FAULT DETECTION OF ROTATING MACHINERY FROM BICOHERENCE ANALYSIS OF VIBRATION DATA

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Abstract: The vibration signal carries the signature of faults in most rotating equipments, and early fault detection is possible by analyzing the signal using different signal processing techniques. In this paper we consider a gearbox as a typical representation of a rotating or cyclo-stationary process. Faults in gearboxes leave their signature on the vibration signal and generally manifest themselves as a non-linear transformation in the signal. Bicoherence analysis detects and quantifies the presence of non-linearity in the signal and thus indicates the severity of the fault in the gearbox. In this work, time synchronous averaging is used to find the proper representation of one period of the cyclo-stationary vibration signal. A pilot scale gearbox case study is presented to demonstrate the practicality and utility of the proposed technique.

Keywords: Rotating Machinery, Fault Detection, Vibration, Bispectrum Analysis, Cyclo-Stationary

1. INTRODUCTION

Vibration signal analysis is widely used to detect early faults in rotating machineries, such as gearboxes, turbines, compressors etc. In this paper we consider gearboxes as a typical representation of a rotating or cyclo-stationary process. There are many techniques that have been developed to detect progressing faults in gearboxes. Many of these methods assume that the signal is ergodic and stationary, and therefore the variance or the power spectra of the signal can serve as indicators of severe faults in the machineries. But these methods may fail to perform properly in noisy industrial environments when the noise encompasses the frequency bandwidth of interest in the signal that carries the fault signature of such equipment (McCormick and Nandi, 1998).

The presence of non-linearity in a vibration signal can also serve as a indicator of fault(s) in the rotating machineries. Failure of a mechanical system is always preceded with changes from linear or weakly non-linear to strong non-linear dynamics. As faults develop in the system the process becomes chaotic and the amount of non-linearity in the system increases. Therefore, a measure of non-linearity in the vibration signal would give a measure of deviation of the process from normal operation to the emergence of a fault in the process. Higher Order Statistics (HOS) can be used to detect and quantify the presence of a non-linearity in the vibration signals (Choudhury *et al.*, 2005). Bicoherence successfully detects the emergence of new frequencies due to generation of faults in the system. But like many other methods, bicoherence requires that process under investigation be stationary.

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In rotating machine vibration analysis, the overall response is a combination of deterministic periodic components dominated by the machine rotation, with stochastic random signals, generated by the surroundings or machine imperfections. Due to the periodic and time varying nature of the response signal, the notion of stationarity does not hold (Antoniadis and Glossiotis, 2001). The best approach to overcome this is to exploit the natural periodicity of the signals by extracting a synchronous average of the signal (McCormick and Nandi, 1998). Assuming the signal is averaged over a large number of rotations, this method removes the stochastic part efficiently and produces a single period of a deterministic periodic signal. Bicoherence can then be applied on this averaged period to detect the presence of non-linearity in the rotating machinery.

This paper proposes the use of time synchronous average to preprocess the cyclo-stationary vibration signal, and the use of bicoherence analysis to detect non-linearity in vibration signals, leading to the detection of the severity of the faults in rotating machineries. The technique was applied on real data from a test rig with two levels of severity of faults.

2. BICOHERENCE ANALYSIS

The first and second order statistics (e.g., mean, variance, autocorrelation, power spectrum) are popular signal processing tools and have been used extensively for the analysis of process data. However second order statistics are only sufficient for describing linear processes. In practice, there are many situations when the process deviates from linearity and exhibits nonlinear behavior. Such type of processes can be conveniently studied using Higher Order Statistics (HOS). There are three main reasons for using Higher Order Statistics (HOS): to extract information due to deviations from Gaussianity, to recover the true phase character of the signals, and to detect and quantify nonlinearities in the time series (Nikias and Petropulu, 1993). Time domain data itself is a good source of information. Many statistical measures, e.g., moments, cumulants, auto-correlation, cross-correlation have been developed to measure the time domain information in such data. Not all the information content of a signal can be necessarily and easily obtained from time domain statistical analysis of the data. Transforming the signal from time to frequency domain can expose the periodicities of the signal, can detect the nonlinearities present in the signal and can also aid in understanding the signal generating process. Just as the power spectrum is the frequency domain counterpart of the second order moment of a signal and represents the decomposition or spread of the signal energy over the frequency channels obtained from the Fast Fourier Transform, the bispectrum is the frequency domain representation of the third order cumulants. It is defined as

