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STANDARDIZED FLEXIBLE END PLATE CONNECTIONS FOR STEEL BEAMS

by

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Abstract

A flexible end plate connection consists of a rectangular plate fastened to the web of the beam, on both sides, by fillet welds. The field connection is made by bolting to the support.

The requirements for the analysis and design of a flexible end plate connection include, in general, the determination of primary shear capacity considering the effect of connection deformation, the determination of the moment developed during connection rotation, and the prediction of the maximum unrestrained connection rotation. The experimental research reviewed addresses only the last two of these points. Connection design handbooks used in several different countries generally neglect the effect of secondary forces that can develop when the connection rotates, and propose that flexible end plates be designed to transfer primary shear only; empirical methods are adopted to ensure that the connection has adequate rotational flexibility.

The proposed analysis and design method employs, with modifications, an existing analytical method for evaluating the moment-rotation behaviour of a flexible end plate connection. An extension of this analytical method is used to determine the magnitude and distribution of secondary forces in all components of the connection, for a given rotation. A series of limit states design (LSD), lower bound resistance equations are proposed for predicting the primary shear strength of a flexible end plate connection. These equations consider the effect that secondary forces, which can be present, have on the primary shear strength of the connection.

The three step connection design approach adopted is used in the development of an interactive computer program for the design of

standardized connections. The program can be used for designing connections for individual beams and also to produce connection design tables. Since the connection design model is completely general, the design program does not restrict users to any particular geometric or material properties, as is currently the case in existing standardized connection design manuals. The program could be of value to both structural engineers and steel fabricators.

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Table of Contents

List of Tables

List of Figures

Figure Page

Figure Page

Figure Page

Figure Page

List of Symbols

a	= distance between the centreline of the bolt and the assumed location of the prying force resultant
A_b	= nominal cross sectional area of the bolt
A_w	= effective throat area of weld
b	= width of the supported beam flange OR length of bearing for a compression bearing specimen
B	= tension force in bolt
B_u	= ultimate bearing capacity of the end plate
C	= compressive force in supported beam web
d	= depth of section
d_b	= nominal bolt diameter
d_e	= distance from the top (bottom) row of bolts to the top (bottom) edge of the end plate
d_h	= bolt hole diameter
d_t	= distance from the top row of bolts to the top surface of the supported beam
d_y	= elemental length of the end plate
e	= distance from the center line of the bolt hole to the nearest edge of the end plate
$e(x)$	= depth from the top or bottom of the supported beam to the fibre strained to ϵ_y

E = modulus of elasticity of supported beam

E_p = modulus of elasticity of end plate

f = fillet weld leg size

g = bolt hole gage

G = elastic shear modulus

h = total depth of end plate

H = the horizontal reaction force acting at the bolts

H_{max} = the maximum horizontal reaction that can develop at the bolts

i = bolt level of interest

I_p = moment of inertia of end plate

k_{sp} = coefficient equal to the ratio of the ultimate shear strength of a bolt through the threads, to the ultimate shear strength of the bolt through the gross shank area

k_v = coefficient equal to the ratio of the ultimate shear strength of a bolt through the gross shank, to the ultimate tensile strength

k_1 = coefficient relating the maximum bolt deformation to the bearing stress developed between the bolt and the plate

k_2 = coefficient relating bolt deformation to the bearing stress developed between the bolt and the plate at load levels of up to 80 percent of the ultimate shear capacity of the bolt

K_{A1} = tensile stiffness of the bolts

K_{A2} = stiffness of the bolt zone parallel to the undeformed plate

K_{A3} = axial stiffness of the gross section of the end plate

K_{R1} = flexural stiffness of the end plate

K_{V1} = shear stiffness of the end plate

ℓ = distance between the inner edge of the bolt hole and the toe of the fillet weld of the undeformed plate OR span of beam

ℓ_b = distance from the neutral axis in the connection to the underside of the bottom flange of the supported beam when bottom flange bearing is imminent

ℓ_{bna} = length of end plate below the neutral axis

M = maximum moment acting in clear span of end plate

M_{end} = beam end moment induced at support

M(x) = bending moment at location 'x'

M_f = factored bending moment

$M_i(x)$ = internal moment at location 'x'

M_p = plastic moment capacity of a rolled shape

M_{pr} = connection moment immediately prior to the occurrence of bottom flange bearing as predicted by analysis

M_{rc} = flexural resistance of coped section assuming no shear force is present

M_t = connection moment immediately prior to the occurrence of bottom flange bearing as observed in test

M_{up} = ultimate flexural capacity of a rectangular section

n = number of pairs of bolts

p = width of an end plate tension specimen OR bolt hole pitch for a full size connection

P = tensile membrane force acting in end plate

P_{max} = maximum membrane force that can develop in an end plate

P_{up} = ultimate tensile capacity of gross section of the end plate

P_{yp} = yield tensile capacity of the gross section of the end plate

Q = prying force resultant

$Q(x)$ = distributed prying force

t = flange thickness

t_p = end plate thickness

t_s = thickness of supporting element

T = tension force in the beam web

T_f = factored tension force in a bolt

T_{max} = limiting tension force that can develop in the web of the beam prior to membrane action in the end plate

T_u = ultimate tensile strength of the net cross section of the end plate

V = shear force in end plate corresponding to end plate moment M

V_c = shear force in web of beam above the top of the end plate

V_f = factored shear force in a bolt

V_{ft} = total factored shear force at connection

V_i = factored shear resistance of connection component at the level of the ith row of bolts

V_L = longitudinal shear force

V_{rb} = factored shear resistance of bolt group

V_{rc} = shear resistance of the beam web at the coped section assuming no flexural stresses are present

V_{rm} = factored shear resistance of the beam web immediately adjacent to the end plate

V_{rmax} = ultimate shear strength of a connection when θ = 0 radians

V_{rp} = factored shear resistance of end plate

V_{rw} = factored shear resistance of fillet weld

$V_{r\theta}$ = the ultimate shear strength of a connection at rotation θ

V_T = transverse shear force

V_u = ultimate shear strength of a bolt through the threads

V_U = ultimate longitudinal shear strength of a fillet weld

V_w = shear transferred from beam web to weld group over length of weld

w = web thickness OR uniformly distributed load

x = distance from the support to a particular location along the beam

X_u = ultimate strength of weld metal as rated by the Electrode Classification Number

α = angle of deformation of endplate assuming hinges adjacent to bolt line and beam web rotate same amount OR load factor

α_D = dead load factor

α_L = live load factor

α_p = angle of deformation of endplate at which H_{max} is first reached

α_y = angle of rotation of hinge when the maximum moment attainable first occurs

α_1 = angle of rotation of the hinge nearest the bolt line

α_2 = angle of rotation of the hinge nearest the web

α_3 = angle of rotation of the edge portion of the end plate

β = interaction factor derived from tests relating shear and tension forces in a bolt

Δ = transverse connection deformation

Δ_{A1} = deformation of spring A1

Δ_{A2} = deformation of spring A2

Δ_{A3} = deformation of spring A3

Δ_{be} = bearing deformation of a flexible end plate between the bolt holes and the edge of the connection plate

Δ_{bs} = shear deformation of a bolt at a load level less than the ultimate shear capacity of the bolt

Δ_{bsmax} = maximum shear deformation of a bolt

Δ_e = maximum elastic end plate deformation

Δ_{ns} = net section tensile deformation for a flexible end plate

Δ_p = maximum tensile deformation of end plate occuring when H_{max} is first reached

Δ_y = maximum end plate deformation occurring when the yield moment is attained at the hinges

ϵ_y = axial yield strain of the material

$\epsilon_{max}(x)$ = maximum flexural fibre strain at a cross-section at location 'x'

γ = shear strain corresponding to shear stress τ

γ_y = yield level shear strain

π = a constant, 3.14

ϕ = resistance factor, taken as 0.67 for bolts and for welds, and 0.90 for plate

$\Phi(x)$ = curvature at location 'x' along the beam span

σ = maximum normal stress that can simultaneously be present with shear stress τ

σ_{bs} = average bearing stress between bolt and plate at a load level less than the ultimate shear capacity of the bolt

σ_{bsmax} = maximum average bearing stress between bolt and plate

σ_c = compressive stress at interface of end plate and beam web

σ_u = ultimate tensile strength of material

σ_{ub} = ultimate tensile strength of the bolt

σ_{up} = ultimate tensile strength of end plate

σ_{uw} = ultimate tensile strength of supported beam web

σ_y = yield tensile strength of material

σ_{yp} = yield tensile strength of end plate

σ_{yw} = yield tensile strength of supported beam web

τ = nominal shear stress

τ_y = nominal shear stress necessary to cause yielding

θ = rotation of connection

θ_b = beam end rotation when bottom flange bearing is imminent

θ_{end} = beam end rotation

θ_{max} = maximum attainable rotation of connection prior to failure

θ_{pr} = connection rotation corresponding to bottom flange bearing of beam on support as predicted by analysis

θ_t = connection rotation corresponding to bottom flange bearing of beam on support as observed in test

1. Introduction

1.1 General

One of the most important steps in the design, fabrication, and erection of structural steel building frames is the design of the connections. Because there is usually a relatively large number of connections required in common steel frames, the economics of a project can be greatly affected by the time spent in selecting and manufacturing the connections.

To minimize the time required to design and detail connections, some steel fabricators have developed so-called 'shop standards', which are simply handbooks containing a summary of preferred, pre-designed connections suitable for use in standard type steel frames. When developing shop standards, fabricators include connection details most convenient to them with preferred geometric and material properties. The details are generally simple, uniform, and easy to manufacture. Because they are specified repeatedly by detailers, the standard connection types are often stockpiled, thus streamlining the fabrication process.

Unfortunately, a good set of shop standards may not be available to all steel fabricators because of the cost to develop them. In some countries, including for example Australia, South Africa, and the United Kingdom, agencies working on behalf of the steel producers, designers, fabricators, and erectors have published standardized connection design handbooks that are available industry wide.

By assisting fabricators in the rapid design of standard connections, these publications encourage industry-wide standardization by tabulating only a limited number of connection details. Opponents to this approach consider that the variation in fabrication equipment, and the individual preferences on such matters as geometric details, make such a high degree of standardization undesirable and not feasible. Ideally, some other means of assisting fabricators in standardized connection design is required.

A prerequisite to the development of standardized or pre-designed connections, is a rational analysis and design procedure to ensure the strength and serviceability of the system. Although most flexible shear type connections common to standard steel frames are designed using simple models, it is recognized that the behaviour of such connections is usually complex. The models have evolved from long standing accepted practice, engineering judgement, and research. Although performance determines whether designs are acceptable or not, connections in the field are rarely loaded above service levels, and the response at overload is generally not well known.

In keeping with the limit states design philosophy now in widespread use in Canada, connection resistance should be determined on such principles to provide adequate but consistent margins of safety. The work of Butler et al. (1972), and Dawe and Kulak (1974), reveals that simple analysis methods, like those based only on elastic response of the connection, can result in connections possessing a considerable range of safety. For these reasons it is desirable to review connection design models and to propose new analysis techniques where necessary.

1.2 Scope and Objectives

The overall objective of this research is to develop a method for the standardization of steel connection design in Canada. To achieve this primary objective, only one type of connection, the flexible end plate, has been investigated. This general method can then be used as a basis for the development of standardized details for other connections.

The work is presented in two parts. The majority of this investigation is found in PART A, where a method is presented for the analysis and design of flexible end plate connections. The method is consistent with the limit states design approach. The development of the model is based soley on existing research.

PART B of the research applies the analysis and design model developed in PART A in the development of a method of standardizing connection design. The method, which employs the computer, does not restrict designers to a limited number of connection parameters, as is encountered in industry-wide standards currently available, but allows the designer to specify the parameters as he sees fit.

PART A: PROPOSED ANALYSIS AND DESIGN METHOD FOR THE FLEXIBLE END PLATE SHEAR CONNECTION

2. Requirements of Analysis and Design Model

2.1 Background

A flexible end plate connection consists of a rectangular plate symmetrically fastened to the supported beam web with fillet welds on both sides of the web, as shown in Figure 2.1. The rectangular plate is fabricated with holes in it so that the field connection can be made by bolting to the support.

Flexible end plate connections are used in steel frame construction to support transversely loaded beams or girders intended to act as simply supported members. Thus, to provide satisfactory performance, the connection must transfer the end shear force from the beam to the support, allow at most only a negligible end moment to develop in the supported beam, and allow virtually unrestrained rotation to occur.

The end rotation of the beam is normally accomodated mostly by the out-of-plane deformation of the end plate. Depending upon the relative stiffnesses of the supported beam, the connection plate, and the supporting element, varying amounts of deformation will also occur in the beam web and in the supporting elements. When the support is relatively stiff, the out-of-plane action causes compatibility or secondary forces to develop in the connection. These secondary forces induce a moment in the supported beam and in the supporting member. In addition, the secondary forces may adversely affect the shear transfer from the beam to the support.

It is usually assumed that simple shear type connections allow unlimited free end rotation of the supported beam to occur. This, however, is not always the case. Flexible end plate connections have a limiting rotation above which considerable restraint to end rotation of the beam occurs. This restraint occurs when the bottom flange of the beam comes into contact with the support, due to beam end rotation, resulting in much changed and undesirable behaviour of the connection.

In recognition of this type of connection behaviour, a general analytical method intended for use in designing a flexible end plate connection should

1. allow for the determination of the shear capacity of the connection in the presence of any secondary forces that can develop,
2. allow for the evaluation of the moment-rotation relationship of the connection, and
3. allow for the estimation of the maximum unrestrained end rotation of a beam supported by a flexible end plate connection.

2.2 Experimental Studies of Flexible End Plate Connections

Flexible end plate connections have been used for about 20 years. Research on the behaviour of these connections has been done by Kennedy (1966), Kennedy (1969), Sommer (1969), Mansell and Pham (1981), Hafez (1982), and Kennedy and Hafez (1984). (The work done by Mansell and Pham (1981) was not obtainable at the time this report was prepared.) In the literature reviewed, the main objective appears to have been the evaluation of the moment-rotation characteristics of the connection; it is generally assumed that a connection designed to transfer shear only will have adequate strength and that this shear strength is unaffected by the moments developed in the connection.

Kennedy (1966) tested a series of identical cantilever beams supported by flexible end plates with varying moment-shear ratios by applying a load at different lever arms. The moment-rotation characteristics for the different moment-shear ratios were essentially the same. Kennedy suggested that the flexibility of the connections was independent of the shear force and, hence, that the shear strength is not significantly affected by the moment. None of the connections, even those with high shear-moment ratios, failed in shear. The results appear to substantiate the common practice of designing 'simple' end connections for shear only, neglecting the end moments developed in them. While this may appear to be justified considering the relatively small moments developed, and the limited rotation required, the effect of the rotation on the ultimate shear strength was not assessed.

The main objective of Sommer's (1969) test program, an extension of Kennedy's work, was to assess the relative effects that geometric parameters have on the connection behaviour. Sommer determined that, in general, flexible end plate connections become stiffer flexurally as the thickness of plate is increased, as the depth of the connection is increased, or as the bolt hole gage is decreased. It was reported that, for the range of flexible end plate connections tested, the end moment developed ranged from 4 percent to 25 percent of the yield moment capacity of the supported beam, with the average being 9 percent. Sommer concluded from his observations that the common practice of designing the connection components to transmit primary shear only, is satisfactory. However, Sommer warns of the potential for premature bolt fracture in deep connections; secondary forces, induced in the bolts at the top of such connections, can be large as a result of supported beam end rotations.

The connections, tested as cantilever connections, failed generally in the connection plate at rotations considerably greater than those encountered for the 'simply supported' ends of beams commonly used in steel construction. None of the 20 tests failed in shear. Coupled with Kennedy's previous tests with high shear-moment ratios, this would indicate that at practical values of end rotations the degradation of shear capacity due to the rotation may not be large. However, an assessment of this degradation was not made.

Indeed, beams designed to carry large shears will likely have only small end rotations as the spans will not be long. In any connection, however, regardless of the supported beam length, the bolts and welds may be selected to carry only the shear that is present. Therefore, if the supports are rigid, under the factored loads, these connection components could reach their capacities prematurely, because no consideration was made of the effects of end rotation on those capacities. Moreover, as shown in Table 2.1, in Sommer's tests the average shear on the bolts at failure had a maximum value of 36 percent of the nominal shear capacity through the threads and the average shear on the welds did not exceed 26 percent of the ultimate capacity. Even though the failures, which occured most often in the connection plate, happened at connection rotations considerably greater than those commonly encountered in steel frames, this condition indicates that some reduction of the ability of these connections to carry shear must occur between no rotation and ultimate rotation levels, as postulated in Figure 2.2.

Hafez (1982) (see also Kennedy and Hafez (1984)) developed an analytical method for predicting the moment-rotation characteristics of flexible end plate connections up to and including the rotation at which the bottom flange of the beam bears against the support. The predicted relationship compared well with test results.

To provide a complete picture of the behaviour of flexible end plates, it is considered necessary to review, and possibly modify, the method of Hafez, and of prime importance to assess quantitatively the effect of beam end rotation on the shear strength of the connection.

2.3 Connection Handbook Analysis and Design Procedures

Even though the Australian Institute of Steel Construction (AustISC, 1978), the American Institute of Steel Construction (AISC, 1980), the Canadian Institute of Steel Construction (CISC, 1980), the British Constructional Steelwork Association (BCSA, 1982), and the South African Institute of Steel Construction (SAISC, 1982) all present unique resistance equations for the design of flexible end plate connections, their methods are, for the most part, similar.

The CISC and the AISC publications are not truly standardized connection design handbooks. They are included, however, because they represent the major industry-wide documents of this type available in North America. The connection design information provided in these handbooks is provided (among other reasons) to assist individual fabricators in developing their own shop standards.

In the design for strength, all the agencies assume that the connection is subject only to a vertical shear force, uniformly distributed throughout the connection, determined assuming the beam to have ideal hinged supports. The limiting strength of the connection is then based on the component having the minimum resistance: the bolt group, the end plate, the fillet weld, or the supported beam web. The effect of coping of the supported beam should also be investigated.

The shear strength of the bolt group is limited to the smallest of the following three values: the shear strength of the bolts, the bearing strength of the end plate, and the bearing strength of the support. Normally, for flexible end plate connections, threads are assumed to exist in the shear plane when evaluating the bolt shear strength.

The AustISC recommends that for long connections, those with more than six rows of bolts, a reduction factor be applied to the calculation of the bolt group shear strength. This reduction factor is intended to account for uneven loading of the bolts, which is common to long connections of tension members. This consideration may not be appropriate for shear type connections where the shear transfer to the bolts is likely more uniform. None of the other publications recommend that such a reduction factor be used.

The AustISC recommends a lower allowable bearing strength for the end plate than for the supporting element. This is to account for the horizontal shear forces induced in the bolts due to the horizontal connection movement that occurs during beam end rotation, and for the relatively small loaded edge distances in the end plates as compared to the supporting material. Since the deformations in long connections, due to beam end rotation, can be relatively large, the secondary forces developed also can be relatively large. The reduced shear strength assigned to bolts in long connections (as discussed in the previous paragraph), rationally, should be assigned to account for the secondary forces and not to account for uneven vertical shear transfer. None of the manuals directly considers the effect of the development of tension and horizontal shear forces in the bolts.

In the four remaining handbooks, the bearing capacity of the support and of the connection end plate are evaluated using only one bearing capacity expression, neglecting the effects of horizontal connection movement. The SAISC manual states, however, that if bolt hole deformations due to high bearing stresses are undesirable, the allowable bearing stress should then be reduced by 25 percent in both the support and the plate.

In the AustISC Handbook, the shear capacity of the end plate is based on its gross section while the BCSA manual uses the net section through the bolt line. The AISC, CISC, and SAISC Manuals do not cover this point.

In the AustISC, AISC, and BCSA Handbooks the longitudinal shear strength of the fillet welds at the supported beam to end plate junction is based on effective weld lengths equal to the specified length minus two times the fillet leg size, to account for the reduced weld profile at the start and stop positions. The SAISC approach appears to be identical. The CISC Handbook simply uses the specified weld length. None of the five handbooks considers the presence of transverse shear components, which can be induced in the fillet welds due to end rotation of the supported beam, when evaluating the strength.

In the AustISC manual, the allowable average shear stress in the web of beams with top flange copes is limited to 89 percent of that for uncoped beams and is limited to 81 percent when both the top and bottom flanges are coped. The AustISC provides guidelines for evaluating the reduced shear capacity of webs with certain size copes only.

The approach for evaluating coped beam web shear strengths in both the BCSA Handbook and the SAISC Handbook is similar to the Australian approach, but it is not explicitly covered in the Canadian or American handbooks. All of the handbooks deal with the shear strength of uncoped webs in the same way, and assume the shear strength of the web is available only over the length of the connection.

Of all of the five handbooks, only the AustISC Handbook provides recommendations regarding the design eccentricity that should be assumed for the flexible end plate when designing the supporting member. In recognition of the partial fixing moment that develops at the connection, the AustISC suggests the design eccentricity be equal to 100 mm greater than the geometric eccentricity. This recommendation, however, is suggested for use only when the support is a column, and no recommendations are given for the case where the support is a beam or girder.

To ensure that the bottom flange of the supported beam does not contact the support, the AustISC has applied the recommendations of Kennedy (1969). Certain geometric constraints must be met. With a maximum expected beam end rotation of 0.030 radians, based on the assumption that the maximum bending moment is the yield moment, and with the further assumptions that placement of the end plate on the web of the beam is typical and that all beam end rotation occurs about the bottom fibre of the end plate, then free rotation occurs if the connection plate length does not exceed 33 times its thickness. This geometric constraint does not, of course, apply if the bottom flange is coped.

The BCSA Handbook follows the AustISC procedure for ensuring adequate rotation, the others have no specific requirements.

To ensure flexibility of the connection, the the top of the end plate must pull away from the supporting member. The AustISC Handbook states that the ratio of the bolt gage to plate thickness must be between 11 and 14 to ensure this. By providing sufficiently flexible end plates, the secondary forces developed at the connection, and therefore the moments developed at the connection, are limited. The other handbooks give limits on the end plate thickness of 6 mm and 10 mm and gage of 89 mm and 150 mm.

Deeper connections tend to induce greater secondary forces (Sommer 1969). Only the SAISC specifically recommends that flexible end plate connections can be used on beams as deep as 2000 mm.

Of the five connection handbooks, only the CISC and the AISC publications provide guidelines for the design of flexible end plate connections for slip-resistant conditions. The flexible nature of this connection, as observed in tests (Sommer 1969, and Hafez 1982), is likely not compatible with slip-resistant bolting procedures because of the deformations that can occur in the connection. Furthermore, it is possible that the connection will slip at load levels less than assumed by the designer, because the designer, when sizing the connection, is not likely to consider the presence of the secondary shear and tension acting in the bolts above the neutral axis of the connection.

In general the handbooks discussed here evaluate the strength of the flexible end plate as if the connection behaved as a perfect hinge. This condition, however, is approached only when the support is very flexible. Also, the analysis and design approach adopted by all of the handbooks seems to ignore the fact that significant inelastic response can occur in the flexible end plate connection even at service level loads, as observed by both Sommer (1969) and Hafez (1982). Even though the CISC Handbook evaluates the resistance of the connection components using a limit states design standard (CAN3-S16.1-M78, CSA 1978), it does not deal with the analysis of the design forces in a compatible way. Some empirical rules are given in the handbooks to limit the secondary forces that develop in these connections. However, no general design approach to account for the secondary forces is given. To step beyond the bounds of empiricism and provide greater consistency, an analytical approach is needed that takes into account the effect of secondary forces that can develop when the supported beam end rotates.

2.4 Proposed Approach

The primary shear force used to design a flexible connection can be directly determined on the basis that the shear distribution in the supported beam is not significantly affected by the relatively small end moments that exist. In the extreme case of a beam supporting a uniformly distributed load, as shown in Figure 2.3, even when the end moments are unequal, and one end moment is taken as 25 percent of the maximum span moment and the other is taken as zero, the end shear is changed from the 'simple' case by only 6 percent. Because the magnitude and distribution of secondary forces that develop in flexible end plate connections depend on the connection geometry and can affect the primary shear strength of the connection, the analysis and design procedure will generally need to be iterative.

The suggested procedure is as follows:

1. select an initial end plate configuration using the simple vertical shear design method proposed in the steel industry handbooks, which neglects secondary effects,
2. if the support is relatively stiff evaluate the moment developed at the rotation associated with the factored load on the supported beam,
3. check to ensure that uninterrupted connection rotation is possible at this load,
4. determine the primary shear capacity considering any secondary effects, and
5. iterate as necessary, as depicted in the flow chart of Figure 2.4.

Table 2.1 Summary of Average Stresses and Forces in Sommer (1969)

Connection Tests at Failure

Test No.	Ult. Load (kips)	Plt. Size Th. x Dep. (inches)	Plt. Yld. Strength (ksi)	No. Of Bolts	Fillet Size (inches)	PLATE Avg. Str. / Yld. Str.	BOLTS [a] Avg. Sh. / Sh. Cap.	WELD [b] Avg. Str. / Ult. Str.	Location Of Rupt.
5	26.9	1/4 x 15	49.3	10	1/4	0.073	0.120	0.110	HAZ Plt. [c]
6	28.5	1/4 x 9	49.3	6	5/16	0.130	0.220	0.155	HAZ Plt.
7	33.3	1/4 x 12	49.3	8	5/16	0.110	0.190	0.136	HAZ Plt.
8	29.8	1/4 x 15	49.3	10	5/16	0.081	0.137	0.097	HAZ Plt.
9	29.9	1/4 x 18	49.3	12	5/16	0.067	0.114	0.081	HAZ Plt.
10	27.0	3/8 x 9	57.2	6	1/4	0.070	0.206	0.184	HAZ Plt.
11	33.0	3/8 x 12	57.2	8	1/4	0.064	0.189	0.168	HAZ Plt.
12	21.5	3/8 x 15	57.2	10	1/4	0.033	0.099	0.088	HAZ Plt.
13	47.0	3/8 x 9	57.2	6	5/16	0.120	0.359	0.256	HAZ Plt.
14	57.3	3/8 x 12	57.2	8	5/16	0.110	0.328	0.234	HAZ Plt.
15	49.1	3/8 x 15	57.2	10	5/16	0.076	0.225	0.160	Bolt
16	54.7	3/8 x 18	57.2	12	5/16	0.071	0.209	0.149	Bolt
17	22.5	1/4 x 12	49.3	8	5/16	0.076	0.129	0.092	HAZ Plt.
18	36.7	1/4 x 15	49.3	10	5/16	0.100	0.168	0.120	BL Plt. [d]
19	51.7	1/2 x 12	52.6	8	5/16	0.082	0.296	0.210	HAZ Plt.
20	47.0	1/2 x 15	52.6	10	5/16	0.060	0.216	0.157	HAZ Plt.
25	17.0	1/4 x 12	49.3	8	1/4	0.057	0.098	0.089	HAZ Plt.
26	13.8	1/4 x 9	49.3	6	1/4	0.062	0.106	0.075	HAZ Plt.
27	7.2	1/4 x 9	49.3	6	3/16	0.033	0.055	0.065	HAZ Plt.
28	8.3	1/4 x 6	49.3	4	3/16	0.056	0.095	0.113	HAZ Plt.

a A325 3/4 inch Diameter Bolts, ULTIMATE STRENGTH = $0.60 \times 0.70 \times \sigma_{ub} \times A_b$

b E7018 Electrodes, ULTIMATE STRENGTH = $0.66 \times X_u \times f \times h \times 2/\sqrt{2}$

c HAZ Plt. : Heat Affected Zone of End Plate.

d BL Plt. : Bolt Line of End Plate.

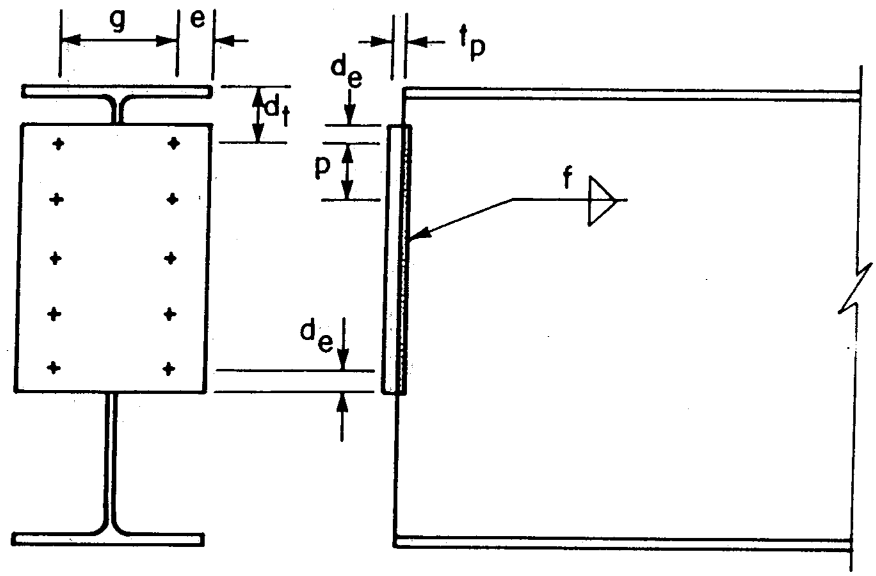


Figure 2.1 Flexible End Plate Connection Detail

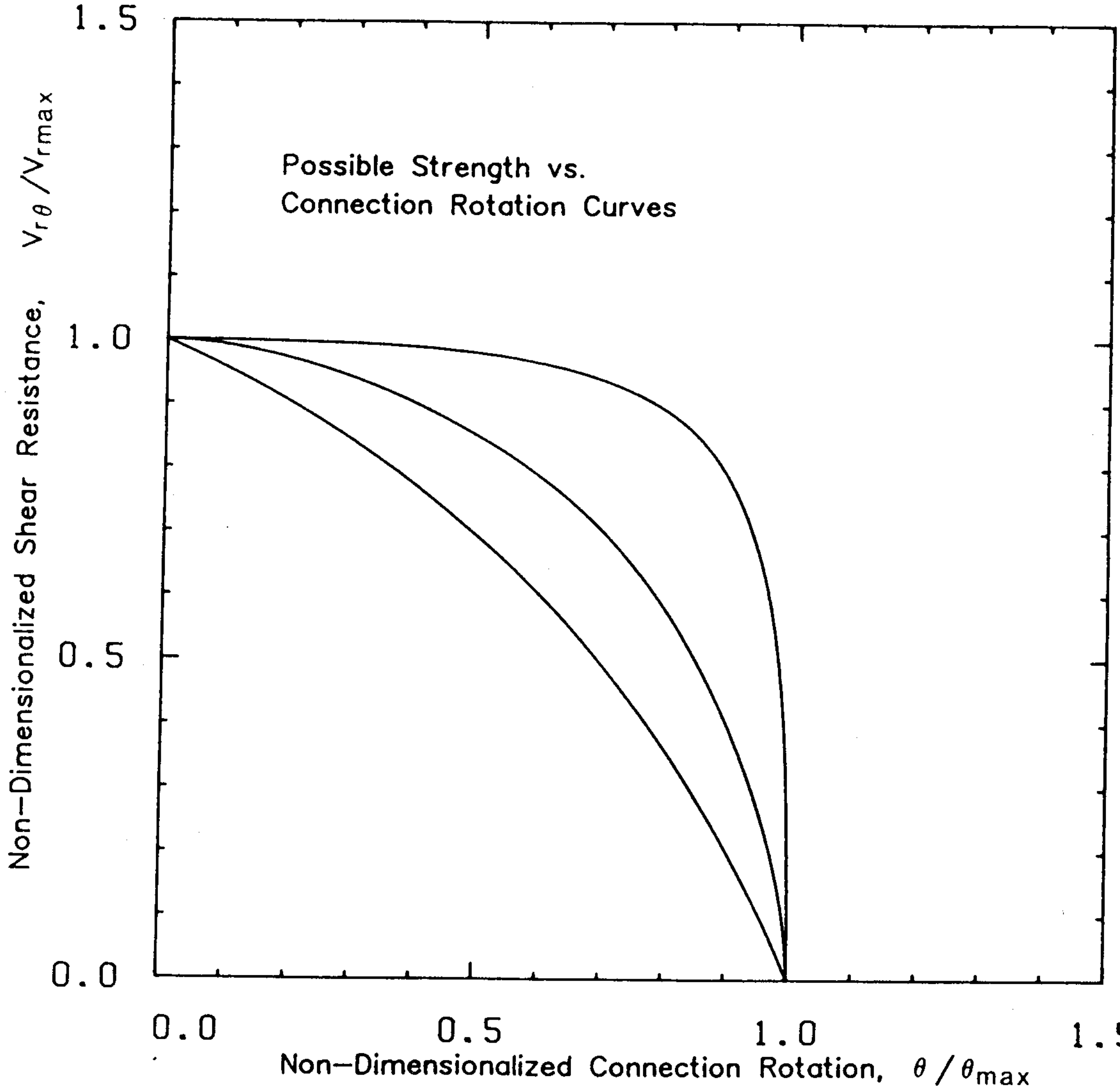


Figure 2.2 Possible Relationship between Flexible End Plate Shear Resistance and Connection Rotation

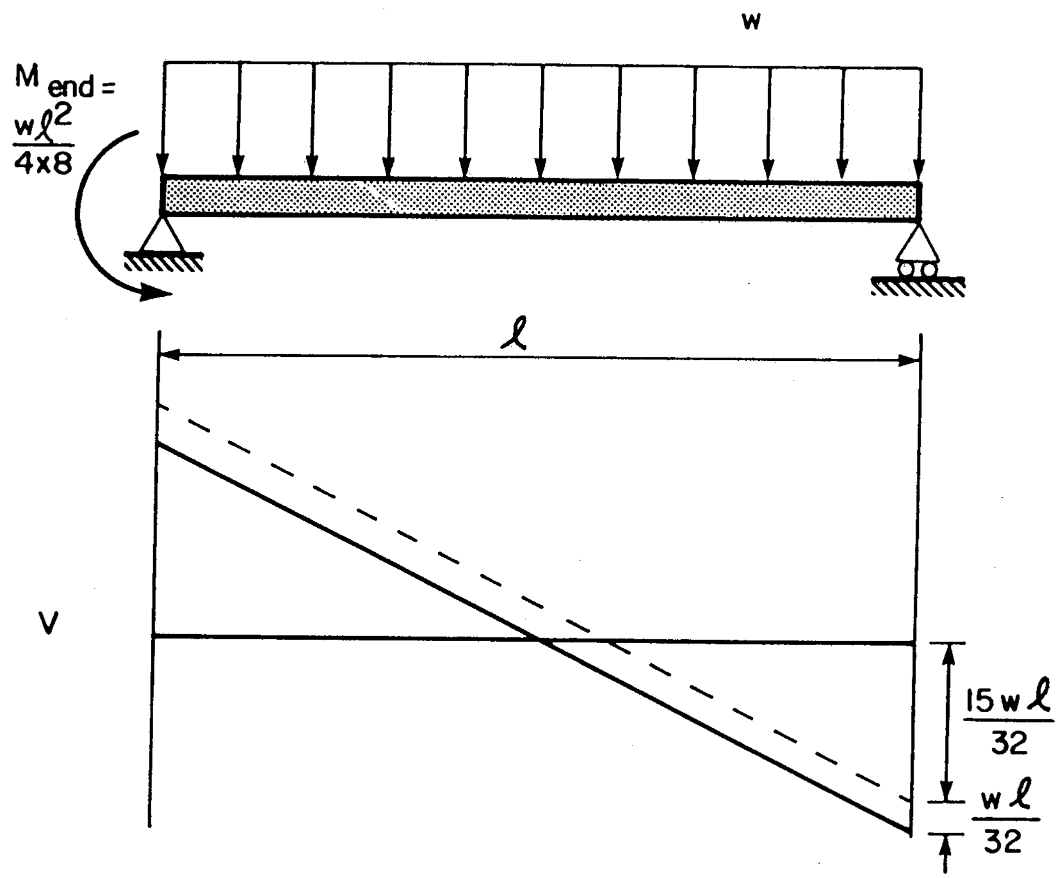


Figure 2.3 Effect of Relatively Small Support Moments on the Shear Distribution in Beams

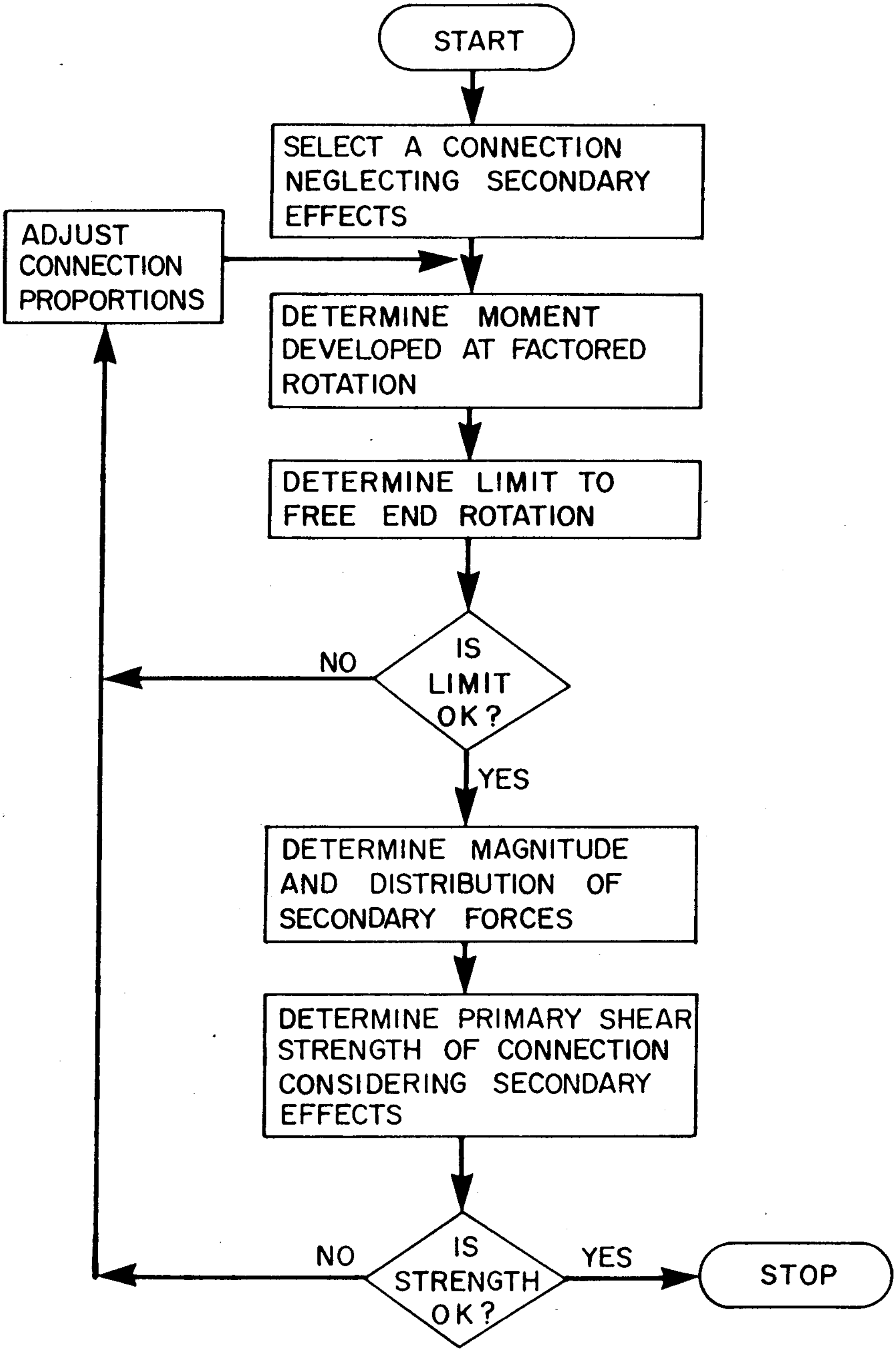


Figure 2.4 Design Process for a Flexible End Plate Connection

3. Evaluation of the Moment-Rotation Relationship of a Flexible End Plate Connection

3.1 Introduction

Sommer (1969) and Hafez (1982) investigated experimentally the moment-rotation characteristics of flexible end plate connections. Sommer evaluated various connection geometries and developed an empirical standardized equation from which the moment-rotation relationship could be predicted. Hafez developed an analytical method to predict the moment-rotation relationship that agreed well with experimental results. This method is extended here by removing some of the simplifications introduced by Hafez.

3.2 Background and Description of Analytical Method

Hafez observed, as had others, that the deformation of the connection varied linearly over its depth and that the location of the neutral axis moved downward as the connection rotation increased, indicating that the stiffness of the compression region differs from that of the tension region. Hafez next developed analytical load-deformation relationships for end plate details, loaded either in tension or compression, that correspond closely with test results.

To calculate the moment developed for a given rotation, a position of the neutral axis is first assumed and the portion of the connection on either side of the axis is divided into a number of elements (for small rotations, the neutral axis is at about mid-height and moves downward with increasing rotation). With the deformations of each element now known, the forces developed in them based on the tension and compression load-deformation relationships are determined, as shown in Figure 3.1. These elemental forces are summed, and the position of the neutral axis is modified until the sum is zero. The sum of the moments of the elemental forces for this position gives the moment in the connection for the assumed rotation.

3.3 Load-Deformation Relationships of Connection

3.3.1 Original Hafez Relationships

3.3.1.1 Tension Region

For the tension region of the end plate, Figure 3.2, Hafez proposed two expressions to describe different portions of the load-deformation response. When the deformation is elastic and small, the relationship between tensile force and plate deformation is given by

$$[3.1] \qquad T = \frac{24\, E_p\, I_p}{\ell^2} \tan \alpha$$

But $\tan \alpha$ equals Δ/ℓ and for a unit width of plate I_p equals $t_p^3/12$. Therefore T can also be determined using

$$[3.2] \qquad T = \frac{2\, E_p\, t_p^3}{\ell^3} \Delta$$

It is assumed that the moment-curvature relationship for the rectangular section is bilinear and that the maximum moment is reached at the ultimate tensile strength. From the free body diagram of Figure 3.2, the limiting value of the tensile force for this behaviour can be expressed as

$$[3.3] \qquad T_{max} = \frac{4\ M_{up}}{\ell}$$

where

$$[3.4] \qquad M_{up} = \frac{\sigma_{up}\ t_p^2}{4}$$

Although it is common to base the moment developed at plastic hinges on the yield strength of the material, Hafez (1982) found that using the ultimate tensile strength gave an analysis more closely fitting test results. This was attributed to the occurrence of significant strain-hardening at the hinge locations. Combining Equations [3.2], [3.3], and [3.4] gives the maximum plate deformation for which Equation [3.2] is valid as

$$[3.5] \qquad \Delta_e = \frac{\sigma_{up}\ \ell^2}{2\ E_p\ t_p}$$

For the second stage of behaviour, it is assumed, based on test observations, that plastic hinges have developed adjacent to the toe of the fillet weld and at the inner edge of the bolt holes. Outward movement of the plate may be large enough so that significant load can be carried in tension, that is by membrane action, as well as by flexural action. From the free body diagrams shown in Figure 3.3 the following relationship can be developed:

$$[3.6] \qquad T = 2\ P \sin \alpha + \frac{4\ M}{\ell} \cos^2 \alpha$$

Assuming the flexural-tension interaction expression for rectangular sections (see "Plastic Design in Steel", ASCE (1971)) is taken as

$$\frac{M}{M_{up}} + \frac{P^2}{P_{up}^2} = 1.0 \qquad [3.7]$$

and combining Equations [3.6] and [3.7] gives T as

$$T = 2\,P \sin\alpha + 4\,\frac{M_{up}}{\ell}\cos^2\alpha - \frac{4\,M_{up}\,P^2}{\ell\,P_{up}^2}\cos^2\alpha \qquad [3.8]$$

Setting the partial derivative of T with respect to P equal to zero results in the value of P being given as

$$P = \frac{\ell\,P_{up}^2 \sin\alpha}{4\,M_{up}\cos^2\alpha} \qquad [3.9]$$

Substituting Equation [3.9] into [3.8] gives T as

$$T = \frac{\ell\,P_{up}^2}{4\,M_{up}}\tan^2\alpha + \frac{4\,M_{up}}{\ell}\cos^2\alpha \qquad [3.10]$$

It was observed after testing end plate specimens in tension that extremely large strains appeared in the plate at plastic hinge locations and that strains in between the hinge locations were considerably less. For this reason Hafez suggested that the maximum moment in the plate be based on the ultimate tensile stress, σ_{up}, but that the maximum axial load in the plate be based on the yield strength, σ_{yp}. The final analytical expression presented by Hafez for determining the inelastic tensile load-deformation response of a flexible end plate is

$$[3.11] \qquad T = \frac{\ell \, P_{yp}^{2}}{4 \, M_{up}} \tan^{2} \alpha + \frac{4 \, M_{up}}{\ell} \cos^{2} \alpha$$

or, in terms of Δ,

$$[3.12] \qquad T = \frac{P_{yp}^{2}}{4 \, M_{up} \, \ell} \Delta^{2} + \frac{4 \, M_{up} \, \ell}{(\ell^{2} + \Delta^{2})}$$

The first part of these expressions is the contribution due to membrane action and the second part is the contribution due to flexural action. When the deflection is zero there is no membrane contribution. However, Equation [3.12] implies that there is a maximum flexural contribution when the deflection is zero, which is clearly not the case. This anomaly arises from the assumption of initial full plasticity. Therefore, Equation [3.12] is to be used only at deformations greater than that given by Eqation [3.5]. As shown by Hafez, the the bi-modal analysis (Equations [3.1] and [3.11] or Equations [3.2] and [3.12]) is in reasonable agreement with the test behaviour provided that the initial flexural behaviour is sufficiently stiff so that the projected elastic line of the first mode intersects the curve described by the second mode.

3.3.1.2 Compression Region

In the compression region, deformations due to beam end rotations occur in the web of the supported beam (this condition is most severe when the support element is relatively stiff), as shown in Figure 3.4.

Hafez conducted compression load-deformation tests on end plate specimens of varying lengths, with the web plate made wider than the end plate. These tests were conducted in order to determine the influence of the supported beam below the connection plate on the strength and deformation characteristics of the compressed region. Empirical elastic and inelastic relationships were developed relating the deformation to the average stress in the supported beam web at 25 mm from the connection plate surface. These relationships consider a 45 degree spreading of the load from the ends of the end plate, as shown in Figure 3.5.

For the material tested, Hafez's proposals in SI units are

[3.13] $\sigma_c = 543\ \Delta$

for $0 < \sigma_c < 166$ MPa

and

[3.14] $\sigma_c = -146 + 12.3\ \Delta - 330\ \Delta^{1/2} + 727\ \Delta^{1/3}$

for $166 \text{ MPa} < \sigma_c$

3.3.2 Modifications to the Hafez Relationships

3.3.2.1 Tension Region

Comparisons of test results with the analytical tensile load-deformation relationships given by Equations [3.1] and [3.11] (Hafez 1982) were in good agreement except for angles of deformation, α, less than about 0.1 radians and greater than about 0.3 radians. For angles less than 0.1 radians, the discrepancy was attributed to the assumption that the moment-curvature behaviour was elastic up to M_{up}, then perfectly plastic. For angles greater than 0.3 radians, Kennedy and Hafez (1984) showed that the discrepancy can be attributed to the incorrect assumption that the gross section of the end plate was available to resist membrane forces. Kennedy and Hafez instead proposed that the net section strength limits the magnitude of these forces. The effect of these discrepancies on the development of moment-rotation relationships is not significant, as will be shown in Section 3.4.

Kennedy and Hafez (1984) rationalized that the maximum developable membrane force in the end plate is not limited by the tensile yield strength of the gross section but by that of the net section. They modified Equation [3.11] to give

$$[3.15] \qquad T = \frac{P_{yp}^{2}\,\ell}{4\,M_{up}} \left(\frac{p - d_h}{p}\right)^2 \tan^2\alpha + \frac{4\,M_{up}}{\ell}\cos^2\alpha$$

Equation [3.15] gave better correlation with test results at angles greater than 0.3 radians than Equation [3.11]. However, modifying the original equation in this way implies that the correct flexural-tension interaction expression is

$$[3.16] \qquad \frac{M}{M_{up}} + \frac{P^2}{P_{yp}^{2}} \left(\frac{p}{p - d_h}\right)^2 = 1.0$$

Equation [3.16] implies that the plastic hinges form through the net section of the end plate for the axial response and at the gross section for the flexural response. This, however, realistically cannot be the case, and in fact the hinge lines were observed to always form through the gross section (Hafez 1982). Equation [3.7] with $P_{up} = P_{yp}$ as originally used by Hafez (1982) is considered correct, provided that a limit on the membrane force is imposed.

From the free body diagram of Figure 3.6, the force P that can develop in the end plate is

$$P = \frac{2\,M}{\ell} \sin\alpha + \frac{H}{\cos\alpha} \tag{3.17}$$

Combining Equation [3.17] with the flexural-membrane interaction expression (Equation [3.7] with $P_{up} = P_{yp}$) gives, in quadratic form, the maximum value of P as

$$P_{max} = \frac{2\,M_{up}}{\ell}\left(1 - \frac{P_{max}^{2}}{P_{yp}^{2}}\right) \sin\alpha + \frac{H_{max}}{\cos\alpha} \tag{3.18}$$

where the force H_{max} is the smallest of the bolt shear capacity, the net section tensile capacity, and the edge bearing capacity, as depicted in Figure 3.6(c). The equations to predict these limiting loads as given in CSA Standard CAN3-S16.1-M78 (CSA 1978), without resistance factors, are respectively

$$V_u = 0.60 \times 0.70\, A_b\, \sigma_{ub} \tag{3.19}$$

$$T_u = (p - d_h)\, t_p\, \sigma_{up} \tag{3.20}$$

and

$$B_u = e\, t_p\, \sigma_{up} < 3\, d_b\, t_p\, \sigma_{up} \tag{3.21}$$

In Equation [3.19] the factor 0.70 accounts for the case when the shear plane passes through the threads, as is likely to be the case for flexible end plate connections. Because of the small edge distances used in end plate connections the upper limit on bearing strength in Equation [3.21] is unlikely to govern.

Modifications to Equations [3.19] and [3.20] could be considered as they may over-estimate the maximum force developable. The shear resistance of the bolts to the force H, as given by Equation [3.19], is reduced because of the tensile force developed in the bolts (Chesson et al. 1965) and because of the primary shear acting on the end plate connection. The tensile resistance as given by Equation [3.20] does not take into account any reduction due to the primary shear on the connection.

Figures 3.7 and 3.8 show for two typical end plate connection details the membrane load-deformation relationships based on the equation used by Hafez (1982), on the equation used by Kennedy and Hafez (1984), and on equations proposed herein. The connection details considered have the following material and geometric properties:

$$\sigma_{yp} = 300 \text{ MPa}$$
$$\sigma_{up} = 450 \text{ MPa}$$
$$\sigma_{ub} = 825 \text{ MPa}$$

$$w = 10 \text{ mm}$$
$$p = 75 \text{ mm}$$
$$e = 30 \text{ mm}$$
$$f = 6 \text{ mm}$$
$$d_b = 20 \text{ mm}$$
$$t_p = 6 \text{ mm or } 12 \text{ mm}$$
$$g = 100 \text{ mm or } 150 \text{ mm},$$

From these two figures, it is seen that both the Hafez and the Kennedy and Hafez equations give greater values than Equation [3.18] for relatively large deformations when H_{max} is limited by the bearing capacity of the plate or by the shearing capacity of the fasteners (a similar condition could likely be developed where the net section capacity of the connection plate limits the value of H_{max}). For small deformations, Equation [3.18] gives larger values for P because it is assumed that H_{max} acts without any deformation. However, Fisher (1965), Wallaert and Fisher (1965), Kato and Aoki (1970), Bahia and Martin (1980), Owens et al. (1981), and Frank and Yura (1981) all show that the ultimate forces are attained only with substantial deformations.

In Figures 3.7 and 3.8 it is seen that the maximum value of P given by Equation [3.18] is not significantly larger than the value given by

$$[3.22] \qquad P_{max} = \frac{H_{max}}{\cos \alpha}$$

Using this value as an approximation for P_{max} and setting this equal to P as given by Equation [3.9] (with P_{up} replaced with P_{yp}) the angle of deformation when H_{max} is reached is given by

$$[3.23] \qquad \alpha_p = \tan^{-1} \left(\frac{4 H_{max} M_{up}}{\ell P_{yp}{}^2}\right)$$

or, in terms of deformation,

$$[3.24] \qquad \Delta_p = \frac{4 H_{max} M_{up}}{P_{yp}{}^2}$$

These limits are marked on Figures 3.7 and 3.8. Equation [3.11] would be considered valid below these deformations and Equation [3.18] or [3.22] above them.

Combining Equations [3.22], [3.6], and [3.7] (with P_{up} replaced with P_{yp}) gives the relationship between the tensile force T and the deformation angle α for angles greater than α_p as

$$[3.25] \qquad T = 2\, H_{max} \tan\alpha + \frac{4\, M_{up}}{\ell} \cos^2\alpha - \frac{4\, M_{up}\, H_{max}^{\;2}}{\ell\, P_{yp}^{\;2}}$$

or, in terms of deformation,

$$[3.26] \qquad T = \frac{2\, H_{max}}{\ell}\, \Delta + \frac{4\, M_{up}\, \ell}{\left(\ell^2 + \Delta^2\right)} - \frac{4\, M_{up}\, H_{max}^{\;2}}{\ell\, P_{yp}^{\;2}}$$

Figures 3.9 to 3.13 compare three analytical methods with the T-section test results of Hafez (1982). For angles of deformation greater than about 0.2 to 0.3 radians, the Hafez relationship (Equation [3.11]) overestimates the tensile force developed. No limit other than yielding of the gross section was placed on the maximum membrane force developable. Equation [3.15], proposed by Kennedy and Hafez, which assumes membrane forces are limited by the net section strength, and which assumes that the plastic hinge lines form through the hole when considering the axial forces, tends to underestimate the results. By using Hafez's Equation [3.11] when H is less than H_{max} and Equation [3.25] when H equals H_{max}, a better prediction of the load-deformation response is obtained. Although in the Hafez T-section tests none of the specimens failed by shear of the bolts, fracture of the net section, or tear out, it is probable that the limiting horizontal force, H_{max}, was near to being reached. This conclusion is corroborated by the significant deformations that were observed for the governing mode (Hafez 1982).

3.3.2.2 Compression Region

The empirical load-deformation equations developed by Hafez (Equations [3.13] and [3.14]) for the compression region of the connection are strictly valid only for steels having the same stress-strain relationships as those tested by Hafez. Owens et al. (1981), investigating the bearing characteristics of bolted plates of two different ultimate strengths, found load-deformation responses similar to that of Hafez, and further proposed that the bearing stresses developed for a given deformation were linearly related to the ultimate strength of the plate.

Because of the very close resemblance between the end plate compression specimen tests conducted by Hafez and the bolt bearing tests conducted by Owens et al., as shown in Figure 3.14, it is proposed that Equations [3.13] and [3.14] be modified for general use by multiplying them by the ratio of the ultimate strength of the plate being analyzed to that tested by Hafez (442 MPa or 64.1 ksi). (However, for the initial linear response, the relationship is probably related to elastic properties.) Equations [3.13] and [3.14] therefore become

[3.27] $$\sigma_c = 543 \left(\frac{\sigma_{uw}}{442}\right) \Delta$$

for $0 < \sigma_c < 166 \left(\frac{\sigma_{uw}}{442}\right)$ MPa

and

[3.28] $$\sigma_c = \left(\frac{\sigma_{uw}}{442}\right)\left(-146 + 12.3\,\Delta - 330\,\Delta^{1/2} + 727\,\Delta^{1/3}\right)$$

for $166 \left(\frac{\sigma_{uw}}{442}\right) \text{ MPa} < \sigma_c$

3.4 Comparison of Moment-Rotation Analysis with Test Results

In Figures 3.15 through 3.22 the analytical relationships developed by Hafez, Kennedy and Hafez, and as proposed herein are compared with the Hafez (1982) test results, while in Figures 3.23 to 3.42 these analyses are compared to the Sommer (1969) results. Each curve uses a different combination of the load-deformation relationships. The Proposed Analysis curves were developed using Equations [3.2], [3.12], and [3.26] in the tension zone of the connection, and Equations [3.27] and [3.28] in the compression zone. The Kennedy-Hafez Analysis curves were developed using Equations [3.2] and [3.15] in the tension zone, and Equations [3.13] and [3.14] in the compression zone. The Hafez Analysis curves were developed using Equations [3.2] and [3.12] in the tension zone, and using Equations [3.13] and [3.14] in the compression zone. The analytical curves are terminated where they predict bearing of the bottom flange of the beam against the support to occur. This point is indicated by a short transverse line at the end of the analytical curves and this phenomenon will be discussed in detail in Chapter 4. In developing the analytical curves the specimens were divided into 100 equal elements over the depth of the connection. The computer program used to generate the curves is given in Appendix A.

All three of the analyses predict the slope of the curves reasonably well. Table 3.1 lists test-to-predicted values of the moment and corresponding rotation developed in the connections at the instant that bearing of the supported beam on the support occurs. Using the Kennedy-Hafez analysis, the mean value of the test to predicted ratio for the moment is 1.18, with a coefficient of variation of 0.11. For the proposed method, the mean value for the ratio is 1.06, with a coefficient of variation of 0.10. The better prediction by the proposed method is attributed to the fact that it takes into account more rationally the limit on the membrane force developable in the tension zone of the connection and because it considers more rationally the compressive forces developable at the bottom of the connection. The comparison of rotations is discussed Chapter 4.

At relatively small rotations, less than about 0.01 radians, the analyses overestimate the moment. As reported by Hafez (1982), this is due to the fact that the tension behaviour has been modelled as elasto-plastic without allowing for gradual yielding.

No deformations of the web of the beam, the fillet welds, or the bolts were considered in the analysis. In the end plate tension tests (Hafez 1982), these deformations were relatively small. Butler and Kulak (1971) found for transversely loaded 6.4 mm fillet welds that the ultimate deformation was between 0.5 and 1.0 mm, which is relatively small as compared to total deformations of the entire end plate assembly observed to be up to 30 mm (Hafez 1982). Rumpf and Fisher (1963) and Sterling et al. (1965) found that elongations of bolts loaded up to 95 percent of the tensile capacity rarely exceeded 1 mm. This substantiates the assumption used, that is, that the deformation takes place chiefly in the end plate itself.

In both the Sommer (1969) and Hafez (1982) tests, the end plate connections were connected to a heavy column flange, as depicted in Figure 3.43. Figures 3.44 through 3.52 show a variety of other details. If the supports were relatively flexible, it is likely that the end moments developed would be less than observed in these tests. Coping of the beam web above or below the connection would also make the connection behaviour more flexible. Depending on the amount of increased flexibility, this may significantly reduce the moment predicted by any of the analyses.

Although Kennedy (1969) reported that large primary shear forces did not affect the moment-rotation behaviour of flexible end plate connections, which is substantiated within the limits of the comparisons here with the Sommer (1969) and Hafez (1982) tests, it is possible that very large primary shears might reduce the end moments developed in a manner similar to the condition observed in steel beams carrying large shear forces and bending moments simultaneously (ASCE 1971).

Based on this discussion it appears that predictions of moments by the method given here would tend to be conservative (that is, overestimate the moments).

Table 3.1 Test-to-Predicted Ratios of the Flexible End Plate Moment and Rotation at the Occurrence of Bottom Flange Bearing

Test No.[a]	TEST		KENNEDY-HAFEZ ANALYSIS				PROPOSED ANALYSIS			
	M_t[b] (in kips)	θ_t[c] (rads)	M_{pr} (in kips)	θ_{pr} (rads)	$\frac{M_t}{M_{pr}}$	$\frac{\theta_t}{\theta_{pr}}$	M_{pr} (in kips)	θ_{pr} (rads)	$\frac{M_t}{M_{pr}}$	$\frac{\theta_t}{\theta_{pr}}$
S5	791	0.0617	609	0.0536	1.30	1.15	729	0.0554	1.09	1.11
S6	190	0.0242	171	0.0164	1.11	1.48	181	0.0167	1.05	1.45
S7	361	0.0218	306	0.0197	1.18	1.11	333	0.0201	1.08	1.08
S8	625	0.0269	503	0.0247	1.24	1.09	580	0.0251	1.08	1.07
S9	971	0.0299	822	0.0319	1.18	0.937	996	0.0327	0.975	0.914
S10	329	0.0260	298	0.0363	1.10	0.716	326	0.0373	1.01	0.697
S11	678	0.0443	548	0.0450	1.24	0.984	619	0.0471	1.10	0.941
S12[d]	-	-	952	0.0594	-	-	1080	0.0623	-	-
S13	425	0.0272	330	0.0229	1.29	1.19	354	0.0234	1.20	1.16
S14	730	0.0316	586	0.0263	1.25	1.20	640	0.0272	1.14	1.16
S15	849	0.0408	665	0.0355	1.28	1.15	752	0.0365	1.13	1.12
S16	1410	0.0533	1090	0.0448	1.29	1.19	1230	0.0468	1.15	1.14
S17	258	0.0264	198	0.0211	1.30	1.25	217	0.0215	1.19	1.23
S18	393	0.0301	340	0.0271	1.16	1.11	398	0.0276	0.987	1.09
S19	752	0.0469	623	0.0354	1.21	1.32	677	0.0366	1.11	1.28
S20	1100	0.0480	995	0.0420	1.11	1.14	1110	0.0436	0.991	1.10
S25	443	0.0486	308	0.0376	1.44	1.29	360	0.0376	1.23	1.29
S26	163	0.0237	159	0.0276	1.03	0.859	173	0.0281	0.942	0.843
S27	207	0.0360	183	0.0728	1.13	0.495	216	0.0728	0.958	0.495
S28	76.4	0.0386	68.1	0.0442	1.12	0.873	74.4	0.0452	1.03	0.854

Table 3.1 Test-to-Predicted Ratios of the Flexible End Plate Moment and Rotation at the Occurrence of Bottom Flange Bearing

Test No. [a]	TEST		KENNEDY-HAFEZ ANALYSIS				PROPOSED ANALYSIS			
	M_t [b] (in kips)	θ_t [c] (rads)	M_{pr} (in kips)	θ_{pr} (rads)	$\frac{M}{M_{pr}}$	$\frac{\theta_t}{\theta_{pr}}$	M_{pr} (in kips)	θ_p (rads)	$\frac{M_t}{M_{pr}}$	$\frac{\theta_t}{\theta_{pr}}$
H1	1260	0.0272	1140	0.0303	1.11	0.898	1290	0.0312	0.977	0.872
H2	940	0.0296	729	0.0229	1.29	1.29	844	0.0233	1.11	1.27
H3	900	0.0395	818	0.0343	1.10	1.15	933	0.0355	0.965	1.11
H4	255	0.0321	194	0.0166	1.31	1.93	206	0.0169	1.24	1.90
H5	365	0.0290	333	0.0238	1.10	1.22	355	0.0241	1.03	1.20
H6	490	0.0335	521	0.0296	0.941	1.13	557	0.0303	0.880	1.11
H7	1780	0.0394	2075	0.0573	0.858	0.688	2270	0.0618	0.784	0.638
H8 [d]	-	-	1570	0.0448	-	-	1710	0.0490	-	-
				Mean Value	1.18	1.11			1.06	1.08
				Standard Dev.	0.126	0.276			0.109	0.273
				Coef. of Var.	0.107	0.248			0.103	0.253

a S refers to tests performed by Sommers (1969) and H refers to tests performed by Hafez (1982).

b The lesser moment observed when the flange did not contact the support simultaneously on both sides of the web.

c The average of the two rotations observed when the flange did not contact the support simultaneously on both sides of the web.

d In this test, the connection failed before the beam came into contact with the support.

