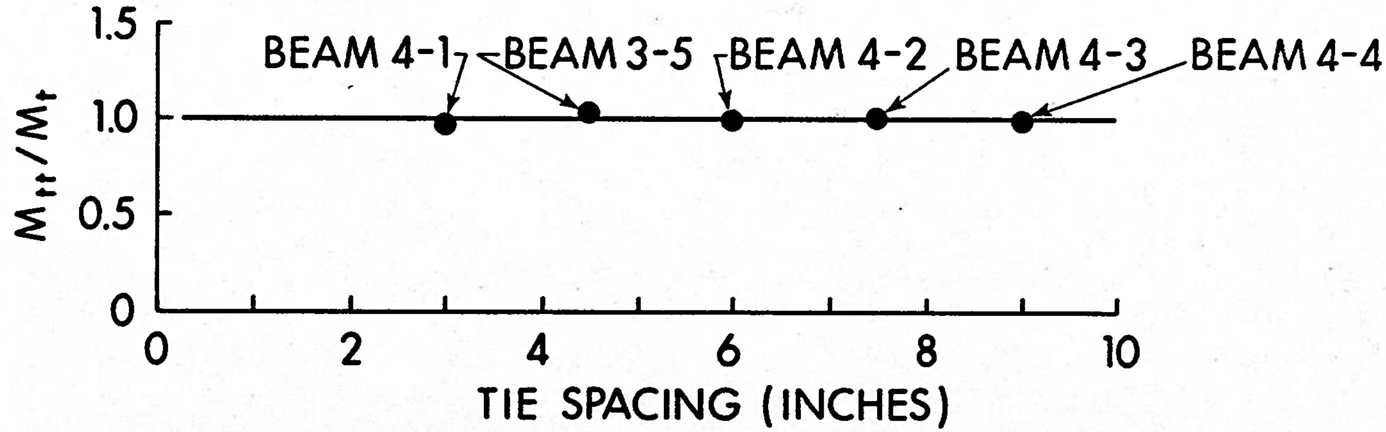


(a) SIMILAR LONGITUDINAL REINFORCEMENT AND DIFFERENT TIE SPACING

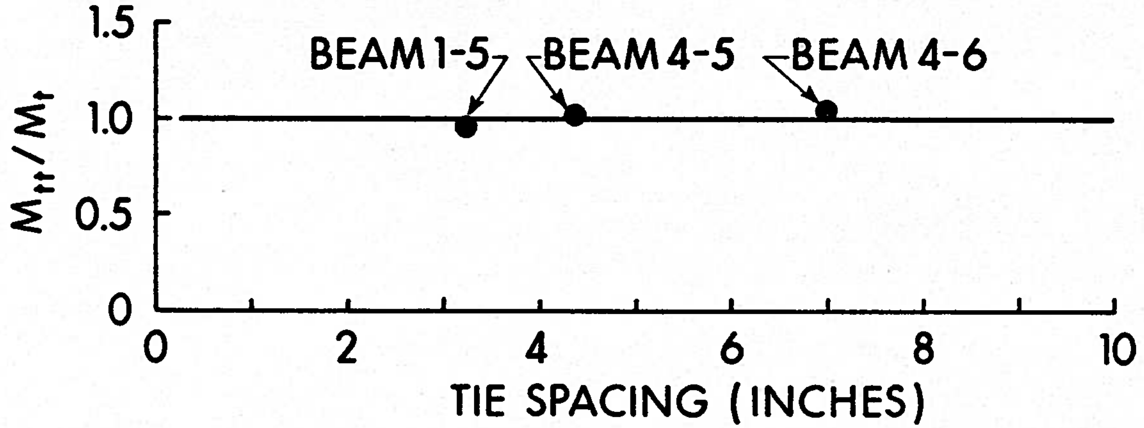


(b) DIFFERENT BOTTOM LONGITUDINAL REINFORCEMENT AND DIFFERENT TIE SPACING

FIGURE 6-3 INTERACTION DIAGRAMS FOR BEAMS WITH DIFFERENT REINFORCEMENT



(a) SIMILAR LONGITUDINAL REINFORCEMENT AND DIFFERENT TIE SPACING



(b) DIFFERENT BOTTOM LONGITUDINAL REINFORCEMENT AND DIFFERENT TIE SPACING

FIGURE 6-4 RATIO BETWEEN EXPERIMENTAL AND THEORETICAL TWISTING MOMENTS AS A FUNCTION OF TIE SPACING

(iii) Beams With Different Longitudinal Reinforcement

Beams 4-5 and 4-6 of Group 4 and Beam 1-5 of Group 1 were similar in all respects except for the spacing of ties and the size of bottom reinforcement. The tie spacing and bottom reinforcement was designed to make the quantity p_1 a constant for the three beams. All three beams were tested with ϕ equal to 0.50.

The interaction diagrams obtained from Equations (5-1), (5-5) and (5-10) for each of these beams are shown in FIGURE 6-3(b). The failure values of twisting moment and bending moment are also shown. FIGURE 6-4(b) shows the ratio of the experimental value of twisting moment to the theoretical value of twisting moment for each beam plotted as a function of tie spacing.

6-3 Beams Tested in Combined Bending, Torsion and Shear

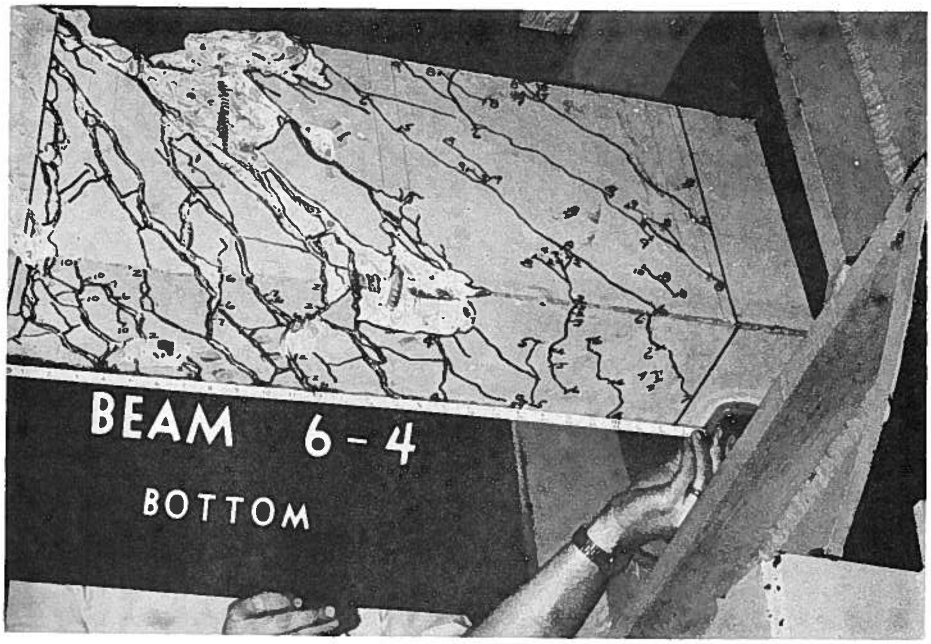
(a) Effect of Variable Bending Moment Within the Gauge Length

In deriving Equations (5-1) and (5-10) the assumption was made that the ratio of twisting moment to bending moment, ϕ , was known. This ratio is constant over the gauge length of a beam subjected to bending and torsion with no transverse shear. However, when transverse shear is present the bending moment and consequently ϕ are variable.

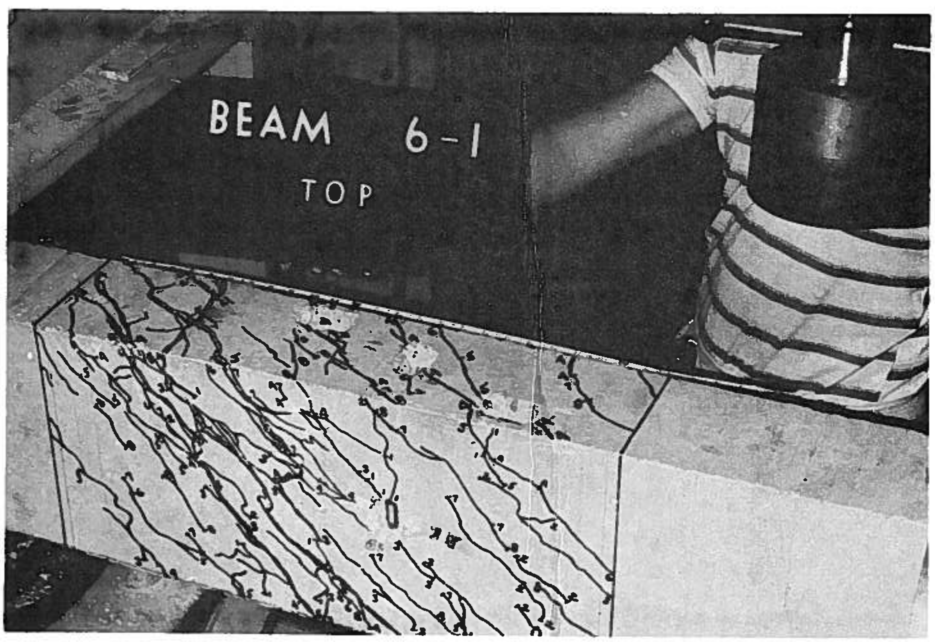
An examination of Equation (5-1) reveals that the minimum value of M_{t1} is obtained for the minimum value of ϕ within the gauge length. In a beam subjected to a constant torsional moment and a linearly varying bending moment, the minimum value of ϕ within the gauge length occurs at the cross section subjected to the maximum bending moment. For the beams of Groups 5, 6 and 7 this cross section was located at the west end of the gauge length.

However, this presupposes that failure occurs at one cross section of the beam. According to FIGURE 5-1, a mode 1 failure occurs on a warped plane extending over a length of beam that is equal to c_1 . The extra reinforcement provided outside of the gauge length forced the failure plane to form within the gauge length. Consequently, it would seem that the failure plane should occur over a length of beam equal to c_1 that is located adjacent to the end of the gauge length subjected to the maximum bending moment. If this is true, the value of $\emptyset$ which should be used in Equation (5-1) is the value at a cross section located a distance $c_1/2$ from the west end of the gauge length. This would be the same as using the average bending moment over the length of the warped failure plane to compute the value of $\emptyset$ to be used in Equation (5-1).

In the beams of Groups 5, 6 and 7 that failed according to mode 1, the failure surface did occur within the region of the gauge length subjected to the highest bending moment. The failure surface appeared to be crowded toward the end of the gauge length and is illustrated by FIGURE 6-5(a). The failure surface in many of these beams appeared to be defined by two sets of spiral cracks. One of these sets contained a crack on the south face which originated at the top of the beam at the west end of the gauge length. This crack progressed down the south face at an angle and continued across the bottom face. The other set



(a) BEAM 6 - 4



(b) BEAM 6 - 1

FIGURE 6 - 5 CRACK PATTERN OF BEAMS 6 - 4 AND 6 - 1

of spiral cracks originated on the bottom face of the beam at the west end of the gauge length and continued up the north face. For a mode 1 failure the reinforcement adjacent to the bottom face of the beam contributes the major portion of the resisting moment. For this reason, there may be justification for considering the critical section to be located a distance equal to $k_{01}c_1/2$ from the cross section within the gauge length that is subjected to the maximum bending moment rather than $c_1/2$ as stated previously.

Equation (5-5), which gives the ultimate twisting moment corresponding to a mode 2 failure, does not contain the term $\emptyset$. Consequently, a linearly varying bending moment within the gauge length does not affect the location of the failure surface.

An examination of Equation (5-10) reveals that the minimum value of M_{t3} is obtained for the maximum value of $\emptyset$ within the gauge length. The maximum value of $\emptyset$ within the gauge length occurs at the cross section subjected to the minimum bending moment. For the beams of Groups 5, 6 and 7 this cross section was located at the east end of the gauge length. Consequently, the warped failure plane for a mode 3 failure should form over a length of the beam equal to c_3 , measured from the east boundary of the gauge length. In the beams of Groups 5, 6 and 7 that failed according to mode 3, the failure surface was located in the east portion of the gauge length. However, there was a tendency for this failure surface to crowd to the east extremity of the gauge length. This is illustrated by FIGURE 6-5(b). As a result, there may be justification for considering the critical section to be located a distance equal to $k_{03}c_3/2$ instead of $c_3/2$ from the cross section within the gauge length that is subjected to the minimum bending moment.

(b) Solution of Equations When the Bending Moment Varies

For beams having a varying bending moment in their gauge length the value of ϕ to be used in Equations (5-1) and (5-10) can not be established until c_1 and c_3 are known but c_1 and c_3 are in turn functions of ϕ. Consequently, an iterative procedure was used to solve Equations (5-1) and (5-10). The value of ϕ was first computed using the experimental value of twisting moment and the bending moment at the west end of the gauge length. Using this value of ϕ, the values of c_1 and c_3 were computed. With these values of c_1 and c_3 a critical section for a mode 1 failure and another critical section for a mode 3 failure were established. Next, the bending moment at each of these critical sections was calculated. Using these bending moments, a new value of ϕ was calculated for each critical section and the process was repeated. This procedure was continued for each of the critical sections until the difference between the values of ϕ computed in two consecutive iterations was less than one tenth of one percent of ϕ.

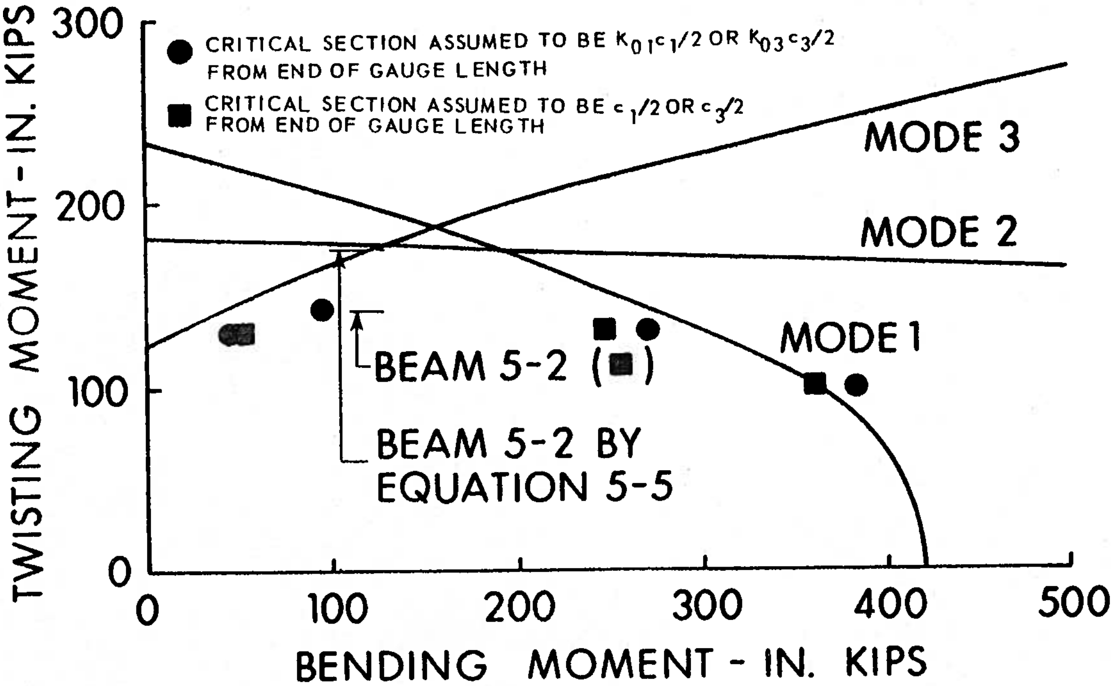
(c) Effect of Transverse Shear

(i) Comparison of Groups 1, 5 and 6

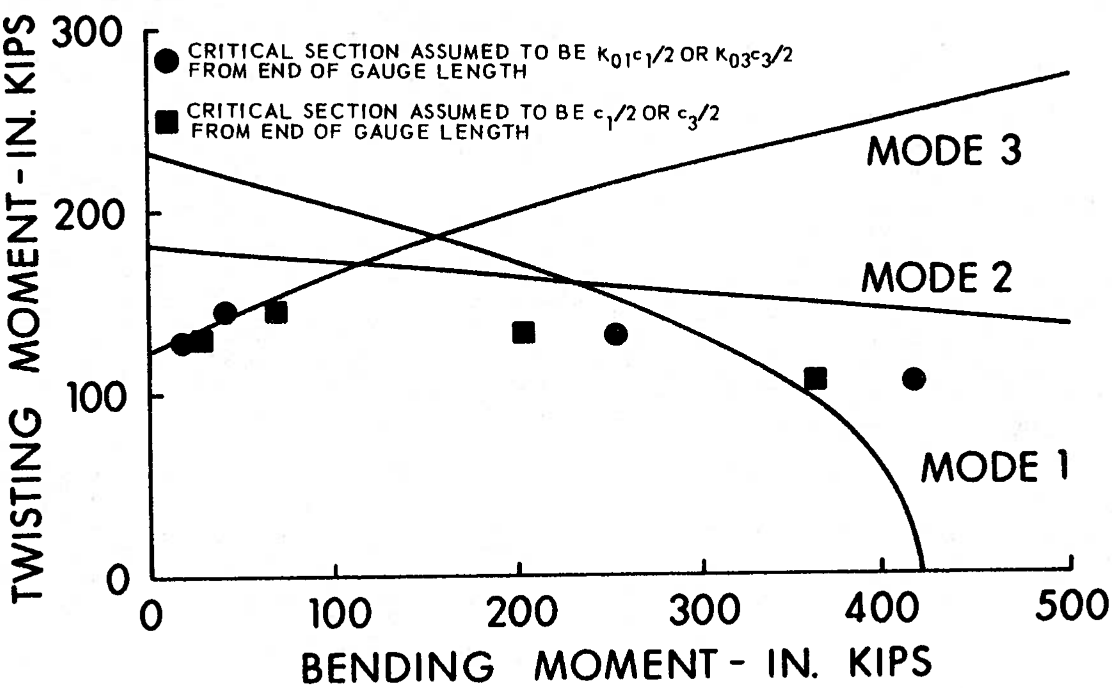
The beams of Groups 1, 5 and 6 were similar in all respects. The beams of Group 1 were subjected to a uniform twisting moment and a uniform bending moment over their gauge length. However, the beams of Groups 5 and 6 were tested with a single transverse load in addition to the twisting moment. Consequently, the beams of these groups were subjected to a uniform twisting moment, a linearly varying bending moment, and a uniform transverse shear. The magnitude of the transverse shear in any beam of Group 6 is approximately twice that of the corresponding beam of Group 5.

The interaction diagram for the beams of Group 1 is shown in FIGURE 6-1(a). The interaction diagrams for the beams of Groups 5 and 6 are shown in FIGURE 6-6. The lines labeled mode 1, mode 2 and mode 3 were obtained by solving Equations (5-1), (5-5) and (5-10) for various values of $\emptyset$. The properties of the beam used in solution of these equations are the average values for the group. Comparison of FIGURES 6-1(a), 6-6(a) and 6-6(b) shows that the curves corresponding to mode 1 and mode 3 are the same for the three interaction diagrams. The reason for this is that the transverse shear does not enter into Equations (5-1) or (5-10). However, the curve corresponding to mode 2, which was a horizontal line in FIGURE 6-1(a), is no longer horizontal in FIGURES 6-6(a) and 6-6(b). This is because the transverse shear does enter into Equation (5-5).

Two sets of failure values of twisting moment and bending moment for each beam have been plotted in FIGURE 6-6. One set of values represents the assumption that the critical section for a mode 1 or mode 3 failure occurs at a distance of $c_1/2$ or $c_3/2$ from the appropriate end of the



(a) GROUP 5



(b) GROUP 6

FIGURE 6 - 6 INTERACTION DIAGRAMS FOR BEAMS OF GROUPS 5 AND 6

gauge length and the other set of values represents the assumption that the critical section for a mode 1 or mode 3 failure occurs at a distance of $k_{01}c_1/2$ or $k_{03}c_3/2$ from the appropriate end of the gauge length.

When the critical section is assumed to be at a distance $k_{01}c_1/2$ or $k_{03}c_3/2$ from the appropriate end of the gauge length, the predicted mode of failure for all beams of Groups 5 and 6 is either mode 1 or 3. When the critical section is assumed to be at a distance $c_1/2$ or $c_3/2$ from the appropriate end of the gauge length, the predicted mode of failure for Beam 5-2 becomes mode 2. This mode of failure is independent of the bending moment and consequently, the failure values of twisting moment and bending moment are no longer represented on the interaction diagram by a point but by a horizontal line. The theoretical value of twisting moment calculated by Equation (5-5) for this beam is shown on the interaction diagram and is also a horizontal line. This line does not correspond to the line labeled mode 2 in FIGURE 6-6(a) because they are not in the same plane. This is further explained in SECTION 6-3(d).

(ii) Group 7

The beams of this group were tested under the same loading conditions as the beams of Group 6. However, in an effort to increase the ratio between the transverse shear force in the beam at failure and the ultimate transverse shear force which would be expected if the beam were not subjected to torsional moment, the size of the bottom longitudinal bars and the tie spacing were increased. This objective was only partially achieved since three of the beams of this group failed according to the mode 3 failure surface at relatively low bending moments. Consequently, the transverse shear in the section at failure was also relatively low.

The interaction diagram for the beams of this group is shown in FIGURE 6-7. The lines labeled mode 1, mode 2 and mode 3 were obtained by solving Equations (5-1), (5-5) and (5-10) for various values of ϕ . The properties of the beam used in solution of these equations are the average values for the group.

Two sets of failure values of twisting moment and bending moment for each beam are plotted in FIGURE 6-7. One set of values represents the assumption that the critical section for a mode 1 or mode 3 failure occurs at a distance $c_1/2$ or $c_3/2$ from the appropriate end of the gauge length and the other set of values represents the assumption that the critical section occurs at a distance of $k_{01}c_1/2$ or $k_{03}c_3/2$ from the appropriate end of the gauge length.

Changing the location of the assumed critical section does not change the predicted mode of failure for Beams 7-1 and 7-2 but does change the predicted mode of failure for Beam 7-3 from mode 3 to mode 2 and for Beam 7-4 from mode 1 to mode 2. Consequently, for Beams 7-3 and 7-4 the failure values and the theoretical values of twisting moment and bending moment corresponding to a critical section assumed to be $c_1/2$ or $c_3/2$ from the appropriate end of the gauge length are represented on the interaction diagram by horizontal lines. The lines representing the theoretical values of twisting moment and bending moment do not

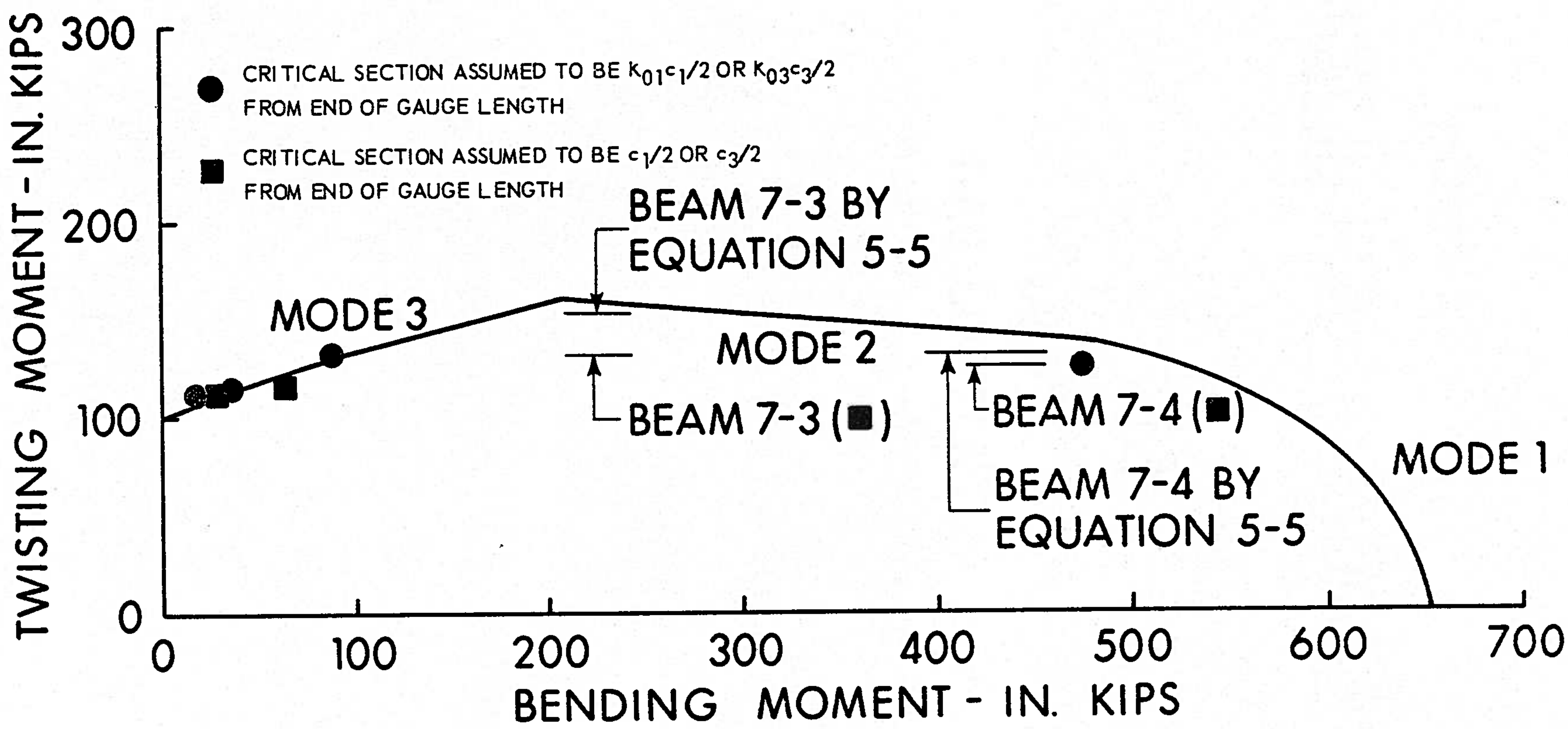


FIGURE 6 - 7 INTERACTION DIAGRAM FOR BEAMS OF GROUP 7

correspond to the line labeled mode 2 in FIGURE 6-7 because they are not in the same plane. This is explained in SECTION 6-3(d).

(d) Interaction Surface for Bending, Torsion and Shear

Part of a typical interaction surface for a beam subjected to bending, torsion and shear is shown in FIGURE 6-8. This surface is composed of three segments which have been labeled mode 1, mode 2 and mode 3. The mode 1 and the mode 3 segments are single curvature surfaces whereas the mode 2 segment is a plane that slopes downward in the direction of the shear axis. A level line in this plane is parallel to the bending moment axis. This interaction surface is intended to be valid only within the range of transverse shear force considered in this investigation and consequently is incomplete. The upper limit of the transverse shear force for which it is valid is discussed in CHAPTER VIII.

In the tests of Groups 5, 6 and 7, the bending moment at any particular cross section of a beam was a function of the transverse shear and since any one test was conducted at a constant value of ϕ , the twisting moment was also a function of the transverse shear. Consequently, the locus of points representing the twisting moment, bending moment and transverse shear at a cross section of the beam from the beginning of the test to failure is a straight line contained in a plane such as plane 0, 1, 2, 3 shown in FIGURE 6-8. An interaction diagram of the type shown in FIGURES 6-6 and 6-7 is obtained by first obtaining the lines of intersection between a plane such as 0, 1, 2, 3 and the interaction surface. These lines of intersection are then projected on the twisting moment-bending

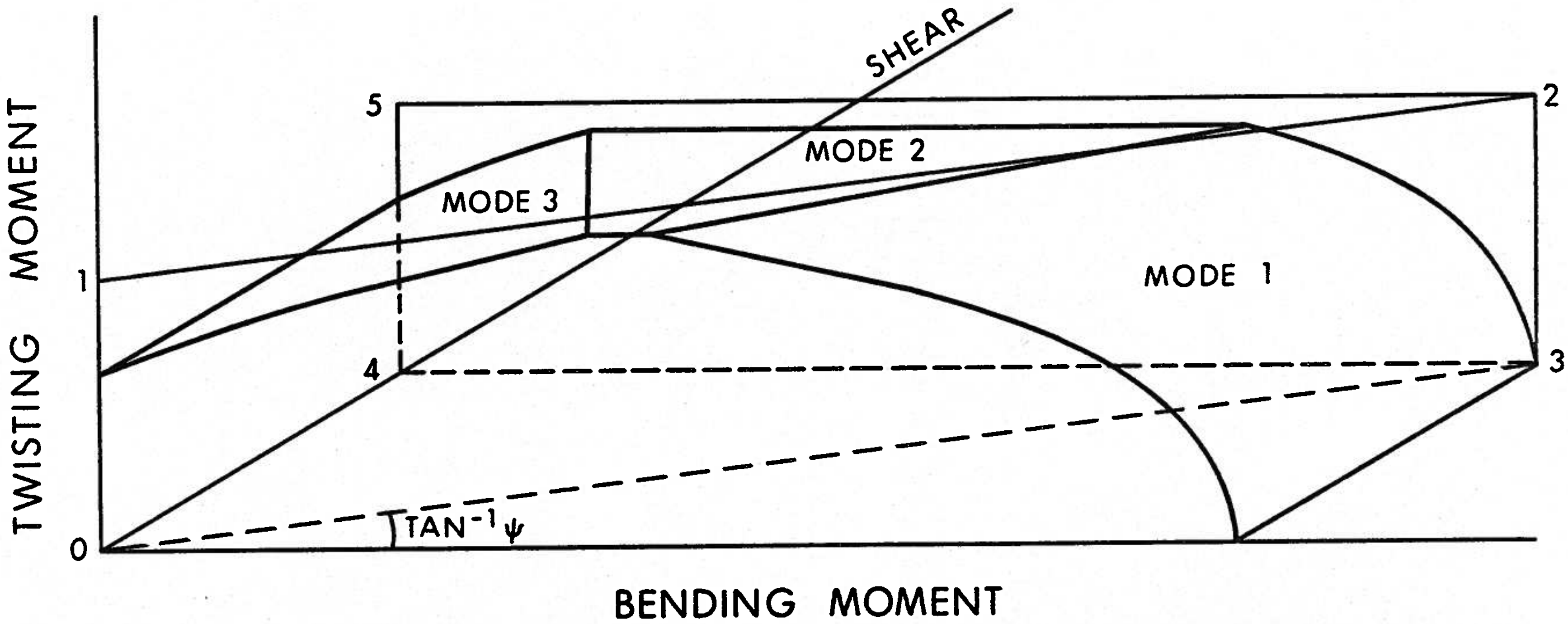


FIGURE 6 - 8 INTERACTION SURFACE FOR BENDING TORSION AND SHEAR

moment plane. The shape of the interaction diagram depends on the value of $\psi = V_t/M_{bt}$. The interaction diagrams shown in FIGURES 6-1, 6-2 and 6-3 correspond to a value of ψ equal to zero.

The beams of Groups 5, 6 and 7 were subjected to a constant twisting moment, a constant shear force and a linearly varying bending moment. Consequently, at any particular stage of a test the twisting moment, bending moment and shear force at all cross sections within the gauge length are represented by points in a plane such as plane 2, 3, 4, 5, shown in FIGURE 6-8. The twisting moment and transverse shear force are the same for all cross sections within the gauge length but the bending moment at each cross section is different. Consequently, changing the location of the cross section that is being considered causes the point representing the twisting moment, bending moment and transverse shear to move in a direction perpendicular to the shear-twisting moment plane.

6-4 Over Reinforced Beams

Assumption four of SECTION 5-3 states that all reinforcing bars which cross the failure plane outside of the compression zone yield at failure. If the amount of longitudinal and transverse reinforcement in a beam is increased sufficiently, the concrete in the compression zone will fail before the tension reinforcement yields. A beam such as this is said to be over reinforced.

Lessig (1961), on the basis of test results, derived two equations to be used to determine if the concrete in the compression zone will fail before the tension reinforcement yields. The first of these equations establishes a maximum value for x_1 and, according to Lessig, is valid for $\emptyset \leq 0.2$. This equation states that if a beam is not over reinforced,

$$x_1 \leq h_{01} \left[0.55 - 0.7(\emptyset)^{1/2}\right] \tag{6-1}$$

Lessig states that the second of these equations is valid for $\emptyset > 0.2$. This equation is:

$$M_{tr} = 0.07\ b^2 h(0.85 k_1 f'_c)$$

$$M_{tr} = 0.06\ b^2 h k_1 f'_c \tag{6-2}$$

where M_{tr} is equal to the maximum torsional moment that can be resisted by a beam that fails due to crushing of the concrete before the reinforcement yields.

6-5 Ratio of Transverse to Longitudinal Reinforcement

Assumption four of SECTION 5-3 states that all reinforcing bars crossing the failure plane outside of the compression zone yield at failure. It is possible that at failure only the longitudinal or only the transverse reinforcement yields, but not both. To ensure that the assumption that both types of reinforcement yield is not violated, it is necessary to restrict the ratio between the transverse and the longitudinal reinforcement within certain limits.

