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THE UNIVERSITY OF ALBERTA

# SEQUENTIAL DEPENDENCIES IN
# MAGNITUDE ESTIMATION

by

© Richard Benner Morris

A THESIS
SUBMITTED TO THE FACULTY OF GRADUATE STUDIES
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FACULTY OF GRADUATE STUDIES

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "Sequential Dependencies in Magnitude Estimation," submitted by Richard B. Morris in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

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# ABSTRACT

The observer in any psychophysical experiment is influenced by the context of the situation within which he makes his judgments. He may take into account the stimulus being presented, the stimuli previously presented, and the prior judgments he made in making each judgment. In order to arrive at a reliable expression of the relationship between a stimulus magnitude and the observer's judgment of that stimulus magnitude, the influence of the context of the situation on the judgment must be fully understood. The experiment reported in this dissertation investigated the degree to which the immediately preceding response influences magnitude estimation judgments.

A measure of judgment sequential dependency was calculated for each observer which reflected the degree to which each judgment was related to the preceding judgment. A Pearson product-moment correlation coefficient was calculated for each observer which was an index of association related to the linear correlation between the ratio of a judgment of a stimulus magnitude to the geometric mean of all of the several judgments made to that stimulus and a like ratio of a

judgment made immediately preceding it in time.

One hundred eighty observers were divided into nine groups of 20 observers each. Each observer made judgments of a series of stimuli from one of three continua using one of three judgment languages. Two of the continua used, numerousness of dots and line length, are prothetic continua. The third continuum used, proportion, is a metathetic continuum. The three judgment languages used were physical unit modulus, no prescribed modulus, and prescribed modulus.

A mean judgment dependency coefficient was determined for each group and each was found to be significantly different from a mean correlation of zero. The dependency coefficients were submitted to a 3 x 3 factorial analysis of variance. The analysis yielded significant continua and judgment language effects. A subsequent analysis with Duncan's <u>multiple range test</u> performed on the main effects indicated that judgment sequential dependency was significantly less for judgments of the proportion continuum than for judgments of the line length and numerousness of dots continua, and that dependence was significantly less for judgments with the physical unit modulus language than for judgments with the no prescribed modulus language and the prescribed modulus language.

In addition to the judgment dependency measure, power law exponents were determined for each observer. A Pearson product-moment correlation was computed between the measure of judgment dependency and the power exponent for observers in each of the conditions to determine if the exponents were related to the dependency coefficients. Results suggested that within each condition, the individual power law exponents were not linearly related to the individual judgment dependency coefficients.

The different results obtained with the physical unit modulus language and the prescribed modulus languages, were interpreted as providing information about some of the ways in which the observer makes his judgments when using these languages. It was noted that the dependency measure is sensitive to a judgment situation in which the observer appears to lose sight of the original prescribed modulus, yet continues to perform according to his instructions. It was suggested that observers may tend to change the modulus in the course of an experiment because of such things as changes in adaptation level, forgetting, and a lack of congruence between the task and task expectation.

# ACKNOWLEDGMENTS

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# INTRODUCTION

As an area of philosophic and scientific concern, the study of psychophysics has a past extending from the early founders of experimental psychology and the Zeitgeist of the mid-nineteenth century to the present. The term psychophysics was coined by Fechner (1860; 1966) and defined as "an exact theory of the functionally dependent relations of body and soul [p. 7]." Empirically, this meant for him a search for the lawful relation between sensation and its physical counterpart. This search resulted in techniques which he assumed measured sensation indirectly by ascertaining the sensitivity or resolving power of the organism and culminated in the derivation of the mathematical expression

$$S = K \log (\phi / \phi_0), \qquad (1)$$

where S represents magnitude of sensation, $\phi$ is the magnitude of the physical stimulus, $\phi_0$ is the absolute threshold of the physical continuum from which $\phi$ is drawn, and K is a constant determined by the Weber fraction.

With the advent of behaviorism and operationism the meaning of S changed but the form of Fechner's law remained virtually unchanged. Sensation became something that was defined operationally. Stevens (1935) noted that "since sensation cannot refer to any private or inner aspect of consciousness which does not show itself in an overt manner, it must exhibit itself to an experimenter as a differential reaction on the part of the organism [p. 524]." It became a construct that intervened between the judgment response of the organism and the physical stimulus (Guilford, 1954, p. 29).

In the years subsequent to the statement of Fechner's law, the procedures used to obtain judgments have proliferated. Contrary to expectations, the results obtained have not always been compatible with results predicted by Fechner's law and differences have not been reconciled. Hence, other psychophysical laws have been proposed. S. S. Stevens (1957) has presented evidence to support the power function as the form of the psychophysical law. The relationship found between physical intensity in Units of Energy and the sone scale of loudness (derived from the composite results of several fractionation experiments) is illustrative of the power law. Loudness in sones is proportional to intensity raised to the power of 0.3 (Stevens, 1936).

In general, the functional relationship between sensation and the physical stimulus has been expressed as

$$\Psi = C\, \phi^{n} \qquad (2)$$

where $\Psi$ represents sensation (operationally defined as a judgment response obtained from an O when using any one of the several psychophysical methods), $\phi$ is the magnitude of the physical stimulus, C is a constant, and n is the power to which the value of the stimulus is raised. The power law indicates that "equal stimulus ratios produce equal subjective ratios [Stevens, 1957, p. 153]."

Although Stevens tentatively considered the power function to be the form of a general psychophysical law holding between all perceptual and physical continua, he was definite about the power function being the form of the psychophysical law for a particular class of perceptual continua called prothetic continua. Stevens (1957; Stevens and Galanter, 1957) stated that perceptual continua can be divided into two classes. Class I was labeled *prothetic*, a derivation from a word meaning "to add", and Class II was labeled *metathetic*, a derivation from a word meaning "to substitute". These labels were suggested to Stevens by the nature of the physiological transducers for pitch and loudness continua. The loudness continuum was labeled prothetic as loudness discrimination seemed to be based on additive mechanisms on the physiological level. The pitch continuum was labeled metathetic as pitch discrimination seems to be based on substitutive mechanisms on the physiological level. That is, it has been suggested that two tones are discriminately different when they activate two separate portions of the basilar membrane (Stevens and Davis, 1937; p. 97).

There are some physical continua such as sound intensity and frequency, for which there are known physiological transducers and for which the nature of the workings of these transducers may be of primary importance. There are other perceptual continua used, such as length, proportion, and numerousness, for which the physiological transducers are of secondary importance and some abstract notion of relationship among elements of the stimulus display becomes primary in importance. Stevens stated that the physiological distinctions drawn between prothetic and metathetic classes were only suggestive and he ultimately relied upon other criteria to distinguish between the two classes. The criteria used were the subjective size of the jnd along the perceptual continuum, the form of the category rating scale for a perceptual continuum, the presence of time-order error, and hysteresis.

Stevens considered the subjective size of the jnd to be the most important criterion as he felt that it explained the other three. He has stated that Fechner's assumption about the equality of the subjective size of the jnd was invalid when applied to prothetic continua and he has based this contention on empirical evidence gathered with work on loudness (Stevens, 1936). He has presented arguments for his position that his results indicated that the subjective size of the jnd increases from the lower end to the upper end of the subjective continuum as the size of the jnd increases along the physical continuum. This situation results in a power relationship between the two continua rather than a logarithmic one. On the other hand, it was noted that the subjective size of the jnd remains relatively constant along a metathetic continuum, implying that Fechner's assumption was valid for this class.

This asymmetry of sensitivity along the prothetic continuum influences the form of its category rating scale. As the sensitivity to differences is less at the upper end of the continuum, the categories tend to be broader at that end. The resulting function relating the category scale to the physical continuum tends to be steep at the lower end of the continuum and then becomes increasingly concave downward. Such a function tends to be a logarithmic function. The form of the category scale for metathetic continua is much less curved and approximates a linear function. This asymmetry of sensitivity also influences judgments in such a manner that prothetic continua are characterized by the presence of time-order error and hysteresis. On the other hand, these systematic biases are supposedly not exhibited by metathetic continua.

