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OPTIMAL LOAD FLOW SOLUTION OF POWER SYSTEMS

by

© AHMED MOHAMED HELMY RASHED

A THESIS

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ABSTRACT

The problem of optimal load flow in power systems is considered. Based on Carpentier's formulation, the problem is reformulated to achieve a reduction in the number of system variables, by treating all generator nodes as swing busses. Reduced system variables are also classified into control and dependent variables. The set of equality constraints is associated with the objective function using Lagrangian multipliers, and functional inequality constraints are included as penalties.

The set of control variables is iteratively adjusted, using Newton's method, to minimize the objective function, while the set of dependent variables is evaluated by solving the set of equality constraints after each such adjustment.

The reduction in the number of system variables and their classification as such provide a great saving in computer storage requirement as compared to other established methods. Furthermore, the use of Newton's method provides an excellent convergence behaviour.

The developed minimization algorithm is applied to the well known minimum generation cost and minimum system loss problem. In addition, the minimum fuel consumption, and combined fuel-cost minimization problems are defined and solved.

An optimal ordering scheme of system nodes, for use with large systems, is also developed and compared to two other effective schemes. The new scheme proved to be generally comparable to both schemes for the cases studied.

The concept of fixed penalty factor, developed in assocation with the minimization algorithm is investigated and compared to the usual concept of monotonically increasing sequence of penalty factors. The former proved to be superior from a practical point of view.

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CHAPTER I

INTRODUCTION

With the rapid growth of demand of electrical energy, power systems have had to expand. Associated with such expansion are problems which have not been encountered before, or encountered only to a minor extent. These problems are related to the question of how to operate or expand a power system reliably and economically. Today, with the vast sizes of power systems that exist, many, if not all, of these problems still have to be faced and dealt with.

While some of these problems, such as stability and associated problems, are related to the transient behaviour of the power system, some others are related to the steady state operation of the system. These latter problems include, among others, the problem of optimal load flow, i.e. the problem of obtaining a load-flow solution which optimizes a certain operating criterion.

The importance of the optimal load flow problem is evident if one considers the very simple case of two generators supplying a common load, and recognizes that cost savings can be effected if generation is shifted from the less efficient generator to the more efficient one. However, an immediate question arises: how much generation is to be shifted to accomplish the most savings? This is the heart of the optimal load flow problem.

1.1 Development of the Optimal Load-Flow Problem

The optimal load flow problem was first recognized in a restricted sense in the form of the economic real power dispatch problem. As mentioned before, the aim was to obtain a generation schedule that will meet the demand most economically. Generator voltages were kept fixed, thus no control on reactive generation could be effected.

First attempts to solve this problem neglected the system's transmission losses. The result was a generation schedule that requires all generators to operate at the same incremental cost[1].

The advent of integrated systems and the interconnection between power companies for the purpose of economy interchange, rendered this approach unsatisfactory. Transmission losses became significant, and their effect had to be taken into account. This led to the modification of the incremental cost method using penalty factors and loss coefficients[2-11].

Although exact coordination of incremental loss and incremental cost was achieved[11], the method suffers from the fact that the methods used to determine these loss coefficients (B - constants) are approximate in nature and depend on particular load levels and generator voltages.

Relatively recently an exact transmission loss formula was developed using the bus impedance matrix of the system, and the results of a load flow solution[12]. Transmission loss was expressed in terms of real and reactive powers at all nodes. Coefficients of the formula are not constants but change as node voltages and angles change. The formula was used in conjunction with the exact co-ordination equations to produce the exact solution of the economic real power dispatch problem.

In the same reference, an extension to the method to economically allocate reactive generation is presented. By moving along the negative of the gradient of transmission loss with respect to reactive generations, these losses can be reduced. By alternating real and reactive power dispatch processes, it is claimed that the more general problem of exact economic dispatch can be solved. However, it has been later shown that although, this extension, will allow the control of reactive generation, it will fail to locate the optimum solution[13].

1.2 The General Optimal Load Flow Problem: Carpentier Formulation

In 1962 a breakthrough in the problem of optimal load flow was obtained by Carpentier[14], who developed an exact and general formulation of the economic dispatch problem.

An N bus power system is considered. At each node there may be generation P_g and Q_g, and consumption C and D, of active and reactive power, respectively. Each node is also characterized by its voltage V, and phase angle δ. These variables are interrelated, in a nonlinear fashion, by the network relations which govern the flow of power in every and all parts of the system.

Of course, total production $\sum_i P_{g_i}$ and $\sum_i Q_{g_i}$, must equal the total consumption $\sum_i C_i$ and $\sum_i D_i$, plus the respective transmission losses. This can be guaranteed by the satisfaction of the above mentioned network relations.

Further, due to engineering and physical considerations, some variables can only change within a specified range. For example, a generator can not produce any power beyond a certain value determined by the capacity of the boiler and turbine driving it.

Finally, the cost of operating the power system is a function of real power generations P_{g_i}, but not of reactive generations Q_{g_i}, which are cost free once the equipment required for their production has been installed.

Thus the problem consists of minimizing the operating cost $f(P_{g_1},\ldots,P_{g_N})$ subject to the following:

1) Satisfaction of equality constraints formed by the power flow equations (network relations).

$$P_{g_i}-C_i = \sum_\alpha V_i\, V_\alpha\, Y_{i\alpha} \cos(\delta_i-\delta_\alpha-\theta_{i\alpha}) \qquad (1.1)$$

$$Q_{g_i}-D_i = \sum_\alpha V_i\, V_\alpha\, Y_{i\alpha} \sin(\delta_i-\delta_\alpha-\theta_{i\alpha}) \qquad (1.2)$$

where $Y_{i\alpha}e^{j\theta_{i\alpha}}$ is the term in the node admittance matrix corresponding to nodes i and α.

2) Satisfaction of inequality constraints imposed by the operating limits of the various variables:

$$P_{i_{min}} \leq P_{g_i} \leq P_{i_{max}} \quad (1.3)$$

$$Q_{i_{min}} \leq Q_{g_i} \leq Q_{i_{max}} \quad (1.4)$$

$$V_{i_{min}} \leq V_i \leq V_{i_{max}} \quad (1.5)$$

Other inequality constraints can also be imposed.

Although Carpentier presented the problem in the form of economic dispatch, changing the function f to any other power system function (e.g. total real generation), and using the appropriate constraints, will result in a different power system optimization problem of the same general form.

1.3 Optimal Load-Flow Solution Methods

Apart from the methods that solve only special cases of the optimal load flow problem, some of which have been described in Section 1.1, several methods have been developed to tackle the problem in its most general form[15-22]. A comparison between some of these methods can be found in reference 13.

In this section two of these methods, due to Dommel and Tinney[15], and Sasson, Viloria and Aboytes[22], will be described in some detail because of their relevance to the work presented in this thesis. However, a brief outline of the other methods will be given first.

Sasson[16,17] and Ramamoorty and Rao[18] employed nonlinear programming methods to solve the problem. Several techniques have been described for incorporating the problem constraints as penalties on the cost function, thus transforming the problem of constrained optimal load flow to an unconstrained problem.

While in reference 18 a first order gradient technique was used for the unconstrained minimization process, references 16 and 17 used the Fletcher-Powell algorithm[23] which represents one of the most powerful techniques in what is known as the "variable metric methods"[24]. Though this method worked quite well for small systems, it developed convergence and computer storage problems as system size increased, and decomposition techniques had to be used[25].

Billinton and Sachdeva[19,20] used a suboptimal technique for solving the problem. With an assumed real power schedule, and keeping generator powers and relative phase angles between system nodes fixed, voltage magnitudes are optimized using transmission losses, a function of the voltages only, as an objective function, reactive power equations as equality constraints, and the limits on reactive generation and voltage magnitudes as inequality constraints. With the resulting voltage magnitudes fixed, a real power dispatch is obtained optimizing the cost of generation under the usual equality and inequality constraints. The process is then repeated until no further improvements can be obtained.

Although the method is similar to that of reference 12, the way the problem is formulated guarantees an optimal solution. However, this optimal solution will depend on the choice of the swing bus[20].

Reid and Hasdorff[21] applied quadratic programming to solve the problem. Since the method requires a quadratic cost function and linear equality constraints, new variables had to be introduced to transform system operating cost into a quadratic function of system variables including those newly defined. These new variables also includes the variables introduced to transform the inequality constraints into equality constraints.

The method worked very nicely for systems of up to 118 busses in size. No mention of storage requirements was given, however. But since the number of variables involved far exceeds those of any other method, storage requirements can jeoparadize the method's success for large realistic systems.

Dommel and Tinney's method[15] centres around ordinary load flow solution by Newton's method[26]. Lagrangian multipliers are used to associate the equality constraints with the objective function. Inequality constraints are included as penalties.

The equality constraints consist of all equations forming the ordinary load-flow problem, i.e. one real power equation for each generator node, and two real and reactive power equations for each load node. Slack node equations are not included.

Apart from fixed parameters and those which can be readily obtained from given equations, e.g. reactive generations, system variables are real power generations, with that of the slack node expressed in terms of the system voltages and angles which form the rest of the variables. The variables are classified as control or independent variables "$\underline{u}$" consisting of generator voltages and real powers, and dependent variables "$\underline{x}$" formed by voltages and angles of load nodes as well as angles of generator nodes.

The Lagrangian function is thus formed as:

$$\mathcal{L}(\underline{u},\underline{x}) = f(\underline{u},\underline{x}) + \underline{\lambda}^T \underline{g}(\underline{u},\underline{x}) \quad (1.6)$$

where $f(\underline{u},\underline{x})$, $\underline{g}(\underline{u},\underline{x})$ and $\underline{\lambda}$ are the objective function, equality constraints and the Lagrangian multipliers, respectively. Conditions for the minimum of the objective function are:

$$\frac{\partial \mathcal{L}}{\partial \underline{\lambda}} = \underline{g}(\underline{u},\underline{x}) = \underline{o} \quad (1.7)$$

$$\frac{\partial \mathcal{L}}{\partial \underline{x}} = \frac{\partial f}{\partial \underline{x}} + [\frac{\partial \underline{g}}{\partial \underline{x}}]^T \underline{\lambda} = \underline{o} \quad (1.8)$$

$$\frac{\partial \mathcal{L}}{\partial \underline{u}} = \frac{\partial f}{\partial \underline{u}} + [\frac{\partial \underline{g}}{\partial \underline{u}}]^T \underline{\lambda} = \underline{o} \quad (1.9)$$

Given an estimate for the control variables "$\underline{u}$", equations (1.7) which are the load-flow equations, are first solved for the dependent variables "$\underline{x}$". The Jacobian is then used in (1.8) to solve for "$\underline{\lambda}$". Equations (1.9) will then give the gradient $\nabla_u \mathcal{L}$ of the Lagrangian function with respect to the control variables $\underline{u}$. A correction is then applied to these control variables by a move along the negative direction of $\nabla_u \mathcal{L}$ which is the direction of maximum decrease of $\mathcal{L}$ at that point. This correction is given by:

$$\Delta \underline{u} = - c \ \nabla_u \quad (1.10)$$

where c is an acceleration factor determining the size of the move.

The process is repeated until no further improvements can be achieved.

The method, although theoretically sound, suffers from the poor convergence characteristics of the steepest descent method and its sensitivity to acceleration c. Storage requirements could also pose a problem for large systems, due to the large number of equality constraints, even if only non-zero elements of the Jacobian matrix are stored. Another problem is that the solutions obtained for some problems, e.g. the minimum loss problem ($f = P_{g_{slack}}(V, \delta)$), are dependent on the location of the slack node.

The major drawback of the method is, however, its inability to handle complex objective functions[27], possibly because of the inadequacy of the steepest descent direction. Complexity of the cost function can result from penalty terms introduced by violated nonlinear inequality constraints which badly deform the hypercontours in the state space. This is possibly why no results involving such constraints have been published using this method.

Sasson[22] did not distinguish between equality and inequality constraints, and penalized the objective function for both types. Generator equations were first removed from the equality constraints and substituted into the objective function and inequality constraints to eliminate generator real and reactive powers as variables. To minimize the cost function, corrections to the voltages and angles of all nodes are applied at the same time using the Hessian matrix of second order partial derivatives of the panalized objective function.

$$\Delta \underline{u} = - \underline{H}^{-1} \nabla_{\underline{u}} f \qquad (1.11)$$

where $\underline{u}$, $\underline{H}$ and $\nabla_{\underline{u}} f$ are the vector of voltages and angles, the Hessian matrix and the gradient of the penalized objective function with respect to the vector $\underline{u}$, respectively.

The method requires even more storage than Dommel and Tinney's method, thus its success could be limited for large systems. Since equality constraints are not satisfied until the last iteration, they are always present as penalties in the objective function with the obvious adverse effects on the method's convergence. This can be seen from the less than adequate satisfaction of these equality constraints after a solution is claimed to be obtained[22].

1.4 Research Objective

As shown in Sections 1.1 and 1.3 every method suffers from one drawback or another. This limits their use in the power industry either due to their storage requirements or poor convergence. Thus it was considered worthwhile to investigate the possibility of developing another method which possesses the advantages of the previous methods, and at the same time does not have, at least, their major shortcommings.

Also investigated, is a new approach to optimal ordering of system nodes to minimize the number of new off-diagonal elements introduced during the elimination process used in Newton's method. The idea is to locate as much off-diagonal element of the original matrix as possible in the lower right hand corner of such matrix.

The goal of this work is, thus, to develop a method and its supporting mechanisms that can be readily implemented by the power industry.

CHAPTER II

PROBLEM REFORMULATION AND ITS SOLUTION

As mentioned in Section 1.4, the objective of the solution method is two fold; first, to reduce computer storage requirement, and second, to obtain better convergence behaviour. The first objective can be achieved by reformulating the problem to eliminate some of the variables, and using a solution method that does not handle all the remaining variables simultaniously. The second objective can be fulfilled by choosing a minimization criterion which provides a better minimization direction.

2.1 Problem Reformation

The basic Carpentier formulation was stated in Section 1.2. It was also mentioned that although Carpentier presented his formulation in the form of the economic dispatch problem, changing the objective function will lead to a different power system optimization problem. This means that the objective function f is not bound to be a function of real power generations only. Therefore, it would be preferrable to restate the problem in the following form, which differs from that of Section 1.2. in only the definition of the objective function:

Minimize the scalar function f of system variables subject to the following constraints:

Equality constraints:

$$\left.\begin{array}{l} P_i(\underline{V},\underline{\delta}) - (P_{g_i} - C_i) = o \\ \\ Q_i(\underline{V},\underline{\delta}) - (Q_{g_i} - D_i) = o \end{array}\right\} \quad i = 1,\ldots,N \qquad \begin{array}{r}(2.1)\\ \\ (2.2)\end{array}$$

Inequality constraints:

$$\left.\begin{array}{l} P_{i_{min}} \leq P_{g_i} \leq P_{i_{max}} \\ \\ Q_{i_{min}} \leq Q_{g_i} \leq Q_{i_{max}} \\ \\ V_{j_{min}} \leq V_j \leq V_{j_{max}} \end{array}\right\} \quad \begin{array}{l} i \text{ for generator} \\ \text{busses, } j \text{ for all} \\ \text{busses} \end{array} \qquad \begin{array}{r}(2.3)\\ \\ (2.4)\\ \\ (2.5)\end{array}$$

where, at a bus i, $P_i(\underline{V},\underline{\delta})$ and $Q_i(\underline{V},\underline{\delta})$ are real and reactive power injections given by the right hand sides of equations (1.1) and (1.2) respectively.

The equality constraints (load flow equations) consist of 2N equations. This number can be reduced if those equations corresponding to generator nodes are removed. This can be accomplished by substituting for P_{g_i}'s and Q_{g_i}'s of generator nodes in the objective function and/or the appropriate inequality constraints. This leaves two equations per each load node to form the equality constraints. This is "N_g-1" equations less than an ordinary load flow equations set, where N_g is the number of generator busses. Note that in an ordinary load flow problem, reactive power equations of generator busses are always excluded due to fixed generator voltages. Also excluded is the real power equation of the slack node due to the fact that its angle is fixed as reference, and its power is left floating to absorb system losses.

Elimination of P_{g_i} and Q_{g_i} in such a way, and the fact that almost all system variables can be expressed in terms of system voltages and angles, transform the problem into the following form.

Minimize the scalar function $f(\underline{V},\underline{\delta})$ subject to the following constraints:

Equality constraints at a load bus i:

$$P_i(\underline{V},\underline{\delta}) + C_i = o \qquad (2.6)*$$

$$Q_i(\underline{V},\underline{\delta}) + D_i = o \qquad (2.7)*$$

Inequality constraints at a generator bus j:

$$P'_{j_{min}} \leq P_j(\underline{V},\underline{\delta}) \leq P'_{j_{max}} \qquad (2.8)$$

$$Q'_{j_{min}} \leq Q_j(\underline{V},\underline{\delta}) \leq Q'_{j_{max}} \qquad (2.9)$$

Inequality constraint at any bus k:

$$V_{k_{min}} \leq V_k \leq V_{k_{max}} \qquad (2.10)$$

---

*Note that P_{g_i} and Q_{g_i} are zero at a load bus.

The prime on the upper and lower limits of generator powers indicates that these limits are modified to account for local loads. It will be omitted from now on,but the foregoing should be emphasized. Note also that the main variables of the system have been reduced to include only the voltage magnitudes and angles.

Elimination of P_{g_i} and Q_{g_i} also means that all generator busses are treated as swing busses in the sense that their powers are determined from the voltage distribution in the system. This will eliminate the dependence of the solution of some problems, e.g. the minimum loss problem, on the choice of a particular swing bus. Any one of these busses can be chosen as a reference bus and its angle set to zero. Note here that the voltage magnitude at any of these busses is free to change (within prescribed limits) and the voltage level in the system is no longer determined by a fixed voltage at a swing bus.

Load flow equations (2.6) and (2.7) are, in vector form:

$$\underline{g}(\underline{V},\underline{\delta}) = \underline{o} \tag{2.11}$$

The order of the vector $\underline{g}$ is 2 N_L, where N_L=N-N_g is the number of load busses. Solution of (2.11) will provide the values of $2N_L$ unknowns out of 2N-1 variables, thus the remaining $2N_g$-1 variables should be assumed. This provides the basis of variable classification into:

a) Control or specified variables; $2N_g$-1 in number,

b) Dependent or unknown variables; $2N_L$ in number.

The most logical and natural choice is that the first set, denoted by the vector $\underline{u}$, should include the voltage magnitude and angles at generator busses. They are exactly $2N_g-1$ in number (note that the angle of the reference bus is fixed at zero). The second set, denoted by the vector $\underline{x}$, will consist of the voltage magnitudes and angles at load busses. The number of these quantities is exactly $2N_L$.

Thus, load flow equations (2.11) become

$$\underline{g}(\underline{u},\underline{x}) = \underline{o} \tag{2.12}$$

and the statement of the problem takes the form:

Minimize the scalar function $f(\underline{u},\underline{x})$ subject to:

Equality constraints:

$$\underline{g}\ (\underline{u},\underline{x}) = o \tag{2.12}$$

Inequality constraints;

$$P_{i_{min}} \leq P_i(\underline{u},\underline{x}) \leq P_{i_{max}} \tag{2.13}$$

$$Q_{i_{min}} \leq Q_i(\underline{u},\underline{x}) \leq Q_{i_{max}} \tag{2.14}$$

$$V_{j_{min}} \leq V_j \leq V_{j_{max}} \tag{2.15}$$

where i is a generator bus, and j is any bus.

2.2 Solution of the Optimal Load Flow Problem

Any constrained nonlinear programming problem, like the one described in the previous section, can be solved by converting it into an unconstrained problem, and then applying one of the numerous methods developed for unconstrained minimization. The conversion is achieved by defining an appropriate auxiliary function, in terms of the original problem functions, and using it as a new unconstrained objective function.

There are two ways to incorporate the problem constraints into the objective function to form this new auxiliary function. The first is to use the Lagrangian multiplier theorem for the equality constraints, and the Kuhn - Tucker theorem[28] for inequality constraints. This requires that the equality constraints are explicitly satisfied. Also required is the satisfaction of what is known as the " exclusion equations" of the Kuhn-Tucker theorem[13]. The minimization process thus moves from one feasible point to another until the minimum of the objective function is located.

The second way is to penalize the objective function for both types of constraints, as Sasson et. al. did[22]. Many forms for the penalty function are available in the literature[24,28,29]. In this case, one moves in the whole space rather than the feasible region until the optimum is obtained.

To choose between these two approaches, one should recognize that unless the equality constraints are explicitly satisfied, they will always be present as penalties with the obvious adverse effects on convergence as mentioned before. Thus the idea of penalizing the cost function for this type of constraint is ruled out. Furthermore, very few inequality constraints are violated simultaneously, so their inclusion as penalties is not as bad as it would be in the case of equality constraints. Moreover, the evaluation of Kuhn-Tucker dual variables and the associated change of node type, which would increase the number of equality constraints, and the order of the load-flow portion, is avoided.

For the problem at hand, the Lagrangian function is defined as:

$$\mathcal{L}(\underline{u},\underline{x}) = f(\underline{u},\underline{x}) + \underline{\lambda}^T \, \underline{g}(\underline{u},\underline{x}) \tag{2.16}$$

where, $f(\underline{u},\underline{x})$ is the objective function which incorporates the penalty terms arising from the violated inequality constraints (if any) as will be discussed later, and $\underline{\lambda}$ is the vector of the Lagrangian multipliers associated with the equality constraints.

It is imperative to say that minimization of the Lagrangian L with the equality constraints satisfied means in fact the minimization of f. Thus at the minimum of $f(\underline{u},\underline{x})$, the following conditions must be satisfied giving the optimal solution:

$$\frac{\partial L}{\partial \underline{\lambda}} = \underline{g}\ (\underline{u},\underline{x}) = o \tag{2.17}$$

$$\frac{\partial L}{\partial \underline{x}} = \frac{\partial f}{\partial \underline{x}} + \left[\frac{\partial \underline{g}}{\partial \underline{x}} \right]^T \underline{\lambda} = o \tag{2.18}$$

$$\frac{\partial L}{\partial \underline{u}} = \frac{\partial f}{\partial \underline{u}} + \left[\frac{\partial \underline{g}}{\partial \underline{u}} \right]^T \underline{\lambda} = o \tag{2.19}$$

Although equations (2.16) - (2.19) are identical to equations (1.6) - (1.9) of Dommel and Tinney[15], one should emphasize the difference in the order of matrices and vectors involved, as well as the differences in the definition of these various quantities. This is what gives this method the advantage as far as computer storage is concerned as will be detailed later.

Equations (2.17) - (2.19) are nonlinear and iterative methods are necessary to solve them. The generalized Newton's method has been known to be the most powerful of all minimization techniques[29], resulting in a superior convergence behaviour. The only theoretical difficulties associated with this method is the amount of storage required for the Jacobian matrix and the expensive efforts to invert it. However, these difficulties are drastically reduced in power system studies because of the extreme sparsity of the Jacobian so that Newton's method has become a standard procedure in the power industry.

Thus, based on this method, the solution algorithm is as follows. A flow chart is also given in Fig. 2.1.

1) An arbitrary set of values is assumed for system voltages and angles.
2) Load flow equations (2.17) are solved using Newton's method[26]. The order of this problem is $2N_L$.
3) Lagrangian multipliers $\underline{\lambda}$ are obtained from equation (2.18) as:

$$\underline{\lambda} = -\left[\frac{\partial g}{\partial \underline{x}}\right]^{T^{-1}} \cdot \left[\frac{\partial f}{\partial \underline{x}}\right] \tag{2.20}$$

The equations are linear in $\underline{\lambda}$ and the Jacobian matrix $[\frac{\partial g}{\partial \underline{x}}]$ is already available from step 2). If the Jacobian is available in factored form (upper and lower triangles)*

---

*This is computationally equivalent to the inverse or transposed inverse[30].

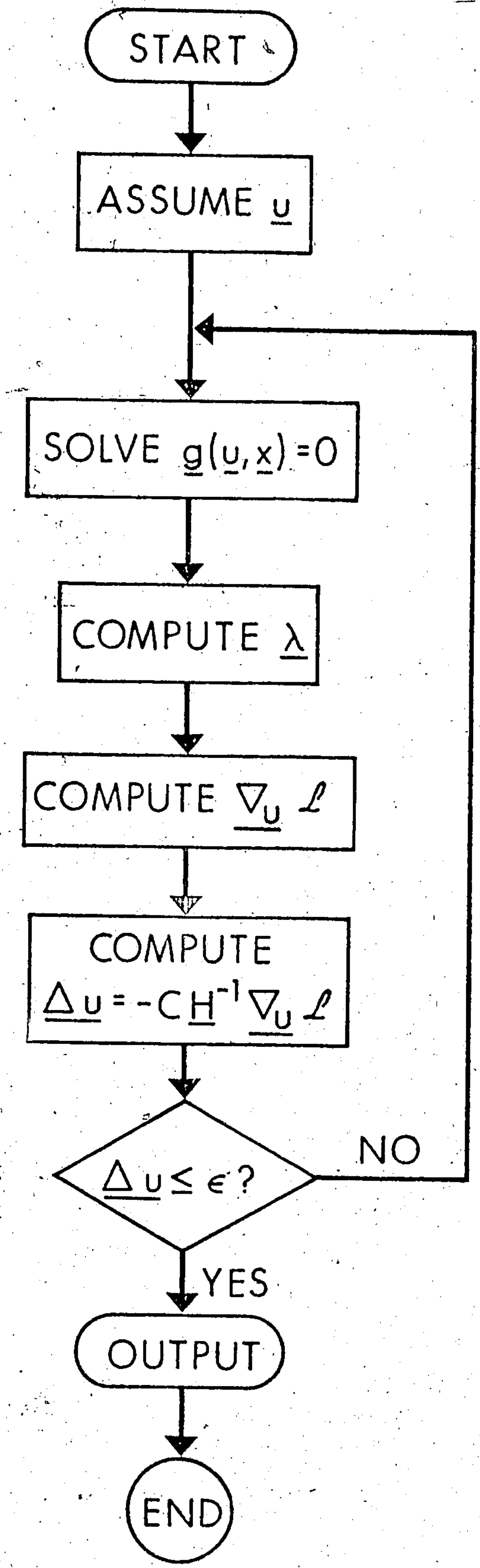

Figure 2.1 Flow-Chart of Solution Algorithm

as is usually the case for large systems, this step represents one repeat solution of a linear system.