Lessig (1961) defined the optimum ratio between transverse and longitudinal reinforcement as the ratio that makes the product of the volume of reinforcement per unit length of beam and the yield stress of the reinforcement a minimum. For a mode 1 failure the following equation was proposed.

$$m_{01} = \frac{1}{1 + \frac{2}{\phi}\left(\frac{b}{2h + b}\right)^{1/2}} \qquad (6\text{-}3)$$

where m_{01} = optimum value of m_1

$m_1 = p_1 b/h$

For a mode 2 failure the following equation was proposed.

$$m_{02} = 1 \qquad (6\text{-}4)$$

where m_{02} = optimum value of m_2

$m_2 = p_2 h/b$

The corresponding equation derived on the basis of a mode 3 failure is:

$$m_{03} = \frac{1}{1 - \frac{2}{\phi}\left(\frac{b}{2h + b}\right)^{1/2}} \qquad (6\text{-}5)$$

where m_{03} = optimum value of m_3

$m_3 = p_3 b/h$

On the basis of experimental evidence, Lessig recommended that the following limits be imposed on the values of m_1 and m_2 .

$$0.5\ m_{01} \leq m_1 \leq 1.5\ m_{01} \tag{6-6}$$

$$0.5\ m_{02} \leq m_2 \leq 1.5\ m_{02} \tag{6-7}$$

Since Lessig did not consider a mode 3 failure, no limits on m_3 were suggested.

6-6 Correlation of Test Results

(a) Beams Tested in this Investigation

The theoretical and experimental results for the 34 beams of this investigation are presented in TABLES 6-1 and 6-2. TABLE 6-1 contains the beams tested in combined bending and torsion and TABLE 6-2 contains the beams tested in combined bending, torsion and shear.

The value of x listed in these tables is the value of x_1 , x_2 , or x_3 corresponding to the predicted mode of failure. That is, if the predicted failure is mode 2, x is taken equal to x_2 . Similarly, the value of m is taken equal to m_1 , m_2 or m_3 and the value of m_o is taken equal to m_{01} , m_{02} or m_{03} for a predicted mode 1, mode 2 or mode 3 failure respectively. When two observed modes of failure are listed, the first one listed is the one that was predominant. These observations are presented in CHAPTER IV.

The value of M_{bt} listed in TABLE 6-2 is the bending moment

TABLE 6-1

COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS

TESTS IN COMBINED BENDING AND TORSION IN THIS INVESTIGATION

Beam	M_{bt} in.kips	M_{tt} in.kips	ϕ	M_{tr} in.kips	m	m_o	$\frac{m}{m_o}$	x inches	M_{t1} in.kips	M_{t2} in.kips	M_{t3} in.kips	M_t in.kips	$\frac{M_{tt}}{M_t}$	Predicted Mode of Failure	Observed Mode of Failure
1-1	0	122	∞	114	0.99	1.00	0.99	0.19	234	184	125	125	0.97	3	3
1-2	47	138	2.94	109	1.02	1.45	0.70	0.19	216	186	151	151	0.92	3	3
1-3	70	140	2.00	106	0.96	1.80	0.53	0.17	202	181	157	157	0.89	3	3
1-4	159	159	1.00	118	0.29	0.52	0.56	0.31	182	186	206	182	0.88	1	2 & 1
1-5	267	131	0.49	102	0.27	0.36	0.75	0.42	138	181	295	138	0.95	1	1
1-6	362	90	0.25	112	0.26	0.22	1.18	0.61	91	180	851	91	0.99	1	1
2-1	0	181	∞	130	0.53	1.00	0.53	0.41	235	230	235	230	0.79	2	2 & 1
2-2	88	172	1.96	119	0.28	0.68	0.41	0.26	205	229	268	205	0.84	1	1
2-3	166	166	1.00	112	0.26	0.53	0.49	0.28	176	223	293	176	0.94	1	1
2-4	267	134	0.50	117	0.28	0.36	0.78	0.39	140	227	404	140	0.96	1	1
2-5	362	90	0.25	123	0.28	0.21	1.33	0.64	92	230	1486	92	0.98	1	1
3-1	0	115	∞	119	0.72	1.00	0.72	0.14	171	141	107	107	1.07	3	3
3-2	111	117	1.05	124	0.29	0.54	0.54	0.21	134	143	160	134	0.87	1	1
3-3	111	120	1.08	126	0.29	0.54	0.54	0.20	135	143	159	135	0.89	1	1
3-4	188	115	0.61	121	0.28	0.40	0.70	0.25	112	141	204	112	1.03	1	1
3-5	265	73	0.28	131	0.29	0.23	1.26	0.41	73	143	562	73	1.00	1	1
4-1	200	120	0.60	119	0.41	0.40	1.03	0.38	123	170	280	123	0.97	1	1
4-2	166	101	0.61	129	0.21	0.40	0.53	0.17	102	123	164	102	0.99	1	1
4-3	153	93	0.61	116	0.16	0.41	0.39	0.13	93	107	132	93	1.00	1	1 & 2
4-4	146	85	0.58	124	0.14	0.39	0.36	0.11	88	100	123	88	0.97	1	1 & 2
4-5	212	103	0.49	127	0.29	0.35	0.83	0.28	102	143	241	102	1.01	1	1
4-6	132	66	0.50	121	0.28	0.36	0.78	0.16	63	95	156	63	1.04	1	1
												Average	0.95		
												Average Deviation	0.055		

TABLE 6-2

COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS

TESTS IN COMBINED BENDING, TORSION AND SHEAR IN THIS INVESTIGATION

Beam	V_t kips	M_{bt} in.kips	M_{tt} in.kips	ϕ	M_{tr} in.kips	m	m_o	$\frac{m}{m_o}$	x inches	M_{t1} in.kips	M_{t2} in.kips	M_{t3} in.kips	M_t in.kips	$\frac{M_{tt}}{M_t}$	Predicted Mode of Failure	Observed Mode of Failure
5-1	0.76	65	128	1.97	124	0.97	1.56	0.62	0.16	206	180	151	151	0.85	3	3
5-2	1.89	143	141	0.99	138	0.83	1.00	0.83	0.31	186	178	184	178	0.79	2	2 & 3
5-3	3.87	278	130	0.47	119	0.26	0.37	0.70	0.34	141	166	262	141	0.92	1	1
5-4	5.51	389	99	0.25	125	0.28	0.23	1.22	0.55	98	158	551	98	1.01	1	1
6-1	1.81	65	129	1.98	126	0.97	1.27	0.76	0.16	212	176	140	140	0.92	3	3
6-2	4.27	149	145	0.97	115	0.93	1.76	0.53	0.15	188	166	153	153	0.95	3	3 & 2
6-3	7.67	264	132	0.50	124	0.28	0.42	0.67	0.32	157	157	210	157	0.84	1	1
6-4	12.45	427	107	0.25	129	0.28	0.25	1.12	0.52	104	137	411	104	1.03	1	1
7-1	1.56	56	112	2.00	116	0.60	1.30	0.46	0.11	226	170	108	108	1.03	3	3
7-2	3.32	115	115	1.00	117	0.63	2.02	0.31	0.11	221	166	125	125	0.92	3	3
7-3	8.11	275	132	0.48	129	0.34	1.00	0.34	0.22	201	154	172	154	0.86	2	3 & 2
7-4	14.97	505	125	0.25	107	0.36	1.00	0.36	0.18	157	132	275	132	0.95	2	1 & 2
													Average	0.92		
													Average Deviation	0.059		

at the west end of the gauge length and is the maximum bending moment within the gauge length. The value of $\emptyset$ listed is based on this bending moment. M_{t1} has been calculated on the basis of the assumption that the critical section is located a distance $c_1/2$ from the west end of the gauge length and M_{t3} has been calculated on the basis of the assumption that the critical section is located a distance $c_3/2$ from the east end of the gauge length. Similarly, m_{01} and m_{03} were calculated using the value of $\emptyset$ existing at the appropriate critical section.

(b) Beams Tested by Other Investigators

The theoretical and experimental results for beams tested by other investigators are presented in TABLES 6-3 to 6-7. The theoretical results are the results obtained using the equations presented in CHAPTER V .

The results for 22 of the beams tested by Pandit (1965) are presented in TABLE 6-3. For those beams tested in combined bending, torsion and shear, the value of M_{bt} listed in this table is the average value of bending moment over the gauge length. The value of $\emptyset$ for each beam has been computed using this value of bending moment. The values of M_t are based on the assumption that the critical section is located a distance $c_1/2$ from the west end of the gauge length for a mode 1 failure and a distance $c_3/2$ from the east end of the gauge length for a mode 3 failure.

Since the analysis presented in CHAPTER V is limited to reinforced concrete beams, the plain concrete beams tested by Pandit are not included in this table. Pandit's Group H beams have also been omitted

TABLE 6-3

COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS

TESTS BY PANDIT (1965)

Beam	b inches	h inches	f'_c psi	Reinforcement Top	Bottom	Ties	V_t kips	M_{bt} in.kips	M_{tt} in.kips	Ø	M_t in.kips	$\frac{M_{tt}}{M_t}$
B-2	6.20	12.20	4640	2#3	2#4	#3@6"	0	195	72	0.37	68	1.06
B-3	6.05	12.20	4690	2#3	2#4	#3@6"	0	110	95	0.86	97	0.98
B-4	6.10	12.20	5070	2#3	2#4	#3@6"	0	0	85	∞	98	0.87
C-1	6.00	12.20	4980	2#3	2#5	#3@4½	0	280	78	0.28	74	1.05
C-2	6.15	12.20	4820	2#3	2#5	#3@4½	0	195	105	0.54	108	0.97
C-3	6.10	12.20	5340	2#3	2#5	#3@4½	0	110	111	1.01	134	0.83
C-4	6.50	12.20	5800	2#3	2#5	#3@4½	0	0	111	∞	116	0.96
D-1	6.20	12.20	4880	2#5	4#6	#3@4½	0	637	99	0.16	116	0.85
D-2	6.20	12.20	5100	2#5	4#6	#3@4½	0	365	164	0.45	204	0.80
D-3	6.20	12.20	4650	2#5	4#6	#3@4½	0	195	156	0.80	237	0.66
D-4	6.20	12.20	5080	2#5	4#6	#3@4½	0	0	146	∞	175	0.84
E-1	6.20	12.20	4910	2#4	2#4	#3@4½	0	195	76	0.39	74	1.03
E-2	6.20	12.20	5100	2#4	2#4	#3@4½	0	110	101	0.92	110	0.92
E-3	6.20	12.20	4700	2#4	2#4	#3@4½	0	0	121	∞	151	0.80
F-1	6.10	12.20	5060	3#3	3#3	#3@8"	9.2	140	58	0.41	56	1.04
F-2	6.10	12.20	5060	3#3	3#3	#3@8"	6.2	95	83	0.87	81	1.02
F-3	6.15	12.20	4650	3#3	3#3	#3@8"	3.2	50	89	1.78	93	0.96
F-4	6.10	12.20	4650	3#3	3#3	#3@8"	0	0	95	∞	103	0.92
G-1	6.20	12.20	4560	3#3	3#3	#3@3½	10.2	155	73	0.47	62	1.18
G-2	6.20	12.20	4560	3#3	3#3	#3@3½	6.2	95	92	0.97	80	1.15
G-3	6.15	12.20	4920	3#3	3#3	#3@3½	3.2	50	103	2.06	88	1.17
G-4	6.20	12.20	4920	3#3	3#3	#3@3½	0	0	115	∞	95	1.21

Average* 0.98

Average Deviation* 0.102

*Not Including Beam D-3

since they were not provided with any top longitudinal reinforcement.

One of the top longitudinal bars and one of the bottom longitudinal bars in Beam D-3 were provided with strain gauges. Strain readings indicated that neither the top nor the bottom reinforcement had yielded when failure occurred. Consequently, it would appear that this beam was over reinforced.

Beams F-1 to G-4 had three longitudinal bars adjacent to each horizontal face. In the computations for mode 2, the bars at the center of the horizontal faces have been assumed to yield and their reduced moment arm has been taken into consideration.

TABLE 6-4 presents the results for 13 of the beams tested by Chinenkov (1959). Only the beams that were provided with transverse reinforcement adjacent to all faces are included. The cylinder strengths listed in this table were obtained by taking 95 percent of the cube strengths given by Chinenkov.

TABLE 6-5 presents the results for the 12 beams tested by Gesund, Schuette, Buchanan and Gray (1964).

TABLE 6-6 presents the results for 17 of the beams tested by Lyalin (1959). Beams that failed by crushing of the concrete before the reinforcement yielded and beams that failed by breaking of the welds between the transverse and longitudinal reinforcement are not included in this table. The concrete cylinder strengths listed were obtained by taking 95 percent of the cube strengths given by Lyalin. The value of M_{bt} listed in this table is the maximum bending moment in the

TABLE 6-4

COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS

TESTS IN COMBINED BENDING AND TORSION BY CHINENKOV (1959)

				Reinforcement										
				Top		Bottom		Ties						
Beam	b inches	h inches	f'_c psi	No. of Bars	Diameter inches	No. of Bars	Diameter inches	Diameter inches	Spacing inches	M_{bt} in.kips	M_{tt} in.kips	$\emptyset$	M_t in.kips	$\frac{M_{tt}}{M_t}$
B-2-8-0.1	7.90	11.80	1460	2	0.79	2	0.79	0.276	3.16	486	49	0.10	47	1.04
B-2-9-0.1a	7.90	12.00	1460	2	0.79	2	0.79	0.276	3.16	469	47	0.10	46	1.02
B-2-8-0.2	7.90	12.20	1220	2	0.79	2	0.79	0.276	3.16	417	83	0.20	84	0.99
B-2-8-0.2a	7.90	12.00	1460	2	0.79	2	0.79	0.276	3.16	417	83	0.20	85	0.97
B-2-8-0.4B	7.90	12.00	1460	2	0.79	2	0.79	0.276	3.16	347	98	0.28	100	0.98
B-2-8-0.4	7.90	12.00	3320	2	0.79	2	0.79	0.276	3.16	365	146	0.40	129	1.13
B-2-8-0.4a	7.90	12.00	3320	2	0.79	2	0.79	0.276	3.16	347	139	0.40	129	1.08
B-2-8-0.4b	7.90	12.00	5140	2	0.79	2	0.79	0.276	3.16	365	146	0.40	129	1.13
B-2-8-0.4c	7.90	11.80	5140	2	0.79	2	0.79	0.276	3.16	382	153	0.40	133	1.15
B-2-8-0.4d	7.90	11.80	2710	2	0.79	2	0.79	0.276	3.16	313	125	0.40	128	0.98
B-2-8-0.4e	7.90	12.00	2710	2	0.79	2	0.79	0.276	3.16	330	132	0.40	129	1.02
B-2-8-0.4f	7.90	11.80	2710	2	0.79	2	0.79	0.394	3.16	347	139	0.40	147	0.94
B-2-8-0.4g	7.90	11.80	2710	2	0.79	2	0.79	0.394	3.16	435	98	0.23	97	1.02
													Average	1.03
													Average Deviation	0.054

TABLE 6-5

COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS

TESTS IN COMBINED BENDING AND TORSION BY GESUND, SCHUETTE, BUCHANAN AND GRAY (1964)

Beam	b inches	h inches	f'_c psi	Reinforcement Top	Bottom	Ties	M_{bt} in.kips	M_{tt} in.kips	Ø	M_t in.kips	$\frac{M_{tt}}{M_t}$
1	8.0	8.0	5030	2#4	3#4	#3@5"	79	79	1.00	93	0.85
2	8.0	8.0	5300	2#4	3#4	#3@2"	102	102	1.00	120	0.85
3	8.0	8.0	5310	2#4	3#4	#3@5"	122	61	0.50	68	0.89
4	8.0	8.0	4680	2#4	3#4	#3@2"	134	67	0.50	78	0.86
5	8.0	8.0	4240	2#4	3#4	#3@5"	147	49	0.33	52	0.94
6	8.0	8.0	4060	2#4	3#4	#3@2"	168	56	0.33	56	1.00
7	8.0	8.0	5280	2#4	3#4	#3@5"	173	43	0.25	42	1.03
8	8.0	8.0	5740	2#4	3#4	#3@2"	176	44	0.25	44	1.00
9	6.0	12.0	4860	2#4	3#4	#3@8"	120	60	0.50	72	0.84
10	6.0	12.0	3900	2#4	3#4	#3@8"	176	44	0.25	53	0.83
11	6.0	12.0	4860	2#4	3#4	#3@4"	138	68	0.49	88	0.77
12	6.0	12.0	3900	2#4	3#4	#3@4"	213	53	0.25	60	0.89

Average 0.90

Average Deviation 0.066

TABLE 6-6

COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS

TESTS IN COMBINED BENDING, TORSION AND SHEAR BY LYALIN (1959)

Beam	b inches	h inches	f'_c psi	Reinforcement Top No. of Bars	Top Diameter inches	Bottom No. of Bars	Bottom Diameter inches	Ties Diameter inches	Ties Spacing inches	V_t kips	M_{bt} in.kips	M_{tt} in.kips	Ø	M_t in.kips	$\frac{M_{tt}}{M_t}$
8-0.1	7.88	12.20	2100	2	0.788	2	0.788	0.315	3.15	12.60	521	52	0.10	55	0.94
8-0.1a	7.88	12.20	2100	2	0.788	2	0.788	0.315	3.15	13.40	555	56	0.10	57	0.98
8-0.2	7.88	11.80	1820	2	0.788	2	0.788	0.315	3.15	11.90	452	90	0.20	109	0.82
8-0.2a	7.88	12.20	2100	2	0.788	2	0.788	0.315	3.15	11.72	486	97	0.20	104	0.93
7-0.2	7.88	12.00	1820	2	0.788	2	0.788	0.262	3.15	11.30	468	94	0.20	112	0.84
7-0.2a	7.88	12.00	2300	2	0.788	2	0.788	0.262	3.15	10.90	452	90	0.20	111	0.81
10-0.2	7.88	12.00	2640	2	0.788	2	0.788	0.394	3.15	12.60	521	104	0.20	102	1.02
10-0.2a	7.88	12.20	2860	2	0.788	2	0.788	0.394	3.15	12.60	521	104	0.20	106	0.98
1	8.66	7.88	4860	2	0.945	2	0.945	0.307	3.15	10.90	451	90	0.20	88	1.02
1a	8.66	7.88	4670	2	0.945	2	0.945	0.307	3.15	12.72	451	90	0.20	93	0.97
2a	8.66	11.80	4860	2	0.945	2	0.945	0.307	3.15	16.77	694	139	0.20	134	1.04
4	8.66	11.80	4410	2	0.945	2	0.945	0.307	3.15	20.20	555	261	0.47	271	0.96
4a	8.66	11.80	4670	2	0.945	2	0.945	0.307	3.15	20.10	555	278	0.50	285	0.98
5	8.66	15.80	4410	2	0.945	2	0.945	0.307	3.15	23.50	973	195	0.20	216	0.90
5a	8.66	15.80	4930	2	0.945	2	0.945	0.307	3.15	23.50	973	195	0.20	193	1.01
6	6.70	15.80	5030	2	0.945	2	0.945	0.307	3.15	20.10	832	166	0.20	176	0.95
6a	6.70	15.80	4930	2	0.945	2	0.945	0.307	3.15	21.80	903	181	0.20	180	1.01
														Average	0.95
														Average Deviation	0.055

beam and the value of $\emptyset$ listed is based on this bending moment. The values of M_t are based on the assumption that for a mode 1 failure the critical section is located a distance $c_1/2$ from the cross section subjected to the maximum bending moment and that for a mode 3 failure the critical section is located a distance $c_3/2$ from the cross section subjected to the minimum bending moment.

The results of 11 of the beams tested by Lessig (1961) are presented in TABLE 6-7. Beams that failed by crushing of the concrete before the reinforcement yielded are not included in this table. The concrete cylinder strengths were obtained by taking 95 percent of the cube strengths given by Lessig. The value of M_{bt} listed in this table is the maximum bending moment in the beam and the value of $\emptyset$ listed is based on this bending moment. The values of M_t listed are based on the assumption that for a mode 1 failure the critical section is located a distance $c_1/2$ from the cross section subjected to the maximum bending moment and that for a mode 3 failure the critical section is located a distance $c_3/2$ from the cross section subjected to the minimum bending moment. In computing M_t for Beams 19 and 19a, the different diameters of the horizontal and vertical legs of the ties were considered.

TABLE 6-7

COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS

TESTS IN COMBINED BENDING, TORSION AND SHEAR BY LESSIG (1961)

Beam	b	h	f'_c	Reinforcement Top No. of Bars	Reinforcement Top Diameter	Reinforcement Bottom No. of Bars	Reinforcement Bottom Diameter	Ties Diameter	Ties Spacing	V_t	M_{bt}	M_{tt}	ϕ	M_t	$\frac{M_{tt}}{M_t}$
	inches	inches	psi		inches		inches	inches	inches	kips	in.kips	in.kips		in.kips	
3	6.69	15.75	2140	2	0.630	2	0.630	0.394	4.92	0	0	175	∞	219	0.80
13a	6.10	11.60	2900	2	0.551	2	1.100	0.394	3.94	23.20	625	125	0.20	159	0.79
14a	5.90	12.00	2960	2	0.551	2	1.100	0.394	3.94	21.20	573	113	0.20	139	0.81
15	7.88	12.20	2840	2	0.551	2	0.788	0.394	3.94	15.50	417	156	0.37	163	0.95
15a	7.88	12.00	3090	2	0.551	2	0.788	0.394	3.94	15.50	417	151	0.36	164	0.92
16	6.10	12.40	3660	2	0.473	2	0.473	0.394	3.94	4.10	156	92	0.59	92	1.00
16a	6.10	12.00	3960	2	0.473	2	0.473	0.394	3.94	4.10	156	83	0.53	86	0.97
17	6.10	12.20	4260	2	0.630	2	0.630	0.394	3.94	7.10	278	83	0.30	88	0.94
17a	6.10	12.20	4440	2	0.630	2	0.630	0.394	3.94	8.00	312	90	0.29	86	1.05
19	6.10	12.00	3780	2	0.630	2	0.630	0.236*	3.94	7.98	312	78	0.25	73	1.07
19a	5.90	12.00	4020	2	0.630	2	0.630	0.236*	3.94	7.98	312	79	0.25	71	1.12
														Average	0.95
														Average Deviation	0.086

*Diameter of horizontal legs of ties = 0.236 inches
Diameter of vertical legs of ties = 0.394 inches

CHAPTER VII

SIMPLIFIED ANALYSIS

7-1 Introduction

Simplified versions of the equations in CHAPTER V are presented in this chapter. An interaction diagram consisting of three straight lines is also presented along with the applicable equations and the procedure to be used in their solution.

The correlation between the experimental results of 109 beams and the simplified analysis is given in TABLES 7-1 and 7-2. Of these beams, 34 were tested in this investigation and 75 were tested by other investigators.

7-2 Simplification of Equations

In order to reduce the number of calculations and to eliminate the iterative procedures required to solve Equations (5-1), (5-5) and (5-10), the following simplifying assumptions are introduced.

(1) The critical section is located a distance equal to b from the cross section of the beam that is subjected to either the maximum or the minimum bending moment within the gauge length.

(2) The transverse reinforcement that is adjacent to the face opposite to the face on which the hinge forms is located at the level of the longitudinal reinforcement.

(3) The force in the legs of the ties at the two faces adjacent to the face on which the hinge forms is neglected.

(4) The depth of the concrete compression zone is zero.

Assumption one defines the cross sections of the beam that should be analysed. If the gauge length is subjected to a uniform bending moment and a uniform twisting moment this assumption does not apply. If a beam has equal top and bottom longitudinal reinforcement, the possibility of a mode 3 failure is precluded and the critical section is assumed to be located a distance equal to b from the cross section subjected to the maximum bending moment. If a beam has less top reinforcement than bottom, the critical section for a mode 1 failure remains unchanged but the critical section for a mode 3 failure is assumed to be located a distance equal to b from the cross section subjected to the minimum bending moment. Since a mode 2 failure is independent of the bending moment it is not necessary to define a critical section for a mode 2 failure for beams subjected to a constant transverse shear.

Assumption two reduces the lever arm of the transverse reinforcement by an amount equal to one half the tie diameter plus one half the diameter of the longitudinal bars. Consequently, it reduces the theoretical ultimate torsional moment slightly.

Assumption three neglects the force in the vertical legs of the ties for the mode 1 and mode 3 failure surfaces and neglects the force in the horizontal legs of the ties for the mode 2 failure surface. The contribution of the force in these legs to the torsional moment is small in comparison to the contribution of the forces in the remainder of the reinforcement. As a result, the theoretical ultimate torsional moment is slightly reduced.

Assumption four increases the lever arm of the force in the longitudinal and transverse reinforcement by a distance equal to one half the depth of the compression zone and therefore increases the ultimate torsional moment. The depth of the compression zone in a beam tested at ϕ equal to zero is significant. However, as the value of ϕ is increased the depth of the compression zone becomes progressively smaller. The depth of compression zone at failure for Beams 1-6 and 2-5 was computed to be 0.47" and 0.50" respectively. These beams were tested at ϕ equal to 0.25. Consequently, increasing the lever arm of the force in the reinforcement by one half the depth of the compression zone increases the theoretical torsional moment by only a small amount.

7-3 Simplified Equations

(a) Mode 1

The simplifying assumptions outlined in SECTION 7-2 transform Equations (5-1), (5-3) and (5-4) into Equations (7-1) and (7-2).

$$M_{t1} = A_{s1} f_{y1} h_{01} \left[\frac{1 + p_1 k_{01} c_1^2 / bh}{\frac{c_1}{b} + \frac{1}{\phi}} \right] \tag{7-1}$$

where
$$c_1 = -\frac{b}{\phi} + b \left[\frac{1}{\phi^2} + \frac{h}{p_1 k_{01} b} \right]^{1/2} \leq 2h + b \tag{7-2}$$

Equations (7-1) and (7-2) can then be transformed into Equations (7-3) and (7-4).

$$M_{t1} = A_{s1} f_{y1} h_{01} b \left[\frac{1 + \dfrac{c_1^2}{k_{y1} \alpha_1 \beta_1}}{c_1 + \dfrac{b}{\phi}} \right] \tag{7-3}$$

where $\beta_1 = 2h + b$

$\alpha_1 = A_{s1}\, S/a_v$

$k_{y1} = f_{y1}/f_{yt}$

and
$$c_1 = -\frac{b}{\phi} + \left[\frac{b^2}{\phi^2} + k_{y1} \alpha_1 \beta_1 \right]^{1/2} \leq \beta_1 \tag{7-4}$$

(b) <u>Mode 2</u>

The simplifying assumptions of SECTION 7-2 transform Equations (5-5), (5-7) and (5-8) into Equations (7-5) and (7-6).

$$M_{t2} = \frac{A_{s2} f_{y2} h b_{02}}{c_2} \left[\frac{1 + p_2 k_{02} c_2^2/bh}{1 + \delta} \right] \tag{7-5}$$

where
$$c_2 = \left[\frac{bh}{p_2 k_{02}} \right]^{1/2} \leq 2b + h \tag{7-6}$$

Equations (7-5) and (7-6) can then be transformed into Equations (7-7) and (7-8).

$$M_{t2} \quad \frac{A_{s2} f_{y2} h b_{02}}{c_2} \left[\frac{1 + \dfrac{c_2^{\,2}}{k_{y2} \alpha_2 \beta_2}}{1 + \delta} \right] \tag{7-7}$$

where $\beta_2 = 2b + h$

$\alpha_2 = A_{s2}\, S/a_v$

$k_{y2} = f_{y2}/f_{yt}$

and $$c_2 = (k_{y2} \alpha_2 \beta_2)^{1/2} \leq \beta_2 \tag{7-8}$$

(c) <u>Mode 3</u>

The simplifying assumptions of SECTION 7-2 transform Equations (5-10), (5-12) and (5-13) into Equations (7-9) and (7-10).

$$M_{t3} = A_{s3} f_{y3} h_{03} \left[\frac{1 + p_3 k_{03} c_3^{\,2}/bh}{\dfrac{c_3}{b} - \dfrac{1}{\emptyset}} \right] \tag{7-9}$$

where $$c_3 = \frac{b}{\emptyset} + b \left[\frac{1}{\emptyset^2} + \frac{h}{p_3 k_{03} b} \right]^{1/2} \leq 2h + b \tag{7-10}$$

Equations (7-9) and (7-10) can then be transformed into Equations (7-11) and (7-12).

$$M_{t3} = A_{s3} f_{y3} h_{03} b \left[\frac{1 + \dfrac{c_3^{\,2}}{k_{y3} \alpha_3 \beta_1}}{c_3 - \dfrac{b}{\emptyset}} \right] \tag{7-11}$$

where $\beta_1 = 2h + b$

$\alpha_3 = A_{s3}\, S/a_v$

$k_{y3} = f_{y3}/f_{yt}$

and
$$c_3 = \frac{b}{\phi} + \left[\frac{b^2}{\phi^2} + k_{y3}\alpha_3\beta_1\right]^{1/2} \leq \beta_1 \qquad (7\text{-}12)$$

7-4 Simplified Interaction Diagram

The interaction diagrams shown in CHAPTER VI can be approximated by three straight lines. This simplified interaction diagram is shown in FIGURE 7-1. Points a, b, c, and d are obtained by solving Equations (7-3), (7-7) and (7-11) for values of ϕ of ∞, 1.0, 0.25 and 0. A value of ϕ equal to infinity represents a beam subjected to pure torsion; ϕ equal to zero represents a beam subjected to pure bending. The values 1.0 and 0.25 have been selected arbitrarily.

The twisting moment at point a is designated M_{ta} and is equal to the smallest of the twisting moments M_{t1}, M_{t2} and M_{t3} obtained from Equations (7-3), (7-7) and (7-11) with ϕ equal to ∞. For this value of ϕ these equations are transformed into Equations (7-13), (7-15) and (7-17) respectively. The twisting moments obtained from these equations are designated M_{t1a}, M_{t2a} and M_{t3a} to indicate they are the values of M_{t1}, M_{t2} and M_{t3} at point a on the simplified interaction diagram.

$$M_{t1a} = A_{s1}f_{y1}h_{01}b\left[\frac{1 + c_1^{\,2}/k_{y1}\beta_1\alpha_1}{c_1}\right] \qquad (7\text{-}13)$$

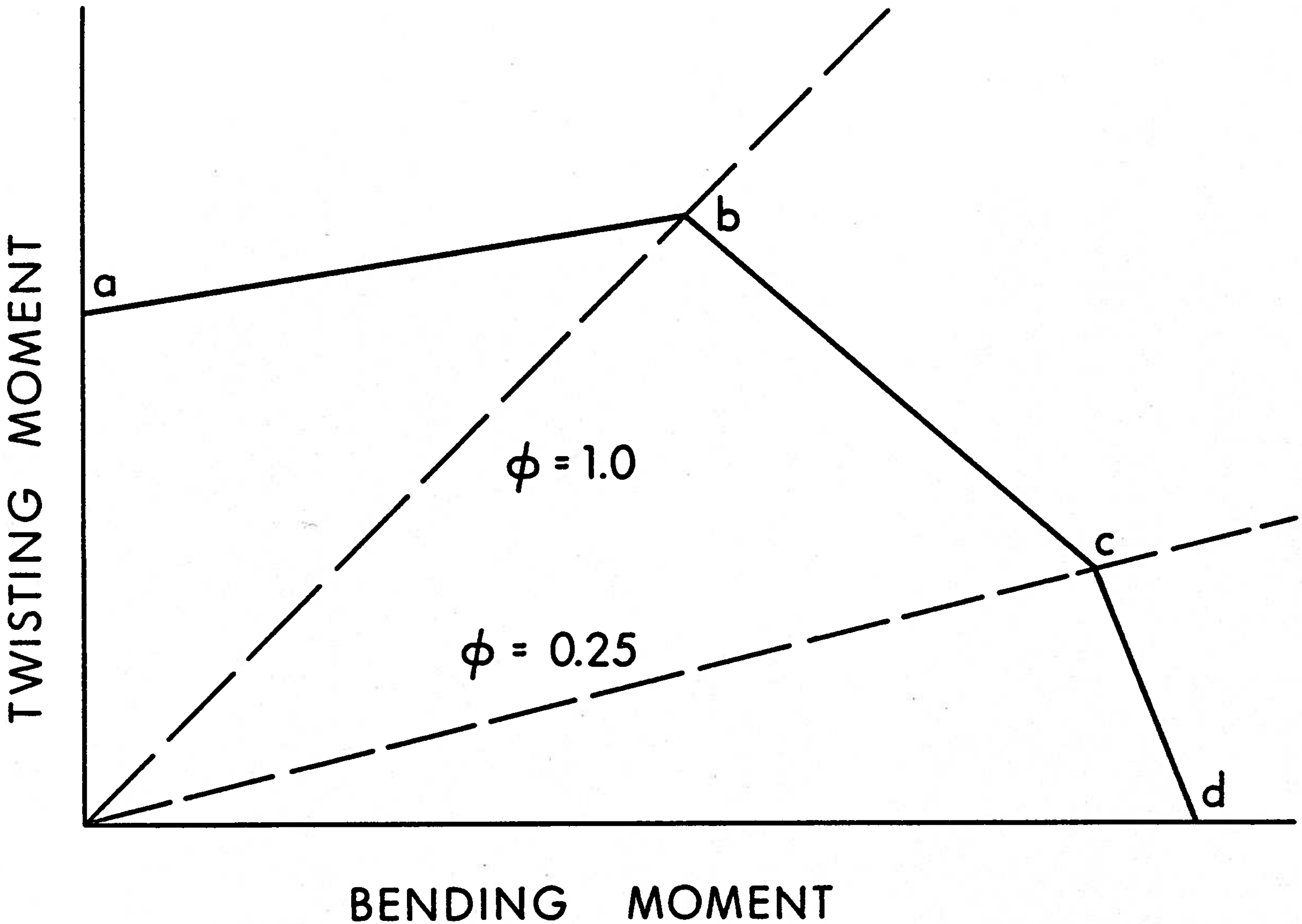


FIGURE 7 - 1 SIMPLIFIED INTERACTION DIAGRAM

where $$c_1 = (k_{y1}\beta_1\alpha_1)^{1/2} \leq \beta_1 \tag{7-14}$$

$$M_{t2a} = \frac{A_{s2}f_{y2}b_{02}h}{c_2}\left[\frac{1 + c_2^{\,2}/k_{y2}\beta_2\alpha_2}{1 + \delta}\right] \tag{7-15}$$

where $$c_2 = (k_{y2}\beta_2\alpha_2)^{1/2} \leq \beta_2 \tag{7-16}$$

$$M_{t3a} = A_{s3}f_{y3}h_{03}b\left[\frac{1 + c_3^{\,2}/k_{y3}\beta_1\alpha_3}{c_3}\right] \tag{7-17}$$

where $$c_3 = (k_{y3}\beta_1\alpha_3)^{1/2} \leq \beta_1 \tag{7-18}$$

The twisting moment at point b is designated M_{tb} and is equal to the smallest of the twisting moments given by Equations (7-3), (7-7) and (7-11) with $\emptyset$ equal to 1.0. For this value of $\emptyset$ these equations are transformed into Equations (7-19), (7-21) and (7-22) and the twisting moments are designated M_{t1b}, M_{t2b} and M_{t3b}. The bending moment at point b is designated M_{bb} and is given by Equation (7-24).

$$M_{t1b} = A_{s1}f_{y1}h_{01}b\left[\frac{1 + c_1^{\,2}/k_{y1}\beta_1\alpha_1}{b + c_1}\right] \tag{7-19}$$

where $$c_1 = - b + (b^2 + k_{y1}\beta_1\alpha_1)^{1/2} \leq \beta_1 \tag{7-20}$$

$$M_{t2b} = M_{t2a} \tag{7-21}$$

$$M_{t3b} = A_{s3}f_{y3}h_{03}b \left[\frac{1 + c_3^2/k_{y3}\beta_1\alpha_3}{c_3 - b} \right] \tag{7-22}$$

where $$c_3 = b + (b^2 + k_{y3}\beta_1\alpha_3)^{1/2} \leq \beta_1 \tag{7-23}$$

$$M_{bb} = M_{tb} \tag{7-24}$$

The twisting moment at point c is designated M_{tc} and is equal to the smallest of the twisting moments given by Equations (7-3), (7-7) and (7-11) with $\emptyset$ equal to 0.25. For this value of $\emptyset$ these equations are transformed into Equations (7-25), (7-27) and (7-28) and the twisting moments are designated M_{t1c}, M_{t2c} and M_{t3c}. The bending moment at point c is designated M_{bc} and is given by Equation (7-30).

$$M_{t1c} = A_{s1}f_{y1}h_{01}b \left[\frac{1 + c_1^2/k_{y1}\beta_1\alpha_1}{4b + c_1} \right] \tag{7-25}$$

where $$c_1 = -4b + (16b^2 + k_{y1}\beta_1\alpha_1)^{1/2} \leq \beta_1 \tag{7-26}$$

$$M_{t2c} = M_{t2a} \tag{7-27}$$

$$M_{t3c} = A_{s3}f_{y3}h_{03}b \left[\frac{1 + c_3^2/k_{y3}\beta_1\alpha_3}{c_3 - 4b} \right] \tag{7-28}$$

where $$c_3 = 4b + (16b^2 + k_{y3}\beta_1\alpha_3)^{1/2} \leq \beta_1 \tag{7-29}$$

$$M_{bc} = 4M_{tc} \tag{7-30}$$

Point d on the simplified interaction diagram represents a beam subjected to pure flexure. With ϕ equal to zero in Equation (7-1), the twisting moment is found to be zero. The corresponding bending moment is given by Equation (7-31) and is designated M_{bu}.

$$M_{bu} = A_{s1}f_{y1}\ (h_{01} - k_1x_1/2) \tag{7-31}$$

where $$x_1 = A_{s1}f_{y1}/0.85\ k_1f'_c\ b \tag{7-32}$$

Equations (7-31) and (7-32) are obtained from Equations (5-1) and (5-2) by rearranging terms and using the fact that c_1 is equal to zero for a beam subjected to pure flexure.