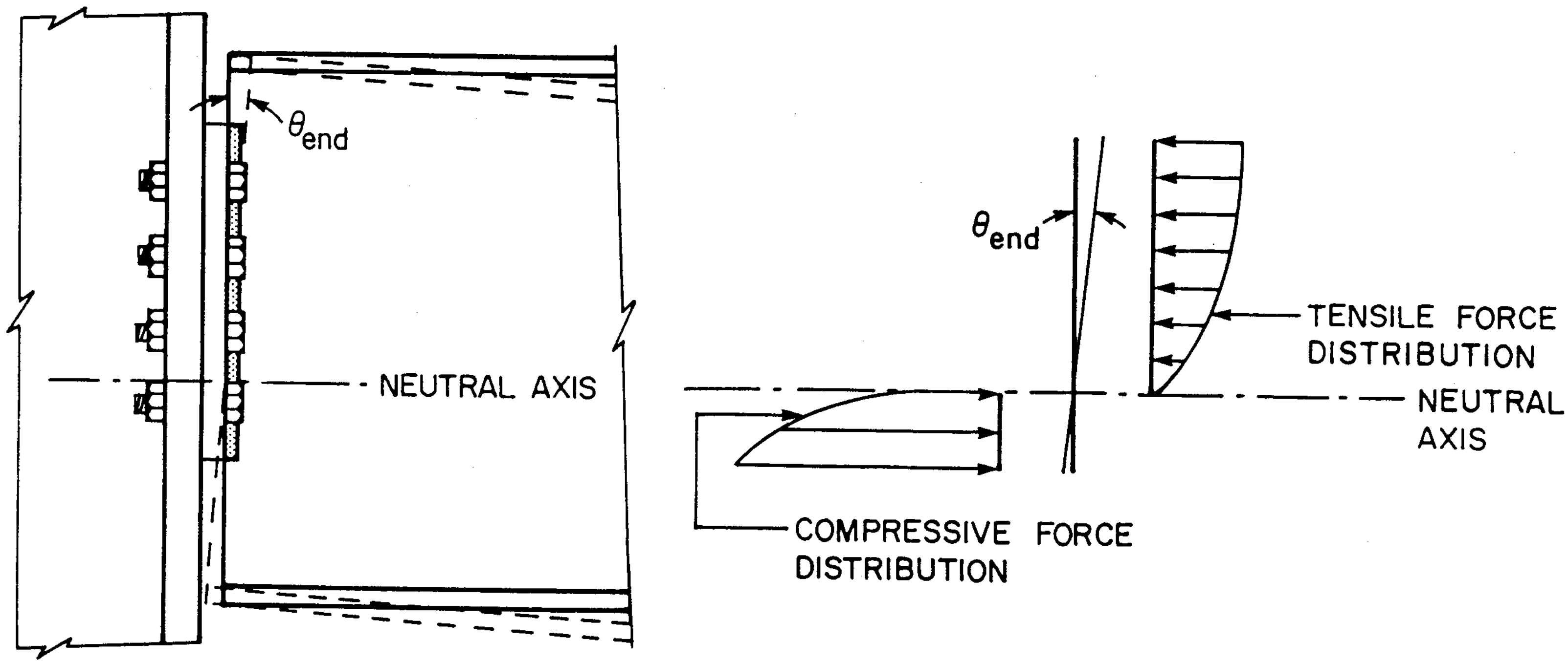


Figure 3.1 Flexural Force Distribution at Flexible End Plate Connection

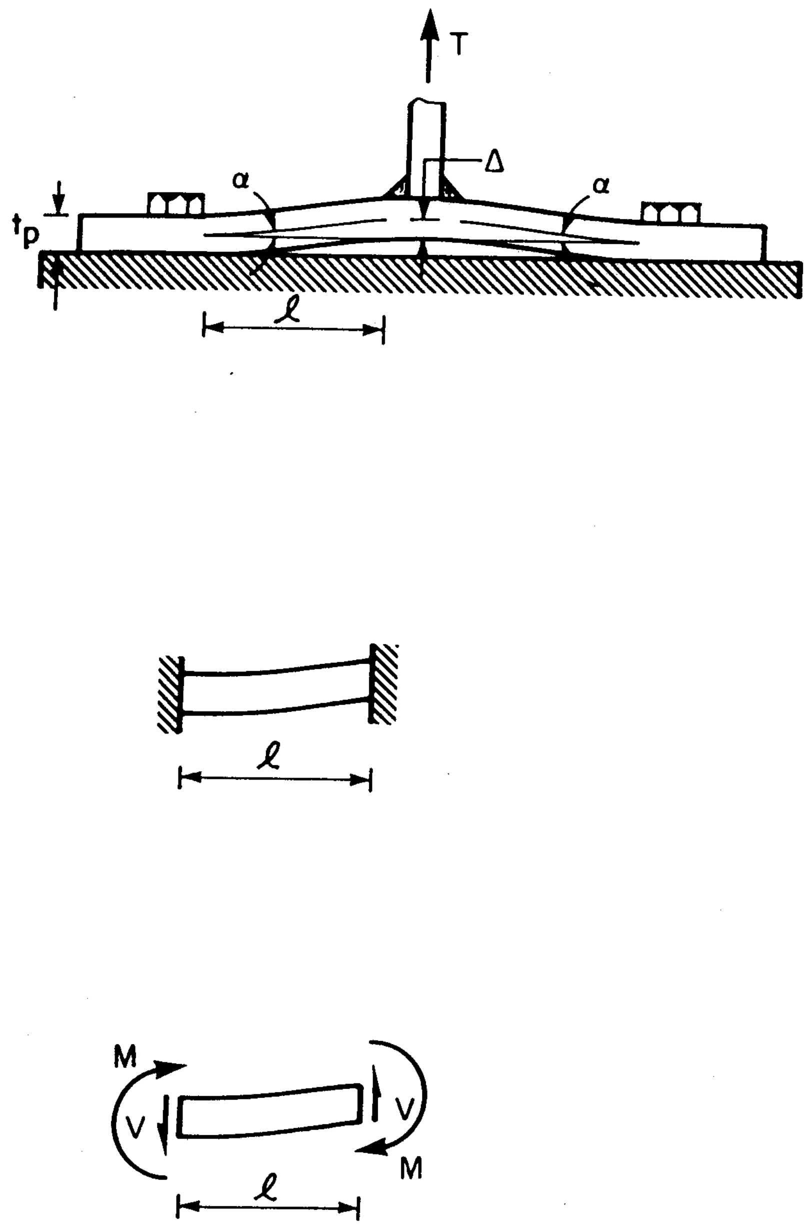
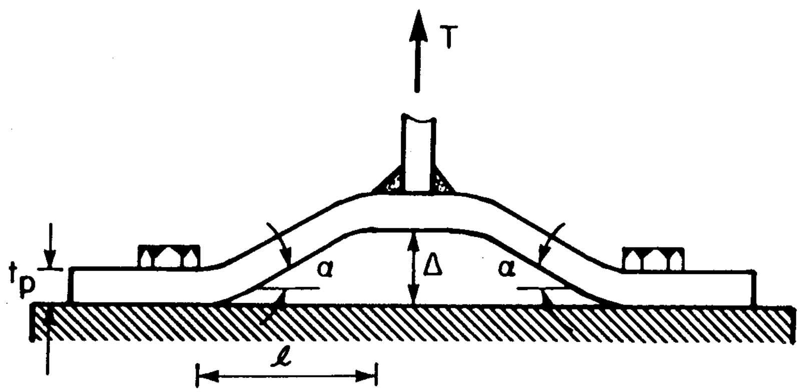
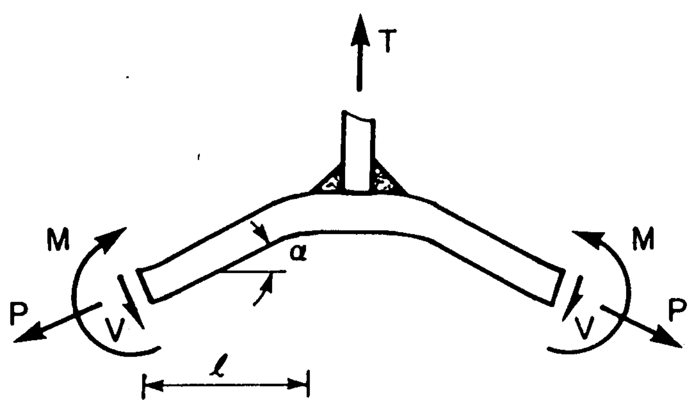


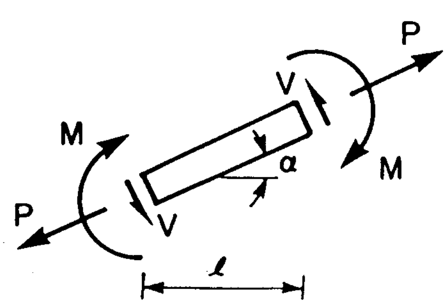
Figure 3.2 Flexible End Plate Connection, Elastic Tension Region Response



a) Deformed Tension Region



b) Free Body Diagram



c) Free Body Diagram

Figure 3.3 Flexible End Plate Connection, Inelastic Tension Region Response

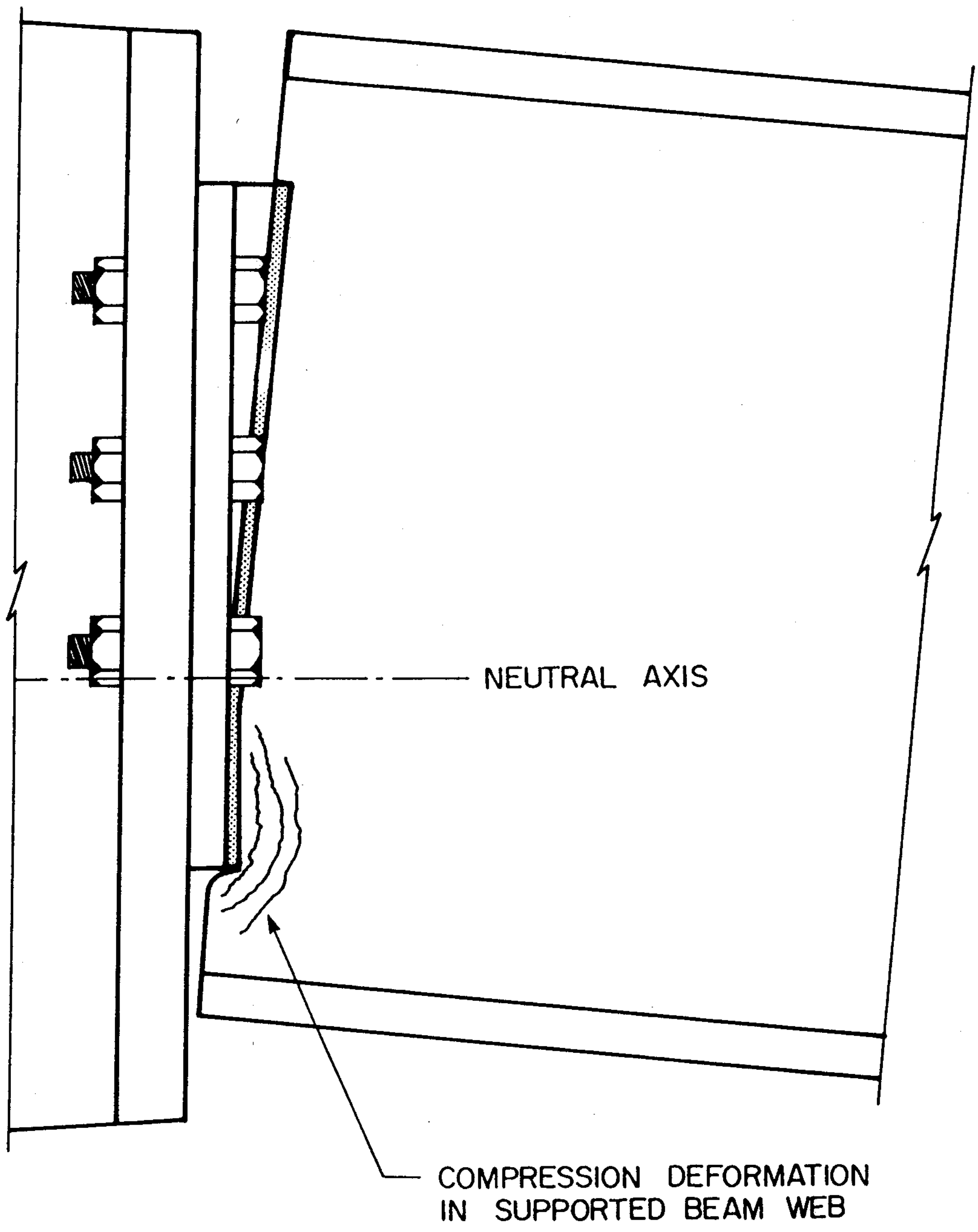
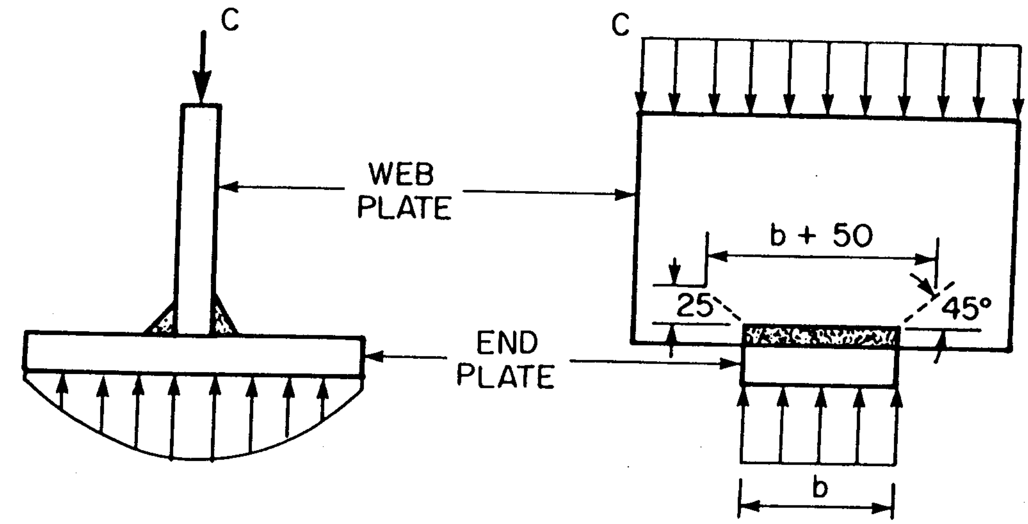
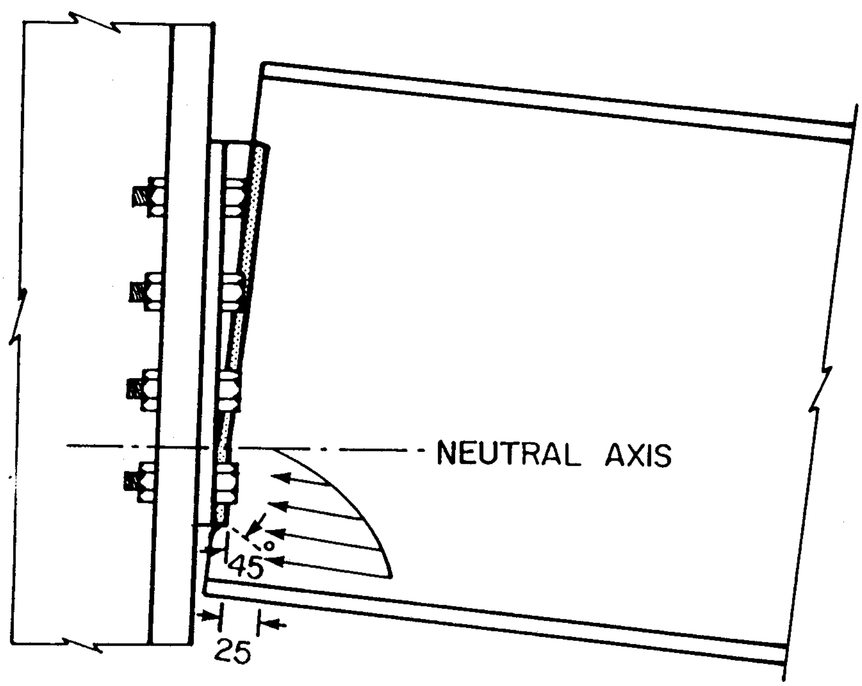


Figure 3.4 Behaviour of Compressed Region of Flexible End Plate Connection

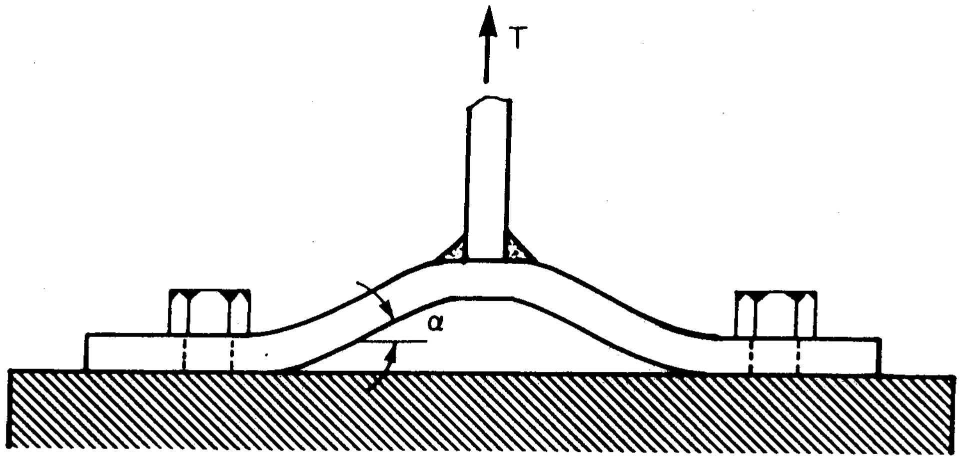


a) END PLATE COMPRESSION LOAD DEFORMATION TEST SPECIMENS

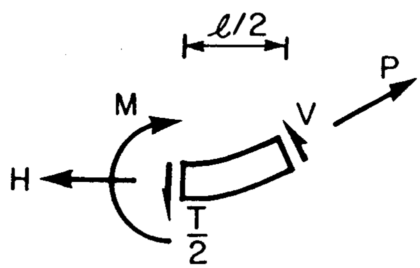


b) SPREAD OUT OF COMPRESSIVE STRESSES IN WEB OF SUPPORTED MEMBER

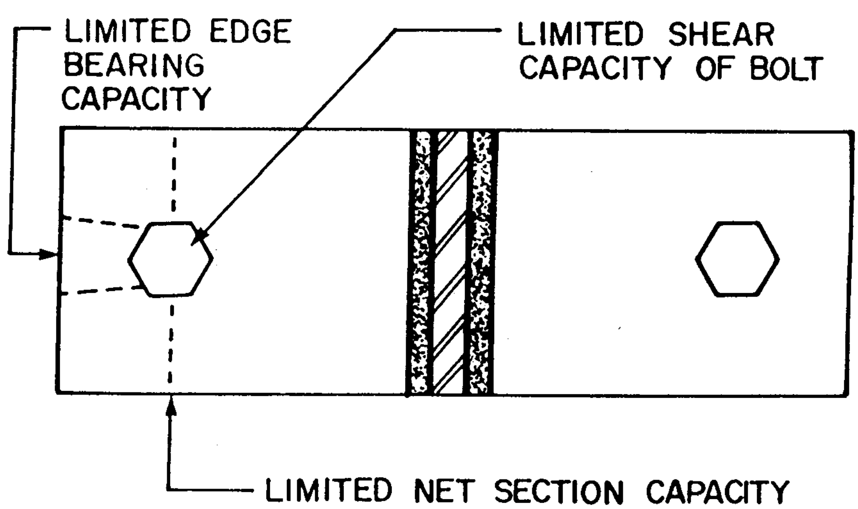
Figure 3.5 Stress Distribution in Compressed Region of Flexible End Plate Connection



a) Deformed Tension Region



b) Free Body Diagram



c) Limits on Force H

Figure 3.6 Factors Limiting the Development of Membrane Forces in Tension Region of Flexible End Plate

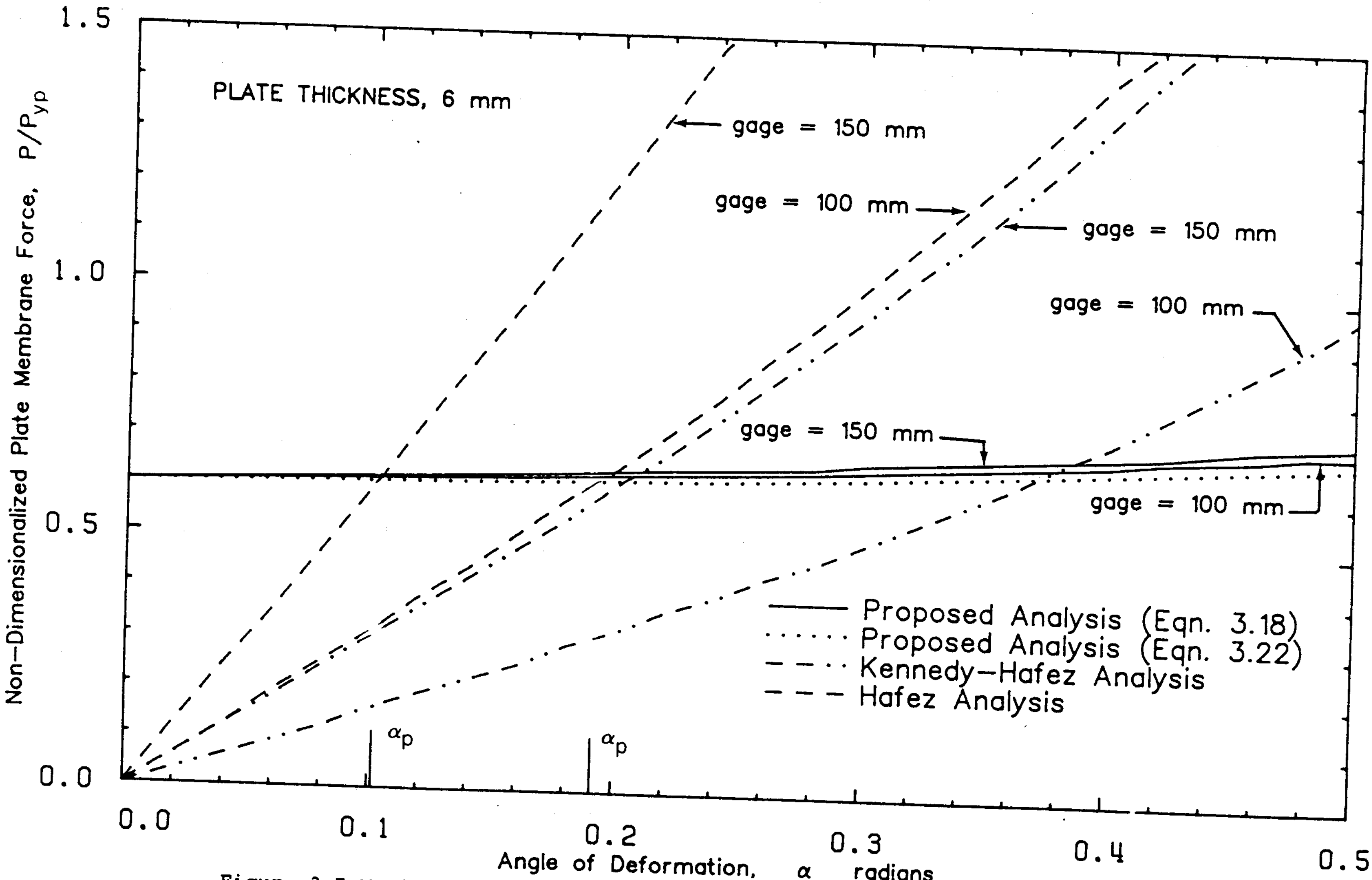


Figure 3.7 Maximum Membrane Force (Limited by Plate Bearing Strength) in a Flexible End Plate vs. Angle of Deformation

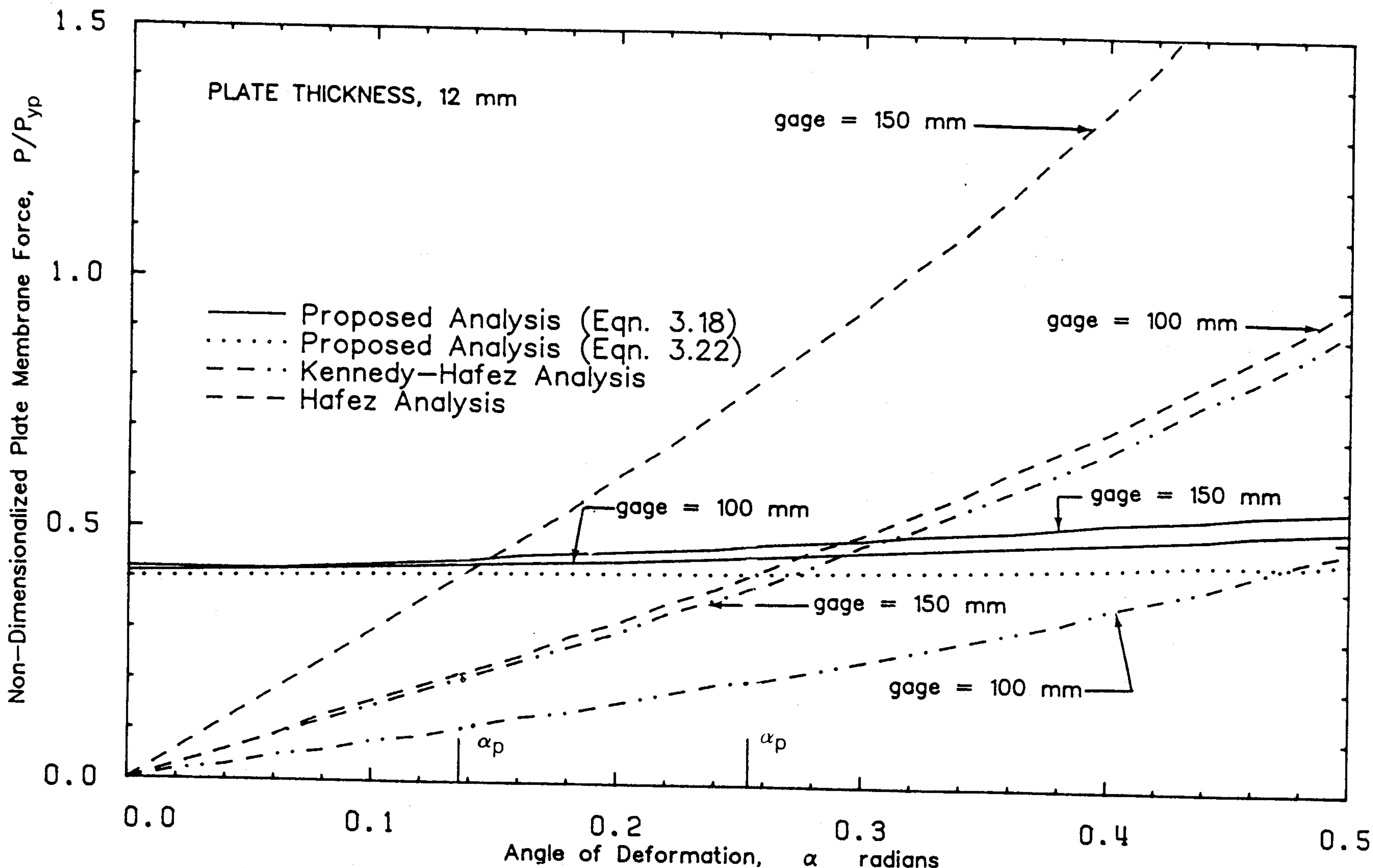


Figure 3.8 Maximum Membrane Force (Limited by Bolt Shear Strength) in a Flexible End Plate vs. Angle of Deformation

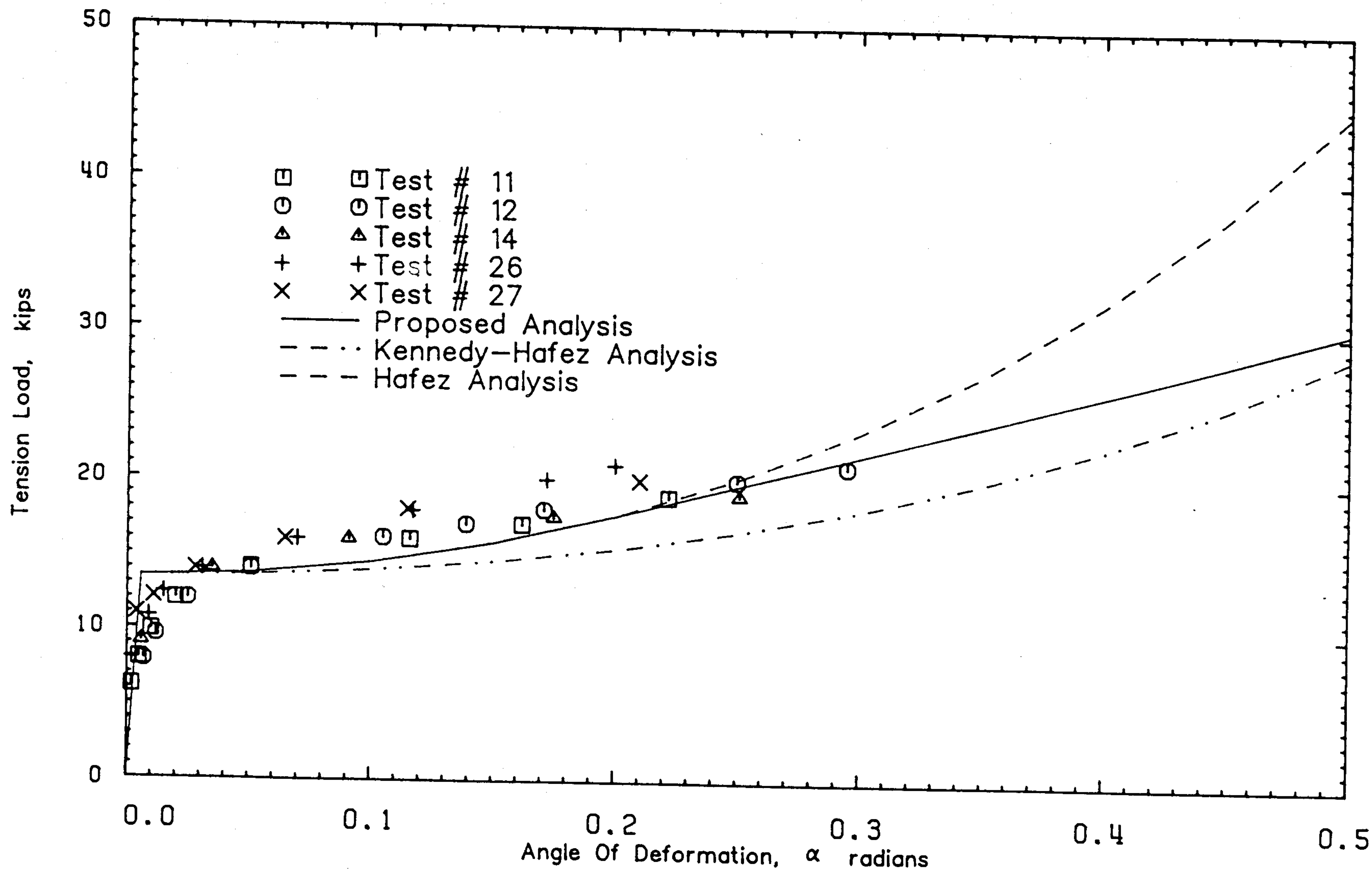


Figure 3.9 Comparison of Analytical Load-Deformation Relationships with Hafez (1982) T-Section Tests, 1/4 inch Plate and 4 inch Gage

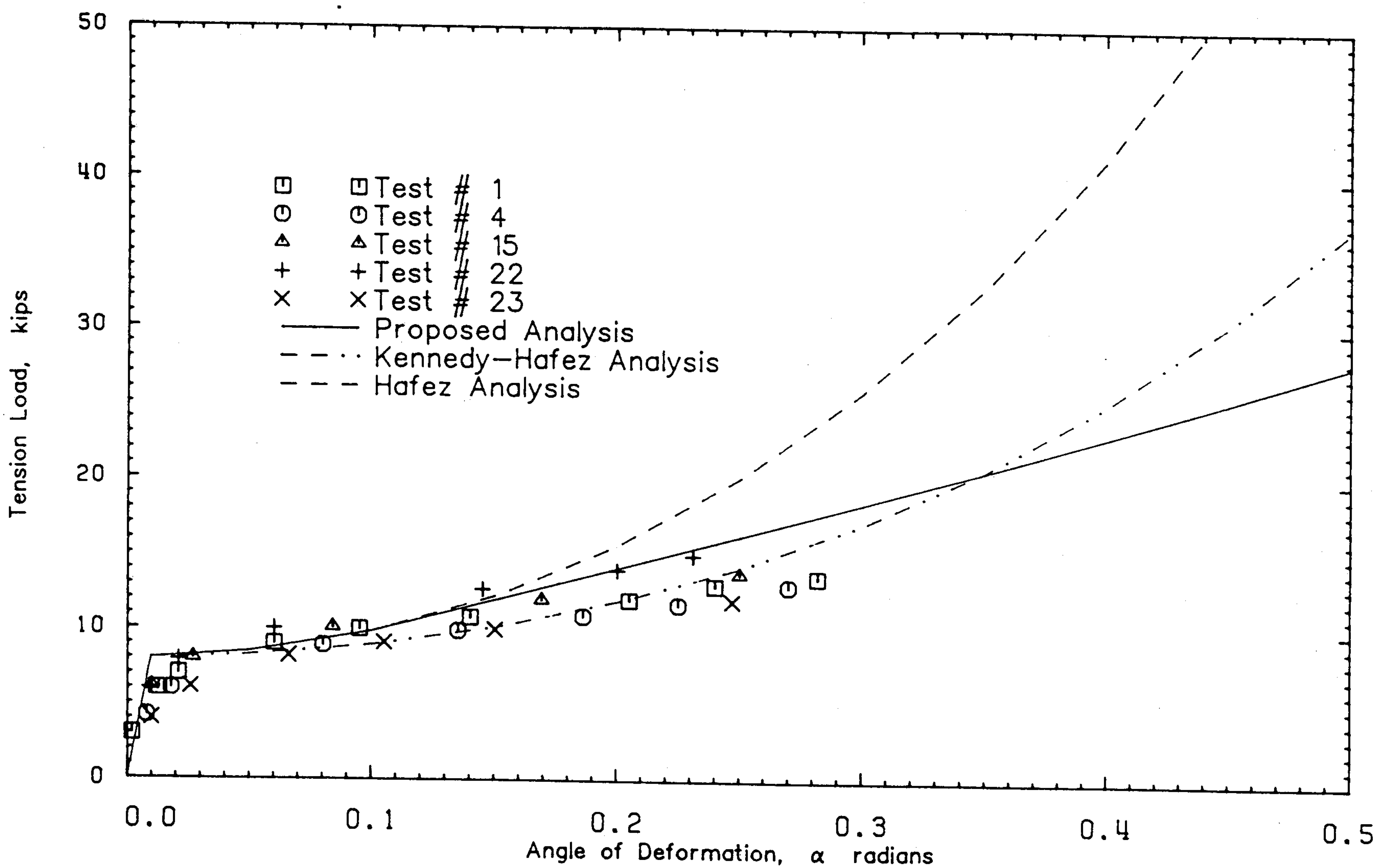


Figure 3.10 Comparison of Analytical Load-Deformation Relationships with Hafez (1982) T-Section Tests, 1/4 inch Plate and 5 1/2 inch Gage

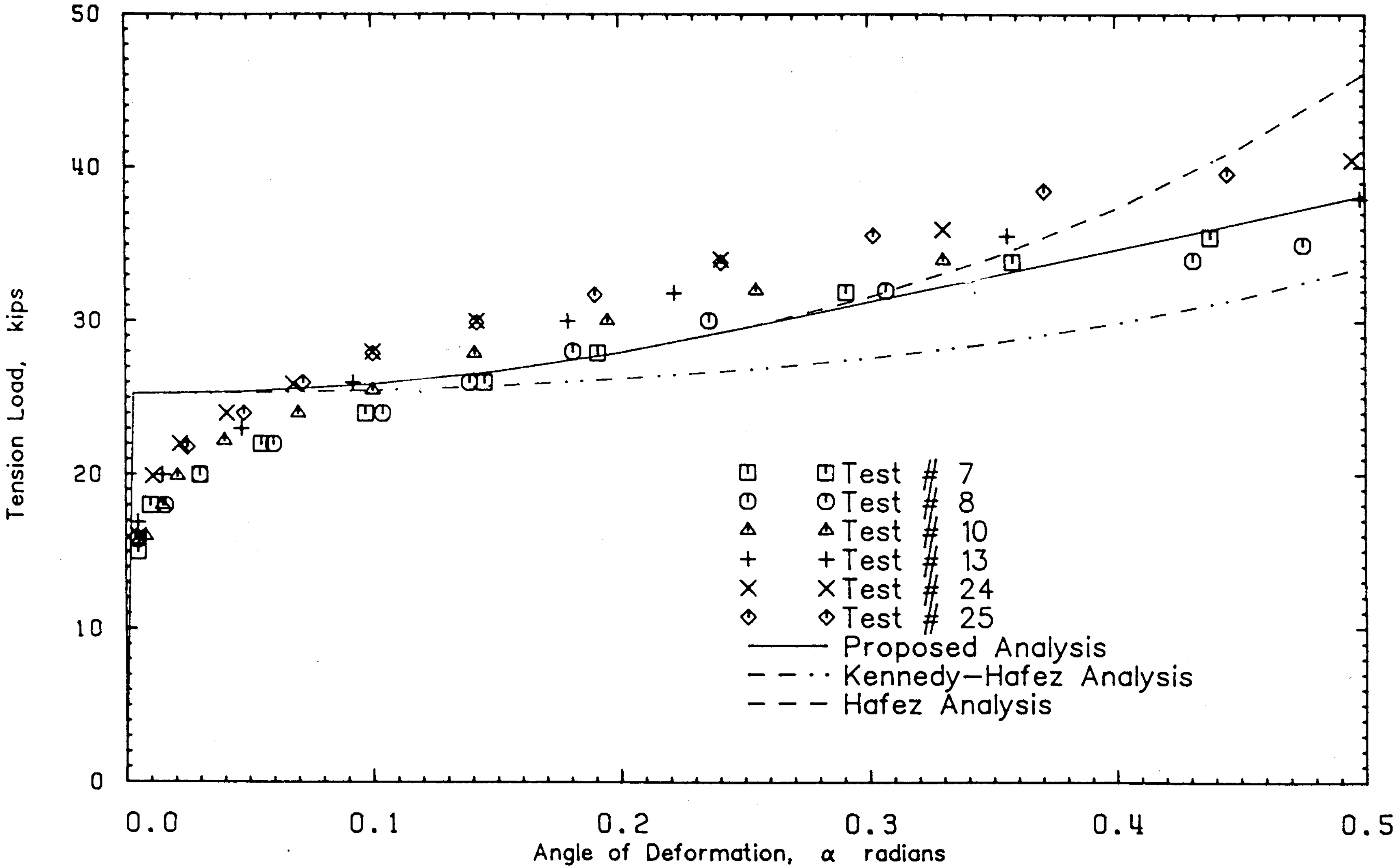


Figure 3.11 Comparison of Analytical Load-Deformation Relationships with Hafez (1982) T-Section Tests, 3/8 inch Plate and 4 inch Gage

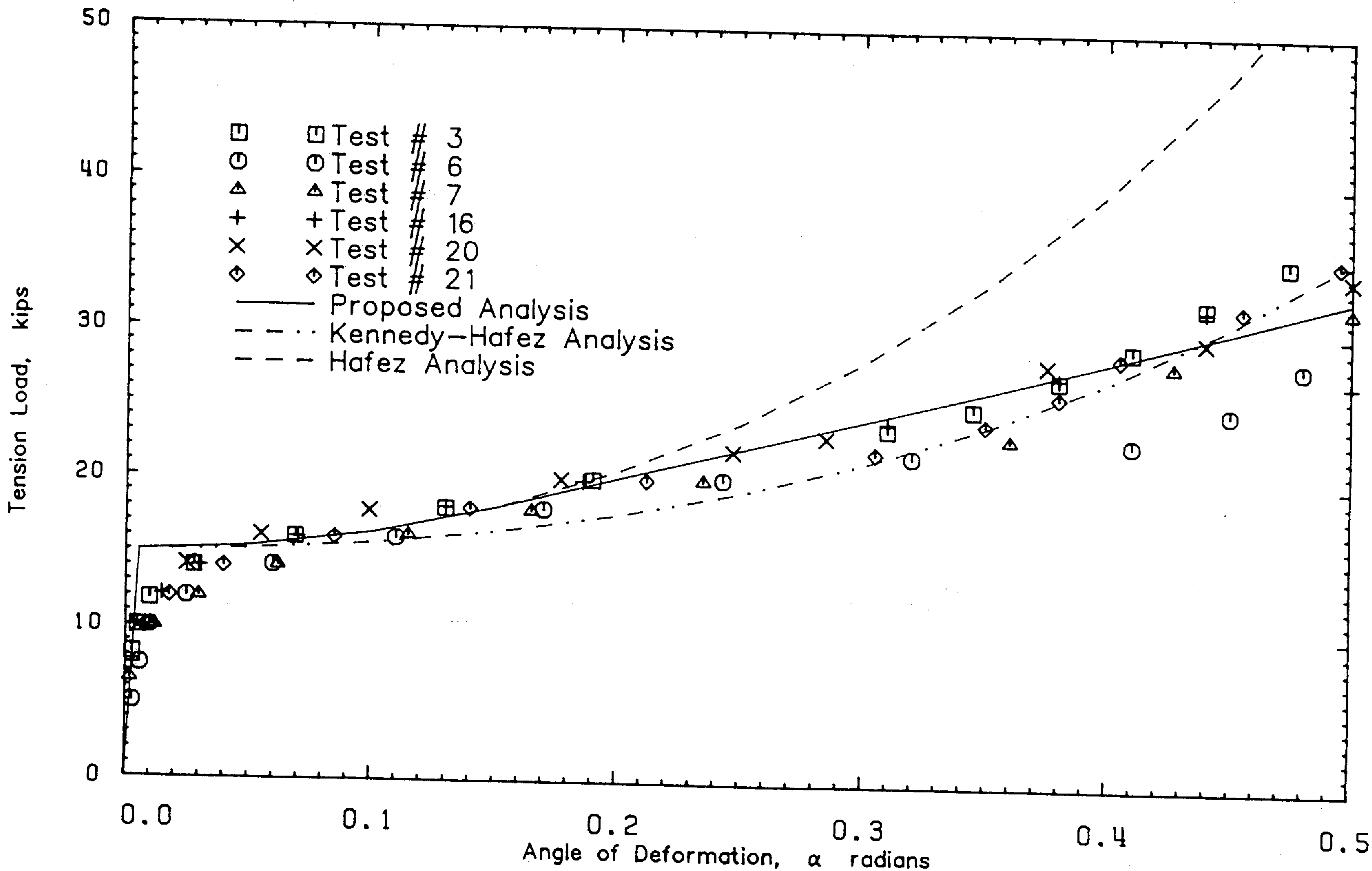


Figure 3.12 Comparison of Analytical Load-Deformation Relationships with Hafez (1982) T-Section Tests, 3/8 inch Plate and 5 1/2 inch Gage

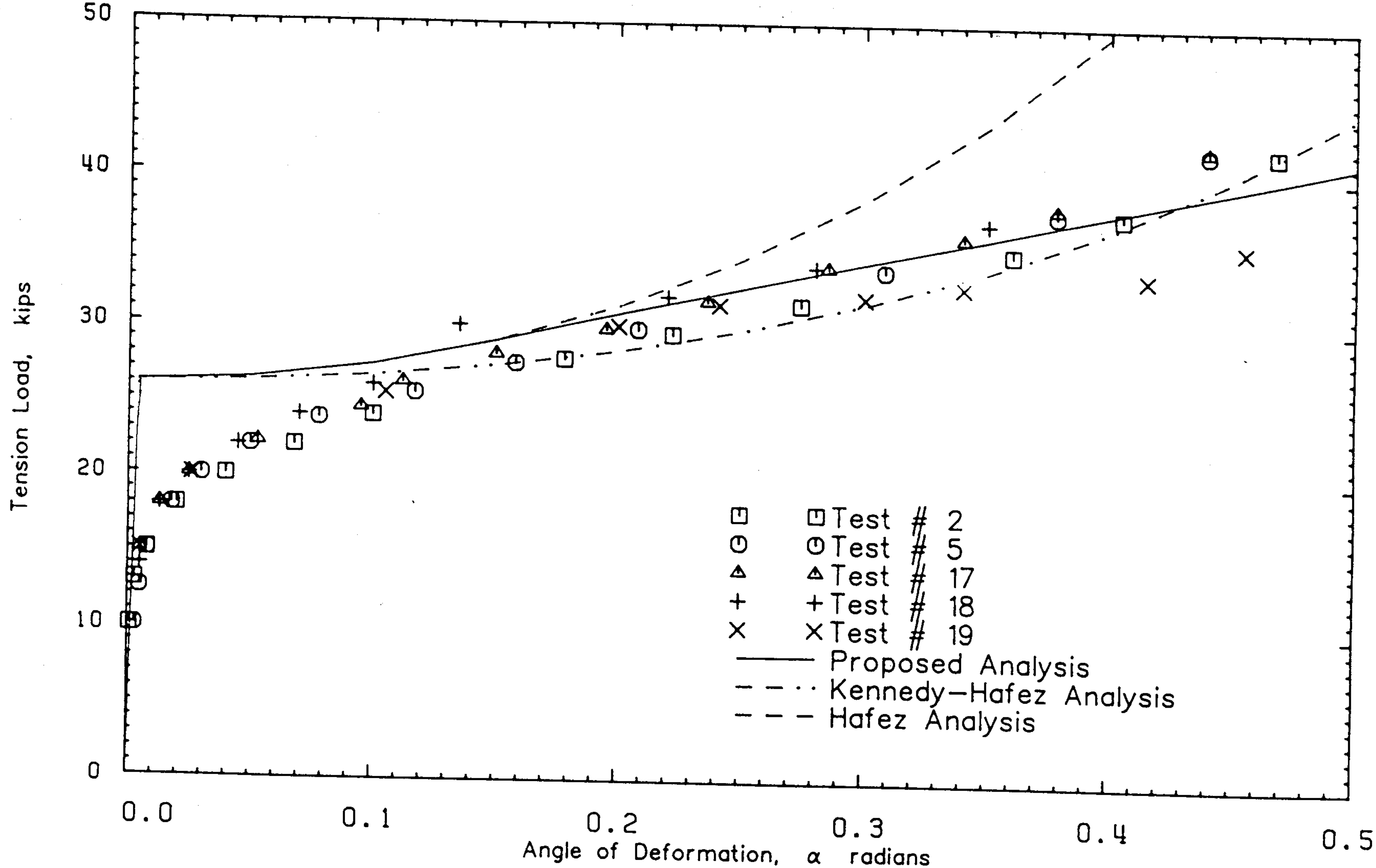


Figure 3.13 Comparison of Analytical Load-Deformation Relationships with Hafez (1982) T-Section Tests, 1/2 inch Plate and 5 1/2 inch Gage

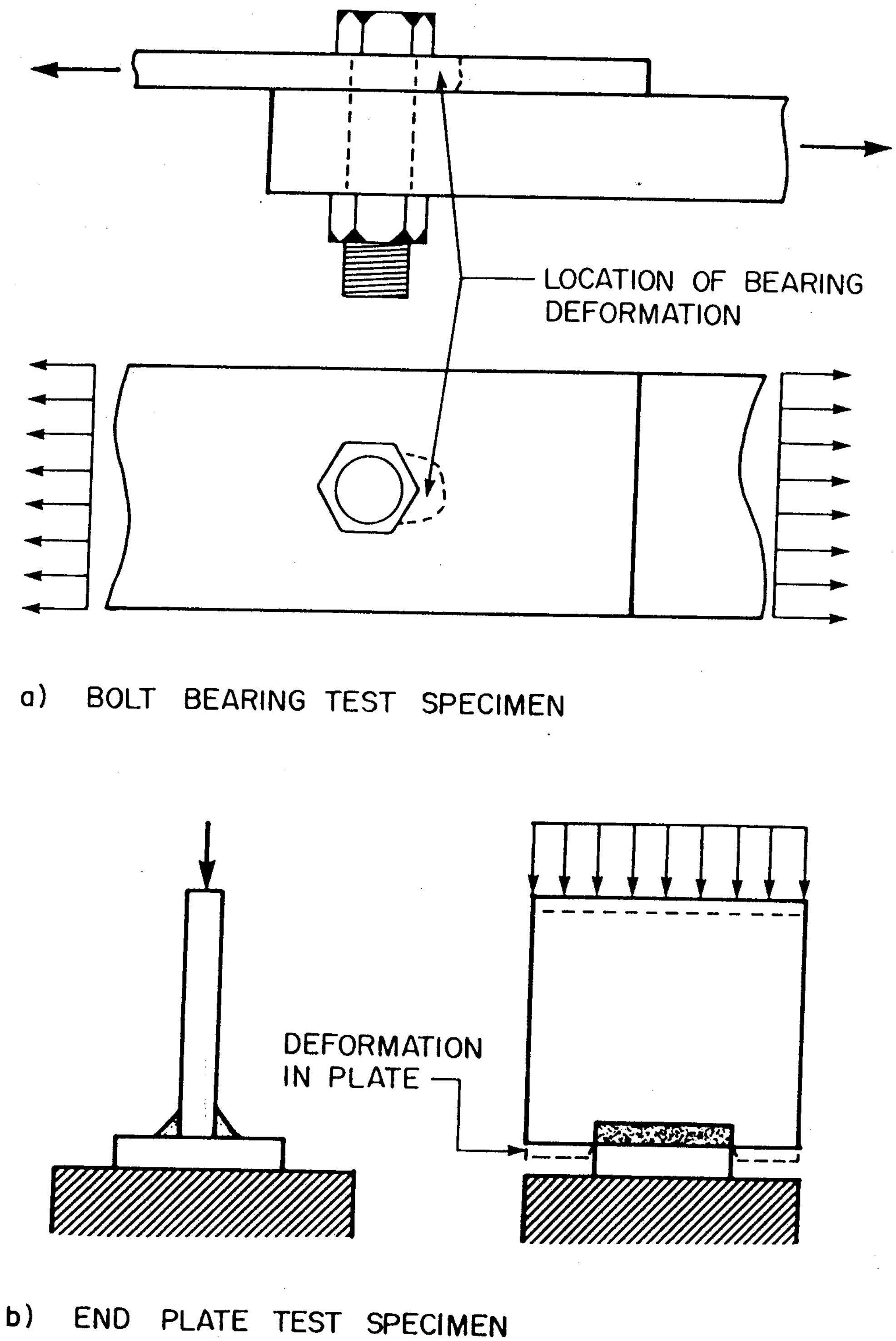


Figure 3.14 Bearing Deformation Specimens

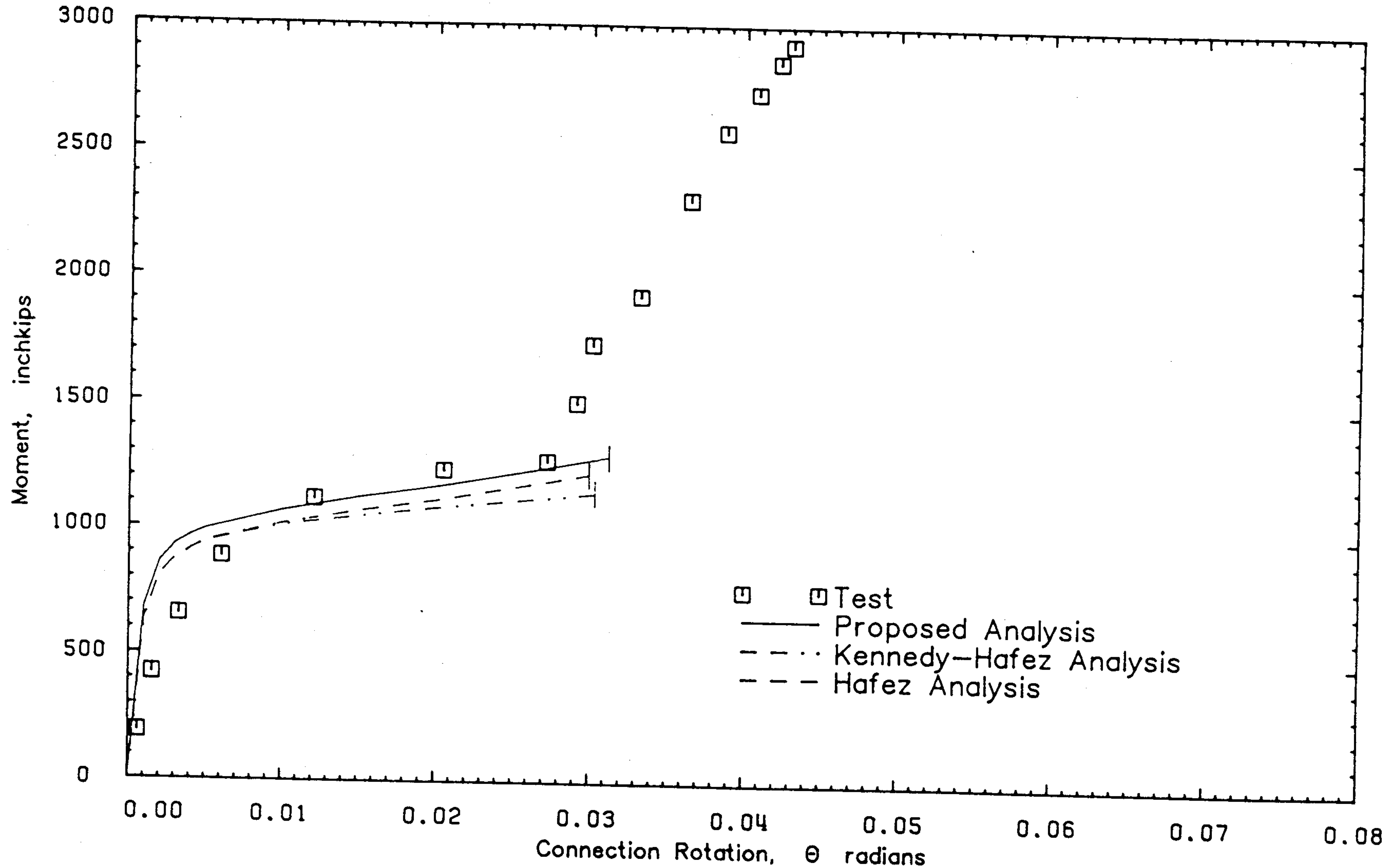


Figure 3.15 Comparison of Analytical Moment-Rotation Relationships with

Hafez (1982) End Plate Test # 1

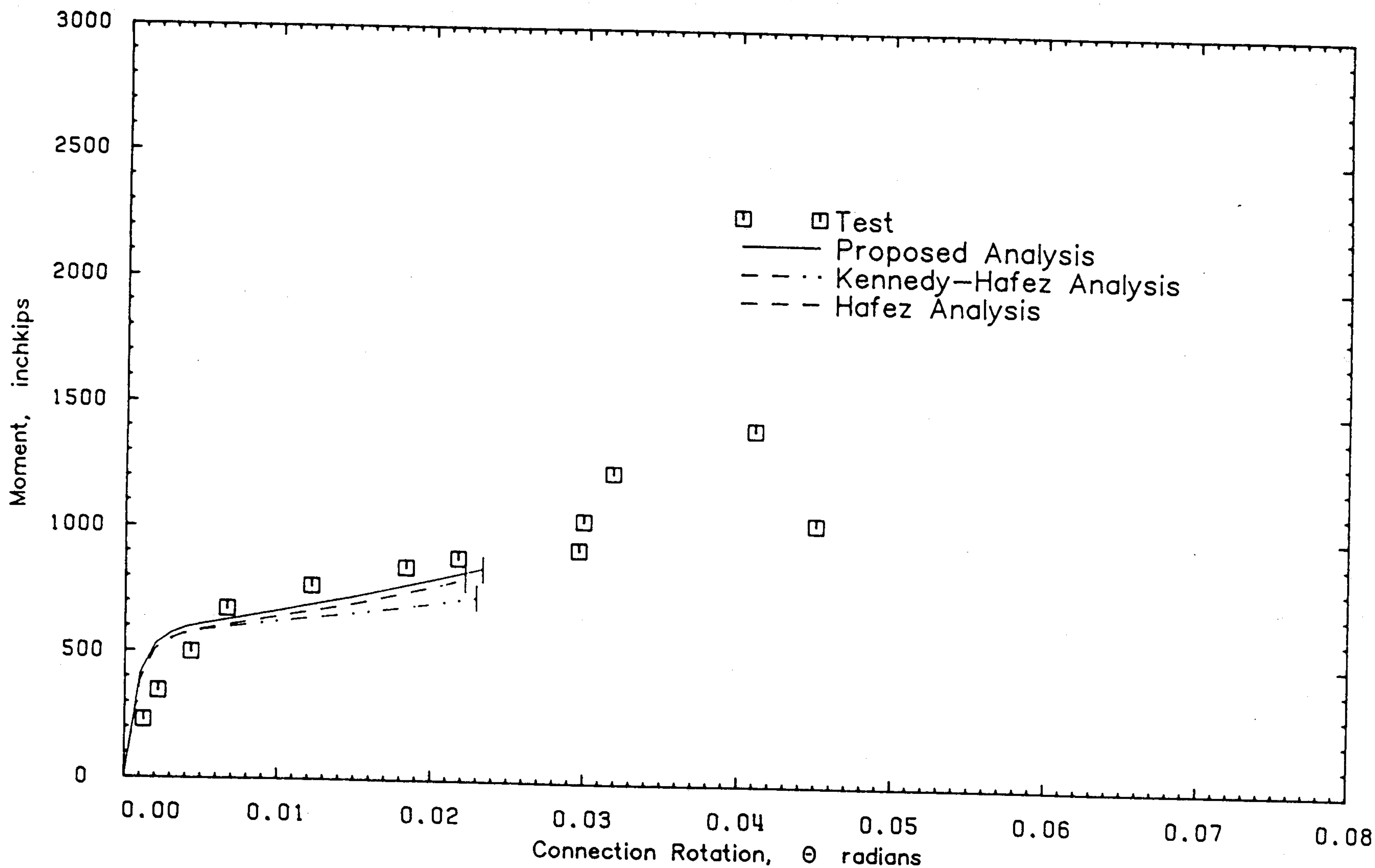


Figure 3.16 Comparison of Analytical Moment-Rotation Relationships with

Hafez (1982) End Plate Test # 2

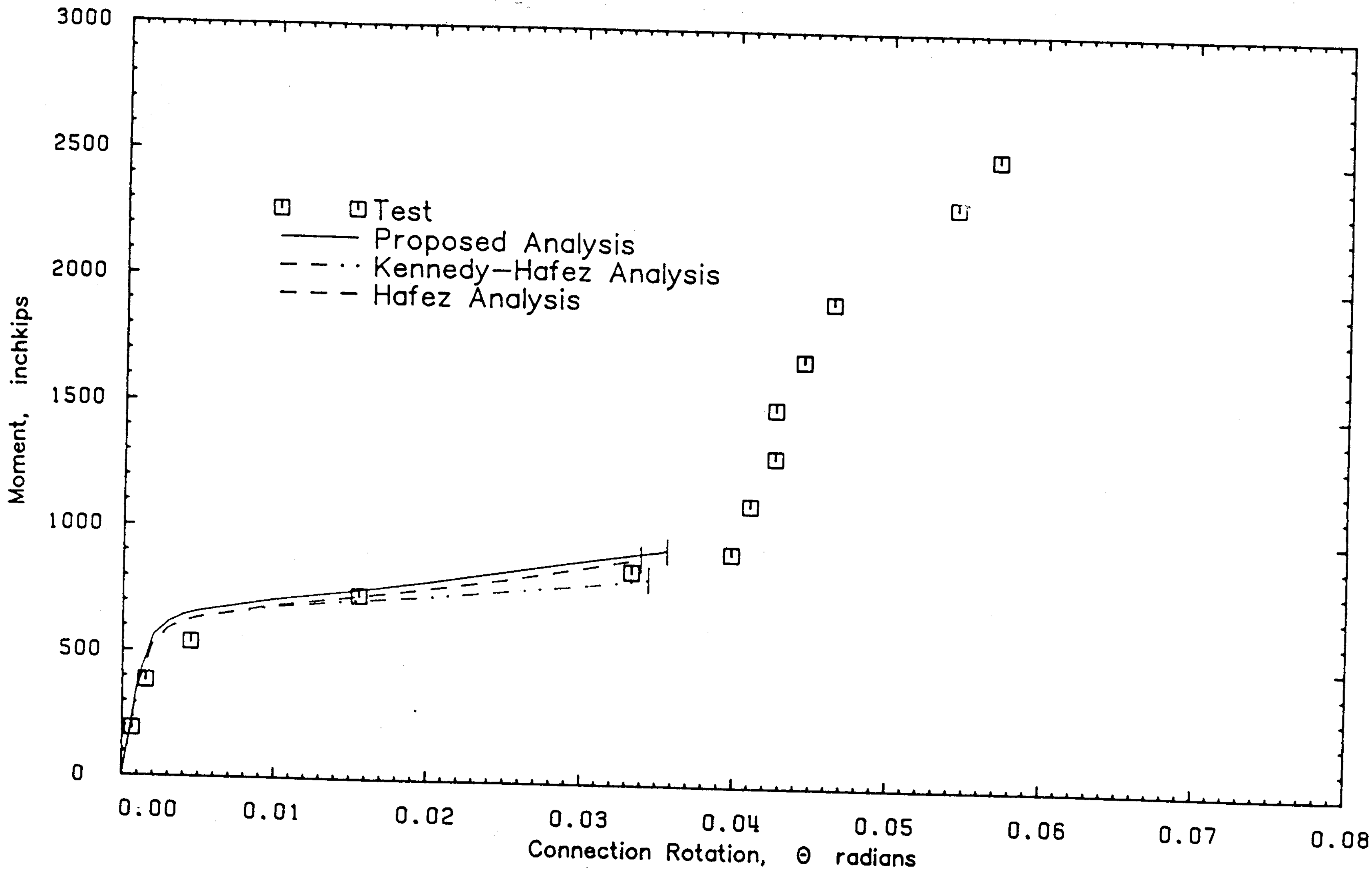


Figure 3.17 Comparison of Analytical Moment-Rotation Relationships with

Hafez (1982) End Plate Test # 3

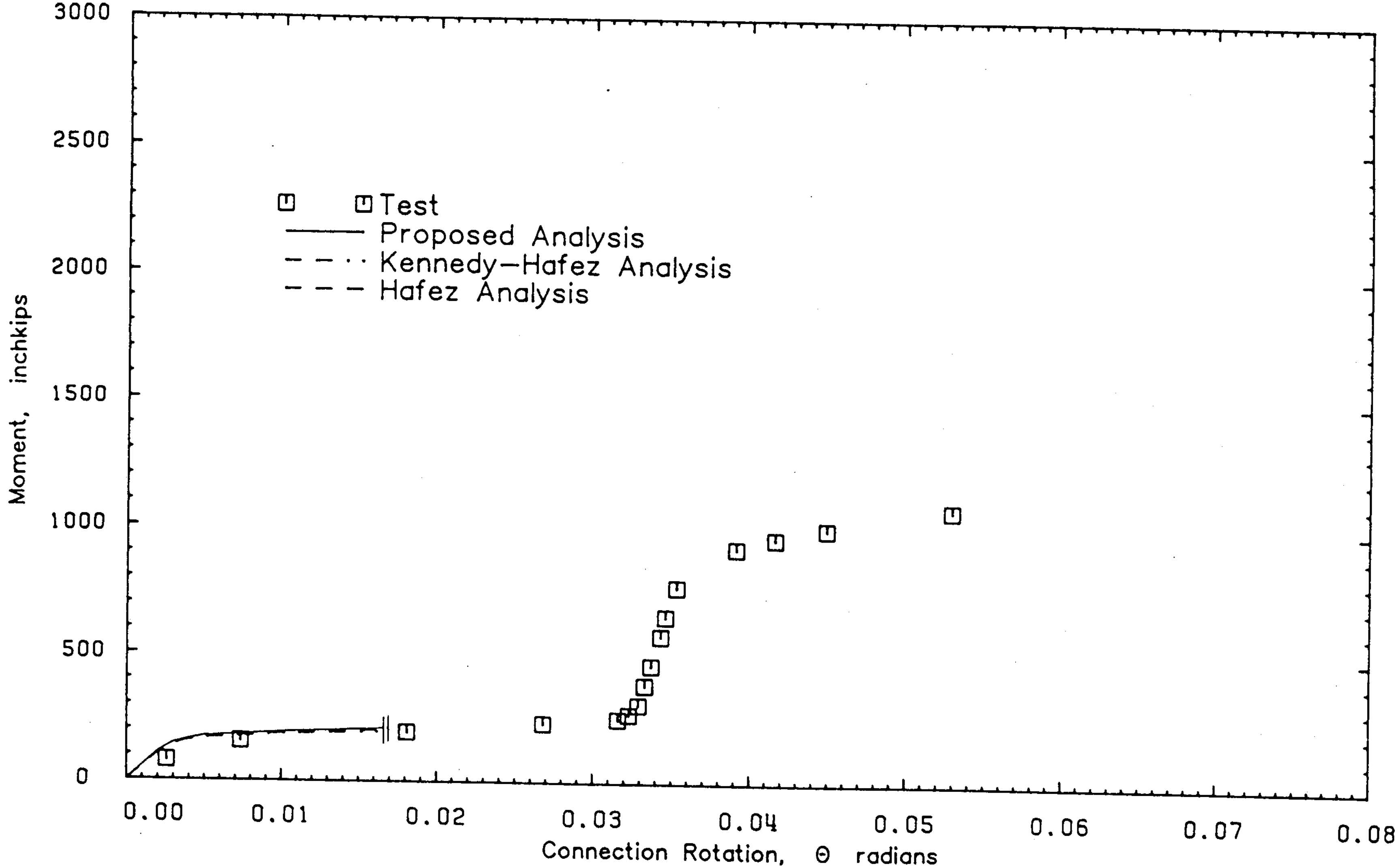


Figure 3.18 Comparison of Analytical Moment-Rotation Relationships with Hafez (1982) End Plate Test # 4

