THE UNIVERSITY OF ALBERTA

EXPERIMENTAL ANALYSIS OF A PERISTALTIC PUMP

by

VIJAY SHUKLA

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

OF MASTER OF SCIENCE

MECHANICAL ENGINEERING

EDMONTON, ALBERTA

SPRING 1989

ISBN 0-315-52815-X

THE UNIVERSITY OF ALBERTA

RELEASE FORM

NAME OF AUTHOR VIJAY SHUKLA

TITLE OF THESIS EXPERIMENTAL ANALYSIS OF A PERISTALTIC PUMP

DEGREE FOR WHICH THESIS WAS PRESENTED MASTER OF SCIENCE

YEAR THIS DEGREE GRANTED SPRING 1989

(SIGNED) V. Shukla

PERMANENT ADDRESS:

H. No. 670, I.I.T. Campus,

Indian Institute of Technology

Kanpur, U.P., INDIA 208016.

DATED 1 April 1989

THE UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled EXPERIMENTAL ANALYSIS OF A PERISTALTIC PUMP submitted by VIJAY SHUKLA in partial fulfilment of the requirements for the degree of MASTER OF SCIENCE.

....

Supervisor

....

....

....

Date April 7/89

DEDICATION

To my Mother

ABSTRACT

In the thesis the problem of peristalsis in a axisymmetric tube has been dealt with experimentally.

First the parameters of this problem were found by nondimensionalizing the Navier-Stokes equations, then experiments were conducted to investigate the relation of nondimensional flow rate to these parameters.

It has been found that the nondimensional flow rate increased with an increase of frequency parameter, amplitude parameter, and number of chambers parameter. The nondimensional flow rate decreased with an increase of pressure parameter. The presence of a one way valve in the pumping apparatus was found to increase pumping by about 20 % for frequency parameter 10 and amplitude parameter 1.0.

The experimental results have been compared with the theory of Barton and Raynor (1968) and are found to agree to within 20% for the case of pumping with 8 chambers and no valve present for frequency parameter less than 8.0.

ACKNOWLEDGEMENT

I would like to acknowledge the supervision of Prof. C. M. Rodkiewicz and Prof. J. S. Kennedy. Discussions with Dr. Z. Sawicki were also helpful. During my stay in the university I have found the support of Prof. M. G. Faulkner encouraging.

Financial support was provided in the form of a teaching assistantship, during the summer of 1988 support was provided by NSERC grant A 4198, both of which are gratefully acknowledged.

The work of all of the support ... f the departm... hanical Engineering is significant. In particular Mr. Alan Muir, Mr. All... n, Mr. Bernie Faulkner, Mr. Tony Van Straten and Mr. Tom Villett have worked hard to help me manufacture the experimental project.

Lastly I would like to acknowledge the support of my father, mother, brother and sister.

Table of Contents

List of Figures

List of Plates

LIST OF SYMBOLS

a - Unstressed radius of tube

A - Amplitude of contraction

c - Wave speed

F_r - Body force in r direction

F_ϕ - Body force in ϕ direction

F_z - Body force in z direction

g - Acceleration due to gravity

L - Length of pump

N - Rotational speed of cam shaft in RPM

p - Pressure

P - Reference pressure

Q - Mean flow rate

r - Radial distance from axis

Re - Reynolds number given by $\frac{ac}{\nu}$

t - Time

T - Time period of revolution of cam shaft

u - Velocity in r direction

U - Reference velocity in r direction

v - Velocity in ϕ direction

w - Velocity in z direction

W - Reference velocity in z direction

z - Axial coordinate

ΔH - Level difference between the two level controlled tanks

Δp - Static pressure difference between the inlet and outlet of pump

λ - Wavelength of contraction

μ - Viscosity

ν - Kinematic viscosity

$\Pi 1$ - Frequency parameter and is given by $a\sqrt{\left(\frac{\omega}{\nu}\right)}$

$\Pi 2$ - Wavelength parameter given by $\frac{a}{\lambda}$

$\Pi 3$ - Amplitude Parameter given by $\frac{A}{a}$

$\Pi 4$ - No of Chambers given by $\frac{L}{\lambda}$

$\Pi 5$ - Pressure ratio given by $\frac{ga^3 \Delta H}{\nu^2}$

ρ - Density

ϕ - Angle ϕ of r, ϕ z coordinate system

ω - Angular frequency

$+$ - Superscript used for nondimensional variables

I. INTRODUCTION

The problem chosen for this thesis is a study of a 'Peristaltic Pump'. The dependence of the flow rate on the operating parameters of the pump was investigated experimentally.

The word peristalsis is derived from the root 'Peristaltikos' which means clasping and compressing. A peristaltic pump is a pump which works because of compressions along its length. This type of pumping occurs often in biological systems where pumping takes place, for example, in the digestive system, Piersol (1930).

The experiment has been designed based on the physical dimensions of the duodenum, the initial portion of the human small intestine. The dimensions are taken from Piersol (1930) and Barton and Raynor (1968). The experimental results have been compared with theoretical results published in literature, Barton and Raynor (1968), Jaffrin & Shapiro (1971).

The objectives of this study were to achieve:

(A) Investigate the characteristics of this pump,

(B) Compare the results with previous experiments,

(C) Investigate the effect of a one way valve on the performance of the pump. This valve simulates the pyloric sphincter in the duodenum.

The results obtained by this experiment were not fully applicable to the duodenum because of the following reasons:

(A) All the motions of the walls were not considered. For example the change in length occurring in the duodenum could not be simulated in the experiment.

(B) The Non-newtonian behaviour of chyme was not considered due to limitations of the experiment.

II. LITERATURE SURVEY

In this thesis the problem of flow through a tube in which peristalsis occurred was chosen. This choice was made as this type of motion occurred often in biological systems and it also may have potential industrial applications.

Peristalsis is the mechanism by which the human body propels and mixes the contents of the intestine. It is partially responsible for flow of bile in the bile duct and urine in the urinary tract, Netter (1966) and Boyarsky (1964).

The theoretical models of peristalsis were idealized, and they represented peristalsis by an infinite train of sinusoidal waves in a two dimensional channel or an axisymmetric tube, Shapiro et. al. (1969), Yin and Fung (1969) and Barton and Raynor (1968). In addition to the above the assumptions of long wavelength and small or moderate inertia were also made. This resulted in a considerable simplification of the equations of motion, and the solution was obtained using perturbation techniques. In the case of small Reynolds numbers the flow was similar to Poiseuille flow if viewed in a frame of reference moving with the contraction wave.

Two important phenomena, Reflux and Trapping, were found to occur in experiments and theory. Reflux is the motion of some parts of the fluid in a direction opposite to that of propogation of the contraction wave. Trapping is the formation of a pocket of fluid that moves in the direction of wave propogation as a bubble of fluid. This occurs when the tube is sufficiently occluded or the flow rates are high. It is believed that Reflux explains the movement of bacteria from the bladder to the kidney against the mean urine flow, Jaffrin & Shapiro (1971).

There have been two previous experimental studies of peristalsis by Weinberg et. al. (1971), and Yin and Fung (1971). They had compared their experimental results with theoretical calculations and the agreement was good for the case of long wavelength and small Reynolds number. At higher Reynolds number (Re larger than 10) the results did not agree with experiment a the theoretical results were not valid for large Reynolds number. This was because the inertia terms of the Navier-Stokes equations had been neglected by assuming that the wavelength was long and Reynolds number small.

In the experiment by Weinberg, Eckstein and Shapiro (1971) the theory of Shapiro et. al. (1969) was confirmed with respect to pressure vs mean flow, pressure vs time, and reflux and trapping. The experiment consisted of a pumping duct rectangular in cross section bounded by a rigid semicircular back wall, a flexible moving wall in which waves were produced by roller cams, and two transparent cover plates. The peristaltic action of the moving wall was produced by means of a cam assembly which was used to produce a travelling sine wave. The dimensions were such that the long wavelength approximation was satisfied, the speeds were choosen so that the Reynolds number was small (i.e. less than 10).

In the experiment by Yin and Fung (1971) the theory of Fung and Yih (1968) was confirmed. This theory was extended and modified to take into account the finite size of channel used in the experiment. The experiment consisted of a rectangular cross-section channel with top and side walls fixed and with a travelling sine wave imposed on the bottom wall. The side walls were made of lucite so that flow visualization could be performed. The result was that the two dimensional theoretical prediction of time average flow rate was in close agreement to experimental data for an amplitude ratio of 0.3.

More recently Takabatake et.al. (1988) did a numerical solution of the problem of peristalsis in circular cylinderical tubes. The fluid mechanics was studied in detail by analyzing reflux and trapping in peristaltic pumping. They found that the Reflux was near the axis of the tube for large Reynolds numbers and near the wall for small Reynolds numbers. The authors defined mechanical efficiency of the pump by the ratio of energy stored in the fluid to the work done on the walls by outside agencies. They found that the efficiency for the tubular configuration was higher compared to the 2-D channel case. The reason for this was that Reflux was smaller for the tubular configuration and Trapping was more effective.

Takabatake and Ayukawa (1982) solved the two dimensional peristalsis problem numerically for moderate Reynolds numbers. They also used flow visualization to verify the velocity profiles that they had predicted numerically. Some other numerical solutions have also been presented. Tong & Vawter (1972) used a finite element approach to the creeping flow problem. Brown and Hung (1977) used a finite difference scheme based on curvilinear coordinates. Brown & Hung (1977) also verified their results experimentally by using flow visualization. Also Pozrikidis (1987) solved the peristalsis problem for a two dimensional channel with sinusoidal waves present. He made use of the boundary integral method for Stokes flow problems. The author concluded that large amplitude peristaltic waves with a slowly varying mean pressure gradient could increase fluid mixing and may be used for design of an efficient heat/mass transfer process.

The effect of peripheral layer viscosity on peristaltic pumping has been investigated by Shukla et. al. (1980). The authors found that the peripheral layer performs a lubricating function in the case of flow of chyme in the intestines. Liron (1976) investigated the efficiency of peristaltic pumping (the definition of efficiency used here was same as in Takabatake et. al.(1988) and Shapiro et. al.(1969).) and found that large amplitude long wavelength waves were best from an mechanical point of view.

Peristaltic pumps have applications in the industry where small flow rates are required. These peristaltic pumps are very low capacity pumps that have a flexible tubing wrapped around a rotating cam. The primary use of peristaltic pumps is for laboratory or pilot scale facilities, Scher (1979). The pump has also been used to pump blood in the heart lung machine, because in this pump the pumped fluid does not come into contact with mechanical components of the pump.

An example of a peristaltic pump used for chromatography is *Pharcia peristaltic pump made by Pharcia Fine Chemicals AB Box 175, S-75104 Uppsala 1 Sweden*. The flow rate range for this pump is 0.6-500 ml/hr, the diameters of tubes used in the pump vary from 1.0 mm to 3.0 mm. The tube material is Silicone Rubber.

As the experiment was based on the physical dimensions of the Duodenum a brief review of relevant literature is included. A sketch of the Duodenum is shown in Figure II.1.

In Christensen et. al. (1978) and Netter (1966), motion of the intestine has been described. A description of anatomy can be found in Piersol (1930). It takes about 4 to 5 hours for the food to pass through the intestine. When the food reaches the stomach peristalsis starts after a lag of about 20 minutes. The food is reduced to a fluid or semifluid type mixture before the pyloric sphincter opens to allow food into the intestines. It takes about four hours for the food to be transferred from the stomach to the intestines.

The Duodenum is the first 20-25 cm of the intestine. The diameter of the Duodenum is approximately 25 mm. The wavelength of the wave is about 4 cm and the frequency of occurrence of the wave can vary from 5 to 12 times a minute. The average flow rates in the Duodenum are of the order of 10 ml per minute, and the mean fluid velocities are about 2 cm/minute. More details can be found in Dillard & Eastman (1965), Kerlin et. al. (1982), Macagno & Christensen (1980,1981,1982), and Melville et. al. (1975). A review of the work done on motility and flow in the small intestine can be found in Christensen et. al. (1978).

