University of Alberta

On Application of a Scoring Procedure for Ordered Categorical Data with WCWL

by

Eunha Yang ©

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Abstract

In many surveys, information is measured in two ways: most items are measured by using an ordinal scale, and the overall characteristics are measured on a numerical scale, for example, 1 to 100. Often, this overall measure is implicitly correlated with other measurements in the study. This thesis uses the ordinal regression approach and the optimal scaling approach to evaluate the method used to arrive at this implicit relationship. This thesis also proposes a new method using constraint regression to estimate the relationship. The methodology proposed in this thesis can be used to estimate this implicit relationship and to assign "optimal scores" to ordinal items that are correlated with the overall measures.

In order to apply this methodology, the data from the General Surgery panel of the Western Canada Waiting List project were used.

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Chapter 1

Introduction

In many statistical studies, the responses are recorded on an ordinal scale, and ordinal scales are common in the social, medical and public health sciences (Agresti, 1984; Agresti and Lang, 1993). For example, respondents can strongly disagree, disagree, be neutral, agree, or strongly agree with a specific opinion, or the severity of pain or injury can be classified as extremely intense, very intense, moderately intense, mild, or none. One such study dealing with ordinal data is the Western Canada Waiting List project, which is briefly described in Section 1.1. The categories in the priority criteria tools developed by the Western Canada Waiting List project are ordinal, and hence, we need to consider the ordinal data's quantitative nature in order to do the analysis. We will focus our study on the General Surgery panel described in Section 1.2. Section 1.3 describes the problem and the data.

1.1 Western Canada Waiting List Project

Over the past several years, in Canada, the waiting time for medical services has been indicated as a major problem in the health care system (WCWL 2001). It was reported that patients were waiting for an increasing length of time for medical services including specialist consultations, diagnostic services, and surgery. An increased length of waiting time can be painful and even life-threatening for the patients on a waiting list. Thus, reducing the length of waiting times and guaranteeing equality in accessing wait-listed services became a major concern. There was, however, no sufficient information on waiting lists and almost no valid tools to determine a patient's priority for medical treatment. Since no universal way existed to measure or define waiting times for medical services, the Canadian health care system did not have accurate waiting list information.

The Western Canada Waiting List (WCWL) project was established to provide tools to manage a waiting list and related issues. The WCWL is a collaborative initiative of 18 regional health authorities, health research centres, medical associations, and ministries of health situated in the four western provinces of Alberta, Manitoba, Saskatchewan, and British Columbia, and the Canadian Medical Association based in Ottawa. The major purpose of the WCWL is to develop clinically valid, reliable, transparent, and useful tools to assist in the management of waiting lists.

The priority criteria tools to manage the waiting lists were developed through several stages, and five clinical areas have chosen to develop priority criteria tools; cataract surgery, children's mental health services, general surgery, hip and knee replacement, and Magnetic Resonance Imaging (MRI) scanning. All five areas have a panel, and panel members are mostly physicians with a specialty or researchers. Major clinical factors and other important factors in predicting the patients' urgency were identified by panel members, and then the five priority criteria tools were drafted. Empirical testing of the five priority criteria tools tests whether the criteria included in the tools are relevant, whether any criteria important in predicting relative urgency are missing, and whether the criteria have a high degree of inter-rater reliability.

1.2 General Surgery Priority Criteria

Among the five panels of the WCWL, we will focus on General Surgery. General Surgery priority tool is used for patients with breast cancer, colorectal cancer, inguinal hernia, Laparoscopic cholecystectomy, etc. The panel members are seven general surgeons, two family physicians, two researchers, and one administrator. The process of developing these priority tools is same as that used by the rest of the panels.

While the General Surgery priority criteria were revised in December 2000, the data available are from the criteria form before its revision. The draft version of the General Surgery tool includes ten questions as well as information about the patient, such as age, gender, the surgical procedure that the patient is waiting for and the name of the physician who diagnosed the patient. Ten questions are included in the General Surgery tool: eight questions with 5 levels of ordered categories, one question with 3 levels of ordered categories, and one question with a ten-centimetre visual analogue scale ranging from not urgent (0) to extremely urgent (100). Questions included in the General Surgery tool have been identified as the major clinical factors needed to rate patients' relative urgency, and the categories in each question have been identified as the appropriate levels reflecting different degrees of severity (WCWL, 2001). Questions 1 to 5 ask about the current state and the predicted state 3-6 months after surgery. Questions 7 and 8 are for patients with a cancer diagnosis only. In the new version of the General Surgery tool, revised in December 2000, 8 questions are included from the draft version and a new question about the maximum waiting time for the patient was has been added.

The priority criteria forms are normally completed by a physician during or immediately after a patient consultation. The data were obtained from 152 patients and 10 physicians involved in the study. Among the 152 patients, 80 were cancer patients, and 72 were non-cancer patients.

1.3 Description of the Problem

Prioritization practice before the WCWL project was for the surgeon or referring clinician to assign patients to one of the following categories: emergent, urgent, semi urgent, and routine. However, it was almost impossible to prioritize patients in different situations such as those involving differences in type of disease, availability of resources, and physicians. In addition, the categories were vague without clear distinctions. This prioritization classification method did not help in reducing the subjectivity of the physicians. As a result, the waiting time of the patients often was not consistent with the urgency of their need for treatment.

The objective of this study is to develop the best method for prioritizing patients based on the available information. Instead of using the above vague method, which assigns patients to one of four categories, the WCWL uses a point-count system to prioritize the patients. The WCWL developed their prioritization method in two stages.

- Step 1: The development of the Priority Criteria form
- Step 2: The Quantification of the ordered categories (Scores were assigned to the categories)

Step 1 involved the development of the Priority Criteria tools by panel members. Step 2 assigned scores to the levels of the questions and will be the focus of this thesis. The Priority Criteria tools developed by Step 1 contain questions with ordered categorical responses. If a question has five levels of categories, then responses are coded as {1,2,3,4,5} for the analysis. However, these responses cannot be treated as a scale because the responses represent levels of the category. The quantification of the levels make categories more specific and interpretable. For example, if the levels of the pain are quantified as scores, it is easier to understand the severity of the pain. The Urgency score is computed by simply adding up the scores of the patient. Then patients are ranked based on their Urgency scores. The patient with a high Urgency score will take priority when accessing the medical services.

Since questions about the predicted state are meaningless unless asked with questions about the current state, we should not treat the five questions pertaining to the predicted state 3-6 months after the operation in the same way as the rest of the questions. Instead, the change of the state (current state minus predicted future state) is closely related to the usefulness of the surgery, and a larger value means that the condition of the patient will be more improved after surgery. Since question 7 and question 8 are for cancer patients only, we will exclude these two questions from the analysis.

Three different sets of the questions will be included in the analysis:

1. Current and Predicted state of Q1-Q5, Q6 and Q10.
2. Current state of Q1-Q5, Current state minus Predicted state of Q1-Q5, Q6 and Q10.
3. Current state of Q1-Q5, Q6 and Q10.

The current prioritization method suggested in the WCWL project is to assign scores with the optimal scaling approach, which will be discussed in Section 2.2. We will apply the current prioritization method to General Surgery data and perform the analysis with the three different sets of questions we mentioned above.

We will suggest a new approach to obtain scores and then will apply this method to the General Surgery data. Even though all questions included in the priority criteria form may be important, they may not have the same importance in predicting urgency. We thus need to give different weights to each question according to their importance in predicting urgency.

Description of the Data and Notation

A brief description of the questions and the scale of measurement is given in Table 1.1. The data are the responses from the draft version of the General Surgery Priority Criteria for 152 patients, and forms were filled out by

10 physicians. Original data were recorded as the responses in the General Surgery tool. Note that the responses for Q1-Q5 were recoded as

- None → 1
- Mild → 2
- Moderately Intense → 3
- Very Intense → 4
- Extremely Intense → 5.

We will denote the vector of ordinal responses of 152 patients from question j as Y_j=($y_{1j}, \ldots, y_{152j}$), where y_{ij} is the response to question j of patient i. The content of question 9 is *Compared with all the patients that you evaluate, how would you rate the urgency or relative priority of this patient?*, and the response is given by a continuous likert scale (Berk, 1979) range from 0 to 100. Let X=($x_1, \ldots, x_{152}$) denote the vector of continuous scale measurements from question 9, where x_i is the scale of patient i.

In the analysis, we need to compute the change of state, the difference of the responses of the current state and predicted state, and these values will range from -4 to 4. A negative value means that after the surgery, the patient's condition will get worse, so surgery is not useful in this case. Zero means that the patient's condition after the surgery will be same as before the surgery. Since all non-positive values mean that the surgery will not improve the patient's condition, we will recode all the negative values to zero and then add one to them so that the values will be consistent with those for the rest of the questions (1,2,3,4,5).

In addition to the responses to the above set of questions, the age and gender of the patients were also obtained. However, since age, gender and other socio-demographic characteristics were not considered to be relevant factors in rating urgency by the WCWL, we also drop these variables for the analysis.

The frequency of the responses of the 152 patients is given in Table 1.2. There are 19 missing data in Q6, while there are almost no missing data in the other questions. The content of Q6 is *History of major complications of condition*, which is rather unclear compared to the content of the other questions, and this problem may be the reason for the missing data. There is no observation for the highest level for some questions such as Q1po, Q2po, Q3po, Q4cu, Q4po, Q5cu, Q5po and Q6. It is natural for Q6 to have no observation for level 4 and level 5 because Q6 has 3 levels only.

Motivation

Ordered categorical variables are quantitative in the sense that there exists an order among categories (Agresti, 1984). Categories can be compared in terms of the magnitude of certain characteristics (Agresti, 1990). Hence, we can tell whether one category has greater or smaller magnitude of a characteristic than another category. However, the numerical distances between two levels are unknown. We were motivated to find the numerical value of each category. Moreover, ordered categorical variables can be assumed to have an underlying continuous scale. This thesis will suggest and apply a method to quantify ordered categories with General Surgery data.