4) At this point the values of $\underline{u}$, $\underline{x}$ and $\underline{\lambda}$ satisfy equations (2.17) and (2.18). They will not, in general, satisfy equations (2.19) which will, then, give the gradient $\underline{\nabla} f_{\underline{u}}$ of the objective function f with respect to the control variables $\underline{u}$ when equality constraints are satisfied[15]. Thus a correction $\underline{\Delta u}$ in $\underline{u}$ is possible using the relation:

$$\underline{\Delta u} = - \underline{H}^{-1} \underline{\nabla f}_{\underline{u}} \qquad (2.21)$$

where $\underline{H}$ is the Hessian matrix of the second order partial derivatives of the Lagrangian function with respect to $\underline{u}$. Its order is $2N_{g-1}$. Formula (2.21) is identical to Newton's formula. It gives not only the direction of the move, but also its size.

5) The new values of the control variables $\underline{u}$ is then given by

$$\underline{u}^{i+1} = \underline{u}^{i} + \underline{\Delta u}^{i} \qquad (2.22)$$

where the superscript indicates the iteration number.

6) If some convergence criterion is satisfied, the solution has been found otherwise return to step 2).

For convergence criterion, one may use one of the following:

a) The change in the value of the objective function is less than a prescribed value. The accuracy of this criterion is doubtful in cases where the objective function happens to be very flat near its minimum resulting in a suboptimal solution.

b) Each component of the correction vector $\underline{\Delta u}$ is smaller than a preassumed value. This will produce the most accurate results because one is checking the movement of the operating point itself.

c) Each component of the gradient vector $\underline{\nabla f_u}$ is less than a set tolerance. This is the practical equivalent to the theoretical requirement of zero gradient which is the condition for an optimal solution. However due to the premultiplication by the inverse Hessian and the fact that, in practice, the gradient will never be identically zero, the correction vector $\underline{\Delta u}$ may not satisfy criterion (b) if it were used instead. Moreover, care must be taken because a component $\frac{\partial L}{\partial u_i}$ of the gradient vector $\underline{\nabla f_u}$ will not be zero if the control variable u_i is on one of its limits[15].

2.2.1 Solution of Equation (2.17)

Newton's method is well known and its development into the most powerful methods of load flow solution by exploiting the sparsity of the Jacobian matrix of load flow equations is well documented[26]. The fact that the Jacobian matrix should be positive definite for assured convergence seems to be satisfied for practical power systems as there is no evidence to the contrary although very many different systems have been studied.

In equations (2.17) the vector $\underline{u}$ is fixed at the initial guess, or the corrected value. The vector $\underline{x}$ is set at an initial estimate, or the value from the previous iteration, and the vector $\underline{g}(\underline{u},\underline{x})$; which is known as the power mismatch, is computed. Generally, it will not equal to zero. Corrections in the vector $\underline{x}$ are then carried out using the relations

$$\underline{\Delta x}^{i} = - \underline{J}^{i^{-1}} \; \underline{g}^{i}(\underline{u}^{i},\underline{x}^{i}) \tag{2.23}$$

$$\underline{x}^{i+1} = \underline{x}^{i} + \underline{\Delta x}^{i} \tag{2.24}$$

where $\underline{J}^{i}$ is the Jacobian matrix evaluated at the i^{th} iteration, and the superscripts indicate the iteration number. The process is repeated until the mismatches $\underline{g}$ are less than a prescribed value (e.g. 10^{-5}).

Equations (2.23) are written in more detail as:

$$\begin{bmatrix} \Delta\,\delta_1 \\ \Delta\,V_1 \\ \vdots \\ \Delta\,\delta_{N_L} \\ \Delta\,V_{N_L} \end{bmatrix} = \begin{bmatrix} \frac{\partial p_1}{\partial \delta_1} & \frac{\partial p_1}{\partial V_1} & \cdots & \frac{\partial p_1}{\partial \delta_{N_L}} & \frac{\partial p_1}{\partial V_{N_L}} \\ \frac{\partial q_1}{\partial \delta_1} & \frac{\partial q_1}{\partial V_1} & \cdots & \frac{\partial q_1}{\partial \delta_{N_L}} & \frac{\partial q_1}{\partial V_{N_L}} \\ \vdots & & & & \vdots \\ \frac{\partial p_{N_L}}{\partial \delta_1} & \frac{\partial p_{N_L}}{\partial V_1} & \cdots & \frac{\partial p_{N_L}}{\partial \delta_{N_L}} & \frac{\partial p_{N_L}}{\partial V_{N_L}} \\ \frac{\partial q_{N_L}}{\partial \delta_1} & \frac{\partial q_{N_L}}{\partial V_1} & \cdots & \frac{\partial q_{N_L}}{\partial \delta_{N_L}} & \frac{\partial q_{N_L}}{\partial V_{N_L}} \end{bmatrix}^{-1} \begin{bmatrix} p_1 \\ q_1 \\ \vdots \\ p_{N_L} \\ q_{N_L} \end{bmatrix} \tag{2.25}$$

where all subscripts belong to the set of load nodes, and p_i and q_i are real and reactive mismatches (forming the vector $\underline{g}$) at load node i. The elements of the Jacobian matrix are given in Appendix A.

Even though the Jacobian matrix of load flow equations is extremely sparse (3.5% full for the IEEE 118-bus test system)* due to the fact that each node is connected to only few adjacent nodes, the computer time required to invert it in the case of a large power system would be prohibitive. Moreover, the inverse proper will be a full matrix requiring a large computer storage area.

These problems have been alleviated by the use of Gaussian elimination resulting in an upper triangular matrix, then using back substitution to obtain the solution $\underline{\Delta x}$. Forward operations may be stored in the lower triangle for the purpose of repeat solutions[30]. It has been shown that for an nxn full matrix the number of operations (multiplication-addition) required for triangulization is of the order $\frac{1}{3}n^3$ compared to n^3 for proper inversion[31]. Back substitution would require the same number of operations, n^2, as the premultiplication with the inverse. So it is evident that large savings in computer time can be effected.

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* see closure of ref. 22.

This approach combined with an optimal ordering scheme of the nodes to preserve the sparsity of the Jacobian during triangulization, and the use of compact storage techniques of sparse matrices, in which only non-zero elements and their positions are stored, will also result in great savings of computer storage which would be required otherwise.

2.2.2 Solution of Equations (2.18)

After equations (2.17) have been solved, a by product is the Jacobian matrix. The vector $\frac{\partial f}{\partial \underline{x}}$ can also be computed. Its components are given in Appendix A for a cost function which is quadratic in real power generations. The availability of these two quantities means that, at the present operating point, equations (2.18) are linear in the Lagrangian multipliers $\underline{\lambda}$. They can be solved by virtue of equation (2.20). Whether the Jacobian is available in factored form or as explicit inverse Jacobian, the determination of $\underline{\lambda}$ amounts to a repeat solution of the transposed system. In this case the vector $\frac{\partial f}{\partial \underline{x}}$ replaces the mismatch vector $\underline{g}$. The number of operations required in either case is n^2 if the matrix is full, or much lower if the sparsity of the Jacobian is taken advantage of, i.e. the factored Jacobian is used instead of the inverse.

2.2.3 The Hessian Matrix

In equations (2.21) the Hessian matrix of the second order partial derivatives of the objective function with respect to the control variables $\underline{u}$, is used to compute the corrections $\Delta\underline{u}$. The matrix is symmetrical, and, thus, only the diagonal and upper triangle elements need to be stored. Moreover the matrix is extremely sparse as shown in Figure 2.2 This is due to the fact that the control variables consist of the voltages and angles of generator nodes only. Unless there is a direct tie between two such nodes, the Lagrangian function will be free from any term involving the variables at these nodes simultaneously. Hence, no cross term (off diagonal), e.g. $\frac{\partial^2 L}{\partial V_i \partial V_j}$, will appear in the Hessian except in the case where there is a tie between generator i and generator j. In practice these ties are rare, and the Hessian reduces almost to 2 x 2 submatrices, along the main diagonal, representing the terms $\frac{\partial^2 L}{\partial \delta_i^2}$, $\frac{\partial^2 L}{\partial \delta_i \partial V_i}$, $\frac{\partial^2 L}{\partial V_i \partial \delta_i}$ and $\frac{\partial^2 L}{\partial V_i^2}$. In Figure 2.2 node j is connected to node k whereas all other nodes are free from any connection with other generator nodes.

Furthermore, the elements of the Hessian matrix are very simple and easy to compute (see Appendix A). This, together with the previous analysis of its structure, shows that equations (2.21) are easy to handle, and no problem as far as storage and time requirements, has to be faced.

As mentioned earlier, equation (2.21) is identical to Newton's relation (2.23). Thus, one expects a convergence behaviour as good as Newton's method. The Hessian matrix should be positive definite to assure such convergence, a condition which was true in all cases studied. The positive definiteness of the Hessian can also be assured if the starting point is close to the optimal solution. This requirement is always satisfied in a flat starting point*. The fact is

* A flat start means all voltages are set to its specified value or at 1.0 p.u. and all angles set at zero.

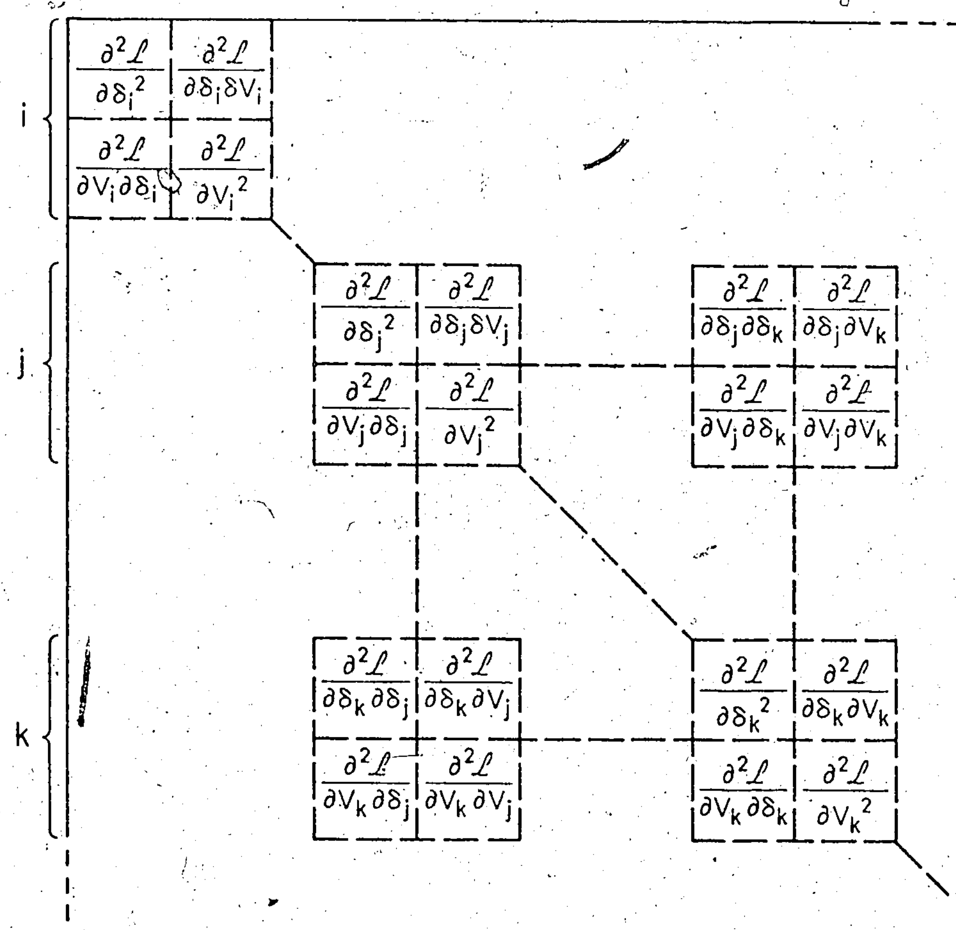

Figure 2.2 Structure of the Hessian Matrix

that in any power system the voltage magnitude at any bus will never change beyond the range 0.9 - 1.1 p.u., and the phase angles amount to a few degrees away from the zero reference.

2.3 Inequality Constraints

In the formulation of section 2.1, system variables were classified into control and dependant variables. As their name indicates, the value of any control variable is under control by virtue of equations (2.21) and (2.22) or any other valid regulating condition. On the other hand, there is no direct control over the dependent variables, as they are completely determined by the load flow solution. Also, control of the value that any generator power can assume is eliminated by treating all generators as swing generators. Therefore, inequality constraints, which determine the operating range of all these variables, can be classified into parameter (or control) and functional constraints.

2.3.1 Parameter or Control Constraints

These are the linear inequality constraints applicable to each individual control variable. Typical constraints are upper and lower limits on voltage magnitudes at generator nodes.

This type of constraints can be easily satisfied by restricting each variable to its operating range, i.e. no control variable is allowed to exceed its limit. Thus equations (2.22) are replaced by:

$$u_j^{i+1} = \begin{cases} u_{j_{max}} & \text{if } u_j^i + \Delta u_j^i > u_{j_{max}} \\ u_{j_{min}} & \text{if } u_j^i + \Delta u_j^i < u_{j_{min}} \\ u_j^i + \Delta u_j^i & \text{otherwise} \end{cases} \qquad (2.27)$$

The use of this approach not only will keep the successive moves within the feasible region as far as control variables are concerned, but will also reduce the number of inequality constraints to be treated by the penalty function approach, thus reducing the chance of introducing penalty terms into the objective function.

2.3.2 Functional Constraints

These consist of all problem constraints except parameter constraints. In general they are the inequality constraints, linear or nonlinear, involving dependent variables and/or two or more of the control variables. Typical constraints are upper and lower limits on generator powers, and upper and lower limits on voltage magnitude at load nodes.

As mentioned before, the penalty function approach is used to handle this type of constraints, particularly because very few such constraints are simultaneously violated. This, in itself is a reason to rule out the use of interior point penalty function[29] which requires that all inequality constraints, whether satisfied or not, be monitored and their derivatives calculated. Another reason to back this decision is the requirement of a feasible point to start an interior penalty function problem. To locate such a point for a high order system is indeed a very tedious task.

This leaves the exterior point methods[29] as the suitable way to handle these functional constraints. For one, they do not need a feasible point to start the minimization process. Secondly, the derivatives of only the violated constraints need to be computed. Finally, in the present practical problem, as may be the case in most practical problems, a functional constraint is seldom a rigid limit in the strict mathematical sense but is, rather, a soft limit. For instance, $V \leq 1.0$ p.u. on a load bus means that V should not exceed 1.0 p.u. by too much, and V=1.01 may still be permissible. Exterior point methods do just that (see Figure 2.3)

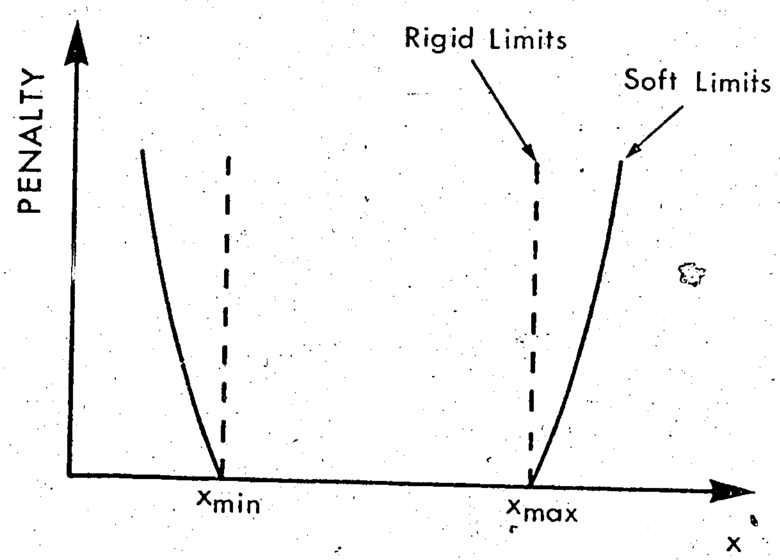

Figure 2.3 Exterior Penalty Function

To handle inequality constraints (2.13) - (2.15), each such relation should be split into two inequality relations, one of which could be active at a time, as:

$$h_{p_i} = P_i(\underline{u},\underline{x}) - P_{i_{min}} \geq 0 \tag{2.28}$$

or

$$\left.\begin{aligned} h_{p_i} &= P_{i_{max}} - P_i(\underline{u},\underline{x}) \geq 0 & (2.29)\\ h_{q_i} &= Q_i(\underline{u},\underline{x}) - Q_{i_{min}} \geq 0 & (2.30)\end{aligned}\right\} \text{ i for generator busses}$$

or

$$h_{q_i} = Q_{i_{max}} - Q_i(\underline{u},\underline{x}) \geq 0 \tag{2.31}$$

$$\left.\begin{aligned} h_{v_i} &= V_i - V_{i_{min}} \geq 0 & (2.32)\\ &\text{or} \\ h_{v_i} &= V_{i_{max}} - V_i \geq 0 & (2.33)\end{aligned}\right\} \text{ i for load busses}$$

A simple and useful form of penalty functions is given by:

$$w = \sum_i r_i \; h_i^2 \tag{2.34}$$

where i stands for violated constraints only

r_i is a penalty factor

h_i is the value of constraint violation given by one of relations (2.28) - (2.33)

Using penalty function (2.34) the objective function is given by:

$$f = f_o + w \tag{2.35}$$

where f_o is the original objective function. The function f is the one to be used in the minimization process. Note that if all constraints are satisfied (which is true at the optimal solution), the function f is in fact the function f_o.

Although proper minimization requires the solution of a sequence of unconstrained minimization problems for monotonically increasing r_i, it was observed that for practical power system problems, two or three such sequences, each consisting of two or three iterations, would be sufficient[22]. Furthermore, as will be shown in this thesis, even one such sequence would be enough. However, to do this the factor r_i should be chosen in such a way that the increase in the value of the objective function due to constraint violations is large enough to be sensed by the solution algorithm, but not so large as to divert the problem into constraint satisfaction rather than cost function minimization. All of this means that in the latter case the factor r_i should depend on the value of the objective function f_o.

2.4 Acceleration

Although relation (2.21) gives both the direction and the size of the minimization step, acceleration would be in order due to the fact that the objective function is not really quadratic in system variables, and that the quadratic approximation that Newton's method assumes, might not be satisfactory to produce the appropriate step size. Relation (2.22) is replaced by:

$$\underline{u}^{i+1} = \underline{u}^{i} + c \, \Delta \underline{u}^{i} \tag{2.36}$$

where $\Delta \underline{u}^{i}$ is given by (2.22), and c is an acceleration factor.

The scheme used here to determine c for any iteration is aimed to minimize overshooting and oscillation around the optimal solution or a constraint boundary. The factor c is always kept at unity until the minimum or a boundary of a functional constraint is approached from inside [illegible] is then reduced in such a way as to damp any possible overshoot [illegible] oscillation. In more detail, this is achieved as follows.

Before applying the correction $\Delta \underline{u}$ to the control variables, a test is made with c=1 to assure that a decrease in the objective function would result. If this is not the case, it means that overshooting the solution or constraint violation would result. c is, thus, halved and the process is repeated as many times as necessary until a decrease in the objective function is achieved or the convergence criterion is satisfied.

To perform the above mentioned test, it would be necessary to solve a load flow problem to obtain the value of the $\underline{x}$ vector corresponding to the value $\underline{u} + c \, \Delta \underline{u}$. Since the value of c is not as critical with Newton's method as it would be with other methods, such as steepest descent, it would suffice to solve a linearized load flow problem rather than the exact one. The linearized solution corresponding to a change $\underline{\Delta u}^i$ in control variables is obtained as:

$$\underline{x}^{i+1} = \underline{x}^i + \underline{\Delta x}^i \tag{2.37}$$

with

$$\underline{\Delta x}^i = \underline{S}\ \underline{\Delta u}^i \tag{2.38}$$

where $\underline{S}$ is the sensitivity matrix given by:

$$\underline{S} = - \left[\frac{\partial \underline{g}}{\partial \underline{x}} \right]^{-1} \cdot \left[\frac{\partial \underline{g}}{\partial \underline{u}} \right] \tag{2.39}$$

Note that the first matrix on the right hand side of equation (2.39) is the inverse Jacobian which is already available either explicitely or in a factored form. Only the second matrix $\left[\frac{\partial \underline{g}}{\partial u} \right]$ needs to be computed. Its elements are very simple as shown in Appendix A.

Substitution of (2.39) into (2.38) gives:

$$\underline{\Delta x}^i = - \left[\frac{\partial \underline{g}}{\partial \underline{x}} \right]^{-1} \left[\frac{\partial \underline{g}}{\partial u} \right] \underline{\Delta u}^i \tag{2.40}$$

A vector $\underline{\mu}$ is then defined as:

$$\underline{\mu} = \left[\frac{\partial \underline{g}}{\partial \underline{u}} \right] \underline{\Delta u}^i \tag{2.41}$$

Note that this product exists since $\left[\frac{\partial \underline{g}}{\partial \underline{u}} \right]$ is a $2N_L \times (2N_g-1)$ rectangular matrix and $\underline{\Delta u}^i$ is a $(2N_g-1)$ order vector.

Equation (2.40) can now be written as:

$$\underline{\Delta x}^i = - \left[\frac{\partial g}{\partial x} \right]^{-1} \underline{\mu} \tag{2.42}$$

which is of the form (2.24) with $\underline{\mu}$ replacing $\underline{g}$. It represents only a linear repeat solution using the same Jacobian matrix.

Although the objective of the acceleration scheme is to eliminate, as completely as possible, overshooting and oscillations, these may still occur due to one or both of the following reasons:

1) The solution of the linearized load flow problem may not quite agree with the exact solution. The former may indicate no overshooting, whereas after applying the latter, at the begining of the next iteration, overshooting does occur. Although, this can be avoided by using the exact solution throughout, the time consumed to obtain one or more such solutions per test would be definitely larger than the time consumed in one or two extra iterations that may be needed otherwise. Moreover, the disagreement between the two solutions will be minimal in the final stages of the process, due to small $\underline{\Delta u}$, when overshooting is more likely to happen.

2) It may happen that overshooting occurs and yet $f_{i+1} < f_i$. This indicates that the overshoot is small; or, if a constraint violation is involved, that r_i is small and a solution well outside the

feasible region will eventually be obtained.

An optimal acceleration factor (in the sense that it minimizes f in the direction $\underline{\Delta u}$) may be used resulting in fewer iterations. However, the extra time required to solve an associated nonlinear load flow problem and the fact that the objective function should, for the purpose of computing this optimal acceleration factor, be approximated to a second or third degree polynomial in c, may lead to almost the same convergence behaviour. Add to that, that the second reason above is general and applies to any acceleration mechanism. Therefore, the extra complexities in programming such optimal acceleration scheme do not appear to be justified.

2.5 Voltage Controlled Busses

A voltage controlled bus is a bus at which the voltage magnitude is controlled through the introduction of reactive power generation. As a result, voltage magnitude should be considered as a specified or control variable, and reactive power at the bus is left to change within the operating limits of the generation equipment. This means that a number of variables (voltage magnitude) corresponding to the number of such busses will be moved from the dependent variable vector $\underline{x}$ to the control variable vector $\underline{u}$. Also introduced at each such bus is a functional constraint of the form (2.14) defining the range in which reactive generation at that node can change. Since the voltage magnitude is no longer a dependent variable and reactive power ceases to be a fixed amount, the reactive power equation corresponding to each of these nodes is removed from load flow equations (2.12).

In other words, a voltage controlled bus is to be treated as a generator (or swing) node as far as reactive power is concerned, and as a load node as far as real power is concerned.

The order of the different vectors in cases involving N_q such nodes will be $2N_g-1 + N_q$ for the control variables $\underline{u}$, and $2N_L-N_q$ for each of the vector $\underline{g}$ representing the load flow equations and the vector $\underline{x}$ of the dependent variables.

2.6 Qualitative Evaluation

The aim of the solution method is to reduce computer storage requirement, and to achieve better convergence characteristics than other methods. Based on these two objectives, a qualitative evaluation of the method can be carried out. The methods of Dommel and Tinney[15], and Sasson et al[22] were considered for comparison purposes. Also given is a rough estimation of computation time requirements as compared to these two methods.

2.6.1 Computer Storage

It is customary, when one speaks of computer storage, to consider large systems, and estimate the amount of storage required for non-zero elements only. Since different matrices differ in their sparsity characteristics, and even the sparsity of the same matrix differs from one system to another, it would be appropriate when comparing storage requirements of different methods to use the concept of a full matrix and then to comment on what effects sparsity will have on each.

Apart from storage requirement of the vectors $\underline{x}$, $\underline{u}$ and other common quantities as power generations and loads, which should be the same for all methods, computer storage requirements of the developed method and the other two are shown in Figure 2.4 and Table 2.1. For simplicity no voltage controlled busses are assumed to be present. Table 2.2 also gives the amount of storage required for five standard test systems. Voltage controlled busses were considered in this case.

As can be seen, the method of Sasson et al[22] requires the most storage for their Hessian matrix. Dommel and Tinney's method may require more or less storage depending on the size of the system and the number of its generator busses. Most of their storage area is for the Jacobian matrix, however. The proposed method requires the least storage of the three methods. Moreover, as in the case of Dommel and Tinney's method, most of the required storage is for the Jacobian matrix.

As for sparsity, it has been shown that for typical power systems the Hessian matrices are more full than the Jacobian matrices*. Therefore, sparsity techniques of storage will give more advantage to those methods where the Jacobian occupies most of the storage. Hence,

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* Closure of Reference 22.

