RURAL ECONOMY

The Inverse Lewbel Demand System

James S. Eales

Staff Paper 92-06

STAFF PAPER

Department of Rural Economy
Faculty of Agriculture and Forestry
University of Alberta
Edmonton, Canada

The Inverse Lewbel Demand System

James S. Eales

Staff Paper 92-06

The author is an Associate Professor, Department of Rural Economy, University of Alberta, Edmonton.

The purpose of the Rural Economy "Staff Paper" series is to provide a forum to accelerate the presentation of issues, concepts, ideas, and research results within the academic and professional community. Staff Papers are published without peer review.

THE INVERSE LEWBEL DEMAND SYSTEM

Lewbel (1989) offered a demand model which nested both the indirect tanslog (ITL) of Christensen, Jorgenson, and Lau (1977) and almost ideal demand system (AIDS) of Deaton and Muellbauer (1980a, 1980b). It has the advantage, then of allowing the applied demand analyst to test the restrictions which imply the ITL and AIDS models directly. In terms of parametric analysis of demand, the increased generality of Lewbel's demand system should minimize the impact of maintained hypotheses on the outcome of the statistical analysis. All of these models have appealing theoretical properties, they correspond to a well defined preference structures, which is convenient for welfare analysis. These so-called PIGLOG preferences also have the property of consistent aggregation from the micro to the market level, while allowing nonlinear Engel curves. Second, the functional form of the preferences is "flexible" in that it can be thought of as a local second order approximation to an unknown preference structure. Third, homogeneity and symmetry restrictions depend only on estimated parameters and so are easily imposed and/or tested.

There are commodities for which the assumption of predetermined prices at the market level may not be viable. Some of the earliest applied work in demand for agricultural products took current supplies as fixed and therefore specified ad hoc inverse demand curves for statistical evaluation. This alternative aggregation story is still employed, especially by those building market models, such as Freebairn and Rausser (1975) and Arzac and Wilkinson (1979). So, for example, if modeling demand for a perishable commodity, the production of which is subject to long biological lags, the researcher might employ inverse demands. Production lags prevent market-level supply response, while perishability requires the commodity be consumed. Thus, price must adjust.

Not all previous studies which have employed inverse demand structures have proceeded in an ad hoc manner. Heien, and Chambers and McConnell developed separable inverse demand systems and applied them to food commodities. Barten and Bettendorf developed an inverse Rotterdam system and applied it to the demand for fish. Christensen, et al. develop the direct translog demand system (as well as the indirect system). Both they and Jorgenson and Lau use the direct translog demand system to test demand restrictions. Huang used the theoretical development of Anderson and the distance function to generate a system of inverse demands, which were applied to composite food and nonfood commodities. Eales and Unnevehr also employed a particular distance function to develop an inverse AIDS model.

In the sections that follow a model which nests both the direct translog (DTDS) and the inverse almost ideal demand system (IAIDS) is developed. This system will be referred to as the inverse Lewbel (ILDS). These three demand systems are then compared and contrasted and used to model Canadian demand for meat.

The Inverse Lewbel Demand System

Following Lewbel, specify the following utility function:

$$\ln U = \Sigma_i \beta_i \ln q_i + \ln (\ln Q) \tag{1}$$

where:

$$\ln Q = \alpha_0 + \Sigma_i \alpha_i \ln q_i + 0.5 \Sigma_i \Sigma_j \gamma_{ij} \ln q_i \ln q_j \tag{2}$$

Share equations may be derived directly, using:

$$w_i = (\partial \ln U / \partial \ln q_i) / \Sigma_j (\partial \ln U / \partial \ln q_j) \tag{3}$$

$$\partial \ln U / \partial \ln q_i = \beta_i + (\alpha_i + \Sigma_j \gamma_{ij} \ln q_j) / \ln Q$$

$$= (\beta_i \ln Q + \alpha_i + \Sigma_j \gamma_{ij} \ln q_j) / \ln Q \qquad (4)$$

$$\Sigma_j (\partial \ln U / \partial \ln q_j) = (\ln Q \, \Sigma_j \beta_j + \Sigma_j \alpha_j + \Sigma_j \Sigma_k \gamma_{jk} \ln q_k) / \ln Q \qquad (5)$$

so:

$$w_i = (\beta_i \ln Q + \alpha_i + \Sigma_j \gamma_{ij} \ln q_j) / (1 + \Sigma_j \Sigma_k \gamma_{jk} \ln q_k) \qquad (6)$$

since adding up implies: $\Sigma_j \beta_j = 0$, $\Sigma_j \alpha_j = 1$, and $\Sigma_j \Sigma_k \gamma_{jk} = 0$; and $\gamma_{ij} = \gamma_{ji}$ for symmetry. If $\beta_i = 0 \;\forall\, i$, the direct translog model results, if $\Sigma_j \gamma_{ij} = 0 \;\forall\, i$ the inverse AIDS model results.

