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On Non-Chord Tone Generalized Interval Systems in Music Analysis

by

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Abstract

Non-chord tone generalized interval systems are a music theoretical tool for analyzing the roles and functions of non-chord tones in music, independent of the methods used to label non-chord tones. Based on David Lewin's GIS, they provide a way to conceptualize changes in textural and motivic patterns of non-chord tones in terms of intervals. The formal framework of transformational theory also provides a number of analytical tools that can be adapted to the analysis of non-chord tones, as well as the capacity to develop new specialized analytical tools to fit analytical requirements. As a practical demonstration of the potential of non-chord tone generalized interval systems and some related transformations, two analytical essays are included: a statistical analysis of Mozart's variations on “Ah! Vous dirai-je, Maman,” K. 265, and a motivic analysis of Brahms's String Quartet No. 2 in A Minor, op. 51, no. 2.

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Chapter 1: Introduction

Although they are a feature of virtually every theory of harmony, non-chord tones have generally received less attention when it comes to the analysis of complete pieces of music. A great deal of energy has been devoted to classifying and identifying non-chord tones in music, and practically every introductory music theory text discusses them in some detail. Nevertheless, as a feature of the music, as something to be discussed and analyzed, they are often overlooked. Perhaps one reason is the plethora of approaches to defining, identifying, and labeling non-chord tones. Even terminology is not standardized, for non-chord tones may also be called figuration tones, non-harmonic tones, unessential tones, or embellishments, to name a few of the more common terms. Out of these many alternatives, I have adopted the term "non-chord tone" for this thesis on entirely pragmatic grounds, since it allows for the easily memorable abbreviation NCT.

Now, it is not the purpose of this thesis to address the problems involved in identifying non-chord tones. Rather, the goal is to provide a tool that can be used to analyze the role that non-chord tones play in a piece of music, regardless of the methodology one uses to identify non-chord tones in the first place. The task of determining whether or not a particular note is a non-chord tone is a piece of groundwork that is left to the individual analyst. The theoretical tool which is to be the subject of this thesis can readily be adapted to work with other theoretical methodologies in whatever manner a theorist might choose. My theoretical starting point is David Lewin's work in *Generalized Musical Intervals and Transformations*. Since my analytical tool falls squarely within the theoretical milieu associated with generalized intervals and transformational theory, this thesis has as its secondary goal an investigation of several aspects of transformational theory that will arise in connection with my approach to non-chord tones. Finally, since the purpose of this thesis is the development of a practical analytical tool, I have devoted two full chapters to analyses of individual pieces of music.

The thesis has six chapters. The remainder of this chapter is devoted to a literature review, examining the various approaches taken by theorists to the problem of identifying non-chord tones and analyzing their role in the music. Chapter 2 provides background information, explaining some of the mathematics involved in *Generalized Musical Intervals and Transformations.* The chapter also contains an introduction to Lewin's work and concludes with a discussion of the concept of a non-chord tone interval system, preparing the way for chapter 3. Chapter 3 is an analysis of Mozart's variations on “Ah! Vous dirai-je, Maman,” K. 265. The analysis employs an approach that might be regarded as a statistical analysis of texture, using non-chord tone interval systems to examine the role of non-chord tones in shaping individual variations and overall form. Chapter 4 introduces a second type of non-chord tone interval system, and examines ways in which the concept can be applied to network analysis. The chapter also develops a number of related concepts in transformational theory that can apply to tonal music in general, but are primarily groundwork for the next chapter. Chapter 5 is a network analysis of Brahms's String Quartet No. 2 in A Minor, demonstrating how non-chord tone interval systems can be integrated within the broader context of transformational theory and network analysis. Like chapter 3, chapter 5 stands as an analysis in its own right, using network analysis to examine themes and motives in Brahms's string quartet. Chapter 6 briefly summarizes the central points made in this thesis and provides further suggestions for the application of non-chord tone interval systems in musical analysis.

Literature Review: The Analysis of Non-Chord Tones

We begin by considering the treatment of non-chord tones at their most basic level as discussed in two textbooks. Walter Piston's *Harmony*, an older and once widely used harmony textbook, gives the following types of non-chord tones: passing tone, auxiliary tone (neighbor tone), appoggiatura, suspension, échappée, cambiata, and anticipation.[1] Piston places great emphasis on rhythmic factors, arguing that accented passing tones and suspensions in which the note is repeated rather than tied are both instances of the appoggiatura.[2] Interestingly, he begins his discussion by arguing for the melodic basis of non-chord tones and

1. Walter Piston, *Harmony*, 3rd ed. (New York: W.W. Norton, 1962), 81–93.

2. Ibid., 81, 87–88.

excusing the term “nonharmonic tones,” which he uses, as an anachronism. He even presents a case, given in figure 1.1, in which the soprano E (in the second measure of the figure) in a C major chord is considered to be an embellishment of a suspended D rather than a chord tone.[3]

A more recent textbook by Kostka and Payne presents a slightly more elaborate approach. They list a number of types of non-chord tones: passing tone, neighbor tone, suspension, retardation, appoggiatura, escape tone, neighbor group, anticipation, and pedal tone. Their criteria for identification is based primarily on how a tone is approached and left, whether by the same tone, by step, or by leap.[4] In addition, they list all the adjectives that may be applied to non-chord tones: accented/unaccented, diatonic/chromatic, ascending/descending, and upper/lower.[5] Interestingly, they include a very telling footnote: “NCT terminology is not standardized, and your instructor may prefer that you use different labels and definitions.”[6]

In both instances, the analysis of non-chord tones means simply their correct identification. The goal is to distinguish between chord tones, which belong to the chord label, and non-chord tones, which have their own appropriate labels. The result is a sort of bookkeeping procedure in which the object is to account for every note. However, even within this context the labels and definitions vary slightly, with different factors receiving different weight. There are, however, theories of harmony that attempt to reduce the number of types of non-chord tones and explain part of their role in the music.

Schoenberg's *Theory of Harmony* uses only four categories: suspensions, passing tones, embellishments (which includes all types of neighbors tones, appoggiaturas, cambiatas, and the like), and anticipations, all of which he treats as

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3. Ibid., 81; all figures are included in the appendix.

4. Stefan Kostka and Dorothy Payne, *Tonal Harmony, with an Introduction to Twentieth-Century Music*, 6th ed. (New York: McGraw-Hill, 2009), 182.

5. Ibid.

6. Ibid.

melodic phenomena rather than purely a matter of consonance and dissonance.[7] He takes a flexible approach to identification, noting that "accidental harmonies sometimes result from the simultaneous appearance of passing tones. These can be regarded as actual chords, as well as combinations of non-harmonic tones."[8] Schoenberg takes great pains to stress that a composer will employ non-harmonic tones with the same foresight as harmonies and rejects exercises in which a student is asked to embellish a chorale by adding non-harmonic tones.[9] He even provides a brief analysis of a Bach chorale in which he argues for the essential role of the non-harmonic tones. Unfortunately, the analysis is rather cursory, omitting any detailed discussion of the non-chord tones and their role as such.

Schenkerian analysis represents another well-established theoretical approach to non-chord tones. In *Harmony*, Schenker only discusses anticipations, suspensions, and the pedal point, which he limits in scope, although he mentions passing tones and neighbor tones.[10] These latter dissonances receive their most comprehensive treatment in *Counterpoint*, where Schenker argues at length for the restrictions imposed on second species counterpoint, including the use of the passing tone and the restrictions on the neighbor tone.[11] In the same passage, he also discusses a variety of passages of music with unconventional non-chord tones and explains how they can be derived from passing tones and neighbor tones. In addition, passing tones, and to a lesser extent neighbor tones, are a key component of Schenker's mature theory. In a passage rejecting the idea that the tones of the fundamental line are overtones, Schenker says that

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7. Arnold Schoenberg, *Theory of Harmony*, trans. Robert Adams (New York: Philosophical Library, 1948), 271.

8. Ibid., 277.

9. Ibid., 280, 282–285.

10. Heinrich Schenker, *Harmony*, ed. Oswald Jonas, trans. Elisabeth Borgese (Chicago: University of Chicago Press, 1954), 302–319.

11. Schenker, *Counterpoint,* bk. 1, *Cantus Firmus and Two-Voice Counterpoint*, ed. John Rothgeb, trans. John Rothgeb and Jürgn Thym (New York: Schirmer Books, 1987), 176–194.

> if, when we hear the tones of the fundamental line in succession, we nevertheless understand <some of> them as fifth or third of a particular bass tone, it is only because we recognize in them the same relationships which establish the octave, fifth, and third of the harmonic series.… Still less should the passing tones in the spaces of the arpeggiation be taken for overtones; they are not contained in the harmonic series at all. It is therefore not permissible to ascribe the same significance to passing tones as to the main bass tone; by definition, a passing tone is dependent upon the consonant tones which surround it.[12]

In the Schenkerian model of the fundamental line and linear progressions the space is outlined by consonant intervals or arpeggiations of triads and filled in by passing tones and neighbor tones. Thus non-chord tones play a significant role in Schenkerian theory.

Although Schenkerian analysis is dependent on non-chord tone analysis, it is not always well adapted to the surface analysis of non-chord tones. Schenkerian analyses tend to cut through the musical surface to examine the background. In doing so, attention is naturally shifted away from the fine details of the extreme foreground, including dissonant non-chord tones. Schenkerian analysis tends to focus on determining which notes are stable consonant elements of the middleground rather than addressing the ways in which dissonant non-chord tones are employed.

One example of non-chord tone analysis that perhaps bears the greatest similarity to my project is Yangkyung Lee's D.M.A. dissertation, which analyzes non-chord tones in Chopin's Op. 28.[13] Lee’s dissertation presents one of the few published examples of detailed non-chord tone analysis. However, Yangkyung Lee's analysis is not systematized in any way. The dissertation examines specific

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12. Schenker, *Free Composition*, trans. and ed. Ernst Oster (New York: Longman, 1979), 12–13.

13. Yangkyung Lee, “Non-Harmonic Tones as Aesthetic Elements in Chopin's Preludes, Op. 28” (D.M.A. diss., University of Cincinnati, 2002).

moments in terms of aesthetic effect, the harmonic ambiguity that non-chord tones can produce, and motivic relationships. The framework for the discussion is technologically ad hoc.

In addition to these more traditional theoretical methodologies, David Huron has investigated the role of non-chord tones in Bach chorales from the perspective of music cognition. Huron takes standard non-chord tone analyses of fifty Bach chorales and uses a statistical model to argue that Bach uses non-chord tones to contribute to the independence of musical parts, with non-chord tones helping to draw attention to different voices in turn.[14] Huron's approach to non-chord tones essentially derives from the standard practice of roman numeral analysis, and involves a complex categorization of dissonances along the lines used by Kostka and Payne in their textbook; however, this issue is not essential in his analysis, since he does not deal extensively with the categories of non-chord tones as such, and his analysis is spread across a sample of fifty chorales without analyzing any individual passage of music.

While it does not really address detailed non-chord tone analysis, there are two points that are worth noting in Huron’s article. In the first place, Huron chooses chorales as the basis of his sample. Because of the simple harmonic and rhythmic texture, non-chord tones can be easily identified according to fixed standards without great difficulty, which is not necessarily the case in complex contrapuntal textures. Secondly, even in this simple context Huron is compelled to deal with the fact that different analyses occasionally arise. Huron asked several independent music theorists to provide analyses of the chorales, and his article discusses an example of disagreement as to which notes are chord tones and which are not; he ultimately concludes that “there may be no resolution for such analytic disputes.… Most theorists would recognize these differences as indicating slightly different analytic styles rather than indicating fundamental theoretical disagreements.”[15] Thus there is always a certain level of potentially insurmountable disagreement in any non-chord tone analysis.

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14. David Huron, “On the Role of Embellishment Tones in the Perceptual Segregation of Concurrent Musical Parts,” *Empirical Musicology Review* 2, no. 4 (2007): 123–139.

In order to demonstrate the potential for disagreement and simultaneously consider what might be said about non-chord tones in a passage of music, let us examine three different analyses of a passage from Bach's Two-Part Invention No. 9 in F Minor. Figure 1.2 provides the first four measures of the invention, whose thin, two-part texture allows for several possible analyses. Figure 1.2a gives an analysis in which very few notes are interpreted as non-chord tones. Passing notes are marked P; neighbor notes are marked N. Along the bottom of the score, the figure provides a conventional Roman Numeral analysis. The harmonic rhythm is very fast, with chords on each sixteenth note in some cases. Although this particular analysis belongs to a class that is now generally considered too fussy and would most likely be frowned upon even in undergraduate theory courses, it is nevertheless a possible analysis of the passage. Figures 1.2b and 1.2c are both more in line with current sensibilities, employing fewer chords and labeling more notes as non-chord tones. Moreover, there are several other analyses that could be produced by combining the features of these three analyses in different ways. In addition, these analyses would change slightly according to the definitions of the non-chord tones. Following the example of Schenker and Schoenberg, I have distinguished only between passing tones, neighbor tones, suspensions and anticipations. However, if we were to distinguish between accented and unaccented passing tones and include the additional categories of escape tones and appoggiaturas, the analysis would change again.

Although it may not be possible to definitively settle on one of these analyses, in each case there is something interesting to say about the non-chord tones. In figure 1.2a it is apparent that the bass voice contains more non-chord tones and that the voices typically do not employ non-chord tones at the same time. This supports David Huron's observation that non-chord tones can contribute to the independence of musical parts. Examining figures 1.2b and 1.2c, we find that, although some non-chord tones now appear simultaneously, they are often of different types. Moreover, examining the analysis in figure 1.2b we find that measures 1 and 2 have exactly the same number of non-chord tones, that

15. Ibid., 127–128.

measure 3 adds one passing tone and one neighbor tone, and that measure four then drops one neighbor tone. This observation suggests an underlying consistency in the non-chord tone texture and that the number of non-chord tones may be increased or decreased as a means of adding design to the phrase. This design is distinct from the underlying combined rhythm of this passage, which consists of steady sixteenth notes throughout. Figure 1.2c is similar, although the design of the phrase is slightly different: each measure contains more non-chord tones than the last. In addition, both figures 1.2b and 1.2c suggest that there are two non-chord tone combinations that might be termed “motivic”: a combination of a passing tone and a neighbor tone in close proximity, which occurs several times in the upper voice, and a scalar passage with several successive passing tones, which appears in both voices. Thus, regardless of the method of identifying non-chord tones, there may be interesting musical observations that can be made for a particular analysis.

Since the problem of identifying non-chord tones has been solved in so many different ways, and since my purpose is the development of an analytical tool that is independent of the criteria for identification, I will not attempt to justify the criteria that I employ in great detail. However, for the sake of analytical consistency the following criteria may serve as a general guide. I follow Schenker and Schoenberg in limiting the types of non-chord tones to passing tones, neighbor tones (including incomplete neighbors), suspensions, and anticipations. Both melodic shape and dissonance will be considered as criteria. Notes that are dissonant relative to the underlying chord will be considered non-chord tones. With the exception of the dominant seventh, sevenths or ninths of chords will usually be analyzed as non-chord tones. A consonant note may also be analyzed as a non-chord tone if it is dissonant relative to an underlying chord that is outlined in the music, particularly if the note is approached and resolved as a non-chord tone. Finally, motivic and thematic consistency will be extremely important in making analytical decisions and will occasionally outweigh other criteria.

Chapter 2: Non-Chord Tone Generalized Interval Systems

Although the concept of a musical interval has traditionally been associated only with distances between pitches and pitch-classes, and perhaps beats and beat-classes, the generalizing power of mathematics allows us to apply the concept in unfamiliar areas. In *Generalized Musical Intervals and Transformations* (GMIT), David Lewin demonstrates that traditional interval systems involving pitches, pitch-classes, beat classes, time-spans, and even differences in the timbre of instruments all share fundamental structural properties, expressed formally using a Generalized Interval System (GIS).[16] This generalizing approach can reveal surprising musical relationships and properties that might otherwise go unnoticed.

In this chapter, we will use GIS structure to construct a non-chord tone interval system, which we will then expand into a number of related systems. The goal of the first non-chord tone GIS is to study the textural effects of filling out a passage by adding or removing non-chord tones of various types. Before examining the conceptual framework and formalism involved in a non-chord tone GIS, however, we should introduce the concept of a Generalized Interval System as developed by David Lewin, along with the mathematics upon which it is based.

A Brief Introduction to Groups

Lewin's work in *Generalized Musical Intervals and Transformations* draws heavily on mathematics, particularly some very basic concepts from group theory. In fact, this mathematical approach is so fundamental a part of Lewin's work that he frequently states his arguments in terms of definitions, theorems, and proofs. In order to work with this framework, we must first explain the basic details of the mathematics involved.

The fundamental mathematical basis for Lewin's work is the mathematical concept of a group. We can best examine this concept by discussing a definition of a group:

> A group $\langle G, * \rangle$ is a set G, closed under a binary operation $*$, such that the following axioms are satisfied:

16. David Lewin, *Generalized Musical Intervals and Transformations*, (Oxford: Oxford University Press, 2007).

G_1: For all a, b, $c \in G$, we have

$$(a * b) * c = a * (b * c).$$ **associativity of** *

G_2: There is an element e in G such that for all $x \in G$,

$$e * x = x * e = x.$$ **identity** *e* for *

G_3: Corresponding to each $a \in G$, there is an element a' in G such that

$$a * a' = a' * a = e.$$ **inverse** *a′* of *a*[17]

In the first place, we must notice that a group consists of a set. The set can often be interpreted as a set of numbers, such as integers, rational numbers, or complex numbers. In addition, functions frequently form the set that is the basis of a group. Second, there must be a binary operation, denoted by *, and the set must also be closed under that binary operation. A binary operation combines two elements to produce a third element. Familiar examples of binary operations are addition, subtraction, and multiplication. An example of a less familiar binary operation is function composition. To say that the set G is closed under a binary operation * means that whenever two elements of G are combined using the binary operation * the third element produced by * will also be an element of G. For example, if a and b are elements of G, G is a group with the binary operation *, and $a * b = c$, then c is also an element of G.

Two familiar examples of groups are the integers under addition, and the rational numbers under multiplication. Both of these are *closed* under their respective operations. For example, consider the equation $a + b = c$, where a and b are integers (positive or negative). Regardless of which integers they are, c will also be an integer. An example of a structure that would not satisfy these criteria would be the positive integers under subtraction. In this mathematical structure $4 - 5 = -1$, which is not a positive integer and is consequently outside the set. In other words, the positive integers under subtraction are not closed.

In addition to this basic consideration, a group must satisfy three axioms. The first of these is associativity, which means that $(a * b) * c = a * (b * c)$. The integers under addition are associative. For example, $(1 + 2) + 3 = 1 + (2 + 3)$.

17. John Fraleigh, *A First Course in Abstract Algebra*, 6th ed. (Reading, MA: Addison-Wesley, 1999), 52.

The integers under subtraction, however, are not. For example, $(7 - 4) - 2 = 1$, while $7 - (4 - 2) = 5$. Secondly, a group must have an identity element e, such that $e * x = x * e = x$. In the integers under addition, the identity element is 0. In the rational numbers under multiplication, it is 1. That is adding 0 to any number or multiplying any number except 0 by 1 will produce the same number. Finally, every element in the group must have an inverse. That is, for every element a in the group there must a corresponding element a', such that $a * a' = a' * a = e$, where e is the identity element. In the example of the integers under addition, the negative numbers are the inverses of the corresponding positive numbers. In the rational numbers under multiplication, the inverse of x takes the form $1/x$.

While groups can serve as a generalization of the principles of addition and multiplication, they can also serve to express relationships among a smaller finite set of elements, where they often correspond to the symmetries of some geometric object. A simple example of a finite group is the integers from 0 to 11 under addition mod 12. This system is cyclic and works by dividing every sum by 12 and taking the remainder. For example, in this system $7 + 10 = 5$, since we obtain the result by adding the second number, dividing by 12, and taking the remainder as follows: first, we take $7 + 10 = 17$; second, $17 \div 12 = 1$, with a remainder of 5; consequently, under addition mod 12, $7 + 10 = 5$. This group corresponds to the rotational symmetry of clockwise movement on a twelve hour clock, but it can also be used to represent other things, such as chromatic pitch-classes. Note that the same principles apply to other examples of modular arithmetic. For example, in the group of integers 0–6 under addition mod 7 the equation $5 + 4 = 2$ would be true.

It is also possible to have groups of functions, with a binary operation of function composition. This is perhaps less familiar than the previous groups we have mentioned. Function composition takes two functions and produces a third function. To make this concrete, suppose that $f(x) = 2x + 1$ and $g(x) = 3x - 2$. Both $f(x)$ and $g(x)$ are functions: if we take any value of x as the input, then these functions will output exactly one number. We can combine the functions to create a third function, which would be equivalent to the result of passing x first through one function and then the other. In our example, the result of passing a value through $f(x)$ and then through $g(x)$ would be a third function $h(x)$. That is, $g(f(x)) = h(x) = 6x + 1$. We could also reverse the order, passing x through $g(x)$ and then $f(x)$ to produce $i(x)$. That is, $f(g(x)) = i(x) = 6x - 3$. Notice that $g(f(x))$ does not equal $f(g(x))$. Function composition is not necessarily commutative, and groups of functions are not necessarily commutative either.

Function composition does bring up an issue of notation. Transforming x by f and then by g might be written as either $gf(x) = h(x)$ or $g(f(x)) = h(x)$, a notation that is known as left orthography, or as $(x)fg = (x)h$, which is an example of right orthography. In both cases, the equation means that f transforms x before g. Because he thinks it is more familiar to music theorists, Lewin uses left orthography in most contexts, but uses right orthography in a few cases.[18] Although I will occasionally use left orthography, I will primarily use right orthography, in a form where each function is separated by square brackets. For example our equations might be written as $[f][g] = [h]$ or $[g][f] = [i]$. In both cases, f then g produces h; g then f produces i. I will primarily employ this method in conjunction with graphs in later chapters, where its use will help to clarify the order in which transformations are applied.

Although some collections of functions form groups, this is not always the case. Some collections of functions form semigroups. A semigroup differs from a group in that it has no identity element and not all elements have an inverse. So the only law of combination that applies is associativity. This situation can often arise when one is working with functions. It is possible for some functions to produce the same output for more than one input value, which means that the function cannot have an inverse, since the inverse function would not produce a unique output for every input. For example, consider a function f, mapping the real numbers into the real numbers, where $f(x) = x^2$. If we apply this function to 1 and −1, we find that $f(1) = 1$, and $f(-1) = 1$. In mathematical terms, this function is not *1-to-1*, because two different arguments, 1 and −1, share the same value (they both produce 1), and it is a mapping *into* rather than *onto* the real numbers, since,

18. Lewin, *Generalized Musical Intervals and Transformations*, 2

although it always produces real numbers, it cannot produce every element in the set of the real numbers (i.e. it cannot produce negative numbers). Because of these features, the function f does not have an inverse. Taking the square root of x will result in two values rather than the single element required. Since it does not have an inverse, this function could not belong to a group. In referring to functions, Lewin distinguishes between two types: *transformations*, which are simply *into* functions, and *operations*, which are 1-to-1 and onto functions.[19] Note that every group of operations is also a semigroup of transformations, since a group automatically satisfies the less stringent requirements of a semigroup. We will return to this distinction later when need arises.

It is also possible to form larger groups from multiple smaller groups through direct product groups. A direct product group combines smaller groups. Its elements are expressed as an ordered list of elements, for example (a, b, c). Each member of the list belongs to a smaller group and is combined with another element from the same group. However, each ordered list is a single element in the larger direct product group, and combining two such lists will produce a third list that is also an element of the direct product group. For example, suppose a and b are elements of the group G, and x and y are elements of group H. Suppose that direct product group of G and H is I. This would be notated $G \times H = I$. The elements of I could be notated as ordered pairs, with each member being an element of G or H. For example, (a, x), (b, x), (a, y), and (b, y) would all be elements of I. Furthermore, if $a * b = c$ in group G and $x * y = z$ in group H, then $(a, x) * (b, y) = (c, z)$ in group I. Let us make this concrete. The integers under addition are often abbreviated as $\mathbb{Z}$. Let us consider $\mathbb{Z} \times \mathbb{Z}$. The elements are ordered pairs. For example, (1, 4) and (3, −2) are both elements of the group, and we can combine them as follows: (1, 4) + (3, −2) = (4, 2). We add 1 + 3 to produce 4, and 4 + −2 to produce 2, so that the direct product being the ordered pair (4, 2). This procedure can easily produce complex groups.

Group theory is a highly developed and very complex branch of pure mathematics. Music theory has only had occasion to use some of the simplest

19. Ibid., 3

groups. Although I have not discussed every aspect of group theory that will be employed in this thesis, this introduction serves as a practical background for Lewin's generalized interval systems.

Generalized Interval Systems

Lewin's generalized interval system uses the concept of a mathematical group to find underlying similarities in the structure of conventional interval systems. Lewin distinguishes between several different musical interval systems, one representing scalar intervals between scale degrees, another for the twelve chromatic pitch classes, one for harmonic space measured in fifths and thirds, and others. In each case, he finds that there is a group involved. He formalizes the general properties of these interval systems as follows:

> A *Generalized Interval System* (GIS) is an ordered triple (S, IVLS, int), where S, the *space* of the GIS, is a family of elements, IVLS, the *group of intervals* for the GIS, is a mathematical group, and int is a function mapping S × S into IVLS, all subject to the two conditions (A) and (B) following.
>
> (A): For all r, s, and t in S, int(r, s)int(s, t) = int(r, t).
>
> (B): For every s in S and every i in IVLS, there is a unique t in S which lies the interval i from s, that is a unique t which satisfies the equation int(s, t) = i.[20]

Let us examine this definition in some detail. As we can see, a GIS has three components. The first of these is the system's *space*, which is a family or set of musical elements. Taking the scalar interval system as an example, the space comprises letter names (excluding accidentals) and octave designations, arranged alphabetically. For example, C4, E5, F7, etc. are all elements of this space.

The second component of a GIS is IVLS, a mathematical group of intervals. In conventional musical interval systems, this group is usually quite simple or familiar. For example, in the case of the scalar interval system described above, the group is integers under addition.

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20. Ibid., 26, Lewin's italics.

The third component of a generalized interval system is the function int(s, t), which maps two elements of the space into the group of intervals. This function means that there is a certain conceptual gap between the elements of the space and the group of intervals. They are two distinct things, a set of elements and a group of intervals, with the int function joining the two. Depending on the interval system in question, the int function can vary widely. In the case of the scalar interval system, the function counts the steps from one letter-name to another. For example, the interval from C4 to E4 is 2 steps, so int(C4, E4) = 2. Notice that, although the interval from C4 to E4 is conventionally called a "third," in Lewin's system the interval is 2. The conventional scalar interval system employs inclusive counting, which counts all the elements in the span from C4 to E4. Inclusive counting has complexities: a third plus a third equals a fifth. To avoid these complexities, Lewin uses standard counting in his int function.

There are two more conditions, (A) and (B), that are necessary for a musical system to qualify as an interval system in Lewin's definition. The first of these, condition A, guarantees that the transitive property will hold true. In other words, combining the interval from s to t and the interval from t to u equals the interval from s to u. In the scalar interval system, the interval from C4 to E4 (2 steps) plus the interval from E4 to G4 (2 steps) equals the interval from C4 to G4 (4 steps). That is, int(C4, E4) + int(E4, G4) = int(C4, G4). The condition ensures that our intervals interact with the elements of the space in an intuitive way.

Condition B states that "For every s in S and every i in IVLS, there is a unique t in S which lies the interval i from s, that is a unique t which satisfies the equation int(s, t) = i."[21] So, take any element in the space. Extend from it any interval in IVLS. Condition B guarantees that there will an element in the space as a result of that extension. In other words, there are no elements in the space that, when combined with any interval in IVLS, will send us outside of the space. This condition can cause problems when the interval system is finite. We will return to a discussion of condition B and its problems shortly.

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21. Ibid., 26

In *Generalized Musical Intervals and Transformations*, Lewin explores two methods of generating a new GIS from an old GIS. First, one can modularize a GIS. For example, the scalar interval system that we have discussed takes no notice of octave equivalence (except that implied in the repeating letter name designations). Thus the interval from C4 to E5 is 9. To model the idea of octave equivalence, Lewin modularizes his system. As an example, let us consider Lewin's interval system for scalar pitch-class intervals.[22] In this interval system, the space consists of the seven pitch-classes denoted by letters. The group consists of the integers mod 7. The function int(s, t) maps the pitch-classes into the intervals by counting the number of steps up necessary to get from one pitch-class to the next. Lewin suggests that this can be visualized by imagining the pitches wrapped around a 7 hour clock and then counting the number of hours of clockwise motion necessary to get from one pitch to another. In this system, the interval from C to F is always 3, while the interval from F to C is always 4, regardless of whether the music moves up or down.

Second, one can combine two GISes to form a third, direct product GIS, in which the elements of S are ordered pairs and the intervals of IVLS belong to a direct product group. We considered an example of such a group earlier when we examined $\mathbb{Z} \times \mathbb{Z}$. In this group, ordered pairs are combined to produce other ordered pairs. For example, equations such as (2, -3) + (4, 1) = (6, -2) simply add each element separately. Lewin applies the same principle to GIS structure, defining a direct product GIS as follows:

> Given $GIS_1 = (S_1, IVLS_1, int_1)$ and $GIS_2 = (S_2, IVLS_2, int_2)$, the *direct product* of GIS_1 and GIS_2, denoted $GIS_1 \otimes GIS_2$, is that $GIS_3 = (S_3, IVLS_3, int_3)$ which is constructed as follows.
> S_3 is $S_1 \times S_2$, the Cartesian product of S_1 and S_2. That is, the elements of S_3 are pairs (s_1, s_2), where s_1 and s_2 are elements of S_1 and S_2 respectively.
> $IVLS_3$ is $IVLS_1 \otimes IVLS_2$, the direct-product group of $IVLS_1$ and $IVLS_2$. That is the members of $IVLS_3$ are pairs (i_1, i_2), where i_1 and

22. Ibid., 17.

i_2 are members of $IVLS_1$ and $IVLS_2$; further, the members (i_1, i_2) and (j_1, j_2) of $IVLS_3$ combine (to form a group) under the rule $(i_1, i_2)(j_1, j_2) = (i_1j_1, i_2j_2)$.

The function int_3, from $S_3 \times S_3$ into $IVLS_3$, is given by the rule

$$int_3((s_1, s_2), (t_1, t_2)) = (int_1(s_1, t_1), int_2(s_2, t_2)).$$[23]

Lewin uses an example of such a product GIS to analyze a piece by Webern, the opening of the third movement of the Piano Variations op. 27. His direct product GIS combines the GIS that measures intervals between equal-tempered chromatic pitch-classes using integers mod 12 with a second GIS that measures the distance between time points by counting the number of beats from time-point s to time-point t. Thus the intervals are pairs representing a pitch interval and a time interval. For instance, in the piece by Webern, the pitch interval of 11, a major seventh, often occurs in conjunction with two successive quarter notes, a beat interval of 1. Thus the combined interval (11, 1) takes on a special significance in this passage, tying together the pitch interval 11 and the beat-defining interval 1.[24] Such direct product intervals can be a powerful analytical tool, tying together multiple features of a passage of music.

Such a direct product GIS provides a sample of the potential that generalized interval systems have. The combination of pitch intervals and temporal intervals into a single entity is something that music theory might discuss, but without a generalized interval system there is no well defined method for conceptualizing it. Consequently, such a combination could easily go unnoticed. The direct product GIS demonstrates that such a combination shares an underlying mathematical similarity with traditional a interval system. In this way, the GIS allows us to map the familiar concept of an interval onto the unfamiliar domain of pitch and rhythm combinations, giving us a means to conceptualize the combination in musical terms. Lewin exploits this feature on several occasions in GMIT, introducing several different ways to conceptualize rhythmic features in

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23. Ibid., 45.

24. Ibid., 38–39.

terms of GIS structure. He even introduces timbral GIS structures that can measure the differences in timbral profile of various instruments.[25]

Transformations

Lewin defines both transpositions and inversions as transformations on the space of a GIS. Following Lewin's distinction between *transformations* and *operations*, transpositions are *operations* in Lewin's terms, because they form a group rather than merely semigroup. Moreover, the group of transpositions is isomorphic to IVLS.[26] That is, the group formed by the transposition operations has the same structure as the group of intervals. Likewise, transpositions and inversions together form a group. In the context of a commutative GIS, a GIS whose group of intervals is commutative, transpositions of collections of musical objects preserve intervals, while inversions reverse intervals.[27] This is the behavior we expect from transpositions and inversions. If we transpose a passage of music, we expect the intervals in the transposed copy to be the same as those in the original passage. Interestingly, in a non-commutative GIS, this does not hold true: in a GIS whose IVLS are non-commutative, the associated transpositions will not preserve intervals.[28] However, since we will deal with only one non-commutative GIS in this thesis, and since we will not use transpositions and inversions in conjunction with it, we will not pursue this distinction further.

After several chapters discussing transformations that generalize musical set theory, Lewin eventually arrives at a structure he calls STRANS, which can replace the idea of a GIS. According to Lewin,

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25. Ibid., 81–85.

26. Ibid., 46–47. Technically, Lewin says that the group is *anti-isomorphic* to the group of intervals, meaning that the order is reversed so that $T_iT_j = T_{ji}$. This distinction between and isomorphism and an anti-isomorphism is only necessary because Lewin employs left orthography. In the construction T_iT_j, T_j is applied before T_i, while T_{ji} is a transposition by the combined interval ji, where j is also applied before i. Thus, in both cases j is applied before i.

27. Ibid., 50, 58.

28. Ibid., 58.

> we can replace the idea of GIS structure by the idea of a space S together with a special sort of operation-group on S. This special sort of group is what mathematicians call *simply transitive* on S. The group STRANS of operations on S is simply transitive when the following condition is satisfied: Given any elements s and t of S, then there exists a unique member OP of STRANS such that OP(s) = t.[29]

When STRANS replaces GIS structure, the group of operations on S are the transpositions. We have already noted that transpositions on the space of a GIS are isomorphic to IVLS. Thus, we can do away with IVLS and int(s, t), and substitute transposition operations acting directly on musical elements. According to Lewin, the reason for introducing GIS structures at all is for the cultural-historical reason that we tend to hear "intervals" between objects, but "transpositions" between Gestalts.[30]

For Lewin, the use of STRANS has two distinct advantages. First, it subsumes intervals under the broader domain of transformational theory, which can include transformations that do not form groups and transformations that have nothing in common with traditional intervals. Transformations form the basis for graphs and networks, which represent transformations in a visual form. Because transposition operations, which correspond to intervals, and more specialized transformations all come under the same heading, he can use graphs to portray either one. Secondly, this shift in thinking toward the "transformational" attitude also belongs to the anti-Cartesian thread in GMIT.[31] For Lewin intervals are static measurements between points that are "out there," while transformations are

29. Ibid., 157.

30. Ibid., 158.

31. Henry Klumpenhouwer, "In Order to Stay Asleep as Observers: The Nature and Origins of Anti-Cartesianism in Lewin's *Generalized Musical Intervals and Transformations*," *Music Theory Spectrum* 28, no. 2 (Autumn 2006): 277–289.

active and can be internalized so that the analyst takes a position *inside* the music.[32]

A Critique of Condition B

As we have seen, STRANS can replace GIS structure. Each interval in IVLS corresponds to a transposition operation in the simply transitive group of operations on S, STRANS. If we examine the definition of a GIS closely, we can see that it actually prepares the way for this shift from GIS structure to STRANS. In particular, conditions A and B in the definition of a GIS must be true in order for STRANS to be a viable replacement for GIS structure. Let us reexamine these conditions:

> (A): For all r, s, and t in S, int(r, s)int(s, t) = int(r, t).
>
> (B): For every s in S and every i in IVLS, there is a unique t in S which lies the interval i from s, that is a unique t which satisfies the equation int(s, t) = i.[33]

Condition A is straightforward. It requires that the int function be transitive and is true of any traditional interval system. This corresponds to the required transitive property of STRANS. Condition B is also necessary for STRANS to apply. As we have already discussed, condition B guarantees that S will hold every element to which one could possibly extend an interval. Because the operations of STRANS act on S, S must necessarily hold every element that could be generated by an operation of STRANS. Consequently, condition B is necessary for STRANS to subsume GIS structure.

Let us suppose, however, that a GIS does not have to lead us to STRANS or to any of the other tools Lewin develops. Is condition B still necessary in order for it to reflect our musical intuitions? As a matter of fact, it is not. The int function could be a mapping that simply generates intervals from notes. There is no need to be able to go backwards from intervals to notes. The int function could go from a finite set of elements to an infinite group of intervals. It would not be

32. Lewin, *Generalized Musical Intervals and Transformations*, 159.

33. Ibid., 26

able to generate every element in that group of intervals, but it could generate some intervals, and the group itself could generate the remaining intervals.

In conventional musical interval systems, the space is constrained by practical limits, such as the number of keys on a piano or what sounds it is possible to produce on a violin. Some of these limits are even built into the theoretical system, as is the case in the medieval gamut system, where the gamma-ut was the lowest note in the space, although its pitch would have varied according to context. On the other hand, intervals have always been a theoretical abstraction which could be infinite, even with a finite gamut of notes. Music theory has been happy to live with this situation for thousands of years. Similar oddities exist in the conventional naming of musical intervals, which employ inclusive counting, lack the number 0, and involve a correspondingly unusual form of arithmetic (so that a third plus a third equals a fifth). All of these disjunctions and oddities are done away with by Lewin's GIS as it stands. Thus condition B is a purely mathematical necessity. In fact, it tends to widen the gap between Lewin's GIS structure and traditional musical intervals.

