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Automaticity of Multiplication Facts and Procedure Use For Multiplication Problems: Evidence For ADAMM

by

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled AUTOMATICITY OF MULTIPLICATION FACTS AND PROCEDURE USE FOR MULTIPLICATION PROBLEMS: EVIDENCE FOR ADAMM submitted by MIKE H. THIBODEAU in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE.

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Abstract

According to associative theories of the representation of arithmetic facts, specific facts may vary in accessibility. To test this hypothesis, subjects in Experiment 1 were required to verify the presence of a target number (e.g., 8) in a previously presented pair (e.g., 5 x 8). Subjects were slower to reject targets that were the product of the cue (e.g., 7 x 3 and 21) than to reject unrelated targets (e.g., 7 x 3 and 20) at brief SOAs, indicating that targets were activated without intention (i.e., automatic). In support of the view that accessibility varies across problems, interference was only found on multiplication problems with products less than 24. The results of Experiment 1 support the view that the problem-size effect in mental arithmetic is due to differences in accessibility of facts in the network. Experiment 2 was designed to evaluate the applicability of Logan's (1988, 1992) definition of automaticity in the domain of multiplication in adults. Logan has specifically defined automaticity as single-step memory retrieval. To test his hypothesis, subjects solved multiplication problems and gave descriptions of their solution procedures on each trial. Further, the number-matching task used in Experiment 1 was employed to test the automaticity of multiplication facts. If automaticity is single-step memory retrieval, as Logan suggested, then subjects who frequently use single-step memory retrieval in the production task should provide evidence for automatic activation of multiplication facts in the number-matching task. In contrast, subjects who infrequently use single-step memory retrieval in the production task should not provide evidence for automatic activation of multiplication facts in the number-matching task. Both the frequent retrievers/automatic pattern and the infrequent retrievers/automatic pattern were identified, indicating that Logan's view of automaticity is adequate for characterizing the performance only of some subjects. The results from Experiment 2 support a model proposed to account for adult performance in both the production task and the number-matching task called the adult distribution of associations multiplication model (i.e., ADAMM). The main assumptions in ADAMM stem from Siegler's (1988) distribution-of-associations model.

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I am especially grateful to Karen Clark for providing me with an abundance of support, encouragement, and love throughout the last two years. Finally, I dedicate this thesis to my mother, Angelique Thibodeau, who was taken from us much too early.

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According to current models of mental arithmetic, arithmetic facts are retrieved from associative networks (Ashcraft, 1987; Campbell & Graham, 1985; Campbell & Oliphant, 1992; Campbell, 1987a, b; Graham & Campbell, 1992). In Campbell and Oliphant's (1992) network-interference model, arithmetic problems are stored according to magnitude codes and physical codes. When a problem is presented (e.g., 6 X 4 = ?), a set of candidate answers is activated. The activation strength of each candidate answer is determined by a feature-matching mechanism. Features consist of operands and the multiplication sign (e.g., 6, 4, X). A candidate answer will be activated if it shares features with the problem (e.g., 6, 4, X, 24). In the retrieval process, the candidate with the highest level of activation is selected. In general, the time required to retrieve the correct answer is a function of the difference between the activation levels of the correct and incorrect candidates.

One assumption of the network-interference model is that activation of the candidate set is automatic. Since Posner and Synder's (1975) work in the mid-seventies, automaticity has been defined as a process that is fast, effortless, unconscious, initiated without intention, and that decays quickly (Logan, 1980; Posner & Snyder, 1975). Recently, Logan (1988, 1992) has specifically redefined automaticity as single-step memory retrieval. One of the intentions of this thesis is to address the applicability of Logan's definition of automaticity in the domain of multiplication in adults.

The problem-size effect, a major empirical phenomenon in mental arithmetic, refers to the finding that problems with smaller products (e.g., 2 X 3) are solved more quickly and accurately than problems with larger products (e.g., 8 X 9; Ashcraft, 1992; Campbell & Graham, 1985; Koshmider & Ashcraft, 1991; Miller, Perlmutter, & Keating, 1984). Experiment 1 was designed to test the hypothesis that the problem-size effect reflects differences in the accessibility of small versus large multiplication facts (Ashcraft, 1992; Campbell, 1987a, b, 1991; Campbell & Graham, 1985; Koshmider & Ashcraft, 1992; Siegler & Shrager, 1984; Thibodeau & LeFevre, 1992). Accessibility is defined as the strength of the connection between the problems and answers in the mental representation of arithmetic facts. Presumably arithmetic facts that are highly accessible are likely to be activated automatically (LeFevre, Bisanz, & Mrkonjic, 1988). Consequently, evidence for automatic activation was used to measure the accessibility of different arithmetic facts. The number-matching task devised by LeFevre et al. (1988) was employed to test the automatic activation of multiplication facts. If the problem-size effect occurs because smaller problems are more accessible than larger problems, then automatic activation should be more evident for smaller than for larger problems.

Experiment 2 was designed to evaluate the applicability of Logan's (1988, 1992) definition of automaticity in the domain of multiplication by having adults perform a production task. In the production task, subjects answered single-digit multiplication problems and described their solution procedures on each problem. Further, the number-matching task used in Experiment 1 was employed to test the automaticity of multiplication facts. If automaticity is single-step memory retrieval, as Logan suggested, then subjects' use of procedures on the production task should correspond with subjects' performance on the number-matching task. More specifically, subjects who frequently use single-step memory retrieval in the production task should provide evidence for automatic activation of multiplication facts in the number-matching task. In contrast, subjects who infrequently use single-step memory retrieval in the production task should not provide evidence for automatic activation of multiplication facts in the number-matching task. Results from the two tasks also serve as evidence for a model proposed to account for adult performance in multiplication.

Experiment 1

Evidence for Automaticity in Simple Arithmetic

Priming paradigms have been used to provide evidence for the automatic activation within the network representation of multiplication facts (Campbell, 1987b, 1991; Koshmider, & Ashcraft, 1991, Meagher, 1992). In a priming task, two stimuli are presented sequentially. Upon presentation of the second stimulus, subjects are required to respond. Depending on the task, subjects' responses are faster and more accurate or slower and less accurate when the second stimulus is related to the first stimulus than when the second stimulus is unrelated to the first stimulus.

In Campbell's (1991) study, subjects were required to answer a multiplication problem preceded 200 ms by a prime. Campbell found that the correct priming condition (e.g., 24 followed by 3 X 8) resulted in faster response times compared to the neutral priming condition (e.g., ## followed by 3 X 8) or the unrelated priming condition (e.g., 14 followed by 3 X 8). There are two possible interpretations of these results. First, the numerical primes may have automatically activated information that facilitated response times in the correct priming condition. Second, subjects may have employed a verification strategy (i.e., a controlled process) that could have resulted in the facilitation. In the verification strategy, subjects would assess the familiarity of the prime with the problem's answer (i.e., is the prime 24 highly familiar with the answer for 4 X 6). If the familiarity was high, as in the correct prime condition, the subject would rapidly state the prime as the answer (i.e., a facilitated response time).

To distinguish between these two interpretations, a range of SOAs can be used. If the facilitation effect found by Campbell (1991) was the result of automatic processes, then the effect should only occur at short SOAs because a traditional assumption about automaticity is that automatic processes occur and decay quickly (LeFevre et al., 1988; Neely, 1977; Posner & Synder, 1975). If the effect was the result of controlled processes (i.e., a verification strategy), then the effect should occur at all SOAs (i.e., there is no time course for controlled processes). Meagher (1992) used the same prime paradigm as Campbell (1991), but employed a range of SOAs. The results from Meagher's experiments led to the conclusion that both automatic processes and controlled processes play a role in subjects' performance in the numerical priming paradigm.

The results from the numerical priming studies (Campbell, 1991; Meagher, 1992) do not offer much insight into the automaticity of multiplication facts because subjects were always intentionally performing mental arithmetic. A traditional assumption about automaticity is that an automatic process is initiated without intention. Hence, a more direct test of the automaticity of multiplication facts requires a task in which mental arithmetic is not likely to be initiated intentionally by subjects.

In an attempt to develop a stringent test of automaticity, LeFevre et al. (1988) devised a Stroop-like task that avoided the potential complication of strategic activation found in Campbell (1989, 1991) and Meagher (1992). In the <u>number-matching task</u>, subjects are presented pairs of numbers (e.g., 2 + 4). After a variable delay, the pair is replaced by a target number. Subjects are required to decide whether the target number is one of the numbers in the initial pair. Note that successful completion of the task does not require mental arithmetic, therefore any effects of arithmetic-based relations could be attributed to the automatic activation of arithmetic facts. When the target presented is the sum (e.g., 6 for the initial pair 2 + 4), LeFevre et al. (1988; Lefevre, Kulak, & Bisanz, 1991) found that subjects were slower to respond "no" than when the target presented was unrelated (e.g., 9 for the initial pair 2 + 4). LeFevre et al. assumed that upon presentation of the number pair, a set of related numbers are activated automatically. The set of activated numbers includes the numbers in the initial pair (e.g., 2 and 4) and associated nodes (e.g., the sum 6, the product 8, etc.). Upon presentation of the target, subjects would compare the target number to the set of activated numbers. LeFevre et al. concluded that the automatic activation of the sum (i.e., an associative node) interfered with the decision process, which resulted in longer reaction times. Hence, these results suggest that sums may be activated in an obligatory fashion in response to presentation of the operands.

Problem-Size Effect and Automaticity

The problem-size effect refers to the finding that problems with smaller products (e.g., 2 X 3) are solved more quickly and accurately than problems with larger products (e.g., 8 X 9; Campbell & Graham, 1985; Koshmider & Ashcraft, 1991; Miller et al., 1984). Currently, associative explanations of the problem-size effect are dominant (Ashcraft, 1992; Campbell, 1987a, 1987b, 1991; Campbell & Graham, 1985; Campbell & Oliphant, 1992; Siegler & Shrager, 1984). Supporters of the associative models assume that the problem-size effect reflects differences in accessibility of specific facts in the network. More specifically, small problems are more accessible than large problems, which results in quicker and more accurate solutions for small problem compared to large problems.

I define accessibility theoretically as the strength of the connection between the problems and answers in the mental representation of arithmetic facts. It is plausible to conclude that arithmetic facts that are activated automatically are highly accessible (LeFevre et al., 1988). Consequently, automatic activation can be used as a measure of accessibility of different multiplication facts.

Thibodeau and LeFevre (1992) employed the same number-matching task as

LeFevre et al. (1988, 1991) to test the hypothesis that the problem-size effect reflects a difference in the accessibility of small versus large multiplication facts. In Thibodeau and LeFevre's experiment, there were three critical cue-target relations. On small-product trials, the target was the product of the cue and was 21 or less (e.g., 4 X 3 followed by 12). On large-product trials, the target was the product of the cue and was 24 or greater (e.g., 5 X 8 followed by 40). On unrelated trials, the target was not divisible by either digit in the cue (e.g., 5 X 9 followed by 12). Thibodeau and LeFevre predicted that the cue would activate a candidate set of related nodes, including the numbers in the cue and the product of the cue. The activation of the product would result in interference in the decision process, which would result in longer reaction times. Further, they predicted that activation of the product would only occur for small products and not large products because small multiplication facts are more accessible than large multiplication facts. Hence, latencies for small-product trials should differ from latencies for unrelated trials, whereas latencies for large-product trials should not differ from unrelated trials.

Thibodeau and LeFevre (1992) found the predicted result. Latencies for small-product trials (i.e., problems with products less than 24; e.g., 3 X 7 followed by 21) were significantly slower than latencies for unrelated trials (e.g., 3 X 7 followed by 16) at 100-ms and 350-ms SOAs. Thibodeau and LeFevre concluded that the interference at the 100-ms SOA was attributed to automatic activation because the interference occurred at a short SOA. Note that a process is only considered automatic when it occurs at short SOAs because at longer SOAs controlled or conscious processing may play a role (Logan, 1980; Neely, 1991; Posner & Snyder, 1975). Thibodeau and LeFevre concluded that small multiplication facts were more accessible than large multiplication facts. The differences in accessibility are consistent with the assumptions made in associative models (Campbell, 1987a, 1987b, 1991; Campbell & Graham, 1985; Campbell & Oliphant, 1992; Siegler & Shrager, 1984).

However, Thibodeau and LeFevre (1992) found an unexpected result: Latencies for the small- and large-unrelated trials were significantly slower than latencies for small- and large-product trials at the 40-ms SOA. The initial explanation for this result was that, because of the difficulty of encoding the cue at the 40-ms SOA, subjects may have relied on backward temporal priming to facilitate processing of the cue (Briand, den Heyer, & Dannenbring, 1988; Kiger & Glass, 1983). For example, if 3 X 7 was followed by 21 (product trial), encoding the target may facilitate the encoding of the cue, resulting in faster latencies for product than for unrelated trials. Subjects may, in essence, use the target information to confirm their encoding of the cue. However, this explanation became disputed when the appropriateness of certain unrelated trials were examined.

Campbell and Oliphant (1992) provided evidence that showed that a high percentage of errors in multiplication problems contain one of the problem's operand (i.e., an operand-intrusion error; e.g., 2 X 5 = 21). Campbell and Oliphant suggested that the operands are physical codes that effect performance on multiplication problems. Hence, an unrelated trial that shared a number in the cue and the target could be classified as a related trial (e.g., 2 X 5 followed by 21). In the stimulus list employed by Thibodeau and LeFevre (1992), one of five large unrelated trial and two of five small unrelated trials shared a number in the cue and the target. It is possible that the common number in the cue and target may have interfered with subjects' decision process at the 40-ms SOA. This operand-intrusion interference could have inflated the mean response time for the unrelated trials.

In Experiment 1 of the present paper, the number-matching task developed by LeFevre et al. (1988) was used to replicate the expected result and to test the hypothesis that operand-intrusion interference caused the unexpected result found by Thibodeau and LeFevre. Because unrelated trials that share a number in the cue and target appeared to cause the unexpected result, the stimulus list employed in Experiment 1 did not contain those types of unrelated trials.

Objectives of Experiment 1

There were two objectives of Experiment 1. The first was to replicate the finding of Thibodeau and LeFevre (1992) that latencies for small-product trials were significantly slower than latencies for unrelated trials at the 100-ms SOA, whereas latencies for large-product trials did not differ from latencies for unrelated trials. This result would provide evidence for automaticity of small multiplication facts and support the assumption that the problem-size effect reflects differences in accessibility of specific facts in the network. The second objective was to test the hypothesis that latencies for the small- and large-product trials were significantly faster than latencies for unrelated trials at the 40-ms SOA in Thibodeau and LeFevre's study because of the operand-intrusion interference effect.

Method

Subjects

Thirty-eight university students (19 males, 19 females) from an introductory psychology course participated. Data from six subjects (3 males, 3 females) were not analyzed because their error rates were 30% or higher in two or more of the critical conditions. Each subject received one experimental credit toward their introductory psychology course. The median age of the subjects was 19:6 (in years:months), with a range of 17:8 to 47:10.

Materials

Each trial included an initial number cue (e.g., 6 X 3) and a subsequent target (e.g., 20). Ties (e.g., 4 X 4) were not used and none of the number cues or targets included 0 or 1. On 50% of trials, the target matched one of the numbers in the cue. The complete set of stimuli is presented in the Appendix A.

Non-Matching Stimuli

Product Stimuli. The 10 product stimuli (e.g., 40 for the pair 5 X 8) were separated into two groups based on the size of their products. The large-problem group consisted of problems with correct products of 27 or greater. The small-problem group consisted of problems with correct products of 21 or less. This distinction between small and large problems was similar but not identical to Campbell's (1987b, 1991; see also Koshmider & Ashcraft, 1991) classification of problems into easy and hard.

Unrelated Stimuli. The 10 unrelated stimuli (e.g., 42 for the pair 5 X 8) were separated into two groups based on the size of their product and median production time (see Miller et al., 1984). The large-unrelated group consisted of problems with correct products of 27 or greater. Furthermore, the 5 large-unrelated stimuli used the same cues as the large-product stimuli. The small-unrelated group consisted of problems with correct products of 21 or less. Three of the small-unrelated stimuli contained the same cues as the small-product stimuli. The remaining two small-unrelated stimuli and two small-product stimuli were matched using median production times found by Miller et al. (1984). This procedure ensured that the time required to activate the correct product for the unmatched small-product stimuli was similar to the time required for the small-unrelated stimuli. Unrelated targets were not divisible by either number in the cue, in order to avoid table-related activation (Campbell, 1987a, b). Furthermore, the single-digit numbers in the unrelated cues did not match either of the numbers in the double-digit targets, in order to avoid operand-intrusion interference. Note that a complete match between small product cues and small unrelated cues was impossible due to the characteristics of the unrelated stimuli.

Non-Matching Filler Stimuli. There were five non-matching filler stimuli with a double-digit number in both the cue and the target (e.g., 46 for the pair 39 X 7). This condition was included so that subjects were presented trials in which the double digit in the cue did not match the target (see target-balancing stimuli below).

Matching Stimuli

Matching stimuli were used to ensure that subjects were not aware of the critical stimuli (product and unrelated) of the experiments and could not predict whether the target matched the cue based on the identity of the cue.

Target-Balancing Stimuli. The 10 target-balancing stimuli had the same ten targets that were used in the product trials, but all targets matched one of the numbers in the cue (e.g., 18 for the pair 18 X 5). These stimuli were included so that subjects saw cases in which the double-digit number in the cue matched the target.

Cue-Balancing Stimuli. The 10 cue-balancing stimuli had the same ten cues which were used in the product trials, but all targets matched one of the numbers in the cue (e.g., 5 for the pair 5 X 8). This condition was included so that subjects saw matching trials that consisted of two single-digit numbers in the cue.

Matching-Filler Stimuli. There were five matching-filler stimuli which had a double-digit number in the cue. The targets matched the single-digit number in the cue (e.g., 8 for the pair 8 X 13). This condition was included so that subjects saw trials in which the sole single digit in the cue matched the target.

Stimulus List

The number of distinct cue-target combinations from the six conditions was 50. Each combination was presented once at SOAs of 40, 100, 220, and 350 ms, for a total of 200 trials. These SOAs were used to replicate the SOAs employed by Thibodeau and LeFevre (1992). To make the stimulus list, a latin square procedure was used to assign the SOAs to the 50 trials. Following the assigning of the SOAs, the trials were ordered unsystematically with the following restrictions: no identical cues or target appeared consecutively; no more than three SOAs of the same value appeared successively; and no more than three consecutive trials of the same responses appeared consecutively ("yes" or "no"). The final stimulus list consisted of an additional 200 trials (for a total of 400) that were identical to the first 200 trials but presented in the reverse order. Two lists of the stimuli used, one the reverse order of the other. List order was counterbalanced across subjects.

Procedure

Stimuli were presented on a microcomputer screen. At the beginning of each trial an asterisk (*) appeared. Subjects pressed the center button of a three-button response panel to begin the trial. Each subject used the index finger of the preferred hand to press the buttons on the response panel. The subject heard a warning signal 200 ms after the center button was pressed. A cue was presented 400 ms after the warning signal. The cue remained on the screen for 40, 100, 220, and 350 ms. Then the target was presented. Subjects were required to decide whether the target matched one of the digits in the cue. The response buttons were the left and right buttons on the response panel. Position of the "no" key was counterbalanced across subjects. If subjects did not respond in five seconds, the target was replaced by the asterisk. Subjects received auditory feedback regarding the accuracy of their response on each trial.

