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Does a Discount Rate Rule Ensure a Pension Plan Can Pay Promised Benefits without Excessive Asset Accumulation?

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Abstract

The choice of discount rate makes a substantial difference to the magnitude of the assets required to ensure a pension plan is fully funded. Finance theory suggests that the discount rate should equal the default-free rate, but pension plan administrators argue for a rate equal to the long run return on plan assets. We evaluate the ability of a fully funded pension plan to meet its promised benefit payments when the plan's liabilities are determined using different discount rate-setting rules. To account for the uncertainty of the return to plan assets and future benefit payments, we employ Monte Carlo techniques and estimates using U.S. data. Due to the volatility of pension fund asset returns and payouts, to generate a high probability of meeting promised pension payments, a plan must use a discount rate that leads, on average, to the accumulation of significant assets in excess of those required to cover promised benefits. The better-performing rules are a function of economic variables, such as the return on government bonds or the inflation rate. Two rules that yield a relatively high probability that pension obligations can be met, combined with the relatively low accumulation of excess assets, set the discount rate equal to a proxy for the corporate bond yield or an inflation forecast plus 3 percent. These rates are greater than the default free rate, but lower than the return on the plan portfolio.

Keywords: Pension plans; discount rate; pension sustainability; defined benefit pension; policy rules.

JEL Codes: H55, H75, H83, J26, J32.

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1. Introduction

Pension plans involve contributions and payouts that occur at different points in time. To evaluate whether the assets of a plan are sufficient to cover promised benefits, plan administrators calculate the present value of expected future payouts, which requires the use of a discount rate. The choice of discount rate can make a substantial difference to the magnitude of the assets required for a pension plan to be fully funded.[1] For example, US public sector pension plans acknowledge a funding shortfall of approximately $1 trillion, but the use of a discount rate equal to the return on Treasury bonds, which is lower than the rate employed by many public sector plans, increases the unfunded liability to over $3 trillion (Gale and Krupkin, 2016; Rauh, 2017).

An extensive literature examines the impact of different discount rates on the *funding status* of specific pension plans.[2] In this paper we investigate a related but different question: How do different rules for setting the discount rate affect the ability of a *fully funded* plan to meet its promised benefit payments, while not accumulating excessive assets? In a world with no uncertainty, a rule that sets the discount rate equal to the realized rate of return on pension plan assets yields a value for a fully funded plan's projected liabilities that just equals the present discounted value of actual future plan benefit payments. The plan will, as a result, be able to pay the pension benefits promised and have no excess assets. In a world with uncertainty surrounding future pension payments and future returns on plan assets, the projected liabilities of the plan may differ significantly from actual future pension payments.

A discount rate setting rule that yields a low discount rate ensures that promised pension payments will be met with high probability, but is more likely to generate excess assets and requires high current contribution rates, which can unnecessarily burden governments, firms, and workers. On the other hand, if a rule sets too high a discount rate, so too few assets are accumulated, pensioners are less likely to receive the pensions they were promised.[3] With a government pension plan, taxpayers may be required to make up any shortfall, implying an intergenerational transfer. Firms and governments that operate pension plans have an incentive to choose a high discount rate since, by reducing the present discounted value of expected future pension payments, a higher discount rate improves the apparent health of a pension plan and, thereby, reduces current plan contributions.

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[1] We define a pension plan as fully funded at time t if the plan's assets equal the present discounted value of forecast future benefit payments for service accrued up to time t, where the present discounted value is calculated using the plan's discount rate.

[2] See, for example, Brown and Wilcox (2009), Munnell, Aubry, Belbase and Hurwitz (2013), Novy-Marx and Rauh (2014), and Farrell and Shoag (2017).

[3] The bankruptcy of Detroit in 2013 led to cuts in the pensions of retired workers.

There is no generally agreed-upon best method for choosing a discount rate. Many pension fund experts argue that pension obligations are similar to risk-free Treasury bonds, so expected future payments should be discounted at a rate that reflects the lower risk associated with the liabilities of the plan, rather than the risk of the assets held by the plan.[4] Following this argument, Novy-Marx and Rauh (2011) argue that the appropriate discount rate for US state pensions is a (tax-adjusted) municipal bond rate. Brown and Wilcox (2009) and Andonov, Bauer and Cremers (2017) go further and argue that the appropriate rate is lower than the municipal bond rate and closer to the nearly default-free zero-coupon US Treasury rate. Their argument is that, due to state constitutional guarantees of pension obligations, states have a lower probability of defaulting on their pension obligations than municipal governments have of defaulting on municipal bonds. In a similar vein, Brown and Pennacchi (2015) maintain that, according to finance theory, the rate used to discount public pension liabilities should be a default-free rate, regardless of a government's likelihood of default, since pension liabilities are riskless. Others have argued that the discount rate should be equal to the yield on an annuity that could be purchased if the fund closed, a rate that is generally lower than the yield on Treasuries (Actuarial Standards Board, 2013). In contrast to these recommendations, the US *National Association of State Retirement Administrators* (*NASRA*) argues that the discount rate should be a reasonable estimate of the long-term rate of return on plan assets, a position *NASRA* notes is supported by the Governmental Accounting Standards Board (*NASRA*, 2010, 2011).

There has been considerable research on the relationship between pension plan discount rates and plan underfunding. Brown and Wilcox (2009), Munnell, Aubry, Belbase and Hurwitz (2013), Novy-Marx and Rauh (2014), Rauh (2017), and Farrell and Shoag (2017) quantify the extent of the underfunding of U.S. state and local pension funds when lower, but more appropriate in their view, discount rates are employed. There has been less discussion of methods to ensure that pension funds have sufficient assets to meet their promised future benefit payments. Studies by Bucciol and Beetsma (2011), Freeman (2013), Beetsma, Lekniute and Ponds (2014), Eldar and Wagner (2015), Boyd and Yin (2016), Turner, et al. (2017) and Lekniute, Beetsma and Ponds (forthcoming) incorporate uncertainty, generally of plan asset returns, and use simulations based on historical data to assess how changes in the discount rate can alter a specific pension plan's funding status. In their studies, Bucciol and Beetsma (2011), Freeman (2013) and Turner, et al.

[4] Specifically, Novy-Marx (2015) argues "the appropriate discount rate for a pension fund's liabilities is the expected rate of return on an optimal 'hedge portfolio', the portfolio that would be held under a liability–driven investment policy (i.e., the portfolio of traded assets that has cash flows that most closely approximates the fund's expected future benefit payments)."

(2017) find, consistent with our results, that a discount rate which varies with economic factors, such as bond yields and the inflation rate, leads to better outcomes.[5]

Our paper differs from earlier work in that we focus on identifying the best discount rate setting *rule* and we evaluate a large number of different types of discount rates. Further, our analysis focuses on a general pension plan, rather than a particular plan for a specific historical period. In addition, we allow for uncertainty with respect to both the return on plan assets and future benefit payments, rather than for return uncertainty only.

The discount rate setting rules we analyze are similar to rules adopted by governments and private pension funds around the world. We examine constant integer discount rates from 3 to 13 percent, since a number of countries stipulate fixed discount rates and use these for long periods (Ponds, Severinson, Yermo (2011)). As several researchers have suggested a discount rate equal to the yield on low risk bonds, and some governments use a rate of this type, we evaluate a rule that sets the discount rate equal to the yield on 10-year U.S. Treasury bonds. To limit the effect of year-to-year yield volatility, we also examine 5, 10, 20 and 30 year moving averages of the 10-year Treasury yield. In addition, we consider discount rates that proxy the return on high quality corporate bonds by augmenting the Treasury yield and the moving averages of the Treasury yield by a fixed percentage. Similarly, to proxy a discount rate that equals the return on annuities, which is typically lower than the return on Treasury bonds, we employ a discount rate that equals the Treasury yield or the moving average of the Treasury yield less a fixed percentage.[6] The U.S. *National Association of State Retirement Administrators* (2010) advocates a discount rate that approximates a pension plan's return on assets, so we evaluate discount rates that equal 10, 20 and 30 year geometric averages of the return on the plan's investment portfolio. Finally, we consider a discount rate rule that augments an estimate of long term inflation with a fixed percentage premium since some plan actuaries recommend the use of a discount rate that equals the sum of an inflation forecast and the long run real return, where the long run real return is proxied by a fixed percentage (Ménard, 2013).

To assess the impact of different discount rate setting rules on the ability of a pension plan to meet its future promised benefits, we specify a plan that faces uncertainty with respect to three variables: (1) the future return on pension plan assets, which depends on both equity and bond returns; (2) the future final salary of current workers; and (3) future inflation, which affects

[5] For example, Bucciol and Beetsma (2011) assess a change in 2007 which required pension funds in the Netherlands to use a market term structure for discounting pension liabilities. They find that the mark-to-market discounting method leads to higher aggregate welfare than the previously employed constant 4 percent discount rate.
[6] We use proxies for the corporate bond yield and the annuity rate to reduce the number of variables to be simulated.

liabilities as a result of benefit indexation. Hence, our model incorporates uncertainty in terms of both asset returns and benefit payments. As regulators and pension fund managers do not know the true path of future plan earnings or liabilities when the discount rate is set, we employ Monte Carlo techniques. These allow us to evaluate the performance of different discount rate setting rules while taking into account the uncertainty associated with future benefit payments and asset returns.

A key result of our analysis is that it is necessary to use a discount rate that, on average, generates significant excess assets if there is to be a high probability of having sufficient assets to make promised future pension benefit payments. More specifically, there are no discount rate setting rules for which excess assets are low – under 20 percent – and the percentage of cases for which assets are sufficient to meet obligations is high – over 90 percent. Even the better rules have relatively large excess assets on average as well as a relatively high probability that accumulated assets will not be adequate to cover promised pension payments.

A second finding is that the better-performing rules vary the discount rate with changes in economic variables, such as the yield on Treasury bonds and the inflation rate. To understand the intuition for this finding, consider the case in which a downward trend in Treasury yields reduces pension plan portfolio returns. Given this change, to meet future obligations, the pension fund must accumulate more assets and contributions to the plan must increase. If the discount rate varies with the Treasury yield, a fall in the Treasury yield causes a fall in the discount rate, which will prompt a rise in projected liabilities and contribution rates. While pension plan sponsors can alter contribution rates when economic conditions change, a rule that makes the discount rate adjust automatically does not require plan sponsors to make a judgement about the need to increase contribution rates, a decision that may be unpopular with plan members.

A third result is that the choice of the "best" discount rate rule depends crucially on the objective function of plan sponsors and members. In particular, it hinges on the weight allocated to the benefit of a higher probability of meeting pension obligations versus the weight given to the cost of holding higher average excess assets. For example, a Treasury bond-based discount rate setting rule, as advocated by finance experts, is preferred if a high weight is given to ensuring that the plan has sufficient assets to meet its future obligations. On the other hand, a rule based on the long term expected return on pension fund assets, espoused by US state pension fund managers (*NASRA*, 2010), performs well if the goal is to set assets equal to future obligations on average, without reference to the probability that assets be sufficient to meet obligations. Two rules that perform well, while giving approximately equal weight to the goals of ensuring a high probability of meeting future benefit obligations and minimizing excess assets, are a rule that proxies the corporate bond yield and a rule based on an inflation forecast plus 3%. The discount rates generated by these two rules are greater than the default free rate, but lower than the return on the plan portfolio.

