University of Alberta

Optimization-Based Benefits Analysis for Process Control Systems

by

Yimin Zhou ©

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment of the requirements for the degree of **Master of Science**.

in

Process Control

Department of Department of Chemical and Materials Engineering

Edmonton, Alberta
Spring 2002

0-612-69788-6

University of Alberta

Library Release Form

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Title of Thesis: Optimization-Based Benefits Analysis for Process Control Systems

Degree: Master of Science

Year this Degree Granted: 2002

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J. Fraser Forbes

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E. Scott Meadows

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J. Horacio Marquez

Date: Nov. 6. 2001

To Yan and my parents

Abstract

A key component of any process control upgrade project is the initial benefits analysis, which evaluates the feasibility of the project based on economic and process information. The core of this analysis is benefits calculation, which is commonly based only on the reduction of variance. In this thesis, an optimization approach was proposed and a structured benefits analysis procedure was discussed.

With the information contained in key process variables, product economics and process operation conditions, the process performance evaluation can be formulated as a chance constrained optimization problem. The solution of the optimization problem is the optimal process operating point and the optimum economic performance, which will allow direct comparison of the different alternatives. This thesis provides an algorithm for the solution of the benefits analysis problem and develops sensitivity expressions for optimal process profitability and operating point.

Acknowledgements

I would like to express my gratitude to Dr. J. Fraser Forbes for his extraordinary support, guidance and encouragement throughout the course of this work.

I would also like to thank the entire CPC group at U of A for welcoming me into their midst and for helping me through out my study. My gratitude goes to Dr. Biao Huang, Dr. Yale Zhang, Zhengang Han and Ivan Trush for their help in this work.

The financial support of Natural Sciences and Engineering Research Council of Canada through Mechanical Wood Pulps Network Centre of Excellence and Alberta Pacific Forest Industries Inc. is fully appreciated.

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Chapter 1

Introduction

With the development of process control technology and its wide application in industry, control system upgrade projects have been recognized to be one of the best ways to improve the profitability of plants. The first step in any control system upgrade project is a benefits analysis study (Marlin *et al.*, 1991). The analysis goals are to identify the control upgrade opportunities and estimate the associated costs and benefits. This study evaluates the efficiency of the project based on economic and process information. The core of the benefits analysis is a benefits calculation. The approach now used to benefits calculation is based only on the variance reduction for key process variables. In this thesis, an optimization-based approach is proposed and a structured benefits analysis procedure is discussed.

The improvement of process economic performance comes from variation reduction for key process variables and selecting operating conditions that best exploit this variance. Process variation is often considered a crucial limitation to process performance. Usually the products are characterized by several attributes, which have specifications associated with them. Without market demand changing, the value of the products depends on whether these product attributes fall within pre-defined ranges of the product specification. Even with the best control, the products attributes cannot be guaranteed to be right on specification. Juran (1980) described two causes of off-specification in product: *Common* causes and *Special* causes. *Common* causes are frequent, short-term, random disturbances that are inherent in every process. *Special or assignable* causes are larger, less frequent disturbances that are identifiable and hopefully preventable. A solution to the variation reduction is process control. Automatic process control (APC), or regulatory control, tries to compensate for common causes or upsets by making frequent adjustments to the process that counteract their effects. Statistical process control (SPC) seeks to identify and remove root causes for the *Special* causes variability (Shunta, 1995).

The result of benefits analysis not only identifies the opportunities and prioritizes them, but also forms the basis for economic justification and subsequent capital investment. Marlin *et al.* (1991) provides several examples of how benefits analysis has been successfully applied in a variety of processes. Such benefits studies are typically carried out in following steps:

- Formulate overall objectives for improving the process performance,
- Identify where control improvements are needed,
- Generate a list of potential improvement opportunities,
- Estimate the potential benefits and costs for the improvements,
- Prioritize and select the best opportunities based on business impact and feasibility,
- Develop design concepts to capture the benefits,
- Report the results and make recommendations.

1.1 Existing Benefits Analysis Methods

Many methods have been posed to carry out the benefits analysis. These methods can be divided into four groups (Harskin, 1996)

- Experience

For budgetary estimates, "rules of thumb" can be used based on experience. Experience provides an order-of-magnitude estimate, but advanced process control projects are seldom funded on this basis because each plant is unique. As a result, this approach is unlikely to produce accurate results. More accurate methods such as best operator and statistical methods are required.

- Best operator

Of the two methods that require plant data, the best operator method is the easiest, because it requires only historical values of key process variables. The average value of a variable is compared with the best value achieved by an operator. The difference between the average and the best operator performance represents the possible improvement with the advanced process control. However, this method assumes that the advanced controller consistently performs like the best operator. It does not consider the fact that the advanced control may do better (or worse) than the best operator. Therefore, the benefits may be underestimated (or overestimated).

- Statistical methods

Statistical methods can be used to analyze the historical data. The means and the standard deviations of the data are calculated. Since it is assumed that improved process control will result in a reduction of standard deviation, a new operating point can be chosen closer to the product specification, which should increase the profitability of the process. The difference between the mean/standard deviation before and after the process control upgrade is used to capture benefit that is expected to arise from the upgrade. A hypothesis test is used to check whether this new operating point should be implemented or resulting change is significant.

- Simulation

Simulation is another attractive approach as it can be used to evaluate operations both within normal operation and outside of historical conditions. However, the cost of obtaining a dynamic model to be used for benefits analysis can be substantial.

Of the four methods discussed above, the experience-based approach is often used for estimating advanced process control benefits for budgetary purposes, and the statistical method is most popular for estimating benefits for project funding appropriations. In practice, these methods are usually combined to carry out a detailed benefits analysis.

Marlin *et al.* (1991) gives a comprehensive framework for justifying control upgrade benefits. They addressed technical issues such as the required calculations and plant tests to predict control benefits. In the study undertaken by the Warren Centre for Advanced Engineering (Marlin *et al.*, 1987), a general method for control benefits analysis was proposed. A key finding in this report is that economic benefits usually result from driving operating process variables closer to their targets than is possible in the base case (*i.e.*, when process is running at the nominal point). The analysis identifies the causes for poor control and selects an appropriate upgrade which leads to tighter control and a smaller standard deviation. The resulting operation is closer to the target. The difference between the averages before and after the control upgrade is the improvement from the control upgrade and can be calculated from the statistical distributions. Marlin proposed an equation for benefits analysis calculation, which summarizes the relevant factors:

$$\begin{aligned} \text{Benefits} &= \text{Improvement} \times \text{Incremental value} \\ &\quad \times \text{Unit throughput} \times \text{Time} \times \text{Service factor} \end{aligned} \tag{1.1}$$

where, the improvement is the difference between the base case and the improved operation. The incremental value is the economic value of the predicted improvement. The service factor is the fraction of the time that the process unit will be in the mode of operation.

Latour (1996) proposed an approach to control benefits calculation called CLIFFTENT, which involves the estimation of the reduction in variability of key process variables resulting from a control system upgrade. Latour made the point that a more "natural" way to view the economic performance of an automation system is in terms of the trade-off between the economic incentives for pushing the process toward the product specifications and the costs associated with violation of these specifications. This is accomplished via an economic model of the "trade-off". In Latour's approach, the probability density function (PDF) for the key process variables is either determined directly from the process operating data or estimated. Given the PDF for the process variables and an economic function that expresses

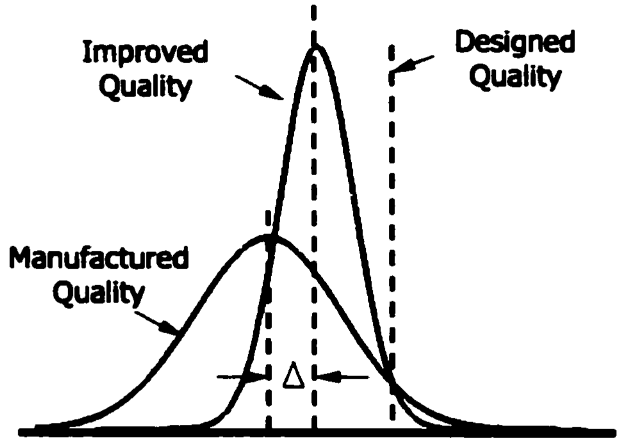


Figure 1.1: Common benefits estimation approach

the cost of deviation from the specification (for deviations that are within specification and those that violate them), the average operating point can be determined that best reflects the economic trade-off within the process variation. This operating point depends not just on process variance, but also on market conditions. Latour used a heuristic approach to determine the "best" operating point.

$$E[P] = \int_{-\infty}^{\infty} \phi(y, s^{*}) \cdot f(y, \tilde{y}, \sigma^2) dy \tag{1.2}$$

Grosdidier (1997) used the similar idea in the study of refinery products blending. He showed the improvement by reducing the standard deviation in the operating data after the control upgrade. By using a simple economic model, he calculated the benefits and associated cost.

Bao *et al.* (2000) proposed a structured approach to benefits analysis. They extended the CLIFFTENT idea to evaluate the control system performance by solving a chance constrained optimization problem. In the same way, the control system performance limit and the proposed control upgrade based on current system and proposed system are evaluated. They also provided a simple tabular form for benefits comparison.

Craig and Henning (2000) provided a new framework for the evaluation of advanced control projects. The approach is to investigate how improvements brought about by advanced control can be measured to a required level of statistical significance, after the controller has been commissioned. They used hypotheses to determine whether a significant difference exists between means of two sets of data (before and after the control upgrade). A statement of their problem is:

$$\begin{aligned} H_0 &: \mu_1 = \mu_2 \\ H_1 &: \mu_1 > \mu_2 \\ \alpha &= 0.05 \end{aligned}$$

To reject H_0 with 95% confidence, the observed difference of the sample means would need to fall sufficiently high in the right trail of the distribution. Then, the hypothesis that $\mu_1 > \mu_2$ cannot be rejected at the specified confidence level. The test assumed that the data were normally distributed and that the sample variances were not significantly different.

Craig and Henning (2000) discussed ways of obtaining controller benefits through not only the reduction of product variations, but also the reduction of downtime (*i.e.*, when the process has to be shut down). Measured improvements were translated into increases in cash flow that resulted from implementing an advanced controller. In the benefit analysis procedure, experimental design and data generation were followed by data analysis and hypothesis testing, then monetary benefits were estimated by simply multiplying the quantity of product by the product value. Finally, an economic project evaluation was made to determine if the expected new control system yields larger benefits.

In the controller comparison phase, instead of doing "before and after" comparisons which are traditionally used, they included two controllers into the control loop and switched them as frequently as possible. But each switching period should be significantly longer than the longest time constant of the process. This is mainly due to the impossibility of ensuring that all factors are identical except for those being tested.

Ossthuizen *et al.* (2000) used the principles described in Craig and Henning (2000), but suggested an optimal on-off time for switching between controllers. It is said that using this strategy, the variations in unmeasured disturbances do not bias the measured plant data significantly. However, the economic evaluation of control systems was not provided.

Lant and Steffens (1998) presented a quantitative statistical tool, which can be used for performing control system benefits analysis. They stated that a major outcome of the Warren Centre study (1987) was the Process Control Opportunities and Benefits Analysis, which has been customized and extensively applied by many process companies around the world. Whilst the Process Control Opportunities and Benefits Analysis is a useful tool in evaluating process control projects, it does require specialist control expertise and an up-front investment of 1-2% of the final project cost. Therefore, a simpler and cheaper means of determining preliminary evaluation of control system performance is proposed. The Process Control Self-Assessment Proforma forms the basis of the benchmark exercise (Brisk and Blackall, 1995). After filling out the table, the scope for improvement in control technology is identified.

The statistical approach cited by Shunta (1995) was used to identify the extent of improvement in terms of reduced variability, which can be expected from improving the process control. The two criteria used in the analysis were the Process Performance Index P_{pk} and Process Capability Index C_{pk}(Kittlitz, 1987) that were defined as:

$$P_{pk} = \frac{|average - nearer\ specification|}{3(s_{tot})} \tag{1.3}$$

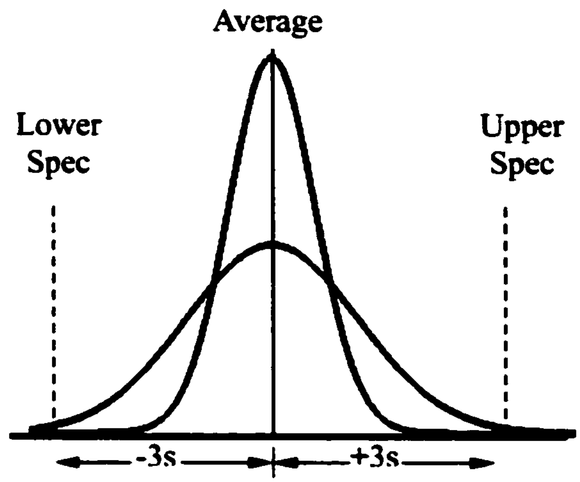


Figure 1.2: Performance index illustration

$$C_{pk} = \frac{|average - nearer\ specification|}{3(s_{cap})} \tag{1.4}$$

where, s_{tot} is the standard deviation of the data under analysis, s_{cap} is the estimated capability standard deviation and is calculated using short periods of data containing only short-term random variability.

P_{pk} indicates how well the current controls can keep the actual variability within the desired range, and C_{pk} represents the smallest variability achievable. By comparing the value of both P_{pk} and C_{pk} in Table 1.1 (Shunta, 1995), a decision can be made if there exists a potential benefit to perform control improvement.

Table 1.1: Performance index

	$P_{pk} < C_{pk}$	$P_{pk} \sim C_{pk}$
($C_{pk} < 1$)	Change Process Improve Control	Change Process
($C_{pk} > 1$)	Improve Control	Little incentive for Improvements

Guidelines for interpreting the process performance indicator are:

- $C_{pk} > 1$ indicates that the process has the capability to meet specifications. The inherent random variability presented is acceptable.
- $C_{pk} < 1$ indicates that the process is not capable of meeting customer specification, even under statistical process control. There is excessive random variability inherent in the process that must be reduced by a change in the process.

- $P_{pk} < C_{pk}$ indicates that the process is not operating up to its capability. There are long term drifts, cycles, shifts, or changes in variability caused by special events that may be reduced by improved process control or eliminated by removing the root causes.

- $P_{pk} \sim C_{pk}$ indicates that the process is operating close to a state of statistical process control.

Shunta (1995) provided a simple tabular form for control performance evaluation. But the performance index only deals with the single variable case, and was not applied to multivariable case. In addition, this method ignores process economics and focuses on ensuring that the product distribution is within the range between the lower and upper specification (99.73%), leaving almost no room for any violation.

Bhullar (1988) studied the opportunities for advanced process controls in various operating units in a case study of a major west coast (U.S.) refinery instrument modernization projects. The benefits of using advanced controls to increase product yield, throughput and quality were expressed in economic terms based on actual plant operating data. In his paper, Bhullar listed a summary of typical economic data as basis for benefits calculation. Also, he suggested that the overall potential benefits analysis were accomplished by breaking down global objectives into lower levels of refinery objectives, unit objectives, section objectives and finally down to equipment level objectives. Once the specific objectives were agreed upon, the plant data was used to analyze how close to the defined objectives the operation already was. The next step was to calculate where they should operate using regulatory controls while taking into consideration of safety, equipment or other physical constraints. The improvements were converted into monetary units and benefit credits expressed in terms of dollars. However, there is no detailed solution method given in the paper.

1.2 Issues & Problems

These existing methods give a basic framework for carrying out control system benefits analysis. It is clear that both experience and statistical methods should be used in any analysis. The key point is to identify the business drivers and the opportunities for improvement. Using process control, a reduction of variation of these key variables may be achieved. There, however, exist some shortcomings in these available methods.

Marlin *et al.* (1991) provided a general framework for benefits analysis. It contained a complete procedure from process information collection to the final ROI estimation. The calculation of benefits used the common approach and was based on variation reduction. Craig *et al.* (2000) modified the framework by suggesting an optimal switch between the current controller and the upgraded controller instead of comparing the result "before" and "after" the controller upgrade. In the control performance comparison, a hypotheses test is used to justify the control improvement. Shunta (1995) used performance index created by Kittlitz (1987) to form a table as a standard for control performance evaluation. This method is simple and easy for an engineer to apply in a real process. However, none of these methods take into account the economic value of different products (or different product grades).

Latour (1996) introduced an economic performance function into the benefits calculation. He proposed a trade-off between profit gain and specification violation, using a piece-wise economic performance function for processes where the variables can be considered independent. Latour's method cannot handle constraints that do not lend themselves to the economic penalty approach. Also, he did not provide a clear, simple method for benefits calculation.

Bao *et al.* (2000) proposed that any comprehensive approach to benefits estimation should accurately estimate: 1) the current economic performance of the process operation; 2) the best achievable economic performance for the existing automation system; 3) the best achievable economic performance for any improvement; and 4) the maximum theoretically achievable economic performance for the system. However, no detailed solution method is given. Also, the sensitivity of process performance with respect to the process parameters is not studied.

To sum up, these existing methods either ignore economic information or fail to give a comprehensive solution approach (*e.g.*, considering multivariable case). Further, none of these methods gives a solution method to deal with the key variable constraints.

1.3 Thesis Objectives & Scope

Marlin *et al.* (1991) provided a good basis for performing the work of this thesis. However, the benefits calculation method proposed in Marlin *et al.* (1991) fails to consider all of the economic information, such as cost of constraint violation. Latour (1996) includes the effect of economic information in the CLIFFTENT integral. Therefore, a modified framework combines Marlin *et al.* and Latour's approaches.

The objective of this thesis is to provide a structured procedure for benefits estimation and complete the procedure of benefits analysis by incorporating process performance sensitivity analysis into it. The new approach frames the benefits calculation as an optimization problem. Upon the completion of benefits analysis procedure, the current performance can be evaluated using the information from both process and market.

The heart of the benefits analysis procedure, the benefits calculation, not only deals with the deterministic constraints but also deals with probability constraints both individually and jointly. This helps to identify the opportunity of improving the profitability of the process by taking appropriate risks.

The proposed method combines four methods (Figure 1.3) discussed by Harskin (1996) and forms a complete benefits analysis procedure based on a stochastic programming ap-

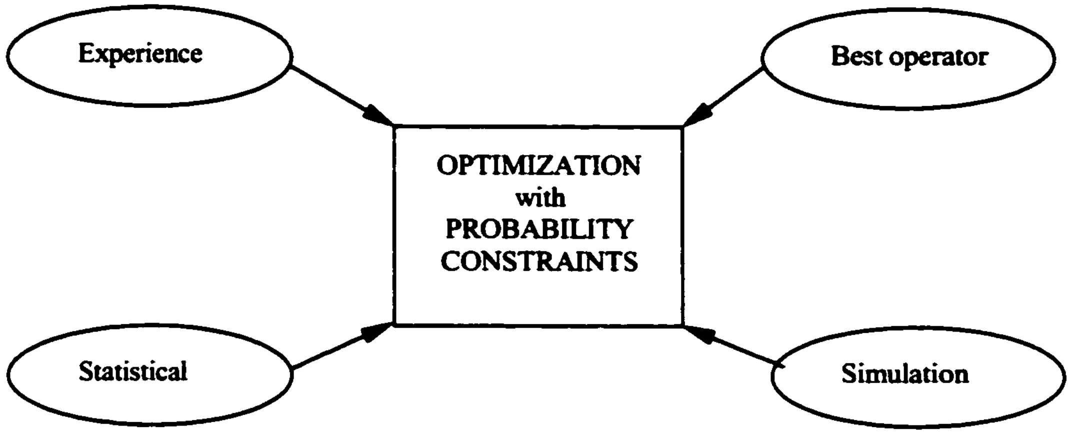


Figure 1.3: Proposed benefits analysis method

proach to benefits calculation. The multivariable case is considered including constraints. The proposed calculation approach is applied to evaluate: 1) the current economic performance of the process operation; 2) the best achievable economic performance for the existing automation system; 3) the best achievable economic performance for any improvement; and 4) the maximum theoretically achievable economic performance for the system. The benefits are compared using the procedure similar to Analysis of Variance (ANOVA). Also, the sensitivity of process performance with respect to the process parameters is studied to justify the results from the performance evaluation problem. Then, decision on whether there is potential process profitability achievable by control system upgrade is made. Finally, performance on the implemented control is monitored.

1.4 Thesis Structure

This thesis begins with an introduction to and motivation for this project, followed by a literature survey of current control system benefits analysis approaches. In Chapter 2, the stochastic programming approach to benefits analysis is introduced. The performance evaluation problem is formulated incorporating an economic performance function, process data probability density function and constraints. An example is given to illustrate the methods used to solve individual probability constraints problem and joint probability constraints problem.

In Chapter 3, a general benefits analysis procedure is proposed including process information collection, performance evaluation. The sensitivity of process performance with respect to the process parameters is studied.

In Chapter 4, proposed benefits analysis procedure is used on two case studies. One is two tank heaters in series. The optimal process profitability of different control systems are compared, and one of them is chosen to be implemented based on the process profitability, implementing cost and sensitivity study. The second is ALPAC's bleach plant D_2 stage. Process data are collected and analyzed, the current control performance and performance limit are evaluated to find whether there is sufficient potential to warrant further study into control system improvement. Chapter 5 contains summary and conclusions for this thesis.

Chapter 2

Optimization-based Performance Evaluation

Marlin *et al.* (1991) provided a comprehensive procedure for benefits analysis, which has been adopted as the framework for the approach proposed in this thesis. However, the heart of any benefits analysis procedure is the performance estimation, which is posed as an optimization problem in this work. Bao *et al.* (2000) proposed that any comprehensive approach to benefits estimation should accurately estimate: 1) the current economic performance of the process operation; 2) the best achievable economic performance for the existing automation system; 3) the best achievable economic performance for any improvement; and 4) the maximum theoretically achievable economic performance for the system.

Latour (1996) proposed performance evaluation in terms of a CLIFFTENT integral.

$$E[P] = \int_{-\infty}^{\infty} \phi(y, s^*) \cdot f(y, \tilde{y}, \sigma^2) dy \tag{2.1}$$

Although Latour's approach provided an effective performance metric, it did not give an effective means for evaluating CLIFFTENT integral. Further, Latour's method does not handle hard constraints that must be enforced; rather it treats all constraints via an economic penalty approach.

In this chapter, a more comprehensive method is proposed for performance evaluation.

2.1 General Problem Formulation

The performance evaluation problem can be expressed in terms of determining the operating point that would provide the "best" economic performance of a given control scenario. Then, it is natural to form the performance evaluation calculation as an optimization problem. This optimization problem should contain constraints that ensure a feasible operating point and must incorporate the uncertainties in both process operation and economics. The inclusion of these uncertainties into the optimization problem result in a stochastic programming problem. Thus, the performance evaluation can be formulated as stochastic programming problem (Zhou *et al.*, 2000):

max Process Profitability

s.t. :

Product Specification
Product Demand
Resources Limitations
Equipment Limitations
Environmental Limitations
Safety Requirements

Uncertainties $\Longrightarrow$ Stochastic Programming Problem (2.2)

The first step of forming the performance evaluation calculation is to set up the objective function listed in Problem (2.2). The key part of the objective function is economic performance function (EPF). This function defines the economic values for different product grades. Since the formulation of this function needs detailed knowledge of process operating costs, it is usually very difficult to obtain an accurate EPF. Latour (1996) proposed a piece-wise linear function, which was adopted as the form for EPF in this thesis.