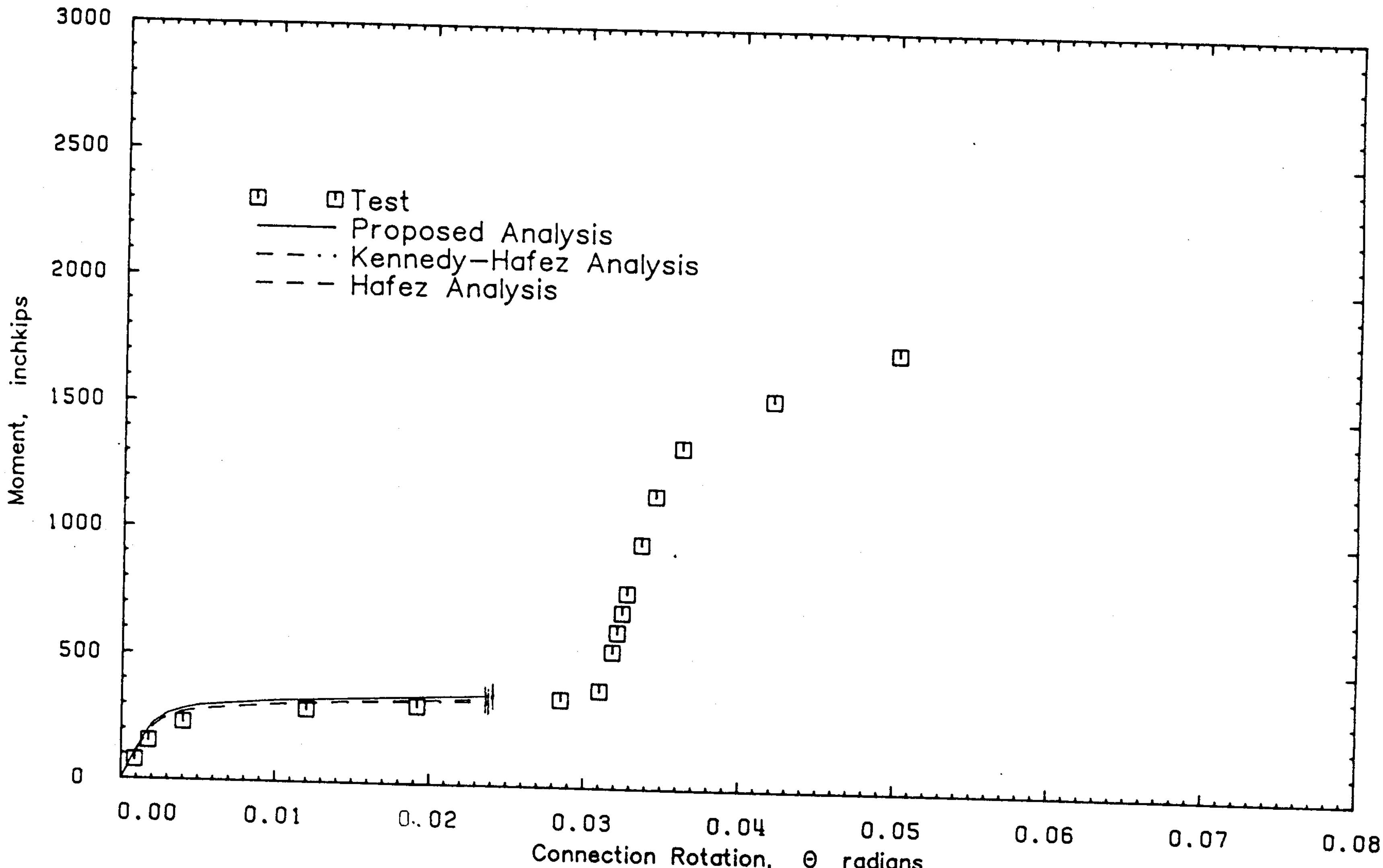


Figure 3.19 Comparison of Analytical Moment-Rotation Relationships with

Hafez (1982) End Plate Test # 5

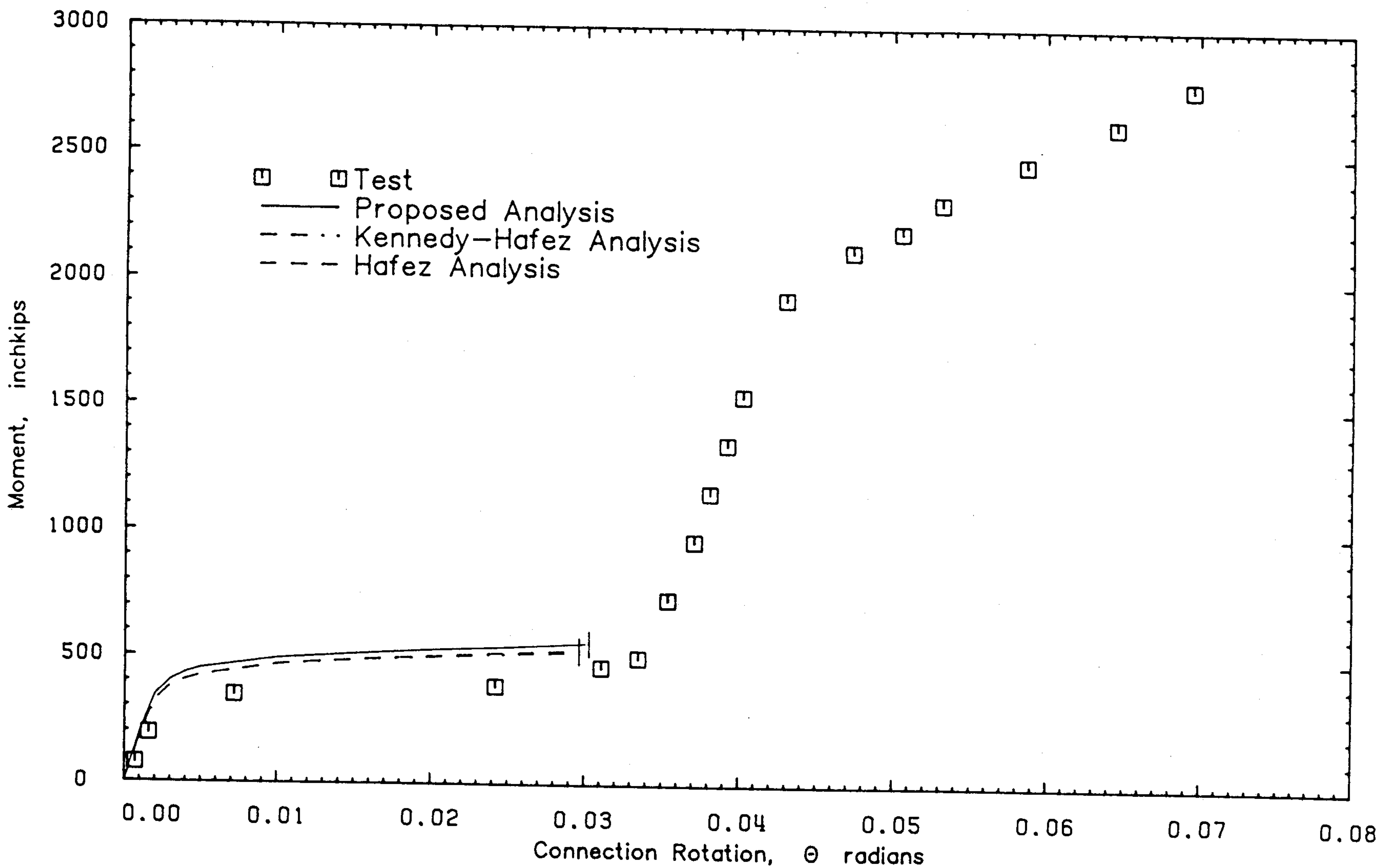


Figure 3.20 Comparison of Analytical Moment-Rotation Relationships with

Hafez (1982) End Plate Test # 6

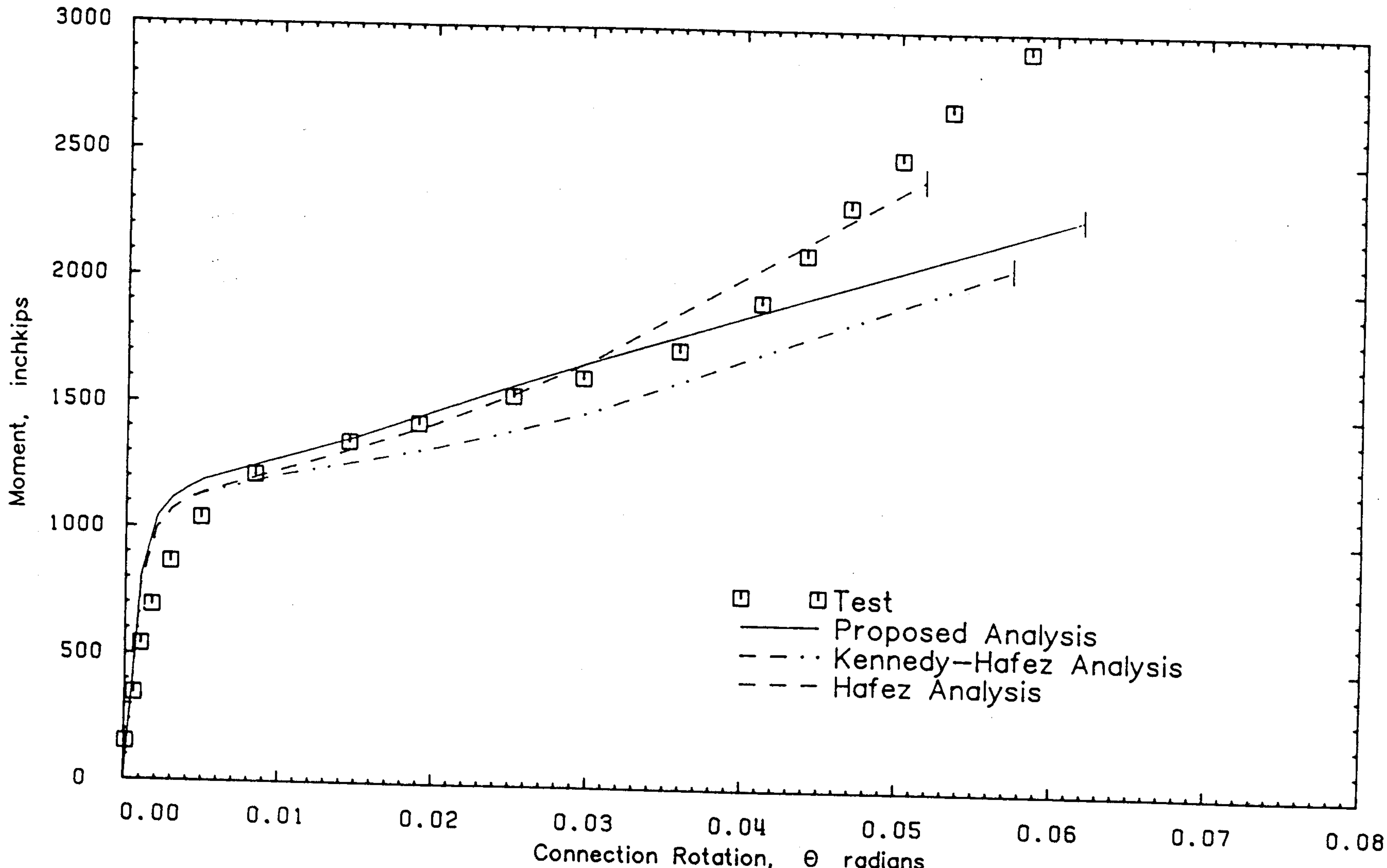


Figure 3.21 Comparison of Analytical Moment-Rotation Relationships with

Hafez (1982) End Plate Test # 7

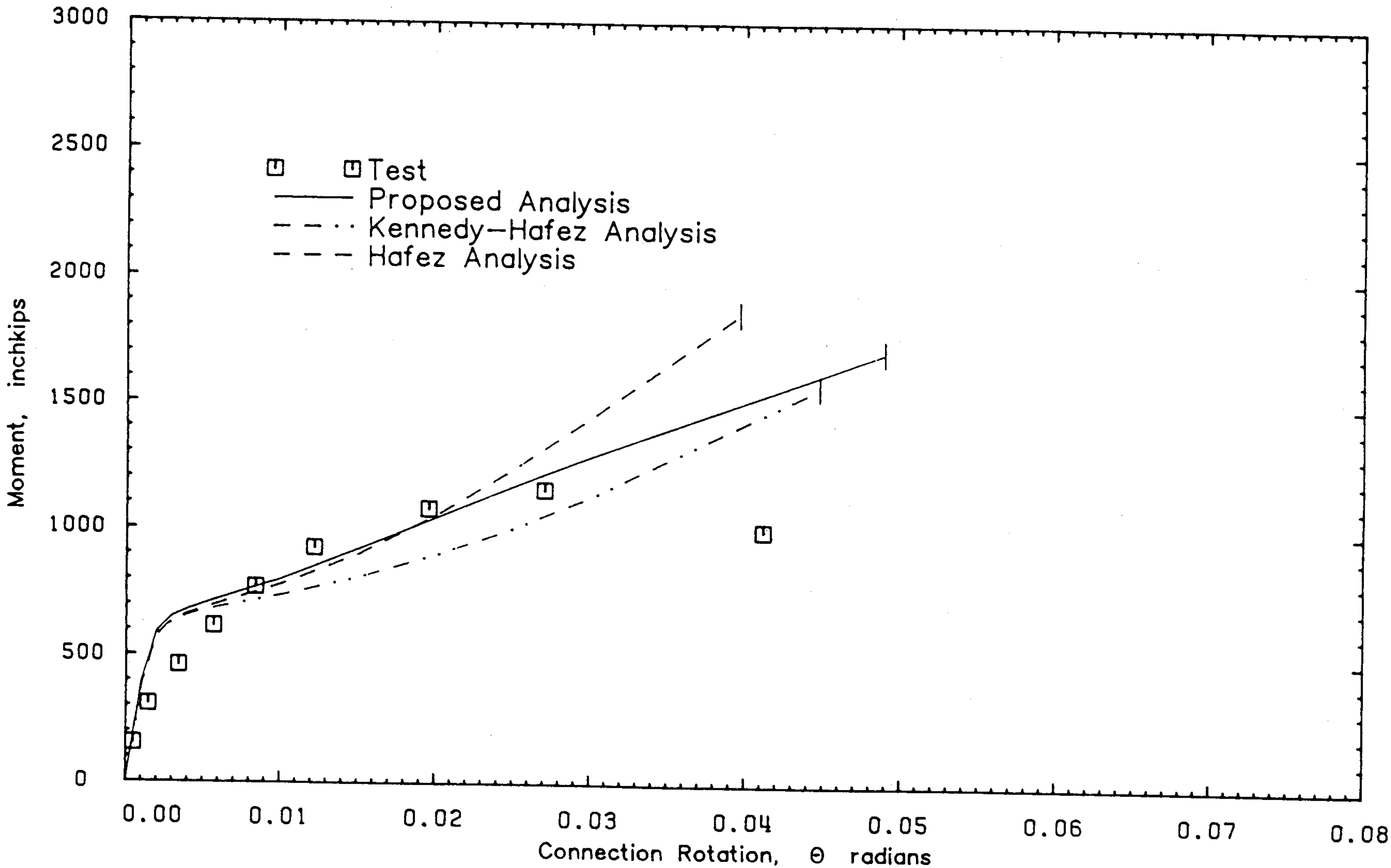


Figure 3.22 Comparison of Analytical Moment-Rotation Relationships with

Hafez (1982) End Plate Test # 8

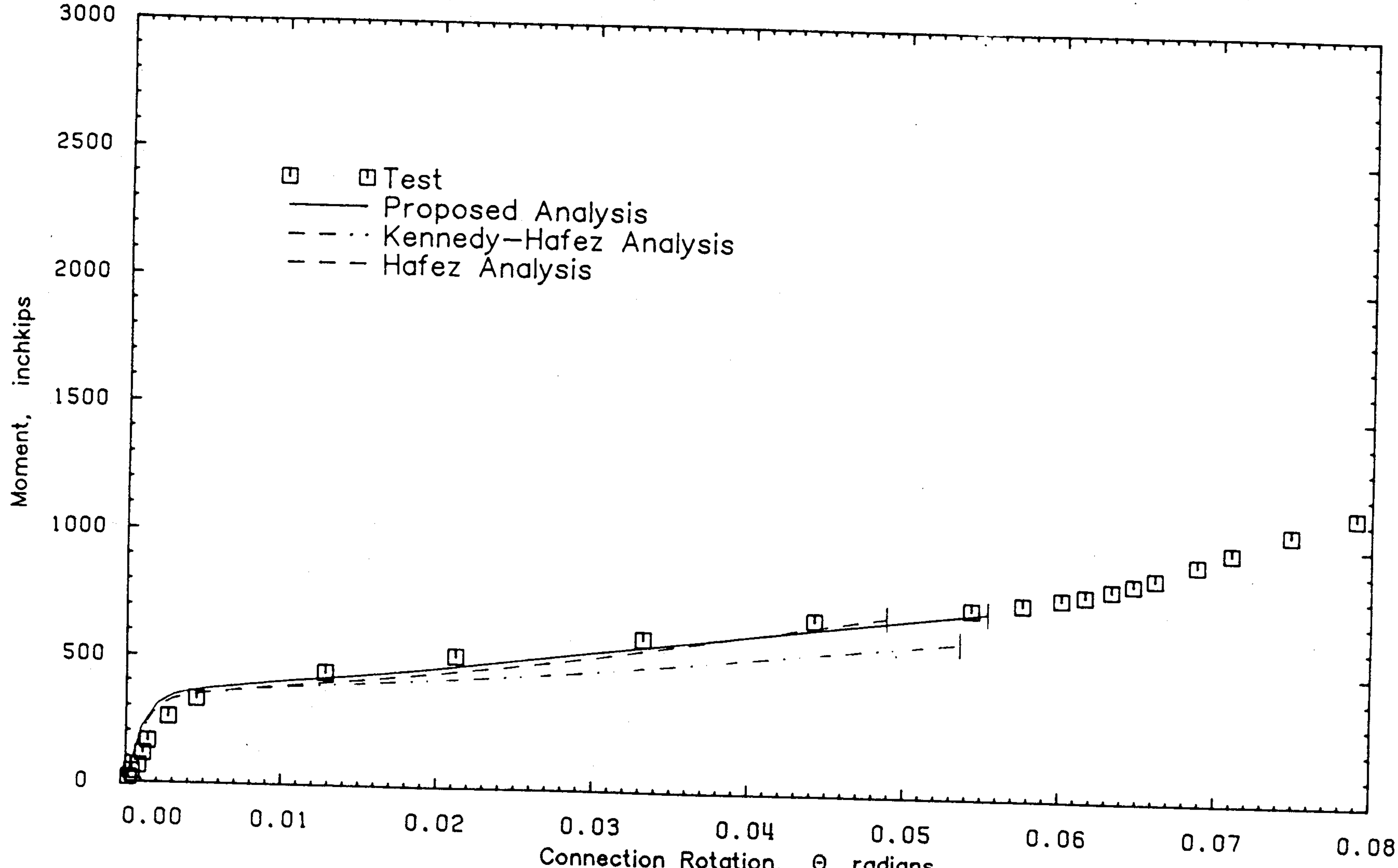


Figure 3.23 Comparison of Analytical Moment-Rotation Relationships with

Sommer (1969) End Plate Test # 5

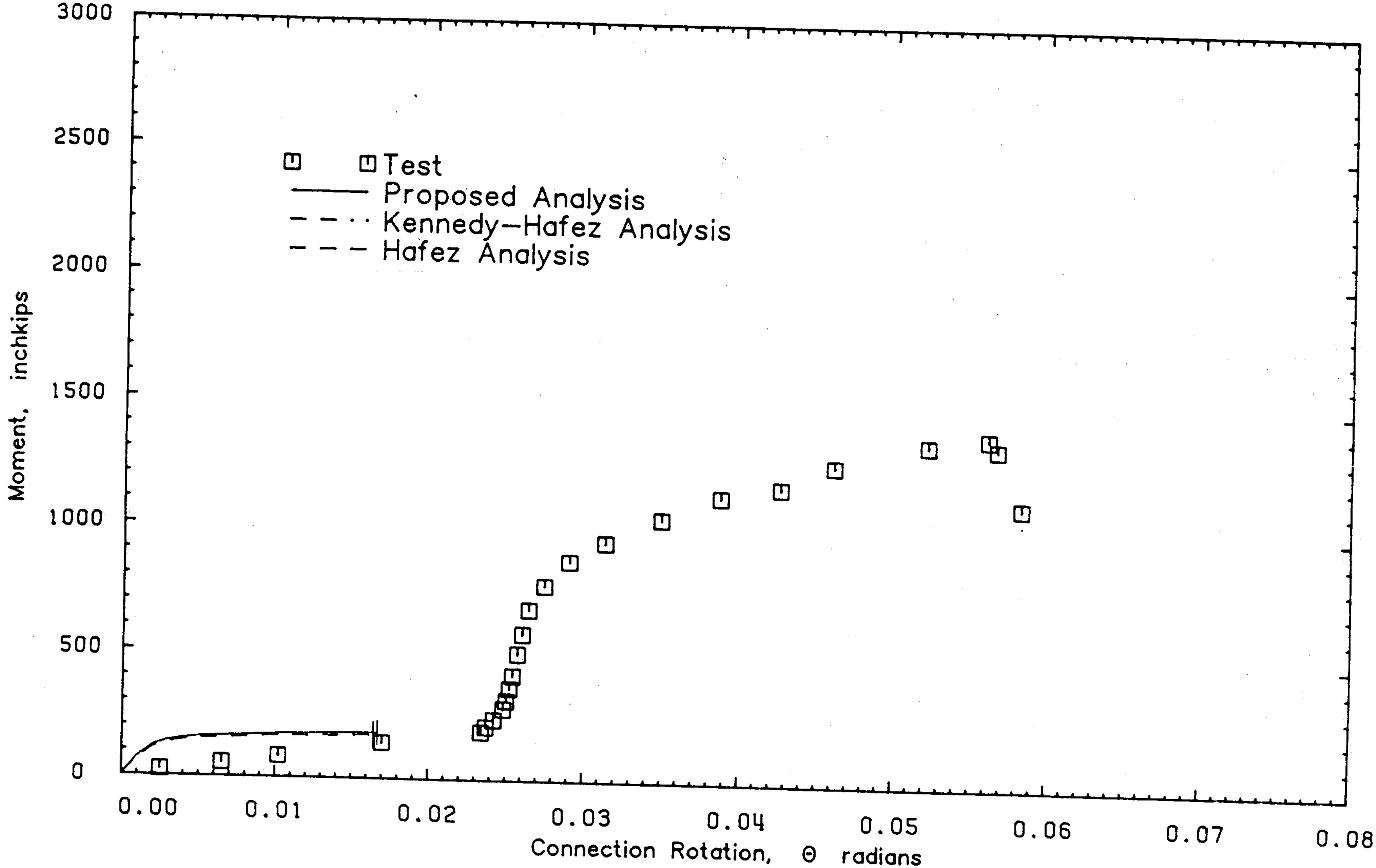


Figure 3.24 Comparison of Analytical Moment-Rotation Relationships with

Sommer (1969) End Plate Test # 6

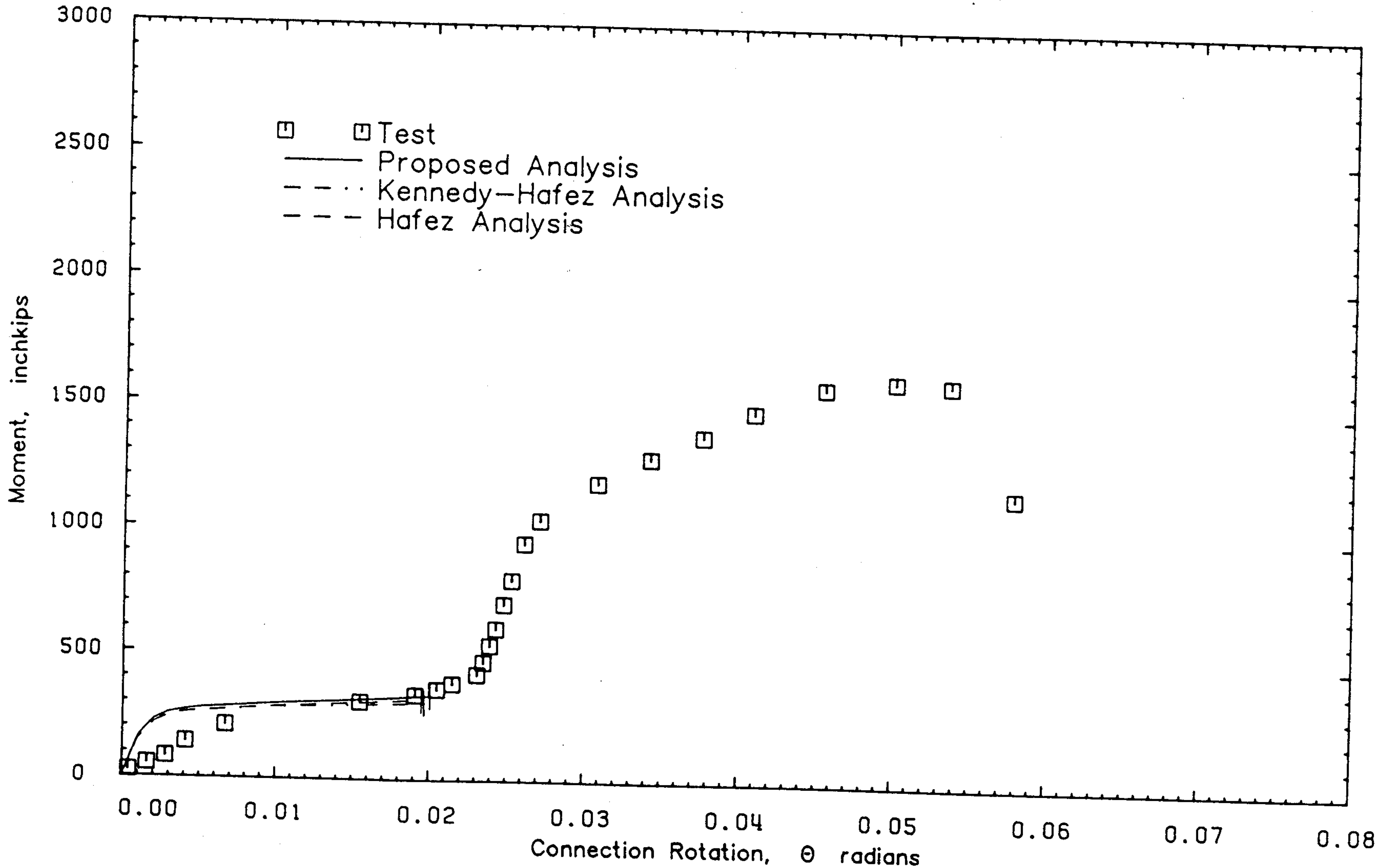


Figure 3.25 Comparison of Analytical Moment-Rotation Relationships with

Sommer (1969) End Plate Test # 7

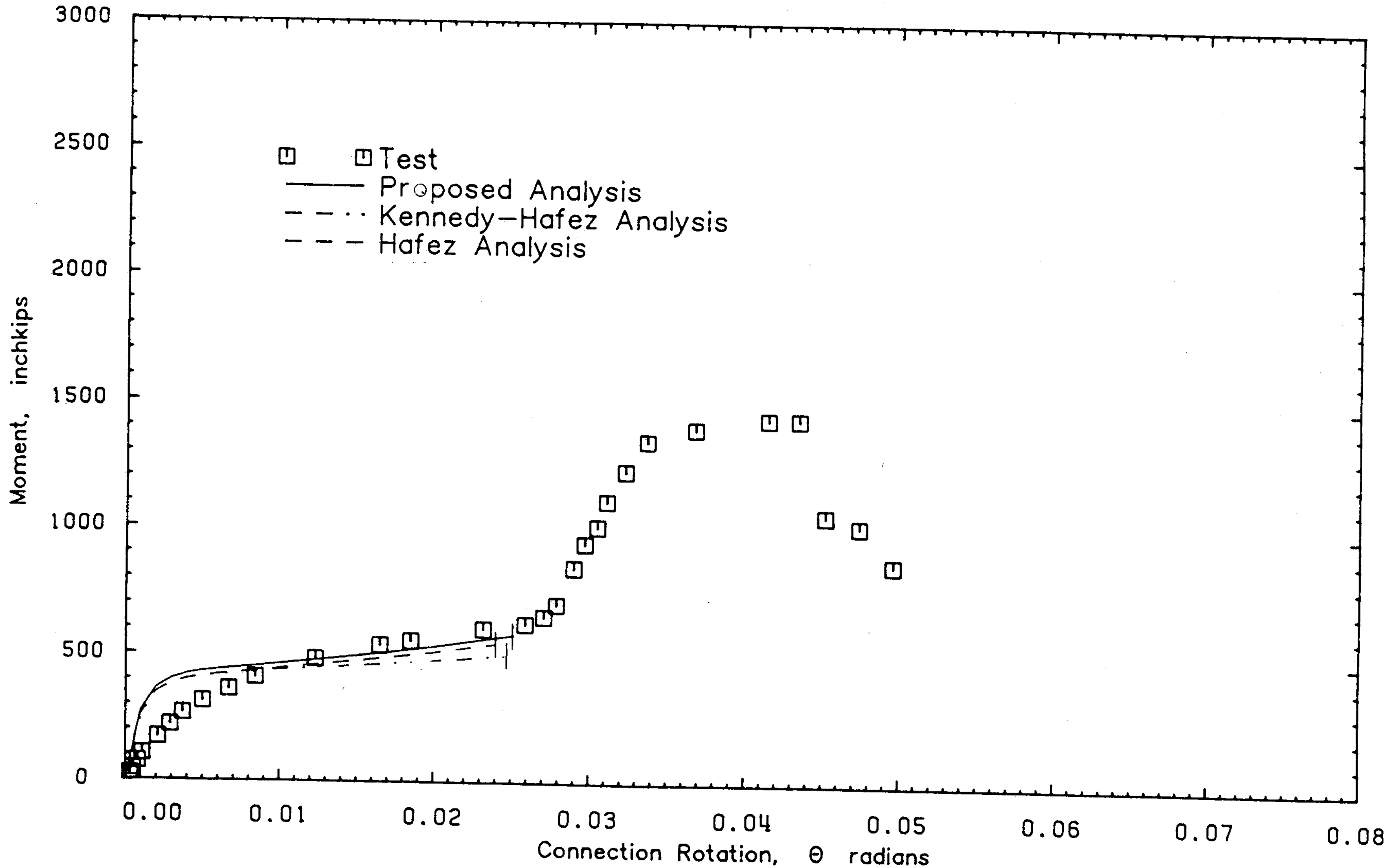


Figure 3.26 Comparison of Analytical Moment-Rotation Relationships with

Sommer (1969) End Plate Test # 8

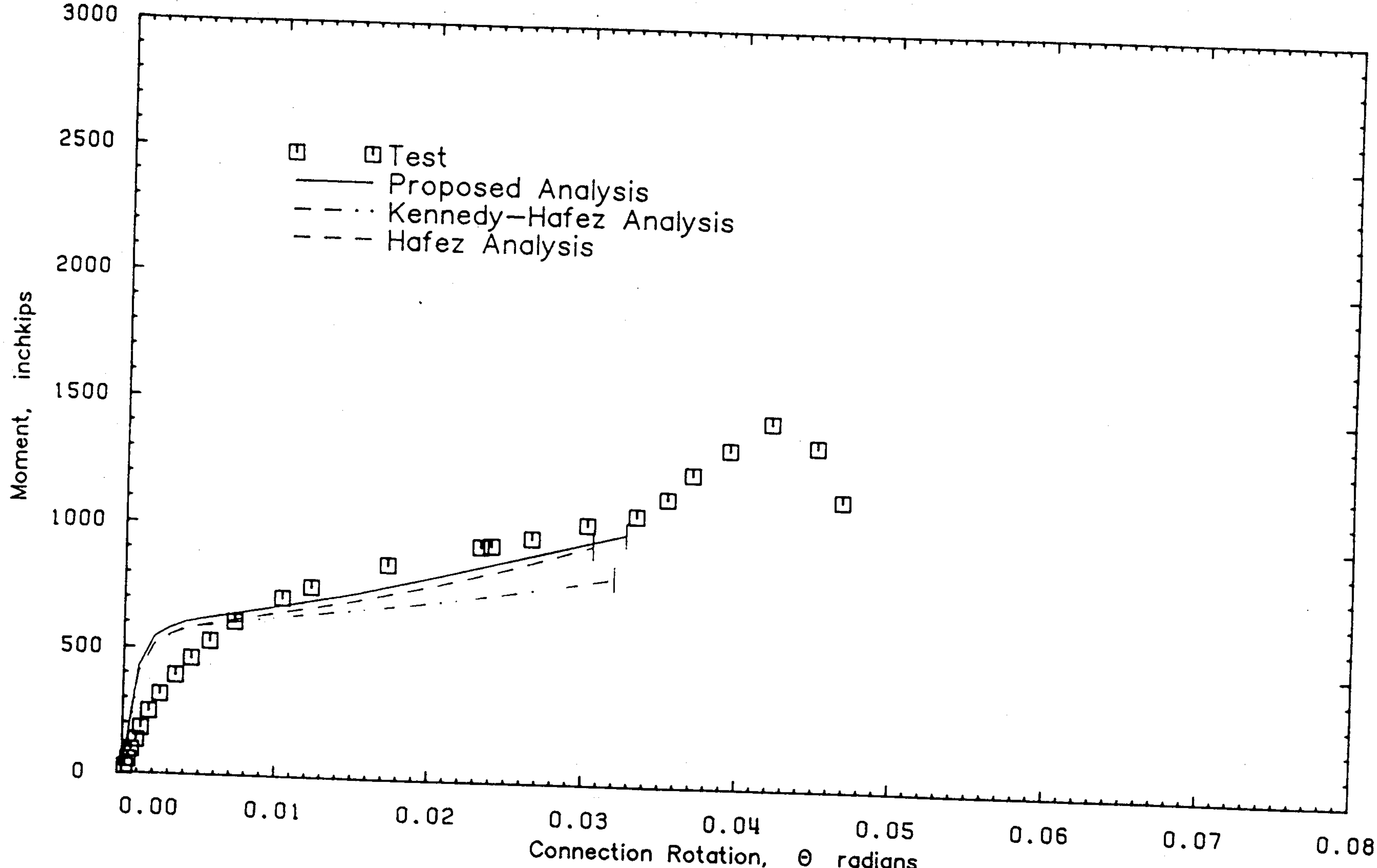


Figure 3.27 Comparison of Analytical Moment-Rotation Relationships with

Sommer (1969) End Plate Test # 9

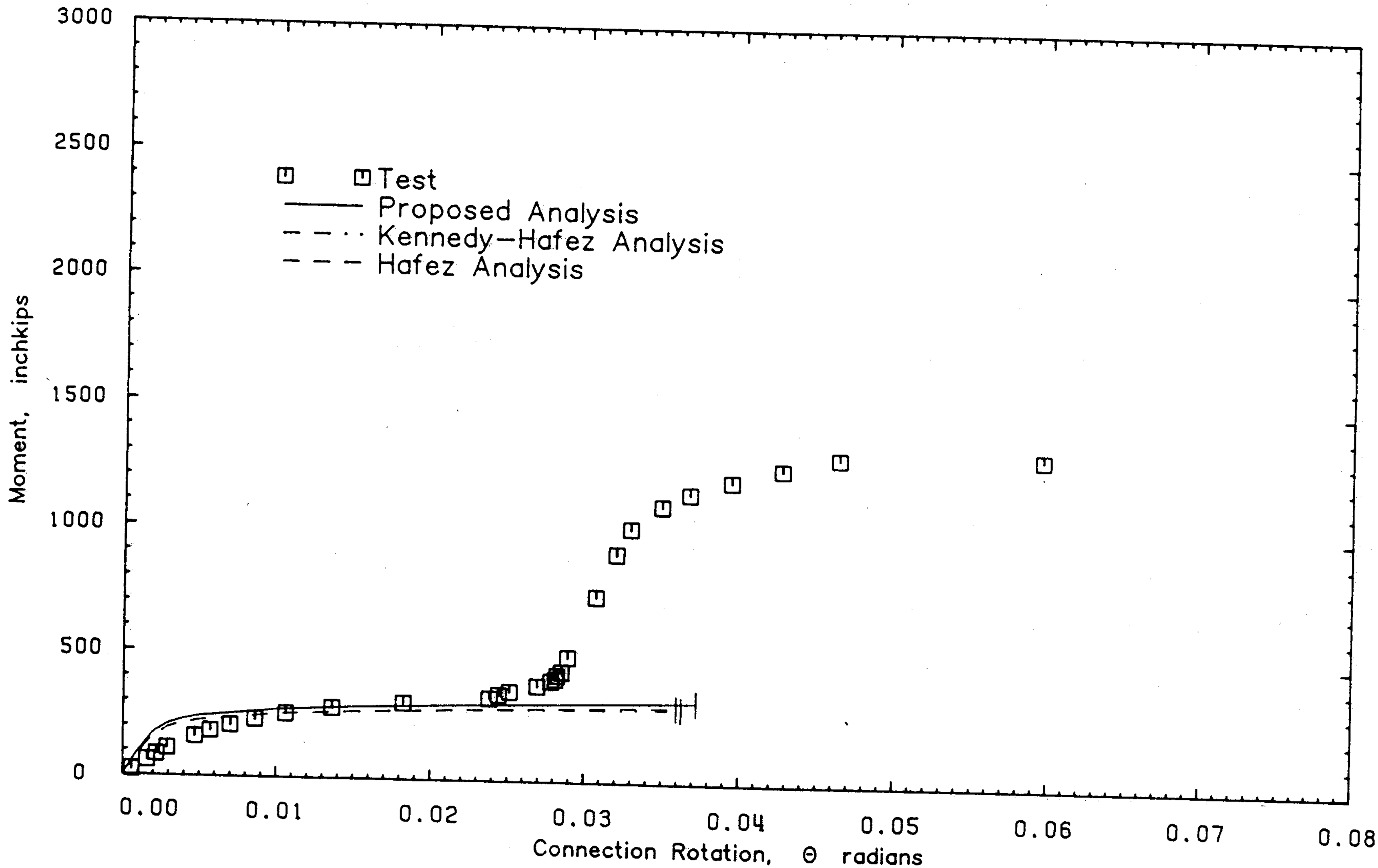


Figure 3.28 Comparison of Analytical Moment-Rotation Relationships with Sommer (1969) End Plate Test # 10

