

**University of Alberta**

Essays on the 2007-08 Financial Crisis and the Global Asset Shortage

by

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Economics

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## **Abstract**

The twenty-year period from 1990 to 2010 exhibited significant and consequential international economic phenomena. Global imbalances, primarily the U.S. current account deficit, were very prominent, along with the development and bursting of a real estate bubble, low interest rates and saving rates, and a brief shock to the oil market in 2008. This thesis addresses these phenomena.

The first paper addresses the question of whether speculative oil inventories were accumulated in response to conditions in asset markets, and whether such inventories were the cause of the oil price shock. Using cointegration techniques, it tests whether the arbitrage condition required for speculative inventories to be accumulated held at any period. It finds evidence that above-ground and below-ground speculative inventories were accumulated, but not at the time of the oil price shock.

The second paper addresses the issue of how to test the dynamic efficiency of an economy. The real estate bubble in the early 2000s and the correspondingly low interest rate suggests the possibility of dynamic inefficiency in the U.S. economy. Previous methods used to test dynamic efficiency are shown to be inadequate for testing dynamic efficiency in this period. Cointegration analysis is introduced as an ideal testing methodology for determining the long-run equilibrium relationship between capital accumulation and profit and whether any bubbles mask that relationship. It finds the U.S. economy became dynamically inefficient in 2000.

The third paper proposes a model to explain the U.S. current account deficit, low interest rate, low saving rate, and the real estate bubble. The model demonstrates how the behaviour of all four variables could have been caused by shocks to the U.S. terms of trade. These shocks initially increased domestic saving and foreign income, creating higher demand for financial assets. Greater demand for assets started the decline of the interest rate. The trade deficit created the current account deficit. The declining interest rate created dynamic inefficiency, causing the emergence of the real estate bubble. Low interest rates encouraged domestic borrowing to finance consumption. Domestic saving rates declined while foreign income continued to increase. The effect of the higher foreign supply of loanable funds was stronger than the increased domestic demand for loanable funds, resulting in a low equilibrium interest rate.

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## **Chapter 1: Introduction**

In 2007 the United States experienced a severe financial crisis that eventually spread across the globe. Some of the principle effects for the U.S. economy were a contraction of wealth, increases in risk spreads, and the decreased functionality of credit markets (Reinhart & Rogoff, 2008). As in other financial crises in the past, the associated asset market collapse has been deep and prolonged (Reinhart & Rogoff, 2009). U.S. real equity prices had declined by 50% by 2009. At the time of the crash U.S. house prices were 110% higher than ten years previously. They have subsequently declined to levels not seen since 2001. This has been a decline of almost 28% - more than twice what occurred during the Great Depression of the 1930s (Reinhart & Rogoff, 2009). The impact of the mortgage defaults that set off the crisis was amplified because of the tranching process, in which mortgages were aggregated in a manner thought to spread risk efficiently (Reinhart & Rogoff, 2008). The result was the freezing of the financial sector. GDP growth in many countries declined and even became negative. U.S. GDP contracted by 2% in the last quarter of 2008. Commodity markets also experienced a lot of turbulence. The price of oil shot up to \$145/bbl in June 2008, then plummeted to \$39/bbl in the next six months.

This thesis consists of three papers concerning the 2007-08 financial crisis and certain economic circumstances leading up to the crisis, including the price of oil and the rate of extraction, U.S. dynamic efficiency, and the U.S. current account deficit. They are related to the asset shortage hypothesis proposed in Caballero et. al. (2008). In brief, this hypothesis asserts a global asset shortage led to increasing demand for U.S. assets, causing the global imbalances and declining

U.S. interest rate. The U.S. economy became dynamically inefficient, creating the circumstances necessary for the real estate bubble to arise. The bursting of the bubble in 2007 worsened the asset shortage and caused a speculative bubble in oil. The global recession caused this bubble to burst in 2008.