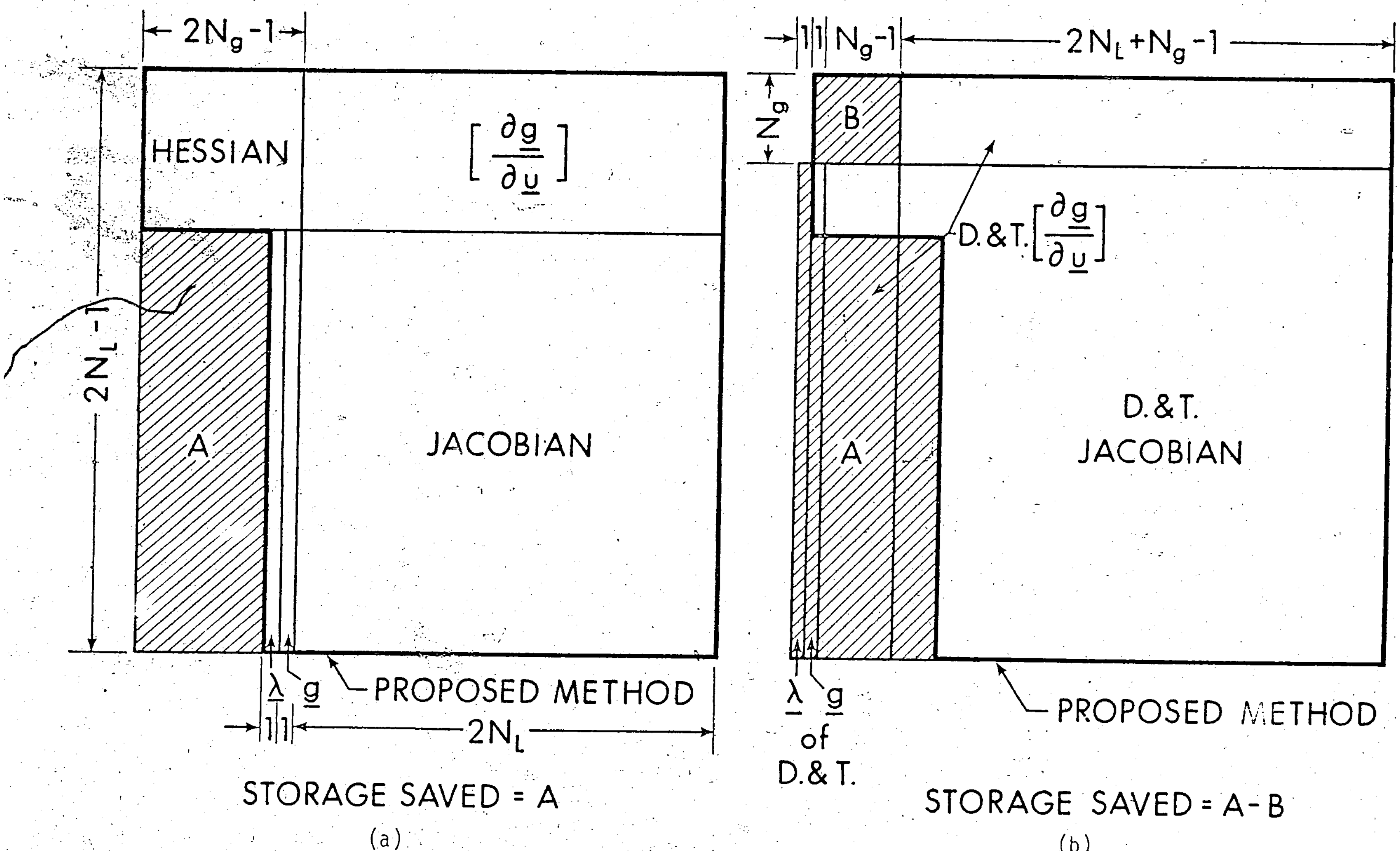

Figure 2.4 Storage Requirement of Proposed Method
(a) Comparison with the method of Sasson et al
(b) Comparison with Dommel and Tinney's method

Method	Storage Required					
	Hessian	Jacobian	$\underline{\lambda}$ Vector	$\underline{g}$ Vector	$[\frac{\partial g}{\partial u}]$ matrix	Total*
Sasson et.al.	$(2N-1)^2$	-	-	-	-	$(2N-1)^2$
Dommel and Tinney	-	$(2N_L+N_g-1)^2$	$2N_L+N_g-1$	$2N_L+N_g-1$	$(2N_L+N_g-1)\cdot(2N_g-1)$	$(2N-1)^2-N_g^2+(2N-N_g-1)$
Proposed Method	$(2N_g-1)^2$	$(2N_L)^2$	$2N_L$	$2N_L$	$2N_L\cdot(2N_g-1)$	$(2N-1)^2-2N_L(2N_g-3)$

* Total storage for the proposed method and Dommel and Tinney's has been reduced to be in terms of (2N-1) for easy comparison.

Table 2.1 Storage Requirement

System	Storage Required		
	Sasson et. al.	Dommel and Tinney	Proposed Method
5-Bus	81	84	75
IEEE 14-Bus	729	682	629
IEEE 30-Bus	3481	3339	3175
IEEE 57-Bus	12769	12508	12045
IEEE 118-Bus	55225	45250	44314

Table 2.2 Storage Requirement for Test Systems

storage savings effected by the proposed method can be emphasized even further.

2.6.2 Convergence

Since the iteration cycle of the proposed method and that of Dommel and Tinney are identical, convergence behaviour as judged by the number of iterations required to obtain the solution, will depend on the way the control variables are modified. Instantly, second order gradient methods, as used in this thesis, emerge as the superior compared to the steepest descent correction as used in Dommel and Tinney's algorithm. Moreover, that latter method is so sensitive to the acceleration factor c, that it will not converge in a reasonable number of iterations, or at all, unless the factor c is carefully chosen This type of sensitivity does not exist in Newton's method..

Sasson et al used Newton's formula as used here. One then should expect that the two methods will have the same convergence characteristics. However, since in their method a search of the whole space rather than the feasible region is required, more iterations will be needed to arrive at the solution.

2.6.3 Computation Time

Accurate assessment of the proposed and the two other methods as far as the computation time needed to obtain a solution is possible only if programs of compatible efficiency were written for each algorithm and run on the same computer. However, a rough comparison is still possible in the following way:

1) While the proposed method uses the Hessian matrix of the second order partial derivatives of the objective function with respect to the control variables to compute the corrections in these control variables, Dommel and Tinney's method[15] uses only the gradient. However, as mentioned before, this Hessian matrix is of low order, extremely sparse and its elements are very easy to compute. Moreover, the load flow problem, which is to be solved in both methods, is of much lower order in the proposed method than in Dommel and Tinney's, thus requiring less computer time. Although part of this time saving will definitely be used in handling the Hessian, in no case will total time per iteration required by the proposed method exceed that required by Dommel and Tinney's method.

2) Although the proposed method and that of Dommel and Tinney require load flow solution each iteration (such solutions usually require one or two Newton iterations), Sasson et al[22] uses a much larger and less sparse matrix, and, thus, will not offer any savings in the time required per iteration.

One can now estimate that the three methods will require about the same amount of time per iteration. The total time required to obtain the solution will then depend on the number of iterations needed to produce that solution. Here the decision will be in favour of the proposed method.

CHAPTER III

OPTIMAL SOLUTIONS OF SMALL SYSTEMS

This chapter deals with the application of the method developed in the last chapter to small power systems. In handling these small systems one does not have to worry about some of the problems encountered with large systems. For instance, the matrices involved are of low order and proper matrix inversion, for which ready routines are available, rather than factorization, can be used. Furthermore, storage requirements are very limited resulting in that the natural order of nodes would be satisfactory, and an optimal ordering scheme can be omitted. The result is a simple computer code and quick assessment of the success or failure of the method.

Two standard test power systems were studied, a 5-bus system[32] and the IEEE 14-bus test system. Data of the two systems is given in Appendix B. Four optimization problems were solved for each system using the University of Alberta IBM 360/67 Computer. The problems are minimum operating cost, minimum losses, minimum fuel comsumption, and combined cost-fuel minimization. The two latter problems are defined and formulated in this thesis for the first time.

3.1 Minimum Operating Cost Problem

This is the exact economic dispatch problem. The objective function is the total cost of generation, which is assumed to be of the form:

$$f = \sum_i (a_i + b_i P_{g_i} + c_i P_{g_i}^2) \quad \$/\text{Hr.} \tag{3.1}$$

The constants a and c_i for both test systems are given in Table 3.1. Since, in the method, generator and load nodes are handled separately, the natural order of nodes in both systems is changed so that generator nodes are numbered first, voltage controlled busses follow, and at the end come load busses.

Tables 3.2 and 3.3 give the solution of the problem, for the 5-bus and 14-bus systems respectively, when voltage magnitudes at generator and voltage controlled busses were kept fixed. In the case of the 5-bus system a flat start was used. However, in the case of the 14-bus system, a different starting point was chosen, since the flat start proved to be too far from the optimal solution, resulting in a slow convergence. This behaviour is typical of any iterative method. It did show, however, that the algorithm can easily handle violated functional constraints. The operating point, which was outside the feasible region for the flat start, was brought inside in two iterations. After that convergence was slow.

Since the optimal solutions were inside the feasible region, and to test the algorithm's ability to handle functional constraints if they are violated near the solution, which is a more sensitive problem, several runs were carried out on the 5-bus system. The results of these runs are given in Tables 3.4 - 3.8.

In Table 3.4, the voltages at generator busses no. 1 and 2

	Bus. No.	a	b	c
5-Bus System	1	44.40	351.00	50.00
	2	40.60	389.00	50.00
14-Bus System	1	105.00	245.00	50.00
	2	44.60	351.00	50.00
	3	40.60	389.00	50.00

Table 3.1 Cost Coefficients of 5- and 14- Bus Systems

Bus No	V_{min}	V	V_{max}	δ	P_{min}	P	P_{max}	Q_{min}	Q	Q_{max}
1	1.02	1.02	1.02	0.0	0.3	0.967	1.2	0.0	0.276	0.6
2	1.04	1.04	1.04	-2.140	0.3	0.685	1.2	0.0	0.531	0.6
3	0.9	0.955	1.05	-6.413		-0.6			-0.3	
4	0.9	0.923	1.05	-9.461		-0.4			-0.1	
5	0.9	0.993	1.05	-4.163		-0.6			-0.2	
f = Total Cost of Generator = 760.95 $/Hr.										

Table 3.2 Optimal Solution of Minimum Cost Problems for the 5-Bus System (fixed Generator Voltages)

Bus No.	V_{min}	V	V_{max}	δ	P_{min}	P	P_{max}	Q_{min}	Q	Q_{max}
1	1.06	1.06	1.06	0.0	0.0	1.571	2.0	-0.2	-0.042	1.0
2	1.045	1.045	1.045	- 3.209	0.2	0.673	1.0	-0.4	0.201	0.5
3	1.07	1.07	1.07	- 7.863	0.2	0.436	1.0	-0.06	0.032	0.45
4	1.01	1.01	1.01	-10.419		-0.942		-0.4	0.207	0.6
5	1.09	1.09	1.09	- 9.778		0.0		-0.06	0.174	0.45
6	0.9	1.024	1.1	- 7.652		-0.478			0.039	
7	0.9	1.027	1.1	- 6.146		-0.076			-0.016	
8	0.9	1.062	1.1	- 9.778		0.0			0.0	
9	0.9	1.053	1.1	-10.890		-0.245			-0.166	
10	0.9	1.048	1.1	-10.644		-0.09			-0.058	
11	0.9	1.055	1.1	- 9.399		-0.035			-0.018	
12	0.9	1.055	1.1	- 8.880		-0.061			-0.016	
13	0.9	1.049	1.1	- 9.124		-0.135			-0.058	
14.	0.9	1.033	1.1	-11.128		-0.149			-0.05	

f = Total Cost of Generation = 1136.14 \$/Hr.

Table 3.3 Optimal Solution of Minimum Cost Problem
for the 14-Bus System
(Fixed Generator Voltages)

Bus No	V_{min}	V	V_{max}	δ	P_{min}	P	P_{max}	Q_{min}	Q	Q_{max}
1	1.0	1.089	1.1	0.0	0.3	0.994	1.2	0.0	0.394	0.6
2	1.0	1.081	1.1	-1.786	0.3	0.651	1.2	0.0	0.386	0.6
3	0.9	1.013	1.05	-5.654		-0.6			-0.3	
4	0.9	0.989	1.05	-8.302		-0.4			-0.1	
5	0.9	1.050*	1.05	-3.657		-0.6			-0.2	
f = Total Cost of Generation = 757.78 $/H.										

* Constraint violation = 9×10^{-5} p.u.

Table 3.4 Optimal Solution of Minimum Cost Problem for the 5-Bus System (Free Generator Voltages) were free to change within the limits shown. In this case the unconstrained optimal solution was outside the feasible region resulting in a voltage constraint violation at load node no. 5. However, the algorithm was able to locate the constrained solution holding the voltage magnitude at that node within 9×10^{-5} p.u. of the constraint boundary. As a result generator voltages could not rise to their maximum limit as one expects in such a case.

In Tables 3.5 - 3.8, node no. 3 was changed into a voltage controlled bus where reactive generation was introduced and the voltage was allowed to change within the limits shown. The intention is to show how nonlinear functional constraints, such as real or reactive power limits, are handled if violated. Two ranges of reactive generation at node no. 3 were assumed: -0.5 to + 0.5, and - 0.4 to + 0.4.

Tables 3.5 and 3.6 give, respectively, the solutions of these two cases when voltage magnitudes of generator nodes were kept fixed. In the first case, the solution is well inside the feasible region, and it represents the unconstrained (as far as functional constraints are concerned) minimum of the objective function. Although this unconstrained solution lies outside the feasible region of the other case, a solution was obtained at which reactive generation at node no. 3 exceeded its maximum limit by only 2.55×10^{-3}, which shows the effective handling of this type of constraint.

Tables 3.7 and 3.8 give the results for the same cases when voltage magnitudes at generator nodes no. 1 and 2 were allowed to change as well. Although in the first case, reactive power generation

Bus No	V_{min}	V	V_{max}	δ	P_{min}	P	P_{max}	Q_{min}	Q	Q_{max}
1	1.02	1.02	1.02	0.0	0.3	0.962	1.2	0.0	0.070	0.6
2	1.04	1.04	1.04	-2.114	0.3	0.682	1.2	0.0	0.234	0.6
3	0.9	1.013	1.1	-6.904		-0.6		-0.5	[illegible]473	0.5
4	0.9	0.960	1.05	-9.437		-0.4			-0.1	
5	0.9	0.993	1.05	-4.150		-0.6			-0.2	

f = Total Cost of Generation = 757.57 \$/Hr.

Table 3.5 Optimal Solution of Minimum Cost Problem for Modified 5-Bus System (Fixed Generator Voltages) $-0.5 \leq Q_3 \leq 0.5$

Bus No	V_{min}	V	V_{max}	δ	P_{min}	P	P_{max}	Q_{min}	Q	Q_{max}
1	1.02	1.02	1.02	0.0	0.3	0.958	1.2	0.0	0.010	0.6
2	1.04	1.04	1.04	-2.055	0.3	0.687	1.2	0.0	0.275	0.6
3	0.9	1.005	1.1	-6.795		-0.6		-0.4	0.403*	0.4
4	0.9	0.955	1.05	-9.413		-0.4			-0.1	
5	0.9	0.993	1.05	-4.120		-0.6			-0.2	

f = Total Cost of Generation = 757.65 \$/Hr.

* Constraint violation = 2.55×10^{-3} p.u.

Table 3.6 Optimal Solution of Minimum Cost Problem for Modified 5-Bus System (Fixed Generator Voltages) $-0.4 \leq Q_3 \leq 0.4$

Bus No	V_{min}	V	V_{max}	δ	P_{min}	P	P_{max}	Q_{min}	Q	Q_{max}
1	1.0	1.089	1.1	0.0	0.3	0.983	1.2	0.0	0.198	0.6
2	1.0	1.082	1.1	-1.700	0.3	0.655	1.2	0.0	0.115	0.6
3	0.9	1.065	1.1	-6.053		-0.6		-0.5	0.442	0.5
4	0.9	1.022	1.05	-8.306		-0.4			-0.1	
5	0.9	1.051*	1.05	-3.612		-0.6			-0.2	

f = Total Cost of Generation = 754.93 $/Hr.

* Constraint violation: 5.19×10^{-4} p.u.

Table 3.7 Optimal Solution of Minimum Cost Problem for Modified 5-Bus System (Free Generator Voltages) $-0.5 \leq Q_3 \leq 0.5$

Bus No	V_{min}	V	V_{max}	δ	P_{min}	P	P_{max}	Q_{min}	Q	Q_{max}
1	1.0	1.092	1.1	0.0	0.3	0.966	1.2	0.0	0.262	0.6
2	1.0	1.077	1.1	-1.354	0.3	0.673	1.2	0.0	0.099	0.6
3	0.9	1.058	1.1	-5.808		-0.6		-0.4	0.394	0.4
4	0.9	1.020	1.05	-8.171		-0.4			-0.1	
5	0.9	1.050*	1.05	-3.441		-0.6			-0.2	

f = Total Cost of Generation = 755.02 $/Hr.

* No Constraint violation

Table 3.8 Optimal Solution of Minimum Cost Problem for Modified 5-Bus System (Free Generator Voltages) $-0.4 \leq Q_3 \leq 0.4$

at node no. 3 stayed well within its limits, the voltage magnitude at node no. 5 exceeded its maximum limit by only 5.19×10^{-4}. In the second case, in which the constraints on reactive generation at node no. 3 and voltage magnitude at node no. 5 were violated during the minimization process, the solution obtained was just inside these constraint boundaries.

3.2 Minimum Loss Problem

This is actually the problem of minimizing total real power generation which is determined by the total load plus system losses. The problem is often termed "optimal reactive power flow" because it is mainly the reactive flow in the transmission system that determines the losses. For lower losses, reactive generations should be close to the loads.

The objective function is given by:

$$f = \sum_i P_{g_i} \tag{3.2}$$

The solutions for the 5- and 14-bus systems are given in Tables 3.9 and 3.10 respectively. In the 5-bus system the only constraint violation was in the voltage magnitude at bus no. 5 which exceeded its maximum limit by only [illegible]85×10^{-4} p.u. thus holding generator voltages from reaching their maximum limits where the unconstrained (with respect to functional constraints) minimum is located. In the 14-bus system, [illegible] generations at generators 1 and 2

Bus No	V_{min}	V	V_{max}		P_{min}	P	P_{max}	Q_{min}	Q	Q_{max}
1	1.0	1.086	1.1		0.2	0.832	1.2	0.0	0.390	0.6
2	1.0	1.084	1.1	0.005	0.3	0.812	1.2	0.0	0.385	0.6
3	0.9	1.014	1.05	-4.588		-0.6			-0.3	
4	0.9	0.990	1.05	-7.684		-0.4			-0.1	
5	0.9	1.051*	1.05	-2.762		-0.6			-0.2	
f = Total System Losses = 0.0438 p.u.										

* Constraint violation = 7.85×10^{-4} p.u.

Table 3.9 Solution of Minimum Loss Problem for the 5-Bus System (Free Generator Voltages)

Bus No.	V_{min}	V	V_{max}	δ	P_{min}	P	P_{max}	Q_{min}	Q	Q_{max}
1	1.06	1.06	1.06	0.0	0.5	0.648	2.0	-0.2	0.160	1.0
2	1.045	1.045	1.045	-1.038	0.2	1.008*	1.0	-0.4	-0.023	0.5
3	1.07	1.07	1.07	0.144	0.2	1.001**	1.0	-0.06	-0.030	0.45
4	1.01	1.01	1.01	-7.598		-0.942		-0.4	0.184	0.6
5	1.09	1.09	1.09	-5.316		0.0		-0.06	0.183	0.45
6	0.9	1.028	1.1	-4.329		-0.478			0.039	
7	0.9	1.033	1.1	-2.852		-0.076			-0.016	
8	0.9	1.060	1.1	-5.316		0.0			0.0	
9	0.9	1.047	1.1	-5.837		-0.295			-0.166	
10	0.9	1.042	1.1	-5.070		-0.09			-0.058	
11	0.9	1.050	1.1	-2.618		-0.035			-0.018	
12	0.9	1.055	1.1	-1.081		-0.061			-0.016	
13	0.9	1.047	1.1	-1.528		-0.135			-0.058	
14	0.9	1.028	1.1	-4.975		-0.149			-0.05	
f = Total System Losses = 0.06678 p.u.										

* Constraint violation = 7.56×10^{-3} p.u. ** Constraint violation = 8.9×10^{-4} p.u.

Table 3.10 Solution of Minimum Loss Problems for

the 14- Bus System

(Fixed Generator Voltages)

exceeded their limits. However, these constraint violations were successfully controlled during the minimization process, such that at the solution maximum violation was 7.56×10^{-3} p.u.

3.3 Minimum Fuel Problem

Although generation cost at a power plant consists mostly of fuel cost, it includes the cost of labour, maintenance, etc. This portion is not the same for all plants within a system. For instance, an old plant needs maintenance more frequently than a new one; a gas turbine power plant needs a different amount of labour than a steam plant of the same capacity. Furthermore, fuel transportation costs are different for different plants. Therefore, generation cost functions are not indicative of the true fuel consumption in a given system, and scheduling generation according to a criterion of minimum generation cost does not necessarily mean that fuel consumption is also at a minimum.

The importance of the minimum fuel problem evolves from environmental and resource conservation grounds. On one hand oxides emission will be held as low as possible*, and on the other hand, by consuming a minimum amount of fuel for a given load condition, it is possible to prolong the life time of depleting fuel resources. The

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* In fact oxides emission does not depend only on the amount of fuel burned but also on the type of fuel and how perfectly it is burned.

idea is to make use of efficient power plants to produce most of the load requirement irrespective of the cost of such production. Thus, in return for the above mentioned advantages one is to pay a premium in the form of more expensive energy.

Let α_i be the proportion of non fuel costs in the generation cost function of generator i. Also assume that generator i receives its fuel at a cost of γ_i dollars/million B.T.U.'s. If generation cost is assumed to be of the form (3.1), then, fuel consumption of generator i in million B.T.U./Hr. is given by

$$f_i = \frac{1-\alpha_i}{\gamma_i} (a_i + b_i P_i + c_i P_i^2) \tag{3.3}$$

The objective function in this case will be

$$f = \sum_i f_i$$

$$= \sum_i \frac{1-\alpha_i}{\gamma_i} (a_i + b_i P_i + c_i P_i^2) \tag{3.4}$$

$$= \sum_i (a_i' + b_i' P_i + c_i' P_i^2) \tag{3.5}$$

The problem was solved for the 5- and 14-bus systems. The coefficients α_i and γ_i are assumed as given in Table 3.11. The coefficients a_i, b_i and c_i are kept as given in Table 3.1. Also shown in Table 3.11 are the coefficients a_i', b_i' and c_i' of the objective function (3.5).

	Bus no.	α	γ	a'	b'	c'
5-Bus System	1 2	0.25 0.2	0.40 0.50	83.25 65.0	658.125 622.5	93.75 80.0
14-Bus System	1 2 3	0.2 0.25 0.2	0.40 0.40 0.50	210.0 83.25 65.0	490.0 658.125 622.5	100.0 93.75 80.0

Table 3.11 Coefficients of the Objective Function of the Minimum Fuel Problem for the 5- and 14-Bus Systems

Tables 3.12 and 3.13 give the solution for the 5-bus system for fixed and free generator voltages respectively. Table 3.14 gives the solution for the 14-bus system.

3.4 Combined Cost-Fuel Minimization

Each of the minimum cost and minimum fuel problems is based on one global minimization criterion governing the whole system, consequently the solution will satisfy this criterion globally rather than at each individual generating plant. Sometimes, that is not quite what is desired. For instance, in addition to scheduling generation to achieve global minimum cost, one plant, a group of plants, or all plants may be required to consume a minimum amount of fuel to overcome a fuel shortage condition, or to achieve a reduction in pollution level in some locality. This means that scheduling of generation would be based on two criteria. In other words, a compromise should be arrived at which may not quite satisfy each criterion individually, but yet it minimizes their sum. One of these two criteria

Bus No	V_{min}	V	V_{max}	δ	P_{min}	P	P_{max}	Q_{min}	Q	Q_{max}
1	1.02	1.02	1.02	0.0	0.3	0.692	1.2	0.0	0.321	0.6
2	1.04	1.04	1.04	1.458	0.3	0.958	1.2	0.0	0.483	0.6
3	0.9	0.955	1.05	-4.265		-0.6			-0.3	
4	0.9	0.923	1.05	-8.197		-0.4			-0.1	
5	0.9	0.993	1.05	-2.346		-0.6			-0.2	
f = Total Fuel Consumption = 1319.00 Million BUT/Hr.										

Table 3.12 Solution of Minimum Fuel Problem for 5-Bus System (Fixed Generator Voltages)

Bus No	V_{min}	V	V_{max}	δ	P_{min}	P	P_{max}	Q_{min}	Q	Q_{max}
1	1.0	1.080	1.1	0.0	0.3	0.695	1.2	0.0	0.347	0.6
2	1.0	1.091	1.1	1.381	0.3	0.950	1.2	0.0	0.432	0.6
3	0.9	1.015	1.05	-3.764		-0.6			-0.3	
4	0.9	0.988	1.05	-7.227		-0.4			-0.1	
5	0.9	1.051*	1.05	-2.070		-0.6			-0.2	
f = Total Fuel Consumption = 1314.27 Million BTU/Hr.										

* Constraint violation = 8.12×10^{-4} p.u.

Table 3.13 Solution of Minimum Fuel Problem for 5-Bus System (Free Generator Voltages)

Bus No.	V_{min}	V	V_{max}	δ	P_{min}	P	P_{max}	Q_{min}	Q	Q_{max}
1	1.06	1.06	1.06	0.0	0.5	1.276	2.0	-0.2	0.001	1.0
2	1.045	1.045	1.045	-2.662	0.2	0.578	1.0	-0.4	0.156	0.5
3	1.07	1.07	1.07	-3.165	0.2	0.814	1.0	-0.06	-0.017	0.45
4	1.01	1.01	1.01	-9.334		-0.942		-0.4	0.191	0.6
5	1.09	1.09	1.09	-7.488		0.0		-0.06	0.179	0.45
6	0.9	1.027	1.1	-6.140		-0.478			0.039	
7	0.9	1.031	1.1	-4.590		-0.076			-0.016	
8	0.9	1.061	1.1	-7.448		0.0			0.0	
9	0.9	1.049	1.1	-8.198		-0.295			-0.166	
10	0.9	1.044	1.1	-7.598		-0.09			-0.058	
11	0.9	1.052	1.1	-5.535		-0.035			-0.018	
12	0.9	1.055	1.1	-4.323		-0.061			-0.016	
13	0.9	1.048	1.1	-4.706		-0.135			-0.058	
14	0.9	1.030	1.1	-7.689		-0.149			-0.05	

f = Total Fuel Consumption = 2117.86 Million BTU/Hr.

Table 3.14 Solution of Minimum Fuel Problem for 14-Bus System (Fixed Generator Voltages) should be global while the other may be only of local concern.

The objective function at a generating node would, then, be:

$$f_i = \sigma_{i_c} f_{i_c} + \sigma_{i_f} f_{i_f} \tag{3.6}$$

where f_{i_c} and f_{i_f} are the cost and fuel functions at node i, and σ_{i_c} and σ_{i_f} are priority factors.

