The Effects of Interleaving on Mathematical Understanding

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Abstract

How does interleaving the mathematics curriculum while promoting productive struggle, improve student understanding in and between mathematical concepts? To clarify, I mean to say that ‘understanding’ is more about knowing ‘why’ and knowing ‘when’ to apply specific concepts and strategies, and not just knowing ‘what’ the mathematical concepts are. Interleaving is a multilayered approach, varying the curriculum to allow students to discriminate between strategies they already know and apply them to new information. It is a way of deepening their understanding, allowing them to productively struggle (Warshauer, 2015), think critically and problem solve to come to their own conclusions. Research shows there is benefit to interleaving practice and intermixing different kinds of practice problems and spacing out the same types of problems across different assignments (Rohrer, Dedrick & Stershic, 2014; Rohrer, Dedrick & Burgess, 2014). Spaced practice interrupts forgetting (Brown et al., 2014). This allows for the creation of a variety of mental models of information and transforms it from short term memory into long term memory and allows for better recall. Experiencing different conditions and circumstances to create our information model broadens mastery, fosters conceptual learning, improves versatility and primes the mind for learning (Brown et al., 2014).

The Effects of Interleaving on Mathematical Understanding

The ideas of engaging students in mathematics, within the frameworks of interleaving, productive struggle and problem solving have always been of interest to me. I have often wondered how to better engage students in mathematical thinking, deepen their understanding, and have them, dare I say, even enjoy math. Traditional methods of teaching mathematics linearly or sequentially, are still used today. These methods include, but are not limited to, introducing students to information, having them learn and practice a unit, process or concept and the specific skills directly related to that mathematical concept or process, one at a time. Unfortunately, by the end of the year, if connections between concepts were not made, it quickly becomes apparent that many students forget what the first unit was all about and how to solve problems related to it. What if teachers used different methods or ways of presenting information for students to learn and practice unique mathematical concepts simultaneously? Playing card, board and dice games not only provide a fun activity to participate in, but also develop critical thinking, problem solving and reasoning skills. Games also provide ways to interleave a variety of mathematical concepts, allow students to analyze the problem and then use their pre-existing knowledge to discern which method, strategy or process is going to best help them find a solution. What if students could learn through the use of a cyclical like structure that intertwines multiple mathematical concepts at once and allows the students to think critically, retrieve information from their own understanding, and problem solve to discern for themselves the solutions? That possible structure is interleaving.

My wondering led to the development of this current research question: How does interleaving the mathematics curriculum while promoting productive struggle, improve student understanding in and between mathematical concepts? To clarify, I mean to say that 'understanding' is more about knowing 'why' and knowing 'when' to apply specific concepts and strategies, not just knowing 'what' the mathematical concepts are about.

Personal Connections to Theories and Frameworks

Looking back at some of the factors that have influenced my interest in interleaving, a few professional development opportunities and resources received in graduate courses stand out as being key to stirring my curiosity about interleaving, productive struggle, complexity theory and how they affect mathematical understanding in students. Currently, Harry Ainlay High School in Edmonton, Alberta uses an interleaved working model with their students and found better mathematical understanding from students participating within this format. The development of their program was based on the book Make it Stick: The Science of Successful Learning (Brown, Roediger III &McDaniel, 2014). Briefly and simply put, this book debunks the myth that re-reading material is an effective way to learn and memorize information. The book promotes alternative ideas that retrieval, through mixed practice (using an interleaving practice approach vs. a traditional blocked practice approach) and productive struggle have lasting effects when learning information and assimilating it into your current understanding. I have attended multiple professional development workshops and presentations done by Fred Kong (Department Head) and the math department at Harry Ainley, and the data they shared is starting to show that interleaving has increased the level of student comprehension between various concepts and strategies across the mathematics curriculum. Another important question that arose for me when looking at an interleaving framework, is what criteria would be used to decide how to interleave the outcomes/concepts/units/ideas within the curriculum? In communicating with Fred Kong, there was some criteria they used to organize and interleave the outcomes and curriculum in a certain way. The more abstract the ideas or concepts were, the earlier they were introduced in the semester so that more time could be spent circling back to those concepts to reinforce them. Using the interleaving model has not come without challenges and push back from stakeholders (parents, students and other teachers) but overall persistence in using an interleaving model has shown generally positive results.

