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Equivalent Uniform Moment Factor for Lateral Torsional Buckling of Steel Beams

by
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to

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Abstract

Lateral torsional buckling is an ultimate limit state that can govern the capacity of laterally unsupported flexural members. The bending moment distribution along the laterally unsupported length of a beam is known to have a significant effect on the stability of laterally unbraced beams. The elastic buckling capacity of laterally unbraced beams is obtained from an equation derived for a constant moment distribution over the unbraced length and modified for the moment distribution within the unbraced length using an equivalent moment factor, ω_2 . The two most commonly used expressions for this equivalent moment factor are attributed to the work of Salvadori (1955) and Kirby and Nethercot (1979). Contrary to the equation proposed by Salvadori, which is applicable for linear moment gradients, the equivalent moment factor proposed by Kirby and Nethercot is applicable to non-linear moment gradients.

A recent investigation by Serna *et al.* (2006) indicated that the equation proposed by Kirby and Nethercot may not be sufficiently conservative for certain moment distributions. A new equation was therefore proposed for the equivalent moment factor. A review of the work of Serna *et al.* by Wong and Driver (2008) lead to the proposition of a new equation for the equivalent moment factor, although close examination of the equation showed that it is very similar to that of Serna *et al.* The work of Wong and Driver was based on the same database of analysis results that were used by Serna *et al.*

In order to assess the various models for equivalent moment factor, a finite element model was developed to investigate the effect of moment distribution on lateral torsional buckling capacity. Of the four equivalent moment factor expressions investigated, the one proposed by Serna *et al.* was found to be in closest agreement with the finite element analysis results. The more widely used expression by Kirby and Nethercot tends to be non-conservative for the case where the end moment ratio is close to 1.0.

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1. Introduction

Buckling and lateral stability are among the key parameters in the design of steel structures. Flexural members subjected to bending about their major axis may develop buckling in the compression flange combined with lateral bending, leading to what is known as lateral torsional buckling. For doubly symmetric, laterally unbraced, slender beams, lateral torsional buckling can be the governing ultimate limit state.

It is customary for beams, girders and joists to have a greater stiffness and strength in the plane of loading. These structural members have a tendency to fail by lateral torsional buckling if not properly restrained against lateral deflection and twisting, especially at the construction stage when the braces are absent or different from the permanent ones.

The lateral torsional buckling capacity depends upon a variety of material and geometric properties, support conditions, location of applied load relative to the shear centre and bending moment distribution along the length of the member. The critical elastic lateral torsional buckling capacity for uniform moment gradient is given by (CSA, 2006):

$$[1] \quad M_u = \frac{\pi}{L} \sqrt{EI_y GJ + \left(\frac{\pi E}{L}\right)^2 I_y C_w}$$

Where M_u is the elastic lateral torsional buckling strength, E is modulus of elasticity, G is the shear modulus, I_y is moment of inertia about weak axis, J is the St. Venant torsional constant, C_w is the warping constant of the section.

Generally, the consideration of non-uniform bending moment diagram is taken into account by means of equivalent uniform moment factor ω_2 . The elastic critical moment of simply supported beam with uniform moment is multiplied by this factor to obtain the elastic critical moment for any bending moment diagram.

This report presents a comparison between different equations for the equivalent moment factor and compares the different equations with finite element analysis results. Of the four models investigated, two have been in use for several years and have been adopted in various design standards, whereas two other models, similar in form and results, have just been recently proposed. The objective of the work presented in this report is to evaluate the equivalent moment factors currently in use and recently proposed

1.1 Lateral torsional buckling:

A short beam with a compact cross-section can reach its full plastic moment capacity without any lateral instability. However, if the beam is slender and the compression flange is not adequately braced in the lateral direction, a different phenomenon occurs. As the beam is loaded in bending about its strong axis, it deforms in the direction of loading.

At some load stage, out-of-plane bending (bending about the weak axis) and twisting of cross-section occur simultaneously, as illustrated in Figure 1. This phenomenon is known as *lateral torsional buckling* and the moment causing it is referred to as *critical moment*. The determination of this critical moment is an eigenvalue problem for geometrically perfect beam. The derivation of the critical moment for a beam with a doubly symmetric cross-section is based on a laterally unsupported beam segment subjected to a constant moment. Under this loading condition, the principal variable affecting the lateral torsional buckling capacity of the beam segment is the length of the segment. Other variables that affect the lateral torsional buckling resistance include the type and position of the loads, the restraints at the ends and at intermediate locations, the type of cross-section, continuity at supports, the presence or absence of devices that restrain warping at critical locations the material properties, the magnitude and distribution of residual stresses, pre-stressing forces, initial geometric imperfections, load variability (orientation and point of application), discontinuities in cross-sections, cross-sectional distortion and interaction between local and overall buckling (Ojalvo & Chambers, 1977).

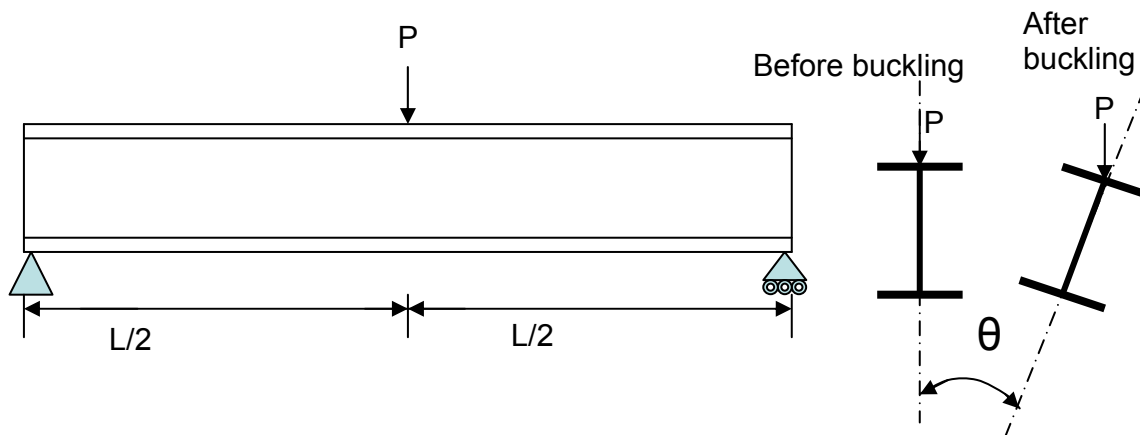


Figure 1– Lateral torsional buckling (section at midspan)

Lateral torsional buckling can generally be avoided by providing sufficient lateral bracing or by using torsionally stiff sections as box sections.

The determination of the lateral torsional buckling strength is quite complex and closed form solutions exist only for simple load cases. However, the solution for the constant moment condition can be modified for more complex moment distribution by using a coefficient known as the *equivalent uniform moment factor*, ω_2 .

1.2 Elastic and Inelastic lateral torsional buckling:

The lateral torsional buckling behaviour is often illustrated graphically by plotting the unbraced length against the critical moment as shown in Figure 2.

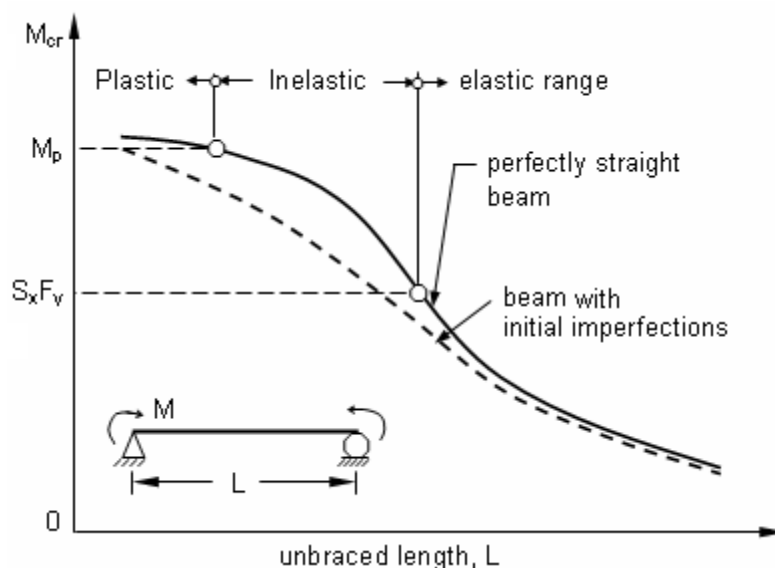


Figure 2– Lateral torsional buckling curves for beams

The solid curve represents the variation of critical load for a perfectly straight beam while the dashed curve characterizes the behaviour of a beam with initial imperfections. The behaviour can be classified into three distinct categories: (1) elastic buckling that occurs for slender members when the cross section is fully elastic; (2) inelastic lateral torsional buckling that occurs in beams of intermediate length where the bending moment at the instant before lateral torsional buckling is sufficient to cause portions of the member to yield; (3) local buckling, which occurs in stocky members where the beam is able to reach its local buckling capacity before failure by lateral torsional buckling. Lateral torsional buckling may occur in elastic or inelastic range, depending upon the laterally unsupported length of the section, while it does not occur for sections of same moment of inertia about the principal axes (box, circular), regardless of the slenderness ratio.