Another part of the objective function is the probability density function (PDF). Because of inherent disturbances in the process operation, the process output data typically is described by a probability distribution. In this thesis, the process output data is assumed to be adequately described by a Normal distribution, and this distribution is independent of operating point. A Normal distribution is characterized by two moments: mean and variance. Both of these can be easily estimated from the process data. The PDF for a Normal distribution is:

$$f(y,\tilde{y},\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-\tilde{y})^2}{2\sigma^2}} \tag{2.3}$$

and the objective function can be expressed as:

$$J = \int_{\mathbf{Y}} \mathrm{EPF} \cdot \mathrm{PDF}\, dy \tag{2.4}$$

Equation (2.4) is Latour's CLIFFTENT integral for the single variable case; however, Latour did not consider the multivariable case. The solution of unconstrained optimization problem that uses Equation (2.4) as the objective would provide the "best" performance for the current system, however, when the process operation must respect equipment limitations, safety requirement, product specifications, and so forth, this "best" solution may be infeasible. Therefore, the performance evaluation problem needs to incorporate constraints such as those listed in Problem (2.2). Some of those constraints can be expressed as deterministic constraints, which are easily handled; while others can be expressed as probabilistic constraints and require special solution methods. These probabilistic constraints change the performance evaluation problem to stochastic programming problem of the form:

$$\begin{aligned} \max_{\tilde{\mathbf{y}}} \quad & J = \int_{\mathbf{Y}} (\boldsymbol{\phi}(\mathbf{y}, \mathbf{s}^*) \cdot f(\mathbf{y}, \tilde{\mathbf{y}}, \mathbf{Q}))d\mathbf{y} \\ \text{subject to:} \quad & \\ & \Pr(\mathbf{H}\mathbf{y} \geq \mathbf{s}^*) \geq \boldsymbol{\alpha} \end{aligned} \tag{2.5}$$

where: J is the process performance; f is the probability density function for the process variables; $\mathbf{H}$ is constraint model coefficients matrix, $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \cdots \mathbf{h}_k]^T$; Pr denotes probability; $\mathbf{s}^*$ represents the constraint limits that must be enforced to some level of confidence $\boldsymbol{\alpha}$; $\mathbf{y}$ is a vector representing the key process variables; $\tilde{\mathbf{y}}$ represents the most desirable operating point; $\mathbf{Y}$ is the space of possible values for the process variables; and ϕ is a economic performance function, which is usually defined in terms of "profit/cost". The economic performance function reflects the relationship between product qualities and product values (*i.e.*, ϕ has a higher value if the product quality meets specification, on the other hand, it has a different value, usually lower, when the specification is violated). A detailed explanation of this is given in Chapter 3. Constraints considered in this thesis are expressed in probabilistic form: individual probability constraints (IPC) and joint probability constraints (JPC). The JPC in Problem (2.5) states that all of the process constraints have to be satisfied at or beyond a specific level of probability, simultaneously. Such joint probability constrained problems have proven very difficult to solve (Prekopa, 1995). As a result, the individually constrained problem:

$$\Pr((\mathbf{H}(i,:)\mathbf{y}) \geq s_i^*) \geq \alpha_i \qquad (i = 1, 2, \cdots, k) \tag{2.6}$$

has received much more attention in the literature (Prekopa, 1995).

Thus, the performance evaluation calculation can be expressed in either IPC problem form:

$$\begin{array}{ll} \max\limits_{\tilde{\mathbf{y}}} & J = \int\limits_{\mathbf{Y}} (\phi(\mathbf{y}, \mathbf{s}^*) \cdot \mathbf{f}(\mathbf{y}, \tilde{\mathbf{y}}, \mathbf{Q}))d\mathbf{y} \\ \text{subject to:} & \\ & \rho_i = \Pr((\mathbf{H}(i,:)\mathbf{y}) \geq s_i^*) \geq \alpha_i \\ & (i = 1, 2, \cdots, k) \end{array} \tag{2.7}$$

or JPC problem form:

$$\begin{array}{ll} \max\limits_{\tilde{\mathbf{y}}} & J = \int\limits_{\mathbf{Y}} (\phi(\mathbf{y}, \mathbf{s}^*) \cdot \mathbf{f}(\mathbf{y}, \tilde{\mathbf{y}}, \mathbf{Q}))d\mathbf{y} \\ \text{subject to:} & \\ & \Pr(\mathbf{H}\mathbf{y} \geq \mathbf{s}^*) \geq \alpha \end{array} \tag{2.8}$$

Although both problems attempt to achieve an optimal solution, which ensures that the inequality constraints are satisfied at some level of probability, there is a distinct difference between them. In the JPC problem, satisfaction of the whole set of probability constraints is represented by one probability constraint with the risk level of α, and it is guaranteed that the solution is feasible with respect to all of the constraints at no less than the specified probability level. Whereas, each individual constraint in IPC problem is satisfied at a certain probability level α_i. However, the probability level at which all of the constraints are satisfied is less than any one of individual probability levels. For example, consider solution of an IPC problem with 10 probability constraints. Each probability constraints are assumed to be satisfied at or beyond the probability level of 90%. Then, the worst case (*i.e.*, lower bound) probability level at which this solution satisfies all of the constraints is equal to: $\alpha = \prod_{i=1}^{10} \alpha_i = (90\%)^{10} \approx 35\%$. A comparison of the IPC and JPC is given at the end of this chapter.

The relationship of PDF, EPF and constraints can be explained in this way: the data of the key variables vary as described by a given probability distribution. The operating point for the process should be set conservatively to guarantee that no constraint violation occurs and to exploit the process economics. Thus, there is a trade-off between constraint violation and process economics.

2.2 Uncertainty in Objective Function

In the problem formulation given in last section, it was assumed that the economic performance function ϕ was deterministic. However, it is possible that this function is stochastic due to the changing market prices and demands. Prekopa (1995) discussed several ways to handle a random objective function.

One approach is the probability maximization formulation. This form is used in situations where a precise performance valuation is given (*e.g.*, $J_{\max}$) and the probability of satisfying this objective should be maximized:

$$\max \quad \Pr(J(\mathbf{x}, \boldsymbol{\xi}) \geq J_{\max}) \tag{2.9}$$

where, J is process performance, $\mathbf{x}$ is a vector of operating conditions and $\boldsymbol{\xi}$ is a set of stochastic parameters. This formulation requires the solution to be such that the probability of performance lower than the specification is the least out of all possible solutions. This approach is not applicable to the benefits analysis problem, as there is no way to a priori set $J_{\max}$.

Another possible formulation is to convert the objective function $J(\mathbf{x}, \boldsymbol{\xi})$ into a deterministic objective function by taking the expectation $E[J(\mathbf{x}, \boldsymbol{\xi})]$. If $J(\mathbf{x}, \boldsymbol{\xi})$ is linear in $\mathbf{x}$, $J(\mathbf{x}, \boldsymbol{\xi}) = \boldsymbol{\xi}^T \mathbf{x}$, then $E[J(\mathbf{x}, \boldsymbol{\xi})] = [E(\boldsymbol{\xi})]^T \mathbf{x}$ is also linear in $\mathbf{x}$. Both IPC and JPC constraints can be incorporated into the problem to make sure the achieved process performance is practically realizable.

$$\begin{aligned} \max \quad & E[J(\mathbf{x}, \boldsymbol{\xi})] \\ s.t.: \quad & IPC \text{ or } JPC \end{aligned} \tag{2.10}$$

Finally, another possible formulation is to take a linear combination of the expectation and the standard deviation of the objective function with constraints:

$$\begin{aligned} \max \quad & E[J] + \gamma \sqrt{Var[J]} \\ s.t.: \quad & IPC \text{ or } JPC \end{aligned} \tag{2.11}$$

where γ is a user specified constant. The first term $E[J]$ represents the expected value of the objective function with respect to the economic performance function ϕ; and the second term $\gamma\sqrt{Var[J]}$ represents the "spread" of the distribution of objective function values about the expected value. The solution of Problem (2.11) results in a trade-off between the expected value and the variance of the objective function (*i.e.*, an operating point that is the best compromise between profitability and expected variation of the profit). The choice of γ is arbitrary and reflects the decision maker's opinion as to the relative importance of the expected value and the variance of the optimum objective function value.

It is clear that the incorporation of stochastic economic parameters into the objective function makes the problem more difficult to solve. Thus, for the purpose of this thesis, the market prices and demands for specific products were assumed to be known. Therefore, the economic performance function is deterministic. The solution methods discussed in the following is based on this assumption.

2.3 Solution Methods

As was discussed in the first section, the process performance evaluation problem can be formulated in either IPC or JPC form (see Problem (2.7) and (2.8)). The solution methods for both problems are discussed below and the solution approaches of these two problems are compared.

2.3.1 IPC Algorithm

The only difference between probabilistic programming (covered in this thesis) and linear/nonlinear programming is the form of the constraints. The common strategy for solving the IPC problem is to convert the probabilistic constraints to deterministic constraints. For each individual constraint, the process key variables can be standardized in terms of the optimal operating point, and the constraint converted to a deterministic form, which allows solution via conventional linear/nonlinear programming.

The individual constraints described in Problem (2.7) can be rewritten as

$$\rho_i = \Pr\left\{(\frac{\mathbf{h}_i\mathbf{y} - \mathbf{h}_i\tilde{\mathbf{y}}}{(\mathbf{h}_i\mathbf{Q}\mathbf{h}_i^T)^{1/2}}) \geq \frac{s_i^* - \mathbf{h}_i\tilde{\mathbf{y}}}{(\mathbf{h}_i\mathbf{Q}\mathbf{h}_i^T)^{1/2}}\right\} \geq \alpha_i \tag{2.12}$$

$$i = 1, 2, \cdots, k$$

where, $\mathbf{Q}$ is the covariance matrix of $\mathbf{y} = [y_1, y_2, \cdots, y_k]^T$, $\mathbf{s}^* = [s_1^*, s_2^*, \cdots, s_k^*]^T$ and α_i is a scalar which denotes the individual probability level.

Let

$$\xi_i = \frac{\mathbf{h}_i\mathbf{y} - \mathbf{h}_i\tilde{\mathbf{y}}}{(\mathbf{h}_i\mathbf{Q}\mathbf{h}_i^T)^{1/2}}$$

then ξ_i is the standardized variable which follows a standard Normal distribution (*i.e.*, $\xi_i \sim N(0, 1)$). This constraint can be rewritten as:

$$\rho_i = \Pr(\xi_i \geq \frac{s_i^* - \mathbf{h}_i\tilde{\mathbf{y}}}{(\mathbf{h}_i\mathbf{Q}\mathbf{h}_i^T)^{1/2}}) \geq \alpha_i \tag{2.13}$$

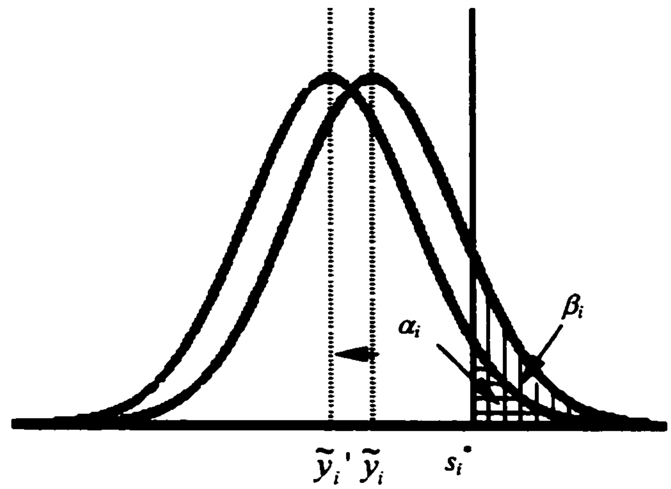


Figure 2.1: IPC algorithm illustration for single variable case

then

$$\rho_i = \Pr(\xi_i \leq \frac{s_i^* - \mathbf{h}_i\tilde{\mathbf{y}}}{(\mathbf{h}_i\mathbf{Q}\mathbf{h}_i^T)^{1/2}}) \leq 1 - \alpha_i$$

Assuming that the cumulative distribution function of ξ_i is given by $\mathcal{F}$,

$$\mathcal{F}(x) = \int_{-\infty}^{x} \frac{1}{2\pi} e^{-\frac{x^2}{2}} dx = \frac{1}{2}\operatorname{erf}(\frac{\sqrt{2}}{2}x) + \frac{1}{2}$$

where "erf" is the error function denoted by

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

then

$$\mathcal{F}(\frac{s_i^* - \mathbf{h}_i\tilde{\mathbf{y}}}{(\mathbf{h}_i\mathbf{Q}\mathbf{h}_i^T)^{1/2}}) \leq 1 - \alpha_i \tag{2.14}$$

$$\mathbf{h}_i\tilde{\mathbf{y}} \geq s_i^* - (\mathbf{h}_i\mathbf{Q}\mathbf{h}_i^T)^{1/2}\mathcal{F}^{-1}(1 - \alpha_i) \tag{2.15}$$

Thus, the probabilistic constraint is converted to deterministic form. This approach can be easily understood using Figure 2.1.

Assuming that the IPC dealt with in Figure 2.1 has the form:

$$\Pr(y_i \geq s_i^*) \leq \alpha_i$$

where, s_i^* is the product specification and α_i is the pre-defined constraint violation level. This constraint requires that the probability of constraint violation (*i.e.*, $y_i \geq s_i^*$) should be no more than α_i. However, it is clear from Figure 2.1 that the current constraint violation is:

$$\Pr(y_i \geq s_i^*) = \beta_i > \alpha_i$$

where β_i is the measured current constraint violation. This means that the product does not satisfy the IPC. Therefore, the corresponding mean $\tilde{y}_i$ of the data should be shifted left to a new point $\tilde{y}_i'$ ensuring that all the data satisfy the IPC.

2.3.2 JPC Algorithm

The solution method for the IPC problem could be used for solving JPC problem; however, due to the distinct characteristic of JPC, more effort is needed. Consider the JPC, which has the form:

$$\Pr(\mathbf{Hy} \geq \mathbf{s}^*) \geq \alpha \tag{2.16}$$

Using the same method as in IPC algorithm:

$$\rho_i = \Pr(\boldsymbol{\xi}_i \geq \frac{s_i^* - \mathbf{h}_i\widetilde{\mathbf{y}}}{(\mathbf{h}_i\mathbf{Q}\mathbf{h}_i^T)^{1/2}}) \geq \alpha_i \tag{2.17}$$

and the JPC can be rewritten as:

$$\Pr\left\{\begin{array}{c} \mathbf{h}_i\widetilde{\mathbf{y}} \geq s_i^* - (\mathbf{h}_i\mathbf{Q}\mathbf{h}_i^T)^{1/2}\xi_i \\ \vdots \end{array}\right\} \geq \alpha$$

$$i = 1, 2, \cdots, k$$

Let

$$\eta_i = s_i^* - (\mathbf{h}_i\mathbf{Q}\mathbf{h}_i^T)^{1/2}\xi_i \tag{2.18}$$

then η_i has Normal distribution

$$\eta_i \sim N(s_i^*, (\mathbf{h}_i\mathbf{Q}\mathbf{h}_i^T)^{1/2})$$

Therefore, the joint constraints can be written as

$$\rho_0 = \Pr\{\mathbf{H}\widetilde{\mathbf{y}} \geq \boldsymbol{\eta}\} \geq \alpha \tag{2.19}$$

where, $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_k]^T$ and $\boldsymbol{\eta} = [\eta_1, \eta_2, \cdots, \eta_k]^T$. In Constraint (2.19), the left hand side of the inequality $\mathbf{H}\widetilde{\mathbf{y}} \geq \boldsymbol{\eta}$ contains the decision variable $\widetilde{\mathbf{y}}$ of the optimization problem and the right hand side of $\mathbf{H}\widetilde{\mathbf{y}} \geq \boldsymbol{\eta}$ is a vector of variables satisfying a Normal distribution.

Note that $\mathbf{y}$ in left hand side of Constraint (2.16) was changed to the decision variable $\tilde{\mathbf{y}}$ in Constraint (2.19). Finally, the original JPC problem was rewritten as:

$$\begin{array}{ll} \max\limits_{\tilde{\mathbf{y}}} & J = \int\limits_{\mathbf{Y}} (\phi(\mathbf{y}, \mathbf{s}^*) \cdot f(\mathbf{y}, \tilde{\mathbf{y}}, \mathbf{Q})) d\mathbf{y} \\ \text{subject to:} & \\ & \Pr\{\mathbf{H}\tilde{\mathbf{y}} \geq \boldsymbol{\eta}\} \geq \alpha \end{array} \tag{2.20}$$

Zhang *et al.* (2000) adopted the JPC algorithm presented by Prekopa (1995) and Szantai (1988), which is based on the Supporting Hyperplane Method. This method was used for solving optimization problem with a linear objective function. In this thesis, this method was expanded to nonlinear objective function case. The JPC algorithm consists of two phases:

Phase 1: Find two initial solutions. One is feasible, that is $\tilde{\mathbf{y}}_{in} \in K^{(1)}$, where $K^{(1)} = \{\tilde{\mathbf{y}} \mid \Pr(\mathbf{H}\tilde{\mathbf{y}} \geq \boldsymbol{\eta}) \geq \alpha\}$; and the other is an infeasible solution, $\tilde{\mathbf{y}}_{out}^{(1)} \notin K^{(1)}$.

To obtain $\tilde{\mathbf{y}}_{in}$, the following linear programming problem is solved:

$$\begin{array}{ll} \min\limits_{\tilde{\mathbf{y}}} & \sum_{i=1}^{k} (\mathbf{h}_{i1}\tilde{y}_1 + \mathbf{h}_{i2}\tilde{y}_2 + \cdots \mathbf{h}_{in}\tilde{y}_n - \mu_{\eta_i})/\sigma_{\eta_i} \\ \text{s.t.:} & \\ & \mathbf{H}\tilde{\mathbf{y}} \geq \boldsymbol{\mu}_{\eta_i} + \tau \boldsymbol{\sigma}_{\eta_i} \end{array} \tag{2.21}$$

where the recommended value for τ is 3, as given by Szantai (1988). A strategy for adjusting the value of τ is also given: if the optimal solution to the problem is out of the feasible domain $K^{(1)}$, then the value of τ should be increased; however, a relatively large τ value may make the problem too restrictive to solve. An effective way to obtain a starting value for $\tilde{\mathbf{y}}_{out}$ (*i.e.*, $\tilde{\mathbf{y}}_{out}^{(1)}$) is to solve the corresponding IPC problem.

Phase 2: There are two steps in Phase 2 for iteration i:

Step(a) Let $\lambda^{(i)}$ be the largest λ $(0 < \lambda < 1)$, for which

$$\Pr(\mathbf{H}(\tilde{\mathbf{y}}_{in} + \lambda(\tilde{\mathbf{y}}_{out}^{(i)} - \tilde{\mathbf{y}}_{in})) \geq \boldsymbol{\eta}) \geq \alpha \tag{2.22}$$

is satisfied. Denote

$$\tilde{\mathbf{y}}_{\lambda}^{(i)} = \tilde{\mathbf{y}}_{in} + \lambda(\tilde{\mathbf{y}}_{out}^{(i)} - \tilde{\mathbf{y}}_{in}) \tag{2.23}$$

If

$$\Pr(\mathbf{H}\tilde{\mathbf{y}}_{\lambda}^{(i)} \geq \boldsymbol{\eta}) - \Pr(\mathbf{H}\tilde{\mathbf{y}}_{out}^{(i)} \geq \boldsymbol{\eta}) < \varepsilon$$

where ε is a user specified tolerance, then $\tilde{\mathbf{y}}_{\lambda}^{(i)}$ is an approximate solution to problem. Otherwise, define

$$K^{(i+1)} = \{\tilde{\mathbf{y}} \mid \tilde{\mathbf{y}} \in K^{(i)} \text{ and } (\nabla_{\tilde{\mathbf{y}}} \Pr(\mathbf{H}\tilde{\mathbf{y}} \geq \boldsymbol{\eta}))_{\tilde{\mathbf{y}}_{\lambda}^{(i)}} (\tilde{\mathbf{y}} - \tilde{\mathbf{y}}_{\lambda}^{(i)}) \geq 0\} \tag{2.24}$$

then go to step (b)

Step (b) Let $\tilde{\mathbf{y}}_{out}^{(i+1)}$ be an optimal solution to the following problem:

$$\begin{array}{ll} \min\limits_{\tilde{\mathbf{y}}} & -J = -\int\limits_{\mathbf{Y}} (\boldsymbol{\phi}(\mathbf{y}, \mathbf{s}^*) \cdot \mathbf{f}(\mathbf{y}, \tilde{\mathbf{y}}, \mathbf{Q})) d\mathbf{y} \\ \text{subject to:} & \\ & \tilde{\mathbf{y}} \in K^{(i+1)} \end{array} \tag{2.25}$$

If

$$\Pr(\mathbf{H}\tilde{\mathbf{y}}_{out}^{(i+1)} \geq \boldsymbol{\eta}) \geq \alpha$$

then $\tilde{\mathbf{y}}_{out}^{(i+1)}$ is an optimal solution to problem; otherwise, go to Step (a), using $(i+1)$ instead of (i).

The algorithm can be speeded up if a fixed $\tilde{\mathbf{y}}_{in}$ in Phase 2 is replaced by an adaptive term. Prekopa (1995) showed that the choice:

$$\tilde{\mathbf{y}}_{in}^{(i+1)} = \tilde{\mathbf{y}}_{in}^{(i)} + \frac{1}{i+1}(\tilde{\mathbf{y}}_{\lambda}^{(i)} - \tilde{\mathbf{y}}_{in}^{(i)}) \tag{2.26}$$

is efficient in many cases. Note that in the Supporting Hyperplane Method it is necessary to calculate the probability values and gradient vector of the probability as a function of the variables $\tilde{\mathbf{y}}$ at the actual boundary point $\tilde{\mathbf{y}}_{\lambda}^{(i)}$. For a multivariate Normal distribution, given the mean values and covariance matrix of the random variables, a Monte Carlo approach is often used for probability calculation and is adopted for the purpose of this work. Care must be exercised in using the Monte Carlo approach to ensure that a sufficient number of simulations are performed to ensure an accurate approximation of the underlying distribution is obtained for optimization purposes. In addition, although this method was used to solve the optimization problem with nonlinear objective function, the limitations are: the EPF was assumed to be a piece-wise linear function and the closed form of integral was used to obtain an analytical expression of the nonlinear objective function.

2.4 Illustrative Example

The Wood-Berry distillation column (Seborg, 1989) was used to illustrate the benefits calculation algorithm. This is a multi-input, multi-output system. Two controlled variables are X_D and X_B. X_D is the distillate composition and X_B is the bottom composition. Two manipulated variables are R and S. R is the reflux flow rate and S is the steam flow rate.

PI controllers are designed and tuned for each loop. At the steady-state conditions, data for X_D and X_B were recorded. The product should satisfy the given product specification with the minimum operating cost (*i.e.*, yield loss and steam cost).

$$\begin{bmatrix} X_D \\ X_B \end{bmatrix} = \begin{bmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-18.9e^{-3s}}{21s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \begin{bmatrix} R \\ S \end{bmatrix} \tag{2.27}$$

Assuming that the probability level is no less than 90% for the overhead product having a minimum purity of 0.87 (mole fraction) and/or the bottom product having a minimum purity of 0.90 (mole fraction). This defines both individual probability constraints (Inequalities (2.28) and (2.29)) and joint probability constraints (Inequalities (2.30)) as follows:

$$\rho_1 = \Pr(X_D \geq 0.87) \geq 90\% \tag{2.28}$$

$$\rho_2 = \Pr(X_B \geq 0.90) \geq 90\% \tag{2.29}$$

$$\rho_0 = \Pr\left\{ \begin{array}{c} X_D \geq 0.87 \\ X_B \geq 0.90 \end{array} \right\} \geq 90\% \tag{2.30}$$

It is reasonable for the real process that we need tighter control on some key variables to guarantee the product quality; however, for those variables that are not as important, the probability can be lower. Joint probability constraints require all of the constraints be satisfied simultaneously and usually the result will be quite different, if there exist strong correlation between key variables. In this example, X_D and X_B have strong correlation.