Moreover, condition B creates problems even for Lewin. In GMIT section 2.2.5 he gives the following example of an interval system:

> The musical space is a family of durations. Int(s, t) is the difference (NB not the quotient) of time units between s and t: Int(s, t) = (t − s) units. So, if r, s, and t are respectively 3, 4 and 8 units long, then int(r, s) = (4 − 3) units = 1 unit, int(s, t) = (8 − 4) units = 4 units, and int(t, r) = (3 − 8) units = −5 units.[34]

When Lewin reexamines this in the light of GIS structure, he finds that it does not lead to a GIS:

> S, IVLS, and int here cannot satisfy Condition (B) of Definition 2.3.1. For instance, try s = 3 units and i = −8 units; then there is *no* duration t in S satisfying int(s, t) = i. … S does not contain "negative durations," and failing some convention not yet specified, it is not clear what intuitions we could possibly be

34. Ibid., 24

modeling, when we stipulate a duration t that lasts not only less than no time at all, but also *measurably* less than no time at all.[35]

Arguably, there is a significant difference between conceptualizing infinitely subsonic pitches and conceptualizing a negative duration, and this is the approach that Lewin takes, allowing GIS structures for absolute pitch, but excluding absolute additive durations. However, it reveals an aspect inherent in generalized interval systems, namely that they are a more significant abstraction from our traditional conceptions of intervals than Lewin's discussion of "intuitions" might suggest. From a purely practical point of view, the concern over whether or not some of the theoretical elements in S can be associated with anything we can actually conceptualize is irrelevant, because we will never encounter them in the music anyway. On the other hand, from a musical point of view, Lewin's GIS of 2.2.5 is a useful means of modeling music, and the fact that it does not satisfy condition B does not make it any less useful.

We have several alternatives to take in regard to condition B. We could drop condition B in the interest of making GIS structure closer to a conventional interval system, thus allowing additional possibilities such as the GIS of section 2.2.5. On the other hand, GIS structure will still be a significant abstraction, and the mathematics necessary for STRANS and many other things will not work out, leaving the entire edifice of GMIT in a fragmented state. Alternatively, we could take Lewin's approach and include or exclude a GIS based on how comfortable we are with conceptualizing impossibilities. Lewin is comfortable with infinitely supersonic and subsonic pitches, but not with measurably negative durations. Finally, we could adopt the following approach: first, retain condition B for the sake of proper mathematics; second, include any kind of impossibility in S by simply defining things such as "negative durations" to exist without worrying about how to conceptualize them, since we will never encounter them in the music anyway; third, acknowledge that a GIS is really a very significant abstraction relative to traditional intervals no matter how we approach it. This is the approach that I take.

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35. Ibid., 29.

The Conceptual Framework

The idea behind a GIS of non-chord tones lies in our musical intuition that if the same chord or note is played with and without some number of non-chord tones, then, while the note is still recognizable as such, the number and type of non-chord tones added will change the musical impression of the note or chord in a distinctive way. Let us consider this in the light of figure 2.1, which provides six distinct musical situations, labeled a to f. If we compare the six examples, we can understand each as a C major triad with the same voicing, more or less. Yet, each one has a distinctive character due to the different arrangements of non-chord tones, the configurations of non-chord tones attached to the C major chord. The differences among these configurations are not all the same. For instance, figures 2.1b and 2.1c sound much "closer" to each other than to any of the others because they both employ one neighbor tone, despite the differences in pitch and rhythm. The relationship of figures 2.1d and 2.1e to figures 2.1b and 2.1c is more complex. Figure 2.1e sounds similar to 2.1b and 2.1c, because there is only one non-chord tone, and because it occurs in the same melodic voice, while 2.1d sounds similar to 2.1b and 2.1c, because it also employs neighbor tones. Yet, while both bear some similarity to 2.1b and 2.1c, figures 2.1d and 2.1e are quite different from each other. If we rephrase this in terms of distance, figures 2.1d and 2.1e are distant from each other, yet each is close to 2.1b and 2.1c in terms of a specific direction. Developing the notion of relational distance, we can conceive of the relationships as a triangle in which 2.1b and 2.1c are at the same point, while the lines to 2.1d and 2.1e form the two shorter sides on either side of an oblique angle and the line from 2.1d to 2.1e forms the third side. While all of these figures would be relatively close to the point represented by 2.1a, figure 2.1f would be far removed from any of the others.

Methodologically, this approach runs into a few difficulties. At what point should we consider two different things to be similar? If we are to have any precise measure of "intervals" between configurations, then we must follow some method of generalization that cuts through the complexities of the music. In the preceding example, we based our consideration of similarity and difference on a number of factors: the type and number of non-chord tones involved and the melodic voice in which the non-chord tones occurred. These distinctions feel relatively natural and adequately capture traditional ways of approaching non-chord tones. We can simplify the distinction even more if we consider the vertical harmony and the melody separately. If we do this in an analytical context, we could consider two chords each containing a single neighbor tone to be identical even though that tone might appear in different voices. We could then analyze the melodic lines separately and point to the differences in the melodic lines created by the distinction that we had previously ignored.

Before we formalize this approach, we should undertake one final consideration. What types of non-chord tones should we take into account? Since there are a variety of approaches to this categorization, we could include quite a few different types. Since this would potentially lead to a more nuanced view of the differences between various configurations we will not exclude the possibility of defining as many types as we wish, but rather build the formal system so as to allow the number of types to change. In this context, however, the focus will primarily be on neighbor tones, passing tones, and suspensions.

Formal Considerations

As we have just seen, it is possible to conceptualize differences between musical entities based on the number and type of non-chord tones without regard to details such as harmony or rhythm. Rhythm and to a lesser extent harmony must still play a role in determining the size of the musical entity under consideration. In figure 2.1, each example consists of a single C major chord lasting one measure. I called each of these examples *configurations* of non-chord tones. The term *configuration* is useful, because it could refer to the non-chord tones associated with a single chord or to a metric consideration such as the non-chord tones in an entire beat, measure, or phrase, regardless of how many chords are in the beat, measure, or phrase. We will retain the term configuration and decide on the scope of that configuration when we analyze a piece of music.

Our first non-chord tone GIS will be very general. It will simply count the number of non-chord tones. This GIS will not distinguish between passing tones, neighbor tones, and suspensions; all of them are simple non-chord tones. However, we will also be developing several more non-chord tone interval systems using the same principles. At that point, we will begin to account for the different types of non-chord tones. Our first non-chord tone GIS is NCT-GIS(NCT); the notation involved in this name will be explained shortly.

NCT-GIS(NCT): Let the space of the system be the family of integers that count non-chord tones associated with some musical entity, such as a note, chord, measure, or phrase. IVLS, our group of intervals, consists of integers under addition. Given two configurations of non-chord tones, s and t, int(s, t) assigns an element of IVLS to the order pair (s, t) by calculating s − t. In other words, the relevant IVLS integer counts the number of non-chord tones one must add to s in order to have the same number of non-chord tones as t.

IVLS is commutative. The GIS satisfies conditions A and B, since both S and IVLS consist of the integers, and int(s, t) simply uses addition to map S into IVLS. It is isomorphic to both Lewin's GISs in sections 2.1.1 and 2.1.2 of GMIT, generalized interval systems for pitch.[36] Notice that S, the space of the GIS does contain negative integers, representing configurations with negative numbers of non-chord tones. We have no way of conceptualizing such configurations; however, they are mathematically necessary for the GIS to satisfy condition B.

Applying the GIS to the examples in figure 2.1, we represent each configuration as a positive integer, by counting the non-chord tones. So, we represent 2.1a as 0, because the configuration in this example has no non-chord tones. The examples in 2.1b, 2.1c, and 2.1e are all represented by 1; likewise, 2.1d is represented as 2; and 2.1f as 8. We calculate the member of IVLS appropriate to the extension from figure 2.1a to 2.1b, that is, from 0 to 1, by subtracting 0 from 1. Accordingly, the GIS assigns the integer 1 as the interval from figure 2.1a to figure 2.1b. In the same way, the interval from 2.1b to 2.1d is 1, and the interval from 2.1a to 2.1d is 2. In other words, int(2.1b, 2.1d) = 1 and int(2.1a, 2.1d) = 2. So far, this GIS is highly intuitive.

36. Ibid., 16–17.

We must emphasize, however, that unless we define negative configurations of non-chord tones, it does violate Lewin's condition B, which states that "for every s in S and every i in IVLS, there is a unique t in S which lies the interval i from s, that is a unique t which satisfies the equation int(s, t) = I."[37] As an example, take the interval from 2.1d to 2.1b, int(2.1d, 2.1b). Since figure 2.1d is 2 and figure 2.b is 1, the interval from 2.1d to 2.1b is −1, int(2.1d, 2.1b) = −1. If we now extend the interval −1 from 0, the configuration in 2.1a, we find that the hypothetical configuration must have −1 non-chord tones. This hypothetical configuration is a member of our space, even though we cannot conceptualize it. Even though we have no suitable conception of a negative number of non-chord tones, we have defined our space that way in order to accommodate the mathematics.

While this does present a technical problem if we wish to invoke Lewin's model, it does not prevent us from obtaining meaningful analytical results, since we will simply never encounter configurations with a negative number of non-chord tones in analysis. As we have already argued, condition B of GIS structure is a mathematical necessity that tends to create a break with our experience. A configuration that has a negative number of non-chord tones may bare a greater similarity to conceptualizing "negative durations," which Lewin does not allow, than it does to conceptualizing sub- and supersonic pitches, which Lewin does allow. However, this does not prohibit practical analysis. The situation is, in fact, no different than many other cases in which we habitually employ negative numbers so as to obtain group structure.

Having established the basic principles, we will now explain the label NCT-GIS(NCT) and develop another GIS, which we will apply in our analysis in Chapter 3. The label NCT-GIS(NCT) is an abbreviation for *non-chord tone generalized interval system*, while the NCT in parentheses indicates the type of non-chord tone that the GIS measures. In this case, we did not distinguish between different types of non-chord tones, so we simply used the abbreviation NCT. We could also substitute a common abbreviation for a specific type of non-

37. Ibid., 26.

chord tone. For example, passing tones are often abbreviated by the letter P. Consequently, NCT-GIS(P) would be a GIS that measured only passing tones, while NCT-GIS(Ant) would count only anticipations. What if we wanted to measure two types of non-chord tones, such as passing tones and neighbor tones? In this case, we would label our GIS as NCT-GIS(P, N) to indicate both passing tones and neighbor tones. There is a distinct difference, however, when we employ more than one non-chord tone. The resulting GIS is now a direct product GIS. For instance, NCT-GIS(P, N) would be the direct product of NCT-GIS(P) and NCT-GIS(N). This method can be used to generate an arbitrarily large direct product GIS. Let us examine a concrete example.

NCT-GIS(P, N, S). Let S be the family of configurations of non-chord tones connected to some musical entity. The elements of S are lists of three integers (P, N, S): P represents the number of passing tones; N, the number of neighbor tones; and S the number of suspensions in the configuration. IVLS consists of lists of three integers <i, j, k>, where i is an integer measuring differences in passing tones, j is an integer measuring differences in neighbor tones, and k is an integer measuring differences in suspensions. Each of these three combines only with the corresponding member under addition. Note that throughout this discussion, intervals (elements in IVLS) will be distinguished from objects (elements in S), by the use of pointed brackets "< >" rather than ordinary parentheses "( )." Given two configurations of non-chord tones, s and t, int(s, t) is the number of non-chord tones of each of the three types that one must add to s in order to have the same number of non-chord tones of each type as t. The GIS is commutative.

This GIS is a direct product GIS created from NCT-GIS(P), NCT-GIS(N), and NCT-GIS(S). Because it distinguishes between three different types of non-chord tones, it is more precise in the information it gives than NCT-GIS(NCT). If we consider figure 2.1 again, we now see that 2.1a, which has no non-chord tones is still equal to (0, 0, 0). Both 2.1b and 2.1c have a single neighbor tone, so 2.1b and 2.1c = (0, 1, 0). Likewise, 2.1d = (0, 2, 0). Because it has one passing tone, 2.1e = (1, 0, 0), while 2.1f = (5, 3, 0), since it has 5 passing tones and 3 neighbor tones. In all cases, the number of suspensions is 0. In addition, we can calculate intervals as follows: 2.1b and 2.1c have the same number of neighbor tones, so int(2.1b, 2.1c) = <0, 0, 0>; int(2.1b, 2.1d) = <0, 1, 0>, because 2.1d has one more neighbor tone that 2.1b; int(2.1b, 2.1e) = <1, −1, 0>, and int(2.1d, 2.1e) = <1, −2, 0>, because in both cases the number of passing tones increases, while the number of neighbor tones decreases.

This GIS now gives us information that more closely matches our intuition regarding the passage as described above, where we compared our conceptualization with the idea of relational distance. Figure 2.2 gives just such a representation on a Cartesian coordinate plane, with the X and Y axes representing passing tones and neighbor tones. Since our GIS includes suspensions, a complete representation would require a three-dimensional graph. However, since we have no suspensions in our example, the graph is limited to two dimensions. The various configurations are plotted on the graph, along with a selection of intervals. As the graph demonstrates, most of the configurations lie relatively close to the origin. Only the configuration for figure 2.1f is relatively distant.

As we have already noted, our system of labeling allows for a large direct-product GIS representing many different types of non-chord tones. The system can easily be altered to suit a different focus. We could generate a complex direct product GIS such as NCT-GIS(P, N, Esc, App, S, R, Ant), which would measure all the common classifications of non-chord tones. Such a GIS would correspond to a representation in seven dimensions and could provide a very nuanced point of view.

There are a number of questions that might be asked in regard to the approach that I have taken. Why, for instance, is it necessary to invoke GIS structure? Within the scope of Lewin's GMIT project, the generalized interval system is ultimately supposed to be superseded by transformations, so why bother with GIS structure at all? In answer to these questions, the obvious advantage of invoking GIS structure is that it provides a solid foundation for more detailed work involving non-chord tone transformations. In addition, GIS structure brings with it a variety of other tools that are associated with GIS structure, and which can be adapted for non-chord tone analysis. Finally, the presentation in terms of a GIS makes it easy to conceive of configurations of non-chord tones in exactly the same way we conceive of pitch. This makes it possible to conceive of features such as thematic sets of non-chord tones that can be transposed, to discuss the non-chord tone “contour” of a passage, and to adopt any of Lewin's tools for the analysis of pitch with the assurance that they will work in this context.

Conclusion

In this chapter, I have introduced the basic concepts involved in Lewin's GMIT project, particularly the idea of a generalized interval system. We have examined some of the features and short-comings of GIS structure, and provided a concrete example of how this concept can be extended in the form of non-chord tone generalized interval systems. Although the methodological and technological framework provided by GMIT is formidable, the basic idea of a non-chord tone generalized interval system which simply counts non-chord tones of various types is relatively simple and intuitive. The formalism inherent in invoking GIS structure helps to provide a means of conceptualizing non-chord tone structures by mapping them onto the more familiar domain of pitch. Moreover, since it brings with it the entire apparatus of GMIT, this method can easily be extended and adapted by the individual analyst. Having sufficiently discussed these formal properties of non-chord tone generalized interval systems, we can now turn to their practical application to musical analysis in chapter 3.

Chapter 3: Analysis of Mozart's Variations on "Ah! Vous dirai-je, Maman"

In the preceding chapter, we introduced the concept of a non-chord tone generalized interval system. As we have seen, our analytical tool carries a substantial theoretical background with it; however, the basic idea of a non-chord tone GIS is relatively simple, since it merely involves counting non-chord tones. We now turn to an analysis of Mozart's variations on "Ah! Vous dirai-je, Maman," K. 265. The analysis is first of all a practical application of a non-chord tone GIS, demonstrating the usefulness of the basic feature of such a GIS, namely counting non-chord tones and conceptualizing the differences between configurations of non-chord tones in terms of intervals. Secondly, it explores some of the possibilities for non-chord tone analysis provided by the theoretical apparatus of Lewin's formalism, including transposition and inversion, retrograde-inversion chains, and interval vectors.

In addition to the practical application of a non-chord tone GIS, this analysis has as its goal a thorough non-chord tone analysis of a complete piece of music. Thus, I have chosen to discuss the entire piece, including the labeling and graphing of non-chord tones in each variation, as well as some features that apply to the overall form. This is necessary partly because I will be taking a statistical approach to the overall form from a non-chord tone perspective; and partly because it also serves as a basis for a brief commentary on interesting features in each variation. As we shall see, non-chord tones in these variations contribute to both local and global compositional design.

Analytical Preliminaries

As we have already noted, there are many approaches to analyzing non-chord tones. Consequently, some basic principles employed in this particular analysis must be outlined. My approach draws fairly heavily on Schenkerian scale-degree theory, by determining the scale-degree governing a given point and then labeling other tones as non-chord tones. Following Schenker (and Schoenberg), we will consider only passing tones, neighbor tones, and suspensions, rather than the more complex designations often employed in basic music theory texts. Since the variations contain no anticipations, we need not consider them in our model. Consequently, the non-chord tone GIS for this analysis will be NCT-GIS(P, N, S), in which both non-chord tone configurations and intervals are expressed as ordered triples.

Before turning to our analysis, we must also briefly define the various non-chord tones that will be analyzed. A passing tone is approached and left in the same direction and brings about a dissonant state. The motion involved need not be stepwise. This means that we can encounter "incomplete" passing tones. For instance, considered against the tonic triad, the succession $\hat{8}$–$\hat{6}$–$\hat{5}$ would contain an incomplete passing tone, $\hat{6}$.[38] A neighbor tone is approached and left in the opposite directions and brings about a dissonant state. Thus, appoggiaturas and escape tones will be considered "incomplete" neighbor tones. A suspension maintains a pitch from the previous consonant state into a dissonant state. Since many of the seventh chords in this piece arise from chains of suspensions and other very clear voice-leading features, sevenths of chords other than the dominant-seventh or fully diminished seventh-chord will generally be analyzed as non-chord tones unless there is some compelling reason not to do so. In the case of the conventional cadential six-four, both components will be treated as chord tones. In some cases, clear instances of compound lines or arpeggiation will be taken into account in analyzing non-chord tones. Further notes will be appended to each variation if anything is unclear.

The final consideration in analyzing non-chord tones is the fact that this piece is a set of variations on a well-known tune. There are a number of cases in which notes could be treated as chord tones, but can also be heard as non-chord tones embellishing the underlying theme. In general, consideration of the theme has the highest priority. Consequently, there will be a few instances of apparent chord tones that will be treated as non-chord tones for thematic reasons.

The form of K. 265 is straightforward, consisting of a theme followed by twelve variations. The underlying theme and harmonic structure of each variation is extremely stable with very few changes between variations. Each variation is in

38. Schenker, *Counterpoint*, 184–185 discusses this type of passing tone in detail.

ternary form, with A sections beginning in measures 1 and 17, and an eight measure B section in measures 9–16. It is constructed in square four-bar phrases with a harmonic rhythm of one or two chords per measure, with clear authentic cadences in measures 7–8 and 23-24 and a half cadence in measures 15–16. Because of the ways in which it is generally treated in the variations, the harmonic rhythm of measures 5–7 and all corresponding passages will generally be considered to be one chord per beat. However, the configurations of non-chord tones will be considered to be the number of non-chord tones in a given measure in both cases, since the harmonic rhythm is still somewhat variable and a measure by measure approach facilitates comparison between different variations. Finally, there is a dominant pedal point implied in measures 9–16 of every variation, which will be ignored when it contradicts the harmonies above it.

For the theme and each subsequent variation, I have provided the score, a graph depicting the non-chord tone profile with lines, and a table giving the relevant non-chord tone configurations and intervals, each as a separate figure. The relevant figures are listed with the heading for each variation. We will have occasion to refer to these extensively throughout the analysis.

The Theme (Figures 3.1–3.3)

The theme appears in figure 3.1. A graph, which appears in figure 3.2, amplifies the pattern of non chord-tone configurations throughout the theme. The bottom of the graph provides measure numbers. The vertical dimension counts the number of non-chord tones in each configuration, drawing lines between successive configurations. These configurations are listed in the table in figure 3.3. The dotted line in the graph represents neighbor tones and indicates that only neighbor tones are present and that they only appear in measures 7, 15, and 23. This line may be thought of as a contour, similar to the contour of a melody, with ascending and descending lines representing intervals.

The table presenting the configurations of non-chord tones in each measure of the theme appears in figure 3.3; it provides the data upon which the graph in figure 3.2 is based. The table presents our GIS objects in the configurations column. The corresponding measure numbers appear to the left of each configuration. The configuration in measure 1 contains no passing tones, no neighbor tones, and no suspensions. Accordingly, it is represented as (0, 0, 0), as are all the measures through measure 6. The table tells us that the next configuration, the configuration in measure 7, contains a neighbor tone. Looking at figure 3.1, we see that E5 in the right hand is enclosed in parentheses and marked with an N. According to the rules we have given, E is an (incomplete) neighbor, since it is a non-chord tone and since it is approached and left in opposite directions. There are no passing tones or suspensions, and so we represent the configuration as (0, 1, 0). The interval that extends from the configuration in measure 6, (0, 0, 0), to the configuration in measure 7, (0, 1, 0), appears in the column to the right of the configuration in measure 6 and slightly offset, indicating that it comes between measures 6 and 7. So applying the interval <0, 1, 0> to (0, 0, 0), the configuration in measure 6, produces (0, 1, 0), the configuration in measure 7. The notation captures the idea of adding no passing tones, one neighbor tone, and no suspension to the configuration in measure 6 to produce the configuration in measure 7, and applying the interval <0, −1, 0>—which represents the idea of "add no passing tones, take away one neighbor tone, add no suspensions"—to configuration 7, produces (0, 0, 0), the configuration in measure 8.

The theme has an extremely simple non-chord tone profile. Mozart embellishes the original tune infrequently and then only by way of neighbor tones. We can also see that the intervals between configurations of non-chord tones also present a "neighboring" profile, motion in one direction followed by a change in direction. There are several examples of this: the intervals from measure 6 to measure 7 and measure 7 to measure 8; measure 14 to measure 15 and measure 15 to measure 16; and measure 22 to measure 23 and measure 23 to measure 24. There is also a neighbor structure among the three configurations themselves, from measure 7 to measure 15 and measure 15 to measure 24.

Variation 1 (Figures 3.4–3.6)

As with the theme, variation 1 and the data for its non-chord tone analysis are given in three figures: figure 3.4, which contains the music with non-chord tones enclosed in parentheses and labeled as either passing tones (P), neighbor tones (N), or suspensions (S); figure 3.6, which gives a table with non-chord tone configurations and intervals; and figure 3.5, which contains a graph representing non-chord tone contours in the variation. Notice that the table in figure 3.6 has an additional column of non-chord tone intervals, giving the interval from a measure in the theme to the corresponding measure in variation 1. For example, the interval from measure 7 of the theme to measure 7 of variation 1 is listed as <0, −1, 0>, because measure 7 of the theme contained one neighbor tone, whereas measure 7 of variation 1 contains no non-chord tones. In addition, the graph in figure 3.5 now contains three clear contours, a passing tone contour represented by a solid line, a neighbor tone contour represented by a dotted line, and a suspension contour represented by a dashed line. The non-chord tone profile as a whole is represented by the simultaneous changes in each of these contours.

The labeling of non-chord tones in this variation presents some difficulty, because of the thin two-part texture. In general, I have analyzed each measure against the projected underlying harmony, even if not all the notes are present. Thus, we consider the A5 in measure 4 a suspension, since we take the harmony in the measure to be C major. In addition, the notes of the original theme are given precedence. In measures 10 and 14, we consider the Gs on the first and last eighth note of each measure as non-chord tones although they are actually consonant relative to the underlying pedal tone G. We do so for two reasons: first, they are clearly dissonant with the F of the theme; and second, the repeated patterns in the measures on either side also treat the corresponding notes as non-chord tones. This suggests that the D and F together should be considered part of a ii chord or vii chord over the dominant pedal rather than an integral part of a dominant-seventh.

This variation presents a non-chord tone profile that is very different from the theme. Whereas in the theme the same interval was consistently employed, this variation, though it may use the same configuration twice in a row, rarely uses the same interval twice in succession. The intervals from the corresponding measures of the theme also feature an interesting situation. While all other intervals are positive, the intervals in the measures before each cadence are negative in the neighbor tone dimension. Accordingly, in those places where the theme had employed non-chord tones, the current variation has now eliminated or decreased the number of non-chord tones, and vice versa. This provides further contrast with the theme, for the only points at which corresponding measures in the theme and the first variation are identical (i.e. form the interval <0, 0, 0>) are the cadences. In addition, while the theme embellished the cadences with non-chord tones, in this variation the largest configurations are in the middle of the phrase in the A section or at the beginning in the case of section B.

An interesting feature of this non-chord tone profile can be found by comparing the passing tone contour in the A sections with the neighbor tone contour in the B section. As figure 3.5 shows, the passing tone contour of measures 1-8 consists of an ascent to 4 passing tones followed by a descent. This is paralleled in the B section by a similar contour.

Variation 2 (Figures 3.7–3.9)

As before, the data for variation 2 is labeled on the score in figure 3.7, graphed in figure 3.8, and listed as a table in figure 3.9. A few choices of non-chord tones deserve commentary. The sevenths in the V chords on the last beats of measures 7 and 23 are treated as passing tones. The seventh-chords in measures 5–7 and 21–23 arise from chains of suspensions and are identified accordingly. The E in measures 12 and 16 is prepared as a suspension despite the intervening neighbor tone and is dissonant against the V chord. Finally, the trill in measure 17 is treated as a single neighbor tone because of its short duration.

Variation 2 contains a number of interesting relationships created by the contour of its non-chord tone profile. There is a recurring pattern in which several pairs of configurations each contain the same interval. That is, measures 1 and 2 are separated by the identity interval <0, 0, 0> because they have the same configuration of non-chord tones. Likewise, measures 3 and 4 and measures 5 and 6 each form a pair outlining the same interval, <0, 0, 0>. Because all of these pairs outline the same interval, they are all “transpositions” of the same underlying configuration.

We can express this formally using transposition operations in the manner that Lewin uses in GMIT, with the symbol $T_i(s)$, where i is an interval in IVLS and s is an element of the space of the GIS.[39] For example, we can notate the transposition of the configuration (0, 3, 0) by the interval <0, 0, 1> to obtain the configuration (0, 3, 1) as follows: $T_{<0, 0, 1>}((0, 3, 0)) = (0, 3, 1)$. This particular transposition applies to the relationship between measures 1 and 3 and between measures 2 and 4. Thus, measures 3–4 are the $T_{<0, 0, 1>}$ of measures 1–2. Likewise, measures 5–6 are the $T_{<0, -1, 0>}$ of measures 3–4. This example is relatively simple, for each pair of measures outlines the identity interval, <0, 0, 0>. Since they both share the same interval, they must be transpositions of each other.[40]

A similar example is provided by measures 14–15 and measures 16–17. Both of these pairs of measures outline the interval <0, 2, −1>. Furthermore, measures 14 and 16 both have the configuration (0, 3, 1), while measures 15 and 17 both have the configuration (0, 5, 1). Since they are the same, the transposition relationship is transposition by identity, or $T_{<0, 0, 0>}$. While transposition by identity is relatively trivial, if we think about the domain of pitch (imaging calling any strict repetition of the same pitches "transposition by identity") such a relationship is more unusual in the domain of non-chord tone intervals, particularly when the melodic and rhythmic content of the pairs of measures does not involve repetition.

Having discussed non-chord tone transposition operations, we can also find non-chord tone inversion operations in this variation. While transposition operations preserve intervals—that is the intervals in a transposed passage will be the same as those in the original—inversion operations reverse intervals.[41] For

39. Lewin, *Generalized Musical Intervals and Transformations*, 46.

40. In this case, the transposition employs all components of the non-chord tone interval; however, it would also be possible to look at this as a partial transposition similar to what Joseph Straus terms relative degrees of *uniformity* and *smoothness* in his article "Uniformity, Balance, and Smoothness, in Atonal Voice-leading," *Music Theory Spectrum* 25, no. 2 (Fall 2003): 305-352.

41. Lewin, *Generalized Musical Intervals and Transformations*, 50, 58; This property only holds true in a commutative GIS, such as the conventional interval system or the non-chord tone interval system under consideration; this would not be true in a non-commutative GIS.

example, the interval from C to E is a third up, while the interval from C to A is a third down. The same feature is true of our non-chord tone intervals. Thus, measures 14–15 (outlining <0, 2, −1>) and 15–16 (outlining <0, −2, 1>) are related by inversion. This is true, because the interval found in measures 14–15 is reversed in measures 15–16; the intervals <0, 2, -1> and <0, -2, 1> are inverses of each other.

We can take this discussion a step further by invoking a phenomenon that Lewin calls a retrograde inversion chain, denoted by a transformation called RICH.[42] A retrograde inversion chain occurs when a brief fragment of music overlaps with its own retrograde inverted form. Taking the domain of diatonic pitch as an example, the retrograde form of the motive C–D–E–A is A–E–D–C. Inverting this motive around C producest E–A–B–C. Since C–D–E–A and E–A–B–C contain the sequence E–A, they can overlap to form the sequence C–D–E–A–B–C. Since the new sequence ends with a stepwise ascent and the original begins with a stepwise ascent, we could overlap a transposed version of the original with the end. Transposing down one step C–D–E–A produces B–C–D–G. Since B–C overlaps with the end of the sequence the whole chain can be extended indefinitely. A depiction of this phenomenon in musical notation appears in figure 3.10.

It is a hallmark of retrograde inversions that their interval succession is a palindrome. This is because the retrograde form reverses the direction of each interval and reverses the order in which intervals appear. Inversion reverses the direction of the intervals. The result is that the direction of each interval is reversed twice, but the order of the intervals is reversed only once.

If we examine variation 2 closely, we can see that measures 15–17 are the retrograde inversion of measures 14–16. The configurations of measures 14–16 are as follows: (0, 3, 1), (0, 5, 0), and (0, 3, 1), with the two intervals <0, 2, −1> and <0, −2, 1>. In retrograde or reverse order these would all be exactly the same, since the sequence forms a palindrome. If we then invert the intervals we arrive at <0, −2, 1> and <0, 2, −1>, the same intervals in reverse order. This is what

42. Ibid., 180–184.

happens in measures 15–17, where we have this sequence of three configurations: (0, 5, 0), (0, 3, 1), and (0, 5, 0), with the intervals <0, −2, 1> and <0, 2, −1>, demonstrating that measures 15–17 are indeed the retrograde inversion of measures 14–16. Since these measures overlap, this presents the beginning of a retrograde inversion chain, Lewin's RICH transformation. This example is very brief and is broken off after only one RICH transformation; in the familiar domain of pitch it would be similar to the retrograde inversion inherent in a series such as C–D–C–D. However, because non-chord tone intervals are much more varied than pitch intervals, even such a simple phenomenon is not necessarily a trivial occurrence. Although it is very simple, even this one instance is suggestive, for retrograde-inversion chains provide forward motion. Thus, this brief retrograde-inversion chain helps to bring us smoothly into the return of the A section in measure 17.

Relative to the theme, variation 2 is very similar to variation 1. The largest configurations occur near the beginning of each phrase, with the configuration (0, 0, 0) occurring only at the end of section A in measures 8 and 24. One feature that does recall the structure of the theme is the introduction of a single passing tone in measure 7 and another measure 23. Since these are the only passing tones in this variation, the passing tone contour of this variation is similar to the neighbor tone contour of the theme. To see this, compare the solid line in figure 3.8 with the dotted line in figure 3.2: both have peaks at measures 7 and 23. This was completely contradicted in variation 1 by the use of (0, 0, 0) in measures 7 and 23. In addition, one of the most interesting features of variation 2, the retrograde inversion chain, occurs at the return of section A, which in the theme was emphasized by the largest configuration of non-chord tones (0, 4, 0), but in variation 1 was broken by a rather sudden instance of (0, 0, 0). Thus, in addition to the obvious similarity of the melody, arising from the fact that the accompaniment now receives most of the embellishment, variation 2 can be seen as a slight move back toward the theme because of the specific effects created by the total configurations of non-chord tones.

Variation 3 (3.11–3.13)

Variation 3 is fairly straightforward. As with the previous variations, the music and data for the analysis can be found in figures 3.11, 3.12, and 3.13. Like the preceding variations, it constantly varies the intervals that are employed. However, it also has a relatively smooth contour and small intervals, demonstrated in figure 3.12 by the relatively low profile. The fact that the intervals from the corresponding measures in variation 2 (shown in the first column of the table in figure 3.13) are almost all negative indicates a substantial decrease in the number of non-chord tones in this variation, suggesting a gradual progression away from variation 1 back toward the theme. This conclusion is further supported by a slight ascent to (0, 0, 2) in the suspension contour at measures 7 and 23 (demonstrated by the peaks of the dashed lines in the graph in figure 3.12), followed by a return to (0, 0, 0) at the cadences in measures 8 and 24, reflecting the neighbor tone contour of the theme and the passing tone contour of variation 2.

The specific features of this variation also connect it with variation 2. As in variation 2, pairs outlining <0, 0, 0> recur several times. In addition, measures 10–13 are the retrograde inversion of measures 12–15, thus creating the beginning of another retrograde inversion chain.

In terms of labeling non-chord tones, the Gs in measures 10 and 14 may be heard as suspensions relative to the underlying theme, a choice that parallels my analysis of variation 1. In addition, the A in the trill in measure 13 must be heard as a neighbor tone, while the A in the last beat should be heard as a chord tone. This hearing involves understanding the second beat of measure 13 as a secondary dominant-seventh leading to a ii chord on the down-beat of measure 14, while the G in the bass is understood as part of a dominant pedal point that underlies the harmonic structure of measures 9–16 in every variation.

Variation 4 (Figures 3.14–3.16)

The non-chord tone profile of Variation 4 is quite interesting, with several phrases outlining very clear contours in which the different non-chord tone contours behave in a more unified fashion. This is demonstrated by the similarities in the lines of the graph in figure 3.15. If we examine the table in figure 3.16, we find that in the first phrase, measures 1 and 2 are paired by the interval <0, 0, 0>. This pair returns, transposed first by <0, 0, 1> in measures 3–4 and then again by <0, 2, 0> in measures 5–6. The energy created by these ascending motions is sustained for another bar, before subsiding with the interval <0, −2, −1> at the cadence in measure 8. This contour reflects the theme much more closely. The high points of the total number of non-chord tones are now concentrated near the cadences of section A at measures 7–8 and 23-24. In our preceding discussion, we found that this same feature was appeared in the contours of a single type of non-chord tone in the theme, in variation 2, and in variation 3; however, in the contour for this variation there is no equivalent phenomenon for the neighbor tones found in measure 15 of the theme, since, as figure 3.15 demonstrates, the high points in measures 9–16 occur in the middle of the phrase rather than at the end.

In terms of non-chord tones, measure 9 represents a key expressive point. Throughout the preceding variations, the configuration (0, 0, 0) has been rare, occurring primarily at cadences. It certainly has not occurred at the beginning of the B section since the original theme. Yet, we find it here in measure 9, prepared by two clear contours. One of these can be traced through measure 9 across successive variations as follows: from the theme to variation 1, <0, 4, 0>; from variation 1 to variation 2, <0, −1, 0>, from variation 2 to variation 3, <0, −2, 0>; and from variation 3 to variation 4, <0, −1, 0>. This represents a large leap followed by a slow steady descent to the arrival at (0, 0, 0). This feature can be found by tracing the interval for measure 9 found in the first column of figures 3.3, 3.6, 3.9, 3.13, and 3.16. This contour appears in figure 3.17, which represents the configurations found in measure 9 of the theme and each variation in a graph similar to that employed within each variation. The other contour is the "thematic" contour of section A, which appears in figure 3.15.

Variation 5 (Figures 3.18–3.20)

Variation 5 continues the process found in variation 4 and approaches quite close to the theme. The profile is thin: no measure contains more than one non-chord tone and there are many instances of <0, 0, 0>. As in variation 4 and the theme, the non-chord tones tend to appear near the end of phrases, and this time a similar feature occurs in the passing tone contour of section B, which we can see in figure 3.19. The key difference that sets this variation apart from all previous variations is that it is the first variation to end with a non-chord tone configuration other than (0, 0, 0); the table in figure 3.20 shows that the configuration in measure 24 is (0, 1, 0), representing the presence of the single neighbor tone. Thus, although its contour strongly connects this variation back to the theme and thus represents the end of a section, the instability of ending on the configuration with a non-chord tone, (0, 1, 0), prepares the listener for a move in a new direction.

Variation 6 (Figures 3.21–3.23)

Variation 6 does, in fact, turn away from the theme. Like variation 1, the high points in the non-chord tone contour occur near the beginning of each phrase, with a descending contour throughout section A. This feature is clearly visible in figure 3.22, which shows that high points of the neighbor tone contour occur around measures 1, 9, and 17. Although many of the same intervals recur in both the A and B sections, this fact is not particularly remarkable because the thematic material of the counter-melody is fairly consistent in both sections.

Perhaps the most striking feature is the fact that <0, 0, 0> appears only in the middle of each phrase and between the B section and the return of the A section in measure 17, thus linking these sections together, even though the motives employed are different. If we refer to the table in figure 3.23, we find the intervals <−1, 0, 0>, <0, 0, 0>, and <1, 0, 0> between successive measures in measures 3–6. The occurrence of the interval <0, 0, 0> between measures 4 and 5 in the middle of the A section links these two phrases (measures 1–4 and 5–8) even more strongly, for the preceding interval, <−1, 0, 0>, in measures 3–4 is the inversion of the following interval, <1, 0, 0>, in measures 5–6. This series of intervals, <−1, 0, 0>, <0, 0, 0>, and <1, 0, 0>, creates a descending/ascending contour of the non-chord tones that is the inverse of the melody's pitch contour at that point.