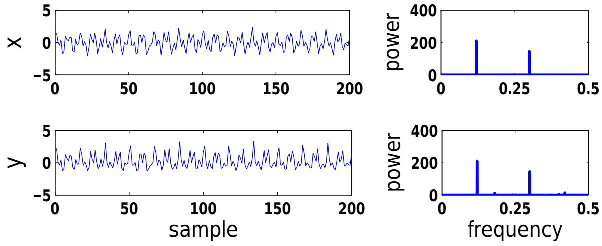


Fig. 1. *Time Trend and Power Spectrum plots of the Linear and Non-Linear Signals.*

$$B(f_1, f_2) = DDFT[c_3(\tau_1, \tau_2)] \equiv E[X(f_1)X(f_2)X^*(f_1+f_2)] \quad (1)$$

where, B(f_1, f_2) is the bispectrum in the bifrequency (f_1, f_2), DDFT stands for Double Discrete Fourier Transformation, $c_3(\tau_1;\ \tau_2)$ is the third order cumulant, τ_1 and τ_2 are the time-lag variables, *X(f)* is the discrete Fourier transform of any time series *x(k)*, and '*' denotes complex conjugate. Equation 1 shows that the bispectrum is a complex quantity having both magnitude and phase. It can be plotted against two independent frequency variables, f_1 and f_2 in a three dimensional (3d) plot.

In order to remove the undesired property of the variance of the estimated bispectrum (Hinich, 1982), the bispectrum can be normalized in such a way that it gives a new measure called bicoherence whose variance is independent of the signal energy (Fackrell, 1996). Bicoherence is defined as:

$$bic^2(f_1, f_2) \triangleq \frac{|B(f_1, f_2)|^2}{E\left[|X(f_1)X(f_2)|^2\right] E\left[|X(f_1+f_2)|^2\right]} \quad (2)$$

where 'bic' is known as the bicoherence function. A useful feature of bicoherence function is that it is bounded between 0 and 1.

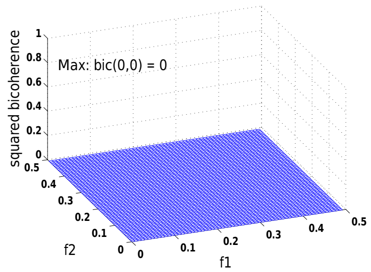
For details of estimating the bispectrum/bicoherence, see (Nikias and Petropulu, 1993; Choudhury *et al.*, 2002).

2.1 Bicoherence of a nonlinear sinusoid signal with noise

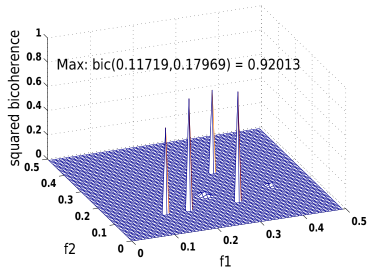
The objective of this example is to demonstrate the power of the bicoherence in the detection of nonlinearity. An input signal was constructed by adding two sinusoids, each having a different frequency and phase. That is,

$$\begin{aligned} x^{'}(k) &= sin(2\pi f_1 k + \phi_1) + sin(2\pi f_2 k + \phi_2) \\ x(k) &= x^{'}(k) + d(k) \\ y(k) &= x^{'}(k) + 0.1x^{'}(k)^2 + d(k) \end{aligned} \quad (3)$$

where, $f_1 = 0.12$, $f_2 = 0.30$ on the normalized frequency scale, and $d(k)$ is a white noise sequence with variance 0.04.



(a) *linear signal x*



(b) *non-linear signal y*

Fig. 2. *Bicoherence Analysis of Linear and Non-Linear Signals.*

The left panel of the figure 1 shows the time series while the right panel shows the power spectrum of the signal x and y, respectively. Neither of these plots help in distinguishing the two signals. However, the use of higher order statistics can successfully detect the nonlinearities present in y. Figure 2 shows the three dimensional squared bicoherence plots of x and y, respectively. For the signal x, the plot shows no peaks and thus clearly indicates that the signal is linear. On the other hand, for the signal y, the plot shows significant peaks indicating the presence of non-linearity in the signal.

