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Interaction Diagrams for Reinforced Masonry

by

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INTERACTION DIAGRAMS FOR REINFORCED MASONRY

by

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ABSTRACT

The object of this paper is to present interaction diagrams for the design of reinforced masonry walls and columns. The development of these diagrams is briefly described. It conforms to the requirements of CSA Standard CAN3-S304-M78. The interaction diagrams can be used either for the determination of the dimensions and reinforcement of the section or for the check of its capacity. Both uses are illustrated with design examples.

ACKNOWLEDGEMENTS

Appreciation is expressed to the Natural Sciences and Engineering Research Council of Canada, to the Alberta Masonry Institute and to the University of Alberta for funding which made possible the development of this report.

INTRODUCTION

The design of masonry walls and columns has been formalized with the presentation of design principles based on engineering analysis in CSA Standard CAN3-S304-M78(1) or its companion Imperial units version. This paper has as its principle objective the presentation of graphical design aids to assist the profession in the area of masonry design. Its primary motivation was to reduce the need for detailed and lengthy sets of calculations especially for those cases of masonry design where the eccentricity ratio exceeds one third the member thickness. For completeness, the design aids developed and presented herein consider all possible eccentricities of vertical load.

In the following sections, a brief discussion of the engineering principles for the design of masonry walls and columns is presented first. Next is a discussion of the development of the design charts and of the important parameters incorporated in these charts. Finally, a set of examples illustrating the use of these charts for the design of various elements is presented.

DESIGN PRINCIPLES FOR MASONRY WALLS AND COLUMNS

For wall and columns subjected to service load levels of combinations of vertical load and bending moment arising either from load eccentricities or from lateral loading the design process includes a calculation of critical stresses and then a comparison of these stresses with limiting allowable stresses. The allowable stresses depend on the type of masonry and the eccentricity ratio. A satisfactory design is one in which the actual stresses from the imposed loading are equal or nearly equal to the limiting stresses.

When the maximum virtual eccentricity does not exceed one third the member thickness, a simplified procedure incorporating factors dependent on loading and slenderness is used. For columns in which the vertical reinforcement is tied as specified the allowable vertical load is given by:

[1] $$P = C_e C_s (f_m + 0.8\, p_n f_s) A_n$$

where

C_e = eccentricity coefficient

C_s = slenderness coefficient

f_m = compressive stress in masonry

p_n = ratio of the area of reinforcement to the net cross-sectional area

f_s = stress in reinforcement

A_n = net cross-sectional area

The slenderness coefficient is given by:

[2] $$C_s = 1.2 - \frac{h/t}{300}\left[5.75 + \left(1.5 + \frac{e_1}{e_2}\right)^2\right] \leq 1.0$$

where

h = effective height of a wall or column

t = effective thickness of a wall or column

e_1 = the smaller virtual eccentricity

e_2 = the larger virtual eccentricity occurring at the top or bottom of a vertical member at lateral supports

For walls, reinforcement is not considered effective for strength and the allowable load is as given above by Equation [1] but with $p_n f_s$ equal zero.

When the maximum virtual eccentricity exceeds one third the member thickness an analysis based on the assumption of a linear strain and stress distribution over the cross section is required. Here the transformed section analysis is a convenient design process. Such an analysis is also required in any case of loading which would produce a tension stress in the reinforcement. Once a vertical load is calculated by this analysis consistent with allowable stresses, it is then modified by the slenderness coefficient to obtain the member allowable load. Reinforcement in compression is neglected if not tied as specified. Additionally, the stress in the compression reinforcement must not exceed the allowable tension stress.

The design process for reinforced masonry walls and columns would likely begin with a selection of thickness and width in the case of columns, and then a determination of the necessary reinforcement. Normally several trials would be necessary to establish suitable dimensions and reinforcements. Such a procedure is relatively straightforward whenever the eccentricity ratio is less than one third; however, for eccentricity ratios larger than one third, it is considerably more involved. The application of the transformed section analysis to walls and columns requires a knowledge of the cross sectional dimensions and material properties before the analysis can be started. Often the first trial section is chosen either on the basis of past experience or is simply an intelligent guess. Normally several iterations involving complex relationships are required to achieve a satisfactory design. The graphical design aids developed and presented in this paper make possible a check on the adequacy of a chosen section or an adjustment and a recheck if not satisfactory in a quick convenient manner. Once a section is considered satisfactory based on a design using these charts it may then be checked finally using the transformed section analysis. This latter step in design would normally not be necessary as it would yield essentially the same solution as that from the charts with possibly small differences attributed to the limited scale accuracy of the charts. It is also noted that the depiction of the design aids in a graphical form permits the designer to visualize better the effects of changing section dimensions, reinforcement amounts, and the material properties and thereby to achieve a proper design more rapidly.

In total, seventeen interaction diagrams were developed for reinforced masonry walls and nineteen for reinforced masonry columns. Computer programs were developed which plotted directly dimensionless interaction diagrams using a CalComp plotter. Essentially, an axial load and moment were first evaluated and then plotted as non dimensional parameters P/f_mA_n versus Pe/f_mA_nt for increments of e/t within a specified range.

For completeness, four interaction diagrams with e less than t/3 are included; one for reinforced wall design, the other three for reinforced masonry columns having e_1/e_2 values of -1, 0 and +1, as shown in Fig. 1. Designs involving values of e_1/e_2 other than -1, 0 or +1 require interpolation. These four interaction diagrams are shown separate from the other interaction diagrams not only for clarity but also because different procedures and parameters were used in the development of interaction diagrams for eccentricity ratios larger than one third.

The remaining thirty-two interaction diagrams are divided equally between walls and columns. They cover the range of eccentricity from tension in the reinforcement to pure bending. Biaxial bending, however, was not included. These interaction diagrams were generated using a program involving a transformed section analysis for cases of solid rectangular columns with reinforcement along two opposite faces and singly reinforced walls.

The three major parameters involved in these interaction diagrams are n, the modular ratio, g, the relative placement of the reinforcement on the cross section and f_s/f_m the ratio of the allowable stress in the reinforcement to the allowable stress in the masonry.

The modular ratio, n, is used in transforming the section. Of the sixteen wall or column interaction diagrams, one half have n equal to 10, the rest have n equal to 20. These values were chosen to represent the practical range of the modular ratio other values are easily interpolated from these.

Each set of eight interaction diagrams for either modular ratio are further subdivided by four values of the parameter, g, the ratio of the distance between the compression face and the centroid of the reinforcement to the total effective thickness in the case of walls as shown in Fig. 2, and the ratio of the distance between the symmetrical reinforcement centroids to the total effective thickness in the case of columns as shown in Fig. 3. Values of g equal to 0.5, 0.6, 0.7 and 0.8 were used for both walls and columns.

In each diagram depicting the interaction of a wall or column, for either modular ratio, and all four values of g, auxiliary diagrams are presented as illustrated in Figs. 2, 3 and 4. The non dimensional parameter f_s/f_m has isolated these diagrams from the main interaction diagrams of these figures.

This ratio was introduced in order to incorporate the large range or masonry strengths while minimizing the number of diagrams required. The lower practical limit of f_s/f_m is equal to 10. For clarity this value and a value of f_s/f_m equal to 20 are plotted separately in those regions which they affect. Values of f_s/f_m equal to 30, 40 and 50 are plotted in the main diagrams and are coincident in most cases but are annotated if they diverge as shown in Figs. 3 and 4. In general, the ratio f_s/f_m is more influential for columns than walls. It becomes more important as bending, the modular ratio, and g increase and as the reinforcement percentage decreases. The value of f_s/f_m equal to 50 was chosen as the upper limit. It diverges only in the case of lightly reinforced columns subjected to large amounts of bending with n = 20. Except in this case, the line represents values greater than 50 as well.

Reinforcement percentages for columns were chosen in order to comply with the allowable limits of CSA Standard CAN3-S304-M78. For simplicity and uniformity these same percentages were used in the interaction diagrams for walls. The family of curves of $p_n(\%)$ equal to 0.3, 0.5, 1, 2, 3 and 4 are used in most cases in conjunction with the family of radial lines which represent different eccentricity ratios, e/t. Since these radial lines are not a necessity it is less confusing to omit them from auxiliary diagrams such as Figs. 2, 3 and 4.

Design Examples

The following design examples illustrate the use of the interaction diagrams. A variety of applications are shown including choice of cross section dimensions, reinforcement, and masonry strength, as well as determining the vertical load carrying capacity of a section. The slenderness coefficient is calculated using Equation [2]. Allowable stresses, moduli and notation are taken from CSA Standard CAN3-S304-M78.

1. Choose the vertical reinforcement for a 5 metre high reinforced brick wall to carry a load of 188 000 N/m at an eccentricity of 125 mm. The wall is to be constructed of two 90 mm thick wythes with a 70 mm thick cavity between them which will contain the reinforcement and grout. Use f'_m = 20 MPa, f_s = 165 MPa and e_1/e_2 = 0.

    t = 90 mm + 90 mm + 70 mm = 250 mm

    e/t = 125 mm/250 mm = 0.5

    e/t is greater than 0.333, therefore compute $n = E_s/E_m$

    E_s = 200 000 MPa

    E_m = 1000 f'_m, but not greater than 20 000 MPa

    = 1000 (20 MPa) = 20 000 MPa

    $n = E_s/E_m$ = 200 000 MPa/20 000 MPa = 10

    h/t = 5 000 mm/250 mm = 20, therefore C_s = 0.667

for a 1 metre length of wall,

$P = 188\ 000\ N/0.667 = 281\ 859\ N$

$f_m = 0.40\ f'_m = 0.40\ (20\ MPa) = 8.0\ MPa$

$$\frac{P}{f_m A_n} = \frac{281\ 859\ N}{8.0\ MPa\ (250\ mm)\ (1000\ mm)} = 0.14$$

With g = 0.6 the reinforcement is located as close as practical to the tension face of the wall.

From Fig. 2, find $p_n = 0.5\%$

$f_s/f_m = 165\ MPa/8.0\ MPa = 21$, not influential here

$A_s = p_n A_n = 0.005\ (250\ mm)(1000\ mm) = 1250\ mm^2$

use 4 - 20M reinforcing bars per metre of wall.

$A_s = 1200\ mm^2$

2. Find the required f'_m for a 5 metre high reinforced brick wall to carry a load of 343 200 N/m at an eccentricity of 210 mm. The wall is 350 mm thick and is reinforced with 20M reinforcing bars spaced at 170 mm. Use $f_s = 165$ MPa, $e_1/e_2 = 0$ and g = 0.6.

$e/t = 210\ mm/350\ mm = 0.6$

e/t is greater than 0.333, therefore require n

assume n = 10

$h/t = 5000\ mm/350\ mm = 14.29$, therefore $C_s = 0.819$

for a 1 metre length of wall,

$A_s = 300\ mm^2\ (1000\ mm/170\ mm) = 1764\ mm^2$

$P_n = A_s/A_n = 1764 \text{ mm}^2/(350 \text{ mm})(1000 \text{ mm}) = 0.00504$ or

0.5%

from Fig. 2 find $\frac{P}{f_m A_n} = 0.11$

$f_m = P/(0.11\ A_n C_s)$

$= 343\ 200 \text{ N}/(0.11)(350 \text{ mm})(1000 \text{ mm})(0.819) = 10.9 \text{ MPa}$

$f_s/f_m = 165 \text{ MPa}/10.9 \text{ MPa} = 15.1$, not influential here

$f'_m = f_m/0.40 = 10.9 \text{ MPa}/0.40 = 27.3 \text{ MPa}$

check n = 10 assumption, $n = E_s/E_m$

$E_s = 200\ 000 \text{ MPa}$

$E_m = 1000\ f'_m$, but not greater than 20 000 MPa

$= 1000\ (27.3 \text{ MPa}) = 27\ 300 \text{ MPa}$, therefore use

$E_m = 20\ 000 \text{ MPa}$

$n = E_s/E_m = 200\ 000 \text{ MPa}/20\ 000 \text{ MPa} = 10$, checks

required $f'_m = 27.3 \text{ MPa}$

3. A 3 metre high reinforced concrete masonry column is to be located within a wall. Choose the dimensions and vertical reinforcement for the column to carry a vertical load of 340 000 N at an eccentricity of 215 mm. Use $f'_m = 10$ MPa, $f_y = 300$ MPa and $e_1/e_2 = +1$.

allowable $h/t = 5(4 - e_1/e_2) = 15$

try a 390 mm by 790 mm section, t = 790 mm

$e/t = 215 \text{ mm}/790 \text{ mm} = 0.272$

h/t = 3000 mm/790 mm = 3.8, therefore C_s = 1.0

$f_m = 0.20\ f'_m = 0.20$ (10 MPa) = 2.0 MPa

$$\frac{P}{f_m A_n} = \frac{340\ 000\ \text{N}}{2.0\ \text{MPa}\ (390\ \text{mm})(790\ \text{mm})} = 0.55$$

from Fig. 1 find $p_n\ f_s/f_m = 0.3$

$f_s = 0.40\ f_y = 0.40$ (300 MPa) = 120 MPa

$p_n = 0.3\ f_m/f_s$ = (0.3) 2.0 MPa/120 MPa = 0.005 or 0.5%

= p_n (minimum) = 0.5% [4.6.8.3.1]

$A_s = p_n A_n$ = 0.005 (390 mm)(790 mm) = 1541 mm^2

use 4 - 20M and 2 - 15M reinforcing bars, A_s = 1600 mm^2

try a 390 mm by 590 mm section, t = 590 mm

e/t = 215 mm/590 mm = 0.36

e/t is greater than 0.333, therefore compute $n = E_s/E_m$

E_s = 200 000 MPa

$E_m = 1000\ f'_m$, but not greater than 20 000 MPa

= 1000 (10 MPa) = 10 000 MPa

$n = E_s/E_m$ = 200 000 MPa/10 000 MPa = 20

h/t = 3000 mm/590 mm = 5.08, therefore C_s = 0.997

P = 340 000 N/0.997 = 341 000 N

$f_m = 0.28\ f'_m = 0.28$ (10 MPa) = 2.8 MPa

$$\frac{P}{f_m A_n} = \frac{341\ 000\ \text{N}}{2.8\ \text{MPa}\ (390\ \text{mm})(590\ \text{mm})} = 0.53$$

assume $g = 0.6$

from Fig. 3 find $p_n = 3.7\%$

$f_s/f_m = 125$ MPa/2.8 MPa = 45, not influential here

Although $p_n = 3.7\%$ is within the limits of 0.5% to 4.0%, the required reinforcement may be difficult to place.

try a 590 mm by 590 section

only A_n changes in above calculations

$$\frac{P}{f_m A_n} = \frac{341\ 000\ \text{N}}{2.8\ \text{MPa}\ (590\ \text{mm})(590\ \text{mm})} = 0.35$$

from Fig. 3 find $p_n = 1.2\%$

$f_s/f_m = 45$, not influential here

$A_s = p_n A_n = 0.012\ (590\ \text{mm})(590\ \text{mm}) = 4178\ \text{mm}^2$

use 6 - 30M reinforcing bars, $A_s = 4200\ \text{mm}^2$

4. Determine the vertical load carrying capacity of a 5.2 metre high reinforced brick column loaded at an eccentricity of 200 mm in the strong direction. The column is symmetrically reinforced with 14 - 15M reinforcing bars and has dimensions 390 mm by 490 mm. Use $f'_m = 10$ MPa, $f_s = 165$ MPa, $e_1/e_2 = +0.25$ and $g = 0.6$.

$e/t = 200\ \text{mm}/490\ \text{mm} = 0.408$

e/t is greater than 0.333, therefore compute $n = E_s/E_m$

$E_s = 200\ 000$ MPa

$E_m = 1000\ f'_m$ but not greater than 20 000 MPa

$= 1000\ (10\ \text{MPa}) = 10\ 000\ \text{MPa}$

$n = E_s/E_m = 200\ 000\ \text{MPa}/10\ 000\ \text{MPa} = 20$

$p_n = A_s/A_n = 14(200\ \text{mm}^2)/(390\ \text{mm})(490\ \text{mm}) = 0.0146$

or 1.46%

from Fig. 3 find $\frac{P}{f_m A_n} = 0.32$

$f_m = 0.32\ f'_m = 0.32(10\ \text{MPa}) = 3.2\ \text{MPa}$

$f_s/f_m = 165\ \text{MPa}/3.2\ \text{MPa} = 52$, not influential here

$h/t = 5200\ \text{mm}/390\ \text{mm} = 13.33$, therefore $C_s = 0.808$.

$P = 0.32\ f_m A_n C_s$

$= 0.32(3.2\ \text{MPa})(390\ \text{mm})(490\ \text{mm})(0.808)$

$= 158\ 115\ \text{N}$ or $P = 158\ \text{kN}$

5. Choose the vertical reinforcement for a reinforced brick column, 390 mm by 690 mm, 7 metres high, to carry a vertical load of 346 000 N at an eccentricity of 585 mm about the strong axis. Assume the column is braced in the weak direction. Use $f'_m = 29$ MPa, $f_s = 140$ MPa, $e_1/e_2 = 0$, and $g = 0.7$.

$e/t = 585\ \text{mm}/690\ \text{mm} = 0.848$

e/t is greater than 0.333, therefore compute $n = E_s/E_m$

$E_s = 200\ 000\ \text{MPa}$

$E_m = 1000\ f'_m$ but not greater than 20 000 MPa

$= 1000\ (29\ \text{MPa}) = 29\ 000\ \text{MPa}$, therefore use

E_m = 20 000 MPa

$n = E_s/E_m$ = 200 000 MPa/20 000 MPa = 10

h/t = 7000 mm/690 mm = 10.14, therefore C_s = 0.930

P = 346 000 N/0.93 = 372 043 N

$f_m = 0.32\ f'_m$ = 0.32 (29 MPa) = 9.28 MPa

$$\frac{P}{f_m A_n} = \frac{372\ 043\ \text{N}}{9.28\ \text{MPa}\ (390\ \text{mm})(690\ \text{mm})} = 0.149$$

From Fig. 4, p_n = 1.8%

f_s/f_m = 140 MPa/9.28 MPa = 15.08, influential here interpolate between diagrams with f_s/f_m = 10 and f_s/f_m = 20

$Pe/f_m A_n t$ = 0.149 (0.848) = 0.126

for f_s/f_m = 10, p_n = 2.0%

for f_s/f_m = 20, p_n = 1.8%

therefore use p_n = 1.9% which is within limits of 0.5% and 4.0%

$A_s = p_n A_n$ = 0.019 (390 mm)(690 mm) = 5113 mm^2

use 10 - 25M bars, A_s provided = 5000 mm^2

SUMMARY AND CONCLUSIONS

The analysis of reinforced masonry walls and columns for strength at service conditions is a lengthy process using methods like the transformed section. An alternate process for analysis and design based on the use of interaction diagrams has been presented in this paper. The interaction diagrams made available can be used conveniently for the design of most types of wall or column sections. They can also be used to determine preliminary cross sections which could then be analyzed using more rigorous analyses. Regardless of the mode of application, the availability of these interaction diagrams should significantly expedite the design process for masonry walls and columns.

REFERENCE

"Masonry Design and Construction for Buildings", Standard No. CAN3-S304-M78, Canadian Standards Association, Rexdale, Ontario.