The ILDS may also be derived from an alternative representation of preferences, the distance function, corresponding to the utility function, above (Deaton, 1979). That is, the distance function is implicitly defined as $U(q / d(u,q)) = u$. Therefore:

$$\ln U = \Sigma_i \beta_i \ln (q_i / d) + \ln (\alpha_0 + \Sigma_i \alpha_i \ln (q_i / d) + 0.5 \, \Sigma_i \Sigma_j \gamma_{ij} \ln (q_i / d) \ln (q_j /d))$$

$$= \Sigma_i \beta_i (\ln q_i - \ln d) + \ln (\alpha_0 + \Sigma_i \alpha_i (\ln q_i - \ln d)$$

$$+ 0.5 \, \Sigma_i \Sigma_j \gamma_{ij} (\ln q_i - \ln d) (\ln q_j - \ln d)) \qquad (7)$$

This can be rewritten:

$$\exp(\ln U - \Sigma_i \beta_i (\ln q_i - \ln d)) = \alpha_0 + \Sigma_i \alpha_i \ln q_i + 0.5 \, \Sigma_i \Sigma_j \gamma_{ij} \ln q_i \ln q_j$$

$$- \ln d (\Sigma_i \alpha_i + 0.5 \, \Sigma_i \Sigma_j \gamma_{ij} (\ln q_i + \ln q_j - \ln d)) \qquad (8)$$

$$U d^{\Sigma_i \beta_i} / (\Pi \, q_i^{\beta_i}) = \ln Q - \ln d (\Sigma_i \alpha_i + 0.5 \, \Sigma_i \Sigma_j \gamma_{ij} (\ln q_i + \ln q_j - \ln d)) \qquad (9)$$

Imposing the restrictions, $\Sigma_j \beta_j = 0$, $\Sigma_j \alpha_j = 1$, $\Sigma_j \Sigma_k \gamma_{jk} = 0$, and $\gamma_{ij} = \gamma_{ji}$, yields:

$$\ln d(U, q) = (\ln Q - U / (\Pi \, q_i^{\beta_i})) / (1 + \Sigma_j \Sigma_k \gamma_{jk} \ln q_k) \qquad (10)$$

Compensated inverse demands are derived by differentiation (where possible, subscripts are suppressed to simplify the notation.):

$$w_i = \partial \ln d / \partial \ln q_i$$

$$= \{ (\alpha + \Sigma\gamma \ln q + \beta_i U / (\Pi q^{\beta}))(1 + \Sigma\Sigma \gamma \ln q)$$

$$- \Sigma \gamma (\ln Q - U / (\Pi q^{\beta})) \} / (1 + \Sigma\Sigma \gamma \ln q)^2$$

$$= \{ \alpha + \Sigma\gamma \ln q + \beta_i U / (\Pi q^{\beta}) - \ln d \, \Sigma \gamma \} / (1 + \Sigma\Sigma \gamma \ln q) \quad (11)$$

Since, at the optimum, ln d(U, q) = 0, the last term in the numerator disappears, and the third term can be rewritten:

$$\ln d(U, q) = 0 = (\ln Q - U / \Pi q_i^{\beta_i}) / (1 + \Sigma_j \Sigma_k \gamma_{jk} \ln q_k) \quad (12)$$

$$\ln Q / (1 + \Sigma_j \Sigma_k \gamma_{jk} \ln q_k) = (U / \Pi q_i^{\beta_i}) / (1 + \Sigma_j \Sigma_k \gamma_{jk} \ln q_k) \quad (13)$$

so the same form for the share equations is obtained.