Although Stevens (1936) suggested that he had an indication of the increase in size of the subjective jnd, it seems clear now that he did not. He did, however, demonstrate the lack of correspondence between a scale derived by the method of fractionation and a scale derived by the integration of jnd's. It is perhaps best to replace Stevens' first criterion with another one, i.e., the relationship between scales derived by direct ratio scaling methods and scales derived by indirect scaling methods.

Stevens (1957; Stevens and Galanter, 1957) considered the direct ratio scaling methods to be the only adequate scaling techniques to use with prothetic continua because the indirect scaling methods which have evolved from the Fechnerian point of view are too prone to distortion from the inadequate control of such second-order variables as stimulus spacing and stimulus order.

Recently, Stevens has relied heavily on the direct scaling method of magnitude estimation which is a variation of the method of fractionation in which the ratio is not specified. The _O_ is asked to assign numbers to a series of stimuli so that they are proportional to the magnitudes of the stimuli in the series. There are several variations of the method of magnitude estimation. In one method, which is called the prescribed modulus method, the _O_ is presented with a "standard" stimulus and instructed to call it a certain number; e.g., 10. He is then instructed to assign numbers to subsequent stimuli which are proportional to this modulus according to his subjective impression of the ratio of the magnitude of the variable stimuli to the magnitude of the standard. For example, if the variable stimulus appears to be twice the size of the standard, then the judgment should be 2 x 10 or 20. In a second method, the _O_ is instructed to respond to the first stimulus in terms of any modulus he wishes. The subsequent judgments should then be made in terms of the modulus used. This method has been appropriately called the no prescribed or no designated modulus method. Finally, a third method has been used which has been designated as the physical unit modulus method by Rule and Markley (1966). In this case the _O_ is instructed to respond in terms of the physical unit of the stimulus display. For example, if an O was presented with a continuum of numerousness of dots and he estimated 50 dots for a particular display, then he should respond with the number 50. Markley and Rule (1965) have suggested that these methods represent situations in which the Os are asked to use a particular judgment language.

With regard to the power function, there is a debate about what factors are responsible for the magnitude of the exponent. Stevens (1961) argued that the power law exponent varies from continuum to continuum and is determined by the characteristics of the physiological transducer involved. He stated that

> It seems rather more probable that the exponents are what they are because of the nature of the sensory transducers. It is likely, for example, that the exponents for light and sound are smaller than 1.0 because these sensory transducers behave essentially as "compressors"--a characteristic that enables them to handle the enormous dynamic ranges of stimulation to which they are subjected...It seems quite improbable that the form of this function was "learned" by the observer [p. 28].

The position taken by Warren (1958) in his physical correlate theory is illustrative of a different point of view. He argued that experience with certain physical attributes of the entire stimulus situation, and not necessarily the more conventional physical attributes, governs the form of the power law function. He noted that it is surprising that the veg scale, a heaviness scale derived using the method of fractionation, is not proportional to physical weight scales because people have so much experience with lifting objects. Warren then argued that this occurs because of the unfamiliar size-weight relationship of the stimuli presented to the _Os_ when comparison stimuli and standard stimuli are all the same size. More specifically, he stated that "the use of comparison-weights the same size as the standard would cause a decrease in their apparent weight [p. 677]." In an experiment to check this hypothesis, he found that when comparison weights were all one-half the size of the standards that half-heaviness judgments were very close to half the standard weights. Warren implied that with this allowance made for the size-weight illusion the veg and physical weight scales would be proportional.

Applying the physical-correlate theory to brightness judgments, Warren predicted the relationship which should be obtained between physical intensity and judged brightness. Brightness should be proportional to the square root of physical intensity. He began his argument by stating that it was reasonable to assume that distance is the physical correlate of brightness judgments. If so, then a relationship between brightness and physical intensity could be predicted by using information provided by the inverse-square law, i.e., that illuminance of an object varies as the inverse of the squared-distance of the object from its source of luminance intensity. Consequently, if the distance from the object to the source of luminance intensity is doubled, then the illuminance of the object is reduced by 25%. He then argued that

> Since doubling the distance of an object from its light-source reduces intensity to approximately one-quarter, the physical-correlate theory predicts that close to 25% of the standard luminance would be judged half as bright for all intensities of the standard. Expressed in a more general form, brightness is proportional to the square root of the stimulus-intensity, or $S = kI^{0.5}$, where S is sensory intensity or brightness, I is physical intensity or luminance, and k is a constant [p. 679].

This relationship should be obtained if the experimental situation is similar to familiar situations. Warren presented evidence indicating that the physical correlate of brightness was distance and Stevens (1957) listed the range of exponents for brightness as 0.3 to 0.5, which includes Warren's prediction of 0.5. In his summary statements Warren stated that "the physical correlate theory is proposed as the basis for judgments of sensory intensity [p. 687]." His results imply that the power law exponent can be changed significantly by the manipulation of the experience which the <u>O</u> has with the entire physical situation, regardless of the sensory transducers most directly involved.

A more or less intermediate position between Stevens and Warren has recently been suggested by the research of Jones and Marcus (1961) and Bruvold and Gaffey (1965). These experimenters were concerned about individual differences which occurred in the power law exponent. Jones and Marcus presented evidence in the form of prescribed modulus magnitude estimation judgments of three continua, weight, taste, and smell, that an individual exponent represents a value obtained by a multiplicative effect of _O_ on continuum. They suggested that the individual exponent should be written as

$$n(i,j) = n_i \cdot n_j, \quad (3)$$

where $n(i,j)$ is the individual exponent of the _i_th person exposed to the _j_th continuum, $n_i$ is the individual component, and $n_j$ is the component particular to a stimulus continuum. On the other hand, Bruvold and Gaffey suggested from their results that an appropriate model would be an additive one, rather than a multiplicative one. They suggested that the individual exponent should be written as

$$n(i,j) = n + n_i + n_{ij}, \quad (4)$$

where $n(i,j)$ and $n_i$ mean the same as in equation (3), $n$ is a population constant, and $n_{ij}$ is a component from the interaction between the individual and the continuum.

Regardless of which model is proven to be the more appropriate, it is clear that these experimenters provide models which might satisfy Stevens or Warren when the nature of the individual component is more fully understood. A question still remains of whether these individual differences are due to an idiosyncratic characteristic of the physiological transducer involved or different experience with the various continua. An explanation for these individual differences, suggested by Jones and Marcus, is that these differences are due to an idiosyncratic use of the number continuum.

Markley and Rule (1965; Markley, 1965; and Rule, 1966) presented evidence which suggested that the individual exponent is composed of at least two components, one, a sensory component and the other, a component which is related to the way in which the _O_ uses the designated judgment language, and hence, very much related to the way the _O_ uses the number continuum. Their procedure consisted of obtaining repeated observations from each _O_ over several different conditions using some of the judgment procedures already discussed above as well as cross-modality matching procedures. In cross-modality matching the _O_ is instructed to match his subjective judgments of one physical continuum with his judgments of another physical continuum. For example, he might be instructed to squeeze a hand dynamometer to represent his estimate of the number of dots in a numerousness display.

Markley and Rule (1965) obtained data from each _O_ over five conditions: (a) no prescribed modulus magnitude estimation of circle size, (b) physical unit modulus magnitude estimation of number of dots, (c) magnitude production of handgrip, (d) cross-modality matching of hand-grip to circle size, and (e) cross-modality matching of hand-grip to number of dots. Individual exponents were obtained for each condition and then correlated. Significant correlations were found between conditions with different physical continua but similar with respect to judgment language; e.g., magnitude production of hand-grip with cross-modality matching of hand-grip to number of dots. Furthermore, the correlations were not significant between conditions with the same stimuli but differing with respect to judgment language; e.g., physical unit modulus magnitude estimation of number of dots with cross-modality matching of hand-grip to number of dots. In addition, a zero correlation was obtained between physical unit modulus magnitude estimation of number of dots and no prescribed modulus magnitude estimation of circle size. In a later study, Rule (1966) found a significant correlation between these same two continua when judgments were made using the prescribed modulus magnitude estimation procedure for both continua.

