Writing Research Articles in Discrete Mathematics:

A Rhetorical, Multimodal Genre Analysis and Pedagogy

by

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A thesis submitted in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

in

Applied Linguistics

Department of Modern Language and Cultural Studies

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Abstract

This dissertation reports my rhetorical, multimodal genre analysis of research articles (RAs) in discrete mathematics and its pedagogical applications. The increasing demand on graduate students to publish their research in English and the need of these students to write in the key genres of their disciplines motivated me to develop this research. I chose mathematics, as the target discipline, because of the existing gap in genre research concerning the discipline of mathematics. Two global research questions guided the study: How do mathematicians write their RAs? Why do they write the way they do?

In order to answer my research questions precisely and to gain a deeper understanding of rhetorical actions of mathematicians, I focused on a manageable corpus of 30 RAs, including RAs with pure and applied orientations in discrete mathematics. I designed three heuristics for my research: 1) Examining the corpus of RAs for their macro-organization, the move structure of Introduction sections, the rhetorical strategies used for identifying research niches, and visual rhetoric in RAs in the corpus. 2) Collecting the discourse community perspectives on the nature of mathematical research and common rhetorical strategies for knowledge creation practiced by the discourse community. 3) Surveying existing literature in philosophy of mathematics, shared values for research and epistemology in the discipline. I then triangulated the findings of the three heuristics to obtain the following results and reach a deep understanding of the links between the discipline and its acceptable rhetorical practices that help to create new knowledge and advance the discipline.

First, my examination of the macro-organizational structure of RAs in the corpus show that RAs in discrete mathematics do not use the traditional Introduction-Methods-Results-Discussion (IMRD) structure for an Introduction-Results model due to the well-established logic-driven induction/deduction research procedure in mathematics which makes

unnecessary having extensive description of the research method and discussions of results as distinct sections.

Second, the findings of pattern-seeking analysis of the rhetorical structure of introductory sections of the corpus articles show that the move structure of RAs in mathematics departed somewhat from patterns identified in other disciplines. A notable departure is that ‘establishing presumptions’ about abstract mathematical objects is an essential constituent of constructing arguments about knowledge claims in mathematics. I proposed that these ontology-driven variations arise out of the hypothetical nature of the mathematical concepts, and the epistemological grounds of mathematics as a logic-driven, argumentation-mediated discipline.

Third, by examining the conventions for ‘Establishing a niche’ in the Introduction sections of the corpus articles, I identified five steps that discrete mathematicians choose from among or combine to establish a niche for their research. Accordingly, I proposed slight modifications to the *Create A Research Space* (CARS) model of RA introductions to accommodate the rhetorical strategies of writers in discrete mathematics and to assist newcomers in understanding the crucial features of RA introductions in this field.

Fourth, through multimodal analysis of images and their links with surrounding texts, I identified ways that the nonverbal contributes to the discipline's intellectual project. I found that visuals perform three functions in the corpus: ontological, argumentative, and epistemological. I also found that visuals initiate three multimodal rhetorical moves in discrete mathematics RAs, suggesting that visual moves go beyond textual considerations by disrupting the RA’s chronological structure and that understanding the crucial associations between the visual representations, disciplinary knowledge, and the rhetorical structure of RAs in disciplines is central to understanding how knowledge is created in the discipline.

Fifth, I designed a writing-in-mathematics course for graduate students in the discipline based on my findings. Using a combined reading-writing genre-based pedagogy, I planned tasks that guide students to examine the multidimensional nature of disciplinary genres and develop an awareness of the interplay between genres and the shared values of the relevant discourse community. A significant feature of my course is that it is applicable to any writing-in-disciplines course with some adjustments in the texts used for genre analysis purposes.

My study thus not only contributes to existing scholarship in multimodal genre analysis in both ESP and Rhetorical Genre Studies in significant ways, but also has developed practical applications to assist graduate students learn how to write in their discipline of study.

Preface

This dissertation is an original work by Shahin Moghaddasi Sarabi. The research project, of which this dissertation is a part, received research ethics approval from the University of Alberta Research Ethics Board, Project Name "Genre Analysis of Research Articles in Discrete Mathematics", No. Pro00069251, July 20th, 2017.

Some of the research conducted for this dissertation started when I was a visiting graduate student at the University of Alberta, supervised fully by Professor Heather Graves from University of Alberta and partially by Professor Azirah Hashim from the University of Malaya. As the primary researcher, I conducted sampling of the research articles constituting the study corpus, designing the research methodology, analyzing and interpreting the data. I am also the primary author of the original draft of the research articles published from this project.

Chapter 2 of this dissertation has been published as Graves, H., Moghaddasi, S., & Hashim, A. (2013). Mathematics is the method: Exploring the macro-organizational structure of research articles in mathematics. *Discourse Studies*, 15, 421-438. https://doi.org/10.1177/1461445613482430. I was responsible for the data collection and analysis as well as the manuscript composition. Professor Heather Graves was the supervisory author and contributed to interpreting the results and manuscript edits. Professor Azirah Hashim contributed by providing some feedback on data collection and analysis.

Chapter 3 of this dissertation has been published as Graves, H., Moghaddasi, S., & Hashim, A. (2014). "Let G¼(V,E) be a graph": Turning the abstract into the tangible in introductions in mathematics research articles. English for Specific Purposes, 36(0), 1-11. https://doi.org/10.1016/j.esp.2014.03.004. I was responsible for the data collection and analysis as well as the manuscript composition. Professor Heather Graves was the supervisory author and contributed to interpreting the results and manuscript edits. Professor Azirah Hashim contributed by providing some feedback on data collection and analysis.

Chapter 4 of this dissertation has been published as Moghaddasi, S., & Graves, H. A. B. (2017). “Since Hadwiger’s conjecture . . . is still open”: Establishing a niche for research in discrete mathematics research article introductions. English for Specific Purposes, 45, 69-85. https://doi.org/10.1016/j.esp.2016.09.003. I was responsible for the data collection and analysis as well as the manuscript composition. Professor Heather Graves was the supervisory author and contributed to interpreting the results and manuscript edits.

Chapter 5 of this dissertation has been published as Moghaddasi, S., Graves, H. A. B, Graves, R., and Gutierrez, X. (2019). “See Figure 1”: Visual moves in discrete mathematics research articles. English for Specific Purposes, 56, 50-67. https://doi.org/10.1016/j.esp.2019.08.001. I was responsible for the data collection and analysis as well as the manuscript composition. Professor Heather Graves was both the supervisory author and collaborated in interpreting the results and manuscript composition and editing. Professor Roger Graves was the supervisory author and provided feedback on the manuscript and the revisions. Professor Gutierrez was the supervisory author.

Chapter 6 of this dissertation is an original work by me. No part of the chapter has been previously published. I have benefited from supervision and feedback from Professor Heather Graves, Professor Roger Graves, and Professor Xavier Gutierrez in improving the scope and quality of the writing course and pedagogical tasks in chapter 6.

To my son Amirhossein and my daughter Mahshid

who suffered but remained patient while I was busy with my graduate study!

Acknowledgements

As I have developed this project, I have benefited greatly from the wisdom and support of many people.

My first and foremost acknowledgement is for Professor Heather Graves, my supervisor, for her unconditional support through my progress in graduate school. She continually held high standards for my research. She has been generous with her time as a meticulous reader and reviewer of drafts of this dissertation. Her experience taught me to become and remain strong in the face of hardships. To be Dr. Heather Graves' student is the greatest chance in my academic and social life.

Similarly, a very special thank you to Professor Roger Graves, my graduate committee member. I have significantly benefited from the depth of his feedback to improve my research skills and outcomes. He adopted me as a member of his research group and an assistant in his writing classes and by doing so gave me the unique opportunity to receive both intellectual and mentoring feedback from him. I learnt from him not to forget about the funny side of life at times when I was challenged by its serious side and to smile in the face of adversities. I am also highly grateful to Professor Graves for his extraordinary human qualities and comprehensive support that made my transition to the University of Alberta possible.

I am also deeply thankful to Professor Xavier Gutierrez. I not only have had the blessing of learning from him in his graduate classes, but also benefited from his precise comments on my dissertation. I can't thank him enough for being supportive of me at the hard moments that I needed his support. I am also grateful to him for holding high standards on this dissertation by requiring me to put my theoretical findings into pedagogical applications. Although I felt extremely out of my comfort zone while developing the pedagogical chapter, it turned into a valuable educational development experience for me as a writing instructor.

This research could not happen without the hospitality, generosity, and wisdom of a group of distinguished research mathematicians who graciously participated in my graduate research as disciplinary informants. With respect to my research ethics and their privacy, I will not disclose their names here, but I will never forget the great favor they did to me by generously investing their wisdom and expertise in this dissertation. I am incredibly thankful to my disciplinary informants for helping me approach and walk in the immense realm of mathematics and to turn the light on for me in this otherwise dark mansion.

Lastly, I acknowledge the incredible support of my family. My two lovely children, Amir and Mahshid suffered a lot as teens for the years that I had to study in my office till late at night but were so patient and barely complained; they are adorable kids! I cannot find words to describe my love and appreciation to the two angels of my life for their understanding of my long research journey. My husband, Karim, has been exceptionally supportive of my graduate work. I also wholeheartedly thank my parents who suffered a lot for not seeing me for my years as a graduate student in Canada but never ceased to pray for me.

Having finished this journey, I am so excited to step forward and start a new chapter in my academic life as a writing studies scholar and instructor with the support of these amazing people and all the others who are too numerous to mention.

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Abbreviations

AR - Announcing Results

CFP - Call for paper

CPP - Conference Presentation Poster

CSH - Content Section Heading

CARS - Creating a Research Space

DV - Defining Visually

DAM - Discrete Applied Mathematics

DM - Discrete Mathematics

D - Discussion

EO - Embodying the Operation

EP - Establishing Presumptions

EN - Establishing a niche

EAP - English for Academic Purposes

ESP - English for Specific Purposes

ET - Establishing Territory

FYC - First-Year Composition Courses

GSH - Generic Section Heading

GSP - Generic Structure Potential

G&C - Graphs and Combinatorics

I**MRD** - Introduction-Method-Results-Discussion

ISI - Institute of Scientific Information

JCO - Journal of Combinatorial Optimization

M or *m* - Move

NCTM - National Council of Teachers of Mathematics

NVM - Non-Verbal Material

PISF - Possible in Some Fields

PPW - Presenting the Present Work

P - Proof

RA(s) - Research Article(s)

RGS - Rhetorical Genre Studies

ROO - Representing the Outcome of an Operation

SIAM - SIAM Journal of Discrete Mathematics

S or **s** - Step

VM - Visual Material

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Chapter 1: Introduction

1.1. Overview of the Study

This dissertation reports on a study which was provoked by questions concerning how and why mathematicians write research articles. In the study, I examined research articles (RAs) in discrete mathematics for their rhetorical outline, the compatibility of their Introduction with the Swales' *Create A Research Space* (CARS) model and the role visuals play in the rhetoric of the RA genre in the discipline. The study is grounded in linguistic and rhetorical traditions in genre research. In particular, the study opts for theories and analytical approaches from English for Specific Purposes (ESP) and Rhetorical Genre Studies (RGS). These traditions have been around for over three decades used by applied linguists and rhetoricians to examine various academic and professional genres for their rhetorical structure as well as lexico-grammatical features. However, two research gaps exist: 1-Academic genres in mathematics have received little attention within genre scholarship. 2- Pedagogy is still an underdeveloped aspect of genre studies. Considering the growing number of graduate students and their urgent need to write and publish research articles (RAs) in mathematics, an analytical (as opposed to descriptive) genre study with clearly articulated pedagogical suggestions in mathematics is warranted. This study attempts to fill these gaps in genre research.

The study began with two main questions: How do mathematicians write their research articles (RAs)? Why do they write the way they do? To answer these questions, I examined the rhetorical structure of mathematical RAs, the interplay between text and visuals in RAs, and the ideological grounds pertaining to the disciplinary epistemology, all of which contribute to writing for establishing new knowledge in mathematics. I thus investigated both the verbal (macro as well as micro levels) and visual modes of argumentation and knowledge creation in the discipline. To reach a deep understanding of the rhetorical practices in the RA genre in mathematics, I used a triangulated research methodology (Candlin and Hyland, 1999) that combines macro- and micro- rhetorical structure analysis of research articles in discrete mathematics with interview data collected through semi-structured interviews with both pure and applied mathematicians. Triangulation was then made between textual and interview data and a third data set obtained through readings in mathematical philosophy and epistemology as well as readings in visual rhetoric.

The findings of this study suggest strong bonds between the rhetorical structure and modalities of RAs in discrete mathematics and the disciplinary context in which they are created, used, and interpreted. Then I used these findings as the theoretical basis to develop a series of writing tasks for a genre-based writing-in-the- disciplines course for mathematics students.

1.2. Research Background

Genre research has flourished over the last four decades connecting researchers and instructors across borders of geographies, disciplines, and target learners' language background and grade level. Geographically, genre research has connected scholars and teachers from North and South America, Australia, and Europe. Genre scholars have also come from diverse disciplinary orientations including various branches of applied linguistics, verbal and visual rhetoric, composition, communication, sociology, education, and literary studies (Bawarshi & Reif, 2010). It has also connected educators with concerns for students from diverse language backgrounds, different grade levels of genre use and teaching, writing in particular, from primary to secondary, post-secondary, public, and professional contexts. Though connected, these diverse orientations have impacted the way genres are defined, examined, and taught by their affiliate researchers and practitioners leading to distinctive approaches in genre scholarship. An overview of the more notable genre traditions including their theoretical premises, analytical approaches, and pedagogical recommendations now introduces my research, which uses a genre approach to the study of genre and disciplinarity.

Following Hyon's (1996) "valuable map-making exercise" (Swales, 2009, p. 3), it is customary to characterize three notable approaches in genre analysis research. These are the Australian work in the tradition of Systemic Functional Linguistics, known as the Sydney School; the UK-originated teaching of English for Specific Purposes, known as ESP approach (recently academic genre studies and pedagogies have differentiated themselves as English for Academic Purposes but I use ESP here to include academic genres studies); and the North American New Rhetoric studies developed in composition and rhetoric contexts, recently known as Rhetoric Genre Studies (RGS).

Other groupings have been suggested for genre scholarship. Flowerdew (2002), for example, dichotomized genre studies into linguistic and non-linguistic camps, positioning ESP approach and Australian school within the linguistic camp, given their use of functional grammar and discourse theories and concentration on the lexico-grammatical and structural realization of communicative purposes embodied in a genre. The non-linguist RGS, in Flowerdew's dichotomy, is more focused on situational context, that is, the purposes and functions of genres and the attitudes, beliefs, values and behaviors of the members of the discourse community in which the genres are situated. A similar dichotomy is suggested and discussed in Bawarshi & Reiff (2010) who outline their overview of genre research in terms of linguistic traditions (systemic functional and corpus linguistics vs. ESP approach) as compared with rhetorical and sociological traditions. These demarcations and some other genre study traditions, however, have not been as influential in genre scholarship as Hyon's (1996) categorization, which is still in fashion. In the following sections, I summarize the three traditions in genre analysis in terms of their initial concerns, epistemological grounds, theoretical conception of genre, trajectories of research, target groups, and writing pedagogy. The summary helps me to provide an overview of the theoretical premises and methodological tools which I have adopted from linguistic and rhetorical genre studies in my research.

Considering the large body of genre research, review of the literature specific to different parts of this study will be provided in the chapters dealing with each part.

1.2.1. ESP Approach

ESP genre approach emerged in the UK out of pedagogical and political concerns with process approaches to teaching writing. It critiqued process-based teaching for over-emphasizing personal meaning, ignoring situational variations in writing, not addressing the requirements of particular writing tasks, and not providing ELL students with a correct understanding of what is required of them in academic settings. In addition to its pedagogical concerns, ESP emerged as a response to shortcomings of applied linguistics and discourse approaches, which did not consider the conventionalized aspects of communicative events. There was thus a need for a model capable of providing a thicker description based on input from a variety of sources, including theoretical and applied linguistics, sociology, ethnomethodology, ethnography, psychology, cognitive studies, and communication research. As Bhatia (1993) argues, a model with this orientation can more effectively answer why conventionalized forms of language produced and used by the specialist communities are written the way they are. ESP genre approach has filled this gap in applied linguistics and language teaching.

After about four decades, genre continues to be a core area of research in ESP. Hyon (2018) points out that the main reason for genre popularity within ESP research lies in its core mission to teach students how to use English effectively in their target contexts. Since these contexts are associated with genres, researching students' target genres and their contexts helps to respond to student needs and teach them effective use of English. Besides an interest in genre contexts, ESP approach is also interested in genre purposes, genre moves, and the lexico-grammatical features of genre following Swales' (1990) influential work in genre analysis of research articles.

Swales established the foundation of his genre analysis approach on three interrelated concepts of genre, discourse community, and task (Swales, 1990). A genre, according to Swales (1990), is

> "a class of communicative events, the members of which share some set of communicative purposes. These purposes are recognized by the expert members of the parent discourse community and thereby constitute the rationale for the genre. This rationale shapes the schematic structure of the discourse and influences and constrains choice of content and style. (p. 58)

Genres thus involve typical events with shared purposes, participants, and rationales. Hyon (2018) clarifies two points with regard to Swales' definition of genre. First, a genre is not itself a text but rather an abstract class of texts. Second, shared communicative purposes or rationales are identifying criteria for categorizing texts as the same genre. These clarifications are important in that they help a deeper understanding of genres; that though genres are represented by their texts, they are more than their texts; they embody agreed-upon communicative purposes and rationales that guide the choice of certain rhetorical organizations and forms over others.

A second pillar of ESP genre theory is Swales' concept of discourse community, initially defined as "sociorhetorical networks that form in order to work towards sets of common goals" (1990, p. 9) or shared communicative purposes. Discourse community members can then achieve communicative purposes through using and responding to genres. Swales (1990) offered a conceptualization of discourse community based on the following six defining characteristics: a broadly-agreed-upon set of common public goals, mechanisms of intercommunication among its members, using its participatory mechanisms primarily to provide information and feedback, utilizing and hence possessing one or more genres in the communicative furtherance of its aims, possessing some specific lexis, and a threshold level of members with a suitable degree of relevant content and discoursal expertise. The relationship between a discourse community and its genres is one of constructing and restructuring, as conceptualized in Swales' (1990) definition of genre. Later, Swales (2017) reconsidered his description of discourse community and added the features of having "a sense of silential relations in doing things without spelling them out and developed horizons of expectations" to his 1990 criteria for identifying a group of genre users as a discourse community.

A further reconsideration in Swales' theoretical conceptions of genre has been the issue of communicative purpose. Originally, Swales (1990) argued that communicative purpose was the key factor in deciding on a text as a member of a genre class. About a decade later, Swales revised his initial ideas based on evidence from the examination of complex contexts in which texts from a named genre (e.g., sales brochure) appear to have different communicative purposes. Accordingly, Askehave & Swales (2001) and Swales (2004) argue that communicative purpose is a slippery notion because the rationale for a genre may change, develop, or shrink over time and across cultures; hence, it cannot always be used as a criterion for categorizing texts as a genre class. As a paradigm modification, Askehave & Swales (2001) suggest that rather than focusing on communicative purpose as the basic criterion for identifying a genre and working out from there, researchers and practitioners need to start at another place by "…designing a more complex context-driven procedure for genre analysis…and repurposing a genre only after having identified the values, goals, material conditions, expectations, and repertoires of a discourse community which values that genre" (p. 208).

The emphasis on contextual analysis assigns a mediatory role to genres with respect to social contexts. As Swales (2009) argues, the work of genre is to mediate between social situations and the texts that respond strategically to the exigencies of these situations, that is, texts perform the genre. “As these performances increase, genres tend to drift through time and geographical space, partly inherently and partly as a result of intertextual acceptances and rejections” (Swales, 2009, p. 14). Genre research, in Swales’ view, should track textual regularities and irregularities and explain them in terms of the pertinent social circumstances and rhetorical demands they provoke.

The third concept in Swales’ (1990) genre theory is task, defined as

> One of a set of differentiated, sequenceable goal-directed activities drawing upon a range of cognitive and communicative procedures relatable to the acquisition of pre-genre and genre skills appropriate to a foreseen or emerging sociocultural situation (p. 74)

As this definition suggests, task is a robust pedagogical construct that works to engage learners in goal-directed social interactions in literacy events with genre acquisition as the learning outcome. Despite its centrality to Swales’ arguments, task, compared to other core notions, remained a less-celebrated concept (Flowerdew, 2015; Johns, 2015) in ESP scholarship, much like ESP genre pedagogy, which Swales left for others to develop (Flowerdew, 2015). Instead, the most celebrated aspect of Swales’ work in genre analysis literature and pedagogy is his *Create A Research Space* (CARS) model of research article introductions, which exemplifies levels of rhetorical (move analysis), lexico-grammatical and contextual analyses. Indeed, ESP genre approach is known for its meticulous move analysis. Swales (2004) defines a ‘move’ as a “discoursal or rhetorical unit that performs a coherent communicative function in a written or spoken discourse (pp. 228-229).” He has further remarked that a move is flexible in terms of its linguistic realization. Move analysis and *Create*

A Research Space model constitute major aspects of my study and will be discussed in detail in chapters 3,4, 5, and 6 in this dissertation.

ESP genre studies also accommodate corpus-based research which interlinks grammatical and stylistic features of genres to their communicative purposes and discourse community values (Hyland, 2000, 2005). Researchers in ESP genre studies have also taken a critical perspective in both scholarship and pedagogy to explore the rationale behind genre regularities and answer the question why members of a discourse community write the way they do (Bhatia, 1993, 2004; Chun, 2016; Hyon, 2018; Johns, 2008). Such critical queries move ESP approach even closer to rhetorical genre examination which has shown notable interest in *what writing does* and *how it does it* (Bazerman & Prior, 2004; Devitt, 2009).

ESP genre methodology has proven flexible over four decades. As Paltridge (2013) notes, it has no linear "hard and fast rule" on the sequence of analysis, allowing for an either “text-first” or “context-first” examination depending on the purpose of the research and what the researcher aims to find (p. 350). The approach is also open to both small data sizes, which are concerned with thick and deep analysis (Bhatia, 1993), and large-corpus data, which look for more generalizable patterns of the rhetorical organization and lexico-grammatical features of genre texts (for example, Hyland 2000, 2005). It has also shown growing attention to precision in deep theorizing by broadening the contextual analysis to include triangulation of diverse data sets from ethnographic to genre-expert perspectives (Bawarshi & Reiff, 2010) to the extent that in some studies “members of the discourse community (along with their physical situation) have become a primary focus of the analysis, equal to if not more important than the text” (Flowerdew & Peacock, 2001, p. 16; Swales, 1998).

The flexibility in methodology is a key advantage of ESP approach over the other genre traditions, and possibly a reason for its being adopted by numerous researchers across the globe. Indeed, the flexibility in the data size and sampling, its analytical sequence and the possibility of collecting different types of data (e.g., text, interview, ethnography, textography data) are its key features. They make ESP approach simple in theory, feasible in research methodology and logistics, rich in the data, and valid in its findings. Overall, a very worthwhile outcome of applying the inclusive ESP research method is an in-depth understanding of disciplinary and professional genres and their discourses. A further characteristic feature of ESP, as emphasized by Bhatia (1993) in its early years of practice, is its pattern-seeking rather than pattern-imposing procedure, which gives the approach a bold heuristic function as both a research tool and a genre-based pedagogical approach (Hyon, 2018).

ESP genre pedagogy has received support as well as criticism. Supporters admire ESP genre pedagogies for their explicit, systematic, needs-based, supportive, empowering, critical, and awareness-raising, context-based genre instruction (Hyland, 2007, 2017; Hyon, 2018; Tardy et al., 2018) with a special focus on teaching English Language Learning (ELL) students. These language learners face the urgent demand to use English in literacy and workplace contexts and are argued to benefit more from explicit genre instruction (Hyon, 2018) than the approaches that "focus on the process of composition, the content of texts, or the abstract prescriptions of disembodied grammars" (Hyland, 2007, p. 148). Genre-based instruction, however, has been critiqued by some composition and rhetoric scholars for being too prescriptive, form-focused, static, and imposing uniformity, hence dangerous (Freedman, 1994).

Despite these criticisms, ESP genre approach has thrived and is predicted to continue to flourish for years to come (Johns, 2013; Partridge, 2014). ESP's ubiquity might be partly due to the broad scope of its genre theory which makes it applicable to various academic and professional communication contexts; as Jordan (2004) argues, "the wider the scope of a theory, the better it is" (p. 97). ESP's success as a research/writing instruction paradigm is also owed to its simplicity in methodology and the empirical support it has received since its emergence. As Jordan (2004) argues, "there are no golden rules for theory assessment, no hard and fast rules, except the obvious requirement that a theory has empirical content" (p. 97). ESP genre approach, and Swales' *Create A Research Space* model, have generated extensive scholarship which has resulted in *Create A Research Space* turning into "a tribal model" (Berretta, 1993, in Ellis, 2010) of genre analysis. These characteristic features support ESP's fruitfulness, simplicity, and falsifiability as a theory in genre research which has been able to work successfully hand in hand with writing pedagogies as its findings are applied to teach students about genre structures and/or inspire writing instructors to conduct genre analysis (Hyon, 2018).

1.2.2. Rhetorical Genre Studies (RGS)

Rhetorical Genre Studies, formerly known as the New Rhetoric tradition, began in North America in the mid-1990s following a period of growth and then decline of product and process approaches in composition studies (Artemeva, 2009). Indeed, an original concern for the New Rhetoric came from first-year composition courses (FYC) that conceived of genres as "formal categories of literature and art" (Tardy et al., 2018, p. 1) and of writing ability as a monolithic skill that can be transferred to other contexts. A second related assumption of FYC was that the types of genres and texts used in composition classes were of secondary importance. They thus used literary texts which, as Russell (2002) remarked, obviously have little in common with genres that most students would encounter in their academic disciplines. These arguments gradually led to the recognition that composition courses have no link to any intellectual discipline and mislead students and writing instructors into conceiving of writing as a generic skill. There was thus a need felt for a more principled, theory-based approach to writing pedagogy (Bawarshi and Reiff, 2010).

With their genesis in ancient Greece, RGS build on postmodern and literary theories, speech act theory in philosophy, and composition research (Freedman & Medway, 1994). One source from which RGS often draws is Activity Theory (Russell, 1997) whose major tenet is that the cognitive cannot be separated from the social. Russell's work in importing Activity Theory into genre research enhances a cornerstone of rhetorical genre theory by convincingly arguing that all aspects of an activity system including the participants, situation, context, and text, are interdependent, hence reciprocally and mutually created.

At the heart of RGS is the conception of genre as social action (Miller, 1984). Miller's 1984 *Genre as social action* is regarded as a defining treatise in RGS (Bawarshi and Reiff, 2010; Devitt, 2004; Martin, 2013; Paltridge, 2007; Swales, 1990). Miller (1984) argues that "a rhetorical[ly] sound definition of genre must be centered not on the substance or the form of discourse but on the action it is used to accomplish" (p. 151), an action which must involve the rhetorical situation, *"a complex of persons, events, objects, and relations presenting an actual or potential exigence" (Bitzer, 1968, p. 6).* Accordingly, we must focus on the rhetorical situation, not on a "materialist" scene (p. 156). Miller further contends that as agents we typify and determine rhetorical situations by finding commonalities, similarities, or analogies among them. She thus proceeds to conceptualize genres as socially-driven, intersubjective, and rhetorical typifications. As she notes, because genre is closely defined by recurrence of social situations and actions, genres are fluid and there is no set list of genres, but rather "an open class with new members evolving, old ones decaying" (p. 153).

Using the biological metaphor of *evolution*, Miller associates three features of living organisms with genres, asserting that "genres evolve, develop and decay" in response to socio-cultural phenomena in their contexts (Miller, 1984, p. 153). These theoretical perspectives encourage adopting a more open strategy to genre classification based on rhetorical practice, that is, the social situations in which genres occur and the social "actions" that genres fulfill within those situations. Bazerman (1997) builds on Miller's argument by arguing that the evolving variety of human circumstances, the creative potential of language, and the cleverness of human action challenge us to know where we are and where we are going in interactions. According to Bazerman, genres, in their shared social attributions, help us and those we communicate with to be on the same page, or close enough for our practical purposes. Like most genre theorists from all three approaches, Bazerman identifies genre as combining many aspects of communication, social arrangements, and human meaning making. He links genres with sequences of thought, styles of self-presentation, author-audience stances and relations, specific contents and organizations, epistemologies and ontologies, speech acts and social accomplishments (Bazerman in his Preface to Bawarshi and Reiff, 2010). Genres, in Bazerman's view, thus shape regularized communicative practices that bind together organizations, institutions, and activity systems.

Genres, however, are not conceived of as overly deterministic in RGS. As Devitt (2004) argues, genres do not limit choices; they allow for choices. Devitt sees people as agents in exploiting genre repertoire and argues that genres do not make texts, people make them; though genres are social action, in the end, people make actions. Devitt also discusses how a genre "reflects, constructs, and reinforces the values, epistemology, and power relationships of the groups from which it is developed and for which it functions" (p. 64). These critical perspectives bring RGS close to discoursal approaches to genre, that is, the Sydney School and ESP, in emphasizing institutional cultures and the social contexts of genres (Bhatia, 1993, 2004; Hyland, 2000; Paltridge, 2013; Swales, 1990).

Despite their shared perspectives on critical genre awareness with ESP approach, RGS's sociological understanding of genre has made some scholars skeptical about the value of genre instruction. An argument has been made that genre knowledge can be acquired unconsciously as students use texts within their natural contexts (Freedman and Medway, 1994). A further argument is that genres as living organisms are closely tied with their context of use and that both genres and their context are dynamic and constantly changing. As such, presenting genre texts in the classroom is indeed locating the study of genres outside their living situations; it limits students' understanding of genres and leads to a distorted view of genres (Freedman, 1993). In view of these criticisms, some genre scholars recommend teaching genres within their contexts of use by employing field research or ethnographic methods.

Notwithstanding the early reservations against genre teaching, the emphasis on ethnography as a research and pedagogy approach has received continual support from RGS, and ESP genre scholars (Bawarshi & Reiff, 2010; Devitt, 2004, 2015; Johns, 2008; Johns et al. 2006; Miller, 1984, 1994; Tardy, 2009). Tardy (2009) argues that ethnography can situate genre analysis and give genre researchers or learners access to authentic sociocultural and institutional contexts of genre use, thus critically engaging them with cultural and textual practices. Similar reasoning for ethnography-based genre pedagogy is advocated by Reiff and Bawarshi in Johns et al. (2006, pp. 242-247), who argue that a genre approach to L2 writing helps to shift the locus of "invention" (that is, exploratory strategies such as free writing and brainstorming in composition classes) "from an interior cognitive process located *within individuals* to a situated cognitive process located *within genres*" (Bawarshi, in Johns et al. 2006, p. 244). Bawarshi proposes that genre be taken as the starting point for the teaching of invention rather than the culmination. The emphasis on situated invention of genres is further extended by a genre awareness pedagogy proposed by Devitt (2009), which emphasises the importance of genre teaching and that understanding the social values embedded within genres and exploring their rationale, that is, genre awareness, should be a critical part of genre pedagogies. The emphasis on ethnographic accounts of the social, cultural, and institutional contexts in RGS, however, does not mean that textual regularities are ignored. As Miller (1984) explains, "a genre becomes a complex of formal and substantive features that create a particular effect in a given situation" (p. 153).

1.2.3. Sydney School Genre Studies

The Sydney school genre work, which emerged in Australia in the 1980s, studies genres in their social context based on two levels of connection between text and context, register and genre (Rose, 2010, p. 235). The Sydney school scholars define genre as "staged, goal-oriented social processes through which social subjects in a given culture live their lives" (Martin, 1984, p. 43) and register as the variety of language associated with a social context, links a situation type with semantic and lexico-grammatical features.

Like the ESP approach, the Sydney school initiated out of concerns about language and writing instructions. A pedagogical model developed out of the Sydney School is the Teaching Learning Cycle (Cope and Kalantzis,1993) and reproduced in Martin (2013) (See Fig. 1).

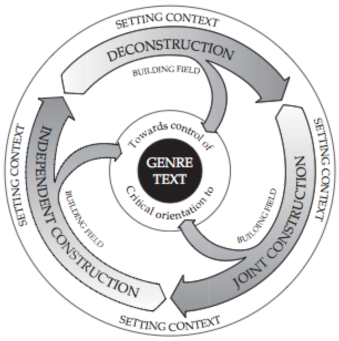

Figure 1.1 Teaching/learning cycle for teaching genre (Martin, 2013).

As Figure 1 shows, teaching a genre starts with setting its context, a stage through which students examine the cultural and situational aspects of the social context of the genre text using ethnographic strategies such as interviews and field trips. Having set the context, students go through the three stages of deconstruction, joint construction, and independent construction of genre texts. The deconstruction stage involves exposing students to target genre text samples and scaffolded analysis of the cultural and situational context of the models, their social purposes, and form-function correlations at both structural and linguistic levels. In stage 2, students practice scaffolded construction of a similar genre text. Students then proceed to the independent genre text production stage. They conduct research to develop content knowledge; draft the genre text; receive feedback from the teacher and peers, revise, evaluate, and submit their draft for a grade. The model thus makes students practice setting genres in their social context and develop explicit knowledge about genres, their staging, and their linguistic realizations. It encourages student agency, expression, and discovery, and emphasizes learning by doing. In simple terms, learning genres requires doing something about them with an expert available for coaching learning on a scaffolded basis. Per long-standing arguments supporting *learning by doing* (Dewey, 1916) the learning that emerges because of applying such an approach is sustained, transferable, and effective in a learner's future genre use and production.

As this quick overview of the three well-known genre approaches suggests, despite the distinctions, more common threads than demarcations bind the dominant genre approaches together in theory, methodology, and pedagogy (Hyon, 2018). Theoretically, the three genre approaches are inspired by social constructivism, a theory that encourages interpreting texts in their social contexts. Methodologically, they share the fundamental characteristic of contextualized analysis of rhetorical patterns and associated linguistic choices of target texts and draw attention to the social actions of genres and the criticality of their rhetorical choices. Pedagogically, they are primarily concerned with teaching writing. These common threads have triggered convergence in genre analysis practices across the three approaches. For example, ESP approach has shown a growing tendency towards RGS in its theory and practice since its emergence, which makes its categorization as a purely linguistic approach imprecise. On the other hand, in RGS researchers are now showing more interest in the forms of genres in addition to their original emphasis on the social and rhetorical contexts of genre emergence and action.

1.2.4. Current Situation

Despite all the criticisms cast at genre approaches from within and outside genre traditions, genre continues to be "ubiquitous" in scholarship and writing pedagogy (Tardy et al., 2018, p. 1) as "[f]ew concepts have had a greater impact on how we understand and teach language than genre" (Hyland, 2015, p. 32). It is undeniable that genres represent crucial sites and strategies that locate writers and guide their rhetorical moves; the writers' knowledge of how to understand genres is thus a valuable tool for entering communities of practice and navigating their cultures. It is also no exaggeration to claim that a writer's engagement in a discourse community's genres provides access to that community and promotes ways of knowing and acting within the community. Considering the common shades as well as the unique exploratory perspectives provided by different genre approaches, exploiting whatever aspect of the dominant approaches in genre theory and research which facilitate navigation of disciplinary cultures and understanding broadly approved ways of communication within disciplinary communities would seem to provide worthwhile resources and activities for advanced students and novice members within those disciplinary communities.

As the present review suggests, ESP and RGS approaches complement one another especially when researchers use text-first or context-first methodologies (Paltridge, 2008). As the research that I report in the following chapters shows, adopting a context-text-context analysis might even yield more reliable results both for research and teaching purposes. To clarify, initiating genre research with the analysis of situation in as early a step as data sampling in this study helped to ensure sampling of representative texts of genres based on their communicative purpose, discourse community, and originality. An ESP-based move-structure analysis then helped to reveal how the communicative purposes have been realized in conventional or specific rhetorical structures or forms of discourse arranged through hierarchies of moves and steps. Such text analysis has proven worthwhile in the present study to answer the following questions: Why are the genre texts in question structured the way they are? How do the structures of the given genre texts contribute to their overall communicative purpose, that is, arguing for and establishing new knowledge in the research article genre? As my research shows, answering such questions requires moving back-and-forth between the texts and their contexts of construction and operation.

Due to the methodological flexibility of ESP approach's as well as its openness to use ideas from RGS for rich rhetorical contextual analysis, my study thus applies a hybrid of ESP genre approach and RGS to the verbal, visual, and contextual analysis of the research article (RA) genre in discrete mathematics as well as to proposing writing-in-discipline modules and tasks. A writing pedagogy informed by both linguistic and rhetorical approaches responds more effectively to the higher-order concerns of advanced students and novice researchers who are under pressure to join their disciplinary discourse communities through writing in the genres of their disciplines.

1.3. Statement of Research Questions

A cursory glance at genre scholarship suggests that the research article genre has received notable attention in ESP genre scholarship. However, not all disciplines have been equally examined by genre scholars, nor are the examined disciplines homogeneous regarding the communicative purposes and argument structure identified in their RAs (Reviews of RA genre studies are presented in the following chapters). Moreover, so far ESP genre scholars have shown little interest in exploring the visual arguments, which are surely part of the argument for new knowledge in RAs in some disciplines. Furthermore, an overview of research in mathematical discourse suggests that much of the available research is micro-level analysis of lexico-grammatical features of texts used in school mathematics (A review of existing research in mathematical discourse is available in the next four chapters.) Accordingly, exploring the communicative purposes, the argument structure, and the text-visual interplays in mathematical RAs remains a research gap for both theoretical research and pedagogical purposes.

This study fills this gap in genre scholarship with the following purposes:

1) Identify the macro-rhetorical structure of discrete mathematics RAs and examine the micro-(move/step) structure of the Introduction sections.

2) Identify consistencies in the rhetorical structure of the RAs in the discipline.

3) Examine the visual moves and the ways they contribute to the argument structure in RAs in the corpus.

4) Explore connections between the rhetorical structure of the RAs examined, the knowledge-making practices of the discourse community, and the epistemology of discrete mathematics as an academic discipline.

5) Based on the research driven by the first 4 aims, propose pedagogical applications through developing some graduate-level modules and tasks for teaching writing in mathematics.

Based on the above purposes, the following research questions were developed:

1. How do discrete mathematicians structure the macro-organization of their RAs in the discipline? Why do they organize them the way they do?

2. How do discrete mathematicians organize their RA Introductions in the discipline? Why do they organize them the way they do?

3. What are the most typical rhetorical strategies for establishing research niches in discrete mathematics? What do they tell us about the discipline's conception of research?

4. How do discrete mathematicians blend visuals into the micro-organization of their RAs in the discipline? Why do they use them the way they do?

5. What practical pedagogical implications can be drawn from the findings of this study?

1.4. Significance of the Study

The research article is the key genre of academia (Hyland, 2006). Examining the rhetorical situation, conventions, and specificities of RAs in disciplines contributes to our understanding of the disciplines, their epistemologies, and the acceptable ways of scholarly communication within their discourse communities. The pedagogical implications of such understanding are straightforward and unquestionable. Writing instructors, as well as graduate students—both as novice researchers and teaching assistants in disciplines—are in urgent need of learning how to write successfully in the context of their discipline or to assist senior professors in their assessment of student writing assignments. To the best of my knowledge, there is little move analysis research on mathematics within ESP and/or rhetorical genre studies. A concurrent research study (Kuteeva & McGrath, 2015) examined the macro structure (IMRD organization) of RAs in pure mathematics. While their study yields interesting findings on some aspects of RAs in mathematics, it does not look at the section-internal rhetorical structures, except for some aspects of the introductions, and they barely include a discussion of visual rhetoric in the RAs examined. Despite recent calls for multimodal genre analysis (Johns, 2013; Tardy and Swales, 2014), to the best of my knowledge, there is little, if any, multimodal genre analysis of RAs in mathematics. My method thus offers a contextualized genre analysis, i.e., a context-based, rhetorically oriented, wide-angle analysis (Berkenkotter, 2009, p. 18) that examines visuals and their related rhetorical moves in context. Therefore, my research makes a significant contribution to existing genre scholarship, workable methodologies in multimodal genre analysis, and approaches to teaching specific genres and writing-in-disciplines.

Mathematics is a vast field which integrates numerous sub-disciplines from both pure and applied orientations (Mathematical subject classification, 2010). In this study, I have tried to show that examining a specific sub-discipline contributes to collecting more focused RA samples, textual and ethnographic data, hence valid results. Due to the significance of discrete mathematics within the broader mathematics discipline, the criticality of its findings to client disciplines such as engineering and health sciences as well as the research logistics of the current research, this study focuses on discrete mathematics, rather than the entire discipline of mathematics.

My research also fills existing gaps in research-informed pedagogical materials to teach novice members of mathematics how to write publishable RAs. In his first sentence in the book, Swales (1990) stated that the aim of his book was "to offer an approach to the teaching of academic and research English" (p. 10). Despite such a clear statement, it is probably true that unlike the discourse analysis side of ESP genre research, pedagogy has always been an under-developed aspect of the genre approach (Flowerdew, 2015) with the exception of practices by some genre scholars (Devitt et al., 2005; Johns, 2008; Hyland, 2006; Paltridge, 2007; Swales & Feak, 2012; Tardy, 2009). Among these sources, Devitt et al. (2004), Hyland (2006) and Swales and Feak's (2012) offer proceduralized tasks for developing students' awareness of different aspects of some academic genres; however, these tasks are mostly within a discipline-neutral context. A recent source, which offers models and tasks for chemistry-specific writing, is Robinson et al.'s (2008) *Write like a chemist*. The book is intended for upper-level chemistry majors as well as graduate students faced with writing in dominant genres in chemistry. To the best of my knowledge, other than this recent publication, there are few disciplinary-specific pedagogical models and materials developed within ESP genre studies. Hence, this research fills this pedagogical gap by developing procedural tasks for teaching RA writing to majors and graduate level students in mathematics.

The present study is also a significant response to recent calls in genre scholarship to examine the relationship between text and image in the multimodal genre texts (Johns, 2013; Tardy and Swales, 2014). Similar calls for research in multimodal forms of communication have been made by semiotists. As Kress and van Leeuwen (2001) note, whether to say something verbally or visually is a key decision in multimodal communication and an important aspect of semiotic studies. Traditionally, the importance of visual information and its centrality to meaning-making in science, technology, and economics, as well as to students' learning, has been identified in academic and technical writing (Johns, 1998; Kress and van Leeuwen, 2001). Much of this identification is owed to studies in rhetoric of science that offer insights into the function of visuals within/across science disciplines, for examples in biology, chemistry, and geology (Gross, 2007); medicine and physics (Idhe, 2007); ecology (Winn, 2009); chemical physics (Wickman, 2010); nanotechnology (Graves, 2014). Roth et al. (2005) discuss the ways in which visuals contribute to the overall purpose of academic texts:

> to present data, illustrate abstract concepts, organize complex sets of information, facilitate the integration of new knowledge with existing knowledge, enhance information retention, mediate thinking processes, and improve problem solving (Roth et al., 2005, pp. 208-9)

Accordingly, Johns (2013) remarks that "it is surprising that so little research has been completed either on the visual/verbal interaction in texts or on academic or on non-academic visual rhetoric" and that the study of visual displays "should be a focus of ESP research" (p. 20). A similar concern arises in Tardy and Swales (2014) who argue that for some genres elements such as visual images are "so essential that it would be impossible to overlook them in an analysis" (p. 173). Accordingly, this study contributes to multimodal genre analysis by filling this gap in genre scholarship. As discussed in chapter 5, by examining the links between visuals and the verbal in the rhetorical structure of discrete mathematics RAs, we can begin to understand what function/s visuals play in argument for new knowledge in the discipline.

This research also contributes to our understanding of communication in academia in significant ways. Academic writing is the primary mode of communication in research universities; hence it is essential to survival in academia. It is also the dominant mode of communication through which student learning and assessment occur. Genre research has established that ways of negotiating academic ideas and new knowledge, hence communication, vary across disciplines and genres. By exploring disciplinary writing including the routinized ways of arguing for knowledge claims, as well as possible variations within and across the disciplinary boundaries, this study adds to existing academic communication research through discussing effective communication practices in certain areas of mathematics.

In sum, this study specifically contributes to the existing research in academic writing in four ways: 1) it partially fills the gap in writing research regarding research mathematics; 2) it shows how data drawn from multiple sources can illuminate why disciplines write the way they do and how they communicate persuasively in and across disciplines; 3), it illustrates why novice writers need both disciplinary epistemic and academic discourse competencies to learn to write well for effective communication in their fields; and 4) it provides evidence for the current arguments that genre knowledge must include understanding of the knowledge construction process in the discipline and how discourse community values framed communications in academic settings.

1.5. Overview of the Dissertation

I have organized this dissertation in seven chapters. Having introduced my research in the current chapter, I proceed as follows:

Chapter 2 answers the first research question regarding the macro-structure of RAs in discrete mathematics. In particular, the chapter reports on the macro-structure and sectioning strategies in RAs in discrete mathematics. It offers a review of influential genre research on the macro-structure of RAs in different disciplines. It also specifies the research design and methodology. A significant part of the chapter presents my textual and interview data and discusses my findings as informed by readings in the philosophy of mathematics and epistemology of the discipline.

Chapter 3 provides an answer to the second research question regarding the micro-structure of RAs in discrete mathematics. The chapter reports on the move structure of Introduction and Complementary Introduction sections of RAs in discrete mathematics. It compares the move structure of these sections in the corpus with Swales' *Create A Research Space* models and discusses the observed variations regarding the values and norms held by the discrete mathematics discourse community and epistemology of the discipline.

Chapter 4 complements chapter 3 by answering research question 3. The chapter focuses on establishing a niche for new research in the RAs in discrete mathematics, identifies the conventional research niches in discrete mathematics and the strategies used by mathematicians to highlight the existing research niches. The chapter also recaps discussions from chapter 2 and 3 and proposes a revised *Create A Research Space* model that fits RA Introductions in discrete mathematics.

Chapter 5 answers the fourth research question. The chapter adds a novel aspect to RA genre research following Tardy and Swales' (2014) identification of the analysis of visual argument as a future trend in ESP genre research. The chapter reports my analysis of visual moves in RAs in discrete mathematics and their mutual contribution to argument structure together with the verbal moves. To the best of my knowledge, no other ESP-oriented genre research has filled this gap in the research article genre so far.

Chapter 6 answers research question 5 regarding the pedagogical implications of this study. In chapter 6, I support genre-based pedagogy by proposing a writing-in-the-disciplines course in research writing for senior undergraduate and graduate students in mathematics. In developing the course, I have used ideas from both ESP and RGS. The proposed course follows a procedure of consciousness-raising, production, and process activities and scaffolded tasks, as suggested in Hyon (2018). The analytical tasks include brainstorming questions for analysis of the rhetorical situation, the schematic structure and communicative purpose of sample RAs and their constituent sections as well as critical thinking in writing and disciplinarity. The emphasis is on developing students' awareness, education as opposed to acquisition (Johns, 2008), of the situational features pertaining to the genre of RA, its overall communicative purpose and the lower-level communicative purposes of different parts of the RA.

Chapter 7 presents the conclusion of the study by recapping highlights of the findings and arguments of the study. It re-situates the findings of this study within the broader area of genre analysis, writing and disciplinarity, and disciplines as social institutions. The chapter concludes by highlighting the key role writing plays in advancing disciplines, their mission for producing new knowledge, and, more importantly, disciplinary writing as a teaching and learning tool in education. I also discuss limitations for my research and how they affect the implications of my study including the generalizability of findings.

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Chapter 2: Mathematics Is the Method: Exploring the Macro-Organizational Structure of Research Articles in Mathematics

2.1. Introduction

Even after three decades, genre analysis of research articles (RAs) continues to be a generative area for new scholarship (see Appendix A). Most studies adopt Swales' ESP approach to genre analysis to determine the communicative purposes of text producers. The communicative purposes are catalogued as hierarchies of obligatory and optional Moves and Steps that suggest a model for a typical RA in that field, a model that is then recommended for pedagogical application by novice writers in the discipline.

Genre study has tended in two directions, one focused on RA sections (that is, introduction sections, method sections, etc.) and the second focused on explaining the link between RA structure, methods used in the discipline, and disciplinary epistemology. The first tendency produces partial knowledge of the genre structure in that field and limits the usefulness of the research from a pedagogical perspective. That is, teachers of academic discourse (for example, English for Academic Purposes (EAP) or English for Specific Purposes (ESP)) and academic writing course content developers need comprehensive knowledge of RA structure so they can help students learn what a well-written RA looks like in their particular discipline. Several studies have contributed to our comprehensive knowledge of RA structure in fields including medicine (Nwogu 1997), computer science (Posteguillo 1999), applied linguistics (Ruiying & Allison 2004), and biochemistry (Kanoksilapatham 2005). These studies are valuable resources for teachers tasked with teaching the application of knowledge from genre analysis.

The second tendency, explaining links between article structure, methodology and disciplinary epistemology, is represented through studies by Brett (1994), Posteguillo (1999), Samraj (2002), Kanoksilapatham (2005) and Parkinson (2011), which make these connections and describe aspects of the RA. An essential contribution to genre analysis, these studies link move structure to disciplinary culture. In contrast, genre studies focused mainly on generating lists of Moves and Steps can imply that learning these typical argument structures equals acquiring genre knowledge. We do know that each discipline defines knowledge in slightly different ways so that new contributions to knowledge do not look the same in or to all disciplines. In addition, wide methodological variations shape how disciplines obtain and claim new knowledge. Therefore, understanding of Move structure must be situated within a student's ethnographic awareness of how and what a particular discipline values as knowledge and research. Indeed, many scholars have emphasized the importance of a multidimensional methodological approach to genre study (Swales 1998, Candlin and Hyland 1999, Hyland 2000 and Bhatia 2004).

Based on these methodological contentions, we have analyzed the schematic structure of mathematics RAs based on a sample of 30 articles embedded within a larger analysis of the writers' macro-framework and major sectioning strategies. Further, we describe, interpret and explore some of the argumentative moves used to present the findings within ethnographic accounts about the texts from four specialists in mathematics.

Our analysis began with the macro structure of RAs. In his overview of their macro structure, Swales (1990) proposed an "hour-glass" structure, established as the Introduction-Method-Results-Discussion (IMRD) structure. Swales noted, however, that Tarone et al. (1981, in Swales, 1990) when studying RAs in Astrophysics, did not find this IMRD structure. In an update, Tarone et al. (1998, p. 115) refer to Swales' hourglass rhetorical structure as an adequate format for describing experimental studies; however they emphasize that it does not apply to RAs in scientific fields where "the subject matter does not lend itself to experimentation." Although they are studying voice in astrophysics papers, Tarone et al.