1.3 Elastic critical moment

For the case of beams with doubly symmetric sections and simply supported ends and subjected to a constant moment over the laterally unbraced length, the elastic lateral torsional buckling strength or elastic critical moment is given by equation [1].

This is conservative for most cases as the actual bending moment is not always uniform. Furthermore the end connections also often provide restraint conditions that are more beneficial than simple support (i.e. free to bend about the weak axis and free to warp, but restrained from twisting and translation in the lateral direction).

The modified form, for any other moment distribution over the laterally unsupported length is given by:

$$[2] \quad M_u = \omega_2 \frac{\pi}{L} \sqrt{EI_y GJ + \left(\frac{\pi E}{L}\right)^2 I_y C_w}$$

where ω_2 , as defined earlier, is a modifier introduced to account for the increased moment resistance as a result of non-uniform moment gradient.

2. Review of equivalent moment factor

This section presents a review of the equivalent moment factor used in various design codes and standards.

2.1 S16-01

The value of ω_2 adopted in S16-01 was derived for a linear moment distribution between the brace points. It is based on the work of Salvadori (1955), and takes the following form:

$$[3] \quad \omega_2 = 1.75 + 1.05\kappa + 0.3\kappa^2 \leq 2.5$$

where κ is the ratio of the smaller factored moment to the larger factored moment at the ends points of lateral support. This ratio is positive for double curvature and negative for single curvature.

Because S16-01 recognizes that this approach may be non-conservative for non-linear moment distributions, a value of ω_2 of 1.0 is suggested if the bending moment within the unbraced length is greater than the larger of the two end moments or when there is no effective lateral support to the compression flange at one end of the unsupported length.

Although the method adopted by S16-01 is simple to implement, the predictions using the value $\omega_2 = 1.0$ can be highly conservative for some load cases as in simply supported beams where the bending moment diagram for different load cases is treated as a constant bending moment. Furthermore, for cases in which the moment is constant over most of the length the values of ω_2 from the above equation may be non-conservative because this loading represents a more severe case. Treatment of some cases as triple curvature is not specifically addressed. These are well documented in Wong and Driver (2008).

2.2 AISC (2005)

A closed form, empirical, expression proposed by Kirby & Nethercot (1979) for ω_2 is given by:

$$[4] \quad \omega_2 = \frac{12}{3(M_1/M_{\max}) + 4(M_2/M_{\max}) + 3(M_3/M_{\max}) + 2}$$

A slightly modified version of this equation was adopted by AISC LRFD, which is valid for any moment distribution and is given by:

$$[5] \quad \omega_2 = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

The upper limit for this expression is specified as 3.

Where M_{\max} is the maximum moment and M_A , M_C , and M_B are the moments at quarter points as shown in Figure 3.

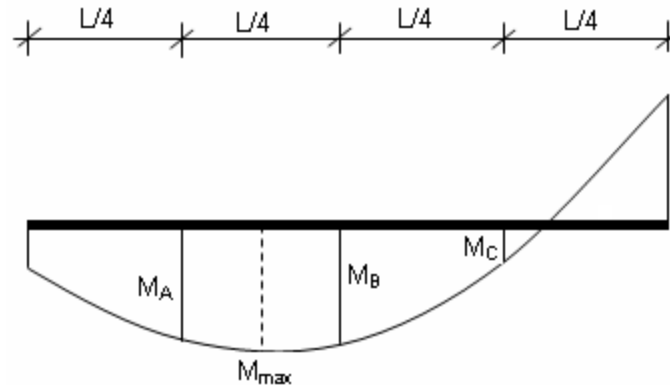


Figure 3 – Moments at quarter and mid points

2.3 Serna *et al.* (2006)

Numerical analysis results were obtained for equivalent uniform moment factor using the finite difference and finite element methods for a wide range of loading and end support conditions. The results also accounted for lateral rotation and warping restraint at the brace points. Both narrow (IPE500) and wide (HEB500) flange sections were used for lengths of 8 and 16 m.

The loadings considered were linear moment distributions, concentrated load with two equal and opposite end moments and with a single end moment, a uniform load with equal end moments and one end moment.

The numerical results were compared to expressions for the equivalent moment factor from different design standards and results of numerical analysis from various research groups. The design standards that were evaluated were AISC (1994), CSA-S16-01, Eurocode 3 (EC3 2005) and BS 5950-1 (2000). Some relevant observations noted were:

- AISC values were conservative for all cases of linear moment distributions.
- For beams with uniformly distributed loading and two equal end moments with simple torsional supports the AISC values were conservative for β (ratio of end moment to simple span moment) > 1.1 while it was non-conservative for $\beta < 1.3$ for prevented lateral bending. Same observations were made for uniform loading and one end moment.

- For midspan concentrated load and two end moments, the AISC values were found to be very conservative or non-conservative, depending upon the support conditions.

A new equation was proposed by curve fitting the numerical results:

$$[6] \omega_2 = \sqrt{\frac{35M_{\max}^2}{M_{\max}^2 + 9M_2^2 + 16M_3^2 + 9M_4^2}}$$

where the moments are defined as shown in Figure 4.

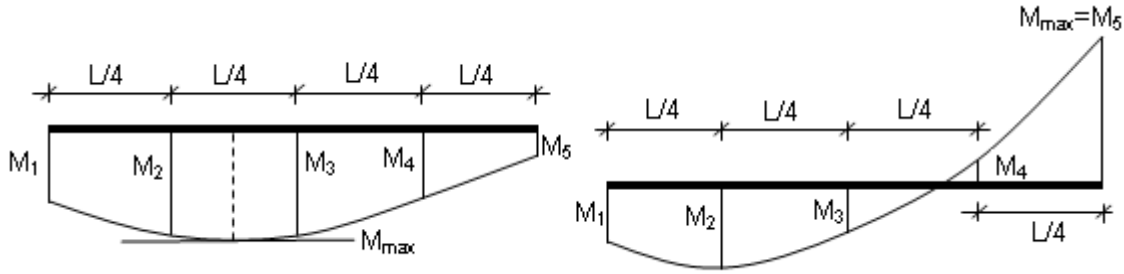


Figure 4 – Quarter and midpoint moments for different moment diagrams

2.4 Wong and Driver (2008)

Wong and Driver (2008) reviewed the work of Serna *et al.* (2006) and added a comparison with the Australian design standard and AASHTO (2005), which were not included in Serna *et al.* comparisons. Additional load cases were also added for their comparisons. Bending moment distributions of a set of 12 loading conditions along with different end supports were considered for this purpose. No new numerical analysis results were added to the database from which the various prediction equations were evaluated.

Some of the observations made by Wong and Driver were:

- As observed by Serna *et al.*, the equation for ω_2 in S16-01 was either very conservative or non-conservative for most of the cases with non-linear moment distribution. This was expected since the S16-01 equation, originally derived by Salvadori (1955), was derived for a linear moment distribution between points of lateral support.
- A comparison of the equivalent moment factor equations with numerical analysis results for a beam segment with a point load at mid length and end moments revealed that the equation proposed by Kirby and Nethercot (1979) and the modified version adopted in AISC (2005) were considerably better than the Salvadori equation, but gave non-conservative values over a small range of β . The

over prediction of the critical moment of these equations for the load case mentioned was as large as 35%.

- The square root format proposed by Serna *et al.* improved the predictions but over predicted some results as compared to numerical results presented by Serna *et al.*, especially for the load case of uniform load with two equal end moments.

These observations created the motivation for the derivation of an alternative equation that could predict the critical moment more conservatively while capturing the trends consistently.

The value for ω_2 proposed by Wong and Driver took the following form:

$$[7] \ \omega_2 = \frac{4M_{\max}}{\sqrt{M_{\max}^2 + 4M_A^2 + 7M_B^2 + 4M_C^2}} \leq 2.5$$

M_{\max} , M_A , M_B and M_C are as defined by AISC.