The results obtained by these various experimenters regarding individual exponents clearly indicate the need for further research in the direction toward a better understanding of the nature of the _O_'s judgments when using the magnitude estimation procedures.

An additional line of research in this direction, initially suggested by Garner (1953), may provide information on judgmental processes. This is research on judgment sequential dependency. As the term suggests, judgment sequential dependency is characterized by findings which indicate that judgments are not independent of one another. In other words, in making a judgment, the _O_ takes into account the stimulus being presented, the stimuli previously presented, and the prior judgments he made. Garner stated that _O_s may tend to repeat the judgment response they last made when the discrimination between stimuli becomes difficult. He pointed out, however, that this was a suggestion not supported with quantitative analysis as it was not possible for him to separate the effects of the preceding stimulus on the judgment from those of the preceding judgment.

Several years earlier, in experiments using the classical psychophysical methods, Fernberger (1920), Arons and Irwin (1932), and Preston (1936), found contrast effects. They noted, using the method of constant stimuli, that Os tended to avoid repetition of categroies; e.g., they tended to alternate from "heavy" to "light" when judging weight stimuli.

Verplanck, Collier, and Cotton (1952), using visual stimuli at threshold luminance and binary responses (i.e., "yes I saw the flash of light" or "no I didn't see the flash of light"), demonstrated that the O's judgments of the stimuli were dependent both on the stimulus being presented and the judgment he made to the preceding stimuli. The dependence decreased rapidly as a function of the "lag" between the judgments being correlated. Lag refers to the number of judgments occurring between judgments correlated. These experimenters ran naive Os in an experiment that combined the use of the method of limits and a second procedure referred to as the method of single luminance. With this latter method the O was presented with a series of 300 flashes of constant luminance; the luminance being selected so that the probability of a "yes" judgment was approximately .50 for each O. This method was used to eliminate the effect of changing intensities of stimulation on the probability of judgment. It is important to note that the Os were never informed of the change from the method of limits to the method of single luminance. It was the latter series of 300 flashes that was submitted to a sequential analysis. From this analysis it was determined that successive judgments were dependent and that the tendency was toward too few runs or a tendency to repeat the last judgment made.

McGill (1957) compared the _O_'s successive judgments of tones at threshold to their successive judgments of a series of "no tone" presentations. The _O_s judged whether a tone was presented or not on each trial. Using uncertainty analysis, he found that preceding judgments had a "marked effect" on judgments made to stimuli at or slightly below the threshold level and that the preceding judgment had its greatest influence when the _O_ thought stimuli were being presented when in fact no stimuli were being presented. He further suggested that the difficulty of making judgments at threshold level contributed to the influence of the preceding judgments. The more difficult the judgment, the more the _O_ relied on the judgment he last made, regardless of the stimulus presented.

The present experiment was intended to determine if such a judgment dependency as was found with the classical psychophysical procedures is present with the new magnitude procedures. Particular interest was focused on the effect of different perceptual continua and different judgment languages on the degree of judgment dependency. The continua of interest included a metathetic continuum, proportion, and two prothetic continua, numerousness and line length. No definite hypothesis was made regarding the difference in magnitude of judgment sequential dependence which might be obtained between prothetic and metathetic continua. However, an hypothesis was put forward regarding the difference which should be obtained among the three judgment languages. It was hypothesized that judgmental dependence found under the physical unit modulus condition would be less than or equal to that found under the prescribed modulus condition. In support of this hypothesis it was argued that judgments in the physical unit modulus would be more stimulus bound than those with the other two moduli and that when no modulus is specified that _O_s tend to respond with a modulus similar to the physical unit modulus.

# METHOD

## Subjects

One hundred eighty students drawn from an introductory psychology course at the University of Alberta participated in the experiment. Participation in an experiment met a course requirement.

## Apparatus and Materials

The stimuli used in this experiment were prepared on 35-mm. slides and when projected appeared white on a black background. They were presented via rear projection on a 23 in. by 28 in. white construction paper screen. The screen was mounted in the center of a dull black partition which divided the experimental room into two parts. Behind the partition was the projector and in front of the partition were situated three small tables separated by low partitions. These tables were placed approximately 5 feet in front of the projection screen. Each table was equipped with a shielded six volt bulb which provided sufficient illumination for the _O_s to write their responses. These bulbs provided the only illumination on the _O_'s side of the partition while the stimuli were being presented.

The slides were projected with a Viewlex, manually operated slide projector. Slide exposure time was controlled by a manually operated Wollensak Alphax shutter mounted at the front of the projector.

The numerousness stimuli were patterns of dots spread randomly over a constant circular area. The continuum consisted of displays containing 9, 12, 17, 23, 31, 43, 59, and 82 dots. The line-length continuum contained a series of stimuli with projected lengths of 0.76, 1.12, 1.60, 2.40, 3.48, 5.32, 7.56, and 11.16 inches. For the proportion continuum the stimuli consisted of a series of displays consisting of lines and dots arranged in eight rows and ten columns. The total number of elements for each display remained constant at 80 while the proportion of lines and dots differed. An example of the proportion stimuli used containing an equal number of lines and dots is presented in Figure 1. The continuum contained the following stimuli, stated in terms of percentage of dots: 6.25, 18.75, 32.50, 43.75, 56.25, 68.75, 81.25, and 93.75.

## Procedure

The Os were divided into nine groups of 20 Os each. Conditions were assigned to the Os on the basis of a 3 x 3 factorial design. The factors were: (a) stimulus continuum with levels numerousness of dots, line-length, and proportion, and (b) magnitude estimation judgment language with levels

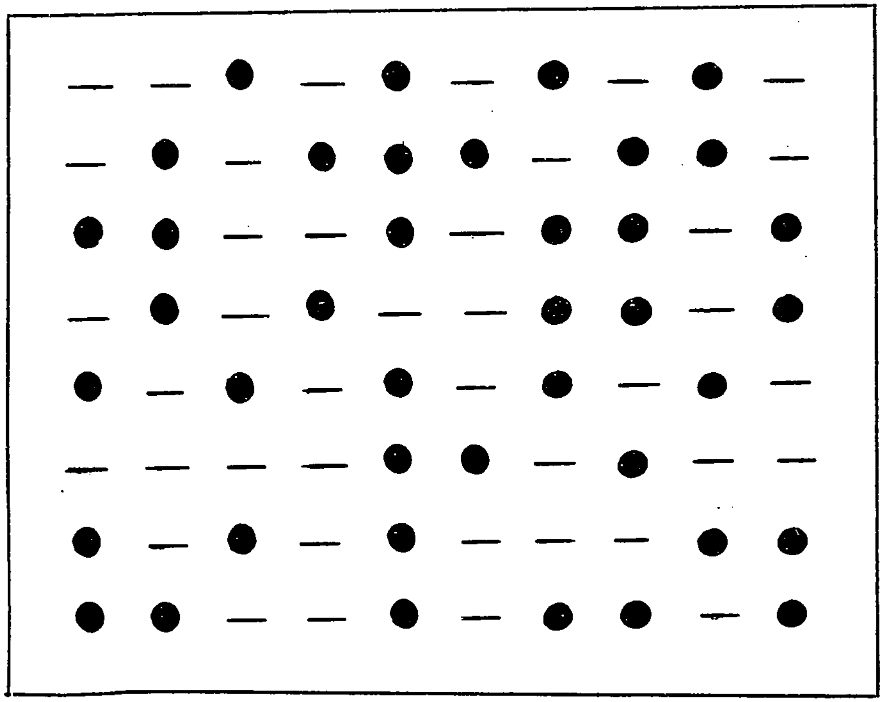

Fig. 1. Example of the stimulus display for proportion.

physical unit modulus, no prescribed modulus, and prescribed modulus.