Variation 7 (Figures 3.24–3.26)

The contour of variation 7 continues to avoid the contour of the theme. In the two A sections, the relatively smooth scalar melody begins with a high number of passing tones—(4, 0, 0)—gradually fades the passing tones out, and introduces a single suspension that is sustained until the cadence, with a brief excursion in the neighbor tone direction smoothing out the transition from passing tones to suspensions. The new melody in section B is even more interesting, for it has a very clear descending passing tone contour.

We can also discover a retrograde inversion chain beneath the surface of this non-chord tone contour. Referring to the table in figure 3.26, we find that, if we ignore the instances of <0, 0, 0> from measures 9–10 and 13–14, considering them as sustaining the configuration (3, 0, 0), that we have the following two sequences of intervals: <3, 0, 0>, <−1, 0, 0>, <−1, 0, 0> and <2, 0, 0> in measures 8–13 and <2, 0, 0>, <−1, 0, 0>, <−1, 0, 0> and <3, 0, 0> in measures 12–17. Since the second sequence of intervals contains the same intervals in reverse order, the measures 12–17 are the retrograde inversion of 8–13, creating the beginning of a retrograde inversion chain. This retrograde inversion chain is more complex than those which we found in preceding variations. As in variation 2, this feature can also be heard as providing continuity with the return of the A section within this variation.

Variation 8 (Figure 3.27–3.29)

Variation 8 is characterized by an imitative texture that produces motivic repetition at the beginning of each line. In terms of the non-chord tone intervals, this results in the initiation of several retrograde inversion chains. As we see in Figure 3.29, measures 1–3 contain the series of configurations (2, 0, 0), (0, 0, 0), and (2, 0, 1), with intervals <−2, 0, 0> and <2, 0, 1>. The pattern overlaps with its own retrograde inversion in measures 2–4, where we find the configurations (0, 0, 0), (2, 0, 1), and (0, 0, 1) with the intervals <2, 0, 1> and <−2, 0, 0>. We can detect the retrograde inversion because the intervallic succession is palindromic. A similar process occurs in the altered repetition in measures 17–20 where <−1, 1, 0> and <1, −1, 1> (found between measures 17–19) overlap with <1, −1, 1> and <−1, 1, 0> (measures 18–20). These relationships could also be explained by modeling the opening motive in measures 1–2 with the pair (2, 0, 0), (0, 0, 0) and considering measures 3–4, containing the pair (2, 0, 1), (0, 0, 1), as a repetition "transposed by one suspension." Using Lewin's designation, the relevant transposition operation is $T_{<0, 0, 1>}$. The same relationship also applies in measures 17–20.

If we view a configuration of two passing tones, (2, 0, 0), as thematic, then the B section of the variation becomes very interesting. In measures 9–11, we have the configurations (0, 1, 0), (2, 0, 0) and (0, 1, 0), with intervals of <2, −1, 0> and <−2, 1, 0>. This overlaps with measures 10–12, which contain the configurations (2, 0, 0), (0, 1, 0) and (2, 0, 0) and intervals <−2, 1, 0>, <2, −1, 0>. Again, the reversal in the order of the intervals indicates the presence of a retrograde inversion chain that oscillates between (0, 1, 0) and the more thematic (2, 0, 0). If it were not for the suspension added to the passing tone in measure 13, creating the configuration (0, 1, 1) rather than (0, 1, 0), the chain would be continued through a third application of the RICH transformation. This final configuration can be profitably be viewed as an extension of the retrograde inversion chain in which the final configuration has been transposed by $T_{<0, 0, 1>}$, which is a thematic transformation drawn from the A section. The B section ends with a climax in which Mozart begins at (0, 0, 0) in measure 14 and applies $T_{<1, 0, 0>}$ twice in order to arrive at (2, 0 0), the central thematic configuration of this variation.

Variation 9 (Figures 3.30–3.32)

The imitative texture of variation 9 associates it with variation 8, yet the non-chord tone profile is now quite reduced. The A sections each maintain a single configuration, (0, 0, 1), for several bars before closing with an emphatic, dynamically emphasized leap from (0, 1, 1) to (0, 0, 0) at the cadences. (See the listing for measures 7–8 and 23–24 in figure 3.32.) The B section provides variety by introducing different configurations and intervals with almost every measure, before closing with a progression to the configuration (2, 0, 0) that reflects a similar progression in Variation 8. Interestingly, the modified return of the A section focuses very heavily on the configuration (0, 0, 0) rather than (0, 0, 1), preparing the way for variation 10. In addition, the neighbor tone contour begins to reflect the theme again, for the largest non-chord tone configurations (0, 1, 1) and (2, 0, 0) occur in measures 7, 14, 16, and 23, just before the cadences of the A sections in measures 8 and 24 and the return of the A section in measure 17.

Variation 10 (Figure 3.33–3.35)

In terms of non-chord tones, Variation 10 is closer to the original theme than any other variation. Like the original theme, it is almost completely lacking in non-chord tones. They are only incorporated at the cadences. As the table in figure 3.35 demonstrates, the only configurations other than (0, 0, 0) are (0, 0, 1) in measures 7 and 23 and (4, 0, 0) in measure 16. There are, of course, some significant differences. In the theme the non-chord tones were all neighbor tones, whereas variation 10 now employs suspensions in the A sections and passing tones in the B section. These dissonances, particularly the passing tones between section B and A, are more pronounced than the neighbor tones in the theme, making these non-chord tone differences quite distinct.

It should be noted that this variation does have a more complex harmonic progression than the theme and that some of these harmonies, such as the secondary dominant in measure 10, could be interpreted in terms of non-chord tones. However, interpreting them as chords in this variation makes more sense. While some seventh-chords such as those that have often occurred on the strong beats of measures 5–7 and measures 21–23 are consistently motivated by a clear chain of suspensions in many variations, the seventh-chords in this variation have clear harmonic explanations. Moreover, these chords produce chromatic alterations of the same scale-degree (for example, F♯ instead of F) rather than different scale-degrees.

Variation 11 (Figure 3.36–3.38)

The non-chord tone profile of Variation 11 is highly varied. Very few patterns or intervals repeat. However, if we consider only one non-chord tone dimension at a time, there are some interesting details in the contour. Examining measures 4–8 and measures 20–24 in figure 3.37, we see that both of the A sections present a clear arch-like contour in the passing tone dimension. The B section opens with a sudden leap to (4, 0, 0), which is paired with a steady, yet rapid descent through repeated reduction by two passing tones, the first member of the ordered triples <−2, 0, 0> and <−2, 4, 0>. This is immediately countered a repeated ascent of 3 passing tones to attain a height of 6 passing tones, which is in turn followed by another even more extended descent that reaches all the way to the return of section A. Since some of the individual motives involved are quite different, this result is not immediately obvious. Since passing tones are very active non-chord tones, and since this is the most extended use of any NCT so far, this variation is quite easily heard as being different than anything that has come before.

Variation 12 (3.39–3.41)

The non-chord tone analysis of Variation 12 requires a few comments. In the table in figure 3.41, there are two intervals in the right-hand column that both extend from measure 23, <4, −5, 0>, which connects measures 23 and 24, and <0, 1, 0>, which connects measures 23 and 24b at the second ending. A few analytical choices also require explanation. Although G on beat 1 of measure 3 could be labeled as a suspension prepared by an implied voice, it can also be heard as an incomplete neighbor tone, a choice which produces more interesting results. As in variations 1 and 3, the initial Gs in measures 10 and 14 are labeled as a suspension for thematic reasons.

In terms of non-chord tones, this variation is saturated with an unusually large number of neighbor notes. The new 3/4 meter provides extra space, which Mozart saturates with neighbor tones. Although most of these are in the accompaniment, Mozart's choice of melodic neighbor tones creates some interesting transpositional affects. For example, the series of configurations in measures 1–3, (0, 7, 0), (0, 7, 0), and (0, 6, 0) which outline the intervals <0, 0, 0> and <0, −1, 0>, is transposed down by one neighbor tone ($T_{<0,\,-1,\,0>}$) and repeated in measures 5–7, creating a slow smooth descent from (0, 7, 0) to (0, 5, 0) across measures 1–7. The relaunch of neighbor tones in section B results in the unusual leap of <−4, 11, 0>, which occurs between measures 8 and 9. Within section B, Mozart also transposes the entire sequence of non-chord tone configurations in measures 9–12, (0, 11, 0), (0, 10, 1), (0, 10, 1), and (0, 10, 1), down by three neighbor tones in measures 13–16. It should be noted that this descending contour bears a greater resemblance to variation 1, 6, or 7 rather than to the theme or variations 5 or 10, which have non-chord tone near the end.

The brief coda begins by suggesting a continuation of the same descending contour. Mozart sustains the energy of this section with beginnings of a repeating retrograde inversion chain in measures 28–31 by alternating the configurations (6, 0, 0) and (0, 5, 0) in a manner similar to several examples which we have already examined. Finally, he gradually dissipates the textural energy of the non-chord tones by bringing the number of neighbor tones down to zero through two descending leaps of <0, −3, 0>.

Interval Vectors and Overall Form

In the preceding discussion, we examined non-chord tone relationships within each variation, demonstrating the ways in which non-chord tones can function on a very local level. In addition, the relationship of each variation relative to the theme was briefly considered in order to chart an overall formal path in which the contours and features of each variation suggest that the theme, variation 5, and variation 10 are all closely tied together by virtue of their non-chord tone profiles. The theme and variation 5 are each followed by a radical departure from that shared contour that is sustained for two variations then gradually returns toward the same contour in variations 5 and 10. Some of these relationships can be examined statistically using the concept of interval vectors.

We have already considered transposition and inversion operations on NCT intervals and have found that they can formally be described in terms of conventional pitch transposition and inversion. These similarities could be used to create a "set theory" of non-chord tones. Lewin employs two functions, CANON and EMB in generalizing musical set theory.[43] The CANON function represents "canonical equivalence" Lewin's term for the various possible rules that might be defined to determine whether or not two sets are considered "equivalent" or not. The EMB function is a generalized equivalent of an interval vector; it counts the

43. Ibid., 106.

number of times different forms of the set X occurs within the set Y. When the set X is a dyad, a set of two elements, it naturally outlines one interval. Thus counting dyads becomes equivalent to an interval vector that counts the number of times an interval occurs within a set. If we consider each variation to be a set, we can use this method to calculate an interval vector for each variation, listing the number of times each interval appears.

A table with interval vectors for each variation in the entire piece is presented in figure 3.42. Note, however, that these are not quite equivalent to interval vectors in the sense that Lewin and Forte's set theory employ them, since I have listed only intervals between consecutive measures, while a traditional interval vector (or Lewin's EMB function) would count the intervals by examining every possible unique pair of elements within a collection. Thus each variation has 23 intervals rather than 276 intervals. My “interval vectors” are similar in that they count the number of times certain intervals occur; however, they are not exactly the same.

I have, however, employed the concept of canonical equivalence. In pitch-class set theory, transposed and inverted forms are considered equivalent. Since we have found non-chord tone features analogous to transposition and inversion the same distinction can be employed. Thus the collection consisting of the configurations (0, 0, 2) and (0, 1, 0) with the interval <0, 1, −2> would be considered equivalent to a collection consisting of configurations (1, 1, 0) and (1, 0, 2), with the interval <0, −1, 2>. The latter collection is the inversion of the former and has been transposed by one passing tone; however, the underlying change within each collection is the interval <0, −1, 2>. All intervals have been converted to a set-class that represents a positive total change. Thus we would use <0, −1, 2> rather than <0, 1, −2>, because the total sum all components of the former interval is 1, while the latter is −1. We could not use <0, 1, 2>, because it is inherent in this direct product interval that neighbor tones and suspensions move in opposite directions. Intervals are sorted from smallest to largest, calculating first the sum total of all changes, then the total number of changes both positive and negative, and finally by considering a change in neighbor tones to be less significant than a change in either suspensions or passing tones. This method is intuitive, for the largest intervals are definitely the most audible or salient.

The modified interval vectors for Mozart's variations are presented as a table in figure 3.42. Each column represents a variation, while the left most column lists all of the different non-chord tone intervals that occur throughout the piece. The numbers indicate the number of times the interval on the left occurs within the variation listed at the top. Thus we can see that the interval <1, 0, 0> occurred 6 times in variation 6. Likewise, it is plain that the intervals <0, 0, 0>, <1, 0, 0>, <0, 1, 0>, and <0, 0, 1> are the most common intervals. The horizontal lines in the table indicate a division between intervals with different cumulative changes. For example, the intervals for the first seven rows all have a total cumulative change of 0, while the next twelve intervals all have a cumulative change of 1. The largest cumulative change is found in a single interval from variation 12, <−4, 11, 0>, which has a cumulative change of 7.

We can use this information to make broad formal divisions. There are three distinct sections, beginning at the theme, at variation 5, and at variation 10 respectively. The progression from the theme to variation 5 is fairly clear. After the rather sudden diversification of intervals in variation 1, there is a gradual reduction in the variety of intervals and general size of the intervals employed. Variations 1 and 2 each employ 10 different intervals. Variation 3 then slightly reduces the number of different intervals to 9, and as noted earlier, the intervals from variation 2 to variation 3 are almost all negative, reflecting a progression back toward the theme. Variation 4 actually employs 10 different intervals, 1 more than variation 3, but 12 of its intervals sum to a total change of 0, while variations 2 and 3 tend to center on intervals that sum to 1. The progression from variation 5 to 10 is similar, but somewhat staggered, since the interval vector of variation 8 is clearly more diverse and focused on larger intervals than variation 7. Finally, variation 9 represents a key step in the process of moving back toward the theme, since, like variation 4, it has 12 intervals that sum to 0. These processes generally reflect a “progressive unification,” which is the inverse of the process of “progressive diversification” that Lewin found in his application of interval vectors to rhythmic motives in Chopin's Sonata in B♭ Minor.[44]

It is important to note that the interval vectors here support a relatively intuitive division of Mozart's set of variations that is supported by other factors. Variations 1–4 are grouped in alternating pairs in which first the melody and then the accompaniment is embellished, and there is a rhythmic slowing down in the use of sixteenth notes in variations 1 and 2 and triplets in variations 3 and 4. This process is broken at variation 5. Variations 6 and 7 can be grouped into another pair in which first the accompaniment and then the melody is embellished by sixteenth note figuration. Variations 8 and 9 can be connected on the basis of their imitative texture, while variation 10 parallels the theme again. Variation 11 then stands as a sort of trio section, inserted as if it were hardly a variation at all, while variation 12 resuscitates the theme in a jubilant coda. This formal division is further supported by shift from a relatively high number of non-chord tones in variations 1 and 6 to a relatively low number of non-chord tones in variations 3 and 4 and 8 and 9. This pattern suggests a sort of large-scale cadential progression of non-chord tones that is repeated across two cycles of five variations each. This is particularly interesting given the fact that the non-chord tones use or avoidance of non-chord tones at the cadences is a key factor in who a particular non-chord tone profile relates to the theme.

The character of the last two variations is also reflected in the interval vectors. Variation 11 has the most diverse interval vector of any variation: 14 intervals with no more than 4 of any one interval. Variation 11 serves to break the patterns set up by the earlier variations, providing a formal contrast to the preceding variations, somewhat in the manner of a trio section in minuet and trio form. The character of its non-chord tone profile gives strong support to an analytical judgement that might otherwise be based only on the *Adagio* tempo marking.

The interval vector of variation 12 is also rather striking. Although it has a much larger interval vector, 32 rather than 23 intervals in all, it is not very diverse.

44. Ibid., 115–116.

In spite of one very large interval, most of its intervals are relatively small, reflecting the strong connection with the original theme. What sets it apart from variations 5 and 10 are its very large configurations of neighbor tones, which create a long descending contour in which the tension of the neighbor tones and non-chord tones in general is brought to a climax and gradually eliminated.

Some Issues and Alternative Methods

One problem with this approach of analyzing intervals between non-chord tones arises in the very fact of labeling all of the non-chord tones in a piece. There are many different systems of labeling non-chord tones, and even within the most rigid frameworks laid out by undergraduate theory textbooks there is usually some flexibility of interpretation. In the preceding analysis I have tried to be relatively consistent, yet even within the conditions I have employed there is room for multiple interpretations. We have already considered this issue in some detail in chapter 1. However, a brief reconsideration of the problem in relation to this piece may serve to shed further light on the issue. Moreover, since some of these possible variant interpretations may significantly alter the data under consideration it would be appropriate to reexamine a few passages from different points of view.

One uncertain passage can be found in variation 1, measure 4 (see figure 3.4). Originally labeled (4, 0, 1) in figure 3.6, this measure could also be labeled as having only two passing tones, (2, 0, 0), by considering the two As as chord tones, as part of a larger seventh-chord. In this case, the C♯ in the left hand would also be considered a chord tone, making a brief V6/5 of ii on the last eighth note of the measure, while only the right-hand D and B would be considered passing tones. This alters the intervals preceding and following this measure from <2, −2, 1> and <−1, 0, 0> to <0, −2, 0> and <1, 0, 0>. While <1, 0, 0> is canonically equivalent to <−1, 0, 0> and changes nothing as far as our interval vector in figure 3.42 is concerned, the interval <0, −2, 0> is already present elsewhere in the variation, thus reducing the diversification of the interval vector. This change would not, however, alter the general point, nor would the changes to contour ultimately be very significant.

On the other hand, some changes could alter interesting features. For instance, suppose we were to consider the E in measure 16 of variation 2 (given in figure 3.7) as a member of an E minor chord in first inversion rather than a suspension against the overall G major triad. In that case, the beginning of a retrograde inversion chain spanning measures 14–17, which we have already discussed, would be altered and shifted to measures 15–18. The relevant intervals from measures 14–18 would be <0, 2, −1>, <0, −2, 0>, <0, 2, 0>, and <0, −2, 0> instead of <0, 2, −1>, <0, −2, 1>, <0, 2, −1>, and <0, −2, 0> as originally listed in figure 3.9.

An alternative approach might involve considering each melody by itself. Although I have avoided this as being outside the central scope of this chapter, it could yield many additional interesting results. Let us consider only the right-hand in measures 1–8 of variation 12 (see figure 3.39). The new configurations are as follows: (0, 2, 0), (0, 2, 0), (0, 1, 0), (0, 2, 0), (0, 2, 0), (0, 2, 0), (0, 0, 0), (0, 0, 0); these yield the following intervals: <0, 0, 0>, <0, −1, 0>, <0, 1, 0>, <0, 0, 0>, <0, 0, 0>, <0, −2, 0>, <0, 0, 0>. The new approach highlights the motivic nature of the trills, setting measure 3, with a configuration of (0, 1, 0), apart as a "neighboring configuration," while the general contour focuses on pairs of configurations outlining <0, 0, 0> that are sustained and then transposed down by two neighbor tones after measure 6. In this case, whether we label the G in measure 3 as a suspension or neighbor hardly affects the results at all. This alternative pattern can be heard as complementing the more complex relationships produced when all non-chord tones are considered.

Conclusion

In this chapter, I have applied the concept of a non-chord tone generalized interval system in order to analyze some interesting features that can be found in Mozart's variations on "Ah! Vous dirai-je, Maman." K. 265. Non-chord tones serve many different musical purposes. In some cases, specific patterns of non-chord tones or changes between measures create continuity by repetition, transposition, and inversion of patterns. Specific configurations may even take on a thematic role. In addition, the general textural contours created by each type of non-chord tone certainly serve a purpose in shaping each variation relative to the theme and contribute to larger formal processes.

In analyzing non-chord tones in this way, I also hope to have demonstrated that the textural features created by non-chord tones can play a key structural role. In a piece such as Mozart's variations on “Ah! Vous dirai-je, Maman,” where the harmony and melody are extremely simple and repeated verbatim several times, where timbre is unified throughout the piece, and even rhythmic variety is relatively low, finding variety can be difficult. Much of that variety can be found in the usage that non-chord tones receive.

By analyzing this variety with a formal model based on GIS structure, I have also demonstrated the ability of the model to explain and highlight many of these features. Furthermore, this analysis has only touched the surface of what the model can show. Because of the flexibility of Lewin's formalism, the model is highly adaptable. As a matter of fact, this analysis has only scratched the surface of the potential results that could be realized in this piece alone by applying other tools that Lewin has developed. Finally, this model has the potential to be combined with other systems in direct product GISes to generate other formal analytical tools that can be used in the broader context network analysis, a topic which will be explored in the following chapters.

Chapter 4: An Alternative Non-Chord Tone GIS

In the preceding chapter, we found that some very intriguing structural properties of non-chord tones in Mozart's variations on “Ah! Vous dirai-je, Maman” could be uncovered simply by counting the non-chord tones in each measure. While a measure by measure approach may often be applicable to variation form and was particularly apt for the example by Mozart, there are many instances in which such an approach might not be as useful. Single chords, notes, or beats are, in many cases, more relevant in the analysis of non-chord tones than large expanses such as measures or phrases. This situation highlights an inadequacy of simply counting non-chord tones, as a basic NCT-GIS does. When considering large configurations such as a whole measures or phrases, the analysis can do little more than represent a statistical approach. On the other hand, if the configurations were to consist of individual chords or notes, with an analytical focus on a single melodic voice, the configurations would so frequently consist of such small numbers of non-chord tones that counting them might almost seem irrelevant.

It is this particular problem of analyzing non-chord tones as local phenomenon that I will address in this chapter and the following chapter. The central focus of this chapter is the development of a new non-chord tone GIS, applicable for motivic analysis. In addition, I will also develop a number of concepts which are not directly concerned with non-chord tones, but which can readily be combined with non-chord tone intervals in analysis. As in chapters 2 and 3, the material in this chapter is intended as formal theoretical groundwork, preparing the way for a detailed network analysis of themes and motives in Brahms's String Quartet No. 2 in A Minor in chapter 5.

Formal Considerations: Non-Chord Tone GIS Structures

In order to clarify the musical situations that our new NCT-GIS models, lets us consider another musical example. Figure 4.1 illustrates some of the most common configurations of non-chord tones that might be attached to a single note. In each case, the note G is followed by some number of passing tones or neighbor tones. These can be considered as “attached” to the G because they occur in the same beat. Accented passing or neighbor tones would be associated with the pitch that follows them. In such cases, it is unlikely that any one chord tone will have many non-chord tones of the same type. Therefore, it might be more relevant to consider whether a specific note does or does not have non-chord tones attached to it. From this point of view, figure 4.1b and figure 4.1c are similar: both involve passing tones only.

Before addressing formal considerations, let us consider exactly which intuitions this GIS will model. The elements of S provide an answer to the question "do I hear non-chord tones associated with this note or chord?" Alternatively, they might model a compositional decision: "what happens when I add non-chord tones to this note or not?" The int(s, t) function then asks whether there is a change from one configuration to the next. It models the compositional decision to continue to repeat the same pattern (i.e. continue to leave notes unadorned or continue to embellish each note) or add variety by doing something different. In other words, it will be a simple yes/no or on/off distinction. Because it models this type of intuition, it will be helpful to notate the intervals in IVLS as <N>, meaning "no change," and <Y>, meaning "yes, there is a change," rather than using 0 and 1, the elements of $\mathbb{Z}_2$, the group which corresponds to this type of distinction.

Let us formalize this approach as a GIS along the lines of NCT-GIS(NCT) from Chapter 2. Our space will consist of configurations of non-chord tones associated with some note, phrase, or other musical unit, and will contain just two elements: 0 and NCT. 0 represents a configuration with no non-chord tones; NCT, represents the presence of at least 1 non-chord tone. IVLS is the group of integers under addition mod 2. The function int(s, t) maps two members s and t of S into a member i of IVLS as follows: if $s = t$ then $i = \langle N \rangle$ and if $s \neq t$ then $i = \langle Y \rangle$. Since S and IVLS both consist of only two elements, it is fairly obvious that Lewin's conditions A and B are satisfied. In addition, the GIS is commutative.

Since this approach to describing non-chord tones may not be immediately intuitive, let us examine some of the intervals in figure 4.1, considering each configuration to be attached to the chord that spans the measure. In the example, the configuration of non-chord tones for the chord in 4.1a is 0, while for 4.1b through 4.1f the configuration is simply NCT. Remember that we are still only considering non-chord tones in general, without reference to any particular type. This distinction is the most basic division that can be made; a particular chord either does or does not have non-chord tones that occur while it is sounding. From this perspective, the interval from the chord in 4.1a to any of the other chords would be <Y>. On the other hand, the interval between any two chords other than 4.1a is always <N>, since no changes take place in the state of the non-chord tones.

We can also consider configurations of non-chord tones attached to a single note instead, a distinction which is perhaps slightly more useful. In figure 4.1b, the G has a passing tone attached to it, while the E has no non-chord tones. We consider the F as attached to G because it occurs in the same beat. Consequently, the configuration of non-chord tones for G is NCT, while it is 0 for the E. Thus, the interval from G to E is <Y>. Since we are still ignoring the type and number of non-chord tones involved, as well as the conventional pitch intervals, in each example from figure 4.1b to 4.1f the configuration of the note on the first beat is always NCT, the configuration for the note on the second beat it is always 0, and the interval from the note on the first beat to the note on the second beat is always <Y>.

In chapter 2, we introduced a general notation for a direct product non-chord tone GIS, NCT-GIS(NCT1, NCT2, … NCTn) where NCT1, NCT2, etc. were different types of non-chord tones. We will now modify this labeling scheme to include the type of group involved, so that we can include our new method of calculating non-chord tone intervals. Each GIS will be labeled NCT-GIS(NCT1, NCT2, … NCTn; G), where NCT specifies a type non-chord tone and G specifies the group that will be used. Thus the GIS employed throughout chapter 3 is NCT-GIS(P, N, S; $\mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$), since we considered passing tones, neighbor tones, and suspensions. The added notation ($\mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$) simply indicates that we were counting the number of each type of non-chord tone. The GIS which we used in this chapter to examine figure 4.1 is NCT-GIS(NCT; $\mathbb{Z}_2$). The notation first indicates that we analyzed non-chord tones in general, and hence we use NCT rather than P, N, or some other abbreviation for a specific type of non-chord tone. Secondly, it indicates that our group of intervals is isomorphic to the integers under addition mod 2 (that is, it has only the two elements <N> and <Y>).

This notation also allows for some more unusual GISes such as NCT-GIS(P, N, S; $\mathbb{Z} \times \mathbb{Z}_2 \times \mathbb{Z}_2$). This would be a direct product GIS that counts the number of passing tones, but not neighbor tones and suspensions. Instead we would simply consider whether neighbor tones and suspensions are present or not. An example of an interval from such a GIS would be <3, Y, N>, which would indicate that the second configuration has three more passing tones than the first configuration, that the configurations of neighbor tones for the two configurations are different (one has them the other does not), and that both configurations either have or do not have suspensions. Although there may be little practical application for some of these GISes, and although we will not be using this GIS in this analysis, it is useful to have a notation which allows us to generate a new GIS which is specially tailored to suit any musical situation we might wish to consider. Bearing this in mind, we will now introduce the NCT-GIS that will be employed in our analysis of Brahms's String Quartet No. 2, and consider some simple musical situations in which it might be useful as an analytical tool by itself.

NCT-GIS(P, N; $\mathbb{Z}_2 \times \mathbb{Z}_2$): This GIS is a direct product formed from NCT-GIS(P; $\mathbb{Z}_2$) and NCT-GIS(N; $\mathbb{Z}_2$) . The space S consists of configurations of non-chord tones modeled by the ordered pairs (0, 0), (P, 0), (0, N), and (P, N), while IVLS is isomorphic to the direct product of two instances of $\mathbb{Z}_2$. The int(s, t) function asks whether there is a change from one configuration to the next. As before, IVLS will employ the Y/N notation.

Before proceeding further let us use this GIS to examine the intervals in figure 4.1. Our direct product GIS renders the musical situations as follows: figure 4.1a = (0, 0); figures 4.1b and 4.1c = (P, 0); figures 4.1d and 4.1e = (0, N); and figure 4.1f = (P, N). Because they both contain passing tones but no neighbor tones, the interval from figure 4.1b to figure 4.1c is <N, N>, the identity element of our IVLS. So is the interval from figure 4.1d to figure 4.1e. The interval from figure 4.1a to figure 4.1b is <Y, N>, because the only change is from no passing tones in figure 4.1a to a configuration with passing tones in figure 4.1b; however, the neighbor tone portion of the interval still indicates no change. The interval from figure 4.1a to figure 4.1d is <N, Y>, indicating the opposite situation, a change in neighbor tones only. Interestingly, the interval from figure 4.1b, to figure 4.1f is <N, Y> as well. This implies the possibility of a progression from figure 4.1a through figure 4.1b to figure 4.1f, first making changes to the passing tones and then to the neighbor tones. The cumulative change from figure 4.1a to figure 4.1f involves both passing tones and neighbor tones, since the interval from figure 4.1a to figure 4.1f is <Y, Y>. At the same time, this system produces some surprising results, since the interval from figure 4.1b to figure 4.1d is <Y, Y> as well. Since both figure 4.1b and figure 4.1d involve non-chord tones, we might not expect that the interval would be the same as that involved in the change from figure 4.1a to figure 4.1f. At the same time this is not as odd as it might appear, since there is a substantial alteration of the configuration of non-chord tones in both cases. The key is that there is a change in both passing tones and neighbor tones in both cases.

Since we will be using this approach extensively, it is worth considering a more complex musical example. However, in this case, rather than considering isolated measures, we look at individual notes and the non-chord tones that may be attached to them. Figure 4.2 shows the first four measures of Bach's Two-Part Invention No. 9 in F Minor with passing tones and neighbor tones labeled in the upper voice The analysis draws on both figures 1.2b and 1.2c from chapter 1. Figure 4.3a presents the upper line as a network of non-chord tone transpositions.

We have not yet discussed the formal properties of networks or graphs, although we will do so shortly; however, even without detailing the formal criteria involved in networks it is plain that figure 4.3a presents a visual representation of the music. Configurations of non-chord tones still appear in parentheses and form the nodes or vertices of a graph. The arrows represent directed relationships, motion from one node to another. The arrows are labeled with intervals designating the relationships that are depicted. In this case, the graphs are depicted in temporal order, with all of the arrows pointing forward in time. Thus, networks are a way of isolating certain elements of the music and highlighting certain relationships between those elements in a visual format.

This example simultaneously demonstrates some of the analytical possibilities of this non-chord tone GIS. The network reveals an interesting property of the first measure: it uses all four possible non-chord tone intervals exactly once each. Moreover, the same feature occurs from C on the last beat of measure 3, which is preceded by an accented passing tone, to A♭ on the last beat of measure 4. This exhaustion of the non-chord tone intervals in such a short span points to the melodic variety and fluency of this passage.

This type of analysis also affects the way in which non-chord tones are analyzed. Notice first of all that the final configuration the network in figure 4.3a is (A♭, 0, 0). We have excluded any non-chord tones that might come after it from the network, because the network presents a closed musical thought, and the neighbor tone is part of the continuation to the next measure. In general terms, this means that when we make networks of non-chord tones, the non-chord tones that occur outside the bounds delimited by the first and last chord tones of the network should be excluded. However, exceptions to this rule might occur in cases where a passage clearly begins with an accented non-chord tone or is clearly bounded by rests. In addition, we must consider whether in the case of complete neighbor tones the return to the original tone after the neighbor tone ought to be considered as a separate tone. In this example we have not done so; however there may be cases in which this would be useful.

Figure 4.3b displays the consequences of analyzing non-chord tones in this way by showing what the melodic line would look like if it were broken into four separate networks, one for each measure. In addition, the resolutions of neighbor tones have been excluded as being part of the same configuration. For example, we count A♭ on the last eighth note of beat one and A♭ on the first eighth note of beat 2 as one note with a neighbor tone attached, while figure 4.3a examines them separately. In the case of the tied notes that span measures 1–2 and 2–3 the networks present them as having no non-chord tones in relation to the preceding measure and as having a single passing tone in relation to the following measure. Although the results in this case are less interesting than the larger network that spans the phrase, this method is more useful for motivic analysis.

Some general methodological points regarding NCT-GIS(P, N; $\mathbb{Z}_2 \times \mathbb{Z}_2$) require comment. The GIS considers only passing tones and neighbor tones. Suspensions and anticipations are excluded. In terms of a general non-chord tone analysis, we may not wish to exclude these classes of non-chord tones. However, in our analysis of Brahms's string quartet it will be more convenient to limit the discussion to passing and neighbor tones. Since the focus of this analysis is to be motivic development, and none of the primary motives under consideration involve prominent suspensions or anticipations, it seems more efficient to exclude them.

In the analysis of the Brahms string quartet, NCT-GIS(P, N; $\mathbb{Z}_2 \times \mathbb{Z}_2$) will not be used by itself. Instead, some of the motives will be modeled using a direct product GIS in which the GIS of diatonic pitch intervals is combined with NCT-GIS(P, N; $\mathbb{Z}_2 \times \mathbb{Z}_2$).[45] Thus, the elements of S will consist of pitches plus any non-chord tones that may be attached to them and will be expressed as ordered triples (I, J, K), where I is a pitch name, J indicates the presence or absence of passing tones, and K indicates the presence or absence of neighbor tones. We have already used this notation for the sake of clarity in figure 4.3. Now our intervals will also be denoted by an ordered triple <X, Y, Z>, where X is the number of scale steps from the first pitch to the second and Y and Z indicate whether or not there is a change in the state of the non-chord tones. As before, the pointed brackets indicate intervals, while parentheses indicate elements.

Since each combined interval outlines a motive in this analysis, the criteria for analyzing non-chord tones are based on melodic shape and motivic association. In general this means that a non-chord tone must be within the bounds of an interval in order to be analyzed as such, as was the case in figure 4.3. However, incomplete neighbors may sometimes occur outside the bounds of the diatonic interval if there is a rest or other feature that clearly associates it with

45. The GIS of diatonic pitch intervals in defined in section 2.1.1, *Generalized Musical Intervals and Transformations*, 16.

an interval. In addition, when there is a complete neighbor tone, the repetition of the chord tone after the neighbor will generally not be considered as separate tone; the entire event is one note plus a neighbor tone. This later consideration should be somewhat flexible. In determining whether the resolution of a neighbor tone is considered independently, rhythmic placement and contextual factors must ultimately decide the case.

Before moving on to the other formal constructs that will be employed in this analysis, let us first examine how this might be applied to the example by Bach and then consider the way it will be applied in Brahms's String Quartet No. 2. Figure 4.4 presents another network representing the first four measures of Bach's Invention No. 9, which demonstrates these new direct product intervals. The intervals are essentially unchanged; only one additional number has been added representing the diatonic pitch interval. In the case of the invention by Bach, it is interesting to note that every single interval is either a third (2 or −2) or a sixth (5 or −5); since we are considering the resolution of a neighbor tone to be a continuation of the same note in this case, even unisons are excluded. On top of this uniform framework, the relatively rich variety is achieved through the non-chord tone components of the intervals which are constantly changing.

Like Bach, Brahms also makes frequent use of melodic thirds in combination with various non-chord tone configurations. In the context of the String Quartet in A minor, this single interval is treated as a motive, as are the second and unison. In our analysis, we will be labeling these motives with letters (motives A, B, and C) in order to distinguish them from some more complex motives which will receive more definitive names. Figure 4.5 gives us a preview of these motives, presenting short musical examples of each motive with a corresponding network. The motives are presented in order of appearance and importance, although the particular examples given are not necessarily the first or most frequently found form of each motive.

Although all of these intervals are common enough, and in the case of motive B almost trivial, this method of presentation actually adds a degree of resistance. For example, one might consider motives A and C to be similar based on a stepwise ascent through a third; however, if we consider whether or not notes have harmonic support, we find that they are actually quite different. Another advantage to this method is that we can consider the underlying pitch interval independently of the non-chord tones with a strong harmonic basis for our choices of intervals. However, in order to take advantage of the opportunities afforded by the potential for considering the boundary interval independently, we must leave the world of non-chord tone GISes and examine several another type of GIS and several non-intervalic transformations which profoundly influence the thematic structure of Brahms's String Quartet.

Formal Considerations: Other GIS Structures

In addition to our direct product GIS, two more GISes will be used in this analysis. One of these is the GIS of diatonic intervals mod 7, which will occasionally be used in standard networks independent of non-chord tones. This GIS is fairly straightforward and has been used by Lewin and others in a variety of contexts. The second GIS is rather less common, though not without precedent. It employs permutations to compare triadic figures based on the temporal order in which the root, third, and fifth appear.

Permutational GIS: The space, S, of our GIS will consist of all possible permutations of the three notes of a triad. The members of S will be represented by ordered triples, with root, third, and fifth (R, 3, 5) each appearing exactly once in some order.[46] IVLS will consist of S_3, the symmetric group on three elements. The function int(s, t) maps s and t into IVLS by finding the permutation that will map the root, third, and fifth of s onto the root, third, and fifth of t respectively. The GIS is not commutative.

This GIS is similar to the system proposed by Klumpenhouwer for studying the registral arrangement of voices, pitches in pitch-class sets, or fugue subjects.[47] The key difference is that this GIS is concerned with the temporal order in which root, third, and fifth appear, rather than registral order. In terms of notation, the

46. This notation bears some similarity to one employed by Rosemary Killam, "An Effective Computer-Assisted Learning Environment for Aural Skill Development," *Music Theory Spectrum* 6 (Spring 1984): 52–62, who uses the notation 1–3–5 to denote the order components of a triad are played in the context of ear training and difficulty of perception of various triadic inversions.

intervals of this GIS will employ both the rounded parentheses often used for permutations and the pointed brackets that indicate intervals in this study. For example, <(R35)> indicates that, in terms of temporal order, the root of the second triad takes the place that the third had occupied in the first triad, the third the place of the fifth, and the fifth the place of the root. That is, it would take the element (R, 3, 5) and turn it into (5, R, 3). Note that the use of a comma separated list does not indicate a direct product group in this context.

