Stochastic Demand Response under Random Renewable Power Generation in Smart Grid

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***Abstract*—Rooftop photovoltaic (PV) generation combined with battery energy storage provides a promising solution for solar energy integration in smart grid. Specifically, the home battery energy storage systems can improve the efficiency and reliability of PV integration while reducing the greenhouse gas emissions. In this paper, we investigate the randomness of home PV generation and the residential random load demand, which may affect the efficiency and reliability of the power grid. A bi-level stochastic programming problem is formulated to provide a pricing strategy to customers for the optimal demand response in smart grid. In particular, the operators model represents the cost minimization of the power system operation, while the customers' model represents the cost minimization of their household energy demand. In the operators model, power loss calculated based on power flow analysis is used as the system loss, while the stochastic model of the household load demand is used instead of the expected value to characterize the human random behaviour. The performance of the proposed stochastic demand response scheme is evaluated through extensive simulations. Simulation results indicate that this novel scheme can help both power system operators and electrical customers to better decide on their operating schedule and energy usage, respectively.**

***Index Terms*—Demand response, PV generation, residential appliances, smart grid, stochastic programming, uncertainty**

NOMENCLATURE

Superscript

b	Human behaviour
ch	Battery charging state
dch	Battery discharging state
d	Devices
PV	PV power generation
L	Power loss

Variables

A	PV panel area
B	Battery state
C	Electrical price
P	Real power
Q	Reactive power
G	Customer electricity cost
F	Utility operation cost
V	Voltage
ψ	Probability distribution
η	Battery charging/discharging efficiency
λ	Battery operation state, binary variable

Set and individuals

t, T	Time set $t \in T$
m, M	House number set $m \in M$
n, N	Node number set $n \in N$

I. INTRODUCTION

Compared with traditional fossil fuels, renewable energy such as solar and wind energy is eco-friendly as a clean energy source, which can help reduce greenhouse gas emissions. Due to the sharp decline in solar panel production cost in recent years, residential solar power systems are reasonably priced to help customers reduce their annual electricity consumption by 20% – 50% [1]. For example, solar energy was not a source of power for utility companies in Canada a decade ago, but in 2016, the installed capacity was 2,310 MW in Ontario [2].

However, the electricity generated by renewable energy sources may affect or disrupt conventional power generation, and due to the random nature of renewable energy sources, it is difficult to predict and integrate variable power sources to the grid. Recent studies discussed the randomness of renewable energy in distribution systems [3–5], small energy consumers (such as buildings [6], marine systems [7] and railway station systems [8]), and home energy systems [9, 10]. Specifically, in [3], the authors propose a semi-Markov model for the stochastic scheduling of photovoltaic power generation in microgrids to reduce fuel consumption. Multiple types of stochastic distributed generation, such as solar and wind power generation, and battery storage systems are considered in [4]. Here, the authors implemented the heuristic moment matching method to generate scenarios from random characters. Randomness is also considered for real-time control of an integrated solar-storage system in [5]. With the high penetration of renewable energy generation, PV panels can also be installed in a building with battery energy storage system [6]. Accordingly, the size of the battery storage and the number of installed PV panels need to be determined based on the consideration of randomness. Similar applications considering randomness are discussed in the sizing problem of a merchant marine [7] and the energy management system of a railway station [8]. In home energy management system (HEMS), authors in [9, 10] propose learning algorithms for scheduling distributed energy resources integrated with home battery storage systems.

Most of the algorithms proposed in recent research works are based on the formulation of stochastic programming problems to handle the randomness. However, the solution methods are based on specific scenarios [3–5, 7–10]. For example, scenarios can be generated by the heuristic moment matching method [4], innovative scenario generation processes

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[7, 8], and heuristic scenario reduction techniques [9, 10]. These algorithms can be categorized as the scenario-based algorithms. Benders decomposition is a well-known method for dealing with large-scale problems, and it is applied in research work [6]. However, this work focuses on the optimization problem in a building, without considering the distribution system operation. Moreover, dynamic programming is an effective method to reduce the computational complexity of stochastic optimization problems. For example, the authors in [11] propose a storage operation problem in distribution systems considering in-house renewables and in-house energy storage devices.