Interpretation of results for inverse demand models is less well understood than that of "normal" demand models. Anderson clarified the issue to a great extent by showing that the appropriate counterpart of the expenditure elasticity is what he termed the scale flexibility.[2] It can be characterized as the percentage change in the marginal value of good i as the scale of consumption is expanded by one percent. Reference for understanding scale flexibilities is established by realizing that if preferences are homothetic, all scale flexibilities are -1 (Barten and Bettendorf; Eales and Unnevehr). Necessities have scale flexibilities which are more negative than -1 and luxuries have scale flexibilities which are greater than -1. A comparison of the share equations and the formulae for price and scale flexibilities from the three models are given in Table 1.

---

[2] Anderson actually calls it a scale elasticity. In keeping with Agricultural Economics literature, it and the "quantity elasticities," will be called scale and price flexibilities, respectively.

Table 1. COMPARISON OF SHARE EQUATIONS AND FLEXIBILITIES

Inverse Lewbel Demand System (ILDS)

$$w_i = (\beta_i \ln Q + \alpha_i + \Sigma_j \gamma_{ij} \ln q_j) / (1 + \Sigma_j \Sigma_k \gamma_{jk} \ln q_k)$$

$$f_{ij} = -\delta_{ij} + \{ \gamma_{ij} + \beta_i (\alpha_j + \Sigma_k \gamma_{jk} \ln q_j) - w_i \Sigma_k \gamma_{jk} \} / \{ w_i (1 + \Sigma_j \Sigma_k \gamma_{jk} \ln q_k) \}$$

$$f_i = -1 + \beta_i / w_i + \{ \Sigma_j \gamma_{ij} \} / \{ w_i (1 + \Sigma_j \Sigma_k \gamma_{jk} \ln q_k) \}$$

$\Sigma_j \gamma_{ij} = 0 \ \forall i$ then ILDS reduces to the Inverse Almost Ideal Demand System (IAIDS)

$$w_i = \beta_i \ln Q + \alpha_i + \Sigma_j \gamma_{ij} \ln q_j$$

$$f_{ij} = -\delta_{ij} + \{ \gamma_{ij} + \beta_i (\alpha_j + \Sigma_k \gamma_{jk} \ln q_j) \} / w_i$$

$$f_i = -1 + \beta_i / w_i$$

if $\beta_i = 0 \ \forall i$ then ILDS collapses to the Direct Translog Demand System (DTDS)

$$w_i = (\alpha_i + \Sigma_j \gamma_{ij} \ln q_j) / (1 + \Sigma_j \Sigma_k \gamma_{jk} \ln q_k)$$

$$f_{ij} = -\delta_{ij} + \{ \gamma_{ij} - w_i \Sigma_k \gamma_{jk} \} / \{ w_i (1 + \Sigma_j \Sigma_k \gamma_{jk} \ln q_k) \}$$

$$f_i = -1 + \{ \Sigma_j \gamma_{ij} \} / \{ w_i (1 + \Sigma_j \Sigma_k \gamma_{jk} \ln q_k) \}$$

In the table: w_is are expenditure shares; f_{ij}s are own- and cross-price flexibilities; f_is are scale flexibilities; ws are expenditure shares; δ_{ij} is the Kronecker delta; ln Q in the ILDS and IAIDS models is the quantity index given in Equation 2 of the text; and αs, βs, and γs are parameters.

Quarterly Demand for Meat in Canada

Now the three demand models are applied to retail demand for meat in Canada. Assuming that beef, pork, and chicken are separable from other consumption goods, a conditional demand system for these three meats is specified.[3] Data employed in this exercise were obtained from Chen and Veeman and from Agriculture Canada. It consists of 96 quarterly observations on per capita retail consumption of beef, pork, and chicken and retail prices from 1967-Q1 through 1990-Q4. Details are in Chen.

Estimation was done using the SHAZAM program (White), which employs a Davidon-Fletcher-Powell algorithm for estimation of the nonlinear sets of share equations for each demand model given in Table 1. Initially, efforts were concentrated on the ILDS model. An equation is omitted during estimation, due to singularity of the covariance matrix for a system of shares and the estimates are invariant to the equation omitted (Barten). Coefficients of the omitted equation can be recovered using the demand restrictions or estimated directly by omitting an alternative equation.