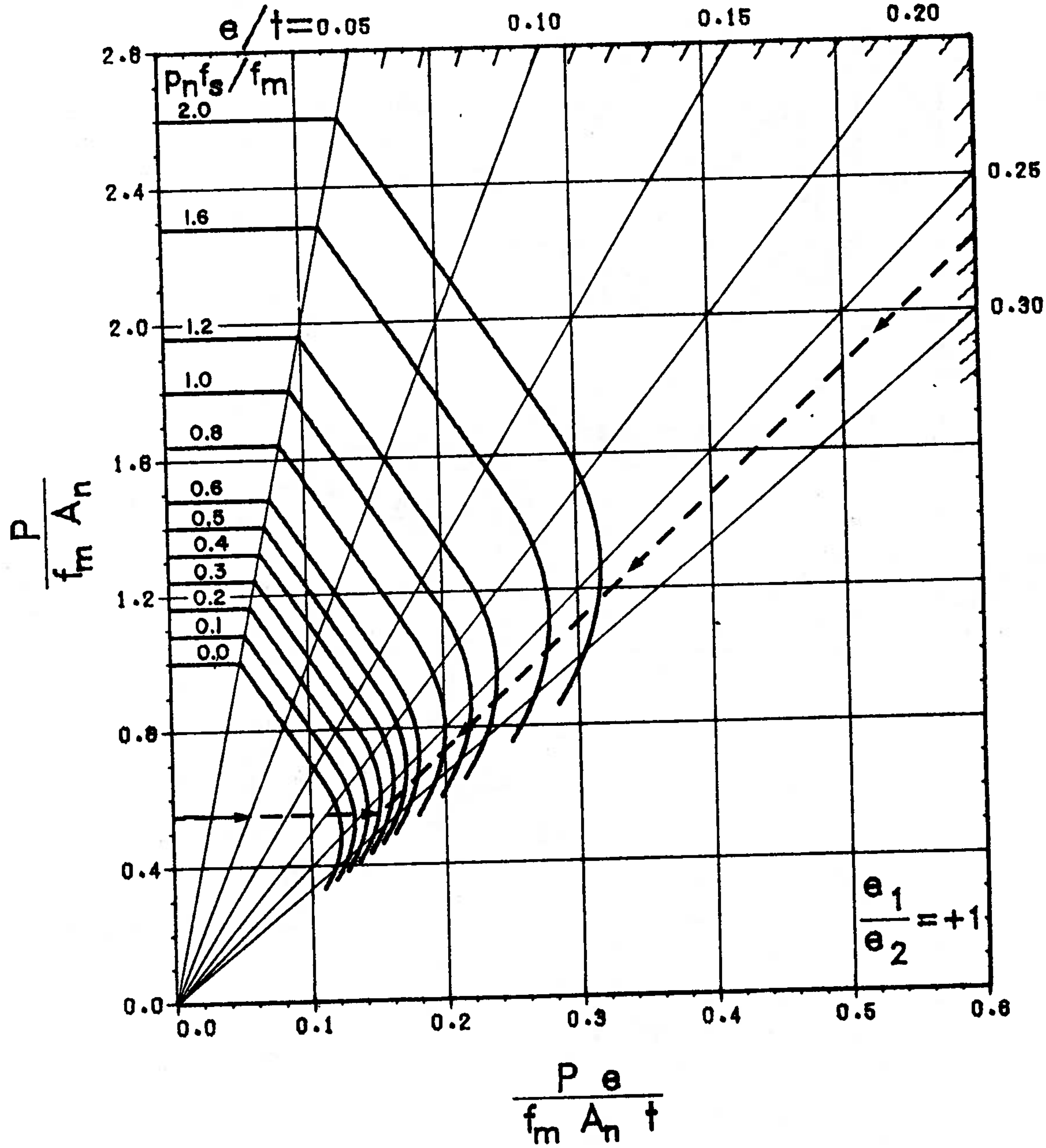


Fig. 1 - Masonry Column Interaction Diagram

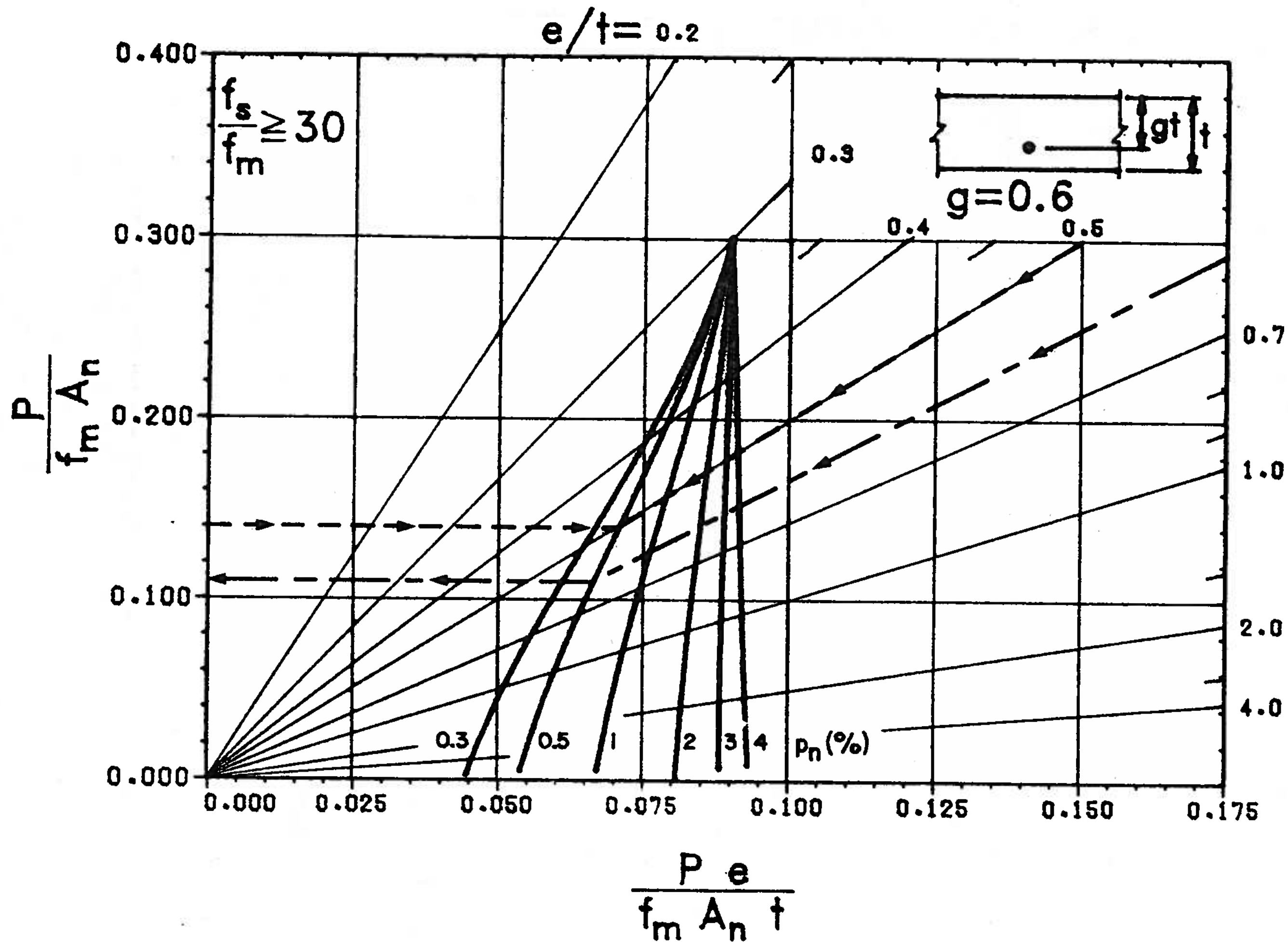


Fig. 2 - Masonry Wall Interaction Diagram, n = 10, g = 0.6

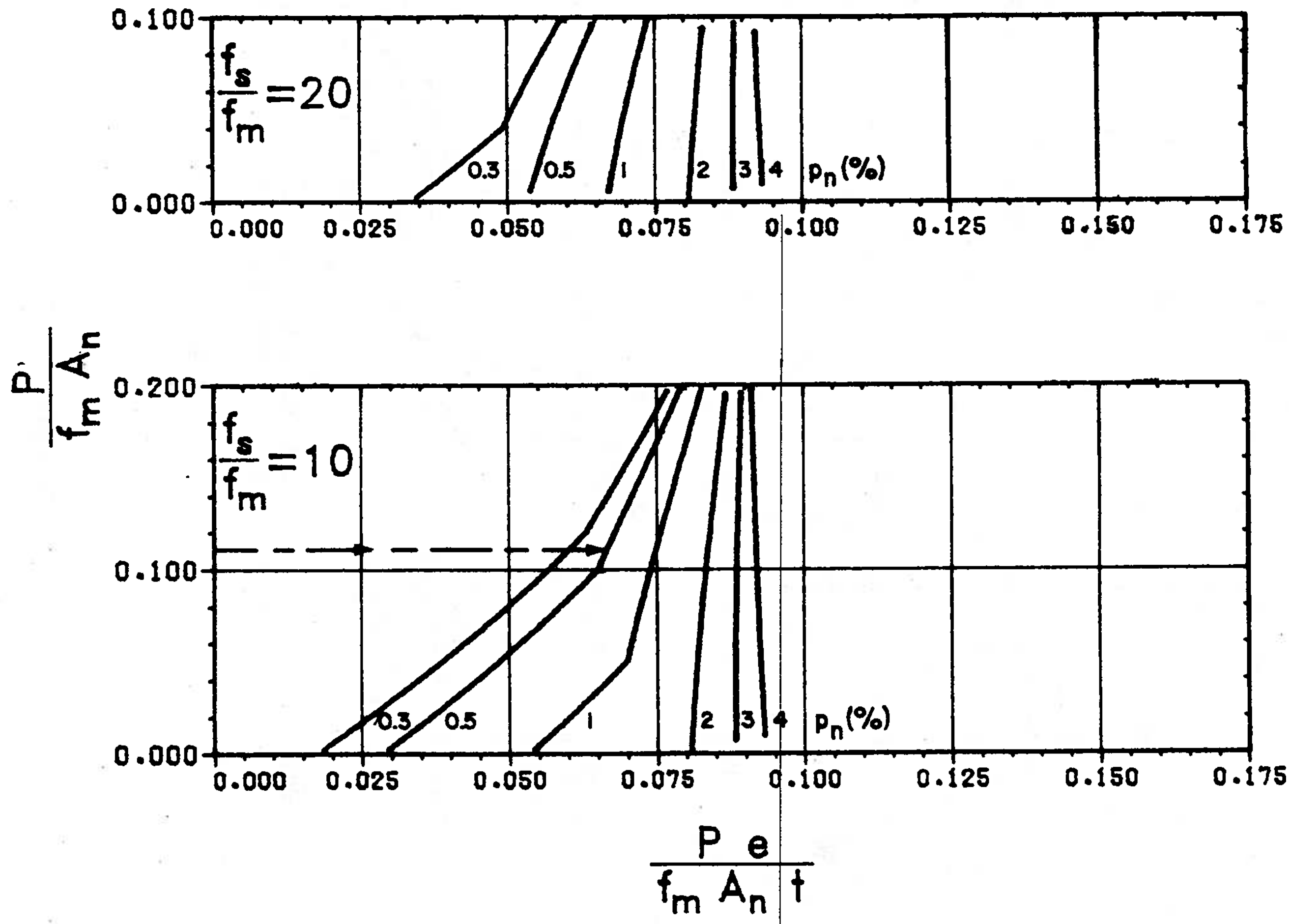


Fig. 2 - Masonry Wall Interaction Diagram, n = 10, α = 0.6

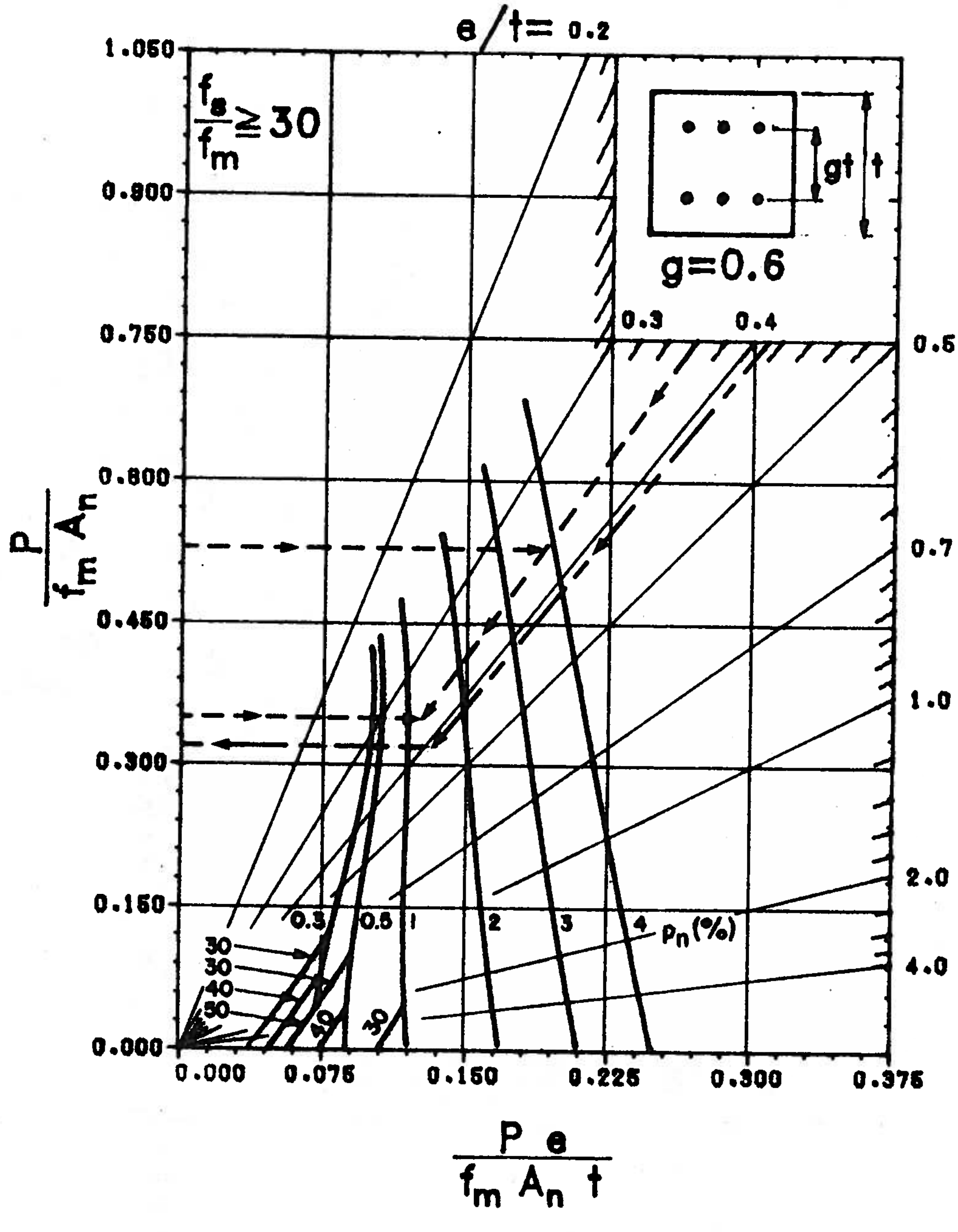


Fig. 3 - Masonry Column Interaction Diagram,
n = 20, g = 0.6

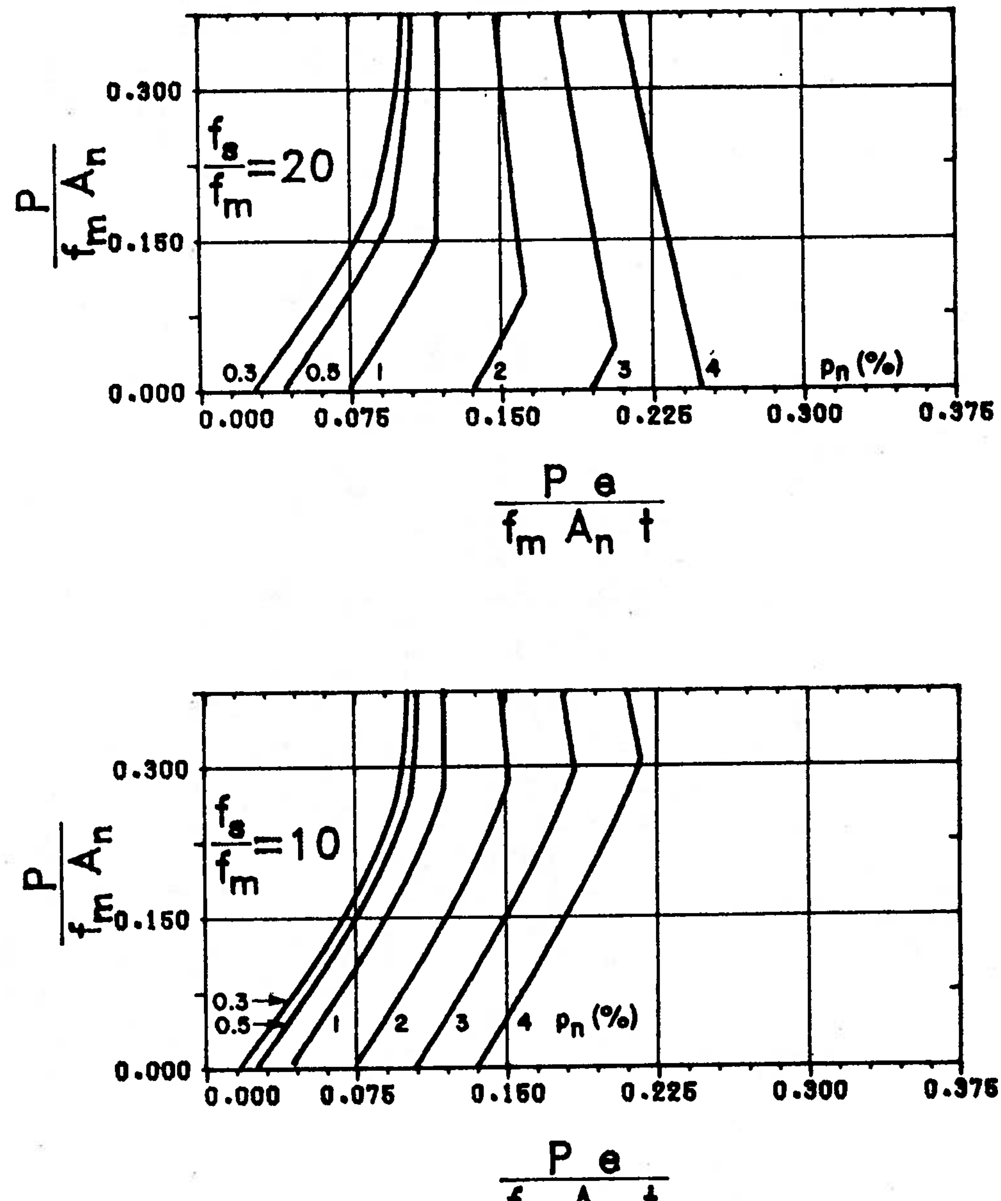


Fig. 3 - Masonry Column Interaction Diagram,
n = 20, g = 0.6

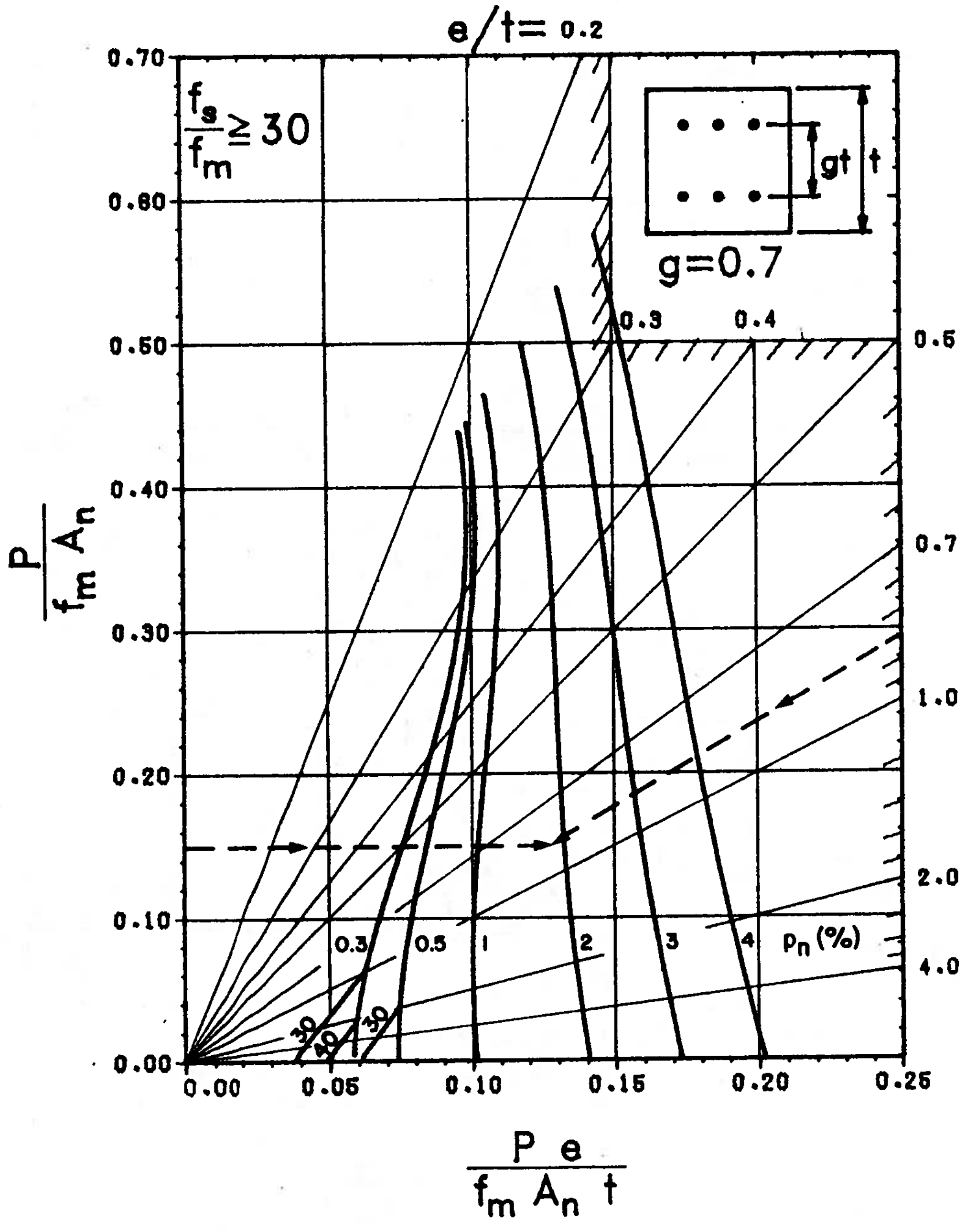
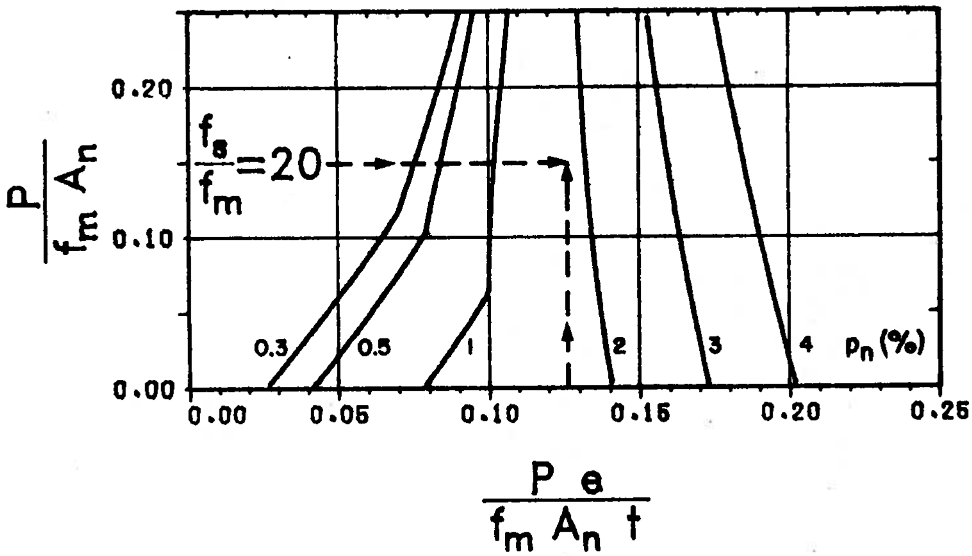


