Momentum Transfer and Friction in the Debris of Rock Avalanches

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Abstract

When a mass of loose, dry, purely frictional material slides down an incline after release at a given velocity, the run out (the distance the centre of gravity of the displaced mass moves from its initial position) depends on momentum transfer within the mass. This can be estimated from the profile of the debris accumulation which also allows more accurate calculation of apparent angles of sliding friction in rock-fall avalanches. The apparent extreme mobility of the Elm and Frank Slides, the typical rock-fall avalanches, is explained by momentum transfer in a loose, dry, purely frictional material with an angle of friction of 30°.

Key words: mass wasting, momentum transfer, landslide, accumulation, rock avalanche, runout distance.

Introduction

Some rock fall-debris flows are extremely mobile, attaining high velocities and having very long run outs. The first quantitative model of these phenomena, the sliding block (Heim, 1932, pp.144-150) required an unrealistically low coefficient of friction to fit the observed behaviour of the Elm Slide. Similar movements have been called rock-fall avalanches (Varnes, 1978) and numerous theories have been proposed to explain their mobility: sliding on a basal mud layer (Heim, 1932); air lubrication (Shreve, 1968); air fluidization (Kent, 1966); and pore vapour fluidization (Habib, 1975). These theories were subject to Howard's (1973) criticism that they fail to explain landslides on the Moon (which take place in a vacuum). Howard (1973) argued for mechanical fluidization and the reduction of friction in the debris at high velocities. Melosh (1979) proposed acoustic fluidization.

Recent laboratory experiments, however, have demonstrated that the angle of friction of a granular material is close to 30° over the range of velocities of natural rock-fall avalanches (Hungr and Morgenstern, 1984). Field observations of the debris of the Frank Slide also cast doubt on the above theories of avalanche motion (Cruden and Hungr, 1986, Cassie, Van Gassen and Cruden, 1988). Sassa (1988) characterized theories such as those mentioned as "sled" theories. In these theories, the displaced mass began moving instantaneously and all the displaced mass stopped moving at the same time after travelling its course. Such theories are obviously only first approximations to the behaviour of moving debris.

Cannon and Savage (1988) proposed a more sophisticated, mass-change model to estimate the distance a debris flow runs before it stops. Their model included, besides viscous and frictional forces, the effect of the changing mass of the debris flow as material is deposited from the flow. A similar formal development (Timoshenko and Young, 1948, pp. 113-118) described the movement of rockets (which lose mass by expelling fuel) and balloons (which discard ballast).

Here, we return to sliding blocks but consider that deposition of rock slide debris occurs gradually, not abruptly. It is suggested that a slide with changing mass is a more appropriate model for the accumulation zone of a rock-fall avalanche. Our model is similar to Cannon and Savage (1988) but it concentrates on frictional forces only. An assumption of 30° for the friction angle of the slide debris reproduces the debris profiles generated by the change of displacing mass during the Elm and Frank Slides.

Terminology for the slope movements follows Varnes (1978) and Cruden (1980).

Equations of motion for rectilinear movement

In a system of particles with changing mass, if the velocity of the gained or expelled particles is zero with respect to the frame of reference selected, Newton's second law may be written as equation 1.

$$\sum \bar{F} = \frac{d(M\bar{v})}{dt} \qquad (1)$$

where $\sum \bar{F}$, M and $\bar{v}$ are the forces acting on the system, the mass and velocity of the system respectively, and t is time (Beer and Johnston, 1976, p. 571).

For a mass moving along an inclined path, equation (1) is:

$$\frac{M}{2} \frac{d(v^2)}{ds} + v^2 \frac{dM}{ds} = Mg \sin\theta - R \qquad (2)$$

where R is a function representing the resistance against movement (friction in this case), g is the acceleration due to gravity, s is the distance measured along the path, and Θ is the inclination of the path from the horizontal. M is the mass of the particles with a velocity different from zero (Perla et al., 1980).

Application to sliding system with changing mass

In this section the run out of a slide with changing mass is compared with the run out of a slide with constant mass. The run out, L, is the distance the centre of gravity of the displaced mass moves from its initial position.