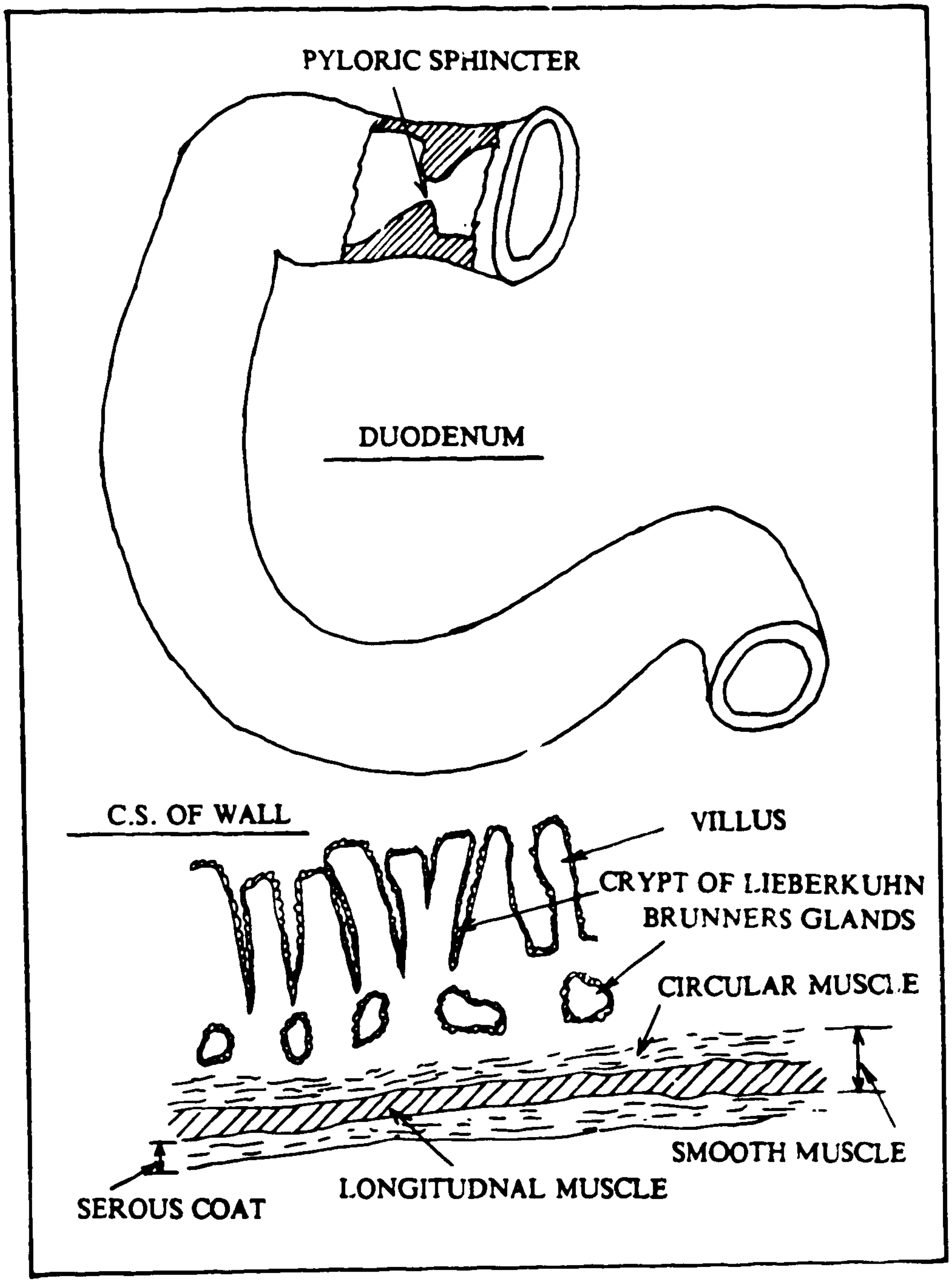


Figure II.1 The shape of the Duodenum and a cross section of the wall.

III. BASIC EQUATIONS

In this chapter the basic equations of motion describing a fluid are reduced to a form suitable for studying flow in a tube with moving walls. The newtonian fluid model was used because before the food enters the duodenum the stomach mixes and pulverises the food to a fluid. This fluid was assumed to be newtonian. The equations of motion in cylindrical polar coordinates are as follows, Schlichting (1968).

r direction momentum -

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \phi} - \frac{v^2}{r} + w\frac{\partial u}{\partial z}\right)$$
$$= Fr - \frac{\partial p}{\partial r} + \mu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \phi^2} - \frac{2}{r^2}\frac{\partial^2 v}{\partial \phi^2} + \frac{\partial^2 u}{\partial z^2}\right) \tag{1}$$

ø direction momentum -

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \phi} + \frac{uv}{r} + w\frac{\partial v}{\partial z}\right)$$
$$= F\phi - \frac{1}{r}\frac{\partial p}{\partial \phi} + \mu\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} - \frac{v}{r^2} + \frac{1}{r^2}\frac{\partial^2 v}{\partial \phi^2} + \frac{2}{r^2}\frac{\partial u}{\partial \phi} + \frac{\partial^2 v}{\partial z^2}\right) \tag{2}$$

z direction momentum -

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + \frac{v}{r}\frac{\partial w}{\partial \phi} + w\frac{\partial w}{\partial z}\right)$$
$$= Fz - \frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \phi^2} + \frac{\partial^2 w}{\partial z^2}\right) \tag{3}$$

Continuity equation -

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \phi} + \frac{\partial w}{\partial z} = 0 \tag{4}$$

The assumptions upon which equations (1) to (4) were based are incompressible flow, isotropic fluid, constant viscosity, and that the continuum hypothesis was valid.

The following additional assumptions were made:

(1) No changes in the ø direction, i.e. the flow is symmetrical about the axis of the tube.

(2) The walls of the tube are assumed to be impermeable.

(3) Body forces are absent.

Application of these assumptions to the equations reduce them to:

r direction momentum -

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2}\right) \tag{5}$$

z direction momentum -

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right) \tag{6}$$

Continuity equation -

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{7}$$

Where

$u = u\ (r,\ z,\ t),\ w = w\ (r,\ z,\ t)$ and $p = p\ (r,\ z,\ t)$

All terms of equations 5 through 7 were nondimensionalized as follows: (plus superscript denotes nondimensional variables)

$$r^{+} = r/a$$

$$z^{+} = z/L$$

$$t^{+} = t/T$$

$$w^{+} = w/W$$

$$u^{+} = u/U$$

$$p^{+} = p/P \tag{8}$$

The continuity equation (7), after nondimensionalization and combining the first two terms reduced to the following form. For convenience the plus superscripts have not been written with the understanding that all the variables (e.g. w, z, u, r, t) in use from now on are nondimensional.

$$\frac{1}{r}\frac{\partial (ru)}{\partial r} + \frac{aW}{LU}\frac{\partial w}{\partial z} = 0$$

Using a similar approach for r direction momentum equation (5) resulted in the following:

$$\rho\left(\frac{U}{T}\frac{\partial u}{\partial t} + \frac{uU^2}{a}\frac{\partial u}{\partial r} + \frac{wWU}{L}\frac{\partial u}{\partial z}\right)$$

$$= -\frac{P}{a}\frac{\partial p}{\partial r} + \mu\left(\frac{U}{a^2}\frac{\partial^2 u}{\partial r^2} + \frac{U}{a^2 r}\frac{\partial u}{\partial r} - \frac{uU}{a^2 r^2} + \frac{U}{L^2}\frac{\partial^2 u}{\partial z^2}\right)$$

After multiplication of both sides by $\frac{a^2}{\mu U}$ the equation became

$$\frac{a^2}{\nu T}\frac{\partial u}{\partial t} + \frac{aU}{\nu}\left(u\frac{\partial u}{\partial r} + \frac{awW}{LU}\frac{\partial u}{\partial z}\right)$$

$$= -\frac{aP}{U\mu}\frac{\partial p}{\partial r} + \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{a^2}{L^2}\frac{\partial^2 u}{\partial z^2}$$

A similar procedure for z momentum equation (6) (Nondimensionalizing and multiplying throughout by $\frac{a^2}{\mu W}$) reduced it to the following:

$$\frac{a^2}{\nu T}\frac{\partial w}{\partial t} + \frac{aU}{\nu}\left(u\frac{\partial w}{\partial r} + \frac{aWw}{LU}\frac{\partial w}{\partial z}\right)$$

$$= -\frac{a^2 P}{\mu LW}\frac{\partial p}{\partial z} + \frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{a^2}{L^2}\frac{\partial^2 u}{\partial z^2}$$

If both terms of continuity were to be retained then

$$\frac{aW}{LU} \simeq O(1) \tag{9}$$

The following were chosen for values of the reference variables :

$$T = \frac{2\pi}{\omega}$$

$$P = \frac{\mu LW}{a^2} = \frac{\mu\nu}{a^2}$$

$$W = \frac{\nu}{L}$$

The equations were thus

$$\frac{1}{r}\frac{\partial(ru)}{\partial r} + \frac{\partial w}{\partial z} = 0 \tag{10}$$

$$\frac{\Pi 1^2}{2\pi}\frac{\partial u}{\partial t} + \frac{\Pi 2^2}{\Pi 4^2}\left(u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} \right)$$

$$= -\frac{\Pi 4^2}{\Pi 2^2}\frac{\partial p}{\partial r} + \frac{\partial^2 u}{\partial r^2} - \frac{u}{r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\Pi 2^2}{\Pi 4^2}\frac{\partial^2 u}{\partial z^2} \tag{11}$$

$$\frac{\Pi 1^2}{2\pi}\frac{\partial w}{\partial t} + \frac{\Pi 2^2}{\Pi 4^2}\left(u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} \right)$$

$$= -\frac{\partial p}{\partial z} + \frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\Pi 2^2}{\Pi 4^2}\frac{\partial^2 w}{\partial z^2} \tag{12}$$

Where

$$\Pi 1 = a\sqrt{\left(\frac{\omega}{\nu}\right)}$$

$$\Pi 2 = \frac{a}{\lambda} \tag{13}$$

$$\Pi 4 = \frac{L}{\lambda}$$

The boundary conditions were checked to see if any new parameters were present.

The motion of the wall was assumed to be:

$$h = a - \frac{A}{2}\sin(\frac{\pi z}{\lambda}) \left(1 - \cos \frac{2\pi t}{T} \right) \tag{14}$$

A sketch of the deflected wall is shown in figure III.1.

The velocity of the wall was therefore

$$\frac{\partial h}{\partial t} = - \frac{A\pi}{T} \sin(\frac{\pi z}{\lambda}) \sin(\frac{2\pi t}{T}) \tag{15}$$

The above in nondimensional form using the previous nondimensionalization scheme and

$$h^{*} = \frac{h}{a} \tag{16}$$

alongwith the fact that the radial velocity was nondimensionalized by $\frac{a\nu}{L^2}$ and the velocity of the fluid and wall should match at the wall the resulting boundary condition for the radial velocity was

$$u\ (h,\ z,\ t) = - \frac{\Pi 3 \Pi 1^2 \Pi 4^2}{2\Pi 2^2} \sin \pi \Pi 4 z \sin 2\pi t$$

where

$$\Pi 3 = \frac{A}{a}$$

$$\Pi 4 = \frac{L}{\lambda} \tag{17}$$

One other parameter was found from the boundary condition on the pressure that was applied on the pump. In the experiment the pressure across the pump

SHAPE OF DEFORMATION IS ASSUMED TO BE

$$h = a - \frac{A}{2}\sin\left(\frac{\pi z}{\lambda}\right)\left(1 - \cos\frac{2\pi t}{T}\right)$$

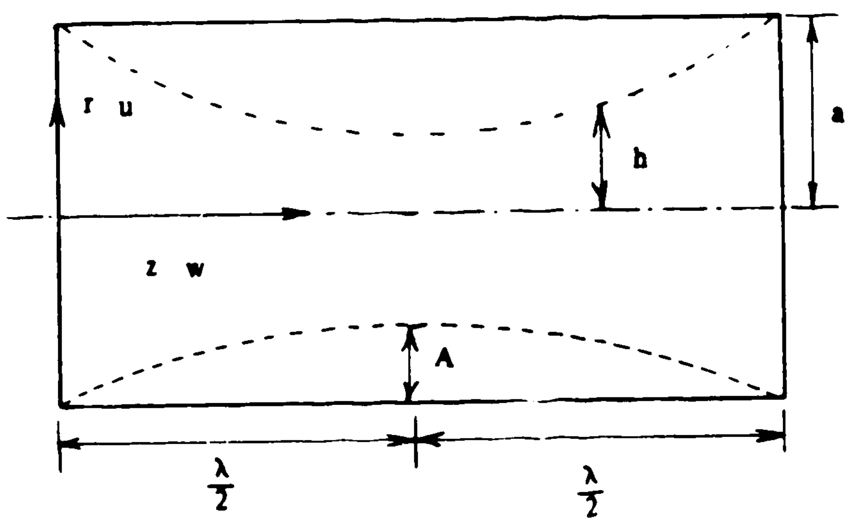


Figure III.1 The shape of the deformation in one chamber and relevant symbols.

was given by

$$\Delta p(t) = p(r, 0, t) - p(r, L, t) \tag{18}$$

$\Delta p(t)$ is shown in figure IV.4. There were difficulties in obtaining $\Delta p(t)$ experimentally, particularly with the presence of the one way valve. This was because there was a tendency for reflux in the pump which caused an error in the measurement of $\Delta p(t)$. To avoid this difficulty $\Delta p(t)$ was replaced by $\rho g \Delta H$ N/m^2 where ΔH was set experimentally as shown in figure IV.1. The reference pressure was $\frac{\mu\nu}{a^2}$ and therefore a new parameter was found to be $\frac{a^3 g \Delta H}{\nu^2}$.
So three new parameters were found in the boundary conditions, they are

$$\Pi 3 = \frac{A}{a}$$
$$\Pi 4 = \frac{L}{\lambda}$$
$$\Pi 5 = \frac{a^3 g \Delta H}{\nu^2}$$

Finally the parameters governing this problem were

(1) Frequency parameter $= \Pi 1 = a\sqrt{(\frac{\omega}{\nu})}$

(2) Wavelength parameter $= \Pi 2 = \frac{a}{\lambda}$

(3) Amplitude Parameter $= \Pi 3 = \frac{A}{a}$

(4) No. of Chambers $= \Pi 4 = \frac{L}{\lambda}$

(5) Pressure parameter $= \Pi 5 = \frac{a^3 g \Delta H}{\nu^2}$

All the above influence the mean flow rate Q. This flow Q was nondimensionalized by the factor $\frac{a^3\nu}{L}$, this factor was found by integration of the axial velocity over the cross-section of the tube and conversion to nondimensional form.