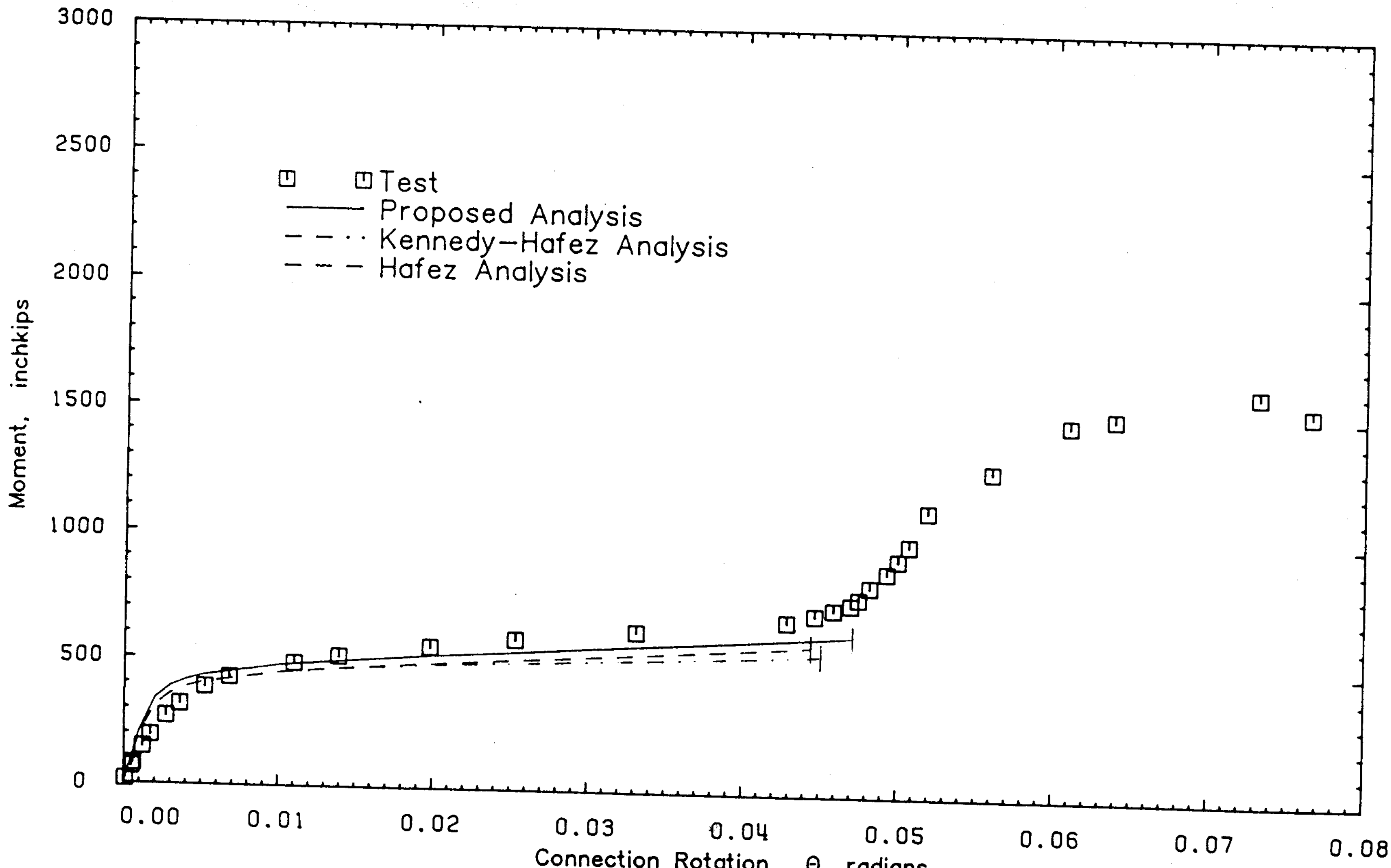


Figure 3.29 Comparison of Analytical Moment-Rotation Relationships with

Sommer (1969) End Plate Test # 11

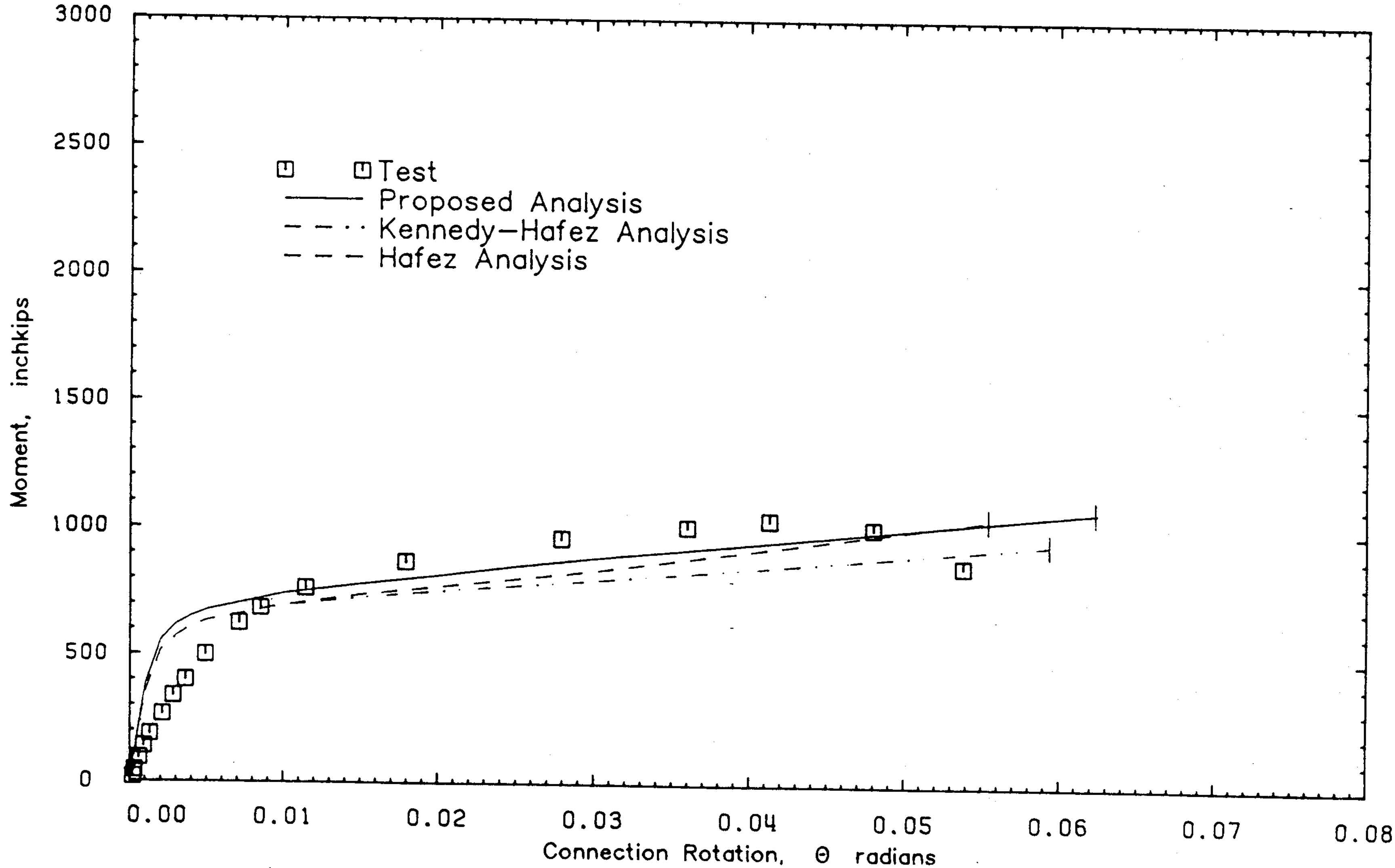


Figure 3.30 Comparison of Analytical Moment-Rotation Relationships with

Sommer (1969) End Plate Test # 12

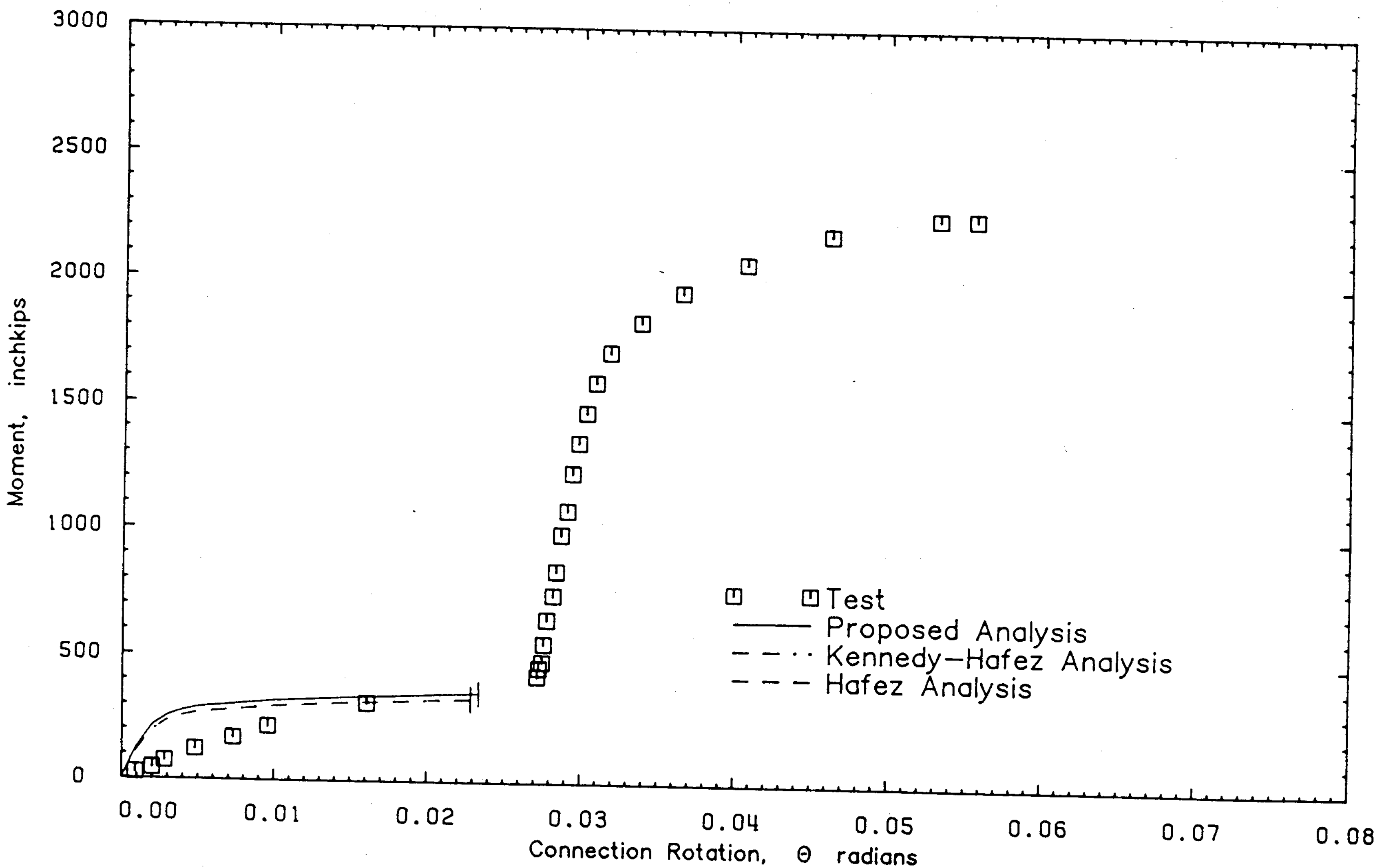


Figure 3.31 Comparison of Analytical Moment-Rotation Relationships with

Sommer (1969) End Plate Test # 13

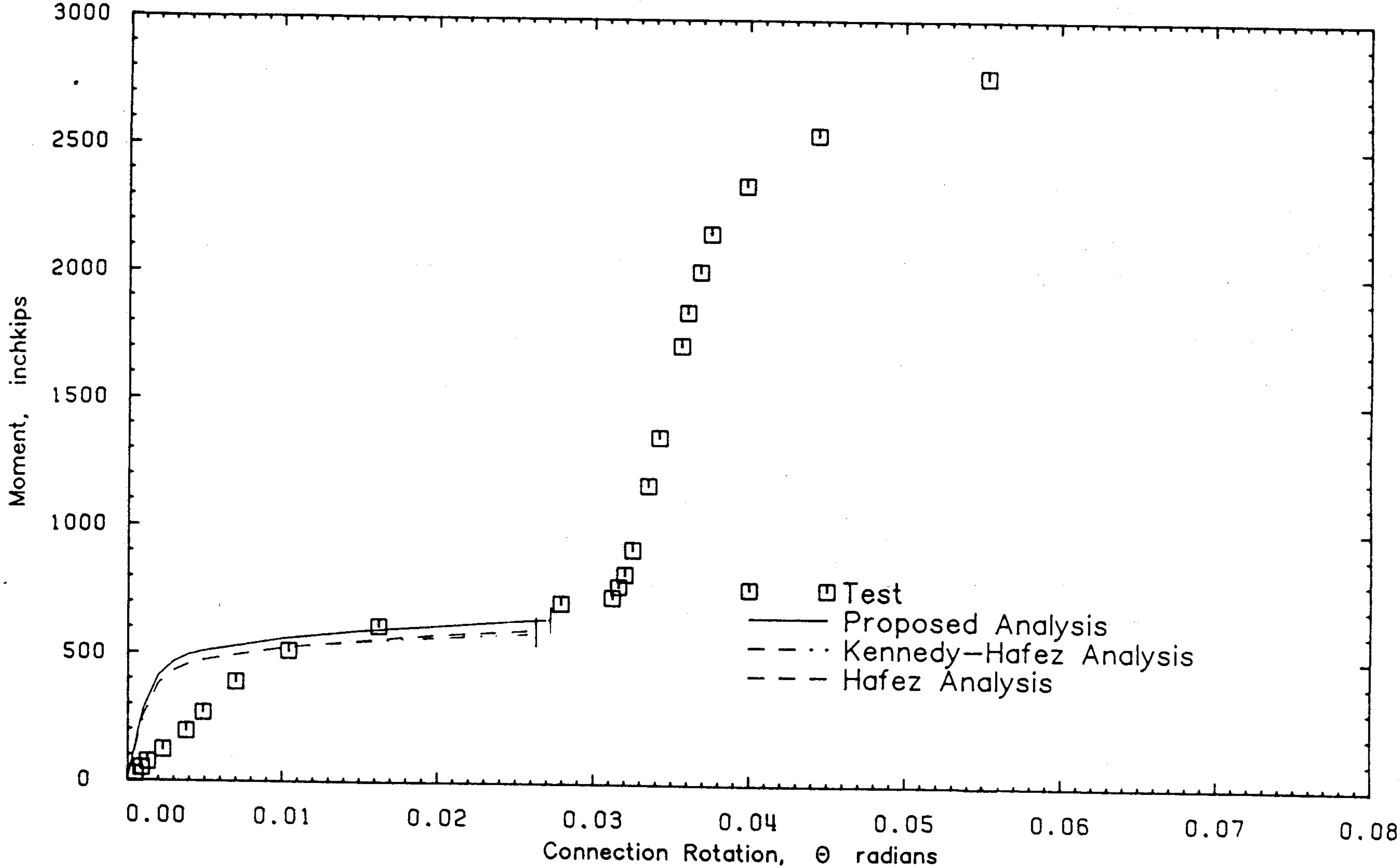


Figure 3.32 Comparison of Analytical Moment-Rotation Relationships with

Sommer (1969) End Plate Test # 14

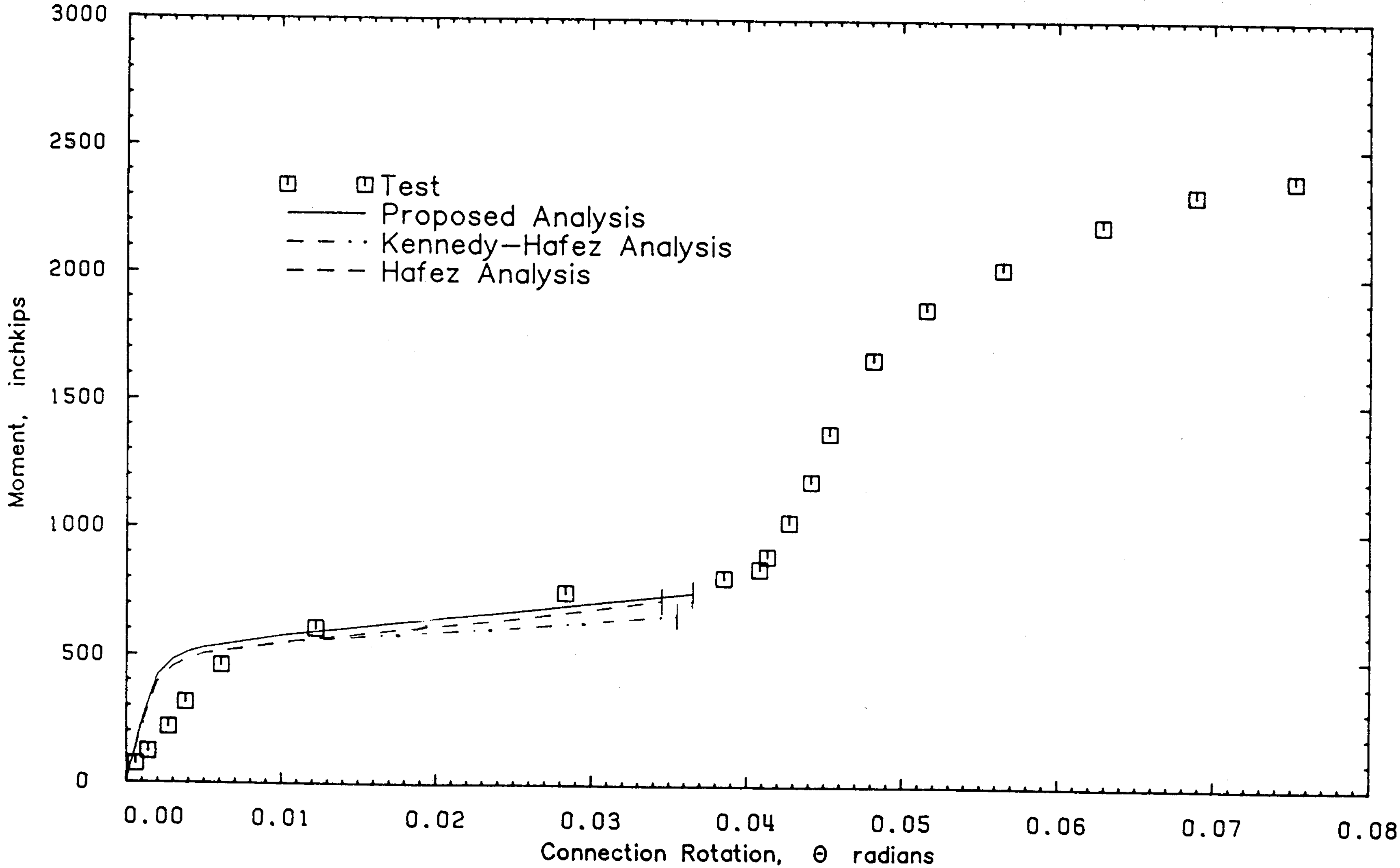


Figure 3.33 Comparison of Analytical Moment-Rotation Relationships with

Sommer (1969) End Plate Test # 15

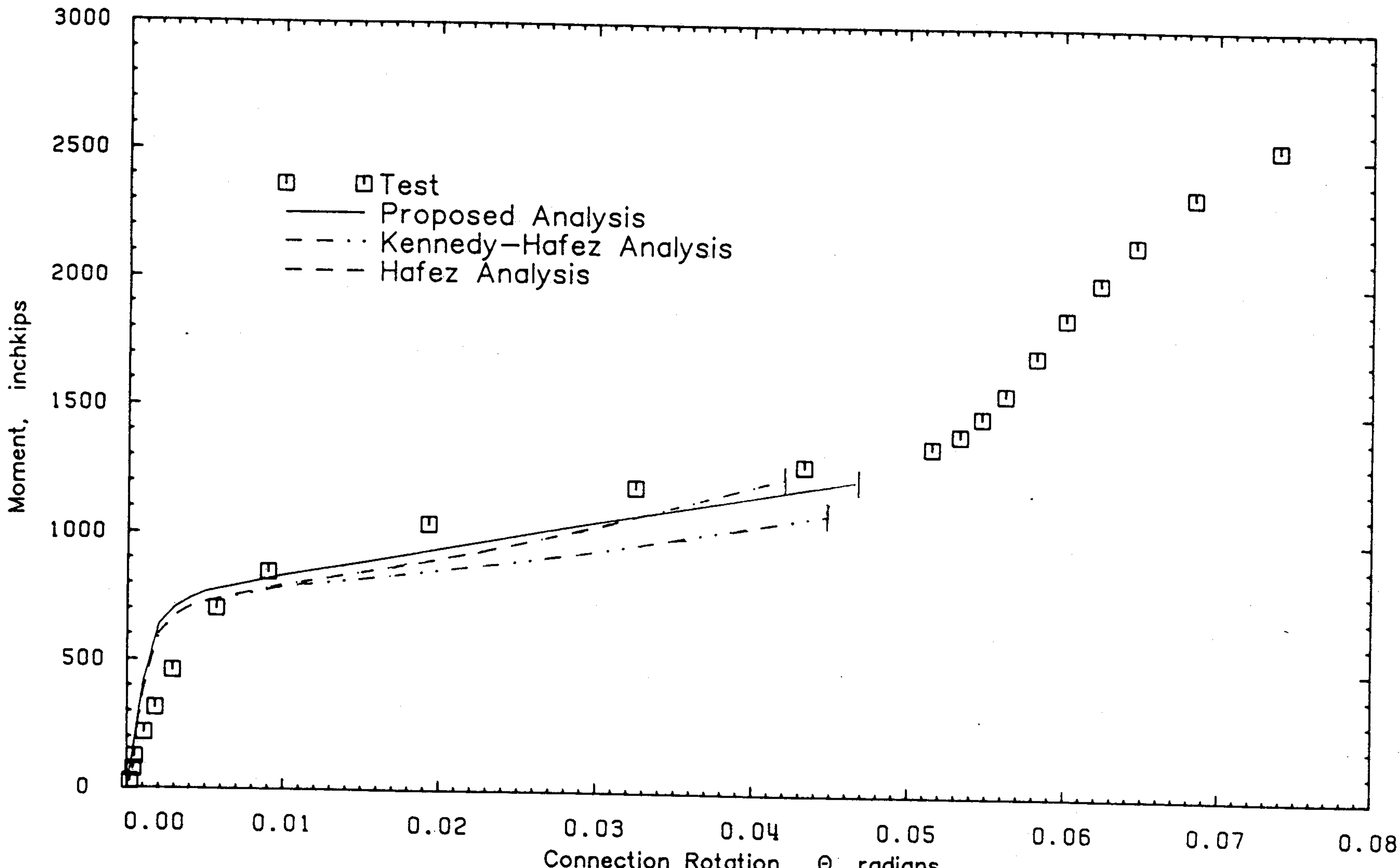


Figure 3.34 Comparison of Analytical Moment-Rotation Relationships with

Sommer (1969) End Plate Test # 16

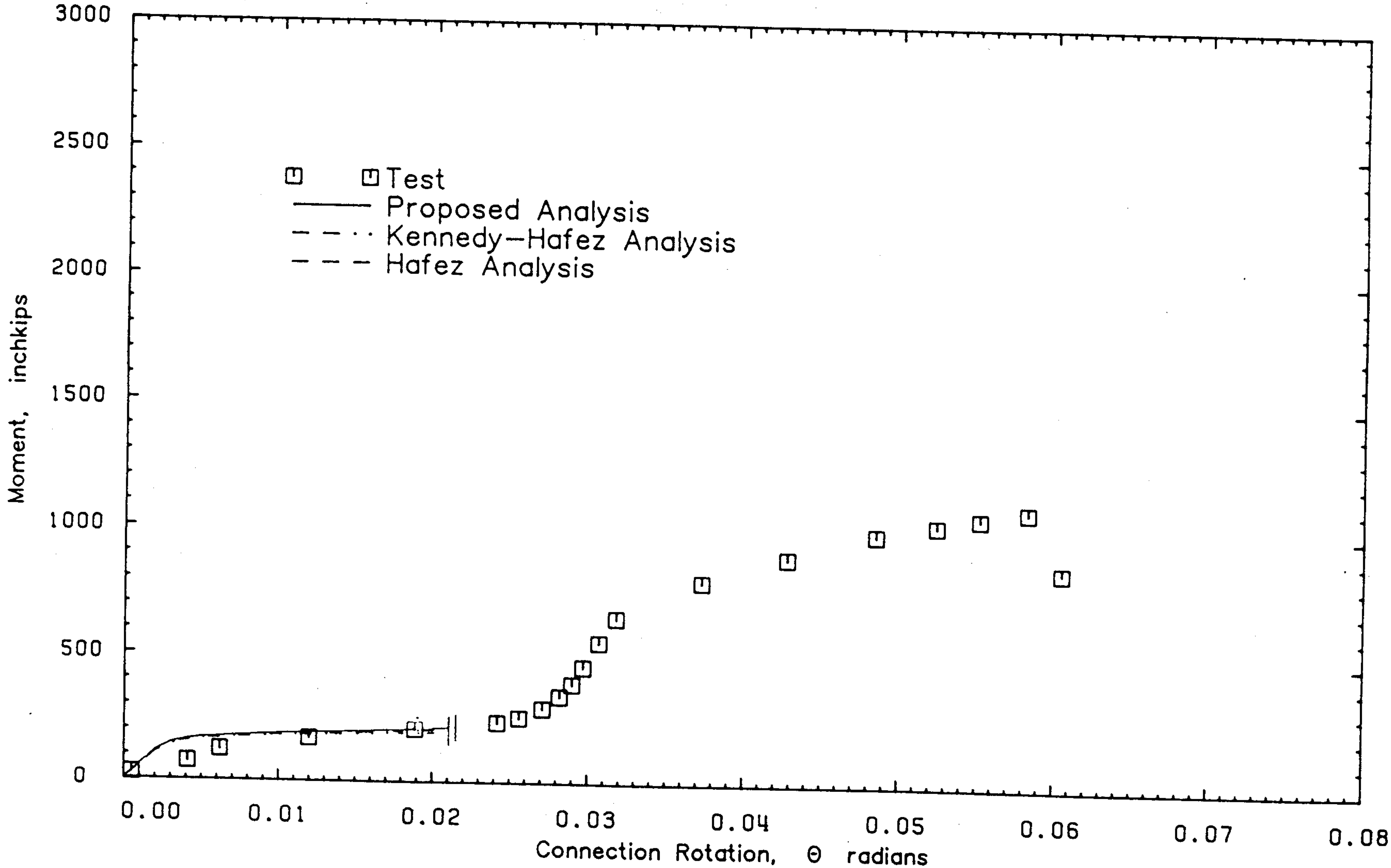


Figure 3.35 Comparison of Analytical Moment-Rotation Relationships with

Sommer (1969) End Plate Test # 17

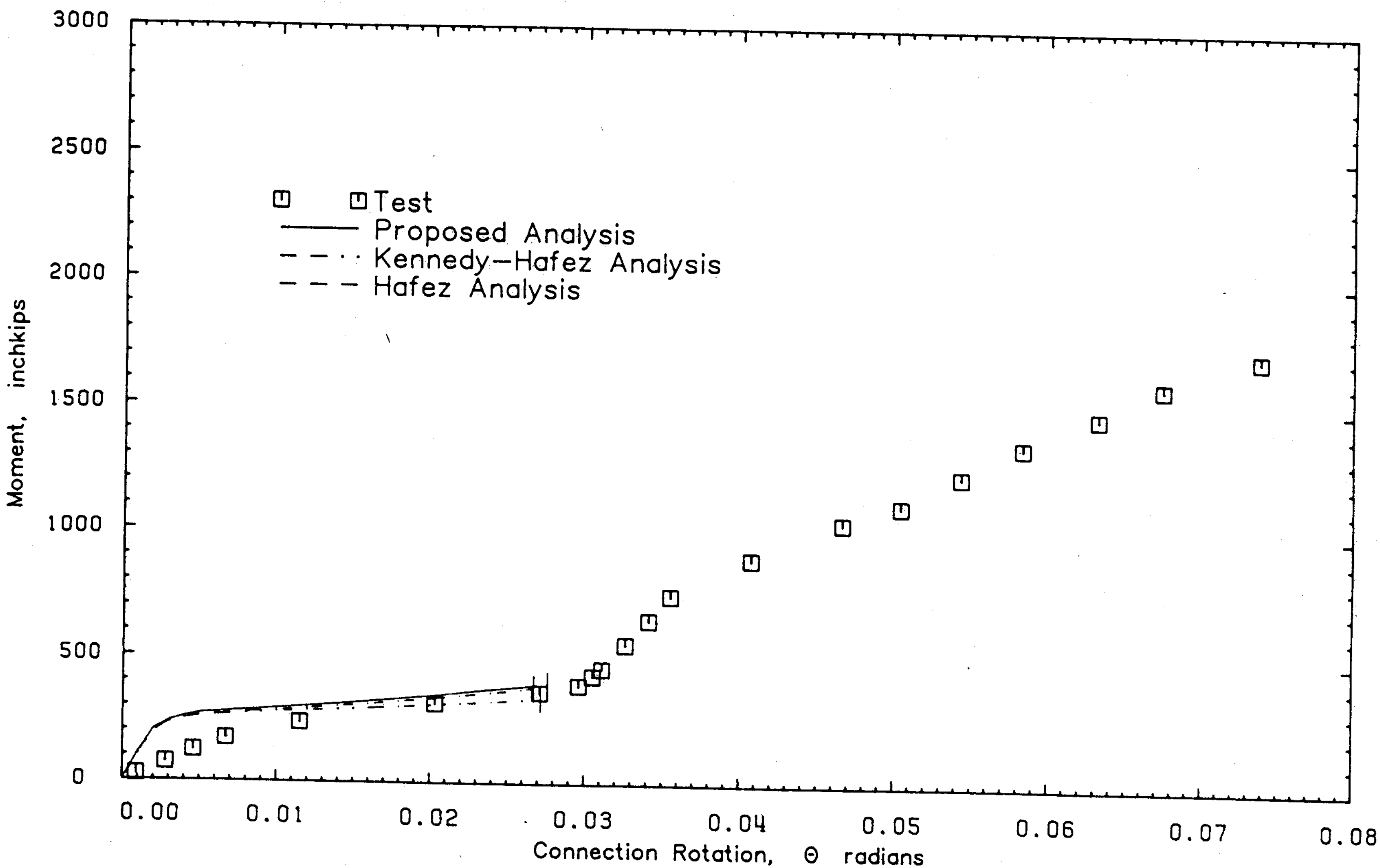


Figure 3.36 Comparison of Analytical Moment-Rotation Relationships with

Sommer (1969) End Plate Test # 18

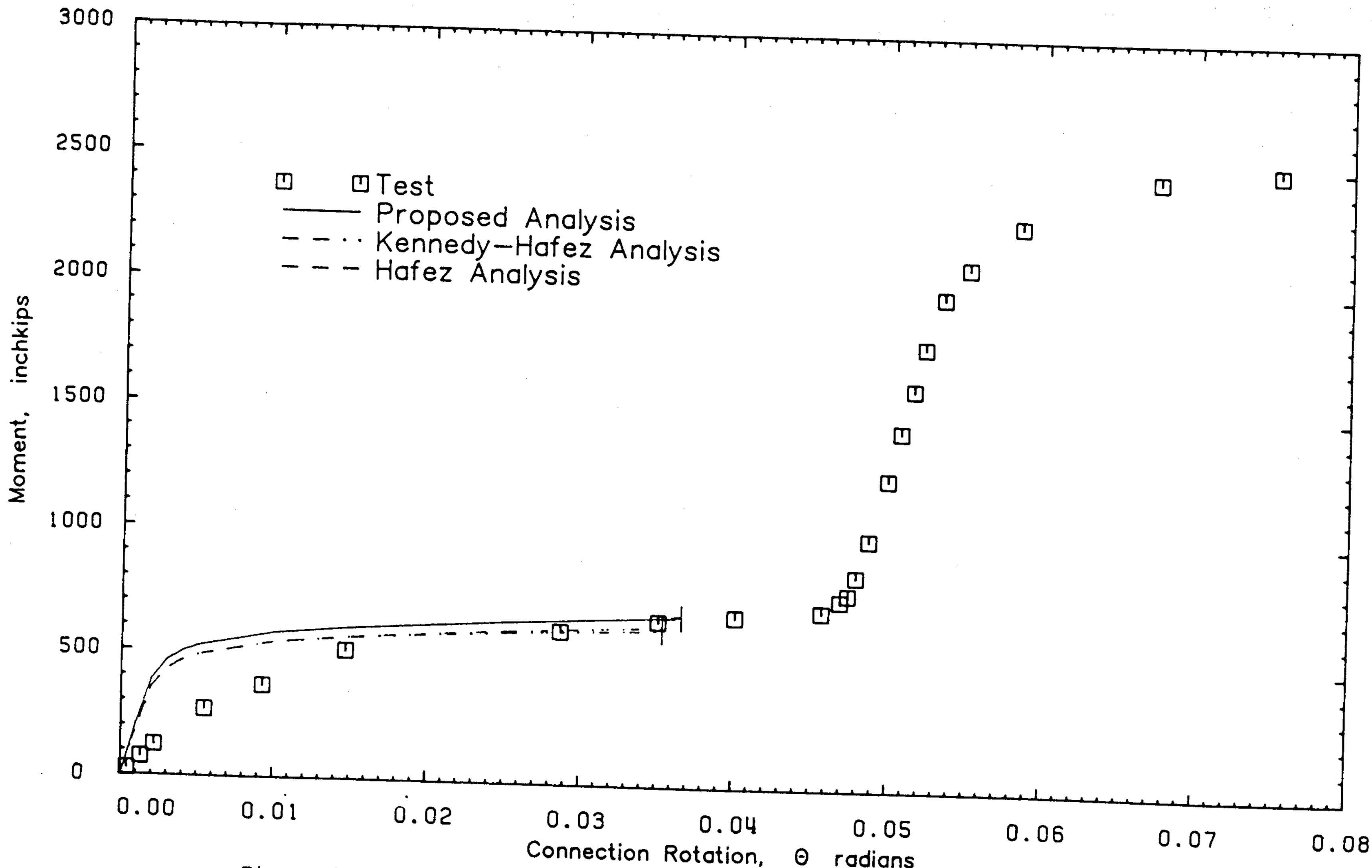


Figure 3.37 Comparison of Analytical Moment-Rotation Relationships with Sommer (1969) End Plate Test # 19

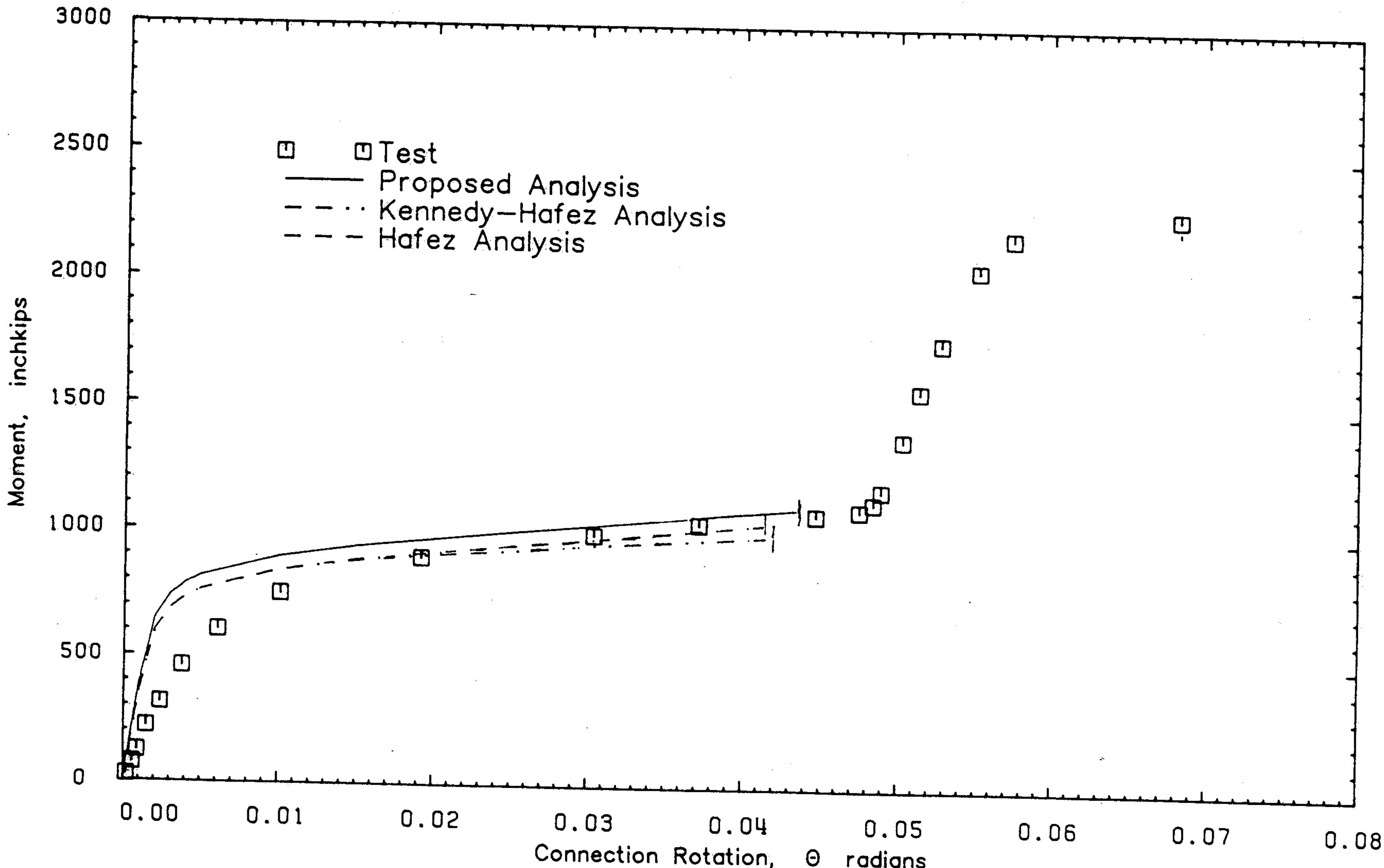


Figure 3.38 Comparison of Analytical Moment-Rotation Relationships with

Sommer (1969) End Plate Test # 20

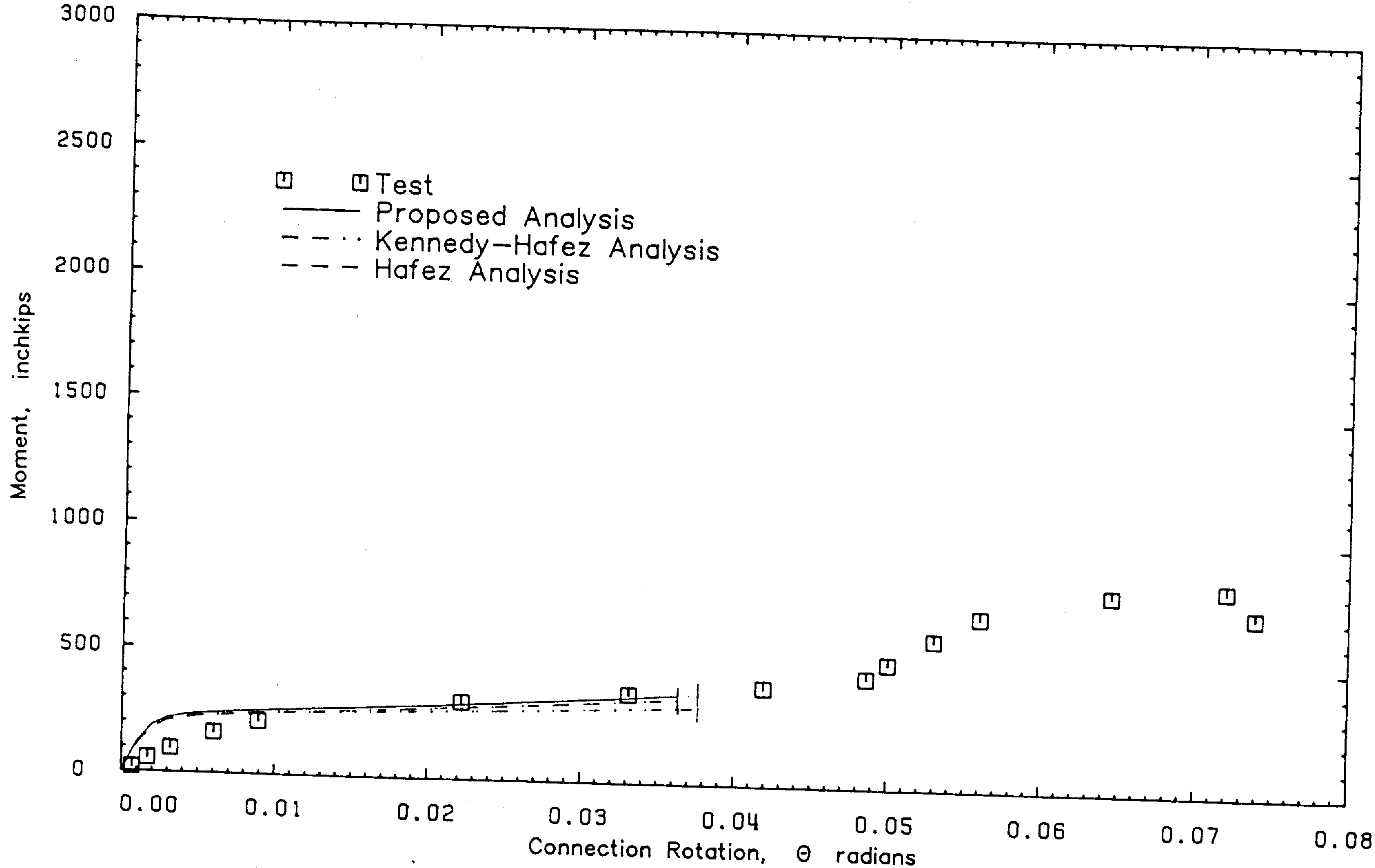


Figure 3.39 Comparison of Analytical Moment-Rotation Relationships with

Sommer (1969) End Plate Test # 25

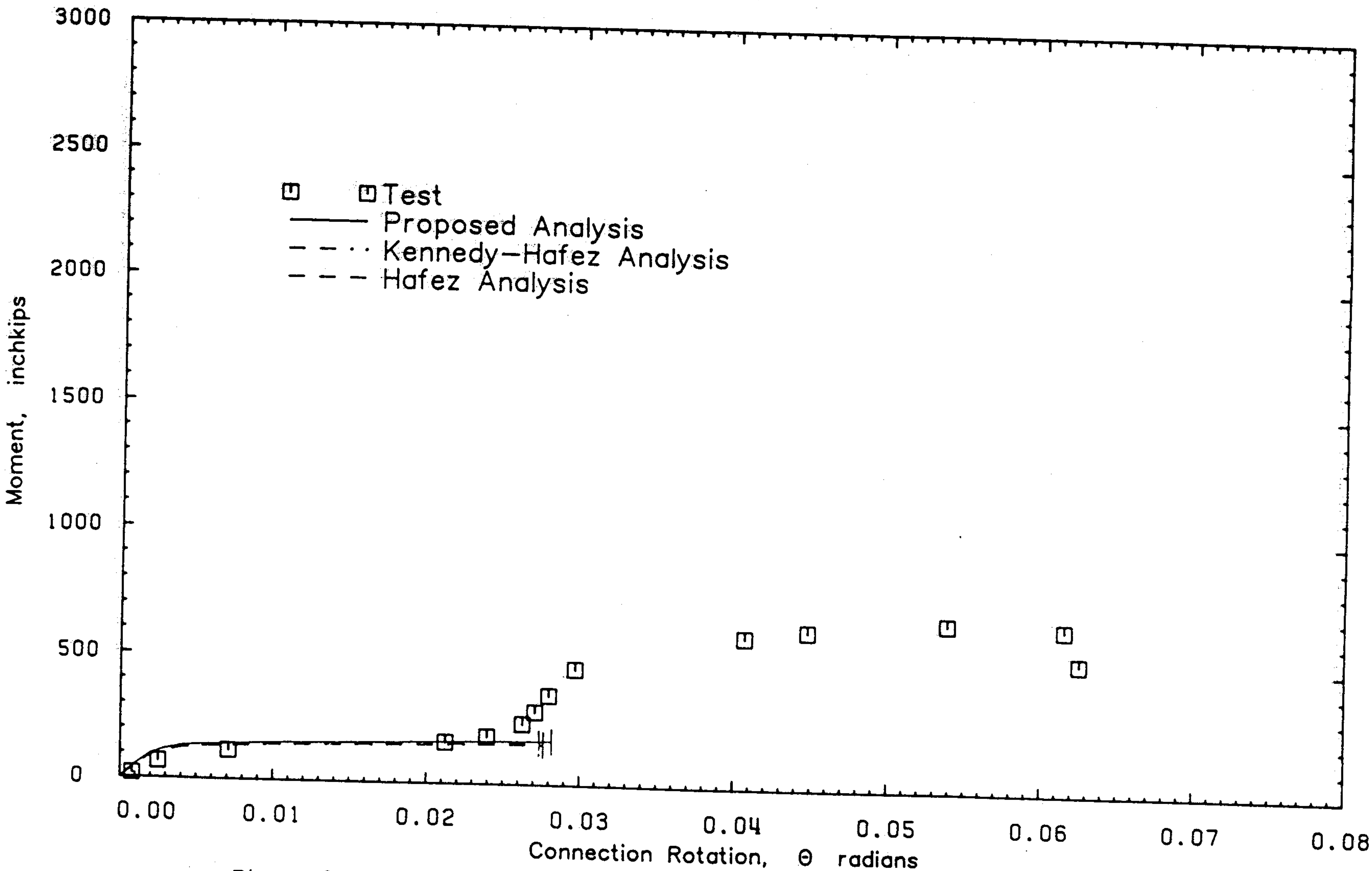


Figure 3.40 Comparison of Analytical Moment-Rotation Relationships with

Sommer (1969) End Plate Test # 26

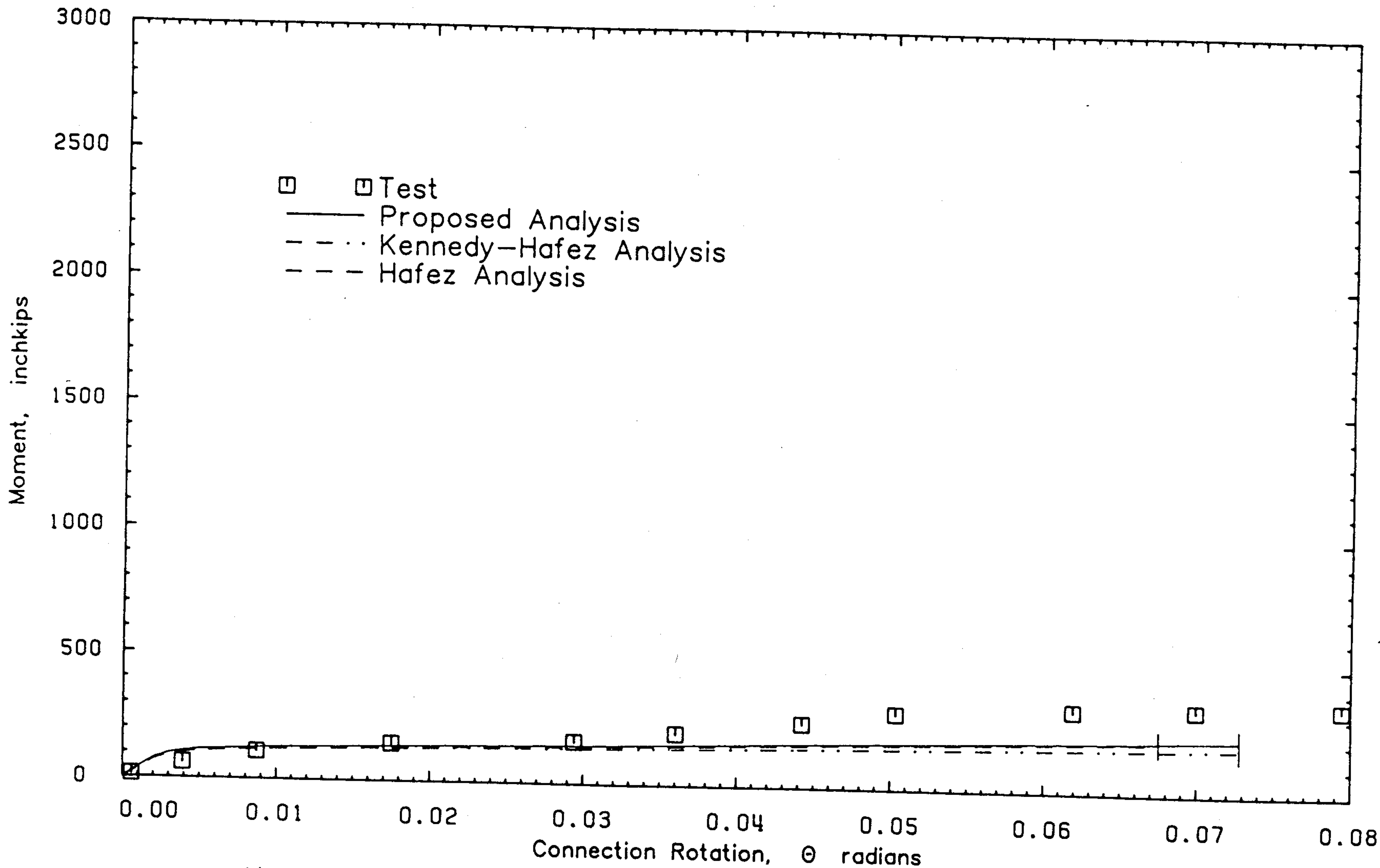


Figure 3.41 Comparison of Analytical Moment-Rotation Relationships with

Sommer (1969) End Plate Test # 27

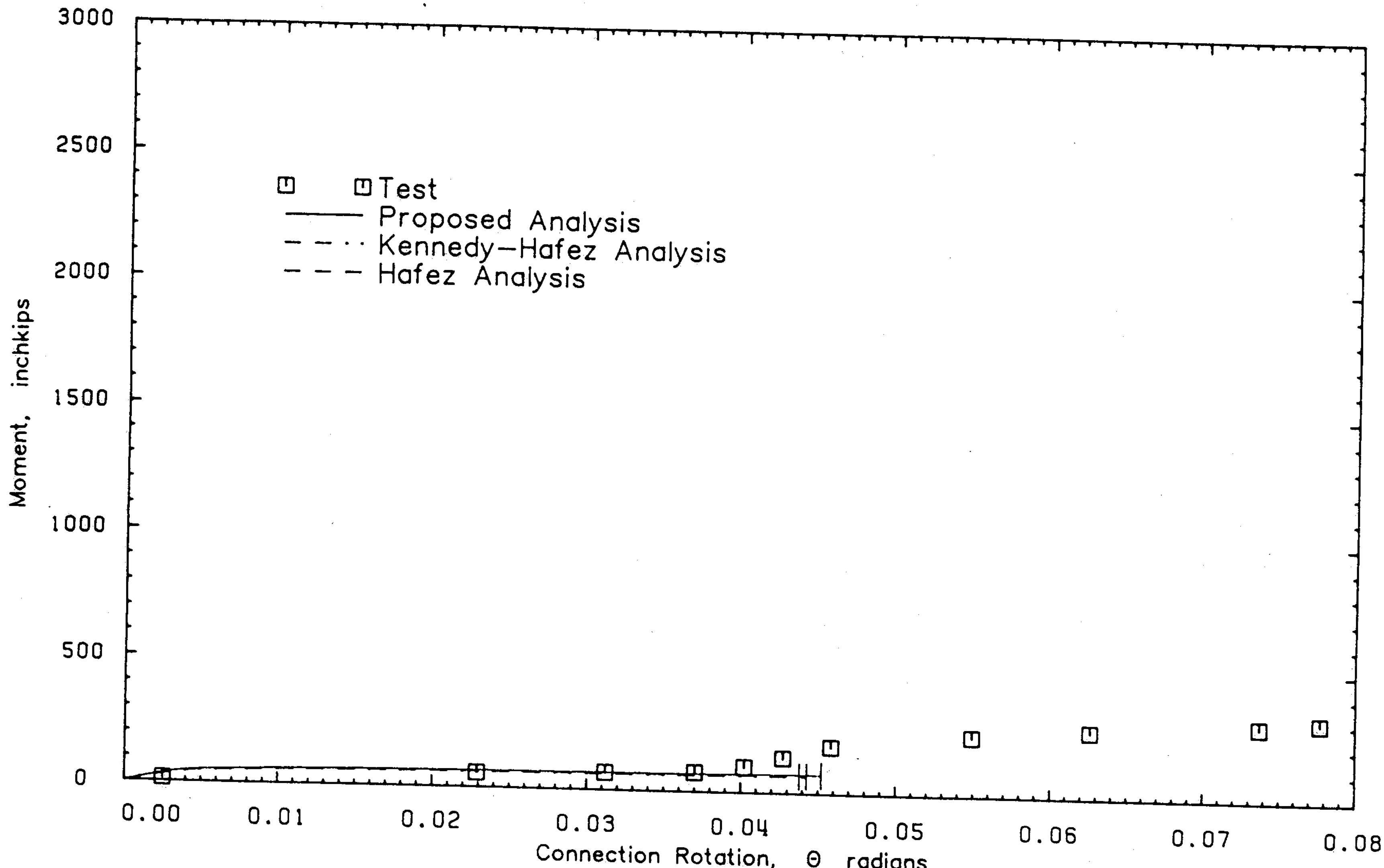


Figure 3.42 Comparison of Analytical Moment-Rotation Relationships with

Sommer (1969) End Plate Test # 28

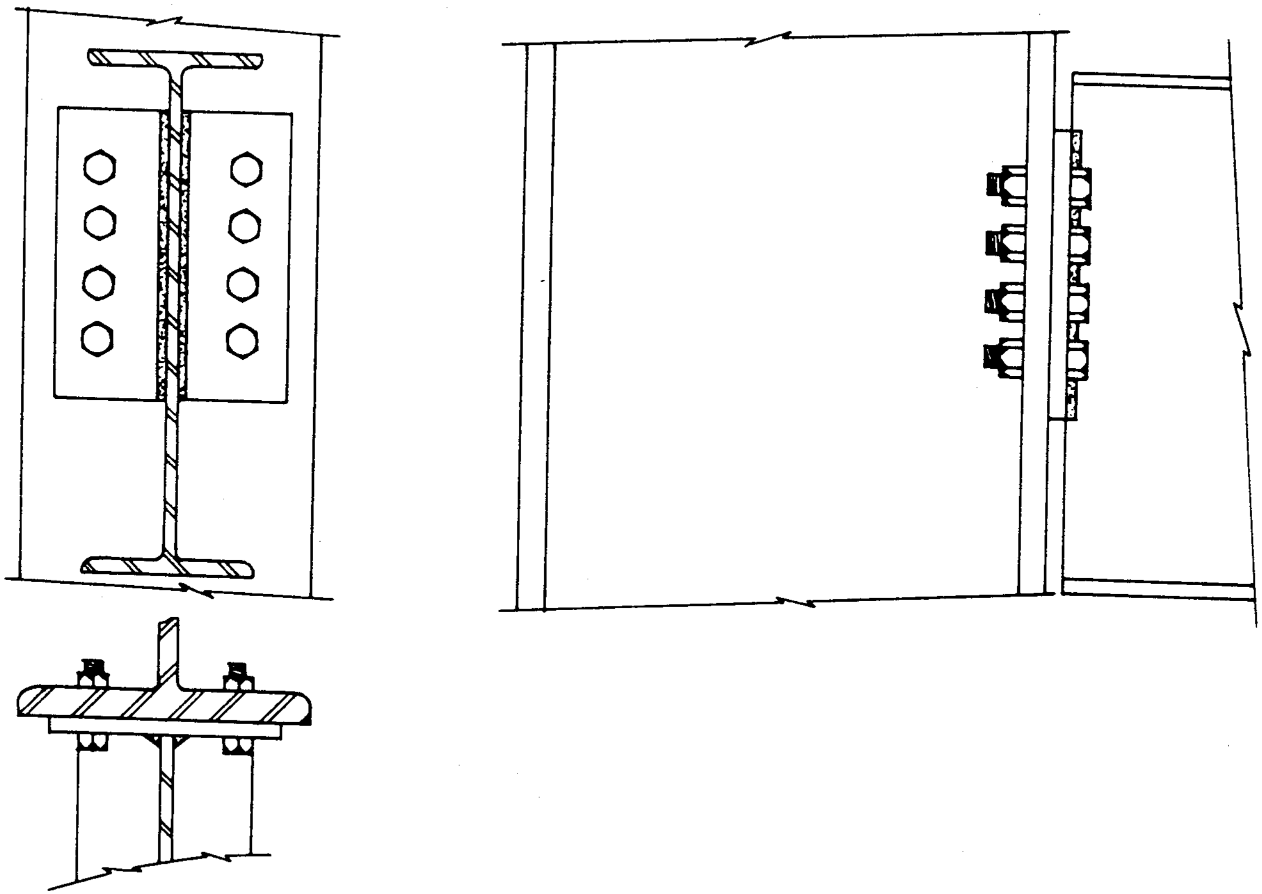


Figure 3.43 Flexible End Plate Connection to Column Flange Detail

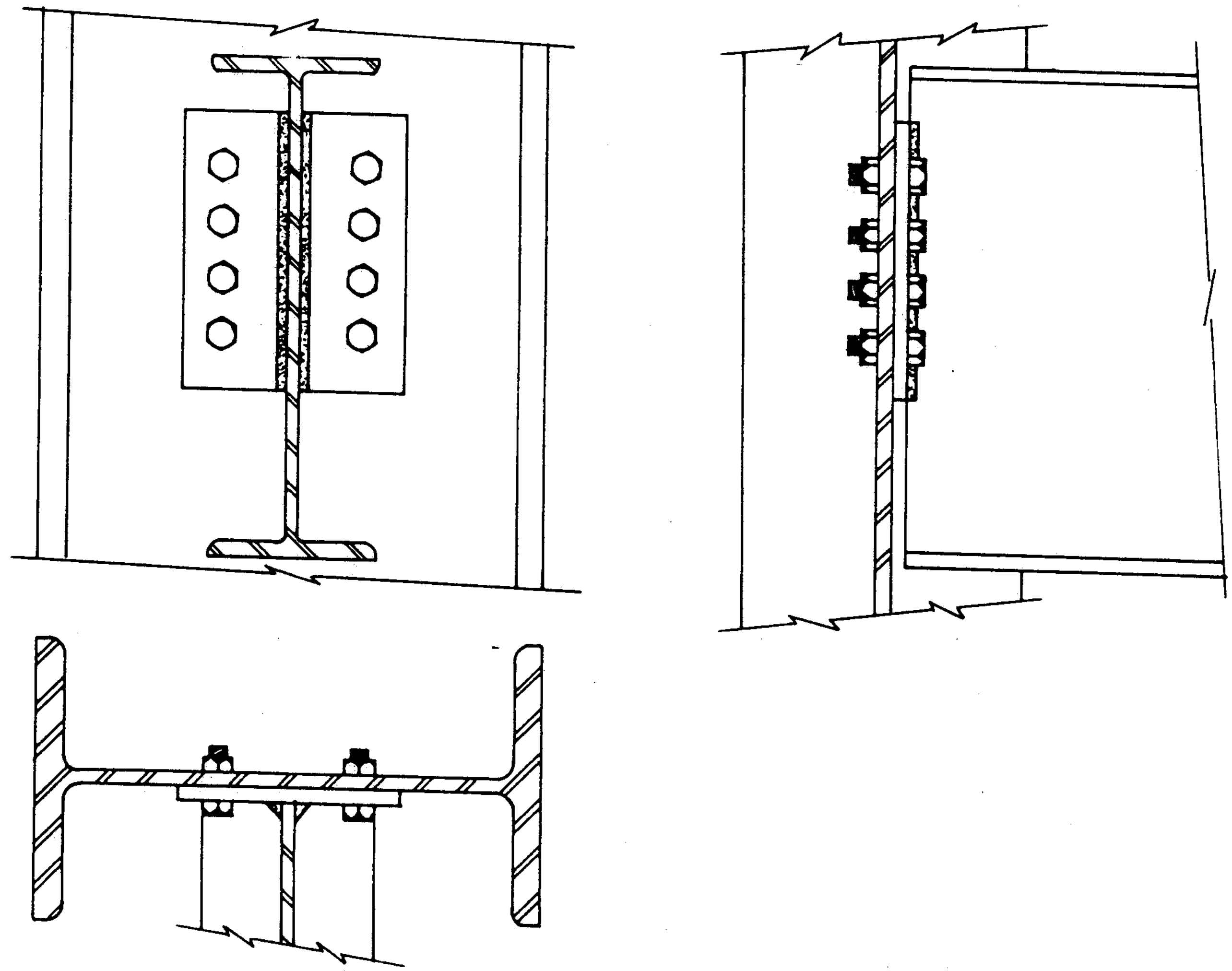


Figure 3.44 Single Flexible End Plate Connection to Column Web Detail

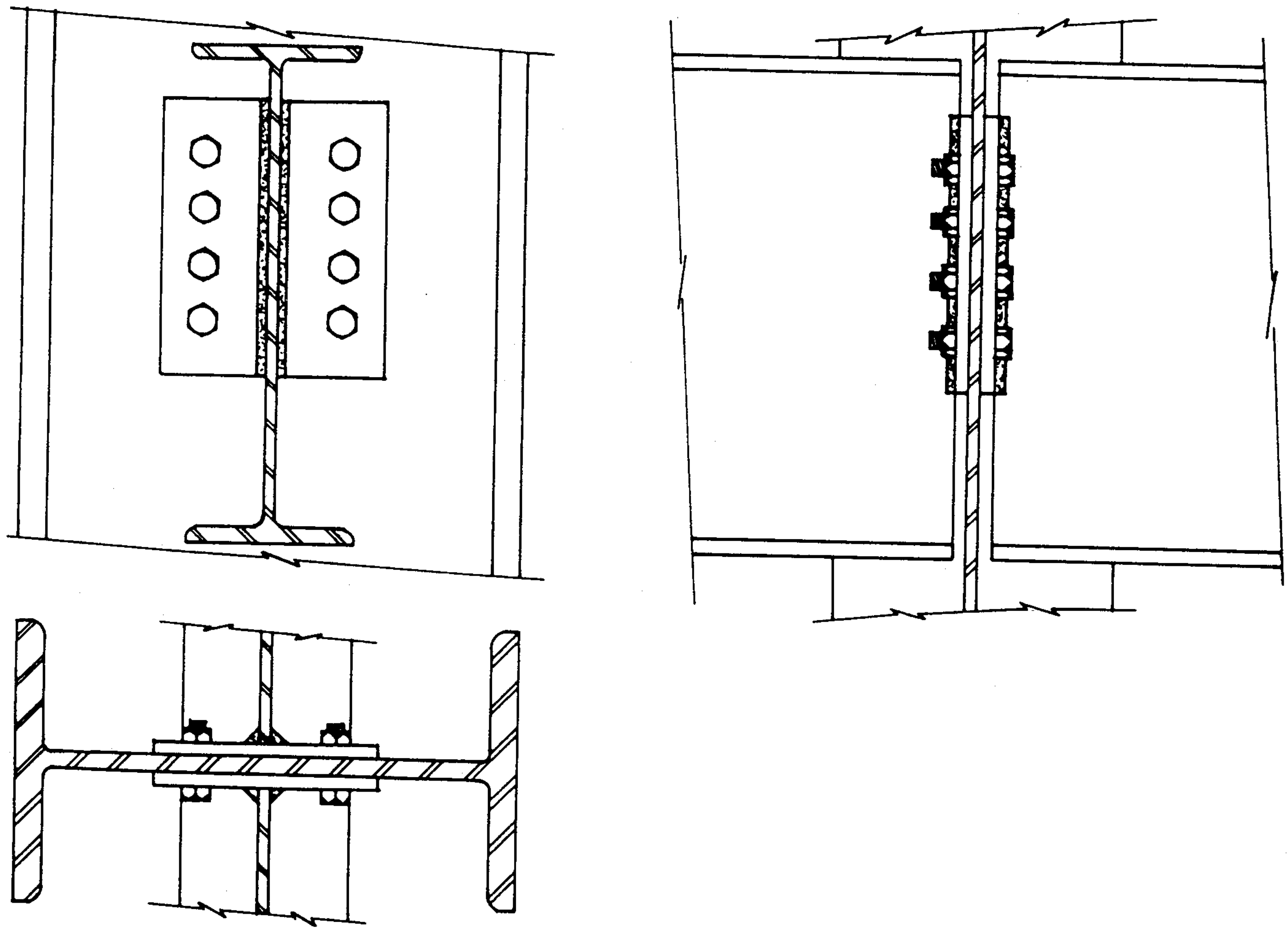


Figure 3.45 Double Flexible End Plate Connection to Column Web Detail

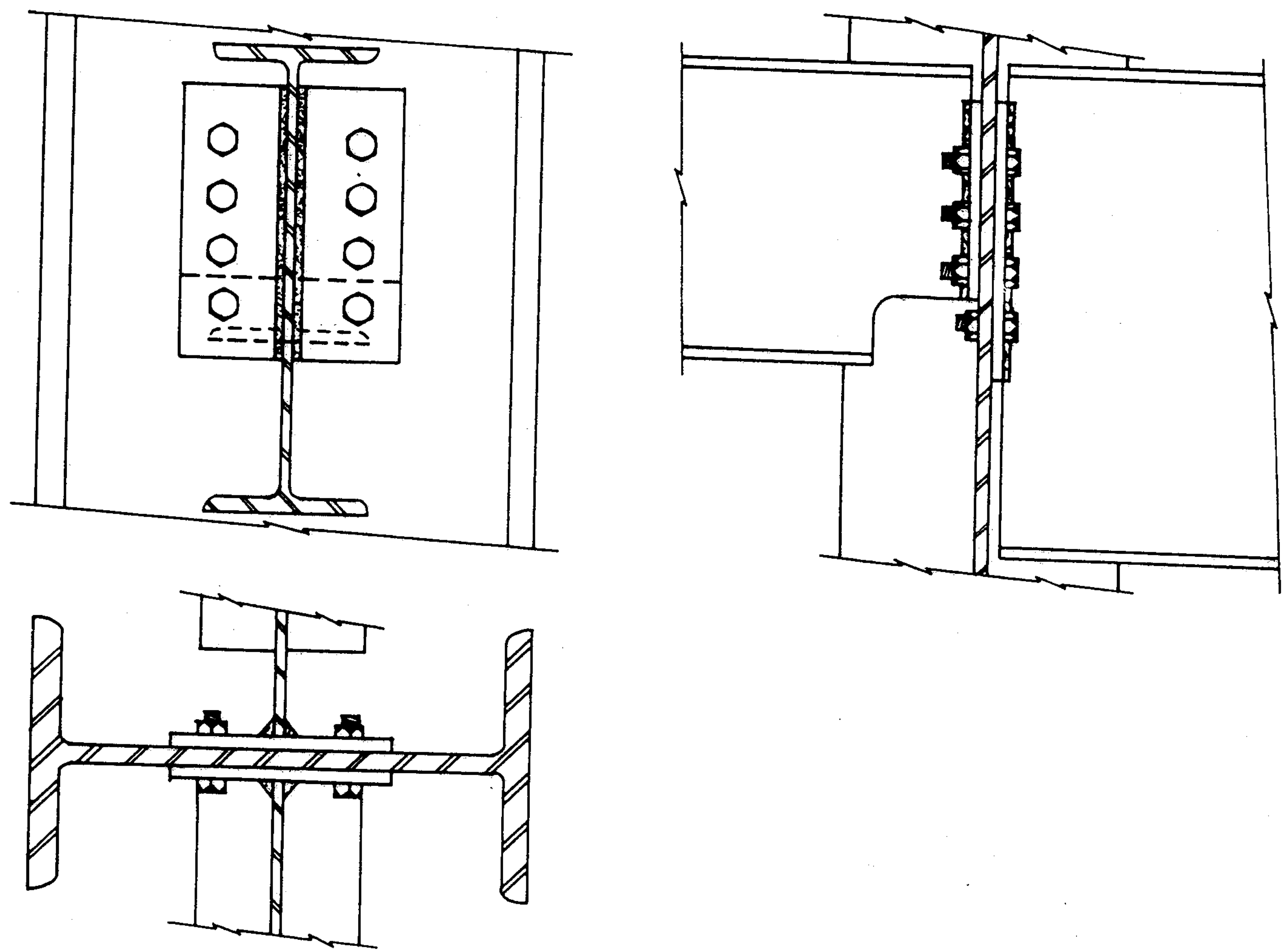


Figure 3.46 Double Flexible End Plate Connection to Column Web Detail

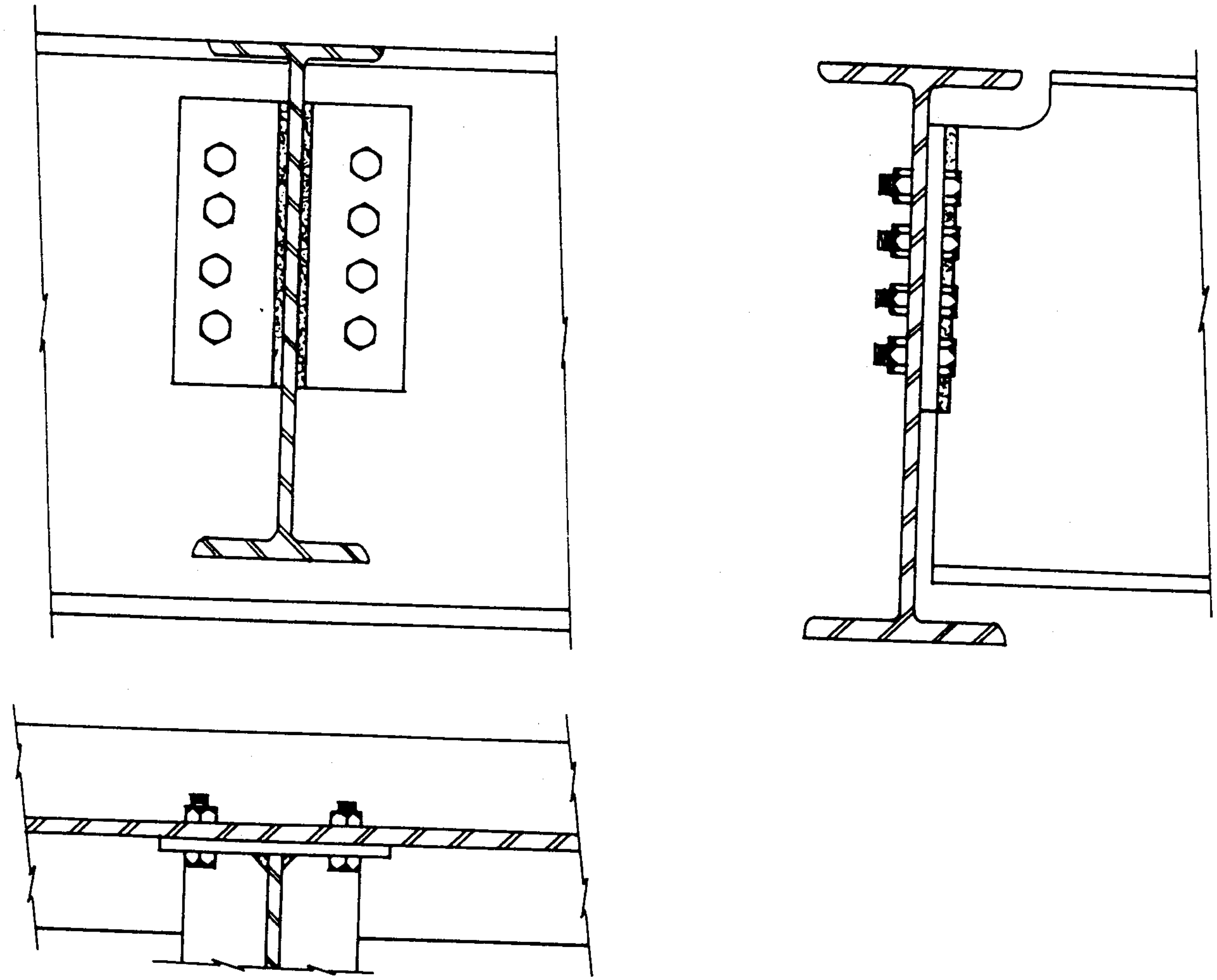


Figure 3.47 Single Flexible End Plate Connection to Beam Web Detail

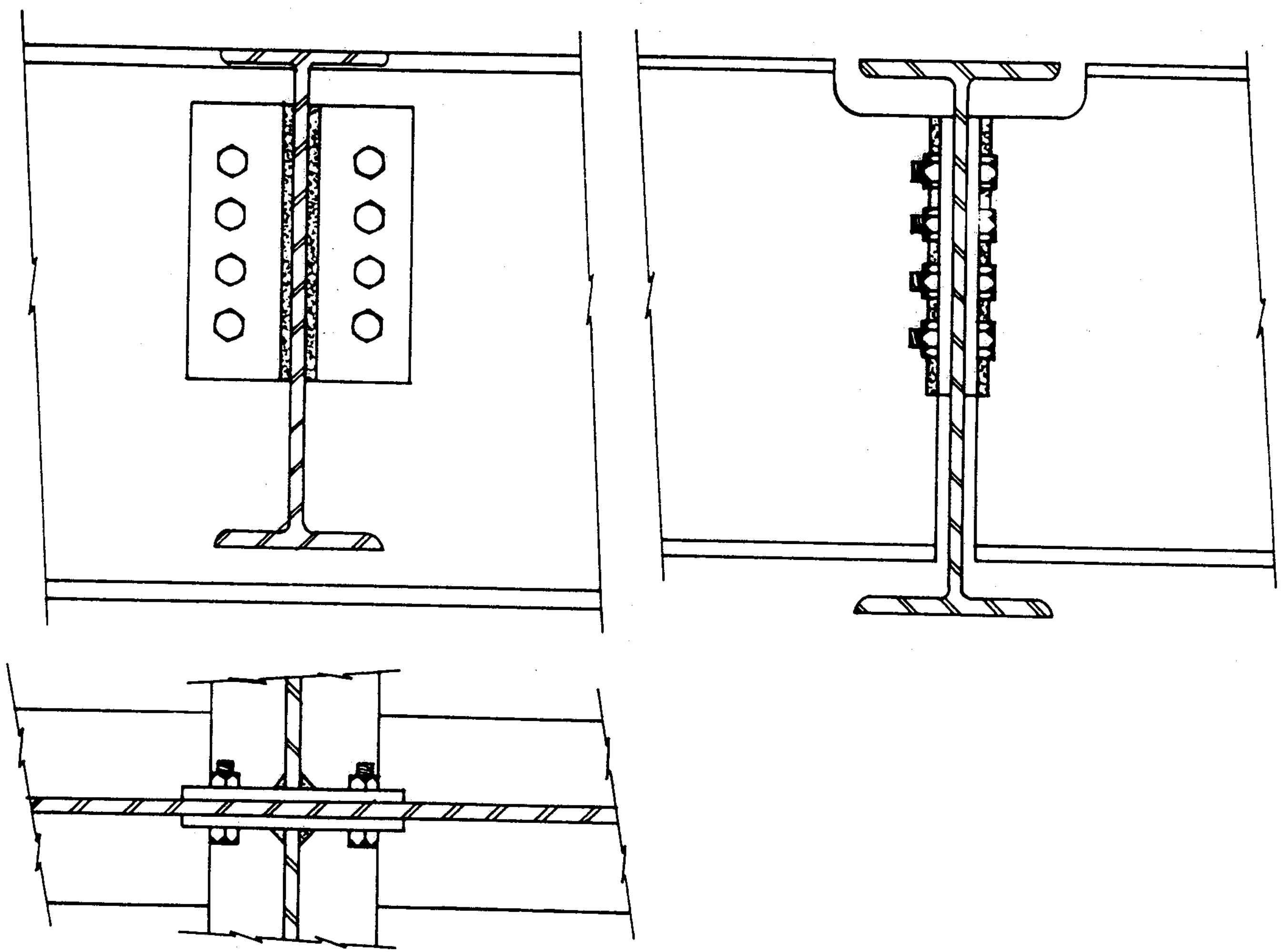


Figure 3.48 Double Flexible End Plate Connection to Beam Web Detail

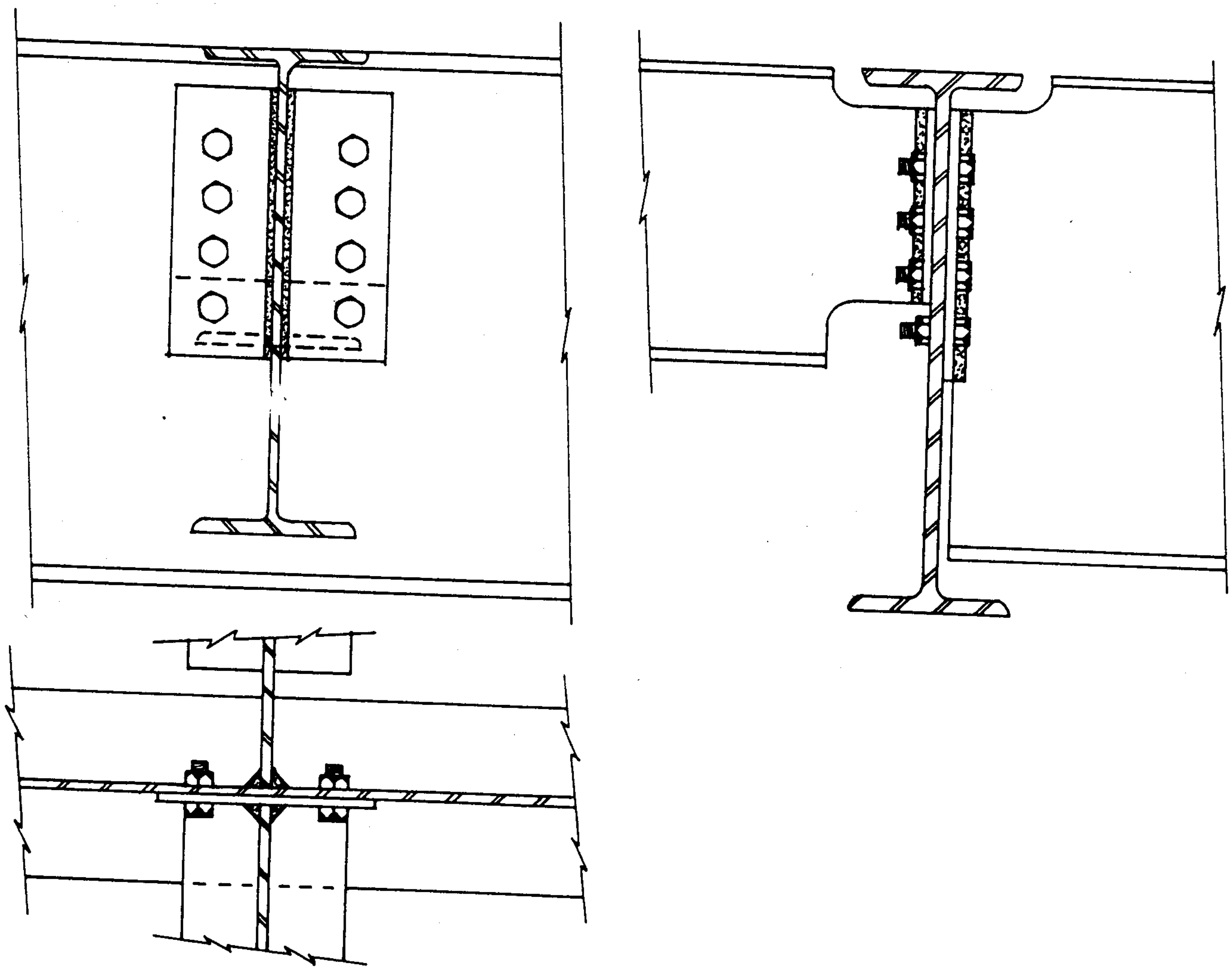


Figure 3.49 Double Flexible End Plate Connection to Beam Web Detail

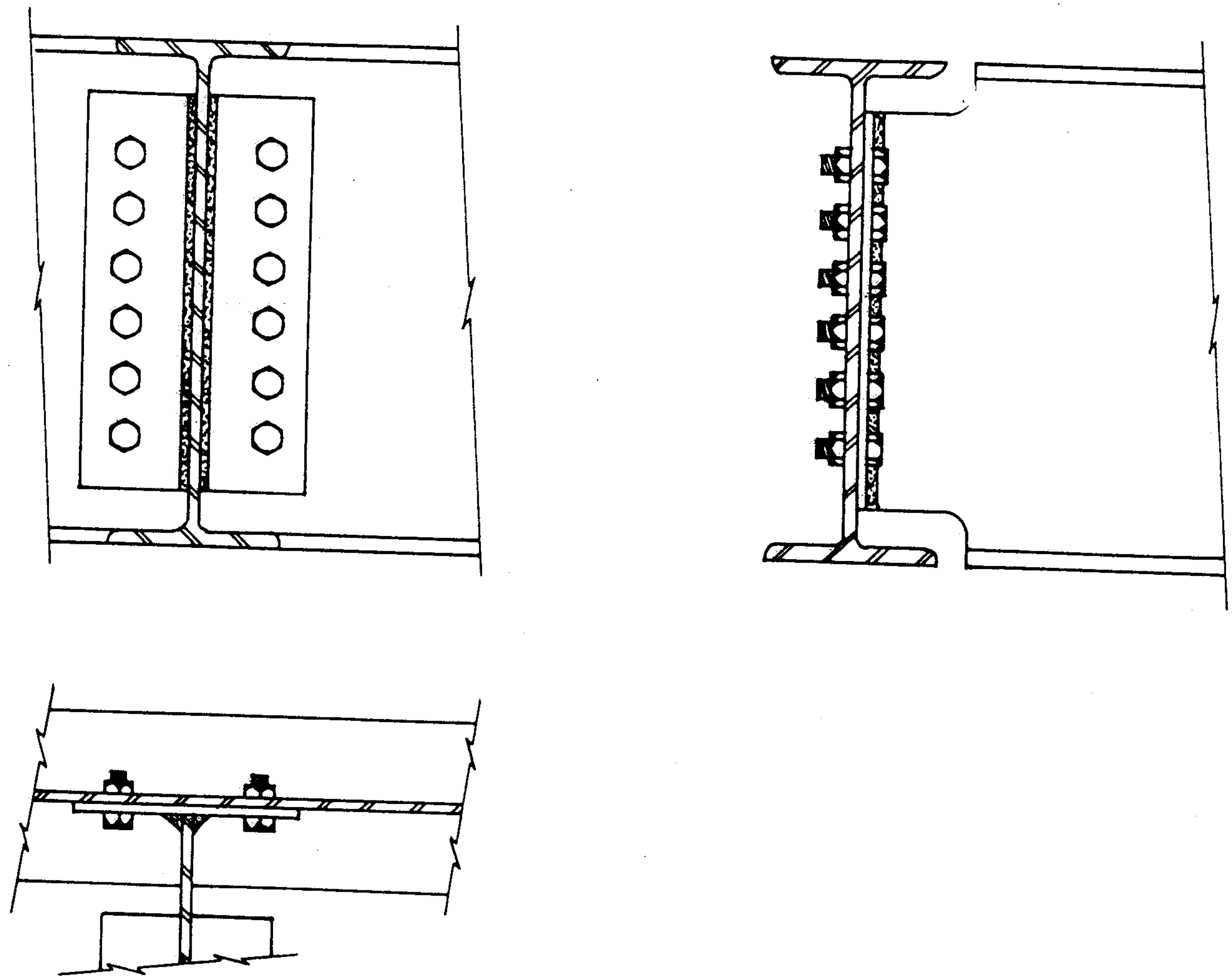


Figure 3.50 Single Flexible End Plate Connection to Beam Web Detail

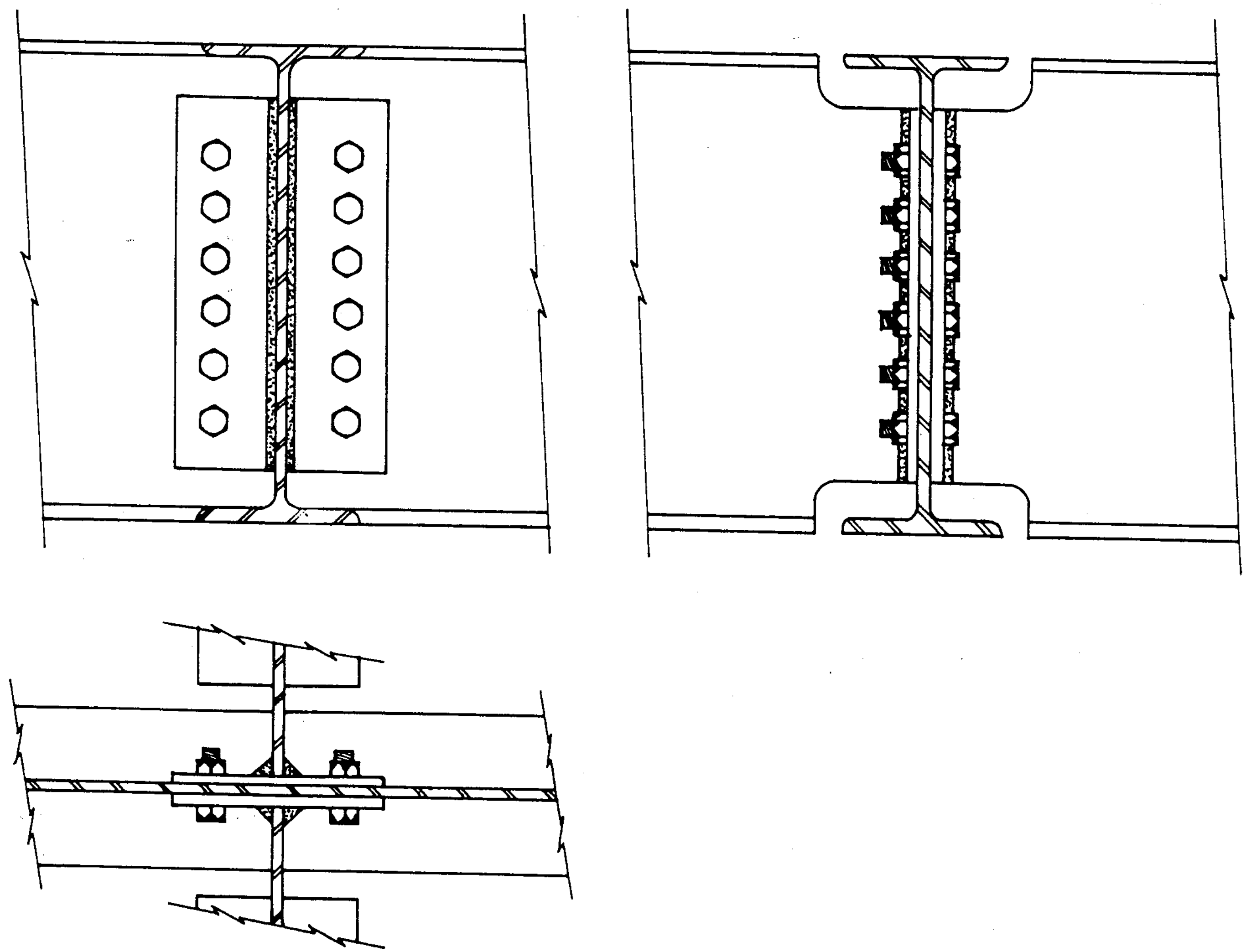


Figure 3.51 Double Flexible End Plate Connection to Beam Web Detail

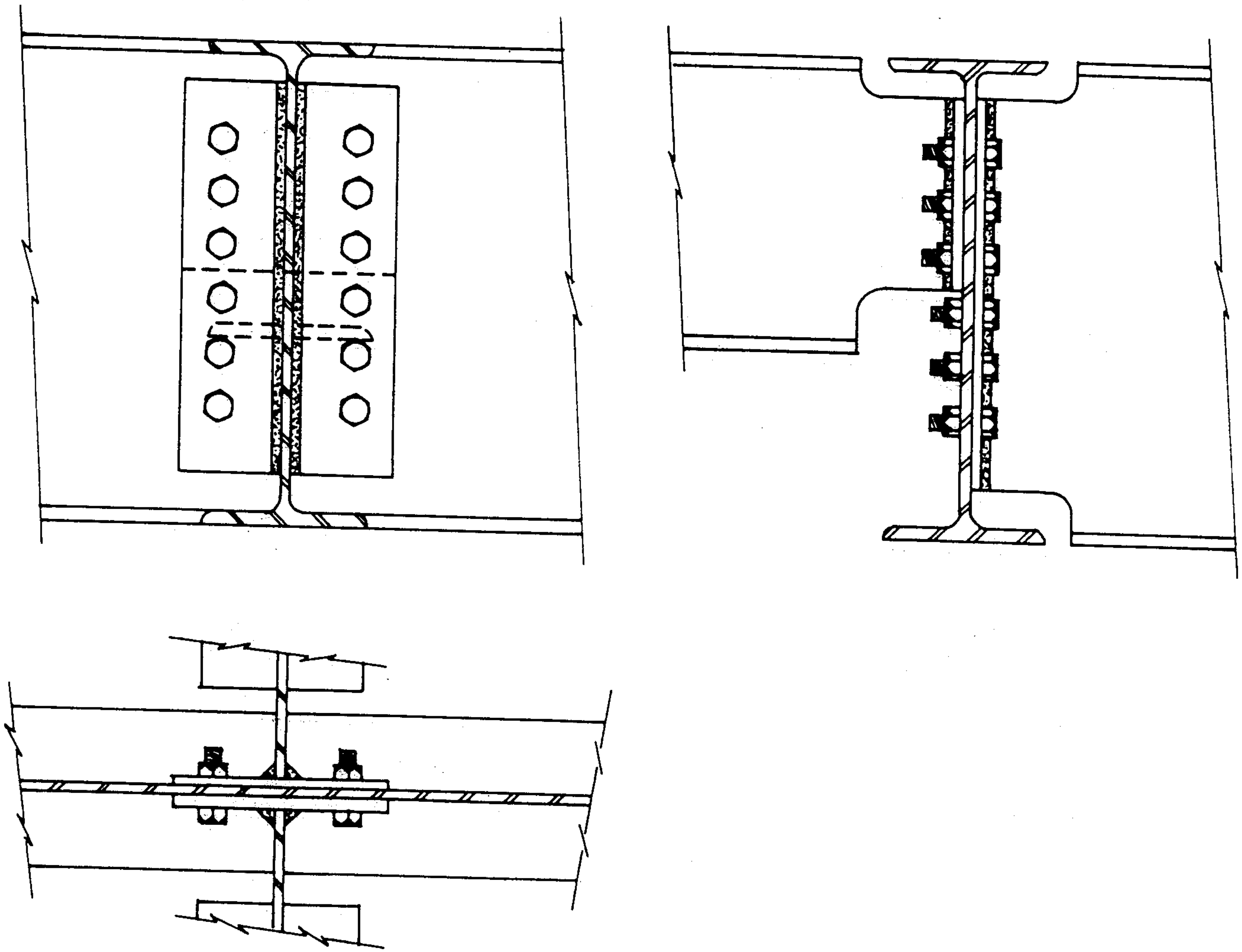


Figure 3.52 Double Flexible End Plate Connection to Beam Web Detail

4. Evaluation of the Unrestrained Rotation Limit of a Flexible End Plate Connection

4.1 Introduction

The flexible end plate connection is selected for its hinge-like characteristics, that is, for the transmission of shear with the development of limited moments. However, if the connection depth is selected only to provide sufficient resistance to primary shear forces, disregarding the rotation that the supported beam end must undergo, then the bottom flange can come to bear against the support, as depicted in Figure 4.1. When bottom flange bearing occurs, and if the support is rigid, the relatively large stiffness of the bottom flange of the supported beam causes the neutral axis of the connection to move to a position near the bottom of the beam, about which subsequent connection rotation occurs. The connection, with increased effective depth, is much stiffer flexurally as observed by both Sommer (1969) and Hafez (1982).