7-5 Computation Procedure

The following procedure can be used to check the torsional strength of beams that have been tested or to design beams for specified bending moments, twisting moments and transverse shear forces. For design, a trial section is selected and ϕ and δ are computed using the design values of bending moment, twisting moment and transverse shear.

After the bending moment and twisting moment corresponding to each of points a, b, c, and d have been calculated, the ultimate torsional moment at any value of ϕ can be obtained by using Equations (7-33), (7-34) and (7-35).

The computation procedure is divided into three parts. The part that is relevant to any particular beam depends on the value of ϕ for that beam. If the top reinforcement of the beam is identical to the bottom, the calculations required by Equations (7-17), (7-18), (7-22), (7-23), (7-28) and (7-29) can be omitted. The three parts of the procedure are:

(a) $0 \leq \phi \leq 0.25$

(b) $0.25 < \phi \leq 1.0$

(c) $\phi > 1.0$

(a) $\underline{0 \leq \phi \leq 0.25}$

1. Compute M_{tc} by Equations (7-25), (7-27) and (7-28)
2. Compute M_{bc} by Equation (7-30)
3. Compute M_{bu} by Equation (7-31)
4. Compute M_t by Equation (7-33)

$$M_t = \frac{M_{bu}}{\frac{M_{bu} - M_{bc}}{M_{tc}} + \frac{1}{\phi}} \tag{7-33}$$

(b) $0.25 < \phi \leq 1.0$

1. Compute M_{tb} by Equations (7-19), (7-21) and (7-22)
2. Compute M_{tc} by Equations (7-25), (7-27) and (7-28)
3. Compute M_{bb} by Equation (7-24)

4. Compute M_{bc} by Equation (7-30)

5. Compute M_t by Equation (7-34)

$$M_t = \frac{M_{tc} + M_{bc}\left[\dfrac{M_{tb} - M_{tc}}{M_{bc} - M_{bb}}\right]}{1 + \dfrac{1}{\phi}\left[\dfrac{M_{tb} - M_{tc}}{M_{bc} - M_{bb}}\right]} \qquad (7\text{-}34)$$

(c) $\phi > 1.0$

1. Compute M_{ta} by Equations (7-13), (7-15) and (7-17)

2. Compute M_{tb} by Equations (7-19), (7-21) and (7-22)

3. Compute M_{bb} by Equation (7-24)

4. Compute M_t by Equation (7-35)

$$M_t = \frac{M_{ta}}{1 - \left[\dfrac{M_{tb} - M_{ta}}{\phi\, M_{bb}}\right]} \qquad (7\text{-}35)$$

7-6 Correlation of Test Results

The theoretical and experimental results for the beams previously presented in TABLES 6-1 to 6-7 are presented in TABLES 7-1 and 7-2. The theoretical value of twisting moment, M_t , listed in these tables was obtained by the procedure presented in SECTION 7-5. For the beams tested in bending, torsion and shear the location of the critical section was established in accordance with assumption one of SECTION 7-2.

TABLE 7-1

COMPARISON OF SIMPLIFIED ANALYSIS AND EXPERIMENTAL RESULTS

THIS INVESTIGATION AND PANDIT (1965)

This Investigation			
Beam	M_{tt} in.kips	M_t in.kips	$\frac{M_{tt}}{M_t}$
1-1	122	115	1.06
1-2	138	132	1.04
1-3	140	134	1.05
1-4	159	169	0.94
1-5	131	129	1.02
1-6	90	90	1.00
2-1	181	209	0.87
2-2	172	187	0.92
2-3	166	164	1.02
2-4	134	132	1.02
2-5	90	91	0.98
3-1	115	99	1.16
3-2	117	124	0.95
3-3	120	123	0.98
3-4	115	105	1.10
3-5	73	71	1.03
4-1	120	116	1.03
4-2	101	96	1.06
4-3	93	85	1.09
4-4	85	80	1.07
4-5	103	96	1.07
4-6	66	59	1.13
5-1	128	130	0.99
5-2	141	147	0.96
5-3	130	124	1.05
5-4	99	96	1.03
6-1	129	123	1.05
6-2	145	124	1.17
6-3	132	126	1.05
6-4	107	98	1.09
7-1	112	95	1.18
7-2	115	107	1.07
7-3	132	132	1.00
7-4	125	118	1.06
		Average	1.04
		Average Deviation	0.051

Pandit (1965)			
Beam	M_{tt} in.kips	M_t in.kips	$\frac{M_{tt}}{M_t}$
B-2	72	64	1.12
B-3	95	90	1.06
B-4	85	89	0.95
C-1	78	72	1.09
C-2	105	101	1.04
C-3	111	124	0.89
C-4	111	109	1.02
D-1	99	109	0.91
D-2	164	184	0.89
D-3	156	199	0.78
D-4	146	160	0.91
E-1	76	70	1.08
E-2	101	103	0.98
E-3	121	139	0.87
F-1	58	52	1.12
F-2	83	70	1.18
F-3	89	82	1.08
F-4	95	94	1.01
G-1	73	53	1.37
G-2	92	68	1.35
G-3	103	77	1.34
G-4	115	86	1.33
		Average *	1.08
		Average Deviation*	0.124

*Not Including Beam D-3

TABLE 7-2

COMPARISON OF SIMPLIFIED ANALYSIS AND EXPERIMENTAL RESULTS

CHINENKOV (1959), GESUND, SCHUETTE, BUCHANAN AND

GRAY (1964), LYALIN (1959) AND LESSIG (1961)

Beam	M_{tt} in.kips	M_t in.kips	$\frac{M_{tt}}{M_t}$
Chinenkov (1959)			
0.1	49	41	1.20
0.1a	47	40	1.17
0.2	83	78	1.06
0.2a	83	80	1.04
0.4B	98	98	1.00
0.4	146	121	1.21
0.4a	139	121	1.15
0.4b	146	121	1.21
0.4c	153	125	1.23
0.4d	125	121	1.04
0.4e	132	122	1.09
0.4f	139	141	0.99
0.4g	98	96	1.02
		Average	1.11
		Average Deviation	0.080
Gesund, Schuette, Buchanan and Gray (1964)			
1	79	90	0.88
2	102	120	0.85
3	61	66	0.93
4	67	78	0.86
5	49	52	0.94
6	56	58	0.96
7	43	43	1.01
8	44	46	0.95
9	60	71	0.85
10	44	54	0.82
11	68	87	0.78
12	53	61	0.87
		Average	0.89
		Average Deviation	0.055

Beam	M_{tt} in.kips	M_t in.kips	$\frac{M_{tt}}{M_t}$
Lyalin (1959)			
8-0.1	52	57	0.92
8-0.1a	56	58	0.96
8-0.2	90	110	0.82
8-0.2a	97	105	0.92
7-0.2	94	101	0.93
7-0.2a	90	99	0.90
10-0.2	104	111	0.94
10-0.2a	104	114	0.91
1	90	97	0.92
1a	90	102	0.88
2a	139	138	1.01
4	261	225	1.16
4a	278	241	1.15
5	195	206	0.95
5a	195	188	1.04
6	166	160	1.04
6a	181	163	1.11
		Average	0.97
		Average Deviation	0.077
Lessig (1961)			
3	175	202	0.87
13a	125	141	0.89
14a	113	122	0.92
15	156	130	1.20
15a	151	129	1.17
16	92	87	1.06
16a	83	81	1.02
17	83	82	1.01
17a	90	81	1.11
19	78	59	1.32
19a	79	57	1.38
		Average	1.09
		Average Deviation	0.136

CHAPTER VIII

DISCUSSION

8-1 General Behavior

The behavior of the test specimens prior to cracking of the concrete was affected very little by the reinforcement provided. However, after cracking occurred the specimens appeared to assume a new configuration of equilibrium that was dependent on the value of ϕ at which the test was being conducted and the reinforcement that was provided. The behavior of the beams has been presented in detail in CHAPTER IV and Appendix B.

Three modes of failure were observed in this investigation and they have been designated mode 1, mode 2, and mode 3. A mode 1 failure is characterized by the formation of a hinge adjacent to the top face of the beam, yielding of the bottom reinforcement and positive vertical deflection of the beam. A mode 2 failure is characterized by the formation of a hinge adjacent to one of the vertical faces of the beam, yielding of the reinforcement adjacent to one or both of the vertical faces and little or no vertical deflection of the beam. A mode 3 failure is characterized by the formation of a hinge adjacent to the bottom face of the beam, yielding of the top reinforcement and negative vertical deflection of the beam. The failure surfaces corresponding to these modes of failure have been idealized and are presented in FIGURES 5-1, 5-2 and 5-3.

The theoretical ultimate torsional moment of a beam has been defined as the smallest of the three moments computed by Equations (5-1), (5-5) and (5-10). Each of these equations has been derived for a different mode of failure. The predicted mode of failure has been designated as the mode that yields the smallest value of theoretical ultimate torsional moment.

The observed mode of failure was determined principally by noting the development and width of cracks and by observing the behavior of the beam at failure. Additional information regarding the mode of failure was obtained from the deflection readings and the strains in the longitudinal reinforcement.

TABLES 6-1 and 6-2 indicate that for most of the beams the observed mode of failure agreed with the predicted mode of failure. When the observed mode of failure did not agree with the predicted mode of failure the beam usually exhibited two failure surfaces corresponding to different modes of failure. The one that appeared to be the best developed was selected as the observed failure surface and the corresponding mode of failure has been recorded in TABLES 6-1 and 6-2. For example, the failure of Beam 1-4 was recorded as a mode 2 failure but it was noted that this beam also had a fairly well developed mode 1 failure surface. The predicted mode of failure for this beam is mode 1. The theoretical torsional moment corresponding to a mode 2 failure surface is 2.6 percent higher than the moment corresponding to a mode 1 failure surface. This is also evident from FIGURE 6-1(a). This interaction diagram indicates that a Group 1 beam tested at ϕ equal to 1.0 would have virtually the same strength according to either a mode 1 or a mode 2 failure surface.

8-2 Effect of Reinforcement

Provision of top longitudinal reinforcement that was less in area than the bottom longitudinal reinforcement did not affect the behavior or the ultimate strength of the beams that were tested at low values of ϕ . Low values of ϕ have been defined as 0.5 or less. Comparison of the test results for Beams 1-5 and 2-4 and Beams 1-6 and 2-5 illustrates this. The observed mode of failure of all four of these beams was mode 1.

Provision of top longitudinal reinforcement that was less in area than the bottom longitudinal reinforcement did affect the behavior and ultimate strength of the beams that were tested at ϕ equal to 1.0. Comparison of the test results of Beams 1-4 and 2-3 illustrates this. The observed mode of failure of Beam 2-3 was mode 1. The observed mode of failure of Beam 1-4 was mode 2 but a fairly well developed mode 1 failure surface was also evident. Although the deflection of both beams was positive, the deflection of Beam 1-4 remained almost constant in the latter stages of the test whereas the deflection of Beam 2-3 continued to increase.

Provision of top longitudinal reinforcement that was less in area than the bottom longitudinal reinforcement significantly affected the behavior and ultimate strength of the beams that were tested at a high value of ϕ . A high value of ϕ has been defined as 2.0 or greater. Comparison of the test results of Beams 1-1 and 2-1 and Beams 1-3 and 2-2 illustrates this. Failure of Beams 1-1 and 1-3 was by mode 3 whereas failure of Beams 2-1 and 2-2 was by mode 2 and mode 1 respectively. After cracking of the concrete occurred, Beams 1-1 and 1-3 exhibited significant negative deflections whereas Beams 2-1 and 2-2 exhibited small negative and small positive deflections respectively.

Increasing the tie spacing while keeping the longitudinal reinforcement and ϕ constant did not affect the behavior of the beams prior to cracking. However, after cracking occured, the slope of the twisting moment-angle of twist relationship decreased with an increase in tie spacing. In addition, the ultimate torsional moment of the beams decreased. The deflection of the beams at a given value of torsional moment was not affected by a variation in the tie spacing.

Comparison of the plots presented in Appendix B for Beams 1-5, 4-5 and 4-6 indicates that the behavior of beams tested at the same value of ϕ but in which the amount of longitudinal and transverse reinforcement was progressively reduced was the same until cracking of the concrete occurred. After cracking occurred, the slope of the twisting moment-angle of twist relationship decreased with a decrease in the amount of reinforcement provided. In addition, the ultimate torsional moment of the beams decreased. The deflection of the beams at a given value of torsional moment increased with a decrease in the amount of reinforcement provided.

TABLES 6-1 and 6-2 indicate that many of the beams tested in this investigation were over reinforced if Equation (6-2) is the criterion used to establish the limit of over reinforcement. However, the results presented in CHAPTER IV and Appendix B indicate that in most cases the reinforcement had yielded at failure of a beam. Consequently, it appears that Equation (6-2) is unnecessarily restrictive and that the coefficient 0.06 should be increased. Strain readings taken after failure of Beams 2-1 and 2-2 indicated that in each of these beams one of the bottom reinforcing bars did not yield. Consequently, it appears that these two beams were slightly over reinforced. If M_{tr} is taken equal to the test value of torsional moment for each of the two beams and the value of the coefficient in Equation (6-2) is calculated, the result is 0.084 and 0.087 for Beams 2-1 and 2-2 respectively. On the basis of the results of these two beams it appears that the coefficient 0.06 in Equation (6-2) could be increased to 0.08.

TABLES 6-1 and 6-2 indicate that all beams tested in this investigation except Beams 2-2, 2-3, 4-3, 4-4, 7-1, 7-2, 7-3 and 7-4 had a value of m between 0.5 and 1.5 times m_o. The value of m for the beams noted above was less than 0.5 m_o. The minimum value of the ratio m/m_o occurred in Beam 7-2 and was equal to 0.31. The maximum value of this ratio occurred in Beam 2-5 and was equal to 1.33. Lessig (1961) suggested that this ratio should be between 0.5 and 1.5. A ratio of less than 0.5 implies that there is insufficient transverse reinforcement present to provide a proper balance between the two types of reinforcement and that the longitudinal reinforcement will not yield. However, in the beams noted above for which this ratio was less than 0.5 the longitudinal reinforcement did yield except in Beam 2-2. Consequently, the suggested lower limit of 0.5 appears to be conservative.

8-3 Effect of Twisting Moment to Bending Moment Ratio

The effect of different ratios of ϕ on the deflection, mode of failure and torsional strength of beams with equal and unequal top and bottom reinforcement has been discussed in SECTION 8-2.

Plots of the twisting moment-angle of twist relationship are presented in Appendix B. The initial straight portion of this relationship represents uncracked behavior of the beam. The decreasing length of this straight portion with decreasing values of ϕ indicates that the presence of bending moment reduced the torsional moment at which cracking of the concrete occurred.

The angle of twist at a particular value of twisting moment for the beams of Group 1 did not show any consistent variation with ϕ and appeared to be influenced by the mode of failure of the beams. For a given value of twisting moment greater than the cracking moment, the angle of twist was greater for the beams that failed by mode 3 than the ones that failed by mode 1. For the beams of Group 2 that failed by mode 1, the angle of twist at a particular value of twisting moment decreased with decreasing values of ϕ .

8-4 Sequence of Loading

Pandit (1965) applied the transverse load first and then twisted the specimens to failure. Cowan and Armstrong (1955) and Gesund, Schuette, Buchanan and Gray (1964) used equipment that increased the bending moment and twisting moment at a constant ratio. Chinenkov (1959), Lyalin (1959), Lessig (1961) and Evans and Sarkar (1965) used equipment that permitted a change in the ratio of twisting moment to bending moment at any stage of a test. However, their tests were carried out by incrementing first one type of load and then the other so that for any one test $\emptyset$ was a constant at the end of each complete load increment. Goode and Helmy (1965) tested beams by first applying a twisting moment and then a bending moment and stated that these beams sustained an ultimate bending moment at least equal to the ultimate bending moment that would be expected in pure flexure.

The interaction diagram for the beams of Group 3 of this investigation and the beams of Group C of Pandit (1965) is shown in FIGURE 6-2. In addition, the failure values of twisting moment and bending moment and the loading paths are shown. All of the Group 3 beams except Beam 3-1 were first subjected to a small bending moment due to the weight of the loading equipment before further load was applied. FIGURE 6-2 indicates that the sequence of loading did not significantly affect the ultimate torsional strength of these beams.

Beam 3-3, which was subjected to a substantial bending moment before it was twisted to failure, exhibited a crack pattern that consisted essentially of two sets of cracks, one set vertical and the other set inclined at approximately 45° to the longitudinal axis of the beam. Beams 3-2, 3-4 and 3-5, which were subjected to a substantial twisting moment before the major portion of the bending moment was applied, exhibited crack patterns that consisted of cracks on all faces at approximately 45°

to the axis of the beam. The major effect of the last bending sequence applied to these three beams was to widen the cracks that had first appeared during the twisting sequence. However, Beams 3-2, 3-3, 3-4 and 3-5 all developed numerous cracks while the loads were at their peak values.. This indicates that the formation of the failure surface was not entirely dependent on the cracks that formed at lower loads and that a redistribution of stress occurred at failure.

Prior to the final bending sequence, Beam 3-4 was subjected to a twisting moment of 115 inch kips and a bending moment of 55 inch kips. At this stage strain readings indicated yield in one of the top bars, both of the bottom bars and the north leg of the tie that was instrumented. However, failure did not occur until the bending moment had been increased to 188 inch kips. This shows that yielding of the reinforcement is not sufficient to indicate that failure is about to occur. Failure does not occur until the concrete in the compression zone has reached its ultimate strength.

It is stated in SECTION 8-3 that the presence of bending moment reduced the torsional moment at which cracking of the concrete occurred. First cracking in beams tested at a constant value of ϕ occurred under the action of combined bending and torsion. At low values of ϕ the tensile stresses in the concrete due to bending were appreciable and consequently the first cracks appeared on the bottom face and lower part of the vertical faces and were inclined to the longitudinal axis of the beam. At high values of ϕ the first cracks developed at approximately mid depth of the vertical faces and were inclined to the longitudinal axis of the beam. An interesting exception to this is illustrated by Beams 3-2 and 3-3. The cracking torque for both of these beams was approximately 45 inch kips. At this stage the bending moment in Beam 3-2 was 24 inch kips whereas in Beam 3-3 it was 111 inch kips. When twisting of Beam 3-3 was commenced, vertical bending cracks were already present. These bending cracks would relieve some of the tensile stress in the uncracked sections of concrete between the cracks. As a result, the first torsional cracks did not appear on the bottom and lower part of the vertical faces of Beam 3-3 but appeared at approximately mid depth of the vertical faces and were inclined to the longitudinal axis of the beam. Consequently, the sequence of loading used by Pandit (1965) appears to be relevant with regard to his finding that the magnitude of flexure had no significant effect on the value of the cracking torque.

8-5 Effect of Transverse Shear

The beams of Groups 1 to 4 were subjected to a uniform twisting moment and a uniform bending moment whereas the beams of Groups 5 to 7 were subjected to a uniform twisting moment, a linearly varying bending moment and a uniform transverse shear. The reinforcement and geometrical details of the beams of Groups 1, 5 and 6 were similar in all respects. The failure surface in the beams that were not subjected to transverse shear was located in different parts of the gauge length in different beams. However, the failure surface in the beams that were subjected to transverse shear generally formed at either the east end or at the west end of the gauge length. For all beams of Groups 5, 6 and 7 the bending moment at the east end of the gauge length was less than at the west end. Failure surfaces occurring at the east end of the gauge length were always of the mode 3 type; those occurring at the west end were always mode 1. Since these beams were subjected to a linearly varying bending moment, ϕ also varied linearly. Equations (5-1) and (5-10) indicate that the ultimate torsional moment of a beam is a function of ϕ . Equation (5-1) indicates that a mode 1 failure will occur in the region subjected to the lowest values of ϕ and Equation (5-10) indicates that a mode 3 failure will occur in the region subjected to the highest values of ϕ . This is in agreement with the observed behavior.

Since ϕ varies linearly in beams subjected to a uniform transverse shear, the question arises as to what value should be used in Equations (5-1) and (5-10). The minimum values of torsional moment are obtained by using the minimum value of ϕ in Equation (5-1) and the maximum value of ϕ in Equation (5-10). However, since the failure surface develops over a length of the beam and not at one cross section, the use of these values is unnecessarily conservative. Another possibility is to use the values of ϕ at distances of $c_1/2$ and $c_3/2$ from the cross sections at which the minimum and maximum values of ϕ occur respectively. Observation of the failure surfaces that formed in these tests indicated that this would sometimes be unconservative since the failure surface often appeared to crowd to one end of the gauge length.

This was most noticeable in those beams subjected to a high bending moment gradient within the gauge length. Some value of $\emptyset$ between these two limiting cases would probably conform more closely with the actual behavior of the beams. Using the values of $\emptyset$ at distances of $c_1/4$ and $c_3/4$ from the cross sections at which the minimum and maximum values of $\emptyset$ occur respectively changes the average of the ratio M_{tt}/M_t for the beams listed in TABLE 6-2 to 0.96 and the average deviation of this ratio to 0.067. The assumption that is made in the simplified analysis is that the values of $\emptyset$ which should be used in the computations are the values occurring at a distance b from the cross sections at which the minimum and maximum values of $\emptyset$ occur. This is a compromise which simplifies the procedure but decreases its accuracy.

Equations (5-1), (5-5) and (5-10) indicate that the transverse shear enters into only a mode 2 failure. Because of the effect of the varying bending moment it is difficult to ascertain if this is entirely true. Although the presence of transverse shear did affect the initial development of the cracks, the observations and test results indicate that transverse shear within the range covered by these tests does not significantly affect the behavior at failure or the ultimate torsional strength of a beam except as provided for by a mode 2 failure surface.

The upper limit of the range of transverse shear covered by these tests can be indicated by using any one of three parameters. The first of these parameters is the ratio V_t/V_u, where V_u is the total ultimate transverse shear capacity of the beam when subjected to only bending and shear. The value of V_u has been computed by the method outlined in Chapter 17 of the American Concrete Institute Building Code (1963). The maximum value of this ratio occurred in Beam 7-4 and was equal to 0.50. The second of these parameters is δ and is defined in CHAPTER V. The maximum value of δ occurred in Beam 6-4 and was equal to 0.37. The third parameter, ψ , relates the transverse shear to the bending moment and is defined in CHAPTER VI. The maximum value of ψ in the beams tested in this investigation occurred in Beam 6-4 at the east end of the gauge length and was equal to 0.123. The value of ψ for this beam at the west end of the gauge length was 0.029.

8-6 Validity of Assumptions

The first and second assumptions of SECTION 5-3 define the failure surface. Observations made during these tests indicate that failure occurs on a warped plane and that at failure the spiral cracks widen on three faces of the beam but not on the fourth face. The cracks on the two faces adjacent to the fourth face decrease in width as they approach the fourth face. These cracks do not always spiral around the beam at a constant angle. The beams tested with ϕ equal to 0.25 exhibited cracks on the vertical faces that were nearly vertical near the bottom and became more horizontal as they progressed upward. The angle that the cracks on the bottom face made with the axis of the beam was generally close to the same as the angle made by the bottom portion of the cracks on the vertical faces. However, for values of ϕ equal to or greater than 0.5 the cracks on all faces were at nearly a constant angle to the longitudinal axis of the beam.

The third assumption of SECTION 5-3 states that the concrete cracks to the level of the neutral axis. This is not entirely true but the moment of the tensile force in the portion of uncracked concrete is very small in comparison to the ultimate torsional moment of the beam.

The fourth assumption neglects the forces in the reinforcement near the face of the beam on which the hinge is located. Comparison of the test results for Beams 1-5 and 2-4 and Beams 1-6 and 2-5 indicates that neither axial nor dowel forces in the top longitudinal reinforcement contributed significantly to the ultimate torsional strength of these beams. Since the lever arm of these forces is small the moment due to these forces is small in comparison to the ultimate torsional moment of the beam.

The sixth assumption states that all reinforcement crossing the failure plane outside of the compression zone yields at failure. The strain readings obtained from gauges mounted on the longitudinal reinforcement indicate that the longitudinal bars adjacent to the face opposite the hinge did yield at failure. The difficulties encountered in measuring the strain in the ties intersecting the failure plane are indicated in SECTION 4-3(c). However, the strain readings obtained from the ties that did intersect the failure plane and the crack widths observed at failure indicate that this assumption is justified with regard to the ties as well as the longitudinal reinforcement.

The seventh assumption of SECTION 5-3 was introduced to simplify the equations. It states that the cross sectional area of transverse reinforcement intersected by the failure plane is constant per unit length of the beam. The area of transverse reinforcement is not constant per unit length of the beam, it is concentrated at the locations of the ties. This assumption is accurate for a tie spacing of zero and becomes progressively more inaccurate as the tie spacing is increased. To allow for the fact that a failure crack can cross only a whole number of ties, Pandit (1965) suggested that the number of ties resisting torsion be taken as the smallest number intersected by a potential crack at 45° to the axis of the beam. Similarly, Lessig (1959) suggested that an efficiency factor of less than 1.0 be applied to the transverse reinforcement.

The beams of Group 4 and Beams 1-5 and 3-4 were used to determine the effect of tie spacing on the torsional strength of beams. The interaction diagrams and the failure values of twisting moment and bending moment for these beams are presented in FIGURE 6-3. The correlation between the experimental and theoretical torsional moments is plotted as a function of tie spacing in FIGURE 6-4. This correlation is good within the range of tie spacing covered by these tests. The ratio of tie spacing to beam depth varied from 0.25 to 0.74; the ratio of tie spacing to beam width varied from 0.48 to 1.44. The value of ϕ at which the beams were tested was 0.61 for Beams 3-4, 4-1, 4-2, 4-3 and 4-4 and 0.50 for Beams 1-5, 4-5 and 4-6.

If the tie spacing of a beam is sufficiently large it should behave in the same manner as a beam with longitudinal reinforcement only. Gesund and Boston (1964) reported tests on beams with longitudinal reinforcement only and stated that most of the beams failed suddenly. In contrast, all beams tested in this investigation exhibited appreciable ductility. This is not surprising for the majority of the beams since the tie spacing was generally less than the beam width. However, even Beam 4-4 which had a tie spacing of 9" exhibited great ductility. The first cracks on the bottom and two vertical faces of this beam were observed after application of load increment 4. At this load stage the bending moment was 44 percent of the ultimate bending moment and the torsional moment was 47 percent of the ultimate torsional moment. The first cracks on the top face were observed after application of load increment 5 when the bending and torsional moments were 54 percent and 56 percent of their ultimate values respectively. The unit angle of twist after application of load increment 9 was 1019×10^{-6} radians per inch. Failure occurred during application of load increment 10. Readings taken subsequent to failure indicated that the beam was supporting 100 percent of the ultimate bending moment, 90 percent of the ultimate torsional moment and that the unit angle of twist was 1777×10^{-6} radians per inch.

The eighth assumption of SECTION 5-3 states that the characteristics of the concrete in the compression zone are known and that the concrete reaches its strength in flexural compression. This assumption is true only for pure bending and becomes progressively more inaccurate as the amount of torsion a beam is subjected to is increased. When a beam is subjected to combined bending and torsion, the compression zone shown in FIGURES 5-1, 5-2 and 5-3 is subjected to a complex state of stress that is further complicated by cracks crossing the compression zone. Subjecting a beam to transverse shear as well as bending and torsion increases the complexity of this problem.

Observation of the beams at failure suggested that the ends of the compression zone were less effective in resisting compression than the central portion. This can be illustrated by referring to FIGURE 5-1. The compression zone shown in this failure surface is intersected by cracks that cross the top face of the beam. These cracks, in conjunction with other cracks that formed shortly before failure, permitted displacement of part of the left half of the compression zone shown in FIGURE 5-1. The other segment of the beam showed similar displacements in the other half of the compression zone. If the ends of the compression zone are less effective in resisting compression than the central portion, the depth of the compression zone should be computed using an effective length of compression zone that is less than the full length shown in FIGURES 5-1, 5-2 and 5-3. In addition, when failure occurred it often appeared that the capacity of the compression zone was exhausted by pieces of concrete in the compression zone breaking away from the remainder of the beam. This is illustrated by FIGURES 4-5 and 4-10. Consequently, it appears that for beams subjected to combined loading the parameters $0.85f_c'$ and k_1 in Equations (5-2), (5-6) and (5-11) should be modified or replaced by other parameters.