(1998, p. 115) noted that in logic-driven fields, the rhetorical shape of an RA features logical arguments, which "cite observations and draw conclusions."

In this paper, we report significant variation from the IMRD structure in mathematics RAs compared to other disciplines. Introduction and Results sections were always present (in abundance), but Method and Discussion sections were brief or absent. Conclusions were used more in applied than in pure mathematics RAs.

Ethnographic data suggests that the macro-structure of math RAs arises out of four factors: the dominance of formalism in research mathematics, the establishment of presumed research methodology, the powerful presence of logic in the discipline, and the complicated nature of the mathematical concepts manipulated. In addition, the Conclusion sections in applied math RAs generally contain promotional discourse, reflecting interdisciplinary demands on this area to produce optimized results of potential use to algorithm-based fields including computer science.

This study contributes in four ways: one, to the best of our knowledge, mathematics has received little study using an ESP approach (a recent study on pure mathematics RAs by McGrath & Kuteeva [2012] discusses stance and engagement); two, it shows how data drawn from multiple sources can illuminate why disciplines write the way they do; three, it illustrates why novice writers need both disciplinary epistemic and academic discourse competencies to learn to write well in their fields; and four, it indicates that genre knowledge must include understanding of the knowledge construction process in the discipline and how discourse community values establish the context for the RA. Students who are situated in the context will gain a much stronger understanding of the disciplinary discourse. This study further suggests that mathematics is a discipline because of the way it argues for new knowledge, an implication that highlights the importance of understanding the role of argumentation in mathematics.

In the following section we briefly explain the data and our methodology. In the third section we present and discuss our findings. The final section summarizes our conclusions.

2.2. Data and Methodology

The 30 RAs used in this study (see Appendix B) were obtained through stratified random sampling from these research journals: *Discrete Mathematics* (DM), *Discrete Applied Mathematics* (DAM), *Journal of Combinatorial Optimization* (JCO), *Graphs and Combinatorics* (G&C) and *SIAM Journal of Discrete Mathematics* (SIAM). These journals publish major research findings in discrete mathematics. The criteria for selection were representativeness, reputation, and accessibility of the journal. DM and DAM are published by ScienceDirect, JCO and G&C by Springer, and SIAM by Society for Applied and Industrial Mathematics; all are indexed by ISI web of knowledge, recommended by disciplinary informants, and subscribed to by most university libraries. Additional criteria for selecting these journals include topic coverage and target readers, information obtained from the journal homepages.

The following principles ensured a stratified random sample: authors (one article per author was sampled), journals (equal numbers of articles were chosen from each journal), issues (one article was sampled from a given issue), year of publication (equal numbers of articles were chosen from a three-year period, 2007-2009). Thus, author, journal, issue and year of publication were the strata used in sampling the data. To ensure a random sample, article topics were selected that span the branches of discrete mathematics (the study of objects that can assume distinct values and are represented by integers [Renze and Weisstein 2012]). Since the study did not compare pure versus applied math papers, we did not use article type as a strata; however, having sampled the corpus, we identified nine applied math papers (DAM2, 3, 4, DM5, D&C5, JCO2, 5, 6, and SIAM2) out of 30. Table 1 summarizes details of the corpus.

Table 2.1.Some details of the corpus.

Number of RAs	Number of pages	Number of authors	Publication date	Number of countries represented
30	345	65	2007-2009	21

In our methodology, we adopted a triangulated approach (Candlin & Hyland, 1999) that integrated textual data (description), ethnographic accounts obtained through interviews and discussions with disciplinary specialists (interpretation), and investigation of the structural and social grounds of the writing practices in the discipline as social institution (exploration). To obtain textual data, we used lexico-grammatical signals and content information to characterize the rhetorical structure. When there were few textual signposts to the rhetorical structure or the contents were too technical to identify communicative purpose, we consulted insider informants. Our insider informants were selected based on their high academic qualifications and numerous research publications. These informants also provided ethnographic information by explaining the problems addressed in each paper as well as the introductory material required for each problem, checking samples of the results, and validating our conclusions (that is, we showed informants the coded passages so that they could agree or disagree. In the event of disagreement, we assessed the reasons for their labels and considered their viewpoints in our final judgments). The informants received a short introduction to the IMRD framework and Swales' ESP approach to genre analysis before checking samples of our rhetorical analysis.

2.3. Results and Discussion

As noted, mathematics RAs do not follow the organizational pattern of the IMRD framework (Introduction, Method, Results, Discussion). In fact, they depart considerably from it (See Figure 2.1). Immediately obvious is the absence of both Method and Discussion sections, suggesting that this framework is not a valid model for math RAs.

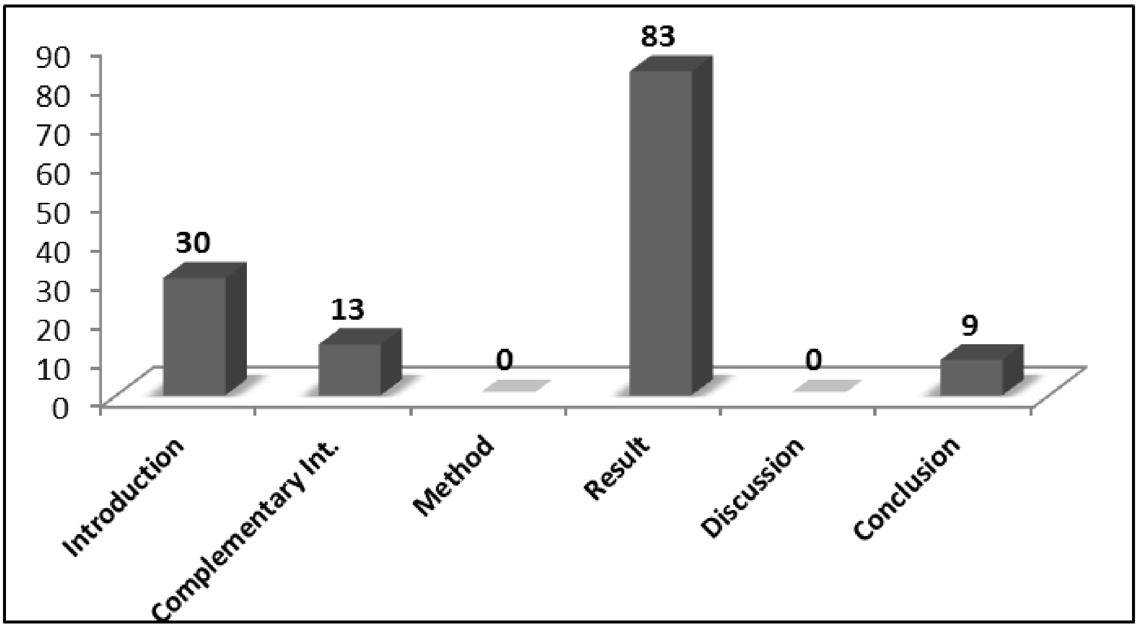

Figure 2.1. Main sections in math RAs

Another notable difference is the use of section headings (See Table 2.2). Math RAs routinely use content (rather than generic) section headings.

Table 2.2. *Frequencies of section headings in the corpus.*

(GSH=Generic Section Heading) (CSH=Content Section Heading)

	Introduction (GSH)	**Complementary Introductions (CSH)**	**Method**	**Results (GSH)**	**Results (CSH)**	**Discussion**	**Conclusion (GCH)**	**Conclusion (CSH)**
RAs	30	13	0	2	81	0	3	6

2.3.1. Introductions

With one exception, all RAs in the corpus used the generic heading of "Introduction" (DAM5 uses "Background" instead). In addition, several articles used compound section headings (for example, "Terminology and introduction" [DM4] and "Introduction and problem description" [JCO2]). These headings identify content and rhetorical function. Two of the journals, DM and DAM, require authors to include an Introduction section; the others (JCO, G&C and SIAM) do not.

The prevalence of Introduction sections in our corpus suggests that writers of mathematics RAs (similar to writers in other disciplines) begin by creating a rhetorical space for their research, space that not only highlights the importance of the research and introduces the new results but also contributes to the argument. In mathematics, persuasion only results when readers and writers share understanding about the concepts being discussed; thus, all introductory sections include clauses that define mathematical concepts and introduce symbols. The act of definition is rhetorical here because it simultaneously identifies topic details, clarifies the gap in knowledge being addressed, and creates shared knowledge between writer and reader from which the results arise. In the following examples, the writers use the first paragraph to define concepts and designate symbols:

> Let $G= (V,E)$ be a graph with vertex set V and edge set E. A *total dominating set*, denoted by TDS, of G with no isolated vertex is a set S of vertices of G such that every vertex is adjacent to a vertex in S. (DM2)
>
> In this paper, G is a simple connected graph with vertex set V(G) and edge set E(G) (briefly V and E). For every vertex vV, the *open neighborhood* N(v) is the set $N(S)= v \in SN(v)$ and the *closed neighborhood* is the set N[v]=N(v). (DAM1)

According to Morgan (2005) and Jamison (2000), definitions are essential constituents of doing mathematics. Our informants also noted that the act of definition is a typical communicative purpose for Introduction sections in math RAs in the sense that new conclusions (Move 3-Step 2, Swales' 1990 *Create A Research Space* model) follow and are deduced from preceding definitions. Definitions, thus, contribute to the communicative purpose of creating a space for research in that they serve as scaffolding upon which new findings are built.

We emphasize that the hypothetical nature of mathematics drives this purpose: the objects of research are abstract concepts that are instantiated through precise definition. Thus, a preliminary step involves mathematicians asking readers to suppose the existence of mathematical objects with specific features. The objects are then transformed into symbols that, among other things, allow mathematical manipulation. For example, "G" above is defined as "a graph with vertex set V and edge set E"; the ten-word definition is condensed into the single letter, "G." G is then used in the formulas and equations that follow.

Although introduced throughout an RA, mathematical concepts and symbols are critical in Introductions where authors must establish agreement with readers on the nature of mathematical concepts and properties. Once agreement is reached, authors can proceed to making and proving their knowledge claims (Foss et. al 2002).

2.3.2. Complementary Introductions

Over one third of the Introductions (36.66 %) include additional sections located before results and labeled with content section headings (13 additional sections), for example, "Definitions," "Known results and more definitions" and "Construction of prism fixers" (three sections in DM3) and "Man-Exchange Stable Marriage," "The Gale-Shapley algorithm" (two in JCO2). Articles from each journal included additional sections, indicating it is a conventional structure.

In these sections, authors define specialist terminology, introduce notations, describe the problem, and review previous research. For example, the section two heading in DM2 states its purpose: “Known results on total domination” (that is, reviewing previous results). It includes seven theorems and citations followed by authors’ comments on those results. The section reviews previously published results, results that two disciplinary informants noted were substantial to “total domination in graphs,” the paper topic. The review contextualizes the authors’ current results on *total domination*, a communicative purpose identified as Move 1, Step 3 in Swales’ (1990) *Create A Research Space* model for RA Introductions. Then the authors incorporate this previous work into the solution that they present for the problem addressed in DM2. When authors in math need to review a substantial body of literature, they present it as a separate section (not part of the general Introduction), and if this previous research is used in solving the current problem, they bold the heading to emphasize the section.

As noted, DM3 presents three intervening sections between Introduction and Results. As the section headings, textual signals and the informants’ data indicate, these sections are prompted by objects under study that require successive definitions and substantial literature review. The complementary introductions allow writers to organize the definitions and symbols while maintaining the clarity of the Introduction. Writers also ensure the results are easily located by presenting them as freestanding sections. Descriptive headings not only allow writers to highlight the research objects’ features; they also enable readers to more easily conceptualize the objects. These sections help establish the context for the mathematical argument and prepare readers for the results that follow, a function that other researchers including Lim (2011) have established.

All 13 complementary introduction sections shared this pattern. General definitions and reviews of well-known results generally appear in the Introduction; however, when the mathematical objects or previous research are foundational to the paper, they appear as separate sections (that is, Complementary Introductions).

2.3.3. Absent Sections

In our corpus, “Method” and “Discussion” sections are absent (see Figure 2.1). This departure from the IMRD framework indicates that, not surprisingly, mathematics lacks an empirical basis. At the same time, exploring why Method and Discussion sections are absent in math RAs helps to articulate a fundamental understanding of mathematics as a discipline. We considered the communicative purposes reported for these sections in genre studies, but little work has been published to date on Method sections. Swales (2004) suggests significant demarcations between disciplines in terms of their research methodologies, noting that the typical structure of Method sections includes a description of the materials used, the procedures adopted, the apparatus employed, and the statistical analyses chosen. He further posits a relationship between discipline and length of Method sections: methods are “clipped” in many hard sciences; elaborated in education, psychology and the social sciences; and of intermediate length in language sciences, public health fields and earth sciences.

The absence of the need to explain methodology in mathematics is interesting because it differentiates this field from both hard and soft sciences. In hard sciences, method sections describe the physical actions performed by researchers prior to and during scientific experiments. These descriptions enable readers to validate (and possibly replicate) the results reported. In contrast, research activities in math are mainly cognitive, a critical difference between math and experimentally oriented fields. The absence of the method section also implies agreement among members of the discourse community on how to solve mathematical problems; that is, to a great extent insiders presume the method. This point supports Brett’s contention (1994) that the extended Method sections in sociology RAs (compared to shorter Method sections in the hard sciences) indicate less agreement on the methodological practice in the discipline. Its absence in math RAs also points towards the extended history of mathematical problem solution, a history with well-established and taken-for-granted methods of logical deduction and induction.

At the same time, absence of a Method section does not mean methods are absent in mathematics RAs. In fact, authors allude to methodology in their proofs. The following example explicitly identifies the authors' method:

> *In our proof we will use* ***the method of cyclic permutations*** *developed by Gyula O.H.Katona in [7].* (Bold in original, underline added, G&C2: 359)

The first sentence of the proof identifies the method but refers readers to a citation for more information. Other examples suggest that mathematicians signal method briefly in their texts:

> *We prove this by induction on the order n of the tree.* (Underline added, DAM1, Proof of Theorem 17).

This statement indicates the proof technique called "mathematical induction." Numerous similar instances are discernible in our corpus.

Another common method is proof by contradiction. Again, this technique is linguistically signaled:

> ***Proof by contradiction***. (Bold in original, underline added, DM1, Proof of Lemma 3.6.)
>
> *We show by contradiction that….* (Underline added, JCO5, Proof of Lemma 2)

In addition to lexemes referring to method, some discourse markers signal procedural descriptions of the problem-solution process. For example, 'first' and 'then' are linguistic markers that guide readers through the procedure of proof. The following example uses 'first' and 'then' to describe a chronological methodology:

> *We will first prove that …. We will then prove that ….* (Underline added, *DAM4,* Proof of Lemma 3)

By signalling the procedure, mathematicians highlight the knowledge and agreement about methodology shared among disciplinary insiders. Since method is presumed in mathematics, it is reduced to a phrase or procedural account embedded in the proof. In other words, oblique references to method are judged adequate by disciplinary insiders. However, different mathematical results within a paper can require different techniques of proof and procedures. That is, each proof may be developed using a unique procedure. However, our data also suggest procedural similarities exist among proofs within a paper. In such cases the authors achieve conciseness by referring to proofs presented earlier, as in the following example:

To complete the colouring we note that … and so the same argument applies.

(Underline added, DAM5, Proof of Theorem 3)

The question arises of whether it is possible to summarize these independent descriptions within a distinct section, and if so, whether it would compromise the readability of the paper, creating ambiguity in the proofs. Our subject specialists suggest not. They note that because the basic methodology is widely assumed in mathematics, brief linguistic cues activate shared knowledge and keep readers on track. As long as mathematicians base their claims on logic and argue inductively or deductively through a piece of proof, readers do not want more detailed information. In other disciplines, the method is used to validate results, but in mathematics, results are valid as long as they are logical deductions from premises. Therefore, the absence of Method sections in math RAs arises from a community-established norm that presumes readers know standard disciplinary research methodology.

Not only are Method sections absent, but Discussion sections are also absent in our corpus. That is, while no separate Discussion section is present, a discussion of results does take place. Discussion-like activities appear within argumentative moves in Results sections: accounting for, commenting on, extending or delimiting, evaluating or exemplifying findings.

These communicative purposes are typical with "Discussion" sections in other disciplines; however, in mathematics RAs the discussion is brief and integrated into the presentation of results. The following passage, very much resembling discussion, succeeds the proof of Theorem 3 in DAM2:

> *In view of the previous result we can say that the diameter … is at most 3. Therefore T(B) must be one of the following:…* (Underline added, DAM2: 2743)

The introductory phrase signals that the authors are elaborating the preceding result. The adverb "therefore" signals that they are drawing conclusions about the first statement. In this passage, the principal communicative purposes are accounting for findings and deducing further results.

In the next example, the writers also evaluate the result that they presented in Corollary 3:

> *Therefore, we conclude that the lower bound of Corollary 3 is better than the information theoretic bound.* (Underline added, *DM5*: 5936)

The lexemes "therefore" and "conclude" and the adjective "better" suggest that the sentence evaluates the knowledge claim established in the corollary. While acts of evaluation typically appear in the Discussion sections of RAs in other disciplines, in mathematics the significance of results is noted immediately. If we consider the communicative function of Discussion sections in general, we can see why discussion is presented with results in mathematics. In other disciplines, results may be understood differently by other researchers; therefore, writers in those disciplines explain their understanding of results—they argue for their interpretation, and this argument appears in the discussion section. In contrast, results in mathematics require little interpretation. The proof of a mathematical hypothesis is demonstrative in itself. Anyone who looks at the proof either accepts or rejects it. If they reject it, it is because of a flaw in the reasoning (that is, the proof is executed incorrectly). If the proofs are based on the right premises and logically argued, the results are accepted; hence they are persuasive (that is, convincing) in themselves and require little, if any, discussion to convince readers. That is, the proof itself persuades readers, not the discussion of the proof: if the proof is not logical, it will not be judged persuasive regardless of the discussion presenting it. An informant confirmed that this applies to all proofs, regardless of their weight. Foss et al (2002) note that most people are inclined to accept claims based on logic. Mathematical findings become persuasive because they are products of logical reasoning. They need little interpretation (or discussion) because they are demonstrative by themselves.

2.3.4. Results

Results sections comprise nearly two-thirds of the total number of sections in the corpus (61.5% of a total of 135 sections) (see Figure 2.1). Results sections were designated based on those labeled as such. Two characteristics were notable: 1) content section headings dominated the presentation of results (see Table 2), and 2) multiple results sections were presented (see Figure 2.2). Only two of 83 Results sections were generically labeled "Main result" (DM2 and SIAM5); the rest used content headings. DM2 contained three Results sections in addition to the first, labeled "Main result." The main result section begins, "We shall prove," and presents a theorem and corollary but no proof: in fact, this section merely announces the main result. Further analysis reveals that the actual proof for the theorem appears two sections later, labeled "Proof of Theorem 8." One section, called "Cost function," separates the "Main result" and its proof. It begins thus:

> *Before presenting a proof of Theorem 8 we introduce the concept of a cost function …* (DM2: 3494)

Clearly, the authors believe that readers must understand the "concept of a cost function" to follow the proof. Our specialist informants identified this concept as a new contribution by the authors: since it is in itself a result, it is highlighted with its own heading. Setting it off emphasizes its importance and draws it to reader attention. These authors use this sectioning strategy rhetorically to highlight the novelty of the concept and the creativity of this work. It also improves readability.

The second case that uses “Main result,” SIAM5, employs its second and third results sections to present solutions to previously published results that have not been solved. That is, the ‘Main result’ section presents the authors’ main solution, but two follow-up results sections announce solutions for problems (that is, fill a gap) that were unsolved hitherto. Disciplinary informants explained that these solutions are new contributions to graph theory research, and it is conventional to present them in independent sections. In addition, the sectioning strategy emphasizes the new results.

As illustrated, math RAs typically include multiple results sections; in our corpus, each RA included 2.8 results sections (on average). With the exception of JCO5 and DM3, all of the papers included more than one section of Results (while DM3 presented a single Results section, JCO5 presented the results as four sub-sections under one heading).

Figure 2.2 shows that 22 out of 30 RAs in our corpus (73.33 %) use two or three Results sections, indicating that presenting results in two or three sections is typical in math RAs. Subdividing results allows mathematicians to highlight their contributions and aid reader comprehension.

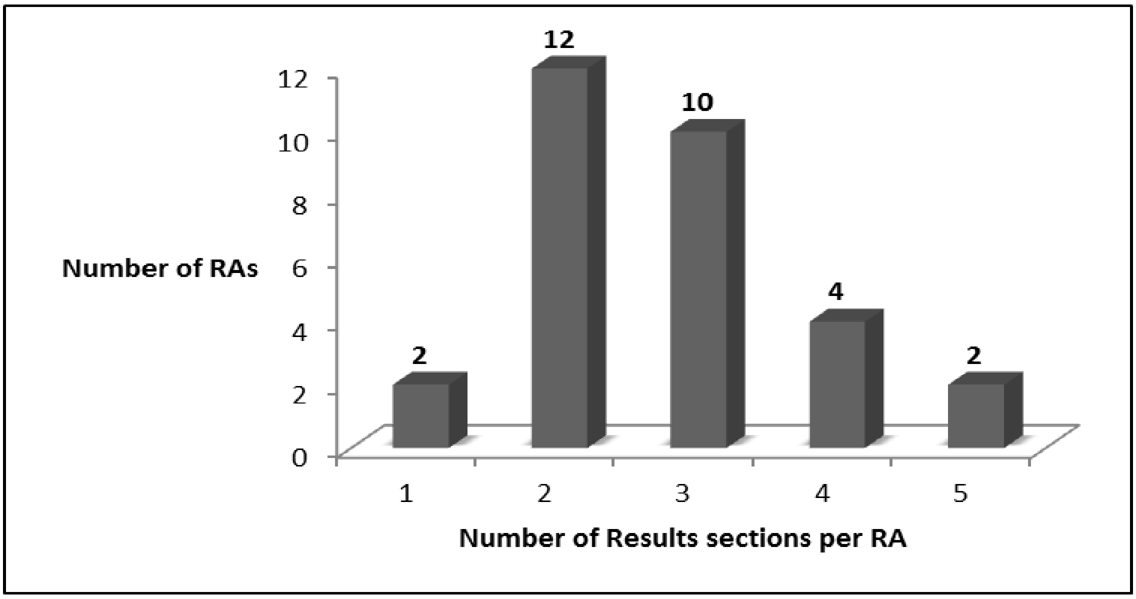

Figure 2.2. *Number of RAs with multiple Results sections.*

As mentioned, the act of defining a novel function is considered a new research finding in mathematics. In such cases, authors devote an independent section to that definition. Researchers also use multiple results sections when they have been able to solve a problem for different quantities. For example, the authors in DM5 present lower and upper bounds for the graphs, as well as solving the problem for some complete graphs; in each case they present that work in its own section. They orient readers to the results' organizational structure:

> *In Section 3 we start with the proof of upper bounds for … Section 4 is devoted to the proof of the sharp upper bound for ... In Section 5 we consider the complete graph* K_n*. We provide a sharp lower bound for … and improve the upper bound of Section 4 for …* (DM5)

The section headings in DM5 declare the rhetorical arrangement. Each heading, "3. Lower bounds," "4. Upper bounds," "5. The complete graph Kn," indicates an autonomous result.

The author of SIAM5, an applied math paper, uses a similar rhetorical structure for his two results sections. In the first one, 'Characterizing theorem,' he characterizes and solves the problem that he regards as the underlying theorem for the algorithm he has developed. He also announces this result in the Introduction section and then announces a further result—the algorithm that arises from the theorem:

Employing this characterization, we present a[n] *... algorithm for ...* (SIAM5: 637)

The algorithm appears in the second Result section, labeled "3. Monte Carlo algorithm."

According to disciplinary insiders, obtaining an algorithm for solving a real-world problem in other disciplines like computer science, electronics, or biology is a characterizing feature of applied mathematics. The algorithm is regarded as the key finding, and as such it normally appears in an independent section.

Mathematicians, who deal with complex or multifaceted mathematical objects, often subdivide their results to communicate more effectively the features of the topic. They probably have additional motives for locating important results in individual results sections; study of a broader corpus would undoubtedly reveal these. Nonetheless, these findings indicate that authors section their results strategically and use section headings to highlight aspects of their work and to aid reader comprehension.

2.3.5. Conclusions

Only nine RAs in the corpus include a Conclusion section (see Figure 2.1). These sections use variations on generic headings: "Conclusion" is used three times (in DM2, DAM4, JCO5). Other headings include "Concluding remarks" (in DAM3, G&C5), "Summary and Conclusion" (in JCO2), "Other remarks and further work" (in SIAM2), "Conclusions and future work" (in SIAM6) and "Comments and Open Problem" (in G&C2). Six of these conclusions appear in applied mathematics RAs (DAM2, 3, 4, JCO2, 5 and

SIAM2). Disciplinary informants noted that Conclusion sections are more common in applied math papers and rare in pure math RAs.

The rhetorical purpose of these sections ranged from reviewing the results, emphasizing the significance of the problem or an application of the results, evaluating and/or interpreting results to suggesting further research on the subject. These purposes resemble those reported for Discussion and Conclusion sections in other disciplines. The following examples illustrate some of the strategies used in the Conclusion sections.

Reviewing the results:

In this paper <u>*we considered*</u> *... graphs.* <u>*We proved*</u> *that…* (Underline added, *DAM2*: 2752)

Emphasizing the significance of the problem or applications:

In this paper we have introduced … motivated by <u>*a significant practical application*</u>*. … In the context of …, this … problem could still be* <u>*significant*</u>*.* (Underline added, *JCO2:* 358)

The concept of a balanced decomposition number f .G/ seems to have <u>*many other applications*</u>. (Underline added, *DAM3*: 3344)

Evaluating results:

This result was <u>*quite surprising*</u> *since disjoint path problems are* <u>*notoriously hard*</u> *in directed graphs. (*underline added, *DAM4*: 96*)*

Interpreting results:

We may be able to regard the concept of … as an extension of …. (DAM3: 3344*)*

Suggesting further research:

<u>*One interesting problem is*</u> *to extend our results to … of these graph families ... Finally, it would be* <u>*an interesting problem to characterize*</u> *… . (*Underline added, *G&C5*: 166*)*

Except for "reviewing the results," these communicative purposes are specific to applied math RAs, which generally serve research in engineering, health science, biology and other scientific disciplines. That is, readers from these disciplines follow research in applied mathematics to identify algorithms and models useful to their own work. Applied mathematicians use a conclusion to highlight the optimality of their algorithms to entice researchers from other disciplines to apply their results in subsequent work. Although disciplinary informants confirm this interpretation, our corpus size is too small to make extensive claims about using conclusions in applied math.

2.4. Conclusion

In this paper we analyzed the organization and rhetorical structure of sectioning in math RAs. This analysis shows that IMRD, the dominant model in empirically oriented disciplines, is (unsurprisingly) **not** the framework for math RAs. This confirms Tarone et al.'s (1998) conjecture that Swales' hourglass model may not apply to mathematics RAs. While researchers in empirical sciences must describe their procedure and explain the rationales for using a particular method (Bazerman, 1988; Swales, 1990), mathematicians do cognitive work that relies on logical rules of deduction and induction. Perhaps also unsurprisingly, the structure of RAs in this discipline reflects this ontological tradition.

In our analysis, we have tried to explain why mathematicians organize and present their knowledge claims in particular ways. As the community of a logic-driven discipline, mathematicians use an Introduction-Results macro-structure in their RAs. The Introduction not only creates space for new work and emphasizes its importance, it also defines the mathematical concepts that constitute the subjects of research. In fact, presenting definitions as the necessary groundwork for arguing knowledge claims is a long-established tradition in mathematics. This same tradition of relying on logic also accounts for the absence of a Methods section. Again, reliance on logic also obviates the need for discussion, hence its absence from the RA framework. Instead, writers in mathematics explain their findings through brief statements integrated directly into the Results sections. When mathematicians wish to highlight the applicability of their work to scholars in other disciplines, they locate this appeal in a brief Conclusion section.

This analysis highlights the link between RA structure in mathematics and its disciplinary traditions; it further shows that the RA is a historically based genre. It also suggests that when teaching academic writing in disciplines we must include explanatory accounts of why disciplines write the way they do (Bhatia, 2004). As Hyland (2000: xiii) explains, academic writing is more than proclaiming research; it is "evidencing a sophisticated awareness of how disciplinary cultures textualize that research into knowledge." This awareness, we believe, can only help novice writers write more successfully in the discipline.

This study suggests that teachers and students of writing must pay much more attention to the structural and rhetorical organization of RAs in each discipline. Students require structural information to help them identify and understand the key markers that constitute an effectively organized argument and structural framework for RAs in their disciplines. This study also shows that macro-rhetorical information is important in learning to write in a particular discipline because it enables writers to connect traditions in the discipline with choices about structure in its genres.

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Chapter 3: "Let G = (V, E) Be A Graph": Turning the Abstract into the Tangible in Introductions to Mathematics Research Articles

3.1. Introduction

Scholars in various fields have extensively studied the generic structure of research articles (RAs) in numerous academic disciplines, analysing their typical lexico-grammatical features and organizational patterns. One aim has been to work out the writing conventions in disciplines as social practices of discourse communities. Scholars have examined article abstracts (in biology, Samraj, 2005; in sciences, Hyland, 2000; in applied linguistics and educational technology, Pho 2008), results sections (in sociology, Brett, 1994; in chemistry, Bruce 2009), methods sections (in management, Lim, 2006), and discussion sections (in social sciences, Holmes, 1997; in physics, Parkinson, 2011; in dentistry, Basturkman, 2009, 2012). The introduction sections have received the most attention following Swales' *Create A Research Space* model (for example, in social sciences, Crookes, 1986; in software engineering, Anthony, 1999; in biology, Samraj, 2002; in applied linguistics, Yang and Allison, 2003; Ozturk, 2007; in sport science and medicine, Zeng, 2009; in agriculture, Del Saz-Rubio, 2011; in civil engineering, Kanoksilapatham, 2011; and in management, Lim, 2012). These studies observed no striking differences in the structure of RA introductions, although they did catalog variations: (1) the absence of a move or step in extended background sections in social sciences; (2) a cycling pattern in biology literature reviews; (3) variations in move patterns in applied linguistics sub-disciplines; and (4) the use of definitions to illustrate difficult concepts in software engineering.

Despite the study of RAs in many disciplines, little attention has focused on mathematics. Existing research includes analyses using functional grammar and discourse analysis (Barton, 2008; Halliday, 2004; Huang & Normanda, 1997; Morgan, 1998, 2005;

O'Halloran, 2005; Pimm, 1984, 2004; Rowland, 1992, 1994, 1999, 2000; and others), focusing largely on textbooks and classroom discourse in school settings. Researchers have examined the use of imperatives, nominalizations, and passive constructions, and algebraic symbolism and discourse markers to express deductive reasoning. Ernest, in his preface to Morgan (1998), offers one reason as to why analyses of move structure in mathematics have been neglected, noting that language is sometimes seen as playing a descriptive rather than a constitutive role in mathematics. He argues that an 'absolutist epistemology' (Morgan, 1998, p. *ix*) in mathematics grants it a 'superhuman real' (Morgan, 1998, p. *ix*) and privileges mathematical thought and cognition over mathematical talk and text. Another very likely reason is the intimidating nature of mathematics RAs for non-specialists, that is applied linguists doing discourse analysis, as opposed to the compelling nature that math RAs have for the math community. Though it is an established strategy in genre analysis to solicit advice from disciplinary informants on the disciplinary conventions as well as RA content, some discourse analysts believe that, ideally, they should understand everything and at a deep level. For researchers who hold this view, the symbolic language that dominates mathematical discourse may add to its intimidating nature. The result is that applied linguists with a background in mathematics might show interest in the topic.

However, interest in the topic is slowly emerging. Lin and Evans (2012), for example, have considered applied mathematics in a cross-disciplinary study of the overall macro-organization of RA structure. A further recent study of math RAs focused on stance and engagement (McGrath & Kuteeva, 2012). Although these studies offer new insights into aspects of mathematics RAs, they do not take into account the rhetorical structure of RAs in the discipline. To address this gap, we undertook a study of RAs in mathematics. In Graves et al. (2013) we reported on the macro-organizational structure of RAs in mathematics. This article reports the variations uncovered by our analysis in move structure and order in introductory sections in mathematics RAs as compared with the 1990 and 2004 *Create A Research Space* models. We speculate that these variations arise out of the hypothetical nature of mathematics as a field, the argumentative requirements of mathematical discourse, and the ideological belief that mathematical findings are assumed to be valid. These variations in structure and move order have implications for EAP, ESP, and graduate writing instruction that uses a genre-based approach and includes mathematics students.

3.2. Methods of Data Collection and Analysis

The sample from which our conclusions are drawn was a corpus of 30 research articles from five journals targeting a readership of research mathematicians and practitioners (see Appendix A for a list of articles and journal titles). The criteria for journal selection were representativeness (defined as publishing outstanding research topics in the discipline), reputation, and accessibility. The research mathematicians consulted in this study recommended seven journals in discrete math; two were excluded because they appear in print only. We selected discrete mathematics for our subject area because this was the area of expertise of several of our informants. The chosen journals are indexed by ISI Web of Knowledge, have high impact factors, and are easily accessible online.

Using a stratified random sampling method, we chose six articles from each journal. The period of publication covered 2007 to 2009. Author, paper type, issue, and year of publication were the strata used to sample the data. To avoid introducing elements of subjectivity based on a given author's style or on the idiosyncrasies of a single journal issue, we selected only one article from an author and only one article per issue. Only original research articles were included. One article in our sample turned out to be a review article. When close examination of the rhetorical structure of the article showed marked structural variation from that of the original RA, we replaced it with an original RA.

We selected mathematics as the focus for this project because of the relative lack of attention it has received in genre analysis in applied linguistics. One of the authors has some background in mathematics, having taken university courses; another author has extensive experience analysing rhetorical structures in science discourse (including mathematical biology and biostatistics) in dissertations and RAs.

Article topics were selected to span the branches of discrete mathematics with pure or applied orientations. We decided not to distinguish between applied and pure mathematics as a major stratum in our sampling in this study based on the rationale that this distinction is often difficult to make and lacks a clearly argued epistemological basis. According to our informants, although many mathematicians are academically affiliated with pure, applied, or even client engineering departments, there is still no consensus as to the borderline between pure and applied math research. Indeed, the term 'applied' in research mathematics is a fuzzy modifier. At one extreme, it seems different from pure mathematics where it refers to actual immediate applications already present at the forefront in subjects like image processing and computer modelling. At the other extreme, it differs very little from pure mathematics when it answers questions posed by particular practical problems, those problems that have been so fascinating to mathematicians and so broadly studied by them that the problems ultimately extend beyond the original application context. Examples of this second extreme are the theory of differential equations and graph theory.

Other arguments point out that mathematics is never a pure and abstract creation—rather any topic can be traced back to some practical problem. A moderate view argues that mathematics is concerned with solving problems to answer questions. The difference, as far as the question is concerned, is that pure mathematics answers questions in mathematics whereas applied mathematics answers questions outside mathematics. However, both disciplines translate the questions, be they abstract or real world, into mathematical problems. In this case, the difference is not one of content or problem, but rather one of motivation.

According to our informants, pure and applied mathematics are not distinct disciplines; rather mathematics is a continuum with pure and applied ends. Based on these ongoing debates about pure versus applied disciplines in mathematics discussed by mathematicians informing this study, we did not distinguish between 'pure' and 'applied' in sampling the 30 RAs that make up the corpus of the present study.

A corpus of 30 RAs seems sufficient to support a critical genre analysis of schematic structure and the communicative purposes in the articles, since our goal was not quantitative analysis. Here we use 'critical' in the sense suggested by Bhatia (1993, 2004). Thus, we seek to both describe writing conventions and the rhetorical structures and lexico-grammatical signals for these structures, and to explain the rationale behind generic regularities, that is, 'why' members of discourse communities write the way they do. Table 3.1 summarizes corpus details. Having decided on the papers comprising the corpus, we assigned each paper a reference code consisting of an abbreviation driven from the journal initials followed by a numerical value. We use this reference code to identify individual articles in the corpus throughout this paper. The list of the papers comprising the corpus, their code, and reference features is presented as Appendix A.

To gain insight into the 'why' of mathematics discourse, we consulted four disciplinary specialists to comment on the discipline, its process of research, and the structure of argument in RAs in mathematics. All informants are university professors: two in pure mathematics, one in applied mathematics, and one in philosophy of mathematics. They are highly qualified, published extensively in their disciplinary specialties (as indicated by their curriculum vitae), and are self-reflective, equipping them to understand, respond to our questions, and comment on the intended purpose of different argumentative strategies. Two of the same informants were asked to act as the inter-rater validating the results from our move analysis. First, they explained the mathematical problems addressed in each paper and the introductory material required for each problem. Then, following a brief introduction to ESP genre theory that focused on the *Create A Research Space* model, the two inter-raters checked samples of the results to validate our conclusions. The sample size checked by the first inter-rater was 20 RAs, and the second inter-rater checked 26 RAs. Therefore, the results from every RA were validated by at least one inter-rater; in addition, two inter-raters checked the samples from 16 RAs to verify their agreement with our conclusions. Agreement among inter-raters was over 90 percent; the source of disagreement lay in two areas: (1) the terminology used to designate 'indicating a gap in previous research' in mathematics RAs, and (2) the prominent role of presumptions and definitions in mathematics RAs (the focus of this article). All of the informants in the interviews (and the inter-raters when verifying the results) disagreed with the researchers' coding of the rhetorical function of presumptions and definitions in math RAs. We had followed Swales (2004) and identified mathematical definitions as part of Move 3 where authors present their work, but our ethnographic informants noted that this inclination minimized the prominent role of this function in mathematics RAs. Their accounts of disciplinary discourse further supported the ethnographic data, leading to our decision not to assign a supplementary function to definitional elements as part of Move 3; this decision is thoroughly explained in the results section. As noted, a second disagreement arose concerning the use of indicating a gap in previous research to establish the research niche. According to specialist informants, the term *gap* in mathematics implies a flaw or mistake, rather than insufficient research in existing problem-solution techniques or proofs. The informants further argued that gaps—in the sense used in applied linguistics—are rarely used in mathematics. As such, the two inter-raters suggested that we reconsider the rhetorical representation of mathematical niches in accordance with disciplinary culture (the manuscript for this discussion is in preparation).

Table 3.1. Details of the corpus

Number of RAs	Number of pages	Number of authors	Publication date	Number of countries represented
30	344	67	2007–2009	21

We used both the 1990 and 2004 *Create A Research Space* (CARS) models as points of departure in this study (see Figure 3.1.) because they are widely used to describe the rhetorical structure of disciplinary genres (Basturkman, 2009, 2012; Bruce, 2008, 2009; Del Saz-Rubio, 2011; Hafner, 2010; Hirano, 2009; Kanoksilapatham, 2011; Lim, 2010, 2012; Zeng, 2009). We also used Swales' definition of a move as "a discoursal … unit that performs a coherent communicative function in a written or spoken discourse" (2004, p. 228). We assigned each discoursal unit in the introductory sections a numerical move (that is, 1, 2, or 3). While our focus was the macrostructure, we also used Swales' concept of 'step' where appropriate. Our labels are drawn from both *Create A Research Space* models.

Swales' CARS model 1990	Swales' CARS model 2004
Move 1: Establishing a territory: **Step 1:** Claiming centrality **Step 2:** Making topic generalization **Step 3:** Reviewing items of previous research	**Move 1**: Establishing a territory (citations required)

Move 2: Establishing a niche: **Step 1A:** Counter-claiming or **Step 1B:** Indicating a gap or **Step 1C:** Question-raising or **Step 1D:** Continuing a tradition	**Move 2**: Establishing a niche (citations possible) via **Step 1A**: Indicating a gap or **Step 1B**: Adding to what is known **Step 2:** (optional) Presenting positive justification
Move 3: Occupying the niche: **Step 1A:** Outlining purposes or **Step 1B:** Announcing present research **Step 2:** Announcing principal findings **Step 3:** Indicating RA structure	**Move 3**: Presenting the present work (citations possible) **Step 1:** (obligatory) Announcing present research descriptively **Step 2***:(optional) Presenting RQs or hypotheses **Step 3:** (optional) Definitional clarifications **Step 4:** (optional) Summarizing methods **Step 5:** (PISF**) Announcing principal outcomes **Step 6:** (PISF) Stating the value of the present research **Step 7:** (PISF) Outlining the structure of the paper (**PISF: Possible in some fields)

Fig. 3.1. We used the 1990 and 2004 Create A Research Space models as points of departure for our analysis.

We used lexico-grammatical signals as well as content information to characterize moves. Occasionally one statement fulfilled two communicative purposes, as in Example 1:

> **Example 1.** In this paper, we extend the Hoffman bound to larger values of *k*. We also propose a convex quadratic upper bound on the size of a *k*-regular induced subgraph and give a characterization of those graphs for which this bound is attained. (JCO1, p. 456)

Both sentences announce the findings. However, 'extending a bound' is a technical phrase meaning 'improve' an existing result (that is, Hoffman bound); therefore, the passage also 'evaluates the findings'. Since both communicative purposes are persuasive, we included both functions in our analysis. By triangulating the textual analysis, input from informants, and the published literature on how mathematicians generate knowledge, we accurately analyzed the move–step structure of the corpus.

3.3. Results

In this section, we report the variations observed in sectioning the introductory material in mathematical RAs. We will show how the variations arise out of authorial preference or topic-driven necessity for highlighting the presumptions and definitions as well as the existing results, which are of immediate application within the RA. We then describe the move structure of Introduction sections by reporting the number and frequency of rhetorical moves identified. This analysis is complemented by a description of move sequence in the third subsection. As is demonstrated, significant variations are observed in Introduction sectioning, move structure, and sequence as compared with the 1990 and 2004 *Create A Research Space* models.

3.3.1. Bordering/Division of Introductions in Mathematics

All of the articles except DAM5 labeled their opening sections "Introduction." These sections introduced the general paper topic and summarized published literature on the topic.

Ten (from all five journals) included a total of 13 additional sections with content headings that appeared between the labeled introduction and the labeled results. These were not Method sections. Subsections between introduction and method sections have been reported in geology RAs (Dressen & Swales, 2000).

We examined all additional sections to determine their function. Here we exemplify the findings by discussing DM2 and DM3. The first example, DM2, used "Known results on total domination" to head the second section. Most of it reviews published results on total domination in graphs (the paper topic), yet the last paragraph (Example 2) uses Moves 2 and 3:

> **Example 2.** It is therefore a natural question to ask whether the upper bound of Theorem 5 can be improved if we restrict G to be a connected claw-free cubic graph of order at least 10. **[Move 2, 'Question raising']** In this paper, we show that under these conditions the upper bound on the total domination number of G in Theorem 5 decreases from one-half its order to five-elevenths its order. **[Move 3, 'Announcing principal findings']** (DM2, p. 3493)

This section, then, reviews the literature on the problem (Move 1, Step 3, 1990 model), raises a question, and announces the finding, all communicative purposes suggested for introduction sections. It appears to complement the 'Introduction', which contains one reference to a published result and several definitions. Regarding this sectioning strategy, one informant said, "Collecting chains of existing results *in a separate section* helps us to highlight the immediate context of research. It also facilitates retrieval of background information by the author and the reader throughout the paper" (personal communication with disciplinary informant, italics added). In DM2, then, the complementary introduction performs the communicative functions typical of introduction sections as suggested by the *Create A Research Space* models.

In contrast, DM3, our second example, presents three sections, 'Introduction', 'Definitions', and 'Known results and more definitions'; the complementary introductions ('Definitions' and 'Known results . . .') highlight some published results and several definitions, topic-specific terminology, and symbols. Our informants noted that the objects studied in DM3 required successive definitions and related notations: Separate sections allow the authors to highlight the objects' important features.

These examples suggest that definitions and review of published results can appear in either the 'Introduction' or independent sections to highlight details foundational to the mathematical objects discussed. Our informants noted that the complementary introductions were driven by the subject matter or author choice. Because these additional sections performed conventional functions reported for introduction sections in the *Create A Research Space* models and were not dictated by journal preference, we concluded that they were meant to complement the 'Introduction', so we treated them as extensions of the RA 'Introduction' in our analysis.

3.3.2. Move Structure in Introductory Material in Mathematics

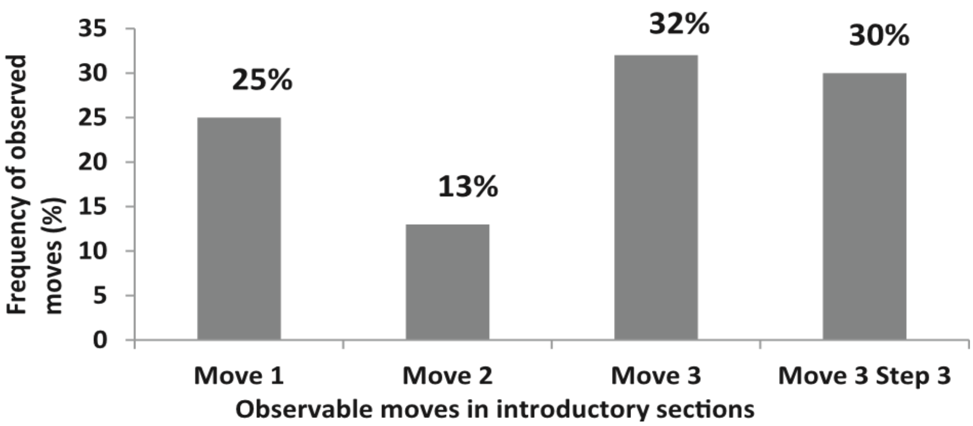

Fig. 3.2. Frequency of Moves 1, 2, 3 & M3S3 in the corpus.

M3S3 appears as a single step in the 2004 *Create A Research Space* model; however, in our corpus, it consists of multiple steps aimed at securing agreement on presumptions related to the topic. These steps occurred in every 'Introduction' and complementary introduction in the corpus, suggesting their importance to the presentation of knowledge in mathematics. Example 4 (DM2) illustrates how Moves 1 and M3S3 are deployed as a series of steps. The first paragraph (M1) consists of two steps: reviewing items of previous research (with citations) and claiming topic centrality and recency ("is now well studied"). The second paragraph illustrates the typical functions of M3S3 executed as three steps: presenting assumptions, introducing notations, and defining objects/terms:

> **Example 4.** Total domination in graphs was introduced by Cockayne et al. [4] **[Reviewing items of previous research]** and is now well studied [**Recency**] in graph theory **[Claiming topic centrality]** (see, for example, 3,7,11]). The literature on this subject has been surveyed and detailed in the two books by Haynes et al. [9,10].**[Reviewing items of previous research]**
>
> Let $G = (V, E)$ be a graph with vertex set V and edge set E. **[1. Presenting assumptions + 2. Introducing notations]** A *total dominating set*, denoted by TDS **[2. Introducing notations]**, of G with no isolated vertex is a set S of vertices of G such that every vertex is adjacent to a vertex in S (other than itself). **[3. Defining objects/terms]** The *total domination number* of G, denoted by $\gamma_t(G)$ **[2. Introducing notations]**, is the minimum cardinality of a TDS. We call a TDS of G of cardinality $\gamma_t(G)$ a $\gamma_t(G)$-set. **[3. Defining objects/terms] (**Italics in original, DM2, p. 3491)

In the second paragraph the authors use the directive cognitive verb, "Let," to prepare readers for the imminent introduction of mathematical concepts including definitions and relevant notations.

In Example 5 the authors initiate their argument using M3S3 and employ four steps: the three steps from Example 4 plus referring to items of previous research:

Example 5. In this paper, G is a simple connected graph with vertex set V(G) and edge set E(G) **[1. Presenting assumption]** (briefly V and E). **[2. Introducing notations]** For every vertex vV, the *open neighborhood* N(v) is the set {uV(G)uvE(G)} and the *closed neighborhood* is the set N[v]=N(v){v}. **[3. Defining objects/terms]** The *open neighborhood* of a set SV is the set N(S)=vSN(v), and the *closed neighborhood* of S is the set N[S]=N(S)S. **[3. Defining objects/terms]** A vertex vV *dominates* itself and its neighbors. A subset S of vertices of G is a *dominating set* if N[S]=V (that is, S dominates V). The domination number γ(G) is the minimum cardinality of a dominating set of G, and a dominating set of minimum cardinality is called a γ-set [8]. A subset S of V is a *double dominating set* of G if S dominates every vertex of G at least twice [5]. The *double domination number dd*(G) is the minimum cardinality of a double dominating set of G. A *dd(G)-set* is a double dominating set of G with cardinality dd(G). **[3. Defining objects/terms]** Throughout this paper when we talk about dd(G) we assume that G has no isolated vertices. **[1. Presenting assumption]** For a more thorough treatment of domination parameters and for terminology not presented here, see [8] and [12]. **[4. Referring to items of previous research]** (DAM1, p. 1700)

In Paragraph 2, seven definitions and notations are presented over six clauses, one presumption is established about the object's features ("we assume that"), and two citations conclude the passage. The detail in the presumptions and the imperative in the final sentence ("see") build a strong argument. To follow the complex reasoning, readers and authors must share understanding of the central concepts, their characterizing features and their corresponding symbols. Nearly half of the papers (13) begin by defining objects, suggesting its importance as a rhetorical strategy in mathematics.

These examples suggest that M3S3 is an important move in mathematics that is independent of, but related to, well-established moves in the *Create A Research Space* models. M3S3 is related to Move 1 in the sense that it establishes presumptions for hypothetical objects of a topic. Move 1 clarifies how the literature relates to the topic and establishes the territory, but M3S3 illuminates those assumptions that undergird and instantiate the abstract concepts manipulated in the research. M3S3 is also related to Move 2 because it further refines the niche. However, Move 2 is absent in 10 RAs, suggesting it is optional in this field. Finally, M3S3 is most strongly related to Move 3 in that it paves the way for or logically proceeds from that move, preparing readers for new results deduced from the definitions. In our data M3S3 could follow or precede any move, suggesting that although it is related, it can be deployed independently.

Further, M3S3 in introductory steps in mathematics RAs arises from a series of steps:

1. presenting assumptions
2. introducing notations
3. defining objects/terms
4. reviewing/referring to items of previous research

The first three steps appear in every instance of the new move in our corpus, while Step 4 seems optional, present in 19 of 65 (29.2%) instances. Based on these steps and the move's role in organizing argument in math RAs, we characterize this new move's communicative purpose as 'Establishing presumptions', represented from this point onwards as 'm*p*'.

