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OPTIMAL HARVEST RATES CONSIDERING THE RISK OF FOREST FIRE

by

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ABSTRACT

Harvest rates for a forest subject to the risk of destruction by fire have previously been estimated through the use of models which simultaneously plan for several decades or more. But the plans are not used for the length of time on which optimality was based. Plans are updated at relatively short periodic intervals, making the use of such models questionable. A computer model using this type of traditional even-flow planning was developed to illustrate its limited use in assessing the effect of fire on harvest levels.

A different model was developed to represent the sequential decision-making process of timber management, using a technique called dynamic programming. Probabilistic destruction by fire was assumed. Optimal harvest rates maximized a function which was the sum of the total expected harvest plus the expected fluctuation in harvest level. In general, it was necessary to relinquish very large amounts of harvest to appreciably reduce fluctuations in harvest. Explanations for this consequence are given. Long-term harvest rates for a forest subject to fire were found to be reduced substantially. However, the policy of reducing current harvest rates to decrease future fluctuations was considered to be of limited value. It is recommended that current allowable cuts not be reduced for fire risk, unless warranted by an evaluation of the importance of fluctuations in harvest.

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1. INTRODUCTION

All managers are faced with the problem of decision making for a future which cannot be completely controlled or predicted. Forest managers generally do not explicitly recognize the risk and uncertainty present. The long planning horizon required in forestry compounds the problem. Timber managers have traditionally assumed that complete information about the future is known, thus allowing the use of deterministic approaches in planning.

One of the major sources of uncertainty about the future state of our forests is fire. Forest fire continues to play a major and unpredictable role in the structure of future forests, even with the advent of sophisticated methods of fire protection. Management agencies recognize the potential impact of fire, and provision for fire losses is frequently made in planning.

Determination of the effect of fire on harvest levels is important in both timber and fire management. Knowledge of expected timber supply will directly affect the current and future forest industry. The long-term effect of fire on harvest rates could also serve as a criterion for investment in forest management such as fire protection. An indirect estimate of the economic value of fire management programs could be made by determining the the changes in timber supply resulting from various fire management levels.

Based on the tenet that an even-flow of annual harvests is desirable, annual allowable cut (AAC) is generally reduced to account for anticipated future fire losses. This, in concept, creates a reserve of timber available for future harvest to make up for fire losses. Methods for assessing the impact of fire on AAC have received very little attention despite the fact that reductions for fire are often significant. Reductions across Canada range from 0% to over 15% of allowable cut (see Chapter 3). Methods in current use are varied; each of the five provinces[1] that reduce AAC for future fire losses uses a different procedure. A typical method is to determine an expected volume loss based on the average area burned yearly. Recently however, considerable effort has been placed on more advanced deterministic techniques such as simulation and linear programming (Van Wagner 1983, Reed and Errico 1986).

Reductions advocated by current research are considerably higher than those previously used (Van Wagner 1983; Reed and Errico 1986). Additional verification of those results is necessary before managers can be expected to adopt them. An important and seemingly overlooked point is that the determination of the effect of fire on long-term timber supply is a different problem than determining optimal harvest levels under the risk of fire.

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[1] British Columbia, Alberta, Saskatchewan, Manitoba, and Nova Scotia (refer to Chapter 3)

Most harvest scheduling models determine AAC based on maintaining an even-flow of annual timber harvests. The models assume that decision making for the entire planning horizon[1] is done only once (at the beginning of the time period). This is not the way planning actually occurs. Plans are updated at periodic intervals. For example, in Alberta replanning will generally occur every 10 years.

Harvest levels will fluctuate over a planning horizon because replanning accounts for changes in management policies, growth prediction and growing stock. Changes in growing stock are caused by both predictable and unpredictable factors. An excess or deficiency of growing stock would cause harvest levels to either gradually decline or increase, respectively. If the possibility of unforseen and irregular losses exists, then it is reasonable to expect that fluctuations in harvest volumes over time would be greater and also irregular. The failure of traditional even-flow models to realistically represent the planning process means that the fluctuations in harvest level are ignored. The ability of such models to accurately determine optimal harvest levels under such circumstances is questionable.

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[1] Planning horizons in western Canada typically range from 60 to 120 years.

1.1 Study Objectives

The primary objective of this study was to develop a methodology to determine optimal harvest levels for a forest subject to the risk of destruction by fire. The secondary objectives were:

1. To determine a method of using historical fire data in timber management planning.
2. To evaluate problems inherent with harvest scheduling models which fail to explicitly recognize the replanning component of timber management.
3. To assess the benefit and cost of reducing AAC in anticipation of future fire losses.
4. To assess the effect of different fire frequencies on the timber resource.

The study can be divided into 3 components. First, an appropriate method for generating hypothetical sequences of annual fire rates[1] was determined. Modelling the interaction between forest harvesting and forest fire was achieved in the other two components.

Two distinct and separate models were developed. The first model used simulation to determine an even-flow of annual timber harvests over an entire planning horizon from a forest subject to probabilistic destruction by fire. The results of this model were used to assess problems caused by assuming that planning is done only once for the entire planning horizon. The model will be referred to as the

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[1] Fire rate, as used in this thesis, refers to the proportion of the total forest area burned annually.

traditional even-flow model. The other model developed was an optimization model which incorporated the replanning aspect of timber management in its formulation. Optimization is based on a technique called dynamic programming (DP) and the model is therefore termed the DP model. Within the model, timber management is treated as a sequence of decisions occurring throughout the planning horizon. However, harvest levels are not determined for the entire planning horizon; only the first decision in the sequence is calculated. The DP model achieves the primary objective. It is also used to accomplish the third and fourth secondary objectives.

2. LITERATURE REVIEW

Prediction of the outcome of forest management activities is difficult because our knowledge of many processes is incomplete and the future is uncertain. Typically we have some expectation of the consequences of our decisions. If probabilities can be assigned to the outcomes, then "decision making under risk" is possible (Nemhauser 1966, Moskowitz and Wright 1979). If we are completely ignorant of these probabilities, we are forced to make decisions "under uncertainty". When we have complete information, we are able to manage with certainty.

Quantitative or scientific techniques used as aids in decision making are commonly know as operations research (OR). The objective of OR is the determination of optimal courses of action in both deterministic and probabilistic systems. A great deal of information is available on OR. Some of the more commonly used books are those by Wagner (1975), Hillier and Lieberman (1980), Bierman *et al.* (1969), Lapin (1981) and Moskowitz and Wright (1979). These books cover both deterministic and probabilistic methods. Information on specific techniques such as simulation, linear programming and dynamic programming is also abundant; references on these can be found in the previously mentioned books. Comprehensive treatment of decision making under uncertainty and risk can be found in Raiffa (1968) and Holloway (1979).

Reviews of the use of OR in forest management and in the manufacture of forest products have been done by Harrison and deKluyver (1984), Johnson and Scheurman (1977), Martin and Sendak (1973) and Field (1976). In addition, Martell (1982) reviewed the application of OR to forest fire management.

The literature pertaining to decision making under risk in forest management is of specific interest to this study. Separation of the research into two broad groups is appropriate:

1. Effect of the risk of fire in timber management
2. Decision making under risk in other forest management applications.

2.1 Risk of Fire in Timber Management

The literature directly related to the effect of the risk of fire in timber management is not extensive. The research can be classified into two distinct groups:

1. That dealing with stand-level analysis.
2. That dealing with forest-level analysis.

In stand-level management, each individual stand is treated as a discrete unit and managed independently of all other stands. Forest-level management is based on the amalgamation of all stands within the forest. Management on a forest basis is generally necessary because there are forest-level constraints which preclude independent stand management.

2.1.1 Stand-Level Analysis

Martell (1980) used a simple probabilistic dynamic programming formulation to determine the optimal rotation of a stand subject to fire. He treated the problem in a fashion similar to the classical equipment replacement problem (Wagner 1975). In any year there was a probability (which remained constant) that the stand would burn, a revenue generated from salvage if the stand burned, and a revenue generated if the stand was cut without a fire occurring. Planting costs were also included. The decision variable was whether or not to cut at the start of each year. The optimal rotation was the cutting age which maximizes expected present value over an infinite planning horizon. Martell found that as the annual probability of fire increased, optimal rotation and expected present value decreased. From this stochastic model Martell was also able to evaluate alternative fire management levels. By subtracting the soil expectation value[1] with no money spent on fire management from the soil expectation value with a certain level of fire management, he was able to calculate the expected present value of fire management activities.

Routledge(1980) also developed a procedure to determine optimal stand rotation when the risk of fire was present. He added an additional term into the Faustmann equation (Faustmann 1849) that included age-dependent probabilities of destruction and the salvageable proportion of timber

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[1] Present value of an infinite series of periodic incomes with the first occurring at the end of the first rotation.

after a fire. The modified Faustmann rule meant that forest growth would be balanced against the interest lost by postponing the harvest as well as the potential loss of revenue from a forest fire. As a result, optimal rotation ages were reduced except where salvage rates were near 100 percent.

Reed (1984) was able to analytically derive a similar modification to the Faustmann equation by assuming that fires occurred in a Poisson fashion. He showed that fire caused an increase in the discount rate by an amount equal to the long-term average rate of fire occurrence. The Faustmann equation thus became:

$$V'(T) = \frac{(\lambda+\delta)(V(T)-C)}{1-e^{(\lambda+\delta)T}}$$

where: T = rotation period

λ = annual probability of fire

δ = discount rate

C = planting cost

V(T) = stumpage value

V'(T) = present value

Foster (1979) in his work on discounting also came to the conclusion that if the annual risk probability was known it should be added directly to the riskless discount rate.

As a further extension, Reed and Errico(1984) determined long-run average yields for various rotation ages. The formula below indicates the procedure:

$$Y'(T) = \frac{\lambda\, V(T)}{e^{\lambda T}-1}$$

where: $Y'(T)$ = long-run average yield at rotation T

λ = annual probability of fire

$V(T)$ = yield at time T with no fire

The resulting "volume-rotation" curve could be determined for different levels of fire probability. Larger fire rates resulted in lower optimal cutting ages and lower long-run average yields. Reed and Errico also demonstrated that ignoring the risk of fire, when in fact it was present, would result in a lower long-run average yield. A simple formulation for salvage was also included to illustrate that salvage reduced the effect of fire on long-run average yield.

2.1.2 Forest-Level Analysis:

Most models used in forest level planning such as Timber RAM (Navon 1971) and MUSYC (Johnson and Jones 1980) are not capable of dealing with uncertainties such as fire. Their reliability in determining cutting policies where forest fire commonly occurs is being questioned. The body of research on how fire affects forest-level planning is relatively small but the results to date have concurred.

Van Wagner (1978) was the first to discuss the potential impact of fire on a forest-level management by examining the how age class distributions were altered by fire. By assuming a constant annual fire-rate and immediate regeneration following fire, he showed that the age class distribution of a forest fit the negative exponential function. Using this distribution, the 'fire cycle'[1] was equal to the mean age of the forest.

Van Wagner (1983) furthered his work in this field by developing a model to simulate the effect of fire on timber supply. The model followed three basic processes: the forest was grown according to a yield curve; an equal area was burned each year with flammability constant with age; the stand of the highest volume was always cut first. Area control was used for harvesting. The basic output of the model was an annual long-term harvest at given levels of fire and cutting. The main conclusions were:

1. The amount by which the maximum sustainable harvest (with no fire) was depressed by fire was substantially greater than the volume of timber lost on the burned area.
2. As the rate of fire increases, the age at which harvesting occurs was gradually reduced.

Reed and Errico (1986) used linear programming to determine the effect of fire on forest level management. Their work was done in two stages. In the first stage a linear programming harvest model was used in a feedback fashion. Given the initial state of the forest, the optimal

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[1] Fire cycle was the number of years required to burn an area equal to the total forest area.

first period harvest was determined. The next state of the forest would then be determined from random fire occurrence. This new state would be specified as the forest state for the second period and the model rerun. The whole process was repeated for all the periods in the planning horizon. In the example used, the fire rates were randomly selected from a frequency distribution of a 34 year fire record.

In the second stage Reed and Errico approximated this feedback mechanism. By assuming that fixed rather than random proportions of the forest were destroyed by fire, a deterministic solution was possible. The rationale behind the assumption was that variations in the proportions burned tended to be small. Their results indicated that although the feedback model shows more fluctuation, the deterministic model with fixed proportions burned in every period, showed the same general tendencies.

The magnitude of the reductions in harvest obtained by Reed and Errico (1986) agree well with those determined by Van Wagner (1983) and those done at stand level (Reed and Errico 1984, Martell 1980, Routledge 1980). The conclusion that these analyses support is that current timber supply estimates which ignore fire losses are too high, perhaps by a considerable margin.

It should be noted that Reed and Errico (1986) and Van Wagner (1983) really only determined estimates of the steady-state or equilibrium harvest. No method for determination of the optimal transition to the steady-state conditions was provided and therefore the optimal current AAC was not determined.

All of the forest level procedures discussed thus far are newly developed simulation or linear programming models. A recent modification to an existing linear programming model, FORPLAN, will allow the incorporation of risk into harvest scheduling (Johnson and Stuart 1984). It will provide for multiple outcomes from differing regeneration success, stand mortality and other decision tree approaches to stand management. A deterministic rate of fire could be incorporated into such a formulation but not a rate that is random. In addition the number of multiple outcomes may be limited because of restrictions on model size.

2.2 Risk in other Forest Management Applications

2.2.1 Stand-Level Analysis:

The idea that forest managers should incorporate the probabilistic nature of forest processes into decision making is not new. In 1966, Hool included probabilistic growth responses in a dynamic programming formulation. The finite stage algorithm was designed to maximize the volume harvested from a stand. He described the stand with two variables, merchantable volume and the number of trees. Management activities examined were clearcutting, selection harvesting and thinning. Optimal policies varied depending upon the planning horizon used.

Lembersky and Johnson (1975) examined a similar stand management problem. Total discounted expected return for a stand was maximized subject to uncertainties in future product markets and in the response of stands to management actions. The problem was modelled as an infinite horizon Markov decision process, with the state of the stand described by average tree size, stocking level, and market condition. A modified version of the bound improvement procedure (Totten 1971) was used to solve the functional equation. The timing and degree of management activities such as thinning, selection harvesting and clearcutting were examined. Lembersky and Johnson were able to determine relationships between forest and market conditions and optimal management activities.

Lembersky (1976) reformulated the problem so that expected sustained yield for each stand was maximized over an infinite planning horizon. The policy improvement algorithm (Hillier and Lieberman 1980) was used to calculate optimal policies. Lembersky was able to develop a "production possibility chart" to show how varying management actions changed the expected sustained yield. A summary of his two studies is found in Lembersky (1978).

Kao (1982) examined optimal stocking levels and rotation under probabilistic growth. He used discrete state dynamic programming with forward recursion. Mean annual increment (MAI) was maximized with growing stock being the state variable. Kao showed that as risk' increases the expected maximum MAI declines and optimal rotation shortens. This relationship between risk and rotation is similar to that described previously in relation to fire (Reed 1984, Routledge 1980, Martell 1980).

2.2.2 Forest-Level Analysis:

Betters and Schaefer (1981) used a Monte Carlo simulation model to examine the relative success of two alternative control strategies for Dutch elm disease. The model used a network approach where several events were possible because each management activity had a probability distribution of success (or failure). Random numbers were used to determine the path taken through the probabilistic network. By generating numerous random paths, a probability density function for each outcome could be derived.

Thompson and Haynes (1971) used simulation in combination with linear programming (LP) to quantitatively evaluate one aspect of uncertainty in forest management. The objective of the LP was to satisfy the wood requirements for a firm through acquisition of timber and land while minimizing cost. Future availabilities of land and timber were considered random variables. Various right hand side coefficients were determined by Monte Carlo simulation techniques. With each set of coefficients a solution to the

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'Risk was defined as a function of the variation in growth rates.

linear programming problem was obtained; thus, a distribution of solutions was produced. Each solution assumed that the decision maker knows in advance what will occur in the future.

Weintraub (1976) also used LP but in a different manner. He formulated a LP problem to determine harvest schedules with probabilistic growth. Probablistic programming was used such that constraints were satisfied with a predetermined probability of success. For example, a constraint might be: non-declining yield for periods 1 to 8 must occur with a probability of 0.80. A "safety factor" added to the coefficients accounted for uncertainty. This factor was determined on the basis of the desired probability of success and the estimated variability in activity levels. The first step was to obtain a solution to the deterministic problem. If the probability of satisfying the constraint was not close enough to the desired probability then, the coefficients were adjusted accordingly and the problem rerun. Weibtraub advocates further testing of his assumptions but suggested that for noncatastrophic losses, his method should approximate the probabilistic problem .

There are several applications of OR in the field of forest fire management involving uncertainty but they are beyond the scope of this review. For further information on these applications refer to Martell (1982) and Cohan *et al.* (1984).

3. CURRENT POLICIES ON FIRE RISK AND AAC

A letter survey was taken of all provincial forest management agencies regarding the effect of fire on AAC[1]. Each was asked if they considered forest fire in the determination of AAC and why. If the agencies did adjust AAC for fire, the procedure used and the average reduction for fire was also requested.

3.1 Newfoundland and Labrador

AAC is not reduced in advance of forest fire occurring. Wood supply well exceeds the demand and fire rates are not considered high (the average for productive land is 0.1% per year) so future fire losses would cause few timber supply problems. The province is reinventoried every 10 years and age class distributions are updated every 5 years to reflect any large depletions. AAC is also recalculated on a 5 year interval.

3.2 New Brunswick

Fire is not considered in the determination of AAC because fire rates are considered too low to warrant it (less than 0.1% of the total forested area is burned annually).

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[1] The information for Alberta was obtained through personal communication with the Timber Management Branch of the AFS.

3.3 Prince Edward Island

The effect of fire is not dealt with in AAC calculations because of the low fire losses (approximately 20 hectares per year are burned) and because 90% of the forested land is privately owned.

3.4 Nova Scotia

AAC is reduced in Nova Scotia for abnormal losses due to insects, disease, wind damage and fire. The impact of fire is not isolated from the other potential losses. A percentage reduction for these abnormal losses is made at the beginning of their simulation to determine AAC (presumably in the yield information). This reduction varies from 10% to 20% depending on the level of management funding.

3.5 Quebec

In Quebec, AAC is not reduced in advance of fire losses occurring. However, burned areas do affect AAC because they alter the land base used in the determination of AAC. Forest development on the burned areas is assessed by sample plots. Depending on the region, a portion of the burned land base is excluded from the AAC calculations[1]. The other portion of the burned land base is considered as either being regenerated or having the ability to regenerate in 5 to 20 years.

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[1] up to 40% is excluded in some regions.

The computer-based inventory is continually updated to reflect depletions from logging, fire and other disturbances. It is estimated that removal of portions of the burned areas from the land base used in AAC calculations, reduces AAC approximately 3%.

3.6 Ontario

AAC is referred to as Maximum Allowable Depletion (MAD) in Ontario. The determination of MAD does not consider future losses due to forest fire. In areas where the level of harvest is at or near MAD, there may be increased emphasis on minimizing fire losses. In such cases, contingency plans are made for alternate sources of wood should it be required. Inventory is updated every 5 years to reflect changes in forest structure and MAD is then recalculated.

3.7 Manitoba

AAC is reduced 12% to provide for no-cut zones, small scattered stands and fire losses. The impact of fire is not separated from the other components but the majority of the reduction is for future fire losses. Yearly volume losses from fire are recorded for each Forest Management Unit. When the losses exceed 12%, AAC is recalculated on an updated age class distribution.

3.8 Saskatchewan

Forest fire is considered in the determination of Saskatchewan's AAC. An average annual reduction factor is calculated from 21 years of forest fire information. AAC is reduced by applying this factor as a percentage reduction. The reduction factor is determined in the following manner. An estimate of productive land burned yearly is made. If one assumes that all productive land is harvested over 90 years and that fire occurs equally in all age classes, then 1/90 of the area burned will occur on land scheduled for harvest each year. Thus, an estimate of the area effectively removed from harvesting by fire can be made:

$$\text{Total mature area lost due to fire over N years} = \sum_{i=1}^{N} \left[\frac{i}{N} \times \text{total area burned yearly} \right]$$

The total mature area lost over the liquidation period divided by the total productive land base yields the reduction factor. The area weighted average reduction for Saskatchewan is 15.1%.

3.9 Alberta

The Alberta Forest Service (AFS) adjusts AAC to reflect future fire losses. Average fire rates for productive and potentially productive land (based on 20 year records) are determined for each provincial Forest. The average yearly burned area for each Management Unit is determined from the fire rates. An average volume per unit area is also calculated for each Management Unit. Multiplication of the average burned area by the average volume per unit area produces the anticipated volume loss on a yearly basis. AAC is reduced by the expected volume loss to a maximum of 10% of the total cut. Currently most of the northern Forests in Alberta receive the maximum reduction allowed.

3.10 British Columbia

AAC is reduced for future fire losses in B.C.. The unsalvageable volume of timber resulting from fire each year is estimated on each Timber Supply Area (TSA) on the basis of historical records. This volume is subtracted from the AAC determined by their forest planning model. Current reductions range from 1% to 10% depending on the TSA. In the example received for the Fort Nelson TSA, only unsalvageable losses on near and medium accessibility classes were considered.

4. METHODS

4.1 Generating Hypothetical Fire Sequences

The first step in the assessment of the effect of fire in timber regulation is to determine which fire rate(s) to use. Classification of possible methods in the manner below is appropriate:

1. Deterministic
2. Stochastic
   a. random selection within the range of past occurrences
   b. random selection from the entire possible range of fire rates

The typical deterministic approach to fire has been to use the average fire rate for some period of historical record. This method is the easiest to use and the most unrealistic since fire rates obviously do vary on a yearly basis. For example the highest fire rate found in the area used in this study[1] from 1961 to 1984 was greater than 13 times the average rate. In order to incorporate this yearly variation, it is necessary to stochastically generate fire rates. In the simplest form, a rate could be randomly selected from those having occurred previously. A cumulative frequency distribution could also be made for the historical record. This distribution would only contain fire rates within the

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[1] Management Unit S1 in Slave Lake Forest, Alberta

range of those previously occurring. The problem with this method and the deterministic approach is that planning is tailored to <u>one</u> historical sequence.

Unless the historical sequence is extremely long, fire rates beyond the range of those having occurred are entirely possible. This is evident even on a provincial basis where yearly variation would tend to be less than for a smaller land unit (Van Wagner 1983). Based on the Alberta record (Murphy 1985) from 1910 to 1980 (71 years) the largest apparent area burned in 1938. Despite the length of the record fire rates outside of the range of those previously occurring are entirely possible as witnessed in 1981 when an area almost two times the size of the previous maximum burned.

Since planning is always concerned with the future it is only reasonable to use the best information we have on possible future occurrences. Restricting all possible future events to those having occurred in the past is unrealistic. Obviously a method to generate hypothetical fire sequences outside of the range of the historical record is required. The method used in this study was to fit a statistical distribution to the historical fire record. This preserves some statistical properties of the original data such as mean and variance. The method is one used by engineers in the development of flood frequency curves.

Frequency analysis is based on three assumptions:

1. That the data analysed describe random events.

2. That the natural processes involved are stationary with respect to time[1]
3. That the population parameters can be estimated from the sample (Kite 1977, Spence 1973).

To assess the validity of the first two assumptions a thorough time series analysis is required which is beyond the scope of this study. Assumption 1 regarding the independence of the events was partially evaluated by examining the autocorrelation coefficients derived from the data. The assumption of stationarity was also tested. Since the level of fire protection has changed over time and vegetation structure and composition is also dynamic, an argument could be made supporting a non-stationary time series. To address this point, the data were divided into two separate time series. A frequency curve for each series was developed and subjectively evaluated for differences.

The second and third assumptions often conflict in frequency analysis. There is a desire to split data series to obtain stationarity, but short records result in poor estimation of the probability associated with the events (Benson 1960, Linsley *et al.* 1975). In general, longer time series result in better parameter estimation. In fact if the series is large enough it will describe the probability distribution without the need of a theoretical function.

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[1] A time series is said to be stationary if there is no systematic change in mean (no trend), if there is no systematic change in variance, and if strictly periodic variations have been removed (Chatfield 1975).

Parameters of the distribution can be estimated by any of several possible methods:

1. Maximum likelihood
2. Method of moments
3. Least squares
4. Graphical

The methods are listed in order of descending efficiency (Kite 1977). The maximum likelihood method is the most difficult to apply. The added complexity of this method was felt to be unwarranted, therefore the method of moments was used to estimate the parameters of the distributions. A more complete description of the methods including programs for several distributions is given by Kite (1977).

A problem in parameter estimation occurs when zero area is burned and the distribution uses logarithms. The logarithm of zero is $-\infty$ which cannot be processed. To overcome this problem 0.1 hectares were assumed to burn in each year where no area was recorded. This was acceptable in light of the fact that small burned areas are not generally recorded and that fires did occur in all years even when no area was recorded as burned.

4.2 Traditional Even-flow Model

The model used to determine the even-flow annual allowable cut is a time-paced simulation model. It essentially follows the same procedure as an area-volume check model (Chapelle 1968) except that provision is made for the loss of timber due to fire. As with all area-volume (A-V) check models, equal yearly cuts are determined such that the entire forest area is harvested in the desired length of time (referred to as liquidation period).

By using random numbers it is possible to generate values from any probability function. Thus random annual fire rates were generated from the fire frequency curve which was based on historical fire data. Normally distributed random numbers were used to produce the annual fire rates since the logarithms of the annual fire rates were found to be normally distributed. For each unique normally distributed random number, a unique fire rate exists.

The Box-Muller method (Box and Muller 1958) was used to obtain normally distributed random numbers. The uniformly distributed random numbers required by that method were produced by URAND a system subroutine[1]. URAND requires a number, referred to as the random number seed, in order to produce a uniformly distributed random number. Only one random number seed is required to produce the fire sequence over the entire liquidation period and the same seed will always produce the same fire sequence.