It is important that both functions be in the same units, otherwise one will dominate the other (see Tables 3.1 and 3.10). One possibility is to transform generation cost into fuel units by dividing the generation cost function at each generator by fuel price at that node. However, since cost minimization is usually of global rather than local interest, and, it is more likely that fuel minimization is of local concern only, it is more appropriate to convert fuel consumption into dollars/Hr. Merely multiplying fuel functions by corresponding fuel prices will not be satisfactory. The resulting functions in this case will not be indicative of relative fuel consumption at system plants, since such prices depend on the type of fuel and include different transportation and handling costs. Thus, a base price should be used instead. The functions f_{i_f} will, then, be:

$$f_{i_f} = \frac{1-\alpha_i}{\gamma_i} (a_i + b_i P_i + c_i P_i^2) \gamma_b \tag{3.7}$$

where γ_b is the selected base fuel price.

The total objective function in this case will, then, be given by:

$$f = \sum_i \sigma_{i_c} (a_i + b_i P_i + c_i P_i^2) + \sum_i \sigma_{i_f} \gamma_b \frac{1-\alpha_i}{\gamma_i} (a_i + b_i P_i + c_i P_i^2) = \sum_i (a_i'' + b_i'' P_i + c_i'' P_i^2) \quad (3.8)$$

If, at a plant, a certain criterion is of no concern, its priority factor is set to zero.

Based on a base fuel price of 0.40 $/million B.T.U. cost-fuel function coefficients for the 5- and 14-bus systems are given in Table 3.15. It should be stressed here that these coefficients are used only when both criteria are active at a given node. Otherwise, the coefficients of Table 3.1 should be used.

	Bus no	a_i''	b_i''	c_i''
5-Bus System	1	77.7	614.25	87.5
	2	66.6	638.0	82.0
14-Bus System	1	189.0	441.0	90.0
	2	77.7	614.25	87.5
	3	66.6	638.0	82.0

Table 3.15 Cost-Fuel Coefficients of the 5- and 14-Bus Systems

Several runs were carried out for different situations. It is assumed that whenever a criterion is active at a plant, it is considered to be as important as the global criterion, i.e. priority factors are taken as unity when their corresponding criteria are of concern.

Two situations were considered for the 5-bus system. Two cases corresponding to fixed and free generator voltages were run for each. In the first situation, it is assumed that the objective is to achieve an overall economy in both generation cost and fuel consumption. The coefficients of the objective function would, then, be those of Table 3.15. The solutions for this situation are given in Tables 3.16 and 3.17.

In the second situation, what is required is to achieve an overall economy in generation cost such that economy in fuel consumption of generator no. 1 is also realized. Therefore, objective function coefficients for generator no. 1 would be drawn from Table 3.15, while those for generator no. 2 from Table 3.1. The solutions for this case are given in Tables 3.18 and 3.19. Observe how generator no. 2 has picked its maximum share of the load to allow the minimization of fuel consumption of generator no. 1.

In the case of the 14-bus system, three different objectives were considered. The first is the same as the first case of the 5-bus system, i.e. an overall economy in cost and fuel is desired. Table 3.20 gives its solution. The second is to achieve an overall cost

Bus No	V_{min}	V	V_{max}	δ	P_{min}	P	P_{max}	Q_{min}	Q	Q_{max}
1	1.02	1.02	1.02	0.0	0.3	0.849	1.2	0.0	0.293	0.6
2	1.04	1.04	1.04	-0.603	0.3	0.801	1.2	0.0	0.508	0.6
3	0.9	0.955	1.05	-5.494		-0.6			-0.3	
4	0.9	0.923	1.05	-8.919		-0.4			-0.1	
5	0.9	0.993	1.05	-3.386		-0.6			-0.2	

f = Total (Cost and Fuel Consumption) = 1292.68 $/Hr
f_c = Total Cost of Generation = 762.75 $/Hr
f_c = Total Fuel Consumption = 1324.78 Million BTU/Hr.

Table 3.16 Solution of Minimum Cost-Fuel Problem for the 5-Bus System (Fixed Generator Voltages)

Bus No	V_{min}	V	V_{max}	δ	P_{min}	P	P_{max}	Q_{min}	Q	Q_{max}
1	1.0	1.086	1.1	0.0	0.3	0.868	1.2	0.0	0.376	0.6
2	1.0	1.086	1.1	-0.447	0.3	0.775	1.2	0.0	0.400	0.6
3	0.9	1.015	1.05	-4.850		-0.6			-0.3	
4	0.9	0.990	1.05	-7.834		-0.4			-0.1	
5	0.9	1.051*	1.05	-2.985		-0.6			-0.2	

f = Total (Cost and Fuel Consumption) = 1287.71 $/Hr.
f_c = Total Cost of Generation = 759.21 $/Hr.
f_f = Total Fuel Consumption = 1321.26 Million BTU/Hr.

* Constraint violation = 1.37×10^{-3} p.u.

Table 3.17 Solution of Minimum Cost-Fuel Problem for the 5-Bus System (Free Generator Voltages)

Bus No	V_{min}	V	V_{max}	δ	P_{min}	P	P_{max}	Q_{min}	Q	Q_{max}
1	1.02	1.02	1.02	0.0	0.3	0.457	1.2	0.0	0.375	0.6
2	1.04	1.04	1.04	4.608	0.3	1.201*	1.2	0.0	0.455	0.6
3	0.9	0.954	1.05	-2.392		-0.6			-0.3	
4	0.9	0.923	1.05	-7.105		-0.4			-0.1	
5	0.9	0.992	1.05	-0.761		-0.6			-0.2	

f = Total Cost and Fuel Consumption at Gen. no. 1 = 956.76 \$/Hr.
f_c = Total Cost of Generation = 794.58 \$?hr.
f_f = f_{1f} = Fuel Consumption of Gen. no. 1 = 405.43 Million BTU/Hr.

* Constraint violation = 1.04×10^{-3}

Table 3.18 Solution of Minimum Total Cost and Fuel Consumption of Generator no. 1 for the 5-Bus System (Fixed Generator Voltages)

Bus No	V_{min}	V	V_{max}	δ	P_{min}	P	P_{max}	Q_{min}	Q	Q_{max}
1	1.0	1.084	1.1	0.0	0.3	0.446	1.2	0.0	0.448	0.6
2	1.0	1.087	1.1	4.506	0.3	1.206**	1.2	0.0	0.359	0.6
3	0.9	1.013	1.05	-1.926		-0.6			-0.3	
4	0.9	0.989	1.05	-6.140		-0.4			-0.1	
5	0.9	1.050*	1.05	-0.512		-0.6			-0.2	

f = Total Cost and Fuel Consumption of Gen. no. 1 = 951.36 \$/Hr
f_c = Total Cost of Generation = 793.20 \$/Hr
f_f=f_{1f} = Fuel Consumption of Gen. no. 1 = 395.37 Million BTU/Hr

* no constraint violation ** constraint violation = 5.77×10^{-3} p.u.

Table 3.19 Solution of Minimum Total Cost and Fuel Consumption of Generator no. 1 for the 5-Bus System (Free Generator Voltages)

Bus No.	V_{min}	V	V_{max}	δ	P_{min}	P	P_{max}	Q_{min}	Q	Q_{max}
1	1.06	1.06	1.06	0.0	0.5	1.452	2.0	-0.2	-0.026	1.0
2	1.045	1.045	1.045	-2.992	0.2	0.630	1.0	-0.4	0.182	0.5
3	1.07	1.07	1.07	-5.932	0.2	0.592	1.0	-0.06	0.009	0.45
4	1.01	1.01	1.01	-9.978		-0.942		-0.4	0.200	0.6
5	1.09	1.09	1.09	-8.839		0.0		-0.06	0.175	0.45
6	0.9	1.025	1.1	-7.035		-0.478			0.039	
7	0.9	1.029	1.1	-5.511		-0.076			-0.016	
8	0.9	1.062	1.1	-8.839		0.0			0.0	
9	0.9	1.052	1.1	-9.785		-0.295			-0.166	
10	0.9	1.046	1.1	-9.393		-0.09			-0.058	
11	0.9	1.054	1.1	-7.812		-0.035			-0.018	
12	0.9	1.055	1.1	-7.007		-0.061			-0.016	
13	0.9	1.049	1.1	-7.308		-0.135			-0.058	
14	0.9	1.032	1.1	-9.715		-0.149			-0.05	

f = Total (Cost and Fuel Consumption) = 1991.42 \$/Hr.
f_c = Total Cost of Generation = 1139.87 \$/Hr.
f_f = Total Fuel Consumption = 2128.88 Million BTU/Hr

Table 3.20 Solution of Minimum Cost-Fuel Problem for the 14-Bus System. (Fixed Generator Voltages)

Bus No.	V_{min}	V	V_{max}	δ	P_{min}	P	P_{max}	Q_{min}	Q	Q_{max}
1	1.06	1.06	1.06	0.0	0.5	1.210	2.0	-0.2	-0.005	1.0
2	1.045	1.045	1.045	-2.618	0.2	0.448	1.0	-0.4	0.168	0.5
3	1.07	1.07	1.07	-0.915	0.2	1.011*	1.0	-0.06	-0.032	0.45
4	1.01	1.01	1.01	-8.991		-0.942		-0.4	0.185	0.6
5	1.09	1.09	1.09	-6.497		0.0		-0.06	0.184	0.45
6	0.9	1.028	1.1	-5.549		-0.478			0.039	
7	0.9	1.033	1.1	-3.958		-0.076			-0.016	
8	0.9	1.060	1.1	-6.497		0.0			0.0	
9	0.9	1.047	1.1	-6.998		-0.295			-0.166	
10	0.9	1.041	1.1	-6.213		-0.09			-0.058	
11	0.9	1.050	1.1	-3.720		-0.035			-0.018	
12	0.9	1.055	1.1	-2.147		-0.061			-0.016	
13	0.9	1.047	1.1	-2.60[illegible]		-0.135			-0.058	
14	0.9	1.028	1.1	-6.0[illegible]		-0.149			-0.05	

Total Cost and Fuel Consumption of Gen. 1 &2 = 1709.28 $/Hr.

Total Cost of Generation = 1171.03 $/Hr.

Fuel Consumption of Generators 1 & 2 = 1345.65 Million BTU/hr.

* Constraint violation = 1.099×10^{-2} p.u.

Table 3.21 Solution of Minimum Total Cost and Fuel Consumption of Generators no. 1 and 2 for the 14-Bus System (Fixed Generator Voltages)

Bus No.	V_{min}	V	V_{max}	δ	P_{min}	P	P_{max}	Q_{min}	Q	Q_{max}
1	1.06	1.06	1.06	0.0	0.5	1.775	2.0	-0.2	-0.114	1.0
2	1.045	1.045	1.045	-3.985	0.2	0.186*	1.0	-0.4	0.318	0.5
3	1.07	1.07	1.07	-5.156	0.2	0.725	1.0	-0.06	0.008	0.45
4	1.01	1.01	1.01	-10.654		-0.942		-0.4	0.195	0.6
5	1.09	1.09	1.09	-8.955		0.0		-0.06	0.178	0.45
6	0.9	1.026	1.1	-7.439		-0.478			0.039	
7	0.9	1.030	1.1	-5.816		-0.076			-0.016	
8	0.9	1.061	1.1	-8.955		0.0			0.0	
9	0.9	1.050	1.1	-9.752		-0.295			-0.166	
10	0.9	1.045	1.1	-9.230		-0.09			-0.058	
11	0.9	1.053	1.1	-7.345		-0.035			-0.018	
12	0.9	1.055	1.1	-6.283		-0.-61			-0.016	
13	0.9	1.048	1.1	-6.636		-0.135			-0.058	
14	0.9	1.031	1.1	-9.406		-0.149			-0.05	

Total Cost and Fuel Consumption of Gen. 2 = 1241.19 \$/Hr.

Total Cost of Generation = 1157.59 \$/Hr.

Fuel Consumption of Generator 2 = 209.01 Million BTU/Hr.

* Constraint violation = 1.385×10^{-2} p.u.

Table 3.22 Solution of Minimum Cost and Fuel Consumption of Generator no. 2 for the 14-Bus System (Fixed Generator Voltages) economy and also to economize in fuel consumption of generators no. 1 and 2. The intention was to show that generator no. 3, whose fuel consumption is of no interest, will provide its maximum generation, and the rest of the load will be scheduled between generators 1 and 2. The solution of this case is given in Table 3.21. The last case is that overall cost economy is sought such that at the same time fuel consumption of only generator 2 is economized. The purpose here is to show that generator 2 will provide its minimum generation and the rest of the load will be scheduled by generators 1 and 3, where fuel consumption is of no concern. Table 3.22 gives the solution.

3.5 Convergence Behaviour

Tables 3.23 and 3.24 show the number of iterations needed to obtain the solution of the cases studied and reported in the previous sections, for the 5- and 14-bus systems, respectively. Also shown is the maximum violation in functional inequality constraints. Parameter constraints were strictly observed by virtue of equation (2.26). Equality constraints were considered to be satisfied when power mismatch of 10^{-5} or less was obtained in the load flow portion of the problem.

In all cases of the 5-bus system, a flat starting point was used. However, in all cases of the 14-bus system it was too far from the optimal operating conditions resulting in slow convergence (e.g. 32 iterations in the minimum cost problem). However, this does not detract from the effectiveness of the method. The flat start is not a realistic operating point as all generators are in phase with the reference

		Proposed Method		Sasson et.al	
		Iter.	Max. Viol.	Iter.	Max.Viol.
Min. Cost	Fixed V_g's	7	-	5	$1.2\text{x}10^{-3}$
	$1.0 \le V_g \le 1.1$	6	$9\text{x}10^{-5}$	-	-
	Fixed V_g's * $-0.5 \le Q_{g_3} \le 0.5$	7	-	-	-
	Fixed V_g's * $-0.4 \le Q_{g_3} \le 0.4$	9	$2.55\text{x}10^{-3}$	-	-
	$1.0 \le V_g \le 1.1$* $-0.5 \le Q_{g_3} \le 0.5$	6	$5.19\text{x}10^{-4}$		
	$1.0 \le V_g \le 1.1$* $-0.4 \le Q_{g_3} \le 0.4$	7	-	-	-
Min. Loss	$1.0 \le V_g \le 1.1$	7	$7.85\text{x}10^{-4}$	-	[illegible]
Min.Fuel	Fixed V_g's	6	-	-	-
	$1.0 \le V_g \le 1.1$	12	$8.12\text{x}10^{-4}$	-	-
Min.(Cost and Fuel	Fixed V_g's	4			
	$1.0 \le V_g \le 1.1$	11	$1.37\text{x}10^{-3}$	-	-
Min.(Cost + Fuel of Gen. no.1	Fixed V_g's	5	$1.04\text{x}10^{-3}$	-	-
	$1.0 \le V_g \le 1.1$	11	$5.77\text{x}10^{-3}$	-	-

Table 3.23 Convergence Characteristics for the 5-Bus System

* For the modified 5-bus system, $0.9 \le V_3 \le 1.1$

	Proposed Method		Sasson et. al	
	Iter.	Max.Viol.	Iter.	Max.Viol.
Min. Cost.	5(32)*	-	6	1.1×10^{-4}
Min. Loss	10	7.56×10^{-3}	-	-
Min. Fuel	3	-	-	-
Min. (Cost and Fuel)	3	-	-	-
Min. (Cost and Fuel of Gen. 1 & 2)	9	1.099×10^{-2}	-	-
Min. (Cost and Fuel of Gen. 2)	4	1.385×10^{-2}	-	-

* For flat start.

Table 3.24 Convergence Characteristics for the 14-Bus System generator. It also does not take into consideration the available information and the experience gained on the system under study, and it is likely that a different starting point will be chosen based on practical judgement of what value each variable is likely to assume. Even with the lack of this information the results of preliminary studies can be used.

Recognizing that, it can be seen that the method possesses an excellent convergence rate. Furthermore, this rate does not depend on the size of the system, since for both systems, solutions are obtained in a comparable number of iterations.

It should be mentioned here that in the cases which required more than 6 or 7 iterations, the objective function was almost flat near the solution, and a large number of iterations were used for little improvement. The minimization process would have terminated much earlier if the improvement was taken as the convergence test.

Tables 3.23 and 3.24 also show the number of iterations required to obtain the solutio o the minimum cost problem, and their associated maximum constraint violations, by the method of Sasson et al[22]. This is the only problem solved in their paper. Since in both cases, the solution is well inside the feasible region, the violations are in the equality constraints. These figures are practically unacceptable and a few extra iterations are needed to improve on them.

Although exact comparison between the two methods is not possible with the lack of the same starting point*, those figures

* no starting point is mentioned in ref. [22].

are a rough indication of the relative effectiveness of the methods.

3.6 Solution Qualification

By the term "solution qualification" is meant the investigation of whether or not a solution of a problem does meet the problem requirements. To achieve this qualification, one should, first investigate the power schedule of each individual solution to see if it agrees with simple intuition provided by the problem itself. Secondly, a comparative study of the solutions of different problems in terms of the value of the different objective functions is carried out. Finally, one is to show that the solutions obtained are indeed the optimum in their respective cases. It should be mentioned that, from the point of view of constraint satisfaction, all solutions, despite the violations involved in some, are quite acceptable for all practical purposes. Consequently, each solution is considered to satisfy its constraint requirements.

Tables 3.25 and 3.26 give power generation schedule, and the values of the different objective-functions involved in each case for the 5- and the 14- bus systems, respectively. For easy reference the different cases were numbered, though their numbers do not agree with the order of their presentation in this chapter. Also included in Table 3.25(a) is the solution of the minimum loss problems of the 5- bus system for fixed generator voltages, which were extremely close to the flat starting point.

Of all cases studied on the two test systems, power generation

		P_1	P_2	Gen. Cost \$/Hr.	Losses p.u.	Fuel 10^6 BTU/Hr.	Fuel of Gen.1
1	Min. Cost	0.967	0.685	760.95	0.05167	1335.99	806.97
2	Min(Cost + Fuel)	0.849	0.801	762.81	0.05021	1324.68	709.82
3	Min.Fuel	0.692	0.958	770.76	0.05083	1319.00	583.94
4	Min(Cost + Fuel of Gen. 1)	0.457	1.201	794.99	0.05756	1331.28	403.24
5	Min. Loss	0.803	0.847	764.49	0.05008	1321.89	672.18
6	Min. Cost $-0.5\le Q_3\le 0.5$	0.962	0.682	757.57	0.04471	1330.02	803.27
7	Min. Cost $-0.4\le Q_3\le 0.4$	0.958	0.687	757.65	0.04426	1329.59	799.36

(a)

		P_1	P_2	Gen. Cost \$/Hr.	Losses p.u.	Fuel 10^6 BTU/Hr.	Fuel of Gen. 1
8	Min. Cost	0.994	0.651	757.78	0.04512	1334.30	830.11
9	Min(Cost + Fuel)	0.868	0.775	759.21	0.04381	1321.26	725.51
10	Min.Fuel	0.695	0.950	767.61	0.04469	1314.27	586.01
11	Min(Cost + Fuel of Gen. 1)	0.446	1.206	793.20	0.05171	1327.28	395.37
12	Min. Loss	0.832	0.812	760.37	0.04379	1318.78	695.93
13	Min. Cost $-0.5\le Q_{g_3}\le 0.5$	0.983	0.655	754.93	0.03874	1328.42	821.05
14	Min. Cost $-0.4\le Q_{g_3}\le 0.4$	0.966	0.673	755.02	0.03868	1326.41	806.35

(b)

Table 3.25 Power Schedule and Value of Objective Functions of the 5-Bus System
(a) Fixed Generator Voltages
(b) Free Generator Voltages

		P_1	P_2	P_3	Gen. Cost $/Hr.	Losses p.u.	Fuel 10^6 BTU/Hr.	Fuel of Gen. 1 & 2	Fuel of Gen. 2
1	Min. Cost	1.571	0.673	0.436	1136.14	0.08964	2146.53	1794.73	568.52
2	Min(Cost + Fuel)	1.452	0.630	0.592	1139.87	0.08838	2128.88	1667.65	535.08
3	Min. Fuel	1.276	0.578	0.814	1153.45	0.07815	2117.86	1493.06	494.98
4	Min(Cost + Fuel of Gen 1 & 2)	1.210	0.448	1.011	1171.03	0.07822	2121.76	1345.65	396.65
5	Min(Cost + Fuel of Gen 2)	1.775	0.186	0.725	1157.59	0.09569	2161.80	1603.46	209.01
6	Min. Loss	0.648	1.008	1.001	1213.71	0.06678	2176.44	1411.24	841.52

Table 3.26 Power Schedule and Values of Objective Functions
of the 14-Bus System
(Fixed Generator Voltages)

schedule of only a few can be readily predicted from the statement of the problem. In all other cases, however, a study of the objective function would produce an idea of how generation is shared by the various generators of the system.

The first category consists of all cases where local criteria were invoked. These are cases no. 4 and 11 of the 5- bus system and cases 4 and 5 of the 14- bus system (see Tables 3.25 and 3.26). In such cases one expects that a particular generator (or a group of generators) will provide minimum generation when it is (they are) required to consume a minimum amount of fuel, provided that the balance of the load can be absorbed by the other generators of the system. If this is not the case, those other generators will provide their maximum share, and the constrained generator(s) must absorb the balance. Investigation of Tables 3.25 and 3.26 shows that this is exactly what has been achieved in the above mentioned cases.

The second category comprises all other cases where minimization criteria were global. Although generation schedules in these cases are less predictable, one should be able to predict the way generation will be shared between various generators by studying their respective contribution to the objective function. A generator contributing less to the objective function will definitely carry a larger share of the load. This is exactly what happened in all cases of this category. Even in case no. 5 of the 14- bus system, generators 1 and 3 shared the balance of the load in such a way as to minimize the total cost of generation, the global criterion.

Investigation of the values of the total cost of generation, system losses, and total fuel consumption as given in Tables 3.25 and 3.26 shows that any of these quantities are minimized at the expense of the two others. A compromise solution can be obtained if all participate in the objective function, as was in the case of the cost-fuel problem.

Invoking the local minimization criteria resulted in a considerable increase in the cost of generation for both systems, and also in fuel consumption in one of the 14-bus system cases. The reason is that generation was forced to shift to the more expensive source, so that the local criterion can be observed. It should be mentioned here that the choice of these local criteria was intended to show such effect. This may or may not be the case in a realistic situation depending on the cost functions associated with the various generators of the system.

Investigation of Table 3.25 will also show that the solutions did not fail in showing the well known effects of increasing the voltage level in a system, and those of introducing reactive generation. Observe that generation cost, system losses and fuel consumption are lower in the cases of free generator voltages than those of fixed generator voltage. Also, they are lower in the cases where there is reactive generation at bus no. 3 than the cases which do not have such generation.

What is left now is to show that these solutions are indeed the optimal solutions and not just a boundary point. First, investigating Tables 3.25 and 3.26 shows that the minimum of each objective function, local or global, does occur opposite the case requiring the minimization of such an objective.

Secondly, the convergence behaviour, as indicated by the value of the acceleration factor, points out that these solutions are the optimal. The acceleration factor in most cases stayed at unity, while in the rest it decreased gradually when the solution was approached. Should it change sharply it would be an indication that a boundary point, rather than the optimal, will be obtained. This will be discussed later.

Thirdly, Figure 3.1 shows the equality constraint curve of the 5- bus system, with fixed generator voltages, on the P_1-P_2 plane. Also shown are some constant total generation cost contours. The diagram represents the minimum generation cost problem. All operating points of the system, including those resulting during the minimization process, must lie on the equality constraint curve. This curve will cross many constant cost contours representing different cost. However, it is the point at which the curve is tangent to one of these contours t[illegible] represents the optimal solution. The optimal operating cost will [illegible]e that represented by that contour. This was indeed the case. For instance [illegible]he contour corresponding to a cost of 700 \$/Hr. represents [illegible] lower [illegible]ost than the optimal. However, any point on that curve does not satisfy the equality constraints, thus, that cost can not be realized.

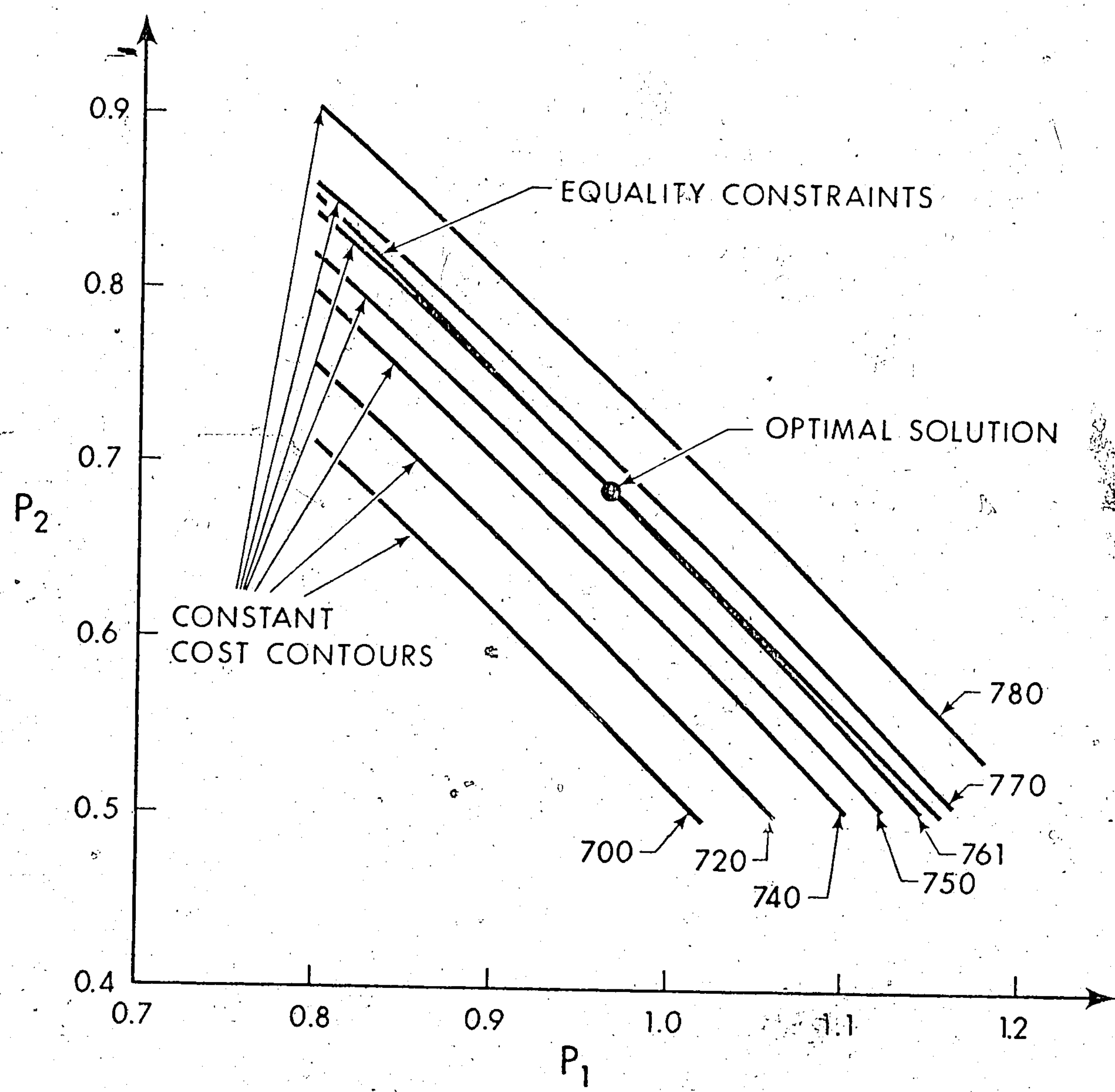

Figure 3.1 Equality Constraints and Constant Cost Contours

The lowest realizable cost is that represented by the 761 $/Hr. contour which touches the curve of the equality constraints at the optimal solution. Higher cost contours will each intersect with that curve at two points resulting in suboptimal solutions.