If a factor k_e is introduced into Equations (5-2), (5-6) and (5-11) to reduce the effective length of the compression zone and to modify the parameters $0.85f'_c$ and k_1 , Equations (8-1), (8-2) and (8-3) are obtained.

$$x_1 = \frac{A_{s1}f_{y1}(b + p_1k_{01}c_1^{2}/h)}{0.85\ k_e k_1 f'_c\ L_1^{2}} \tag{8-1}$$

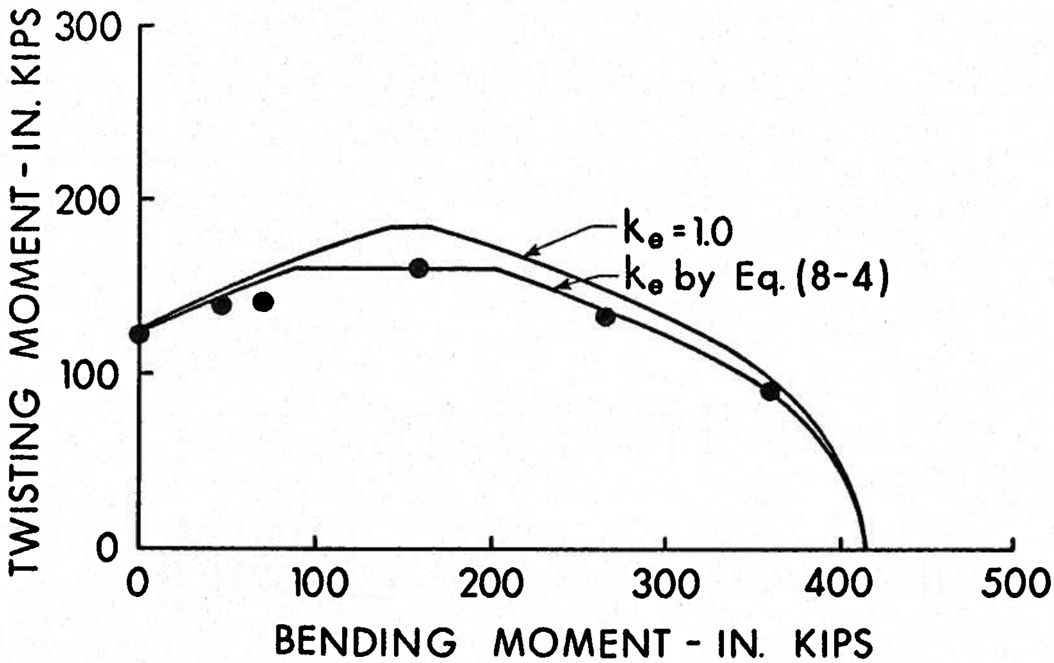
$$x_2 = \frac{A_{s2}f_{y2}(h + p_2k_{02}c_2^{2}/b) - V\ c_2}{0.85\ k_e k_1 f'_c\ L_2^{2}} \tag{8-2}$$

$$x_3 = \frac{A_{s3}f_{y3}(b + p_3k_{03}c_3^{2}/h)}{0.85\ k_e k_1 f'_c\ L_3^{2}} \tag{8-3}$$

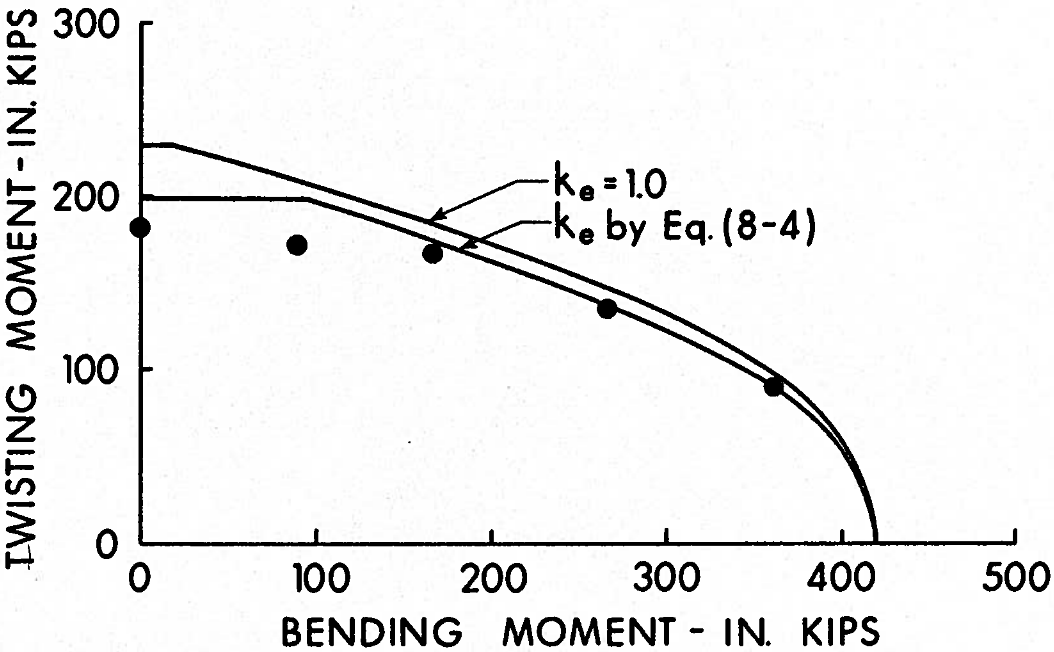
To show the effect of this factor on the theoretical ultimate torsional moment of a beam, the interaction diagrams for the beams of Groups 1 and 2 are shown in FIGURE 8-1. In addition, the failure values of twisting moment and bending moment are shown. The interaction diagrams for $k_e = 1.0$ are the same as the ones shown in FIGURE 6-1. The other interaction diagrams in FIGURE 8-1 are plotted with the value of k_e defined by Equation (8-4).

$$k_e = 1.0 - 1.6\ c/c_m \geq 0.2 \tag{8-4}$$

For values of c/c_m greater than 0.5 this equation assigns a value to k_e of 0.2. For values of c/c_m between zero and 0.5 the value assigned



(a) GROUP 1



(b) GROUP 2

FIGURE 8 - 1 MODIFIED INTERACTION DIAGRAMS FOR BEAMS OF GROUPS 1 AND 2

to k_e by this equation varies linearly from 1.0 to 0.2. The selection of the lower limit of k_e as 0.2 and the upper limit of c/c_m as 0.5 is completely arbitrary. The linear variation is assumed in order to provide a simple but smooth transition in the range $0<c<0.5\ c_m$. Equation (8-4) greatly oversimplifies the determination of k_e and is intended only to show the effect of this factor on the interaction diagram.

8-7 Simplified Analysis

A simplified analysis and a simplified interaction diagram are presented in CHAPTER VII. Comparison of the values of the ratio M_{tt}/M_t listed in TABLES 6-1 to 6-7 with the values listed in TABLES 7-1 and 7-2 indicates that the simplified analysis is generally conservative compared to the analysis presented in CHAPTER V.

The analysis presented in CHAPTER VII has the disadvantage of requiring numerous computations to determine the torsional strength of a beam. However, the number of calculations required is decreased if top reinforcement equal to the bottom reinforcement is provided. In addition, the parameters in the equations are grouped in a manner such that if adjustments to the trial section are required, a minimum number of quantities must be recalculated. The principal advantage of this type of analysis is that it is adaptable to tabulation of M_{ta}, M_{tb}, M_{tc} and M_{bu} for various beam cross sections and reinforcement combinations.

This analysis is applicable only to rectangular, concrete beams that are provided with both longitudinal and transverse reinforcement adjacent to all faces. The amount of reinforcement provided must be sufficient to ensure that failure does not occur when first cracking of the concrete occurs. If this requirement is not satisfied the beam should be analysed as a plain concrete beam. This analysis should not be used if the magnitude of transverse shear exceeds the limits discussed in SECTION 8-5.

CHAPTER IX

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

9-1 Summary

The experimental phase of this investigation consisted of testing 34 reinforced concrete beams subjected to various combinations of load. All beams had a nominal 6" x 12" cross section and a nominal concrete strength of 5000 psi. Various combinations of reinforcement were provided but except for the beams of Group 4 all beams within one group had similar reinforcement. All beams were provided with both longitudinal and transverse reinforcement.

The 34 beams were divided into 7 groups. The beams of Groups 1, 2, 3 and 4 were subjected to various combinations of bending and torsion and the beams of Groups 5, 6 and 7 were subjected to various combinations of bending, torsion and shear.

All beams were tested to failure by applying load in a series of increments. Each increment consisted of increasing to a predetermined level the transverse load or the twisting load or both, depending on the type of test. In the cases where both types of load were applied in the same increment, the transverse load was applied first. All beams except those of Group 3 and Beam 1-2 were subjected to loads such that for any one test the ratio of twisting moment to bending moment at the end of each increment was a constant. The loading sequences for the beams of Group 3 and Beam 1-2 have been outlined in SECTION 3-4.

Based on the observations made in the experimental phase of this investigation three idealized failure surfaces have been presented, two of which were first suggested by Lessig (1959). Equations for the ultimate torsional strength of a beam based on each of the three failure surfaces have been developed and their method of solution has been presented. These equations have then been simplified and an interaction diagram consisting of three straight lines has been presented along with the applicable equations.

The correlation between the experimental results of 109 beams and the results of both the analysis presented in CHAPTER V and the simplified analysis presented in CHAPTER VII has been given. Of these beams, 34 are the beams tested in this investigation and 75 are beams that have been tested by other investigators.

9-2 Conclusions

The following conclusions are based on the results of this investigation and are limited to rectangular, concrete beams with both longitudinal and transverse reinforcement.

(1) Reinforced concrete beams that are subjected to bending, torsion and moderate amounts of transverse shear can fail by three different modes. These modes of failure are characterized by the formation of a hinge adjacent to one face of the beam and yielding of the reinforcement adjacent to the face opposite to the hinge. The modes of failure predicted by the analysis agree with the observed modes of failure.

(2) In beams subjected to a constant twisting moment and a linearly varying bending moment a mode 3 failure occurs in the region subjected to the lowest bending moment and a mode 1 failure occurs in the region subjected to the highest bending moment.

(3) A beam that is provided with less top longitudinal reinforcement than bottom longitudinal reinforcement and is subjected to a small bending moment in addition to a torsional moment exhibits a greater torsional strength than if the same beam is subjected to pure torsion.

(4) A beam that is provided with less top longitudinal reinforcement than bottom longitudinal reinforcement and is subjected to either pure torsion or torsion in combination with a small bending moment deflects upwards after cracking of the concrete has occurred and exhibits an upward deflection at failure.

(5) The presence of flexure does not increase the torsional strength of a beam provided with equal top and bottom reinforcement.

(6) A small or moderate amount of transverse shear does not affect the torsional strength of a beam except as provided for by a mode 2 failure surface.

(7) The top longitudinal reinforcement does not contribute significantly to the torsional strength of a beam that fails by mode 1.

(8) The assumption that at failure the full length of the concrete compression zone reaches its flexural strength gives an upper bound of the torsional strength of a beam.

(9) Increasing the tie spacing decreases the torsional strength of a beam.

(10) The sequence of loading does not significantly affect the torsional strength of a beam.

(11) The behavior of a beam prior to cracking of the concrete is not significantly affected by the reinforcement provided. After cracking occurs, the behavior depends on the reinforcement and on the ratio of twisting moment to bending moment.

(12) In beams tested at a constant ratio of twisting moment to bending moment the presence of bending moment reduces the torsional moment at which cracking of the concrete occurs.

(13) The limit of over reinforcement suggested by Lessig for ratios of twisting moment to bending moment greater than 0.2 is conservative.

(14) The lower limit of the ratio of transverse to longitudinal reinforcement suggested by Lessig is conservative.

9-3 Recommendations

The scope of this investigation is necessarily limited and before a comprehensive design procedure can be established further research is necessary. In particular, the following aspects of the problem should be investigated.

(1) Determination of the effective length of the concrete compression zone and the criterion of failure for the concrete in the compression zone is necessary.

(2) The amount of reinforcement necessary to cause failure of a beam before the reinforcement yields must be determined.

(3) The minimum amount of reinforcement necessary to prevent failure from occurring when first cracking of the concrete occurs must be determined.

(4) The maximum tie spacing for which this analysis applies must be determined.

(5) The effect of placing the longitudinal bars at various locations within the cross section should be investigated.

(6) Beams with height to width ratios of less than 1.0 should be tested.

(7) The location of the critical plane in beams subjected to a torsional moment and a variable bending moment must be more accurately established.

(8) Beams subjected to torsion, bending and high transverse shear should be tested. Also, it is necessary to develop a method to predict the torsional strength of a beam subjected to only torsion and transverse shear. This is necessary not only as a limiting case but to permit determination of the torsional strength of a beam at an inflection point.

(9) Beams with cross sections other than rectangular or square should be tested. Equations for the torsional strength of these beams could be developed on the basis of the failure surfaces that are observed.

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APPENDIX A

PROPERTIES OF MATERIALS AND FABRICATION OF SPECIMENS

APPENDIX A

PROPERTIES OF MATERIALS AND FABRICATION OF SPECIMENS

A-1 Materials

(a) Sand

A sieve analysis of the sand used is given in TABLE A-1. The average moisture content of the sand was found to be 4.5 percent.

TABLE A-1
Sieve Analysis of Sand

Sieve Size	Weight Retained (gms.)	% Retained	Cumulative % Retained	A.S.T.M. Standard
#4	17.5	3.0	3.0	0 - 5
#8	85.2	14.7	17.7	
#16	54.6	9.5	27.2	20 - 55
#30	60.0	10.3	37.5	
#50	208.4	35.8	73.3	70 - 90
#100	122.9	21.1	94.4	90 - 98
Pan	17.8	3.1	--	
Silt	14.4	2.5	--	
Total	580.8	100.0	253.1	

Fineness Modulus 2.53

(b) Coarse Aggregate

The coarse aggregate was 3/4" maximum size crushed rock. The average moisture content was 1.7 percent. The results of a sieve analysis are presented in TABLE A-2.

TABLE A-2

Sieve Analysis of Coarse Aggregate

Sieve Size	Weight Retained (lbs.)	% Retained	Cumulative % Retained
3/4"	0.30	1.1	1.1
3/8"	15.63	58.4	59.5
#4	10.03	37.5	97.0
Pan	0.80	3.0	100.0
Total	26.76	100.0	257.6

(c) Concrete

The mix design used for all specimens was:

(1) Cement 114 lbs.

(2) Sand 294 lbs.

(3) Coarse Aggregate 428 lbs.

The quantity of water used was approximately 57 lbs. but minor adjustments were made as required to produce a mix of approximately 3" slump. The yield of this mix was about six cubic feet.

High early strength cement supplied in paper bags was used.

(d) Reinforcement

All reinforcement used in the specimens was of the deformed type. The reinforcement of one size was all from the same heat except for the ties. These were received in two batches and were designated as lot number 1 and lot number 2. All ties within one lot were from the same heat. Coupon tests were performed on three specimens from each group in order to determine the yield and ultimate strength of the reinforcement. The results of these tests are shown in TABLE A-3.

TABLE A-3
Strength Properties of Reinforcement

Group	Average Yield Stress (k.s.i.)	Average Ultimate Stress (k.s.i.)
#3 Bars	53.0	79.5
#4 Bars	47.0	73.9
#5 Bars	48.9	76.1
#6 Bars	46.9	76.7
#8 Bars	43.8	81.8
#3 Ties Lot #1	55.0	80.4
#3 Ties Lot #2	53.7	78.4

A-2 Fabrication of Specimens

The longitudinal reinforcement was first cut to the required length and then the reinforcement cages were fabricated by wiring the ties to the longitudinal bars at each intersection. At each point where an electrical strain gauge was required, the reinforcement was ground to a smooth finish and covered with a piece of rigid insulation which was securely taped to the bar. Before concrete was cast, the forms were oiled and the reinforcing cages were placed and properly positioned in them.

The concrete was mixed in a nine cubic foot capacity laboratory concrete mixer. One batch was sufficient for each beam along with its control cylinders. Before mixing began, a one cubic foot butter mix was used to condition the mixer. All concrete was mixed for approximately five minutes. The water content was adjusted until a slump of 3" was obtained. When the proper slump was obtained the concrete was deposited in the forms and vibrated with a high frequency internal vibrator.

Four six by twelve inch concrete cylinders were cast with each beam. These cylinders were compacted by rodding and were cured and stored under the same conditions as the beams. Two cylinders were tested in compression and the other two were subjected to the splitting tensile test. A beam and the cylinders corresponding to it were tested on the same day.

The beams and cylinders were cured for three days by covering them with wet burlap and a plastic sheet. Following this, the forms were stripped and the specimens were stored in air until the day of test.

Before the day of test, the pieces of insulation covering the smooth areas of the reinforcement were removed in order to expose the prepared surfaces of the bar. These surfaces were then cleaned and SR-4 electrical strain gauges were glued to them.

APPENDIX B

OBSERVED DATA

APPENDIX B

OBSERVED DATA

B-1 Test Results

The test results for each beam are given in TABLES B-1 to B-34. Reinforcement stresses that are enclosed in parentheses indicate that the reinforcement had yielded before the strain began to decrease. For computation of these stresses it has been assumed that the maximum recorded strain was equal to the maximum strain that occurred.

B-2 Deformation Characteristics and Reinforcement Stresses

The angle of twist, deflection and reinforcement stress for each beam are plotted against twisting moment in FIGURES B-1 to B-18. No deflections were measured for Beams 3-1 or 3-4. Upward (negative) deflections are plotted on the left side of zero and downward (positive) deflections on the right side. Compressive (negative) stresses are plotted on the left side of zero and tensile (positive) stresses on the right side. The unusual shape of the plots for the beams of Group 3 is due to the sequence of loading that was used. The sequence of loading for the beams is presented in CHAPTER III.

The plots of twisting moment versus deflection and reinforcement stress have been selected as representative of the results obtained. The word east, west or center beside a plot of deflection indicates the deflection gauge to which the plot corresponds. The number beside a plot of reinforcement stress indicates the strain gauge to which the plot corresponds. The location of the deflection gauges and the numbering system of the strain gauges are presented in CHAPTER III.

TABLE B-1

TEST RESULTS FOR BEAM 1-1

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in. x 10^6	Deflection inches x 10^2 West	Center	East
0	0	--	--	0	0	0	0
1	23	--	--	30	0	- 0.5	- 0.5
2	46	--	--	65	0	0	- 0.5
3	55	--	--	90	0	0	- 0.5
4	65	--	--	243	- 1.0	- 1.5	- 1.5
5	79	--	--	503	- 4.0	- 5.0	- 4.5
6	98	--	--	813	- 9.0	-10.5	- 9.0
7	107	--	--	960	-11.0	-12.5	-11.0
8	117	--	--	1233	-16.0	-19.0	-16.5
9*	122	--	--	--	--	--	--

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	+0.1	+0.2	+0.3	+0.3	+0.6	+0.3	-0.1	+0.2
2	0.3	0.7	1.0	0.9	1.7	2.7	-0.3	0.5
3	0.9	1.5	1.9	1.7	3.0	7.8	-0.1	1.0
4	18.8	15.3	8.1	7.0	10.8	26.4	+10.5	6.0
5	35.8	38.5	15.0	11.6	18.7	38.2	15.3	12.9
6	52.7	Yield	21.5	17.3	28.5	48.6	18.6	24.8
7	Yield	Yield	22.8	19.7	32.2	52.5	21.0	32.4
8	Yield	Yield	23.5	23.8	37.2	Yield	23.7	39.0
9*	--	--	--	--	--	--	--	--

*Failure

TABLE B-2

TEST RESULTS FOR BEAM 1-2

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in. x 10^6	Deflection inches x 10^2 West	Center	East
0	0	0	0	0	0	0	0
1	49	24	2.04	85	+ 0.5	+ 1.0	+ 1.0
2	60	27	2.22	183	1.0	0.5	1.5
3	71	30	2.37	363	0.5	0	0
4	81	32	2.53	535	- 1.0	- 1.5	- 1.0
5	91	35	2.60	683	- 2.0	- 3.5	- 3.0
6	102	38	2.69	838	- 4.0	- 5.5	- 4.0
7	107	39	2.74	905	- 5.0	- 6.0	- 4.5
8	112	40	2.80	990	- 5.5	- 7.0	- 6.5
9	117	42	2.79	1073	- 6.0	- 8.0	- 7.0
10	123	43	2.86	1173	- 7.0	- 9.0	- 8.0
11	127	44	2.89	1328	- 8.5	-11.0	- 9.0
12	133	46	2.90	1435	-11.0	-13.0	-10.5
13*	138	47	2.94	--	--	--	--

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	-0.1	+0.2	+2.5	+2.7	+2.7	+4.6	-0.7	+0.2
2	+3.3	2.9	9.8	7.1	6.6	14.1	+2.0	2.9
3	17.1	13.3	15.6	13.1	11.1	25.1	5.3	13.3
4	29.5	26.6	20.7	16.2	14.4	30.9	9.0	26.6
5	38.7	34.5	23.7	18.6	17.4	36.5	12.6	34.5
6	48.7	42.2	26.3	20.8	21.2	41.1	16.0	42.2
7	Yield	45.2	27.3	21.8	23.1	42.9	17.7	45.2
8	Yield	48.5	28.6	23.0	25.1	44.9	19.5	48.5
9	Yield	51.9	30.0	24.3	27.5	47.6	21.8	51.9
10	Yield	Yield	31.1	25.6	30.0	49.6	24.3	Yield
11	Yield	Yield	32.2	27.3	32.5	51.2	26.7	Yield
12	Yield	Yield	34.0	29.1	35.6	52.6	30.0	Yield
13*	Yield	Yield	36.2	31.0	38.4	Yield	34.6	Yield

*Failure

TABLE B-3

TEST RESULTS FOR BEAM 1-3

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in. x 10^6	Deflection inches x 10^2 West	Center	East
0	0	0	--	0	0	0	0
1	48	24	2.00	110	1.5	0.5	0.5
2	60	30	2.00	288	1.0	0	0.5
3	71	35	2.03	495	1.0	- 0.5	-0.5
4	80	40	2.00	633	0	- 1.5	-1.0
5	91	46	1.98	813	-0.5	- 3.0	-2.5
6	102	51	2.00	995	-1.5	- 4.0	-4.0
7	113	57	1.98	1200	-2.5	- 6.0	-5.0
8	124	62	2.00	1410	-3.5	- 7.5	-6.0
9	129	65	1.99	1570	-4.5	-10.0	-7.5
10	134	67	2.00	1730	-5.5	-11.5	-8.5
11*	140	70	2.00	--	--	--	--

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	0	1.3	2.6	3.3	0.5	1.6	-0.4	5.7
2	14.5	9.9	10.4	11.2	4.8	21.2	+9.1	21.5
3	23.7	21.0	14.7	15.6	8.9	26.7	15.8	27.9
4	29.7	27.3	17.5	19.2	12.1	30.0	20.7	33.0
5	37.5	36.5	20.2	22.6	18.0	33.1	27.3	38.4
6	43.6	45.2	22.8	25.8	23.2	36.9	31.0	42.6
7	50.5	Yield	25.5	29.4	29.8	39.8	34.1	47.0
8	Yield	Yield	28.2	32.6	36.0	43.0	37.4	49.9
9	Yield	Yield	29.3	34.5	39.4	45.5	38.9	51.1
10	Yield	Yield	31.2	36.4	44.5	46.3	40.9	51.0
11*	Yield	Yield	33.3	38.9	49.3	45.3	--	--

*Failure

TABLE B-4

TEST RESULTS FOR BEAM 1-4

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in. x 10^6	Deflection inches x 10^2 West	Center	East
0	0	0	--	0	0	0	0
1	23	24	0.96	28	1.0	0.5	0
2	38	38	1.00	50	1.5	1.5	1.0
3	52	51	1.02	100	3.5	3.5	3.0
4	65	65	1.00	195	5.5	6.0	5.0
5	78	78	1.00	315	7.5	7.0	6.0
6	98	98	1.00	553	8.0	7.5	6.5
7	118	119	0.99	813	8.5	8.0	7.0
8	132	132	1.00	1013	8.0	7.5	6.5
9	145	146	0.99	1258	8.5	7.0	6.0
10*	159	159	1.00	--	-	-	-

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	-0.6	-0.5	+1.0	+0.8	+0.1	+0.3	-0.2	+0.4
2	-2.0	-1.6	3.2	3.7	0.6	1.1	-0.5	1.1
3	-3.3	-2.5	6.6	8.7	1.8	3.1	-1.0	5.8
4	-3.4	-1.9	13.6	16.6	5.3	20.1	+1.5	20.7
5	+2.7	+3.6	21.6	23.8	7.9	26.7	10.4	28.2
6	11.1	14.1	30.0	32.7	13.5	34.8	15.8	37.8
7	18.4	27.0	38.1	40.8	21.0	42.9	17.9	45.3
8	24.8	31.5	44.0	45.3	25.8	45.3	18.9	49.4
9	33.4	37.2	Yield	Yield	31.8	47.7	20.1	53.4
10*	--	--	--	--	--	--	--	--

*Failure

TABLE B-5

TEST RESULTS FOR BEAM 1-5

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in. x 10^6	Deflection inches x 10^2 West	Center	East
0	0	0	--	0	0	0	0
1	13	24	0.54	18	0.5	0.5	0
2	27	51	0.53	43	2.5	2.0	1.5
3	39	78	0.50	80	5.0	5.0	4.5
4	53	105	0.50	153	8.0	9.5	7.0
5	67	132	0.51	250	11.0	12.5	10.5
6	86	173	0.50	443	15.5	17.0	15.0
7	107	213	0.50	695	19.0	21.0	17.5
8	113	227	0.50	805	20.5	22.5	19.5
9	120	240	0.50	898	22.0	24.0	19.5
10	128	254	0.50	1210	26.0	29.0	24.5
11*	131	267	0.49	--	--	--	--
12**	111	267	0.42	2325	57.0	62.5	50.5

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	-0.6	-0.4	+0.6	+0.9	+0.5	+0.3	+0.3	+0.2
2	-3.1	-2,9	3.2	4.5	1.6	1.0	0.5	0.8
3	-6.6	-5.1	9.1	10.7	3.9	2.8	0.4	9.9
4	-8.3	-5.7	16.8	18.7	6.5	9.5	0.6	17.7
5	-9.2	-4.8	23.0	26.6	8.4	15.7	1.7	25.8
6	-5.4	-0.9	34.7	41.4	11.1	25.7	10.8	34.3
7	-3.8	+3.2	44.8	Yield	19.9	32.3	15.9	45.9
8	-3.9	3.0	Yield	Yield	23.7	34.7	18.3	49.5
9	-4.2	3.6	Yield	Yield	27.7	36.3	20.2	53.6
10	-1.2	5.7	Yield	--	35.9	39.8	26.7	Yield
11*	--	--	--	--	--	--	--	--
12**	-8.7	-5.9	--	--	Yield	Yield	37.5	Yield

*Failure
**Past peak loads

TABLE B-6

TEST RESULTS FOR BEAM 1-6

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in. x 10^6	Deflection inches x 10^2 West	Center	East
0	0	0	--	0	0	0	0
1	10	38	0.26	13	1.0	1.5	1.0
2	19	78	0.24	30	3.5	5.0	4.0
3	29	119	0.24	55	7.5	9.0	8.0
4	39	159	0.25	90	11.5	13.0	14.0
5	49	200	0.25	135	15.5	17.5	15.5
6	60	240	0.25	188	19.5	22.0	19.0
7	70	281	0.25	243	23.0	26.5	23.0
8	80	321	0.25	315	27.0	31.0	27.0
9*	90	362	0.25	-	--	--	--
10**	76	362	0.21	-	173.0	207.5	177.0

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	- 1.6	- 0.9	+1.5	+1.7	+0.4	+0.1	+0.4	+0.1
2	- 6.6	- 4.3	7.8	7.1	0.9	0	0.9	0
3	-11.3	- 7.5	15.9	16.0	3.2	-0.1	1.2	-0.3
4	-15.5	-10.1	23.6	24.3	4.2	+0.3	1.7	+0.1
5	-19.6	-12.3	30.3	32.4	5.2	0.9	2.0	0.8
6	-24.3	-14.4	36.6	40.5	6.2	2.5	2.4	1.9
7	-28.4	-15.9	42.9	Yield	7.4	3.9	2.7	8.4
8	-32.9	-17.6	Yield	Yield	9.1	8.7	3.6	15.6
9*	-37.2	-17.8	Yield	Yield	21.0	--	--	--
10**	Yield	-44.3	--	--	Yield	Yield	42.7	46.5

*Failure
**Past peak loads

TABLE B-7

TEST RESULTS FOR BEAM 2-1

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in. x 10^6	Deflection inches x 10^2 West	Center	East
0	0	--	--	0	0	0	0
1	20	--	--	30	-0.5	0	0
2	40	--	--	58	0	0	0
3	60	--	--	258	+0.5	0	-1.5
4	80	--	--	463	-0.5	-1.5	-1.0
5	100	--	--	668	-1.5	-2.0	-2.0
6	120	--	--	895	-2.0	-3.0	-3.0
7	140	--	--	1145	-3.0	-4.0	-3.5
8	161	--	--	1483	-4.0	-4.5	-4.0
9*	181	--	--	--	--	--	--
10**	162	--	--	2748	-5.0	-16.5	-5.5

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	+0.1	0	0	+0.2	+0.1	+0.3	0	-0.2
2	0.6	+0.5	+0.4	0.7	1.0	0.9	-0.2	+1.5
3	8.9	10.4	8.7	6.0	10.5	18.5	+10.6	27.2
4	15.7	16.8	14.7	12.3	16.6	28.4	16.1	32.4
5	20.1	21.3	17.4	18.6	22.6	37.6	21.0	40.4
6	23.6	24.5	20.8	22.8	29.7	45.9	25.6	46.1
7	27.3	27.7	24.2	26.6	37.9	Yield	30.0	51.9
8	32.2	32.5	30.0	30.2	46.9	Yield	34.9	Yield
9*	40.6	42.0	36.7	37.8	Yield	Yield	--	--
10**	Yield	Yield	Yield	34.4	Yield	--	Yield	Yield

*Failure
**Past peak loads

TABLE B-8

TEST RESULTS FOR BEAM 2-2

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in. x 10^6	Deflection inches x 10^2 West	Center	East
0	0	0	--	0	0	0	0
1	48	24	2.00	208	1.0	1.5	3.5
2	59	30	1.97	320	2.0	1.0	1.5
3	70	35	2.00	408	2.5	2.5	2.0
4	81	40	2.02	520	2.5	2.0	0.5
5	99	50	1.98	733	3.0	3.5	3.0
6	119	59	2.02	988	3.5	4.0	4.0
7	138	69	2.00	1233	4.0	4.0	3.5
8	157	78	2.01	1593	4.0	4.5	4.0
9*	172	88	1.96	--	-	-	-
10**	154	88	1.58	2190	7.0	4.5	7.0

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	4.9	5.6	6.8	10.8	11.0	22.3	11.1	17.2
2	9.9	11.7	17.7	13.9	14.1	26.9	14.6	23.0
3	12.3	13.5	21.6	16.0	16.4	31.2	17.3	25.5
4	14.7	15.0	25.8	18.0	19.6	35.7	20.0	30.7
5	17.1	16.4	31.4	23.6	28.8	41.3	28.0	43.8
6	21.3	18.9	37.5	30.0	35.4	45.2	34.4	Yield
7	24.8	21.9	43.8	34.6	42.8	50.1	39.9	Yield
8	27.8	26.0	Yield	39.9	53.0	Yield	43.2	(51.7)
9*	--	--	--	--	--	--	--	--
10**	33.6	29.4	Yield	42.9	Yield	Yield	48.3	Yield

*Failure
**Past peak loads

TABLE B-9

TEST RESULTS FOR BEAM 2-3

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in. x 10^6	Deflection inches x 10^2 West	Center	East
0	0	0	--	0	0	0	0
1	25	24	1.04	33	0.5	0	0.5
2	38	38	1.00	90	1.5	2.0	2.0
3	51	51	1.00	200	4.0	4.5	4.0
4	64	65	0.98	333	5.0	6.0	5.5
5	78	78	1.00	478	6.5	7.0	6.5
6	97	98	0.99	753	8.0	8.5	7.5
7	118	119	0.99	1055	10.5	11.5	9.5
8	132	132	1.00	1240	11.0	12.0	10.5
9	145	146	0.99	1485	13.0	14.0	11.5
10	159	159	1.00	1850	16.5	17.5	14.5
11*	166	166	1.00	--	--	--	--
12**	149	166	0.90	2703	29.5	27.0	27.5

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	-0.5	-0.1	+0.6	+0.7	+0.5	+0.2	-0.3	0
2	-0.8	+1.6	2.9	4.6	6.3	-0.9	+1.2	17.2
3	+3.9	3.3	12.7	10.4	7.8	+1.1	3.0	24.9
4	6.0	11.5	18.0	18.7	9.9	4.6	5.5	30.3
5	7.6	15.0	22.0	23.5	13.3	24.0	8.5	32.7
6	9.3	18.4	27.9	30.3	20.4	37.2	13.5	40.8
7	14.7	21.2	35.1	36.0	28.7	46.1	19.9	49.5
8	16.2	22.6	39.9	40.1	33.1	51.7	23.5	Yield
9	17.4	24.1	45.0	44.3	37.4	Yield	27.3	Yield
10	20.0	27.0	Yield	Yield	46.2	Yield	32.4	Yield
11*	--	--	--	--	--	--	--	--
12**	13.8	27.3	Yield	Yield	Yield	(40.0)	47.2	Yield

*Failure
**Past peak loads

TABLE B-10

TEST RESULTS FOR BEAM 2-4

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in. x 10^6	Deflection inches x 10^2 West	Center	East
0	0	0	--	0	0	0	0
1	12	24	0.50	15	0	0.5	0.5
2	25	51	0.49	30	1.0	2.5	1.5
3	38	78	0.49	60	4.0	5.0	4.0
4	52	105	0.50	148	8.0	9.0	7.5
5	66	132	0.50	250	11.0	12.5	10.5
6	86	173	0.50	428	15.0	17.5	14.5
7	106	213	0.50	623	18.5	22.0	19.0
8	120	240	0.50	780	21.0	24.5	21.0
9	126	254	0.50	875	23.0	26.0	22.0
10*	1.34	267	0.50	--	--	--	--
11**	112	267	0.42	2030	55.0	59.5	48.5