The combination of two intervals is also slightly unusual. Consider the combination of <(R35)> and <(R3)>. In combining them, we first map R to 3, according to the first permutation. We then map 3 back to R according to the second permutation. Since R maps to itself, it is excluded from the final permutation. We now map 3 to 5 according to the first permutation and 5 to 5 according to the second permutation (the absence of 5 indicates that 5 maps to itself). Finally, we map 5 to R in the first permutation and R to 3 in the second. We now have the permutation <(35)>, which indicates that R is mapped to R, 3 is mapped to 5 and 5 is mapped to 3.

Let us consider a few examples of how this GIS works. Figure 4.6 shows a transitional passage from the last movement of Beethoven's Piano Sonata Op. 14 No. 1. The figuration in the right hand consists entirely of triads. Even in cases where the full harmony is a seventh-chord the melodic figuration represents a diminished triad, which we will consider independently. Throughout this analysis, foreground seventh-chords which cannot be explained as melodic dissonances will be analyzed as the overlapping or combination of two triads. Although this approach is slightly unusual, it facilitates permutational analysis. Moreover, it is not without historical precedent, being rooted in nineteenth-century German Harmonic theory. Consequently, each figure can be represented as a triad according to the order in which root, third, and fifth enter. Figure 4.7 models this passage as two networks of triadic permutations.

From an analytical standpoint these networks demonstrate that Beethoven uses primarily 2-member permutational cycles such as <(R3)>, <(R5)>, and

47. Henry Klumpenhouwer, "A Generalized Model for Voice-leading for Atonal Music" (PhD diss., Harvard University, 1991), chap. 2:14, 4:12.

<(35)>. He employs the 3-member cycles <(R35)> and <(R53)> and the identity interval <()> primarily in preparation for the strong V–I progression at the end of each pair of bars.

This brings up some interesting properties of permutational groups in general and S_3, the group of intervals under consideration, in particular. In a symmetric group all permutations can be written as the product of one or more 2-members cycles, which mathematicians designate as transpositions. Because of this property, it is possible to make a distinction between permutations that are the product of an even number of transpositions (2-member cycles) and permutations that are the product of an odd number of transpositions.[48] Obviously <(R3)>, <(R5)>, and <(35)> can all be written as the product of an odd number of transpositions since each consists of exactly one 2-member cycle. On the other hand <()>, <(R35)>, and <(R53)> are all even permutations, since they can be written as the product of two transpositions (for example <(R3)><(35)> = <(R53)>). This is slightly counter-intuitive since in this case the odd permutations <(R3)>, <(R5)>, and <(35)> all have 2 numbers, while the even permutations <()>, <(R35)>, and <(R53)> have either 1 or 3 numbers. However, this property can easily be seen by examining the Cayley table for S_3 in figure 4.8. This table lists every element of the group along both the top and the side. We can find the combination of any two elements in the group by finding the first element in the list of rows along the left side of the table and then moving along that row until we arrive at the column labeled by the second element; the element that occurs at this intersection is the combination of these two elements.

As the Cayley table demonstrates, the even permutations <()>, <(R35)>, and <(R53)> form a subgroup within S_3. That is, if we take only these elements from the group and use the same protocol for combining them the result is also a group. This subgroup is the alternating group A_3. In fact it is a general property of a symmetric group S_n that all of the even permutations will form a subgroup, which is the alternating group A_n.[49] This way of partitioning S_3 is particularly

48. Fraleigh, *A First Course in Abstract Algebra*, 111–113.

49. Ibid., 114.

interesting, because it creates a factor group or quotient group isomorphic to $\mathbb{Z}_2$.[50] This factor group is highlighted by the shading of the table in figure 4.8. The elements in the area with the white background are the even permutations, which form the subgroup A_3. The table includes all possible combinations, demonstrating the group properties of associativity, the identity element, and inverses for each element. The elements in the shaded area are all odd permutations.

Practically, this means that combining any two even permutations produces an even permutation. On the other hand, combining an even permutation and an odd permutation will produce an odd permutation (i.e. either <(R3)>, <(R5)>, or <(35)>). This can be seen visually by examining the different combinations in the table in figure 4.8.

While this may seem rather abstract, it can be very concretely related to triadic arpeggiations and inversions. The alternating group A_3 which contains the even permutations <()>, <(R35)>, and <(R53)>, corresponds to the traditional model of triadic inversion. If each triadic figure appears as an ascending or descending arpeggiation, then as long as the permutations between successive triads belongs to A_3 the arpeggiations will all continue to ascend or descend. However, if the composer introduces one of the odd permutations from the coset (<(R3)>, <(R5)>, or <(35)>) then the direction of the arpeggiations will be reversed.

This is demonstrated in figures 4.6 and 4.7. Although Beethoven does not use strict ascending and descending arpeggiations, we can relate each triad to an ascending or descending arpeggiation by raising or lowering some notes by one octave. If we were to do so, we would find that he consistently alternates ascending and descending arpeggiations on each beat except at the V–I progressions, where he repeats descending arpeggiations. This hidden reversal of shape lends a sense of balance to each figure and intensifies each cadential

50. Fraleigh, *Abstract Algebra*, provides an interesting discussion of the relationship between subgroups and cosets with S_3 as an example (120–124), and the properties of factor groups (Lewin's quotient groups), particularly the factor group S_n/A_n (170–175, 181).

progression. It is this feature of the passage that the permutational networks in figure 4.7 highlight. As we shall see in our analysis, Brahms also exploits this permutational feature of triadic arpeggiation. However, he does so in a very different way, employing permutations that are members of A_3 to lend consistency to the shape of a triadic motive, and using odd (2-member) permutations for contrast and conflict.

Lewin's Transformation Graphs and Networks

In addition to the GISes used in this study there are two non-intervalic transformations that will be employed in our analysis of Brahms's String Quartet. Both transformations are functions that apply to the same type of operation graphs. Since they both involve the formal properties of networks and graphs as outlined by Lewin, let us briefly examine the formal properties of operation graphs.

Lewin distinguishes between three structures, which he designates node/arrows systems, graphs, and networks respectively. In doing so he adopts slightly different terminology than that normally employed in graph theory. In Lewin's approach the most basic level is a node/arrow system which he defines as follows:

> By a *node/arrow system* we shall mean an ordered pair (NODES, ARROW), where NODES is a family (i.e. set in the mathematical sense), and ARROW is a subfamily of NODES × NODES, i.e. a collection containing some ordered pairs (N_1, N_2) of NODES. We say that N_1 and N_2 are "in the arrow relation" if the pair (N_1, N_2) is a member of the collection ARROW. For present purposes, we shall stipulate that every node is in the arrow relation with itself. That is, we assume that (N, N) is a member of ARROW for every node N.[51]

This definition is essentially equivalent to what would be termed a directed graph or *digraph* in graph theory.[52] In graph theory, what Lewin calls nodes are called *vertices*, while arrows are called *arcs*. Note that a graph or

51. Lewin, *Generalized Musical Intervals and Transformations*, 193.

node/arrow system can actually be represented as two sets, without any visual representation. However, it is conventional to draw pictures in which the nodes or vertices are represented by dots and the arrows or arcs by arrows leading from one dot to another. Notice that the nodes or vertices are simply dots, they do not inherently have any meaning. Likewise, the arrows or arcs represent relationships, without specifying what those relationships are. Finally, note that Lewin's node/arrow system is directed. In graph theory, there are undirected graphs as well as directed graphs. In undirected graphs, vertices are connected by *edges*, which are *unordered pairs* (i.e. it does not matter in what direction you approach the line connecting two dots of the graph). Every directed graph can be converted into an undirected graph, called the *underlying graph*, by ignoring the direction in which the arrows point. This distinction is useful, because we may occasionally wish to consider the structure of the graph without reference to the direction of the arrows.

Lewin's second category is a transformation graph. He provides the following definition:

> A *transformation graph* is an ordered quadruple (NODES, ARROW, SGP, TRANSIT) satisfying criteria (A), (B), (C), and (D) below.
>
> (A): (NODES, ARROW) is a node/arrows system.
>
> (B): SGP is a semigroup.
>
> (C): TRANSIT is a function mapping ARROW into SGP.
>
> (D): Given nodes N and N', suppose that N_0, N_1,…, N_j is an arrow chain from N to N'. Suppose that M_0, M_1,..., M_k is also an arrow chain from N to N'. For each j between 1 and J inclusive, let x_1 = TRANSIT(N_{j-1}, N_j). For each k between 1 and K inclusive, let y_k = TRANSIT(M_{k-1}, M_k). Then the semigroup product $x_J \dots x_2x_1$ is equal to the semigroup product $y_K \dots y_2y_1$.[53]

52. W. D. Wallis, *A Beginners Guide to Graph Theory*, (Boston: Birkhäuser, 2000), 4–6.

53. Lewin, *Generalized Musical Intervals and Transformations*, 195.

This definition essentially adds one feature to a node/arrow system: it labels every arrow in the graph with an element of a semigroup. Lewin refers to it as a transformation graph because the elements of the semigroup will usually be transformations, functions that satisfy semigroup properties. As we recall from chapter 2, Lewin distinguishes between semigroups of transformations and groups of operations. Thus an *operation graph* is a transformation graph in which the semigroup of transformations is a group of operations. Thus, many of the graphs used by music theorists to describe intervallic relationships are actually operation graphs, although the term transformation graph is more frequently used.

In spite of the complex description, Lewin's condition D is actually very simple. It guarantees that if you take two nodes, N and N', and have two different paths, or *arrow chains* as Lewin calls them, leading from one to the other, then the transformations labeling the arrows will add up to the same thing. This prevents situations where one could, for instance, label arrows along one path with 3 and 5 and arrows along the other path with 2 and 4. Doing so would violate condition D because 3 + 5 does not equal 2 + 4. Rather, if the arrows on one path are labeled with 3 and 5, the arrows on the other path would have to be labeled by some combination that adds up to 8, such as 1 and 7 or 2 and 6. We will have occasion to deal with this issue shortly.

Lewin's final structure is a *transformation network* (and a corresponding *operation network*), which adds contents from a set to each node of the graph, along with appropriate conditions guaranteeing that the transformations labeling the arrows and objects in the nodes will interact properly.[54] Lewin also defines isomorphisms and homomorphisms on each of these types of graph. An isomorphism guarantees that the node/arrow system will have exactly the same structure via a function called NODEMAP, while an isomorphism of a graph adds additional conditions guaranteeing that there is also an isomorphism (SGMAP or GMAP) for the transformations or operations.[55] When it comes to networks,

54. Ibid., 196–197.

55. Ibid., 199–200.

Lewin uses the term *isographic* to refer to networks that have the same graph (i.e. the graphs of the two networks are isomorphic).

Lewin's node/arrow and graph homomorphisms guarantee that two node/arrow systems or graphs will have similar structures, rather than precisely the same structure. His key distinctions are between homomorphisms that are either *into*, *onto*, or *1-to-1*. To this end he gives the following definition:

> Given node/arrow systems (NODES, ARROW) and (NODES', ARROW'), a mapping NODEMAP of NODES into NODES' is a *homomorphism* of the first system *into* the second if (NODEMAP(N_1), NODEMAP(N_2)) is in the ARROW' relation whenever (N_1, N_2) is in the ARROW relation. NODEMAP is a *homomorphism onto* if it maps NODES onto NODES' in a special way: Whenever N'_1 and N'_2 are in the ARROW' relation, there exist N_1 and N_2 *in the ARROW relation* such that N'_1 = NODEMAP(N_1) and N'_2 = NODEMAP(N_2). A homomorphism NODEMAP is *1-to-1* as a homomorphism between systems if it is 1-to-1 as a map of NODES into NODES'.[56]

Although Lewin uses a significant amount of formalism to define these graphs, networks, and functions, the end result is usually relatively intuitive if we are considering actual graphs. The visual presentation of labeled arrows leading between named objects is something that we find in many domains of experience, such as maps, charts, and diagrams. The formalism serves to give the application of such graphs to music a solid basis for analysis, allowing definite statements regarding the properties of some musical structure.

Non-Intervalic Transformations

Having discussed the formal properties of graphs and networks as defined by Lewin, we can now proceed to discuss the non-intervallic transformations which we are developing with a view to the analysis of Brahms's string quartet. This discussion of the formal properties of graphs was necessary, since both of

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56. Ibid., 201.

these transformations act on operation graphs, altering them in various ways. Let us define and discuss each of them in turn.

Interval Alteration Function [ALT]: Suppose we have two operation graphs G and G' with the following properties:

(A): The group of operations consists of the diatonic transpositions mod 7.

(B): The node/arrow systems are connected and have exactly one path of forward or backward pointing arrows between any two nodes, provided each arrow from any node N to itself is ignored. In terms of graph theory, condition B states that the underlying graph—the graph considered without reference to the direction of the arrows—will be a tree.[57] A graph with tree structure is one in which, given any two vertices or nodes, there will be exactly one path that can be traced between those two nodes. When considering whether or not a graph conforms to tree structure, we ignore the direction of the arrows, since the directionality of the arrows has no effect on the function we are considering. We must also ignore the loops on each node of the graph, which are assumed to be present, since Lewin stipulates that every node is in the arrow relation with itself.[58] This feature of Lewin's networks is useful, but it means that no graph will have tree structure. Since these loops do not cause any problems in our model, we will simply ignore them when considering whether or not a graph has tree structure.

The interval alteration function ALT(G) = G' maps the operation graph G onto G' according to these two conditions:

(C): The mapping of nodes and arrows from G onto G' is identity;

(D): The mapping GMAP mapping family of the integers mod 7 into itself in the following manner: 0 maps to 0, 1 maps to 3, 2 maps to itself, 3 maps to 5, 4 map to 2, 5 maps toitself, and 6 maps to 4. This function is presented as a table in figure 4.9. Note that this mapping is simply a mapping from the integers mod 7 into itself; it is not an automorphism.

57. Wallis, *A Beginner's Guide to Graph Theory*, 43.

58. Lewin, *Generalized Musical Intervals and Transformations*, 193.

This transformation has some interesting properties. First of all, it maps non-triadic diatonic pitch networks into triadic networks by simply altering some of the intervals. In order to see how this works, let us examine the music in figure 4.10 and corresponding networks in figure 4.11. Figures 4.10a and 4.11a show a typical and easily recognizable use of ALT: an incomplete seventh-chord is mapped to a triad, which is in turn reduced to a simple third. Interestingly, this example partially reflects common patterns by which notes are occasionally omitted from seventh-chords and triads in four-part voice-leading. Figures 4.10b and 4.11b demonstrates the application of ALT to a musical passage which includes all the mod 7 intervals in no particular order, while in figures 4.10c and 4.11c we apply ALT to a diatonic scale above its tonic. Although they are certainly less musically relevant, these later examples demonstrate three properties of ALT quite clearly: it can only be applied twice before any further application would be redundant; the mapping will always produce intervals of a third or sixth; and, depending on the arrangement of arrows in the graph, the final result can be somewhat haphazard, although it often produces a triad or third.

It is this behavior of ALT, its capacity for indiscriminately exchanging one interval for another without regard to the whole network, that necessitates condition B regarding graph structure. By stipulating that the graph have a tree structure, we avoid confronting a problem posed by condition D in Lewin's definition of a transformation graph, which requires that for two nodes N and N' the semigroup product of two different arrow chains must be equal.[59] If we were to apply ALT to an operation graph that has more than one path between any two nodes, we could easily create an impossible graph. Figure 4.12 demonstrates this by presenting two pairs of graphs each representing the result of ALT when applied to an operation graph. The second graph in 4.12a is clearly impossible, while the graphs in 4.12b which lack the <5> arrow are both possible. Note that the tree graphs upon which ALT operates do not violate Lewin's definition. On the contrary, one could hypothetically add additional arrows to any graph to which it

59. Ibid., 195.

applies or to any graph which it produces and thus demonstrate that it meets Lewin's condition D.

While ALT may seem to be a unusual transformation, it has definite analytical significance in relation to Brahms's String Quartet No. 2. As we shall see, there are several clear instances in which the same motive appears two or more times in such a manner that the intervals in the successive instances of the motive are altered according to this transformation. Moreover, it has more general application to tonal music, since the transformation of a non-triadic motive into some form of triad is not an uncommon musical phenomenon.

The second transformation that we will consider is the Graph Reduction Function, REDUCE for short.

Graph Reduction Function [REDUCE]: Given two operation graphs G and G', the graph reduction function REDUCE(G) = G' maps G into G' such that GMAP is the identity mapping of the group of operations onto itself and NODEMAP has the following properties:

(A): Given any nodes N_a, N_b, and N_c in G, suppose that any of the following properties are true:

1. (N_a, N_b) is a member of ARROW and TRANSIT(N_a, N_b) = identity;
2. N_a and N_b are not in the arrow relation but (N_a, N_c) and (N_b, N_c) are members of ARROW and TRANSIT(N_a, N_c) = TRANSIT(N_b, N_c);
3. N_a and N_b are not in the arrow relation, but (N_c, N_a) and (N_c, N_b) are both members of ARROW and TRANSIT(N_c, N_a) = TRANSIT(N_c, N_b);

In any of these cases, NODEMAP will map the two nodes N_a and N_b into a single node N' in G'.

(B): Suppose NODEMAP maps two nodes N_a and N_b into N', then for all nodes besides N_a and N_b, NODEMAP is an identity mapping of nodes and arrows into corresponding nodes in G' other than N'.

(C): Suppose that N_y is either N_a or N_b, that N_z is a node other than N_a or N_b, that NODEMAP(N_y) = N', that NODEMAP(N_z) = N_z', and that (N_y, N_z) is a member of ARROW. Then (N', N_z') is a member of ARROW'.

(D): The graph reduction function always acts repeatedly, so that conditions A, B and C are applied as many times as possible. Thus, we might imagine a series of intermediate graphs between G and G'.

One of the primary features of the graph reduction function is that when it is applied to a graph G, G' will be a homomorphic image of G. Specifically, it will be a homomorphism of G into G'. This is relatively easy to prove. Lewin's definition of a homomorphism of a node/arrow system states that a mapping NODEMAP is a homomorphism *into* if (NODEMAP(N_1), NODEMAP(N_2)) is in the ARROW' relation whenever (N_1, N_2) is in the ARROW relation.[60] If nodes N_a and N_b are already in the ARROW relation, mapping both nodes into N' will continue to maintain the ARROW relation, since Lewin considers every node to be in the arrow relation with itself.[61] Condition B requires an identity mapping of nodes that are not reduced. Condition C requires that whenever two nodes are mapped into one node the arrows to any other nodes be preserved. Essentially, this generalizes a type of homomorphism between graphs that Lewin occasionally uses in his analyses.[62] Rather than stating how a specific instance of NODEMAP works and examining whether it is a homomorphism, the graph reduction function outlines criteria that shape NODEMAP differently in each case.

We can see what this function does by examining figure 4.13. Graph A is a complex graph of mod 7 transpositions in which many nodes contain the same scale-degrees. We do not know what scale-degrees they contain, but we can identify instances of the same scale-degree by looking for places in which a T_0 arrow could be added. Graph B is the output of REDUCE(Graph A) and is a homomorphic image of Graph A. It eliminates many of the nodes whose content was the same by mapping them into the same node. However, it does not eliminate every instance of repeated scale degrees, but only those that are related to each other or that are related to another node by the same arrow. Thus nodes A_2

60. Ibid., 201.

61. Ibid., 193.

62. Ibid., 235.

and A_3 contain the same scale-degree and are both mapped into node B_2, but A_1 and A_7 also contain the same scale-degree and are mapped into nodes B_1 and B_6 respectively. If A_1 and A_7 were in the arrow relation with a <0> arrow, Graph B would look quite different. Thus, the choice of musical relationships that are placed in the arrow relationship has a key effect on the nature of the graph. In musical terms, the graph reduction function eliminates closely related repeated notes from the graph.

Having noted that the arrow relationships displayed in a graph have a profound effect on this transformation, let us examine the application of REDUCE to a more complex graph of a concrete musical situation. Figure 4.14 presents a chord progression in a very thick texture, and several related musical examples which can be generated by REDUCE depending on the construction of the graph of figure 4.14a, while figures 4.15 and 4.16 present the corresponding graphs. Figure 4.15a presents the musical example as a complex graph in which every note generates both the note in the voice above and the next note in the same voice, creating a graph with a total of 45 nodes and 76 arrows. Let us apply REDUCE to this graph in two stages to produce the graph in figure 4.15b. The first stage removes all <0> arrows, producing a simpler graph which we can easily work with. Now if we apply conditions B and C, we find that we eliminate any duplicate nodes. If we start at the large C representing the middle voices, we find that the Gs on either side of the A must be mapped to the same node, because they both connect by <3>. Now we find that the last C in the middle voice belongs with the first C, since both are generated from the G. Furthermore, the bass Cs belong together and, since we have a <3> arrow representing the final descent of the bass, all of the Cs belong in one node connected to G by a bidirectional <3>/<4> arrow. Since both Fs are generated by <3> from C, and all Es by <2>, these must also be mapped to single nodes. The result is that each note occurs exactly once, and several of the arrows become bidirectional arrows representing <1>/<6> or <3>/<4>. This graph is difficult to represent musically, but figure 4.14b gives one possible realization: a sustained C major chord, with other notes appearing on top of it.

REDUCE makes a very dramatic change in the graph of figure 4.15a primarily because figure 4.15a relates each node to several others, with several different paths between any two nodes. Consequently, pitches that have the same letter name are all directly related. If we present the music in figure 4.14a as a graph with fewer arrows, however, REDUCE will not eliminate so many features. Figure 4.16 presents another graph of figure 4.14a along with its reduction. The graph primarily connects adjacent voices within chords. It also connects the notes of the bass line and the upper voices in the cadential six-four chord, but otherwise avoids voice-leading connections. If we apply REDUCE to this graph, the resulting graph eliminates repeated chords and doublings, but leaves the progression intact. This reduction is represented in musical notation in figure 4.14c. Note that the graph in figure 4.16 is one of the few ways to leave this progression intact after the application of REDUCE. For example, if we had chosen to connect the G to the C within the cadential six-four chord rather than connecting C and E to the following B and D, then we would have had to place that C and the final C in the bass in the same node, since both are connected to the G by a <3> arrow. Consequently, the placement of arrows in a graph has a significant impact on the way that REDUCE will alter it.

In the graphs of figure 4.16 both graphs conformed to tree structure; they had only one path between any two nodes. This highlights another property of REDUCE. Whenever the REDUCE is applied to a graph with tree structure (ignoring the loop from each node to itself) then the resulting graph will also have tree structure. This is true because every subgraph of a tree is a tree, and the only alterations that REDUCE makes to a graph involve first deleting an arrow (edge) and then joining the remaining subgraphs so that they share one node (vertex). Every edge in a tree is a bridge, and the deletion of a bridge by definition produces a graph with two components that are not connected.[63] When each of these components is joined by REDUCE, they are joined at only one vertex, so the result can have no cycles and must therefore also be a tree.

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63. Wallis, *Beginner's Guide to Graph Theory*, 35.

This has profound consequences. If we limit our operation graphs to graphs with tree structure, then the sets on which both the interval alteration function (ALT) and the graph reduction function (REDUCE) operate will be the same. Together these two functions form a semigroup of transformations under function composition operating on the set of operation graphs with tree structure. It should be noted that they do not form a group of operations, since neither function is 1-to-1 or onto and neither function has an inverse. Nevertheless, they both have significant musical implications and form a semigroup, which allows them to be used in transformation networks.

Furthermore, we could combine this semigroup with the group of automorphisms of these networks. Automorphisms are functions that map a group into itself so as to preserve the group product. In the context of mod 12 pitch-class space, automorphisms have frequently been used in Klumpenhouwer networks to create recursive networks. As Lewin demonstrates, the automorphisms of the T/I group used in K-nets are multiplication by 1, 5, 7, or 11 mod 12 combined with a second number that is added to inversions.[64] If we limit the multiplication operations to multiplication by 1 or 11, the automorphisms are isomorphic to the T/I group. Thus, most analyses have limited the automorphisms to these two numbers and have, in fact, used notation that corresponds to the T/I group. However, multiplication by 5 or 7 would be equally possible. This derives from a well-known fact of group theory, which states that "if a is a generator of a finite cyclic group G of order n, then the other generators of G are the elements of the form a^r, where r is relatively prime to n."[65] In the case of $\mathbb{Z}_{12}$ this means that 1, 5, 7, and 11 are generators of the group and consequently, that multiplication by these four numbers mod 12 will produce a group of automorphisms on $\mathbb{Z}_{12}$. If we apply this same fact to $\mathbb{Z}_7$, we find that 1, 2, 3, 4, 5, and 6 are all generators. Consequently, these elements form a cyclic multiplicative group of automorphisms on the diatonic transpositions mod 7. As a result, we can establish

64. David Lewin, "Klumpenhouwer Networks and Some Isographies That Involve Them," *Music Theory Spectrum* 12, No. 1 (Spring 1990): 88–89.

65. Fraleigh, *A First Course in Abstract Algebra*, 82.

isographies between some of our operation networks using these automorphisms, and we can combine them with ALT and REDUCE in a transformation semigroup.[66]

Although they are useful, these automorphisms do pose a problem, for most of them have little conventional musical meaning, beyond their ability to preserve group product. The exceptions are multiplication by 1, which corresponds to identity, and multiplication by 6, which corresponds to inversion. Although ALT and REDUCE do not commute with each other, both of them do commute with inversion, since ALT maps each pair of transpositions related by inversion into another pair related by inversion and REDUCE does not alter intervals. Using these transformations, we can make transformation graphs of our operation graphs in order to describe relationships between different variations of a motive within a larger theme.

The final methodological issue involves the distinction between processes that operate between different motives and processes that operate between different variations of the same motive. As we shall see, ALT, REDUCE, and the automorphisms of the diatonic intervals mod 7 all occur primarily between different motives. The intervals of our direct product GIS and permutational GIS operate exclusively between different variations of the same motives. While it might be possible to incorporate non-chord tones into the larger thematic model, it would hinder the clarity of presentation for both the larger theme and the individual motives.

Moreover, if we had decided to use transpositions mod 7 rather than the absolute distance up or down in our direct product GIS, it would be too easy to find isographic relationships between networks with only two nodes. If we use group automorphisms to create recursive networks of networks that use the direct product GIS, this will limit the possible automorphisms to identity and inversion for the pitch component and Y or N for each type of non-chord tone. If we were to use mod 7 transpositions in the direct product GIS, we could theoretically find an isography between any two two-node networks, which would become virtually

66. Ibid., 58. Technically, the algebraic structure is a monoid rather than a semigroup, since it includes identity.

meaningless. In any case, we can still establish a network homomorphism between any network of direct product transpositions and a network of mod 7 transpositions with an isomorphic graph if we can find a homomorphism between the two groups.[67] This would be relatively easy, since there is always a group homomorphism from $\mathbb{Z}$ into $\mathbb{Z}_n$, and we could use the trivial homomorphism to exclude the non-chord tone intervals.[68] Thus, clarity of presentation demands a sharp distinction between these two types of processes, while the requirement that a given motive must be able to be represented in two different ways adds a slight degree of resistance that will help to establish connections in empirical and phenomenological terms.

Conclusion

In this chapter we have outlined an alternative non-chord tone GIS, as well as a permutational GIS and a number of non-intervallic transformations. Although these latter tools do not directly relate to non-chord tones, they are all necessary groundwork for the analysis of Brahms's String Quartet in Chapter 5. However, even taken by themselves these transformations form a useful unit for musical analysis. By taking non-chord tones into account with our direct product GIS the underlying diatonic intervals between consonant chord tones will often be thirds or fourths belonging to some triadic structure. Our permutational GIS can model relationships between different triads, while the ALT function will transform any motive into a triad and ultimately a third, thus making a connection between non-triadic and triadic forms of a motive. The REDUCE function is of general use, since it simply generalizes a transformation that Lewin uses in particular cases. Thus, although their primary goal is to facilitate the analysis in the following chapter, these transformations may be useful in other contexts as well.

67. Lewin, *Generalized Musical Intervals and Transformations*, 202.

68. Fraleigh, *Abstract Algebra*, 162, 164.

Chapter 5: Analysis of Brahms's String Quartet No. 2 in A Minor

In chapter 4 we outlined a new type of non-chord tone GIS along with several indirectly related transformations. Although there may be a more general application to tonal music, all of these formal tools are directly related to the thematic structure of Brahms's String Quartet No. 2. We now turn to a detailed analysis of that quartet.

As a whole, Brahms's String Quartet in A minor has not received much attention from analysts in recent years. However, there are some detailed analyses of particular passages. Schoenberg's analysis of the beginning of the second movement has drawn a great deal of attention.[69] In addition, Arnold Whittall has offered a formal and harmonic analysis of the last movement, comparing it with the finale of Op. 51, no. 1.[70] Rainer Wilke's motivic analysis is the only detailed analysis of the entire piece.[71] As a general rule, Wilke's analysis is fairly eclectic and informal. He focusses on literal repetition of some combination of pitch, rhythm, contour, and even dynamics. Thus, he finds a total of six different motives in the first 12 measures and adds to that number from later portions of the work. There is a certain amount of positivism in Wilke's analysis, a rejection of anything that could be called "subjective," which has been the subject of criticism by Friedhelm Krummacher.[72] However, in spite of its defects, as a description of

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69. Arnold Schoenberg, "Brahms the Progressive," in *Style and Idea* (New York: Philosophical Library, 1950), 88–89; the responses to this analysis include: Walter Frisch, *Brahms and the Principle of Developing Variation* (Berkeley, University of California Press, 1984), 6–9; John Rothgeb, review of *Brahms and the Principle of Developing Variation*, by Walter Frisch, *Music Theory Spectrum* 9 (Spring 1987): 205–208; Pieter van den Toorn, "What's in a Motive? Schenker and Schoenberg Reconsidered," *The Journal of Musicology* 14, no. 3 (Summer 1996): 384–388.

70. Arnold Whittall, "Two of a kind? Brahms's Op. 51 finales," in *Brahms 2: Biographical, Documentary and Analytical Studies*, ed. Michael Musgrave (Cambridge: Cambridge University Press, 1987), 145–164.

71. Rainer Wilke, *Brahms. Reger. Schönberg. Streichquartette: Motivisch-thematische Prozesse und formale Gestalt* (Hamburg: Karl Dieter Wagner, 1980), 33–62.

thematic and motivic processes which encompasses many factors, Wilke's analysis is useful, and his treatment of several of the motives is quite thorough.

The analysis that follows takes a slightly different perspective and differs from Wilke's approach in several respects. First, I have strictly limited the analysis to pitches and intervals. Part of the problem with an informal motivic analysis such as Wilke's is that, although it is easy to find relationships between themes, it is very difficult to describe the differences in concrete terms that can relate the thematic structure to form in a meaningful way. Consequently, the analysis has difficulty in showing, for example, why the second theme belongs where it is. By taking a transformational approach I hope to provide concrete grounds for comparison of similarities and dissimilarities between themes.

Second, I make no claim to objectivity. Although the transformations introduced in the preceding chapter have general application, they ultimately derived their inspiration from a particular hearing of this piece. Thus, the theoretical technology itself is in some sense a component of the analysis. The basic goal of this analysis is to first show how the opening motive can be transformed into two motivic tonal events in the first theme and second to show how these motives are transformed throughout the form of the string quartet.

Analysis: First Movement, Primary Theme

Having outlined the formal properties of the transformations which we will be considering in chapter 4, we will now proceed to examine the ways in which they shape the motivic structure of Brahms's String Quartet No. 2. The primary theme and the beginning of the transition are given in figure 5.1. Three motives are labeled on the score: M.S., which represents the Motto Subject; T.M., which represents the Triad Motive; and motive A, which is characterized by the interval of a third plus varying configurations of non-chord tones. Although some of these motives and an additional new motive also appear after the cadence in measure 12, we will confine our attention to the first twelve measures, since the motivic material of the cadential section after measure 12 is primarily rhythmic, and the focus of this analysis is on pitch factors.

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72. Friedhelm Krummacher, "Reception and Analysis: On the Brahms Quartets, Op. 51, Nos. 1 and 2," *19th-Century Music* 18, no. 1 (Summer 1994): 28.

In addition to the labels for the motives, motive A has been marked by an arrow leading from one configuration to another and labeled with the appropriate direct product GIS interval. Taking the first instance in measure 3 as an example, we find an arrow pointing from F to D, labeled with the interval <−2, Y, N>. This indicates that the interval from F to D is −2 (two steps down) and that there is a change in the number of passing tones from F to D but not in the number of neighbor tones. The single passing tone in question is the E between F and D. Because it occurs in the same beat with F, E is considered "attached" to it, making the configurations (F, P, 0) and (D, 0, 0). Because we will always be dealing with such small intervals and configurations, many of which are the same, and to avoid cluttering the score unnecessarily, we have not indicated the configurations on the music, since they can easily be spotted by looking at the point in the music where the arrow begins and ends. When detailed discussion of these motives and the non-chord tone intervals is necessary, the configurations and intervals will appear in separate graphs such as that found in figure 5.7.

Returning to a discussion of the motives, we notice that the first of these three motives is labeled as the "Motto Subject," so called because three of its notes, F–A–E, suggest Joseph Joachim's motto, "Frei, aber einsam."[73] This motive in particular has been the subject of much attention in motivic analyses of this work. Rainer Wilke, in his motivic analysis of this quartet, demonstrates that this motive can be found in the contour of almost every theme in the first movement and that it can also be connected with the opening themes of the third and fourth movement.[74] Wilke's method involves isolating specific notes that correspond to this motive or some variation of it and identifying them in each theme. In considering this motive, Wilke addresses only variations in which all four notes

73. Friedhelm Krummacher, "Reception and Analysis: On the Brahms Quartets, Op. 51, Nos. 1 and 2," *19th-Century Music* 18, no. 1 (Sum. 1994): 31; Edwin Evans, *Handbook to the Chamber and Orchestral Music of Johannes Brahms*, vol. 1 (London: William Reeves, 1933), 222, 224. I have adopted Evan's term in order to distinguish this motive from the other motives which are the focus of this analysis.

74. Rainer Wilke, *Brahms. Reger. Schönberg. Streichquartette*, 214–218.

are present, although he does briefly touch on one example in which he finds both a shortened and full length version of the motive in measures 299–300 of the first movement.[75] Although he does discuss other motives, particularly in the first movement, Wilke is generally more concerned with the rhythmic component of these motives. In particular, he often draws attention to the series of eighth notes in the pickup to measure 5.

In this analysis, we will consider the Motto Subject (M.S.) and other motives from the perspective of pitch. Figure 5.2 represents the motive as an operation network. The operations in this case are diatonic intervals. The motive is presented as a four-node network (5.2a), reflecting the first four notes as they first appear in the music, and as three-node network (5.2b), which excludes the initial A and represents a graph reduction of the four-node network (REDUCE(5.2a) = 5.2b). In this figure the arrow labeled by REDUCE is effectively creating a network of networks, with figures 5.2a and 5.2b as the node contents. The transformation is musically relevant, for there are a number of instances in which the first note and interval are clearly omitted or repeated, particularly in measures 291–304, all of which can be reduced to 5.2b. Moreover, this focuses more attention on the key motto notes, "F–A–E."

Although other arrangements of nodes and arrows would be possible, such as an arrow chain reflecting the temporal order of the notes, the particular arrangement of the nodes and arrows in the graph has been chosen for both formal an interpretive reasons. The formal reason is that the graph must have tree structure in order to comply with our transformations. In terms of interpretation, the arrangement suggests that F moves to both A and E, with the high A somewhat detached from the E. In addition, the high A in the four-node network is generated by the initial A through octave transposition. This suggests that the F and E belong to a different, lower voice than A; that the lower voice is temporarily superimposed above the F, and that key motion from one voice to the other occurs from the F to the A within the "F–A–E" motto. This interpretation has the advantage of bringing out a connection just below the musical surface, as well as

75. Ibid., 34.

highlighting the letters “F–A–E.” It represents a more dynamic and interesting reading than a simple temporal arrangement. Furthermore, this interpretation can actually be realized in performance by playing the high A as a harmonic and changing bow direction on the E, rather than at the barline as many performers do.

The second motive we will consider initially appears as a fairly obvious transformation of the Motto Subject, appearing in retrograde motion with the intervals altered so that all of the notes belong to a B♭ major triad. This form appears in measures 7–8 and is labeled as the “Triad Motive” (T.M.). A four-node network representing this motive is presented in figure 5.3a. In addition, two clear triadic figures also appear in the first violin line in measures 10 and 12 and are represented as three-node networks in figure 5.3b and 5.3c. As with the Motto Subject, the three-node networks are reductions of the four-node network. In all cases, the networks are represented so that the third and fifth of the triad are generated by the root, and in the case of the four-node graph, the high D is also generated by the low D. It should be noted that although its notes are part of a larger dominant-seventh chord, the motive in measure 10 is still modeled as a diminished triad in accordance with its melodic shape.

The third motive we will consider consists of a third surrounding a passing tone. In other words, it consists of the single interval <−2, Y, N>, together with “variations” of that interval, such as <2, Y, N>. It appears in each of measures 3–6. It appears again with both a neighbor tone and a passing tone in measures 9 and 11, creating the interval <2, Y, Y>. This motive is labeled as motive A. Throughout this analysis motives that consist of a single direct product interval will be labeled by letters, as a opposed to the more complex motives, the Motto Subject and Triad Motive, which have more descriptive names. Although the direct product intervals are marked on the score, and are in fact the substance of motive A, we will ignore the non-chord tones for the present and focus our attention on several networks of diatonic transpositions mod 7 that can also be used to model this motive. By focusing our attention on the pitch interval and ignoring the non-chord tone component, we can relate motive A to the other motives; we will return to the direct product intervals when we discuss different variations of motive A.