In both cases, the economic performance functions are the same and can be expressed as:

$$\phi_{X_D} = \left\{ \begin{array}{cc} -0.01 & X_D \leq 0.85 \\ 0.4X_D - 0.35 & X_D \leq 0.90 \\ -X_D + 0.95 & X_D \leq 0.96 \\ -0.01 & X_D \geq 0.96 \end{array} \right. \tag{2.31}$$

$$\phi_{X_B} = \left\{ \begin{array}{cc} -0.02 & X_B \leq 0.90 \\ X_B - 0.92 & X_B \leq 0.95 \\ -2.5X_B + 2.455 & X_B \leq 0.99 \\ -0.02 & X_B \geq 0.99 \end{array} \right. \tag{2.32}$$

As shown in Figure 2.2, the product is of the highest value at the desired product specification ($X_D = 0.90$). To the left of this specification, the product is off-grade and therefore some discount may apply on the product price until the point when the quality is too low and must be sold at a loss. On the other hand, if the product is of higher composition than the specification, the market price will not be higher while the production cost typically increases with an increase in the amount that the specification is exceeded.

In the case of individual probability constraints, the desired operating point for X_D or X_B are 0.9158 and 0.9606. Both of them are slightly higher than the product specification. The constraint satisfaction for X_D and X_B are 90.02% and 90.43%. However, the probability of satisfying both constraints jointly is only approximately 85%. This is due to the correlation between the constraints and the inability of the IPC algorithm to handle this correlation.

$$R = \left[\begin{array}{cc} 1 & 0.7096 \\ 0.7096 & 1 \end{array} \right]$$

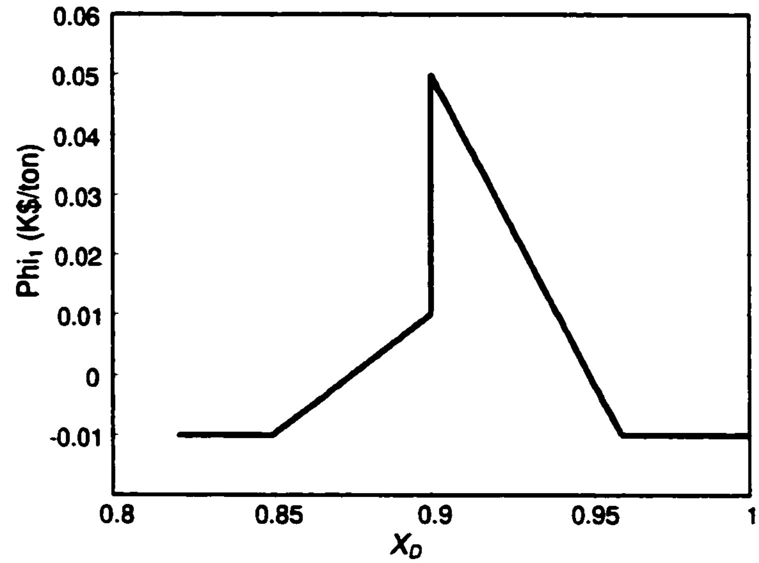


Figure 2.2: Economic performance function for X_D

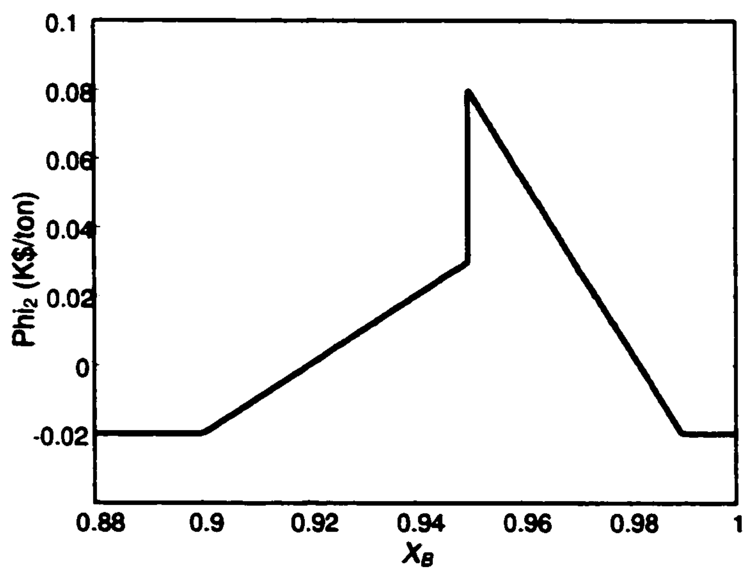


Figure 2.3: Economic performance function for X_B

In this example, the joint constraint satisfaction probability delivered by the IPC approach is only slightly less than the specified level of 90%. This is due to the comparatively small size of the problem. In general, the joint constraint satisfaction probability level obtained using the IPC algorithm will depend on two main factors: the number of probability constraints and the amount of correlation among them. To satisfy both constraint simultaneously, the probability level of either X_D or X_B or both can be increased, this leads to a large profit loss and it is hard to get values of X_D and X_B that satisfy the constraints. Further, this is a "trial and error" type procedure and is time consuming.

In the JPC case, the desired operating point for X_D and X_B are 0.9269 and 0.9711. The joint probability constraint is satisfied at the expense of reduced profit (*i.e.*, from 17 $\$/ton$ to 14.7 $\$/ton$). The comparison of two cases is listed in Table 2.1.

Table 2.1: Comparison of IPC and JPC

	IPC	JPC
$\bar{X}_D$	0.9158	0.9269
$\bar{X}_B$	0.9606	0.9711
ρ_1	90.02	93.99
ρ_2	90.43	93.55
ρ_0	84.88	90.01
Profitability ($K\$/ton$)	0.0170	0.0147

The JPC algorithm can be illustrated in Figure 2.4 and Figure 2.5. The vertical line is the overhead composition specification ($X_D = 87\%$), and the horizontal line is the bottom composition specification ($X_B = 90\%$). The ellipses are constant probability contours for the joint normal distribution for X_D and X_B with 99.73% confidence. A is the current operating point. Assuming that at this point, the IPC constraints are satisfied (*i.e.*, $\rho_1 = 90.02\%$ and $\rho_2 = 90.43\%$); while the JPC constraint is not satisfied (*i.e.*, $\rho_0 = 84.88\%$). The problem is how to locate the new operating point which gives the best process performance without violating the pre-defined probability level (*i.e.*, $\rho_0 = 90\%$). Although the final goal is to satisfy the joint probability level, the individual probability constraints should be increased appropriately to achieve this final goal.

A simple way is to keep one constraint probability level fixed, and increase the other constraint probability level by shifting the operating point. For this example, it equals to either keeping the probability level of bottom composition satisfaction fixed (*i.e.*, $\rho_2 = \Pr(X_B \geq 0.90) = 90\%$), and increase the probability level of head composition satisfaction (*i.e.*, $\rho_1 = \Pr(X_D \geq 0.87) = 99\%$); or, keeping the ρ_1 fixed and increase ρ_2. This can be illustrated from Figure 2.4: either the operating point for X_B can be changed, which may lead to operating point B, or the operating point for X_D can be changed, which may lead to operating point D. However, the benefits derived from B or D will be smaller because this change does not consider the correlation between these two variables and the resulting

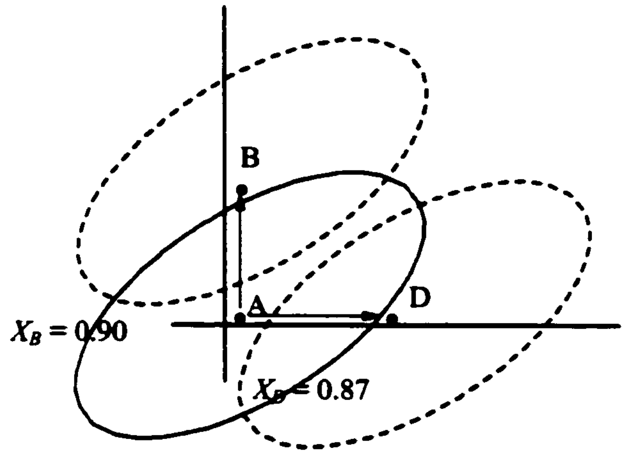


Figure 2.4: IPC approach to JPC problem

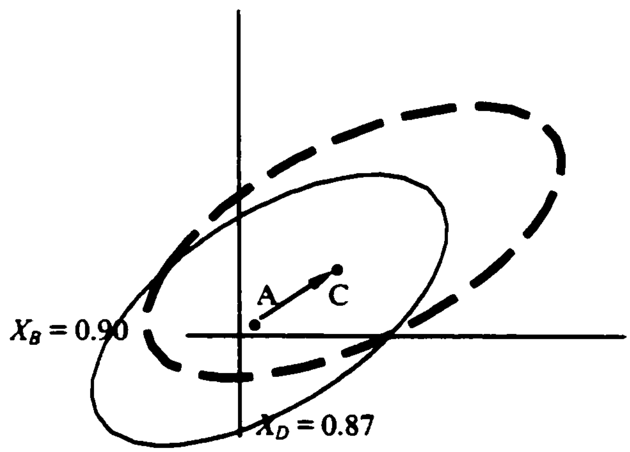


Figure 2.5: Illustration of JPC algorithm

increase in off-specification rate.

Another way is "trial and error" method, which tries to increase ρ_1 and ρ_2 sequentially. It, however, can be time consuming even for this two constraints problem. The JPC algorithm discussed in this section locates the optimal operating point (C in Figure 2.5), which gives the best performance by increasing ρ_1 and ρ_2 appropriately.

Chapter 3

Benefits Analysis

As was discussed in Chapter 2, process control system performance can be evaluated using an optimization-based approach. Results of this evaluation provide the optimal operating conditions for the process and the optimal process profitability. This is a crucial part of benefits analysis. However, a complete benefits analysis includes a number of additional steps (or phases). Marlin *et al.* (1991) provided a framework, which was adopted as the basis of benefits analysis in this thesis.

In the benefits analysis procedure (see Figure 3.1) proposed by Marlin *et al.* (1991), the opportunity for control upgrade was identified and prioritized by a team performing the benefits study. However, the method they used for benefits estimation is variance reduction, which is a commonly used approach. An optimization-based approach was discussed in Chapter 2 and will be used in the modified benefits analysis procedure. Also missed in the existing methods is a sensitivity analysis of the benefits results to the information (*i.e.*, data) used in calculating benefits.

In this Chapter, a modified benefits analysis is proposed (see Figure 3.2). This analysis includes: information collection, current performance evaluation, opportunity identification, sensitivity analysis, decision making and performance monitoring.

3.1 Information Collection

The benefits analysis study begins with information collection, which typically includes information about: the process, process control system(s), product(s) and economics. Marlin *et al.* (1991) suggested that a team be assembled for the benefits analysis study and should include: engineers experienced in plant operation, economics, instrumentation and control engineering, and senior operator. Also, a member who is not familiar with the day-to-day operation of the plant should be included to bring a fresh view to the benefits analysis study.

The team begins with reviewing the plant operation. This review includes a brief familiarization of the plant, its operation and control equipment, *etc.* The team should then prepare a detailed questionnaire for the plant personnel. An in-depth review of the plant

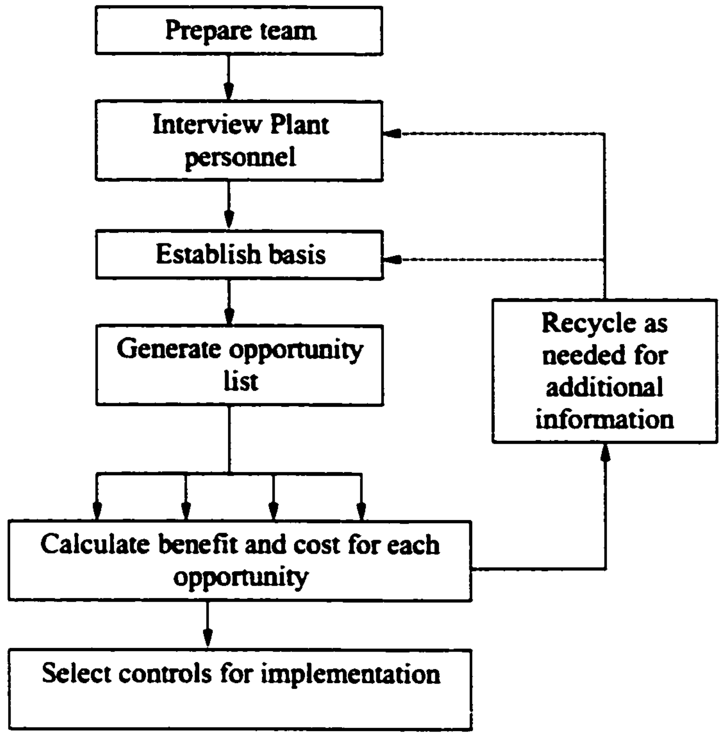


Figure 3.1: Major activities in Marlin *et al.* (1991) benefits study

then begins with gathering information from process engineers, control engineers, operators, economics and lab personnel. Before the information collection can commence, a questionnaire should be developed. An outline for such a questionnaire, as given by Marlin *et al.* (1991), is listed in Table 3.1.

Table 3.1: Outline of questionnaire for control upgrade review (Marlin *et al.*, 1991)

Process flow diagram, and P&ID review
Statement of operating goals
Brief economics overview
List of key operating variables and product qualities
List of troublesome or missing measurements, controllers, alarms and final control elements
List of parameters used to monitor the unit performance
List of potential process equipment changes

The first interviews might not achieve all of the goals for this phase of the study, or some new issues might arise during the interview process. Therefore some follow-up discussions are usually necessary. The results of the interview should give the team not only general plant information on which the benefits analysis is to be performed, but also detailed information regarding the key process variables and relationships between process performance

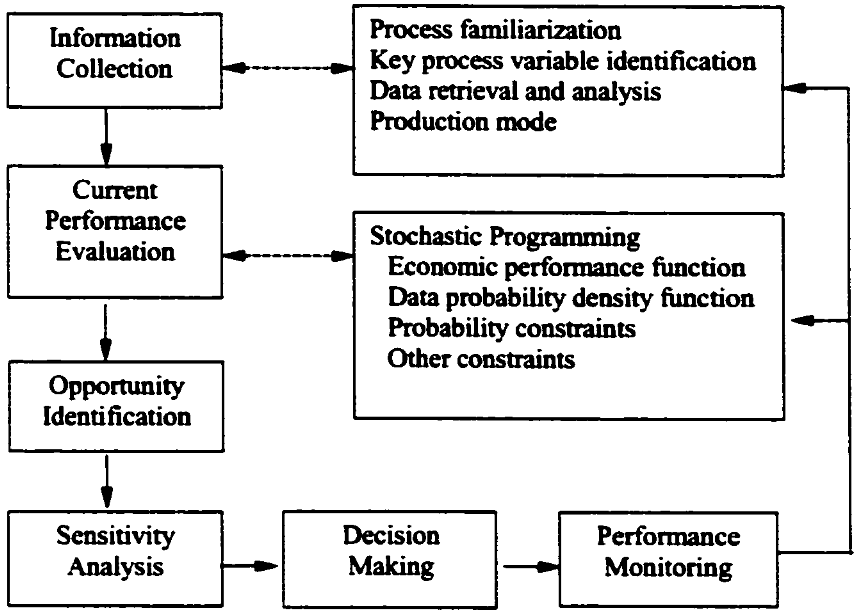


Figure 3.2: Modified benefits analysis procedure

and these key variables.

In the benefits study questionnaire, there is a list of key operating variables and product qualities. These variables are of great importance and related to the quality of the product. Data for these key variables should be recorded or retrieved from historical data. Also, the economics of the process should be discussed in detail to develop the EPF, which is a crucial component of the benefits analysis. Also, through the interview process, production constraints and process bottlenecks should be identified, and the violation frequency of these constraints should be recorded for the use in the probability constraints of the performance calculation. Thus, sufficient information is collected to set up the performance evaluation problem which is discussed in the next section.

3.2 Optimization-Based Performance Evaluation

A detailed explanation of optimization-based performance evaluation was given in Chapter 2. An example was also presented to illustrate: the proposed performance evaluation approach; the methods used to deal with the probabilistic constraints (IPC and JPC); the solutions obtained for both IPC and JPC cases and the relationship between the solutions.

3.2.1 Key Process Variables Data Analysis

During the information collection, the key variables are identified and data for these variables are recorded or retrieved from historical databases. Data analysis includes checking data trends, checking data distributions, calculating means and variances and calculating auto-correlations and cross-correlations.

If the key variables are independent of each other, each variable has an independent Normal distribution; however, in many cases, the key process variables are correlated. Then, setpoint changes for one variable influence the other correlated variables. This correlation can be calculated from process data of these variables and a joint distribution is used to describe the data distribution. For the purposes of this thesis, correlated data will be assumed to follow a joint Normal distribution, then for a single variable or independent variables the Normal distribution can be characterized by mean and variance, and for correlated variables the joint Normal distribution that is characterized by a mean and covariance matrix $\mathbf{Q}$.

3.2.2 Economic Performance Function

As introduced in Chapter 2, EPF is an important part in forming the objective function for the performance evaluation problem. In the example in Chapter 2, an economic performance function is set up for the key variables, which was a piece-wise linear function with negative value on both ends. The function was explained in the example. In this section, guidelines are given for what information should be obtained and how the information should be used to set up the EPF.

The information required for developing the EPF is often difficult to gather. Market prices for both products and feedstocks can be quite volatile and can vary substantially with market conditions (*i.e.*, economic recessions, *etc.*). Thus, some basis set of economic information must be assumed for the purpose of a benefits analysis. Not only should the price of the products and the raw materials be considered, but also the market demands for the products and the availability of the raw materials must be considered. In addition, the demand for different product grades also needs to be considered. Price and demand are two elements that affect the plant economic performance, and it is these two factors that dictate the direction for improving process profitability. When the process produces different products grades, which each have individual prices and market demands, the total effect of different production modes must be integrated into the EPF.

3.2.3 Constraints

The last step in the benefits calculation is to determine the constraints that must be considered for the purpose of benefits analysis. Possible process constraints include: product specifications, product demands, raw materials availability, equipment limitation, environmental limitation, *etc.* The only constraints considered in this thesis are product specifications and some other key variables specifications. The constraints can be divided into linear constraints and nonlinear constraints, and in another way, deterministic constraints and stochastic constraints.

Some of the constraints must be satisfied absolutely, such as safety constraints; some of them are less restrictive, such as product specification and can be violated with some frequency. In most cases, the product key variables specification is allowed to be violated with some degree of freedom. For more important variables, the probability of constraint satisfaction should be higher; while for those that are less important, the probability of constraint satisfaction might be lower. When the variables are correlated, a joint probability may be assigned and the constraints should not be treated as independent.

Some constraints should be treated as deterministic constraints (*e.g.*, safety). Since safety constraints can limit process profitability, they should be carefully investigated (and specified in the benefits calculation) to ensure they are appropriate and not too restrictive.

IPC is used when the constraints are to be satisfied at different probability levels. As illustrated in Chapter 2, the IPC problem is easier to solve. Therefore, the formulation of the performance evaluation problem in IPC form is often preferred; the IPC formulation fails to deal with the variables with correlations between each other, which is common in multi-input and multi-output system. JPC is used when the constraints are to be satisfied simultaneously with the same probability.

To sum up, the three components of the performance evaluation are: probability density function for the process variables, the economic performance function and the process constraints. Given this information for a specific process, a performance evaluation problem can be formulated, the solution of which yields an optimal operating point and the maximum process performance that can be expected.

3.3 Current System Performance Evaluation

One objective for evaluating the current control system performance is to confirm if the current operating point is optimal (*i.e.*, the mean values of the key process key variables are as close as possible to the product specification). This is done by checking where the location of current operating point, the variance of process key variables data distribution is and percentage of product specification satisfaction.

Since, for the purpose of this work, the key process variables are assumed to be Normally distributed, the measured variance is used in the performance evaluation problem for the current control system. The decision variable in the optimization problem is the mean of the key variables. Thus, the calculated mean (optimal operating point) is compared with the current operating point and product specification. Results fall into three cases: 1) the calculated operating point is closer to the specification than the current operating point; 2) the calculated operating point is the same as the current one; 3) the current operating point is closer to the specification than the calculated one.

In the first case, the current operating point is not as close to the specification as possible, potential profitability is lost. This lost profit can be reduced by simply shifting the current operating point to the optimal operating point that is calculated from the performance evaluation problem. In the second case, the current operating point is optimal, shifting the operating point will not create more profitability. In the last case, the current operating point is running too close to the specification, which will make the operating cost higher than expected. Thus, shifting the current operating point farther back should improve the profitability of the operation.

3.4 Performance Limits

Generally automation improvements are credited with increasing process performance by reducing the variance of key process variables, which allows the mean operating point to be shifted closer to the specification. It is not clear, however, how much it is possible to reduce the variance and as a result, how close the operating point can be moved to the specification. A standard or benchmark needs to be adopted for the performance evaluation, against which various automation upgrade options can be compared. Minimum variance control is used as a benchmark for controller performance assessment in this thesis. The calculated minimum variance is used as the theoretically achievable minimum variance in the performance evaluation problem. Then, the maximum theoretically achievable improvement for process performance is calculated based on this minimum variance. It, however, does not mean that this potential can be fully achieved. Nevertheless, three choices exist to improve the current performance. One is tuning of the existing control system, the second is to design a more advanced control system, the third is redesigning the process. Only the first two can be addressed via automation and will be focus of this thesis.

It has been shown (Harris, 1989) that for a single input single output (SISO) system with time delay d, a portion of the output variance is feedback control invariant and can be estimated from routine operating data. This is the minimum variance portion. To separate this invariant term, the closed-loop output data needs to be modeled by an infinite-order moving average (MA) process (Huang and Shah, 1999):

$$y_t = \underbrace{f_0 a_t + f_0 a_t + ... + f_{d-1} a_{t-(d-1)}}_{e_t} + f_d a_{t-d} + f_{d+1} a_{t-(d+1)} + \cdots \tag{3.1}$$

where, a_t is a white noise sequence and e_t is the portion of the output that is independent of feedback control (Harris, 1989). The minimum variance of the invariant portion of the output can be estimated by time series analysis of routine closed-loop operating data, and can be used subsequently as a benchmark for the theoretically achievable absolute lower bound of output variance to assess control loop performance.

The FCOR (Filtering and Correlation analysis) algorithm was used to calculate the minimum variance of a stable closed-loop process (Huang and Shah, 1999). Multiplying Equation (3.1) by $a_t, a_{t-1}, \cdots, a_{t-d+1}$ respectively and then taking the expectation of both sides of the equation yields

$$\begin{aligned}
r_{ya}(0) &= E[y_t a_t] = f_0 \sigma_a^2 \\
r_{ya}(1) &= E[y_t a_{t-1}] = f_1 \sigma_a^2 \\
r_{ya}(2) &= E[y_t a_{t-2}] = f_2 \sigma_a^2 \\
&\vdots \\
r_{ya}(d-1) &= E[y_t a_{t-d+1}] = f_{d-1} \sigma_a^2
\end{aligned} \tag{3.2}$$

where, σ_a^2 is the variance of a_t. Therefore the minimum variance or the invariant portion of output variance is

$$\sigma_{mv}^2 = (f_0^2 + f_1^2 + f_2^2 + \cdots + f_{d-1}^2)\sigma_a^2 \tag{3.3}$$

Although a_t is unknown, it can be replaced by the estimated innovations sequence $\hat{a}_t$. The estimate $\hat{a}_t$ is obtained by pre-whitening the process output variable y_t via time series analysis (Huang and Shah, 1999). Thus, σ_a^2 can be estimated from $\hat{a}_t$, and σ_{mv}^2 is then achieved.

Minimum variance control benchmark may or may not be achievable in practice depending on process invertability and other physical constraints on the processes; however, as a benchmark, it provides useful information as to the quality of the current controller performance and how much "potential" there is to improve controller performance.