The Os were run in groups of three or less. They were escroted into the experimental room, seated at the tables with pencils and record sheets in front of them. When they were seated the E read the instructions which were appropriate for the condition. The instructions varied for the three judgment languages. For each judgment language the instructions were approximately the same across continua. Instructions, of course, varied with regard to continua used.

For the respective judgment languages the instructions used were:

Physical unit modulus. This is an experiment on how people judge numerousness, line-length, proportion). You will be presented a series of slides containing (patterns of dots, lengths of line, both lines and dots). Your task will be to judge the (number of dots, length of line in inches, proportion of dots) on each slide. Please judge each slide as it is presented and make sure you have a judgment for each slide on your papers. Try to make each judgment as quickly as possible. Each slide will be presented briefly at 10 second intervals. Consequently you will be given only 10 seconds to make your judgment. Are there any questions?

No prescribed modulus. This is an experiment on how people judge (numerousness, line-length, proportion). You will be presented a series of slides containing (patterns of dots, lengths of line, both lines and dots). Your task will be to judge the (number of dots, length of line, proportion of dots) on each slide. I want you to assign a number to the first slide of the series. You may use any number which seems appropriate. Your task is to then assign numbers which are proportional to your subjective impression for the remaining slides. For example, if one of the slides presented seems to contain (three times as many dots as, a line three times as long as that on, three times the proportion of dots as) the first slide, assign a number three times as large as that assigned to the first. If it seems to contain (one-fifth the number of dots, a line one-fifth as long, one-fifth the proportion of dots), assign a number one-fifth as large, and so on. Please judge each slide as it is presented and make sure you have a judgment for each slide on your papers. Try to make each judgment as quickly as possible. Each slide will be presented briefly at 10 second intervals. Consequently you will be given only 10 seconds to make your judgment. Are there any questions?

<u>Prescribed modulus</u>. This is an experiment on how people judge (numerousness, line-length, proportion). You will be presented a series of slides containing (patterns of dots, lengths of line, both lines and dots). Your task will be to judge the (number of dots, length of line, proportion of dots) on each slide. I want you to assign the number 10 to the first slide. Your task is to then assign numbers which are proportional to your subjective impression for the remaining slides. For example, if one of the slides presented is judged to contain (three times as many dots as the standard, a line that is three times as long as that on the first slide, three times the proportion of dots as the standard), assign a number three times as large or 30. If it seems to contain (one-fifth as many dots, a line that is one-fifth as long, one-fifth the proportion of dots), assign a number one-fifth as large or 2, and so on. Please judge each slide as it is presented and make sure you have a judgment for each slide on your papers. Try to make each judgment as quickly as possible. Each slide will be presented briefly at 10 second intervals. Consequently you will be given only 10 seconds to make your judgment. Are there any questions?

After the instructions were read the room lights were extinguished and the E presented the stimuli.

Each stimulus was exposed for one second and approximately 10 seconds elapsed between each presentation. The stimuli were presented in random order with the restriction that each stimulus preceded every stimulus including itself an equal number of times. This restriction was necessary in order that the stimuli presentations not be linearly correlated. There were 65 presentations in all, therefore each stimulus preceded every other stimulus once. The stimulus presented first was also presented last, hence this stimulus was presented one more time than each of the others.

# RESULTS

A measure of judgment sequential dependency was calculated for each *O* which reflected the degree to which each judgment was related to the preceding judgment. The dependency measure was a product-moment correlation coefficient obtained by pairing a transformation of each judgment with a transformation of the preceding judgment. These pairs may be illustrated as follows:

$$
\begin{array}{ccccccccccccccc}
w_1 & w_2 & w_3 & w_4 & . & . & . & . & . & . & w_{63} & w_{64} & w_{65} & \\
 & w_1 & w_2 & w_3 & . & . & . & . & . & . & w_{62} & w_{63} & w_{64} & w_{65}
\end{array}
$$

The correlation coefficient was computed between the values in the first row and those in the second row. The first value is paired with the second, the second with the third, the third with the fourth, and so on. Notice that $w_1$ in the first row is not paired with any value in the second row, and $w_{65}$ in the second row is not paired with any value in the first row. Hence the coefficient was computed on 64 pairs.

The correlation coefficient was not computed on the *O*'s judgments *per se*, but on a transformation of the judgments. The transformation was necessary (a) to partial out the effect of stimulus magnitude on *O*'s judgments and (b) to provide values which were comparable regardless of judgment language, moduli, and continuum class.

The transformation procedure is illustrated in the diagram in Figure 2. Each judgment, $X_{ij}$, corresponding to the _i_th judgment of the _j_th stimulus, where _i_ ranges over 8 or 9 depending upon _j_ and _j_ ranges over 8, was transformed to a common logarithm, log $X_{ij}$. The mean of the transformed values was calculated for each stimulus by: (a) summing over the _i_ values for each _j_th stimulus,

$$\sum_{i=1}^{m_j} \log X_{ij} \ , \qquad m_j = \begin{cases} 9 \text{ for } j = \text{5th stimulus} \\ 8 \text{ for } j \neq \text{5th stimulus} \end{cases} \tag{5}$$

and (b) dividing by $m_j$

$$\overline{\log X.j} = \sum_{i=1}^{m_j} \log X_{ij}/m_j . \tag{6}$$

Note that the 5th stimulus was presented first and last for each condition.

A deviation value, $w_{ij}$, was determined by subtracting the mean of the logs, $\overline{\log X.j}$, for the _j_th stimulus from log $X_{ij}$

$$w_{ij} = \log X_{ij} - \overline{\log X.j} . \tag{7}$$

The w's were then arranged in the order of the occurrence of the stimuli and the correlation coefficient was calculated