Table 1.1: Notations of the Questions included in the General Surgery Priority Criteria

Notation	Contents	Type
Q1 cu	Usual intensity of pain(current)	ordinal
Q1 po	Usual intensity of pain(post operative)	ordinal
Q2 cu	Usual intensity of other forms of suffering(current)	ordinal
Q2 po	Usual intensity of other forms of suffering (post operative)	ordinal
Q3 cu	Usual frequency of painful episodes/ suffering(current)	ordinal
Q3 po	Usual frequency of painful episodes/ suffering(post operative)	ordinal
Q4 cu	Usual degree of impairment in role function due to surgical condition (current)	ordinal
Q4 po	Usual degree of impairment in role function due to surgical condition (post operative)	ordinal
Q5 cu	Usual degree of impairment in social activities (current)	ordinal
Q5 po	Usual degree of impairment in social activities (post operative)	ordinal
Q6	History of major complications of condition	ordinal
Q7	Life-expectancy implications of condition without procedure	ordinal
Q8	Expected degree of improvement (EDI) in life expectancy with surgery	ordinal
Q9	Compared with all the patients that you evaluate, how would you rate the urgency or relative priority of this patient	Likert scale
Q10	Compared with all the patients that you evaluate, how would you rate the urgency or relative priority of this patient	ordinal

Table 1.2: Frequency Table of the General Surgery Data with 152 Patients

Question	1	2	3	4	5	Missing
Q1 cu	43	35	50	16	8	0
Q1 po	118	31	2	1	0	0
Q2 cu	54	36	42	14	6	0
Q2 po	116	28	6	2	0	0
Q3 cu	48	57	29	15	3	0
Q3 po	119	31	1	1	0	0
Q4 cu	37	69	31	14	0	0
Q4 po	117	30	3	1	0	1
Q5 cu	53	67	22	9	0	1
Q5 po	123	26	2	1	0	0
Q6	112	21	0	-	-	19
Q10	12	34	48	46	11	1
Q1 ch	46	53	31	15	7	0
Q2 ch	71	35	31	14	1	0
Q3 ch	56	58	25	12	1	0
Q4 ch	49	66	31	5	0	1
Q5 ch	65	63	19	4	0	1

Chapter 2

Ordinal Regression and the Current Scoring Method

This chapter reviews ordinal regression models and the current prioritization method used by the WCWL. A description of ordinal regression model is presented in Section 2.1. Section 2.2 deals with the current scoring method which utilizes the optimal scaling procedure.

2.1 Regression Models for Ordered Categorical Data

Suppose Y is the response variable from a question with ordered categories. Since Y is an ordered categorical response, we need to discuss regression models for ordered categorical data. These models have been discussed by several authors (McCullagh, 1980; Anderson, 1984).

The responses can be arranged in an $r \times c$ table, where r and c are the number of rows and columns, respectively. When ordered categorical data are observed as the row variable, a log linear row effects model or logit row effects model can be used when the column variable is nominal. When the column variable is ordinal, a linear by linear association model or global odds ratio model can be used (Agresti, 1984).

Now we will review the ordinal regression methods which is widely used when Y is ordinal response and covariate X is interval or categorical. However, for simplicity of the model, we will consider the continuous covariate. Let $y_1, y_2, \ldots, y_N$ be the ordered categorical responses and $x_1, x_2, \ldots, x_N$ be the continuous covariates for N patients. Let y_i take one of J ordered categories $1, 2, \ldots, J$ for patient i, $i = 1, \ldots, N$. Ordered categories $1, \ldots, J$ can be thought of as contiguous intervals on some continuous scale, and we can assume there exists an underlying latent variable (Formann, 1982; Clogg and Goodman, 1984; Formann, 1992; Menezes and Bartholomew, 1996). Let z_i be the latent variable representing the underlying continuous scale of y_i. The value of y_i is decided by z_i and the cut points $\gamma_1 \leq \gamma_2 \leq \ldots \leq \gamma_J$ such that $y_i = k$ is observed if z_i is in (γ_{k-1}, γ_k). Since z is continuous, z can be formulated as

$$z = \beta x + \varepsilon,$$

where ε has the density function f (Albert and Chib, 1993). The probability that y_i is observed to be k is given by

$$\begin{aligned} P(y_i = k \mid x) &= P(\gamma_{k-1} < z_i \leq \gamma_k) \\ &= P(\gamma_{k-1} - \beta x < z_i - \beta x \leq \gamma_k - \beta x) \\ &= F(\gamma_k - \beta x) - F(\gamma_{k-1} - \beta x), \end{aligned}$$

where $F = \int f$ (Johnson, 1996; Johnson and Albert, 1999). Then the cumulative probability of y_i is

$$\begin{aligned} \theta_k(x_i) &= P(y_i \leq k) \\ &= F(\gamma_k - \beta x). \end{aligned}$$

The above model can have the general form as follows

$$\text{link}(\theta_k(x)) = \gamma_k - \beta x,$$

where the link function can be any monotone increasing function mapping the unit interval (0,1) onto $(-\infty, \infty)$. Commonly used links are the logit function, complementary log-log function, or inverse of cumulative normal function. For example, the proportional odds model

$$\log\left(\frac{\theta_k(x)}{1-\theta_k(x)}\right) = \gamma_k - \beta x$$

is obtained if we use the logit link, and the proportional hazards model

$$\log(-\log(1-\theta_k(x))) = \gamma_k - \beta x$$

is obtained if we use the complementary log-log function as the link. If we use the inverse of cumulative normal function as a link, then the probit model

$$\Phi^{-1}(\theta_k(x)) = \gamma_k - \beta x$$

is obtained (McCullagh and Nelder, 1989; McCullagh, 1980), where Φ is a cumulative normal function.

The degree of contribution of each question in predicting the urgency of a patient can be judged by the R-square values of the constructed ordinal regression model. Hence, the R-square value can be used to obtain the relative importance of the question in predicting a patient's urgency. However, scores of the categories cannot be obtained from the ordinal regression models. The use of an ordinal regression model does not give the ability to rank patients based on their urgency. So we will review the optimal scaling method used by WCWL.

The likelihood from the above model is given by

$$\begin{aligned} \text{L} &= \Pi_{i=1}^{N} \text{P}(y_i|\beta, \gamma, z) \\ &= \Pi_{i=1}^{N} \text{P}(\gamma_{y_i-1} \le z_i < \gamma_{y_i}) \\ &= \Pi_{i=1}^{N} f(z_i - x_i\beta)\text{I}(\gamma_{y_i-1} \le z_i < \gamma_{y_i}). \end{aligned}$$

Jansen (1991) discussed the iterative weighted least squares algorithm as a method for obtaining the maximum likelihood estimator for the ordinal regression models. For estimating β and γ, iterative weighted least squares algorithm will be used.

A discussion of an application of this method to the General Surgery data will be given in Chapter 3.

2.2 Current Scoring Method with Optimal Scaling

Hadorn (2000) discussed the point-count method to prioritize patients. The WCWL adapted the point-count method from New Zealand and the U.K (Hadorn, 2000) where the point-count method is used to rate the urgency and priority of the patient based on the severity of the patient's condition and the extent of potential benefit from clinical services. This method assigns scores to patients, based on the severity of their pain and the degree to which it limits their lives. The use of a point-count method helps manage the waiting lists relatively transparently and provides accuracy in prioritizing patients. The point-count method used by the WCWL starts with assigning scores to the various levels for all questions. The scores to be assigned to the various question levels are decided by the transformed value obtained from the optimal transformation of the ordinal responses (Bradley, Katti, and Coons, 1962; Snell, 1964).

The scores to be assigned to the levels of the questions are decided by using the scales obtained from the optimal transformation. The Urgency score for the patient is computed by adding up all the scores of the patient. The patients with higher Urgency scores should be given priority over the patients with lower Urgency scores.

Optimal scaling is obtained by using the optimal transformations. The objective of the transformations is to find the best-fitting regression model.

Suppose the response variable Y takes values in $\{1,2,3,4,5\}$, then ordered categories of Y can be transformed into scales by using the optimal transformation $\theta(Y)$.

A simple iterative algorithm suggested by Brieman and Friedman (1985) is used to estimate optimal transformation $\theta(Y)$ and $\phi(X)$. The basic method to obtain estimates of $\theta(Y)$ and $\phi(X)$ is to find θ^* and ϕ^*, which minimize

$$e^2(\theta, \phi) = \frac{\mathrm{E}[\theta(Y) - \phi(X)]^2}{\mathrm{Var}(\theta(Y))}.$$

Hence, θ^* and ϕ^* are obtained as

$$e^2(\theta^*, \phi^*) = \min_{\theta,\phi} e^2(\theta, \phi).$$

Brieman and Friedman (1985) showed that a simple iterative algorithm exists for optimal transformations that converge to an optimal solution. Without loss of generality, we can assume that $\mathrm{E}(\theta(Y)^2)$ =1 and $\mathrm{E}(\theta(Y)) = \mathrm{E}(\phi(X))$ =0; then

$$e^2(\theta, \phi) = \mathrm{E}[(\theta(Y) - \phi(X))^2]. \tag{2.1}$$

The solution of $\theta(Y)$, which minimizes (2.1) given $\phi(X)$, is

$$\theta(Y) = \frac{\mathrm{E}(\phi(X)|Y)}{\| \mathrm{E}(\phi(X)|Y) \|}, \tag{2.2}$$

with $\| \cdot \| = \sqrt{[\mathrm{E}(\cdot)^2]}$ and the solution of $\phi(X)$, which minimizes (2.1) given $\theta(Y)$, is

$$\phi(X) = \mathrm{E}(\theta(Y)|X). \tag{2.3}$$

Since Y is a categorical variable, the conditional expectation can be computed as

$$\mathrm{E}(\phi(X)|Y = y) = \frac{\sum_{Y_i = y} \phi(X_i)}{\sum \mathrm{I}(Y_i = y)}$$

where I is an indicator function. When (2.2) and (2.3) are used alternately, the optimal transformations converge to the optimal solution. The alternating conditional expectations (ACE) algorithm discussed by Brieman and Friedman (1985) is given by the following steps:

1. Initialize $\theta(Y) = \frac{Y}{\|Y\|}$.

2. Repeat the following step until $e^2(\theta, \phi)$ fails to decrease

$$\phi_1(X) = \mathrm{E}(\theta(Y)|X)$$

   replace $\phi(X)$ with $\phi_1(X)$

$$\theta_1(Y) = \frac{\mathrm{E}(\phi(X)|Y)}{\| \mathrm{E}(\phi(X)|Y) \|}$$

   then replace $\theta(Y)$ with $\theta_1(Y)$.