The tensile deformation of the top part of the end plate also increases more rapidly after bottom flange bearing occurs. If the magnitude of the tensile forces in the connection become too large, the primary shear capacity of either the welds, the plate, or the bolts may be diminished so that shear failure of the connection is imminent. Unrestrained connection rotation is essential if the flexible end plate is to perform as intended.

4.2 Analytical Method

Bottom flange bearing of the supported beam is imminent when the bottom surface of the flange has approached the support by a distance equal to the thickness of the end plate (Hafez 1982). From Figure 4.1, this condition occurs when the beam end rotation is

$$[4.1] \qquad \theta_b = \tan^{-1} \left(\frac{t_p}{\ell_b}\right)$$

For the angles of end rotation encountered in standard steel framing members, it is generally sufficient to calculate θ_b as

$$[4.2] \qquad \theta_b = \frac{t_p}{\ell_b}$$

To establish the rotation at which bottom flange bearing will occur, a connection rotation θ_b (as given in Equation [4.2]) is first assumed. Next, the corresponding neutral axis position is calculated. By applying the end plate load-deformation relationships over the entire depth of the connection, it is possible to check whether or not statics is satisfied. This procedure is repeated until the horizontal forces sum to zero.

4.3 Comparison of Analytical Method with Test Results

Even though the analytical methods presented in Chapter 3 predict the shape of the moment-rotation curves well, the angle of rotation of the connection at bottom flange bearing, θ_b, is not predicted as well, as can be deduced from Figures 3.15 to 3.42.

In two of the eight tests performed by Hafez (Figures 3.15 to 3.22) bottom flange bearing occurred at rotations less than predicted by analysis, and in five of the tests the analysis underestimates the limiting rotation. Test # 8 (Figure 3.22) cannot be considered valid since premature weld fracture rendered its behaviour to be non-typical.

In Sommer's test series (Figures 3.23 to 3.42) four of twenty tests experienced bottom flange bearing prior to the point predicted by analysis, and in fifteen tests the reverse was true. Test # 12 (Figure 3.30) failed prematurely in the heat-affected zone of the plate adjacent to the welds and should not be included in analysis.

From Table 3.1, the mean test-to-predicted ratio of the rotation at bottom flange bearing by the Kennedy-Hafez analysis is 1.11 with a coefficient of variation of 0.25, while the proposed method gives a ratio of 1.08 and a coefficient of variation of 0.25. The dispersion as reflected by the coefficient of variation is considerably greater for the rotations than the moments.

To account for the greater dispersion and to preclude, insofar as possible, premature bearing of the bottom flange of the beam on the support, Hafez (1982) suggests using only 2/3 of the predicted rotation when the method is used in design. Only one test, Test # 27 (Sommer 1969), falls below this value at a test-to-predicted value of 0.50. However, because of the very flexible nature of this particular connection, the use of the 2/3 rule would not have resulted in too excessive an end moment, as may be deduced from Figure 3.41.

Bottom flange bearing is chiefly a problem when the support details are similar to those used in the tests of Sommer (1969) and Hafez (1982), as depicted in Figure 3.43. Bolt deformations, or other unaccounted for tensile deformations, would tend to lower the position of the neutral axis and, therefore, from Equation [4.2], increase the rotation at which bottom flange bearing would occur. However, if the supporting member deforms locally, near the bottom of the end plate, as shown in Figure 4.2, bearing would occur sooner than predicted.

In connection details which have the bottom flange of the supported beam coped, as depicted in Figures 3.46, 3.49, 3.50, 3.51, and 3.52, connection rotation will not be restrained by bottom flange bearing for usual proportions and this condition need not be checked for such details.

Although generally not considered in design, the deflection of a beam at ultimate load levels causes a shortening of the beam to occur, as shown in Figure 4.3. If the supports are restrained from moving, the compressed region of the end plate connection would be reduced. This would tend to reduce the possibility of bottom flange bearing.

The presence of shear stresses in steel members loaded into the plastic range is known to 'soften' the normal or flexural stress-strain response of such members (ASCE 1971). It is likely that some degree of softening occurs in the web of the supported beam and in the end plate, because significant plastic behaviour is observed to occur in these regions (Sommer 1969, Hafez 1982). Since no account is made of this reduced stiffness, the analytical method would not be expected to predict θ_b accurately in all cases. Depending upon the distribution of shear stresses in the connection, the softening effect of the shear stresses may result in either or both a compression region or a tension region that does not follow exactly the load-deformation behaviour of the end plate details without such shear stresses.

It is apparent that quantifying the value of θ_b can be complicated by several factors associated with a connection detail. To account for these factors, which are not considered directly in the analysis, the proposal of Hafez (1982) to use 2/3 of the value of θ_b predicted by the analysis, is recommended for design.

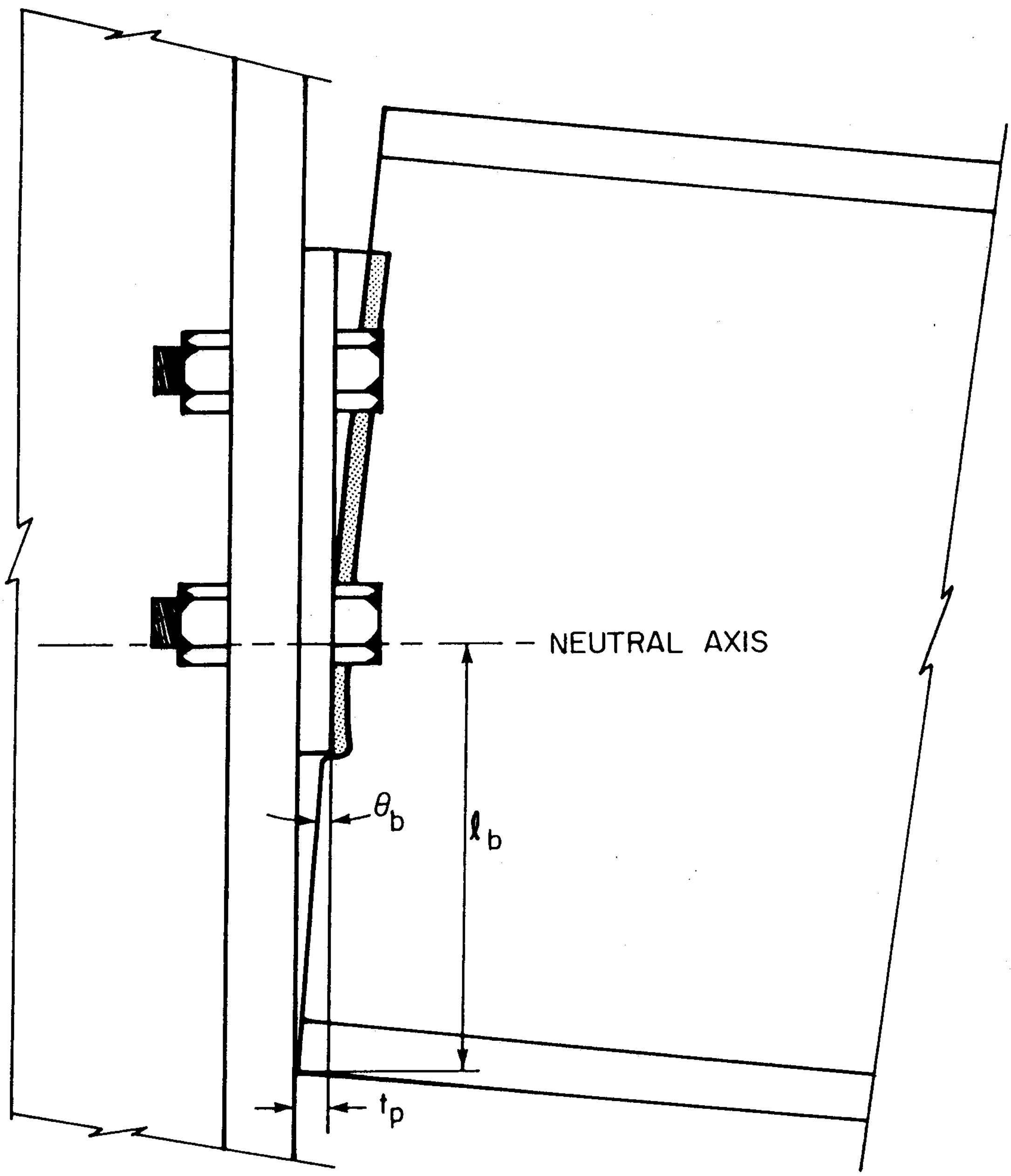


Figure 4.1 Bottom Flange Bearing of Beam on Support

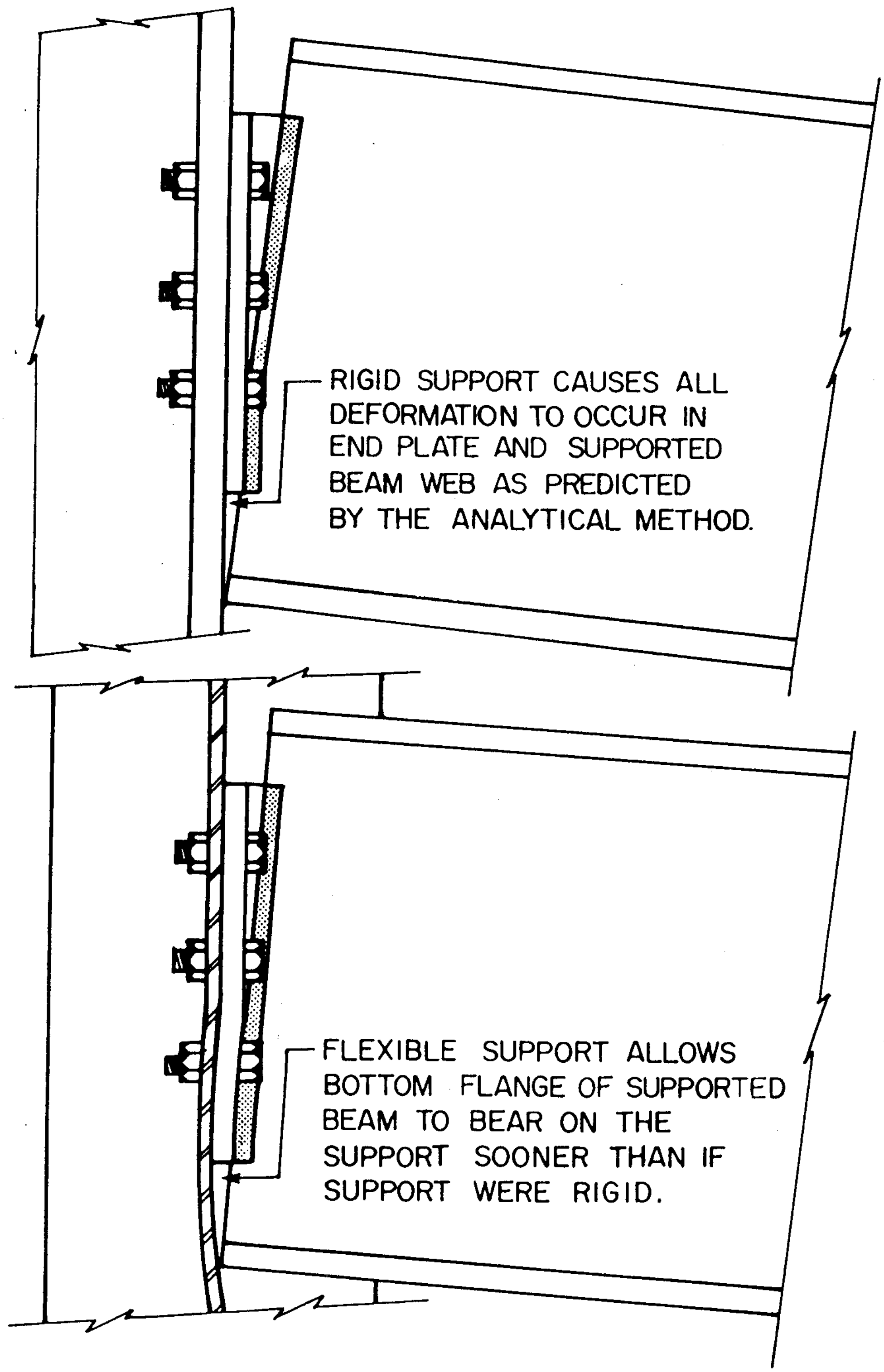


Figure 4.2 Effect of Support Stiffness on Bottom Flange Bearing

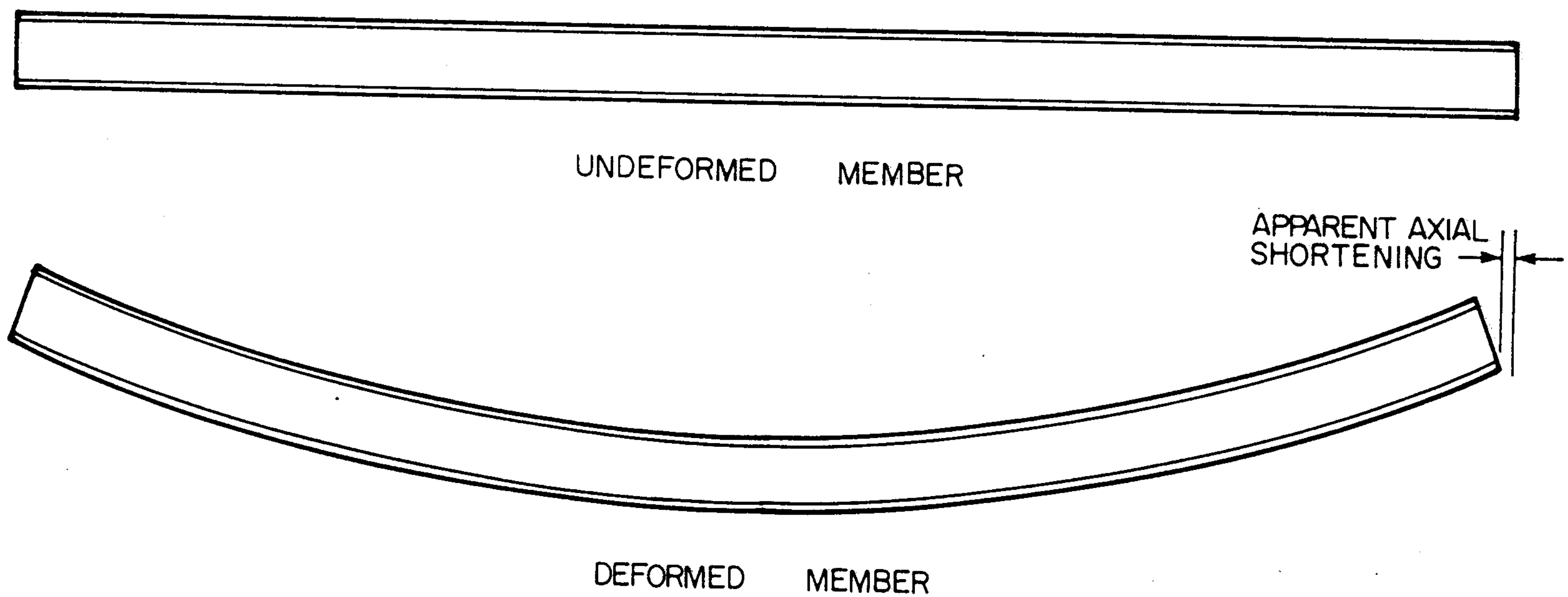


Figure 4.3 Apparent Axial Shortening of Supported Beam Due to Large Transverse Deflections

5. Evaluation of the Secondary Forces that Develop in the Bolts of a Flexible End Plate Connection

5.1 Introduction

A prerequisite to determining the primary shear capacity of a flexible end plate connection is an analytical method to determine the magnitude and distribution of secondary forces that develop in the connection when it deforms. The method presented in Chapter 3 to evaluate the moment-rotation relationship can be used to estimate the distribution of tensile and compressive forces that develop at the interface between the end plate and beam web. In this chapter consideration is given to the secondary tensile and shear forces which develop in the bolts.

5.2 Description of Model

Although not common in flexible end plate connections the possibility of bolt failure in deep connections exists (Sommer 1969). To estimate the reduced primary shear strength of the bolts, an analysis must consider the limiting deformation that the connection can undergo due to limited bolt strength and deformability. To permit a rigorous analysis, the tension zone of a typical end plate shown in Figure 5.1(a) is modelled as shown in Figure 5.1(b).

Two collinear, rigid, horizontal, elements model the plate between the bolts and the plate edge. The two portions of the end plate between the fillet welds and the bolt lines are modelled by axial springs A3 and by shear springs V1. The axial springs A1 oriented perpendicular to the plate at the center line of bolt hole locations model the tensile stiffness of the bolts. Axial springs A2 located in the plane of the undeformed connection at the bolts represent the collective stiffnesses of the bolt in shear, the net cross-section of the plate in tension, and the bearing stiffness of the plate between the edge of the plate and the outer edge of the bolt hole. Hinges with rotational springs R1 are located where plastic hinges were generally observed to form. The flexural stiffness of the plate is modelled by the rotational springs. Between the two central hinges is the very stiff junction of the connection plate and the supported beam web, represented by an undeformable element with a length equal to the thickness of the supported beam web plus two times the size of one of the fillet welds. The entire connection plate system rests on a frictionless surface supported by a rigid base, representing the supporting member. The width of the model is taken as one bolt hole pitch to allow a direct evaluation of secondary bolt forces at each bolt location.

The four hinges are idealized to be undeformable and capable of transferring axial and shear forces only. Because the plastic hinges at both the bolt lines and at the toes of the fillet welds formed in the gross section of the plate (Hafez 1982), all four hinges are assumed to have identical stiffnesses. For flexible end plate connections tested by Sommer (1969) and by Hafez (1982), the simultaneous formation of four plastic hinges in the plate was always observed, and is therefore assumed to be the case for this model.

Deformations of the fillet welds, the supported beam web, and the support, all assumed here to be rigid, would tend to reduce secondary forces in the connection plate and bolts. Neglecting these deformations is therefore conservative. Because bolt holes are necessarily larger than the bolts, there is always the possibility of slip in a real connection. However, no consideration of this is made herein. Connection rotations and deformations calculated based on this assumption will result in prediction of secondary forces which are conservatively large.

When the supporting material is thicker than the end plate, which was the case in the Sommer (1969) and Hafez (1982) tests, the bearing deformations in the end plate will predominate. Secondary forces due to bearing resistance are thereby limited by the bearing stiffness of the end plate. Ignoring bearing deformations in the support, which can be significant if the support is as thin or thinner than the end plate, results in a conservative overestimate of the secondary horizontal forces at the bolts. This approach is adopted herein.

Both Kennedy et al. (1981), and Sawa et al. (1982), considered the flexural stiffness of the bolts when modelling bolted T-joints. While this flexural stiffness is not considered directly here, the shear stiffness used for describing the bolts implicitly considers it.

To use the model, appropriate spring stiffnesses must be developed.

5.3 Analytical Relationships

Figure 5.2 shows the deformations and resulting external force system that develops when out-of-plane deformations occur. Various free body diagrams are shown in Figure 5.3. The distributed prying force Q(x) is replaced with a discrete prying force Q such that

[5.1] $$Q = \int_{x=0}^{x=x'} Q(x)\, dx$$

The location of the prying force resultant is taken as the lesser of two values given in the CISC Handbook (CISC 1980), that is, either at the extreme edge of the plate or at a distance of 1.25 (ℓ + f) measured from the centerline of the bolt hole towards the plate edge.

From the free body diagram shown in Figure 5.3(a), the value of the prying force Q is given as

$$[5.2] \qquad Q = \frac{1}{a}\left(M + \frac{V\ d_h}{2\ \cos\ \alpha_1} + \frac{H\ d_h}{2}\ \tan\ \alpha_i\right)$$

If the angles α_3 are assumed to be zero, (see Figure 5.2) then $\alpha_1 = \alpha_2 = \alpha$ and Equation [5.2] can be written as

$$[5.3] \qquad Q = \frac{1}{a}\left(M + \frac{V\ d_h}{2\ \cos\ \alpha} + \frac{H\ d_h}{2}\ \tan\ \alpha\right)$$

Assuming α_3 equals zero is tantamount to assuming that the axial deformations of the bolts, that is the A1 springs, are generally small compared to the maximum plate deformation Δ. This is generally the case, unless the connection geometry is chosen to create a very stiff connection detail. When connection proportions are selected so that α_3 is not small, that is four hinges do not form simultaneously in the end plate, then the connection is unfavourably stiff and should not be used. Using this assumption for overly stiff connections results in the tensile forces in the bolts being overestimated (and tensile bolt forces would be predicted to be greater than the tensile strength of the bolts). Such connections would be rejected on the basis of strength (see Chapter 6) and therefore the four hinge assumption not only simplifies the analysis but also prevents undesirably stiff details from being prescribed.

From the free body diagram in Figure 5.3(c), the shear force V is given as

$$[5.4] \qquad V = \frac{2\ M}{\ell} \cos \alpha$$

Combining Equation [5.4] with Equation [5.3] gives Q as

$$[5.5] \qquad Q = \frac{1}{a} \left(M + \frac{M\ d_h}{\ell} + \frac{H\ d_h}{2} \tan \alpha \right)$$

where the value of M is

$$[5.6] \qquad M = K_{R1}\ \alpha$$

and the value of H is

$$[5.7] \qquad H = K_{A2}\ \Delta_{A2} + K_{A3}\ \Delta_{A3} \cos \alpha$$

From the free body diagram of Figure 5.3 (a), the value of the tension force in a bolt, B, is

$$[5.8] \qquad B = Q + V \cos \alpha + P \sin \alpha$$

Using Equations [5.4], [5.5], and [3.17] to evaluate V, Q, and P respectively gives B as

$$[5.9] \qquad B = H \tan \alpha \left(1 + \frac{d_h}{2\ a}\right) + \frac{M}{a} \left(1 + \frac{d_h}{\ell} + \frac{2\ a}{\ell}\right)$$

5.4 Spring Stiffness Characteristics

5.4.1 Axial Spring A1

The springs with stiffness K_{A1} represent the tensile load-deformation characteristics of the bolts which, in Canada, are likely to be high-strength bolts.

From the studies done by Rumpf and Fisher (1963) on A325 bolts and by Sterling et al. (1965) on A490 bolts it is apparent that load-deformation characteristics of high-strength bolts are primarily a function of the length of the thread in the grip and the bolt installation technique.

As the length of the threaded portion in the grip is reduced, the ultimate deformation capacity of the bolts decrease. Sterling et al. also noted that this reduction in the length of the threaded portion caused an increase in the ultimate tensile strength of the bolt. Rumpf and Fisher reported that for bolts with grip lengths up to about 200 mm, the total grip had no appreciable effect on the load deformation characteristics of the bolt beyond the proportional limit; the deformation occurring in the length of thread within the grip governed the maximum attainable bolt elongation.

Rumpf and Fisher also noted that, for A325 bolts tightened 1/2 to 2/3 of a turn beyond snug, the ultimate elongation of a bolt is reduced to about 60 percent of the ultimate elongation attainable for a similar bolt installed finger-tight only. The shear strains induced in the bolt when it is tightened beyond the snug position reduce the ultimate axial strain attainable. However, as reported by Sterling et al., the effect of torquing A490 bolts beyond snug did not appear to cause such a great reduction in ultimate bolt elongation. For both A325 and A490 bolts the ultimate tensile strength of the bolts was not affected by the installation procedure.

For finger-tightened bolts, the ultimate elongation attainable ranges from about 3 mm to 4 mm, when the thread length in the grip is 3 mm, up to 9 mm when the thread length in the grip is 25 mm (Rumpf and Fisher). At the present time in Canada, bolts are to be tightened beyond snug and therefore would only have about 60 percent of these ultimate deformations, that is, 2 mm to 5 mm, if A325 or A325M bolts were used. Except for very stiff connections, this deformation is relatively small compared to the total deformation occurring at end plate connections at ultimate conditions. Therefore, neglecting bolt elongation is conservative and reduces the complexity of the model.

Furthermore, the bolts in flexible end plate connections are provided mainly to transfer beam end shear. The presence of shear strain in the bolt due to this shear force likely acts to reduce the ultimate tensile strain of the bolts even further, in a manner similar to that caused by torquing the bolts beyond snug at installation time. Therefore, it appears reasonable to neglect bolt elongation, that is, springs A1 are assumed to be infinitely stiff.

5.4.2 Axial Spring A2

The axial spring with stiffness K_{A2} represents the collective stiffness of the bolt in shear, the reduced cross-section in tension, and the plate between the bolt and the plate edge in bearing.

5.4.2.1 Shear Stiffness of the Bolts

The load-deformation characteristics of high-strength bolts loaded in shear have been investigated by Fisher (1965), Wallaert and Fisher (1965), Crawford and Kulak (1971), and Bahia and Martin (1980), among others. The tests conducted by Bahia and Martin resemble most closely the characteristics of the bolts of the flexible end plate connection when they undergo horizontal deformation due to out-of-plane movement of the end plate.

The test results of interest were obtained from a series of tests on single bolt specimens loaded in single shear where the bolt threads intercepted the shear plane (which may be the case in practice). The lap plates were manufactured from plates having nominal thicknesses of 12 mm, 14 mm, 17 mm, and 20 mm. To maintain a uniform yield strength in all of the lap plates, all specimens were fabricated from the same lot of 20 mm thick plate, and then the three thinner specimen groups were obtained by machining. The mean yield strength of the plate material was reported as 272 MPa. The high strength bolts had nominal diameters of 20 mm and had a mean ultimate tensile strength of 906 MPa.

For the range of test specimens evaluated, it was concluded that the total deformation of a bolt at ultimate load is directly related to the average bearing stress that exists between the bolt and the plate. No empirical relationship was developed, however, a plot is provided which compares the average bearing stress to the total shear deformation of a bolt over the entire loading range of the test specimens. From this it is possible to develop the following expression relating average bearing stress to ultimate shear deformation:

$$[5.10] \qquad \Delta_{bsmax} = k_1 \, \sigma_{bsmax}$$

For this particular case k_1 = 0.0139 when SI units are used. The coefficient k_1 is likely a function of the material properties of the connected plates, the bolt grade, the bolt diameter, and the location of the shear plane in the bolt. Investigations made by Wallaert and Fisher appear to substantiate this.

The maximum shear deformation of a bolt can also be related directly to the geometric and material properties of the bolt and the connected plates. The maximum average bearing stress that can be developed if the bolt fails in shear prior to plate failure can be expressed as

$$[5.11] \qquad \sigma_{bsmax} = \frac{V_u}{d_b \, t_p}$$

where the ultimate shear strength of the bolt is

$$[5.12] \qquad V_u = k_v \, k_{sp} \, A_b \, \sigma_{ub}$$

Substituting Equations [5.11] and [5.12] into Equation [5.10] gives

$$[5.13] \qquad \Delta_{bsmax} = k_1 \, k_v \, k_{sp} \, \frac{\pi \, d_b}{4 \, t_p} \, \sigma_{ub}$$

The coefficient k_v is a ratio of the shear strength of a bolt through the shank to its tensile strength and is usually observed to be between 0.6 and 0.7 in tests. The value of k_v observed by Bahia and Martin was

0.67. The coefficient k_{sp} is a ratio of the cross-sectional area of the bolt through the threads to the area at the shank. It normally ranges from about 0.70 to 0.80 and the value observed by Bahia and Martin was 0.78.

Equation [5.13] gives the maximum deformation of the bolt when it fails in shear. Maximum deformations, using the values of k_1, k_v and k_{sp} obtained by Bahia and Martin, are presented in Figure 5.4 as the upper limits of bilinear load-deformation relationships. Test results of Bahia and Martin are also plotted in Figure 5.4. In examining the test results it appeared appropriate to use a bilinear expression to model the load-deformation relationship, as shown in the figure.

Up to about 80 percent of the ultimate shear capacity, the deformation-shear relationship is given by

$$[5.14] \qquad \Delta_{bs} = \frac{k_2 \; H}{d_b \; t_p}$$

where k_2 has a value equal to 0.0074 when SI units are used. It is interesting to note that the bolt deformation is inversely proportional to the plate thickness; the thicker the plate, the smaller is the bending and local bearing component of the overall deformation.

The upper portion of the bilinear expression is obtained by joining the point representing both the deformation and 80 percent of the ultimate bolt shear to that for the maximum deformation given by Equation [5.14] and the ultimate bolt shear. The equation is

$$[5.15] \qquad \Delta_{bs} = \left(5\,k_1 - 4\,k_2\right) \frac{H}{d_b \; t_p} - k_v \, k_{sp} \, \pi \, \frac{d_b}{t_p} \, \sigma_{ub} \left(k_1 - k_2\right)$$

As shown in Figure 5.4, the bilinear relationships correlate reasonably well with the test results. Because coefficients in Equation [5.15] have been derived empirically, it is strictly valid only for connections with material and geometric properties similar to those in the Bahia and Martin tests. It is, however, considered valid for most end plate connections of usual proportions composed of mild or medium strength steel and high-strength bolts.

Throughout the load-deformation response, the deformation is inversely proportional to the plate thickness. The thicker plates provide more restraint than do the thinner ones. Hawkins (1971) suggested that the major factor that influences the shear deformation response of a bolt is the amount of restraint that is provided by the fastened plates and that the restraint provided by the connected plates is affected greatly by the thickness of the adjoining plates as depicted in Figure 5.5

Further, most flexible end plates are between 6 mm and 10 mm in thickness. This range lies outside the range tested by Bahia and Martin. It is assumed, because of the good inverse correlation obtained between deformation and plate thickness, that the relationship developed can be extended to the normal range of flexible end plate thicknesses.

In addition, the end plate is likely to be considerably thinner than the supporting element. In the limit, with a very thin end plate, rotational restraint of the bolt would only be provided by the thick supporting element and the bolt is bent in single curvature rather than in double curvature. To account for this, the bolt deformation in each plate can be computed independently by taking one half of the deformations of each plate as calculated using either Equation [5.14] or

Equation [5.15], and adding them together.

Moreover, in flexible end plate connections the bolts resisting large horizontal shears also resist tensile forces. These tensile forces are likely to increase the bolt deformation and therefore the relationships developed not considering this increased deformation would overestimate the shear force for a given bolt deformation (and hence are conservative). The primary shear forces will have a similar effect and neglecting them would again be conservative.

5.4.2.2 Tensile Stiffness of the Net Section

Fisher (1965), in conducting tests on mild strength tension specimens with holes, observed the development of a triangular yield pattern on both sides of the hole. For specimens with a gross plate-width to hole-diameter ratio of from 3.0 to 7.5, the ultimate load was equal to the net cross-sectional area multiplied by the ultimate tensile strength.

A finite element analysis done by Kato and Aoki (1970) showed, as would be expected, for the same type of specimen that the greatest strains occur near the net section. This analysis also indicated that the triangular zones of yielding around the holes form at angles of 45 degrees to the axis of loading, as shown in Figure 5.6(a). This zone of excessive straining persisted until the net section ruptured.

Tests conducted by Iwankiw and Schlafly (1982) on single bolt double lap joints designed to fail on the net section indicated that it is possible to reach the ultimate net section tensile capacity.

A plausible model, therefore, could consist of a series of parallel springs as shown in Figure 5.6(b), each with a load-deformation response based on the uniaxial stress-strain characteristics of the plate material. Because of the different spring lengths, the load-deformation response for the member as a whole would be similar to the stress-strain curve but rounded due to the uneven straining. A simpler model consists of two springs, one on either side of the hole, located midway between the center of the hole and the edges of the specimen with an ultimate capacity equal to the ultimate tensile strength multiplied by the net cross-sectional area. However, ultimate deformations should be based on strains at the edge of the hole which are proportional to the strain of the two springs used in the simple model.

Using the two spring model with the bi-modal stress-strain curve shown in Figure 5.7 gives a reasonable analytical model of the net section tensile stiffness. As seen in Figure 5.7 the proposed stress-strain relationship does not include a yield plateau common to hot rolled steels. Because of the uneven tensile straining in the vicinity of the net section coupled with the presence of secondary flexural and primary shear stresses, this simplified curve is considered reasonable.

Based on the stress-strain curve of Figure 5.7 and using the simple two spring model, equations describing the load-deformation response of the net section are

[5.16] $$H = 2\left(p - d_h\right) \frac{t_p E_p}{p} \Delta_{ns}$$

for $$0 < \Delta_{ns} < \frac{\sigma_{yp}\, p}{2\, E_p}$$

and

[5.17] $$H = (p - d_h)\, t_p\, \sigma_{yp} \left\{0.984 + 0.0177 \left(\frac{\varepsilon}{\varepsilon_y}\right) - 0.000182 \left(\frac{\varepsilon}{\varepsilon_y}\right)^2 + 0.63 \times 10^{-6} \left(\frac{\varepsilon}{\varepsilon_y}\right)^3\right\}$$

for $\dfrac{\sigma_{yp}\, p}{2\, E_p} \leq \Delta_{ns}$

where $\dfrac{\varepsilon}{\varepsilon_y} = \dfrac{2\, E_p\, \Delta_{ns}}{p\, \sigma_{yp}}$

5.4.2.3 Bearing Stiffness of the Plate

de Back and de Jong (1968) observed in single bolt, double shear, lap splice tests of mild steel specimens that for plates having gross width to loaded edge distance ratios greater than or equal to about 1.60, failure occurred in a bearing or tear-out fashion. As the bolt pitch to edge distance ratio for end plate connections falls within the range between 1.8 to 3.3, bearing deformations are likely to be an important consideration.

The test series of Owens et al. (1981) performed on single bolt, single shear lap plate splices with 20 mm diameter high strength bolts in 6 mm thick by 72 mm wide lap plates with a 22 mm diameter hole centered in the plate located 40 mm from the loaded edge replicate well the characteristics of end plates. The steel was Grade 43 with a nominal yield strength of about 300 MPa.

Based on non-dimensionalized experimentally obtained bearing stress vs. deformation curves of Owens et al., the approximate bilinear relationships of

[5.18] $H = \sigma_{up}\, t_p\, d_b\, \Delta_{be}$

for $0 < \Delta_{be} < \frac{e}{d_h}$

and

[5.19] $H = \sigma_{up}\, t_p\, e$

for $\frac{e}{d_h} < \Delta_{be} < 0.40\, d_h$

are proposed. The upper limit is assigned because bearing rupture occurred at a deformation level, Δ_{be}, of about 40 percent of the bolt diameter.

Equations [5.18] and [5.19] and experimental data of Owens et al. are shown in Figure 5.8. Thomas and Bennetts (1982), Chung Wing and Harris (1983), and CAN3-S16.1-M78 (CSA 1978) all use Equation [5.19] to predict the ultimate bearing strength.

The load-deformation response of specimens tested by Chung Wing and Harris was similar to that observed by Owens et al., with the exception that the connections appeared stiffer at load levels below that causing a bearing stress of σ_{up}. This response is unexpected as the loaded edge distance to bolt hole diameter ratio of the tests of Chung Wing and Harris were about the same as Owens et al.. It is attributed to the fact that the bolts were installed to 1/2 of a turn past snug, whereas Owens et al. installed bolts to finger-tightness only.

Frank and Yura (1981) observed that as lap plate width increases, all other details remaining unchanged, the ultimate bearing deformation of bearing specimens increase with no significant change in ultimate strength. This greater ultimate deformation was also observed by Thomas and Bennetts for one test with a large plate width to hole diameter ratio. The Chung Wing and Harris tests, having relatively large plate width to bolt hole diameter ratios, also appeared to have greater ultimate deformations than the tests of Owens et al.. Although none of these researchers discussed this point, it appears that the ultimate deformability of bearing specimens is a function of the gross plate width to bolt hole diameter ratio.

Richard et al. (1980) tested single high-strength bolt, single shear, lap plate specimens of mild steel. They report initial stiffnesses greater than that reported by Owens et al.. These greater stiffnesses are attributed to the clamping force of the pretensioned bolts. In flexible end plate connections, the bolt tensions that occur with end plate distortion will reduce the clamping force. Therefore, the simple model based on the Owens et al. tests is considered valid.

5.4.2.4 Application

Axial spring A2 has a load-deformation response composed of three parts. For a given horizontal force H, the resulting spring deformation Δ_{A2} is

[5.20] $$\Delta_{A2} = \Delta_{bs} + \Delta_{ns} + \Delta_{be}$$

Because the magnitudes of the three component deformations of Δ_{A2} are not known initially, an iterative procedure using a computer or a programmable calculator is used to determine the deformations corresponding to the force H (as given by Equation [5.7]).

5.4.3 Axial Spring A3

The springs with stiffness K_{A3} represent the tensile stiffness of the gross section of the end plate between the inner edge of the bolt holes and the toe of the fillet welds. Since the net section axial springs (that is springs A2) are assumed to extend a distance equal to 1/4 of the pitch (p) from the center of the bolt hole towards the fillet welds, it is reasonable to assume the length of the A3 springs to be equal to $(\ell - p/4 + d_h/2)$. By applying the stress-strain relationships shown in Figure 5.7 (which is reasonable because the primary shear and secondary flexural behaviour in the plate likely results in a 'rounded' tensile stress-strain response), the following load-deformation relationships for the gross section of the plate can be developed:

[5.21] $$H = \frac{p\, t_p\, E_p}{(\ell - \frac{p}{4} + \frac{d_h}{2})} \Delta_{A3}$$

for $$0 < \Delta_{A3} < \frac{\sigma_{yp} (\ell - \frac{p}{4} + \frac{d_h}{2})}{E_p}$$

and

[5.22] $$H = p\, t_p\, \sigma_{yp} \left\{0.984 + 0.0177 \left(\frac{\varepsilon}{\varepsilon_y}\right) - 0.000182 \left(\frac{\varepsilon}{\varepsilon_y}\right)^2 + 0.63 \times 10^{-6} \left(\frac{\varepsilon}{\varepsilon_y}\right)^3\right\}$$

for $$\frac{\sigma_{yp}(\ell - \frac{p}{4} + \frac{d_h}{2})}{E_p} < \Delta_{A3}$$

where $$\frac{\varepsilon}{\varepsilon_y} = \frac{E_p \, \Delta_{A3}}{(\ell - \frac{p}{4} + \frac{d_h}{2}) \; \sigma_{yp}}$$

5.4.4 Shear Spring V1

The springs with stiffness K_{V1} represent the shear stiffness of the end plate perpendicular to its plane. It is unlikely that the gage to plate thickness ratio for flexible end plates will ever be less than 8. As with the analysis of any plate element having proportions shallower than this, this analysis will ignore the contribution of shear deformations to the total deformations occuring. In other words, springs V1 will be assumed to be infinitely rigid.

5.4.5 Rotational Spring R1

Rotational springs with stiffness K_{R1} represent the flexural stiffness of the end plate. The hinge rotation α as a function of the plate deformation and geometry is

[5.23] $$\alpha = \tan^{-1} \frac{\Delta}{\ell}$$

For elastic behaviour the plate deflection in terms of the moment M at the hinges is

[5.24] $$\Delta = \frac{2 \; M \; \ell^2}{E_p \; p \; t_p^3}$$

Combining Equations [5.23] and [5.24] gives the elastic M-α relationship

$$[5.25] \qquad M = \frac{p\, t_p^3\, E_p}{2\, \ell} \sin \alpha$$

Assuming a perfectly elasto-plastic moment-curvature relationship and that the maximum moment obtainable is $M_{up} = pt_p^2 \sigma_{up}/4$, then the limiting value of the angle α for elastic behaviour is

$$[5.26] \qquad \alpha_y = \tan^{-1} \left(\frac{\sigma_{up}\, \ell}{2\, E_p\, t_p}\right)$$

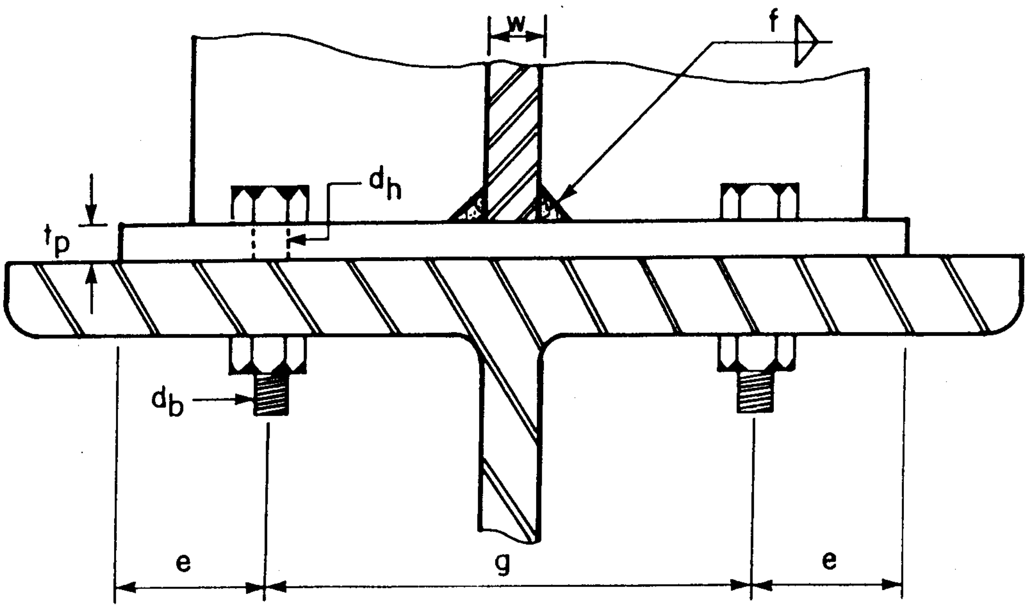
In the inelastic region, as the deflection Δ becomes significant, it is necessary to consider the reduction in moment M due to the presence of axial load as given by Equation [3.7]. Combining Equations [3.7] (with $P_{up} = P_{yp}$) and [3.17] (ignoring the flexural term), the M-α relationship is

$$[5.27] \qquad M = M_{up} \left(1 - \frac{H^2}{\cos^2 \alpha\, P_{yp}^2}\right)$$

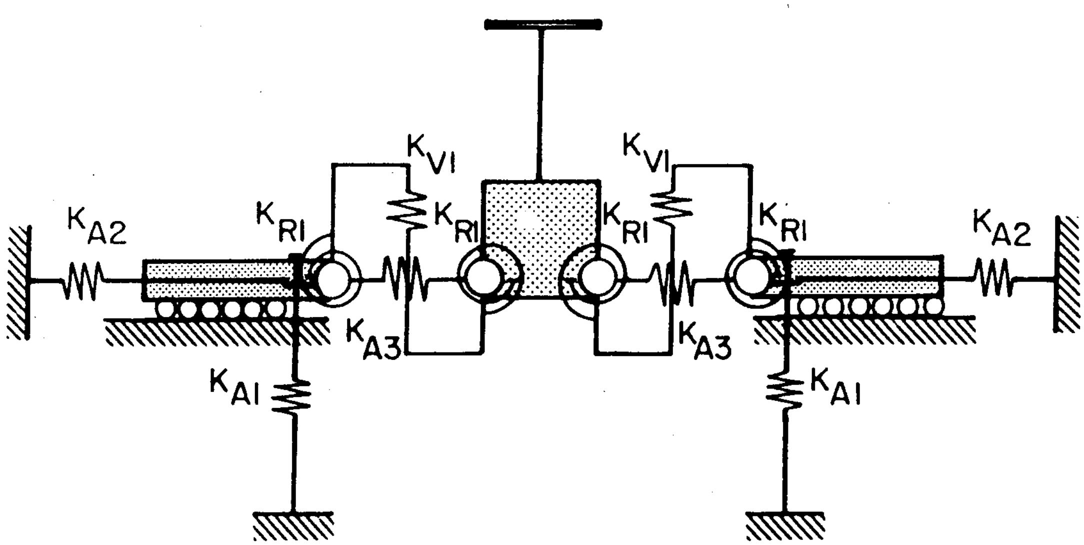
Table 5.1 shows for the 27 T-section tests performed by Hafez (1982), that when 6.4 mm (1/4 inch) plate is used the maximum value of α was 0.25 radians on the average, and when 9.5 mm (3/8 inch) and 12.7 (1/2 inch) plate was used it was about 0.45 radians. These values should be used as limits in any analyses performed using the model described herein.

Table 5.1 Ultimate Rotation of Plastic Hinges in End Plate Specimens, Hafez (1982)

Test No.	Plate Thickness (inches)	Bolt gage (inches)	Rupture Angle (rads)	Average Rupture Angle (rads)
11 12 14 26 27	1/4	4	0.222 0.295 0.251 0.200 0.210	0.236
1 4 15 22 23	1/4	5.5	0.282 0.270 0.250 0.231 0.247	0.256
8 9 10 13 24 25	3/8	4	0.475 0.438 0.330 0.498 0.495 0.445	0.447
3 6 7 16 20 21	3/8	5.5	0.374 0.480 0.500 0.500 0.500 0.495	0.475
2 5 17 18 19	1/2	5.5	0.468 0.440 0.440 0.350 0.455	0.431



a) Connection Detail



b) Connection Model

Figure 5.1 Model of Flexible End Plate Connection

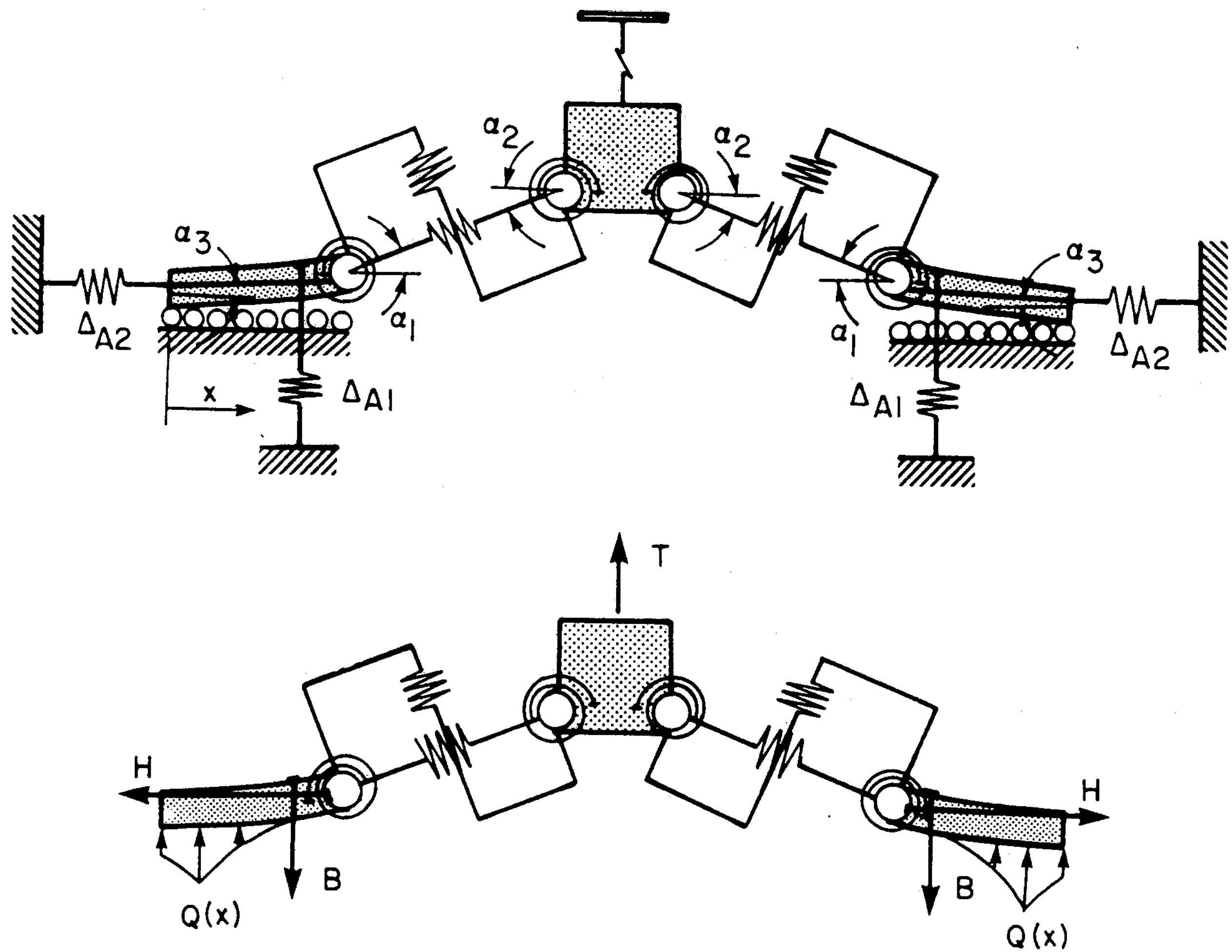


Figure 5.2 Model of Flexible End Plate Connection In Deformed Position

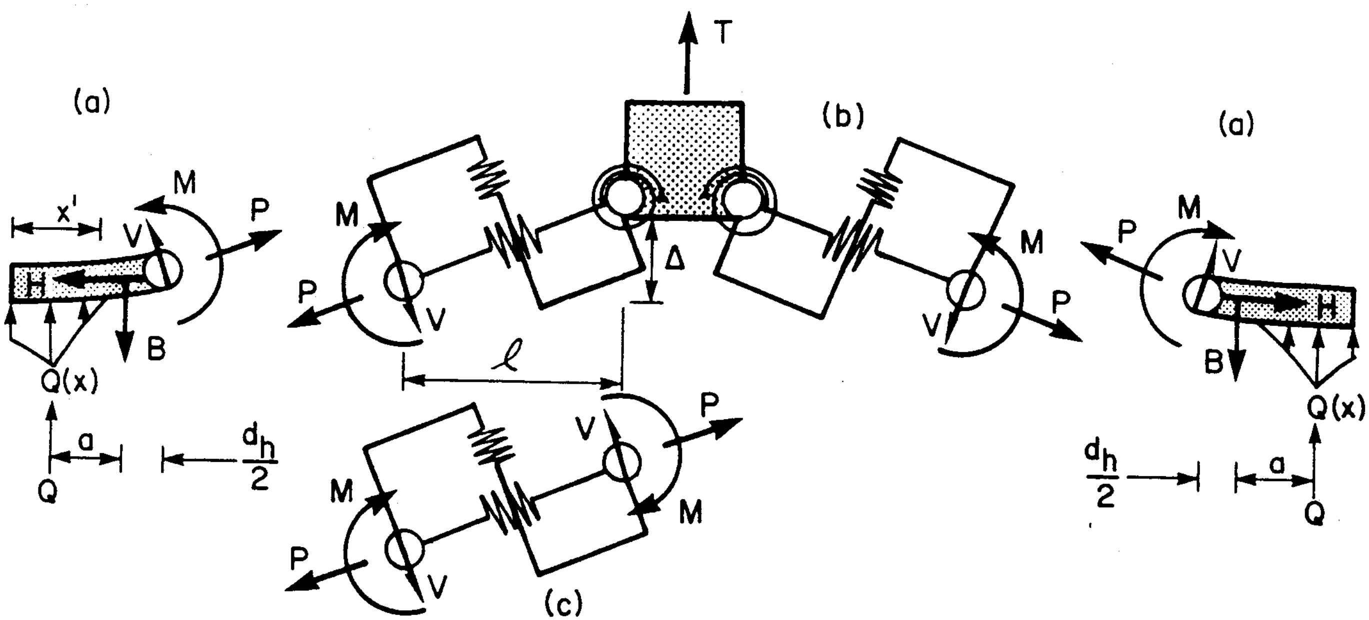


Figure 5.3 Free Body Diagrams of Flexible End Plate Connection Model

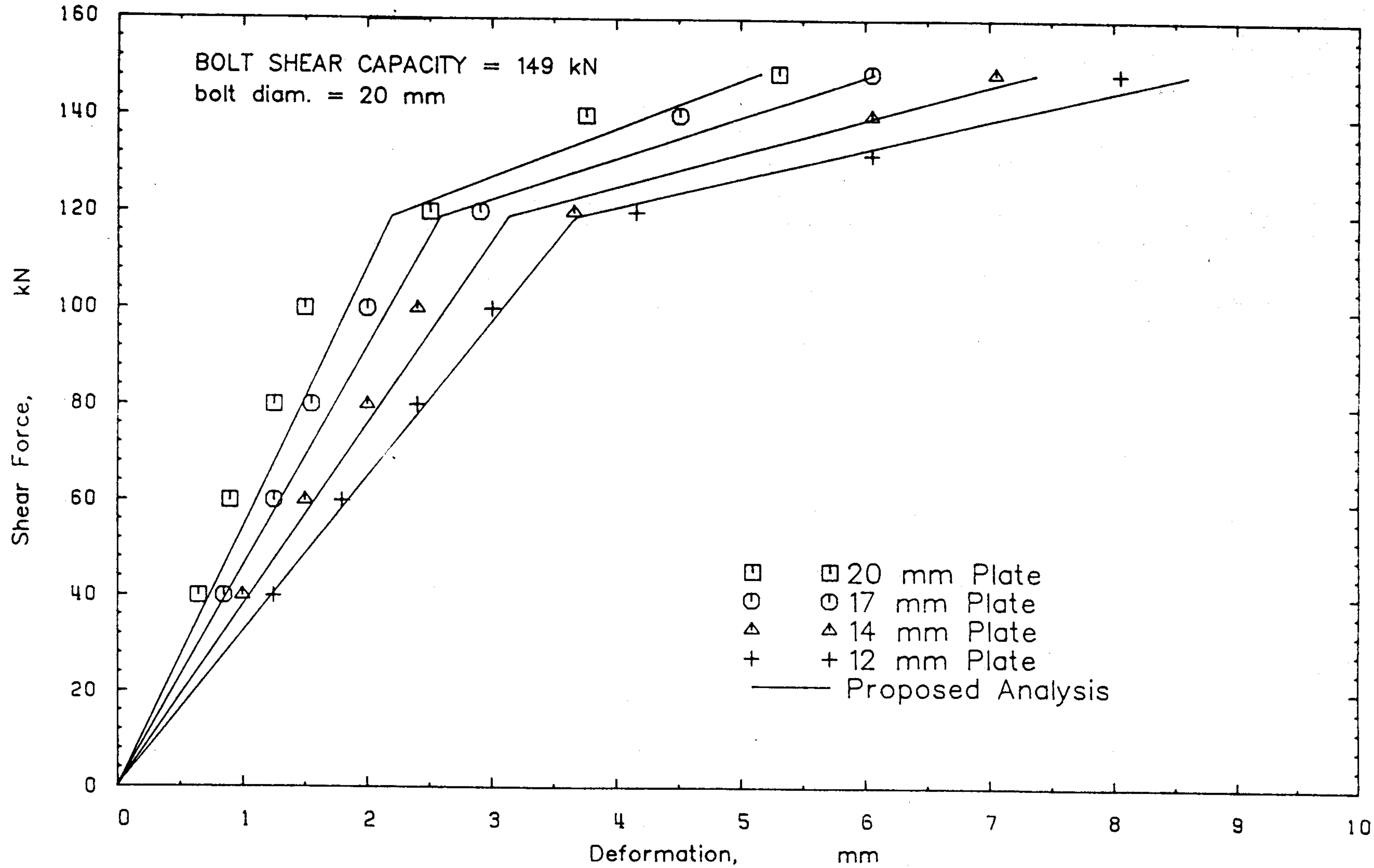


Figure 5.4 Comparison of Load-Deformation Relationship for High-Strength Bolts (Single Shear and Threads in Shear Plane) with Test Results of Bahia and Martin (1980)