The key information needed for minimum variance estimation algorithm discussed above is the dead-time d of the SISO process. The difference between SISO and MIMO (multi-input, multi-output) is the characterization of dead-time. For SISO case, dead-time is a scalar quantity. For MIMO case, one way to characterize the dead-time is the interactor matrix D (Huang and Shah, 1999). To obtain the minimum variance for MIMO system, the interactor matrix D was introduced by Wolovich and Falb (1976), Wolovich and Elliott (1983), as well as Goodwin and Sin (1984), which is the generalization of the SISO time delay for the MIMO case. The definition and calculation of interactor matrix D, and the algorithm for MIMO system minimum variance estimation are available (Huang and Shah, 1999).

3.5 Best Performance for Existing Control System

The different process variance between the current system and the minimum variance control bench mark is the driving force for further control system upgrades. Since the current controller may not be optimally tuned, the first approach to control system upgrade is retuning the existing control system. Some of the following steps are typically involved in this approach: small changes will be made to controller configurations; sensors need to be checked and upgraded; control valves should be checked. The product trends after the retuning procedure will be analyzed. The process variance after retuning is σ^2_{retune} or $\mathbf{Q}_{retune}$.

The difference between the current process variance and variance after retuning is the basis for the performance evaluation of the retuned system. Using the same algorithm as discussed in Chapter 2, the optimal operating point $\bar{\mathbf{y}}_{retune}$ and the best achievable process performance J_{retune} are estimated.

3.6 Best Performance for Proposed Control System

If the retuned performance is still far less than the theoretically achievable performance, the potential process profitability may be achieved through designing an advanced control system for the existing process. Since the performance of the proposed system needs to be known before it is implemented, simulation is used to examine the potential process variance reduction. Using the same algorithm as above, the optimal operating point $\bar{\mathbf{y}}_{redesign}$ and the best achievable process performance $J_{redesign}$ are estimated.

3.7 Benefits Comparison

Performance evaluation algorithm is applied on four different control systems: current control system, retuned, redesigned and minimum variance control system. The key character of these systems performance is the process variance. The performance of different control systems is due to the process variance resulting from use of different control systems. This can be illustrated from Figure 3.3.

As shown in the diagram, A, B, C and D represent the single process key variable data distribution achieved under different control systems. The associated process performance can be symbolized as J_A, J_B, J_C and J_D. The means of all these curves are located at the estimated optimal operating points. Also, all of these distributions satisfy the same specification violation frequency. Because of the different process variance, the distances between the operating point and the product specification are different for these four systems, which yield different economic performance.

The mean of A distribution is the farthest from the specification while the mean of D distribution is the closest to the specification. The difference between the J_A and J_D represents the maximum available profitability improvement that can be achieved through control system upgrade. Improvement of the existing control system or designing a new advanced control system leads to better control performance (represented by J_B, and J_C). Process variation can be reduced and the operating point can be pushed closer to the specification. Comparison of performance for these four systems are given in Table 3.2.

Benefits analysis not only evaluates the performance of the proposed control system, but also considers the cost of implementing these upgrades. Thus, the implementation and maintenance costs of different control systems should be deducted from the according

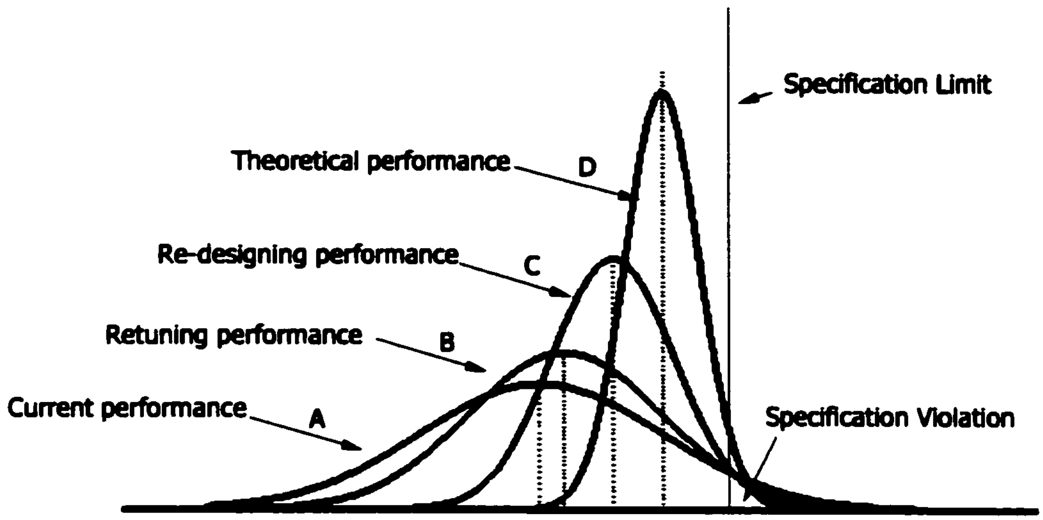


Figure 3.3: Performance comparison (single variable)

Table 3.2: Performance comparison

$J_B - J_A$	Profitability available for retuning
$J_C - J_A$	Profitability available for proposed changes
$J_D - J_B$	Uncaptured profitability after retuning
$J_D - J_C$	Uncaptured profitability after proposed changes
$J_C - J_B$	Profitability for proposed changes beyond retuning

performance. The net benefits of different systems can then be compared (see Table 3.3).

Table 3.3: Benefits comparison

$J_B - J_A - (Cost_B - Cost_A)$	Benefits available for retuning
$J_C - J_A - (Cost_C - Cost_A)$	Benefits available for proposed changes
$J_D - J_B - (Cost_D - Cost_B)$	Uncaptured benefits after retuning
$J_D - J_C - (Cost_D - Cost_C)$	Uncaptured benefits after proposed changes
$J_C - J_B - (Cost_C - Cost_B)$	Net benefits for proposed changes beyond retuning

3.8 Sensitivity Analysis

As discussed in Chapter 2, the optimal operating point is calculated by solving the performance evaluation problem. Due to uncertainties and/or variations of the problem parameters, an optimal solution gained from a deterministic optimization problem may not by itself be entirely useful (Ganesh and Biegler, 1987). Therefore, post-optimality analysis (or sensitivity analysis) becomes necessary to ascertain how parametric variations affect the optimal results under normal operations. An efficient and rigorous strategy for evaluating the first-order sensitivity of the optimal solution to changes in process parameters is proposed by Ganesh and Biegler (1987), and was adopted for sensitivity analysis of the performance evaluation problem.

Consider the general performance evaluation problem:

$$\begin{aligned} \max_{\tilde{\mathbf{y}}} \quad & J = \int_{\mathbf{Y}} (\phi(\mathbf{y}, \mathbf{s}^*, \mathbf{c}) \cdot f(\mathbf{y}, \tilde{\mathbf{y}}, \mathbf{Q}))d\mathbf{y} \\ s.t.: & \\ & \mathbf{h}_i\tilde{\mathbf{y}} \geq s_i^* - (\mathbf{h}_i\mathbf{Q}\mathbf{h}_i)^{1/2}F(1-\alpha_i) \end{aligned} \tag{3.4}$$

where,

$$\begin{aligned} \mathbf{c} &\equiv \text{economics} \\ \mathbf{Q} &\equiv \text{estimated variance matrix of key variable data} \\ s_i^* &\equiv \text{product specifications} \\ \alpha_i &\equiv \text{constraints probability levels} \\ \mathbf{h}_i &\equiv \text{model coefficients} \end{aligned}$$

The problem can be written in the general form:

$$\begin{aligned} \max_{\tilde{\mathbf{y}}} \quad & J(\tilde{\mathbf{y}}, \mathbf{p}) \\ s.t.: & \\ & \mathbf{g}(\tilde{\mathbf{y}}, \mathbf{p}) \leq 0 \end{aligned}$$

where, $\mathbf{g}$ is a vector of inequality constraints, and $\mathbf{p} = \begin{bmatrix} \mathbf{Q} & s_i^* & \alpha_i & \mathbf{h}_i & \mathbf{c} \end{bmatrix}^T$. With the assumptions discussed in Ganesh and Biegler (1987), the KKT conditions at the optimum $\tilde{\mathbf{y}}_0$ are:

$$\begin{aligned} \nabla_{\tilde{\mathbf{y}}} L(\tilde{\mathbf{y}}_0, \mathbf{p}_0) &= 0 \\ \mathbf{g}_A(\tilde{\mathbf{y}}_0, \mathbf{p}_0) &= 0 \end{aligned} \tag{3.5}$$

where $L = J + \mathbf{u}^T\mathbf{g}$ (Lagrangian function) and $\mathbf{g}_A$ are the active inequality constraints. Using the Implicit Function Theorem, the first-order variation of the constraints is:

$$\begin{aligned} d\left[\nabla_{\tilde{\mathbf{y}}} L(\tilde{\mathbf{y}}_0, \mathbf{p}_0)\right] &= \nabla_{\tilde{\mathbf{y}}\tilde{\mathbf{y}}} L^0 d\tilde{\mathbf{y}} + \nabla_{\tilde{\mathbf{y}}} \mathbf{g}_A^0 d\mathbf{u} + \nabla_{\mathbf{p}\tilde{\mathbf{y}}} L^{T0} d\mathbf{p} = 0 \\ d\left[\mathbf{g}_A(\tilde{\mathbf{y}}_0, \mathbf{p}_0)\right] &= \nabla_{\tilde{\mathbf{y}}} \mathbf{g}_A^{T0} d\tilde{\mathbf{y}} + \nabla_{\mathbf{p}} \mathbf{g}_A^{T0} d\mathbf{p} = 0 \end{aligned} \tag{3.6}$$

Rearranging these expressions in the form of linear system of equations:

$$\begin{bmatrix} \nabla_{\mathbf{p}\tilde{\mathbf{y}}} L^{T0} \\ \nabla_{\mathbf{p}} \mathbf{g}_A^{T0} \end{bmatrix} = -\begin{bmatrix} \nabla_{\tilde{\mathbf{y}}\tilde{\mathbf{y}}} L^0 & \nabla_{\tilde{\mathbf{y}}} \mathbf{g}_A^0 \\ \nabla_{\tilde{\mathbf{y}}} \mathbf{g}_A^{T0} & 0 \end{bmatrix} \begin{bmatrix} \nabla_{\mathbf{p}} \tilde{\mathbf{y}} \\ \nabla_{\mathbf{p}} \mathbf{u} \end{bmatrix} \tag{3.7}$$

The sensitivity of the setpoint for process variables to the parameters can be obtained by solving Equation (3.7). However, not all of the parameters are of interests in this thesis. It is assumed that the EPF is fixed during the benefits analysis period, as are the economic parameters $\mathbf{c}$. The product specifications (s_i^*) are also assumed to be known and fixed during the benefits study. The model parameters $\mathbf{h}_i$, used in the probabilistic constraints, will affect the solution to the performance evaluation problem; however, for the purpose of this study, $\mathbf{h}_i$ was assumed to be fixed while the key variables were assumed to follow a Normal distribution.

The remaining parameters are $\mathbf{Q}$ and α_i. For each particular control system, the covariance matrix $\mathbf{Q}$ is assumed to be fixed in this work. Then, only the parameter α_i, which are specified (somewhat arbitrarily) during the benefits analysis, are considered here. Therefore, the changes of process profitability J and process operating point $\tilde{\mathbf{y}}$ with respect to probability levels α_i is the focus of this sensitivity analysis study.

It is clear that from Equation (3.7),

$$[\nabla_{\tilde{\mathbf{y}}}\mathbf{g}_A^{T0}]\,\nabla_{\boldsymbol{\alpha}}\tilde{\mathbf{y}} = -\nabla_{\boldsymbol{\alpha}}\mathbf{g}_A^{T0} \tag{3.8}$$

and has a unique solution when $\nabla_{\tilde{\mathbf{y}}}\mathbf{g}_A^{T0}$ is invertible. This assumption is satisfied if any of following two cases applies:

Case 1: If there is one independent active constraint for each $\tilde{y}_i$ at the optimum (*i.e.*, $\mathbf{g}_\mathbf{A} = \mathbf{g}$), since $\frac{d\mathbf{g}}{d\tilde{\mathbf{y}}} = \mathbf{H}$, the sensitivity of $\tilde{\mathbf{y}}$ with respect to $\boldsymbol{\alpha}$ is:

$$\nabla_{\boldsymbol{\alpha}}\tilde{\mathbf{y}} = -\mathbf{H}^{-1}\nabla_{\boldsymbol{\alpha}}\mathbf{g}_A$$

where

$$\begin{aligned}\frac{dg_j}{d\alpha_j} &= (\mathbf{h}_j\mathbf{Q}\mathbf{h}_j^T)^{1/2}\frac{dF^{-1}(1-\alpha_j)}{d\alpha_j}\\ &= -(\mathbf{h}_j\mathbf{Q}\mathbf{h}_j^T)^{1/2}\frac{dF^{-1}(\alpha_j)}{d\alpha_j}\\ &= -(\mathbf{h}_j\mathbf{Q}\mathbf{h}_j^T)^{1/2}\frac{1}{pdf_j}\end{aligned}$$

and pdf_j is the probability density function of $\mathbf{h}_j\mathbf{y}$. Therefore,

$$\frac{d\tilde{y}_i}{d\alpha_j} = (\mathbf{H}^{-1})_i(\mathbf{h}_j\mathbf{Q}\mathbf{h}_j^T)^{1/2}\frac{1}{pdf_j} \tag{3.9}$$

where, $(\mathbf{H}^{-1})_i$ denotes the i^{th} row of $\mathbf{H}^{-1}$. Then,

$$\frac{d\tilde{y}_i}{d\alpha_j} = (\mathbf{H}^{-1})_i\frac{(\mathbf{h}_j\mathbf{Q}\mathbf{h}_j^T)^{1/2}}{pdf_j} = \sqrt{2\pi}(\mathbf{H}^{-1})_i(\mathbf{h}_j\mathbf{Q}\mathbf{h}_j^T)e^{\frac{(\mathbf{h}_j\mathbf{y}-\mathbf{h}_j\tilde{\mathbf{y}})^2}{2(\mathbf{h}_j\mathbf{Q}\mathbf{h}_j^T)^2}} \tag{3.10}$$

At the optimum operating point,

$$\frac{d\tilde{y}_i^*}{d\alpha_j} = \sqrt{2\pi}(\mathbf{H}^{-1})_i(\mathbf{h}_j\mathbf{Q}\mathbf{h}_j^T)e^{\frac{(\mathbf{h}_j\tilde{\mathbf{y}}^*-\mathbf{h}_j\tilde{\mathbf{y}}^*)^2}{2(\mathbf{h}_j\mathbf{Q}\mathbf{h}_j^T)^2}} = \sqrt{2\pi}(\mathbf{H}^{-1})_i(\mathbf{h}_j\mathbf{Q}\mathbf{h}_j^T) \tag{3.11}$$

It is clear that the first-order change in the i^{th} optimal operating condition with respect to the j^{th} probability level is a multiplication of i^{th} row of $\mathbf{H}^{-1}$ and variance which characterize the distribution of $\mathbf{h}_j\mathbf{y}$. Therefore, small change $\Delta\boldsymbol{\alpha}_j$ leads to a change of $\Delta\tilde{\mathbf{y}}_i$ in the same direction.

The sensitivity of the process profitability with respect to the change in probability level is:

$$\frac{dJ}{d\boldsymbol{\alpha}} = \frac{dJ}{d\tilde{\mathbf{y}}^*}\nabla_{\boldsymbol{\alpha}}\tilde{\mathbf{y}}^* \tag{3.12}$$

Case 2: If not all of the probability constraints are active at the optimum, the optimal solution can be partitioned as:

$$\tilde{\mathbf{y}} = [\ \tilde{\mathbf{y}}_c\,|\tilde{\mathbf{y}}_u]$$

where, $\tilde{\mathbf{y}}_c$ are constrained process variables and $\tilde{\mathbf{y}}_u$ are unconstrained process variables. Since the constraints are of the form:

$$\mathbf{h}_i\tilde{\mathbf{y}} \geq s_i^* - (\mathbf{h}_i\mathbf{Q}\mathbf{h}_i)^{1/2}\mathcal{F}(1-\alpha_i)$$

which can be rewritten as:

$$\tilde{y}_i \geq s_i^* - (\mathbf{h}_i\mathbf{Q}\mathbf{h}_i)^{1/2}\mathcal{F}(1-\alpha_i) - \bar{g}(\tilde{y}_j) \quad i \neq j \tag{3.13}$$

Then, $\tilde{\mathbf{y}}_c$ and $\tilde{\mathbf{y}}_u$ can be identified through the optimization results. Equation (3.8) can be written as:

$$\nabla_{\boldsymbol{\alpha}}\tilde{\mathbf{y}}_c = -\left[\nabla_{\tilde{\mathbf{y}}_c}\mathbf{g}_A^{T0}\right]^{-1}\nabla_{\boldsymbol{\alpha}}\mathbf{g}_A^{T0} \tag{3.14}$$

and Equation (3.11) becomes:

$$\frac{d\tilde{y}_{ci}^*}{d\alpha_j} = \sqrt{2\pi}(\mathbf{H}^{-1})_i(\mathbf{h}_j\mathbf{Q}\mathbf{h}_j^T) \tag{3.15}$$

If the relationship between each key variables can be determined, the constraint parameter matrix $\mathbf{H}$ can then be developed. It, however, needs thorough understanding of the process dynamics and process models, which is not the purpose of this thesis. In many cases, it is easy to define each process variable specification individually, which leads $\mathbf{H}$ matrix to be an identity matrix. Further, when the individual probability constraint levels are assumed to be identical:

$$\alpha_1 = \alpha_2 = \alpha_j = \cdots = \alpha \tag{3.16}$$

$$\mathbf{H} = \mathbf{I} \tag{3.17}$$

Since

$$\frac{d\tilde{y}_i^*}{d\alpha_j} = \sqrt{2\pi}(\mathbf{H}^{-1})_i(\mathbf{h}_j\mathbf{Q}\mathbf{h}_j^T) \tag{3.18}$$

$$(\mathbf{h}_j\mathbf{Q}\mathbf{h}_j^T) = \sqrt{2\pi}\sigma_j^2 \tag{3.19}$$

at the optimum:

$$\frac{d\bar{y}_i^*}{d\alpha_j} = \frac{d\bar{y}_i^*}{d\alpha} = \sqrt{2\pi}\sigma_i^2 \tag{3.20}$$

To this point sensitivity analysis discussions have focused only on the IPC case. For the JPC case, the sensitivity of the process profitability with respect to the joint probability level α can be obtained if the change of each individual probability level were known. For example, the individual probability constraints are:

$$\rho_{Ii} = \Pr(y_i \geq s_i^*) \geq \alpha_{Ii}$$

and the joint probability constraint is:

$$\rho_{J0} = \Pr\begin{pmatrix} y_i \geq s_i^* \\ \vdots \end{pmatrix} \geq \alpha_J$$

The solution of the JPC problem not only finds ρ_{J0} but also the associated ρ_{Ji}

$$\rho_{Ji} = \Pr(y_i \geq s_i^*) \geq \alpha_{Ji}$$

It turns out that $\rho_{Ji} \geq \rho_{Ii}$, which means the individual probability level should be all or partially increased to satisfy the joint probability level. The final probability level is achieved through a combination of each individual probability movement. The basis $\vec{n}$ of the joint probability movement can be calculated if each individual moving direction and length is determined. Then, from ρ_{Ii} and ρ_{Ji}, the length is determined to be $\triangle\rho_i = \rho_{Ji} - \rho_{Ii}$. Then, the basis is calculated:

$$\vec{n} = \frac{[\ \triangle\rho_1 \ \ \triangle\rho_2 \ \ \cdots \ \ \triangle\rho_k\ ]^T}{\left\| [\ \triangle\rho_1 \ \ \triangle\rho_2 \ \ \ \triangle\rho_k\ ]^T \right\|_2}$$

It also can be seen from the illustration in Figure 3.4 that the basis is the combination of each movement of individual probability level. Then the sensitivity of process performance with respect to the JPC probability level α_J is:

$$\frac{d\bar{y}_i^*}{d\alpha_J} = \frac{d\bar{y}_i^*}{d\boldsymbol{\alpha}_I}\vec{n}$$

where $\boldsymbol{\alpha}_I = [\ \alpha_{I1} \ \ \alpha_{I2} \ \ \cdots \ \ \alpha_{Ik}\ ]^T$. $\frac{d\bar{y}_i^*}{d\boldsymbol{\alpha}_I}$ can be calculated following the same procedure as that of IPC case.

Once the sensitivity of process profitability and process operating conditions with respect to the change in probability level (*i.e.*, $\frac{dJ}{d\alpha}$ and $\frac{d\bar{\mathbf{y}}}{d\alpha}$) is obtained, from small changes in probability level $\triangle\alpha$, the corresponding change in the process profitability and process operating conditions (*i.e.*, $\triangle J$ and $\triangle\bar{\mathbf{y}}$) can be estimated:

$$\begin{aligned} \triangle J &= \frac{dJ}{d\alpha}\triangle\alpha \\ \triangle\bar{\mathbf{y}} &= \frac{d\bar{\mathbf{y}}}{d\alpha}\triangle\alpha \end{aligned}$$

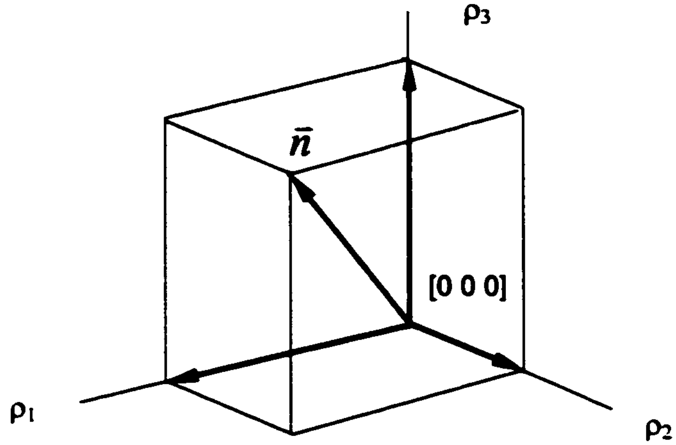


Figure 3.4: Illustration of basis of senstivity analysis for JPC case

Then, $\triangle J$ for different control systems are calculated (*i.e.*, $\triangle J_A, \triangle J_B, \triangle J_C$ and $\triangle J_D$, representing the performance of current system, retuned system, redesigned system and minimum variance control system respectively) at the same probability level α. From Table 3.3, if it is assumed that based on nominal analysis, a decision is made to choose SYS_i as the result of the benefits analysis study. The sensitivity of the performance of SYS_i can be easily seen from $\frac{dJ_i}{d\alpha}$. There is also an alternative solution to this benefits analysis SYS_j, which yields the performance slight lower than the proposed system. The sensitivity of the performance of SYS_j can be easily seen from $\frac{dJ_j}{d\alpha}$. Comparing SYS_i and SYS_j, it is easy to see that the profitability difference is $\triangle P = J_i - J_j - (Cost_i - Cost_j) > 0$. However, this conclusion is based on a specific probability level α_0. The sensitivity of performance of different systems with respect to α is different, which means the change of α may affect the conclusion that SYS_i yields higher profitability than SYS_j. A key issue is :" how much α must change to affect the decision (*i.e.*, SYS_i yields no more profit than SYS_j)?". The first-order Taylor series approximation of the net profitability of each system at the probability level α is:

$$J_i(\alpha) - Cost_i \approx J_i(\alpha_0) - Cost_i + \left.\frac{d(J_i - Cost_i)}{d\alpha}\right|_{\alpha_0} (\alpha - \alpha_0)$$

$$J_j(\alpha) - Cost_j \approx J_j(\alpha_0) - Cost_j + \left.\frac{d(J_j - Cost_j)}{d\alpha}\right|_{\alpha_0} (\alpha - \alpha_0)$$

Let $\Delta\alpha = (\alpha - \alpha_0)$, $Cost_i$ and $Cost_j$ are assumed to be constants. Assuming that there exists an α such that

$$J_i - Cost_i = J_j - Cost_j$$

which means SYS_i yields the same profit as that of SYS_j. Then,

$$J_i(\alpha_0) - J_j(\alpha_0) - (Cost_i - Cost_j) = \frac{dJ_i - dJ_j}{d\alpha}\Delta\alpha$$

$\Delta\alpha$ is then determined by:

$$\Delta\alpha = \left[- \left. \frac{dJ_i - dJ_j}{d\alpha} \right|_{\alpha_0} \right]^{-1} [J_i(\alpha_0) - J_j(\alpha_0) - (Cost_i - Cost_j)]$$

The results fall within three cases:

Case 1: $|\Delta\alpha| > \Delta\alpha_0$, where $\Delta\alpha_0$ is a user defined constant. For the purpose of this thesis, $\Delta\alpha_0$ is determined to be: $\Delta\alpha_0 = 20\%$, which means if increasing or decreasing the probability satisfaction level by 20% will not lead to $\Delta P = J_i - J_j - (Cost_i - Cost_j) = 0$. The original decision (*i.e.*, SYS_i yields higher profitability than SYS_j) is reliable. The evaluated profitability for SYS_i is not sensitive to the probability level. Therefore, implementing SYS_i is appropriate.