3.3.3. Order of Moves in Introductory Material in Mathematics Ras

A third notable feature of the introductory material in mathematics RAs is that moves are often used multiple times and not in the numerical order represented by the *Create A Research Space* models. Only two articles follow the chronological pattern: SIAM1 uses the three moves in order (twice): [1-2-3-m*p*-3-1-2-3-m*p*-3-m*p*-3]; SIAM4 uses it once [1-2-3-m*p*-3-m*p*-3-1]. Neither article uses a complementary introduction, but they both contain repeated uses of the moves. Most RAs in the corpus contain between five and nine moves.

To initiate arguments for new knowledge claims, mathematicians primarily use M1 (n=14, 46.7%) or m*p* (n=13, 43.3%). Moves 2 and 3 were used once (3.3%) and twice (6.7%), respectively, as initiating moves (see Figure 3.2). Clearly mathematicians prefer to begin arguments by either establishing the territory or introducing presumptions to build reader comprehension (m*p* also implies the territory). This preference suggests that m*p* is essential to understanding the territory of research (M1) and describing the problem (M2). It can also form the foundation for generating new results (M3). Based on its independent status in the rhetorical organization, its rhetorical relationship to surrounding moves (including deriving new deductions), and its step structure, we suggest that m*p* attains move status in mathematics RAs.

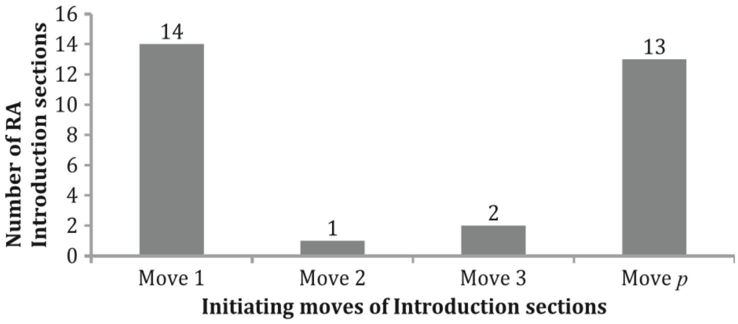

Figure 3.3. Frequency of the initiating move in the corpus

Moves 1, *p*, and 3 are used to conclude introductory material in our corpus (M1=2n, M*p*=7n, M3=21n). M1 is used as the closing move when published results are immediately relevant to the problems targeted in the RA (6.6%). When collected into a separate section before the ‘Results’, they create a transition from given-to-new results (deduction). Sometimes m*p* serves as the closing move (23.3%). Not surprisingly, M3 is the most frequently used closing move (76.6%) to propel readers to the rest of the paper where new deductions are presented as solutions to the problem discussed in the introduction.

Our analysis of the move structure of introductions in the corpus shows variations in move order and frequency from the *Create A Research Space* models. The absence of M2 and relative underuse of M1 in some RAs suggest that establishing agreement on mathematical concepts (m*p*) is valued over establishing a territory or a niche. And while mathematicians may begin ‘Introductions’ with M1 to establish a territory (46.7%), they nearly as often begin by establishing presumptions (43%) to explain hypothetical objects and secure agreement on terminology and notation (see Figure 3.3).

3.4. Discussion

Three significant variations to the *Create A Research Space* models were identified in this analysis of move structure in introductory material in mathematics RAs: (1) the complementary introductions that authors use to emphasize particular information; (2) the prevalence of ‘establishing presumptions’ in all introductory material; and (3) the lack of an established order for moves in mathematics RAs. The 10 articles with complementary introductions use them to cite published results pertinent to the new results and to establish presumptions about the mathematical objects treated. The remaining articles accomplished these functions in one ‘Introduction’. Our informants explained that authors separate these sections to highlight information that readers need to grasp the authors’ new results.

In addition to using a separate introductory section for citing the important published results following the start of the introduction section, a prevalent activity in all introductory material is ‘establishing presumptions’, leading us to hypothesize both activities as central to mathematical argument. Anthony (1999) originally proposed incorporating ‘definition’ into the *Create A Research Space* model (Swales did so in his 2004 revision). However, Anthony characterized definition as parenthetical explanation in his work on software engineering, suggesting that this activity performs different roles among disciplines. Our data indicates that in mathematics definition is crucial to constituting the abstract concepts at the heart of this field, making it central to contextualizing research and contributing to comprehension and persuasion in this field. As such, we argue that definition (in its broader manifestation, 'establishing presumptions') warrants status as a 'move' in the mathematics RAs based on four points. First, the prevalence of m*p* in the corpus, as well as its use as an initiating move, suggests it is central in mathematics RAs. Second, it enables mathematicians to share understanding of the objects discussed. Third, it surpasses shared understanding by affording the basis for new insights in mathematics (that is, novel contributions). And fourth, by establishing agreement about mathematical concepts it enables argument, making it central to constructing knowledge in mathematics.

The prevalence of m*p* in the corpus should be examined in light of disciplinary assumptions about knowledge in mathematics. First, M3 is the most frequently used move in the corpus (32%) (Figure 3.2), suggesting that mathematicians prefer presenting their work to either establishing its importance and generality or highlighting the need for new research. This observation aligns with Morgan's (1998, p.2) view that because theoretical mathematicians work on original problems, they expect their audiences "to be genuinely interested in knowing the results and to need to be persuaded of the correctness of the results". But m*p* is the second most prevalent move (30%), as well as the second most prevalent *initiating* move (43.3%) (see Figure 3.3). Jamison explains the historical precedent of m*p* as an initiating move (2000, p.46):

> There is … a nearly universally accepted logical and rhetorical structure to mathematical exposition. For over two millennia serious mathematics has been presented following a format of definition–theorem–proof. Euclid's *Elements* from

circa 300 BC codified this mode of presentation which, with minor variations in style, is still used today in journal articles and advanced texts.

History reinforces the contention that establishing presumptions/definitions is a central move in mathematics RAs, perhaps more important than M2. The optional nature of M2 can be understood within the context of the discipline's goals and traditions. Hardy (1940, p.9) states that mathematical calculation "has a certain character of permanence". Others including Lakatos (1977) and Dieudonné (1992) have noted the infallibility of mathematical findings. Lakatos (1977, p. 142) describes how mathematics transforms itself into infallibility:

> Mathematics is presented as an ever-increasing set of eternal, immutable truths. Counterexamples, refutations, criticism cannot possibly enter. An authoritarian air is secured for the subject by beginning with disguised monster-barring and proof-generated definitions …, and by suppressing the primitive conjecture, the refutations, and the criticism of the proof. The whole story vanishes … while the end result is exalted into sacred infallibility.

When mathematics becomes infallible, activities encompassed in M2, such as counterclaiming (M2S1A, 1990 model) or indicating a gap in previous findings (M2S1B, 1990 model), are rarely necessary.

Many philosophers, scholars, and teachers of mathematics have attested to the importance of establishing presumptions to the activity of the discipline. Philosophers (including one informant) have characterized it as a central element to any mathematical situation (Dieudonné, 1992; Byers, 2007). Mathematics educators including Morgan, Jamison, and our informants note that an abstract concept must be introduced and its properties defined before it can be manipulated: that is, definition catalogs basic properties: "ideally those that are simply stated and have immediate intuitive appeal" (Jamison, 2000, p.

48). Our informants called definitions the building blocks of mathematical practice at any level and of any genre.

But definition also initiates activity: It sets out assumptions that mathematicians use to create new abstract concepts that generate new results (Morgan, 2005). Borasi (1992, p.17), among other scholars, has noted the constitutive nature of definition in mathematics: "the choice of definition makes a significant difference to the process of solution". Morgan (2005, p. 4) calls definition "a deliberate creative act" in mathematics. They note that in mathematics definition goes beyond enumerating a concept's properties to actually *bringing it into existence*: Definition can create new abstract entities. Our informants concurred, noting that definition and notation make it possible to conceptualize a discipline as abstract as mathematics: "without definitions there would be no mathematics" (personal communication with disciplinary informants). Consequently, m*p* (and its steps) is a critical tool for mathematicians to create new knowledge.

'Establishing presumptions' is also central from a rhetorical perspective. Although it contains elements of demonstration and formal logic, mathematical discourse is inherently argumentative (Morgan, 1998). It aims not only to demonstrate calculations but also to gain the readers' adherence to a thesis that presupposes a "meeting of minds" (Foss et al., 2002). Presumptions and facts are essential to establish the grounds for agreement before argument can begin (Foss et al., 2002). These presumptions focus audience attention by making concrete and present concepts that are otherwise abstract and absent. Foss et al. (2002, p.95) emphasize that "to use intangible and abstract starting points successfully, an arguer needs to endow them with presence, which involves selecting certain techniques of presentation". Defining abstract mathematical concepts and introducing them symbolically serve as important presentation techniques in mathematics: Thus they build the argument.

The third variation to the *Create A Research Space* model lies in the lack of an established order for moves in the corpus. Instead, writers arrange their move structure based primarily on the mathematics problem being solved. Because no pattern was present and to avoid the chronology implied by numbers, we designated this new move as *p* ('Establishing presumptions') rather than as a number that would imply order of occurrence.

We should note that this relatively small corpus (30 RAs) limits the extent of our claims, but it is suggestive. Broader generalization of our claims would require further study with larger corpora covering more fields in mathematics.

3.5. Conclusion

This study explored the organization and rhetorical function of the moves in RA introductions in mathematics. It noted several departures from the structures outlined in Swales' *Create A Research Space* models, including the repeated moves and the dynamic and flexible move order that organize arguments in discrete mathematics RAs: Authors organize moves based on the nature of the problem being solved and their goals in persuading readers. We also propose that in mathematics m*p* is a freestanding, frequently used move. It should supplement the move options described in Swales' *Create A Research Space* models for genre specialists working on mathematics.

Writing in mathematics is no different from other disciplines where a sophisticated awareness is required of communicative acts typical within that discourse community. Our conclusions have three implications for teachers of academic writing in classrooms that contain students in mathematics disciplines. Hyland (2002), Bazerman (2001), and Johns (2001) have argued for the importance of scholarship and pedagogy remaining flexible in the face of wide variations among disciplinary discourse conventions. Hyland (2002, p. 87) cautions that

> Research offers no universal solution to many of the issues raised by practice, and implies no single method of teaching or learning writing. We cannot simply apply some neat body of conclusive research or comprehensive theory of writing to ensure success in our classrooms. But theory and research both shape instructional practices in important ways, and it is enlightening to reflect on examples of how research has been best applied.

Our study suggests that existing models for generic structure of RAs need revision to accurately incorporate the conventions and practices of mathematics. Some assumptions that underlie the IMRD structure (the basis for current models) and dictate the focus and the chronology of moves are not reflected in mathematics RAs as a genre. For example, the necessity of establishing a niche (by counterclaiming or overtly indicating a gap) is optional in mathematics because readers assume the gap and that the logic-driven results are valid. Moreover, establishing presumptions (to both turn abstract concepts into tangible ones and to secure common understanding for elaborating new knowledge) is obligatory in mathematics.

A second pedagogical implication arises from the absence in mathematics of the move sequence reflected in existing models. Instructors would want to make mathematics students aware of the flexibility possible when deploying the *Create A Research Space* structure in math RA introductions. Twice the moves in our corpus followed the chronological sequence but more often they appeared 'out of order' and cyclically. Thus, moves were used in any order and often repeated in varying orders throughout the introductory material. This suggests that in spite of the prevalent fallacy, writing math is not a de-personalized absolutist convention; rather variations are discernible within disciplinary conventions. There are lessons for writing classes in turn. Bazerman (2001, p. 28) notes that if writing teachers "support the right opportunities and create the space for students to work through the tensions of their discursive transformation, [they] can witness their integration of knowledges and selves, and their development of complex discursive resources and presentations". Bazerman proposes that the role for writing teachers is to support students in their process of discovering and emulating the discursive and argumentative conventions of their disciplines; students will become members of their field as they integrate this knowledge into their writing practices. The results of our analysis highlight how existing theory might be adjusted to ensure it accounts for the variations in conventions in non-IMRD-based disciplines such as mathematics that some students would need to become successful participants in these fields.

Third, the cycled move structure suggests that instructors must help students understand the need to recognize the move sequence dictated by the nature of the problem in their research work. Bazerman (2001, p.25) notes that "[i]f students want the power of the discipline, they have to develop the appropriate ways of expressing and using the knowledge. Students need to learn to speak with the voices recognizable as legitimate, warrantable, and powerful within the disciplines and professions", part of which requires recognizing the move sequence being dictated by the research problem. While the latter knowledge is probably best gained by students working with research supervisors, writing instructors should ensure that their students understand how generic conventions in any field are always driven by disciplinary assumptions about knowledge. Bazerman further argues that students must develop their critical faculties, "for learning academic writing entails learning to wield tools of symbolic power for immediate rhetorical purposes" (2001, p. 25). This study has applied a critical lens to genre analysis to highlight how the processes of knowledge creation in mathematics shape the structure and argument of its RAs.

Our concluding remark addresses the power of genre studies in exploring disciplinary discourses. This research shows that genre analysis is a strong methodological approach that can reveal the epistemological conventions even in disciplines that appear particularly impenetrable to members of other disciplinary communities including discourse analysts, provided the textual data is complemented and triangulated by ethnographic data acquired from disciplinary experts. At the same time, in the progress of studying a discipline with challenging subject matter, researchers gain some understanding of the discussion that makes the discipline more accessible and less intimidating. In addition, if researchers can look past the intimidating content to the natural language used to express ideas, they find the grammatical structures work in similar ways to all language.

3.6. Chapter References

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Appendix A: Articles in the corpus

DM1. Hoffmann-Ostenhof, A. (2007). A counterexample to the bipartizing matching conjecture. *Discrete Mathematics, 307*(22), 2723–2733.

DM2. Favaron, O., & Henning, M. A. (2008). Bounds on total domination in claw-free cubic graphs. *Discrete Mathematics, 308*(16), 3491–3507.

DM3. Gibson, R. G. (2008). Bipartite graphs are not universal fixers. *Discrete Mathematics, 308*(24), 5937–5943.

DM4. Meierling, D., & Volkmann, L. (2009). On the number of cycles in local tournaments. *Discrete Mathematics, 309*(8), 2042–2052.

DM5. Kelarev, A., Ryan, J., & Yearwood, J. (2009). Cayley graphs as classifiers for data mining: The influence of asymmetries. *Discrete Mathematics, 309*(17), 5360–5369.

DM6. Bielak, H. (2007). Chromatic properties of Hamiltonian graphs. *Discrete Mathematics, 307*(11–12), 1245–1254.

DAM1. Atapour, M., Khodkar, A., & Sheikholeslami, S. M. (2007). Characterization of double domination subdivision number of trees. *Discrete Applied Mathematics, 155*(13), 1700–1707.

DAM2. Bjlint, V. (2008). A distance approximating trees. *Discrete Applied Mathematics, 156*(14), 2740–2752.

DAM3. Fujita, S., & Nakamigawa, T. (2008). Balanced decomposition of a vertex-colored graph. *Discrete Applied Mathematics, 156*(18), 3339–3344.

DAM4. Jarry, A., & Pérennes, S. (2009). Disjoint paths in symmetric digraphs. *Discrete Applied Mathematics, 157*(1), 90–97.

DAM5. Feder, T., Hell, P., & Huang, J. (2009). Extension problems with degree bounds. *Discrete Applied Mathematics, 157*(7), 1592–1599.

DAM6. Chen, W. Y. C., Li, N. Y., & Shapiro, L. W. (2007). The butterfly decomposition of plane trees. *Discrete Applied Mathematics, 155*(17), 2187–2201.

G&C1. Gould, R., & Whalen, T. (2007). Subdivision extendibility. *Graphs and Combinatorics, 23*(2), 165–182.

G&C2. Bernáth, A., & Gerbner, D. (2007). Chain Intersecting Families. *Graphs and Combinatorics, 23*(4), 353–366.

G&C3. Chandran, L. S., Kostochka, A., & Raju, J. K. (2008). Hadwiger Number and the Cartesian Product of Graphs. *Graphs and Combinatorics, 24*(4), 291–301.

G&C4. Ghebleh, M., Goddyn, L. A., Mahmoodian, E. S., & Verdian-Rizi, M. (2008). Silver cubes. *Graphs and Combinatorics, 24*(5), 429–442.

G&C5. Bonomo, F., Durán, G., Maffray, F., Marenco, J., & Valencia-Pabon, M. (2009). On the coloring of cographs and p4-sparse graphs. *Graphs and Combinatorics, 25*(2), 153–167.

G&C6. Cui, Q., Haxell, P., & Ma, W. (2009). Packing and covering triangles in planar graphs. *Graphs and Combinatorics, 25*(6), 817–824.

JCO1. Cardoso, D., Kamiński, M., & Lozin, V. (2007). Maximum k-regular induced subgraphs. *Journal of Combinatorial Optimization, 14*(4), 455–463.

JCO2. Irving, R. (2008). Stable matching problems with exchange restrictions. *Journal of Combinatorial Optimization, 16*(4), 344–360.

JCO3. Goddard, W., & Henning, M. (2007). Restricted domination parameters in graphs. *Journal of Combinatorial Optimization, 13*(4), 353–363.

JCO4. Vanetik, N. (2009). Path packing and a related optimization problem. *Journal of Combinatorial Optimization, 17*(2), 192–205.

JCO5. Liu, C., Song, Y., & Burge, L. (2008). Parameterized lower bound and inapproximability of polylogarithmic string barcoding. *Journal of Combinatorial Optimization, 16*(1), 39–49.

JCO6. Messinger, M., & Nowakowski, R. (2009). The robot cleans up. *Journal of Combinatorial Optimization, 18*(4), 350–361.

SIAM1. Berman, K., A. (2007). Locating servers for reliability and affine embeddings. *SIAM Journal on Discrete Mathematics, 21*(3), 637–646.

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Chapter 4: 'Since Hadwiger's Conjection . . . Is Still Open': Establishing a Niche for Research in Discrete Mathematics Research Article Introductions

4.1. Introduction

The genre of research article (RA) continues to motivate much research following Swales' (1990) *Genre Analysis* and his proposal of the *Create A Research Space* model for research article introductions (RAIs). Since then, research has focused on either verifying the *Create A Research Space* structure in RAIs from different disciplines (e.g., Anthony, 1999; Ozturk, 2007; Samraj, 2002) or examining sections other than the Introductions to create similar models (Basturkmen, 2012; Bruce, 2008).

Descriptions of the rhetorical structures and accounts of the lexico-grammatical features representing such structures have been offered; explanations of these aspects have emerged more slowly. Existing explanatory genre analysis aims to answer this fundamental question: why do disciplines write the way they do? Various genre theory scholars explore how institutions and disciplines connect with and shape the nature and structure of genres (Bhatia, 1993, 2004; Hyland, 2000; Swales, 1998). Researchers studying argument structure in mathematics RAs have shown how these authors deploy the RA resources differently to argue their results. Among them, Lin and Evans (2012, p. 157) report that applied mathematics RAs lack the "frequently-used major structural patterns." Similarly, Graves et al. (2013) and Kuteeva & McGrath (2015) report that mathematicians eschew the traditional "hour-glass structure" of Introduction-Methods-Results-Discussion (IMRD) used in empirical disciplines as described by Swales (1990), instead favouring logical arguments similar to Tarone et al (1998) regarding astrophysics RAs. Graves et al. (2014) and Kuteeva & McGrath (ibid) also note multiple introductory sections where authors establish and explain their presumptions in the RA and cycle through Swales' (1990, 2004) moves to *Create A Research Space* (CARS).

Briefly, the *Create A Research Space* model describes RAI structure as having three moves: establish a territory, establish a niche, and occupy the niche. Each move contributes to the argument structure of the RA (See Appendix A for Swales' 1990 and 2004 *Create A Research Space* models). Lyda and Warchal (2014) highlighted the move of 'establishing a niche', hereafter move EN, in RAs as an important concept in academic argument with varying prevalence across disciplines. Research established that biology (Samraj, 2002), biochemistry (Kanoksilapatham, 2005), and second language writing (Ozturk, 2007) employ this move 70-95% of the time. In some of this and more recent research, move EN is also identified, often recursively, with move 1. Table 1 summarizes research since 2008 on 'establish a niche' in eleven disciplines (excluding intercultural rhetoric).

Table 4.1. Establishing a Niche (EN) across disciplines: summary of recent research

Discipline	**Reference**	**EN**
Computer science	Shehzad (2008)	EN: obligatory (94.64%) Steps: 1*- gap indications: - short (26%) - lengthy (25%) - embedded within move 1 (23%) 2- multiple act statements (10%) 3- reported in previous research (5%) 4- research questions/rationale (5%) 5- extension of previous work (3%) 6 - contrasting ideas (3%)

Agricultural science	Del saz Rubio (2011)	EN: obligatory (100%) Steps: 1- Gap signalling (the most prominent step) 2- Present positive justification (the 2nd frequent step) 3- combine the two steps	Alterations between move 1 and 2 were common; fronted move 3 embedded within move 1 was observed; This implies that move 2 was a deferred move in the samples.
Education	Chang and Schleppegrell (2011)	EN: obligatory (100%) Variations in realization of move 2 through deploying different linguistic devices were noted.	
Information system (2 sub-disciplines)	Kwan et al. (2012)	EN: obligatory (100%) NE strategies: 1- Suggest a solution 2- Counter-claim 3- Provide a positive appraisal 4- Raise a question	5- Indicate a gap 6- Indicate a requirement 7- Claim relevancy of importance of a concept 8- Make inferences Sub-disciplinary preferences were reported in the deployment of certain strategies informed by the dominant research paradigms in each field.
Management	Lim (2012)	EN: obligatory (100%) Steps: 1- Indicate a gap (96.7%)	2- Add to what is known (20%)

		-Highlight the complete absence of research - Stress insufficient research - Reveal a limitation in previous research - Contrast conflicting previous research Findings	Cyclicity was identified especially in deployment of gap indications.
Chemistry	Stoller and Robinson (2013)	Identify the gap(s): obligatory (100%) Steps: No step hierarchy is suggested. However, some identified examples are: - an unanswered question - a poorly understood or understudied area	– a step that needs to be take – a procedure that needs to be improved – a hypothesis/observation that requires validation
Applied linguistics and psychology	Bruce (2014)	NES: present in all Applied Linguistics sub-corpora and in 14 out of 15 RAs in Psychology sub-corpora.	– highlight conflicting findings – point to the lack of certain studies Move 2 was identified mostly recursively with move 1.

		Steps: No step hierarchy is suggested. However, some identified examples are: − indicate a gap in the previous research − question the generalizability of previous studies
Discrete mathematics	Graves et al. (2014)	EN: identified in 20 RAs. Steps: No step is reported Recursive deployment of move 2 identified.
Forestry	Joseph et al. (2014)	EN: obligatory (100%) Steps: 1- Indicate a gap (95%) a) indicate total gap, b) indicate insufficient research, c) lack of sufficient number of research, d) limitation in the present knowledge 2- Present positive justification (»50%) a) highlight the need for research in b) highlight the merits of the species chosen for the study, c) further knowledge in the field Cyclic patterning was considerably common; the commonest cyclic pattern consisted of '1-2' sequences observed in 12 out of the 20 RAs; the recurrence of Move 2 was observed in 70% of the corpus.

		relation to real world problems,	
Engineering (3 sub-disciplines)	Kanoksilapatham (2015)	EN: conventional (72% to 86%) Steps: - Indicate a gap (the most frequent) - Add to what is known (less frequent)	- Present positive justification (the least frequent in two sub-disciplines but more frequent in another)
Pure mathematics	Kuteeva & McGrath (2015)	EN: (identified in 10 RAs) Steps: 1- Question raised (in 4 RAs); could be a) a question by the authors, or b) one raised in past research.	2- Indicate a gap (in 6 RAs) by a) point to the limited generalizability of a theorem
Law	Tessuto (2015)	A) EN in Introduction sections: almost compulsory (69%) Steps: 1- Indicate a gap (48%) 2- Add to what is known (34%) 3- Present positive justification (18%)	B) EN in Background review sections: quasi-obligatory (88%) Steps: 1- Indicate a gap (37%) 2- Add to what is known (34%) 3- Present positive justification (29%)

* Numbers are assigned to steps/strategies for reader convenience of in-text referencing; they are not necessarily used in the original source.

Table 1 compares cross-disciplinary frequencies of move EN and the diversity of functional labels used to identify the rhetorical steps (as available from the original research). The move's frequency across disciplines varies from 100%, suggesting obligatory status, to about 45%, suggesting optional status. While move EN appears to be conventional in many disciplines, it is less salient in pure mathematics, where it was identified in 10/22 RAs (Kuteeva & McGrath, 2015). The authors suggest it is optional in this field.

These studies also employ diverse functional labels to identify the rhetorical steps of move EN (Table 1), ranging from two in management (Lim, 2012), forestry (Joseph et al., 2014), and pure mathematics (Kuteeva & McGrath, 2015) to eight in information systems (Kwan et al., 2012). This range suggests that, despite Swales' (2004) decision to condense these labels in move EN, other researchers prefer the steps, possibly because they afford clearer distinctions among cross-disciplinary differences. *Indicate a gap* is reported in all the disciplines, although with varying frequency. Stoller and Robinson (2013) found '*indicate a gap'* was the sole option for move EN. However, other studies identify sub-steps (Joseph et al., 2014) and detail their linguistic representation (Lim, 2012; Shehzad, 2008). Multiple researchers note move EN recurrence (Bruce, 2014; Del saz Rubio, 2011; Graves et al., 2014; Joseph et al., 2014; Lim, 2012). In agricultural science, Del saz Rubio (2011) reported instances of move 3 embedded within move 1, suggesting the possibility that sometimes move EN is deferred.

Shehzad (2008) and Lim (2012) both focused on the linguistic indicators of move EN in computer science and management, respectively. Shehzad (2008) notes gap indication as the most prominent step and identifies three categories of linguistic indicators. Lim (2012) found that Swales' 2004 model best fit RAIs in management. He notes that although *indicate a gap* is prevalent, writers sometimes strategically combine it with *add to what is known*. Lim (2012) also describes the nuances of citing past research to set up gap indications: where much research exists, writers may critique flaws among previous findings to identify a gap.

Evaluating past scholarship is also noted as move EN in both Kwan et al. (2012) and Kanoksilapatham (2015). Kwan et al. found the move deployed in information systems using eight possible strategies with varying degrees of frequency, and they argue that the fields' dominant research paradigms drive these variations. Similarly, Kanoksilapatham (2015) notes variations across three engineering sub-disciplines in deploying three prototypical steps (See Table 1). She argues that these variations are motivated by their contextual features including researchers' goals and objects of study, which together shape the sub-disciplines' preferred ways of arguing for new knowledge—a conclusion suggested by Bazerman et al. (2005) and confirmed by genre scholarship. These findings suggest that in disciplines composed of markedly different sub-disciplines (e.g., mathematics), any findings about one sub-discipline should not be easily generalized to the larger discipline unless verified by further research. This scholarship has identified sufficient variation to warrant further investigation into how different disciplines and subdisciplines handle niche establishment.

As noted, recent scholarship on the *Create A Research Space* model in mathematics RA introductions (Graves et al., 2013, 2014; Kuteeva and McGrath, 2015) has revealed insights into linguistically salient aspects of move EN but has not examined this move in less salient or "textually silent" cases (Tardy and Swales, 2014, p. 173). Consequently, these researchers may have overlooked subtler constructions by over-relying on linguistic indicators or informant data. Since niche establishment justifies new research and demonstrates its significance, we argue the need to re-examine this move in disciplines where it seems less present. Further, any claim about move EN's status in mathematics generally (or discrete mathematics specifically) requires detailed analysis of its RAs, given the challenging nature of genre analysis applied to the field's highly technical and linguistically compact ways of arguing for new knowledge (Graves et al., 2013).

The findings reported here are part of a larger study of argument structure in RAs in discrete mathematics. Here, however, we report on 1) the strategies that discrete mathematicians use to “establish a niche”, 2) how disciplinary culture shapes how research niches are motivated and instantiated in its RA introductions, and 3) the theoretical implications of these results for genre practitioners and writing pedagogy in mathematics-related disciplines.

4.2. Methodology

4.2.1. The field of Discrete Mathematics

Mathematics as a discipline was chosen for the larger study because in ESP genre literature it has received limited attention (In Graves et al., 2014, we summarize reasons why mathematics has been understudied). Even recent findings related to the rhetorical structure of mathematics RAs (e.g., Graves et al., 2013, 2014; McGrath and Kuteeva, 2012; Kuteeva and McGrath, 2015), although promising, are not easily generalizable due to the discipline’s massive territory. As one of the oldest disciplines in the history of human enquiry, mathematics has generated numerous sub-disciplines, branches, and subjects (Mathematical subject classification, 2010). While different mathematical fields generally agree on the epistemological categories that underlie research mathematics’ foundations, disagreements and counterarguments do exist (See Burton’s (1999) interviews with 70 mathematicians). Trowler (2012) has shown that conflicts and paradigm wars exist within all disciplines. The lesson for genre analysis is that researchers must examine several sub-disciplines before they can generalize claims to the entire discipline. Accordingly, we delimit our project to discrete mathematics, not the general field.

Discrete mathematics was chosen because 1) it forms the backbone of many mathematics sub-disciplines and beyond (Rosen, 2012, p. xviii); 2) it is considered the ‘gateway’ to advanced mathematics; 3) it is known for its rich arguments (Rosen, 2012), ideally suiting it to genre analysis; and 4) three discrete mathematicians agreed to act as both disciplinary informants and inter-raters of text analysis results. Their participation allowed us to triangulate our data, that is, “to provide multiple perspectives of what is being examined . . . [to] enable the researchers to gain a more complete understanding of the topic being investigated” (Paltridge and Starfield, 2016, p. 219).

4.2.2. The corpus

These results are based on analysis of a 30-research article corpus from five leading journals in discrete mathematics, coded here as DM, DAM, JCO, G&C, and SIAM (See Appendix B). These journals were judged prestigious in the field based on their impact factors, their being indexed in ISI Web of Knowledge, their presence in reputable databases (e.g., ScienceDirect), and their identification by our disciplinary informants as top journals in their field.

Having selected the journals, we created a stratified random sample, the strata being the authors (one article per author), number of articles selected per issue (one), and year of publication (three-year sample, equal number of articles from each year). This sampling generated a corpus of nine applied mathematics and 21 pure mathematics papers. We did not distinguish between pure and applied mathematics RAs based on discussions with disciplinary informants who argued that discrete mathematics is a continuum with theoretical math at one end and applied math at the other. See Table 2 for details of the corpus.

Table 4.2. Details of the corpus

Number of RAs	**Number of pages**	**Number of authors**	**Publication date**	**Number of countries represented**
30	344	67	2007-2009	21

As Table 2 shows, our corpus represents 67 authors from 21 countries from English- and non-English-speaking institutional backgrounds. We did not filter non-English speaking authors because English is considered the academic *lingua franca*, and we believe that "[a]cademic English has no native speakers" (Mauranen et al., 2016, p. 52).

4.2.3. Coding

The corpus was analyzed by combining data from thick descriptions of sample texts, informant perspectives and literature on epistemology in mathematics. Textual data were examined using both lexico-grammatical signals and content information to describe rhetorical structures. In instances where the rhetorical structure was unclear or the subject matter too technical to judge a segment's communicative purpose, we consulted at least two disciplinary specialists to clarify (See 2.4.).

This article reports findings of the rhetorical strategies that discrete mathematicians use to establish a research niche. Swales' two models (1990, 2004) (See Appendix A) were our points of departure. Our analysis was driven by his definition of move as "a discoursal . . . unit that performs a coherent communicative function in a written or spoken discourse" (2004, p. 228). Thus, rather than take a specific linguistic unit (e.g., a sentence) as our criterion, we examined the coherent communicative functions (e.g., question raising) fulfilled by different discourse units.

Once instances of move EN were identified, each was further analyzed using Swales' concept of step, as appropriate. We adapted labels from whichever *Create A Research Space* model best fit our data and characterized moves and steps using a combination of lexico-grammatical signals and content information. When we encountered discoursal units that served multiple communicative functions, we recorded all functions (See Example 9, for instance). Following the analysis, two disciplinary informants acted as inter-raters to check samples to validate our conclusions. One inter-rater checked our analysis of 20 RAs; the second inter-rater checked 26 RAs. Therefore, at least one inter-rater validated the results of each RA analysis. The main disagreement between our coding and the inter-raters arose regarding the *indicate a gap* step (1990 model). How we negotiated the disagreement is addressed in the Results section (see 3.2).

4.2.4. Informant Data

Informant data came from semi-structured interviews and communications through in-person meetings and Skype connections with five mathematics professors. Three professors have a discrete mathematics background; one had published over 100 RAs in ISI-indexed journals. The fourth informant's research focuses on philosophy of mathematics. The fifth informant was not a discrete mathematics specialist but was chosen after being named a distinguished mathematician with an outstanding publication and citation record. Informants A and B met, in-person and through Skype, respectively, weekly or bi-weekly for six months with one of the authors. They also answered any questions or clarified discrepancies arising from our corpus analysis. Informants C and D were interviewed three times, each for a total of 3.5 hours. Informant E participated in one semi-structured interview and some personal communications on a casual basis. Interview questions included mathematics' foundations as a discipline, research motivations, researching and writing processes in mathematics, and topics initiated by informants.

To prepare informants for the interviews and to check our rhetorical analysis' accuracy, one of the authors presented to each informant a one-session (75 minute) introduction to Swales' ESP approach to genre analysis and his *Create A Research Space* model (CARS) for RAIs based on Swales and Feak (2012). Prior to introducing the CARS model, it was explained that in genre analysis this model is used as a guide in pattern-seeking not as a rigid model to impose on the analysis. Informants were also informed that Swales has revised the CARS model in response to studies showing where it did not accurately reflect a particular discipline's practice. These efforts aimed to provide informants with concepts and vocabulary, so they didn't have to "reinvent the wheel" without constraining the outcomes of their analysis.

We also explored the broader context of the articles in the corpus by identifying and reading seminal sources suggested by informants on ways of thinking, approaches to research, and disciplinary writing practices in discrete mathematics and mathematics in general.

4.3. Results and Discussion

Swales and Feak (2012, p. 348) have argued that *establish a niche* is a 'key move' in RAs, the 'hinge' that connects move 1 to move 3. Its demonstrated presence in science, engineering, social science and law (See Table 1) reflects this importance. In contrast, our corpus analysis identified strategies of move EN in two-thirds (20/30) of the RAs, a 66.7% occurrence, and Kuteeva and McGrath (2015) reported it in fewer than 50% (10/22) of their examples in pure mathematics which is, as far as we know, the lowest occurrence reported so far. However, the RAs in their corpus were written by just five authors, so its relatively low occurrence in their corpus may reflect individual author argument styles rather than disciplinary trends. This variation may also be sub-discipline-based, reflecting conventional approaches in pure mathematics. Our informants emphasized that discrete mathematics has a broad readership both in and outside the discipline; consequently, some authors may more explicitly establish their niche to accommodate such extra-disciplinary readers. This broader audience may account for the variation between our findings and those of Kuteeva and McGrath (2015) in pure mathematics.

Our data suggest five options with which authors in discrete mathematics may establish a niche for their research:

1a *retrieve a problem,*

1b indicate absence of/insufficient research,

1c raise a question,

1d add to what is known, and

1e counter-claim.

Scholarship on move EN in other disciplines generally uses *indicate a gap* for strategy 1b; however, for reasons we discuss in 3.2, we rename it here *indicate absence of or insufficient research*.

Figure 4.1 summarizes the frequency of use of move EN. It shows that strategies a to c (*retrieve a problem*, *indicate insufficient research*, and *raise a question*) are used most often (nine and ten times); strategies d and e (*add to what is known* and *counterclaim*) were used four times each.

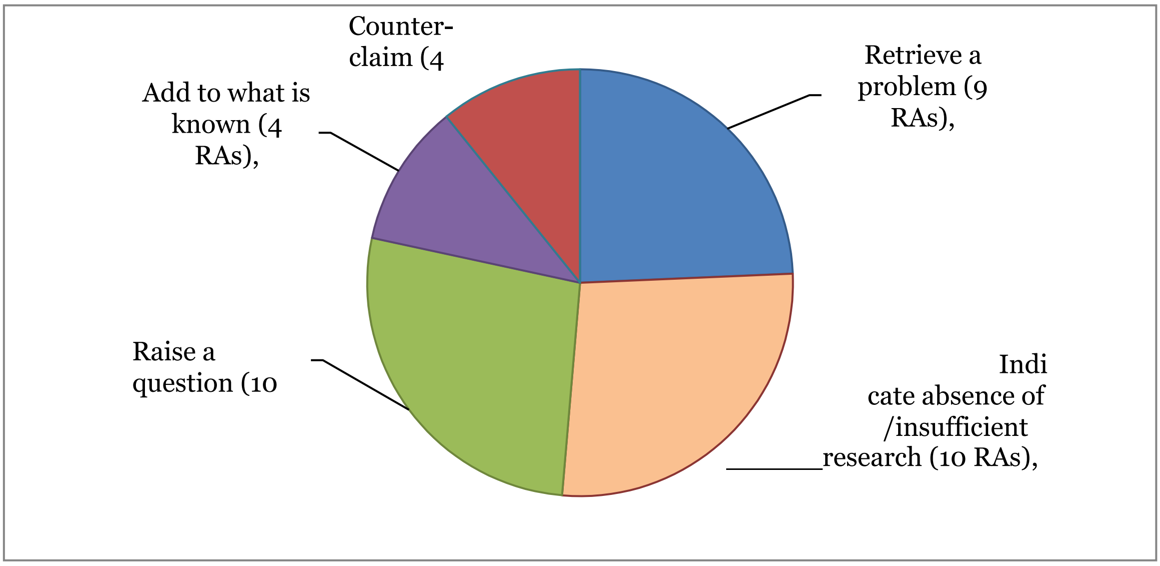

Fig. 4.1. Strategies in establishing a niche in discrete mathematics

Of these steps, *retrieve a problem* is new and not represented in either the *Create A Research Space* model or in research in other disciplines. The second option, *indicate absence of/insufficient research*, resembles Swales' *indicate a gap*. The last option, *counterclaim*, is used infrequently in our corpus but under specific circumstances. These textual data are supported by interview data. All five mathematicians considered solving open problems, conjectures and questions posed in published RAs as primary motivations for new research; hence Step 1a is a significant method for establishing research niches in discrete mathematics. Similarly, informants A, B, C and E suggested that counter-claiming would be an unlikely strategy for most mathematicians because published mathematical findings are regarded as valid and infallible, especially in pure mathematics. However, informant D remarked that the infallibility of mathematical findings is under debate and taking a position on it depends on one's ideological view of disciplinary epistemology. This latter view is supported by Lakatos (1976), who argues that "[i]t was the infallibilist philosophical background of Euclidean method that bred the authoritarian traditional patterns in mathematics […] that made impossible the rise of mathematical criticism" (p. 147).

In total, 37 instances of move EN appeared in 20 RAs, ranging from apparent absence in 10 RAs to one strategy in 11 RAs to multiple strategies in 9 RAs (DM3, DAM4, G&C6, JCO2, 4, 5, 6, SIAM 1, 2) These variations are presented in Fig. 2.

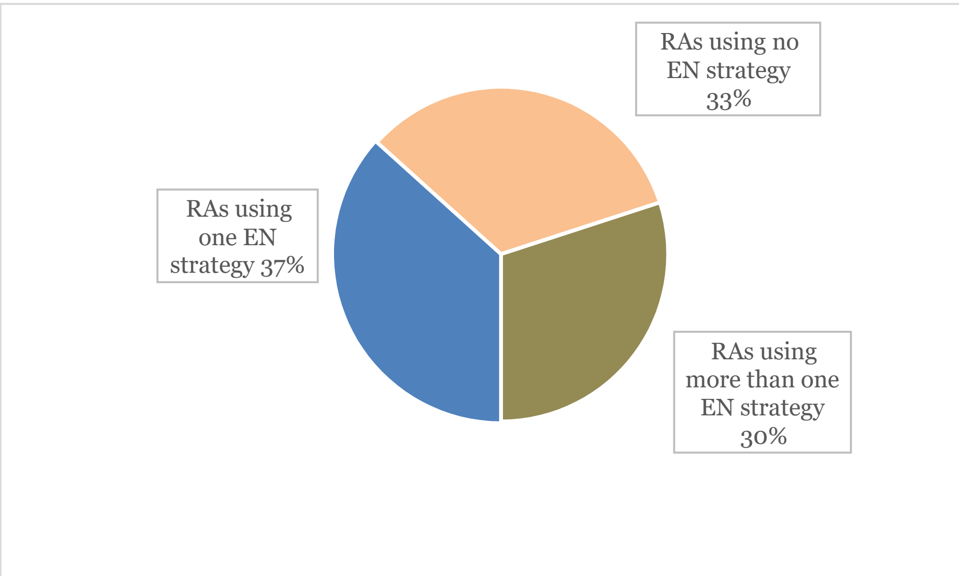

Fig. 4.2. Variations in using EN strategies in RAs

Figure 4.2 suggests that these authors feel little pressure to use a particular quantity of move EN. Moreover, we argue that the availability of three options to use EN once, more, or not at all is meaningful because it implies the importance of these authors' strategic decisions when they employ these rhetorical structures.

4.3.1. Step 1a: Retrieve a Problem

As noted, a prevalent method of niche establishment is new: highlight an existing unsolved problem or retrieve a(n existing) problem. Usually these problems are well known, so writers need only cite the problem posed in the published literature. They do not have to invent the problem as researchers in other disciplines must do (See, for example, 'reported' gaps in computer science (Shehzad, 2008)). Two variations of retrieving a problem exist in our corpus: open problem and existing problem. Open problems are clearly phrased statements or hypotheses in the literature which are well-known for their old age, that is, they have remained unsolved for decades or even centuries. Such problems are considered weightier and more important the longer they have remained unsolved. Open problems are thus distinguished from the existing problems (that is, mostly conjectures) based on their age. Existing problems are recent, while open problems are challenging ones that mathematicians have worked on for years without solving. For example, Fermat's last theorem remained an *open problem* for 300 years until a successful proof was published for it in 1995. Such historical open problems are rare; however, several RAs in our corpus did refer to open problems, including Example 1:

Example 1:

> Whether or not it is possible to satisfy requests on a network translates into a routing **problem** on a graph with capacities, **a classical graph theory problem**. (DAM4, p. 90) (Emphasis added)

Here the author emphasizes the existing open problem's significance by using linguistic resources including the noun, *problem*, and the qualifier denoting age, *classical*.

One of two subspecies of open problem present in our corpus was the *conjecture*. In mathematical terms, conjectures are evidence-based answers to as-yet unproven questions. Thus, proofs that solve unproven conjectures are, by definition, significant results. Conjectures are linguistically indicated by a possessive construction (that is, the originator's name) and the term *conjecture*, as in example 2:

Example 2:

> To determine the worst case complexity c(d, n) is **an unexpectedly hard problem** and is **open** for $d \geq 2$. Since **Hadwiger's conjecture** in the general case **is still open**, researchers have **shown interest** to **derive** lower bounds for Hadwiger number . . .. (G&C3, p. 292) (Emphasis added)

The authors describe an *unexpectedly hard problem* and *conjecture* that *is open* for some quantities and *still open* in general. They use nouns (*problem*, *conjecture*), adjectives denoting difficulty (*hard*) and unsolvedness (*open*), adverbs denoting unforeseen (*unexpectedly*) or unchanged state (*still*), and verbs denoting mathematical actions (*shown interest*, *derive*) to signal it is a significant existing mathematical problem. They avoid explicitly negative linguistic indicators and use multiple steps to create move EN, an option discussed later in this section.

A second sub-species of retrieving a problem is identifying existing problems (distinguished from 'open' problems) that writers refer to in general terms:

Example 3:

> Although this subject has been very well studied over many decades, ... interesting **problems** and approaches **still emerge**. (DAM6, p. 2187) (Emphasis added)

Here the emergence of problems in general is identified as the niche to be filled. The writers locate the work's exigency in the 'interesting' nature of the problem, not in the published literature, invoking mathematicians' shared value for solving difficult problems.

Since open problems are well known, writers need only name their problem (e.g., *Hadwiger's conjecture*) to establish the niche and the work's significance. When the open problem is not well-known, they may employ similar locutions to signal to readers the niche and its importance. Similarly, when asked about their sources for interesting research problems, our disciplinary informants all listed 'open problems and conjectures' first.

4.3.2. Step 1b: Indicate an Absence of or Insufficient Research

An equally frequently used strategy was to indicate the absence of or insufficient research on the topic. This step parallels Swales' move 2, Step 1b (1990 model), *indicate a gap*. However, our disciplinary informants cautioned that referring to an absence of literature as a 'gap' or pointing out inconsistencies in previous work are inappropriate as methods of niche establishment in discrete mathematics:

> That [word 'gap'] implies flaws in existing mathematical proofs. Sometimes, when I review papers for publication in journals, I come across proofs which miss key steps. I mean, nothing is wrong with skipping some steps in proofs. We all do that. We leave some labour for the reader to infer the skipped steps. It should read like a poem: concise, challenging, and beautiful. However, the missing steps should be retrievable by peers. When major steps are missing, we consider them to invalidate proofs by leaving them incomplete. So, I comment on the gap in the proof and send the paper back for revision. Such gaps must be filled before the paper is publishable. (Interview with Informant A)

This point was reiterated by all of our disciplinary informants, who stated that we should avoid the term 'gap' when we talk about niche establishment because in mathematics it indicates a fatal flaw in a proof. Informant B explained that because mathematics results are viewed as logical conclusions of thorough arguments, established proofs should not include a mistake or omit an important step, a Platonic view that has been widely debated (Burton, 1999). In response to this feedback and to avoid "over-reliance on specialist informants" (Swales, 1990, p. 129), we examined scholarship on the mathematical 'gap,' which confirmed and clarified the informant data. For example, Azzouni (2005) distinguishes ordinary gaps (that is, steps that are 'deliberately or inadvertently skipped') from gaps that 'invalidate' traditional proofs (p. 21). He argues that ordinary gaps are not problematic:

> [f]irst there are the […] allowable basic mathematical inferences that can be replaced with explicit formal derivations. Second, there are the gaps due to common understandings among mathematicians that certain details may be left out because everyone is familiar with them. Third, there is the capacity to recognize patterns in forms of proofs—so that details and even sub-proofs may be left out (pp. 21-22).

Azzouni describes invalidating gaps as "places where effective recognizability fails between steps in a proof" (2005). Skipping lower-level steps constitutes ordinary practice in mathematical proofs, while skipping higher order steps is regarded as gaps in the proofs that invalidate the results. Thus, researchers in mathematics do not consider indicating 'gaps' in previous work as a valid means of justifying their work. We acknowledge that *indicate a gap* and *indicate insufficient research* may be synonymous from the genre scholar's perspective, but we use the latter to avoid importing technical language from genre analysis that may interfere with established usage and discourage uptake in mathematics.

When using this strategy, authors describe limitations ranging from 'highlighting the complete absence' to 'stressing . . . insufficient research on a certain topic' (Lim, 2012, p. 243). In our corpus, adversative conjunctions (e.g., *however*) usually signal this step. This step can also use verbs of negation or a 'full negative' (Swales and Feak, 2012, p. 350) to emphasize the lack. In addition, present tense verbs are typically used to foreground the lack of knowledge and contribute syntactically to niche establishment, as in Example 4:

Example 4:

> **Although** we know of **no existing literature** (for d>2), silver (n, d)-cubes … appear to be connected with classical combinatorics. . .. (G&C4, p. 430) (Emphasis added)

These authors use a negative quantitative noun phrase (*no existing literature*) and the simple present tense to emphasize the absence. Lim (2012) argues that present and present perfect verb tenses foster the impression that a thorough review produced this conclusion. Other uses of Step 1b focus on the limited research conducted on a topic.

The frequency of use of steps 1a and 1b suggest that they are ‘probable’ steps in discrete mathematics. This result contrasts those of Kuteeva and McGrath (2015), who argue that move 2 in 6/10 RAs in their corpus was structured through indicating a gap by emphasizing a theorem’s limited generalizability.

4.3.3. Step 1c: Raise a Question

Step 1c, *raise a question*, is identical to Swales’ 1990 model and was used as frequently as step 1b. Writers usually located it after move 1, step 3, *review items of previous research*, to serve two purposes: 1) situate the research within a tradition, and 2) provide a rationale for new research. When using this step, writers either imply a question that requires an answer or explicitly pose the question that the article answers. Neither method involves authorial voice; instead, questions are framed as spontaneous research demands, as in Example 5:

Example 5:

> The fact that every edge in a graph will be traversed **raises several questions**.
>
> Question 2.4 Over all … for a graph G = (V,E),
>
> 1. **What is** the maximum number of times an edge $e \in E$ can be traversed …?
> 2. **What is** the maximum number of times the robot can be located …?
> 3. **What is** the minimum or maximum number of time steps before …? (JCO6, p.

353-4) (Emphasis added)

Still other authors use a combination of metaphor and authorial voice to state questions indirectly to reinforce the mathematical problem's self-sponsored nature. In example 6, the authors use the metaphor of the self-sponsored question spontaneously arising. This question is characterized as arising within the authors and readers simultaneously in the first use of "we." However, the second use of "we" refers exclusively to the authors:

Example 6:

> For the string barcoding problem, **a natural question arises** when **we** consider …. In particular, **we would like to know whether** it is possible to . . .. (JCO5, p. 41) (Emphasis added)

These authors imply the significance of their answer (to a "raised question") by presenting the questions as both spontaneously germinating (in readers and the authors) and as personal interests. These findings support Kuteeva and McGrath (2015), who identified question raising as a step in move 2 (EN), which, they noted, could be a question by the authors or one raised in past research.

While *raise a question* may resemble *retrieve a problem*, we see the two as separate because writers in our corpus use them differently. Authors who retrieve a problem merely allude to it, assuming that readers can supply the context; in contrast, authors who raise questions establish their niche more precisely by posing clearly articulated research questions. For instance, in Example 5, the questions clearly elaborate the focus of the inquiry, while in example 2 readers are expected to supply all details of Hadwiger's conjecture.

4.3.4. Step 1d: Add to What is Known

The fourth method, *add to what is known*, appeared in four articles. Our analysis suggests that writers use this step after a reference to the literature without criticizing, questioning, or counter-claiming it. They use it to acknowledge existing research on the topic and their plan to add to it.

In example 7, we include the preceding paragraph to highlight the relationship between the two passages:

Example 7:

> Several fundamental graph problems consist in finding …. In this paper, we study the problem of finding …. Both problems … are known to be NP-hard. We show that, **not surprisingly, the same conclusion holds** for any ….
>
> The NP-hardness of the problem in question suggests to look for. . .. In this paper, we **extend** the Hoffman bound to.… Finding an independent set of … is known to be an NP-hard problem (Garey and Johnson 1979; Karp 1972). It is also known that finding a … is NP-hard (Cameron 1989; Stockmeyer and Vazirani1982). The next section **generalizes** these two results. (JCO1, p. 455-456) (Emphasis added)

In paragraph one, the writers acknowledge past results on the topic. The bolded statements in paragraph two show step 1d. The writers then announce that they deduce the *same conclusio*n, *extend* the existing results and *generalize* them. They do not critique the previous studies. Example 7 supports Lim's (2012, p. 240) contention that "overt acknowledgement of the intention to proceed with the current research tradition" is a criterion that differentiates *add to what is known* from *indicate a gap*.

