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A STUDY OF THE RELATIONSHIPS OF ABILITY, PERSONALITY, AND MOTIVATION FACTORS WITH ACHIEVEMENT IN CALCULUS VIA TWO INSTRUCTIONAL TREATMENTS

by

© PATRICK NEIL WEBBER

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled "A Study of the Relationships of Ability, Personality, and Motivation Factors with Achievement in Calculus via Two Instructional Treatments," submitted by Patrick Neil Webber in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Secondary Education.

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ABSTRACT

The main purpose of the study was to examine the importance of student aptitude, personality, and motivation factors with respect to calculus achievement scores in each of two learning settings--independent study and lecture. The secondary purpose was to compare the independent study and lecture methods in terms of student achievement scores and dropout ratios.

Two classes of first-year university transfer students at Mount Royal College participated in the experiment. The students were randomly assigned to their classes, resulting in 67 students in the independent study group and 62 students in the lecture group. At the beginning of the 1973 spring semester, the students were administered a test battery consisting of aptitude tests, a personality questionnaire, and motivation scales. Calculus achievement scores were obtained during the semester using four term tests, a final examination, and a standardized calculus test. Stepwise multiple regression, Pearson correlation, and *t* test procedures were carried out on the data collected to test the major hypotheses of the study.

The results of the study were as follows:

1. Aptitude variables played the most important predictive role with respect to calculus achievement scores in the lecture group;
2. Motivation and personality variables were the most important predictor variables in the independent study group;

3. An aptitude variable was the most important factor distinguishing the lecture dropout from the lecture non-dropout;

4. A personality factor was the most important factor distinguishing the independent study dropout from the independent study non-dropout;

5. There was no significant difference in the final examination and the standardized test mean scores between the two groups; and

6. There were significantly more dropouts in the independent study group.

The main conclusion of the study was that there were identifiable student characteristics that were related to achievement in calculus via the two instructional methods used.

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CHAPTER I

THE PROBLEM

INTRODUCTION

During the past decade educators have given increased attention to the areas of mathematics curriculum development and methodology, not only in our schools but in our colleges and universities as well. Particular consideration has been given to individualized instruction. The causes of the attention to individualized instruction in mathematics may be attributed to the change in our view of education, the increasing importance of mathematics in the world today, and, in part, to student dissatisfaction with university and college education.

View of Education

For hundreds of years education has been viewed as a selection process. At certain stages of the educational program, those learners who met certain criteria were allowed to continue to the next stage. Those who failed at the different stages were essentially ignored by educators. These "failures" were to find their appropriate niche in society, while the "graduates" of the educational system were to become the professional elite and the contributors to society.

Education was conceived as a set of learning experiences which presumably became more difficult as the learner proceeded through his schooling. It was assumed that fewer and fewer learners had the necessary gift of intelligence to succeed as they moved from the lower grades to the higher grades and from high school into college.

Today, many of the countries of the world are demanding more and more education for their citizens. The reasons for these demands are not clear. Increased economic growth, increased social and occupational mobility, and desire to enlarge our knowledge of nature and society and to strengthen our culture have been given as reasons. However, the Commission of Post-Secondary Education in Ontario (1972) gave evidence to indicate that it is difficult to discover any single set of objective social, economic, or demographic criteria that can be said to account for the rapid growth in the area of post-secondary education. The commission concluded that we are ". . . left only with the unsatisfactory and unsatisfying conclusion that post-secondary education seems to expand when a society desires it to be expanded" (1972:2). Whatever the reasons, societies are demanding more education for their citizens and consider any educationally disadvantaged segment of society as a source of social problems. Bloom (1971) indicated that, in the United States, about 75 per cent of the 18-year olds were finishing high school and, in Japan, about 60 per cent of the 18-year olds were completing secondary education. In Canada, nearly 60 per cent of the 18-year olds are completing high school. More and more of these high school graduates are moving into colleges and universities.

In Canada, our society is insisting upon educational institutions that will serve a broad spectrum of the community needs. Some of these needs have been outlined by Fisher (1967) and Kristjanson (1969). It appears that the educational ideal is to provide <u>all</u> citizens an opportunity to obtain an education. At present, many Canadians seem to be much attracted to the relatively new post-secondary educational institution called the community college. In 1972, there were approximately 130 Canadian community colleges serving a total student population of approximately 100,000. The provinces of Quebec, Ontario, Alberta, and British Columbia have shown remarkable growth in the community college movement. The result is that colleges and universities, particularly community colleges, are enrolling ever more heterogeneous groups of students. This heterogeneity exists both in the domain of aptitude and motivation. It is seen as the task of the institution to maximize for each of this broad spectrum of students his ability to function effectively in a complex society.

This change in the view of education poses many problems in the general area of curriculum development and methodology. Educators are looking at alternate methods of instruction, particularly individualized instruction, to help as many students as possible learn to cope effectively with their environment.

Importance of Mathematics

Mathematical research is advancing so rapidly that the quantity of known mathematics is doubling approximately every fifteen years, which implies an eight-fold increase in the working lifetime of today's students. In the past fifty to seventy years man has seen the fantastic development of subjects in mathematics, such as topology, theories of integration, functional analysis, measure theory, abstract algebra, probability and statistics, and game theory. These subjects have found important and extensive applications in the physical and engineering sciences, biological sciences, and in the social sciences. Game theory is a mathematical theory of games of strategy and has applications in economics, war, and the study of human relations. "Linear programming" has provided an important tool for more efficient management of large-scale industrial and governmental operations. "Operations research" employs many mathematical and statistical methods and is used by industrial firms to make their operations more efficient and more productive. "Quality control" uses statistical techniques for efficient control of quality in large-scale manufacturing processes.

These changes in the importance of mathematics and the effects of the electronic computer and automation on our lives are almost beyond our comprehension. Mathematics has become basic to the fabric of our social order. Thus, it has become necessary for greater numbers of people to learn mathematics. This places demands upon research in curriculum and instruction so that these increased numbers may achieve the objectives of mathematics instruction.

Student Dissatisfaction

Student dissatisfaction with university and college education has reached crisis proportions. More and more students are becoming unhappy with impersonal mass lectures and irrelevant course content. At the same time, little encouragement seems to be offered to faculty for high-quality teaching and for the development of efficient learning methods. Academic rewards continue to be almost exclusively for discipline-related research contributions. McLeish (1968) indicated that the student discontent has helped educators focus attention on curriculum development and methodology.

Hopefully, this study has been a small contribution to the revolutionary cause of improving instruction in the area of college mathematics. The study was concerned with mathematics curriculum development and methodology in the teaching of calculus. Two modes of instruction were used--the traditional lecture method and an independent study method. The purpose of the investigation was to assess the relationships between student characteristics and mathematics achievement in each of the instructional modes.

BACKGROUND TO THE PROBLEM

In the past few years many educators at the community college level in Canada and the United States have concerned themselves with individualized instruction and other instructional techniques. They have known that there are great differences in how each student learns, and have recognized the need for instructional systems which can make higher education available to large numbers of students and, at the same time, offer an individualized learning experience. Still, few modifications have departed significantly from the traditional lecture approach to instruction. This is not to say that the lecture approach is ineffective. However, educational research (see R. and D. Perspectives, 1968) has demonstrated that other instructional methods, such as programmed instruction, computer-assisted learning, audio-tutorial, and independent study methods are at least as effective. Some students, according to Bloom (1968), can learn through independent learning experiences while others need highly structured situations.

We have to recognize that no <u>one</u> instructional method will prove effective for <u>all</u> students. Alternative learning methods need to be developed, and, as Goldschmid (1969) said, these instructional methods ". . . need to be optimally matched with student characteristics including personality, prior learning, interests, and aspirations" (1969:4).

In the past (and frequently today) adapting instruction to individual differences meant fixing the curriculum and method of instruction and adjusting through initial selection and through allowing for dropouts. Cronbach and Snow (1969) stated that a more responsible method of adapting is to choose different educational modes. They stated: "Our concern, then, is with adaptations in method that will fit instruction to the relevant characteristics individuals bring to the classroom" (1969:175). They could see no short-term solution to the problem of individual differences "save artistic design of alternative instructional schemes" (1969:177). The long-term requirement is for an understanding of the factors that cause a student to respond to one instructional plan rather than another.

THE PROBLEM

The main purpose of the research was to examine the importance of student characteristics with respect to calculus achievement scores in each of two learning settings. It was surmised that student characteristics, including student aptitude, personality, and motivation factors, may be related to calculus achievement scores. With respect to the main purpose of the study, the following questions were asked:

1. What student characteristics can be used to predict calculus achievement scores in a lecture setting?
2. What student characteristics can be used to predict calculus achievement scores in an independent study setting?
3. What are the characteristics of the dropouts in the lecture setting?
4. What are the characteristics of the dropouts in the independent study setting?

These questions related to each setting separately and were not comparative in nature.

The secondary purpose of the study was to compare the independent study and lecture methods in terms of student achievement scores and dropout ratios. The specific questions asked were:

5. Is there any difference in the mean achievement scores of the independent study and lecture groups?
6. Is the dropout ratio of the independent study group different from the dropout ratio of the lecture group?

DEFINITION OF TERMS

1. Mastery learning. Learning defined in terms of the achievement of specific sets of objectives.

2. Independent study. An expression most difficult to describe since it can take on many different forms. In this study, the term was used in the following sense: A teacher-structured study program designed for mastery learning with each student having independence

in terms of designing his or her own study times and in the choice of instructional aids. The terms "independent study" and "individualized instruction" were considered synonymous.

3. Formative evaluation. Diagnostic testing used to provide immediate and frequent feedback to the student regarding his progress during instruction.

4. Summative evaluation. Testing used to assess the students' achievement at the end of instruction.

OUTLINE OF THE REPORT

The present chapter serves to stress the factors that make it necessary for college educators to focus their attention on curriculum development and on the methodology of teaching mathematics, gives a background to the problem and need for adapting instruction to student characteristics, and outlines the problem. Chapter II is devoted to a review of the literature relevant to this study. The design of the study, outlining the instructional settings and the student characteristics used, is presented in Chapter III. Chapter IV includes the analysis of data and the results of the investigation. The summary, conclusions, and recommendations are found in Chapter V.

CHAPTER II

REVIEW OF THE LITERATURE

The central purpose of this study was to assess the importance of student characteristics with respect to calculus achievement scores when an independent study approach and a lecture approach to instruction are used. The independent study method used in this research is based upon features of the mastery learning approach to instruction. Thus, in the first part of this chapter, the history and theory of mastery learning is reviewed along with a review of the lecture method. A description of research findings related to these learning approaches is also included since the secondary purpose was to compare the independent study and lecture methods with respect to achievement scores and dropout ratios.

The latter part of this chapter gives a review of the literature pertaining to the relationships between student characteristics and academic achievement. This review is considered under the three general areas of aptitude or intellectual factors, personality factors, and motivation factors. The literature related to these areas is treated separately from the review associated with mastery learning and the lecture approach for a specific reason. The history and development of the research related to the three areas was concerned with academic achievement in general and was not primarily associated with the relationship of student characteristics with methods of instruction. The review of the literature associated with the student characteristics is generally too broad to be classified under either method of instruction used in this study. Thus, a perspective of the relationships between student characteristics and academic achievement is given in the latter part of this chapter. The study of the relationships between student characteristics and calculus achievement scores when learning occurs via the independent study approach or the lecture approach is reported in the following chapters.

MASTERY LEARNING

History of Mastery Learning

The idea of learning for mastery is not new. In the 1920's there were two major attempts to produce mastery in students' learning. One attempt was led by Carleton Washburne in 1922 and the other by Henry C. Morrison in 1926 at the University of Chicago's Laboratory School. The major features of the approaches of Washburne and Morrison as summarized from Block (1971a) were:

1. Mastery was defined in terms of particular educational objectives each student was expected to achieve.
2. Instruction was organized into well-defined learning units.
3. Complete mastery of each unit was required of students before proceeding to the next.
4. An ungraded, diagnostic test was administered at the completion of each unit to provide feedback on the adequacy of the students' learning.
5. On the basis of this diagnostic information, each student's original instruction was supplemented with appropriate

learning correctives.

6. Time was used as a variable in individualizing instruction.

These early attempts at mastery learning disappeared in the 1930s "due to the lack of technology required to sustain a successful strategy" (Block, 1971a:4). The idea arose again in the late 'fifties and early 'sixties as a result of the popularity of programmed instruction. It was in 1963 when John B. Carroll published a paper entitled Model of School Learning that provided a useful model for mastery learning. It is important to note that Carroll did not view his model as a "learning theory" but as a description of the "economics" of the school learning process; it took the fact of learning for granted. His model outlined the major factors influencing student learning and indicated how these factors interact. It was Bloom (1968) who transformed this conceptual model into an effective working model for mastery learning.

Block (1971a) felt that the most recent approaches to mastery learning have advantages over previous efforts in two important respects:

1. The feedback instruments are much improved; and
2. A greater variety of instructional correctives are used.

Theory of Mastery Learning

In 1963 Carroll's model was based on the concept of a learning task which he defined as:

> The learner's task of going from ignorance of some specified fact or concept to knowledge or understanding of it, or of proceeding from incapability of performing some specified act to capability of performing it. [1963:723]

Carroll assumed that most learning objectives can be expressed in the form of learning tasks or a series of learning tasks, each of which can be described (behavioural objectives) and that means can be found for making a valid judgment as to when the learner has accomplished the learning task.

Basically, Carroll's model claimed that the learner will succeed in learning a given task to the extent that he spends the amount of time that he needs to learn the task. If the learner is not allowed enough time, the degree to which he could be expected to learn is a function of the ratio of the time actually spent in learning to the time needed:

$$\text{Degree of learning} = f\left(\frac{\text{time actually spent}}{\text{time needed}}\right)$$

The model proposed that the <u>time needed</u> is a function of:

1. <u>Aptitude for learning</u>--measured by the amount of time needed to learn the task under optimal conditions.
2. <u>Ability to understand instruction</u>--measured by some combination of "general intelligence" and "verbal ability."
3. <u>Quality of instruction</u>--the degree to which the presentation and ordering of the learning task's elements approached the optimum for each student.

The model further proposed that the <u>time spent</u> is a function of:

1. <u>Time allowed</u> for learning.
2. <u>Perseverance</u>--the time the learner is willing to spend in learning.

Thus, the Carroll model can be summarized by the following relation:

$$\text{Degree of learning} = f\left(\frac{\text{1. Time allowed} \quad \text{2. Perseverance}}{\begin{array}{l}\text{3. Aptitude} \quad \text{4. Quality of instruction}\\ \text{5. Ability to understand instruction}\end{array}}\right)$$

In his 1968 paper, Bloom considered the variables of Carroll's model and the ways these variables may be used in a strategy for mastery learning.

> A learning strategy for mastery may be derived from the work of Carroll (1963), supported by the ideas of Morrison (1926), Bruner (1966), Skinner (1954), Suppes (1966), Goodlad and Anderson (1959) and Glaser (1968). [Bloom, 1968:3]

Bloom argued that, if students are normally distributed with respect to aptitude for some subject and if they are provided the same instruction in terms of amount, quality, and learning time, achievement would be normally distributed. Also, the relationship between aptitude and achievement would be high. However, if each learner is provided with optimum learning conditions in terms of kind and quality of instruction and the learning time he required, then the majority of students could be expected to achieve mastery. The relationship between aptitude and achievement should approach zero. Thus, to achieve mastery fix the degree of learning and manipulate the other variables in Carroll's model.

Bloom proposed a strategy for mastery learning where the time for learning is relatively fixed, i.e., semester or period of calendar year in which the course is usually taught. Basically, the strategy consisted of the following:

1. Define mastery in terms of a specific set of objectives the student is expected to achieve.
2. Divide the course into small learning units (one or two weeks' instruction) with unit objectives.
3. Use teaching methods appropriate to learner needs.
4. Prepare short diagnostic tests (formative tests) to determine

whether the student has mastered the unit and what, if anything, the student must do to master it.

5. Prepare tests (summative tests) to appraise student's competence with regard to content and objectives.

The remainder of this section will be devoted to an analysis of Bloom's strategy for mastery learning and of the variables in Carroll's model. Since Bloom's 1968 publication extensive mastery learning research has been carried out and some of this will be reviewed as well.

<u>Aptitude</u>. It is clear that some students have characteristics that give them special advantages over other students in learning some tasks. Whether or not these "aptitudes" are innate gifts or the result of previous training has been a controversial issue. It seems that some "aptitudes" are the result of endowment but at the same time, the rate of learning tasks can be altered by environmental conditions or prior learning experiences.

> The amount of time that a student needs to learn a given task under optimal conditions is a reflection of some basic characteristic or characteristics of the student that may be called "aptitude." Often one can use various tests and other indicators to predict learning time and the use of aptitude predictors will sometimes help in dealing with variations in learning time. Learning time for a given task is often a complex function of a number of basic aptitudes--verbal ability, memory ability, spatial ability and so forth as they have been identified by factor analysis studies. . . . [Carroll, 1970:31]

How does one measure "rate of learning under optimal conditions"? Carroll (1970) said that the true amount of time that a student needs to learn something is a variable that cannot be observed directly since it assumes that the student is well-motivated and that instruction is optimal. However, one can measure the time to reach criterion. He also reported that there is some evidence that aptitude interacts with the kind of instruction offered. Davis (1967) found that certain factors of Guilford's "structure of intellect" interacted with type of instruction in mathematics such that students with good abilities in "cognition of semantic classes" were much better off when they were taught with "semantic methods" as opposed to "symbolic" methods. Conversely, students with good "symbolic" abilities were much better off under instruction that stressed symbolic content. One of the major research problems now is to identify those aptitudes most relevant for a given learning task.

As far as teaching for mastery learning is concerned, the teacher must recognize the differences in the learning rates of students and allow for these differences in the instructional process.

Ability to understand instruction. Carroll isolated this variable from those considered under "aptitude." He assumed it to interact with the quality of instruction. As one may expect, Block (1971a) claimed that research evidence indicates that students with a high ability to understand (as usually measured by verbal intelligence tests) and profit from instruction are little affected by variations in quality of instruction, while students with low ability to understand instruction are much more affected by variations in quality. Thus, the different instructional modes should be used in such a way so as to take advantage of the student's non-verbal abilities (i.e., spatial, symbolic). There is little evidence, however, that this would produce significant changes in student behaviour.

<u>Quality of instruction</u>. There are many aspects of "quality of instruction" but Carroll (1970) emphasized those that have to do with sequencing the order of presentation of material from simple to more complex, with making sure each stage is properly mastered before the next one is taken up, and with making sure that the pupil understands exactly what the objectives of instruction are. He also listed the quality of the instructional materials, the teacher's knowledge of the subject matter, and ability to diagnose student difficulties as important factors.

In a summary of mastery learning research pertaining to the quality of instruction, Block (1971a) stated:

> The research results suggest that quality of instruction is best defined by (a) the clarity and appropriateness of the instructional cues for each student; (b) the amount of participation in and practice of the learning by each pupil; (c) the amount and types of reinforcements given to each learner.
>
> . . . .
>
> Related to these variables are the findings that the variety of instructional modes and materials, teacher verbal ability, the type of feedback available to both teacher and student, and the frequency and variety of teacher reinforcements are all predictive of student achievement. [1971a:93]

Obviously, it would be extremely difficult to establish a measure of the quality of instruction.

<u>Perseverance</u>. This, as previously defined, is the time the learner is willing to spend in learning. Carroll (1963) surmised that perseverance is a function of many variables including the desire to learn, frustrations encountered in learning, prior experience of success or failure with similar tasks. It seems reasonable that frequent feedback and positive reinforcement leads to greater perseverance.

Time. In many college courses material is presented at such a rapid pace that only those students with the "higher aptitudes" are able to keep up while the others fall behind, sometimes never to catch up. It does not follow that mastery learning is the result of allowing students sufficient time. However, if they are not allowed the time their learning will be incomplete. If a course is presented as a sequence of learning units, Merrill, Barton, and Wood (1970) indicated more time than usual may have to be allowed for early units for corrective and review purposes but the time spent here usually pays off in terms of less time for later units.

Evaluation. In mastery learning the role of the teacher is that of a "manager." His role is to specify what is to be learned (objectives), provide students with instructional materials, administer instruction at a rate suitable to each student, motivate the student, monitor students' progress, diagnose deficiencies in the students' learning, provide remediation for them, and provide review and practice. Thus, one of the more important aspects of a mastery learning strategy is that of evaluation.

Airasian (1971) claimed that an effective mastery strategy requires two types of evaluation--formative evaluation and summative evaluation. Summative evaluation is directed toward an assessment of the students' achievement of the course objectives or some substantial part of the objectives. Summative examinations usually occur infrequently, two or three times for one course, and focus more on the "broader" objectives of the course. Those students who attain a predetermined mastery level receive an A grade (or equivalent) and those who fail to attain the mastery level receive appropriately lower grades. Bloom et al. (1971) indicated how the technical characteristics of validity, reliability, and scoring objectively need to be carefully considered when using summative testing.

The term "formative evaluation" was first used by Scriven (1967) in connection with curriculum improvement. In his view, formative evaluation involved the collection of data during the construction and trying out of a new curriculum in such a way that revisions of the curriculum could be based on this data. However, formative evaluation has now been extended for use in the teaching and learning process for the purpose of improvement of that process. The most important use of formative evaluation in mastery learning is in providing immediate and continuous information regarding a student's progress. This information can be used to direct or correct subsequent learning. Airasian (1971) believed formative evaluation should occur frequently so as to identify unmastered objectives early enough to permit their correction before the summative evaluation. This is especially important in those sequential courses where objectives learned in the early stages of instruction form the basis for learning later objectives.

One technique of formative evaluation is to evaluate all the objectives of a unit in terms of mastery or non-mastery. Bloom et al. (1971) have used accuracy levels of 80 to 85 per cent on each formative test as an indication of mastery. Block (1971a) reported a study involving eighth graders where different mastery levels were established for six different groups. Each group had a pre-established level of mastery. These were 0, 65, 75, 85, and 95 per cent mastery. The results are rather interesting. First, the 95 per cent mastery level group produced maximal cognitive learning (achievement, transfer and retention) but, had long run negative effects on student interest and attitudes. Maintenance of the 85 per cent level produced maximal interest and attitudes, but slightly less than optimal cognitive learning.

Airasian (1971) made the important point that the formative test should be scored in terms of item response patterns so that each unmastered objective can be identified for the purpose of the student. That is, the student must be able to recognize from his marked formative test those objectives he has not mastered and what he needs to do to master those objectives.

What corrective procedures can be used to help a student master the "unlearned" objectives? There is a wide variety of instructional correctives that can be used, such as:

1. individual tutoring,
2. small group sessions, and
3. alternative learning materials.

Probably the most effective method is individual tutoring, if it is feasible in terms of cost and time. At the college level, it is possible to use para-professionals and/or paid student tutors. The small group sessions usually involve two to four students, with or without the teacher, who meet at specific times to discuss specific learning problems. For those students who have the perserverance and interest to correct their learning problems themselves, the use of alternative learning materials can be effective. These alternative materials can involve alternative textbooks, programmed workbooks, audio-visual materials, such as motion pictures, film strips, cassette tapes, and games.

Previous research has indicated that the use of feedback/correction procedures can significantly improve student achievement. One study worth noting is that of Collins (1970), who investigated the effectiveness of the different variables in Bloom's mastery learning strategy for teaching modern mathematics at the grade eight level. The course was divided into units with a list of objectives for each unit. Each list indicated the objectives to be covered per class session and assignment. Group A was given only the lists of objectives. Group B was given the lists plus a problem during each session testing the objectives covered in the previous session and assignment. After working on the problem, it was discussed and questions were answered. Specific corrective directions were then provided for using the textbook, notes, and handouts to learn the objectives not mastered. Group C received the lists of objectives, the diagnostic problems, and the review prescriptions. In addition, learning resources such as textbooks, workbooks, games, and SRA instructional kits were given. Group D received only the problems and review prescriptions. Group E received only the problems. Group F received none of the extra materials and had only their classwork and assignments. All classes were given the same tests based on the objectives. Of Groups B and C, 80 per cent achieved mastery (Grade A or B). Groups D, A, and E had 70, 60, and 50 per cent respectively achieved mastery. Collins suggested that these results indicate the great importance of specifying objectives and the major effects that diagnostic testing can have on student achievement when used with a corrective prescription.

The use of criterion-referenced tests (comparing students' performance in terms of predetermined standards) has created recent controversy. (Formative tests may be classified as criterion-referenced.) Ebel (1971) attacked not only criterion-referenced tests but mastery learning as well. His arguments against criterion-referenced tests were:

1. They do not tellus all we need to know about achievement.
2. They are difficult to obtain on any sound basis.
3. They are necessary for only a small fraction of important educational achievements.

Block (1971b) defended the use of criterion-referenced tests and pointed out weaknesses in Ebel's arguments. In his paper, Block emphasized the importance of criterion-referenced tests and objectives in the feedback/correction aspect of master learning:

> The findings to date suggest that prespecified instructional objectives provide a key to maximally effective classroom instruction when put into operation in the form of criterion-referenced measurements for use in a feedback/correction system. [Block, 1971b:294]

<u>Affective consequences of mastery learning</u>. Bloom (1971b) traced the process by which the present educational system destroys a large number of students' interest in learning, creates in the student a negative attitude toward himself and the school, and infects a number of students with mental health problems. Both Block and Bloom indicated that research findings reveal a clear relationship between a student's

academic performance and both his self-concept and his mental health. They also claimed that if a student can be provided with successful experiences in a given task then his confidence in his ability to perform similar and related tasks will increase. Further continued success over a long period of time may constitute a form of immunization against anxiety and emotional problems. Block (1971a) claimed that research shows that mastery learning methods produce

> . . . greater interest in and better attitudes toward the material learned than more conventional approaches. They seem to help most students overcome feelings of defeatism and passiveness brought to learning. Their powerful affective consequences may be attributed to many factors, the most important of which seem to be the co-operative rather than the competitive learning conditions, successful and rewarding learning experiences, personalized attention to each student's learning problems and the use of certain correctives (for example, student tutors and small group study sessions) which add a personal-social aspect to the learning not typical of group-based instruction. [1971a:97]

The independent study method used in the present study adopted mastery learning features and is described in Chapter III.

THE LECTURE METHOD

The lecture method has been the basic method of instruction in higher education for centuries. Most edueators have been brought up with the lecture and many are disenchanted with the method as the only method for all courses, all students, and all teachers. However, the lecture has survived and has advantages as well as disadvantages as a mode of instruction.

The traditional lecture method is usually considered as the approach to instruction whereby the instructor gives a verbal presentation to a passively receptive audience. Student participation, if any at all, is generally limited to asking questions of the lecturer. There can be variations in the amount of student-teacher discussion in a lecture setting. Most lecture settings in undergraduate mathematics programs are supplemented by a weekly group tutorial session. This tutorial session can take the form of a teacher meeting with a subset of the students to answer questions and help students with the solution of problems.

McLeish (1968) questioned the worth of the lecture approach when he wrote:

> The new orientation of the universities towards technology and modern science raise doubts about the efficiency of the transmission of academic materials by the traditional method of an uninterrupted discourse, often without benefit of rest pauses, of variety of presentation and pace, of visual materials--sometimes devoid of human warmth and intelligibility. [1968:3]

McLeish also indicated that lecturing is an <u>art</u> and the lecturer must possess a variety of skills which are not easily attained. One of the main disadvantages is that it pays little attention to the individual differences among the students. Another disadvantage of the lecture is that it tends to encourage undesirable attitudes to learning. McLeish indicated this last point when he said:

> Where there are compulsory lectures buttressed by examination pressures, it is more or less inevitable that students, and to some extent their teachers, learn to regard knowledge as a closed system. [1968:47]

The lecture method does have virtues as a mode of instruction. It can be used effectively in introducing new topics or new material to the student or in motivating the students to a new topic. The lecture can be advantageous in synthesizing previous learned ideas to provide an overview and in relating currently topical events. McLeish stressed this advantage in the following quote:

> By the use of the lecture technique the scholar can readily inspire an audience with his own enthusiasm; he can capture the imagination of his auditors with the relationship of his special field to human destiny and human purposes; he can communicate the latest results of the painstaking efforts of his fellow-scholars on man's present and future estate. The lecture method enables him to achieve these ends with the utmost of means. [1968:47]

McLeish carried out a series of experiments from 1964 to 1966 to determine the effectiveness of the lecture method. In one study on attitudes to methods of instruction he found that older and mature students strongly disfavour the lecture. They favoured more student-centred teaching methods. The academically superior students disliked the lecture method more than other students. The students involved in the study were education students attending the Cambridge Institute of Education. McLeish (1968) reported that research has shown a sequence in student "performance" during a sixty-minute lecture situation. The sequence consists of three steps:

1. An initial spurt which lasts approximately five minutes. Student "efficiency" is at a maximum during this period.
2. A middle sag which clearly results from a combination of boredom and fatigue. This sag reaches its lowest point after approximately forty minutes.
3. An end spurt which continues to improve to the end of the lecture but does not reach the initial level.

He indicated that there is seldom a radical departure from this typical pattern.

With regard to student retention, McLeish reported:

> Students listening to an uninterrupted discourse within their range of understanding and taking notes in their normal fashion, carry away something of the order of 40 per cent of the factual data, the theoretical principles stated, and the general applications referred to by the lecturer. A week later they have forgotten at least half of this material. But there are considerable individual differences, of the order 3:1, between the best and the worst case. [1968:12]

Many of the studies referred to later in this chapter have used the lecture method (or variations of it) as a mode of instruction. Entwistle and Entwistle (1970), King (1972), and Domino (1971) reported results relating student characteristics to academic achievement at the university level when one of the instructional methods used was the lecture method. A study related to effects of students choosing their preferred teaching mode (including the lecture method) is reported here.

Pascal (1971) investigated the effects of offering three instructional options (lecture, lecture-discussion, and independent study) to students in a psychology course. In addition to studying whether or not students did better when given their preferred method of learning, the study investigated the possible differential effectiveness of the three methods on different instructional outcomes. The subjects were 185 students enrolled in a psychology course at the University of Michigan during 1969. The criterion test consisted of items covering the course material and was prepared by the researcher. Two levels of items were included in the test: Level 1 contained items corresponding to the Knowledge and Comprehension objectives of Bloom's Taxonomy, whereas Level 2 contained items corresponding to the Applications objectives of the same taxonomy. The Attitude Toward Psychology Questionnaire, prepared by Carrier, was given to the students at the beginning and end of the course. In addition, the students evaluated a seven-page novel article toward the end of the semester and a score was given for the resulting evaluation. An essential criterion for the evaluation was a succinct integration of course material. Receiving one's preferred option had no significant effect on the criterion outcome; however, those students receiving their preferred option had a more positive attitude towards psychology than the other students. Students in the lecture-discussion and lecture options performed significantly better than the independent study students on the Level 1 items of the criterion test. For the evaluation of the novel article the independent study students scored significantly better than those students in the other two options.

A review of the research relating aptitude, personality, and motivation factors to academic achievement is now given.

APTITUDE FACTORS

Intelligence has long been recognized as a crucial factor in the academic success of man. For about the first thirty years of this century, it was generally accepted (due primarily to the work of Spearman) that one single general factor of intelligence accounted fairly satisfactorily for all the manifestations of human ability. In fact, this still remains the viewpoint of the layman insofar as general intelligence tests and measurements of I.Q. are still widely used. Thurstone (1947) established a set of primary abilities. This did not contradict the concept of a general intelligence as further analysis of his primary factors resulted in a second-order factor of general ability. In more recent years Guilford and associates (see Guilford and

Hoepfner, 1971) have, by the use of factor analytic methods, isolated nearly 100 primary factors of intelligence. Cattell (1963) suggested that the Guilford abilities are relatively specific and combinable to form minor group factors, then major group factors, and then still more basic general factors, forming a hierarchical structure. Cattell has theorized about the existence and nature of two general factors: fluid intelligence and crystallized intelligence. Cronbach and Snow (1969) pointed out that some form of hierarchical structure is endorsed by nearly all recent theorists and that, "at the peak of the system is something called g, or fluid intelligence which is now being distinguished from crystallized; . . . " (1969:53).

Aiken (1971) reviewed recent research associated with the effect of intellectual variables on achievement in mathematics. He stated:

> The view that performance in mathematics depends upon a number of abilities is reflected in prediction studies (e.g., Hills, 1957; Guilford, Hoepfner, and Peterson, 1965; Wampler, 1965). These investigations demonstrated that composites of "factor pure" aptitude measures can improve the prediction of achievement in college algebra and calculus. [1971:202]

Aiken pointed out that recent factor analytic studies of mathematical abilities have resulted in a "modest degree of agreement" in the identification of factors. Some of the factors identified were numerical ability, verbal comprehension, and deductive reasoning.

What is the relationship between abilities and mathematical achievement for different methods of instruction? It is generally agreed that some form of general intelligence factors are strongly related to learning in many instructional situations. For the idea of aptitude-treatment interaction (ATI) it is assumed that, for a student having a particular pattern of abilities, certain techniques of instruction in a particular subject area are more effective than others. Aiken (1971) listed only two clear-cut cases where "disordinal interactions" have been reported. These are the studies of King, Roberts, and Kropp (1969) and Behr (1967). Behr's study involved college students and significant interactions between aptitude factor (figural or semantic) and method of instruction (figural or verbal) were obtained.

Leidtke (1971) reported a study of the relationship of student characteristics and mathematics learning in a self-directed, partially teacher-directed, and teacher-directed setting. The subjects were grade five students from the Edmonton Public School Board: 51 in the self-directed group, 53 in the partially-directed group, and 37 in the teacher-directed group. A criterion test was given at the end of the study and again four weeks later to obtain a measure of initial learning and retention, respectively. Five predictor variables were used. The first was a measure of intelligence based upon the California Short-Form Test of Mental Maturity (Level 2). The second was a measure of reading ability as determined by the Paragraph Meaning Test of the Stanford Achievement Test-Reading (Intermediate I). Two variables (personal adjustment, social adjustment) were measured by the California Test of Personality (Elementary-Form AA). The fifth variable was a measure of socio-economic status as determined by matching the father's occupation against occupations listed in Blishen's Revised Occupational Class Scale. No significant correlations existed between the predictor variables and scores on the initial learning and retention test for the subjects in the self-directed group. Significant relationships existed between intelligence, personal adjustment, social adjustment, reading ability and the initial learning and retention test scores for the students in the partially-directed group. For the teacher-directed subjects, significant correlations existed between intelligence, reading ability, and the initial learning and retention scores. Leidtke concluded that in a teacher-directed setting the most important predictor variable of mathematics achievement is intelligence. However, in the self-directed group, changes in the leadership style of the teacher resulted in different behaviour reactions of the students; even with intellectual ability failing to show up as a reliable predictor of mathematics achievement. Other studies involving aptitude factors will be reported in later parts of this chapter since these factors are studied in conjunction with personality and motivation factors.