Fig. 4 - Masonry Column Interaction Diagram, n = 10, g = 0.7



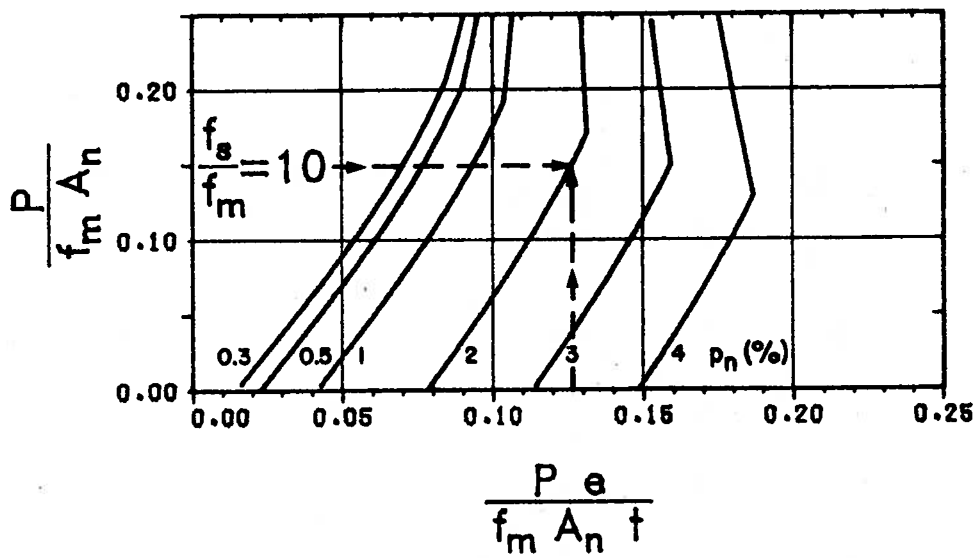


Fig. 4 - Masonry Column Interaction Diagram,
$n = 10$, $g = 0.7$

APPENDIX "A"

to

Interaction Diagrams for Reinforced Masonry

by

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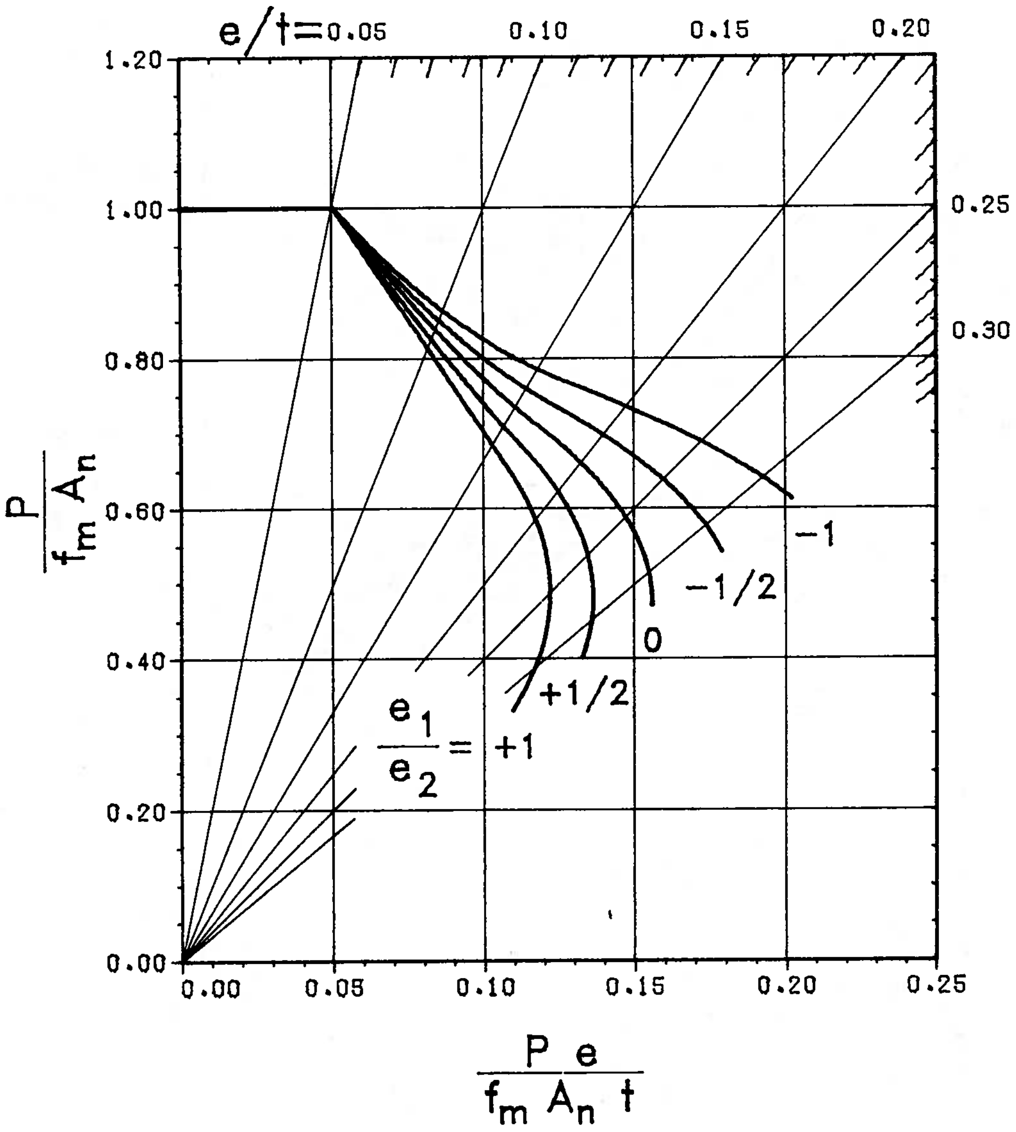
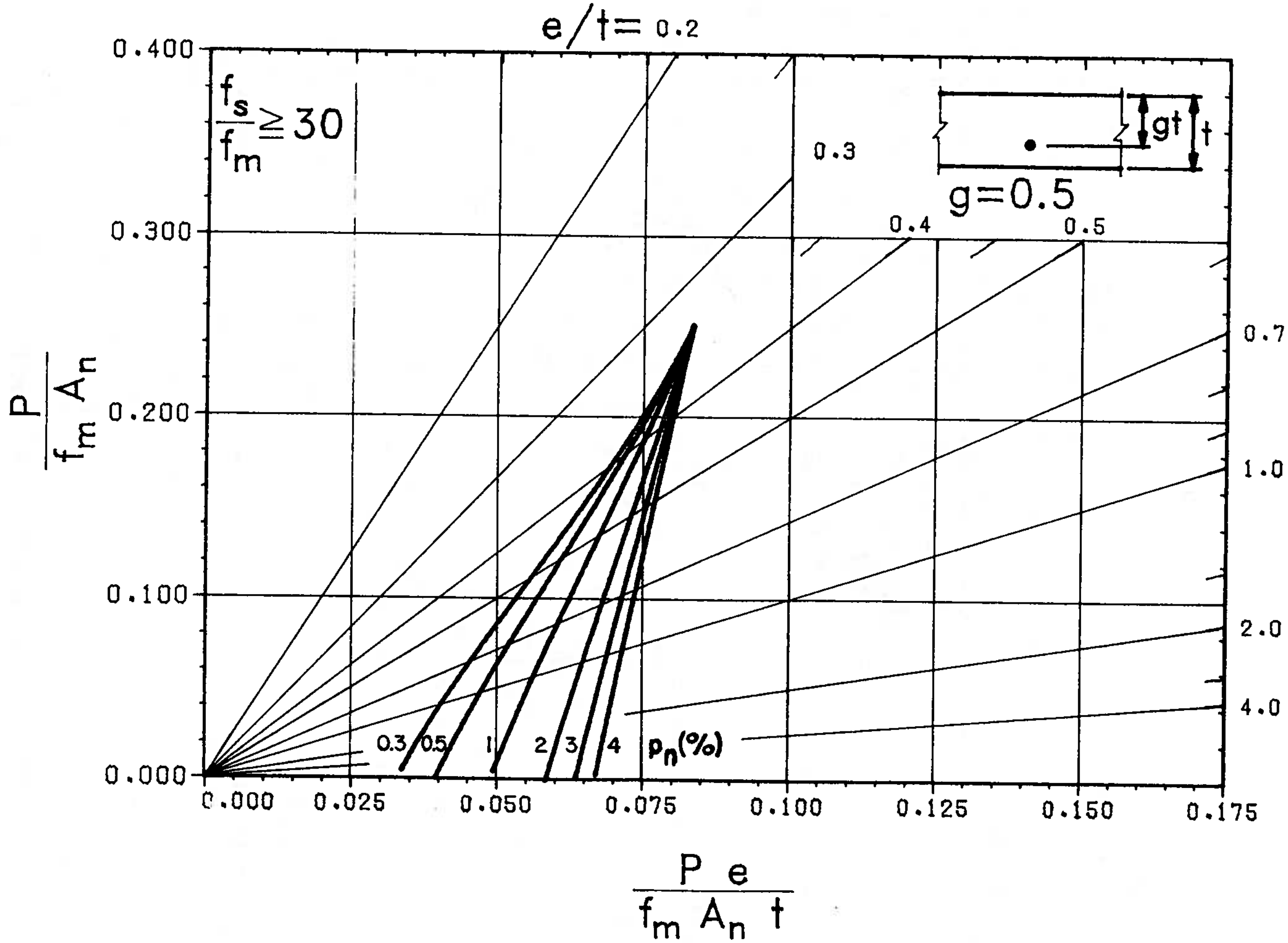
Masonry Wall Interaction Diagrams