The first paper empirically tests for speculative oil inventories. To date, there has not been an econometric investigation of whether there was a link between the 2007 crisis in financial markets and the subsequent oil price shock. This paper uses cointegration analysis to test for such a link.

Caballero et. al. (2008) proposes that the bursting of the real estate bubble in 2007 incited speculative accumulation of oil inventories. The plummeting value of mortgage-backed securities and related assets created a new asset shortage. The reduction in the asset supply caused a low interest rate, which caused investors to hoard speculative inventories. Thus, conditions in financial markets created a bubble in commodities. So far, the literature has not empirically tested for speculative inventories. Some economists have considered speculation in futures markets, but not in inventories (Interagency Task Force on Commodity Markets, 2008; Hamilton, 2009b; Smith, 2009). Others considered but rejected speculative inventories based only on the observation that physical inventories didn't increase over the relevant time period (Kilian, 2010; Turner et. al., 2011). This ignores the possibility of below-ground speculative inventories.

This first paper empirically tests for speculative inventories by testing whether the arbitrage condition held (see Pindyck, 1993; Chambers & Bailey, 1996; Deaton & Laroque, 1996; Caballero et. al., 2008; Hamilton, 2009a). Cointegration analysis determines speculative inventories were accumulated above-ground and below-ground. However, the timing of such inventories indicates they were not the cause of the 2007-08 oil price shock.

Another implication of the analysis is that one cannot test for speculation in oil inventories simply by observing changes in physical inventories. The ability of oil extractors to “store” oil below ground makes speculative inventories an attractive investment. Such investment is overlooked by analysis of physical storage only.

The second paper concerns testing economies for dynamic efficiency. In the past, econometric evaluation of theoretical conditions for dynamic efficiency has not been rigorous. This paper shows how cointegration methods can be used to rigorously test dynamic efficiency. These methods enable a researcher to test the long run stability of dynamic efficiency conditions, to test for breaks in the long run relationship, and to find evidence for the cause of any such breaks. Such depth of analysis has not been conducted in the past.

Previous work on dynamic efficiency has focused on the development of testable conditions rather than on the testing itself (see Tobin, 1965; Solow, 1970; Feldstein, 1977; Feldstein et. al. 1977; Tirole, 1985; Abel et. al., 1989; Santos & Woodford, 1997). Three methods have been used in the past. The first is simply to observe whether income appears consistently greater than or less than investment (Abel et. al., 1989). The second is to compare the return on capital with

the economic growth rate at a single point in time (Feldstein, 1977; Feldstein et. al., 1977). The third method calculates the mean of the series  $\ln[(1 + r_t)/(1 + g_t)]$  and tests whether it is smaller than zero (Barbie et. al., 2004). The paper in this thesis points out the problems with these approaches to testing and demonstrates how using cointegration overcomes these shortcomings. An economy can change equilibrium growth paths. The analysis finds the U.S. economy shifted to a dynamically inefficient path in 2000.

This is the first paper to assess U.S. dynamic efficiency during the period relevant to the financial crisis. Its primary contribution, however, is in using cointegration methods to assess efficiency and any changes to efficiency over time. Previous analyses of dynamic efficiency have not considered the possibility of change.

The third paper presents a model to explain the behaviour of the interest rate and of the U.S. saving rate. It also explains the current account deficit and the rise and fall of the real estate bubble. Its contribution is to demonstrate how all these phenomena can be explained by negative shocks to the U.S. terms of trade. These shocks created the large negative trade balance. This was the driving force behind the current account deficit. Increased expenditure on imports was financed by borrowing from foreign sources. The inflows of capital were not used to invest in the creation of capital. This is evidenced by the development of the asset bubble in real estate, which is often considered an unproductive asset. Thus, the U.S. was not attracting investment in its economy, but rather sought financing for consumption.