In view of the major role aptitude factors usually play in predicting mathematics achievement, selected aptitude factors were a subset of the student characteristics selected in Chapter III. Aiken (1970) recommended that future studies related to predicting mathematical achievement should include non-intellectual as well as intellectual variables since only about half of the variance in mathematical achievement can be accounted for by differences in abilities. Cattell et al. (1966) have gone as far as to suggest that 25 per cent of the total variance may be accounted for by personality factors.

PERSONALITY FACTORS

In the early part of this century the study of personality was of secondary importance to the study of intellectual abilities as far as predicting academic success was concerned. Cattell and Butcher (1968) summarized the approaches that have helped our understanding of personality over the years. The first approach comprises the literary insights of writers who built up a body of intuitive knowledge before psychology appeared as a science in the late nineteenth century. This approach has more recently been absorbed into the psychology and is represented by such psychologists as Allport, Murphy, and Klages. The second approach is a purely experimental attack originating in the laboratories of Wundt in Germany and J. McKeen Cattell in America. Thirdly, the clinical approach made great strides with the work of Freud, Adler, and Jung.

Cattell (relative of J. McKeen Cattell) and Butcher (1968) stated that the experimental approach made relatively little progress until the last twenty years. The reason for the recent success has been the switch to multi-variate experimental work from two-variable experimental work. Around 1930 investigators such as Cattell and Guilford started investigating personality by the methods of correlation and factor analysis that were being used in the study of abilities. In the last few years results have come fairly quickly due mainly to the efforts of Cattell and his associates in America and by Eysenck and his associates in England. However, Eysenck has concentrated more on the needs of clinical psychologists whereas Cattell has aimed at the whole personality sphere in normal adults and children. At present there are some twenty to forty primary factors of personality that have been found. Cattell and Nichols (1972) stated, "Although disputes about the number and nature of primary personality factors in questionnaire data are still rife, there is promise of some agreement about the second order" (1972:187).

Many studies of the relationships between personality factors and academic achievement have been concerned with the second-order factors of neuroticism and extraversion. The results show some conflicting findings but there is a fair amount of agreement that both neuroticism and extraversion are relevant to academic success. Three recent studies that included these two factors follow.

Elliott (1972) investigated the relationship between reading attainment, intelligence, extraversion, and neuroticism in groups of primary children in England. Three groups, totalling 322 students, were involved in the study. The Schonell Graded Word Reading Test and the Moray House Picture Intelligence Test were administered to obtain reading and intelligence scores, respectively. A Junior Eysenck Personality Inventory was used to obtain extraversion and neuroticism scores. A high positive correlation between extraversion and reading attainment was found. Neuroticism was negatively correlated with scholastic attainment.

The aim of the Brown (1970) study was to examine the neuroticism-performance relationships in school children, using a task with unlimited time. The sample consisted of 147 third-year students randomly selected from a single comprehensive school in the Midlands of Scotland. The mean age was 13.7 years. The task involved learning a mathematics unit dealing with number bases. Success was judged by a score on a test at the completion of the task. Neuroticism was measured by the Junior Eysenck Personality Inventory. The conclusion of the study was that the results gave strong support to the curvilinear relationship, in the form of an inverted "U," between neuroticism and success.

Entwistle and Entwistle (1970) examined the relationship between personality, study methods, and academic performance. The subjects consisted of 257 first-year university students who were volunteers for the study. The students completed a student attitude questionnaire which included a section on study habits and completed either Form A or Form B of the Eysenck Personality Inventory. The questionnaire and inventory yielded four scores: motivation, study methods, extraversion, and neuroticism. Previous academic achievement was assessed as the sum of A-level grades and a criterion of academic performance at the end of the first year was obtained. The criterion was the sum of marks from three subjects. Motivation and study methods were related to academic performance, though the correlations were not consistently significant. The relationship between neuroticism and academic performance was not significant; however, the relationship between extraversion and academic achievement was significantly correlated in the negative direction. A multiple correlation of 0.36 was obtained using study methods, extraversion, and motivation as predictor variables. The authors concluded that the successful student tended to have below average scores on extraversion, together with high scores on the study methods and motivation scales.

Eysenck (1972) stated:

> Much work has been done in relation to anxiety, neuroticism, and extraversion-introversion, both with school children and with students, but results, although suggestive, have not been consistent and correlations between personality and attainment have not usually been very high. There is a fair amount of agreement that both

> neuroticism and extraversion are relevant to success (Warburton, 1968), but the relationships observed seem to depend on the age of the subjects (or possibly on the formal nature of the teaching, or the selection policies employed--these three factors are so closely interwoven that it is difficult to separate them out). At primary school, extraversion and stability seem to predispose the child to success; at secondary school, introversion and stability; at University, introversion and neuroticism . . . . [1972:40]

Entwistle (1972) pointed out that research using Eysenck's inventories have necessarily concentrated on the factors of extraversion and neuroticism. Factor analysis designed to produce unrelated factors allowed Eysenck to choose items for his inventory which measured the extraversion and neuroticism factors. Cattell preferred to use an alternative method of factor analysis which produces inter-related factors. Investigations into the relationship between personality and academic attainment have used both types of inventory. Entwistle reported that with respect to extraversion the research, using Cattel's tests, indicated the same age trend as noted by Eysenck. Stable extraversion is related to success in the primary school. By age eighteen, introversion and possibly emotional instability seem to be related to academic success. He also claimed the existence of convincing evidence for the superiority of the introvert in higher education.

A study concerned with the effects of intellectual and personality factors on academic achievement using Cattell's instruments follows.

Barton, Dielman, and Cattell (1971) assessed the relative importance of ability and personality variables in the prediction of school achievement in a variety of areas. The subjects consisted of 169 sixth-grade and 142 seventh-grade students enrolled in a junior high school in Illinois. The students completed the Culture Fair Intelligence

Test (CFIQ) and the High School Personality Questionnaire (HSPQ) in January, 1970. In March, 1970, four standardized achievement tests (Educational Testing Service) in the areas of mathematics, science, social studies, and reading were written. Correlations between any given personality factor and achievement score were of the same general magnitude and sign over all four types of achievement tests within any grade (ignoring non-significant correlations). Factors that were significantly related to all four measures of achievement in both grades were the two measures of I.Q. (Factor B of the HSPQ and the CFIQ) and Factor G of the HSPQ, a measure of conscientiousness. In the sixth grade, Factor A (warm-hearted participation) and Factor H (adventurousness) were significantly related to achievement in mathematics. In the seventh grade, Factor C (emotional stability), Factor E (dominance), Factor H, Factor I (tough-mindedness), Factor J (desire of group action), Factor O (self-assuredness), and Factor Q_3 (exacting will power) were significantly correlated with achievement in mathematics. Note that Factor H was one factor related to mathematics achievement in both grades. By the use of multiple regression analysis the following results were found:

1. The HSPQ accounted for 44 per cent and 43 per cent of the variance in mathematical achievement in grades six and seven respectively.
2. The CFIQ test accounted for 36 per cent and 43 per cent of the variance in mathematical achievement in grades six and seven respectively.
3. The personality factors (excluding Factor B) of the HSPQ

accounted for 19 and 27 per cent of the variance in mathematical achievement in grades six and seven respectively.

4. The use of both the HSPQ and CFIQ tests accounted for 53 per cent and 55 per cent of the total variance in mathematical achievement in grades six and seven respectively.

Barton et al. (1971) found evidence that personality factors seem to have differential importance with respect to grade level. Thus, one would not necessarily expect the important factors of the above study to be important for the prediction of mathematics achievement at the college level; however, it seems reasonable to expect personality factors to have a significant contribution to the prediction of mathematical achievement at that level.

What is the relationship between personality factors and academic achievement for different modes of instruction? Research dealing with this question appears to be scarce; however, several ongoing studies are listed in Inventory of Current Research on Post-Secondary Education--1972 by Hefferlin, Bloom, Gaff, and Longacre.

King (1972) studied associations between cognitive and affective changes and psychological traits in two teaching settings. The open learning method was a student-centred mode in which students were randomly assigned to small groups of five or six but were free to change from one group to another or to do independent study whenever they desired to do so. Students in the open learning group planned their own learning experiences, within the confines of a departmental course outline, and contributed to their own evaluation. The other group was taught using the traditional lecture method. There were 49 college mathematics students in the open learning group and 43 college mathematics students in the lecture group. Pre-test and post-test data were collected on two cognitive variables (structure of real numbers and statistics-probability) and four affective variables (attitude, self-concept, anxiety, and interest). Structure of the Number System, Form A, developed by Educational Testing Service and a statistics and probability test, designed by the researcher, gave measures of the two cognitive variables. Affective trait scores were obtained from Ideas and Preferences, Form 2151, from the National Longitudinal Study of Mathematical Abilities Z-Population Test Batteries. Gough's California Psychological Inventory was used to measure the psychological traits. Significant positive correlations were found with cognitive changes under the lecture method on the factors of responsibility, sociability, socialization, and femininity. Significant positive correlations between the cognitive changes and self-control were found under the open learning system. Significant negative correlations with cognitive changes and social presence were also found under the open learning method. The study also reported that attrition in the open learning classes was considerably lower than in the lecture classes.

Domino (1971) reported a study of the interaction between student achievement orientations and teaching styles. One purpose of the study was to ascertain the effect of this interaction on academic achievement and the student's expressed satisfaction with his scholastic environment. The subjects involved 900 university freshman students. In 1968, they were administered the California Psychological Inventory (CPI). From a frequency distribution of scores on the Achievement-via-Conformance (Ac) and Achievement-via-Independence (Ai) scales of the CPI, 50 High Ac-low Ai and 50 low Ac-high Ai students were identified. In 1969, these 100 students were assigned to four introductory psychological sections, with the same instructor, and in such a way so that each section contained 25 students, with relatively equal sex composition and comparable mean S.A.T. scores. One group of high Ai students and one group of high Ac students were taught in a lecture situation or a "conforming" manner. One group of high Ai students and one group of high Ac students were taught in an "independent" manner. All students wrote the same final examination, consisting of two parts: one part of multiple-choice questions, and one part of essay questions. Students also evaluated the course and instructor. Domino concluded:

> The results indicated that students taught in a manner consonant with their achievement orientation obtained significantly higher means on the multiple-choice items, on factual knowledge ratings of their essay answers and on their ratings of teacher effectiveness and course evaluation than their peers taught in a dissonant manner. [1971:427]

In view of the reported relationships between personality factors and academic achievement, personality factors were a subset of the student characteristics selected in Chapter III.

Other studies involving personality factors follow in the next section since these studies also involved motivational factors.

MOTIVATIONAL FACTORS

The term "motivation" is probably used in education as commonly as the term "ability" as an explanation as to why students do or do not learn in academic situations. Clark (1970) said:

> While "ability" summarizes observations in such a fashion as to denote what an organism can do or is able to do, "motivation" tends to summarize observations as to what an organism wants to do. [1970:17]

During the past twenty years there have been many studies in the area of motivation (achievement motivation and need achievement are usually synonymous with motivation). The pioneering work of McClelland and Atkinson provided the basis for a large body of information pertaining to the topic of motivation. Much of this information has, according to Herrenkohl (1972), " . . . tended to confuse and obscure rather than simplify and clarify" (1972:314). One of the most difficult problems seems to be the defining and measuring of achievement motivation. Some authors, such as Frymier (1970) and Maehr and Sjogren (1971), used the term "academic motivation" as motivation related to learning in an academic setting. Regardless of the term used, motivation is recognized (see Herrenkohl, 1972) as being amazingly complex with biological, personality, community, and social class factors involved.

Bower, Boyer, and Scheirer (1970) gave a review of research related to academic motivation carried out from 1960 to 1970. They claimed that research in this area was often carried out in the absence of a precise definition of the motivation construct or factors presumed to bear a causal relationship to motivation. A great volume of work, much of which has been summarized by Atkinson and Feather (1966) and Heckhausen (1967), has shown interaction of need achievement with treatment conditions in experiments but Cronbach and Snow (1969) pointed out that these interactions have usually occurred in studies of risk taking and not instruction. However, some authors, including Kight and

Sassenrath (1966) and Hartley, Holt, and Hogarth (1970), have studied the relationship between achievement motivation and programmed learning performance.

The purpose of the Kight and Sassenrath study was to investigate the influence of achievement motivation and test anxiety on performance in programmed instruction. The subjects included 139 undergraduates in educational psychology at Indiana University. A score for achievement motivation labelled "achievement imagery" was obtained from the Iowa Picture Interpretation Test (Form RK). The test anxiety score was obtained from the Test Anxiety Questionnaire (TAQ). Three performance measures were used:

1. Time required to complete the course material;
2. Number of frames answered incorrectly; and
3. Proficiency on an achievement test administered at the end of the course.

The high-achievement-motivated students performed significantly better on all three performance measures than low-achievement-motivated students. The high-test-anxiety students performed better on criteria (1) and (2) but failed to score higher on the achievement test.

Hartley et al. (1971) investigated interrelationships between academic motivation and programmed learning when that learning was conducted in pairs. The pairing was based upon scores on a motivational test designed by the researchers. Three groups of pairs were required: two sets of homogeneous pairs (High-High, Low-Low) and one set of heterogeneous ones (High-Low). Taking each student as an individual, a comparison was made between highly motivated students, H, in a High-Low pairing, H(L), with highly motivated students working together, HH. Also, comparison was made between less motivated students in L(H) pairs with less motivated students working together, LL. The subjects were 64 grammar school boys aged 12-13 years. The criterion variables were post-test scores on a six-week programmed science unit and times required to complete the different parts of the course. The results of the study showed no significant difference between H(L) and HH students and between L(H) and LL students on the criterion variables. In fact, there was no significant difference between the highly motivated students and less motivated students, taken overall.

With respect to the relationship between academic motivation and programmed learning, Hartley et al. have failed to find any significant relationship in various studies they have carried out. They indicated that these findings reflect those of other researchers except for the study of Kight and Sassenrath (1966). Based upon a review of the literature, Hartley et al. claimed there is no clear cut evidence to support the view that achievement motivation (however it is measured) is related to academic achievement. Evidence for a positive relationship between motivation and performance has been substantiated in rather less than one half of these studies and these positive relationships have usually been small. A recent study of Farley and Truog (1971) showed no significant relation between achievement motivation and academic achievement measured over four university subject areas. Maehr and Sjogren (1971) have summarized studies concerned with the relationships between achievement motivation and academic achievement in an independent study environment. They reported that evidence suggests that high "need to achieve" students in classrooms outperform low "need to achieve" students in classrooms which allow for or are more dependent on self-motivation.

Cattell and Butcher (1968) pointed out that, until rather recently there has been missing from the personality factors what we think of as interest, attitude, and motivation, or what the clinician calls "dynamic psychology." They referred to this as the area of "dynamic calculus" and indicated that factor analysis and similar methods have revealed "dynamic structure factors." Cattell and Butcher grouped these factors as either ergs (instinctive patterns comparable with drives observed in nonhuman mammals) or sentiments (aggregate of attitudes that focus on a common social institution), with attitudes as the underlying bricks out of which ergs and sentiments are built.

Much of the early work carried out by Cattell and associates aimed at determining how attitude measures could add to prediction of academic achievement. The results were somewhat disappointing as the great majority of correlations were not statistically significant; however, with the improvement of measuring instruments, such as objective tests, evidence of the last five years has shown that motivation tests can add a substantial quota of predictive power in forecasting school achievement. Cattell and Butcher (1968) reported that ability, personality, and motivation factors accounted for approximately 60 per cent of the variance of school achievement prediction in one study. In the same publication, Cattell hypothesized that the accountable variance in achievement could be divided about equally among ability, personality, and motivation factors, each accounting for 25 per cent. Two recent reports of Cattell and associates involving the same students follow.

The purpose of the Dielman, Barton, and Cattell (1971) study was to test the above-mentioned hypotheses of Cattell with respect to ability and motivation factors. In addition, they wanted to develop regression equations, predicting achievement in different areas .n ability and motivation. The subjects were 299 sixth- and seventh-grade students in Illinois. Thirty motivation scores were obtained from the School Motivational Analysis Test (SMAT) and criterion scores came from the Educational Testing Service mathematics, social studies, English, and science achievement tests. The Culture Fair Intelligence Test (CFIQ) was also administered. The three scores for each of ten scales (ergs and sentiments) were:

1. "integrated" (I) scores which may be thought of as the psycho-analytic concept of conscious motivation;
2. "unintegrated" (U) scores which may be thought of as unconscious motivation; and
3. "total" (T) score where T = I + U.

The CFIQ accounted for 15 per cent to 36 per cent of the total variance in the criterion scores for the sixth grade and 22 per cent to 43 per cent for the seventh grade. The ten integrated SMAT scores added a significant increase in the variance with the increase ranging from 12 to 33 per cent. The integrated Super-Ego scale appeared as the best single SMAT predictor.

Cattell, Barton, and Dielman (1972) reported a variation of the foregoing study involving the same students. Their paper presented the results pertaining to the hypotheses that ability, personality, and motivation factors each account for about 25 per cent of the total variance in academic achievement. The School Motivational Analysis Test (SMAT), High School Personality Questionnaire (HSPQ), and Culture Fair I.Q. Test (CFIQ) measured motivation, personality, and ability factors respectively. In the sixth grade, 74 per cent of the integrated (I) scores of the SMAT and 90 per cent of the I scores were significantly correlated with an achievement score. In the area of mathematics achievement, 61 per cent and 69 per cent of the variance in the sixth grade and seventh grade, respectively, was accounted for by a combination of all three tests. Individually, the SMAT, HSPQ, and CFIQ scores, respectively, accounted for 38 per cent, 44 per cent, and 36 per cent of the total variance in grade six mathematics achievement, and 45 per cent, 43 per cent, and 43 per cent in grade seven. The addition of HSPQ to the combination of CFIQ and SMAT scores resulted in a significant increase in the variance accounted for in mathematics achievement at both the grade six and grade seven levels. The addition of SMAT to the CFIQ-HSPQ combination added significantly to the mathematics variance at the grade seven level but not at the grade six level. The addition of CFIQ to the SMAT-HSPQ combination added significantly to the mathematics variance at both grade levels. The researchers pointed out that the [illegible] of SMAT U scores are virtually useless in the prediction of achievement in most areas.

In spite of the varied results and views regarding the relationships between motivation factors and academic achievement, motivation factors were included in the present study along with ability and personality factors. The importance of the "need to achieve" factor in an independent study environment reported by Maehr and Sjogren and the results of Cattell and associates are encouraging and helped in the decision to include motivation factors in this study.

The review of research related to mastery learning demonstrates that features such as the use of specific behavioural objectives, small learning units, formative and summative evaluation techniques, and alternate learning aids can improve student achievement. It may generally be said that mastery learning techniques, lecture, and other learning methods have some benefits for different kinds of students. The purpose of the present study is to assess what kinds of students, in terms of ability, personality, and motivation factors, benefit from the lecture approach and independent study approach to the learning of calculus.

Studies pertaining to the effect of intellectual factors on academic achievement indicated the importance of specific aptitude factors as well as a general intelligence score. Some of Guilford's factors (see studies of Davis, King et al., and Behr), the factors of numerical ability, verbal ability, and deductive reasoning (see review of Aiken) have been identified as important. Liedtke's study was interesting in that a general intelligence score and reading ability score failed to show up as reli[illegible]e predictors in an independent study situation.

In the personality sphere, the factors of neuroticism and extraversion have emerged as usually important in predicting achievement. There is support for a curvilinear relationship between neuroticism and academic success in the form of an inverted "U". There is a tendency for stable extraverts to be more successful in primary grades while introverts tend to be predominant among the better students at university. The studies of King, Cattell and associates, and Domino have demonstrated the importance of personality factors to the prediction of achievement.

Evidence does not clearly support the view that motivation is related to academic achievement. However, the report of Maehr and Sjogren suggested that high "need to achieve" students perform significantly better than low "need to achieve" students in independent study situations. The study of Cattell, Barton, and Dielman showed that motivational scores can add significantly to the prediction of achievement.

CHAPTER III

DESIGN OF THE STUDY

This study was designed first to assess the importance of student characteristics with respect to the learning of calculus in each of two learning settings, and second, to compare the achievement scores and dropout ratios of the two groups. This chapter will describe how the study strived toward the indicated purposes. An overview of the study is followed by a description of the sample, the methods of instruction, the testing program, and the analysis of the data.

OVERVIEW OF THE STUDY

This study was made during the 1973 spring semester at Mount Royal College, located in Calgary, Alberta. The subjects were enrolled in a first-year university calculus course of a one-semester duration. Two methods of instruction, under two different instructors, were used: one method was an independent study approach and the other a lecture approach. At the beginning of the semester, the students wrote several tests and answered a number of questionnaires related to the areas of ability, personality, and motivation. The criteria for determining academic achievement were scores on a number of achievement tests. The questionnaire scores and criterion scores were analysed with the statistical analysis being mainly methods of correlation and regression.

THE SAMPLE

The research subjects were Mount Royal College students registered in a calculus course, Pure Mathematics 201 (PMAT 201), for the 1973 spring semester. Mount Royal is a two-year comprehensive community college located in a suburban area of Calgary, Alberta, enrolling approximately 3,000 full-time equivalent students. The programs offered include courses in technical, vocational skills and manpower training, community service, general education, remedial and upgrading, and university transfer. Mount Royal College is affiliated with The University of Calgary and, thus, offers a number of first-year university courses, including PMAT 201.

One week prior to the beginning of the spring semester, over 129 students registered for PMAT 201. The students were informed of the nature of the experiment and then randomly assigned into two groups, using a table of random numbers. Some students failed to appear for the initial testing and the beginning of the semester. The final sample sizes for each of the groups are given in Table I, where IS refers to the independent study group and L to the lecture group.

Table I

Final Experimental Population Sizes

	Group	
Sex	IS	L
Male	47	52
Female	20	10
Total	67	62

METHODS OF INSTRUCTION

Independent Study Method

The features of Bloom's strategy for mastery learning, as outlined in the last chapter, were adopted in the independent study method used in this study. The strategy consisted of the following features:

1. Define mastery in terms of a specific set of objectives the student is expected to achieve.
2. Divide the course into small learning units (one or two weeks' instruction) with unit objectives.
3. Use teaching methods appropriate to learner needs.
4. Prepare short diagnostic-progress tests (formative tests) to determine whether the student has mastered the unit and what, if anything, the student must do to master it.
5. Prepare tests (summative tests) to appraise student's competence with regard to content and objectives.

These features will be referred to in the following description of the independent study method used in this study.

The calculus course (PMAT 201) was divided into nineteen instructional units and specific behavioural objectives were written, by the researcher, for each unit. A unit was considered as approximately the course content that could be learned in one week in order to complete the course in a semester of fifteen weeks' duration. The overall objective for each unit was the mastery of the specific objectives of that unit (see features 1 and 2 of Bloom's strategy).

A self-instructional package, in the form of a written handout, was prepared for each unit. Each package contained the objectives of the unit, notes on the unit, problems and solutions to the problems, and a formative post-test. In many of the packages references were made to appropriate textbooks, cassette tapes, and in one case to a sound-slide presentation. Each package was designed to guide the student's learning through the corresponding unit. The packages, cassette tapes, and sound-slide presentation were prepared by the researcher.

The calculus course was scheduled in the student's timetable as three one-hour classes per week. One hour (5:00 p.m. Thursdays) was intended as time for a lecture and/or a general question period, as well as the writing of term tests. The other two hours were to be spent in the mathematics section of the open-area learning library. Reference books, cassette recorders, cassette tapes, slides, and slide projectors were available, from 8:00 a.m. to 5:00 p.m. each day (except weekends), at the mathematics resource island located in the mathematics section of the learning library. The student could also check materials out overnight or over weekends. At almost any time, the students could use the learning library to study, work in small groups, or use course materials. The faculty offices were also located in the learning library near the mathematics resource island. The independent study instructor was available to the students during posted office hours of approximately 10 to 15 hours per week. The mathematics department had the services of a full-time para-professional, and during the day, he was available to the students for tutoring purposes.

The preceding two paragraphs relate to feature 3 of Bloom's strategy. With regard to the appropriateness of the method, one of the aims of this study was to determine what kinds of students are best suited to the independent study setting.

A major aspect of the independent study method was the "feedback-correction" procedure. Upon completion of a unit, the student could write the post-test (formative test) contained in his package. He could then take the test to the instructor. They could discuss the answers and note those unit objectives the student had not mastered. Even though no grade was marked on the post-test, the student was to obtain a score of approximately 80 per cent or better before proceeding to the next unit. This was the feedback part of "feedback-correction"; the student received feedback regarding the objectives he had or had not mastered.

The instructor used several corrective procedures to help the student master the unmastered objectives. These procedures included the discussion of the appropriate topics with the student, assigning appropriate sections of the self-instructional package for the student to review, referring the student to specific sections of the resource textbooks, assigning specific problems, and referring the student to the para-professional for special tutoring. After completing his corrective procedure the student could then write another formative test, answering those questions corresponding to the unmastered objectives (see feature 4).

One of the following six grades were awarded at the end of the semester: A for superior, B for excellent, C for average, D for pass, I for incomplete, and W for withdrawal. The grades were based upon four term tests and a final examination. The student had to write the term tests and final examination by certain fixed dates, with exceptions made for students having special problems with the course. An examination schedule was prepared and is given in Table II. An I grade was awarded to a student if he did not complete the course by the end of the semester. He could have the I removed upon completing the course and passing a special final examination within a period of one month from the end of the semester. No student was allowed to have this privilege unless he had completed at least ten units and had written two term tests during the semester (see feature 5).

Table II

IS Test Schedule

Test Number	Units Covered	Date
1	1 and 2	February 15
2	1 through 7 inclusive	March 8
3	1 through 11 inclusive	March 29
4	1 through 16 inclusive	April 19
Final	1 through 19 inclusive	May 22

The preceding remarks basically outline the independent study method used in this study. For further details, see Appendix A.

<u>Lecture Method</u>

The calculus course was scheduled in the student's timetable as three one-hour lectures per week, 5:00 p.m. Mondays, Wednesdays, and Fridays; in addition, there was a scheduled one-hour tutorial session, 5:00 p.m. Thursdays. The tutorial hour was intended as time for students to ask questions, receive assistance in solving problems, and write the term tests. The lectures and tutorial were held in a lecture theatre adjacent to the learning library. The instructor had posted office hours (five hours per week) when he was available to his students. The students also had the tutoring services of the para-professional.

The lecture group studied the same calculus topics (see Appendix B) as the independent study students but had a different instructor. The instructor developed concepts and theorems as close as possible to the manner and sequence displayed in the self-instructional packages of the independent study group. Calculus and Analytic Geometry (4th edition) by R. Johnson and F. Kiokemeister was used as the textbook for the course.

The letter grades A, B, C, D, I, and W were awarded to the lecture students in the same way as was done for the independent study students; however, there was no procedure for having an I grade changed after the end of the semester. An examination schedule was prepared and is given in Table III.

Table III

Lecture Test Schedule

Test Number	Date
1	February 15
2	March 8
3	March 29
4	April 19
Final	May 22

THE TESTING PROGRAM

Determination of Student Characteristics

The instruments used to gather data pertaining to student characteristics are listed as follows:

1. Differential Aptitude Tests, (DAT), (1963 edition): Form L, Booklet 1.
2. Sixteen Personality Factor Questionnaire: Form A.
3. Motivation Analysis Test, (1964 edition): Form A.
4. Costello's Achievement Motivation Scales I and II (see Appendix C).
5. Van Wagenen Rate of Comprehension Scale: Form A.
6. Wonderlic Personnel Test: Form A.
7. Questionnaire for PMAT 201 Students (see Appendix D).

Part of the entrance testing program at Mount Royal College involved the administration of the Van Wagenen and Wonderlic tests. These tests were administered to the new students during the registration period of both the fall and spring semesters. The other instruments were administered during the first week of the spring semester with each student having three one-hour sessions.

The remainder of this section is devoted to a description of the use of each instrument with a rationale for using that instrument.

Differential Aptitude Tests

Differential Aptitude Tests, (DAT), (1963 edition): Form L, Booklet 1 has four tests: Verbal Reasoning, Numerical Ability, Abstract Reasoning, Clerical Speed and Accuracy. The first two tests measure ability factors referred to in the review of the literature as important in the prediction of achievement in mathematics. Nunnally (1970) indicated that the DAT is a suitable instrument for adults.

The Numerical Ability and Abstract Reasoning tests were used for this study. Since the students were subjected to considerable testing, the researcher decided to use the already available Van Wagenen and Wonderlic test scores as a measure of reading ability and general ability respectively. The two DAT tests used were hand scored and then converted to percentile norms by using the grade 12 male and female percentile norms for Form L, contained in Directions for Administration and Scoring and Norms, Forms L and M.

Sixteen Personality Factor Questionnaire

Sixteen Personality Factor Questionnaire, (16 PF), (1967-1968 edition): Form A was developed by R. B. Cattell and associates using "factor analytic" procedures. It is a widely used instrument that is purported to measure the major dimensions of personality. Form A, consisting of 187 questions, yields 16 "primary" factor scores and 4 "second order" factor scores for individuals aged sixteen years and older. Appendix E lists the 16 primary factors which are described in detail by Cattell, Eber, and Tatsuoka (1970). While the verbal names associated with each factor convey meaning to persons unfamiliar with the 16 PF, Cattell (1973) preferred the letter designations. Any verbal label gets tied to a complex of everyday meanings and nuances that are different for different persons; moreover, the labels rarely capture the full meaning and content of the factor. The second order factors are:

1. Introversion versus Extraversion (EXVIA).

2. Low Anxiety (Adjustment) versus High Anxiety.

3. Tender-minded Emotionality versus Tough Poise.

4. Subduedness versus Independence.

Most of the personality factors, including extraversion and neuroticism (anxiety), considered in the review of the literature are included in the primary and secondary factors as measured by the 16 PF.

Cattell, Eber, and Tatsuoka (1970) considered the psychometric properties of reliability and validity, and subdivide reliability into dependability (short-term test-retest correlations) and stability (test-retest correlations over a longer period of time). The Manual for the 16 PF (1972) indicates that for Form A the dependability coefficients (N = 243) range from 0.72 to 0.92 with 11 coefficients above 0.80. The stability coefficients (N = 44) range from 0.35 to 0.85 with 10 coefficients above 0.60. The concept validity (how well the scale correlates with the concept which the scale is to measure) is given in the manual as direct concept validity (concept validity determined by factor analysis). For Form A these coefficients (N = 958) range from 0.35 to 0.92 with 11 coefficients above 0.60. Buros (1970) stated:

> Split-half reliabilities (N = 450) range from 0.71 to 0.93, ten coefficients being above 0.80. This is quite good; but even more pleasing is the fact that validities (based on factor loadings) range from 0.73 to 0.96 with eleven coefficients exceeding 0.80. For a multi-dimensional test of this kind one could not hope for much more. Evidently, despite the reputation of questionnaire methods as being unreliable, this test does succeed. [1970:819]

According to a review by Maurice Lorr in Buros (1970), the 16 PF appears to be the best factor-based personality inventory available but it still should be used primarily as a research instrument.

The 16 PF answer sheets were scored by hand using the stencil scoring keys. The raw scores were used in the statistical analysis of this study. The raw scores were converted to sten scores in order to obtain second-order factor scores. Sten scores (the term comes from "standard ten") are normally distributed over ten equal interval standard score points from 1 to 10, with the population mean fixed at 5.5. Stens 5 and 6 extend respectively one-half standard deviation above and below the mean. This conversion was accomplished by using "Table 22: Norms for College Students" published in the Tabular Supplement. By applying the weights and constants (in Table 10.9 of the Handbook for the 16 PF) to the sten scores of the primary factors, the four second-order factor sten scores were obtained. This last step was accomplished by using the computer program shown in Appendix F. Thus, for each student, raw scores were obtained for each of the sixteen primary personality factors and sten scores were obtained for the four second-order factors.

Motivation Analysis Test

Projective techniques have been the principal devices used to determine the strength of achievement motivation with McClelland's Thematic Apperception Test (TAT), (McClelland et al., 1953), being a favourite. For a number of reasons given by Hermans (1970), Herrenkohl (1972), and West and Uhlenberg (1970), the TAT is not satisfactory for use in assessing motivation in academic settings. A number of questionnaires, rating scales, and checklists have been devised but they have several disadvantages including deliberate distortion by the examinee.

West and Uhlenberg (1970), in a review of techniques of measurement as applied to motivation, stated:

> The disadvantages of projective testing . . . have led educators and some psychologists . . . to devise "objective" tests of motivation. By objective tests we mean here assessment procedures which obtain consistent scoring from different scorers and which also conceal from the examinee the nature of the scoring and interpretation of his responses. [1970:53]

Factor analytic studies by Cattell et al. have resulted in several objective tests including the Motivation Analysis Test (MAT) (1964 edition) Form A.

The MAT is an adult test which measures ten motivation factors, giving an "integrated" score (I) and an "unintegrated" score (U) for each factor. As shown in the literature review, an integrated score may be thought of as the psychoanalytic concept of conscious motivation, whereas an unintegrated score may be thought of as unconscious motivation. The ten motivation factors are made up of five primary factors called ergs and five primary factors called sentiments (see Table IV).

Table IV

Primary Factors Measured by MAT

Ergs	Sentiments
Mating (sexual love)	Sentiment to self 1. social reputation 2. control and understanding
Assertiveness (achievement)	Superego
Fear (escape)	Career
Narcism (comfort, sensuality)	Sweetheart-Spouse
Pugnacity (sadism)	Home-Parent

See Appendix G for a brief description of each factor. Sweney (1969) gave an interpretation for each of the I and U scales.

The answer sheets were hand scored using the stencil scoring keys. There were four raw scores for each of the ten factors. Two raw scores were converted into an I sten score and the other two were converted into a U sten score by using the norms in the Handbook for the Motivation Analysis Test. The MAT Dynamic Structure Profile sheet was used to assist with the computation. Ten Total Motivation and ten Conflict sten scores were obtained from the U and I stens using norms in the handbook. Five other derivation scores were obtained from the profile sheet and the norms. These five scores are optional and experimental in nature and are:

1. Total Integration.
2. Total Personal Interest (sum of the ten Total Motivation scores and then converted to a sten score).
3. Total Conflict (sum of the ten Conflict scores and converted to a sten score).
4. General Autism-Optimism.
5. General Intelligence-Information.

The Handbook for the Motivation Analysis Test gives reliability and validity coefficients for the ten MAT scales. The dependability coefficients (N = 156) range from 0.51 to 0.81 with six coefficients above 0.65. The stability coefficients (N = 101) range from 0.39 to 0.69 with five coefficients above 0.50. Split-half reliability coefficients (N = 151) range from 0.33 to 0.70 with five coefficients above 0.50. Concept validity coefficients determined by correlating primary factors scored over all items in a developmental study with the factors measured by the items actually selected for the test range from 0.52 to 0.76 with eight coefficients above 0.60. Reviews in Buros (1972) indicated that the MAT offers much promise as an experimental instrument but does not fare too well, as yet, with regard to meeting the usual psychometric criteria.