Different from previous recent works, in this study, we propose a stochastic bi-level demand response scheme for operator optimal pricing scheme in a distribution system. We assume that the residence is equipped with various electrical devices, rooftop PV power generation and battery energy storage system. Residential load demand as the lower level is modelled by the customer's uncertain behaviour, and PV power generation is determined by the probability distribution of random solar irradiance. Next, considering the random load demand response of the houses in the distribution system, we revolve these customers random demand scenarios in the operator economic model as the upper level. Therefore, we solve the proposed problem through stochastic programming, and an acceleration strategy is implemented to improve the algorithm efficiency. The main contributions of this work are summarized as follows:

- In this work, a bottom-up stochastic model is developed for both residential electrical appliances and PV power generation in the HEMS.
- We proposed a stochastic bi-level demand response scheme aiming at seeking the optimal pricing scheme for operator minimum system loss. The problem is solved by simplex and mixed integer linear programming (MILP) algorithms.
- The proposed algorithm is implemented by acceleration strategy to improve its efficiency.

The paper is structured as follows: Section II describes the system model including both customer's model and operator's model. The proposed stochastic demand response scheme formulation and solution method are introduced in Section III. The simulation results are presented in Section IV, and Section V concludes the paper.

II. System Model

A. Customer Model – Lower Level

This subsection describes the customer's electrical equipment model. We assume that all of these devices, such as renewable energy generation, household appliances and energy storage units, are equipped in house m. To simplify the formulation, we omit the subscript m in the following subsection.

1) PV Power Generation Probabilistic Model: We can derive the PV power generation probabilistic distribution by the solar irradiance s, as follows [12]:

$$P^{PV} = sA\eta^{PV}, \tag{1}$$

where A and η^{PV} refer to the area of the PV array and PV panel efficiency, respectively. As the solar irradiance is variant by time, the PV power output distribution can be written as:

$$P_t^{PV} = g^s(s_t)A\eta^{PV}, \tag{2}$$

and the corresponding probability distribution can be derived as:

$$\psi^{PV} = g^{PV}(P_t^{PV}). \tag{3}$$

An example of PV power output distribution is shown in Fig. 1.

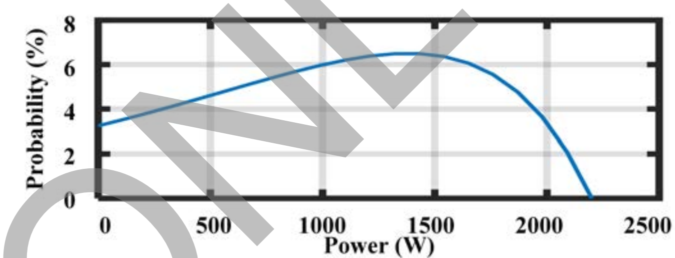

Fig. 1: An example of PV power output distribution, with a $20m^2$ solar panel at 10am in Ontario, July 2017.

2) House Appliances Load Demand Probabilistic Model: The probabilistic home demand model can be derived from our previous work [13], in which we assume that the probabilistic use of all appliances is randomly controlled by the customer. By considering the customer's behavioural probability distribution, the devices usage probability distribution can be determined. For example, customer entertainment related appliances such as television and stereo set have the same distribution as entertainment behaviour. Therefore, the appliance usage time distribution $\boldsymbol{\phi}_t^d$ belongs to the same behaviour ϕ_t^b can be derived as:

$$\boldsymbol{\phi}_t^d = \phi_t^b, \quad \forall d \in b. \tag{4}$$

Here, we define the probability distribution of human behaviour as ϕ^b, and the time of use distribution for the specific behaviour b related appliance d is represented by ϕ^d. Therefore, we can derive the power consumption distribution for each time period from the probability distribution of all devices and their rated power $\tilde{P}^d$ as follows:

$$\phi_t = \sum_d \phi_t^d, \quad \forall P_t^d = \tilde{P}^d, \tag{5}$$

where ϕ_t refers to the power distribution probability, and the corresponding power is P_t^d. Therefore, the probability distribution of household electricity demand is:

$$\psi_t^d = g^d(P_t^d). \tag{6}$$

3) Home Energy Storage Model: The limits on battery charging power B^{ch} and discharging power B^{dch} can be described as:

$$0 \leq B_t^{ch} \leq B^{ch}(\max),$$
$$0 \leq B_t^{dch} \leq B^{dch}(\max). \tag{7}$$