The data requirements include an age class distribution for the forest, a yield curve, a desired liquidation period, a random number seed, parameters describing the fire frequency curve and an initial cut estimate. A simplified

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[1] available at the University of Alberta Computing Centre

flow-chart of the even-flow simulation model is shown in Figure 4.1.

The simulation begins at year 1 by cutting the oldest timber first at a rate set by the user. A random fire rate is generated and the area in all age classes is reduced by the fire rate. If all of the standing timber has not been cut or burned, the year is incremented and the forest grows according to the yield curve supplied. After all of the standing timber has been cut or burned, a check is made to see if the length of time required equals the desired liquidation period. The cut rate is adjusted according if the two times do not match. This adjusted cut rate is then used to rerun the model. The same sequence of fire rates is used for each iteration. The run is complete when the length of time required to cut or burn the forest equals the desired time.

There are several assumptions and conditions which warrant further clarification. Volume control is used in the harvesting portion of the model but fire alters the age class distribution on an areal basis. The probability of destruction by fire is constant for all age classes. This does not necessarily imply that flammability is constant. It is also assumed that all fires are lethal; no salvage volume is allowed.

Fire frequency remains constant regardless of the state of the forest. If probability of destruction varied significantly with age, then fire frequency could be

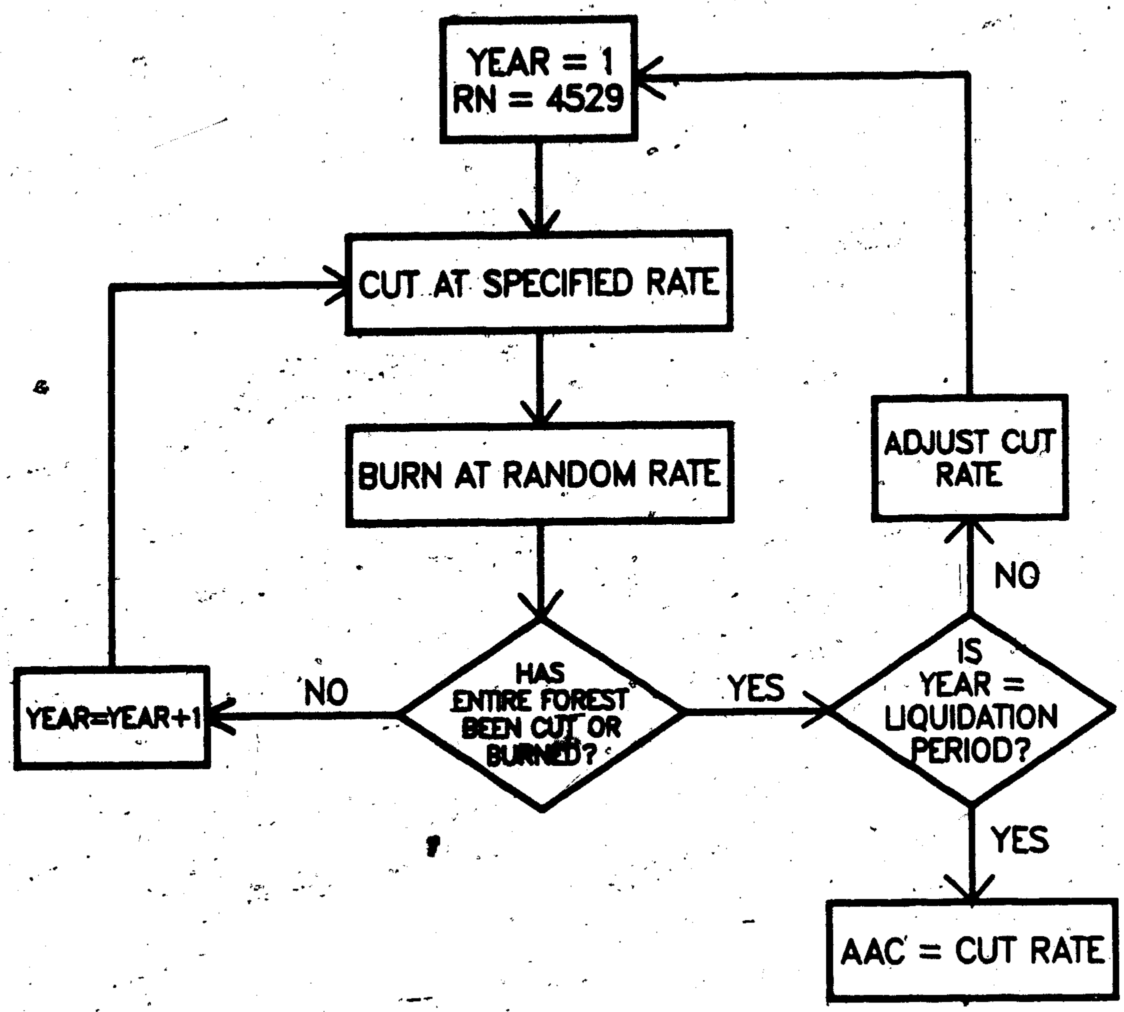

Figure 4.1: Flow-chart of Even-flow Simulation Model

expected to change as the age class distribution was altered. The fire frequency curve has no maximum value, so it is possible that a rate exceeding 100% could be randomly selected. If this occurs a new random fire rate is generated. This slightly increases the probability of selecting rates less than or equal to 100%. However, the effect is not critical because the probability of selecting a rate greater than 100% is very small.

The priority for harvesting is based on greatest age although the program could be modified to select some other criterion such as highest volume. All of the standing timber must either be cut or burned before the simulation compares the harvesting time with the desired liquidation time. This is different than the area equivalent to the forest because the total area burned will contain some regenerated timber as well as standing timber.

Every complete run of the model will yield an annual cut rate that is sustainable over the liquidation period given the unique fire sequence that occurred. By running the model 1200 times, a distribution of sustainable cuts (even-flow AACs) was produced thus representing a broad range of potential fire sequences.

4.3 Dynamic Programming Model

4.3.1 Introduction

The need for a model to realistically represent the planning process of timber management has been discussed previously. When the replanning component is considered, a sequence of decisions occurs during the planning horizon. In addition, the model should make the harvest level decision at each replanning time dependent on the structure of the forest at each particular point in time.

An optimizing solution technique known as dynamic programming is able to handle both of these components. It is useful in determining an optimal policy of a sequential decision process and the policy will depend on the state of the system. Unlike linear programming, there is no standard mathematical formulation in dynamic programming (DP). Equations are developed to suit each individual application. DP can be used in either a deterministic or probabilistic manner.

The major difference between the two approaches is that there is a probability associated with each return in the probabilistic model. That is, each return is weighted by its frequency of occurrence. With discrete random variables, the sum of the probabilities multiplied by the corresponding return is referred to as the expected value or return. Expected value can be thought of as the long-term average value.

DP generally uses a backward recursive (step by step) solution technique. That is, the optimal decision for the last period is determined first; then the next to last time period is solved and so on until the optimal decision for the first period is calculated. This technique is based on Bellman's Principle of Optimality (Hillier and Lieberman 1980):

> Given the current state of the system, the optimal set of decisions for the remaining stages is independent of the decisions used to reach the present state.

A simple deterministic example will illustrate the type of problem for which DP can be used and the terminology associated with it. The problem is illustrated in Figure 4.2. Suppose that various age class distributions are represented by the various ACDs in the figure. The numbers represent cut levels. So if 300 units are cut yearly from a forest with ACD1 for 10 years, it will still have the same age class distribution at year 10. However cutting at 400 or 500 units each year for 10 years will put the forest into different age class distributions. The problem can be divided into <u>stages</u>, with a decision required at each stage. In this example, the stages are points in time; year 10 is stage 1 and year 0 is stage 2[1]. Year 20 is not a stage because no decision is required. Each stage has a number of <u>states</u> associated with it; the various age class distributions are states. The objective in this problem

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[1] The convention in DP is usually to count stages backwards, with the first stage being the last decision point.

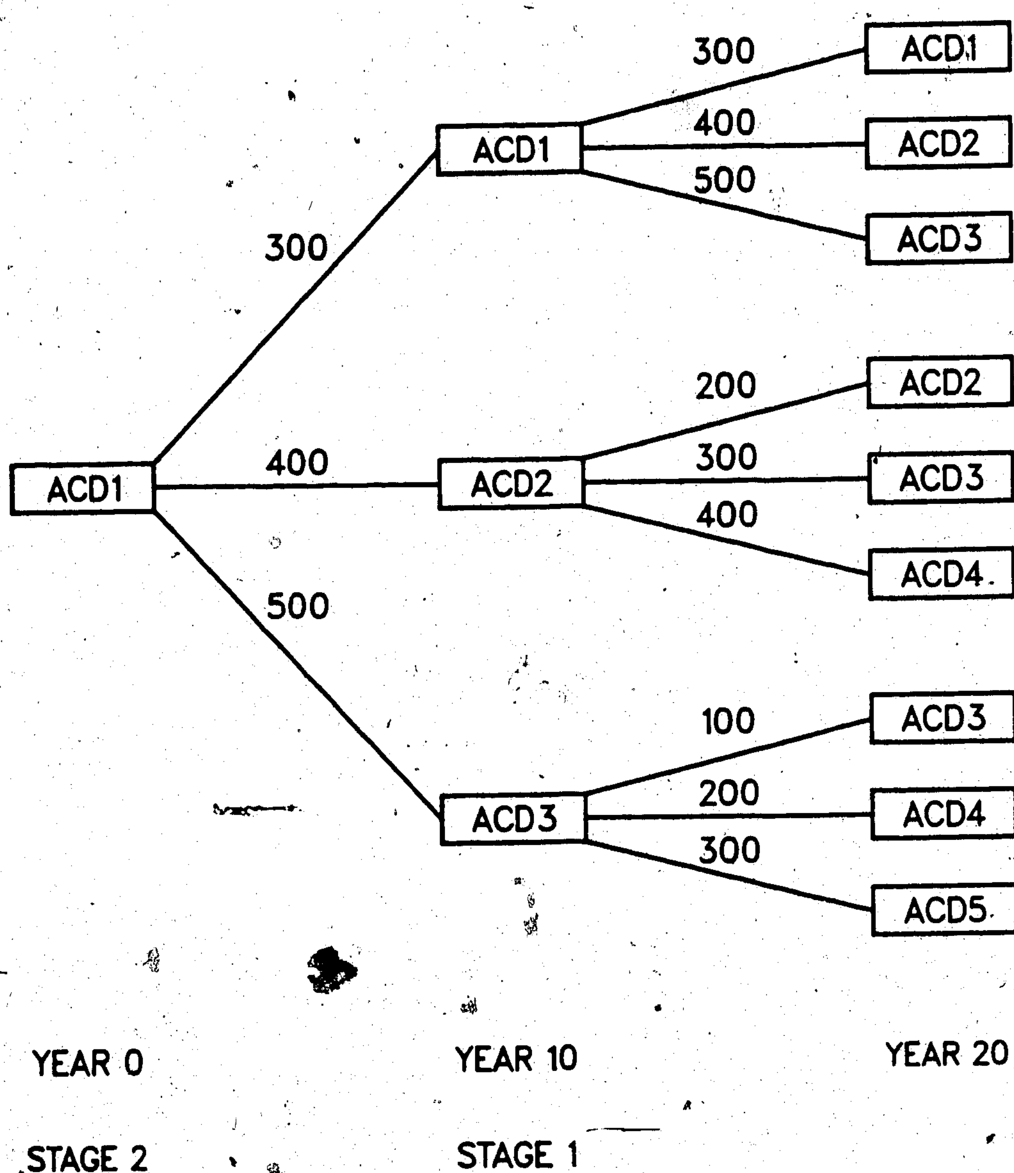

Figure 4.2: Example of a Deterministic DP Problem

might be to maximize the cut over 20 years, so the <u>decision variable</u> is the amount of cut.

A <u>recursive relationship</u> can be developed to mathematically identify the optimal policy for each state at a given stage. In this problem the recursive relationship is:

$$f_n(s) = \max_d \left[r_n + f_{n-1} \right]$$

This simply means that the value associated with the optimal decision at stage n and state s ($f_n(s)$) is the value associated with the decision (d) that will maximize the reward from the current cut (r_n) plus the reward from the future cut (f_{n-1}). When using the backward solution technique, the future cut is determined first. Calculations for subsequent stages then use the values determined from those future time periods. It is acceptable to determine the future solutions first because of Bellman's Principle of Optimality.

In the simple example shown in Figure 4.2, three optimal solutions are present; each one obtains an accumulated cut of 800 units over the 20 year period. This simple problem could be solved by exhaustive enumeration; that is, determining the total cut obtained over the 20 years for every possible alternative. Exhaustive enumeration is not efficient, so DP is used to decrease the number of calculations.

Figure 4.3 is an example of a probabilistic problem. The fractional numbers represent probabilities; they might

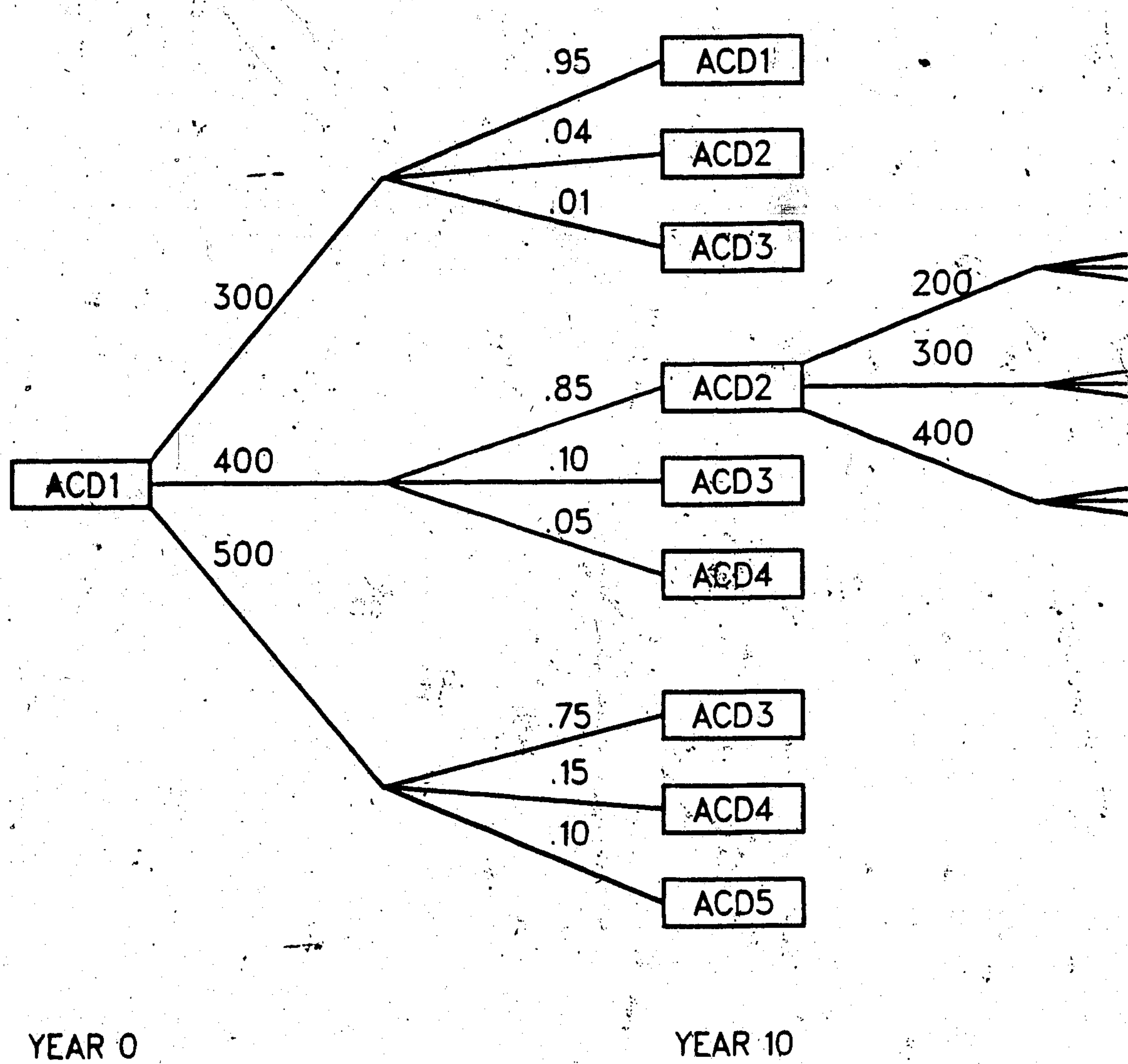

Figure 4.3: Example of a Probabilistic DP Problem

reflect different fire rates that occur during the 10 year planning intervals. So for example, cutting at a rate of 300 units each year and ending up in state ACD1 after 10 years would reflect a very commonly occurring fire sequence (occurring 95% of the time). However ending up in state ACD3 would be relatively rare, perhaps indicating a severe fire sequence. Therefore the transition from one state to another is governed both by the cutting rate and the rate of burning.

The probabilities of going from one state to another in one time period are referred to as one-step transition probabilities or Markovian transition probabilities. If the objective is to maximize the cut, then the recursive relationship for the probabilistic problem is:

$$f_n = \max_d \{ \text{ current cut + expected future cut } \}$$

or

$$f_n(i) = \max_d \left[r_n(i)^d + p_{ij}^d \; f_{n-1}(j) \right]$$

where: $f_n(i)$ = expected reward over the next n stages given that the the current state of the system is state i.

$r_n(i)^d$ = reward from choosing decision d in state i.

p_{ij}^d = transition probability from state i to state j using decision policy d.

$f_{n-1}(j)$ = the expected reward from being in state j at stage n-1.

It is possible to have different objective functions for the same problem structure. In the examples shown previously, the objective was to maximize the amount of harvest over a specified time period. The objective could also have been to minimize the harvest or alternatively minimize the variation in harvest levels period to period. Several different objective functions were used in this study.

The problem formulated in this study is similar to the problem in Figure 4.3. The development of the DP model can be separated into two sections. The first section dealt with structuring forest regulation into a problem that dynamic programming could solve. The second component consisted of the determination of the one-step transition probabilities.

4.3.2 Dynamic Programming Formulation

The problem shown in Figure 4.3 begins at time 0 with one age class distribution, namely ACD1. Use of backward recursion permits a series of optimal harvest levels to be determined over a range of possible initial forest states. Figure 4.4 illustrates the structure of this problem. Since the relationship between initial forest state and optimal harvest levels was desired, the problem structure in Figure 4.4 was adopted for this study.

In order to use DP the stage, state and decision variables must be defined. The state variable or the descriptor of the age class distributions was the even-flow

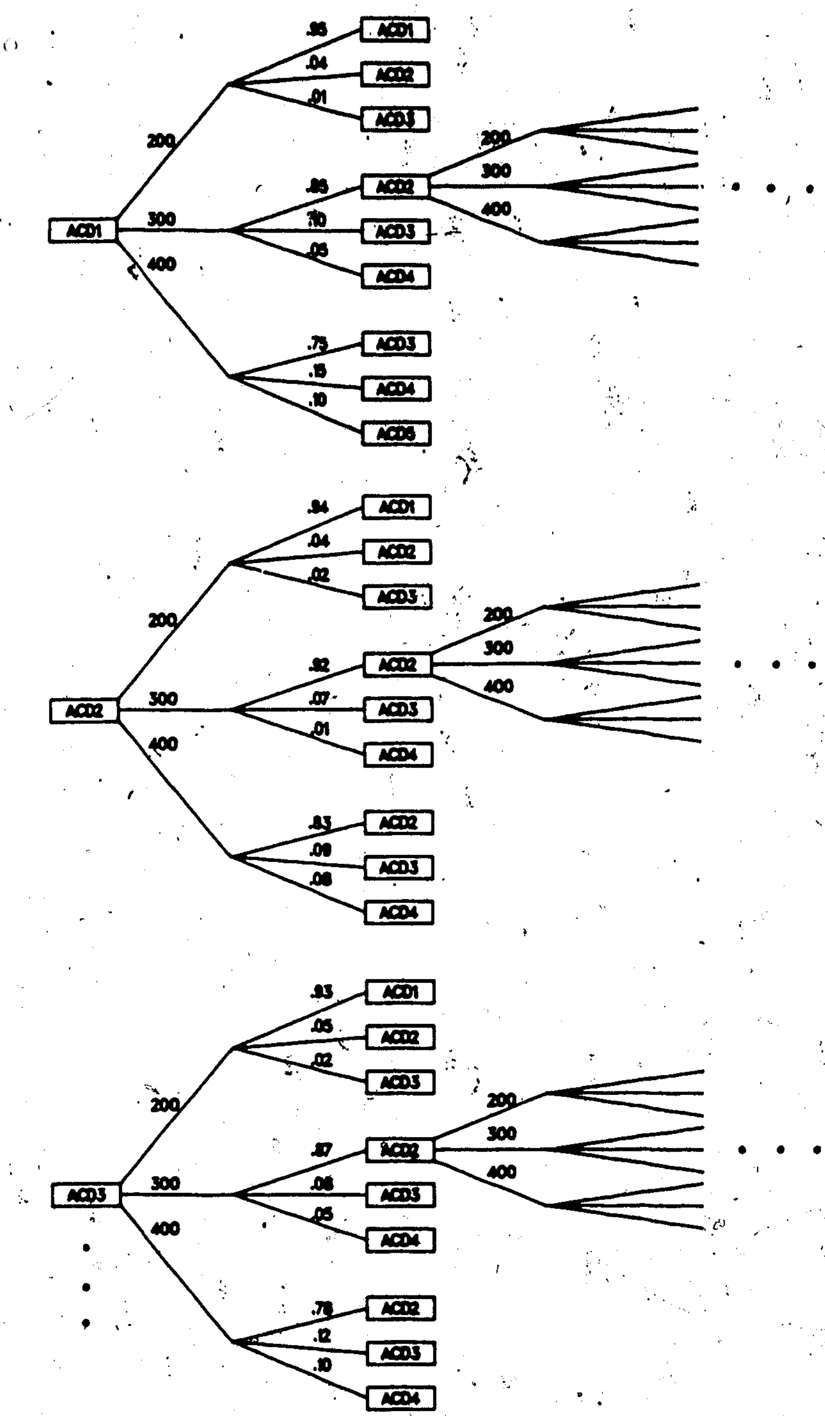

Figure 4.4: Problem Structure Used in the DP Model

sustainable yield as determined by an area-volume check. The decision variable was the cut level. Stages were 10 year time intervals to simulate a 10 year replanning interval.

Regenerated timber originates on land which is either cut or burned. Regeneration following cutting is generally assumed to grow better than standing timber or regeneration following fire reflecting such factors as stocking and genetic improvement. Age class distributions with different proportions of standing and regenerated timber, and the same even-flow AAC respond differently to harvesting and fire. This means that the state variable, even-flow AAC, does not fully describe the state of the forest. It might be possible to identify two or three state variables that are able to more fully describe the state of the forest, but dimensionality becomes a problem. That is, the required number of calculations can increase drastically when additional state variables are introduced (Hillier and Lieberman 1980). So rather than introduce additional state variables, it was assumed that both standing timber and regenerated timber grew according to the standing timber yield curve.

The planning horizon used in finite-stage dynamic programming is controlled by the number of stages. Problems using probabilistic dynamic programming where an infinite planning horizon is desired, are referred to as Markovian decision processes (Hillier and Lieberman 1980). There are several solution methods for Markovian decision processes.

The one chosen for this study is referred to as the method of successive approximations (Hillier and Lieberman 1980). The method is based on the premise that as the number of stages is increased the optimal activity chosen for a given state will eventually stabilize or converge to one activity (steady state conditions are reached). There is no procedure to determine exactly how many stages are required to reach convergence. In addition, not all Markovian decision processes will exhibit this behavior (Lembersky and Johnson 1975). The advantage that the method of successive approximations has over other techniques is that it does not require the solution of a system of simultaneous equations. DP algorithms must be written to suit the problem so one was developed for this study.

4.3.2.1 Model Constraints

Traditional methods for determination of harvest rates such as linear programming are constrained maximization models (Duerr *et al*. 1975). Dynamic programming can be used for both constrained and unconstrained problems. To model uneven-flow, the constraint of equal annual harvests must be lifted. In fact, because fire is capable of reducing harvest rates to zero, there can be no lower bound. An upper bound to harvesting was set in the dynamic programming formulation used in this study, perhaps as a throwback to more traditional approaches and also because an unconstrained problem would have required more simulations. Harvest rates greater than the even-flow AAC at any stage were not permitted.

Often it is desirable to have a lower limit on harvest rates for administrative and other reasons. To test this, a limit was set at 225,000 m^3 per year. At these low initial state levels, the age of the timber was often lower than the minimum harvest age of 60 years used by the AFS and this seemed a reasonable way to set this restrict harvesting of these low initial states. The state variable, even-flow AAC, would indicate harvest rates greater than zero in such cases because the yield table has merchantable volume in young age classes even if the harvesting of such volume is not permitted by policy. In retrospect, a yield table reflecting only harvestable volume would be a better method to deal with this problem. Obviously not all of the timber in all age class distributions with an even-flow AAC of 225,000 m^3 would be less than 60 years, but some restriction was necessary. In reality, harvest levels would probably more gradually decrease and the abrupt, artificially imposed decrease does cause some model anomalies as will be seen. When the state is less than 225,000 m^3 per year, the harvest level is set to zero only for 10 years; cutting will resume when the state of the forest exceeds 225,000 m^3 per year.