For a slide with a constant mass, M, initial velocity v_o, and coefficient of friction, μ, on a surface inclined at an angle, Θ, Newton's second law leads to:

$$\frac{d(v^2)}{ds} = 2g(\sin\Theta - \mu\cos\Theta) \quad (3)$$

and the run out distance,

$$L = - \frac{v_o^2}{2g(\sin\Theta - \mu\cos\Theta)} \quad (4)$$

$$v^2 = - 2g(\sin\Theta - \mu\cos\Theta) \; (L - s) \quad (5)$$

which can be transformed into:

$$v = v_o \sqrt{1 - \frac{s}{L}} \quad (6)$$

Consider a slide of loose material entering its accumulation zone with a velocity v_o. Friction slows particles at the slide toe to a velocity $(v_o - \Delta v)$ after a short time, while the material behind is still entering the run out zone at a velocity, v_o. So the material at the toe and the back interacts. The leading particles are propelled forward, while the material behind is slowed down and some of it is deposited. This process continues throughout the accumulation zone. The properties of the material and the geometry of the accumulation zone determine the shape of the resulting deposit. Here some simple shapes of the final deposit are assumed and the corresponding run out is calculated.

If a slide has a linearly changing mass (Figure 1)

$$M(s) = M \frac{s_f - s}{s_f} \qquad (7)$$

and therefore a constant depositional rate, s_f is the distance at which both velocity and mass become zero; note that one implies the other. Equation (2) may now be written as:

$$\frac{d(v^2)}{ds} - \frac{2v^2}{s_f - s} = 2g(\sin\Theta - \mu\cos\Theta) \qquad (8)$$

with solution,

$$v^2 = \left[v_0^2 + \frac{2}{3} g(\sin\Theta - \mu\cos\Theta)s_f\right] \frac{s_f^2}{(s - s_f)^2} \qquad (9)$$

$$+ \frac{2}{3} g(\sin\Theta - \mu\cos\Theta) (s - s_f)$$

Multiplying both sides of equation (9) by $(s - s_f)^2$, it then becomes

$$v^2(s - s_f)^2 = \left[v_0^2 + \frac{2}{3} g(\sin\Theta - \mu\cos\Theta)s_f\right]s_f^2 +$$

$$\frac{2}{3} g(\sin\Theta - \mu\cos\Theta)\ (s - s_f)^3 \qquad (10)$$

with the boundary condition, $v = 0$ at $s = s_f$:

$$s_f = -\frac{3}{2} \frac{v_o^2}{g(\sin\Theta - \mu\cos\Theta)} \qquad (11)$$

As a rectangular depositional profile has its centre of gravity at its midpoint:

$$L = \frac{s_f}{2} = -\frac{3}{2} \frac{v_0^2}{2g(\sin\Theta - \mu\cos\Theta)} \qquad (12)$$

and,

$$v^2 = -\frac{2}{3} g(\sin\Theta - \mu\cos\Theta)\ (s_f - s) \qquad (13)$$

which can be transformed into:

$$v = v_0 \sqrt{1 - \frac{s}{s_f}} \qquad (14)$$

Equations (4) and (12) show that the run out of the sliding block and the slide with changing mass, can be expressed by similar formulae. However the run out of the slide with changing mass is one and one half times larger than for the sliding block.

It is also possible to calculate the runout for a slide with exponentially decreasing mass:

$$M(s) = Me^{-s/L_0} \tag{15}$$

and initial deposition rate, M/L_0. Since this profile extends to infinity it needs to be truncated at some finite point (Figure 2), say it is cut off at $s = 5L_0$. For the exponential profile the centre of gravity is located at $L = L_0$, for the profile truncated at $s = 5L_0$, the centre of gravity is at $L = 0.966L_0$. Inserting (15) and solving equation (2) for v^2 leads to:

$$v^2 = C\, e^{2s/L_0} - gL_0(\sin\Theta - \mu\cos\Theta) \tag{16}$$

and introducing the initial condition $v = v_o$ at $s = 0$,

$$v^2 = \left[v_o^2 + gL_0(\sin\Theta - \mu\cos\Theta)\right]e^{2s/L_0} - gL_0(\sin\Theta - \mu\cos\Theta) \tag{17}$$

and with $v = 0$ at $s = 5L_0$:

$$0 = \left[v_o^2 + gL_0(\sin\Theta - \mu\cos\Theta)\right]e^{10} - gL_0(\sin\Theta - \mu\cos\Theta) \tag{18}$$