So the problem could be stated as (in terms of nondimensional variables)

$$\frac{1}{r}\frac{\partial(ru)}{\partial r} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\Pi 1^2}{2\pi}\frac{\partial u}{\partial t} + \frac{\Pi 2^2}{\Pi 4^2}\left(u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right)$$

$$= -\frac{\Pi 4^2}{\Pi 2^2}\frac{\partial p}{\partial r} + \frac{\partial^2 u}{\partial r^2} - \frac{u}{r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\Pi 2^2}{\Pi 4^2}\frac{\partial^2 u}{\partial z^2}$$

$$\frac{\Pi 1^2}{2\pi}\frac{\partial w}{\partial t} + \frac{\Pi 2^2}{\Pi 4^2}\left(u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right)$$

$$= -\frac{\partial p}{\partial z} + \frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\Pi 2^2}{\Pi 4^2}\frac{\partial^2 w}{\partial z^2}$$

(19)

with boundary conditions

(1) $u\,(h,\ z,\ t) = -\dfrac{\Pi 3 \Pi 1^2 \Pi 4^2}{2\Pi 2^2}\sin \pi\Pi 4 z \sin 2\pi t$

(2) $w\,(h,\ z,\ t) = 0$

(3) $\dfrac{\partial w}{\partial r}\Big|\,(0,\ z,\ t) = 0$

(4) $u\,(0,\ z,\ t) = 0$

(5) $\dfrac{\partial u}{\partial r}\Big|\,(0,\ z,\ t) = 0$

(20)

Also fluid velocities were finite at all times. For pressure a pressure differential Δp across the pumping unit is assigned. This differential corresponds to the level difference ΔH between the two level controlled tanks.

In the chapters ahead the effect of the parameters that have been found by nondimensionalizing the equations will be investigated experimentally.

IV. APPARATUS

In order to investigate the effect of the parameters found in the chapter on equations two peristaltic pumps were constructed, they consisted of four and eight chambers respectively.

The inlet side of the pump was connected to a fixed head tank. This tank could be moved up and down so that the pressure against which the pump operated could be changed. Between this tank and the pump a one way valve was connected to simulate the pyloric sphincter. After this valve and before the pump a tank was kept to bleed the air out of the system. The outlet of the pump was connected to another fixed head tank from which the flow due to the pump could be collected through a overflow pipe into a measuring flask. A schematic of the system is shown in IV.1.

The main pumping unit consi flexible silicone rubber tube of 1 cm radius. This tube was surrounded ur or eight plexiglass chambers of 5 cm length and 3 cm diameter. The chambers were constructed so that they were sealed completely and could be pressurized to make the tube contract. The tube was made by pouring a solution of silicone rubber compound and toluene on a pipe of 1 cm radius that was rotating at a low speed of about 5 RPM. It took 4 to 5 hours for the tube to solidify. Several coats of the above material were poured so that a tube wall thickness of 2 mm was achieved. Later 5 or 9 washers were made by pouring the above solution on a glass sheet and then 1 cm holes were made in these silicone rubber sheets. The washers were then glued to the tube so that in effect four or eight sealed chambers could be created on assembly of the unit, see figure IV.2.

These four or eight chambers were pressurized in sequence using cams, limit switches and 3-way solenoid valves so that pumping occurred due to passage of a contraction wave. The cams were such that they allowed contractions over one-third of one revolution. The sequence of contractions used to produce

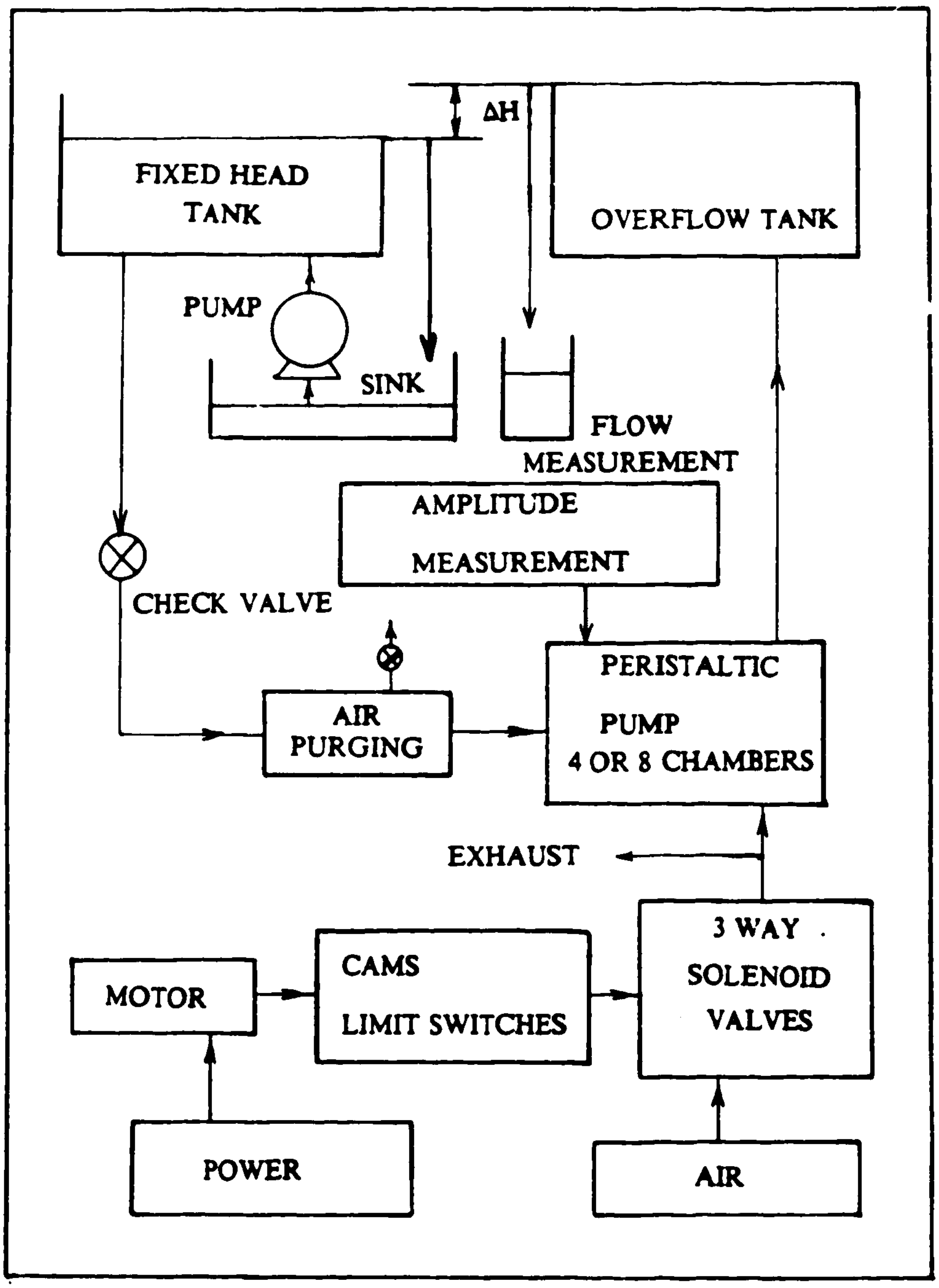


Figure IV.1 Schematic diagram of the experimental setup.

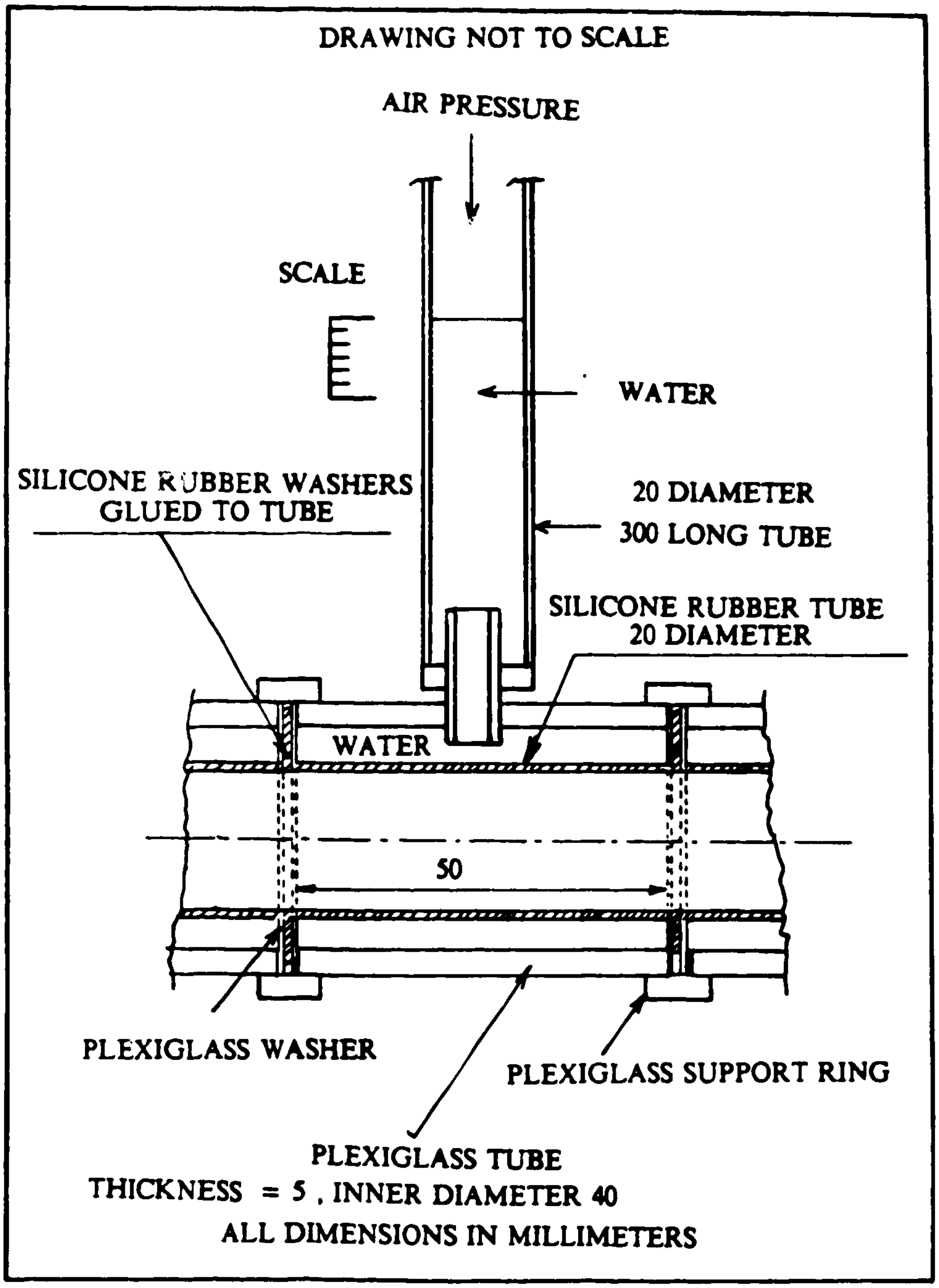


Figure IV.2 One chamber of the pump.

pumping is shown in figure IV.3.

The experiment was performed by measuring the flow rate while controlling amplitude, pressure, frequency, and number of chambers parameters. These parameters were a result of nondimensionalization of the Navier-Stokes equations of motion. The pressure difference between the inlet and outlet of the pump Δp was measured using a capacitive differential transducer, a validyne bridge circuit and an HP strip chart recorder. It was found that the pressure rise at the entrance of the pump, which was caused by the presence of the one way valve, affected the measurement of Δp adversely. For this reason Δp was not measured directly but was replaced by $\rho g \Delta H$. For presentation of experimental results the parameter $\frac{ga^3\Delta H}{\nu^2}$ was used.

A plot of pressure difference across the pump when the pump is operating is shown in figure IV.4. This plot is intended to be illustrative of the pressure variations and the variations have been copied from the chart by hand.

The amplitude parameter was determined by using a vertical water column that moved in proportion to the amplitude of contraction of the chamber that was contracting. Such columns were part of each [illegible] ;e chambers. The displacement of the water column was related to the amplitude parameter by assuming that the contraction was sinusoidal in shape. This assumption was made so that the experimental results could be compared to theories that assumed sinusoidal contractions, Barton & Raynor (1968).