Costello's Achievement Motivation Scales I and II

The two scales, developed by C. G. Costello in 1967, are self-reporting checklists taking about five minutes to complete. Scale I and Scale II have ten and fourteen items respectively and were developed by factor analytic methods. According to Costello (1967), Scale I measures the motivational dispositions of the individual who wants to do a job well through his own efforts, whereas Scale II measures the disposition or need of a person to be a success defined in terms of the emulation of successful people. The split-half reliability for Scale I was 0.82 and for Scale II was 0.73 with a sample of 264 college students. The intercorrelation of the two scales was -0.17. Costello also reported that Scale II scores were significantly related to scores measuring neuroticism and anxiety.

Costello (1968) carried out a study designed to establish the predictive validity of the two "need to achieve" scales. The subjects were 198 freshmen college students. Four groups of ten students were selected on the basis of their scores. Group 1 consisted of those students high (above the upper quartile point) on Scale I and low (below the lower quartile point) on Scale II. Group 2 students were high on I and II; Group 3 students were low on I and high on II; Group 4 students were low on I and II. Criterion scores were the mean examination scores for all the final examinations at the end of the year. The overall mean examination marks were: Group 1, 70.07; Group 2, 66.98; Group 3, 56.46; and Group 4, 53.84. An analysis of variance indicated that the main effect of Scale I was significant.

Van Wagenen Rate of Comprehension Scale

The Van Wagenen Rate of Comprehension Scale: Form A measures the rate an individual can read with comprehension in words per minute. It consists of 56 thirty-word paragraphs and is a four-minute test. The number of paragraphs correctly read are converted into words per minute by a scoring key on the front of the scale. The student scores were obtained from the Educational Development Services, Mount Royal College.

Wonderlic Personnel Test

The Wonderlic Personnel Test: Form A is an adaptation of the Otis Self-Administration Test of Mental Ability. It was designed for testing adults in business and industrial situations. Form A is a twelve-minute test of problem-solving ability.

The percentile scores of this test were obtained from the Educational Development Services of Mount Royal College. Fourth-year high school male and female norms based on a 1960-1961 study were used. The norms are printed in the Wonderlic Personnel Test Manual by E. F. Wonderlic.

Questionnaire for PMAT 201 Students

This brief questionnaire was devised by the researcher for the purpose of being able to keep in contact with the student and obtaining some background information on each student.

Measurement of Achievement

During the spring semester, criterion measures were obtained from four term tests, a final examination, and a standardized examination called the Co-operative Mathematics Test (Calculus), (1963): Form A.

Term Tests and Final Examination

Each term test was administered during a fifty-five-minute class according to the schedules given earlier in this chapter. Students in both groups received the same amount of time to complete each test. There was a six-hour spread in the testing times of the lecture and independent study groups; thus, it was necessary for each group to have a different term test. The final examination was administered to both groups (at the same time) during a one and one half-hour session. The instructors marked the term tests according to a scoring key prepared by the researcher, whereas the final examination was scored by the researcher. The term tests and final examination are contained in Appendix H.

The tests were constructed by the researcher according to the specific behavioural objectives of the course. The correlations between the final examination scores and standardized Co-operative Mathematics Test scores were 0.71 and 0.86 for the independent study and lecture groups respectively. Thus, the <u>content validity</u> is assumed to be satisfactory. Both instructors of the two groups reviewed these examinations. They did not formally evaluate the tests but recommended revision of some items. This review provided further face validity of these tests. No empirical estimates of the reliability of the term tests and final examination were calculated.

The Co-operative Mathematics Test

The Co-operative Mathematics Test (Calculus), (1963): Form A was designed to assess achievement in terms of the students' comprehension of the basic concepts, techniques, and unifying principles in the area of calculus and analytic geometry. It is indicated in the Handbook: Co-operative Mathematics Test that, where possible, many new trends and emphases in mathematics are represented in the test but that the content had been selected carefully to ensure the appropriateness of the test for most students. Ability to apply understanding of mathematical ideas to new situations and to reason with insight are emphasized. Factual recall and computation are minimized. The test covers work in analytic geometry and calculus through differential and integral calculus of algebraic and transcendental functions including work on topics such as the theorem of the mean and limits of indeterminate forms. As is stated in the Handbook, "The test is appropriate for students who have completed work in separate courses in these content areas or at least a full year of work in these combined areas" (1964:8).

The test has two forty-minute parts of 30 items. The answer sheets were hand scored using "Table 26" of the Handbook and the raw scores were used in the statistical analysis. With respect to content validity, the Handbook indicated that the test construction was entrusted to well-qualified persons capable of judging the relationship of test content to teaching objectives. Estimates of the reliability coefficients were made using the Kuder-Richardson Formula 20. With two samples of college students (N = 450 and N = 350) the reliability coefficients were 0.87 and 0.84 respectively. A reviewer, W. E. Kline in Buros (1972) stated: "This reviewer knows of no standardized test designed to measure achievement at the close of a year's course in calculus that is better than the Co-operative Mathematics Tests: Calculus" (1972:521).

NULL HYPOTHESES

The data gathered from the testing program described in the preceding pages were used in statistical analyses designed to test the following null hypotheses.

Null Hypothesis 1. The correlation coefficient between a given student characteristic and achievement scores is not significantly different from zero at the .05 per cent level of confidence.

In each setting (lecture and independent study) this hypothesis was repeated for each student characteristic (each student aptitude, personality, and motivation factor) and stated separately for six sets of achievement scores (four term tests, a final examination, and the Co-operative Mathematics Test).

Null Hypothesis 2. The square of the multiple correlation coefficient between a combination of predictor variables (student characteristics) and the criterion achievement variable is not significantly different from zero at the .05 per cent level of significance.

In each setting (lecture and independent study) this hypothesis will be repeated for each student characteristic (each aptitude, personality, and motivation factor).

Null Hypothesis 3. The square of the multiple correlation coefficient between a combination of predictor variables (student characteristics) and the criterion variable of whether a student is or is not a dropout is not significantly different from zero at the .05 per cent level of significance.

This hypothesis was tested for one group of all students (students in the lecture and independent study settings combined to form one group) and repeated for each setting separately.

Null Hypothesis 4. There is no significant difference in the mean scores for a given student characteristic between the student dropouts and the students completing the course.

In each setting (lecture and independent study) this hypothesis was repeated for each student characteristic (each aptitude, personality and motivation factor).

The foregoing null hypotheses were tested to fulfill the main purpose of the investigation which was to examine the importance of student characteristics with respect to calculus achievement scores in each of the two learning settings. The following null hypotheses were tested to fulfill the secondary purpose of the study which was to compare the independent study and lecture methods in terms of student achievement and dropout ratios.

Null Hypothesis 5. There is no significant difference in the mean criterion achievement scores between lecture and independent study groups.

This hypothesis was repeated for each of the six sets of achievement scores (four term tests, a final examination, and the Co-operative Mathematics Test).

Null Hypothesis 6. There is no significant difference in the dropout ratios between the lecture and independent study groups.

THE ANALYSIS

Each student who had written the tests of the study was assigned an identification number. The identification number, age, test scores, and dropout code number for each student were punched on IBM cards in preparation for data processing.

Tables of Pearson product-moment correlations between test scores and the criterion achievement scores were prepared for each of the independent study and lecture groups. The tables allow an inspection of the "closeness" of linear relationships between the predictor variables and the criterion achievement scores.

The correlation coefficients and the corresponding probabilities that the correlations in the population from which the sample was drawn are equal to zero were used to test Null Hypothesis 1.

Multiple regression analysis was used to construct a linear combination of independent variables (student characteristics) which "best" predicts the dependent or criterion variable (achievement scores). A "stepwise" regression procedure was used to arrive at a prediction equation of the form:

$$y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 \ . \ . \ . + b_nx_n + e$$

in which

y is the dependent variable;

$x_1, x_2, x_3, \ldots, x_n$ are the independent variables;

$b_0, b_1, b_2, \ldots, b_n$ are the coefficients that produce the "best" equation; and

e is the error term (the difference between the predicted and actual values of the dependent variable). The "best" equation is defined by the coefficients, $b_0, b_1, \ldots, b_n$, that make the sum of e^2 a minimum for a particular series of criterion values and predictor values obtained from a given sample.

"Stepwise" regression procedures produce a series of regression equations of the form:

$$y = b_0^{(1)} + b_1^{(1)}x_1 + e^{(1)}$$

$$y = b_0^{(2)} + b_1^{(2)}x_1 + b_2^{(2)}x_2 + e^{(2)}$$

$$y = b_0^{(3)} + b_1^{(3)}x_1 + b_2^{(3)}x_2 + b_3^{(3)}x_3 + e^{(3)}$$

$$\vdots \qquad \vdots \qquad \vdots$$

At each successive step, a new variable is added to make an improvement in the prediction equation. Usually one adds the variable which accounts for the greatest reduction in the sum of the squared error terms or which accounts for the greatest proportion of the remaining variance of the dependent or criterion variable. New coefficients are found at each step to produce the "best" equation in terms of the specific variables included in the equation at that step.

The stepwise procedure used in this study was programmed for computer use and is outlined by Nie, Bent, and Hull (1970). With this procedure, the user enters into regression the variables he wishes included and in the order he wishes them included. One may first enter the variable most highly correlated with the criterion variable. The procedure then computes, for each of the variables not in the equation, an F ratio giving a test of the hypothesis that the variable would make no significant contribution to the prediction if it were to enter the regression equation next. This F ratio also measures the significance of the partial correlation of that variable with the criterion variable. If the variable enters the equation at the next step, the F value will remain the same. At each step of the regression analysis the F ratios of the remaining variables not in the equation are computed.

At each step of this regression program the following values are output:

1. The regular regression coefficients, $b_0, b_1, b_2, \ldots, b_n$, listed as B values and normalized regression coefficients listed as BETA values.
2. The standard error, S, associated with each regular coefficient estimate.
3. The F value measuring the relative size of B and S.

$$F = \left(\frac{B}{S}\right)^2.$$

This value tests the significance of the coefficient for each variable in the best prediction equation.

4. The multiple correlation coefficient, R, between the criterion and the predictor variables.

5. The squared multiple correlation, R^2, measuring the proportion of variance shared by the criterion and predicted scores. A phrase commonly used in place of "variance shared by the criterion and predicted scores" is "variance accounted for by the regression equation."

6. The standard deviation of the residual which is a measure of the standard error of the predicted y.

7. The sum of squares about regression which is the sum of squares about the predicted values of the criterion (SSR).

8. The sum of squares due to regression or the error sum of squares (SSE).

9. The regression mean square (MSR) and the residual mean square (MSE).

10. An *F* ratio, F_R, distributed with m and N-m-1 degrees of freedom testing the significance of the regression equation representing more than mere chance or testing the significance of R^2 being different from zero. N is the number of subjects and m is the step number.

$$F_R = \frac{SSR/m}{SSE/(N-m-1)} = \frac{MSR}{MSE}.$$

This *F* value can be shown to be equivalent to the *F* value for testing the significance of R^2 as given by Cooley and Lohnes (1962), namely,

$$F = \frac{\bar{R}^2(N-m-1)}{(1-\bar{R}^2)m}$$

Using the fact that $R^2 = \frac{SSR}{SSE+SSR}$, one can show the equivalence by substitution.

11. The partial correlation between each independent variable not in the equation and the dependent variable when the independent variables in the regression equation are controlled for.
12. The F ratio for each prospective variable giving a test of the hypothesis that the variable would make no significant contribution if it were to enter the regression equation at the next step.
13. The "tolerance" of each prospective variable indicating whether or not a new "dimension" is being added to the prediction equation. If the tolerance is small, then that variable is nearly a linear combination of variables already in the equation. "Tolerance" values range between 0 and 1.

A summary table is given as output as well. The table contains the multiple R, the squared multiple correlation, R^2, the change in the R^2 from the value of the previous step, the correlation coefficients between each predictor variable and the criterion variable, and the regular and normalized regression coefficients.

In this study, a predictor variable was retained in the prediction equation if the F value of the entering variable and the F value testing the significnace of R^2 exceeded the 0.05 level of confidence.

Null Hypothesis 2 and Null Hypothesis 3 were tested using the stepwise regression analysis decribed in the preceding pages.

For each factor obtained from the tests of this experiment, a mean and a standard deviation were determined for the following groups:

1. Independent study students.
2. Lecture students.
3. Lecture dropouts (lecture students not writing the final examination and Co-operative Mathematics Test).
4. Independent study dropouts (independent study students not writing the final examination and Co-operative Mathematics Test).
5. Independent study students writing the final examination and the Co-operative Mathematics Test.
6. Lecture students writing the final examination and the Co-operative Mathematics Test.

Tables of means and standard deviations prepared for each of the above groups allow a comparison of the relative performance of each.

Null Hypothesis 4 was tested by using a t test to compare the mean student characteristic scores of (4) and (5), and of (3) and (6). Null Hypothesis 5 was tested by using a t test to compare the mean achievement scores of (1) and (2). In addition, the t test was used to compare the student characteristic mean scores of (1) and (2) to check whether the process of random sampling used in this study resulted in any significant differences between the groups. The t ratio used was:

$$t = \frac{m_2 - m_1}{\sqrt{\dfrac{N_1 s_1^2 + N_2 s_2^2}{N_1 + N_2 - 2}\left(\dfrac{N_1 + N_2}{N_1 N_2}\right)}}$$

where

m_1 and m_2 are the sample means;

s_1 and s_2 are the sample standard deviations; and

N_1 and N_2 are the sample sizes.

The significance of the $\underline{t}$ value was determined by referring to a $\underline{t}$ distribution table and using $N_1 + N_2 - 2$ degrees of freedom.

Null Hypothesis 6 was [illegible] a z ratio (Guilford, 1965) to compare the dropout ratios of the [illegible] and independent study group. The z ratio used was:

$$z = \frac{p_1 - p_2}{\sqrt{\bar{p}_e \bar{q}_e \left(\frac{N_1 + N_2}{N_1 N_2} \right)}}$$

where

p_1 and p_2 are the two sample proportions;

N_1 and N_2 are the sample sizes;

$\bar{p}_e = \frac{N_1 p_1 + N_2 p_2}{N_1 + N_2}$ is the estimated population proportion; and

$\bar{q}_e = 1 - \bar{p}_e$ is the variance of estimated population proportion.

The present chapter has consisted of a description of the experimental design including the statistical procedures followed in analysing the data collected in the study being reported. The results of the analysis are presented in the following chapter.

CHAPTER IV

RESULTS OF THE STUDY

INTRODUCTION

It is the purpose of this chapter to present detailed results of the study described in the preceding chapter. To facilitate reporting results, each null hypothesis will be re-stated, followed by an analysis of the data, and then a decision to accept or reject the null hypothesis.

The scores on the tests described in Chapter III, which form the basis for the results of this chapter, are to be found in Appendix I. Notice that certain data are missing for some students. Eighteen of the 129 students failed to complete all of the testing instruments used to gather data pertaining to the student characteristics. Twelve of these 18 students eventually dropped out of the course. The available scores for these 18 students were included in the analysis of this study except for the multiple regression analysis. The Wonderlic and Van Wagenen scores were available for only 60 students, since many students apparently failed to complete the Mount Royal College entrance testing program.

Before considering the results related to the null hypotheses, note that Table V displays the means and standard deviations of student characteristic scores obtained from the tests of this experiment for the 67 independent study and 62 lecture students. The number of students for which scores are available are indicated in the table. The *t* ratios, which were used to test for significance of a difference between the group

Table V

Means, Standard Deviations (S. D.), and Numbers of Students (N) for Test Scores of Independent Study (IS) and Lecture (L) Students

Variable	Treatment Group	Mean	S. D.	N	t ratio
X_2 : Age	IS	20.35	2.03	64	.05
	L	20.23	2.35	60	
X_3 : Numerical Ability	IS	79.77	16.79	61	.96
	L	82.54	14.10	54	
X_4 : Abstract Reasoning	IS	74.00	21.63	60	.16
	L	73.30	24.83	54	
X_5 : Wonderlic	IS	81.58	17.44	31	.29
	L	82.91	18.40	32	
X_6 : Van Wagenen	IS	222.50	51.86	30	.04
	L	221.94	66.58	31	
X_7 : Costello I	IS	7.37	2.08	62	1.75
	L	6.66	2.35	58	
X_8 : Costello II	IS	6.82	2.61	62	.12
	L	6.76	2.82	58	
X_9 : 16 PF Factor A	IS	8.65	2.96	61	1.92
	L	7.55	3.26	58	
X_{10}: 16 PF Factor B	IS	8.66	1.60	61	1.54
	L	9.12	1.66	58	
X_{11}: 16 PF Factor C	IS	14.72	3.67	61	1.02
	L	14.02	3.80	58	
X_{12}: 16 PF Factor E	IS	12.68	4.13	61	1.51
	L	13.90	4.66	58	
X_{13}: 16 PF Factor F	IS	14.00	4.25	61	.41
	L	14.36	5.19	58	
X_{14}: 16 PF Factor G	IS	11.39	3.56	61	.12
	L	11.47	3.81	58	
X_{15}: 16 PF Factor H	IS	11.87	6.19	61	.96
	L	12.93	5.89	58	
X_{16}: 16 PF Factor I	IS	9.98	3.40	61	.06
	L	10.02	4.18	58	
X_{17}: 16 PF Factor L	IS	9.44	3.01	61	.07
	L	9.48	3.21	58	
X_{18}: 16 PF Factor M	IS	12.33	2.79	61	.02
	L	12.34	3.44	58	
X_{19}: 16 PF Factor N	IS	8.75	2.92	61	1.53
	L	7.90	3.14	58	
X_{20}: 16 PF Factor O	IS	12.33	3.90	61	1.54
	L	11.21	4.01	58	
X_{21}: 16 PF Factor Q_1	IS	10.89	2.91	61	1.09
	L	10.29	3.08	58	
X_{22}: 16 PF Factor Q_2	IS	11.59	3.26	61	1.32
	L	12.41	3.52	58	

Table V (continued)

	Variance	Treatment Group	Mean	S. D.	N	$\underline{t}$ ratio
X_{23}:	16 PF Factor Q_3	IS	11.74	2.90	61	.22
		L	11.86	3.05	58	
X_{24}:	16 PF Factor Q_4	IS	13.52	4.54	61	.06
		L	13.47	4.74	58	
X_{25}:	16 PF Exvia	IS	4.51	1.69	61	.61
		L	4.72	2.05	58	
X_{26}:	16 PF Anxiety	IS	5.95	1.72	61	.30
		L	5.86	1.60	58	
X_{27}:	16 PF Cortertia	IS	5.40	1.95	61	1.65
		L	5.99	1.94	58	
X_{28}:	16 PF Independence	IS	5.75	1.70	61	.84
		L	6.03	1.91	58	
X_{29}:	MAT Career I	IS	5.07	1.47	60	1.36
		L	4.70	1.45	56	
X_{30}:	MAT Home I	IS	4.63	1.62	60	.68
		L	4.86	1.99	56	
X_{31}:	MAT Fear I	IS	4.58	1.87	60	1.61
		L	5.14	1.87	56	
X_{32}:	MAT Narcism I	IS	5.52	2.25	60	1.47
		L	4.96	2.23	56	
X_{33}:	MAT Superego I	IS	4.55	1.61	60	.53
		L	4.39	1.62	56	
X_{34}:	MAT Self-concept I	IS	5.13	1.90	60	0.00
		L	5.13	1.78	56	
X_{35}:	MAT Mating I	IS	5.92	2.00	60	.28
		L	5.82	1.80	56	
X_{36}:	MAT Pugnacity I	IS	5.85	2.04	60	.21
		L	5.77	2.06	56	
X_{37}:	MAT Assertiveness	IS	4.22	1.83	60	.29
		L	4.32	1.86	56	
X_{38}:	MAT Sweetheart I	IS	5.10	1.92	60	.09
		L	5.07	1.80	56	
X_{39}:	MAT General Autism	IS	4.77	1.84	60	.98
		L	4.45	1.66	56	
X_{40}:	MAT General Intelligence-Information	IS	6.00	1.46	60	.79
		L	5.79	1.42	56	
X_{41}:	MAT Total Integration	IS	5.23	1.83	60	.14
		L	5.09	1.55	56	
X_{42}:	MAT Total Interest	IS	5.08	1.56	60	.27
		L	5.16	1.66	56	
X_{43}:	MAT Total Conflict	IS	7.55	1.32	60	.34
		L	7.63	1.25	56	

*$p < 0.05$

means, are included. The t ratios of the student characteristic means indicate there are no significant differences between the two randomly assigned groups as far as student characteristics are concerned.

NULL HYPOTHESIS 1

> The correlation coefficient between a given student characteristic and achievement scores is not significantly different from zero at the .05 per cent level of confidence.

This hypothesis was repeated in each setting (lecture and independent study) for each student characteristic and stated separately for six sets of achievement scores (four term tests, final examination, and the Co-operative Mathematics Test).

Appendix K gives the Pearson correlation coefficients showing the "closeness" of a linear relationship between the criterion achievement tests and the factors measured by the data gathering instruments of this study. Table VI gives the correlations for those factors which have significant (at the 0.05 level) correlations with one or more achievement tests for the lecture group. The symbols, X_3, X_4, etc. represent the same variables as indicated in Table V.

The factor most consistently highly correlated with achievement tests in Table VI was X_6 (<u>Van Wagenen Rate of Comprehension Scale</u>). X_6 was significantly correlated in a positive sense with Test 2, Test 3, Test 4, the CMT, and the Final. This implies that the students with higher scores on this reading comprehension instrument tended to score higher calculus test scores in the lecture setting.

Table VI

Correlations of Selected Student Characteristic Scores with the Term Test Scores, Co-operative Mathematics Test (CMT) Scores, and Final Examination Scores For LECTURE Students

Test	X_3	X_4	X_5	X_6	X_{13}	X_{14}	X_{15}
Test 1	0.493*	0.355*	0.130	0.095	-0.183	0.108	-0.158
Test 2	0.425*	0.211	0.272	0.483*	-0.304*	-0.103	-0.427*
Test 3	0.434*	0.359*	0.324	0.528*	-0.368*	0.097	-0.259
Test 4	0.316	0.043	0.248	0.532*	-0.425*	0.188	-0.304
CMT	0.335*	0.317	0.448*	0.528*	-0.344*	0.330*	-0.290
Final	0.246	0.282	0.379	0.562*	-0.326	0.204	-0.180

Test	X_{17}	X_{19}	X_{21}	X_{25}	X_{39}	X_{40}	X_{42}
Test 1	-0.100	0.023	0.436*	-0.080	-0.295*	0.061	-0.042
Test 2	-0.169	0.033	0.092	-0.321*	-0.550*	0.027	-0.311*
Test 3	-0.225	0.174	0.200	-0.235	-0.012	-0.131	-0.120
Test 4	-0.360*	0.254	0.160	-0.295	-0.391*	-0.396*	-0.530*
CMT	-0.387*	0.346*	0.099	-0.268	-0.474*	-0.134	-0.369*
Final	-0.484*	0.252	0.128	-0.240	-0.301	-0.111	-0.310

* $p < 0.05$

X_3 (DAT Numerical Ability Score), X_4 (DAT Abstract Reasoning Score), and X_5 (Wonderlic Score) were all positively related with all the criterion achievement tests. Significant relationships existed between [illegible] Test 1, Test 2, Test 3, and the CMT; between X_4 and Test 1 and Test 3; between X_5 and the CMT. The importance of the aptitude factors agrees with the review of the literature pertaining to aptitude factors. In particular, these results agree with the findings of Leidtke (1971) where intelligence scores and reading ability were significantly related to the academic success of grade five students in a teacher directed setting.

There were consistent negative correlations between X_{13} (16 PF Factor F--Sober versus Happy-go-lucky) and the criterion tests. Significant correlations occurred with Test 2, Test 3, Test 4, and the CMT. Factor F is one of the most important components of the second-order factor, Exvia (Introversion versus Extraversion). The negative correlations indicate that the sober, serious, concerned, or cautious student tended to achieve higher scores in the lecture setting than the talkative, happy-go-lucky, enthusiastic, or heedless person.

X_{14} (16 PF Factor G--Expedient versus Conscientious) was significantly related to the Co-operative Mathematics Test. This same positive relationship held with the other tests except for Test 2; however, the correlations were low. Cattell et al. (1970) indicated, as one would expect, a positive relationship between Factor G and academic achievement.

X_{15} (16 PF Factor H--Shy versus Venturesome) was negatively correlated with all the achievement tests and had a significant negative relationship with Test 2. This relationship implies that the shy, restrained student tended to achieve higher scores than the adventurous, responsive, friendly, impulsive, or carefree student in the lecture situation.

X_{17} (16 PF Factor L--Trusting versus Suspicious) was negatively correlated with all the achievement tests. Factor L was significantly negatively correlated with Test 4, the CMT, and the Final. Cattell et al. (1970) reported that in group dynamic experiments the high scoring L person was rated unpopular, and groups averaging high on L were significantly less cohesive, and low on morale. The lecture setting is basically a group setting and the negative correlations indicate that students scoring low on L tended to achieve higher calculus scores than those with higher L scores.

X_{19} (16 PF Factor N--Forthright versus Shrewd) and X_{21} (16 PF Factor Q_1--Conservative versus Experimenting) were both positively correlated with all the criterion tests. Factor N was significantly related to the CMT and Factor Q_I was significantly related with Test 1.

X_{25} (16 PF Exvia--Introversion versus Extraversion) was negatively correlated with all the achievement tests. The relationship was significant with Test 2.

The MAT General Autism-Optimism score, X_{39}, was significantly related (negatively) to four of the six achievement tests. This score is a measure of wishful thinking of a person as applied to his own cognitions. High scores indicate a general feeling of well being and a rosy outlook on life. Sweney (1969) pointed out that a depressed score on this factor is an indicator of the "loser's syndrome." However, in the lecture setting of this study, students with lower scores tended to achieve higher calculus scores than those with higher General Autism scores.

Negative correlations occurred between X_{42} (MAT Total Personal Interest) and achievement tests. X_{42} was significantly related to Test 2, Test 4, and the CMT. The relationship with the Final was nearly significant. Sweney (1969) indicated that this score is, operationally, a measure of total motivation and life interest.

In the lecture setting, Null Hypothesis 1 is rejected for the 32 cases in Table VI where significant correlations occurred.

Table VII gives the correlations for those factors which had significant (at the 0.05 level) correlations with one or more achievement tests for the independent study group.

The factor most consistently highly correlated with the achievement tests in Table VII was X_8: Costello Scale II. X_8 scores were significantly related to Test 2, Test 4, and the Co-operative Mathematics Test scores. Scale II measures the disposition or need of a person to be a success defined in terms of the emulation of successful people.

Consistent negative correlations occurred between X_{13} and the achievement tests. X_{13} is the 16 PF Factor F (Sober versus Happy-go-lucky). According to Cattell et al. (1970), Factor F is one of the most important components in the second-order factor of Exvia (Introversion versus Extraversion). The negative correlations for Factor F (significant with the Final) indicate that the sober, serious, concerned, or cautious student tended to achieve higher scores in the independent study setting than the talkative, happy-go-lucky, enthusiastic, or heedless

Table VII

Correlations of Selected Student Characteristic Scores with the Term Test Scores, Co-operative Mathematics Test (CMT) Scores, and Final Examination Scores for INDEPENDENT STUDY Students

Test	X_7	X_8	X_{13}	X_{21}
Test 1	-0.340*	0.279	-0.104	0.020
Test 2	-0.081	0.318*	-0.182	-0.126
Test 3	-0.176	0.214	-0.306	0.016
Test 4	0.042	0.477*	-0.141	-0.297
CMT	0.132	0.421*	-0.178	-0.351
Final	0.328	0.359	-0.555*	-0.417*

Test	X_{22}	X_{24}	X_{26}	X_{42}
Test 1	0.339*	0.246	0.276	-0.201
Test 2	0.126	0.406*	0.357*	-0.318*
Test 3	0.335	0.036	0.079	-0.305
Test 4	-0.105	0.274	0.248	-0.338
CMT	0.345	0.219	0.193	0.041
Final	0.201	0.097	0.160	-0.354

*$p < 0.05$

person.

X_{21} (16 PF Factor Q_1--Conservative versus Experimenting) was significantly correlated (negatively) with the Final. There was also a negative correlation with Test 4 and the CMT. The negative correlations are in contrast to the positive correlations noted in the lecture setting.

X_{22} (16 PF Factor Q_2--Group Dependent versus Self-sufficient) was significantly correlated (positively) with Test 1 and was nearly significant with Test 3 and the Co-operative Mathematics Test. Cattell et al. (1970) claimed that Factor Q_2 is a major component of the second-order factor, Exvia.

X_{24} (16 PF Factor Q_4--Relaxed versus Tense) was positively related with achievement scores; the relationship with Test 2 was significant.

The second-order factor X_{26} (16 PF Anxiety--Low versus High) was positively related with the achievement tests; there was a significant relationship with Test 2.

As with the lecture group, the MAT Total Personal Interest factor, X_{42}, was negatively correlated with all achievement tests except the CMT. The relationship was significant with Test 2 and nearly significant with Test 3, Test 4, and the Final.

Notice that X_7 (Costello Scale I) was significantly correlated in a negative sense with Test 1, whereas the relationship became positive (nearly significant) with the Final.

In the independent study setting, Null Hypothesis 1 is rejected for the 11 cases in Table VII where significant correlations occurred.

The following remarks relate to the correlations for the independent study students as seen in Appendix K.

There were consistent negative correlations between X_{11} and the achievement tests, although none of the correlations was significant. X_{11} is the 16 PF Factor C (Affected by Feelings versus Emotionally stable). The relationships indicate that the emotionally less stable student tended to achieve higher scores in the calculus achievement tests.

X_{15} (16 PF Factor H--Shy versus Venturesome) was negatively correlated, although very low, with all the achievement tests. In the lecture group, this factor also correlated negatively with the achievement scores. The probability of observing 12 negative correlations by chance alone would be $(1/2)^{12}$, which is quite small.

The second-order factor X_{25} (16 PF Exvia--Introversion versus Extraversion) correlated negatively with all the achievement tests. These results agree with the review of the literature claim for the academic superiority of the introvert in higher education. The same negative correlations were noted in the lecture setting.

The MAT Integrated (I) Mating factor, X_{35}, was negatively correlated (low) with the achievement tests. Negative relationships also existed between the MAT Integrated (I) Sweetheart-Spouse factor, X_{38}, and four achievement tests. These relationships may indicate a tendency for students having a lower mating drive and weaker sentiments toward a wife (husband) or sweetheart to achieve higher scores in calculus in an independent study setting.

NULL HYPOTHESIS 2

The square of the multiple correlation coefficient between a combination of predictor variables (student characteristics) and the criterion achievement variable is not significantly different from zero at the .05 per cent level of significance.

In each setting (lecture and independent study) this hypothesis was repeated for each of the achievement variables.

In each instructional setting, the stepwise regression analysis outlined in Chapter III resulted in prediction equations for each of the achievement tests. Information pertaining to each regression equation is summarized in a table. Each table, Tables VIII through XI, lists the variables that made a significant (0.05 level) contribution to the prediction of the criterion variable (achievement test scores) in the order they were entered into the regression equation. Included, as well, were the corresponding multiple correlations (R) with the criterion, F ratios (F_R) for testing the significance of the squared multiple correlations (R^2), the percentage of the criterion variance accounted for by the predicted scores, the F value testing the significance of the contribution from the variable added at each step, the best prediction equation, the F values testing the significance of the coefficients for each variable in the prediction equation, and the simple correlation coefficient (r) between the predictor variable entered and the criterion variable. A predictor variable was retained in the prediction equation if the F value of the entering variable and the F value (F_R) testing the significance of R^2 exceeded the 0.05 level of confidence.

The Wonderlic and Van Wagenen variables were omitted from the regression analysis because of the low number of scores.

The results pertaining to each regression equation are first given for the lecture students and then reported for the independent study students.

Table VIII relates to the significant predictors of the final examination scores for 36 students in the lecture group. The variables included were:

1. X_{17}: 16 PF Factor L--Trusting versus Suspicious.

2. X_4 : DAT Abstract Reasoning.

The multiple correlation, R, between the two predictor variables and the Final was 0.575, indication that 33 per cent of the variance of the criterion was accounted for by the predictors. The squared multiple correlation was significantly different from zero since the F_R value of 8.16 is greater than the critical F value of 3.29 (0.05 level) for 2 and 33 degrees of freedom and, thus, Null Hypothesis 2 is rejected in this case.

The Final correlated significantly with X_6: Van Wagenen Rate of Comprehension and X_{17}: 16 PF Factor L (see Table VI). The correlation between the Van Wagenen scores and Factor L was -0.052. Thus, if the Van Wagenen could have been included in the regression equation, variance accounted for may have been considerably more than the 33 per cent.

Factor F was correlated significantly with most of the criterion variables but did not appear in the regression equation because of its

Table [illegible]

Significant Predictors of LECTURE FINAL Examination Scores in the Order Entered During Stepwise Regression (N = 36)

Step m	Predictors in Regression Equation	Multiple Correlation R	F_R	Percentage of Variance Accounted For	F	r
1	X_{17}	0.5006	11.37	25.1	11.37	-0.5016*
2	X_{17}, X_4	0.5752	8.16	33.1	3.96	0.2908
Best Prediction Equation			$Y_{FINAL} = 62.75 - 4.26\ X_{17} + 0.33\ X_4$			
Coefficient F Ratios				12.15	3.96	

*$p < 0.05$

X_{17}: 16 PF Factor L

X_4 : DAT Abstract Reasoning

significant correlation (0.311) with Factor L.

The regression equation would predict high Final scores for students with low Factor L scores (more trusting) and high Abstract Reasoning scores.

Significant predictors of the Co-operative Mathematics Test scores are given in Table IX for 36 lecture students. The variables entered were:

1. X_{39}: MAT General Autism.
2. X_{17}: 16 PF Factor L--Trusting versus Suspicious.
3. X_{14}: 16 PF Factor G--Expedient versus Conscientious.
4. X_{28}: 16 PF Independence.
5. X_{11}: 16 PF Factor C--Affected by Feelings versus Emotionally Stable.

The multiple correlation, R, between the four predictor variables and the CMT was 0.746, indication that 56 per cent of the variance of the criterion was accounted for by the predictors. The squared multiple correlation was significantly different from zero since the F_R value of 7.52 is greater than the critical F value of 2.68 (0.05 level) for 4 and 31 degrees of freedom, and, thus, Null Hypothesis 2 is rejected in this case.

Lecture students with low scores on General Autism, Factor L and Factor C along with high scores on Factor G and Independence would achieve high scores on the Co-operative Mathematics Test according to this prediction equation. These students would tend to be trusting, conscientious, emotionally unstable, dependent and pessimistic.