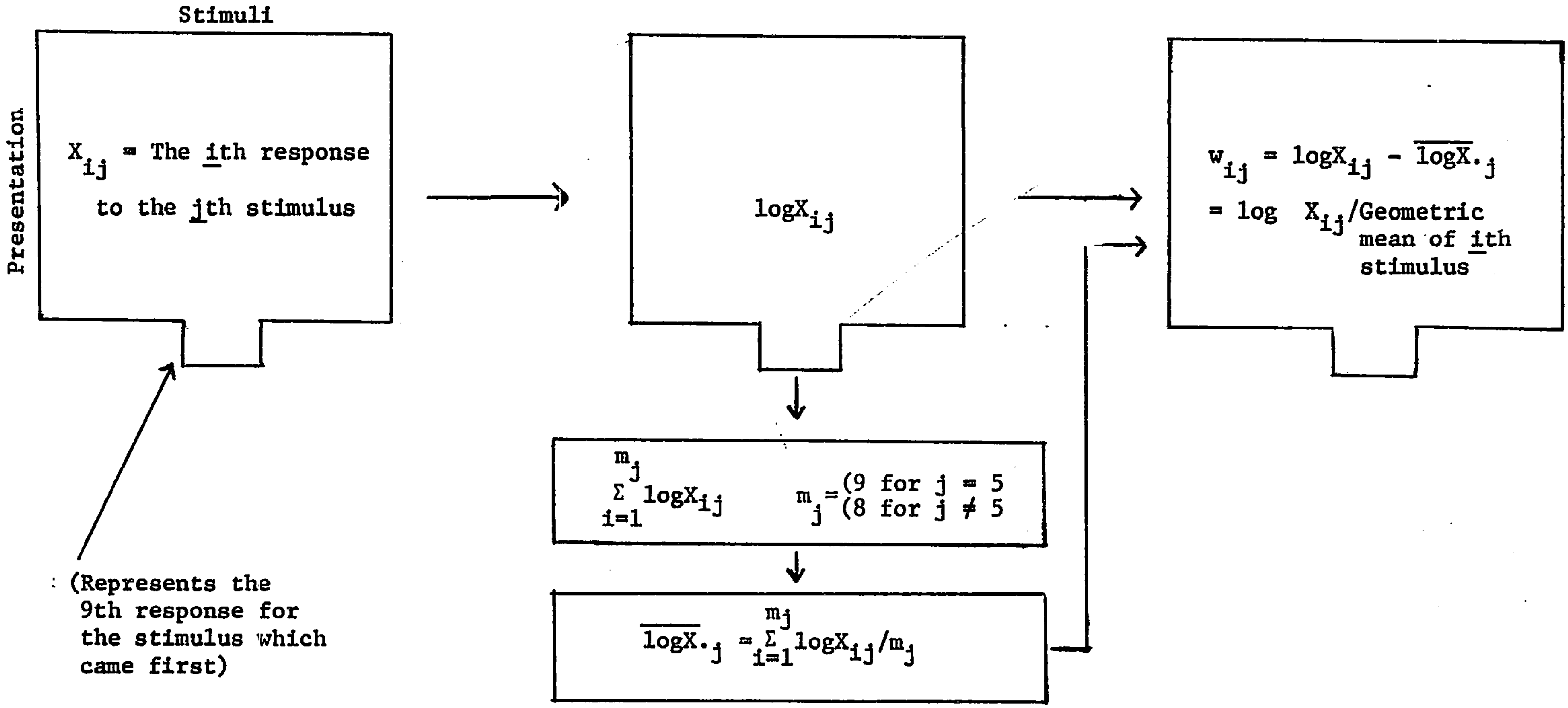

Fig. 2. A diagram of the procedures used to transform the judgments to values used in the calculation of the measure of sequential dependency.

on the pairs of w's,

$$r_{w_{ij}(1,64),\, w_{ij}(2,65)} = \frac{\mathrm{Cov}[w_{ij}(1,64),\, w_{ij}(2,65)]}{\sigma_{w_{ij}}(1,64)\ \sigma_{w_{ij}}(2,65)} . \quad (8)$$

As a result of the transformation, the dependency measure represents an index of association which is related to the linear correlation between the ratio of a judgment of a stimulus magnitude to the geometric mean of all the judgments made to that stimulus and a like ratio of a judgment made immediately preceding it in time.

To illustrate the relationship between the judgments _per se_ and the transformed values as well as to more clearly visualize what the dependency measure represents, hypothetical data were determined which yields a dependency coefficient of unity. These data are for four stimuli. For each stimulus to follow every stimulus including itself an equal number of times a total of at least 17 presentations is required. The hypothetical judgments and the corresponding transformations are presented in Figure 3. The stimuli are indicated by the letters A, B, C, and D, in rank order from smallest to largest. The judgments are presented in Figure 3A. It can be seen that each judgment to a particular stimulus increased by a constant proportion on each subsequent presentation of that stimulus. For example, the first judgment to stimulus

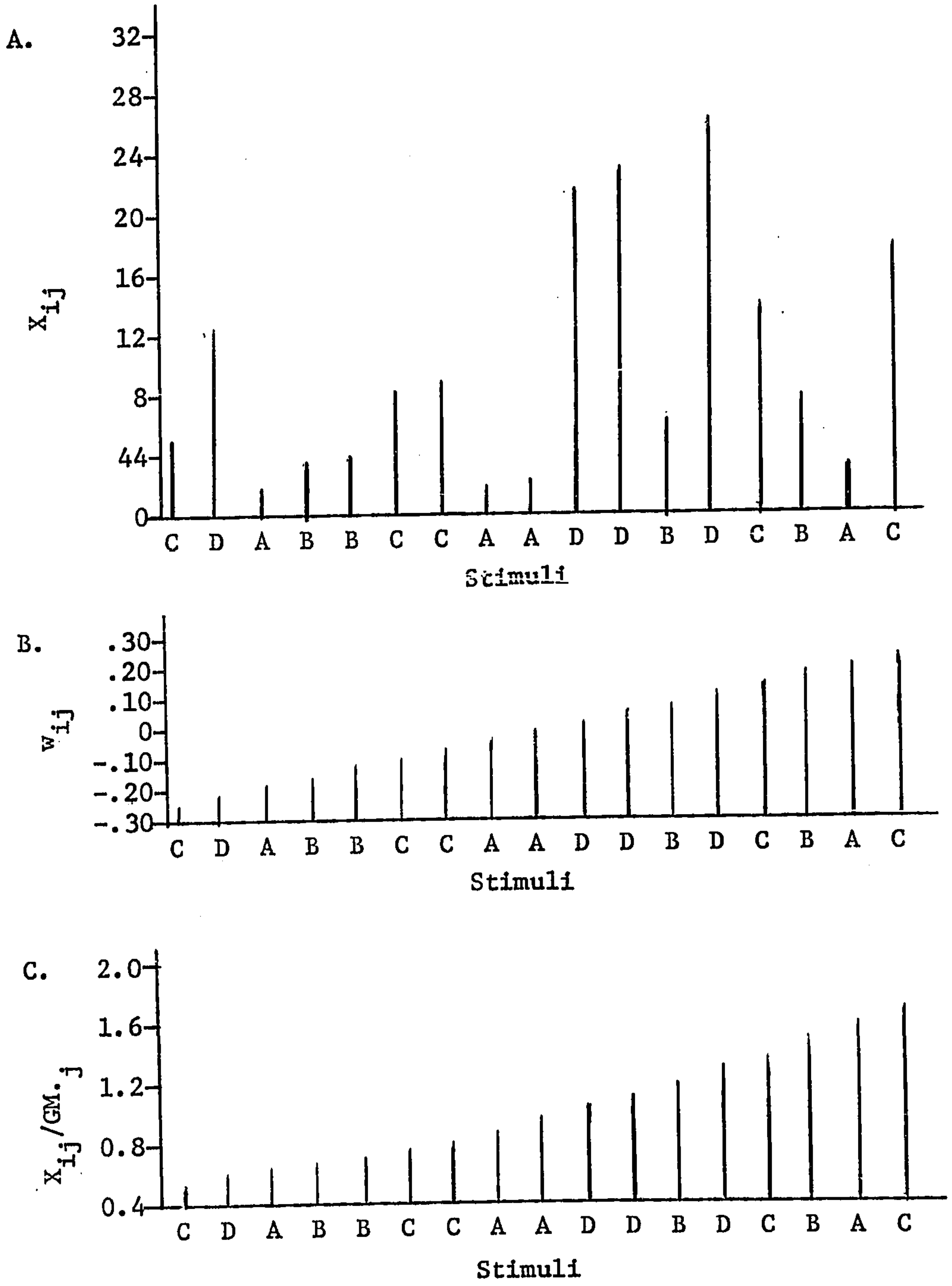

Fig. 3. Hypothetical data which yield a dependency coefficient of unity.

C was 5.75 and the final judgment to stimulus C was 17.38. Figure 3B presents the respective values of $w_{ij}$ for each of the stimulus presentations. It can be seen that there is a definite linear increase in the size of the difference between log $X_{ij}$ and $\overline{\log X}.j$. In Figure 3C is presented the antilog transformation of $w_{ij}$ or the ratio of a judgment of a stimulus magnitude to the geometric mean of all the judgments made to that stimulus.

It should be noted that if the O's judgments are consistent for each stimulus, i.e., if the values remain the same throughout the entire experiment, then this dependency measure would be indeterminate.

Turning from the hypothetical example to the results actually obtained, the mean, range, and standard deviation of the dependency coefficients for each group are presented in Table 1.