Thus,

$$L_0 = \frac{e^{10}}{1 - e^{10}} \; \frac{v_o^2}{g(\sin\Theta - \mu\cos\Theta)} \simeq - \frac{v_o^2}{g(\sin\Theta - \mu\cos\Theta)} \tag{19}$$

and since $L = 0.966L_0$,

$$L = -1.93\, \frac{v_o^2}{2g(\sin\Theta - \mu\cos\Theta)} \tag{20}$$

It can be shown that, as the exponential profile is truncated at higher values of the travelled distance (e.g. $s = 10L_0$), the factor 1.93 in equation (20) trends to 2.

Comparison of equations (4) and (12) shows that if mass is deposited during the movement, the deposition has a marked influence on the run out. Comparison of equations (12) and (20) (Figures 2 and 3) shows that the shape of the depositional profile also has an influence on the runout.

An important conclusion from the calculations is that, if a slide with changing mass is modelled as a sliding block with constant mass, the run out is grossly underestimated. So when the coefficient of friction in a slide with changing mass (such as a rock-fall avalanche) is calculated with the formula for a sliding block with constant mass, friction will be underestimated. For instance, when the displacing mass is linearly decreasing and has a coefficient of friction $\mu = \tan 30° = 0.577$, on a horizontal surface, the formula for the sliding block with constant mass yields a coefficient of friction $\mu = 0.385 = \tan 21°$. When the slide's mass changes exponentially, the difference between the calculated and the actual friction angle is even more pronounced, the coefficient of friction estimated assuming a constant mass is $\mu = 0.289 = \tan 16°$.

Hsu (1975) showed that the interpretation of the fahrboschung (or the inclination of the line from the crown to the slide tip) as the friction angle of the sliding material is not correct. He stated that the angle of friction of the sliding material is given by the inclination of the line connecting the centres of gravity of the pre- and postslide masses. It has been shown here that the run out of a slide depends upon the shape of the deposit and the rate of deposition. Hsu 's assertion is correct only when the mass moves as a rigid block with constant mass. So it is not surprising to find that the inclination of the line connecting the centres of gravity is, in general, considerably smaller than plausible angles of friction of the slide mass (Hsu, 1975).

Examples

To illustrate the model, we apply it to the two typical rock-fall avalanches identified by Varnes (1978, p. 21), the events at Elm, Switzerland in 1881 and Frank, Canada in 1903.

Elm

Excellent evidence of the kinematics of the movement at Elm provides comparatively tight constraints on the interpretation of its dynamics. The entire event was observed by numerous inhabitants of the village of Elm and they were interviewed by Heim immediately following the disaster (Heim, 1882, 1932; Hsu, 1978). A translation of Heim (1932) by Nigel Skermer has now been published by Bitech Press, Vancouver, B.C.

The velocity of the avalanche can be estimated from the trajectory it followed through the air, as it was deflected from a horizontal rockshelf. An eyewitness described the jump (Heim,1882, p. 89):

"Then I saw the mass jump away from the ledge. The lower part of the block was squeezed by the pressure of the rapidly falling upper part, became disintegrated and burst forth into the air.

... The debris was shot with unbelievable speed northward towards the hamlet of Untertal and over and above the creek, for I could see the alder forest by the creek under the stream of shooting debris."

(Translation from Hsu, 1975, p. 131)

The geometry of the jump may be deduced from Figs. 19 (reproduced herein as

Figure 4) and 20 in Heim (1932) which are in agreement with the present topographic map.

Assuming that the final horizontal velocity equals the initial horizontal velocity, and that the avalanche shot away horizontally and dropped 250 m while traversing a horizontal distance of 475 m, the velocity at the toe of the slope was approximately 66 m/s (based on the trajectory of a projectile).

The profile of the deposited mass in Elm is presented in Fig. 4. The debris was deposited on a incline of 4°, the thickness varied from 50 m at the proximal end to 5 m at the distal end, and the width of the debris was constant over the whole length. The length of the accumulation zone from the point where the avalanche touched down to its tip was 1360 m.