3. θ and ϕ are the solutions of θ^* and ϕ^*, respectively.

Brieman and Friedman (1985) proved that the solutions from the ACE algorithm converge to an optimal solution. The above algorithm can be used for both continuous and categorical variables. An implementation of the algorithm will be done by using regression with optimal scaling in SPSS 10.0. We assign continuous variable as a independent and categorical variable as a dependent and choose types and range of variables. The regression with optimal scaling quantifies the categorical variable by using ACE algorithm.

From the ACE algorithm, the optimal transformations are obtained, and the categories of Y are quantified as optimal scales. The scores of the levels are obtained by rescaling the optimal scales according to the weight of the question. The weight given to each question should reflect its importance. From a regression with $\phi(X)$ and $\theta(Y)$, the R-square is obtained. The R-square is a measure of the amount of variability of $\theta(Y)$ explained by the regression model; hence, a large value of R-square implies that the regression model is good. Since the R-square seems to reflect the importance of the question, weights are obtained by rescaling the R-square values. Scores are obtained by the transformation of optimal scales.

The same procedures to assign scores are repeated for all questions in the General Surgery priority criteria. The Urgency score of the patient can be computed by adding up the patient's scores. Prioritizing the patients will be done according to the patient's Urgency scores.

The validity of the method can be determined by looking at the correlation between the Urgency score and the overall urgency rating (scale from question 9). However, a low R-square value does not always mean a lack of validity because the overall urgency rating is subjective and a large variation could exist among physicians (WCWL, 2001). Despite this flaw, conducting this analysis can improve the procedure of judging a patient's priority and help identify the factors which can be useful for future judgements.

Chapter 3

Application of the Ordinal Regression Model

In this chapter we apply ordinal regression model to the General Surgery data. Among ten questions in the General Surgery tool, nine questions have responses with ordered categories and one question has response with continuous scale. Let Y_j be the vector of the ordered responses from question j, and y_{ij} be the ordered response of patient i to question j. Let X be the vector of scales from Q9, and x_i be the scale of patient i from Q9. We will construct the univariate ordinal regression model. Several ordinal regression models have been discussed in Section 2.1. The ordinal regression models assuming the latent variable have the general form given as

$$\text{link}(\theta_k(x)) = \gamma_k - \beta x.$$

The common links used are the inverse of cumulative normal function, logit function and complementary log-log function. The ordinal regression model with the above link functions are the probit model,

$$\Phi^{-1}(\theta_k(x)) = \gamma_k - \beta x;$$

the proportional odds model,

$$\log\left(\frac{\theta_k(x)}{1-\theta_k(x)}\right) = \gamma_k - \beta x;$$

and the proportional hazards model,

$$\log(-\log(1-\theta_k(x))) = \gamma_k - \beta x,$$

respectively (McCullagh, 1980).

We constructed these three ordinal regression models for the responses to each question separately. From the R-square value of the fitted ordinal regression model, we will determine how important the question is in assessing the relative urgency.

First, we use a probit model, and the results are displayed in Table 3.1. Note that R-square values from the probit models are in a low range from 0.001 to 0.245, except for Q10. The model with Q1cu has the lowest R-square value of 0.001. We next used the proportional odds model, and results are presented in Table 3.2. The proportional odds model shows almost the same result as the probit model. Lastly, we use the proportional hazards model, and results are presented in Table 3.3. The R-square values of the proportional hazards model with Q4cu, Q5cu and Q10 are higher compared to the R-square values of the two other models. The R-square value for the model with Q10 is 0.891, which is the highest of all.

The R-square values of the regression model of Q10 are high and range from 0.727 to 0.891, and those of the regression model of Q2cu range from 0.243 to 0.274. However, the R-square values of the regression model of the rest of the questions are in a very low range, 0.001 to 0.137. All three models give almost the same result. Q10 seems to be the most important question for assessing the relative urgency of the patient. The rest of the questions, judging by R-square values, do not seem to be as useful in identifying the patient's urgency. The R-square values of the ordinal regression model can be used to obtain a weight of the question. However numerical values of the categories cannot be obtained from the ordinal regression method. From this method, we may not be able to obtain the Urgency score of the patient. To overcome this deficiency, we will apply the optimal scaling method used by the WCWL in next chapter.

Table 3.1: Estimates and P-values from Ordinal Regression Model with Inverse of Cumulative Normal Link Function

Question		γ_1	γ_2	γ_3	γ_4	β	R-square
Q1 cu	Estimate	-0.505	0.102	1.077	1.696	0.001	0.001
	P-value	0.012	0.607	0.000	0.000	0.674	
Q1 po	Estimate	1.459	2.832	3.291	-	0.012	0.054
	P-value	0.000	0.000	0.000	-	0.005	
Q2 cu	Estimate	0.787	1.494	2.569	3.340	0.024	0.245
	P-value	0.000	0.000	0.000	0.000	0.000	
Q2 po	Estimate	1.742	2.750	3.432	-	0.018	0.109
	P-value	0.000	0.000	0.000	-	0.000	
Q3 cu	Estimate	-0.309	0.669	1.372	2.271	0.03	0.007
	P-value	0.123	0.001	0.000	0.000	0.301	
Q3 po	Estimate	1.454	2.969	3.251	-	0.012	0.049
	P-value	0.000	0.000	0.000	-	0.007	
Q4 cu	Estimate	-0.135	1.131	2.001	-	0.011	0.070
	P-value	0.505	0.000	0.000	-	0.001	
Q4 po	Estimate	1.790	3.091	3.702	-	0.018	0.099
	P-value	0.000	0.000	0.000	-	0.000	
Q5 cu	Estimate	0.345	1.645	2.459	-	0.015	0.108
	P-value	0.096	0.000	0.000	-	0.000	
Q5 po	Estimate	1.649	2.918	3.386	-	0.014	0.058
	P-value	0.000	0.000	0.000	-	0.004	
Q6	Estimate	1.893	-			0.016	0.065
	P-value	0.000	-			0.004	
Q10	Estimate	0.593	2.549	4.735	6.943	0.075	0.727
	P-value	0.017	0.000	0.000	0.0000	0.000	

Table 3.2: Estimates and P-values from Ordinal Regression Model with Logit Link Function

Question		γ_1	γ_2	γ_3	γ_4	β	R-square
Q1 cu	Estimate	-0.762	0.222	1.856	3.077	0.004	0.003
	P-value	0.023	0.502	0.000	0.000	0.524	
Q1 po	Estimate	2.435	5.152	6.271	-	0.022	0.049
	P-value	0.000	0.000	0.000	-	0.008	
Q2 cu	Estimate	1.313	2.509	4.344	5.766	0.041	0.243
	P-value	0.000	0.000	0.000	0.000	0.000	
Q2 po	Estimate	3.007	4.849	6.308	-	0.032	0.103
	P-value	0.000	0.000	0.000	-	0.000	
Q3 cu	Estimate	-0.582	0.994	2.213	4.122	0.04	0.003
	P-value	0.083	0.004	0.000	0.000	0.462	
Q3 po	Estimate	2.437	5.525	6.229	-	0.021	0.045
	P-value	0.000	0.000	0.000	-	0.011	
Q4 cu	Estimate	-0.277	1.802	3.319	-	0.019	0.066
	P-value	0.416	0.000	0.000	-	0.002	
Q4 po	Estimate	3.026	5.500	6.924	-	0.031	0.092
	P-value	0.000	0.000	0.000	-	0.000	
Q5 cu	Estimate	0.547	2.688	4.185	-	0.025	0.103
	P-value	0.114	0.000	0.000	-	0.000	
Q5 po	Estimate	2.789	5.309	6.431	-	0.024	0.052
	P-value	0.000	0.000	0.000	-	0.007	
Q6	Estimate	3.297	-			0.029	0.063
	P-value	0.000	-			0.006	
Q10	Estimate	1.167	4.693	8.644	12.527	0.135	0.728
	P-value	0.008	0.000	0.000	0.0000	0.000	

Table 3.3: Estimates and P-values from Ordinal Regression Model with Complementary log-log Link Function