Before the test trials, subjects were given instructions and practice trials and were instructed to respond as quickly and accurately as possible to each trial. Subjects were permitted to do as many practice trials as they wanted in multiples of ten. Most subjects did 10 practice trials.

There were two lists of the stimuli used, one the reverse order of the other. The stimulus list of 400 trials was administered in two sessions of 200. Each session lasted approximately ten minutes. The order of the stimuli was counterbalanced with gender and the "yes" key across subjects. Experimental sessions lasted approximately 35 minutes.

Results and Discussion

Median correct latencies and percentage errors for the product and unrelated trials were analyzed in a 4 (SOA: 40, 100, 220, 350 ms) X 2 (problem size: large, small) X 2 (problem type: product, unrelated) analyses of variance, with repeated measures on all factors. Unless otherwise indicated, reported results were significant with an alpha level of .05. Data from matching trials and from nonmatching filler trials do not bear on the hypotheses and are not discussed further. Mean correct latencies and percentage errors for all conditions are presented in Appendix B.

Latencies

Latencies decreased with increasing SOA (from 855 ms at the 40-ms SOA to 689 ms at the 350-ms SOA), $F(3,93) = 105.74$, $MSe = 6665$. There were significant linear, $F(1,93) = 12.09$, and quadratic trends, $F(1,93) = 5.79$. The decline in latencies with SOA presumably occurs because at shorter SOAs the latency includes encoding and processing of the initial pair of digits. At longer SOAs, encoding and processing of the initial pair is completed before the target is presented and therefore do not contribute to the latency.

Latencies for small-problem trials were significantly slower than for large-problem trials (765 vs. 745 ms), $\underline{F}(1,31) = 10.96$, $\underline{MS}e = 4895$. A plausible explanation is that large problems have larger differences between the magnitude of the operands and target than small problems, resulting in a slower reaction times for small problems (i.e., distance effect; LeFevre et al., 1988, 1991). Researchers have proposed that the distance effect is the result of spreading activation and the mental representation of a number-line (Campbell & Oliphant, 1992; LeFevre et al., 1988). If the numbers 6 and 3 are encoded, then numbers such as 2, 4, 5, and 7 receive similar levels of activation because these numbers are closely connected to 6 or 3. In contrast, when 5 and 8 are encoded a number such as 42 does not receive similar activation because 42 is not closely connected to 5 or 8. Consequently, the distance effect implies that it was more difficult to compare the target to the cue in small problems than to compare the target to the cue in large problems because of the closeness of the numbers in the small problems. It is important to note that the distance effect is independent of any interference effect caused by automatic activation. If the distance effect were the only significant effect measured in the number-matching task, then latencies should only vary as a function of problem size. However, if latencies also vary as a function of problem type (e.g., small-product vs. small-unrelated), then the interference effect, not the distance effect, is a plausible explanation.

Problem size interacted with SOA, $\underline{F}(3,93) = 2.78$, $\underline{MS}e = 3422$. Latencies for small trials were significantly slower than latencies for large trials at the 40-ms SOA (878 vs. 833 ms), $\underline{F}(1,124) = 17.5$, $\underline{MS}e = 3790$. This finding may reflect the difficulty of encoding the cue at the 40-ms SOA (see the next section on errors), conjoined with the interpretation that small-problem trials were more difficult than large-problem trials (i.e., distance effect; LeFevre et al., 1988, 1991).

Latencies also varied as a function of problem type (product vs. unrelated), $F(1,31) = 10.33$, $MSe = 6895$. Latencies on product trials were significantly slower than on unrelated trials (767 vs. 713 ms). This finding supports the hypothesis that the cue activates a candidate set of related nodes, including the product of the cue, which results in slower latencies for product trials than for unrelated trials.

There was no interaction of SOA, problem type, and problem size, $F(3,93) = 1.34$, $MSe = 3063$, $p = .268$. Thibodeau and LeFevre (1992), however, found a three-way interaction and test of simple effects indicated that latencies for small-product trials were significantly slower than latencies for small-unrelated trials at the 100-ms and 350-ms SOAs. Further, latencies for large-product trials were not significantly slower than latencies for large-unrelated trials at any SOA. Hence, tests of simple effects were used to compare latencies for product and unrelated trials for all combinations of product size and SOA (see Figure 1). If the problem-size effect is due to the difference between accessibility of specific facts, then latencies for small-product trials would be slower than latencies for small-unrelated trials (i.e.,evidence for automatic activation) and latencies for large-product trials would not differ from latencies for large-unrelated trials (i.e., no evidence for automatic activation). Test of simple effects indicated that at the 100-ms and 220-ms SOAs, latencies for small-product trials were significantly slower than latencies for small-unrelated trials, $F(1,248) = 8.24$, and $F(1,248) = 6.81$, $MSe = 4093$. No comparisons between large-product and large-unrelated trials were significant. Thus, evidence for automatic activation was found for the small multiplication facts at the 100-ms and 220-ms SOAs. Further, the absence of evidence for automatic activation of large multiplication facts provides evidence for the hypothesis that the problem-size effect is due to the difference between accessibility of specific facts (Thibodeau & LeFevre, 1992).

Insert Figure 1 about here

The interference effect found at the 220-ms SOA in this experiment was not found in Thibodeau and LeFevre (1992). A plausible explanation for the discrepancy between the results of the two experiments is that the present experiment employed a different stimulus list than Thibodeau and LeFevre. Only two of the five small-product trials used in the present experiment were used in Thibodeau and LeFevre. Hence, the different trials used in the present experiment may have resulted in the interference effect at the 220-ms SOA. More specifically, the different trials used may have possessed very strong links between problems and facts in the mental representation which resulted in the maintenance of automatic activation of these small-product facts at the 220-ms SOA. Further, the parameters of automatic activation of multiplication facts are still relatively unexplored as compared to those of addition facts. LeFevre et al. (1991) concluded that addition facts were obligatorily activated with SOAs of under 100 ms and that obligatory activation declines at a SOA of 120 ms. The present results suggest that small-multiplication facts are activated more slowly than addition facts (see also Zbrodoff & Logan, 1986).

Recall that one of the objectives of this experiment was to test the hypothesis that latencies for the small- and large-product trials were significantly faster than latencies for unrelated trials at the 40-ms SOA in Thibodeau and LeFevre (1992) because of the operand-intrusion interference effect. Because removing operand-intrusion trials (i.e., unrelated trials that share a number in the cue and the target; e.g., <u>2</u> X 5 followed by <u>2</u>1) from the stimulus list in this experiment resulted in an absence of the operand-intrusion interference at the 40-ms SOA, it can be concluded that in a "truly" unrelated trial, the cue cannot share any similar features with its target (Campbell & Oliphant, 1992).

The operand-intrusion interference effect experienced by subjects on unrelated trials in Thibodeau and LeFevre's (1992) study could be explained in the following manner. Because the effect only occurred at the 40-ms SOA and error rates at the 40-ms SOA were significantly higher than those at the 100-, 220- and 350-ms SOAs, it is possible that the cue presentation of 40 ms was not sufficient time for subjects to properly distinguish between the numbers in the cue and in the target. The inadequate distinction between the numbers in the trial may have produced confusion for subjects. The confusion may have became detrimental when one of numbers in the double-digit target matched one of the single-digit numbers in the cue (e.g., 2 X 5 followed by 21). To support the notion that unrelated trials containing physical matches contributed to a high level of confusion at the 40-ms SOA, the error rate for unrelated trials at the 40-ms SOA decreased to 4.1% in the present experiment from 7.5% in Thibodeau and LeFevre's experiment. Recall that the stimulus list employed in the present experiment omitted unrelated trials containing physical matches. Hence, the operand-intrusion interference effect may have been produced by a high level of confusion experienced by subjects, resulting in slower overall reaction times for the unrelated trials at the 40-ms SOA in the study conducted by Thibodeau and LeFevre.

Errors

Errors were low, averaging 2.0% across subjects. The analysis of errors should be interpreted cautiously because the low overall error rate suggests a floor effect. Errors varied with SOA, $F(3,93) = 8.78$, $MSe = 20.7$. Error rates at the 40-ms SOA were significantly higher than those at the 100-, 220- and 350-ms SOAs, (3.8 vs. 1.7, 1.3, 1.3%; Tukey HSD, $p < .05$). No other main effect or interaction was significant.

Summary

The first objective of Experiment 1 was to replicate previous findings (Thibodeau & LeFevre, 1992) and to garner further support for the view that the problem-size effect reflects differences in accessibility of specific facts. The results of Experiment 1 were consistent with the previous findings. Evidence for automatic activation was found for small multiplication facts at the 100-ms and 220-ms SOAs, whereas no evidence for automatic activation was found for large multiplication facts at any SOA. The absence of evidence for automatic activation of large multiplication facts provides evidence for the hypothesis that the problem-size effect maybe due to differences among accessibilities of specific facts, with small problem-answer dyads being more accessible than large problem-answer dyads. The second objective was to test the hypothesis that latencies for the small- and large-product trials were significantly faster than latencies for unrelated trials at the 40-ms SOA in Thibodeau and LeFevre's study because of the operand-intrusion interference effect. The results of the present experiment confirmed this interpretation: When the opportunity for operand confusion was removed, no significant difference was found between the latencies for product trials and for unrelated trials at the 40-ms SOA.

Experiment 2

Logan (1988) proposed an instance theory of automatization. In this theory, novices solve a problem (e.g., 3 + 4) with a general algorithm (e.g., counting). With practice, novices begin to learn the solution (e.g., 7) to the specific problem and begin to use single-step memory retrieval upon repeated presentation of the problem. With sufficient experience, individuals will begin to solve the problem via single-step memory retrieval and bypass the algorithm completely. Logan proposed that at this point the process used to obtain the solution for this specific problem is automatic. One of the intentions of this thesis is to address the applicability of Logan's definition of automaticity in the domain of multiplication in adults.

The Instance Theory Definition of Automaticity

Since Posner and Synder's (1975) work in the mid-seventies, automaticity has been defined as a process that is initiated without intention and that is fast, effortless, unconscious, and subject to rapid decay. In Logan's theory (1988), an automatic process is initiated with intention. He proposed that individuals exploit automaticity as a retrieval process (i.e., initiating an automatic process to achieve a goal). In addition, Logan suggested that certain traditional assumptions about automaticity (i.e., that automatic processes are fast, effortless and unconscious) are by-products of single-step memory retrieval being the underlying process of automaticity. More specifically, single-step memory retrieval is fast relative to algorithmic computation. Single-step memory retrieval is effortless relative to the algorithmic solution in the sense that facts obtained through single-step memory retrieval appear to "pop into mind" easily. Single-step memory retrieval is unconscious relative to the algorithmic solution in the sense that there is enough time in the algorithmic solution for introspection by the individual's "mind's eye" (i.e., consciousness), whereas single-step memory retrieval may not be available to the "mind's eye". Logan does not discuss the characteristic that an automatic process decays quickly.

In Logan's (1988) theory, automaticity is specifically defined as follows: "Automaticity is memory retrieval: Performance is automatic when it is based on single-step direct-access retrieval of past solutions from memory" (p. 493). More recently, Logan (1992) has continued to define automaticity in the same manner: "Performance is automatic when it is based on single-step, direct-access retrieval of solutions from memory" (p. 884).

In Logan and Klapp (1991), subjects were required to estimate the percentage of alphabet arithmetic trials (e.g., A X 2 = C) they solved by counting and the percentage of trials they solved by retrieval at the end of each session of trials. As the number of sessions increased, the estimated percentage for counting decreased and the estimated percentage for retrieval increased. Along with the increase use of retrieval, there were quantitative changes in reaction time and standard deviation that suggests that automatic processes were involved in solving the alphabet arithmetic problems. Logan and Klapp used this finding to conclude that single-step memory retrieval is the underlying mechanism of automaticity.

There was a major methodological problem in Experiment 1 of Logan and Klapp (1991). From the verbal protocol data collected, Logan and Klapp could not determine which problems were solved by single-step memory retrieval and which were solved by an algorithmic solution. Logan and Klapp concluded that in session 12 (the last session) subjects' performance was automatic. Subjects reported counting on 28% of the trials and memory retrieval on 64%. Under the strict interpretation of Logan's (1988, 1992) definition of automaticity, subjects' performance was only automatic on 64% of the trials. However, the reaction time and standard deviation data from all trials meet the quantitative assumptions of automaticity in Logan's instance theory. A more appropriate test of the theory would have been to analyze the data of the trials where subjects reported using retrieval compared to the trials where subjects reported using counting. If Logan's theory is correct, then only the reaction times and standard deviations from the retrieval trials would have provided evidence for the quantitative assumptions of the instance theory. In general, a study that records procedure use on each trial and tests automaticity on each trial would properly evaluate the applicability of Logan's definition of automaticity.

The applicability of Logan's (1988, 1992) definition of automaticity in the domain of multiplication in adults is addressed in Experiment 2. Adult multiplication was considered a suitable domain because Bisanz, LeFevre, and Sadesky (1993) found that adults do not solely rely on retrieval to solve multiplication problems. To address the applicability of Logan's definition of automaticity in the domain of multiplication, subjects solved multiplication problems and gave descriptions of their solution procedures on each trial. Further, the number-matching task used in Experiment 1 was employed to test the automaticity of multiplication facts. If automaticity is single-step memory retrieval, as Logan suggested, then subjects' use of procedures on the production task should correspond with subjects' performance on the number-matching task. More specifically, subjects who frequently use single-step memory retrieval in the production task should provide evidence for automatic activation of multiplication facts in the number-matching task. In contrast, subjects who infrequently use single-step memory retrieval in the production task should not provide evidence for automatic activation of multiplication facts in the number-matching task.

Four possible patterns of results could occur in Experiment 2. First, subjects could frequently use single-step memory retrieval (i.e., automatic processing by Logan's definition) to solve the multiplication problems and experience interference on the number-matching task (i.e., frequent retrieval/automatic pattern). Second, subjects might infrequently use single-step memory retrieval (i.e., nonautomatic processing, in Logan's terms) to solve the multiplication problems and experience no interference on the number-matching task (i.e., infrequent retrieval/nonautomatic pattern). Third, subjects could frequently use single-step memory retrieval to solve the multiplication problems and experience no interference on the number-matching task (i.e., frequent retrieval/nonautomatic pattern). Fourth, subjects might infrequently use single-step memory retrieval to solve the multiplication problems and experience interference on the number-matching task (i.e., infrequent retrieval/automatic pattern). Note that there could be individual differences in the results (i.e., some subjects might fall into one pattern and other subjects into another pattern).

The first two possible patterns of results would support Logan's (1988, 1992) definition of automaticity. The latter two patterns, however, would be problematic. Logan's (1988, 1992) definition would be challenged if the frequent retrieval/nonautomatic pattern occurred because, under the strict interpretation of Logan's definition, the retrieval process is an automatic process. However, Logan could argue that these subjects were no different than the frequent retrieval/automatic subjects except in their ability to control the initiation of automatic processes. In Logan's theory, initiation of automatic processes is controlled. Logan could argue that certain individuals were able to control the initiation of automatic processes more effectively than other individuals. If this is plausible, then the frequent retrieval/nonautomatic result would not contradict Logan's definition of automaticity. The validity of this argument could be qualified because the experimental data that provided evidence for control of automatic processes involved overt behaviors, such as speaking and typing (e.g., Levelt, 1983; Logan, 1982). Hence, it may be more difficult to inhibit automatic processes that involve less overt behaviors like the behaviors in the number-matching task. Consequently, the potential difference between more and less overt behaviors could raise questions about the plausibility of Logan's possible explanation of the frequent retrieval/nonautomatic pattern.

In contrast, Logan (1988, 1992) could not easily provide an adequate explanation for an infrequent retrieval/automatic pattern for two reasons. First, Logan only considers retrieval as an automatic process. Therefore, Logan would predict that subjects who infrequently use retrieval to solve multiplication problems would show no evidence of automatic activation of multiplication facts.

Second, the infrequent retrieval/automatic pattern may reflect a different criterion level for retrieval in the production task compared to the criterion level for automaticity in the number-matching task. For example, suppose that in the production task the threshold level for retrieval of the answer to 6 X 4 is 1.0 (i.e., for retrieval to occur 24 must receive activation of 1.0 or greater) and in the number-matching task the threshold level for automatic activation that results in interference for the cue 6 X 4 is 0.7. The threshold level for retrieval in the production task would be higher than the threshold level in the number-matching task because in the production task the number (i.e., 24) must be retrieved to a higher level of cognitive awareness (i.e., subjects must articulate 24). Consequently, it is possible that when the subject was presented 6 X 4 in the number-matching task, 24 was activated to the level of 0.8. Hence, the subject experienced interference in the number-matching task. In contrast, when the subject was presented 6 X 4 in the production task, 24 was again activated to the level of 0.8. However, because the threshold for memory retrieval in the production task was 1.0, the answer to the problem was not retrieved. The hypothesis that different levels of automatic activation correspond to different outcomes goes beyond Logan's definition of automaticity because, according to Logan, automaticity and memory retrieval are indistinguishable.

Objectives of Experiment 2

There were three objectives of Experiment 2. The first objective was to test the applicability of Logan's definition of automaticity in the domain of multiplication in adults. The first objective was accomplished by having subjects solve multiplication problems, give descriptions of their solution procedures on each trial, and perform a number-matching task. The second objective was to provide an empirical basis for a new model of adult performance in multiplication.

The third objective was to determine whether masking the cue would alter the interference effect in the number-macthing task. A mask was added because the interference effects found in the number-matching task in previous research tended to be small (LeFevre et al., 1988; Thibodeau, 1992; Thibodeau & LeFevre, 1992). Masking the cue could moderate the automatic activation of multiplication facts or enhance the interference effect caused by automatic activation of multiplication facts in the number-matching task. I assumed the latter would occur. I believe that the steps involved in the number-matching task are as follows. First, upon presentation of the cue, a set of associated numbers are activated automatically (Ashcraft, 1983, 1987; Campbell & Graham, 1985; Campbell & Oliphant, 1992; LeFevre et al., 1988, 1991; Logan, 1988, 1992). The set of associated numbers would include the numbers in the cue and associated nodes (i.e., the product, sum, etc.). Upon presentation of the target, subjects must compare the target to the set of activated numbers. If the target is the product or sum of the cue and automatic activation activates associative nodes, then the subject experiences interference, which results in longer response times (LeFevre et al., 1988; Thibodeau & LeFevre, 1992).