Interleaving is an idea that challenges the traditional blocked practice approach (given specific strategies and only practicing one skill at a time) and allows students to discriminate between problems to effectively choose a strategy to solve them. Interleaving the curriculum allows multiple entry points for students to be given the opportunity to engage in productive struggle and participate in problem solving activities. A potential roadblock for this method of instruction is that it is self-proclaimed by students as being a slower learning process. To them, it feels as though the process is not as efficient or effective in developing their procedural or conceptual understanding in mathematics. It takes longer for students to connect and learn different concepts which can be frustrating for the student and seem as though the learning and understanding of information is not happening, or not happening quick enough. Being able to memorize information, retrieve it and apply it to new situations, is one way for students to prove they are able to reason and communicate their mathematical understanding. Focusing on one concept at a time, as in a blocked practice approach, feels like you are learning and remembering the material quicker because in the initial stages of learning and remembering new material and information, you are placing the ideas, concepts and strategies into your short term memory. Once the ideas, concepts and strategies have made it into short term memory it can give the learner the illusion that the content is fully understood and available for retrieval in the long term. If the material is not transferred into long term memory, it may not be retrievable in the future (Brown et al. 2014).

Productive struggle is a framework that is based on the idea that students should be given learning opportunities to deepen their mathematical understanding through participating in a variety of cognitively demanding tasks and opportunities. The activities students participate in should be open ended, allowing multiple entry points for all ability levels, developing persistence in students and moving their learning forward. Productive struggle is as much about teacher response as the student's participation in it (Warshauer, 2015a; Hiebert & Grouws, 2007). It is important for the teacher to quickly identify the kind of productive struggle the student is experiencing and respond appropriately. Asking good quality questions is critical to ensure students are able to work through their own thinking without making it so difficult that they give up or that the solution is shared too quickly. This can be a fine line sometimes and requires the teacher to be able to identify what kinds of challenges the student is experiencing in order to respond accordingly. Professional noticing addresses how the teacher should respond to the struggle the student is experiencing. Noticing what kind of struggle it is, what mathematical concepts are being addressed, and making connections between the two are key components into how the teacher responds to effectively support student learning (Mason & Davis, 2013).

Complexity theory (Davis & Simmt, 2003), related to mathematics education, is the idea that "...learning is understood in terms of the adaptive behaviors of phenomena that arise in the interactions of multiple agents" (p.137). Complexity theory is related to interleaving because by interleaving the curriculum, you are providing 'interactions of multiple agents' as there are multiple concepts and ideas to consider simultaneously. The learning of material allows students to discern between strategies and processes to find solutions on their own, which deepens their understanding. Providing opportunities for students to gain knowledge and understanding from their peers, not just receiving information from the teacher or a textbook, is also a valuable practice in participating in the multiple interactions of ideas. It is important to recognize that students today still struggle in understanding mathematics and if we, as educators can continue to research methods and strategies to support mathematical understanding in students and improve learning experiences, then we are fulfilling our purpose.

Theory of Experience

How does Educational Theory, and more specifically John Dewey's Theory of Experience (1938), support the ideas provided within the interleaving framework? More explicitly, when we interleave the mathematics curriculum, how does it affect student perception of the learning experience, and how might it influence how they approach their future learnings? The ideas surrounding 'experience' are broad and bring about many facets that make up ideas of experience. For clarity, when referring to experience in this paper, it is referring to how we as learners perceive our own learning experiences, and how our perceptions of experience influences how we approach new learning opportunities.