4.3.2.2 Objective Functions

Fight and Bell (1977) discuss a conceptual framework for making consistent decisions in timber management when risk and uncertainty are explicitly recognized. The dynamic programming problem formulated in this study conforms well with their framework, though not by design. They realized that changes in harvest levels over time are inevitable and that current harvest levels influence the magnitude and direction of future adjustments. That relationship may seem obvious, but no currently operational harvest scheduling model incorporates the trade-off between harvest level and smoothness of flow directly into its objective function.

It may be easiest to think of the objective in the problem described herein as maximizing net social gain; that is, social gain minus social loss. Social gain is the value associated with harvesting, and social loss occurs when deviations from the desired harvest schedule occurs. Decreases in harvest level may produce social costs such as reduced mill efficiency, bankruptcy, relocation of people and facilities and unemployment. Increases in harvest level may also be considered social costs. Development of the additional resource may occur faster than the factors of production can efficiently adjust. For example, wages and the cost of goods and services may be unnaturally high for a period of time.

The social loss function may take several forms. Figure 4.5 shows a symmetrical loss function'; both reductions and increases in harvest volumes over time are equally important. In this case, the magnitude of the social loss is directly proportional to the magnitude of the deviation in flow. Figure 4.6 is a non-linear symmetrical loss function, meaning that large deviations in flow are relatively more critical than small deviations.

The units for social gain and loss must be the same in order to determine net social gain. The units could have been monetary, but for simplicity, units of timber volume were chosen. Figure 4.7 gives hypothetical numbers for a possible loss function. The linear function shows that a decrease of 1000 m^3 from one period to another has a social cost equivalent to 2000 m^3. If the value of one m^3 of harvested wood is equivalent to $10 in terms of social benefit, then a decrease of 1000 m^3 has a social cost of $20,000. In Figure 4.7, there is no loss associated with even-flow, therefore the slope of each side of the loss function will always determine its shape.

It is easy to understand that social gain, the amount of wood harvested over some time period, can be represented by units of volume. However, the units for deviations from even-flow are more difficult to grasp.

' adapted from Fight and Bell (1977)

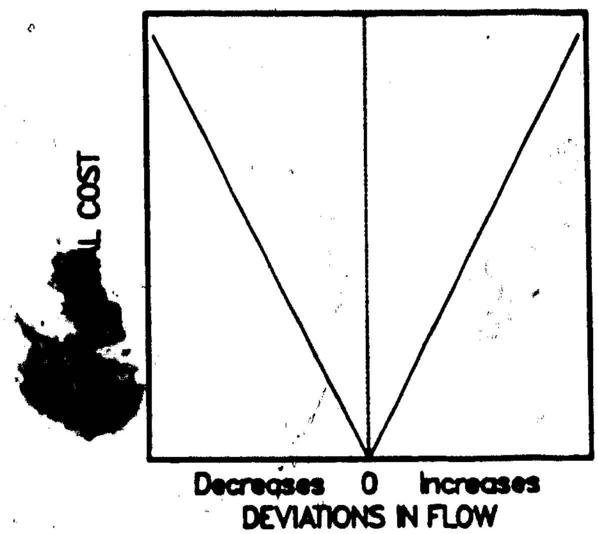

Figure 4.5: Symmetrical, Linear Social Loss Function

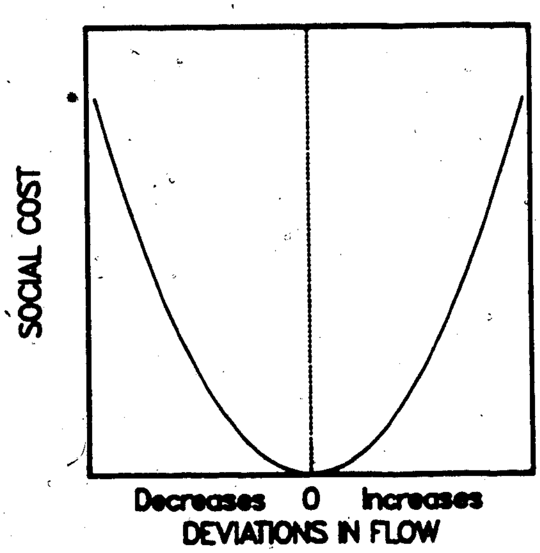

Figure 4.6: Symmetrical, Non-linear Social Loss Function

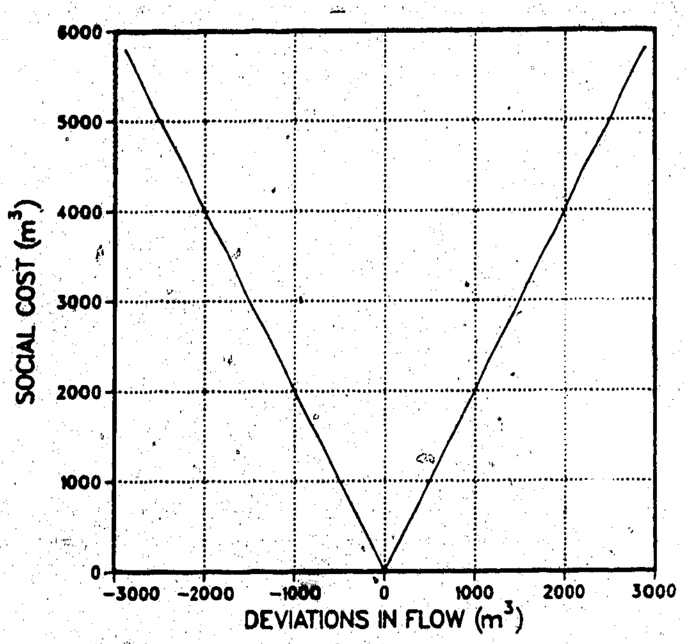

Figure 4.7: Social Loss Function with Units

In this study, the units for the social cost of fluctuations in harvest rate are equivalent to the social benefit associated with harvesting one m^3 of wood.

Non-symmetrical loss functions are entirely possible. In this study the majority of the runs were done with a loss function for only reductions in flow. In such cases, the absolute value of the slope of the function will be referred to as the <u>cut level reduction cost</u>. The social cost associated with both increases and decreases in cut level will be referred to as <u>cut level change cost</u>. Figure 4.8 indicates various loss functions and their associated cut level reduction costs. When the

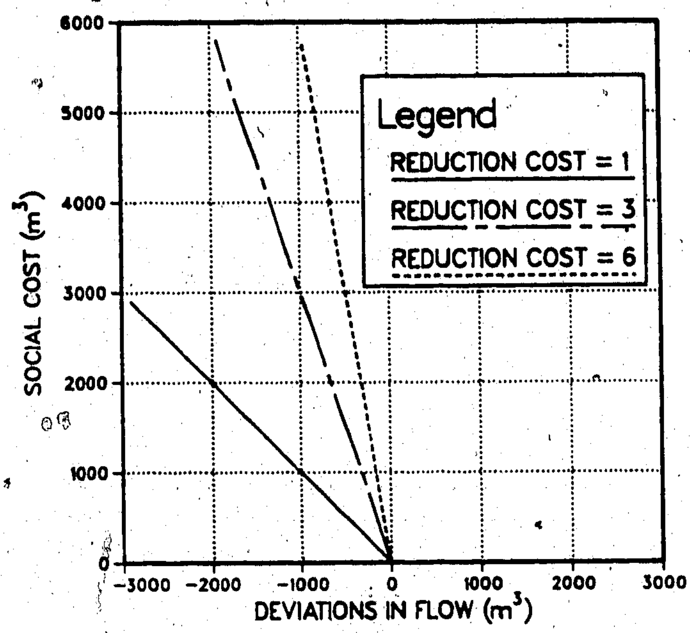

Figure 4.8: Social Loss Function Showing Cut Level Reduction Cost

reduction cost is zero, there is no social loss associated with deviations in flow.

Time-preference is an important element to consider in social decision making. Therefore, the effect of varying degrees of time-preference on optimal decisions was tested in this study. Because discount rates were used, the social loss function measures the present value of deviations in flow against the present value of harvested wood.

As previously mentioned, objective functions in dynamic programming take the form of recursive relationships. For the problem structure used herein, net social gain is described by the following recursive relationship:

$$f_n(i) = \max_d \left[r_n(i)^d + \alpha \, p_{ij}^d \, f_{n-1}(j) \right]$$

<u>where:</u>

$f_n(i)$ = expected reward over the next n stages given that the current state of the system is state i.

$r_n(i)^d$ = reward from choosing decision d in state i.
- if a decrease in flow occurs: $r_n(i)^d = d_n - KFALL(d^*_{n-1}(j) - d_n)$
- if a increase in flow occurs: $r_n(i)^d = d_n - KRISE(d_n - d^*_{n-1}(j))$

KFALL=absolute value of the slope of the social loss function for decreases in flow

KRISE=absolute value of the slope of the social loss function for increases in flow

$d^*_{n-1}(j)$= optimal cut decision in state j at stage n-1

α = discount factor = $[1/(1+i)]^{10}$

p_{ij}^d = transition probability from state i to state j using decision policy d.

$f_{n-1}(j)$= the expected reward from being in state j at stage n-1.

4.3.2.3 Model Runs

Several series of runs of the DP algorithm were made to assess the effects of fire frequency, interest rate and shape of the social loss function on optimal harvest rates. Only linear loss functions were used. Table 4.1 indicates the trials done.

The number of stages, number of states and number of decision levels were held constant for all of the runs done. The optimal decision did converge to one activity after a number of stages, generally by stage 45 or 450 years. So the steady-state optimal values reported are those that occurred at stage 45.

4.3.3 One-step Transition Probabilities

A simulation model was used to determine the one-step transition probabilities. The simplified flow-chart (Figure 4.9) illustrates the procedure. An age class distribution[1] was used as the starting forest structure. Based on this distribution and on the data required by an area-volume check, an initial sustainable even-flow AAC was determined (hereafter referred to as the initial AAC). This initial forest structure was altered on a yearly basis by cutting at a constant rate of volume specified by the user and by burning at random rates according to the fire frequency curve. The cutting and burning occurred for 10 years. At the end of the simulation an AAC was determined based on the

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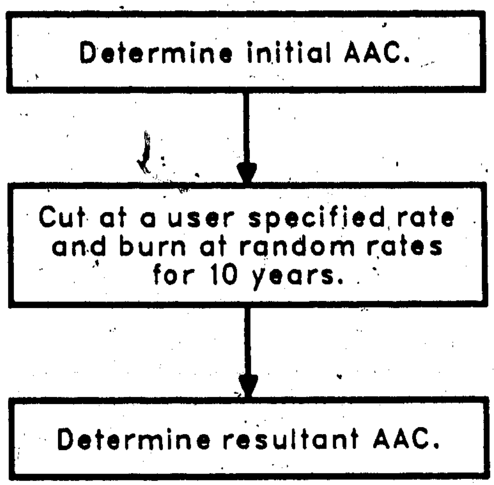
[1] Several hypothetical age class distributions were used, including the actual one for the case study area.

Table 4.1: TYPES OF COMPUTER RUNS

<table>
<tr><th>Parameter(s) Evaluated</th><th>Computer Runs</th></tr>
<tr><td>Cut Level Reduction Cost, Fire Frequency, and Interest Rate</td><td>- For each series indicated below, six runs were done with cut level reduction costs of 0,1,2,3,6 and 9. Only decreases in flow were treated as social costs.
<table>
<tr><th></th><th></th><th colspan="3">Average Fire Rate (%)</th></tr>
<tr><th></th><th></th><th>0.10</th><th>0.15</th><th>0.20</th></tr>
<tr><td rowspan="3">Interest Rate</td><td>0.02</td><td></td><td>X</td><td></td></tr>
<tr><td>0.05</td><td>X</td><td>X</td><td>X</td></tr>
<tr><td>0.08</td><td></td><td>X</td><td></td></tr>
</table></td></tr>
<tr><td>Cut Level Change Cost</td><td>- The effect of a symmetrical loss function where both increases and decreases in harvest rate are considered a social loss was assessed. A series of six runs was done.</td></tr>
<tr><td>Discount Factor of 1.0</td><td>- One run was done with a cut level reduction cost of 0.0, a fire rate of 0.15% and a discount factor of 1.0, so that the effect of no time-preference could be assessed.</td></tr>
<tr><td>A Cut Level Reduction Cost of 50.</td><td>- One run was done with a cut level reduction cost of 50, a fire rate of 0.15% and an interest rate of 5%, to determine the effect of an high emphasis on smoothness of flow.</td></tr>
</table>

Figure 4.9: Flow-chart for Simulation of Transition Probabilities



resulting age class distribution. The AAC calculated after the 10 years will be referred to as the resultant AAC. The probability of destruction by fire was constant for all ages unless the area has been burned within the previous three years[1].

By simulating numerous 10-year random fire sequences it was possible to determine the probability associated with achieving a particular resultant AAC. A range of initial AACs and cut levels were simulated to obtain the complete matrix of transition probabilities needed for a problem structure like that in Figure 4.4. Three fire frequency curves were evaluated. Each curve produced a unique set of transition probabilities.

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[1] Three years was the time suggested before reburning could occur (Slave Lake Forest staff).

5. CASE STUDY AREA

The procedure for using historical fire data in timber management and the timber regulation models developed in this study were tested on Management Unit S1 in Slave Lake Forest[1] (Figure 5.1). An AAC is determined for each Management Unit. S1 was chosen since forest fire and harvesting are both common in the region. The average annual fire rate for Slave Lake Forest is 0.36% (1961-1980)[2] and nearly 97% of the coniferous AAC in S1 has been allocated. The total forested area of S1 is 379,945 hectares. The coniferous land base used in AAC calculations is 181,889 hectares.

5.1 Fire Data

The period of record used in this study was 1961 to 1984[3]. This was the extent of the computerized fire records. All fires starting in Management Unit S1 were sorted from the fire record. Large fires that moved both into and out of S1 were examined and only the area burned in S1 was included.

The AFS currently classifies land as productive, potentially productive and non-productive. Fire rates were determined for the land base comprising all productive and potentially productive areas. This was done for two reasons:

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[1] The forest region of Alberta is divided into 10 Forests. Each Forest is then subdivided into several Management Units

[2] The average of all ten Forests for this same period of record is 0.361%.

[3] courtesy of the Alberta Forest Service (AFS), Department of Forestry, Lands and Wildlife

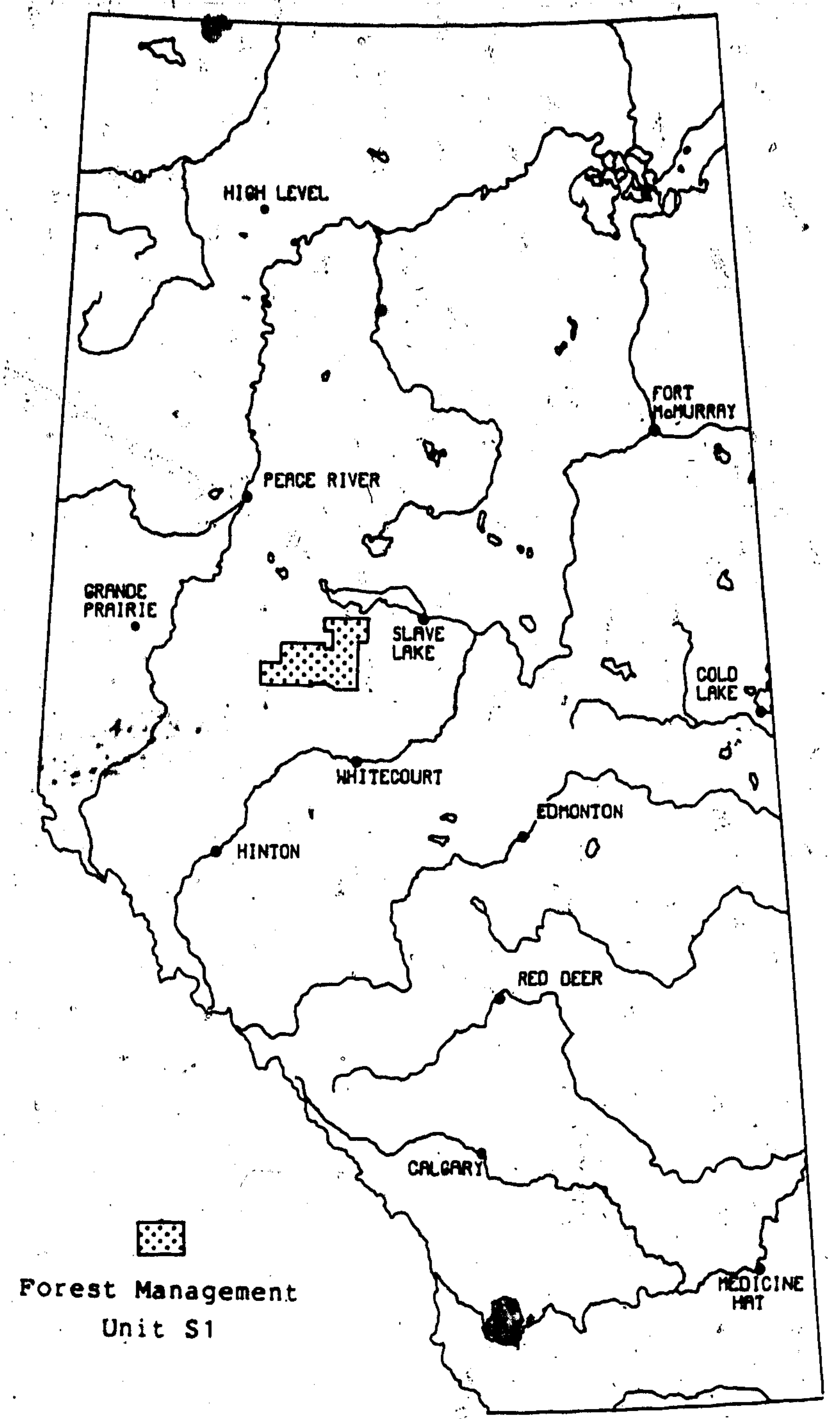

Figure 5.1: Map of Case Study Area

1. Potentially productive land includes some clearcut and burned area, both of which were to be modelled.
2. The area comprising productive and potentially productive land in S1 was thought to remain relatively constant during the period of record compared to the productive land base. The productive land base may have varied substantially over the 24 year period because burned areas are removed from the productive land base and then classed as potentially productive.

Prior to 1971, classification of burns into the three productivity classes was generally not done. Therefore, an alternative method was used for 1961 to 1970. An the average ratio of area burned on productive plus potentially productive land to the total burned area for 1971 to 1984 was determined. This ratio was then applied to all pre-1971 fires in order to approximate productive plus potentially productive burn rates.

5.2 Timber Data

The data required for both the traditional and DP models were obtained from the AFS. The data requirements for the two models were very similar and can be summarized as follows:

1. An age class distribution - Figure 5.2 illustrates the current age class distribution for S1.
2. A standing timber yield curve - The tabular yield table used by the AFS was transformed into a non-linear

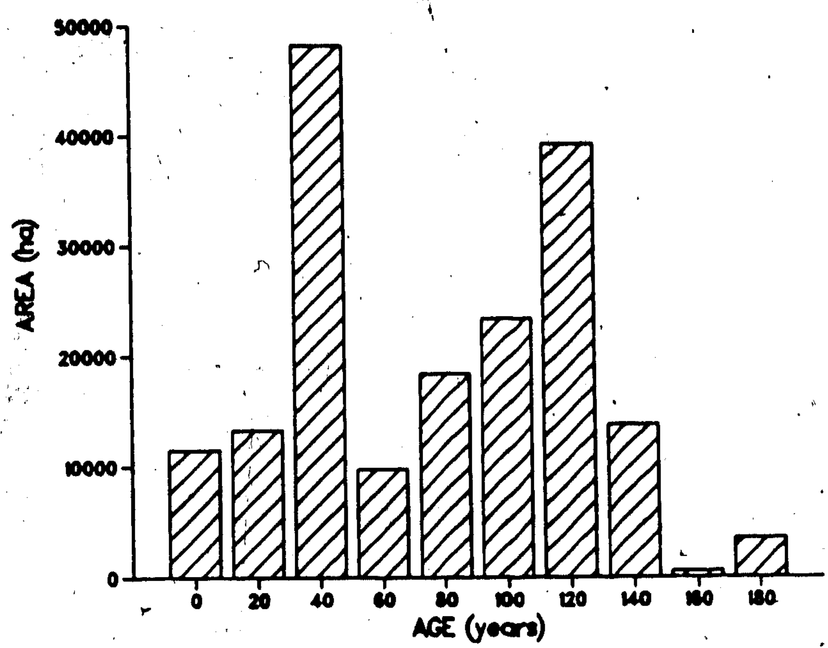

Figure 5.2: Age Class Distribution for S1

regression equation for use in the models.

3. Liquidation period and regeneration (regen) lag - The rotation or liquidation period was 90 years including a regen lag of 4 years for harvested timber.

The DP model determines an optimal harvest level for any age class distribution, providing the total land base remains constant. In addition to a regen lag for harvested area, the DP model also required a lag for burned areas. This was set, somewhat arbitrarily at 7 years, since the AFS were reluctant to estimate it.

6. RESULTS AND DISCUSSION

6.1 Generating Hypothetical Fire Sequences

The data extracted from the 1961-1984 fire record for FMU S1 are listed in Table 6.1. The mean rate is 0.15% per year. Determination of the mean fire rate directly from the age class distribution using the method of Murphy (1986) is inappropriate because the young age classes strongly reflect recent harvesting. The following statistical distributions were fit to the data: extreme value, log-Pearson Type 3, 3 parameter lognormal, 2 parameter lognormal and pareto. All of these distributions have some prior basis for modelling rare or infrequent events (Kite 1977, Cooper and Weekes 1983). Goodness of fit was assessed by the Chi-square test (Cooper and Weekes 1983). The distribution with the lowest Chi-square was the 2 parameter lognormal, so it was chosen to model fire frequency. It was judged to be a valid model at the 95% level of confidence. The equation derived was:

$$\text{Fire rate } (\%) = \exp(-6.18 + Z\sqrt{11.725})$$

where: Z = standard normal deviate

Thus the natural logarithms of the annual fire rates are normally distributed.

This relationship is illustrated in Figure 6.1. The discrete points indicated are all of the non-zero fire rates for S1 from 1961-1984. Since the probability of obtaining each fire rate is uncertain, the probability used to plot

Table 6.1: FIRE RATES FOR PRODUCTIVE AND POTENTIALLY PRODUCTIVE LAND IN S1

Year	Fire Rate (%)	Year	Fire Rate (%)
1961	0.17722	1973	0.00068
1962	0.00864	1974	0.00113
1963	0.00149	1975	0.00000
1964	0.00011	1976	0.00034
1965	0.00000	1977	0.02406
1966	0.00000	1978	0.00000
1967	0.00205	1979	0.00139
1968	1.05350	1980	0.03125
1969	0.00107	1981	2.01430
1970	0.00033	1982	0.10881
1971	0.00011	1983	0.00000
1972	0.00119	1984	0.13[illegible]

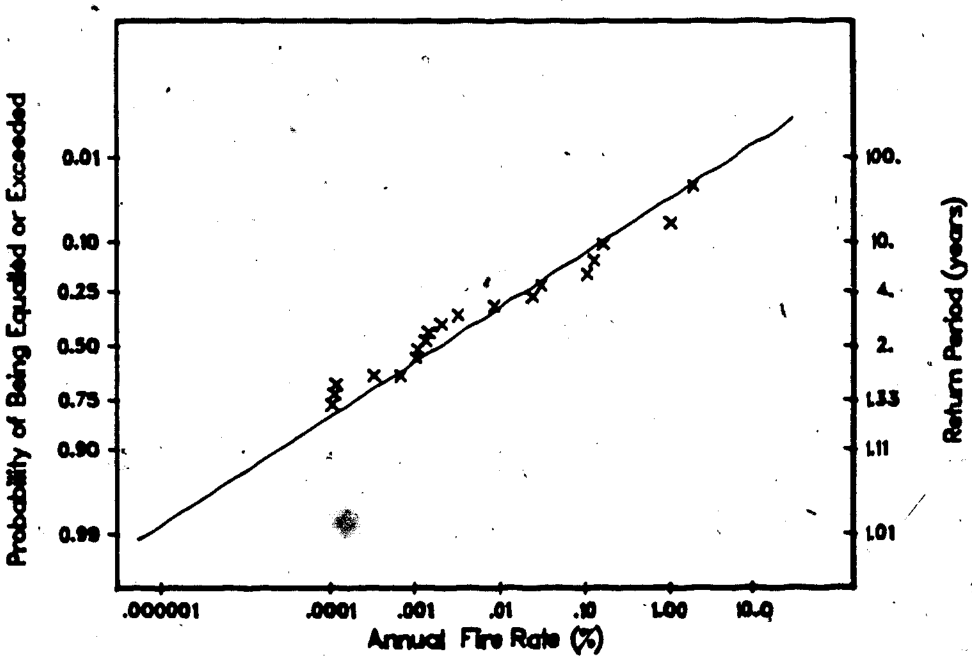

Figure 6.1: Fire Frequency Curve for S1

the points was obtained by the formula suggested by Cunnane (1978):

$$\text{plotting position} = (i - 0.4)/(N + 1 - 2(0.4))$$

where: i = rank of the event

N = total number of events

The fire frequency curve indicates the probability of any fire rate being equalled or exceeded in any given year. For example, there is approximately a 10% chance of equalling or exceeding a fire rate of 0.10%. This is equivalent to a return period of 10 years' . Return period is the long-term average interval between occurrences of a particular fire rate. Periodicity should not be associated with return period; for example, it is entirely possible that a fire rate with a return period of ten years could occur in two consecutive years.

The conditions assumed in any frequency analysis were discussed in Methods. The first assumption requiring that the events be random was evaluated by determining the autocorrelation coefficients for the time series. These coefficients are a measure of the correlation between successive observations. For example, a year with a low fire rate may tend to be followed by another low fire rate. Such persistence reduces the random component in the process. The distance in time between observations is referred to as lag. So the autocorrelation coefficient for lag 10 measures the correlation between events 10 years apart. If the time

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' return period is the reciprocal of probability series is large and completely random then the correlation coefficients should be approximately zero. But even if the series is truly random, a correlation coefficient that is significant is not unusual (Chatfield 1975).