X_3: Numerical Ability was significantly correlated with the

Table IX

Significant Predictors of LECTURE CO-OPERATIVE MATHEMATICS TEST Scores in the Order Entered During Stepwise Regression (N = 36)

Step m	Predictors in Regression Equation	Multiple Correlation R	F_R	Percentage of Variance Accounted For	F	r
1	X_{39}	0.4777	10.05	22.80	10.05	-0.4777*
2	X_{39}, X_{17}	0.5732	8.07	32.90	4.93	-0.3938*
3	X_{39}, X_{17}, X_{14}	0.6519	7.88	42.50	5.37	0.2995
4	X_{39}, X_{17}, X_{14}, X_{28}	0.7087	7.82	50.22	4.81	-0.1211
5	X_{39}, X_{17}, X_{14}, X_{28}, X_{11}	0.7458	7.52	55.62	3.65	0.0515

Best Prediction Equation $Y_{CO\text{-}OP} = 31.59 - 1.91\ X_{39} - 1.77\ X_{17} + 1.06\ X_{14} + 2.05\ X_{28} - 0.58\ X_{11}$

Coefficient F Ratios	10.49	15.23	13.77	7.92	3.65

*$p < 0.05$

X_{39}: MAT General Autism; X_{17}: 16 PF Factor L; X_{14}: 16 PF Factor G; X_{28}: 16 PF Independence;

X_{11}: 16 PF Factor C

Co-operative Mathematics Test (see Table VI). However, Numerical Ability does not appear in the regression equation since General Autism and Numerical Ability were highly correlated.

The regression analysis tables corresponding to the term tests for the lecture group are in Appendix L. Null Hypothesis 2 is rejected for each of the tests since the critical F value was exceeded in every case.

The predictor variables entered into regression for Test 4 were:

1. X_{42}: MAT Total Personal Interest
2. X_3 : DAT Numerical Ability
3. X_{32}: MAT Integrated Narcism

Students with low Total Personal Interest scores, high Numerical Ability, and high Narcism scores would achieve high scores on Test 4 according to the resultant regression equation.

The predictor variables entered into regression for Test 3 were:

1. X_3 : DAT Numerical Ability
2. X_{13}: 16 PF Factor F--Sober vs. Happy-go-lucky
3. X_{30}: MAT Integrated Home
4. X_{18}: 16 PF Factor M--Practical vs. Imaginative
5. X_{10}: 16 PF Factor B--Less Intelligent vs. More Intelligent
6. X_{25}: 16 PF Exvia--Introversion vs. Extraversion.

Students with high Numerical Ability and Factor B (more intelligent) scores along with low Factor F (sober) MAT Home and Factor M (practical) scores would achieve high scores on Test 3 according to the regression equation. The positive contribution of X_{25} to the regression equation is strange in view of the negative correlation between X_{25} and Test 3 scores.

The predictor variables entered into regression for <u>Test 2</u> were:

1. X_{39}: MAT General Autism.
2. X_3 : DAT Numerical Ability.
3. X_{13}: 16 PF Factor F--Sober versus Happy-go-lucky.

Students with low General Autism and Factor F (sober) scores along with high Numerical Ability scores would achieve high Test 2 scores according to the regression equation.

The predictor variables entered into regression for <u>Test 1</u> were:

1. X_3 : DAT Numerical Ability.
2. X_{21}: 16 PF Factor Q_1--Conservative versus Experimenting.
3. X_{13}: 16 PF Factor F--Sober versus Happy-go-lucky.
4. X_{38}: MAT Integrated Sweetheart.
5. X_{35}: MAT Integrated Mating.
6. X_{32}: MAT Integrated Narcism.
7. X_{11}: 16 PF Factor C--Affected by Feelings versus Emotionally Stable.
8. X_{14}: 16 PF Factor G--Expedient versus Conscientious.

Students with low Factor F (sober), MAT Sweetheart, and MAT Narcism scores along with high Numerical Ability, Factor Q_1 (experimenting), MAT Mating, Factor C (emotionally stable), and Factor G (conscientious) would achieve high Test 1 scores according to the regression equation.

Table X gives information relating to the significant predictors of the <u>final examination</u> scores for 27 students in the <u>independent study</u> groups. The variables entered were:

1. X_{13}: 16 PF Factor F--Sober versus Happy-go-lucky.
2. X_8 : Costello Scale II.

Table X

Significant Predictors of INDEPENDENT STUDY FINAL Examination Scores in the Order Entered During Stepwise Regression (N = 27)

Step m	Predictors in Regression Equation	Multiple Correlation R	F_R	Percentage of Variance Accounted For	F	r
1	X_{13}	0.5551	11.13	30.8	11.13	-0.5551*
2	X_{13}, X_8	0.6568	9.10	43.1	5.20	0.3590
Best Prediction Equation			$Y_{FINAL} = 69.66 - 3.40\ X_{13} + 3.05\ X_8$			
Coefficient F Ratios			12.77		5.20	

*$p < 0.05$

X_{13}: 16 PF Factor F

X_8: Costello Scale II

The multiple correlation, R, between the two predictor variables and the Final was 0.657, indication that 43 per cent of the variance of the criterion was accounted for by the predictors. The squared multiple correlation was significantly different from zero as the critical F value at the 0.05 level for 2 and 24 degrees of freedom is 3.40, and, thus, Null Hypothesis 2 is rejected in this case.

Students with low Factor F (sober) scores and high Costello Scale II scores would achieve high Final scores according to the regression equation.

The Final correlated significantly with X_{21}: 16 PF Factor Q_1, as can be seen from Table VII; however, this factor did not make a significant contribution to the regression equation after the above two variables had entered.

Significant predictors of the Co-operative Mathematics Test is given in Table XI for 27 independent study students. The variables entered were:

1. X_8 : Costello Scale II.
2. X_{22}: 16 PF Factor Q_2--Group Dependent versus Self-sufficient.
3. X_{21}: 16 PF Factor Q_1--Conservative versus Experimenting.
4. X_{32}: MAT Integrated Narcism.

The multiple correlation, R, between the four predictors and the CMT was 0.759, indicating that 58 per cent of the variance of the criterion variable was accounted for by the predictors. The squared multiple correlation was significantly different from zero since the F_R value of 7.46 is greater than the critical F value of 2.82 (0.05 level) for 4 and 22 degrees of freedom and, thus, Null Hypothesis 2 is rejected in this

Table XI

Significant Predictors of INDEPENDENT STUDY CO-OPERATIVE MATHEMATICS TEST Scores in the Order Entered During Stepwise Regression (N = 27)

Step m	Predictors in Regression Equation	Multiple Correlation R	F_R	Percentage of Variance Accounted For	F	r
1.	X_8	0.4213	5.40	17.8	5.40	0.4213*
2.	X_8, X_{22}	0.6409	8.36	41.1	9.49	0.3450
3.	X_8, X_{22}, X_{21}	0.7043	7.55	49.6	3.89	-0.3505
4.	X_8, X_{22}, X_{21}, X_{32}	0.7586	7.46	57.6	4.12	0.3012
Best Prediction Equation	$Y_{CO-OP} = 6.84 + 1.03 X_8 + 0.92 X_{22} - 0.75 X_{21} + 0.91 X_{32}$					
Coefficient F Ratios		7.16	11.41	4.55		4.12

*$p < 0.05$

X_8 : Costello Scale II X_{22}: 16 PF Factor Q_2

X_{21}: 16 PF Factor Q_1 X_{32}: MAT Integrated Narcism case.

Students with high scores on Costello Scale II, Factor Q_2, and Narcism, along with a low score on Factor Q_1, would achieve high scores on the Co-operative Mathematics Test according to the prediction equation. These students would tend to be self-sufficient and conservative with a high need to achieve (as measured by the Costello Scale II) and a desire for the good life.

X_8 is the only variable significantly correlated to the CMT scores and accounts for about 18 per cent of the variance of the CMT scores.

The regression analysis tables corresponding to the term tests for the independent study group are in Appendix L. Null Hypothesis 2 is rejected for each of the term tests for the independent study group.

The predictor variables entered into regression for Test 4 were:

1. X_8 : Costello Scale II.
2. X_{19}: 16 PF Factor N—Forthright versus Shrewd.
3. X_{29}: MAT Integrated Career.
4. X_{21}: 16 PF Factor Q_1—Conservative versus Experimenting.
5. X_{37}: MAT Integrated Assertiveness.
6. X_{17}: 16 PF Factor L—Trusting versus Suspicious,

Students with high scores on Costello Scale II, Factor N (shrewd), Factor Q_1 (experimenting), and MAT Assertiveness along with low scores on the MAT Career and Factor L (trusting) would achieve high scores on Test 4 according to the regression equation.

The predictor variables entered into regression for Test 3 were:

1. X_{18}: 16 PF Factor M—Practical versus Imaginative.
2. X_{34}: MAT Integrated Self-concept.

3. X_{38}: MAT Integrated Sweetheart.

Students with high Factor M (imaginative) scores along with low MAT Self-concept and MAT Sweetheart scores would achieve high scores on Test 3 according to the regression equation.

The predictor variables entered into regression for Test 2 were:

1. X_{24}: 16 PF Factor Q_4--Relaxed versus Tense.
2. X_{42}: MAT Total Personal Interest.
3. X_{29}: MAT Integrated Career.
4. X_{17}: 16 PF Factor L--Trusting versus Suspicious.
5. X_{37}: MAT Integrated Assertiveness.

Students with high Factor Q_4 (tense), MAT Career, and MAT Assertiveness scores along with low MAT Total Personal Interest and Factor L (trusting) scores would achieve higher Test 2 scores according to the regression equation.

The predictor variables entered into regression for Test 1 were:

1. X_7 : Costello Scale I.
2. X_{22}: 16 PF Factor Q_2--Group Dependent versus Self-sufficient.
3. X_8 : Costello Scale II.

Students with low Costello Scale I scores along with high Costello Scale II and Factor Q_2 (self-sufficient) scores would achieve high scores on Test 1 according to the regression equation.

NULL HYPOTHESIS 3

The square of the multiple correlation coefficient between a combination of predictor variables (student characteristics) and the criterion variable of whether a student is or is not a dropout is not significantly different from zero at the .05 per cent level of significance.

This hypothesis was tested for one group of all students and repeated for the lecture and independent study groups separately.

By using the stepwise regression procedure outlined in Chapter III on the data from 112 students (from both the lecture and independent study groups), the following predictor variables were selected:

1. X_3 : DAT Numerical Ability.
2. X_{22}: 16 PF Factor Q_2--Group Dependent versus Self-sufficient.
3. X_9 : 16 PF Factor A--Reserved versus Outgoing.

The criterion variable, y, had a value of 1 if the student was a dropout and a value of 0 if the student was not a dropout. Table XII lists the variables that made a significant (0.05 level) contribution to the prediction of y in the order they were entered into the regression equation. Also included is the same information that was given in the preceding tables for the prediction of achievement scores.

The multiple correlation, R, between the predictor variables and the criterion was 0.405, indication that 16 per cent of the variance of the criterion was accounted for by the predicted scores. The squared multiple correlation was significantly different from zero since for 3 and 108 degrees of freedom the critical value of F at the 0.05 level is 2.70, and thus, Null Hypothesis 3 is rejected in this case.

Students with low scores on Numerical Ability, Factor Q_2 (group dependent) and Factor A (reserved) tended to drop out according to the dropout prediction equation. However, considering that only 16 per cent of the total variance is accounted for by the equation, one can say that

Table XII

Significant Predictors of DROPOUTS in the Order Entered During Stepwise Regression (N = 112)

Step m	Predictors in Regression Equation	Multiple Correlation R	F_R	Percentage of Variance Accounted For	F	r
1	X_3	0.2565	7.75	6.6	7.75	-0.2565*
2	X_3, X_{22}	0.3205	6.24	10.3	4.49	-0.2132*
3	X_3, X_{22}, X_9	0.4052	7.07	16.4	7.94	-0.1288
Best Prediction Equation	$Y_{DROPOUT} = 1.927 - 0.007\,X_3 - 0.047\,X_{22} - 0.045\,X_9$					
Coefficient F Ratios		6.37		10.72	7.94	

*$p < 0.05$

X_3 : DAT Numerical Ability

X_{22}: 16 PF Factor Q_2

X_9 : 16 PF Factor A

the prediction equation is of little use.

Table XIII gives the results of the stepwise regression analysis for predicting <u>dropouts</u> among 58 <u>independent study</u> students. Those variables that made a significant contribution to the prediction of y (y = 1 if the student was a dropout; y = 0 if the student was not a dropout) follow:

1. X_{18}: 16 PF Factor M--Practical versus Imaginative.
2. X_{10}: 16 PF Factor B--Less Intelligent versus More Intelligent.

The multiple correlation, R between the predictor variables and the criterion was 0.408, indicating that 17 per cent of the variance was accounted for by the predicted scores. The squared multiple correlation was significantly different from zero since the critical *F* value for 2 and 55 degrees of freedom, at the 0.05 level, is 3.17 and thus, Null Hypothesis 3 is rejected for the independent study group.

Students with low Factor M (practical) scores and high Factor B (more intelligent) tend to drop out according to the regression equation.

Table XIV gives the results of the stepwise regression analysis for predicting <u>dropouts</u> among 54 <u>lecture</u> students. The variables making a significant contribution are:

1. X_3 : DAT Numerical Ability.
2. X_{35}: MAT Integrated Mating.

The multiple correlation, R, was 0.306, indication that 14 per cent of the variance was accounted for by the prediction equation. The squared multiple correlation was significantly different from zero since the critical *F* value, for 2 and 51 degrees of freedom, at the 0.05 level, is 3.18, and thus, Null Hypothesis 3 is rejected for the lecture group.

Table XIII

Significant Predictors of INDEPENDENT STUDY DROPOUTS in the Order Entered During Stepwise Regression (N = 58)

Step m	Predictors in Regression Equation	Multiple Correlation R	F_R	Percentage of Variance Accounted For	F	r
1	X_{18}	0.3134	6.10	9.8	6.10	-0.3134
2.	X_{18}, X_{10}	0.4087	5.52	16.7	4.54	0.2316
Best Prediction Equation		$Y_{DROPOUT} = 0.588 - 0.063\ X_{18} + 0.081\ X_{10}$				
Coefficient F Ratios			7.49	4.54		

X_{18}: 16 PF Factor M

X_{10}: 16 PF Factor B

Table XIV

Significant Predictors of LECTURE DROPOUTS in the Order Entered During Stepwise Regression (N = 54)

Step m	Predictors in Regression Equation	Multiple Correlation R	F_R	Percentage of Variance Accounted For	F	r
1	X_3	0.3056	5.36	9.34	5.36	-0.3056
2	X_3 X_{35}	0.3772	<u>4.23</u>	14.23	2.91	-0.2509
Best Prediction Equation		$Y_{DROPOUT} = 1.448 - 0.009\ X_3 - 0.057\ X_{35}$				
Coefficient F Ratios			4.72	2.91		

X_3 : DAT Numerical Ability

X_{35}: MAT Integrated Mating

Students with low scores on both Numerical Ability and Mating tended to drop out according to the regression equation.

NULL HYPOTHESIS 4

> There is no significant difference in the mean scores for a given student characteristic between the student dropouts and the students completing the course.

In each setting (lecture and independent study) this hypothesis was repeated for each student characteristic (each aptitude, personality, and motivation factor).

Means and standard deviations of scores from the tests of the student characteristics for the independent study dropouts and the independent study students who wrote the final examination and the Co-operative Mathematics Test (independent study non-dropouts) were computed. In order to test for significance of a difference between the group means the *t* ratio described in Chapter III was used. Table XV lists the student characteristics which had significant group mean differences.

The *t* ratios indicate a significant difference in the means of X_{18}: 16 PF Factor M--Practical versus Imaginative and X_{35}: MAT Integrated Mating.

The independent study dropouts had a lower mean score on Factor M than the independent study non-dropouts. According to the characteristics of Factor M, one can say the independent study dropout tended to be more conventional, more concerned with his or her immediate interests and issues, whereas the student who stayed in the independent study setting tended to be unconventional, absorbed in ideas, imaginative,

Table XV

Means, Standard Deviations (S. D.), and Numbers of Students (N) for Selected Student Characteristic Scores of INDEPENDENT STUDY DROPOUTS and INDEPENDENT STUDY NON-DROPOUTS

Variable	Group	Mean	S. D.	N	t ratio
X_{18}: 16 PF Factor M	Dropout	11.69	3.03	32	2.39^*
	Non-dropout	13.30	2.12	27	
X_{35}: MAT Mating I	Dropout	6.41	1.93	32	2.23^*
	Non-dropout	5.30	1.88	27	

$^*p < 0.05$

and interested in art, theory, and basic beliefs. Cattell et al. (1970) stated:

> In terms of criteria, high M individuals in groups tend to feel unaccepted, but unconcerned . . . . They express significantly more dissatisfaction with the group unity and the group's regard for rules of procedure . . . . Occupationally, high M occurs in artists, researchers, some planning executives, and many editors; low M in occupations requiring mechanical sense, realism, and alertness. [1970:98]

The independent study dropout had a higher MAT Integrated Mating mean score than the independent study non-dropout. This scale measures heterosexual interest directed toward the sexual act itself and, according to Sweney (1969), a high score will indicate in most cases an active participative interest in sexual intercourse. Thus, one might say that the independent study dropout seemed to be more interested in pursuing sex than in studying calculus.

There were no significant differences between the group mean "aptitude" scores such as Numerical Ability, Abstract Reasoning, Wonderlic, Van Wagenen, 16 PF Factor B, and MAT General Intelligence-Information scores. Also, there were no significant differences in the Costello Scales measuring "need to achieve."

Means and standard deviations of scores from the tests of the student characteristics for lecture dropouts and the lecture students who wrote the final examination and the Co-operative Mathematics Test (lecture non-dropouts) were computed. The t ratios in Table XVI show significant differences in the group mean scores for variable DAT Numerical Ability and MAT Integrated Sweetheart.

The lecture dropouts had a lower mean Numerical Ability score than the lecture non-dropouts. There were no significant differences

Table XVI

Means, Standard Deviations (S. D.), and Numbers of Students (N) for Selected Student Characteristic Scores of LECTURE DROPOUTS and LECTURE NON-DROPOUTS

Variable	Group	Mean	S. D.	N	t ratio
X_3 : DAT Numerical Ability	Dropout	75.24	14.70	17	2.51^*
	Non-dropout	85.58	12.48	36	
X_{38}: MAT Sweetheart I	Dropout	5.74	1.71	19	2.02^*
	Non-dropout	4.75	1.77	36	

$^*p < 0.05$

for the other aptitude variables.

The lecture dropouts had a higher MAT Integrated Sweetheart mean score than the lecture non-dropouts. This scale measures more specifically the affectional needs of the subject in relation to a person of the opposite sex. It operates independently of the mating erg. On the basis of the difference observed between the two groups, one might say that the lecture dropouts had, compared to their friends going to lectures, a higher conscious drive for romantic love relationships.

On the basis of the t ratios, one can only reject Null Hypothesis 4 in the independent study setting with regard to the 16 PF Factor M and the MAT Integrated Mating. In the lecture setting, one can reject Null Hypothesis 4 with respect to the DAT Numerical Ability and the MAT Integrated Sweetheart.

NULL HYPOTHESIS 5

There is no significant difference in the mean criterion achievement scores between the lecture and independent study groups.

This hypothesis was repeated for each of the six sets of achievement scores (four term tests, final examination, and the Co-operative Mathematics Test).

Means and standard deviations of achievement test scores for the independent study and lecture groups are given in Table XVII; t ratios, which were used to test for significance of a difference between the group means, are included.

On the basis of the t ratios, one must reject Null Hypothesis 5 for Term Test 2 and Term Test 3. The lecture group had higher mean

Table XVII

Means, Standard Deviations (S. D.), and Number of Students (N) for Achievement Test Scores of the Independent Study and Lecture Groups

Variable	Group	Mean	S. D.	N	$\underline{t}$ ratio
X_{44}: Term Test 1	IS	59.77	22.52	53	1.12
	L	64.28	19.48	57	
X_{45}: Term Test 2	IS	55.43	21.80	47	3.11*
	L	67.32	14.61	47	
X_{46}: Term Test 3	IS	54.40	27.41	30	2.82*
	L	70.48	17.72	42	
X_{47}: Term Test 4	IS	62.25	23.16	28	0.42
	L	60.03	18.53	40	
X_{48}: Co-operative Mathematics Test	IS	21.46	7.02	28	0.94
	L	23.18	7.85	39	
X_{49}: Final Examination	IS	44.50	26.81	28	0.33
	L	46.69	27.16	39	

*$p < 0.05$

scores on all tests except for Term Test 4.

NULL HYPOTHESIS 6

There is no significant difference in the dropout ratios between the lecture and independent study groups.

Table XVIII gives the numbers of students involved in the successive stages of the calculus course.

Table XVIII

Number of Independent Study (IS) Students, Number of Lecture (L) Students, and Total Number of Students Involved in the Successive Stages of the Calculus Course

	1	2	3	4	5	6	7
		Test 1	Test 2	Test 3	Test 4	CMT and Final	Dropouts
IS	67	53	47	30	28	28	39
L	62	57	47	42	40	39	23
Total	129	110	94	72	68	67	62

Column 1 shows the final experimental population sizes resulting from the random assignment of the students into the two groups.

Column 2 shows the number of students who wrote the first term test or Test 1. Notice that 14 independent study students failed to write Test 1 whereas only 5 lecture students did not write this test. Some of the students failed to show up because they acquired jobs and others because of changes in their educational plans. Most of the students in the independent study group had previously experienced only the lecture approach or variations of the lecture approach to instruction; thus, some of these students dropped the course because they felt they could not learn or were not prepared to attempt to learn in this new environment. This last statement partially explains the large number of independent study students failing to write Test 1.

Columns 3 to 6 show the numbers of students writing Test 2, Test 3, Test 4, the Co-operative Mathematics Test (CMT), and the final examination (Final). There were a large number of dropouts in the independent study group between Test 2 and Test 3. This drastic drop was probably due to several factors which will be explained in the next chapter.

Column 7 gives the number of dropouts for each of the as well as the total number of dropouts. The number of dropouts is simply the total number in Column 1 minus the number of students who wrote the CMT and the Final in Column 6. Sixty-two out of 129 (48 per cent) of the students became dropouts. The percentage of IS students who dropped out was 58, as compared to 37 per cent of the lecture students. Using a z ratio to compare the dropout ratios of the two groups, a z value of 2.4 was found, which is significant at the 0.05 level of confidence. Thus, Null Hypothesis 6 is rejected.

CHAPTER V

SUMMARY, CONCLUSIONS, LIMITATIONS, AND IMPLICATIONS

INTRODUCTION

This study has been concerned with mathematics curriculum development and methodology of the teaching of calculus. The researcher has adapted mastery learning features of Carroll (1963) and Bloom (1968) to a one-semester, first-year university calculus course resulting in an independent study approach to the learning of calculus. An implicit attitude which led to the present study was that there is no one best method of instruction for all students. In fact, it was assumed that certain teaching methods were best for certain kinds of students defined in terms of capabilities, personalities, aspirations, and backgrounds. Thus, the primary purpose of this investigation was to determine student characteristics important to the learning of calculus in a traditional lecture setting and to contrast these with student characteristics important to the learning of calculus in the independent study setting. Student characteristics were classified under the three general areas of aptitude, personality, and motivation.

SUMMARY OF THE STUDY

Two calculus classes of randomly assigned, first-year university students participated in the study at Mount Royal College during the

1973 spring semester. One class of 67 students received their instruction by an independent study approach and the other class of 62 students by a lecture approach. The study was designed to assess the importance of student characteristics with respect to the learning of calculus in each of the two learning settings and to describe the learning outcomes of the two learning methods.

At the beginning of the semester, the students were administered a test battery consisting of the Numerical Ability and Abstract Reasoning tests from the Differential Aptitude Tests (DAT) battery, Cattell's Sixteen Personality Factor Questionnaire (16 PF), Cattell's Motivation Analysis Test (MAT), and Costello's Achievement Motivation Scales I and II. Scores from the Wonderlic Personnel Test and the Van Wagenen Rate of Comprehension Scale were obtained from the Educational Development Services of Mount Royal College. Mathematical achievement scores were obtained during the semester by four term tests, a final examination, and the Co-operative Mathematics Test (Calculus).

SUMMARY OF RESULTS AND CONCLUSIONS

The results and conclusions are summarized as answers to the questions posed in Chapter I.

1. What student characteristics can be used to predict calculus achievement scores in a lecture setting?

One of the most important factors for predicting calculus achievement in the lecture setting was DAT Numerical Ability. This factor was significantly correlated with 4 of the 6 criterion achievement variables and appeared in 4 of the 6 regression equations. The DAT Abstract Reasoning was significantly correlated with Numerical Ability and the two factors were never in a regression equation together. DAT Abstract Reasoning was one of the two factors in the regression equation for the Final. The Van Wagenen Rate of Comprehension was significantly correlated with all achievement tests except for Test 1; however, due to the small number of available scores, this factor was not entered into the regression analysis. Thus, aptitude factors played a strong predictive role with respect to achievement in a lecture setting.

The 16 PF Factor F (Sober versus Happy-go-lucky) appeared as the other most important factor in the lecture setting. Factor F was significantly correlated with 4 of the 6 achievement tests and appeared in three regression equations. As mentioned earlier, Factor F is a major component of the 16 PF second-order factor, Exvia. Exvia was negatively correlated with all the achievement tests. Thus, the students with low scores on Factor F and Exvia tended to achieve higher scores in calculus. These students tended to be serious, introspective, and concerned or more generally introverted, as measured by the 16 PF scales.

In the lecture group, the MAT General Autism factor seemed to be an important factor. It was significantly correlated in a negative way with 4 of the 6 achievement tests and was a predictor variable in two regression equations. General Autism was significantly correlated (positively) with another important factor, MAT Total Personal Interest. This factor appeared in the regression equation for Test 4, while General Autism was absent. Students with low scores on both General Autism and Total Personal Interest tended to achieve higher calculus achievement scores. This is the opposite of what one may expect from the description of these two factors given by Sweney (1969). General Autism or Optimism is a "measure of wishful thinking applied to one's cognitions" (Sweney, 1969:21), with high scores indicating greater optimism or a rosier outlook on life. Sweney (1969) indicated that low scores on this factor is an indication of the "loser's syndrome." A high Total Personal Interest score is, supposedly, indication of high motivation and life interest.

The 16 PF Factor L (Trusting versus Suspicious) was significantly correlated (negatively) with three achievement tests and appeared in two regression equations, being the most important factor in the regression equation for the Final. Factor L was significantly correlated (0.311) with Factor F and the factors never appeared in a regression equation together, although one or the other appeared in 5 of the 6 regression equations predicting achievement scores. A low score on Factor L indicated a trusting, friendly, easy-going, and relaxed person, according to the verbal description of this factor. Such a person tended to achieve higher scores in calculus in this study.

16 PF Factor G (Expedient versus Conscientious) appeared as a factor in two regression equations for the lecture group. Students with high scores (conscientious) tended to achieve higher achievement scores.

Thus, factors from the aptitude, personality, and motivation domains played predictive roles with respect to calculus achievement in this study. DAT Numerical Ability, DAT Abstract Reasoning, and the Van Wagenen Rate of Comprehension were the important aptitude predictor variables. The more important personality and motivation predictors were Factors F, L, and G of the 16 Personality Factor Questionnaire and General Autism and Total Personal Interest of the Motivational Analysis Test.

On the basis of this investigation, it would be predicted that students with high aptitude scores, low Factor L and Factor F scores, high Factor G scores, and low General Autism and Total Personal Interest scores should achieve the higher calculus achievement scores in the lecture setting. The percentage of variance accounted for by the predictor variables of the regression equation ranged from 33 per cent for the Final to 67 per cent for Test 1.

2. <u>What student characteristics can be used to predict calculus achievement scores in an independent study setting?</u>

The most important factor for predicting calculus achievement scores in the independent study setting was <u>Costello Scale II</u>. This factor was significantly correlated with 3 of the 6 criterion achievement variables and appeared in 4 of the 6 regression equations. According to Costello (1967) this scale measures the disposition or need of a person to be a success defined in terms of the emulation of successful people. In both the lecture and independent study groups this factor correlated significantly with 16 PF Factor G (Expedient versus Conscientious), 16 PF Factor C (Affected by Fellings versus Emotionally Stable), 16 PF Anxiety, 16 PF Factor Q_4 (Relaxed versus Tense), and MAT Total Personal Interest. On the basis of these correlations one may be able to say that students scoring high on the Costello Scale II tended to be more conscientious, more affected by feelings or emotionally unstable, more tense and more anxious with less interest in the external world than students scoring low on this scale. Two of the above variables (Factor C and Factor C) did not appear in the regression equations which was probably due to suppression by the Costello Scale II.

Two factors that are major components of the 16 PF second-order factor of Exvia (Introversion versus Extraversion) emerged as important factors in the prediction of calculus achievement. These factors were Factor F (Sober versus Happy-go-lucky) and Factor Q_2 (Group Dependent versus Self-sufficient). Factor F was in the regression equation for the Final and was negatively correlated with all the achievement tests. Factor Q_2 was in the regression equation for Test 1 and the Co-operative Mathematics Test. Students with low scores on Factor F and high scores on Factor Q_2 tended to achieve higher scores in calculus. The verbal description of these factors implies that these students tended to be serious, introspective, and self-sufficient. Low scores on Factor F and high scores on Factor Q_2 are also typical for introverts or low scorers on the Exvia factor.

16 PF Factor Q_1 (Conservative versus Experimenting) appeared as an important factor being significantly correlated (negatively) with the Final and appearing in the regression equation for the CMT. Students with low Factor Q_1 scores (conservative) tended to achieve higher calculus scores.

It is notable that there were no aptitude variables in the regression equations predicting achievement scores in the independent study group. The absence of significant positive correlations between the aptitude variables and achievement scores for the independent study group agreed with the results of a preliminary report in Appendix J. The importance of the Costello Scale II in the independent study setting was also in agreement with the preliminary study.

Notice that, apart from the Costello Scale II factor, the most important predictor variables were from the personality domain, namely, Factors F, Q_2, and Q_1. The percentage of variance accounted for by the regression equations ranged from 31 per cent for Test 1 to 76 per cent for Test 4.

3. What are the characteristics of the dropouts in the lecture setting?

In the lecture setting, there was a significant difference in the mean DAT Numerical Ability and MAT Integrated Sweetheart scores of the dropouts and non-dropouts. The dropouts had a lower mean Numerical Ability score and a higher mean Sweetheart score. The dropout prediction equation had DAT Numerical Ability and MAT Integrated Mating as independent variables; however, the equation accounted for only 15 per cent of the variance in the criterion scores.

It is tentatively concluded that lower Numerical Ability scores was the most important characteristic of the lecture dropout. Other characteristics appeared to be a lower conscious sex drive (Mating factor) and stronger sentiments for romantic love relationships (Sweetheart factor).

4. What are the characteristics of the dropout in the independent study setting?

In the independent study setting, there was a significant difference in the mean 16 PF *Factor M* (Practical versus Imaginative) and MAT *Integrated Mating* scores of the dropouts and non-dropouts. The dropouts had a *lower* mean score on Factor M and a *higher* mean score on the Mating factor. The dropout prediction equation had 16 PF *Factor M* and 16 PF *Factor B* (Less Intelligent versus More Intelligent) as independent variables; however, the equation accounted for only 17 per cent of the variance in the criterion scores.

It is concluded, again tentatively, that lower Factor M scores was the most important characteristic of the independent study dropouts. The verbal description of Factor M indicates that the dropout tended to be more practical, conventional, and concerned with his or her immediate interests than the non-dropout. A higher heterosexual drive and higher intelligence score appeared to be other characteristics of the independent study dropout. The experience of the researcher in teaching calculus via the independent study approach would confirm results that the independent study dropouts tended to be more intelligent. It seems that the more intelligent student may be able to pass a calculus course with less effort and hard work by the lecture approach.

It is interesting to note that a *personality* factor (M) was the most important factor distinguishing the independent study dropout from the non-dropout whereas an *aptitude* factor (Numerical Ability) was the most important factor distinguishing the lecture dropout from the non-dropout. This finding concurs in part with the achievement prediction findings discussed earlier in this chapter.

In contrasting lecture and independent study dropout groups, significant differences were found for two variables, MAT Mating and

Narcism, with the independent study group having higher scores on both variables. From this one could conclude that the independent study drop-outs had a stronger preference for the "finer things" in life, and had a stronger drive toward self-indulgence including sex than did the lecture dropouts.

5. Is there any difference in the mean achievement scores of the independent study and lecture groups?

There was a significant difference in the mean scores of the two groups with respect to Test 2 and Test 3. The lecture students achieved the higher mean scores. It must be acknowledged that the two groups wrote different term tests and each instructor marked the test papers of his group. However, the two groups wrote the same Co-operative Mathematics Test and had a common final examination, with the researcher marking these tests. There was no significant difference in the mean scores on either the Co-operative Mathematics Test or the Final.

The importance of formative evaluation and feedback/correction as stressed by Airasian, Bloom, and Block, was cited in the review of the literature chapter. If these two procedures of mastery learning had been more strictly adhered to in this study, then the mean achievement test scores for the independent study students may have been higher. Indeed, on the basis of mastery learning studies and the mean achievement scores of the preliminary study (see Appendix J), the researcher is convinced that the mean achievement scores of the independent study group on the final examination and the Co-operative Mathematics Test could have been significantly higher.

6. Is the dropout ratio of the independent study group different from the dropout ratio of the lecture group?

Defining a dropout as a student who did not take the final exam, there was a significant difference in the percentage of dropouts in the lecture and independent study groups. The independent study group had a dropout ratio of 58 per cent as compared with 37 per cent in the lecture group. The high dropout ratio in the independent study setting is in accord with the high dropout ratio found in the preliminary study and reported in Appendix J.

It was found that very few students in either group dropped out after Test 3. In the lecture group, the largest dropout numbers occurred after Test 1. In the independent study group, the largest dropout numbers occurred prior to Test 1 and between Test 2 and Test 3. The large initial drop in the independent study setting is probably due to the fact that the students had no previous independent study experience and some of these students felt incapable of learning or were not prepared to attempt learning in this new environment.

Test 3 tested the objectives of units 7 to 11 inclusive. These units involve applications of the derivative concept and cause, in the researcher's opinion, greater difficulties for most students than the other units. It was in this portion of the course that seventeen independent study students dropped out. While studying these units, many students need encouragement and frequent feedback on their progress which some students claimed was not readily available from their instructor in this experiment.

LIMITATIONS OF THE STUDY

There are a number of limitations that should be considered in interpreting the results from this study. The first limitation concerns the size of the samples. As a result of failure to acquire data for all students and the large number of dropouts, the sample sizes decreased to the point where data for only 36 and 27 students in the two groups were available for the analysis of the relationships between student characteristics and achievement on the final examination and the Co-operative Mathematics Test.

The second limitation concerns the interaction effects of the two groups, although the extent of such effects is difficult to ascertain. There was interaction to the extent that it was possible for students from both groups to study together and exchange or lend study materials. The independent study students could not attend the lectures of the lecture group but the lecture students could obtain independent study packages from the independent study students; thus, the interaction effect was probably most beneficial for the lecture students. To reduce this study possibility for the lecture students, it was not possible for them to obtain copies of the packages from the instructors. Only one copy of each package was handed out to each independent study student.