Throughout my childhood education, learning experiences in math class; doing and understanding math was not always easy for me. I had to work hard to understand concepts and shed many tears (especially in high school) because I did not understand the material or concepts that were covered in class. I struggled to make connections between ideas, largely due to the fact that we learned one unit or concept at a time without making some connections to previous ideas or concepts. After learning one unit, and participating in a blocked practice approach, as soon as one unit was done, we moved onto the next. When it came time to write the final exam, much of what I learned at the beginning of the semester was forgotten. I went into the final exam with a surface level of understanding of some concepts and ideas we learned throughout the year, and struggled with applying and communicating the 'how' and 'why' of concepts and strategies I was expected to know. In John Dewey's (1938) words, "Every experience is a moving force. Its value can be judged only on the ground of what it moves toward and into" (p. 38). Our experiences influence who we are, what we think, value and how we learn. In my experience, I feel I did not make enough connections between concepts and believe that was largely due to learning mathematics in a linear and sequential way. Learning the same way for so many years impacted how I looked and experienced future learnings about math and it was not until I took an undergraduate math course, that I realised math could be taught differently. That knowledge and understanding are not just found in textbooks but exists in our relationships and experiences, within multiple areas, people, places and things.

I wanted to delve further into the conditions of experience and the impact on student learning. Most times, when students experience challenges or struggle with material in junior high, it stems from years of frustrations with math, and because now gaps in their learning have developed, they find they are further and further behind and at times for them, it seems they cannot catch up. Certainly many factors influence learning. How students perceive their past learning experiences affect how they approach new ones. Some factors to learning are external like teaching methods and the delivery of material. Parts of the learning environment such as class sizes and class culture also impact the ability to learn for every student. Other external factors such as the home environment and access to support systems outside the classroom also affect the child's behavior and attitude towards learning. Internal factors such as learning disabilities, and mental/emotional challenges require different strategies from both the student as well as the teacher to manage and improve the learning environment. As a whole, it is a fluid, complicated process and all factors that influence learning are either perceived to be in our locus of control or they are not. Perhaps that is why the purpose of my research focuses on factors that are in my control so that I can better develop the understanding of math for my students. In my nine years as an educator, I have seen the same difficulties and frustrations in my own students today that I experienced years ago and because of that, my hope for teaching mathematics in a better way emerged. I hope to also encourage educators to reflect on their own practice and consider teaching in a way that encourages students to engage in problem solving, critical thinking and making connections within and throughout the entire mathematics curriculum.

There are multiple aspects of teaching and learning that educators want to improve on or see change in and multiple studies have been done that focus on improving teaching practice. Participants in these kinds of studies play an important role as researchers because the 'change' they want to see "...takes ongoing interactions between pedagogues, learners and context." (Casey et al., 2017, p. 15). It is these interactions, learning experiences and opportunities that drive my research and action forward. John Dewey's (1938) ideas of educational experience are not new but are worth revisiting and considering when looking at creating change, because "...every experience lives on in further experiences" (p. 9). Researchers are learners and the learning experiences we have in the past, shape our current epistemologies and ontologies and impact how we will plan, act and reflect in future experiences. We cannot move forward into the future without recognizing and reflecting on our past. When we are able to weave together the past, present and future, we are in a way challenging a linear pattern by circling back, or layering ideas by interleaving and connecting our experiences together.

Interleaving and the effects on learning

How we see our learning experiences in the past affects our future perceptions on how we learn. Traditionally mathematics has been taught in a linear fashion. Educators taught one unit in its entirety and then moved to the next one when complete. For example, students in Alberta who work on squares and square roots as well as pythagorean theorem in grade 8, in the first month of school, do not see any other questions pertaining to these concepts until the end of the year for the final exam. In my own experience (within my math department), and for the sake of consistency, we all teach units in the same order, and administer the same common assessments for all classes. On one level, I believe it is good practice for a few reasons. As educators, we are able to see the concepts and ideas most students struggle with and can identify as a department what we need to change in our lessons as well as for assessments. Students have the opportunity to work together with their friends in other classes because they are all learning the same material at the same time. However, the problem with teaching one unit at a time is that when students learn a new concept often the practice that is done revolves around that specific idea and involves using the same strategy to solve the same types of problems. For example, if students are working on multiplying fractions and they know the strategy (multiply the numerators, multiply the denominators and reduce the fraction to simplest form), when they get to a word problem, often they are not even reading it, they are just scanning the problem for two fractions to multiply together. Generally students do not need to discriminate between different kinds of strategies to solve different kinds of problems because they have already been given the strategy to execute before they start to decide how to solve the problem (Rohrer, Dedrick & Stershic, 2015). That being said, when we assess our student’s learning, we usually start with assessing one skill or concept at a time and then as we continue through a unit or concept, we increase the difficulty by assessing more than one concept at a time and give multi step problems (multiplication and division of fractions, and then moving towards all four operations). By the time we get to the unit test, students need to decide what strategy to use to execute or solve the problem that is asked. In a minor way, by doing this, we have incorporated some interleaving principles.