Masonry Column Interaction Diagrams



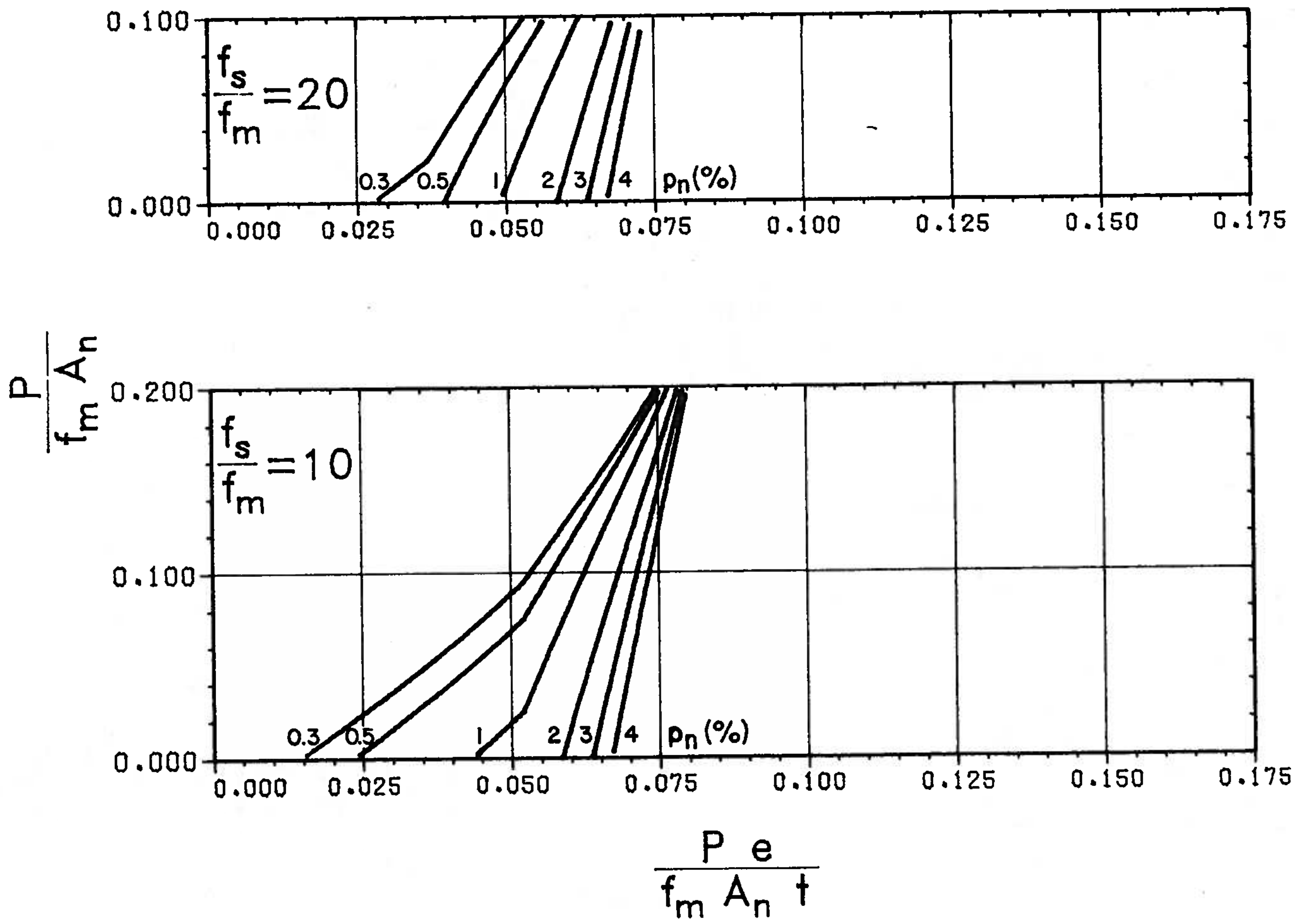
Masonry Wall Interaction Diagram

A2



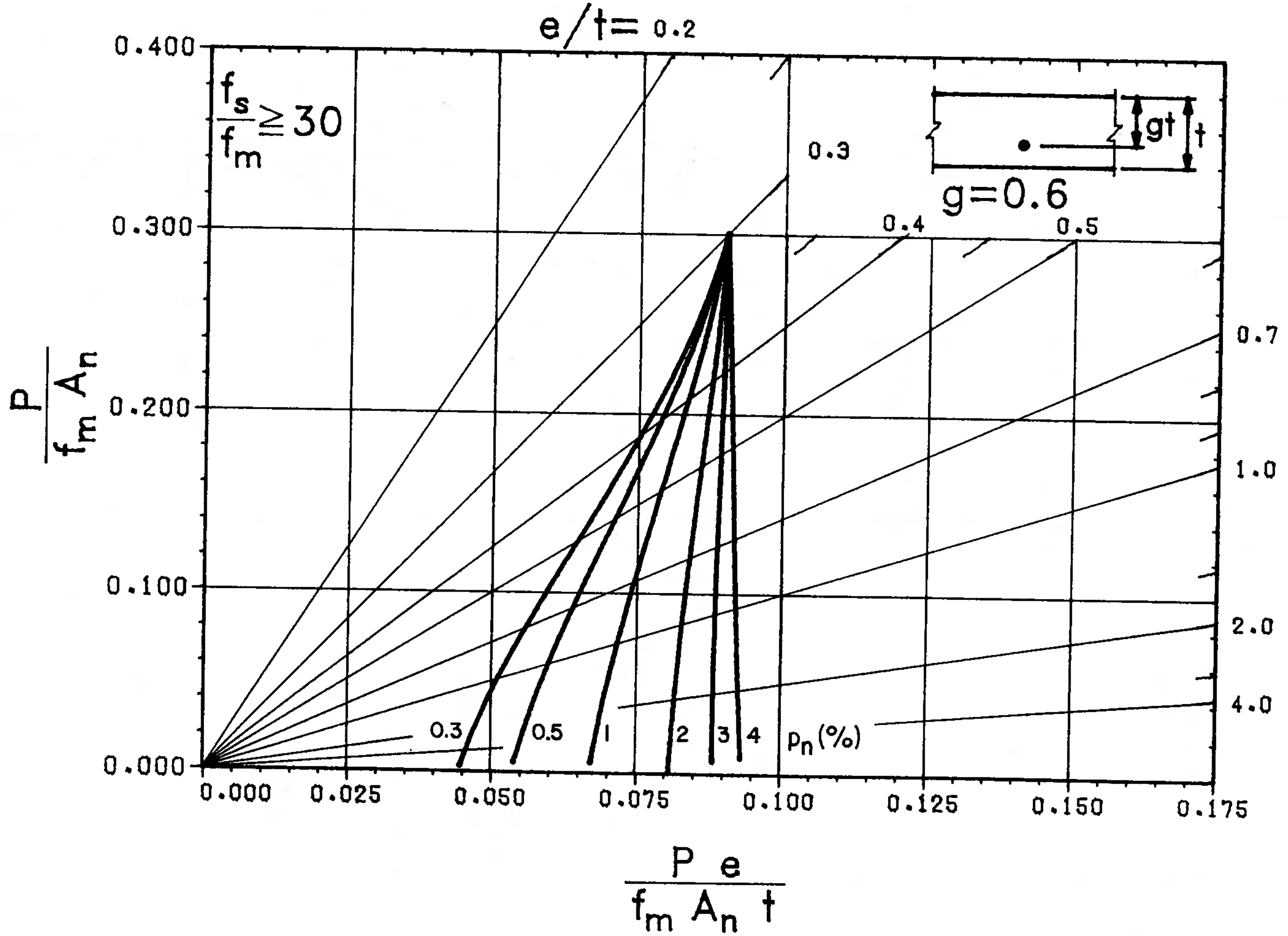
Masonry Wall Interaction Diagram

n=10 g=0.5



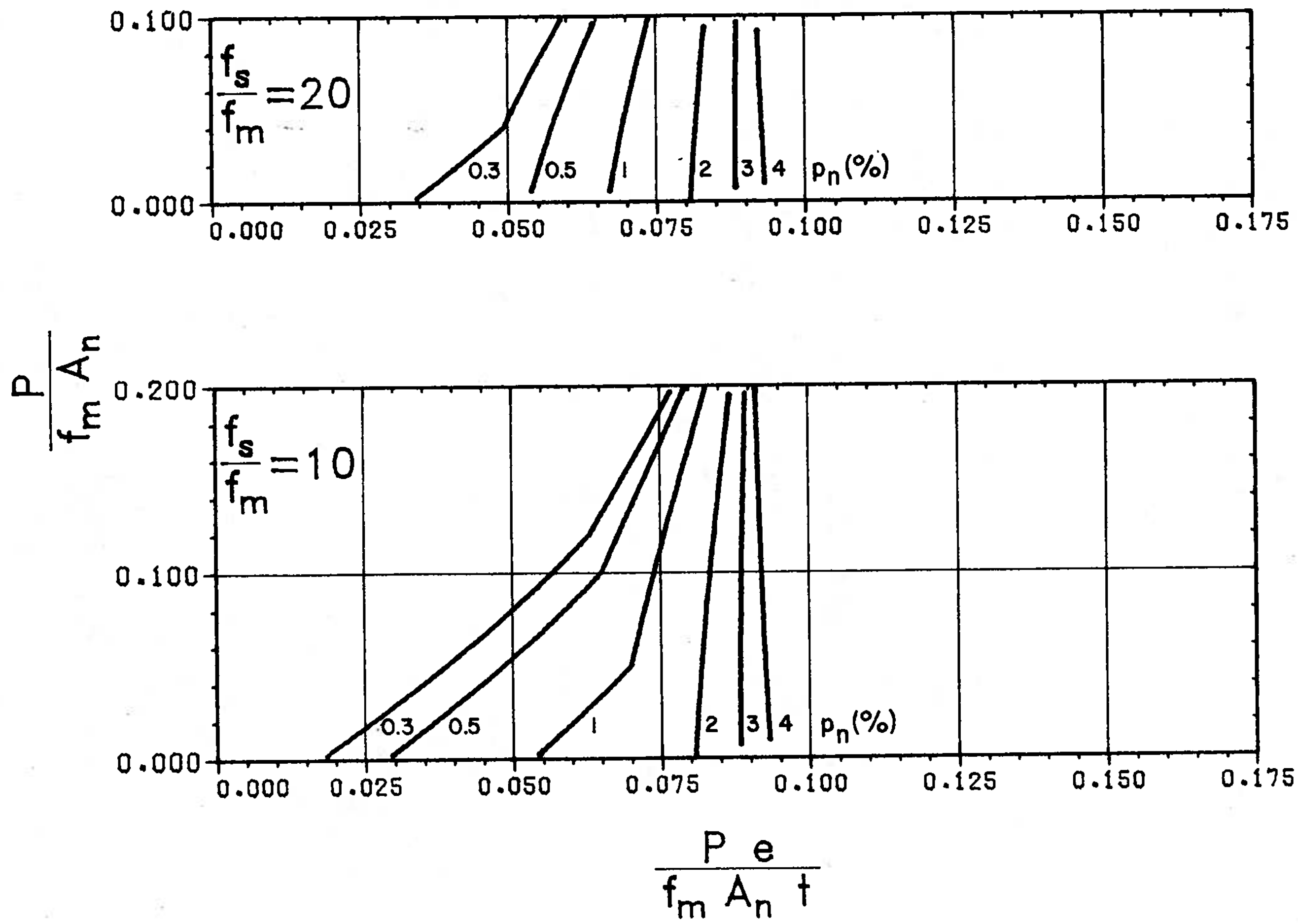
Masonry Wall Interaction Diagram

n=10 g=0.5



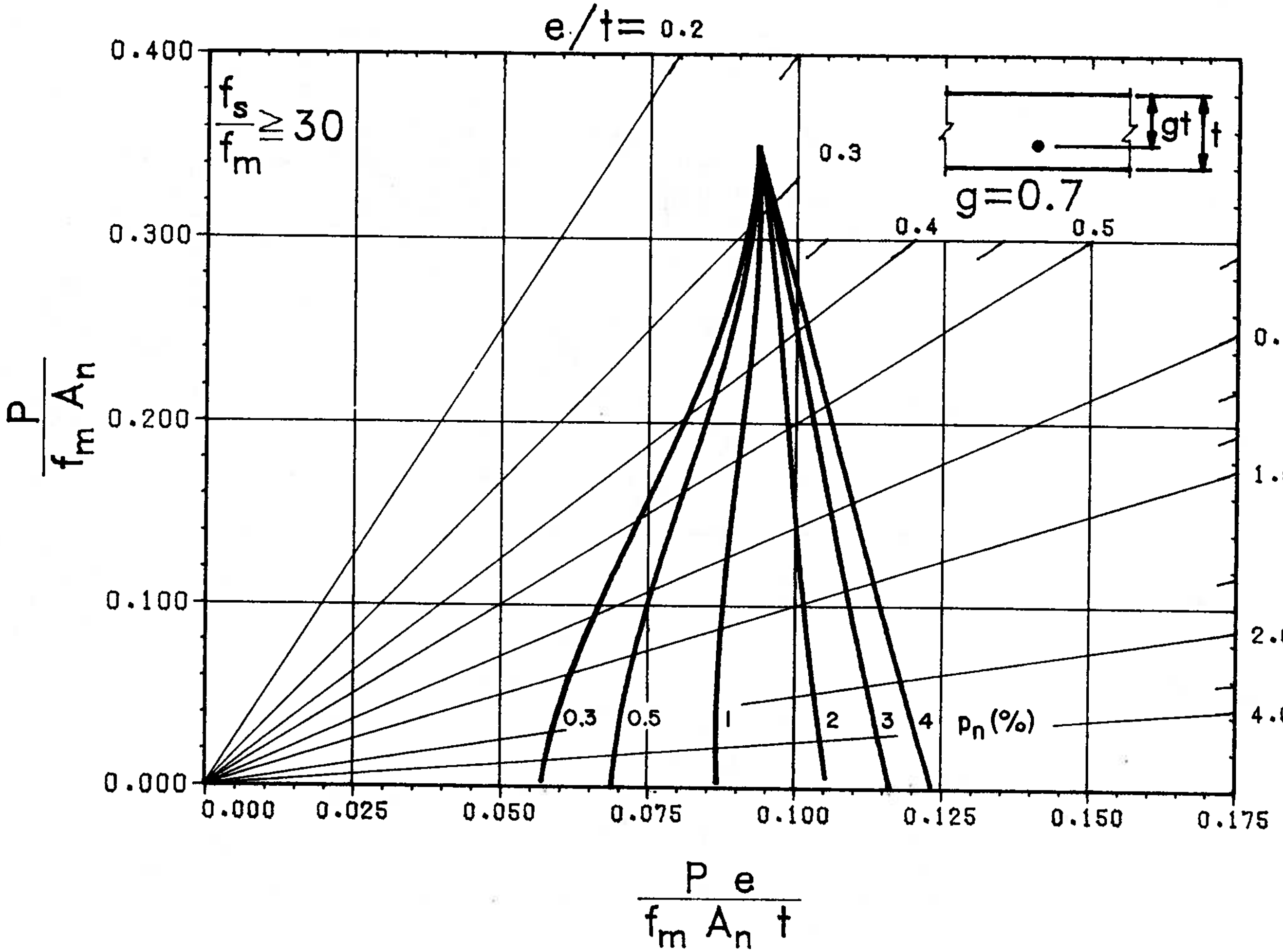
Masonry Wall Interaction Diagram

$n=10 \quad g=0.6$



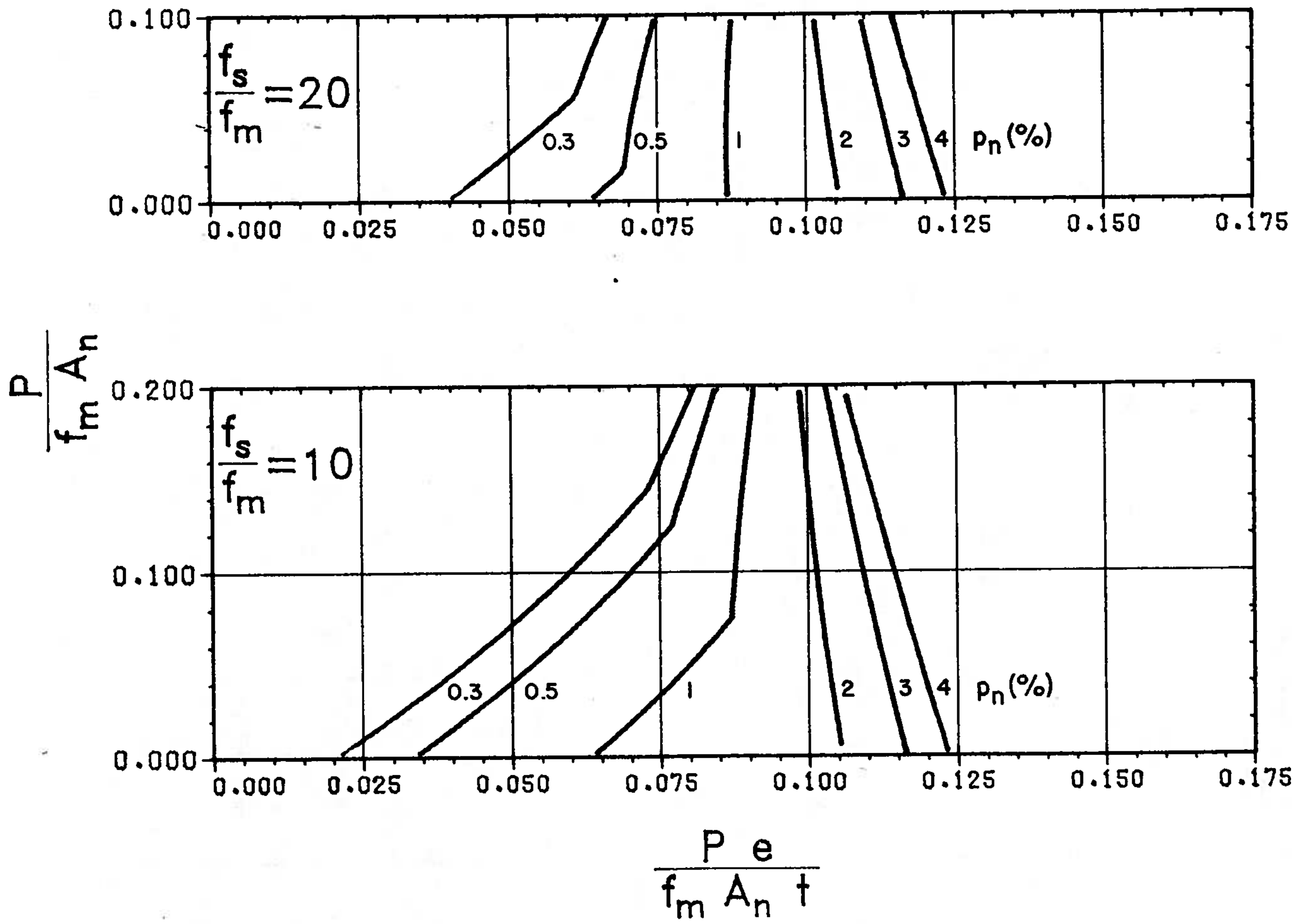
Masonry Wall Interaction Diagram

n=10 g=0.6



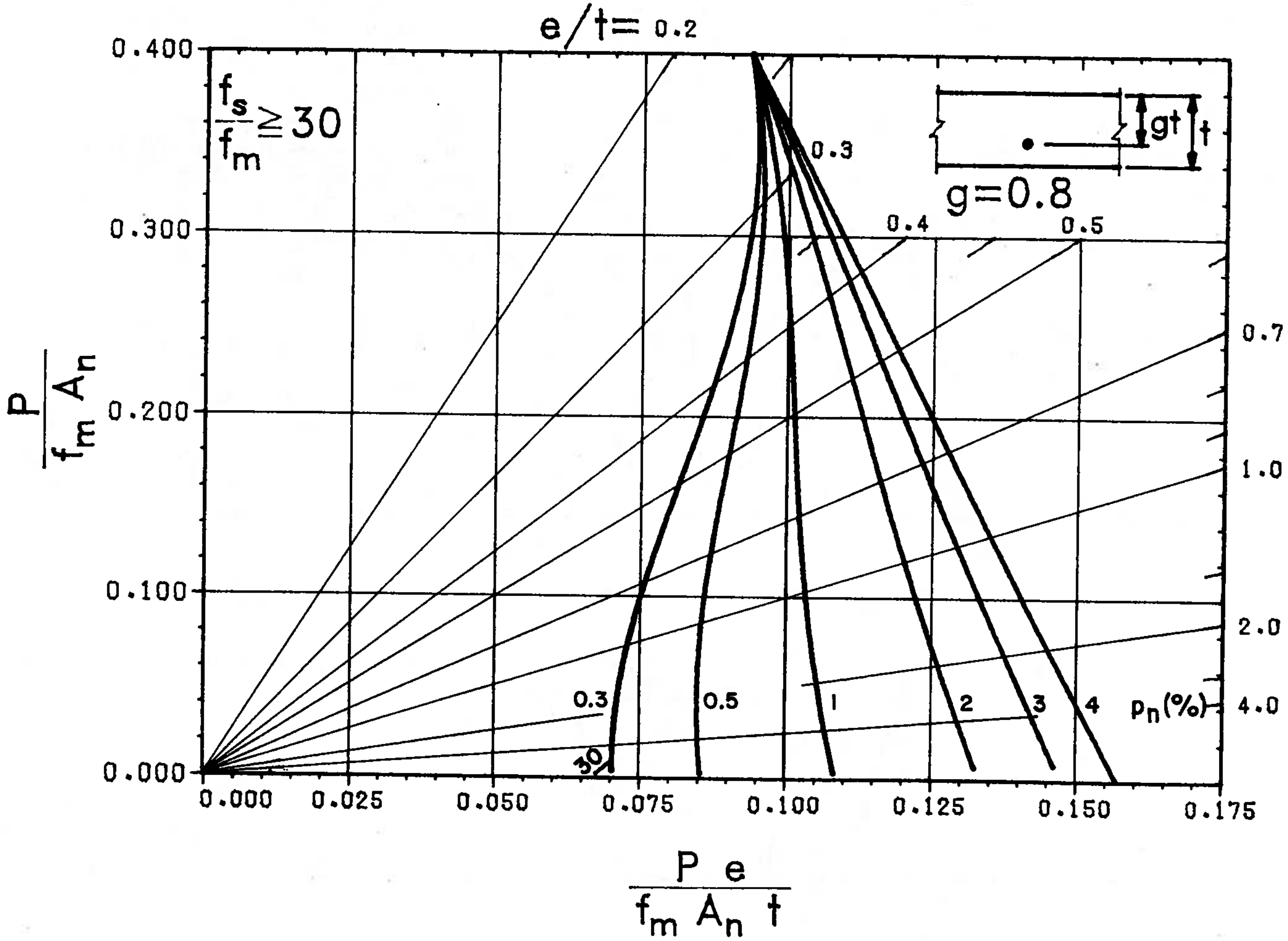
Masonry Wall Interaction Diagram

n=10 g=0.7



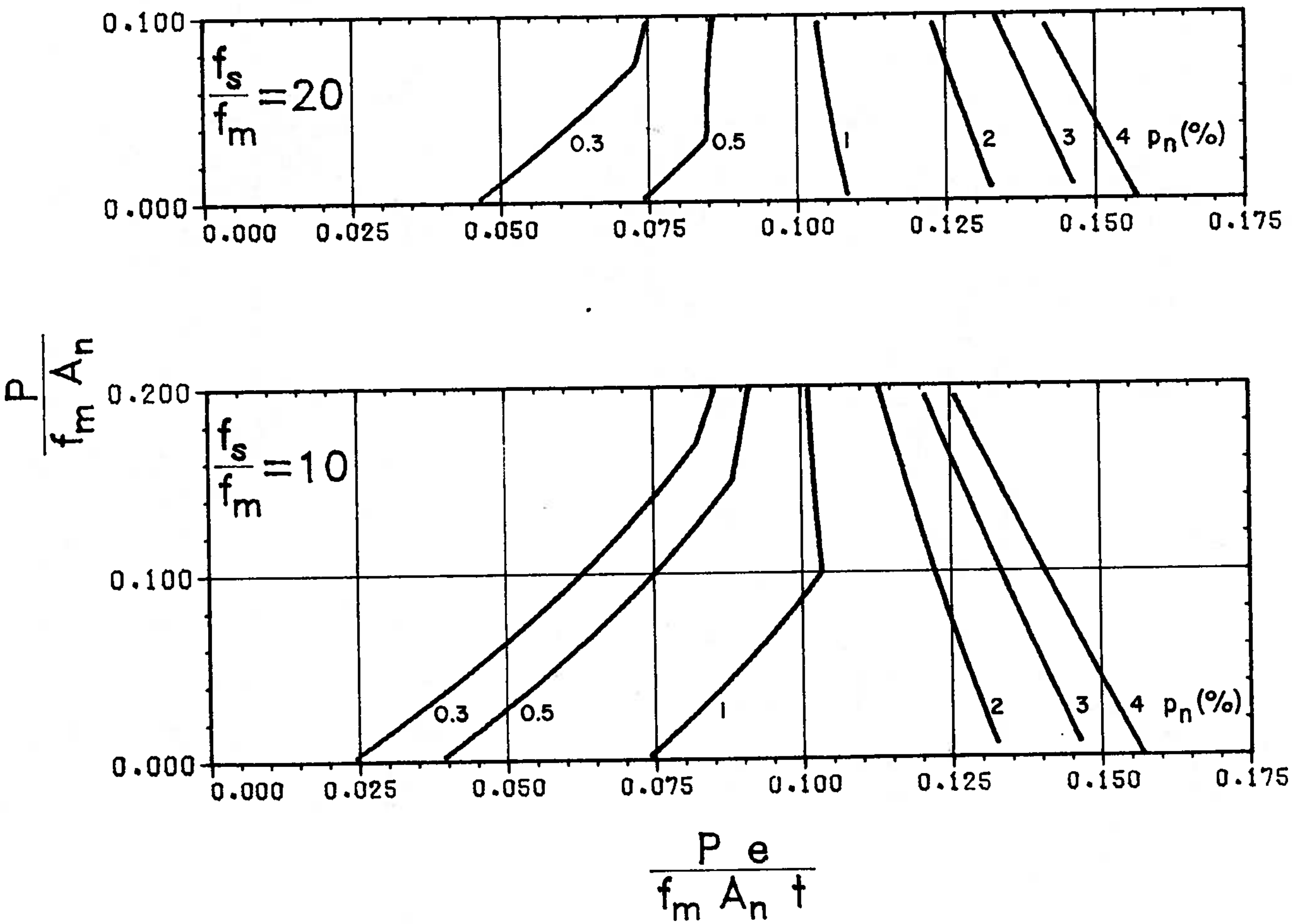
Masonry Wall Interaction Diagram

n=10 g=0.7



Masonry Wall Interaction Diagram