A t test was used to determine if the mean dependency coefficient for each group was significantly different from a mean correlation of zero. It was found that each of the mean dependency coefficients was significantly different from a mean of zero at a level beyond the .01 level. These results indicate that in every condition there was in fact a tendency for each judgment to depend on the preceding judgment. It

TABLE 1

Mean, Standard Deviation, and Range of the Group Judgment Dependency Coefficients

| Judgment Language | Continua | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Prothetic | | | | | | Metathetic | | |
| | Numerousness of Dots | | | Line Length | | | Proportion | | |
| | Mean | SD | Range | Mean | SD | Range | Mean | SD | Range |
| Physical Unit Modulus | 0.23 | 0.15 | 0.56–0.02 | 0.24 | 0.14 | 0.56–0.01 | 0.19 | 0.10 | 0.38–(–0.15) |
| No Prescribed Modulus | 0.35 | 0.16 | 0.64–0.05 | 0.35 | 0.23 | 0.74–0.02 | 0.21 | 0.11 | 0.47–0.02 |
| Prescribed Modulus | 0.27 | 0.13 | 0.49–0.07 | 0.31 | 0.24 | 0.78–(–0.18) | 0.27 | 0.15 | 0.58–0.12 |

may be noted that only three Os out of the 180 showed a correlation of zero or less.

In addition to the judgment dependency measure, power law exponents were determined for each O using the method of least squares for the equation

$$\log \Psi = \log k + n \log \phi \qquad (9)$$

where $\Psi$ is the geometric mean of the judgments made to each stimulus, $\phi$ is the physical value of the stimulus, and n, the slope of the line of best fit, is the exponent of the power law function. Group exponents were determined by summing over the individual O exponents in their respective groups. The group exponents and two indications of the variability of the exponents within each group, the standard deviation and the range, appear in Table 2. The individual O exponents, together with their respective dependency coefficients are listed in Appendix A.

To determine if the exponents were related to the dependency coefficients a Pearson product-moment correlation was computed between the measure of judgment dependency and the power exponent for Os in each of the conditions. The correlation for each condition is presented in Table 3. The results indicated that the average r for all conditions was -0.06 and that in all but one case the correlations obtained were not

## TABLE 2

### Mean, Standard Deviation, and Range of Group Power Law Exponents

| Judgment Language | Continua | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Prothetic | | | | | | Metathetic | | |
| | Numerousness of Dots | | | Line Length | | | Proportion | | |
| | Mean | SD | Range | Mean | SD | Range | Mean | SD | Range |
| Physical Unit Modulus | 0.77 | 0.18 | 1.38-0.55 | 0.97 | 0.10 | 1.11-0.78 | 1.08 | 0.18 | 1.47-0.82 |
| No Prescribed Modulus | 0.91 | 0.19 | 1.32-0.67 | 0.98 | 0.12 | 1.18-0.76 | 0.99 | 0.24 | 1.50-0.69 |
| Prescribed Modulus | 0.98 | 0.21 | 1.46-0.67 | 1.04 | 0.15 | 1.33-0.80 | 0.96 | 0.20 | 1.33-0.47 |

TABLE 3

Correlation Between Exponents and the

Judgment Dependency Coefficients

| Judgment Language | Continua | | |
|---|---|---|---|
| | Prothetic | | Metathetic |
| | Numerousness of Dots | Line Length | Proportion |
| Physical Unit Modulus | -0.39 | -0.47* | 0.14 |
| No Prescribed Modulus | 0.05 | -0.15 | 0.11 |
| Prescribed Modulus | -0.11 | -0.01 | 0.30 |

* $P<0.05$

## TABLE 4

## Analysis of Variance of Judgment Dependency Coefficients

| Source of Variation | df | Mean Square | F |
|---|---|---|---|
| Continua | 2 | 0.09375 | 3.31* |
| Language | 2 | 0.12037 | 4.25* |
| C x L Interaction | 4 | 0.02985 | 1.05 |
| Within | 171 | 0.02834 | |
| Total | 179 | | |

*$P < 0.05$

## TABLE 5

## Results of Duncan's Multiple Range Test

| Continua | Numerousness of Dots | Line Length | Proportion |
|---|---|---|---|
| Means | *0.285 | 0.298 | 0.224 |
| Judgment Language | Prescribed Modulus | No Prescribed Modulus | Physical Unit Modulus |
| Means | *0.282 | 0.305 | 0.219 |

*means underlined by the same single line were not significantly different.

significantly different from a zero correlation using a two-tailed t test of significance. Using an experimentwise protection level it was determined that one or more significant correlations out of nine could occur by chance about 40% of the time, (Balaam and Federer, 1965). Assuming that the one significant correlation was due to chance, these results suggest that within each condition, the individual O power law exponents were not linearly related to the individual judgment dependency coefficients.

The dependency coefficients were submitted to a 3 x 3 factorial analysis of variance. The factors were: (a) stimulus continuum with levels numerousness of dots, line-length, and proportion, and (b) magnitude estimation language with levels physical unit modulus, no prescribed modulus, and prescribed modulus. The analysis, summarized in Table 4, yielded significant continua and judgment language effects beyond the .05 level with Fs = 3.31 and 4.25 respectively, both with df = 2/171.

A subsequent analysis with Duncan's multiple range test performed on the main effects indicated that judgment sequential dependency was significantly less for judgments of the proportion continuum than for judgments of the line-length and numerousness of dots continua, and that dependence was significantly less for judgments with the physical unit modulus language than for judgments with the no prescribed modulus language and the prescribed modulus language. In both cases the significance was beyond the .05 level. The results of the Duncan's test are summarized in Table 5.

# DISCUSSION

In the present experiment it was found that judgments with the method of magnitude estimation were subject to a particular kind of sequential dependency. The measure of judgment sequential dependency used was an index of association which is related to the linear correlation between the ratio of a judgment of a stimulus magnitude to the geometric mean of all the judgments made to that stimulus and a like ratio of a judgment made immediately preceding it in time. The degree of dependency was significant for all conditions. In addition, it was found that the amount of dependency for judgments of the proportion continuum was significantly less than that for judgments of the line-length and numerousness of dots continua and that the amount of dependency for judgments with the physical unit modulus was significantly less than that for judgments with the prescribed modulus and no prescribed modulus languages.

It is not yet certain whether these results will be found with other representatives of prothetic and metathetic continua; but if so, they provide further evidence that <u>O</u>s respond differently when presented with continua from these two classes.

The difference in degree of dependency found with the judgment languages provide support for the position that Os respond differently when using the various judgment languages in addition to the evidence provided by differences in power law exponents. That this is additional support is indicated by the finding that, with the exception of one group, the dependency measures and the power law exponents were not linearly related to a significant degree.

The results obtained provide data for speculations on the manner in which the O makes his judgments using the various judgment languages. Stevens (1966) has described the judgment process as an act in which the O "selects from one domain an item that matches, in some respect or other, an item drawn from another domain [p. 385]." The O in the psychophysical experiment acts as a measuring instrument as he attempts to match a value from one perceptual continuum with a corresponding value from another perceptual continuum. With the magnitude estimation procedures used in this experiment it is assumed that the O was matching his subjective impression of items from the number continuum with his subjective impression of the magnitudes of stimuli presented. Compared with many measuring instruments the human organism is relatively unreliable. For several reasons he responds differently to the same stimulus presented at different times. This variability may be due to fluctuations in sensory transducers and/or what might be referred to as cognitive processes. In this latter somewhat amorphous category one might include the <u>O</u>'s use of the number continuum and his judgment criteria. Fluctuations can occur with either or both of the perceptual continua the <u>O</u> is attempting to match. It is difficult however, to see what role sensory transducers would play in fluctuations of judgments made with the number continuum.

Variability in judgments to the same stimuli was evident under all three judgment language conditions, regardless of the continuum presented. Inspection of the group means of the product of standard deviations used as the denominator in the calculation of the dependency measure

$$\sigma_{w_{ij}}(1,64)\sigma_{w_{ij}}(2,65), \qquad (10)$$

indicated that variability was greater under the prescribed and no prescribed modulus language conditions than under the physical unit modulus language condition (Table 6).