Finally, for almost all the rules examined, there is a greater than 60 percent probability that plan assets will either fall short of obligations or exceed obligations by more than 20 percent. Thus, there is a strong likelihood of a large difference between accumulated assets and promised benefit payments.

In the next section we describe the methodology, the discount rate rules and the data. In section 3 we outline our results, while we summarize and discuss the implications of our findings in Section 4.

2. Methodology

2.1 The Pension Fund Model

To evaluate the impact of different discount rate setting rules on the ability of a pension to meet its promised pension benefit obligations, it is necessary to specify a model of a pension plan. To avoid transition and implementation effects, we assume a mature plan, so the ratio of retirees to workers is not changing and all retirees have the maximum number of pensionable years of service. Every year one person enters work, one person retires and one person dies, so there is one person of each age and the plan is in a steady state with respect to the number of workers and retirees. Workers retire at age R, and age is normalized so that age at the end of the first year of work is 1, which implies that retirees have worked for R years. In each year t, a worker earns a salary, accumulates pensionable service, and pays contributions to the pension plan at the end of the year. All workers earn the same wage (W_t), no matter their age, and movements in the wage through time are the same for all workers.[7] A retiree receives a pension based on the number of years they have worked and the replacement rate of salary (p) for each year of pensionable service. The pension has a defined benefit with payments based on final year salary, where the final salary for each retiree differs since each retiree stops working at the end of a different year. The pension is paid at the end of the year for all ages $R+1,...,T$, where T is the age at end of life.

The pension plan is fully funded at the end of each period. That is, given the assumed

[7] The alternative is to include an age-dependent wage gradient in the model. If this gradient is constant, nothing important changes, only an additional parameter is added to the model. If the gradient is stochastic, this adds one more element of uncertainty to the model, which would give this issue more prominence than seems warranted relative to the other forms of uncertainty.

discount rate and the forecast future path of plan benefit obligations, the assets accumulated by the plan are sufficient to meet projected benefit payments for all service accrued up to that point in time.

Our goal is to determine how best to set the pension plan's discount rate so the plan accumulates sufficient assets to meet its *actual* promised future pension benefits while not accumulating excess assets. Thus, the important comparison in our analysis is between the *projected* liabilities of the plan, which depend on the discount rate chosen as well as forecasts of future benefit payments, and *actual* future pension payments. To make this comparison, we must first determine the projected liabilities of the plan at a point in time and then compare these to the actual future pension benefits the plan has promised to pay for accrued service up to that same point in time.

At the end of each year t, the pension plan sponsor calculates a forecast of the present discounted value of the future pension payments (payments to be made in $t+1$ and beyond) that have been promised to each current worker and retiree for pensionable service up to and including period t. Let these projected liabilities of the plan be represented by PL_t. To calculate this forecast of the pension liability, it is necessary to determine the pension benefits that will be paid to current retirees for the remainder of their lifetimes and to current workers once they retire. Since each worker and retiree has a different final year salary, and the workers each have different years of pensionable service in year t, the liability for workers and retirees of each age are calculated separately and then summed to arrive at the estimate of the total projected pension liability.

The current retiree pension liability is the liability for individuals who have ages $R+1$ to $T-1$ in period t. A retiree of age T has reached the last year of life at the end of period t, so entails no future liability. The key differences between retirees and workers is that all retirees have R years of service, the final salaries of retirees are known, and the inflation rate used to index the pensions of retirees up to period t is also known. At the end of year t, a retiree of age $R+k$, for $k = 1,\ldots,T-R-1$, generates a pension liability, due to the future pension benefits they will receive in periods $t+1$ and beyond, equal to:

$$PL^R_{R+k,t} = [pRW_{t-k}]\left[\prod_{i=0}^{k}(1+I\pi_{t-i})\right]\left[\sum_{j=1}^{T-R-k}\frac{\left(1+I\pi^F_t\right)^{j-1}}{(1+\delta_t)^j}\right], \quad \text{for all } k = 1,\ldots,T\text{-}R\text{-}1, \qquad (1)$$

where W_{t-k} is the final year salary of a retired person of age $k+R$ in period t, where their last year of work was in period t-k; p is the replacement rate of final year salary per year of pensionable service; I is the rate at which pension payments are indexed to inflation, $0 \leq I \leq 1$; π_t is the inflation rate in period t, so the price increase from the end of $t-1$ to the end of t; π_t^F is the forecast in period t of future inflation, which is required to forecast the increase in indexed benefits; and δ_t is the discount rate used in period t to discount future projected pension payments. The analysis below considers different rules for determining the value of δ_t. All variables are measured in nominal terms so it is easier to keep track of indexation and inflation effects.

Given equation (1), the total pension liability associated with future pension payments to all current retirees is projected at time t to be:

$$PL_t^R = \sum_{k=1}^{T-R-1} PL_{R+k,t}^R \ . \tag{2}$$

The accrued pension liability of each current worker at the end of period t is based on the worker's pensionable years of service and a forecast of the salary the worker will earn in their final year of work at age R.[8] Thus, at the end of period t, a current worker with k years of pensionable service, where $k=1,\ldots,R$, generates a pension liability of:

$$\begin{aligned} PL_{kt}^W &= [pkW_t(1+\pi_t^{WF})^{R-k}]\left[\sum_{j=1}^{T-R} \frac{(1+I\pi_t^F)^j}{(1+\delta_t)^{R-k+j}}\right], \\ &= \left[pkW_t\left(\frac{1+\pi_t^{WF}}{1+\delta_t}\right)^{R-k}\right]\left[\sum_{j=1}^{T-R}\left(\frac{1+I\pi_t^F}{1+\delta_t}\right)^j\right], \end{aligned} \tag{3}$$

where π_t^{WF} is the forecast in period t of future wage inflation, which is used to predict the worker's final year salary, and W_t is the wage of all workers working in period t.

The total projected pension liability of all current workers at the end of period t is:

$$\begin{aligned} PL_t^W &= \sum_{k=1}^{R} PL_{kt}^W, \\ &= \left[\sum_{j=1}^{T-R}\left(\frac{1+I\pi_t^F}{1+\delta_t}\right)^j\right]\left[\sum_{k=1}^{R} pkW_t\left(\frac{1+\pi_t^{WF}}{1+\delta_t}\right)^{R-k}\right] \\ &= \left[\frac{\alpha(1-\alpha^{T-R})}{1-\alpha}\right]\left[\sum_{k=1}^{R} pkW_t\left(\frac{1+\pi_t^{WF}}{1+\delta_t}\right)^{R-k}\right] \end{aligned} \tag{4}$$

where $\alpha = \frac{1+I\pi_t^F}{1+\delta_t}$.

The total pension liability of current workers and current retirees that result from accrued pensionable service up to the end of period t (for pensions to be paid in $t+1$ and beyond) is projected to be:

[8] The benefits promised to a worker as of the end of period t, for service accrued up to the end of t, depend on the final year salary of the worker (which is the salary of the worker at age R). These benefits will begin to be paid when the worker reaches age $R+1$. An alternative would be to base the benefits on the salary of the worker in period t, but this would not be consistent with the final year salary pension promised to the worker.

$$PL_t = PL_t^W + PL_t^R \,. \tag{5}$$

The variable PL_t represents the projected liabilities associated with a *forecast* at the end of period t of the pension payments to be made in $t+1$ and beyond, so may be different from the discounted value of *actual* future pension benefit payments. While pension plan administrators know the formula that determines future pension benefits, the present discounted value of *actual* future pension payments is not known with certainty at the time pension contributions are collected due to uncertainty with respect to wage growth, inflation and the return on fund assets. In order to focus the analysis on the choice of the discount rate in the face of these three types of uncertainty, we assume there are no survivor benefits and there is no uncertainty with respect to the number of workers or retirees, years of service, or the mortality rate.[9]

At the end of every period, the pension plan has assets given by:

$$A_t = (1+r_t)A_{t-1} + c_tS_t - B_t \,, \tag{6}$$

where r_t is the return on the assets in the plan from the end of period $t-1$ to the end of period t, with r_t observed at the end of period t. The return on pension plan assets in period t is given by:

$$r_t = \theta r_t^S + (1-\theta) r_t^B, \tag{7}$$

where θ is the share of assets invested in equities, r_t^S is the return on equities in period t, and r_t^B is the return on bonds in period t.

The variable S_t is the annual salary bill of all workers, $S_t = RW_t$; where R is the number of workers because there is one worker of each age and R equals the age at retirement (and age at the end of the first year of work is normalized to 1). The pension contribution rate, c_t, is the percentage of salary contributed to the pension plan at the end of each year. The variable B_t is the aggregate pension benefit payment made to all retirees at the end of period t and is given by:

$$B_t = \sum_{i=R+1}^{T} B_{it}, \tag{8}$$

where B_{it} is the pension benefit paid at the end of period t to a retiree of age i:

$$B_{it} = pRW_{t-(i-R)} \prod_{j=1}^{i-R} (1 + I\pi_{t-j}) \qquad \text{for } i = R+1, \ldots, T, \tag{9}$$

where $W_{t-(i-R)}$ is the final year salary of a retired person of age i in period t.

The contribution rate is set so that the pension plan is fully funded at the end of each

[9] We also do not allow for early retirement, quits and deferred pensions, all of which would introduce more uncertainty. Further, we assume that the pension contribution rate does not affect the wage and consider only one method of specifying the salary on which pension benefits are based (the salary in the final year of work). It seems unlikely that use of the average of the salary in the last five years of work (employed by some pension plans) would have a significant effect on the results.

period t. Although known, the values of A_{t-1}, r_t, S_t and B_t are out of the control of the plan at the end of period t, so fully funded status is accomplished by setting the contribution rate (c_t) such that the value of the plan's assets (A_t) equals projected liabilities (PL_t). While contributions are set so assets equal projected liabilities, which ensures the plan is always fully funded *ex ante*, the uncertain nature of future portfolio returns, wages and prices means that the projected liability calculated at the end of period t may not equal the present discounted value of *actual* future pension benefit payments for service accrued up to the end of t.[10]

In order to determine whether the assets of the fully funded pension plan are adequate to cover *actual* promised future pension benefit payments, we must calculate the present value of the *actual* future payments to be made by the plan. These are calculated using formulas similar to those given in equations (1) through (5) above with two important modifications. First, rather than employing forecasts of inflation and wage growth, the actual future payments are calculated using the realized future values of inflation and wage growth. Second, the present value of the actual future benefit payments is calculated using the realized return on the assets of the plan as the discount rate, where this return can differ in every future year. It is the differences between actual inflation and forecast inflation, actual wage growth and forecast wage growth, and the actual return on the portfolio and the discount rate (δ_t), that cause the assets required to meet actual future pension payments to differ from the assets accumulated by the fully funded pension plan. Detailed examples of the formulas used to calculate the projected liabilities (PL_t) and the actual future pension benefit payments (APP_t) for workers of different ages are given in *Appendix 1*.

