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ON THE RISE OF BUOYANT PLUMES IN TURBULENT ENVIRONMENTS

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ABSTRACT

A plume rise model is derived from the equations of turbulent motion, retaining the turbulent flux terms. The solutions are similar to those proposed earlier by Csanady but containing an exponential decay term. The model finds particular value in predicting a leveled-off plume trajectory in neutral atmospheric conditions. In unstable atmospheric conditions the ultimate mode of behavior depends on whether the atmospheric turbulence or the unstable stratification finally dominate the plume motion.

ON THE RISE OF BUOYANT PLUMES IN TURBULENT ENVIRONMENTS

Current Modeling Practices

Historically, in describing the motion of buoyant chimney plumes, two rather different theoretical models have been employed. In both models the mixing with the environment has been found to have an important effect on the motion, although this mixing is supposed to occur in different ways. The first theory considers a "thermal." In this case mixing is the result of turbulent motion produced by buoyancy forces within the plume and the plume grows by entraining external fluid. This theory demands that the entrainment velocity is everywhere proportional to the upward plume velocity and so applies exactly in a quiescent environment. The approach was first suggested by Morton et al (1956). The second theory regards the buoyant plume element as a region which is mixing fluid with its surrounding at a rate governed by the level of turbulence in the environment. In this case plume growth or plume dispersion depends on such quantities as the size of the plume element and the environmental turbulence intensity (Batchelor, 1950).

Environmental turbulence produced independently of the motion of buoyant plumes will almost certainly have an effect on this motion and should therefore be included in the description of plume motion in a turbulent environment. Each of these theories could be relevant at a different stage in the development of a buoyant plume. When the velocity of the plume is high the environmental turbulence will be relatively unimportant and the first theory is then an appropriate model. But when the plume and environmental turbulence velocities are of similar magnitude the second theory provides the more accurate predictions. One way of describing the complete history of a buoyant plume would be to superpose the two separate theories. In fact this is the method used by among others, Slawson and Csanady (1967), Briggs (1969) and Bringfelt (1969). However, this approach necessitates the introduction of rather arbitrary transition points, the concept of which only adds to the complexity of the problem. Furthermore the approach never accounts for the leveled-off plume trajectory anticipated in near-neutral and neutral atmospheric conditions. In the present paper a comparable theory will be developed but one which describes the entire history of plume motion in a single formulation.

Governing Equations

The present theory encompasses the two plume phases described above, but in the case of the atmospheric phase the plume element is regarded as a region of constant size or an "open parcel." This concept has previously been used by Priestly (1953) in describing the motion of natural thermals.

Following Csanady (1973) the approximate equations of vertical momentum and potential temperature for the mean component of turbulent flow are:

$$\frac{\partial w}{\partial t} + \underline{V} \cdot \nabla w - K_m \nabla^2 w = g \frac{\theta'}{\theta_a} \tag{1}$$

$$\frac{\partial \theta}{\partial t} + \underline{V} \cdot \nabla \theta - K_h \nabla^2 \theta = 0 \tag{2}$$

where θ' is the excess plume potential temperature and K_m and K_h are the turbulent transfer coefficients for momentum and heat respectively. Since the Laplacian expresses a difference in quantity between neighboring points the turbulent flux terms may be written as (Priestly (1953), Csanady (1973)):

$$K_m \nabla^2 w = - \frac{C_1 K_m}{R^2} w$$

$$K_h \nabla^2 \theta = - \frac{C_2 K_h}{R^2} \theta'$$

where C_1 and C_2 are constants depending only on the shape of the distribution of w and θ. R is the characteristic constant size (radius) of the plume element. If it is assumed that momentum and heat are transported in the same way with the same effectiveness, by the turbulence one would expect that $K_m/K_h = 1$. Also assuming, for the purpose of this study, constancy of $K_m = K_h = K$ as well as identical shapes of the distribution of w and θ i.e. $C_1 = C_2 = C$ we obtain:

$$\frac{CK}{R^2} = k = \text{constant.}$$

In view of this expression Eq.'s (1) and (2) now become:

$$\frac{\partial w}{\partial t} + \underline{V} \cdot \nabla w + kw = g\,\frac{\theta'}{\theta_a} \tag{3}$$

$$\frac{\partial \theta'}{\partial t} + \underline{V} \cdot \nabla\theta' + k\theta' + w\,\frac{\partial \theta_a}{\partial z} = 0 \tag{4}$$

where instead of studying the variation in plume potential temperature we selected to study the variation in plume excess potential temperature i.e., $\theta = \theta_a + \theta'$. In addition it was assumed that θ_a varies essentially only in the vertical direction.

Consider a buoyant chimney plume rising in a constant cross-wind U. The change in total momentum and buoyancy (in kinematic units) is now obtained by integrating Eq.'s (3) and (4) over the entire plume cross-section. Thus:

$$\frac{d}{dt}\iint_F w\,dydz + k\iint_F w\,dydz = \iint_F g\,\frac{\theta'}{\theta_a}\,dydz \tag{5}$$

$$\frac{d}{dt}\iint_F \theta'\,dydz + k\iint_F \theta'\,dydz = -\frac{\partial \theta_a}{\partial z}\iint_F w\,dydz \tag{6}$$

where a linear profile of ambient potential temperature is assumed. Following Csanady (1973) it is now convenient to introduce the flux of total momentum and buoyancy respectively;

$$M/U = \iint_F w\,dy\,dz$$

$$F/U = \iint_F g\theta'/\theta_a\,dy\,dz$$

Then multiplying Eq. (6) by the ratio g/θ_a, considered constant, and making use of the above definitions, Eq.'s (5) and (6) finally become:

$$\frac{dM}{dt} = F - kM \tag{7}$$

$$\frac{dF}{dt} = -N^2M - kF \tag{8}$$

where $N^2 = (g/\theta_a)\partial\theta_a/\partial z$ = constant (N = Brunt-Vaisala frequency).

Plume Motion

By eliminating F from Eq.'s (7) and (8) the equation governing the vertical motion of the plume is arrived at. Thus:

$$\frac{d^2M}{dt^2} + 2k\frac{dM}{dt} + (N^2 + k^2)M = 0. \tag{9}$$

This is a second order linear differential equation with constant coefficients the solution of which depends on the sign of N^2. Different signs of N^2 correspond to different atmospheric stability conditions.

Suppose that the atmosphere is stably stratified, $N^2 > 0$. In this case the solution of Eq. (9) is:

$$M = \frac{F_o}{N}\sqrt{1 + \frac{M_o^2N^2}{F_o^2}}\ \sin\left[Nt + \arctan\frac{M_oN}{F_o}\right]e^{-kt} \tag{10}$$

for the initial conditions $M = M_o$, $F = F_o$ at $t = 0$. This solution is similar to that given by Csanady (1973) with the exception of the term

e^{-kt}. The motion is oscillatory, the plume rising to a global maximum when M = 0 after which it executes damped harmonic oscillations about a lower equilibrium height. As $k \to 0$ the motion becomes strictly periodic corresponding to the solution proposed by Csanady (1973).

For neutal atmospheric conditions, $N^2 = 0$, the solution of Eq. (9) is:

$$M = (M_o + F_o t)e^{-kt}. \tag{11}$$

The motion is asymptotic, i.e. a final rise is observed as $M \to 0$.

For unstable stratification, $N^2 < 0$, the solution of Eq. (9) is:

$$M = \frac{F_o}{N}\sqrt{1 - \frac{M_o^2 N^2}{F_o^2}} \sinh\left[Nt + \operatorname{artanh} \frac{M_o N}{F_o}\right]e^{-kt} \tag{12}$$

where $N = |N^2|^{\frac{1}{2}}$. There are two different modes of behavior depending on whether the exponential of hyperbolic term ultimately dominates. For sufficiently large k the exponential term is finally dominant and the plume approaches its final height asymptotically. On the other hand when k is sufficiently small the plume's vertical momentum flux will ultimately increase exponentially with time, i.e. the plume becomes absolutely buoyant.