What are the obvious differences between the prescribed modulus languages and the physical unit modulus language that might contribute to variability as well as degree of sequential dependency? First of all, the nature of the task imposed on the <u>O</u> by the prescribed modulus conditions is such that he could be variable in his number assignments by

## TABLE 6

### Group Means of the Product of

### $\sigma_{w_{ij}}(1,64)$ and $\sigma_{w_{ij}}(2,65)$

| Judgment Language | Continua | | |
|---|---|---|---|
| | Prothetic | | Metathetic |
| | Numerousness of Dots | Line Length | Proportion |
| Physical Unit Modulus | 0.30 | 0.26 | 1.45 |
| No Prescribed Modulus | 0.62 | 0.59 | 1.46 |
| Prescribed Modulus | 0.74 | 0.67 | 1.66 |

being inconsistent in his impressions of the ratio between the standard stimulus and a variable stimulus. Or, if memory of the standard fails him the O̲ might use other stimuli as a basis for his judgments and might be inconsistent in his impressions of the ratio between two variable stimuli. The O̲ might also be inconsistent in his impression of the absolute magnitude of the stimuli. Secondly, although it sounds contradictory, the O̲ might be inconsistent in his number assignments by being relatively consistent in his impression of the relationship between the stimuli presented. Such a situation could occur for several different reasons; but the simple example presented in Figure 4 should clarify what is suggested here.

| | | | | | | |
|---|---|---|---|---|---|---|
| Stimuli | $\phi_A$ | | $\phi_B$ | | $\phi_A$ . . . | $\phi_A$ |
| Judgment | 10 | | 22 | | 11 . . . | 20 |
| Ratio of stimuli | 1 | : | 2 | : | 1 . . . . | . |

Fig. 4. Hypothetical series of inconsistent judgments

On the first presentation of $\phi_A$ the O̲ responds with "10", on the first presentation of $\phi_B$ he responds as if he judges the ratio between $\phi_A$ and $\phi_B$ to be 1 : 2.2 or as if the stimulus appeared to be larger than it was. On the second presentation of $\phi_A$ his judgment is in accordance with what was presented. His judgments are in accordance with the stimuli presented for several more presentations and then other shifts occur so that on the final presentation of $\phi_A$ he responds with "20". It can be seen that there is a gradual shift upward in his numerical assignments to $\phi_A$. The measure of dependency used in this experiment is sensitive to such shifts in judgments.

Under the physical unit modulus instructions the *O* needs only to consider the absolute magnitudes of the stimuli presented. He is not instructed to be concerned about the ratio between a standard stimulus and variable stimuli. He is also probably not as concerned about the relationship between successive stimuli. Furthermore, as he has no standard stimulus to lose sight of, he is perhaps not as concerned about the relationship between successive stimuli when making his judgments. While there will of course be inconsistencies, the task set by these instructions appears to be relatively easier.

It is difficult to say why variability occurred. It seems easiest to eliminate the possibility of fluctuations in sensory transducers since the visual system was involved in all conditions. There are other possibilities. Poulton

(1968) stated in his learned-calibration theory that Os "learn to allocate numbers to sensory magnitudes [p. 16]." Because Os learn to perform this matching, their judgments will be subject to variation with variation in such things as context, time, and adaptation level. Poulton noted that Os seem to re-calibrate during the course of a psychophysical experiment. Using Poulton's term, when an O selects a new modulus in the course of an experiment for one reason or another, he is re-calibrating. The O's judgments are influenced by his expectations. The naive O's expectations are usually that the experimenter is presenting a series of different stimuli, not the same stimuli repeated several times. If and when the O realizes that the same stimuli have been presented, he usually reports that he is aware that only a few have been repeated. In the prescribed modulus conditions the O or the experimenter assigns a value to the standard stimulus which is not always used as the standard stimulus and is not always recognized if presented during the course of the experiment. In looking over the O's judgments it is found that Os frequently assign different numbers to the standard stimulus when it is presented several times during the course of the experiment.

Fluctuations might occur because of the inability of the

O to use the number continuum consistently. If, as Rule (1969) suggested, the number continuum is a prothetic continuum, then there is perhaps as little subjective difference between numbers like 30 and 35 as there is between 30 dots and 35 dots.

It is not clear what was happening with judgments of the proportion continuum as the results indicate that Os were relatively more variable in their judgments under all judgment language conditions (Table 6). It appears that Os were making their judgments relatively less dependent of one another regardless of the judgment language. The nature of the proportion stimuli may provide a possible reason why these results were obtained. Each proportion stimulus appears to provide its own frame-of-reference or more of a frame-of-reference than found with the numerousness and line-length continua. The O made his judgments in terms of proportion of dots to lines for each stimulus and was perhaps not as concerned about the relationship between stimuli.

# SUMMARY

One hundred eighty Os were divided into nine groups of 20 Os each. Conditions were assigned to the Os on the basis of a 3 x 3 factorial design. The factors were: (a) stimulus continuum with levels numerousness of dots, line-length, and proportion, and (b) magnitude estimation judgment language with levels physical unit modulus, no prescribed modulus, and prescribed modulus.

A measure of judgment sequential dependency was calculated for each O which reflected the degree to which each judgment was related to the preceding judgment. The measure used was an index of association which is related to the linear correlation between the ratio of a judgment of a stimulus magnitude to the geometric mean of all of the judgments made to that stimulus and a like ratio of a judgment made immediately preceding it in time. In addition to the judgment dependency measure, power law exponents were determined for each O.

Analysis of the results indicated that: (a) judgment sequential dependency was significantly less for judgments of the proportion continuum than for judgments of the line-

length and numerousness of dots continua, (b) judgment sequential dependency was significantly less for judgments with the physical unit modulus language than for judgments with the no prescribed modulus language and the prescribed modulus language, and (c) within each condition, the individual power law exponents were not linearly related to the individual judgment dependency coefficients.

The different results obtained with the physical unit modulus language and the prescribed modulus languages were interpreted as providng information about some of the ways in which the _O_ makes his judgments when using these languages.

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# APPENDICES

# APPENDIX A

# Appendix A

## Individual Intercept Coefficients, Exponents, and Judgment Dependency Coefficients

Numerousness of Dots Continuum

| | Physical Unit Modulus | | | Prescribed Modulus | | | No Prescribed Modulus | | |
|---|---|---|---|---|---|---|---|---|---|
| Observer | k* | n | r | k* | n | r | k* | n | r |
| 1 | 0.34 | 0.68 | 0.56 | 0.32 | 0.69 | 0.17 | -0.63 | 1.32 | 0.59 |
| 2 | 0.22 | 0.84 | 0.15 | -0.11 | 0.98 | 0.49 | 0.32 | 0.69 | 0.21 |
| 3 | -0.38 | 1.38 | 0.02 | -0.82 | 1.13 | 0.41 | 0.02 | 0.73 | 0.58 |
| 4 | 0.34 | 0.73 | 0.34 | 0.22 | 0.79 | 0.42 | 0.23 | 0.78 | 0.36 |
| 5 | 0.21 | 0.75 | 0.15 | -0.40 | 1.04 | 0.21 | 0.16 | 0.93 | 0.05 |
| 6 | 0.37 | 0.66 | 0.10 | -0.52 | 1.01 | 0.14 | -0.80 | 1.26 | 0.15 |
| 7 | -0.06 | 1.14 | 0.14 | -0.29 | 0.84 | 0.18 | 0.15 | 0.83 | 0.39 |
| 8 | 0.26 | 0.77 | 0.40 | -0.33 | 0.94 | 0.07 | -0.81 | 1.22 | 0.30 |
| 9 | 0.50 | 0.55 | 0.41 | 0.31 | 0.67 | 0.33 | 0.27 | 0.85 | 0.33 |
| 10 | 0.25 | 0.76 | 0.12 | 0.03 | 0.67 | 0.46 | 0.43 | 0.69 | 0.40 |
| 11 | 0.27 | 0.77 | 0.22 | -0.35 | 0.88 | 0.17 | 0.08 | 0.87 | 0.11 |
| 12 | 0.51 | 0.58 | 0.31 | -0.83 | 1.19 | 0.20 | 0.37 | 0.73 | 0.52 |
| 13 | 0.34 | 0.68 | 0.27 | -0.97 | 1.28 | 0.32 | 0.22 | 0.85 | 0.32 |
| 14 | 0.25 | 0.81 | 0.26 | -0.18 | 0.79 | 0.26 | 0.06 | 0.93 | 0.27 |
| 15 | 0.43 | 0.68 | 0.38 | -1.28 | 1.46 | 0.13 | 0.11 | 0.97 | 0.55 |
| 16 | 0.28 | 0.73 | 0.03 | -1.06 | 1.19 | 0.48 | -0.16 | 1.11 | 0.64 |
| 17 | 0.29 | 0.79 | 0.20 | 0.36 | 0.71 | 0.29 | -0.29 | 1.06 | 0.20 |
| 18 | 0.31 | 0.75 | 0.02 | -0.49 | 1.03 | 0.07 | 0.28 | 0.67 | 0.28 |
| 19 | 0.48 | 0.60 | 0.12 | -0.79 | 1.11 | 0.36 | -0.11 | 0.92 | 0.45 |
| 20 | 0.28 | 0.80 | 0.48 | -0.38 | 1.00 | 0.23 | -0.34 | 0.82 | 0.34 |