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	-0.4	-0.2	+0.6	+0.6	+0.2	0	0	+0.3
2	-2.1	-1.5	2.6	2.7	0.6	0	+0.1	0.6
3	-4.5	-3.1	7.1	7.9	2.0	0	-0.3	6.9
4	-4.4	-1.6	14.8	22.9	4.3	+8.2	-1.0	12.3
5	-3.9	-1.2	22.2	32.1	6.8	22.5	-0.1	16.5
6	-3.4	-1.2	35.5	44.2	11.3	29.6	+4.9	27.3
7	-5.0	-3.0	45.9	Yield	16.5	37.9	14.7	36.4
8	-5.4	-4.7	Yield	Yield	22.0	42.8	19.8	41.1
9	-5.7	-5.5	Yield	Yield	25.5	45.8	22.1	43.1
10*	--	--	--	--	--	--	--	--
11**	-10.9	-10.7	Yield	Yield	Yield	34.0	36.9	49.4

*Failure
**Past peak loads

TABLE B-11

TEST RESULTS FOR BEAM 2-5

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in. x 10^6	Deflection inches x 10^2 West	Center	East
0	0	0	--	0	0	0	0
1	9	38	0.24	8	1.5	1.5	1.5
2	20	78	0.26	33	5.0	6.0	5.0
3	30	119	0.25	58	8.5	10.5	8.5
4	40	159	0.25	105	13.0	14.5	12.5
5	51	200	0.25	158	17.0	19.5	17.0
6	61	240	0.25	208	21.0	23.5	20.5
7	70	281	0.25	275	24.5	27.5	24.0
8	80	321	0.25	335	28.0	32.0	28.0
9*	90	362	0.25	--	--	--	--
10**	79	362	0.22	1923	99.0	123.0	104.5

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	- 1.4	- 1.0	+ 1.6	+ 1.5	+0.4	+0.1	+0.1	+0.3
2	- 5.0	- 2.8	+10.6	+13.7	+3.1	0	-0.3	+1.7
3	- 8.1	- 5.1	19.9	20.1	4.8	+1.6	-1.1	3.6
4	-10.5	- 7.6	27.8	26.2	6.4	+2.6	-1.8	6.4
5	-13.2	-10.1	34.1	33.1	6.9	3.3	-2.2	10.5
6	-15.9	-11.9	40.0	39.6	7.6	4.0	-2.3	14.2
7	-18.6	-13.6	46.5	46.1	7.8	5.5	-1.8	17.5
8	-21.0	-15.3	Yield	Yield	8.3	7.0	-1.0	21.0
9*	--	--	--	--	--	--	--	--
10**	-29.4	-42.8	Yield	--	Yield	46.8	+20.0	Yield

*Failure
**Past peak loads

TABLE B-12

TEST RESULTS FOR BEAM 3-1

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in. x 10^6
0	0	--	--	0
1	22	--	--	25
2	45	--	--	65
3	50	--	--	75
4	55	--	--	85
5	59	--	--	172
6	64	--	--	258
7	73	--	--	593
8	82	--	--	733
9	92	--	--	922
10	102	--	--	1140
11	106	--	--	1270
12	111	--	--	1440
13*	115	--	--	--

Load	Reinforcement Stresses (ksi)							
	Gauge							
Stage	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	-0.1	-0.1	+0.4	+0.1	+0.2	+0.3	+0.2	+0.1
2	+0.2	+0.5	+1.1	+0.8	+0.5	+1.2	+0.5	+1.2
3	+0.6	+0.9	1.7	1.2	0.9	2.1	0.6	1.8
4	0.8	1.0	2.1	1.5	1.0	2.9	0.7	2.3
5	9.2	11.8	8.4	14.9	6.0	18.7	8.4	25.8
6	16.1	14.7	11.6	18.9	7.7	27.2	10.2	31.4
7	30.0	33.7	25.8	25.8	11.7	34.4	17.0	41.8
8	38.9	42.2	30.8	31.1	14.0	39.1	20.0	47.8
9	49.7	51.9	37.3	36.6	18.2	44.1	22.9	53.0
10	Yield	Yield	45.1	41.5	22.1	47.7	24.8	Yield
11	Yield	Yield	48.8	44.2	24.0	49.3	26.1	Yield
12	Yield	Yield	Yield	47.2	25.9	51.2	28.0	Yield
13*	--	--	--	--	--	--	--	--

*Failure

TABLE B-13

TEST RESULTS FOR BEAM 3-2

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in. x 10^6	Deflection inches x 10^2 West	Center	East
0	0	0	--	0	0	0	0
1	0	24	0	0	+0.5	+0.5	0
2	24	24	1.00	25	+1.0	+0.5	+0.5
3	45	24	1.87	75	+1.0	+0.5	+1.0
4	54	24	2.25	330	+1.0	0	+0.5
5	68	24	2.83	613	-2.0	-2.5	-2.5
6	85	24	3.54	1020	-5.5	-8.0	-7.0
7	85	51	1.67	1035	-3.5	-4.5	-5.0
8	85	78	1.09	1048	0	-1.5	-2.0
9	85	111	0.77	1050	+4.0	+3.5	+2.0
10	103	111	0.93	1238	+5.0	+4.5	+3.0
11	112	111	1.01	1733	+6.0	+4.5	+3.0
12*	117	111	1.05	--	--	--	--

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7
0	0	0	0	0	0	0	0
1	- 0.5	- 0.5	+0.4	+0.5	0	0	0
2	- 0.3	- 0.4	+0.7	+1.1	+0.3	+0.6	+0.1
3	+ 0.1	+ 0.2	1.7	4.5	+0.9	+1.7	+0.2
4	+17.7	+26.8	17.2	29.3	7.3	-0.5	+15.8
5	29.0	49.5	28.6	37.7	13.5	+0.6	21.0
6	50.4	Yield	38.7	Yield	22.8	+4.9	29.0
7	47.4	(48.4)	42.0	Yield	23.3	5.1	28.9
8	40.8	(42.2)	45.2	Yield	23.2	4.8	28.8
9	33.4	(34.4)	Yield	Yield	22.9	4.3	28.5
10	39.2	(44.2)	Yield	Yield	29.7	6.9	35.5
11	48.6	Yield	Yield	Yield	39.9	10.6	46.7
12*	--	--	--	--	--	--	--

*Failure

TABLE B-13 (continued)

TEST RESULTS FOR BEAM 3-2

Load Stage	Reinforcement Stresses (ksi) Gauge 8	9	10	11	12	13
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	+0.8	0	+0.3	-0.2	+0.3	-0.3
3	+8.0	+0.3	+1.1	-0.8	+0.7	-0.3
4	45.4	+4.8	3.6	+1.7	1.8	+1.3
5	Yield	10.3	6.7	+1.2	2.9	+6.6
6	Yield	16.4	11.0	2.4	2.5	10.8
7	Yield	17.0	10.7	1.8	1.5	10.1
8	Yield	17.3	10.5	1.5	1.2	10.0
9	Yield	17.4	10.4	1.7	1.2	10.1
10	Yield	23.4	13.7	2.1	2.0	13.4
11	Yield	31.7	19.7	8.2	3.3	18.8
12*	--	--	--	--	--	--

*Failure

TABLE B-14

TEST RESULTS FOR BEAM 3-3

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in. x 10^6	Deflection inches x 10^2 West	Center	East
0	0	0	--	0	0	0	0
1	0	51	0	0	+0.5	+2.0	+1.5
2	0	72	0	0	+2.5	3.0	2.5
3	0	92	0	0	5.0	6.0	5.5
4	0	111	0	0	7.0	9.0	8.0
5	22	111	0.20	28	8.5	9.5	8.5
6	43	111	0.39	73	9.5	10.5	10.0
7	52	111	0.47	110	9.0	12.0	10.0
8	56	111	0.50	140	9.5	11.5	10.5
9	61	111	0.55	170	10.0	12.5	10.5
10	70	111	0.63	273	11.0	12.5	11.5
11	79	111	0.71	398	11.5	13.0	11.5
12	88	111	0.79	580	11.0	13.0	10.5
13	97	111	0.87	780	10.5	11.5	9.5
14	106	111	0.95	1023	9.5	10.0	8.0
15	115	111	1.04	1430	--	--	--
16*	120	111	1.08	--	--	--	--

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	- 2.0	- 1.7	+2.0	+1.9	+0.6	+0.1	+0.4	+0.2
2	- 3.9	- 3.3	+3.9	+4.1	+1.6	+0.3	+0.8	+0.6
3	- 6.3	- 5.6	10.8	10.5	5.0	1.3	1.2	2.3
4	- 8.4	- 7.3	16.4	15.6	5.7	1.2	1.6	3.0
5	- 9.5	- 7.5	19.1	20.2	5.7	+0.5	1.9	2.3
6	-10.4	- 6.7	22.4	25.2	6.4	0	2.0	3.6
7	-10.2	- 6.3	24.5	28.2	7.5	+1.8	1.8	6.7
8	- 8.7	- 6.5	25.5	31.2	8.4	11.0	2.3	8.1
9	- 7.9	- 6.1	26.3	33.0	9.5	14.1	2.8	9.7
10	- 1.2	+ 1.7	30.2	38.9	12.6	24.9	9.3	15.9
11	+ 8.4	+11.4	34.5	43.7	16.1	35.9	16.0	25.1
12	+15.6	15.6	39.3	48.0	19.4	43.9	18.9	34.2
13	23.1	21.0	Yield	Yield	25.0	49.8	22.0	42.6
14	30.9	27.9	Yield	Yield	29.5	Yield	28.4	52.2
15	44.1	38.1	Yield	Yield	43.2	Yield	34.7	Yield
16*	--	--	--	--	--	--	--	--

*Failure

TABLE B-15

TEST RESULTS FOR BEAM 3-4

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in. x 10^6
0	0	0	--	0
1	0	24	0	0
2	22	24	0.92	20
3	45	24	1.87	53
4	49	24	2.04	63
5	54	24	2.25	75
6	59	24	2.46	95
7	63	24	2.63	150
8	68	24	2.83	215
9	77	24	3.21	415
10	86	24	3.58	698
11	86	40	2.15	723
12	86	55	1.57	728
13	96	55	1.75	790
14	106	55	1.93	938
15	110	55	2.00	1005
16	115	55	2.09	1110
17	115	65	1.77	1133
18	115	78	1.48	1140
19	115	92	1.25	1150
20	115	105	1.10	1155
21	115	119	0.97	1168
22	115	132	0.87	1170
23	115	153	0.75	--
24	115	173	0.67	1253
25*	115	188	0.61	--

*Failure

TABLE B-15 (continued)

TEST RESULTS FOR BEAM 3-4

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	-0.8	-0.8	+0.8	+1.0	+0.2	+0.1	+0.2	+0.2
2	-0.8	-0.8	+1.1	+1.0	+0.3	+0.3	0	+0.2
3	-0.5	-0.5	1.8	1.7	0.6	1.1	-0.1	1.5
4	-0.5	-0.4	2.1	2.0	0.8	1.4	0	2.3
5	-0.4	-0.1	2.3	2.6	0.8	1.7	-0.1	3.4
6	+0.7	+1.7	5.0	18.3	1.8	11.1	-1.1	16.7
7	+10.0	+3.4	8.0	24.4	4.0	26.6	+0.1	21.1
8	17.5	11.6	11.6	25.9	6.6	36.3	+1.1	22.4
9	29.3	22.6	21.7	32.1	11.9	46.4	10.4	30.9
10	41.4	34.9	35.6	37.4	17.0	Yield	19.8	40.3
11	41.0	34.6	38.0	39.2	17.2	(54.0)	20.1	40.2
12	38.3	32.5	40.2	41.6	17.1	(53.6)	20.0	40.1
13	41.3	34.9	43.1	44.9	18.8	Yield	21.9	43.4
14	48.2	40.6	48.1	48.8	21.6	Yield	25.2	48.2
15	51.7	43.6	Yield	Yield	22.9	Yield	26.7	49.4
16	Yield	47.1	Yield	Yield	24.6	Yield	28.5	51.8
17	Yield	46.9	Yield	Yield	24.6	Yield	28.9	51.5
18	(50.9)	44.7	Yield	Yield	24.5	(54.4)	28.9	51.1
19	(48.8)	42.4	Yield	Yield	24.4	(54.2)	29.0	51.1
20	(46.1)	39.5	Yield	Yield	24.2	(54.1)	29.2	51.0
21	(43.4)	36.7	Yield	Yield	24.1	(53.8)	29.1	50.9
22	(40.7)	33.9	Yield	Yield	23.9	(53.5)	29.3	51.2
23	(36.9)	29.1	Yield	Yield	24.5	(53.3)	29.6	52.7
24	(33.3)	24.4	Yield	Yield	25.1	(53.2)	31.1	Yield
25*	--	--	--	--	--	--	--	--

*Failure

TABLE B-16

TEST RESULTS FOR BEAM 3-5

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in. x 10^6	Deflection inches x 10^2 West	Center	East
0	0	0	--	0	0	0	0
1	0	24	0	0	+0.5	0	+0.5
2	21	24	0.88	25	+0.5	0	+0.5
3	43	24	1.79	60	+0.5	0	+0.5
4	55	24	2.29	228	+1.0	+1.0	+0.5
5	73	24	3.04	680	-2.5	-4.5	-3.0
6	73	92	0.79	708	+4.0	+2.5	+3.5
7	73	159	0.46	718	+9.5	+11.0	+10.5
8	73	227	0.32	738	+19.5	+21.0	+29.0
9*	73	265	0.28	-	--	--	--

Reinforcement Stresses

Load Stage	Gauge 1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0
1	-0.8	-0.3	+0.4	+0.5	0	+0.2	+0.2	+0.2	-0.3	-0.3
2	-0.8	-0.4	+0.5	+0.6	0	0.2	0	+0.3	-0.2	-0.3
3	-1.1	-0.5	1.0	1.1	+0.1	1.8	-0.3	1.4	-0.1	-0.4
4	--	+4.5	10.0	22.3	9.4	21.3	+2.5	34.7	+0.5	+0.3
5	--	40.1	26.1	33.6	19.1	51.0	13.8	54.2	+23.6	43.5
6	--	29.3	36.6	43.7	20.0	51.7	13.8	53.8	17.5	31.7
7	--	15.0	47.7	Yield	20.2	51.5	13.3	53.4	9.0	18.0
8	--	2.3	Yield	Yield	20.1	51.6	12.4	54.1	0.8	5.9
9*	--	--	--	--	--	--	--	--	--	--

Load Stage	Gauge 11	12	13	14	15	16	17	18	19	20
0	0	0	0	0	0	0	0	0	0	0
1	+0.6	+0.6	+0.1	+0.2	+0.3	+0.2	-0.2	+0.6	+0.1	+0.2
2	+0.6	0.8	0.3	0.3	0.3	0.1	-0.3	0.6	0	0.3
3	0.9	1.5	0.8	0.9	0.4	0.7	-0.5	5.7	-0.3	8.1
4	1.9	3.7	2.3	2.2	0.8	4.3	+2.5	--	+4.2	46.9
5	27.6	31.3	19.6	49.0	18.1	54.7	38.9	--	13.8	Yield
6	38.4	41.7	20.0	48.3	18.0	53.0	25.7	--	16.8	Yield
7	Yield	Yield	20.0	46.8	17.1	51.7	+10.5	--	18.6	Yield
8	Yield	Yield	19.6	44.7	16.4	51.5	-3.3	--	19.9	Yield
9*	--	--	--	--	--	--	--	--	--	--

*Failure

TABLE B-17

TEST RESULTS FOR BEAM 4-1

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in. x 10^6	Deflection inches x 10^2 West	Center	East
0	0	0	0	0	0	0	0
1	14	24	0.58	15	-0.5	+0.5	+0.5
2	31	51	0.61	40	+1.5	2.0	2.0
3	39	65	0.60	58	3.0	3.0	3.5
4	47	78	0.60	93	5.0	5.5	5.0
5	64	105	0.61	215	9.5	11.0	10.0
6	80	132	0.61	355	14.0	16.5	14.5
7	97	159	0.61	548	17.5	20.0	18.5
8	113	186	0.61	818	22.0	24.5	22.0
9*	120	200	0.60	--	--	--	--
10**	96	200	0.48	2028	52.0	61.0	109.0

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	-0.5	-0.6	+0.5	+0.6	+0.1	+0.2	+0.1	+0.2
2	-3.3	-3.0	+3.9	+3.8	+0.9	+0.5	+0.4	0.6
3	-4.9	-4.4	9.0	6.1	1.8	0.9	0.6	1.5
4	-6.9	-6.1	14.6	11.4	3.3	1.5	0.8	5.5
5	-9.8	-7.9	26.6	24.9	8.1	5.7	1.9	18.8
6	-10.2	-5.2	36.1	38.0	11.9	9.6	7.5	26.3
7	-8.1	-2.4	44.9	Yield	17.2	15.3	18.3	29.4
8	0	+2.6	Yield	Yield	23.1	22.2	25.6	33.4
9*	--	--	--	--	--	--	--	--
10**	-5.4	-6.2	Yield	Yield	34.5	33.6	24.1	24.6

*Failure
**Past peak loads

TABLE B-18

TEST RESULTS FOR BEAM 4-2

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in. x 10^6	Deflection inches x 10^2 West	Center	East
0	0	0	--	0	0	0	0
1	14	24	0.58	22	0	+0.5	0
2	31	51	0.61	48	+2.0	2.5	+2.0
3	39	65	0.60	70	3.0	4.0	3.0
4	48	78	0.62	115	5.5	7.0	5.5
5	63	105	0.60	288	11.5	13.0	11.0
6	72	119	0.61	383	12.5	15.0	12.5
7	80	132	0.61	495	14.5	17.0	14.5
8	89	146	0.61	693	16.0	18.0	15.0
9	97	159	0.61	935	18.0	20.0	17.0
10*	101	166	0.61	--	--	--	--
11**	88	166	0.53	2092	31.0	30.0	29.5

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	-0.7	-0.5	+0.5	+0.4	-0.1	0	0	0
2	-3.3	-2.3	3.5	3.0	+0.1	+0.9	+0.1	+0.6
3	-5.0	-3.5	6.6	5.1	1.1	+1.9	+0.1	+3.5
4	-6.9	-4.7	16.0	9.6	3.1	4.2	-0.3	10.2
5	-8.5	-3.0	40.3	26.1	5.9	7.1	+5.0	25.9
6	-6.9	-1.1	47.2	31.4	7.2	8.4	8.2	33.6
7	-1.2	+1.8	Yield	37.2	9.4	10.9	10.6	44.7
8	+1.7	+6.6	Yield	Yield	12.8	31.9	16.2	51.3
9	+4.6	+9.0	Yield	Yield	18.0	39.8	22.8	Yield
10*	--	--	--	--	--	--	--	--
11**	+7.8	+28.5	Yield	--	+42.3	+6.8	Yield	Yield

*Failure
**Past peak loads

TABLE B-19

TEST RESULTS FOR BEAM 4-3

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in. x 10^6	Deflection inches x 10^2 West	Center	East
0	0	0	--	0	0	0	0
1	14	24	0.58	17	+1.0	+0.5	+0.5
2	31	51	0.61	45	2.5	2.5	2.0
3	39	65	0.60	77	3.5	4.5	3.5
4	47	78	0.60	155	7.0	8.0	6.5
5	56	92	0.61	245	10.0	11.0	9.5
6	65	105	0.62	388	12.0	11.5	11.0
7	73	119	0.61	559	13.5	15.0	12.5
8	81	132	0.61	775	14.5	16.0	14.0
9	89	146	0.61	1094	16.0	18.5	15.0
10*	93	153	0.61	--	--	--	--
11**	83	153	0.54	1998	20.5	19.5	17.5

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	-0.5	-0.3	+0.6	+0.8	+0.3	+0.4	+0.4	0
2	-3.2	-2.4	2.4	3.2	1.1	1.3	+0.5	+0.3
3	-4.9	-3.3	4.3	9.6	2.2	5.0	+0.3	6.0
4	-5.5	+0.2	20.7	16.7	6.8	14.0	-0.3	20.1
5	-7.9	+3.0	28.0	25.3	8.1	25.4	+0.1	25.7
6	-6.8	6.4	35.9	35.0	9.9	35.7	+9.2	36.0
7	+4.2	10.1	45.1	40.4	11.1	42.4	14.1	42.8
8	+12.5	8.3	Yield	46.3	13.9	51.7	21.0	49.1
9	+19.5	6.7	Yield	Yield	21.7	Yield	32.0	Yield
10*	--	--	--	--	--	--	--	--
11**	+33.1	+4.2	Yield	Yield	51.3	Yield	Yield	Yield

*Failure
**Past peak loads

TABLE B-20

TEST RESULTS FOR BEAM 4-4

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in. x 10^6	Deflection inches x 10^2 West	Center	East
0	0	0	--	0	0	0	0
1	15	24	0.62	22	+0.5	+0.5	+0.5
2	24	38	0.63	37	1.5	1.0	1.0
3	31	51	0.61	47	2.0	2.5	2.0
4	40	65	0.62	80	4.5	4.5	4.0
5	47	78	0.60	158	7.0	8.0	7.0
6	56	92	0.61	307	9.5	11.0	10.0
7	64	105	0.61	487	12.5	14.0	12.0
8	73	119	0.61	713	14.5	16.0	13.5
9	81	132	0.61	1019	16.0	17.5	14.0
10*	85	146	0.58	--	--	--	--
11**	77	146	0.53	1777	21.5	21.5	19.0

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	-0.6	+0.7	+0.9	+0.8	+0.3	+0.2	0	0
2	-1.7	-0.2	1.9	2.2	+0.9	+0.5	-0.1	0
3	-3.0	-1.4	3.0	4.5	2.4	+0.7	-0.4	+0.1
4	-5.5	-2.5	6.3	13.4	4.9	-0.8	-0.9	+7.5
5	-6.7	-3.1	23.1	20.5	8.2	+7.6	-1.6	23.7
6	-6.0	-0.1	31.2	--	18.3	11.7	-0.9	32.7
7	+2.7	+7.9	38.7	--	24.7	18.8	+5.5	41.5
8	11.0	15.2	46.5	--	31.2	27.9	9.6	50.1
9	16.6	21.7	Yield	--	40.8	37.1	15.3	Yield
10*	--	--	--	--	--	--	--	--
11**	+17.7	+30.7	Yield	--	Yield	45.8	29.9	(48.5)

*Failure
**Past peak loads

TABLE B-21

TEST RESULTS FOR BEAM 4-5

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in. x 10^6	Deflection inches x 10^2 West	Center	East
0	0	0	--	0	0	0	0
1	12	24	0.50	15	+0.5	0	+1.5
2	22	45	0.49	27	1.5	+2.0	2.0
3	32	65	0.49	47	3.5	3.5	4.0
4	41	85	0.48	95	6.0	7.5	7.5
5	52	105	0.50	170	10.0	12.0	11.0
6	62	126	0.49	252	14.0	15.5	14.5
7	72	146	0.49	335	16.5	18.0	16.5
8	83	166	0.50	445	19.0	21.5	19.5
9	93	186	0.50	588	22.0	25.0	21.5
10	100	200	0.50	717	24.5	27.0	24.0
11*	103	212	0.49	--	--	--	--
12**	82	212	0.39	1923	54.0	68.5	62.0

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	-0.4	-0.5	+0.7	+0.9	+0.3	+0.2	+0.2	+0.2
2	-1.8	-2.2	2.4	2.9	0.9	0.5	+0.2	+0.3
3	-3.6	-4.2	5.4	6.6	3.6	1.5	+0.2	+0.5
4	-6.1	-4.8	12.9	19.0	6.6	2.1	0	-0.3
5	-7.6	-4.4	21.9	26.2	7.9	5.7	-0.3	-0.3
6	-8.6	-3.4	28.9	35.9	10.9	9.3	-0.3	-0.2
7	-7.9	-1.5	35.9	44.0	13.7	12.1	-0.1	+1.2
8	-6.9	-0.2	43.3	Yield	17.1	15.1	+4.4	3.9
9	-5.4	+0.8	Yield	Yield	21.4	19.3	10.2	6.0
10	-2.6	+2.0	Yield	(47.2)	24.3	22.2	17.2	6.9
11*	--	--	--	--	--	--	--	--
12**	-6.6	+8.0	Yield	Yield	28.5	39.1	26.1	6.6

*Failure
**Past peak loads

TABLE B-22

TEST RESULTS FOR BEAM 4-6

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in. x 10^6	Deflection inches x 10^2 West	Center	East
0	0	0	--	0	0	0	0
1	12	24	0.50	19	+1.0	+1.0	+0.5
2	19	38	0.50	30	1.5	2.0	1.5
3	26	51	0.51	45	2.5	3.0	2.0
4	32	65	0.49	65	4.5	5.0	4.0
5	39	78	0.50	128	8.0	8.5	7.0
6	46	92	0.50	202	12.0	13.5	11.0
7	53	105	0.50	347	15.0	18.0	15.0
8	56	112	0.50	405	17.0	20.0	16.0
9	59	119	0.50	453	19.0	21.0	17.5
10	63	126	0.50	567	21.5	24.5	19.5
11*	66	132	0.50	--	--	--	--
12**	58	132	0.44	2260	57.0	63.0	47.0

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	-0.6	-0.4	+0.4	+0.6	+0.2	0	+0.1	+0.1
2	-1.9	-1.3	1.5	1.8	0.8	+0.1	+0.1	+0.2
3	-3.3	-2.3	2.8	4.6	1.7	0.3	0	+0.4
4	-5.2	-3.5	5.3	9.6	4.2	1.6	-0.2	-0.2
5	-8.1	-5.0	26.5	24.3	7.8	4.2	-0.4	-0.8
6	-9.6	-1.2	38.2	33.1	10.4	7.3	-0.5	+0.2
7	-8.9	+1.4	Yield	44.5	13.6	19.3	+2.8	+5.9
8	-7.2	+3.4	Yield	Yield	15.5	22.4	5.0	8.1
9	-6.3	4.5	Yield	Yield	16.6	24.7	6.2	9.7
10	+1.8	6.5	Yield	Yield	20.4	28.9	7.5	12.0
11*	+34.0	6.4	--	--	39.4	--	--	--
12**	+37.0	3.3	(46.3)	(43.8)	43.5	43.2	31.6	32.1

*Failure
**Past peak loads

TABLE B-23

TEST RESULTS FOR BEAM 5-1

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in.x10^6	Shear Force kips	Deflection inches x 10^2		
						West	Center	East
0	0	0	--	0	0	0	0	0
1	28	15	1.87	51	0	0	0	0
2	42	21	2.00	82	0.09	+0.5	0	0
3	54	27	2.00	169	0.19	+1.0	+0.5	0
4	66	34	1.94	420	0.28	0	-1.5	-1.0
5	80	40	2.00	649	0.38	-2.0	-3.0	-3.5
6	92	46	2.00	926	0.47	-4.5	-5.5	-5.5
7	105	53	1.98	1182	0.57	-6.5	-8.0	-8.5
8	117	59	1.98	1528	0.66	-10.0	-12.0	-11.0
9*	128	65	1.97	--	0.76	--	--	--
10**	103	65	1.59	2602	0.76	-17.0	-22.0	-19.5

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	0	+0.1	+0.3	+0.8	+0.6	+0.1	0	+0.3
2	-0.6	0.6	1.4	3.1	1.1	0.4	0	4.2
3	-0.3	2.1	6.6	10.2	3.1	7.0	+4.6	20.9
4	+6.0	20.2	11.7	15.1	7.9	22.6	9.2	32.0
5	+16.5	27.4	17.4	20.3	14.1	30.8	18.3	38.4
6	33.0	39.7	23.4	25.2	19.9	36.3	23.6	43.7
7	46.0	50.7	27.5	28.8	25.0	41.5	28.4	49.2
8	Yield	Yield	31.9	33.7	30.9	47.3	36.0	53.0
9*	--	--	--	--	--	--	42.0	--
10**	Yield	Yield	33.6	34.9	31.8	45.0	36.4	53.1

*Failure
**Past peak loads

TABLE B-24

TEST RESULTS FOR BEAM 5-2

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in.x10^6	Shear Force kips	Deflection inches x 10^2 West	Center	East
0	0	0	--	0	0	0	0	0
1	15	15	1.00	18	0	+0.5	0	0
2	31	32	0.97	44	0.25	1.0	+0.5	+0.5
3	48	49	0.98	129	0.50	3.0	2.5	1.0
4	65	66	0.99	377	0.76	4.0	3.5	2.5
5	82	83	0.99	634	1.01	3.5	3.0	1.5
6	100	100	1.00	895	1.26	3.0	2.0	+0.5
7	116	117	0.99	1215	1.51	2.5	1.5	0
8	124	126	0.98	1372	1.64	3.0	+0.5	-1.5
9	134	134	1.00	1746	1.76	+1.0	-1.5	-3.0
10*	141	143	0.99	--	1.89	--	--	--
11**	117	143	0.82	3131	1.89	-3.0	-17.5	-8.0

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	0	0	+0.1	+0.2	+0.3	0	-0.1	+0.1
2	-1.3	-1.0	1.7	2.0	1.6	+0.1	-0.3	+0.8
3	-3.2	+0.2	7.4	11.0	4.7	1.3	-0.7	5.7
4	+4.9	16.1	18.1	20.8	14.8	14.5	+7.8	27.6
5	12.9	23.5	24.9	24.3	21.0	21.6	13.7	37.4
6	25.3	29.1	31.8	28.5	29.5	30.9	19.3	45.8
7	35.4	36.8	37.8	32.9	37.5	38.4	25.2	Yield
8	40.8	42.0	40.5	35.2	40.5	40.8	28.6	Yield
9	49.0	Yield	45.6	40.2	43.2	45.3	36.3	Yield
10*	--	--	--	Yield	--	--	--	--
11**	52.0	Yield	Yield	(44.8)	Yield	47.7	Yield	Yield

*Failure
**Past peak loads

TABLE B-25

TEST RESULTS FOR BEAM 5-3

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians Per in.x10^6	Shear Force kips	Deflection inches x 10^2 West	Center	East
0	0	0	--	0	0	0	0	0
1	15	27	0.56	25	0.19	+0.5	+1.0	+0.5
2	29	55	0.53	51	0.60	2.0	2.0	1.5
3	42	83	0.51	102	1.01	4.5	4.5	3.5
4	55	111	0.50	215	1.42	7.5	8.0	6.5
5	69	139	0.50	377	1.83	10.0	10.0	8.5
6	83	167	0.50	559	2.23	11.5	12.0	10.0
7	97	194	0.50	787	2.64	14.0	13.5	11.0
8	110	222	0.50	995	3.05	15.0	14.0	12.0
9	125	250	0.50	1390	3.46	19.0	17.5	13.5
10*	130	278	0.47	--	3.87	--	--	--
11**	111	278	0.40	2734	3.87	50.5	47.0	40.5

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	-0.9	-0.6	+0.9	+0.8	+0.3	+0.3	-0.1	+0.3
2	-3.5	-2.3	4.9	3.8	1.3	1.0	-0.3	1.8
3	-7.4	-3.7	12.6	12.0	3.1	3.1	-0.6	12.7
4	-9.8	-2.7	21.1	19.5	4.9	12.5	+1.2	20.2
5	-9.6	+1.5	27.5	26.8	7.3	19.9	10.5	31.3
6	-7.4	+7.0	32.7	33.2	9.9	29.2	23.7	38.4
7	-6.0	11.3	37.8	39.0	16.4	36.3	30.1	46.6
8	-3.2	15.6	43.8	44.7	23.1	41.9	36.3	Yield
9	-0.5	24.5	Yield	Yield	32.5	48.9	43.0	Yield
10*	--	--	--	--	--	--	--	--
11**	-14.4	22.0	Yield	Yield	Yield	42.2	52.5	Yield

*Failure
**Past peak loads

TABLE B-26

TEST RESULTS FOR BEAM 5-4

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians Per in.x10^6	Shear Force kips	Deflection inches x 10^2 West	Center	East
0	0	0	--	0	0	0	0	0
1	10	36	0.28	15	0.31	+1.0	+1.0	+0.5
2	18	72	0.25	22	0.85	3.0	2.5	2.0
3	28	109	0.26	54	1.39	5.5	5.5	4.5
4	36	145	0.25	90	1.92	9.0	8.5	7.5
5	45	182	0.25	128	2.46	13.0	12.5	10.0
6	54	218	0.25	185	2.99	16.5	16.5	14.0
7	64	254	0.25	246	3.53	20.0	19.0	17.0
8	72	291	0.25	315	4.06	23.0	22.5	19.5
9	81	327	0.25	403	4.60	26.5	26.0	22.5
10	91	364	0.25	1815	5.13	77.5	75.0	62.5
11*	98	389	0.25	--	5.51	--	--	--

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	-1.3	-0.9	+1.3	+1.3	+0.2	0	+0.5	+0.3
2	-4.2	-3.5	6.3	4.2	1.1	+0.1	0.7	0.5
3	-7.9	-6.3	13.5	9.4	3.1	+0.3	0.9	1.7
4	-11.4	-8.1	19.6	20.9	4.6	0	1.1	3.3
5	-15.1	-9.3	25.9	29.8	6.0	0	1.1	4.5
6	-18.6	-10.3	32.0	37.5	7.0	+0.2	1.6	6.6
7	-22.0	-11.1	38.4	45.0	7.8	1.7	3.4	8.9
8	-25.5	-10.6	44.7	Yield	9.0	6.3	5.1	10.9
9	-29.3	-10.2	Yield	Yield	11.7	9.1	7.7	13.5
10	-49.2	-12.0	--	Yield	Yield	18.7	27.5	22.8
11*	--	--	--	--	--	--	--	--

*Failure

TABLE B-27

TEST RESULTS FOR BEAM 6-1

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians Per in.x10^6	Shear Force kips	Deflection inches x 10^2 West	Center	East
0	0	0	--	0	0	0	0	0
1	29	14	2.07	21	0.30	0	0	0
2	41	20	2.05	42	0.49	+0.5	0	0
3	53	27	1.96	149	0.68	+0.5	0	+0.5
4	67	33	2.03	419	0.87	0	-0.5	-0.5
5	79	40	1.97	600	1.06	-1.5	-1.5	-1.5
6	92	46	2.00	832	1.24	-2.0	-3.0	-2.0
7	105	52	2.02	1077	1.43	-3.5	-4.0	-3.0
8	118	59	2.00	1397	1.62	-4.5	-6.5	-4.0
9*	129	65	1.99	--	1.81	--	--	--
10**	107	65	1.65	3035	1.81	-16.0	-21.0	-14.0