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## Chapter 2: Cointegration Analysis of Speculative Inventories

### 1. Introduction

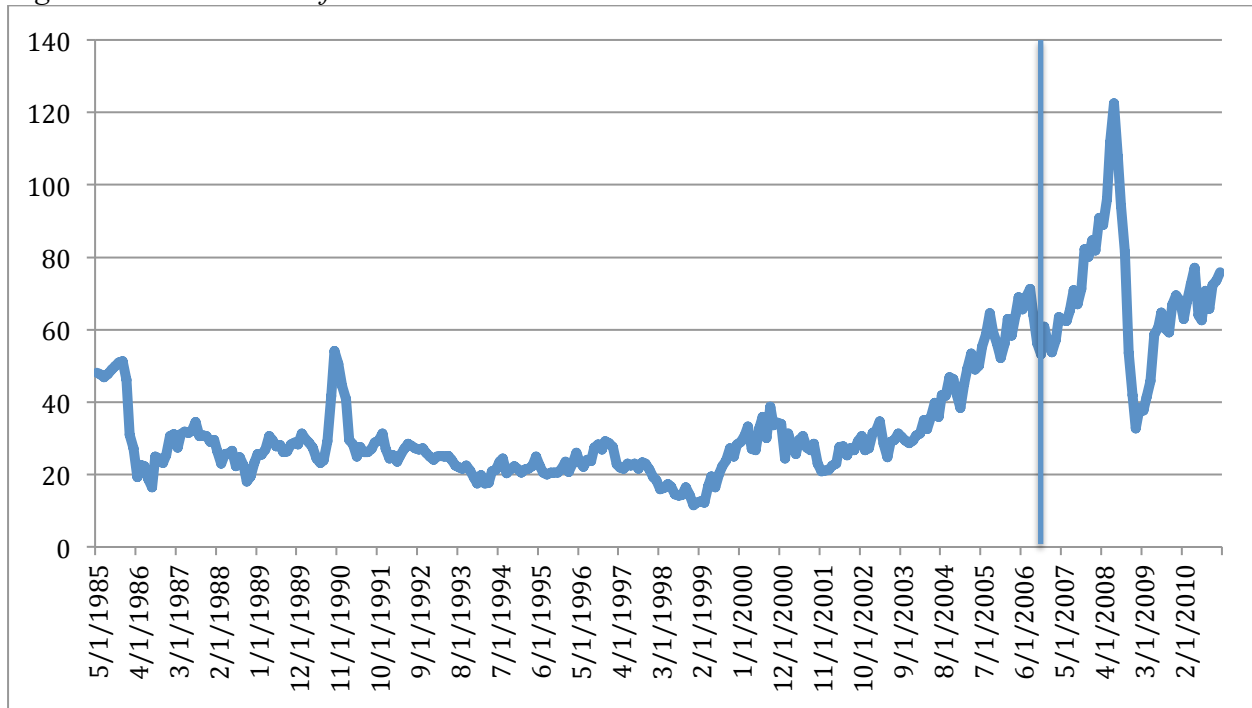
The global financial crisis of 2007-08 was accompanied by one of the biggest oil price shocks in history (Hamilton, 2009b). After reaching a high of \$145/BBL in June 2008, the price of oil plummeted to \$39/BBL in December of the same year. The beginning of this shock coincides with the increase in U.S. subprime mortgage defaults in February 2007. The mortgage defaults marked the bursting of the real estate bubble, which has been noted as the trigger of the subsequent financial crisis (Brunnermeier, 2009). Hamilton (2009b) proposes that the speed and magnitude of the collapse of the oil price requires one to consider whether it was a speculative price bubble that burst. To date, the question of speculative oil inventories has not been empirically tested. In this paper, I use cointegration analysis to test whether such inventories were accumulated during this period and whether they caused the oil price shock.