This is similar in form to the equation proposed by Serna *et al.* It follows the trends well over its full range. An upper limit of 2.5 was recommended in the absence of more data to justify the elimination of this limit.

It should be noted that some of the numerical analysis results presented by Serna *et al.* for beams loaded with a uniformly distributed load and end moments were not consistent with the expected trend. Indeed, when comparing beams 8 m long to 16 m long beams, a significant difference in capacity between the two was noted for a certain range of β values. Since ω_2 is not a function of the beam length, (considering that it is determined from an elastic analysis), it is surprising that such difference was observed between the 8 m and the 16 m long beams. This issue was not mentioned by Wong and Driver, although this particular load case formed the basis for proposing a new equation for ω_2 .

Given the suspicious nature of some of the numerical analysis results from Serna *et al.* as well as the fact that these analysis results were used as the basis for the new proposed expression for ω_2 for CSA-S16-09, an investigation of this problem is warranted. A finite element investigation of the lateral torsional buckling capacity of beams loaded with either a point load at midspan or a uniformly distributed load, coupled with end moments, was conducted and the results are presented in the following.

3. Finite Element Analysis

S-Frame, structural analysis software, was used for the analysis. A Eigen value analysis was used to get the deflected shape (mode shape or eigenvector) and the associated load factor (eigenvalue).

3.1 Model for the analysis

A W460x89 section was chosen for this investigation. This section is equivalent to the IPE500 section used by Serna *et al.* (2006). The beam was modeled as three plates representing the two flanges and the web. Four-node quadrilateral shell elements were used to model the web and flanges. The flanges are modeled with 8 shell elements over the width and 160 elements along the beam length. The web is modeled with 8 shell elements over the height. A general diagram and related details are shown in Figure 5. The finite element model in S-Frame is shown in 5 (a) while the cross-section dimensions are shown in 5 (b).

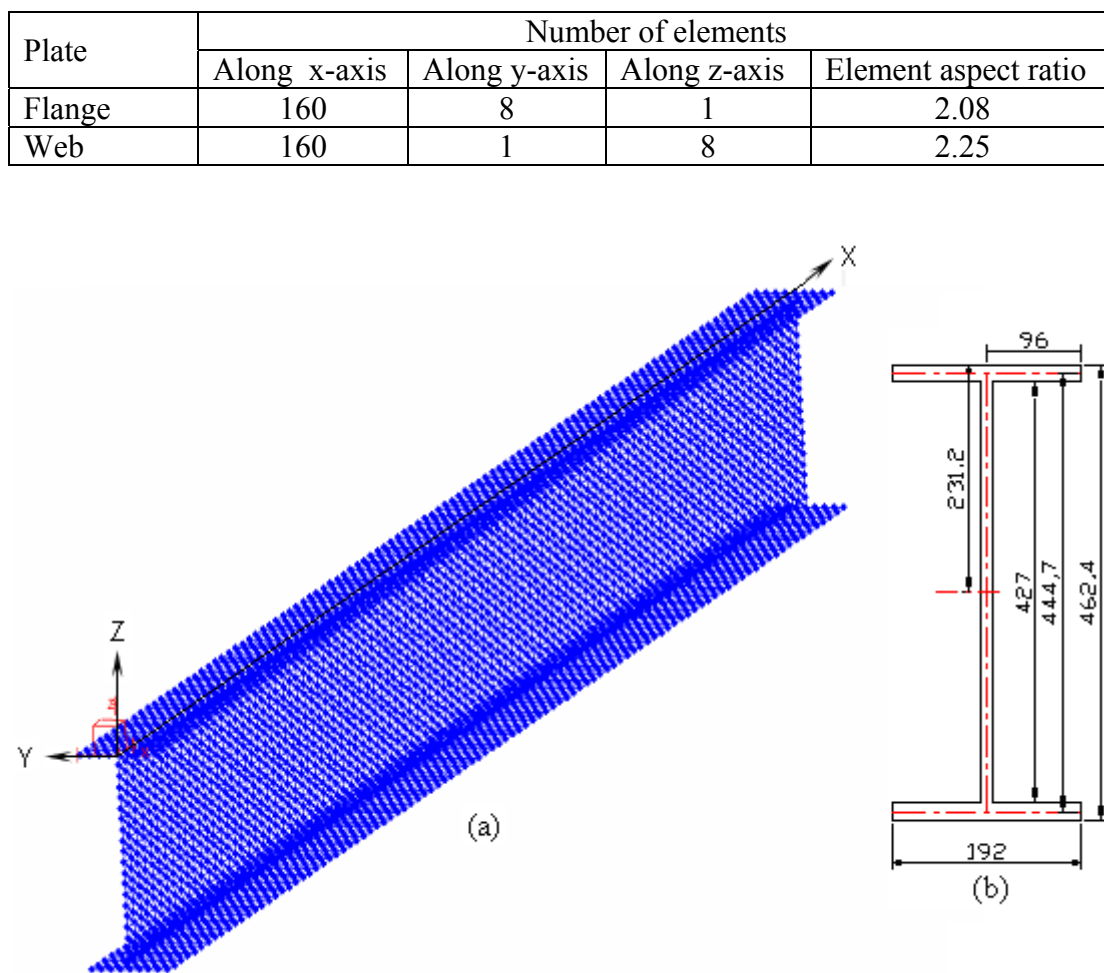


Figure 5 – Cross section and principal axes

Element details

The shell elements used for this investigation have six degrees of freedom per node; rotation and translation in three orthogonal directions. In order to model the flange to web fillets the thickness of the web elements at the flange to web junction was increased to provide the same area as the fillet and web in that zone. This is illustrated in Figure 6.

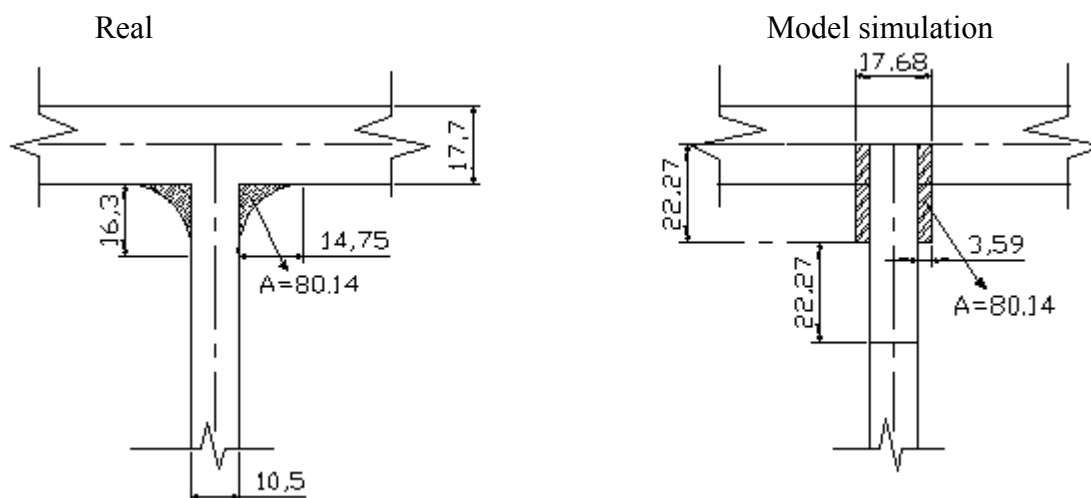


Figure 6 – Flange to web junction (all dimensions in mm)

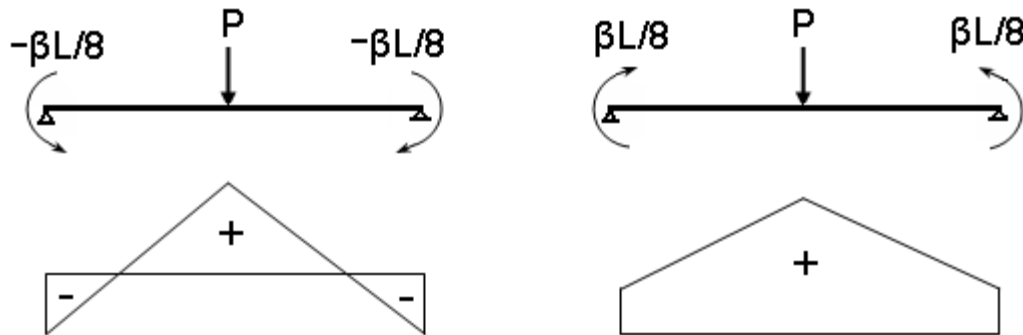
3.2 Support conditions

In order to model simple supports at both ends of the beam, the central web-node at each end is supported in the z-direction and rotation about the axis of the member (the x-axis) is restrained at both ends. The rotation about the y- and z- axes are released at both ends while an additional x-axis translation is released at the far end. All degrees of freedom are released, except y-axis translation for corner nodes of the flanges thus establishing lateral and torsional supports at both ends. The orientation of the reference axes is shown in Figure 5 (a).