Initial estimates indicated three difficulties with ILDS as specified in Equation 6. First, the parameter, α_0, proved impossible to estimate. The algorithm would not converge. In their original paper on the AIDS model Deaton and Muellbauer (1980a) suggest that estimation of this

---

[3] Implicitly, it is assumed that within a calendar quarter the quantities of beef, pork, and chicken are fixed, due to production lags, and because meats are perishable, prices adjust so that the available quantities are consumed. Note, that if this is so, then all the right-hand-side variables in Equation 6 are predetermined. Thus, nonlinear seemingly unrelated regressions is an appropriate estimator.

parameter may be "problematic." Deaton (1986, p. 1784) is even stronger.[4] Second, there is strong seasonality in meat consumption. Demand for beef tends to be strongest in the second and third quarter, demand for pork in the first and fourth quarters, and chicken demand in the first quarter. To incorporate this in Equation 6, the α_is were augmented with three seasonal dummy variables for the second, third, and fourth quarters of the year. Note that if this is done in ln Q (Equation 2), the model derived will be similar to Equation 6, but with the seasonal dummies appearing as intercept shifters in w_i (Equation 6) and as slope shifters in ln Q (Equation 2).[5] The third difficulty encountered was autocorrelation of the estimated residuals. This suggested the estimation ought to include a first-order autocorrelation correction, where the coefficient associated with the autocorrelation, ρ, should be the same for all equations, in order that the system still adds up (Berndt and Savin). SHAZAM employs an approach to such problems due to Pagan.

---

[4] Deaton and Muellbauer (1980a) suggest determining α_0 (for the AIDS model), a priori, by noting it is the "outlay required for a minimal standard of living" in the base year, when all prices are one (p.316). As an alternative, α_0 is fixed at one. Examination of Equation 1 shows that this scaling of the conditional utility function for meat has the effect of setting it to one in the base year, when all quantities are one. Then the sensitivity of the other parameter estimates is investigated by varying α_0 in the range, 0.1 to 10. This had some impact on the αs, but little on the γs or the βs.

[5] To account for seasonality of demand for meats, the α_js in Equation 2 are augmented with three seasonal dummy variables; D_k k=2,3,4; whose associated coefficients must sum to zero over i for adding up. This results in:

$$w_i = \alpha_i + \sum_k \theta_{ik} D_k + \sum_j \gamma_{ij} \ln q_j + \beta_i \ln Q^s \qquad (6')$$

with ln Q^s given by:

$$\ln Q^s = \alpha_0 + \sum_j (\alpha_j + \sum_k \theta_{jk} D_k) \ln q_j + 0.5 \sum_i \sum_j \gamma_{ij} \ln q_i \ln q_j \qquad (2')$$

Results for the ILDS model are given in the top third of Table 2. The share equations fit well and show no evidence of continued difficulties with autocorrelation. The constant and dummy variables associated with quarters 2 and 3 are significant for both beef and pork, while the fourth quarter dummy is significantly negative for chicken, supporting the seasonality noted earlier. The own-quantity effect for chicken is significant at the 10% level as is cross-quantity effects between pork and chicken. As noted earlier, both the DTDS and the IAIDS are nested within the ILDS. Wald tests of the parameter restrictions associated with DTDS and IAIDS were: 1.034 and 1.116, respectively. Each is distributed chi-square with 2 degrees of freedom. Thus, neither the DTDS nor the IAIDS is rejected and so each of these models was estimated, as well. Results are given in the bottom two-thirds of Table 2. The results are similar for all three specifications. The DTDS and IAIDS models show more significance of the quantity effects, because imposition of either set of restrictions dramatically improves the efficiency of the estimators (standard errors of estimated quantity effects in the ILDS model are from four to ten times the size of those in the other two models).