In order to calculate explicit comparisons between the assets of the fully funded pension fund and the actual future benefit payments to be made by the plan, we must choose specific values for the parameters of the model. The retirement age R is set equal to 40 and the total years of life, T, is 60, so the worker is retired for 20 years. The age at the end of the first year of work is 1, so end of life occurs 60 years after the worker reaches working age.[11] We consider two portfolios, an equity-dominant portfolio with θ equal to .65, and a bond-dominant portfolio with θ equal to .35.[12] We set the replacement rate, p, so that 60 percent of final salary is replaced. This implies that $p = .015$ (40 years of work × 1.5% = 60%). The indexation rate, I, is set to 100

[10] Since we impose full funding of the pension plan each year, we do not address the decision of whether to fully fund the plan in a particular year.
[11] A 20-year retirement period was chosen as it is similar to the life expectancy at retirement in the U.S., which for females is 23.8 years and for males is 21.6 years (Society of Actuaries, 2014).
[12] Common equity share targets range from 50% to 70% for *National Association of State Retirement Administrators* survey participants (Callan Associates, 2010, p. 10). Munnell, Aubry, and Hurwitz (2013) note that a portfolio invested 65 percent in equities and 35 percent in bonds corresponds to the portfolio of a typical U.S. public pension plan. Aubry, Chen and Munnell (2017) find that the equity/bond ratio has remained relatively stable.

percent.[13] If the inflation rate is negative, benefits are reduced. In Section 3.4, we gauge the robustness of our results to variations in these parameter choices.

The formula used to calculate projected liabilities (PL_t) involves forecasts of future wage growth and inflation. We assume the plan uses forecasts of the future inflation rate and wage growth rate that equal a 20-year moving average (using the current and 19 previous periods) of the inflation and wage growth rates, respectively. This formula allows the forecast to change through time, but the 20 year average constrains the magnitudes of these changes.[14]

2.2 Discount rate rules

Our purpose is to determine how different rules for setting the discount rate (δ_t) affect the likelihood that a fully funded pension plan will have adequate assets to meet its actual future benefit obligations, while minimizing the accumulation of excess assets. We analyze the following discount rate setting rules, which are similar to those that have been employed by pension funds worldwide:

i) *Geometric average rule:* Rolling 10, 20 and 30 year geometric averages of the returns on the pension fund's asset portfolio. This rule is examined because some pension plans are required to use a discount rate based on the expected return on the plan's asset portfolio.[15]

ii) *10-year Treasury yield rule*: The 10-year Treasury bond yield. This yield is an approximation to the risk-free return and is recommended by some financial economists due to the low risk associated with pension liabilities.[16] We also consider the 5, 10, 20 and 30 year moving averages of the 10-year Treasury yield to determine whether it is beneficial to remove some of the volatility from yields.

iii) *Corporate bond yield rule*: An approximation to the high quality corporate bond yield (proxied by the 10-year Treasury yield plus 1.5 percentage points).[17] Some US private sector employers are required to use a discount rate equal to a high-quality corporate bond yield.[18] We also examine the performance of 5, 10, 20 and 30 year moving averages of this corporate bond yield proxy.

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[13] Indexation is lagged one year, so benefits are increased from year-to-year depending on the actual inflation rate calculated at the end of the previous year. Therefore, the percentage increase in benefits in period t as a result of indexation is $I \times \pi_{t-1}$.

[14] Results change by only a small amount when we use a 10-year (rather than 20-year) moving average to calculate the wage growth and inflation rate forecasts.

[15] For U.S. state and local government employers, the discount rate used for defined benefit pension plans "should be based on an estimated long-term investment yield for the plan" (Governmental Accounting Standards Board, 1994, p.6).

[16] See Novy-Marx (2015), Brown and Wilcox (2009), Munnell, et al. (2010).

[17] The basis for this approximation is that 1.5 percentage points is the average difference between the annual 10-year Treasury yield and the annual yield on high quality corporate bonds over the 20 year period from 1996-2015 using *Moody's Yield on Seasoned Corporate Bonds – All Industries, AAA* and the constant maturity 10 year Treasury yield, both taken from the Federal Reserve Board H15 database (downloaded 24 February 2017).

[18] Under the *Employee Retirement Income Security Act* (*ERISA*), private sector single-employer pension plan sponsors are required to use high-quality corporate bond yields (United States Government Accountability Office, 2014).

iv) *Annuity rule*: The 10-year Treasury yield minus one percentage point as a proxy for an annuity rate since annuity rates are generally lower than the yield on Treasuries.[19] We also consider 5, 10, 20 and 30 year moving averages of these rates.

v) *Inflation forecast plus a constant integer rule*: An inflation forecast plus 1, 2, 3, 4, 5 or 6 percentage points. This rule is analyzed because some actuaries calculate the discount rate by adding a fixed real return to an inflation forecast.[20]

vi) *Constant discount rate rule*: Constant integer discount rates from 3 to 13 percent. A number of countries stipulate fixed discount rates for long periods. An advantage of a fixed discount rate is that a variable rate may result in volatile movements in the value of pension fund liabilities even though the underlying benefit cash flows may not change (Bucciol and Beetsma, 2011[21]; Ponds, Severinson and Yermo, 2011, p. 24).[22]

2.3 Modelling Pension Plan Uncertainty

Our goal is to determine whether some discount rate setting rules improve the likelihood that a fully funded pension plan will accumulate sufficient assets (which equal *projected* liabilities given the plan is fully funded) to cover future *actual* promised pension benefit payments, without accumulating significant excess assets. Projected liabilities are based on known past values of wages, prices, returns and yields, but actual future pension payments are unknown when *projected* liabilities are calculated because they depend on the *future* paths of wages, prices and the portfolio return. Thus, to compare projected liabilities and actual pension obligations, we undertake a Monte Carlo simulation. This simulation calculates 50,000 different paths for the variables that have uncertain future values – prices, wages, bond yields, the return on bonds, and the return on equities. We use these to simulate 50,000 paths for actual future pension benefits (APP_t), employing the simulated values of future inflation, future wage growth and the future return on the fund's portfolio. We also simulate 50,000 paths for the projected liabilities of the pension plan (PL_t), using the discount rate rules and lagged simulated values of inflation and wage growth to determine the inflation and wage growth forecasts. To determine the probability that the fund's assets will be sufficient to cover promised benefits, for each of the 50,000 simulations, we compare the assets held by the fund (which equal projected liabilities because the plan is fully

[19] The Actuarial Standards Board (2013) argues that the discount rate implicit in annuity prices can be used to determine the value of assets needed today to fund future pension plan liabilities.
[20] The Ontario Teachers' Pension Plan determines the discount factor by separately choosing a real rate of return and an inflation forecast (Ontario Teachers' Pension Plan, 2012). See also Ménard (2013).
[21] Bucciol and Beetsma (2011) note that, in the Netherlands, a 2007 requirement for pension funds to move from a constant 4% discount rate to a market-based discount rate created concern that this would unduly increase the uncertainty about future pensions and hurt specific groups that find it difficult to respond to such adjustments (say by working longer).
[22] Ponds, Severinson and Yermo (2011, p. 24) suggest the use of a fixed discount rate "related to some long-term average of the rate of interest on long government bonds or related to some assumed value as a good proxy for the interest rate on government bonds. Such a rate should also be consistent with long-term trends in economic growth, which ultimately determines the government's capacity to finance pensions."

funded) to the present value of the actual future pension benefit payments.[23]

To calculate the 50,000 paths for the variables used in the simulations, we use U.S. data to estimate a four-variable VAR with two lags of each variable. The four variables in the VAR are the CPI-based inflation rate, wage inflation, the yield on 10-year Treasury bonds and the return on equities (see *Appendix 2: Data and Sources* for precise definitions and sources). Two lags are chosen because one lag is not sufficient to eliminate serial correlation in the residuals.[24] Impulse response functions given by the VAR estimates are shown in Figure 1. The signs of the movements in the variables given in the impulse responses are generally as would be expected. Further, as shown in *Appendix 3*, a dynamic simulation of the VAR, when all shocks are set equal to zero, converges to values that are very close to the means of the data used to estimate the VAR.

The 50,000 simulated series for inflation, wage inflation, the bond yield and the return on equities are derived using a dynamic simulation of the estimated VAR incorporating 50,000 series of random shocks for each of the four variables. The random shocks for each variable are drawn from a normal distribution with standard deviation equal to the standard deviation of the corresponding estimated structural error from the VAR. Noting that the reduced form errors in a VAR can be written as linear combinations of the VAR structural errors, linear combinations of the random shocks are used to derive 50,000 simulated series for each of the four reduced form VAR errors.[25] These simulated reduced form error series are appended to the corresponding VAR equation and the VAR is simulated dynamically 50,000 times to create 50,000 simulated series for the inflation rate, the wage growth rate, the 10-year Treasury yield, and the return on equities.[26] The 50,000 simulated bond return series are calculated from the simulated Treasury yields using the methodology described in *Appendix 2*. As can be seen from *Appendix 3*, the means and standard deviations of the simulated series are close to the means and standard deviations of the data used to estimate the VAR.[27]

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[23] There were no significant changes to the results when we used 100,000, rather than 50,000, replications.

[24] Given the large number of parameters in the VAR, some of the lagged parameters are estimated very imprecisely. Since the VAR estimates are important to the simulation, to improve the precision of the estimates, we sequentially constrained the parameters in the VAR with t-statistics less than one to be zero, beginning with the parameter with the smallest t-statistic, until the estimated parameters associated with the longest lags of each explanatory variable in the VAR had a t-statistic greater than one. Using this process, of the 32 coefficients on lagged variables in the VAR, 12 were constrained to be zero. The imposition of these constraints ensured that the means of the simulated values and the values to which the VAR converges are close to the means of the data used to estimate the VAR (see *Appendix 3*).

[25] The estimated structural errors and the weights in the linear combinations of the structural errors that yield the reduced form errors are identified using a Cholesky decomposition with the following variable ordering: inflation, the wage growth rate, the 10-year Treasury yield, and the return on equities.

[26] As starting values for all simulations, we use the values to which the estimated VAR converges when there are no shocks (with the starting values for the non-stochastic VAR given by the means of the data used to estimate the VAR).

[27] To determine whether the results were sensitive to the method used to simulate the random shocks, we also generated the random shocks using a uniform distribution. This process involved first retrieving the four series of

2.4 Modelling a Pension Plan's Ability to Meet Promised Benefits

A pension plan can meet its commitments if the plan's projected liabilities (which determine the quantity of assets the plan must accumulate to be fully funded) equal or exceed the present value of the actual future pension payments of the plan, where the actual future payments are discounted by the actual return on the plan's portfolio. We simulate 50,000 projected pension plan liability (PL_t) series using the 50,000 simulated series for bond yields, equity returns, bond returns, inflation and wage growth as well as equations (1) to (5) from section 2.1 and the discount rate setting rules described in section 2.2.

The present value of future actual pension payments (APP_t) is calculated for each of the 50,000 simulations using the method described in Section 2.1 and detailed in Appendix 1. The key difference between the actual future pension payment calculation and the projected liability calculation is that the actual payment series use the future values of simulated inflation, wage growth and returns while the projected liabilities series use forecasts based on lagged simulated values and the discount rate determined by the rule. Since the discount rate is only used to calculate projected liabilities, it does not alter the future actual pension payment (APP_t) calculation.