Case 2: $|\Delta\alpha| < \Delta\alpha_1$, where $\Delta\alpha_1$ is also a user defined constant, $\Delta\alpha_1 < \Delta\alpha_0$. For the purpose of this thesis, $\Delta\alpha_0$ is determined to be: $\Delta\alpha_0 = 5\%$, which means if increasing or decreasing the probability satisfaction level by 5% will lead to $\Delta P = J_i - J_j - (Cost_i - Cost_j) = 0$. The original decision is not reliable considering the change in probability level. Therefore, implementing SYS_i is not the recommended solution to this benefits analysis, and SYS_j should be considered as a candidate for control system upgrade.

Case 3: $\Delta\alpha_1 \leq |\Delta\alpha| \leq \Delta\alpha_0$, the decision should be made based on both α and $|\Delta\alpha|$. For example, if $|\Delta\alpha| = 10\%$, and the current probability level α_0 is 80%, at which SYS_i yields higher profitability than SYS_j. If probability level is raised up to $\alpha = \alpha_0 + |\Delta\alpha| = 90\%$, the profitability of two systems drops to the same level. If 80% is good enough for the production, then SYS_i should be implemented; however, if more than 90% is often required, then SYS_j should be considered as a candidate.

To sum up, two parameters are defined as lower bound and higher bound to justify whether a decision should be kept. If $|\Delta\alpha| \geq 20\%$, the decision should be kept; if $|\Delta\alpha| \leq 5\%$, the decision is sensitive to α and therefore should be discarded; if $5\% \leq |\Delta\alpha| \leq 20\%$, the decision can be kept based on the values of both $|\Delta\alpha|$ and α.

Note that ΔP is a random variable, then $\Delta P = 0$ is reliable only within a statistical confidence interval. When the data used in the benefits evaluation is small, the variance of ΔP should be calculated (*i.e.*, $\sigma_{\Delta P}$), and $\Delta P \pm \sigma_{\Delta P} = 0$ is used in the sensitivity calculation. However, in this thesis, the data set is large enough, ΔP can be taken as an expected value $\Delta\hat{P}$.

3.9 Decision Making

Based on the information of benefits comparison and sensitivity study, a decision can be made by choosing one of the options: keep the current control system; retune the current

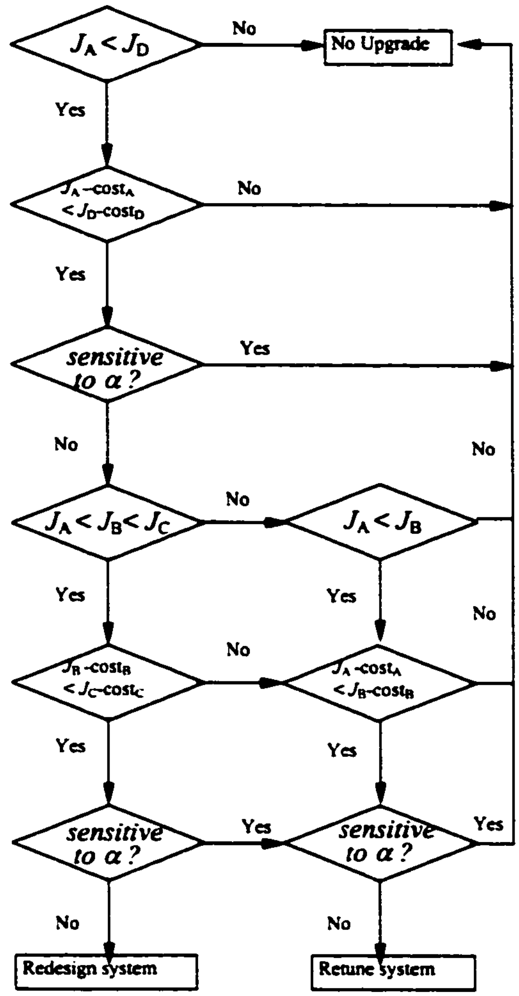


Figure 3.5: Decision making flow chart

control system; redesign an advanced control system. The procedure can be illustrated in Figure 3.5. The symbols used in this diagram agree with those in Figure 3.3 and Tables 3.2 and 3.3.

3.10 Performance Tracking

The proposed control system is implemented because it yields the best performance for the existing system. It, however, will not always be optimal because the assumptions made in the performance evaluation (*e.g.*, the market prices do not change, the operating mode is fixed, and so forth). In addition, the controller performance will degrade with time, which will also make the product deviate from the expected value. Therefore, it is necessary to perform performance tracking.

The reasons of the unexpected poor performance should be analyzed, and the performance of the system should be reevaluated based on the changed informations.

3.11 Summary of Benefits Analysis Procedure

A complete control system benefits analysis procedure was discussed in this chapter, which is based on the study by Marlin *et al.* (1991). The key part of benefits analysis, benefits estimation, is performed with an optimization approach. Also, this procedure incorporates the sensitivity analysis of process performance with respect to the probability level, which is not considered in Marlin *et al.* (1991).

The modified benefits analysis procedure is illustrated in Figure 3.6. Work enclosed by solid lines are discussed and performed in this thesis; work enclosed by dash lines are recommended for future study.

The top part of this diagram, which is enclosed in a dash rectangle, denotes a flow chart of a process. Products from this process are sold according to different market prices and demands. At the same time, key process variables distributions are examined to evaluate the product quality. The smaller the variance, the better the product quality. From the product distribution and market prices, PDF (characterized by $\bar{\mathbf{y}}$ and $\mathbf{Q}$) and EPF are developed, which are used to formulate the performance evaluation problem. Incorporating the product specifications as IPC or JPC into the problem, the performance evaluation problem is formulated as a stochastic programming problem. The solutions to this problem are the maximized process performance and the optimal operating points. Sensitivity analysis of process performance with respect to the change of probability level is performed at the optimal operating points. If process performance is sensitive to the probability level, the proposed control system upgrade should not be implemented.

The performance of four different control systems alternative are determined. The implementation cost should be deducted from the performance of associated control system. The resulting benefits of four systems are then compared. Combining the results of benefits comparison and sensitivity study, a decision can be made on whether a control system upgrade can improve the process performance, and if yes, which proposed control system (*i.e.*, retuned control system or redesigned control system) should be implemented. The final step is performance tracking.

Note that the work not discussed in this thesis are: modeling between process key variables and product demands, raw materials availability, equipment limitations, environmental limitations, and so forth; sensitivity analysis of process performance with respect to the process or model parameters other than probability level; control upgrade auditing.

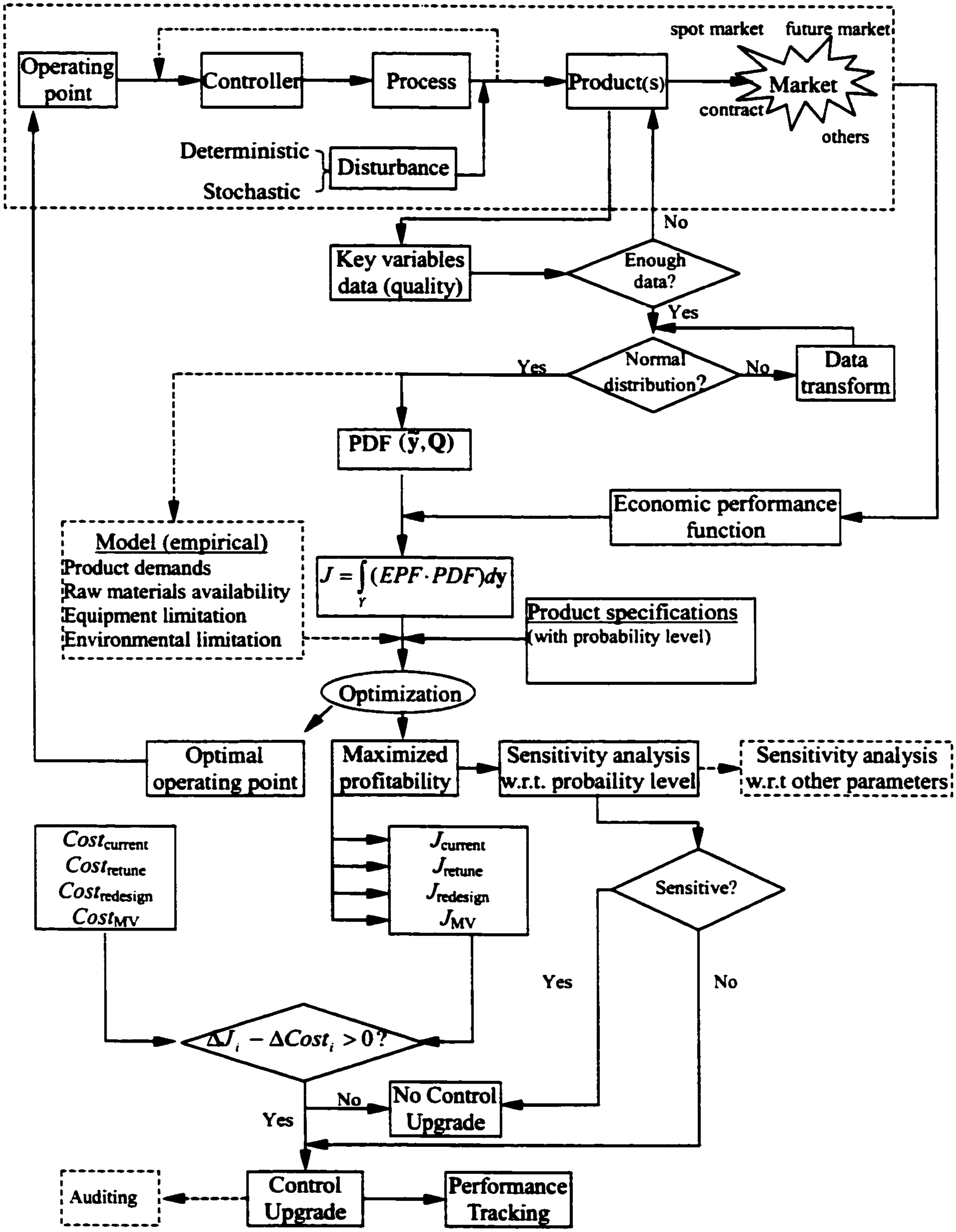


Figure 3.6: Complete benefits analysis procedure

Chapter 4

Case Studies

The benefits analysis procedure outlined in this thesis was used on two case studies. In the first case study, a benefits analysis was performed on a simulation of a pilot plant in the Computer Process Control Lab at the University of Alberta. This pilot plant consists of two stirred tank heaters in series. In the second case study, a similar benefits analysis procedure was performed on a final stage of bleaching plant at Alberta Pacific Forest Industries Inc.(ALPAC). However, different from the first case study, real data was used to formulate the performance evaluation problem. The current control system performance was evaluated against the performance limit calculated using the real data. The purpose of the second case study is to determine whether there is a sufficiently large potential economic benefit for ALPAC to warrant further study of the possible automation improvements for this part of their bleach plant.

4.1 Two Stirred Tank Heaters in Series

The objective of the this case study is to illustrate the proposed benefits analysis procedure using a simulation of two stirred tank heaters in series. In this study, the optimal process profitability of different control systems was compared, and one of them was chosen to be implemented based on the results of the benefits analysis and sensitivity study.

4.1.1 Process Description

The process considered in this case study is two stirred tank heaters in series. For the purpose of this study, the cold water and steam flow rates are the manipulated variables (see Figure 4.1) . Inlet temperature T_2 and second tank level h_2 are the controlled variables. The control objective is to keep T_2 and h_2 at the specified values $h_2 = 40$ cm and $T_2 = 35$ $^{\circ}C$. This process may be affected by disturbances such as the ambient temperature and by the manipulated variables. Both the process model and the disturbance model are given in Figure 4.2.

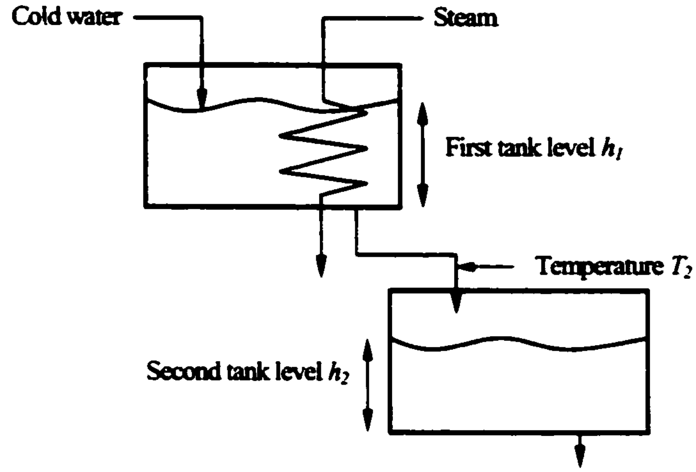


Figure 4.1: Two stirred tank heaters in series

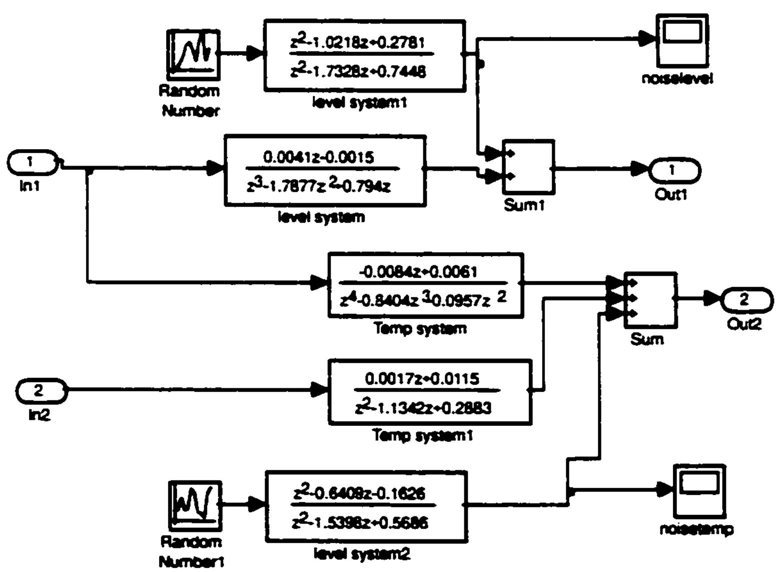


Figure 4.2: Two stirred tank heaters in series model

Controllers

The Simulink diagram of this process is shown in Figure 4.3. Current control system for

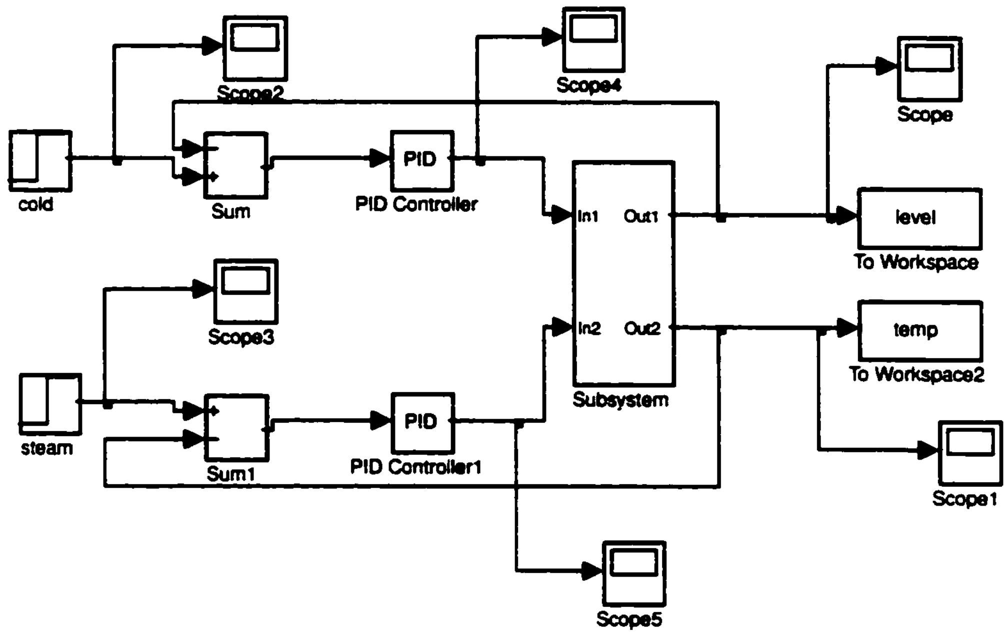


Figure 4.3: Simulink diagram for two stirred tank heaters in series

this MIMO system are PI controllers, whose parameters are:

Table 4.1: PI controllers parameters

	P	I
$Controller_1$	1	0.08
$Controller_2$	2	0.6

Since the current performance will be compared with the retuned and redesigned (MPC) control systems, the implementation costs of these two systems were assumed to be $0.08M\$/year$ and $0.2M\$/year$ respectively (These values were chosen arbitrarily for illustration purposes, but a representative relative costs).

EPF

The key variables for this process are defined to be the controlled variables, T_2 and h_2. The EPF for both key variables are expressed as following:

$$\phi_1 = \begin{cases} -2 & h_2 \le 35cm \\ 1.4h_2 - 51 & 35cm \le h_2 \le 40cm \\ -0.6h_2 + 25 & 40cm \le h_2 \le 45cm \\ -2 & h_2 \ge 45cm \end{cases} \tag{4.1}$$

$$\phi_2 = \begin{cases} -2 & T_2 \le 25°C \\ -0.7T_2 - 19.5 & 25°C \le T_2 \le 35°C \\ -0.6T_2 + 22 & 35°C \le T_2 \le 40°C \\ -2 & T_2 \ge 40°C \end{cases} \tag{4.2}$$

The expressions for both ϕ_1 and ϕ_2 are formulated based on the process profitability at different product qualities. The expressions were arbitrarily chosen for illustrative purpose. At the product specifications, ϕ_1 and ϕ_2 are of the highest values, $\phi_1 = 5\$/ton$, $\phi_2 = 5\$/ton$. To the left, process profitability goes down gradually to $\phi_1 = -2\$/ton$ and $\phi_2 = -2\$/ton$ at the points $h_2 = 35\ cm$ and $T_2 = 25\ °C$; to the right, process profitability falls down dramatically to $\phi_1 = 1\$/ton$ and $\phi_2 = 1\$/ton$ at the points $h_2 = 40\ cm$ and $T_2 = 35°C$, then goes down slowly to $\phi_1 = -2\$/ton$ and $\phi_2 = -2\$/ton$ at the points $h_2 = 45\ cm$ and $T_2 = 40°C$. It is clear from the expressions that each EPF is a function of only one key variable. Since it was assumed that the process profitability can be obtained individually from the values of h_2 and T_2, ϕ_1 and ϕ_2 are independent to each other.

PDF

At current operating condition, the data for h_2 and T_2 were recorded and plotted in Figure 4.4. A *Q-Q* plot was used to check the normality (Johnson and Wichern, 1992) of the data distribution (see Figure 4.5 and Figure 4.6). As seen the *Q-Q* plot is approximately linear (Normally distributed data should produce a linear *Q-Q* plot). For the purpose of this case study, the data was recognized to satisfy Normal distribution. The standard deviations were calculated from the data and found to be, $\sigma_1 = 1.0577$, $\sigma_2 = 0.7346$. Thus, the probability density function for each set of data can be expressed as

$$pdf_i = f(y_i, \tilde{y}_i, \sigma_i) = \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{(y_i - \tilde{y}_i)^2}{2\sigma_i^2}} \qquad i = 1, 2 \tag{4.3}$$

Note that $\tilde{y}_i$ denotes the mean value for one of process variables and σ_i denotes the corresponding standard deviation.