Next the formulae in Table 1 were used along with the sample means of the data to calculate flexibilities for all three demand specifications. Findings are in Table 3. The results show that all three models produce similar results. All meats are more own-price flexible than would be expected based on previous work with "normal" demand systems. Chicken, in particular, is very flexible. The own-price flexibilities from the ILDS model are larger in absolute value than those from the DTDS and IAIDS models by as little as 3% for pork (IAIDS) to almost 25% for chicken (IAIDS). In all cases, the scale flexibilities are close to -1. The

Table 2. COMPARISON OF DEMAND MODELS FOR CANADIAN MEATS.[1]

	BFQ	PKQ	CKQ	Q	CONST	Q2	Q3	Q4	R^2/DW
	Inverse Lewbel								
BEEF	.266	-.060	-.076	-.109	.625 *	.012*	.009*	.000	.917
	(.215)	(.077)	(.058)	(.176)	(.183)	(.002)	(.003)	(.002)	1.770
PORK	-.060	.070	-.063*	.030	.270*	-.013*	-.008*	.002	.911
	(.077)	(.087)	(.024)	(.161)	(.185)	(.002)	(.002)	(.002)	1.548
CHK	-.076	-.063*	.062*	.087	.105	.001	-.001	-.002*	.987
	(.058)	(.024)	(.023)	(.086)	(.098)	(.001)	(.002)	(.001)	2.183
	Inverse AIDS								
BEEF	.117*	-.067*	-.049*	.027	.483*	.012*	.009*	.000	.917
	(.018)	(.014)	(.011)	(.035)	(.063)	(.002)	(.003)	(.002)	1.755
PORK	-.068*	.102*	-.034*	-.032	.329*	-.013*	-.008*	.002	.910
	(.014)	(.014)	(.009)	(.031)	(.051)	(.002)	(.002)	(.002)	1.538
CHK	-.049*	-.034*	.083*	.006	.187*	.001	-.001	-.002*	.987
	(.011)	(.009)	(.013)	(.021)	(.050)	(.001)	(.002)	(.001)	2.181
	Direct Translog								
BEEF	.146*	-.075*	-.041*		.509*	.012*	.009*	.000	.917
	(.032)	(.015)	(.012)		(.050)	(.002)	(.003)	(.002)	1.764
PORK	-.075 *	.083*	-.039*		.296*	-.013*	-.008*	.002	.910
	(.015)	(.024)	(.012)		(.042)	(.002)	(.002)	(.002)	1.537
CHK	-.041*	-.039*	.082*		.195*	.001	-.001	-.002*	.987
	(.012)	(.012)	(.016)		(.042)	(.001)	(.002)	(.001)	2.189

* Indicates significance at the 0.05 level.

1. Asymptotic standard errors are in parentheses. All 3 systems were corrected for first-order autocorrelation, as indicated in the text. Estimates of ρ were: ILDS, .975 (.024); IAIDS, .978 (.022); and DTDS, .978 (.022); respectively. Standard errors are obtained for all coefficients by re-estimating with an alternative equation omitted.

Table 3. **COMPARISON OF PRICE AND SCALE FLEXIBILITIES**[1]

	BEEF	PORK	CHICKEN	SCALE
	Inverse Lewbel			
BEEF	-.843	-.088	-.046	-.978
PORK	-.225	-.723	-.126	-1.074
CHICKEN	-.102	-.244	-.586	-.931
	Inverse AIDS			
BEEF	-.757	-.111	-.084	-.951
PORK	-.279	-.701	-.125	-1.105
CHICKEN	-.300	-.210	-.455	-.964
	Direct Translog			
BEEF	-.761	-.108	-.077	-.946
PORK	-.273	-.700	-.127	-1.099
CHICKEN	-.297	-.221	-.473	-.992

1. Flexibilities are calculated for each model using the formulae in Table 1, using the sample means.

DTDS and the IAIDS estimates of all flexibilities are quite close, none differing by more than 3%. Both produce similar characterizations of consumer preferences. Finally, all three models show meats to be gross q-substitutes (negative cross-price flexibilities), as one would expect of meats (Hicks).

Two recent studies, Reynolds and Goddard and Chen and Veeman, have analyzed quarterly Canadian meat demand. Both employed AIDS models to examine the issue of whether consumer preferences for meats had undergone a structural shift. Marshallian elasticities were calculated in each case at sample means of the data. The ranges of own-price elasticities reported were: -1.06 to -0.74 (beef); -0.82 to -0.67 (pork); and -0.95 to 0.17 (chicken). Based on these results one would expect all the own-price flexibilities to be less than -1.[6]

CONCLUSIONS

A new model of consumer preferences at the market level is introduced. It is similar to a demand system proposed by Lewbel, except the share equations are derived from the primal specification. That is, starting from a utility or distance function the inverse Lewbel system (ILDS) is derived. The demand equations relate expenditure shares to quantities and an inverse