Interleaving, according to the Merriam-Webster's (2020) dictionary is: ‘to arrange in or as if in alternate layers’. Foster et al., (2019) offer this definition of interleaving: “Interleaved practice involves studying exemplars from different categories in a non-systematic, pseudorandom order under the constraint that no two exemplars from the same category are presented consecutively” (p. 1088). Keeping these definitions in mind, the following question arose for me. To what extent do you interleave or ‘arrange in alternate layers’ within the classroom environment? Multiple studies have been done that include, but are not limited to, interleaving types of mathematical problems, interleaving different units or concepts of study, and interleaving practice vs. a blocked practice approach. Many mathematics textbooks used today offer a blocked practice approach. A concept and/or strategy is learned and then practice questions are given that require the student to use the same strategy over and over again. Research shows there is benefit to interleaving practice and intermixing or rearranging different kinds of practice problems within tasks given. In addition to presenting different kinds of problems there are also benefits to spacing out the same types of problems across different activities (Rohrer, Dedrick & Stershic, 2015; Rohrer, Dedrick & Burgess, 2014). Spaced practice interrupts forgetting (Brown et al., 2014). It allows students to see material, practice it, and continue assimilating new knowledge into what they already know. Said another way, students are spiralling back, connecting past ideas to extend into new ideas, continuing to practice with a variety of mixed questions over multiple days/weeks. Distributing mathematical practice and spacing out testing and assessment benefits retention and recall (Barzager & Ebersbach, 2019). The interleaving process challenges students and increases difficulty in that it requires students to think about, and retrieve from their prior knowledge, rather than depending on short term memory and rote responses. It strengthens their conceptual knowledge because it forces students to discriminate between problems and strategies in order to solve them. By interleaving practice and testing, students demonstrate better long term retention. This is very important, especially in math, as many new concepts build on previous ones and if students are able to retain material longer, it strengthens their understanding and ability to apply new material and/or concepts to ones learned in the past.

Foster et al., (2019) describe the interleaving process in a couple of different ways. The discriminative-contrast hypothesis refers to interleaving (varying or layering) different types of problems and ensuring there is a mix of different types of questions such as multiple choice, short answer, long answer, fill in the blank, and others. The distributed-practice hypothesis refers to the idea of spacing out practice and distributing different concepts and ideas throughout the practice. Both hypotheses support why interleaving is successful but the study done by Foster et al., (2019) also brought to light that the degree to which each hypothesis influences the success of interleaving changes depending on the content or material students are studying. Depending on how similar and/or different the concepts were (calculating volume of different shapes) within the bigger idea (geometry) it impacted how much of each, discriminative-contrast or distributed-practice, influenced the success of the interleaving process. In the end both hypotheses revealed that interleaving in both ways produced higher recall and retention of material, but in this particular experiment the distributed-practice hypothesis provided more substantial positive results.

Interleaving provides the opportunity to access long term memory. Moving prior knowledge into long term memory requires more work and might take longer, but the benefits are well worth the time and effort. When we put in the effort and use spaced practice we reconstruct our memories, relearn and strengthen connections of present knowledge with new knowledge and information. Spaced practice provides opportunities for similar material to be presented in different conditions and circumstances. This allows for the creation of a variety of mental models of information and transforms it from short term memory into long term memory. Experiencing different conditions and circumstances to create our information model broadens mastery, fosters conceptual learning, improves versatility and primes the mind for learning (Brown et al., 2014).