Assuming a sliding system with linearly decreasing mass and a coefficient of friction, $\mu = \tan 30°$, the initial velocity required to reach a length of 1360 m is given by equation (12), $v_o = 67$ m/s. Similarly, fitting an exponential profile truncated at 2.3 L_0, such that the thickness at the beginning and the end have a proportion of 10 to 1, the initial velocity required, calculated with the equivalent of equation (20), is 54 m/s. Equation (4) for a sliding block with constant mass yields 82 m/s.

Assuming that the required velocity was available after touchdown, the duration of the event from the moment the mass became airborne until standstill can be calculated for the models. The duration for the linearly decreasing mass model is 48 seconds, and for the exponentialy decreasing mass model is 41 seconds. Adding a few seconds for the initial stages of the slide, these values compare well with a duration of between 45 and 50 seconds quoted by Heim (1932, p. 93).

An alternative approach is to assume the velocity at the toe of the slope was 66 m/s and to calculate the angles of friction. Using equation (12), the linearly-decreasing mass model yields $\mu = \tan 29.7°$; using $s_f = 2.3L_0$ in equation (17), the exponentially-decreasing mass model yields $\mu = \tan 39.7°$.

Frank

The Frank Slide is a more typical rock-fall avalanche; its velocity is poorly constrained by the evidence of eyewitnesses. "It is difficult to arrive at any definite conclusion in regard to the time occupied by the slide as the estimates of eyewitnesses range all the way from 20 seconds to 2 minutes ..... The slide occurred about 4:10 a.m. ... when most of the inhabitants of the valley were asleep and before full daylight" (McConnell and Brock 1904, p. 8, p. 6). However, the topography of the debris is exceptionally well described. The Frank Slide debris was used as a training ground for the newly-formed Topographical Division of Geological Survey of Canada (Zaslow, 1975, pp. 273-274) which contributed a 1:9600 topographic map made by plane-tabling and supplemented by 1:6000 sections to Daly et al. (1912). Cruden (1980) added information on the thickness of the debris to one of the Daly sections to produce a detailed profile of the Slide (Cruden, 1980, Figure 2).

Figure 5 is Cruden's profile modified west of the Crowsnest River to include Hungr's (1981) mapping. It suggests that the accumulation of the Frank Slide can be approximated by a rectangular profile which slopes back at 2 degrees from the tip of the slide debris to the Crowsnest River, 1700 metres away.

Cruden and Krahn (1978) measured friction angles ranging from 14 to 52 degrees on natural discontinuities from the limestone debris. Cruden and Hungr (1986, p. 425) documented vertical sorting in the debris, the base of the debris is "crushed limestone, mainly of sand and gravel size and contains rounded pebbles from tile and alluvial deposits on the surface of separation". We have taken 30 degrees as a reasonable estimate of the friction angle of the (loose) debris and substitute this, with the topographic information, in equation (14). The resulting estimate of the velocity of the rockfall at the Crowsnest River and the start of the accumulation of the debris is 82 metres per second. This velocity is produced by a fall of less than 350 metres.

Figure 5 gives the vertical fall from the toe of the surface of rupture of the Slide to the lake as about 400 m. So, for the debris to attain sufficient velocity, a large portion of its travel down the east flank of Turtle Mountain should have been through the air. There have been observations which suggest this was the case.

First, McConnell and Brock (1904, p. 8) commented "A shelf of rock in the basin of the slide seems to have hurled most of the material over the coal mine at the base of the mountain into the river bed, or beyond. The movement of the broken rock mass cannot be characterized as a slide in the ordinary sense of the word. The blocks must have travelled to their destination largely by a succession of great leaps or ricochets, probably accompanied by a certain amount of rolling and sliding. The character of the movement is clearly shown in the gradually lessening bounds ending in a short roll of a number of fragments, which were thrown forward beyond the main mass. The progress of these can be distinctly traced by the indentations made in the surface by the bounding rocks. While the movements of the individual fragments consisted of a succession of bounds from the surface and caroms from flying rocks, the movement of the mass, taken as a whole suggests that of a viscous fluid".