For each of the 50,000 simulations, we compare the present value of future actual pension payments (APP_t) to the assets the pension fund would have accumulated if it had accumulated assets just equal to the projected liabilities (PL_t). That is, for each simulation, we calculate the assets that would be required to make the actual promised pension payments and compare this value to the assets the fully funded plan has accumulated under each discount rate setting rule. Effectively, this comparison determines whether, if the plan closes in period t, the assets of the plan would be sufficient to meet promised benefit payments for all service accrued through period t.[28] If projected liabilities (PL_t) are greater than actual pension payment obligations (APP_t), the

estimated reduced form residuals from the estimated VAR. Using a uniform distribution, we drew random integers with replacement from the set of integers from 1 up to the number of observations (61) used to estimate the VAR. With this process, we create 50,000 different vectors of randomly chosen integers from 1 through 61. Matching the estimated four reduced form residuals for a particular observation to the randomly drawn integer that has the same number as that observation, we create 50,000 matrices with four columns consisting of randomly ordered reduced form residuals from the estimated VAR. This yields 50,000 series of four random shocks based on the reduced form errors of the VAR. The VAR is then simulated 50,000 times by appending each of the four simulated reduced form random shocks to the appropriate equation of the VAR. The conclusions of our comparison of discount rate setting rules is unaffected by use of this alternative random shock generation methodology.

[28] If the plan closes at the end of period t, payments will be made to retirees in periods $t+1$ through $t+59$. The payments extend for 59 years since a worker with one year of service at the end of period t will work for 39 more years and then receive a pension for 20 years based on their one year of accrued service and their wage in their 40th year of work. After year 59, the lives of all the workers who accrued service in the plan have ended.

pension plan will have accumulated assets in excess of those required to pay the benefits promised.

3. Comparison of the Discount Rate Rules

3.1 Metrics for comparison

For each discount rate setting rule and each of the 50,000 different paths for the stochastic variables – the inflation rate, the wage growth rate, the Treasury yield, the return on equities and the return on bonds – we compare the assets accumulated by the pension plan at time *t* (which equal the projected pension liabilities since the plan is fully funded) to the present value of actual future pension benefit payments for service accrued up to time *t*, where the present value of actual promised benefits is determined by the characteristics of the defined benefit plan and the future movements in asset returns, wages and inflation. We use the following metrics to assess whether the discount rate setting rules lead the pension plan to have sufficient assets to meet benefit obligations, without accumulating significant excess assets:

i) The average percentage by which accumulated assets exceed the assets required to meet accrued pension benefit obligations.

ii) The median percentage by which accumulated assets exceed the assets required to meet accrued pension benefit obligations.

iii) The percentage of the 50,000 simulations for which assets are less than pension benefit obligations.

iv) The percentage of the 50,000 simulations for which assets are less than 80% of pension benefit obligations.

v) The percentage of the 50,000 simulations for which assets are greater than 120% of pension benefit obligations.

In all cases, these metrics are calculated for the 100th year after the start of the simulation of the VAR. This allows for the effect of the starting values on the simulation to be dissipated and leaves a history of adequate length to determine the final wages of current retirees and to derive the wage and inflation forecasts used to calculate projected liabilities.[29]

For all five metrics, the preferred value is zero, since a zero value would mean the plan is able to meet all its future pension obligations, but does not collect contributions in excess of those

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A key assumption we make is that, if the pension closes at the end of period *t*, current workers receive a pension based on the final salary they would earn if they had continued working, not their salary when the plan closes (although they only receive credit for the number of actual years they have worked up to the closing of the plan). We use this assumption because it more accurately reflects the pension promised to workers as of time *t* and, thus, better reflects the promised benefit obligations of the plan.

[29] The results are similar when observations 80 or 120 are employed.

required. Given that some variables are stochastic – inflation, wage growth, bond yields and asset returns – projected liabilities (and, thus, plan assets) are unlikely to equal actual future benefit payments exactly, so the five metrics will generally not be zero. Also, the metrics are not likely to move independently. For example, a smaller share of cases in which assets are not sufficient to cover obligations, so smaller values for metrics (iii) and (iv), would be expected to accompany higher mean and median accumulated assets, and larger values for metrics (i), (ii), and (v).

3.2 Comparison of the Discount Rate Rules

The Non-Stochastic Case

To improve understanding of the stochastic case, it is useful to first examine several characteristics of the model when the world is non-stochastic; that is, when all the random shocks in the VAR are restricted to be zero. In this non-stochastic case, the forecasts of inflation and wage growth equal actual inflation and actual wage growth, respectively. Also, the return on the pension plan's asset portfolio is constant and known. If the discount rate is set equal to the return on the portfolio, the plan always has exactly the correct level of assets to meet its future pension benefit obligations and no excess assets are accumulated.[30] If the discount rate is set below the return on assets, with a fully funded plan, assets will be accumulated in excess of those required to meet the benefit obligations of the plan. The greater the deviation between the discount rate and the return on the portfolio, the larger the difference between the assets accumulated and the assets required to meet future pension obligations.

Even in a non-stochastic world, seemingly small deviations between the discount rate and the return on assets can lead to large funding imbalances due to the long time span between pension benefit accrual and pension benefit payments. For example, with the portfolio invested 65% in equities and 35% in bonds, the use of a discount rate of 8 percent, rather than 9.7 percent (the 10-year moving average of the return on the portfolio), leads to the accumulation of 23 percent more assets than required to meet promised benefits.[31]

Comparison of the Discount Rate Setting Rules in the Stochastic Case

As discussed in Section 2, to model a stochastic world, we generate 50,000 future paths for

[30] To model this case, we assume that all variables take on the values to which the VAR converges and remain constant through time. For this non-stochastic case, the equity-dominant portfolio (65% equity, 35% bonds) would have a constant return of 9.68% and the contribution rate out of salary would be 8.1%.

[31] The 10-year average portfolio return of 9.68% (rounded to 9.7%) differs somewhat from the 10-year moving average portfolio return of 9.25% given in Table 1 as the calculation in Table 1 is a geometric mean moving average, averaged over the 50,000 simulations.

inflation, wage growth, the bond yield and the returns on bonds and equities. The liabilities projected by the pension plan at a point in time can differ from actual future pension plan payments due to differences between the actual future inflation rate and the forecast rate, the actual future wage and the forecast wage, and the actual future return on the fund's portfolio and the assumed discount rate. We focus mainly on a portfolio consisting of 65% equities and 35% bonds but, as a comparison, in Section 3.4 we consider a portfolio consisting of 65% bonds and 35% equities. The values of the five metrics for each of the different discount rate setting rules are presented in columns (2) to (6) of Tables 1 and 2 for the equity-dominant and bond-dominant portfolios, respectively.

A first result is that, for the rules that set the discount rate equal to the average portfolio return (*10-year geometric average portfolio return*, *20-year geometric average portfolio return*, *30-year geometric average portfolio return*), in approximately 50 percent of the 50,000 simulations, accumulated assets are not sufficient to meet promised pension benefit payments (Table 1.A, column (4), rows 1 - 3). Since these rules set the discount rate equal to an *average* portfolio return, in half the cases the discount rate is higher than the *actual* future portfolio return. As a consequence, in half the cases, the plan accumulates assets that are lower than those required to cover actual future benefit payments.

A more general result is that there are no discount rate setting rules for which both mean and median excess assets are low, say under 20 percent (Table 1, columns (2) and (3)), and the percentage of cases for which assets are less than obligations is small, say under 10 percent (column (4)).[32] Thus, to have a relatively high likelihood of meeting future pension obligations (a small value in column (4)), a plan must, on average, use a discount rate that leads to large median and mean excess assets.

Fewer than half of the discount rate rules yield a 90 percent or greater probability of being able to meet future benefit obligations (or, equivalently, have less than a 10 percent chance that assets are less than obligations (Table 1, column 4)). To achieve this 90 percent probability, a rule must generate a discount rate that is below 6 percent even though the average return on the portfolio is over 9 percent. For all the rules that meet the 90 percent criteria, mean excess assets exceed 60 percent (Table 1, column 2).

Whether a pension fund's assets meet or exceed benefit obligations is likely to be less of

[32] Other studies also find that the volatility of pension plan liabilities and asset returns imply that plans require significant excess assets to have a high probability of meeting obligations. For example, Elder and Wagner (2015) find, using data from Pennsylvania's two largest public pension plans, that the plans would need assets equal to 181 percent of the present value of liabilities to have a 90% chance of holding sufficient assets to pay all future liabilities. For the plan to achieve a 95 or 99 percent level, the funding ratio "explodes" to 210% and 282%, respectively (p. 22).

an issue for both pension plan sponsors and beneficiaries if any shortfall or excess is small. A notable feature of the pension discount rate rules examined is that they all yield a large number of cases for which there is either a funding shortfall of more than 20% (Table 1, column 5) or excess assets of more than 20% (Table 1, column 6). For example, for the three average return rules, in approximately one-third of cases accumulated assets are less than 80 percent of obligations, while in another one-third of cases, assets are more than 120 percent of obligations. Similarly, assets exceed 120 percent of obligations in over half of cases for the *10-year Treasury yield* rules, the *Corporate bond yield* rules and the *Annuity* rules. Further, as can be seen in Table 1, Part A, 21 of the 24 variable discount rate rules have at least 60 percent of cases for which assets either exceed or fall short of obligations by more than 20 percent (the values in columns 5 and 6 sum to more than 60). The three rules for which this is not the case – the inflation forecast plus 4, 5 and 6 percent rules – have a relatively high percentage of cases for which assets are less than obligations as well as less than 80 percent of obligations (Table 1, columns 4 and 5, rows 22 to 24), which may be of greater concern than large excess assets (column 6).

The constant discount rate rules generally do not perform as well as the rules that vary the discount rate with the yield on assets or the inflation rate. For example, the *20-year moving average of the Treasury bond yield - 1%* rule has similar median excess assets as the *constant 5%* rule (81.6% and 79.8%, respectively). However, the percentage of cases for which assets are less than obligations is 2.5 times as large with the *constant 5%* rule, and the percentage of cases for which assets are less than 80% of obligations is four times as large. Similar comparisons hold for other constant and variable rate rules. This suggests there is a benefit to choosing a discount rate that varies with the economic factors that affect pension payouts and asset accumulation. As an example, suppose a fall in inflation accompanies a fall in the nominal return on the pension plan portfolio. To the extent that the fall in inflation leads to a fall in the inflation forecast, under the *Inflation forecast + 1%* rule the discount rate also falls, triggering a rise in projected liabilities and the accumulation of more assets. As the rise in assets accompanies the declining portfolio return, underfunding of the pension plan is less likely.[33]

For the four classes of rules that involve moving averages (Table 1, Part A, rows 1 through 18), a longer moving average, at least up to 10 or 20 years, smooths the discount rate and generates lower mean excess assets, lower median assets, fewer cases of assets that are insufficient to meet obligations, fewer cases of assets below 80% of obligations, and more cases of assets

[33] Freeman (2013) and Abourashchi (2013) also find that when modeling pension fund sustainability, it is important to consider co-movement between economic variables that affect the fund and the discount rate.

exceeding 120% of obligations. Thus, smoothing movements in the discount rate over 10 or 20 years makes it more likely that sufficient assets will be held to meet future pension payment obligations, while accumulating fewer excess assets on average.

3.3 Ranking the Discount Rate Rules

As is clear from Table 1, the choice of a "best" discount rate rule depends on the relative importance of the probability of not being able to meet promised pension obligations (columns 4 and 5) versus the cost of holding excess assets (columns 2, 3 and 6). One way to compare rules when there are multiple objectives such as this is to employ a loss function.

We specify two loss functions. The first loss function includes two criteria – squared median excess assets and the squared frequency of negative excess assets (from columns 3 and 4 of Table 1, respectively). The weights on these two criteria in the loss function sum to one with the weight on squared median excess assets represented by ω. We vary ω from 0 to 1 in increments of 0.1.