Productive Struggle and the Effect on Learning

Interleaving is a form of differentiated instruction and a tool designed to deepen mathematical understanding by enhancing the potential for students to experience productive struggle - one result of interleaving. “If understanding is defined as the mental connections among mathematical facts, ideas and procedures, then struggling is viewed as a process that reconfigures these things” (Hiebert & Grouws, 2007, p. 388). Based on the work by Yuichi Handa (2003), Hiebert and Grouws (2007) go on to further define understanding and struggle as follows:

> Struggle results in restructuring one’s mental connections in more powerful ways. If understanding is defined as participating in a community of people who are becoming adept at doing and making sense of mathematics, then struggle is vital because it can be an essential aspect of personal meaning-making within the community. (p. 388)

Productive struggle is not merely giving students extra time to work out challenging math problems. Its central theme surrounds equity. Equity for all students to be able to contribute, engage in, and make sense of the math. Differentiated instruction (Tomlinson, 1999) is not a new idea. It comes from the premise that instruction can be modified by either changing the content, the process of information the student uses to engage in the content, and/or changing the product or way the student communicates their understanding. When students are able to struggle productively, they engage in critical thinking, problem solving and meaningful cognitive demand while maintaining mathematical rigor. Students are creatively thinking through challenging tasks, making mistakes and developing resiliency to work through the challenge and not giving up. Ensuring that tasks or activities have appropriate cognitive demand is just as important as ensuring the teacher is appropriately responding to the challenges or struggles the students experience. Warshauer (2015b, p. 392) describes the following strategies (paraphrased) to ensure teachers are supporting students through the process of productive struggle.

1. Ask purposeful questions that guide students through their own thinking, without giving a solution.
2. Encourage critical thinking in students and creative problem solving that help them focus on the process of cognition rather than only finding a solution or end result.
3. Give plenty of time for students to work through their thinking.
4. Create a classroom culture that values struggle, and where making mistakes is acceptable and encouraged because it improves the learning experience.

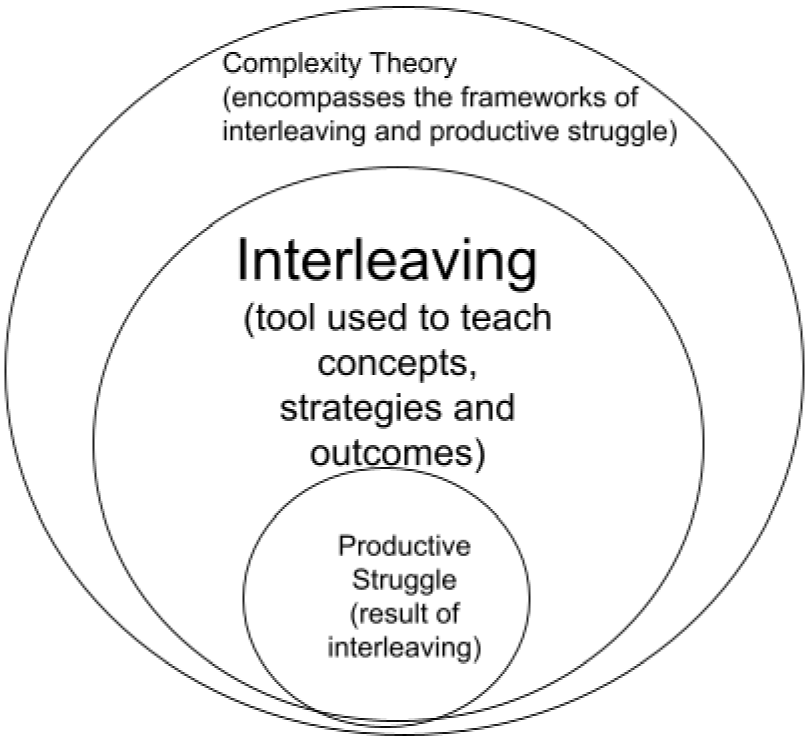
Participation in productive struggle does not start when students are given the tasks, it starts when educators create and plan a task or activity and consider how productive struggle could be experienced by the students. Tasks should be rich in cognitive demand and allow for scaffolding and multiple entry points for all students to be able to participate and be engaged. Teachers should constantly notice and evaluate what their students are saying and doing during the task or activity and decide when they need to intervene with support and motivation to keep their students on track. This awareness - professional noticing - is easier said than done and improved over time and through experience. It is a necessary skill teachers should continue to develop throughout their career (Mason & Davis, 2013; Jacobs et al., 2010). Personally, I find I reflect often on the kinds of questions I ask my students to ensure they remain purposeful and create meaningful learning for my students. Unfortunately, because it is easier, I do at times slip into asking too many leading questions that require surface level thinking and unfortunately give up the solution or strategy too quickly which undermines deep understanding in students' learning. Asking questions that allow the student to move forward in their own thinking and creating opportunities for the student to come up with the solution or strategy themselves is both rewarding and best for rich and deep mathematical understanding.