Question		γ_1	γ_2	γ_3	γ_4	β	R-square
Q1 cu	Estimate	-0.895	-0.119	0.842	1.312	0.004	0.010
	P-value	0.000	0.553	0.000	0.000	0.198	
Q1 po	Estimate	1.005	2.072	2.369	-	0.011	0.058
	P-value	0.000	0.000	0.000	-	0.003	
Q2 cu	Estimate	0.397	1.233	2.297	2.970	0.026	0.274
	P-value	0.055	0.000	0.000	0.000	0.000	
Q2 po	Estimate	1.182	2.021	2.506	-	0.015	0.110
	P-value	0.000	0.000	0.000	-	0.000	
Q3 cu	Estimate	-0.458	0.707	1.392	2.101	0.011	0.061
	P-value	0.031	0.000	0.000	0.000	0.001	
Q3 po	Estimate	0.997	2.146	2.326	-	0.011	0.052
	P-value	0.000	0.000	0.000	-	0.005	
Q4 cu	Estimate	-0.430	1.155	2.004	-	0.018	0.144
	P-value	0.057	0.000	0.000	-	0.000	
Q4 po	Estimate	1.278	2.332	2.744	-	0.016	0.105
	P-value	0.000	0.000	0.000	-	0.000	
Q5 cu	Estimate	-0.022	1.416	2.093	-	0.017	0.137
	P-value	0.916	0.000	0.000	-	0.000	
Q5 po	Estimate	1.148	2.127	2.434	-	0.012	0.063
	P-value	0.000	0.000	0.000	-	0.002	
Q6	Estimate	1.336	-			0.014	0.066
	P-value	0.000	-			0.003	
Q10	Estimate	-0.197	1.730	3.649	5.703	0.065	0.891
	P-value	0.528	0.000	0.000	0.0000	0.000	

Chapter 4

Application of Point Count Method with Optimal Scaling

We applied the ordinal regression method in Chapter 3 and this method provides relative weight of each question in obtaining the Urgency score. However, the ordinal regression method cannot provide the score for each category within the question. The optimal scaling method described in Section 2.2 can be used to assign scores to the categories within the question. In this chapter, we will illustrate the application of the optimal scaling method to the General Surgery priority data. We obtain scores for all levels of the questions by optimal transformation and then compute the Urgency score of the patient by the point count method.

We follow the current WCWL prioritization method and include all the questions to prioritize the patient. The scores for the levels of each question are obtained by using optimal transformation. Let Y_1 be the responses from the first question with five ordered categories: None (N), Mild (M), Moderately Intense (MI), Very Intense (VI) and Extremely Intense (EI), and X be the scale from Q9. Estimates of the optimal transformation are based on the data $\{y_1, \dots, y_N\}$ and $\{x_1, \dots, x_N\}$, which are the observed values from Y and X, respectively, and N is the number of patients. Let θ be the transformation function of Y and ϕ be the transformation function of X. The transformation functions do not need to be known, parameterized functions. Optimal transformations are estimated by finding the functions θ and ϕ to minimize the following,

$$e^2(\theta, \phi) = \frac{\mathrm{E}[\theta(Y) - \phi(X)]^2}{\mathrm{Var}(\theta(Y))}.$$

The algorithm used to obtain the optimal transformations was discussed in Section 2.2. The optimal transformation finds the best regression model by using transformation functions ϕ and θ of X and Y, respectively. The ordered categories of Y can be transformed to continuous scale by using the optimal transformation. From the ACE algorithm, the optimal transformation functions ϕ and θ are estimated, and the optimal scales which are the transformation of Y by function θ can be obtained. The optimal scales of Y are given in Table 4.1. We obtained the optimal scales from the regression with optimal scaling, also called the categorical regression using SPSS 10.0. The categorical regression quantifies the categorical data by using numerical values to the categories. The values are obtained by the ACE algorithm.

Now we need to assign scores for each level of the questions. The scores for different levels of the questions are computed by linear transformation of the optimal scale in Table 4.1. The optimal scales for level 1 given in Table 4.1 are negative. For example, the optimal scale of level 1 (Y=none) of Q1cu is -0.809. It is natural to assume that the score of the first level is zero insuring that the Urgency score is zero when all responses are at the baseline. Therefore, we will add the same quantity to all levels in a question so that the score for the first level is zero. For example, if we add 0.809 to the optimal scale of all levels in Q1cu, then the score of level 1 becomes zero.

We next to assign weights to the questions. The weight should represent a question's contribution in determining the urgency of patients. Weights are decided from the regression model with $\phi(X)$ and $\theta(Y)$, where $\phi(X)$ and $\theta(Y)$ are the optimal transformations of X and Y, respectively. Questions with a better model fit are regarded as more important, so any statistic which reflects a good model fit can be used to decide weights. The WCWL uses the R-square of the regression model with $\phi(X)$ and $\theta(Y)$ to obtain a weight. Weights can be computed by rescaling the value of the R-square so that sum of the weights for the questions included to prioritize the patients is 100. The R-square values and the weights appear in Table 4.2, and the weights are obtained by multiplying 52.41 by the R-square values, ensuring that the sum of the weights is 100. The weights obtained in Table 4.2 are used to assign scores. The score of the highest level of a question is assumed to be same as a weight. Thus, the sum of the scores of the highest level of all questions will be 100.

To obtain the scores by using optimal scales and weights, the following procedures will be undertaken:

1. Transform the optimal scales so that the score of the first level is zero
2. Rescale the transformed scales so that the score of the highest level is the same as a weight.

For example with Q1cu, we add 0.809 to the optimal scales of all levels; thus, the score of the first level becomes 0, and the score of the highest level becomes 2.813. We multiply the transformed scales of all levels by 0.633 ($= 1.78 \div 2.183$) so that the score for the highest level is the same as the weight of Q1cu given in Table 4.2.

Since we consider three different sets of questions in Section 1.3, we perform the same scoring method on the three sets of questions, and the scores are given in Tables 4.3-4.5.

Some questions have no observation for the highest level (Y=5). In this case, the optimal scale cannot be obtained for level 5, and thus the score cannot be computed. We will thus assume the score for level 5 to be the same as the score for level 4.

The Urgency score of the patient can be obtained by a point count method, which adds up the scores of the patients. Since Q7 and Q8 are for cancer patients only, we exclude these questions in computing the Urgency score. The Urgency score ranges from 0 to 100, where a minimum score of 0 is obtained if a patient's responses are 1 for all questions, and a maximum score of 100 is obtained if a patient's responses are 5 for all questions.

The WCWL prioritizes the patients by using the Urgency score. The patients with higher Urgency scores are given priority over the patients with lower Urgency scores. We will test the usefulness of this scoring method by using the Urgency scores. We use the regression model with the Urgency scores and continuous scale from Q9 to test the performance of the optimal scaling method:

$$X = \rho_0 + \rho_1 \text{Urgency score},$$

where X is a scale from Q9 (rating of the urgency), and ρ_0 and ρ_1 are coefficients. The usefulness of the scoring method decided by the R-square value which leads to the conclusion that the Urgency score accurately reflects the urgency of the patient. This conclusion is based on the assumption that the rate of urgency (a scale from Q9) is correct and provides objective information about the patient's urgency. The R-square of the model fit appears in Table 4.6, and values are given as 0.655, 0.511 and 0.642.

The regression model for the Urgency score obtained from the responses to the question about the current state and the change of the state after the surgery has the lowest R-square value of 0.511. Contrary to our expectation, the Urgency score computed from the current and post-state performs better than the Urgency score computed from the current state and change of the state.

The scores are obtained by transforming the optimal scales to nonnegative values and then rescaling the transformed scales according to the weights. That is, scores are obtained by linear transformations of the optimal scales. However, the optimal scales given in Table 4.1 are obtained from optimal transformations which are not necessarily linear transformations. If optimal transformations are nonlinear, the linear transformation used to generate the scores may not produce the optimal scores. The procedure of linear transformations may affect the optimality. That is, the scores given in Table 4.3 through Table 4.5 may not be optimal scores. To avoid this problem, we

Table 4.1: Scales from the optimal transformation

Question	1	2	3	4	5
Q1 cu	-0.809	-0.809	0.300	2.004	2.004
Q1 po	-0.492	1.480	3.115	6.008	*
Q2 cu	-0.865	-0.640	0.823	1.388	2.628
Q2 po	-0.518	1.304	2.527	4.182	*
Q3 cu	-0.517	-0.517	0.262	2.516	2.996
Q3 po	-0.485	1.570	2.782	6.310	*
Q4 cu	-0.612	-0.612	1.018	2.378	*
Q4 po	-0.512	1.538	3.018	4.700	*
Q5 cu	-0.660	-0.379	1.868	2.146	*
Q5 po	-0.436	1.516	4.153	5.863	*
Q6	-0.433	2.309	*	-	-
Q10	-1.714	-1.714	-0.010	0.947	1.583

NOTE: ⋆ indicates no estimate due to
no observation

propose an alternative method which will be described in next chapter. The proposed method will introduce nonnegative scores.

Table 4.2: Weights based on R-square value of optimal regression

Question	R square	weight
Q1 cu	0.034	1.78
Q1 po	0.055	2.88
Q2 cu	0.269	14.10
Q2 po	0.109	5.71
Q3 cu	0.117	6.13
Q3 po	0.050	2.62
Q4 cu	0.165	8.65
Q4 po	0.088	4.61
Q5 cu	0.147	7.70
Q5 po	0.059	3.09
Q6	0.062	3.25
Q10	0.753	39.47

Table 4.3: Scores of the first set of questions obtained from the optimal transformation

Question	1	2	3	4	5
Q1 cu	0	0	1	2	2
Q1 po	0	1	2	3	3
Q2 cu	0	1	7	9	14
Q2 po	0	2	4	6	6
Q3 cu	0	0	1	5	6
Q3 po	0	1	1	3	3
Q4 cu	0	0	5	9	9
Q4 po	0	2	3	5	5
Q5 cu	0	1	7	8	8
Q5 po	0	1	2	3	3
Q6	0	3	3	-	-
Q10	0	6	20	32	39

Table 4.4: Scores of the second set of questions obtained from the optimal transformation

Question	1	2	3	4	5
Q1 cu	0	0	1	2	2
Q2 cu	0	1	6	8	12
Q3 cu	0	0	1	5	5
Q4 cu	0	0	4	7	7
Q5 cu	0	1	6	7	7
Q6	0	3	3	-	-
Q10	0	0	14	26	34
Q1 ch	0	3	3	3	3
Q2 ch	0	2	4	10	10
Q3 ch	0	0	1	6	6
Q4 ch	0	0	4	6	6
Q5 ch	0	1	6	6	6

Table 4.5: Scores of the third set of questions obtained from the optimal transformation

Question	1	2	3	4	5
Q1 cu	0	0	1	2	2
Q2 cu	0	1	8	11	17
Q3 cu	0	0	2	7	8
Q4 cu	0	0	6	11	11
Q5 cu	0	1	9	10	10
Q6	0	4	4	-	-
Q10	0	8	25	39	49

Table 4.6: R-square value of $X = \rho_0 + \rho_1$Urgency score

Questions used	R-square
Current and Post	0.655
Current and Change	0.511
Current only	0.642

Chapter 5

Constraint Regression Approach

In Chapter 4, we applied the optimal scaling method to prioritize the patients by using General Surgery data. We also mentioned the shortcoming of the optimal scaling method in obtaining scores. To overcome this shortcoming, we propose an alternative method to obtain scores by using a linear model in this chapter. We apply this method to the General Surgery Priority data to quantify the ordinal levels for each question. We also compare the scores obtained from the proposed method and the optimal scaling method. These scores are computed with five different weights.