The flow rate was measured by observing the time it took for collection of a given volume of fluid in a measuring flask. Care was taken so that at least five contraction waves had passed through the pump. This was necessary as the flow varied slightly in one cycle of operation and the mean flow was required for plotting of results.

A photograph of the pump is shown in plate IV.1. The whole experimental apparatus can be seen in plate IV.2. In plate IV.1 the various

ANGLE IN DEGREES	CHAMBER I	CHAMBER I+1
0-90	CONTRACTED	OPEN
90-120	CONTRACTED	CONTRACTED
120-210	OPEN	CONTRACTED
210-360	OPEN	OPEN

PUMPING →

CHAMBER NUMBER

1 2 3 4 5 6 7 8

The above shows the contraction sequence between the I and I+1 chamber assuming the pump starts such that I chamber begins contracting. The angle is the amount the cam shaft rotates.

Figure IV.3 Explanation of contraction sequence.

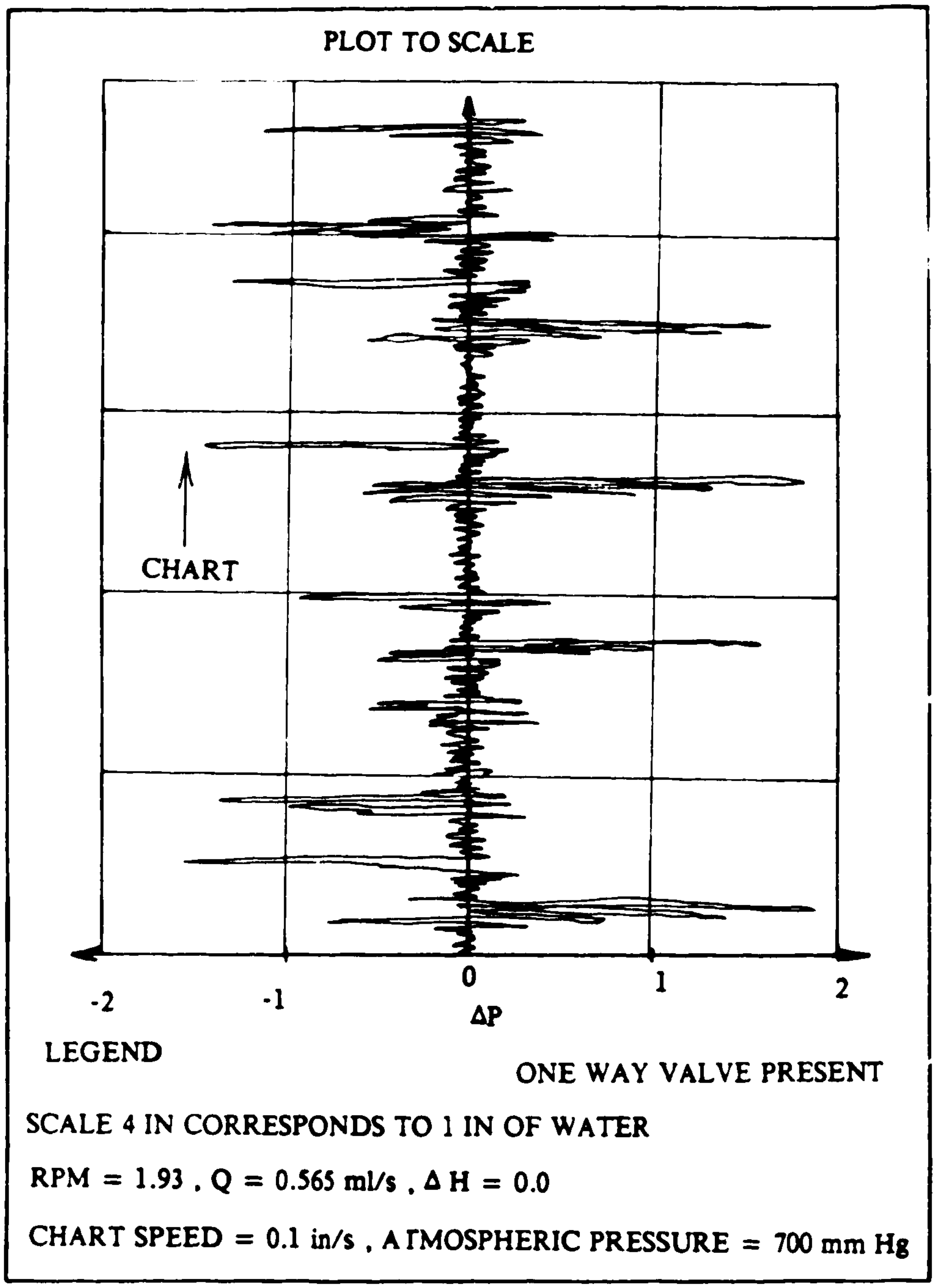


Figure IV.4 Plot of pressure difference between the inlet and outlet of the pump.

numbers represent the following:

(1) The pump with four chambers. The vertical water columns can also be seen.

(2) The cam shaft driven by a continuously variable speed gear box.

(3) The cams.

(4) Limit switches.

(5) The counter.

In plate IV.2 the numbers represent the following parts of the system:

(1) The pump.

(2) Differential pressure transducer.

(3) Solenoid valves.

(4) Manometers.

(5) Downstream fixed head tank.

(6) Strip chart recorder.

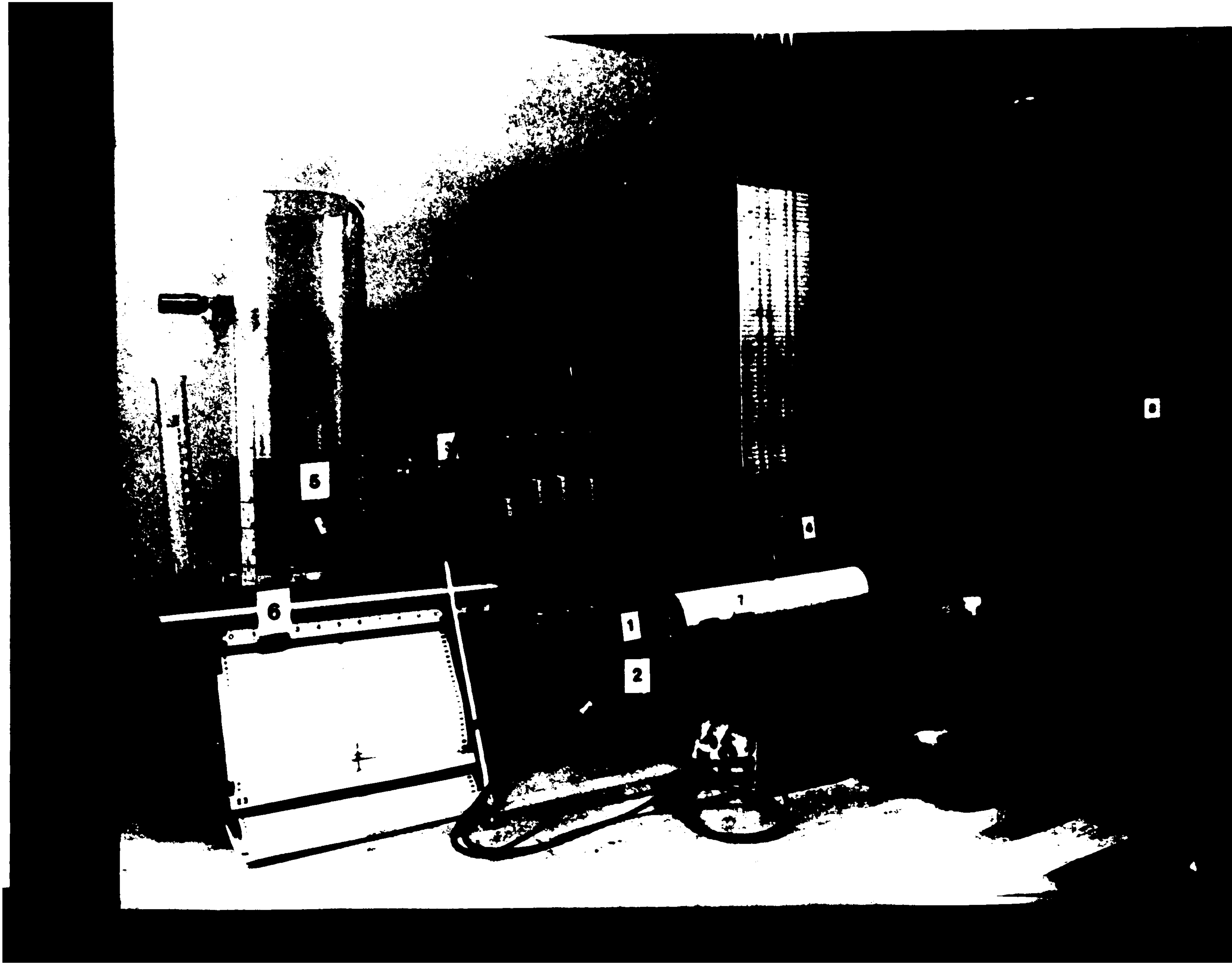
(7) Air bleeding tank.

(8) Supply tank (may be raised or lowered).

Plate IV.1 Closeup of peristaltic pump


4
3
2
5
1

Plate IV.2 Overall view of experimental apparatus


5
6
1
2

V. DISCUSSION

Experiments were carried out with the peristaltic pump described in the earlier chapters. The fluid used for experiments was water, the temperature was kept between 20 - 22 °C. The value of ν used for plotting of results was .01 cm^2/s, Schlichting (1968).

The experimental apparatus was such that the error involved in setting up of various parameters was as follows:

(1) $\frac{A}{a} \simeq 5\ \%$

(2) $\Delta H \simeq 0.1$ cm

(3) $Q \simeq 1\ \%$

(4) $N \simeq 2\ \%$

The experiments were done for the following ranges of operating parameters:

$$2 < \Pi 1 < 14$$

$$\Pi 2 = 0.2$$

$$0.0 < \Pi 3 < 1.0$$

$$3 < \Pi 4 < 8$$

$$0 < \Pi 5 < 3.43 \cdot 10^{8}$$

The main reason for operation in the above range was the limitation of the apparatus being used.

The length of the pump was changed by following the same sequence/timing of contractions as for the 8 chambered pump but by disconnecting 1, 2, 3, 4, or 5 chambers. This caused a time interval during which there were no contractions present in the pump. This did not cause any error in the case when the one way valve was present. When the valve was not present the error due to backflow (Reflux) was a maximum of 20 % in the case of pumping with 3 chambers.

The results of the experiments are shown in figures V.1 through V.9. The discussion of experimental results is divided into two main parts. In the first part the experiments with the pump operating with a one way valve present are described, in the second part the results of pumping without the valve are described.

A. Experiments with one way valve present

The two pumps were operated with a one way valve present. The following were the results of the experimental study:

(1) The nondimensional flow rate increased with the increase in frequency parameter. This increase seems to be linear in the experimental range A similar result was seen when experiments were done with the 8 chambered pump. See figure V.1 & V.2.

(2) The nondimensional flow rate increased with the increase in amplitude parameter. This increase was close to parabolic. See figure V.3.

(3) The nondimensional flow rate increased with the increase in number of chambers operating. The experiment in this part was done by making 3, 4, 5, 6, 7, or 8 chambers of the pump operational. The sequence of contractions was as described in figure IV.3. This type of experiment was done for 4 & 8 chambered pump and results were similar. See figure V.4.

(4) The nondimensional flow rate decreased as the pressure parameter increased. This was expected as the pumping should decrease when the head against which the pump works increases. See figure V.5.

From the above experiments the experimental relation of the nondimensional flow rate with all parameters except wavelength parameter was clear. The experimental results showed similar trends whether done on the 4 or 8 chambered pump.