Linking Productive Struggle to the Interleaving or layering of Memories

Research has shown that productive struggle increases complexity in learning and while it is more challenging for students to engage in, it increases their understanding of mathematical processes, concepts and ideas as well as builds on life skills such as resiliency and creative problem solving. Said another way, productive struggle or desired difficulties, can “trigger encoding and retrieval processes that support learning, comprehension, and remembering.” (Bjork & Bjork, 2011, p. 58). Memory, retrieval and forgetting are linked. Easier, faster ways to study like massed practice or cramming the night before might seem like the most beneficial and efficient way to learn and memorize material. However, when you do this, you are using your short term memory, cramming as much as you can within that space to hopefully retrieve and apply it in the exam or assessment the next day. Although it is easier to store the material, it is also easier to forget and easier is not always better. Cramming for an exam the night before or massed practice is all too familiar and has often been used by students to study and learn material. The desired effect is to remember material and information, with the ability to recall information and apply knowledge, understanding and interleaving of ideas in tests or exams. In exams our memory is tested in two ways: storage strength versus retrieval strength. Storage strength reflects our ability to relate our memory representation with skills and knowledge. Retrieval strength refers to the ability to access that representation through cues or exposure to opportunities (Bjork & Bjork, 2011). 'Desirable difficulties' (Bjork & Bjork, 2011) or learning through productive struggle, ensures that both the ability to effectively learn and remember, and the ability to retrieve information is achieved.

Desirable difficulties or productive struggle also incorporate the acceptance of failure or making mistakes as being part of the learning process and that they are needing to happen for our experiences to become productive. Making errors, and recognizing them, can lead us to mastery over material. It becomes a key component in learning. When we are learning new material we encode it and initially place it in short term memory. It starts to move to long term memory when we start to consolidate and reorganize new information with prior knowledge. This consolidation strengthens connections and gives meaning to the information we are reorganizing. Finally we can retrieve what is in our long term memory and apply it to new experiences and opportunities. Developing powerful retrieval strategies strengthens long term memory and allows us to make more meaningful connections when we continue to reorganize our current knowledge and understanding and apply to new information and experiences. When we access long term memory, we are practicing interleaving by discriminating between our own layers of ideas, strategies and thinking and pulling out what we need to apply in current learning opportunities. Experiencing productive struggle adds complexity. The following diagram illustrates my understanding of the relationship between interleaving, productive struggle and complexity theory.

Figure 1: *Interleaving, Productive Struggle and Complexity Theory*



Complexity Theory and its connection to Interleaving

Ton Jörg (2009) suggests that complexity in thinking has its focus on "that which is interwoven" (p. 1). Learning is a complex structure or system linked to experiences and relationships. We do not learn alone, but often learn in and from communities. Learning provides complexity and is dynamic and nonlinear, producing a weblike multilayered structure. When we interact with others we generate ideas, we interleave ideas and construct meaning together. The more we interact with others and look at different ways of knowing, the more complex the ideas become which can cause struggles and its own challenges because we sometimes feel it is easier to think about things or do them on our own. Although interacting with other perspectives and introducing multiple ways of knowing might be more labor intensive, it is well worth the time and effort. This generative action can be powerful as it adds reciprocal learning to the experience. As more perceptions and ideas are expressed, new interpretations are created and become rich and diverse and everyone involved in the process is better for it because everyone learns from each other.