Figure 5.4 presents the first versions of motive A, found in measures 3 and 4, as networks of two to five nodes, including an alternative segmentation (figures 5.4c and 5.4d). In general, the segmentation into networks of five nodes or two nodes seems the most sensitive reading, although the four-node and three-node networks also have advantages. Again, the arrangement of arrows has been chosen so as to produce tree structure; however, it is not without interpretive value. For example, in the five-node network representing measures 3 and 4 the arrows suggest that the F is somewhat independent of the first D and that the second D receives its impulse from both the preceding D and the F. Meanwhile, the absence of arrows between the third, fourth, and fifth nodes highlights the staccato articulation, and the arrows from the F to the third and fourth D suggest that they still have their origin in that note. This is a much more dynamic reading than figure 5.4e, which has too many arrows, or figure 5.4f, which is not particularly interesting. Finally, it should be noted that all of the graphs reduce to the two-node graph (figure 5.4b) through some combination of the REDUCE function and inversion.

We can now examine the relationships of the theme as they appear in figure 5.5. The figure is a network of networks, with individual operation networks depicting each motive and transformations between networks indicating relationships between different motives. It is important to distinguish between the transformations that employ solid arrows and are labeled with pointed brackets, such as <1>, and those using dashed arrows and square brackets, such as [E]. The former are part of operation graphs transforming pitches, while the latter are part of a single transformation graph, whose nodes consist of the operation networks. Although it would be possible to enclose each network in a circle indicating that it is a node in the larger network, this would clutter the graph and significantly increase the space required. The visual distinction is further clarified by the fact that transformations between networks do not touch individual nodes of the operation networks. Notice too, that all of the operation graphs have tree structure, while the larger transformation network does not. Finally, notice that some of the transformations are labeled with multiple transformations, such as [A][R]. These are functions combined using right orthography. In other words, the transformation [A][R] indicates that we first perform ALT and then REDUCE.

Figure 5.5 demonstrates the application that the ALT and REDUCE functions have to Brahms's string quartet. If we apply ALT to the graphs of either the four-node graph representing the Motto Subject or the three-node graph it will produce graphs that are identical to the four-node and three-node graphs of the Triad Motive. In addition, if we apply ALT again, it will produce two more graphs, a four-node graph and a three-node graph, which are identical to the four-node and three-node graphs of motive A in figure 5.4. Since ALT, REDUCE, inversion and identity form a semigroup, we can model the first 12 measures of Brahms's string quartet as the transformation network given in figure 5.5. Although there are several other ways of representing the primary theme through these transformations, particularly if we include the alternative segmentation of motive A, the network in figure 5.5 captures the formal symmetry and antecedent/consequent relationship within the opening 12 measures while remaining quite close to the most intuitive segmentation. In particular, this graph highlights the way in which each motive is derived from the Motto Subject. While motive A' is derived from the Motto Subject, it is also derived from motive A. In this reading, the initial appearance of motive A represents a separate entity distinct from the Motto Subject. It is only in motive A', when both motives have gone through the REDUCE function, that they generate a common motive. Hence there is no arrow leading to the first appearance of motive A. In addition, there are strong parallels between measures 1–6 and 7–12 which are emphasized by the relatively simple transformations that link these two parts.

The variation between different forms of motive A goes beyond what the transformation graph in figure 5.5 can capture. It involves the treatment of non-chord tones, which we have ignored in presenting the larger theme and the relationships between different motives. We can now consider their effect on different variations of the same motive by modeling motive A through the direct product GIS of diatonic pitch intervals and NCT-GIS(P, N; $\mathbb{Z}_2 \times \mathbb{Z}_2$). Figure 5.6 presents the variations of the reduced form of this motive as simple networks using the direct product GIS and as networks of diatonic pitch intervals. Examining the formal properties of these networks, we see that there are some clear advantages to representing this motive using a direct product GIS that incorporates non-chord tones. While the networks using direct product intervals are all isographic, the same clearly cannot be said of the networks that use only diatonic operations. The network derived from the direct product GIS takes the neighbor tone into account and reflects the subtle difference that it makes, while maintaining an isographic relationship with the networks for the other motives. Moreover, it highlights the connection between motive A and its presentation in the larger network of the theme, in which non-chord tones were excluded.

We can also examine the relationships between different variations of motive A within the primary theme by creating a network of these networks. This is presented in figure 5.7, which further emphasizes the formal development that takes place between the antecedent and consequent through the changes that take place in the non-chord tones. While the first instances of the motive use only passing tones, the motives in the consequent add neighbor tones. As in figure 5.5, the dashed arrows and transformations in square brackets indicate transformations between networks. In this case, the transformations are automorphisms of our direct product GIS. For example, the transformation [E, N, Y], which connects two networks with the intervals <2, Y, N> and <2, Y, Y> respectively, compares the intervals in the two networks and indicates that in the second network the diatonic component and the passing tone components are unchanged, while the neighbor tone component has changed.

In addition to the variation among different forms of motive A found within the primary theme, the Triad Motive is also subject to substantial variation. As we have already seen, it can be modeled as a network of diatonic intervals. However, the number of networks representing this motive that would be isographic is too high to properly capture the most significant transformations that it undergoes, since the distinctions would primarily be captured by the shape of the graphical representation. In order to capture the permutational effect of the transformations on the motive, we will apply the permutation GIS instead.

The permutational GIS does not model the intervals within motives but rather the relationship between variations of the triadic motive. Thus its intervals parallel the automorphisms between the networks for different forms of motive A. The network of variations of this motive presented in figure 5.8 suggests a subtle connection among all three forms of the motive. In terms of group theory, the three transformations presented in the network, <()>, <(R35)>, and <(R53)>, all belong to A_3, the unique subgroup of order three in S_3. If we consider the temporal order in which the root, third, and fifth of a triad appear in the music, this means that a series of triads separated by permutation intervals that belong to this subgroup will either consist of only ascending arpeggiations or only descending arpeggiations; a mix of ascending and descending arpeggiations would require other permutation intervals. In this case, however, not all the triads descend; the form which appears in measures 7 and 8 combines both. Thus, the connection between the three forms of the Triad Motive goes deeper than the fact that the figures in measures 10 and 12 both descend.

Analysis: Transition

As Wilke notes, the transition contains a new motive, whose metric pattern is derived from the closing bars of the primary theme, and which is combined with the motive from measure 4 (motive A'), which he considers as a series of eighth notes.[76] The transition motive and motive A', which appear repeatedly in this section, are marked on the score in figure 5.1. While the rhythmic connections are fairly obvious, there is a deep underlying connection between the first instance of the transition motive and the opening, which is demonstrated by the network of networks in figure 5.9. The double stops in the second violin strongly connect the A with F♯ and E to create a variation of the Motto Subject, while the repeated A at the beginning of motive A suggests a retrograde version of motive A rather than the inversion A'. Thus, the transition motive does have a

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76. Ibid., 37–39.

strong connection with the primary theme but is treated with considerable freedom, leading away from the principle thematic material to create variety.

The Triad Motive also appears extensively in the transition. Beginning in measure 30, the first violin plays descending arpeggiations of a diminished-seventh chord, while the second violin and viola use both motive A and the Triad Motive in an accompanying stream of eighth notes. Figure 5.10 shows this pattern as it first appears in the violins in measure 30–31. Interestingly, almost every triadic figure that appears throughout the entire primary theme and transition appears in descending form, and those few that do not tend to have an accompanimental role. Thus almost every possible permutational interval within the primary theme and transition is one of <()>, <(R35)>, or <(R53)>. Even some ascending accompanimental figures such as the cello figure in measures 40–41 (figure 5.11) maintain this pattern by appearing in open form.

The solo violin line in measures 43–45 leading into the second theme is particularly interesting. Figure 5.11 provides the relevant excerpt. The descending figures in the violin combine motive A with the Triad Motive, for one of the thirds in each triadic figure is filled in with a passing tone. In addition both motives are presented in their original form, namely, <−2, Y, N> for motive A, and (5, 3, R) for the triad. In terms of its thematic role, the passage helps gradually to bring back thematic material in preparation for the second theme. In his analysis, Wilke makes a similar argument. He points to measure 38–39 as the starting point, noting several rhythmic factors that connect this passage with the theme.[77] Although in terms of the pitch factors that are the focus of this analysis measures 38–42 have little connection with the theme, my analysis finds these in measures 43–45 which have little rhythmic connection with the theme. Consequently, the transition can be heard as dissecting the theme into its separate components and preparing the way for new combinations in the second theme.

Analysis: Secondary Theme

Figure 5.11 also contains the secondary theme, which appears in the first violin in measures 46–61. The viola repeats it in measure 62. As the labels on the

77. Ibid., 38–39.

score indicate, many of the same motives appear in both the primary theme and secondary theme. In fact, the only new motive is labeled motive B, and consists of a repeated pair of lower neighbor tones outlining the identity interval <0, N, N>. Since motive B always appears in conjunction with motive A in this context, and since the graph reduction function eliminates identity intervals, we can consider motive A and motive B to be part of the same network relative to the Triad Motive and Motto Subject. Consequently we can represent the secondary theme with several different networks, including a network, given in figure 5.12, that models the connections between motives found in this passage and networks given in figures 5.13 and 5.15, modeling the transformation of individual motives.

Figure 5.12 demonstrates how the secondary theme can be derived using the same transformations and motives found in the primary theme. In this example, measures 54–57 are ignored since they are derived entirely from motive A and require a more detailed explanation. There are striking similarities and differences between these networks and that presented in figure 5.15. In the first place, the Motto Subject is completely absent from the first part of the secondary theme, although it is outlined within the contour.[78] Secondly, the transformations are simpler and more loosely organized. Whereas the motives in the primary theme could more easily be connected with other motives, most of the motives in the second theme are transformed directly into the reduced form of motive A with few other possible connections. This reflects the fact that the secondary theme focuses on one basic shape, which it varies in length, while the primary theme employs several short, clear motives that are repeated only a few times and varied in more complex ways.

The second theme also includes a four measure closing phrase (measures 58–61) that strongly contrasts with the previous material. This is presented as a network in figure 5.12b. Interestingly, it is at this point that we find a reduced variation of the Motto Subject in inversion, followed by several repetitions of the Triad Motive. Since the preceding measures employed motive A almost exclusively, this sudden shift contributes to a sense of a new beginning. If we

78. Ibid., 35.

remember that there are no transformations in our semigroup that will lead us to the Motto Subject, only transformations leading from the Motto Subject toward motive A, this is particularly interesting.

The treatment of the Triad Motive within the second theme also demonstrates the importance of the factors that we have discussed so far. Figure 5.13 shows the transformations that this motive undergoes from the end of the transition to measure 61. While the transformations of the Triad Motive in the primary theme and transition were derived almost exclusively from <()>, <(R35)>, and <(R53)>, at the beginning of the second theme on the anacrusis to measure 46 we find the transformation <(R5)> leading to a motive of the form (R, 3, 5). Furthermore, the interval to the following motive (5, R, 3) in measure 53 is <(R35)>. In terms of group theory, this means that, as in the primary theme and transition, the intervals within this theme all belong to A_3, the alternating subgroup of S_3, which includes the permutations <()>, <(R35)>, and <(R53)>, while the intervals from any motive in primary theme to any motive in the second theme will be odd permutations (that is, they will be one of the two member cycles <(R3)>, <(R5)>, or <(35)>).

However, the second theme also sets up a certain degree of opposition within itself, since the motives in measures 59–60 are all separated from the rest of the second theme by 2-member cycles and consequently are implicitly united with the primary theme and transition. At the same time, these motives also employ an overall ascending shape that connects them with the second theme, although the order of root, third, and fifth is derived from the first theme. This joint connection further supports our argument that the closing phrase of the second theme appears almost as a new beginning and represents an opposing musical impulse.

The division at measure 58 is further borne out by the treatment of motive A. Figure 5.14 presents the underlying voice-leading of the first violin line in measures 54–57, while networks for motives A and B appear in figure 5.15. In terms of basic features, the main differences besides organizational structure between the graph of motive A in the secondary theme (figure 5.15a) and the corresponding graph of the primary theme, given earlier in figure 5.7, are that the second theme uses inversion only once, and the changes in the non-chord tone component of the intervals are more prominent. In addition, the network for motive B (figure 5.15b) is quite simple; we could find similar networks in the primary theme by examining the repeated notes independently. Consequently, from one perspective, the differences between the two themes are not great. At the same time, the phrase in measures 54–57 presents an anomaly. We can find the structure of motive A in measures 55–56 by analyzing the violin melody as a compound line with two instances of the motive. However, if we attempt to analyze measure 54 in the same way, we find that each interval is broken off or interrupted by a repetition. We could analyze this measure as a neighbor tone figure instead; however, this would not capture the complexity of the situation, nor would it reflect the fact that the theme has been eliminating repeated neighbor tones and reducing motive A. Rather, measure 54 poses a problem that is only partially solved by measure 55. Thus, in spite of the smooth lyrical character of the line, measures 54–57 create a break in thematic continuity that sharply distinguishes this theme from the primary theme. This may also help to explain why the repetition of this theme by the viola expands the corresponding passage in measures 70–76 by two bars.

Although they are decidedly different themes, the second theme shares many motives and transformations with the primary theme. Our network approach allows us to see exactly how the two themes are related, demonstrating both similarities and differences. In the process of comparing the two themes in this way, we have also isolated several factors that appear to have a significant bearing on the overall structure of the movement and the piece as a whole. The way in which the Motto Subject is used, the permutations of the Triad Motive, and treatment of motive A all have structural meaning that will be further examined throughout the course of the piece.

Analysis: Exposition, Measures 81–119

The remainder of exposition comprises two sections, a continuation of the second theme, which extends to measure 104 and appears in figure 5.16, and a third codetta theme in figure 5.19. Finally, at measure 120, a short passage consisting of the Motto Subject in canon connects the exposition with its own repetition and the beginning of the development.

Although there are other interesting features that we could discuss, the thematic aspect that most stands out in the continuation of the second theme is the striking treatment of triadic permutations. Two brief passages, measures 81–83 and 94–95, are sufficient to demonstrate this, since they are subjected to considerable repetition and variation. Permutational networks of these passages appear in figures 5.17 and 5.18. Both networks visually distinguish different instruments by placing permutations of the Triad Motive at different heights. In measure 81, the first violin's accompanimental pattern in eighth notes, which had previously employed a variety of intervallic patterns, is transformed into a triadic pattern that bears a certain resemblance to the motive in triplet quarter notes from the second theme, which appears in the viola part in measure 81. The transformed motive is subsequently passed around among the various parts and juxtaposed against the original triplet version. It is the contrast between two motives that is at the heart of the conflict in this passage.

Although there is an obvious rhythmic conflict of 4 against 3, there is a more subtle conflict based on the permutation that each motive employs. The motive in eighth notes always introduces an extra note on its ascent, producing a clear ascending arpeggiation, while the triplet motive continues to maintain an underlying descending contour. This is portrayed in figure 5.17. The permutation <(R3)> appears twice between successive motives in the first violin part, once at the transition to a triadic pattern and again when the violin adopts the triplet motive. Likewise <(R5)> appears in the viola part between the triplets of measure 81 and the eighth notes of measure 83. In addition, the arrows connecting the two parts contain mostly 2-member cycles. As in the contrast between the first and second themes, the use of 2-member cycles can be read in terms of conflict, underscoring the sense of rhythmic conflict and connecting it with other processes in the exposition.

The same feature can be found in measures 94–95, represented in figure 5.18. Here the first violin and viola are unified, having agreed on descending permutations. It is only the second violin, projecting a permutation of (5, R, 3) with its quarter notes, that weakly struggles against the overwhelming force of the descending permutations. Given the tumultuous ascending arpeggiations between measures 83 and 94, and the ultimate adoption of descending permutations by the second violin at the cadence in measures 102–103, it is easy to read this passage as a failed attempt on the part of the second theme to achieve a true space for itself and its ascending permutations against the first theme and its descending permutations, which had even ventured to invade the second theme with descending permutations in measures 58–60. (figures 5.11 and 5.13).

While the first and second themes are clear cut and opposed, the codetta theme that begins in measure 104 is characterized by a profound ambiguity which is derived from several aspects of the theme. In the first place, although the theme clearly begins and ends in C major, there is a pronounced shift toward A minor and D minor within each phrase. However, an even more pronounced ambiguity arises in the analysis of non-chord tones. In addition to figure 5.19, which contains the full score of this passage, two phrases of the first violin line been isolated in figure 5.20, with non-chord tone intervals presented on the music. Within the ascending line in measures 105 and 111, each tone is supported by a triad, thus we do not have to hear non-chord tones at all. However, the melodic shape suggests that B and, perhaps, A could be heard as passing tones within the overall outline of an A minor or C major triad. B is supported by a dissonant diminished triad and can easily be heard as passing tone. A on the other hand is supported by an F major triad and, given the fact that the C and E are supported by A minor and E major triads respectively, it is possible to hear either a C major or A minor triad underlying the melody here.

Whether the underlying triad is C major or A minor, the appropriate permutation of the underlying triad, (R, 3, 5) for A minor or (5, R, 3) for C major, ascends, contradicting the conclusion of the struggle between ascending and descending permutations that led to the cadence in measure 104. By promptly opposing the descending arpeggiations, this theme helps to open up a space for the development. Note that this is not the case when this theme reappears at the end of the recapitulation. Although a phrase corresponding to measures 104–109 does appear in measures 272–277, it is immediately contradicted by a passage in which the ascending line is inverted and transformed into a clear descending arpeggiation.

The final feature of the codetta theme that deserves note is the ambiguity of the non-chord tone intervals in measures 107–109 (figure 5.20a) and the change that takes place in measures 113–115 (figure 5.20b). At first glance, the G in measure 108 would appear to be stable, the goal of an ascending third with a passing tone, <2, Y, N>, which is then complemented by a descending third and passing tone <–2, Y, N> from (F, 0, 0) to (D, P, N). However, a hearing that takes all voices into account (figure 5.16) would find that the G and C♯ in measure 108 are part of a C♯ diminished-seventh chord juxtaposed against the bass D and that these resolve to a D minor chord on the eighth note anacrusis to measure 109. If we hear the preceding F in measure 107 as anticipating this resolution, then the G becomes a large-scale neighbor tone and the underlying interval is <1, N, Y> from (E, 0, 0) to (F, 0, N). This latter interval is much more clearly articulated in measures 113–115, where the high B♭ is a dissonant neighbor tone and the A is clearly stable. Moreover, the interval <1, N, Y> is repeated in measures 114–115 from (A, 0, 0) to (B, 0, N), provided we hear the C in the cadential six-four as a dissonant neighbor.

By this rather subtle shift, presenting three ascending notes first as the interval <2, Y, N> and then as part of <1, N, Y>, Brahms introduces the interval <1, N, Y> as a new motive, which we will designate as motive C. From a motivic standpoint, motive C actually introduces a musical problem. All previous motives appeared in some form in the primary theme and were the result of the transformations ALT and REDUCE. Even motive B, which contains the identity interval <0, N, N>, could be found in the repeated notes in the theme, although it did not gain any prominence until the second theme. Motive C, however, cannot be so easily found; it is something foreign to the basic substance of the theme. Moreover, this problem is not musically resolved in this movement. Although it does reappear occasionally, Brahms tends to ignore this motive. Even in the recapitulation, although the rhythmic component and contour reappear they tend to be presented as examples of motive A, the interval <2, Y, N> rather than <1, N, Y> which distinguishes the melodic substance of motive C.

Analysis: Development and Recapitulation

Although the motives which we have been considering constantly appear in interesting and suggestive ways throughout the entire first movement, it is neither possible nor desirable to treat each in detail. Moreover, in the development it is relatively easy to discern the connections between motives and variations, since they usually retain other features of the three primary themes such as rhythm and articulation. In addition, many portions of the recapitulation are transposed repetitions of the exposition. Thus we will briefly discuss only two passages, the retransition and the coda.

Figure 5.21 shows measures 177–184, the end of the development and the first two measures of the recapitulation. The Motto Subject appears every two bars in first violin part, while the viola responds with an inverted form of the same motive. Likewise, the second violin and cello exchange a small accompaniment figure. The structure of this figure is particularly interesting, especially in the three instances that appear in measure 181–184, which are represented as networks in figure 5.22. Figure 5.22a employs direct product intervals to distinguish between chord tones and non-chord tones, while figure 5.22b employs a network of mod 7 networks to represent the transformations between the different motives. As the networks demonstrate, although from the perspective of contour and rhythm each of the accompanying motives is essentially the same, each of these motives is different when we take harmonic stability into account. Underlying the motive in the second violin part is the Motto Subject, which is altered to become the Triad motive in measures 183–184. In the cello part, on the other hand, the E is stable, creating motive A instead. The harmonic flexibility of this accompaniment motive allows it to be associated with either the Motto

Subject or motive A, and Brahms takes full advantage of its harmonic possibilities. By choosing this harmonically flexible motive, Brahms prepares the recapitulation by bringing back the accompaniment pattern from the exposition and simultaneously creates a densely motivic texture which we can associate with the primary themes and motives.

The coda contains some particularly interesting features that further support the argument we have been making regarding motivic processes. Figure 5.23 shows the final portion of the coda, measures 304–335. Briefly, the thematic features are as follows: in measures 304–307, the first violin plays a rhythmically condensed version of the opening theme in which many of the pitches are repeated exactly; starting in the first violin line at measure 312 and in the other parts at measure 315, the primary motive is the Triad Motive, which consistently appears in ascending form according to the underlying permutation, even when the overall line descends; finally, the Motto Subject appears in canon in measure 321, accompanied by ascending triads. The final phrase in the first violin part is particularly important. Figure 5.24 presents two networks for this passage, a direct product network of variations of motive A for measures 327–332, and a network of motives from measure 325–332 in figure 5.24b.

One analytical question regarding these networks concerns the analysis of non-chord tones in measures 329 and 331. Although both G and F are harmonically supported on the most local level and the first G in measure 329 is part of a six-four chord, it is preferable to designate G as the stable tone. Not only does this analysis reflect the local melodic contour, but points to the underlying voice-leading. Beginning in measure 321, A is reinforced and supported as the highest note in the underlying harmony. G appears as a lower neighbor through the tonicization of C major in measures 323 and 324 and again in measure 327. The final progression involves a chromatic shift from G, supported by C major or its dominant, through the leading tone G♯ and the dominant of A minor. Thus, Brahms maintains a harmonic conflict between A minor and C major until the very end, which is also evident in the overlapping Triad Motives in measures 327–328.

In terms of thematic content (figure 5.24b), this passage thoroughly rounds off the piece. The three motives appear in order, connected by a combination of ALT and REDUCE. In addition, the overlapping Triad Motives in measures 327–328 both descend, contradicting the ascending triads that predominated in the preceding section. However, while this passage can be heard as an appropriate resolution to the thematic conflicts of the movement, it also introduces something which we have not heard before, an overlapping of a neighbor-tone variant of motive A (the interval <−2, N, Y>) and the descending Triad Motive. This combination of a descending arpeggiation with neighbor tones packs the thematic material of the Triad Motive and motive A into a very small space and is of great significance for the entire work. By introducing it at the end of the first movement, Brahms leaves the work open for continuation. We need to hear that combined motive played out to its logical conclusion. As we shall see, Brahms reserves this for the last movement.

Analysis: Second Movement

Although we will not discuss each movement in great detail, it is worth examining the primary themes of each movement to show how they connect to the first movement and consequently how each movement is positioned within the overall process of the piece. Consequently, we will examine the opening 8 bars of the movement, which are presented in figure 5.25. This particular passage has received a great deal of attention in Schoenberg's analysis in “Brahms the Progressive,” to which a number of authors, including Walter Frisch, John Rothgeb, and Pieter van den Toorn, have responded. Wilke, on the other hand, does not discuss the second movement in great detail; however, he does note that the opening motive of the Second Movement is derived from measure 113 in the First Movement (motive C in our terms), a connection which we will discuss shortly.[79]

Schoenberg's analysis can be summed up as follows.[80] All the motives in the theme can be derived from the interval of a second, found in the first two

79. Ibid., 49, 216.

80. Arnold Schoenberg, “Brahms the Progressive,” 88–89.

notes. Each subsequent motive is explained as the combination of one or more seconds, or as derived from the previous motives. Thus, Schoenberg finds a motive that consists of a fourth, which combines two successive seconds, in the descent from E to B in the second measure. He then derives other motives from this interval as well. In total, he finds six different motives, all derived from the interval of a second.

John Rothgeb, in a review of *Brahms and the Principle of Developing Variation* by Walter Frisch, objects to Schoenberg's analysis and Frisch's apparently uncritical reception of it.[81] Rothgeb's Schenkerian account of the passage argues that the passage is based on neither the interval of a second nor the interval of a fourth. Instead, he calls the D in measure 1 a passing tone within a third and labeling the E in measure 2 as a neighbor tone within a D–E–D motion, while the descent from D to B is another third. Rothgeb's account is primarily concerned with giving priority to the stable underlying tonal events that form the basis of a Schenkerian account. Thus he observes that "Brahms's music, although it undeniably involves developing variation (in a way different from that claimed by Schoenberg), is *founded on* the horizontal unfolding, by voice-leading and diminution of triads."[82]

While Rothgeb's concern for harmonic stability and melodic unfolding is highly relevant for our discussion of non-chord tones, his assertion that the E in measure 1 is stable while the initial D is a passing tone obscures a clear motivic association with measure 3. A slightly more balanced approach can be found in Pieter van den Toorn's commentary.[83] Van den Toorn presents a modified Schenkerian reading in which D in measure 1 is stable while E is a neighbor tone. He then argues in favor of certain aspects of Schoenberg's analysis, particularly

81. John Rothgeb, review of *Brahms and the Principle of Developing Variation*, 205–208; Walter Frisch, *Brahms and the Principle of Developing Variation*, 6–9.

82. Rothgeb, review of *Brahms and the Principle of Developing Variation*, 207 (Rothgeb's italics).

83. Pieter van den Toorn, "What's in a Motive?" 384–388.

the neighbor tone figures D–E–D in measures 1–2 and E–F♯–E in measure 3 that Schoenberg identifies as motivic repetition.

While it may not be desirable to derive this passage from the interval of a second in exactly the way that Schoenberg does, it is possible to find an highly organized motivic reading of this passage using our direct product networks. In this way, we can take tonal stability and diminution into account. Furthermore, although we cannot derive every motive from one interval as Schoenberg does, we can show how this passage is connected to the first movement. Figure 5.26 presents two networks, a direct product network of the entire theme and a corresponding network of unconnected subnetworks that highlight each motive as heard in isolation. As these networks demonstrate, this passage involves a very tight overlapping of each of motives A, B, and C. Some of these motives do appear slightly behind the foreground, such as motive C in measures 2–3, the interval <1, N, Y> from (D, 0, 0) to (E, 0, N); however, such motions are relatively obvious and appear in both the Rothgeb's and Schoenberg's accounts of the passage.

In terms of motivic process this passage also reveals the fundamental substance of the second movement. The first movement in its continuous progression from the Motto Subject to motive A presented a constant reduction of motives, in which the theme was essentially compressed into its smallest elements. Further, we must recall that there is no transformation that will take us back to the Motto Subject, since both ALT and REDUCE have no inverse. In addition, Brahms had introduced motive C, which had no precedent, and was left hanging without appropriate development. In the second movement, Brahms reverses this trend by taking those same small elements and tightly knitting them together to form a larger theme. He avoids motivic triads in the melody at the beginning of the second movement, and the Motto Subject is nowhere to be found, a fact which partly explains Wilke's scant treatment of this movement. This construction can also explain the fact that Brahms gradually introduces more and more descending arpeggiated triads in the repetition of the A section until we find that they saturate the texture in measures 116–120, just before the final chords (figure 5.27). In short, the second movement has two goals, the resolution of motive C through a more thorough treatment and the gradual reconstruction of the Triad Motive from smaller elements.

Analysis: Third Movement

That the third movement lays hold on the descending Triad Motives found at the end of the second movement and develops them should come as no surprise. Having restored the triad as a viable motive, Brahms does not again abandon it. Instead, the third movement begins to deal with the potential to integrate the smaller motives, particularly motives A and B, into the Triad Motive. This possibility appeared occasionally in the first movement and appeared prominently at the end of the coda. Nevertheless, the combination was left hanging, without significant development. The third movement explores one possible way of integrating these motives.

Examining the opening theme in figure 5.28, we see that both the non-chord tone motives and the Triad motive are present. Descending A minor triads support the beginnings of each of the phrases in measures 1–3 and 4–6. Within these triads the non-chord tones create several different variants of motives A and B. Networks for both types of motive appear in figure 5.29. In figure 5.29a, both triads are descending, and the connecting interval naturally belongs to the alternating subgroup A_3.

Even more importantly, the networks in figure 5.29b demonstrate that the predominating direct product intervals are <−2, Y, Y> and <−2, N, N>. This represents a shift from the first two movements, which tended to emphasize <−2, Y, N> and <2, Y, N>. In addition, the interval <−2, N, Y>, which occurred prominently in the coda of the first movement, is also absent. In other words, the motivic thirds in the first movement tended to involve a single passing tone. In the coda of the first movement, thirds which encompassed a single neighbor tone also appeared, posing a problem for the motivic consistency of the piece. The third movement avoids a return to the primary form of motive A by including both neighbor tones and passing tones or by employing unadorned thirds. Both of these intervals clarify the underlying triad by leaving a gap and emphasizing the stable third. The intervals <−2, Y, N> and <2, Y, N> do not clarify the underlying third in the same way. In fact, they are potentially ambiguous and can easily be interpreted as <1, N, Y>, as was the case at the introduction of motive C in the first movement. The interval <−2, N, Y> strongly emphasizes the third and was used in the first movement to create the overlapping with C major and A minor triads in the final bars of the coda. However, Brahms continues to leave that possibility unexplored in this movement. Instead, we find the combination of motive A with the Triad motive explored through <−2, Y, Y> and <−2, N, N>.

The third movement needs little further commentary. As is often the case with Minuets, there is a great deal of literal repetition. Moreover, the interested reader who takes a cursory glance through the score will quickly see that there many different rhythmic variations on the phrase from measures 1–3 and that motive A, as defined by the interval <−2, Y, Y>, saturates the movement.

Analysis: Fourth Movement

The finale brings the quartet to a logical close by grappling with the structural problem posed at the close of the first movement, namely, the integration of the Triad Motive and motive A, particularly motive A as defined by the direct product interval <−2, N, Y>. Although there are other processes at work in the last movement as well, we cannot fully discuss them in this context. However, since the movement is in sonata-rondo form, a brief examination of the primary theme will suffice to show the primary ways in which the same motives and transformations are still at work in this movement.

Figure 5.30 shows the primary theme, which extends to measure 13, where it is repeated by the viola, while figure 5.31 presents three networks that represent the theme from different perspectives. Note that the anacrusis figure has been analyzed as a third rather than a fourth; this reading considers these figures as belonging to implied secondary dominants each leading to the following bar, which is particularly apt for the pickups to measures 4 and 7. As these networks demonstrate, the theme consists entirely of triads and intervals of a third, which may encompass some type of non-chord tone. In many cases, the Triad Motive and motive A overlap, producing a synthesis that can be connected with the preceding movement but more directly reflects the end of the first movement by employing the interval <−2, N, Y>. An additional factor that connects this theme with the first movement is its tight construction; practically every motive can be connected with those around it. As in the first movement, the primary theme employs only descending triads, which the network in figure 5.31b demonstrates by including only permutation intervals from A_3.

Figure 5.32 shows how the processes that have operated in this piece are finally resolved in the last eight bars. For six bars before the final chords, there is nothing but the repetition of thematic synthesis, the overlapping of the Triad Motive with motive A as defined by <−2, N, Y>. The eighth note C on the last beat of each bar from measure 350 to 353 creates a bare ascending triad that leads into the next bar. As a result, the descending triad (5, 3, R) is juxtaposed with the ascending triad (R, 3, 5), thus creating the odd permutation interval <(R5)>, until in measures 354–355 all the parts join in a two octave descent through the synthesized motive. Thus Brahms maintains thematic tension until the very end, employing the processes which we have discussed until they drive the thematic material into the tightest possible combination, arriving at a point from which no further development would be possible.

Conclusion

The analytical approach which we have taken in examining Brahms's string quartet is one that can be loosely characterized as Schoenbergian. Schoenberg's analytical thought is not a fixed system, but consists of a number of key ideas and ways of thinking which appear piecemeal in Schoenberg's writings. In my analysis there are several points of contact with Schoenberg's ideas, and in discussing the overall significance of my analysis it will be worthwhile to examine the ways in which it reflects Schoenberg's ideas.

One of the central aspects of Schoenberg's musical thought is the concept of *Grundgestalt* or “basic shape.” The *Grundgestalt* is itself intimately connected with the concept of the idea in music. According to Schoenberg, idea can be synonymous with theme or motive but ultimately can be considered to be the totality of a piece. He goes on to clarify this statement by defining idea as the method by which balance is restored in a composition, over against the imbalance naturally created by tonal and rhythmic forces.[84] Patricia Carpenter treats the *Grundgestalt* as “the concrete technical aspect of the idea,” a memorable shape that encompasses harmony, motive, and rhythm.[85] Likewise, David Epstein argues that “the *Grundgestalt* denotes a configuration of musical elements that is significant to the form and structure of a work and is manifested throughout the work in differing guises and on various structural levels.”[86]

In my analysis, the various motives and the transformations which can be applied to them may be taken as a partial description of the thematic and motivic aspects of the *Grundgestalt*. It is in this context that the abstraction inherent in network analysis is helpful. We need not take the musical passage in bars 1–12 of the first movement as the *Grundgestalt*. Instead, the combination of the Motto Subject, Triad Motive, and Motives A, B, and C, together with the semigroup of transformations that mediates between motives (ALT, REDUCE, and inversion), as well as the operations that apply to individual motives (permutations and direct product automorphisms) may all collectively be taken as the *Grundgestalt* as it applies to melodic and intervallic features of the music.

The treatment of motives in Brahms's quartet as described in this analysis can also be compared with Schoenberg's discussion of the motive: “The *motive* generally appears in a characteristic and impressive manner at the beginning of a piece. … Inasmuch as almost every figure within a piece reveals some relationship to it, the basic motive is often considered the 'germ' of the idea.”[87]

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84. Schoenberg, “New Music, Outmoded Music, Style and Idea,” in *Style and Idea*, 49.

85. Patricia Carpenter, “*Grundgestalt* as Tonal Function,” *Music Theory Spectrum* 5, no. 1 (Spring 1983): 15.

86. David Epstein, *Beyond Orpheus*: *Studies in Musical Structure* (Cambridge, MA: MIT Press, 1979), 19.

87. Arnold Schoenberg, *Fundamentals of Musical Composition* (London: Faber and Faber, 1967), 8.

Musically, the Motto Subject fits this description, while our analysis takes it as a point of origin from which other motives are derived. In this context the fact that ALT and REDUCE are transformations with no inverse is helpful, because it clearly describes the Motto Subject as a source—input rather than output. The transformation of the Motto Subject into simpler tonal motives also exemplifies what Schoenberg calls liquidation: "*Liquidation* consists in gradually eliminating characteristic features, until only uncharacteristic ones remain, which no longer demand continuation. Often only residues remain, which have little in common with the basic motive."[88] This is locally true within many of the themes we examined, in which motives A, B, and C often appeared after the more complex motives. Liquidation also applies to the entire piece, with the gradual elimination of the Motto Subject in favor of the tight combination of motives found in the final bars, producing a tendency toward closure. This reflects Schoenberg's statement that "the purpose of liquidation is to counteract the tendency toward unlimited extension."[89] This also resonates with the characterization of idea and *Grundgestalt* as forces that provide balance and logical closure.

A final aspect of my analysis that reflects Schoenberg's thematic thinking is the characterization of the music in terms of dialectical oppositions, which Michael Cherlin argues are a significant aspect of Schoenberg's musical thought, including the *Grundgestalt* concept.[90] Schoenberg often describes musical processes in terms of problems that must be solved arguing that "every succession of tones produces unrest, conflict, problems. … Every musical form can be considered as an attempt to treat this unrest either by halting or limiting it, or by solving the problem."[91] It is exactly this kind of problem solving aspect of form that I have tried to explicate in my analysis, presenting oppositions between

88. Ibid., 58.

89. Ibid.

90. Michael Cherlin, "Dialectical Opposition in Schoenberg's Music and Thought," *Music Theory Spectrum*, 22, no. 2 (Autumn 2002): 170.

91. Schoenberg, *Fundamentals of Musical Composition*, 102.

themes, motives, and variations of motives in terms of oppositions and problems that must be resolved, and demonstrating the ways in which the music solves those problems.

As my analysis has also demonstrated, non-chord tone GISes can easily be employed in network analysis. They can be integrated with other types of operation networks and more complex transformations in the context of a musical analysis. The analysis itself demonstrates the applicability of these types of networks to motivic analysis, allowing us to examine both the motives and process in Brahms's quartet. Although the networks can only present a portion of the thematic aspect of the *Grundgestalt*, and even these could be examined in other ways, this particular method of description nevertheless offers several advantages including clear graphical presentation, the flexibility and precision of mathematical definitions, and the ability to capture dynamic aspects of the *Grundgestalt* through the use of non-intervallic transformations. Even the level of abstraction inherent in the approach is useful, for it allows us to present this thematic aspect of the *Grundgestalt* in terms of a set of motives (graphs) and several associated groups and semigroups of transformations rather than linking the *Grundgestalt* to one particular manifestation in the music.