The second loss function adds a third criteria – the squared frequency for which assets are less than 80% of obligations (from Table 1, column 5). To reduce the number of weight permutations, this third criteria receives the same weight as the second criteria, with the weights on the three criteria restricted to sum to one.

The loss function methodology makes it clear how the choice of the best discount rate setting rule depends on the weights chosen for the different objectives in the loss function. Table 3 reports the best performing rules for the two loss functions and the eleven different values of the weights.

As can be seen in Table 3, the two loss functions yield similar results. If all the weight is allocated to minimizing the quantity of excess assets (ω equal to 1.0), the best discount rate rule is the *30-year geometric average of the portfolio return*. As more weight is given to the percentage of cases for which assets are less than obligations, rules with a lower average discount rate, such as the *10-year moving average of the Treasury yield + 1.5%* are preferred. This trade-off is evident in Figure 2, which depicts median excess assets and the frequency of negative excess assets for selected rules.[34] As the optimal point is at the origin, to highlight the best rules, an envelope line has been added to connect the points closest to the origin. This line shows that some discount rate setting rules are clearly dominated by other rules. These clearly dominated rules

[34] To improve the readability of the figure, outlier cases have been excluded — specifically, where median excess assets exceed 160% and the frequency of negative excess assets exceed 50%. Note that the scales of the two axes differ.

include the constant discount rates of from 4 to 8 percent, and all the rules based on the Treasury bond yield other than the 10 year moving averages.

As can be seen from Figure 2 and Table 3, when approximately equal weight is given to minimizing both median excess asset holdings and the frequency that assets are less than future benefit payment obligations, the *Inflation forecast + 3%*, and the corporate bond yield proxy rule (*10-year moving average of the TB yield + 1.5%*) dominate the other rules. These two rules minimize the loss function for a majority of the preference weight choices, and are the best rules except when preferences heavily weight one criteria over the other. The only other rule that minimizes losses for more than one value of the preference weights is the *10-year moving average of the Treasury bond yield*, and then only in the case of the first loss function. While the *Inflation forecast + 3%*, and *10-year moving average of the TB yield + 1.5%* rules may be the best rules for many loss function weight combinations, these rules yield median excess assets of more than 24 percent and a probability that plan assets will not be sufficient to meet actual plan obligations of at least 14 percent. Thus, even the best rules have high median excess assets, but fail to meet promised benefit obligations in a large share of cases.

3.4 Robustness of the Results to Model Variations

Our analysis has assumed that the investment portfolio consists of 65% equities and 35% bonds. As an alternative to this equity-dominant portfolio, Table 2 provides results for a bond-dominant portfolio in which the pension plan invests 65% of its assets in bonds and 35% in equities. This bond dominant portfolio is less volatile than the equity dominant portfolio, but earns a lower return on average (7.8% compared to 9.2%).

The switch from the equity dominant to bond dominant portfolio does not have an impact on the values of the discount rates generated by any of the discount rate setting rules except for the three rules based on the geometric average of past portfolio returns (Table 2, Part A, rows 1-3). In all but these three cases, because the discount rates do not change, the projected liabilities also do not change, and the quantity of assets accumulated to meet the projected obligations of the plan is unchanged. With the same quantity of assets and a portfolio that yields a lower return on average, there is a higher probability that the accumulated assets of the plan will be insufficient to meet future benefit obligations. This is evident from a comparison of column (4) in Tables 1 and 2, where the percentage of cases for which assets are less than obligations is greater with the bond-dominant portfolio. Further, column (5) shows that the percentage of cases for which assets are less than 80% of obligations is also greater for a majority of the bond-dominant portfolio cases.

Thus, even though the volatility of the bond-dominant portfolio is lower, the higher average return of the equity-dominant portfolio leads to a greater likelihood that the pension fund will be able to meet its promised future obligations.

As the equity-dominant portfolio performs better than the bond-dominant portfolio, we also considered a portfolio consisting of 100 percent equities. To keep the results succinct, Table 4 examines this case, and the other model variations discussed below, using only two rules: the *10-year moving average of the Treasury bond yield* and the *Inflation forecast + 3%* rule. Neither of these two discount rate rules is affected by the switch from the equity dominant to the pure equity portfolio. Since the discount rates have not changed, projected liabilities do not change, so the total assets accumulated do not change. Compared to the results for the 65% equity portfolio given in Table 1, use of a 100 percent equity portfolio reduces the likelihood that accumulated assets will not be sufficient to cover promised benefits. On the other hand, mean and median excess assets rise by approximately 50%, and the percentage of cases for which assets exceed obligations by more than 120% rises by 3-12% (Table 4, rows 2 and 6). In addition, the percentage of cases for which assets are less than 80% of obligations also rises. Thus, with the greater volatility of the 100% equity portfolio, there is greater risk relative to a more balanced portfolio.[35]

Robustness of the results is also assessed using two further variations of the pension model. First, the indexation rate was reduced to 50% from 100%. Since this variation causes both projected liabilities and actual future benefit payments to fall, there is little change in any of the metrics, so the level of indexation has only a small effect on the results (Table 4, rows 3 and 7). One significant change, however, is that a lower indexation rate causes the magnitude of the assets accumulated by the pension fund to fall (by almost 20 percent) because fewer assets are required to pay benefits when the indexation rate is lower.

The second variation considered is a reduction in the replacement rate to 40% from 60%. As shown in rows 4 and 8 of Table 4, this has effectively no impact on the relative metrics of the different discount rate rules because both projected liabilities and actual promised benefit payments fall commensurately with the decline in the replacement rate. Of course, the one third drop in the replacement rate yields a one third drop in the assets held by the pension fund since the pension payouts to retirees will be smaller by a third in this case (Table 4, column 7).

When we simulated the Treasury yields, we did not restrict the simulated yields to be

[35] In their analysis of measures to improve the sustainability of U.S. state pension funds, Lekniute, Beetsma and Ponds (forthcoming) find similar results in that a full-equity strategy improves the median and upside of the funding ratio in the long run, but the probability of pension fund default also increases.

positive. To determine whether a zero-lower bound for the Treasury yield would affect our results, we restricted the simulated Treasury yields to be no lower than .001. A comparison (not reported) of the unrestricted results to the results that use these restricted yields for the *10-year moving average of the Treasury bond yield* discount rate found that this restriction caused only small changes in the results.

4. Conclusions and Discussion

Financial economists argue that the rate used to discount pension liabilities should be a default-free rate, while pension plan administrators maintain that the discount rate should be the long run rate of return on plan assets. We develop a model of a defined benefit pension plan to evaluate the ability of a fully funded plan to meet its actual future benefit obligations under six types of discount rate-setting rules. Our purpose is to identify rules that yield a high probability that a fully funded plan will have sufficient assets to meet benefit payments while amassing few excess assets on average. As the future path of pension liabilities and asset returns is uncertain when the discount rate is set, we employ Monte Carlo simulation techniques.

A first key result is that none of the discount rate setting rules we examine yield both low average excess assets and a high probability that the assets in the fund will be adequate to meet future pension benefit obligations. The rules that yield low levels of excess assets (for example, average excess assets under 20 percent of obligations), have a high probability (over 40 percent) that the assets accumulated by the fund will not be adequate to cover future promised benefits. Rules that generate a high likelihood that a plan will have sufficient assets to meet promised obligations (for example, a probability of over 90 percent) accumulate, on average, a large quantity of assets in excess of obligations (over 60 percent).

Another important result is that the rules we evaluate have a wide range of outcomes and there is a high probability that plan assets will be insufficient or excessive by a large amount. For example, almost all the rules generate a 60 percent or greater probability that plan assets will either exceed or fall short of obligations by at least 20 percent.

The better performing rules incorporate variable discount rates. Compared to constant rate rules, the variable discount rate rules have a lower probability that assets will be insufficient to cover obligations for a given level of excess assets. The advantage of rules that cause the discount rate to vary with economic factors, such as asset yields and inflation, is that they can move the discount rate in the direction of change of the portfolio return.

Many pension fund managers recommend setting the discount rate equal to the pension fund's historical average portfolio return. This strategy generates assets that are just sufficient to meet obligations on average, but implies that assets cannot cover promised pension payments in 50 percent of cases. A rule must generate a discount rate lower than the portfolio return on average to achieve a greater than 50 percent probability that assets will be sufficient to cover future benefits. In our simulations, to ensure there is no more than a 10 percent probability that assets accumulated are less than obligations, the discount rate must be less than 6 percent, a value that is significantly below the average portfolio return of over 9 percent. All the rules that generate a discount rate under 6 percent amass assets that are, on average, at least 60 percent greater than required to meet future pension obligations.

We find that the choice of the "best" discount rate setting rule depends on the weight allocated to the benefit of a higher probability of being able to meet promised pension benefit payments relative to the cost of holding excess assets. Using a loss function that allocates roughly equal weight to these two factors, we identify the best performing rules to be the *10-year moving average of the TB yield + 1.5%* rule, a proxy for the corporate bond yield, and the *Inflation forecast + 3%* rule. These rules have average discount rates in the 6.7 to 7.4 percent range, which is lower than the average portfolio return, the discount rate choice of pension fund managers (*NASRA*, 2010). A discount rate equal to the average portfolio return is preferred only if little weight is given to the probability that assets are less than future pension obligations. As well, the *10-year moving average of the TB yield + 1.5%* rule and the *Inflation forecast + 3%* rule yield discount rates that are higher than the relatively low default free discount rates recommended by finance theorists, such as the *10-year moving average of the TB yield*. A low default-free rate is preferred only if ensuring that the pension plan can meet its obligations is given a large weight in the loss function.

Even with the "best" discount rate setting rules, substantial excess assets are accumulated on average and there remains a significant probability that plan assets will not be adequate to meet future obligations. For example, the probability of underfunding is 22 and 14 percent for the *10-year moving average of the TB yield + 1.5%* rule and the *Inflation forecast + 3%* rule, respectively. Further, these two rules generate substantial median excess assets of, respectively, 24 and 37 percent.

The pension plan we analyze is fully funded, so has a 100 percent funding ratio, with contribution rates changed each year to ensure accumulated assets equal projected liabilities. Given our focus on the discount rate, we have not examined the impact of different funding ratios, although the funding ratio is used as a policy tool in some countries.[36] For example, the Netherlands requires pension plans with a funding ratio under 105 percent to implement a recovery plan, and restricts payouts if the ratio is below 130 percent (PEW, 2017). For a given discount rate setting rule, a higher funding ratio increases the magnitude of the average assets accumulated by the plan and, thus, the likelihood that assets will be adequate to cover promised pension payments.[37] From a policy perspective, however, an advantage of focusing on the discount rate (while requiring full funding) is that a funding ratio above 100 percent "can lead to significant political pressure to raise pension benefits, which can exacerbate the pension funding problems." (Elder and Wagner (p 5, 2015)).