A sub-species of step 1d evolves from the disciplinary value of improving past results (that is, mathematical generalization). Informant B explained:

> [It]'s a category of research math, although we practice generalizing existing results in all areas and levels of doing math. Sometimes we modify the definition of a concept to apply it to more situations, sometimes we extend a bound and get a stronger result. (Interview)

Harel and Tall (1991) also identify categories of generalization to emphasize its importance in advanced mathematics.

Regarding linguistic indicators, our data suggest that some authors directly state their goal of generalization: “[t]he initial motivation for the present work was to **improve** on the above lower bound’ (SIAM4, p. 489, emphasis added), while others announce their intention to offer a general result: “We prove a **general** result which gives sharp bounds for several domination-like parameters” (JCO3 p. 353, emphasis added). However, generalizing and extending past results do not imply negative evaluation of past research; as a second informant emphasized, “sometimes it is the same results extended to another situation, and sometimes it is a matter of good and better: generalized results are better.” These informant data that extensions and generalizations add to past research rather than criticize it as a gap are consistent with Swales and Feak’s (2012, p. 349) point that extensions or refinements of previous work are cases of “adding another brick to the wall of knowledge” metaphor not gap indication. Our description of niche establishment strategies and our interpretation, however, contrast with Kuteeva and McGrath (2015), who identify similar strategies as gap indications.

4.3.5. Step 1e: Counter-claim

Counter-claiming is least frequently used to establish a niche in our data, occurring in three RAs. Disciplinary specialists concurred, noting this step is ‘rare’ because mathematical findings are logical and assumed to be correct. Informant D added that “mathematics is not about opinions; it is about truth, which stands independent of the mathematician” (Personal communication).

Example 8 below uses this step: the authors judge existing knowledge as inadequate. They convey judgment using a negative evaluation (*no improvements*) and hedge with an adverbial to convey doubt (*unlikely*):

Example 8:

This, along with the fact that there have been **no improvements** on Fredman’s and

Khachiyan's … algorithm [13], suggests that the Sequential Method is **unlikely** to list Tr(H) …. Answering a question posed by Hirsh [21], we show that …, namely, that the Sequential Method is **inefficient** no matter how the edges are ordered. (SIAM2 p. 937) (Emphasis added)

The authors use present and present perfect tenses to link past research with the present to problematize existing results to imply the need for further research. This RA examines an applied math problem, and the method counter-claimed has applications in algorithmic fields.

Disciplinary specialists A, B, C, and E suggest that refuting existing research is more likely in applied mathematics where rapid technological development in fields such as computer programming drives new methods and algorithms; thus, authors may counter-claim to emphasize their work's significance. In pure mathematics, counter-claims typically appear with conjectures (that is, unproven hypotheses), not established proofs; the counter may highlight an unverified part of the hypothesis to undermine the original conclusion.

Since step 1e appears four times in three RAs in our corpus (twice in one applied mathematics RA and twice in two pure mathematics RAs to disprove conjectures), we cannot reflect more broadly on the use of this step. Additional research is necessary to offer greater insight into how discrete mathematicians employ counter-claiming to establish a niche, especially in more applied areas of discrete mathematics.

4.3.6. Using Multiple Steps to Establish a Research Niche

No pattern emerged for the steps in those RAs where move EN was present, leading us to conclude that all steps were possible but none obligatory. Consequently, a model of niche establishment in discrete mathematics RAs would require the insertion of 'and/or' between the steps. Variation ranged from using no to one to multiple steps (See Fig. 2). In our data, the authors of nine RAs use multiple steps to establish a niche for their research in cycles with other Introduction moves. These data accord with Swales and Feak's (2012, p. 352) report on "cumulative and recycling" move 2s in RA introductions in some disciplines.

Where multiple steps were present, they ranged from using two (6 RAs) to three (1 RA) to four (1 RA) or more (1 RA) step strategies. Example 9 uses multiple steps:

Example 9:

> The **question** whether Tr(H) can be … [15, 22, 34, 35] **is a longstanding open problem** for enumeration algorithms. [Step 1c, *raise a question* embedded within Step 1a, *retrieve a problem*]
>
> While the approach, … is well known and appears often in the literature (see ... [4 … 31]), **there is little theoretical information** about its behavior. [Step 1b, *indicate insufficient research*]
>
> This, along with the fact that **there have been no improvements** on Fredman's and Khachiyan's quasipolynomial algorithm [13], suggests that **the Sequential Method is unlikely to list** Tr(H) …. **Answering a question posed by** Hirsh [21], we show that this is the case in a rather strong sense, namely, that **the Sequential Method is inefficient** no matter how the edges are ordered. [Step 1b, *indicate absence of research*, Step 1e, *counter claim*, and Step 1c, *raise a question*] (SIAM2, pp. 936-927. Emphasis added)

In the first paragraph, the authors initiate move EN using step 1c when they invoke the preliminary notion of *The question whether*. They end this paragraph with step 1a to highlight their topic as *a longstanding open problem*, the most highly valued mathematical problem. In the second paragraph, the authors note in an adversative clause (*While . . . there is little*) that much research has been done on their topic but emphasize that one area—its theoretical behavior—has been neglected (Step 1b, *insufficient research*). In the third paragraph the authors counter claim (*no improvements, unlikely*, *inefficient*) and respond to the question they raised earlier (*raise a question*).

These authors argue strongly for their work's significance as they create move EN. They recursively use seven strategies to indicate the niche and to develop the context. They emphasize the open problem's longevity (its importance); they frame it as a question that they answer (advance knowledge), and they refute previous researchers' results (reinforce the significance). They emphasize their knowledge contribution's strength by showing its importance on multiple levels. Likewise, in six of nine RAs the authors combine the challenging nature of the problem with a lack of knowledge to argue the need for their solution. These variations in step combinations and move cycling, according to informants A, B and C, are driven by the complexity of the RA topics; the authors use the steps to achieve their objectives. In addition, informant E noted that increased competition to publish motivates writers to emphasize their work's significance as they establish its niche.

Example 9 also exhibits the rhetorical strategy of step and move embedding. That is, in paragraph one, *raise a question* is embedded within *retrieve a problem*. In paragraph three (sentence one), *indicate absence of research* is embedded within *counter claim*. Finally, in the last sentence, move EN (*counter claim*) merges with move 3 (*announce research findings*), suggesting that counter-claiming in discrete mathematics may both establish and occupy the niche simultaneously, especially where accompanied by linguistic indicators of move 3 (e.g., *we show*). This example supports Swales and Feak's (2012) argument that EN is a 'key move'. We further argue that in discrete mathematics strategies of EN, however, can function as more than a 'hinge' by serving multiple communicative purposes concurrently (that is, *establish a niche* and *occupy the niche*), thus increasing conciseness. In addition, the argument for move embedding might explain the apparent absence of move EN in some RAs.

4.3.7. The Absence of Move EN: Merging/Implying the Niche and Readers Infer the Niche

In 10/30 RAs in our corpus, the writers do not appear to establish a niche (that is, DM4, 6, DAM1, 2, 3, 5, G&C1, 2, 5 and SIAM 3). This finding contrasts with reports of move 2 in other fields where it is identified as a crucial move (Swales, 1990, 2004; Shehzad, 2008; Lim 2012). Only two-thirds of the RAs in our corpus explicitly establish a niche. However, our number is greater than that reported by Kuteeva and McGrath (2015) in pure mathematics. The absence of move EN in part of our corpus may be explained by Swales and Feak's (2012, p. 336) contention that 'in some fields (e.g., engineering) the rationale for research is not explicitly stated in move 2, it is rather implied'. As a reviewer of this paper noted, 'when you are dealing with the inner workings of a proof, the reason for the new research exists in the math itself', suggesting that the mathematics component subsumes the linguistic component in some part of the argument for new knowledge. Implying move EN is clearly a strategy used in some RAs in our corpus; however, in others move 'merging' is preferred.

In our data, half (five) of those RAs lacking move EN merged it with move 1 or 3, and in the remaining half readers must infer it from move 3. In Example 10, the authors merge the niche by summarizing central problems from existing research and then announcing their problem:

Example 10:

> A number of variants of the following basic problems have been considered. EXT_H, called the extension problem for H, is …. Thus the extension problem … asks whether or not … can be extended to ….
>
> ∆ CLHOMH, the connected list homomorphism problem to H, is the restriction of. …

In this paper we will focus on **the problems** EXT_H and CLHOM_H. (DAM5, p. 1592-3) (Emphasis added)

Paragraphs one and two define the original problem; paragraph three announces the paper's focus. While the rhetorical function of paragraph three is to *announce the present research*, (indicated by the lexemes that signal the new results (*in this paper*) and the scope of the work (*focus*)), it also identifies *the problem* (that is, the niche) that the paper addresses. In summarizing the general problem, the authors merge their niche into the research announcement. Informants A and B noted that this tactic enhances the economy of mathematical discourse. Other articles in this group similarly embed a niche within move 1 or move 3 without distinctly establishing it.

In the second group, readers must infer the niche from the description of the present research, move 3. Example 11 uses this strategy. In the first two paragraphs, the writers review existing research to establish topic centrality (move 1). We reproduce the first paragraph of this review:

Example 11:

In 2006, Kotani [7] investigated how many non-separating vertices a tournament … has …. Inspired by this article, Meierling and Volkmann [9] generalized her results …

In Section 3 **we characterize** all strongly connected local tournaments ….

In Section 4 we further investigate **the following problem**.

Problem 1.7. Given a strong local tournament D . . . How many cycles … exist in D? (DM4, p. 2043) (Emphasis added)

Following move 1, the authors jump from move 3 to the paper's structure (see italicized type) and identify the problem from past research. By juxtaposing the citations and their results, the authors imply, 'the problem has been studied, but we prove new aspects to it.' Thus, readers must infer move 2 from the information in move 3. This example supports

Swales and Feak's (2012, p. 334) argument that RAIs in some disciplines do not explicitly state the study's rationale: 'Rather, the study seems to emerge as a natural and rational response to some kind of gap in the literature. In other words, RA introductions do not show how the study started at all'.

We see implying a niche as an option for writers in discrete mathematics but not a method *per se* of niche establishment. The question then arises as to why a writer might choose not to establish a niche explicitly. Research on argument suggests that when readers must actively connect stated points their commitment increases because they find those arguments most persuasive that they have constructed themselves (Aristotle, 1984). Insights from Informant C support this explanation:

> It's like poetry; we are concise in drafting our research. It's indeed the art of the mathematician to decide which details to include and which to omit; and as readers, we enjoy working out the missing parts. (Interview)

Hardy (1940/2005) has long emphasized the importance of beauty in doing mathematics by placing it closer to the arts side of the continuum than to the sciences. Therefore, precedent exists in mathematics for omitting information that readers can supply themselves, perhaps explaining why one third of the authors do not explicitly establish a niche.

Our findings contrast with some existing results in mathematics and in disciplines outside of mathematics. The status of move EN in discrete mathematics departs significantly from its obligatory status in many disciplines including agriculture (Del Saz Rubio, 2011), chemistry (Stoller and Robinson, 2013), computer science (Shehzad, 2008), engineering (Kanoksilpatham, 2015), management (Lim, 2012), and law (Tessuto, 2015). We acknowledge that these cross-disciplinary variations are informed by discipline-specific epistemologies established through ongoing practices of constructing and arguing for new knowledge. However, we consider epistemological influence to be one among many influences on disciplinary discourse. Indeed, following Trowler (2012), we argue that epistemological determinism should be treated with some skepticism. Trowler (2012) contends that disciplinary influences are less powerful than they were, no longer being the primary force shaping academic practice. Other forces including demands from allied disciplines or client organizations are at play; in addition, researchers' concerns, the topics they study, and the ways they talk may all contribute towards reconfiguring epistemological influences and reshaping academic practices. Accordingly, we argue that niche establishment is argued more markedly in RAs that address interdisciplinary audiences and less markedly in RAs in pure areas of discrete mathematics focused on self-instigated topics.

Regarding variations in how disciplines use rhetorical strategies, although distinctive ways of communicating in each discipline are identifiable in broader terms, as Trowler (2012) argues, diversity and even paradigm wars exist within the same discipline: researchers in the same field may disagree on what to do and how to do it. This insight has two important implications for genre research. The first regards use of informant data. While informants are essential to explain data from textual analysis, variations in perspective should be expected from different informants within the same discipline. Burton's (1999) ethnographic study of 70 research mathematicians yielded startling differences among them on five fundamental epistemological criteria in mathematics. Trowler's (2012) work implies, then, that genre analysis should use all informant data cautiously.

A second implication relates to the generalizability of findings. If, following Trowler (2012), dynamic influences constantly reshape disciplines and their practices, even the most generalizable findings may soon be outdated. The ephemeral nature of research findings, we argue, may shift the nature of genre research to focus more on identifying how different disciplinary, cross-disciplinary and extra-disciplinary influences (and their inter-dynamic relationships) might be reflected in the argument structures of disciplinary texts. Accordingly, this focus has important implications for genre research methodology: current trends are moving towards analyzing huge corpora to yield generalized findings that may obscure these dynamic influences. Instead, researchers might consider selecting sample sizes that allow detailed, in-depth analysis and explanations. This implication, understandably, may raise concerns about the 'representativeness' of a genre sample, but we would argue that as disciplines subdivide into sub-disciplines and (inter)disciplines emerge from allied disciplines (e.g., nanotechnology, biological mathematics, quantum biology), 'representativeness' as a notion may become obsolete in the same way that the 'disciplinary tribes and territories' metaphor has lost much of its validity over time (Trowler, 2012).

4.4. Conclusion

Our analysis identified six options in discrete mathematics for establishing a niche: 1) retrieve a problem, 2) indicate an absence of or insufficient research, 3) raise a question, 4) add to what is known, and 5) counterclaim. Writers may use one or more of these strategies, or they may not, leading to the sixth option—readers infer the niche. This option, *readers infer a niche*, was noted by Swales and Feak (2012) but has received little attention in genre analysis. We also identified two methods used to establish a niche that appear to be unique (that is, not yet described in genre studies scholarship): readers infer the niche and retrieve a problem. This analysis also yields two insights regarding linguistic indicators for EN strategies. First, mathematicians use clearly defined technical language (e.g., *conjecture*, *open problem*, *extension*, and *generalization*) that is well known within the discipline (but less so to genre analysts) to establish a niche. Second, some generic labels (that is, *gap indication*), may interfere with disciplinary-specific labels and hence should be adjusted.

The findings of this study may be limited by the size of our corpus and the range of sub-disciplines within mathematics as well as the diversity of topics and problems that the

RAs target. Additional investigation of RAs representing different sub-disciplines of mathematics are required to substantiate and/or generalize results of this study. A second limitation to this study is its focus on text and natural language; symbolic language and visuals are essential components to mathematical research (and knowledge), yet we have glossed over these areas in favour of the written text because, as writing instructors, we are charged with helping novice disciplinary members create the natural language to explain their results. As one reviewer pointed out, “I read the math first and then look at the text in articles of this type”. Our discussion, therefore, should be viewed with the understanding that it is partial and not comprehensive.

Nonetheless, these insights into how niches are established textually are important because they build on previous findings that the organizational and rhetorical structures of discrete mathematics RAs depart from discursive conventions in other disciplines. Previous scholarship (Grave et al. 2013; 2014; Lyda and Warchal, 2014, Kuteeva and McGrath, 2015, etc.), coupled with this analysis, indicates that enough aspects of the *Create A Research Space* model for Introduction sections fit discrete mathematics to make it relevant as an instructional tool for novice writers, yet to improve its ‘fit’ and relevance two crucial adaptations are necessary. First, move numbers might change to remove the implications of a required chronology and stipulated single usage. We propose the possibility of replacing the numbers with acronyms. As represented in Fig. 3, we have adapted the *Create A Research Space* model to reflect accurately discrete mathematicians’ practices. This adaptation results in a model that has four moves for RAIs in discrete mathematics: move ET (establish a territory), move P (establish presumptions), move PPW (present the present work), and move EN (establish a niche). These moves are drawn from findings in previously published work on RAs in discrete mathematics (Graves, 2013; 2014) and in this article. Second, the first three moves (P, ET, and PPW) would be designated obligatory, and the fourth move, EN, possible. Further, we propose ‘and/or’ linkages between the steps in move EN because writers in discrete mathematics may use multiple steps to establish the niche (or none at all).

Swales’ CARS model 1990	Swales’ CARS model 2004	CARS model for discrete mathematics RAs
Move 1: Establishing a territory Step 1: Claiming centrality Step 2: Making topic generalizations Step 3: Reviewing items of previous research	Move 1: Establishing a territory (citations required)	Move ET*: (obligatory) Establishing a territory (citations required)
Move 2: Establishing a niche Step 1A: Counter-claiming or Step 1B: Indicating	Move 2: Establish a niche (citations possible) vis Step 1A: Indicating a gap Step 1B: Adding to what is known Step 2: (optional)	Move P: (obligatory) (Graves et al 2014) Establishing presumptions Step 1: Presenting assumptions Step 2: Introducing notations Step 3.

a gap or Step 1C: Question- raising or Step 1D: Continuing a tradition	Presenting positive justification	Defining objects/terms Step 4. (optional) Revising/referring to items of previous research
Move 3: Occupying the niche Step 1A: Outlining purposes or Step 1B: Announcing present research Step 2: Announcing principal findings Step 3: Indication RA structure	Move 3: Presenting the present work (citations possible) Step 1: (obligatory) Announcing present research descriptively Step 2*: (optional) Presenting RQs or hypotheses Step 3: (optional) Definitional clarification Step 4: (optional) Summarizing methods Step 5: (PISF**) Announcing principal	Move PPW: (obligatory) Presenting the present work (citations possible) Step 1: (obligatory) Announcing present research descriptively Step 2: (optional) Presenting RQs or hypotheses Step 5: (optional) Announcing principal outcomes Step 6: (optional) Stating the value of the present research Step 7: (optional) Outlining the structure of the paper

	outcomes Step 6: (PISF) Stating the value of the present research Step 7: (PISF) Outlining the structure of the paper	
		Move EN: (optional) Establish a niche Step 1a: Retrieve a problem (and/or) Step 1b: Indicate an absence of or insufficient research (and/or) Step 1c: Instigate a problem (and/or) Step 1d: Add to what is known (and/or) Step 1e: Counter-claim
	(**PSIF: Possible in some fields)	(*In mathematics, these moves can appear in any order and they are often

		cycled through, so initials rather than numbers are used in this model to reflect accurately mathematicians' practices).

Figure 4.3. We have adapted the CARS model to reflect accurately discrete mathematicians' practices.

Given that move structure in discrete mathematics RAIs eschews a chronology, numbering moves in the *Create A Research Space* model to indicate an established sequence may be misleading in some disciplines. This proposal has significant implications for EAP instructors, who need to be aware that numbering the moves in the *Create A Research Space* model may mislead some students. While the *Create A Research Space* model as described by Swales (1990, 2004) does accurately reflect the organizational structure of RA introductions in many disciplines, making it something of a conventional model for writing instruction in *those* disciplines, it requires modifications to fit discrete mathematics. Writers in discrete mathematics would benefit from learning a revised model (that is, Figure 4.3) that emphasizes its flexibility, rather than representing the moves as linear and non-recursive.

The mediating role of combined writing and talking aloud as a pedagogical genre (that is, chalk talk) in undergraduate mathematics instruction has been emphasized by Artemeva and Fox (2011). A study of academic writing instruction for mathematics students is still 'open' to future research. However, we are aware that most course-based writing instruction offered at the graduate level, at least in North American academic contexts, usually has students from across the disciplines, making it impractical to tailor instructional models to individual students' disciplines (only a few of whom might be writing in mathematics—even fewer in discrete mathematics). However, offering multiple models to represent the range of move structure in RAIs across disciplines seems potentially valuable to all members of a multi-disciplinary writing course. Highlighting variations between the models can sensitize students to the existence and nature of potential differences. When they study the move structure in sample RAs in their own discipline, they will be more likely to perceive how their samples vary from those models. Introducing students to differing models can help instructors to initiate classroom conversations about the 'invisible discourse' that students must learn to become successful writers in their fields (Brandt, 1990, p. 119; Hirvela, 1997, p. 84).

Tardy (2009, p. 281) has noted that generic writing classes for graduate students provide lower exigence-based assignments and therefore more limited (genre) knowledge-building opportunities related to disciplinary discourse. She applauds courses that expose students to "source texts, samples, models, and peer writing" and notes that these "textual interactions [serve to] broaden learner's exposure to and engagement in genres—both important resources for genre learning." Instructors can boost the learning opportunities by offering more complex models that require students to engage with multiple approaches so that they may develop a more nuanced conception of the move structure in RAIs in their field. Tardy contends that learners (and she intends here 'both multilingual and monolingual writers' (p. 283)) 'can benefit greatly from seeing how a single genre may be approached by different writers in unique rhetorical contexts' (p. 284). We argue that presenting different *Create A Research Space* models in mixed-discipline graduate writing classes could enable learners to see how the unique context of each discipline (within dynamic influences) creates the conventions of their discipline's RA introduction rhetorical structure.

Accordingly, and considering Swales' (1990) argument for adopting a consciousness-raising, task-based approach to genre instruction, we argue that writing-in-the-disciplines instructors should draw students' attention to generic features, including rhetorical actions and structures and the linguistic resources that help to accomplish those actions. We argue that a problem-solving scaffolding approach, originated by Vygotsky (1987), elaborated by Wood et al. (1976) and explained in relation to Swales' genre research by Flowerdew (2015), can benefit from being informed by generic structure (moves, steps, linguistic indicators) described for RAs in different disciplines at the modelling stage. These generic descriptions of different genres of various disciplines provide rich resources for modelling the target genres in writing classes. However, in modelling such genres, students' attention should be drawn to not only shared but also unique features in both communicative events and individual language-users (Devitt, 2015, p. 44) as "genre-in-use is simultaneously unique and shared". To use Devitt's re-articulation of a competence vs. performance metaphor, instruction and assessment should address not only genre competence but also individual users' genre performance. Genre research, in this sense, offers a rich data pool to model in writing classes.

4.5. Chapter References

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Chapter Appendices

Appendix A: Swales' 1990 and 2004 CARS models

Swales' CARS model 1990	Swales' CARS model 2004
Move 1: Establishing a territory: **Step 1:** Claiming centrality **Step 2:** Making topic generalization **Step 3:** Reviewing items of previous research	**Move 1**: Establishing a territory (citations required)
Move 2: Establishing a niche: **Step 1A:** Counter-claiming or **Step 1B:** Indicating a gap or **Step 1C:** Question-raising or **Step 1D:** Continuing a tradition	**Move 2**: Establishing a niche (citations possible) via **Step 1A**: Indicating a gap or **Step 1B**: Adding to what is known **Step 2:** (optional) Presenting positive Justification

Move 3: Occupying the niche:	**Move 3**: Presenting the present work (citations possible)
Step 1A: Outlining purposes or	**Step 1:** (obligatory) Announcing present research descriptively
Step 1B: Announcing present research	**Step 2***: (optional) Presenting RQs or hypotheses
Step 2: Announcing principle findings	**Step 3:** (optional) Definitional clarifications
Step 3: Indicating RA structure	**Step 4:** (optional) Summarizing methods
	Step 5: (PISF**) Announcing principle outcomes
	Step 6: (PISF) Stating the value of the present research
	Step 7: (PISF) Outlining the structure of the Paper
	(**PISF: Possible in some fields)

Appendix B: Articles in the Corpus

DM1. Hoffmann-Ostenhof, A. (2007). A counterexample to the bipartizing matching conjecture. *Discrete Mathematics, 307*(22), 2723-2733.

DM2. Favaron, O., & Henning, M. A. (2008). Bounds on total domination in claw-free cubic graphs. *Discrete Mathematics, 308*(16), 3491-3507.

DM3. Gibson, R. G. (2008). Bipartite graphs are not universal fixers. *Discrete Mathematics, 308*(24), 5937-5943.

DM4. Meierling, D., & Volkmann, L. (2009). On the number of cycles in local tournaments. *Discrete Mathematics, 309*(8), 2042-2052.

DM5. Kelarev, A., Ryan, J., & Yearwood, J. (2009). Cayley graphs as classifiers for data mining: The influence of asymmetries. *Discrete Mathematics, 309*(17), 5360-5369.

DM6. Bielak, H. (2007). Chromatic properties of hamiltonian graphs. *Discrete Mathematics, 307*(11-12), 1245-1254.

DAM1. Atapour, M., Khodkar, A., & Sheikholeslami, S. M. (2007). Characterization of double domination subdivision number of trees. *Discrete Applied Mathematics, 155*(13), 1700-1707.

DAM2. Bjlint, V. (2008). A distance approximating trees. *Discrete Applied Mathematics, 156*(14), 2740-2752.

DAM3. Fujita, S., & Nakamigawa, T. (2008). Balanced decomposition of a vertex-colored graph. *Discrete Applied Mathematics, 156*(18), 3339-3344.

DAM4. Jarry, A., & Pérennes, S. (2009). Disjoint paths in symmetric digraphs. *Discrete Applied Mathematics, 157*(1), 90-97.

DAM5. Feder, T., Hell, P., & Huang, J. (2009). Extension problems with degree bounds. *Discrete Applied Mathematics, 157*(7), 1592-1599.

DAM6. Chen, W. Y. C., Li, N. Y., & Shapiro, L. W. (2007). The butterfly decomposition of plane trees. *Discrete Applied Mathematics, 155*(17), 2187-2201.

G&C1. Gould, R., & Whalen, T. (2007). Subdivision extendibility. *Graphs and Combinatorics, 23*(2), 165-182.

G&C2. Bernáth, A., & Gerbner, D. (2007). Chain Intersecting Families. *Graphs and Combinatorics, 23*(4), 353-366.

G&C3. Chandran, L. S., Kostochka, A., & Raju, J. K. (2008). Hadwiger Number and the Cartesian Product of Graphs. *Graph. Comb., 24*(4), 291-301.

G&C4. Ghebleh, M., Goddyn, L. A., Mahmoodian, E. S., & Verdian-Rizi, M. (2008). Silver Cubes. *Graph. Comb., 24*(5), 429-442.

G&C5. Bonomo, F., Durán, G., Maffray, F., Marenco, J., & Valencia-Pabon, M. (2009). On the b-

Coloring of Cographs and P4-Sparse Graphs. *Graphs and Combinatorics, 25*(2), 153-167.

G&C6. Cui, Q., Haxell, P., & Ma, W. (2009). Packing and Covering Triangles in Planar Graphs. *Graphs and Combinatorics, 25*(6), 817-824.

JCO1. Cardoso, D., Kamiński, M., & Lozin, V. (2007). Maximum k -regular induced subgraphs. *Journal of Combinatorial Optimization, 14*(4), 455-463.

JCO2. Irving, R. (2008). Stable matching problems with exchange restrictions. *Journal of Combinatorial Optimization, 16*(4), 344-360.

JCO3. Goddard, W., & Henning, M. (2007). Restricted domination parameters in graphs. *Journal of Combinatorial Optimization, 13*(4), 353-363.

JCO4. Vanetik, N. (2009). Path packing and a related optimization problem. *Journal of Combinatorial Optimization, 17*(2), 192-205.

JCO5. Liu, C., Song, Y., & Burge, L. (2008). Parameterized lower bound and inapproximability of polylogarithmic string barcoding. *Journal of Combinatorial Optimization, 16*(1), 39-49.

JCO6. Messinger, M., & Nowakowski, R. (2009). The robot cleans up. *Journal of Combinatorial Optimization, 18*(4), 350-361.

SIAM1. Berman, K., A. . (2007). Locating Servers for Reliability and Affine Embeddings. *SIAM Journal on Discrete Mathematics, 21*(3), 637-646.

SIAM2. Takata, K. (2007). A Worst-Case Analysis of the Sequential Method to List the Minimal Hitting Sets of a Hypergraph. *SIAM Journal on Discrete Mathematics, 21*(4), 936-946.

SIAM3. Dankelmann, P., Simon, M., & Henda, C. S. (2008). Average Distance and Edge-Connectivity I. *SIAM Journal on Discrete Mathematics, 22*(1), 92-101.

SIAM4. Korner, J., Claudia, M., & Gabor, S. (2008). Graph-Different Permutations. *SIAM Journal on Discrete Mathematics, 22*(2), 489-499.

SIAM5. Krotov, D., S., & Vladimir, N. P. (2009). n-Ary Quasigroups of Order 4. *SIAM Journal on Discrete Mathematics, 23*(2), 561-570.

SIAM6. McClosky, B., & Illya, V. H. (2009). The Co-2-plex Polytope and Integral Systems. *SIAM Journal on Discrete Mathematics, 23*(3), 1135-1148.

Chapter 5: "See Figure 1": Visual Moves in Discrete Mathematics Research Articles

5.1. Introduction

Through more than three decades of research by applied linguists and rhetoricians, genre analysis has grown into an interdisciplinary field of study. The field integrates theories and analytical tools from both schools in the study and teaching of genres with an emphasis on the relationship between the context, linguistic, rhetorical, and social features of genres (Tardy and Swales, 2014). The integrated approach in genre analysis has shown increasing interest in academic discourses and genres of disciplines revealing disciplinarity in their discourses, rhetorical organization and argument (see for example, Bazerman, 1987; Hyland, 2000; Prior, 1998; Swales, 1990, 2004).

From among the disciplinary genres, the research article (RA) has attracted much interest, being associated with Swales' *Create A Research Space* model (1990, 2004) and Hyland's corpus studies of disciplinary discourses. As "the pre-eminent genre of the academy" (Hyland, 2010, p. 117), the RA requires novice researchers and an increasing number of graduate students to master it to achieve academic success, making it a focus of research by scholars tasked with helping these populations. Since 2010, RA genre research has extended Swales' model of move structure in introductions to other sections or similar genres across disciplines. These studies have included genres in science and technology disciplines—traditionally known for their multimodal discourse—yet most researchers analyze the textual components in RAs and ignore the visual ones (Hyland, 2006; Johns, 2013; Tardy and Swales, 2014). Genre research pioneers have argued that visual components equally deserve attention because they contribute to the RA's rhetoric. Johns, for example, proposed research in multimodalities as a future ESP topic in 2013. She recalled her 1998 work on visual and verbal interactions in economics thus: "It is surprising that so little research has been completed either on the visual/verbal interaction in texts or on academic or nonacademic visual rhetoric" (Johns, 2013, p. 20).

Tardy and Swales (2014) also identify multimodal/visual genre analysis as a new direction. They argue that in some genres multimodal elements are "so essential that it would be impossible to overlook them in an analysis" (p. 173). In addition, Hyland (2006) points out, the fact that they "can occupy up to a half a science research article testifies to the significance of visuals in academic genres" (p. 53). Summarizing Miller (1998), Hyland (2006) also points out that "while arguments are based on plausible, and well-constructed, interpretations of data, they ultimately rest on findings, and these are often presented in visual form" (p. 54). Furthermore, many engineers and scientists assess the visual data first, when either reading an RA or writing one, to determine the argument and, in the latter case, distribute these data to structure their argument (Graves, 2014; Kresta et al, 2011). From a literacy perspective, ignoring the visual components of academic genres compromises student learning; hence EAP/ESP practitioners must attend to visual literacy and teach students, especially those in science and technical disciplines, "to read visuals as much as texts" (Hyland, 2006, p. 53).

As part of a larger study, this paper uses visual rhetoric (Gross and Harmon, 2014) to explore verbal-visual interaction in research articles in discrete mathematics. Unlike rhetorical analysis, which is mainly concerned with the verbal components of argument, visual rhetoric in science analyzes nonverbal displays to understand their role in scientific practices, knowledge creation, and communication. The discipline of study is discrete mathematics, a major sub-discipline in mathematics with inter- as well as cross-disciplinary links to other fields, e.g., optimization, computer sciences, and engineering. The generic structure of research articles in the discipline has already been examined (Graves, Moghaddasi, and Hashim, 2017). Based on the results from our analysis of the rhetorical structure from the larger project, we sought to answer the following questions:

1. What roles do visuals play in research articles in discrete mathematics?
2. Which rhetorical structures in research articles in discrete mathematics rely more/less on visuals? Why?
3. How can the results of the study be applied to the teaching of academic writing in mathematics and related disciplines?

To answer these questions, we used triangulation (Candlin and Hyland, 1999), which allows both collecting data from multiple sources and using multiple approaches to analyze the data (Salkind, 2010). Triangulation "provide[d] multiple lines of sight and multiple contexts" (Salkind, 2010) to examine and enrich our understanding of the move-visual interactions in research articles in discrete mathematics. Our results show notable associations between the move structure and the visuals used in the articles in ways that contribute to the central rhetorical purpose of the articles, namely establishing facts (that is, new knowledge). Here we first summarize some existing research on visuals in academic genres. Next, we briefly describe the study design. We then present results including the roles that visuals play in RAs in discrete mathematics as well as examples of move-visual associations in their rhetorical structure. Lastly, we discuss the implications of our findings, including the pedagogical implications for academic writing classes in mathematics and related disciplines.

5.2. Studies of visuals in academic genres

The study of visuals in multimodal genres has a tradition in linguistic-oriented ESP research. In 1980 Dubois studied presentation slides in biomedical speeches, becoming probably the first ESP researcher to demonstrate the potential of visuals to make meaning. Other researchers followed thereafter, mainly drawing on Halliday's Systemic Functional

Linguistics in their analyzes (Morell, 2015; O'Halloran, 2005; Rowley-Jolivet, 2002, 2004). Rowley-Jolivet (2002), for example, analyzed videos of scientific conference presentations to develop a taxonomy for visuals used in conference presentations that facilitates descriptive analysis of visuals in academic genres in combination with systemic-functional analysis (Tardy and Swales 2014).

Subsequent studies have linked visuals to disciplinary epistemologies and arguments. For example, Rowley-Jolivet (2004) views a discipline as a 'social microcosm' with conventions of use for visuals and 'visual ergonomics' (p. 148). Epistemological requirements—data type, research methodology, and warrants used to argue knowledge claims—govern a discipline's use of visuals. Visuals also provide evidence for new knowledge (Charles and Ventola, 2002; Morell, 2015). Morell (2015) assigns three functions to non-verbal (that is, visual) material (NVM)[1] in her analysis of conference presentations: illustrative, decorative, and expository. Illustrative NVM contains a verbal component, such as illustrating a process using a flow chart. Decorative NVM creates backgrounds and usually appears in social sciences. Expository[2] NVM fills evidence-providing roles and appears more commonly in sciences and engineering. Morrell's categories identify dominant functions, yet she notes the functions are mixed. For example, a 'decorative' flow chart of a process might also serve as evidence to support a claim, functioning equally as expository, a finding that supports existing research on the role of visuals in arguments in academic texts (Miller, 1998). Gross (2007), however, argues that visuals in science are always multifunctional: they both illustrate and argue for theory.

[1] Morell de-emphasizes the visual in her label, choosing 'non-verbal material' [NVM] over 'visual material' [VM]. This label implies that visuals are, if not subordinate, at least always linked to verbal language.
[2] In the field of writing studies and rhetoric, 'expository' refers to explanation and not argument (i.e., in the modes of discourse, exposition is distinguished from persuasion), yet Morell describes this category as capable of contributing to persuasion by acting as evidence.

Regarding visuals in articles in biology, Miller (1998) concludes that visuals in RAs both 'prove' and 'clarify': they 'provide the foundation of the argument' and condense new information for the 'informed and potentially skeptical reader' (p. 43). He argues that, by condensing new information, visuals maximize academic genres' persuasiveness. He also notes that visuals surpass text in science RAs because 'many scientists write their article . . . to highlight the visuals' and 'expert readers often read the visuals first' (Miller, 1998, pp. 29-30). Thus, 'visuals [in RAs] not only buttress the argument but also attract the reader to the argument in the first place' (Miller, 1998, p. 44).

Other scholarship assigns complementary status to visuals in research genres. For example, Hemais (2014), studying RAs in service marketing, examines visual-verbal relations by applying a visual grammar and concludes that they work together to make meaning and support research facts. She argues that verbal elements refine the meanings of theories, while visuals make tangible complex verbally-realized concepts, a finding important to the study of how visuals function in RAs. However, Hemais does not connect the visuals to the rhetorical moves of genre analysis. We would argue that identifying visual-move associations at the micro-rhetorical level of argument is essential because the data presented visually often constitute the new knowledge and are, of necessity, a central element of the RA argument. Consequently, how visuals contribute to the argument for new knowledge in this genre has important pedagogical implications.

Applied linguists studying multimodality in mathematical texts have mostly used Systemic Functional Linguistics as their analytical approach. For example, when analyzing a school mathematics lesson, O'Halloran (2010) applies Halliday's meta-functions to explore how mathematical knowledge accumulates across the three semiotic resources of language, images, and mathematical symbols, which together are the building blocks of mathematical knowledge. O'Halloran (2010) argues that each semiotic resource fulfills particular functions:

images 'provide an intuitive overview of the relations between mathematical participants,' language and images 'introduce and conceptualize mathematical concepts and problems' (p. 4), and symbolism 'formalise[s] those relations and solve[s] the problem' (p. 5). She further argues that the resources' inter-semiotic integration allows for 'immense semantic expansion' in mathematical texts, 'beyond the sum of the meaning potentials of the three resources' (p. 5). The linguistic studies summarized here emphasize the importance of visuals to the argument.

In visual rhetoric and rhetoric of science, scholars have explored the role of visuals in arguments in science. Much of this work has focused on how visuals help scientists communicate complex concepts to non-academic (that is, general and lay) readers (Northcut, 2011; Dyehouse, 2011; Walsh, 2010; and others). How scientists communicate with academic audiences through visuals has received some attention (e.g., Gross, 2007; Graves, 2014; Gross and Harmon, 2014). Another focus of existing work in visual rhetoric is how visualization works as evidence to support verbal arguments (Reeve, 2011; Kimball, 2006; Northcut, 2007). Some rhetorical studies have addressed this role in RAs, focusing on how visuals and text interact to depict theories and argue for them, increasing the persuasive power of the overall text and knowledge generation (Gross, 2007; Graves, 2014; Gross and Harmon, 2014). Others have examined how visual proofs support knowledge claims (Winn, 2009).

Gross and Harmon (2014) identify the stages in argument that show visual presence and the meanings it fulfills. They develop a method of analyzing examples of verbal-visual interactions from multiple areas in science. They suggest that visuals enact "the En-framing of the world, turning it into mathematization," which helps scientists to represent the world as "a calculable nexus of forces" (2014, p. 17). They argue that this function enables visuals potentially to change their epistemic status by contributing to different stages in an argument from problem selection and hypothesis generation to experimentation for discovery to argument justification.

Gross and Harmon (2014) also propose that throughout various argument stages visuals fulfill different semiotic meanings: 1) iconic (that is, they represent the world); 2) symbolic (that is, they stand for aspects of the world); and 3) indexical (that is, they show causal relationships in the world). Gross and Harmon argue that viewers interpret this meaning by placing the visuals in the context of argumentative structures. Unlike Morrell (2015), who implies an author-stipulated meaning for visuals, Gross and Harmon emphasize the role of readers in assigning meanings/functions to visuals. Their theory also highlights that meaning construction is a process of contextualized, rather than de-contextualized, reading of visuals. Gross and Harmon (2014) also emphasize the synergistic interaction of verbal and visual as indispensable to building a persuasive argument in natural sciences, although they assign a subordinate position to visuals, stating that they cannot be arguments—a view not shared in all scientific disciplines. For example, in the fields of physics, nanotechnology, and engineering, researchers start their argument with the visuals and then write the verbal to support the visuals (Graves, 2014; Kresta et al, 2011).

Mathematization, the primary purpose of some visuals, is defined by Goodwin (2001) as those contextually-driven practices aimed at transforming intractable phenomena into mathematically tractable visuals such as graphs and diagrams. Being contextually-driven, mathematization thus shapes, and is shaped by, professionality and disciplinarity (Graves, 2014). As a disciplinary practice, developing and reading mathematically visualized information requires advanced disciplinary knowledge.

Studying visuals in nanotechnology RAs, Graves (2014) shows how influence from parent disciplines can guide researchers to select and mathematize more complex evidence as visuals in this interdisciplinary field than within each contributing discipline. Graves (2014) argues that visual data in nanotechnology accomplish complex activities from informational to rhetorical to ontological to epistemological: visual data can constitute the evidence that develops and supports the claims, but it can also be the fact itself or simultaneously form the foundation and the structure for new knowledge.

These rhetorical studies of visuals in research genres show us that visuals sometimes simultaneously generate and argue for new knowledge. Hence, we contend that rhetorical and linguistic approaches together can better articulate the complex roles that visuals play in disciplines. Here we explore some of these roles by examining how the visuals in discrete mathematics RAs interact with the rhetorical moves. But first, we briefly describe our research design, methodology, and our strategic decisions to ensure validity of our findings.

5.3. Research Design and Methodology

This research builds on earlier research analyzing the move structure of discrete mathematics RAs (Graves et al, 2013, 2014; Moghaddasi and Graves, 2017). Our corpus comprises five high impact discrete mathematics journals. Journal selection criteria included their being listed in two prestigious citation indexing service platforms (Thomson Reuters Web of Science and Scopus); their high impact factors; their coverage of a wide range of topics from both pure and applied orientations within different sub-disciplines of discrete mathematics; and their being recommended by experts in the field. We then selected a stratified random sample of 30 RAs (See Moghaddasi and Graves, 2017, for specific titles). The relatively small size of our corpus was intended to support a focused and targeted contextual analysis (Tardy, 2013). Our sample included six RAs from each journal; the strata included the authors (one article per author), the number of articles selected per issue (one), and the year of publication (equal number of articles chosen from three consecutive publication years). For convenience of referring to the corpus, we coded the articles by abbreviating the home journal title for each article followed by a number that represents the number of the article sampled from the journal. Abbreviations are *Discrete Mathematics*: DM; *Discrete Applied Mathematics*: DAM; *Graphs and Combinatorics*: G&C; *Journal of Combinatorics and Optimization*: JCO; and *SIAM Journal on Discrete Mathematics*: SIAM. The corpus formed by the articles from these journals reflect topics from across the sub-disciplines of discrete mathematics, the study of mathematical objects represented by integers (Renze and Weisstein, 2012). Table 5.1 presents some details of the corpus.

Table 5.1. Details of the corpus.

Number of RAs	Number of Authors	Publication date range (Yrs)	Number of Countries Represented
30	67	3	21

Within our sample, nine RAs address applied mathematics problems and 21 have a mostly pure mathematics orientation. However, we did not distinguish between pure and applied mathematics RAs in our analysis based on discussions with disciplinary informants who argued that in discrete mathematics the orientation of some RAs could better be categorized on a more/less pure/applied continuum than a binary categorization. The larger project of which this article is a part received approval from the Research Ethics Board 1 at the University of Alberta, Edmonton, Alberta, Canada.

To avoid a 'circumscribed' analysis, 'heavy on description, but light on interpretation and explanation' (Swales, 2019, p. 77), we used a triangulated approach (Candlin and Hyland, 1999) to analyze, interpret, and explain our data. Our approach was also guided by our research questions, which required investigating the generic context of the visual material, the argumentative and rhetorical purposes of the research articles, and conventions of knowledge creation in mathematics as a hypothetical discipline. Van Leeuwen and Jewitt

(2011) suggest that aspects of different visual analysis approaches might be integrated depending on the project requirements. We used two sets of data including textual-visual and informant data. We also used metadisciplinary research in mathematics to analyze our data sets as well as to interpret our findings. By metadisciplinary we mean existing literature about mathematics, the history of the discipline, mathematical philosophy, research procedures and foundations of knowledge creation and epistemology. These readings as well as the informant data, when triangulated with the textual-visual data, facilitated deeper understanding of disciplinarity in the RA genre in mathematics.

Studies in genre analysis have traditionally focused on *textual* discursive moves; that is, 'a discoursal unit that performs a coherent communicative function' in the text (Swales, 2004, p. 228). However, in this article, while exploring the contribution of visuals to RAs in discrete mathematics, it became necessary to differentiate between textual and verbal signs. The former pertains to both linguistic and mathematical symbolism, while the latter solely refers to the linguistic signs. Our focus is, thus, on verbal language.

Therefore, to collect verbal-visual data, we analyzed the corpus for moves, visuals, and move-visual associations (the moves that use and mention the visuals); we identified only moves which pointed to visuals. We then analyzed the rhetorical function/s of each visual and the move/s with which it was associated. For example, where a definition was followed by a statement such as "See Fig. 1," we classified the visual as part of the 'Establishing presumptions' move, a move that defines the abstract objects of study in mathematics (See Graves et al, 2014). Likewise, a visual associated with a post-definition statement such as "[A]s Fig. 3 shows, this property does not hold in general" was identified as an argumentative visual supporting a 'Discussion' move. Identifying these move-visual associations enabled a clear understanding of the episodes of argument and knowledge generation that motivated the presence of the papers' visuals.

To code the visual data, we treated each figure as a single visual regardless of the number of images it included. This decision was inspired by Swales' (2004) definition of a move as a unit, a linguistic unit in the case of textual moves, that performed a coherent function. We realized that visuals in a single figure, much like words in a textual move, collaboratively perform a coherent function, so we coded instances of such visual sets as one visual. In six cases, where visuals were integrated into verbal statements (not as labeled figures), we counted them as visual data and as a single visual. For example, SIAM5 includes 10 images distributed among two figures and two proposition statements. That is, figures 1 and 2 include three and two images, respectively. However, since the images are integrated with symbolic signs and make two visual-symbolic statements, we considered them as two rather than five visuals. Regarding the proposition statements, one image has no label and is integrated into proposition 4.4. In proposition 4.5 (See Example 4), one statement and four images are integrated into two statements with two images in each statement (and no label). Based on the number of statements and the coherent rhetorical functions these images performed, we counted them as three visuals not five; therefore, we coded five rather than 10 visuals for SIAM 5. Applying the same strategy throughout, we coded the 245 individual images in our data as 108 visual units.

As well as using verbal-visual data, we also used both interview data and data obtained from credible secondary sources in meta-mathematics and mathematical philosophy. These latter sets of data helped to enrich our understanding of the social norms and discourse community values that constitute discrete mathematics as a sub-discipline and differentiate it from other disciplines in its use of visuals (exploration). Our method thus offers a *contextualized genre analysis*, that is, a context-based, rhetorically oriented, "wide-angle" analysis (Berkenkotter, 2009, p. 18) that examines visuals and their related rhetorical moves in context. In analysing the broader, *supra-contextual* meaning potential of visuals, where words are attached to visuals, we "[saw] word and image as one indivisible unit of analysis" (Van Leeuwen and Jewitt, 2011, p. 7) and aimed to achieve a more sophisticated understanding of genre shaping factors and the learning of writing for specific purposes (Tardy, 2013).

Our interview data come from semi-structured interviews with five mathematicians from different areas of discrete mathematics. The interview questions focused on the role of visuals in discrete mathematics in different stages of research, drafting research, and arguing for new knowledge in the field. Interview sessions were transcribed and coded. We identified themes/concepts repeated and emphasized as important by different interviewees. We also identified themes which were either clearly linked with our verbal or visual data or as being similar to what we had read about in philosophy of mathematics publications or visual rhetoric literature. We then categorized our codes and labeled them based on their conceptual similarities and, most importantly, on the stages of argument they linked with. We then assessed the relative importance of each category.

We also shared samples of our textual data (that is, move-visual associations and classification of visual functions) with the disciplinary informants for their validation of our conclusions; at least one disciplinary informant validated the results of our analysis of the verbal moves and visuals in each RA. In seven cases, a disciplinary coder disagreed with the functions we identified for the visuals (94% agreement). To resolve these coding disagreements, we took the cases to two different disciplinary informants and asked for their ideas about the functions of the disputed visuals. We then assigned the role with the most votes (including the votes of the original coder) to the visuals.

We also performed a second coding, both to verify the reliability of the data from our first coding (intracoder reliability) but also because we realized that there were stand-alone visual moves in the RA figures—visuals that had no textual description. This second coding allowed us to determine the prevalence of the stand-alone visual moves. Our results are thus supported by both intercoder and intracoder reliability. We then triangulated data from the verbal-visual analysis and informant data with existing scholarship in mathematical discourse and epistemology.

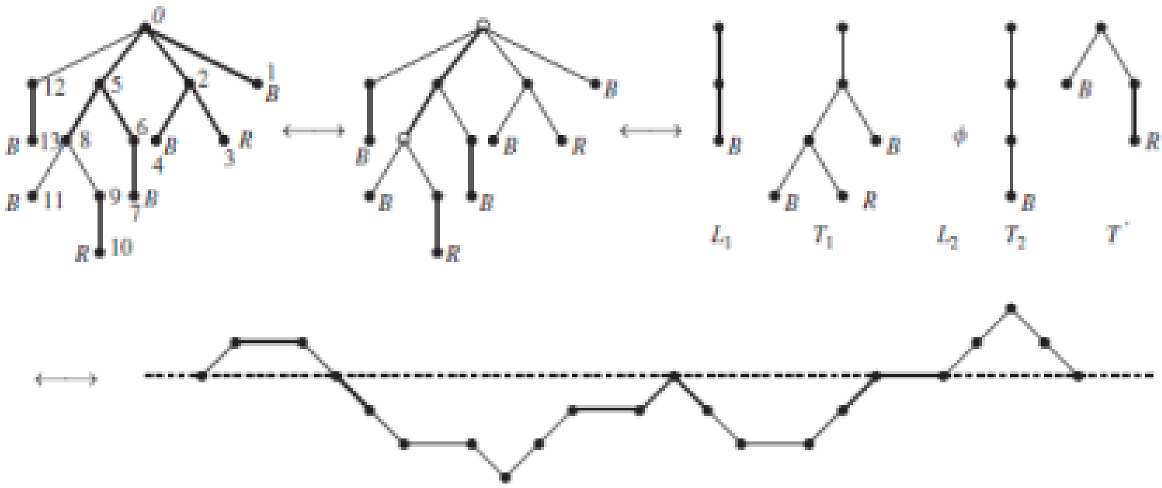
Having classified the visuals based on verbal-visual analysis and our interview data, we randomly sampled four visuals from each functional category, comprising a total of 12 visuals, and emailed our analysis of those visuals to the corresponding/first authors of the original RAs where the visual appeared for their validation and feedback. All the authors validated the identified functions and our interpretations of the visuals in their RAs. The example visuals presented in this article are from among the verified samples.