And the battery current state can be derived as follows:

$$B_t = B_{t-1} + \eta^{ch} B_t^{ch} - \eta^{dch} B_t^{dch},$$
$$B_t(\min) \leq B_t \leq B_t(\max), \tag{8}$$

where the battery current state B_t is related to the previous state B_{t-1}, and is also related to the amount of charged or discharged energy. Moreover, only one operation can be performed between the charging and discharging process at the same time. Accordingly, we impose the following constraint:

$$\lambda_t^{ch} + \lambda_t^{dch} \leq 1, \quad \forall \begin{cases} \lambda_t^{ch} = 1, & \text{if} \quad B_t^{ch} \neq 0, \\ \lambda_t^{dch} = 1, & \text{if} \quad B_t^{dch} \neq 0, \end{cases} \tag{9}$$

where we define the variables λ_t^{ch} and λ_t^{dch} as the binary variables for the purpose of charging and discharging operation constraints. Thus, we can ensure that only one battery storage operation takes place.

Therefore, household power consumption from the grid is given by

$$P_t = P_t^{PV}(\psi^{PV}) + P_t^d(\psi^d) + B_t, \tag{10}$$

and reactive power can be derived from this real power and power factor $\cos\theta$ as follows:

$$Q_t = P_t \sqrt{1/\cos^2\theta - 1}. \tag{11}$$

Then, the customer's electrical cost is given by

$$G = \mathbb{E} \sum_t C_t \cdot P_t. \tag{12}$$

Also, the constraints introduced previously and the following power limit should be taken into account:

$$P(\min) \leq P_t \leq P(\max). \tag{13}$$

B. Operator Model – Upper Level

For operators, the problem is to set the optimal price and indirectly control the usage period of the customer's load demand, to achieve minimum system loss and maintain system stability. In this work, we consider linear power flow analysis [14] for analytical tractability. And the power loss is chosen as the system cost, since other costs such as investment and maintenance costs are typically charged as a fixed rate, which does not affect the results. The piecewise linearized power loss [15] between node i and node j is given by

$$P_{ij}^L = (G_{ij}/B_{ij}^2) \sum_{k=1}^{K} \lambda(k) \Delta P_{ij}(k), \tag{14}$$
$$\lambda(k) = (2k-1) P_{ij}(\max)/L, \tag{15}$$

where G_{ij} and B_{ij} are transmission line conductance and admittance from node i to node j, respectively. And P_{ij} refers to the power flow from node i to node j. The basic idea of the linearized line loss modelling is to approximated the loss by K linear sections.

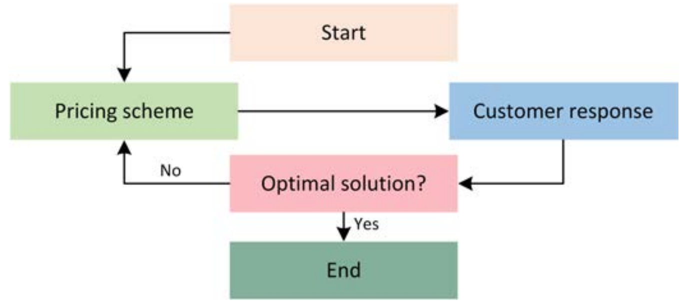

Fig. 2: Basic structure of the proposed problem.

Therefore, operator's system cost can be formulated as:

$$F = \sum_{n,t} C_t \cdot P_{n,t}^L + \mathbb{E}\Big(\sum_{n,m,t} C_t \cdot P_{n,m,t} \Big), \tag{16}$$

where the electrical price C_t refers to the optimal pricing as the decision variable for the operator's model. And the following constraints are considered:

$$P_{n,t} = \sum P_{m,t}, \tag{17}$$
$$P_{n,t}(\min) \leq P_{n,t} \leq P_{n,t}(\max), \tag{18}$$
$$V_{n,t}(\min) \leq V_{n,t} \leq V_{n,t}(\max). \tag{19}$$

Here, $P_{n,t}$ refers to the node power, which can be achieved from the house power consumption $P_{m,t}$. The following two constraints (18) and (19) refer to the nodal power limits and nodal voltage limits, respectively.