Masking the cue could moderate the automatic activation of multiplication facts if the mask resulted in the cue not being encoded sufficiently. To avoid this problem, a pilot study was conducted to ensure that subjects could encode the cue. In the pilot study, subjects were presented the cue for 60 ms, followed by a mask (#######) for 40 ms. After an ISI of either 0, 20, 120, or 250 ms, a target was presented for 100 ms and then followed by a mask for 40 ms. Subjects were required to name the two numbers in the cue and the number in the target. Subjects were presented 10 practice trials and 50 test trials. Six subjects participated and the cue was named accurately on 82.6% of all trials. When the data from the worst subject was taken out (an accuracy score of 53.7%), the accuracy of the remaining five subjects was 89.6%. Further, 84.6% of the errors (22 out of 26) consisted of cues that contained a double-digit number. Note that in the number-matching task the critical trials (i.e., product and unrelated trials) never contain double-digit numbers in the cue. The accuracy of the six subjects on all trials with two single-digit numbers in the cue was 88.0% (with the worst subject taken out, the accuracy increases to 98.7%). Hence, it was concluded that subjects were able to encode the critical cues when the presentation time was 60 ms.

Masking the cue could enhance the interference effect by reducing the activation level of the numbers in the cue. Without the mask, the activation level of the numbers in the cue presumably would be higher than the activation level of associated numbers. Consequently, the cue numbers in the activated set would be more discriminable than the associated numbers. With a mask, the activation level of the numbers in the cue would presumably be substantially lower because presentation time of the cue has been reduced and activation would not benefit from visual persistence. Hence, the cue numbers in the activated set would be less discriminable from the associated numbers. This decrease in the discriminability could have resulted in more interference in the decision process, which could have resulted in longer response times for product trials.

Method

Subjects

Forty university students (20 males, 20 females) from an introductory psychology course participated. Data from eight subjects (3 males, 5 females) were not analyzed because their error rates were 30% or higher in two or more of the critical conditions in the number-matching task. Each subject received one experimental credit toward their introductory psychology course. The median age of the subjects was 18:11 (in years:months), with a range of 17:10 to 23:6.

Materials

In the number-matching task, the materials used in Experiment 2 were the same as in Experiment 1. In the production task, the problem set included 40 combinations of single-digit multiplication problems. Ten of the multiplication problems consisted of the product cues used in the number-matching task. The remaining thirty multiplication problems included 10 problems randomly selected from the 0 X 1 to 3 X 9, 10 problems randomly selected from 4 X 0 to 6 X 9, and 10 problems randomly selected from 7 X 0 to 9 X 9. The stimulus list was ordered unsystematically with the following restrictions: no operand or product was repeated on successive trials, and no problem's inverse (e.g., 4 X 2 and 2 X 4) appeared in the same half of the list. Two lists of the stimuli used, one the reverse order of the other. The order of the stimuli was counterbalanced with gender and task order across subjects.

Procedure

In the number-matching task, the procedure was the same as Experiment 1 with the following changes. The cue remained on the screen for 60 ms. Then the cue was replaced by a 40-ms mask consisting of seven octothorpes (i.e., #######). Following the mask, a blank screen was presented during inter-stimulus intervals (ISI) of 0, 20, 120, or 240 ms resulting in stimulus onset asynchronies (SOAs) of 100, 120, 220 and 350 ms. Then the target was presented. The SOAs of 100, 220 and 350 were used to replicate the SOAs employed in Experiment 1 and Thibodeau and LeFevre (1992). A SOA of 120 ms was employed to maximize the potential of finding an interference effect because from previous research it appears that the effect occurred around SOAs of 100 ms.

Before beginning the production task, subjects were told that the purpose of this task is to examine how adults solve simple multiplication problems. Examples of possibilities were provided:

> What do people do when asked to multiply 8 X 4? You could just retrieve the sole answer, 32. This is called single retrieval. You could also remember a number of different answers that you think are correct. This could be done in two different ways: sequentially or simultaneously. When this is done sequentially, numbers are retrieved one at a time. For example, 24 pops into your head then 36, then 32 and you determine the answer is 32. This is called sequential multiple retrieval. When it is simultaneous, a group of numbers are retrieved all at once. For example, 24, 36 and 32 would be retrieved upon presentation of 8 X 4. This is called simultaneous multiple retrieval. The difference between single retrieval and the multiple retrievals is the number of retrieved answers. In single retrieval you only retrieve one number and in multiple retrieval you retrieve more than one. The similarity between the single retrieval and the multiple retrievals is the fact that no calculations are done to arrive at the correct answer. You could use a derived facts. For example, you could remember that 8 X 5 equals 40, so 8 X 4 has to be 8 less. You could figure the answer out by multiple counting. For example, you could count by 8 four times; 8, 16, 24, 32. You could also figure the answer out by adding. You could add 8 + 8 equals 16 plus 8 equals 24 plus 8 equals 32. Or you could solve it in some other way. Subjects were told that each problem consisted of two parts.

First, subjects were to answer the problem as quickly and accurately as possible. Second, subjects were to describe how they solved the problems. Subjects' self-reports were prompted by a menu of procedures which appeared on the screen after each problem. On 10 of the 40 problems, subjects were asked to explain in detail why they had chosen the particular strategy. Subjects' protocols were noted by the experimenter. Ten practice trials preceded the test trials.

The multiplication problems were presented on a microcomputer. Subjects were required to provide answers for the presented multiplication problems. A voice-operated relay was interfaced with the microcomputer to record reaction times. Accuracy of the subject's responses was entered into the computer by the experimenter. Before each trial, an asterisk appeared on the screen. When the subject was ready he or she said "go" to begin the trial. Once "go" was said the asterisk was replaced be the problem and the timer started, following a variable delay. The subject's vocal response triggered the relay and the computer recorded the subject's latency for each trial. Experimental sessions lasted approximately 1 hour.

Results and Discussion

Task order did not effect performance on either task. Consequently, the data from both orders were analyzed together. The results from the number-matching task are described first, followed by the results from the production task. Finally, patterns of results from both tasks are discussed.

Number-Matching Task

Median correct latencies and percentage errors for the product and unrelated trials were analyzed in a 4 (SOA: 100, 120, 220, 350 ms) X 2 (problem size: large, small) X 2 (problem type: product, unrelated) analyses of variance, with repeated measures on both variables. Unless otherwise indicated, reported results were significant with an alpha level of .05. Data from matching trials and from nonmatching filler trials do not bear on the hypotheses and are not discussed further. Mean correct latencies and percentage errors for all conditions are presented in Appendix B.

Latencies. Latencies decreased with increasing SOA, $F(3,93) = 77.38$, $MSe = 3950$ (from 814 ms at the 100-ms SOA to 702 ms at the 350-ms SOA). There was a significant linear trend, $F(1,93) = 57.6$. The decline in latencies with SOA presumably occurs because at shorter SOAs the latency includes encoding and processing of the initial pair of digits. At longer SOAs, encoding and processing of the initial pair is completed before the target is presented and therefore, does not contribute to the latency.

Latencies also varied as a function of problem type (product vs. unrelated), $F(1,31) = 19.15$, $MSe = 44523$. Latencies on product trials were significantly slower than on unrelated trials (770 vs. 744 ms). This finding supports the hypothesis that cue activates a candidate set of associated nodes, including the product of the cue, which results in slower latencies for product trials than for unrelated trials.

The interaction of problem type and SOA approached significance, $F(3,93) = 2.51$, $MSe = 3561$, $p = .064$. Tests of simple effects indicated that latencies for product trials were significantly slower than latencies for unrelated trials at the 100-ms and 120-ms SOAs, $F(1,124) = 4.87$ and $F(1,12) = 21.06$, $MSe = 3802$. This finding supports the hypothesis that the cue automatically activates the product, which results in slower latencies for product trials than for unrelated trials at the 100-ms and 120-ms SOA.

There was no interaction between the problem type, SOA and problem size, $F(3,93) = 1.12$, $p = .345$. However, Thibodeau and LeFevre (1992) found a three-way interaction in which latencies for small-product trials were significantly slower than latencies for small-unrelated trials at the 100-ms and 350-ms SOAs. Further, latencies for large-product trials were not significantly slower than latencies for large-unrelated trials at any SOA. Hence, tests of simple effects were used to compare latencies for product and unrelated trials for all combinations of product size and SOA (see Figure 2).

If small-multiplication facts are automatically activated, then latencies for small-product trials would be slower than latencies for small-unrelated trials. At the 120-ms SOA, latencies for small-product were significantly slower than latencies for small-unrelated trials, $F(1,248) = 13.25$, $MSe = 3766$, and at the 220-ms SOA the difference between latencies for small-product trials and small-unrelated trials was marginally significant, $F(1,248) = 2.84$, $MSe = 3766$, $p < .10$. Thus, as in Experiment 1, evidence for automatic activation was found for small-multiplication facts. If large-multiplication facts are automatically activated, then latencies for large-product trials would be slower than latencies for large-unrelated trials. Tests of simple effects indicated that at the 100-ms and 120-ms SOAs, latencies for large-product trials were significantly slower than latencies for large-unrelated trials, $F(1,248) = 6.80$, and $F(1,248) = 8.29$, $MSe = 3766$. Thus, unlike Experiment 1, evidence for automatic activation was found for large-multiplication facts.

Insert Figure 2 about here

The interference effect found for large-product trials at the 100-ms and 120-ms SOAs in this experiment was not found in Experiment 1. A plausible explanation for the discrepancies between the results of the experiments is that the cue was masked in the present experiment. Masking the cue may have reduced the activation level of the numbers in the cue, which would result in less discriminability between the numbers in the cue and the automatically activated associated numbers (i.e. products) in the subjects' activated set. This decrease in discriminability could have resulted in more interference in the decision process for the large-product trials, which could have resulted in longer response times for large-product trials.

The data from the present experiment are consistent with the view that masking the cue increased latencies for large-product trials and enhanced the interference effect caused by the automatic activation of multiplication facts. Latencies for large-products trials in the present experiment increased 65 ms at the 100-ms SOA, 15 ms at the 220-ms SOA, and 17 ms at the 350-ms SOA compared to Experiment 1. Further, by comparing the data from both experiments, it appears that the mask enhanced the interference effect by 27 ms for large trials at the 100-ms SOA (12 ms vs. 39 ms). Note that no comparison can be made for large trials at the 120-ms SOA because the 120-ms SOA was not employed in Experiment 1. Consequently, the interference effect found for large-product trials at the 100-ms and 120-ms SOAs in the present experiment could be explained by the fact that the cue was masked.

Errors. Errors were low, averaging 3.5% across subjects. The analysis of errors should be interpreted cautiously because the low overall error rate suggests a floor effect. Error rates decreased with increasing SOA, $F(3,93) = 3.90$, $MSe = 49.72$ (from 5.16% at the 100-ms SOA to 2.27% at the 350-ms SOA).

Error rates also varied as a function of problem type (product vs. unrelated), $F(1,31) = 8.43$, $MSe = 75.30$. Errors rates on product trials were significantly higher than on unrelated trials (4.57 vs. 2.30%). This finding supports the hypothesis that the cue activates a candidate set of associated nodes, including the product of the cue, which results in a higher error rate for product trials than for unrelated trials.

These main effects were qualified by the three-way interaction of problem type, SOA, and problem size, $F(3,93) = 2.79$, $MSe = 20.04$ (see Figure 3). If multiplication facts are automatically activated, then error rates for product trials should be higher than error rates for unrelated trials. Tests of simple effects indicated that error rates for small-product trials were significantly higher than for small-unrelated trials at the 120-ms and

220-ms SOAs, $F(1,248) = 4.26$, and $F(1,248) = 5.25$, $MSe = 29.74$. Thus, consistent with the latency data, evidence for automatic activation was found for small-multiplication facts at the 120-ms and 220-ms SOAs. Test of simple effects indicated that error rates for large-product trials were significantly higher than for large-unrelated trials at the 100-ms and 120-ms SOAs, $F(1,248) = 7.56$, and $F(1,248) = 8.88$, $MSe = 29.74$. Thus, consistent with the latency data, evidence for automatic activation was found for large-multiplication facts at the 100-ms and 120-ms SOA.

Insert Figure 3 about here

Finding evidence for automatic activation in the error data in this experiment was nc ound in Experiment 1. This discrepancy may be due to the use of a mask in the present experiment. With a mask the activation level of the numbers in the cue presumably would be substantially lower than the numbers in an unmasked cue. Consequently, with a mask, cue numbers in the activated set would be less discriminable than the associated numbers (e.g., the product of the cue). This decrease in discriminability could have resulted in more interference in the decision process, which could have resulted in a higher error rate for product trials.

The data from the present experiment are consistent with the view that masking the cue resulted in a higher error rate for product trials. The error rate for all product trials in the present experiment was significantly higher than the error rate for all product trials in Experiment 1 (4.57 vs. 2.34%; $t(62) = 2.56$, $p < .01$). Further, the error rate for all unrelated trials in the present experiment was not significantly higher than the error rate for all unrelated trials in Experiment 1 (2.34 vs. 1.64, $t(62) = 0.98$).

Summary. The results of the number-matching task were consistent with the view that multiplication facts are automatically activated (Campbell & Graham, 1985; Campbell, 1987a; Koshmider & Ashcraft, 1991; Yu, 1990; Zbrodoff & Logan, 1986). This view was supported by the fact that latencies for small-product trials were significantly slower than latencies for small-unrelated trials at the 120-ms SOA and approaching significance at the 220-ms SOA. Further, error rates for small-product trials were significantly higher than error rates for small-unrelated trials at the 120-ms and 220-ms SOAs. Latencies for large-product trials were significantly slower than latencies for large-unrelated trials at the 100-ms and 120-ms SOAs. Further, error rates for large-product trials were significantly higher than error rates for large-unrelated trials at the 100-ms and 120-ms SOAs.

Results for small-product trials from the present experiment did not completely replicate the results from Experiment 1. Specifically, interference effects were found at the 100-ms SOA for small-product trials in Experiment 1 and at the 120-ms SOA for small-product trials in Experiment 2. The onset discrepancy of the interference effect between the experiments does not appear to have a straightforward explanation. It could be argued that the masked cue in the present experiment resulted in a delay of the interference effect. The consistent result for small-product trials between the two experiments was that the interference effect was absence at the 350-ms SOA. Consequently, from the results of the two experiments it can be concluded that small multiplication facts are automatically activated at the 100-ms SOA (i.e., without a mask) and at the 120-ms SOAs (i.e., with a mask), and that automatic activation declines at the 350-ms SOA (i.e., with or without a mask). This time course of automatic activation is consistent with current research. Zbrodoff and Logan (1986) have suggested that multiplication facts are activated more slowly than additions facts. LeFevre et al. (1991) found that automatic activation of addition facts occurred with SOAs of under 100 ms and that automatic activation declines at the SOA of 120 ms (see LeFevre et al., 1988). Note that from the results of the present experiments, it can be concluded that with a masked cue, large multiplication facts are automatically activated at the 100-ms SOA and that automatic activation declines at the 220-ms SOA. This time course of automatic activation is also consistent with current research.

An interesting discrepancy in the results of Experiment 2 was that the interference effect was found earlier for large-product trials (i.e., at the 100-ms SOA) compared to small-product trials (i.e., at the 120-ms SOA). The distance effect phenomenon may be a plausible explanation for the discrepancy. The distance effect refers to the phenomenon that it takes more time to evaluate numbers that are numerically close on the mental number-line compared to numbers that are numerically distant on the mental number-line (LeFevre et al., 1988, 1991). At the 100-ms SOA, latencies for small-product trials and large-product trials did not differ (824 vs. 828 ms). However, at the 100-ms SOA, latencies for small-unrelated trials were slower than latencies for large-unrelated trials (816 vs. 789 ms). Note that the numbers used in the small-unrelated trials were numerically close compared to the numbers used in the large-unrelated trials. Hence, the absence of the interference effect at the 100-ms SOA for small-product trials may have stemmed from a distance effect that resulted in inflated latencies for small-unrelated trials. Note that this explanation is problematic because it would be expected that the distance effect should also have caused the latencies for small-product trials to be longer than the latencies for large-product trials at the 100-ms SOA.

The fact that evidence was found for automatic activation of large multiplication facts in this present experiment should not affect the interpretation of the results of Experiment 1. Instead, the present results could be seen as offering further evidence for the interpretation of the results of Experiment 1. Recall that evidence for automatic activation was found for small multiplication facts, whereas no evidence for automatic activation was found for large multiplication facts in Experiment 1. These results were interpreted as supporting evidence for the associative models assumption that the problem-size effect reflects differences in accessibility of specific facts in the network. More specifically, answers to small multiplication problems are more accessible than answers to large multiplication problems, which results in quicker solution times and greater accuracy for small multiplication problems.

The results of the present experiment could be seen as additional evidence for the interpretation of the results of Experiment 1 if the methods and results of the two experiments are compared. Recall that both experiments used the same number-matching task and stimulus list. In the present experiment the cue was masked, whereas the cue was unmasked in Experiment 1. I have previously suggested that masking the cue may have reduced the activation level of the numbers in the cue, which would result in less discriminability between the numbers in the cue and the automatically activated associated numbers (i.e. products) in the subjects' activated set. This decrease in discriminability could have resulted in more interference in the decision process. The results of the two experiments implies that for the interference effect to be significant for the large multiplication problems, the activation level of the numbers in the cue must be reduced. This suggest that when the cue is unmasked, as in Experiment 1, large multiplication facts are not activated enough to create interference in the decision process of subjects. In contrast, when the cue is unmasked, small multiplication facts were accessible enough that the activation level of products was sufficient to cause interference in the decision process of subjects. Therefore, the fact that evidence was found for automatic activation of small-multiplication facts in both experiments, whereas evidence for automatic activation was only found for large multiplication facts in the present experiment, suggests that large multiplication facts are not as highly accessible as small-multiplication facts.

Production Task

The purpose of the production task was to assess the procedures that adults use to solve simple multiplication problems. The main issues concerning self-reports are (a) whether the self-reports accurately reflect procedures subjects used to solve the problems and (b) whether the verbalization resulted in any changes in the normal processing involved in solving the problem (Russo, Johnson, & Stephens, 1989). These two issues are termed veridicality and reactivity respectively. It is essential to deal with the issues of veridicality and reactivity because attempting to make inferences about the cognitive processes based on nonveridical self-reports or processes that were reactive to the verbalization of the self-reports is inappropriate. In the following section, an overview of what was found will be presented first. Then the issues of veridicality and reactivity will be discussed.

Overview. Error rates were low, averaging 3.75% across subjects. Trials involving failure of the voice-operated relay accounted for another 1.05% of trials. Mean solution time for all correct trials was 1.2 seconds.

In the present experiment, subjects reported using three types of retrieval (single retrieval, simultaneous multiple retrieval, and sequential multiple retrieval) and four main algorithmic procedures (derived facts, multiple counting, adding, and rule based) to solve simple multiplication problems (Table 1). Subjects reported that single retrieval involved the retrieval of a sole answer from memory with no use of an algorithm. Subjects reported that simultaneous multiple retrieval required retrieval of a group of numbers from memory all at once. Subjects reported serial multiple retrieval involved retrieval of a group of numbers from memory one at a time. Note that both multiple retrieval procedures were reported as accomplished without the use of an algorithm procedure. The derived-fact procedure involved using an intermediate fact to derive the desired answer. For example, a subject would report solving 8 X 4 by using the fact 8 X 5 equals 40 and then subtracting 8 to determine the desired answer of 32. Multiple counting involved counting by increments of one of the operands. For example, a subject would report counting by 8 four times (8, 16, 24, 32) to solve the problem 8 X 4. Adding involved continually adding one of the operands until the problem was solved. For example, a subject would report adding 8 + 8 equals 16 plus 8 equals 24 plus 8 equals 32 to solve the problem 8 X 4. Rule-based procedures involved the use of unbreakable rules in multiplication. For example, subjects reported using _a_ X 0 equals 0 or _b_ X 1 equals _b_.