While the choice of the discount rate setting rule can improve the trade-off between the accumulation of excess assets and the likelihood that assets will not be sufficient to meet promised obligations, the long time horizon inherent in any pension plan, plus the volatility associated with pension plan asset returns and benefit payments, means that, in many cases, the assets accumulated will be either inadequate to meet promised benefit payments or far more than required. When a plan does not accumulate enough assets to meet its benefit obligations, prospective pension recipients will not receive the payments they have been promised or current workers will be asked to bail out the plan, which would impose a cost on the current generation. If a plan amasses excess assets, these assets are likely to benefit the current generation, through lower contribution rates or higher benefits, but were costly for previous generations to accumulate. A public policy implication of our results is that intergenerational transfers may be unavoidable regardless of the discount rate setting rule chosen.[38]

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[36] Funding ratios have been a concern in the US where in 2015 only 2.5 percent of state and local government plans had a funding ratio above 100 percent, and the aggregate funding ratio was just 74 percent (Munnell and Aubry, 2016).

[37] While our results are generated with full funding imposed, they give some sense of how changing the funding ratio would affect pension plan sustainability. Assuming a 20 percentage point increase in the required funding ratio increases mean excess assets by 20 percent, a 120 percent funding ratio, combined with a rule that generates a relatively low discount rate, such as the *10-year moving average of the TB yield* rule, would reduce the probability that assets will be insufficient to cover future obligations to almost zero (from 7.3 to 1.5 percent), but would raise the quantity of excess assets from approximately 60 percent to 80 percent, which imposes a significant cost on current contributors to the plan. Our findings also imply that if a funding ratio of 120 percent is required, a higher discount rate may be preferred. For example, with the discount rate given by the *10-year moving average of the TB yield + 1.5%* rule and a 120 percent funding ratio, the probability that assets are less than promised payment obligations would fall from over 22 percent to under 7 percent, but average excess assets would rise from just under 30 percent to almost 50 percent. If this accumulation of assets is deemed to be too costly, a higher discount rate would be required.

[38] This is an important finding for policymakers, given the large magnitude of defined pension plan obligations, valued at $23 trillion worldwide (Economist, 2014).

Appendix 1: Pension Benefit Payments for Selected Workers and Retirees

This appendix gives examples of the formulas used to calculate the projected pension liability (*PL*) for workers and retirees of different ages as well as the present discounted value (PDV) of the actual pension payments that have been promised to workers and retirees (*APP*). In total, in any period *t*, there are workers of 40 different ages and retirees who will continue to receive a pension of 19 different ages. We have only provided formulas for workers of three different ages and retirees of two different ages, but the formulas for the workers and retirees of the other ages take a similar form. All variables are defined in sub-section 2.1 of the text.

Case 1: Worker with 1 year of service

Projected liabilities:

$$PL_{1t}^{W} = [pW_t(1+\pi_t^{WF})^{39}]\left[\sum_{j=1}^{20}\frac{(1+I\pi_t^F)^j}{(1+\delta_t)^{39+j}}\right]$$
$$= \left[pW_t\left(\frac{1+\pi_t^{WF}}{1+\delta_t}\right)^{39}\right]\left[\sum_{j=1}^{20}\left(\frac{1+I\pi_t^F}{1+\delta_t}\right)^j\right]$$

PDV of future actual promised pension payments:

$$APP_{1t} = [pW_{t+39}]\left[\frac{1}{(1+r_{t+1})(1+r_{t+2})\dots(1+r_{t+40})}\right](1+I\pi_{t+39})$$
$$+ [pW_{t+39}]\left[\frac{1}{(1+r_{t+1})(1+r_{t+2})\dots(1+r_{t+41})}\right]\left[\prod_{j=39}^{40}(1+I\pi_{t+j})\right]$$
$$+ [pW_{t+39}]\left[\frac{1}{(1+r_{t+1})(1+r_{t+2})\dots(1+r_{t+42})}\right]\left[\prod_{j=39}^{41}(1+I\pi_{t+j})\right]$$
$$+ \dots$$
$$+ [pW_{t+39}]\left[\frac{1}{(1+r_{t+1})(1+r_{t+2})\dots(1+r_{t+59})}\right]\left[\prod_{j=39}^{58}(1+I\pi_{t+j})\right]$$

Note: $W_{t+39} = W_t(1+\pi_{t+1}^W)(1+\pi_{t+2}^W)\dots(1+\pi_{t+39}^W)$

Case 2: Worker with 30 years of service

Projected liability:

$$PL_{30t}^{W} = [p30W_t(1+\pi_t^{WF})^{10}]\left[\sum_{j=1}^{20}\frac{(1+I\pi_t^F)^j}{(1+\delta_t)^{10+j}}\right]$$
$$= \left[p30W_t\left(\frac{1+\pi_t^{WF}}{1+\delta_t}\right)^{10}\right]\left[\sum_{j=1}^{20}\left(\frac{1+I\pi_t^F}{1+\delta_t}\right)^j\right]$$

PDV of future actual promised pension payments:

$$APP_{30t} = [p30W_{t+10}]\left[\frac{1}{(1+r_{t+1})(1+r_{t+2})\ldots(1+r_{t+11})}\right](1+I\pi_{t+10})$$

$$+ [p30W_{t+10}]\left[\frac{1}{(1+r_{t+1})(1+r_{t+2})\ldots(1+r_{t+12})}\right]\left[\prod_{j=10}^{11}(1+I\pi_{t+j})\right]$$

$$+ [p30W_{t+10}]\left[\frac{1}{(1+r_{t+1})(1+r_{t+2})\ldots(1+r_{t+13})}\right]\left[\prod_{j=10}^{12}(1+I\pi_{t+j})\right]$$

$$+ \ldots$$

$$+ [p30W_{t+10}]\left[\frac{1}{(1+r_{t+1})(1+r_{t+2})\ldots(1+r_{t+30})}\right]\left[\prod_{j=10}^{29}(1+I\pi_{t+j})\right]$$

Note: $W_{t+10} = W_t(1+\pi_{t+1}^W)(1+\pi_{t+2}^W)\ldots(1+\pi_{t+10}^W)$

Case 3: Worker with 40 years of service (entering first year of retirement in the next year)

Projected liability:

$$PL_{40t}^W = [pRW_t]\left[\sum_{j=1}^{20}\frac{(1+I\pi_t^F)^j}{(1+\delta_t)^j}\right]$$
$$= [pRW_t]\left[\sum_{j=1}^{20}\left(\frac{1+I\pi_t^F}{1+\delta_t}\right)^j\right]$$

PDV of future actual promised pension payments:

$$APP_{40t} = [pRW_t]\left[\frac{1}{1+r_{t+1}}\right](1+I\pi_t)$$

$$+ [pRW_t]\left[\frac{1}{(1+r_{t+1})(1+r_{t+2})}\right]\left[\prod_{j=0}^{1}(1+I\pi_{t+j})\right]$$

$$+ [pRW_t]\left[\frac{1}{(1+r_{t+1})(1+r_{t+2})(1+r_{t+3})}\right]\left[\prod_{j=0}^{2}(1+I\pi_{t+j})\right]$$

$$+ \ldots$$

$$+ [pRW_t]\left[\frac{1}{(1+r_{t+1})(1+r_{t+2})\ldots(1+r_{t+20})}\right]\left[\prod_{j=0}^{19}(1+I\pi_{t+j})\right]$$

Case 4: Retiree with ten years to live (aged 50)

Projected liability:

$$PL^{R}_{R+10,t} = [pRW_{t-10}][\prod_{i=0}^{10}(1 + I\pi_{t-i})]\left[\sum_{j=1}^{10}\frac{(1+I\pi_t^F)^{j-1}}{(1+\delta_t)^j}\right]$$

PDV of future actual promised pension payments:

$$APP_{R+10,t} = \left[pRW_{t-10}\prod_{j=0}^{10}(1 + I\pi_{t-j})\right]\left[\frac{1}{1+r_{t+1}}\right]$$

$$+ \left[pRW_{t-10}\prod_{j=0}^{10}(1 + I\pi_{t-j})\right]\left[\frac{1}{(1+r_{t+1})(1+r_{t+2})}\right](1 + I\pi_{t+1})$$

$$+ \left[pRW_{t-10}\prod_{j=0}^{10}(1 + I\pi_{t-j})\right]\left[\frac{1}{(1+r_{t+1})(1+r_{t+2})(1+r_{t+3})}\right]\left[\prod_{j=1}^{2}(1 + I\pi_{t+j})\right]$$

$$+ \ldots$$

$$+ \left[pRW_{t-10}\prod_{j=0}^{10}(1 + I\pi_{t-j})\right]\left[\frac{1}{(1+r_{t+1})(1+r_{t+2})\ldots(1+r_{t+10})}\right]\left[\prod_{j=1}^{9}(1 + I\pi_{t+j})\right]$$

Case 5: Retiree with one year to live (aged 59)

Projected liability:

$$PL^{R}_{R+19,t} = [pRW_{t-19}][\prod_{i=0}^{19}(1 + I\pi_{t-i})]\left[\frac{1}{1+\delta_t}\right]$$

PDV of future actual promised pension payments:

$$APP_{R+19,t} = \left[pRW_{t-19}\prod_{j=0}^{19}(1 + I\pi_{t-j})\right]\left[\frac{1}{1+r_{t+1}}\right]$$

Appendix 2: Data and Sources

The VAR employs data from 1954 – 2016. As the VAR has two lags, the estimation period for the VAR is 1956-2016. Two lags are chosen because tests for first and second order serial correlation of the reduced form residuals could not reject the hypothesis of no serial correlation when the VAR included two lags, but not when it included only one lag.

Inflation: The inflation rate given by CRSP (downloaded 6 March 2017). Note that this inflation rate is the same as the December to December inflation rate calculated using the cpi data from FRED - "Consumer Price Index for All Urban Consumers: All Items, Index 1982-1984=100, Monthly, Not Seasonally Adjusted".

Wage inflation: The year-to-year change in salaries per full time state and local government employee. The salary data is downloaded from the BEA "Wages and Salaries Per Full-Time Equivalent Employee by Industry" except for 2016 where we used the percentage change in the BLS State and local government *Total compensation; Cost per hour worked* where the percentage change is the average of Q1-Q3 2016 to the average Q1-Q3 2015. Downloaded from BEA on 6 March 2017.

Treasury Bond yield: The December value of the Constant maturity 10-year Treasury bond yield from the H15 historical data series of the US Federal Reserve Board. "Market yield on U.S. Treasury securities at 10-year constant maturity, quoted on investment basis" Downloaded 10 March 2017.

The methodology used to create the return from the yield is taken from Aswath Damodaran, Department of Finance, Stern School, New York University (www.stern.nyu.edu/~adamodar/pc/datasets/histretSP.xls). The bond yield (y) is used to construct the return (r) on bonds as follows:

$$r_t = \left(\frac{y_{t-1} * \left(1 - (1 + y_t)^\wedge(-10)\right)}{y_t} + \frac{1}{(1 + y_t)^\wedge 10} \right) - 1 + y_{t-1}$$

Equity return: CRSP annual value-weighted return (including distributions) on the S&P 500. This is the geometric average of the monthly returns. Downloaded 10 March 2017.