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	-0.1	+0.1	+0.6	+0.6	+0.6	-0.6	+0.1	+0.9
2	-0.6	0.3	1.5	1.7	1.0	-0.1	0.3	2.3
3	-1.0	12.6	4.6	9.3	1.6	+2.6	2.4	17.2
4	+5.1	33.3	14.1	16.5	13.2	30.3	14.9	33.9
5	16.9	42.3	17.4	18.8	18.4	35.1	16.0	40.4
6	28.2	Yield	20.4	22.7	26.1	39.4	16.8	43.9
7	36.9	Yield	23.6	25.8	32.4	42.9	18.0	49.4
8	47.0	Yield	27.9	28.2	40.8	45.8	24.6	Yield
9*	--	--	--	--	--	--	--	--
10**	51.3	--	31.2	28.8	49.0	36.6	34.5	Yield

*Failure
**Past peak loads

TABLE B-28

TEST RESULTS FOR BEAM 6-2

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in. x 10^6	Shear Force kips	Deflection inches x 10^2 West	Center	East
0	0	0	--	0	0	0	0	0
1	15	14	1.07	19	0.30	0	0	0
2	30	29	1.03	43	0.74	+1.0	+0.5	+0.5
3	51	44	1.16	128	1.18	2.0	1.5	1.0
4	59	59	1.00	203	1.62	2.5	2.0	1.0
5	74	74	1.00	477	2.06	4.5	2.0	+0.5
6	89	89	1.00	747	2.50	3.5	1.5	0
7	103	104	0.99	987	2.94	2.5	+1.0	-0.5
8	119	119	1.00	1326	3.39	1.5	-0.5	-2.0
9	134	134	1.00	1821	3.83	+0.5	-2.5	-3.0
10*	145	149	0.97	--	4.27	--	--	--
11**	133	149	0.89	2837	4.27	-4.0	-8.5	-9.5

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	0	+0.1	+0.1	+0.1	+0.1	+0.2	+0.1	+0.2
2	-0.9	-0.1	1.1	0.9	0.3	0.6	0	0.7
3	-3.0	+0.6	5.8	9.3	1.9	3.3	+2.7	1.8
4	-2.6	+1.9	9.6	18.6	3.4	10.2	10.1	0.3
5	+3.1	18.4	17.0	25.6	10.2	24.9	19.7	20.8
6	8.8	34.0	22.8	30.1	18.3	35.5	21.9	32.7
7	18.6	44.1	27.1	33.7	26.0	42.1	24.3	42.4
8	31.1	Yield	33.7	39.2	33.9	48.0	27.5	47.4
9	49.3	Yield	41.4	43.7	43.6	52.6	33.3	49.2
10*	--	--	--	--	--	--	--	--
11**	Yield	Yield	47.4	43.0	47.9	Yield	37.4	47.5

*Failure
**Past peak loads

TABLE B-29

TEST RESULTS FOR BEAM 6-3

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in. x 10^6	Shear Force kips	Deflection inches x 10^2 West	Center	East
0	0	0	--	0	0	0	0	0
1	15	27	0.56	24	0.68	+0.5	0	+0.5
2	27	55	0.49	43	1.50	1.5	+1.0	1.0
3	41	82	0.50	83	2.31	4.0	2.5	1.5
4	56	110	0.51	187	3.13	6.0	4.0	2.0
5	69	138	0.50	315	3.95	8.0	5.5	2.5
6	83	166	0.50	498	4.77	11.0	7.0	3.0
7	97	194	0.50	725	5.59	12.0	7.5	3.5
8	111	222	0.50	995	6.41	13.5	8.5	3.0
9	125	249	0.50	1459	7.23	15.5	9.5	2.0
10*	132	264	0.50	--	7.67	--	--	--
11**	113	264	0.43	2400	7.67	25.0	18.5	6.5

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	-0.7	-0.5	+0.7	+0.6	+0.3	0	+0.2	+0.1
2	-3.4	-2.1	2.1	1.9	0.1	-0.3	0	0.3
3	-6.0	-2.2	5.9	6.8	0.7	0	+0.8	8.1
4	-9.4	+4.4	12.3	17.0	4.2	+0.7	4.8	33.6
5	-9.7	15.4	17.3	22.6	6.5	5.4	7.4	46.6
6	-8.4	19.2	23.0	29.3	10.3	14.1	18.1	51.6
7	-7.2	22.8	29.4	34.1	16.6	21.4	24.1	Yield
8	-6.5	29.3	35.8	40.9	24.1	26.6	30.0	Yield
9	+0.3	43.2	Yield	Yield	40.3	31.5	37.1	Yield
10*	--	--	--	--	Yield	--	--	--
11**	+5.7	46.0	Yield	Yield	(51.2)	38.1	47.9	(48.3)

*Failure
**Past peak loads

TABLE B-30

TEST RESULTS FOR BEAM 6-4

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in. x 10^6	Shear Force kips	Deflection inches x 10^2 West	Center	East
0	0	0	--	0	0	0	0	0
1	8	35	0.23	14	0.93	+0.5	+0.5	+0.5
2	18	72	0.25	21	2.00	2.0	1.5	1.5
3	27	108	0.25	51	3.07	4.5	3.0	2.0
4	36	144	0.25	80	4.14	7.0	5.5	3.5
5	45	181	0.25	120	5.21	10.5	7.5	5.0
6	54	217	0.25	182	6.28	13.5	9.5	5.5
7	63	254	0.25	248	7.35	16.5	11.5	6.0
8	72	290	0.25	312	8.42	19.5	13.5	7.5
9	81	326	0.25	408	9.49	23.0	16.5	8.5
10	90	363	0.25	718	10.56	46.0	33.0	16.0
11	95	380	0.25	1006	11.07	59.0	42.0	20.0
12*	107	427	0.25	--	12.45	--	--	--
13**	89	427	0.21	***	12.45	157.5	113.5	54.0

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	-1.6	-0.8	+1.0	+0.6	+0.4	0	0	+0.1
2	-5.2	-2.4	4.1	2.4	1.5	0	0	0.6
3	-10.2	-3.8	9.3	7.3	3.0	0	+0.1	4.4
4	-15.6	-4.7	16.1	14.7	5.3	-0.4	-0.1	13.0
5	-20.4	-5.3	21.9	20.1	6.7	-0.6	+0.3	21.4
6	-25.6	-3.3	27.5	25.8	7.9	-0.3	2.0	28.7
7	-29.7	-1.8	33.0	31.3	9.9	+0.9	6.5	34.8
8	-33.6	+0.9	38.5	36.9	11.7	2.1	10.3	39.6
9	-38.4	+11.7	Yield	42.8	14.6	4.5	12.1	44.1
10	-52.7	+10.8	Yield	Yield	25.0	5.2	14.5	51.0
11	Yield	+12.3	Yield	Yield	39.3	7.4	13.8	Yield
12*	--	--	--	--	--	--	--	--
13**	Yield	-17.8	--	--	Yield	22.6	53.5	--

*Failure
**Past peak loads
***Twist exceeded capacity of twistmeters

TABLE B-31

TEST RESULTS FOR BEAM 7-1

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in. x 10^6	Shear Force kips	Deflection inches x 10^2 West	Center	East
0	0	0	--	0	0	0	0	0
1	28	14	2.00	40	0.30	-0.5	-0.5	-0.5
2	36	18	2.00	61	0.43	-0.5	-0.5	-0.5
3	44	22	2.00	98	0.55	0	-0.5	-0.5
4	52	26	2.00	203	0.68	0	-0.5	-0.5
5	61	31	1.97	472	0.80	-2.0	-1.5	-3.5
6	70	35	2.00	586	0.93	-2.0	-2.5	-2.5
7	82	41	2.00	808	1.12	-5.0	-5.0	-4.5
8	95	47	2.02	1069	1.31	-6.5	-6.5	-6.5
9	103	52	1.98	1240	1.43	-6.0	-7.5	-6.5
10*	112	56	2.00	1592	1.56	--	--	--
11**	97	56	1.73	2237	1.56	-14.0	-16.0	-14.0

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	0	-0.2	+0.3	+0.6	+0.4	+0.2	+0.4	+0.9
2	-0.1	-0.3	0.9	1.0	0.6	0.6	0.6	1.5
3	-0.3	-0.5	1.4	2.5	0.4	1.1	1.0	3.1
4	+3.6	+2.5	3.8	11.6	1.4	4.3	11.5	3.6
5	+24.9	29.9	11.1	16.0	3.8	30.7	12.0	28.6
6	33.1	35.8	12.6	18.9	5.6	35.9	12.9	35.1
7	46.2	46.4	15.0	21.9	10.3	41.9	16.7	47.7
8	Yield	Yield	17.0	25.0	18.5	49.9	23.7	Yield
9	Yield	Yield	18.1	26.4	24.3	Yield	28.3	Yield
10*	Yield	Yield	19.7	27.3	30.9	Yield	34.2	Yield
11**	(50.1)	(43.7)	16.8	23.2	34.7	(47.6)	33.0	Yield

*Failure
**Past peak loads

TABLE B-32

TEST RESULTS FOR BEAM 7-2

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in. x 10^6	Shear Force kips	Deflection inches x 10^2 West	Center	East
0	0	0	--	0	0	0	0	0
1	13	14	0.93	16	0.30	+0.5	0	0
2	26	26	1.00	40	0.68	1.0	-0.5	+0.5
3	39	39	1.00	61	1.06	1.0	+0.5	0
4	52	52	1.00	131	1.43	2.0	+0.5	+1.0
5	65	64	1.02	301	1.81	2.5	+1.0	0
6	77	77	1.00	490	2.19	1.5	0	0
7	90	90	1.00	683	2.57	+1.0	-1.0	-1.0
8	102	102	1.00	955	2.94	0	-2.5	-2.0
9*	115	115	1.00	--	3.32	--	--	--
10**	96	115	0.84	2466	3.32	-7.0	-12.5	-17.0

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	0	0	+0.1	+0.2	+0.3	+0.1	+0.1	+0.3
2	+2.1	-0.2	1.2	0.9	0.7	0.2	0.3	0.6
3	-1.1	-0.2	2.3	1.8	1.2	0.8	0.8	1.9
4	-0.3	+0.8	7.8	6.1	2.1	9.3	2.9	15.0
5	+0.6	24.9	13.0	11.1	7.0	26.4	4.9	30.3
6	11.2	41.9	16.3	14.7	11.7	37.0	12.6	36.7
7	25.9	52.7	20.0	16.7	16.9	43.8	12.9	42.3
8	36.3	Yield	23.5	19.6	28.0	51.0	19.1	47.9
9*	43.6	Yield	25.1	24.4	--	--	--	--
10**	46.2	Yield	15.3	27.4	48.0	50.9	48.7	Yield

*Failure
**Past peak loads

TABLE B-33

TEST RESULTS FOR BEAM 7-3

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in. x 10^6	Shear Force kips	Deflection inches x 10^2 West	Center	East
0	0	0	--	0	0	0	0	0
1	16	29	0.55	18	0.74	0	+1.0	0
2	29	56	0.52	45	1.56	+1.0	1.5	+0.5
3	42	83	0.51	86	2.38	2.5	2.0	1.0
4	55	111	0.50	173	3.20	4.5	3.5	1.5
5	69	138	0.50	304	4.01	5.5	4.0	1.5
6	82	166	0.49	520	4.83	6.5	4.5	1.5
7	97	193	0.50	701	5.65	8.0	4.5	1.5
8	110	220	0.50	933	6.47	8.5	4.5	+0.5
9	124	248	0.50	1504	7.29	8.5	2.0	-7.5
10*	132	275	0.48	--	8.11	--	--	--
11**	105	275	0.38	2477	8.11	10.0	2.0	-15.5

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	-0.7	-0.4	+0.6	+0.5	+0.1	0	+0.3	+0.6
2	-2.4	-1.1	2.2	1.5	0.6	+0.1	0.6	1.8
3	-4.5	-0.6	5.4	7.5	0.8	1.8	1.3	4.1
4	-4.3	+1.3	10.5	14.5	2.1	11.1	10.8	4.7
5	-3.6	15.0	14.2	21.3	5.1	21.6	16.3	10.8
6	-1.6	30.5	18.0	25.6	10.1	31.5	19.2	43.3
7	-0.6	39.0	21.1	27.4	14.4	39.8	23.3	Yield
8	+2.3	51.6	24.6	30.0	22.4	49.5	29.8	Yield
9	8.5	Yield	27.9	34.1	35.0	Yield	46.3	Yield
10*	--	--	--	41.8	--	--	--	--
11**	7.5	Yield	30.0	38.5	39.4	(45.3)	Yield	--

*Failure
**Past peak loads

TABLE B-34

TEST RESULTS FOR BEAM 7-4

Load Stage	Torque in.kips	Bending Moment in.kips	Ø	Twist Radians per in. x 10^6	Shear Force kips	Deflection inches x 10^2 West	Center	East
0	0	0	--	0	0	0	0	0
1	13	50	0.26	19	1.37	+0.5	+1.0	+0.5
2	25	100	0.25	40	2.88	3.5	2.5	1.5
3	38	151	0.25	88	4.39	6.0	5.0	2.5
4	50	201	0.25	162	5.90	9.0	7.0	4.0
5	62	252	0.25	240	7.41	13.0	10.0	4.0
6	76	303	0.25	360	8.93	15.5	11.5	6.5
7	88	353	0.25	496	10.44	18.5	13.5	7.5
8	101	404	0.25	661	11.95	21.5	15.5	8.5
9	114	455	0.25	870	13.46	24.5	17.5	8.5
10*	125	505	0.25	--	14.97	--	--	--
11**	103	505	0.20	2653	14.97	54.0	39.5	19.0

Load Stage	Reinforcement Stresses (ksi) Gauge 1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	-2.1	-1.2	+1.9	+1.5	+0.4	0	+0.7	+0.6
2	-5.7	-3.0	5.1	3.1	0.3	-0.3	1.2	2.0
3	-9.9	-3.8	9.3	10.1	0.7	-1.1	2.1	5.6
4	-13.8	-3.1	14.0	18.5	2.8	-2.3	4.1	11.2
5	-17.1	+3.4	18.3	26.5	5.1	-1.1	9.1	17.2
6	-20.4	11.8	22.7	33.6	8.0	+4.0	14.9	24.4
7	-23.5	18.6	27.7	39.0	12.5	10.6	23.7	Yield
8	-26.7	27.4	32.7	Yield	17.4	21.0	29.9	Yield
9	-28.9	36.0	37.5	Yield	23.5	32.3	37.0	Yield
10*	--	39.7	--	--	--	--	--	--
11**	-26.3	20.1	Yield	Yield	51.7	33.1	Yield	--

*Failure
**Past peak loads

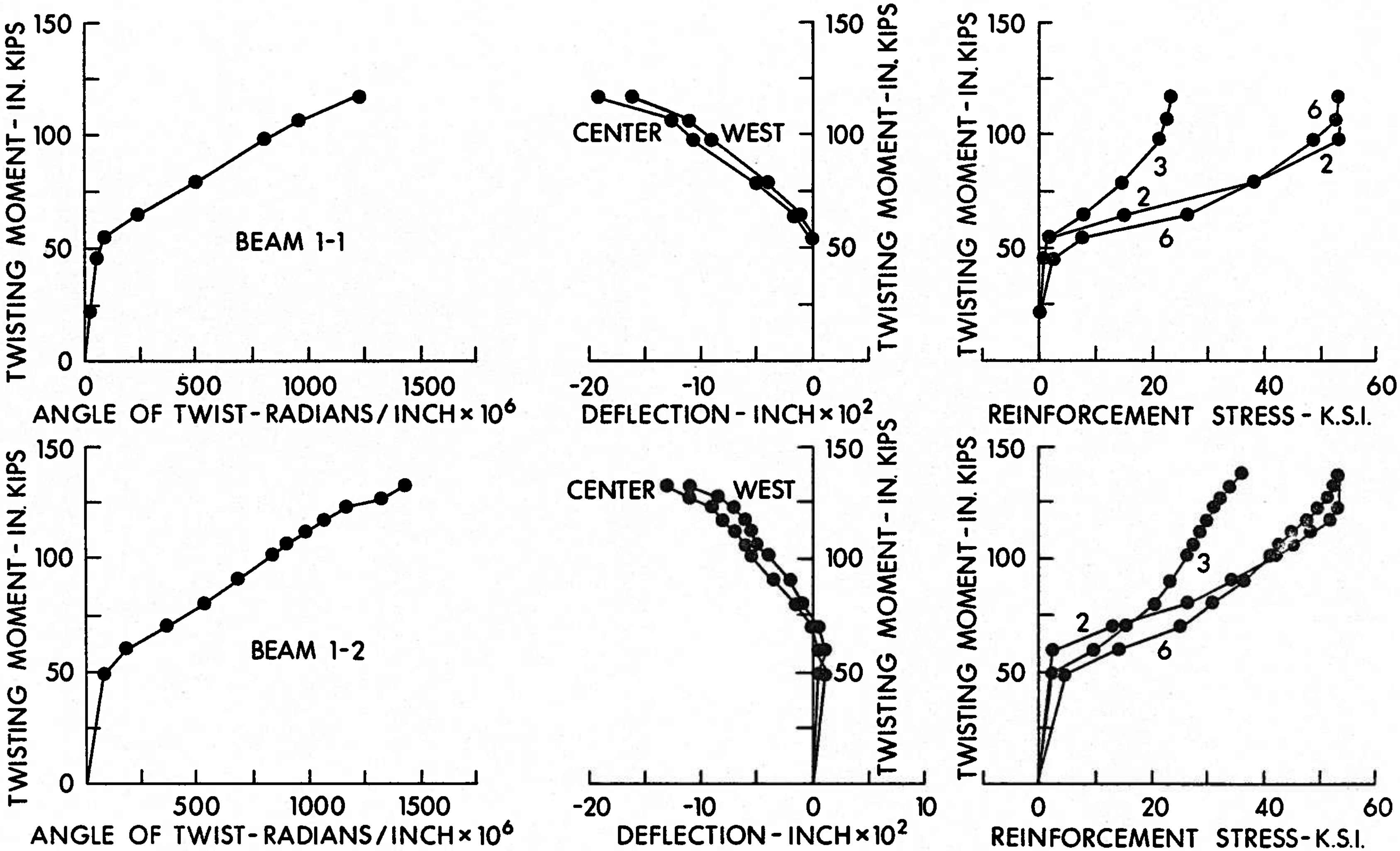

FIGURE B-1 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 1-1 AND 1-2

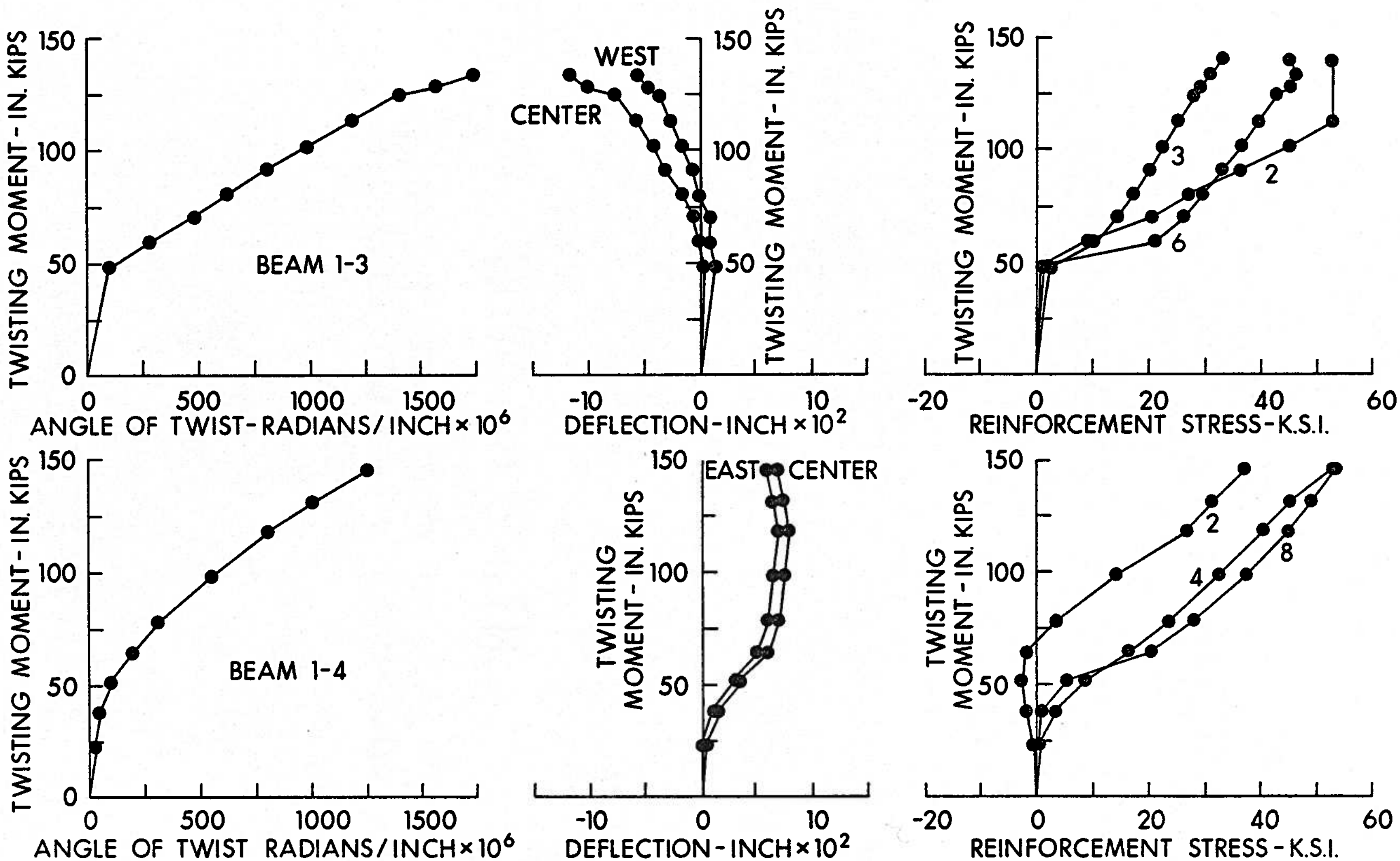

FIGURE B-2 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 1 - 3 AND 1 - 4

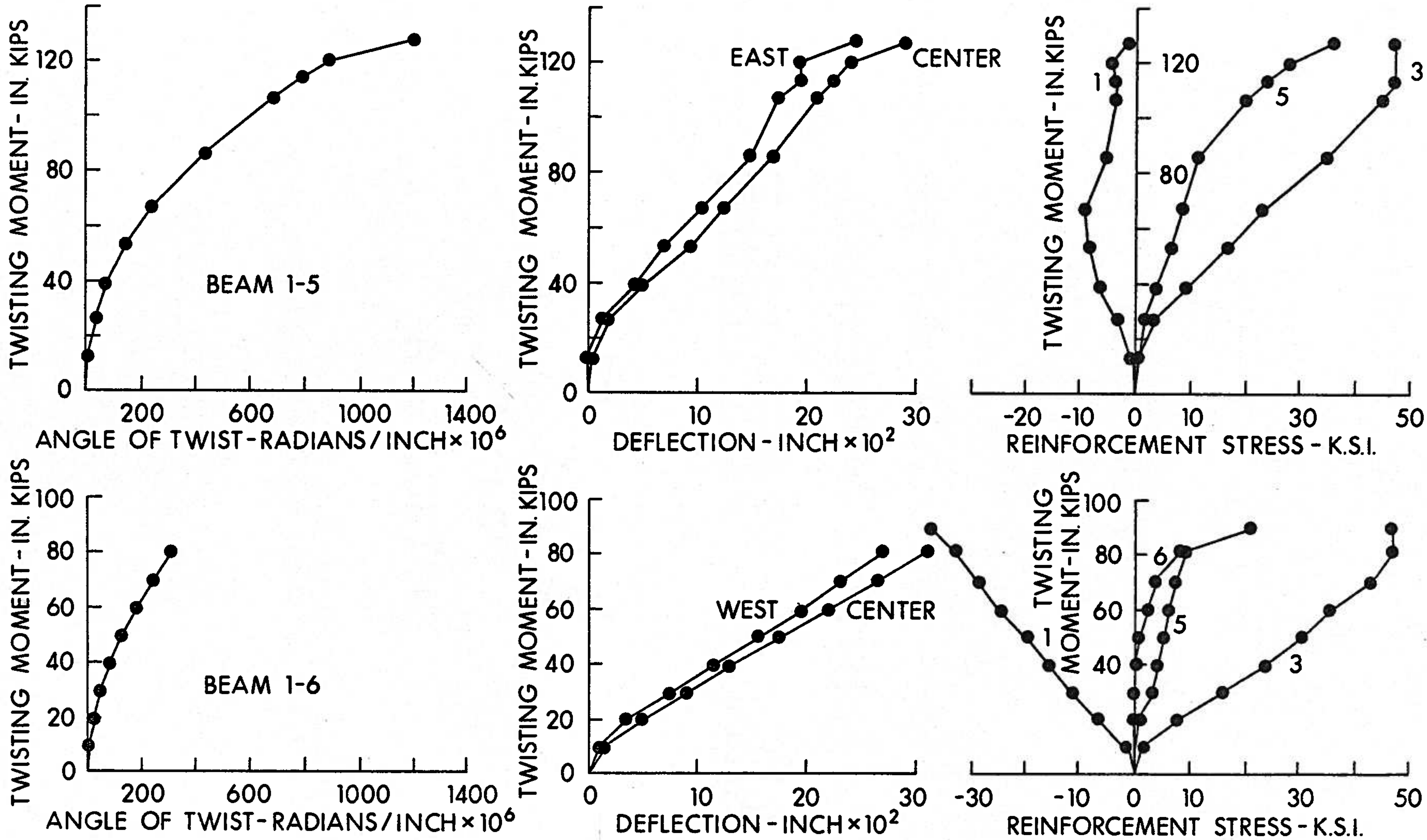

FIGURE B-3 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 1 - 5 AND 1 - 6

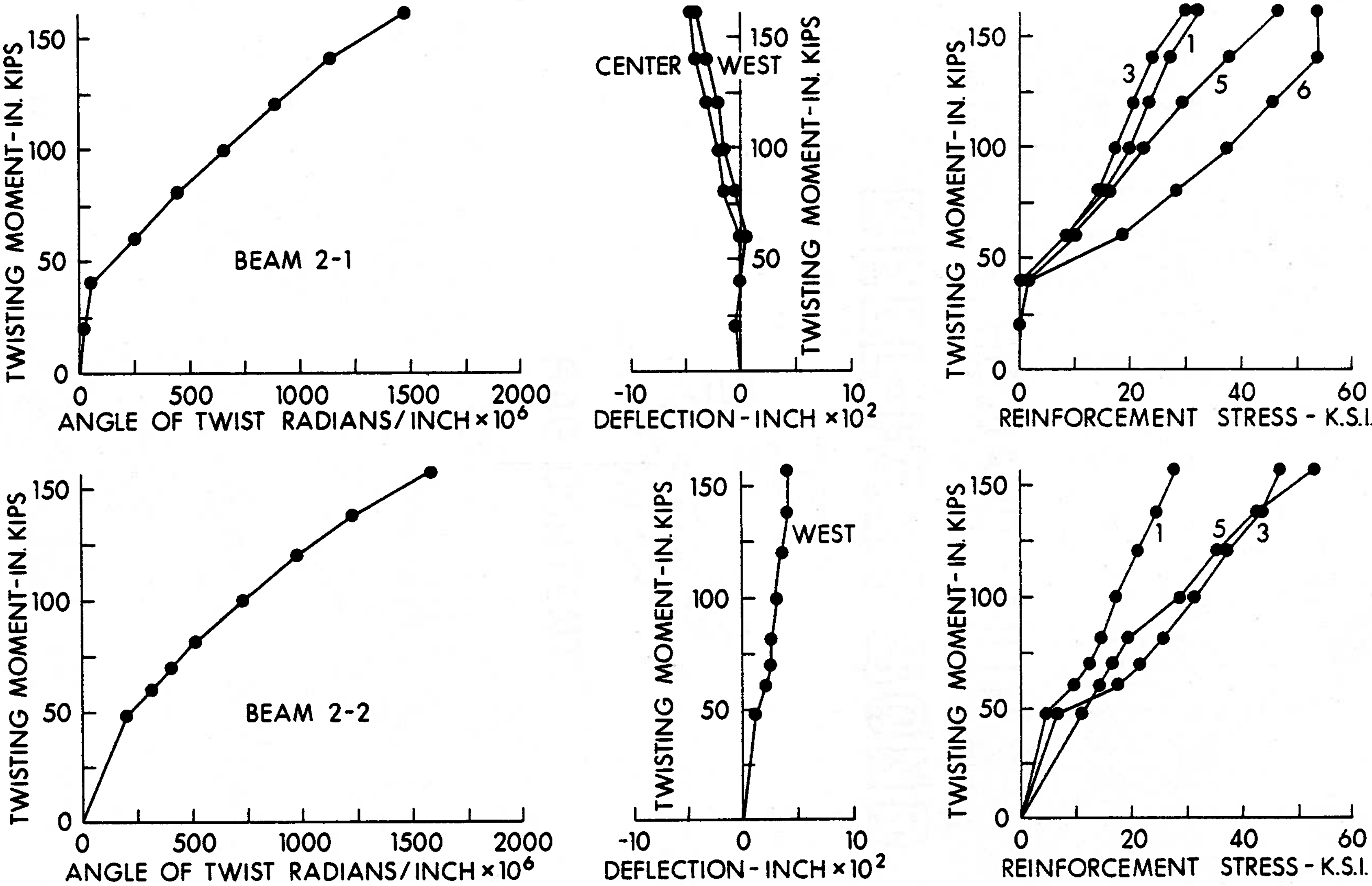

FIGURE B-4 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 2 - 1 AND 2 -2

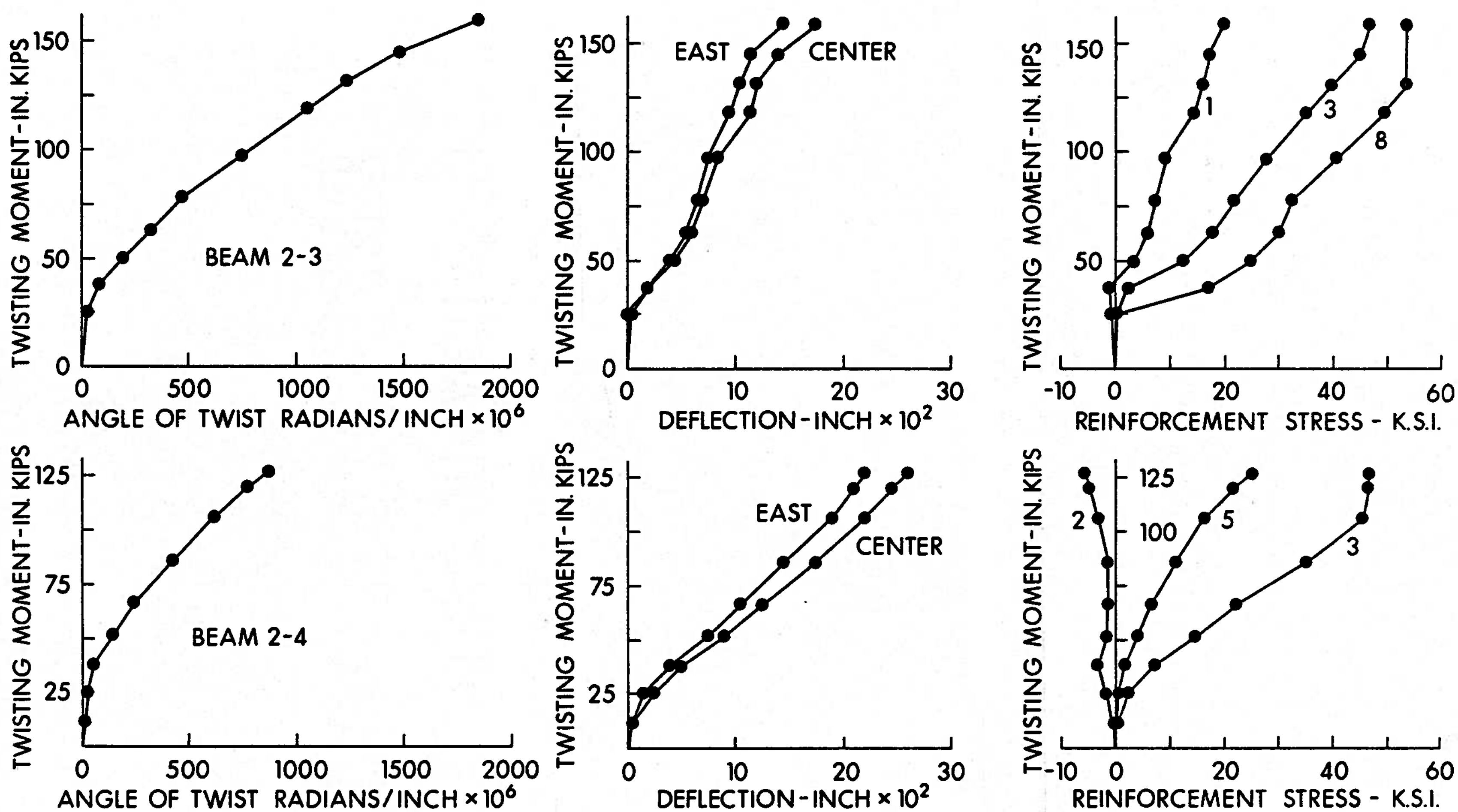

FIGURE B - 5 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 2 - 3 AND 2 - 4