Caballero et. al. (2008) proposes international macroeconomic conditions from the 1990s through the time of the oil price shock were driven by a global asset shortage. It presents a hypothesis<sup>1</sup> that explains how the asset shortage created global imbalances, low interest rates, and ultimately the oil price shock. When the real estate bubble burst, excess asset demand needed a new source of assets. Conditions in commodity markets made oil inventories an attractive investment. The subsequent speculation in oil inventories was responsible for the 2007-08 oil price shock.

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<sup>1</sup> Hereafter called the “asset shortage hypothesis”.

Figure 2-1: Real Price of Crude Oil



Price of West Texas Intermediate Crude in USD/BBL

Data Source: Datastream

Figure 2-1 plots the real price of crude oil from the beginning of 1990 through August 2011. The vertical line marks February 2007, the point where subprime mortgage defaults increased (Brunnermeier, 2009). It serves as a possible marker for the bursting of the real estate bubble. Just prior to this point, the price of oil stopped its persistent upward trend with a brief downturn. It then began a rapid increase shortly before the indicated month. The asset shortage hypothesis proposes this was a bubble that migrated from real estate to oil inventories in order to meet the demand for assets.



For the bubble to have migrated from real estate to oil inventories there must have been an accumulation of speculative oil inventories. The cointegration analysis in this paper tests whether such accumulation occurred.

Though speculation in oil inventory markets has been considered in previous research, it has not been empirically tested for. Furthermore, no one has used cointegration to analyze oil inventories in any context. Most literature regarding speculation has analyzed futures markets (Interagency Task Force on Commodity Markets, 2008; Hamilton, 2009b; Smith, 2009). These have uniformly rejected speculation as a cause of the oil price shock. Speculative inventories during the 2007-08 oil price shock have been dismissed based on only the observation of physical inventories (Kilian, 2010; Turner et. al., 2011). This ignores the potential for speculative “below-ground” inventories – speculative inventories in the form of un-extracted oil. As Caballero et. al. (2008) points out, a speculative reason for crude oil price increases “raises the effective opportunity cost of resource extraction for producers, since there is now an asset opportunity cost, as in Hotelling’s model, which reduces extraction incentives for commodity producers. The latter response means that, in equilibrium, there need not be any rise in measured inventories...”. The results of this paper establish below-ground speculative inventories as significant. Such a result has not been illustrated in any other research to date.

Not all below-ground oil reserves are speculative inventories. This paper defines below-ground inventories as speculative if they are left below ground because of speculation of future price increases. Un-extracted oil due to capacity constraints or changes in market fundamentals is not speculative. For example, in their discussion of speculation in the oil market, Kilian and Murphy

(2011) point out, “an alternative view of speculation is that OPEC in anticipation of even higher oil prices held back its production after 2001, using oil below ground effectively as inventories.” The benefit of such a practice is that there is no explicit storage cost as there is for above-ground inventories.

This paper extends the literature in several ways. Firstly, it directly tests for speculation in inventories and does not rely solely on observations of physical inventories. Secondly, the focus is on speculative inventories (above and below ground) rather than speculation in futures markets. Thirdly, the methodology recognizes the link between asset markets and speculative inventories pointed out in the asset shortage hypothesis: If investors choose to invest in oil inventories as financial assets, the oil inventory market must be efficient, meaning the arbitrage condition relating the interest rate to the oil spot price must hold (see Pindyck, 1993; Chambers & Bailey, 1996; Deaton & Laroque, 1996; Caballero et. al., 2008; and Hamilton, 2009a).

This paper tests the efficiency of the oil inventories market by using cointegration methods to determine whether the arbitrage condition holds. I also analyze data on world oil production to supplement the results. If the arbitrage condition does not hold, it means the inventory market is inefficient. In such a circumstance, speculative oil inventories were not accumulated and they were not viewed as an alternative to financial assets. Otherwise, arbitrage would occur between the financial market and the oil inventory market until no more profit could be made by such a practice. If the arbitrage condition does hold, it suggests that financial assets and oil inventories were viewed as interchangeable and that investors would have changed inventory holdings in response to a crisis in financial markets.