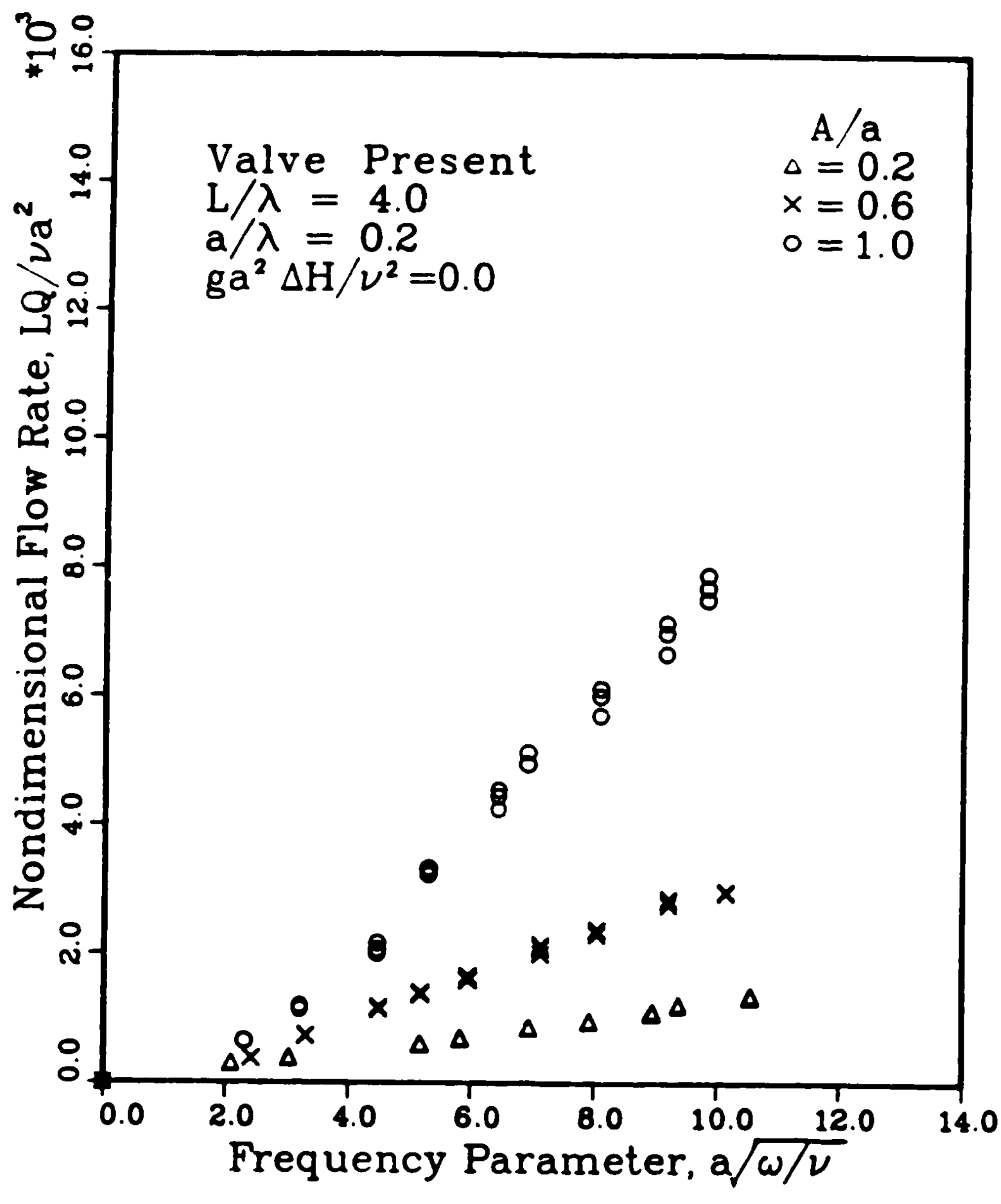


Figure V.1 Nondimensional flow rate vs frequency parameter, with effect of amplitude parameter. (Old pump, valve present)

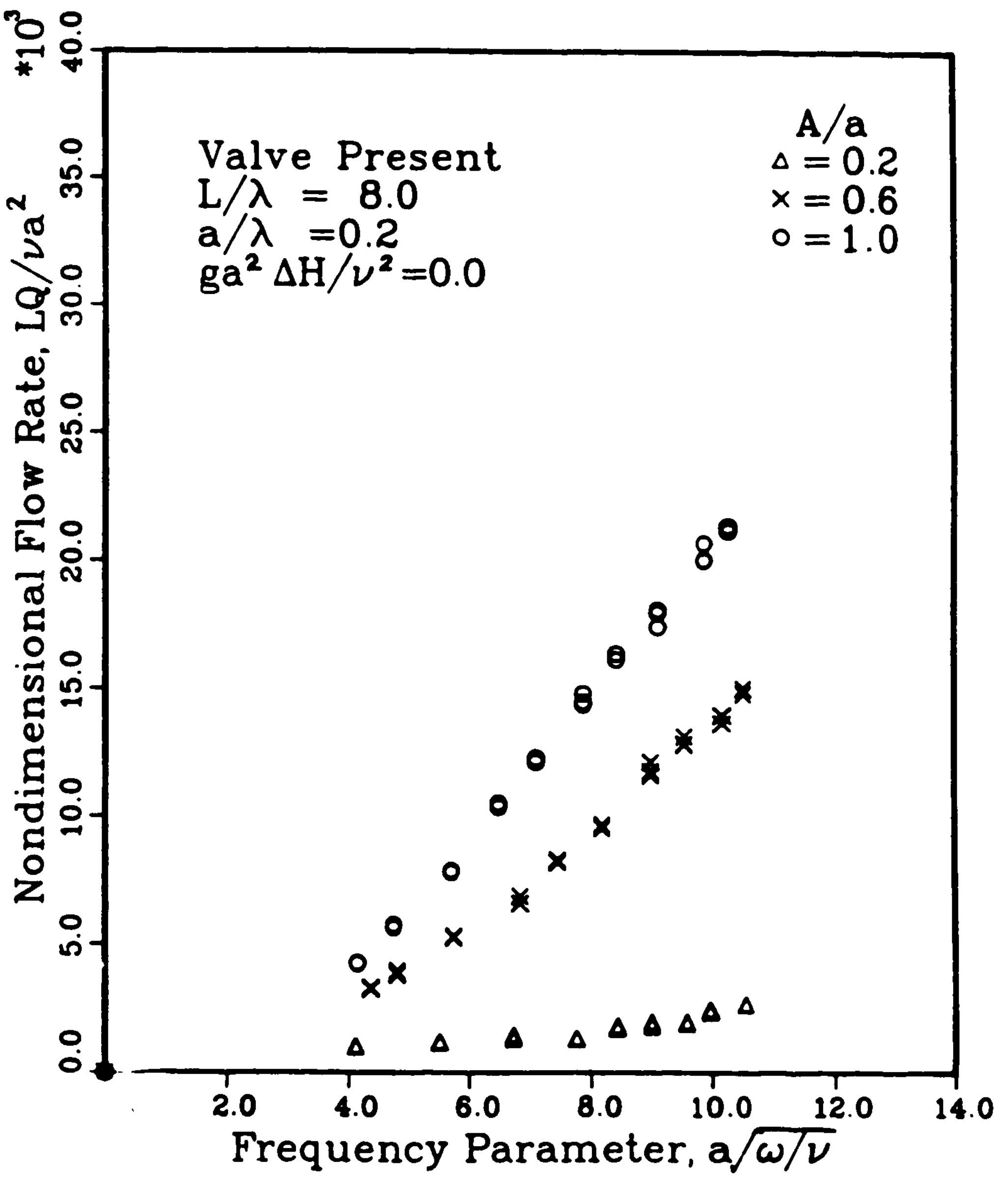


Figure V.2 Nondimensional flow rate vs frequency parameter, with effect of amplitude parameter.(New pump, valve present)

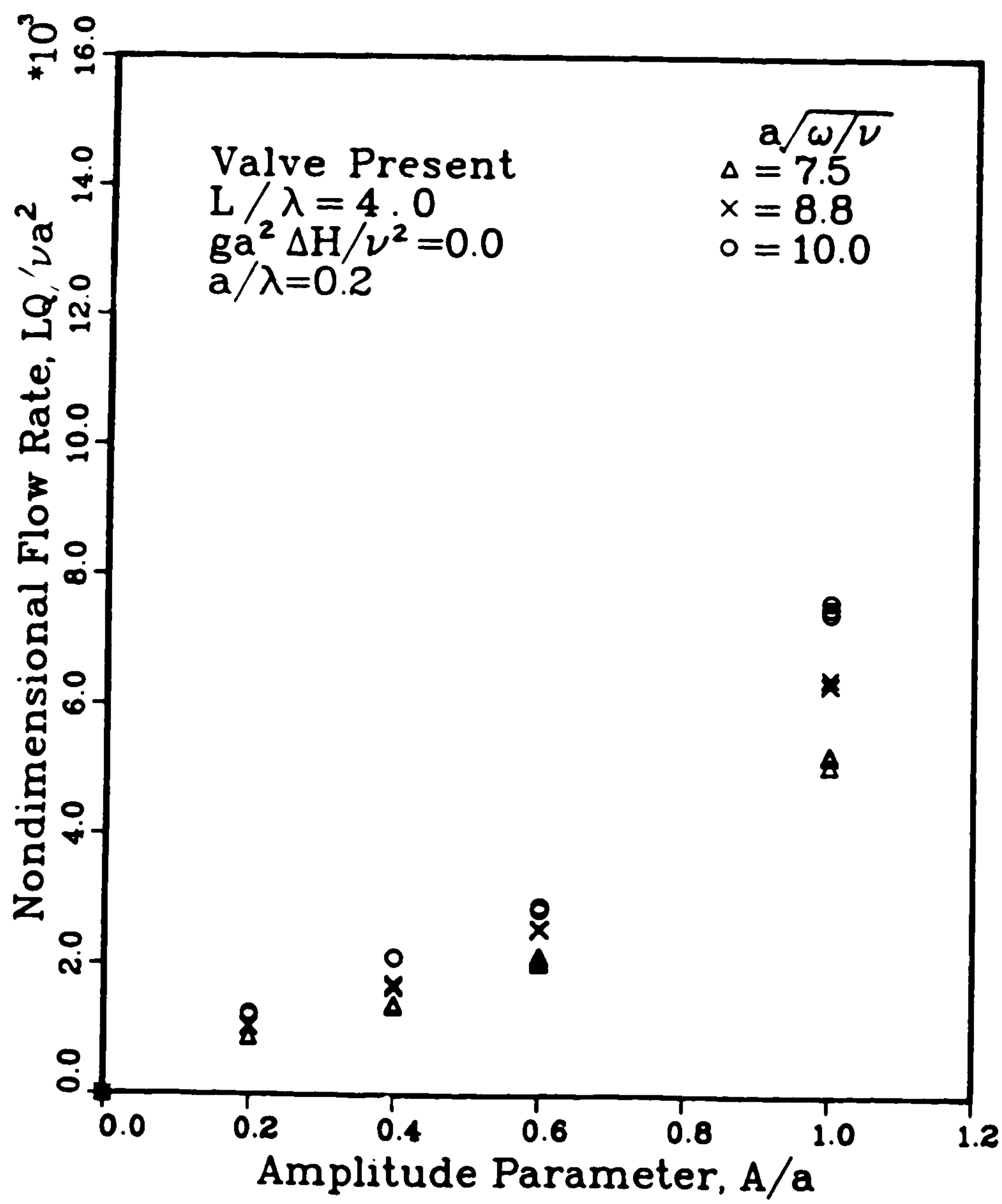


Figure V.3 Nondimensional flow rate vs amplitude parameter, with effect of frequency parameter. (valve present)

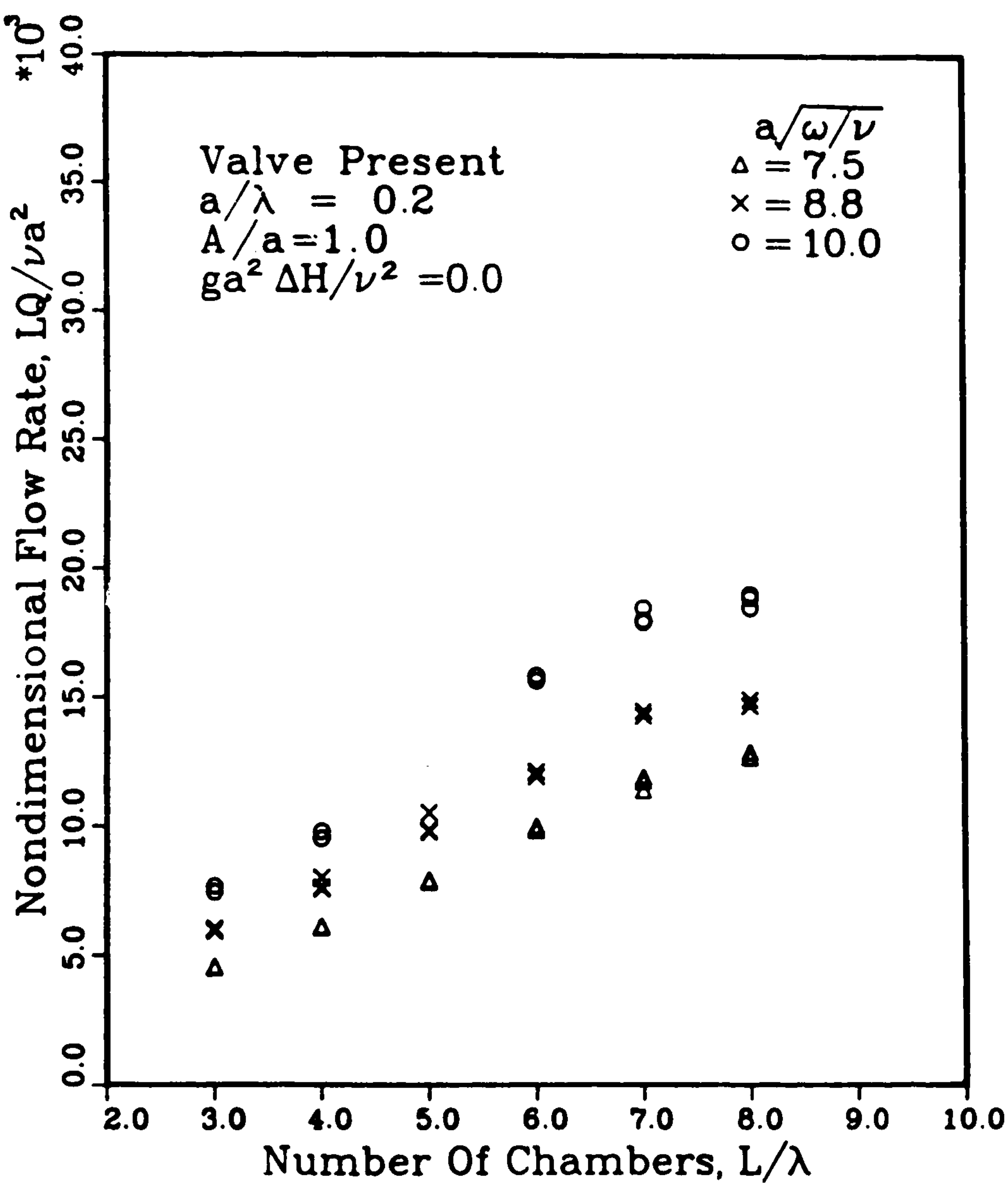


Figure V.4 Nondimensional flow rate vs number of chambers, with effect of frequency parameter. (valve present)