The feedback/correction feature of the independent study method was not adhered to during the course of instruction. This could be considered a limitation of the study in that possibly different student characteristics would have emerged in the regression equations if the feedback/correction procedure had been used. The low mean criterion test scores for the independent study group can probably be attributed to lack of feedback/correction.

A fourth limitation is related to the lack of involvement of the instructors in the preparation and selection of course materials. The selection of texts and preparation of objectives, packages, and tests were performed by the researcher. More involvement by the instructors appears desirable to encourage their identification with and commitment to the methods used.

A fifth limitation of the study is in the area of generalizability. The study population can be considered representative of calculus students at Mount Royal College over a time period of several years. Generalizations beyond this population are dangerous. Despite this, it is possible to use the results as the basis of further research and evaluation hypotheses in other settings.

A sixth limitation may be that predictor variables other than aptitude, personality, and motivation factors of the present study should have been used. The aptitude factors used in the regression analysis were restricted to the DAT Numerical Ability and the DAT Abstract Reasoning. Using further aptitude factors may have significantly increased the percentage of variance of the criterion variables accounted for by the prediction equations for the lecture group.

The method of linear regression analysis is a standard technique in predictive studies in education; however, appropriateness of the technique is dependent upon the absence of a non-linear relationship in the regression of any variable upon the criterion as well as among variables. Acceptance of this assumption for the absence of non-linear relationships in this study may be questioned in this investigation. However, the statistical significance of the correlation coefficients indicate that linear relationships exist among the variables.

IMPLICATIONS FOR INSTRUCTION

The main conclusion from this study was that there are student characteristics related to achievement in calculus in the lecture setting that are not so important in an independent study setting, and vice versa. The dropouts in each setting also had distinguishing characteristics. The implications from these results relate to the advising and guidance of the prospective calculus student into one of the two methods of instruction that may be best suited for him or her. Although the prediction equations need to be improved upon, there were certain emergent factors from this study that one can look to with regard to helping students make a choice.

Prior to the advisement of students, the following data-gathering instruments could be administered:

1. Costello Scales I and II. Time: 5 to 6 minutes.
2. Van Wagenen Rate of Comprehension. Time: 4 minutes.
3. Cattell's Sixteen Personality Factor Questionnaire. Time: 60 minutes.
4. Numerical Ability, Abstract Reasoning, and Verbal Ability tests of the Differential Aptitude Test battery. Time: 90 minutes.

Use of these testing instruments would provide student characteristic scores in the three realms of aptitude, personality, and motivation.

The three specific aptitude scores from the DAT, as well as a general aptitude score, which is the sum of the Numerical Ability and Verbal Ability scores, would be available. The Van Wagenen Rate of Comprehension is included because of the significant correlations with achievement in this present study. Personality factor scores would be available from the 16 PF. Motivation scores measuring certain aspects of "need to achieve" would come from the Costello scales.

Notice that Cattell's Motivation Analysis Test was omitted, even though some MAT Factors were related to achievement. General Autism and Total Personal Interest made some significant contributions to the prediction of achievement in the lecture setting; however, these contributions seemed contrary to what one might expect. The instrument requires sixty minutes for administration and the scoring can be very time consuming. Thus, the researcher recommends further research with this instrument before using it to guide students into either a lecture or independent study setting.

Using the data from the recommended measuring instruments, the advisor may be able to assist students by keeping in mind the review of the literature results and the results of this study. Some particular points to consider include:

1. Persons with low 16 PF Factor M (Practical versus Imaginative) scores and high aptitude scores tended to drop out in the independent study setting.
2. Persons with lower aptitude scores tended to drop out in the lecture setting.
3. Persons with high Costello Scale II and 16 PF Factor Q_2

(Group Dependent vs. Self-sufficient) scores tended to achieve higher Calculus scores in the independent study setting.

4. Persons with low 16 PF Factor F (Sober vs. Happy-go-lucky), 16 PF Exvia (Introversion vs. Extraversion), and 16 PF Factor C (Affected by Feelings vs. Emotionally Stable) scores tended to obtain higher calculus achievment scores in the independent study setting.
5. Persons with low 16 PF Factor Q_1 (Conservative vs. Experimenting) scores tended to obtain higher calculus scores in the independent study setting, whereas persons with high 16 PF Factor Q_1 scores tended to obtain higher calculus scores in the lecture setting.
6. Persons with high aptitude scores, including high Van Wagenen scores, tended to obtain higher calculus scores in the lecture setting.
7. Persons with low 16 PF Factor F (Sober vs. Happy-go-lucky) and 16 PF Factor L (Trusting vs. Suspicious) scores tended to obtain higher calculus scores in the lecture setting.

IMPLICATIONS FOR FURTHER RESEARCH

It is recommended that research be continued to further determine what individual differences are relevant to particular instructional methods. Studies similar to the present one are needed, but involving larger samples and different post-secondary institutions. Such studies would possibly justify generalizations to larger populations.

This study was concerned with only two teaching methods and involved measures of forty-two factors determined by specific testing instruments. Other instructional approaches, cognitive and non-cognitive factors, measuring instruments, and analysis procedures need to be included in future studies in order to carry on the task of determining relevant characteristics of the students associated with learning via different instructional modes.

In view of the emergence of personality variables and the Costello Scale II factor as predictor variables in the independent study setting of this investigation, further research involving these variables is necessary. For years, work concentrated on aptitude variables as the explanatory variables for academic achievement; however, personality variables seem to play an important predictive role as well. The fact that the Costello Scales require so little time to administer makes it imperative that these instruments receive more attention.

A more important and detailed problem than the problem of this study is one of identifying, for each student, the most effective sequence and combination of methods for achievement of each of the specific course objectives. We need to think of instruction in terms of the selection of teaching strategies, chosen to fit optimally with specific objectives and student characteristics. Ultimately, we want to be able to say that for this particular student and this particular instructor and this particular course objective we have the best instructional strategy in terms of sequencing and combination of methods. Although this may be an impossible goal, studies which lead to decisions in this realm should prove valuable to students and teachers alike.

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APPENDICES

APPENDIX A

INTRODUCTION TO PURE MATHEMATICS 201 AND SAMPLE SELF-INSTRUCTIONAL PACKAGE* USED BY THE INDEPENDENT STUDY GROUP

*The self-instructional package was prepared by the author and is under copyright by the author.

INTRODUCTION TO PURE MATHEMATICS 201

1. General Objectives

   (a) You will find the study of mathematics enjoyable because of the success you will have in mastering calculus. Failure will not occur if you are motivated to spend the necessary time.

   (b) You will become acquainted with a process of studying the properties of functions through the study of the derivative and the integral.

   (c) You will come to an understanding of the interaction between calculus and "real world" problems.

   (d) You will learn that mathematics is a human activity and its development has come about from sensitive human beings.

   (e) You will learn that mathematics is built upon intuitive understandings and agreed conventions that are not "God-given" or eternally fixed.

   (f) You will learn that complex things are sometimes simple and simple things are sometimes complex and that, in mathematics as in other fields, it pays to study something to detailed to study and to study something which seems hopelessly intricate.

2. Method of Study

   The calculus course has been divided into 19 units with a "package" prepared for each unit. A "package" is a set of notes consisting essentially of:

   (a) Specific objectives: exactly what is expected of you upon completion of the unit.

   (b) References to texts, cassette tapes, and other materials.

   (c) Written explanation of the concepts and procedures involved in each unit.

   (d) Problems for you to do with the solutions provided.

   (e) Post-test.

   The package is designed to <u>guide</u> your learning.

The teaching method is structured so that you will have a combination of lectures, self-study, small group study, and individualized attention.

There will be one lecture per week to provide all students with general instructions, explanation of concepts and an opportunity for asking questions. The lecture sessions will progress at a pace whereby the entire course will be covered in one semester.

Now what do you do besides going to one lecture per week? Well, mainly self-study with help and guidance provided from your instructor and any other mathematics instructor in the mathematics section of the learning library. The learning library is designed so that you can study independently, work in small groups, listen to cassette tapes, use other audiovisual aids, write tests and ask for HELP. You will be able to receive cassette tapes, cassette recorders, reference texts and other materials from the mathematics resource island.

One important feature of the method of instruction is something called "feedback-correction." At the end of each package is a post-test for you to write. Upon completion of this test, you will ask an instructor to mark it for you. You will not receive a grade for this test. You and the instructor will mark the test together. As it is marked, you will discuss the answers and the unit objectives that you have mastered. For the unmastered objectives, the instructor may use several procedures that will enable you to master them. This feature is called feedback-correction because you are given immediate feedback on those objectives you have mastered and at the same time you are given a corrective procedure to master those objectives you missed. Even though no grade is given on the post-test, you must obtain approximately 80% or better mastery before proceeding to the next unit. If you do not achieve this level of mastery, then you will write another test (answering those questions corresponding to the unmastered objectives) after the correction procedures are complete. One important aspect of feedback-correction is the personal contact between the student and instructor. It is important that the instructor know you as a person and you know him as a person if learning is to take place.

In essence, the method of instruction enables you to progress through the course at a rate suited to your mathematical background, your abilities, and your personality. We realize that each one of you is a unique, different human being with different backgrounds, different abilities, different personalities, different motivations, and different personal problems. It is our aim to provide you with the opportunity to achieve a mastery of calculus at a rate best suited to you.

3. Grading Procedure

One of six grades will be awarded at the end of the semester, based upon:

(a) four term tests, and

(b) a final examination.

The grades are: A for superior
B for excellent
C for average
D for low pass
I for incomplete
W for withdrawal.

Your achievement on the term tests and final exam will determine your final grade. The term tests and final exam will count approximately 50% each toward your final grade, with exceptions made depending upon your specific situation. You may challenge the term tests and final exam at almost any time.

On the term tests and the final exam, you will score above 80% for an A, between 65% and 80% for a B, between 50% and 65% for a C, between 40% and 49% for a D, and below 40% for an I.

An I grade may be awarded to you if you do not complete the course by the end of the semester. You may have the I removed by completing the course within a specified period at the end of the semester or by passing the final exam. If you received an I and did not pass the final, then you will have an opportunity to write another final exam on a date within the specified period. IT IS IMPORTANT TO NOTE THAT NO STUDENT WILL BE ALLOWED TO REMOVE HIS/HER I AFTER THE FINAL UNLESS HE/SHE HAS COMPLETED AT LEAST TEN UNITS AND TWO TERM TESTS. Any student not removing an I may register for the course again the following semester and proceed from where he left off in the preceding semester.

Term Test #1 will cover units 1 to 2 inclusive and will take place at the end of the second week.

Term Test #2 will cover units 1 to 7 inclusive and will take place at the end of the fifth week.

Term Test #3 will cover units 1 to 11 inclusive and will take place at the end of the eighth week.

Term Test #4 will cover units 1 to 16 inclusive and will take place at the end of the eleventh week.

If you are unable to write a Term Test, then it is your responsibility to contact your instructor.

Please note that <u>you can take the term tests and final exam earlier than the specified times</u> if you <u>progress at a faster rate</u>. In fact, if you have the motivation to complete the course in half the semester, then you can take the rest of the semester off as far as mathematics is concerned.

4. <u>General Comments</u>

(a) You will be given a complete set of packages. Thus, a large three-ring binder should be purchased to contain the packages.

(b) Cassette tapes and recorders must not be removed from the learning library; however, you may exchange a blank C-60 cassette tape for one "full" cassette tape for your own use.

(c) You may purchase any of the following texts from the bookstore:

(i) R. Johnson and F. Kiokemeister, <u>Calculus and Analytic Geometry</u>, 4th ed. Boston: Allyn and Bacon.

(ii) Protter and Morrey, <u>Calculus with Analytic Geometry: A First Course</u>, 2nd ed. Addison Wesley.

(iii) Serge Lang, <u>A First Course in Calculus</u>, 2nd ed. Addison Wesley.

(iv) Fisher and Ziebur, <u>Calculus and Analytic Geometry</u>, 2nd ed. Prentice Hall.

(v) Lowengrub and Stampfli, <u>Topics in Calculus</u>. Ginn-Blaisdell.

(vi) Louis Leithold, <u>The Calculus Book: A First Course with Applications and Theory</u>. Harper and Row.

There are copies of each of the above texts for your use at the mathematics resource island.

(d) If you are having problems, ask for help <u>as soon as possible</u>. Remember, you have paid your fees and thus have a right to expect help. Don't think of your questions as stupid questions.

(e) <u>Discuss</u> your work with your classmates and/or friends. Two heads are often better than one and trying to expalin a concept to someone helps to clarify it in your mind.

(f) Use a pencil and paper when studying mathematics. You will be amazed (sometimes) at what you can do if you would but write down what is given and any ideas you have pertaining to the question.

(g) Work like hell and you'll enjoy the course more.

PURE MATHEMATICS 201

SELF-INSTRUCTIONAL PACKAGE #8

MAXIMA AND MINIMA

MAXIMA AND MINIMA

A major problem of a manufacturer is how to minimize the cost of production and how to maximize profits. An engineer wants to maximize the strength of structures. A landlord wants to know what rent to charge in order to maximize profits. These are examples of <u>many</u> practical maxima and minima problems that arise in our lives. The derivative is a powerful tool used to help us solve these problems.

Cost, profit, strength, etc. are variables, and variables may be related by means of <u>functions</u>. The study of <u>maxima and minima</u> problems thus entails the study of <u>functions</u> and in particular the study of the <u>derivative of functions</u>. In this package, you will study a number of concepts so that you will be able to attack applications of maxima and minima in the next package.

Objectives

1. You will be able to define:

   (a) increasing function,

   (b) decreasing function,

   (c) relative maximum,

   (d) relative minimum,

   (e) maximum,

   (f) minimum,

   (g) critical point.

2. You will be able to find the intervals on which given functions are increasing and decreasing.

3. You will be able to determine the critical points of given functions.

4. You will be able to determine the points of relative minimum and relative maximum of given functions.

5. You will be able to determine the points of minimum and maximum of given functions.

MAXIMA AND MINIMA

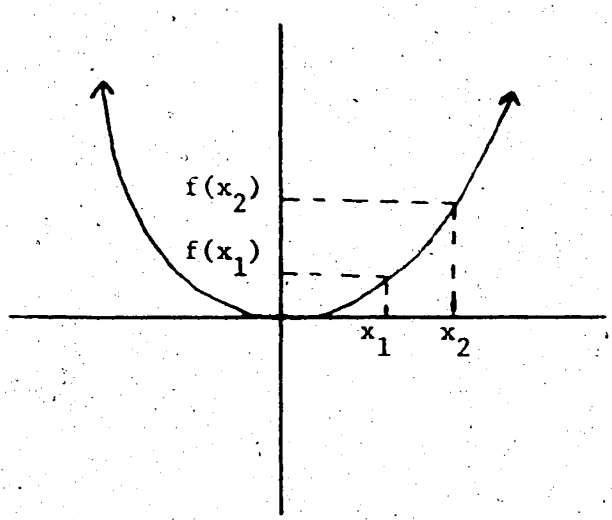
Increasing and Decreasing Functions

Definition 1

A function f is said to be increasing on an interval I if $x_2 > x_1$ implies $f(x_2) > f(x)_1$, where $x_1, x_2 \in I$

(see objective 1a).

Ex. 1



$f(x) = x^2$ is increasing on $[0, \infty)$ since for $x_1, x_2 \in [0, \infty)$ $x_2 > x_1 \Rightarrow f(x_2) > f(x_1)$

Note: x_1 and x_2 are simply values of x along the x-axis. Subscripts are used to distinguish one value of x from another value of x. x_1 reads "x sub one" and is not $x = 1$.

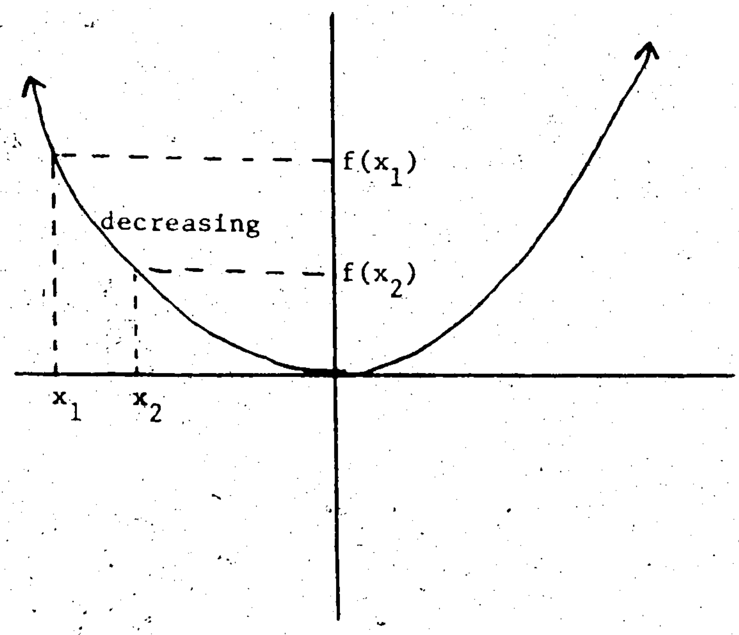
Definition 2

A function f is said to be decreasing on an interval I if for $x_1, x_2 \in I$.

$x_2 > x_1$ implies $f(x_2) < f(x_1)$

(see objective 1b)

Ex. 2



$f(x) = x^2$ is decreasing on the interval $(-\infty, 0]$ since for $x_1, x_2 \in (-\infty, 0]$ $x_2 > x_1 \Longrightarrow f(x_2) < f(x_1)$.

If a function is neither increasing nor decreasing on an interval, it is said to be ______________________________.

constant

ANSWER

Find the intervals on which f is increasing and those on which it is decreasing for each of the following:

(a) $f(x) = x^2 - 4$

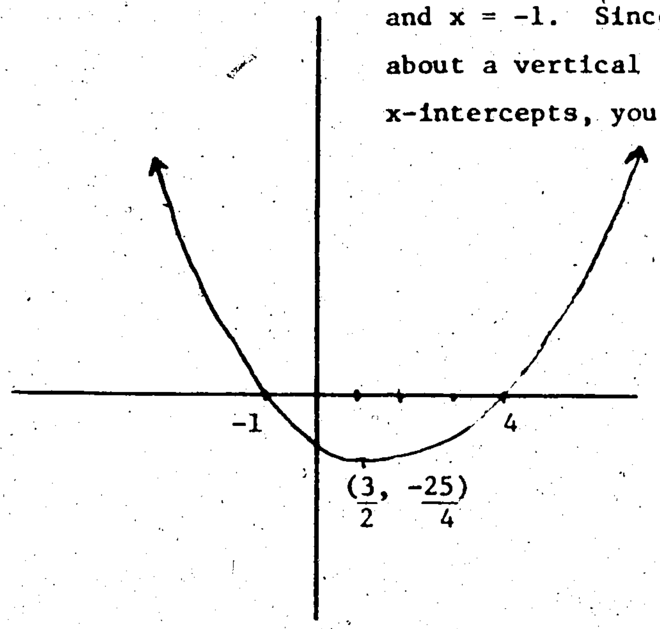
(b) $f(x) = (x - 1)^2 (x + 2)$

(see objective 2)

ANSWERS

(a) $f(x) = x^2 - 3x - 4$
$= (x - 4)(x + 1)$

The graph of the given quadratic function is a parabola crossing the x-axis at $x = 4$ and $x = -1$. Since the parabola is symmetric about a vertical line halfway between the x-intercepts, you can determine the axis of symmetry as $x = \frac{3}{2}$.

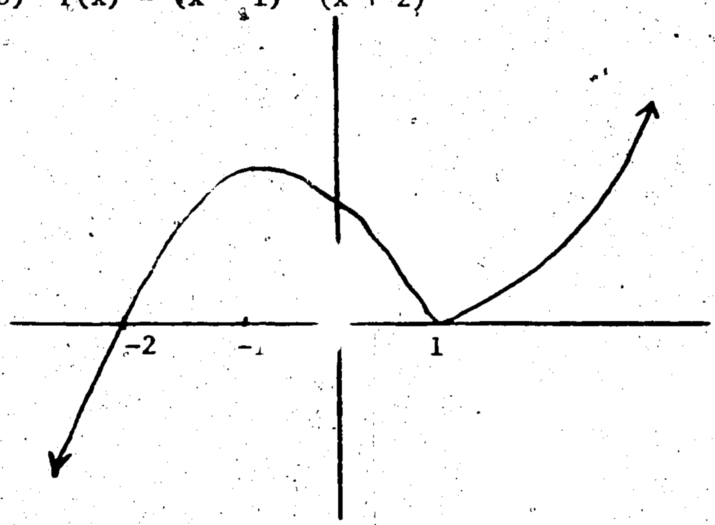


From the graph, we see that

$f(x)$ is increasing on $[3/2, \infty)$

and $f(x)$ is decreasing on $(-\infty, 3/2]$

(b) $f(x) = (x - 1)^2 (x + 2)$



From sketching the graph, we observe that

$f(x)$ is increasing on $(-\infty, -1]$ and $[1, \infty)$

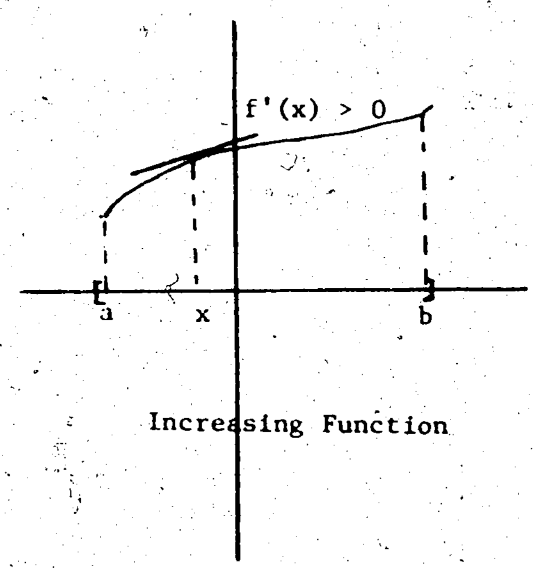
$f(x)$ is decreasing on $[-1, 1]$

Is there an easier way to determine where a function is increasing or decreasing? Let's see.

Refer to the graph of $y = x^2$ in example 1. You can observe that when the slope of the tangent line is positive, the function is increasing and when the slope of the tangent line is negative, the function is decreasing. Since $f'(x)$ is the slope of the tangent line, you have:

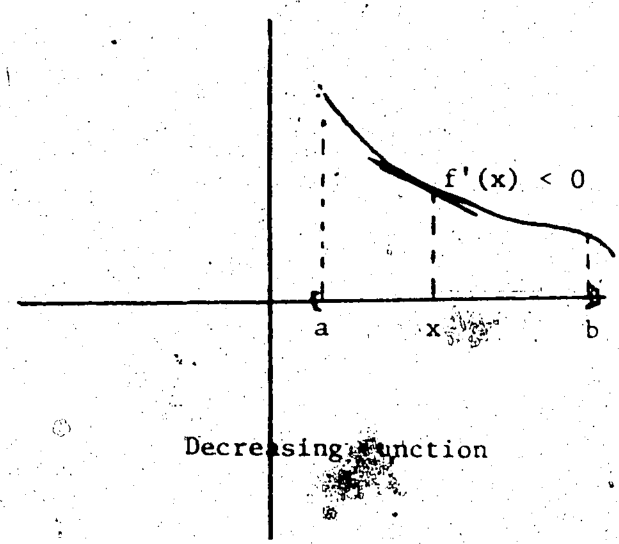
f is <u>increasing</u> on $[a, b]$ if <u>$f'(x) > 0$</u> for all $x \in (a, b)$
f is <u>decreasing</u> on $[a, b]$ if <u>$f'(x) < 0$</u> for all $x \in (a, b)$

<u>Ex. 3</u>



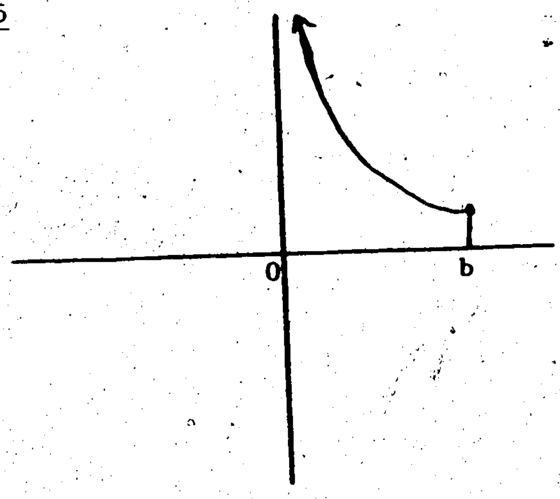
(slopes are positive)

<u>Ex. 4</u>



(slopes are negative)

Ex. 5



$f(x) = \frac{1}{x} \quad 0 < x \geq b$

Here f is decreasing on $(0, b]$ and $f'(x) < 0$ for all $x \in (0, b$

Let's solve the problems on page 140 now by considering the sign of $f'(x)$.

(a) $f(x) = x^2 - 3x - 4$

$f'(x) = 2x - 3$

$f'(x) < 0 \Longrightarrow 2x - 3 < 0$

or $2x < 3$

or $x < 3/2$

Therefore, f is decreasing on $(-\infty, 3/2]$

Also, $f'(x) > 0$ for $x > 3/2 \Longrightarrow f$ is increasing on $[3/2, \infty)$

(b) $f(x) = (x - 1)^2 (x + 2)$

$f'(x) = (x - 1)^2 \cdot 1 + (x + 2) \cdot 2(x - 1)$

$= 3(x - 1)(x + 1)$

$f'(x) < 0 \Longrightarrow (x - 1)(x + 1) < 0$ (You remember, of course, how to solve inequalities.)

One factor is positive and the other negative

$S = x - 1$, and $L = x + 1$

$S < 0$ and $L > 0$

$x - 1 < 0$ and $x + 1 > 0$

$\Longrightarrow x < 1$ and $x > -1$

Therefore, f is decreasing on $[-1, 1]$

Also, $f'(x) > 0 \Longrightarrow x < -1$ or $x > 1$

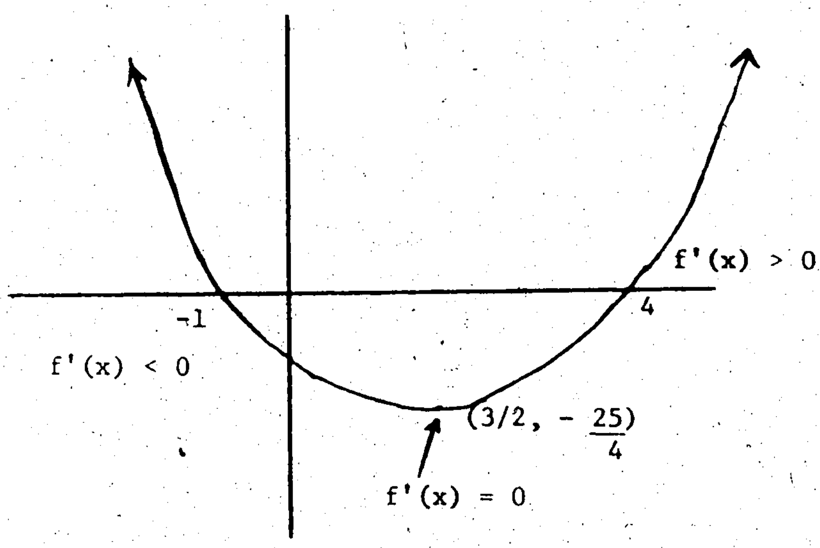
Therefore, f is increasing on $(-\infty, -1] \cup [1, \infty)$

(see objective 2)

Maximum, Minimum, Relative Maximum, Relative Minimum

In the case of $f(x) = x^2 - 3x - 4$, where is $f'(x) = 0$?

$f'(x) = 2x - 3$, and so $f'(x) = 0$ at $x = 3/2$.



Is there a minimum or maximum value of f at $x = 3/2$? ____________

Minimum is correct!

Where do you think $f'(x) = 0$ for the function $f(x) = (x - 1)^2 (x + 2)$? See graph on page 141.

____________________________. Answer: $x = -1, 1$

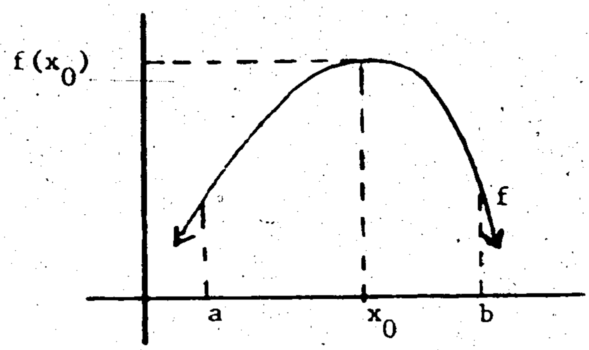
Is there a minimum or maximum value of the function at $x = 1, -1$? The function has neither a minimum nor maximum, but it is said to have a local or relative maximum at $x = -1$ and a local or relative minimum at $x = 1$. You will see the definitions of relative minimum and relative maximum shortly, but first the definitions of maximum and minimum (sometimes referred to as absolute maximum and absolute minimum) will be given.

Definition

A function f is said to have a <u>maximum on an interval I</u> if there exists $x_o \in I$ such that $f(x_o) \geq f(x)$ for all $x \in I$.

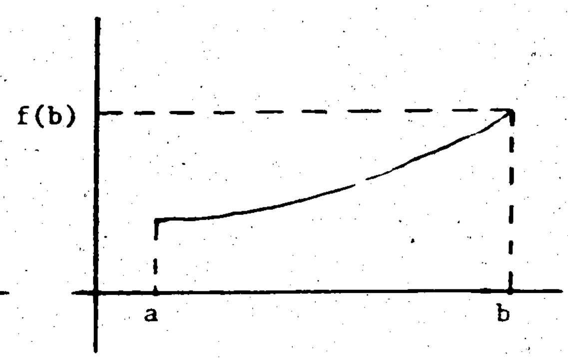
(see objective 1c)

Ex. 6



f has a maximum of [a, b] at x_0.

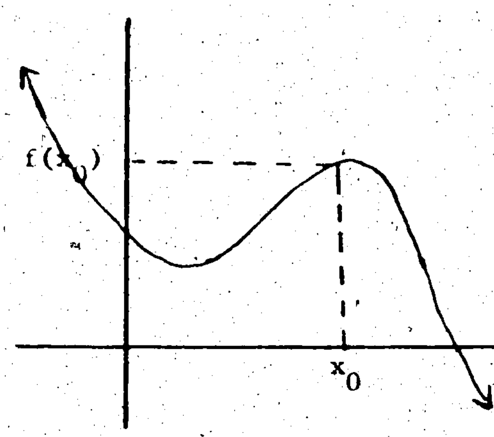
Ex. 7



f has a maximum on [a, b] at b.

f has no maximum on [a, b).

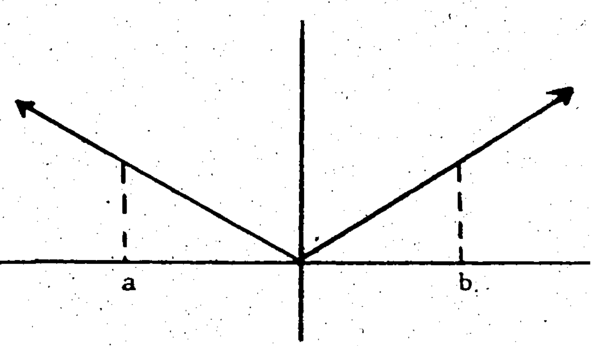
Ex. 8



f has no maximum on $(-\infty, \infty)$.

f has a maximum on $[0, \infty)$ at x_0.

Ex. 9



f has a maximum on [a, b] at both a and b.

As seen in the examples, f may or may not have a maximum on a given interval.

A definition for minimum is:

Definition 4

A function f is said to have a minimum on an interval I if there exists $x_0 \in I$ such that $f(x_0) \leq f(x)$ for all $x \in I$.

(see objective 1f)

Question

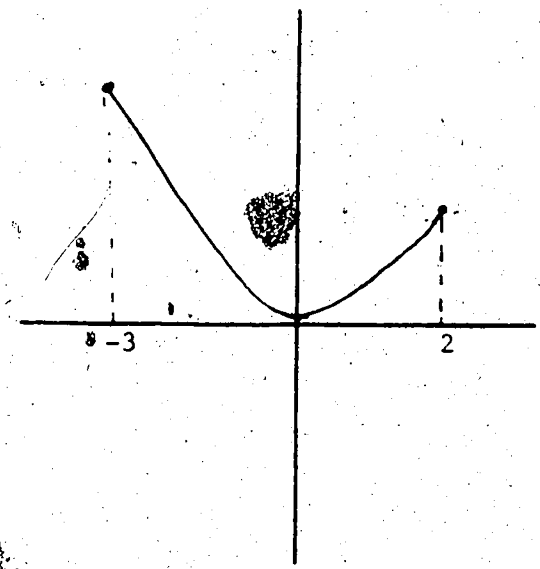
1. If $f(x) = x^2$, find the maximum and minimum of f on $(-3, 2]$ if there are any.

Answer

By examining the graph of $f(x) = x^2$ on (-3, 2], you see that:

(i) f has no maximum

(ii) f has a minimum value at x = 0, f(0) = 0.



Now for relative maxima and relative minima.

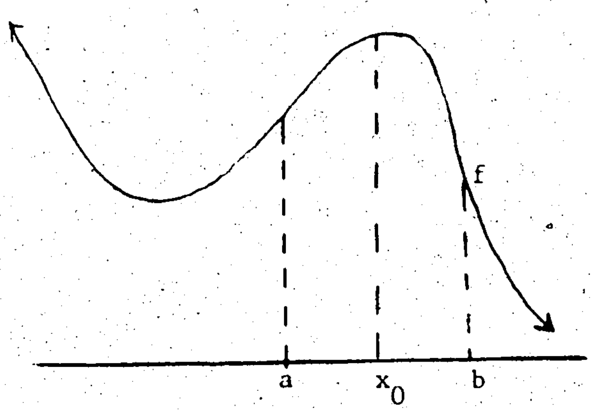
Definition 5

A function f is said to have a relative or local maximum at a point x_0 if there exists an open interval (a, b) containing x_0 such that $f(x_0) \geq f(x)$ for all x in (a, b).

Note: x_0 reads "x sub zero" and is not x = 0.

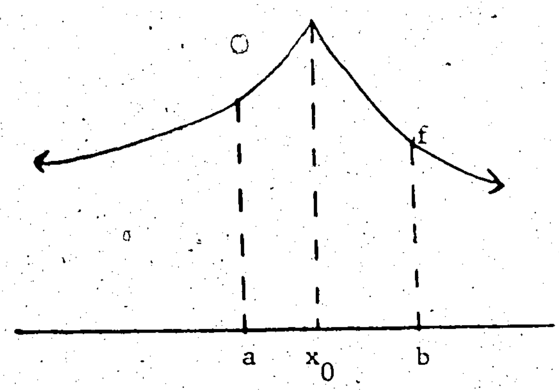
(see objective 1c)

Ex. 10



f has a relative maximum at x_0.
$f(x_0) \geq f(x)$ for all $x \in (a, b)$.