In brief, the results show that there were below-ground and above-ground speculative oil inventories during part of the sample period analyzed. No speculative inventories were held during the period relevant to the oil price shock and financial crisis. I thus conclude that speculation in inventories did not drive the shock, and the bubble in real estate did not migrate to oil inventories.

In the next section I derive the arbitrage condition and explain the underlying theory. I also describe how I use the condition to test for speculative inventories and whether they were a substitute for financial assets. Section 3 describes the data and presents the results of the tests. Section 4 concludes.

## **2. Oil Inventory Arbitrage and Testing for Speculation**

There are two possible motivations for holding oil inventories (Miao et. al., 2011). First, a firm may hold inventories because oil is a necessary part of the firm's production process. I call these "inventories of convenience". Second, an investor may hold inventories as an asset based on the expectation that the future price will compensate for the present cost of buying and storing oil. I call these "speculative inventories". Speculative inventories can either be above ground or below ground. Above-ground speculative inventories (AGSI) are composed of extracted oil that is bought and stored in storage facilities for future sale. Oil producers that forgo present extraction in order to extract more in the future, when they speculate the price will be higher, are accumulating below-ground speculative inventories (BGSI).

Inventories of convenience generate a convenience yield – the benefit the firm receives from holding inventories, such as increased efficiency in production and sales. For example, producers and refiners of crude oil hold inventories because they are essential for transporting, refining and delivering oil (Hamilton, 2009a; Kilian & Murphy, 2011). Inventories ensure against interruptions to production. All such inventories are held because they have a positive impact on firm profits. They are not bought with the intent to re-sell at a higher price, and are held even if the price is expected to decline. The convenience yield alone is the return on these oil inventory assets. The level of these inventories would be affected very little, if at all, by the rate of return on financial assets. Financial assets cannot substitute for the production benefits received by inventories of convenience. The amount of such inventories to hold is solely determined by the net marginal convenience yield (NMCY), defined as the marginal convenience yield less the marginal cost of storage. So long as the NMCY is positive, inventories are increased. Accumulation stops where the marginal benefit equals marginal cost – NMCY is zero.

If speculative inventories are held, equilibrium must be such that there is no arbitrage between oil inventories and financial assets (Pindyck, 1993; Chambers & Bailey, 1996; Deaton & Laroque, 1996; Caballero et. al., 2008; Hamilton, 2009a). If the rate of return on oil inventories is greater than the return on financial assets, investors will short financial assets in order to buy oil inventories. In the next period, they can sell the inventories and use a part of the proceeds to buy the financial assets they shorted. Such arbitrage would decrease the current price of financial assets (due to increase supply) and increase the current price of oil inventories (due to increased demand). The rate of return on financial assets would increase, whereas the return on oil

inventories would decrease. This process would continue until the expected return on both assets is equivalent. When markets are efficient this equilibrium is achieved very quickly.

Whether or not this arbitrage condition holds is therefore indicative of whether or not there are speculative inventories. It remains, then, to specify this condition mathematically. I use the condition as developed in Hamilton (2009a).

The basic point of the condition is that the expected return on oil inventories should equal the expected return on financial assets. A hypothetical investor is faced with the choice of either buying and storing a barrel of oil, or else buying a financial asset with a real rate of return equal to  $r$ . This is not necessarily the risk-free asset. Using  $r$  as the discount rate, the investor is better off buying the barrel of oil if the discounted expected future price of oil is greater than the present cost of buying and storing the oil:

$$(2-1) \quad \frac{E_t p_{t+1}}{1+r_t} > p_t + C_t$$

Here,  $p_t$  is the real price of a barrel of oil in period  $t$ ,  $C_t$  is the real cost of storing a barrel of oil one period, and  $E_t$  indicates it is the period  $t$  expectation of what the variable will be in a later period. The responses of supply of and demand for oil and financial assets to this inequality would be such that prices and expectations would change to make equation (2-1) an equality:

$$(2-2) \quad E_t p_{t+1} = (1 + r_t)(p_t + C_t)$$

Agents who produce oil or use it in their production processes also hold inventories of convenience. In this case, the inventories have a convenience yield in addition to the storage cost. Letting  $d$  represent the NMCY, the arbitrage condition becomes:

$$(2-3a) \quad E_t p_{t+1} = (1 + r_t)(p_t - d_t)$$

This arbitrage condition will hold *if* there are speculative inventories. When oil inventories are not a substitute for financial assets, and there are no speculative inventories, equation (2-3a) needs not hold with equality. All inventories are inventories of convenience. They are held regardless of the relationship between expected price, the current price, and the interest rate. The equilibrium condition is simply that the NMCY equals zero.

The potential for BGSI presents a special situation. Such inventories have no convenience yield or explicit storage cost. The arbitrage condition for BGSI excludes  $d$ , the NMCY. If BGSI are the only speculative inventories accumulated, the arbitrage condition is:

$$(2-3b) \quad E_t p_{t+1} = (1 + r_t)p_t$$

Equation (2-3b) means the extractors will delay extraction of oil if the price of oil is expected to grow at a rate (greater than or) equal to the rate of return on financial assets.

It should be noted that this is not a Hotelling-type rule. The Hotelling model considers the dynamically optimal rate of extraction of an exhaustible resource. This arbitrage condition looks

only one period ahead and applies to any kind of asset. It does not suppose dynamic efficiency and does not consider profit-maximization over time. It assumes extractors are looking only at the marginal unit of oil.

The issue of whether or not speculative inventories were accumulated can thus be settled by testing whether these arbitrage conditions held. If equation (2-3a) holds, AGSI were accumulated. If only equation (2-3b) holds, there were no AGSI, but there were BGSIs. If neither arbitrage condition holds, no speculative inventories were accumulated.

Testing whether the arbitrage conditions hold is the same as testing market efficiency. I use a cointegration test of the same kind used in previous tests of market efficiency and of relationships between markets (for example, see Meese, 1986; Bollerslev & Hodrick, 1992; Shiller, 1992; Crowder & Hamed, 1993; Herbert, 1993; Moosa & Al-Loughani, 1994; Walls, 1995; Switzer & El-Khoury, 2007). Cointegration is deemed a necessary condition for market efficiency (Switzer & El-Khoury, 2007), as it can properly account for non-stationarity in the variables (Lai & Lai, 1991). Efficiency requires an equilibrium relationship, and cointegration tests for such a relationship. Market efficiency implies the terms are cointegrated and that a specific cointegrating vector holds. Both conditions must be met (Lai & Lai, 1991). Defining market efficiency in terms of a cointegrating relationship precludes markets from exhibiting persistent risk-free arbitrage profit opportunities (Crowder & Hamed, 1993).

As in previous tests, I make the arbitrage condition linear by taking natural logarithms:

$$(2-4a) \quad \ln E_t p_{t+1} = \alpha + \ln(1 + r_t) + \ln(p_t - d_t) + \varepsilon_t$$

$$(2-4b) \quad \ln E_t p_{t+1} = \beta + \ln(1 + r_t) + \ln p_t + \varepsilon_t$$

The parameters  $\alpha$  and  $\beta$  are included to allow for factors such as risk premia and transportation costs. There are two steps to finding whether the arbitrage condition holds. First, all variables in the condition must be integrated of the same order. This is a pre-requisite for a long-run relationship between them. Second, the variables must be cointegrated with the cointegrating relationship as predicted by the arbitrage condition.

The analysis uses monthly data from October 1985 through December 2010. For the interest rate, I use the return on 3-month U.S. treasury bills. The analysis was conducted with other interest rates and achieved similar results. The spot price of oil is the price of Brent crude oil. The NMCY is estimated using the methodology in Pindyck (1993) (explained below).