The statistical package SPSS^{x} which uses the Box-Jenkins procedure (Box and Jenkins 1976) was used to determine the correlogram' for the fire data time series. The results are shown in Figure 6.2. All of the correlation coefficients except that for lag 13 were non-significant as indicated by the standard error limits. The coefficient for lag 13 is caused by the high fire rates occurring in 1968

' a plot of the correlations coefficients for various lags

AUTOCORRELATION FUNCTION FOR VARIABLE SIZE
AUTOCORRELATIONS *
TWO STANDARD ERROR LIMITS .

LAG	AUTO. CORR.	STAND. ERR.
1	-0.046	0.188
2	-0.107	0.183
3	-0.056	0.179
4	-0.043	0.174
5	-0.056	0.170
6	-0.060	0.165
7	-0.027	0.160
8	-0.037	0.155
9	-0.036	0.150
10	-0.047	0.144
11	-0.050	0.139
12	-0.049	0.133
13	0.391	0.127
14	-0.040	0.120
15	-0.070	0.113
16	-0.044	0.106
17	-0.045	0.098
18	-0.049	0.090
19	-0.050	0.080
20	0.016	0.069
21	0.005	0.057
22	-0.000	0.040

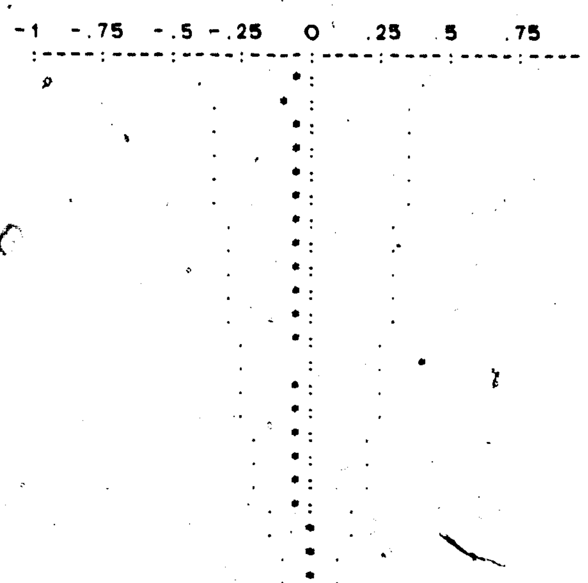

Figure 6.2: Correlogram for Fire Data Time Series

and 1981 (a lag of 13 years). Any other time span between between the two events would also result in a high coefficient for that particular lag. Based only on two events, it would be inappropriate to conclude that fires rates for any given year are related to the rate occurring 13 years ago. Anomalies such as this are not sufficient evidence to disprove that the process is random (Chatfield 1975).

The second assumption that frequency analysis implies is stationarity. If a time series contains a trend then the value for the correlation coefficient will not come down to zero except for very large lags. This is because an observation on one side of the overall mean tends to be followed by a large number of further observations on the same side of the mean. Figure 6.2 indicates no such pattern. Stationarity was also subjectively evaluated by comparing the frequency curve for 1961-1970 with that obtained from the 1971-1984 time series. Figure 6.3 illustrates the differences. Considering the very short record lengths used, the frequency curves are remarkably similar. Therefore, stationarity can be reasonably assumed even though fire protection efforts and fire incidence have not remained constant during the period of record.

The age class distribution shown in Figure 5.2 indicates the effect of variability in fire season severity. In the past, extremely severe fire years have occurred, but it cannot be concluded, on the basis of the age class

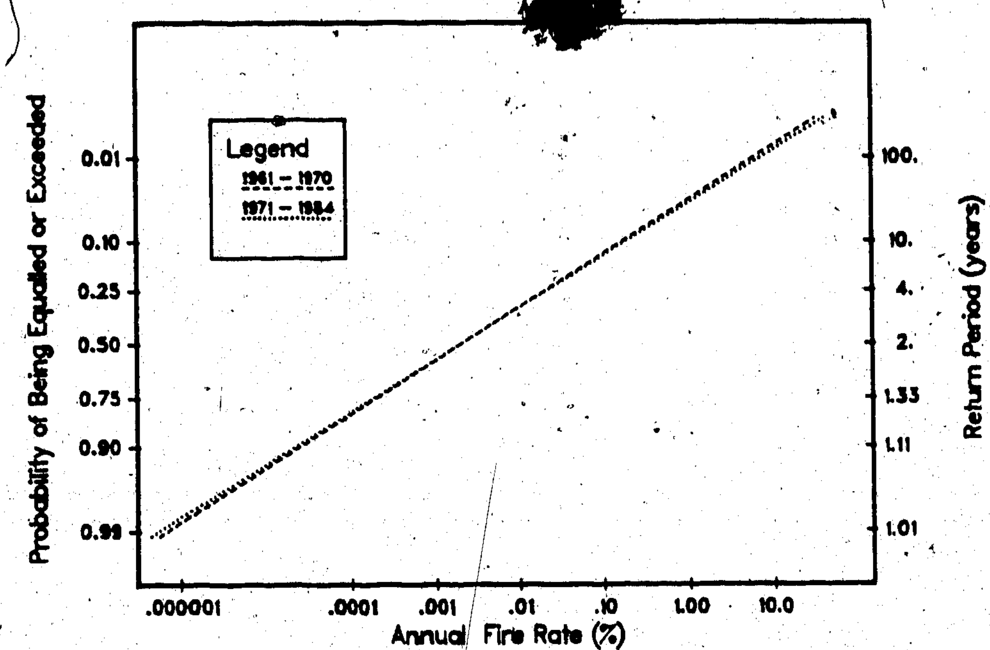

Figure 6.3: Fire Frequencies for 1961-70 and 1971-84

distribution, that such rates represent a different frequency distribution than that shown in Figure 6.1.

The third assumption made in any frequency analysis is that the available data can provide good estimates of the population statistics. In hydrology, record lengths shorter than 20 years have been found to cause considerable error in parameter estimation (Benson 1960, Linsley *et al*. 1975), but often there are no alternatives. The 24 years of data used in this study is probably acceptable, but development of regional frequency curves rather that local curves would improve the accuracy of the parameter estimates.

Regional curves are developed by assembling data from several areas which are suspected of having similar fire frequency. The use of frequency analysis in fire management is new, but several techniques have been developed in hydrology for the construction of regional frequency curves (Kite 1977, Benson 1960, Linsley *et al*. 1975). These could undoubtedly be adapted for use with fire data.

The development of fire frequency curves for this study has indicated the possibility and need of further research. First, fire frequency curves can be a powerful tool in succinctly analysing regional difference in fire behavior. Frequency curves avoid some of the shortcomings common to other statistics. Arithmetic averages when used on data with large variability are often misleading; for example in S1 only 3 annual fire rates exceeded the average during 24 years of record. Some preliminary work has shown that frequency analysis could also be used in the development of the relationship between fire size and frequency. Both types of frequency analysis should be of use in fire management.

The need for a thorough time series analysis of fire records was alluded to earlier. Time series analysis is capable of detecting both trends and cycles even if there is extreme variability in the time series, as is the case for most fire records. Evaluation of the effectiveness of fire protection might be possible using these techniques. Trend can be also be removed from a time series, perhaps allowing longer fire records to be used in frequency analysis. Methods are also available for examining point processes such as the frequency of severe fire years or the frequency of conflagrations (Cox and Lewis 1966). Time series analysis is a powerful tool, yet to be used in the examination of fire data.

6.2 Traditional Even-flow Model

The product of the traditional model was 1200 AACs each of which was sustainable for the liquidation period, given the unique sequence of fire rates which occurred during the period. The AACs were separated into classes and a cumulative form of the Weibull distribution was fit to the data[1]. The coefficient of determination for the relationship was 0.991.

Using the Weibull relationship the probability density curve shown in Figure 6.4 was constructed. The maximum AAC shown is slightly less than 395,000 m^3 per year which is the sustainable cut rate with no fire. The pronounced peak at approximately 388,000 m^3 per year indicates the most probable occurrence; fire rates causing this peak would be relatively mild and thus AAC would be reduced only a small amount. Lower AACs reflect relatively more severe fire sequences.

One run of the even-flow simulation model was done with annual fire rate held constant at the mean of 0.15%. The sustainable AAC under this fire scenario was 370,000 m^3 per year, as seen in Figure 6.4. The average fire rate is commonly used to model the effect of fire on timber supply

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[1] The non-linear regression used the package NREG, which is supported by the Department of Forest Science.

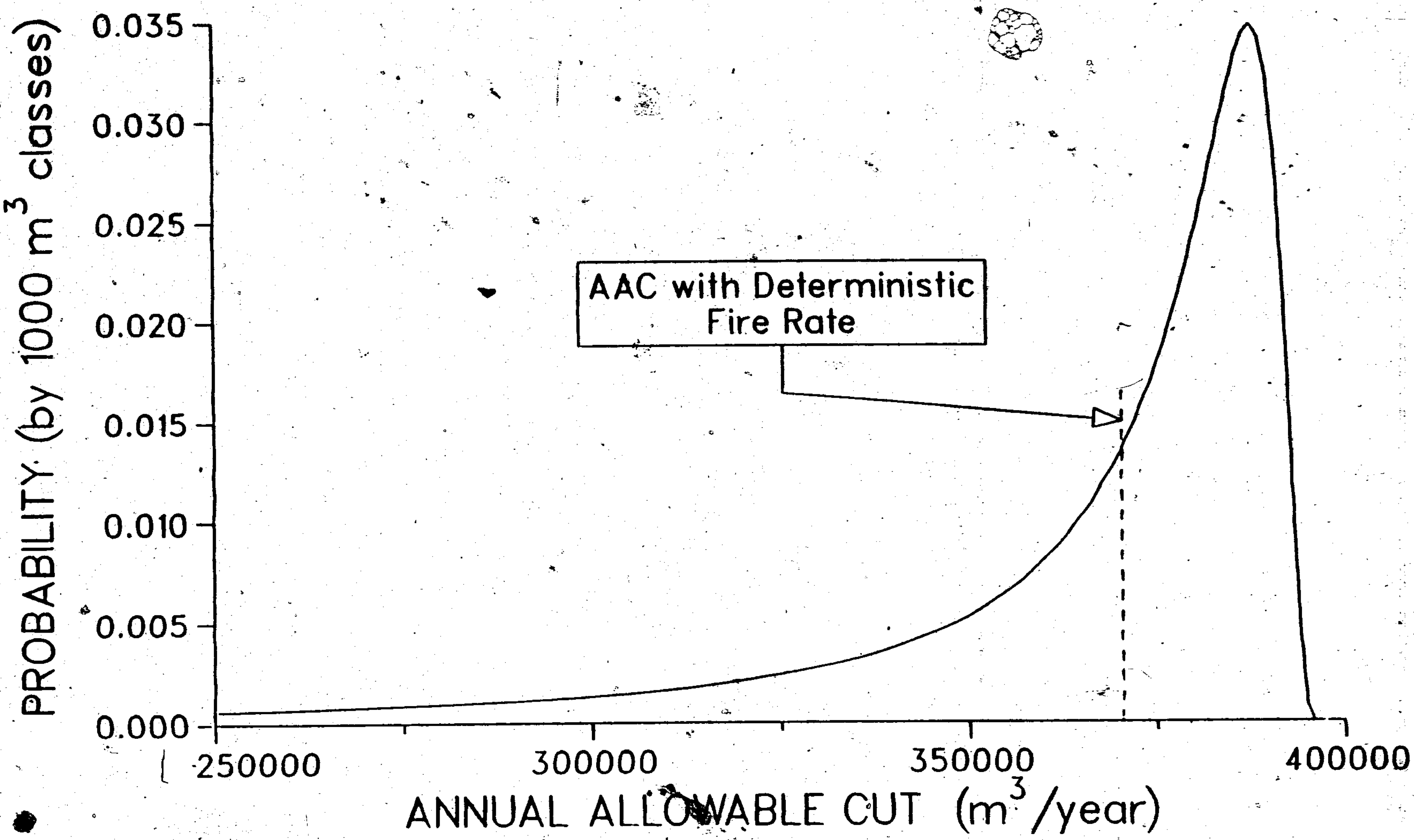


Figure 6.4: Distribution of Possible Even-flow AACs

(Van Wagner 1983, Reed and Errico 1986). The rationale is that the effect of the average fire rate should approximate the effect produced by stochastic rates.

Clearly, use of the deterministic rate gives no indication of the variation in possible AACs but it may not even represent the average AAC. The expected or average AAC obtained from the distribution of possible AACs is 347,150 m^3 per year. The actual average obtained is lower than 370,000 because of two reasons. First, use of the mean fire rate implies that the distribution of possible fire rates is symmetrically distributed about the mean; whereas, the natural logarithms of the fire rates were found to be normally distributed. The second reason is that the upper limit to the range of possible AACs is restricted by the even-flow AAC without the presence of fire. For example, suppose that the fire rates were symmetrically distributed and the AAC obtained from using the deterministic fire rate was 350,000 m^3. Then the median AAC would equal 350,000 m^3 as well. Thus 50% of the AACs would fall between 0 and 350,000 m^3; and the other 50% would fall between 350,000 and 395,000 m^3. Because the upper limit is more restricted, the arithmetic mean would not necessarily equal 350,000 m^3 per year. Therefore, the use of the mean fire rate is inappropriate for determining the average effect of fire on AAC.

Use of the relationship shown [illegible]igure 6.4 for setting current AAC is problematic. Every AAC has a probability of being sustained over the liquidation period. For example, if 375,500 m^3 per year (the median) were selected, there is a 50% chance of being able to sustain that cut over the 90 year liquidation period. Lower cut rates would have greater probabilities of being sustained but increased probability of underestimating the potential harvest. There is no logical basis for selecting an optimal AAC using Figure 6.4. Determination of the consequences of not being able to sustain a particular cut rate is not possible with this type of analysis. If a management agency decided that an AAC must have a certain probability of being sustained, then the decision is an arbitrary one, and not necessarily optimal.

A large part of the problem in using this procedure is intrinsic to all models which use traditional even-flow planning. The planning represented by such models does not coincide with the way planning actually occurs. In reality, AAC is recalculated at periodic intervals. AAC will not remain constant over the entire planning horizon because of changes in policy, growth predictions and growing stock. Some of the changes in harvest level are predictable; others are not. Changes in harvest rate over time are not accurately assessed by traditional even-flow planning. Predictable changes, such as those caused by harvesting excess growing stock, will occur gradually as replanting occurs, not solely at the end of the planning horizon.

The use of even-flow models for determining optimal harvest rates under the risk of forest fire is extremely questionable. The models completely ignore the manner in which all uncertainties in management are ultimately handled, namely through revised planning. It is widely acknowledged that even-flow will not occur, especially under the risk of destruction by fire. However, even-flow models restrict the choice of "optimal" harvest rate to one which can be maintained throughout the planning horizon, ignoring replanning. There is no rational basis for restricting the selection of optimal harvest rates in this manner.

6.3 Dynamic Programming Model

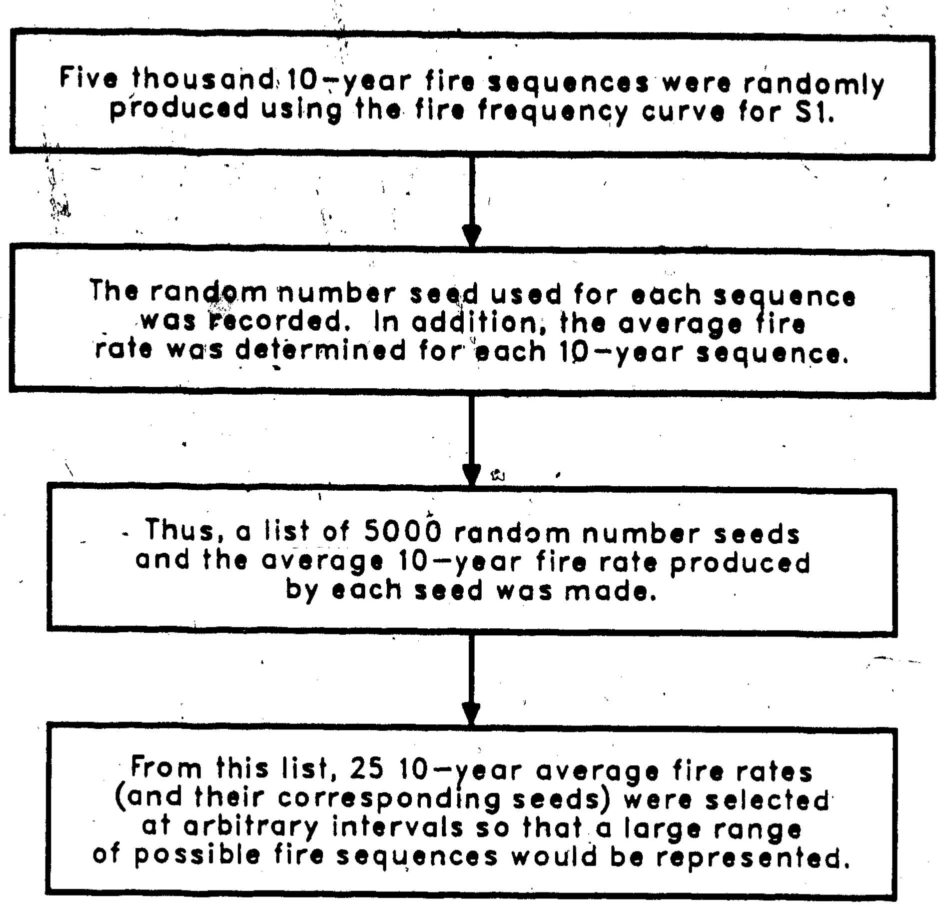
6.3.1 One-step Transition Probabilities

Ten different hypothetical age class distributions were created representing a broad range of initial AACs. Several rates of cut (3 to 5) were simulated on each age class distribution. For every cut rate a broad range of possible fire sequences was simulated. Table 6.2 indicates the initial AAC derived from the ten hypothetical age class distributions and the cut rates that were simulated on each. Determination of the fire sequences was accomplished in the manner shown in Figure 6.5. Five thousand 10-year fire sequences were generated from the fire frequency curve for the study area. The fire rates were averaged over each 10 year sequence and recorded. The random seed used to generate each 10 year sequence was also recorded. Twenty five random seeds were selected so that a broad range of possible fire

Table 6.2: SIMULATION RUNS

Initial AAC (m^3/year)	Cut Rates (m^3/year)	Range of 10-year Ave. Fire Rates Simulated (%)	No. of 10-year Fire Sequences Simulated
102720	0	0.0045 to 7.5	25
	10000	0.0045 to 6.1	24
	100000	0.0045 to 2.4	18
195462	0	0.0045 to 7.5	25
	100000	0.0045 to 6.1	24
	200000	0.0045 to 3.8	21
228219	0	0.0045 to 7.5	25
	100000	0.0045 to 7.5	25
	200000	0.0045 to 4.6	22
	300000	0.0045 to 3.8	21
265540	0	0.0045 to 7.5	25
	100000	0.0045 to 7.5	25
	200000	0.0045 to 7.5	25
	300000	0.0045 to 6.0	23
288809	0	0.0045 to 7.5	25
	100000	0.0045 to 7.5	25
	200000	0.0045 to 7.5	25
	300000	0.0045 to 6.1	24
350580	10000	0.0045 to 7.5	25
	100000	0.0045 to 7.5	25
	200000	0.0045 to 7.5	25
	300000	0.0045 to 7.5	25
	400000	0.0045 to 7.5	25
359286	10000	0.0045 to 7.5	25
	100000	0.0045 to 7.5	25
	200000	0.0045 to 7.5	25
	300000	0.0045 to 7.5	25
	400000	0.0045 to 7.5	25
378130	10000	0.0045 to 7.5	25
	100000	0.0045 to 7.5	25
	200000	0.0045 to 7.5	25
	300000	0.0045 to 7.5	25
	400000	0.0045 to 7.5	25
395765	10000	0.0045 to 7.5	25
	100000	0.0045 to 7.5	25
	200000	0.0045 to 7.5	25
	300000	0.0045 to 7.5	25
	400000	0.0045 to 7.5	25
403868	10000	0.0045 to 7.5	25
	100000	0.0045 to 7.5	25
	200000	0.0045 to 7.5	25
	300000	0.0045 to 7.5	25
	400000	0.0045 to 7.5	25

Figure 6.5: Determination of the Fire Rates for Use in the Simulation Model



sequences would be simulated. Thus, for every initial AAC and cut level combination, 25 simulations were done. Each simulation had a different 10-year fire sequence.

Problems in determining the resultant AAC occurred with some age class distributions especially when high fire rates were simulated. Age class distributions that are heavily skewed to the zero age class cause the simulation model problems in approximating AAC. Very small differences in cut rate can cause large changes in the time required for cutting. Therefore, the range of fire rates simulated is smaller on age class distributions with low initial AACs (refer to Table 6.2).

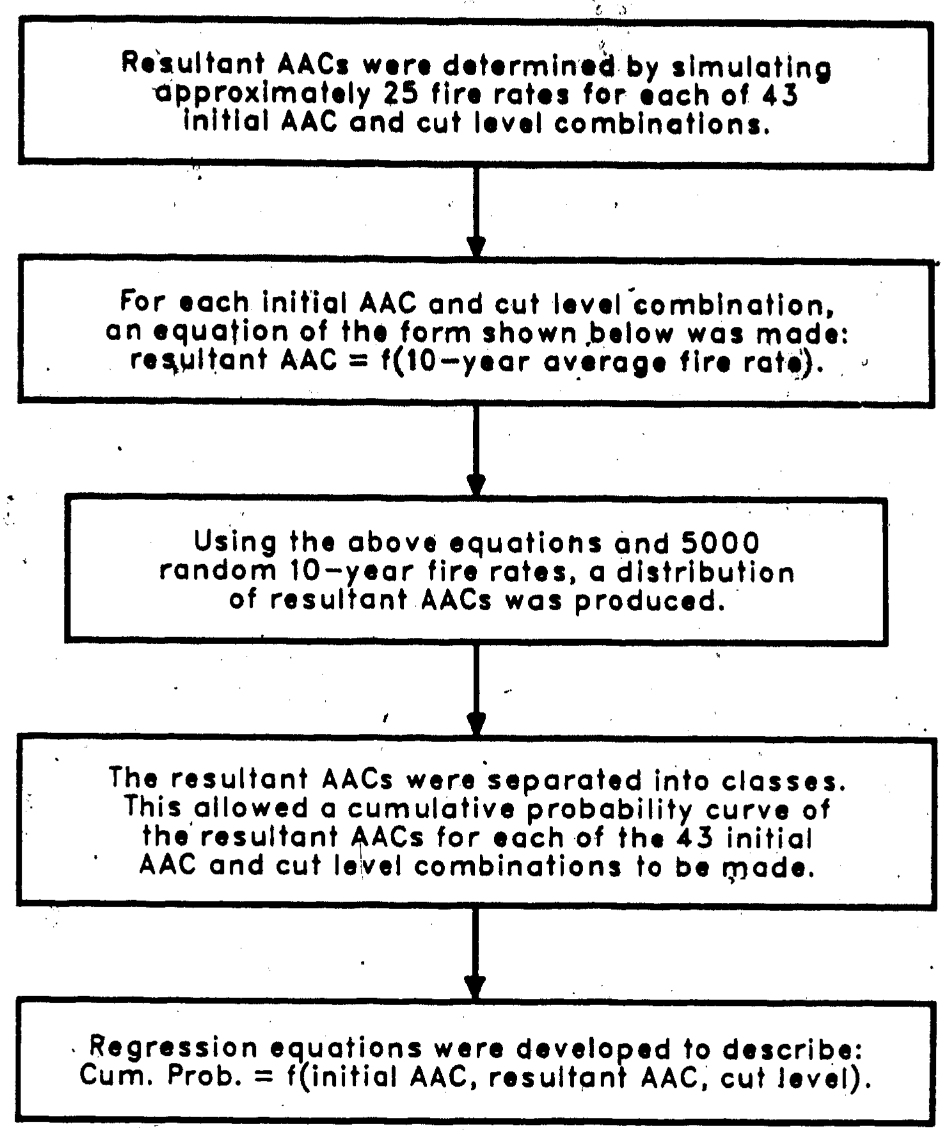
Based on the simulations, probabilities were derived. The procedure is somewhat complicated; the flow-chart in Figure 6.6 indicates the major steps. For each age class distribution and cut level combination, a linear regression was done in the form:

$$\text{Resultant AAC} = b_0 + b_1(\text{10-year fire rate}) \qquad (1)$$

Twenty five pairs of data were used for each equation, except where the range of fire rates being simulated was restricted. In total 43 such equations were obtained. The average coefficient of determination for these 43 equations was 0.990. These equations were used to determine the probability of occurrence of any resultant AAC.

As mentioned previously, 5000 10-year fire sequences were randomly generated from the fire frequency curve for the study area. An average rate for each 10-year period had been determined. These fire rates were used as the independent variable in the regression equations. The result was that 5000 potential resultant AACs were produced for each of the 43 age class distribution and cut level combinations. Each AAC represented a possible age class distribution after 10 years of cutting and burning. Separation of the 5000 possible AACs into classes meant that

Figure 6.6: Estimation of Transition Probabilities



a probability density curve of the resulting AACs could be made.