The peaks in the bifrequency plane can be explained by rewriting the expression for y as:

$$\begin{aligned} y(k) &= \sin(2\pi f_1 k + \phi_1) + \sin(2\pi f_2 k + \phi_2) \\ &+0.1[1 - \cos(2(2\pi f_1 k + \phi_1)) - \cos(2(2\pi f_2 k + \phi_2)) \\ &+ \cos(2\pi(f_2 - f_1)k + \phi_2 - \phi_1) \\ &- \cos(2\pi(f_1 + f_2)k + \phi_1 + \phi_2)] + d(k) \end{aligned} \quad (4)$$

The nonlinearities are caused by the interactions of any two of the signals with frequencies f_1, f_2, $2f_1$, $2f_2$, f_2-f_1, and $f_1 + f_2$. For the output signal y, the squared bicoherence plot shows peaks at (0.12,0.12), (0.12,0.18), (0.30,0.30), and (0.12,0.30) bifrequencies. These bifrequencies correspond to (f_1, f_1),(f_1, f_2-f_1), (f_2, f_2),and (f_1, f_2), respectively. Therefore, the bicoherence plot correctly identifies the frequency interactions that resulted from the presence of nonlinearity in the signal.

3. CYCLOSTATIONARITY

Vibration signals from a gearbox are a combination of periodic signals with random noises and the combination of these two components produces a signal that have a periodically time-varying statistics. For a stationary signal the statistics does not change with time and the moments of the signal remain constant. If the statistics of the signal has a periodically time varying component it is identified as a cyclostationary signal. The weak or wide sense cyclostationarity of a signal refers only to the variations of the mean and autocorrelation of the signal.

3.1 Definition:

A random process $x(t)$ is cyclostationary of order N with period T, if for every $n = 1, N$ and time instants t_1, t_2,...,t_n, the probability distribution function $P_{x(t)}$ is periodic with period T:

$$\begin{aligned} P_{x(t)} &= P_{x(t+T)} \\ P_{x(t)} &= Prob\{x(t+t_1) \le X_1, x(t+t_2) \\ &\le X_2, ..., x(t+t_n) \le X_n\} \end{aligned} \quad (5)$$

As a direct consequence of Equation 5, the moments and cumulants of $x(t)$ also vary periodically with time:

$$E\{\prod_{i=1}^{N} x(t_i)\} = E\{\prod_{i=1}^{N} x(t_i + T)\} \quad (6)$$

where N denotes the order of the statistic function and $E\{.\}$ denotes the statistical expectation operator (Antoniadis and Glossiotis, 2001). If the process is assumed *cycloergodic*, the statistical expectation operator $E\{.\}$ in Equation 6 can be replaced by the time average operator $\langle . \rangle$ which can be defined as:

$$Continuous : \langle x(t) \rangle \equiv \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t) dt \quad (7)$$

$$Discrete : \langle x(n) \rangle \equiv \lim_{M \to \infty} \frac{1}{(2M+1)} \sum_{j=-M}^{M} x(j) \quad (8)$$

A first order cyclostationary process, $N = 1$, can be represented by the time periodical mean (first order moment):

$$m(t) = E\{x(t)\} = m(t+T) \quad (9)$$

An example of this process is a sinusoidal signal with added white noise.

3.2 Time Synchronous Averaging

The vibration signal of a gearbox can be categorized as a weak or wide sense cyclostationary signal. If only Equation 6 is valid, the random signal $x(t)$ is cyclostationary in a weak or wide sense. Techniques that require the assumption of stationarity cannot be used to analyze this signal as these methods may produce erratic results if a periodic disturbance is present within the frequency range at which the fault of the gearbox is being investigated. To generate a stationary signal that can represent the deterministic part of the original signal, the natural periodicity of the signal can be extracted and organized by extracting a synchronous average of the signal. If the signal is synchronously averaged over a large number of rotations of a gear, it can remove the stochastic part of the signal keeping only the periodic deterministic part of the signal harmonically related to the rotational period of the gear. It is assumed that the stochastic or noise component has a zero mean. This would produce a single period representation of the vibration signal and is known as the Time Synchronous Average of the signal (McCormick and Nandi, 1998). First order cyclostationarity is exploited in condition monitoring applications through the use of time synchronous averaging. According to this method, a vibration signal $x(t)$ is averaged for one rotation period by calculating the mean of the samples that have been measured for a number of rotations N separated by a time interval T of one period of rotation:

$$m(t) = \frac{1}{(N-1)} \sum_{l=0}^{N-1} x(t + lT) \tag{10}$$

From the time synchronous average, non-linearity in the vibration signal of the rotating machine can be identified using bicoherence.