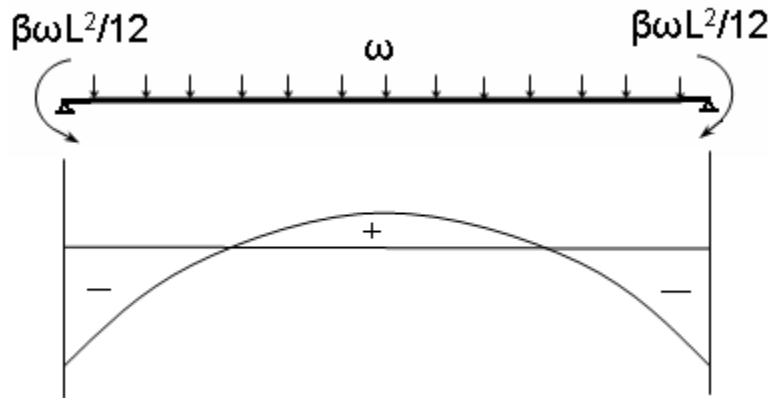
3.3 Loading

Serna *et al.* (2006) and Wong and Driver (2008) indicated two load conditions in which the equations demonstrated discrepancies in predicting the ω_2 value. The first loading condition for which the AISC equivalent moment factor was found to lead to non-conservative strength predictions includes a midspan point-load applied at the shear centre (central node between the beam ends and flanges) and concentrated equal

moments at the beam ends. The negative end moments are a fraction β of the simple span moment as shown in Figure 7a.



(a) Beam segment with point load at mid length



(b) Beam segment with a uniformly distributed load

Figure 7 – Loads and parameter β for point load and equal end moments.

The second load condition is uniform load with two equal end moments for which the equation given by Serna *et al.* was reported to be non-conservative over a certain range of β . The loading condition and parameter β are shown below in Figure 7b.

3.4 Validation

In order to validate the finite element model developed for this investigation, a buckling analysis was run for the model with concentrated end moments only and the predicted buckling moment was compared with the theoretical value of the lateral torsional buckling capacity.

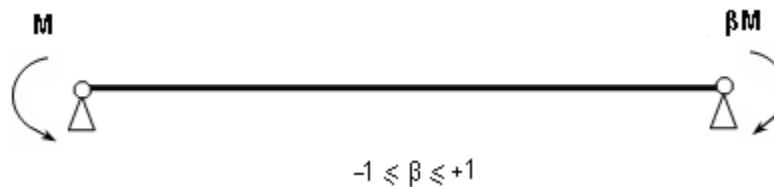


Figure 8 – Member with end moments only

P- Δ buckling analysis was run in S-Frame for the beam segment subjected to a constant moment ($\beta = 1.0$). The load factor determined by the analysis was 251. Hence lateral torsional buckling capacity predicted from the analysis is 251kN-m. A graphical representation of the beam in the deformed and un-deformed configurations is illustrated in Figure 9. Since the applied end moments in the finite element model were 1 kN·m, it was concluded that the lateral torsional buckling capacity of the analysis specimen is 251 kN·m.

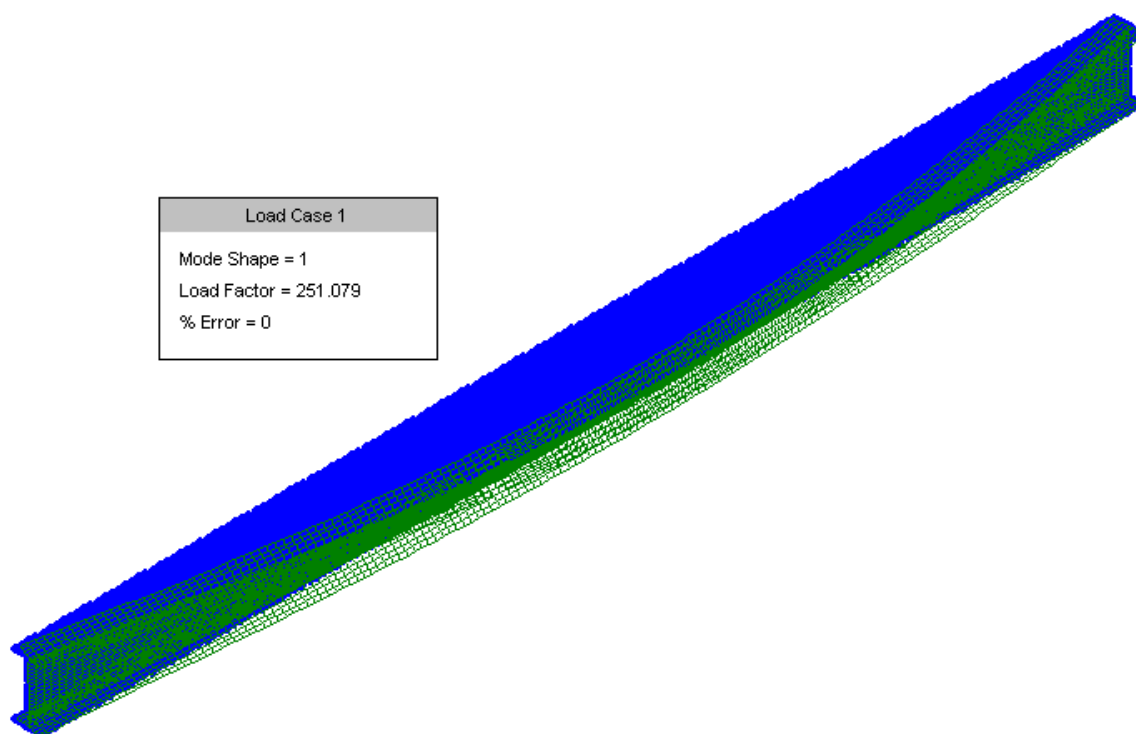


Figure 9 – Buckled shape and load factor for equal end moments only.

The finite element results for the case of end moments are only plotted and compared with the four equations shown below in Figure 10. The finite element result seems to be in close agreement with the equation for ω_2 presented by Serna *et al.*

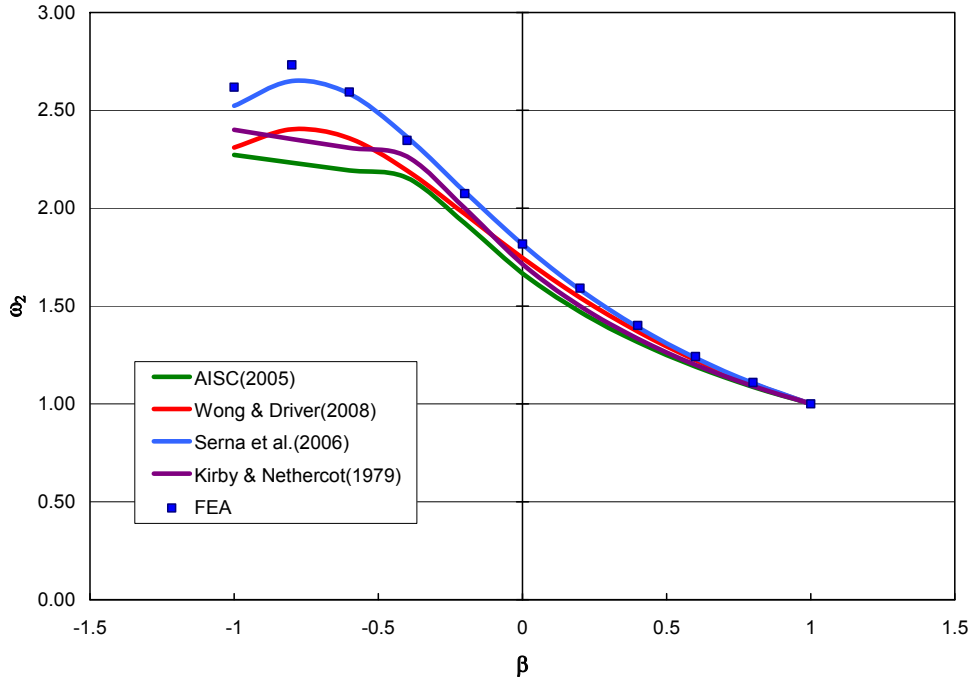


Figure 10 – ω_2 versus β for the case of end moments only

The theoretical moment capacity is calculated using equation [1] as:

$$M_u = \frac{\pi}{L} \sqrt{EI_y GJ + \left(\frac{\pi E}{L}\right)^2 I_y C_w}$$

$$EI_y GJ = 2 \times 10^5 \times 20.9 \times 10^6 \times 77000 \times 907 \times 10^3 = 2.92 \times 10^{23}$$

$$\left(\frac{\pi E}{L}\right)^2 I_y C_w = \left(\frac{\pi \times 200000}{8000}\right)^2 \times 20.9 \times 10^6 \times 1040 \times 10^9 = 1.34 \times 10^{23}$$

$$M_u = \frac{\pi}{8000} \sqrt{2.92 \times 10^{23} + 1.34 \times 10^{23}} = 256 \text{ kN} \cdot \text{m}$$