Table 6.3 shows a simplified example of how the distributions might appear. Regression equations were derived to represent the cumulative form of these curves for

Table 6.3: PROBABILITY MATRIX

Initial AAC	Cut Level	Resultant AAC 200000	250000	300000	350000
300,000	200,000	.01	.02	.07	.90
	300,000	.03	.07	.15	.75
350,000	300,000	.02	.03	.09	.86
	350,000	.05	.08	.18	.69
400,000	300,000	.01	.02	.09	.88
	400,000	.10	.13	.20	.57

use in the DP algorithm. The equations had to represent a four dimensional relationship in the form:

Cumulative Probability = f(initial AAC, resultant AAC, cut level)

As mentioned previously, three fire frequency curves were evaluated. The three frequency curves used are parallel curves. The average fire rate is changed but not the variance. Since the effect of any particular fire rate remains the same, it was not necessary to redo the simulation component. But altering the fire frequency will obviously change how often a given fire rate can be expected to occur. Therefore, it was necessary to generate another 5000 10-year fire sequences based on the altered frequency curve and then go through the regression procedures again.

After many trials, the regression procedures below were chosen:

1. For each age class distribution and cut level combination, a 3 parameter Weibull equation was obtained such that:

$$\text{Cum. Prob.} = 1 - \exp[-((P1-AAC)/P2)^{P3}] \qquad (2)$$

where: AAC = resultant AAC
Cum. Prob. = cumulative probability
Pi = parameter i

This equation estimates the cumulative probability of obtaining the resultant AAC. Parameter P1 is the upper limit for the x-axis. Parameter P2 is referred to as the scale parameter and P3 is the shape parameter (Hastings and Peacock 1974). The Weibull function is very flexible and different sets of parameter values may produce the same cumulative function. The first parameter (P1) in this equation was determined prior to running the non-linear regression package. Parameter P1 is the highest possible resultant AAC for a given age class distribution and cut level combination. Therefore, it is the AAC that occurs when the average fire rate for the 10 year period is 0.0%. This means that parameter b_0 from equation (1) equals parameter P1. A multiple linear regression was used to estimate P1 as a function of initial AAC and cut level. This equation is used for all three fire frequencies that were simulated. The adjusted correlation coefficient was 0.993.

2. The first series of non-linear regressions used equation 2, with a known P1, to determine 43 values for parameter P2 and parameter P3. Attempts were made at estimating P2 and P3 from initial AAC, resultant AAC and cut level but the regressions were poor. This was because changes in either P2 or P3, for the particular data sets used, produced similar changes in function shape. It was therefore necessary to hold one parameter (either P2 or P3) constant. The choice of which parameter to hold constant was not critical since the parameter allowed to vary would compensate for the one held constant.

3. The 43 values derived for parameter P2 (from above) were averaged. The average value was then used as a constant and every age class distribution and cut level combination was rerun in the same form as above. With this series of runs, only values of P3 were produced since P1 and P2 were known. The average of the 43 coefficients of determination (r^2) for each of the fire frequencies is listed below:

Average Fire Rate (%)	r^2
0.10	0.986
0.15	0.992
0.20	0.994

4. A multiple linear regression to estimate the values for P3, which were derived above, was obtained such that:

P3 = f(initial AAC, cut level)

The average adjusted coefficients of determination are:

Average Fire Rate (%)	Adjusted r^2
0.10	0.615
0.15	0.694
0.20	0.761

Thus a series of three regression equations must be used to obtain the cumulative probability for use in the dynamic programming algorithm. One for parameter P1, another for parameter P3 and finally the Weibull equation estimates the cumulative probability. Parameter P2 is held constant.

The AAC variable indicated in the Weibull function has a range from $-\infty$ to the value of P1. Suppose that the current state of a forest is at 200,000 m^3 per year and that any annual fire rate greater than 40% in the next 10 years would reduce the state of the forest to 0 m^3 per year. The total probability of reaching state 0 would be slightly greater than the probability of reaching a state just above zero because more fire rates could cause a transition to state 0. This increased probability of reaching state zero is the cumulative probability from states $-\infty$ to zero. It was felt that the probability of reaching state zero was slightly overestimated because of the lower range of the Weibull function being $-\infty$. As a compromise the cumulative probability from state -10,000 to 0 was set equal to the probability of reaching state 0. The cumulative probability from $-\infty$ to -10,000 was then distributed equally amongst all other state transition probabilities. The effect of this modification was small and in retrospect, may not have been necessary.

6.3.2 Model Results and Discussion

For discussion purposes, three different variables determined by every DP run are considered:

1. Optimal cut level: The optimal cut level is the decision which produces the highest expected reward over 450 years given the current state of the system. The optimal cut is the harvest rate for the first period only. Optimal harvest rates for subsequent periods depend on the state of the forest at the beginning of each period.
2. Present value (PV) of harvests: This variable is the PV of the expected future harvest over the next 450 years given the current state of the system. It is obtained by following the optimal cut levels at each stage of the problem and is one portion of the objective function. PV of harvests is one measure of long-term timber supply.
3. Present value of falldowns: PV of falldowns is the PV of the expected decreases in harvest over 450 years given the current state of the system. It is the other component of the objective function.

The present value of falldowns and harvests could have been determined in several ways as indicated below:

1. PV of harvests is calculated as the stream of annual harvest over 450 years. The impact of a falldown is assumed to occur only in one year of the 10-year period.

$$\text{PV of harvests} = \sum_{n=1}^{45} \left[\frac{(1+i)^{10}-1}{(1+i)^{10}i} \right] \left[\frac{1}{(1+i)^{10}} \right]^{n-1} HR_n$$

$$\text{PV of falldowns} = \sum_{n=1}^{45} \left[\frac{1}{(1+i)^{10}} \right]^{n-1} F_n$$

HR_n = annual harvest rate for period n.

F_n = falldown occurring at end of period n.

2. PV of harvests is calculated as in 1 above, but the impact of a falldown is assumed to occur every year in the period affected. Both PV of harvest and PV of falldowns are, therefore, determined on a yearly basis.

$$\text{PV of harvests} = \sum_{n=1}^{45} \left[\frac{(1+i)^{10}-1}{(1+i)^{10}i} \right] \left[\frac{1}{(1+i)^{10}} \right]^{n-1} HR_n$$

$$\text{PV of falldowns} = \sum_{n=1}^{45} \left[\frac{(1+i)^{10}-1}{(1+i)^{10}i} \right] \left[\frac{1}{(1+i)^{10}} \right]^{n-1} F_n$$

3. The same relative proportion between PV of harvests and PV of falldowns found in 2 above can be retained by eliminating the first term in each equation. This produces the equations below. Thus, both PV of harvests and PV of falldowns are determined on a period basis.

$$\text{PV of harvests} = \sum_{n=1}^{45} \left[\frac{1}{(1+i)^{10}} \right]^{n-1} HR_n$$

$$\text{PV of falldowns} = \sum_{n=1}^{45} \left[\frac{1}{(1+i)^{10}} \right]^{n-1} F_n$$

There is no single correct method to use; any of the viewpoints are valid. The third method was chosen because of its simplicity.

In addition to the 3 variables described above, it is possible to determine the steady-state AAC and the steady-state falldown. The steady-state quantities are the long-term averages that can be expected after a large number of transitions have occurred. Steady-state does not imply that the process settles down to one state; transitions from state to state will still occur but the long-term average will equal the steady-state quantity. The steady-state condition is independent of initial state.

The steady-state AAC is calculated by subtracting the accumulated harvest at stage 44 from the accumulated harvest at stage 45. The accumulated harvest at stages 44 and 45 are the averages of all possible harvests up to each particular point in time. They reflect all possible fire occurrences and their associated probabilities of occurrence. Steady state fluctuation was determined similarly. This procedure assumes that steady-state conditions have been reached by stage 44. Theoretically, the steady-s[illegible]es for all initial states are equal. However, [illegible]ount of fluctuation was found, probably at[illegible] to the discrete nature of the prob[illegible]e steady-state values reported are the medians of th[illegible]-state values found for 10 initial states. The steady[illegible]e quantities were only determined for selected runs.

6.3.2.1 Effect of Cut Level Reduction Cost

To illustrate the eff[illegible]t that cut level reduction cost has on optimal harvest policies, the results from the runs using an interest rate of 5% and a fire rate of 0.15% will be discussed. Figure 6.7 shows that as cut level reduction cost increases, optimal cut levels are decreased. Reducing the cut produces less severe decreases in flow (Figure 6.8), because the harvest rates are closer to the steady-state conditions and because higher levels of growing stock are maintained to buffer fluctuations. However, the increased emphasis on avoiding falldowns causes a reduction in PV of harvests (Figure 6.9). PV of harvest has been forsaken for lower PV of falldowns.

PV of expected falldowns will always be reduced to the point where further reductions would exceed the

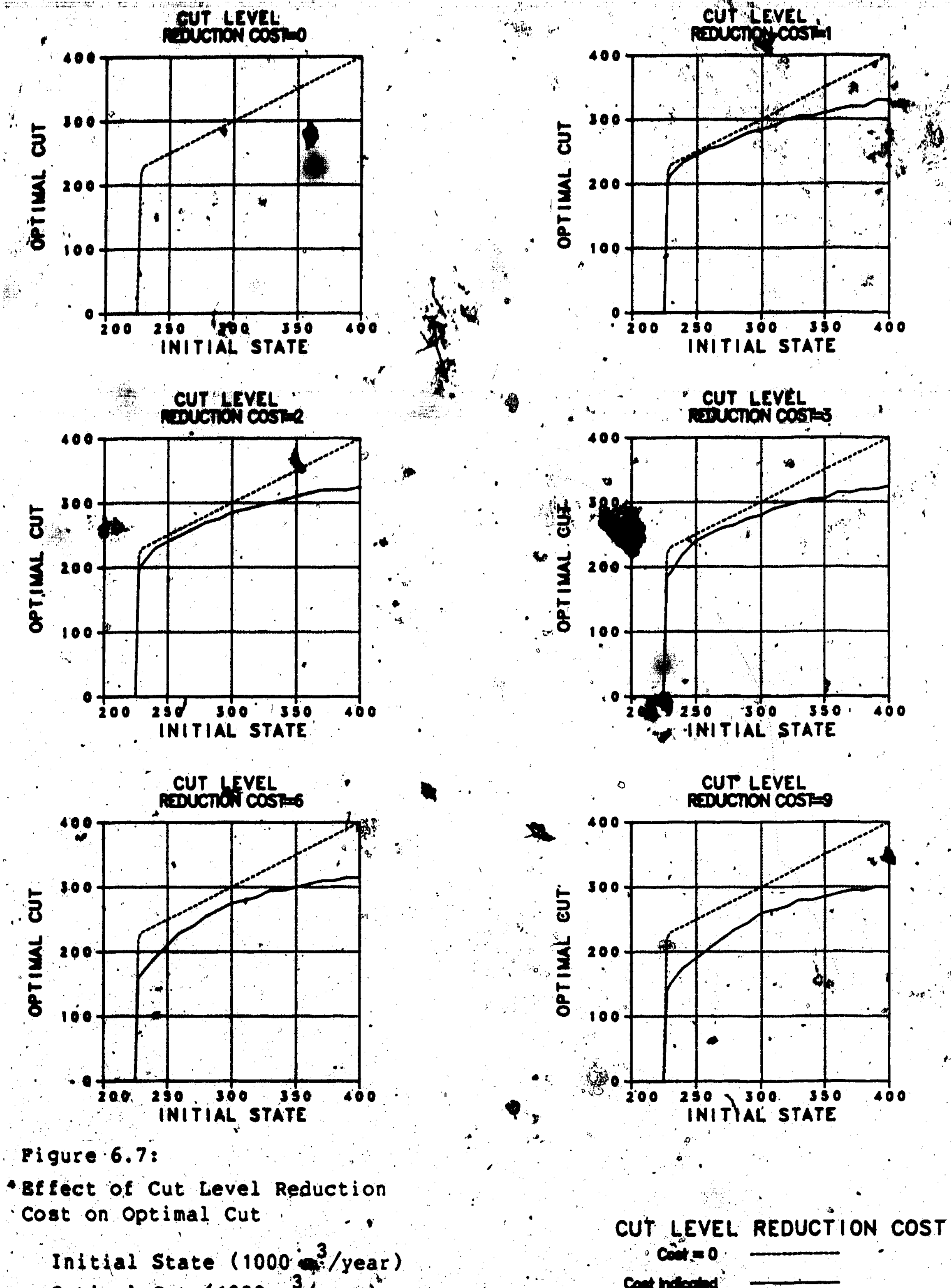


Figure 6.7:
Effect of Cut Level Reduction Cost on Optimal Cut

Initial State (1000 m^3/year)
Optimal Cut (1000 m^3/year)

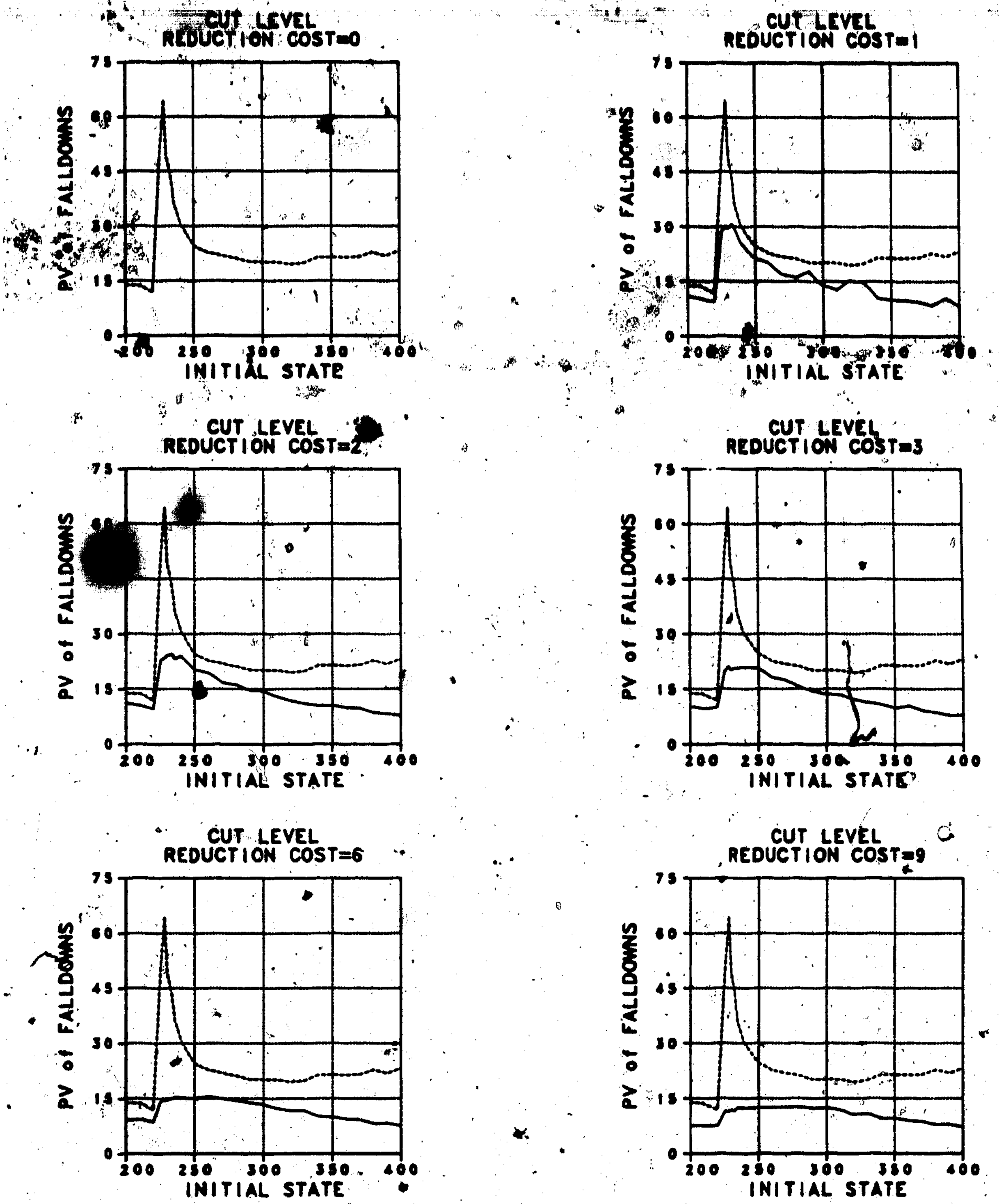
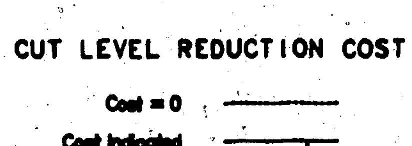


Figure 6.8:

Effect of Cut Level Reduction Cost on PV of Falldowns

Initial State (1000 m^3/year)

PV of Falldowns (1000 m^3)



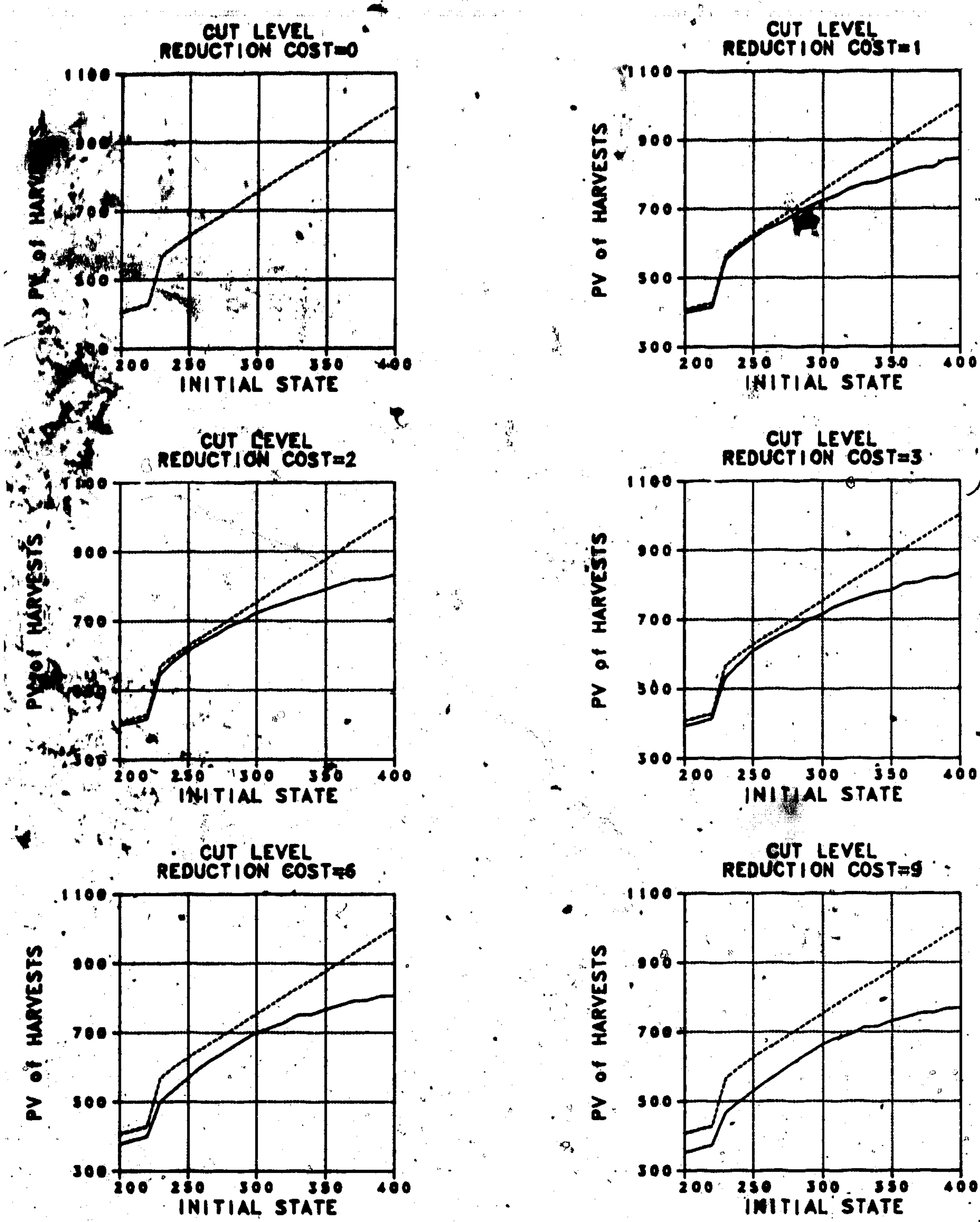
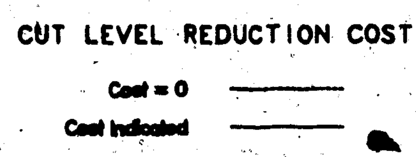


Figure 6.9:
Effect of Cut Level Reduction Cost on PV of Harvests

Initial State (1000 m^3/year)
PV of Harvests (1000 m^3)



benefit gained. This point is where the value associated with smoothness of flow plus the value of the harvest over the planning horizon is maximized.

Referring to Figure 6.7, it is apparent that the reduction in optimal cut levels is not proportional. The cut levels for high states were reduced substantially. In addition, states slightly above state 225,000 m^3 per year received relative greater reductions in harvest level. Reduction of optimal cut is based on the unit cost of reducing PV of falldowns. Cost is measured in terms of PV of harvest. The variable unit costs of reducing PV of falldowns are based on two principles:

1. For falldowns of equal magnitude, the higher the probability of occurrence, the lower the unit cost.
2. For falldowns of equal probability, the higher the magnitude, the lower the unit cost is to avoid them.

The relatively low cost of reducing PV of falldown in high initial states is caused by non-linear growth rates. The lower the state, the higher the growth rate. For example, a forest with an even-flow AAC of 300,000 m^3 might become a forest with an AAC of 320,000 m^3 in 10 years if no harvesting occurred. Whereas, a forest of state 380,000 m^3 per year might only increase to state 385,000 m^3 per year. The non-linear growth rate causes the effect of fire and harvesting to vary depending on the initial state. Generally for a given fire rate and harvest rate, the probability of a transition to a lower state will increase as the initial state increases. Thus, the relative cost of reducing PV of falldowns is low for high initial states.

The disproportionate reduction in optimal cut around state 225,000 m^3 per year is caused by the restriction of harvest level to zero for states less than 225,000 m^3 per year. The constraint causes the magnitude of the expected falldown, produced by transitions to lower states, to [illegible]high. The high magnitude of the expected falld[illegible]akes reduction of PV of falldown cost-effective.

The shape of the falldown curve in Figure 6.8, when the cut level reduction cost is zero, reflects the influence of the constraint at 225,000 m^3 per year and the non-linear growth rate. A slight trend to higher falldowns at high state levels indicates that the slow growth rate cannot sustain the cut. The increase in PV of falldown as the state approaches 225,000 m^3 per year reflects the increasing expected value because of the harvest rate being reduced to zero at state 225,000 m^3 per year.

When the reduction cost exceeds zero, the shape of the falldown curve is largely affected by the proximity to state 225,000 m^3 per year . The influence of the non-linear growth rate appears to be removed. When the falldown at state 225,000 m^3 per year is large', the

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' which happens when the reduction cost is low slope of the line (to state 225,000) is higher. If there were no constraint at state 225,000 m^3 per year and the cut level reduction cost exceeded zero, the curve of PV of falldowns should be a horizontal line indicative of the equal probabilities of destruction by fire for all states.

The optimal harvest rates when some emphasis is placed on smoothness of flow has strong implications for the harvesting of excess growing stock. Even when the social cost of a falldown is small, the harvesting of excess growing stock is not optimal. Falldowns caused by the harvesting of excess growing stock are certain to occur. Their probability of occurrence is one, whereas the probability of a falldown created by fire will always be less than one. Not harvesting excess growing stock has two benefits:

1. The certain falldowns are avoided.
2. The excess growing stock is useful in buffering fluctuations caused by fire.

The first benefit is the primary cause of the reduction in optimal harvest level when excess growing stock is present; the second benefit is largely a by-product. If the second benefit was a primary cause, then substantial reductions in optimal harvest level would occur for all states because surplus growing stock is useful in reducing transition probabilities for all initial states. Intuitively, the model's result is logical. If smoothness of flow is important, it makes sense to avoid falldowns which will be certain to occur before consideration is given to falldowns which are unpredictable.

6.3.2.2 Effect of Fire Frequency

When no value is placed on smoothness of flow, optimal harvest rate does not vary with fire frequency[1] (Figure 6.10). But higher fire frequencies do produce higher levels of PV of falldowns (Figure 6.11) and lower levels of PV of harvests[2] (Figure 6.12). As the social cost of falldowns increases, it becomes cost-effective to reduce the higher PV of falldowns caused by high fire frequencies by reducing optimal cut rates. This occurs at the expense of lower PV of harvests.

As cut level reduction cost increases, the change in optimal cut occurs first at state levels just above 225,000 m^3 per year. This is because the magnitude of the differences in PV of falldown is greatest at state 225,000 m^3 per year.

The number of cut levels in the model is discrete; divisions of 5000 m^3 are used. If more cut levels were present, small differences in optimal cut rates would be

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[1] In this study the upper limit to harvest rates constrains optimality. Without this constraint, higher harvest rates are anticipated with high fire frequencies (Reed and Errico 1985).

[2] PV of harvests of inversely proportional to fire frequency but the differences are small at low cut level reduction costs and therefore may not be apparent in Figure 6.12.