For a lossy system the curve of equality constraints will be concave upwards, whereas the constant cost contours are concave downwards. This will ensure that a single minimum occurs. In this case, however, since system losses are small (typical for power systems) and there is little difference between the light load costs at the two generators, this curvature is small. The result is that the true minimum point is difficult to locate with any realistic convergence criterion. However, in practice, this is unimportant as the cost (or the operating point depending on the convergence criterion) will be within the preset tolerance of the actual minimum.

Similar diagrams can be drawn for the other objective functions of this case*. However, although the cases of the 5-bus system with free generator voltages, and those of the 14- bus system

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* In the minimum loss problem, the constant loss contours will be discontinuous, and each will reduce to two points bracketing the optimal point. This is because system losses are defined only on the curve of equality constraints.

follow the same theory, it is difficult to draw such diagrams as they are multidimensional.

Finally, other methods[21,22,33] have treated two of the problems presented in this chapter, namely, the minimum cost problem of the 5- and 14- bus systems. For all practical purposes their solutions agree with the solutions obtained by the proposed method.

CHAPTER IV

OPTIMAL ORDERING OF LARGE SYSTEMS

As with any solution method, in applying the proposed method to a large system, one has to deal with several problems that are not involved in the case of small systems. For instance, the computer storage required to store the system matrix would be prohibitive and the efforts of inverting such a matrix very expensive. However, as mentioned before, these problems have been greatly eased in the case of power systems due to the extreme sparsity of the matrices involved, e.g. the Jacobian matrix. Only non zero elements of the matrix would be stored, but then, one has to keep track of their positions in the original matrix. Matrix inversion as such must be avoided leading to the use of an elimination process (e.g. Gaussian elimination) to triangulize the matrix and then using back substitution to obtain the solution. An optimal ordering scheme to order the rows of such a matrix would be in order to preserve its sparsity during the elimination process, thus reducing the extra storage that would be needed.

In this chapter a new method of optimal ordering is described, and compared to two other established methods Three standard test systems are used. They are the IEEE 30-, 57-, and 118- bus test systems. Data of these systems is given in Appendix B. Optimal cost solution of the 30-bus system is also presented as a representative of optimal solutions of large systems.

4.1 Optimal Ordering

An absolute optimal ordering scheme of the elimination process, through reordering system nodes, would result in the least possible terms in the table of factors*. An efficient algorithm for determining such absolute optimal order has not been developed, and it appears to be a practical impossibility. However, several effective schemes to determine a near-optimal order have been developed. Although these schemes do not produce the true optimal order, they are known, and hereafter are referred to, as optimal ordering schemes. An extensive comparative study of these schemes is found in reference 34.

There are two basic approaches to optimal ordering. The first is preordering, and it amounts to renumbering system nodes, i.e. the rows and columns of the system matrix, according to the order required in the elimination process. Solutions can then be obtained by choosing successive pivots down the leading diagonal. The main advantages of such schemes are that they are simple to program (consequently they are fast to execute),and that the only information needed to perform them is a node-branch connection list.

The best known ordering scheme using this approach is a one in which system nodes are numbered starting with that having the

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* The table of factors refers to the triangulized matrix when forward operations are stored in the lower triangle.

fewest number of connected branches, and ending with that having the most. It does not, however, take into account anything that happens during the elimination process[26,27]. This scheme will be referred to as scheme I for the purpose of comparison later in the chapter.

The second approach is known as dynamic ordering. System nodes are ordered according to the way the table of factors develops during the elimination process, rather than from the structural properties of the original system matrix. Programming of such methods is, therefore, more complicated, as ordering takes place during the actual elimination process or at least using its simulation. Execution of such schemes is slower than preordering schemes; however, they do produce an order closer to the absolute optimal.

Of t[illegible]ring schemes, two are the most familiar. In the fi[illegible]scheme II, at each step, the next node to be el[illegible]th the least number of connected lines in the r[illegible]next variable to be eliminated is that asso[illegible]nd column of the reduced matrix containing [illegible]st non-zero off diagonal elements. In the second scheme, the next node to be eliminated is the one which will introduce the fewest new equivalent lines. This scheme not only requires the simulation of the elimination process according to the natural order of the remaining nodes, but of every feasible alternative at each step. This scheme is referred to as scheme III.

The choice of scheme is a trade-off between speed of execution and the number of times the result is to be used. Scheme I, for instance, is good for problems requiring a single solution with no iterations. Scheme III, on the other hand, is good for problems requiring a large number of iterations to justify the extra time used in its execution. For load flow studies, scheme II was found to be the best of the three[26,27,30,34].

4.2 The Proposed Method

The main disadvantage of preordering schemes is that they do not take into account the changes that occur in the table of factors during the elimination process. This makes such schemes unsuitable for iterative solutions. On the other hand, dynamic ordering schemes require longer time to execute which makes them relatively unsuited to solutions that require only few iterations.

The proposed method falls within the category of pre-ordering schemes. However, ordering is carried out in such a way to reduce, as much as possible, the structural changes in the table of factors, thus reducing the number of the new elements introduced during the elimination process.

It would be instructive to show that in a sparse symmetric, or incident symmetric, matrix, if the rows are ordered such that off-diagonal elements become more and more dense as one moves down along the major diagonal, that the chance of introducing new elements during the elimination process becomes more and more unlikely. A simple reason is that very few openings are left for fill-in terms. The elimination process will merely modify already existing elements. In power systems, each row has at least one off-diagonal element*, and it is unavoidable for such terms to appear in every row. The strategy will, then, be to restrict these elements to the triangular portion below the minor diagonal, and, as before, to make them more concentrated in the lower right hand corner of the matrix. This would be the ideal structure for the elimination process in such cases[27,35].

An ordering scheme of system nodes to achieve such a structure is given as follows:

1) If there are any nodes with no connected branches, they are numbered first. As a matter of fact these nodes will not affect the elimination process, and can take any order. However, they are numbered first for simplicity.

2) Numbered next is the node connected to the least number of branches. If more than one qualify, the choice could be either arbitrary, or acc[illegible] a descending order of the number of branches connec[illegible] other end nodes.

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* For the present formulation of the optimal load-flow problem, it could happen that a node, particularly a generator node, may not have any connected branch leading to another node of the same set of nodes, because load nodes and generator nodes are treated separately.

3) The node (or nodes) which is (are) connected to the node just ordered, and which is (are) not previously numbered, is (are), then given the highest order(s) available, according to a descending order of the number of branches connected to each.

4) Of the remaining nodes, the node having the least number of branches is picked up. If no more than one qualify, steps 2) and 3) are repeated. However, if more than one are found, they are searched for a node connected to the highest ordered node. If one is found, it is given the lowest available order, and step 3) is repeated. If no one is found, the search is repeated for a connection with the second highest ordered node, and so on. When these highest ordered nodes are covered, steps 2) and 3) are repeated. The process is continued until all nodes have been numbered.

It can be seen that once a low order node is numbered, all its connecting nodes are moved away to the minor diagonal and beyond. Furthermore, such connecting nodes are numbered such that the node having the highest number of connected branches is given the highest order. This will ensure that the larger the number of the off-diagonal elements, the lower they will appear in the matrix being ordered.

For relatively small systems, the method may not work as well as described. The number of nodes which are connected to only one other node is quite small. Some of these nodes may be connected to the same node and the chances are that one may have to order a node connected to two ore more nodes of which none has been previously numbered. The result will be quite a few off-diagonal elements appearing to the left of th minor diagonal. The opposite is true for large systems. The number of those nodes which are connected to only one node is relatively large and scattered around the system, such that when nodes connected to two or more nodes have to be ordered some of these connecting nodes would have been already numbered and the off-diagonal elements will be located about the minor diagonal and to its right. The number of the off-diagonal elements to the left of the minor diagonal will be very small, however.

This is illustrated in Figure 4.1. The shaded area represents the area of the ordered matrix where off-diagonal elements will appear. For large systems, an area to the right of the minor diagonal will have no elements. It corresponds to those nodes with only one connected branch which will be located on the minor diagonal itself. This area may not exist for small systems due to the very small number of such nodes. Note also that elements located to the left of the minor diagonal will appear high in the matrix structure in the case of small systems as opposed to the case of large systems.

The shaded area is not full of off-diagonal elements. Rather, the concentration of these elements increases as one moves down along the main diagonal. This is because increasing row number almost corresponds to an increasing number of off-diagonal elements

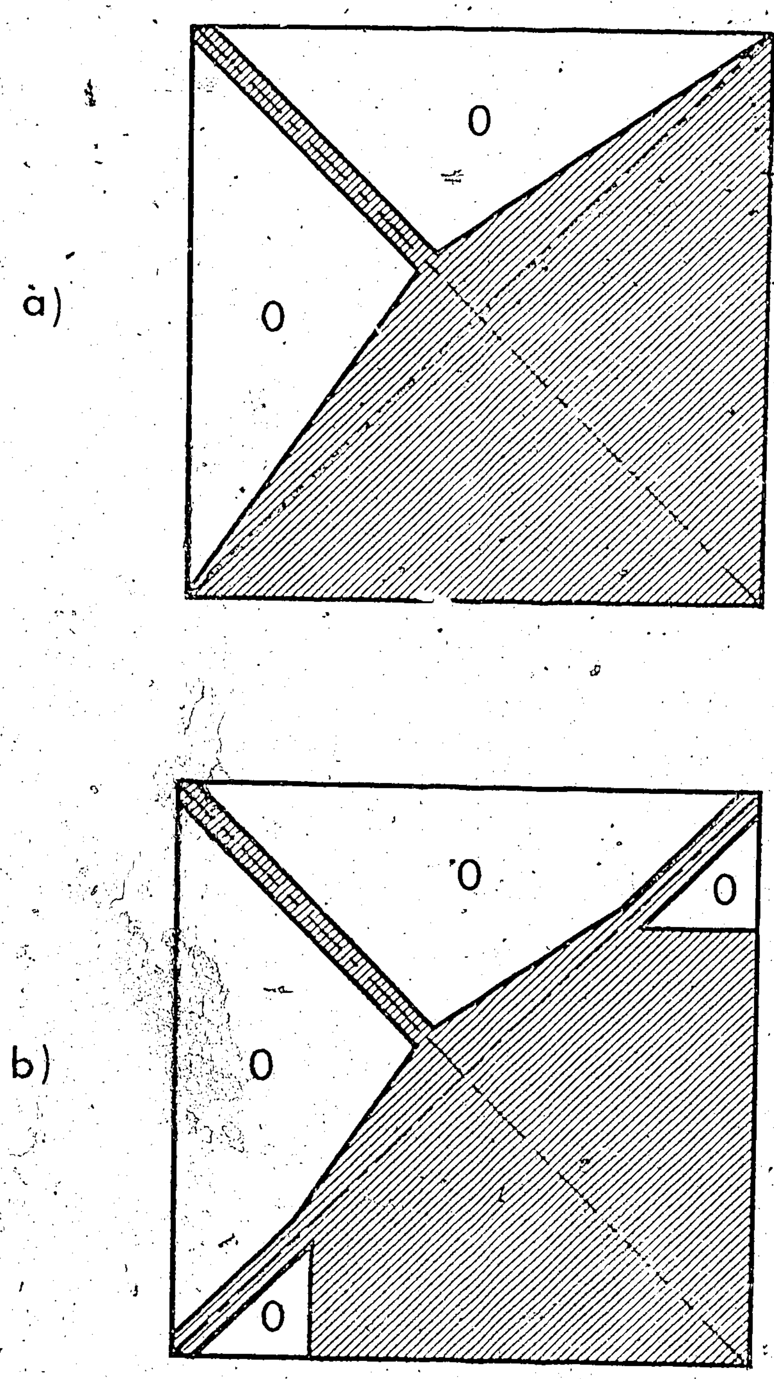

Figure 4.1 Matrix Structure After Ordering

a) Small Systems

b) Large Systems

An exception would be a case in which lightly connected nodes are connected together (e.g. chain circuit). In this case, heavily connected nodes will appear in the middle rows, resulting in a large number of fill-in terms during the elimination process. The success of the method will, thus, depend on system structure, a characteristic of any practical ordering schemes[30].

4.3 Storage Bookkeeping Technique

Should system nodes be numbered as one unit, special programming techniques, apart from compact storage, would have not been necessary. System data, e.g. the bus admittance matrix, could be rearranged in the new order, and used directly. However, in the present method of solution, generator nodes and load nodes are handled separately, and, thus, should be so numbered. Voltage controlled busses have dual roles. They are control nodes since their voltage is a control variable. They are also dependent nodes, since their real powers are fixed, and the voltage angles are determined from the load-flow solution. Therefore, these nodes will participate in both node groups, and each of them will end up with two numbers to determine its order within each group. Rearranging the bus admittance network, say, to meet such a situation, would be impossible.

To solve this problem, system data are left in the original arrangement. Correspondence arrays are formed. These arrays determine the original order of a given node, based on which group it belongs to, and its order within that group. Once the original order of a node is determined, the original data arrays can be readily used. Furthermore, the original node-branch connection array is split in two. The first, which is kept only for generator nodes*, gives, in the optimal order, the numbers of the generator nodes connected to each one. This array is used in computing and storing the first and second order partial derivatives of the objective function (excluding the associated equality constraints) with respect to the control variables.

The second consists of two separate arrays, one of which gives the list of the node-branch connections, in optimal order, within the group of load nodes, while the other gives the list of load nodes, in terms of their optimal order within their own group, connected to each generator node. The former array is used for computing and storing the Jacobian matrix, and the latter in computing and storing the first derivative of the cost function (excluding the associated equality constraints) with respect to the dependent variables. It is also used in computing the matrix $[\partial \underline{g}/\partial \underline{u}]$ and its contribution in the Hessian matrix.

To illustrate the just described technique, consider the IEEE 14-bus test system shown in Figure 4.2. Note that each of nodes

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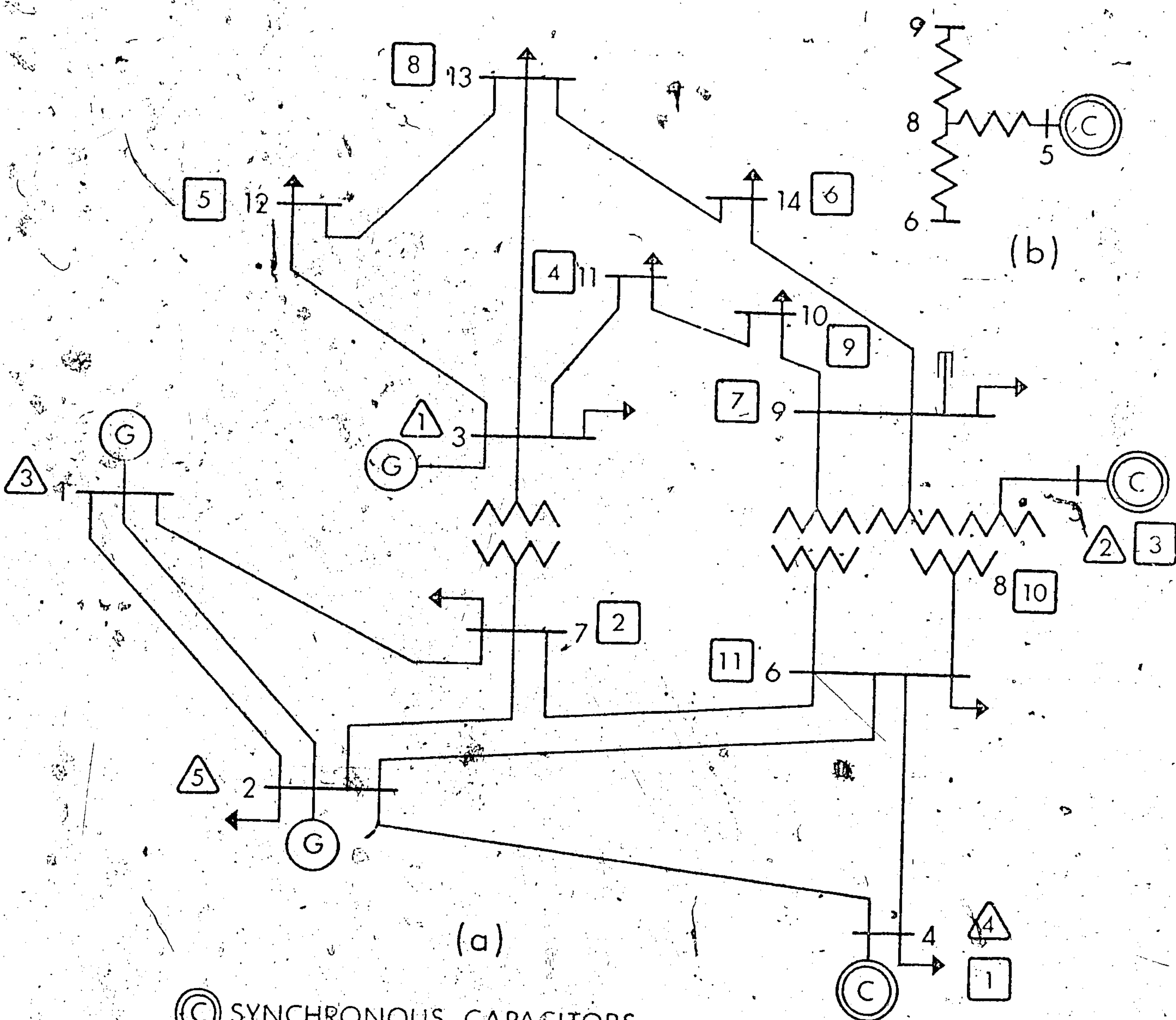
* It should be understood that reference to generator nodes, or load nodes, means the collection of such nodes plus voltage controlled nodes.



(C) SYNCHRONOUS CAPACITORS

(G) GENERATORS

(a) BUS-CODE DIAGRAM

(b) 3-WINDING TRANSFORMER EQUIVALENT

△ OPTIMAL ORDER OF GENERATORS

□ OPTIMAL ORDER OF LOADS

Figure 4.2 IEEE 14-Bus Test System Optimally Ordered

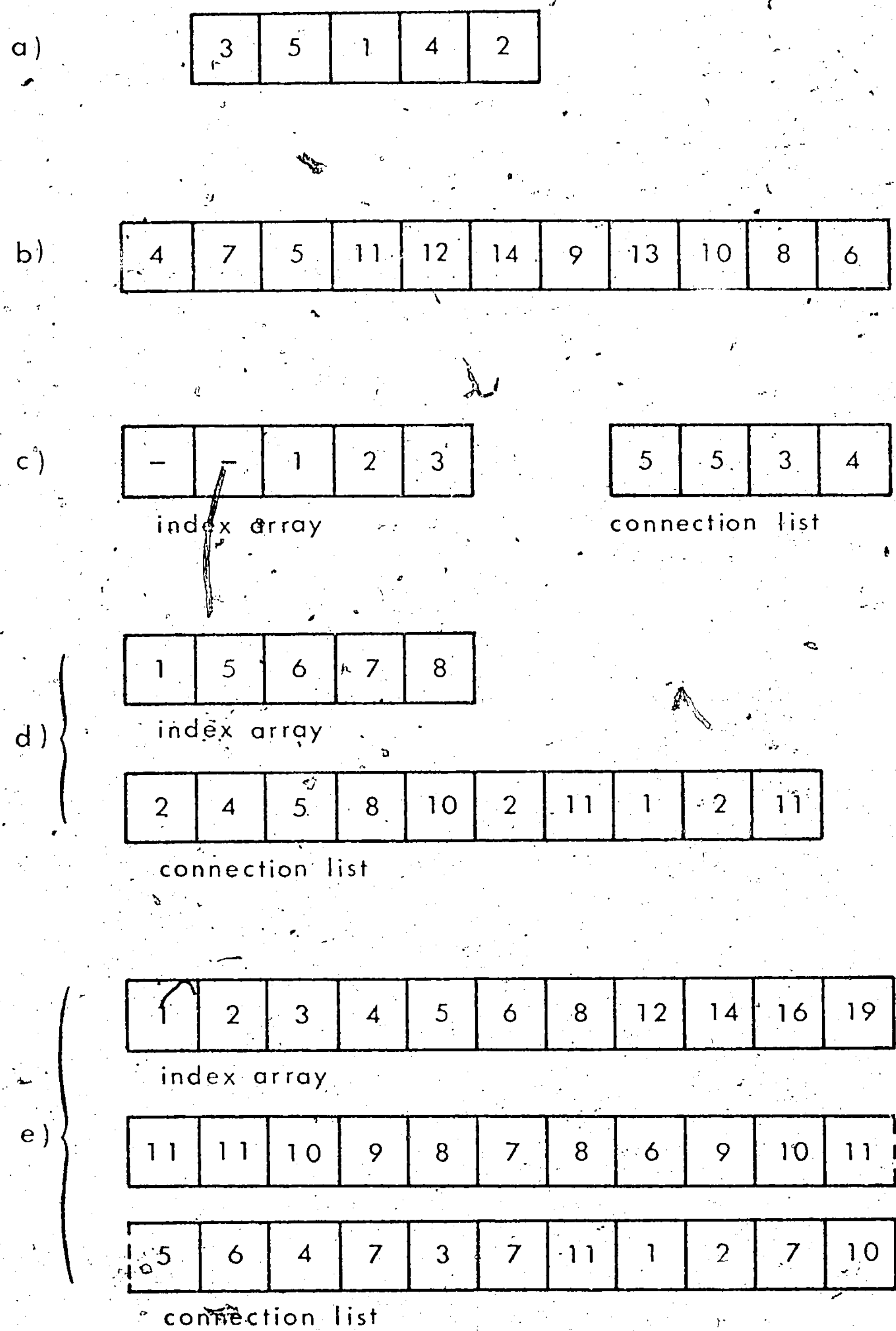


Figure 4.3 Storage Bookkeeping Arrangement

no. 4 and 5 (original order), which are voltage controlled nodes, has assumed two optimal orders. For instance, the node originally no. 5 is the second node in the group of generator nodes, while it is the third in the load nodes group. The correspondence arrays will, then, be given as in Figure 4.3 (a and b). Figures 4.3(a) gives the correspondence for generator nodes, while Figure 4.3(b) is for load nodes. In both arrays, the position of an entry determines the node number in optimal order, while the entry is its number in the original order.

The difference node-branch connection lists are shown in Figure 4.3 (c, d and e). Associated with each list is an index array which gives the starting position, in the respective connection list, of the list of nodes connected to each node of the node group concerned. For example, referring to Figure 4.3(c), which gives the node-branch connections within generator nodes, generator node no. 1 (optimal order; no. 3 in the original order) has no entry in the connection list in view of its isolation from other generator nodes. On the other hand, the list of generator nodes connected to generator node no. 5 (optimal order) starts at position 3 of the connection list. This means that node no. 5 is connected to nodes no. 3 and 4.

4.4 Comparison of Ordering Schemes

A comparison between the proposed method and schemes I and II of optimal ordering (see section 4.1) was carried out. The results of this comparison are presented in Tables 4.1 - 4.4.

System	Time in Sec Scheme I	Scheme II	Proposed Method
30-bus	0.84	1.053	0.95
57-bus	1.46	2.33	1.57
118-bus	3.42	6.15	6.43

Table 4.1 Ordering Times

System	Required Storage Natural	Scheme I & II	Proposed Method
30-bus	859	475	603
57-bus	1855	1059	1211
118-bus	2794	1125	1371

Table 4.2 Storage of the Table of Factors

Table 4.1 gives the times required for ordering the three systems by the three methods. The times shown consist of the time required to order the two groups of nodes mentioned before, as well as all times required to load the program and read system data. These latter portions are the same for the three methods.

It can be seen that scheme I is definitely the fastest of the three schemes tested. This is a well established result[26]. While the proposed method and scheme II required about the same amount of time in the cases of the 30- and 118- bus systems, the proposed method was much faster in the 57- bus system case. This means that the speed of either the proposed scheme or scheme II is not only dependent on system size only, as in the case of scheme I, but also on system configuration. This should be expected since in both schemes ordering very much depends on how the system nodes are tied together.

Table 4.2 gives the amount of storage required for the table of factors of the Jacobian matrix of the three systems using the three methods, as well as for the natural order. Storage required for the Hessian matrix is not shown since it represents a very small fraction of that required by the above mentioned table of factors*.

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* In the case of natural order, which usually requires the largest amount of storage of all cases, the Hessian required only 26 locations.

System	Storage Savings % Schemes I & II	Storage Savings % Proposed Method
30-bus	44.7	29.8
57-bus	42.9	34.7
118-bus	59.7	50.9

Table 4.3 Percentage Savings in Storage Over that of Natural Order

	Proposed Scheme	Scheme II
30-bus	572.85	500
57-bus	1901.27	2467.47
118-bus	8815.53	6918.75

Table 4.4 Product of Ordering Time by Storage Required for Ordering Schemes

Storage requirement for the $[\frac{\partial g}{\partial u}]$ matrix is the same in all cases.

Although the proposed method required between 14 and 27% more storage than that required by schemes I or II, the non-monotonic nature of such extra storage, backed up by the fact that scheme II was not better than scheme I in these cases, indicates that the required storage by an optimal scheme is dependent on system configuration, as mentioned before.

Based on the amount of storage required by the natural order, the savings effected by the three schemes are shown in Table 4.3. These figures, coupled with the relative speed of each ordering scheme suggests that the proposed scheme is, in general, comparable to scheme II, which is well accepted for load flow studies[26,27,30,34].