5.4. Results and Discussion

Seventy percent (70%) of our corpus contains visuals, suggesting they are conventional in RAs in discrete mathematics. However, in 8 RAs (DAM3, DM3, G&C2, JCO5, SIAM1, 2, 3, 4), they are absent. Our informants suggested that the mathematical problems addressed in these RAs either are supported by existing objects or deal with simpler abstract objects, making visuals inessential. However, where present, the visuals project both disciplinarity and specificity.

5.4.1. Disciplinarity and Specificity in Using Visuals

Our data suggest that discrete mathematics shows a high preference for graphical culture and uses a specific ‘visual genre’ (Kostelnick, 2007), namely graphical visuals (See Example 1). The visuals are similar, “bare, skeleton-like, or diagrammatic” (Peirce, 2010, p.91). They do not capture real-world objects or relations but create simple visual objects combining dots and lines, suggesting disciplinary convention in using ‘graphical visuals’ (Rowley-Jolivet, 2002). The visual design unfolds mathematical hypotheses as “always the conception of a system of relations” (Peirce, 2010, p.30), particularly examining distinct values between any two points in discrete mathematics. This characteristic design suggests that disciplinary perspectives shape visual data (Graves, 2014) or, in Tardy's (2005) terms, visuals project disciplinarity.



Example 1. Sample visuals from DAM6 representing the diagrammatic appearance of visuals in discrete mathematics RAs. Reprinted from Discrete Applied Mathematics, 155/7, Chen, W. Y. C., Li, N. Y., & Shapiro, L. W., The butterfly decomposition of plane trees, 2187-2201, Copyright (2007), with permission from Elsevier.

A second area where these graphical visuals project disciplinarity is the value placed in mathematics on aesthetics. Kostelnick (2007, p. 283) explains:

> Brasseur claims members of the "graphing culture," particularly mathematicians, often celebrate the aesthetics of data design [3, pp. 27–28]. Although he derides graphics that foreground the artistic element, Tufte the social scientist waxes aesthetic about the "wonder," "beauty," and "graphical elegance" of well-designed displays [1, pp. 121, 137, 177]. By eliciting a subjective response from readers, this ancillary aesthetic element engenders two rhetorical effects. First, it makes displays more inviting to readers because readers are naturally drawn to elegant displays; and second, it bolsters their credibility because beauty and truth are cognate qualities.

Kostelnick highlights the emotional appeal of visuals for mathematicians (and others). Our informants also mentioned the disciplinary value of brevity: “From among the visual, symbolic and verbal resources, visuals are the most concise way of communicating ideas.” (Disciplinary Informant A, Personal Communication).

Visuals in discrete mathematics are constructed, suggesting they represent mathematicians’ concepts of intangible objects and that construction grants them physical ‘presence’ (Perelman and Olbrechts-Tyteca, 1991). As we show, these graphical visuals represent powerful mathematical concepts and operations, taking readers to the heart of the mathematical reasoning and theories (Angot-Pellissier, 2015). They embody mathematical relations resulting from mathematicians’ thought experiments with abstract mathematical concepts. From a philosophical perspective, these graphs create phenomena in discrete mathematics—an observable entity (that is, fact) of mathematical abstractions (Graves et al, 2014); they make concrete the RAs’ abstract ideas.

Historically, the nature of graphical visuals in discrete mathematics has remained stable, unlike visualization in other scientific disciplines which have experienced radical changes in the gloss and nature of their visuals with the advent of computer and other technologies and/or shifts in research paradigms (Disciplinary Informant D, Personal Communication; Rowley-Jolivet, 2002; Rhyne and Chen, 2018). While the visuals in discrete mathematics may appear similar in style, they differ in number and the rhetorical moves with which they are associated in each RA, suggesting specificity in their employment and rhetorical functions.

The number of visuals in RAs that include them ranges from one in DAM1 and DAM2 to 28 in DM2. The large number of visuals in DM2 was explained by our informants as arising from the fact that this article generates novel families of graphs, so the authors present samples of each graph family. All but one visual in DM2 define mathematical concepts and objects of study, and 25 of 28 visuals include no verbal definition. Thus, in 25 cases, visuals alone clarify the concepts and act as testimony for these new contributions. Disciplinary Informant D stated that these visual definitions contribute significantly to this article's brevity, allowing readers to explore the topic, understand the mathematical knowledge more easily, and become engaged with it (Personal Communication).

Visuals also formed integral parts of the RAs' arguments for new knowledge. Table 5.2 presents details about the visuals, including the numbers of pointers to visuals and the numbers of visuals associated with different moves. Note that it shows more pointers than visuals in the corpus, indicating that some visuals are pointed to multiple times. As we demonstrate, some visuals are called on multiple times to support different parts of the argument.

Table 5.2. Visuals, pointers and rhetorical moves in the corpus (Visual-Verbal moves: EP: Establishing Presumptions; AR: Announcing Result; P: Proof; D: Discussion.)

RAs	**RAs including visuals**	**Total number of Images**	**Total number of Visuals**	**Pointers to visuals**	**Visual-Verbal Moves with visual pointers**			
					EP	**AR**	**P**	**D**
30	**21**	**245**	**108**	**176**	**31**	**14**	**43**	**11**

5.4.2. Rhetorical Functions of Visual-Verbal Moves

Table 5.2 shows that visuals, when pointed to by verbal language, are frequently associated with specific moves including 'Establishing presumptions' (31 times) and 'Proof' (43 times). They are also associated with the moves 'Announcing Results,' and 'Discussion,' though less frequently. These associations suggest that visuals participate in constructing the areas of the argument developed by the moves throughout the RA.

1- Establishing presumptions: Mathematicians use this move to explain their assumptions about the abstract mathematical objects they study, define the objects, introduce notations, or refer to published research for already defined objects (Graves et al, 2014).

2- Announcing results: This move presents statements of results (that is, new knowledge), labeled to indicate their degree of significance—a theorem is most important, a lemma is a minor result, and corollaries are drawn from theorems or proofs.

3- Proof: This move is a concise logical argument aimed at convincing readers that the proposed fact is true. Like results, proofs are labeled and employ the vocabulary and grammar particular to mathematical proofs.

4- Discussion: This move usually follows a result statement or proof. It may present examples as evidence to validate proposed facts, link results, argue for deducing some following results, or highlight a bottom line.

Based on our analysis, we identified three areas in which visuals in RAs in discrete mathematics play a role: ontology, argument and epistemology. These functions arose out of the data that we analyzed. Other functions may be present in different areas in mathematics research. We adopt these labels (Graves, 2014) to facilitate distinguishing one function from another while analyzing examples. But we also emphasize that in each instance the visual performs multiple functions simultaneously. For example, a visual that plays an ontological role by creating a mathematical object also plays an argumentative role if the writer uses it as evidence to support part of the argument (this will be further illustrated in section 5.4.2.4).

The visuals in our corpus are multimodal because they comprise two parts: the figure (that is, graph and (verbal/symbolic) caption) and the verbal/symbolic material related to the figure. Therefore, in our examples we reproduce the figure/caption in the upper panel and the related verbal/symbolic text in the lower panel, following the convention used in discrete mathematics. We emphasize that the rhetorical functions for visuals overlap in an RA and that a visual can simultaneously support, illustrate or create a claim; instantiate a new or existing object, concept, or outcome of a process relevant to the argument; and **be** the new fact the authors contribute to knowledge in discrete mathematics. In the next section, we present some examples to show how the visuals in our corpus performed the rhetorical functions we identified.

5.4.2.1. Ontological Function in Visuals: Definitions and Operations.

Many of the visuals in our corpus were located in sections of the RAs where the authors referred to them to highlight assumptions, define concepts, or construct complex mathematical objects. They generally appeared in RA Introductions, as part of the 'Establishing presumptions' move, and in the Results sections, sometimes in the introductory material, preceding labeled definitions, and/or in statements of results and proofs to help construct specific concepts. In performing these functions, the visuals *made* the abstract mathematical objects: these objects became 'present' and 'tangible.' In these examples, the visuals function ontologically, that is, they create the fundamental objects studied in the RAs. They depict processes that bring new material into the world, materials that do not occur naturally (Wickman, 2010). According to Graves (2014), this function for the visual both instantiates the object (ontological) and serves as evidence that it exists (argumentative). One informant explained that this link between visuals and definitions reflects the nature of the discipline:

> Our area of study is imaginary discrete objects and the relations among them. Once we have images of such structures, they are no longer imaginary; they are real-world objects, much like a painting, which was first an idea but once it is drawn it exists out there, and everybody can touch it. (Disciplinary informant D, Personal communication).

Here Informant D compares the role of the visual to that of a painting to describe how it transforms the mathematical idea into an object or entity—"it exists out there"—and she emphasizes the visual's ontological role. She also implies that visuals transform readers from outsiders, trying to imagine the mathematicians' idea, to insiders who share the mathematicians' idea. In rhetorical terms, if readers share an understanding of the nature of the author's mathematical object, they are more likely to be persuaded not only that it exists but that the argument is sound.

Another important ontological function that visuals in our corpus perform is to enable authors to illustrate the outcome of the operations presented in their calculations. Example 2 from G&C1 occurs as part of the 'Establishing presumptions' verbal move in the Result section. Panel 1 (Fig. 2) presents two graphs; its caption explains the two operations illustrated, 'insertion' and 'uniform insertion'. Panel 2 verbally (and symbolically) describes the authors' presumptions for these operations.

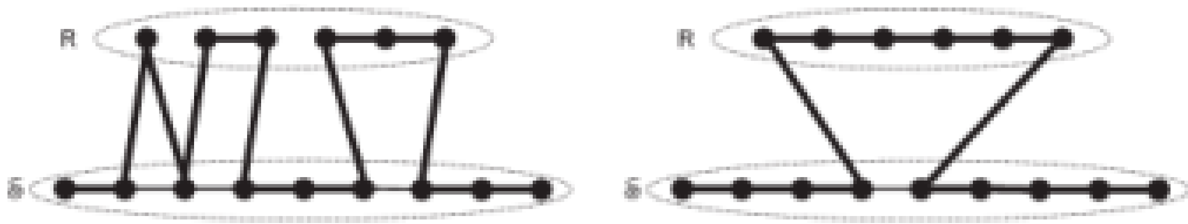


Fig. 2. On the left: R is *inserted* in $\mathcal{S}$. On the right: R is *uniformly inserted* in $\mathcal{S}$

Let $\mathcal{S}$ be an H-subdivision on S. If, for $u, v \in V(G) - f_{\mathcal{S}}(V(H))$, P is a proper $[u, v]$-path of $G - \mathcal{S}$, we say that P is *uniformly insertible* in $\mathcal{S}$ if there exists an edge $xy \in E(\mathcal{S})$ such that $ux, vy \in E(G)$. When P *uniformly inserts* in $\mathcal{S}$ we obtain a larger H-subdivision $\mathcal{S} \triangleleft P$ on S, namely

$$\mathcal{S} \triangleleft P = \mathcal{S} - xy \cup xu \cup P \cup vy.$$

If $R \subset V(G - \mathcal{S})$ we say that R is *insertible* in $\mathcal{S}$ if there is a sequence $\mathcal{S}_0, \ldots, \mathcal{S}_t$ of H-subdivisions on S where $\mathcal{S}_0 = \mathcal{S}$, $V(\mathcal{S}_t) = V(\mathcal{S}) \cup V(R)$, and for all $i \in [t]$, $\mathcal{S}_i$ is obtained by uniformly inserting a subpath of $G[R] - \mathcal{S}_{i-1}$ into $\mathcal{S}_{i-1}$: We say that $\mathcal{S}_t$ was obtained by *inserting* R into $\mathcal{S}$. Figure 2 illustrates insertion and uniform insertion.

Example 2. Ontological visuals create manipulated objects in G&C1. This example also constitutes a move, 'representing the product of an operation,' that is associated with the move, 'establishing presumptions,' but that conveys information that neither the verbal or symbolic language can communicate. Reprinted by permission from Springer: Springer Nature. Graphs and Combinatorics, Subdivision extendibility, Gould, R., & Whalen, T, COPYRIGHT (2007)

The graphs function ontologically in the sense that they create and illustrate the outcomes of the two operations. They translate the verbs, '*is insertible in* [sic, italics in original]' and '*is uniformly insertible in* [sic, italics in original]', the products that the two processes generate. That is, the visuals 'construct' the entities that result when R is insertable in S and uniformly insertable in S. Mathematical philosophy discusses the ontological function that some visuals perform in mathematics research:

> Thus, the necessary reasoning of mathematics is performed by means of observation and experiment, and its necessary character is due simply to the circumstance that the subject of this observation and experiment is a diagram of our own creation, the conditions of whose being we know all about (Peirce, 2010, p. 19).

These constructions and operations are thus hypothesis-laden (Kant, in Peirce 2010), which in Peirce's terms is 'always the conception of a system of relations,' (Peirce 2010, p. 30) and make concrete the researcher's mental experiment, submitted to reader scrutiny. Being hypothesis-laden also enables some visuals to play explicit epistemological roles, as Gross and Harmon (2014) noted.

In Example 2, Fig. 2 illustrates the outcomes of two operations, helps readers visualize the concepts to differentiate them, and encourages readers to experiment with the concepts to discover new relations among their parts not stated in the text (Peirce, 2010). This visual does, in fact, indicate something not stated in the text; it initiates the move, 'Representing the Outcome of an Operation' (ROO). This visual move is associated with the verbal move, 'Establishing presumptions,' in that the verbal and symbolic language create the context that makes the visual move comprehensible (that is, without the verbal context the graph's meaning is unclear). However, this visual also accomplishes unique work in representing the outcome of the operation that is not communicable through either the verbal or symbolic language. This example illustrates how visuals create new theoretical objects. It also suggests that in discrete mathematics visuals can *initiate* moves that accomplish work not communicable through verbal or symbolic language.

5.4.2.2. Argument Functions in Visuals: Introduction, Result, Discussion.

All visuals function as part of the argument in discrete mathematics RAs. As noted, visuals not only create the object of study or depict the product of a described process, they can also serve as evidence in the argument that the concept or object actually exists. The example in this section highlights the visual's role as evidence, explicitly supporting a stated claim (and in the following section, as instantiating the claim). Such visuals generally appear in the Introductions or the Results—in Introductions to constitute and support arguments for definitions, in Results to support claims regarding cause and effect among objects of study, policies/strategies proposed for solutions, or in evaluation of the results in discussion moves.

Example 3 from DM2 reproduces a proof, which in research mathematics is an argument for a Result—for example, a conditional statement that claims causal relationships between properties and objects. Proofs incorporate symbolic and natural language and may include visuals. Visuals can perform various functions in the proof argument: for example, raise methodological concerns (e.g., the mathematicians' strategic decisions) or rebut potential reader objections. Example 3 illustrates both these uses. The authors explicitly identify this visual as an argument ("repeating the argument") and point to the figure twice ("in Fig. 10") to emphasize that the visual completes this proof.

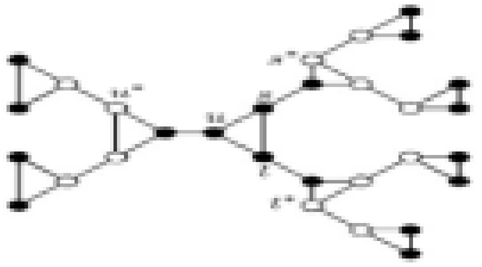


Fig. 10. A subgraph of G.

Repeating the argument with the vertex u replaced by s or t shows that the graph shown in Fig. 10 is a subgrap of G. But then with the vertices s^*, t^* and u^* as indicated in Fig. 10, $(S\setminus\{u, s, t\}) \cup \{u^*, s^*, t^*\}$ is a TDS of G tha satisfies condition (1) but induces fewer K_3's than does $G[S]$, contradicting our choice of S. □

Example 3. Visual evidence for problem-solution strategy in Proof of Lemma 10 in DM2. The visual not only presents counter-examples to support the authors' claim, it also disrupts the linear flow of the argument by calling into question the textual discussion of the claim. Reprinted from Discrete Mathematics, 308 (16), Favaron, O., & Henning, M. A., Bounds on total domination in claw-free cubic graphs, 3491-3507, Copyright (2008), with permission from Elsevier.

The whole proof (only one part of which is reproduced in Example 3) presents a series of claims about different parameters in graph G. Example 3 reproduces the second claim in this series, which articulates a procedure for manipulating the graph. A disciplinary informant noted that the authors argue that the vertex from which the procedure starts is critical to the solution. The authors explain:

> Repeating the argument with the vertex u replaced by s or t show that the graph shown in Fig. 10 is a subgraph of G. But then with the vertices s^*, t^* and u^* . . . , $(S\setminus\{u, s, t\})$ U (u^*, s^*, t^*) is a TDS of G that satisfies condition (1) but induces fewer K_3's than does $G[S]$, contradicting our choice of S.

That is, starting the procedure with vertex u gives a better result than starting it with vertex s or t. The visual illustrates the graphs produced from all possible outcomes based on the strategic choice of starting vertex, and the verbal elements explain why some choices are bad.

Assuming a linear reading strategy (that, we realize, not all readers may follow), readers 1) examine the graph,[3] 2) presumably read the caption identifying the topic ('A subgraph of G'), 3) read the restatement of the argument and the claim, and 4) return to examine the graph more carefully. This argument is organized for maximum persuasive effect. If readers have studied Fig. 10, they may have observed for themselves that vertex *u* is superior to *s* or *t ('contradicting our choice of S')*. Here readers, in effect, participate in constructing the authors' argument as they process its parts. The visual plays a key role because it enables readers to perceive and perhaps even arrive at the claim *before* the authors state it. This multimodal argument, then—what Lemke (1998, p. 87) has called a 'semantic hybrid'—builds the case for audience adherence, a process that transforms the claim from argument into a fact and shared knowledge (Perelman and Olbrechts-Tyteca, 1991).

Example 3 (as did Example 2) initiates the move, 'representing the outcome of the operation.' This visual move contributes to the verbal/symbolic move of 'proof.' In these examples, the visual move is critical to the argument because it illustrates the outcome of the mathematical operation, serving as evidence that supports the argumentative claim (Winn, 2009), a device to illustrate for readers how the operation works and a rebuttal to a potential reader objection. The visual move presents these actions (support, illustrate, rebut) while readers cognitively process the argument's nuances. Thus, the visual in Example 3 illustrates the outcome of the wrong (and correct) choices, thereby contributing to the argument's emotional appeal and inviting readers to accept the writers' claim and increasing readers' confidence in its validity.

[3] The disciplinary specialists in our study noted that they always read the visuals first.

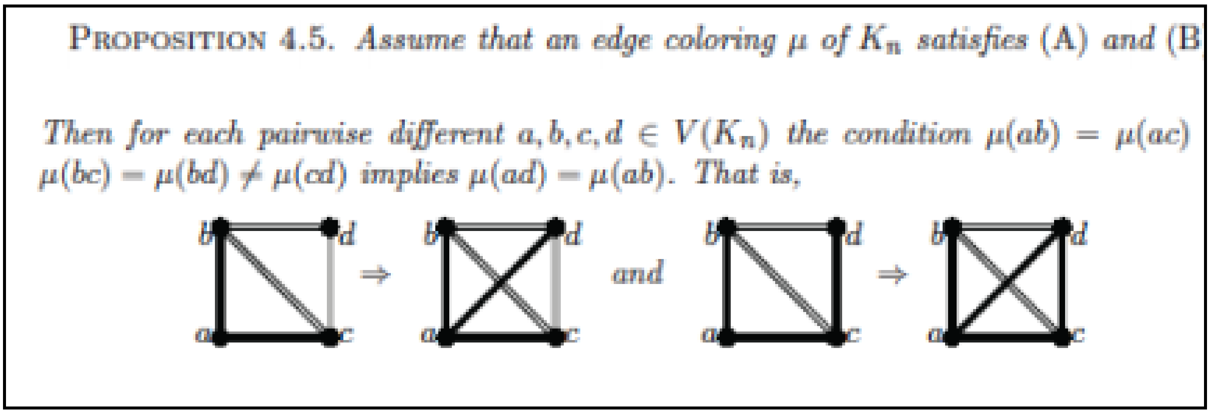
In analyzing the moves in Example 3, we have also highlighted the argument's organizational structure. As noted, the visual appears before discussion in the verbal/symbolic language, and it offers a counter-example/rebuttal before readers see the argument. If we consider the verbal/symbolic language alone (as is traditional in genre analysis), we observe a linear, chronological narrative progression in the argument organization. However, when the visual moves are factored into the analysis, a different organizational structure emerges. The visual rebuts potential reader objections in advance, disrupting the argument's linear organization. Readers will know the counterexamples (*s* or *t*) as they process the verbal/symbolic explanation. Their foreknowledge shapes how they respond to the verbal language, allowing them to participate in the argument as it unfolds from the authors, creating a powerfully persuasive process in which readers are motivated to accept the claim because they themselves have contributed to creating it (Barnes, 1984).

We would argue that the structure of visual followed by verbal and symbolic language-based explanation is a strategic persuasive device used in some RAs in discrete mathematics. The visuals draw readers into the mathematical manipulations, and the readers must make sense of the images' roles in the arguments about knowledge. Disciplinary informant D notes that using visuals as counterexamples is a routine strategy in mathematics: "[Visuals] are quick, brief, and transparent, hence preferred over verbal counterexamples" (Personal communication), suggesting that, in discrete mathematics, visuals are an integral—in fact, irreplaceable—component of the argument. In addition to being an effective persuasive strategy, this structure also disrupts conventional thinking about the structure of RA arguments more generally as an entirely linear chronological verbal narrative. The disruption is important in inviting RA users to reformulate their meta-rhetorical knowledge about argument structure and construction. It also reveals the power of multimodal rhetorical-visual genre analysis in deepening our understanding of "the visual heart" of disciplines and their arguments (Gross and Harmon, 2014, p. 264).

5.4.2.3. Epistemological Function in Visuals: Results, Proofs.

Argument claims in research mathematics can be assembled to create epistemological facts (that is, new knowledge). In our data, epistemological facts, and the visuals associated with them, appear mainly in announcements of results and proofs. These visuals are generally integrated as part of the result statement/claim itself or separate from the claim but cited in-text.

As noted, results in research mathematics are typically labeled (e.g., "Proposition 4.5.", "Lemma 10"). Results statements concisely state the author's proposed fact. In our data, visuals sometimes appear within the argument for the proposed fact itself, as in Example 4 from SIAM5.



PROPOSITION 4.5. *Assume that an edge coloring μ of K_n satisfies* (A) *and* (B

Then for each pairwise different $a, b, c, d \in V(K_n)$ *the condition* $\mu(ab) = \mu(ac)$ $\mu(bc) = \mu(bd) \neq \mu(cd)$ *implies* $\mu(ad) = \mu(ab)$. *That is,*

Example 4. Epistemological visuals integrated into knowledge claim in SIAM5. Here the verbal/symbolic explanation of the proposition appears first. The textual discussion presents the outcome of the operation while the visual depicts the process of reaching that outcome. Reprinted from SIAM Journal on Discrete Mathematics, 23(2), Krotov, D. S., & Vladimir, N. P., n-Ary quasigroups of order 4., 561-570. Copyright ©2009 Society for Industrial and Applied Mathematics. Reprinted with permission. All rights reserved.

The proposed fact is labeled Proposition 4.5 and pertains to a series of relations (equalities and inequalities) in a parameter (edge coloring) imposed on parts of the object studied ("Assume that an edge coloring μ of K_n satisfies (A) and (B). Then for each pairwise different a, b, $c \in V\ (Kn)$ the condition $\mu(ab) = \mu(ac) =\mu(bc) = \mu(bd) = \mu(cd)$ implies $\mu(ad) = \mu(ab)$," italics in original). The verbal claim highlights relationships between the complex entities ($\mu(ab), \mu(ac), \mu(bc), \mu(bd), \mu(cd), \mu(ad)$) using mathematical symbols =, ≠, ∈ to indicate states of equality, inequality, and set membership, respectively. Four graphs then follow, identified as a restatement of the operations in the verbal claim ("That is"). Each operation features two square graphs linked by arrow symbols (meaning 'result in'), communicating that the two operations are connected ("and"). The authors organize the argument for their proposed fact by rendering the objects and the relations within and among them verbally, symbolically, and visually (Roth et al., 2005, in Johns, 2013), creating what Lemke (1998, p. 87) calls a "semiotic hybrid" that combines "verbal, mathematical, visual-graphical and actional-operational elements."

Example 4 departs from the organizational structure of previous examples by presenting the verbal/symbolic explanation first and then presenting the images (that is, the visual is not labeled as a figure). In this case the verbal language explains the context for "Proposition 4.5. *Assume an edge coloring* μ *of* K_n *satisfies (A) and (B)*", and the visual depicts the implication, $\mu(ad) = \mu(ab)$. Here the visual enables a complete discussion of the proposition, creating a persuasive case for its validity as an 'established' fact. A second departure from previous examples is the function of the verbal and the visual: the verbal discussion describes the operation outcome ("*the condition* $\mu(ab) = \mu(ac) \neq \mu(bc) = \mu(bd) \neq \mu(cd)$ *implies* $\mu(ad) = \mu(ab)$", italics in original), while the visuals depict the process. The authors characterize this visual as restating the verbal claim, which, according to a disciplinary informant 'is conventionally dense and terse':

> I mean that's how a labeled claim statement is supposed to be, short. Short and beautiful! In mathematics, we are not a fan of restating an idea unless it helps us

clarify without being redundant' (Disciplinary informant A, Personal communication).

Our informant notes that redundant repetition of a point is discouraged in discrete mathematics. We thus argue that the visual in Example 4 does not reproduce the verbal but, in fact, contributes something new. It initiates the move, 'Embodying the operation' (EO): it depicts the otherwise absent "actional-operational elements" of Lemke's definition. In the EO move, the visual illustrates the process (indicated by the arrows and '*and*'), while the move, ROO depicts the product. Therefore, the visual in Example 4 accomplishes the verbal move of 'Announcing result' by depicting a process that verbal/symbolic language cannot communicate.

Epistemological claims also appear in proofs. Example 5 reproduces part of a proof, also from SIAM5. This example, unlike Example 4 from the same RA, presents the visual as a figure followed by the verbal/symbolic language. The figure caption orients readers, noting that the graphs depict a sequence from the previous result, Proposition 4.8, which the authors develop into a "Claim (*)". Given the premium accorded to brevity in mathematical discourse, these authors display the visual component of their argument first to set the stage for the verbal/symbolic components (Schriver, 1997) that follow.

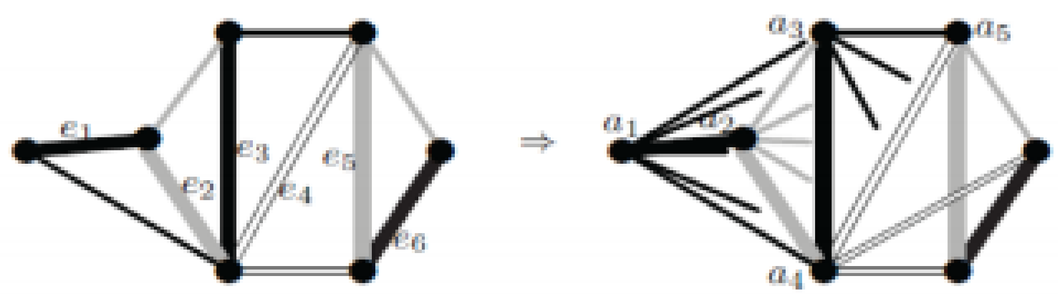


FIG. 2. *An example of a sequence* $e_1, \ldots, e_k$ *from the proof of Proposition 4.8*

Claim (*). We claim that for every i, j, $1 \le i < j \le k$, and $d \in e_j$ the vertices a and d are different and $\mu(a_i d) = \mu(e_i)$ (see Figure 2). We will show this by inductio on $j - i$.

Example 5. Epistemological visuals reproduce knowledge claims in SIAM5. Reprinted from SIAM Journal on Discrete Mathematics, 23(2), Krotov, D. S., & Vladimir, N. P., n-Ary quasigroups of order 4., 561-570, Copyright ©2009 Society for Industrial and Applied Mathematics. Reprinted with permission. All rights reserved.

We suggest that readers are intended to examine the graphs before they read the verbal explanation. The authors assume that readers will provisionally accept the claims for new facts in the verbal/symbolic language based on their initial analysis of the graphs, from which it will be apparent that: 1) the authors identify a difference between two entities (vertices), and 2) they claim the equality of an edge coloring parameter in the objects studied. The authors clearly don't expect readers to be persuaded of the validity of these claims based on the visuals alone because, immediately following the pointer to the visuals ("see Figure 2"), the authors state that they use induction to generate evidence to support this claim. We argue that the presence of evidence from induction indicates that the visuals not only support the claim but also elaborate the proposed facts. These visuals create an actional-operational instantiation of the process represented in the symbolic language (the visual move of EO). They help the authors generate their argument for their proposed facts (Gross and Harmon, 2014). Thus, this claim for new facts depends on the visuals functioning epistemologically. The authors have used the visuals to frame the mathematical objects and their claim as a mathematized "calculable nexus of forces" (Gross and Harmon, 2014, p. 17). As in earlier examples, the structure of Example 5 is strategic, organized to strengthen the persuasive capacity of the linguistic resources: the visuals prepare readers for the claim, illustrating important background information from Proposition 4.8. and embodying the key features that make the Claim comprehensible.

In this section, we have discussed how the authors in our corpus used visuals in their arguments for new knowledge. As noted, visuals play multiple roles in the mathematicians' arguments. In the next section we analyze the three functions being performed simultaneously in an example.

5.4.2.4. Multi-Functional Visuals.

In the previous sections we distinguished among ontological, argumentative and epistemological functions for visuals in our corpus; however, as we have noted these distinctions are artificial because in many cases the visuals fulfill multiple roles, as in Example 6 from DM6. Figure 7 appears first in the article, followed by an explicitly labeled "Proof" to which it connects. As we have argued, the ordering of visual/verbal/symbolic language must be seen as strategic choices by authors to develop their arguments. Figure 7 presents three graphs: the top graph is labeled D_k; the second, D_{k-1}; and the third, D_k/e. The caption explains briefly what each graph depicts. D_k pictures the central concept, while the other two graphs illustrate operations.

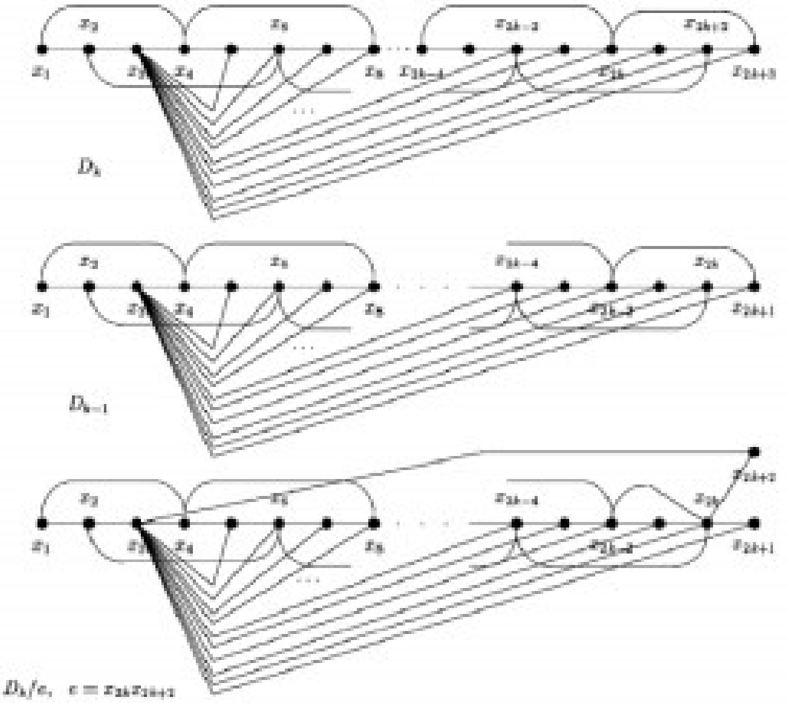


Fig. 7. Reduction of D_k. Note that D_k/e is obtained by gluing to D_{k-2} two copies of K_3 at x_3, x_{2k-1} and K_4 at x_3, x_{2k-2}, x_{2k-1}. Similarly $D_k + e$ is obtained by gluing to D_{k-1} two copies of K_4 at x_3, x_{2k}, x_{2k+1}.

Proof. Let p_k denote the chromatic polynomial of D_k. We apply the reduction formula of Whitney (2) for the nonedge $e = x_{2k}x_{2k+2}$ in D_k and then the gluing formula (3) (see Fig. 7).

Thus

$$p_k = \begin{cases} 0, & k < 0, \\ t(t-1)(t-2), & k = 0, \\ (t-2)(t-3)p_0 + (t-1)p_0, & k = 1, \\ (t-3)^2 p_1 + (t-2)^3 p_0, & k = 2, \\ (t-3)^2 p_{k-1} + (t-3)(t-2)^2 p_{k-2}, & k \geqslant 3. \end{cases}$$

Hence, we get

$$p_k = (t-3)^2 p_{k-1} + (t-3)(t-2)^2 p_{k-2} + [k=0]t(t-1)(t-2) + [k=1]2t(t-1)(t-2)^2 + [k=2]t(t-1)(t-2)^3.$$

Example 6. Multifunctional visuals in DM6.Reprinted from Discrete Mathematics, 307/11-12, Bielak, H., Chromatic properties of hamiltonian graphs, 1245-1254, Copyright (2007), with permission from Elsevier.

Following the caption is the verbal discussion of the proof where the author introduces the central concept, “the chromatic polynomial of D_k”, but does not define it. Assuming readers examined the visual first, they will have recognized that the top graph defines this concept and thus creates the object (ontological function). This visual move is not associated with any verbal or symbolic explanation that follows.

The author then states her decision to apply a procedure, “reduction of D_k”, which consists of applying two formulae, ‘the reduction formula of Whitney” and “the gluing formula.” To illustrate the application of these formulae, the author refers readers to Fig. 7, “(see Fig. 7)” (argumentative function). The second and third graphs initiate the move ROO in association with the verbal/symbolic language in the figure caption for the middle graph and in the explanation of the proof for the bottom graph. Once she has performed the reduction on the visuals, the author obtains the new object, represented in the bottom graph (ontological and epistemological function). She then translates the features of this new object into equations that become the result (new knowledge). As the discourse marker ‘thus’ (used before the equations) indicates, the equations are deduced from the procedure and the graph.

As noted, the images in Fig. 7 perform the three functions: 1) they define and construct the objects, 2) they provide evidence of the procedure’s validity, and 3) they illustrate and support the equations (new results) deduced from the procedure. The multi-functions of these graphs support Gross and Harmon’s (2014) contention that visuals can change their epistemic status as they contribute to different aspects of the argument from problem selection/hypothesis generation to experimentation for discovery to argument justification. Comments by a disciplinary informant support our argument for the multi-functionality of the visuals used in Example 6 specifically and in discrete mathematics generally:

> Depending on the area and topic, sometimes using images for multiple purposes is the most efficient strategy to develop our argument. It might not be the only choice, but it could be the most efficient and engaging one. And we're quite sure that the reader can catch up. I mean, in mathematics, we're used to reading, imagining and understanding the concepts and arguments visually. We record and recall, or better say, draw a representation of the visuals in our brain when double checking the concepts and the calculations. We might do it subconsciously, but we definitely do it. (Disciplinary Informant D, Personal Communication).

Here the disciplinary informant reiterates disciplinary values such as efficiency, brevity and reader engagement in explaining why visuals may be multipurpose. Since visuals in RAs in discrete mathematics contribute in multiple roles, they facilitate doing and reporting research in the discipline, especially in areas such as Combinatorics, where visuals are essential for constructing mathematical objects (DM1, DM2, DM4, DAM1, JCO3 and G&C4).

In the next section we examine more closely the visual moves that were identified during our analysis of the visual-verbal moves in the corpus.

5.4.3. Visual Moves

Initially, we assumed that all visuals in the corpus would be associated with verbal rhetorical moves. We did indeed find numerous instances of visuals that were associated with established verbal moves in discrete mathematics (Graves et al, 2014; Moghaddasi and Graves, 2017). However, we also found several instances where visual moves *replaced* verbal moves. In these cases, verbal language (e.g., elaborating or restating the meaning of the visual) was absent. This finding required us to rethink our conceptual framework for what constitutes a rhetorical move, specifically the idea of it being a 'discoursal unit,' because our authors were also constructing their arguments using visuals.

We return now to Example 6, focusing on the top graph labeled 'D_k,' which we noted defines the central concept 'the chromatic polynomial of D_k.' Here the graph D_k is not associated with a verbal/symbolic definition (beyond the pointer "(see Fig. 7)"); instead, it *replaces* it. The graph alone defines the object (D_k); readers must "hermeneutically read" the object's definition from the image (Ihde, 2007). Several of our disciplinary informants noted this discursive strategy where authors leave out important information in their argument to be filled by the disciplinary-insider reader, a strategy that builds authorial credibility on the one hand and enhances reader engagement on the other. Disciplinary informant A elaborated on why authors might choose to define objects through a visual:

> There could be a number of motivations for doing this [using visuals for defining objects]. First, it's a cultural . . . demand in mathematics for brevity. From among the visual, symbolic and verbal resources, visuals are the most concise way of communicating ideas. If we can say something visually, we'd rather not say it verbally or even symbolically . . .. Sometimes visual language is the most effective . . . way of communicating. A second reason is that this [a research article] is an expert-to-expert communication [where] part of our credibility comes from guessing what our audience can supply. Depending on the topic, we sometimes assume that, with a certain amount of shared background, our audience can read the definition from the visual. Putting some labour on the audience to fill in the information holes engages them in interactive reading. Third, it is an implicit authority practice; we control our audience by the amount of information we give them and the amount we require them to supply. (Personal communication)

This disciplinary informant identifies three reasons for using visuals in discrete mathematics: to enhance brevity, engage readers, and strengthen writers' authority. The idea of creating an interactive reading experience by leaving gaps in the explanation for readers to fill is intriguing. It (plus the idea that visual and verbal information is never redundant) also raises the possibility that visuals in mathematics may perform activities independent of verbal or symbolic language.

In her analysis of the moves in Canada's Food Guide, Rachul (2016, 125) identifies a move, "summarizing visually," that is not associated with a verbal move. In her analysis, the food guide summarizes as a picture the information detailed in the rest of the brochure. Rachul's assumption that visuals can initiate moves in textual documents is relevant here. Our example suggests that visuals can also accomplish moves in discrete mathematics RAs. A visual counterpart of the verbal move, 'Establishing presumptions,' Example 6 *visually defines* graph D_k, replacing verbal language and creating a new mathematical object.

A disciplinary informant explained the lack of a verbal definition as achieving two goals:

> An image of [the concept] is better appreciated than a description . . .. [the image] is an exact, concise, and quick way to equalize background knowledge between the author and readers. (Informant D, Personal Communication).

Informant D notes that the graph presents the pertinent details of the object and succinctly reproduces information to situate readers in the scholarship needed to follow the mathematical argument. Example 6 illustrates how visuals can contribute unique information, both instantiating the object and defining it. This example supports Lemke's (1998) contention that visuals in mathematics do not *illustrate* the text: 'they complement the main text and, in many cases, they complete it' (p. 105), or they replace it. This example also shows that visuals can initiate moves without being associated with verbal components.

As we have noted, these two findings (that visual moves can replace verbal moves in discrete mathematics, and they can represent dimensions inexpressible using verbal or symbolic language) both raise the possibility that moves may not be exclusively verbal; our results suggest that while 'rhetorical moves' in discrete mathematics are often verbal, they can also be visual-verbal (associated with verbal move) or visual (independent of verbal).

Table 5.3 shows the number of visual moves in our corpus.[4] Most of the visual moves were associated with pointers. Occasionally, authors referred briefly to the visual in the verbal discussion instead (e.g., 'where G is the graph in figure x'). We decided to treat the pointers associated with visual moves as directives (Hyland, 2005) not textual associations because they refer readers to the figure but contribute little to its meaning. We also consider the figure captions, although they contain verbal language, to be part of the visual move because, in these examples, the visuals are not explicitly linked to verbal moves in the body of the article.

Table 5.3. Visuals, pointers and rhetorical moves in the corpus (Visual-Verbal moves: EP: Establishing Presumptions; AR: Announcing Result; P: Proof; D: Discussion. Visual Moves: DV: Defining Visually; ROO: Representing the Outcome of the Operation; EO: Embodying the Operation.)

RAs	**RAs including visuals**	**Number of Visuals**	**Pointers to visuals**	**Visual-Verbal Moves**				**Visual Moves**		
				EP	**AR**	**P**	**D**	**DV**	**ROO**	**EO**
30	**21**	**109**	**179**	**31**	**13**	**43**	**13**	**56**	**12**	**4**

Our data presented three visual moves:

[4] We thank an anonymous reviewer for asking the question that sparked this insight.

1- Defining visually: The visual equivalent of the verbal move 'establishing presumptions,' the visual defines an object, but no text in the verbal explanation (beyond a pointer) refers to it.

2- Representing the Outcome of the Operation: This move presents a visual that illustrates the outcome or product of a mathematical operation that cannot be communicated using verbal or symbolic language. This move highlights how visual language can communicate unique information, therefore rendering it equal to verbal or symbolic language.

3- Embodying the Operation: In this move the visual depicts an action-operation that cannot be rendered through verbal or symbolic language. As does ROO, this move undermines assumptions that visual language is subordinate to verbal (that is, words are required to clarify the visual's meaning). In this move, the visual speaks when verbal/symbolic language is mute.

Most notable of the figures in Table 5.3 is the number of times that the move 'defining visually' was used, 56, which is higher than the most frequently used verbal-visual move, 'Proof,' at 43. These data suggest that 'Defining Visually' is an important and prevalent move among discrete mathematicians, yet previous scholarship has overlooked this function by not considering the contributions of visuals to the rhetorical structure of this field's RAs. The other two visual moves, although less frequently employed, also have intriguing implications for genre research because they offer discrete mathematicians options for communicating what verbal and symbolic language cannot: products and processes.

5.5. A Model for Multimodal-Move Analysis

This analysis has highlighted how visuals can participate in creating the argument in discrete mathematics in RAs where they are present. We identified three possible functions and the rhetorical moves with which they are generally associated, Establishing

Presumptions, Proof, Discussion, and Announcing Results. We also identified three possible visual moves, Defining Visually (DV), Representing the Outcome of the Operation (ROO), and Embodying the Operation (EO), that authors in our corpus used to build their argument for new knowledge in the RAs. Notably, these visual moves sometimes replaced textual moves—that is, they contributed to the authors' argument independently of the verbal/symbolic resources used. Of the visual moves, DV was not associated with verbal moves (that is, definitions using verbal or symbolic language or the verbal move, 'Establishing presumptions') in the examples in which it was present. Both ROO and EO were associated with verbal moves, but they contributed information inexpressible using verbal/symbolic language. Our analysis highlights how many of the visuals perform multiple functions independently and in concert with the textual moves. Figure 5.1 presents a model of our multimodal move analysis. It illustrates the relationships between the rhetorical functions and the textual/verbal and visual moves in the examples in our corpus. Sometimes these moves are associated with verbal moves (e.g., ROO and EO) such as EP, AR, P, and D, but they may also be independent of verbal moves, as in the case of DV, a visual version of EP.

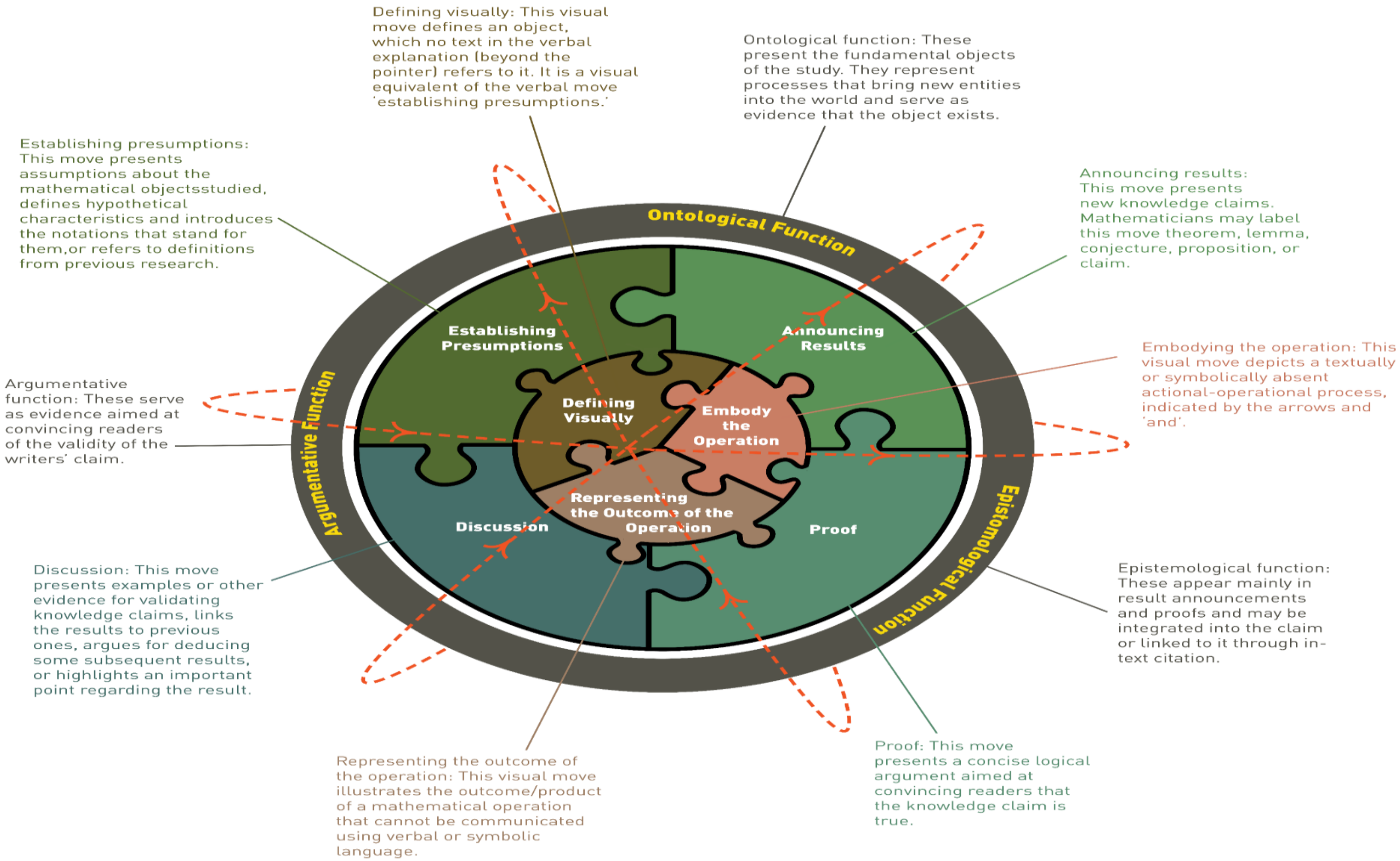


Figure 5.1. Model of rhetorical functions for visual and verbal rhetorical moves in discrete mathematics research articles.

We have represented DV as interacting with the other visual and verbal moves in Figure 5.1 because this move could be associated with the verbal moves, although the authors in our corpus tended to opt for concision, using only the visual to define a concept. We argue that the interactions among verbal and visual moves *enable* mathematicians to create compelling arguments for the new facts that they present in RAs. In numerous cases, without the visuals these authors would be unable to communicate important aspects of their arguments. As our model makes clear, visuals, where present, construct the kernel (or in Miller's (1998, p.44) terms, the "nucleus") of the mathematical argument by defining the key concepts and objects of study, representing the outcome of critical operations or embodying them. Angot-Pellissier (2015) argues that visuals lie at the heart of mathematical notions, theories, and reasoning. Fig. 1 also depicts the relationship among the visuals' functions and the verbal and visual moves. Our examples show how the various visual and verbal moves can be deployed to accomplish the RAs' purposes. The model illustrates how verbal and visual elements in discrete mathematics provide resources that discrete mathematicians use to create and argue for new knowledge.

Lastly, we examined the organizational structure of these examples to explore how the authors arranged the visuals and the verbal/symbolic language to construct a persuasive argument for the new knowledge they present. We showed how visuals placed to precede verbal moves served to inform readers' responses to the verbal/symbolic explanations that followed. We have argued that when they reach the authors' proposed insight ahead of the verbal unveiling of it, readers are contributing to the argument by anticipating it, a process that encourages them to invest emotionally in the outcome of the argument. In addition, we have shown how authors in discrete mathematics arrange the visuals and verbal/symbolic language to achieve complex persuasive effects. One implication, then, is that when scholars of genre analysis factor them into their understanding of the argument structure, the visuals synthesize or scramble the linear verbal moves in ways that complicate our conventional assumptions about the “narrative” structure of RAs.[5] This analysis suggests that RAs in discrete mathematics are less ‘narratives of research’ and more carefully constructed arguments.

5.5. Conclusion

In this study, we have examined how mathematical argument initiates, develops, and establishes new knowledge through structuring the verbal and visual rhetoric in interactive ways. We showed how the visuals collaborate rhetorically with the verbal to create tangible objects from abstract concepts (ontological function), supply the reasoning behind mathematical relations (argumentative function), and integrate to create and support arguments for new knowledge (epistemological function). One implication of this study is that visuals, when used in discrete mathematics, participate in many aspects of the authors’ argument, from framing new concepts to developing arguments to announcing new facts. Therefore, visuals are not ‘add-ons’ but can perform the intellectual work of the discipline. In fact, by identifying three *visual moves*, which authors use to develop unique aspects of their argument for new knowledge **not** repeated in the verbal or symbolic language, we have highlighted the importance of including visuals as part of move analyses in rhetorical genre studies in fields where visual data contribute to the results presented in the RA. Otherwise researchers may overlook important aspects of the article’s argument and the rhetorical strategies available to writers in that discipline.

A second implication of these findings is that move structure in RAs in discrete mathematics is not located solely in this genre’s verbal language. The idea that visual components can initiate moves is intriguing because it suggests that other researchers perhaps

[5] We wish to thank an anonymous reviewer for pointing out this implication of our analysis and encouraging us to explore it further.

should incorporate the study of visuals into their study of disciplinary discourses that also use visuals. That work would clarify whether visual moves are more widespread or unique to discrete mathematics. It may be that researchers in these disciplines may need to move to multimodal move analysis (which would include analysis of potential 'symbolic moves', although that topic was beyond the scope of our project here) so that they build models of disciplinary discourse that incorporate extra-verbal contributions, without which their conclusions may be incomplete. Multimodal move analysis also has implications for verbal-based models of move structure such as *Create A Research Space*.