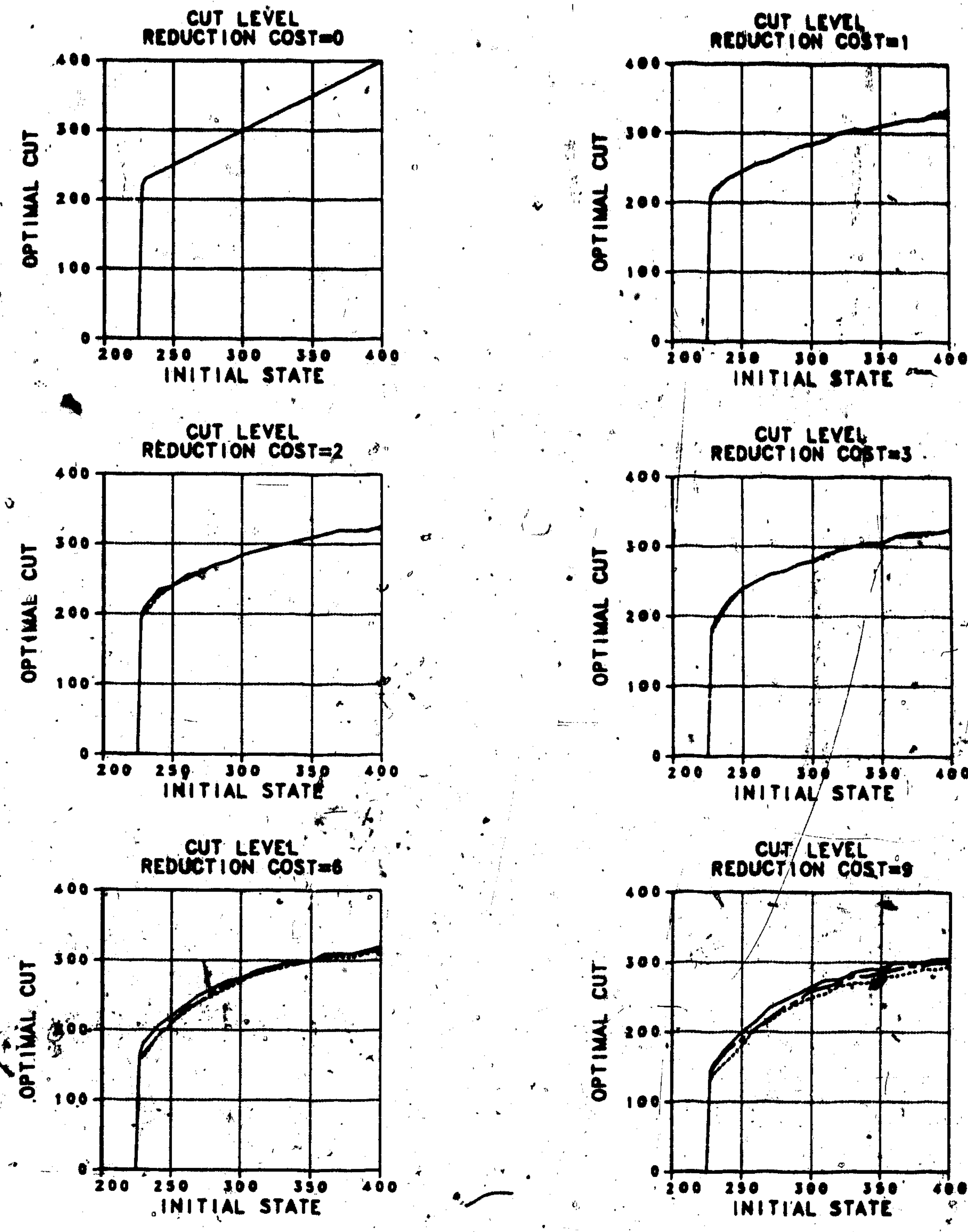

Figure 6.10: Effect of Fire Frequency on Optimal Cut

Initial State (1000 m^3/year)
Optimal Cut (1000 m^3/year)

AVERAGE FIRE RATE

0.10 %
0.15 %
0.20 %

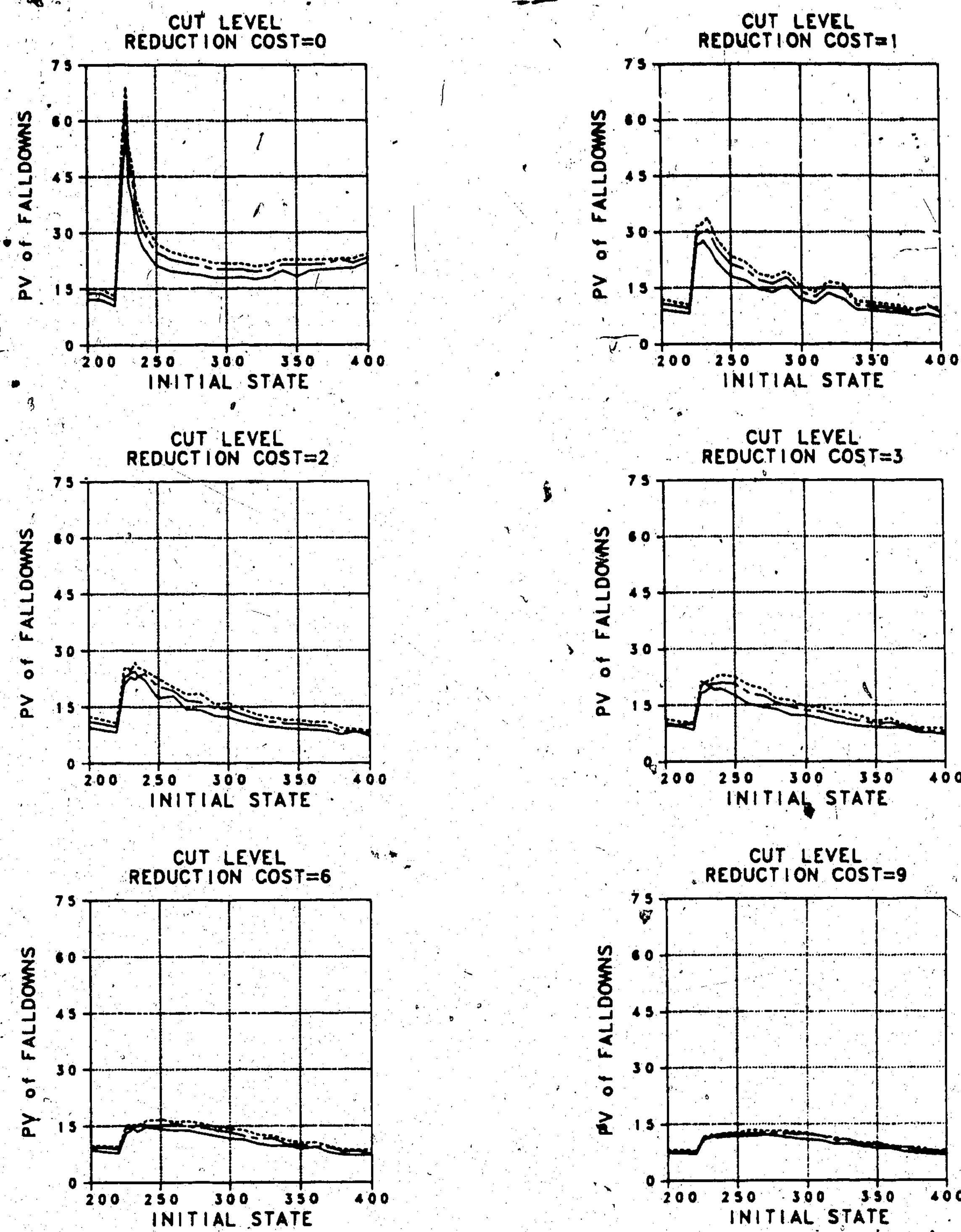

Figure 6.11: Effect of Fire Frequency on PV of Falldowns

Initial State (1000 m^3/year)
PV of Falldowns (1000 m^3)

AVERAGE FIRE RATE

0.10 % ————
0.15 % —·—·—
0.20 % ··········

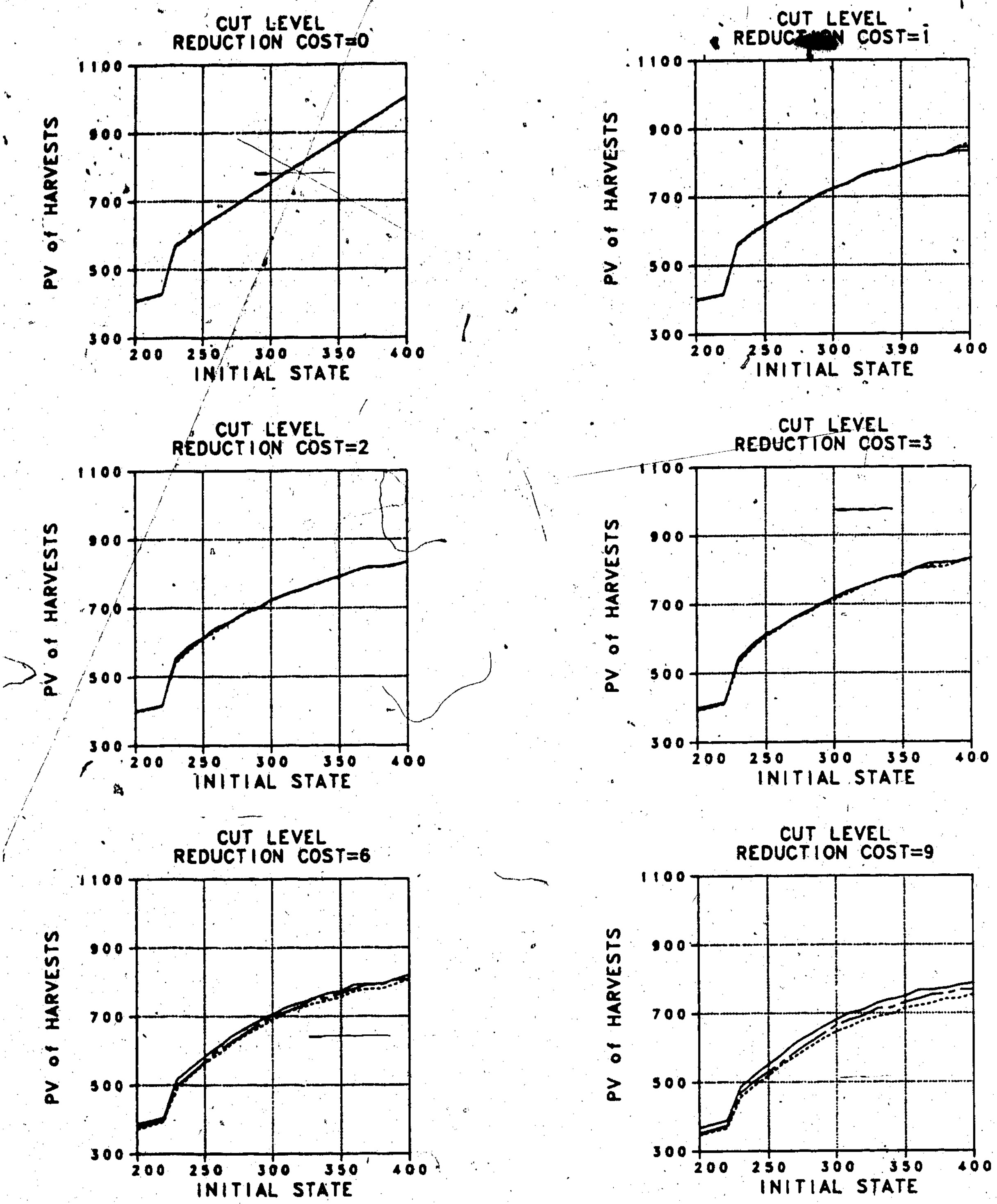

Figure 6.12: Effect of Fire Frequency on PV of Harvests

Initial State (1000 m^3/year)

PV of Harvests (1000 m^3)

AVERAGE FIRE RATE

0.10 % ————

0.15 % —·—·—

0.20 % ·········

expected to occur when the cut level reduction cost is small. In fact, some fluctuation in optimal cut levels can be seen at low reduction costs, indicating some tendency to lower optimal cut.

6.3.2.3 Effect of Interest Rate

No Interest Rate:

As indicated in Table 4.1, one run was done to illustrate the effect of no time-preference. The cut level change cost was zero, average fire rate was 0.15% and the discount factor was 1.0. Therefore the objective was simply to maximize total harvest over 450 years. Figure 6.13 shows the optimal cut levels. For any state less than 340,000 m^3 per year, the optimal decision is not to cut, even though there would be harvestable timber in many of these states. This means that more timber can be harvested over 450 years, by not harvesting in relatively low states; but rather, letting the forest grow until it reaches a state greater than 340,000 m^3 per year and then harvesting. This somewhat surprising result is caused by the previously discussed non-linear growth rate.

The policy indicated in Figure 6.13 is not particularly realistic. A regular annual or periodic harvest is the basis of "sustained yield" which is a policy widely adopted in public forestry (Duerr *et al.* 1977). Traditional harvest scheduling models assure this policy by imposing various forms of flow constraints.

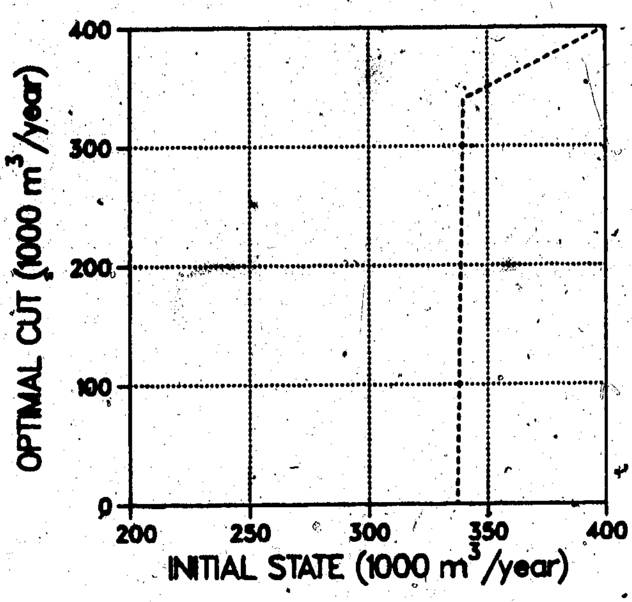

Figure 6.13: Optimal Cut Curve With No Discounting

Even though more timber could by attained in the long run by not harvesting for certain periods, the traditional models effectively stipulate that it is more important to maintain regular periodic harvests. Such a policy implies that either there is a value associated with smoothness of flow or a time-preference for harvested wood or a combination of both factors.

Variable Interest Rates

Altering the interest rates causes dramatic changes in optimal cut rates, PV of harvests and PV of falldowns (Figures 6.14, 6.15, 6.16). As interest rate decreases, the optimal solution tends to move towards that shown in

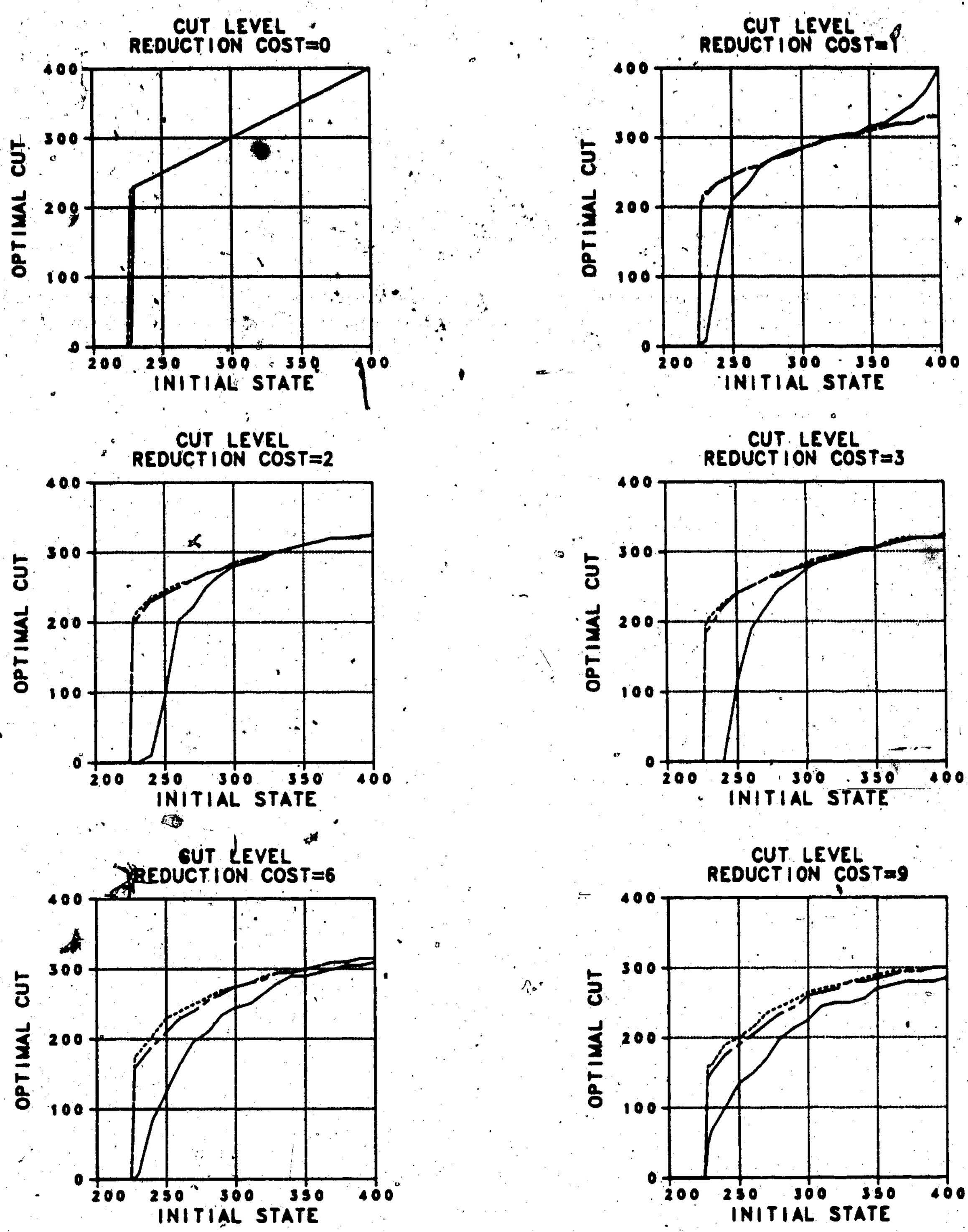

Figure 6.14: Effect of Interest Rate on Optimal Cut

Initial State (1000 m^3/year)
Optimal Cut (1000 m^3/year)

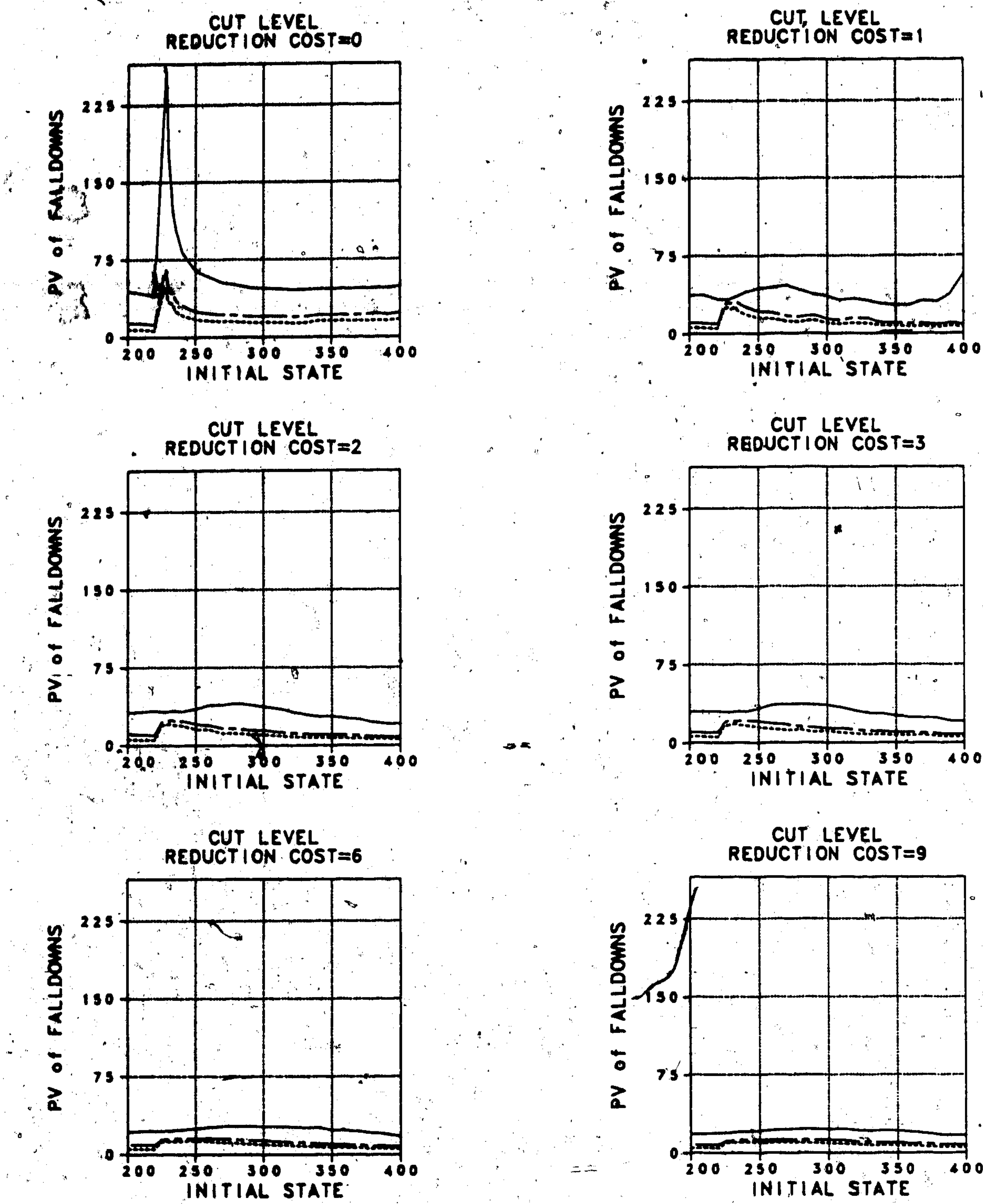

Figure 6.15: Effect of Interest Rate on PV of Falldowns

Initial State (1000 m^3/year)

PV of Falldowns (1000 m^3)

INTEREST RATE

2 % ————

5 % —·—·—

8 % ---------

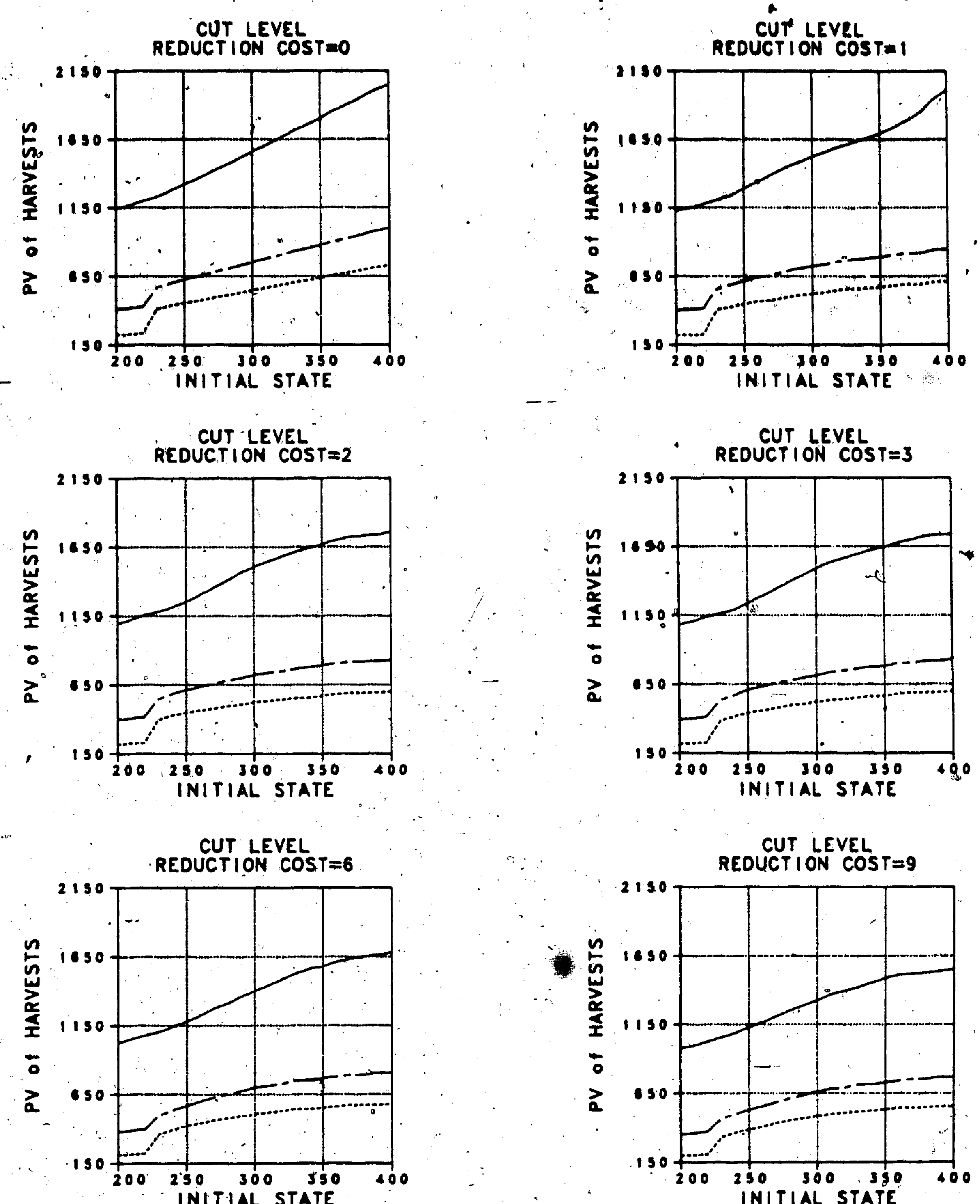

Figure 6.16: Effect of Interest Rate on PV of Harvests

Initial State (1000 m^3/year)
PV of Harvests (1000 m^3)

• INTEREST RATE

2 % ————
5 % —·—
8 % ········

Figure 6.13. That is, the optimal harvest levels, shown in Figure 6.14, for low interest rates will tend toward the solution for the maximization of undiscounted harvests. Differences in optimal cut rates [illegible] by interest rate, are greatest when the state is [illegible] above 225,000 m^3 per year, because of the non-linear growth rate. Low state levels have the highest growth rate and as time-preference decreases, it becomes optimal to defer the cutting at these low states until growth slows.