III. The Proposed Stochastic Demand Response Scheme

We formulated the optimal pricing problem under customers random load demand as a bi-level stochastic programming problem as follows:

$$\min_C F = \sum_{n,t} C_t \cdot P_{n,t}^L + \mathbb{E}\Big(\sum_{n,m,t} G_m \Big), \tag{20}$$
$$\min_B G_m = \mathbb{E} \sum_t C_t \cdot P_t, \tag{21}$$
$$\text{s.t. } (1) - (12),\ (13) - (15),\ (17) - (19), \tag{22}$$

where the decision variable of the operator objective function is the electrical pricing scheme C, and for the customer objective function it is the energy storage charging/discharging process B.

As we can see, the formulated problem is linearly constrained in both upper level and lower level. Therefore, we implemented the simplex algorithm for the upper level optimization, and mixed-integer linear programming for the lower level. Since there exists a large amount scenarios in household random load consumption model, we implement parallel computing to accelerate the algorithm. The details are shown in Algorithm 1 and a flowchart shown in Fig. 2.

Algorithm 1 Parallel stochastic demand response scheme

procedure UPPER LEVEL
  for iteration $i = 1$ **do**
    for $t \in T$ **do**
      Generate renewable power P_t^{PV} by (3)
      Publish pricing scheme for users
      procedure LOWER LEVEL
    end for
    Evaluate the results using (20) and
    if feasible **then** STOP
    else Go to the next iteration, $i = i + 1$
    end if
  end for
procedure LOWER LEVEL
  for house $m \in M$ **do**
    Calculate house load demand distribution for each t from equation (6), and generate the home demand scenario set
    Parallel computing optimal solution (12) for each scenario using MILP and saving the results
  end for

Once the amount of renewable power generation is determined, our proposed pricing scheme can be utilized to find the optimal operations for both operators and customers. Moreover, customer's optimal operation under the published pricing scheme considers all potential scenarios that are derived from the distribution of household probabilistic load demand.

IV. NUMERICAL RESULTS

In this section, we evaluate the proposed pricing scheme under stochastic demand response based on IEEE 33-bus test distribution system. The one-line diagram is shown in Fig. 3, and the system parameter can be found in [16]. The simulations are implemented with MATLAB linear programming toolbox on a Windows desktop with an Intel i7-4790 CPU at 3.60 GHz with 16 GB RAM.

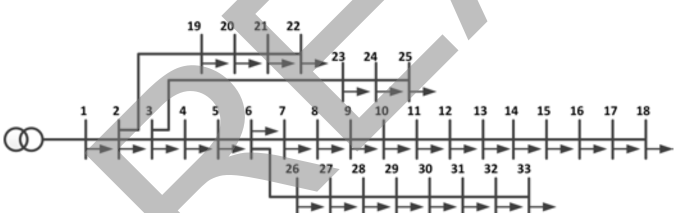

Fig. 3: One-line diagram of IEEE 33-bus test distribution system.

In this simulation, the home energy system consists of a 5 kW rooftop PV system, a 6 kWh energy storage system and several kinds of electrical devices. Specifically, the maximum and minimum energy storage states are 600W and 5400W, respectively. The typical home appliance parameters can be found in [17]. We assume that PV power is randomly generated by Monte Carlo simulation to simulate the real cases. Therefore, the optimal pricing for each simulation varies, and we use Time-of-Use (TOU) price in Ontario for pricing scheme comparison. The details of TOU price is introduced in TABLE I. In addition, the minimum costs of the operator and the customers are compared under these different pricing schemes. For the voltage magnitude, the lower and upper limits are 0.96 $p.u.$ and 1.04 $p.u.$, respectively.

TABLE I: Time of Use price, Ontario (2018 – 2019).

	Period	Price
Peak hours	11:00 – 17:00	13.2 ¢/kWh
Mid-peak hours	17:00 – 19:00, 7:00 – 11:00	9.4 ¢/kWh
Off-peak hours	19:00 – 7:00	6.5 ¢/kWh

Firstly, we test our proposed home energy storage system with the TOU price, and the results are shown in Fig. 4. In this figure, we compare our proposed method with the highest probability method, the random scenario selection method, and the scenario selection technique proposed in reference [7]. The blue line indicates the average cost by implementing the average value of home load demand instead of scenarios. The proposed scenario based algorithm and other comparison methods correspond to the red lines in the results. Furthermore, the daily total electricity cost are: 9.667\$, 11.637\$, 11.322\$, 10.269\$, respectively.