Chapter 6: Conclusions

In this thesis we have examined a new approach to the analysis of non-chord tones using the theoretical work in Lewin's *Generalized Musical Intervals and Transformations* as the underlying methodology. In doing so, we have not only been exploring non-chord tone analysis but also the technical features of Lewin's system. Because the approach we have taken involves so many small digressions into the formal properties inherent in the GMIT project as well as a great deal of attention to the details of Mozart's variations on "Ah! Vous dirai-je, Maman," K. 265 and Brahms's String Quartet No. 2, we can perhaps best conclude our study by enumerating some of the essential points.

First, the non-chord tones in music are worthy of analytical attention as non-chord tones. Music theory has a tendency to label non-chord tones and then omit them from the discussion of the music. Even Schenkerian analysis, which does occasionally discuss the role of non-chord tones as part of the counterpoint, often does this by focusing on the middleground without dealing with the details of the foreground. However, as our analyses have demonstrated, non-chord tones can play a key role in themes and motives and even in overall formal structure, particularly in a set of variations.

Second, our system of non-chord tone analysis is designed to be adaptable to many different types of analysis. We explored a statistical approach in chapter 3 and a Schoenbergian motivic approach in chapter 5. However, non-chord tone generalized interval systems could be applied in other ways as well. Although we have avoided Schenkerian analysis, since it does already present a way of dealing with non-chord tones, non-chord tone interval systems could easily be combined with Schenkerian graphs. Since most of the prominent features of Schenkerian analysis, such as the *Zug,* include notes that are technically expanded passing tones, we could find application for non-chord tone intervals on a background level. It is a key feature of non-chord tone interval systems that they do not depend on any one way of labeling non-chord tones and can thus be adapted to other analytical methods. Moreover, they build extensively on Lewin's work in transformational theory, which is itself a framework of theoretical tools that can be applied in almost any music theoretical context. The thesis ultimately expands that framework to include a sophisticated method for dealing with non-chord tones, along with other tools for analyzing tonal music.

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Appendix

Figures for Chapter 1

Figure 1.1: Consonant embellishment from Piston, *Harmony*, 81.

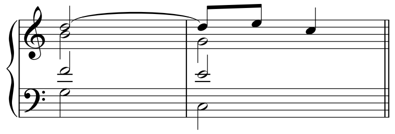
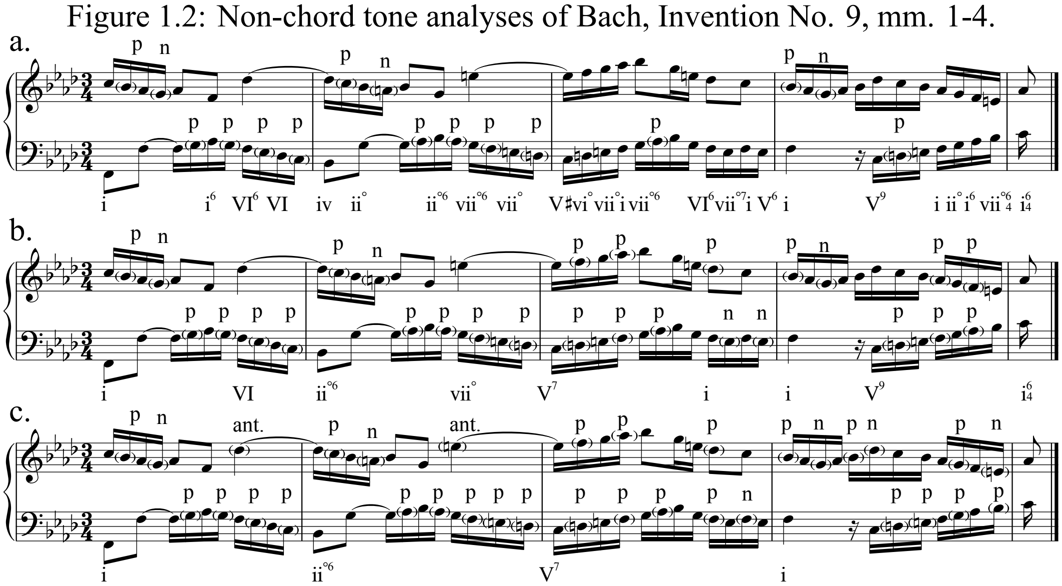


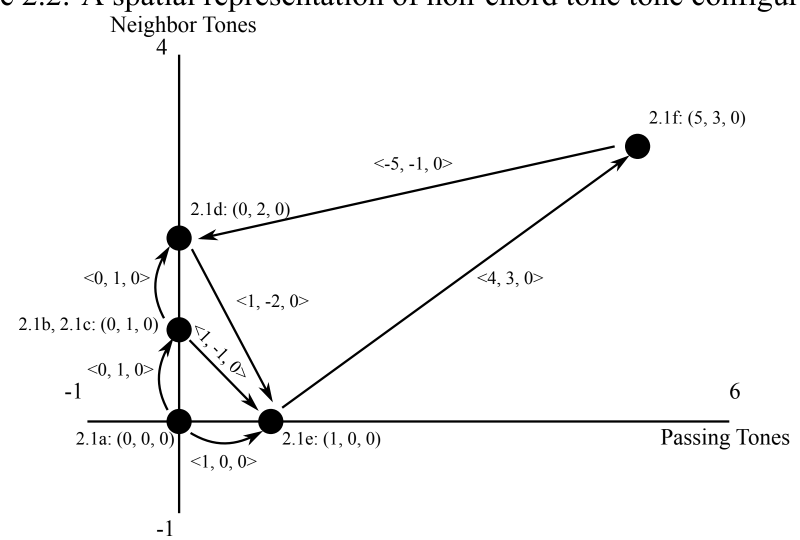
Figure 1.2: Non-chord tone analyses of Bach, Invention No. 9, mm. 1-4.



Figures for Chapter 2

Figure 2.1: Non-chord tone configurations attached to a C major chord.

Figure 2.2: A spatial representation of non-chord tone tone configurations.



Figures for Chapter 3

Figure 3.1: Mozart, Variations on ”Ah! Vous dirai-je, Maman,” Theme.

Figure 3.2: Theme: graph of non-chord tone contours

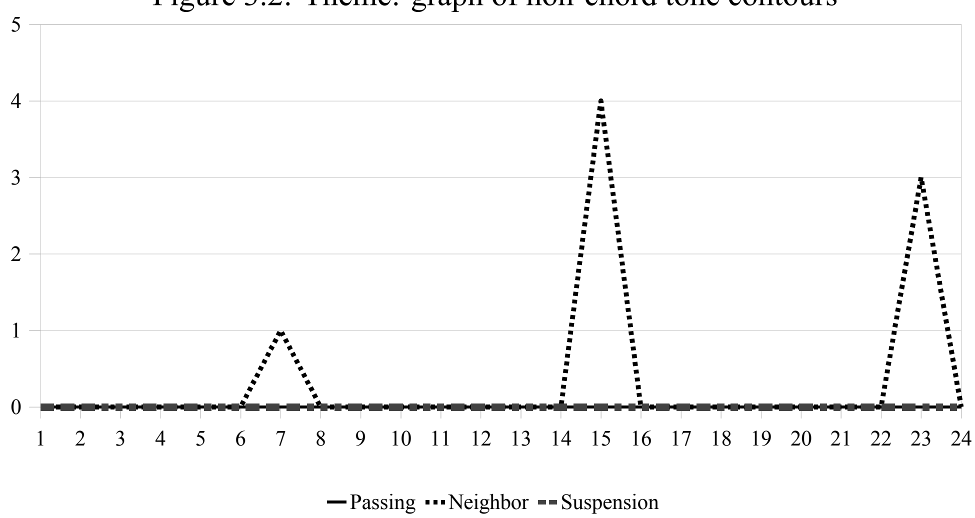


Figure 3.3: Theme: table of configurations and intervals.

Measure Numbers	Theme: Configurations	Intervals Between Measures
M. 1	(0, 0, 0)	
		<0, 0, 0>
M. 2	(0, 0, 0)	
		<0, 0, 0>
M. 3	(0, 0, 0)	
		<0, 0, 0>
M. 4	(0, 0, 0)	
		<0, 0, 0>
M. 5	(0, 0, 0)	
		<0, 0, 0>
M. 6	(0, 0, 0)	
		<0, 1, 0>
M. 7	(0, 1, 0)	
		<0, -1, 0>
M. 8	(0, 0, 0)	
		<0, 0, 0>
M. 9	(0, 0, 0)	
		<0, 0, 0>
M. 10	(0, 0, 0)	
		<0, 0, 0>
M. 11	(0, 0, 0)	
		<0, 0, 0>
M. 12	(0, 0, 0)	
		<0, 0, 0>
M. 13	(0, 0, 0)	
		<0, 0, 0>
M. 14	(0, 0, 0)	
		<0, 4, 0>
M. 15	(0, 4, 0)	
		<0, -4, 0>
M. 16	(0, 0, 0)	
		<0, 0, 0>
M. 17	(0, 0, 0)	
		<0, 0, 0>
M. 18	(0, 0, 0)	
		<0, 0, 0>
M. 19	(0, 0, 0)	
		<0, 0, 0>
M. 20	(0, 0, 0)	
		<0, 0, 0>
M. 21	(0, 0, 0)	
		<0, 0, 0>
M. 22	(0, 0, 0)	
		<0, 3, 0>
M. 23	(0, 3, 0)	
		<0, -3, 0>
M. 24	(0, 0, 0)	

Figure 3.4: Mozart, Variations on ”Ah! Vous dirai-je, Maman,” Variation 1.

Figure 3.5: Variation 1: graph of non-chord tone contours

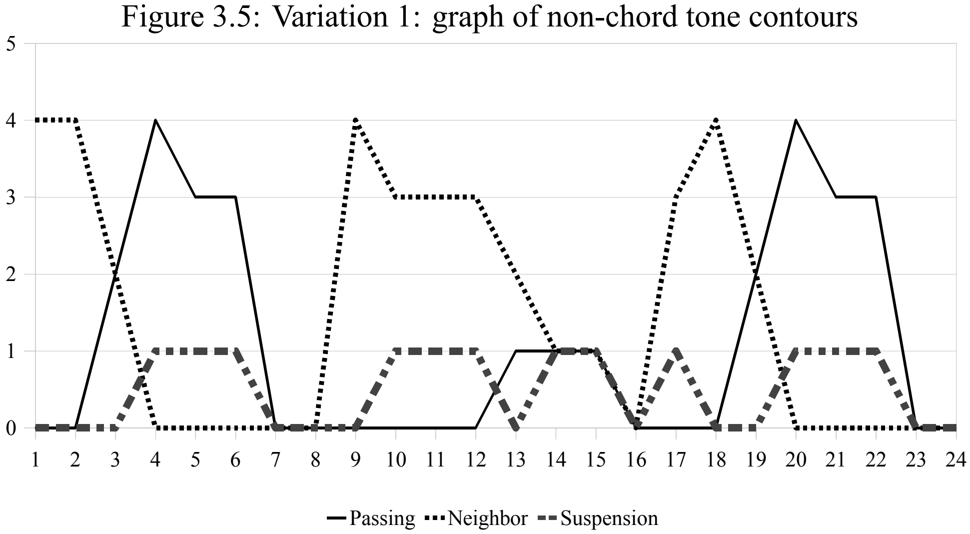


Figure 3.6: Variation 1: table of configurations and intervals.

Measure Numbers	Intervals from the Theme	Variation 1: Configurations	Intervals Between Measures
M. 1	<0, 4, 0>	(0, 4, 0)	<0, 0, 0>
M. 2	<0, 4, 0>	(0, 4, 0)	<2, -2, 0>
M. 3	<2, 2, 0>	(2, 2, 0)	<2, -2, 1>
M. 4	<4, 0, 1>	(4, 0, 1)	<-1, 0, 0>
M. 5	<3, 0, 1>	(3, 0, 1)	<0, 0, 0>
M. 6	<3, 0, 1>	(3, 0, 1)	<-3, 0, -1>
M. 7	<0, -1, 0>	(0, 0, 0)	<0, 0, 0>
M. 8	<0, 0, 0>	(0, 0, 0)	<0, 4, 0>
M. 9	<0, 4, 0>	(0, 4, 0)	<0,-1, 1>
M. 10	<0, 3, 1>	(0, 3, 1)	<0, 0, 0>
M. 11	<0, 3, 1>	(0, 3, 1)	<0, 0, 0>
M. 12	<0, 3, 1>	(0, 3, 1)	<1, -1, -1>
M. 13	<1, 2, 0>	(1, 2, 0)	<0, -1, 1>
M. 14	<1, 1, 1>	(1, 1, 1)	<0, 0, 0>
M. 15	<1, -3, 1>	(1, 1, 1)	<-1, -1, -1>
M. 16	<0, 0, 0>	(0, 0, 0)	<0, 3, 1>
M. 17	<0, 3, 1>	(0, 3, 1)	<0, 1, -1>
M. 18	<0, 4, 0>	(0, 4, 0)	<2, -2, 0>
M. 19	<2, 2, 0>	(2, 2, 0)	<2, -2, 1>
M. 20	<4, 0, 1>	(4, 0, 1)	<-1, 0, 0>
M. 21	<3, 0, 1>	(3, 0, 1)	<0, 0, 0>
M. 22	<3, 0, 1>	(3, 0, 1)	<-3, 0, -1>
M. 23	<0, -3, 0>	(0, 0, 0)	<0, 0, 0>
M. 24	<0, 0, 0>	(0, 0, 0)	

Figure 3.7: Mozart, Variations on ”Ah! Vous dirai-je, Maman,” Variation 2.

Figure 3.8: Variation 2: graph of non-chord tone contours

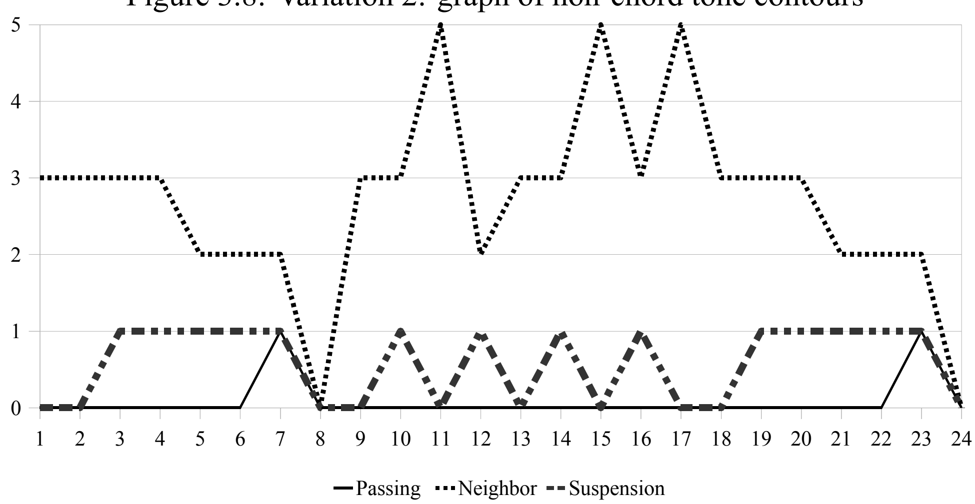


Figure 3.9: Variation 2: table of configurations and intervals.

Measure Numbers	Intervals from Variation 1	Variation 2: Configurations	Intervals Between Measures
M. 1	<0, -1, 0>	(0, 3, 0)	
			<0, 0, 0>
M. 2	<0, -1, 0>	(0, 3, 0)	
			<0, 0, 1>
M. 3	<-2, 1, 1>	(0, 3, 1)	
			<0, 0, 0>
M. 4	<-4, 3, 0>	(0, 3, 1)	
			<0, -1, 0>
M. 5	<-3, 2, 1>	(0, 2, 1)	
			<0, 0, 0>
M. 6	<-3, 2, 0>	(0, 2, 1)	
			<1, 0, 0>
M. 7	<1, 2, 1>	(1, 2, 1)	
			<-1, -2, -1>
M. 8	<0, 0, 0>	(0, 0, 0)	
			<0, 3, 0>
M. 9	<0, -1, 0>	(0, 3, 0)	
			<0, 0, 1>
M. 10	<0, 0, 0>	(0, 3, 1)	
			<0, 2, -1>
M. 11	<0, 2, -1>	(0, 5, 0)	
			<0,-3, 1>
M. 12	<0, -1, 0>	(0, 2, 1)	
			<0, 1, -1>
M. 13	<1, 1, 0>	(0, 3, 0)	
			<0, 0, 1>
M. 14	<-1, 2, 0>	(0, 3, 1)	
			<0, 2, -1>
M. 15	<-1, 4, -1>	(0, 5, 0)	
			<0, -2, 1>
M. 16	<0, 3, 1>	(0, 3, 1)	
			<0, 2, -1>
M. 17	<0, 2, 0>	(0, 5, 0)	
			<0, -2, 0>
M. 18	<0, -1, 0>	(0, 3, 0)	
			<0, 0, 1>
M. 19	<-2, 1, 1>	(0, 3, 1)	
			<0, 0, 0>
M. 20	<-4, 3, 0>	(0, 3, 1)	
			<0, 1, 0>
M. 21	<-3, 2, 0>	(0, 2, 1)	
			<0, 0, 0>
M. 22	<-3, 2, 0>	(0, 2, 1)	
			<1, 0, 0>
M. 23	<1, 2, 1>	(1, 2, 1)	
			<-1, -2, -1>
M. 24	<0, 0, 0>	(0, 0, 0)	

Figure 3.10: Retrograde-Inversion Chain

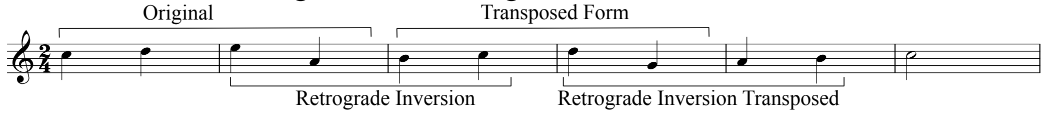


Figure 3.11: Mozart, Variations on ”Ah! Vous dirai-je, Maman,” Variation 3.

Figure 3.12: Variation 3: graph of non-chord tone contours

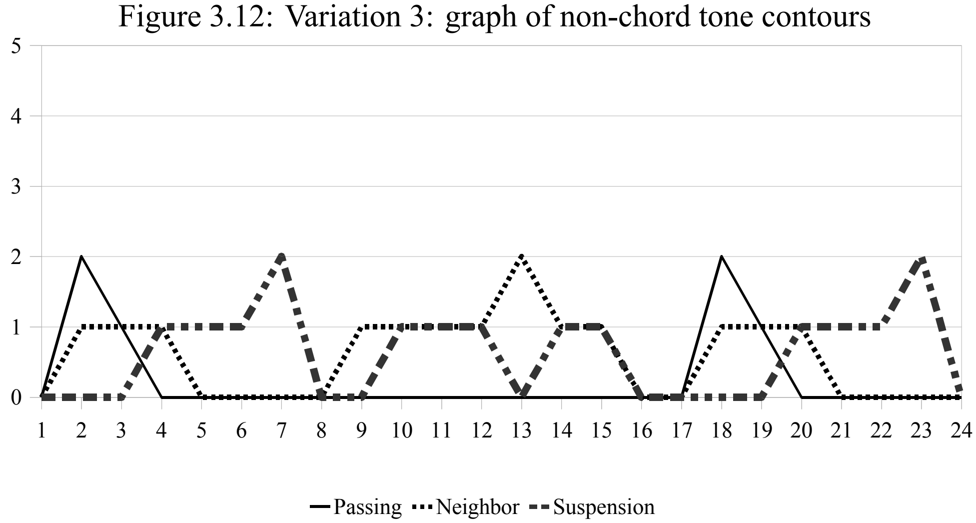


Figure 3.13: Variation 3: table of configurations and intervals.

Measure Numbers	Intervals from Variation 2	Variation 3: Configurations	Intervals Between Measures
M. 1	<0, -3, 0>	(0, 0, 0)	<2, 1, 0>
M. 2	<2, -2, 0>	(2, 1, 0)	<-1, 0, 0>
M. 3	<1, -2, -1>	(1, 1, 0)	<-1, 0, 1>
M. 4	<0, -2, 0>	(0, 1, 1)	<0, -1, 0>
M. 5	<0, -2, 0>	(0, 0, 1)	<0, 0, 0>
M. 6	<0, -2, 0>	(0, 0, 1)	<0, 0, 1>
M. 7	<-1, -2, 1>	(0, 0, 2)	<0, 0, -2>
M. 8	<0, 0, 0>	(0, 0, 0)	<0, 1, 0>
M. 9	<0, -2, 0>	(0, 1, 0)	<0, 0, 1>
M. 10	<0, -2, -1>	(0, 1, 1)	<0, 0, 0>
M. 11	<0, -4, -1>	(0, 1, 1)	<0, 0, 0>
M. 12	<0, -1, 0>	(0, 1, 1)	<0, 1, -1>
M. 13	<-1, -2, 0>	(0, 2, 0)	<0, -1, 1>
M. 14	<0, -2, -1>	(0, 1, 1)	<0, 0, 0>
M. 15	<0, -4, -1>	(0, 1, 1)	<0, -1, -1>
M. 16	<0, -3, -1>	(0, 0, 0)	<0, 0, 0>
M. 17	<0, -5, 0>	(0, 0, 0)	<2, 1, 0>
M. 18	<2, -2, 0>	(2, 1, 0)	<-1, 0, 0>
M. 19	<1, -2, -1>	(1, 1, 0)	<-1, 0, 1>
M. 20	<0, -2, 0>	(0, 1, 1)	<0, -1, 0>
M. 21	<0, -2, 0>	(0, 0, 1)	<0, 0, 0>
M. 22	<0, -2, 0>	(0, 0, 1)	<0, 0, 1>
M. 23	<-1, -2, 1>	(0, 0, 2)	<0, 0, -2>
M. 24	<0, 0, 0>	(0, 0, 0)	

Figure 3.14: Mozart, Variations on "Ah! Vous dirai-je, Maman," Variation 4.

Figure 3.15: Variation 4: graph of non-chord tone contours

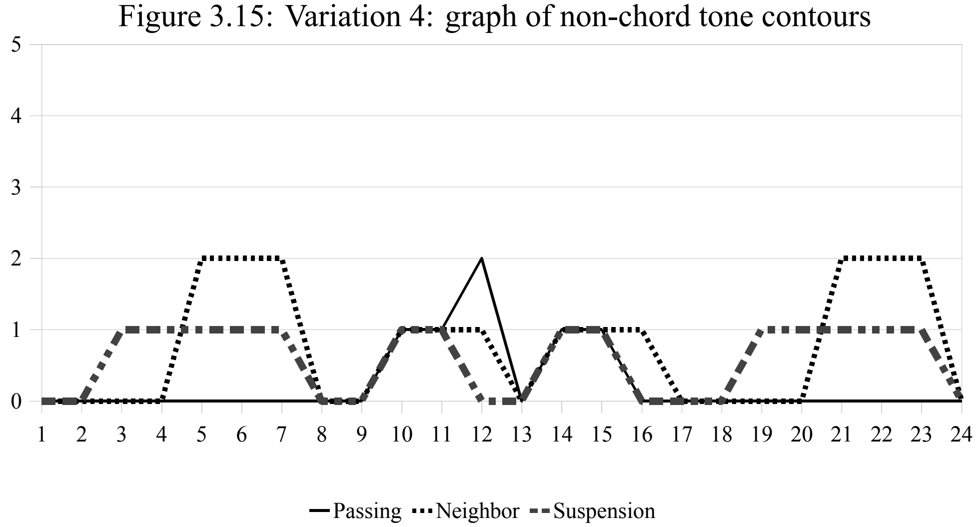


Figure 3.16: Variation 4: table of configurations and intervals.

Measure Numbers	Intervals from Variation 3	Variation 4: Configurations	Intervals Between Measures
M. 1	<0, 0, 0>	(0, 0, 0)	
			<0, 0, 0>
M. 2	<-2, -1, 0>	(0, 0, 0)	
			<0, 0, 1>
M. 3	<-1, -1, 1>	(0, 0, 1)	
			<0, 0, 0>
M. 4	<0, -1, 0>	(0, 0, 1)	
			<0, 2, 0>
M. 5	<0, 2, 0>	(0, 2, 1)	
			<0, 0, 0>
M. 6	<0, 2, 0>	(0, 2, 1)	
			<0, 0, 0>
M. 7	<0, 2, -1>	(0, 2, 1)	
			<0, -2, -1>
M. 8	<0, 0, 0>	(0, 0, 0)	
			<0, 0, 0>
M. 9	<0, -1, 0>	(0, 0, 0)	
			<1, 1, 1>
M. 10	<1, 0, 0>	(1, 1, 1)	
			<0, 0, 0>
M. 11	<1, 0, 0>	(1, 1, 1)	
			<1, 0, -1>
M. 12	<2, 0, 1>	(2, 1, 0)	
			<-2, -1, 0>
M. 13	<0, -2, 0>	(0, 0, 0)	
			<1, 1, 1>
M. 14	<1, 0, 0>	(1, 1, 1)	
			<0, 0, 0>
M. 15	<1, 0, 0>	(1, 1, 1)	
			<-1, 0, -1
M. 16	<0, 1, 0>	(0, 1, 0)	
			<0, -1, 0>
M. 17	<0, 0, 0>	(0, 0, 0)	
			<0, 0, 0>
M. 18	<-2, -1, 0>	(0, 0, 0)	
			<0, 0, 1>
M. 19	<-1, -1, 1>	(0, 0, 1)	
			<0, 0, 0>
M. 20	<0, -1, 0>	(0, 0, 1)	
			<0, 2, 0>
M. 21	<0, 2, 0>	(0, 2, 1)	
			<0, 0, 0>
M. 22	<0, 2, 0>	(0, 2, 1)	
			<0, 0, 0>
M. 23	<0, 2, -1>	(0, 2, 1)	
			<0, -2, -1>
M. 24	<0, 0, 0>	(0, 0, 0)	

Figure 3.17: Non-chord tone contour for measure 9 up to Variation 4.

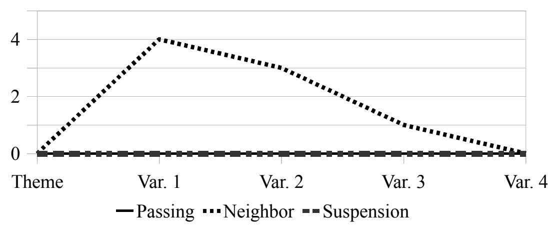


Figure 3.18: Mozart, Variations on ”Ah! Vous dirai-je, Maman,” Variation 5.

Figure 3.19: Variation 5: graph of non-chord tone contours

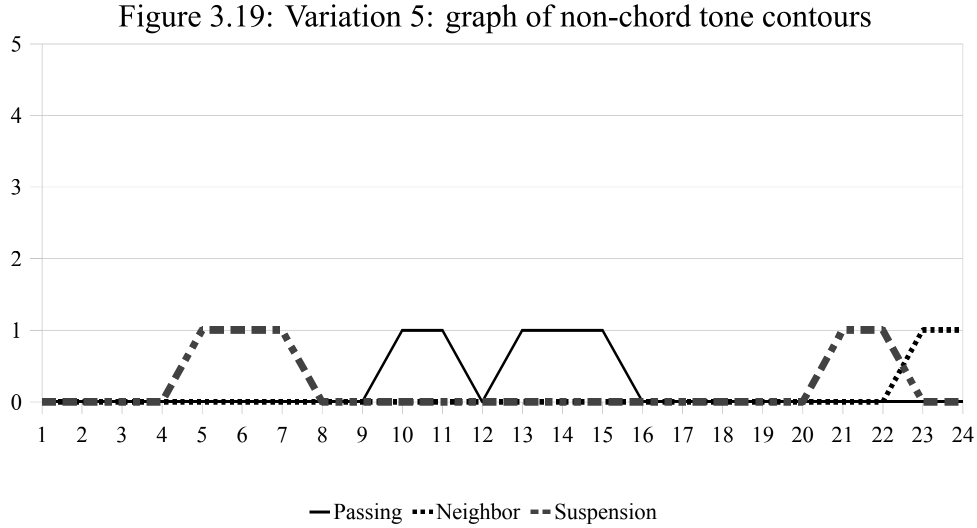


Figure 3.20: Variation 5: table of configurations and intervals.

Measure Numbers	Intervals from Variation 4	Variation 5: Configurations	Intervals Between Measures
M. 1	<0, 0, 0>	(0, 0, 0)	
			<0, 0, 0>
M. 2	<0, 0, 0>	(0, 0, 0)	
			<0, 0, 0>
M. 3	<0, 0, -1>	(0, 0, 0)	
			<0, 0, 0>
M. 4	<0, 0, -1>	(0, 0, 0)	
			<0, 0, 1>
M. 5	<0, -2, 0>	(0, 0, 1)	
			<0, 0, 0>
M. 6	<0, -2, 0>	(0, 0, 1)	
			<0, 0, 0>
M. 7	<0, -2, 0>	(0, 0, 1)	
			<0, 0, -1>
M. 8	<0, 0, 0>	(0, 0, 0)	
			<0, 0, 0>
M. 9	<0, 0, 0>	(0, 0, 0)	
			<1, 0, 0>
M. 10	<0, -1, -1>	(1, 0, 0)	
			<0, 0, 0>
M. 11	<0, -1, -1>	(1, 0, 0)	
			<-1, 0, 0>
M. 12	<-2, -1, 0>	(0, 0, 0)	
			<1, 0, 0>
M. 13	<1, 0, 0>	(1, 0, 0)	
			<0, 0, 0>
M. 14	<0, -1, -1>	(1, 0, 0)	
			<0, 0, 0>
M. 15	<0, -1, -1>	(1, 0, 0)	
			<-1, 0, 0>
M. 16	<0, -1, 0>	(0, 0, 0)	
			<0, 0, 0>
M. 17	<0, 0, 0>	(0, 0, 0)	
			<0, 0, 0>
M. 18	<0, 0, 0>	(0, 0, 0)	
			<0, 0, 0>
M. 19	<0, 0, -1>	(0, 0, 0)	
			<0, 0, 0>
M. 20	<0, 0, -1>	(0, 0, 0)	
			<0, 0, 1>
M. 21	<0, -2, 0>	(0, 0, 1)	
			<0, 0, 0>
M. 22	<0, -2, 0>	(0, 0, 1)	
			<0, 1, -1>
M. 23	<0, -1, -1>	(0, 1, 0)	
			<0, 0, 0>
M. 24	<0, 1, 0>	(0, 1, 0)	

Figure 3.21: Mozart, Variations on "Ah! Vous dirai-je, Maman," Variation 6.

Figure 3.22: Variation 6: graph of non-chord tone contours

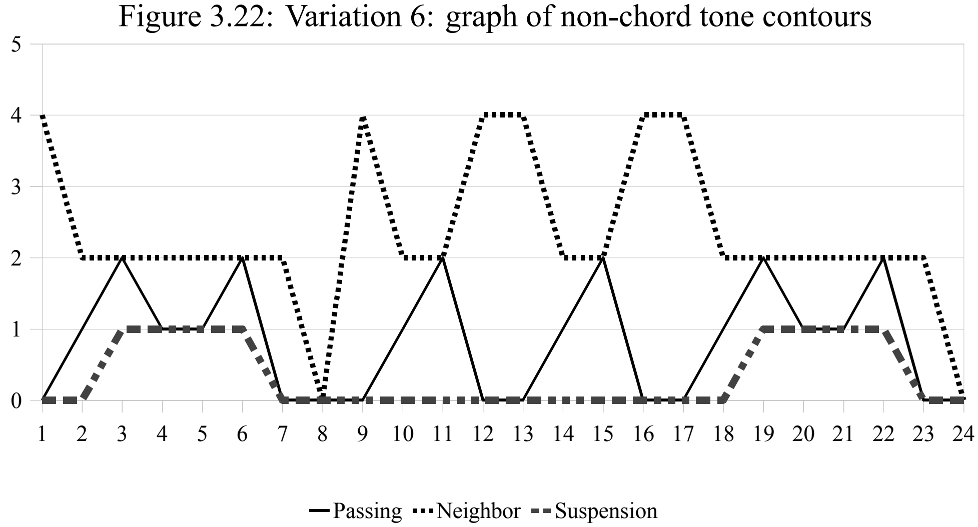


Figure 3.23: Variation 6: table of configurations and intervals.

Measure Numbers	Intervals from Variation 5	Variation 6: Configurations	Intervals Between Measures
M. 1	<0, 4, 0>	(0, 4, 0)	
			<1, -2, 0>
M. 2	<1, 2, 0>	(1, 2, 0)	
			<1, 0, 1>
M. 3	<2, 2, 1>	(2, 2, 1)	
			<-1, 0, 0>
M. 4	<1, 2, 1>	(1, 2, 1)	
			<0, 0, 0>
M. 5	<1, 2, 0>	(1, 2, 1)	
			<1, 0, 0>
M. 6	<2, 2, 0>	(2, 2, 1)	
			<2, 0, -1>
M. 7	<0, 2, -1>	(0, 2, 0)	
			<0, -2, 0>
M. 8	<0, 0, 0>	(0, 0, 0)	
			<0, 4, 0>
M. 9	<0, 4, 0>	(0, 4, 0)	
			<1, -2, 0>
M. 10	<0, 2, 0>	(1, 2, 0)	
			<1, 0, 0>
M. 11	<1, 2, 0>	(2, 2, 0)	
			<-2, 2, 0>
M. 12	<0, 4, 0>	(0, 4, 0)	
			<0, 0, 0>
M. 13	<-1, 4, 0>	(0, 4, 0)	
			<1, -2, 0>
M. 14	<0, 2, 0>	(1, 2, 0)	
			<1, 0, 0>
M. 15	<1, 2, 0>	(2, 2, 0)	
			<-2, 2, 0>
M. 16	<0, 4, 0>	(0, 4, 0)	
			<0, 0, 0>
M. 17	<0, 4, 0>	(0, 4, 0)	
			<1, -2, 0>
M. 18	<1, 2, 0>	(1, 2, 0)	
			<1, 0, 1>
M. 19	<2, 2, 0>	(2, 2, 1)	
			<-1, 0, 0>
M. 20	<1, 2, 0>	(1, 2, 1)	
			<0, 0, 0>
M. 21	<1, 2, -1>	(1, 2, 1)	
			<1, 0, 0>
M. 22	<2, 2, -1>	(2, 2, 1)	
			<2, 0, -1>
M. 23	<0, 1, 0>	(0, 2, 0)	
			<0, -2, 0>
M. 24	<0, -1, 0>	(0, 0, 0)	

Figure 3.24: Mozart, Variations on "Ah! Vous dirai-je, Maman," Variation 7.

Figure 3.25: Variation 7: graph of non-chord tone contours

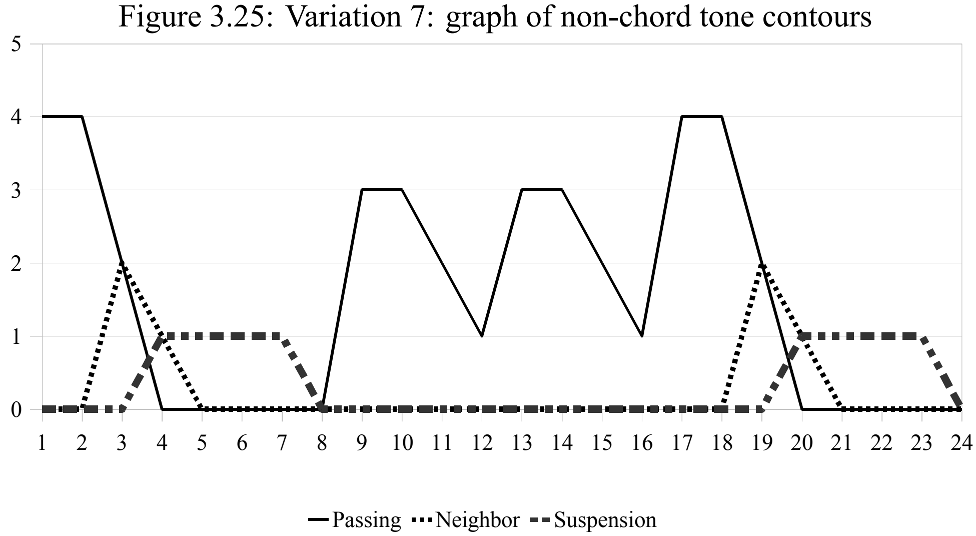


Figure 3.26: Variation 7: table of configurations and intervals.

Measure Numbers	Intervals from Variation 6	Variation 7: Configurations	Intervals Between Measures
M. 1	<4, -4, 0>	(4, 0, 0)	<0, 0, 0>
M. 2	<3, -2, 0>	(4, 0, 0)	<-2, 2, 0>
M. 3	<0, 0, -1>	(2, 2, 0)	<-2, -1, 1>
M. 4	<1, -1, 0>	(0, 1, 1)	<0, -1, 0>
M. 5	<-1, -2, 0>	(0, 0, 1)	<0, 0, 0>
M. 6	<-2, -2, 0>	(0, 0, 1)	<0, 0, 0>
M. 7	<0, -2, 1>	(0, 0, 1)	<0, 0, -1>
M. 8	<0, 0, 0>	(0, 0, 0)	<3, 0, 0>
M. 9	<3, -4, 0>	(3, 0, 0)	<0, 0, 0>
M. 10	<2, -2, 0>	(3, 0, 0)	<-1, 0, 0>
M. 11	<0, -2, 0>	(2, 0, 0)	<-1, 0, 0>
M. 12	<1, -3, 0>	(1, 0, 0)	<2, 0, 0>
M. 13	<3, -3, 0>	(3, 0, 0)	<0, 0, 0>
M. 14	<2, -2, 0>	(3, 0, 0)	<-1, 0, 0>
M. 15	<0, -2, 0>	(2, 0, 0)	<-1, 0, 0>
M. 16	<1, -4, 0>	(1, 0, 0)	<3, 0, 0>
M. 17	<4, -4, 0>	(4, 0, 0)	<0, 0, 0>
M. 18	<3, -2, 0>	(4, 0, 0)	<-2, 2, 0>
M. 19	<0, 0, -1>	(2, 2, 0)	<-2, -1, 1>
M. 20	<1, -1, 0>	(0, 1, 1)	<0, -1, 0>
M. 21	<-1, -2, 0>	(0, 0, 1)	<0, 0, 0>
M. 22	<-2, -2, 0>	(0, 0, 1)	<0, 0, 0>
M. 23	<0, -2, 1>	(0, 0, 1)	<0, 0, -1>
M. 24	<0, 0, 0>	(0, 0, 0)	N/A

Figure 3.27: Mozart, Variations on ”Ah! Vous dirai-je, Maman,” Variation 8.