As the cut level reduction cost increases the differences caused by interest rate tend to be more uniform across all states (Figure 6.14). This reflects the tendency for uniformity in optimal harvest rates when a high value is placed on smoothness of flow. In fact at very high cut level reduction costs, the optimal cut curve will be flat regardless of interest rate, although the level of the curves would be different. One run was done with a reduction cost equal to 50 and interest rate of 5%; the optimal cut rate was 35,000 m^3 per year for every state where harvesting was allowed.

The curve when interest rate is 2% and cut level reduction cost equals one is unusual. It illustrates one of the drawbacks to dynamic programming. As the number of state levels is increased, accuracy improves but the computing cost increases (Norman 1975). When the number of states levels is too small, convergence to an optimal solution may not occur. In this case, the solution failed to converge for states greater than 350,000 m^3 per year. Therefore, the cut rates shown are probably not optimal.

The graphs of present value for both falldowns and expected harvest reflect two factors: the optimal cut rates (shown in Figure 6.14) and the discount factor. In Figure 6.15 and 6.16, the effect of the optimal cut rate is over-shadowed by the influence of the discount factor on present value. For example, in Figure 6.14 when the cut level reduction cost is 9, the optimal cut rates for 2% interest rate are lower in every state than the optimal rates for 5%. One would expect the lower harvest rates to produce less expected PV of harvests in most states. But exactly the opposite trend is shown in Figure 6.16. The cause is the discount factor.

The effect of the variable discount factors on the PV quantities can be removed by determining PV on a common interest rate. The transformations are made by determining the equal periodic payments required to obtain the PV of harvests and PV of falldowns for 2% and 8%; then this periodic payment is converted back to a PV quantity using a 5% interest rate. Figure 6.17 shows the transformation when the cut level reduction cost is 9. Lower PV of harvests occurs for states greater than 250,000 m^3 per year when optimal cut rates are based on a 2% interest rate. For states less than 250,000 m^3 per

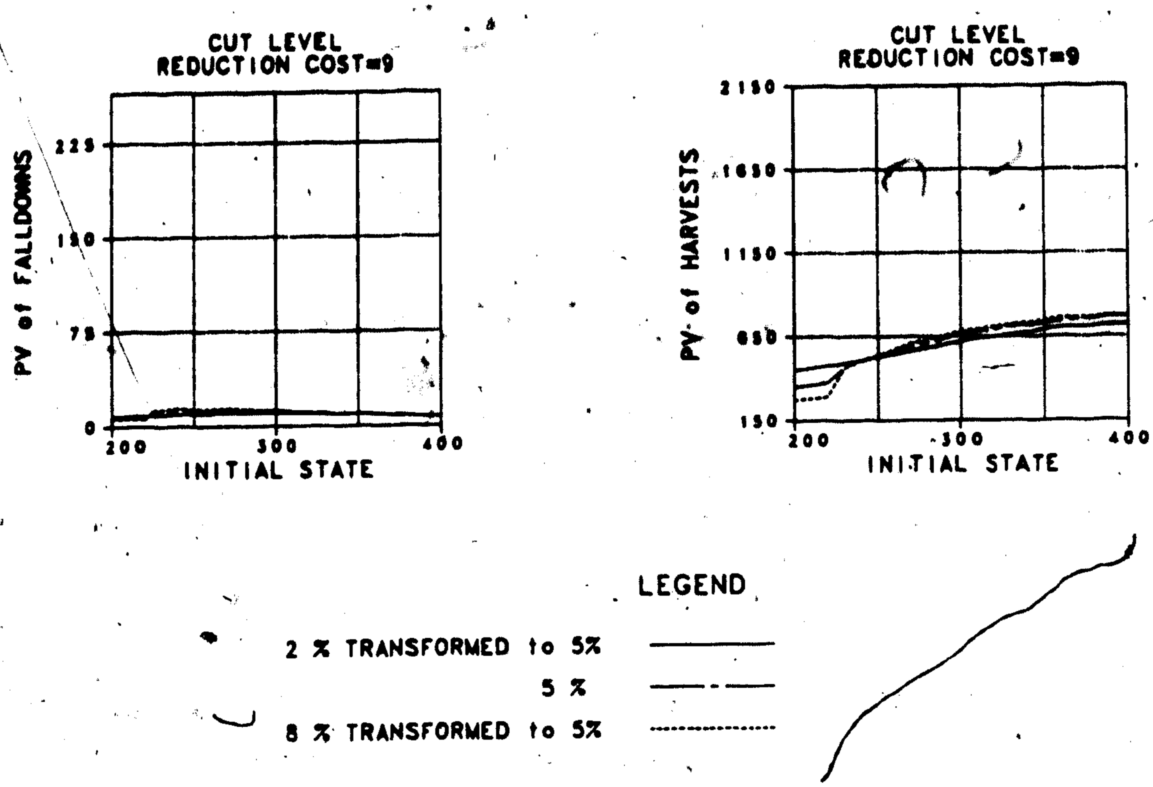

Figure 6.17: PV of Harvests and Falldowns Using a Common Interest Rate,

year, the trend of PV of harvests with interest rate is reversed. Low optimal harvest rates when the initial state is relatively low, will defer cutting until the state level is higher and the growth rate is lower.

6.3.2.4 Effect of the Shape of the Social Loss Function

Only two basic forms of the social loss function were tested. The first form where social loss occurs only from decreases in harvest level (non-symmetrical) has been discussed. The second form of the social loss function is linear and symmetrical; social loss occurs with any deviation in harvest level and is described as the cut level change cost. It should be realized that when the cut level change cost is zero, the symmetry of the social cost function is irrelevant. Therefore, the curves for optimal cut, PV of harvests and PV of falldown will be identical when change cost is zero.

The most striking feature of a linear symmetrical loss function is that optimal cut level remains virtually the same, regardless of the slope of the loss function (Figure 6.18). The same trend holds true for the PV of harvests and the PV of falldowns curves (Figures 6.19 and 6.20). This behavior is produced because the probability of a decrease in flow is proportional to the initial state but the probability of an increase in flow is inversely proportional to the initial state. Reductions in cut level will decrease the probability of a falldown, but they are generally ineffective in reducing PV of increases in flow because they cause an increase in the probability of a transition to a higher state. There is a trade-off caused by the opposing trends in probability.

Optimal cut is reduced relatively large amounts for high initial states when a social value is placed on smoothness of flow (Figure 6.18). Falldowns in these high initial state regions are virtually certain to occur[1]. Reduction of cut in these regions does not cause significant increases in the probability of increase in

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[1] These are falldowns from excess growing stock.

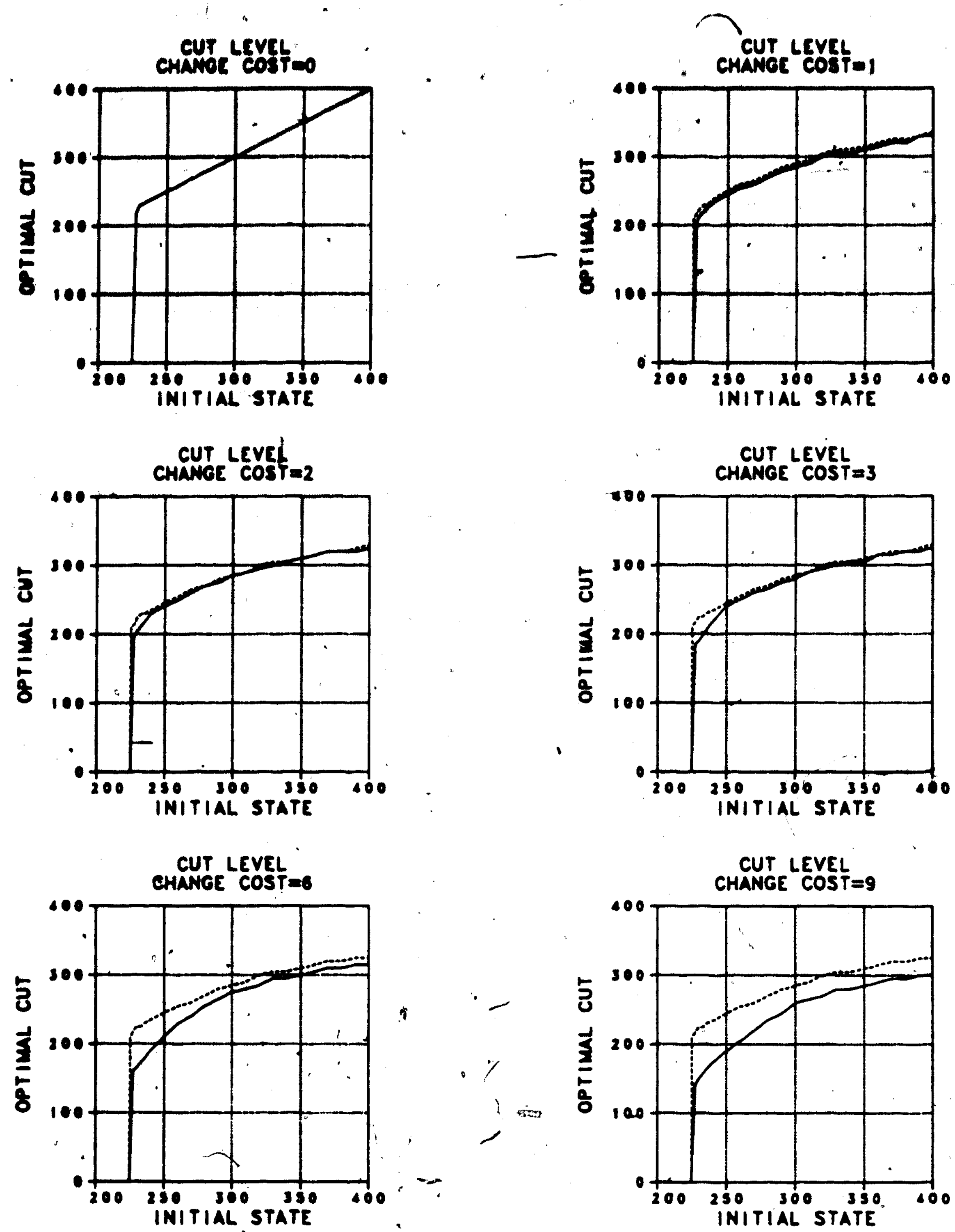


Figure 6.18:

Effect of Cut Level Change Cost on Optimal ~~Cut~~

Initial State (1000 m^3/year)

Optimal Cut (1000 m^3/year)

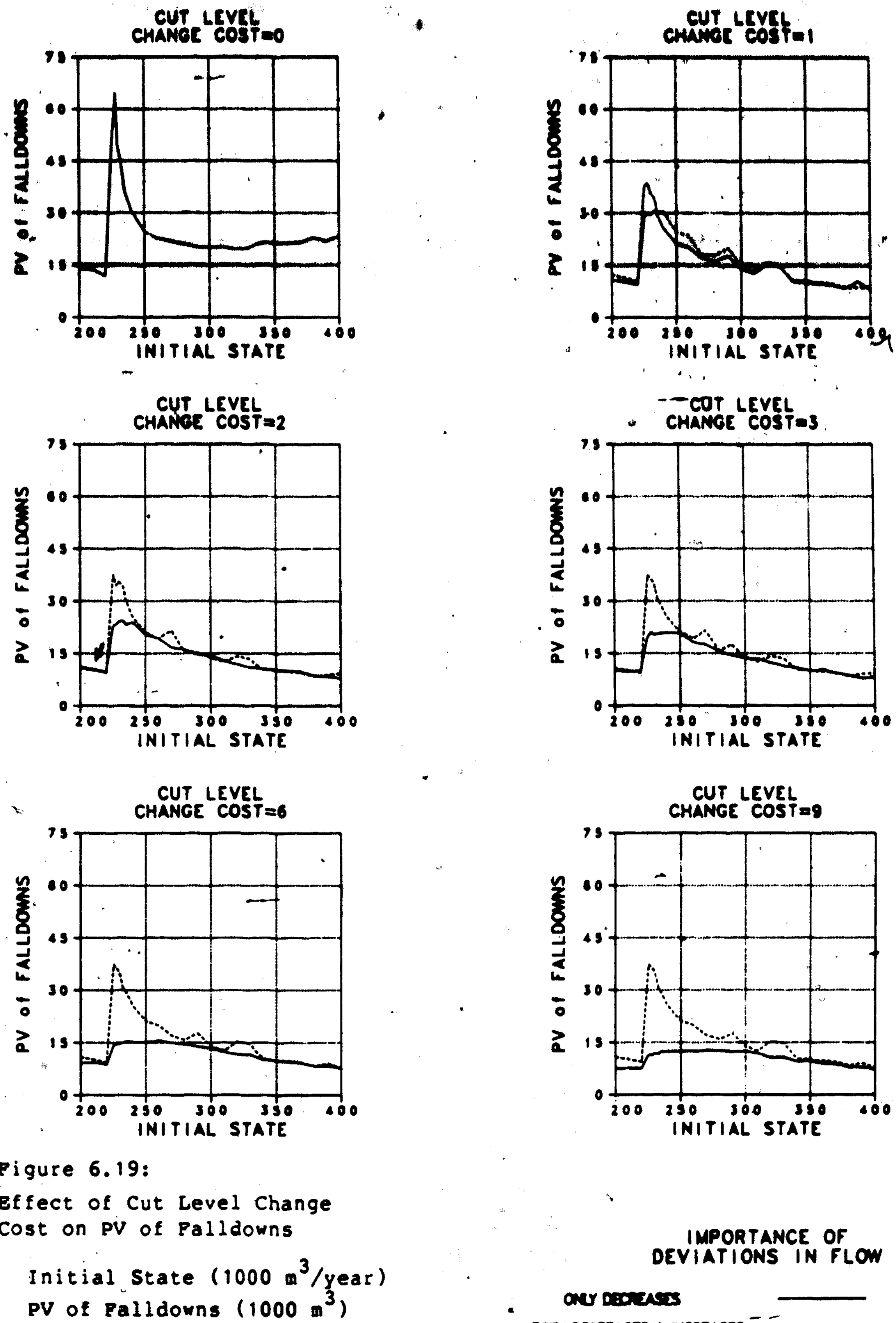


Figure 6.19:

Effect of Cut Level Change Cost on PV of Falldowns

Initial State (1000 m^3/year)
PV of Falldowns (1000 m^3)

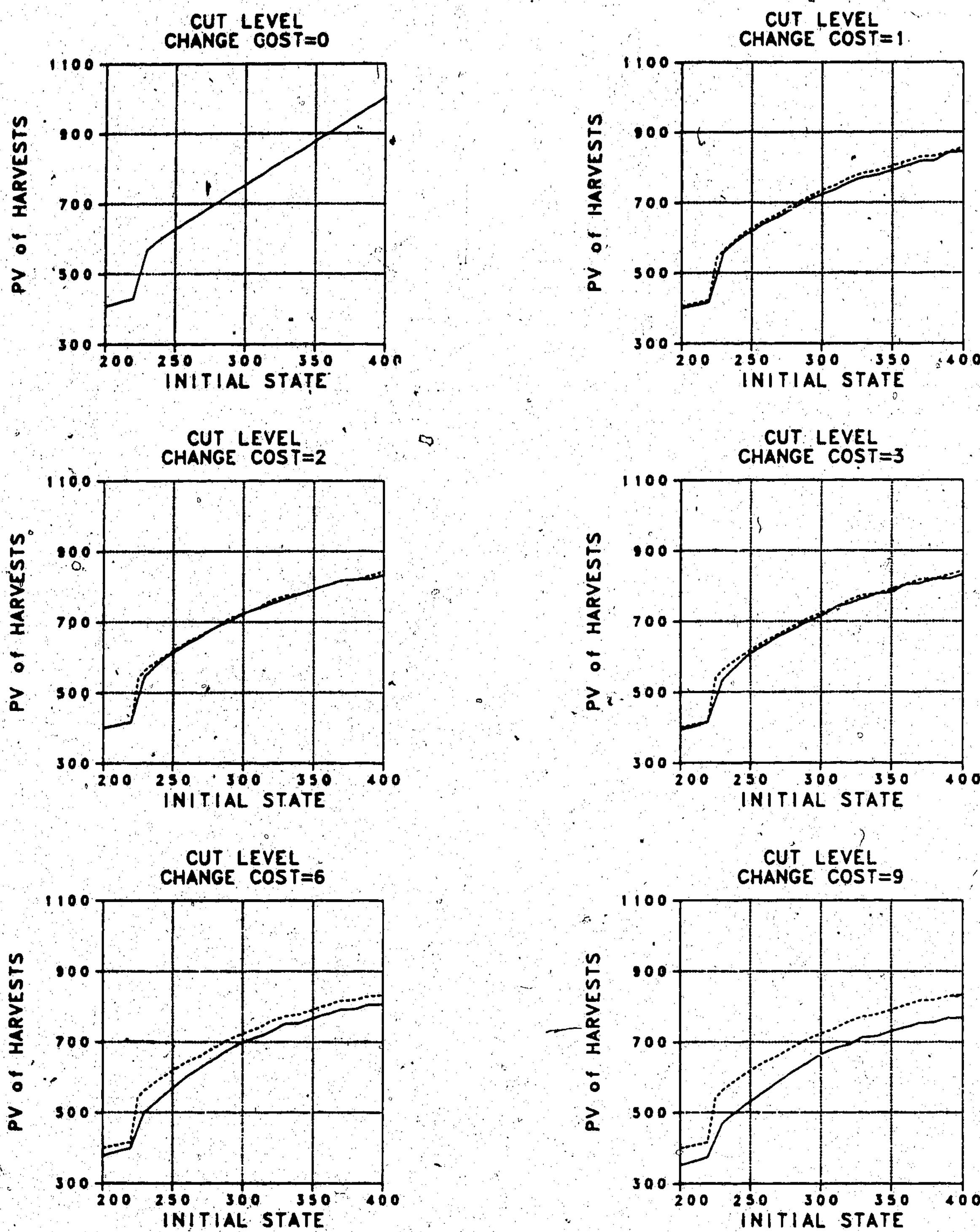


Figure 6.20:
Effect of Cut Level Change Cost on PV of Harvests

Initial State (1000 m^3/year)
PV of Harvests (1000 m^3)

IMPORTANCE OF DEVIATIONS IN FLOW

ONLY DECREASES ————
BOTH DECREASES & INCREASES - - - - - -

cut level because the states have very low or no probability of sustaining a higher cut.

The fluctuations in PV of falldowns, shown in Figure 6.19, are probably caused by an insufficient number of states. The discrete nature of the state and cut variables does cause some fluctuation in nearly all of the optimal solutions but the objective function for the symmetrical loss function is especially sensitive.

6.3.2.5 Steady-state Conditions

The steady-state condition in a forest subject to the risk of fire is one of fluctuation. The average harvest and the average amount of fluctuation period to period were determined for a few selected runs. Using the optimal harvest rates when the cut level reduction cost was 0, interest rate was 5% and fire rate was 0.15%, produced a steady-state harvest of 232,728 m^3 per year. The steady-state harvest or long-run sustained yield average when fire is not present was calculated to be 365,000 m^3 per year. So, the influence of fire on steady-state conditions is severe.

The level of steady-state harvest with fire is lower than anticipated in light of other research results (Reed and Enrico 1986, Van Wagner 1983). A cause was found in the regression equations used to determine the transition probabilities. As discussed previously, the upper limit to the resultant AAC (for each initial AAC and cut level combination) was determined from equation 1 (page 67) by setting the 10-year fire rate to zero. Unfortunately, this procedure slightly underestimated the upper limit. An alternative procedure was developed where the upper limit to the resultant AACs were taken directly from the simulations done for each initial AAC and cut level combination. The resultant AACs obtained using the lowest 10-year fire rate (0.0045%) simulated was assumed to be the upper limit to the resultant AAC. Revised estimates of these upper limits led to the development of different coefficients for the equation for parameter P1 (page 70).

This new equation was used for the steady-state conditions reported below, but was not used for any of the runs previously described. The new equation would not alter any of the relationships or patterns found in the optimal cut curves or the PV curves. However, the magnitude of some of the quantities would be affected slightly. For example, the optimal cut for state 400,000 m^3 per year with a cut level reduction cost of 3 and and interest rate of 5%, using the revised regression was 355,000 m^3 per year, whereas previously it would have been 325,000 m^3 per year.

The steady-state conditions for the three fire frequencies evaluated are shown below. These quantities could be expected to occur by following the optimal harvest rates for a reduction cost of 0 and an interest rate of 5%.

Average Fire Rate %	Steady-state Harvest m^3/year	Steady-state Fluctuation m^3/year
0.10	270,980	±5,504
0.15	269,054	±6,484
0.20	265,527	±7,200

The trend shown is expected; higher fire rates cause lower steady-state harvests and increased fluctuation about that rate. The steady-state harvest rate for an average fire rate of 0.15% is approximately 26% lower than if fire was non-existent. The steady-state harvests found in this study should be slightly lower than those expected from methods which use a constant annual rate of fire (Reed and Errico 1986, Van Wagner 1983). This is the result of allowing for the possibility of very severe fire rates which would substantially reduce AAC for short periods of time. It is important to note that the steady-state conditions are the expected long-term averages. The most commonly occurring fluctuation and harvest rate would be less severe because most fire rates will be relatively mild.

6.3.3 Implications for Timber Management

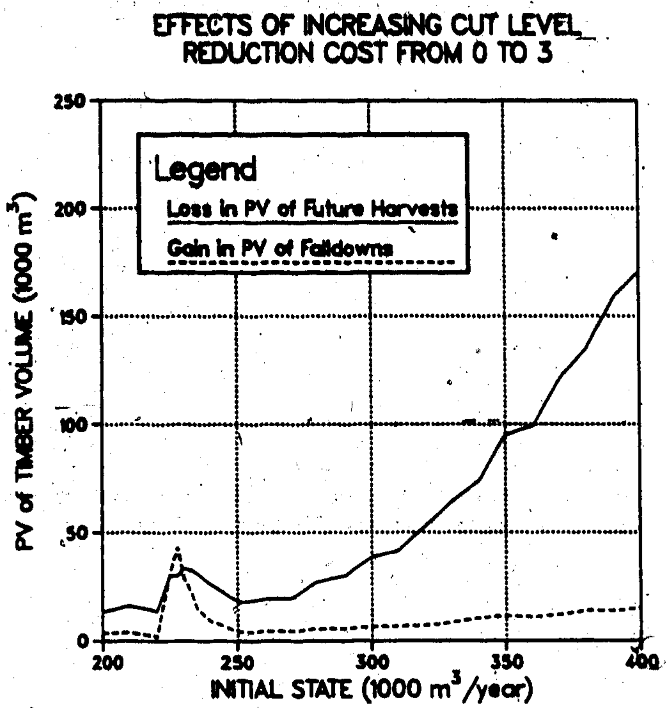
The DP model developed in this study is necessarily complex. Failure to model key elements such as the sequential decision process of timber management or the probabilistic nature of fire would probably have led to erroneous conclusions. However, application of the results does not require determination of values for all of the variables contained in the model. The benefits and costs of reducing harvest rates in anticipation of future fire losses, as determined in this study, can be discussed in general terms.

6.3.3.1 Benefits and Costs of Reducing AAC for Fire

The DP model quantifies the relationship between smoothness of flow and total harvest, thus providing a rational basis for decision making. The benefit obtained by reducing harvest rates is greater smoothness of flow and can be measured in terms of PV of falldowns. The cost is a reduced amount of PV of harvests. As mentioned previously, the use of the m^3 unit for PV of harvest is appropriate because harvest can be measured in volume units. In this study, the units for social cost should be thought of as the value equivalent to the social benefit of harvesting one m^3 of wood. Figure 6.21 shows these benefits and costs for two different cut level reduction costs. The upper curve shows the impact of a reduction cost of 3. The curve for gain in PV of falldowns was determined from:

$$\text{PV of falldowns at reduction cost}=0 \; - \; \text{PV of falldowns at reduction cost}=3$$

The reduction in falldowns has occurred because of the



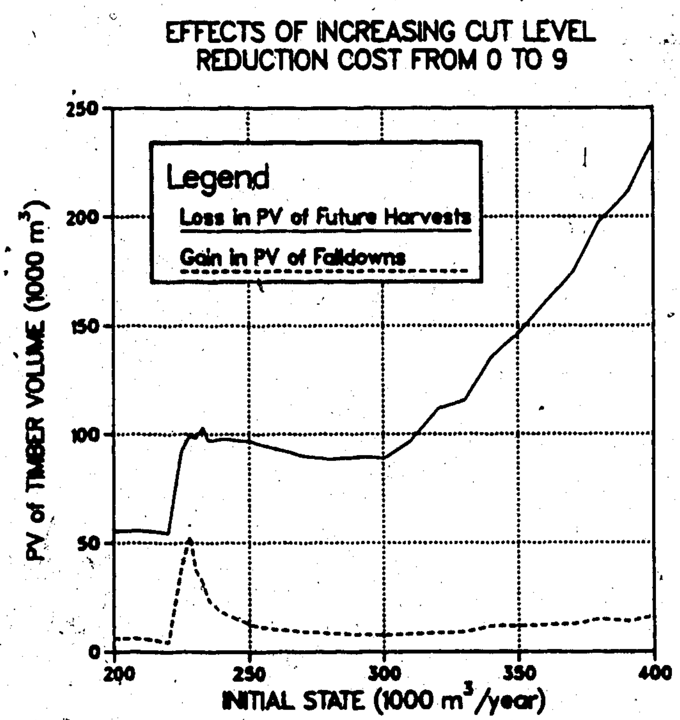


Figure 6.21: Gain in PV of Falldowns Compared to Loss of PV of Harvests for Reduction Costs of 3 and 9 (i=0.05)

increased value associated with them. It is a benefit of setting the cut level reduction cost at 3, so it is labelled as "gain". The curve for loss in PV of harvests was determined as:

PV of harvests at reduction cost=0 − PV of harvests at reduction cost=3

Using the upper graph in Figure 6.21, if a forest had an initial state of 400,000 m^3 per year, then over 170,000 m^3 of expected PV of harvests would have to be given up in order to decrease the expected PV of falldowns by 15,000 m^3 (approximate ratio of 11:1). The trade-off is not as extreme for lower states, but generally very large amounts of harvest must be forsaken to reduce the expected PV of falldowns.