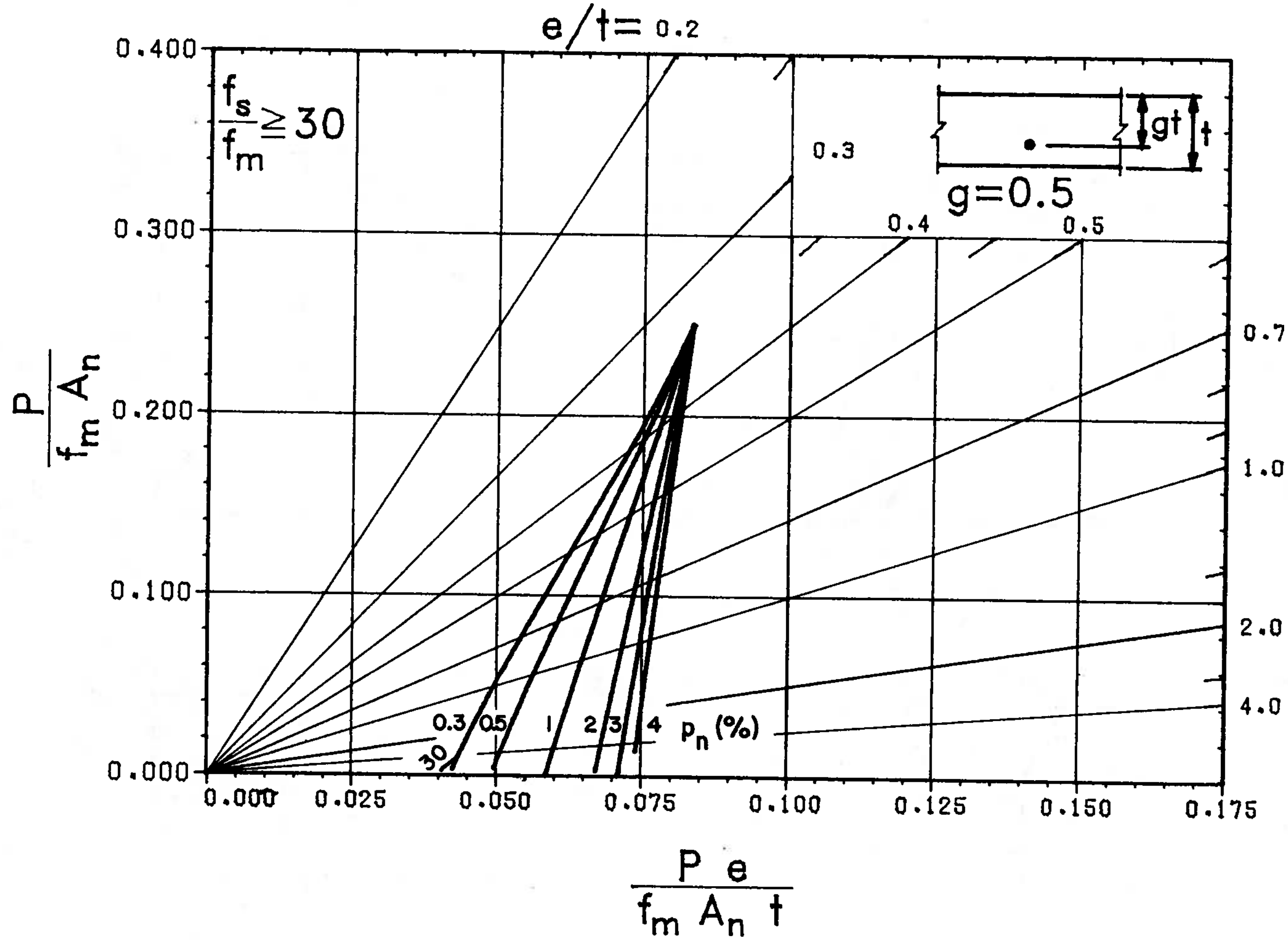
n=10 g=0.8



$$\frac{P\,e}{f_m\,A_n\,t}$$

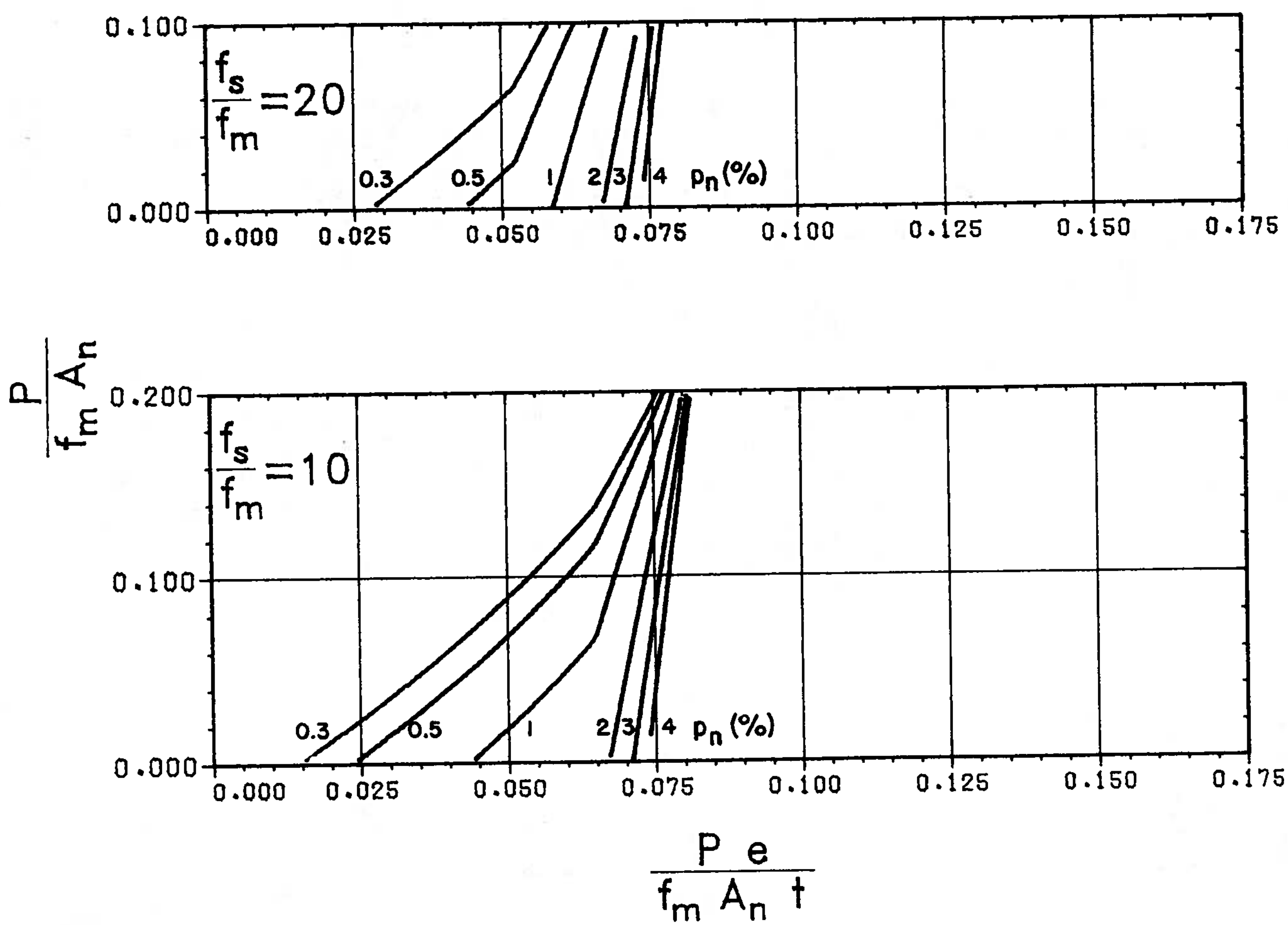
Masonry Wall Interaction Diagram

n=10 g=0.8



Masonry Wall Interaction Diagram

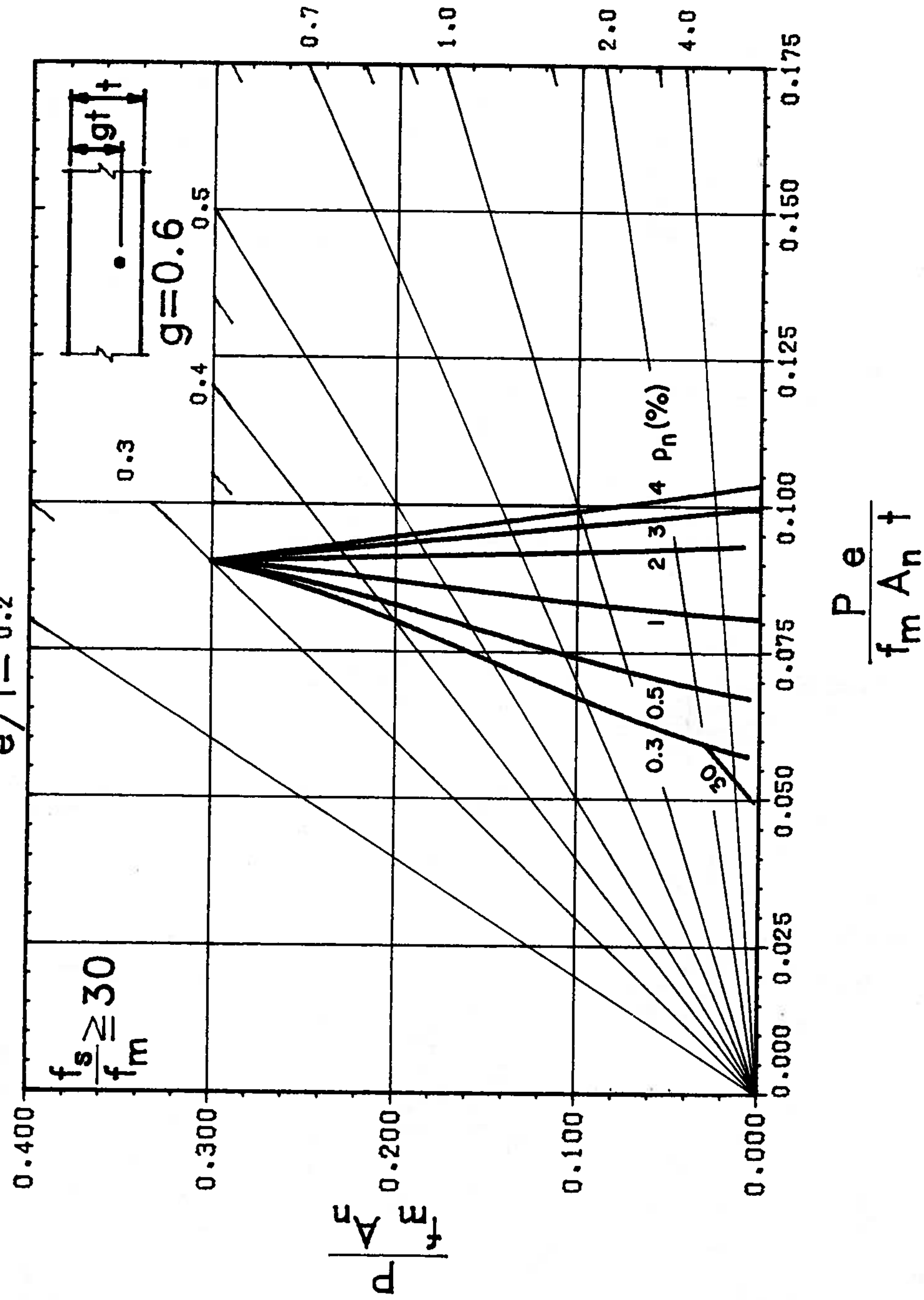
n=20 g=0.5



Masonry Wall Interaction Diagram

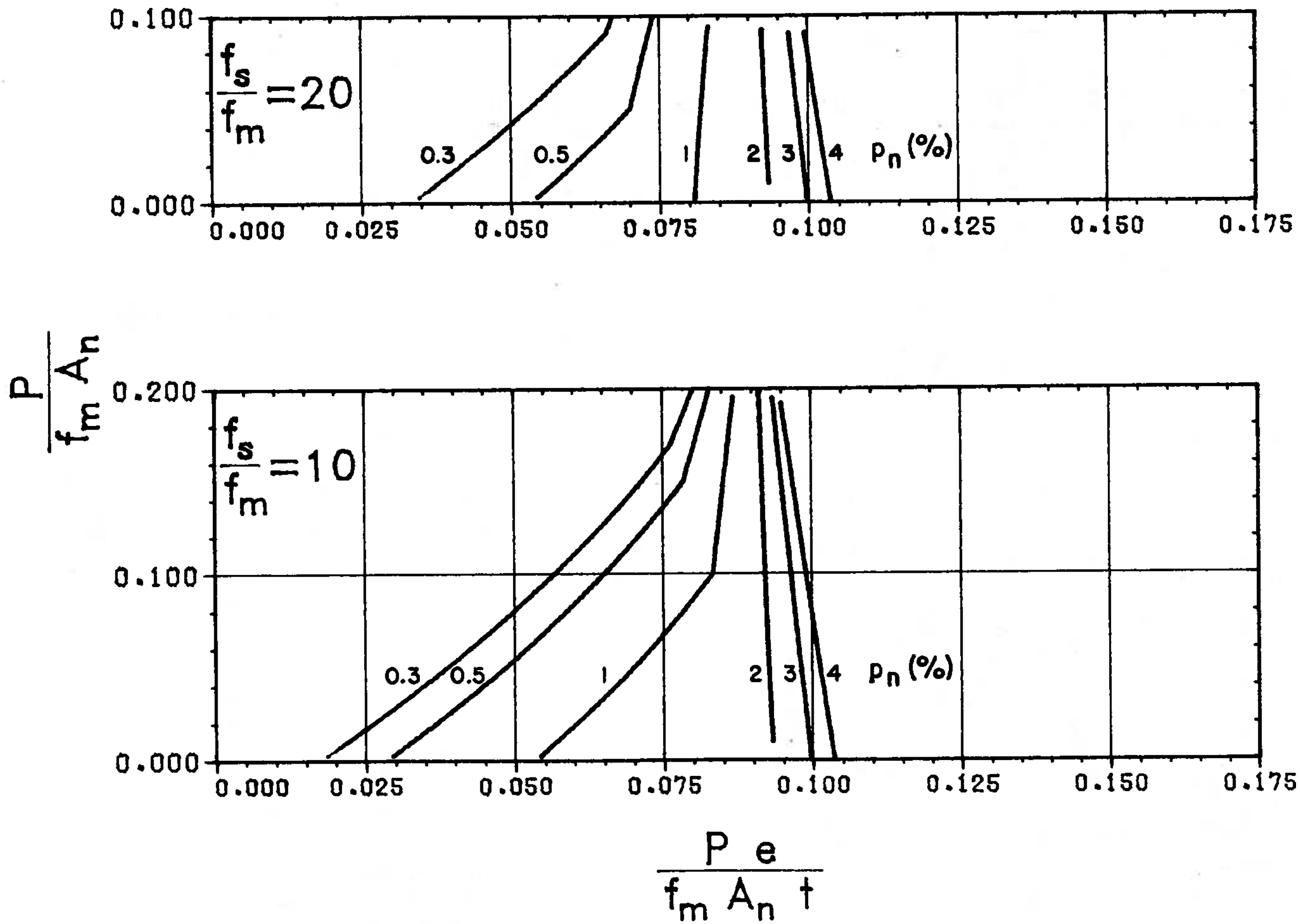
$n=20 \quad g=0.5$

A12



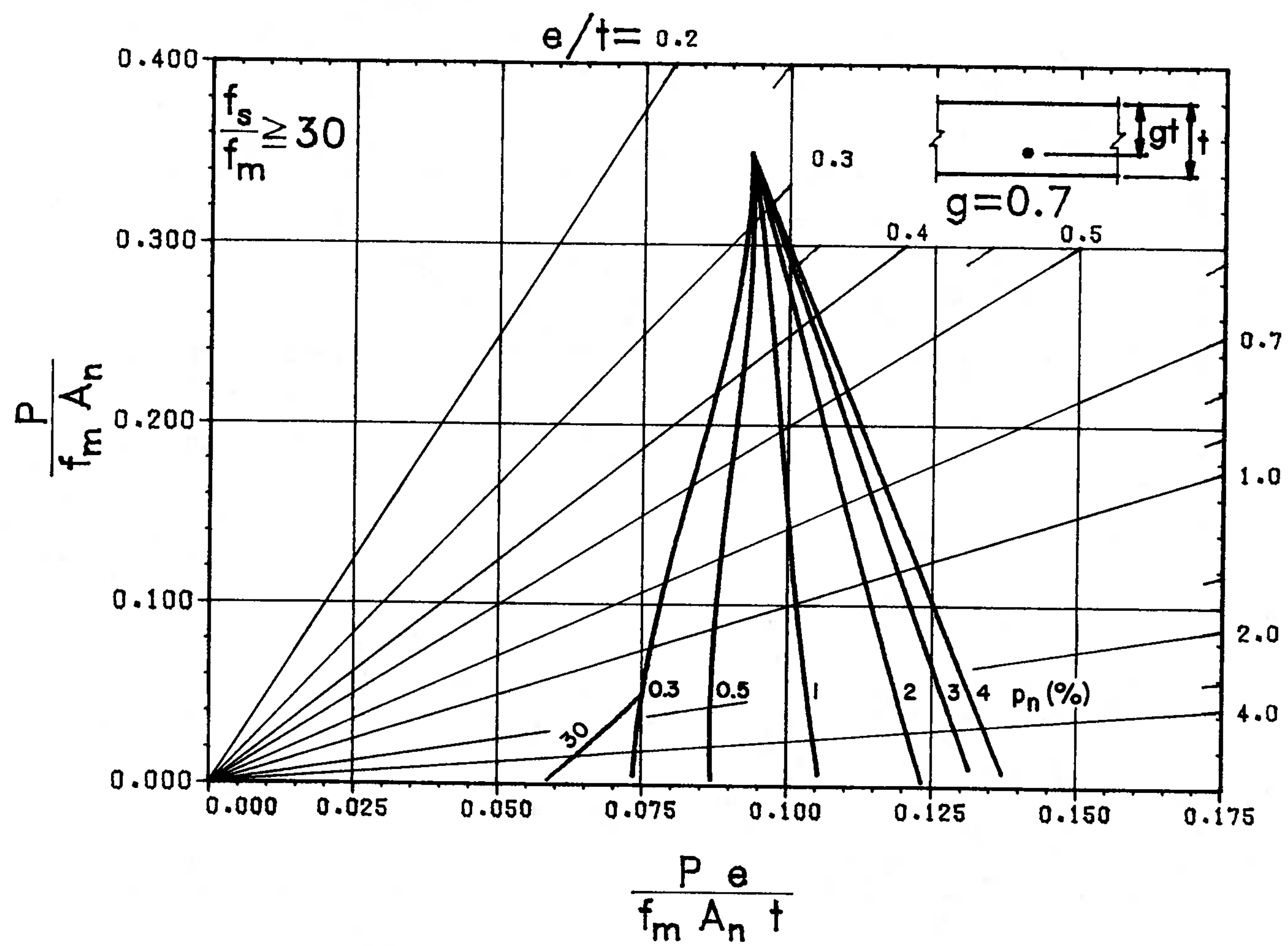
Masonry Wall Interaction Diagram

n=20 g=0.6

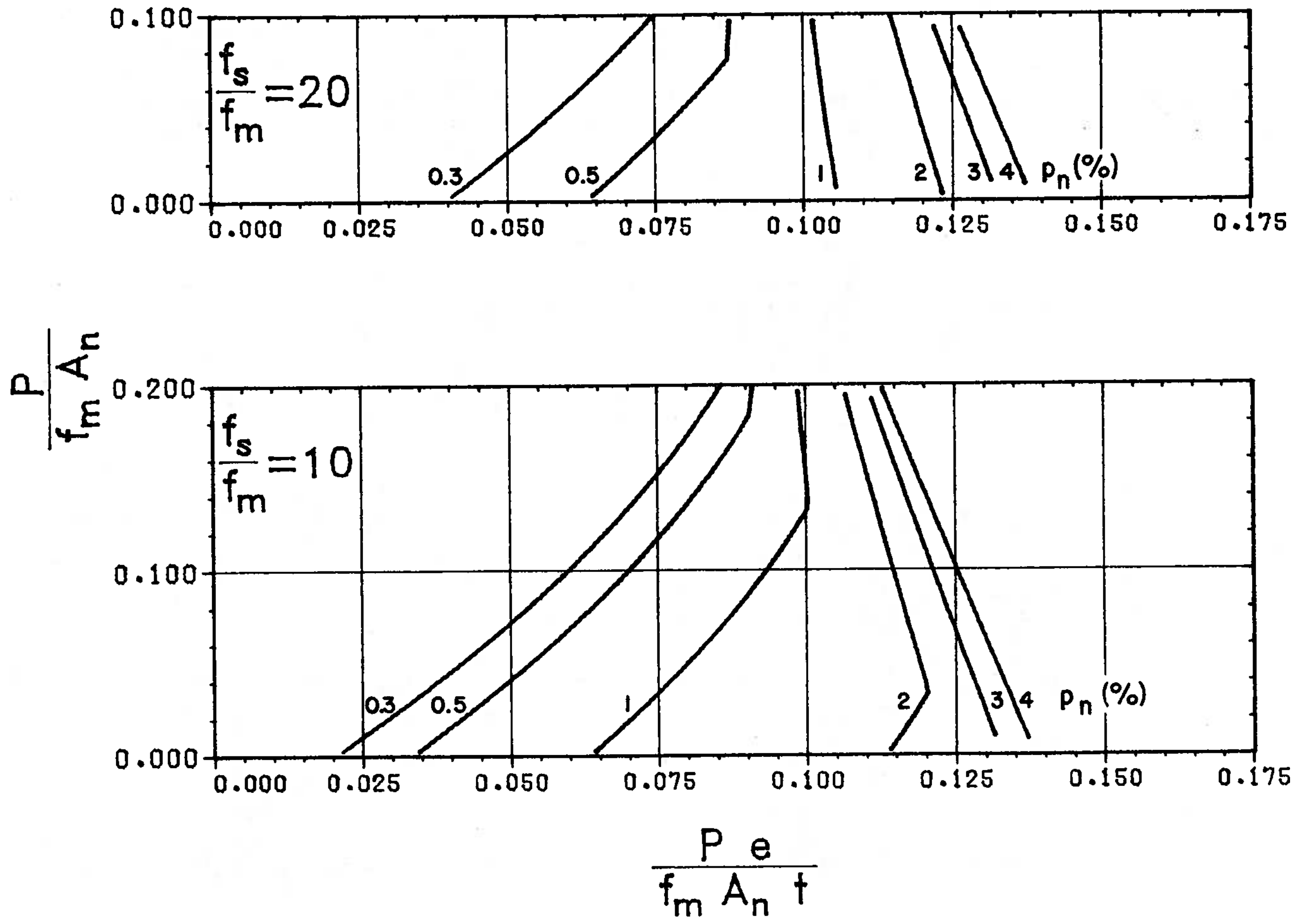


Masonry Wall Interaction Diagram

$n=20 \quad g=0.6$

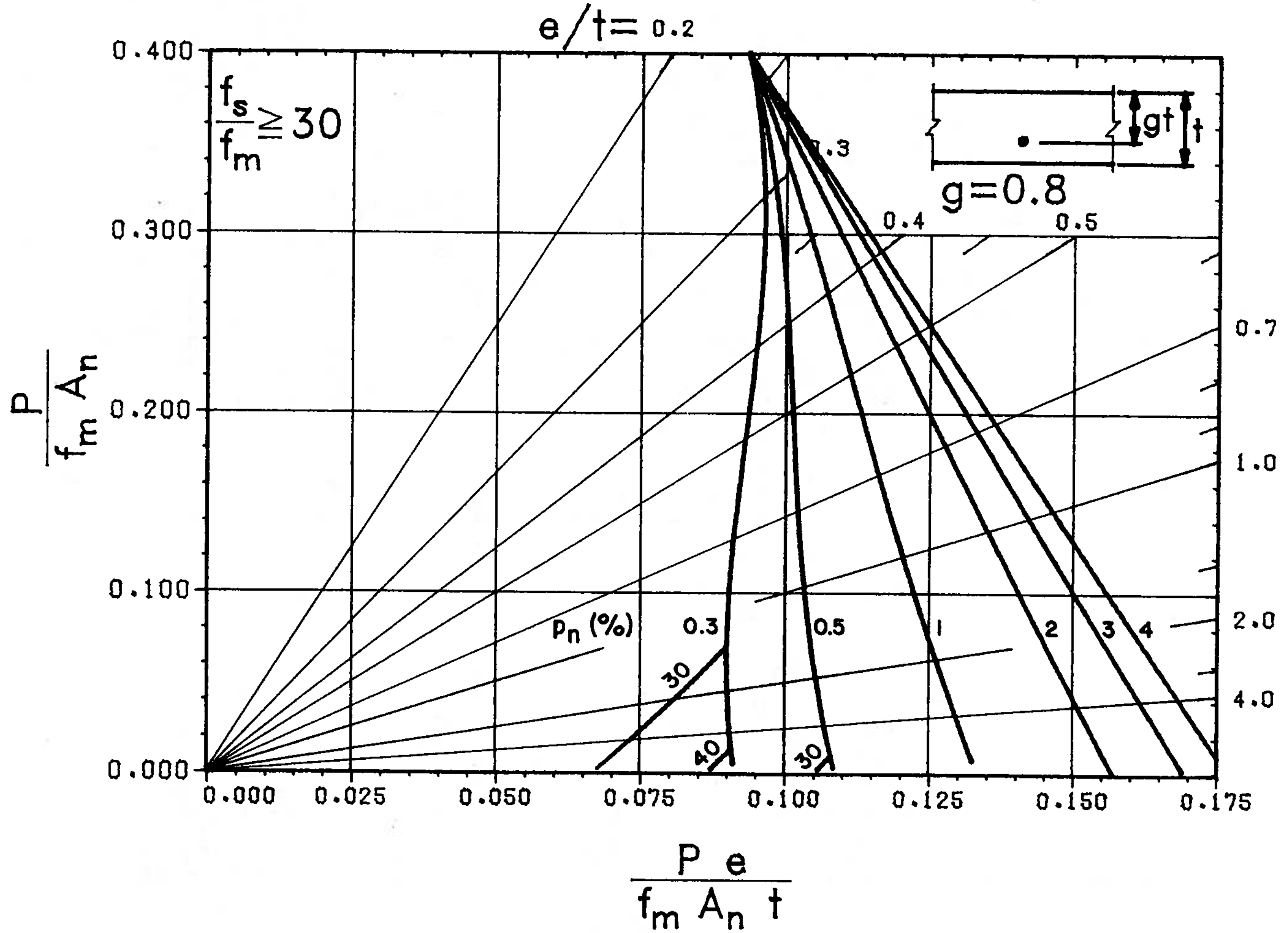


Masonry Wall Interaction Diagram
n=20 g=0.7



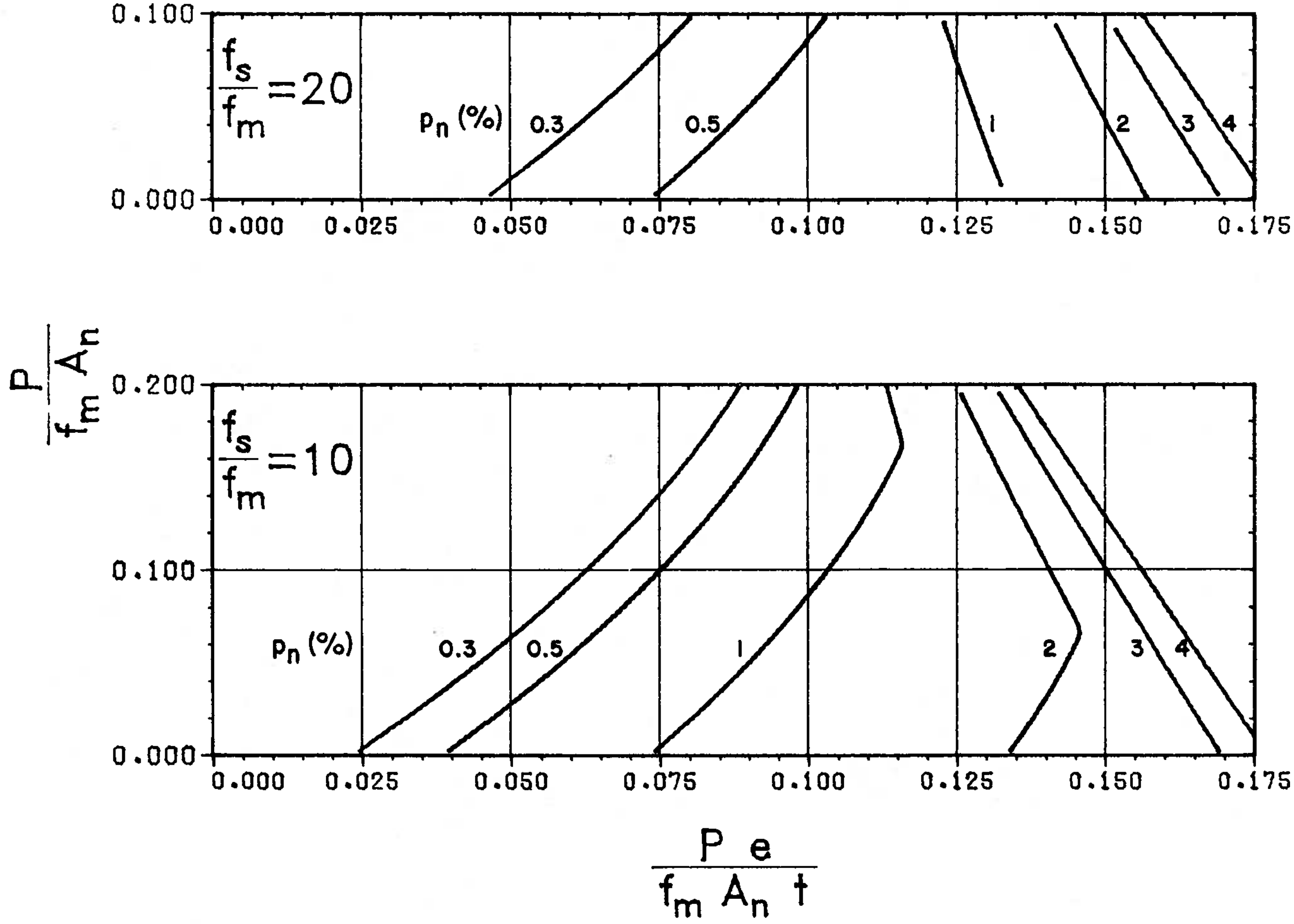
Masonry Wall Interaction Diagram

n=20 g=0.7