The analysis result is only 2.04% less than the theoretical value, which indicates that the proposed finite element approach and model are valid.

Validity of the same model was also confirmed using the commercial finite element software ANSYS. The element used was BEAM 188. It is a quadratic three-dimensional beam element and is suitable for analyzing slender to moderately stocky beams. It possesses warping degrees of freedom in addition to the conventional six degrees of freedom. The cross-section details are provided as part of the input parameters. A section is associated with the beam elements by specifying the section ID number. Eigen-

buckling analysis was run for an Element size of 30mm. The result of the buckling analysis is shown in Figure 11, where the buckled shape and the load factor (FREQ), expressed in $\text{kN} \cdot \text{mm}$, are indicated. The buckling capacity predicted using the beam element BEAM 188 from ANSYS is within 0.6% of the theoretical value.

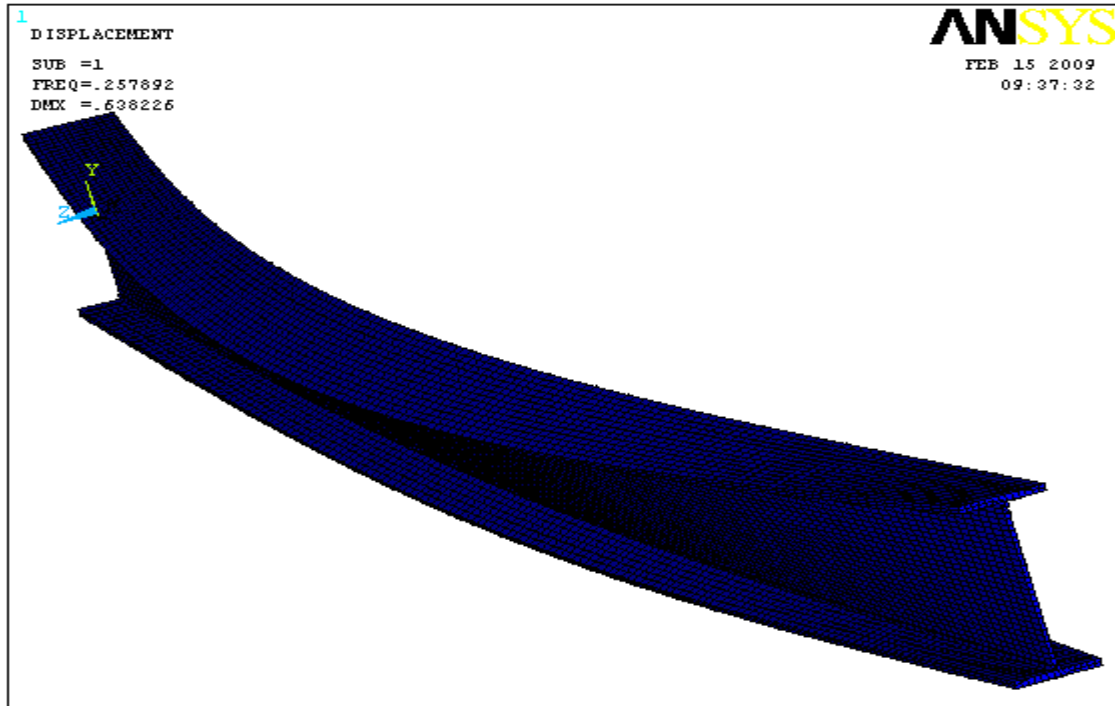


Figure 11– Buckled shape and load factor for end moments only

4. Parametric Study

4.1 Description of parameters

Although the finite element analysis performed using ANSYS indicated better correlation with the theoretical value than the shell element model, the latter was used for a parametric study.

The analysis was performed for two load cases, namely, for a point load at mid length and at the shear centre with two equal end moments as shown in Figure 7a and a uniform load with two equal end moments as shown in Figure 7b. The value of load (P for point load and w for uniform load) is taken as 1.0 for simplicity in subsequent calculations. The value of β is varied so as to make different load cases, each different from the other in parameter β .

After multiplying the analysis loads with the appropriate load factors moments and values for equivalent uniform moment factor were determined, as shown in appendix A and appendix B, for both load cases.

4.2 Analysis results

The buckled shape and load factor for a typical case are shown in Figure 12.

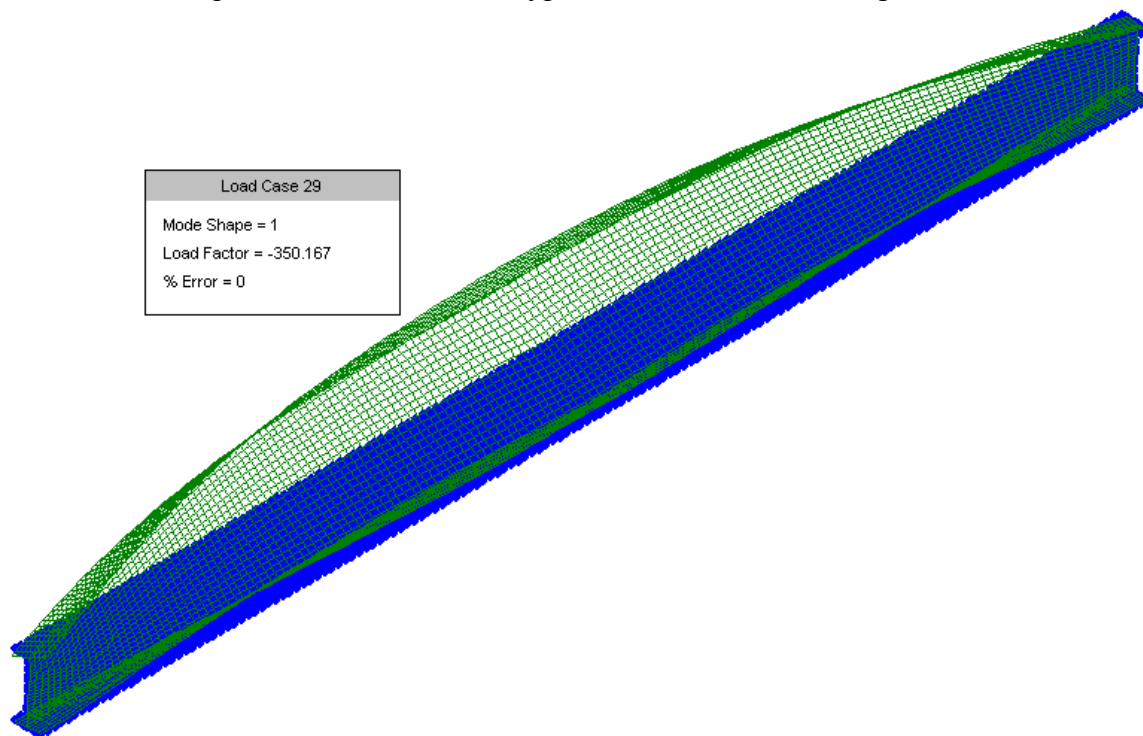


Figure 12 – Buckled shape and load factor for a point load at midspan with end moments

The analysis results and the results of the calculation for point load with two equal end moments are presented in Table 1. Figure 13 also presents a comparison between the finite element analysis results with the various design equations presented in section 2 for the same load case.