---

[6] Reynolds and Goddard report four sets of elasticities, two before and two after a shift in preferences; so the ranges reported in the text are over all five sets reported. If one estimates the entire matrix of elasticities then one can invert these to calculate the flexibilities (Houck). In conditional demand systems; such as those employed by Reynolds and Goddard, Chen and Veeman, and in the present study; this is not possible as it would require that all the off-diagonal blocks of the elasticity or flexibility matrix be zero. Thus, it is only possible to broadly characterize expectations about the conditional flexibilities given in Table 3. The inelastic demands found by the previous studies lead one to expect inflexible (own-price flexibility < -1) demands. However, note that at least one (and possibly all) of these sets of elasticities or flexibilities may be contaminated by simultaneity problems, that is, either prices and/or quantities are endogenous.

AIDS quantity index (Eales and Unnevehr). The ILDS model nests both the direct translog (DTDS) and inverse almost ideal demand system (IAIDS). This makes it possible to test whether either set of restrictions corresponding to the DTDS or IAIDS is consistent with the data at hand. The derived model is appropriate when one can assume that at the market level, quantities are fixed and that prices adjust so that the fixed quantities are consumed. One would expect the ILDS (or DTDS, or IAIDS) to be appropriate when modeling perishable commodities where supply response lags for either biological or other reasons.

On the dual side, it is often advantageous to estimate conditional demand systems, where included commodities are assumed separable from those which are excluded. It has been pointed out, that even if prices are predetermined in such systems, expenditures on those commodities can not be (LaFrance). Any of the three systems, discussed above, has the advantage that if quantities are predetermined then all RHS variables will be. In terms of relative advantages amongst the three demand systems, the DTDS does not require the researcher to cope with α_0 in the IAIDS quantity index. It does, however, require nonlinear estimation. The IAIDS model has been shown to be well approximated, in practice, by substitution of a Stone's quantity index for the IAIDS index (Eales and Unnevehr). This avoids both the necessity of dealing with α_0 and nonlinear estimation. The substitution of the Stone's quantity index could be employed for estimation of the ILDS model, as well. However, the remaining model, while simplified, is still nonlinear. The ILDS does allow for increased flexibility over either the DTDS or the IAIDS, in that it nests them both. Restrictions required for homogeneity, symmetry, DTDS, or IAIDS all depend only on unknown parameters and are, therefore, easy to test or impose.

As an example, all three demand systems are applied to the problem of modeling quarterly Canadian meat consumption. All three models fit well and gave qualitatively similar characterizations of Canadian preferences for meats. Wald tests of the ILDS results suggested neither the DTDS nor the IAIDS models could be rejected. Results of all three specification were similar with respect to beef and pork quantities and scale effects. All were more own-price flexible for beef and pork than would be suggested by previous studies, which took prices and expenditures as fixed.

Of course, an obvious question is: are prices or quantities of meats predetermined in quarterly data. Certainly the majority of previous research on meat demand has employed demand models which implicitly assumes prices and expenditures are predetermined. Previous work on this issue has employed either ad hoc demand systems (Thurman) or looked at only one side of the issue (Wahl and Hayes). The existence of inverse Lewbel system should make it possible to test endogeneity symmetrically, using theoretically consistent, flexible demand systems to test the endogeneity of both prices and quantities. However, several issues remain to be addressed before such a symmetric treatment can be employed. First, application of Wu-Hausman tests to nonlinear systems requires nonlinear simultaneous systems estimation, which is less well understood than either nonlinear or simultaneous systems estimation. Also, confounding the problem is the issue raised by LaFrance. If expenditures are not predetermined in conditional demand systems, then one must account for potential enodogeneity of expenditures, as well, as prices. This adds an additional layer of complexity to an already thorny problem.

BIBLIOGRAPHY

Anderson, R.W. 1980. Some Theory of Inverse Demand for Applied Demand Analysis. *European Economic Review* 14: 281-290.

Arzac, E.R. and M. Wilkinson. 1979. A Quarterly Econometric Model of United States Livestock and Feed Grain Markets and Some of Its Policy Implications. *American Journal of Agricultural Economics* 61: 297-308.

Barten, A.P. 1969. Maximum Likelihood Estimation of a Complete System of Demand Equations. *European Economic Review* 1: 7-73.