Insert Table 1 about here

Similar to children (Siegler, 1988), adults used multiple procedures to solve simple multiplication problems: 97% of all subjects reported using two or more procedures, 82% reported using three or more, 56% reported using four or more and 31% reported using five or more. The diversity of procedure used in adult multiplication found in the present study contradicts the belief that adults solely rely on retrieval to solve simple arithmetic problems (Ashcraft, 1982, 1992; Siegler, 1987). Across all subjects, single retrieval was the most frequent procedure employed (73%) and the derived-fact procedure was the most frequently reported algorithmic procedure (8%).

Solution times varied with procedure (Table 1). Across all subjects, single retrieval, multiple counting and rule-based procedures were among the fastest procedures. These were followed by adding, simultaneous multiple retrieval, sequential multiple retrieval, derived facts and finally by other (i.e., guessing, using fingers, etc.).

There was a large discrepancy in accuracy between the single retrieval (98%) and the two multiple retrieval procedures (89%). Further, procedures categorized as "other" (i.e., guessing, using fingers, etc.) were considerably less accurate (60%) than all other procedures reported.

Veridicality. Self-reports are veridical when they accurately reflect procedures subjects used to solve the problems. Nonveridical self-reports would reflect errors of omission or comission in the reporting of procedures. To determine the veridicality of the self-reports of retrieval in the present study, mean solution times and consistency with previous research were examined. To determine the veridicality of the self-reports of algorithmic procedures, mean solution times and the type of problems that elicited the algorithmic procedure were examined. Note that there is no standard test of veridicality of self-reports (Russo et al., 1989).

To determine whether the three types of retrieval reported (single retrieval, simultaneous multiple retrieval and sequential multiple retrieval) were veridical, solution times were compared. Recall that the three types of retrieval involved no algorithmic procedures. Further, the single retrieval involved retrieval of one number, whereas multiple retrieval involved retrieval of two or more numbers. It is plausible that retrieving two or more numbers when attempting to solve a multiplication problem may result in the solution time being longer than retrieving one number because the retrieval of two or more numbers would require the selection of the correct answer. Consequently, mean solution time for single retrieval was expected to be faster than mean solution times of the multiple retrievals. To test this hypothesis, mean solution time for single retrieval was compared to the two multiple retrievals. To avoid unstable estimates of solution times, mean solution times were computed for subjects who had used all three types of retrieval on at least two problems. As expected, mean solution time for single retrieval (990 ms) was significantly faster than simultaneous multiple retrieval (1488 ms), $t(190) = 5.19$, and than sequential multiple retrieval (1622 ms), $t(185) = 6.76$. Mean solution time and accuracy rate for simultaneous multiple retrieval was not significantly different than mean solution time and accuracy rate for sequential multiple retrieval. The implications and examination of the multiple-retrieval strategies will be discussed further in the General Discussion.

The cumulative relative frequency of the three types of retrieval are consistent with previous research. Bisanz et al. (1993) employed a multiplication production task and verbal protocol task with adult subjects. The procedure description in Bisanz et al. did not discriminate between the single and multiple retrieval. Consequently, it would be expected that the cumulative relative frequency of the three types of retrieval in the present study should be consistent with the percent frequency of the retrieval reported in Bisanz et al. The expected consistency between studies was found; in the present experiment the relative frequency of the three types of retrieval totalled 84%, and the frequency of retrieval in Bisanz et al. was 85%. Consequently, from the mean solution times and consistency with previous research, it was concluded that subjects' self-reports of retrieval were veridical in the present experiment. To determine the veridicality of the self-reports of algorithmic procedures, mean solution times and the type of problems which elicited the algorithmic procedure were examined.

The derived-fact procedure involved using an intermediate fact to derive the desired answer (e.g., solving 5 X 4 by using the fact 4 X 4 equals 16 and then adding 4 to determine the desired answer of 20). Because the derived-fact procedure involves the retrieval of a reliable intermediate fact, it could be expected that the intermediate facts would be tie facts (e.g., 4 X 4 = 16). This assumption is made because tie problems are answered more accurately and quickly than non-tie problems (Campbell & Graham, 1985; Miller et al., 1984). Consequently, it was expected that a high percentage of multiplication problems that elicited the derived-fact procedure would contained operands which differ by 1 (e.g., 5 X 4). However, only a small percentage (29% or 7 out of 24 problems) of the multiplication problems on which subjects' reported using a derived-fact procedure contained operands which differed by 1. Further, these 7 problems which differed by 1 only accounted for 27% of the reported use of the derived-fact procedure.

Of the remaining 17 problems on which subjects reported using a derived-fact procedure, 14 (82%) were classified as "difficult" multiplication problems according to Campbell (1987b), based on Campbell and Graham's (1985, Appendix B) normative production data for adults. Presumably a problem is difficult because the individual cannot readily retrieve the answer from memory. Hence, it would be expected that on these "difficult" problems, subjects would have to rely on an algorithmic procedure (e.g., derived fact) to produce the answer. It is also informative to note that Campbell classified multiplication problems as "difficult" or "easy" based on mean solution times (i.e., the slowest problems were classified as "difficult"). Because Campbell and Graham did not record procedure use, the reason why "difficult" multiplication problems possessed lengthy solution times was unclear. However, from the present results, it could be proposed that the length of the solution times for "difficult" multiplication problems was the result of some subjects using the derived-fact procedure to solve the "difficult" problems. The present experiment supports the hypothesis: The overall mean solution time of problems solved by the derived-fact procedure was longer than any of the procedures reported (2175 ms).

For the rule-based procedure, 100% of the reported uses occurred on multiplication problems which contained 0 or 1 as an operand. Recall that a rule-based procedure involved the use of characteristic rules of multiplication (i.e., a X 0 equals 0 or b X 1 equals b). Further, mean solution time for the problems reportedly solved by the rule-based procedure was consistent with the belief that the solution times rule-based procedures are fast (Baroody, 1985, 1987). In the present experiment, mean solution time for rule-based procedures was the fastest of all procedures (1050 ms).

For the multiple counting procedure, 87% of the reported uses occurred on multiplication problems that contained 2 or 3 as an operands. Recall that a multiple counting procedure involved counting by increments of one of the operands (e.g., count by 5 two times (5, 10) to solve the problem 2 X 5). Because the majority of the multiplication problems contained 2 or 3 as an operand, mean solution time for using the multiple counting procedure should be relatively fast because the number of counts would be small. In the present experiment, mean solution time for the multiple counting procedure was the second fastest (1080 ms) of all procedures.

Similar to the multiple counting procedure, 90% of the reported uses of the adding procedure occurred on multiplication problems which contained 2 or 3 as an operand. Recall that the adding procedure involved continually adding one of the operands until the problem was solved (e.g., adding 5 + 5 equals 10 plus 5 equals 15 to solve the problem 5 X 3). The main discrepancy between the multiple counting and adding procedures is that the latter involved more elaborate steps, which should result in a longer mean solution time for the adding procedure versus the multiple counting procedure. To test this hypothesis, mean solution time for the adding procedure was compared to the multiple counting procedure. To avoid unstable estimates of solution times, mean solution times were derived from subjects who had used both the adding procedure and the multiple counting procedure. As expected, mean solution time for the multiple counting procedure (1011 ms) was significantly faster than the adding procedure (1389 ms), t(23) = 1.86. Consequently, from the mean solution times and the type of pr ems which elicited an algorithmic procedure, it was concluded that subjects' self-reports of the algorithmic strategies were veridical in the present experiment.

Reactivity. Reactivity occurs when the verbalization of the self-reports changes the normal processing involved in solving the problem. If the normal processes were reactive to the verbalization, then making inferences about the cognitive processes involved in solving multiplication problems is inappropriate. The change in normal processing, caused by reactivity, can result in differences in solution time and/or accuracy compared to a silent control group (i.e., the same task done with no self-reports). Note that prolonging mean solution time is seldom seen as evidence for the altering of normal processes because it cannot be determined whether the normal processes were altered or whether the normal processes were simply slowed down (Payne, Braunstein, & Carroll, 1978; Russo et al., 1989). To determine whether the self-reports affected the error rate or mean solution time, the results of the present experiment are compared to previous research that did not employ self-reports.

In Miller et al. (1984), subjects were presented the 100 combinations of the single-digit multiplication problems. From their results, mean solution time for the 40 problems (calculated from Table A1 of Miller et al.) used in the present experiment was 0.8 seconds. Recall that in the present experiment, mean solution time was 1.2 seconds. Consequently, there is a discrepancy of 0.4 seconds between the mean solution times for the 40 problems of the present experiment and Miller et al. In Campbell and Graham (1985), subjects were presented all the problems from 2 X 2 to 9 X 9 with problems containing 0 or 1 as operands excluded. In the present experiment, nine problems contained 0 or 1 as operands. Hence, only 31 of the problems from the present experiment can be compared to results reported by Campbell and Graham. Of these 31 problems, the mean solution time was 1.3 seconds in the present experiment and 0.8 seconds in Campbell and Graham's study (calculated from Appendix A of Campbell and Graham). Again there is a discrepancy between the mean solution times of the present experiment and previous research. However, as previously mentioned, prolongation of mean solution time is not conclusive in determining whether the normal processes were altered by the verbalization because it cannot be determined whether the normal processes were altered or whether the normal processes were simply slowed down (Payne et al., 1978; Russo et al., 1989).

The error rate for subjects' performance on the multiplication problems in Miller et al.'s (1984) study was not reported. In Campbell and Graham (1985), error rate on the multiplication problems was 7.65%. On the 31 problems used in both experiments, the error rate was 4.5% in the present experiment and 7.5% in Campbell and Graham's study (calculated from Appendix A of Campbell and Graham). The initial interpretation of the lower error rate in the present experiment could be that the self-reports altered the normal processes involved in solving multiplication problems (i.e., reactivity). The lower error rate found in the present experiment could be attributed to a methodological discrepancy between the present experiment and Campbell and Graham's study. In present study, presentation of the multiplication problems was self-paced. In Campbell and Graham's study, problems were computer paced with a 5-second interval between the end of one trial and the beginning of the next. It is plausible that the self-pacing done in the present experiment affected the error rate. It could be argued that subjects in the present experiment were better prepared for each problem (i.e., subjects could initiate the next problem when they felt completely ready) compared to subjects in Campbell and Graham's study. Consequently, the lower error rate in the present experiment could have been the result of the subjects in the present study being able to control presentation rate.

Comparisons of mean solution times and error rates across studies may reflect a speed-accuracy tradeoff: Subjects in the present experiment were slower but more accurate than subjects in the study by Campbell and Graham (1985). Such a tradeoff might imply that the normal processes involved in solving multiplication problems were influenced by the requirement of self-report. If normal processes were altered, then direct measures of those processes should differ. In previous research on multiplication in adults, variables such as prod (the product of the operands) were assumed to reflect the time required for retrieval processes and were used to account for differences across problems in solution latency. Across several studies (Campbell & Graham, 1985; Geary, Widaman, & Little, 1986; Miller et al., 1984), the amount of variance accounted for by prod ranged from 60% to 72%. In the present study, prod accounted for 72% of the variance. The consistency of these values across studies would not be expected if the requirement of self-reports changed the normal processes involved in solving multiplication problems.

Summary. Based on the self-reports, subjects reported using seven main procedures to solve simple multiplication problems: single retrieval, sequential multiple retrieval, simultaneous multiple retrieval, derived facts, multiple counting, adding, and rule-based procedures. Of the seven procedures, single retrieval was the most frequently used procedure. Of the algorithmic procedures, the derived fact procedure was the most frequently used procedure.

The purpose of the production task was to access the procedures that adults use to solve simple multiplication problems. Consequently, it was imperative to provide evidence that the procedures reported were veridical. From the mean solution times and consistency with previous research, it was concluded that subjects' self-reports of retrieval were veridical. From the mean solution times and by analyzing what types of problems elicited the algorithmic procedure, it was concluded that subjects' self-reports of the algorithmic procedures were veridical.

Based on previous research on adult simple multiplication (Geary et al., 1986; Miller et al., 1984) the present experiment has replicated the result that prod is the best predictor of the solution times for adult multiplication. Consequently, it was concluded that the verbalization of the self-reports did not change the normal processing involved in solving the multiplication problems.

From the evaluation of the data from the production task, it can be concluded that categorizing subjects into groups based on their procedure use appears to be legitimate. This approach enables data from the number-matching task to be analyzed with subjects categorized as frequent retrievers and infrequent retrievers. These analyses can be used to test the applicability of Logan's (1988, 1992) definition of automaticity in the domain of multiplication in adults.

Number-Matching Task and Production Task

One of the objectives of the present experiment was to test the applicability of Logan's (1988, 1992) definition of automaticity in the domain of multiplication in adults. To test the applicability of Logan's definition, the pattern of results between subjects' procedure use on the multiplication production task and subjects' performance on the number-matching task were analyzed. Recall that Logan defined automaticity as single-step, direct-access memory retrieval. In Logan's elaboration of his definition of automaticity, he stated that performance is automatic when individuals respond "with a solution from memory on every trial and abandon the algorithm entirely" (emphasis added; 1988, p. 493). Consequently, Logan appears to have proposed that an automatic process is one that involves only single-step, direct-access memory retrieval to obtain a single solution.

In the production task employed in this experiment, subjects reported using single retrieval, sequential multiple retrieval, simultaneous multiple retrieval, derived facts, multiple counting, adding, and rule-based procedures to solve simple multiplication problems. Derived facts, multiple counting, adding, and rule-based procedures would not be considered automatic processes under Logan's (1988, 1992) definition of automaticity because these procedures are algorithms. Sequential multiple retrieval and simultaneous multiple retrieval could not be considered automatic processes under the strict interpretation of Logan's definition of automaticity because these retrievals result in two or more solutions. Hence, the only procedure that clearly fits Logan's definition of an automatic process would be single retrieval. Single retrieval is defined as a retrieval process that results in one solution.

There are four possible patterns of results that can occur in the present experiment between the results of the number-matching task and production task. First, subjects might frequently use single retrieval to solve the multiplication problems and experience interference on the number-matching task (i.e., frequent single retrievers/automatic pattern). Second, subjects might infrequently use single retrieval to solve the multiplication problems and experience no interference on the number-matching task (i.e., infrequent single retrievers/nonautomatic pattern). Third, subjects might frequently use single retrieval to solve the multiplication problems and experience no interference on the number-matching task (i.e., frequent single retrievers/nonautomatic pattern). Fourth, subjects might infrequently use single retrieval to solve the multiplication problems and experience interference on the number-matching task (i.e., infrequent single retrievers/automatic pattern). The first two possible patterns of results would be consistent with Logan's (1988, 1992) definition of automaticity. The latter two patterns of results would be problematic for Logan's definition of automaticity.

Frequent single retrievers versus infrequent single retrievers based on all 40 problems in the production task. Subjects were separated into Frequent Single Retrievers (FSR) and Infrequent Single Retrievers (ISR) according to the relative frequency with which the single retrieval procedure was used in all 40 problems in the production task. A subject was considered a FSR if his or her relative frequency use of the single retrieval procedure was greater than or equal to 80%, and an ISR if his or her relative frequency use of the single retrieval procedure was less than or equal to 60%. These cutoffs resulted in 14 subjects being categorized as FSR and 10 subjects being categorized as ISR. Note that other cutoffs were used to categorize subjects and the pattern of results found were consistent with the present reported results. To determine the patterns of results and test the applicability of Logan's (1988, 1992) definition of automaticity in the domain of adult multiplication, median correct latencies for the product and unrelated trials were analyzed in a 2 (group: FSR, ISR) X 4 (SOA: 100, 120, 220, 350 ms) X 2 (problem size: large, small) X 2 (problem type: product, unrelated) analyses of variance with repeated measures on the last three factors. To avoid redundancy with the previous analyses, only effects involving groups are reported.

The difference between latencies for FSR (786 ms) and ISR (692 ms) approached significance, $F(1,22) = 3.27$, $p = .08$. This finding could be the result of FSR having stronger associations between problems and answers than ISR. This discrepancy in activation levels is plausible because FSR used single retrieval more often than ISR. Upon presentation of the cue in the number-matching task, subjects activate a set of possible targets that includes the actual numbers presented (e.g., 4, 5), as well as associated nodes (e.g., 9, 20). Activation of associated nodes makes the decision about the target more difficult, which slows response times. It is plausible that the activation set of associated nodes for FSR consisted of highly activated numbers which resulted in their increased response time.

To determine the pattern of results between procedure use and performance on the number-matching task, tests of simple effects were used to compare the small- and large-product trials to unrelated trials at each SOA for each retrieval group (see Figure 4). If small- or large-multiplication facts are automatically activated, then latencies for small- and large-product trials would be slower than their respective latencies for unrelated trials. For the FSR, tests of simple effects indicated that at the 100-ms SOA, latencies for large-product trials were significantly slower than latencies for large-unrelated trials, $\underline{F}(1,352) = 11.34$. At the 120-ms SOA, difference between latencies for small-product trials and small-unrelated trials was substantial (42 ms) but not significant, $\underline{F}(1,352) = 3.37$, $\underline{MS}e = 3585$, $\underline{p} < .10$, perhaps due to reduced power because of the relatively small set of subjects in these analyses. Thus, FSR experienced some interference on the number-matching task at brief SOAs. Consequently, the frequent single retrievers/automatic pattern was found. As previously mentioned, the frequent single retrievers/automatic pattern is consistent with Logan's (1988, 1992) definition of automaticity.

Insert Figure 4 about here

For ISR, tests of simple effects indicated that at the 120-ms SOA, latencies for large-product trials were significantly slower than latencies for large-unrelated trials, $\underline{F}(1,352) = 4.79$, and at the 120-ms and 220-ms SOAs, latencies for small-product trials were significantly slower than latencies for small-unrelated trials, $\underline{F}(1,352) = 8.70$, and $\underline{F}(1,352) = 5.59$, $\underline{MS}e = 3585$. Thus, it could be concluded that ISR also experienced some interference on the number-matching task at brief SOAs. Hence, the infrequent single retrievers/automatic pattern was found. This pattern challenges Logan's definition of automaticity for two reasons. First, the only procedure that clearly fits Logan's definition of an automatic process is single retrieval. Consequently, according to the most straightforward intrepretation of Logan's definition, subjects who are categorized as ISR should not show evidence of automaticity. Second, the infrequent single retrievers/automatic pattern implies that the activation level required for single retrieval is different and/or higher than the activation level required to cause interference in the number-matching task. The different levels of activation corresponding to different outcomes goes beyond Logan's definition of automaticity. Logan implies that automaticity and single retrieval are indistinguishable. An infrequent single retrieval/automatic pattern would imply that automaticity and single retrieval are separable. More specifically, there could be a different criterion level for single retrieval in the production task compared to the criterion level for automaticity in the number-matching task. The implications of these findings are outlined in the General Discussion.