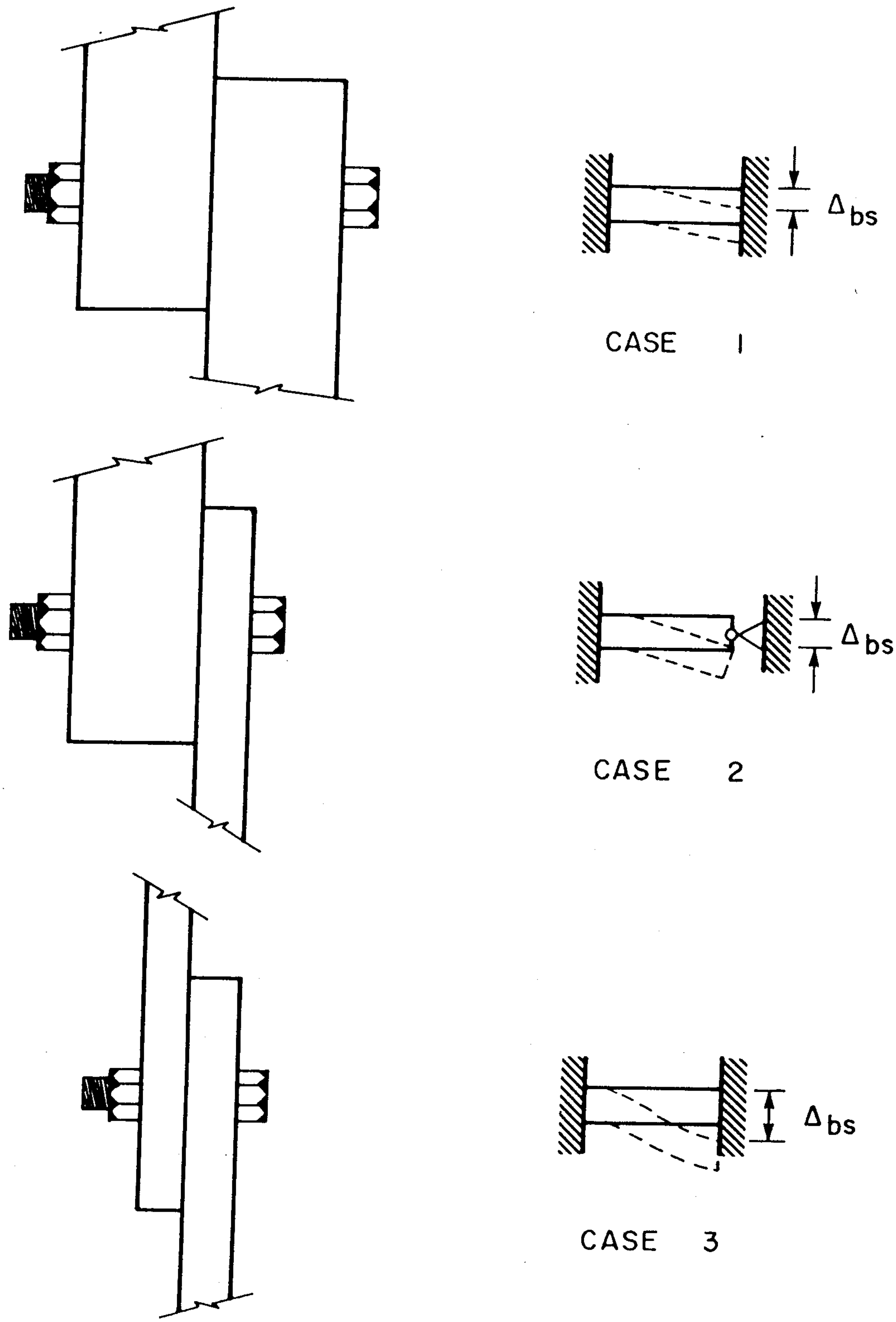


Figure 5.5 Effect of Lap Plate Thickness on Shear Deformation Characteristics of High-Strength Bolts

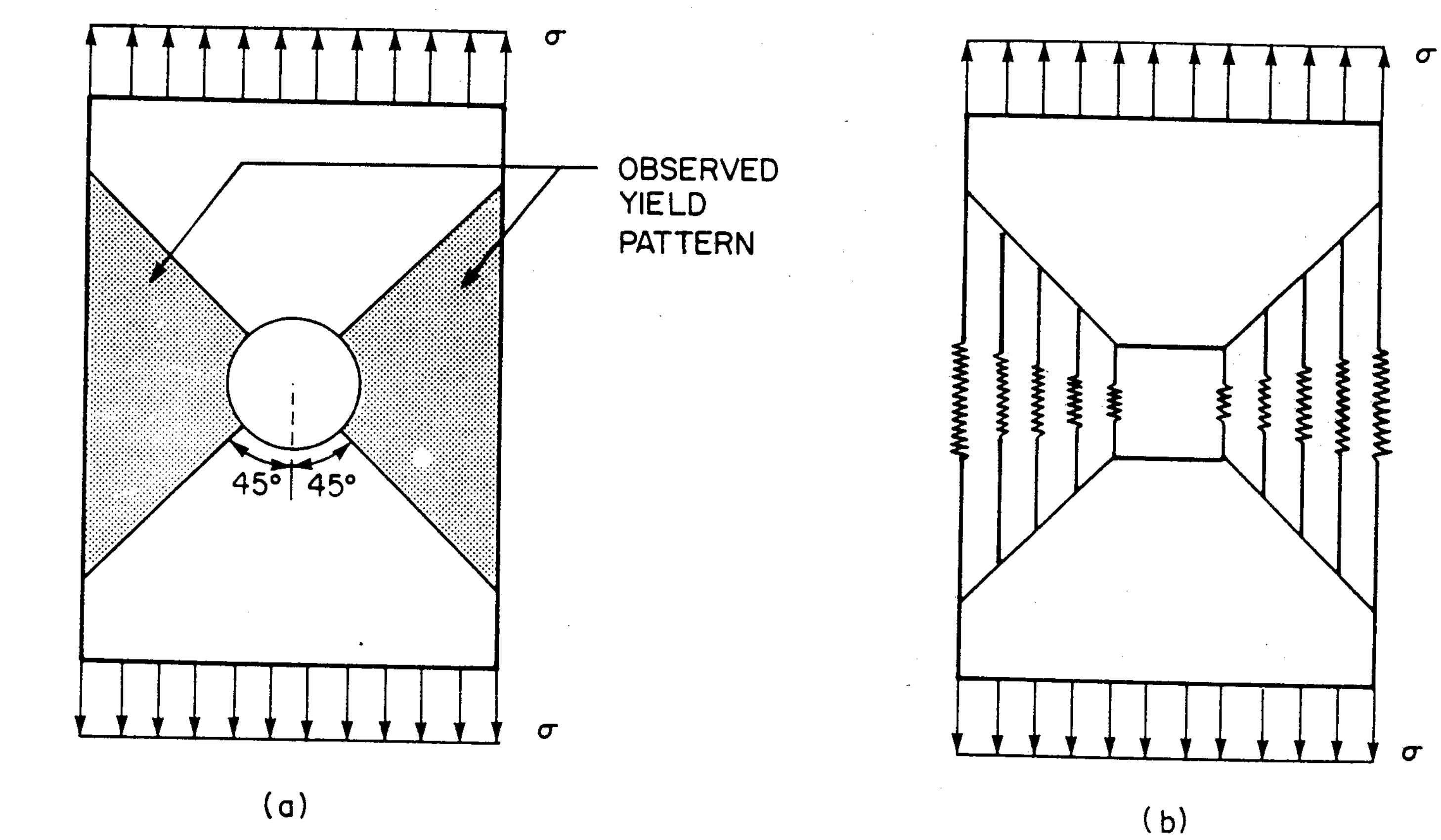


Figure 5.6 Model of a Tension Specimen with a Hole

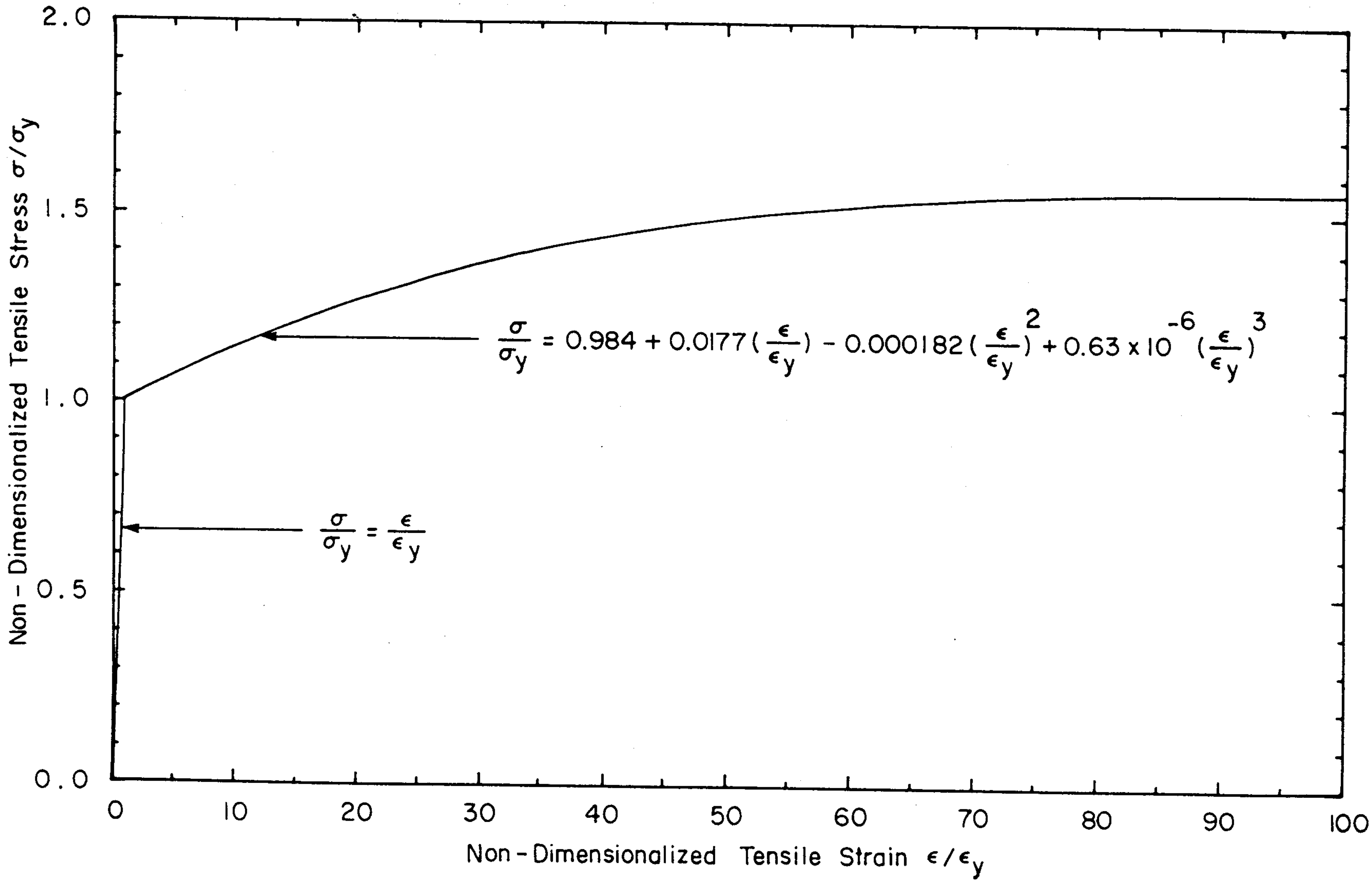


Figure 5.7 Idealized Stress-Strain Curve for Flexible End Plate Material

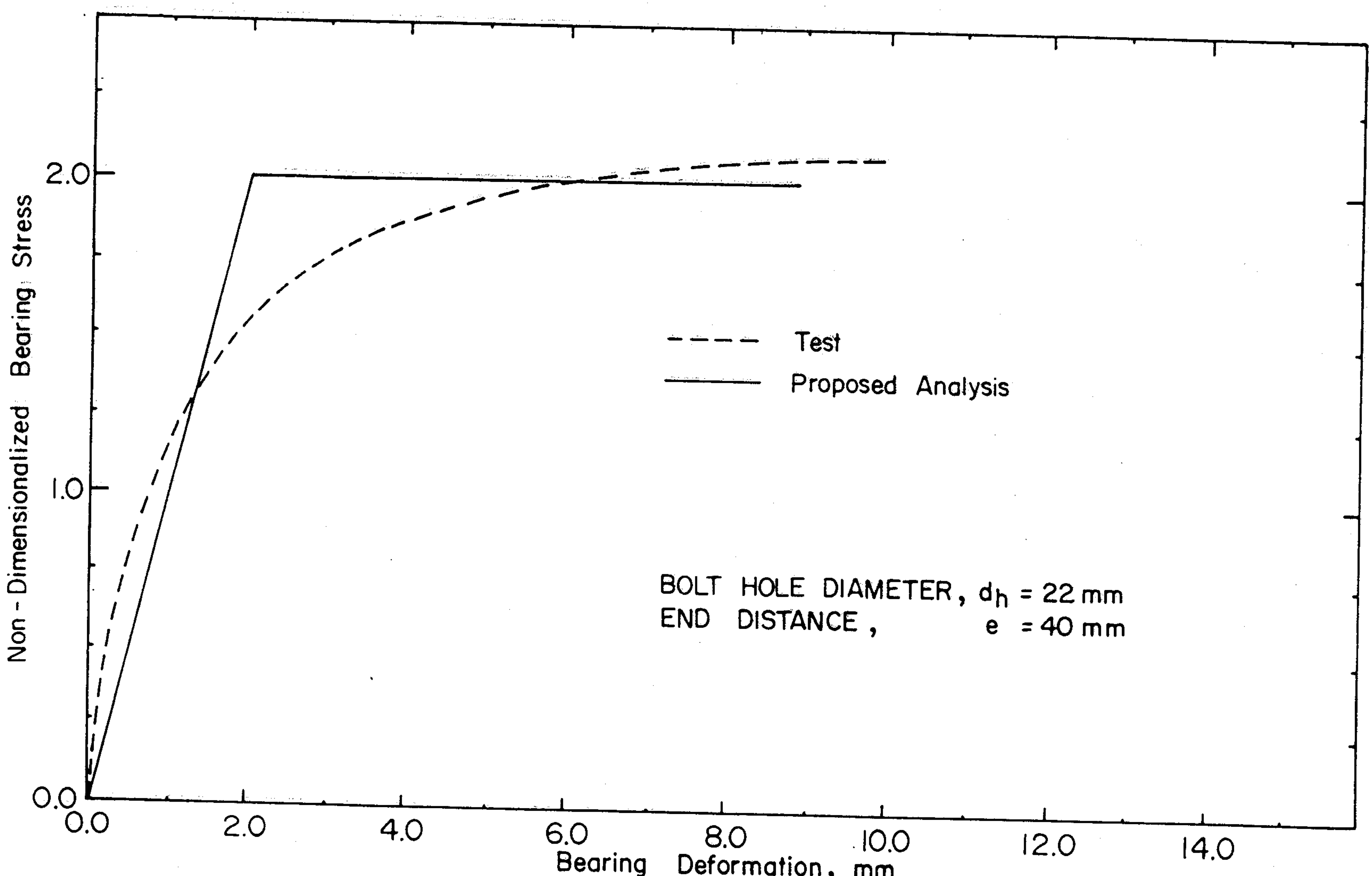


Figure 5.8 Comparison of Bearing Load-Deformation Relationship for Flexible End Plate at Bolts with Test Results of Owens et al. (1981)

6. Evaluation of the Primary Shear Capacity of a Flexible End Plate Connection

6.1 Introduction

The current design procedure for end connections assumed to transmit shear only is to design the components of the connection only for this shear. However, if secondary forces develop in the connection due to its distortion relative to the support, the capacity of the components to carry the primary shear will be reduced and the resulting margins of safety may be less than those implied in the governing design standards. By including the effects of secondary forces on the primary shear capacity at the factored load level (a procedure consistent with limit states design), it is possible to determine a more rational factored shear resistance of the connection. This reduced factored shear resistance must be equal to or greater than the factored shear.

Figure 2.4 presents a flow chart for the design process of a flexible end plate connection. For a beam of given size, length, and loading configuration, the factored end shears are computed assuming the beam to be simply supported. The connection at each end of the beam is sized for the factored shear neglecting, at this stage, any secondary forces that may exist. Next, the end rotations are calculated at the factored load level considering the restraining effect of the end moments developed by the connections (when the supports are relatively flexible, e. g. spandrel girders, it may be reasonable to ignore the restraining effect of the connections). Using the model developed in Chapter 5, the secondary forces in the various elements of the proposed connection can then be established. In this chapter, methods are given for estimating the capacity of the elements to carry primary shear in the presence of the secondary forces (if the supports are relatively flexible it may be reasonable to assume no secondary forces develop). If the shear resistance of the critical component, e. g. the bolt group, is less than the factored shear, the size of the connection is increased and the new connection is analyzed.

When the support is relatively stiff the general approach taken in arriving at the total primary factored shear resistance of each of the components in a connection is as follows:

1. compute the secondary forces over the depth of the connection,
2. at several discrete levels of the connection (for example, at each bolt line) determine the primary shear force that can be carried in conjunction with the secondary forces present, and
3. integrate the primary shear forces over the connection depth to arrive at the total primary shear resistance.

The validity of this lower bound approach depends upon the elements having sufficient ductility so that the ultimate capacity can be integrated over the depth of the connection.

Since the secondary forces to be used are computed for factored level loads, no further factoring is required. It is always possible that some elements of the connection assembly may have capacities greater than the nominal values. The secondary forces that would be developed in such elements would, of course, also be greater than those corresponding to nominal capacities. It is considered that this latter possibility is adequately covered by using factored resistances for the strengths of the connection components.

6.2 Bolts

Above the neutral axis of the connection the bolts are subject to horizontal shears and tensions as well as primary shears. The tension and horizontal shear forces increase with distance from the neutral axis. The factored resistance of bolts subject to shear and tension, based on the work of Chesson et al. (1965) and as given in CSA Standard CAN3-S16.1-M78 (CSA 1978) is

$$[6.1] \qquad V_f^{\,2} + \beta\, T_f^{\,2} < 0.56\, \phi^2\, \beta\, (A_b\, \sigma_{ub})^2$$

The factored shear force, V_f, is the vector sum of the vertical shear and the horizontal shear H. Considering this and using Equation [6.1], the primary shear that can be carried by the pair of bolts at the ith level is calculated by

$$[6.2] \qquad V_i = 2\sqrt{[(0.75\, \phi\, A_b\, \sigma_{ub})^2 - B_i^{\,2}]\, \beta - H_i^{\,2}}$$

in which the factored tensile force T_f is represented by the tensile force in the bolts, B_i, arising from the secondary forces in the connection.

The capacity of bolts may be limited to the bearing capacity of the end plate and therefore the shear at the ith level for two bolts should not exceed

$$[6.3] \qquad V_i = 2\,\phi\, t_p\, e\, \sigma_{up} < 6\,\phi\, t_p\, d_b\, \sigma_{up}$$

based on the bearing resistance expression given in CAN3-S16.1-M78.

Equation [6.3] should also be used to check the bearing capacity of the bolts on the supporting member using, of course, the appropriate geometric and material properties.

Assuming that sufficient ductility exists, the shear strength of the entire bolt group is obtained using

$$[6.4] \qquad V_{rb} = \sum_{i=1}^{i=n} V_i$$

When the shear strength at a given level is evaluated by Equation [6.2], in some instances it will be found to be zero. This indicates that the capacity of the bolts at that level has been exhausted by the secondary forces alone. Because this condition indicates that the connection is unfavourably stiff, it appears reasonable to reject such a connection.

The formulation for the primary shear resistance of the bolts does not take into consideration the frictional force that could be developed on the faying surface between the end plate and supporting member due to the compressive force acting on the bottom portion of the connection. For those connections carrying large shears, where this force would have the maximum benefit, calculations indicate that the shear carried by the frictional component is only about 5 to 10 percent of the factored shear.

6.3 Plate

The three modes of failure to be considered for the end plate subject to in-plane shear forces are: maximum shear capacity, excessive deformation, and shear buckling.

Even neglecting post buckling strength with the formation of a tension field and considering the greatest slenderness ratios likely to be encountered, the shear buckling mode will not be critical. For example, for a plate thickness of 6 mm and a 150 mm bolt gage, the critical elastic shear buckling stress is some twenty times the tensile yield stress. This mode need not be considered further.

Eyre (1973) extended the earlier work of Hall and Newmark (1957), and investigated the influence of shear deformation on the total deformation of steel beams. He established the non-dimensionalized shear stress-strain relationships shown on Figure 6.1. (The empirically derived parameters shown in Figure 6.1 are those for SI units.) The maximum shear stress obtained is about 1.4 times the tensile yield strength of the material, or slightly more than two times the value specified in CSA Standard CAN3-S16.1-M78 (CSA 1978) for webs of stocky beams. With the shear carried equally by the plate extending from both sides of the supported beam web, and considering the critical section to be that through the bolt holes, the shear resistance of the end plate, neglecting the secondary forces developed in the plate, is

$$[6.5] \qquad V_{rp} = 2\,\phi\,\left(h - n\,d_h\right)\,t_p\,1.4\,\sigma_{yp}$$

The Commentary on CSA Standard CAN3-S16.1-M78 (CISC 1980) indicates that the limit on shear stress to 0.66 times the tensile yield strength is more to prevent excessive deformation rather than to prevent rupture. Based on Eyre's (1973) equations, setting a shear deformation limit in the plate of 1 mm for the minimum pitch of 3 bolt diameters the net section is 67 percent of the gross section and the average maximum shear stress is about 80 percent of the tensile yield strength. This is about 19 percent greater than the average maximum shear stress specified in CAN3-S16.1-M78. The corresponding shear stress on the gross section is about 0.55 times the tensile yield stress. For a depth of connection equal to the bolt pitch the shear carried at the ith level is

$$[6.6] \qquad V_i = 2\,\phi\,p\,t_p\,0.55\,\sigma_{yp}$$

Equation [6.6] does not recognize that in the upper part of the connection, above the neutral axis, the deformation generates horizontal forces H_i at the ith level of each pair of bolts. Using the Huber-von Mises yield criterion and taking the comparative stress to be equal to the ultimate tensile strength of the plate, the effect of these forces on the vertical shear capacity can be considered. For a depth of connection equal to the bolt pitch giving an area for the two sides of the plate of $2t_p(p - d_h)$, the shear carried at the ith level is

$$[6.7] \qquad V_i = 2\,t_p\,(p - d_h)\sqrt{\frac{1}{3}\left\{\phi^2\,\sigma_{up}^{\;2} - \frac{H_i^{\;2}}{(p - d_h)^2\,t_p^{\;2}}\right\}}$$

However, the value given by Equation [6.6] should not be exceeded and will control the lower levels of the connection.

The total shear resistance of the end plate is the sum of the V_i values, [6.8] $$V_{rp} = \sum_{i=1}^{i=n} V_i$$

This analysis does not directly take into account the effect of the out-of-plane flexural deformations at the toe of the fillet welds and on a line at the inner edge of the bolt holes. It is recognized that significant straining occurs at these locations and in section 5.4.5 a deformation limit was imposed to preclude the rupture that may result. Thus, the ultimate tensile strength used as the comparative stress for the Huber-von Mises criterion is not exceeded.

Equation [6.7] is based on a shear rupture through the net section, a mode of failure not observed in the tests of Sommer (1969) and Hafez (1982), where failures generally occurred in the gross section because of excessive out-of-plane deformations.

6.4 Fillet Welds

The capacity of the fillet welds to carry primary shear must be evaluated separately above and below the neutral axis. In addition to primary shear, the fillet welds above the neutral axis must carry tension T, as shown in Figure 6.2. Provided that all components have sufficient ductility, a lower bound solution can be obtained by assigning the horizontal forces H and the moments M to the end plate, leaving the welds to carry the tension and shear. If after considering the secondary forces the welds do not have sufficient capacity to resist the vertical shear, the weld size must be increased to safeguard against rupture.

The approach currently specified in CSA Standard CAN3-S16.1-M78 (CSA 1978) for fillet welds loaded by tension and shear, indicates that the transverse shear strength of a weld is equal to the longitudinal strength of that weld and that the two shear stress components should be added vectorially. This is equivalent to saying that the interaction diagram is circular. Butler and Kulak (1971) investigated fillet welds loaded with transverse and longitudinal shear components and showed that the strength of the welds varied with the angle of loading. For example, welds loaded only by transverse forces were about 45 percent stronger than those loaded only by longitudinal forces. The strength established by Butler and Kulak has been incorporated in the CISC design tables for eccentrically loaded welds (CISC 1980). Kennedy and Kriviak (1985) evaluated the experimental results of Kato and Morita (1969), Butler and Kulak (1971), and Clark (1971) and proposed the parabolic interaction equation

$$[6.9] \qquad 1.2 \left(\frac{V_T}{V_U}\right)^2 - \left(\frac{V_T}{V_U}\right) + \left(\frac{V_L}{V_U}\right) = 1.0$$

The value of the ultimate shear, V_U, for shear through the throat of the weld as given in CAN3-S16.1-M84 (CSA 1984) is

$$[6.10] \qquad V_U = 0.67\ \phi\ A_w\ X_u$$

The coefficient 0.67 used in this expression appears to be quite conservative. Higgins and Preece (1969) found values of 0.85 to 0.90, except for high strength steels and electrodes where they observed a value of about 0.70. Butler and Kulak (1971) reported 1.0; Dawe and

Kulak (1972) reported 0.90; and Swannell (1981) reported 1.0. Kato and Morita (1969) give a value of 0.66, the only results consistent with CAN3-S16.1-M84. A statistical evaluation of these and further test results may show that a larger coefficient can be used while still maintaining the required reliability index.

Kennedy and Kriviak (1985) pointed out that shear at the fusion face does not govern the fillet weld strength for commonly used combinations of base metal and electrodes. In Equation [6.9], when dealing with a depth of plate equal to the bolt pitch at the ith level, the longitudinal shear, V_L, is the ith level primary shear force V_i ; the tensile force, V_T, is the ith level tensile force T_i. Making these substitutions and rearranging Equation [6.9] gives

$$[6.11] \qquad V_i = 1.34\ \phi\ p \frac{f}{\sqrt{2}} X_u + T_i - \frac{1.2\ T_i^2}{0.67\ \phi\ A_w\ X_u}$$

where the throat area of the two fillet welds for one bolt pitch is

$$[6.12] \qquad A_w = \frac{2}{\sqrt{2}}\ p\ f$$

For a lower bound solution, the compressive force below the neutral axis can be considered to be transmitted directly into the web of the supported beam. The fillet welds are required to carry primary shear only. The last two terms of Equation [6.11], for this reason, are zero. This approach satisfies the three lower bound solution requirements in that all the internal and external forces are in equilibrium, the internal forces assumed in no case exceed the relevant force capacity, and ductile behaviour in compression is assured as is noted in both the compression T-tests and the full scale connection tests of Hafez (1982).

The total shear resistance of the fillet welds is

$$[6.13] \qquad V_{rw} = \sum_{i=1}^{i=n} V_i$$

6.5 Beam Web

Because the end plate does not extend over the full depth of the beam web, the shear capacity of the beam web at the connection will be less than that of the full web. The supported beam may also require coping of either or both the top or bottom flanges. Thus, the shear strength of the web must be determined for uncoped beams, top flange coped beams, and top and bottom flange coped beams.

Consider the uncoped beam shown in Figure 6.3. In addition to the shear transferred to the web by the fillet welds, the web must also carry the tensile and compressive forces generated by the deformation of the end plate connection. Consider first the web above the neutral axis. The tensile stresses at each and every level due to the connection deformation are known. The capacity of the web to carry shear stresses can be determined using some yield criterion such as the Huber-von Mises criterion. Because the ultimate shear strength in beam webs is taken to be $0.66\sigma_{yw}$, as given by CAN3-S16.1-M78, rather than the Huber-von Mises shear yield value of $\sigma_{yw}/\sqrt{3}$, the comparative stress using the yield criterion can be increased by $0.66\sqrt{3} = 1.14$ times.

For a member subjected to uniaxial tension and shear, the Huber-von Mises criterion with this adjustment is:

$$[6.14] \qquad \tau = \sqrt{\frac{1}{3}\left[(1.14\ \sigma_{yw})^2 - \sigma^2\right]}$$

For an elemental length of dy, the primary shear resistance is

$$[6.15] \qquad V_i = w\ dy \sqrt{\frac{1}{3}\left[(1.14\ \phi\ \sigma_{yw})^2 - \left(\frac{T_i}{w\ dy}\right)^2\right]}$$

As shown in Figure 6.3, column action is developed above the top of the fillet welds in the short section of web due to the upward acting shear. The force developable in this portion is limited by the shear stress of $0.66\sigma_{yw}$, resulting in a maximum shear resistance of

$$[6.16] \qquad V_c = 0.66\ \phi\ d_t\ w\ \sigma_{yw}$$

but is not greater than a force causing the yield stress on the area of the web enclosed by the welds, that is

$$[6.17] \qquad V_c = \phi\ f\ w\ \sigma_{yw}$$

Below the neutral axis, the web subject to compressive stresses acts in a manner similar to the portion above. The shear capacity for an elemental length is as given by Equation [6.15]. The total shear capacity of the web, taking into account the forces due to the deformation of the connection, is then the sum of the three portions described above, that is,

$$[6.18] \qquad V_{rm} = \sum_{i=1}^{i=\frac{h}{d_y}} V_i + V_c$$

where the summation is over the full depth of the end plate.

For beams with top flange copes, Equation [6.18] would apply except that the term for column action equals zero. At the end of the cope, as shown in Figure 6.4, the interaction of shear and moment should be considered. The ASCE publication "Plastic Design in Steel" (ASCE 1971) gives the following moment-shear interaction relationship for rectangular sections which could conservatively be applied here, with the assumption that the bottom flange does not exist:

$$[6.19] \qquad \frac{M_f}{M_{rc}} + \left(\frac{V_{ft}}{V_{rc}}\right)^4 = 1.0$$

When both the bottom and top flanges are coped the relationships developed for beams with only top flanges coped are applicable as well. Of prime concern for coped beams is the possibility of web instability. Therefore, this should also be investigated (see Cheng (1984)).

6.6 Other Considerations

It is necessary that the supporting member must be adequate to carry the primary and secondary forces delivered to it by the connection. Therefore, in addition to the evaluation of the shear capacity, the effect of end moment on the supporting member should also be investigated.

Where beams with end plate connections frame into spandrel girders it would seem reasonable to use as flexible an end plate connection as possible in order to limit the amount of torsional moment developed in the girder. In any event, it is recognized that the twisting is self-limiting.

When flexible end plate connections are used on laterally unsupported beams it is important that the torsional stiffness of the connection be proven adequate. Bennetts et al. (1982) have shown that the torsional stiffness can be relatively small. This condition is especially severe for end plates of small depth. The amount of twist would, of course, be dependent primarily upon the portions of the web not connected to the end plate. Milek (1980) warns that the torsional stiffness of beams and girders with copes is further reduced from that of uncoped members supported by flexible end plates.

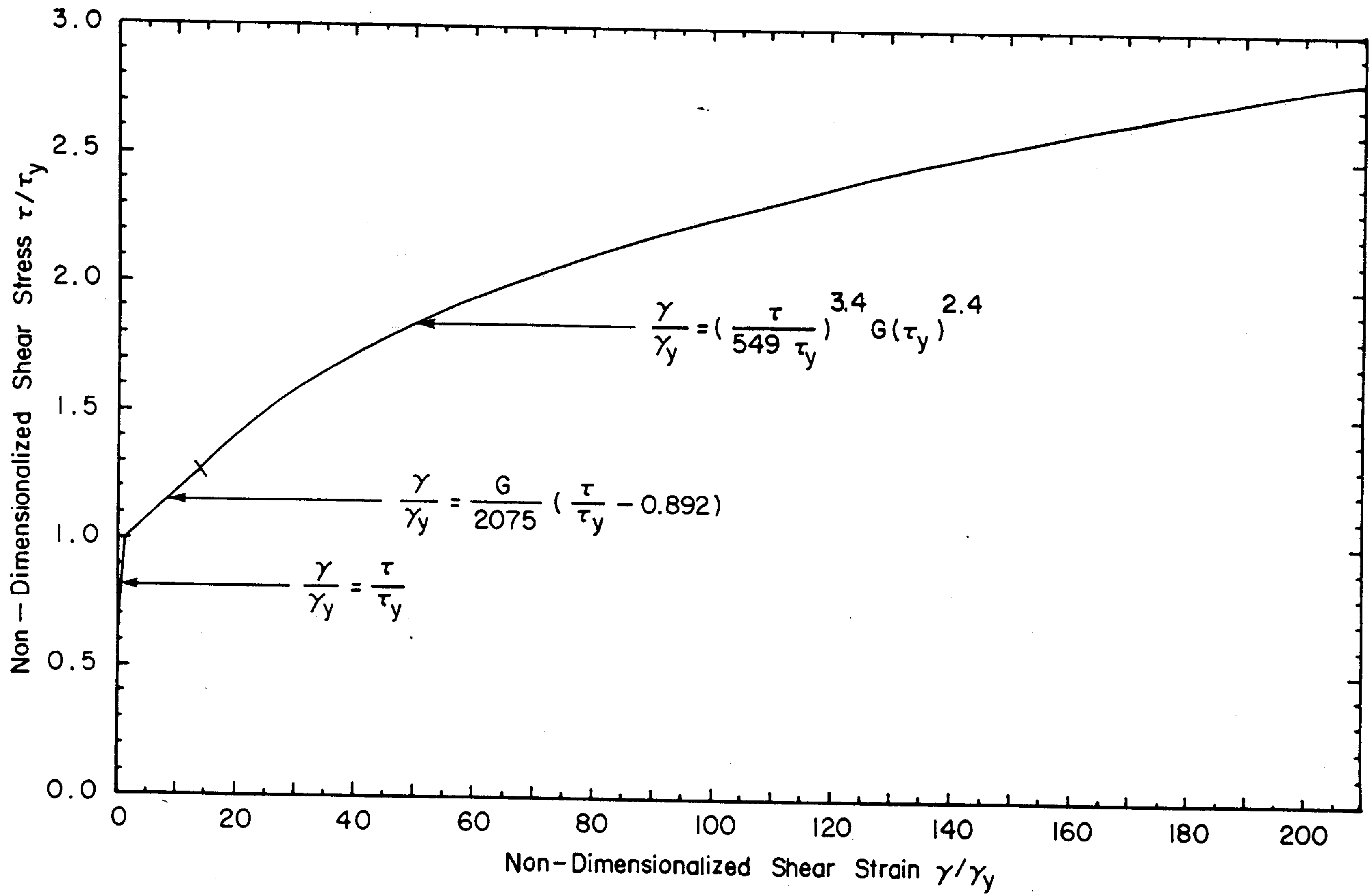


Figure 6.1 Shear Stress-Strain Relationship of Eyre (1973)

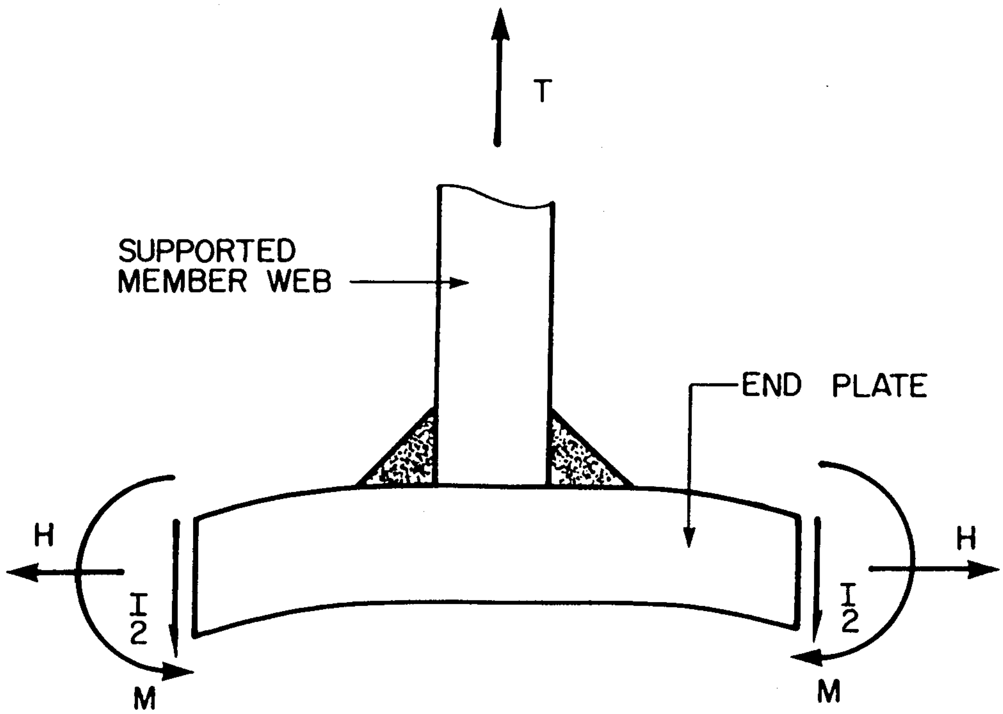


Figure 6.2 Secondary Forces Near the Fillet Welds in the Tension Region of an End Plate Connection

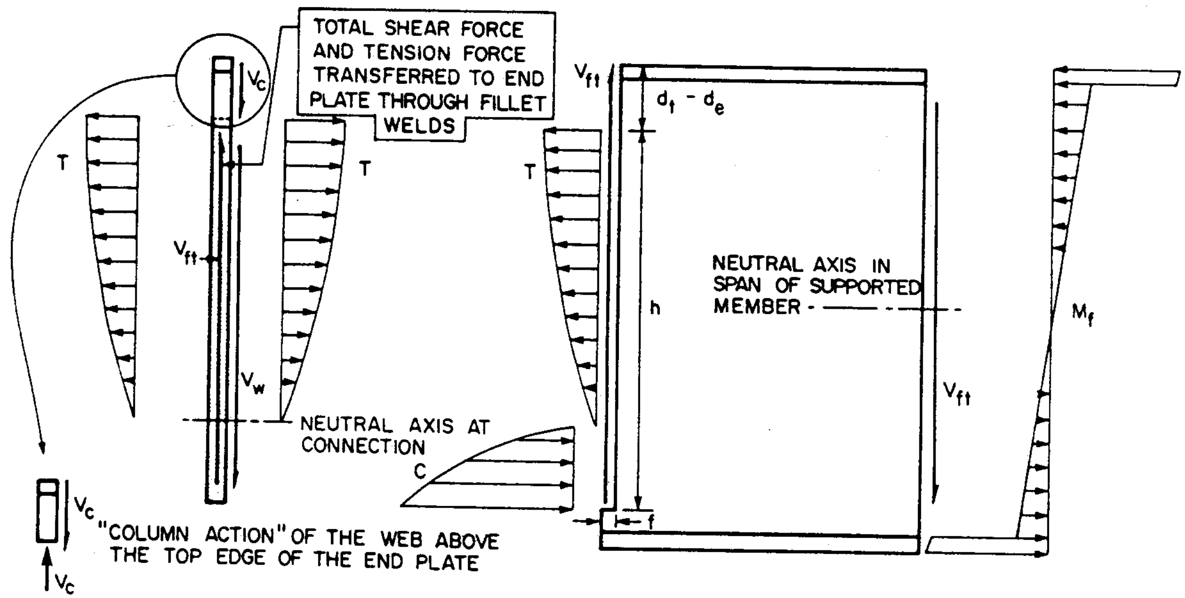


Figure 6.3 Shear Transfer From the Beam Web to the Flexible End Plate

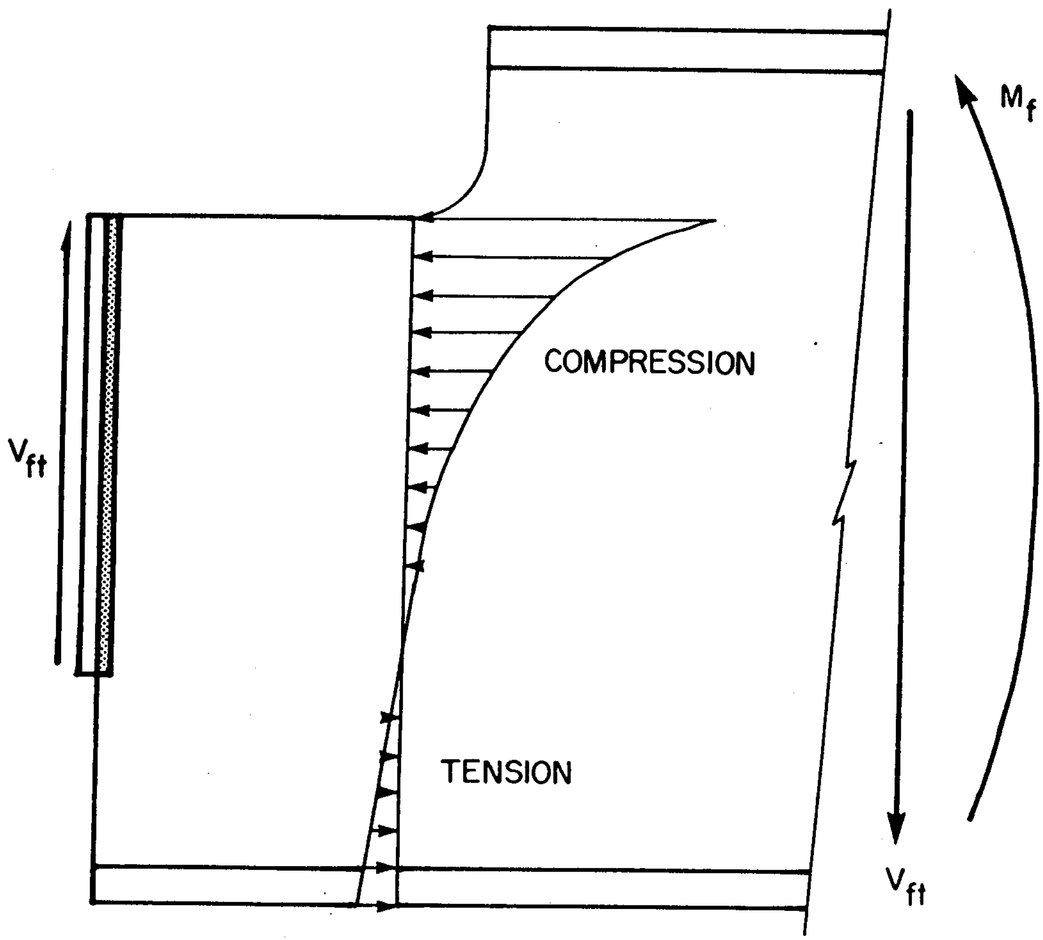


Figure 6.4 Possible Flexural Stress Distribution at Coped Region of a Simply Supported W-Shape

PART B: DEVELOPMENT OF STANDARDIZED FLEXIBLE END PLATE CONNECTIONS

7. Requirements for Standardized Connections

7.1 Introduction

Standardized connections can be described as a collection of pre-designed connections with prescribed geometric details and material characteristics and with specified strengths or capacities. Although industry-wide standardized connections currently available meet these criteria, the use of the high-speed computer with an adequate model of the connection behaviour allows a much broader range of details to be considered and indeed,

1. connections can be standardized to meet the shop practice of a particular fabricator,
2. connection costs can be developed based on the operational costs of a particular fabricator, and
3. more accurate design models can be used.

7.2 Limitations of Current Approach to Standardized Connections

Designers use handbooks such as the "Handbook of Steel Construction" (CISC 1980) to select steel members once factored forces acting on the members have been determined. The handbooks give the factored resistances of the members based on the relevant design standard. Because steel shapes available in a given country are universal, these data are equally beneficial to all users. Similarly, when selecting connection details, designers can choose appropriate connections from a published list of pre-designed connections. Unfortunately, these connection design tables are not likely to reflect the fabrication techniques preferred by all fabricators. Therefore, unlike the published data on the standard shapes, this information may be unduly restrictive and can result in increased costs.

A multitude of combinations of plates, bolts, welds, gages, and pitches exist, even for one type of connection. Up to the present time, only a limited number of combinations of connection components for flexible end plate connections have been proposed. Some of these are shown in Table 7.1 (AustISC 1978, BCSA 1982, SAISC 1982).

In North America both the Canadian Institute of Steel Construction (CISC) and the American Institute of Steel Construction (AISC) provide a slightly different approach. Instead of publishing tables of pre-designed connections, they provide tables of connection component strengths that can be used as 'building-blocks' to develop shop standards. Even though these tables are based on a limited number of possible component geometries and material strengths, a large number of different connection details can be developed. However, it is expensive, even with these data, for fabricators to develop and update standardized connection details.

A preferred alternative to using published standard connection details is for fabricators to be able to develop their own sets of pre-designed details using the high-speed computer with adequate design programs based on rational connection design models. Also, by using computers the variation in fabrication costs can be considered when optimizing connection details. The advent of limit states design, which recognizes serviceability limit states and ultimate limit states, provides additional impetus to develop models that can describe connection behaviour throughout the loading range. Again, this is facilitated by high-speed computational devices.

A review of the connection selection process using current design handbooks identifies other limitations and the considerable work involved. The steps the designer takes are as follows:

1. Determine the type of supported beam, its end preparations such as copes, end shear force, and the type of support.
2. From tables choose connection details of sufficient capacity.
3. Make a series of supplementary connection design checks, including the evaluation of the bearing capacity of the supporting element at the bolts, the reduced shear and flexural capacity of the web due to coping, and the rotational serviceability of the connection. These may be aided by other design tables.
4. Ensure that the chosen connection will fit the support conditions.

By developing standardized connections which consider beam size, span length, and loading, the limiting rotation of the connection can be evaluated directly. This eliminates the need for a supplementary check, as is currently made in the Australian connection design handbook (AustISC 1978). The other supplementary design checks can also be included in a computer method, thereby reducing not only the design time, but also the possibility of errors.

Thus, it would seem most promising to produce computer software that could be used to generate standardized connection details of the form discussed here. Fabricators would have standardized connections meeting their specific preferences. To date this approach has not been implemented.

7.3 Proposed Interactive Computer Program for Standardized Connections

Computer software can be developed to provide 'customized' connection design tables, one possible example of which is given in Appendix B for flexible end plates (the limit states design model developed in PART A is used). The program is written in Fortran IV language. In addition it has certain system commands that are compatible with the Michigan Terminal System (MTS) operating at the University of Alberta.

Input data is introduced to the program during a user friendly interactive input session. The program uses metric units exclusively and meets, for the most part, the requirements of CSA Standard CAN3-S16.1-M84, "Steel Structures For Buildings - Limit States Design" (CSA 1984).

The program can be used in either one of the following two modes:

1. Connection design for individual beams - when this mode is used the program designs connections for single beams of any span length loaded by any combination of uniform and up to six concentrated loads.
2. Connection design table preparation - when this mode is used the program designs connections for a specified steel shape carrying a

specified percentage of its maximum factored load. The user specifies the load type either as uniformly distributed or as mid span concentrated, and the load level up to 100 percent the maximum factored level. Connections are designed for ten different span lengths of the supported beam, ranging from three times the beam depth to thirty times the beam depth in increments of three times the beam depth.

The program is linked with data files of standard rolled and welded shapes prepared by the Canadian Institue of Steel Construction (CISC). Therefore, only a minimum of data concerning the geometry of the beam and the support needs to be input by the user. Connections can be designed for WWF, W, HP, M, S, C, and MC shapes, as listed in the "Handbook of Steel Construction" (CISC 1980). Material properties of the supported and supporting members are selected by the program user. Cope geometry can be specified by the user as well.

From the beam length and the specified loading condition the program computes the factored end shear forces. The end rotations of the beam are also required for the evaluation of secondary force effects and are therefore computed also.

The factored shear and moments are compared with the factored resistance of the beam to ensure that the specified loading does not overload the beam. All calculations are based on the assumptions that continuous lateral support is present and that the beam is simply supported.

Geometric and material properties for the connection plate, the bolts, and the fillet welds are selected by the user, with the exception of the connection plate length which is determined by the program based on the other parameters. The user can assign cost factors to the three basic connection components in order to determine total connection costs.

The program output is extensive. Complete records of the geometric and material properties of both the supported beam and the connection are given. The loading and resulting beam deformations assuming simple supports are tabulated together with complete connection design information.

Table 7.1 Geometric and Material Property Limitations on Standardized Flexible End Plate Connections

	CHOICES AVAILABLE		
	AustISC (1978)	BCSA (1982)	SAISC (1982)
END PLATE:			
thicknesses	1	2^a	2
material grades	1	1	1
gages	1	not set	2
pitches	1	1	2^c
end distances	1	1	1
edge distances	1	2^b	1
BOLTS:			
diameters	1	1	1
material grades	2	2	2
WELDS:			
sizes	1	2^c	3^d
material grades	1	1	1
SUPPORTED BEAM:			
material grades	1	2	1

Depends on: a size of the supported member,

b plate cutting process,

c bolt grade used,

d supported member web thickness.

8. Interactive Computer Program: Internal Operations

8.1 Introduction

The interactive Connection Design Program (CDP) given in Appendix B designs flexible end plate connections using the limit states design (LSD) model of PART A. The model from PART A cannot be used effectively without the use of a high-speed computational device because numerical iterative procedures are essential in the design process. The general flow charts of the operations followed in CDP are shown in Figures 8.1 and 8.2.

8.2 Input Data

The CDP program first reads all the user input data from a computer terminal during the interactive data input mode (see Figure 8.1). This interactive session obviates the need for an explanatory 'user manual'. The interactive mode has sufficient explanations to aid the user and 'error traps' to notify the user if input data is unacceptable.

The necessary input data required for each program operation is as follows:

A. Load and resistance factors and elastic moduli -

1. if not specified load factors default to the CAN3-S16.1-M84 (CSA 1984) values as follows:
   - i. live load factor, α_L, 1.50,
   - ii. dead load factor, α_D, 1.25,

2. if not specified resistance factors default to the CAN3-S16.1-M84 values as follows:

   i. for the main members, ϕ, 0.90,

   ii. for the plate, ϕ, 0.90,

   iii. for the bolts, ϕ, 0.67,

   vi. for the welds, ϕ, 0.67,

3. if not specified elastic moduli default to the following values:

   i. for the main members, E, 200,000. MPa,

   ii. for the end plate, E_p, 200,000. MPa.

B. Supported and supporting member description, OPERATIONAL MODE # 1, connection design for individual beams -

1. selection of support stiffness at both ends of beam:

   i. rigid, recommended when support is stiff, e. g. column flange,

   ii. flexible, recommended when support is flexible, e. g. spandrel girder,

2. selection of the material properties of the beam and supports (it is assumed that the beam and supports have the same material properties):

   i. tensile yield strength, σ_y, MPa,

   ii. tensile ultimate strength, σ_u, MPa,

3. supported beam type in one of two options:

   i. a standard steel shape, e. g. a W410x46 shape,

ii. a doubly symmetric I-shape plate girder (flange width, flange thickness, total depth, and web thickness must be specified),

4. supported beam length and loading condition:

   i. length, mm,

   ii. loading - any combination of uniformly distributed load (kN/m) and up to six concentrated loads (kN) at any location on the beam (mm), in any proportion of dead to live load,

5. the top and bottom flange cope dimensions of the supported beam, if any (mm),

6. identification of the supporting element, in one of four options:

   i. flange of a column, e. g. W360x79,

   ii. web of a column, e. g. W360x79,

   iii. web of a beam, e. g. W530x82,

   iv. a steel plate element of a specified thickness,

7. a connection design title.

C. Supported and supporting member description, OPERATIONAL MODE # 2, connection design tables -

1. selection of support stiffness (assumed to be the same at both ends of beam):

   i. rigid, recommended when supports are stiff, e. g. column flanges,

ii. flexible, recommended when supports are flexible, e. g. spandrel girders,

2. selection of the material properties of the beam and supports (it is assumed that the beam and supports have the same material properties):

i. tensile yield strength, σ_y , MPa,

ii. tensile ultimate strength, σ_u , MPa,

3. supported beam type which must be a standard steel shape, e. g. a W410x46 shape,

4. supported beam loading type and percentage of beam strength to be mobilized:

i. load types - either uniformly distributed or mid span concentrated,

ii. percentage of beam strength to be mobilized ranges from 1 to 100 percent,

5. the top and bottom flange cope dimensions of the supported beam, if any (mm) (assumed to be identical at both ends of beam),

6. identification of the supporting element (assumed to be identical at both ends of the beam), in one of four options:

i. flange of a column, e. g. W360x79,

ii. web of a column, e. g. W360x79,

iii. web of a beam, e. g. W530x82,

iv. a steel plate element of a specified thickness,

7. a connection design table number.

D. Connection material and geometric properties -

1. specified bolt strength (MPa),
2. specified bolt diameter (mm),
3. bolt hole diameter (mm),
4. bolt cost factor ($/bolt),
5. specified electrode strength (MPa),
6. specified fillet weld leg size (mm),
7. weld cost factor ($/mm of weld),
8. specified yield strength of the plate (MPa),
9. specified ultimate strength of the plate (MPa),
10. plate cost factor ($/kg),
11. bolt hole gage (mm),
12. bolt hole pitch (mm),
13. bolt hole end distance (mm),
14. bolt hole edge distance (mm),
15. distance from top row of bolts to top of the supported beam (mm),
16. end plate thicknesses (from one to four can be specified (mm)).

Each time the program is used any or all of the input data from the previous program operation, which is stored within a data file attached to CDP, can be reused to reduce the amount of data that must be input for each new program operation.

Prior to commencement of connection design the program also requires a complete list of the geometric properties of the supported beam and the thickness of the support if it is a standard steel shape. This information is obtained automatically from a data file containing all of the geometric properties of all of the standard steel shapes as given in the "Handbook of Steel Construction" (CISC 1980).

8.3 Classification and Factored Resistance of Supported Beam

The shear and flexural resistances of the supported beam are calculated using the appropriate resistance equations given in CSA Standard CAN3-S16.1-M84 (CSA 1984). It is assumed that the beam is continuously laterally braced. In this process the section class is computed and included in the connection design output for reference.

When the program is being used to design connections for individual beams (OPERATIONAL MODE # 1), the factored load effects are compared with the factored resistance of the supported beam. If the resistance is not great enough, the program user is requested to adjust the input data.

8.4 Determination of Service Level Deformations of Supported Beam

8.4.1 OPERATIONAL MODE # 1 - Connection Design for Individual Beams

Based on the magnitude and distribution of the specified dead and live loads the program computes, assuming simple supports, the service level moments at one hundred discrete, evenly spaced, locations along the beam. The corresponding curvature at each location is calculated by dividing the moment by the product of the modulus of elasticity and moment of inertia of the beam. The curvatures are numerically summed, according to the two moment-area theorems (see, for example, Popov 1978) to determine the beam end rotations and corresponding maximum span deflection. This information, not specifically required for the design of connection details, is considered useful to designers and is therefore included in the CDP output.

8.4.2 OPERATIONAL MODE # 2 - Connection Design Tables

For each of the ten span lengths, the specified load type, specified percentage of maximum factored loading, and the factored beam strength, the program calculates the corresponding magnitude of the factored load. Using this factored load and a dead load factor of 1.25 (CAN3-S16.1-M78 (CSA 1978)) the magnitude of the unfactored or specified 100 percent dead load is calculated.

Beam end rotations and maximum span deflections at the 100 percent specified dead load level are calculated using closed form expressions developed with the moment-area method (see, for example, Popov (1978)) assuming the beam is simply supported. This information, not specifically required for the design of connection details, is considered useful to designers and is therefore included in the CDP output tables.

8.5 Design of Flexible End Plate Connections

CDP first selects connection details assuming that only uniformly distributed vertical shear acts on the connection. The connection size is governed by the component with the least strength, that is, either the bolt group in shear or bearing, the end plate in shear, the fillet weld group in longitudinal shear, or the supported beam web in shear. If the support is flexible, and significant secondary forces are not likely to develop in the connection, then the connection selected in this manner is probably adequate. (However, if the connection is very shallow it may not provide adequate torsional restraint to the beam.) When the support is relatively rigid the connection design is not necessarily complete and in general an iterative procedure must be followed to complete the design. The necessary steps (as depicted in Figure 8.2) are as follows:

1. For the factored loads and span length the beam end rotations are calculated assuming simple supports with no rotational stiffness. Depending on the magnitude and distribution of the factored loads, the beam deformations may be inelastic. A complete description of the method used to predict the factored level deformations is given subsequently in Section 8.6.
2. For the connection initially selected and using the end rotation just calculated at the factored load level, the moment developed at the connection is computed using the proposed LSD moment-rotation model given in Chapter 3. These moments reduce the beam end rotations and the program iterates until the beam end rotation is compatible with the connection moment-rotation response. (Consideration of the rotational stiffness of the connection in calculating the beam end rotations affects most significantly the magnitude of the rotations when the beam is loaded beyond its yield moment capacity.)
3. Next, using the method described in Chapter 4, the program checks

whether or not bottom flange bearing of the beam on the support occurs for the given connection and beam end rotation. If bottom flange bearing is predicted to occur then the connection depth is increased, if possible, and step 2 is re-entered. When the connection depth cannot be increased a message is given in the design tables indicating that a connection design is not possible because of flange bearing. (When bottom flange bearing is predicted to occur the program does not attempt to design the connection for the reasons outlined in PART A).

4. When flange bearing does not occur the program predicts the primary shear strength of the connections considering the secondary forces developed due to rotations. If a connection is understrength the depth is increased, if possible, and step 2 is re-entered. If the connection is understrength and the depth cannot be increased because of geometric limitations, a message is given in the design output indicating that connection design is not possible.

8.6 Determination of Supported Member Deformations at the Factored Load Level

The limit states design (LSD) model proposed in PART A requires, in general, the connection end rotation at the factored load level because, except when supports are flexible, connection strength is affected by this rotation. Although not required for design the maximum span deflection is also calculated under the factored load.

When the factored load does not cause flexural yielding the end rotations and span deflections are determined using the methods discussed previously for service level beam deformations, except that the reduced deflections due to the end moments developed are now considered. In all calculations the effects of shear deformations are ignored.

If the factored load causes flexural yielding to occur, numerical methods are used so that those portions of the beam where some yielding has occurred can be adequately accounted for. For yielded portions it is assumed that the stress-strain relationship is elasto-plastic and residual stress effects do not exist. The curvature at a distance 'x' along the beam is given by

$$[8.1] \qquad \Phi(x) = \frac{2\ \varepsilon_{max}(x)}{d}$$

which can be rearranged (see Figure 8.3) to

$$[8.2] \qquad \Phi(x) = \frac{2}{\left(d - 2\ e(x)\right)}\ \frac{\phi\ \sigma_y}{E}$$

Calculation of $\Phi(x)$ using Equation [8.2] requires a value of $e(x)$. Since it is inconvenient to calculate $e(x)$ given $M(x)$, the method used in CDP to determine $e(x)$ is an iterative one. Knowing $M(x)$, a series of different $e(x)$ values are assumed until the internal bending moment $M_i(x)$ matches the external bending moment $M(x)$. For W and C shapes (see Figure 8.4) the inelastic moments, when yielding is confined to the flanges and when it extends into the web, are respectively

$$[8.3] \qquad M_i(x) = \phi\ \sigma_y \left(d - e(x)\right)\ b\ e(x) + \left(\frac{2\ \phi\ \sigma_y}{d - 2\ e(x)}\right) \left\{\frac{(d - 2\ t)^3\ w}{12} + \frac{b}{2}\left(t - e(x)\right)\left(d - t - e(x)\right)^2\right\}$$

for $0 < e(x) < t$

and

$$[8.4] \qquad M_i(x) = \phi\, \sigma_y\, (d - t)\, b\, t + \phi\, \big(e(x) - t\big)\big(d - e(x) - t\big)\, w\, \sigma_y + \phi\, \big(d - 2\, e(x)\big)^2 \frac{w\, \sigma_y}{6}$$

for $t < e(x)$

In recognition of strain-hardening, $\epsilon_{max}(x)$ has been taken as $10\epsilon_y$ when M_p is reached. This level of strain corresponds to that attainable by class 1 and 2 shapes when the onset of local buckling is imminent ("Plastic Design in Steel", ASCE 1971).

The curvatures are calculated taking into account the beam end moments and are summed to determine slopes, and the slopes are integrated to determine the maximum beam end rotations and span deflection.

8.7 Program Output

8.7.1 OPERATIONAL MODE # 1 - Connection Design for Individual Beams

The output generated for standardized connection details for one beam is extensive, as shown in Table 8.1. The first page of output contains input data summaries and beam analysis results. The second page contains the standardized connection details.

In the first block at the top of the first page a complete description is given of the beam, including geometric and material properties of the section as well as the class and the strength of the section. Width-to-thickness classifications for the flange and for the web of the supported beam are given, for flexure designated with a 'B' and for shear with a 'V'.

In the second block of information shown on the first page of Table 8.1 a complete description is given of the geometric and material properties of the connection components: the plate, the bolts, and the welds. The cost factors and unit resistances for each of these components is also given. At the top of this connection data summary, the expression

FEP(Fyp-Dh-g-p-Dend-Dedg-Dtop-tp-L) B(Fub-Db-#) W(Xu-f)

completely describes the flexible end plate connection once the connection is designed. On page 2 of Table 8.1 these symbols are replaced, in general, by their numeric values.

Based on the assumption that the beam is simply supported information is given in the third block at the bottom of the first page pertaining to the loading and deformation of the supported beam at service and ultimate levels as follows:

1. beam length and length-to-depth ratio,
2. specified service level loads and load factors,
3. ratio of the maximum applied factored moment to the beam moment

resistance and ratios of the applied factored shear force to the beam shear resistance, for both ends of the beam,

4. maximum beam span deflection at factored and service level loading, the locations of these deflections, and ratios of the beam length to the maximum deflections, and

5. beam end rotation at factored and service level loads.

Page 2 of the output contains the actual connection designs. Up to four connection plate thicknesses can be considered separately by CDP. Except for the plate length (L), the plate thickness (tp), and the number of bolts (#), all geometric and material properties are identical for all eight connection designs. These properties are summarized at the heading of each design table in the short form expression described earlier. In addition, the support type and relative stiffness, and the cope dimensions are listed at the heading of each table of connection details.

For each of the plate thicknesses specified, the following connection design information is provided in tabular form:

1. connection identification number,
2. plate thickness,
3. end plate length,
4. number of bolts,
5. connection cost,
6. condition which governs connection depth,
7. factored end shear force,

8. ratios of the connection component resistances to the factored shear force,
9. factored end moment,
10. ratio of the factored end moment to the factored moment resistance of the supported beam,
11. maximum beam deflection at the factored load, dead load, and live load levels considering the rotational stiffnesses of the connections,
12. beam end rotations at the factored load, dead load, and live load levels considering the rotational stiffnesses of the connections,
13. the displacement of the top of the connection away from the support at the factored load level considering the rotational stiffnesses of the connections,
14. the displacement of the top of the beam away from the support at the dead and live load levels considering the rotational stiffnesses of the connections,
15. the ratio of the maximum factored beam moment to the factored moment resistance of the beam, considering the end moments developed by the connections.

8.7.2 OPERATIONAL MODE # 2 - Connection Design Tables

The output generated for the Connection Design Tables, for one beam, is extensive, as shown in Table 8.2. The first page of output contains input data summaries and beam design and analysis information. The second and third pages contain the standardized connection details.

The first two blocks of output on page number one are identical to the information provided when OPERATIONAL MODE # 1 is used, as discussed previously.

Based on the assumption that the beam is simply supported, information is given pertaining to the loading and deformation of the beam at service and factored levels in the third block of output at the bottom of the first page in tabular form for each beam length as follows:

1. beam identification number,
2. beam length and length-to-depth ratio,
3. factored and 100 percent service dead load,
4. ratios of the beam shear resistance to the applied factored shear force and beam moment resistance to the maximum factored moment,
5. maximum beam deflection at factored and 100 percent service dead load and ratios of the beam length to these maximum deflections at factored and 100 percent service dead load, and
6. beam end rotations at factored and 100 percent service dead load.

The remaining pages of the output contain the actual connection designs. Up to four connection plate thicknesses can be considered during each use of CDP. For each end plate thickness, connection details are tabulated for each of the ten span lengths. Except for the plate length (L) and the number of bolts (#), all geometric and material properties are identical for all ten span lengths. These properties are summarized at the heading of each design table in the short form expression described earlier. In addition, the support type and relative stiffness, and the cope dimensions are listed at the heading of each table of connection details.

For each of the ten beam lengths, the following connection design information is provided in tabular form:

1. beam identification number,
2. beam length and length-to-depth ratio,
3. end plate length,
4. number of bolts,
5. connection cost,
6. condition which governed the connection size,
7. factored end shear force,
8. ratios of the connection component resistances to the factored shear force,
9. factored end moment,
10. maximum beam deflection considering the rotational stiffnesses of the connections at both the factored and 100 percent dead load levels,
11. ratios of the beam length to maximum beam deflections at the factored and 100 percent dead load levels,
12. beam end rotations considering the effect of the rotational stiffnesses of the connections at both the factored and 100 percent dead load levels,
13. displacement of the top of the connection away from the support at the factored load level including the effect of the end moments developed by the connections,
14. displacement of the top of the beam away from the support at the 100 percent dead load level including the effect of the end moments

developed by the connections,

15. ratio of the beam moment resistance to the maximum factored beam moment including the effect of the end moments developed by the connections, and

16. ratio of the beam end moment at the factored level to the factored resistance of the beam.