Plume Trajectories

The definition of vertical momentum flux provides a suitable expression from which to calculate the plume trajectory. Writing $\pi M/U = \iint_F w\,dy\,dz$, the integration is carried out over a circular plume region to yield:

$$M = UR^2\overline{w} = UR^2 dz/dt \tag{13}$$

where $\overline{w}$ is the vertical plume velocity averaged over the cross-sectional plume area. Given an appropriate relationship for the plume radius R, Eq. (13) can be integrated to yield the time-dependent trajectory. The present theory requires that the plume grows during its self-structured phase only, by entraining external fluid. Then in accordance with earlier plume rise models (see eg. Briggs, 1969) we may adopt the simple linear growth relationship, $R = \alpha z$. The derivation of the complete plume trajectory is now a relatively straightforward matter. Here it will suffice to illustrate the different trajectories graphically. For this purpose Figures 1 and 2 were prepared. The trajectories were calculated assuming a buoyancy-dominated plume, i.e. one where $M_o N/F_o << 1$. Of particular interest is the leveled-off plume trajectory in neutral atmospheric conditions ($N^2 = 0$) as $t \to \infty$. Also noteworthy is the asymptotic rise in unstable conditions ($N^2 < 0$) when k is sufficiently large, a point to be discussed later.

In dispersion calculations we are not so much interested in the plume trajectory as in the final rise of the plume. This rise may be defined as the height above the physical source when the plume centerline is in equilibrium. This concept applies in all instances of oscillatory and asymptotic motion since then w and dw/dt tend to zero as t tends to infinity. The final plume rise is obtained from Eq.'s (9) and (13) and is:

$$z_f = \left[\frac{3(F_o + kM_o)}{U\alpha^2(N^2 + k^2)}\right]^{1/3} . \tag{14}$$

Eq. (14) holds for all values of N^2. However in unstable atmospheric conditions, $N^2 < 0$, a final plume rise can be attained only when the denominator in Eq. (14) is greater than zero. Of special interest is the global maximum rise obtained in stable atmospheric stratification ($N^2 > 0$), i.e.

$$Z_m/Z_f = (1 + e^{-\pi k/N})^{1/3} \quad \text{when } t = \pi/N. \tag{15}$$

Eq. (15) is strictly valid for a buoyancy-dominated plume but is also an excellent approximation to a plume with relatively large exit velocity.

Unstable Plume Behavior

In unstable atmospheric conditions the ultimate mode of behavior depends on whether the atmospheric turbulence level (represented by k) or the unstable stratification (represented by $-N^2$) finally dominates the plume motion. For high atmospheric turbulence level, $k > |N^2|^{1/2}$, the plume breakup may be so vigorous that both buoyancy and vertical momentum are rapidly dissipated and further rise is prevented. This corresponds to the asymptotic case. On the other hand when the atmospheric turbulence level is low, $k < |N^2|^{1/2}$, the plume remains coherent and the effects of the unstable atmosphere dominate the motion of the plume. The plume becomes absolutely buoyant.

In order to use the present model the mixing ratio, k, must be known or estimated. Future work will therefore concentrate on establishing relationships for this parameter, the value of which presumably depends on the stability of the atmosphere, the level of atmospheric turbulence, and some characteristic plume size.

FIGURE 1.

ILLUSTRATION OF PLUME TRAJECTORIES WHEN $MoN/Fo << 1$ IN NEUTRAL AND STABLE ATMOSPHERIC CONDITIONS.