Constraints

Although many constraints might be incorporated into the process, only key variable specifications are constrained with an assumed probability level in this case study. The constraints

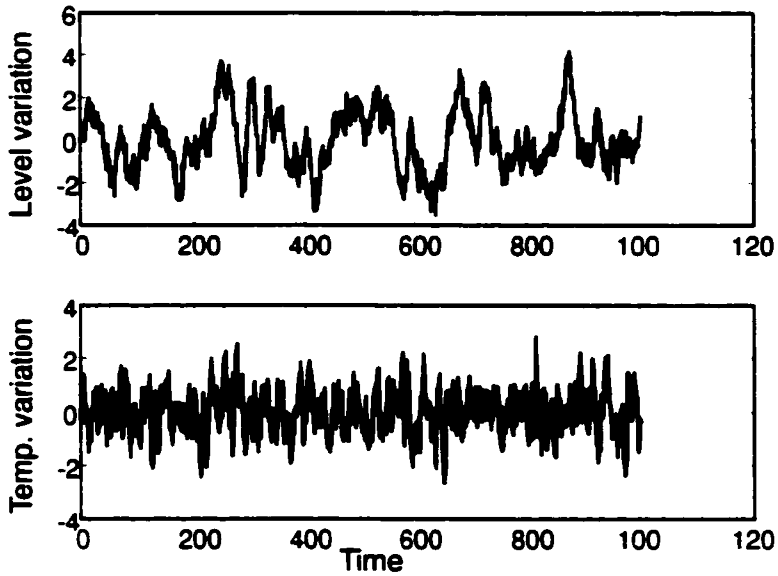


Figure 4.4: h_2 and T_2 time series

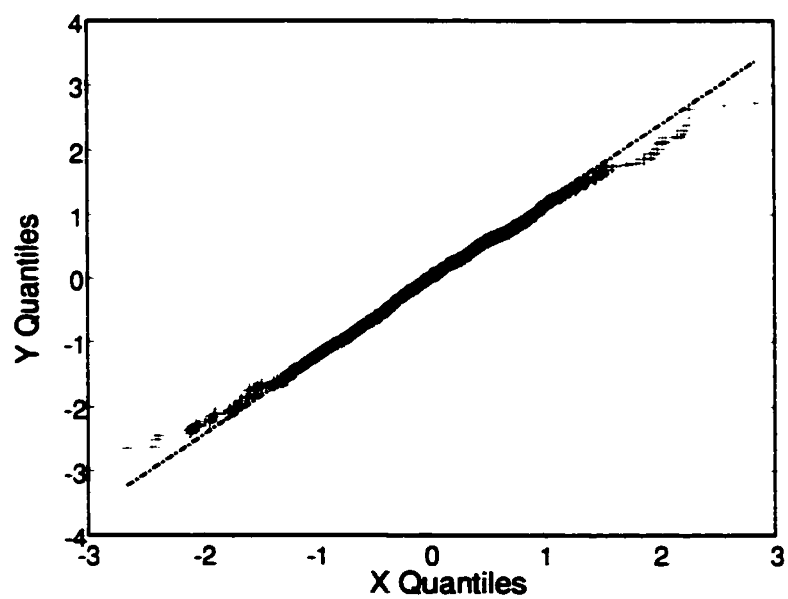


Figure 4.5: Q-Q plot of h_2 data

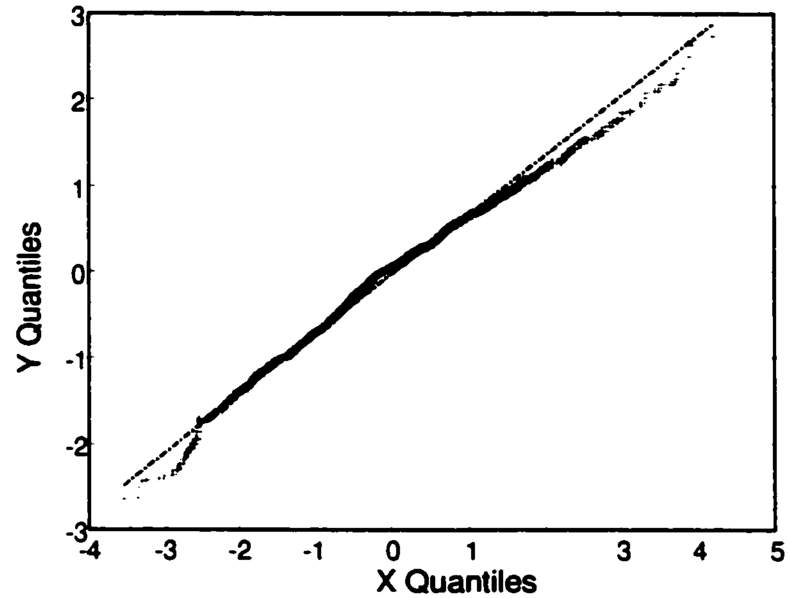


Figure 4.6: *Q-Q* plot of T_2 data

were formulated as

$$\rho_1 = \Pr(h_2 \geq 37\ cm) \geq 90\%$$
$$\rho_2 = \Pr(T_2 \geq 33\ ^\circ C) \geq 90\%$$

4.1.2 Case Study Procedure

This case study was performed in the following steps:

1. Current control system performance evaluation
2. Performance limits (Minimum variance control system evaluation)
3. Retuned control system performance evaluation
4. Redesigned control system performance evaluation (MPC)
5. Performance comparison
6. Benefits comparison
7. Sensitivity analysis on the proposed control system upgrade

Current System Performance Evaluation

As discussed in Chapter 3, with the information of EPF, PDF and constraints, the performance evaluation problem for this case study is formulated as

$$\begin{array}{ll} \max\limits_{\bar{\mathbf{y}}} & J = \int\limits_{\mathbf{Y}_1} (\phi_1 \cdot f_1) dy_1 + \int\limits_{\mathbf{Y}_2} (\phi_2 \cdot f_2) dy_2 \\ \text{subject to:} & \\ \text{IPC} & \begin{array}{l} \rho_1 = \Pr(y_1 \geq 37\ cm) \geq 90\% \\ \rho_2 = \Pr(y_2 \geq 33\ ^\circ C) \geq 90\% \end{array} \end{array} \tag{4.4}$$

or

$$\begin{array}{lll} \max\limits_{\tilde{\mathbf{y}}} & J = \int\limits_{\mathbf{Y}_1} (\phi_1 \cdot f_1) dy_1 + \int\limits_{\mathbf{Y}_2} (\phi_2 \cdot f_2) dy_2 & \\ \text{subject to:} & & \\ \text{JPC} & \rho_0 = \Pr \begin{pmatrix} y_1 \geq 37\ cm \\ y_2 \geq 33\ ^\circ C \end{pmatrix} \geq 90\% & \end{array} \tag{4.5}$$

where, $\mathbf{y} = [y_1, y_2]'$, y_1 denotes h_2 and y_2 denotes T_2.

Current performance was evaluated using the variance from the current data. An optimal operating point was obtained by solving the IPC and JPC problem. The individual probabilistic constraints can be converted to:

$$\begin{array}{l} \tilde{y}_1 \geq 37 + 1.36\ cm \\ \tilde{y}_2 \geq 33 + 0.94\ ^\circ C \end{array}$$

The optimal operating points were found at the constraints:

$$\begin{array}{l} \tilde{y}_1^* = 38.36\ cm \\ \tilde{y}_2^* = 33.94\ ^\circ C \end{array}$$

The total optimized profitability was $J = 4.25\ \$/ton$.

In the above calculation, it was assumed that two controlled variables are independent. In fact, they are correlated to some degree. The correlation between the two variables was found to be:

$$R = \begin{bmatrix} 1 & -0.024 \\ -0.024 & 1 \end{bmatrix}$$

It is easily seen that the correlation between two variables is not large, however, for the purpose of illustration in solving JPC optimization problem, the correlation is considered. The joint probability constraints are specified as:

$$\rho_0 = \Pr \begin{bmatrix} y_1 \geq 37\ cm \\ y_2 \geq 33\ ^\circ C \end{bmatrix} \geq 90\%$$

Performance Limits

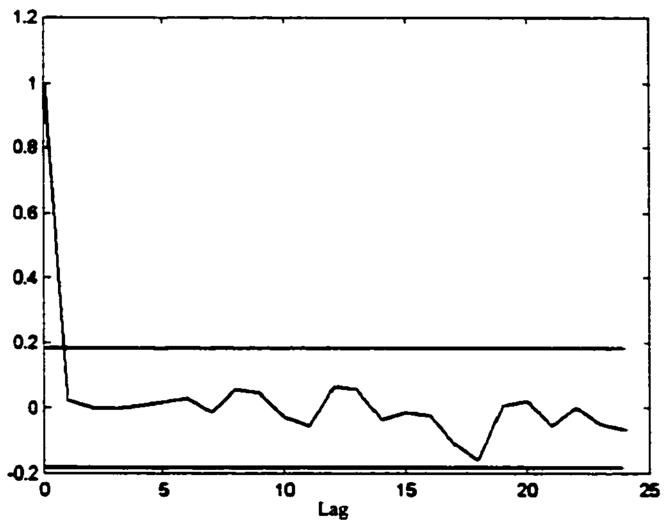
The model used in this simulation is specified:

$$\begin{bmatrix} h_2 \\ T_2 \end{bmatrix} = \begin{bmatrix} \frac{0.0041q^{-2} - 0.0015q^{-3}}{1 - 1.7877q^{-1} + 0.7941q^{-1}} & 0 \\ \frac{-0.0084q^{-3} + 0.0061q^{-4}}{1 - 0.8404q^{-1} - 0.0957q^{-2}} & \frac{0.0017q^{-2} + 0.0115q^{-3}}{1 - 1.1342q^{-1} + 0.2883q^{-2}} \end{bmatrix} \begin{bmatrix} \text{cold water flow rate} \\ \text{steam flow rate} \end{bmatrix}$$

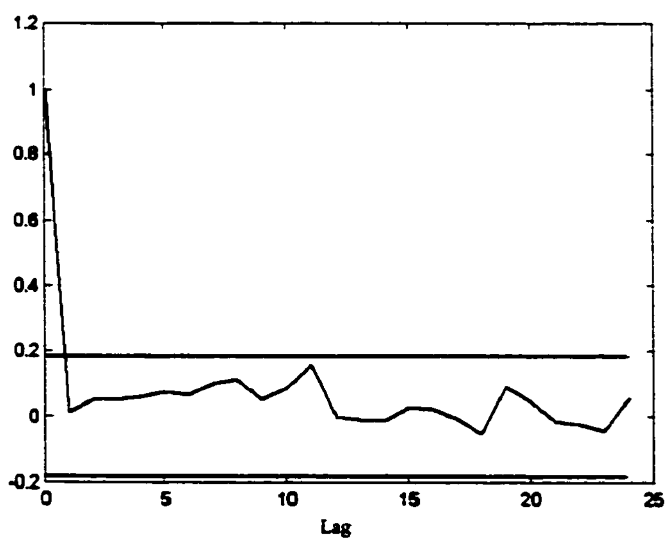
The interactor matrix D was calculated using the algorithm by Huang and Shah (1999). The calculations yield:

$$D = \begin{bmatrix} q^2 & 0 \\ 0 & q^2 \end{bmatrix}$$

The minimum variance for this MIMO system could be calculated using the algorithm by Rogozinski *et al.* (1987) and Peng and Kinnaert (1992) or, since the interactor matrix is a *simple* matrix, the minimum variance of each variable can be calculated individually from each output data (*i.e.*, treated as two independent *SISO* systems respectively) (Huang and Shah, 1999).



Correlation function of residuals for h_2



Correlation function of residuals for T_2

Retuned system and Redesigned system

The current controllers were tuned and the parameters of retuned controllers are given in Table 4.2. A Model Predictive Controller was used as the redesigned system for this case study. This controller was designed using the MPC Toolbox in Matlab (Version 5.3). The parameters for this controller are given in Table 4.3.

Table 4.2: PI controllers parameters for retuned system

	P	I
$Controller_1$	5	0.13
$Controller_2$	2	0.4

Table 4.3: MPC controller parameters

Parameters		Value
r	=	$\begin{bmatrix} 0 & 0 \end{bmatrix}$
M	=	2
P	=	6
ywt	=	$\begin{bmatrix} 1 & 1 \end{bmatrix}$
uwt	=	$\begin{bmatrix} 3 & 1 \end{bmatrix}$
u_0	=	$\begin{bmatrix} 0 & 0 \end{bmatrix}$
u_{sat}	=	$\begin{bmatrix} -10 & -10 & 10 & 10 & 10 & 2 & 2 \end{bmatrix}$

4.1.3 Results

Current System Performance

The simulation results for both IPC and JPC are compared in Table 4.4.

Table 4.4: Comparison of IPC and JPC results

	IPC	JPC
$\tilde{y}_1$ cm	38.36	39.37
$\tilde{y}_2$ $°C$	33.94	33.98
ρ_1 %	90.18	98.72
ρ_2 %	90.14	91.25
ρ_0 %	81.16	90.11
Process Profitability ($\$/ton$)	4.26	4.04

As can be seen from Table 4.4, the individual probability constraints were satisfied, with one probability reaching 90.18% and the other 90.14%. However, the probability of satisfying both constraints was only approximately 81%. This was due to the correlation between the constraints and the inability of the IPC algorithm to handle this correlation. The JPC algorithm delivered the specified 90% constraint satisfaction frequency at the expense of decreasing the objective profits to 4.04 $\$/ton$, which is a little smaller than the value of 4.26 $\$/ton$ delivered by the IPC algorithm.

Process Variation Comparison for Four Control Systems

The complete results for process data variation are listed in Table 4.5.

Table 4.5: Comparison of different control systems

	Current	Retune	Redesign	MV Limit
σ_1	1.0577	0.9251	0.9117	0.6377
σ_2	0.7346	0.7187	0.7090	0.5114

The process variation reduction is shown in Table 4.6. It is seen from Table 4.6 that there exists large potential process improvement opportunities, which can be partially achieved by retuning or redesigning the system. The second choice, however, seems not encouraging because the process variation delivered by redesigned system is only marginally smaller than that of retuned system.

Table 4.6: Process variation reduction

	Current *vs* MV Limit	Current *vs* Retune	Current *vs* Redesign	Retune *vs* Redesign
$\Delta\sigma_1/\sigma_1$ (%)	65.90	12.54	13.80	0.46
$\Delta\sigma_2/\sigma_2$ (%)	30.38	2.16	2.56	0.40

Performance Comparison

Results of performance evaluation for four different control systems were compared in three cases: unconstrained optimization problem; IPC problem; JPC problem. They were given in Tables 4.7, 4.8 and 4.9 respectively.

Table 4.7: Comparison of different control systems - no constraints

Unconstrained optimization	Current	Retune	Redesign	*MV* Limit
$\tilde{y}_{1u}$ cm	36.82	36.65	39.40	38.50
$\tilde{y}_{2u}$ $^{\circ}C$	33.84	33.86	34.91	34.67
J_u ($\$/ton$)	6.55	6.71	6.73	7.29

It can be seen from Tables 4.7, 4.8 and 4.9 that $J_{current} < J_{retune} < J_{MPC} < J_{MV}$. For the purpose of this case study, JPC results were used for further comparison. The maximum potential profitability for current control system is $5.51 - 4.00 = 1.51$ $\$/ton$. The best achievable profitability for existing system is $4.14 - 4.00 = 0.14$ $\$/ton$, compared with the best achievable profitability for proposed advanced control system, which is $4.19 - 4.00 = 0.19$ $\$/ton$, the former profitability is just slightly smaller than the latter one. Since

$$process\ net\ benefits = process\ profitability - costs$$

Table 4.8: Comparison of different control systems with IPC constraints

IPC	**Current**	**Retune**	**Redesign**	***MV* Limit**
$\tilde{y}_1$ *cm*	38.36	38.19	38.17	37.82
$\tilde{y}_2$ °*C*	34.93	34.92	34.90	34.66
ρ_1 %	89.93	90.12	89.92	89.81
ρ_2 %	90.02	90.14	90.04	90.24
ρ_0 %	80.93	81.17	90.16	81.03
J (\$/*ton*)	4.72	4.88	4.93	5.97

Table 4.9: Comparison of different control systems with JPC constraints

JPC	**Current**	**Retune**	**Redesign**	***MV* Limit**
$\tilde{y}_1$ *cm*	39.56	39.31	39.40	38.50
$\tilde{y}_2$ °*C*	34.94	34.95	34.91	34.67
ρ_1 %	99.14	99.36	99.44	99.21
ρ_2 %	90.84	91.16	90.69	90.67
ρ_0 %	90.01	90.46	90.16	90.01
J (\$/*ton*)	4.00	4.14	4.19	5.51

The throughput of the system was $2Mton/year$. Then, benefits for retuning the control system were 0.14 $\$/ton \times 2Mton/year - 0.08M\$/year = 0.20M\$/year$. The benefits from redesigning a MPC controller for the process were 0.19 $\$/ton \times 2Mton/year - 0.2M\$/year = 0.18M\$/year$. The process profitability/benefits for different systems are compared and given in Table 4.10.

Table 4.10: Benefits comparison for Two Tank Heaters in Series

Profitability/Benefits comparison ($M\$/year$)			Description
$J_{retune} - J_{current}$	=	0.28	Profitability available for retuning
$J_{MPC} - J_{current}$	=	0.36	Profitability available for proposed changes
$J_{MV} - J_{retune}$	=	2.74	Uncaptured profitability after retuning
$J_{MV} - J_{MPC}$	=	2.64	Uncaptured profitability after proposed changes
$J_{MPC} - J_{retune}$	=	0.10	Net profitability for proposed changes beyond retuning
$J_{retune} - J_{current} - Cost_{retune}$	=	0.20	Net benefits after retuning
$J_{MPC} - J_{current} - Cost_{MPC}$	=	0.18	Net benefits after proposed changes

Since the cost of implementing the proposed control system was much higher than retuning the system, the total benefits for designing advance control system was less than that of retuned system. Therefore, the proposed MPC was not economically acceptable in this case study. Before the decision of retuning the current control system is made, a sensitivity analysis of process operating condition with respect to the probability constraint level should be performed.

Sensitivity Analysis

The sensitivities of optimal operating point for controlled variables $\tilde{\mathbf{y}}$ ($\tilde{T}_2$ and $\tilde{h}_2$) with respect to the change of probability level α can be determined using the developments of Chapter 3. The sensitivity results for the retuned system were given in Table 4.11.

Table 4.11: Sensitivity analysis of process profitability with respect to α

	IPC	*JPC*
Opt. Process Profitability($\$/ton$)	4.88	4.14
$\frac{dJ}{dy_1^*}$	-0.5961	-0.6
$\frac{dJ}{dy_2^*}$	-2.0194	-2.1281
$\frac{dJ}{d\alpha}$	-4.3863	-1.5825
$\Delta\alpha$	+1%	+1%
Predicted Opt. Profitability($\$/ton$)	4.76	3.98
Calculated Opt. Profitability($\$/ton$)	4.83	4.12

As can be seen from Table 4.11, at the optimal operating point, the process profitability for IPC and JPC problems are 4.88 $\$/ton$ and 4.14 $\$/ton$ respectively. The first-order change of process profitability with respect to the change of process key variables — h_2 and T_2 are -0.59, -0.60 and -2.02, -2.13. This implies that the increase in h_2 and T_2 operating point leads to the decrease of total process profitability for both IPC and JPC problems.

The first-order change of process profitability with respect to the individual probability level was calculated to be -4.39. This means when probability level increase from 90% to 91%, the total process profitability will decrease by 4.39%. The predicted profitability and calculated one were given in the table. For JPC, $\overline{\pi}^*$ was calculated to be $\begin{bmatrix} 0.9836 & 0.1801 \end{bmatrix}^T$, the predicted optimal process profitability after the 1% change in probability level was calculated to be 3.98 $\$/ton$. The simulation result was also given in the last row of the table.

For IPC case, if there is a small change in the probability level, say $\Delta\alpha = +1\%$, the associated process profitability will decrease by $4.3863 \times 1\%/4.88 = 0.9\%$. For JPC case, the predicted optimal process profitability changes from 4.14 $\$/ton$ to 3.98 $\$/ton$ after the probability level increased from 90% to 91%.

Following the same procedure, the sensitivity results for different systems were obtained and given in Table 4.12. Based on the results in Table 4.12, a decision was made to implement the retuned system. However, is the retuned system really reliable compared to the current system? Following the method discussed in Chapter 3, $\Delta\alpha$ for the current system and the retuned system were calculated to be 45.97%. It is clear that to achieve the same benefit as the current system, the probability satisfaction level of the retuned system should be increased by 45.97% > 20%. Therefore, the performance yielded by the retuned system is reliable according to the sensitivity analysis discussed in Chapter 3.

Table 4.12: Sensitivity analysis of process profitability with respect to α for different systems

JPC	Current	Retune	Redesign	MV Limit
Opt. Process Profitability($\$/ton$)	4.00	4.14	4.19	5.51
Operating Cost ($\$/ton$)	0	0.04	0.1	0.5
$\frac{dJ}{d\alpha}$	-1.8005	-1.5825	-1.6120	-1.3872

4.1.4 Discussion

Combining the results from the performance evaluation and comparison of different control systems, the conclusion can be drawn: 1) There is substantial opportunity to improve the process profitability for this case study; 2) The retuned control system is recommended to be implemented based on the control system performance, implementation cost and sensitivity to the probability level.

4.2 ALPAC Bleach Plant D_2 Stage

In the first case study, the proposed benefits procedure was applied on a simple simulated process. In this section, the proposed procedure was applied on a real process, a bleaching plant D_2 stage, and real process data was used in the benefits analysis. The objective of the study is to determine whether there is sufficient potential to warrant further study into control system improvement.

4.2.1 Bleaching Plant Description

"*Bleaching is a chemical process applied to cellulosic materials to increase their brightness. Brightness is the reflectance of visible light from cellulosic cloth or pulp fibers formed into sheets. Bleaching increases the capacity of paper for accepting printed or written images and so increases its usefulness.*" (Dence and Reeve, 1996) gives the simple definition and objective for bleaching.

In the bleaching process, chemicals (including oxidants such as Cl_2, ClO_2, O_2, H_2O_2, and alkali such as $NaOH$) are mixed with pulp suspensions. The required brightness can be sequentially achieved by controlling the retention time, temperature, pH and so forth. The costs associated to the bleaching process include those of chemicals, steam, and electric power, *etc.*

ALPAC is one of the first kraft pulp mills specifically designed to incorporate 100% chlorine-dioxide bleaching. The pulp mill is designed to produce a minimum of 1,500 ADt (Air Dry metric tones) per day of bleached hardwood pulp or 1,250 ADt of bleached softwood pulp. In ALPAC's bleaching system, chemicals are charged sequentially with intermediate washing between treatments (stages), because it is not possible to achieve sufficient removal or decolorization of lignin by the action of any one chemical in a single stage. Four stages D_N, D_0, *Eop*, and D_2 were designed to achieve the required product specification. However, D_N stage is only used as a pre-washer because of the good satisfaction of the product.

D_N stage This stage was originally designed as the third stage in the whole bleaching series. Now, only the washer in this stage is used as a pre-washer of the feed stock coming from digester.

D_0 stage In this stage, the pre-washed pulp was mixed with H_2SO_4 and steam to adjust the pH, followed by the charge of ClO_2. As ClO_2 bleaching proceeds, the concentration of chemical structures in the pulp that cause the color in the pulp decreases. The pulp brightness increases fast initially, then it slows down and another stage is needed to achieve higher brightness.

***Eop* stage** In this stage, three chemicals are charged sequentially. $NaOH$ is used to remove the lignin made potentially soluble by the previous acidic oxidizing stage. The addition of O_2 and/or H_2O_2 result in a reduced requirement for ClO_2 dosage in later stages to attain the same brightness target. Or, certain brightness can be reached with fewer stages.

D_2 stage This is the final stage of bleaching process. According to the pH and brightness of the incoming feed from the Eop stage, H_2SO_4 and ClO_2 are charged into the pulp. After almost two hours of reaction time, the high brightness pulp goes through the final washer, then goes to the pulp machine system.

This stage plays a very important role in the whole bleaching plant. The measurement of some key variables are included in this stage. Therefore, D_2 stage is chosen as the object for benefits analysis.

4.2.2 D_2 Stage Process Description

The mixture from *Eop* stage is retained at a prescribed pH, temperature, and concentration for a specified time period. The progress of the bleaching reaction is monitored by measuring pulp lignin content (Kappa Number), pulp brightness, and residual chemical. Bleaching processes are monitored by on-line sensors and process control algorithms that have been devised to achieve product quality targets (*i.e.*, Brightness) with efficient use of chemicals and energy.

The principal objective of pulp bleaching is to achieve a high brightness with several other secondary objectives. Such secondary objectives are usually end use specific and can include high brightness stability, pulp cleanliness (freedom from colored particles), and cellulose content. These objectives must be achieved without compromising the strength of the final product.

Bleaching costs include those for ClO_2, H_2SO_4, steam, electrical power and so forth. Capital costs and operating costs associated with the bleach plant must also be taken into consideration.

Typical bleach plant control systems incorporate both feedback and feedforward control loops.

Bleach Plant Variables

The process variables considered in D_2 stage are divided into manipulated, disturbance and controlled variables (see Tables 4.13 and 4.14).

Table 4.13: ALPAC bleaching plant D_2 stage process variables

Manipulated Variables	Disturbances
ClO_2 flow	Incoming pulp flow
H_2SO_4 flow	Incoming consistency
Dilution water flow	Incoming brown stock kappa number
Steam flow	Incoming temperature
	Bleach chemical strength

Table 4.14: ALPAC bleaching plant D_2 stage process variables cont'd.

Controlled Variables	Sensors
Brt.	*Brt.*
Kappa (Lignin Content)	Chemical Residual
pH	*pH*
Viscosity	Temperature
Chemical Residual	Level
Dirt	Consistency
	Flow

Disturbances may occur prior to the bleach plant or may occur between the bleaching stages. For example, washer disturbances can alter the consistency and flow into each stage. In addition, the concentration of the bleach chemical solution being charged to each stage can vary with time. Other disturbances may include the production mode switching or equipment failure, *etc.*

Key Variables Analysis

Key variables in D_2 stage are chosen from the 15 process variables listed in Tables 4.13 and 4.14. These variables are related directly or indirectly to the product quality and/or process cost. Thus, product type and associated specifications need to be identified.

The main products include high brightness pulp, normal brightness pulp (hardwood) and normal brightness pulp (softwood). The product changes based on market demand. Normal brightness pulp (hardwood) has the largest demand, while high brightness pulp has the lowest demand. The brightness (*Brt.*), tower temperature (*Temp.*) and *pH* after the retention tower are the key variables considered in this thesis.

Feeds to the D_2 stage include pulp from the *Eop* stage (disturbance), H_2SO_4, steam, ClO_2 and water into the washer are manipulated variables. Key variables in D_2 Stage are listed in Table 4.15.