Ex. 11



f has a relative maximum at x_0. The relative maximum is also a maximum for f.

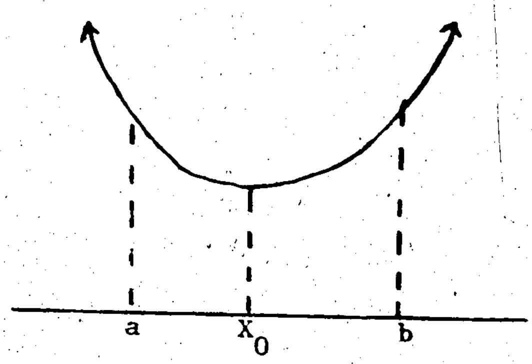
<u>Definition 6</u>

A function f is said to have a <u>relative</u> or <u>local minimum</u> at a point x_0 if there exists an open interval (a, b) containing x_0 such that $f(x_0) \leq f(x)$ for all x in (a,b)

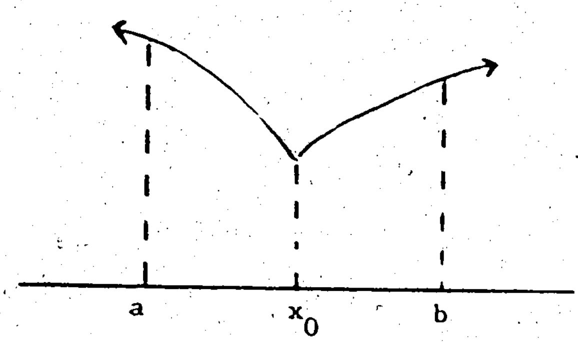
(see objective 1d)

The following graphs have relative minimums at x_0.

<u>Ex. 12</u>



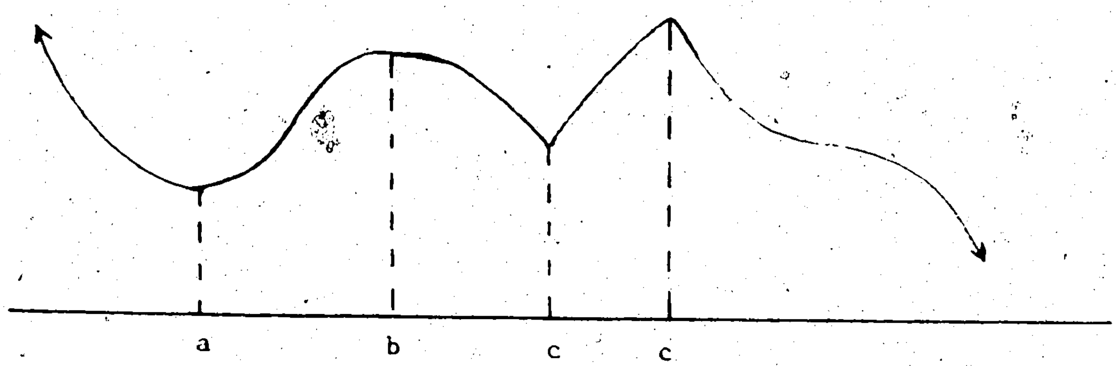
Ex. 13



<u>Note</u>: If $x \in (a, b)$, then $f(x_0) \leq f(x)$.

Consider the following graph.

Ex. 14.



There are relative minimums at ______________.

There are relative maximums at ______________.

ANSWERS

a, c relative minimum

b, c relative maximum

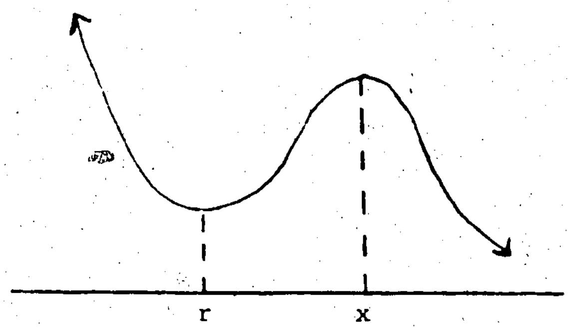
Perhaps you have noticed that, at a point of relative maximum or minimum, the derivative is either zero or does not exist. Thus, if you are to find where relative minimum and maximum values of a given function f occur, you would find where $f'(x) = 0$ and where $f'(x)$ does not exist.

Definition 7

If x_0 is a number in the domain of f and if either $f'(x_0) = 0$ or $f'(x_0)$ does not exist, then x_0 is called a critical point of f.

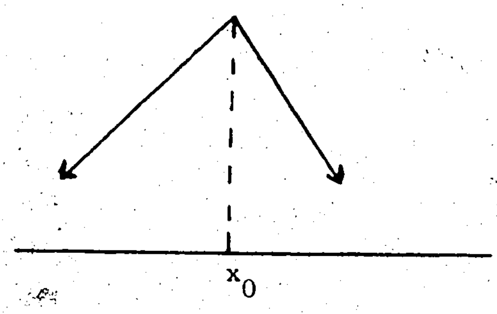
(See objective 1g)

Ex. 15



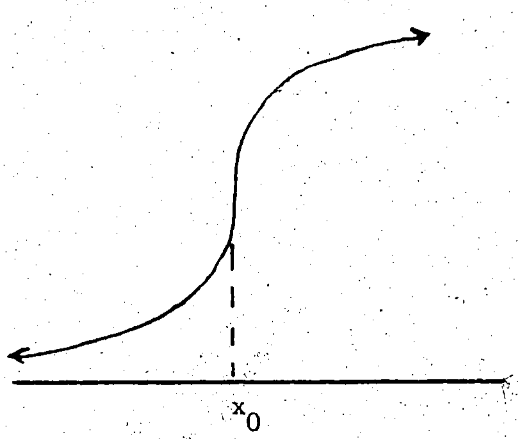
r and x are critical points
$f'(r) = 0$, $f'(x) = 0$

Ex. 16



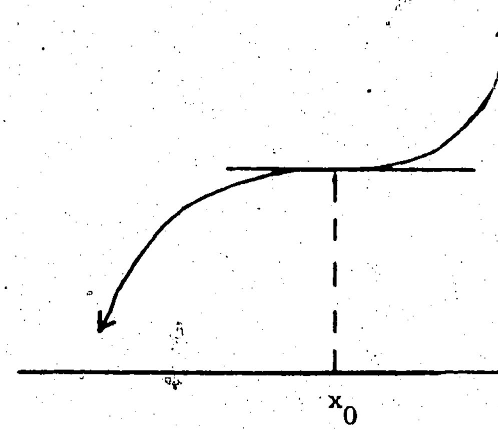
x_0 is a critical point
$f'(x_0)$ does not exist.

Ex. 17



x_0 is a critical point.
$f'(x_0)$ does not exist.

Ex. 18



x_0 is a critical point.
$f'(x_0) = 0$.

The last two examples illustrate that, at a critical point, there need not be a relative minimum or relative **maximum**.

Consider the following examples of how to find critical points.

Ex. 19

Find the critical points of $f(x) = x^3 + 7x^2$.

Solution: We want to find where $f'(x) = 0$, or $f'(x)$ is undefined

$f'(x) = 3x^2 + 14x$

$f'(x)$ exists for all values of x and thus is not undefined

$f'(x) = 0 \Longrightarrow 3x^2 + 14x = 0$

or $x(3x + 14) = 0$

or $x = 0,\ x = -\frac{14}{3}$

Thus the critical points are 0 and $-\frac{14}{3}$.

Ex. 20

Find the critical points of $f(x) = x^{5/3} + 5x^{1/3}$.

Solution:

$f'(x) = \frac{5}{3}x^{2/3} + \frac{5}{3}x^{-2/3}$ (factor out $x^{-2/3}$)

$= \frac{5}{3}x^{-2/3}(x^{4/3} + 1)$ (remember when multiplying powers with like bases, you add the exponents.)

$= \frac{5(x^{4/3} + 1)}{3x^{2/3}}$

$f'(x)$ is undefined at $x = 0$, and so $x = 0$ is a critical point.

$f'(x) = 0 \Longrightarrow x^{4/3} + 1 = 0$, but this is **never** zero since $x^{4/3} = (x^{1/3})^4$ is always > 0, and so there are no critical points where $f'(x) = 0$.

Thus, the only critical point is 0.

Problems

Find the critical points of each of the following:

1. $f(x) = \frac{1}{3}x^3 - x$

2. $f(x) = \frac{1}{x + 1}$

3. $f(x) = 2x^{7/6} - 14x^{1/6}$

Solutions

1. $f(x) = \frac{1}{3}x^3 - x$

$f'(x) = x^2 - 1$

$f'(x) = 0 \Rightarrow x^2 - 1 = 0$

$\Rightarrow x = 1, -1$

$f'(x)$ exists everywhere. Thus, critical points are 1 and -1.

2. $f(x) = \frac{1}{x + 1} = (x + 1)^{-1}$

$f'(x) = -(x + 1)^{-2}$

$f'(x) = \frac{-1}{(x + 1)^2}$

$f'(x)$ is never zero and $f'(x)$ is defined for all values of x in the domain of f. Thus there are no critical points.

3. $f(x) = 2x^{7/6} - 14x^{1/6}$

$f'(x) = \frac{14x^{1/6}}{6} - \frac{14x^{-5/6}}{6}$

$= \frac{14x^{-5/6}}{6}(x - 1)$

$= \frac{7(x - 1)}{3x^{5/6}}$

$f'(x) = 0$ when $x = 1$

$f'(x)$ is undefined at $x = 0$.

Thus, the critical points are 0, 1.

(see objective 3)

The technique for finding points $(x, f(x))$ of relative maximum and relative minimum of a function f is:

(1) Find the critical points.

(2). Check the sign of the derivative on either side of each critical point.

(3) Find the values of f at the critical points.

Ex. 21

Find the points of relative maximum and relative minimum of

$$f(x) = -2x^2 + 5x + 6.$$

Solution:

(1) Critical points

$f'(x) = -4x + 5$

Now $f'(x) = 0 \Longrightarrow x = 5/4$

and $f'(x)$ is defined everywhere.

Thus the only critical point is $5/4$.

(2) Sign of derivative

Let us check the sign of the derivative on either side of the critical point.

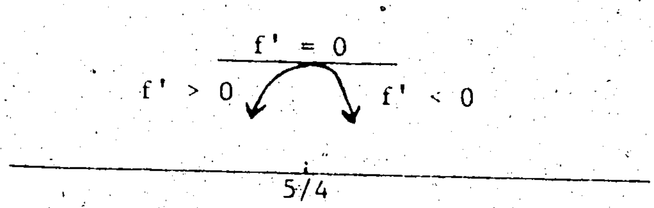
If $x > 5/4$, then $f'(x) < 0$.

If $x < 5/4$, then $f'(x) > 0$.

Recall $f'(x) < 0$ implies f is decreasing, and $f'(x) > 0$ implies f is increasing.

With f decreasing for $x > 5/4$, and f increasing for $x < 5/4$, there is a relative maximum at $x = 5/4$.

(3) The relative maximum is $f(5/4) = -2(5/4)^2 + 5 \cdot 5/4 + 6 = 73/8$.



Thus, $\left(\frac{5}{4}, \frac{73}{8}\right)$ is a point of relative maximum.

The technique for finding points of relative maximum and relative minimum invoved three steps.

First, the critical points (or values of x where $f'(x) = 0$ or $f'(x)$ is undefined) were found. In Ex. 21, the critical point is $x = \frac{5}{4}$, which is the x-coordinate of the point $(x, f(x))$ of relative maximum or relative minimum.

Second, by checking the sign of f' on either side of the critical point, you are able to determine whether there is a relative maximum or relative minimum. In Ex. 21, there is a relative maximum at $x = \frac{5}{4}$.

Third, find the values of f at the critical points and the resulting values are the y-coordinates of the points $(x, f(x))$ of relative maximum or relative minimum.

Let's look at another example.

Ex. 22

Find the points of relative minimum and relative maximum of

$$f(x) = (x - 2)(x - 1)^3$$

Solution:

(1) Critical points

$$f'(x) = (x - 2) \cdot 3(x - 1)^2 + (x - 1)^3 \cdot 1$$

$$= (x - 1)^2 [3(x - 2) + x - 1]$$

Therefore, $f'(x) = (x - 1)^2 \cdot (4x - 7) = (x - 1)^2 \cdot 4(x - 7/4)$

Now $f'(x) = 0 \Rightarrow x = 1, 7/4$

And $f'(x)$ is defined everywhere.

Thus, the critical points are 1 and 7/4

(2) Sign of derivative

Let us check the sign of the derivative on either side of each of the critical points.

$x > \frac{7}{4} \Rightarrow f'(x) > 0$

$1 < x < \frac{7}{4} \Rightarrow f'(x) < 0$

$\Rightarrow$ relative minimum at $\frac{7}{4}$ as the graph is increasing on the right of $\frac{7}{4}$ and decreasing on the left of $\frac{7}{4}$.

$x < 1 \Rightarrow f'(x) < 0$

$\Rightarrow$ neither relative minimum nor relative maximum at 1. Since the sign of the derivation on either side of x = 1 is the same.

There is a point of relative minimum at $x = \frac{7}{4}$.

(3) The relative minimum is $f(7/4) = (7/4 - 2)(7/4 - 1)$

$$= \frac{-1}{4} \cdot \left(\frac{3}{4}\right)^3$$

$$= \frac{-27}{256}$$

Thus, $\left(\frac{7}{4}, -\frac{27}{256}\right)$ is a point of relative minimum.

Problems

Find the points of relative maximum and relative minimum, if they exist, for each of the following:

1. $f(x) = x^2 - 2x - 3$
2. $f(x) = x^3 - 6x^2 + 9x - 8$
3. $f(x) = (2 - x)^3$
4. $f(x) = x^3 + \frac{48}{x}$

Solutions:

1. $f(x) = x^2 - 2x - 3$

(1) Critical points

$f'(x) = 2x - 2$

Now $f'(x) = 0$ implies $x = 1$

And $f'(x)$ is defined for all real numbers.

Thus the only critical point is $x = 1$.

(2) Sign of derivative

$$\left.\begin{array}{l} x > 1 \Longrightarrow f'(x) > 0 \\ x < 1 \Longrightarrow f'(x) < 0 \end{array}\right\} \Longrightarrow \text{relative minimum at } x = 1$$

(3) The relative minimum is $f(1) = -4$, and thus $(1, -4)$ is a point of relative minimum.

2. $f(x) = x^3 - 6x^2 + 9x - 8$

(1) Critical points

$$f'(x) = 3x^2 - 12x + 9$$
$$= 3(x^2 - 4x + 3)$$
$$= 3(x - 3)(x - 1)$$

Now $f'(x) = 0 \Longrightarrow x = 3, 1$

and $f'(x)$ is defined everywhere (for all reals).

Thus, the critical points are 1 and 3.

(2) Sign of derivative

$$\left.\begin{array}{l} x > 3 \Longrightarrow f'(x) > 0 \\ 1 < x < 3 \Longrightarrow f'(x) < 0 \end{array}\right\} \Longrightarrow \text{relative minimum at } x = 3$$

$$\left.\begin{array}{l} x < 1 \Longrightarrow f'(x) > 0 \end{array}\right\} \Longrightarrow \text{relative maximum at } x = 1$$

(3) $f(3) = 3^3 - 6(3)^2 + 9.3 - 8 = -8$

$f(1) = 1^3 - 6(1)^2 + 9.1 - 8 = -4$

Thus, $(3, -8)$ is a point of relative minimum.

And $(1, -4)$ is a point of relative maximum.

3. $f(x) = (2 - x)^3$

(1) Critical points

$f'(x) = -3(2 - x)^2$

Now $f'(x) = 0 \implies x = 2$

And $f'(x)$ is defined everywhere.

Thus, the only critical point is $x = 2$.

(2) Sign of derivative

$$\left.\begin{aligned} x < 2 &\implies f'(x) < 0 \\ x > 2 &\implies f'(x) < 0 \end{aligned}\right\} \text{ no relative maximum or relative minimum}$$

4. $f(x) = x^3 + \dfrac{48}{x}$

(1) Critical points

$$\begin{aligned} f'(x) &= 3x^2 - \frac{48}{x^2} \\ &= \frac{3x^4 - 48}{x^2} \\ &= \frac{3(x^4 - 16)}{x^2} \\ &= \frac{3(x^2 + 4)(x - 2)(x + 2)}{x^2} \end{aligned}$$

Now $f'(x) = 0 \implies x = 2, -2$

$f'(x)$ is undefined at $x = 0$, but f is undefined at $x = 0$ as well.

Thus, the critical points are 2 and -2.

(2) Sign of derivative

$$\left.\begin{aligned} x > 2 &\implies f'(x) > 0 \\ 0 < x < 2 &\implies f'(x) < 0 \end{aligned}\right\} \implies \text{relative minimum at } x = 2$$

$$\left.\begin{aligned} -2 < x < 0 &\implies f'(x) < 0 \\ x < -2 &\implies f'(x) > 0 \end{aligned}\right\} \implies \text{relative maximum at } x = -2$$

(3) $f(2) = 2^3 + \frac{48}{2} = 8 + 24 = 32$

$f(-2) = (-2)^3 + \frac{48}{-2} = -8 - 24 = -32$

Thus, (2, 32) is a point of relative minimum, and (-2, -32) is a point of relative maximum.

(see objective 4)

In the examples and problems following Definitions 3 and 4, you observed that a function may or may not have a maximum and minimum on an interval. However, there is a theorem (it will not be proven here) commonly referred to as the Extreme-value Theorem which states:

> If a function f is continuous on the closed interval [a, b], then f has both a maximum and a minimum of [a, b].

You have probably observed that a maximum or minimum value may occur at a critical point. See Ex. 6 and Ex. 16.

Also, a maximum or minimum may occur at an endpoint of the closed interval [a, b]. See Ex. 7 and Ex. 9.

As a result of these observations, the following technique is outlined:

> The technique for finding maximum and minimum values on a closed interval [a, b] is:
>
> (1) Find the critical points of f on [a, b].
>
> (2) Calculate the value of the function at each critical point.
>
> (3) Calculate the values of f(a) and f(b).
>
> (4) The largest of the values from steps (2) and (3) is the maximum and the smallest of the values if the minimum.

Ex. 23

Find the maximum and minimum values of $f(x) = x^4 - 8x^2$ on the interval $[-1, 3]$.

Solution:

(1) Critical points

$f'(x) = 4x^3 - 16x$

$= 4x(x^2 - 4)$

$= 4x(x - 2)(x + 2)$

$f'(x) = 0$ when $x = 0, 2, -2$, but $-2 \notin [-1, 3]$

$f'(x)$ is defined everywhere.

Critical points of f on $[-1, 3]$ are 0, 2.

(2) $f(0) = 0$

$f(2) = -16$

(3) $f(a) = f(-1) = -7$

$f(b) = f(3) = 9$

(4) The maximum value on $[-1, 3]$ is 9 at $x = 3$.

The minimum value on $[-1, 3]$ is -16 at $x = 2$.

Note:

The problem of finding a maximum or minimum (if any exist) on any interval I is similar to the preceding example, except that one may not have to worry about endpoints. That is, if $I = (a, b)$, for example, there are no endpoints to worry about, so you just have to find the critical points and check them out for maximum and minimum values.

Problems

Find the maximum and minimum of each of the following functions in the intervals indicated.

1. $f(x) = x^2 - 2x + 5$, $I = [-3, 4]$
2. $f(x) = x^3 + 2x - 4$, $I = [-1, 2]$

3. $f(x) = x^3 - 2x^2 + x - 1$, $I = (-3, 3)$

4. $f(x) = \dfrac{x^2}{x - 3}$, $I = [2, 3]$

Solutions

1. $f(x) = x^2 - 2x + 5$, $I = [-3, 4]$

Critical Points

$f'(x) = 2x - 2$

$f'(x) = 0 \Longrightarrow x = 1$, which is a member of I

$f'(x)$ is defined everywhere.

Thus the only critical point is $x = 1$.

$f(1) = 4$.

Endpoints

$f(-3) = 20$

$f(4) = 13$

The maximum value on $[-3, 4]$ is 20 at $x = -3$.

The minimum value on $[-3, 4]$ is 4 at $x = 1$.

2. $f(x) = x^3 + 2x - 4$, $I = [-1, 2]$

Critical Points

$f'(x) = 3x^2 + 2$

$f'(x) = 0 \Longrightarrow 3x^2 + 2 = 0$, which is impossible since 2 is added to the positive number $3x^2$.

$f'(x)$ is defined everywhere.

Thus there are no critical points.

Endpoints

$f(-1) = -7$

$f(2) = 8$

The maximum value on $[-1, 2]$ is 8 at $x = 2$.

The minimum value on $[-1, 2]$ is -7 at $x = -1$.

3. $f(x) = x^3 - 2x^2 + x - 1$, $I = (-3, 3)$

Critical Points

$f'(x) = 3x^2 - 4x + 1$

$= (3x - 1)(x - 1)$

$f'(x) = 0 \Longrightarrow x = \frac{1}{3}, 1$, which are in I.

$f'(x)$ is defined everywhere.

Thus, the critical points are $x = \frac{1}{3}, 1$, which are in $(-3, 3)$

$f(\frac{1}{3}) = \frac{23}{-27}$

$f(1) = -1$

There are no endpoints.

Therefore, we can only conclude that $x = \frac{1}{3}, 1$, are possible points of maximum and minimum.

To check that there are no maximum and minimum values on $(-3, 3)$, look at the function as x approaches 3 and -3.

As $x \to 3$, $f(x) \to 11$

As $x \to -3$, $f(x) \to -49$

4. $f(x) = \frac{x^2}{x - 3}$, $I = [2, 3]$

$f'(x) = \frac{(x - 3)\,2x - x^2}{(x - 3)^2}$

$= \frac{2x^2 - 6x - x^2}{(x - 3)^2}$

$= \frac{x^2 - 6x}{(x - 3)^2}$

$= \frac{x(x - 6)}{(x - 3)^2}$

$f'(x) = 0 \Longrightarrow x = 0, 6$, neither of which are in $[2, 3]$

$f'(x)$ is defined for all x in $[2, 3]$,

thus there are no critical points in $[2, 3]$

The only endpoint is 2 and $f(2) = -4$.

Also, $f'(x) < 0$ for $x \in [2, 3]$ and thus the graph is decreasing.

As $x \to 3$, $f(x) \to -\infty$

There is a maximum value of -4 at $x = 2$, and there is no minimum.

(see objective 5)

New Vocabulary

1. increasing function
2. decreasing function
3. relative maximum
4. relative minimum
5. maximum
6. minimum
7. critical point

POST-TEST ON MAXIMA AND MINIMA

1. Define: (a) critical point
   (b) relative minimum

2. Find the critical points of the function $f(x) = x^3 - 3x$.

3. Find the minimum and maximum values of the function $f(x) = x^3 - 3x$ on the interval $[-3, 3]$.

4. Find the points of relative maximum and relative minimum for the function in question 3.

5. Find the intervals on which $f(x) = x^3 - 3x$ is increasing, and the intervals on which $f(x)$ is decreasing.

APPENDIX B

TOPICAL OUTLINE OF PMAT 201

TOPICAL OUTLINE OF PMAT 201

1. Number systems, sets, and functions.
2. Inequalities.
3. Limits.
4. Infinite limits.
5. Continuous functions.
6. The derivative.
7. Differentiation of algebraic functions.
8. Maxima and minima.
9. Applications of maxima and minima.
10. Rates of change and implicit differentiation.
11. Curve sketching.
12. Areas.
13. The definite integral.
14. The fundamental theorem of calculus.
15. Methods of integration.
16. Applications of integration.
17. Differentiation and integration of trigonometric functions.
18. Logarithmic and exponential functions.
19. Integration by parts and partial fractions.

APPENDIX C

COSTELLO'S ACHIEVEMENT MOTIVATION SCALES I AND II

QUESTIONNAIRE

DIRECTIONS: The little questionnaire you are being asked to complete is to be used for research purposes only. Please answer all questions honestly by checking the yes box or no box for each question. It will only take a few minutes.

SCALE I

		YES	NO
1.	Are you incli[illegible]of the successes of others rather than do[illegible]k of making yourself a success?		
2.	Would you describe yourself as an amibitous person?		
3.	Do you work for success rather than daydream about it?		
4.	Would you describe yourself as being lazy?		
5.	Do you usually work to do more than just get through an examination?		
6.	Will days often go by without your having done a thing?		
7.	Do you do things "today" rather than putting them off to do "tomorrow"?		
8.	Are you inclined to take life as it comes without much planning?		
9.	Do you work hard at a job?		
10.	Do you, or did you, do little preparation for examinations?		

SCALE II

	YES	NO
1. Do you grow excited when telling someone about the work you are doing?		
2. Do you usually remain free from boredom when on hollidays?		
3. Are you very interested in the lives of successful people?		
4. Do you remain relaxed at the thought of a difficult task you are about to undertake?		
5. Are you usually unimpressed by how hard others work?		
6. Are you usually able to sleep even when engaged in an exciting job?		
7. Are you usually awed in the presence of very successful people?		
8. Can you usually concentrate on what people are saying to you even when an important job is finished?		
9. Does the great achievement of others sometimes make you feel small?		
10. Have you at any time tried to model your life on that of a successful person?		
11. Do you readily forget your work when you are on holiday?		
12. Are you influenced by those around you in the amount of work you do?		
13. Do you usually remain free from envy when others are successful?		
14. Do you often compare how well you can do something with how well others can do it?		

APPENDIX D

QUESTIONNAIRE FOR PMAT 201 STUDENTS

QUESTIONNAIRE FOR PMAT 201 STUDENTS

Name: ______________________________ Age: ______________

Address: ____

Sex: Male_____ Female_____ Telephone Number: ______________

Last School Attended: ______________________ Year __________

Last Mathematics Course Taken: ______________ Year __________

Grade Obtained in Last Mathematics Course: ____________________

Reason for Attending Mount Royal College: _____________________

____

____

Educational Plans: ______________________________________

____

____

Father's Occupation: ____________________________________

Mother's Occupation: ____________________________________

APPENDIX E

THE SIXTEEN PRIMARY FACTORS OF THE 16PF

THE SIXTEEN PRIMARY FACTORS OF THE 16PF

Factor	Low Score		High Score
A	Reserved, Detached, Critical, Cool	vs.	Outgoing, Warmhearted, Easy-going, Participating
B	Less Intelligent, Concrete-thinking	vs.	More Intelligent, Abstract-thinking
C	Affected by Feelings, Emotionally less stable, Easily upset	vs.	Emotionally Stable, Faces Reality, Calm, Mature
E	Humble, Mild, Accommodating, Conforming	vs.	Assertive, Independent, Aggressive, Competitive, Stubborn
F	Sober, Prudent, Serious, Taciturn	vs.	Happy-go-lucky, Impulsively lively, Enthusiastic
G	Expedient, Evades rules, Feels few obligations	vs.	Conscientious, Persevering, Staid, Rule-bound
H	Shy, Restrained, Diffident, Timid	vs.	Venturesome, Socially-bold, Uninhibited, Spontaneous
I	Tough-minded, Self-reliant, Realistic, No-nonsense	vs.	Tender-minded, Dependent, Over-protected, Sensitive
L	Trusting, Adaptable, Free of jealousy, Easy to get on with	vs.	Suspicious, Self-opinionated, Hard to fool
M	Practical, Careful, Conventional, Proper	vs.	Imaginative, Wrapped up in inner urgencies, Careless of practical matters
N	Forthright, Natural, Artless, Sentimental	vs.	Shrewd, Calculating, Worldly, Penetrating
O	Placid, Self-assured, Confident, Serene	vs.	Apprehensive, Worrying, Depressive, Troubled
Q_1	Conservative, Respecting established ideas, Tolerant of traditional difficulties	vs.	Experimenting, Critical, Liberal, Analytical, Free-thinking

Factor	Low Score		High Score
Q_2	Group-dependent, A "joiner" and sound follower	vs.	Self-sufficient, Prefers own decisions, Resourceful
Q_3	Undisciplined, Self-conflict, Careless of protocol, Follows own urges	vs.	Controlled, Socially precise, Following self-image
Q_4	Relaxed, Tranquil, [illegible], Unfrustrated	vs.	Tense, Frustrated, Driven, Overwrought

A low raw score on a 16 PF factor should not be interpreted as a "poor" score, but simply as a score corresponding to the behaviour described on the left. Similarly, a high raw score corresponds to the description on the right.

APPENDIX F

COMPUTER PROGRAM FOR COMPUTING 16 PF SECOND ORDER SCORES

16PF SECOND ORDER SCORES FOR MRC STUDENTS 1973

```
      DIMENSION A(4,16), C(4), X(16), Y(16)
      DO 1  I = 1,4
      READ (5, 100) (A(I,J), J = 1, 16), C(I)
      WRITE (6, 200) (A(I,J), J = 1, 16), C(I)
    1 CONTINUE
      DO 6  I = 1,90
      READ (5, 101) ID, (X(L), L = 1, 16)
      WRITE (6, 201) ID, (X(L), L = 1, 16)
      DO 4  K = 1,4
      TOTAL = 0
      DO 2  M = 1, 16
      Y(M) = A(K,M)*X(M)
      TOTAL = TOTAL + Y(M)
    2 CONTINUE
      TOTAL = TOTAL + C(K)
      WRITE (6, 202) TOTAL
    4 CONTINUE
    6 CONTINUE
  100 FORMAT (16F4.2, F5.2)
  101 FORMAT (I3, 16F2.0)
  200 FORMAT (5X, 16(F5.2, 2X), 3X, F5.2)
  201 FORMAT (5X, I4, 2X, 16F5.2)
  202 FORMAT (5X, F8.2)
      END
```

DO 1 loop read in the four sets of weights and constants from Table 10.9, <u>Handbook For the 16PF</u>. (There were separate male and female weights and constants.)

DO 6 loop read in each student's 16 primary factor sten scores, calculated and printed out the four second-order scores in stens.

In this particular program, there were 90 male students (I = 1, 90).

APPENDIX G

THE TEN PRIMARY FACTORS MEASURED BY MAT

THE TEN PRIMARY FACTORS MEASURED BY MAT

1.	Mating Erg	Strength of the normal, heterosexual or mating drive.
2.	Assertiveness Erg	Strength of the drive to self-assertion, mastery, and achievement.
3.	Fear (Escape) Erg	Level of alertness to external dangers.
4.	Narcism-Comfort Erg	Level of drive to sensuous, self-indulgent satisfactions.
5.	Pugnacity-Sadism Erg	Strength of destructive, hostile impulses.
6.	Self-Concept Sentiment	Level of concern about the self-concept, social repute, and more remote rewards.
7.	Superego Sentiment	Strength of development of conscience.
8.	Career Sentiment	Amount of development of interests in a career.
9.	Sweetheart-Spouse Sentiment	Strength of attachment to wife(husband) or sweetheart.
10.	Home-Parental Sentiment	Strength of attitudes attaching to the parental home.

APPENDIX H

TERM TESTS AND FINAL EXAMINATION USED IN THIS STUDY

INDEPENDENT STUDY GROUP

PMAT 201 TERM TEST #1

Answer all of the questions. Show your work for each question.

1. Which of the following numbers are rational numbers?

   $2,\ 1.7,\ 16^{\frac{1}{2}},\ \frac{0}{3},\ \sqrt[3]{9},\ \sqrt{8},\ \frac{\pi}{2},\ \frac{1}{0}$

2. Which of the following functions are rational functions.

   (a) $f(x) = x^2 - 4x + 5$

   (b) $g(x) = \frac{x^4 - 4}{x}$

   (c) $L(x) = x^2 - \frac{1}{2}$

   (d) $f(t) = t + t^{\frac{1}{2}}$

3. Suppose $f(x) = x^{\frac{1}{2}}$ and $g(x) = \frac{x^3 + 1}{x^2}$

   (a) State the domain of f.

   (b) State the domain of g.

   (c) Find $f(4) / g(2)$.

   (d) Write the composition function $(g \circ f)(x)$.

   (e) State the domain of $g \circ f$.

4. If $A = [-3,6]$ and $B = (-2,3) \cup [4,5]$, find:

   (a) $A \cap B$

   (b) B'

   (c) $A \cup B$

   (d) $B' \cap A$

5. Sketch the graph of each of the following functions:

   (a) $y = \frac{1}{2 - x}$

   (b) $y = |-x|$

6. Solve the following inequalities:

   (a) $\frac{3 + x}{x - 4} > 0$

(b) $|3x + 4| \geq 6$

(c) $\frac{x(x - 1)^2}{x + 2} \leq 0$

7. If $a^2 < b^2$ does it follow that $a < b$? Why?

LECTURE GROUP

PMAT 201 TERM TEST #1

Answer <u>all</u> of the questions. <u>Show</u> your work for each question.

1. (a) Which of the following numbers are <u>not</u> rational numbers?

   $5,\ \frac{0}{4},\ 7.\dot{3}\dot{3},\ \sqrt[3]{9},\ 8\tfrac{1}{2},\ \frac{\pi}{3},\ 2.63,\ \frac{6}{3}$

   (b) Why is the expression $-6 \geq x \geq 3$ incorrect?

2. Suppose $f(x) = \dfrac{1}{\sqrt{4 - x^2}}$ and $g(x) = \dfrac{1}{x}$.

   (a) State the domain of f.

   (b) Write the composition function (gof)(x).

   (c) State the domain of gof.

   (d) Find (f.g)(1).

3. If $A = [-2, 3) \cup [4, 5)$ and $B = (3, 6]$ find:

   (a) $A \cup B$

   (b) A'

   (c) $A \cap B$

   (d) $B' \cap A$

4. Which of the following functions are rational functions?

   (a) $f(x) = x^{2/3} - 1$

   (b) $g(x) = \dfrac{x^5 - 1}{x}$

   (c) $L(r) = 2r^2 - 3r + 1$

   (d) $g(t) = t - \dfrac{1}{t^2}$

5. Solve the following inequalities:

   (a) $\dfrac{x - 4}{2 - x} \leq 0$

   (b) $|2x - 5| \geq 3$

   (c) $\dfrac{(x + 2)(x - 4)}{x(x - 1)} \leq 0$

6. Sketch the graph of:

(a) $y = -|1 - x|$

(b) $y = \frac{1}{2 - x}$

INDEPENDENT STUDY GROUP

PMAT 201 TERM TEST #2

Answer each of the following questions showing your work.