Again (op.cit. p. 10) "Just west of the lower lake at the south end of the slide, a boulder clay terrace is partially buried under and partially cut away by the slide. The cutting appears to have been done by huge flying boulders, which shot through it".

Mapping by Hungr (1981) confirmed that debris was thin or absent immediately below the toe of the rupture surface of the slide, Cruden and Hungr (1986, Figure 1) showed the "bare patch" presumably passed over by the falling rock.

Finally, bedrock mapping (Cruden and Krahn, 1978, Figures 5-8) demonstrated that the steep rupture surface of the Slide flattened at its toe. Displaced material may then have been projected horizontally from the toe of the rupture surface after acceleration down the steep upper portion of the slide rupture surface.

Our model then is compatible with many of the observed feature of the Frank Slide. The velocities indicate that transport from the toe of the rupture surface to the tip of the debris would take about 50 seconds. The perceived duration of the event may be longer if precursory phenomena are included.

Discussion

The concept of momentum transfer within a rock avalanche is not new; Heim (1882) visualised the movement of a rockfall avalanche as:

"The uppermost block at the very rear of the stream would attempt to get ahead. It hurried but struck the block slightly ahead, which was in the way. The kinetic energy, of which the first had more than the second, was thus transmitted through impact. In this fashion, the uppermost block could not overtake the lower block and had to stay behind. This process was repeated a thousandfold, resulting eventually in the preservation of the sequential order in the debris stream. This does not mean that the energy of the falling blocks from originally higher positions was lost; rather the energy was transmitted through impact. The whole body of a sturzstrom was full of kinetic energy to which each single stone contributed its part. No stone was free to work in any other way."

(Translation from Hsu, 1975, p. 133)

Newton's second law embraces the mathematical form of the phenomenon here described by Heim.

Attention has been drawn to the importance of momentum transfer in the dynamics of mass wasting processes. As an example of the concept, its implications for the mobility of rock avalanches have been described.

The frictional model which includes momentum transfer due to mass change quite accurately predicts the accumulation zone of rock-fall avalanches and the problem of the extreme mobility of some dry rock-fall debris flows appears to be solved.

Other problems remain. Factors influencing the form of the debris profile have yet to be quantified. Unfortunately, the velocity, v_o, can be estimated for only a few rock avalanches. The details of how the displaced rock mass evolves into a high velocity debris flow are nebulous, as Johnson (1978) noted, and may be prerequisites to the complete solution of the dynamics of rock-fall avalanches.

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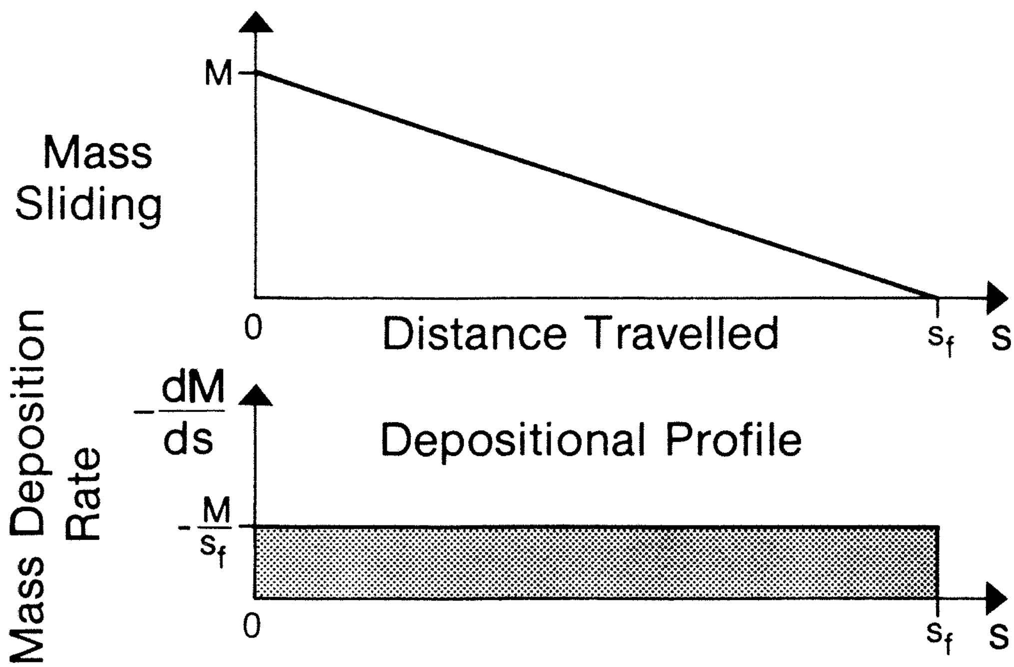