Table 8.1 Flexible End Plate Connection Design Summary for a Single, Simply Supported, Continuously Braced, Standard Steel Beam

1

W610X101 (Available In Canada And U.S.A.), Normally Used As A Beam.

SECTION PROPERTIES

Nominal Mass,	m =	101 kg/m
Cross Sectional Area,	A =	13000 mm2
x-Axis Mom. Of Inertia,	Ix =	764.0x10**6 mm4
x-Axis Section Modulus,	Sx =	2530.0x10**3 mm3
x-Axis Plastic Modulus,	Zx =	2900.0x10**3 mm3
y-Axis Mom. Of Inertia,	Iy =	29.5x10**6 mm4
y-Axis Section Modulus,	Sy =	259.0x10**3 mm3
y-Axis Plastic Modulus,	Zy =	404.0x10**3 mm3
Torsional Constant,	J =	781.0x10**3 mm4
Warping Constant,	Cw =	2550.0x10**9 mm6
Overall Section Depth,	d =	603.0 mm
Flange Width,	b =	228.0 mm
Flange Thickness,	t =	14.9 mm
Clear Flange Distance,	fT =	91.0 mm
Web Height,	h =	573.2 mm
Web Thickness,	w =	10.5 mm
Clear Web Distance,	wT =	535.0 mm

MATERIAL PROPERTIES

Yield Strength,	Fy =	300.0 MPa
Modulus of Elasticity,	E =	200000. MPa

SECTION CLASS & SECTION STRENGTH

Flange Width/Thickness,	b/t =	7.7, CLASS 1
Web Width/Thickness,	h/w =	54.6, CLASS 1B CLASS 1V
Resistance Factor,	φ =	0.90
Fact. Yld. Moment Cap.,	My =	683.1 kNm
Fact. Moment Capacity,	Mr =	783.0 kNm
Fact. Shear Capacity,	Vr =	1128.3 kN
Max. Shear Strength,	Fs =	198.0 MPa = 0.66xFy

FLEXIBLE END PLATE CONNECTION FEP(Fyp-Dh-g-p-Dend-Dedg-Dtop-tp-L) B(Fub-Db-#) W(Xu-f)

PLATE PROPERTIES

Yield Strength,	Fyp =	300.0 MPa
Ultimate Strength,	Fup =	450.0 MPa
Modulus of Elasticity,	Ep =	200000.0 MPa
Resistance Factor,	φp =	0.90
Bolt Hole Diameter,	Dh =	21.0 mm
Bolt Hole Gage,	g =	140.0 mm
Bolt Hole Pitch,	p =	75.0 mm
Bolt Hole End Dist.,	Dend =	35.0 mm
Bolt Hole Edge Dist.,	Dedg =	35.0 mm
Bolt Hole Top Dist.,	Dtop =	100.0 mm
Material Cost,	COSTP =	$ 3.00/kg

BOLT PROPERTIES

Ultimate Strength,	Fub =	825.0 MPa
Resistance Factor,	φb =	0.67
Nominal Diameter,	Db =	19.0 mm
Fact. Tensile Cap.,	Trb =	117.5 kN/Bolt
Fact. Shear Capacity,	Vrb =	65.8 kN/Bolt
Bolt Cost,	COSTB =	$ 10.00/Bolt

WELD PROPERTIES

Ultimate Strength,	Xu =	480.0 MPa
Resistance Factor,	φw =	0.67
Fillet Weld Leg Size,	f =	6.0 mm
Fact. Long. Sh. Cap.,	Vrw =	0.91 kN/mm
Fillet Weld Cost,	COSTW =	$ 0.10/mm

SIMPLE BEAM ANALYSIS

Beam Span = 10000.0 mm Span/Depth = 16.6

SPECIFIED LOADS:

Uniform: Unf. Dead = 26.0 kN/m & Unf. Live = 20.0 kN/m
NO CONCENTRATED LOADS.

Live Load Factor = 1.50, Dead Load Factor = 1.25.

Factored Resistance mobilized: Mf/Mr = 1.00 Left, Vf/Vr = 0.28 Right, Vf/Vr = 0.28

Maximum Factored Moment located 5000. mm from left support.

Factored Load Deflection = 179.1 mm @ 5000. mm from left support, Span/Deflection = 55.8
Dead Load Deflection = 21.8 mm @ 5000. mm from left support, Span/Deflection = 458.7
Live Load Deflection = 16.8 mm Span/Deflection = 596.2
Maximum total load deflection located 5000. mm from left support.

Factored Load Rotations,	θend Left = 0.0448 rads	θend Right = 0.0448 rads
Dead Load Rotations,	θend Left = 0.0071 rads	θend Right = 0.0071 rads
Live Load Rotations,	θend Left = 0.0055 rads	θend Right = 0.0055 rads

Table 8.1 Flexible End Plate Connection Design Summary for a Single, Simply Supported, Continuously Braced, Standard Steel Beam

2

W610X101 (Available In Canada And U.S.A.), Normally Used As A Beam.

CONNECTION DETAIL FEP(300-21-140- 75-35-35-100-tp-L) B(825-19-#) W(480- 6)

LEFT Support, Flange of a W360X79 Column. RIGHT Support, Web of a W610X101 Girder.
LEFT Support assumed to be RIGID. RIGHT Support assumed to be FLEXIBLE.
Support Material as per Supported Member
LEFT Copes: None.
RIGHT Copes: TOP 50.0 mm deep and 125.0 mm long, BOTTOM 50.0 mm deep and 125.0 mm long.

#	Plate Thick. (mm)	Plate Length (mm)	# of Blts Reqd	Conn Cost ($)	Govern Condit	FACT SHEAR (kN)	Vbolt/Vf	Vweld/Vf	Vplt/Vf	Vbeam/Vf	FACT Mend (kNm)	Mend/Mr
1L 1R	6.0 6.0	445.0 220.0	12 6	222.17 110.51	Rote Plat	312.5 312.5	2.43 1.26	2.83 1.29	2.54 1.25	2.61 1.32	44. 0.	0.06 0.0
2L 2R	8.0 8.0	370.0 220.0	10 6	188.60 112.68	Rote Bolt	312.5 312.5	2.11 1.26	2.29 1.29	2.81 1.67	2.12 1.32	43. 0.	0.05 0.0
3L 3R	10.0 10.0	370.0 220.0	10 6	192.25 114.85	Rote Bolt	312.5 312.5	1.96 1.26	1.96 1.29	3.52 2.09	1.95 1.32	59. 0.	0.07 0.0
4L 4R	12.0 12.0	370.0 220.0	10 6	195.90 117.02	Weld Bolt	312.5 312.5	1.59 1.26	1.44 1.29	4.22 2.51	1.78 1.32	76. 0.	0.10 0.0

#	Max FL Defl (mm)	Max DL Defl (mm)	Max LL Defl (mm)	θend FL (rads)	θend DL (rads)	θend LL (rads)	Disp Top Conn FL (mm)	Disp Top Beam DL (mm)	Disp Top Beam LL (mm)	Mr/Mf
1L 1R	71.2 -	20.5 -	16.6 -	0.0206 -	0.0064 -	0.0054 -	8.1 -	2.8 -	2.6 -	1.03 -
2L 2R	72.1 -	20.4 -	16.7 -	0.0208 -	0.0063 -	0.0054 -	6.5 -	2.2 -	2.1 -	1.03 -
3L 3R	64.3 -	19.8 -	16.6 -	0.0189 -	0.0060 -	0.0054 -	5.3 -	1.9 -	1.9 -	1.04 -
4L 4R	59.0 -	19.2 -	16.6 -	0.0175 -	0.0057 -	0.0053 -	4.5 -	1.7 -	1.8 -	1.05 -

Table 8.2 Flexible End Plate Connection Design Table for a Simply Supported, Continuously Braced, Standard Steel Beam, With a Uniformly Distributed Load of 100 % of the Ultimate Level

1

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W610X101        ( Available In Canada And U.S.A. ), Normally Used As A Beam.

SECTION PROPERTIES                                        MATERIAL PROPERTIES

Nominal Mass,                 m  = 101    kg/m            Yield Strength,           Fy = 300.0     MPa
Cross Sectional Area,         A  = 13000  mm2             Modulus of Elasticity,    E  = 200000.   MPa
x-Axis Mom. Of Inertia,       Ix =    764.0x10**6 mm4
x-Axis Section Modulus,       Sx =   2530.0x10**3 mm3
x-Axis Plastic Modulus,       Zx =   2900.0x10**3 mm3
y-Axis Mom. Of Inertia,       Iy =     29.5x10**6 mm4     SECTION CLASS & SECTION STRENGTH
y-Axis Section Modulus,       Sy =    259.0x10**3 mm3
y-Axis Plastic Modulus,       Zy =    404.0x10**3 mm3     Flange Width/Thickness,b/t =  7.7, CLASS 1
Torsional Constant,           J  =    781.0x10**3 mm4     Web Width/Thickness,    h/w = 54.6, CLASS 1B
Warping Constant,             Cw =   2550.0x10**9 mm6                                         CLASS 1V
Overall Section Depth,        d  = 603.0 mm               Resistance Factor,        ϕ  = 0.90
Flange Width,                 b  = 228.0 mm
Flange Thickness,             t  =  14.9 mm               Fact. Yld. Moment Cap., My =  683.1 kNm
Clear Flange Distance,        fT =  91.0 mm               Fact. Moment Capacity,  Mr =  783.0 kNm
Web Height,                   h  = 573.2 mm               Fact. Shear Capacity,   Vr = 1128.3  kN
Web Thickness,                w  =  10.5 mm
Clear Web Distance,           wT = 535.0 mm               Max  Shear Strength,    Fs = 198.0 MPa
                                                                                     =   0.66xFy
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FLEXIBLE END PLATE CONNECTION        FEP(Fyp-Dh-g-p-Dend-Dedg-Dtop-tp-L)  B(Fub-Db-#)  W(Xu-f)

PLATE PROPERTIES                                          BOLT PROPERTIES

Yield Strength,            Fyp =    300.0 MPa             Ultimate Strength,        Fub =  825.0 MPa
Ultimate Strength,         Fup =    450.0 MPa             Resistance Factor,        ϕb  =    0.67
Modulus of Elasticity,     Ep  = 200000.0 MPa             Nominal Diameter,         Db  =   19.0 mm
Resistance Factor,         ϕp  =      0.90                Fact. Tensile Cap.,       Trb =  117.5 kN/Bolt
                                                          Fact. Shear Capacity,     Vrb =   65.8 kN/Bolt
Bolt Hole Diameter,        Dh   =    21.0 mm
Bolt Hole Gage,            g    =   140.0 mm              Bolt Cost,              COSTB = $ 10.00/Bolt
Bolt Hole Pitch,           p    =    75.0 mm
Bolt Hole End Dist.,       Dend =    35.0 mm              WELD PROPERTIES
Bolt Hole Edge Dist.,      Dedg =    35.0 mm
Bolt Hole Top Dist.,       Dtop =    75.0 mm              Ultimate Strength,        Xu  =  480.0 MPa
                                                          Resistance Factor,        ϕw  =    0.67
Material Cost,          COSTP = $  3.00/kg                Fillet Weld Leg Size,     f   =    6.0 mm
                                                          Fact. Long. Sh. Cap.,     Vrw =   0.91 kN/mm

                                                          Fillet Weld Cost,       COSTW = $  0.10/mm
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SIMPLE BEAM ANALYSIS

UNIFORMLY DISTRIBUTED LOAD mobilizing 100. % of the FACTORED RESISTANCE of this member.
Dead Load Factor = 1.25 .

#	Beam Length (mm)	Length / Depth	FACT LOAD (kN/m)	SPEC 100%DL (kN/m)	Vr / Vf	Mr / Mf	Max FL defl (mm)	Max DL defl (mm)	Length / FLdefl	Length / DLdefl	θend FL (rads)	θend DL (rads)
1	1809	3.0	1247.4	997.9	1.00	1.53	1.1	0.9	1589.	1986.	0.0020	0.0016
2	3618	6.0	478.5	382.8	1.30	1.00	25.1	5.6	144.	647.	0.0171	0.0049
3	5427	9.0	212.7	170.1	1.96	1.00	56.5	12.6	96.	432.	0.0256	0.0074
4	7236	12.0	119.6	95.7	2.61	1.00	100.4	22.4	72.	324.	0.0342	0.0099
5	9045	15.0	76.6	61.3	3.26	1.00	157.0	34.9	58.	259.	0.0427	0.0124
6	10854	18.0	53.2	42.5	3.91	1.00	226.1	50.3	48.	216.	0.0513	0.0148
7	12663	21.0	39.1	31.3	4.56	1.00	307.9	68.5	41.	185.	0.0598	0.0173
8	14472	24.0	29.9	23.9	5.21	1.00	402.4	89.4	36.	162.	0.0684	0.0198
9	16281	27.0	23.6	18.9	5.87	1.00	509.5	113.2	32.	144.	0.0769	0.0222
10	18090	30.0	19.1	15.3	6.52	1.00	628.4	139.7	29.	129.	0.0854	0.0247

Table 8.2 Flexible End Plate Connection Design Table for a Simply Supported, Continuously Braced, Standard Steel Beam, With a Uniformly Distributed Load of 100 % of the Ultimate Level

2

W610X101 (Available In Canada And U.S.A.), Normally Used As A Beam.

CONNECTION #1 : FEP(300-21-140- 75-35-35- 75- 6-L) B(825-19-#) W(480- 6)

Supports, Flanges of W360X79 Columns. Material Properties as per Supported Member.
Supports assumed to be RIGID.
Supported member is uncoped.

#	Beam Length (mm)	Length / Depth	Plate Length (mm)	# of Blts Reqd	Conn Cost ($)	Govern Condit	FACT SHEAR (kN)	Vbolt / Vf	Vweld / Vf	Vplt. / Vf	Vbeam / Vf	FACT Mend (kNm)
1	1809	3.0	Bolt	Bolt	Bolt	Bolt	1128.3	0.79	0.89	0.82	0.87	Bolt
2	3618	6.0	520.0	14	259.39	Bolt	865.7	1.02	1.19	1.07	1.12	44.
3	5427	9.0	370.0	10	184.95	Bolt	577.1	1.10	1.27	1.14	1.20	24.
4	7236	12.0	370.0	10	184.95	Rote	432.8	1.46	1.69	1.52	1.60	26.
5	9045	15.0	445.0	12	222.17	Rote	346.3	2.19	2.55	2.29	2.37	40.
6	10854	18.0	445.0	12	222.17	Rote	288.6	2.63	3.06	2.75	2.83	42.
7	12663	21.0	445.0	12	222.17	Rote	247.3	3.07	3.57	3.21	3.28	45.
8	14472	24.0	520.0	14	259.39	Rote	216.4	4.10	4.78	4.28	4.33	67.
9	16281	27.0	520.0	14	259.39	Rote	192.4	4.61	5.38	4.82	4.84	71.
10	18090	30.0	520.0	14	259.39	Rote	173.1	5.12	5.98	5.35	5.32	75.

#	Max FL defl (mm)	Max DL defl (mm)	Length / FLdefl	Length / DLdefl	θend FL (rads)	θend DL (rads)	Disp Top Conn FL (mm)	Disp Top Beam DL (mm)	Mr / Mf	Mend / Mr
1	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt
2	7.6	5.1	476.	703.	0.0063	0.0045	2.8	2.1	1.06	0.06
3	21.3	12.0	254.	451.	0.0114	0.0070	3.7	2.5	1.03	0.03
4	37.0	21.4	196.	339.	0.0148	0.0093	4.8	3.3	1.03	0.03
5	49.2	32.6	184.	278.	0.0163	0.0113	6.4	4.9	1.05	0.05
6	69.2	46.7	157.	232.	0.0192	0.0135	7.5	5.8	1.06	0.05
7	92.3	63.4	137.	200.	0.0220	0.0157	8.6	6.8	1.06	0.06
8	106.5	79.6	136.	182.	0.0225	0.0171	10.3	8.5	1.09	0.09
9	131.7	100.1	124.	163.	0.0248	0.0190	11.4	9.5	1.10	0.09
10	160.8	122.5	112.	148.	0.0273	0.0209	12.5	10.4	1.11	0.10

CONNECTION #2 : FEP(300-21-140- 75-35-35- 75- 8-L) B(825-19-#) W(480- 6)

Supports, Flanges of W360X79 Columns. Material Properties as per Supported Member.
Supports assumed to be RIGID.
Supported member is uncoped.

#	Beam Length (mm)	Length / Depth	Plate Length (mm)	# of Blts Reqd	Conn Cost ($)	Govern Condit	FACT SHEAR (kN)	Vbolt / Vf	Vweld / Vf	Vplt. / Vf	Vbeam / Vf	FACT Mend (kNm)
1	1809	3.0	Bolt	Bolt	Bolt	Bolt	1128.3	0.82	0.88	1.10	0.86	Bolt
2	3618	6.0	520.0	14	264.52	Bolt	865.7	1.06	1.15	1.43	1.09	70.
3	5427	9.0	370.0	10	188.60	Bolt	577.1	1.14	1.24	1.52	1.17	38.
4	7236	12.0	295.0	8	150.64	Bolt	432.8	1.22	1.31	1.62	1.25	25.
5	9045	15.0	370.0	10	188.60	Rote	346.3	1.90	2.06	2.54	1.92	41.
6	10854	18.0	370.0	10	188.60	Rote	288.6	2.28	2.48	3.05	2.30	42.
7	12663	21.0	445.0	12	226.56	Rote	247.3	3.19	3.48	4.27	3.11	63.
8	14472	24.0	445.0	12	226.56	Rote	216.4	3.65	3.97	4.89	3.51	66.
9	16281	27.0	445.0	12	226.56	Rote	192.4	4.11	4.46	5.50	3.90	68.
10	18090	30.0	445.0	12	226.56	Rote	173.1	4.56	4.95	6.11	4.38	70.

#	Max FL defl (mm)	Max DL defl (mm)	Length / FLdefl	Length / DLdefl	θend FL (rads)	θend DL (rads)	Disp Top Conn FL (mm)	Disp Top Beam DL (mm)	Mr / Mf	Mend / Mr
1	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt
2	6.5	4.9	554.	743.	0.0055	0.0041	2.2	1.8	1.10	0.09
3	18.0	11.7	302.	464.	0.0089	0.0068	2.9	2.2	1.05	0.05
4	37.0	21.3	196.	339.	0.0149	0.0093	3.6	2.6	1.03	0.03
5	48.8	32.3	185.	280.	0.0162	0.0112	4.9	3.8	1.05	0.05
6	69.2	46.5	157.	233.	0.0192	0.0134	6.0	4.6	1.06	0.05
7	82.7	60.8	153.	208.	0.0200	0.0149	7.3	6.0	1.09	0.08
8	106.9	79.1	135.	183.	0.0226	0.0168	8.2	6.9	1.09	0.08
9	133.6	99.7	122.	163.	0.0251	0.0189	9.2	7.7	1.10	0.08
10	163.4	122.6	111.	148.	0.0277	0.0208	10.4	8.5	1.10	0.09

Table 8.2 Flexible End Plate Connection Design Table for a Simply Supported, Continuously Braced, Standard Steel Beam, With a Uniformly Distributed Load of 100 % of the Ultimate Level

3

W610X101 (Available In Canada And U.S.A), Normally Used As A Beam.

CONNECTION #3 : FEP(300-21-140- 75-35-35- 75-10-L) B(825-19-#) W(480- 6)

Supports, Flanges of W360X79 Columns. Material Properties as per Supported Member.
Supports assumed to be RIGID.
Supported member is uncoped.

#	Beam Length (mm)	Length / Depth	Plate Length (mm)	# of Blts Reqd	Conn Cost ($)	Govern Condit	FACT SHEAR (kN)	Vbolt / Vf	Vweld / Vf	Vplt. / Vf	Vbeam / Vf	FACT Mend (kNm)
1	1809	3.0	Bolt	Bolt	Bolt	Bolt	1128.3	0.80	0.83	1.37	0.84	Bolt
2	3618	6.0	520.0	14	269.65	Bolt	865.7	1.01	1.04	1.78	1.05	99.
3	5427	9.0	370.0	10	192.25	Bolt	577.1	1.08	1.11	1.90	1.11	54.
4	7236	12.0	295.0	8	153.55	Weld	432.8	1.14	1.14	2.02	1.20	35.
5	9045	15.0	295.0	8	153.55	Rote	346.3	1.43	1.42	2.53	1.48	37.
6	10854	18.0	370.0	10	192.25	Rote	288.6	2.13	2.13	3.81	2.14	58.
7	12663	21.0	370.0	10	192.25	Rote	247.3	2.48	2.48	4.44	2.49	59.
8	14472	24.0	370.0	10	192.25	Rote	216.4	2.83	2.83	5.08	2.80	60.
9	16281	27.0	445.0	12	230.95	Rote	192.4	3.79	3.80	6.87	3.67	89.
10	18090	30.0	445.0	12	230.95	Rote	173.1	4.19	4.21	7.63	4.04	91

#	Max FL defl (mm)	Max DL defl (mm)	Length / FLdefl	Length / DLdefl	θend FL (rads)	θend DL (rads)	Disp Top Conn FL (mm)	Disp Top Beam DL (mm)	Mr / Mf	Mend / Mr
1	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt
2	5.9	4.6	609.	791.	0.0050	0.0038	1.8	1.5	1.14	0.13
3	16.0	11.3	340.	479.	0.0090	0.0065	2.4	1.9	1.07	0.07
4	32.8	20.9	221.	346.	0.0135	0.0091	3.0	2.3	1.05	0.05
5	50.7	32.6	179.	278.	0.0167	0.0113	3.7	2.9	1.05	0.05
6	62.5	44.9	174.	242.	0.0175	0.0128	4.9	4.0	1.08	0.07
7	84.6	61.1	150.	207.	0.0204	0.0150	5.9	4.8	1.08	0.08
8	106.6	78.6	132.	182.	0.0231	0.0170	6.7	5.5	1.08	0.08
9	123.9	94.9	131.	172.	0.0233	0.0178	8.1	6.7	1.13	0.11
10	152.1	116.7	119	155.	0.0258	0.0196	8.9	7.4	1.13	0.12

CONNECTION #4 : FEP(300-21-140- 75-35-35- 76-12-L) B(825-19-#) W(480- 6)

Supports, Flanges of W360X79 Columns. Material Properties as per Supported Member.
Supports assumed to be RIGID.
Supported member is uncoped.

#	Beam Length (mm)	Length / Depth	Plate Length (mm)	# of Blts Reqd	Conn Cost ($)	Govern Condit	FACT SHEAR (kN)	Vbolt / Vf	Vweld / Vf	Vplt. / Vf	Vbeam / Vf	FACT Mend (kNm)
1	1809	3.0	Weld	Weld	Weld	Weld	1128.3	0.72	0.71	1.64	0.82	Weld
2	3618	6.0	Weld	Weld	Weld	Weld	865.7	0.82	0.75	2.14	0.99	Weld
3	5427	9.0	520.0	14	274.78	Weld	577.1	1.22	1.14	3.21	1.44	134.
4	7236	12.0	370.0	10	195.90	Weld	432.6	1.15	1.04	3.05	1.37	71.
5	9045	15.0	370.0	10	195.90	Weld	346.3	1.43	1.30	3.81	1.65	74.
6	10854	18.0	295.0	8	156.46	Rote	288.6	1.26	1.08	3.64	1.60	49.
7	12663	21.0	295.0	8	156.46	Rote	247.3	1.47	1.26	4.25	1.85	49.
8	14472	24.0	370.0	10	195.90	Rote	216.4	2.25	2.03	6.09	2.53	77.
9	16281	27.0	370.0	10	195.90	Rote	192.4	2.49	2.23	6.85	2.80	79.
10	18090	30.0	370.0	10	195.90	Rote	173.1	2.71	2.41	7.62	3.07	80.

#	Max FL defl (mm)	Max DL defl (mm)	Length / FLdefl	Length / DLdefl	θend FL (rads)	θend DL (rads)	Disp Top Conn FL (mm)	Disp Top Beam DL (mm)	Mr / Mf	Mend / Mr
1	Weld	Weld	Weld	Weld	Weld	Weld	Weld	Weld	Weld	Weld
2	Weld	Weld	Weld	Weld	Weld	Weld	Weld	Weld	Weld	Weld
3	12.5	9.5	434.	573.	0.0069	0.0051	2.4	1.9	1.21	0.17
4	26.0	19.4	278.	373.	0.0110	0.0083	2.8	2.3	1.10	0.09
5	40.2	30.2	225	300.	0.0136	0.0102	3.4	2.9	1.10	0.09
6	66.1	45.8	164.	237.	0.0184	0.0132	3.8	3.2	1.07	0.06
7	89.7	62.3	141.	203.	0.0214	0.0153	4.6	3.8	1.07	0.06
8	101.8	76.7	142.	188.	0.0216	0.0162	5.7	4.9	1.11	0.10
9	127.8	96.7	127.	168.	0.0241	0.0182	6.4	5.4	1.11	0.10
10	156.8	119.1	115.	152.	0.0266	0.0202	7.1	6.2	1.11	0.10

Table 8.3 Flexible End Plate Connection Design Table for a Simply Supported, Continuously Braced, Standard Steel Beam, With a Uniformly Distributed Load of 90 % of the Ultimate Level

1

W610X101 (Available In Canada And U.S.A.), Normally Used As A Beam.

SECTION PROPERTIES

Nominal Mass,	m =	101 kg/m
Cross Sectional Area,	A =	13000 mm2
x-Axis Mom. Of Inertia,	Ix =	764.0x10**6 mm4
x-Axis Section Modulus,	Sx =	2530.0x10**3 mm3
x-Axis Plastic Modulus,	Zx =	2900.0x10**3 mm3
y-Axis Mom. Of Inertia,	Iy =	29.5x10**6 mm4
y-Axis Section Modulus,	Sy =	259.0x10**3 mm3
y-Axis Plastic Modulus,	Zy =	404.0x10**3 mm3
Torsional Constant,	J =	781.0x10**3 mm4
Warping Constant,	Cw =	2550.0x10**9 mm6
Overall Section Depth,	d =	603.0 mm
Flange Width,	b =	228.0 mm
Flange Thickness,	t =	14.9 mm
Clear Flange Distance,	fT =	91.0 mm
Web Height,	h =	573.2 mm
Web Thickness,	w =	10.5 mm
Clear Web Distance,	wT =	535.0 mm

MATERIAL PROPERTIES

Yield Strength,	Fy =	300.0 MPa
Modulus of Elasticity,	E =	200000. MPa

SECTION CLASS & SECTION STRENGTH

Flange Width/Thickness,	b/t =	7.7, CLASS 1
Web Width/Thickness,	h/w =	54.6, CLASS 1B CLASS 1V
Resistance Factor,	ϕ =	0.90
Fact. Yld. Moment Cap.,	My =	683.1 kNm
Fact. Moment Capacity,	Mr =	783.0 kNm
Fact. Shear Capacity,	Vr =	1128.3 kN
Max. Shear Strength,	Fs =	198.0 MPa = 0.66xFy

FLEXIBLE END PLATE CONNECTION FEP(Fyp-Dh-g-p-Dend-Dedg-Dtop-tp-L) B(Fub-Db-#) W(Xu-f)

PLATE PROPERTIES

Yield Strength,	Fyp =	300.0 MPa
Ultimate Strength,	Fup =	450.0 MPa
Modulus of Elasticity,	Ep =	200000.0 MPa
Resistance Factor,	ϕp =	0.90
Bolt Hole Diameter,	Dh =	21.0 mm
Bolt Hole Gage,	g =	140.0 mm
Bolt Hole Pitch,	p =	75.0 mm
Bolt Hole End Dist.,	Dend =	35.0 mm
Bolt Hole Edge Dist.,	Dedg =	35.0 mm
Bolt Hole Top Dist.,	Dtop =	75.0 mm
Material Cost,	COSTP =	$ 3.00/kg

BOLT PROPERTIES

Ultimate Strength,	Fub =	825.0 MPa
Resistance Factor,	ϕb =	0.67
Nominal Diameter,	Db =	19.0 mm
Fact. Tensile Cap.,	Trb =	117.5 kN/Bolt
Fact. Shear Capacity,	Vrb =	65.8 kN/Bolt
Bolt Cost,	COSTB =	$ 10.00/Bolt

WELD PROPERTIES

Ultimate Strength,	Xu =	480.0 MPa
Resistance Factor,	ϕw =	0.67
Fillet Weld Leg Size,	f =	6.0 mm
Fact. Long Sh. Cap.,	Vrw =	0.91 kN/mm
Fillet Weld Cost,	COSTW =	$ 0.10/mm

SIMPLE BEAM ANALYSIS

UNIFORMLY DISTRIBUTED LOAD mobilizing 90. % of the FACTORED RESISTANCE of this member.
Dead Load Factor = 1.25 .

#	Beam Length (mm)	Length / Depth	FACT LOAD (kN/m)	SPEC 100%DL (kN/m)	Vr / Vf	Mr / Mf	Max FL defl (mm)	Max DL defl (mm)	Length / FLdefl	Length / DLdefl	θend FL (rads)	θend DL (rads)
1	1809	3.0	1122.7	898.1	1.11	1.71	1.0	0.8	1766.	2207.	0.0018	0.0014
2	3618	6.0	430.7	344.5	1.45	1.11	6.5	5.0	560.	719.	0.0057	0.0044
3	5427	9.0	191.4	153.1	2.17	1.11	14.5	11.3	373	479.	0.0085	0.0067
4	7236	12.0	107.7	86.1	2.90	1.11	25.8	20.1	280.	360.	0.0113	0.0089
5	9045	15.0	68.9	55.1	3.62	1.11	40.4	31.4	224.	288.	0.0142	0.0111
6	10854	18.0	47.9	38.3	4.34	1.11	58.1	45.3	187.	240.	0.0170	0.0133
7	12663	21.0	35.2	28.1	5.07	1.11	79.1	61.6	160.	205.	0.0198	0.0156
8	14472	24.0	26.9	21.5	5.79	1.11	103.4	80.5	140.	180.	0.0227	0.0178
9	16281	27.0	21.3	17.0	6.52	1.11	130.8	101.9	124.	160.	0.0255	0.0200
10	18090	30.0	17.2	13.8	7.24	1.11	161.5	125.8	112.	144	0.0283	0.0222

Table 8.3 Flexible End Plate Connection Design Table for a Simply Supported, Continuously Braced, Standard Steel Beam, With a Uniformly Distributed Load of 90 % of the Ultimate Level

2

W610X101 (Available In Canada And U.S.A.), Normally Used As A Beam

CONNECTION #1 : FEP(300-21-140- 75-35-35- 75- 6-L) B(825-19-#) W(480- 6)

Supports, Flanges of W360X79 Columns. Material Properties as per Supported Member.
Supports assumed to be RIGID.
Supported member is uncoped.

#	Beam Length (mm)	Length / Depth	Plate Length (mm)	# of Blts Reqd	Conn Cost ($)	Govern Condit	FACT SHEAR (kN)	Vbolt / Vf	Vweld / Vf	Vplt. / Vf	Vbeam / Vf	FACT Mend (kNm)
1	1809	3.0	Bolt	Bolt	Bolt	Bolt	1015.4	0.87	0.99	0.91	0.96	Bolt
2	3618	6.0	520.0	14	259.39	Bolt	779.1	1.14	1.32	1.19	1.24	43.
3	5427	9.0	370.0	10	184.95	Bolt	519.4	1.22	1.41	1.27	1.34	23.
4	7236	12.0	295.0	8	147.73	Bolt	389.6	1.30	1.49	1.36	1.44	15.
5	9045	15.0	370.0	10	184.95	Rote	311.6	2.03	2.35	2.12	2.22	25.
6	10854	18.0	370.0	10	184 95	Rote	259.7	2.44	2.82	2.54	2.66	26.
7	12663	21.0	445.0	12	222.17	Rote	222.6	3.41	3.97	3.56	3.68	41.
8	14472	24.0	445.0	12	222.17	Rote	194.8	3.90	4.54	4.07	4.18	43.
9	16281	27.0	445.0	12	222 17	Rote	173.1	4.39	5.11	4.58	4.68	46.
10	18090	30.0	520.0	14	259.39	Rote	155.8	5.69	6.64	5.95	5.99	69.

#	Max FL defl (mm)	Max DL defl (mm)	Length / FLdefl	Length / DLdefl	θend FL (rads)	θend DL (rads)	Disp Top Conn FL (mm)	Disp Top Beam DL (mm)	Mr / Mf	Mend / Mr
1	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt
2	5.8	4.6	620.	789.	0.0051	0.0040	2.2	1.8	1.18	0.05
3	13.6	10.8	399	503.	0.0079	0.0063	2.5	2.2	1.15	0.03
4	24.7	19.5	293.	371.	0.0108	0.0086	2.8	2.5	1.14	0.02
5	37.6	29.9	240.	303.	0.0132	0.0104	4.3	3.8	1.15	0.03
6	54.1	42.9	201.	253.	0.0158	0.0125	5.1	4.6	1.15	0.03
7	71.7	56.7	177.	223.	0.0178	0.0140	7.0	6.1	1.18	0.05
8	93.2	73.8	155.	196.	0.0202	0.0159	7.9	6.9	1.18	0.06
9	117.4	93.0	139.	175.	0.0226	0.0178	8.8	7.7	1.19	0.06
10	138.8	109.6	130	165.	0.0237	0.0187	10.9	9.3	1.23	0.09

CONNECTION #2 : FEP(300-21-140- 75-35-35- 75- 8-L) B(825-19-#) W(480- 6)

Supports, Flanges of W360X79 Columns. Material Properties as per Supported Member.
Supports assumed to be RIGID.
Supported member is uncoped

#	Beam Length (mm)	Length / Depth	Plate Length (mm)	# of Blts Reqd	Conn Cost ($)	Govern Condit	FACT SHEAR (kN)	Vbolt / Vf	Vweld / Vf	Vplt. / Vf	Vbeam / Vf	FACT Mend (kNm)
1	1809	3.0	Bolt	Bolt	Bolt	Bolt	1015 4	0.91	0.98	1.22	0.96	Bolt
2	3618	6.0	445.0	12	226.56	Bolt	779.1	1.01	1 10	1.36	1.04	51.
3	5427	9.0	295.0	8	150.64	Bolt	519.4	1.01	1.09	1.35	1.05	24.
4	7236	12.0	220.0	6	112.68	Bolt	389.6	1.01	1.08	1.34	1.06	14.
5	9045	15.0	220.0	6	112 68	Bolt	311.6	1.27	1.35	1.68	1.32	14
6	10854	18.0	295.0	8	150.64	Rote	259 7	2.03	2.19	2.70	2.08	26
7	12663	21.0	370 0	10	188.60	Rote	222.6	2.96	3.21	3.95	2.97	42
8	14472	24.0	370.0	10	188.60	Rote	194.8	3.38	3.67	4.51	3.40	42.
9	16281	27.0	445.0	12	226.56	Rote	173.1	4.56	4.97	6.11	4.41	65.
10	18090	30.0	445.0	12	226.56	Rote	155.8	5.07	5.51	6.79	4.64	67.

#	Max FL defl (mm)	Max DL defl (mm)	Length / FLdefl	Length / DLdefl	θend FL (rads)	θend DL (rads)	Disp Top Conn FL (mm)	Disp Top Beam DL (mm)	Mr / Mf	Mend / Mr
1	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt
2	5.7	4.5	630	803.	0.0050	0.0039	1.7	1.4	1.20	0.06
3	13.6	10.8	400.	504.	0.0079	0.0063	1.8	1.7	1.15	0.03
4	24.8	19.6	282.	370.	0.0109	0.0086	1.9	1.9	1.13	0.02
5	38.7	30.5	234.	296.	0.0136	0.0107	2.4	2.3	1.13	0.02
6	54.1	42.9	201.	253	0.0158	0.0125	3.8	3.5	1.15	0.03
7	71.6	56.4	177.	224.	0.0177	0.0139	5.4	4.8	1.18	0.05
8	93.4	73 5	155	197.	0.0202	0.0159	6.3	5.5	1.18	0.05
9	113.3	88.8	144	183.	0.0216	0.0168	7.9	6.8	1.22	0.08
10	139.3	108.2	130.	166	0.0238	0.0186	8.7	7.5	1.23	0.09

Table 8.3 Flexible End Plate Connection Design Table for a Simply Supported, Continuously Braced, Standard Steel Beam, With a Uniformly Distributed Load of 90 % of the Ultimate Level

3

W610X101 (Available In Canada And U.S.A.), Normally Used As A Beam

CONNECTION #3 : FEP(300-21-140- 75-35-35- 75-10-L) B(825-19-#) W(480- 6)

Supports, Flanges of W360X79 Columns. Material Properties as per Supported Member.
Supports assumed to be RIGID.
Supported member is uncoped.

#	Beam Length (mm)	Length / Depth	Plate Length (mm)	# of Blts Reqd	Conn Cost ($)	Govern Condit	FACT SHEAR (kN)	Vbolt / Vf	Vweld / Vf	Vplt. / Vf	Vbeam / Vf	FACT Mend (kNm)
1	1809	3.0	Bolt	Bolt	Bolt	Bolt	1015.4	0.89	0.93	1.52	0.94	Bolt
2	3618	6.0	520.0	14	269.65	Bolt	779.1	1.13	1.16	1.98	1.16	98.
3	5427	9.0	370.0	10	192.25	Bolt	519.4	1.21	1.23	2.12	1.25	52.
4	7236	12.0	295.0	8	153.55	Weld	389.6	1.27	1.27	2.25	1.34	34.
5	9045	15.0	220.0	6	114.85	Weld	311.6	1.20	1.19	2.10	1.28	20.
6	10854	18.0	220.0	6	114.85	Rote	259.7	1.44	1.43	2.52	1.52	21.
7	12663	21.0	295.0	8	153.55	Rote	222.6	2.22	2.21	3.94	2.28	37.
8	14472	24.0	370.0	10	192.25	Rote	194.8	3.15	3.15	5.64	3.12	59.
9	16281	27.0	370.0	10	192.25	Rote	173.1	3.54	3.54	6.35	3.52	60.
10	18090	30.0	445.0	12	230.95	Rote	155.8	4.68	4.70	8.48	4.55	88.

#	Max FL defl (mm)	Max DL defl (mm)	Length / FLdefl	Length / DLdefl	θend FL (rads)	θend DL (rads)	Disp Top Conn FL (mm)	Disp Top Beam DL (mm)	Mr / Mf	Mend / Mr
1	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt
2	5.2	4.0	691.	899.	0.0044	0.0033	1.6	1.3	1.29	0.13
3	12.9	10.1	421.	537	0.0074	0.0058	2.0	1.7	1.20	0.07
4	23.7	18.7	306.	387.	0.0103	0.0081	2.3	2.0	1.17	0.04
5	38.1	30.1	237.	300.	0.0133	0.0105	2.2	2.1	1.14	0.03
6	54.7	43.3	198.	250.	0.0160	0.0126	2.7	2.6	1.14	0.03
7	72.2	56.9	175.	222.	0.0179	0.0141	4.0	3.7	1.17	0.05
8	90.5	70.8	160.	204.	0.0195	0.0151	5.5	4.9	1.21	0.08
9	114.4	89.4	142.	182.	0.0219	0.0170	6.3	5.4	1.21	0.08
10	133.7	103.3	135.	175.	0.0226	0.0173	7.8	6.5	1.27	0.11

CONNECTION #4 : FEP(300-21-140- 75-35-35- 75-12-L) B(825-19-#) W(480- 6)

Supports, Flanges of W360X79 Columns. Material Properties as per Supported Member.
Supports assumed to be RIGID.
Supported member is uncoped.

#	Beam Length (mm)	Length / Depth	Plate Length (mm)	# of Blts Reqd	Conn Cost ($)	Govern Condit	FACT SHEAR (kN)	Vbolt / Vf	Vweld / Vf	Vplt. / Vf	Vbeam / Vf	FACT Mend (kNm)
1	1809	3.0	Bolt	Bolt	Bolt	Bolt	1015.4	0.81	0.82	1.83	0.92	Bolt
2	3618	6.0	Weld	Weld	Weld	Weld	779.1	0.91	0.83	2.38	1.11	Weld
3	5427	9.0	445.0	12	235.34	Weld	519.4	1.18	1.08	3.05	1.39	98.
4	7236	12.0	295.0	8	156.46	Weld	389.6	1.08	1.02	2.70	1.26	45.
5	9045	15.0	295.0	8	156.46	Weld	311.6	1.23	1.08	3.37	1.55	46.
6	10854	18.0	295.0	8	156.46	Weld	259.7	1.41	1.20	4.05	1.84	47.
7	12663	21.0	220.0	6	117.02	Rote	222.6	1.29	1.13	3.52	1.64	28.
8	14472	24.0	295.0	8	156.46	Rote	194.8	1.87	1.60	5.40	2.33	49.
9	16281	27.0	295.0	8	156.46	Rote	173.1	2.10	1.80	6.07	2.62	49.
10	18090	30.0	370.0	10	195.90	Rote	155.8	3.10	2.79	8.46	3.48	78.

#	Max FL defl (mm)	Max DL defl (mm)	Length / FLdefl	Length / DLdefl	θend FL (rads)	θend DL (rads)	Disp Top Conn FL (mm)	Disp Top Beam DL (mm)	Mr / Mf	Mend / Mr
1	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt
2	Weld	Weld	Weld	Weld	Weld	Weld	Weld	Weld	Weld	Weld
3	11.8	9.1	460.	600.	0.0066	0.0050	1.9	1.6	1.29	0.12
4	23.2	16.2	312.	397.	0.0100	0.0079	2.0	1.8	1.19	0.06
5	36.2	28.4	250.	318.	0.0125	0.0098	2.5	2.3	1.19	0.06
6	52.1	40.9	209.	266	0.0150	0.0117	3.1	2.8	1.19	0.06
7	73.4	58.1	172	218.	0.0183	0.0145	2.8	2.7	1.16	0.04
8	92.2	72.4	157.	200.	0.0199	0.0156	4.1	3.8	1.19	0.06
9	116.6	91.4	140.	178.	0.0224	0.0175	4.8	4.3	1.19	0.06
10	136.3	105.5	133.	171.	0.0232	0.0178	6.2	5.3	1.25	0.10

Table 8.4 Flexible End Plate Connection Design Table for a Simply Supported, Continuously Braced, Standard Steel Beam, With a Uniformly Distributed Load of 80 % of the Ultimate Level

1

W610X101 (Available In Canada And U.S.A.), Normally Used As A Beam.

SECTION PROPERTIES

Nominal Mass,	m =	101	kg/m
Cross Sectional Area,	A =	13000	mm2
x-Axis Mom. Of Inertia,	Ix =	764.0x10**6	mm4
x-Axis Section Modulus,	Sx =	2530.0x10**3	mm3
x-Axis Plastic Modulus,	Zx =	2900.0x10**3	mm3
y-Axis Mom. Of Inertia,	Iy =	29.5x10**6	mm4
y-Axis Section Modulus,	Sy =	259.0x10**3	mm3
y-Axis Plastic Modulus,	Zy =	404.0x10**3	mm3
Torsional Constant,	J =	781.0x10**3	mm4
Warping Constant,	Cw =	2550.0x10**9	mm6
Overall Section Depth,	d =	603.0	mm
Flange Width,	b =	228.0	mm
Flange Thickness,	t =	14.9	mm
Clear Flange Distance,	fT =	91.0	mm
Web Height,	h =	573.2	mm
Web Thickness,	w =	10.5	mm
Clear Web Distance,	wT =	535.0	mm

MATERIAL PROPERTIES

Yield Strength,	Fy =	300.0	MPa
Modulus of Elasticity,	E =	200000	MPa

SECTION CLASS & SECTION STRENGTH

Flange Width/Thickness,	b/t =	7.7,	CLASS 1
Web Width/Thickness,	h/w =	54.6,	CLASS 1B
			CLASS 1V
Resistance Factor,	ϕ =	0.90	
Fact. Yld. Moment Cap.,	My =	683.1	kNm
Fact. Moment Capacity,	Mr =	783.0	kNm
Fact. Shear Capacity,	Vr =	1128.3	kN
Max. Shear Strength,	Fs =	198.0	MPa
	=	0.66xFy	

FLEXIBLE END PLATE CONNECTION FEP(Fyp-Dh-g-p-Dend-Dedg-Dtop-tp-L) B(Fub-Db-#) W(Xu-f)

PLATE PROPERTIES

Yield Strength,	Fyp =	300.0	MPa
Ultimate Strength,	Fup =	450.0	MPa
Modulus of Elasticity,	Ep =	200000.0	MPa
Resistance Factor,	ϕp =	0.90	
Bolt Hole Diameter,	Dh =	21.0	mm
Bolt Hole Gage,	g =	140.0	mm
Bolt Hole Pitch,	p =	75.0	mm
Bolt Hole End Dist.,	Dend =	35.0	mm
Bolt Hole Edge Dist.,	Dedg =	35.0	mm
Bolt Hole Top Dist.,	Dtop =	75.0	mm
Material Cost,	COSTP =	$ 3.00/kg	

BOLT PROPERTIES

Ultimate Strength,	Fub =	825.0	MPa
Resistance Factor,	ϕb =	0.67	
Nominal Diameter,	Db =	19.0	mm
Fact. Tensile Cap.,	Trb =	117.5	kN/Bolt
Fact. Shear Capacity,	Vrb =	65.8	kN/Bolt
Bolt Cost,	COSTB =	$ 10.00/Bolt	

WELD PROPERTIES

Ultimate Strength,	Xu =	480.0	MPa
Resistance Factor,	ϕw =	0.67	
Fillet Weld Leg Size,	f =	6.0	mm
Fact. Long. Sh. Cap.,	Vrw =	0.91	kN/mm
Fillet Weld Cost,	COSTW =	$ 0.10/mm	

SIMPLE BEAM ANALYSIS

UNIFORMLY DISTRIBUTED LOAD mobilizing 80. % of the FACTORED RESISTANCE of this member.
Dead Load Factor = 1.25

#	Beam Length (mm)	Length/Depth	FACT LOAD (kN/m)	SPEC 100%DL (kN/m)	Vr/Vf	Mr/Mf	Max FL defl (mm)	Max DL defl (mm)	Length/FLdefl	Length/DLdefl	θend FL (rads)	θend DL (rads)
1	1809	3.0	997.9	798.3	1.25	1.82	0.9	0.7	1986.	2483	0.0016	0.0013
2	3618	6.0	382.8	306.3	1.63	1.25	5.6	4.5	647	809.	0.0049	0.0040
3	5427	9.0	170.1	136.1	2.44	1.25	12.6	10.1	432.	539.	0.0074	0.0059
4	7236	12.0	95.7	76.6	3.26	1.25	22.4	17.9	324.	405.	0.0099	0.0079
5	9045	15.0	61.3	49.0	4.07	1.25	34.9	27.9	259.	324.	0.0124	0.0099
6	10854	18.0	42.5	34.0	4.89	1.25	50.3	40.2	216.	270.	0.0148	0.0119
7	12663	21.0	31.3	25.0	5.70	1.25	68.5	54.8	185.	231.	0.0173	0.0138
8	14472	24.0	23.9	19.1	6.52	1.25	89.4	71.5	162.	202.	0.0198	0.0158
9	16281	27.0	18.9	15.1	7.33	1.25	113.2	90.6	144.	180.	0.0222	0.0178
10	18090	30.0	15.3	12.3	8.15	1.25	139.7	111.8	129.	162.	0.0247	0.0198

Table 8.4 Flexible End Plate Connection Design Table for a Simply Supported, Continuously Braced, Standard Steel Beam, With a Uniformly Distributed Load of 80 % of the Ultimate Level

2

W610X101 (Available In Canada And U.S.A.), Normally Used As A Beam.

CONNECTION #1 . FEP(300-21-140- 75-35-35- 75- 6-L) B(825-19-#) W(480- 6)

Supports. Flanges of W360X79 Columns Material Properties as per Supported Member.
Supports assumed to be RIGID.
Supported member is uncoped.

#	Beam Length (mm)	Length/Depth	Plate Length (mm)	# of Blts Reqd	Conn Cost ($)	Govern Condit	FACT SHEAR (kN)	Vbolt/Vf	Vweld/Vf	Vplt./Vf	Vbeam/Vf	FACT Mend (kNm)
1	1809	3.0	Bolt	Bolt	Bolt	Bolt	902.6	0.98	1.11	1.03	1.09	Bolt
2	3618	6.0	445.0	12	222.17	Bolt	692.5	1.10	1.27	1.15	1.21	30.
3	5427	9.0	295.0	8	147.73	Bolt	461 7	1.10	1.26	1.14	1.22	14.
4	7236	12.0	220.0	6	110.51	Bolt	346.3	1.10	1.25	1.13	1.22	8.
5	9045	15.0	295.0	8	147.73	Rote	277.0	1.83	2 10	1.90	2.02	15.
6	10854	18.0	370.0	10	184.85	Rote	230.8	2 74	3.17	2.86	3.00	25.
7	12663	21.0	370.0	10	184 95	Rote	197.9	3.20	3 70	3.33	3.48	26.
8	14472	24.0	445 0	12	222.17	Rote	173.1	4.39	5.10	4 58	4 73	41
9	16281	27 0	445.0	12	222.17	Rote	153.9	4.94	5.74	5 15	5.30	43.
10	18090	30.0	445.0	12	222 17	Rote	138.5	5.49	6.38	5.73	5.86	45.

#	Max FL defl (mm)	Max DL defl (mm)	Length/FLdefl	Length/DLdefl	θend FL (rads)	θend DL (rads)	Disp Top Conn FL (mm)	Disp Top Beam DL (mm)	Mr/Mf	Mend/Mr
1	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt
2	5.3	4.2	687.	870.	0.0046	0.0036	1.7	1.5	1.31	0.04
3	12.2	9.7	444.	558.	0.0072	0.0057	1.8	1.6	1.28	0.02
4	22.0	17.6	329.	412.	0.0097	0.0077	1.8	1.8	1.27	0.01
5	33.9	26.9	267.	336.	0.0119	0.0094	3.1	2.8	1.28	0.02
6	47.9	37.9	227.	286.	0.0139	0.0110	4.5	4.0	1.30	0.03
7	65.0	51.5	195	246.	0.0162	0.0128	5.3	4.7	1.30	0.03
8	82.4	65.1	176.	222.	0.0178	0.0140	7.0	6.1	1.30	0.03
9	103.8	82.1	157.	198.	0.0199	0.0157	7.8	6.8	1.34	0.05
10	127.6	101.0	142.	179.	0.0220	0.0174	8.6	7.5	1.35	0.06

CONNECTION #2 : FEP(300-21-140- 75-35-35- 75- 8-L) B(825-19-#) W(480- 6)

Supports, Flanges of W360X79 Columns. Material Properties as per Supported Member.
Supports assumed to be RIGID.
Supported member is uncoped.

#	Beam Length (mm)	Length/Depth	Plate Length (mm)	# of Blts Reqd	Conn Cost ($)	Govern Condit	FACT SHEAR (kN)	Vbolt/Vf	Vweld/Vf	Vplt./Vf	Vbeam/Vf	FACT Mend (kNm)
1	1809	3.0	520.0	14	264.52	Bolt	902.6	1.02	1.11	1.37	1.08	48.
2	3618	6.0	445.0	12	226.56	Bolt	692 5	1.14	1.23	1.53	1.18	50.
3	5427	9.0	295.0	8	150.64	Bolt	461.7	1.14	1.23	1.52	1.19	23.
4	7236	12.0	220.0	6	112.68	Bolt	346.3	1.14	1.22	1.51	1.20	14.
5	9045	15.0	220.0	6	112.68	Bolt	277 0	1.43	1.52	1.89	1.50	14.
6	10854	18 0	295 0	8	150.64	Rote	230.8	2.28	2.46	3.04	2.35	25.
7	12663	21.0	295 0	8	150.64	Rote	197.9	2.66	2.87	3.54	2.72	26.
8	14472	24.0	370.0	10	188.60	Rote	173.1	3.80	4.13	5.08	3.81	42.
9	16281	27.0	370 0	10	188.60	Rote	153.9	4.28	4.64	5.71	4.31	42.
10	18090	30.0	445.0	12	226.56	Rote	138 5	5 70	6.21	7.63	5.53	64

#	Max FL defl (mm)	Max DL defl (mm)	Length/FLdefl	Length/DLdefl	θend FL (rads)	θend DL (rads)	Disp Top Conn FL (mm)	Disp Top Beam DL (mm)	Mr/Mf	Mend/Mr
1	0.8	0.6	2319.	2935.	0.0013	0.0010	0.5	0.4	2.18	0.06
2	5.1	4.0	715	813.	0.0044	0.0034	1.5	1.3	1.36	0.06
3	12.0	9.5	452.	570.	0.0070	0.0055	1.6	1.5	1.30	0.03
4	21.8	17.3	332.	418.	0.0096	0.0076	1.7	1.6	1.28	0.02
5	34.0	27.0	266.	335	0 0119	0.0095	2.2	2.0	1.28	0.02
6	47.9	37.9	227	287.	0.0138	0.0110	3 4	3.0	1.30	0.03
7	65.1	51.5	195	246.	0.0162	0.0128	3.9	3.6	1.30	0.03
8	82.3	64.8	176.	223.	0.0178	0 0138	5.4	4.8	1.34	0.05
9	104.0	81.8	166.	189.	0.0200	0.0156	6.2	5.4	1.34	0.05
10	122.6	95.8	148.	188.	0.0209	0 0162	7.6	6.6	1.39	0.08

Table 8.4 Flexible End Plate Connection Design Table for a Simply Supported, Continuously Braced, Standard Steel Beam, With a Uniformly Distributed Load of 80 % of the Ultimate Level

3

W610X101 (Available In Canada And U.S.A), Normally Used As A Beam

CONNECTION #3 : FEP(300-21-140- 75-35-35- 75-10-L) B(825-19-#) W(480- 6)

Supports, Flanges of W360X79 Columns. Material Properties as per Supported Member.
Supports assumed to be RIGID.
Supported member is uncoped.

#	Beam Length (mm)	Length / Depth	Plate Length (mm)	# of Blts Reqd	Conn Cost ($)	Govern Condit	FACT SHEAR (kN)	Vbolt / Vf	Vweld / Vf	Vplt. / Vf	Vbeam / Vf	FACT Mend (kNm)
1	1809	3.0	520.0	14	269.65	Bolt	902.6	1.01	1.06	1.71	1.06	68.
2	3618	6.0	445.0	12	230.95	Bolt	692.5	1.09	1.12	1.91	1.14	70.
3	5427	9.0	295.0	8	153.55	Bolt	461.7	1.09	1.13	1.90	1.15	33.
4	7236	12.0	220.0	6	114.85	Weld	346.3	1.08	1.07	1.89	1.16	19.
5	9045	15.0	220.0	6	114.85	Weld	277.0	1.35	1.34	2.36	1.44	20.
6	10854	18.0	145.0	4	76.15	Weld	230.8	1.09	1.09	1.87	1.18	9.
7	12663	21.0	220.0	6	114.85	Rote	197.9	1.89	1.88	3.30	1.98	21.
8	14472	24.0	295.0	8	153.55	Rote	173.1	2.85	2.85	5.06	2.93	37.
9	16281	27.0	370.0	10	192.25	Rote	153.9	3.99	3.99	7.14	3.96	59.
10	18090	30.0	370.0	10	192.25	Rote	138.5	4.42	4.43	7.93	4.42	59.

#	Max FL defl (mm)	Max DL defl (mm)	Length / FLdefl	Length / DLdefl	θend FL (rads)	θend DL (rads)	Disp Top Conn FL (mm)	Disp Top Beam DL (mm)	Mr / Mf	Mend / Mr
1	0.7	0.6	2491.	3133	0.0012	0.0010	0.4	0.3	2.30	0.09
2	4.8	3.6	747.	963.	0.0041	0.0032	1.2	1.1	1.41	0.09
3	11.8	9.3	461.	584.	0.0068	0.0054	1.4	1.3	1.32	0.04
4	21.5	17.1	336.	424	0.0094	0.0075	1.5	1.4	1.29	0.02
5	33.6	26.6	269.	339.	0.0118	0.0093	1.9	1.8	1.29	0.03
6	49.4	39.4	220	276.	0.0145	0.0115	1.6	1.7	1.27	0.01
7	65.8	52.1	193.	243.	0.0164	0.0130	2.7	2.6	1.29	0.03
8	83.1	65.4	174.	221.	0.0180	0.0141	4.0	3.7	1.33	0.05
9	100.4	78.4	162.	208.	0.0191	0.0148	5.4	4.8	1.38	0.08
10	123.9	96.5	146	187.	0.0212	0.0164	6.1	5.3	1.38	0.08

CONNECTION #4 : FEP(300-21-140- 75-35-35- 75-12-L) B(825-19-#) W(480- 6)

Supports, Flanges of W360X79 Columns. Material Properties as per Supported Member.
Supports assumed to be RIGID.
Supported member is uncoped.

#	Beam Length (mm)	Length / Depth	Plate Length (mm)	# of Blts Reqd	Conn Cost ($)	Govern Condit	FACT SHEAR (kN)	Vbolt / Vf	Vweld / Vf	Vplt. / Vf	Vbeam / Vf	FACT Mend (kNm)
1	1809	3.0	Bolt	Bolt	Bolt	Bolt	902.6	0.93	0.95	2.05	1.05	Bolt
2	3618	6.0	Weld	Weld	Weld	Weld	692.5	1.06	0.99	2.68	1.26	Weld
3	5427	9.0	445.0	12	235.34	Weld	461.7	1.35	1.25	3.44	1.59	95.
4	7236	12.0	295.0	8	156.46	Weld	346.3	1.21	1.16	3.04	1.43	44.
5	9045	15.0	295.0	8	156.46	Weld	277.0	1.51	1.43	3.80	1.75	46.
6	10854	18.0	220.0	6	117.02	Weld	230.8	1.25	1.09	3.40	1.61	27.
7	12663	21.0	220.0	6	117.02	Weld	197.9	1.45	1.27	3.96	1.87	27.
8	14472	24.0	220.0	6	117.02	Rote	173.1	1.66	1.45	4.53	2.11	28.
9	16281	27.0	295.0	8	156.46	Rote	153.9	2.37	2.02	6.83	2.96	49.
10	18090	30.0	295.0	6	156.46	Rote	138.5	2.63	2.25	7.59	3.30	49.