4. GEARBOX FAULT DETECTION

The vibration signal of a gearbox carries the signature of the faults in the gears. Faults in gearbox are associated with some non-linear mode of operation. A fault free machine running smoothly in normal operation would generate linear periodic vibrations. Faults manifest themselves as non-linear elements, typically due to the presence of new frequencies and interactions between these frequencies. This non-linearity would increase as the process deviates more from its normal operation. Therefore, a measure of non-linearity of the process would give a measure of deviation of the process from normal operation and the emergence of a fault in the process. Bicoherence analysis can be used to detect non-linearity in the vibration signal of a gearbox and a measure of the non-linearity can be obtained. But since vibration signals from a gearbox are cyclostationary, direct application of bicoherence technique would lead to unpredictable results. The cyclo-stationarity and zero-mean noise components are first removed using Time Synchronous Averaging technique over one period of rotation. The resulting averaged signal should then be analyzed using bicoherence analysis to detect the amount of non-linearity within the system.

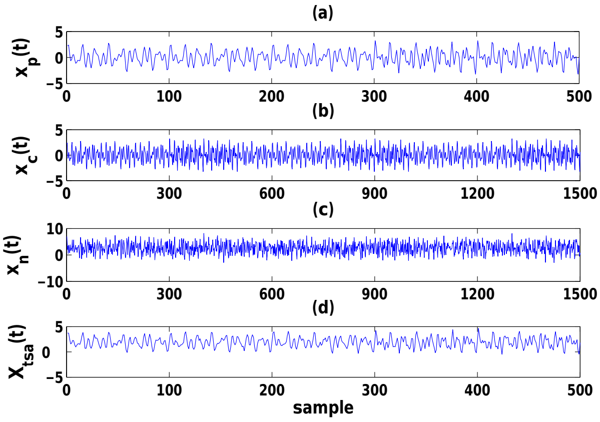
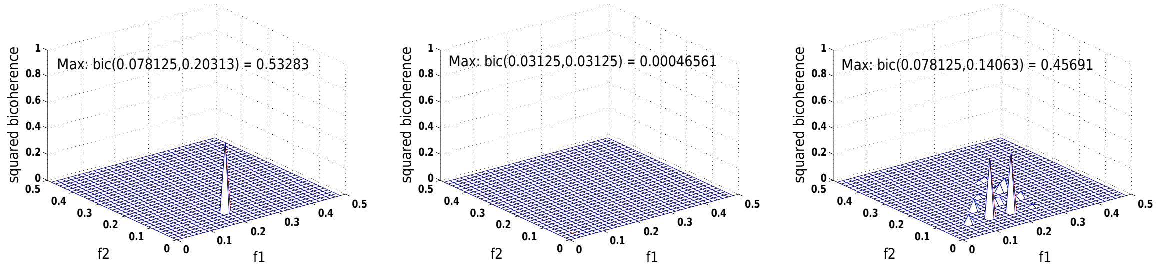


Fig. 3. *Plot showing (a) one period of the simulated time series (b) simulated time series before adding noise (c) simulated time series after adding noise with a SNR of 1 (first 1500 samples have been shown) (d) time synchronous average of the simulated signal.*