Brent Davis and Pratim Sengupta (2020) describe complexity in learning revolving "...around such terms as emergent, noncompressible, multi-level, self-organizing, context-sensitive, and adaptive" (p.113). Is that not also what interleaving and productive struggle offer? There is an epistemological discourse rooted in complexity; how we learn and where we find knowledge is complex. How we make sense or meaning of information and knowledge is also complex. Interleaving is complex because it is a layering of concepts and strategies. Historically mathematics has been taught linearly, and because calculations were done by hand and for pragmatic reasons, efficiency was the priority in learning and doing math. Technology continually is changing how we do mathematical calculations, and therefore how we look at learning math and how to incorporate technology, should also change. Changes in technology has also opened up different ways of teaching and learning, and has allowed interleaving to have potential benefits to both how we teach and how students learn. The ideas of interleaving, productive struggle and complexity are not easy and require a change in the mindset of those that wish to apply these principles. It includes debunking the myth of errorless learning and creating a culture that accepts making mistakes as an important part of learning. It is encouraging both self reflection and working collaboratively when making sense of information, knowledge and creating new models and the restructuring of ideas. It is also considering different ways to find knowledge and also opening up our ideas of what is considered knowledge.

How does complexity theory create a mathematical community, why is it important and how does it relate to interleaving?

Davis and Simmt (2003) add further insight into complexity science and its relation to constructing a mathematical community. By creating a mathematical community you are incorporating multiple agents to form nesting learning systems. Thinking back on interleaving and productive struggle, interleaving is in a way taking multiple layers and various concepts and intertwining them. Said another way, interleaving is a way to nest different systems together. Productive struggle or desirable difficulties create sense making opportunities and complexity science because it "...includes the co-implicated processes of individual sense-making and collective knowledge-generation" (Davis & Simmt, 2003, p.142). Not only can productive struggle and interleaving be seen as an individual experience, but it can be applied to a broader sense; the classroom collective or mathematical community.

According to Davis and Simmt (2003) a successful mathematical community or learning system is built with the following factors: internal diversity, redundancy, decentralized control, organized randomness and neighbor interactions. Internal diversity describes the differences each person brings to the classroom. Each person will bring with them different strategies and responses to emergent circumstances in the learning environment. As students experience productive struggle, each one will react differently depending on their past experiences and understanding. When we allow students to individually work through their challenges and allow them to share their unique ideas, we open up learning possibilities because the students are learning from each other. Collectively they are interleaving their ideas and offering further thinking into the mathematical concepts and generating new knowledge together. While internal diversity offers creativity to practice math, redundancy offers a sense of stability or a firm foundation. Establishing common standards of engagement as well as common experiences creates a starting point for developing collaborative and collective understanding. There is a need to find balance in both, defining parameters (stability) and having sufficient organization, and allowing for a certain amount of randomness to promote a variety of responses (creativity). Organized randomness is what helps keep this balance. Decentralized control should be looked at as middle ground and interplay between a teacher-centered and a student-centered approach. It is not an either/or approach but a complex unity which shares ideas and understandings to collectively generate knowledge. From a teaching perspective, when we disperse control in our classrooms, and focus more on idea making over not making mistakes, we strengthen the whole class culture and the mathematical community. From a student perspective, when they have some control and decision making over their learning and interactions, math becomes more meaningful for them. Neighbor interactions do not include physical objects such as a textbook, whiteboard, lectures etc, nor do they include actual physical social groupings. It is the interaction and purposeful knitting together of ideas, perspectives and interpretations that create an effective mathematical community.