A third implication relates to our finding regarding the 'narrative' structure of an RA in discrete mathematics. While the verbal language of the RA might create a chrono/logical account of the research/topic that supports unexamined assumptions about academic argument structure, the visual that precedes the verbal may, in reality, undermine that logical account, priming readers to critique traditional assumptions about the topic. In fact, several of the visuals used in our corpus disrupted the narrative and, in so doing, created an emotional appeal that enhanced the authors' argument for their contributions to knowledge. If RAs in discrete mathematics use an argumentative rather than narrative structure, so might other disciplines that incorporate visuals into their RAs.

A third implication of these findings is that knowing how to construct and integrate visuals effectively into an argument is a crucial skill for young researchers of disciplines that use visuals, not only to create authorial credibility, but also to learn, know and communicate in the discipline. Rowley-Jolivet (2002) notes that "an important part of the socialization of young researchers into their discipline is mastering the visual conventions used in their field" (p. 22). We have shown that mastering these conventions is important because the visuals, when present in research mathematics, do unique work in the field that contributes to its growth and intellectual development.

Whereas school mathematics uses visuals mainly to reproduce established knowledge or to characterize and illustrate mathematical objects (e.g., to help learners conceptualize and more precisely understand the concepts), visuals in research mathematics are integral to proving claims; that is, they make concrete hypothetical concepts and play a variety of argumentative roles to persuade the readers of the claims' validity. Visuals in research mathematics go beyond presenting concepts about mathematics: they also contribute to the ontological process of creating new concepts and relations, of developing mathematical arguments, and of generating new knowledge. While the ontological function may be fulfilled when the authors define a new concept or comment on the relations between newly created mathematical concepts, the latter functions are performed when authors use visuals as evidence for their problem-solution procedure and/or their arguments for new knowledge. As such, while they *may* employ visuals to illustrate (a common semiotic resource in many genres of mathematics), research mathematicians generally use visuals for generative purposes.

Our findings, although based on a relatively small sample size in one area of mathematics, are grounded in a thorough analytical approach; hence, they have pedagogical value. The pedagogical implication for these findings is straightforward and significant: Raising student awareness of the functions of visuals in knowledge creation and argumentation matters. Do we do that? Probably not. Despite Miller's (1998) call for more attention to 'the visual nucleus of the argument' (p. 44) in ESP classes and encouraging students to read visuals as critically as they read texts, genre-based writing classes still seem to downplay or even ignore the role of visuals in structuring genres and contributing to their communicative purposes.

Assuming that writing instructors are willing to heed various genre scholars' calls to teach visuals (Hyland, 2006; Johns, 2013; Miller, 1998; Tardy and Swales, 2014) as part of disciplinary writing instruction, the following questions arise: what are the best ways to help mathematics students learn about visual rhetoric in their research-based writing? What should instructors consider as they develop practical materials and in-class writing tasks? We suggest that the research questions and analytical approaches employed in this study can be applied flexibly to sample genres in the classroom context. Instructors can ask students to analyze the verbal and visual rhetoric in sample genres to identify the aspects of the argument, including any multimodal moves, used in the visuals and to draw connections between the visual and the verbal (Hyland, 2006). While first and second-year undergraduate students in a math class might find it too difficult to analyze how visuals contribute to the overall argument, advanced undergraduate and graduate students might benefit from just this type of study. At the same time, we acknowledge that limitations exist for non-disciplinary writing instructors who try to teach visual rhetoric to students, a reality that justifies disciplinary and writing studies specialists working together to design, implement, and teach this aspect of writing-in-disciplines courses. An alternative could be training students as ethnographers (Johns, 2013; Tardy, 2006) to discuss the rhetorical functions of visuals with their disciplinary instructors. While these recommendations may be read as “applied aspirations” (Cheng, 2019), an extensive discussion of the pedagogical implications of our results is beyond the scope of this paper, and we leave it to a future pedagogically oriented article.

5.6. Chapter References

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Chapter 6: From Research to Pedagogical Practice: Combined Reading-Writing Genre-Based Instruction for Mathematics Students

This chapter aims to apply the findings of the present graduate research to develop academic writing tasks for mathematics students at upper undergraduate and lower graduate levels from any language background who meet the generally accepted English language requirements in North America. The chapter begins with an overview of genre analysis as an instructional approach as well as a state of the art in writing instruction research/resources in mathematics. Then, a series of lessons and writing tasks will be proposed for engaging the target students in the social practice of writing research in the discipline.

6.1. Theoretical Background: Genre pedagogy

Writing pedagogy at a postsecondary level has been significantly influenced by genre scholarship during the last three decades. A notable influence came from Swales' (1990) *Genre Analysis* which emphasizes an understanding of how language is structured to accomplish specific social actions in particular contexts. The approach introduced genre-based writing pedagogies, especially for academic, research, and professional writing purposes and contexts. The approach emerged out of 'a pragmatic concern' to help language users, irrespective of their language background, develop communicative competence, mostly their writing competence, for specific purposes in specific contexts (Swales, 1990, p. 9). The emphasis on writing purpose and context sets genre-based writing pedagogies apart from process writing or purely linguistic approaches (Flowerdew, 2015). In academic and research contexts, genre pedagogy aims to enhance post-secondary students and novice researchers' understanding of academic discourse, its shapes, and purposes.

The notional core of Swales' (1990) genre analysis is composed of "a trio interlocking concepts" (Swales, 2017, p. 2): discourse community, genre, and language-learning task. While the notion of genre has received notable attention in writing instruction, the concept of discourse community has remained a less practiced concept in writing pedagogy particularly in writing-in-disciplines classes, (Johns, 2015). As well, the concept of learning-task remains generic and less researched. A possible reason for undermining, rather than centralizing, the concept of discourse community in genre pedagogy is discourse communities' dilemma, as argued by Hirvela (2013). According to Hirvela, while students need to learn their target community discourse, writing instructors do not usually have a background in the various target disciplines of students. This lack of sufficient knowledge about disciplines may discourage them from teaching about discourse communities in writing classes.

A second, and a more important, reason, for under-attending to the concept of discourse community in writing classes might be the pragmatic intractability and complexities involved in identifying the boundaries of discourse communities, especially in the era of globalization (Swales, 2017). As Swales himself remarks, the concept has developed fuzzier boundaries as the world has changed. Discourse community, according to Swales (2017), is "a largely heterogeneous, socio-rhetorical assemblage of people who broadly share occupational or recreational experiences, goals, and interests (p. 4). However, Swales argues that the heterogeneous boundaries of discourse communities do not harm their shared communicative practices, hence should not distract us from the central role the concept deserves to have in writing pedagogy. According to Swales, although "it is doubtful [...] that the concept is a robust social construct [, …] this probably does not matter as long as our focus is on rhetorical principles of organization, on discoursal expectations, on significative linguistic tokens, and on intriguing textual extracts. Such attention on these more surface features provides insight into what at first sight might seem standard, ordinary and predictable" (Swales 2017, p. 9). A foregrounding of the concept of discourse community in genre and writing instruction, especially in writing classrooms using the heuristic critical genre awareness approach (Devitt, 2009) is thus tenable given the role discourse communities play in setting the shared expectations for genre construction and use. Acknowledging these arguments and following his three-decades of involvement with the concept, Swales (2017) refines his 1990 conceptualization of discourse community and argues for the following defining characteristics: a broadly-agreed-upon set of common public goals, mechanisms of intercommunication among its members, using its participatory mechanisms to initiate actions and activities and provide information, utilizing one or more genres in the communicative furtherance of its aims, possessing some specific lexis, a threshold level of members with a suitable degree of relevant content and discoursal expertise, a sense of silential relations in doing things without spelling them out, and developed horizons of expectations. These defining features of discourse community are useful in configuring and foregrounding the connections between writer, audience, and text when teaching university-level writing.

A second underpinning concept in Swales's approach is genre. Swales (1990) defines genre as a class of communicative events with textual regularities. Although textual regularities is a defining characteristic of genres, Swales and Feak (2012) encourage students to practice and understand more than possible prototypicality by analyzing writer's potential purposes, stances and discourse community involvement as well as other factors that influence how a single text is processed and produced.

The relationship between a discourse community and the genres it uses is one of constructing and restructuring, as conceptualized in Swales' (1990) definition of genre:

> A genre is a class of communicative events, the members of which share some set of communicative purposes. These purposes are recognized by the expert members of the parent discourse community and thereby constitute the rationale for the genre. This rationale shapes the schematics structure of the discourse and influences and constraints choice of content and style. (p. 58)

The concept of communicative purpose in identifying genres, however, experienced a paradigm moderation later influenced by research findings, from both ESP and rhetorical genre studies (Askehave and Swales, 2001; Bawarshi and Reif, 2010; Johns, 2015). These studies suggest that a communicative purpose cannot be attributed to a genre category without thoroughly examining the broader context surrounding the genre. Askehave and Swales (2001, p. 208) propose that rather than focusing on communicative purpose as the basic criterion for identifying a genre and working out from there, researchers and practitioners should design "a more context-driven procedure" (p. 208). The procedure initiates with identifying "the values, goals, material conditions, expectations, [and] repertoires of a discourse community which values that genre" (P. 208) and repurposing the genre later.

The emphasis on context analysis brought ESP pedagogy much closer to New Rhetoric that emphasized extensive text-in-context that is rhetorical situation, inquiry in the categorization of genres over straightforward textual or transcriptual scrutiny or introspective system building (Bawarshi and Reiff, 2010). Hence the activities developed in this chapter emphasize interlinks between genres and their rhetorical situations. The analysis of rhetorical situation is encouraged as a prerequisite for genre analysis tasks and activities.

A second identifying feature of genre is the prototypicality of its member texts. Originally, Swales (1990) emphasized 'family resemblances' (p. 49) or repeated textual and structural features as key criteria in genre analysis research and pedagogy. According to Swales, genre research and pedagogy should track these textual regularities and irregularities and explain them in terms of the pertinent social circumstances and rhetorical demands they provoke. This perspective is reflected in Swales and Feaks's (2012) *Academic Writing for Graduate Students* in which they develop writing tasks as they offer prototypical samples of genres while emphasizing specific factors which influence how a text is structured including writers' potential purpose, stances, and discourse community involvement. Swales' turn to social contexts of genres and his emphasis on explanatory genre analysis is owing to earlier arguments by rhetorical genre scholars, especially Carolyn Miller's (1984) outstanding article in which she defines genres as forms of social action (A summary of Miller's argument is available in Chapter 1).

Following Miller (1984), Swales (2009) argues that the work of genre is to mediate between social situations and the texts that respond strategically to the exigencies of these situations, that is, texts perform the genre. "As these performances increase, genres tend to drift through time and geographical space, partly inherently and partly as a result of intertextual acceptances and rejections" (Swales, 2009, p. 14). Swales' emphasis on accommodating both textual regularities and irregularities in genre analysis as interlinked with socio-rhetorical situations (that is, genre as social action), seems to have a clear pedagogical implication: that instructors should emphasize genre's potential for variation as much as they emphasize their commonalities (that is, genres represent both shared and specific socio-cultural features); so do the texts presenting genres. Hence, presenting genre samples as static one-model-fits-all artefacts should be avoided as it offers a fixed conception of genre and restricts students' appreciation of dynamism in texts belonging to the same genre. Instead, instructors should encourage students to adapt their responses to writing demands made on them by first analyzing the local circumstances surrounding the target genre and the texts representing them. Examples of such demands are specific topic requirements, assignment prompts and instructions, journal requirements in the case of research article genres, the specific disciplinary cultural restrictions, and even individual voices in writing the same genre. I will use these arguments to design some disciplinary-writing tasks and assignments in the following sections of this chapter. The designed tasks will have students explore the shared as well as specific contextual features of sample genre texts including the disciplinary norms, research topic, audience, and authorial preferences which put the social context of writing in constant flux. Together with emphasizing patterns of rhetorical, linguistic, and structural practices, genre samples in this chapter are analyzed as writers' practice of values at a local context while responding to a rhetorical situation

As I will show, the above argument on genre as social action applies equally to writing instruction, a point supported in Russell and Foster (2002), and also resonates in Graves and Graves (2006): "If Russell and Foster (2002) are right, local conditions and local cultures do more than shape writing instruction--they determine it" (p. 1). The view is as much, if not more, valid today as it was over a decade ago. Writing instruction and practice, as it was argued, is instigated, and formed by the sociocultural, institutional, and technological features of the context in which it is developed and practiced. In developing the analytical reading and writing tasks proposed in this chapter, then, I have considered the specific disciplinary conditions, namely writing research in discrete mathematics. While I would argue that most of the activities developed in this chapter are adaptable to fit writing instruction in other disciplines, some specific tasks might be more exclusive to mathematics and a few sister disciplines, or they might require some alterations to fit writing instruction in some other disciplines.

The third central concept in Swales' (1990) genre approach is language-learning task, which has been ignored to a great extent in genre research (Flowerdew, 2015; Johns, 2015). Swales' designing of challenging classroom tasks for students goes back to the 1970s and are mostly devoted to noticing, discussing, and comparing/contrasting visual and print texts. Perhaps the most remarkable task-based ESP work is Swales and Feak's tasks published as an academic series (Swales & Feak, 2000, 2004, 2011, 2012). While the tasks designed by Swales and Feak are still mostly text and language-based, there is increasing emphasis on the pragmatics, that is, the social-linguistic actions, of a situation, particularly in terms of writers' ethos as competent members of the chosen discipline and appropriate textual approaches for entering academic communities, as announced by Swales and Feak (2012, p. 1). For example, they suggest ways of moderating a claim (p.159), identifying the differences between plagiarizing and legitimate quotation (p.198), and a writer's evaluative stance (p. 242), all assessed as well-crafted tasks by writing scholars (Johns, 2015).

A related, less-developed topic in pedagogy research is modelling genre instruction; modeling writing introductions is a notable exception. Perhaps the best-known ESP genre model, which has boosted genre research, is Swales' (1990) *Create A Research Space* (CARS) model of research article introductions. *Create A Research Space* exemplifies levels of rhetorical (move analysis), lexico-grammatical and contextual analyses. The analytical approach inspired by Swales (1990, 2004) and CARS scholarship will be applied in developing a variety of tasks about writing different elemental genres comprising mathematical research articles in this chapter. However, as I have tried to show in preceding chapters, the CARS model, as any other rhetorical model, is mostly useful as a kind of heuristic rather than a step-by-step guide about research article introductions. So, the proposed tasks and activities would encourage a pattern seeking rather than pattern imposing approach (Bhatia, 1993) and aim at enhancing students' critical thinking about genre variations across and within disciplines.

In addition to examining the structural organization of research and professional genres through move analysis, ESP accommodates a wide range of corpus-based research on sentence-level grammatical and stylistic features, such as tense, reporting verbs, voice, modality, and identity features that contribute to communicative purposes of disciplinary discourses and communities (Hyland, 2000, 2005a, 2005b). As noted, ESP has also shown a growing interest in developing an explanatory, or in Bhatia's terms, a critical, dimension both in research and pedagogy (Bhatia, 1993, 2004; Candlin & Hyland, 1999; Johns, 2008;

Swales, 2004) to answer the questions of rationale behind genre regularities, namely, why do members of the discourse community write the way they do? How is knowledge constructed in the discipline? These questions are important to raise in writing-in-disciplines classes. As Bazerman (1992) argues, “[b]y understanding how knowledge is constructed, they [students] can judge what knowledge it is they wish to construct (p. 68).” Accordingly, I have designed some of the tasks in this chapter to explicitly address these and similar questions and to engage students in critical thinking about epistemological grounds of mathematics as a discipline of study.

I should note at this juncture that despite existing teaching material and books in genre-based pedagogy, to my best knowledge, little resources are available about existing disciplinary-specific learning tasks and activities. Among the few notable published works is *Write Like a Chemist* (Robinson, 2008) which proposes a series of exercises and tasks for writing four major genres in chemistry including the research article, scientific poster, conference abstract, and research proposal. Regarding mathematics, while some existing research addresses writing instruction in mathematics (https://wac.colostate.edu), few instructional materials are apparently available for teaching writing university-level research in mathematics. The present chapter engages ideas from ESP genre pedagogy, Robinson’s (2008) *Write Like a Chemist* in particular, to fill this gap in genre pedagogy in research mathematics. I develop a series of reading-writing genre-based tasks and activities as modules for writing research articles in discrete mathematics. Before proceeding to the learning modules, I review existing pedagogical practices and material for writing in mathematics in the next section. My intention is to offer an overview of the current situation in writing pedagogy in mathematics and to point out the pedagogical gaps which motivated me to develop the sample material for teaching writing in university-level mathematics in this chapter.

6.2. Writing Pedagogy in Mathematics: An Overview of Existing Research and Teaching Resources

A cursory glance at the published research in mathematics education suggests that there is a consensus on the important role writing plays in doing and learning mathematics (Estes, 1989; Knox, 2017; Kosco and Zimmerman, 2017; Martin, 2015; Martin and Polly, 2016; Martin et al., 2016; Morgan, 1998; [American] National Council of Teachers of Mathematics [NCTM], 2000; Norenes and Ludvigsen, 2016; Pugalee, 2001; Reiter, 1995; Russek, 1998; Sundstorm, 2014; Teledahl, 2017). The educators reporting these studies argue that writing in mathematics matters because it helps improve students' mathematical reasoning, consolidate their thinking, reflect on doing mathematics, improve communication, develop their argumentation skills, and enhance their learning. For example, Teledahl (2017) rejects the knowledge transmission view of writing in mathematics and argues that, rather, there is a close relationship between what students write in mathematics, what they know, understand, and learn, and how they are assessed by the teachers. Her findings illustrate that while students' writing demonstrates their understanding of the subject, it accounts for their mathematical problem-solving processes and strategies. She also remarks that in her study student drafts show their awareness of audience and text organizing, that is they write to an imaginary audience and take the audience into account in organizing their texts. These findings are important to note because they suggest that student writing in mathematics classes share features of most writing classes at both school and college/university level.

Despite the expressed centrality of writing in learning mathematics, most available scholarship is limited in its scope, mainly focusing on writing in school mathematics. Moreover, it barely reports sample writing tasks and activities used for promoting writing in mathematics classes. On a more generic basis, a rough search for available instructional resources using popular search engines like Google usually yields links to several quick/short/brief guides to writing mathematics. A common feature of most such guides is their context-free and genre-empty hints for writing in mathematics with a focus on sentence structure, grammar, formatting, style, and use of mathematical symbols.

Among the few resources on university-level writing in mathematics are *A short Guide to Writing Mathematics, a guide for undergraduates* by Maurer (n.d.) and *Writing a Research Paper in Mathematics* by Reiter (1995). Both sources provide brief guidelines to writing mathematics. The content list of Maurer's book is available at http://www.swarthmore.edu/NatSci/smaurer1/WriteGuide/index.html; I don't have much to say about the first source as my efforts looking for a copy of the book did not produce one. In the latter source, Reiter (1995) likens writing mathematics to conducting the performance of a piece of music and emphasizes that it is not sufficient to simply write mathematics, but one should write it well to communicate the beauty of the mathematics one has done. Perhaps the hallmark of Reiter's guideline is a series of questions which, as she suggests, would help a student writer to situate and identify their work within existing research in the field. Reiter's questions are interesting in that they prompt important exposition about the specific aspects of the research to be drafted by the students. However, a shortcoming of Reiter's guideline is that it does not point out how these questions relate to the rhetoric and argumentation for new knowledge in mathematics, nor does it provide clear instruction about which questions should be answered in which section in the research article and how.

Another concern is the distinction Reiter, following Streenrod et al. (1983), makes between formal and informal exposition (Reiter, 1995) in mathematics research articles. Reiter groups definitions, theorems, and proofs as formal exposition, while she refers to examples, analogies, explanations, motivations and metamathematical material as informal writing. Reiter, however, does not clarify in what sense the two categories vary in the level of formality or style, except that the logical structure for the former should be clearly articulated, nor does she give clear instructions on how to draft the formal vs. informal parts. Reiter, however, reiterates the advice by Streenrod et al. (1983) who suggest "[s]ince the formal structure does not depend on the informal, the author can write up the former in complete detail before adding any of the latter (p. 2)." It appears that Streenrod et al.'s distinction between formal vs. informal structures is more about the rhetorical rigidity in phrasing the aforementioned parts. In other words, while there seems to be more conventionality, hence less variation, in phrasing the definitions, theorems, and proofs, there appears to be less rigidity in verbalizing the introductions and discussions over results in mathematics research articles. In this sense, Sundstrom's (2014) textbook on how to construct and write proofs in mathematics is an instruction on formal exposition in mathematics. Although the research reported in earlier chapters of this dissertation confirms variations in rhetorical organization of the so-called informal structures, it has shown that patterns are discernible in the rhetorical organization of these expositions in the research articles in the corpus of this study. Hence, the analytical activities developed in this chapter focus on both patterns and variations in different rhetorical structures in research articles in mathematics.

Moreover, my research suggests that the formal structures, as differentiated by Streenrod et al., do depend on the informal structures, as they call it, for their contribution to the argumentation and creation of new knowledge. However, Streenrod et al.'s idea to develop/draft the formal structures first and integrate them into the broader structure of argument at a later stage seems interesting and practical, especially as it helps to split the broader argument into manageable pieces and to synthesize the pieces into a unified argument at later stages. From a pedagogical perspective, Streenrod et al.'s idea also aligns well with scaffolding student learning as a series of manageable writing tasks that add up in an accumulative way. So, I will use Streenrod et al's idea in developing the analytical as well as synthetic in-class writing tasks in this chapter.

The guidelines reviewed briefly above, and similar pedagogical advice published by most North American universities on how to write mathematics (e.g., University of Alberta's guide on *How to write mathematics*; See http://web.cs.du.edu/~mkinyon/mathwrite.html for more examples) are valuable in that they emphasize the role writing plays in doing, learning, and creating new knowledge in mathematics. The guidelines also hint at the existing need for writing pedagogy in mathematics. They are also appreciated in that they give general, and at times specific, advice on *what to do,* or not to, in writing mathematics.

There are, however, three shortcomings with such guides. A notable shortcoming is that these guidelines are small scale rather than institution wide; hence they may have less pedagogical impact, if any. A second drawback is that they provide few scaffolded tasks and activities to instruct the students *how to* write in mathematics. In other words, while they tell the students what to do, they don't show them how to do it. The third, and the most important, shortcoming is that most of these guidelines are generic and not genre-specific; an exception is Reiter (1995). In other words, most of these guidelines fall short of identifying a target genre and analyzing its rhetorical and language features. This is while writing scholars increasingly emphasize the importance of teaching genres (e.g., Devitt, 2009; Hyon, 2018; Johns, 2015; Tardy, 2009). As Devitt (2009) declares, "[p]ractically … teachers cannot escape genres, even if they want to (p. 341)." According to Devitt, in classrooms where teachers ignore genres, students will still use their prior genre knowledge to interpret what teachers ask for in a writing assignment. She also points out the reality that knowing some genres in academia, disciplines, and professions is not a choice but a requirement. Despite this emphasis, as it was shown in this section, genre-based writing-in-mathematics pedagogies which can respond to students' research-based writing needs largely remain a curriculum gap which requires attention from genre researchers and writing-in-disciplines scholars.

6.3. Student Needs and Writing Support

Writing is inarguably an indispensable part of doing a university degree. Anecdotal evidence from students, at both undergraduate and graduate levels, suggests that students are assigned complex and highly diversified writing assignments for notable grade components in most core and elective courses through their program of study. In a more systematic effort, Graves and Hyland (2017) observed that papers, presentations, reports, and essays were among the most required genres at the undergraduate level across university disciplines in Canada. At the graduate level worldwide, students as 'junior researchers' seem to be more focused on research writing. Example genres include research reports, journal articles, conference papers and presentations, proposals, and dissertations (Swales, 2000; Swales and Feak, 2012). More directly, these written assignments are used to assess students' learning of the subjects of their disciplines. But while writing is utilized as a "core gatekeeping and assessment" tool in higher education (Hyland, 2009, p. 5), it also promotes student learning and disciplinary membership. It is this latter function that is the objective of the modules developed in this chapter.

Writing in its various forms promotes students' understanding of authorized knowledge in their discipline. In the process of writing, students engage in several cognitive and rhetorical actions including but not limited to "trying on the peculiar ways of knowing, selecting, evaluating, reporting, concluding, and arguing" (Bartholomae, 1986, in Hyland, 2009, p.6) in ways that are conventional to their discipline and are seen as understanding knowledge by disciplinary insiders. Writing and understanding of disciplinary subjects thus takes place synchronically. Through writing, students develop both theories about ways of doing things in their discipline and rhetorical skills to operationalize their theoretical knowledge. Learning takes place as students use and revise their theories and skills in the process of writing. Disciplinary writing thus performs key functions in learning the knowledge content where it is a substantial component of a course at both undergraduate and graduate levels. Learning disciplinary writing is however not an easy game for students and novice researchers regardless of their language background.

The fact that learning disciplinary genres goes beyond their linguistic form makes disciplinary writing development a challenging and complicated side of students' scholarly endeavors (Tardy, 2009, p. 5). Among the factors which complicate learning disciplinary genres are learning what counts as content knowledge, how it is constructed and distributed in the discipline, the commonly used genres of the discipline and their communicative purposes, the rhetorical and sociocultural contexts of genres, their standardized rhetorical organization and lexico-grammatical features, and the cultural values embedded in genres (Tardy, 2009). These challenges are experienced, though on a varying level, by students from all language backgrounds including speakers of English as a native language (Hyland, 2018; Tardy, 2009; Wingate, 2018). As the genre scholarship reviewed in this dissertation and a more recent argument by Hyland (2018) emphasize, students' writing difficulties should not be attributed to grammar deficiency. Rather they are challenges in the process of acquiring new discourse practices, which cannot be addressed by "traditional models of teacher prescriptivism and grammatical reverence, focusing on study skills and isolated texts" (p. 384). Hyland (2018) argues that students need to be engaged in writing of contextualized communicative genres as a social practice in ways similar to what discourse community members do. As most discourse community members in disciplines are engaged in communicating research, teaching students how to write and argue for their research in their discipline assumes importance.

A key research genre of academia is the research article. An increasing number of students, especially international graduate and senior undergraduate students around the world, face program or career requirements to write and publish their research as articles, mostly in English. Despite much exposition on the increasing needs for writing by these students, there is little systematic and consistent institutional support from within disciplinary communities and programs to address these students' disciplinary writing needs (Graves and Hyland, 2017; Wingate, 2018). The lack of systematic writing support from within disciplinary programs is partly due to a belief by some lecturers who hold that students should learn writing before entering the university. This fallacy is not tenable as knowledge of writing in disciplines has been proven to be tightly connected with the disciplinary values and conventions of doing things as well as with the knowledge content of the discipline. So, expecting students to have mastered writing conventions of the discourse community in advance is not justified.

A second reason for lack of discipline-specific writing support might be that instructors in disciplines lack explicit knowledge of discourse conventions in their communities. So, they might hope that students gradually develop tacit knowledge of writing conventions (Jacobs, 2005). This could be true in the long run, however, as Wingate (2018) remarks, while most students may eventually acquire academic literacy capabilities, "the process could be much accelerated and made easier if explicit support was given within the discourse community, or more specifically, within the curriculum" (p. 349). Examples of such support may include explicit genre instruction courses, training teaching assistants and peer tutors to provide disciplinary-informed feedback on students drafts and developing curricular print and media resources. The curricular gap regarding explicit support for disciplinary genre/writing instruction has also been identified by writing scholarship.

Writing scholars and educationists have not been monolithic in supporting a disciplinary genre/writing curriculum. Devitt (2009) reviews some counter arguments against teaching genres, disciplinary discourses, writing across the curriculum and academic writing. As Devitt remarks, some composition and writing teachers argue that genres carry with them ideologies, hence, teaching genre is formulaic, constraining, assimilationist, and therefore inhibits creativity. These critics contend that teaching disciplinary genres and discourses can stagnate students' critical thinking, sense of identity, and agency, hence promote adherence to the disciplines' assumptions and social power structures. In response, Devitt (2009) proposes a critical genre awareness curriculum, a type of rhetorical awareness, which promotes conscious participation rather than disengagement in genres and critique rather than assimilation.

Another criticism reviewed by Devitt (2009) is that writing instructors might lack insider knowledge of the enormous genre assortment across disciplines so their teaching of those genres will be incomplete. Fair enough! However, teaching of any kind of writing might be subject to the same criticism; there is no perfection in writing pedagogy. As Devitt (2009) argues, "[i]f we teach a genre explicitly, we will inevitably teach it incompletely, but students will understand more about it than they would have if we had taught them nothing about it at all" (p. 341). Despite existing strong arguments for teaching disciplinary genres and discourses, writing-in-disciplines remains a curricular gap in most universities.

To fill this gap in writing-in-disciplines pedagogy I have proposed sample rhetorical awareness-raising tasks and scaffolded and interpretive production practices (Hyon, 2018) for writing in mathematics in Appendix C and briefly discuss them here. The aim is to involve the upper-undergraduate and lower-graduate students in *collaborative* (Golfin et al., 2005) *active learning* (as suggested by the Conference Board of Mathematical Sciences 2016 statement) with a focus on writing research in mathematics. I have done this by focusing on the rhetorical context of the genre (Devitt, 2009; Hyland, 2007; and Hyon, 2018) and by making explicit what is to be learned, providing a coherent framework for studying both rhetoric and language in context, and developing tasks for students to understand and challenge the structure of the argument and valued discourses in their discipline.

The genre targeted in the main modules and various writing tasks in Appendix C is the research article (RA). The choice of the genre is motivated by increasing demand on graduate students, as novice researchers, to write the results of their research and create new knowledge to graduate. Indeed, at the graduate level, students as 'junior researchers' seem to be more concerned with different kinds of research writing including research reports, journal articles, conference papers and presentations, proposals, and dissertations (Swales, 2000; Swales and Feak, 2012). As the preceding chapters in this dissertation suggest, a prototypical research article in mathematics, as a research genre, comprises a number of elemental genres such as introduction, literature review, result sections, proofs, and conclusion. I used some of these elemental genres in designing learning tasks in Appendix C.

Focusing on a single genre might seem a narrow scope for the suggested lessons and activities. However, I re-emphasize that the main pedagogical goal used throughout the proposed activities is education rather than training (Hyland, 2018) that aims to promote genre awareness and not just genre acquisition (Johns, 2015). In other words, while the lessons and writing tasks revolve around analysis of and writing research articles (that is acquisition of genre knowledge), the objective is to promote students' critical awareness of writing as a social practice rather than a language skill. The skills and capabilities developed through these activities are meant to be transferable to understanding and writing other unforeseen academic and research genres students may encounter, or be required to write in, in their program of study.

6.4. Rationale Behind the Designed Modules and Writing Tasks

The tasks and activities designed in Appendix C aim to channel between the goals of a genre-based writing-in-mathematics course targeted at upper-undergraduate and graduate level student learning. I use a genre approach to teaching writing and provide vivid material including authentic genre examples as well as tasks which use the authentic material as opportunities for the students to analyze, critique, and use genres for both genre acquisition and genre awareness purposes within a writing course and beyond it at the program level.

Some current writing instruction approaches, five-paragraph essay instruction being a notable example, are text-bound, that is, they are prescriptive about the text, its stages or organization sequences and language. These pedagogies require students to reproduce some organized model text types in predictable ways. A major drawback of these approaches is that students develop limited awareness of the role of context in forming the text; hence they may perceive model texts as stable prototypes rather than dynamic entities which are subject to evolution and change depending on contextual changes. They thus give students a limited grasp of the notion of genre and writing skills, which barely go beyond the limits of the genres practiced in the class. In such genre acquisition pedagogies, students master one or a limited set of text types. So, it is important that writing instructors integrate genre awareness approaches into their writing classes.

Genre awareness pedagogies assist students in adapting to the evolving and dynamic contexts of genre construction and use (Devitt, 2009; Johns, 2015; Russell and Fisher, 2010). It should be noted that these approaches do not discard attention to the shared surface features of genres such as their rhetorical organization, discoursal expectations and lexico-grammatical tokens. As Swales (2017) remarks, such attention "provides insight into what at first sight might seem standard, ordinary and predictable". Genre awareness pedagogies, as Johns (2015) argues, assist students in developing the "rhetorical flexibility" for adapting their socio-cognitive genre knowledge to future situations (p. 116). These approaches are thus freeing and nonrestrictive (Devitt, 2009), as opposed to a solely genre acquisition approach. Educated through a genre awareness approach, students build genre schemata by performing in-depth analysis of genres and their contexts, reassess their built-in schemata for genres on the go, and revise their theories for the examined genres as needed. So, the goal is to raise student awareness of genre standards and variations and of the fact that “genres predict--but do not determine--the nature of a text that will be produced in a situation” (p. 116). As such, the activities proposed in this chapter should be implemented with flexibility and adjustments based on the features of the immediate context of use. More importantly, the suggested activities should be used to enhance students’ understanding that notwithstanding their shared features, neither texts nor their reading and writing processes, (that is, literacy processes), remain determined. Hence, it is important that students do sophisticated analysis to identify genre variations by examining text circumstances, writer and audience purposes, and other contextual features.

The variety of activities suggested in this chapter and presented in Appendix C addresses students’ learning needs by promoting narrow-angle genre acquisition initially and cultivating wide-angle genre awareness gradually. The activities can be grouped into three categories of tasks advocated in genre-based ESP pedagogies: rhetorical consciousness-raising, text production, and process (Hyon, 2018). Rhetorical consciousness-raising tasks have received the most extensive support from genre scholarship, both old and recent, as the key activities in awareness-raising ESP genre-based courses (Hyland, 2008, 2016; Hyon, 2018; Paltridge, 2001; Swales, 1990, 2011). The rationale, as Hyon (2018) argues, is that the more conscious a person is of how a genre works, the better prepared s/he is to participate in it. Consciousness-raising tasks are in line with the metacognitive approach to teaching and learning which regards learners as thinking beings who notice features in genres and apply their cognitive power to distil rules from the available data. The tasks are thus inductive in nature (Hyon, 2018) as they provide students with examples of genres and assign students the roles of genre analysts by engaging them in ‘noticing’ the genre context, purpose, its rhetorical moves, move organization, and lexico-grammar (Hyon, 2018, p. 131).

A second type of activity that I will introduce here are genre production tasks. As many genre scholars have recommended, students' practice in producing the target genre is necessary for genre mastery (Artemeva and Fox, 2010; Hyon, 2018; Tardy, 2009). Production tasks indeed complement rhetorical consciousness-raising tasks. Research by Artemeva and Fox (2010) showed that genre-awareness may not necessarily lead to effective genre production if students do not have prior production experience. In other words, mere genre exposure and identification is not sufficient to function in a genre. Instead, 'situated performance' in a genre is the primary vehicle for genre acquisition (Artemeva and Fox, 2010, p. 497). This brings us to the heart of the rationale for this chapter, which is about the importance of designing a variety of situated genre production tasks and modules.

Hyon (2018) proposes different kinds of genre production tasks, which as she notes, vary in how controlled they are:

- controlled production tasks require that students mimic certain features of genres in their production;
- scaffolded production tasks provide guided preparation for genre production but are less controlled and require more decision-making on the part of the students;
- interpretive production tasks require that students analyze the writing situation and independently interpret which moves and linguistic features best fit their situation and use them in developing their text.

The scaffolded production and interpretive production tasks, thus, both require students to decide independently on the context, content and language of their texts; hence these tasks promote more autonomous genre production (Hyon, 2018). The major difference between the two activities is that in scaffolded activities, students receive assistance from the instructor to acquire skills and knowledge. The support, however, diminishes gradually, in the process of performing similar tasks. The idea is to have students move from being otherregulated to self-regulated genre participants. For example, students first analyze samples of the target genre with the instructor coaching them. They then produce a sample of the target genre independently. On the other hand, interpretive production tasks include less assistance from the instructor so that students must figure out the context of their writing and the genre features they are required to produce. In designing the modules in this chapter, I use a combination of the three production tasks with the objective of moving the students towards becoming more autonomous genre participants.

A third major category of tasks designed here following Hyon (2018) are process tasks. These tasks follow Rhetorical awareness-raising and production tasks and focus on the processes for producing and receiving texts belonging to different genres. The notable examples of process tasks are peer commenting and revising/redrafting. Genre-based process tasks thus have students recognize the importance of the revision process as well as teach them how to respond to the texts they receive and, more importantly, how to revise their texts.

As Hyon (2018) remarks, all three types of tasks can facilitate both genre acquisition and genre awareness; that is, they help students acquire genre features and use them effectively. Moreover, they help students build awareness of the genre context. While the rhetorical consciousness-raising tasks are more direct in this mission, the less controlled scaffolding and interpretive genre production tasks do so by having students more autonomously decide on the genre element they might include (or exclude) in specific situations. Given the level of target students for the lessons and writing tasks developed in this chapter, the production activities are predominantly designed as scaffolded. Once the students grasp the specific genre knowledge, they will then be required to undertake the interpretive production tasks.

I have designed the major modules, writing activities and tasks with upper-undergraduate and graduate students in mathematics in mind. I have chosen these groups, especially the graduate students, because of the increasing demands on them to disseminate the results of their graduate research by publishing in research journals. Choosing academically senior students as the focus group is also motivated by the fact that this group of students have already mastered some level of content knowledge in their discipline, so they do not find themselves in alien lands when exposed to the mathematical content of the tasks.

The choice of research article as the focus genre for the pedagogical activities in Appendix C is justified by the genre's centrality to academic and research communities and the increasing need of graduate students to grasp the genre of research article. So, the activities developed here aim to assist these student groups in learning about the genre of research article and how to write research for knowledge creation. The pragmatic/rhetorical activities involved in writing research articles are designed to assist senior undergraduate and graduate students in positioning themselves as credible writers and (near-)competent members of their chosen disciplinary discourse community. The activities teach students how to participate in the more subtle pragmatic aspects of disciplinary writing including making knowledge claims, moderating claims, adopting a proper stance, engaging readers, and constructing persuasive arguments to establish their claims and create new knowledge.

In the rest of this chapter, I will introduce the modules and writing tasks that I have designed for the mathematics students and discuss the rationale behind choosing them. These discussions are geared to the original audience of this dissertation and writing instructors. You will find the modules and learning activities in Appendix C. As you will see, I have shifted my tone to a student audience in the module/task descriptions and instructions. I also used italic typeface for the texts that address a student audience.

6.5. Modules and Writing Tasks

6.5.1. Module 1: Analysis of Rhetorical Situation

The first module that I have developed (See Appendix C) aims to introduce the students to the threshold concept that every writing happens in a rhetorical situation. Rhetorical situation, introduced by Bitzer (1968), is central to students' understanding of the context in which they create their writing and its impact on their writing decisions and strategies. I have designed the module by first briefly introducing the concept of rhetorical situation and its key components, namely audience, purpose, and genre. The introduction serves as a short reading that gives the students the conceptual background that will help them complete the learning activities. I have then moved to the analytical activities that give the students the chance to develop skills, knowledge, and awareness of the rhetorical situation in writing.

I have designed the activities as pre-task, main task, and post-task to maximize student engagement and learning experience. The purpose of the pre-task in Module 1 is to familiarize students with concepts pertaining to a rhetorical situation through analysis of the less technical texts, which students normally encounter in their daily life outside of the classroom. Using texts that students are familiar with helps them to connect with the rhetorical situation of those genres more comfortably. It also helps students to focus on the rhetorical situation of the text without worrying about understanding its information content. Using short non-academic popular texts to demonstrate analysis of rhetorical situations also helps to minimize the effect of the assignment and task situation on students' understanding of rhetorical situations.

The pre-task is followed with two main tasks (Task 1 and 2, Appendix C). Task 1 and 2 both aim at consciousness-raising of the elements of rhetorical situations in which texts are produced and used. Task 1 initiates a focus on the pragmatic functions of genres in different social contexts and then switches to a more elaborate analysis of those rhetorical situations. By completing Task 1 students should develop an overview of how genres are inevitable to performing some social actions. They also analyze and develop an awareness of the rhetorical situation of genres. Task 2, on the other hand, follows a narrower scope by asking students to focus on the genres of their discipline. By completing task 2, students put their theoretical knowledge of rhetorical situations, as well as their gain from the pre-task, into the context of their discipline and group texts based on their target audience, their specific purpose, and the overall look of the texts. The task also assists students in identifying their status within (or around) the discourse community (undergraduate vs. graduate vs. expert members) in terms of the reading and writing that they do. Therefore, the task helps students understand that their role in the program is defined by what they do, including mastering school and research genres.

Besides engaging students with *what* questions, both tasks pose *why* questions so as to trigger students' critical thinking about perceived boundaries between texts and their rhetorical context. Students also develop an awareness of how each criterion puts both restrictions on and opportunities for the choice of content and form of genre. For example, a research article and a grant proposal are two documents aimed at an expert audience in mathematics. Both genres aim to construct a persuasive argument to convince their expert audience about a claim. However, the texts have a different thesis and eventual purposes, that is, the former seeks to establish a knowledge claim (that is, create new knowledge) while the latter aims to convince a committee of experts in the field of the proposed project's research worthiness and to gain funding approval. The striking difference in the purpose controls the organization, content, and formal features of the two texts and results in two different documents. Both tasks thus help to develop students' critical awareness of the role of rhetorical situation in framing, grouping, and differentiating genres. Tasks 1 and 2 are then followed with a post-task. The post task gives the students the opportunity to validate their understanding of the concepts of the rhetorical situation, get the instructor and peer feedback on their practice from the main tasks and to revise, if needed, their understanding of the rhetorical situation for texts. Through sharing their responses with peers, students may be introduced to genres that they were not familiar with, hence extending their repertoire of academic and/or research genre categories. By completing the tasks in the module students will be ready to start working on the individual major assignment for the module which asks them to write a rhetorical situation analysis report of a disciplinary writing genre.

In Module 1, I developed activities which aimed to help students understand the different components of rhetorical situations for writing. Understanding rhetorical situations is key to the writing process as it helps students, as novice writers, make informed rhetorical choices in planning and drafting their work. The next module, Module 2, builds on module 1 by providing more rhetorical situation analysis practices. The major goal of module 2, however, is to teach students about genre analysis as the core of my combined reading-writing genre-based approach to teaching writing-in-mathematics.

6.5.2. Module 2: Genre Analysis

The main objective of Module 2 (See Appendix C) is to facilitate students' genre learning. Following Hyon (2018), I use a genre-based pedagogy through which I propose three categories of genre analysis activities including consciousness-raising, process, and production that are scaffolded through read-analyze-write steps (Robinson, 2008). The tasks which I have developed help students read and analyze texts from select genres for their context, organization, and social action. I opted for this approach based on the premise that many successful writers develop their capabilities in disciplinary-specific writing by reading and analyzing good samples of the kind of writing which they aim to produce (Johns, 2013; Hyon, 2018; Tardy, 2009).

Moreover, I avoid an assimilative and accommodationist conception of genre and genre analysis in my teaching approach and the proposed tasks. Indeed, I am aware that practicing genre analysis by focusing only on formal features of genre texts could be restrictive and might lead students to develop a distorted conception of genre. In order to avoid such a shortcoming, I use discussion questions as part of preparatory tasks. I tailor my proposed discussions to questions that help to engage students in thinking about the rationale behind rhetorical choices in genre texts and exploring the links between texts and disciplines. My goal is to promote critical thinking in students through such discussions and help them develop awareness about the ideological power of genres (Bazerman, 1997; Bhatia, 2004; Johns, 2008; Devitt, 2009; Tardy, 2009).

Similar to Module 1, I started Module 2 with a short reading that provides a theoretical background on genre analysis and introduces it as an approach to understanding how texts are organized and written and why they are written the way they do. The reading is followed by a series of learning activities including a pre-task and two main tasks, each followed by a post-task (See Module 2 in Appendix C). The purpose of the pre-task is to demonstrate how to analyze the rhetorical situation and organizational structure in sample nontechnical texts and to encourage the students to notice the links between the context and text. The use of nontechnical texts would guarantee that the students connect with the topic and content of the demo texts with minimum challenge and thus can focus on the main goal of the activity which is the analysis of the rhetorical organization.

For the pre-task, I have used math tutoring ads which are easy to read and understand (Figure C.1). They are also short and concise enough to be covered in a pre-task. A more important reason for choosing this particular genre is that ads belong to the main category of posters. However, as I have shown in the subsequent tasks in Module 2 (Figures C.2, C.3., C.4), different posters represent different genres and show notable variations in their macro and micro-organization depending on their audience, purpose, and information content. The pre-task will prepare the students for the subsequent two main tasks which ask them to analyze the organization and rhetorical structure of two different types of posters which, due to their rhetorical situation, represent two different genres than the genre of a job ad poster used in the pre-task. It is important to note that practicing the rhetorical structure analysis as starting with nontechnical texts (pre-task) and moving to scholarly and academic texts (major task) helps students gradually recognize organizational variations across texts that carry similar labels. Students will also learn that there are usually conventions for organizing texts and that these conventions vary according to the rhetorical situation and context of texts. Through these activities, students also develop analytical skills for understanding both the general principles that apply to most contexts of writing and specific conventions which group texts are a certain genre. The goal is for students to develop an awareness of genre, as well as the knowledge and skills that they can then transfer to future writing situations.

Following Task 3, I have developed a post-task discussion that aims at consciousness-raising about the differences in the purpose (informing about services vs. informing about events) and the audience (school students vs. experts or near experts) across the two genres reproduced in Task 3. The goal of the post-task is to raise student awareness about how variations in audience and purpose across these posters interlinks with the organization of the information in each poster and eventually results in two different genres.

Task 4 then builds on the post-task discussion of how the purposes of the two posters in Task 3 (informing the audience about a service and persuading them to act (that is to buy) vs. informing the audience about a research event and persuading them to act (that is participate and/or present in the conference). It is an upper-introductory activity, which asks students to analyze a conference presentation poster (CPP) (See Task 4, Appendix C, for the learning objectives and the task instruction.) The choice of conference presentation poster for this activity was motivated by the following rationales:

1. It represents a research genre.
2. As an undergraduate research genre, the content of a CPP is less challenging for the upper-undergraduate and graduate students. So, they can focus on the analysis of the organization of the CPP without worrying about how to understand its content and argument.
3. It shows notable similarities in structure and content to the genre of research article, which is the main writing activity targeted in the modules developed in the rest of this chapter.
4. It is concise and short, limited to a single page, hence is more manageable for in-class analysis.
5. It allows for comparative analysis with an ad and a CFP poster.
6. It assists a smooth transition from the analysis of a non-academic genre (an ad) to examining a quasi-academic/professional genre (conference call for paper) to the analysis of an academic/research genre (conference presentation poster). It is hoped that this gradual transition across the three genres, the first two of which are easy for students to read and understand, helps them focus on the purpose of the lesson, which is to analyze the rhetorical situation and genre organization, without having to worry about understanding the content.

Task 4 is also followed by a post-task class discussion. The purpose of the post-task is for the class to receive peer and instructor feedback on their analysis of the rhetorical structure of a research genre. The feedback will help the class to revise and/or establish their developing theory of genre and genre features. Students will also learn introductory information about a key research genre, that is, the research article, and start building genre knowledge about it. The post-task discussion will also aim at consciousness-raising of both the context and textual forms that students used to identify the rhetorical structure of the poster. The discussion gives students an opportunity to compare/contrast their analysis and understanding of the rhetorical organization of the two posters and receive instructor feedback regarding the concept of conventional vs. optional moves in genre organizations. The instructor may also briefly refer to the similarities and differences in the rhetorical structures between a CPP and a research article.

The major assignment that I have designed for Module 2 is a genre analysis report (See Appendix C, Module 2 major writing assignment:Analytic report). The assignment asks students to analyze two samples from select academic genres and draft their analysis as a report. The genre samples include, a student research proposal, an infographic, a book blurb, a short communication, a book review, a student problem-solution essay, etc. A brief introduction by the instructor would give the students some background about the communicative purpose, audience, and the discourse community who uses the genre. The purpose of the major writing assignment is to promote write to learn by having students individually analyze samples of genres from their program of study. A second main purpose is for students to practice writing an analytical report which is a much-required student assignment, especially in the context of Canadian universities (Graves and Hyland, 2017). The assignment is also intended to give the students a first chance to consciously analyze their rhetorical situation as a writer and be aware of the potential constraints of writing a student genre.

Providing a hands-on opportunity for students to practice genre analysis is important. It is equally important to give them a chance to reflect on their genre analysis experience. The reflection helps students to think deeply about their learning experience, develop their knowledge and even locate gaps in their knowledge and come up with new questions, which may trigger more learning. Therefore, I decided to wrap up module 2 with a reflective activity designed by Robinson (2008) that asks students to reflect on their genre analysis experience and draft their reflection as a journal.

Modules 1-2 were introductory modules which aimed to prepare students for module 3, which is the major module of my suggested course. Module 3 focuses on the RA genre and aims to prepare students to start drafting a research article Introduction for their research findings.

6.5.3. Module 3: Research Article Genre

The third module gives hands-on practice in writing research. The goal is to teach writing a research article through a number of analytical tasks (See Appendix C, Module 3). I have planned the sections in this module in s way to involve the students in both accumulating genre knowledge, both the organization and argumentation, and applying their knowledge in constructing an argument for new knowledge and structuring their draft as a research article. Students thus should be able to develop a priori as well as a posteriori genre knowledge which they can put forward to explore other genres in future. I designed Module 3 into five parts. Each part scaffolds student learning of one key aspect of writing-in-mathematics.

Part 1 starts with a short reading about research article as a genre, its rhetorical situation, and writing process (See Module 3 (Part1) in Appendix C) followed by a pre-task, a main task, and a post-task. The pre-task has students brainstorm their existing knowledge of the audience, think critically about the characteristics of an expert audience and in so doing prepares them for Task 5. Task 5 is a revised reproduction of Exercise 2.1 from Robinson (2008, p. 35). The revisions tailor the task for mathematics students. The purpose of the main task (Task 5 in Appendix C) is to engage students in analysis of expert discourse in their discipline and require them to consciously focus on the formal features of expert writing. The practice helps to raise students' consciousness of the importance of lexico-grammatical choices in research article writing and draw their attention to the critical point that part of the credibility to write to an expert audience relies on special lexico-grammatical and structural choices from the language system. By credibility I mean trustworthiness of a writer as an expert in the field s/he writes in. Having and demonstrating the quality of being trustworthy is a rhetorical strategy which contributes to the persuasiveness of a writer's argument. A formative feedback post-task wrapps up Part 1. It gives students a chance to receive oral peer and instructor feedback regarding the formal features of expert writing, confirm/revise their understandings of disciplinary expert discourse and move them towards building an internalized concept and theory of writing to an expert audience in their discipline.