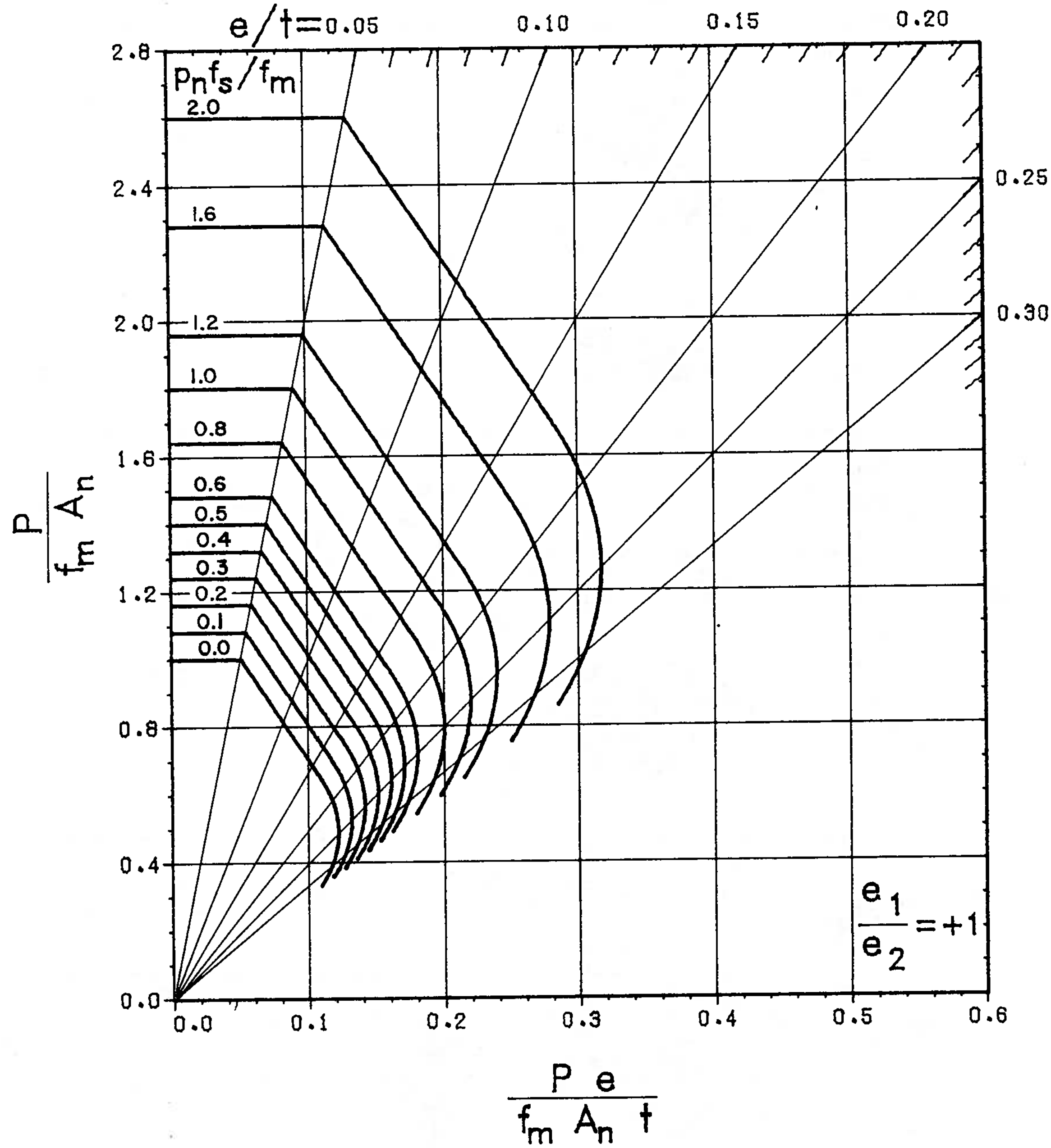
Masonry Wall Interaction Diagram

n=20 g=0.8

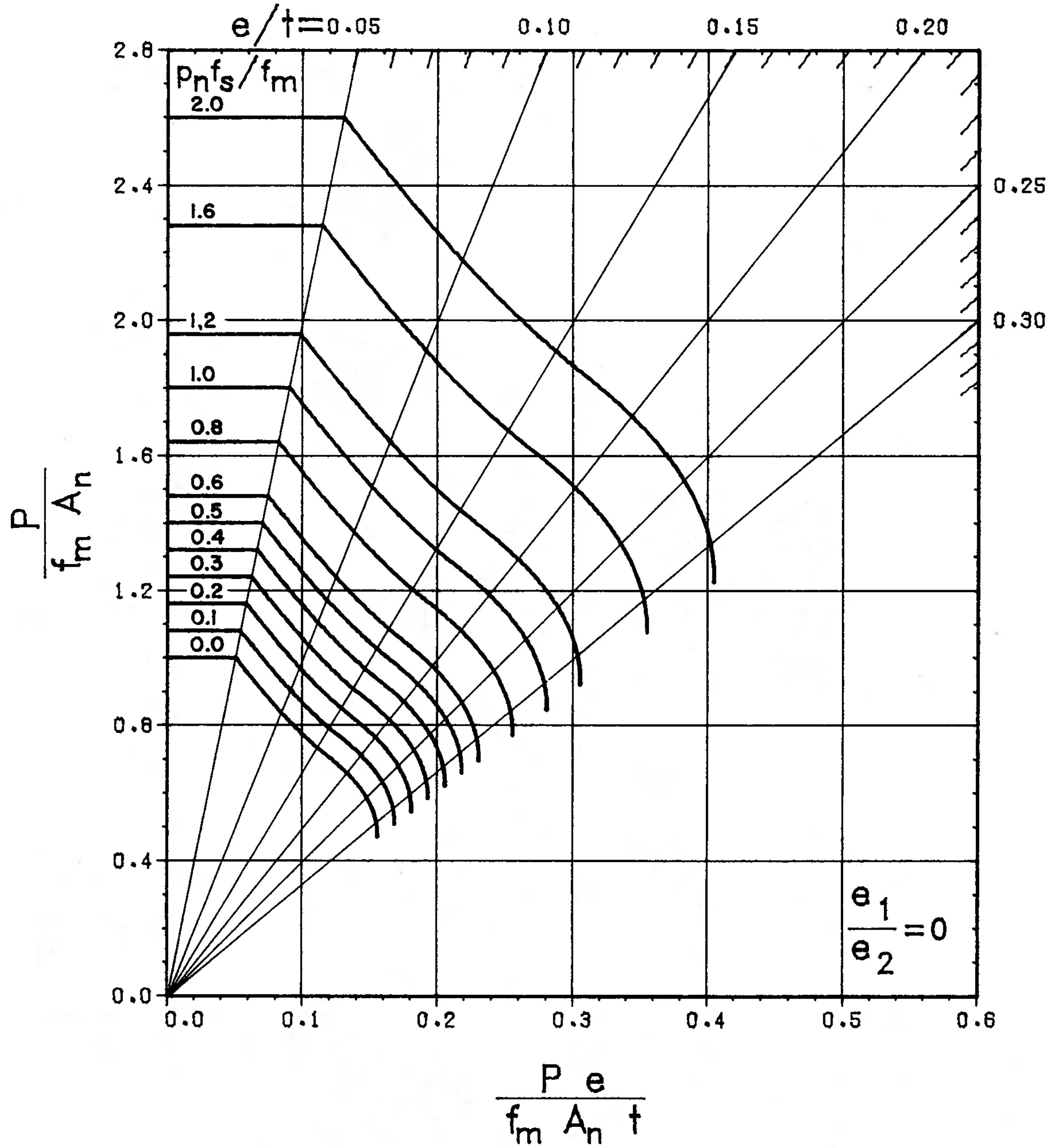


Masonry Wall Interaction Diagram

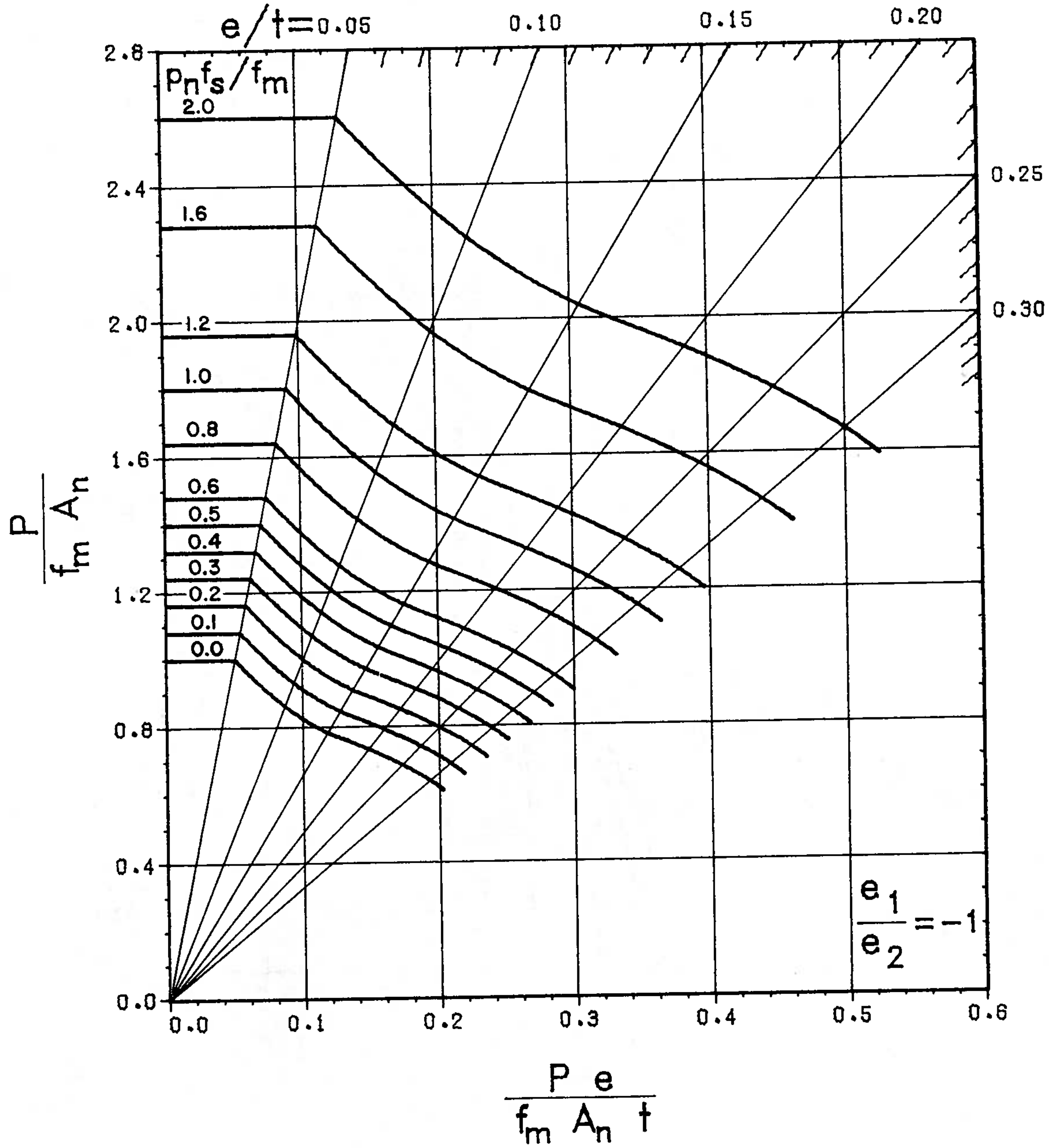
n=20 g=0.8



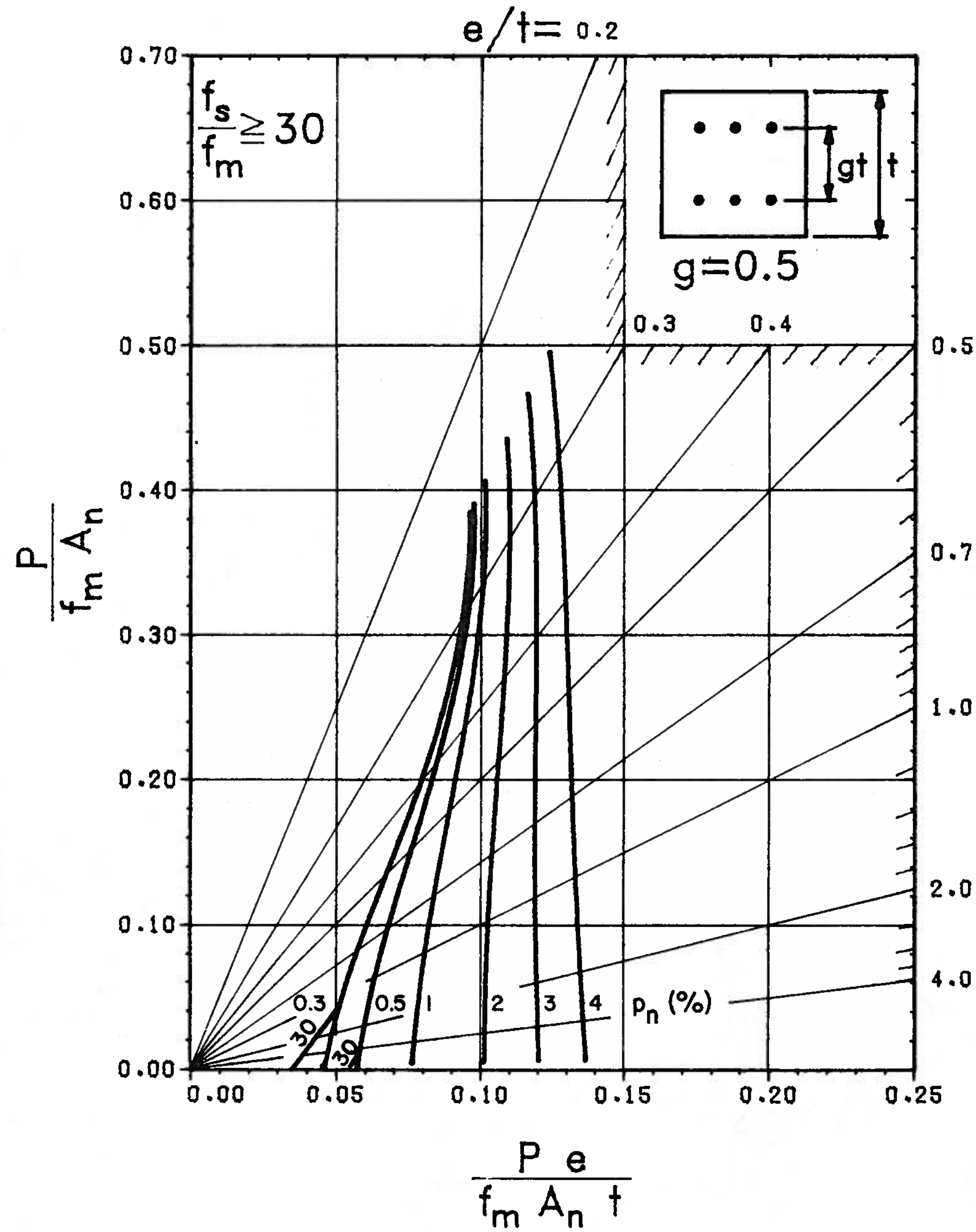
Masonry Column Interaction Diagram



Masonry Column Interaction Diagram

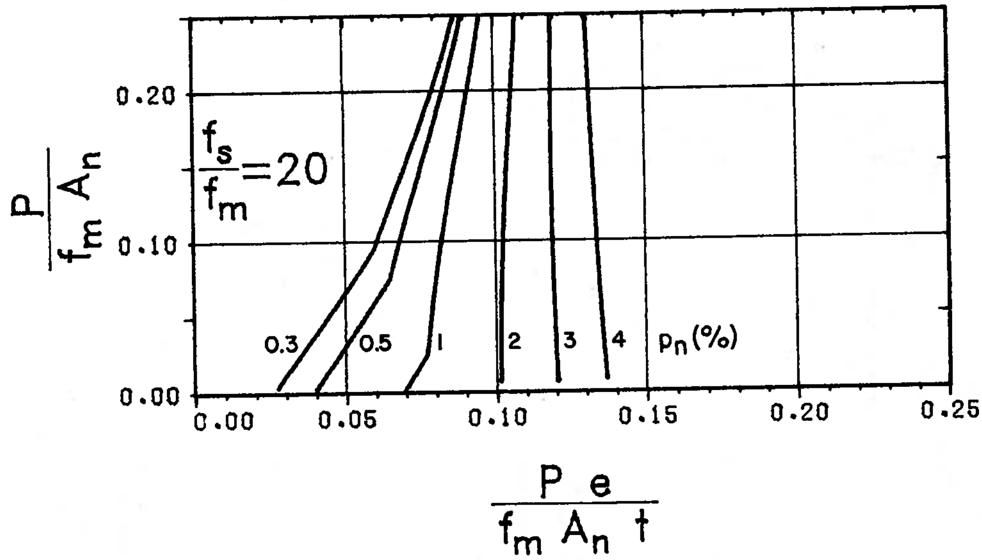


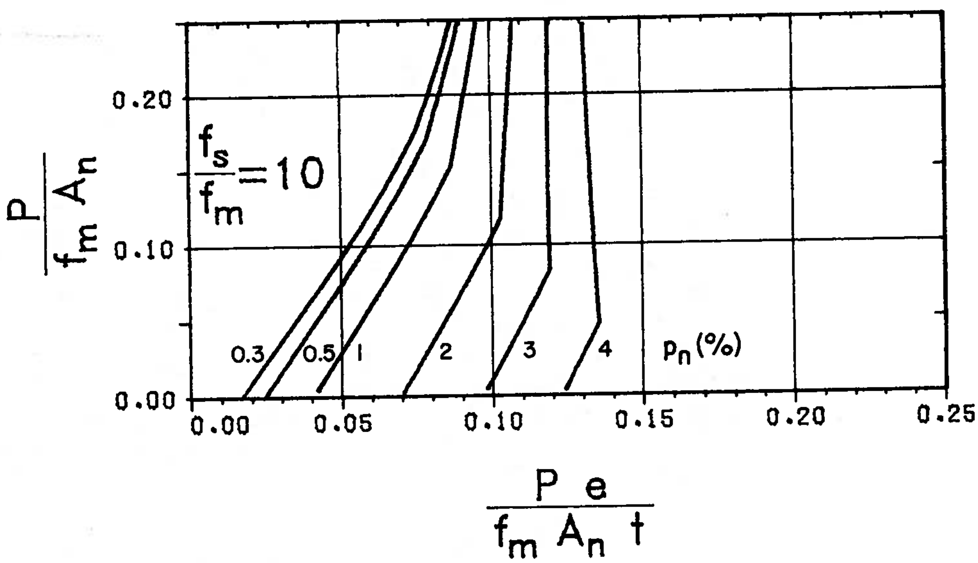
Masonry Column Interaction Diagram



Masonry Column Interaction Diagram

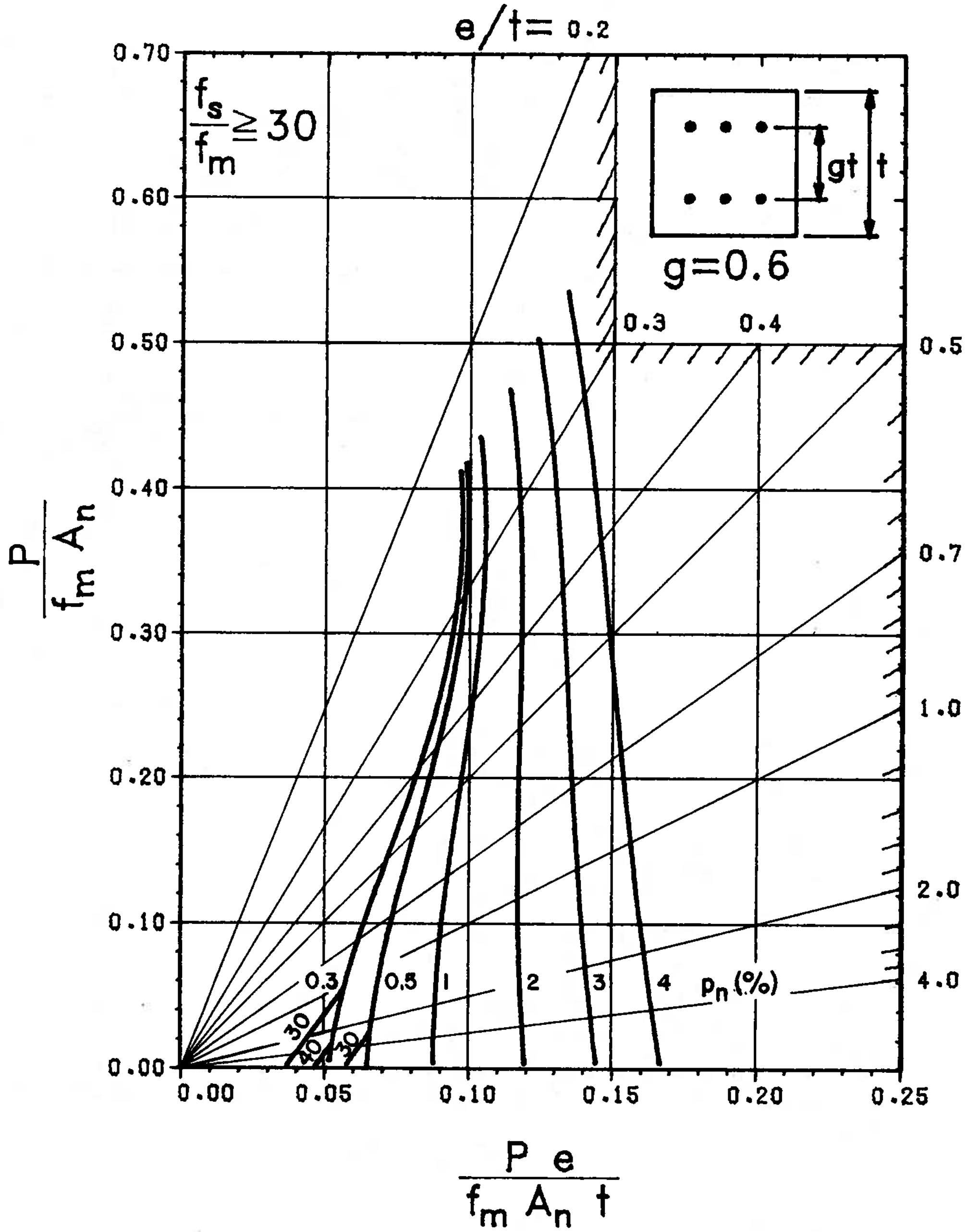
n=10 g=0.5





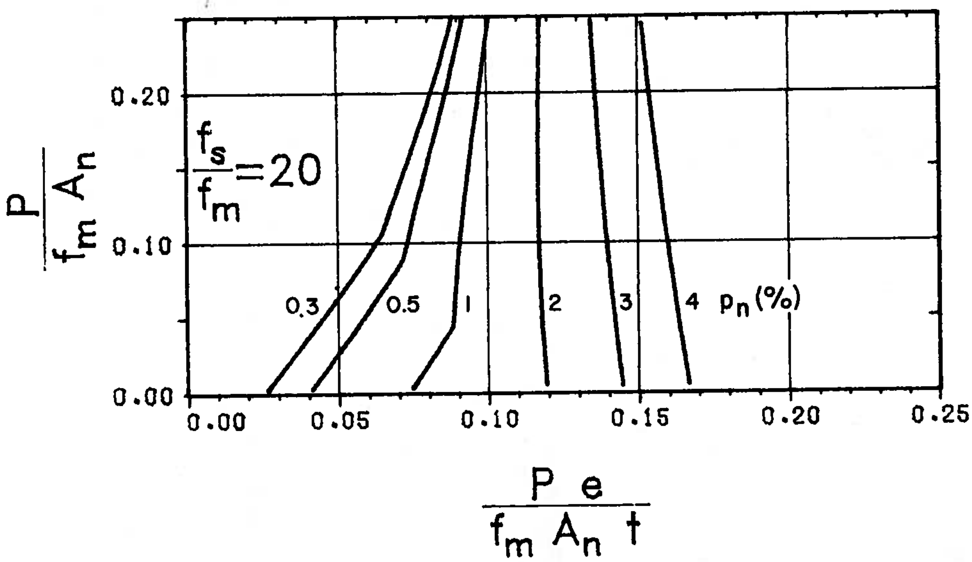
Masonry Column Interaction Diagram

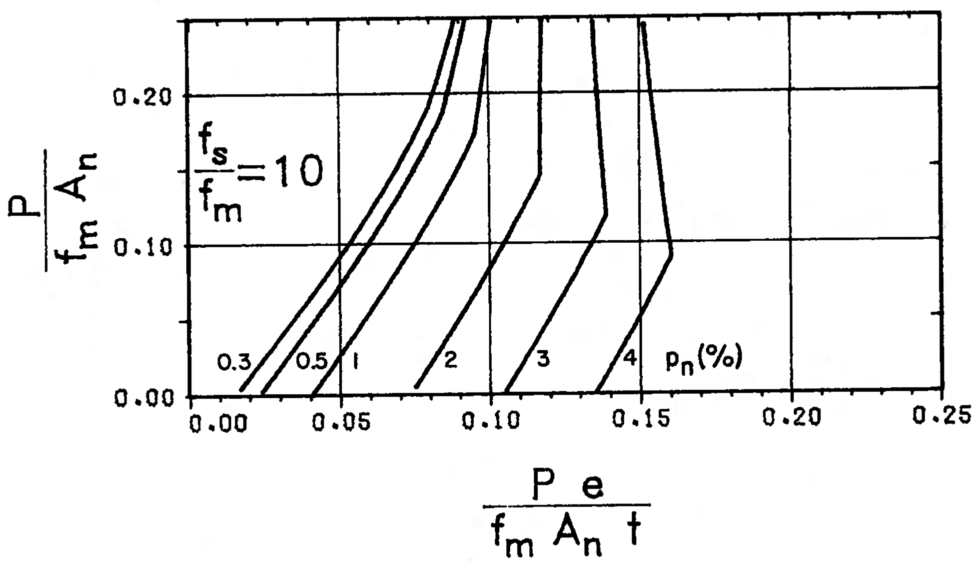
n=10 g=0.5



Masonry Column Interaction Diagram