1. Solve $\dfrac{2x-3}{x-2} > 1$

2. State the meaning of each of the following:

   (a) $\lim_{x\to 2} f(x) = -3$

   (b) $\lim_{x\to 1^+} g(x) = -1$

3. Find the following limits, if they exist.

   (a) $\lim_{x\to 3} (-2) =$

   (b) $\lim_{x\to -2} \dfrac{x^2-4}{x+2} =$

   (c) $\lim_{t\to 0} \dfrac{f(3-t)-f(3)}{t}$ where $f(t) = t^2+1$

4. Find the following limits:

   (a) $\lim_{x\to\infty} \dfrac{2x+2}{x} =$

   (b) $\lim_{x\to 0^-} \dfrac{1}{x^3} =$

   (c) $\lim_{x\to\infty} \dfrac{x-4}{2x^2-5} =$

5. Find the vertical and horizontal asymptotes (if any) of each of the functions in question 4. SHOW WORK!

6. (a) Define continuity of a function g at a point b.

   (b) Sketch the graph of the function

$$f(x) = \begin{cases} 2 & \text{if } x < -1 \\ x^2 & \text{if } -1 \le x < 2 \\ 2x & \text{if } x \ge 2 \end{cases}$$

   (c) Indicate the points where f is not continuous and the reasons why f is not continuous.

7. (a) Using the DEFINITION of derivative, find the derivative of:

$$f(x) = 3x^2 - 2$$

(b) Find the equation of the tangent line to the graph of

$$f(x) = 3x^2 - 2 \text{ at } x = -1$$

8. Find the derivative of each of the following functions:

(a) $f(x) = \frac{1}{2}x^4 - 2x$

(b) $f(t) = (2t - 1)(t + 1)^2$

(c) $g(r) = r^{\frac{1}{2}} - 2r^{4/3}$

LECTURE GROUP

PMAT 201 TERM TEST #2

Answer each of the following questions showing your work.

1. Solve $\frac{2x - 3}{x - 2} > 1$

2. State the meaning of each of the following:

   (a) $\lim_{x \to 2} f(x) = -3$

   (b) $\lim_{x \to 1^+} g(x) = -1$

3. Find the following limits, if they exist.

   (a) $\lim_{x \to 3} (-2) =$

   (b) $\lim_{x \to -2} \frac{x^2 - 4}{x + 2} =$

   (c) $\lim_{t \to 0} \frac{f(3 - t) - f(3)}{t}$ where $f(t) = t^2 + 1$

4. Find the following limits:

   (a) $\lim_{x \to \infty} \frac{2x + 2}{x} =$

   (b) $\lim_{x \to 0^-} \frac{1}{x^3} =$

   (c) $\lim_{x \to \infty} \frac{x - 4}{2x^2 - 5} =$

5. Find the vertical and horizontal asymptotes (if any) of each of the functions in question 4. SHOW WORK!

6. (a) Define continuity of a function g at a point b.

   (b) Sketch the graph of the function

$$f(x) = \begin{cases} 2 & \text{if } x < -1 \\ x^2 & \text{if } -1 \leq x < 2 \\ 2x & \text{if } x \geq 2 \end{cases}$$

   (c) Indicate the points where f is not continuous and the reasons why f is not continuous.

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$$f(x) = 3x^2 - 2 \text{ at } x = -1$$

8\. Find the derivative of each of the following functions:

(a) $f(x) = \frac{1}{2}x^4 - 2x$

(b) $f(t) = (2t - 1)(t + 1)^2$

(c) $g(r) = r^{\frac{1}{2}} - 2r^{4/3}$

INDEPENDENT STUDY GROUP

PMAT 201 TERM TEST #3

1. Find the equation of the tangent line to the curve of
   $f(x) = x^{2/3} + x^{-1}$ at $x = 1$

2. Find the derivatives of each of the following functions:
   (a) $f(x) = \sqrt[4]{x} + \sqrt[7]{x^3}$
   (b) $f(x) = (4x^2 - x^3)^5$
   (c) $f(x) = (x^{1/2} + 4)^3(1 - 6x + 4x^3)^2$

3. The volume of a cube is increasing at 100 cu. in./sec. How fast is the edge increasing when it is 12 in.?

4. A function f is defined by $f(x) = \frac{x^3}{3} - x^2 - 15x$

   (a) Find the intervals in which $f(x)$ is increasing.
   (b) Find the values of x where the relative maximum and relative minimum occur.
   (c) Find the second derivative of $f(x)$.
   (d) Find the points of inflection.
   (e) Determine the intervals in which f is concave downward and concave upward respectively.
   (f) Sketch the graph of the function.

5. Find the altitude of the cylinder of maximum volume that can be inscribed in a sphere of radius R.

LECTURE GROUP

PMAT 201 TERM TEST #3

1. Find the derivatives of each of the following functions:
   (a) $f(x) = x^{1/2} + 4x^{3/2} + x^0$
   (b) $f(x) = (2 - 3x^2)^6$
   (c) $f(x) = \dfrac{(8x - 4x^2)^3}{(x^3 - 1)^4}$

2. The equation of a circle with center (0, 0) and radius 5 is $x^2 + y^2 = 25$. Show that the line tangent to the circle at (3, 4) is perpendicular to the line through (0, 0) and (3, 4).

3. Show that the square has the greatest area among all rectangles of a given perimeter.

4. A function f is defined by $f(x) = x^3 - 3x$.
   (a) Find the intervals in which f(x) is increasing.
   (b) Find the values of x where the relative maximum and relative minimum occur.
   (c) Find the second derivative of f(x).
   (d) Find the points of inflection.
   (e) Determine the intervals in which f is concave downward and concave upward respectively.
   (f) Sketch the graph of the function.

5. When the radius is 1 ft., find the rate of change of the radius of a sphere whose volume is changing at 8π cubic ft. per minute.

INDEPENDENT STUDY GROUP

PMAT 201 TERM TEST #4

1. Evaluate $\int_1^3 x^2\,dx$ using the definition of the definite integral.

2. (a) State the fundamental theorem of calculus.

   (b) Find the area of the region bounded by the graphs of $y = x^2 - 4$ and $y = 4 - x^2$

3. Find the volume generated by revolving the region bounded by the graphs of $y = \sqrt{4 + x}$, $x = 0$, $y = 0$, about the x axis.

4. (a) Given that $D_x(\cos x) = -\sin x$, prove that $D_x(\sec x) = \sec x \tan x$.

   (b) Find:

      (i) $\int (3x^2 + x - 2)\,dx$

      (ii) $\int (2x + 3)(x^2 + 3x + 5)\,dx$

      (iii) $\int x\sqrt{1 - 3x^2}\,dx$

      (iv) $\int \sec^5 x \tan x\,dx$

LECTURE GROUP

PMAT 201 TERM TEST #4

1. Find: $\int (\sec^2 u + \csc^2 u + 2)\, du$

$\int \frac{x\, dx}{(x^2 + a^2)^3}$

$3 \int x \left(\frac{3}{2} x^2 - 7\right)^4 dx$

$\int (2x + x^3 - 7)\, dx$

2. Given that $\tan x = \frac{\sin x}{\cos x}$, show that

$D_u (\tan u) = \sec^2 u$

3. Find the area of the region bounded by the graphs of $y = x^2 - 1$; $y = x + 1$.

4. Find the volume generated by revolving the region bounded by the graphs of $y = \sqrt{4 - x}$, $x = 0$, $y = 0$, about the x-axis.

5. One student worked an indefinite integral problem and got:

$\tan^2 x + \frac{1}{6} + C,$

while a fellow student used a slightly different approach to the same problem, and got:

$\tan^2 x + C.$

Are the answers different? Explain briefly.

6. Evaluate $\int_{-1}^{1} (-x^2)\, dx$ using the definition of the definite integral.

MOUNT ROYAL COLLEGE

DEPARTMENT OF MATHEMATICS

PMAT 201 FINAL - MAY 22, 1973

Answer ANY FIVE of the following problems. SHOW your work.

1. Listing all relevant information (i.e., intersection with axes, critical points, horizontal and vertical asymptotes), sketch the graph of $y = \frac{x^2 + 4}{x^2 - 1}$

2. Differentiate the following functions with respect to the variable x.

   (a) $y = 2x^3 \ln x$

   (b) $y = 3x^{\frac{1}{2}} - \frac{2}{3}x^{3/5}$

   (c) $y = \frac{\sin x}{x^2 + 1}$

   (d) $xy^2 - y + x = 0$

3. Sand is falling in a pile, always having the shape of a cone, at the rate of 5 cu. ft./min. Assume that the diameter at the base of the pile is always three times the altitude. At what rate is the altitude increasing when the altitude is 9 ft.? (Volume of a cone whose base has diameter "d" and altitude "h" is $\frac{d^2h}{12}$)

4. Prove: If f is continuous at t, and g is continuous at t, and $g(t) \neq 0$, show that $\frac{f}{g}$ is continuous at t.

5. By showing $\frac{d}{dx}$ $(\ln(ax) - \ln x) = 0$, where a is some constant, prove that $\ln(ax) - \ln x = c$, where c is some constant. What is the value of c?

6. State and prove the product rule for differentiation.

7. Find the following integrals:

   (a) $\int_1^8 (\frac{1}{2}x + x^{-2/3})\, dx$

(b) $\int_1^2 \frac{x\,dx}{1 + x^2}$

(c) $\int x \cos x\,dx$

(d) $\int \frac{x + 2}{x + 1}\,dx$

8. Using the definition of derivative, find the derivative of the function $f(x) = \frac{1}{x + 3}$ from first principles.

9. Find the height of the right circular cylinder of maximum volume V which can be inscribed in a sphere of radius a.

APPENDIX I

THE DATA GATHERED IN THIS STUDY

THE DATA GATHERED IN THIS STUDY

<u>Code</u>

X_1: Student Identification Number

X_2: Student Age in Years

X_3: Numerical Ability Percentile Score (DAT)

X_4: Abstract Reasoning Percentile Score (DAT)

X_5: Wonderlic Percentile Score

X_6: Van Wagenen Rate of Comprehension Score in Words per Minute

X_7: Costello Scale I Raw Score

X_8: Costello Scale II Raw Score

X_9: Factor A raw Score (16 PF)

X_{10}: Factor B Raw Score (16 PF)

X_{11}: Factor C Raw Score (16 PF)

X_{12}: Factor E Raw Score (16 PF)

X_{13}: Factor F Raw Score (16 PF)

X_{14}: Factor G Raw Score (16 PF)

X_{15}: Factor H Raw Score (16 PF)

X_{16}: Factor I Raw Score (16 PF)

X_{17}: Factor L Raw Score (16 PF)

X_{18}: Factor M Raw Score (16 PF)

X_{19}: Factor N Raw Score (16 PF)

X_{20}: Factor C Raw Score (16 PF)

X_{21}: Factor Q_1 Raw Score (16 PF)

X_{22}: Factor Q_2 Raw Score (16 PF)

Code

X_{23}: Factor Q_3 Raw Score (16 PF)

X_{24}: Factor Q_4 Raw Score (16 PF)

X_{25}: Exvia (Introversion-Extraverion) Sten Score (16 PF)

X_{26}: Anxiety (Low anxiety-High anxiety) Sten Score

X_{27}: Cortertia (Tenderminded Emotionality-Tough Poise) Sten Score

X_{28}: Independence (Subduedness-Independence) Sten Score

X_{29}: Integrated Career Sten Score (MAT)

X_{30}: Integrated Home-Parental Sten Score (MAT)

X_{31}: Integrated Fear Sten Score (MAT)

X_{32}: Integrated Narcism-Comfort Sten Score (MAT)

X_{33}: Integrated Superego Sten Score (MAT)

X_{34}: Integrated Self-Concept Sten Score (MAT)

X_{35}: Integrated Mating Sten Score (MAT)

X_{36}: Integrated Pugnacity-Sadism Sten Score (MAT)

X_{37}: Integrated Assertiveness Sten Score (MAT)

X_{38}: Integrated Sweetheart-Spouse Sten Score (MAT)

X_{39}: General Autism-Optimism Sten Score (MAT)

X_{40}: General Intelligence-Information Sten Score (MAT)

X_{41}: Total Integration Sten Score (MAT)

X_{42}: Total Personal Interest Sten Score (MAT)

X_{43}: Total Conflict Sten Score (MAT)

X_{44}: Term Test #1 Percent Score

X_{45}: Term Test #2 Percent Score

X_{46}: Term Test #3 Percent Score

X_{47}: Term Test #4 Percent Score

Code

X_{48}: Cooperative Mathematics Test Raw Score

X_{49}: Final Examination Percent Score

Note

Students with identification numbers from 1 to 67 inclusive were in the independent study group while those with identification numbers from 68 to 129 inclusive were in the lecture group.

Missing data in the computer printout is indicated by zeros except for the two cases where a dagger (†) appears with the zero.

X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15	X16	X17
1	18	97	97	96	322	10	6	9	9	14	6	7	8	6	16	[illegible]
2	22	70	90	64	157	9	9	2	10	16	15	11	14	12	3	11
3	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
4	22	55	55	-0	-0	7	8	15	9	15	17	16	14	19	8	13
5	21	97	45	82	157	8	6	9	9	12	15	10	16	17	13	7
6	21	85	55	88	277	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
7	19	75	80	54	232	9	9	4	9	16	15	18	12	9	4	7
8	21	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
9	20	95	85	-0	-0	4	6	5	9	5	10	16	5	8	17	8
10	22	70	40	-0	-0	9	6	9	9	18	11	13	16	14	10	13
11	19	90	99	-0	-0	4	12	10	10	12	9	16	14	4	11	7
12	23	95	50	100	240	-0	-0	5	7	11	13	10	12	11	6	9
13	19	70	40	-0	-0	8	6	8	10	18	8	22	11	17	8	13
14	22	50	55	-0	-0	3	8	8	7	12	11	15	12	5	10	11
15	20	97	95	97	315	3	8	6	9	13	11	21	7	22	2	6
16	21	75	85	-0	-0	10	9	9	7	18	16	13	17	22	16	10
17	18	97	50	-0	-0	7	5	12	5	14	16	18	6	19	7	9
18	28	95	45	43	157	6	8	10	5	8	9	9	11	3	12	12
19	20	60	75	64	195	10	7	7	9	14	13	8	14	8	12	12
20	19	97	65	100	262	6	12	7	10	7	14	12	11	5	5	14
21	19	80	90	82	202	7	2	7	6	19	12	17	9	7	9	7
22	20	90	90	-0	-0	4	9	6	10	10	18	17	12	18	12	9
23	21	-0	-0	93	225	9	12	9	7	15	14	11	13	13	12	13
24	21	99	95	-0	-0	8	8	10	10	10	13	10	14	7	15	10
25	18	80	75	82	195	6	8	10	10	11	5	11	12	5	9	12
26	17	80	85	-0	-0	7	7	13	8	23	8	16	14	23	13	6
27	20	80	75	-0	-0	4	4	8	10	17	15	19	13	16	9	9
28	19	95	90	-0	-0	7	3	-0	-0	-0	-0	-0	-0	-0	-0	-0
29	22	60	85	-0	-0	10	1	6	11	13	14	9	13	5	4	10

X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15	X16	X17
30	20	65	20	-0	-0	6	10	9	7	9	10	9	13	9	17	13
31	22	80	55	-0	-0	10	7	14	8	18	19	20	12	22	8	8
32	22	90	60	-0	-0	9	10	12	8	13	15	6	18	9	10	6
33	22	85	85	-0	-0	5	6	5	8	12	10	9	9	12	10	12
34	19	95	99	-0	-0	8	6	12	6	9	7	4	10	2	9	9
35	20	-0	-0	-0	-0	6	8	13	9	16	15	11	18	13	8	9
36	19	30	85	64	187	9	8	9	8	17	5	10	12	9	17	7
37	19	95	85	90	247	9	8	6	11	16	10	12	11	5	11	8
38	27	97	70	97	307	10	5	6	8	16	15	15	16	6	12	7
39	20	70	35	64	225	9	5	7	8	18	7	14	12	13	10	11
40	19	70	75	-0	-0	8	7	7	10	17	13	20	10	18	9	13
41	20	85	90	73	150	6	8	8	10	16	15	16	4	15	12	12
42	20	50	75	-0	-0	7	2	10	7	15	5	13	13	3	12	[illegible]
43	19	95	97	-0	-0	9	5	9	10	13	14	15	4	13	15	10
44	-0	-0	-0	-0	-0	6	9	-0	-0	-0	-0	-0	-0	-0	-0	-0
45	23	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
46	21	75	50	82	120	10	4	10	9	21	14	15	16	21	10	9
47	21	70	85	-0	-0	8	9	6	10	14	14	13	7	12	11	13
48	18	90	97	100	292	7	3	10	11	14	20	24	7	24	8	12
49	17	90	85	99	277	10	9	13	9	12	22	21	10	23	6	12
50	21	90	97	73	240	7	4	8	9	17	15	13	10	12	8	9
51	20	95	99	99	-0	6	4	6	10	19	18	17	8	17	9	9
52	22	35	35	-0	-0	5	12	15	8	14	16	17	11	22	13	11
53	19	75	50	64	172	7	6	7	7	14	7	18	8	15	9	9
54	18	90	97	-0	-0	8	3	8	7	10	22	17	4	12	6	17
55	18	70	80	82	210	7	7	12	8	16	11	16	15	17	9	10
56	18	97	97	-0	-0	6	8	14	8	18	9	15	15	4	12	5
57	18	70	20	97	172	10	7	8	7	20	14	12	13	20	9	13
58	18	90	99	-0	-0	9	8	13	8	18	6	11	17	10	12	6

X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15	X16	X17
59	19	70	97	98	225	2	7	11	9	15	15	15	9	2	7	6
60	19	60	80	73	240	9	8	13	6	18	12	15	10	5	7	6
61	21	45	65	88	270	5	6	6	10	9	18	21	9	7	4	10
62	19	70	85	32	180	6	6	5	8	18	6	14	9	9	10	7
63	19	90	95	99	225	5	2	6	9	17	16	10	6	8	9	14
64	24	99	85	-0	-0	10	9	10	12	17	9	11	16	12	13	7
65	21	60	55	-0	-0	9	11	5	10	12	12	11	14	6	10	8
66	22	97	55	-0	-0	0	7	5	7	18	14	9	13	7	8	12
67	20	97	85	-0	-0	10	4	6	13	22	16	20	6	15	14	4
68	22	97	85	100	340	8	2	8	10	14	13	7	9	9	14	10
69	20	-0	-0	-0	-0	4	7	6	7	15	12	11	13	10	14	9
70	18	85	90	94	202	6	8	10	11	11	12	16	10	15	12	6
71	20	-0	-0	88	105	3	1	7	6	18	13	12	7	14	15	8
72	23	65	40	-0	-0	3	6	10	9	9	8	6	10	8	15	12
73	20	99	85	-0	-0	10	4	7	8	17	14	8	18	16	4	11
74	22	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
75	19	85	90	-0	-0	8	2	11	8	9	16	11	6	5	5	12
76	21	90	80	73	232	7	4	11	8	15	11	15	12	13	19	5
77	18	70	90	99	262	9	6	7	8	20	17	21	14	20	16	6
78	19	70	75	-0	-0	9	12	2	10	12	17	21	16	17	9	16
79	19	75	60	96	300	5	8	6	9	9	22	18	11	17	4	13
80	22	85	75	47	120	3	9	7	8	20	20	18	4	14	4	12
81	18	90	95	-0	-0	5	8	8	6	9	7	10	13	10	11	11
82	23	90	65	-0	-0	7	10	4	11	10	8	16	13	7	9	5
83	18	70	15	73	202	4	8	13	9	21	20	20	10	24	8	12
84	20	60	60	54	195	8	7	7	10	12	15	17	12	24	14	10
85	24	50	55	82	285	5	4	0	9	12	11	12	6	4	10	8
86	20	90	95	-0	-0	5	6	6	9	3	23	22	16	14	5	17
87	22	50	20	43	157	10	9	8	11	12	9	16	7	13	15	10

X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15	X16	X17
88	18	90	90	82	180	4	9	7	12	11	17	18	5	17	6	12
89	17	97	97	99	232	9	7	8	10	15	17	17	12	19	16	14
90	28	97	85	-0	-0	6	7	4	9	11	6	4	10	8	9	13
91	20	95	99	97	377	3	3	6	12	13	24	16	4	9	2	12
92	20	85	90	99	292	10	10	11	7	15	16	20	19	15	8	14
93	17	99	99	-0	-0	8	8	8	11	14	18	12	12	5	9	9
94	18	85	85	88	262	5	4	9	8	17	16	10	9	10	13	7
95	19	90	90	93	247	8	7	6	9	17	22	17	13	18	17	15
96	18	97	95	97	300	9	12	10	8	9	18	23	12	19	10	14
97	23	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
98	19	75	95	-0	-0	5	1	4	11	15	16	15	7	18	14	10
99	19	97	90	96	210	6	8	16	10	15	10	21	14	20	9	1[illegible]
100	21	99	60	-0	-0	7	11	8	10	14	12	10	11	5	16	12
101	20	80	40	-0	-0	8	4	5	5	21	14	18	12	17	4	9
102	20	60	15	-0	-0	7	10	12	9	15	16	14	13	16	14	10
103	18	80	90	82	210	6	7	9	8	16	9	19	7	15	10	5
104	28	97	45	-0	-0	9	4	6	8	16	9	14	19	10	6	11
105	20	90	60	88	187	5	8	4	6	14	14	15	6	11	13	13
106	21	80	65	96	225	3	10	4	9	11	16	11	8	6	8	10
107	18	65	35	-0	-0	3	8	8	9	8	20	22	6	21	11	14
108	20	95	95	-0	-0	10	10	9	11	9	9	22	17	14	12	6
109	20	85	85	-0	-0	7	8	5	8	10	17	18	16	15	10	10
110	24	75	40	-0	-0	7	2	9	10	20	18	21	13	17	14	9
111	20	90	35	-0	-0	10	7	12	8	9	6	10	11	9	12	2
112	19	65	90	-0	-0	8	4	12	12	21	7	17	9	20	14	5
113	18	80	65	88	225	3	5	9	11	15	19	19	8	19	5	14
114	25	50	15	-0	-0	7	2	4	8	18	16	12	11	13	4	9
115	18	60	95	-0	-0	9	5	6	8	19	5	5	13	3	8	7
116	18	-0	-0	73	-0	5	7	7	11	13	11	15	13	18	9	2

X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15	X16	X17
117	23	85	90	99	187	7	6	6	12	18	19	16	10	17	13	9
118	-0	-0	-0	73	187	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
119	21	-0	-0	-0	-0	9	9	12	9	14	13	8	14	13	9	13
120	19	75	55	54	180	10	7	8	8	14	14	21	18	16	12	12
121	22	90	95	99	157	9	8	16	9	15	14	20	14	24	9	11
122	18	95	99	97	322	6	7	10	10	16	15	7	16	5	7	7
123	22	90	95	92	270	1	1	1	11	17	11	9	8	1	6	4
124	18	90	95	-0	-0	4	7	3	12	9	12	11	13	3	16	4
125	19	97	90	-0	-0	9	8	9	10	14	4	10	10	4	14	7
126	21	90	60	-0	-0	9	9	9	8	16	13	6	11	11	10	5
127	20	60	60	33	45	9	10	5	8	15	10	5	17	13	6	11
128	19	97	99	-0	-0	7	11	7	7	16	15	8	17	2	6	4
129	-0	-0	-0	93	225	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0

X1	X18	X19	X20	X21	X22	X23	X24	X25	X26	X27	X28	X29	X30	X31	X32	X33
1	12	10	17	12	9	14	20	2.4	7.6	.6	4.1	4	6	5	4	6
2	16	6	8	16	13	14	16	4.3	5.6	9.6	8.0	6	6	4	6	6
3	-0	-0	-0	-0	-0	-0	-0	*.0	*.0	*.0	*.0	-0	-0	-0	-0	-0
4	10	11	14	17	9	12	10	6.1	5.5	2.6	6.8	5	7	3	8	5
5	12	13	7	10	15	15	5	3.8	3.3	6.5	6.8	5	9	3	1	6
6	-0	-0	-0	-0	-0	-0	-0	*.0	*.0	*.0	*.0	-0	-0	-0	-0	-0
7	9	7	9	11	14	11	15	4.9	5.4	9.6	5.5	8	3	7	7	1
8	-0	-0	-0	-0	-0	-0	-0	*.0	*.0	*.0	*.0	-0	-0	-0	-0	-0
9	14	9	12	9	13	9	16	2.8	7.1	4.8	6.5	4	5	5	3	2
10	8	6	11	7	12	17	16	4.8	6.2	4.2	4.7	6	8	1	9	7
11	14	8	14	11	14	11	20	3.7	7.9	4.5	3.5	5	4	6	7	7
12	7	7	19	10	10	11	16	3.7	7.7	6.6	4.7	5	5	5	9	4
13	12	2	14	9	8	12	15	6.7	6.1	7.3	4.8	4	4	4	5	7
14	9	8	19	11	9	13	18	4.0	8.7	4.3	3.4	4	4	4	5	5
15	11	9	11	7	8	10	15	7.0	5.8	9.9	4.3	8	6	3	8	5
16	12	8	9	9	10	17	11	5.8	4.6	3.2	6.2	7	6	3	5	5
17	10	5	11	12	7	9	9	6.7	5.3	5.2	6.3	5	4	5	8	5
18	10	11	21	10	13	10	21	2.6	9.7	5.1	4.4	6	6	5	4	6
19	13	6	10	10	13	9	13	3.3	6.4	4.7	5.8	6	5	7	5	7
20	15	5	11	14	15	10	18	3.9	7.8	[illegible].7	7.8	8	4	9	4	9
21	12	14	8	13	15	13	9	3.5	4.2	5.5	5.6	5	6	1	3	7
22	15	6	13	10	13	10	16	6.1	7.0	7.2	7.4	5	5	5	3	6
23	8	5	15	9	9	11	9	4.3	6.4	3.4	5.1	-0	-0	-0	-0	-0
24	13	12	14	8	8	9	16	4.0	7.3	4.9	5.7	3	4	6	4	5
25	13	7	18	8	10	8	20	3.0	9.2	3.5	2.5	4	4	6	4	5
26	16	12	9	11	3	11	4	6.4	1.9	5.3	4.9	4	6	3	4	5
27	13	6	9	16	18	10	12	5.4	4.8	7.2	8.3	8	6	6	5	4
28	-0	-0	-0	-0	-0	-0	-0	*.0	*.0	*.0	*.0	3	2	9	8	4
29	8	6	18	13	13	12	10	3.0	6.3	6.7	5.7	5	5	3	5	5

X1	X18	X19	X20	X21	X22	X23	X24	X25	X26	X27	X28	X29	X30	X31	X32	X33
30	8	17	20	8	10	12	15	3.4	9.8	4.8	3.4	6	5	3	2	4
31	12	8	6	17	8	14	8	9.0	3.3	4.9	7.7	6	1	1	3	6
32	14	13	12	6	14	15	9	2.9	5.2	1.9	4.5	3	4	3	4	5
33	15	14	17	12	19	8	18	1.1	7.2	6.3	6.4	8	6	3	3	2
34	10	7	20	14	14	13	17	2.2	8.7	2.6	4.9	4	4	6	4	[illegible]
35	11	8	9	9	13	10	6	4.4	4.5	3.7	5.7	-0	-0	-0	-0	-0
36	14	10	14	9	8	14	17	3.3	6.5	2.9	3.9	5	5	3	6	7
37	13	9	12	13	9	12	15	3.0	5.6	4.5	6.2	5	3	6	9	7
38	16	14	8	14	16	13	17	3.5	5.7	5.8	6.2	6	4	4	7	4
39	13	11	11	12	17	13	4	3.1	3.9	4.7	5.6	5	3	4	5	5
40	6	7	10	13	11	15	16	6.3	6.2	5.8	5.9	4	4	2	3	5
41	9	9	16	10	9	13	17	5.1	6.7	5.4	5.6	6	4	6	5	2
42	15	11	16	11	14	16	8	2.1	5.1	2.6	2.5	4	4	6	9	5
43	16	8	15	10	13	7	13	4.2	6.5	5.3	7.7	4	8	1	7	4
44	-0	-0	-0	-0	-0	-0	-0	*.0	*.0	*.0	*.0	-0	-0	-0	-0	-0
45	-0	-0	-0	-0	-0	-0	-0	*.0	*.0	*.0	*.0	-0	-0	-0	-0	-0
46	12	9	3	9	8	19	3	6.6	1.4	4.2	6.5	4	3	6	9	5
47	14	5	15	13	12	6	18	4.4	7.8	7.3	7.1	5	3	6	9	8
48	8	6	[illegible]	11	11	8	15	9.5	5.2	8.4	8.2	2	4	5	4	4
49	14	4	10	12	16	10	15	9.1	6.0	7.3	9.5	4	4	7	10	7
50	12	9	13	17	13	9	16	4.4	6.5	6.4	6.	6	6	3	5	6
51	16	7	7	10	15	12	6	4.6	2.1	8.0	8.5	4	6	3	10	4
52	15	6	11	13	7	9	15	7.4	6.0	3.8	7.1	5	1	6	2	2
53	7	12	9	9	11	10	11	4.3	4.7	7.0	3.6	8	5	1	6	7
54	15	10	12	10	12	6	18	5.8	7.8	7.9	9.0	3	6	4	6	4
55	8	14	12	10	10	14	13	5.0	5.8	2.6	3.9	4	3	4	6	5
56	12	11	14	9	7	17	10	4.6	5.2	3.3	3.5	8	6	5	4	5
57	15	8	10	13	10	10	9	5.5	4.5	5.7	7.5	4	5	5	6	3
58	12	7	8	9	8	15	14	4.7	4.6	4.2	4.2	3	5	4	6	2

⋮

X1	X18	X19	X20	X21	X22	X23	X24	X25	X26	X27	X28	X29	X30	X31	X32	X33
59	12	9	10	12	14	10	21	3.8	7.3	5.6	5.2	6	5	6	5	5
60	16	8	15	6	9	12	18	4.7	7.0	4.6	3.7	4	5	6	4	2
61	12	4	17	13	11	8	20	6.0	8.8	9.5	6.5	4	2	6	3	5
62	12	10	12	12	13	12	12	3.7	4.9	6.2	4.2	4	5	9	6	4
63	16	12	8	17	18	11	10	2.6	4.6	6.2	9.5	4	1	5	9	7
64	12	14	14	5	8	13	8	3.4	5.1	4.4	3.3	5	6	4	2	2
65	16	12	12	4	16	9	15	2.4	6.4	6.1	4.8	8	5	3	8	4
66	16	10	11	12	14	18	17	3.2	5.3	5.5	6.8	5	2	4	5	7
67	16	6	16	8	8	15	15	5.1	4.4	5.3	6.3	4	5	6	9	4
68	24	6	13	12	10	9	15	3.2	6.2	5.0	6.8	4	7	4	9	7
69	16	7	10	10	10	13	12	3.4	5.6	4.7	5.8	6	3	7	6	2
70	14	9	16	9	8	12	9	4.7	5.7	5.2	5.7	6	9	3	7	5
71	16	9	6	14	15	10	8	3.7	3.5	6.9	7.5	-0	-0	-0	-0	-0
72	8	15	14	5	15	13	9	1.4	6.7	.2	3.4	3	8	1	2	5
73	14	8	10	9	14	13	8	4.1	5.0	6.2	6.2	6	6	6	1	4
74	-0	-0	-0	-0	-0	-0	-0	*.0	*.0	*.0	*.0	-0	-0	-0	-0	-0
75	12	11	12	13	15	14	14	3.1	6.3	4.3	7.2	7	3	6	6	2
76	14	3	6	8	11	15	10	4.9	3.9	3.2	4.9	3	6	5	3	9
77	12	8	7	10	12	13	11	7.1	4.0	6.1	6.5	5	5	6	6	6
78	11	6	10	9	8	10	19	7.2	7.8	8.3	6.3	5	5	7	4	7
79	10	3	16	16	13	9	24	6.6	8.9	9.2	8.6	5	4	7	7	6
80	10	6	14	16	12	8	8	5.9	4.8	8.7	8.8	6	6	7	5	2
81	11	12	17	10	16	6	13	2.0	8.0	4.9	4.8	6	5	1	5	4
82	8	5	15	9	13	13	24	3.7	8.8	7.2	3.0	5	6	8	4	1
83	8	4	11	12	13	14	10	8.1	4.0	6.0	8.3	5	2	3	6	5
84	15	7	12	7	9	9	17	7.3	6.5	6.3	6.0	2	5	9	6	6
85	7	8	15	5	14	4	17	2.4	8.0	7.7	2.9	2	4	3	6	5
86	6	10	16	12	17	8	23	6.7	9.9	7.8	7.4	5	5	6	[illegible]	7
87	12	4	13	4	13	7	14	4.0	6.9	5.2	4.1	6	3	6	4	4

X1	X18	X19	X20	X21	X22	X23	X24	X25	X26	X27	X28	X29	X30	X31	X32	X33
88	10	6	19	12	15	8	18	5.3	8.4	8.2	6.5	6	1	6	3	9
89	13	5	7	9	12	13	8	6.[illegible]	4.6	5.4	7.6	6	3	4	5	[illegible]
90	14	6	14	11	12	14	10	1.9	5.7	7.8	5.0	5	5	7	5	1
91	12	5	5	16	18	11	18	4.2	4.7	8.9	9.9	5	6	6	7	4
92	8	9	9	14	4	14	13	7.6	6.1	4.3	5.1	4	3	6	7	3
93	14	9	2	11	13	12	15	4.4	4.4	6.7	7.6	5	3	4	8	4
94	15	5	10	10	14	12	19	4.[illegible]	6.3	5.6	6.6	5	4	6	6	4
95	13	5	10	15	13	15	10	6.4	4.9	6.2	9.9	5	2	1	6	3
96	10	8	8	8	13	13	23	7.8	8.3	5.8	6.6	6	1	6	8	5
97	-0	-0	-0	-0	-0	-0	-0	*.0	*.0	*.0	*.0	-0	-0	-0	-0	-0
98	17	8	6	14	15	10	17	4.8	3.3	7.4	9.2	6	2	7	5	4
99	12	7	10	10	8	12	14	7.8	5.9	3.4	4.6	5	7	5	4	5
100	8	7	18	8	13	16	18	3.3	7.5	6.2	5.5	1	4	3	8	3
101	12	9	4	7	14	15	11	5.7	3.3	8.8	6.3	5	4	5	8	4
102	17	10	14	11	7	13	15	5.8	6.0	5.7	7.0	4	2	7	1	3
103	12	6	12	11	7	13	13	6.1	5.4	6.5	4.0	5	6	5	5	3
104	8	12	10	4	15	18	11	3.6	5.6	5.3	3.2	3	6	9	3	5
105	12	6	11	13	14	8	10	4.2	5.6	6.7	7.3	4	5	7	4	7
106	16	10	12	9	17	12	18	2.8	7.2	7.5	6.9	4	5	7	5	3
107	8	0	11	12	8	15	20	8.3	7.9	7.0	7.8	3	5	3	5	4
108	8	9	14	8	4	18	18	7.1	7.2	6.8	3.0	3	6	6	3	3
109	14	10	14	11	8	15	16	6.1	7.0	5.9	5.5	3	6	5	5	3
110	15	5	7	10	9	11	8	7.3	3.8	6.5	7.5	3	6	3	4	5
111	10	12	14	5	13	8	11	2.7	6.2	2.8	1.5	7	4	3	3	2
112	12	10	11	10	10	14	7	5.3	3.2	4.3	4.8	3	6	5	5	5
113	13	6	7	15	8	9	7	7.3	4.0	7.7	8.5	5	6	5	9	5
114	18	8	7	9	19	15	15	7.9	4.5	9.3	8.3	6	6	7	5	4
115	14	12	8	3	19	15	4	[illegible]	2.9	4.2	3.3	4	4	4	9	6
116	13	6	15	10	9	8	10	5.3	6.1	6.1	5.1	-0	-0	-0	-0	-0