5. ILLUSTRATIVE EXAMPLE

A simulation example is presented here to illustrate the use of bicoherence technique on cyclostationary vibration signal from a gearbox. A time series signal $x_p(t)$ of 500 samples has been generated with the 3 frequencies 0.2, 0.08, and 0.15 Hz of the same amplitude. Assuming that non-linearity exists only for half of the time of one period, a signal with frequency 0.28 (sum of 0.2 and 0.08) Hz is added to the signal $x_p(t)$ for the first 250 samples only to introduce non-linearity. Figure 3(a) shows the signal $x_p(t)$. This short time series of 500 samples has been repeated 30 times to generate the time series $x_c(t)$ so that $x_p(t)$ represents a period of the periodic signal $x_c(t)$. Figure 3(b) shows the signal $x_c(t)$, clearly depicting the periodic nature of the time series. White noise ε, such that the signal to noise ratio is 1, has been added to $x_c(t)$ to generate the noise corrupted signal $x_n(t)$ shown in Figure 3(c). The signal $x_p(t)$, which is a period of the deterministic part of the signal $x_n(t)$, is a stationary signal. Application of bicoherence analysis on this signal generates the plot shown in Figure 4(a). The figure confirms that bicoherence detects the presence of non-linearity in the signal. However if bicoherence analysis is applied to the generated noisy signal $x_n(t)$, it fails to detect the presence of any non-linearity. Figure 4(b) shows the plot for bicoherence analysis on the simulated noisy signal $x_n(t)$. The failure of the technique is due to the fact that signal $x_n(t)$ is cyclostationary and it does not meet the requirements for bicoherence analysis. To detect the non-linearity present in the simulated signal $x_n(t)$, the signal is first treated with time synchronous averaging technique. At least 10 periods of data samples is required in the signal to compute the time synchronous average that can properly represent one period of the signal. The time synchronous average $x_{tsa}(t)$ of the signal $x_n(t)$ is calculated by the equation



(a) *single period* (b) *NOT pre-processed by TSA* (c) *pre-processed by TSA*

Fig. 4. *Bicoherence analysis of the simulated signal.*

$$x_{tsa}(t) = \frac{1}{(30-1)} \sum_{l=0}^{30-1} x_n(t + l \times 500)$$

where, $t = 1, 2, \ldots, 500$ as each period has 500 samples. Figure 3(d) shows the time synchronous average $x_{tsa}(t)$ of the simulated signal $x_n(t)$. Bicoherence analysis can now be applied on the stationary signal $x_{tsa}(t)$ and the resulting plot is shown in Figure 4(c). The plot clearly shows significant peaks, indicating the presence of non-linearity in the time synchronous averaged signal. It confirms that bicoherence analysis applied on the time synchronous average of a cyclostationary signal can clearly identify non-linearity within the cyclostationary signal, whereas the technique fails if the time synchronous averaging is not performed before bicoherence is applied.

6. PILOT PLANT CASE STUDY

A pilot plant case study was performed to assess the effectiveness of the proposed technique in early detection of gear faults. Data was generated using a test rig that could simulate single and multiple faults. The rig is located in the Reliability Lab in the Mechanical Engineering Building at the University of Alberta, Canada (Tian *et al.*, 2002). The configuration of the test rig is shown in Figure 5. The gearbox had 3 shafts with a total of 4 gears a, b, c and d. A brake was used to create the desired load of operation. Normal gear a was later replaced by the damaged gear a', which had a chipped tooth. Similarly, normal gear d was later replaced by the damaged gear d', which had a missing tooth. Both damaged gears were also used at the same time to simulate multiple faults. Shafts 1, 2 and 3 were rotating at 10, 3.3 and 5 Hz respectively during data collection. The gear meshing frequency was 160 Hz or 0.125 Hz in the normalized frequency.

6.1 Data Description

A total of three data sets were collected from the test rig. The data sets were collected as accelerometer measurement from sensors. Every time series had 8192 samples collected at 1280

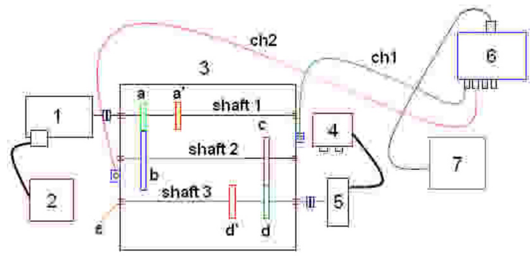


Fig. 5. *Configuration of the test rig used to generate data for the case study. 1-Motor, 2-Variable Speed Motor Controller, 3-Gearbox, 4-Brake Controller, 5-Brake, 6-Siglab Vibration Analyzer, 7-Computer.*