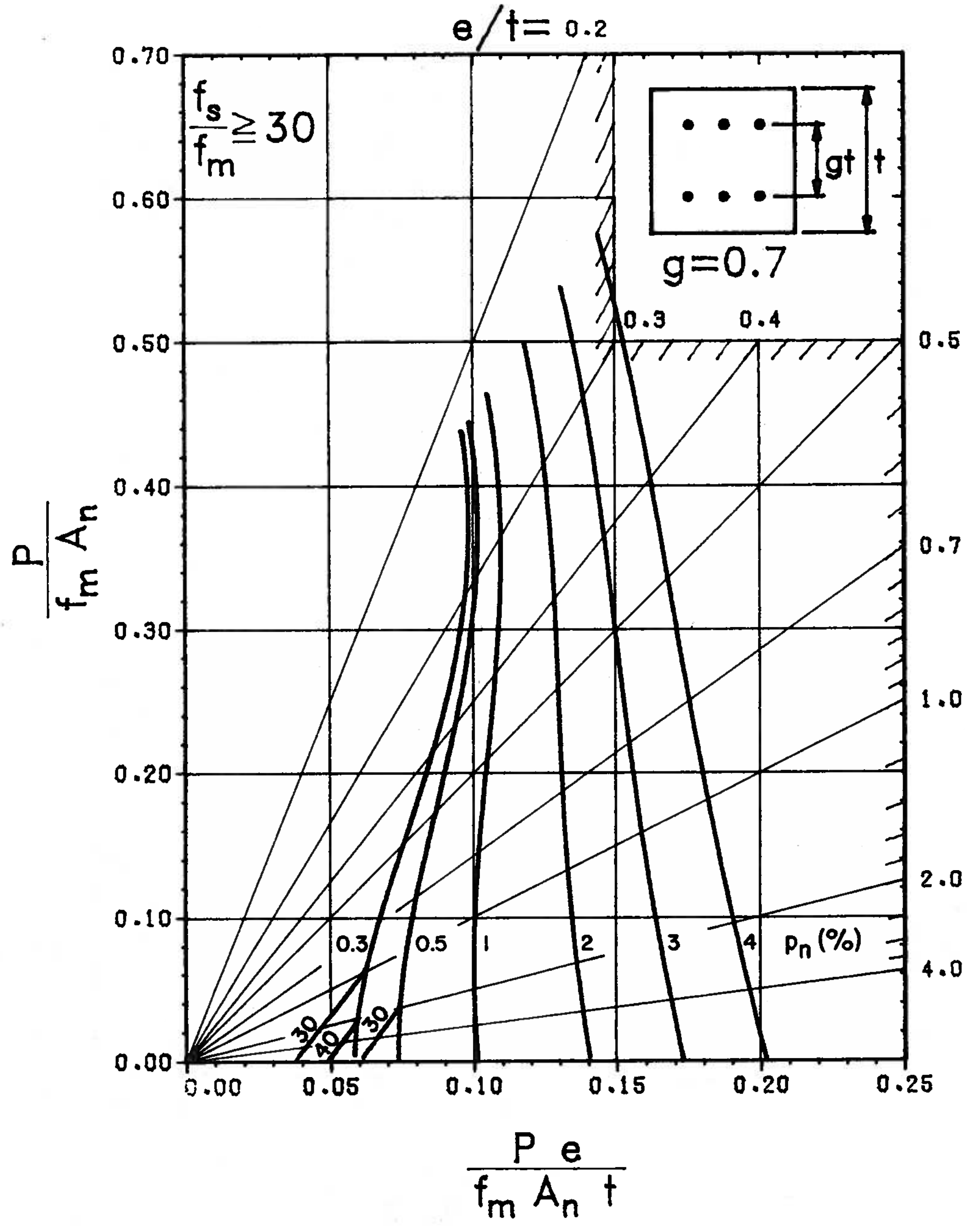
$n=10 \quad g=0.6$





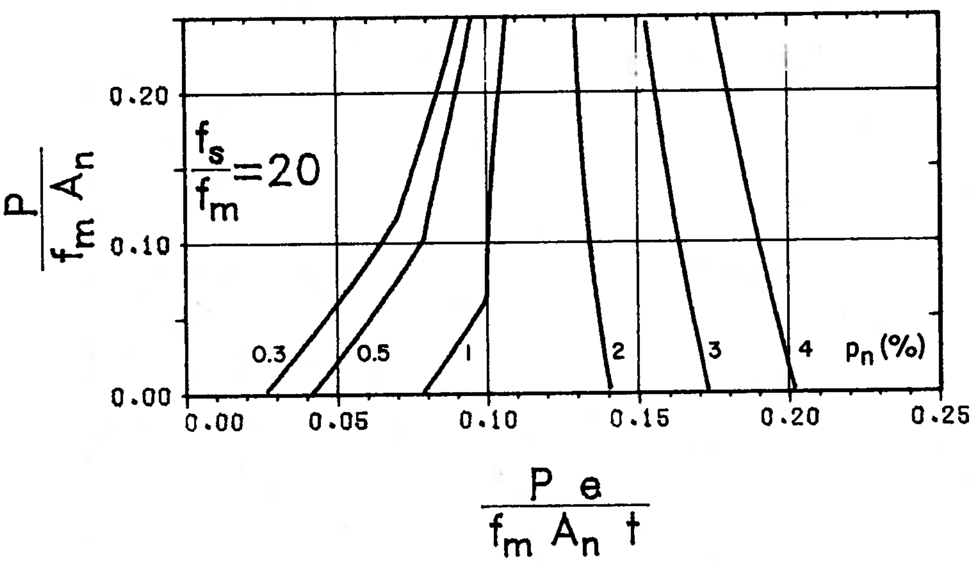
Masonry Column Interaction Diagram

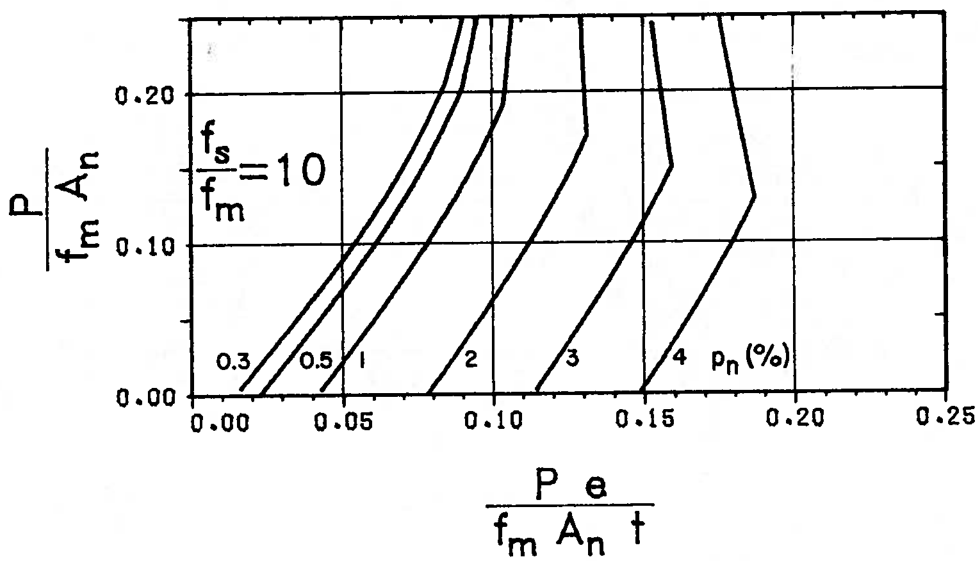
$n=10 \quad g=0.6$



Masonry Column Interaction Diagram

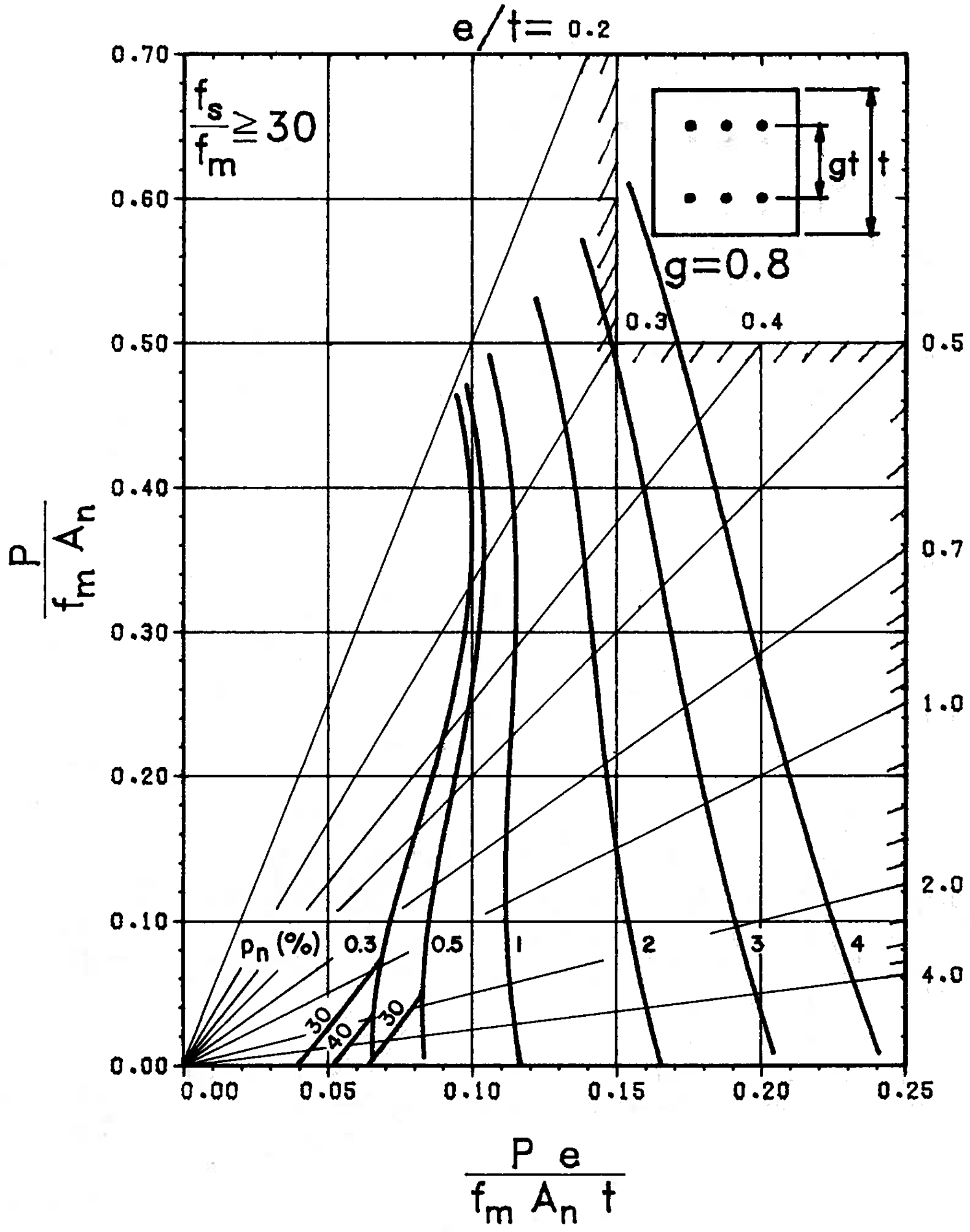
$n=10 \qquad g=0.7$





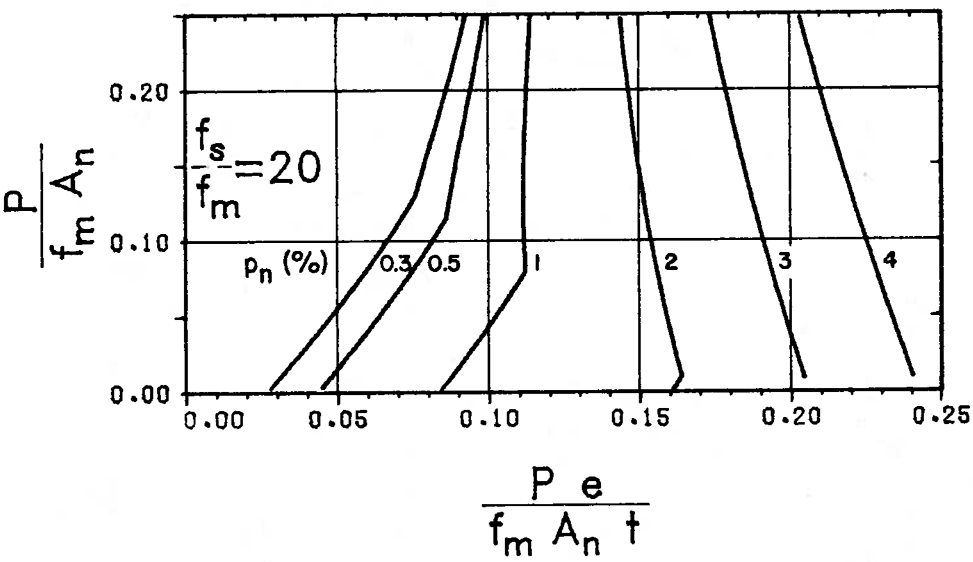
Masonry Column Interaction Diagram

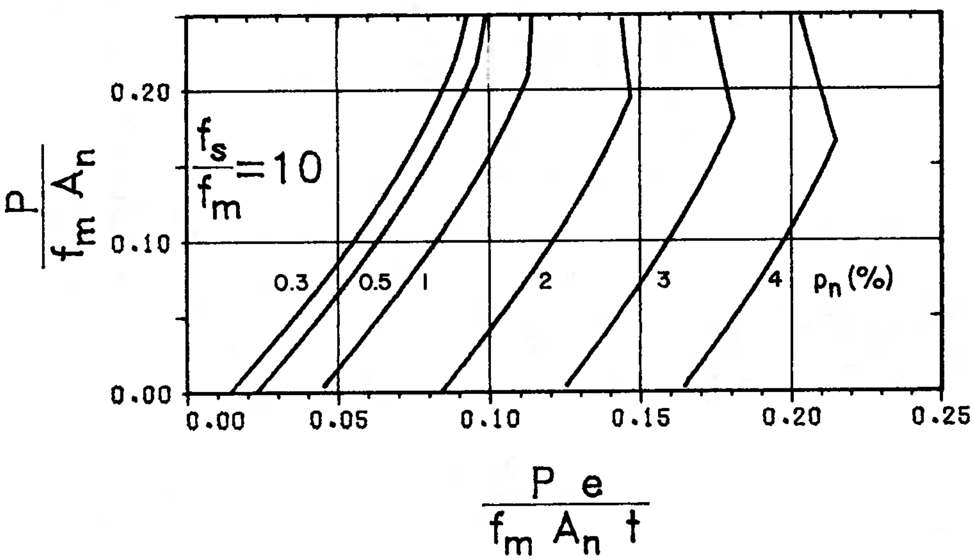
$n=10$ $g=0.7$



Masonry Column Interaction Diagram

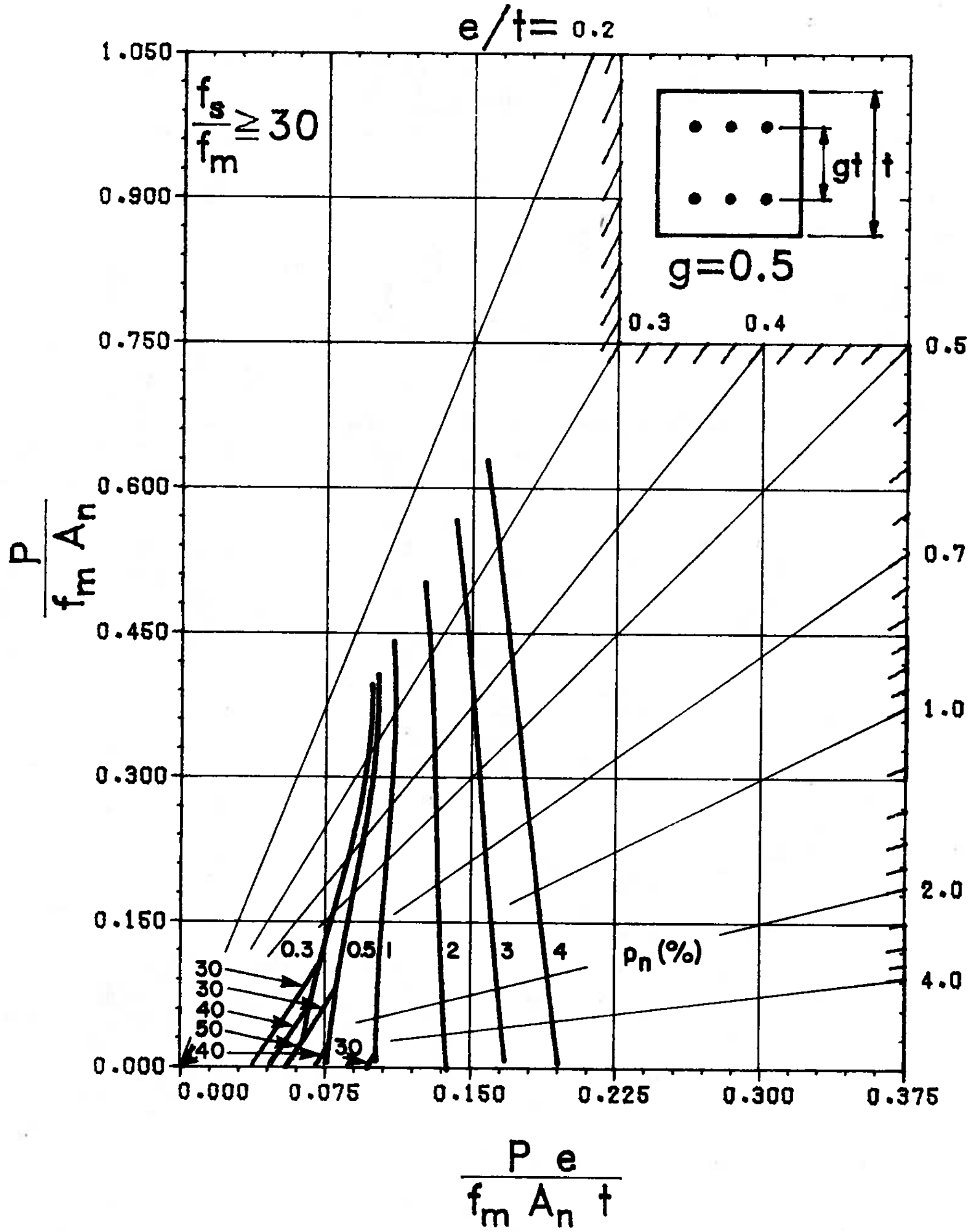
n=10 g=0.8





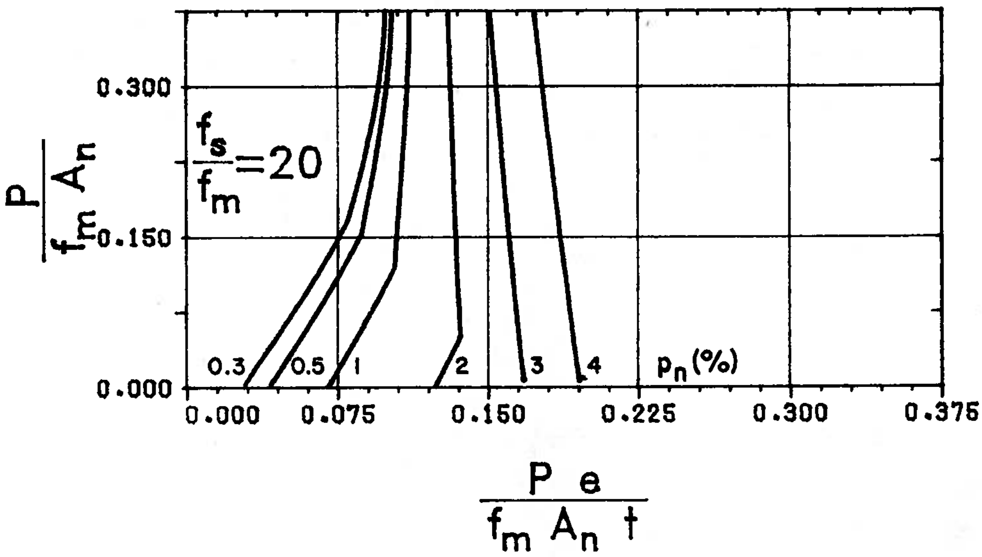
Masonry Column Interaction Diagram

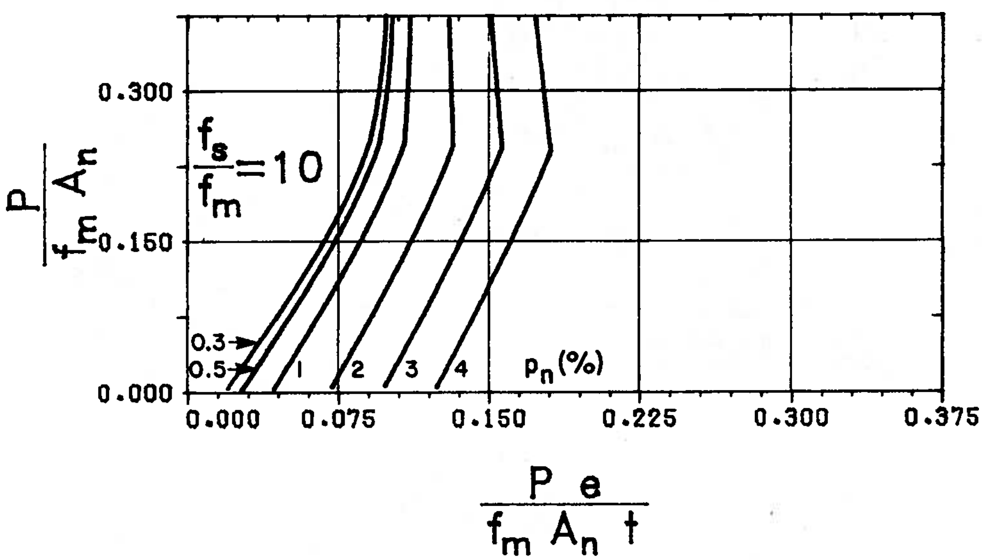
n=10 g=0.8



Masonry Column Interaction Diagram

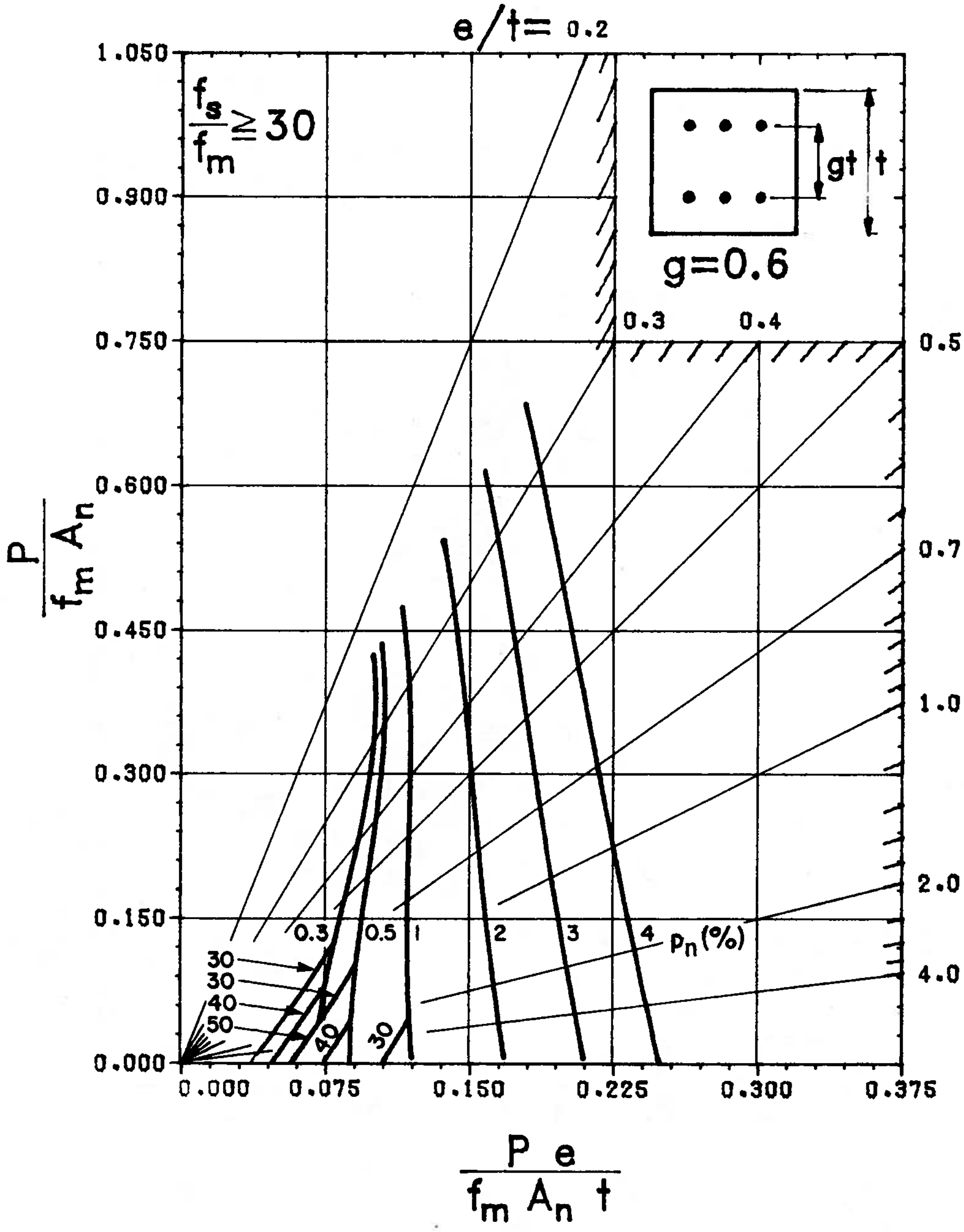