Two arguments could be put forth that suggest that the previous data analysis was not a direct or stringent test of the applicability of Logan's (1988, 1992) definition of automaticity in the domain of adult multiplication. First, it could be argued that the cutoff levels used to categorize subjects as FSR or ISR were too liberal. More specifically, using a cutoff level of 80% or higher for FSR may be considered too low and using a cutoff level of 60% or less for ISR may be considered too high. Second, categorizing subjects based on all 40 multiplication problems in the production task is problematic if one of the goals of the research is to test the applicability of Logan's definition of automaticity. Logan's definition of automaticity is part of an instance theory which emphasizes that each problem is different based on previous exposures. Hence, a more appropriate test of the applicability of Logan's definition would be to compare the data of the subjects after they were categorized in groups based on their procedure use on the 10 critical products trials which were used in the production task and the number-matching task.

FSR and ISR based on the 10 critical problems in the production task. Subjects were separated into FSR and ISR according to the relative frequency with which the single retrieval procedure was used on the 10 critical problems in the production task. Note that the reported use of the different procedures for the ten critical problems was no different than the reported use of the different procedures for all 40 problems. In both cases, retrieval accounted for 84% (73% was single retrieval and 11% were the multiple retrievals) of the relative frequency use. A subject was considered a FSR if the subjects relative frequency use of the single retrieval procedure was 100%, and a ISR if the subjects relative frequency use of the single-retrieval procedure was less than or equal to 40%. Note that the cutoff levels of 100% and 40% were more conservative compared to the previous cutoff levels of 80% and 60%. These cutoffs resulted in 10 subjects being categorized as FSR and 7 subjects being categorized as ISR. Note that other cutoffs were used to categorize subjects and the pattern of results found were consistent with the present reported results. Median correct latencies for the product and unrelated trials were analyzed in a 2 (group: FSR, ISR) X 4 (SOA: 100, 120, 220, 350 ms) X 2 (problem size: large, small) X 2 (problem type: product, unrelated) analyses of variance, with repeated measures on the last three factors.

The difference between latencies for FSR (769 ms) and ISR (699 ms) approached significance, $F(1,15) = 2.23$, $p = .16$. As previously mentioned, this finding could be the result of FSR having stronger associations between problems and answers than ISR.

To determine the pattern of results between procedure use and performance on the number-matching task, tests of simple effects were used to compare the small- and large-product trials to unrelated trials at each SOA for each retrieval group (see Figure 5). For the FSR, tests of simple effects indicated that at the 100-ms and 120-ms SOAs, latencies for large-product trials were significantly slower than latencies for large-unrelated trials, $F(1,240) = 5.08$, and $F(1,240) = 4.45$, and at the 120-ms SOA, latencies for small-product trials were significantly slower than latencies for small-unrelated trials, $F(1,240) = 6.38$, $MSe = 4117$. Thus, the frequent single retrievers/automatic pattern was found.

Insert Figure 5 about here

For ISR, tests of simple effects indicated that at the 120-ms SOA, latencies for large-product trials were significantly slower than latencies for large-unrelated trials, $F(1,240) = 4.11$, and at the 120-ms and 220-ms SOAs, latencies for small-product trials were significantly slower than latencies for small-unrelated trials, $F(1,240) = 4.89$, and $F(1,240) = 4.27$, $MSe = 4117$. Thus, the infrequent single retrievers/automatic pattern was found. As previously discussed, the infrequent single retrievers/automatic pattern challenges Logan's (1988, 1992) definition of automaticity.

To further add to the finding that ISR experienced interference (i.e., automaticity of multiplication facts) on the number-matching task, the median correct latencies from the only subject who did not use the single retrieval to solve any of the five critical large problems, and from the only subject who did not use the single retrieval to solve of the five critical small problems, were examined. From the latencies on the number-matching task, it was obvious that the subject who did not use single retrieval on any of the large problems in the production task experienced interference. At the 120-ms SOA, response time for the large product trials took 311 ms longer than the response times for the large unrelated trials. From the latencies on the number-matching task, it was obvious that the subject who did not use single retrieval on any of the small problems in the production task also experienced interference. At the 120-, 220- and 350-ms SOAs, response times for the small product trials took 221, 149, and 130 ms longer than the response times for the small unrelated trials at the respective SOAs. Consequently, in the most stringent test of the applicability of Logan's (1988, 1992) definition of automaticity (i.e., subjects with 0% use of single retrieval), a nonsingle retrievers/automatic pattern was found. Therefore, this results, joined with the previous analysis, suggests that Logan's definition of automaticity may not be adequate for the domain of adult multiplication.

A plausible argument could be put forth suggesting that only equating single retrieval with automaticity is too severe for the applicability of Logan's (1988, 1992) definition. When researchers have discussed the use of retrieval, different types of retrieval have never been mentioned (at least in research on mental arithmetic, see Ashcraft, 1992). Consequently, when Logan defined automaticity as memory retrieval, he most likely was not concerned with different types of retrieval. Because Logan never explicitly stated the nature of retrieval when he defined automaticity as memory retrieval, he could argue that any form of retrieval (i.e., single or multiple) is automatic.

Because all the previous tests of the applicability of Logan's definition of automaticity categorized subjects by their use of single retrieval, the conclusions could be questioned by the argument that all retrievals are considered automatic. Consequently, subjects were separated into retrievers and infrequent retrievers based on the frequency with which they use the three retrieval procedures on the 10 critical problems in the production task. A subject was considered a retriever if their frequency use of the three retrieval procedures was equal to 100% and an infrequent retriever if their frequency use of the three retrieval procedure was equal to or less than 60%. These cutoffs resulted in 16 subjects categorized as retrievers and 7 subjects categorized as infrequent retrievers. Note that the frequency use of the three retrieval procedures was equal to or less than 50% for 5 out of 7 infrequent retrievers. The median correct latencies for the product and unrelated trials were analyzed in a 2 (group: retrievers, infrequent retrievers) X 4 (SOA: 100, 120, 220, 350 ms) X 2 (problem size: large, small) X 2 (problem type: product, unrelated) analyses of variance, with repeated measures on the last three factors. The patterns of results between procedure use and performance on the number-matching task of the present ANOVA were consistent with the patterns of results found when subjects were categorized by their frequency use of single retrieval: The retrievers/automatic and infrequent retrievers/automatic patterns were found.

Summary. Logan (1988, 1992) defined automaticity as single-step, direct access memory retrieval. Under strict interpretation of Logan's definition of automaticity, automaticity is a retrieval process that results in a single solution and the process does not involve any algorithms. Furthermore, Logan's definition of automaticity implies that automaticity and memory retrieval are indistinguishable.

There were two consistent patterns in all the different analyses of the number-matching data with the subjects categorized into groups based on the procedure use: the frequent single retrievers/automatic pattern and the infrequent single retrievers/automatic pattern. The frequent single retrievers/automatic pattern supports Logan's (1988, 1992) definition of automaticity, whereas the infrequent single retrievers/automatic pattern challenges Logan's definition for two reasons. First, according to the most straightforward intrepretation of Logan's definition, only single retrieval should be considered an automatic process. Hence, subjects who are categorized as infrequent single retrievers of multiplication facts should not show evidence of automaticity of multiplication facts. Second, the infrequent single retrieval/automatic pattern implies that automaticity and retrieval are separable. In contrast, under Logan's definition of automaticity, automaticity and retrieval are indistinguishable. With the consistent finding that infrequent single retrievers experienced automatic activation of multiplication facts, it could be concluded that Logan's definition of automaticity was not applicable to the domain of adult multiplication.

General Discussion

Experiment 1 was designed to test the hypothesis that the problem-size effect reflects differences in accessibility of specific facts in the network (Thibodeau & LeFevre, 1992). To test the hypothesis, the number-matching task was employed. Experiment 2 was designed to evaluate the applicability of Logan's (1988, 1992) definition of automaticity in the domain of multiplication by having adults solve multiplication problems, give descriptions of their solution procedures on each trial, and perform a number-matching task.

Implications of the results from the present experiments can be addressed by discussing (a) the implications of the results from the number-matching task (Experiment 1 and 2), (b) the implications of the results from the production task (Experiment 2), and (c) the implications of the patterns of results between the number-matching task and the production task (Experiment 2).

Implications of the Results From the Number-Matching Task

The results from the number-matching task reported in this paper are consistent with the view that multiplication facts are automatically activated in the sense that a product is activated even when such activation is irrelevant to the task (Campbell & Graham, 1985; Campbell, 1987a; Koshmider & Ashcraft, 1991; Yu, 1990; Zbrodoff & Logan, 1986). More importantly, however, the results accord with the notion that small problems are more accessible than large problems. A question that needs to addressed is why are small problems more accessible than large problems. It is plausible that the differences in accessibility is the result of the acquisition history of arithmetic knowledge (Graham, 1987). Because small problems are learned before large problems (Hamann & Ashcraft, 1986; Siegler & Shrager, 1984), Graham has suggested that when a child learns large problems there is proactive interference from the previously learned small problems. This proactive interference results in weaker accessibility for large problems than for small problems.

The current results extend the findings of LeFevre et al. (1988) addition-based study to multiplication. This extension is important because the interference effect found by LeFevre et al. was open to an alternative explanation. It could be argued that latencies for sum trials were significantly slower than latencies for neutral trials as a result of obligatory counting or rule-based activation rather than automatically activated addition fact (Baroody, 1985, 1987). This argument could be made because nine of the ten sum trials used in LeFevre et al. involved problems with addends of 1 or 2 (e.g., 1 + 2 followed by 3). However, the counting-based activation explanation is not plausible for the present multiplication-based experiments. Consequently, findings from the number-matching tasks with multiplication stimuli increase the plausibility that the interference effects are the result of automatic activation of arithmetic facts.

As previously mentioned, the results reported in this paper are consistent with the notion that small problems are more accessible than large problems. This finding supports the view that the problem-size effect reflects the accessibility of specific facts in the network (Campbell, 1987b, 1991; Koshmider & Ashcraft, 1991). Differences in accessibility are consistent with the assumptions made in associative models. Further, these findings are inconsistent with the assumptions made in structural models (e.g., Ashcraft, 1982, 1987; Widaman et al., 1989). According to structural models, the problem-size effect reflects the architecture of the network. For example, Widaman et al. (1989) suggested that addition and multiplication facts are represented in a two-dimensional table. The addends and operands are arranged numerically (from 0 to 9) along the sides of each table. Hence, representation in structural models is assumed to be a function of the characteristics of the number system, such that numbers are arranged from smallest to largest. This assumption was explicitly implied by Widaman et al. when they concluded that "the memory network is bounded by nodal values from 0 through 9 and does not include larger nodal values" (p. 918). In structural models, as presently formulated, accessibility would not vary among facts because structural models do not include a mechanism that would predict varying strengths among the problems and answers. Thus, in structural models, the main determinant of performance is the organization of information. For example, Widaman et al. (1989) proposed that "retrieval time is related to the area of a tablelike network that must be traversed in order to obtain the correct sum of two single-digit addends" (p. 917). Although structural models do predict differences in response time to multiplication problems, they do not predict directly that the links among operands and answers vary in strength. Structural models could include assumptions about accessibility, but, such assumptions make a rigid numerical arrangement of nodes unnecessary.

Widaman et al. (1989) stated that "the choice between tabular [i.e., structural models] and nontabular models [i.e., associative models] must be determined on bases such as which type of model leads to maximal goodness of fit with empirical data and to the confirmation of the greatest number of unique, testable hypotheses" (p. 903). The present results of the number-matching task can be easily accounted for using the assumptions of the associative models, specifically that the operands and answers of large problems are less strongly linked than those of small problems. If the problem-size effect is due to variation in accessibility, then the strict organizational assumptions of structural models are unnecessary. Further, the strict organizational assumptions of structural models make it difficult to account for other findings in the arithmetic area, such as the effects (i.e., relative speed of solution) of ties in addition (Ashcraft & Battaglia, 1978; Groen & Parkman, 1972), ties in multiplication and fives in multiplication (Campbell & Oliphant, 1992). It is also generally assumed that problems involving ones and zeros are unique and are usually solved by rules, rather than by retrieval from a table-like mental representation (Ashcraft, 1992, Experiment 2 of the present paper). Thus, evidence is accumulating to support the view that structural models are not accurate representations for the mental organization of arithmetic knowledge.

Implications of the Results From the Production Task

Results from the production task are inconsistent with the general belief that adults' rely solely on retrieval to solve simple arithmetic problems (Ashcraft, 1982, 1992; Siegler, 1987). The diversity of procedure use in adult multiplication found in the present study is similar to the diversity of strategy use in multiplication done by children. Siegler (1988) found that 92% of the children (8 to 10 years old) reported using two or more strategies, 65% reported using three or more and 23% reported using four. In Experiment 2 of the present study, 97% of adults reported using two or more strategies, 82% reported using three or more, 56% reported using four or more and 31% reported using five or more. Another significant result of the present study was that subjects reported using three types of retrieval: single retrieval; simultaneous multiple retrieval; and sequential multiple retrieval. This finding is significant because no other researcher has proposed that there are different types of retrieval in adult mental multiplication. Consequently, models of mental multiplication designed to explain adult performance should account for the diversity of procedures used.

Currently, the dominant mental arithmetic model that deals with procedure use in multiplication is Siegler's (1988) distribution-of- associations model. Note that Siegler's model is proposed to account for children's performance, however it can also be useful for explaining adults' performance on multiplication problems.

In the distribution-of-associations model (Siegler, 1988; Siegler & Shrager, 1984), the representation of a multiplication problem consists of distribution of associative strengths between the problem and possible answers (i.e., correct and incorrect). For each problem, the distribution of associative strengths can be categorized by its peakedness. For example, in a peaked distribution one of the links between the problem and answer dominates the associative strength (see Figure 6). In a flat distribution, the associative strength is distributed among many answers. In this model, the retrieval of a number is proportional to the associative strength of the particular number relative to the associative strength of the all numbers in the representation. In Siegler's model, the total associative strength of all the numbers representing a problem is 1.00. Hence, if the associative strength of a number is .30 to a particular problem, then the probability that the number will be retrieved is .30.

Insert Figure 6 about here

Siegler (1988; Siegler & Shrager, 1984; Siegler & Shipley, in press) hypothesized that a child sets a confidence criterion and a search length when given a multiplication problem. The confidence criterion determines the minimum level that an associative strength of a retrieved number must exceed for the number to be stated as the answer. The search length determines the number of times that the retrieval will be used to produce the answer. The confidence criterion and search length are set randomly at the onset of each multiplication problem. For example, a child could set the confidence criterion at .35 and the search length at two for the problem 5 X 3. If the associative strength of the first retrieved number was .30, then the retrieved number would not be stated as the answer and the second retrieval would take place. If the associative strength of the second retrieved number was .32, then again the retrieved number would not be stated as the answer and the retrieval process would stop because the search length was reached. Hence, the child would then employ a different approach (i.e., an algorithmic procedure) to produce the answer.

One of the assumptions Siegler (1988) makes with his distribution-of-associations model is that the retrieval process results in the retrieval of one number. This assumption has been explicitly stated by Siegler on many occasions (Siegler & Shrager, 1984; Siegler, 1988; Siegler & Shipley, in press). For example, Siegler (1988) stated that "Once these two parameters [i.e., confidence criterion and search length] are set, the child retrieves <u>an answer</u>" (emphasis added; p.260).

Siegler's (1988) distribution-of-associations model can account for single retrieval reported in the present study. Recall that single retrieval involved the retrieval of one number from memory. Further, no algorithms were used to produce the answers. In the distribution-of-associations model, single retrieval would begin with subjects setting the confidence criterion and search length to the multiplication problem presented and then engaging in the retrieval process. In the retrieval process, the number that was retrieved had an associative strength higher than the confidence criterion which resulted in the number being stated as the answer.

A problem for Siegler's (1988) distribution-of-associations model is accounting for simultaneous multiple retrieval. Recall that simultaneous multiple retrieval involved the retrieval of two or more numbers all at once with no algorithms used to produce the answer. Hence, simultaneous multiple retrieval violates Siegler's assumption that the retrieval process results in the retrieval of one answer.

To account for the present violation, I propose modifications to Siegler's distribution-of-associations model. The new model is called ADAMM (Adult Distribution of Associations Multiplication Model). The main modification to Siegler's model is the addition of an awareness criterion. The awareness criterion is the value that the activation level of a number must exceed for the number to became consciously activated. Unlike the confidence criterion, the awareness criterion is fixed across all problems.

In ADAMM, Siegler's retrieval process is called the search/selection process. This change in terminology allows for the clarification of what retrieval is. In the search/selection process all the numbers related to the multiplication problem are activated automatically. The search process consists of identifying the related number with the highest activation. If this number has an activation level that exceeds the confidence criterion, then the number will be selected as the answer. This procedure would be classified as retrieval. If the search results in a number that has an activation level that does not exceed the confidence criterion, then the number will not be selected. Like Siegler's model, ADAMM proposes that a confidence criterion and a search length is set before each problem.

In ADAMM, simultaneous multiple retrieval would begin with subjects setting the confidence criterion and search length, and then engaging in the search/selection process. The search/selection process would result in more than one number with an activation level that exceeds the awareness criterion, but typically only one number will have an activation level that exceeds the confidence criterion (see Figure 7). The number that exceeds the confidence criterion would be stated as the answer.

Insert Figure 7 about here

ADAMM has a procedure choice component to account for the diversity of procedures used by adults and for the finding that mean solution times for some algorithmic procedures were as fast as the mean solution times for single retrieval. For example, mean solution times for rule based (1050 ms), and multiple counting procedures (1080 ms) were no different than single retrieval (1093 ms). Consequently, from the mean solution time data, it does not appear that subjects first went through the search/selection process and then used an algorithmic procedure to solve the problem. Instead it appears that the subjects used an algorithmic procedure at the onset of solving the problem. From the mean solution times of the present experiment, it appears that the multiple counting, adding and rule based strategies were selected by the subjects' procedure choice component at the onset of solving the problem.

I propose that the procedure choice component uses the features (i.e., operands, multiplication sign) of the multiplication problem to initiate the algorithmic procedure and that the features are extracted during the encoding of the problem. Hence, the procedure choice component possesses a feature-detector mechanism that activates algorithmic procedures. For example, when an individual is presented the problem 2 X 0, the feature- detector mechanism extracts the multiplication sign and 0. These features activate the rule-based procedure that a X 0 equals 0 and the individual uses this algorithmic procedure to solve the problem. As previously mentioned, the mean reaction times for the multiple counting and adding procedures indicate that these procedures were initiated by the feature-detector mechanism of the procedure choice component.

Because the majority of the multiplication problems that were solved by the multiple counting or adding procedures included the operands 2 or 3, the important features for activation of these algorithms appear to be the multiplication sign and the operands 2 or 3.