Part 2 (See Module 3 in Appendix C) focuses on understanding the purpose and characterizations of journal publications as genre. The tasks in part 2 aim to help students learn about the interconnections between sub-disciplinary trends, library research, and some preliminary literacy practices in mathematics. The pre-task is designed to engage students in the topic and prepare them for the main task. It aims to activate students' prior, possibly subconscious, knowledge about their major area of research and the disciplinary associations they belong to as well as their library research practices. In particular, it gives students a chance to recall their prior meta-disciplinary and meta-genre knowledge about journal publications. Helping students to become conscious of the connection between the classroom activities and their real-world disciplinary knowledge and activities will help them better see the value of these tasks and engage in them more attentively.

The brainstorming pre-task in Part 2 makes students conscious of their metacognitive knowledge of research articles in their discipline of study how it interlinks with their research reading and genre knowledge. The pretask is followed by Task 6, which builds on the activated metacognitive information from the pre-task. Task 6 asks the students to identify different types of academic journal articles. It sets up a preliminary step in academic genre identification. Identifying the type of journal articles is important because it activates meta genre information such as the purpose, information content of the article, the organization, and the type of argument typically made in different genres of journal article. Such insight facilitates reading and writing in genres. Moreover, since the task requires library research, it creates its own dynamic of engagement. It also involves the students in social interaction through group work. I have wrapped up part 2 with a formative feedback post-task. The feedback assists the students to establish, or otherwise revise, their conceptions of different journal article types.

In Part 3, I ask students to examine two important stylistic features of research writing in mathematics, namely concision and nominalizations. I have used examples of wordy sentences and their concise revisions. I have also designed scaffolded activities that engages students in active reading and comparison of wordy sentences and their concise versions as revised by writing experts (pre-task). The pre-task prepares students for the cognitive demand of Task 7, which asks students to revise sample wordy sentences for concision. Students will finally discuss their strategies as a class in the post-task. Shortening longer structures and sentences is usually part of the revision process for effective academic communication, especially for research journal submissions. Task 7 draws students' attention to this step of the writing process. Through this task, students practice how to look for and cut the unnecessary words from a group of sentences written for mathematicians without detracting from the original ideas. Task 7 is followed by a post-task that gives the class a chance to learn about the strategies used in other groups and receive peer and instructor feedback on the clarity and efficiency of their revision.

Task 8 then asks students to apply the knowledge and skills from the previous task to their analytical report from Module 1 and improve it. Completing individual activities allows students to not only practice their knowledge and skills and learn more, but also to develop into more autonomous writers. The purpose of Task 8 as a focused individual activity is to engage students in their own writing and to make them practice concise writing strategies and enhance their revision skills. Because one tends to treat one's own writing as clear and understandable, I have used a peer feedback task, namely task 9, to complement the self-revision activity.

Task 9 aims to help students develop awareness of the fact that effective conciseness and clarity in academic writing are obtained through interaction with the audience during the drafting and revision processes. Sadly, writers have little opportunity to access audience feedback before their manuscript is accepted for a review and not until they hear back from reviewers. However, there are some strategies which help to receive some audience voice before finalizing a piece of writing. A typical strategy is to have cognitive interactions with a tentative audience while drafting a work. An example of such cognitive interactions is asking questions such as "is this example/evidence strong/valid enough to persuade my audience about the claim which I make in this paragraph?" Another strategy is to ask for peer feedback on a first draft. A more advised strategy for novice writers is to read their draft out loud to catch sentences which sound awkward and incoherent. The instructor can suggest these and/or other effective strategies and use them to compensate for the absence of the target audience in the drafting and revision stages. I have developed similar activities for teaching about nominalizations in Task 10. See Task 10 and the associated post-task in Appendix C for more information on teaching about nominalizations.

Part 4 then teaches about the macro-organization of research articles using the traditional IMRD model and current findings about organization of research articles in mathematics. The tasks in this part have students develop understanding of the discernible patterns and potential variations in their macro-organization within and across the disciplines.

The scaffolding of the learning activities in part 4 includes two major tasks, namely task 11 and 12 (Appendix C) that have students practice identifying the macro-organization of journal articles in their discipline of study and to reflect on the identified organization. As a pre-task, I have suggested that the instructor stimulates a short class conversation about students' prior knowledge regarding macro-organization of research articles in their field of research and the type of information that section headings offer when they read research articles in the field. The pre-task has students brainstorm what they already noticed/know about the macro-organization of the research articles in their field of research. It prepares students for active engagement in the following main task.

Task 11 then builds on the brainstorming from the pre-task and has students scan for the macro-organization of research articles in their field. Students learn a) the macro-organization of research articles in mathematics, b) how to scan research articles for their content outline based on their macro-organization, c) how to use section headings to give their writing better organization and clarity of structure. Having completed the main task, the class will proceed to the post-task. The post-task opens up an opportunity for more interactive learning and for confirming or reformulating students' tacit theories about the macro-organization of research articles in mathematics. These tacit genre theories guide students in the process of drafting and revising their individual research for journal submission.

The group activity in Task 11 is followed by an individual activity (Task 12 in Appendix C) that asks students to use their learning of the macro-organization of RAs in mathematics in practice. By completing Task 12 students not only develop a macro-organization for their research article and use it as a map in drafting their research article, but also have the opportunity to receive feedback and improve their draft. A second reflection journal will end part 4. The purpose of the reflection journal is to engage students more deeply in the concepts and activities of parts 1-4 by thinking critically about what they have read, practiced, and learned through those parts. The cognitive tasks and questions are verbatim from an Exercise suggested by Robinson (2008, p. 29) in *How to Write Like a Chemist*. I decided to use Robinson's exercise as the journal prompt because it clearly instigates guided reflection in 3 main respects which match the purpose and activities of this section.

The fifth, and indeed the last, part in Module 3 introduces a step-by-step procedure to analyzing and later writing an introduction for their would-be journal manuscript (Appendix C, Module 3, Part 5). I have started this part with a brief reading about introduction sections in mathematics research articles and the models proposed by recent research (Figures C.5, C.6. in Appendix C) about writing introductions in mathematics research articles. Following the reading, I have designed three tasks (Tasks 13-15 in Appendix C) and a manor writing assignment. Tasks 13-15 are rhetorical consciousness-raising tasks that ask students to do a rough analysis of the Introduction section of a journal article to develop an awareness of the rhetorical context, organization, and language features of the section. Task 13 is composed of four major sections (A-D), each section focusing on a number of dimensions of the genre in question. The aim of grouping these different dimensions of analysis under one task title is to demonstrate the multidimensionality of genre production and reception, an awareness of which is an important learning goal of the lessons developed in this chapter.

Obviously, task 13 is not a one-shot activity; it presents a sequence of analytical tasks. I suggest that the task be introduced to the class and practiced section-by-section, with a class discussion following each activity section. With the focus of each section shifting to a different aspect of genre production, students should be able to develop awareness of the multi-dimensional nature of texts. Completing these tasks will also help students to develop analytical glosses on how to focus on a different dimension of genre at a time while reading or writing samples of the genre.

I believe the various consciousness-raising, process, and production tasks I designed to teach the organization of RAs and Introduction sections would apply, with modifications and adaptations, to the analysis and learning of other sections of RAs in mathematics and many other disciplines. I thus refrain from repeating similar tasks for writing other sections of RAs in mathematics and move on to some concluding remarks about the suggested pedagogical activities.

6.6. Concluding Remarks

In this chapter, I tried to operationalize some of the pedagogical implications, which I already suggested for my research in the previous chapters. My main goal in designing the tasks and activities in this chapter was to help writing instructors and students develop analytical and critical thinking skills regarding genres and writing in disciplines. My other goal was to demonstrate how genre pedagogies can enhance student learning of both disciplinary communication and knowledge.

As with any research and curriculum design, there are limitations to my work. In this chapter, I demonstrated genre analysis of only one section of RA. As such, no claim is made as for the completeness or adequacy of the tasks; I believe no course or dissertation would claim to teach genres completely and thoroughly. Moreover, I have not tested the designed modules and tasks in a real-world class, so it might happen that they slip in performance in the context of real-world classes and with real world students. Hence, future action research is required to confirm the tasks or the efficiency of the suggested modules and tasks. Furthermore, the basic idea and/or the actual format of most of the analytical/writing tasks come from genre scholarship and curricular material, so no claim whatsoever is made to the originality and uniqueness of the modules and tasks proposed in this chapter.

Notwithstanding the limitations, I have tried to enforce genre analysis for both training and educating students as heuristic genre analysts in my writing pedagogy in this chapter. My decision to adopt this approach is motivated by an intention to empower students to develop substantial knowledge and skills which they can transfer and apply to their writing situations to understand the context of their writing and the restrictions and opportunities of writing in that context.

The pedagogical activities developed in this chapter effectively show that genre analysis is a valuable and powerful model that can be used in writing classes both as a teaching approach and a learning tool. As a teaching approach, genre analysis facilitates designing genre-based courses and creating concrete materials which are closely connected with students' immediate writing needs. As a learning tool, genre analysis would help students not only develop analytical and critical skills to respond to their writing needs in the context of their discipline but also learn about their discipline through writing.

6.7. Chapter References

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Chapter 7: Conclusion

The aim of this study was to explore the RA genre in discrete mathematics by applying Swalesian genre analysis as well as perspectives from RGS and visual rhetoric. This chapter concludes the dissertation by presenting an overview of the study and the research questions which instigated it. I then summarize the key findings of the study concisely and discuss the significance of the study and its contribution to genre scholarship and writing studies research. Finally, I point out the limitations of the study and conclude with some suggestions for future research.

7.1. The study in a Nutshell

This dissertation reported multimodal genre analysis of research articles (RAs) in discrete mathematics. I began this study with the global research question of how and why mathematicians write the way they do. To get more concrete results, I refined my global questions into the following questions:

1. How do discrete mathematicians structure the macro-organization of their RAs in the discipline? Why do they organize them the way they do?
2. How do discrete mathematicians structure the micro-organization (Move-Step organization) of their RA Introductions in the discipline? Why do they organize them the way they do?
3. What are the most typical rhetorical strategies for establishing research niches in discrete mathematics? What do they tell us about the discipline's conception of research?
4. How do discrete mathematicians blend visuals into the micro-organization of their RAs in the discipline? Why do they use them the way they do?
5. What practical pedagogical implications can be drawn from the findings of this study?

To answer these questions, I analyzed a corpus of 30 RAs from both pure and applied orientations in discrete mathematics. This provided me with an overview of the macro (sectioning) rhetorical organization of RAs in the corpus, the micro (Move-Step) rhetorical organization of the Introduction sections, strategies for niche establishment, visual moves in the corpus and their associations with the verbal moves. In analyzing the multidimensional nature of the RA genre, I relied on both ESP genre analysis approach and Rhetorical Genre Studies (RGS) as well as visual rhetoric. To understand the rhetorical organization of RAs in my corpus, I triangulated data from the textual and visual analysis with interview data gathered from 5 research mathematicians as well as literature in the philosophy of mathematics. Triangulating different data sets helped me to reach a deeper understanding of the argumentative and epistemological rationale behind the observed generic features of RAs in discrete mathematics (Candlin and Hyland, 1999).

7.2. Highlights of Findings

Over the last four decades, genre scholarship has shown that the rhetorical and discourse structures of genre texts are shaped by a genre's communicative purpose, its social actions in Miller's (1984) terms, and the values held by the discourse communities that use genres (Hyon, 2018). Academic and disciplinary genres have been seen as "frames for social actions" (Bazerman 1997, p. 17) and the textual product of such genres have been shown to be responsive to the disciplinary norms and practices (Hyland, 2015). It has been argued that the shape of RAs is framed by "the knowledge-constructing practices and ideologies of [each] discipline" (Hyland, 2015, p. 114). The findings of this study not only support these arguments but also show that ontological practices underlie and support the epistemological practices in both doing mathematics and framing the macro and microstructure of its RAs in ways which are distinct from many other disciplines.

In response to the first research question, this study accentuates the links between the rhetorical organization of RAs in discrete mathematics and the historical traditions of the discipline. First, the findings suggest that RAs in discrete mathematics follow the Introduction-Results framework, which remarkably (yet unsurprisingly) deviates from the dominant IMRD model in empirical disciplines. As shown in chapter 2, a lack of Method section is due to the dominance of logic-driven induction and deduction as the cognitive work of mathematicians, which makes a separate Method section rhetorically and stylistically unnecessary. Similarly, the disciplinary tradition of relying on logic for the purposes of persuading the audience rules out the need for a Discussion section. Instead, where required, concise explanations are integrated into the Results sections. Furthermore, a Conclusion section is used on an occasional basis when the findings suggest applications that might be of interest to scholars from client disciplines such as computer science, engineering, applied biology and medical sciences.

On the other hand, the findings showed that it is conventional for RAs in discrete mathematics to include a number of Introduction and Results sections depending on the topic of study, its history of research, the number of definitions and symbolic notations required to conceptualize the abstract objects of study and the number of major results of the study.

In addition to the overall schematic (macro rhetorical) structure of RAs in discrete mathematics, I also focused on the move (micro rhetorical) structure of Introduction sections (cf. chapter 3) as inspired by my second research question. I showed how Introduction sections in RAs in discrete mathematics depart from the two versions of the *Create A Research Space* model (Swales, 1990, 2004) both in terms of the moves, their recurrence, and the flexibility of their order as guided by the problem being solved. As suggested, a notable departure from *Create A Research Space* is the presence of Establishing Presumptions, a free-standing and frequently used move which is used to define the hypothetical mathematical concepts of study. The move initiates assumptions that mathematicians use to create new hypothetical objects of study and give them physical presence as notations. Establishing Presumptions thus initiates a rhetorical activity which is critical to turning the abstract mathematical concepts into tangible objects which can then be manipulated to generate new results. The move also highlights the highly argumentative nature of mathematical discourse which strives to gain the readers' adherence to a thesis by building on shared assumptions. Furthermore, the findings of this study suggested variations in occurrence and chronology of moves in Introduction sections of RAs in discrete mathematics as compared to the *Create A Research Space* model.

My third research question was concerned with strategies for establishing research niches in the discipline. As shown in chapter 4, a notable variation from the *Create A Research Space* model was noted in my verbal data regarding the *Establishing a Niche* move. As my findings suggested, mathematicians may opt for a combination or any of the strategies of retrieving a problem, indicating an absence of or insufficient research, raising a question, adding to what is known, and counterclaiming or they may decide not to use any strategies for niche establishment leaving it to the reader to *infer a niche*. From among the identified strategies, retrieving a problem and inferring a niche seems to be unique to this study and, to the best of my knowledge, are not reported in other genre scholarship. Linguistically, Establishing a Niche, where present, often appears with particular mathematical jargon. For example, the study findings suggest this move is associated with technical language such as *conjecture*, *open problem*, *extension*, and *generalization*. On the other hand, some labels used in the *Create A Research Space* model such as 'indicating a gap' connotate negative meaning in mathematics. To clarify, the term 'gap' in mathematics is used when there is a flaw in a proof. So, 'indicating a gap' from *Create A Research Space* model interferes with mathematical jargon and the discipline's worldview on the infallibility of its findings unless in exceptional cases.

A further novel finding of this study concerns the fourth research question regarding visual moves in RA genre in discrete mathematics. As shown in Chapter 5, some discrete mathematicians use visuals to initiate and develop their arguments and to establish new knowledge. I showed that visuals interact with the verbal moves as either free-standing moves or in association with the verbal moves through a combination of ontological (to create objects from abstract concepts), argumentative (to provide evidence for mathematical relations), and/or epistemological (integrate to create and support new knowledge claims) actions. My findings also suggested that in some cases visuals nailed down the pillars of the argument before the verbal to help the readers anticipate, invest in, and contribute to the author's verbal argument. By showing that visuals can scramble the linear synthesis of the verbal moves, these findings violated the traditional assumptions about the linear narrative structure of RAs and supported the conception of RA genre as a carefully constructed piece of argument.

In sum, my multimodal genre analysis in this study suggested that mathematicians fabricate their argument for new knowledge in tandem with the conventions and norms guided by the philosophy of mathematics and approved by its discourse community. My findings support the claims that "the contexts that genres carry include ideologies, norms, and values" (Devitt, 2009, pp.338-339), that academic knowledge is "a product of the situations in which it is created, rooted in disciplinary arguments, affiliation, and agreement-making" (Hyland, 2009, p. 11), that we cannot view disciplinary writing as simply a medium of knowledge presentation "without consideration of its deeper cultural and epistemological underpinnings" (Hyland, 2013, p.97) and that disciplinary writing is "evidencing a sophisticated awareness of how disciplinary cultures textualize that research into knowledge" (Hyland, 2000, p. iii).

7.3. Significance and Implications of Findings

This study contributes to existing scholarship in academic genre research and pedagogy in important ways. First, it shows the power of linguistic-rhetorical genre approaches as a research method to understand disciplines, their socio-rhetorical actions, and the values blended into their genre texts and discourses. The analysis of the RA genre in this study unpacked how mathematicians perform the social action of creating new knowledge in tandem with both the discipline's values and norms and discourse community members' sense of identity and agency. I believe unpacking the social actions and their reciprocal links with the disciplinary norms would not have happened should I have not used a multidisciplinary theoretical foundation including applied linguistics, rhetoric, visual rhetoric, and philosophy of mathematics, as well as a triangulation approach for my research. The study thus provides evidence for existing arguments that research and education into disciplinary discourses require "significant reallocation of resources and priorities both within departments of literary and language studies and within the many other disciplines of the academy" (Bazerman, 1992, p. 68).

Second, this study contributes to existing genre analysis methodologies by showing that even when the findings are contextually situated within limited data, adopting multidisciplinary theoretical perspectives and triangulating different data sets facilitates a depth of understanding of disciplines and their rhetorical actions. This makes me suspect that the global claims that arise out of my study may also be relevant to other areas of mathematics and similar disciplines, although I admit that we need more research to reach certainty about this claim.

Third, this study lends support to ESP and RGS by responding to recent calls for multimodal genre analysis in these trends (Johns, 2013; Miller, 1998; Tardy and Swales, 2014). To the best of my knowledge, no earlier research is available on verbal-visual move analysis of the RA genre. Move analysis of the visual data in this study and examining their interaction with the verbal moves showed the undeniable role of visual elements in the RA genre in creating mathematical objects of study, providing evidence for arguments about the logical relationships between objects, and solidifying new knowledge. One implication, then, is that in disciplines where visuals are used in the RA genre, and indeed in any genre which uses visuals, rhetorical analysis of visuals should be part of the study design, otherwise a significant dimension of the genre will be missed, leading to a distorted understanding of the genre, its cultural norms and social actions. The pedagogical implication is significant: Genre-based curriculum should raise student awareness of the visual rhetoric in disciplinary texts by encouraging them to understand how and the extent to which the visuals in their field contribute to creating arguments (Hyland, 2006) and by collaboratively unpacking the visual core of the argument (Miller, 1998).

Finally, this study contributes to genre pedagogy by proposing advanced level instructional tasks and practices for a tentative writing-in-mathematics course. This contribution is important as it exemplifies a genre-based writing course (Hyon, 2018). Through developing a series of consciousness-raising, production, and process writing tasks (cf. chapter 6), I tried to show that genre analysis has the potential to help students and novice researchers to develop and use "rhetorical perception [used] as a means to distance [them]selves from the everyday practice [of their discipline] in order to reveal and evaluate the hidden mechanisms of [its] life" (Bazerman, 1992, p. 62) and in doing so provide “the means for more informed and thoughtful participation” (Bazerman, 1992, p. 64) in the life of their academic disciplines. As I have tried to show in practice, one pedagogical implication which “can help build the intellectual foundations for courses” would be to use genre-based pedagogies to engage students in the key genres and discourse of their discipline in ways that facilitate an understanding of “the dynamics of each field and the state of play into which each new participant enters” (Bazerman, 1992, p. 67). A similar argument has recently been made about teachers learning about genre to be able to teach it (Tardy et al., 2018).

The development of the genre-based instructional content in this study is also important in the sense that it distinguishes my work from theoretical genre studies. Indeed, while it is common to studies of academic genres to propose pedagogical implications based on their findings, few studies functionalize their pedagogical aspirations through developing practical tasks and activities for a specific group of students. This study thus takes one step forward in suggesting how genre analysis as a theoretical and research tradition works as both a teaching approach and learning tool in the context of a writing-in-disciplines program. An example of such a program is the Writing-Enhanced Curriculum (WEC) at the University of Minnesota (wec.umn.edu) that integrates writing into mathematics curriculum to teach effective writing and communication to mathematics students in the context of their discipline. The curricular material I developed in this dissertation can help WEC and similar curriculum to teach mathematics students develop writing skills and learn about their discipline as they practice writing.

7.4. The study Limitations and Recommendations for Future Research

As with any research, there are some limitations to the research shared here. First, only one sub-discipline of mathematics was examined. Therefore, the study findings are shaped by the local context of the examined sub-discipline. Similar research in other branches of mathematics would contribute to making generalizations of the findings of this study to the entire discipline of mathematics.

Second, the study used a sample of 30 RAs from both pure and applied orientations in discrete mathematics. While 30 RAs make a decent size for this study to offer a deep understanding of the rhetorical and epistemological practices in research writing in discrete mathematics, analysis using two sets of distinct and larger corpora from pure and applied mathematics RAs may yield more representative findings regarding the macro and micro schematic organization of RAs in pure and applied mathematics.

Third, the logistics of this study limited my access to only 5 research mathematicians as the disciplinary informants who provided insider perspective and validated samples of the textual data. I was also fortunate to receive validating feedback over my visual data from the original authors of the RAs. In addition to the interview data and author feedback, I relied on available scholarly publications about the philosophy of mathematics, disciplinary norms and values in researching and writing mathematics, and general cognitive processes of mathematicians in research. Future research may benefit from recruiting a bigger cohort of disciplinary informants from a variety of sub-fields in discrete mathematics. This study can also be complemented with a future ethnographic study of mathematicians in their immediate context of research and writing for deeper theorizing of research findings.

Fourth, this multimodal genre analysis research examined only the verbal and visual modes to explain how they contributed to the disciplinary culture of brevity and clarity as well as to facilitate and support mathematical argument for new knowledge. Future research may include an examination of mathematical symbolism and the ways symbols interact with the verbal and visual moves in RAs to promote the disciplinary culture of concision and transparency and/or its rhetorical functions.

Fifth, although I built on genre scholarship and the findings of this research to develop curriculum material for advanced research writing in mathematics (cf. chapter 6), I certainly make no claim regarding the uniqueness of the pedagogical approach nor the developed activities. Indeed, my main goal in developing the curriculum activities has been to show how, as writing instructors, we can effectively benefit from genre research and findings in planning our writing pedagogy and class activities. Moreover, I have not tried the developed tasks in a real-world class with real mathematics students, nor have I done any action research to measure their effectiveness. So, it's possible that my curriculum *unsurprisingly* "slips as it encounters real students with real intentions and reactions" (Devitt, 2009, p. 349).

Finally, this study builds only on ESP and RGS approaches to genre analysis. It is clear that using a combination of these approaches affected my research questions and my research methodology. There are other trends such as the Sydney School and the Brazilian approach to genre study not touched in this study. It is possible that opting for certain genre analysis approaches and refusing others affect the study findings and the interpretation of results. Hence, future research applying any of the latter approaches to the study of RA genre in mathematics may come up with similar, complementary, or different findings than the ones reported in this study.

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Appendices

Appendix A. Some recent research on RA genres

Author(s)	RA section studied	Disciplines	Language(s)
Basturkmen (2012)	Discussion	Dentistry	English
Del Saz-Rubio (2011)	Introduction	Agricultural sciences	English
Kanoksilapatham (2011)	Introduction	Civil engineering	English
Parkinson (2011)	Discussion	Physics	English
Soler (2011)	Titles	Biology and Social science	English/Spanish
Lim (2010)	Discussion	Applied Linguistics and education	English
Hirano (2009)	Introduction	Applied Linguistics	Brazilian Portuguese and English
Basturkman (2009)	Discussion	Language	English
Bruce (2009)	Results	Sociology and Organic chemistry	English
Bruce (2008)	Method	Physical Sciences/ Social sciences	English
Pho (2008)	Abstract	Applied linguistics and Educational technology	English

Ozturk (2007)	Introduction	Language	English
Van Bonn and Swales (2007)	Abstract	Language	English and French
Monreal et al (2006)	Section headings	Computer Sciences	English
Lim (2006)	Method	Human Resource Management	English
Kanoksilapatham (2005)	All sections	Biochemistry	English
Samraj (2005)	Abstract and Introduction	Biology	English

Appendix B. Articles in the Corpus

DM1. Hoffmann-Ostenhof A (2007) A counterexample to the bipartizing matching conjecture. *Discrete Mathematics 307*(22) 2723-2733.

DM2. Favaron O and Henning MA (2008) Bounds on total domination in claw-free cubic graphs. *Discrete Mathematics 308*(16) 3491-3507.

DM3. Gibson RG (2008) Bipartite graphs are not universal fixers. *Discrete Mathematics 308*(24) 5937-5943.

DM4. Meierling D and Volkmann L (2009) On the number of cycles in local tournaments. *Discrete Mathematics 309*(8) 2042-2052.

DM5. Gerzen T (2009) Edge search in graphs with restricted test sets. *Discrete mathematics 309(20)* 5932–5942.

DM6. Bielak H (2007) Chromatic properties of hamiltonian graphs. *Discrete Mathematics 307*(11-12) 1245-1254.

DAM1. Atapour M, Khodkar A and Sheikholeslami SM (2007) Characterization of double domination subdivision number of trees. *Discrete Applied Mathematics 155*(13) 1700-1707.

DAM2. Blint V (2008) A distance approximating trees. *Discrete Applied Mathematics 156*(14) 2740-2752.

DAM3. Fujita S and Nakamigawa T (2008) Balanced decomposition of a vertex-colored graph. *Discrete Applied Mathematics 156*(18) 3339-3344.

DAM4. Jarry A and Pérennes S (2009) Disjoint paths in symmetric digraphs. *Discrete Applied Mathematics 157*(1) 90-97.

DAM5. Feder T, Hell P and Huang J (2009) Extension problems with degree bounds. *Discrete Applied Mathematics 157*(7) 1592-1599.

DAM6. Chen WYC, Li NY and Shapiro LW (2007) The butterfly decomposition of plane trees. *Discrete Applied Mathematics 155*(17) 2187-2201.

G&C1. Gould R and Whalen T (2007) Subdivision extendibility. *Graphs and Combinatorics 23*(2) 165-182.

G&C2. Bernáth A and Gerbner D (2007) Chain Intersecting Families. *Graphs and Combinatorics 23*(4) 353-366.

G&C3. Chandran LS, Kostochka A and Raju JK (2008) Hadwiger Number and the Cartesian Product of Graphs. *Graphs and Combinatorics 24*(4) 291-301.

G&C4. Ghebleh M, Goddyn LA, Mahmoodian ES and Verdian-Rizi M (2008) Silver Cubes. *Graphs and Combinatorics 24*(5) 429-442.

G&C5. Bonomo F, Durán G, Maffray F, Marenco J and Valencia-Pabon M (2009) On the b-Coloring of Cographs and P4-Sparse Graphs. *Graphs and Combinatorics 25*(2) 153-167.

G&C6. Cui Q, Haxell P and Ma W (2009) Packing and Covering Triangles in Planar Graphs. *Graphs and Combinatorics 25*(6) 817-824.

JCO1. Cardoso D, Kamiński M and Lozin V (2007) Maximum k -regular induced subgraphs. *Journal of Combinatorial Optimization* 14(4) 455-463.

JCO2. Irving R (2008) Stable matching problems with exchange restrictions. *Journal of Combinatorial Optimization 16*(4) 344-360.

JCO3. Goddard W and Henning M (2007) Restricted domination parameters in graphs. *Journal of Combinatorial Optimization 13*(4) 353-363.

JCO4. Vanetik N (2009) Path packing and a related optimization problem. *Journal of Combinatorial Optimization 17*(2) 192-205.

JCO5. Liu C, Song Y and Burge L (2008) Parameterized lower bound and inapproximability of polylogarithmic string barcoding. *Journal of Combinatorial Optimization 16*(1) 39-49.

JCO6. Messinger M and Nowakowski R (2009) The robot cleans up. *Journal of Combinatorial Optimization 18*(4) 350-361.

SIAM1. Berman KA (2007) Locating Servers for Reliability and Affine Embeddings. *SIAM Journal on Discrete Mathematics 21*(3) 637-646.

SIAM2. Takata K (2007) A Worst-Case Analysis of the Sequential Method to List the Minimal Hitting Sets of a Hypergraph. *SIAM Journal on Discrete Mathematics 21*(4) 936-946.

SIAM3. Dankelmann P, Simon M and Henda CS (2008) Average Distance and Edge-Connectivity I. *SIAM Journal on Discrete Mathematics 22*(1) 92-101.

SIAM4. Korner J, Claudia M and Gabor S (2008) Graph-Different Permutations. *SIAM Journal on Discrete Mathematics 22*(2) 489-499.

SIAM5. Krotov DS and Vladimir NP (2009) n-Ary Quasigroups of Order 4. *SIAM Journal on Discrete Mathematics 23*(2) 561-570.

SIAM6. McClosky B and Illya VH (2009) The Co-2-plex Polytope and Integral Systems. *SIAM Journal on Discrete Mathematics 23*(3) 1135-1148.

Appendix C. Modules and writing task

Module 1: Analysis of Rhetorical Situation

This module aims to introduce you to a preliminary step in reading and writing in disciplines, namely, analysis of the rhetorical situation. By the end of this module, you should be able to:

- *Identify the elements of the rhetorical situation within which a piece of writing is generated.*
- *Explain what is meant by audience and differentiate between types of audiences.*
- *Explain what is meant by purpose.*
- *Identify common/specific rhetorical situations for most student assignments.*
- *Identify common/specific rhetorical situations of expert writing.*
- *Identify different genres in your discipline of study.*
- *Explain why awareness of the rhetorical situation is critical to effective writing in your discipline of study.*
- *Think critically about some choices you have as you negotiate the rhetorical situation of an assignment.*

In addition to analyzing samples of genres, the module uses a 'write to learn' approach by having you write both reports and reflections on your genre analysis. The module also gives you a chance to consciously analyze your rhetorical situation as a writer and be aware of the potential constraints of writing a student genre.

Theoretical Concepts

Disciplinary writing, similar to other kinds of rhetoric, happens in a context. Bitzer (1968) called the context a rhetorical situation and defined it as "a complex of persons, events, objects, and relations presenting an actual or potential exigence" (Bitzer, 1968, p. 6). Exigence, namely the rhetorical reason for which a person uses language and creates discourse, is a key constituent of a rhetorical situation because it necessitates communication. Exigence triggers a reason(s) and purpose(s) for writing (See the part about "Purpose" for more information). Other key constituents of a rhetorical situation are audience and the genre constraints. The three constituents collaboratively influence the choice of content, organization, style and language in the process of writing. Therefore, analyzing and developing awareness of the rhetorical situation within which you write contributes to effective writing. Understanding the rhetorical situation of your writing, however, is not always easy. While most disciplinary and professional writings happen in an authentic rhetorical situation of genre production and use, writing them as course requirements situates them in the predominant rhetorical situation of assignments.

As senior students, many of you are already familiar with the constraints of writing assignments in general. My emphasis here is to make you even more aware of the artificial nature of writing a research article as a "school" assignment versus writing the research article for a journal submission. As you can imagine, the rhetorical situation of a writing assignment can distract from the authentic rhetorical situation of research article writing. On the one hand it puts you as student writers at risk of partial, or even mis-, understanding of genres of writing as social action, for example, 'doing mathematics'. On the other hand, by being predominantly preoccupied with the assignment purpose of writing, you might foreground the instructor as the key component of the rhetorical situation and therefore your understanding of other features of the rhetorical situation including yourself and your agency as the author may be undermined. The ignorance of the authentic rhetorical situation of *genres thus might result in developing distorted conceptions of a genre of writing and failing to prepare you to write in real world contexts of genres. While I admit that the major modules and tasks developed here are inevitably framed within the rhetorical situation of "assignment writing," I have tried to work around/minimize/address this situation by choosing excerpts of authentic genres and authentic tools and process for the analysis, understanding, and production of the target genre. This approach will, I hope, help you to see and practice the kind of writing that is real outside the classroom. The rest of this section focuses on this mission by defining the key features of rhetorical situations.*

Audience- *Each text is written for an intended reader or a group of readers as the audience. The audience for the mathematics 10 textbook, for example, is high school students in grade 10. The audience for a job application letter is an employer or members of a search committee. The more you know about the background and needs of your audience, the better you can plan for your writing. You, as the writer, must understand the audience, the member (or members') level of background knowledge as well as any knowledge gaps, their specific interests in and expectations of the target text. Having or knowing this information helps you to plan the content, organization, language, and mode of the text you are about to produce so that it matches the intended audience. In the case of the mathematics 10 textbook, for example, the writer is an expert in mathematics, while the audience (that is, the students in grade 10) are less knowledgeable in the field. So the writer adjusts the level of content and its presentation in structure and language to a level which is only slightly above the background knowledge of the intended readers so that they can connect with, follow, and learn from the content of the textbook.*

Understanding the audience and their characteristic features is a challenging undertaking. Audience is, indeed, a fairly loose term and does not necessarily entail a homogeneous group of readers especially in their background knowledge and expectations. The audience for local *news published in the* Edmonton Metro*, for example, is all Edmontonians from any background in gender, race, culture, education, career, individual and collective experiences, and interest! In contrast, the audience for review articles published in* Nature Review Neuroscience *are experts in neuroscience. However, as a highly multidisciplinary field, neuroscience attracts experts from a variety of disciplines including various sub-disciplines in biology, discrete mathematics, computer science, and psychology. These experts bring with them specific background knowledge, expectations, and ways of understanding and responding to texts. Understanding the audience in this case entails being mindful of the potential readers of the article and their specificities.*

As you are probably already aware, writing assignments at the university level usually ask you to write for a dual audience: the primary, also called perceived audience and the secondary, or the stipulated, audience. The primary audience is the individual, or a group of individuals, whom you are conscious of in the immediate context of your writing and who will be reading your work and responding to it. For example, as a student in this writing course, you would perceive writing a research article introduction (in the last module) as just an assignment, which is meant to be read (and marked) by me as your instructor rather than, a research journal editor, reviewers, and reader. The secondary audience is an imaginary figure that is usually defined by the course instructor and included in the assignment description. For example, the secondary audience for a briefing note writing assignment is usually defined as a government official in the field. In planning your writing, it is important to be conscious of the dual audienceship and the challenges it might create for you.

Purpose- *All writing has a purpose. For example, while the purpose of a fairy tale is usually to entertain, the purpose of a mathematics textbook is to transfer mathematics knowledge to the students. The purpose of a job application letter is to allow the candidate to make an argument that s/he is qualified for the position and to persuade the readers to invite the* *candidate for an interview. Two points should be considered when identifying the purpose of a piece of writing. First, some genre texts may have more than one purpose. For example, a student creating a writing portfolio may have the dual purpose of fulfilling a course assignment as well as using it as a document in his/her job application to support the writer's claim that he/she can teach writing or has excellent communication skills. Second, that the purpose of writing is interconnected with other components of the rhetorical situation especially with audience and genre. For example, other than writing to fulfill course requirements, some typical purposes for student writing are to inform, express, persuade, call to action, entertain, and create new knowledge.*

Genre- *Simply defined, a genre is a kind of writing. A more elaborate definition is "[a] type of writing that is distinguished from other types of writing because of differences in content, organization, style and vocabulary choice, audience, purpose, and context of presentation (Robinson, 2008, p. 6)." Examples of genre are poems, novels, short stories, lab reports, essays, editorials, textbooks, conference presentations, research articles, sales promotion letters, job application letters. Genres are also defined as social actions (Miller, 1984); that is, they are produced and used to do things in social contexts. For example, a cover letter performs the social action of applying for a job. So, one way to identify a group of texts as belonging to a genre would be to identify the kind of social action which a text performs once produced and used. Note that the content, structure and language of texts grouped as a certain genre may show similarities as well as variations, which suggests that every rhetorical situation presents certain constraints as well as opportunities. It is thus critical that students, as writers, analyze and understand the elements and requirements of their own rhetorical situation early in their writing process. The next section presents some tasks to practice rhetorical situation analysis and to prepare the students for Module 1 writing assignment.*

Learning Tasks: Rhetorical situation analysis

Pre-task: *Modeling the rhetorical situation*

As a pre-task activity, let's analyse the rhetorical situation of a couple of non-academic texts such as a recipe, a shopping list, an email message, an ad, or even a joke.

Task 1. *Grouping genres as social actions*

Below is a list of some genres.

1. *Together with a partner, mark the genres with which you are familiar or that you have already produced.*

Literacy narrative	*Recommendation report*	*Annotated bibliography*	*Research article*	*Briefing notes*	*Job fact sheet*
Textbook	*Grant proposal*	*PhD dissertation*	*Infographic s*	*Case report*	*Book blurb*
Letter to the editor	*Research report*	*Policy analysis essay*	*Reprint requests*	*Short communication*	*Cover letter*
Book review	*Reflection essay*	*Conference abstract*	*A federal bill*	*Scholarly book*	*Lab reports*

1. Now identify the social actions which each genre performs. Group the genres based on their social action into the 3 main categories of school, research, and professional genres. Justify your choices by identifying the purpose, audience, and rhetorical context of each genre.

Genre category	*Exampl es*	*Purpo se*	*Audien ce*	*Modality (text/images)*
School Genres (assignments)				
Research dissemination Genres				
Professional communication Genres				

1. Which genres apply to your discipline? Do you know of any other kinds of writing in your discipline which are not included in the list?

1. How are the texts similar/different in terms of their audience, purpose, modality and the overall appearance? Why are they different? Apply ideas from the instruction material and the pre-task to figure out the similarities/differences. Explain your understanding in a few argumentative paragraphs in 250-300 words.

Task 2: *Grouping texts by their author/audience*

Together with a partner:

1. *Make a list of five texts that undergraduate students majoring in mathematics might be asked to read or write.*
2. *Now, make a second list of three texts that a graduate student might read or write.*

3. *As a third list, identify three texts that a mathematician in academia (a professor in mathematics) might read or write.*
4. *How are the texts similar/different in terms of their audience, purpose, modality and the overall appearance? Why are they different? Apply ideas from the instruction material and pre-task to figure out the similarities/differences and explain your understanding in 250-300 words.*

Post-task: *Discuss (compare/contrast) your answers to Task 1 & 2 as a class.*

Module 1 major assignment: ***Rhetorical situation analysis report***

Select a text from your recent readings or writings in your discipline. Read the text. Analyze the rhetorical situation of the text as follows: Identify the author, intended audience, social action, rhetorical purpose, medium and modalities of communication which together constitute the context of the text in its production process. Think critically about the features of the context which you analyzed. Describe the results of your analysis and explain possible reasons for the observed features as a rhetorical situation analysis report in 750 words.

The following tasks help to breakdown the assignment into concrete steps.

Task a. *Pick a text (or part of a text) with which you are familiar. Make sure you understand the content information of the text. Read through the text for a grasp of the topic, content, and what's going on in the text.*

Task b. *Craft a concept map of the rhetorical situation in which, to the best of your understanding, the text was generated. Your concept map should include information that* *answer who, what, why, where, when kinds of questions, that is your concept map informs us about the author, the audience, the subject, and the purpose of writing. For an example of a rhetorical situation concept map see https://images.app.goo.gl/eKvv4GeSgLERJ94E8). Use the items of the concept map, for example, the discipline, topic, author, audience, purpose, etc. and add as many notes and details about each item as you can. For example, the author, his/her credentials and affiliation, his/her role relationship with the intended audience, etc.*

Task c. *Now that you have some notes about the items of the rhetorical situation, step back and think critically about whether and how the specific details from each item interconnect with other items of the rhetorical situation and with the way the text has been written. As a final critical step, situate your analysis within the context of your discipline by answering the question of how all these features of the rhetorical situation relate to your understanding of your discipline and the ways things are done in your discipline of study.*

Task d. *Use your notes and critical analysis from Task 2 and 3 to draft your rhetorical situation analysis report. Organize your work as a report including introduction, body and conclusion sections. Add full citation information about the source text to the end of you report.*

Your audience: Instructor and other students in class

Your purpose: To demonstrate your practical skills for and understanding of rhetorical situation analysis

Genre of writing: Analytical report

Module 2: Genre Analysis

This module aims to introduce you to key concepts in genre theory in reading and writing in disciplines and help you develop analytical skills to identify the genre components, their functions and forms.. By the end of this module, you should be able to:

- *Identify the rhetorical purpose of sample academic genres*
- *Analyse a piece of academic genre into its rhetorical structure*
- *Analyse the lexico-grammatical features of a given genre*
- *Explain the rhetorical structure of a disciplinary genre*
- *Develop an understanding of how academic genres are written*
- *Develop critical awareness of why sample academic genres are written the way they do.*

Theoretical Concepts

Genre analysis is a way of examining samples of similar kinds of writing with the purpose of understanding how texts belonging to a category of writing are written. In particular, genre analysis examines the purposes of different textual (and visual) units, rhetorical conventions, and the style of writing that are common in a genre. we call these textual (and visual, if any,) units moves. Move analysis usually yields an outline for the organization of texts at both the section and paragraph level. Together these elements represent the move organization of a genre text. Note that genre analysis involves more than a description of the generic features of a text. It aims at heuristic and explanatory purposes. Accordingly, this module asks you to think critically about the perceived generic features through both exploring the rhetorical situation of each kind of writing in the context of your disciplines and talking to the expert members of the discourse community who use the genre. The approach thus encourages you to connect with the members of the discourse community in your discipline and learn about their perspectives, as gatekeepers, about disciplinary norms in writing. You are expected to *systematically describe the features of sample texts as you try to understand the rationale behind those features in your discipline, and you do this before you attempt to write samples of those genres for yourself. Through such individual/group genre analysis activities, you examine, identify, and procedurally develop explicit genre knowledge which you can then use to write your major assignment. It also helps you develop an awareness of the role rhetorical situations and disciplinary norms play in genre texts. As a result, you make more informed decisions in your choice of organization, form, and content in your writing.*

In analyzing the genre components of the sample texts in mathematics you may use the analytical items from the following list.

Audience	*Purpose*	*Organization*	*Language and Style (concision, word choice, formality, grammar, mechanics)*	*Modality*
Undergraduate students vs. graduate student vs. expert vs. public audience	*Create knowledge (Research article, conference poster; demonstrate learning (student assignment); request funding (grant proposal); receiving feedback*	*Macro organization (e.g., sectionings) vs. micro organization ('moves')*	*tense, voice, pronouns, abbreviations and acronyms, level of formality, punctuation, typographical*	*Text, graphics and visuals, hyperlinking, etc.*

	(first student draft); communicate with colleagues (email);teach or instruct (textbook); etc.		*features like using boldface characters, italicization, bullet and numbering, etc.*	

From among the concepts introduced in the above chart, audience and purpose belong to the rhetorical situation of writing genres. An important concept which remains for further unpacking is genre organization. In the following paragraphs, I expand on the concept of organization and the two levels of organization analysis.

Organization: *Having identified the audience and purpose of the intended communication, you need to decide on the organization or the shape of your message. However, note that the organization of a message is not detached from its audience and purpose. Indeed, when writing, you address audiences as members of the social group in a social situation. The members of a social group use shared forms of communication associated with those situations and adjusted to the purpose of their communication. Effective communication in social groups and disciplinary communities thus entails organizing your writing in ways which are recognizable by your audience as addressing the right social situation and fulfilling the purpose it aims to. So, by agreeing to write in a certain genre, you implicitly agree to structure your writing in certain ways which are prototypical of that genre and approved by the members of the discourse community which uses the genre.*

There are two levels of analysis of texts for their generic structure. One is the macro-organization of the text. The macro-organization of a text is usually outlined in terms of the *major sections/heading it includes. For example, the following outline represents a traditional macro-organization structure of research articles in science and some social science disciplines: Introduction- Methods- Results- Discussion- Conclusion. Recently, authors of research articles in some scientific disciplines, however, use an altered macro-organization in terms of the type and order of sections. Some 2019 RAs published in Nature Neuroscience, for example, represent a Main-Results-Discussion-Methods macro organization in their main body. Note that the alteration of organizational structure of a genre happens over time as disciplinary practitioners try to respond to perceived needs in the field. Regarding RAs in science, for example, with the proliferation of scientific findings, readers want to know the main results first and foremost; then they want the background to help them assess the value and validity of the results. It is thus important to focus on and think critically about the macro-organization of the academic texts which you approach in your discipline of study.*

A second level of analysis of the organization focuses on the micro rhetorical components, or the communicative purposes developed within paragraphs and sentences of each section. A well-known concept used in the genre analysis approach to identify and name micro rhetorical purposes is move. I already defined move in simple language. Here is a more technical definition of move in genre analysis:

> *A 'move' in genre analysis is a discoursal or rhetorical unit that performs a coherent communicative function in a written or spoken discourse. Although it has sometimes been aligned with a grammatical unit such as a sentence, utterance, or paragraph …, it is better seen as flexible in terms of its linguistic realization. At one extreme, it can be realized by a clause; at the other by several sentences. It is a functional, not a formal, unit. (Swales, 2004, pp. 228-229).*

As I explained, move analysis is usually reported as an outline or flowchart called move structure. A move structure usually informs readers about the more common - sometimes also the less common-moves, their possible recurrence, as well as their sequencing in the text. Each move serves an important communicative purpose and contributes to the global communicative purpose of the genre text. For example, the communicative purpose of the RA genre is usually to create new knowledge. To fulfill this purpose, the author should make a persuasive argument to establish his/her knowledge claim. The author thus organizes his/her argument into different sections, each serving a specific purpose and consists of a number of rhetorical moves. As with the sections of RA, the moves comprising each section may vary in RAs across disciplines.

Note that, as we shall see later in this chapter, both the macro and micro-organizations of RAs in disciplines may hint at important information about a discipline including the values held by its discourse community, its acceptable ways of doing research, constructing an argument, and creating new knowledge. Before we proceed to the analysis of the organization structure of genre texts, and as a preliminary analytical task, we start with practicing the rhetorical situation of some simple non-technical texts. We then practice analysis of the organizational structure of a conference call for paper (CFP) as an academic-professional genre.

Learning Tasks: Analysis of audience, purpose, and organization

Pre-task: *Macro vs. micro rhetorical organization analysis*

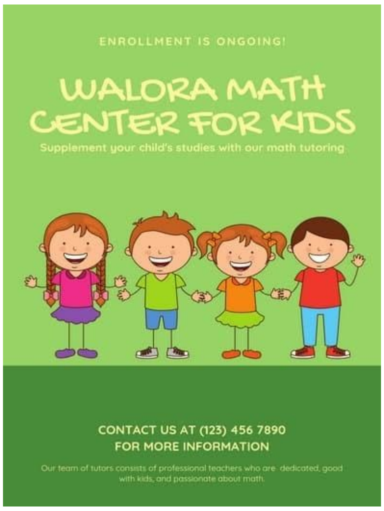
As a pre-task activity, let's focus on audience, purpose, and the macro (main sections) vs. micro (move) organization structure in the three non-technical tutorial posters presented in Figure C.1.