The product of the time required for ordering by the amount of storage required is often used as a criterion to compare ordering schemes[35]. This product is given in Table 4.4 for the three systems for scheme II and the proposed method. It backs up the above conclusion.

A definite conclusion on the superiority of one scheme over the other should involve the consideration of a large number of power systems. This was not possible in the present study.

4.5 Minimum Cost Solution of the 30- Bus System

The optimal solution of the minimum cost problem for the IEEE 30- bus test system is shown in Table 4.5. The system includes

3 generating nodes whose generation cost is the same as those of the 14- bus system and are given in Table 3.1. Several starting conditions have been used for which the solution has been obtained in 5 to 12 iterations. The 12 iterations figure refers to the flat start which, as mentioned before, is not a realistic operating point. This convergence rate confirms the earlier conclusion that it is independent of system size.

Table 4.6 shows a comparison between the proposed solution method and the method of reference 22. Although the minimum generation cost as obtained by that latter method is lower, it will probably increase if better satisfaction of equality constraints achieved.

Table 4.5 Minimum Cost Solution of the IEEE 30-Bus Test System

Bus No.	V_{min}	V	V_{max}	δ	P_{min}	P	P_{max}	Q_{min}	Q	Q_{max}
1	1.06	1.06	1.06	0.0	0.5	1.660	3.0	-0.6	-0.023	0.6
2	1.045	1.045	1.045	-3.285	0.2	0.748	1.5	-0.6	0.305	0.6
3	0.9	1.025	1.1	-5.417		-0.024			-0.012	
4	0.9	1.017	1.1	-6.505		-0.076			-0.016	
5	1.01	1.01	1.01	-11.376		-0.942		-0.6	0.336	0.6
6	0.9	1.015	1.1	-7.652		0.0			0.0	
7	0.9	1.005	1.1	-9.704		-0.228			-0.109	
8	1.01	1.01	1.01	-8.310		-0.3		-0.6	0.243	0.6
9	0.9	1.053	1.1	-7.202		0.0			0.0	
10	0.9	1.050	1.1	-9.984		-0.058			-0.02	
11	1.082	1.082	1.082	-1.655	0.2	0.530	1.5	-0.6	0.175	0.6
12	0.9	1.058	1.1	-10.760		-0.112			-0.075	
13	1.071	1.071	1.071	-10.760		0.0		-0.6	0.097	0.6
14	0.9	1.044	1.1	-11.494		-0.062			-0.016	
15	0.9	1.041	1.1	-11.441		-0.082			-0.025	
16	0.9	1.048	1.1	-10.711		-0.035			-0.018	
17	0.9	1.045	1.1	-10.410		-0.09			-0.058	
18	0.9	1.034	1.1	-11.650		-0.032			-0.009	
19	0.9	1.034	1.1	-11.582		-0.095			-0.034	
20	0.9	1.039	1.1	-11.264		-0.22			-0.007	
21	0.9	1.038	1.1	-10.546		-0.175			-0.112	
22	0.9	1.038	1.1	-10.572		0.0			0.0	
23	0.9	1.030	1.1	-11.601		-0.032			-0.016	
24	0.9	1.025	1.1	-11.467		-0.087			-0.067	

25	0.9	1.019	1.1	-11.617		0.0			0.0	
26	0.9	1.001	1.1	-12.035		-0.035			-0.023	
27	0.9	1.024	1.1	-11.454		0.0			0.0	
28	0.9	1.010	1.1	-8.180		0.0			0.0	
29	0.9	1.004	1.1	-12.682		-0.024			-0.009	
30	0.9	0.993	1.1	-13.564		-0.106			-0.019	

f^* = Total Cost of Generation = 1245.25 $/Hr.

Table 4.6 Comparison Between the Proposed Method and the Method of Reference 22 for the 30-Bus System

	No. of Iter	Maximum Mismatch	Minimum Cost
Proposed Method	6	$<0.22\times10^{-5*}$	1245.25
Method of Ref.22	5	1.5×10^{-3}	1242.5

* Typical Mismatch: 0.3×10^{-13} - 0.2×10^{-15}

CHAPTER V

EFFECT OF PENALTY AND ACCELERATION FACTORS

In this chapter, the effect of penalty and acceleration factors on the convergence of the proposed method is discussed. Since, practically, it is always possible, but not necessarily essential, to start the minimization process with a feasible operating point, only the effect of these two factors when a boundary of the feasible region is encountered from inside is considered.

Furthermore, the concept of a fixed penalty factor developed with the method, is compared to the usual concept of a monotonically increasing sequence of penalty factors.

Since the effects of penalty and acceleration factors complement one another, no attempt has been made to consider each separately.

5.1 Effect of Penalty and Acceleration Factors on Convergence

Referring to Figure 5.1, which represents schematically

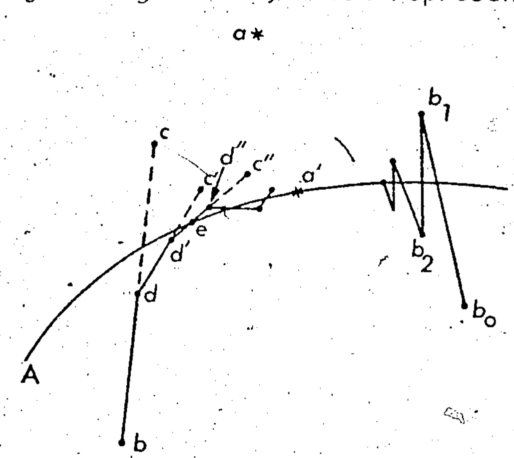

Figure 5.1 Effect of Penalty and Acceleration Factors

the minimization process, let curve A represent a boundary of the feasible region due some functional constraint. Let points a and a' represent, respectively, the unconstrained, and constrained optimal points. Also, let point b represent the operating point at the start of a particular iteration.

Assume that the unaccelerated move at that iteration moves the operating point b to a new point c outside the feasible region. If the penalty factor is small, point c is likely to have a lower cost (plus penalty) than point b, thus becoming the starting point of the next iteration. Since the penalty factor is kept fixed, point c will continue to move toward point a, with very little attention to the satisfaction of functional constraints. No acceleration will likely be introduced, and eventually an unfeasible solution will be obtained.

As long as the value of the penalty factor is not high enough to fully activate the acceleration mechanism, described in Chapter II, the minimization pattern will be similar to the one discussed above. Convergence, as measured by the number of iterations, will become worse as the penalty factor is increased.

Suppose, now, that the value of the penalty factor is high enough, so that point c is of higher cost than point b. The acceleration mechanism will start functioning and the minimization move will be bd rather than bc. This pattern is continued, bd d'd"..., until the optimal point a' is located. In this case, constraint violation may be allowed, as long as the decrease in the objective function overcomes such violation. Furthermore, a move from a point such as d" will try to produce a decrease in both the objective function and constraint violation. In such a way the operating point will stay close to the boundary, resulting in a better convergence rate.

If the penalty factor is higher than necessary, the minimization process may converge to a point such as e, which will, then, be a suboptimal solution. However, it will be characterized by a sudden or continued decrease in the value of the acceleration factor, indicating a premature convergence. The reason is that any slight violation of a constraint will result in a large increase in the objective function, and the acceleration factor will continue to decrease to avoid such violation, and to keep the operating point within the feasible region. If the operating point at the start of the iteration is close to the boundary, it is likely that the value of the acceleration factor will drop to a very low level and could result in the satisfaction of the convergence criterion, without actually locating the optimal point. On the other hand, if constraint violation does occur in the early stages of the process, when the objective function changes rapidly, the emphasis will be on the satisfaction of the violated constraint, and very little attention is given to the minimization of the original objective function. The operating point will be pulled well inside the feasible region, far from the optimal solution. The result is a very slow convergence to a suboptimal solution. This is shown by trajectory $b_0 b_1 b_2 \ldots$ in Figure 5.1.

This behaviour is demonstrated in Tables 5.1 and 5.2, which refer to two different cases of the 5- bus system. Similar behaviour is also experienced in the cases of the 14- bus system.

In Table 5.1, it is clear that, acceleration did not take full effect upto $r_V=10^4$, and thus, as expected convergence became worse as the penalty factor is increased. Constraint violation was smaller, however. In the case of $r_V=10^5$, acceleration took full action resulting in an excellent convergence behaviour in both rate and constraint violation.

Similar behaviour is experienced in Table 5.2. However, in the case of $r_Q=5 \times 10^3$, acceleration took effect during the iteration process rather than at its end. The fact that the process converged with unity acceleration factor and larger cost shows, as have also been discussed in Chapter III, that the cost contours are very flat and the satisfaction of a practical convergence criterion, $\max \Delta u_i \leq 10^{-3}$, can occur in a region rather than a point. In this particular case a range of values between 10^3 and 5×10^3 for the penalty factor is satisfactory.

The behaviour of the iteration process for $r_Q=10^4$ and 10^5 was typical of one with higher than necessary penalty factor. With the former value the operating point was repeatedly moved well inside the feasible region whenever constraint violation occur, whereas with the latter, it was not allowed to leave the feasible region in the

Table 5.1 Convergence Behaviour for Different Penalty Factors: Min. Cost Problem, 5- Bus System, Free Gen. Voltages

Penalty factor	No. of Iter.	Acc. Factor	Violation of V_s Constraint	Cost at Solution
$r_v=10^2$	9	1.0	1.582×10^{-2}	757.17
$r_v=10^3$	10	1.0	1.351×10^{-2}	757.22
$r_v=10^4$	Oscillatory Solution. Acc. Factor = 0.5			
$r_v=10^5$	6	0.125	0.90×10^{-4}	757.78

Table 5.2 Convergence Behaviour for Different Penalty Factors: Min. Cost Problem, Modified 5-Bus System, $-0.4\leq Q_3\leq 0.4$ Fixed Gen. Voltages

Penalty Factor	No. of Iter.	Acc. Factor	Violation of Q_3 Constraint	Cost at Solution
$r_Q=0.0$	7	1.0	7.30×10^{-2}	757.57
$r_Q=10^2$	7	1.0	1.03×10^{-2}	757.63
$r_Q=10^3$	9	1.0	2.55×10^{-3}	757.65
$r_Q=2\times10^3$	10	1.0	2.06×10^{-3}	757.67
$r_Q=5\times10^3$	9	1.0	1.67×10^{-3}	758.31
$r_Q=10^4$	17	0.0625	1.72×10^{-3}	758.86
$r_Q=10^5$	3	0.0625	-	759.42

first place. Both solutions are suboptimal.

5.2 Factors Affecting the Choice of the Penalty Factor

The main factor that determines whether or not acceleration will take place is the value of the penalty function due to violated constraints, and whether or not, when added to the objective function, it increases the value of such objective function over the previous value. Too large a penalty will certainly eat up any decrease that would be effected in the original objective function by the minimization process, forcing a premature convergence. On the other hand, too small a penalty will contribute a little, and a solution outside the feasible region would be obtained.

Since the decrease in the objective function, particularly at the last stages of the process, is a very small fraction of that function, the penalty factor, which determines the value of the penalty function, should be chosen according to such a change. This means that it is dependent on the value of the objective function itself. The larger this value is, the larger the penalty factor should be.

Table 5.3 shows the convergence behaviour of two objective functions for varying penalty factor. Part (a) of the Table refers to the minimum cost problem of the 5- bus system when generator voltages are free. It is drawn from Table 5.1. Part (b) is obtained for the same case except the objective function was divided by a factor of 100. This factor should not affect the minimization process or the solution since it will cancel in the equation:

Penalty factor	Normal Objective Function			Scaled Objective Function		
	No. of Iter.	Acc. factor	Max. Violation	No. of Iter.	Acc. factor	Max. Violation
1					1.0	1.582×10^{-2}
10				10	1.0	1.351×10^{-2}
100	9	1.0	1.582×10^{-2}	Oscillatory Solution		
10^3	10	1.0	1.351×10^{-2}	6	0.125	9×10^{-5}
10^4	Oscillatory Solution					
10^5	6	0.125	9×10^{-5}			

Table 5.3 Dependence of Penalty Factor on the Objective Function

$$\underline{\Delta u} = - \underline{H}^{-1} \underline{\nabla f}_{\underline{u}} \qquad (5.1)$$

which gives the minimization move.

It can be seen that parts (a) and (b) would be identical if the factor of 100 is also taken into consideration in the value of the penalty factor. The same convergence behaviour is obtained for both objective functions when the penalty factor in both cases are related by this factor of 100.

It should also be mentioned here that a penalty factor of 200 was found acceptable for the minimum loss problem of the 5- bus system (Table 3.9) as compared to 1000 and 10^5 for the two cases presented here. The value of the objective function in the three cases is 1.644, 7.59 and 759.78 respectively. Correlation between the values of the objective function and the penalty factor is evident. The penalty factor is about 100 times larger than the objective function. This ratio was also satisfactory for all other cases for this type of constraint.

Another factor affecting the choice of the penalty factor is the type of constraint. It can be observed from Tables 5.1 and 5.2 that while a value of 10^5 was acceptable for the voltage constraint at load node no. 5, a value of 10^3 to 5×10^3 was satisfactory for reactive generation constraint at voltage controlled node no. 3. This difference can be explained in the following way.

One should first recognize that the former type of constraint is a linear constraint of only one dependent variable, i.e. the voltage at a load node, whereas the latter is a nonlinear constraint which is a function of both control and dependent variables. These are the two types of functional constraints involved in the optimal load flow problem. In view of the equations

$$\underline{\nabla f}_{\underline{x}} = \frac{\partial (f+w)}{\partial \underline{x}} + \left[\frac{\partial \underline{g}}{\partial \underline{x}} \right]^T \underline{\lambda} = o \tag{5.2}$$

$$\underline{\nabla f}_{\underline{u}} = \frac{\partial (f+w)}{\partial \underline{u}} + \left[\frac{\partial \underline{g}}{\partial \underline{u}} \right]^T \underline{\lambda} = o \tag{5.3}$$

which represent the gradients of the penalized objective function with respect to dependent and control variables respectively*, it can be seen that the first type of constraint, which has only a first order derivative with respect to a particular dependent variable, will have only an indirect contribution, through the vector $\underline{\lambda}$, to the gradient $\underline{\nabla f}_{\underline{u}}$ of equation (5.3). On the other hand, the second type of constraint, will have both such indirect contribution and a direct one through the derivative $\frac{\partial w}{\partial \underline{u}}$. A lower penalty factor for that latter type should, then, be expected. An acceptable value for the penalty factor in this case is about 2 to 10 times the value of the objective function.

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* These equations are identical to equations (2.18) and (2.19) except a penalty function w is added to the objective function f.

5.3 Comparison of Penalty Factor Schemes

A proper mathematical way to solve a constrained minimization problem using penalty function approach is to start the minimization process with a small penalty factor and then increase its value after convergence is achieved. This process is continued until an acceptable solution is obtained. This process will, then, require a large number of iterations as it consists of several minimization subproblems. This is in contrast with only one such minimization problem, when the concept of a fixed penalty factor is used.

If the objective function has a distinctive constrained optimal solution, it could be argued that the former method will be definitely the superior. However, this is not the case in minimization problems of power systems. The contours of the objective function are very flat, and any practical convergence criterion will locate a neighbourhood of the optimal solution rather than the solution point itself. Within such neighbourhood cost variation is minimal and, from a practical point of view, the extra effort required to locate the true optimum is not worthwhile.

Consider Table 5.4 which shows the number of iterations required to obtain the solution of the minimum cost problem for the 5- bus and the modified 5- bus systems using both penalty factor approaches. In the first case, generator voltages were free, while in the second case they were fixed and $-0.4 \leq Q_3 \leq 0.4$.

System	Approach		No. of Iter.	Max.Violation	Cost at Solution
5-Bus System	Fixed Penalty Factor		6	9×10^{-5}	757.78
5-Bus System	$r_o=10$, $r_f=10^4$	x10	44	7.4×10^{-5}	757.76
5-Bus System	$r_o=10$, $r_f=10^4$	x100	Did not converge in 35 iterations		
Modified 5-Bus System	Fixed Penalty Factor		9	2.55×10^{-3}	757.65
Modified 5-Bus System	$r_o=10$, $r_f=10^5$	x10	14	1.12×10^{-3}	757.88
Modified 5-Bus System	$r_o=10$, $r_f=10^5$	x100	13	1.19×10^{-3}	759.16

Table 5.4 Comparison of Penalty Factor Approaches

It is quite clear that, for all practical reasons, the concept of fixed penalty factor is more acceptable. In the first case the difference in optimal cost indeed does not justify the extra iterations. In the second case a varying penalty factor approach was not acceptable with regard to both convergence rate and the solution obtained. It does point out, however, the sensitivity and cruciality of the growth rate of the penalty factor.

CHAPTER VI

CONCLUSIONS

In this thesis the problem of optimal load flow of power systems is investigated. A brief review of the development of the problem and its solution methods is given. The Carpentier formulation of the exact economic dispatch problem, which is the basic starting point of all recent developments, is also presented.

The solution method proposed in this thesis, which is also based on Carpentier formulation, is characterized by:

1) All generators are treated as swing busses.
2) Newton's method, using the Hessian matrix of the objective function, is used to compute the adjustments to the control variables.
3) A simple acceleration scheme is used.
4) A fixed penalty factor is used throughout the iterative process.

By treating all generators as swing busses, several advantages have been achieved:

a) The sensitivity of some problems to the location of one particular swing bus is eliminated.
b) Storage requirement is greatly reduced, as compared to other methods, due to the elimination of generator equations from the set of load flow equations. The Jacobian matrix of load flow equations is much smaller, resulting also in a large reduction

in computation efforts required to solve the load flow portion of the problem.

c) In addition, generator voltage magnitudes are kept free to adjust themselves within the prescribed range, to achieve true minimization. Although this is possible with other methods, the only results that have been reported are those of Billinton and Sachdeva[20] using a suboptimum technique of sequential real/reactive power dispatch.

The use of Newton's method to adjust the set of control variables, as defined in this thesis, provided excellent convergence rate. The Hessian matrix needed for this purpose is of a low order and is extremely sparse, thus posing no problems in its handling or storage.

By using the simple acceleration scheme described in this thesis, the complexities and efforts needed to evaluate an optimal acceleration factor is eliminated. Yet both may produce the same results.

The fixed penalty factor approach, complimented by the acceleration scheme, proved to be superior to the usual way in which penalty factors are used. The process converges in fewer iterations with little or no sacrifice in the final cost. This latter point is practically of no concern because of the flatness of the cost contours and the impossibility of implementi exactly, the true mathematical optimal solution. By relating the choice of s fixed penalty factor to the objective function and the type constraint, a rough guide to estimate its value was possible.

The method is extensively tested using two standard test systems. Four basic problems are solved: the minimum cost problem, or the exact economic dispatch, the minimum loss problem, the minimum fuel problem, and the combined cost-fuel minimization problem. The first two problems are well established, whereas the last two are newly defined and formulated for power systems.

An optimal ordering scheme, for use with large systems, is also developed in this thesis. Near optimal order is achieved by moving off diagonal elements to the right of the minor diagonal and as far below it as possible. The scheme was found to be comparable with two of the well accepted schemes.

For future considerations, it is worthwhile investigating the feasibility of applying the method to the problem of hydro-thermal dispatch, in which optimization should be carried out over a time period, rather than at a particular instant. The method can also be investigated from the point of view of system planning and expansion. More work also need to be done on the fixed penalty factor approach to establish a firm guideline for the choice of the proper value.

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* This paper is based on material contained in this thesis.

Appendix A

Given here are the elements of the different matrices and vectors involved in the solution method presented in Chapter II.

A.1 The Jacobian Matrix

Load flow equations at a load node are given by:

$$p_i(\underline{u},\underline{x}) = V_i \sum_{j=1}^{N} V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}) + C_i = o \tag{A.1}$$

$$q_i(\underline{u},\underline{x}) = V_i \sum_{j=1}^{N} V_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij}) + D_i = o \tag{A.2}$$

The elements of the Jacobian are the derivatives of these quantities with respect to the dependent variables $\underline{x}$. Self derivatives are given by:

$$\frac{\partial p_i}{\partial \delta_i} = - V_i \sum_{\substack{j=1 \\ j \neq i}}^{N} V_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij})$$

$$= D_i - V_i^2 Y_{ii} \sin(\theta_{ii}) \tag{A.3}$$

$$\frac{\partial p_i}{\partial V_i} = 2V_i Y_{ii} \cos(\theta_{ii}) + \sum_{\substack{j=1 \\ j \neq i}}^{N} V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij})$$

$$= - C_i/V_i + V_i Y_{ii} \cos(\theta_{ii}) \tag{A.4}$$

Similarly

$$\frac{\partial q_i}{\partial \delta_i} = - C_i - V_i^2 Y_{ii} \cos(\theta_{ii}) \tag{A.5}$$

$$\frac{\partial q_i}{\partial V_i} = - D_i/V_i - V_i Y_{ii} \sin\theta_{ii} \tag{A.6}$$

Mutual derivatives are given by:

$$\frac{\partial p_i}{\partial \delta_j} = V_i V_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij}) \tag{A.7}$$

$$\frac{\partial p_i}{\partial V_j} = V_i Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}) \tag{A.8}$$

$$\frac{\partial q_i}{\partial \delta_j} = - V_i V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}) \tag{A.9}$$

$$\frac{\partial q_i}{\partial V_j} = V_i Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij}) \tag{A.10}$$

In equations (A.3) - (A.10) i and j stand for load busses.

A.2 The Cost Function and its derivatives

It is assumed that the cost function is a quadratic function in generated real powers as follows:

$$f = \sum_i a_i + b_i P_{g_i} + c_i P_{g_i}^{2} \tag{A.11}$$

where i is for a generator bus, a_i, b_i and c_i are constants, and P_{g_i}, the real power generated at bus i, is given by:

$$P_{g_i} = \sum_{j=1}^{N} V_i V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}) + C_i \quad (A.12)$$

A.2.1 Elements of the derivative vector $\frac{\partial f}{\partial \underline{x}}$

$$\frac{\partial f}{\partial \delta_j} = \sum_i (b_i + 2c_i P_{g_i}) V_i V_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij}) \quad (A.13)$$

$$\frac{\partial f}{\partial V_j} = \sum_i (b_i + 2c_i P_{g_i}) V_i Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}) \quad (A.14)$$

where j is for a load bus.

A.2.2 Elements of the vector $\frac{\partial f}{\partial \underline{u}}$

$$\frac{\partial f}{\partial \delta_k} = \sum_i (b_i + 2c_i P_{g_i}) \frac{\partial P_{g_i}}{\partial \delta_k} \quad (A.15)$$

$$\frac{\partial f}{\partial V_k} = \sum_i (b_i + 2c_i P_{g_i}) \frac{\partial P_{g_i}}{\partial V_k} \quad (A.16)$$

A.2.3 Second order partial derivatives of the cost function

$$\frac{\partial^2 f}{\partial \delta_k^2} = \sum_i 2c_i \left(\frac{\partial P_{g_i}}{\partial \delta_k} \right)^2 - \sum_{\substack{i \\ i \neq k}} (b_i + 2c_i P_{g_i}) V_i V_k Y_{ik} \cos(\delta_i - \delta_k - \theta_{ik})$$

$$- (b_k + 2c_k P_{g_k}) [P_{g_k} - C_k - V_k^2 Y_{kk} \cos\theta_{kk}] \quad (A.17)$$

$$\frac{\partial^2 f}{\partial V_k^2} = \sum_i 2c_i \left(\frac{\partial P_{g_i}}{\partial V_k} \right) + 2(b_k + 2c_k P_{g_k}) \, Y_{kk} \cos\theta_{kk} \qquad (A.18)$$

$$\frac{\partial^2 f}{\partial \delta_k \partial V_k} = \sum_i 2c_i \left(\frac{\partial P_{g_i}}{\partial \delta_k} \right) \left(\frac{\partial P_{g_i}}{\partial V_k} \right)$$

$$+ \sum_{\substack{i \\ i \neq k}} (b_i + 2c_i P_{g_i}) \, V_i Y_{ik} \sin(\delta_i - \delta_k - \theta_{ik})$$

$$- (b_k + 2c_k P_{g_k}) [(Q_{g_k} - D_k)/V_k + V_k Y_{kk} \sin\theta_{kk}] \qquad (A.19)$$

$$\frac{\partial^2 f}{\partial \delta_k \partial \delta_\ell} = \sum_i 2c_i \left(\frac{\partial P_{g_i}}{\partial \delta_k} \right) \left(\frac{\partial P_{g_i}}{\partial \delta_\ell} \right)$$

$$+ V_k V_\ell Y_{k\ell} [(b_k + 2c_k P_{g_k}) \cos(\delta_k - \delta_\ell - \theta_{k\ell})$$

$$+ (b_\ell + 2c_\ell P_{g_\ell}) \cos(\delta_\ell - \delta_k - \theta_{\ell k})] \qquad (A.20)$$

$$\frac{\partial^2 f}{\partial V_k \partial V_\ell} = \sum_i 2c_i \left(\frac{\partial P_{g_i}}{\partial V_k} \right) \left(\frac{\partial P_{g_i}}{\partial V_\ell} \right)$$

$$+ Y_{k\ell} [(b_k + 2c_k P_{g_k}) \cos(\delta_k - \delta_\ell - \theta_{k\ell})$$

$$+ (b_\ell + 2c_\ell P_{g_\ell}) \cos(\delta_\ell - \delta_k - \theta_{\ell k})] \qquad (A.21)$$

$$\frac{\partial^2 f}{\partial \delta_k \partial V_\ell} = \sum_i 2c_i \left(\frac{\partial P_{g_i}}{\partial \delta_k} \right) \left(\frac{\partial P_{g_i}}{\partial V_\ell} \right)$$

$$+ V_k Y_{k\ell} [-(b_k + 2c_k P_{g_k}) \sin(\delta_k - \delta_\ell - \theta_{k\ell})$$

$$+(b_\ell + 2c_\ell P_{g_\ell}) \sin(\delta_\ell - \delta_k - \theta_{\ell k})] \qquad (A.22)$$

In Equations (A.15) - (A.22), k and ℓ are for generator busses, and the derivatives of generated powers P_{g_i}, with respect to generator variables δ_i, V_i, δ_j and V_j, are given by:

$$\frac{\partial P_{g_i}}{\partial \delta_i} = - [Q_{g_i} - D_i + V_i^2 Y_{ii} \sin\theta_{ii}] \qquad (A.23)$$

$$\frac{\partial P_{g_i}}{\partial V_i} = (P_{g_i} - C_i)/V_i + V_i Y_{ii} \cos\theta_{ii} \qquad (A.24)$$

$$\frac{\partial P_{g_i}}{\partial \delta_j} = V_i V_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij}) \qquad (A.25)$$

$$\frac{\partial P_{g_i}}{\partial V_j} = V_i Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}) \qquad (A.26)$$

A.3 The Matrix $[\partial \underline{g}/\partial \underline{u}]$

Equations (A.7) - (A.10) give the elements of the matrix $[\partial \underline{g}/\partial \underline{u}]$ provided that subscript j is for a generator node rather than a load node.