Figure 3.28: Variation 8: graph of non-chord tone contours

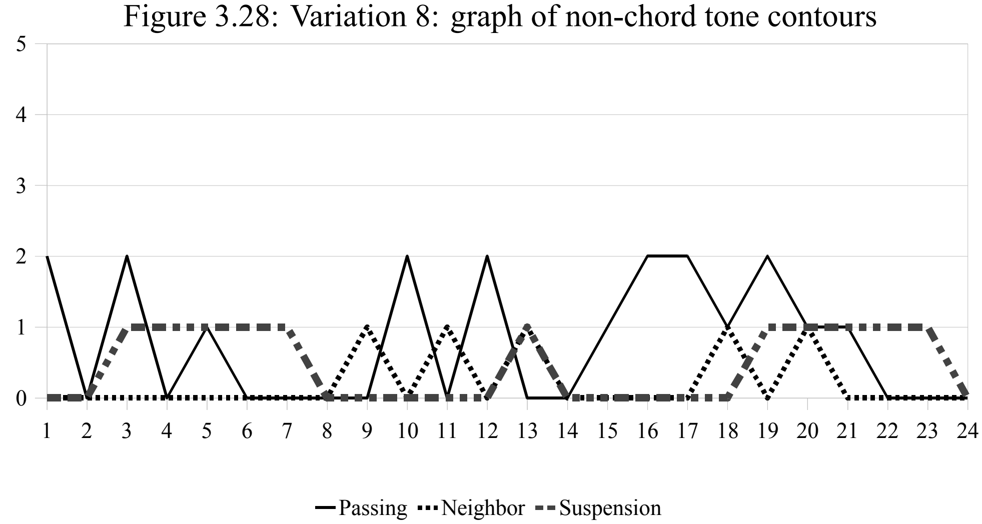


Figure 3.29: Variation 8: table of configurations and intervals.

Measure Numbers	Intervals from Variation 7	Variation 8: Configurations	Intervals Between Measures
M. 1	<-2, 0, 0>	(2, 0, 0)	<-2, 0, 0>
M. 2	<-4, 0, 0>	(0, 0, 0)	<2, 0, 1>
M. 3	<0, -2, 1>	(2, 0, 1)	<-2, 0, 0>
M. 4	<0, -1, 0>	(0, 0, 1)	<1, 0, 0>
M. 5	<1, 0, 0>	(1, 0, 1)	<-1, 0, 0>
M. 6	<0, 0, 0>	(0, 0, 1)	<0, 0, 0>
M. 7	<0, 0, 0>	(0, 0, 1)	<0, 0, -1>
M. 8	<0, 0, 0>	(0, 0, 0)	<0, 1, 0>
M. 9	<-2, 1, 0>	(0, 1, 0)	<2, -1, 0>
M. 10	<-1, 0, 0>	(2, 0, 0)	<-2, 1, 0>
M. 11	<-2, 1, 0>	(0, 1, 0)	<2, -1, 0>
M. 12	<1, 0, 0>	(2, 0, 0)	<-2, 1, 1>
M. 13	<-3, 1, 1>	(0, 1, 1)	<0, -1, -1>
M. 14	<-3, 0, 0>	(0, 0, 0)	<1, 0, 0>
M. 15	<-1, 0, 0>	(1, 0, 0)	<1, 0, 0>
M. 16	<1, 0, 0>	(2, 0, 0)	<0, 0, 0>
M. 17	<-2, 0, 0>	(2, 0, 0)	<-1, 1, 0>
M. 18	<-3, 1, 0>	(1, 1, 0)	<1, -1, 1>
M. 19	<0, -2, 1>	(2, 0, 1)	<-1, 1, 0>
M. 20	<1, 0, 0>	(1, 1, 1)	<0, -1, 0>
M. 21	<1, 0, 0>	(1, 0, 1)	<-1, 0, 0>
M. 22	<0, 0, 0>	(0, 0, 1)	<0, 0, 0>
M. 23	<0, 0, 0>	(0, 0, 1)	<0, 0, -1>
M. 24	<0, 0, 0>	(0, 0, 0)	

Figure 3.30: Mozart, Variations on "Ah! Vous dirai-je, Maman," Variation 9.

Figure 3.31: Variation 9: graph of non-chord tone contours

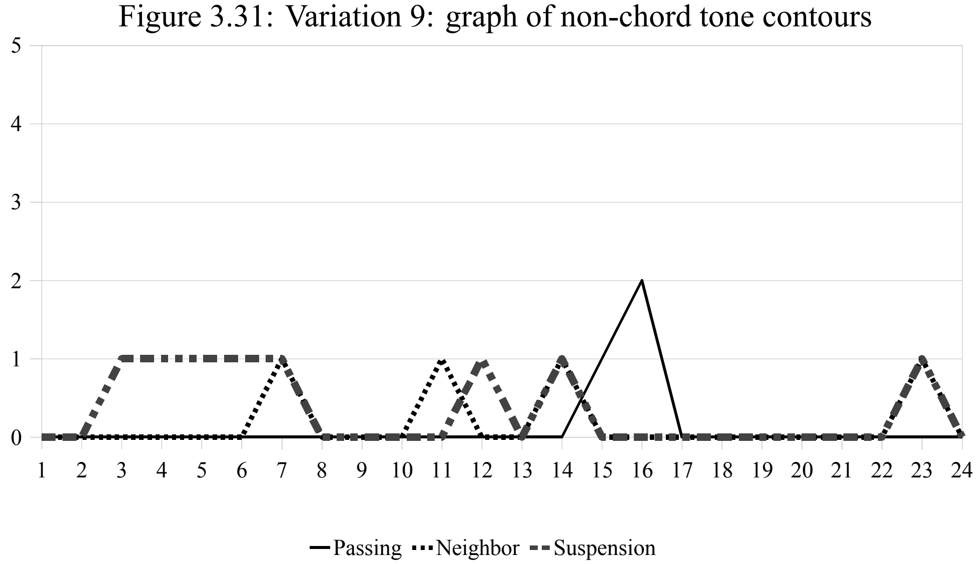


Figure 3.32: Variation 9: table of configurations and intervals.

Measure Numbers	Intervals from Variation 8	Variation 9: Configurations	Intervals Between Measures
M. 1	<-2, 0, 0>	(0, 0, 0)	<0, 0, 0>
M. 2	<0, 0, 0>	(0, 0, 0)	<0, 0, 1>
M. 3	<-2, 0, 0>	(0, 0, 1)	<0, 0, 0>
M. 4	<0, 0, 0>	(0, 0, 1)	<0, 0, 0>
M. 5	<-1, 0, 0>	(0, 0, 1)	<0, 0, 0>
M. 6	<0, 0, 0>	(0, 0, 1)	<0, 1, 0>
M. 7	<0, 1, 0>	(0, 1, 1)	<0, -1, -1>
M. 8	<0, 0, 0>	(0, 0, 0)	<0, 0, 0>
M. 9	<0, 1, 0>	(0, 0, 0)	<0, 0, 0>
M. 10	<-2, 0, 0>	(0, 0, 0)	<0, 1, 0>
M. 11	<0, 0, 0>	(0, 1, 0)	<0, -1, 1>
M. 12	<-1, 0, 0>	(0, 0, 1)	<0, 0, -1>
M. 13	<0, -1, -1>	(0, 0, 0)	<0, 1, 1>
M. 14	<0, 1, 1>	(0, 1, 1)	<1, -1, -1>
M. 15	<0, 0, 0>	(1, 0, 0)	<1, 0, 0>
M. 16	<0, 0, 0>	(2, 0, 0)	<-2, 0, 0>
M. 17	<-2, 0, 0>	(0, 0, 0)	<0, 0, 0>
M. 18	<1, -1, 0>	(0, 0, 0)	<0, 0, 0>
M. 19	<-2, 0, -1>	(0, 0, 0)	<0, 0, 0>
M. 20	<0, -2, -1>	(0, 0, 0)	<0, 0, 0>
M. 21	<-1, 0, -1>	(0, 0, 0)	<0, 0, 0>
M. 22	<0, 0, -1>	(0, 0, 0)	<0, 1, 1>
M. 23	<0, 1, 0>	(0, 1, 1)	<0, -1, -1>
M. 24	<0, 0, 0>	(0, 0, 0)	

Figure 3.33: Mozart, Variations on ”Ah! Vous dirai-je, Maman,” Variation 10.

Figure 3.34: Variation 10: graph of non-chord tone contours

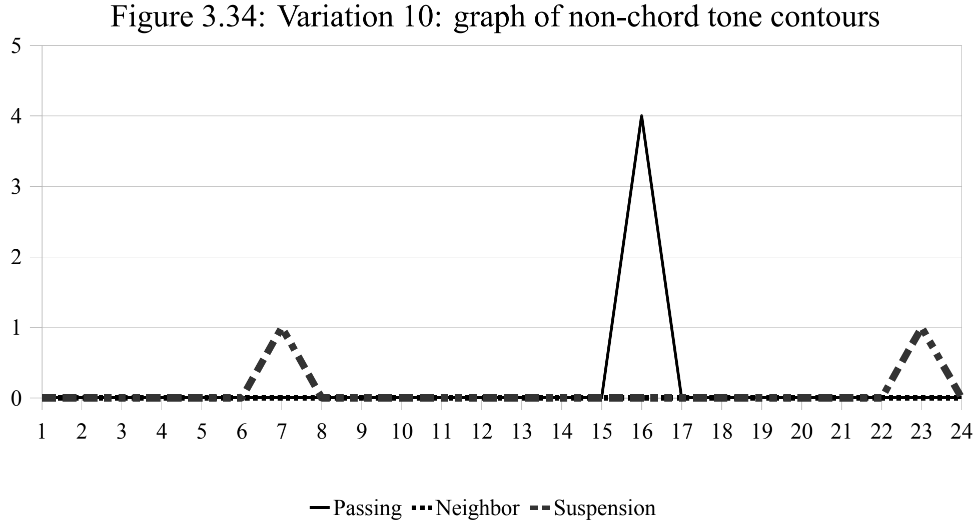


Figure 3.35: Variation 10: table of configurations and intervals.

Measure Numbers	Intervals from Variation 9	Variation 10: Configurations	Intervals Between Measures
M. 1	<0, 0, 0>	(0, 0, 0)	<0, 0, 0>
M. 2	<0, 0, 0>	(0, 0, 0)	<0, 0, 0>
M. 3	<0, 0, -1>	(0, 0, 0)	<0, 0, 0>
M. 4	<0, 0, -1>	(0, 0, 0)	<0, 0, 0>
M. 5	<0, 0, -1>	(0, 0, 0)	<0, 0, 0>
M. 6	<0, 0, -1>	(0, 0, 0)	<0, 0, 1>
M. 7	<0, -1, 0>	(0, 0, 1)	<0, 0, -1>
M. 8	<0, 0, 0>	(0, 0, 0)	<0, 0, 0>
M. 9	<0, 0, 0>	(0, 0, 0)	<0, 0, 0>
M. 10	<0, 0, 0>	(0, 0, 0)	<0, 0, 0>
M. 11	<0, -1, 0>	(0, 0, 0)	<0, 0, 0>
M. 12	<0, 0, -1>	(0, 0, 0)	<0, 0, 0>
M. 13	<0, 0, 0>	(0, 0, 0)	<0, 0, 0>
M. 14	<0, -1, -1>	(0, 0, 0)	<0, 0, 0>
M. 15	<-1, 0, 0>	(0, 0, 0)	<4, 0, 0>
M. 16	<2, 0, 0>	(4, 0, 0)	<-4, 0, 0>
M. 17	<0, 0, 0>	(0, 0, 0)	<0, 0, 0>
M. 18	<0, 0, 0>	(0, 0, 0)	<0, 0, 0>
M. 19	<0, 0, 0>	(0, 0, 0)	<0, 0, 0>
M. 20	<0, 0, 0>	(0, 0, 0)	<0, 0, 0>
M. 21	<0, 0, 0>	(0, 0, 0)	<0, 0, 0>
M. 22	<0, 0, 0>	(0, 0, 0)	<0, 0, 1>
M. 23	<0, -1, 0>	(0, 0, 1)	<0, 0, -1>
M. 24	<0, 0, 0>	(0, 0, 0)	

Figure 3.36: Mozart, Variations on "Ah! Vous dirai-je, Maman," Variation 11.

Figure 3.37: Variation 11: graph of non-chord tone contours

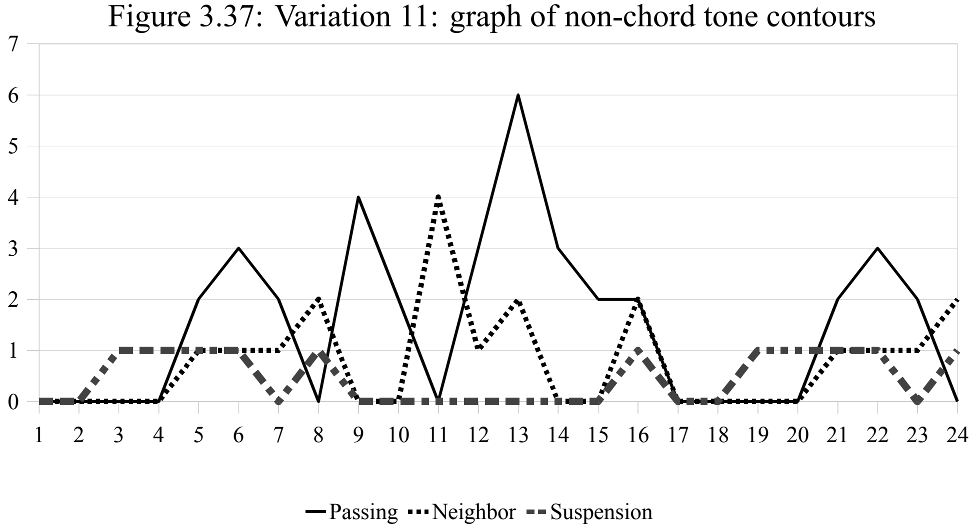


Figure 3.38: Variation 11: table of configurations and intervals.

Measure Numbers	Intervals from Variation 10	Variation 11: Configurations	Intervals Between Measures
M. 1	<0, 0, 0>	(0, 0, 0)	<0, 0, 0>
M. 2	<0, 0, 0>	(0, 0, 0)	<0, 0, 1>
M. 3	<0, 0, 1>	(0, 0, 1)	<0, 0, 0>
M. 4	<0, 0, 1>	(0, 0, 1)	<2, 1, 0>
M. 5	<2, 1, 1>	(2, 1, 1)	<1, 0, 0>
M. 6	<3, 1, 1>	(3, 1, 1)	<-1, 0, -1>
M. 7	<2, 1, -1>	(2, 1, 0)	<-2, 1, 1>
M. 8	<0, 2, 1>	(0, 2, 1)	<4, -2, -1>
M. 9	<4, 0, 0>	(4, 0, 0)	<-2, 0, 0>
M. 10	<2, 0, 0>	(2, 0, 0)	<-2, 4, 0>
M. 11	<0, 4, 0>	(0, 4, 0)	<3, -3, 0>
M. 12	<3, 1, 0>	(3, 1, 0)	<3, 1, 0>
M. 13	<6, 2, 0>	(6, 2, 0)	<-3, -2, 0>
M. 14	<3, 0, 0>	(3, 0, 0)	<-1, 0, 0>
M. 15	<2, 0, 0>	(2, 0, 0)	<0, 2, 1>
M. 16	<-2, 2, 1>	(2, 2, 1)	<-2, -2, -1>
M. 17	<0, 0, 0>	(0, 0, 0)	<0, 0, 0>
M. 18	<0, 0, 0>	(0, 0, 0)	<0, 0, 1>
M. 19	<0, 0, 1>	(0, 0, 1)	<0, 0, 0>
M. 20	<0, 0, 1>	(0, 0, 1)	<2, 1, 0>
M. 21	<2, 1, 1>	(2, 1, 1)	<1, 0, 0>
M. 22	<3, 1, 1>	(3, 1, 1)	<-1, 0, -1>
M. 23	<2, 1, -1>	(2, 1, 0)	<-2, 1, 1>
M. 24	<0, 2, 1>	(0, 2, 1)	

Figure 3.39: Mozart, Variations on "Ah! Vous dirai-je, Maman," Variation 12.

Figure 3.40: Variation 12: graph of non-chord tone contours

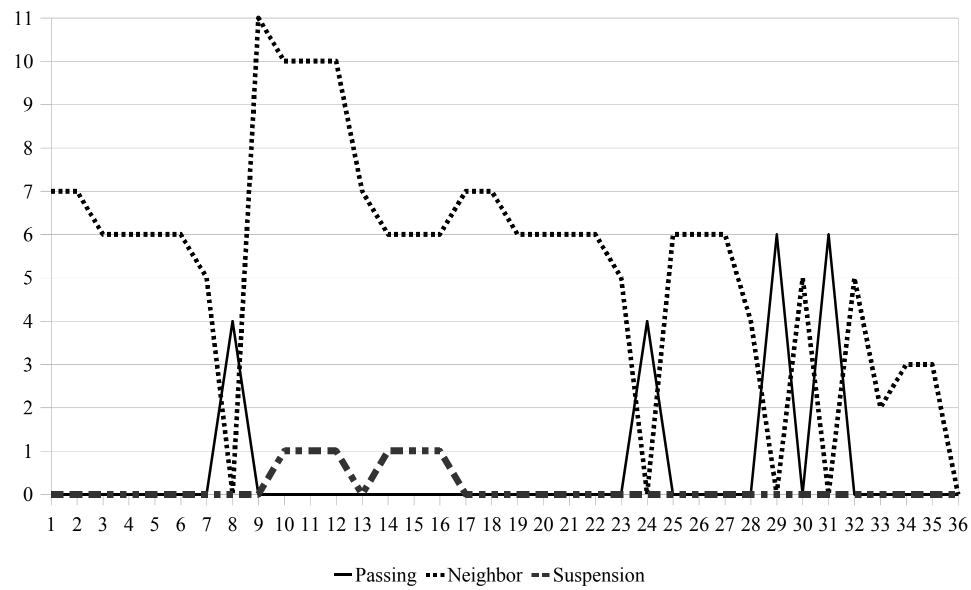


Figure 3.41: Variation 12: table of configurations and intervals.

Measure Numbers	Intervals from Variation 11	Variation 12: Configurations	Intervals Between Measures
M. 1	<0, 7, 0>	(0, 7, 0)	<0, 0, 0>
M. 2	<0, 7, 0>	(0, 7, 0)	<0, -1, 0>
M. 3	<0, 6, -1>	(0, 6, 0)	<0, 0, 0>
M. 4	<0, 6, -1>	(0, 6, 0)	<0, 0, 0>
M. 5	<-2, 5, -1>	(0, 6, 0)	<0, 0, 0>
M. 6	<-3, 5, -1>	(0, 6, 0)	<0, -1, 0>
M. 7	<-2, 4, 0>	(0, 5, 0)	<4, -5, 0>
M. 8	<4, 0, 0>	(4, 0, 0)	<-4, 11, 0>
M. 9	<-4, 11, 0>	(0, 11, 0)	<0, -1, 1>
M. 10	<-2, 10, 1>	(0, 10, 1)	<0, 0, 0>
M. 11	<0, 10, 1>	(0, 10, 1)	<0, 0, 0>
M. 12	<-3, 9, 1>	(0, 10, 1)	<0, -3, -1>
M. 13	<0, 7, 0>	(0, 7, 0)	<0, -1, 1>
M. 14	<-3, 6, 0>	(0, 6, 1)	<0, 0, 0>
M. 15	<-2, 6, 1>	(0, 6, 1)	<0, 0, 0>
M. 16	<-2, 4, 0>	(0, 6, 1)	<0, 1, -1>
M. 17	<0, 7, 0>	(0, 7, 0)	<0, 0, 0>
M. 18	<0, 7, 0>	(0, 7, 0)	<0, -1, 0>
M. 19	<0, 6, -1>	(0, 6, 0)	<0, 0, 0>
M. 20	<0, 6, -1>	(0, 6, 0)	<0, 0, 0>
M. 21	<-2, 5, 1>	(0, 6, 0)	<0, 0, 0>
M. 22	<-3, 5, -1>	(0, 6, 0)	<0, -1, 0>
M. 23	<-2, 4, 0>	(0, 5, 0)	<4, -5, 0>
M. 24	<4, 0, 0>	(4, 0, 0)	<0, 1, 0>
M. 24b	N/A	(0, 6, 0)	<0, 0, 0>
M. 25	N/A	(0, 6, 0)	<0, 0, 0>
M. 26	N/A	(0, 6, 0)	<0, -2, 0>
M. 27	N/A	(0, 4, 0)	<6, -4, 0>
M. 28	N/A	(6, 0, 0)	<-6, 5, 0>
M. 29	N/A	(0, 5, 0)	<6, -5, 0>
M. 30	N/A	(6, 0, 0)	<-6, 5, 0>
M. 31	N/A	(0, 5, 0)	<0, -3, 0>
M. 32	N/A	(0, 2, 0)	<0, 1, 0>
M. 33	N/A	(0, 3, 0)	<0, 0, 0>
M. 34	N/A	(0, 3, 0)	<0, -3, 0>
M. 35	N/A	(0, 0, 0)	

Figure 3.42: Interval vectors.

Interval	Theme	Var. 1	Var. 2	Var. 3	Var. 4	Var. 5	Var. 6	Var. 7	Var. 8	Var. 9	Var. 10	Var. 11	Var. 12
<0, 0, 0>	17	8	5	5	10	15	4	9	3	11	17	4	13
<0, 1, -1>		3	1	2		1				1			1
<1, -1, 0>									2				
<-1, 0, 1>				2	2								
<-2, 1, 1>									1			2	
<2, -2, 0>		2					2	2					
<3, -3, 0												1	
<0, 1, 0>	2		2	3	1			2	2	2			7
<0, 0, 1>			4	5	2	3		2	2	2	4	2	2
<1, 0, 0>		2	2	2	2	4	6	4	5	1		3	
<-1, 2, 0>							4						
<0, 2, -1>			4										
<-1, 1, 1>		1								1			
<1, -1, 1>									1				
<2, -1, 0>									3				
<2, 0, -1>							2						
<4, -2, -1>												1	
<-4, 5, 0>													2
<6, -5, 0>													3
<0, 2, 0>			1		2		2						
<0, 1, 1>									1	4			
<1, 0, 1>					1		2						
<0, 0, 2>				2									
<2, 0, 0>									2	1		1	
<0, 3, -1>			1	1									
<2, 1, -1>								2					
<-2, 4, 0>												1	
<0, 3, 0>	2		1	1									2
<0, 2, 1>		1			2							1	
<1, 1, 1>					2								
<2, 1, 0>					1							2	
<2, 0, 1>									1				
<3, 0, 0>								2					
<2, -2, 1>		2											
<0, 4, 0>	2	1					1						
<0, 3, 1>		1											1
<3, 1, 0>												1	
<1, 2, 1>			2										
<3, 0, 1>		2											
<4, 0, 0>											2	2	
<2, 2, 1>												1	
<3, 2, 0>												1	
<-4, 11, 0>													1

Figures for Chapter 4

Figure 4.1: Non-chord tone configurations.

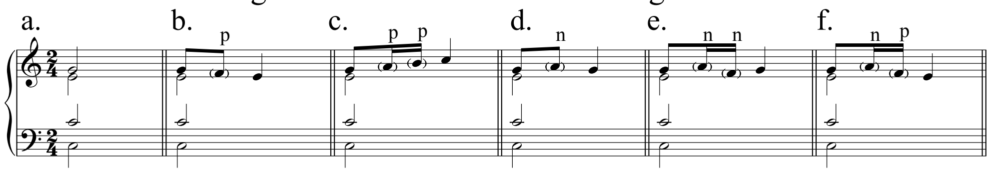


Figure 4.2: Non-chord tone analysis of Bach, Invention No. 9, mm. 1-4.

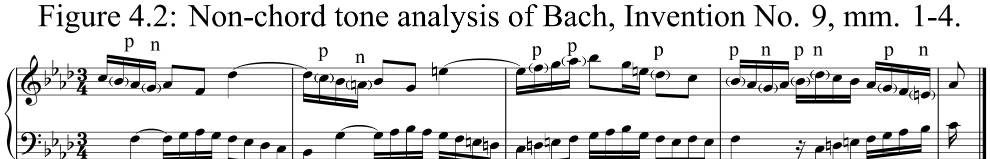
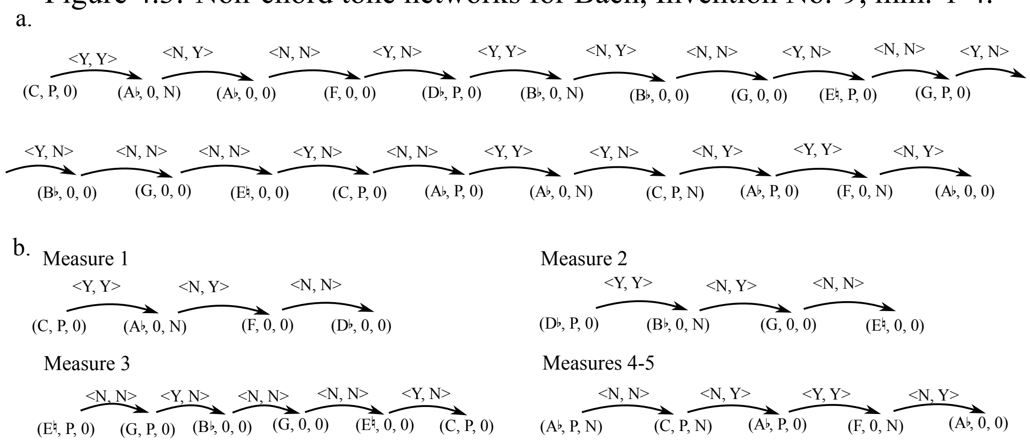


Figure 4.3: Non-chord tone networks for Bach, Invention No. 9, mm. 1-4.



For ease of identification, the chord tones to which non-chord tone configurations are attached have been included as the first element in each configuration.

Figure 4.4: Direct product networks for Bach, Invention No. 9, mm. 1-4.

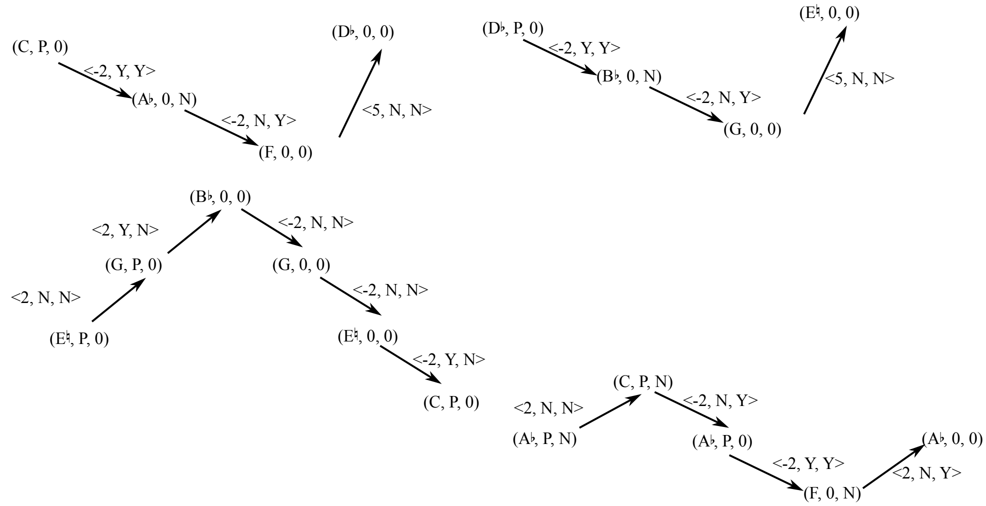


Figure 4.5: Motives and NCT intervals in Brahms, String Quartet No. 2

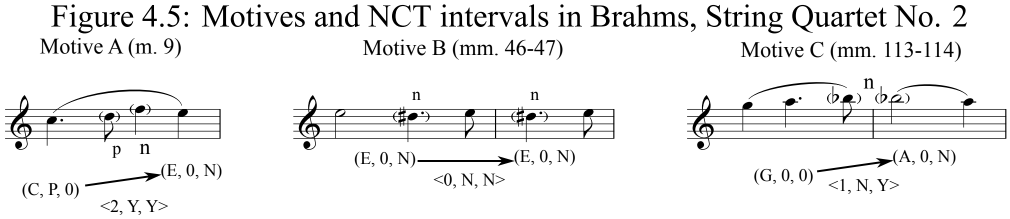


Figure 4.6: Beethoven, Piano Sonata Op. 14, no. 1, movt. 3, mm. 47-50.

Figure 4.7: Permutation networks.

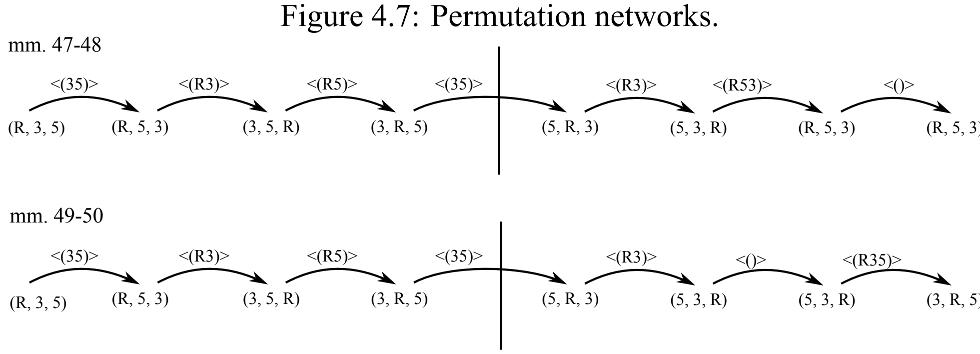


Figure 4.8: Cayley table for the symmetric group on three letters, R, 3, 5.

	()	(R35)	(R53)	(R3)	(R5)	(35)
()	()	(R35)	(R53)	(R3)	(R5)	(35)
(R53)	(R53)	()	(R35)	(R5)	(35)	(R3)
(R35)	(R35)	(R53)	()	(35)	(R3)	(R5)
(R3)	(R3)	(R5)	(35)	()	(R35)	(R53)
(R5)	(R5)	(35)	(R3)	(R53)	()	(R35)
(35)	(35)	(R3)	(R5)	(R35)	(R53)	()

Figure 4.9: The Interval Alteration Function represented as a table.

Input	Output
0	0
1	3
2	2
3	5
4	2
5	5
6	4

Figure 4.10: Musical application of ALT

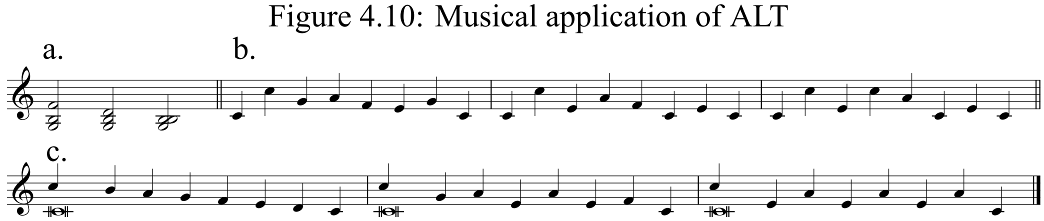


Figure 4.11: Network representation of ALT.

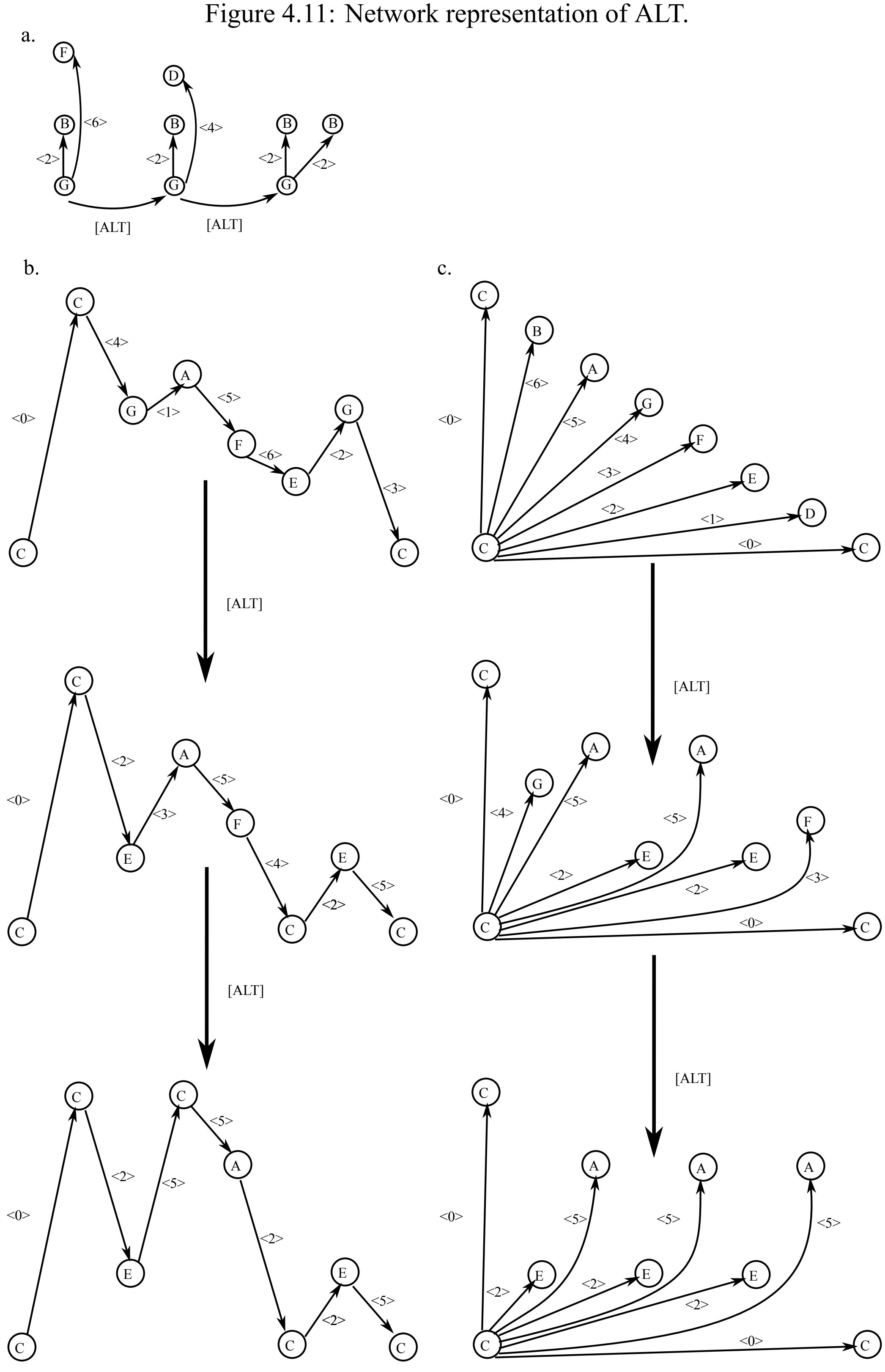
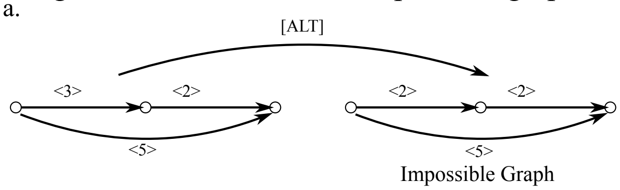


Figure 4.12: Possible and impossible graphs.

a.



b.

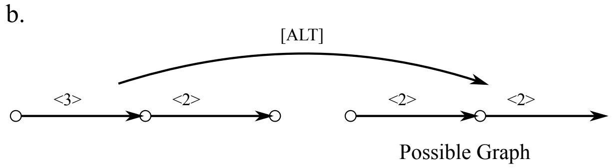


Figure 4.13: Network representation of REDUCE.

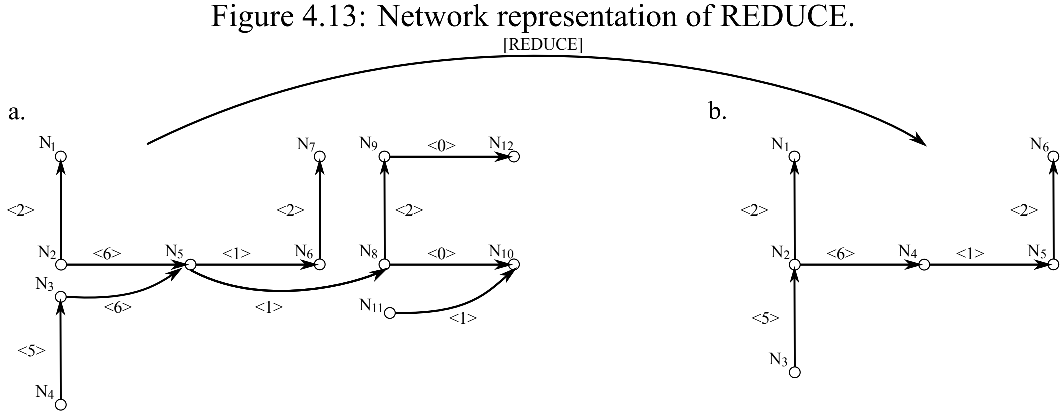


Figure 4.14: Musical application of REDUCE.

a.

b.

c.