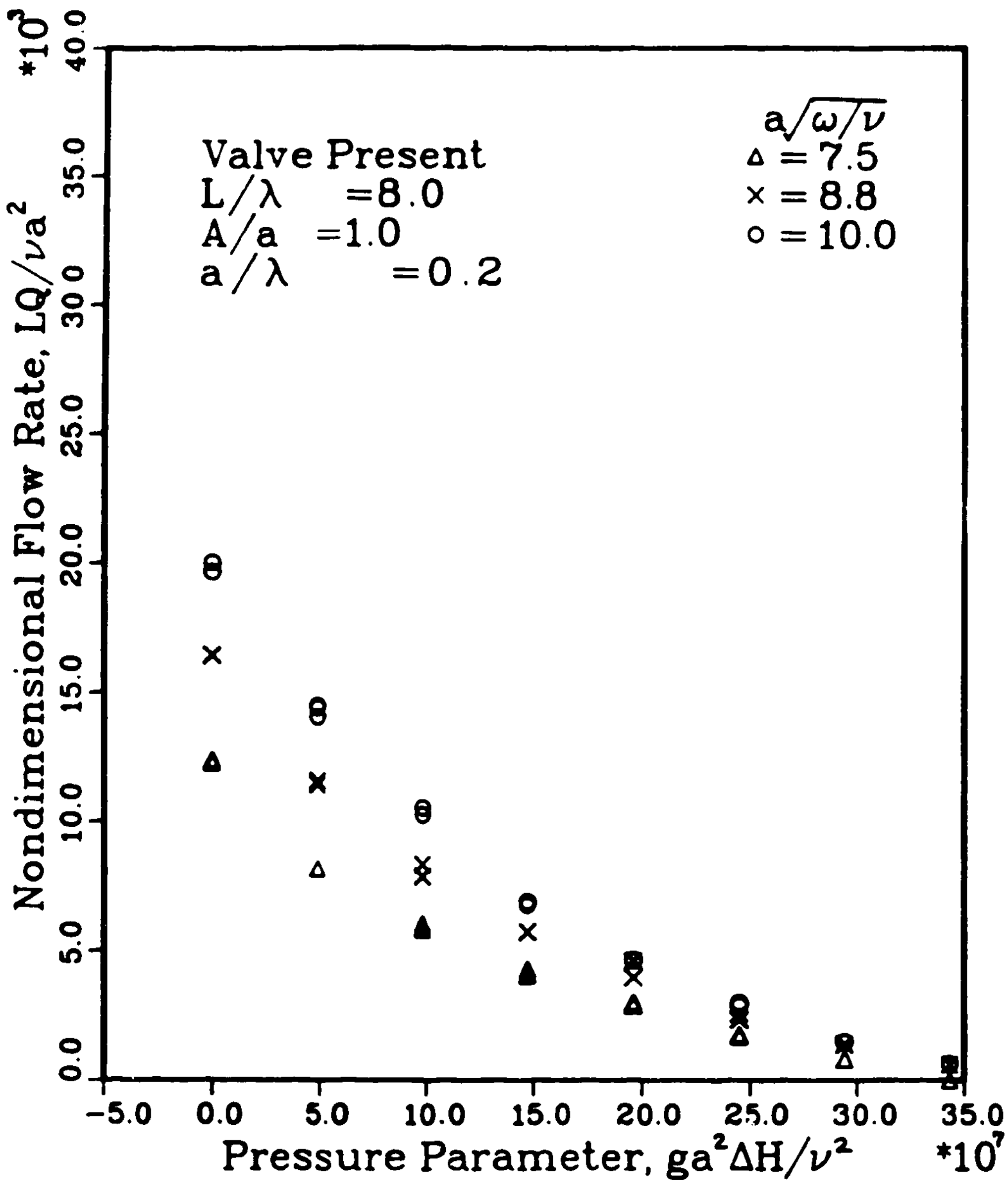


Figure V.5 Nondimensional flow rate vs pressure parameter, with effect of frequency parameter. (valve present)

B. Experiments without one way valve

In this section the effect of removal of the one way valve has been investigated. The case of one way valve was important in that it is present in the case of Duodenum. The case of pumping with no valve may be important in the case of human beings with a weak pyloric sphincter. In the case of pumping without one way valve the following was observed:

(1) Nondimensional flow rate increased with increase in frequency parameter. It was interesting to note that the pump was unable to operate when the amplitude ratios were small. It was also seen that for amplitude ratio of 1.0 the nondimensional flow rate was reduced by about 20-30% compared to that of pumping with the valve present. See figure V.6 for the details.

(2) The nondimensional flow rate increased with an increase of amplitude parameter. This can be seen in figure V.7. This was similar to the case of pumping with the valve present except that no pumping occurred when the amplitude parameter was less than 0.4.

(3) The nondimensional flow rate increased with increase in number of chambers operating. It was seen that pumping was significant only when more than 5 chambers operated. See figure V.8.

(4) The nondimensional flow rate decreased with increase in pressure parameter. This can be seen in figure V.9. The effect was similar to the case of pumping when valve was present except that pumping occurred for a smaller range of pressure parameter.

As can be seen from the above experiments the mean flow rate has been investigated with respect to all the parameters describing the pump except wavelength parameter.

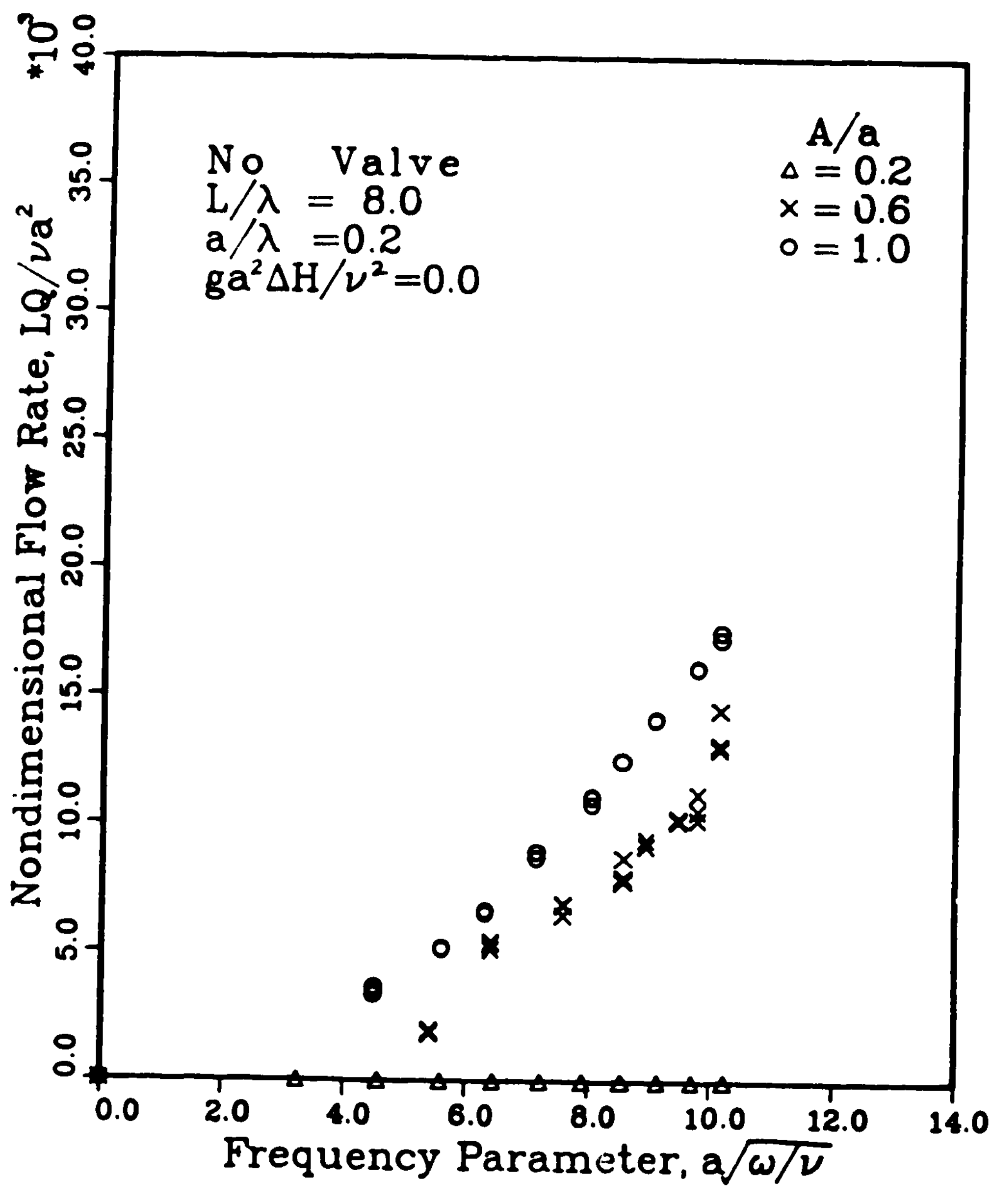


Figure V.6 Nondimensional flow rate vs frequency parameter, with effect of amplitude parameter. (no valve)

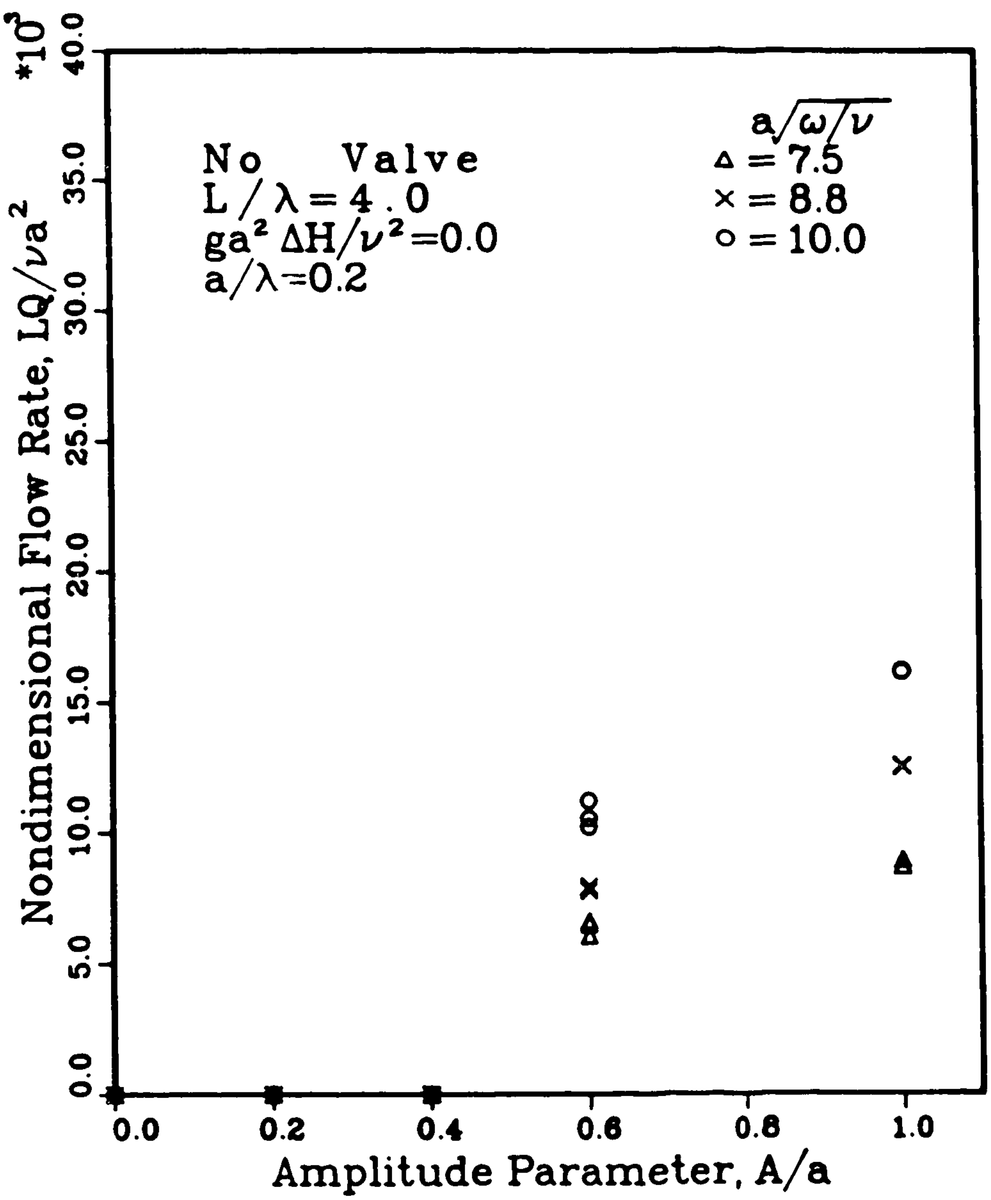


Figure V.7 Nondimensional flow rate vs amplitude parameter, with effect of frequency parameter. (no valve)