Figure 1. A sliding system with linearly decreasing mass

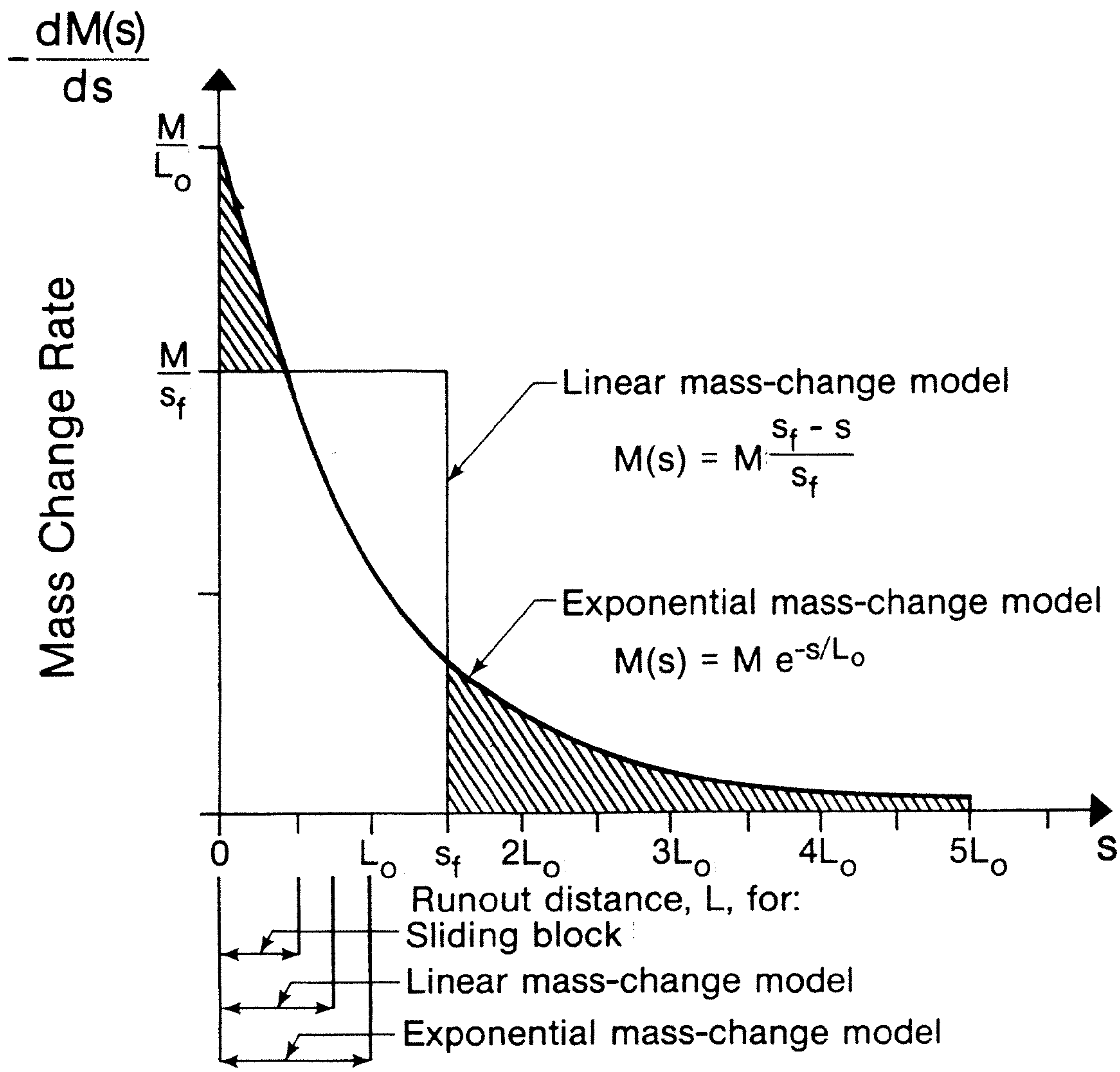

Figure 2. Comparison of depositional profiles and runout

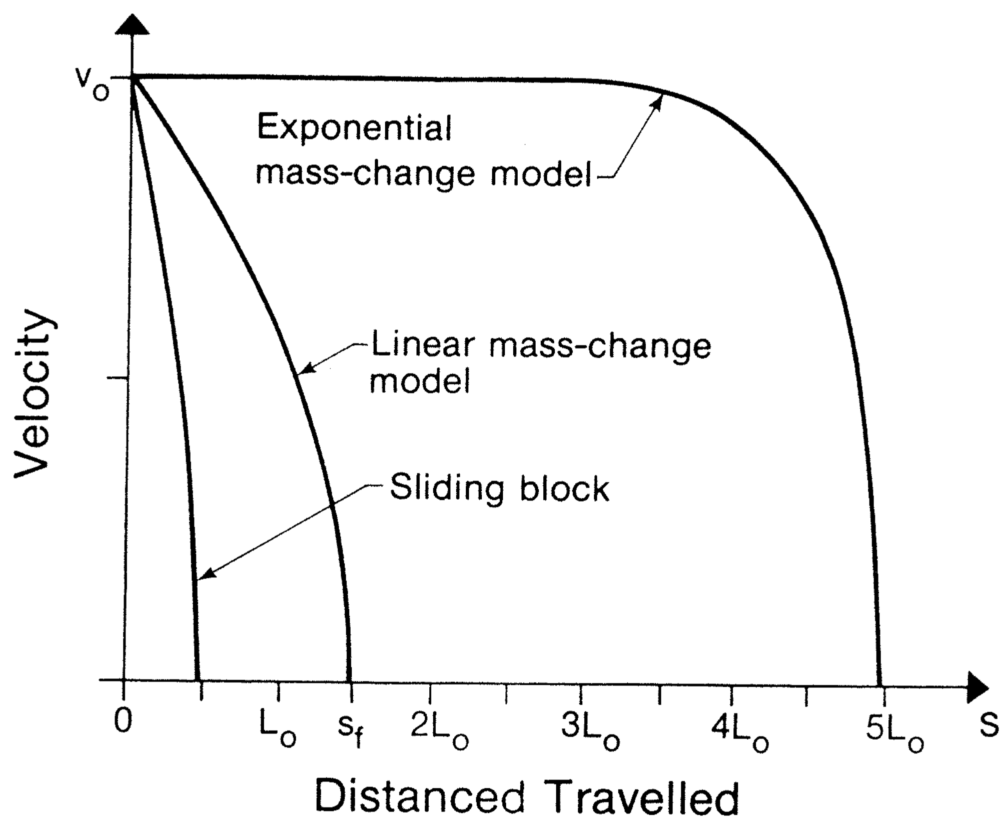

Figure 3. Comparison of velocity profiles

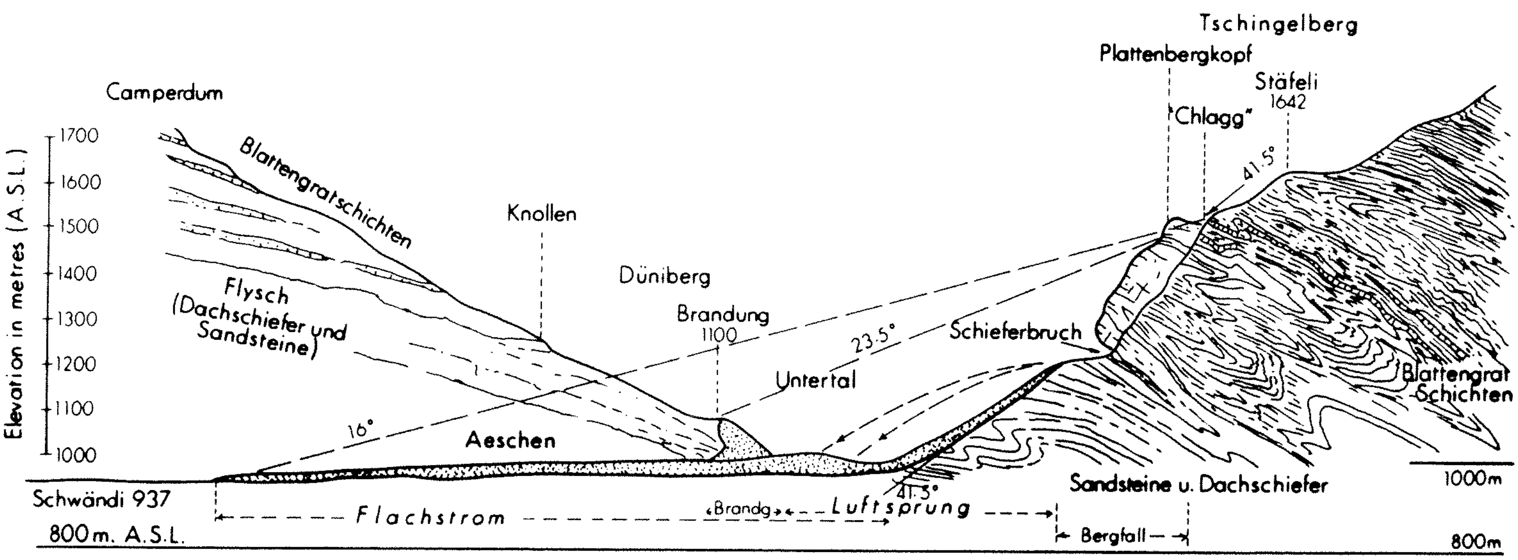