* Intercept Coefficients are expressed in $\log_{10}$ units

Appendix A (Cont'd.)

Length of Line Continuum

| | Physical Unit Modulus | | | Prescribed Modulus | | | No Prescribed Modulus | | |
|---|---|---|---|---|---|---|---|---|---|
| Observer | k* | n | r | k* | n | r | k* | n | r |
| 1 | -0.02 | 1.00 | 0.20 | 0.67 | 0.89 | 0.46 | 0.03 | 0.91 | 0.20 |
| 2 | -0.08 | 1.07 | 0.28 | 0.57 | 0.88 | 0.02 | 0.78 | 0.89 | 0.55 |
| 3 | 0.17 | 0.89 | 0.25 | 0.21 | 1.09 | 0.57 | 0.22 | 1.02 | 0.74 |
| 4 | 0.02 | 1.00 | 0.29 | 0.44 | 0.82 | 0.72 | 0.18 | 1.00 | 0.39 |
| 5 | 0.02 | 1.07 | 0.37 | 0.41 | 1.06 | 0.24 | 0.70 | 0.78 | 0.13 |
| 6 | 0.12 | 0.86 | 0.37 | 0.42 | 1.11 | 0.13 | 0.22 | 0.98 | 0.54 |
| 7 | 0.05 | 1.06 | 0.29 | 0.43 | 0.90 | 0.59 | 0.47 | 0.85 | 0.05 |
| 8 | 0.26 | 0.78 | 0.39 | 0.56 | 1.16 | 0.36 | 1.29 | 1.05 | 0.14 |
| 9 | 0.06 | 1.00 | 0.24 | 0.23 | 1.05 | 0.25 | 1.17 | 1.00 | 0.70 |
| 10 | 0.17 | 0.94 | 0.35 | 0.52 | 0.80 | -0.18 | -0.24 | 1.03 | 0.11 |
| 11 | 0.06 | 0.98 | 0.16 | 0.72 | 1.13 | 0.34 | 0.72 | 0.76 | 0.41 |
| 12 | 0.17 | 0.79 | 0.28 | 0.19 | 1.32 | 0.33 | 0.45 | 0.98 | 0.25 |
| 13 | -0.10 | 1.11 | 0.15 | 0.32 | 1.18 | 0.00 | 0.49 | 1.13 | 0.30 |
| 14 | 0.16 | 0.80 | 0.56 | 0.59 | 0.99 | 0.26 | 1.62 | 1.14 | 0.05 |
| 15 | 0.09 | 1.06 | 0.01 | 0.04 | 1.16 | 0.78 | 0.24 | 1.15 | 0.53 |
| 16 | 0.19 | 1.01 | 0.02 | 0.45 | 0.89 | 0.49 | 0.47 | 1.18 | 0.26 |
| 17 | 0.19 | 0.99 | 0.12 | 0.42 | 1.10 | 0.31 | 0.36 | 0.86 | 0.71 |
| 18 | 0.12 | 0.91 | 0.06 | 0.43 | 1.01 | 0.13 | 0.02 | 0.88 | 0.48 |
| 19 | 0.00 | 1.04 | 0.28 | 0.39 | 1.33 | 0.17 | -0.50 | 0.93 | 0.48 |
| 20 | 0.15 | 1.03 | 0.01 | 0.46 | 0.97 | 0.14 | 0.43 | 1.09 | 0.02 |

* Intercept Coefficients are expressed in $\log_{10}$ units

Appendix A (Cont'd.)

Proportion Continuum

| | Physical Unit Modulus | | | Prescribed Modulus | | | No Prescribed Modulus | | |
|---|---|---|---|---|---|---|---|---|---|
| Observer | k* | n | r | k* | n | r | k* | n | r |
| 1 | -0.87 | 1.44 | 0.26 | -0.03 | 0.85 | 0.24 | -0.45 | 0.71 | 0.31 |
| 2 | -0.02 | 1.01 | 0.15 | -0.55 | 0.99 | 0.24 | -0.43 | 0.73 | 0.27 |
| 3 | -0.44 | 1.20 | 0.25 | -0.18 | 0.74 | 0.03 | -0.28 | 1.11 | 0.14 |
| 4 | 0.04 | 0.92 | 0.29 | -0.65 | 1.15 | 0.31 | -0.03 | 0.86 | 0.18 |
| 5 | -0.28 | 1.11 | 0.17 | -0.54 | 0.76 | 0.22 | -0.18 | 0.98 | 0.13 |
| 6 | -0.02 | 0.91 | 0.27 | -0.95 | 1.15 | 0.08 | -0.26 | 0.77 | 0.24 |
| 7 | 0.28 | 0.83 | 0.10 | -0.89 | 1.16 | 0.31 | -0.64 | 0.94 | 0.08 |
| 8 | -0.05 | 1.02 | -0.05 | -0.56 | 1.10 | 0.27 | -0.97 | 1.15 | 0.28 |
| 9 | -0.30 | 1.12 | 0.14 | -1.02 | 1.07 | 0.56 | -0.07 | 0.86 | 0.04 |
| 10 | -0.37 | 1.18 | 0.06 | 0.03 | 0.47 | 0.02 | -1.56 | 1.26 | 0.33 |
| 11 | 0.10 | 0.91 | 0.21 | -0.94 | 1.09 | 0.19 | -0.26 | 0.74 | 0.32 |
| 12 | -0.25 | 1.08 | 0.18 | -0.37 | 0.80 | 0.55 | -0.24 | 0.85 | 0.36 |
| 13 | -0.20 | 1.04 | 0.38 | -0.49 | 0.89 | 0.20 | -1.83 | 1.04 | 0.09 |
| 14 | 0.09 | 0.93 | 0.31 | -1.35 | 1.33 | 0.23 | -1.06 | 1.39 | 0.22 |
| 15 | -0.85 | 1.39 | 0.10 | -0.40 | 0.87 | 0.23 | -2.52 | 1.50 | 0.12 |
| 16 | -0.07 | 1.02 | 0.07 | -0.53 | 0.80 | 0.20 | -0.51 | 0.88 | 0.15 |
| 17 | 0.32 | 0.82 | 0.12 | -0.62 | 0.93 | 0.35 | 0.01 | 0.69 | 0.20 |
| 18 | -0.50 | 1.20 | 0.26 | 0.15 | 0.75 | 0.39 | -0.47 | 0.94 | 0.28 |
| 19 | -0.39 | 1.18 | 0.16 | -0.66 | 1.14 | 0.23 | -0.24 | 0.89 | 0.02 |
| 20 | -0.96 | 1.47 | 0.31 | -0.91 | 1.17 | 0.58 | -0.67 | 1.41 | 0.47 |

* Intercept Coefficients are expressed in $\log_{10}$ units