A.4 The Hessian Matrix

The elements of the Hessian matrix are the derivatives of relations (2.19) with respect to the control variables. Bearing in mind equation (A.7) - (A.10) and the provision of section A.3, these elements reduce to:

$$\frac{\partial^2 L}{\partial \delta_k^2} = \frac{\partial^2 f}{\partial \delta_k^2} - V_k \sum_j V_j Y_{jk} [\lambda_{p_j} \cos(\delta_j - \delta_k - \theta_{jk})$$

$$+ \lambda_{q_j} \sin(\delta_j - \delta_k - \theta_{jk})] \qquad (A.27)$$

$$\frac{\partial^2 L}{\partial V_k^2} = \frac{\partial^2 f}{\partial V_k^2} \qquad (A.28)$$

$$\frac{\partial^2 L}{\partial \delta_k \partial V_k} = \frac{\partial^2 f \; L}{\partial \delta_k V_k} + \sum_j V_j \; Y_{jk} [\lambda_{p_j} \sin(\delta_j - \delta_k - \theta_{jk})$$

$$- \lambda_{q_j} \cos(\delta_j - \delta_k - \theta_{jk})] \qquad (A.29)$$

$$\frac{\partial^2 L}{\partial \delta_k \partial \delta_\ell} = \frac{\partial^2 f \; L}{\partial \delta_k \partial \delta_\ell} \qquad (A.30)$$

$$\frac{\partial^2 L}{\partial \delta_k \partial V_\ell} = \frac{\partial^2 f \; L}{\partial \delta_k \partial V_\ell} \qquad (A.31)$$

$$\frac{\partial^2 L}{\partial V_k \partial V_\ell} = \frac{\partial^2 f \; L}{\partial V_k \partial V_\ell} \qquad (A.32)$$

where, in (A.27) - (A.32), j is for a load bus, and k and ℓ are for generator busses.

A.5 Derivatives of the Penalty Function

The penalty function (2.33) can be written, in more detail, as:

$$w = \sum_i r_{p_i} h_{p_i}^2 + \sum_j r_{q_j} h_{q_j}^2 + \sum_k r_{v_k} h_{v_k}^2 \qquad (A.33)$$

where i and j stand for generator nodes, k is for a load node, and r_p, r_q, and r_v are penalty factors. Again, only violated functional constraints, given by inequalities (2.27) - (2.32), will appear in (A.33).

A.5.1 The Vector $\frac{\partial w}{\partial \underline{x}}$

$$\frac{\partial w}{\partial \delta_k} = 2V_k[\sum_i \pm r_{p_i} h_{p_i} V_i Y_{ik} \sin(\delta_i-\delta_k-\theta_{ik})$$

$$- \sum_j \pm r_{q_j} h_{q_j} V_j Y_{jk} \cos(\delta_j-\delta_k-\theta_{jk})] \qquad (A.34)$$

$$\frac{\partial w}{\partial V_k} = 2[\sum_i \pm r_{p_i} h_{p_i} V_i Y_{ik} \cos(\delta_i-\delta_k-\theta_{ik})$$

$$+ \sum_j \pm r_{q_j} h_{q_j} V_j Y_{jk} \sin(\delta_j-\delta_k-\theta_{jk})$$

$$\pm r_{v_k} h_{v_k}] \qquad (A.35)$$

The positive sign is used if violated constraint is a lower inequality, and the negative sign if it is an upper inequality.

A.5.2 The Vector $\frac{\partial w}{\partial \underline{u}}$

$$\frac{\partial w}{\partial \delta_\ell} = 2\Big[\sum_i \pm r_{p_i} h_{p_i} \frac{\partial P_i(\underline{u},\underline{x})}{\partial \delta_\ell} + \sum_j \pm r_{q_j} h_{q_j} \frac{\partial Q_j(\underline{u},\underline{x})}{\partial \delta_\ell} \Big] \tag{A.36}$$

$$\frac{\partial w}{\partial V_\ell} = 2\Big[\sum_i \pm r_{p_i} h_{p_i} \frac{\partial P_i(\underline{u},\underline{x})}{\partial V_\ell} + \sum_j \pm r_{q_j} h_{q_j} \frac{\partial Q_j(\underline{u},\underline{x})}{\partial V_\ell} \Big] \tag{A.37}$$

A.5.3 The Matrix $\frac{\partial^2 w}{\partial \underline{u}^2}$

$$\frac{\partial^2 w}{\partial \delta_\ell^2} = 2\Big[\sum_i r_{p_i} \Big(\frac{\partial P_i(\underline{u},\underline{x})}{\partial \delta_\ell} \Big)^2 + \sum_j r_{q_j} \Big(\frac{\partial Q_j(\underline{u},\underline{x})}{\partial \delta_\ell} \Big)^2$$

$$- \sum_{\substack{i \\ i \neq \ell}} \pm r_{p_i} h_{p_i} V_i V_\ell Y_{i\ell} \cos(\delta_i - \delta_\ell - \theta_{i\ell})$$

$$- \sum_{\substack{j \\ j \neq \ell}} \pm r_{q_j} h_{q_j} V_j V_\ell Y_{j\ell} \sin(\delta_j - \delta_\ell - \theta_{j\ell})$$

$$- \{\pm\, r_{p_\ell} h_{p_\ell} (P_{g_\ell} - C_\ell - V_\ell^2\, Y_{\ell\ell} \cos\theta_{\ell\ell})\}$$

$$- \{\pm\, r_{q_\ell} h_{q_\ell} (Q_{g_\ell} - D_\ell + V_\ell^2\, Y_{\ell\ell} \sin\theta_{\ell\ell})\}] \qquad \text{(A.38)}$$

$$\frac{\partial^2 w}{\partial V_\ell{}^2} = 2[\sum_i r_{p_i} \left(\frac{\partial P_i(\underline{u},\underline{x})}{\partial V_\ell}\right)^2 + \sum_j r_{q_j} \left(\frac{\partial Q_j(\underline{u},\underline{x})}{\partial V_\ell}\right)^2$$

$$\pm\, 2 r_{p_\ell} h_{p_\ell} Y_{\ell\ell} \cos\theta_{\ell\ell} \pm 2 r_{q_\ell} h_{q_\ell} (-Y_{\ell\ell} \sin\theta_{\ell\ell})] \qquad \text{(A.39)}$$

$$\frac{\partial^2 w}{\partial \delta_\ell \partial V_\ell} = 2[\sum_i r_{p_i} \left(\frac{\partial P_i(\underline{u},\underline{x})}{\partial \delta_\ell}\right)\left(\frac{\partial P_i(\underline{u},\underline{x})}{\partial V_\ell}\right)$$

$$+ \sum_j r_{q_j} \left(\frac{\partial Q_j(\underline{u},\underline{x})}{\partial \delta_\ell}\right)\left(\frac{\partial Q_j(\underline{u},\underline{x})}{\partial V_\ell}\right)$$

$$+ \sum_{\substack{i \\ i\neq\ell}} \pm\, r_{p_i} h_{p_i} V_i Y_{i\ell} \sin(\delta_i - \delta_\ell - \theta_{i\ell})$$

$$- \sum_{\substack{j \\ j\neq\ell}} \pm\, r_{q_j} h_{q_j} V_j Y_{j\ell} \cos(\delta_j - \delta_\ell - \theta_{j\ell})$$

$$- [\pm\, r_{p_\ell} h_{p_\ell} \left(\frac{Q_{g_\ell} - D_\ell}{V_\ell} + V_\ell Y_{\ell\ell} \sin\theta_{\ell\ell}\right)]$$

$$\pm\, r_{g_\ell} h_{g_\ell} \left(\frac{P_{g_\ell} - C_\ell}{V_\ell} - V_\ell Y_{\ell\ell} \cos\theta_{\ell\ell}\right)] \qquad \text{(A.40)}$$

$$\frac{\partial^2 w}{\partial \delta_\ell \partial \delta_m} = 2[\sum_i r_{p_i} (\frac{\partial P_i(\underline{u},\underline{x})}{\partial \delta_\ell})(\frac{\partial P_i(\underline{u},\underline{x})}{\partial \delta_m})$$

$$+ \sum_j r_{q_j} (\frac{\partial Q_j(\underline{u},\underline{x})}{\partial \delta_\ell})(\frac{\partial Q_j(\underline{u},\underline{x})}{\partial \delta_m})$$

$$+ V_\ell V_m Y_{\ell m}\{ \pm r_{p_\ell} h_{p_\ell} \cos(\delta_\ell - \delta_m - \theta_{\ell m})$$

$$\pm r_{p_m} h_{p_m} \cos(\delta_m - \delta_\ell - \theta_{m\ell})\}$$

$$+ V_\ell V_m Y_{\ell m}\{\pm r_{q_\ell} h_{q_\ell} \sin(\delta_\ell - \delta_m - \theta_{\ell m})$$

$$\pm r_{q_m} h_{q_m} \sin(\delta_m - \delta_\ell - \theta_{m\ell})\}] \tag{A.41}$$

$$\frac{\partial^2 w}{\partial V_\ell \partial V_m} = 2[\sum_i r_{p_i} (\frac{\partial P_i(\underline{u},\underline{x})}{\partial V_\ell})(\frac{\partial P_i(\underline{u},\underline{x})}{\partial V_m})$$

$$+ \sum_j r_{q_j} (\frac{\partial Q_j(\underline{u},\underline{x})}{\partial V_\ell}) (\frac{\partial Q_j(\underline{u},\underline{x})}{\partial V_m})$$

$$+ Y_{\ell m}\{ \pm r_{p_\ell} h_{p_\ell} \cos(\delta_\ell - \delta_m - \theta_{\ell m}) \pm r_{p_m} h_{p_m} \cos(\delta_m - \delta_\ell - \theta_{m\ell})\}$$

$$+ Y_{\ell m}\{ \pm r_{q_\ell} h_{q_\ell} \sin(\delta_\ell - \delta_m - \theta_{\ell m}) \pm r_{q_m} h_{q_m} \sin(\delta_m - \delta_\ell - \theta_{m\ell})\}] \tag{A.42}$$

$$\frac{\partial^2 w}{\partial \delta_\ell \partial V_m} = 2[\sum_i r_{p_i}(\frac{\partial P_i(\underline{u},\underline{x})}{\partial \delta_\ell})(\frac{\partial P_i(\underline{u},\underline{x})}{\partial V_m})$$

$$+ \sum_j r_{q_j}(\frac{\partial Q_j(\underline{u},\underline{x})}{\partial \delta_\ell})(\frac{\partial Q_j(\underline{u},\underline{x})}{\partial V_m})$$

$$+ V_\ell Y_{\ell m}\{-(\pm r_{p_\ell} h_{p_\ell} \sin(\delta_\ell - \delta_m - \theta_{\ell m})) \pm r_{p_m} h_{p_m} \sin(\delta_m$$

$$- \delta_\ell - \theta_{m\ell})\} + V_\ell Y_{\ell m}\{\pm r_{q_\ell} h_{q_\ell} \cos(\delta_\ell - \delta_m - \theta_{\ell m})$$

$$- (\pm r_{q_m} h_{q_m} \cos(\delta_m - \delta_\ell - \theta_{m\ell}))\}] \qquad \text{(A.43)}$$

In (A.36) - (A.43), ℓ and m are for generator busses, and the sign convention is as given in conjunction with equations (A.34) and (A.35). The derivatives $\frac{\partial P_i(\underline{u},\underline{x})}{\partial \underline{u}}$ and $\frac{\partial Q_i(\underline{u},\underline{x})}{\partial \underline{u}}$ are given by equations (A.23) - (A.26) and the following equations, respectively.

$$\frac{\partial Q_j(\underline{u},\underline{x})}{\partial \delta_j} = P_{g_j} - C_j - V_j^2 Y_{jj} \cos\theta_{jj} \qquad \text{(A.44)}$$

$$\frac{\partial Q_j(\underline{u},\underline{x})}{\partial V_j} = \frac{Q_{g_j} - D_j}{V_j} - V_j Y_{jj} \sin\theta_{jj} \qquad \text{(A.45)}$$

$$\frac{\partial Q_j(\underline{u},\underline{x})}{\partial \delta_i} = - V_j V_i Y_{ji} \cos(\delta_j - \delta_i - \theta_{ji}) \qquad \text{(A.46)}$$

$$\frac{\partial Q_j(\underline{u},\underline{x})}{\partial V_i} = V_j Y_{ji} \sin(\delta_j - \delta_i - \theta_{ji}) \qquad \text{(A.47)}$$

APPENDIX B

DESCRIPTION OF TEST SYSTEMS

Given below are the line diagrams and the bus admittance matrices, in polar form, of the systems used in this thesis. Also given are their loading conditions. In calculating the bus admittance matrices, off-nominal transformer tap ratios, and static capacitors have been taken into account. Regulated busses (generator and voltage controlled busses) in each system are indicated in the solutions given in Chapters 3 and 4.

B.1 The 5-Bus Test System

Table B.1

DIAGONAL AND NON-ZERO UPPER DIAGONAL ELEMENTS OF THE BUS ADMITTANCE MATRIX

BUS	TO	BUS	MAGNITUDE	ANGLE
1	-	1	8.8930	-75.96
1	-	3	2.4254	104.04
1	-	4	1.6169	104.04
1	-	5	4.8507	104.04
2	-	2	9.7014	-75.96
2	-	3	4.8507	104.04
2	-	5	4.8507	104.04
3	-	3	9.7014	-75.96
3	-	4	2.4254	104.04
4	-	4	4.0423	-75.96
5	-	5	9.7014	-75.96

Table B.2

SYSTEM LOADS
(IN P.U. ON A 100 MVA BASE)

BUS	REAL	REACTIVE	BUS	REAL	REACTIVE
1	0.0	0.0	4	0.400	0.100
2	0.0	0.0	5	0.600	0.200
3	0.600	0.300			

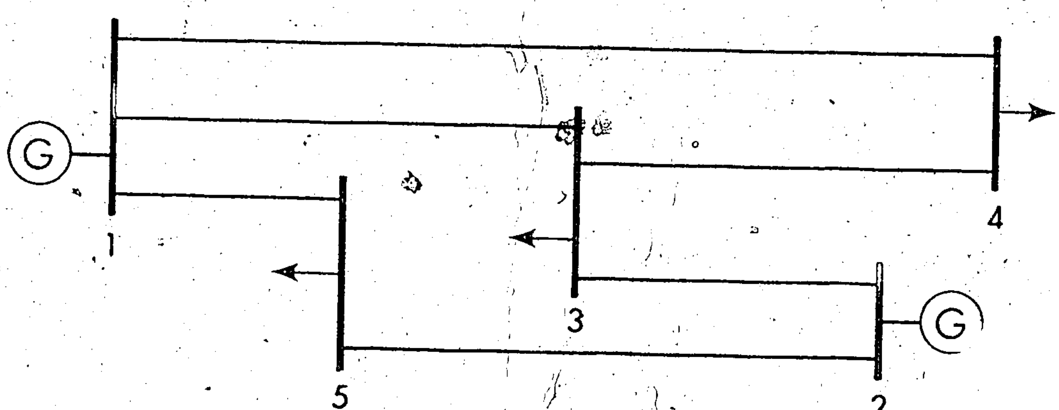

Figure B.1 Line Diagram of the 5-bus Test System

B.2 The IEEE 14-Bus Test System

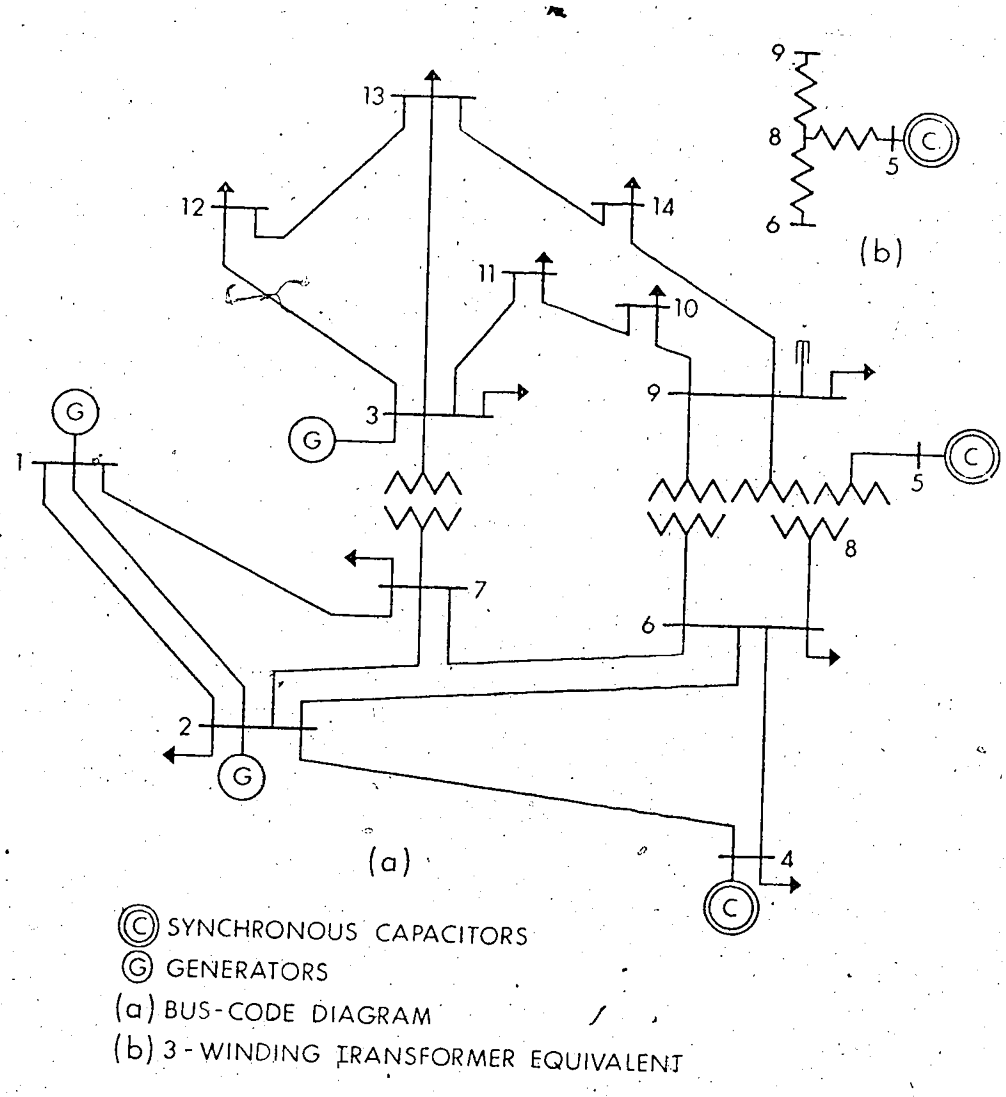

Figure B.2 Line Diagram of the IEEE 14- Bus Test System

Table B.3 DIAGONAL AND NON-ZERO UPPER DIAGONAL ELEMENTS OF THE BUS ADMITTANCE MATRIX

BUS	TO	BUS	MAGNITUDE	ANGLE
	-	1	20.3590	-72.79
	-	2	16.0609	108.14
1	-	7	4.3575	103.62
2	-	2	31.7328	-72.54
2	-	4	4.9147	103.35
2	-	6	5.3865	108.24
2	-	7	5.4654	108.13
3	-	3	18.5471	-69.22
3	-	7	4.2574	90.00
3	-	11	4.5369	115.53
3	-	12	3.5235	115.66
3	-	13	6.8445	116.92
4	-	4	10.2959	-72.35
4	-	6	5.4440	111.40
5	-	5	5.6770	-90.00
5	-	8	5.6770	90.00
6	-	6	40.0400	-74.78
6	-	7	22.6370	107.59
6	-	8	4.8895	90.00
6	-	9	1.8555	90.00
7	-	7	36.7934	-74.93
8	-	8	19.5490	-90.00
8	-	9	9.0901	90.00
9	-	9	24.6742	-77.53
9	-	10	11.0755	110.63
9	-	14	3.3471	115.18
10	-	10	15.8602	-68.62
10	-	11	4.7879	113.13
11	-	11	9.3227	-65.70
12	-	12	6.7515	-53.51
12	-	13	3.3566	137.86
13	-	13	12.6122	-57.78
13	-	14	2.5791	116.16
14	-	14	5.9260	-64.39

Table B.4 SYSTEM LOADS (IN P.U. ON A 100 MVA BASE)

BUS	REAL	REACTIVE	BUS	REAL	REACTIVE
1	0.0	0.0	8	0.0	0.0
2	0.217	0.127	9	0.295	0.166
3	0.112	0.075	10	0.090	0.058
4	0.942	0.190	11	0.035	0.018
5	0.0	0.0	12	0.061	0.016
6	0.478	-0.040	13	0.135	0.058
7	0.076	0.016	14	0.149	0.050

B.3 The IEEE 30-Bus Test System

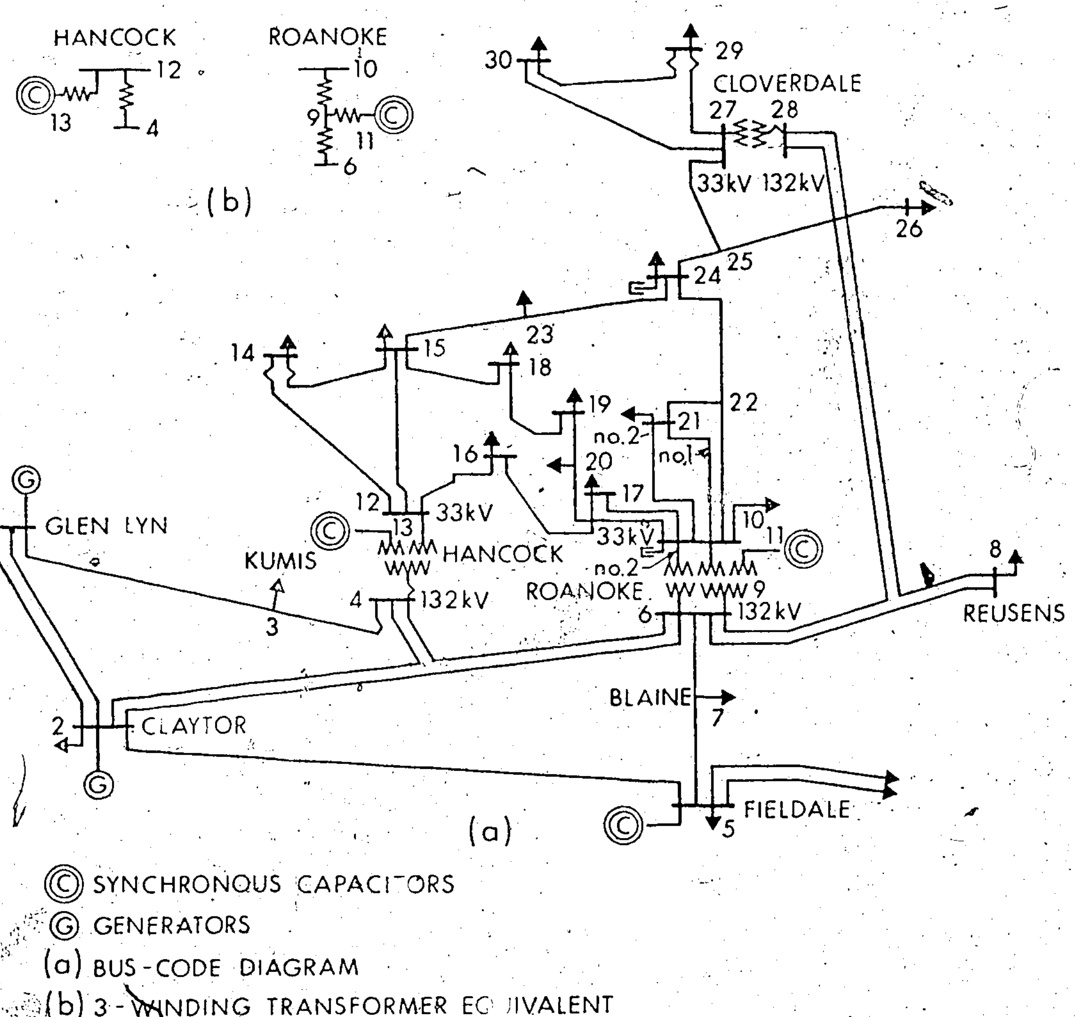

Figure B.3 Line Diagram of the IEEE 30-Bus Test System

Table B.5 DIAGONAL AND NON-ZERO UPPER DIAGONAL ELEMENTS OF THE BUS ADMITTANCE MATRIX

BUS TO BUS		MAGNITUDE	ANGLE
1 -	1	21.6832	-72.64
1 -	2	16.4959	108.46
1 -	3	5.2456	103.72
2 -	2	32.1629	-72.35
2 -	4	5.4701	108.17
2 -	5	4.9058	103.39
2 -	6	5.3872	108.24
3 -	3	30.1196	-71.74
3 -	4	24.9172	109.20
4 -	4	57.8571	-73.62
4 -	6	23.2146	106.04
4 -	12	4.1913	90.00
5 -	5	12.8584	-71.45
5 -	7	8.0136	111.63
6 -	6	85.7893	-74.90
6 -	7	11.5959	108.04
6 -	8	22.8934	105.95
6 -	9	4.9158	90.00
6 -	10	1.8561	90.00
6 -	28	16.0673	105.76
7 -	7	19.5826	-70.48
8 -	8	27.6317	-73.75
8 -	28	4.7649	107.64
9 -	9	18.7063	-90.00
9 -	10	9.0909	90.00
9 -	11	4.8077	90.00
10 -	10	43.8522	-73.37
10 -	17	11.0498	110.98
10 -	20	4.7010	100.73
10 -	21	12.1080	114.92
10 -	22	6.0024	115.87
11 -	11	4.8077	-90.00
12 -	12	25.2934	-74.94
12 -	13	7.1428	90.00
12 -	14	3.5215	115.69