5.1 Proposed Method with Constraint Regression

Let Y be the vector of the ordered categorical response, which can be one of None(N), Mild(M), Moderately Intense(MI), Very Intense(VI) and Extremely Intense(EI) and coded as 1,2,3,4 and 5, respectively. From the observed data where Y takes values in $\{1,2,3,4,5\}$, we can see the order of the categories, but the distance between the categories or the differences among the same levels of different questions are unknown. We need to quantify the categories to find the numerical differences between the categories. We propose a method using constrained regression with X as a dependent variable and using four dummy variables for each question to quantify each level of Y. For each question, we create four dummy variables, Z_1, Z_2, Z_3 and Z_4:

$$z_{i1} = \begin{cases} 1 & \text{if } y_i = \text{EI,VI,MI,M}, \\ 0 & \text{otherwise.} \end{cases}$$

$$z_{i2} = \begin{cases} 1 & \text{if } y_i = \text{EI,VI,MI}, \\ 0 & \text{otherwise.} \end{cases}$$

$$z_{i3} = \begin{cases} 1 & \text{if } y_i = \text{EI,VI}, \\ 0 & \text{otherwise.} \end{cases}$$

$$z_{i4} = \begin{cases} 1 & \text{if } y_i = \text{EI}, \\ 0 & \text{otherwise.} \end{cases}$$

Note that these dummy variables have the order $Z_1 \geq Z_2 \geq Z_3 \geq Z_4$ since

$$\begin{aligned} Z_1 &= \text{I}(Y \geq 2) \\ Z_2 &= \text{I}(Y \geq 3) \\ Z_3 &= \text{I}(Y \geq 4) \\ Z_4 &= \text{I}(Y \geq 5), \end{aligned}$$

where I is an indicator function.

We develop a multiple linear regression model with $z_{1i}, z_{2i}, z_{3i}, z_{4i}$ and X_i for patient i:

$$X_i = \beta_0 + \beta_1 z_{1i} + \beta_2 z_{2i} + \beta_3 z_{3i} + \beta_4 z_{4i} + \varepsilon_i, \tag{5.1}$$

where $\varepsilon_i \sim iid\, \text{N}(0, \sigma^2)$ and X_i represent likert scale values as given in question 9. The above model can be rewritten as follows:

$$X_i = \beta_0 + \beta_1 \text{I}(y_i \geq 2) + \beta_2 \text{I}(y_i \geq 3) + \beta_3 \text{I}(y_i \geq 4) + \beta_4 \text{I}(y_i \geq 5) + \varepsilon_i. \tag{5.2}$$

The expected value of X_i given y_i can be computed as follows:

$$\begin{aligned}
\mathrm{E}(X_i|y_i = N) &= \beta_0, \\
\mathrm{E}(X_i|y_i = M) &= \beta_0 + \beta_1, \\
\mathrm{E}(X_i|y_i = MI) &= \beta_0 + \beta_1 + \beta_2, \\
\mathrm{E}(X_i|y_i = VI) &= \beta_0 + \beta_1 + \beta_2 + \beta_3, \\
\mathrm{E}(X_i|y_i = EI) &= \beta_0 + \beta_1 + \beta_2 + \beta_3 + \beta_4.
\end{aligned}$$

Since the categories of y_i are ordered, the expected value of X_i given y_i should have an order such as

$$\begin{aligned}
&\mathrm{E}(X_i|y_i = N) \leq \mathrm{E}(X_i|y_i = M) \leq \mathrm{E}(X_i|y_i = MI) \\
&\qquad\qquad \leq \mathrm{E}(X_i|y_i = VI) \leq \mathrm{E}(X_i|y_i = EI).
\end{aligned}$$

To satisfy the above conditions, we need to constrain the parameters so that $\beta_0, \beta_1, \beta_2, \beta_3$ and β_4 are all nonnegative, and hence,

$$\beta_0 \leq \beta_0 + \beta_1 \leq \ldots \leq \beta_0 + \beta_1 + \beta_2 + \beta_3 + \beta_4. \tag{5.3}$$

The estimates of the β's are considered as numerical distances between two categories. If any of the β's are estimated to be zero, then the relevant categories are not distinct. For example, if β_4 is estimated to be zero then no difference exists between level4 and level5. Parameter estimates can be obtained by using the nonnegative least square method. Levels can be quantified by the expected value of X given Y by using estimates of the parameters. For example, the expected value of X given Y=1 is the quantification of the ordered categorical response Y=1.

Now, we will quantify each level of Y with the estimates of the expected value of X given Y. Since X is the rate of urgency or scale of relative priority, it is reasonable to assume the expected value of X given the first category of

Y to be zero; that is, when all responses are at the baseline, the urgency of the patient is expected to be zero. For example, we can assume the expected rate of urgency for the patients with no usual pain (y=None for question 1) to be zero. To satisfy the above assumption, we should fix $\beta_0 = 0$, and the estimates of expected values of X given Y for the rest of the level should be computed by assuming $\beta_0 = 0$. The expected values of X_i given Y_i, assuming $\beta_0 = 0$, are given by

$$
\begin{aligned}
&\mathrm{E}(X_i|y_i = N) = 0,\\
&\mathrm{E}(X_i|y_i = M) = \beta_1,\\
&\mathrm{E}(X_i|y_i = MI) = \beta_1 + \beta_2,\\
&\mathrm{E}(X_i|y_i = VI) = \beta_1 + \beta_2 + \beta_3,\\
&\mathrm{E}(X_i|y_i = EI) = \beta_1 + \beta_2 + \beta_3 + \beta_4.
\end{aligned}
$$

For subsequent analysis, we will consider these estimates as predictors in fitting a linear regression model to evaluate the importance of Y_j in predicting X, where Y_j is a vector of the responses to question j. That is, by letting S_{ij} denote the estimate for $\mathrm{E}(X|y_{ij})$, we consider the following model,

$$X_i = \delta_0 + \delta_1 S_{i1} + \ldots + \delta_l S_{il} + \varepsilon_i,$$

where y_{ij} is a response from $j-th$ question of $i-th$ patient. Responses from all questions in the General Surgery Priority criteria form will be included to compute the Urgency score. However, the questions may not have the same importance; hence, we assign a weight to each question according to its importance, so that the weight represents the relative contribution of each question to the Urgency score. We will use statistics that reflect the model fit between X and Y_j to obtain the weight.

We will construct the regression model of X and Y_j. First, we construct the regression model with X as a response and $Y_1, \ldots, Y_l$ as predictors, where

l is the number of questions. That is, the underlying model is given by

$$X_i = \delta_0 + \delta_1 S_{i1} + \ldots + \delta_l S_{il} + \varepsilon_i, \tag{5.4}$$

where $\delta_0, \delta_1, \ldots, \delta_1$ are coefficients, and S_{ij} is the estimate of the expected value of X given $y_{ij}(\mathrm{E}(X|y_{ij}))$. Second, we construct the regression model with X as a response and Y_j as a predictor for $j = 1, \ldots, l$; that is,

$$X_i = \delta_0 + \delta_1 S_{ij} + \varepsilon_i. \tag{5.5}$$

We used 5 measures to obtain weights for question j:

1. Absolute value of the t statistic for testing H_0:δ_1=0 for model $X_i = \delta_0 + \delta_1 S_{ij}$ (weight1).
2. Inverse of the variance of δ_1 for model $X_i = \delta_0 + \delta_1 S_{ij}$ (weight2).
3. R square for the model $X_i = \delta_0 + \delta_1 S_{ij}$ (weight3).
4. Absolute value of the t statistic for testing H_0:δ_j=0 for model $X_i = \delta_0 + \delta_1 S_{i1} + \ldots + \delta_l S_{il}$ (weight4).
5. Inverse of the variance of δ_j for model $X_i = \delta_0 + \delta_1 S_{i1} + \ldots + \delta_l S_{il}$ (weight5).

The weight based on the above measures will be denoted as weight1, weight2, weight3, weight4 and weight5, respectively. Furthermore, all the measures will be rescaled so that sum of the weights for all questions is 100; that is, $\sum_{j=1}^{l} W_j$=100, where W_j is the weight for the question j, and l is the number of questions.

Scores for the levels of the question j are determined by estimates of the expected value of X given y_j and W_j. The expected value of X given $y_j(S_j)$, should be rescaled so that the score for the highest level is the same as W_j; that is, multiply by W_j in all levels and divide by $\mathrm{E}(X \mid y_j = 5)$ in all levels. Scores are proportional to the expected values of X given the level of Y.

The Urgency score for each patient can be computed by adding up his/her scores for all questions. In the next section, we consider applications of this method to the General Surgery data.