There are three methods of estimating expected price in the literature: (1) Use the cointegration of the futures price and the spot price of oil (Campbell & Shiller, 1987; Spilimbergo, 2001); (2) Use a weighted average of prices from the preceding 6 months (Farrow, 1985); and, (3) Assume perfect foresight (Farrow, 1985). In this paper I use the first two methods, for robustness. I do not use perfect foresight because, if the price of oil is  $\sim I(1)$ , the difference between the expected price and the spot price is stationary simply because of the nature of the spot price.



The “cointegration” expected price series is estimated as follows. Under the hypothesis of the present value model of asset pricing, if the futures price of oil and the spot price of oil are both integrated of order one, they are also cointegrated with a cointegrating vector  $(1, \rho - 1)$  (Campbell & Shiller, 1987). Then expected price can be estimated by:

$$(2-5) \quad E_t p_{t+1} = f_{1,t} + \rho p_t$$

In this equation,  $f_{1,t}$  is the real futures price at time  $t$  for a contract that expires in one period. Consistent with the findings in Campbell and Shiller (1987), GLS-detrended ADF tests of the data for this sample period show both the real futures price of oil and the real spot price of oil are  $\sim I(1)$ . Tests of the first difference of the two variables rejected the null hypothesis of a unit root. Using the Johansen methodology, I found the two variables are cointegrated, with  $\rho$  equal to -0.00540. This value was used with equation (2-5) to estimate the first series of expected prices. Estimations done using this method are denoted by “cointegration”.

The second method sets the expected price equal to a weighted average of prices from the preceding six months. The weights are found by OLS regression of

$$p_t = a_0 + \sum_{i=1}^6 a_i p_{t-i} + e_t$$

The expected price is then estimated by using the estimated coefficients in this manner:

$$E_t p_{t+1} = \hat{a}_0 + \sum_{i=0}^5 \hat{a}_{i+1} p_{t-i}$$

Estimations made when this method has been used are denoted by “weighted average”.

The NMCY is estimated using the method in Pindyck (1993). According to this method, the return from holding a barrel of oil from  $t$  to  $t + T$  is  $(p_{t+T} - p_t) + d_{t,T}$  (variables are defined as above). If one instead shorts a forward contract at time  $t$ , the return is  $(F_{T,t} - p_t) + d_{t,T}$  where  $F_{T,t}$  is the price of a forward contract for oil made at time  $t$  for delivery at time  $t + T$ . Pindyck (1993) establishes that, since no outlay is required for the forward contract and the total return is non-stochastic, the total return must equal  $r_T p_t$ , where  $r_T$  is the risk-free interest rate on an asset that matures at time  $t + T$ :

$$r_T p_t = (F_{T,t} - p_t) + d_{t,T}$$

Rearranging:

$$(2-6) \quad d_{t,T} = (1 + r_T) p_t - F_{T,t}$$

Because futures prices and forward prices usually differ only marginally, futures prices can be used instead (Pindyck, 1993). Futures are traded more actively than forwards, so the futures market is likely more efficient. I use the 1-month futures price on Brent crude oil.

The base data for this analysis are the spot price of crude oil, the futures price of crude oil, and the interest rate. These series were converted to real values using the U.S. CPI as reported monthly by the U.S. Bureau of Economic Analysis, with 1983 as the base year. They are used to derive the remaining variables needed: the weighted average expected price series, the cointegration expected price series, and the NMCY. All series then have the seasonal component removed using the moving averages method. The interest rate is a monthly rate rather than an annual rate. Table 2-1 provides some summary statistics, and Figures 2-2 through 2-4 illustrate the expected price series, the real interest rate, and the NMCY.