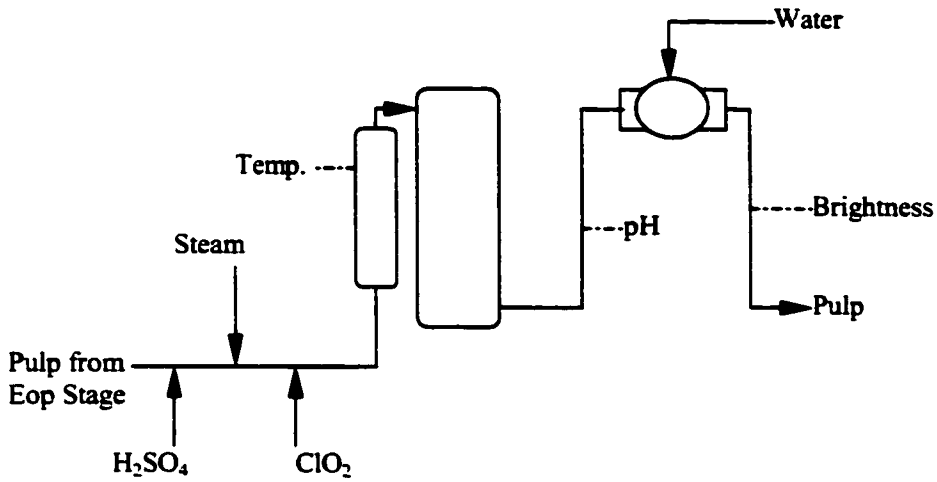


Figure 4.7: D_2 stage flow chart

Table 4.15: Key variables in D_2 Stage

Key variables	Target	Current
Brt. %ISO	90.5	
pH	4.8	5.37
Residual, g/L	0	0
Acid charge, Kg/ton		7.4
ClO_2 charge, Kg/ton	< 2.0	1.0
Steam		
Tower *Temp.*, ^oC	70	75
Tower *level*, %	70	67

EPF

Economic information such as market price for the product and operating cost were collected. For the purposes of this thesis, only chemical cost and steam cost are considered. The pulp was valued according to its brightness and dirt percentage.

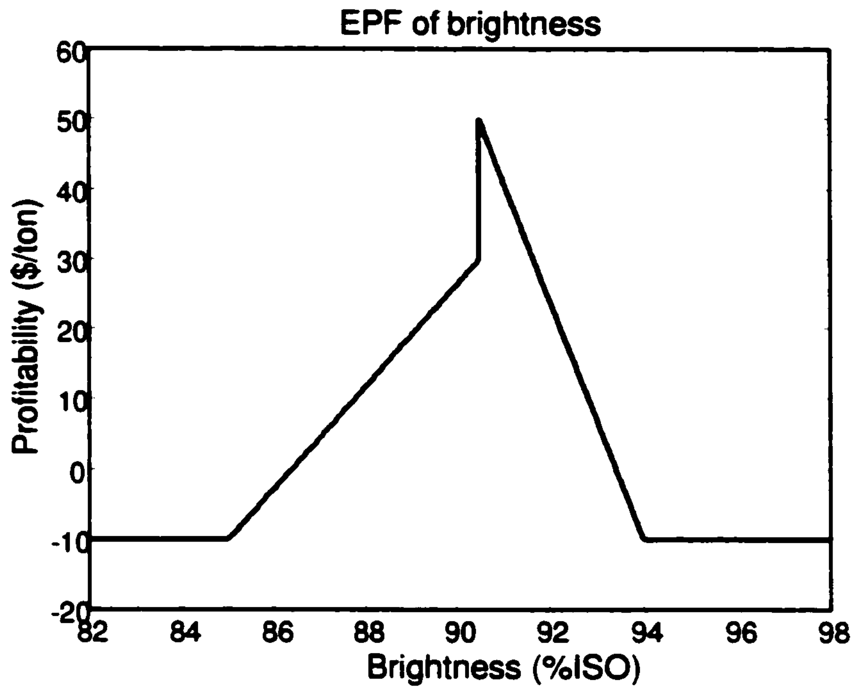


Figure 4.8: Economic performance function for brightness

Product Price The price of the normal brightness (hardwood) is $500\$/ton$, while the brightness is a little lower ($88\%ISO \sim 90.5\%ISO$), it will be repulped with higher brightness pulp, therefore, the estimated price is $300\$/ton$. The pulp with very high brightness requires a lot of chemical charge, which dramatically reduces the profitability of the pulp. On the other hand, if the pulp brightness is too low, it requires too much high brightness pulp to repulp it, which makes the production economically unacceptable. Considering the cost of ClO_2 charge, royalty for producing the chemicals on line and other costs (including sale cost), the profitability for the pulp at the specification is $50\$/ton$. The profitability for off-specification pulp is no more than $30\$/ton$. The profitability for producing very high and very low brightness pulp is $-10\$/ton$ Based on these consideration, the economic performance function for brightness was defined as:

$$\phi_{brt}(\$/ton) = \begin{cases} -10 & Brt \leq 85\%ISO \\ 7.27Brt - 628.18 & 85\%ISO \leq Brt \leq 90.5\%ISO \\ -17.14Brt + 1601.43 & 90.5\%ISO \leq Brt \leq 94\%ISO \\ -10 & Brt \geq 94\%ISO \end{cases} \tag{4.6}$$

Operating Cost

ClO_2 cost was included in the brightness economic performance function. Thus, only steam cost and H_2SO_4 cost are considered here. If the effect of the temperature and pH on pulp brightness is ignored, the cost function is linear. A low operating temperature reduces

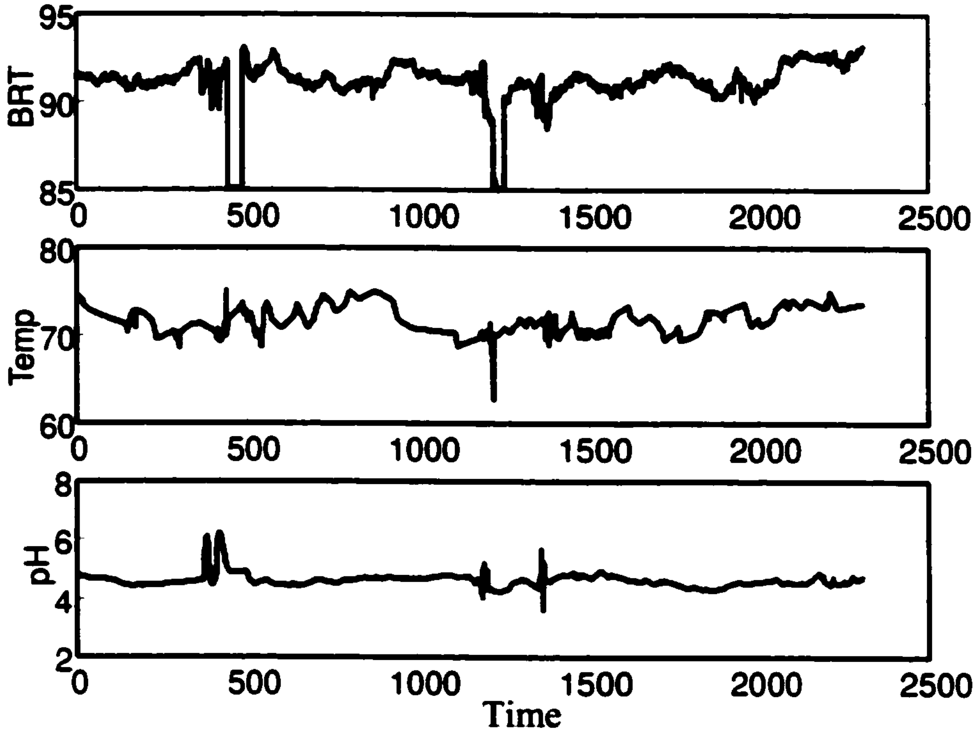


Figure 4.9: Bleach plant D_2 stage key variables data

steam costs and a high *pH* reduces H_2SO_4 charge. The temperature, however, must be kept within a range where a specific brightness can be achieved. For a very precise calculation, a model of the relationship between temperature and brightness, *pH* and brightness would be required. This level of accuracy should not be required for the purposes of this work and is not possible given the inaccuracies of some of the problem data (*e.g.*, economics). Then, the cost functions relating steam and temperature, as well as acid cost and *pH* are:

$$C_{Temp} = 0.1667 * Temp - 4.8333$$
$$C_{pH} = -0.7565 * pH + 10.2756$$

In this case study, data was retrieved from the same operation mode —normal brightness and hardwood. The data was recorded in 12 minute interval, which yielded 2300 data that were used for benefits analysis. Raw data for *Brt.*, *Temp.* and *pH* are shown in Figure 4.9.

From Figure 4.9, it can be seen clearly that brightness is approximately 91.5 ISO%, but there are two large spikes in the data. The spikes stay far below the specification (90.5 ISO%) for 70 minutes before it is changed back to the normal value. This is due to a disturbance from an upstream process. Note that it takes approximately 70 minutes to observe the effect before operator can compensate for off-specification product. For the purpose of performance evaluation, the obvious outliers were discarded.

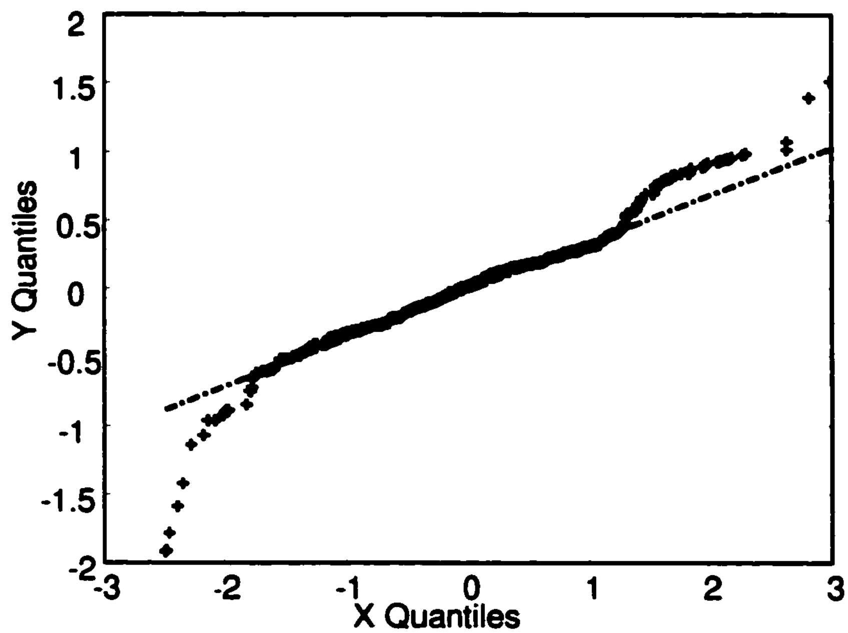


Figure 4.10: *Q-Q* plot of *Brt* data

The data distribution was checked for Normal distribution and the mean, variance and the correlation between key variables were calculated. (The mean, variance and correlation between key variables are given in Table 4.16).

Table 4.16: Key variables data analysis

	mean	standard deviation	minimum	maximum
Brt.	91.3822	0.7287	87.0649	93.2278
Temp.	71.8553	1.5077	62.6125	75.1828
pH	4.6254	0.2212	3.5946	6.2232

The covariance matrix of the key variables was calculated as:

$$\mathbf{Q} = \begin{bmatrix} 0.5310 & 0.2413 & 0.0144 \\ 0.2413 & 2.2731 & -0.0240 \\ 0.0144 & -0.0240 & 0.0489 \end{bmatrix}$$

The correlation matrix was determined to be:

$$\mathbf{R} = \begin{bmatrix} 1.0000 & 0.2196 & 0.0895 \\ 0.2196 & 1.0000 & -0.0721 \\ 0.0895 & -0.0721 & 1.0000 \end{bmatrix}$$

As can be seen that the correlation between *Brt* and *Temp* was 0.2196, slightly larger than that between *Brt* and *pH*, which was 0.0895, and *pH* and *Temp*, which was -0.0721.

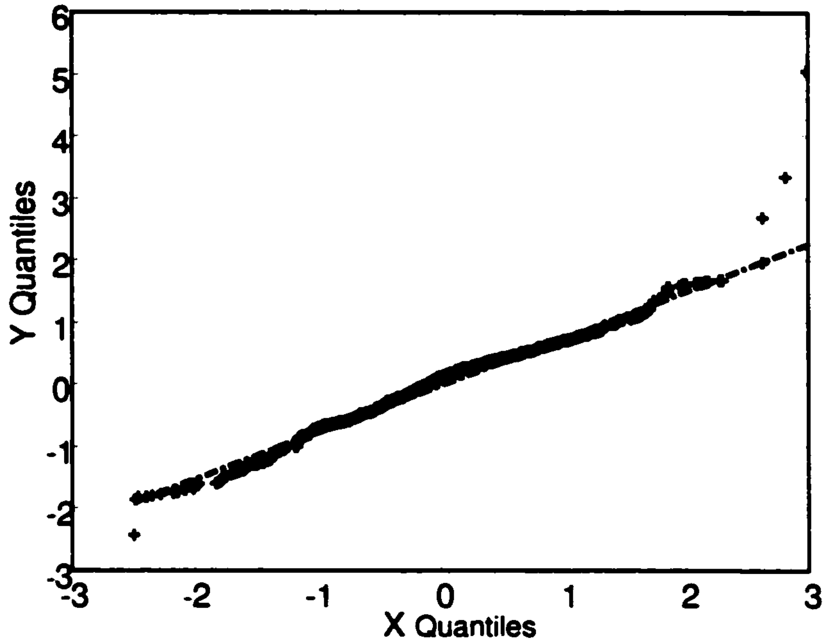


Figure 4.11: *Q-Q* plot of *Temp* data

Constraints

As discussed in Chapter 3, a lot of constraints could be considered for a comprehensive benefits analysis. However, for the purpose of this case study, only product specification and operating limits are considered. Product demands, throughput, feed availability and so forth were not considered in this study. This is usually the most important constraint in the performance evaluation calculation.

The objective is to maximize the product profitability with the minimum cost; however, based on the market prices and buyers' requirements, it is not wise to produce as high quality product as the process can without considering the chemical charge and other costs. Therefore, a reasonable probability level for product specification satisfaction is made either by the customer or engineers. Similar to the first case study, 90% was chosen as the probability level. The constraint was specified as:

$$\rho_{brt} = \Pr(y_{Brt} \geq 90.5\ \%ISO) \geq 90\%$$

To achieve specified brightness, the key variables (*e.g.*, reactor temperature and *pH*) are limited to a small range. It was assumed that within this specific range, *Temp* or *pH* changes do not impact the brightness. These two constraints were specified as:

$$\rho_{Temp} = \Pr(68\ ^{\circ}C \leq y_{Temp} \leq 72\ ^{\circ}C) \geq 90\%$$
$$\rho_{pH} = \Pr(4.2 \leq y_{pH} \leq 5.2) \geq 90\%$$

Some constraints that were not considered in this case study include product demands, plant limitation, feed availability, production rate limitation and so forth. Product demands can have a substantial impact on the process operation (*i.e.*, operating mode). In this thesis, operation mode changes are not included and only one operation mode was used through the whole benefits analysis. The designed throughput of ALPAC bleaching plant is 1800 ton/day and was used as a constant in the case study. Wood chips are fed to the digester continuously, chemical are produced on site. Therefore, no feed limitation is considered.

4.2.3 Case Study Procedure

This case study was performed in the following steps:

1. Current control system performance evaluation
2. Performance limits (minimum variance control system evaluation)
3. Performance comparison
4. Benefits comparison
5. Sensitivity analysis on the minimum variance control system upgrade

Performance Evaluation Problem

Based on the information collected during the first phase of the benefits study, the performance evaluation problem was specified as:

$$\max_{\bar{y}} \quad J = \int_{\mathbf{Y}_1} (\phi_{Brt} \cdot f_{Brt}) dy_{Brt} - \int_{\mathbf{Y}_2} (C_{Temp} \cdot f_{Temp}) dy_{Temp} - \int_{\mathbf{Y}_2} (C_{pH} \cdot f_{pH}) dy_{pH}$$

subject to:

$$\text{IPC} \qquad \begin{aligned} \rho_{Brt} &= \Pr(y_{Brt} \geq 90.5\% ISO) \geq 90\% \\ \rho_{Temp} &= \Pr(68^{\circ}C \leq y_{Temp} \leq 72^{\circ}C) \geq 90\% \\ \rho_{pH} &= \Pr(4.2 \leq y_{pH} \leq 5.2) \geq 90\% \end{aligned}$$

or

$$\text{JPC} \qquad \rho_0 = \Pr \begin{pmatrix} y_{Brt} \geq 90.5\% ISO \\ 68^{\circ}C \leq y_{Temp} \leq 72^{\circ}C \\ 4.2 \leq y_{pH} \leq 5.2 \end{pmatrix} \geq 90\% \qquad (4.7)$$

In solving this performance evaluation problem, the expected profitability is maximized by integrating the economic performance over the product key variable distribution and deducting the associated steam and acid cost, subject to the appropriate set of constraints.

Performance Limits

To calculate the minimum variance, the process model was identified using Identification Toolbox in Matlab Version 5.3:

$$T = \begin{bmatrix} T_{Brt} \\ T_{Temp} \\ T_{pH} \end{bmatrix} = \begin{bmatrix} \frac{-0.0753+0.0602q^{-1}}{1-1.9164q^{-1}+0.9353q^{-2}} q^{-4} & 0 & 0 \\ 0 & \frac{-0.0264+0.0267q^{-1}}{1-1.8668q^{-1}+0.8669q^{-2}} q^{-2} & \frac{-0.0059}{1-1.9990q^{-1}+0.9998q^{-2}} q^{-2} \\ \frac{0.0481}{1-0.5898q^{-1}-0.4096q^{-2}} q^{-2} & \frac{-0.0002}{1-1.5050q^{-1}+0.5162q^{-2}} q^{-2} & \frac{0.0351-0.0379q^{-1}}{1-1.6842q^{-1}+0.6887q^{-2}} q^{-1} \end{bmatrix}$$

This process has a diagonal interactor matrix

$$D = \begin{bmatrix} q^4 & 0 & 0 \\ 0 & q^2 & 0 \\ 0 & 0 & q \end{bmatrix}$$

that

$$\lim_{q^{-1}\to 0} DT = \lim_{q^{-1}\to 0} \widetilde{T} = K$$

Since the interactor matrix of this MIMO system is a diagonal matrix, the minimum variance of each variable can be calculated from each individual output.

For brightness, the system time delay is estimated from Figure 4.12.

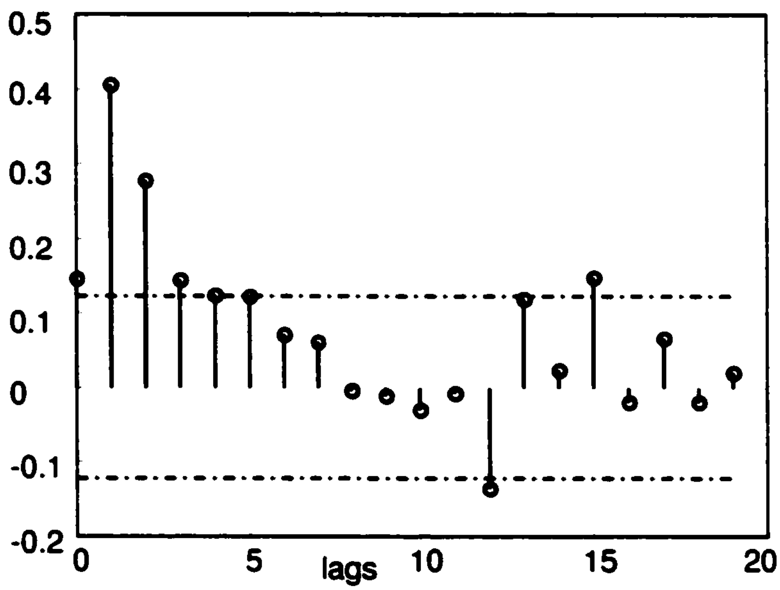


Figure 4.12: Impulse response estimate (*Brt.*)

4.2.4 Results

Current Operation Performance Analysis

The current control strategy is to keep the product as far above the specification as possible without much consideration of the cost of chemical charge. Current performance is analyzed by checking both constraint satisfaction and the profit flow at the mean operating point.

It can be seen from Table 4.17 that the two of three key variables are satisfied, while the specification satisfaction of *Temp.* is only 57.24%, which leads to the JPC satisfaction only to be 49.96%.

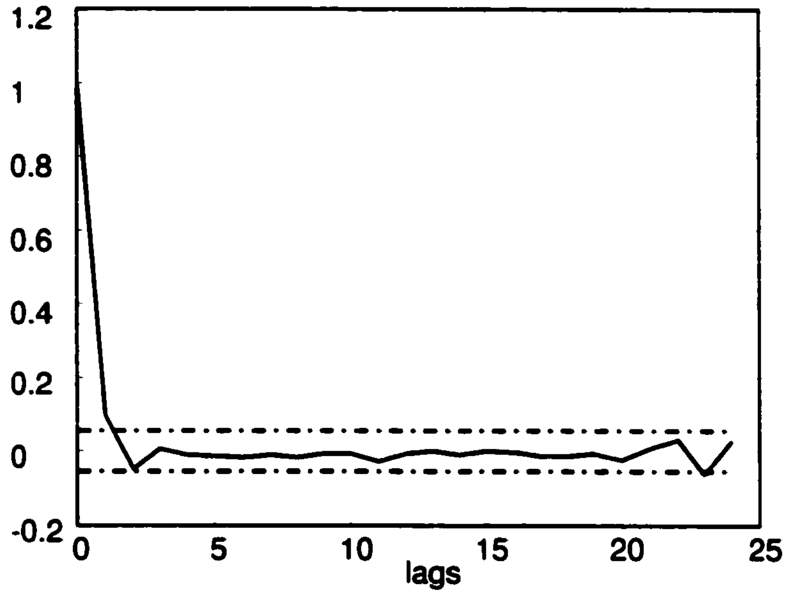


Figure 4.13: Correlation analysis for residuals (*Brt.*)

Table 4.17: Current performance evaluation

Variables	Operating Point	IPC	JPC	Profitability ($\$/ton$)
Brt.	91.3822 $\%ISO$	92.47%		
Temp.	71.8533 ^oC	57.24%		
pH.	4.6254	97.76%		
			49.96%	17.7211

Current Performance Improvement

The optimal operating point for current system was calculated and the profitability was compared with the current one. The improvement was listed in Table 4.18.

Comparing the results in Tables 4.17 and 4.18, it is clear that the optimal operating points for three key process variables are located. The associated IPC profitability is 17.7709 $\$/ton$, which is larger than the current process profitability, 17.7211 $\$/ton$. The JPC profitability is less than that of IPC case, which mainly because the number of the variables in the constraints and the correlation involved in these variables.

Table 4.18: Current performance improvement

		IPC	JPC
Operating Point	*Brt.*	91.2439%ISO	91.6827%ISO
	Temp.	69.9322°C	71.3793°C
	pH.	4.9165	4.7179
Spec satisfaction (%)	*Brt.*	89.94	94.35
	Temp.	90.17	97.82
	pH.	89.96	97.83
Spec satisfaction (%)	Joint	73.71	90.20
Profitability ($\$/ton$)		17.7709	14.5201

Performance Limits

Using the FCOR algorithm (Huang and Shah, 1999), the standard deviations of these variables were:

$$\sigma_{mv\_Brt} = 0.3129$$
$$\sigma_{mv\_Temp} = 1.2526$$
$$\sigma_{mv\_pH} = 0.0436$$

The same performance evaluation problem is solved using the minimum variance. The results are listed in Table 4.19.