Table 2 and Figure 14 present calculation and graphical comparison of the finite element results and different code equations for the case of uniform load with two equal end moments.

Table 3 presents ratio of the values of ω_2 from the finite element analysis to the values from different equations given in section 2 for the case of point load and equal end moments.

For the load case investigated in this report, Figure 13 clearly indicates that the equation used by AISC-2005 is un-conservative for values of β between 0.2 and 1.15 while it is too conservative for values of β between 1.15 and 2.0. The equation by Serna *et al.* (2006) seems to be in close agreement with the finite element analysis results with only three data points at values of β between 1.85 and 2.5 falling below a FEA to predicted value of 1.0. It is generally accurate with a mean FEA to predicted value of 1.03 and a coefficient of variation (COV) of 0.044. On the other hand, the equation currently used in the AISC design specification yields a mean FEA/predicted value of 1.02 and a COV of 0.11. The equation proposed by Wong and Driver is the most conservative with a mean FEA/predicted value of 1.06 and COV of 0.035.

Although the equation proposed by Wong and Driver on gives better results than AISC-2005, it is not as accurate as the equation proposed by Serna *et al.* (2006).

Table 1

Values of β , the corresponding load factor from analysis and equivalent uniform moment factor compared to different sources for the case of point load with equal end moments..

β	$\beta \cdot p \cdot L/8$	Finite element analysis				AISC(2005)					Wong and Driver	Serna <i>et. al.</i>
		Load Factor	Moments		$\frac{M}{\mu}$	Moments at quarter points				ω_2		
			Support	mid span		M_{max}	M_A	M_B	M_C		Ω_2	ω_2
4	-4	94.1	-376.2	-188.1	1.50	376	282	188	282	1.39	1.49	1.52
3	-3	147.3	-441.9	-147.3	1.76	442	295	147	295	1.60	1.73	1.80
2.5	-2.5	202.1	-505.3	-101.1	2.01	505	303	101	303	1.81	1.96	2.08
2	-2	305.0	-610.0	0.0	2.43	610	305	0	305	2.27	2.31	2.52
1.85	-1.85	350.2	-647.9	52.5	2.58	648	298	53	298	2.24	2.42	2.67
1.75	-1.75	383.3	-670.7	95.8	2.67	671	287	96	287	2.22	2.47	2.75
1.5	-1.5	455.2	-682.8	227.6	2.72	683	228	228	228	2.14	2.45	2.71
1.25	-1.25	450.4	-563.0	337.8	2.24	563	113	338	113	2.05	2.04	2.16
1.1	-1.1	408.2	-449.0	367.4	1.79	449	41	367	41	1.98	1.67	1.72
1.15	-1.15	424.0	-487.6	360.4	1.94	488	64	360	64	2.00	1.80	1.87
1	-1	374.8	-374.8	374.8	1.49	375	0	375	0	1.92	1.41	1.43
0.9	-0.9	341.9	-307.7	376.1	1.50	376	34	376	34	1.77	1.41	1.43
0.8	-0.8	311.6	-249.3	373.9	1.49	374	62	374	62	1.67	1.39	1.41
0.7	-0.7	284.5	-199.1	369.8	1.47	370	85	370	85	1.59	1.38	1.40
0.6	-0.6	260.6	-156.4	364.8	1.45	365	104	365	104	1.52	1.36	1.38
0.5	-0.5	239.7	-119.8	359.5	1.43	360	120	360	120	1.47	1.34	1.36
0.4	-0.4	221.4	-88.6	354.2	1.41	354	133	354	133	1.43	1.32	1.34
0.2	-0.2	191.2	-38.2	344.2	1.37	344	153	344	153	1.36	1.29	1.30
0	0	167.7	0.0	335.5	1.34	335	168	335	168	1.32	1.26	1.28
-0.2	0.2	149.1	29.8	327.9	1.31	328	179	328	179	1.28	1.24	1.25
-0.4	0.4	133.9	53.6	321.4	1.28	321	187	321	187	1.25	1.22	1.23
-0.5	0.5	127.4	63.7	318.6	1.27	319	191	319	191	1.24	1.21	1.22
-0.6	0.6	121.5	72.9	315.9	1.26	316	194	316	194	1.23	1.20	1.21
-0.8	0.8	111.1	88.9	311.2	1.24	311	200	311	200	1.21	1.19	1.20
-1	1	102.3	102.3	306.9	1.22	307	205	307	205	1.19	1.18	1.18
-1.25	1.25	93.1	116.3	302.4	1.20	302	209	302	209	1.17	1.16	1.17
-1.5	1.5	85.3	128.0	298.6	1.19	299	213	299	213	1.16	1.15	1.16
-1.75	1.75	78.7	137.8	295.2	1.18	295	217	295	217	1.15	1.14	1.15
-2	2	73.1	146.2	292.3	1.16	292	219	292	219	1.14	1.13	1.14

Table 2

Values of β , the corresponding load factor from analysis and equivalent uniform moment factor compared for the case of uniform load and two equal end moments.

β	$\beta\omega l^2/12$	Finite element analysis for uniform load and two end moments						Serna <i>et al.</i>	Wong and Driver
		Load Factor	Moment			M/Mo			
			End	quarter	mid	8M	16M		
-2	-10.67	61.87	-659.9	-288.7	-165.0	2.628	2.657	2.535	2.322
-1.9	-10.13	70.34	-712.8	-290.7	-150.1	2.839	2.871	2.728	2.461
-1.8	-9.60	81.10	-778.6	-292.0	-129.8	3.101	3.137	2.967	2.626
-1.7	-9.07	94.95	-860.8	-291.2	-101.3	3.429	3.468	3.266	2.820
-1.6	-8.53	112.70	-961.7	-285.5	-60.1	3.830	3.874	3.635	3.039
-1.5	-8.00	134.58	-1077	-269.2	0.0	4.288	4.342	4.058	3.266
-1.4	-7.47	158.19	-1181	-232.0	84.4	4.704	4.799	4.439	3.450
-1.35	-7.20	168.24	-1211	-201.9	134.6	4.825	4.961	4.541	3.497
-1.3	-6.93	174.75	-1212	-163.1	186.4	4.826	5.012	4.531	3.494
-1.2	-6.40	170.03	-1088	-68.0	272.0	4.334	4.547	4.112	3.301
-1.1	-5.87	146.11	-857.2	19.5	311.7	3.414	3.529	3.347	2.879
-1	-5.33	120.40	-642.1	80.3	321.1	2.558	2.605	2.574	2.359
-0.5	-2.67	53.56	-142.8	178.5	285.6	1.138	1.199	1.207	1.199
0	0.00	35.65	0.0	213.9	285.2	1.136	1.139	1.136	1.131
0.5	2.67	25.35	67.6	219.7	270.4	1.077	—	1.101	—
1	5.33	20.39	108.7	231.1	271.8	1.083	—	1.080	—
1.1	5.87	19.55	114.7	232.0	271.0	1.080	—	1.077	—
1.2	6.40	18.77	120.1	232.8	270.3	1.077	—	1.074	—
1.3	6.93	18.06	125.2	233.5	269.6	1.074	—	1.071	—
1.4	7.47	17.39	129.9	234.2	269.0	1.071	—	1.069	—
1.45	7.73	17.08	132.1	234.5	268.7	1.070	—	1.067	—
1.48	7.89	16.90	133.4	234.7	268.5	1.069	—	1.067	—
1.5	8.00	16.78	134.2	234.9	268.4	1.069	—	1.066	—
1.6	8.53	16.20	138.2	235.4	267.8	1.067	—	1.064	—
1.7	9.07	15.66	142.0	236.0	267.3	1.065	—	1.062	—
1.8	9.60	15.16	145.5	236.5	266.8	1.063	—	1.060	—
1.9	10.13	14.69	148.9	237.0	266.4	1.061	—	1.058	—
2	10.67	14.25	152.0	237.4	265.9	1.059	—	1.057	—