5.2 Applications

In this section, we describe an application of the proposed method given in Section 5.1 to compute the Urgency score by using the General Surgery data. From Q1 to Q5, responses are given for both the current state and the predicted state after surgery. Since these two responses are about the state before surgery and the state after surgery, respectively, these two responses should be treated differently. We then use three different sets of questions, as we did in Chapter 4. We will drop Q7 and Q8 from the analysis because we are dealing with both cancer patients and non-cancer patients. The three sets of questions used for applying our methods are given as follows:

1. Current and Predicted state of Q1-Q5, Q6 and Q10
2. Current state of Q1-Q5, Current state minus Predicted state of Q1-Q5, Q6 and Q10
3. Current state of Q1-Q5, Q6 and Q10.

We will construct the regression model as in (5.1), with the constraint that all coefficients are nonnegative. The estimates of the β's thus obtained are given in Table 5.1. Some estimates of the β's are zero, which means that the numerical distances between two categories are zero. For example, a zero estimate for β_1 implies that level 1 and level 2 are not distinguishable. If no observation exists for a level, then estimates cannot be obtained. For this reason, there are no estimates for β_4 for Q1po, Q2po, Q3po, Q4cu, Q4po, Q5cu, Q5po, Q4ch and Q5ch.

The estimates of the expected value of X given Y assuming β_0=0, are given in Table 5.2. The expected value of X when a level of Y has no observation is replaced by the estimates of the expected value of X for the previous level of Y. For example, the estimate of $E(X|Y = 5)$ is assumed to be the same as the estimate of $E(X|Y = 4)$ in Q1po, Q2po, Q3po, Q4cu, Q4po, Q5cu, Q5po, Q4ch and Q5ch.

The weights based on the five different measures mentioned in Section 5.1 appear as Table 5.3, Table 5.9 and Table 5.15. The weights given in these tables denote the degree of the question's contribution in deciding the urgency of the patient. For example, Q10 contributes 32.3 percent to the Urgency score when weight1 is used.

The scores for the current and predicted state of Q1-Q5, Q6 and Q10, based on five weights, appear in Tables 5.4 through 5.8. The scores of the questions are closely related to the urgency of patients because these scores are obtained based on the scale of urgency (scale from Question 9). Since Q6 has only three levels, the scores for level 4 and 5 are not defined. The scores are proportional to the estimates of the expected value of X given Y, assuming β_0=0. The scores of Q10 are the highest among the scores of all questions, so Q10 is the most important question for deciding the urgency of the patients. Scores of Q1cu obtained by weight4 are all zero; that is, Q1cu has no effect on deciding the urgency of patients. When weight2 is used to obtain scores, the highest score of 1 is given to Q1cu, Q1po, Q3po and Q6, whereas when weight1 is used, the highest score of 4, 5, 4 and 5 is given to Q1cu, Q1po, Q3po and Q6, respectively. The scores given to questions vary according to which weight was used to compute the score.

The scores for the current state of Q1-Q5 and the change of state of Q1-Q5, Q6 and Q10 are given in Tables 5.10 through 5.14. The question Q10 seems to be the most important question for judging the urgency of patients because the scores of Q10 show the highest score of 69 in Table 5.14 when weight5 is used. The scores vary according to the weight used to obtain a score; for example, the scores of Q3ch are zero for all levels when weight4 is used, whereas the score of the highest level of Q3ch is 6 when weight 1 is used.

The scores for the current state of Q1-Q5, Q6 and Q10 are given in Tables 5.16 through 5.20. The scores in these tables are higher than the previous scores because only six questions are included in this set of questions.

Regardless of which set of questions we use, Q10 proves to be the most important question for deciding the urgency of patients. In the next section, we will discuss which method that we have applied so far gives the best Urgency score of patients and which set of questions is the most useful for predicting the urgency of the patients.

5.3 Discussion of the Proposed Method and Comparison with the Optimal Scaling Method

In Sections 5.1 and 5.2, we discussed and applied the proposed method by using weights based on five measures. This thesis aims to find the best method for prioritizing the patients. To accomplish our aim, we need to obtain the Urgency score of the patient and test its performance.

The Urgency score can be computed by adding up the scores of the patient. We then fit the regression model with the Urgency score and the scale from question 9 as follows:

$$X_i = \rho_0 + \rho_1 \text{Urgency score}_i. \tag{5.6}$$

We will use the R-square values to compare the performance of the Urgency score, and the R-square values from the model fits are given in the Table 5.21. The proposed method with weight2 shows high R-square values of 0.739 and 0.729 when the first and the third sets of questions are used. The proposed method with weight4 and weight5 gives high R-square values for all three sets of questions. Among the five weights we used for the proposed method, weight4 and weight5 are recommended. The use of weight2 is not recommended because the R-square value of the model with the second set of the question is

0.476.

Now we will compare the proposed method with the optimal scaling method. The optimal scaling method discussed in Chapter 4 may be affected by linear transformation, so we proposed the alternative method, which is not affected by linear transformation. In Table 5.21, the R-square values obtained from the proposed method are higher than those obtained from the optimal scaling method. Thus, the proposed method seems to perform better than the optimal scaling method. Hence, the proposed method by using weight4 and weight5 is recommended to obtain the scores.

Table 5.1: Estimates and Confidence Intervals of β's from the Regression Model

Question	β_0	β_1	β_2	β_3	β_4
Q1cu	47.63	0.00	5.12	8.47	0.00
	(39.77,55.54)	(-11.77,11.77)	(-6.27,16.52)	(-6.38,23.33)	(-22.40,22.40)
Q1po	48.48	12.03	9.48	19.00	⋆
	(43.77,53.19)	(1.71,22.36)	(-27.84,46.81)	(-43.66,81.66)	⋆
Q2cu	39.78	2.97	19.92	7.62	17.00
	(35.65,45.90)	(-6.71,12.65)	(9.70,30.14)	(-6.27,21.51)	(-5.08,38.88)
Q2po	47.00	15.90	10.27	14.30	⋆
	(42.38,51.60)	(5.45,26.36)	(-12.06,32.61)	(-26.21,54.88)	⋆
Q3cu	46.87	0.00	6.86	20.40	4.00
	(39.73,54.00)	(-9.68,9.68)	(-4.42,18.13)	(4.69,36.13)	(-27.06,35.46)
Q3po	48.65	12.03	6.32	22.00	⋆
	(43.95,53.35)	(1.69,22.37)	(-45.78,58.42)	(-50.52,94.52)	⋆
Q4cu	45.16	0.00	17.26	11.20	⋆
	(37.17,53.15)	(-9.90,9.90)	(6.75,27.77)	(-4.47,26.83)	⋆
Q4po	47.54	16.56	4.10	0.00	⋆
	(42.86,52.21)	(6.22,26.91)	(-26.51,34.71)	(-58.38,58.38)	⋆
Q5cu	44.72	2.70	22.58	2.67	⋆
	(38.08,51.36)	(-6.19,11.59)	(10.70,34.46)	(-16.46,21.79)	⋆
Q5po	48.72	12.31	17.46	10.50	⋆
	(44.12,53.32)	(1.30,22.33)	(-19.99,54.91)	(-52.00,73.00)	⋆
Q6	46.62	17.85	⋆	-	-
	(41.82,51.43)	(5.75,29.95)	⋆	-	-
Q10	12.42	12.23	26.56	21.80	12.00
	(4.77,20.07)	(3.33,21.13)	(20.62,32.50)	(16.34,27.28)	(3.55,20.74)
Q1 ch	49.74	0.00	0.00	12.10	0.00
Q2 ch	43.62	6.84	9.77	12.40	7.00
Q3 ch	47.99	0.00	7.21	19.80	0.00
Q4 ch	47.50	0.00	15.50	14.00	⋆
Q5 ch	46.52	3.27	18.90	0.00	⋆

Table 5.2: Estimates of expected value of X given Y assuming $\beta_0 = 0$

Question	$E(X \mid Y = 1)$	$E(X \mid Y = 2)$	$E(X \mid Y = 3)$	$E(X \mid Y = 4)$	$E(X \mid Y = 5)$
Q1cu	0.00	0.00	5.12	13.59	13.59
Q1po	0.00	12.03	21.51	40.51	40.51
Q2cu	0.00	2.97	22.89	30.51	47.39
Q2po	0.00	15.9	26.17	40.50	40.50
Q3cu	0.00	0	6.86	27.27	31.47
Q3po	0.00	12.03	18.35	40.35	40.35
Q4cu	0.00	0	17.26	28.44	28.44
Q4po	0.00	16.56	20.66	20.66	20.66
Q5cu	0.00	2.70	25.28	27.95	27.95
Q5po	0.00	12.31	29.77	40.27	40.27
Q6	0.00	17.85	17.85	-	-
Q10	0.00	12.23	38.79	60.60	72.74
Q1 ch	0.00	0.00	0.00	12.08	12.08
Q2 ch	0.00	6.84	16.61	28.96	36.39
Q3 ch	0.00	0.00	7.21	27.01	27.01
Q4 ch	0.00	0.00	15.50	29.50	29.50
Q5 ch	0.00	3.27	22.17	22.17	22.17

Table 5.3: Weights for the first set of the questions based on five measurements

Question	weight1	weight2	weight3	weight4	weight5
Q1 cu	3.5	0.8	1.8	0.4	1.8
Q1 po	4.5	1.4	2.9	1.5	1.3
Q2 cu	11.3	8.7	14.1	9.6	16.3
Q2 po	6.5	2.9	5.7	3.3	4.6
Q3 cu	6.8	3.2	6.1	4.1	5.7
Q3 po	4.3	1.3	2.3	4.4	0.9
Q4 cu	8.2	4.0	8.6	3.6	6.5
Q4 po	5.8	1.9	4.7	9.6	2.5
Q5 cu	7.7	4.0	7.7	7.0	5.3
Q5 po	4.7	1.5	3.1	0.9	1.8
Q6	4.5	1.4	3.2	3.9	4.4
Q10	32.3	68.8	39.4	51.7	49.0