n=20 g=0.5





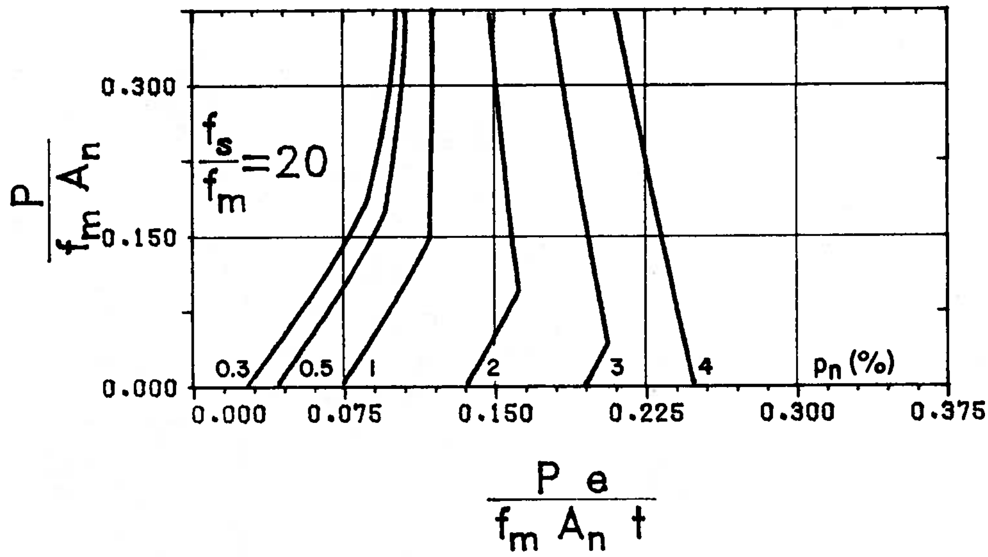
Masonry Column Interaction Diagram

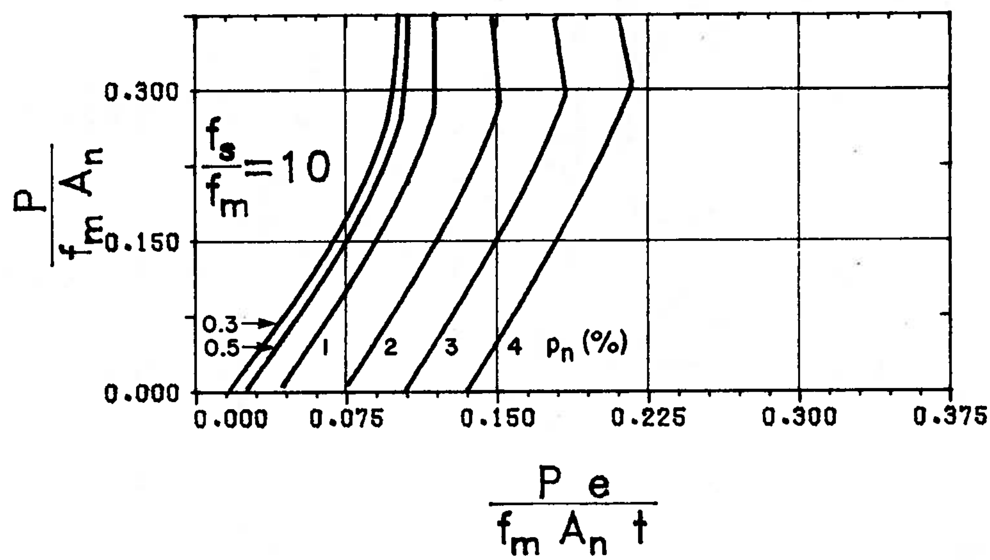
n=20 g=0.5



Masonry Column Interaction Diagram

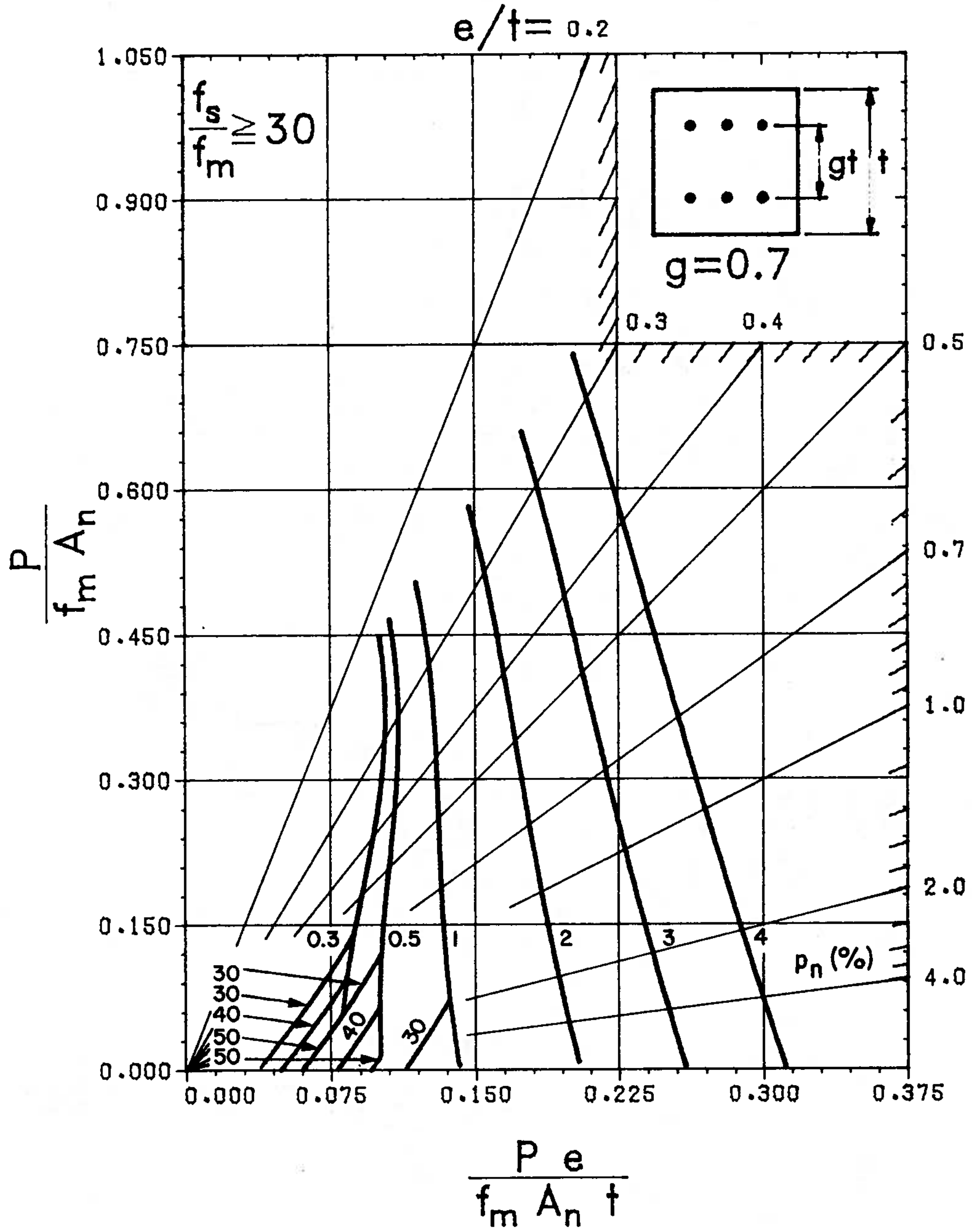
n=20 g=0.6





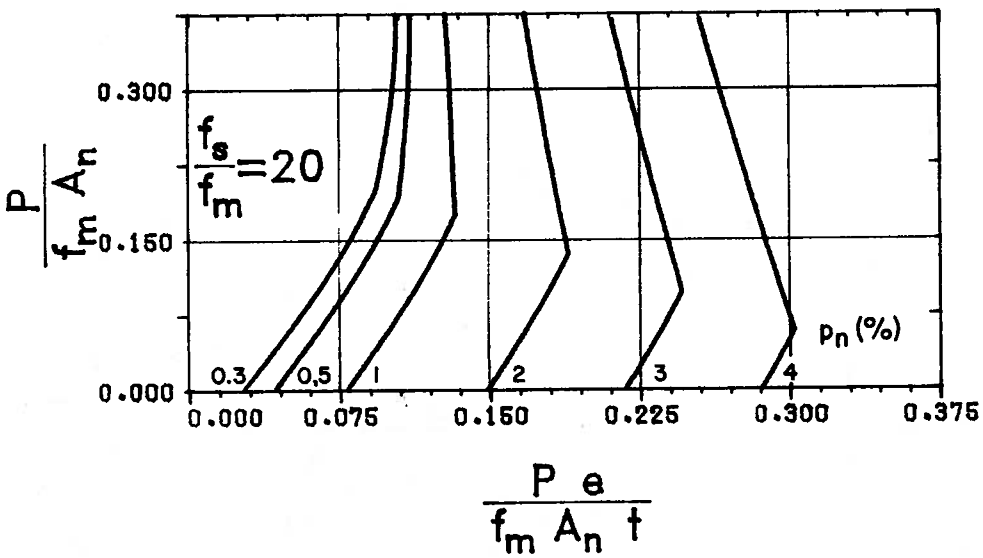
Masonry Column Interaction Diagram

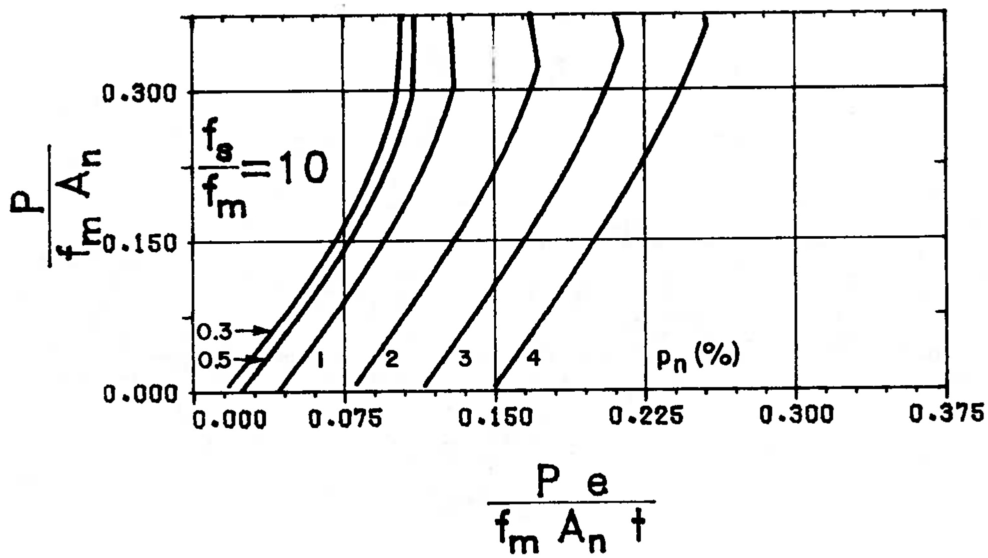
n=20 g=0.6



Masonry Column Interaction Diagram

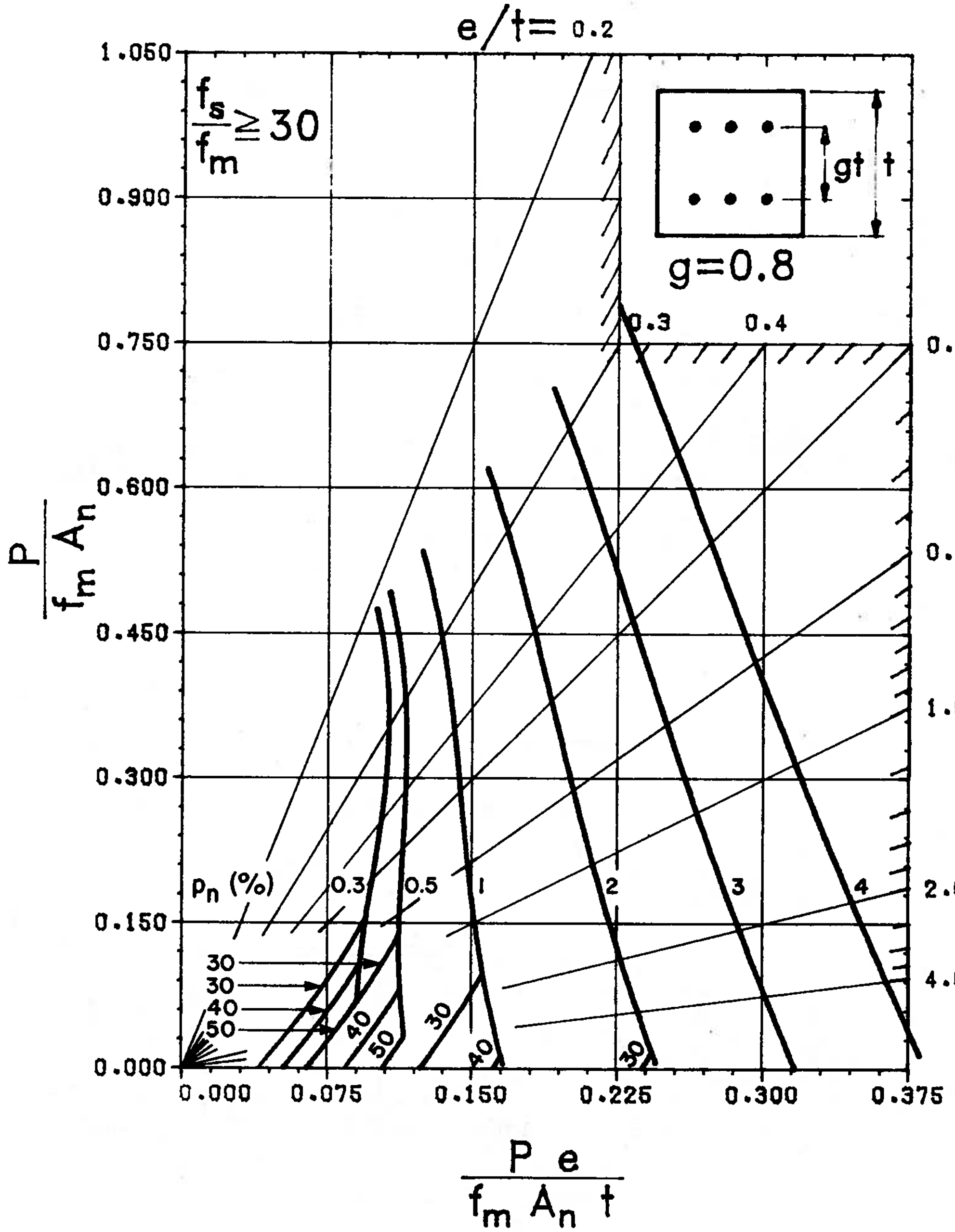
n=20 g=0.7





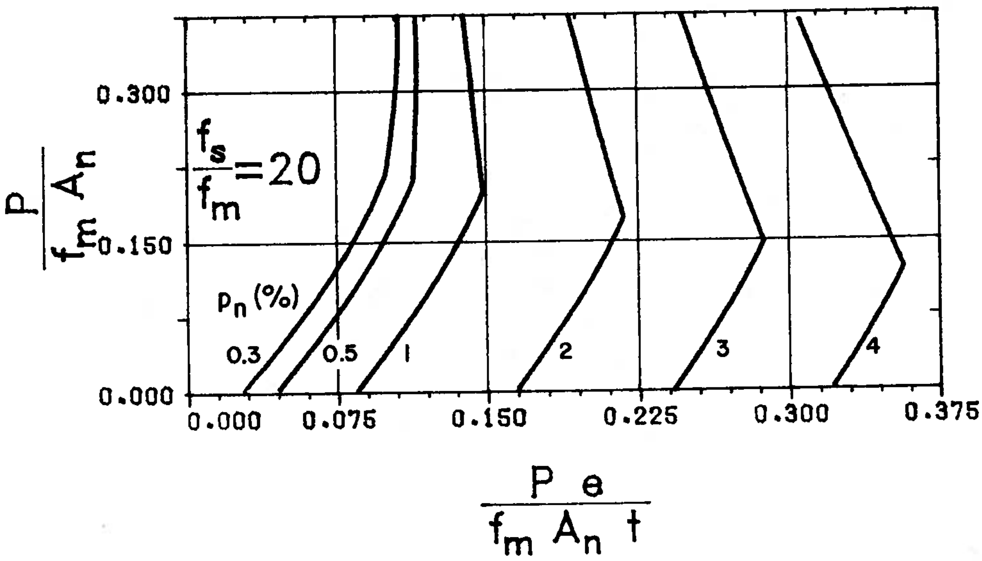
Masonry Column Interaction Diagram

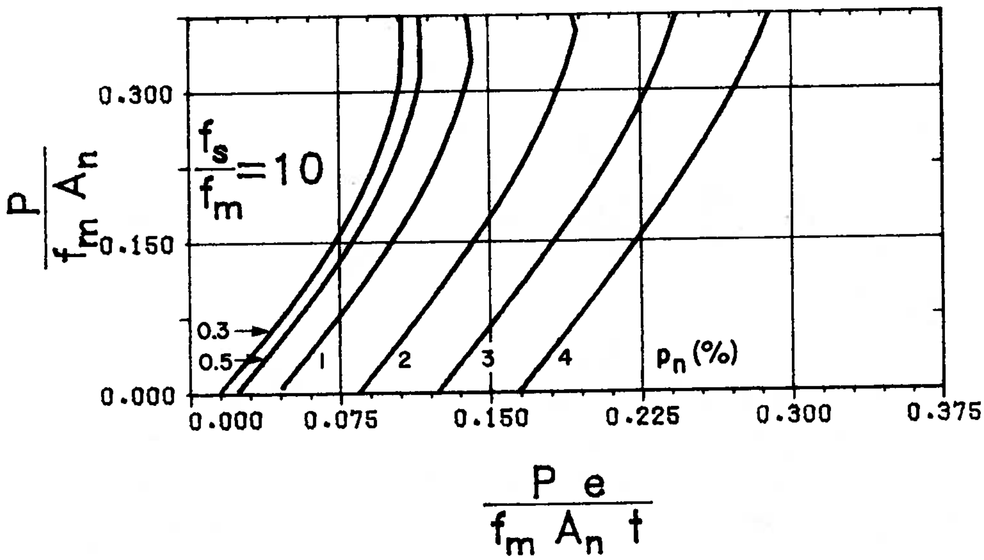
n=20 g=0.7



Masonry Column Interaction Diagram

n=20 g=0.8





Masonry Column Interaction Diagram

n=20 g=0.8