X1	X18	X19	X20	X21	X22	X23	X24	X25	X26	X27	X28	X29	X30	X31	X32	X33
117	20	5	8	14	13	9	8	5.5	4.1	8.0	9.5	5	8	7	6	7
118	-0	-0	-0	-0	-0	-0	-0	*.0	*.0	*.0	*.0	-0	-0	-0	-0	-0
119	9	14	12	7	10	11	15	3.7	6.5	2.0	4.4	5	7	5	3	4
120	13	10	19	8	13	13	11	5.9	7.0	5.0	5.0	7	8	6	8	5
121	11	11	7	10	8	12	13	8.4	5.7	3.2	5.9	7	2	5	2	6
122	13	14	6	12	15	13	17	2.9	4.8	5.6	6.6	4	4	4	8	4
123	15	7	5	15	18	5	18	1.9	5.6	9.0	6.6	2	4	7	5	5
124	17	2	14	11	18	13	19	2.2	7.2	7.1	6.2	3	5	3	1	3
125	12	12	10	11	14	16	9	1.5	4.9	2.2	3.2	5	2	6	3	6
126	14	12	16	10	11	13	10	2.8	5.0	4.3	5.4	3	9	3	1	5
127	8	12	13	13	16	15	12	2.1	5.8	4.6	5.3	7	7	6	1	4
128	8	8	18	10	15	11	17	2.2	7.3	5.6	3.6	6	9	3	1	4
129	-0	-0	-0	-0	-0	-0	-0	*.0	*.0	*.0	*.0	-0	-0	-0	-0	-0

X1	X34	X35	X36	X37	X38	X39	X40	X41	X42	X43	X44	X45	X46	X47	X48	X49
1	8	5	7	1	7	3	7	8	3	6	-0	-0	-0	-0	-0	-0
2	4	5	6	6	4	1	7	9	4	5	-0	-0	-0	-0	-0	-0
3	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	40	-0	-0	-0	-0	-0
4	5	5	6	1	1	8	5	1	6	9	-0	10	-0	-0	-0	-0
5	1	9	5	4	7	4	5	5	5	8	44	30	83	66	25	79
6	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	78	-0	-0	-0	-0
7	5	9	9	3	7	6	9	5	8	6	38	24	-0	-0	-0	-0
8	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	44	54	-0	79	14	-0
9	8	9	7	6	2	5	7	5	5	8	-0	-0	-0	-0	-0	-0
10	5	5	6	7	7	8	7	6	8	7	28	46	-0	-0	-0	-0
11	5	5	4	1	7	4	6	7	5	6	86	74	70	93	37	30
12	6	9	4	3	1	4	6	5	5	7	22	-0	-0	-0	-0	-0
13	6	8	6	3	3	2	6	6	4	7	88	58	50	-0	-0	-0
14	6	8	8	7	7	6	8	6	8	7	-0	-0	-0	-0	-0	-0
15	1	5	5	3	7	6	6	4	7	9	50	-0	-0	-0	-0	-0
16	6	4	3	3	6	6	6	4	5	8	75	60	-0	50	23	35
17	6	5	5	6	8	5	7	6	8	7	54	42	13	33	19	28
18	5	5	6	6	1	5	5	5	4	8	62	66	53	50	16	35
19	8	9	4	3	6	5	8	7	6	6	64	46	-0	14	-0	-0
20	4	1	2	2	4	3	5	6	2	8	76	84	58	56	16	40
21	1	5	10	3	8	4	5	5	4	7	78	52	5	34	15	18
22	7	3	6	5	7	6	7	6	5	8	-0	-0	-0	-0	-0	-0
23	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	80	68	73	50	-0	-0
24	5	3	6	1	6	4	5	4	3	8	38	36	55	82	27	70
25	8	6	4	6	5	4	7	6	5	7	76	62	30	-0	-0	-0
26	3	3	9	2	6	6	4	3	4	8	40	38	-0	-0	14	28
27	5	5	8	5	3	4	6	9	5	5	-0	-0	-0	11	16	20
28	7	7	4	4	5	6	7	5	7	10	46	60	-0	-0	-0	-0
29	3	5	5	3	6	6	4	3	5	9	76	-0	-0	-0	-0	-0

X1	X34	X35	X36	X37	X38	X39	X40	X41	X42	X43	X44	X45	X46	X47	X48	X49
30	3	6	1	3	1	6	3	1	2	10	68	48	-0	-0	-0	-0
31	6	7	8	5	4	5	4	3	5	0	26	-0	-0	-0	-0	-0
32	4	5	7	2	6	2	4	6	2	7	88	70	85	100	24	83
33	6	7	7	7	6	3	7	8	5	6	92	85	93	84	32	100
34	1	3	8	6	4	4	4	4	3	8	90	96	85	79	27	95
35	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	10	-0	-0	-0	-0
36	5	3	2	5	7	4	6	5	4	8	34	-0	-0	-0	-0	-0
37	8	6	6	6	6	4	8	7	8	6	36	20	-0	-0	-0	-0
38	8	5	5	4	7	9	7	4	7	9	92	66	63	66	22	35
39	5	8	6	4	4	5	6	5	5	8	12	30	-0	-0	-0	-0
40	5	7	9	4	6	3	6	6	4	7	66	48	-0	-0	-0	-0
41	5	8	5	7	7	4	6	6	7	7	86	60	3	57	23	37
42	6	6	4	4	6	3	7	7	5	6	64	78	97	44	17	30
43	5	6	3	5	3	3	5	4	4	8	-0	-0	-0	-0	-0	-0
44	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
45	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	32	-0	-0	-0	-0	-0
46	7	2	5	2	5	10	5	2	7	10	38	-0	-0	-0	-0	-0
47	9	8	8	3	4	5	10	9	7	5	-0	-0	-0	-0	-0	-0
48	5	10	6	6	6	4	6	5	6	8	62	58	35	-0	-0	-0
49	9	5	3	6	4	3	10	9	5	5	92	70	45	82	33	65
50	6	7	5	7	6	4	7	7	6	6	44	74	73	82	13	43
51	5	4	6	6	2	8	5	4	6	8	-0	16	-0	-0	-0	-0
52	4	4	8	6	5	4	4	5	3	8	72	84	90	83	21	44
53	4	5	2	6	6	5	5	5	4	8	50	74	-0	-0	-0	-0
54	2	7	7	3	4	4	4	4	4	9	36	14	-0	-0	-0	-0
55	4	8	8	2	5	4	5	5	4	8	44	-0	-0	-0	-0	-0
56	2	4	7	3	8	7	5	5	5	8	90	58	-0	-0	-0	-0
57	5	6	9	6	3	4	6	6	4	7	-0	-0	-0	-0	-0	-0
58	6	3	2	4	4	6	4	2	2	9	52	56	25	62	18	58

Y1	X34	X35	X36	X37	X38	X39	X40	X41	X42	X43	X44	X45	X46	X47	X48	X49
59	4	8	6	6	2	3	6	6	5	6	82	70	85	-0	-0	-0
60	5	8	6	3	7	4	6	5	5	8	58	76	48	70	17	45
61	4	6	8	2	6	7	5	3	5	9	66	40	38	-0	15	10
62	6	7	7	3	4	4	7	5	6	7	86	78	67	82	24	48
63	5	8	7	8	7	2	7	7	7	5	96	-0	38	44	20	10
64	6	5	5	3	6	6	5	3	4	9	72	78	40	86	17	38
65	3	3	6	3	2	8	4	2	5	9	90	92	85	74	43	100
66	5	6	9	3	6	5	7	6	5	9	64	48	60	30	18	25
67	6	5	5	6	6	6	7	7	6	9	28	60	-0	-0	-0	-0
68	8	5	6	4	4	8	7	5	7	8	-0	60	100	-0	-0	-0
69	6	7	5	7	6	4	7	7	6	6	40	52	-0	-0	-0	-0
70	5	4	4	2	8	5	6	6	4	8	20	-0	-0	-0	-0	-0
71	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
72	4	5	3	3	9	6	5	3	4	9	48	-0	-0	-0	-0	-0
73	4	1	6	1	7	3	4	4	3	9	80	88	95	97	38	100
74	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	52	-0	-0	-0	-0	-0
75	3	9	7	6	4	3	6	5	6	6	68	68	55	57	16	26
76	5	6	10	2	4	6	7	4	6	8	72	84	60	27	13	36
77	6	2	5	5	7	6	7	5	7	8	-0	-0	-0	-0	-0	-0
78	6	6	7	4	4	3	6	6	5	8	64	72	40	30	-0	-0
79	5	6	10	3	4	6	7	7	6	6	48	64	85	84	23	49
80	4	6	7	5	1	4	5	5	5	8	88	68	60	51	15	10
81	3	3	5	1	6	6	3	1	4	10	40	-0	-0	-0	-0	-0
82	8	8	4	6	6	3	7	7	6	7	52	52	60	72	25	75
83	5	5	7	4	6	4	5	5	4	8	48	-0	-0	-0	-0	-0
84	5	6	8	2	2	7	6	3	7	9	52	40	-0	-0	-0	-0
85	6	6	5	4	6	4	5	5	5	8	16	-0	-0	-0	-0	-0
86	6	6	7	5	4	6	7	5	8	9	64	52	95	27	18	28
87	6	5	7	4	4	6	6	5	5	8	28	64	60	33	17	16

X1	X34	X35	X36	X37	X38	X39	X40	X41	X42	X43	X44	X45	X46	X47	X48	X49
88	6	7	5	6	8	3	8	6	8	6	92	72	90	36	20	58
89	7	7	7	6	2	6	7	5	6	8	88	68	50	57	26	46
90	7	7	2	3	7	3	6	6	4	7	84	96	95	75	23	50
91	6	7	8	8	5	1	9	9	7	4	76	80	70	69	23	41
92	5	10	4	4	4	4	5	6	5	7	92	68	85	78	25	70
93	9	9	3	6	2	1	7	5	4	6	100	100	100	81	48	100
94	7	5	5	4	3	3	6	5	4	8	56	60	75	66	21	56
95	8	7	8	6	4	4	7	5	6	8	68	-0	-0	-0	-0	-0
96	4	7	8	3	6	6	6	5	7	8	68	60	70	66	17	74
97	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	68	72	45	78	17	33
98	6	9	6	8	4	6	7	6	7	7	80	72	85	42	22	66
99	5	7	4	6	9	4	7	8	5	6	76	68	55	42	18	20
100	4	3	4	3	6	2	3	4	2	10	56	-0	-0	-0	13	-0
101	4	8	5	6	4	6	5	5	7	8	44	48	50	45	13	15
102	3	3	5	3	3	4	3	3	1	9	72	56	55	69	17	20
103	7	7	3	2	7	4	6	5	5	8	48	64	-0	-0	-0	-0
104	6	7	6	3	6	6	7	6	8	7	72	80	70	54	20	36
105	7	5	9	3	5	2	8	4	5	5	64	68	35	-0	-0	-0
106	7	5	5	6	6	7	6	4	7	8	48	60	60	51	18	18
107	6	10	5	7	4	6	7	5	6	7	64	80	65	51	17	33
108	4	7	7	6	6	6	5	6	5	8	56	48	-0	-0	-0	-0
109	1	4	7	6	5	2	4	5	3	[illegible]	40	80	75	57	40	80
110	7	6	8	3	7	5	7	5	6	8	44	-0	-0	-0	-0	-0
111	3	5	6	4	2	4	4	3	2	9	76	80	90	84	33	98
112	8	5	3	1	4	7	5	3	4	9	64	-0	70	75	29	91
113	5	5	6	7	4	4	6	6	7	7	-0	-0	-0	-0	-0	-0
114	6	5	10	7	5	3	9	9	7	5	56	52	-0	-0	-0	-0
115	3	5	4	6	4	4	5	5	4	8	44	68	55	45	21	26
116	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	76	40	70	54	19	13

X1	X34	X35	X36	X37	X38	X39	X40	X41	X42	X43	X44	X45	X46	X47	X48	X49
117	5	7	7	1	8	5	7	7	7	6	68	56	50	45	20	38
118	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	68	68	75	57	22	46
119	3	5	4	2	3	4	4	3	3	9	52	-0	-0	-0	-0	-0
120	4	6	2	5	4	3	6	7	6	6	64	72	40	78	24	45
121	3	5	5	5	6	7	4	7	6	9	44	40	80	57	18	19
122	5	6	8	6	4	4	6	6	6	7	96	88	85	69	33	65
123	5	6	4	4	6	3	5	6	5	7	100	84	80	78	33	99
124	6	6	2	5	7	4	4	3	3	9	72	-0	-0	-0	-0	-0
125	5	5	3	3	6	2	5	6	3	7	80	80	95	78	28	51
126	2	5	8	3	5	3	4	5	3	8	92	88	70	81	30	86
127	1	3	5	2	7	7	4	2	4	9	76	40	-0	0+	-0	-0
128	2	4	9	4	4	4	4	5	3	8	100	72	95	81	28	73
129	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	72	65	24	26	40

APPENDIX J

PRELIMINARY STUDY

PRELIMINARY STUDY

This is a brief summary of a study the researcher made during the 1971-72 academic year at Mount Royal College in Calgary, Alberta.

In September, 1971, a one-semester calculus course, PMAT 201, was offered on an independent study mastery learning approach similar to that described in Chapter III. There was a difference in that the students wrote weekly tests that counted towards their final grade and there was no formative testing. At the beginning of the semester, scores were obtained from the General Aptitude Test Battery (GATB). The GATB yields a general intelligence score (G) and subscores for verbal ability (V), numerical ability (N), spatial ability (S), form perception (P), and clerical perception (Q). These six scores were compared with the final achievement scores (in terms of grade point) of the students. Means, standard deviations, and correlations for 37 students are shown in Table XIX.

Table XIX

Means, Standard Deviations (SD) and Correlations (r) for 37 Fall (1971) Independent Study Students

	G	V	N	S	P	Q	Achievement Score
Mean	71.2	61.6	77.0	70.0	71.8	81.6	1.7
SD	18.2	21.4	15.5	23.3	20.6	15.8	1.4
r	-0.344*	-0.303	0.087	-0.269	0.157	-0.330*	

*significant at the 0.05 level

A comparison of achievement scores with scores on the Wonderlic Test for 41 students showed a nonsignificant correlation of -0.156. The Wonderlic Test is an adaptation of the Otis Self-Administering Test of mental ability. A nonsignificant correlation of -0.087 between achievement scores and scores on the Van Wagenen reading test showing rate of comprehension in words per minute for 43 students was also found.

In the 1972 spring semester, the calculus course was again offered on an independent study mastery learning approach as described in Chapter III, except the course was completely self-pacing. Scores were obtained from Costello's scales of need achievement. Means, standard deviations, and correlations are as shown in Table XX for 37 students:

Table XX

Means, Standard Deviations (SD), and Correlations (r) for 37 Spring (1972) Independent Study Students

	Costello Scale I	Costello Scale II	Achievement Score
Mean	6.1	6.2	2.0
SD	2.3	2.3	1.3
r	0.325*	0.483**	

*significant at the 0.05 level

**significant at the 0.01 level

The absence of a significant positive relationship between general ability (G score) and achievement in calculus in the fall semester, and the presence of significant positive relationships between need to achieve scores and achievement in calculus is interesting. The latter relationship makes sense in that the course was highly structured and involved considerable work for the student, regardless of his intellectual powers. However, the former situation is indeed puzzling.

The dropout ratio for the 1971 fall and 1972 spring groups were 52 per cent and 54 per cent respectively. When PMAT 201 was taught by the lecture method during the 1970 academic year at Mount Royal College, the dropout ratio was 38 per cent.

The mean grades on the final examination for the fall and spring groups were 44.4 per cent and 76.6 per cent respectively. The difference in the method of instruction for the two groups was with respect to formative testing and self-pacing. The spring group had formative testing and self-pacing, whereas the fall group did not have these two aspects of independent study. The lecture group of 1970 had a mean final examination score of 45.6 per cent. All final examinations were very similar in nature.

APPENDIX K

CORRELATION TABLES FOR THE LECTURE AND INDEPENDENT STUDY GROUPS

Table XXI

Correlations of Student Test Scores with the Term Test Scores, Co-operative Mathematics Test (CMT) Scores, and Final Examination Scores For LECTURE Students

Test	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9
Test 1	-0.127	0.493*	0.355*	0.130	0.095	0.018	0.033	0.030
Test 2	0.039	0.425*	0.211	0.272	0.483*	-0.056	-0.139	-0.045
Test 3	0.006	0.434*	0.359*	0.324	0.528*	-0.040	-0.136	0.074
Test 4	0.019	0.316	0.043	0.248	0.532*	0.181	0.028	0.095
CMT	-0.197	0.335*	0.317	0.448*	0.528*	0.171	-0.085	-0.140
Final	-0.160	0.246	0.282	0.379	0.562*	0.163	-0.233	-0.084

Test	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}
Test 1	0.130	0.140	0.142	-0.183	0.108	-0.158	-0.237	-0.100
Test 2	0.051	0.034	-0.038	-0.304*	-0.103	-0.427*	-0.135	-0.169
Test 3	0.168	-0.197	-0.033	-0.368*	0.097	-0.259	-0.264	-0.225
Test 4	-0.124	0.128	-0.147	-0.425*	0.188	-0.304	-0.206	-0.360*
CMT	0.119	0.061	-0.082	-0.344*	0.330*	-0.290	-0.127	-0.387*
Final	0.054	0.105	-0.095	-0.326	0.204	-0.180	-0.059	-0.484*

* $p < 0.05$

Table XXI (continued)

Test	X_{18}	X_{19}	X_{20}	X_{21}	X_{22}	X_{23}	X_{24}	X_{25}
Test 1	0.060	0.023	-0.189	0.436*	0.128	0.057	-0.041	-0.080
Test 2	0.020	0.033	-0.167	0.092	0.166	0.069	-0.164	-0.321*
Test 3	-0.022	0.174	0.055	0.200	0.097	-0.089	0.167	-0.235
Test 4	0.008	0.254	0.085	0.160	-0.005	0.132	-0.003	-0.295
CMT	0.150	0.346*	-0.173	0.099	0.041	0.004	-0.067	-0.268
Final	0.138	0.252	-0.140	0.128	0.038	-0.080	-0.120	-0.240

Test	X_{26}	X_{27}	X_{28}	X_{29}	X_{30}	X_{31}	X_{32}	X_{33}
Test 1	-0.166	0.076	0.187	0.093	-0.050	0.130	-0.103	-0.025
Test 2	-0.201	-0.038	-0.012	-0.165	0.097	-0.269	0.065	-0.061
Test 3	0.114	-0.053	-0.097	0.029	-0.224	-0.140	-0.069	0.036
Test 4	-0.050	-0.120	-0.167	0.035	0.131	-0.076	-0.046	-0.319
CMT	-0.154	-0.054	-0.134	-0.052	0.144	-0.166	-0.066	-0.121
Final	-0.184	-0.080	-0.134	-0.042	0.143	-0.169	-0.197	0.009

*$p < 0.05$

Table XXI (continued)

Test	X_{34}	X_{35}	X_{36}	X_{37}	X_{38}	X_{39}	X_{40}	X_{41}
Test 1	0.006	0.213	0.042	0.111	-0.213	-0.295*	0.061	0.025
Test 2	0.122	0.086	-0.117	-0.019	-0.064	-0.550*	0.027	0.159
Test 3	0.069	-0.085	-0.107	-0.031	0.037	-0.012	-0.131	-0.060
Test 4	-0.110	-0.237	-0.208	-0.306	-0.071	-0.391*	-0.396*	-0.040
CMT	0.055	-0.125	-0.139	-0.047	-0.100	-0.474*	-0.134	0.015
Final	0.044	-0.076	-0.034	-0.211	-0.095	-0.301	-0.111	-0.063

Test	X_{42}	X_{43}	X_{44}	X_{45}	X_{46}	X_{47}	X_{48}	X_{49}
Test 1	-0.042	-0.237	1.000	-	-	-	-	-
Test 2	-0.311*	-0.250	0.537*	1.000	-	-	-	-
Test 3	-0.120	0.171	0.406*	0.296	1.000	-	-	-
Test 4	-0.530*	-0.045	0.352*	0.339*	0.372*	1.000	-	-
CMT	-0.369*	-0.103	0.419*	0.610*	0.537*	0.576*	1.000	-
Final	-0.310	-0.060	0.498*	0.687*	0.595*	0.593*	0.864*	1.000

*$p < 0.05$

Table XXII

Correlations of Student Test Scores with the Term Test Scores, Co-operative Mathematics Test (CMT) Scores, and Final Examination Scores For INDEPENDENT STUDY Students

Test	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9
Test 1	-0.039	0.054	0.187	0.184	0.132	-0.340*	0.279	0.063
Test 2	0.137	0.030	0.044	-0.072	0.097	-0.081	0.318*	0.019
Test 3	0.294	-0.132	-0.174	0.087	0.154	-0.176	0.214	0.046
Test 4	0.085	0.088	0.179	-0.048	0.274	0.042	0.477*	0.291
CMT	-0.073	0.023	0.029	0.119	-0.013	0.132	0.421*	-0.119
Final	0.008	0.131	-0.028	-0.127	-0.170	0.328	0.359	0.107

Test	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}
Test 1	-0.053	-0.225	-0.020	-0.104	-0.047	-0.257	0.004	0.092
Test 2	-0.075	-0.194	-0.255	-0.182	-0.056	-0.206	0.086	-0.100
Test 3	-0.070	-0.218	-0.135	-0.306	0.325	-0.183	0.135	-0.282
Test 4	0.262	-0.266	-0.122	-0.141	0.162	-0.023	0.098	-0.228
CMT	0.225	-0.357	0.052	-0.178	0.094	-0.026	0.121	0.080
Final	0.029	-0.381	-0.149	-0.555*	0.261	-0.054	0.160	-0.067

*$p < 0.05$

Table XXII (continued)

Test	X_{18}	X_{19}	X_{20}	X_{21}	X_{22}	X_{23}	X_{24}	X_{25}
Test 1	0.214	0.059	0.119	0.020	0.339*	-0.127	0.246	-0.171
Test 2	0.088	-0.004	0.279	-0.126	0.126	-0.056	0.406*	-0.179
Test 3	0.344	0.247	0.190	0.016	0.335	0.123	0.036	-0.323
Test 4	0.121	0.254	0.285	-0.297	-0.105	-0.098	0.274	-0.036
CMT	0.157	0.110	0.081	-0.351	0.345	-0.166	0.219	-0.155
Final	0.023	0.214	0.203	-0.417*	0.201	-0.067	0.097	-0.301

Test	X_{26}	X_{27}	X_{28}	X_{29}	X_{30}	X_{31}	X_{32}	X_{33}
Test 1	0.276	-0.084	-0.023	0.125	-0.059	0.203	-0.169	0.139
Test 2	0.357*	-0.125	-0.263	0.252	-0.134	0.174	-0.257	0.040
Test 3	0.079	-0.229	-0.116	0.183	0.061	-0.068	-0.071	0.133
Test 4	0.248	-0.173	-0.265	-0.296	-0.008	-0.025	-0.151	-0.059
CMT	0.193	-0.036	0.006	0.163	0.006	0.018	0.301	-0.038
Final	-0.160	-0.243	-0.136	0.054	0.299	-0.128	-0.128	-0.138

* $p < 0.05$

Table XXII (continued)

Test	X_{34}	X_{35}	X_{36}	X_{37}	X_{38}	X_{39}	X_{40}	X_{41}
Test 1	-0.025	-0.119	0.051	0.077	0.004	-0.255	0.048	0.228
Test 2	-0.009	-0.242	-0.161	0.206	0.003	-0.250	-0.083	0.190
Test 3	-0.282	-0.197	0.148	-0.033	-0.316	-0.134	-0.250	-0.009
Test 4	-0.062	-0.187	-0.096	-0.118	-0.044	0.062	-0.254	-0.233
CMT	0.031	-0.053	-0.158	-0.019	-0.206	0.081	0.091	0.030
Final	-0.175	-0.169	-0.114	0.038	-0.307	-0.019	-0.208	-0.125

Test	X_{42}	X_{43}	X_{44}	X_{45}	X_{46}	X_{47}	X_{48}	X_{49}
Test 1	-0.201	-0.301*	1.000	-	-	-	-	-
Test 2	-0.318*	-0.066	0.657*	1.000	-	-	-	-
Test 3	-0.305	0.022	0.147	0.394*	1.000	-	-	-
Test 4	-0.338	0.185	0.151	0.528*	0.562*	1.000	-	-
CMT	-0.041	-0.087	0.530*	0.402*	0.372	0.452*	1.000	-
Final	-0.354	0.115	0.239	0.440*	0.557*	0.532*	0.710*	1.000

*$p < 0.05$

APPENDIX L

STEPWISE REGRESSION TABLFS FOR TERM TESTS OF THE LECTURE AND INDEPENDENT STUDY GROUPS

Table XXIII

Significant Predictors of LECTURE TEST 4 Scores in the Order Entered During Stepwise Regression (N = 36)

Step m	Predictors in Regression Equation	Multiple Correlation R	F_R	Percentage of Variance Accounted For	F	r
1	X_{42}	0.5300	13.28	28.1	13.28	-0.5300*
2	X_{42}, X_3	0.6507	12.12	42.3	8.15	0.3163
3	X_{42}, X_3, X_{32}	0.6979	10.13	48.7	3.97	-0.0456
Best Prediction Equation			$Y_{TEST\ 4} = 36.49 - 7.36\ X_{42} + 0.60\ X_3 + 2.34\ X_{32}$			
Coefficient F Ratios			24.11	10.31	3.97	

*$p < 0.05$

X_{42}: MAT Total Personal Interest

X_3: DAT Numerical Ability

X_{32}: MAT Integrated Narcism

Table XXIV

Significant Predictors of LECTURE TEST 3 Scores in the Order Entered During Stepwise Regression (N = 37)

Step m	Predictors in Regression Equation	Multiple Correlation R	F_R	Percentage of Variance Accounted For	F	r
1	X_3	0.4127	7.19	17.0	7.19	0.4127
2	X_3, X_{13}	0.4850	5.23	23.5	2.88	-0.3271
3	X_3, X_{13}, X_{30}	0.5652	5.16	32.0	4.09	-0.1981
4	$X_3, X_{13}, X_{30}, X_{18}$	0.6323	5.33	40.0	4.28	-0.2051
5	$X_3, X_{13}, X_{30}, X_{18}, X_{10}$	0.6835	5.44	46.7	3.92	0.1566
6	$X_3, X_{13}, X_{30}, X_{18}, X_{10}, X_{25}$	0.7442	<u>6.20</u>	55.4	5.82	-0.2114

Best Prediction Equation $Y_{TEST\ 3} = 81.38 + 0.38\ X_3 - 4.05\ X_{13} - 2.18\ X_{30} - 3.09\ X_{18} + 3.71\ X_{10} + 6.33\ X_{25}$

Coefficient F Ratios						
	4.15	13.38	4.00	11.71	7.11	5.82

X_3: Numerical Ability; X_{13} 16PF Factor F; X_{30}: MAT Integregated Home-Parent;

X_{18}: 16PF Factor M; X_{10}: 16PF Factor B; X_{25}: 16PF Exvia

Table XXV

Significant Predictors of LECTURE TEST 2 Scores in the Order Entered During Stepwise Regression (N = 41)

Step m	Predictors in Regression Equation	Multiple Correlation R	F_R	Percentage of Variance Accounted For	F	r
1	X_{39}	0.5680	18.57	32.3	18.57	-0.5680
2	X_{39}, X_3	0.6406	13.22	41.0	5.65	0.4448
3	X_{39}, X_3, X_{13}	0.6830	<u>10.78</u>	46.7	3.89	-0.3614

Best Prediction Equation: $Y_{TEST\ 2} = 64.78 - 3.57\ X_{39} + 0.35\ X_3 - 0.68\ X_{13}$

Coefficient F Ratios: 10.07   6.62   3.89

X_{39}: MAT General Autism; X_3: Numerical Ability; X_{13}: 16PF Factor F

Table XXVI

Significant Predictors of LECTURE TEST 1 Scores in the Order Entered During Stepwise Regression (N = 51)

Step m	Predictors in Regression Equation	Multiple Correlation R	F_R	Percentage of Variance Accounted For	F	r
1	X_3	0.4926	15.70	24.3	15.70	0.4926
2	X_3, X_{21}	0.5895	12.78	34.8	7.72	0.4370
3	X_3, X_{21}, X_{13}	0.6459	11.21	41.7	5.61	-0.2226
4	$X_3, X_{21}, X_{13}, X_{38}$	0.7016	11.15	49.2	6.80	-0.2224
5	$X_3, X_{21}, X_{13}, X_{38}, X_{35}$	0.7301	10.27	53.3	3.93	0.2265
6	$X_3, X_{21}, X_{13}, X_{38}, X_{35}, X_{32}$	0.7646	10.32	58.5	5.46	-0.1028
7	$X_3, X_{21}, X_{13}, X_{38}, X_{35}, X_{32}, X_{11}$	0.7947	10.53	63.2	5.48	0.1530
8	$X_3, X_{21}, X_{13}, X_{38}, X_{35}, X_{32}, X_{11}, X_{14}$	0.8158	10.44	66.6	4.26	0.1237

Best Prediction Equation $Y_{TEST\ 1} = -4.71 + 0.53\ X_3 + 2.05\ X_{21} - 1.54\ X_{13} - 3.05\ X_{38} + 4.88\ X_{35} - 2.88\ X_{32} + 1.12\ X_{11} + 1.04\ X_{14}$

Coefficient F Ratios	14.41	9.86	15.68	8.68	14.59	8.66	5.33	4.26

X_3: Numerical Ability; X_{21}: 16PF Factor Q_1; X_{13}: 16PF Factor F; X_{38}: MAT Integrated Sweetheart; X_{35}: MAT Integrated Mating; X_{32}: MAT Integrated Narcism; X_{11}: 16 PF Factor C; X_{14}: 16 PF Factor G

Table XXVII

Significant Predictors of INDEPENDENT STUDY TEST 4 Scores in the order Entered During Stepwise Regression (N = 26)

Step m	Predictors in Regression Equation	Multiple Correlation R	F_R	Percentage of Variance Accounted For	F	r
1	X_8	0.5340	9.58	28.5	9.58	0.5340*
2	X_8, X_{19}	0.6724	9.49	45.2	7.00	0.2378
3	X_8, X_{19}, X_{29}	0.7615	10.12	58.0	6.69	-0.2961
4	X_8, X_{19}, X_{29}, X_{21}	0.8136	10.28	66.2	5.10	-0.3116
5	X_8, X_{19}, X_{29}, X_{21}, X_{37}	0.8342	9.16	69.6	2.23	-0.1175
6	X_8, X_{19}, X_{29}, X_{21}, X_{37}, X_{17}	0.8669	9.57	75.8	4.24	-0.2106

Best Prediction Equation $Y_{TEST\ 4} = -34.21 + 8.71\ X_8 + 4.45\ X_{19} - 6.12\ X_{29} + 2.88\ X_{21} + 3.86\ X_{37} - 2.51\ X_{17}$

Coefficient F Ratios	42.99	18.19	9.88	5.49	4.74	4.24

X_8: Costello Scale II; X_{19} 16PF Factor N; X_{29}: MAT Integrated Career; X_{21}: 16PF Factor Q_1; X_{37}: MAT Integrated Assertiveness; X_{17}; 16PF Factor L

* $p < 0.05$

Table XXVIII

Significant Predictors of INDEPENDENT STUDY TEST 3 Scores in the Order Entered During Stepwise Regression (N = 29)

Step m	Predictors in Regression Equation	Multiple Correlation R	F_R	Percentage of Variance Accounted For	F	r
1	X_{18}	0.3940	4.96	15.5	4.96	0.3940*
2	X_{18}, X_{34}	0.5276	5.02	27.8	4.44	-0.2822
3	X_{18}, X_{34}, X_{38}	0.6349	5.63	40.3	5.22	-0.3158
Best Prediction Equation		$Y_{TEST\ 3} = 31.95 + 6.01\ X_{18} - 5.21\ X_{34} - 5.53\ X_{38}$				
Coefficient F Ratios			9.31	5.37	5.22	

*$p < 0.05$

X_{18}: 16PF Factor M; X_{34}: MAT Integrated Self-Concept;

X_{38}: MAT Integrated Sweetheart-Spouse

Table XXIX

Significant Predictors of INDEPENDENT STUDY TEST 2 Scores in the Order Entered During Stepwise Regression (N = 42)

Step m	Predictors in Regression Equation	Multiple Correlation R	F_R	Percentage of Variance Accounted For	F	r
1	X_{24}	0.3780	6.67	14.3	6.67	0.3780*
2	X_{24}, X_{42}	0.4981	6.43	24.8	5.46	-0.3300*
3	X_{24}, X_{42}, X_{29}	0.5835	6.54	34.1	5.32	0.2650
4	$X_{24}, X_{42}, X_{29}, X_{17}$	0.6592	7.11	43.5	6.16	-0.1299
5	$X_{24}, X_{42}, X_{29}, X_{17}, X_{37}$	0.7193	<u>7.71</u>	51.7	6.18	0.2068

Best Prediction Equation $Y_{TEST\ 2} = 49.04 + 1.88\ X_{24} - 7.16\ X_{42} + 4.44\ X_{29} - 2.39\ X_{17} + 3.75\ X_{37}$

*$p < 0.05$

X_{24}: 16PF Factor Q_4; X_{42}: MAT Total Personal Interest; X_{29}: MAT Integrated Career;

X_{17}: 16PF Factor L; X_{37}: MAT Integrated Assertiveness

Table XXX

Significant Predictors of INDEPENDENT STUDY TEST 1 Scores in the Order Entered During Stepwise Regression (N = 47)

Step m	Predictors in Regression Equation	Multiple Correlation F	F_R	Percentage of Variance Accounted For	F	r
1	X_7	0.3632	6.84	13.2	6.84	-0.3632*
2	X_7, X_{22}	0.4905	6.97	24.1	6.30	0.3520*
3	X_7, X_{22}, X_8	0.5566	6.43	31.0	4.31	0.2439
Best Prediction Equation		$Y_{TEST\ 1} = 43.32 - 3.35\ Y_7 + 2.49\ X_{22} + 2.25\ X_8$				
Coefficient F Ratios			5.41	8.77	4.31	

*$p < 0.05$

X_7: Costello Scale I; X_{22}: 16PF Factor Q_2; X_8: Costello Scale II