To be successful, ADAMM must account for the use of the other procedures reported by subjects. In ADAMM, all procedures begin with a subject setting the confidence criterion, search length and then encoding of the problem. If there are features in the problem that activate a particular algorithm (via the feature-detector mechanism), the required steps of the algorithm are performed and then the answer is stated. Note that even when the individual opts for the algorithm at the onset, the search/selection process is still carried out (i.e., the search/selection process is automatic; for evidence see the number-matching task from Experiment 1 and 2 of the present study). If there are no features in the problem that activate a particular algorithm, the individual will proceed with the search/selection process.

In single retrieval, there would be no features in the problem that activate an algorithmic procedure. Further, in the search/selection process only one number's activation level would exceeded both the awareness criterion and the confidence criterion (see Figure 8). Hence, this number would be stated as the answer.

Insert Figure 8 about here

Sequential multiple retrieval reported in the present study could be explained by ADAMM by proposing that subjects did a series of single retrievals. However, it is important to note that sequential multiple retrieval may be the same as simultaneous multiple retrieval because the mean solution times and accuracy rates were similar between the procedures. It is plausible that the only difference between the multiple retrieval procedures is that the discrepancy between the activation level of the number that exceeded the confidence criterion and the activation level of the number that exceeded the awareness criterion was much larger for sequential multiple retrieval than for simultaneous multiple retrieval (see Figure 9). It is possible that when a problem is presented, there is a race between the activation of the potential answers. Further, it is plausible to propose that there is a relation between activation level and time. More specifically, the stronger the activation level of a number, the faster the number is activated. Consequently, a number with a large activation level may be perceived as being retrieved first, followed by numbers that had activation levels that were much lower (i.e., subjects perceive the numbers as being retrieved one at a time). Therefore, it is plausible that sequential multiple retrieval is the same procedure as simultaneous multiple retrieval.

Insert Figure 9 about here

In an algorithmic procedure, there may be numbers in the search/selection process that exceeded the awareness criterion but none of the numbers exceeded the confidence criterion (see Figure 10). Consequently, an algorithmic procedure would be implemented to obtain the answer. From the mean solution times for problems solved by algorithmic procedures in the present experiment, the derived fact procedure was longer than any of the other algorithmic procedures (2175 vs. 1227, 1080, 1050 ms) and single retrieval (2175 vs. 1093 ms). Hence, it is plausible that subjects first attempted the search/selection process and then opted for the derived fact procedure, whereas the other algorithmic procedures were elicited by the procedure choice component during encoding.

Insert Figure 10 about here

Mean solution time for single retrieval was faster than mean solution times for the multiple retrieval procedures in the present study. Further, the accuracy rate was higher for single retrieval compared to the accuracy rate for the multiple retrieval procedures. These findings cannot be accounted for by ADAMM in its current state. The discrepancies between single retrieval and multiple retrieval procedures can be accounted for if ADAMM incorporates the lateral inhibition mechanism that is used in Campbell and Oliphant's (1992; Campbell, 1987a, 1990, 1991) network-interference model. Campbell and Oliphant proposed that during the retrieval process, each node receives excitatory input from the featural and magnitude characteristics of the problem, and simultaneously receives inhibitory input from other nodes (for a full description of featural and magnitude characteristics see Campbell & Oliphant, 1992). The retrieval efficiency and/or retrieval time is determined by the activation level (i.e., excitation) of the correct and incorrect nodes. If a problem has incorrect nodes that is strongly activated then the incorrect nodes will significantly inhibit the correct node. This would result in longer latencies and lower accuracy than a problem that has incorrect nodes that are weakly activated. Implementing the lateral inhibition mechanism in the retrieval process of ADAMM makes it easy to account for the discrepancies between single retrieval and the multiple retrieval procedures. More specifically, the multiple retrieval procedures would yield longer mean solution times and lower accuracy rates than single retrieval because in the multiple retrieval procedures there were incorrect nodes that were strongly activated (i.e., above the awareness criterion) that inhibited the correct node. Note that in single retrieval no incorrect node had an activation level that exceeded the awareness criterion, hence the retrieval efficiency was high (i.e., fast solution times and highly accurate).

To conclude, the steps involved in ADAMM to solve a simple multiplication problem for adults are shown in Figure 11. When an individual is presented a simple multiplication problem, the individual sets a confidence criterion, search length and then encodes the problem. If there are features in the problem that activate a particular algorithm (i.e., via the feature-detector mechanism in the procedure choice component), the individual will perform the required steps of the algorithm and state the answer. Note that even when the individual opts for the algorithm at the onset, the numbers in an individual's mental representation are activated through automatic activation. If there are no features in the problem that activate a particular algorithm, the individual will proceed with the search/selection process. In the search/selection process all the numbers related to the multiplication problem are activated automatically but the individual is aware of only the numbers with activation levels exceeding the awareness criterion. However, an answer will only be stated if one of the numbers also exceeds the confidence criterion. The retrieval efficiency is determined by the activation levels of the incorrect and correct answers (i.e., via the lateral inhibition mechanism). If no number exceed the confidence criterion, the individual will opt for an algorithm procedure.

Insert Figure 11 about here

<u>Implications of the Patterns of Results Between the Number-Matching Task and the Production Task</u>

Logan (1988) has defined automaticity as "single-step direct-access retrieval of past solutions from memory" (p. 493). This implies that automaticity and memory retrieval are undistinguishable. The patterns of results reported in Experiment 2 of the present study are inconsistent with Logan's view that automaticity is strictly memory retrieval. The pattern of results that challenges Logan's definition of automaticity is the infrequent retrieval/automatic pattern. The infrequent retrieval/automatic pattern implies that automaticity and memory retrieval are separable. This pattern of results leads to the hypothesis that there could be a different criterion level for memory retrieval in the production task compared to the criterion level for automaticity in the number-matching task.

The notion that the infrequent retrieval/automatic pattern reflects different criterion levels for retrieval and automaticity is explainable by ADAMM. In ADAMM there are two criterion levels: an awareness criterion and a confidence criterion. The confidence criterion is the level that the activation level of a number must exceed for the number to be retrieved as the answer to a particular multiplication problem. The awareness criterion is the level that the activation level of a number must exceed for the number to become consciously activated. Note that in ADAMM, the confidence criterion would not be set in the number-matching task because subjects were not attempting to retrieve an answer from long-term memory. However, the awareness criterion would still be set in the number-matching task because the awareness criterion is fixed and unchanging across all problems and situations. Recall that in ADAMM, numbers are activated through automatic activation when a stimulus is presented. Consequently, when a stimulus was presented in the number-matching task, the numbers of the stimulus and the related numbers would be activated. Hence, it is plausible that the interference effect (i.e., evidence for automatic activation), in the number-matching task, only occurs when the activation level of a number (i.e., the product) exceeds the awareness criterion. Hence, the confidence criterion determines retrieval in the production task, whereas the awareness criterion determines automaticity in the number-matching task.

The suggestion that ADAMM can account for the infrequent retrieval/automatic pattern is shown explicitly in Figure 12. In ADAMM there are two possible scenarios that could lead to a subject not reporting a retrieval procedure. First, the subject's feature detecor mechanism in the procedure choice component could have extracted a feature or features in the problem that activated an algorithmic procedure (see top left of Figure 12). Note, that activation of all related number still occurs when an algorithmic procedure is used from the onset of solving a problem. Second, the subject engages in the retrieval process and there is no number with an activation level that exceeded the confidence criterion (see top right of Figure 12). However, in both of these cases, there could have been numbers that had activation levels which exceeded the awareness criterion (see bottom left and bottom right of Figure 12). Consequently, in the number-matching task, the infrequent retrievers had products with activation levels that exceeded the awareness criterion (i.e., resulting in the interference effect), whereas in the production task, the products' activation levels did not exceed the confidence criterion or subjects used an algorithm procedure from the onset of the problem.

Insert Figure 12 about here

The major implication from the pattern of results between the number-matching task and the production task for Logan's (1988, 1992) definition of automaticity is that it is now problematic to propose that the underlying mechanism of automaticity is memory retrieval. From the present results, it appears that the underlying mechanism of automaticity in adult multiplication is activation, which results in activation levels exceeding an awareness criterion. In contrast, the underlying mechanism of memory retrieval is activation that results in activation levels exceeding a confidence criterion. Because of the results of Experiment 2 in the present study, it could be concluded that Logan's definition of automaticity was not exceedingly applicable to adults' multiplication.

Concluding Remarks

The results from the present experiments have helped clarify some of the issues concerning adults' performance in multiplication. For example, the results from the number-matching task reported in this paper are consistent with the view that the problem-size effect reflects the accessibility of specific facts in the network. Differences in accessibility are consistent with the assumptions made in associative models (e.g., Campbell, 1987b, 1991; Koshmider & Ashcraft, 1991), but these differences are also inconsistent with the assumptions made in structural models (e.g., Ashcraft, 1982, 1987; Widaman et al., 1989). This inconsistency leads to questions about the representation of multiplication knowledge in structural models.

From the presents results, some beliefs regarding adults' performance in multiplication can be questioned. For example, results from the production task are inconsistent with the belief that adults' solely rely on retrieval to solve simple arithmetic problems (Ashcraft, 1982, 1992; Siegler, 1987). Consequently, the diversity of procedure use in adult multiplication found in the present study suggests that models of mental multiplication that are designed to explain adult performance should account for the diversity of procedure use. In the model proposed in the present paper, the adult distribution of associations multiplication model (i.e., ADAMM), the diversity of procedure use by adults was accounted for by the procedure choice component in the encoding process and the confidence criterion in the retrieval process. In addition to accounting for the diversity of procedure use by adults, ADAMM was able to account for the results from the number-matching task. The next step in demonstrating ADAMM's validity would be to generate a computer simulation, with ADAMM's assumptions, that could accurately reproduce performance of adults' mental multiplication (i.e., reaction times, errors, procedure use).

In Logan's (1988) instance theory of automatization, automaticity is specifically defined as single-step direct-access memory retrieval. However, Logan's assumption that automaticity is single-step memory retrieval had never been properly tested. In Experiment 2 of the present study the applicability of Logan's definition of automaticity was properly tested in the domain of adults' multiplication. From the present results, it was concluded that Logan's definition of automaticity was not applicable. More specifically, memory retrieval does not appear to be the underlying process of automaticity.

The major contribution of the present research is the proposed model of adult multiplication called ADAMM. As previously mentioned, ADAMM can account for adult performance in both the production task and the number-matching task. However, it should be noted that ADAMM is a static model of multiplication performance by young adults. What should not be forgotten is the importance of developing models which attempt to discover the mechanism of change (Klahr, 1989). For example, how does a child develop a representation of multiplication facts or how does a child learn and use new procedures? Proper answers to these and other questions could be valuable to prove the authenticity of ADAMM and for developing a model which can account for performance in mental multiplication from childhood to adulthood.

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Table 1

Procedure Use, Mean Solution Times and Accuracy

Procedure	Procedure Use	Mean Solution Times	Accuracy
Single Retrieval	73%	1093 ms	98%
Simultaneous Multiple Retrieval	7	1561	89
Sequential Multiple Retrieval	4	1923	89
Derived Fact	8	2175	94
Adding	3	1227	100
Multiple Counting	1	1080	93
Rule Based	2	1050	97
Other (guessing, using fingers, etc.)	1	2654	60
Overall	99%	1229 ms	95.2%

Note. Overall procedure use totals 99%; 1% of the trials were discarded because of problems with the voice-operated relay.

Appendix A

Stimulus List Used in the Number-matching Task in Experiment 1 and Experiment 2

	Nonmatching Trials	
Product Cue	Target	Unrelated Cue
	Large Problems	
5 X 8	40	7 X 6
7 X 6	42	5 X 8
7 X 4	28	3 X 9
5 X 6	30	7 X 4
3 X 9	27	5 X 6
	Small Problems	
2 X 5	10	6 X 3
6 X 3	18	5 X 4
5 X 4	20	7 X 3
7 X 3	21	4 X 6
2 X 7	14	3 X 5
	Nonmatching Filler Trials	
39 X 7	46	
4 X 26	37	
22 X 5	19	
9 X 17	13	
24 X 7	38	

Matching Trials

Cue	Target
	Target-balancing Trials
7 X 40	40
42 X 8	42
5 X 28	28
30 X 4	30
6 X 27	27
3 X 10	10
18 X 5	18
3 X 20	20
21 X 4	21
14 X 6	14
	Cue-balancing Trials
5 X 8	5
7 X 6	7
7 X [illegible]	4
5 X 6	6
3 X 9	3
2 X 5	5
6 X 3	6
5 X 4	5
7 X 3	3
2 X 7	7

Matching Filler	
5 X 26	5
47 X 3	3
2 X 19	2
31 X 7	7
8 X 13	8

Appendix B

Mean Median Correct Latencies (in msec) and Percentage of Errors (in Parentheses) in Experiment 1 as a Function of SOA, Problem Size and Problem Type

	Stimulus Onset Asynchrony (in msec)			
Product Type	40	100	220	350
Nonmatching Trials				
Small				
Product	883 (3.8)	797 (3.1)	738 (2.5)	699 (1.9)
Unrelated	874 (4.1)	751 (1.6)	696 (0.9)	686 (0.6)
Large				
Product	848 (3.1)	763 (1.3)	719 (0.6)	688 (2.5)
Unrelated	817 (4.1)	751 (0.9)	688 (0.9)	684 (0.0)
Nonmatching filler	872 (5.6)	830 (3.4)	727 (1.6)	703 (2.5)
Matching Trials				
Target-balancing	795 (13.2)	749 (6.7)	693 (12.7)	656 (4.7)
Cue-balancing	731 (2.3)	692 (1.6)	641 (1.4)	611 (1.9)
Matching filler	790 (7.5)	747 (4.7)	685 (4.1)	652 (2.5)

Appendix C

Mean Median Correct Latencies (in msec) and Percentage of Errors (in Parentheses) in Experiment 2 as a Function of SOA, Problem Size and Problem Type

	Stimulus Onset Asynchrony (in msec)			
Product Type	100	120	220	350
Nonmatching Trials				
Small				
Product	824 (6.9)	808 (4.1)	754 (5.0)	712 (3.4)
Unrelated	816 (6.3)	752 (1.3)	729 (1.9)	695 (1.6)
Large				
Product	828 (5.6)	795 (5.3)	734 (3.4)	705 (2.8)
Unrelated	789 (1.9)	751 (1.3)	724 (3.4)	697 (1.3)
Nonmatching filler	798 (5.6)	831 ([illegible])	730 (2.8)	704 (1.3)
Matching Trials				
Target-balancing	754 (16.1)	762 (13.[illegible])	6[illegible]7 (18.8)	652 (1[illegible].6)
Cue-balancing	693 (5.3)	688 (3.0)	6[illegible]3 (2.7)	605 (3.1)
Matching filler	755 (10.0)	742 (5.0)	698 (7.1)	642 (2.2)

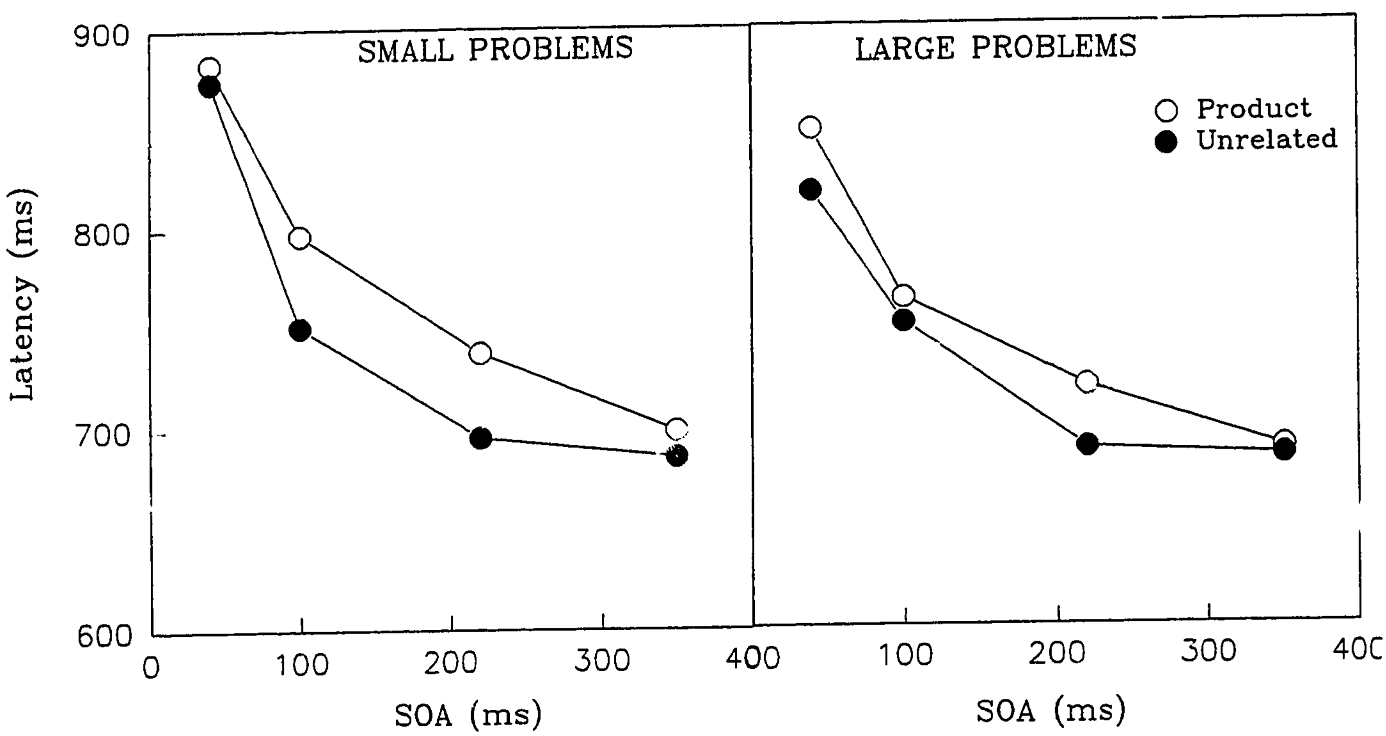

Figure : Mean median reaction times on product and unrelated trials in the number-matching task as a function of SOA and problem size in Experiment 1.