Figure 4. The Elm rockfall-debris avalanche (after Heim, 1932, Figure 19). Heim distinguished 4 stages in the event, "Bergfall", mountain-fall, "Luftsprung", literally lift-spring or jumps, "Brandung", literally the surf zone or run-up and "Flachstrom", literally flatstream or flow.

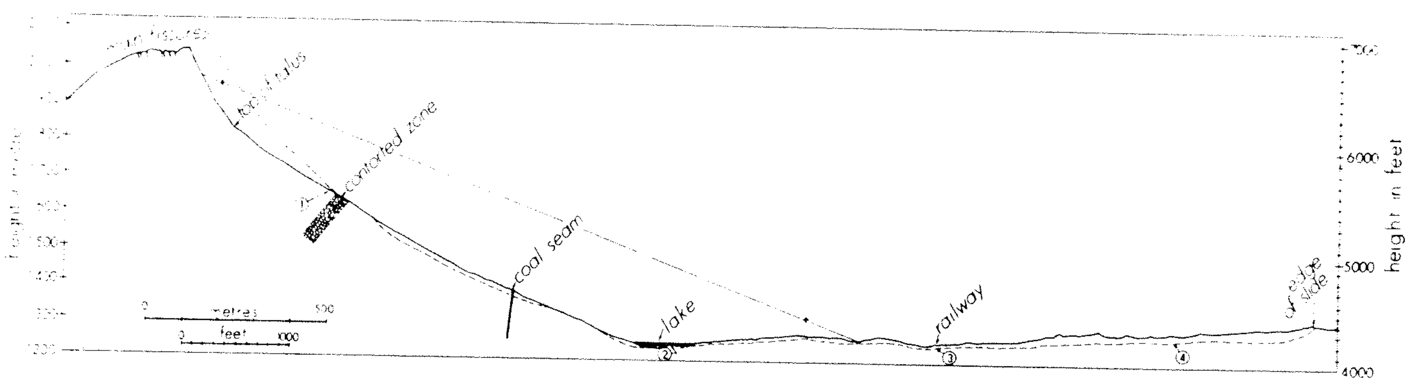

Figure 5. A depositional profile of the Frank Slide along the topographic profile given by Daly et al. (1912, Figure 8). Solid line indicates the present land surface, dashed line shows the pre-Slide topography.