Table 3
Values of ω_2 from the analysis and FEA-to-predicted ratios

β	ω_2				FEA-to-predicted ratio		
	FEA	AISC (2005)	Wong and Driver	Serna <i>et al.</i>	AISC (2005)	Wong and Driver	Serna <i>et al.</i>
4	1.498	1.389	1.486	1.521	1.079	1.009	0.985
3	1.760	1.596	1.732	1.802	1.103	1.016	0.977
2.5	2.012	1.812	1.961	2.076	1.111	1.026	0.969
2	2.429	2.273	2.309	2.523	1.069	1.052	0.963
1.85	2.581	2.240	2.419	2.671	1.152	1.067	0.966
1.75	2.671	2.215	2.475	2.749	1.206	1.079	0.972
1.5	2.720	2.143	2.449	2.707	1.269	1.110	1.005
1.25	2.242	2.049	2.041	2.163	1.094	1.099	1.037
1.1	1.788	1.978	1.668	1.718	0.904	1.072	1.041
1.15	1.942	2.003	1.796	1.866	0.969	1.081	1.040
1	1.493	1.923	1.414	1.435	0.776	1.055	1.040
0.9	1.498	1.774	1.408	1.429	0.844	1.064	1.049
0.8	1.489	1.667	1.395	1.414	0.894	1.068	1.053
0.7	1.473	1.585	1.378	1.396	0.929	1.069	1.055
0.6	1.453	1.522	1.360	1.377	0.955	1.069	1.056
0.5	1.432	1.471	1.342	1.357	0.974	1.067	1.055
0.4	1.411	1.429	1.324	1.339	0.987	1.065	1.054
0.2	1.371	1.364	1.292	1.305	1.005	1.061	1.051
0	1.336	1.316	1.265	1.276	1.015	1.056	1.047
-0.2	1.306	1.279	1.242	1.251	1.021	1.052	1.044
-0.4	1.280	1.250	1.222	1.230	1.024	1.048	1.040
-0.5	1.269	1.238	1.213	1.221	1.025	1.046	1.039
-0.6	1.258	1.226	1.204	1.212	1.026	1.045	1.038
-0.8	1.239	1.207	1.190	1.197	1.027	1.042	1.036
-1	1.222	1.190	1.177	1.183	1.027	1.039	1.033
-1.25	1.204	1.173	1.163	1.169	1.027	1.036	1.031
-1.5	1.189	1.159	1.151	1.156	1.026	1.033	1.029
-1.75	1.176	1.147	1.140	1.145	1.025	1.031	1.027
-2	1.164	1.136	1.131	1.136	1.025	1.029	1.025
Mean					1.02	1.05	1.03
C.O.V.					0.098	0.022	0.030

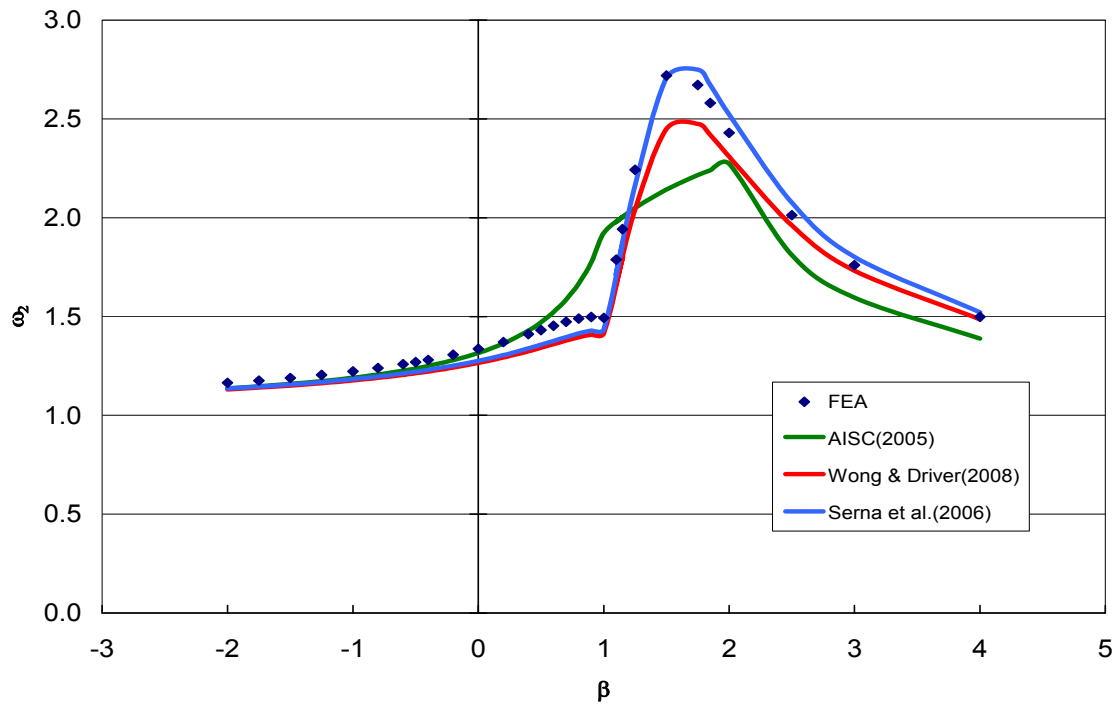


Figure 13 – Graphical comparison of ω_2 for point load and two end moments.

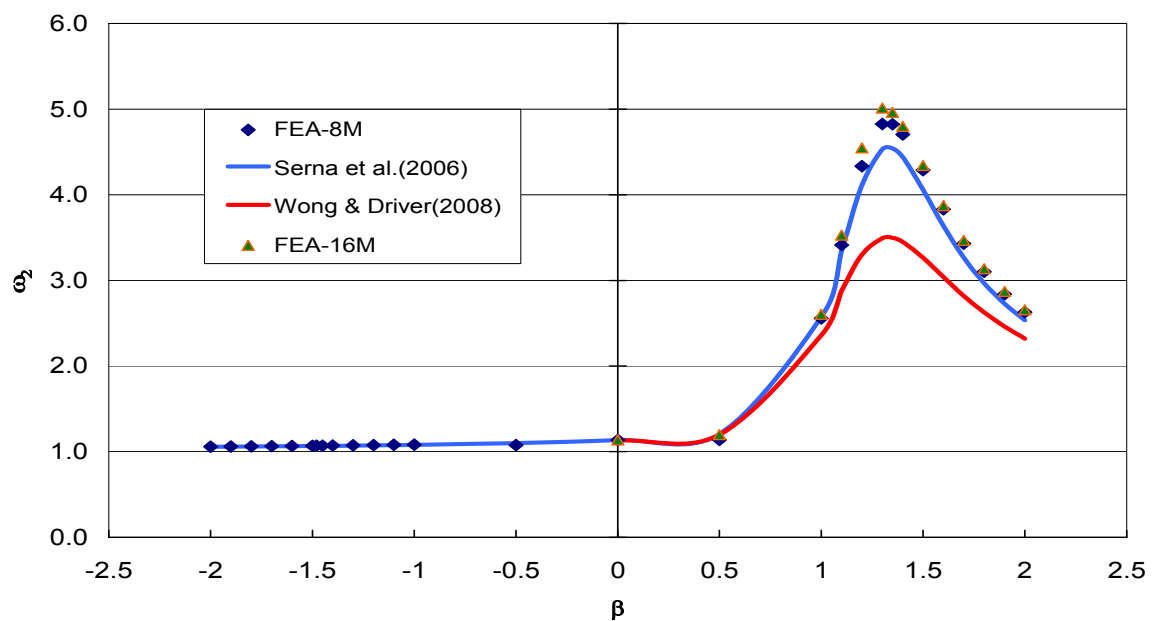


Figure 14 – Graphical comparison of ω_2 for uniform load and two end moments.

Contrary to the findings of Serna *et al.* (2006), the numerical analysis results presented in Figure 14 do not show significantly different values of ω_2 for beams 8 m and 16 m long.

Figure 15 presents the ratio of finite element analysis results to the values predicted by the equation currently in use in AISC (2005), the equation proposed by Serna *et al.*, and the equation proposed by Wong and Driver for the loading condition consisting of a point load and two end moments. It is clear that the equation currently used in AISC is not doing a good job of predicting the finite element analysis results; it can be either very unconservative or very conservative. The equation proposed by Wong and Driver is conservative over most of the range of b values investigated, with FEA to predicted ratio reaching a value of 1.13. The equation proposed by Serna *et al.* is not as conservative and is slightly unconservative with FEA to predicted ratio reaching a minimum of 0.95.