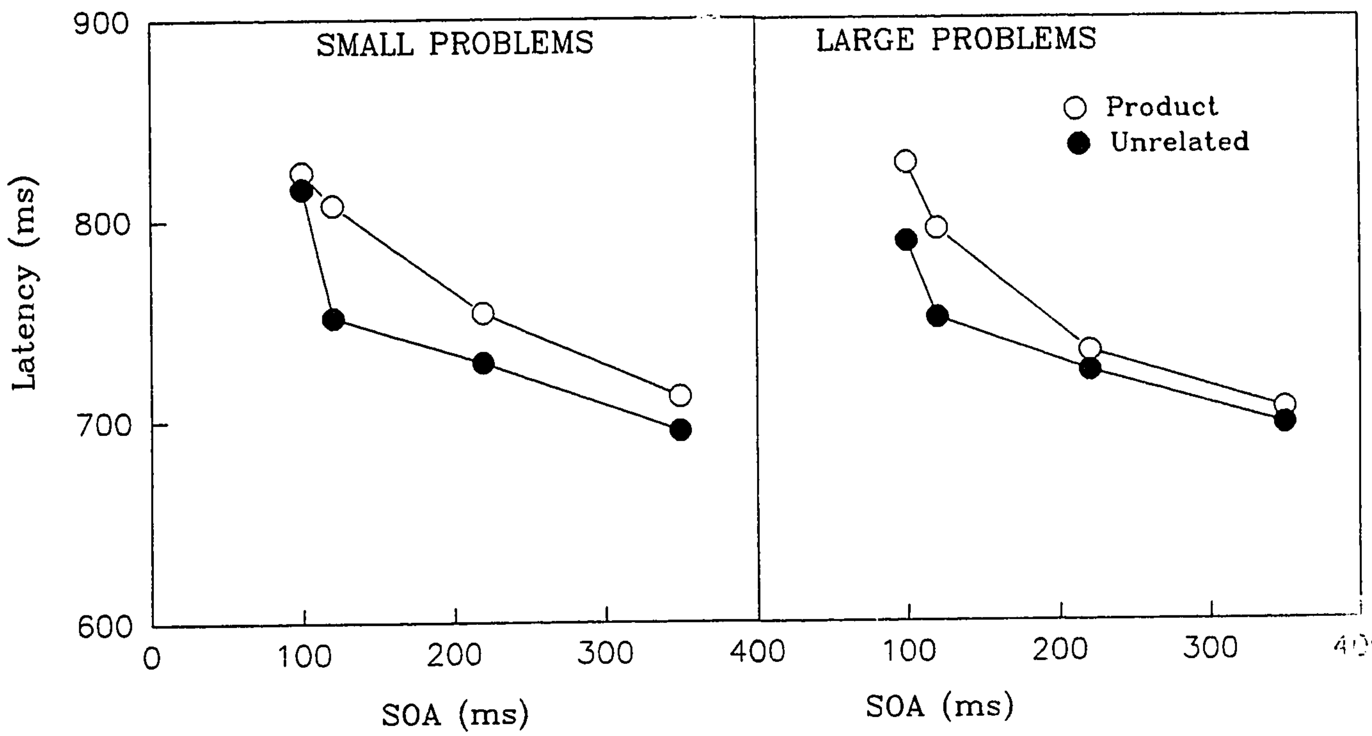

Figure 2. Mean median reaction times on product and unrelated trials in the number-matching task as a function of SOA and problem size in Experiment 2.

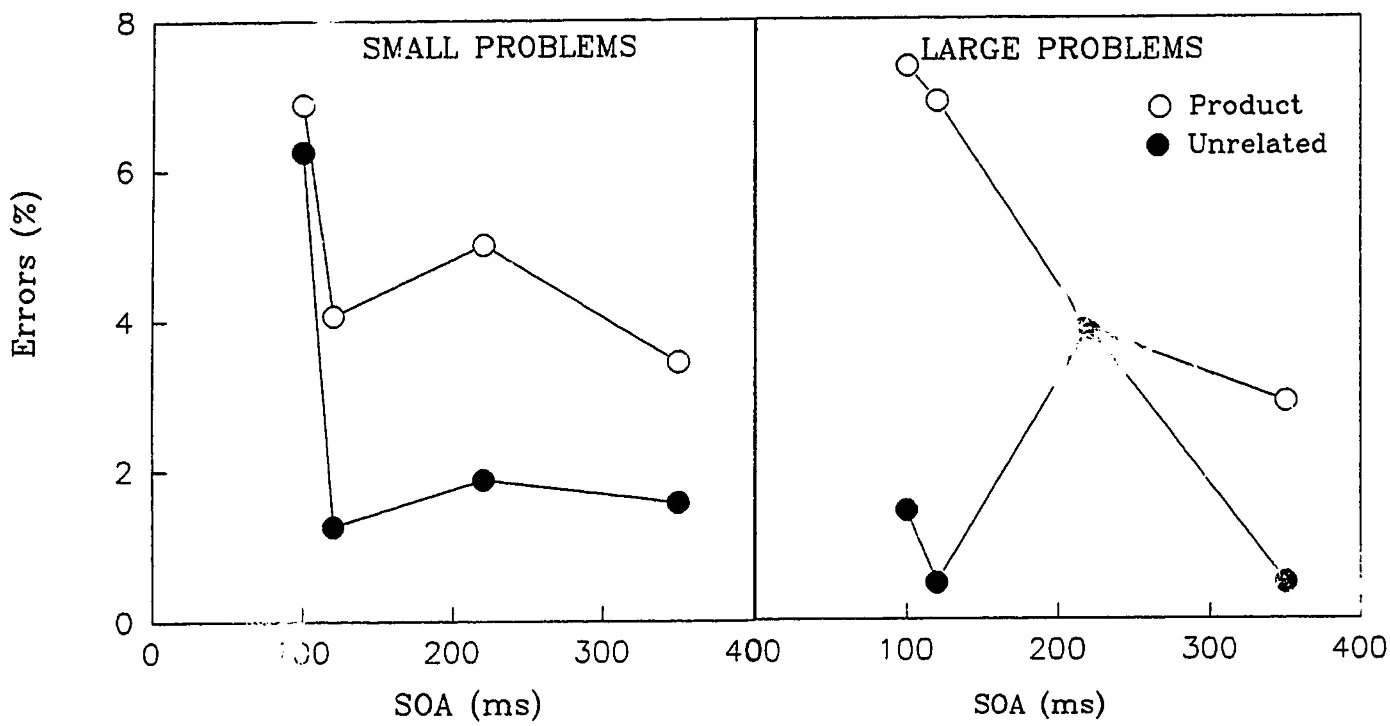

Figure 3. Mean error rate on product and unrelated trials in the number-matching task as a function of SOA and problem size in Experiment 2.

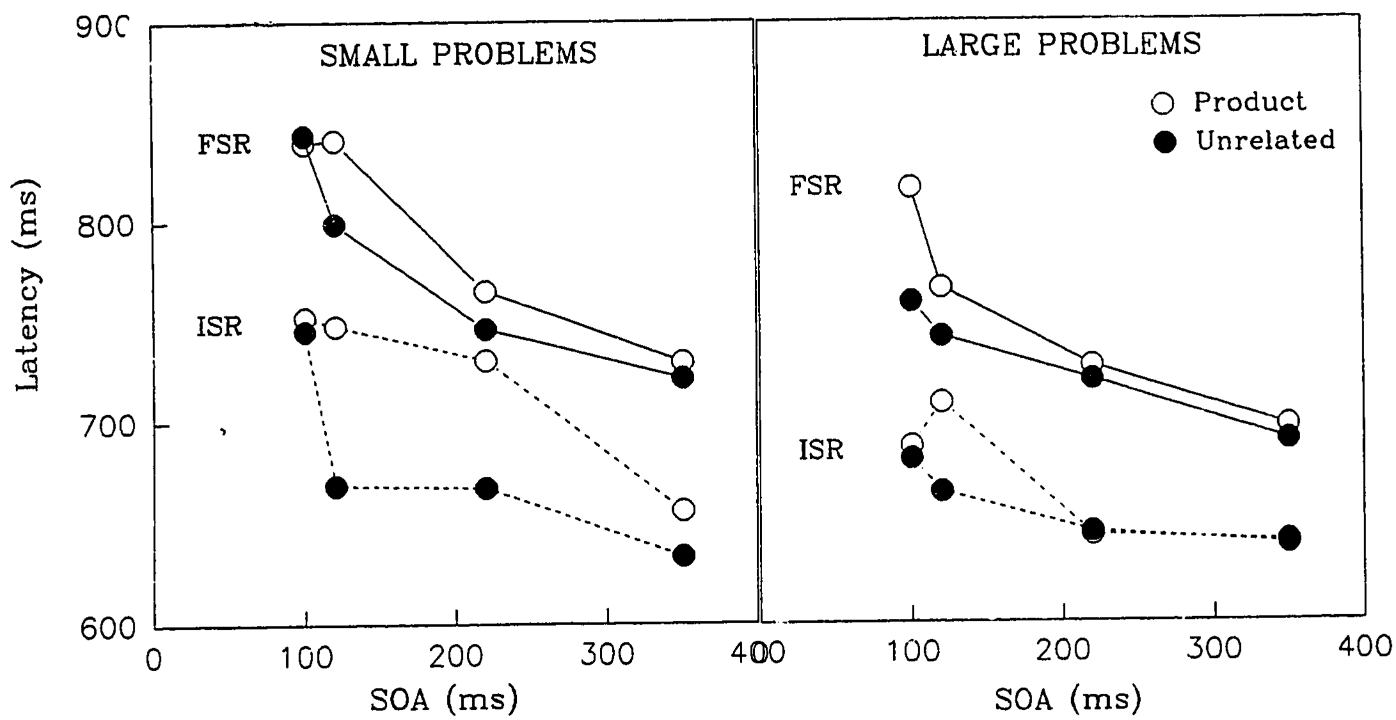

Figure 4. Mean median reaction times on product and unrelated trials in the number-matching task as a function of SOA and problem size for FSR (Frequent Single Retrievers) and ISR (Infrequent Single Retrievers) in Experiment 2. Subjects categorized by the self-reports collected for 40 problems in the production task.

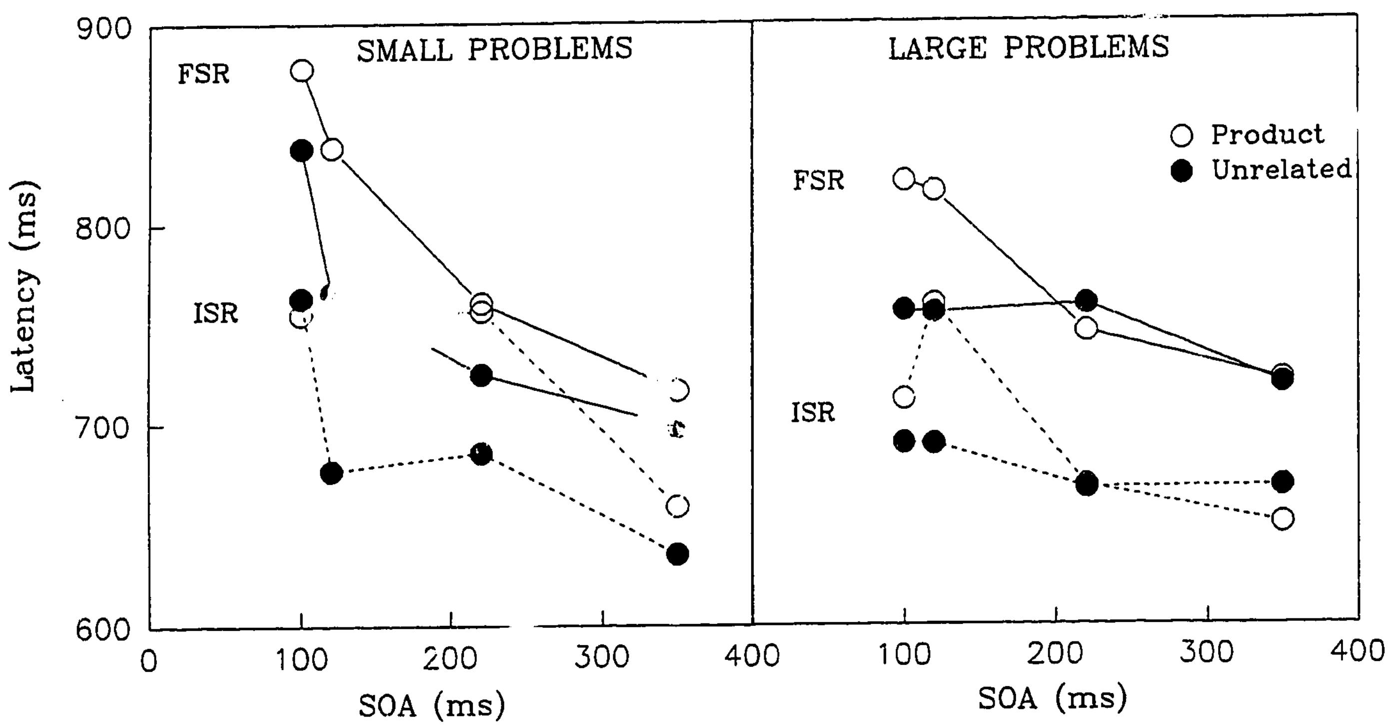

Figure 5. Mean median reaction times on product and unrelated trials in the number-matching task as a function of SOA and problem size for FSR (Frequent Single Retrievers) and ISP (Infrequent Single REtrievers) in Experiment 2. Subjects categorized [illegible] e self-reports collected for the 10 critical problems in the product[illegible] ask.

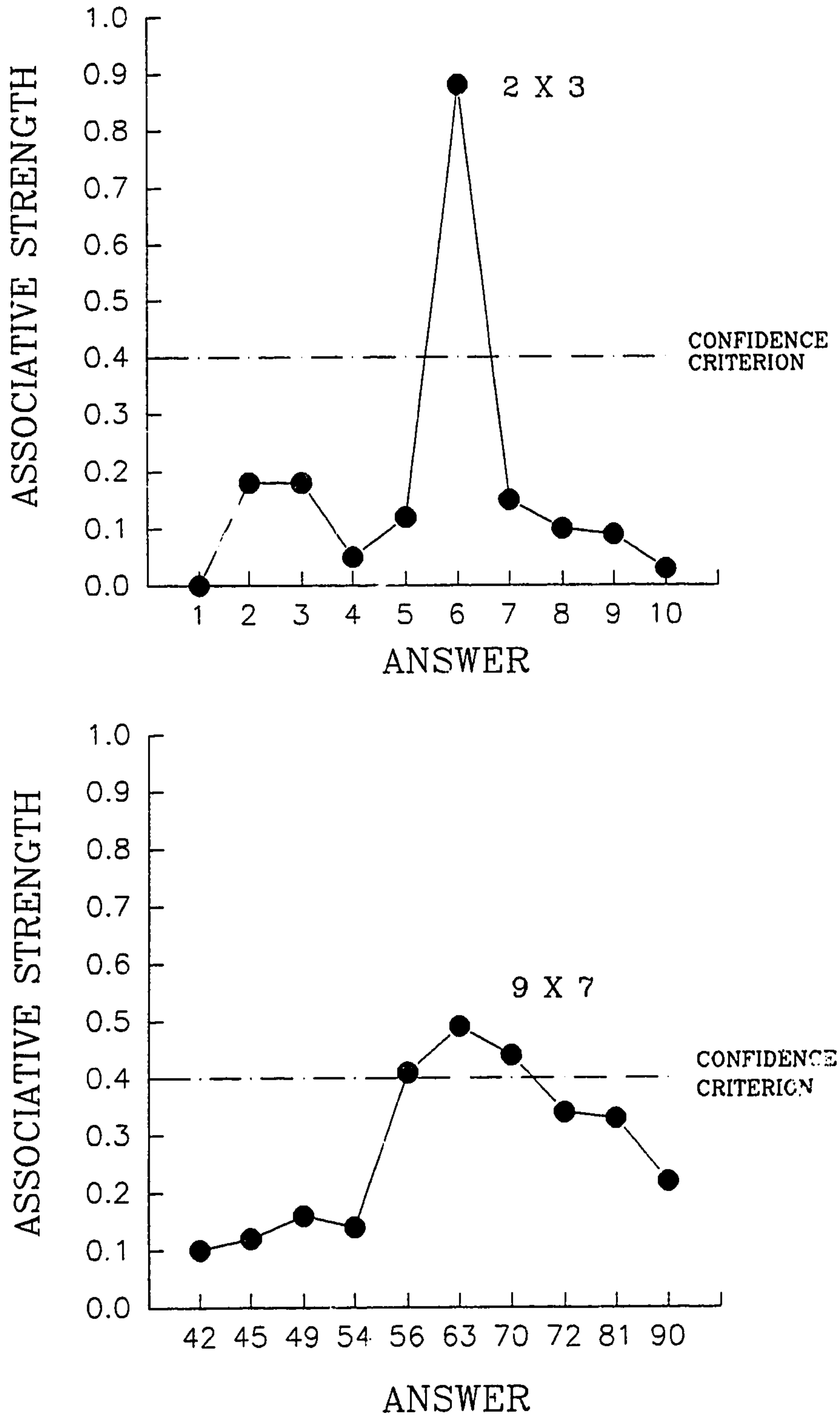

Figure 6. A peaked distribution of asso[illegible]ons (to[illegible]) and a flatter distribution of association (bottom).

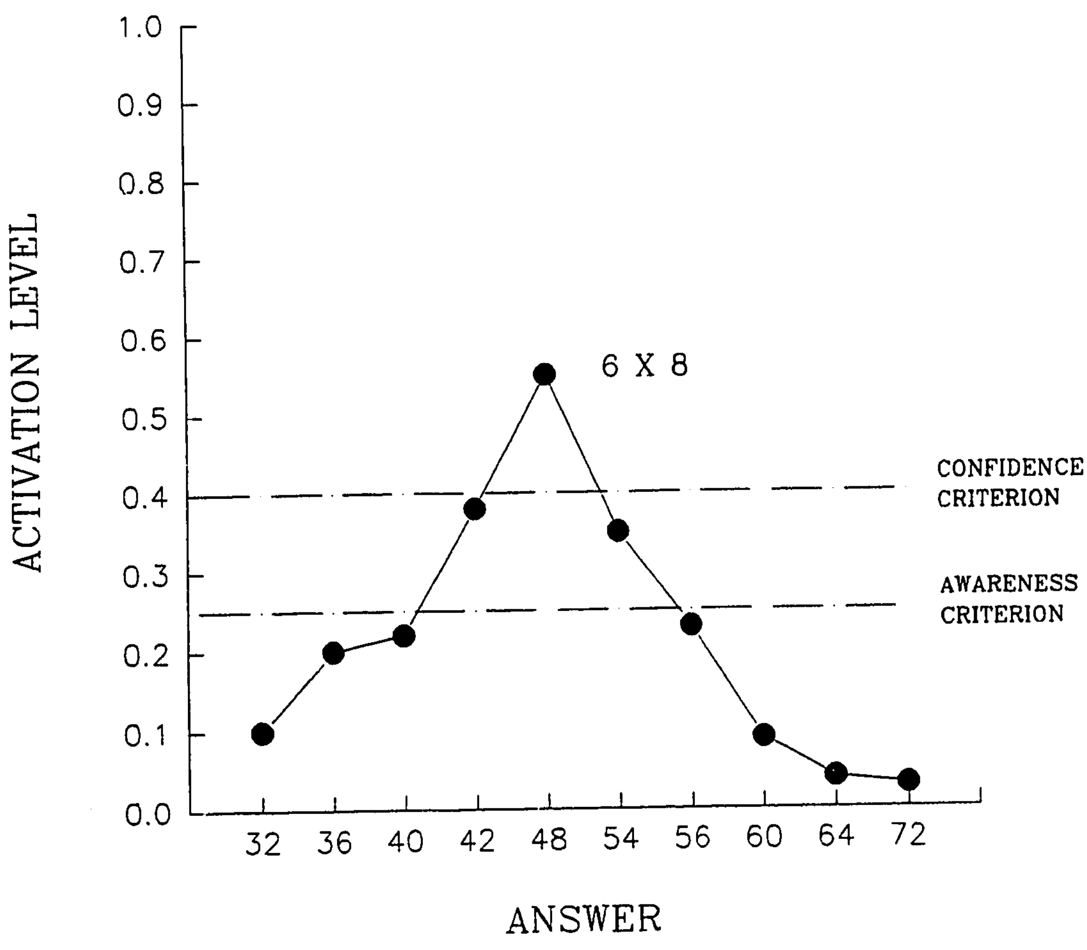

Figure 7. A hypothetical distribution of associations for simultaneous multiple retrieval.