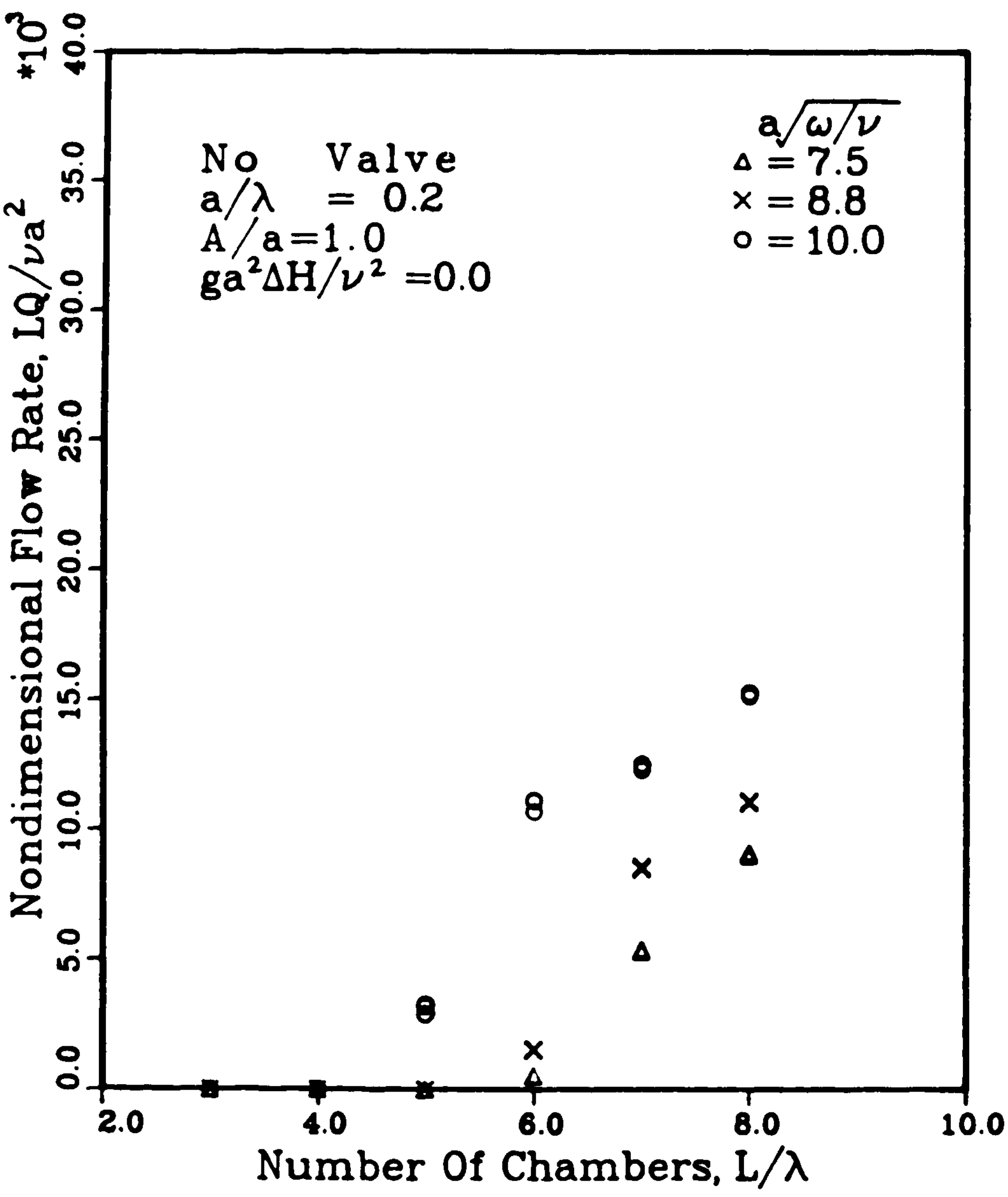


Figure V.8 Nondimensional flow rate vs number of chambers, with effect of frequency parameter. (no valve)

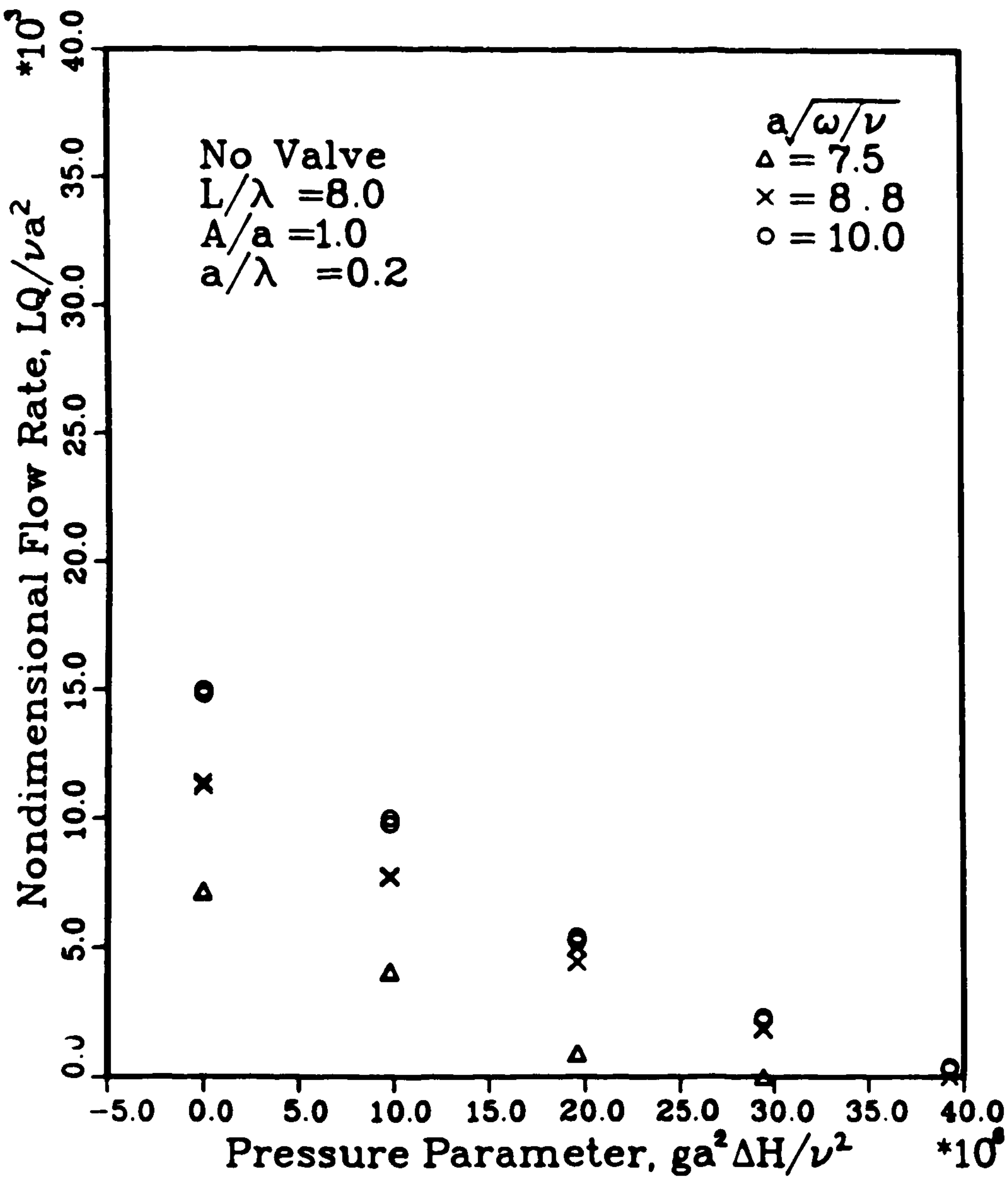


Figure V.9 Nondimensional flow rate vs pressure parameter, with effect of frequency parameter. (no valve)

C. **Comparison of results with theory**

Many theories have been put forth to explain peristaltic pumping. They include, Yin & Fung (1971), Weinberg et. al. (1971), Barton & Raynor (1968), Pozrikidis (1987), Takabatake (1988) et. al. and Jaffrin & Shapiro (1971).

The theories of Pozrikidis (1987), Takabatake et. al. (1988) were not suitable for comparision with this experiment as they did not consider dependence of mean flow rate on the parameters of the the problem of peristalsis in a tube. The theories of Yin & Fung (1971) and Weinberg et. al. (1971) were not considered for comparison as they were related to peristalsis in 2-dime... channels and not tubes.

The theories chosen for comparison were Jaffrin & Shapiro (1971) and Barton & Raynor (1968). Both of these theories considered axisymmetric tubes with peristaltic waves. Upon analysis both theories resulted in the following equation for mean flow rate:

$$Q = \frac{\pi c A^2}{2} \left(\frac{16a^2 - A^2}{2a^2 + 3A^2} \right) \tag{21}$$

The above result is valid for pumping in an infinite tube with $\Delta H = 0$, $\frac{a}{\lambda} \ll 1$ and small Reynolds number. In the experiment $\frac{a}{\lambda} = 0.2$ and Re=2 corresponded to $\Pi 1 = 3.5$.

The above equation, after conversion to the nondimensional form used in experiments, became

$$Q \text{ (Nondimensional)} = \Pi 4 \, \frac{\Pi 1^2 \Pi 3^2}{4.\Pi 2^2} \left(\frac{16 - \Pi 3^2}{2 + 3.\Pi 3^2} \right) \tag{22}$$

In forming the above equation it was assumed that the equivalent wave velocity for the experiment was

$$c = \frac{\lambda}{T} \text{ where } T = \frac{2\pi}{\omega} \tag{23}$$

The results of the comparison can be seen in figures V.10 and V.11. It was seen that the flow rates were in good agreement for the case of pumping without a valve. Also the flow for the case of pumping with 8 chambers was closer to the theory because the pump was longer and the theory was for a infinite pump. It was also seen that the theory predicted the order of magnitude of pumping for both the cases. The main causes of differences in theory and experiment are:

(1) There was no wave in the experiment, the motion was caused by discrete timed contractions in a particular sequence.

(2) The shape of contractions was not sinusoidal in the experiment.

Barton & Raynor (1968) have used the above theory of pumping in an axisymmetric tube to predict the flow rate of chyme in the intestine to within 28 %.

D. Limitations of the experiment and suggestions for future work

The limitations of the program were that the experiments could be done only within a small range of parameters. In particular the effect of wavelength parameter on the flow rate may be an interesting area for further investigation. The wavelength parameter was fixed for both of the pumps.

It was observed that the nondimensional flow rate increased with increasing frequency parameter for frequency parameter less than 20. It would be interesting to see the flow relationship with all of the parameters for a wider range of experiments.