Descriptive Statistics of the Data Used to Estimate the VAR (1954-2016)

	Mean	Standard Deviation	Skewness	Kurtosis
CPI Inflation	.0358	.0289	1.67	5.82
Wage Inflation	.0470	.0204	.38	2.67
Yield on 10-yr Treasuries	.0588	.0276	.82	3.26
Return on 10-yr Treasuries	.0608	.0861	.76	3.51
Return on the S&P 500	.1235	.1751	-.36	3.05

Appendix 3: Data vs Simulated Values

	Mean (Standard Deviation) of the Data 1954-2016	Values to which the VAR estimates converge[a]	Mean (Standard Deviation) of the Simulated Values[b]
CPI Inflation	.0358 (.0289)	.0370	.0367 (.0279)
Wage Inflation	.0470 (.0204)	.0468	.0465 (.0205)
Yield on 10-yr Treasuries	.0588 (.0276)	.0592	.0589 (.0272)
Return on 10-yr Treasuries	.0608 (.0861)	.0592	.0627 (.0919)
Return on the S&P 500	.1235 (.1751)	.1171	.1149 (.1641)
Inflation Forecast			.0369 (.0151)
Wage Inflation Forecast			.0467 (.0126)
Equity Dominant Portfolio Return (equity weight, θ=.65)	.1016 (.1169)	.0968	.0983 (.1106)
Bond Dominant Portfolio Return (equity weight, θ=.35)	.0827 (.0819)	.0795	.0813 (.0823)

Notes:

[a] These are the values to which a dynamic simulation of the VAR converges when all the shocks are set equal to zero.
[b] The means and standard deviations of the simulated values are the means and standard deviations of the 50,000 simulated values for observation 100 after the beginning of the simulation (to remove any influence of the starting values on the simulation). If we calculate the standard deviation over a time series for each simulation, say from observation 51 through observation 150, and then take the mean of these standard deviations across the 50,000 simulations, the results are almost identical to those given above for the 100th observation.

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Table 1: Simulation Results – Equity Dominant Portfolio (65% Equities, 35% Bonds)

Discount Rate Rule	Average discount rate, δ (standard deviation) (1)	Mean assets in excess of obligations (percentage) (2)	Median assets in excess of obligations (percentage) (3)	Percentage of cases assets < obligations (4)	Percentage of cases assets < 80% of obligations (5)	Percentage of cases assets > 120% of obligations (6)
A. Variable Discount Rate Rules						
i) *Geometric average rule*						
1. 10-yr geometric average portfolio return	9.25 (3.50)	17.0	-0.6	50.5	32.5	35.0
2. 20-yr geometric average portfolio return	9.21 (2.58)	11.0	-0.5	50.5	31.0	33.7
3. 30-yr geometric average portfolio return	9.20 (2.14)	9.2	-0.5	50.5	30.3	32.7
ii) *10-year Treasury bond (TB) yield rule*						
4. TB yield	5.92 (2.74)	67.5	55.1	10.0	2.7	77.2
5. 5-yr MA TB yield	5.92 (2.58)	63.2	55.1	8.0	1.8	79.7
6. 10-yr MA TB yield	5.92 (2.41)	60.7	54.8	7.3	1.5	80.7
7. 20-yr MA TB yield	5.92 (2.09)	60.3	54.4	8.7	2.1	79.2
8. 30-yr MA TB yield	5.92 (1.85)	62.5	54.7	11.0	3.3	76.5
iii) *Corporate bond yield rule*						
9. TB yield + 1.5%	7.42 (2.74)	32.7	24.7	24.6	8.6	54.8
10. 5-yr MA TB yield + 1.5%	7.42 (2.58)	30.0	24.8	22.8	7.1	55.2
11. 10-yr MA TB yield + 1.5%	7.42 (2.41)	28.4	24.4	22.3	6.8	55.1
12. 20-yr MA TB yield + 1.5%	7.42 (2.09)	28.3	24.1	24.1	8.2	54.7
13. 30-yr MA TB yield+ 1.5%	7.42 (1.85)	29.9	24.4	26.0	10.2	54.3
iv) *Annuity rule*						
14. TB yield – 1%	4.92 (2.74)	99.4	82.4	4.7	1.0	87.8
15. 5-yr MA TB yield – 1%	4.92 (2.58)	93.4	82.5	3.1	0.6	90.4
16. 10-yr MA TB yield – 1%	4.92 (2.41)	89.8	82.0	2.6	0.4	91.5
17. 20-yr MA TB yield – 1%	4.92 (2.09)	89.1	81.6	3.4	0.7	90.2
18. 30-yr MA TB yield – 1%	4.92 (1.85)	92.0	82.1	5.1	1.3	87.6
v) *Inflation forecast plus a constant integer rule*						
19. Inflation forecast + 1%	4.71 (1.53)	93.1	88.1	2.1	0.4	93.2
20. Inflation forecast + 2%	5.71 (1.53)	63.6	59.3	6.2	1.4	83.3
21. Inflation forecast + 3%	6.71 (1.53)	40.5	36.9	14.5	4.0	67.6
22. Inflation forecast + 4%	7.71 (1.53)	22.2	19.1	27.5	9.3	48.9
23. Inflation forecast + 5%	8.71 (1.53)	7.5	4.7	43.5	18.1	31.4
24. Inflation forecast + 6%	9.71 (1.53)	-4.5	-6.9	60.0	30.2	17.5

Discount Rate Rule	Average discount rate, δ (standard deviation)	Mean assets in excess of obligations (percentage)	Median assets in excess of obligations (percentage)	Percentage of cases assets < obligations	Percentage of cases assets < 80% of obligations	Percentage of cases assets > 120% of obligations
	(1)	(2)	(3)	(4)	(5)	(6)
B. Constant Discount Rate Rules						
3%	3 (0)	184.1	159.6	1.9	0.6	95.5
4%	4 (0)	132.7	114.3	4.3	1.4	90.7
5%	5 (0)	93.0	79.8	8.6	3.1	83.0
6%	6 (0)	64.0	53.0	15.3	6.2	72.4
7%	7 (0)	40.7	32.0	24.5	10.9	59.5
8%	8 (0)	22.3	15.2	35.8	17.7	45.5
9%	9 (0)	7.5	1.6	48.3	26.6	32.4
10%	10 (0)	-4.5	-9.4	61.1	36.9	21.3
11%	11 (0)	-14.4	-18.6	72.3	48.1	13.2
12%	12 (0)	-22.7	-26.3	81.3	59.5	7.5
13%	13 (0)	-29.6	-32.8	88.3	69.6	3.9
Average of 30-year portfolio return	9.2 (0)	4.9	-0.8	50.8	28.6	29.9

Notes:
"TB yield" is the yield on 10-year U.S. Treasury bonds.
MA denotes "moving average".

Table 2: Simulation Results – Bond Dominant Portfolio (65% Bonds, 35% Equities)

Discount Rate Rule	Average discount rate, δ (standard deviation) (1)	Mean assets in excess of obligations (percentage) (2)	Median assets in excess of obligations (percentage) (3)	Percentage of cases assets < obligations (4)	Percentage of cases assets < 80% of obligations (5)	Percentage of cases assets > 120% of obligations (6)
A. Variable Discount Rate Rules						
i) *Geometric average rule*						
1. 10-yr geometric average portfolio return	7.82 (3.11)	12.8	-0.1	50.1	30.4	33.7
2. 20-yr geometric average portfolio return	7.79 (2.34)	9.7	-0.2	50.2	29.6	32.7
3. 30-yr geometric average portfolio return	7.78 (1.95)	8.9	-0.0	50.0	29.2	32.5
ii) *10-year Treasury bond (TB) yield rule*						
4. TB yield	5.92 (2.74)	37.0	30.6	14.5	2.4	63.0
5. 5-yr MA TB yield	5.92 (2.58)	34.2	30.4	11.6	1.4	64.6
6. 10-yr MA TB yield	5.92 (2.41)	32.9	29.9	11.9	1.5	64.5
7. 20-yr MA TB yield	5.92 (2.09)	33.4	29.9	16.6	4.0	61.7
8. 30-yr MA TB yield	5.92 (1.85)	35.7	30.0	20.6	7.0	60.3
iii) *Corporate bond yield rule*						
9. TB yield + 1.5%	7.42 (2.74)	8.8	5.0	42.0	12.2	29.0
10. 5-yr MA TB yield + 1.5%	7.42 (2.58)	7.1	4.8	41.3	10.2	25.6
11. 10-yr MA TB yield + 1.5%	7.42 (2.41)	6.4	4.5	41.9	11.4	25.3
12. 20-yr MA TB yield + 1.5%	7.42 (2.09)	6.9	4.4	43.5	15.9	29.0
13. 30-yr MA TB yield+ 1.5%	7.42 (1.85)	8.5	4.4	44.2	19.6	32.2
iv) *Annuity rule*						
14. TB yield – 1%	4.92 (2.74)	62.8	53.4	5.2	0.6	81.9
15. 5-yr MA TB yield – 1%	4.92 (2.58)	58.8	53.4	3.1	0.2	85.6
16. 10-yr MA TB yield – 1%	4.92 (2.41)	56.9	52.8	3.1	0.3	85.9
17. 20-yr MA TB yield – 1%	4.92 (2.09)	57.3	52.8	6.3	1.1	81.3
18. 30-yr MA TB yield – 1%	4.92 (1.85)	60.3	53.0	10.2	2.8	77.0
v) *Inflation forecast plus a constant integer rule*						
19. Inflation forecast + 1%	4.71 (1.53)	60.7	58.2	3.8	0.5	86.9
20. Inflation forecast + 2%	5.71 (1.53)	36.2	34.0	12.1	2.5	67.7
21. Inflation forecast + 3%	6.71 (1.53)	17.0	15.2	28.1	7.8	43.3
22. Inflation forecast + 4%	7.71 (1.53)	1.8	0.2	49.6	18.5	21.6
23. Inflation forecast + 5%	8.71 (1.53)	-10.4	-11.8	70.5	34.6	8.5
24. Inflation forecast + 6%	9.71 (1.53)	-20.4	-21.6	85.7	53.3	2.6

Discount Rate Rule	Average discount rate, δ (standard deviation)	Mean assest in excess of obligations (percentage)	Median assets in excess of obligations (percentage)	Percentage of cases assets < obligations	Percentage of cases assets < 80% of obligations	Percentage of cases assets > 120% of obligations
	(1)	(2)	(3)	(4)	(5)	(6)
B. Constant Discount Rate Rules						
3%	3 (0)	139.5	118.1	4.4	1.5	90.7
4%	4 (0)	96.1	80.1	8.9	3.3	82.5
5%	5 (0)	63.2	51.1	16.3	6.8	71.0
6%	6 (0)	38.0	28.6	26.7	12.2	56.8
7%	7 (0)	18.3	10.9	39.6	20.5	41.8
8%	8 (0)	2.8	-3.2	53.5	30.8	27.8
9%	9 (0)	-9.7	-14.6	67.1	42.9	16.9
10%	10 (0)	-19.8	-23.9	78.4	55.6	9.3
11%	11 (0)	-28.2	-31.6	87.0	67.8	4.6
12%	12 (0)	-35.1	-38.0	92.8	77.8	2.0
13%	13 (0)	-40.9	-43.5	96.3	85.8	0.9
Average of 30-year portfolio return	7.78 (0)	6.0	-.4	50.4	28.3	30.7

Notes:
"TB yield" is the yield on 10-year U.S. Treasury bonds.
MA denotes "moving average".