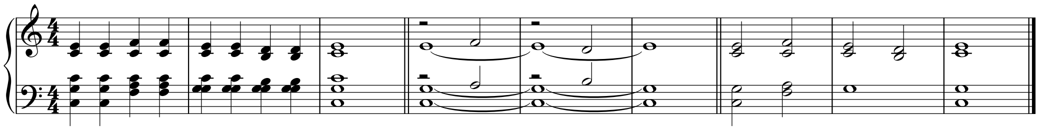


Figure 4.15: Effects of REDUCE on a graph without tree structure.

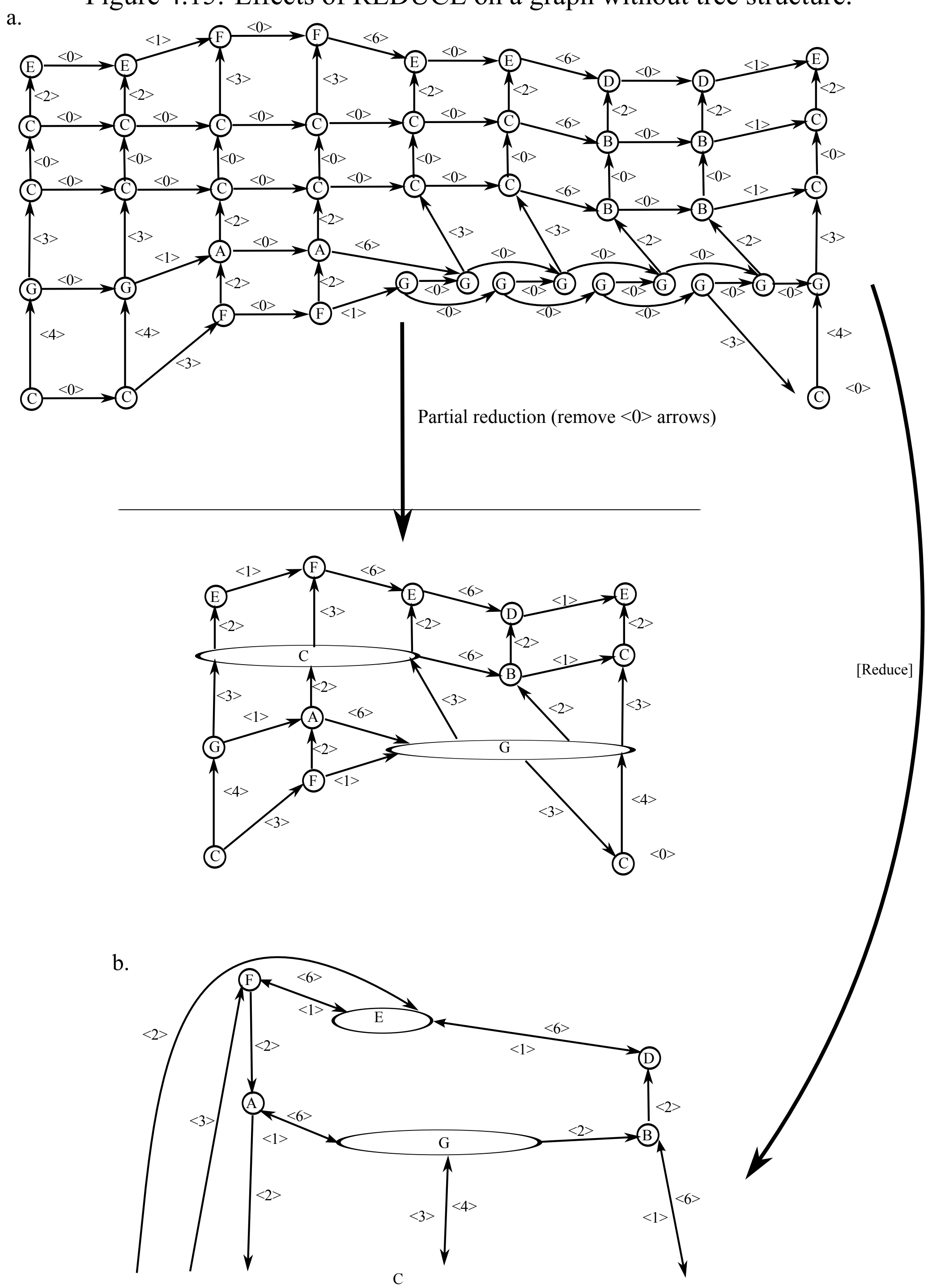
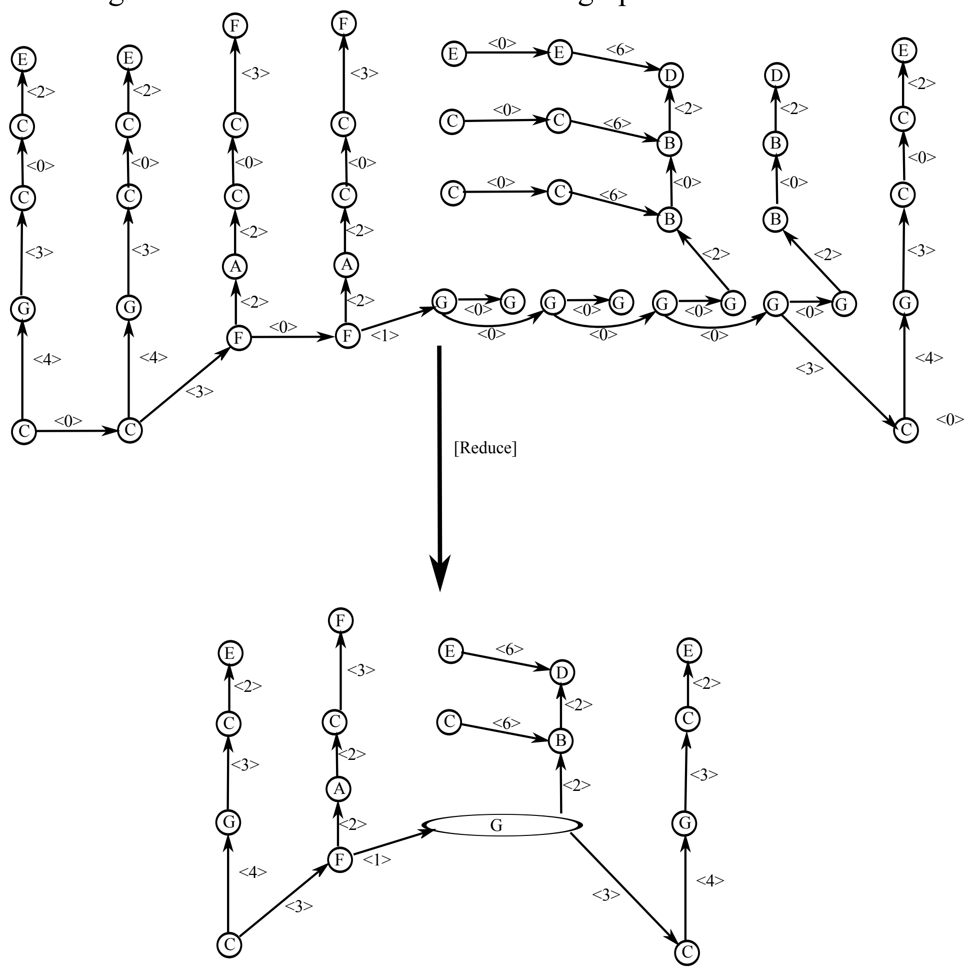


Figure 4.16: Effects of REDUCE on a graph with tree structure.



Figures for Chapter 5

Figure 5.1: Brahms, String Quartet No. 2, movt. 1, mm. 1-25.

Figure 5.2: Diatonic network representations of the Motto Subject.

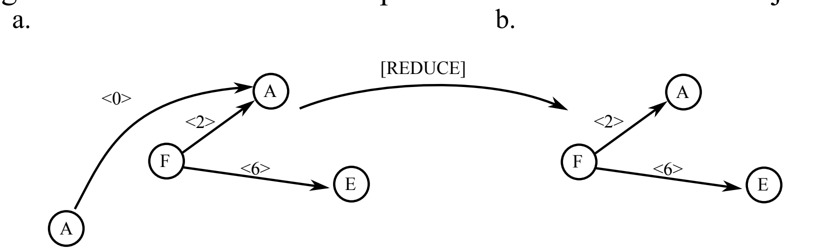


Figure 5.3: Diatonic network representations of the Triad Motive.

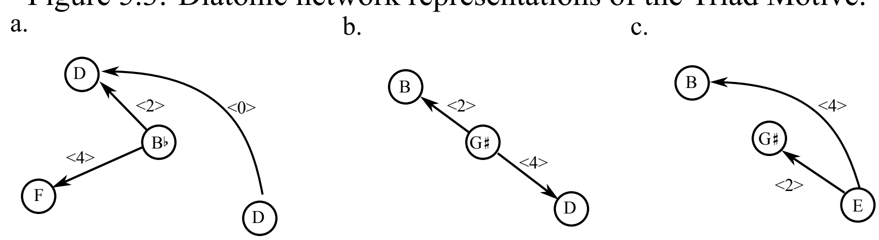


Figure 5.4: Diatonic network representations of motive A.

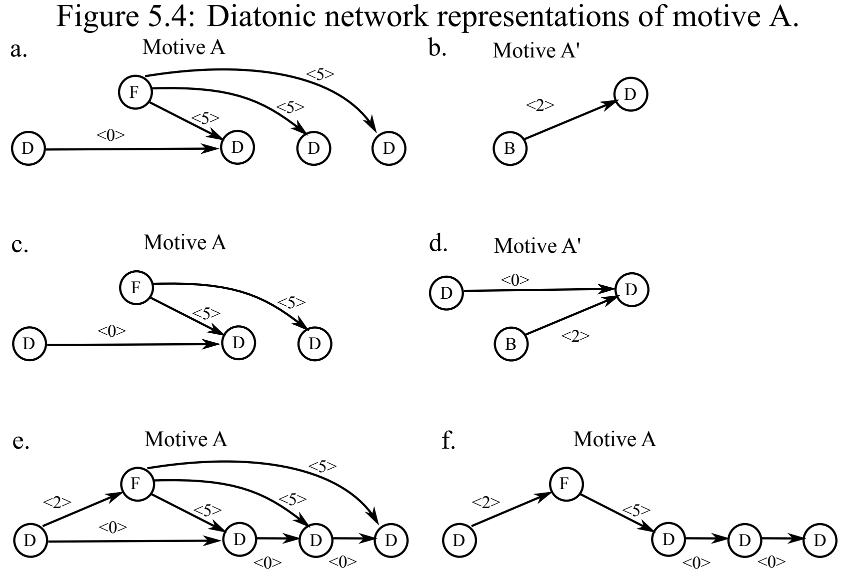
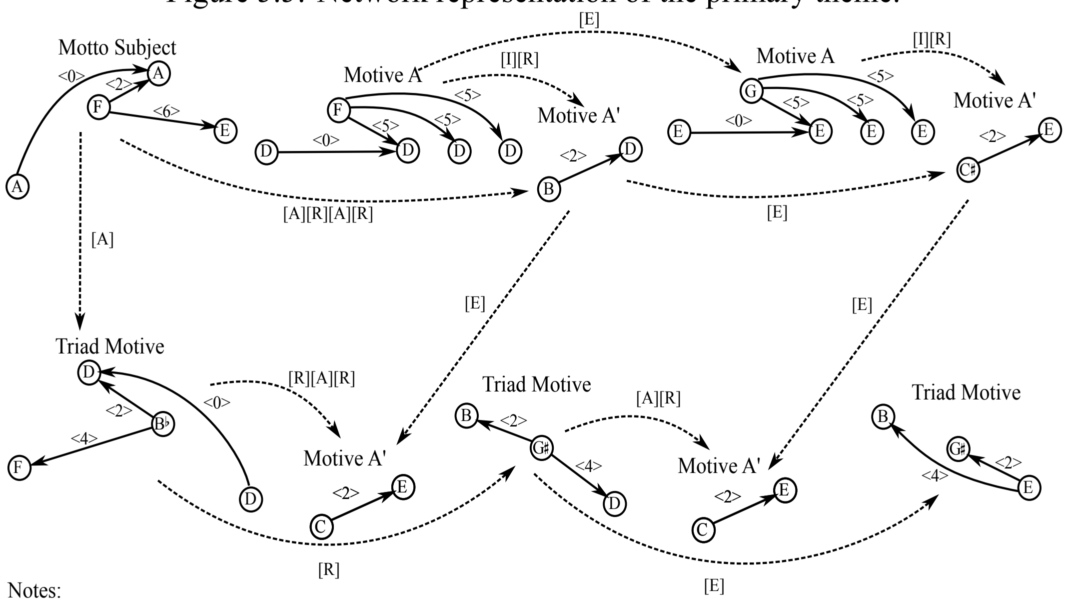


Figure 5.5: Network representation of the primary theme.



Notes:
[E] = Identity
[I] = Inversion
[A] = Interval Alteration Function
[R] = Graph Reduction Function
Square brackets and dashed arrows indicate transformations of graphs,
while pointed brackets and solid arrows indicate diatonic transpositions mod 7.
Combined transformations should be read using left-to-right orthography,
although function composition is the understood composition.

Figure 5.6: Network representations of motive A.

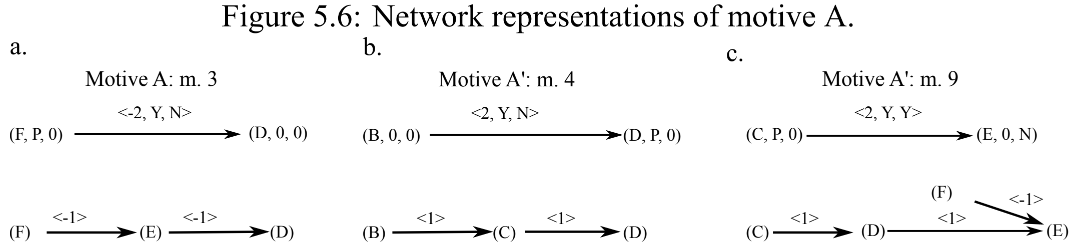
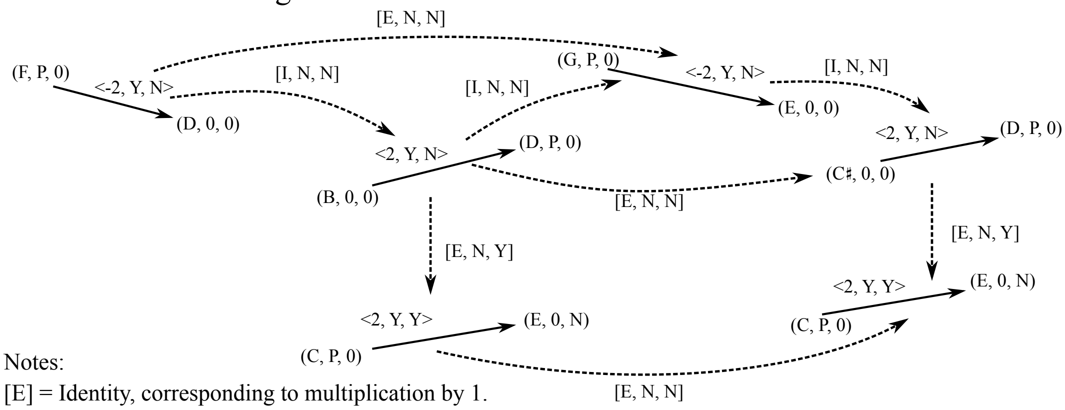


Figure 5.7: The transformation of motive A.



Notes:

[E] = Identity, corresponding to multiplication by 1.

[I] = Inversion, corresponding to multiplication by -1.

Square brackets and dashed arrows occur between subnetworks and indicate automorphisms of direct product intervals, while pointed brackets and solid arrows indicate direct product intervals

Figure 5.8: Permutation network for the Triad Motive.

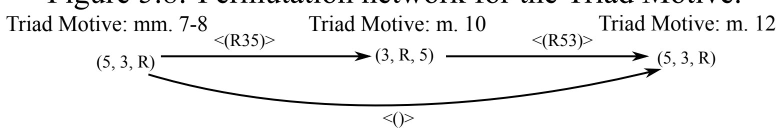
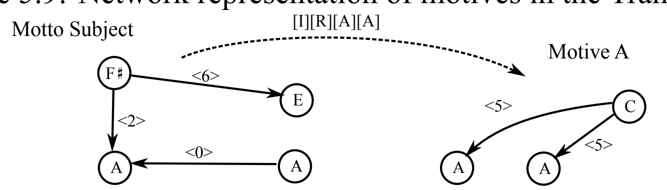


Figure 5.9: Network representation of motives in the Transition.



Notes:

[E] = Identity

[I] = Inversion

[A] = Interval Alteration Function

[R] = Graph Reduction Function

Square brackets and dashed arrows indicate transformations of graphs, while pointed brackets and solid arrows indicate diatonic transpositions mod 7.

Combined transformations should be read using left-to-right orthography, although function composition is the understood composition.

Figure 5.10: The Triad Motive in the transition, mm. 30-31.

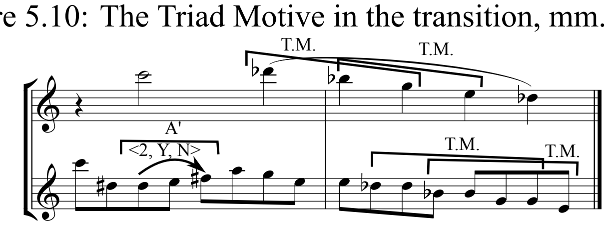
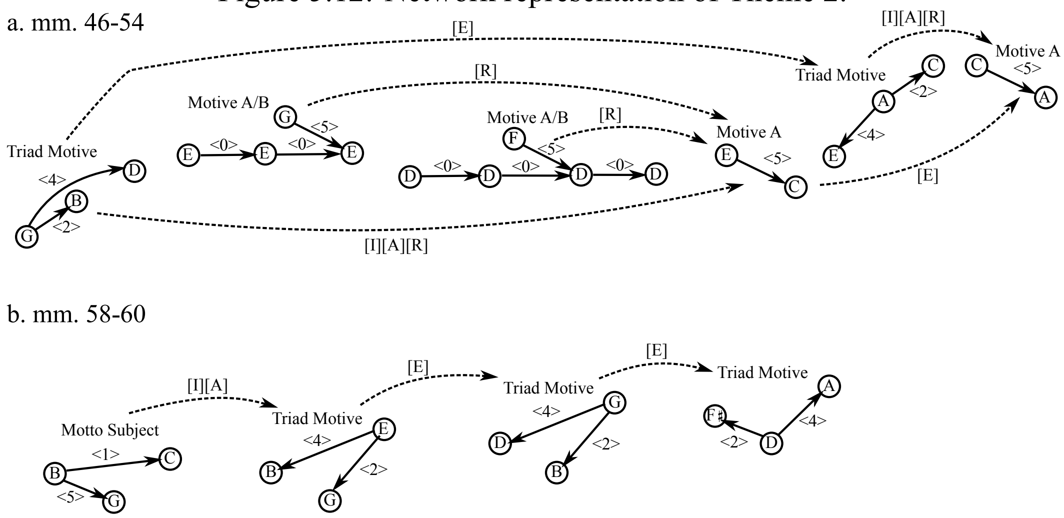


Figure 5.11: Brahms, String Quartet No. 2, transition to Theme 2, mm. 38-66.

Figure 5.12: Network representation of Theme 2.



Notes:
[E] = Identity
[I] = Inversion
[A] = Interval Alteration Function
[R] = Graph Reduction Function
Square brackets and dashed arrows indicate transformations of graphs,
while pointed brackets and solid arrows indicate diatonic transpositions mod 7.
Combined transformations should be read using left-to-right orthography,
although function composition is the understood composition.

Figure 5.13: Transformation of the Triad Motive at Theme 2.

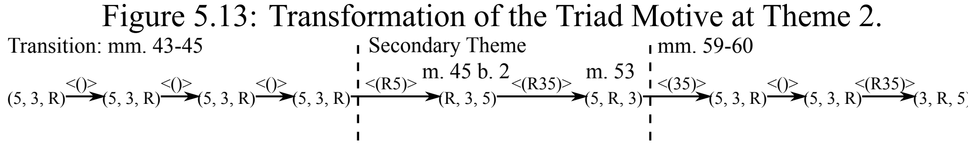


Figure 5.14: Voice leading and motive A, mm. 54-59.

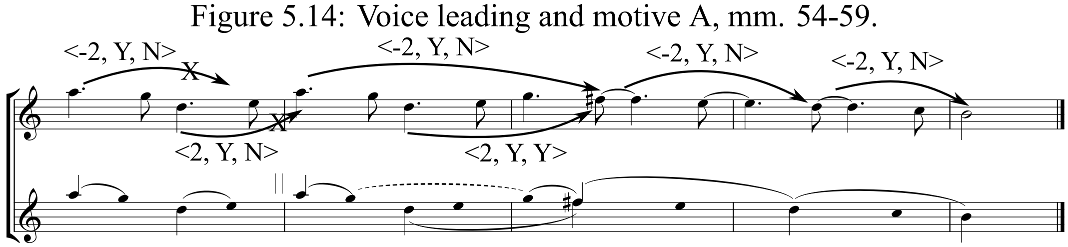
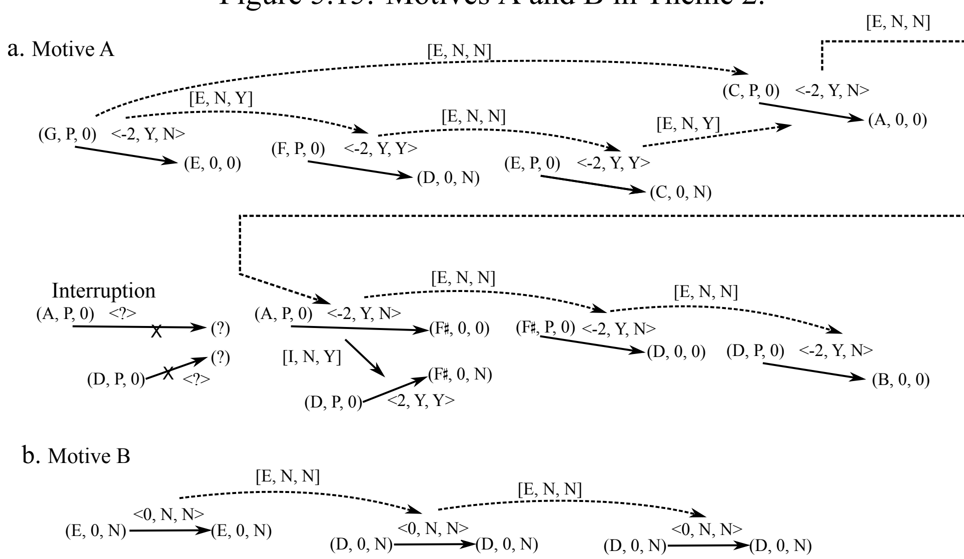


Figure 5.15: Motives A and B in Theme 2.



Notes:
[E] = Identity, corresponding to multiplication by 1.
[I] = Inversion, corresponding to multiplication by -1.

Figure 5.16: Brahms, String Quartet No. 2, mm. 81-104.

Figure 5.17: Transformation of the Triad Motive, mm. 81-83.

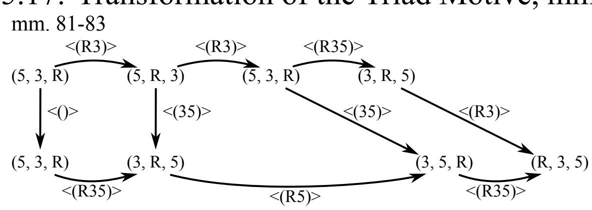


Figure 5.18: Transformation of the Triad Motive, mm. 94-95.

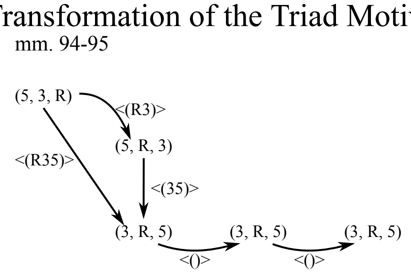


Figure 5.19: Brahms, String Quartet No. 2, Codetta, mm. 104-115.

Figure 5.20: Ambiguity in direct product intervals.

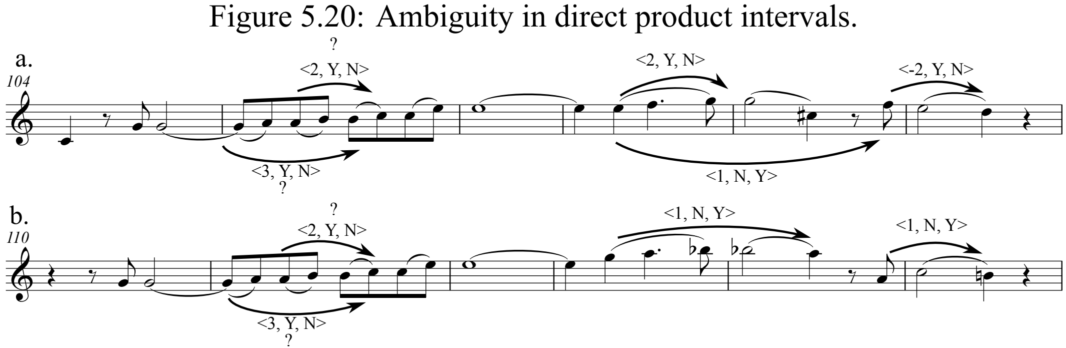
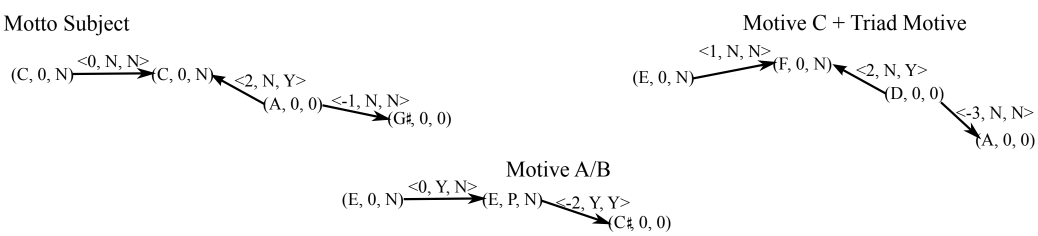


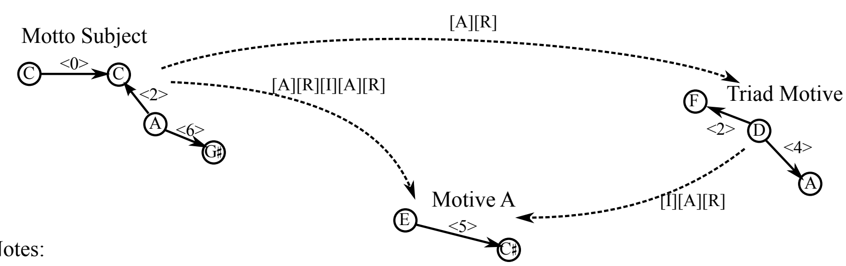
Figure 5.21: Brahms, String Quartet No. 2, Retransition, mm. 177-184.

Figure 5.22: Network representation of the Retransition, mm. 177-184.

a.



b.



Notes:

[E] = Identity

[I] = Inversion

[A] = Interval Alteration Function

[R] = Graph Reduction Function

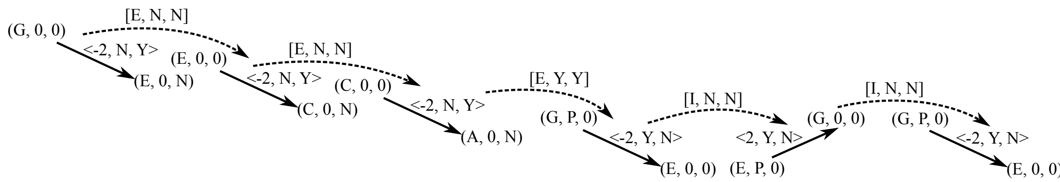
Square brackets and dashed arrows indicate transformations of graphs, while pointed brackets and solid arrows indicate diatonic transpositions mod 7.

Combined transformations should be read using left-to-right orthography, although function composition is the understood composition.

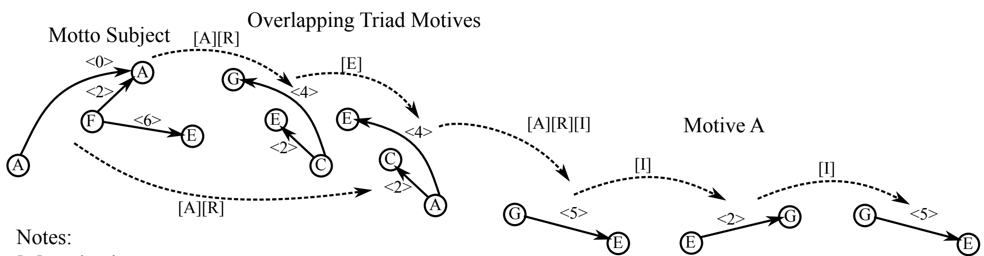
Figure 5.23: Brahms, String Quartet No. 2, Coda, mm. 304-335.

Figure 5.24: Networks in the Coda.

a. Motive A in mm. 327-332



b. Motives in mm. 325-332



Notes:

[E] = Identity

[I] = Inversion

[A] = Interval Alteration Function

[R] = Graph Reduction Function

Square brackets and dashed arrows indicate transformations of graphs such as automorphisms, while pointed brackets and solid arrows indicate transformations within graphs.

Combined transformations should be read using left-to-right orthography, although function composition is the understood composition.

Figure 5.25: Brahms, String Quartet No. 2, movt. 2, mm. 1-6.

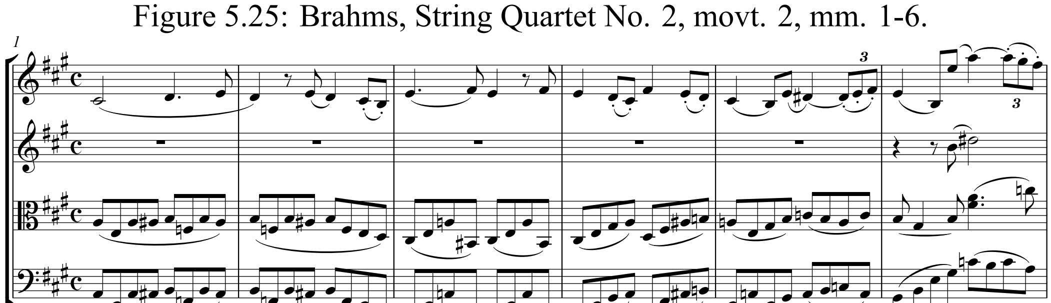


Figure 5.26: Networks for motive A in movt. 2, mm. 1-6.

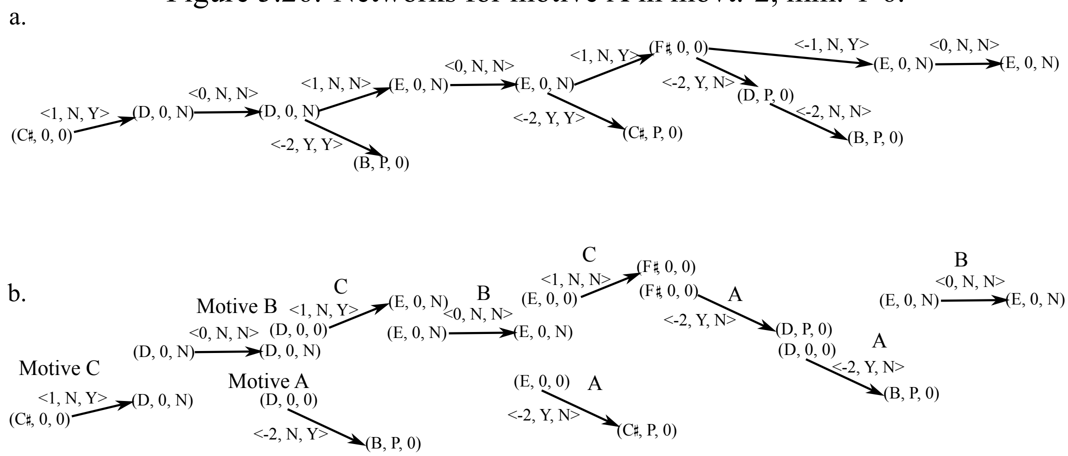


Figure 5.27: Brahms, String Quartet No. 2, movt. 2, mm. 115-124.

Figure 5.28: Brahms, String Quartet No. 2, movt. 3, mm. 1-7.

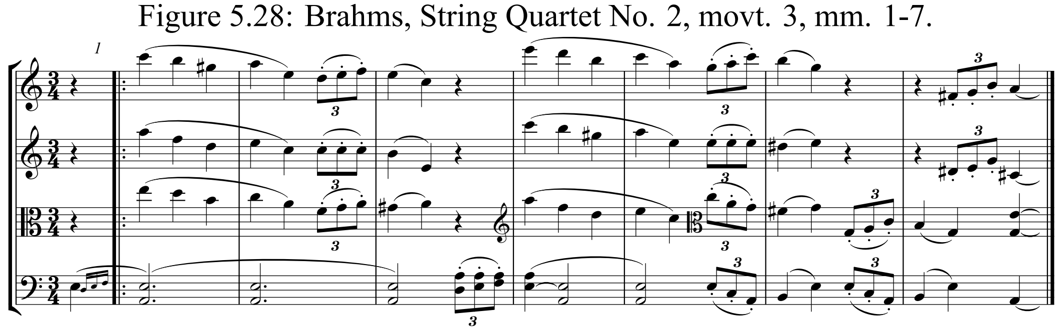


Figure 5.29: Networks in movt. 3, mm. 1-7.

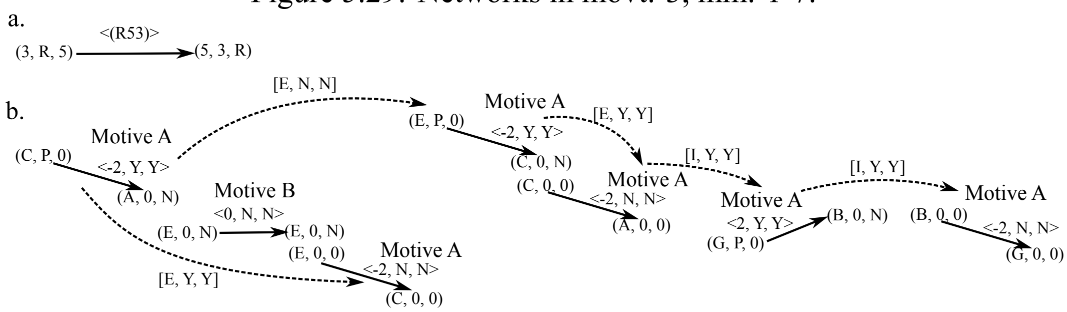


Figure 5.30: Brahms, String Quartet No. 2, movt. 4, mm. 1-17.

Figure 5.31: Networks in movt. 4, mm. 1-10.

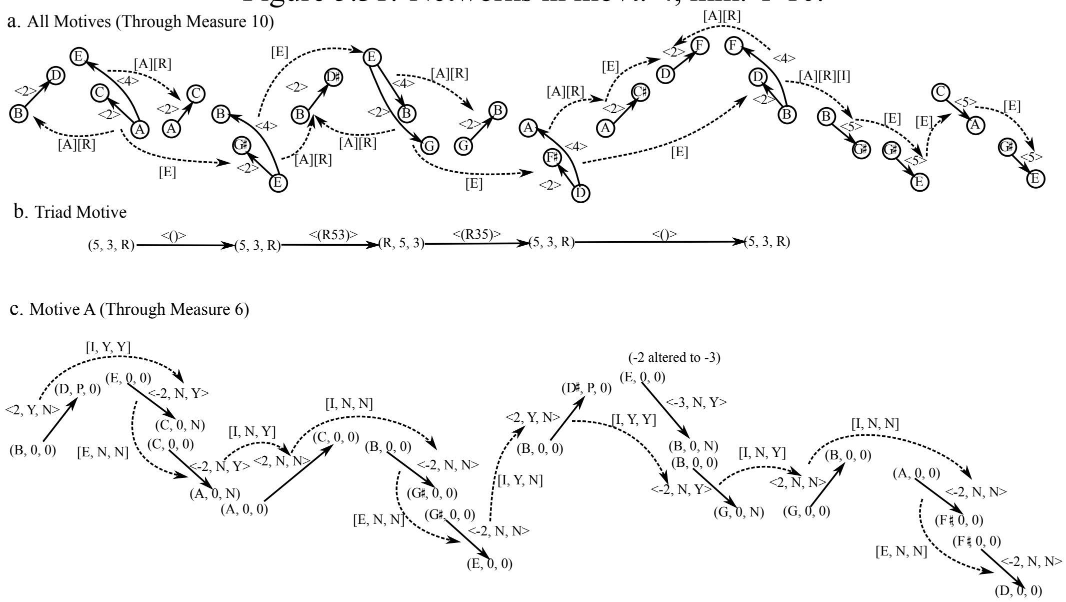


Figure 5.32: Brahms, String Quartet No. 2, movt. 4, mm. 350-359.