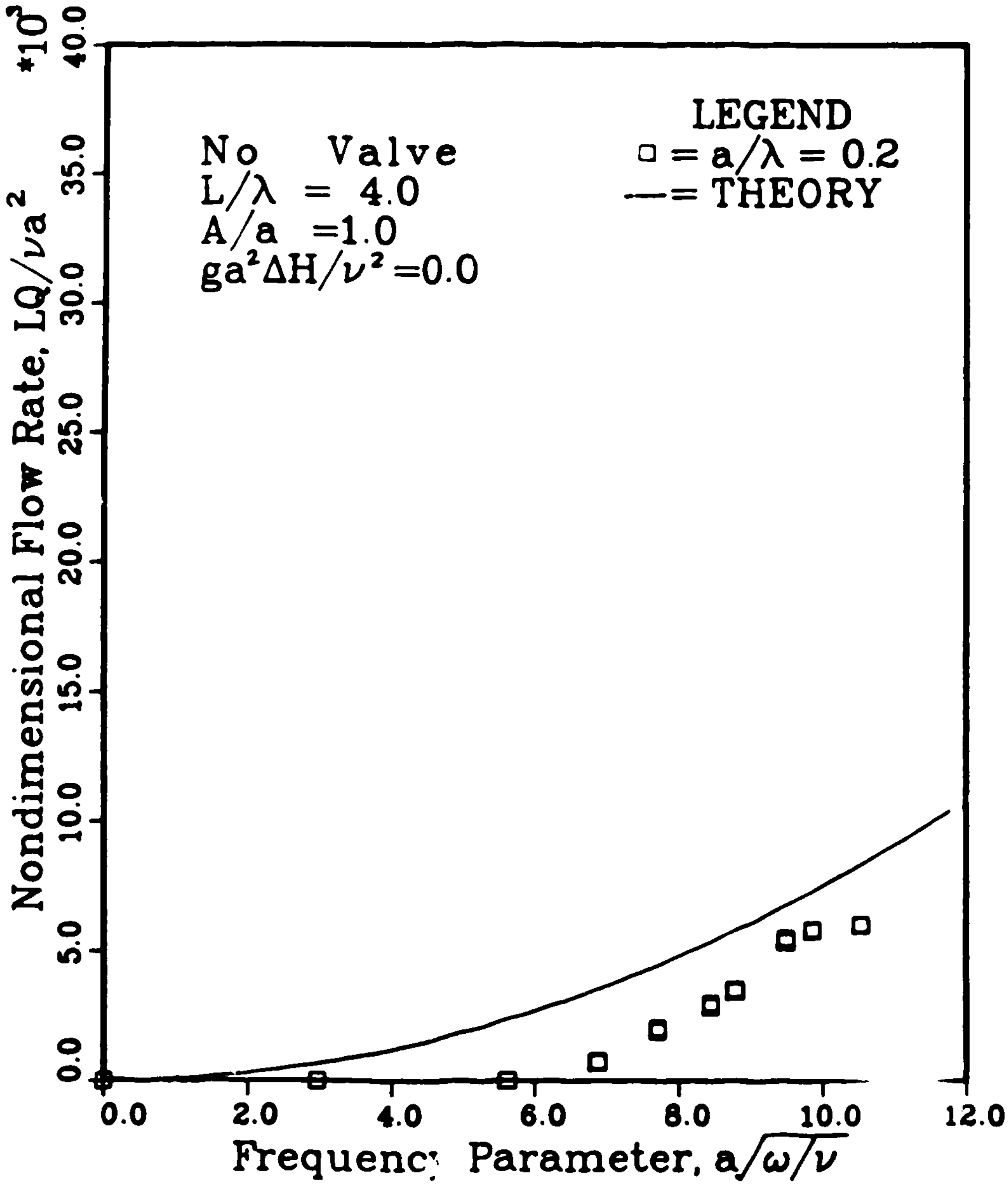


Figure V.10 Nondimensional flow rate vs frequency parameter, comparison with theory for no valve and short pump.

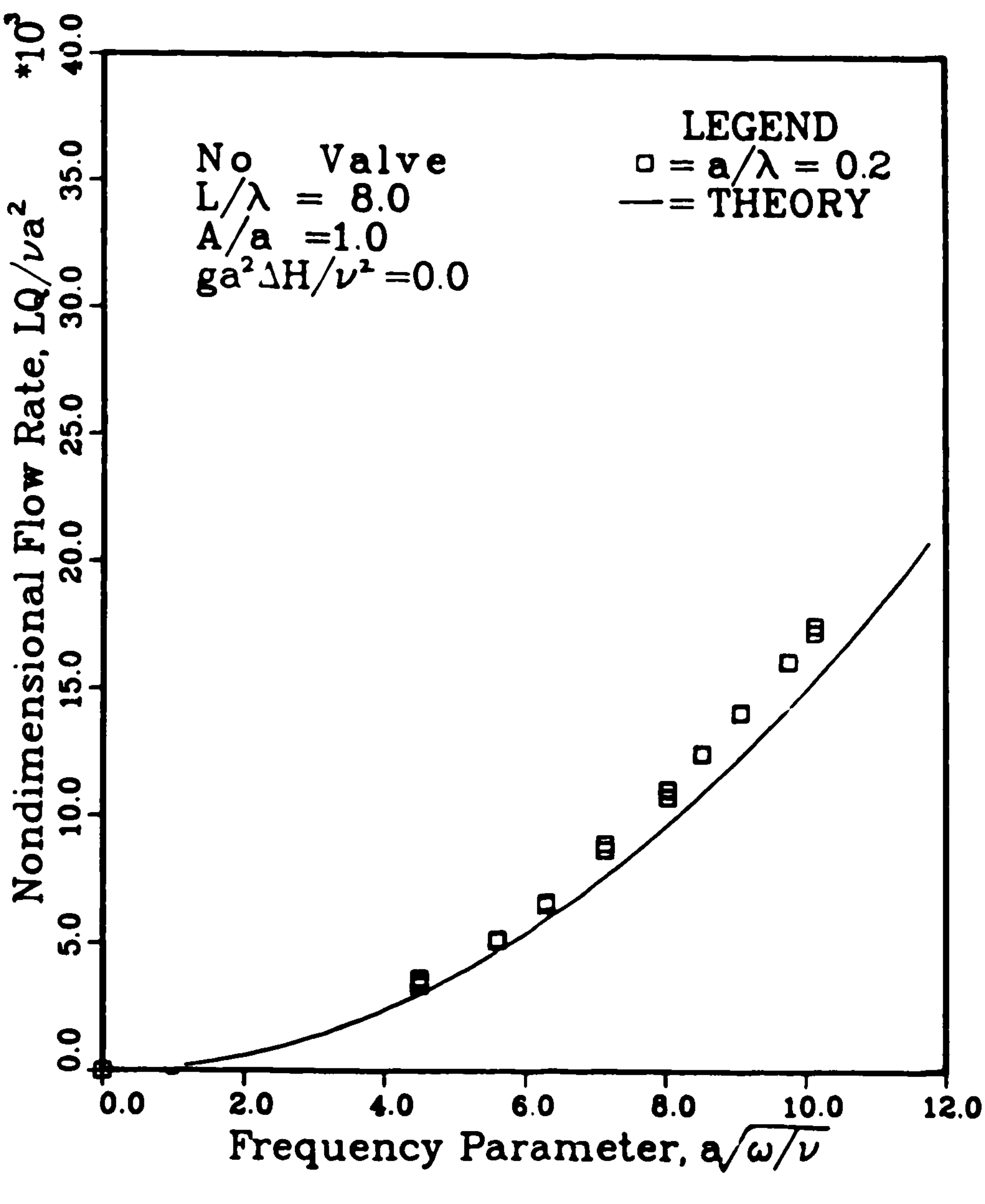


Figure V.11 Nondimensional flow rate vs frequency parameter, comparison with theory for no valve and long pump.

The effect of assumptions made in deriving the parameters of the problem was to make the experimental results amenable to comparisons with previous theories.

Further work could be done in the following areas:

(1) Modification of experiment to include the effect of expansion and other motions of the duodenum.

(2) The effect of gravity on the peristaltic flow could be an interesting area for further work.

(3) The pumping characteristics with a non-newtonian fluid should be tested. In this experiment it was not possible as the silicone rubber of the pump material reacted adversely to the oils that were used.

(4) A more suitable theory could be formulated for explaining the results of this experiment. This could be done by accounting for non sinusoidal shape of the contractions. The fact that the contractions were not in the form of a wave but discrete in nature could also be considered in the development of a theoretical explanation of the experimental results.

VI. CONCLUSIONS

After experimentation on the two peristaltic pumps of length 20 and 40 cm the following conclusions were drawn:

(1) The nondimensional flow rate increased with increase of amplitude parameter, frequency parameter and no of chambers parameter.

(2) The nondimensional flow rate decreased with increase of pressure parameter.

(3) The comparision of experiment and theory showed agreement to within 20% in the range of experimentation for the case of the 40 cm pump.

BIBLIOGRAPHY

Barton, C. & Raynor, S. (1968) " Peristaltic flow in tubes." Bull. Mathematical Biophysics vol 30:663-680.

Boyarsky, S. (1964) "Surgical physiology of the renal pelvis." Monogr. Surg. Sci. vol 1:173-213.

Brown, T. D. & Hung, T. K. (1977) "Computational and experimental investigation of two dimensional nonlinear peristaltic flows." J. Fluid Mech. vol 83:249-272.

Burns, J. C. & Parkes, T. (1967) " Peristaltic motion." J. Fluid Mechanics vol 29:731-743.

Christensen, J., Macagno, E. O. & Melville, J. G. (1978) " Motility and flow in the small intestine." J. of Engineering Mechanics Division vol 104:11-29.

Dillard, R. L., Eastman, H. & Fordtran, J. S.(1965) " Volume flow relationship during transport of fluid through human small intestine. " Gastroenterology vol 49:58-66.

Fung, Y. C. & Yih, C. S. (1968) " Peristaltic transport." Trans. ASME, J. of Applied Mechanics vol 35:669-675.

Jaffrin, M. Y. & Shapiro, A. H. (1971) " Peristaltic pumping. " Ann. Rev. Fluid Mech. vol 3:13-36.

Kerlin, P., Zinmeister, A. & Phillips, S.(1982) " Relationship of Motility to flow contents in the human small intestine. " Gastroentrology vol 82:701-706.

Liron, N. (1976) " On peristaltic flow and its efficiency." Bull. of Mathematical Biology vol 38:573-596.

Macagno, E. O. & Christensen, J. (1980) " Fluid mechanics of the Duodenum." Ann. Rev. Fluid Mechanics, vol 12:139-158.

Macagno, E. O. & Christensen, J. (1981) "Fluid mechanics of gastrointestinal flow." in Physiology of the Gastrointestinal tract, edited by Johnson, L. R., Raven Press New York:335-358.

Macagno, E. O., Christensen, J. & Lee, C. L. (1982) " Modelling the effect of wall movement on absorbtion in the intestine." Am. J. Phsiol., vol 243:541-550.

Melville, J. , Macagno, E. O. & Christensen, J. (1975) " Longitudinal Contractions in the Duodenum: their fluid mechanical function." Am. J. Phsiol. vol 228:1887-1892.

Netter, F. H. (1966) " The CIBA collection of Medical illustrations." vol 3 part I and II, Digestive system, CIBA pharmaceutical products, Montreal.

Piersol, G. A. (1930) " Human Anatomy." J. B. Lippincott Company.

Pozrikidis, C. (1987) " A study of peristaltic flow." J. Fluid Mech. vol 180:515-527.

Scher, J. A. (1979) " Industrial waste water pumps. " Fluid movers pumps and compressors, Fans and Blowers, Edited by Matley, J. McGraw Hill.

Schlichting, H. (1968) " Boundary layer theory. " 6 (Ed), McGraw Hill, New York.

Shapiro, A. H., Jafrin, M. Y. & Weinberg, S. L. (1969) "Peristaltic pumping with long wavelengths at low reynolds numbers." J. Fliud Mech. vol 37:799-825.

Shukla, [illegible], Parihar, R. S. & Rao, B. R. (1980) "Effects of peripheral [illegible] viscosity on peristaltic transport of a biofluid." J. Fluid Mech. vol 97:225-23[illegible].

Takabatake, S. & Ayukawa, K. (1982) "Numerical study of two dimensional peristaltic flows." J. Fluid Mech. vol 122:439-465.

Takabatake, S., Ayukawa, K. & Mori, A. (1988) " Peristaltic pumping in circular cylindrical tubes: a numerical study of fluid transport and its efficiency." J. Fluid Mech. vol 193:267-283.

Tong, P. & Vawter, D. (1972) "An analysis of peristaltic pumping." J. Appl. Maths vol 33:857-869.

Weinberg, S. L., Eckstein, E. C. & Shapiro, A. H. (1971) "An experimental study of peristaltic pumping." J. Fluid Mech. vol 49:461-479.

Yin, Y. C. & Fung, Y. C. (1971) "Comparison of theory and experiment in peristaltic transport." J. Fluid Mech. vol 47:93-112.

Zien, T. F. & Ostrach, S. (1970) "A long wave approximation to peristaltic motion." J. Biomechanics vol 13:63-75.

VITA

NAME: Vijay Shukla

PLACE OF BIRTH: Gonda, Uttar Pradesh, INDIA.

YEAR OF BIRTH: 1963

POST SECONDARY EDUCATION:

(1) Bachelor of Technology in Mechanical Engineering from Indian Institute of Technology, Kanpur.

(2) Master of Science in Mechanical Engineering from University of Alberta, Edmonton. (Spring 1989).

HONOURS AND AWARDS:

(1) Awarded one year exemption at Indian Institute of Technology Kanpur on the basis of an all India competitive examination and thus completed the five year programme in four years.

(2) Awarded teaching assistantship at the Department of Mechanical Engineering, University of Alberta, Edmonton.

RELATED WORK EXPERIENCE:

(1) Worked for two and a half years as an engineer at Maruti Udyog Limited, an automobile manufacturer in Delhi, INDIA.

(2) Worked as a teaching assistant, during September 1987 to April 1989, in Department of Mechanical Engineering and helped with lab courses MEC E 401, MEC E 403 & MEC E 431.