Barten, A.P. and L.J. Bettendorf. 1989. Price Formation of Fish: An Application of an Inverse Demand System. *European Economic Review* 33: 1509-1525.

Berndt, E.R. and N.E. Savin. 1975. Estimation and Hypothesis Testing in Singular Equation Systems with Autoregressive Disturbances *Econometrica* 43: 937-957

Chambers, R.G. and K.E. McConnell. 1983. Decomposition and Additivity in Price Dependent Demand Systems. *American Journal of Agricultural Economics* 65: 596-602.

Chen, P.Y. 1991. *Analysis of Market Demand for Meats in Canada.* unpublished Ph.D. Thesis. University of Alberta, Department of Rural Economy. 107 p.

Chen, P.Y. and M.M. Veeman. 1991. An Almost Ideal Demand System Analysis for Meats with Habit Formation and Structural Change. *Canadian Journal of Agricultural Economics* 39: 223-235.

Christensen, L.R., D.W. Jorgenson and L.J. Lau. 1975. Transcendental Logarithmic Utility Functions. *American Economic Review* 65: 367-83.

Dahlgran, R.A. 1988. Changing Meat Demand Structure in the United States: Evidence from a Price Flexibility Analysis. *North Central Journal of Agricultural Economics* 10: 165–76.

Deaton, A. 1979. The Distance Function in Consumer Behavior with applications to Index Numbers and Optimal Taxation. *Review of Economic Studies* 46: 391-405.

Deaton, A. 1986. Demand Analysis in *Handbook of Econometrics. vol.3* edited by Z. Griliches and M.D. Intriligator (Elsevier Science Publishers B.V. Amsterdam).

Deaton, A. and J. Muellbauer. 1980a. *Economics and Consumer Behavior* (Cambridge University Press, Cambridge).

Deaton, A. and J. Muellbauer. 1980b. An Almost Ideal Demand System. *American Economic Review* 70: 312-326.

Eales, J. S.. and L. J. Unnevehr. 1991. The Inverse Almost Ideal Demand System. *Proceedings of the NCR-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management*. 402-17.

Freebairn, J.W. and G.C. Rausser. 1975. Effects of Changes in the Level of US Beef Imports. *American Journal of Agricultural Economics* 57: 676-688.

Heien, D.M. 1982. The Structure of Food Demand: Interrelatedness and Duality. *American Journal of Agricultural Economics* 64: 213-21.

Hicks, J.R. 1956. *A Revision of Demand Theory* (Oxford University Press, Oxford)

Houck, J.P. 1965. The Relationship of Direct Price Flexibilities to Direct Price Elasticities. *Journal of Farm Economics* 47: 301-21.

Huang, K.S. 1988. An Inverse Demand System for US Composite Foods. *American Journal of Agricultural Economics* 70: 902-8.

Huang, K.S. 1990. An Inverse Demand System for US Composite Foods: Reply. *American Journal of Agricultural Economics* 72: 239-40.

Jorgenson, D.W. and L.J. Lau. 1975. The Structure of Consumer Preferences. *Annals of Economic and Social Measurement* 4: 49-101.

LaFrance, J. T. 1991. When is Expenditure 'Exogenous' in Separable Demand Models? *Western Journal of Agricultural Economics*. 16:49-62.

Lewbel, A. 1989. Nesting the AIDS and Translog Demand Systems. *International Economic Review* 30: 349-356.

Pagan, A. 1974. A Generalized Approach to Treatment of Autocorrelation *Australian Economic Papers* 13: 267-280.

Reynolds, A. and E. Goddard. 1991. Structural Change in Canadian Meat Demand. *Canadian Journal of Agricultural Economics* 39: 211-222.

Thurman, W. 1987. The Poultry Market: Demand Stability and Industry Structure. *American Journal of Agricultural Economics*. 69: 30-37.

Wahl, T.I. and D.J. Hayes. 1990. Demand System Estimation with Upward-Sloping Supply. *Canadian Journal of Agricultural Economics*. 38: 107-22.

White, K.J. 1978. A General Computer Program for Econometric Methods - SHAZAM. *Econometrica* 46: 239-40.

Young, T. 1990. An Inverse Demand System for US Composite Foods: Comment. *American Journal of Agricultural Economics* 72: 237-8.