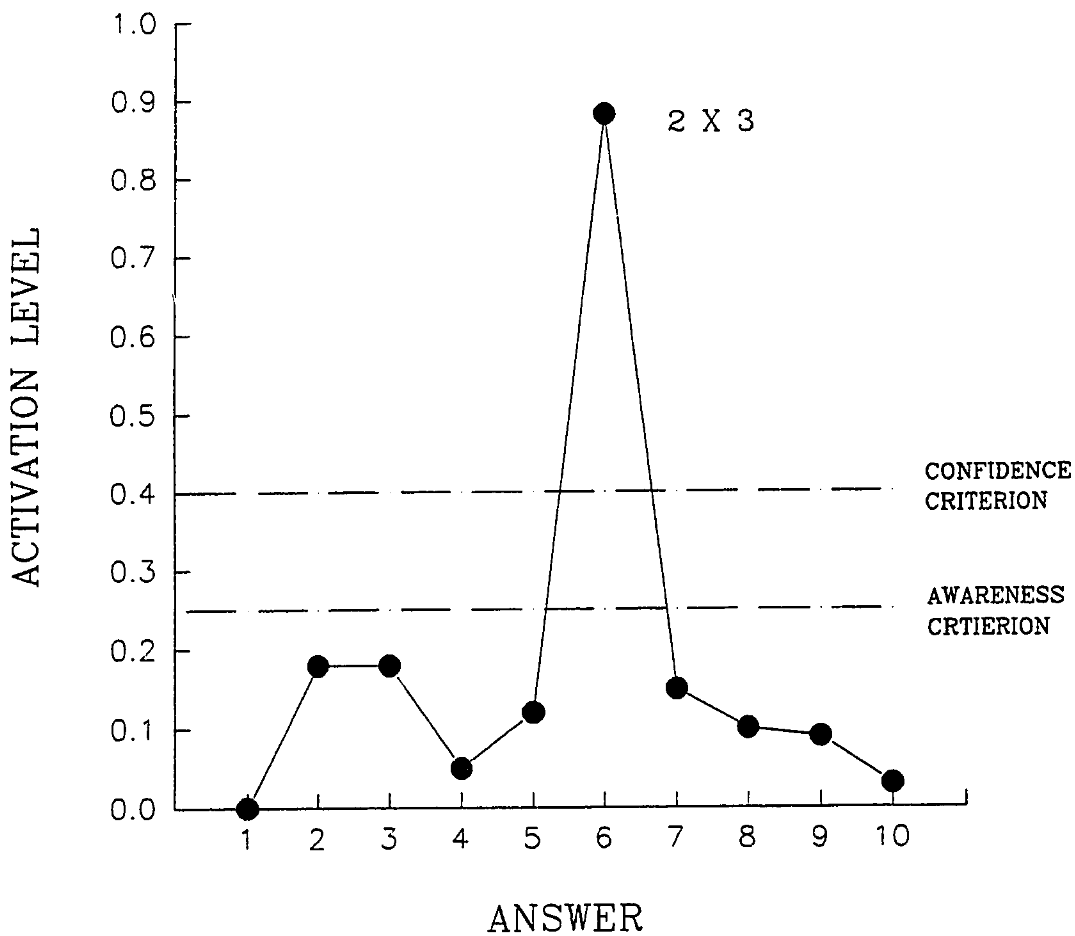

Figure 8. A hypothetical distribution of associations for single retrieval.

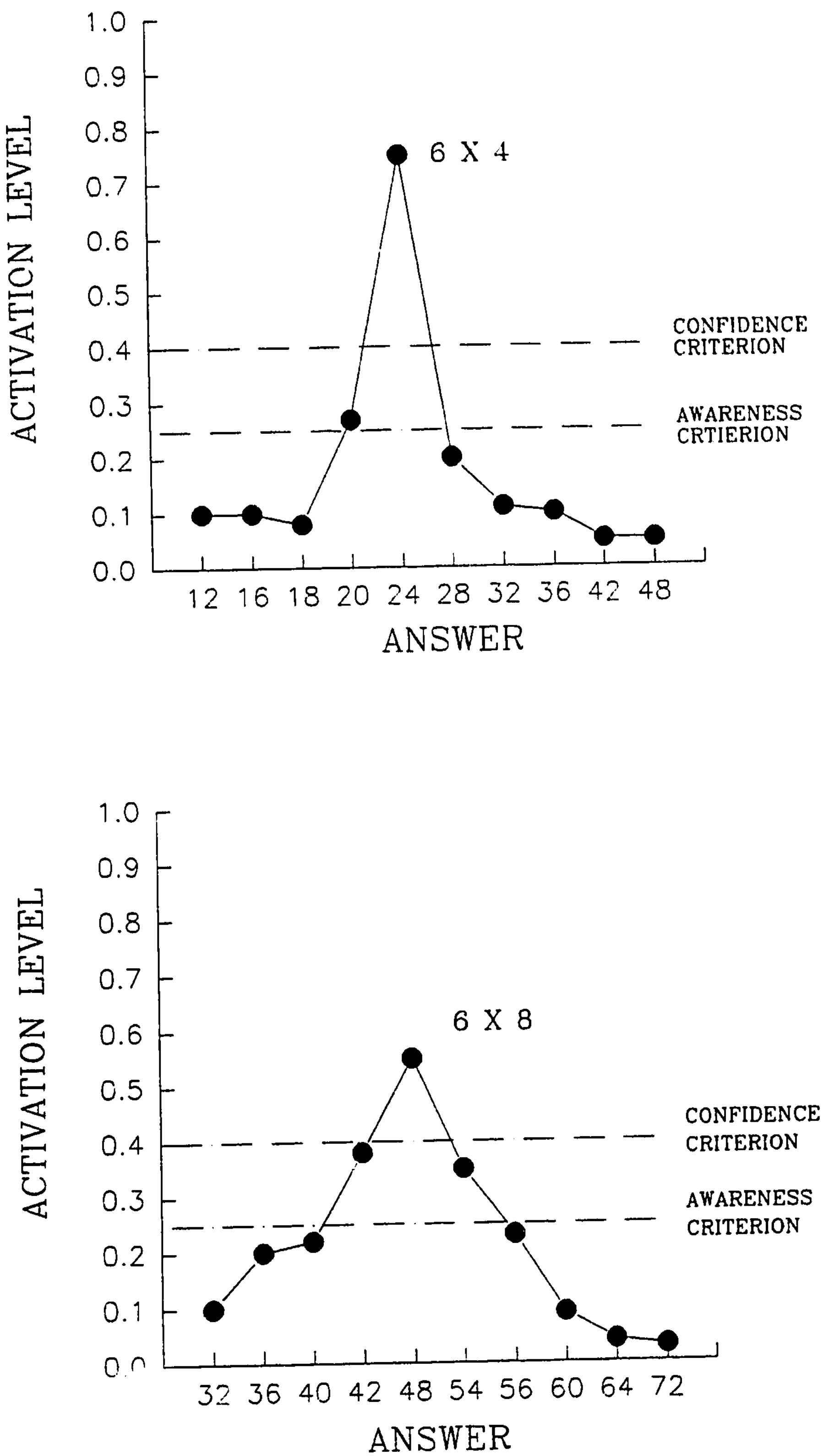

Figure 9. Hypothetical distributions of associations for sequential multiple retrieval (top) and simultaneous multiple retrieval (bottom).

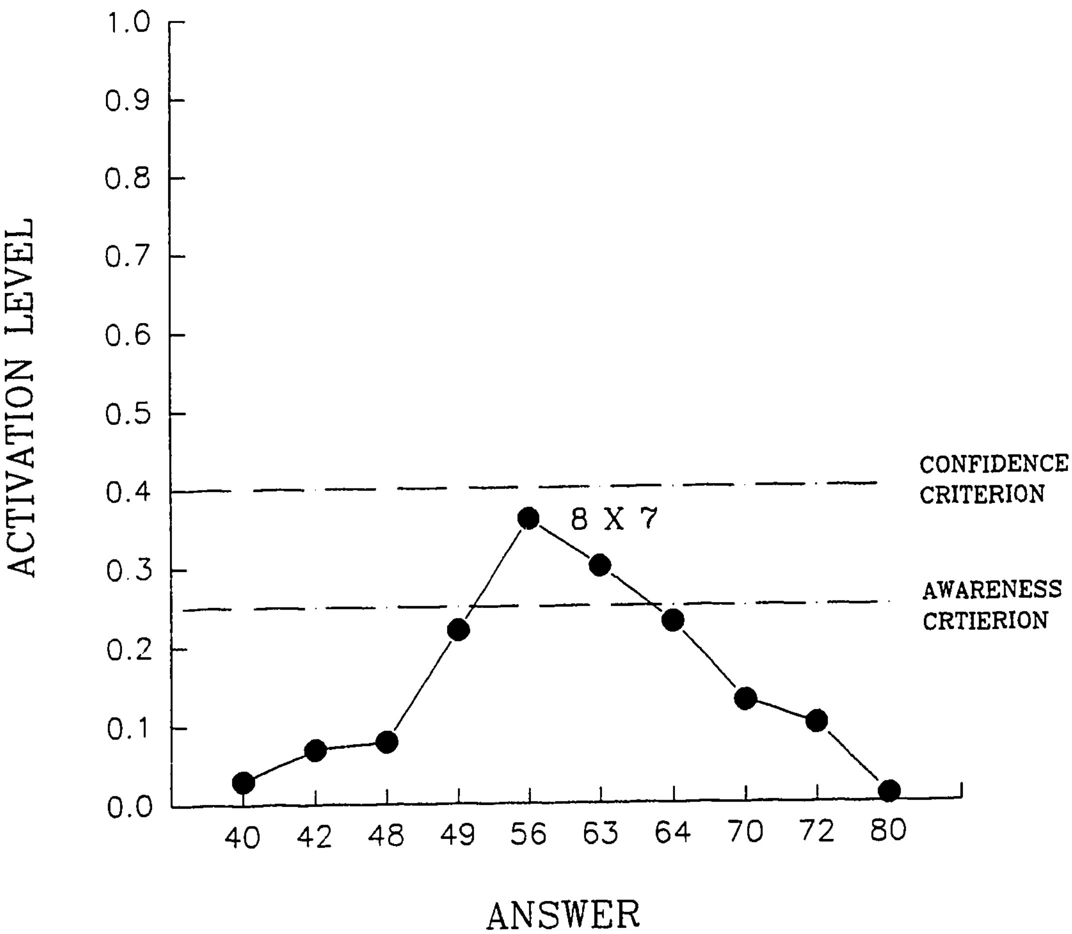

Figure 10. A hypothetical distribution of associations for algorithmic procedures that are not elicited by the procedure choice component.

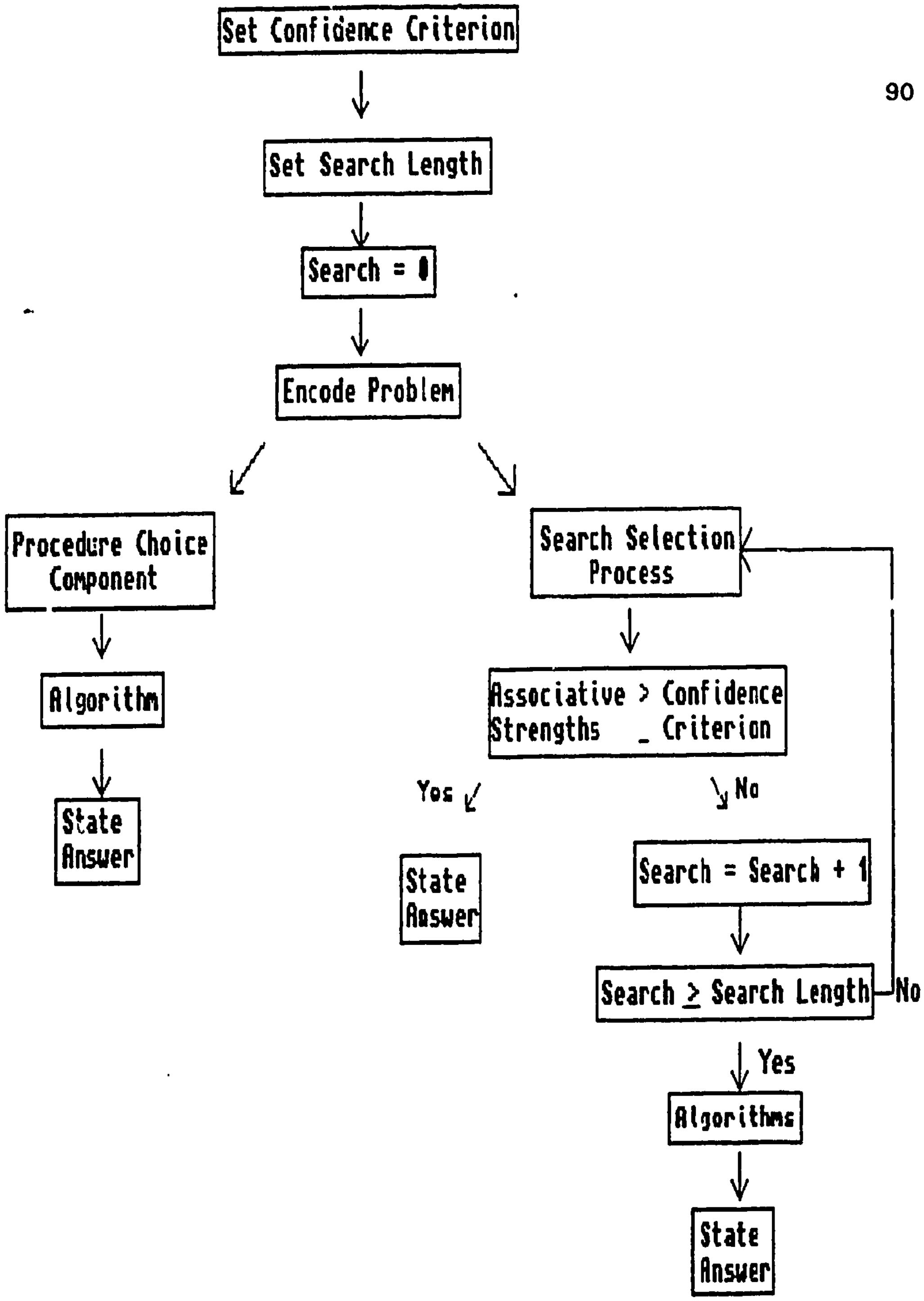

Figure 11. Overview of ADAMM.

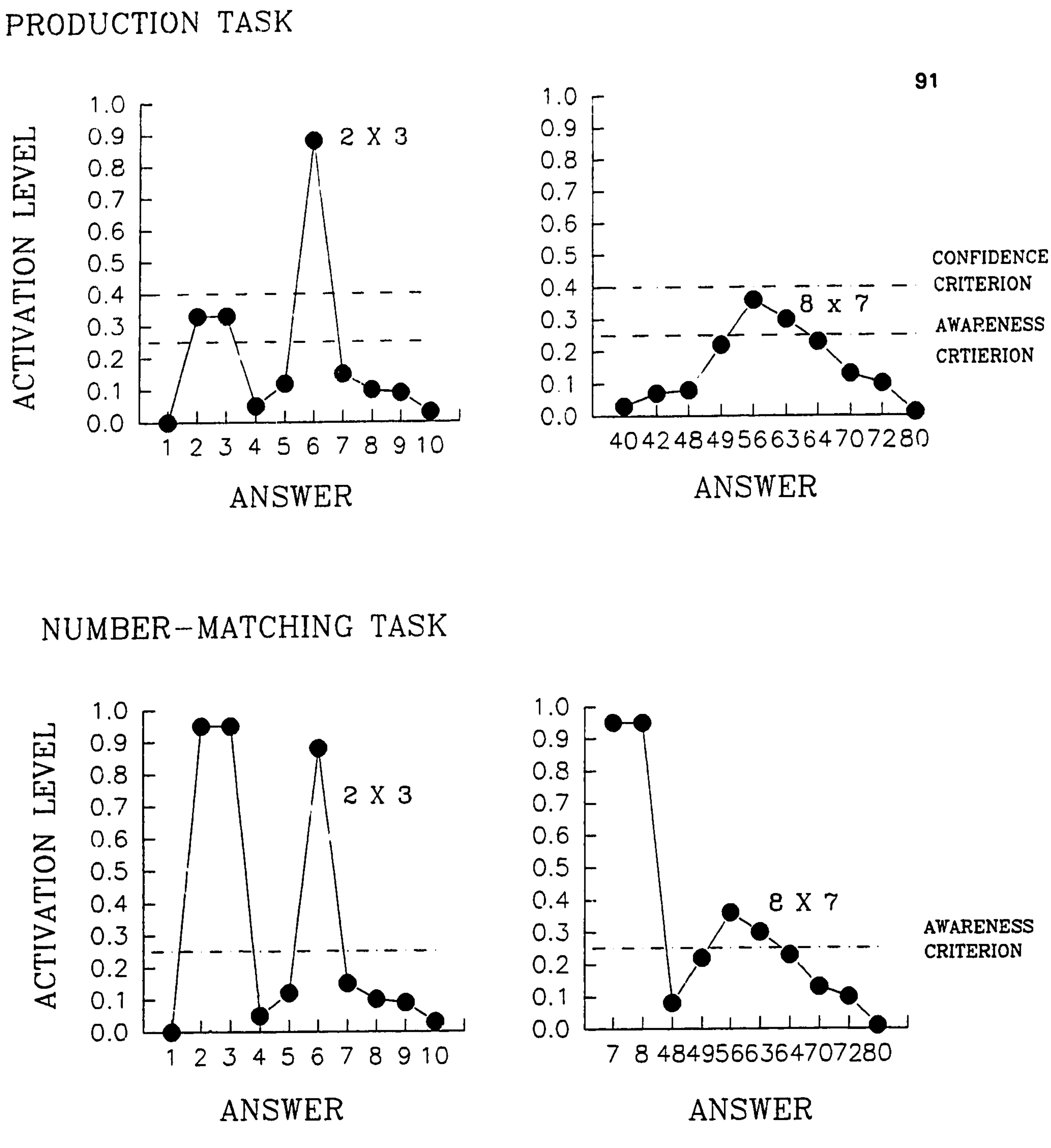

Figure 12. Hypothetical distributions of associations for algorithmic procedures in the production task (top left and top right). Hypothetical distributions of associations for infrequent retrievers in the number-matching task (bottom left and bottom right).