Table 3: Loss Function Discount Rate Rule Comparison

A: Loss Function 1 = ω(median excess assets)^2 + (1-ω)(percentage of cases assets < obligations)^2

ω	Rule with the smallest loss	Average discount rate	Median excess assets	Frequency of negative excess assets
1.0	30-year geometric average portfolio return	9.20	-0.5	50.5
0.9	Inflation forecast + 4%	7.71	4.7	43.5
0.8	10-year MA of TB yield + 1.5%	7.42	24.4	22.3
0.7	10-year MA of TB yield + 1.5%	7.42	24.4	22.3
0.6	10-year MA of TB yield + 1.5%	7.42	24.4	22.3
0.5	Inflation forecast + 3%	6.71	36.9	14.5
0.4	Inflation forecast + 3%	6.71	36.9	14.5
0.3	Inflation forecast + 3%	6.71	36.9	14.5
0.2	10-year MA of TB yield	5.92	54.8	7.3
0.1	10-year MA of TB yield	5.92	54.8	7.3
0.0	Constant 3%	3.00	159.6	1.9

B. Loss Function 2 = ω(median excess assets)^2 + .5(1-ω)[(percentage of cases assets < obligations)^2 + (percentage of cases assets < 80% of obligations)^2]

ω	Rule with the smallest loss	Average discount rate	Median excess assets	Frequency of negative excess assets	Frequency of assets less than 80% of obligations
1.0	30-year geometric average portfolio return	9.20	-0.5	50.5	30.3
0.9	Inflation forecast + 5%	8.71	4.7	43.5	18.1
0.8	Inflation forecast + 4%	7.71	19.1	27.5	9.3
0.7	10-year MA of TB yield + 1.5%	7.42	24.4	22.3	6.8
0.6	10-year MA of TB yield + 1.5%	7.42	24.4	22.3	6.8
0.5	10-year MA of TB yield + 1.5%	7.42	24.4	22.3	6.8
0.4	Inflation forecast + 3%	6.71	36.9	14.5	4.0
0.3	Inflation forecast + 3%	6.71	36.9	14.5	4.0
0.2	Inflation forecast + 3%	6.71	36.9	14.5	4.0
0.1	10-year MA of TB yield	5.92	54.8	7.3	1.5
0.0	Constant 3%	3.00	159.6	1.9	0.6

Notes:
"TB yield" denotes the 10-year US Treasury bond yield.
"MA" denotes "moving average."

Table 4: Selected Variations of the Model

	Average discount rate, δ (standard deviation) (1)	Mean assets in excess of obligations (percentage) (2)	Median assets in excess of obligations (percentage) (3)	Percentage of cases assets < obligations (4)	Percentage of cases assets < 80% of obligations (5)	Percentage of cases assets > 120% of obligations (6)	Assets (percentage of pension fund assets in the Base Case) (7)
A. Discount Rate: *10-yr MA TB yield*							
1. Base Case	5.92 (2.41)	60.7	54.8	7.3	1.5	80.7	100.0
2. 100% equity portfolio	5.92 (2.41)	88.4	77.2	8.2	2.9	83.5	100.0
3. Inflation indexation 50%	5.92 (2.41)	60.4	54.3	6.7	1.2	81.1	81.8
4. Replacement rate of 40%	5.92 (2.41)	60.7	54.8	7.3	1.5	80.7	66.7
B. Discount Rate: *Inflation forecast + 3%*							
5. Base Case	6.71 (1.53)	40.5	36.9	14.5	4.0	67.6	100.0
6. 100% equity portfolio	6.71 (1.53)	63.6	57.1	12.6	4.9	76.0	100.0
7. Inflation indexation 50%	6.71 (1.53)	39.9	36.8	12.8	3.0	69.0	81.2
8. Replacement rate of 40%	6.71 (1.53)	40.5	36.9	14.5	4.0	67.6	66.7

Notes:
"TB yield" is the yield on 10-year US Treasury bonds.
MA denotes "moving average".
In the "Base Case", the portfolio consists of 65% equities and 35% bonds, inflation indexation is 100% and the replacement rate is 60%.

Figure 1: Responses to a One Standard Deviation Structural Shock

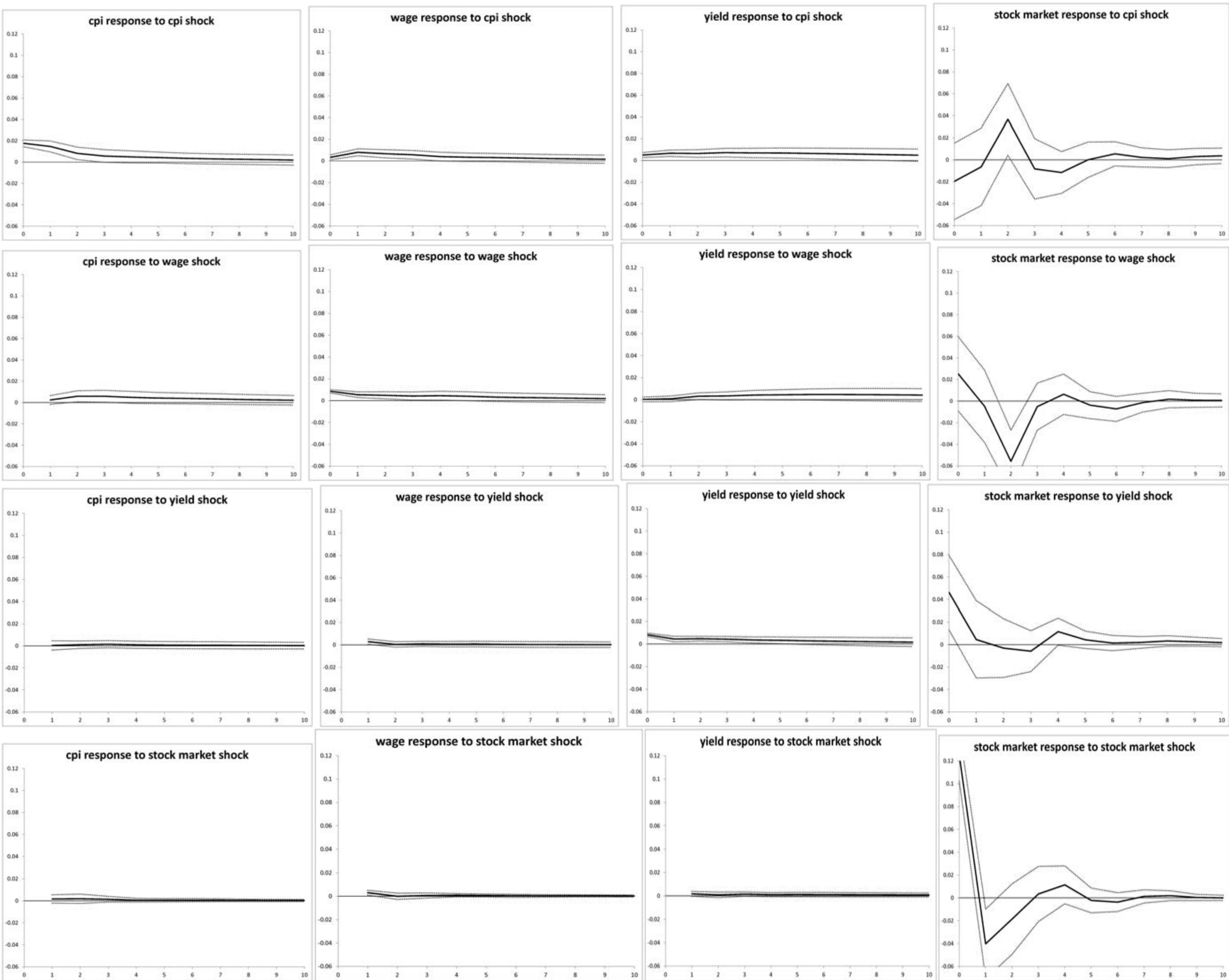
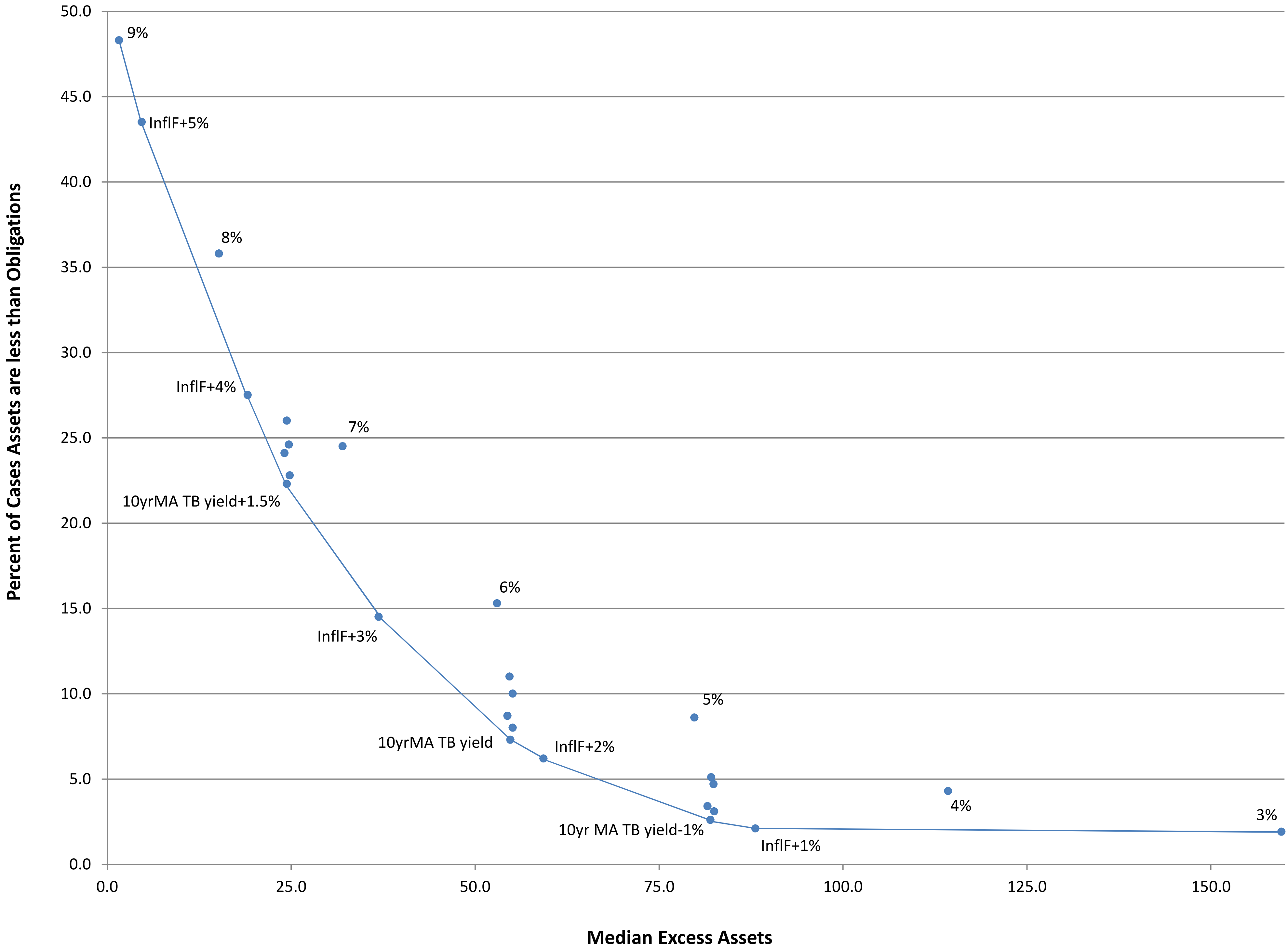


Figure 2: Median Excess Assets and Frequency of Negative Excess Assets



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