The ratio of PV of harvests to PV of falldowns is not restricted to the cut level reduction cost. Higher ratios can be expected because completely different optimal paths for each reduction cost have been developed which produce different PV quantities. Therefore, the PV quantities obtained by harvesting at a given rate when the cut level reduction cost is 0 are not possible alternatives when the reduction cost is 3.

The benefit in PV of falldowns shown in Figure 6.21 is somewhat higher around state 225,000 m^3 per year than might be expected in reality. The high benefit shown is largely produced because the harvest is constrained to zero at states less than 225,000 m^3 per year resulting in a very high PV of falldowns, when the cut level reduction cost is zero. In reality, a reduction to a harvest rate of zero would probably occur gradually as the amount of harvestable timber was reduced.

Comparison of the upper curve for a cut level reduction cost of 3 with the lower curve for a reduction cost of 9 shows that the cost[1] for reducing falldowns increases dramatically as the cut level reduction cost increases. When the reduction cost is 9, very large amounts of harvest are required to reduce falldowns even at low initial states.

The present values associated with falldowns and harvests are somewhat difficult to interpret since such quantities are not commonly used in forestry. An example of a more conventional method of portraying harvest schedules will be shown to clarify the present value quantities and to further show the effect of reducing AAC for fire. This example is produced by a variation of the even-flow simulation model discussed earlier. The model is not related to the DP formulation and should not be construed as an analytical tool. It is used only to demonstrate the effect of different policies in one hypothetical situation.

Replanning was simulated at 10 year intervals. Both regenerated and standing timber were assumed to grow according to the standing timber yield curve of S1. The

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[1] in terms of harvest forgone

fire frequency curve, regen lags and liquidation period were those used previously for S1. One 140-year sequence of fire rates was randomly produced from the fire frequency curve. This sequence is only one of a infinite number of possible sequences. Three harvest schedules were produced by using three strategies for handling the risk of fire.

1. One of the schedules was produced by assuming that no reduction for anticipated fire losses would be made. Thus the AAC for each period was set at the maximum even-flow AAC which was based on the age class distribution of the forest at the time of replanning. For all schedules produced, the age class distribution was continually updated to reflect the effect of fire and harvesting.
2. The second harvest schedule was produced by assuming that AAC should be reduced 10% in advance of the fire actually occurring. This is the current AFS policy on fire for all management units in Slave Lake Forest.
3. The third harvest schedule assumed that the harvest rates should follow those produced by the dynamic programming model when the cut level reduction cost is 3, and the interest rate is 5% (Figure 6.7).

The policies used to produce the first and third schedules are consistent in their treatment of risk. That is, they treat the risk of decreases of flow in the same manner. In policy 1, no regard is given to any falldowns; in the third policy decreases in flow are avoided if the benefit gained exceeds the harvest that must be forgone. The second policy is not a consistent policy in handling the task of falldowns. When the forest is at a high initial state the probability of a falldown is very large (1.0 or slightly less). When the forest is at a low initial state, the probability of a falldown is small. But policy 2 reduces AAC 10% regardless of the state of the forest.

The schedules produced by each policy are shown in Figure 6.22. Reductions in harvest level occur because of fire and because of the depletion of excess growing stock by harvesting. Decade 7 contained a particularly bad fire sequence; when replanning occurred at the end of the decade, the harvest level was reduced considerably. Very slight or no reductions in harvest level represent relatively mild fire sequences. The magnitude of the falldowns is damped when harvest levels are reduced, because higher levels of growing stock are maintained. Increases in harvest level which may occur would also be damped when AAC is reduced in anticipation of future fire losses'.

The schedules show that very large amounts of AAC have to be given up to reduce the magnitude of

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' This behavior is not clearly visible in the harvest schedules because of the scale of the graph and because of the inherent variability in AAC when determined by an A-V check.

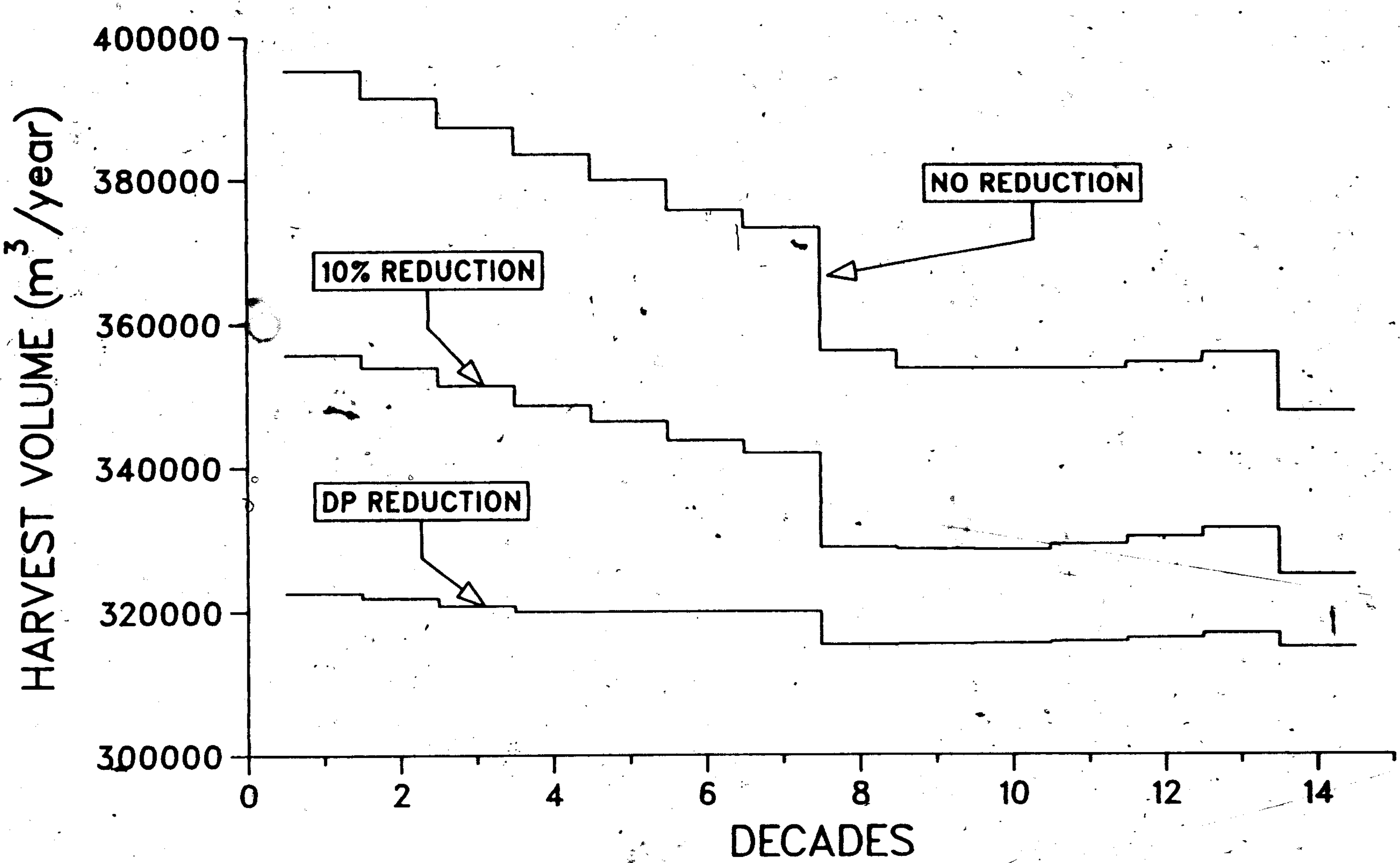

Figure 6.22: Harvest Schedules Produced Using Three Policies for Dealing with the Risk of Fire

falldowns. Thus, the social cost of a falldown must be considerable before reductions in AAC should occur.

By comparing the "reduction" policies to the "non-reduction" policy, the ratios of loss of PV of harvest to gain in PV of falldowns were calculated.

$$\text{10\% reduction: } \frac{\text{loss of PV of harvest}}{\text{gain in PV of falldowns}} = 22.7$$

$$\text{DP reduction: } \frac{\text{loss of PV of harvest}}{\text{gain in PV of falldowns}} = 20.6$$

Falldowns reduced by 10% reduction policy actually cost more on a per unit basis. This may seem surprising, but it should be expected because the 10% reduction policy gives no consideration to the cost of reducing falldowns. Whereas, the DP policy will always maximize the gain from PV of harvest minus any social loss due to decreases in flow. It should be noted that the schedule produced by following the DP policy assumed a cut level reduction cost of 3. Other harvest policies can be derived from the DP model by altering the emphasis on smoothness of flow.

In the upper graph in Figure 6.21, the quantity of PV of harvests lost was an average of approximately 11 times greater than the PV of falldowns gained for state 400,000 m^3 per year. The comparable proportion just calculated for Figure 6.22 was 20.6. The difference between the two numbers is because the actual fire occurrence differed from the expected or average fire conditions.

The harvest schedule that is most desirable depends entirely on how much social cost is associated with decreases in flow. If a manager is interested in only maximizing PV of harvest, then a "no reduction" policy is the best choice. The schedule produced in Figure 6.22 by the DP reduction will obviously give the fewest deviations in flow.

The benefits obtained by reducing AAC, shown in Figures 6.21 and 6.22, are probably significantly less than those previously supposed. Very large amounts of annual harvest must be relinquished for relatively small changes in smoothness of flow.

6.3.3.2 Factors Contributing to the High Cost of Smoothness of Flow

Three factors which contribute to the high cost of smoothness of flow are the continual maintenance of "reserved timber", the burning of that reserved volume and the influence of frequent replanning. These factors are not generally considered in the determination of the effect of fire on AAC, but are actually very important.

The process of reserving volume in anticipation of fire losses is often misunderstood. Figure 6.23 illustrates the benefit generally attributed to reducing harvest rates. Based on the best information available at time 0, AAC is set at level A. Unforeseen circumstances cause a reduction to level C when replanning occurs at time 1. If the knowledge gained at

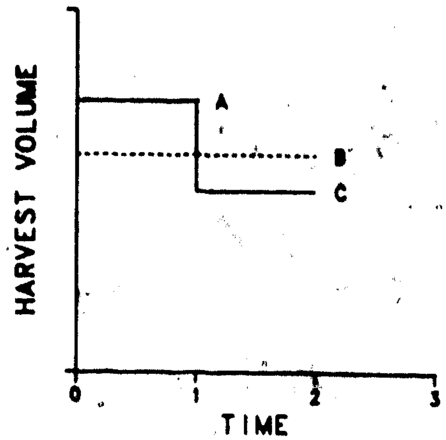

Figure 6.23: Hypothetical Harvest Schedule under Uncertainty

time 1 were available at time 0, then the optimal cut would be level B. Generally, level B will be lower than 50% of the difference between A and C because the reserve timber set aside at time 0 is subject to fire during period 1.

However, the process shown in Figure 6.23 is not how a policy of reducing harvest rates for fire would actually function. In Figure 6.23, most of the volume reserved in period 1 is used in period 2. However, if the policy of reducing harvest rates were to continue, volume would again be reserved in period 2 and every period thereafter. The cut level in period 2 would, therefore fall below level C. Over time such a policy, means that the majority of the reserved volume is never utilized, making it relatively ineffective in decreasing fluctuations in harvest rate. The fact that volume continues to be reserved with each plan revision is a very important cause of the high cost of smoothness of flow.

The ineffectiveness of reserving growing stock is magnified because the reserved volume is continually subject to fire losses. Substantially more volume will be burned by following a policy of reducing AAC for fire because a forest with a higher average age is produced. Consequently more volume will be lost for every hectare burned. Another serious problem may also be created. If the probability of destruction by fire increases with stand age (Philpot 1977, Martell 1980), then a forest produced by the policy of reducing AAC will have a higher fire frequency.

The importance of the reserve timber burning and the ineffectiveness of maintaining this reserve is illustrated in Figure 6.24. This figure shows two harvest schedules produced by different policies for the same forest. Suppose that at the end of decade 13, the "DP reduction" policy was switched to a "no reduction" policy. All of the growing stock reserved for 130 years would become available for harvest, so AAC would be expected to increase beyond the harvest rate indicated for the "no reduction" policy. AAC does increase, but only to a level 5.6% over the "no reduction" AAC in decade 14. This increase is probably smaller than expected considering the harvest volume forgone (the

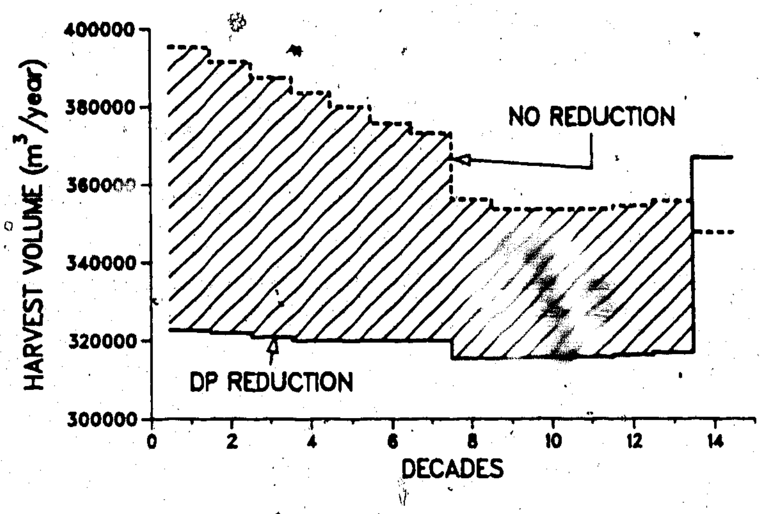

Figure 6.24: Impact of the Reserve Timber Burning

shaded area in Figure 6.24), and is a direct result of the reserve volume continually being depleted by fire. In fact, the total volume burned over the first 130 years by following the "DP reduction" policy is 25% greater than the volume burned by following the "no reduction" policy. The additional volume burned was calculated assuming the probability of destruction was constant with age. A greater difference in volume burned would result if the probability of destruction increased with age.

Because frequent replanning is a good method of handling risk and uncertainty, it contributes to the high cost of smoothness of flow. In Figure 6.23, overcutting occurs in period 1. The severity of the overcutting increases as the period continues. If a severe fire occurred early in period 1, the amount of overcutting for the rest of the period would be high. If the AAC were recalculated immediately after a severe fire year, the amount of overcutting would be small. Thus shorter replanning intervals will reduce the severity of overcutting.

The frequency of replanning is a very important element in determining the adequacy of replanning to handle risk. In Alberta, a trend to more frequent replanning is present. The current policy is to review AACs every 10 years (AFS 1985). If a falldown of 5% or more is anticipated because of fire or any other factor, a new AAC would be immediately recalculated. In addition, management units in which the AAC is nearly fully allocated may have updates every 5 years. Elsewhere in Canada, replanning also occurs frequently. For example, Ontario updates their plans every 5 years; B.C. hopes to update every 3 to 5 years. Frequent replanning assures that the harvest level generally does not substantially exceed the "sustainable" AAC.

The use of frequent replanning to handle the risk of destruction requires that flexibility for change is incorporated in planning. In Alberta, specification of quota volumes as a percentage of AAC in each management unit is an example. Flexibility could also be assured by having leases with different lengths of tenure within each management unit. The same type of flexibility is required if excess growing stock is to be harvested. In Alberta, the additional harvest possible, when excess growing stock exists, is sometimes allocated to short term leases. This extra volume is not part of the "official" AAC for the management unit and as such the quota holders are not entitled to it. The need to maintain options does not only apply to the policy of not reducing for future fire losses. As Figure 6.22 shows, flexibility would also be required even if AAC were reduced 10%.

6.3.3.3 Problems with Current Policies on Reducing AAC for Fire

The major drawback to current methods of dealing with fire risk is that the benefit of reducing AAC is never quantified. All current methods attempt in various ways to determine level B shown in Figure 6.23. If fire rates were entirely predictable, then it would be possible to determine an optimal AAC from such methods. Unfortunately, fire rates are not predictable and in fact, vary widely. As a result deviations in flow are certain to occur. However, a traditional model cannot quantify the deviations in flow because replanning is not part of the model formulation and because fire is not always treated as a stochastic process. The benefit of reducing AAC cannot be determined using such models.

In order to determine optimal cut rates, the relationship between expected deviations in flow and expected harvest is necessary. Such a relationship can only be derived by modelling timber management as a sequential decision process.

Another problem with current timber management policy is that uncertainty and risk for factors other than fire are usually not explicitly considered. Reductions for "fire losses" are often used as a catch-all for any uncertainty. Uncertainty exists with growth rates, regeneration lags, current stand volumes, accessibility of stands, the influence of seismic lines and a myriad of other factors. A reduction in AAC also allows for greater ease in the layout of cut blocks. If reductions for anticipated fire losses are not warranted, then it is time to rename the reduction for these other assorted elements.

6.3.4 Implications for Fire Management

In the past, damage to the timber resource has been estimated by using the area burned as the criterion of damage (Van Wagner 1979, Gorte and Gorte 1979). The loss to the timber resource is more correctly assessed by determining the change in present value of an infinite series of periodic cash flows produced by forest harvesting (Martell 1980, Reed 1984, Van Wagner 1979). But as shown in this study, expressing changes in PV of harvest without also showing the accompanying changes in PV of falldown is meaningless. Deviations in flow are a social cost and need to be included in the assessment of the economic consequences of forest fire.

Nearly all current economic analyses of fire management use marginal analysis, where the incremental change in fire effects between program levels is evaluated, not the total effect of fire protection (Schweitzer *et al*. 1982). The upper limit to protection expenditures is determined as the difference in the value of the timber resource between two different levels of fire protection[1].

Different levels of fire protection have previously been represented by the average annual fire rates. But in fact, fire protection may alter the both the shape and scale of the fire frequency curve. It is possible, for example that a particular fire management program may decrease the probability of severe fire years at the expense increasing the frequency of moderate fire years. The average annual rate is not necessarily altered, but the change in fire frequency could still alter PV of harvests and PV of falldowns. Fire frequency curves give a more accurate assessment of the effect of fire management than average fire rates.

Table 6.4 indicates PV of harvests and PV of falldowns which were obtained for the three frequency curves used in this study. The interest rate used for these runs was 5% and

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[1] Additional expenditures may be justified when other resource values and public safety are considered.

Table 6.4: EFFECT OF FIRE FREQUENCY ON TOTAL SOCIAL BENEFIT

Ave. Fire Rate (%)	Initial State (m^3/year)	PV of Harvests (m^3)	PV of Falldowns (m^3)	Total Social Benefit (m^3)
.10	200,000	398,884	9,622	370,018
	300,000	719,262	12,276	682,404
	400,000	833,418	7,109	812,091
.15	200,000	394,139	10,258	363,365
	300,000	715,014	13,606	674,196
	400,000	832,643	8,006	808,625
.20	200,000	358,512	11,391	358,512
	300,000	711,651	14,606	667,833
	400,000	831,531	8,949	804,684

the cut level reduction cost was three[1].

Generally the change in fire frequency appears to have more effect on PV of harvests for low initial states. This is because low initial states have a high probability of having harvest rates of zero in the near future and this probability is sensitive to fire frequency. Caution should be taken in interpreting small differences in these numerical values too literally, as there is some fluctuation in the solutions state to state.

Using the total social benefit[2] figures from Table 6.4, the upper level to protection expenditures that can be

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[1] Different PV quantities would be obtained if the emphasis on smoothness of flow was different than three.

[2] Total social benefit = PV of harvests - reduction cost × PV of falldowns

justified for reducing fire rates from 0.15% to 0.10% when the state of the forest is 200,000 m^3 per year is 6,653 m^3. That is, the total additional expenditure should not exceed the value that society places on the PV of 6,653 m^3 of harvested wood. Another interesting feature indicated by Table 6.4 is that the justifiable expenditure will vary depending upon the initial state of the forest. In the example shown here, more expenditure is warranted to reduce the fire frequency from 0.15% to 0.10% in state 300,000 m^3 per year than in any other state.

In order to determine an optimal level of fire management two components are required:

1. The first one is the benefit obtained from changing fire frequency. The benefits to the timber resource could be determined from this study.
2. The second necessary factor is the change in fire frequency produced by expenditure in fire protection.

With regard to the second component, it is conceivable that certain changes to the fire frequency curve might be more cost-effective than others. For example reducing the frequency of severe fire years may produce the most benefit to the timber resource per unit of expenditure on fire protection.

7. SUMMARY AND CONCLUSIONS

The dynamic programming model developed in this study can be used to determine optimal harvest rates for a forest subject to the risk of fire. The model determines the optimal harvest rate on the basis of the value associated with smoothness of flow and the value associated with total harvest. The major innovations contained in the model are the treatment of timber management as a sequential decision process and the use of fire frequency curves to estimate the probabilities of all possible fire rates.

The results of the study can be described in terms of the secondary objectives:

1. **To determine a method of using historical fire data in timber management.**

   Frequency analysis of historical fire data is a good method to describe fire history. It is useful for generating random fire sequences and allows for the possibility of fire rates outside of the range of those having previously occurred. Use of a constant annual rate of fire to determine the effect of fire on current AAC is probably inappropriate largely because it leads to the false assumption that deterministic approaches in planning are capable of determining optimal harvest levels. The use of average rates of fire for determining steady-state harvest rates will probably result in overestimates. In addition, use of constant fire rates makes the determination of the steady-state fluctuation in harvest impossible as seen in Reed and Errico (1986) and Van Wagner (1983).

2. **To evaluate the problems inherent with harvest scheduling models which fail to explicitly recognize the replanning component of timber management.**

The failure of most harvest scheduling models to represent the sequential decision-making process of timber management is a major shortcoming. Such models cannot quantify expected fluctuations in harvest level which result from forest fire or from an excess or deficiency of growing stock. The magnitude of such fluctuations can be influenced by harvest rates, but unless replanning is modelled, the relationship between smoothness of flow and total harvest cannot be assessed. Without knowledge of this relationship, determination of optimal harvest rates is not possible.

Recognition of the sequential decision-making process is especially important in assessing the effect of fire on harvest rates because fire rates are highly variable and unpredictable. But even in a forest not subject to fire, the use of models which simultaneously plan for the entire planning horizon is questionable.

3. **To assess the benefits and costs of reducing AAC in anticipation of future fire losses.**

In this study, the benefit of reducing AAC is a reduced level of expected falldowns discounted to the present time. The cost is a lower total expected harvest, also discounted to the present. In general, very substantial amounts of harvest must be forgone in order to reduce expected falldowns appreciably. This is especially true if large amounts of excess growing stock exist (i.e. the initial state is high).

Three factors which contribute to the high cost of smoothness of flow are the continual maintenance of reserved growing stock, the burning of reserved volume and the role of frequent replanning. Any timber volume set aside in anticipation of future fire losses is itself subject to fire losses. Frequent replanning ensures that the current harvest rate does not significantly exceed the sustainable rate.

4. To assess the effect of different fire frequencies on the timber resource.

Optimal harvest rate does not change substantially with fire frequency unless a very high value is associated with smoothness of flow. This does not imply that fire frequency has no effect on the timber resource. Lower levels of harvest and increased fluctuation in flow can be expected when fire frequency is high. This relationship is evident in the steady-state conditions and in the present value quantities presented.

If smoothness of flow has value, then the benefit of reducing fire rates through fire protection should be assessed in terms of both the expected harvest and the expected fluctuation in harvest.

Determination of the optimal AAC depends to a large degree on the importance associated with smoothness of flow. Substantial reductions in AAC and therefore in total harvest are necessary to increase smoothness appreciably. Because of the very high cost of smoothness of flow, it is recommended that current AAC not be reduced for fire risk, unless warranted from a determination of the value of smoothness of flow. A policy of reducing AAC in anticipation of future fire losses is relatively ineffective for reducing the magnitude of fluctuations in harvest level. Frequent replanning is capable of ensuring that overcutting will never be severe; this is particularly true if plans are updated after severe fire years. Not reducing harvest rates will mean that fluctuations in harvest rates will be greater, but it should be clear that fluctuations will occur regardless.

The problem of changes in harvest level over time is a serious one. But attempting to solve the problem by reserving growing stock and thus allowing it to burn is clearly not desirable. Any reserve volume that is burned is timber which could have been utilized. Solutions to the problem of variable harvest rates may require changes in timber management policies but surely such changes are justified.

The steady-state harvest can be expected to be substantially lower when a forest is subject to fire. The steady-state harvests presented in this study may be slightly lower than those expected by the use of other methods because fire rates outside of the historical range are considered possible. But generally the steady-state harvests are of the same magnitude as those found elsewhere (Reed and Errico 1986, Van Wagner 1983). However, it is anticipated that the steady-state condition will be subject to relatively large fluctuations in harvest due to the variability of fire rates on the size of area for which AACs are determined and the unpredictability of fire.

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