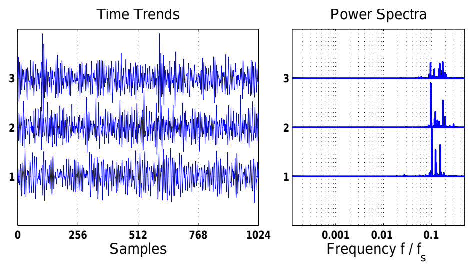


Fig. 6. *Time Trend and Power Spectrum plots of the data sets generated from the test rig for case study (only first 1024 samples have been shown).*

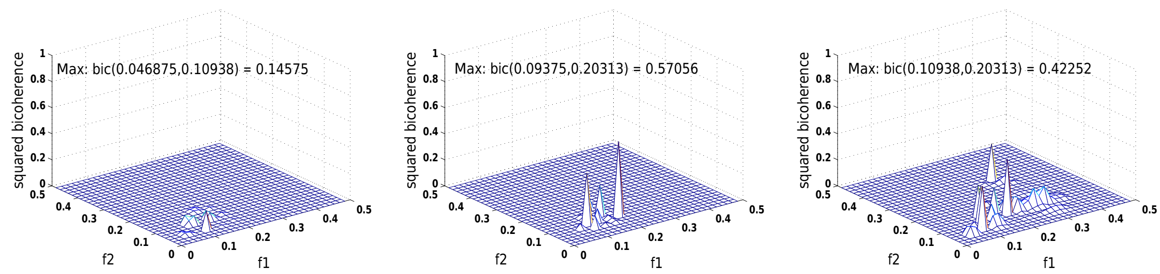
Hz. The three data sets were collected under the following conditions:

(1) All normal gears used
(2) One damaged gear with a Chipped tooth used - gear a was replaced by a'
(3) One damaged gear with a Chipped tooth and another damaged gear with a Missing tooth used - both gear a and d were replaced by a' and d'

The time-trend plots (with only the first 1024 data samples) and the power spectra of the three data sets are shown in Figure 6. Clearly, the data are noisy and it is hard to conclude anything from power spectrum.

6.2 Bicoherence Analysis

Figure 7 shows the bicoherence analysis of the three data sets after time synchronous averaging.



(a) *no damaged gear* (b) *one damaged gear* (c) *two damaged gears*

Fig. 7. *Bicoherence analysis of real data from rig.*

The Least Common Multiple (LCM) of the periods of the shafts under investigation was 0.2 seconds (LCM of 0.1 and 0.2 seconds corresponding to 10 and 5 Hz respectively). This LCM was used as the period to calculate the time synchronous average of the signals. Therefore each time synchronous average represents 2 periods of the gear a and 1 period of the gear d. Brief explanations of the plots are given below.

6.2.1. Normal Gears Figure 7(a) depicts the bicoherence analysis plot for the data set generated from normal gears. No significant peaks can be observed in the plot since the gears had no fault in their teeth. The maximum bicoherence is 0.15 which in this case can be taken as negligible.

6.2.2. One damaged Gear Figure 7(b) shows the bicoherence analysis plot for the data set generated from one damaged gear. The 2 large peaks in the plot indicate the non-linearity present in the data set. The maximum bicoherence is 0.57 which is significant. It should be noted that the peaks usually show up at the gear meshing frequency of the gearbox.

6.2.3. Two damaged gears Figure 7(c) shows the bicoherence analysis plot for the data set generated from two damaged gears. This time the plot has multiple significant peaks and they are spread over a wider range of frequencies. The extent of non-linearity has clearly increased in this data set. Though the maximum bicoherence is 0.42 (less than that of one damaged gear) the larger number of peaks indicate the non-linearity in the data set is higher than the one for one damaged gear.

7. CONCLUSION

The application of bicoherence analysis combined with time synchronous averaging has been proposed here to detect the severity of faults present in gearboxes. The presence of faults in rotating machineries are accompanied by the increased presence of non-linearity in the vibration signal. Bicoherence successfully detects and quantifies the amount of non-linearity present in the signal provided the signal is stationary. Since the vibration signal from a gearbox is cyclo-stationary it is first transformed to a clean signal representing one period of rotation by time synchronous averaging. The peaks in the plots of bicoherence analysis indicates the presence of non-linearity which in turn indicates the presence of faults. The number of significant peaks in the plots increases with an increase in the number of faults present. Therefore bicoherence analysis successfully detects the severity of the faults present in the gearbox. The proper application of the technique on real vibration data from the rig demonstrates the strength and efficacy of the technique.

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