By interleaving the curriculum and nesting complex learning systems together, you create opportunities for students to engage in productive struggle. By dispersing control you open up to communal learning, variety in interpretation and creativity in problem solving. When students develop resiliency and are able to seek and find solutions on their own (within some organized randomness) they understand what it is they are supposed to learn at a conceptual level and not just at a procedural level. Taking the time to wrestle with ideas and spacing practice between ideas allows for the interruption of forgetting and enables stronger connections enabling short term memory to transition to long term memory. Transitioning information into long term memory allows for the opportunity of communicating deeper understanding of mathematics.

Conclusion

Delving deeper into the concepts of interleaving has influenced my current research by adding subsequent complexities and rabbit holes in which I can enter. I have only scratched the surface of what interleaving and productive struggle is, how they relate to complexity theory and see my desire to delve further into understanding the relationship and connections between them. Through restating my research question: How does interleaving the mathematics curriculum and promoting productive struggle improve student understanding in and between mathematical concepts, I see the undertaking of a PhD! Taking the mathematics curriculum and restructuring it within an interleaving model is an exciting, massive project that deserves future consideration because of its potential benefits and effects on mathematical understanding.

The first part of putting interleaving into practice would be to rearrange the order of how the specific outcomes and/or units of study of the curriculum are taught. Rearranging the curriculum would be a huge undertaking that has potential benefit but also creates challenges and barriers. Because of the magnitude of the change in how an interleaved curriculum might be taught, many questions arise. Specifically a concern might be how assessment and evaluation might be addressed. Currently, in my own position, we have common assessments developed and used by our math department. How would assessments have to change to evaluate the new framework of how concepts, strategies and ideas are taught? Would you get buy in from all those involved in the department affected by this change? Not only are you expecting teachers to teach differently, you are also expecting them to shift their thinking about how assessment might be done. How would the degree of success in interleaving the curriculum be affected if you did not have total buy in from colleagues? Again, this leads to further questions of what we identify as in our locus of control. Focusing on what I can control, I can interleave the curriculum for my own classroom lessons and assessments, without having the entire department change over to an interleaved approach. It would mean some sacrifice of consistency between all classes but would allow the teachers to choose how and when they teach the various mathematical concepts.

I see value in continuing to understand frameworks and theories related to teaching and learning such as interleaving, complexity theory and how they create generative mathematical communities. I continue to see the importance of the expansive theory of experience (Dewey, 1938) and how it affects both an effective teaching and the learning environment. Creating an environment that includes interleaving and productive struggle, incorporating the nesting of complex learning systems, takes a substantial amount of effort from all stakeholders. It is a change in mindset and perhaps might require a change in how one teaches (and how one learns), which brings some people out of their comfort zone. If I want to make this change for myself and my own teaching, how do I avoid the same barriers I worry about for others? I suppose in creating a possible action plan for myself I am engaging in the very idea that I want my students to engage in. I will need to be resilient and engage in productive struggle. I will need to interleave ideas and problem solve, seeking out knowledge with and from others to produce new ideas. Through this paper I have looked more closely at what interleaving is as well as productive struggle which has created more questions. Is there a tipping point when interleaving has negative results? To clarify, the research I have read surrounds the interleaving of types of problems and/or spaced practice and testing but what if we apply interleaving at a bigger level? What if instead of interleaving within units or concepts of study, we interleave the entire math curriculum across all the units of study across grades? What if we layer further and interleave at a cross curricular level? It certainly would increase complexity and allow for opportunities of productive struggle, but would it make the learning too complex and result in counterproductive learning? Do traditional methods still have value and if so how could we include them within the interleaved model?

Again, as I ask these questions, I see more rabbit holes develop. Complexity in individual versus collective learning and their connectedness are also factors in creating a mathematical community and have an impact on improving mathematical understanding in students. Just when I see the direction of my research getting a little clearer, more ideas surface to muddy the waters. Research is messy, exciting, infuriating, confusing and very rewarding all in one. Curiosity caused Alice to fall down the rabbit hole into Wonderland. I look forward to remaining curious, leading me to 'fall' down many a rabbit hole into the Wonderland of research.

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