NOTE : Weights are based on the following measures

Weight1 : Absolute value of the t statistic for testing H_0:δ_1=0 for model $X_i = \delta_0 + \delta_1 S_{ij}$

Weight2 : Inverse of the variance of δ_1 for model $X_i = \delta_0 + \delta_1 S_{ij}$

Weight3 : R-square for the model $X_i = \delta_0 + \delta_1 S_{ij}$

Weight4 : Absolute value of the t statistic for testing H_0:δ_j=0 for model $X_i = \delta_0 + \delta_1 S_{i1} + \ldots + \delta_l S_{il}$

Weight5 : Inverse of the variance of δ_j for model $X_i = \delta_0 + \delta_1 S_{i1} + \ldots + \delta_l S_{il}$

Table 5.4: Scores from the first set of the questions with weight1 based on the absolute value of t statistic for testing H_0:δ_1=0 for model $X_i = \delta_0 + \delta_1 S_{ij}$

Question	1	2	3	4	5
Q1 cu	0	0	1	4	4
Q1 po	0	1	2	5	5
Q2 cu	0	1	5	7	11
Q2 po	0	3	4	7	7
Q3 cu	0	0	1	6	7
Q3 po	0	1	2	4	4
Q4 cu	0	0	5	8	8
Q4 po	0	5	6	6	6
Q5 cu	0	1	7	8	8
Q5 po	0	1	3	5	5
Q6	0	5	5	-	-
Q10	0	5	17	27	32

Table 5.5: Scores from the first set of the questions with weight2 based on the inverse of the variance of δ_1 for model $X_i = \delta_0 + \delta_1 S_{ij}$

Question	1	2	3	4	5
Q1 cu	0	0	0	1	1
Q1 po	0	0	1	1	1
Q2 cu	0	1	4	6	9
Q2 po	0	1	2	3	3
Q3 cu	0	0	1	3	3
Q3 po	0	0	1	1	1
Q4 cu	0	0	2	4	4
Q4 po	0	2	2	2	2
Q5 cu	0	0	4	4	4
Q5 po	0	0	1	2	2
Q6	0	1	1	-	-
Q10	0	12	37	57	69

Table 5.6: Scores from the first set of the questions with weight3 based on the R-square of the model $X_i = \delta_0 + \delta_1 S_{ij}$

Question	1	2	3	4	5
Q1 cu	0	0	1	2	2
Q1 po	0	1	2	3	3
Q2 cu	0	1	7	9	14
Q2 po	0	2	4	6	6
Q3 cu	0	0	1	5	6
Q3 po	0	1	1	3	3
Q4 cu	0	0	5	9	9
Q4 po	0	4	5	5	5
Q5 cu	0	1	7	8	8
Q5 po	0	1	2	3	3
Q6	0	3	3	-	-
Q10	0	7	21	33	39

Table 5.7: Scores from the first set of the questions with weight4 based on the absolute value of t statistic for testing H_0:δ_j=0 for model $X_i = \delta_0 + \delta_1 S_{i1} + \ldots + \delta_l S_{il}$

Question	1	2	3	4	5
Q1 cu	0	0	0	0	0
Q1 po	0	0	1	2	2
Q2 cu	0	1	5	6	10
Q2 po	0	1	2	3	3
Q3 cu	0	0	1	4	4
Q3 po	0	1	2	4	4
Q4 cu	0	0	2	4	4
Q4 po	0	8	10	10	10
Q5 cu	0	1	6	7	7
Q5 po	0	0	1	1	1
Q6	0	4	4	-	-
Q10	0	9	28	43	52

Table 5.8: Scores from the first set of the questions with weight5 based on the inverse of the variance of δ_j for model $X_i = \delta_0 + \delta_1 S_{i1} + \ldots + \delta_l S_{il}$

Question	1	2	3	4	5
Q1 cu	0	0	1	2	2
Q1 po	0	0	1	1	1
Q2 cu	0	1	8	10	16
Q2 po	0	2	3	5	5
Q3 cu	0	2	3	6	6
Q3 po	0	0	0	1	1
Q4 cu	0	0	4	7	7
Q4 po	0	2	3	3	3
Q5 cu	0	0	5	5	5
Q5 po	0	1	1	2	2
Q6	0	4	4	-	-
Q10	0	8	26	41	49

Table 5.9: Weights for the second set of the questions based on five measurements

Question	weight1	weight2	weight3	weight4	weight5
Q1 cu	3.4	0.8	1.7	5.8	1.0
Q2 cu	11.1	8.5	13.7	11.8	7.7
Q3 cu	6.7	3.1	6.0	2.2	1.9
Q4 cu	8.1	4.0	8.4	0.9	2.2
Q5 cu	7.6	4.0	7.5	2.5	2.6
Q6	4.4	1.4	3.2	3.1	6.3
Q10	31.8	67.5	38.3	55.6	68.9
Q1 ch	3.0	0.6	1.3	7.5	0.9
Q2 ch	7.2	3.6	6.8	6.8	3.8
Q3 ch	5.6	2.2	4.4	0.2	1.4
Q4 ch	5.7	2.3	4.5	2.0	1.7
Q5 ch	5.5	2.1	4.2	1.6	1.6

NOTE : Weights are based on the following measures

Weight1 : Absolute value of the t statistic for testing H_0:δ_1=0 for model $X_i = \delta_0 + \delta_1 S_{ij}$

Weight2 : Inverse of the variance of δ_1 for model $X_i = \delta_0 + \delta_1 S_{ij}$

Weight3 : R-square for the model $X_i = \delta_0 + \delta_1 S_{ij}$

Weight4 : Absolute value of the t statistic for testing H_0:δ_j=0 for model $X_i = \delta_0 + \delta_1 S_{i1} + \ldots + \delta_l S_{il}$

Weight5 : Inverse of the variance of δ_j for model $X_i = \delta_0 + \delta_1 S_{i1} + \ldots + \delta_l S_{il}$

Table 5.10: Scores from the second set of the questions with weight1 based on the absolute value of t statistic for testing H_0:δ_1=0 for model $X_i = \delta_0 + \delta_1 S_{ij}$

Question	1	2	3	4	5
Q1 cu	0	0	1	3	3
Q2 cu	0	1	5	7	11
Q3 cu	0	0	1	6	7
Q4 cu	0	0	5	8	8
Q5 cu	0	1	7	8	8
Q6	0	4	4	-	-
Q10	0	5	17	27	32
Q1 ch	0	0	0	3	3
Q2 ch	0	1	3	6	7
Q3 ch	0	0	1	6	6
Q4 ch	0	0	3	6	6
Q5 ch	0	1	6	6	6

Table 5.11: Scores from the second set of the questions with weight2 based on the inverse of the variance of δ_1 for model $X_i = \delta_0 + \delta_1 S_{ij}$

Question	1	2	3	4	5
Q1 cu	0	0	0	1	1
Q2 cu	0	1	4	5	9
Q3 cu	0	0	1	3	3
Q4 cu	0	0	2	4	4
Q5 cu	0	0	4	4	4
Q6	0	1	1	-	-
Q10	0	11	36	56	68
Q1 ch	0	0	0	1	1
Q2 ch	0	1	2	3	4
Q3 ch	0	0	1	2	2
Q4 ch	0	0	1	2	2
Q5 ch	0	0	2	2	2

Table 5.12: Scores from the second set of the questions with weight3 based on the R-square of the model $X_i = \delta_0 + \delta_1 S_{ij}$

Question	1	2	3	4	5
Q1 cu	0	0	1	2	2
Q2 cu	0	1	7	9	14
Q3 cu	0	0	1	5	6
Q4 cu	0	0	5	8	8
Q5 cu	0	1	7	8	8
Q6	0	3	3	-	-
Q10	0	6	20	32	38
Q1 ch	0	0	0	1	1
Q2 ch	0	1	3	5	7
Q3 ch	0	0	1	4	4
Q4 ch	0	0	2	5	5
Q5 ch	0	1	4	4	4

Table 5.13: Scores from the second set of the questions with weight4 based on the absolute value of t statistic for testing H_0:δ_j=0 for model $X_i = \delta_0 + \delta_1 S_{i1} + \ldots + \delta_l S_{il}$

Question	1	2	3	4	5
Q1 cu	0	0	2	6	6
Q2 cu	0	1	6	8	12
Q3 cu	0	0	0	2	2
Q4 cu	0	0	1	1	1
Q5 cu	0	0	3	3	3
Q6	0	3	3	-	-
Q10	0	9	30	46	56
Q1 ch	0	0	0	8	8
Q2 ch	0	1	3	5	7
Q3 ch	0	0	0	0	0
Q4 ch	0	0	1	2	2
Q5 ch	0	0	2	2	2

Table 5.14: Scores from the second set of the questions with weight5 based on the inverse of the variance of δ_j for model $X_i = \delta_0 + \delta_1 S_{i1} + \ldots + \delta_l S_{il}$

Question	1	2	3	4	5
Q1 cu	0	0	0	1	1
Q2 cu	0	0	4	5	8
Q3 cu	0	0	0	2	2
Q4 cu	0	0	1	2	2
Q5 cu	0	0	3	3	3
Q6	0	6	6	-	-
Q10	0	12	37	57	69
Q1 ch	0	0	0	1	1
Q2 ch	0	1	2	3	4
Q3 ch	0	0	0	1	1
Q4 ch	0	0	1	2	2
Q5 ch	0	0	2	2	2

Table 5.15: Weights for the third set of the questions based on five measurements