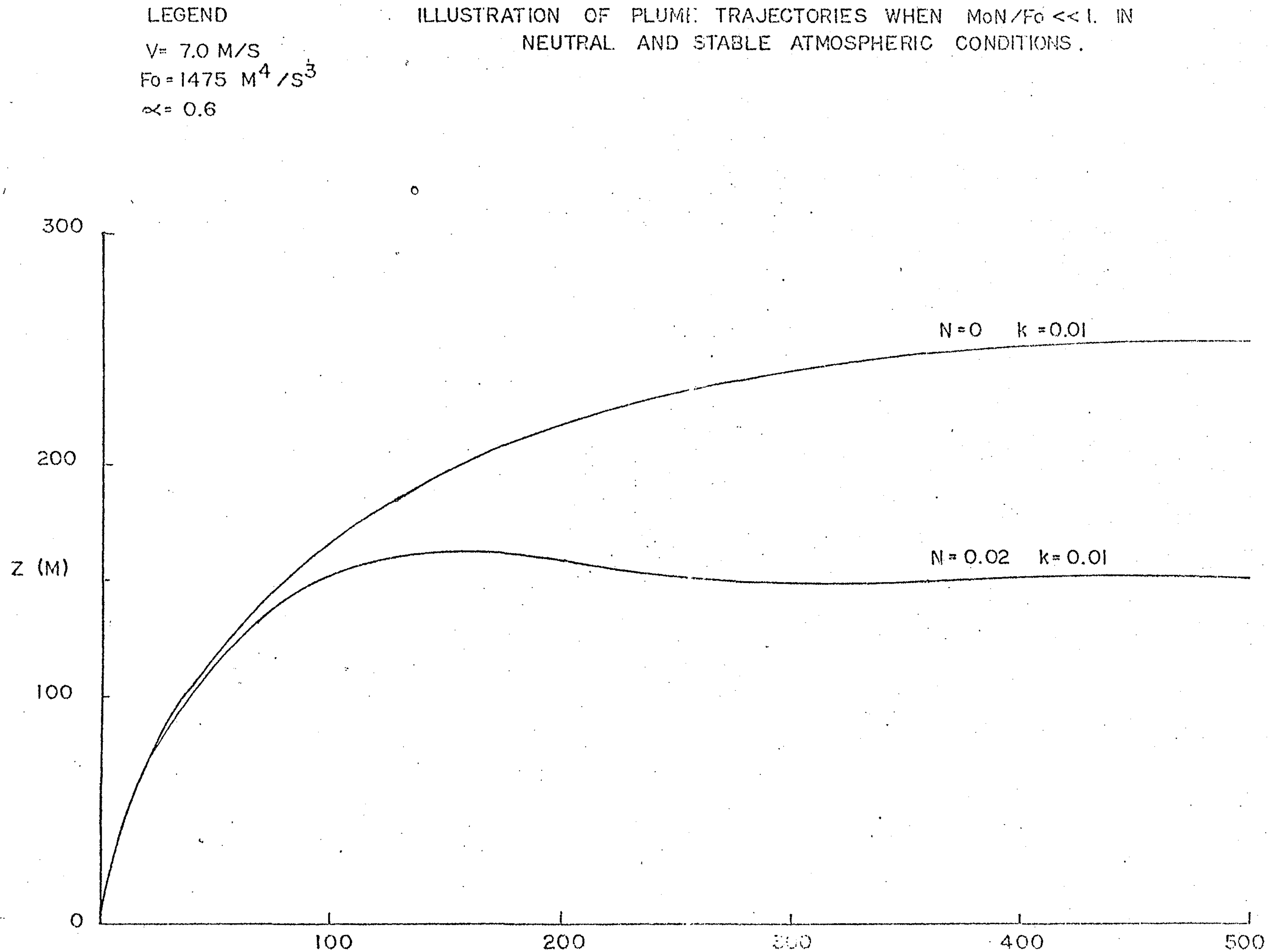
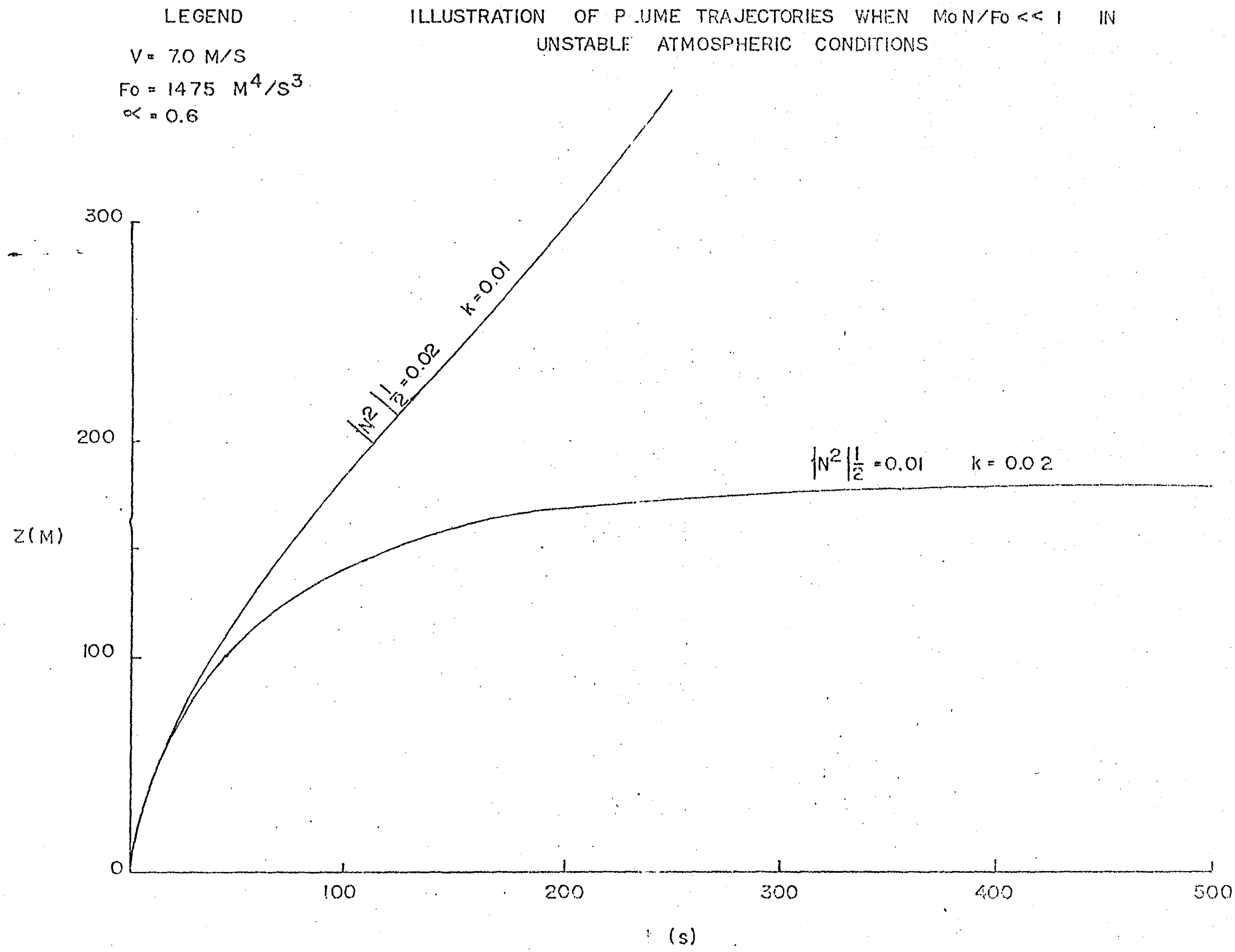


FIGURE 2.

ILLUSTRATION OF PLUME TRAJECTORIES WHEN $M_0 N / F_0 << 1$ IN UNSTABLE ATMOSPHERIC CONDITIONS



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