#	Max FL defl (mm)	Max DL defl (mm)	Length / FLdefl	Length / DLdefl	θend FL (rads)	θend DL (rads)	Disp Top Conn FL (mm)	Disp Top Beam DL (mm)	Mr / Mf	Mend / Mr
1	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt	Bolt
2	Weld	Weld	Weld	Weld	Weld	Weld	Weld	Weld	Weld	Weld
3	10.3	7.9	528.	691	0.0057	0.0043	1.6	1.4	1.47	0.12
4	20.5	16.0	354.	451.	0.0088	0.0069	1.7	1.6	1.35	0.06
5	31.8	25.0	284.	362.	0.0110	0.0086	2.1	2.0	1.35	0.06
6	47.7	37.7	227	288.	0.0139	0.0109	2.1	2.0	1.31	0.03
7	64.9	51.3	195.	247	0.0162	0.0128	2.5	2.4	1.31	0.03
8	84.7	66.9	171.	216	0.0185	0.0145	2.8	2.8	1.31	0.04
9	102.5	80.3	159	203.	0.0196	0.0153	4.1	3.8	1.36	0.06
10	126.6	99.0	143	183.	0.0218	0.0169	4.6	4.2	1.36	0.06

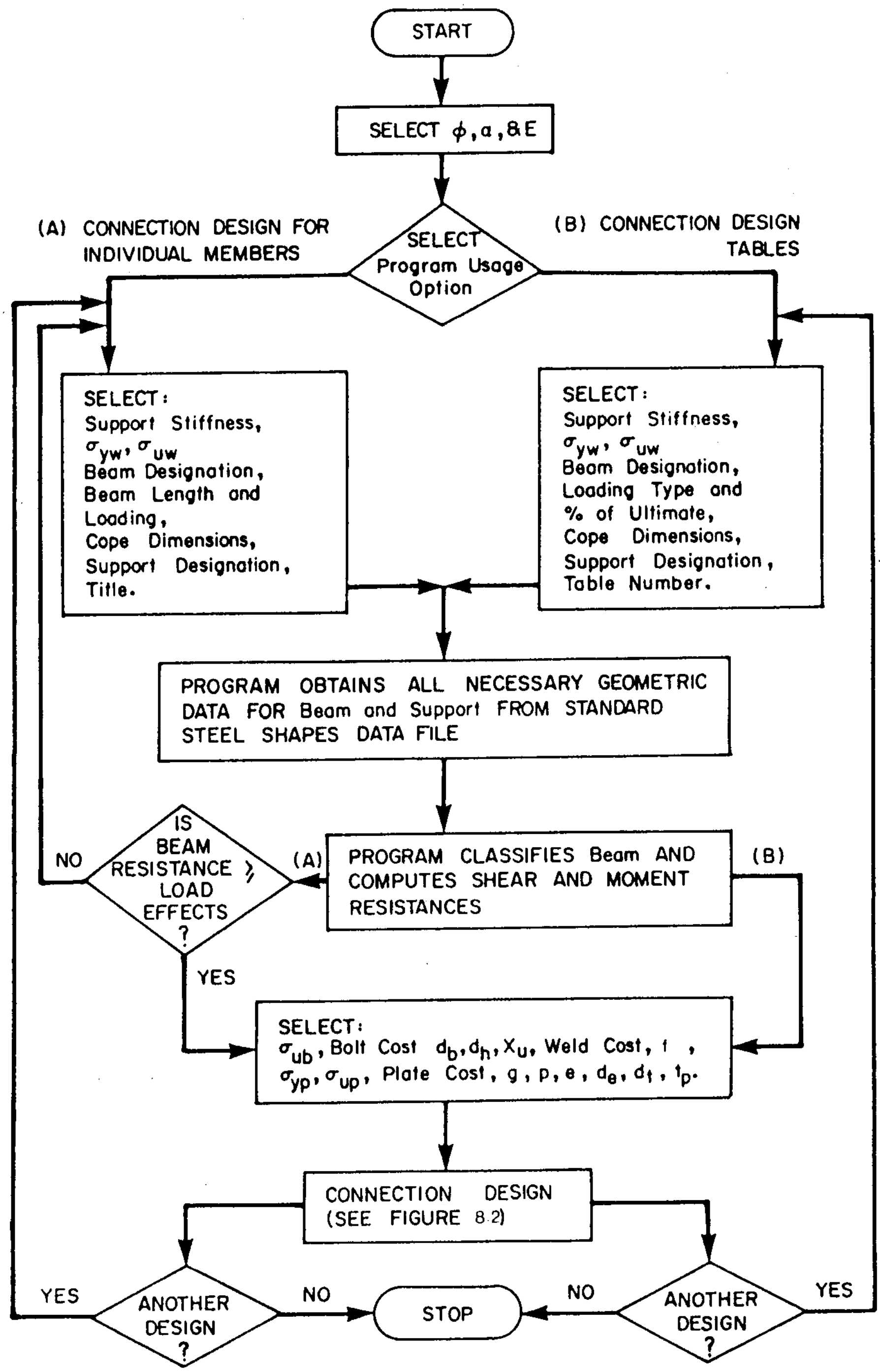


Figure 8.1 Flow Chart of Data Input Sequence for CDP

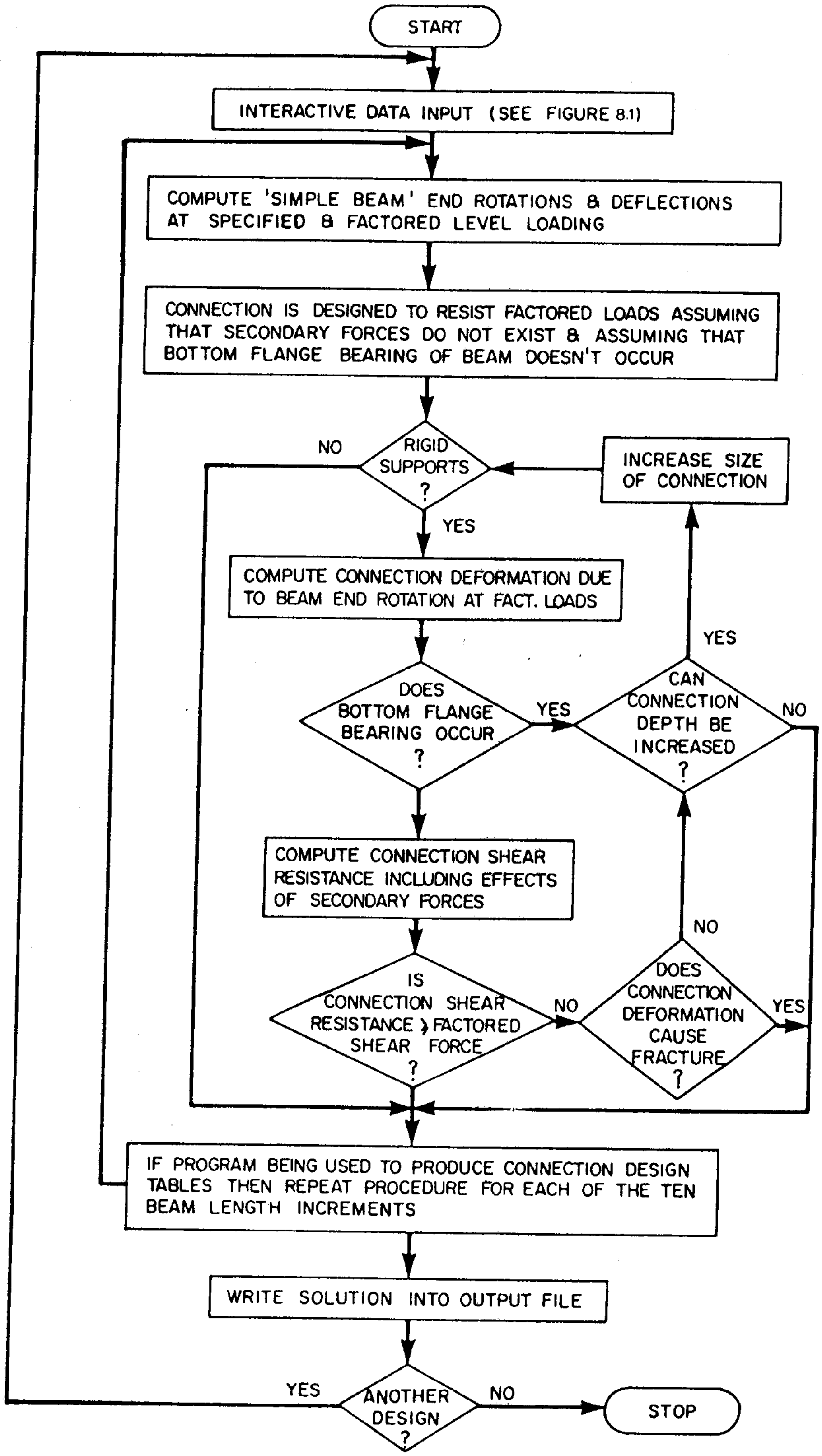
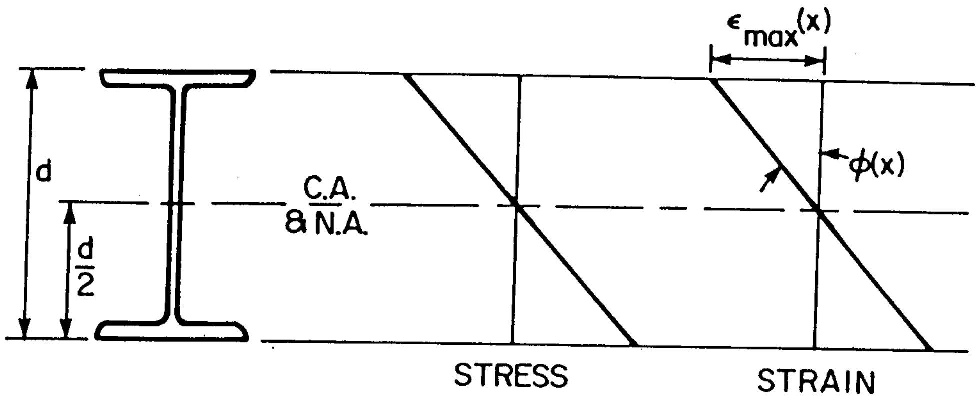
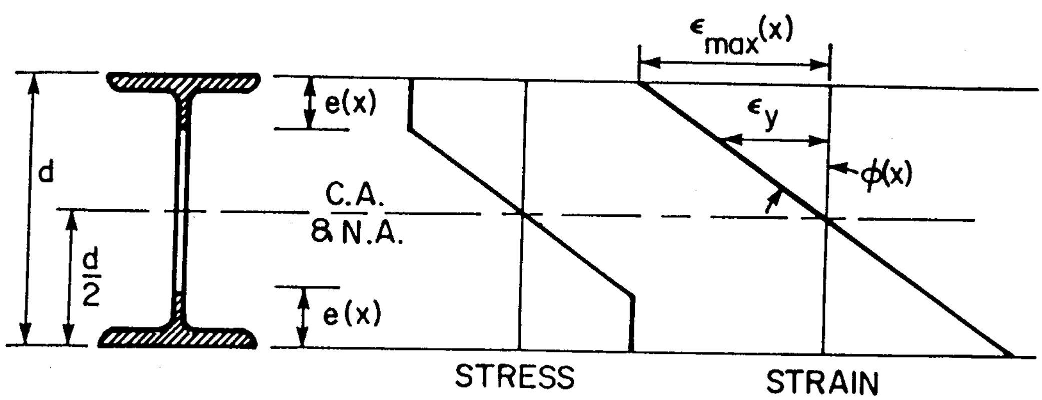


Figure 8.2 Flow Chart of Connection Design Process for CDP

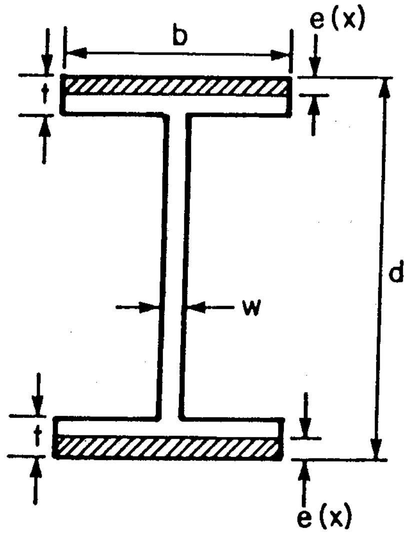


a) ELASTIC LEVEL, $\epsilon_{max} < \epsilon_y$



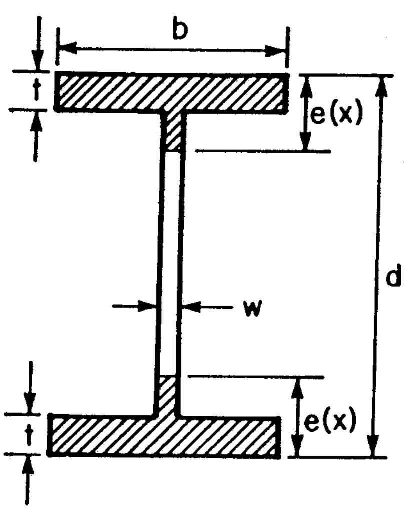
b) INELASTIC LEVEL, $\epsilon_{max} > \epsilon_y$

Figure 8.3 Determination of the Curvature at a Cross-Section



$$M_i(x) = \phi(d-e(x))\, b\, e(x)\, \sigma_y + \frac{2\,\phi\,\sigma_y}{(d-2e(x))} \left\{(d-2t)^3 \frac{w}{12} + (t-e(x)) \frac{b}{2}(d-t-e(x))^2\right\}$$

i) YIELDING CONFINED TO FLANGES



$$M_i(x) = \phi(d-t)\, b\, t\, \sigma_y + \phi(e(x)-t)(d-e(x)-t)\, w\, \sigma_y + \phi\,(d-2e(x))^2 \frac{w\sigma_y}{6}$$

ii) YIELDING INTO THE WEB

Figure 8.4 Calculation of the Internal Moment at a Cross-Section Which is Partly Inelastic

9. Interactive Computer Program: Use and Application of Program Output

9.1 Introduction

Standardized connection designs and design tables developed using CDP can be used by both fabricators and consulting engineers. Connection design output for a single beam (OPERATIONAL MODE # 1) is given in Table 8.1. Three connection design tables (OPERATIONAL MODE # 2) are given in Tables 8.2 to 8.4.

Table 8.1 was developed for a W610x101 beam, 10000. mm long, supporting a uniformly distributed dead and live load which results in about 100 % of the factored resistance of the beam (simply supported) to be mobilized. The support at the left end is the flange of a W360x79 column, assumed to be rigid, and the support at the right end of the beam is the web of a W610x101 spandrel girder, assumed to be flexible. The right end of the beam is coped. Four different end plate thicknesses are considered.

Tables 8.2 to 8.4, each of which is 3 pages long, were developed for uncoped W610x101 beams supporting uniformly distributed loads. The loads range from 80 to 100 percent of ultimate levels in increments of 10 percent. Thus, each of the tables provides connection designs for the supported beam at a factored load level causing a different percentage of the factored resistance of that beam to be developed. The supports, W360x79 column flanges, are assumed to be rigid.

The remaining portion of this chapter describes some of the possible applications for the standardized connection design summaries and the standardized connection design tables. A numeric example using the connection design tables is given in Appendix C.

9.2 OPERATIONAL MODE # 1 - Connection Design for Individual Beams

Even though there are some differences in the output generated for the two OPERATIONAL MODES (i. e. either connection design for individual beams or connection design table preparation) the types of design problems which can be solved using the output from either MODE are identical. The major difference between the output generated by the two MODES is that MODE # 1 gives connection design information for one beam length where as MODE # 2 gives design information for a series of ten beam lengths. MODE # 1 also allows completely general loading whereas MODE # 2 allows only uniformly distributed and mid span concentrated loading. Because MODE # 2 design information is for a series of incremental beam lengths interpolation is generally required when using the design tables to solve problems; interpolation is of course not required when MODE # 1 is used.

Six different design problems that can be solved using the output from CDP are demonstrated in Section 9.3 for MODE # 2. The solution processes given are generally applicable for MODE # 1 output.

9.3 OPERATIONAL MODE # 2 - Connection Design Tables

9.3.1. SELECTION OF A CONNECTION DETAIL WHEN THE BEAM LENGTH AND FACTORED LOAD ARE KNOWN. The designer enters the design tables, e. g. Tables 8.2 to 8.4, that give the correct beam size, loading pattern, material properties, and then

i. determines the two beam lengths that bracket the specified length, given at the bottom of the first page of each table, under the column heading

Beam
Length
(mm)

ii. for these two span lengths, determines which of the series of tables contains a factored load level closest to the specified factored load level, as given at the bottom of the first page under the column heading

FACT
LOAD
(kN/m)

iii. from the second and third pages of the correct table, determines which of the four connection plate thicknesses is preferred, based on any one of several different conditions. In any case the strength ratios given on these pages, corresponding to the bracketing span lengths, must exceed a value of 1.00, which indicates the intended margin of safety is provided. The strength ratios are found under the column headings

$$\frac{Vbolt}{Vf} \qquad \frac{Vweld}{Vf} \qquad \frac{Vplt.}{Vf} \qquad \frac{Vbeam}{Vf}$$

When selecting a connection for a span which is not close in length to either of the bracketing lengths, selection of a connection detail specified for the shorter of the two spans, which carries a larger total load, results in a detail designed for a shear force greater than required. The connection associated with the longer span, designed for a larger rotation, will definitely be deep enough to prevent bottom flange bearing from occurring. With connection strength ensured, the designer may wish to select the preferred connection detail based on one of the following items:

a. a preferred plate thickness,

b. minimum cost as given in the table under the column heading

Conn
Cost
($)

c. minimum end moment as listed in the table under the column heading

FACT
Mend
(kNm)

In some instances, for certain span lengths, all or some of the connection design information in the tables is replaced with a word or a four letter abbreviation. This occurs when a connection detail cannot be adequately designed for the factored load imposed upon it using the specified connection material and geometry; the displacement of the numeric data with the word serves as a warning to the designer to investigate other solutions. When any one of the connection component (e. g. the bolt group) shear strength ratios is less than 1.00 an abbreviated word identifying the minimum understrength component is printed in all columns of the connection design table except the shear strength columns. In some cases secondary forces exhaust completely the strength of one or more of the connection components in any one group (e. g. the bolts near the top of the connection), leaving in these locations of the connection no longitudinal shear strength. When this latter condition occurs all of the information in the design tables is overstruck by the abbreviation referring to the component in question. The word or abbreviation that can occur signifies the problem condition as follows:

Beam - supported beam web is understrength,

Bolt - components in the bolt group are understrength,

Cope - beam web at the cope is understrength,

Flng - bottom flange of the supported beam will likely bear on the support prior to attaining the factored level rotation,

Plat - end plate is understrength,

Weld - fillet weld is understrength,

These six words or abbreviations, and one other abbreviation, can also appear in the table under the column heading

Govern
Condit

In this column these words are intended to notify the designer which of seven conditions governs the connection design. The words and abbreviations have the following meanings:

Beam - beam web is the weakest connection component,

Bolt - bolt group is the weakest connection component, Cope - indicates that for a coped beam, the coped section is weaker than any of the connection components,

Flng - bottom flange of the beam will bear on the support prior to attaining the factored level rotation and therefore the connection is unacceptable,

Plat - end plate is the weakest connection component,

Rote - connection depth is not governed by component strength but rather selected to ensure that bottom flange bearing does not occur,

Weld - fillet weld group is the weakest connection component.

iv. once the preferred connection detail is selected, the designer records all of the pertinent design information for the connection detail according to the following symbolic system, replacing the symbols with the appropriate numbers from the table:

FEP(Fyp-Dh-g-p-Dend-Dedg-Dtop-tp-L) B(Fub-Db-#) W(Xu-f)

9.3.2. SELECTION OF A CONNECTION DETAIL WHEN ONLY THE END SHEAR IS KNOWN. The problem with this situation is that the effect of beam end rotation on the primary shear capacity of the connection is not known directly and must be estimated. (Of course when the supports are flexible the effect of beam end rotation on the connection shear strength is not significant and therefore this problem is of no concern.) This situation may arise in composite construction where it is not possible to arrive at the factored end rotation of the supported beam using CDP in its existing form. Once the designer has the set of design tables, e. g. Tables 8.2 to 8.4, that give the correct beam size and the correct material properties, the suggested approach for the designer to take is

i. assume that the supported beam is loaded by a uniform load to 90 or 100 percent of the maximum possible factored level,

ii. enter page two and page three of the appropriate design table and locate the factored shear force nearest the required value, under the column heading

FACT
SHEAR
(kN)

iii. select one of the four possible connection details, as described in 9.3.1.iii above, and

iv. record connection detail information as in 9.3.1.iv above.

This approach results in connection designs which consider the maximum rotation likely to occur because factored loads are selected which would cause relatively large end rotations. Conservatively large secondary forces are also considered.

9.3.3. DETERMINATION OF DEAD LOAD CAMBER FOR A BEAM WHEN THE LENGTH AND SERVICE LEVEL LOADING ARE KNOWN. The designer enters the design tables, e. g. Tables 8.2 to 8.4, that give the correct beam size, loading pattern, material properties, and then

i. determines the two beam lengths that bracket the specified beam length from page one of any one of the tables under the column heading

Beam
Length
(mm)

ii. for these two span lengths, determines the corresponding maximum span deflection for 100 percent dead loading, given at the bottom of the first page under the column with the heading

Max DL
defl
(mm)

which is the required dead load camber assuming that the entire load is dead and assuming that the supports are simple and offer no restraint to beam end rotation. Multiply this value by the ratio of the specified dead load to the 100 percent specified dead load, found on page one under the column heading

SPEC
100%DL
(kN/m)

to give the required dead load camber.

Since the dead load deflections used in the above procedure were developed assuming simple supports, ignoring the finite rotational restraint of the connection, the dead load camber calculated will be larger than necessary. An alternative to this is to repeat the procedure just outlined except use the maximum dead-load deflection values given on pages two and three of the tables, where the rotational stiffnesses of the connections are considered.

9.3.4. ESTIMATION OF SUPPORT MOMENTS DEVELOPED IN SIMPLE FRAMES. Knowing the loading and span lengths, engineers can investigate the range of support moments which could develop for various connection details. This information, as described previously, is available on pages two and three of the design tables. By evaluating the magnitude of end moments which may develop, engineers can ensure that supports are designed adequately.

9.3.5. EVALUATION OF THE DEFLECTION SERVICEABILITY OF BEAMS AND GIRDERS. The designer enters the design tables, e. g. Tables 8.2 to 8.4, that give the correct beam size, the correct loading pattern, and the correct material properties, and then

i. determines the two beam lengths that bracket the specified beam length given at the bottom of the first page of any one of the tables under the column heading

Beam
Length
(mm)

ii. determines the corresponding magnitude of the maximum span deflection for 100 percent service dead load, listed at the bottom of the first page under the column heading

Max DL
defl
(mm)

Multiply this value by the ratio of the given specified load to the 100 percent specified dead load, found on page one under the column heading

SPEC
100%DL
(kN/m)

to get the maximum service level deflection for the given loading.

iii. In order to assess the relative magnitude of the beam deflection, enter on page one of the table, the column under the heading

<u>Length</u>
DLdefl

which gives the ratio of the beam length to the 100 percent dead load deflection. As before, multiply this by the ratio of the service load to the 100 percent dead load listed in the table, to get the desired relative deflection.

As described under 9.3.3 if the supports are rigid there is likely to be some restraining effect by the connections and therefore service level deflections estimated in this way could be conservatively high.

9.3.6. EVALUATION OF THE END ROTATION SERVICEABILITY OF BEAMS AND

GIRDERS. The movement of the top surface of the beam away from the support may be of concern to designers when beams are relatively deep, or when the live load is relatively large. To evaluate this, once the end detail has been selected, the designer

i. determines the two beam lengths that bracket the specified beam length, as given on page two or page three of the appropriate table under the column heading

Beam
Length
(mm)

ii. determines in the second row of tabular information, the corresponding magnitude of the horizontal displacement at the top of the supported beam due to 100 percent service dead load. This is listed under the column heading

Disp Top
Beam DL
(mm)

Multiply this value by the ratio of the specified load to the 100 percent dead load, found on page one under the column heading

SPEC
100%DL
(kN/m)

to estimate the maximum service level movement at the top of the beam, adjacent to the support. (NOTE: Because the rotational stiffness of flexible end plate connections is non-linear, calculations, like this one, based on linear interpolation will not necessarily be exact.)

10. Summary and Conclusions

10.1 Summary and Conclusions

1. A two part study has been made in which a limit states design (LSD) model for flexible end plate connections has been developed and then used in the development of an interactive connection design program for design of standardized connections.

PART A

2. A review of literature on experimental research indicated that flexible end plate connections need only be designed to transfer shear. This was based on the premise that relatively small moments were developed in the connections and that the rotational characteristics were independent of the shear-to-moment ratio. However, no assessment of the ultimate shear strength of the connections was made.

3. Connection design handbooks from five different countries indicate that it is current general practice to design flexible end plate connections to transfer primary shear only and to disregard the secondary forces which develop in the connection when it rotates. However, the AustISC (1978) handbook includes some empirical rules which are intended to compensate for the secondary forces. Empirical rules are also used to ensure adequate flexibility of these connections. Evaluation of the maximum unrestrained rotation of the connection is attempted only in the Australian (AustISC 1978) and British (BCSA 1982) handbooks. However, the method used is known to be unconservative.

4. A complete design model, in order to ensure a consistent margin of safety, should evaluate

i. the primary shear strength of the connection in the presence of secondary forces,

ii. the end moment developed at the factored load level, and

iii. the maximum unrestrained rotation of the connection at which bottom flange bearing occurs.

5. With modifications to the load-deformation relationships originally developed by Hafez (1982) and by Kennedy and Hafez (1984), the analytical method of Hafez (1982) has been used to predict the moment-rotation relationship of 28 full-scale connections. The predictions correlate well with test results obtained prior to the occurrence of bottom flange bearing of the beam on the support.

6. The analytical prediction of the moment immediately prior to bottom flange bearing is very good, with a mean value of the test-to-predicted ratio of the 28 tests being 1.06 and the coefficient of variation being 0.103. However, the corresponding test-to-predicted ratio for connection rotation and the coefficient of variation when bottom flange bearing is imminent are not as good, and were calculated to be, respectively, 1.08 and 0.253.

7. While the analytical method was developed for uncoped beams with stiff supports, it will likely result in accurate or conservative overestimates of the connection moment at a given rotation when applied to other types of connection details.

8. When the support is not rigid, as is assumed in the analytical method developed, it is possible that bottom flange bearing could occur at smaller rotations than those predicted by the analysis.

9. To prevent the undesirable situation of premature bottom flange bearing from occurring, due to conditions discussed above in items 6 and 8, it is suggested that the unrestrained rotation limit predicted using the analysis be reduced by a factor of 2/3 for design purposes.

10. An analytical model based on experimental studies reported in the literature was developed to predict the magnitude and distribution of secondary forces in the bolts. The model also allows the prediction of end plate and bolt rupture due to excessive connection deformation. The Hafez (1982) analytical method, with modifications, is used to predict the secondary forces in the beam web, the fillet welds, and the end plate.

11. By applying lower bound plasticity theory, a series of resistance equations were developed for the prediction of the primary shear resistance of each of the connection components (the bolts, the end plate, the welds, and the beam web), taking into account any secondary forces acting in the connection. In some cases the resulting resistance equations are less conservative (but possibly more rational) than the current guidelines set out in the governing design standard, CAN3-S16.1-M84 (CSA 1984).

12. The flexible end plate connection, depending upon its size relative to the depth of the supported beam and upon whether or not the beam end is coped, may or may not provide significant torsional restraint. The three-part design model proposed herein, which does not

consider this point, is therefore recommended for use only when the supported beam has sufficient bracing along its length so that this condition is of no concern.

PART B

13. Existing connection design handbooks limit users to a limited number of geometric and material properties. One alternative is for fabricators to develop their own 'shop standards'.

14. Computer programs can be developed to allow the rapid design of standard connection details possessing any combination of geometric and material properties provided that the connection design models are general enough in nature to encompass a wide range of applications.

15. An interactive computer program for the design of standardized flexible end plate connections, based on the design model developed herein, is developed to provide one possible software solution.

16. The computer program designs connections at the factored load level. In some instances this may cause inelastic response of the supported beams. Ignoring this condition can result in inadequate connection details at the factored load level.

17. The computer program can be used in two modes. It can be used to design connections for individual beams with any span length and loaded with any combination of uniformly distributed load and up to six concentrated loads. It can also be used to develop connection design tables for single beams carrying either a uniformly distributed load or a mid span concentrated load (at any level between 1 and 100 percent of

the maximum factored level) for ten span lengths ranging from three times the beam depth to thirty times the beam depth.

18. The computer program is useful to both fabricators and consulting engineers and can be applied to the solution of several different design problems.

10.2 Areas of Further Research

Areas for suggested further study are:

PART A

1. An experimental program to verify the connection resistance equations proposed in this report as there have been no experimental studies conducted specifically for the evaluation of the ultimate shear capacity of flexible end plates. This would require that a series of connection details be designed to fail in the bolts, in the plate, in the welds, and in the supported beam web at factored load levels that could be attained in standard steel building frames.

2. An experimental and analytical investigation into the behaviour of skewed flexible end plate connection details. Such a study would determine whether or not the design model proposed herein is generally applicable or not, and would identify any unique behavioural characteristics of skewed connections.

3. An experimental program to verify or improve on the load-deformation models developed in Chapter 5 for use in determining the magnitude and distribution of secondary forces in the bolts.

4. An experimental and analytical investigation into the behaviour of flexible end plate connections subjected to dynamic loading or load reversals. Such a study woulddetermine the value of using this type of connection detail in seismic regions.

5. An analytical or statistical study investigating the minimum shear force for which shear type connections should be designed. It is not clear whether or not a minimum size connection detail should be specified when the factored shear force is relatively small, in order to account for torsional and other effects at the connection not normally considered.

PART B

6. Adjust CDP so that connection details framing back-to-back into a common support can be considered. This would require an evaluation of the geometric considerations and of the strength of such a detail.

7. Add a graphics capability to CDP so that connection details could be designed and detailed in one step.

8. Incorporate the connection design model from PART A into a frame design program, along with other connection design models, so that beam-girder-column framing systems could be designed and connections detailed simultaneously.

9. With the development of a method for predicting factored level connection rotations for composite beams, incorporate the design model developed in PART A, or a modification of it, into CDP for use in connection design for composite construction.

10. Modify CDP so that it can design connections for built-up members.

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Appendix A

Computer Program for the Analysis of the Moment in a Flexible End Plate Connection

```
C23456789012345678901234567890123456789012345678901234567890123456789
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     File name MOMENT
C
C     This program calculates the moment-rotation
C     relationship of a flexible end plate connection.
C     The method originally proposed by HAFEZ (1982) is
C     used with three sets of different load-deformation
C     relationships as follows: the HAFEZ models, the
C     KENNEDY-HAFEZ models, and the PROPOSED models.  The
C     output from the program compares the results of the
C     three analyses.
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     To use this program simply create a data file
C     conforming to the description below (as many end
C     plate details as you choose can be evaluated in one
C     run simply by entering three cards of data for each
C     unique detail; to stop the program execution a zero
C     entry must be placed on the last three cards
C     in the data file ).
C
C     After the data file is created use the following
C     commands while in MTS mode ,
C
C
C     RUN *FORTGTEST SCARDS=MOMENT SPRINT=-LIST
C    &PAR=ID,SOURCE
C
C     DEBUG -LOAD# 5=DATA 6=-OUTPUT T=2
C
C
C     If you copy -LOAD# to a permanent file then only one
C     command there after needs to be issued to obtain
C     results.  If -LOAD# was copied to permanent file
C     MOMENT.C then to execute the program you would issue
C     following command,
C
C
C     DEBUG MOMENT.C 5=DATA 6=-OUTPUT T=2
C
C
C     To retrieve the program analysis LIST -OUTPUT.
C     The analysis prints out in the form of a Table, one
C     Table to the page, with thesis margins if -OUTPUT is
C     copied to *PRINT* using FORMAT=FMTP2.
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     DEFINITION OF INPUT VARIABLES :
C
```

```
C     Card # 1,
C
C     TITLE  = a title of up to 40 characters in length
C              can be used to describe the specimen
C
C     Card # 2,
C
C     G      = bolt hole gage
C     P      = bolt hole pitch for full size connection
C              specimens and the plate width for tension
C              specimens
C     E      = the edge distance of end plate measured from
C              the center line of bolt hole to nearest edge
C              of plate
C     T      = end plate thickness
C     F      = leg size of fillet weld joining the end
C              plate to supported beam web
C     W      = thickness of the supported beam web
C     DEPTHP = the total depth of the end plate
C     DEPTHB = the total depth of the supported member
C     TOPDIS = the distance from the top of the supported
C              member to the top of the end plate
C     DH     = bolt hole diameter
C     DB     = nominal bolt diameter
C
C     Card # 3,
C
C     FYP    = tensile yield strength of the end plate
C     FUP    = tensile ultimate strength of the end plate
C     EP     = Young's Modulus of elasticity of the end
C              plate
C     FUB    = tensile ultimate strength of the bolt
C     FY     = tensile yield strength of the supported
C              beam
C     FU     = tensile ultimate strength of the supported
C              beam
C
C     UNITS  = 1 for IMPERIAL, 2 for SI (when IMPERIAL
C              units are used all dimensions must be in
C              inches and all material strengths in ksi,
C              for SI units, dimensions must be in mm
C              and material strengths in MPa)
C
C     TABLE  = insert the table number or identification,
C              up to 4 characters in each of the two
C              variables, an example table heading is:
C
C                                  TABLE 5.12
C
C              to obtain this you would input  5.12  in
C              variable TABLE(1) and leave variable
C              TABLE(2) blank.
C
C     NTYPE  = set equal to 1 if tabular output is desired
```

```
C              and set equal to 2 if plot compatible
C              output is desired.
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      REAL MOMEN1,MOMEN2,MOMEN3,MUP,NULT
C
      DIMENSION CARM1(200),CARM2(200),CARM3(200),CF1(200),
     &          CF2(200),CF3(200),DELTC1(200),DELTC2(200),
     &          DELTC3(200),DELTT1(200),DELTT2(200),
     &          DELTT3(200),ENDM1(20),ENDM2(20),ENDM3(20),
     &          TABLE(5),TITLE(20),TARM1(200),TARM2(200),
     &          TARM3(200),TEN1(200),TEN2(200),TEN3(200)
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C*****READ STATEMENTS:
C
C
    1 CONTINUE
      READ(5,10)TITLE(1),TITLE(2),TITLE(3),TITLE(4),
     &          TITLE(5),TITLE(6),TITLE(7),TITLE(8),
     &          TITLE(9),TITLE(10)
   10 FORMAT(10A4)
      READ(5,20)G,P,E,T,F,W,DEPTHP,DEPTHB,TOPDIS,DH,DB,
     &          FYP,FUP,EP,FUB,FY,FU,UNITS,TABLE(1),
     &          TABLE(2),NTYPE
   20 FORMAT(11F7.2,/,7F9.2,2A4,I1)
      IF(G.LE.0)STOP
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C*****CONVERSION OF SI DIMENSIONS TO IMPERIAL:
C
C
      IF(UNITS.GE.2)CALL CONV1(DB,DEPTHB,DEPTHP,DH,E,EP,
     &              F,FY,FYP,FU,FUB,FUP,G,P,T,TOPDIS,W    )
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C*****CALCULATION OF CONSTANTS:
C
C
      PI = 3.1416
      CLEARL = (G - W - DH - 2*F)/2.
C
      DELTAE = (CLEARL**2*FUP)/(2.*EP*T)
      ALPHAE = ATAN2(DELTAE,CLEARL)
C
      VULT = (0.60*0.70*PI*(DB**2/4)*FUB)/P
      HMAX = VULT
      NULT = (P - DH)*T*FUP/P
```

```
      IF(NULT.LT.HMAX)HMAX = NULT
      BULT = E*T*FUP/P
      IF(E.GT.(3.*DB))BULT = 3.*DB*T*FUP/P
      IF(BULT.LT.HMAX)HMAX = BULT
C
      MUP = 0.25*(T**2)*FUP
      PYP = T*FYP
C
      DELTAP = 4.*HMAX*MUP/(PYP**2)
      ALPHAP = ATAN2(DELTAP,CLEARL)
C
      BOTTOM = DEPTHB - DEPTHP - TOPDIS
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C*****CALCULATION OF LIMITING ROTATION & MOMENT OF
C     CONNECTION:
C
C
      PART = DEPTHP/100
      I = 0
 2050 I = I + 1
C
      IF(I.GE.100)CODE = 10
      IF(I.GE.100)CALL ERROR(CODE)
C
      TLENGT = PART*(100 - I)
      CLENGT = PART*I
      ADJACE = CLENGT + BOTTOM
      THETA = ATAN2(T,ADJACE)
C
      IF(I.GT.1.AND.CFORC1.GT.TFORC1)GO TO 2010
      IMAX1 = I
      THETA1 = THETA
      CALL COMP1(CARM1,CF1,CFORC1,CLENGT,DELTC1,FU,I,
     &           PART,THETA,W)
C
      CALL TENS1(CLEARL,DELTAE,DELTAP,DELTT1,EP,HMAX,
     &        I,MUP,PYP,PART,T,TARM1,TEN1,TFORC1,THETA)
C
 2010 IF(I.GT.1.AND.CFORC2.GT.TFORC2)GO TO 2020
      IMAX2 = I
      THETA2 = THETA
      CALL COMP2(CARM2,CF2,CFORC2,CLENGT,DELTC2,FU,I,
     &           PART,THETA,W)
      CALL TENS2(CLEARL,DELTAE,DELTAP,DELTT2,DH,EP,HMAX,
     &        I,MUP,P,PYP,PART,T,TARM2,TEN2,TFORC2,THETA)
C
 2020 IF(I.GT.1.AND.CFORC3.GT.TFORC3)GO TO 2030
      IMAX3 = I
      THETA3 = THETA
      CALL COMP2(CARM3,CF3,CFORC3,CLENGT,DELTC3,FU,I,
     &           PART,THETA,W)
C
```

```
      CALL TENS3(CLEARL,DELTAE,DELTAP,DELTT3,EP,HMAX,I,MUP,
     &                  PYP,PART,T,TARM3,TEN3,TFORC3,THETA )
C
      GO TO 2050
C
 2030 IF(CFORC1.GT.TFORC1.AND.CFORC2.GT.TFORC2.AND.
     &CFORC3.GT.TFORC3)GO TO 2060
      GO TO 2050
C
 2060 CALL MOMENT(CARM1,CF1,IMAX1,PART,TARM1,TEN1,TOTALM)
      MOMEN1 = TOTALM
C
      CALL MOMENT(CARM2,CF2,IMAX2,PART,TARM2,TEN2,TOTALM)
      MOMEN2 = TOTALM
C
      CALL MOMENT(CARM3,CF3,IMAX3,PART,TARM3,TEN3,TOTALM)
      MOMEN3 = TOTALM
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C*****CALCULATION OF MOMENT ROTATION RELATIONSHIP:
C
C
      DO 3000 N = 1,10
      IF(N.LE.5)THETA = 0.001*N
      IF(N.GT.5)THETA = 0.005 + (N - 5)*0.005
C     THETA = 0.0025*N
      I = 0
 3050 I = I + 1
C
      IF(I.GE.100)CODE = 20
      IF(I.GE.100)CALL ERROR(CODE)
C
      TLENGT = PART*(100 - I)
      CLENGT = PART*I
C
      IF(THETA.GT.THETA1)STOP1 = N
      IF(THETA.GT.THETA1)GO TO 3010
C
      IF(I.GT.1.AND.CFORC1.GT.TFORC1)GO TO 3010
      IMAX1 = I
      CALL COMP1(CARM1,CF1,CFORC1,CLENGT,DELTC1,FU,I,
     &            PART,THETA,W)
      CALL TENS1(CLEARL,DELTAE,DELTAP,DELTT1,EP,HMAX,
     &           I,MUP,PYP,PART,T,TARM1,TEN1,TFORC1,THETA)
C
 3010 IF(THETA.GT.THETA2)STOP2 = N
      IF(THETA.GT.THETA2)GO TO 3020
      IF(I.GT.1.AND.CFORC2.GT.TFORC2)GO TO 3020
      IMAX2 = I
      CALL COMP2(CARM2,CF2,CFORC2,CLENGT,DELTC2,FU,
     &            I,PART,THETA,W)
      CALL TENS2(CLEARL,DELTAE,DELTAP,DELTT2,DH,EP,
     &  HMAX,I,MUP,P,PYP,PART,T,TARM2,TEN2,TFORC2,THETA)
```

```
C
 3020 IF(THETA.GT.THETA3)STOP3 = N
      IF(THETA.GT.THETA3)GO TO 3030
      IF(I.GT.1.AND.CFORC3.GT.TFORC3)GO TO 3030
      IMAX3 = I
      CALL COMP2(CARM3,CF3,CFORC3,CLENGT,DELTC3,FU,
     &            I,PART,THETA,W)
C
      CALL TENS3(CLEARL,DELTAE,DELTAP,DELTT3,EP,HMAX,
     &         I,MUP,PYP,PART,T,TARM3,TEN3,TFORC3,THETA)
C
      GO TO 3050
C
 3030 IF(THETA.GT.THETA1.OR.CFORC1.GT.TFORC1)GO TO 3061
      GO TO 3050
 3061 IF(THETA.GT.THETA2.OR.CFORC2.GT.TFORC2)GO TO 3062
      GO TO 3050
 3062 IF(THETA.GT.THETA3.OR.CFORC3.GT.TFORC3)GO TO 3060
      GO TO 3050
C
 3060 IF(THETA.GT.THETA1)ENDM1(N) = 0.0
      IF(THETA.GT.THETA1)GO TO 3070
      CALL MOMENT(CARM1,CF1,IMAX1,PART,TARM1,TEN1,TOTALM)
      ENDM1(N) = TOTALM
C
 3070 IF(THETA.GT.THETA2)ENDM2(N) = 0.0
      IF(THETA.GT.THETA2)GO TO 3080
      CALL MOMENT(CARM2,CF2,IMAX2,PART,TARM2,TEN2,TOTALM)
      ENDM2(N) = TOTALM
C
 3080 IF(THETA.GT.THETA3)ENDM3(N) = 0.0
      IF(THETA.GT.THETA3)GO TO 3090
      CALL MOMENT(CARM3,CF3,IMAX3,PART,TARM3,TEN3,TOTALM)
      ENDM3(N) = TOTALM
C
 3090 CONTINUE
C
 3000 CONTINUE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C*****PRESENTATION OF OUTPUT:
C
C
      IF(UNITS.GE.2)
     &CALL CONV2(DB,DEPTHB,DEPTHP,DH,E,ENDM1,ENDM2,
     &            ENDM3,EP,F,FY,FYP,FU,FUB,FUP,G,MOMEN1,
     &            MOMEN2,MOMEN3,P,T,TOPDIS,W   )
C
      IF(NTYPE.EQ.2)GO TO 13
      CALL WRITE(DB,DEPTHB,DEPTHP,DH,E,ENDM1,ENDM2,ENDM3,
     &     EP,F,FY,FYP,FU,FUB,FUP,G,MOMEN1,MOMEN2,MOMEN3,
     &     P,T,TABLE,THETA1,THETA2,THETA3,TITLE,TOPDIS,
     &     W)
```

```
      GO TO 14
   13 CONTINUE
      CALL WRITE2(DB,DEPTHB,DEPTHP,DH,E,ENDM1,ENDM2,ENDM3,
     &     EP,F,FY,FYP,FU,FUB,FUP,G,MOMEN1,MOMEN2,MOMEN3,
     &     P,T,TABLE,THETA1,THETA2,THETA3,TITLE,TOPDIS,
     &     W)
   14 CONTINUE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C*****RETURN TO EVALUATE ANOTHER END PLATE OR TO STOP
C     EXECUTION
C
C
      GO TO 1
      END
C
C***********************************************************
C***********************************************************
C
C                     SUBROUTINE LISTINGS
C
C***********************************************************
C***********************************************************
C
C     SUBROUTINE CONV1:
C
C     Used to convert SI data to IMPERIAL data.
C
C
      SUBROUTINE CONV1(DB,DEPTHB,DEPTHP,DH,E,EP,F,FY,FYP,
     &                 FU,FUB,FUP,G,P,T,TOPDIS,W  )
C
      DB     = DB/25.4
      DEPTHB = DEPTHB/25.4
      DEPTHP = DEPTHP/25.4
      DH     = DH/25.4
      E      = E/25.4
      EP     = EP/6.895
      F      = F/25.4
      FY     = FY/6.895
      FYP    = FYP/6.895
      FU     = FU/6.895
      FUB    = FUB/6.895
      FUP    = FUP/6.895
      G      = G/25.4
      P      = P/25.4
      T      = T/25.4
      TOPDIS = TOPDIS/25.4
      W      = W/25.4
C
      RETURN
      END
C
```

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     SUBROUTINE CONV2:
C
C     Used to convert IMPERIAL data to SI data.
C
C
      SUBROUTINE CONV2(DB,DEPTHB,DEPTHP,DH,E,ENDM1,
     &                 ENDM2,ENDM3,EP,F,FY,FYP,FU,FUB,
     &         FUP,G,MOMEN1,MOMEN2,MOMEN3,P,T,TOPDIS,W)
C
      REAL MOMEN1,MOMEN2,MOMEN3
C
      DIMENSION ENDM1(20),ENDM2(20),ENDM3(20)
C
      DB     = DB*25.4
      DEPTHB = DEPTHB*25.4
      DEPTHP = DEPTHP*25.4
      DH     = DH*25.4
      E      = E*25.4
      EP     = EP*6.895
      F      = F*25.4
      FY     = FY*6.895
      FYP    = FYP*6.895
      FU     = FU*6.895
      FUB    = FUB*6.895
      FUP    = FUP*6.895
      G      = G*25.4
      P      = P*25.4
      T      = T*25.4
      TOPDIS = TOPDIS*25.4
      W      = W*25.4
C
      MOMEN1 = MOMEN1*0.1130
      MOMEN2 = MOMEN2*0.1130
      MOMEN3 = MOMEN3*0.1130
C
      DO 1000 N = 1,10
      ENDM1(N)  = ENDM1(N)*0.1130
      ENDM2(N)  = ENDM2(N)*0.1130
      ENDM3(N)  = ENDM3(N)*0.1130
 1000 CONTINUE
C
      RETURN
      END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     SUBROUTINE COMP1:
C
C     Calculates the compressive force distribution in a
C     flexible end plate connection using the PROPOSED
```

```
C     compression model.
C
C
      SUBROUTINE COMP1(CARM1,CF1,CFORC1,CLENGT,DELTC1,
     &                 FU,I,PART,THETA,W)
C
      DIMENSION CARM1(200),CF1(200),DELTC1(200),SIGMAC(200)
C
      CFORC1 = 0
      DO 1000 N = 1,100
      IF(N.GT.I)GO TO 1100
      DELTC1(N) = N*PART*TAN(THETA)
      CARM1(N)  = N*PART
      SIGMAC(N) = 2000.*(FU/64.1)*DELTC1(N)
      IF(SIGMAC(N).GT.((FU/64.1)*24.))
     &SIGMAC(N) = (FU/64.1)*(- 21.2 + 45.4*DELTC1(N)
     &                        - 241.0*DELTC1(N)**0.5
     &                        + 310.0*DELTC1(N)**0.333 )
C
      CF1(N) = SIGMAC(N)*PART*W
      CFORC1 = CFORC1 + CF1(N)
      IF(N.EQ.I)CFORC1 = CFORC1 + (1- 0.5*PART)*SIGMAC(N)*W
      GO TO 1000
C
 1100 CONTINUE
      CF1(N) = 0.0
      CARM1(N) = 0.0
C
 1000 CONTINUE
C
      RETURN
      END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     SUBROUTINE COMP2:
C
C     Calculates the compressive force distribution in a
C     flexible end plate connection using the HAFEZ
C     compression model.
C
C
      SUBROUTINE COMP2(CARM2,CF2,CFORC2,CLENGT,DELTC2,
     &                 FU,I,PART,THETA,W)
C
      DIMENSION CARM2(200),CF2(200),DELTC2(200),SIGMAC(200)
C
      CFORC2 = 0
      DO 1000 N = 1,100
      IF(N.GT.I)GO TO 1100
      DELTC2(N) = N*PART*TAN(THETA)
      CARM2(N)  = N*PART
      SIGMAC(N) = 2000.*DELTC2(N)
```

```
       IF(SIGMAC(N).GT.24.)
      &SIGMAC(N) = - 21.2 + 45.4*DELTC2(N)
      &              - 241.0*DELTC2(N)**0.5
      &              + 310.0*DELTC2(N)**0.333
C
       CF2(N) = SIGMAC(N)*PART*W
       CFORC2 = CFORC2 + CF2(N)
       IF(N.EQ.I)CFORC2 = CFORC2 + (1- 0.5*PART)*SIGMAC(N)*W
       GO TO 1000
C
 1100 CONTINUE
       CF2(N) = 0.0
       CARM2(N) = 0.0
C
 1000 CONTINUE
C
       RETURN
       END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C      SUBROUTINE ERROR:
C
C      Prints out an error message in the output file if a
C      logic error is encountered within the program
C      execution at certain locations.
C
C      CODE = 10 = iteration problem encountered when
C                  calculating the limiting rotation and
C                  moment for a given connection
C
C      CODE = 20 = iteration problem encountered when
C                  calculating the moment-rotation
C                  relationship of a given connection
C
C
       SUBROUTINE ERROR(CODE)
C
       WRITE(6,100)CODE
  100  FORMAT('0',/////,T25,
      &       'LOGIC ERROR, CODE # ',F3.0,' !!!!',////)
C
       STOP
       END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C      SUBROUTINE TENS1:
C
C      Calculates the tensile force distribution in a
C      flexible end plate connection using the PROPOSED
C      model.
```

```
C
C
      SUBROUTINE TENS1(CLEARL,DELTAE,DELTAP,DELTT1,EP,
     &      HMAX,I,MUP,PYP,PART,T,TARM1,TEN1,TFORC1,THETA)
C
      REAL MUP
C
      DIMENSION DELTT1(200),TARM1(200),TEN1(200)
C
      J = 100 - I
      TFORC1 = 0
      DO 1000 N = 1,100
C
      IF(N.GT.J)GO TO 1100
      DELTT1(N) = N*PART*TAN(THETA)
      TARM1(N)  = N*PART
      IF(DELTT1(N).LE.DELTAE)TEN1(N) = 2.*EP*PART*T**3*
     &                                  DELTT1(N)/CLEARL**3
      IF(DELTT1(N).GT.DELTAE.AND.DELTT1(N).LE.DELTAP)
     &TEN1(N) = (PYP**2*DELTT1(N)**2/(4.*MUP*CLEARL)
     &    + 4.*MUP*CLEARL/(CLEARL**2 + DELTT1(N)**2))*PART
      IF(DELTT1(N).GT.DELTAP)TEN1(N) = (2.*HMAX*DELTT1(N)/
     &CLEARL  + 4.*MUP*CLEARL/(CLEARL**2 + DELTT1(N)**2)
     &        - 4.*MUP*HMAX**2/(CLEARL*PYP**2))*PART
C
      TFORC1 = TEN1(N) + TFORC1
      GO TO 1000
 1100 CONTINUE
      TARM1(N) = 0.0
      TEN1(N) = 0.0
 1000 CONTINUE
C
      RETURN
      END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     SUBROUTINE TENS2:
C
C     Calculates the tensile force distribution in a
C     flexible end plate connection using the
C     KENNEDY-HAFEZ model.
C
C
      SUBROUTINE TENS2(CLEARL,DELTAE,DELTAP,DELTT2,DH,EP,
     &   HMAX,I,MUP,P,PYP,PART,T,TARM2,TEN2,TFORC2,THETA)
C
      REAL MUP
C
      DIMENSION DELTT2(200),TARM2(200),TEN2(200)
C
      J = 100 - I
      TFORC2 = 0
```

```
      DO 1000 N = 1,100
      IF(N.GT.J)GO TO 1100
C
      DELTT2(N) = N*PART*TAN(THETA)
      TARM2(N)  = N*PART
      IF(DELTT2(N).LE.DELTAE)TEN2(N) = 2.*EP*PART*T**3*
     &                          DELTT2(N)/  CLEARL**3
      IF(DELTT2(N).GT.DELTAE)
     &TEN2(N) = (PYP**2*DELTT2(N)**2*((P - DH)/P)**2/
     &  (4.*MUP*CLEARL)  +  4.*MUP*CLEARL/(CLEARL**2 +
     &  DELTT2(N)**2))*PART
C
      TFORC2 = TEN2(N) + TFORC2
      GO TO 1000
 1100 CONTINUE
      TARM2(N) = 0.0
      TEN2(N) = 0.0
 1000 CONTINUE
C
      RETURN
      END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     SUBROUTINE TENS3:
C
C     Calculates the tensile force distribution in a
C     flexible end plate connection using the HAFEZ model.
C
C
      SUBROUTINE TENS3(CLEARL,DELTAE,DELTAP,DELTT3,EP,
     & HMAX,I,MUP, PYP,PART,T,TARM3,TEN3,TFORC3,THETA)
C
      REAL MUP
C
      DIMENSION DELTT3(200),TARM3(200),TEN3(200)
C
      J = 100 - I
      TFORC3 = 0
      DO 1000 N = 1,100
      IF(N.GT.J)GO TO 1100
C
      DELTT3(N) = N*PART*TAN(THETA)
      TARM3(N)  = N*PART
      IF(DELTT3(N).LE.DELTAE)TEN3(N) = 2.*EP*PART*T**3*
     &                      DELTT3(N)/CLEARL**3
      IF(DELTT3(N).GT.DELTAE)
     &TEN3(N) = (PYP**2*DELTT3(N)**2/(4.*MUP*CLEARL)
     & + 4.*MUP*CLEARL/(CLEARL**2 + DELTT3(N)**2))*PART
C
      TFORC3 = TEN3(N) + TFORC3
      GO TO 1000
 1100 CONTINUE
```

```
      TARM3(N) = 0.0
      TEN3(N)  = 0.0
C
 1000 CONTINUE
C
C
      RETURN
      END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     SUBROUTINE MOMENT:
C
C     Calculates the moment at the flexible end plate for a
C     given distribution of compression and tension forces.
C
C
      SUBROUTINE MOMENT(CARM1,CF1,IMAX1,PART,TARM1,
     &                  TEN1,TOTALM)
C
      DIMENSION CARM1(200),CF1(200),CM1(200),
     &         TARM1(200),TEN1(200),TM1(200)
C
      CMOM1 = 0.0
C
      DO 1000 N = 1,IMAX1
      IF(IMAX1.EQ.1)CMOM1 = 0.25*CARM1(N)*(0.5*0.5*CF1(N))
     &                 + 0.75*CARM1(N)*(0.5*CF1(N))
     &                 + (0.5 + CARM1(N))*(1/PART)*CF1(N)
      IF(IMAX1.EQ.1)GO TO 1001
C
      IF(N.LT.IMAX1)CM1(N) = CARM1(N)*CF1(N)
      IF(N.EQ.IMAX1)CM1(N) = (CARM1(N-1) + PART*0.75)
     &   *(CF1(N)*0.5)  + (CARM1(N) + 0.5)*(1/PART)*CF1(N)
      CMOM1 = CMOM1 + CM1(N)
 1001 CONTINUE
C
 1000 CONTINUE
C
      TMOM1 = 0.0
      JMAX1 = 100 - IMAX1
      DO 2000 N = 1,JMAX1
      IF(N.LT.JMAX1)TM1(N) = TARM1(N)*TEN1(N)
      IF(N.EQ.JMAX1)TM1(N) = (TARM1(N-1) + PART*0.5)*
     &                       TEN1(N)*0.5
      TMOM1 = TMOM1 + TM1(N)
 2000 CONTINUE
C
      TOTALM = TMOM1 + CMOM1
      RETURN
      END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
```

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     SUBROUTINE WRITE:
C
C     When the TABLE option for output is selected this
C     routine is called to generate the output.
C
C
      SUBROUTINE WRITE(DB,DEPTHB,DEPTHP,DH,E,ENDM1,
     &            ENDM2,ENDM3,EP,F,FY,FYP,FU,FUB,FUP,G,
     &            MOMEN1,MOMEN2,MOMEN3, P,T,TABLE,THETA1,
     &            THETA2,THETA3,TITLE,TOPDIS,W)
C
      REAL MOMEN1,MOMEN2,MOMEN3
C
      DIMENSION ENDM1(20),ENDM2(20),ENDM3(20),
     &          TABLE(5),TITLE(20)
C
      WRITE(6,100)TABLE(1),TABLE(2),TITLE(1),TITLE(2),
     &            TITLE(3),TITLE(4),TITLE(5),TITLE(6),
     &            TITLE(7),TITLE(8),TITLE(9),TITLE(10)
  100 FORMAT('1',////,T42,'TABLE ',2A4,//,
     &T19,'COMPARISON OF ANALYTICAL PREDICTIONS OF',
     &' MOMENT-ROTATION',/,
     &T19,'RELATIONSHIPS FOR A FLEXIBLE END PLATE ',
     &'CONNECTION    ',/,//,
     &T19,'CONNECTION DESCRIPTION :',/,'+',T19,22(''),
     &//,T19,10A4)
C
      IF(UNITS.GE.2)GO TO 5000
C
      WRITE(6,200)G,P,E,T,F,DEPTHP,TOPDIS,DEPTHB,W,DH,
     &            DB,FYP,FUP,EP,FUB,FY,FU
  200 FORMAT(' ',/,T19,'Bolt Hole Gage,          G = ',
     &F7.1,' inches',
     &/,T19,'Bolt Hole Pitch,          P = ',F7.1,' inches',
     &/,T19,'Bolt Hole Edge Dist.,     E = ',F7.1,' inches',
     &/,T19,'End Plate Thickness,      T = ',F7.2,' inches',
     &/,T19,'Fillet Weld Leg Size,     F = ',F7.2,' inches',
     &/,T19,'End Plate Depth,     DEPTHP = ',F7.1,' inches',
     &/,T19,'Top Distance,        TOPDIS = ',F7.1,' inches',
     &/,T19,'Beam Depth,          DEPTHB = ',F7.1,' inches',
     &/,T19,'Beam Web Thickness,       W = ',F7.2,' inches',
     &/,T19,'Bolt Hole Diameter,      DH = ',F7.2,' inches',
     &/,T19,'Bolt Diameter,           DB = ',F7.2,' inches',
     &/,T19,'End Plate Yield Str., FYP = ',F7.1,' ksi',
     &/,T19,'End Plate Ult. Str.,  FUP = ',F7.1,' ksi',
     &/,T19,'End Plate Mod. Elast., EP = ',F7.1,' ksi',
     &/,T19,'Bolt Ult. Str.,       FUB = ',F7.1,' ksi',
     &/,T19,'Beam Yield Str.,       FY = ',F7.1,' ksi',
     &/,T19,'Beam Ult. Str.,        FU = ',F7.1,' ksi')
C
      GO TO 6000
C
```

```
 5000 CONTINUE
C
      WRITE(6,250)G,P,E,T,F,DEPTHP,TOPDIS,DEPTHB,W,DH,
     &            DB,FYP,FUP,EP,FUB,FY,FU
  250 FORMAT(' ',/,T19,'Bolt Hole Gage,          G = ',
     &F7.1,' mm',
     &/,T19,'Bolt Hole Pitch,          P = ',F7.1,' mm',
     &/,T19,'Bolt Hole Edge Dist.,     E = ',F7.1,' mm',
     &/,T19,'End Plate Thickness,      T = ',F7.2,' mm',
     &/,T19,'Fillet Weld Leg Size,     F = ',F7.2,' mm',
     &/,T19,'End Plate Depth,     DEPTHP = ',F7.1,' mm',
     &/,T19,'Top Distance,        TOPDIS = ',F7.1,' mm',
     &/,T19,'Beam Depth,          DEPTHB = ',F7.1,' mm',
     &/,T19,'Beam Web Thickness,       W = ',F7.2,' mm',
     &/,T19,'Bolt Hole Diameter,      DH = ',F7.2,' mm',
     &/,T19,'Bolt Diameter,           DB = ',F7.2,' mm',
     &/,T19,'End Plate Yield Str., FYP = ',F7.1,' MPa',
     &/,T19,'End Plate Ult. Str.,  FUP = ',F7.1,' MPa',
     &/,T19,'End Plate Mod. Elast., EP = ',F7.1,' MPa',
     &/,T19,'Bolt Ult. Str.,       FUB = ',F7.1,' MPa',
     &/,T19,'Beam Yield Str.,       FY = ',F7.1,' MPa',
     &/,T19,'Beam Ult. Str.,        FU = ',F7.1,' MPa')
C
 6000 CONTINUE
C
      IF(UNITS.GE.2)GO TO 5500
C
      WRITE(6,300)THETA1,MOMEN1,THETA2,MOMEN2,
     &            THETA3,MOMEN3
  300 FORMAT(' ',/,T19,'MOMENT-ROTATION RELATIONSHIP :',
     & /,'+',T19,28(''),
     &//,T19,'PROPOSED ROT. LIMIT,          THETA1 = ',
     & F7.4,' radians',/
     &  T19,'PROPOSED MOM. LIMIT,         MOMENT1 = ',
     & F7.2,' inchkips',/
     &  T19,'KENNEDY-HAFEZ ROT. LIMIT,     THETA2 = ',
     & F7.4,' radians',/
     &  T19,'KENNEDY-HAFEZ MOM. LIMIT,   MOMENT2 = ',
     & F7.2,' inchkips',/
     &  T19,'HAFEZ ROT. LIMIT,             THETA3 = ',
     & F7.4,' radians',/
     &  T19,'HAFEZ ROT. LIMIT,           MOMENT3 = ',
     &F7.2,' inchkips',// )
      WRITE(6,301)
  301 FORMAT(' ',
     & T19,'     CONNECTION     PROPOSED     KENNEDY',
     &'      HAFEZ  ',/,
     & T19,' #     ROTATION       MOMENT       HAFEZ ',
     &'      MOMENT ',/,
     & T19,'                                   MOMENT ',
     & '              ',/,
     & T19,'       (radians)    (inchkips)   (inchkips)',
     &'   (inchkips)',/)
C
```

```
      GO TO 6500
C
 5500 CONTINUE
C
      WRITE(6,350)THETA1,MOMEN1,THETA2,MOMEN2,
     &              THETA3,MOMEN3
  350 FORMAT(' ',/,T19,'MOMENT-ROTATION RELATIONSHIP :'
     & ,/,'+',T19,28(''),
     &//,T19,'PROPOSED ROT. LIMIT,        THETA1 = ',
     &F7.4,' radians',/
     &   T19,'PROPOSED ROT. LIMIT,       MOMENT1 = ',
     &F7.2,' kNm  ',/
     &   T19,'KENNEDY-HAFEZ ROT. LIMIT,   THETA2 = ',
     &F7.4,' radians',/
     &   T19,'KENNEDY-HAFEZ MOM. LIMIT,  MOMENT2 = ',
     &F7.2,' kNm',/
     &   T19,'HAFEZ ROT. LIMIT,           THETA3 = ',
     &F7.4,' radians',/
     &   T19,'HAFEZ MOM. LIMIT,          MOMENT3 = ',
     &F7.2,' kNm',//)
      WRITE(6,351)
  351 FORMAT(' ',
     &  T19,'       CONNECTION      PROPOSED      KENNEDY',
     &'       HAFEZ   ',/,
     &  T19,' #      ROTATION        MOMENT        HAFEZ ',
     &'       MOMENT ',/,
     &  T19,'                                      MOMENT',
     &'              ',/,
     & T19,'        (radians)       (kNm)         (kNm)   ',
     &'     (kNm)   ',/)
C
 6500 CONTINUE
C
      DO 1000 I = 1,10
      IF(I.LE.5)THETA = 0.001*I
      IF(I.GT.5)THETA = 0.005 + (I - 5)*0.005
C     THETA = I*0.0025
C
      WRITE(6,400)I,THETA,ENDM1(I),ENDM2(I),ENDM3(I)
  400 FORMAT(' ',T19,I2,6X,F6.4,7X,F7.2,6X,F7.2,6X,F7.2)
C
 1000 CONTINUE
C
      RETURN
      END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     SUBROUTINE WRITE2:
C
C     This output routine prints into a file the
C     rotation-moment data in a form that is compatible
C     for a PLOTTING program.
```

```
C
C
      SUBROUTINE WRITE2(DB,DEPTHB,DEPTHP,DH,E,ENDM1,ENDM2,
     & ENDM3,EP,F,FY,FYP,FU,FUB,FUP,G,MOMEN1,MOMEN2,
     & MOMEN3,P,T,TABLE,THETA1,THETA2,THETA3,
     & TITLE,TOPDIS,W)
C
      REAL MOMEN1,MOMEN2,MOMEN3
C
      DIMENSION ENDM1(20),ENDM2(20),ENDM3(20),TABLE(5),
     &          TITLE(20),ENDMO(20)
C
      XX = 0.
      YY = 0.
      DO 1500 N = 1,3
      WRITE(6,450)TITLE(1),TITLE(2),TITLE(3),TITLE(4),
     &            TITLE(5),TITLE(6),TITLE(7),TITLE(8),
     &            TITLE(9),TITLE(10),N
  450 FORMAT(10A4,I1)
      WRITE(6,400)XX,YY
      DO 1000 I = 1,10
      IF(I.LE.5)THETA = 0.001*I
      IF(I.GT.5)THETA = 0.005 + (I - 5)*0.005
C     THETA = I*0.0025
C
      IF(N.EQ.1)ENDMO(I) = ENDM1(I)
      IF(N.EQ.2)ENDMO(I) = ENDM2(I)
      IF(N.EQ.3)ENDMO(I) = ENDM3(I)
      WRITE(6,400)THETA,ENDMO(I)
  400 FORMAT(' ',F7.5,F7.2)
C
 1000 CONTINUE
      IF(N.EQ.1)WRITE(6,400)THETA1,MOMEN1
      IF(N.EQ.2)WRITE(6,400)THETA2,MOMEN2
      IF(N.EQ.3)WRITE(6,400)THETA3,MOMEN3
 1500 CONTINUE
C
      RETURN
      END
```

Appendix B

Connection Design Program CDP

The listing of this program is lengthy and is therefore not reproduced here. A copy of it is on record at The Department of Civil Engineering at The University of Alberta, Edmonton, Alberta, Canada.

Appendix C

Design Example Using Standardized Connection Design Tables 8.2 to 8.4

Example - SELECTION OF A FLEXIBLE END PLATE CONNECTION WHEN THE SUPPORTED BEAM LENGTH AND FACTORED LOAD ARE KNOWN.

Given:

W610x101 beam, 10000 mm long.

Factored load, 55 kN/m.

Solution:

i. From the bottom of page 1 of Table 8.2 the beam lengths which bracket the given length are

9045 mm & 10854 mm

ii. Calculating the maximum factored moment for the case at hand

$$M_f = w\,\ell^2/8$$
$$= 55 \times (10)^2/8$$
$$= 687.5 \text{ kNm}$$

and dividing this value by the factored moment resistance of a W610x101, found at the top of page 1 of Table 8.2 gives the loading ratio to be

$$M_f/M_r = 687.5/783.0 = .878$$

therfore it is likely that Table 8.3 is appropriate for use here with a uniformly distributed load set at 90 percent of ultimate. From the bottom of page 1 of Table 8.3 the factored loads associated with the two bracketing lengths are

68.9 kN/m & 47.9 kN/m

which, of course, also bracket the given factored load.

iii. From the pages 2 and 3 of Table 8.3, for the two span lengths, the appropriate connection lengths for the 4 possible plate thicknesses are:

CONNECTION #1, plate thick. 6 mm, 370 mm & 370 mm,
CONNECTION #2, plate thick. 8 mm, 220 mm & 295 mm,
CONNECTION #3, plate thick. 10 mm, 220 mm & 220 mm,
CONNECTION #4, plate thick. 12 mm, 295 mm & 295 mm,

If the design criteria required a connection detail with a depth equal to about one half of the beam depth or greater and having the least cost, then CONNECTION #2 with a plate length of 295 mm would be the choice because its cost at $150.64 is the least, while meeting the depth criteria. The possibility of premature bottom flange bearing with this, the longer of the two possible connection details, does not exist.

If the design criteria were to use a connection detail of a depth equal to about one half of the beam depth or greater and having the least moment, then either the 6 mm thick end plate (CONNECTION #1) or the 8 mm thick end plate (CONNECTION #2) would be selected because both of these details cause end moments of about 26 kNm and both meet the depth criteria. The other connection details either do not meet the depth criteria or cause significantly larger end moments.

iv. If the 8 mm thick end plate (CONNECTION #2) detail is selected the symbolic reference, as taken from page 2 of Table 8.3 is

FEP(300-21-140-75-35-35-75-8-295) B(825-19-8) W(480-6)