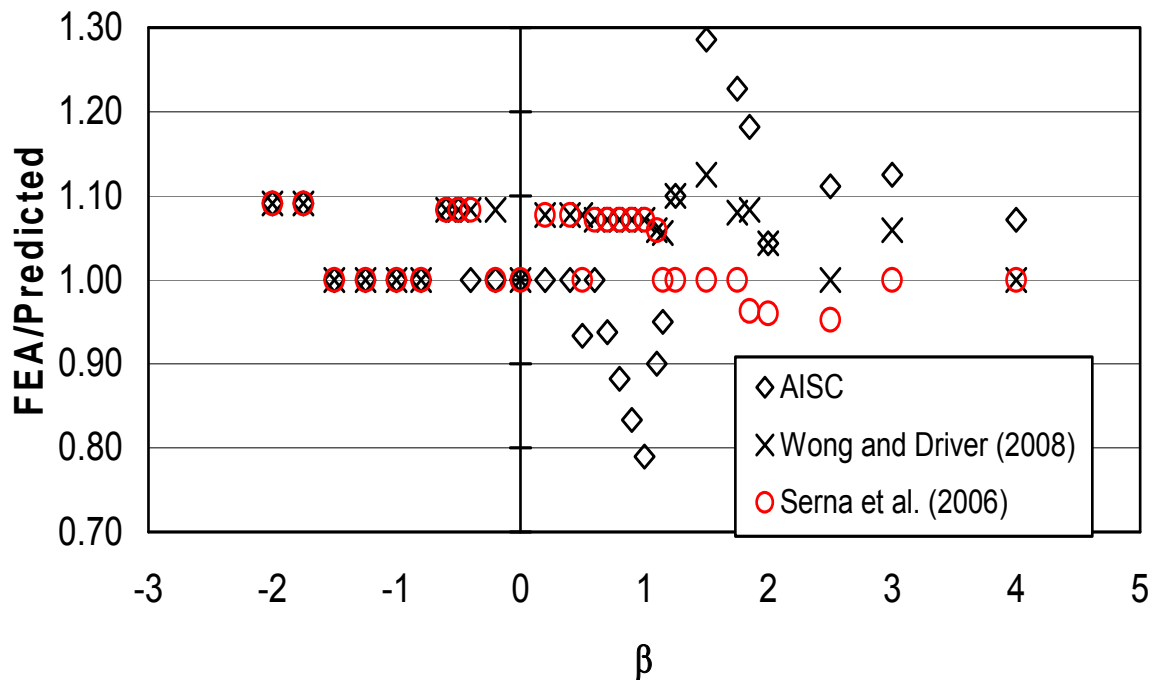


Figure 15 – FEA to predicted values for three prediction equations

5. Summary, conclusions and recommendation

Comparison of the finite element results with the various methods in section 2 of this report reveals the following observations:

- As evidenced by Figure 13 and as indicated by Serna *et al.* the AISC (2005) equation is not safe to evaluate equivalent uniform moment factor for values of β between 1.15 and 2.0, for the case of point load with two equal end moments.
- Figure 14 shows that the equation proposed by Wong and Driver (2008) tends to be over conservative for the case of uniform loading with two equal end moments.
- Figures 10, 13 and 14 show that the equation proposed by Serna *et al.* is more consistent and in better agreement with the numerical results except for a very small range of β in Figure 13 where it is slightly over-predicting the lateral torsional buckling capacity. The over prediction reaches a maximum of 3.6 %. The close agreement of this equation with test and analysis results was also demonstrated in some of the figures presented by Wong and Driver (2008).

On the basis of these observations, the equation proposed by Serna *et al.* is recommended for evaluating the equivalent uniform moment factor for laterally unsupported steel beams.

A reliability analysis is recommended to form a basis for acceptance of an equation for the determination of the equivalent uniform moment factor.

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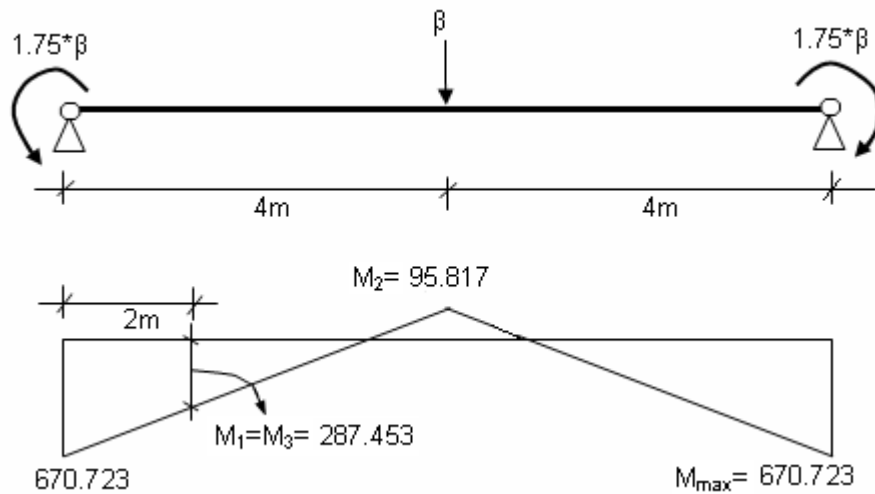
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Appendix A

Sample calculation for moment at quarter points and equivalent uniform moment factors to verify the spreadsheet calculations for point load and two equal end moments:

For $P=1$ and $L = 8\text{m}$, $\beta \cdot P \cdot L/8 = \beta$.



The load factor obtained from the finite element analysis is 383.27.

$$\beta \times M = 1.75 \times 383.27 = 670.723$$

$$\text{Midspan Moment} = 383.27 \times \frac{8}{4} - 670.723 = 95.817$$

$$\text{Quarter pt. Moment} = (95.817 + 670.723) \times \frac{2}{4} - 95.817 = 287.453$$

Theoretical value of critical moment (M_u) = 251.0797

Analysis:

$$M/M_u = \frac{670.723}{251.0797} = 2.671$$

AISC:

$$\omega_2 = \frac{12.5 \times 670.723}{(2.5 \times 670.723 + 3 \times 287.453 + 4 \times 95.817 + 3 \times 287.453)} = 2.215$$

Wong and Driver:

$$\omega_2 = \frac{4 \times 670.723}{\sqrt{670.723^2 + 4 \times 287.453^2 + 7 \times 95.817^2 + 4 \times 287.453^2}} = 2.474$$

Serna *et al.*:

$$\omega_2 = \sqrt{\frac{35 \times 670.723^2}{670.723^2 + 9 \times 287.453^2 + 16 \times 95.817^2 + 9 \times 287.453^2}} = 2.748$$

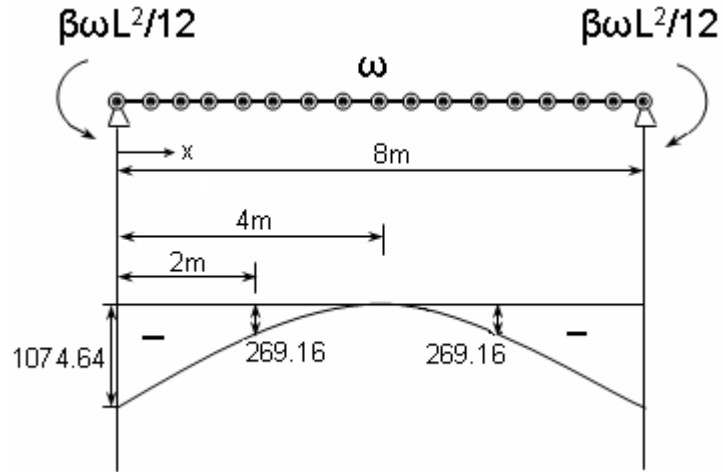
Appendix B

These calculations are for uniform load and two equal end moments.

The model has 161 longitudinal nodes. Each node is loaded in negative Z direction by 0.05 kN to come up with approximately uniform load of 1 kN/m.

Calculations shown are for value of $\beta = 1.5$

$$\beta\omega \frac{L^2}{12} = 1.5 \times 1 \times \frac{8^2}{12} = 8$$



Load factor obtained from the analysis is:
L.F = 134.58

The end moment = $8 \times 134.58 = -1076.64$ kN-m

Moment equation along the length of the beam is given by:

$$M_x = \frac{\omega}{2} \left(Lx - \beta \frac{L^2}{6} - x^2 \right)$$

From the moment equation the quarter point and mid point moments -269.16 kN-m and zero respectively.

$$\text{Then } \frac{M}{M_u} = \frac{1076.64}{251.0797} = 4.288$$

The value obtained by equation given by Serna *et al.* is:

$$\omega_2 = \sqrt{\frac{35 \times 1076.64^2}{1076.64^2 + 9 \times 269.16^2 + 16 \times 0 + 9 \times 269.16^2}} = 4.059$$

This shows that FEA value is greater than the value obtained by the equation proposed by Serna *et al.*