Question	weight1	weight2	weight3	weight4	weight5
Q1 cu	4.7	0.9	2.2	2.9	2.2
Q2 cu	15.2	9.6	17.4	13.6	19.3
Q3 cu	9.1	3.5	7.6	5.8	6.2
Q4 cu	11.1	4.4	10.7	4.0	7.2
Q5 cu	10.3	4.4	9.5	8.5	6.3
Q6	6.0	1.5	4.0	3.0	5.0
Q10	43.5	75.6	48.6	62.3	53.8

NOTE : Weights are based on the following measures

Weight1 : Absolute value of the t statistic for testing $H_0{:}\delta_1{=}0$ for model $X_i = \delta_0 + \delta_1 S_{ij}$

Weight2 : Inverse of the variance of δ_1 for model $X_i = \delta_0 + \delta_1 S_{ij}$

Weight3 : R-square for the model $X_i = \delta_0 + \delta_1 S_{ij}$

Weight4 : Absolute value of the t statistic for testing $H_0{:}\delta_j{=}0$ for model $X_i = \delta_0 + \delta_1 S_{i1} + \ldots + \delta_l S_{il}$

Weight5 : Inverse of the variance of δ_j for model $X_i = \delta_0 + \delta_1 S_{i1} + \ldots + \delta_l S_{il}$

Table 5.16: Scores from the third set of the questions with weight1 based on the absolute value of t statistic for testing H_0:δ_1=0 for model $X_i = \delta_0 + \delta_1 S_{ij}$

Question	1	2	3	4	5
Q1 cu	0	0	2	5	5
Q2 cu	0	1	7	10	15
Q3 cu	0	0	2	8	9
Q4 cu	0	0	7	11	11
Q5 cu	0	1	9	10	10
Q6	0	6	6	-	-
Q10	0	7	23	36	44

Table 5.17: Scores from the third set of the questions with weight2 based on the inverse of the variance of δ_1 for model $X_i = \delta_0 + \delta_1 S_{ij}$

Question	1	2	3	4	5
Q1 cu	0	0	0	1	1
Q2 cu	0	1	5	6	10
Q3 cu	0	0	1	3	4
Q4 cu	0	0	3	4	4
Q5 cu	0	0	4	4	4
Q6	0	2	2	-	-
Q10	0	13	40	63	76

Table 5.18: Scores from the third set of the questions with weight3 based on the R-square of the model $X_i = \delta_0 + \delta_1 S_{ij}$

Question	1	2	3	4	5
Q1 cu	0	0	1	2	2
Q2 cu	0	1	8	11	17
Q3 cu	0	0	2	7	8
Q4 cu	0	0	6	11	11
Q5 cu	0	1	9	10	10
Q6	0	4	4	-	-
Q10	0	8	26	40	49

Table 5.19: Scores from the third set of the questions with weight4 based on the absolute value of t statistic for testing H_0:δ_j=0 for model $X_i = \delta_0 + \delta_1 S_{i1} + \ldots + \delta_l S_{il}$

Question	1	2	3	4	5
Q1 cu	0	0	1	3	3
Q2 cu	0	1	7	9	14
Q3 cu	0	0	1	5	6
Q4 cu	0	0	2	4	4
Q5 cu	0	1	8	9	9
Q6	0	3	3	-	-
Q10	0	10	33	52	62

Table 5.20: Scores from the third set of the questions with weight5 based on the inverse of the variance of δ_j for model $X_i = \delta_0 + \delta_1 S_{i1} + \ldots + \delta_l S_{il}$

Question	1	2	3	4	5
Q1 cu	0	0	1	2	2
Q2 cu	0	1	9	12	19
Q3 cu	0	0	1	5	6
Q4 cu	0	0	4	7	7
Q5 cu	0	1	6	6	6
Q6	0	5	5	-	-
Q10	0	9	29	45	54

Table 5.21: Table of R-squares

Method	Current and Post	Current and Change	Current only
Optimal Scaling Method	0.655	0.511	0.642
Proposed method with weight 1	0.614	0.506	0.604
Proposed method with weight 2	0.739	0.476	0.729
Proposed method with weight 3	0.660	0.579	0.645
Proposed method with weight 4	0.711	0.662	0.698
Proposed method with weight 5	0.676	0.718	0.683

Chapter 6

Concluding Remarks and Future Research

In many surveys, information is measured in two ways: most items are measured by using an ordinal scale, and the overall characteristics are measured on a numerical scale, for example, 1 to 100. Often, this overall measure is implicitly correlated with other measurements in the study. The methodology proposed in this thesis can be used to estimate this implicit relationship and to assign **optimal scores** to the ordinal items that are correlated with the overall measures.

Now we will summarize the results by comparing the current prioritization method and the proposed method using constraint regression. The optimal scaling method produces negative scales. A shortcoming of the optimal scaling method results from the linear transformation of the scales. To overcome the shortcoming of the optimal scaling method, we suggested the proposed method. The proposed method using constraint regression also allows the user to decide whether some categories can be combined within questions.

We compared the performance of the proposed method using constraint regression with the optimal scaling method by using the R-square values. Overall, the proposed method with the recommended weights produced higher R-square values than the optimal scaling method. Therefore, the proposed method with weights based on the t statistics and the inverse of the variance of the coefficients from the regression model with multiple covariates is recommended as a method of prioritization.

Finally, we will discuss the shortcomings of the study and suggest directions for future research. The last question used as one of the predictors in the analysis elicited information about the urgency of the patient. This information is related to the rate of urgency used as a dependent. Hence, a higher correlation was observed between the last predictor and dependent. This finding resulted in assigning a higher weight to the last predictor. However, in the present analysis, this question was included based on the physician's opinion. So in further research, we will investigate the effect of removing this question and assigning weights to other questions for predicting the Urgency score.

We also suggest modifying the optimal scaling method so that the nonnegative values can be obtained.

Ten different physicians were involved in the study, and significant variations in their responses to the question related to the overall rating of urgency were noted. Thus, we propose to use the multilevel model in order to obtain optimal scores by incorporating this additional source of variation.

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Western Canada Waiting List Project

GENERAL SURGERY PRIORITY CRITERIA

PLEASE PRINT CLEARLY
Provincial Health Care Number: ______________________

Patient Age: ________ Sex: [circle one] **M** **F**

Check all surgical procedures for which this patient is currently waiting:

☐ Gallbladder Disease ☐ Hernia ☐ Other: ______________________

☐ Cholecystitis ☐ Pancreatitis ☐ Cancer Body Part: ______________________

Stage: ______________________

Ratings assume patients are receiving appropriate medical therapy (e.g. analgesics)

PLEASE CHECK THE BOX WHICH MOST ACCURATELY DESCRIBES THE PATIENT'S CURRENT SITUATION AND THE PATIENT'S EXPECTED SITUTATION 3-6 MONTHS AFTER SURGERY

	Current	Predicted 3-6 months (post-operative)
1. Usual intensity of pain:		
Extremely intense	☐	☐
Very intense	☐	☐
Moderately intense	☐	☐
Mild	☐	☐
None	☐	☐
2. Usual intensity of other forms of suffering (e.g. nausea or vomiting, shortness of breath, itching, psychological stress such as anxiety or depression):		
Extremely intense	☐	☐
Very intense	☐	☐
Moderately intense	☐	☐
Mild	☐	☐
None	☐	☐
3. Usual frequency of painful episodes/suffering:		
Constant, never pain-free	☐	☐
Only short episodes pain-free	☐	☐
In pain about half the time	☐	☐
Infrequent episodes of pain	☐	☐
No pain	☐	☐
4. Usual degree of impairment in role function (e.g. job, housework, independence) due to surgical condition:		
Unable to perform any role function	☐	☐
Able but very difficult and at much reduced level	☐	☐
Able but difficult and/or somewhat impaired	☐	☐
Mildly impaired	☐	☐
Not impaired at all	☐	☐

5. **Usual degree of impairment in social activities (e.g. mobility, visiting friends/family, hobbies):**

Unable to perform any social activity	☐	☐
Able but very difficult and at much reduced level	☐	☐
Able but difficult and/or somewhat impaired	☐	☐
Mildly impaired	☐	☐
Not impaired at all	☐	☐

6. **History of major complications of condition:**
   - ☐ Yes, recently
   - ☐ Yes, but not recently
   - ☐ No

COMPLETE QUESTIONS 7 AND 8 FOR PATIENTS WITH A CANCER DIAGNOSIS ONLY

7. **Life-expectancy implications of condition without procedure:**
   - ☐ Patient has condition that is likely to be fatal within six months
   - ☐ Patient has condition that is likely to be fatal between six months and two years
   - ☐ Patient faces substantially reduced life expectancy
   - ☐ Patient faces somewhat reduced life expectancy
   - ☐ Minimal threat to life

8. **Expected degree of improvement (EDI) in life expectancy with surgery:**
   - ☐ Surgery is almost certain to restore normal life expectancy
   - ☐ Surgery is very likely to restore normal life expectancy or almost certain to substantially increase life expectancy
   - ☐ Surgery is somewhat likely to restore normal life expectancy or very likely to substantially increase life expectancy
   - ☐ Surgery is not likely to restore normal life expectancy but somewhat more likely to substantially increase life expectancy
   - ☐ Surgery is unlikely to substantially increase life expectancy

9. **Compared with all the patients that you evaluate, how would you rate the urgency or relative priority of this patient? Please circle one. [0 = not urgent at all; 10 = most urgent, virtually an emergency]**

   0 1 2 3 4 5 6 7 8 9 10

10. **Compared with all the patients that you evaluate, how would you rate the urgency or relative priority of this patient? Please check only one box.**
    - ☐ Much more urgent than the average patient
    - ☐ More urgent than the average patient
    - ☐ About as urgent as the average patient
    - ☐ Less urgent than the average patient
    - ☐ Much less urgent than the average patient

Comments: ____

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