Table 4.19: Performance limits

		IPC	JPC
Operating Point	*Brt.*	90.9010%ISO	90.9208%ISO
	Temp.	69.6099°C	70.8948°C
	pH.	5.1441	5.0451
Individual Spec Satisfaction (%)	*Brt.*	89.91	94.22
	Temp.	90.01	97.98
	pH.	89.82	97.97
Joint Spec Satisfaction (%)		73.31	90.20
Profitability ($\$/ton$)		20.9012	20.5962

As can be seen from Table 4.19, with the reduction of variance, the operating point can be moved closer to the variable specification without violating the probability constraints. The profitability could be increased 18% to 20.9012 $\$/ton$ with individual probability constraints. If the joint probability constraints are used in the optimization, the profitability could be increased 44% to 20.5962 $\$/ton$. If the throughput was taken as 1800 ton/day, the total potential profitability for current production is:

$$(20.5962 - 14.5201)\ \$/ton \times 1800\ ton/day = 10.8\ K\$/day$$

However, this is theoretically achievable profitability without considering the cost and practicality of implementing the minium variance controller. Nevertheless, a significant improvement can still be expected from using retuning controllers or redesigning controllers.

Sensitivity Analysis

The sensitivities of operating condition of controlled variables $\tilde{\mathbf{y}}$ ($\widetilde{Brt}$, $\widetilde{Temp}$ and $\widetilde{pH}$) with respect to the change of probability level $\boldsymbol{\alpha}$ follow the relationship developed in Chapter 3.

The sensitivity analysis was performed on Minimum Variance control system to justify the potential achievable process profitability improvement. The results of sensitivity study were given in Table 4.20.

Table 4.20: Sensitivity analysis of ALPAC bleaching plant D_2 stage profitability with respect to probability level change

	IPC	JPC
Opt. Process Profitability ($\$/ton$)	20.90	20.596
$\frac{dJ}{dy^*_{Brt}}$	0.1982	−0.1315
$\frac{dJ}{dy^*_{Temp}}$	−0.1667	−0.1667
$\frac{dJ}{dy^*_{pH}}$	0.7565	0.7565
$\frac{dJ}{d\alpha}$	−0.6070	−0.4594
$\Delta\alpha$	+1%	+1%
Predicted Opt. Profitability ($\$/ton$)	20.895	20.591
Calculated Opt. Profitability ($\$/ton$)	20.872	20.530

As can be seen from Table 4.20, at the optimal operating point, the ALPAC bleaching plant D_2 stage profitability for IPC and JPC problems are 20.901 $\$/ton$ and 20.596 $\$/ton$ respectively. The first-order change of process profitability with respect to the change of process key variables — ***Brt***, ***Temp*** and ***pH*** are 0.1982, −0.1667, 0.7565 and −0.1315, -0.1667, 0.7565. The result shows the different direction on process profitability with respect to the change in ***Brt*** for IPC and JPC cases.

For IPC case, the sensitivity of the process profitability with respect to the change of probability constraints level was calculated to be -4.3863. This means that if there is a small change in the probability level, say $\Delta\alpha = +1\%$, the associated process profitability will decrease by $0.607 \times 1\%/20.596 = 0.003\%$. For JPC case, $\vec{\pi}$ was calculated to be $\left[\ 0.0984 \quad 0.6958 \quad 0.7114\ \right]^T$, the predicted optimal process profitability changes from 20.596 to 20.591 after the probability level increased from 90% to 91%. The percentage of decrease is $(20.596 - 20.591)/20.596 = 0.0024\%$.

The sensitivity of process profitability with respect to probability level for the current system is also calculated to be: $\frac{dJ_{cu}}{d\alpha} = -1.0869$. The assumed implementation cost for minimum variance control system is 5$/ton.

Based on the profitability yielded by the minimum control system, there is large potential benefit between the current system and the minimum control system. However, is this result reliable if there is some change in probability level α? According to the sensitivity analysis procedure discussed in Chapter 3, to change the decision that there is obvious potential benefit for the current system upgrading, $|\triangle\alpha|$ is:

$$\begin{aligned} |\triangle\alpha| &= (-1.0869 + 0.4594)^{-1}(20.596 - 14.52 - 5) \\ &= 171.2\% >> 20\% \end{aligned}$$

Therefore, the potential profitability between the current system and the minimum variance control system is reliable.

4.2.5 Discussion

It was shown from the results of sensitivity study that the first-order change of process profitability with respect to the change in probability level is very small. Also, to achieve the same benefits, the current control system should decrease the probability level to $90\% - 171.2\% = -81.2\%$, which means the probability change will not affect the conclusion that the minimum variance control system yield much more benefits than the current system. Combining the result of benefits comparison and sensitivity analysis, the conclusion can be drawn: There is substantial opportunity to improve the process profitability for ALPAC bleaching plant D_2 stage. Further information about controller tuning and advanced controller can be collected and future work on improving the process profitability is recommended.

Chapter 5

Conclusions & Future Work

Process control upgrades have been recognized as an efficient way to improve process profitability. Benefits analysis studies are performed to justify the potential profitability that can be delivered through a control system improvement. Control system benefits analysis uses process information, customer requirements, as well as economic data to evaluate the process performance. Current approaches to benefits analysis do not make full use of available information in a systematic manner. An optimization approach to benefits analysis was proposed in this thesis, which efficiently uses the available information to provide an estimate of both the maximum performance that can be expected from a given automation system and the operating conditions that yield this performance.

The optimization problem at the heart of the proposed method consists of process economic performance function EPF, probability density function for the key process variables and individual or joint probability constraints. This problem is solved for the expected process variation resulting from different proposed control systems. The expected process performance for different systems was evaluated by solving the performance estimation problem, and compared to prioritize the best opportunity. Sensitivity analysis was performed to ensure the estimated performance is reliable. Then, a final selection of the appropriate control system upgrade is made.

The proposed benefits analysis procedure was used on two case studies. One was a simulation of a pilot plant in Computer Process Control Lab at the University of Alberta. This pilot plant consists of two stirred tank heaters in series. The other was ALPAC bleaching plant D_2 stage. For the first case study, the expected performance of different control systems were compared. Based on the benefits study results, retuning the existing controllers is expected to yield the highest process profitability. For the ALPAC bleaching plant D_2 stage case study, an economic performance function was developed based on the market price for different pulp products; and probability constraints for key variables were defined according to the customer requirements. The current control system was evaluated against a minimum variance benchmark, and based on this economic performance comparison, it was found that there exists a considerable opportunity to improve the current control system and achieve higher process profitability.

5.1 Thesis Contributions

One of the main contributions of this thesis is the modification of algorithm used to solve control system performance evaluation problem. Zhang *et al.* (2000) modified the algorithm (originally proposed by Prekopa (1995)) for solving the JPC problem with a linear objective function. This algorithm was modified in this thesis to solve the JPC optimization problem with nonlinearities in the objective function.

The other contribution in this thesis is the sensitivity analysis of process profitability and process operating point with respect to the change in constraint satisfaction probability level. It was shown that the first-order change of process profitability with respect to the change of constraint satisfaction probability level is a function of model parameter matrix **H** and the probability density function of the joint Normal distribution. Typically, if the process performance is not sensitive to the changing parameter (*i.e.*, $|\triangle\alpha| > 20\%$), the proposed control system upgrade should be implemented.

The proposed optimization-based process performance evaluation and sensitivity analysis were integrated into a comprehensive benefits analysis procedure.

5.2 Future Work

In the objective function formulation, the process data were assumed to satisfy normal distribution. Use of a non-Normal distribution in the performance estimation should be fully investigated.

Also, the EPF was considered to be a linear piece-wise function, which was also assumed to be deterministic in the performance evaluation problem. Further work on multivariable, nonlinear stochastic EPFs is required to encompass the range of possibilities for operating plants.

Probability constraints used in the performance evaluation problem were only for process key variables specification. Approximate modelling (*i.e.*, development of **H** matrix) methods for probabilistic constraints should be considered in the future work.

In the sensitivity study, $\triangle P = J_C - J_B - (Cost_C - Cost_B)$ is a random variable, then $\triangle P = 0$ is reliable only within a statistical confidence interval. When the data used in the benefits evaluation containing significant stochastic behavior, the variance of $\triangle P$ should be calculated (*i.e.*, $\sigma_{\triangle P}$) using appropriate statistical methods. Further sensitivity study needs to be done for these data.

The JPC problem required substantially more computation than that of IPC problem. The increased computation load was primarily due to Monte Carlo simulations required to characterize joint probability density function and the cutting plane method used for optimization. More efficient methods in these two areas would reduce the computation expense.

The benefits analysis was proposed for the control system upgrade, however, it may be possibly expanded to any automation system upgrade (*i.e.*, instrumentation, RTO, *etc.*). It, again, requires more accurate model for EPF and faster JPC problem solving methods.

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Appendix A

Introduction to Stochastic Programming

Stochastic programming deals with situations where some or all the parameters of the problem are described by random variables rather than by deterministic quantities. Such cases seem typical of real-life problems where it is difficult to determine the values of the required parameters exactly. In the linear programming, sensitivity analysis can be used to study the effect of changes in the program. This, however, represents a partial answer to the problem especially when the parameters are actually random variables. The objective of stochastic programming is to consider these random effects explicitly in the solution of the model.

We will use a branch of Stochastic Programming (*i.e.*, probability programming) to solve the proposed problem in this paper. The basic idea of probability programming models is to convert the probabilistic nature of the problem into an equivalent deterministic model.

"Stochastic Programming handles mathematical programming problems where some of the parameters are random variables..." (Prekopa, 1995) is one of the simpler definitions given for Stochastic Programming.

A.1 Optimization Under Uncertainty

Consider the general optimization problem

$$\begin{aligned} \min_{\tilde{y}} \quad & f(\phi, \tilde{y}) \\ \text{subject to:} \quad & \\ & h(y) \geq s^* \end{aligned} \tag{A.1}$$

where: f is the objective function, ϕ is the economic performance function, $\tilde{y}$ is the operating point for key variable vector, y is the data set for key variable, s^* is the product specification vector. Equality constraints are not considered in the above formulation as it has been assumed that any equality constraints have already been used to reduce the dimensionality of the optimization problem (*i.e.*, the optimization problem is considered in its reduced space).

The constraint in the above problem has a stochastic variable y. Assume y satisfies a Normal distribution. Then, y can be split into two parts:

$$y = \widetilde{y} + \xi \tag{A.2}$$

where, $\xi \sim N(0, Q)$. Q is the variance matrix of data set y. The problem can be converted to

$$\begin{array}{ll} \min\limits_{\widetilde{y}} & f(\phi, \widetilde{y}) \\ \text{subject to:} & \\ & h(\widetilde{y}, \xi) \geq s^* \end{array} \tag{A.3}$$

Stochastic programming problems can be split into three categories based on the location of uncertainty in the optimization problem:

- uncertainty in economics performance function ϕ
- uncertainty in product specifications s^*
- uncertainty in the constraint function ξ

These categories are illustrated by showing the effect of uncertainty on an LP problem in two dimensions. All the arguments presented below can be generalized to an n-dimensional NLP problem.

Uncertainty in economic data translates to uncertainty in the gradient of the profit function. Figure A.1 shows that the uncertainty in the slope of the economic performance function can cause the apparent optimum to shift to a sub-optimal operating point. Uncertainty in product specifications can cause the feasible region expand or shrink, which can be easily seen from Figure A.2. The change of feasible region will definitely affect the optimal operating point. Uncertainty in the last category is in the data set y. This is also the most important problem that we have to tackle in the benefits analysis. As seen in the Figure A.3, the random variable ξ will make the operating point change. This change will be much more complex than the changes occurred in the former two categories.In this thesis, the third category is the main concern, only a simple discussion was given to the uncertainty in the first category.

A.2 Simple Example of IPC Problem

A chance-constrained model is defined generally as:

$$\begin{array}{ll} \max\limits_{x} & \sum_{j=1}^{n} c_j x_j, \\ \text{subject to} & \Pr\{\sum_{j=1}^{n} a_{ij} x_j \leq b_i\} \geq \alpha_i, \qquad i = 1, 2, ..., m, \qquad x_j \geq 0. \end{array} \tag{A.4}$$

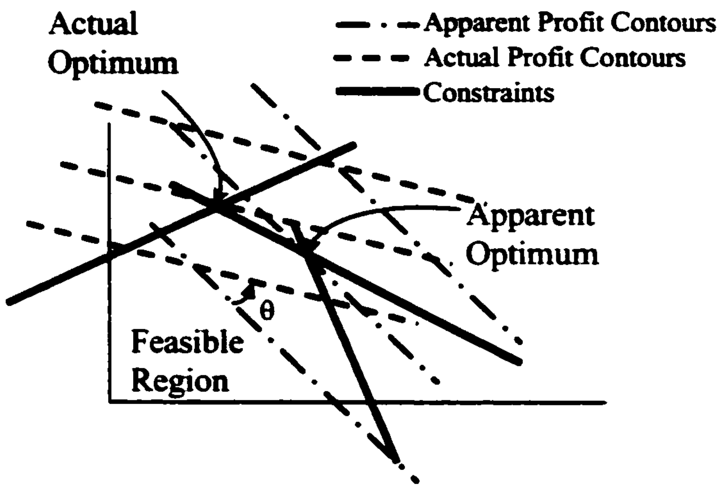

Figure A.1: Uncertainty in economics function parameters

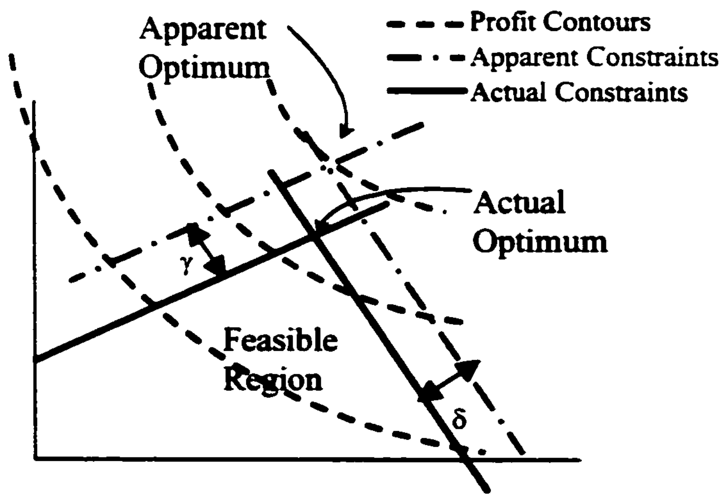

Figure A.2: Uncertainty in constraint parameters (RHS)

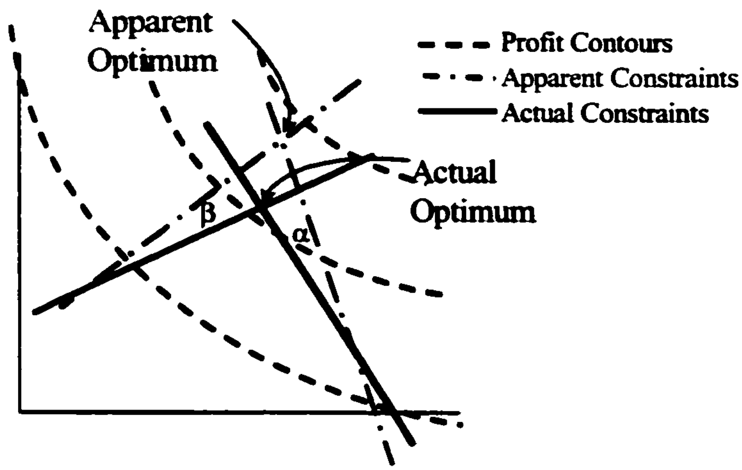

Figure A.3: Uncertainty in constraint parameters (LHS)

The name "chance-constrained" follows from the fact that each constraint,

$$\sum_{j=1}^{n} a_{ij}x_j \leq b_i, \tag{A.5}$$

is realized with a minimum probability of $\alpha_i, 0 < \alpha_i < 1$.

In the general case, it is assumed that c_j, a_{ij},and b_i are all random variables. The fact that c_j is a random variable may be treated by replacing it by its expected value. The cases in which a_{ij} or b_i is treated as random variables in the constraint are analyzed as follows. In both cases it will be assumed that the parameters are Normally distributed with known means and variances.

Case 1:

In this case it is assumed that each a_{ij} is normally distributed with mean $E\{a_{ij}\}$ and variances $Var\{a_{ij}\}$. It is further assumed that the covariance between a_{ij} and $a_{i'j'}$ is given by $Cov\{a_{ij}, a_{i'j'}\}$.

Consider the i^{th} constraint,

$$\Pr\{\sum_{j=1}^{n} a_{ij}x_j \leq b_i\} \geq 1 - \alpha_i, \tag{A.6}$$

and define

$$h_i = \sum_{j=1}^{n} a_{ij}x_j. \tag{A.7}$$

Then h_i is normally distributed with mean

$$E\{h_i\} = \sum_{j=1}^{n} E\{a_{ij}\}x_j. \tag{A.8}$$

and variance

$$Var\{h_i\} = \mathbf{X}^T \mathbf{D}_i \mathbf{X} \tag{A.9}$$

where

$$\begin{aligned}
\mathbf{X} &= (x_1, ..., x_n)^T, \\
\mathbf{D}_i &= i^{th} \text{ covariance matrix} \\
&= \begin{pmatrix} Var\{a_{i1}\} & \cdots & Cov\{a_{i1}, a_{in}\} \\ \cdots & & \cdots \\ Cov\{a_{1n}, a_{i1}\} & \cdots & Var\{a_{in}\} \end{pmatrix}.
\end{aligned}$$

Now,

$$P\{h_i \le b_i\} = P\left\{ \left(\frac{h_i - E\{h_i\}}{\sqrt{Var\{h_i\}}} \le \frac{b_i - E\{h_i\}}{\sqrt{Var\{h_i\}}} \right) \right\} \ge 1 - \alpha_i, \tag{A.10}$$

where $\left(\frac{h_i - E\{h_i\}}{\sqrt{Var\{h_i\}}} \right)$ is Normally distributed with zero mean and unit variance. This means that

$$\Pr\{h_i \le b_i\} = \Phi \left(\frac{b_i - E\{h_i\}}{\sqrt{Var\{h_i\}}} \right) \tag{A.11}$$

where represents the cumulative density function of standard normal distribution.

Let K_{α_i} be the standard Normal value such that

$$\Phi(K_{\alpha_i}) = 1 - \alpha_i \tag{A.12}$$

Then the statement, $\Pr\{h_i \le b_i\} \ge \alpha_i$. is realized if and only if

$$\frac{b_i - E\{h_i\}}{\sqrt{Var\{h_i\}}} \ge K_{\alpha_i} \tag{A.13}$$

This yields the following nonlinear constraint

$$\left(\sum_{j=1}^{n} E\{a_{ij}\} x_j + K_{\alpha_i} \sqrt{\mathbf{X}^T \mathbf{D}_i \mathbf{X}} \right) \le b_i \tag{A.14}$$

which is equivalent to the original stochastic constraint.

For the special case where the Normal distributions are independent, then $Cov\{a_{ij}, a_{i'j'}\} = 0$ and the last constraint reduces to

$$\left(\sum_{j=1}^{n} E\{a_{ij}\} x_j + K_{\alpha_i} \sqrt{\sum_{j=1}^{n} Var\{a_{ij}\} x_j^2} \right) \le b \tag{A.15}$$

This constraint can now be put into the separable programming form (Taha, 1971) using the substitution

$$y_i = \sqrt{\sum_{j=1}^{n} Var\{a_{ij}\}x_j^2} \tag{A.16}$$

Thus the original constraint is equivalent to

$$\left(\sum_{j=1}^{n} E\{a_{ij}\}x_j + K_{\alpha_i}y_i\right) \leq b_i \tag{A.17}$$

and

$$\left(\sum_{j=1}^{n} Var\{a_{ij}\}x_j^2 - y_i^2\right) = 0 \tag{A.18}$$

Where $y_i \geq 0$. The constraint is now properly separable.

Case 2:

In this case it is assumed that only b_i is a Normal random variable with mean $E\{b_i\}$ and variance $Var\{b_i\}$. The analysis in this case is very similar to that of Case 1 above. Consider the stochastic constraint.

$$\Pr\{b_i \geq \sum_{j=1}^{n} a_{ij}x_j\} \geq \alpha_i \tag{A.19}$$

Thus, as in Case 1,

$$\Pr\left\{\left(\frac{b_i - E\{b_i\}}{\sqrt{Var\{b_i\}}} \geq \frac{\sum_{j=1}^{n} a_{ij}x_j - E\{b_i\}}{\sqrt{Var\{b_i\}}}\right)\right\} \geq \alpha_i \tag{A.20}$$

This can only hold if,

$$\frac{\sum_{j=1}^{n} a_{ij}x_j - E\{b_i\}}{\sqrt{Var\{b_i\}}} \leq K_{\alpha_i} \tag{A.21}$$

This means that the stochastic constraint is equivalent to the deterministic linear constraint,

$$\sum_{j=1}^{n} a_{ij}x_j \leq \left(E\{b_i\} + K_{\alpha_i}\sqrt{Var\{b_i\}}\right) \tag{A.22}$$

This shows that in Case 2 the chance-constrained model can be converted into an equivalent linear programming problem.

The solution method can be illustrated using the following example. Consider the chance-constrained problem:

$$\begin{array}{ll} \max & 5x_1 + 6x_2 + 3x_3 \\ \text{subject to} & \\ & \Pr\{a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \leq 8\} \geq 0.95 \\ & \Pr\{5x_1 + x_2 + 6x_3 \leq b_2\} \geq 0.10 \end{array} \tag{A.23}$$

with all $x_j \geq 0$. Suppose a_{1j}'s are independent Normally distributed random variables with the following mens and variance.

$$\begin{array}{ccc} E\{a_{11}\} = 1 & E\{a_{12}\} = 3 & E\{a_{13}\} = 9 \\ Var\{a_{11}\} = 25 & Var\{a_{12}\} = 16 & Var\{a_{13}\} = 4 \end{array}$$

The parameter is Normally distributed with mean 7 and variance 9.

From the standard normal tables

$$K_{\alpha_1} = K_{0.5} \simeq 1.645$$

$$K_{\alpha_2} = K_{0.1} \simeq 1.285$$

Now, for the first constraint, the equivalent deterministic constraint is given by

$$\left(x_1 + 3x_2 + 9x_3 + 1.645\sqrt{25x_1^2 + 16x_2^2 + 4x_3^2}\right) \leq 8$$

and for the second constraint

$$[5x_1 + x_2 + 6x_3] \leq [7 + 1.295(3)] = 10.855$$

Let

$$y = \sqrt{25x_1^2 + 16x_2^2 + 4x_3^2}$$

the complete problem then becomes:

$$\begin{array}{ll} \max & 5x_1 + 6x_2 + 3x_3, \\ \text{subject to} & \\ & x_1 + 3x_2 + 9x_3 + 1.645y \leq 8, \\ & 25x_1^2 + 16x_2^2 + 4x_3^2 - y^2 = 0 \\ & 5x_1 + x_2 + 6x_3 \leq 10.855 \\ & x_1, x_2, x_3, y \geq 0 \end{array} \tag{A.24}$$

which can now be solved by separable programming (Taha, 1971).

Appendix B

Benefits Analysis Interview Checklist

Table B.1: Interview list

	Responsibility	Questionnaire/Duty
Operator	Process Operation	Review historical data to determine normal and unusual operations Determine the process disturbances (repeated or infrequent)
Process Engineer	Process flow diagram Operating goals Products pecification	Equipment limitation & potential changes Product supply & demand Feed quality, availability Environmental limitations
Control Engineer	Process control strategies/objectives	Process model Important parameters Controllers, sensors, alarms, final Quallity of control good (based on variance?) Controller maintenance requirements maintained (related cost)?
Finance /Marketing	Process Economics	Required benefits analysis frequency and period Market data Economic performance function