12	15	6.8380	116.92
12	16	4.5449	115.44
13	13	7.1429	-90.00
14	14	6.7501	-53.47
14	15	3.3573	137.90
15	15	18.5514	-59.69
15	18	4.1103	116.09
15	23	4.4366	116.34
16	16	9.3230	-65.71
16	17	4.7799	113.19
17	17	15.8273	-68.35
18	18	11.0480	-63.77
18	19	6.9378	116.32
19	19	20.0911	-63.52
19	20	13.1533	116.57
20	20	17.7225	-67.59
21	21	50.1333	-64.13
21	22	38.0275	116.18
22	22	48.7020	-63.23
22	24	4.7002	122.72
23	23	7.7639	-63.78
23	24	3.3274	116.05
24	24	10.5132	-59.97
24	25	2.6361	119.80
25	25	9.0592	-60.25
25	26	2.1868	123.80
25	27	4.2447	117.64
26	26	2.1868	-56.20
27	27	10.1410	-68.89
27	28	2.6087	90.00
27	29	2.1282	117.89
27	30	1.4652	117.98
28	28	23.4033	-75.63
29	29	4.0780	-62.11
29	30	1.9498	117.89
30	30	3.4151	-62.07

Table B.6 SYSTEM LOADS
(IN P.U. ON A 100 MVA BASE)

BUS	REAL	REACTIVE	BUS	REAL	REACTIVE
1	0.0	0.0	16	0.035	0.018
2	0.217	0.127	17	0.090	0.058
3	0.024	0.012	18	0.032	0.009
4	0.076	0.016	19	0.095	0.034
5	0.942	0.190	20	0.022	0.007
6	0.0	0.0	21	0.175	0.112
7	0.228	0.109	22	0.0	0.0
8	0.300	0.300	23	0.032	0.016
9	0.0	0.0	24	0.087	0.067
10	0.058	0.020	25	0.0	0.0
11	0.0	0.0	26	0.035	0.023
12	0.112	0.075	27	0.0	0.0
13	0.0	0.0	28	0.0	0.0
14	0.062	0.016	29	0.024	0.009
15	0.082	0.025	30	0.106	0.019

B.4 The IEEE 57-Bus Test System

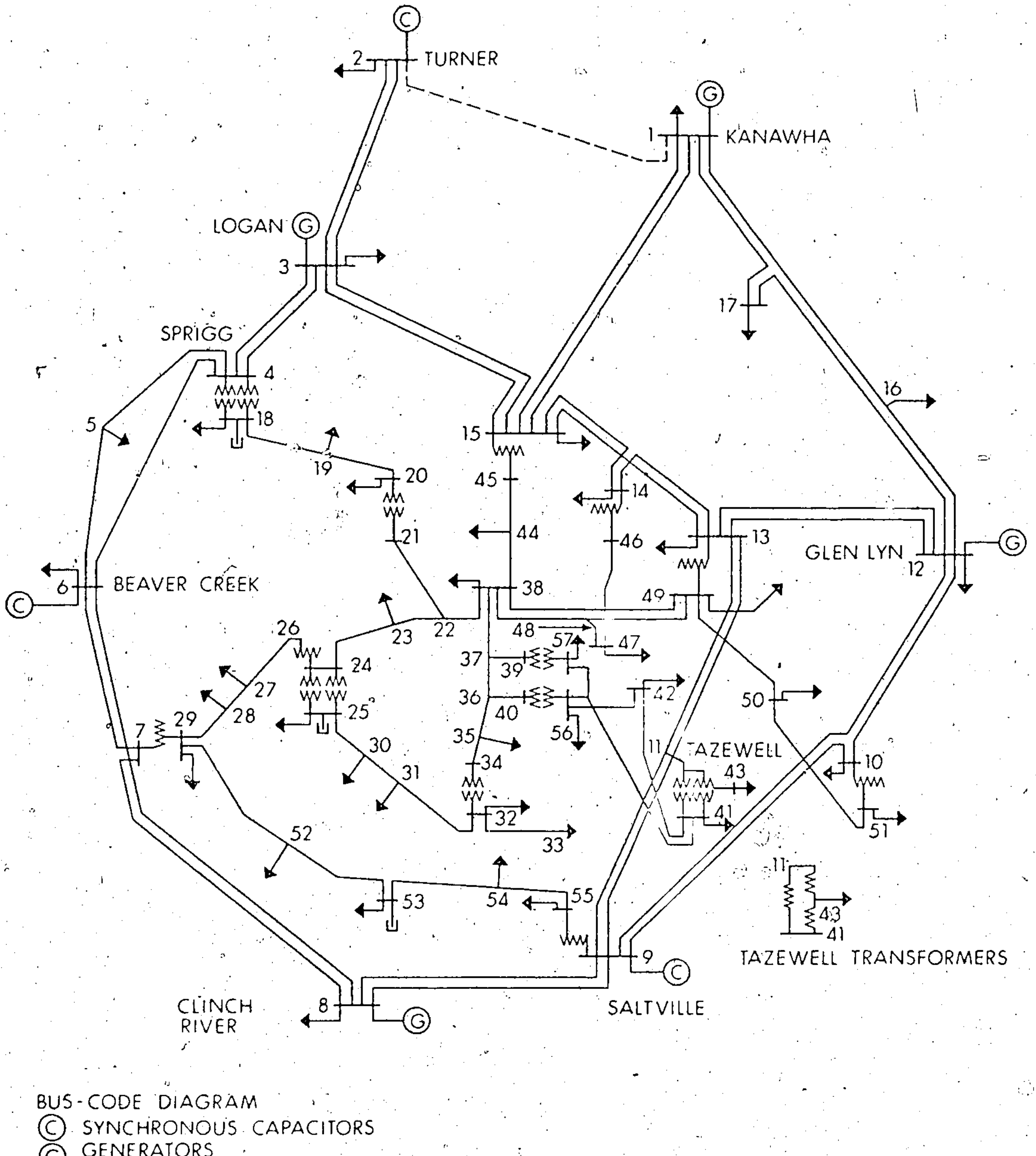

Figure B.4 Line Diagram of the IEEE 57-Bus Test System

Table B.7 DIAGONAL AND NON-ZERO UPPER DIAGONAL ELEMENTS OF THE BUS ADMITTANCE MATRIX

BUS	TO	BUS	MAGNITUDE	ANGLE
1	-	1	58.6091	-75.41
1	-	2	34.2416	106.51
1	-	15	10.7847	101.07
1	-	16	4.7406	102.43
1	-	17	9.0423	102.43
2	-	2	45.2330	-72.76
2	-	3	11.1022	109.32
3	-	3	55.1821	-72.50
3	-	4	26.1264	107.01
3	-	15	18.0438	107.00
4	-	4	43.4997	-73.46
4	-	5	6.8470	115.34
4	-	6	6.4884	106.20
4	-	18	4.2354	90.00
5	-	5	20.9425	-64.71
5	-	6	14.1128	115.23
6	-	6	35.6110	-72.40
6	-	7	9.6207	101.09
6	-	8	5.6725	101.09
7	-	7	39.7051	-83.50
7	-	8	13.7847	101.05
7	-	29	15.9587	90.00
8	-	8	38.8298	-78.91
8	-	9	19.4321	101.09
9	-	9	54.9045	-78.71
9	-	10	5.8171	102.40
9	-	11	11.2819	106.92
9	-	12	3.3109	102.39
9	-	13	6.0548	106.93
9	-	55	8.8285	90.00
10	-	10	29.5849	-84.36
10	-	12	7.7397	102.38
10	-	51	15.1021	90.00
11	-	11	32.6394	-77.45
11	-	13	13.0682	106.94
11	-	41	1.3980	90.00

11 - 43	6.8225	90.00
12 - 12	44.8465	-75.82
12 - 13	16.4827	107.06
12 - 16	12.0093	102.48
12 - [illegible]	5.4541	102.51
13 - [illegible]	74.8435	-74.44
13 - [illegible]	22.0444	106.92
13 - [illegible]	10.9928	107.20
13 - 49	5.8498	90.00
14 - 14	55.7529	-77.97
14 - 15	17.4488	107.36
14 - 46	15.1172	90.00
15 - 15	67.3035	-76.42
15 - 45	10.0491	90.00
16 - 16	16.7127	-77.50
17 - 17	14.4592	-77.51
18 - 18	5.0774	-82.35
18 - 19	1.2111	123.94
19 - 19	3.1411	-56.57
19 - 20	1.9301	123.11
20 - 20	3.0896	-70.05
20 - 21	1.2344	90.00
21 - 21	8.2605	-62.20
21 - 22	7.2346	122.17
22 - 22	90.7722	-57.00
22 - 23	55.1276	123.08
22 - 38	28.4108	123.06
23 - 23	58.4016	-56.93
23 - 24	3.2775	122.96
24 - 24	23.9058	-85.72
24 - 25	1.6590	90.00
24 - 26	20.2700	90.00
25 - 25	5.5183	-65.52
25 - 30	4.1159	123.76
26 - 26	23.9778	-85.70
26 - 27	3.3015	123.01
27 - 27	12.0991	-57.04
27 - 28	8.7976	122.94

28	28	22.6693	-55.52
28	29	13.8769	125.45
29	29	31.9137	-70.53
29	52	4.2348	127.64
30	30	5.7983	-56.39
30	31	1.6824	123.26
31	31	2.7820	-56.49
31	32	1.0996	123.88
32	32	20.5936	-45.43
32	33	18.7890	137.44
32	34	1.0762	90.00
33	33	18.7890	-42.56
34	34	11.6005	-59.33
34	35	10.6673	123.69
35	35	25.1781	-53.43
35	36	14.5360	128.69
36	36	53.9334	-53.41
36	37	21.4149	128.39
36	40	18.0434	122.77
37	37	51.9909	-55.14
37	38	8.3279	122.83
37	39	22.3182	122.24
38	38	39.3558	-9.23
38	44	15.3259	116.29
38	48	17.4165	122.92
38	49	4.7376	123.01
39	39	22.9718	-58.79
39	57	0.7531	90.00
40	40	18.8166	-58.73
40	56	0.8735	90.00
41	41	7.1108	-72.38
41	42	2.4489	120.46
41	43	2.4272	90.00
41	56	1.2833	135.21
42	42	4.8708	-59.28
42	56	2.4220	120.98
43	43	8.9631	-90.00
44	44	22.5176	-63.58
44	45	7.1945	116.68

45	-	45	16.3460	-78.60
46	-	46	27.1691	-80.54
46	-	47	13.9306	108.69
47	-	47	47.1941	-57.61
47	-	48	33.8230	127.99
48	-	48	57.6922	-54.11
48	-	49	6.5099	122.88
49	-	49	22.4121	-64.56
49	-	50	6.6227	122.04
50	-	50	10.4685	-57.90
50	-	51	3.8459	122.21
51	-	51	17.4199	-83.24
52	-	52	12.2698	-52.29
52	-	53	8.0350	127.75
53	-	53	11.3353	-51.69
53	-	54	3.3503	128.99
54	-	54	6.8567	-51.82
54	-	55	3.5071	127.40
55	-	55	11.2876	-79.12
56	-	56	7.5760	-58.71
56	-	57	3.1964	123.79
57	-	57	3.8318	-62.36

Table B.8 SYSTEM LOADS
(IN P.U. ON A 100 MVA BASE)

BUS	REAL	REACTIVE
1	0.550	0.170
2	0.030	0.880
3	0.410	0.210
4	0.0	0.0
5	0.130	0.040
6	0.750	0.020
7	0.0	0.0
8	1.500	0.220
9	1.210	0.260
10	0.050	0.020
11	0.0	0.0
12	3.770	0.240
13	0.180	0.023
14	0.105	0.053
15	0.220	0.050
16	0.430	0.030
17	0.420	0.080
18	0.272	0.098
19	0.033	0.006
20	0.023	0.010
21	0.0	0.0
22	0.0	0.0
23	0.063	0.021
24	0.0	0.0
25	0.063	0.032
26	0.0	0.0
27	0.093	0.005
28	0.046	0.023
29	0.170	0.026
30	0.036	0.018
31	0.058	0.029
32	0.016	0.008
33	0.038	0.019
34	0.0	0.0
35	0.060	0.030
36	0.0	0.0
37	0.0	0.0
38	0.140	0.070
39	0.0	0.0
40	0.0	0.0
41	0.063	0.030
42	0.071	0.044
43	0.020	0.010
44	0.120	0.018
45	0.0	0.0
46	0.0	0.0
47	0.297	0.116
48	0.0	0.0
49	0.180	0.085
50	0.210	0.105
51	0.180	0.053
52	0.049	0.022
53	0.200	0.100
54	0.041	0.014
55	0.068	0.034
56	0.076	0.022
57	0.067	0.020

B.5 The IEEE 118-Bus Test System

A line diagram of the IEEE 118-Bus test system can be found elsewhere[36]. Only the bus admittance matrix and loads of the system are given here.

Table B.9. DIAGONAL AND NON-ZERO UPPER DIAGONAL ELEMENTS OF THE BUS ADMITTANCE MATRIX

BUS TO	BUS	MAGNITUDE	ANGLE
1 -	1	32.1255	-73.08
1 -	2	9.5791	106.87
1 -	3	22.5637	106.92
2 -	2	25.0932	-73.10
2 -	12	15.5338	106.89
3 -	3	37.5250	-74.12
3 -	5	9.0370	102.58
3 -	12	5.9823	106.83
4 -	4	136.2323	-77.11
4 -	5	122.3723	102.44
4 -	11	13.9073	106.90
5 -	5	200.5713	-79.59
5 -	6	18.0846	102.43
5 -	8	38.0235	90.00
5 -	11	14.0534	106.58
6 -	6	65.0224	-77.56
6 -	7	46.9474	102.44
7 -	7	75.4417	-76.88
7 -	12	28.5098	104.23
8 -	8	90.1423	-87.27
8 -	9	32.6824	94.57
8 -	30	19.7691	94.89
9 -	9	62.4470	-85.33
9 -	10	30.9567	94.58
10 -	10	30.3437	-85.33
11 -	11	89.8400	-73.15
11 -	12	48.8204	106.89
11 -	13	13.0871	106.93
12 -	12	130.8460	-74.11
12 -	14	13.5324	106.91
12 -	16	11.6208	104.26
12 -	117	6.9534	103.22
13 -	13	16.9624	-73.03
13 -	15	3.9143	106.93

14	- 14	18.4046	-73.04
14	- 15	4.9050	106.97
15	- 15	62.5971	-73.07
15	- 17	21.9058	106.81
15	- 19	24.2796	106.84
15	- 33	7.6879	106.99
16	- 16	16.9719	-75.75
16	- 17	5.3840	104.15
17	- 17	109.3548	-77.75
17	- 18	19.2395	103.69
17	- 30	26.8471	90.00
17	- 31	6.1226	106.87
17	- 113	31.7923	106.87
18	- 18	38.9453	-76.37
18	- 19	19.7177	103.57
19	- 19	56.1379	-74.94
19	- 20	8.3554	102.15
19	- 34	3.8731	106.93
20	- 20	19.8444	-77.82
20	- 21	11.5141	102.16
21	- 21	21.5695	-77.83
21	- 22	10.0780	102.16
22	- 22	16.1949	-77.82
22	- 23	6.1487	102.14
23	- 23	46.2112	-76.17
23	- 24	19.6007	105.34
23	- 25	12.2689	101.03
23	- 32	8.3627	105.37
24	- 24	26.8117	-74.97
24	- 70	2.3585	103.95
24	- 72	4.9509	103.98
25	- 25	44.1378	-85.45
25	- 26	27.2588	90.00
25	- 27	6.0215	101.04
26	- 26	39.4939	-88.45
26	- 30	11.5780	95.31
27	- 27	43.1403	-76.36
27	- 28	11.4137	102.61
27	- 32	12.6748	106.87
27	- 115	13.1764	102.48

28	–	28	21.6744	-76.67
28	–	29	10.2846	104.11
29	–	29	38.9724	-72.97
29	–	31	28.7213	108.07
30	–	30	76.7746	-86.76
30	–	38	18.4505	94.91
31	–	31	44.5239	-72.35
31	–	32	9.7173	106.83
32	–	32	51.2850	-74.72
32	–	113	4.7145	106.85
32	–	114	15.9563	102.44
33	–	33	14.4143	-73.30
33	–	37	6.7595	106.29
34	–	34	147.5528	-74.09
34	–	36	35.4863	108.00
34	–	37	102.6445	105.23
34	–	43	5.7770	103.80
35	–	35	115.3949	-77.60
35	–	36	95.7573	102.39
35	–	37	19.6453	102.48
36	–	36	131.1152	-76.10
37	–	37	169.7107	-77.20
37	–	38	28.5205	90.00
37	–	39	9.0290	106.85
37	–	40	5.6130	109.44
38	–	38	58.2636	-87.54
38	–	65	10.0999	95.22
39	–	39	24.8224	-73.09
39	–	40	15.8137	106.92
40	–	40	46.6152	-72.78
40	–	41	20.0197	106.87
40	–	42	5.2293	106.87
41	–	41	27.0851	-73.11
41	–	42	7.0877	106.89
42	–	42	18.2287	-74.46
42	–	49	6.0456	102.48
43	–	43	9.6825	-76.08
43	–	44	3.9554	103.92
44	–	44	14.5890	-75.92

44	-	45	10.7709	103.96
45	-	45	22.7256	-73.80
45	-	46	7.0733	106.44
45	-	49	5.0460	110.19
46	-	46	19.5091	-73.04
46	-	47	7.5436	106.66
46	-	48	5.0422	107.64
47	-	47	26.2325	-73.08
47	-	49	15.3014	106.99
47	-	69	3.4443	106.90
48	-	48	23.5344	-70.74
48	-	49	18.6642	109.52
49	-	49	94.9713	-73.50
49	-	50	12.5314	109.55
49	-	51	6.8792	109.53
49	-	54	6.6461	105.39
49	-	66	21.3570	101.08
49	-	69	2.9530	106.91
50	-	50	19.5424	-70.45
50	-	57	7.0355	109.48
51	-	51	36.0317	-70.67
51	-	52	16.0757	109.05
51	-	58	13.1082	109.53
52	-	52	21.9691	-72.32
52	-	53	5.9368	103.91
53	-	53	13.9130	-77.06
53	-	54	8.0127	102.17
54	-	54	133.1372	-74.58
54	-	55	13.7567	103.44
54	-	56	100.6233	106.06
			4.2598	102.37
55	-	55	81.2115	-73.14
55	-	56	63.0160	107.91
55	-	59	4.5261	102.39
56	-	56	90.7772	-72.87
56	-	57	9.7553	109.55
56	-	58	9.7553	109.55
56	-	59	7.7510	108.39
57	-	57	16.7637	-70.45
58	-	58	22.8436	-70.45

59	59	55.1035	-82.54
59	60	6.7374	102.33
59	61	6.5128	102.33
59	63	26.9862	90.00
0	60	96.8088	-78.61
0	61	72.6971	101.06
60	62	17.4117	102.37
61	61	141.9151	-81.53
61	62	25.9792	102.36
61	64	37.8817	90.00
62	62	56.1611	-77.61
62		4.4790	102.47
62		8.3465	102.44
63		77.7528	-86.85
63	64	49.8161	94.92
64	64	120.8597	-86.59
64	65	32.9820	95.09
65	65	135.1451	-86.10
65	66	28.9059	90.00
65	68	62.2688	94.93
66	66	62.0991	-83.39
66	67	9.6207	102.44
67	67	17.9390	-77.54
68	68	387.6479	-85.53
68	69	28.9059	90.00
68	81	49.3202	94.95
68	116	246.0481	94.80
69	69	57.4749	-81.15
69	70	7.6631	103.29
69	75	7.7793	108.36
69	77	9.4678	107.01
70	70	51.2183	-75.31
70	71	27.3379	103.95
70	74	7.2336	106.86
70	75	6.7864	106.89
71	71	54.3154	-77.30
71	72	5.3925	103.92
71	73	21.6363	100.80
72	72	10.2982	-75.99
73	7[illegible]	21.6305	-79.20

74	-	74	30.6702	-73.07
74	-	75	23.5725	106.85
75	-	75	62.7207	-72.96
75	-	77	4.7907	106.73
75	-	118	19.9052	106.78
76	-	76	24.0476	-73.23
76	-	77	6.4718	106.70
76	-	118	17.6000	106.78
77	-	77	137.4072	-72.66
77	-	78	77.1752	106.87
77	-	80	28.6161	108.14
77	-	82	11.0674	109.26
78	-	78	117.0878	-74.58
78	-	79	39.9945	102.61
79	-	79	53.6552	-77.37
79	-	80	13.8681	102.49
80	-	80	98.3485	-79.49
80	-	81	28.9059	90.00
80	-	96	5.3923	101.07
80	-	97	10.5069	101.09
80	-	98	9.0423	102.43
80	-	99	4.7371	102.44
81	-	81	79.7612	-86.94
82	-	82	54.9244	-72.46
82	-	83	26.0939	106.99
82	-	96	18.0438	107.00
83	-	83	39.2249	-71.62
83	-	84	6.8470	115.34
83	-	85	6.4884	106.20
84	-	84	20.9425	-64.71
84	-	85	14.1127	115.23
85	-	85	43.4252	-73.13
85	-	86	7.8197	105.88
85	-	88	9.6207	101.09
85	-	89	5.7260	97.87
86	-	86	12.5321	-77.16
86	-	87	4.7774	97.76
87	-	87	4.7553	-82.20
88	-	88	23.3824	-78.92
88	-	89	13.7847	101.05

89	-	89	59.7732	-78.15
89	-	90	14.8820	104.11
89	-	92	25.5646	101.78
90	-	90	26.2326	-74.63
90	-	91	11.4451	106.90
91	-	91	18.9405	-73.07
91	-	92	7.5212	106.92
92	-	92	71.0080	-76.21
92	-	93	11.2819	106.92
92	-	94	6.0548	106.93
92	-	100	3.3109	102.39
92	-	102	17.4711	102.41
93	-	93	24.3307	-73.05
93	-	94	13.0682	106.94
94	-	94	68.5692	-72.98
94	-	95	22.0444	106.92
94	-	96	10.9928	107.20
94	-	100	16.4826	107.06
95	-	95	39.4805	-72.88
95	-	96	17.4488	107.36
96	-	96	62.8181	-74.40
96	-	97	11.0895	101.06
97	-	97	21.5722	-78.91
98	-	98	14.4592	-77.51
98	-	100	5.4541	102.51
99	-	99	16.7127	-77.50
99	-	100	12.0093	102.48
100	-	100	71.9737	-75.18
100	-	101	7.7397	102.38
100	-	103	18.2202	106.95
100	-	104	4.7864	102.47
100	-	106	4.2220	104.80
101	-	101	16.4300	-77.59
101	-	102	8.7207	102.39
102	-	102	26.1703	-77.59
103	-	103	35.4084	-73.62
103	-	104	6.0565	106.39
103	-	105	5.8452	108.22
103	-	110	5.3920	102.16
104	-	104	36.3712	-75.27

104	-	105	25.5852	104.73
105	-	105	67.4139	-73.78
105	-	106	17.7106	104.36
105	-	107	5.2488	106.15
105	-	109	13.3354	110.37
106	-	106	27.1197	-75.19
106	-	107	5.2488	106.15
107	-	107	10.3946	-73.68
108	-	108	45.9447	-69.87
108	-	109	32.6218	110.03
109	-	109	44.9372	-69.96
109	-	110	12.3285	110.04
110	-	110	44.8242	-71.58
110	-	111	12.7162	106.25
110	-	112	14.5771	111.10
111	-	111	12.7066	-73.74
112	-	112	14.5481	-68.85
113	-	113	36.4783	-73.12
114	-	114	109.8323	-77.53
114	-	115	93.8853	102.47
115	-	115	107.0508	-77.53
116	-	116	245.9664	-85.20
117	-	117	6.9360	-76.74
118	-	118	37.4930	-73.22

Table B.10 SYSTEM LOADS
(IN P.U. ON A 100 MVA BASE)

BUS	REAL	REACTIVE
1	0.510	0.270
2	0.200	0.090
3	0.390	0.100
4	0.390	0.120
5	0.0	0.0
6	0.520	0.220
7	0.190	0.020
8	0.280	0.0
9	0.0	0.0
10	0.0	0.0
11	0.700	0.230
12	0.470	0.100
13	0.340	0.160
14	0.140	0.010
15	0.900	0.300
16	0.250	0.100
17	0.110	0.030
18	0.600	0.340
19	0.450	0.250
20	0.180	0.030
21	0.140	0.080
22	0.100	0.050
23	0.070	0.030
24	0.130	0.0
25	0.0	0.0
26	0.0	0.0
27	0.710	0.130
28	0.170	0.070
29	0.240	0.040
30	0.0	0.0
31	0.360	0.270
32	0.590	0.230
33	0.230	0.090
34	0.590	0.260
35	0.330	0.090
36	0.310	0.170
37	0.0	0.0
38	0.0	0.0
39	0.270	0.110
40	0.660	0.230
41	0.370	0.100
42	0.960	0.230
43	0.180	0.070
44	0.160	0.080
45	0.530	0.220
46	0.090	0.100

BUS	REAL	REACTIVE
60	0.780	0.030
61	0.0	0.0
62	0.770	0.140
63	0.0	0.0
64	0.0	0.0
65	0.0	0.0
66	0.390	0.180
67	0.280	0.070
68	0.0	0.0
69	0.0	0.0
70	0.660	0.200
71	0.0	0.0
72	0.120	0.0
73	0.060	0.0
74	0.680	0.270
75	0.470	0.110
76	0.680	0.360
77	0.610	0.280
78	0.710	0.260
79	0.390	0.320
80	1.300	0.260
81	0.0	0.0
82	0.540	0.270
83	0.200	0.100
84	0.110	0.070
85	0.240	0.150
86	0.210	0.100
87	-0.040	0.0
88	0.480	0.100
89	0.0	0.0
90	1.630	0.420
91	0.100	0.0
92	0.650	0.100
93	0.120	0.070
94	0.300	0.160
95	0.420	0.310
96	0.380	0.150
97	0.150	0.090
98	0.340	0.080
99	0.420	0.0
100	0.370	0.180
101	0.220	0.150
102	0.050	0.030
103	0.230	0.160
104	0.380	0.250
105	0.310	0.260

47	0.340	0.0
48	0.200	0.110
49	0.870	0.300
50	0.170	0.040
51	0.170	0.080
52	0.180	0.050
53	0.230	0.110
54	0.650	0.320
55	0.630	0.220
56	0.840	0.180
57	0.120	0.030
58	0.120	0.030
59	2.770	1.130

106	0.430	0.160
107	0.500	0.120
108	0.020	0.010
109	0.080	0.030
110	0.390	0.300
111	-0.360	0.0
112	0.680	0.130
113	0.060	0.0
114	0.080	0.030
115	0.220	0.070
116	1.840	0.0
117	0.200	0.080
118	0.330	0.150