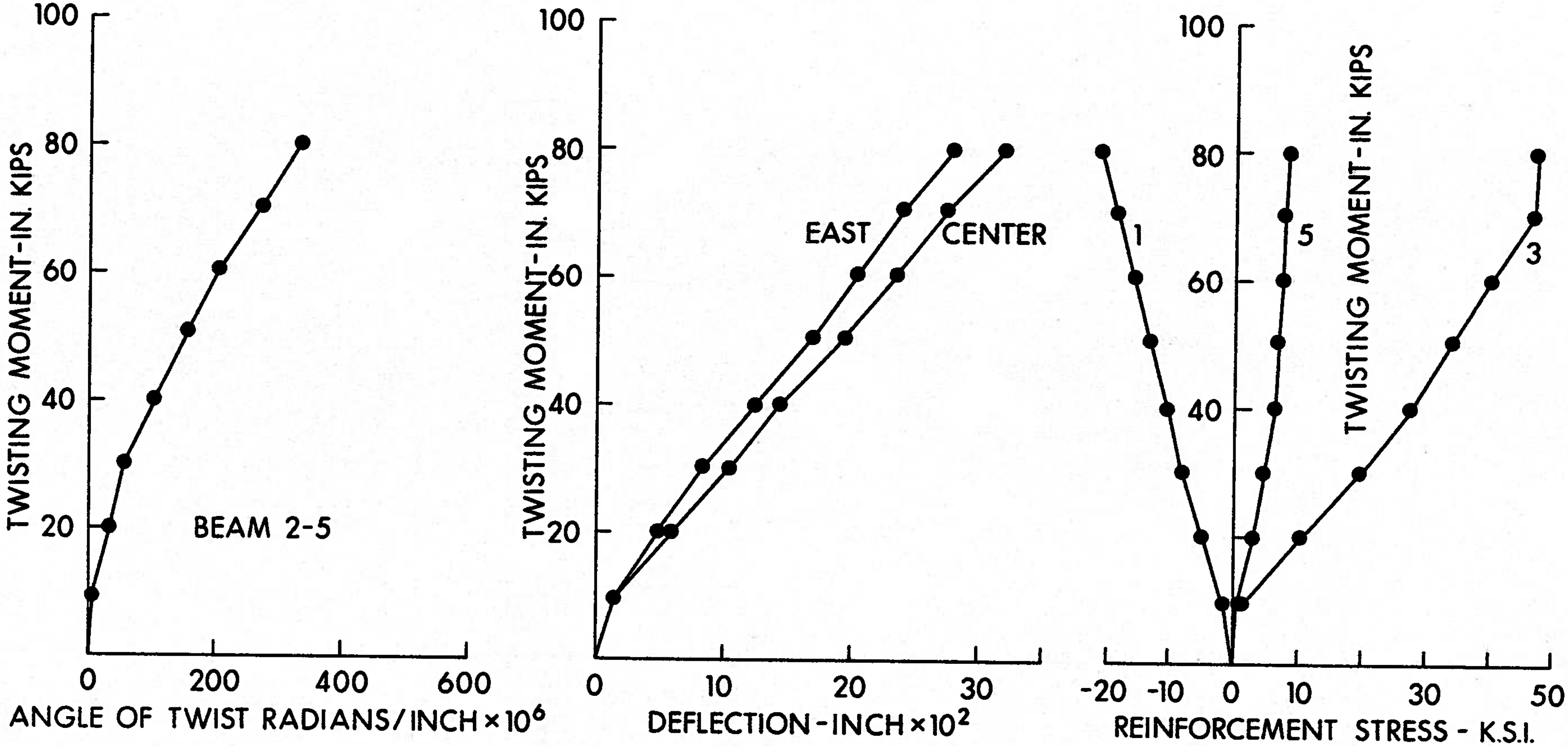

FIGURE B - 6 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAM 2 - 5

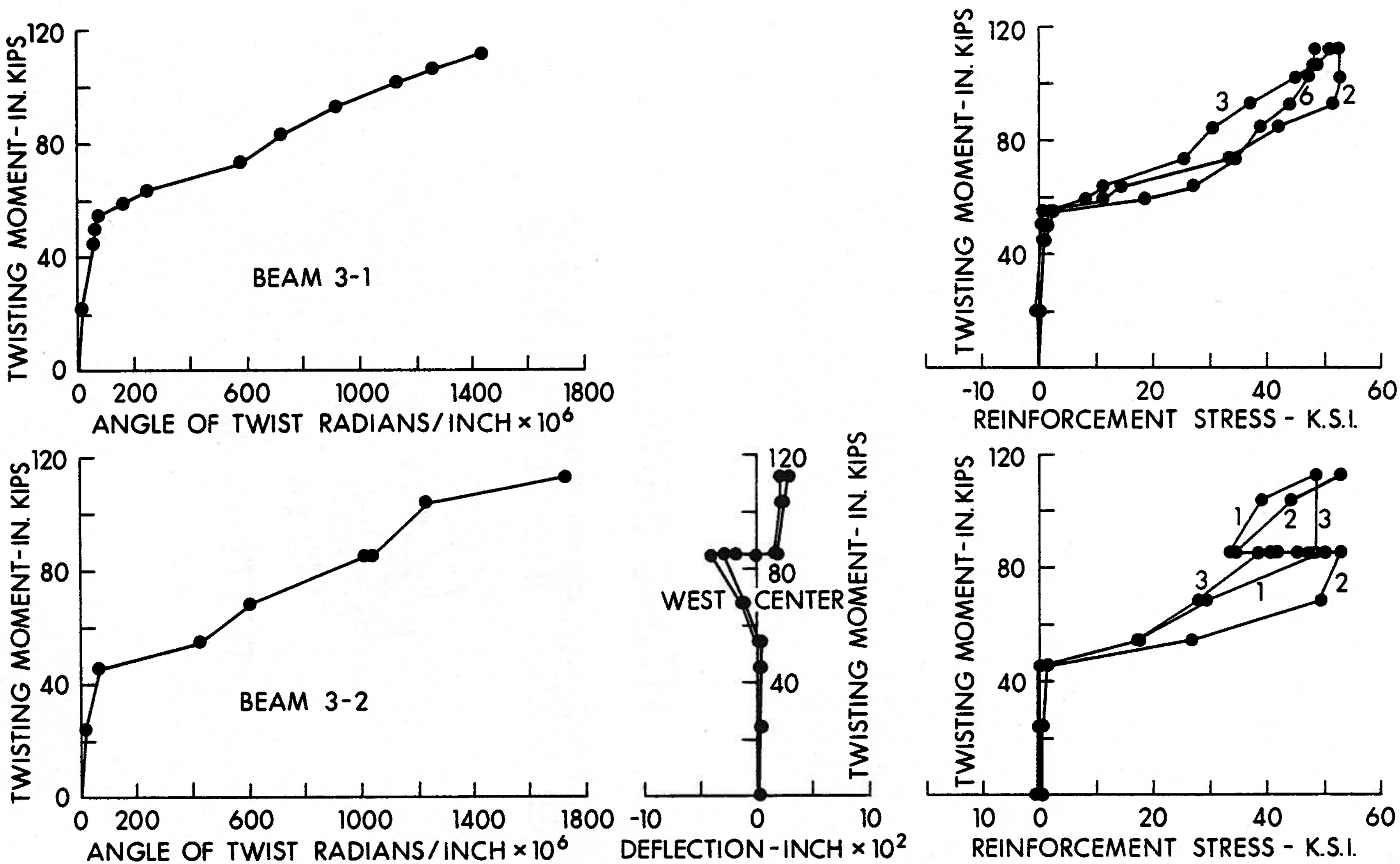

FIGURE B-7 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 3 - 1 AND 3 - 2

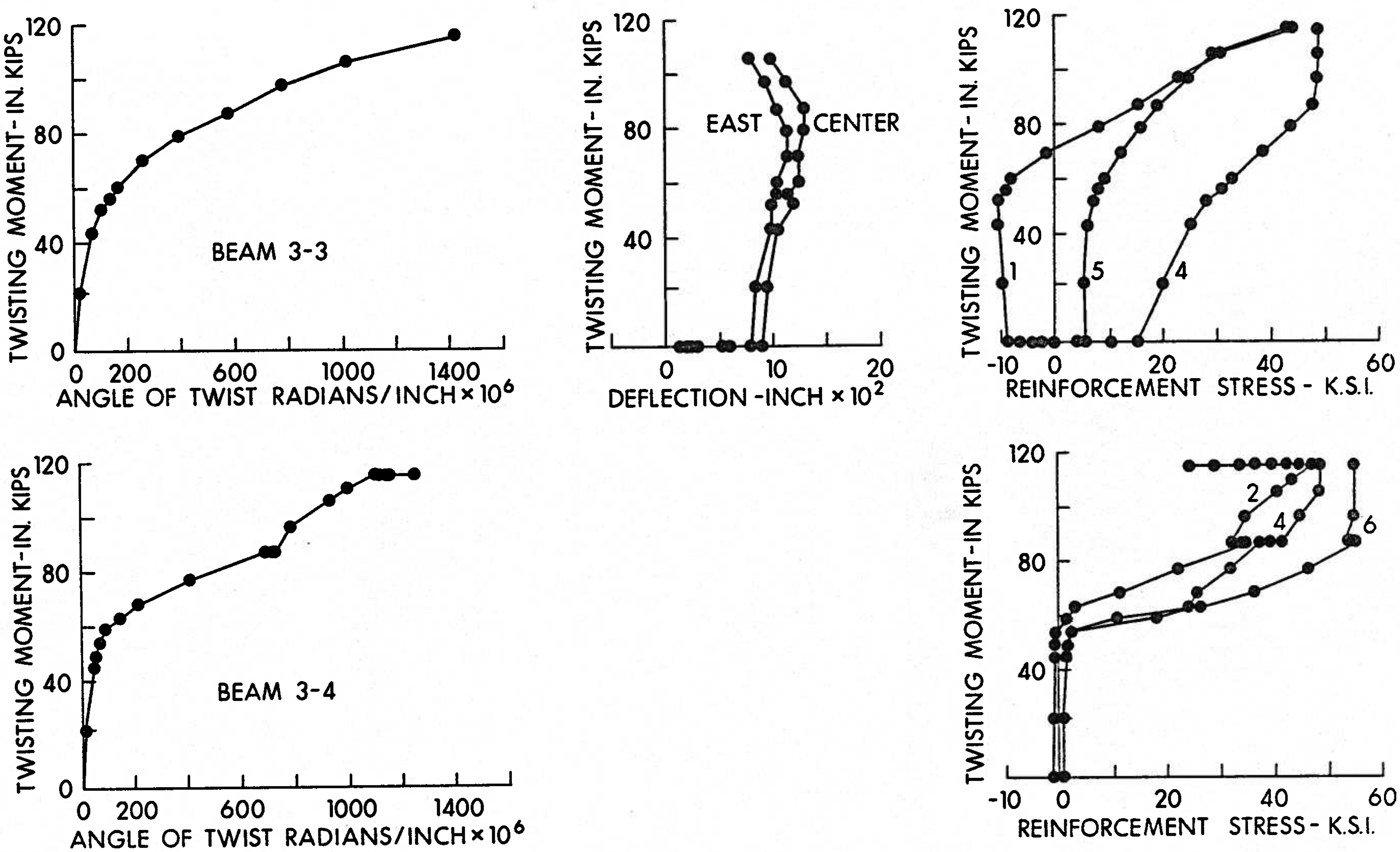

FIGURE B-8 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 3 - 3 AND 3 - 4

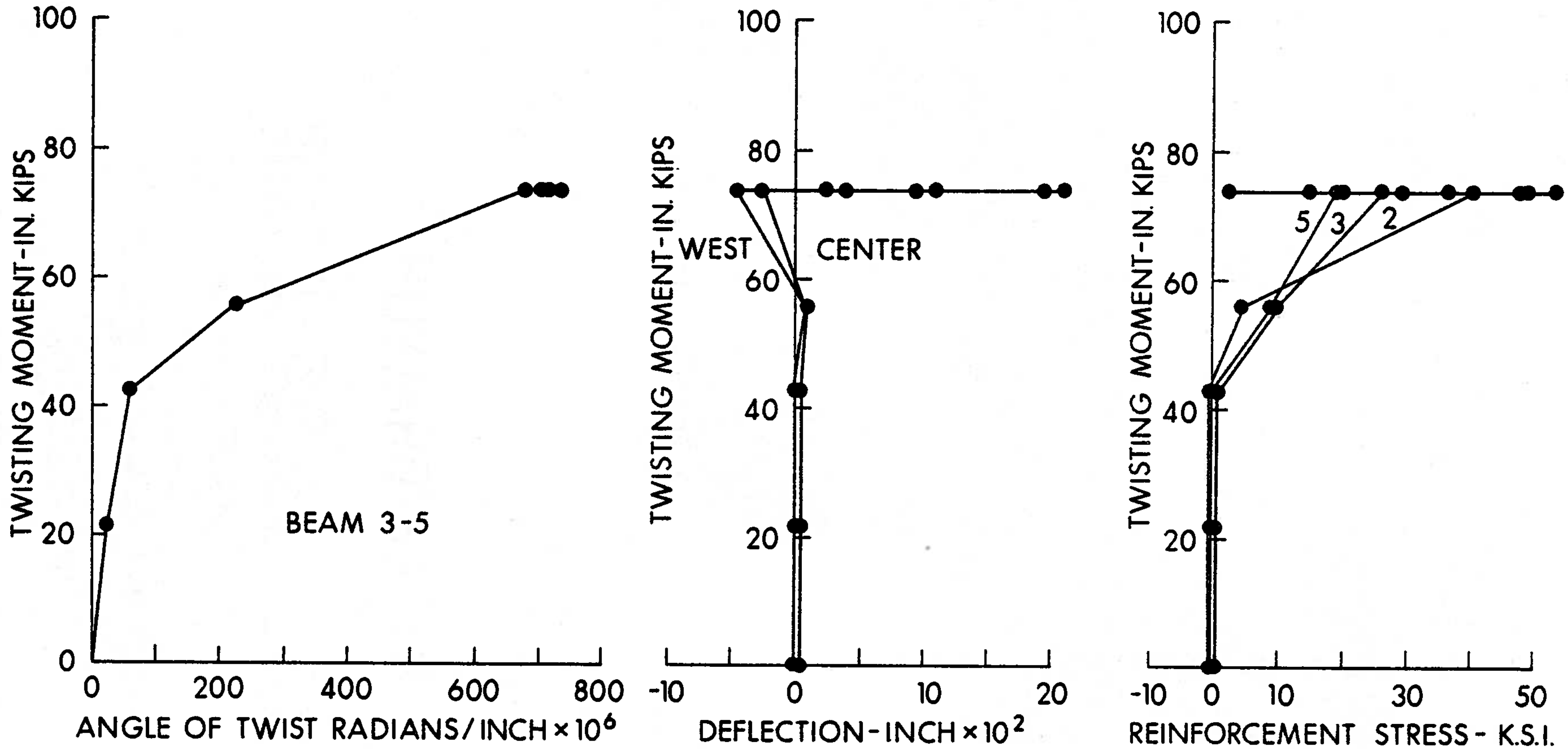

FIGURE B - 9 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAM 3 - 5

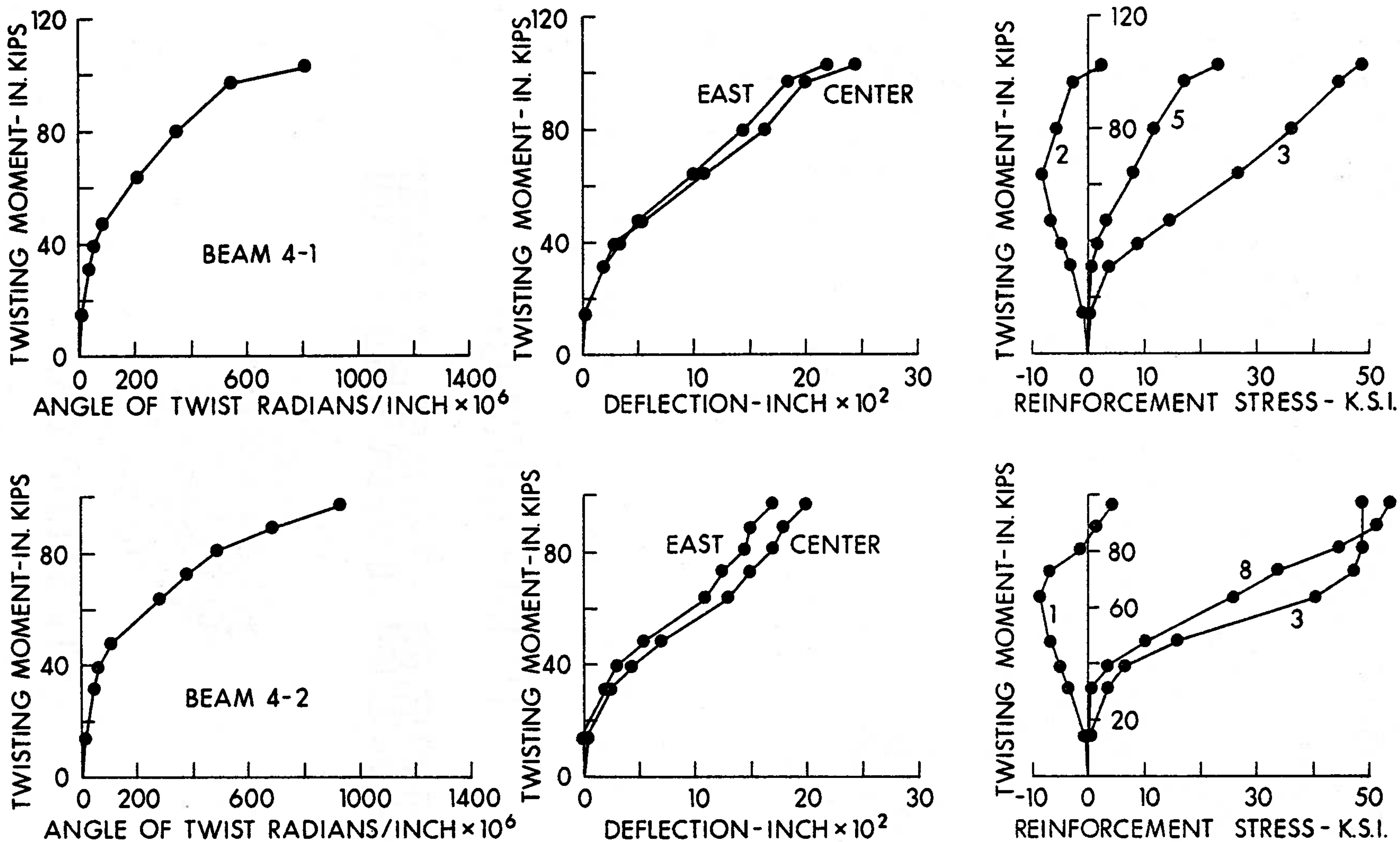

FIGURE B - 10 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 4 - 1 AND 4 - 2

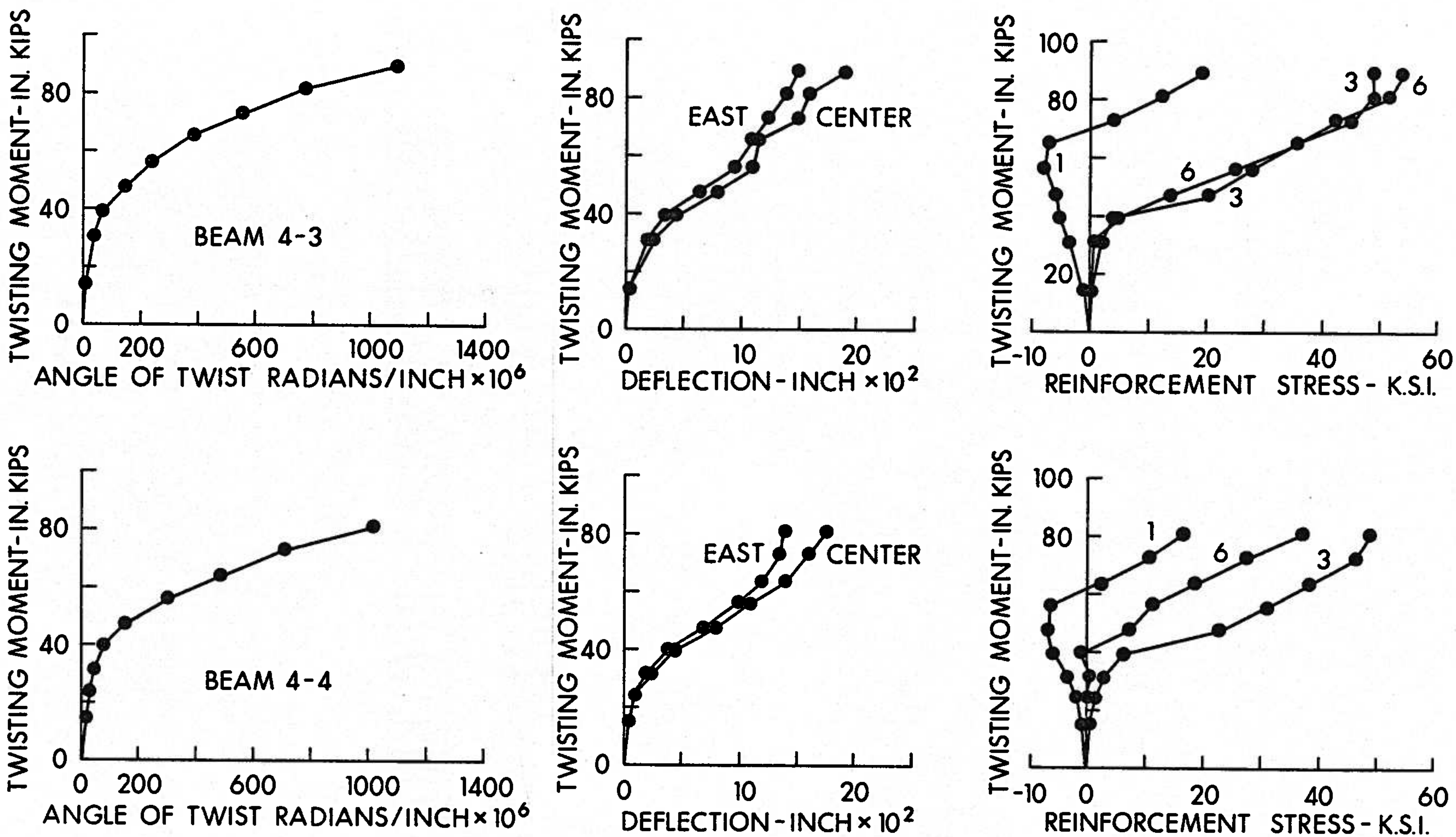

FIGURE B - 11 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 4 - 3 AND 4 - 4

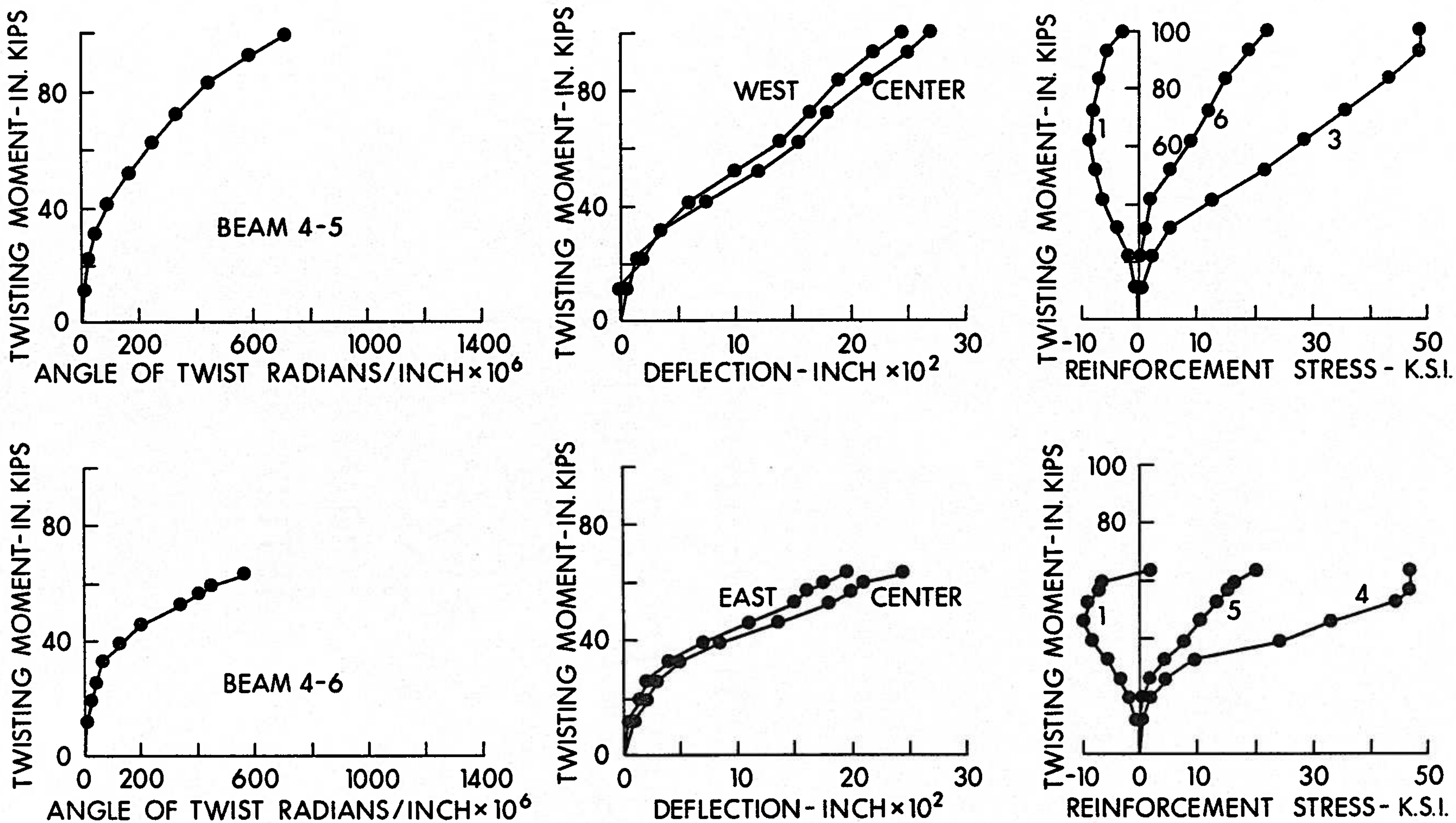

FIGURE B-12 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 4 - 5 AND 4 - 6

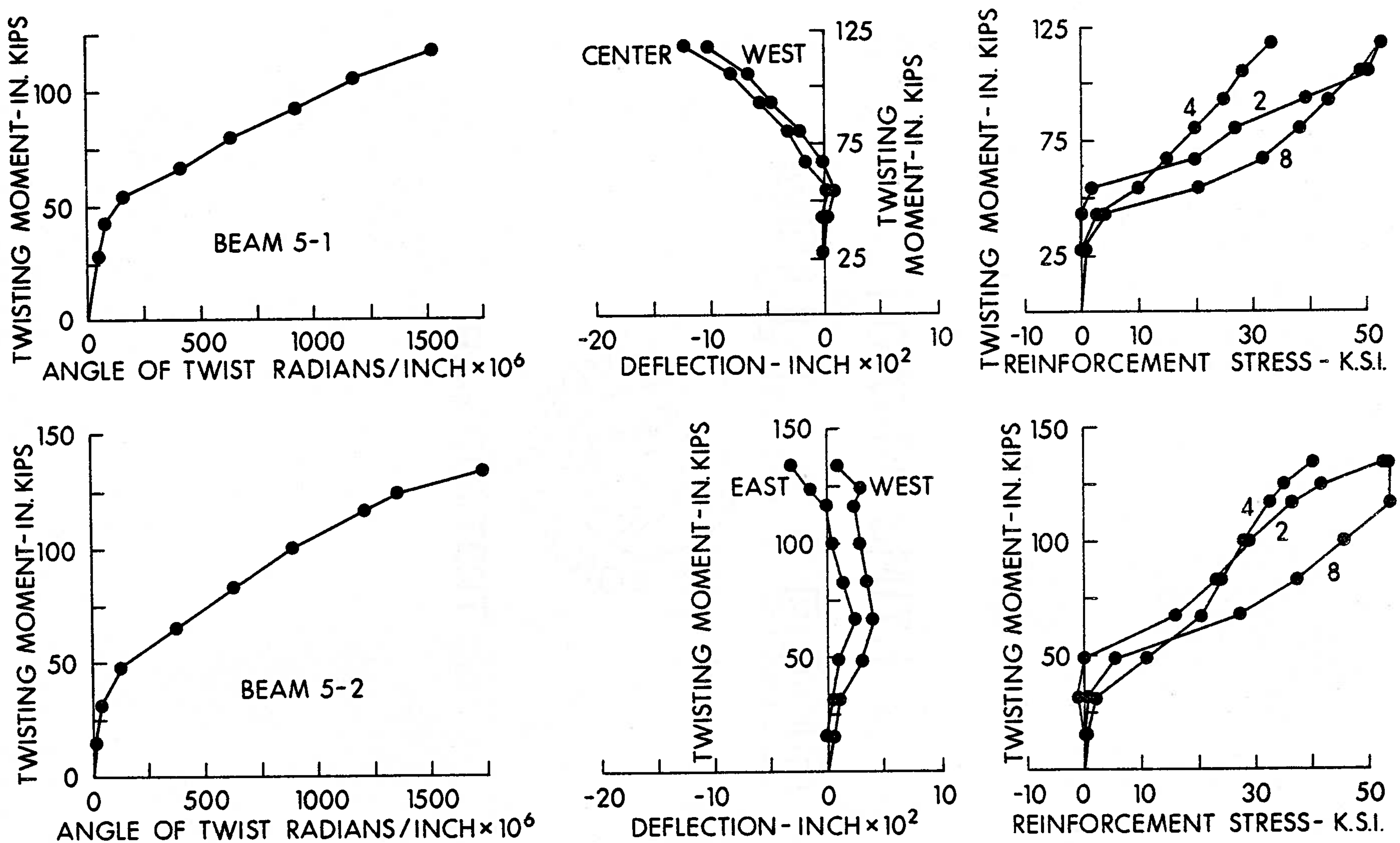

FIGURE B - 13 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 5 - 1 AND 5 - 2

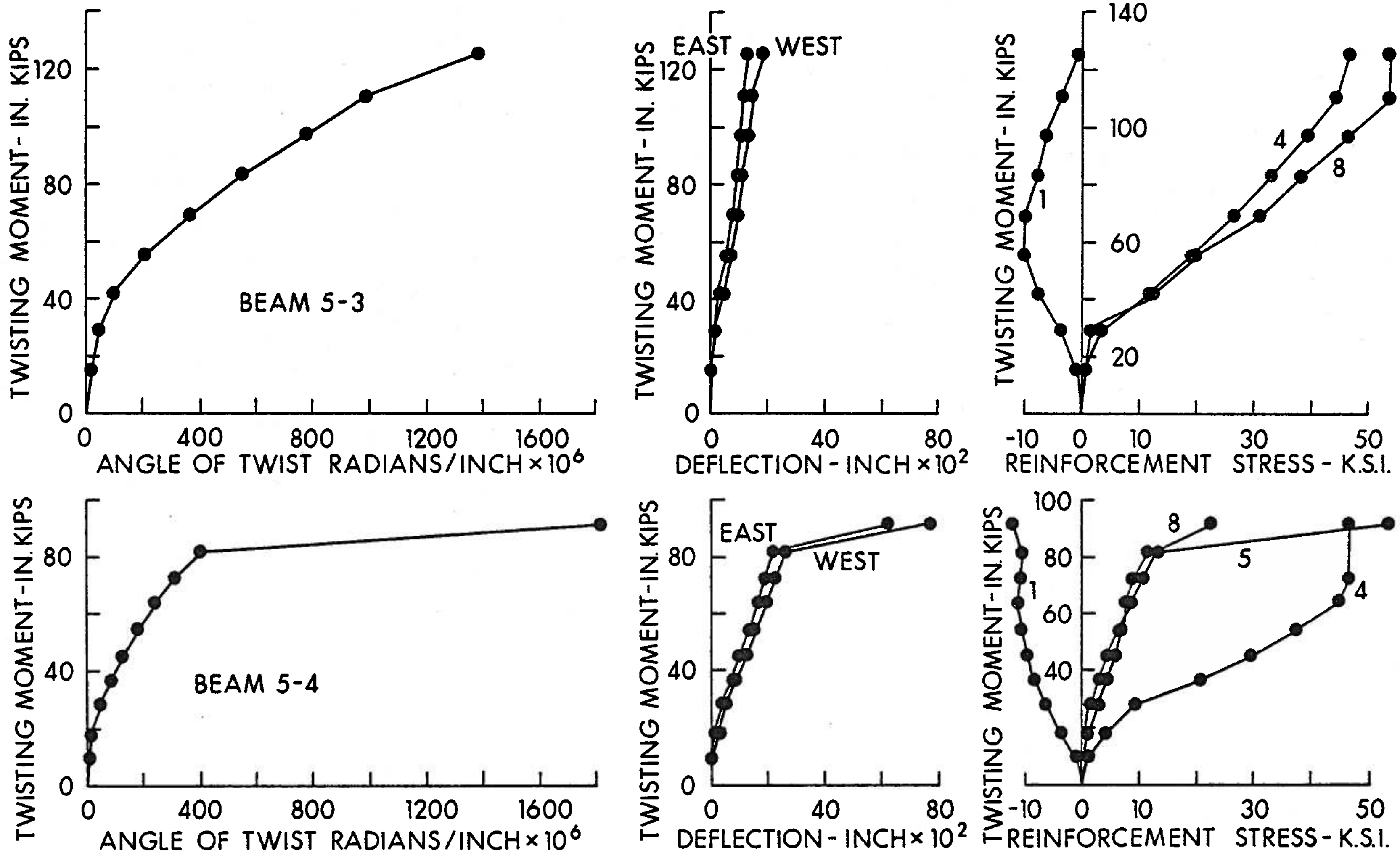

FIGURE B - 14 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 5 - 3 AND 5 - 4