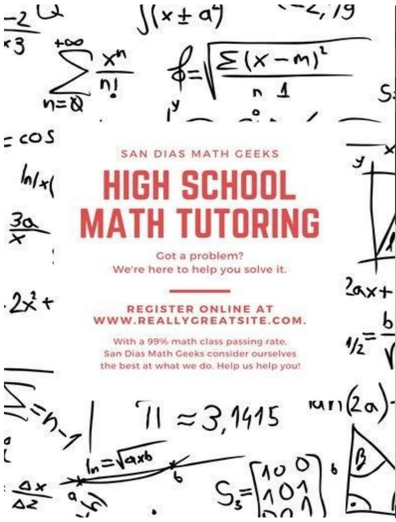
The following questions may help you in your rhetorical situation analysis:

- *What is the purpose of the posters?*
- *Who is most likely the audience for each poster?*
- *How is the organization of information similar/different across the three posters (compare/contrast)? What could be the potential reasons for perceived similarities and differences?*
- *What are the shared features (text, image, colour) of the three posters?*
- *Which textual/visual segments (that is, moves) directly contribute to each poster's purpose? Why?*
- *Which poster offers more moves? Why?*
- *Which poster does a better job of organizing the information? Why?*
- *Could the author/designer achieve his/her goal if the text was missing some of the identified moves? Why?*
- *To what extent would an ad poster achieve its purpose if it contains all of the identified moves? Why?*
- *What would you do differently, for example, add or drop, if you were the author/designer of these posters?*

It is important to note that:

- *these questions are meant to make you think about the rhetorical situation and structure of texts,*
- *an absolute 'yes' or 'no' answer is desirable;*
- *several right answers are possible as long as you persuasively argue for your answer.*





<u>Poster 1</u> <u>Poster 2</u> <u>Poster 3</u>

Figure C.1. The images of posters are reprinted with permission from <u>*Canva.com*</u>*; © 2018 Copyright Canva.*

Task 3: *Organizational analysis of a conference call for paper (CFP)*

Poster 4 (Figure C.2) reproduces a CFP poster. Together with a partner,

- *first decide about the topic, purpose, and intended audience for the CFP poster.*
- *then analyze the macro-organization (major sections of information) of the CFP.*
- *Once you have an outline of the macro-organization, try to analyze each section for its micro rhetorical structure, that is, moves. In identifying the move structure, try to focus on both the verbal and visual information in each section.*
- *develop a move structure outline which illustrates the macro and micro rhetorical structures of the CFP.*

Figure C.2. Poster 4: An example of conference call for paper poster. Reprinted with permission from the conference organizing committee; © University of Manchester)

***Post-task**: Discuss your answers to Task 3 as a class.*

***Task 4**: Organization & Move analysis of a CPP*

Task 4 aims the following learning objectives:

- *Analyze an academic/research genre*
- *Highlight the audience, purpose and organization of CPP*
- *Increase awareness about variations in genre organization triggered by components of the rhetorical situation across the 3 types of posters, namely ad, call for paper, and conference presentation posters*
- *Develop transferable genre knowledge and skills*

Poster 5 (Figure C.3) and Poster 6 (Figure C.4) below reproduce two undergraduate conference presentation posters (CPP). Together with a partner, choose one of the CPPs:

- *Decide on the intended audience and purpose of the CPP.*
    - *In identifying the audience and purpose, use your background knowledge, lecture and class discussions. Also, consider the levels of detail, formality, conciseness, and word choice*
- *Make an outline of the macro-organization (major sections) of the CPP.*
- *Then, analyze each section for its micro rhetorical structure, that is, moves. Assign move labels that reveal the actions taken by the writers.*
- *Develop a move structure flowchart that illustrates the macro and micro rhetorical structures of the CPP.*
- *Draft 1-2 paragraphs (250 words) in which you reflect on (explain) the links among the rhetorical structure of this particular sample CPP and its purpose and intended audience.*

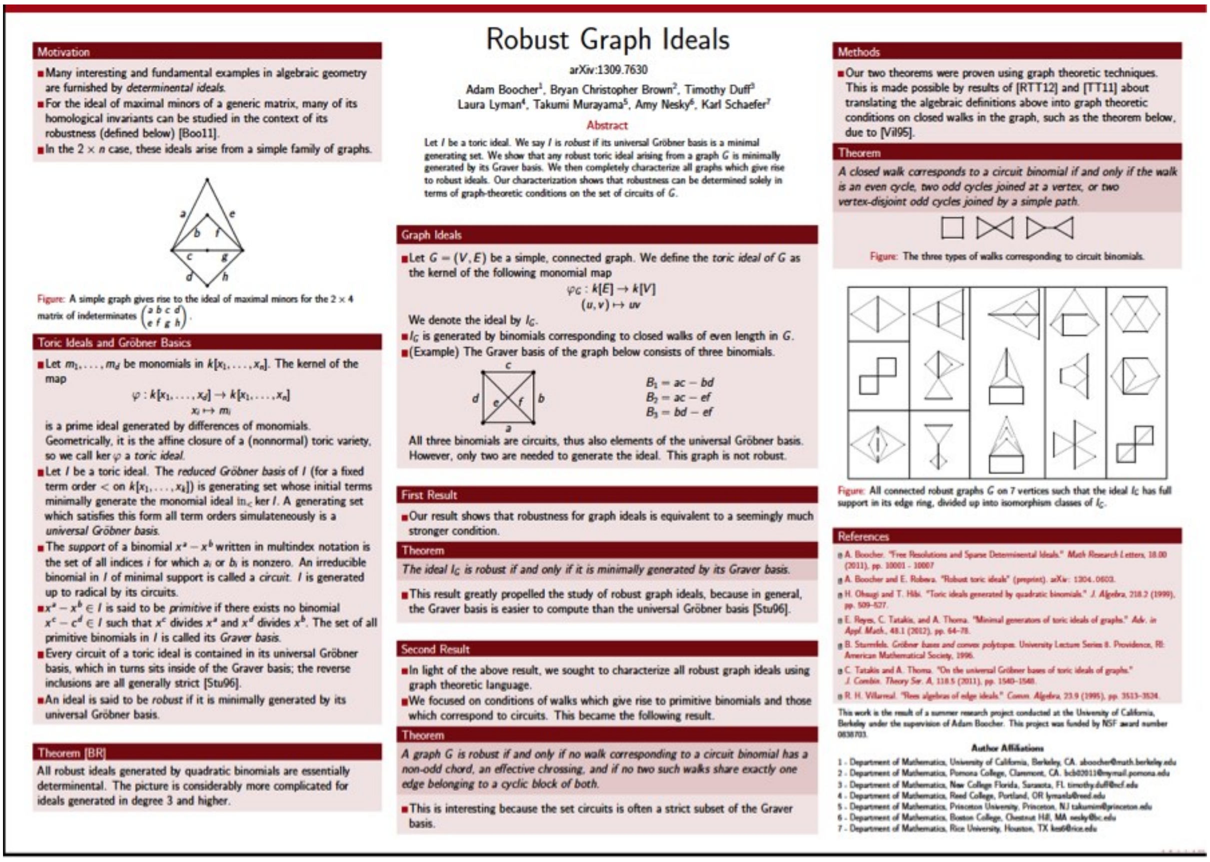
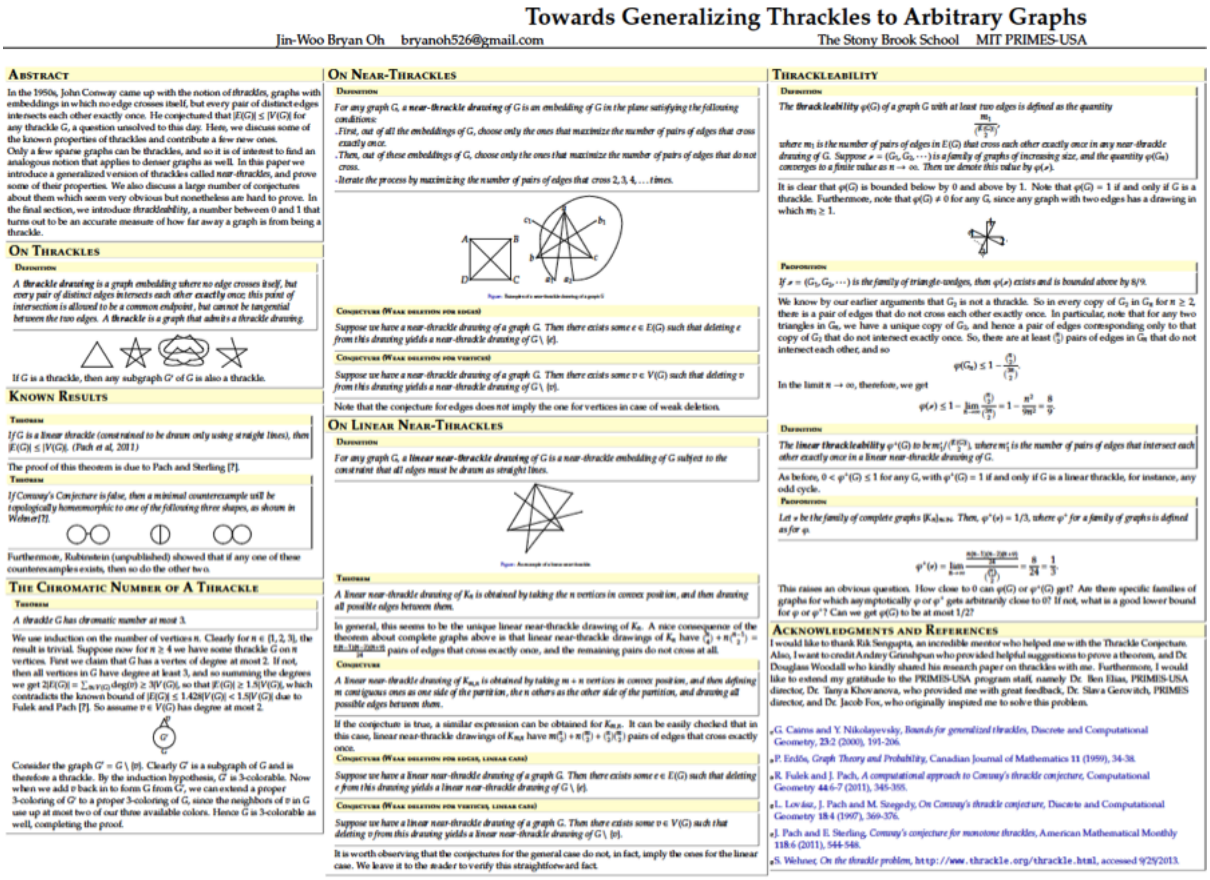


Figure C.3: Poster 5: Boocher et al. (2014). Reprinted with permission from Media Relations and Marketing, Mathematical Association of America (Retrieved June 6 2019 from https://www.maa.org/sites/default/files/DuffSchaeffer_poster2014.pdf)



Towards Generalizing Thrackles to Arbitrary Graphs

Jin-Woo Bryan Oh bryanoh526@gmail.com The Stony Brook School MIT PRIMES-USA

ABSTRACT

ON THRACKLES

KNOWN RESULTS

THE CHROMATIC NUMBER OF A THRACKLE

ON NEAR-THRACKLES

ON LINEAR NEAR-THRACKLES

THRACKLEABILITY

ACKNOWLEDGMENTS AND REFERENCES

Figure C.4. Poster 6: Oh (2014). Reprinted with permission from Media Relations and Marketing, Mathematical Association of America (Retrieved June 6, 2019 from https://www.maa.org/sites/default/files/DuffSchaeffer_poster2014.pdf)

> ***Post-task**: Discuss their answers to Task 3 and 4 as a class.*

In this module, we practiced analysis of the rhetorical situation and organizational structure of three different non-technical, semi-technical, and technical genre texts. We saw how the authors organize their texts based on the rhetorical situation, including the purpose of communication, their intended audience and their needs. We also saw how texts belonging to the same genre have both similarities and differences in their organizational structure. These findings, however, are not yet generalizable to similar genres. We need to analyze more *sample texts from the same genre before developing a theory about the organizational structure of the given genre.*

The next activity is a major assignment which makes you practice analysis of some select academic genres and draft your work as an analytic report. The assignment involves 5 different genre samples (e.g., a student research proposal, an infographic, a book blurb, a short communication, a book review, a student problem-solution essay, etc).

Module 2 major writing assignment: Analytic report

Assignment instruction:

- *Choose two texts from among the genre samples provided by the instructor.*
- *Analyze the audience, purpose, and genre organization pertaining to each text.*
- *Write a short report (750 words) in which you compare/contrast the two sample texts in terms of the rhetorical situation and organizational features for each genre.*

Hint 1: *In analyzing the audience, you may also think about the role relationship between the author and the target reader; do they share the same background knowledge about the topic and content of the text?*

Hint 2: *In analyzing the purpose of student genres, you want to locate the specific rhetorical purposes of the kind of writing beyond the immediate purpose of writing as handing in an assignment. For example, whether the purpose of writing is to persuade readers to read a book or to accept a knowledge claim.*

Hint 3*: In analyzing the organization and layout of each text, you may want to examine whether the author uses headings as signposts to inform about the content of different parts of his/her text. If so, whether s/he uses generic section headings or content section headings and how it affects your reading.*

Optional component: *You have the option to compare/contrast the sample texts for their visuals. For example, comment on which text uses more visuals, how the visuals are similar/different across the two texts, what purposes do they serve in each text, etc.*

Drafting note*: Before you start drafting your report, consider* **your rhetorical situation** *as a writer. Think about your audience and their expectations/needs. You must also think about your purpose for writing, that is, the internal purpose of your text vs. the assignment* *purpose. Be aware that the choice of audience and purpose will influence the content, formality level, style concision, and word choices in your writing.*

***Formatting note**: Organize your draft as a report including a number of sections. You may decide to choose audience, purpose, organization, and visuals as the heading for your major sections. Try to explain how did you decide on the audience, purpose, and the organization structure for each text; what clues you used in your analysis.*

Once you are done with your main sections, go back and draft a short introduction section in which you briefly introduce your work and your purpose of writing. You also draft a short conclusion section in which you summarize the highlights of your analysis and (re-)emphasize its significance (take home message).

Audience: The instructor and other students in the class

Purpose: To categorize disciplinary genres and describe how they compare and contrast

Genre: Short analytical report (750 words)

Reflection journal 1

The purpose of a reflection journal is to engage with the concepts and activities of the module so as to reach deep insights and awareness of genres and writing as a learning tool. You are asked to compare/contrast your gain from the first two modules with their previous reading and writing situations.

*Select **one** of the following reflection tasks (a, b, c, or d) and write a thorough response to the questions under the selected task as a journal entry (400 words).*

a. Reflect on the idea of audience in [...] writing.

■ *What did you know about audience before participating in this lesson? Where did you learn it?*

■ *What audiences have you written for in the past? What types of writing did you do for these audiences (e.g., essays, research reports, posters, journal articles)?*

■ *Have you written for an expert audience before? What challenges do you think you'll encounter writing for an expert audience?*

b. Reflect on the relationship between reading and writing [in mathematics].

■ *How might your writing improve by reading authentic examples of [...] writing (e.g., journal articles, posters, research proposals)?*

■ *How might your reading improve by learning to write in professional[/experts] Genres [of mathematics]?*

■ *What aspects of your reading and writing do you hope will improve? Why?*

c. Reflect on the value of genre analysis activities and their role in the read-analyze-write approach to writing.

■ *What are you likely to learn from engaging in genre analysis activities?*

■ *How might your reading and writing abilities improve as a result of genre analysis?*

■ *Which focal points of genre analysis (audience and purpose, organization, [etc.] do you think will be most useful to you when you attempt to write for an expert audience? Why?*

d. Reflect on the value of move structures.

■ *What is the value of depicting the fine organization of [mathematics] genres through move structures?*

■ *How might move structures help you with your writing?*

(From Robinson, 2008, p. 30; bold facing, italicizing and some slight content and formatting revisions are mine.)

In this module, we practiced genre analysis of some non-technical and academic texts. We practiced which aspects of the texts and their contexts to focus on and what kind of questions to use when approaching the texts to maximize our understanding of the rhetorical actions of genres and their texts. Through these practices, you have started developing skills and genre knowledge, which you continue to use and further develop in the next module. As you may have noticed, modules 1-2 were introductory modules which aimed to prepare you for module 3, which is the major module of this course. Module 3 focuses on the RA genre and prepares you to start drafting the results of your research as a research article manuscript.

Module 3: Research Article Genre

This module provides opportunity for hands-on practice on writing a research manuscript. The goal is to promote learning how to write a research article by having you practice various analytical tasks. The sections in this module involve both accumulating genre knowledge (both the organization and argumentation) and applying your genre knowledge to construct an argument for new knowledge and structure your draft as a research article. You thus should be able to develop a priori as well as a posteriori genre knowledge which you can then put forward to explore other genres in the future.

By completing the five parts of this module, you should be able to:

- *Differentiate between types of journal publications in mathematics.*
- *Demonstrate understanding of the rhetorical situation and disciplinary context of research articles by explaining why mathematicians write research articles the way they do and how new knowledge is created in the discipline.*
- *Demonstrate understanding of some organizational and formal features of research articles in their discipline.*

- *Apply genre awareness and skills to craft part of a genre, namely an Introduction section for your research article manuscript.*
- *Give feedback on peer drafts.*

Theoretical Concepts

This module presents both a bird's eye view and a worm's eye view of the research article (RA) genre in mathematics. It expands on the concepts of audience and purpose of research articles. It also introduces different genres of journal publications.

The module consists of five parts. Part 1 provides some background knowledge about the audience for RA genre and helps you develop knowledge of the audience through practicing analytical tasks. Part 2 familiarizes you with different types of journal publications and their purposes. Part 3 unpacks some common stylistic features of RA texts, an awareness of which is a prerequisite of effective writing and getting published in the journals. Part 4 provides hands-on practice to identify the macro-organization of RAs in their field of study and develop awareness about it. The last part, part 5, offers practices to analyze and draft some sections of RAs, namely Introduction and complementary Introduction sections. Through these practices you not only understand the micro-organization, namely rhetorical moves, of some sections in RAs, but also develop move analysis skills and use them to understand other sections of RAs. The module thus gives you a chance to use your knowledge and skills to perform in the genre.

Part 1 Analysis of Rhetorical Situation: Audience

Research articles are usually written to be read by the expert, or near expert, members in the disciplinary discourse community. For example, the audience for a research article in combinatorics is the group of mathematicians, from novice to expert, who study or are interested in combinatorics. Moreover, depending on the topic and pure vs. applied orientation of a research article, it might attract readers from some client disciplines. For *example, a research article that disseminates research in combinatorics and proposes applications in communication systems or computer science might attract readers from these client disciplines. As we shall see, such extended scope in the audience will influence the broad and fine rhetorical organization as well as the argumentation in such research articles.*

Here, a research article is defined as a kind of text which reports research findings in peer-reviewed, also called refereed, academic journals. These articles usually go through meticulous review processes by 2 or more expert members in the discourse community of the discipline. The review process in most top-tier academic journals takes about 6 months or over. The information about the review process of an academic journal is usually available in journal homepages under headings such as 'Review Process' or 'Instructions to the Authors'. Having reviewed a manuscript, the reviewers provide critical comments and feedback which decide whether the manuscript is publishable in the target journal and if so, what revisions are required. The comments usually fall into two categories: the originality and quality of the research being reported as well as the argumentation/ writing quality of the manuscript.

As reviewers are among the first and key readers of a manuscript, it is important that students as novice authors have ongoing cognitive interaction with the tentative reviewers while drafting and revising their manuscripts. As well, it is important to have students practice giving and receiving feedback on sample texts and peer drafts in this module both to improve their draft and to prepare them for giving and/or responding to real-world feedback in their future journal submissions. The following tasks thus make students practice interacting with intended readers and giving and receiving feedback.

Learning Tasks for Part 1: Writing for an Expert Audience

Pre-task: *The following brainstorming questions will engage you in focusing on the concept of expert audience:*

- *What kind of texts do you read during your leisure time? (Depending on the topic of the texts which students name, the instructor might name expert texts on the same topic). Why don't you read X (X represents an expert texts) which is about the same topic?*
- *Do you have experience reading a journal article in fields other than mathematics? If no, what has made you disinterested in RAs from other disciplines? If yes, have you found reading those articles challenging? What made the experience challenging?*
- *Imagine you are given a textbook and a research article in mathematics. Which text might be more challenging to read and understand? Why?*

Discuss your answers as a class.

Task 5: *Together with two partners, browse through three research articles from three different discrete mathematics journals (one article from each journal). Choose from among the following journals: Journal of Discrete Mathematics, Journal of Applied Discrete Mathematics, SIAM Journal of Discrete Mathematics, and Journal of Combinatorial Optimization. Perform these tasks:*

1. *Find two or three sentences that are easy to read and understand in each section of the article (abstract, Introduction, Experimental, Results, and conclusion).*
2. *Next, find two or three sentences that are difficult to understand in each section.*

3. *What differences, if any, do you notice in the readability of these sections? Which sections are the easiest to read and understand? Why do you think some sections are easier to understand than others? What does make them challenging?*
4. *Read one of your three articles carefully. What makes the authors sound like experts? Jot down at least 10 examples (sentences) of expert-like writing.*

** By readability, I mean how easy the text is to read. To get an impression about the readability of a text, you want to focus on the number of words per sentence, complexity of syntax, voice and style, unfamiliar abbreviations which are used by the author but not introduced in full words, unfamiliar jargon and technical language, and any lexico-grammatical features of a text which makes reading and understanding the text easy or otherwise challenging. Note that readability of a text varies across individuals. So it is very possible that you come up with different answers than your partners.*

***Post-task**: Each group reports their answers to the class.*

Part 2 Analysis of Rhetorical Situation: Purpose and Genre

Broadly speaking, the purpose of most research articles appearing in academic journals is to create new knowledge. To accomplish this goal, research articles propose a knowledge claim(s) and persuasively argue to establish their claim(s) as new knowledge. Moreover, authors of research articles organize their argument in ways which are compatible with, and representative of, disciplinary epistemology. As such, the rhetorical organization of a research article provides important information about its purpose and how knowledge is conventionally defined by the discipline and its discourse community.

Both the purpose and organization of a research article are interconnected with the type of research article. A well-known disciplinary typification is pure vs. applied mathematics research articles. The distinction usually originates from the orientation of the research topic/question on a theoretical-applied continuum. Generally speaking, pure mathematics is argued to answer questions which originate from within the discipline of mathematics while applied mathematics is claimed to answer questions cast from the real world, that is, questions originating from outside the discipline. Note that the separation between pure and applied mathematics is an ongoing controversy with some mathematicians arguing for the separation and others counterarguing the opinion (See Rabajante, 2013, for example). To temporarily settle the dispute in this section, let's consider the research articles in mathematics on a pure-applied continuum; that is to say, while some articles may be absolutely pure or applied, others might be more pure than applied and vice versa. In the following pages, I have reproduced content from articles which lean towards the pure end of the continuum as well as those which fall on the applied end.

A second categorization for articles is discernible in academic journals. Some notable members of this genre category are original research article, note, perspective, review article, and book review. The criteria for deciding whether an article belongs to a certain category is the research method used in the article, the rhetorical purpose, and sometimes the length of the manuscript. Information about research articles is usually provided by academic journals. For example, the following information about the types of articles is available through the Guide for authors link in the Discrete Mathematics journal homepage.

Contribution

Full length original research article of more than 7 journal pages, or about 10 full pages of a typical manuscript.

Note

A short self-contained original research article of at most 7 journal pages, or about 10 full pages, of a typical manuscript.

Corrigendum

Concise correction to article by the same author published in the journal. The full publication data of the original paper should be included. ScienceDirect will provide a link to the Corrigendum in the original article.

Perspectives

Expository articles of broad interest that cast a new light on well-known or insufficiently-known topics, including survey papers and papers that present the authors' unifying points of view on problems, trends, or methods. Perspectives papers are intended to be accessible to a broad cross-section of the readership.

(From Guide for authors, Discrete Mathematics journal (2019). Elsevier.

As the above information suggests, journal publications come in different types depending on their purposes and shaped by journal guidelines. The following activities help you to put your theoretical knowledge of RA genre into practice, develop your genre analysis skills further and get a chance to share your ideas with peers and learn about their ideas.

Learning Tasks for Part 2: Identifying types of journal articles

Pre-task:

A. *Read the following questions and formulate your answers as notes:*
    1. *Do you affiliate yourself with pure or applied mathematics? why?*
    2. *What kind of journal articles you have read so far?*

3. *Is there a reason for choosing a specific kind of article to read?*
4. *What kind of information do you look for in each article type?*
5. *What do you find specific or interesting about each kind of article you read?*

B. Discuss your answers as a class.

Task 6*: This is a group activity (3 students)*

Search for the Journal of Discrete Mathematics and Journal of Applied Discrete Mathematics. Browse through several issues of the two journals by examining their Table of Contents.

1. *Find 3 different kinds of published item. Skim through enough of each item to decide whether it deals with a pure or an applied problem as well as to decide whether it is a research article, a review article, a note, or other type of journal publication.*
2. *For each item, write down its title, the name of the journal, type of article, and the method, if any, used to investigate the topic.*
3. *Write a two-paragraph description (200- 250 words) about the specific purpose, the audience (interdisciplinary vs. intradisciplinary), the investigation method, the type of article, and the macro organization of each article. Briefly explain how you see these features as being connected.*

Post-task: Groups report their answers to the class.

Part 3 Stylistic features of research writing in mathematics

Concision and Brevity *Mathematicians writing for mathematicians value concision and brevity (Bullock and Millman, 1992). As Bullock and Millman (1992) remark, among the rhetorical principles valued by mathematicians are: "brief, concise, and low on elaboration or repetition" (P. 336). It has also been shown that "the more advanced the mathematics, the more concise the writing" (Bullock and Millman, 1992, P. 335). So, it is not an exaggeration to claim that one of the skills which differentiates expert mathematicians from non-experts in the field is the ability to phrase mathematical content in concise sentences.*

This section focuses on tasks that aim to raise your awareness of concision in writing in academic and research mathematics--why it is essential and how to do it. You will soon recognize concision as a key requirement of journals and some genres in the discipline. The awareness-raising will be attained through hands-on practice in text-analysis and revision of sample sentences and short texts that model the target writing.

A glance at samples of the various types of mathematical texts introduced in this chapter will show you that both their authors and readers prefer crisp sentences and avoid wordy writing. Indeed, precedent exists in mathematics for omitting information that readers can supply themselves (Hardy, 1945; Gonzales, n.d.).

Some strategies for concise writing are:

- *Eliminate unnecessary words*
- *Reduce wordy phrases*
- *Simplify tangled sentences*
- *Use positive constructions*
- *Simplify sentence structure*
- *Using nominalizations*

In the following example, a wordy sentence and a concise version of it is presented. The example also offers a brief note about the revision.

Example: *Wordy: Bruce and Smith (2017) in a paper published in Discrete Mathematics solved the problem and presented an example which is contrary to Theorem number 3 in Baker (2015).* *Concise: Bruce and Smith (2017) proved a counterexample to Baker's (2015) Theorem 3.* *Note: It is clear by the in-text citation that the work has been published and that the publication information is available in the Reference list which generally appears at the end of the article. It is also obvious that examples are used to solve a mathematical problem. Moreover, anybody familiar with mathematical jargon knows that examples which disprove existing results are referred to as counterexamples.*

The following section offers more examples of wordy sentences and their concise revisions. First, you will read sample sentences revised for concision by writing experts. You will then practice revising sample wordy sentences for concision and discuss your experience as a class. You will also revise a piece of your own work and exchange feedback with peers.

Pre-task: *Together with a partner skim through examples from Faigley, Graves, & Graves (2018, pp. 348-353). Pick 10 example wordy sentences and their revised partners.* 1. *Count the number of words in each paired example sentence.* 2. *What is the difference in wordcount between the sentences in each set?* 3. *Is the concise version clear enough to comprehend compared to the wordy version in each set of examples?*

4. *Is there any important information in the original sentences which is missing in the revised sentences?*

Task 7: *Together with a partner, read the following sentences. Identify the parts that you consider wordy. Re-write those parts to shorten them. Remember: Never sacrifice clarity for concision.*

1. *Matrix games belonging to this class, as a matter of fact, are observed to be a complication of statistical games which we reviewed earlier in this paper in Subsection 3.2.*
2. *Work has been done by researchers to enumerate the many different vertex triangulations belonging to this kind of a product.*
3. *A vast number of research studies has been devoted to the problem of finding upper and lower bounds for this number, but they are not devoted to determining it or to characterizing dominating sets.*
4. *As reported in the research literature, the strong metric dimension problem is a NP-hard decision problem in general case.*
5. *A survey paper that has been published in Discrete Mathematics [12] reviews a number of results on the strong metric dimension of Cartesian graphs up to 2015.*
6. *There is a satisfactory approach according to which we need to assume that the mentioned particles exist in a stochastic flow.*

7. *Due to the fact that the cluster existence time is dependent on how fast the distance between two particles grows, the predictability limit is known to be the reciprocal of the TLE.*

Post-task*: Report (examples of) your group answers to the class. Describe the strategies you used and receive feedback about the efficiency of your revision from the instructor and the rest of the class.*

Criteria for efficiency: clarity of the revised sentence, preservation of the main idea from the original, and the number of words cut.

Below are possible solutions to Task 7:

1. *Matrix games of this class, in fact, are a complication of statistical games reviewed in Subsection 3.2.*
2. *Research has enumerated the many vertex triangulations of this product.*
3. *Many research examine finding upper and lower bounds for this number, but few studies determine it or characterize dominating sets.*
4. *The strong metric dimension problem is NP-hard in general case.*

5. *A survey paper [12] reviews results on the strong metric dimension of Cartesian graphs up to 2015.*
6. *A satisfactory approach is to assume that the particles exist in a stochastic flow.*

7. *Because the cluster existence time depends on how fast the distance between two particles grows, the predictability limit is the reciprocal of the TLE.*

Task 8 (Individual activity): Revising your writing for concision

Apply the strategies in this section to your own writing. Re-read your analytical report from the Introduction module. Using a marker, highlight the sentences, words or phrases that could be revised or removed to make your writing more concise. Be careful, though, not to sacrifice important information or the meaning for conciseness.

Task 9 (Individual activity):Peer feedback activity

Swap your revised draft from Task 8 (individual activity) with the student next to you and ask for concision feedback.

Once you have received peer comments on your draft, go through the comments; put a checkmark by the comments/revisions you agree with. Negotiate the comments that you disagree with with the peer.

Nominalizations *A dominant stylistic strategy in writing mathematics, science, and technology is to use nominalization. Nominalization refers to the nouns created from other parts of speech, especially verbs and adjectives, through the addition of suffixes. Some well-known nominalizing suffixes include -tion, -sion, -ence, -ment, and -sis. Consider the following example:*

*Without nominalization: One of the crowning results that microeconomic theory **has achieved** during the past century is that it **has formally treated** the issues that were pertaining to social welfare (26 words).*

*With nominalization: "One of the crowning **achievements** of microeconomic theory during the past century is the formal **treatment** of issues pertaining to social welfare." (22 words) (From: An Algorithmic View of Voting by Fagin et al., 2016; italisization, underlining and boldface added.)*

As the numbers in brackets suggest, by using two nominalizations in the second sentence the authors have cut four words from the sentence length. Note also that the authors have cut 2 words, namely 'that were' from the qualifying clause that describes 'issues'.

There are other motives in using nominalizations than improving concision. A main reason for the popularity of nominalization in science and mathematics is that it helps omit unimportant information such as the actor (when it is irrelevant) and instead foregrounds more important information such as a scientific/mathematical action, process or event. Using nominalizations also helps to create variety in your writing by making it unnecessary to repeatedly use the same verb. Below are some common nominalizations used in mathematical discourse.

definition	*assumption*	*coordination*	*configuration*	*arrangement*
subtraction	*multiplication*	*addition*	*modification*	*analysis*
construction	*solution*	*contradiction*	*replacement*	*division*
computation	*combination*	*replacement*	*permutation*	*deletion*
dependence	*induction*	*deduction*	*supposition*	*observation*

generalization	*extension*	*creation*	*operation*	*demonstration*
calculation	*relation*	*presentation*	*estimation*	*collection*
characterization	*decision*	*application*	*independence*	*completion*
concentration	*determination*	*equation*	*preference*	*optimization*

Task 10: *The following sentences are from research articles in mathematics. Identify the nominalizations used in each sentence. Use examples from the above list of nominalizations as a reference.*

1. *Our classification is based on the more general problem of rank aggregation in which, beyond electing a winner, we also seek to compute an aggregate ranking of all the candidates; moreover, our classification is offered from a computational perspective based on whether or not the voting method generalizes to an aggregation algorithm guaranteed to produce solutions that are near optimal in minimizing the distance of the aggregate ranking to the voters' rankings [...]. (From Fagin et al., 2016)*
2. *This paper originated from the following Lagrangian prediction problem: to evaluate the position of a current-following particle in an ill-known flow, given its initial position and observations of several other particles released at approximately the same time. (From Piterbarg, 2001)*
3. *By replacing Fi in M by these two copies of Y , we get a -tiling in H larger than M , a contradiction. (From Han, 2018)*

4. *However, Brandt [4] proved that the existence of a Hamiltonian cycle is not related with the relationship between the density of a graph and the distribution of cycle lengths.(From Chen et al., 2018)*
5. *Thus the above approach results in violation of the two-to-one correspondence that requires an agreement between the one-particle motion statistics obtained from the one- and two-particle models, respectively. It is important to note that the well mixed condition is not to blame for this trouble, but rather the Gaussian conjecture for the two-point Eulerian distribution. (From Piterbarg, 2001)*

Post task: *Discuss your answers to task 10 as a class.*

Part 4 Macro organization of journal articles

In Part 2, we learned that texts from the same genre have similarities in both their macro- and micro-organization. In this section we learn about similarities in the organization of journal articles in mathematics. We will see how RAs in mathematics are similar and/or different within and across the disciplines in terms of their macro-organization.

The IMRD structure*Journal articles in most disciplines including science, engineering, and social science streams are generally organized into the following 4 sections:* ***I****ntroduction,* ***M****ethod,* ***R****esults,* ***D****iscussion, known as the IMRD structure. However, some journals or authors opt for slightly different section titles but they still have similar purposes to the*

IMRD sections. For example, some use Background instead of Introduction or omit a title for the opening section, presuming that the experienced reader can infer the title.

Each section within a research article aims at a certain rhetorical/communicative purpose. For example, the introduction section introduces the topic of research (what), highlights its importance (why), gives an overview of existing research on the topic (what, who, where, when of existing information) and introduces the research reported in the article (how, what of new information). The method section describes the design and research procedure and argues for their validity. The results section reports the data obtained through research, and the discussion section explains what those results mean from a disciplinary perspective and how they should be treated. In addition to these sections, most journal articles in science and technology include a conclusion section in which authors re-emphasize their main claim, suggest what practical or conceptual implications their results might have in broader situations, and what still remains to be done about the topic. As it can be seen from this simple description of the IMRD structure, the different sections of a research article together perform a coherent rhetorical function, that is to establish a knowledge claim through constructing a persuasive argument. The argument aims to convince the audience of the worthiness of the research topic, the validity of the research method, and the reliability of the findings and the main claims drawn from findings.

Despite the traditional popularity of the IMRD structure in most science, social science and technology-driven disciplines, a cursory glance at journal articles in various disciplines suggests that: 1) not all disciplines follow an IMRD structure in their research article organization, 2) some disciplines merge some sections, for example, they combine Results and Discussion and split the content of the Introduction section into two or three sections before the Method or Results section, 3) variations in the number and order of sections in the research articles appear even within the same discipline depending on the topic, journal *guidelines, and individual author preferences. For example, research articles in English literature and humanities often omit methods or results sections, while research articles in chemistry, biology, and education usually include method and results sections, and research articles in credible science journals such as Nature present results first and the method second.*

In mathematics, depending on whether your problem statement arises from within the discipline (pure mathematics) or is triggered by a real world problem and/or client disciplines such as engineering (applied mathematics), and depending on the target journal, you might opt for different macro organizations.

Note that journal articles might use content section headings, rather than generic headings to characterize their macro-organization. Whether they choose generic or content headings, it is important to note that the macro-organization and section headings respond to the readers' need by informing about the section content, that is they provide a guide to the reader, and hence support targeted reading. This means that readers do not need to read material in the order that it has been presented. Rather they can choose which section to read depending on the type of information they seek. This is how some expert members in disciplines read journal articles; they read selectively rather than linearly. A second related note is that having a sequence of sections, whether IMRD or a different sequence, does not indicate the order in which they have been drafted. For example, in most experimental fields in science, authors draft their method section first and the introduction section last.

Macro-organization of journal articles in mathematics *There are two macro-organizations for journal articles in mathematics suggested in recent research. The first has been suggested by Graves et al. (2013) for research articles in discrete mathematics. According to Graves et al. (2013) research articles in the field represent the following macro-organization sectioning:*

- *Introduction (GSH)*
- *Complementary Introductions (CSH)*
- *Results (both GSH and CSH)*
- *Conclusion (both GSH and CSH)*

*(**GSH**: Generic Section Heading; **CSH**: Content Section Heading)*

Graves et al. (2013) suggest that journal articles in discrete mathematics include neither a Methods section nor a Discussion section. Rather, clarifications about methods and discussions over results both appear in the Results sections.

Here is a brief overview of each section: The Introduction section of a research article sets the scene for the argument for new knowledge claim. It identifies the research topic, highlights its importance, defines the concepts, provides an overview of existing research and informs about the findings to be presented. A complementary introduction section might provide a more substantial review of existing research and/or mathematical definitions. A Results section presents authors' arguments, that is, proofs, for mathematical knowledge claims, that is, theorems, lemmas, propositions, conjectures, claims, etc. The section may also include short discussions about results. Three features are notable about the Results sections in mathematics, as shown in Graves et al. (2013):

Use of Content section headings *Using content section headings is a typical organization strategy in mathematics journal articles. A content section heading signals the main purpose of the section, its specific findings, or the variables which are core to the result. As such, a content section heading contributes to highlighting the key ideas which the authors consider critical and which the readers might find informative as well as fascinating about the section. It is also reasonable to assume that using content section headings helps to better establish the relationship between different categories of results in a journal article.*

Multiplicity of Results sections *Mathematics journal articles usually include a number of Results sections. Most research mathematicians opt for presenting their major findings, the preliminaries, the related proof, and discussion, as different Results sections. Variations in the number of Results sections mostly correlate with the problem being solved in the paper, the parameters of the problem and the refining effect the solution may have on the previous results, that is, perhaps the results obtained refine previous results in the literature, so the refinements are announced as further results in addition to the main result, either in an independent Results section or as complementary results in the same section.*

Multiplicity of labeled results within a single section *Another outstanding feature of Results sections in mathematics journal articles is the presentation of chains of results within a single section. These are immediately discernible from the bold face labels which are used for every statement of result, for example, a theorem, a lemma, or other kinds of result statements. The labels not only signal the appearance of rhetorical Moves that introduce result statements but also help the reader to differentiate between the results for their specific rhetorical purpose and their contribution to the overall knowledge production procedure.*

Finally, a research article may include a Conclusion section which recaps the argument for new knowledge, evaluates results and suggests applications/further research. The macro-organization of RAs in mathematics will be focused on in fuller detail in the following tasks.

A second model has been suggested by Kuteeva and McGrath (2015) for macro-organization of journal articles in pure mathematics. The model suggests that journal articles in the field represent the following macro-organization:

- *Introduction*
- *Results*
- *Concluding remarks (optional)*

Learning Tasks: Identifying macro-organization of mathematics journal articles

Pre-task: *Think about a couple of journal articles that you have read recently.*

1. *What kind of macro-organization structure have you noticed in the articles?*
2. *What sections were more common?*
3. *Did the articles use generic section headings or content section headings?*
4. *What kind of information did each section present?*

Task 11: *In groups of 3, browse through the following 4 journals.*

1. *Journal of Discrete mathematics*
2. *Siam Journal of Discrete Mathematics*
3. *Journal of Optimization Theory and Applications*
4. *Proceedings of the American Mathematical Society*

(If the above journals are not related to your area of research, you can also choose peer-reviewed research journals from an area in mathematics which best fits your research topic.)

Select a research article from the most recent issues of each journal. Scan the articles for their macro-organization and fill in the following chart for each journal article.

Type of articles (pure/applied):	*Article 1:* *Article 2:* *Article 3:* *Article 4:*

Identify the macro-organization of each article (How many main sections are discernible in each article?)	*Article 1:* *Article 2:* *Article 3:* *Article 4:*
How many results sections does the article include?	*Article 1:* *Article 2:* *Article 3:* *Article 4:*
Scan the longest Results section from each article. How many main and subsidiary results labels are discernible in each Results section? (Examples of such labels include theorem, lemma, proposition,	*Article 1:* *Article 2:* *Article 3:* *Article 4:*

corollary algorithm, etc.)	
What variations in sectional divisions are noticeable? Does the article use the IMRD structure?	*Article 1:* *Article 2:* *Article 3:* *Article 4:*
Does the article use generic or content section headings?	*Article 1:* *Article 2:* *Article 3:* *Article 4:*
Extract a content outline for the research article using its macro-organization	*Article 1:* *Article 2:* *Article 3:* *Article 4:*

Post-task: *Report your answers from task 11 to the class by writing the identified macro-organizational structures on the wall pads. The whole class decides whether a pattern is discernible for the macro-organization of research articles in mathematics.*

Task 12 (Individual activity):

Propose a tentative macro-organization for the first draft of your research article. It does not need to be a polished outline. It is just a tentative map to begin with. You can make changes to it in the process of drafting your work.

Once you have a tentative macro-organization for your research article, think about your answers to the following questions:

- *Which section are you more likely to begin drafting first?*
- *Which section are you more likely to draft last?*

Note: If you already know which journal you are going to submit your work to, consult the journal submission guidelines before developing a macro-organization for your research article. Such guidelines sometimes offer information on the journals' preference for the organization of research articles.

Reflection journal 2

Select one of the following reflection tasks (a, b, or c) and write a thorough response to the questions as a journal entry (400 words).

a. Reflect on the level of professionalism in published journal articles.

■ What are the predominant characteristics of published journal articles that make them appear so professional?

■ What aspects of this professional writing will you try to emulate?

b. Reflect on your own writing abilities.

■ What aspects of your writing will you need to improve to move toward more professional writing?

■ How will you go about making these improvements?

■ What do you think will be most challenging about learning to write for expert audiences?

c. Reflect on your reading habits.

■ Based on what you've learned in these sections, how might you change the way in which you approach journal articles in the future to improve (1) your understanding of the articles and (2) your writing abilities?

(From Robinson, 2008, p. 29; bold facing and some slight content and formatting revisions are mine)

Part 5: Constructing Introduction and Complementary Introduction Sections

The activities in this section offer a step-by-step procedure to read Introduction sections analytically, build knowledge of Introductions as a partial genre, (that is, part of a bigger genre), and put your knowledge into practice in developing an Introduction section for your manuscripts for a journal submission.

By completing the following activities, you would be able to:

- *analyze the rhetorical organization, that is, move structure, of Introduction sections from different journal articles,*
- *review, that is, summarize and critically assess, existing literature on a research topic;*
- *identify and phrase gaps in existing research about a topic,*
- *use disciplinary-conventional rhetorical strategies and language,*

- *outline a space for your research,*
- *draft an Introduction section for your journal-quality manuscript submission.*

Introduction sections *An Introductions section is the first section to appear in a journal article in most disciplines. However, it is usually the last section drafted by authors. The reason is that Introduction gives an overview of research literature about the topic, how the article builds on, connects with, responds to, or fills knowledge gaps in the existing research about the topic, and how the article has been structured. So, it makes sense that writers defer writing an Introduction section for their article to the time when they have developed the other sections of the article and have a clear sense of what they have reported/argued in the rest of the article. Having a sense of the content and the various rhetorical sections of the rest of the article, however, does not make writing an Introduction section for the article easy; rather writing Introduction sections have been reported to be more troublesome than initially assumed by novice writers. Genre analysis scholarship has widely addressed this writing challenge by examining the rhetorical organization of Introduction sections of RAs from various disciplines.*

A well-known model of research article Introductions is John Swales' Create A Research Space (CARS) model (Swales, 1990). The model proposes an outline of the most common rhetorical moves and their lower level constituent steps which constitute the Introduction sections of RAs in many disciplines. Since its dissemination, many genre scholars have used Swales' CARS model in a pattern seeking way to figure out the rhetorical organization of Introduction sections of RAs in disciplines or sub-disciplines not examined by Swales. These studies reported both similarities and variations to the CARS model to the extent that Swales (2004) revised the 1990 CARS model to a slightly more general model. Due to the popularity and applicability of the CARS model in academic writing instruction, genre analysis scholarship continued analysis of Introduction as well as other sections of research articles *in various disciplines, including mathematics, for both theoretical and pedagogical purposes.*

Examples of recent studies that explore and report on the rhetorical structure of both the macro-organization and the Introduction sections of research articles in mathematics are Kuteeva and McGrath (2015), and Graves et al. (2013, 2014), and Moghaddasi and Graves (2017). Below, I reproduce the two notable models proposed by these studies (Figure C.5 and Figure C.6).

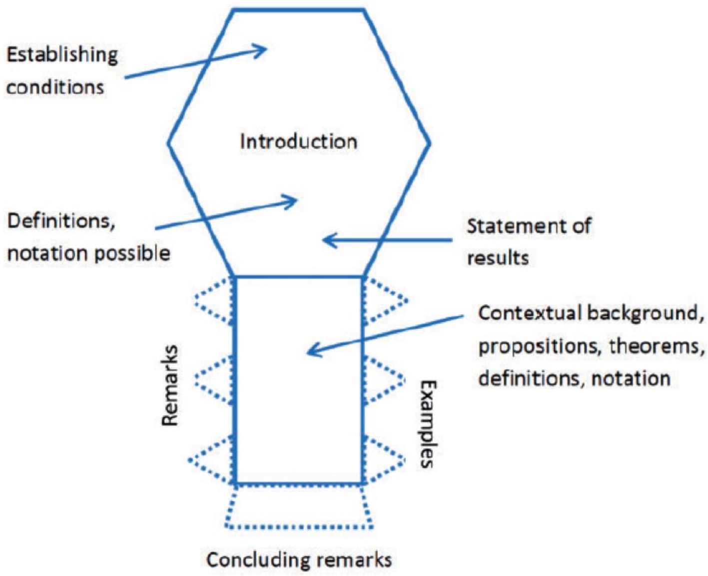


Figure C.5. Model A: Rhetorical organization of RAs in pure mathematics proposed in Kuteeva and McGrath (2015)

Swales' CARS model 1990	Swales' CARS model 2004	CARS Model for Discrete Mathematics RAs
Move 1: Establishing a territory Step 1: Claiming centrality Step 2: Making topic generalizations Step 3: Reviewing items of previous research	Move 1: Establishing a territory (citations required)	Move ET*: (obligatory) Establishing a territory (citations required)
Move 2: Establishing a niche Step 1A: Counter-claiming or Step 1B: Indicating a gap or Step 1C: Question-raising or Step 1D: Continuing a tradition	Move 2: Establish a niche (citations possible) vis Step 1A: Indicating a gap Step 1B: Adding to what is known Step 2: (optional) Presenting positive justification	Move P: (obligatory) (Graves et al 2014) Establishing presumptions Step 1: Presenting assumptions Step 2: Introducing notations Step 3. Defining objects/terms Step 4. (optional) Revising/referring to items of previous research
Move 3: Occupying the niche Step 1A: Outlining purposes or Step 1B: Announcing present research Step 2: Announcing principal findings Step 3: Indication RA structure	Move 3: Presenting the present work (citations possible) Step 1: (obligatory) Announcing present research descriptively Step 2*: (optional) Presenting RQs or hypotheses Step 3: (optional) Definitional clarification Step 4: (optional) Summarizing methods Step 5: (PISF**) Announcing principal outcomes Step 6: (PISF) Stating the value of the present research Step 7: (PISF) Outlining the structure of the paper	Move PPW: (obligatory) Presenting the present work (citations possible) Step 1: (obligatory) Announcing present research descriptively Step 2: (optional) Presenting RQs or hypotheses Step 5: (optional) Announcing principal outcomes Step 6: (optional) Stating the value of the present research Step 7: (optional) Outlining the structure of the paper
		Move EN: (optional) Establish a niche Step 1a: Retrieve a problem (and/or) Step 1b: Indicate an absence of or insufficient research (and/or) Step 1c: Instigate a problem (and/or) Step 1d: Add to what is known (and/or) Step 1e: Counter-claim
	(**PSIF: Possible in some fields)	(*In mathematics, these moves can appear in any order and they are often cycled through, so initials rather than numbers are used in this model to reflect accurately mathematicians' practices).

Figure C.6. Model B: Swales' CARS model of RA Introductions (1990, 2004) and an adapted CARS model proposed for RA Introductions in discrete mathematics (Moghaddasi and Graves, 2017)

As the two models show, both models propose ideas about the rhetorical structure of RAs in mathematics. While model A offers an overview of the schematic organization of the RAs, model B presents a more detailed outline for the move structure of Introduction sections in comparison to Swales 1990 and 2004 CARS models. In completing the upcoming tasks and the major writing assignment for this module, you will benefit from such recent studies in writing research articles in mathematics. However, it is important to note that these models, although presenting some patterns, are not static; they are to some extent dynamic and subject to change depending on the sub-discipline, topic of RA, and authorial preferences and choices. It is thus of critical importance that in completing the genre analysis tasks, writing the major assignment for this module, and in any future genre analysis activities, you use ideas from Model A and B and the suggested sources in a pattern seeking rather than pattern imposing way. It is also important that you pay equal attention to both patterns and specificities of genre organization and form as well as to think critically about potential motivations for both.

Learning Tasks: Practicing genre analysis of Introduction sections

Task 13. *Make groups of 3. Search the university library website for the following research article and download a copy on your device.*

McClosky B and Illya VH (2009) The Co-2-plex polytope and integral systems. SIAM Journal on Discrete Mathematics 23(3): 1135–1148.

Identify and scan the Introduction section as well as any section before the Results section/s in the article. Answer the questions, and complete the discussion activities for A, B, C, and D.

A. Rhetorical situation analysis:

1. *Who is the audience for the text?*

2. *What is the role relationship between the authors and their audience?*
3. *What is the rhetorical purpose of the authors in drafting the text?*
4. *What kind of genre is it?*

Class discussion: Share and discuss your answers with the rest of the class.

B. Genre analysis

Annotate the introduction section(s). Use the following guiding questions:

Note: *The assigned article includes two introductory sections (Introduction and Preliminaries). Repeat the following questions for each section separately.*

1. *How do the authors begin the Introduction/Preliminaries section?*
2. *Which statement introduces the topic of the article?*
3. *Do the authors highlight the importance of the topic/their research? If so, annotate the related statement.*
4. *Do the authors introduce new concepts, notations, assumptions? Annotate the related statements.*
5. *Do the authors give a summary of the existing research about the topic? If so, annotate the related statement.*
6. *Do the authors raise any questions about the summarized research or highlight any gaps or shortcomings in the research they summarized? If so, annotate the related statement.*
7. *Do the authors announce their findings and results? If so, annotate the related statement.*

8. *Outline a list of the most salient rhetorical moves you identified in the Introduction section (s).*

Class discussion: Share and discuss your answers with the rest of the class.

C. Language focus

1. *Are there any specific word, adjective, or adverb choices that state the significance of the topic/research? Highlight them in green..*
2. *Which words guide you to identify the statement that announces the authors' research? Highlight them in blue.*
3. *Highlight the verbs in yellow. What kinds of verbs and verb tenses are more dominant?*
4. *How do you identify summaries of past research? Highlight the guiding words and citations in pink.*
5. *Which language choices helped you to identify a statement of gap indication? Highlight them in red.*
6. *Do the writers use transitions to connect paragraphs and sentences? Highlight the transition words in orange.*

Class discussion: Share and discuss your answers with the rest of the class.

D. Outlining the rhetorical organization of the Introduction section

1. *Use your annotations from Section B and the answers shared by the class discussion to develop a rhetorical outline for the journal article Introduction sections in this sample article.*

   *(**Important note:** The research article you examined in this activity is just one sample and may not be reflective of the whole discipline's practice.)*

1. *Use the answers to A, B, C, and D (1) to reflect on possible connections between the rhetorical situation, rhetorical organization,and language features of the Introduction section that you just analyzed.*
2. *Discuss your reflection results from (2) with the rest of the class.*

Module 3 major assignment: Constructing the Introduction section for your research article manuscript

Use your outline from task 12, your knowledge of Introduction sections in mathematics journal articles from task 13, and other tasks from Modules 1, 2, and 3 to craft an introduction section for a journal manuscript based on your research project.

Audience: Decide on a target journal before planning your draft; familiarize yourself with the journal requirements for research article manuscripts and follow the guidelines when planning and drafting your work. Your primary audiences are the journal editor and two reviewers who are experts in the topic of your research. Your broader audience are expert or near-expert discourse community members in the specific field of your topic in mathematics. Depending on your topic and the orientation of your research, members of client disciplines such as engineering might also be among your audience.

Purpose: Your purpose is to create a space for your research within existing research in the field. A more immediate purpose is to convince the journal editor and reviewers of the worthiness of your research and its knowledge claim compared to other highly competitive journal submissions. So your goal should be to convince the editor and reviewers to take on the work of helping you revise your manuscript so it can be accepted for publication. Note that if your manuscript is too under-developed or requires too much work to make it *acceptable, it will likely be rejected. Your immediate goal is then to receive a 'revise and resubmit' response from the journal editor.*

Genre: Research article Introduction (part-genre)

Length: 700-900 words.

Task 14 (individual activity): Peer feedback activity

As a writing process activity, you are required to share your Introduction draft with other students and the instructor in an online platform identified by the instructor. Each student should provide feedback (a minimum of 2 comments) on 3 peer drafts. The comments should address different dimensions of the Introduction part-genre which have been highlighted in the module instruction, practice tasks, and class discussions.

Go through the peer feedback; put a checkmark by the comments/revisions you agree with. Negotiate the comments, which you disagree with, with the peer.

Task 15 (individual activity): Reflection activity

Go through the feedback that you received from peers and the instructor on your introduction draft. Put a checkmark by the comments/revisions you agree with. Put a cross out mark by the comments you disagree with. In about 250 words, draft your reflections on the received comments; explain what kinds of comments you received and why you find some more helpful than the others.