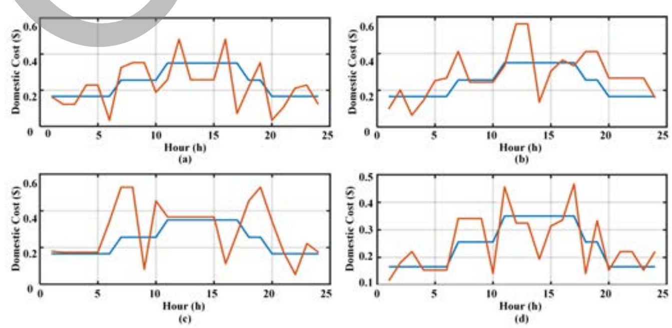

Fig. 4: A comparison of the domestic cost: (a) The proposed method; (b) The highest probability method; (c) The random scenario selection method; (d) The scenario selection technique proposed in reference [7].

As we can see, our battery storage system can effectively reduce the peak-hour cost in comparison with the other methods. The voltage profiles of these cases are shown in Fig. 5. From the figure we can see that our proposed algorithm can improve the voltage profile, while some other scenario selection methods may violate the lower limit of voltage requirement (0.95 p.u.). The power loss versus iterations is shown in Fig. 6. It can be observed that all the methods can converge to the optimal value within 30 iterations, and our proposed method converges faster. It is worth noting that the convergence of the active power loss also indicates that the proposed pricing scheme converges to the optimal value under bottom-up stochastic models of both residence electrical appliances and PV power generation, which is different from the existing research works.

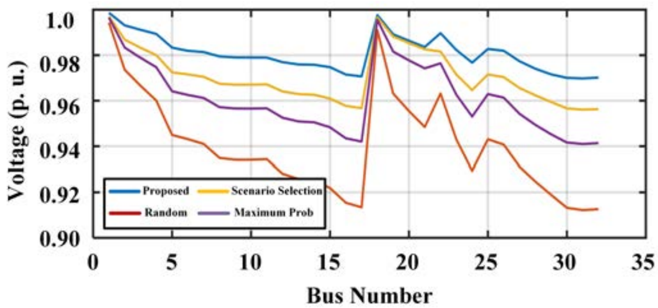

Fig. 5: Voltage profiles obtained based on different methods.

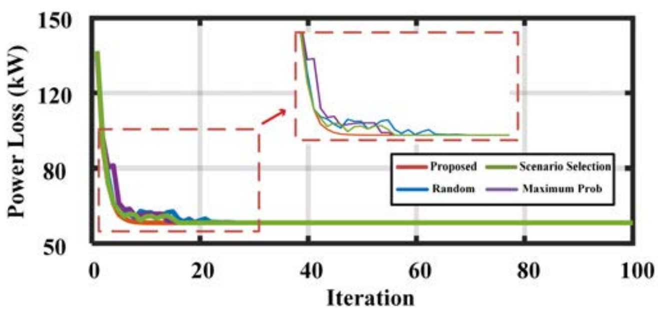

Fig. 6: Convergence of power loss based on different methods.

To further demonstrate the effectiveness of the proposed parallel process, several cases with different numbers of appliances are designed to compare the proposed parallel and sequential computing. The results are shown in the TABLE II. The number of home electrical appliances and their corresponding scenarios are also listed in this table. We can observe that for a single core, when the number of scenarios is small, the execution time is shorter than multi-core parallel computing. However, as the number of appliances increases, the execution time of the single core sequential process is dramatically increased compared to the parallel process.

TABLE II: Execution time (s).

Number of appliances	4	6	8	12	24
Number of scenarios	16	48	83	198	520
Sequential process (s)	97.33	296.41	528.75	1294.46	3525.66
Parallel process (s)	309.51	390.04	467.77	735.38	1046.52

V. Conclusion

In this paper, we propose an optimal pricing scheme under user's random load demand to achieve the optimal demand response in the smart grid. In the residence model, PV power generation, household appliances and energy storage unit are considered. Besides, PV power and electrical appliance demand are modelled based on a probabilistic model. The simplex and MILP algorithms are utilized to find the optimal pricing scheme under user's random demand response, while parallel computing technique is embedded in the algorithm to accelerate the computational process due to a large amount random scenarios. The proposed method has been evaluated through the simulations. Comparing several scenario selection technique that implemented in most research works, the proposed scheme is more effective and efficient in terms of cost reduction and voltage regulation.

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