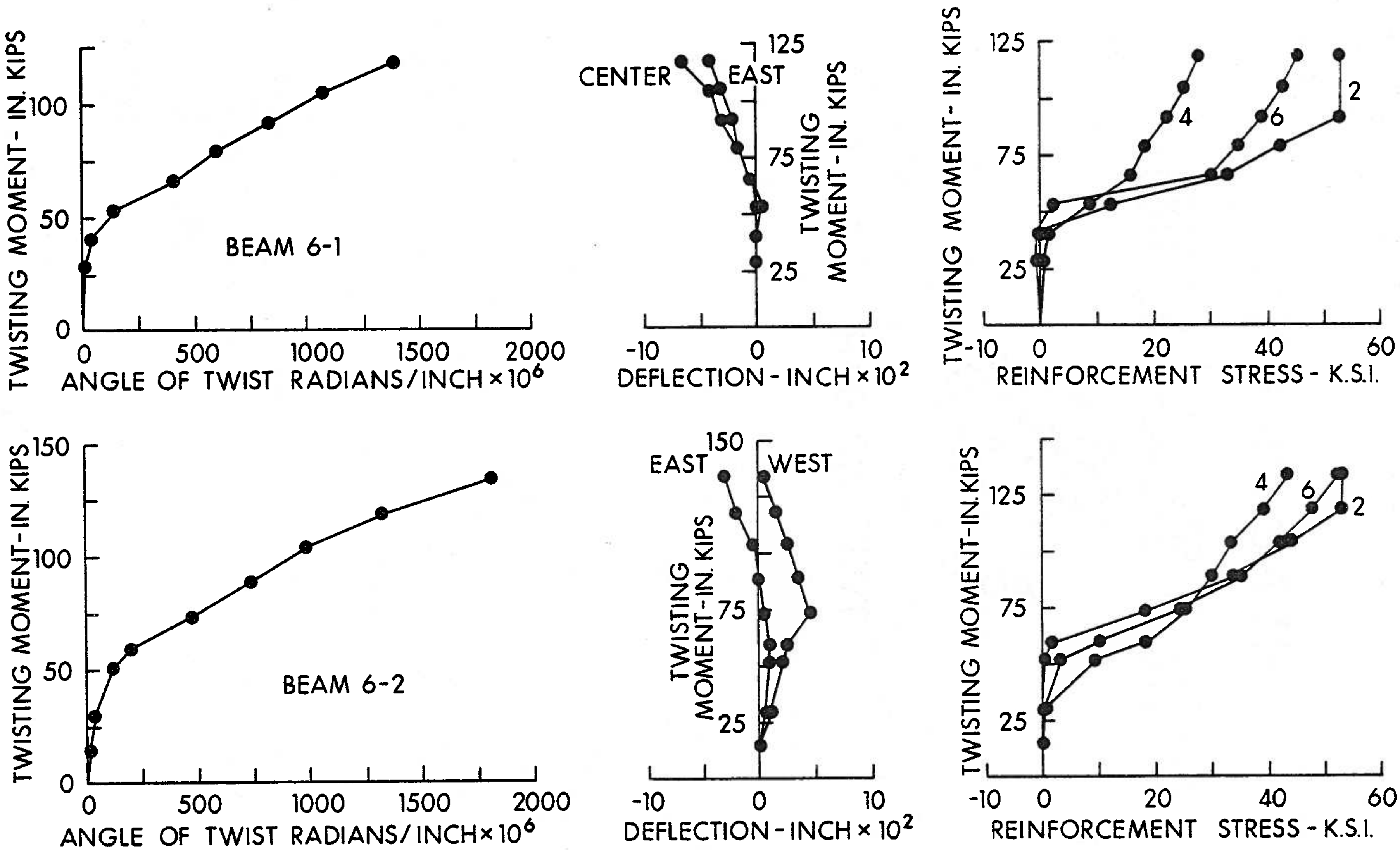

FIGURE B - 15 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 6 - 1 AND 6 - 2

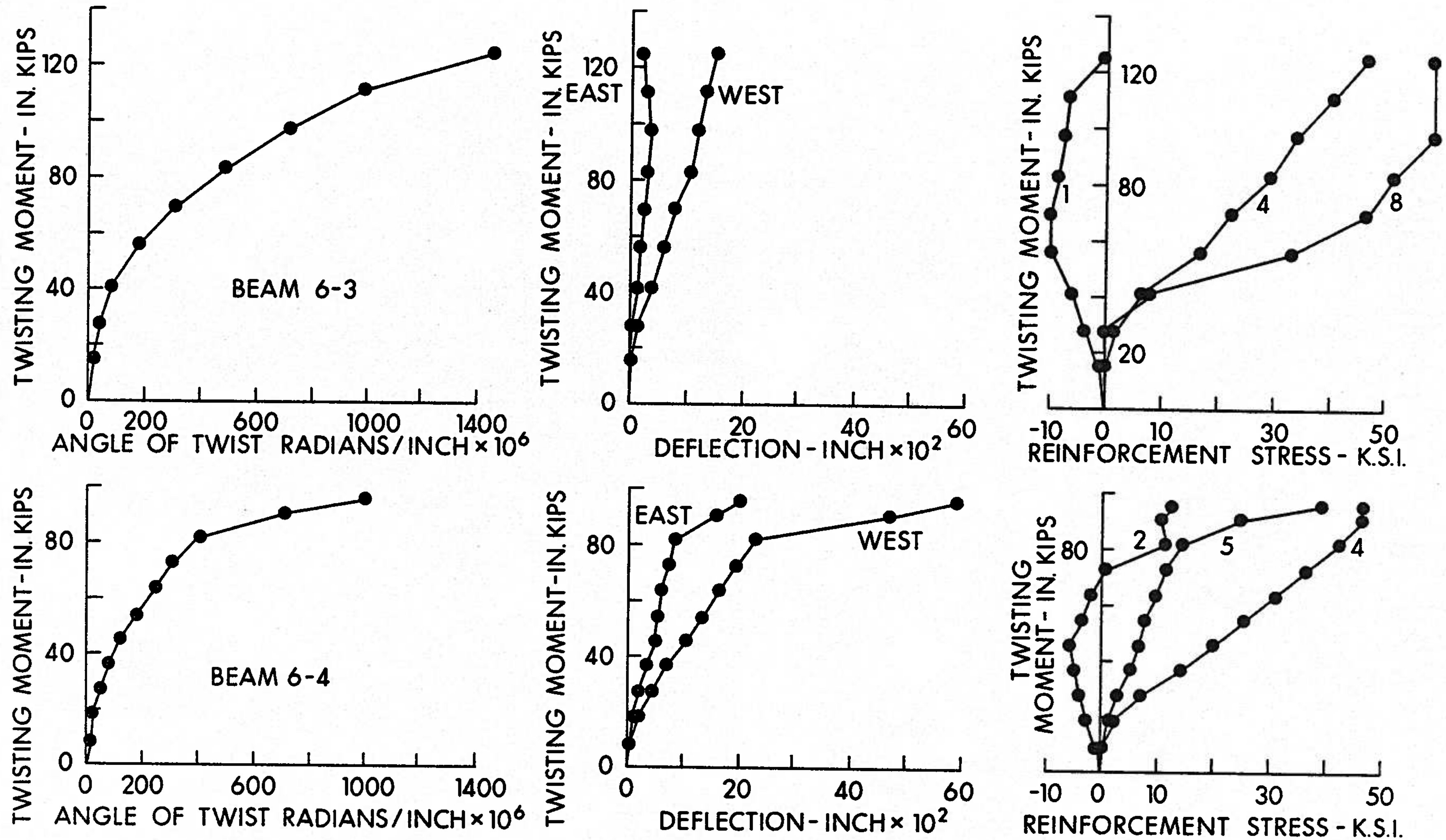

FIGURE B - 16 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 6 - 3 AND 6 - 4

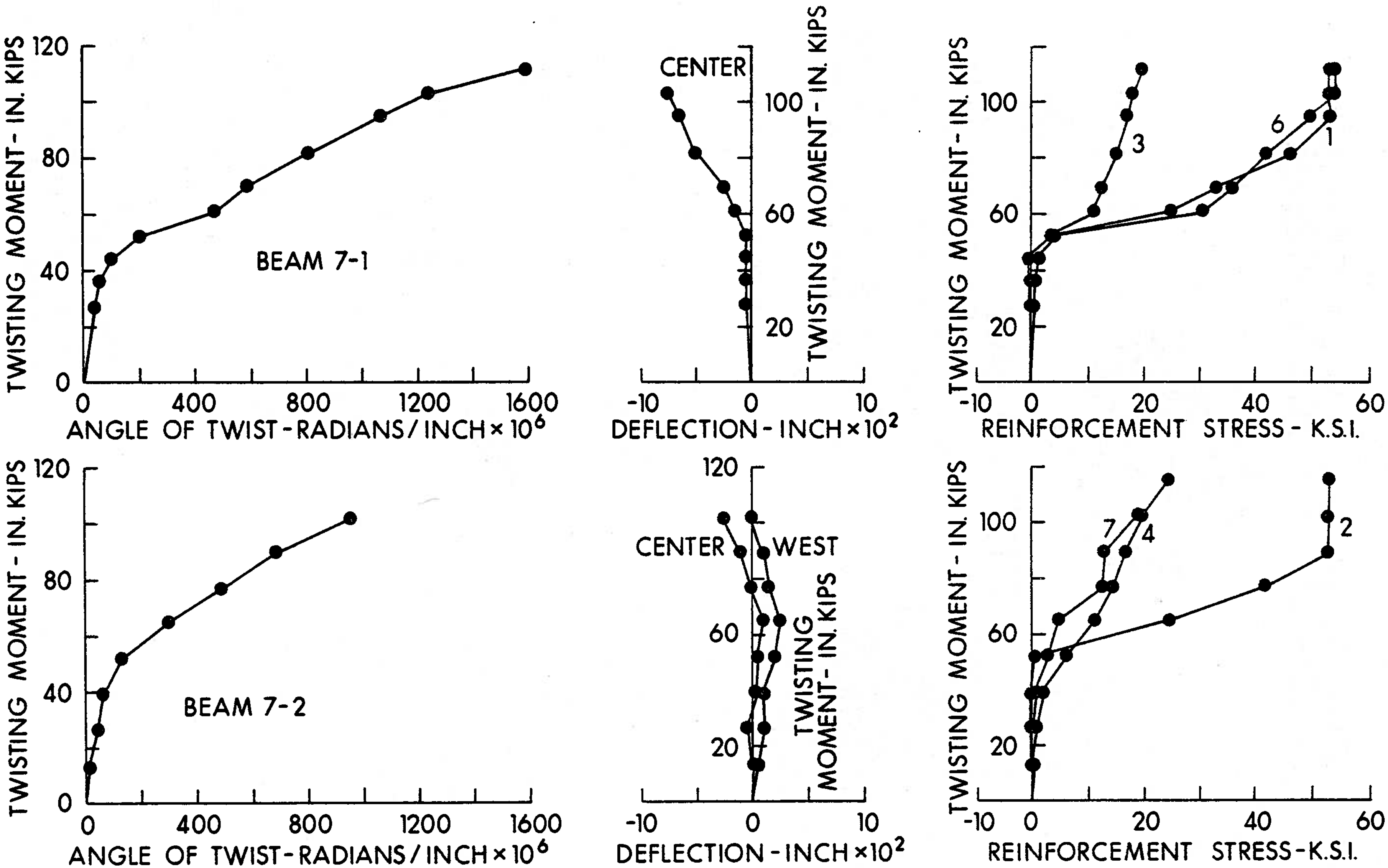

FIGURE B-17 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 7-1 AND 7-2

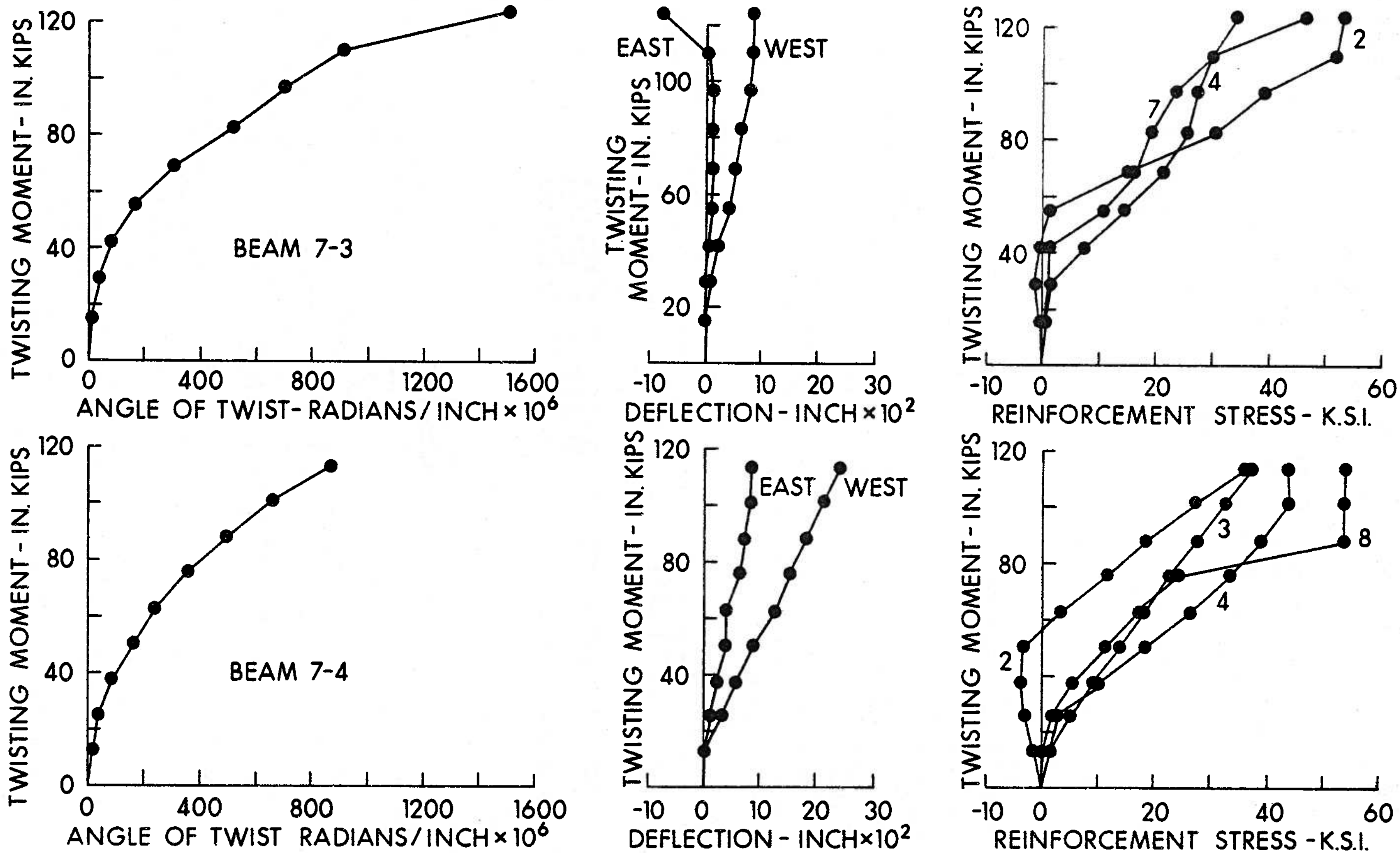

FIGURE B-18 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 7 - 3 AND 7 - 4

APPENDIX C

DERIVATION OF EQUATIONS

APPENDIX C

DERIVATION OF EQUATIONS

C-1 Mode 1 Failure Surface

The assumptions referred to below are presented in SECTION 5-3 of CHAPTER V. The symbols are defined in SECTION 5-4(a) and FIGURE 5-1 of CHAPTER V.

Moments are taken about an axis parallel to the neutral axis and located a distance $k_1x_1/2$ from the top face of the beam.

The external moments are:

Component of bending moment = $M_{b1}b/L_1$

Component of twisting moment = $M_{t1}c_1/L_1$

Moment due to transverse shear = 0

The internal moments are:

Moment due to force in longitudinal reinforcement

$$= A_{s1}f_{y1}\frac{b}{L_1}(h_{01} - \frac{k_1x_1}{2})$$

Moment due to force in horizontal legs of ties

$$= \frac{k_{01}c_1{}^2a_vf_{yt}}{SL_1}(h - a_{1t} - \frac{k_1x_1}{2})$$

Moment due to force in vertical legs of ties

$$= \frac{a_v f_{yt}(1 - k_{01})\ c_1}{S}\left[\frac{(1 - k_{01})b}{4} - a_{2t}\right]\frac{c_1}{L_1}$$

Equating external and internal moments gives:

$$\frac{M_{b1}\ b}{L_1} + \frac{M_{t1}c_1}{L_1} = \frac{A_{s1}f_{y1}b}{L_1}(h_{01} - \frac{k_1x_1}{2}) + \frac{k_{01}c_1^2 a_v f_{yt}}{SL_1}(h - a_{1t} - \frac{k_1x_1}{2})$$

$$+ \frac{a_v f_{yt}(1 - k_{01})c_1^2}{SL_1}\left[\frac{(1 - k_{01})b}{4} - a_{2t}\right]$$

$$M_{b1}b + M_{t1}c_1 = A_{s1}f_{y1}b(h_{01} - \frac{k_1x_1}{2})$$

$$+ \frac{a_v f_{yt}c_1^2}{S}\left[k_{01}\ (h - a_{1t} - \frac{k_1x_1}{2}) + (1 - k_{01})\left[\frac{(1 - k_{01})b}{4} - a_{2t}\right]\right]$$

Let $\emptyset = \dfrac{M_{t1}}{M_{b1}}$ and $p_1 = \dfrac{f_{yt}\ a_v\ h}{f_{y1}\ A_{s1}\ S}$

$$M_{t1}\left[\frac{c_1}{b} + \frac{1}{\emptyset}\right] = A_{s1}\ f_{y1}(h_{01} - \frac{k_1x_1}{2})$$

$$+ \frac{p_1A_{s1}f_{y1}c_1^2}{bh}\left[k_{01}(h - a_{1t} - \frac{k_1x_1}{2}) + (1 - k_{01})\frac{b}{4}(1 - k_{01} - \frac{4a_{2t}}{b})\right]$$

$$M_{t1}\left[\frac{c_1}{b}+\frac{1}{\emptyset}\right] = A_{s1}f_{y1}\left[(h_{01}-\frac{k_1x_1}{2}) + \frac{p_1c_1^2}{bh}\left[k_{01}(h-a_{1t}-\frac{k_1x_1}{2})\right.\right.$$

$$\left.\left.+\frac{b}{4}(1-k_{01})(1-k_{01}-\frac{4a_{2t}}{b})\right]\right]$$

Let $z_1 = h_{01} - k_1x_1/2$

and $y_1 = k_{01}(h - a_{1t} - \frac{k_1x_1}{2}) + \frac{b}{4}(1 - k_{01})(1 - k_{01} - \frac{4a_{2t}}{b})$

$$M_{t1} = A_{s1}f_{y1}\left[\frac{z_1 + \frac{p_1y_1c_1^2}{bh}}{\frac{c_1}{b} + \frac{1}{\emptyset}}\right] \qquad (5\text{-}1)$$

The sum of the forces acting perpendicular to a plane that contains the neutral axis and is perpendicular to the top face of the beam must equal zero. These forces are:

Force in longitudinal reinforcement $= \frac{A_{s1}f_{y1}b}{L_1}$

Force in horizontal legs of ties $= \frac{a_vf_{yt}k_{01}c_1^2}{S\ L_1}$

Force in concrete compression zone $= 0.85\ k_1x_1L_1f'_c$

Summing of forces gives:

$$\frac{A_{s1}f_{y1}b}{L_1} + \frac{a_vf_{yt}k_{01}c_1^2}{S\ L_1} - 0.85\ k_1x_1L_1f'_c = 0$$

$$x_1 = \frac{A_{s1} f_{y1} (b + p_1 k_{01} c_1^2 / h)}{0.85 \; k_1 f'_c \; L_1^2} \tag{5-2}$$

Using assumption two and neglecting the depth of the compression zone:

$$k_{01} = \frac{b}{2h + b}$$

In order to solve Equations (5-1) and (5-2) the value of c_1 must be determined. The value of this parameter that should be used is the one which gives the minimum value of twisting moment. The value of x_1, which depends on c_1, has a relatively small effect on z_1 and y_1. Therefore, in the following operation it is assumed that z_1 and y_1 are not functions of x_1.

Differentiating Equation (5-1) with respect to c_1, equating the first derivative to zero and solving for c_1 yields:

$$c_1 = -\frac{b}{\emptyset} \pm b \left[\frac{1}{\emptyset^2} + \frac{z_1 h}{p_1 y_1 b} \right]^{1/2} \tag{5-3}$$

The positive sign for the second term is used since negative values of c_1 have no physical significance. Similarly, the minimum value that c_1 can have is zero. The maximum value that c_1 can have is obtained by neglecting the depth of the compression zone and using assumption two. The resulting equation is:

$$c_{m1} = 2h + b \tag{5-4}$$

C-2 Mode 2 Failure Surface

The assumptions referred to below are presented in SECTION 5-3 of CHAPTER V. The symbols are defined in SECTION 5-4(b) and FIGURE 5-2 of CHAPTER V.

Moments are taken about an axis parallel to the neutral axis and located a distance $k_1x_2/2$ from the vertical face of the beam that is adjacent to the compression zone.

The external moments are:

Component of bending moment = 0

Component of twisting moment $= \dfrac{M_{t2}\,c_2}{L_2}$

Moment due to transverse shear $= V\left[\dfrac{b}{2} - \dfrac{k_1x_2}{2}\right]\dfrac{c_2}{L_2}$

The internal moments are:

Moment due to force in longitudinal reinforcement

$$= A_{s2}f_{y2}\;\frac{h}{L_2}\;(b_{02} - \frac{k_1x_2}{2})$$

Moment due to force in vertical legs of ties

$$= \frac{a_v f_{yt} k_{02} c_2^{\,2}}{S\;L_2}\;(b - a_{2t} - \frac{k_1x_2}{2})$$

Moment due to force in horizontal legs of ties

$$= \frac{a_v f_{yt}(1 - k_{02})\ c_2}{S} \left[\frac{(1 - k_{02})\ h}{4} - a_{1t} \right] \frac{c_2}{L_2}$$

Equating external and internal moments yields:

$$\frac{M_{t2}c_2}{L_2} + \frac{Vc_2}{L_2} \left(\frac{b}{2} - \frac{k_1 x_2}{2}\right) = \frac{A_{s2} f_{y2} h}{L_2} \left(b_{02} - \frac{k_1 x_2}{2}\right)$$

$$+ \frac{a_v f_{yt} k_{02} c_2^{\,2}}{S\ L_2} \left(b - a_{2t} - \frac{k_1 x_2}{2}\right)$$

$$+ \frac{a_v f_{yt}(1 - k_{02}) c_2^{\,2}}{S\ L_2} \left[\frac{(1 - k_{02})\ h}{4} - a_{1t} \right]$$

$$M_{t2}c_2 + Vc_2 \left(\frac{b}{2} - \frac{k_1 x_2}{2}\right) = A_{s2} f_{y2} h \left(b_{02} - \frac{k_1 x_2}{2}\right)$$

$$+ \frac{a_v f_{yt} c_2^{\,2}}{S} \left[k_{02}\left(b - a_{2t} - \frac{k_1 x_2}{2}\right) + (1 - k_{02})\frac{h}{4} \left(1 - k_{02} - \frac{4a_{1t}}{h}\right) \right]$$

Let $\delta = \dfrac{Vb}{2M_{t2}}$; $p_2 = \dfrac{f_{yt}\ a_v\ b}{f_{y2}\ A_{s2}\ S}$; $z_2 = b_{02} - \dfrac{k_1 x_2}{2}$;

$$y_2 = k_{02}\left(b - a_{2t} - \frac{k_1 x_2}{2}\right) + (1 - k_{02})\ \frac{h}{4}\ \left(1 - k_{02} - \frac{4a_{1t}}{h}\right)$$

$$M_{t2}\left(1 + \delta - \frac{\delta k_1 x_2}{b}\right) = \frac{A_{s2} f_{y2} h}{c_2} \left[z_2 + \frac{p_2 y_2 c_2^{\,2}}{bh} \right]$$

$$M_{t2} = \frac{A_{s2}f_{y2}h}{c_2}\left[\frac{z_2 + \dfrac{p_2y_2c_2^2}{bh}}{1+\delta-\dfrac{\delta k_1x_2}{b}}\right] \tag{5-5}$$

The sum of the forces acting perpendicular to a plane that contains the neutral axis and is perpendicular to the vertical face of the beam on which the compression zone is located must equal zero. These forces are:

Force in longitudinal reinforcement $= \dfrac{A_{s2}f_{y2}h}{L_2}$

Force in vertical legs of ties $= \dfrac{a_vf_{yt}k_{02}c_2^2}{S\ L_2}$

Force in concrete compression zone $= 0.85\ k_1x_2L_2f'_c$

Transverse shear force $= V\ c_2/L_2$

Summing of forces yields:

$$\frac{A_{s2}f_{y2}h}{L_2} + \frac{a_vf_{yt}k_{02}c_2^2}{S\ L_2} - 0.85\ k_1x_2L_2f'_c - \frac{V\ c_2}{L_2} = 0$$

$$x_2 = \frac{A_{s2}f_{y2}\left[h + \dfrac{p_2k_{02}c_2^2}{b}\right] - V\ c_2}{0.85\ k_1f'_c\ L_2^2} \tag{5-6}$$

Using assumption two and neglecting the depth of the compression zone:

$$k_{02} = \frac{h}{2b + h}$$

In order to solve Equations (5-5) and (5-6) the value of c_2 must be determined. The value of this parameter that should be used is the one which yields the minimum value of twisting moment. The value of x_2, which depends on c_2, has a relatively small effect on z_2 and y_2. Therefore, in the following operation it is assumed that z_2 and y_2 are not functions of x_2.

Differentiating Equation (5-5) with respect to c_2, equating the first derivative to zero and solving for c_2 yields:

$$c_2 = \pm \left[\frac{h\ z_2\ b}{p_2 y_2}\right]^{1/2} \tag{5-7}$$

The positive sign is used since negative values of c_2 have no physical significance. The maximum value that c_2 can have is obtained by neglecting the depth of the compression zone and using assumption two. The resulting equation is:

$$c_{m2} = 2b + h \tag{5-8}$$

The non-dimensional quantity δ that was used in deriving Equation (5-5) was defined as:

$$\delta = \frac{Vb}{2M_{t2}}$$

This equation is rearranged to obtain Equation (5-9).

$$V = \frac{2M_{t2}\delta}{b} \tag{5-9}$$

C-3 Mode 3 Failure Surface

The assumptions referred to below are presented in SECTION 5-3 of CHAPTER V. The symbols are defined in SECTION 5-4(c) and FIGURE 5-3 of CHAPTER V.

Moments are taken about an axis parallel to the neutral axis and located a distance $k_1 x_3/2$ from the bottom face of the beam.

The external moments are:

Component of bending moment $= M_{b3} \dfrac{b}{L_3}$

Component of twisting moment $= M_{t3} \dfrac{c_3}{L_3}$

Moment due to transverse shear = 0

The internal moments are:

Moment due to force in longitudinal reinforcement

$$= \frac{A_{s3} f_{y3} b}{L_3} (h_{03} - \frac{k_1 x_3}{2})$$

Moment due to force in horizontal legs of ties

$$= \frac{a_v f_{yt} k_{03} c_3^{\,2}}{S\ L_3} (h - a_{3t} - \frac{k_1 x_3}{2})$$

Moment due to force in vertical legs of ties

$$= \frac{a_v f_{yt}(1 - k_{03})\ c_3}{S} \left[\frac{(1 - k_{03})\ b}{4} - a_{2t}\right] \frac{c_3}{L_3}$$

Equating external and internal moments gives:

$$M_{t3}\frac{c_3}{L_3} - \frac{M_{b3}\ b}{L_3} = \frac{A_{s3}f_{y3}\ b}{L_3}\left(h_{03} - \frac{k_1x_3}{2}\right) + \frac{a_vf_{yt}k_{03}c_3^{\ 2}}{S\ L_3}\left(h - a_{3t} - \frac{k_1x_3}{2}\right)$$

$$+ \frac{a_vf_{yt}(1 - k_{03})\ c_3^{\ 2}}{S\ L_3}\left[\frac{(1 - k_{03})\ b}{4} - a_{2t}\right]$$

$$M_{t3}\ c_3 - M_{b3}\ b = A_{s3}f_{y3}\ b\left(h_{03} - \frac{k_1x_3}{2}\right)$$

$$+ \frac{a_vf_{yt}c_3^{\ 2}}{S}\left[k_{03}\left(h - a_{3t} - \frac{k_1x_3}{2}\right) + (1 - k_{03})\left[\frac{(1 - k_{03})\ b}{4} - a_{2t}\right]\right]$$

Let $\emptyset = \dfrac{M_{t3}}{M_{b3}}$ and $p_3 = \dfrac{a_vf_{yt}h}{A_{s3}f_{y3}S}$

$$M_{t3}\left[\frac{c_3}{b} - \frac{1}{\emptyset}\right] = A_{s3}f_{y3}\left(h_{03} - \frac{k_1x_3}{2}\right)$$

$$+ \frac{p_3A_{s3}f_{y3}c_3^{\ 2}}{bh}\left[k_{03}\left(h - a_{3t} - \frac{k_1x_3}{2}\right) + (1 - k_{03})\frac{b}{4}\left(1 - k_{03} - \frac{4a_{2t}}{b}\right)\right]$$

$$M_{t3}\left[\frac{c_3}{b} - \frac{1}{\emptyset}\right] = A_{s3}f_{y3}\left[\left(h_{03} - \frac{k_1x_3}{2}\right) + \frac{p_3c_3^{\ 2}}{bh}\left[k_{03}\left(h - a_{3t} - \frac{k_1x_3}{2}\right)\right.\right.$$

$$\left.\left. + \frac{b}{4}(1 - k_{03})\left(1 - k_{03} - \frac{4a_{2t}}{b}\right)\right]\right]$$

Let $z_3 = h_{03} - \frac{k_1 x_3}{2}$

and $y_3 = k_{03}(h - a_{3t} - \frac{k_1 x_3}{2}) + \frac{b}{4}(1 - k_{03})(1 - k_{03} - \frac{4a_{2t}}{b})$

$$M_{t3} = A_{s3} f_{y3} \left[\frac{z_3 + \frac{p_3 y_3 c_3^2}{bh}}{\frac{c_3}{b} - \frac{1}{\emptyset}} \right] \qquad (5\text{-}10)$$

The sum of the forces acting perpendicular to a plane that contains the neutral axis and is perpendicular to the bottom face of the beam must equal zero. These forces are:

Force in longitudinal reinforcement $= \frac{A_{s3} f_{y3}\, b}{L_3}$

Force in horizontal legs of ties $= \frac{a_v f_{yt} k_{03} c_3^2}{S\, L_3}$

Force in concrete compression zone $= 0.85\, k_1 x_3 L_3 f'_c$

Summing of forces yields:

$$\frac{A_{s3} f_{y3} b}{L_3} + \frac{a_v f_{yt} k_{03} c_3^2}{S\, L_3} - 0.85\, k_1 x_3 L_3 f'_c = 0$$

$$x_3 = \frac{A_{s3} f_{y3} \left[b + \frac{p_3 k_{03} c_3^2}{h} \right]}{0.85\, k_1 f'_c\, L_3^2} \qquad (5\text{-}11)$$

Using assumption two and neglecting the depth of the compression zone:

$$k_{03} = \frac{b}{2h + b}$$

Before Equations (5-10) and (5-11) can be solved, the value of c_3 must be determined. The value of this parameter that should be used is the one that yields the minimum value of twisting moment. The value of x_3, which depends on c_3, has a relatively small effect on z_3 and y_3. Therefore, in the following operation it is assumed that z_3 and y_3 are not functions of x_3.

Differentiating Equation (5-10) with respect to c_3, equating the first derivative to zero and solving for c_3 yields:

$$c_3 = \frac{b}{\phi} \pm b \left[\frac{1}{\phi^2} + \frac{z_3 h}{p_3 y_3 b}\right]^{1/2} \qquad (5\text{-}12)$$

Since the value of the terms within the brackets is always greater than $\frac{1}{\phi}$ and since negative values of c_3 have no physical significance, the positive sign is used. The maximum value that c_3 can have is obtained by neglecting the depth of the compression zone and using assumption two. The resulting equation is:

$$c_{m3} = 2h + b \qquad (5\text{-}13)$$