*Table 2-1: Summary Statistics*

	<b>Spot Price</b>	<b>Cointegration Method</b>	<b>Weighted Average</b>	<b>Interest Rate</b>	<b>NMCY</b>
<b>Average</b>	18.77	18.64	18.60	0.002503	0.5187
<b>St. Dev.</b>	10.15	10.17	9.689	0.002109	0.8301
<b>Maximum</b>	63.50	63.54	63.64	0.008937	4.279
<b>Minimum</b>	6.510	6.669	6.658	-0.004082	-2.377

The average is the mathematical mean of each series over the entire time period. The standard deviation is the square root of the variance. The maximum and the minimum are the highest value and the lowest value, respectively.

The two expected price series are very similar and highly correlated. The weighted average series seems late with its predictions. It doesn't anticipate the peak price will occur until January, whereas the cointegration series expects it to occur in August 2008. The interest rate is cyclical, but seems to have an overall downward trend, even excluding the high rates of the early 1980s.

Figure 2-2: Expected Price Series

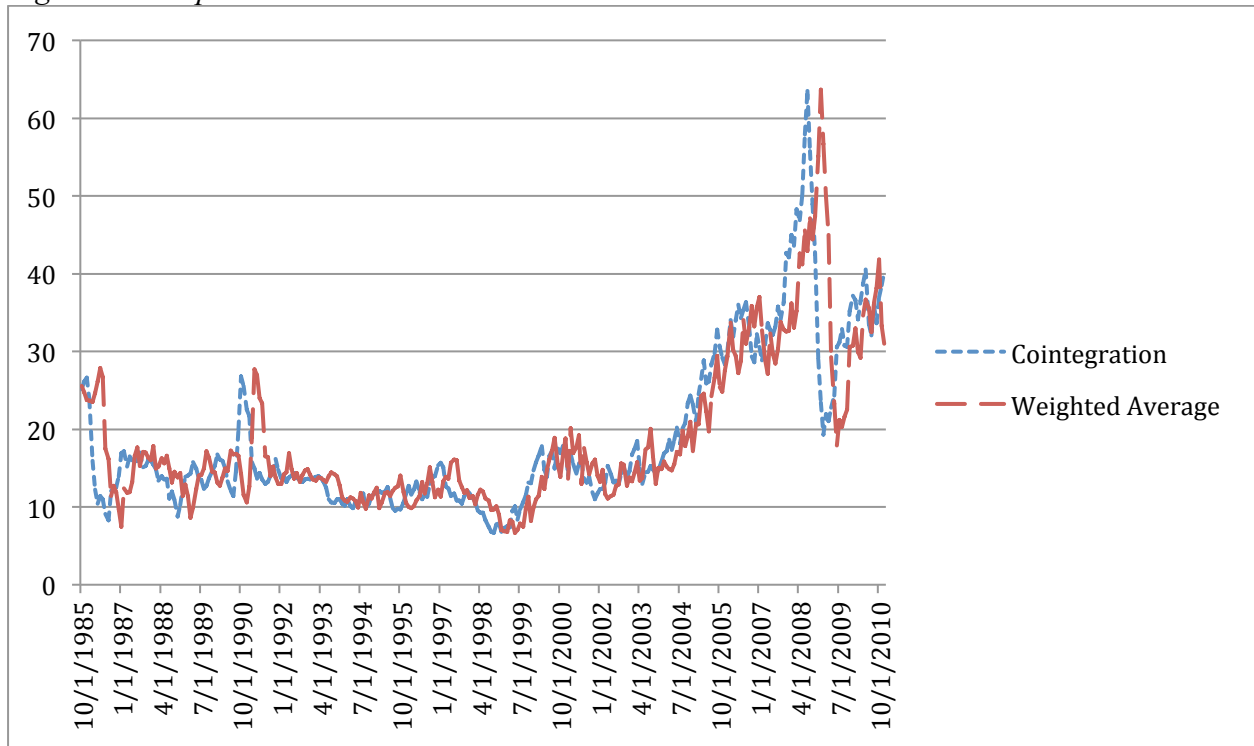
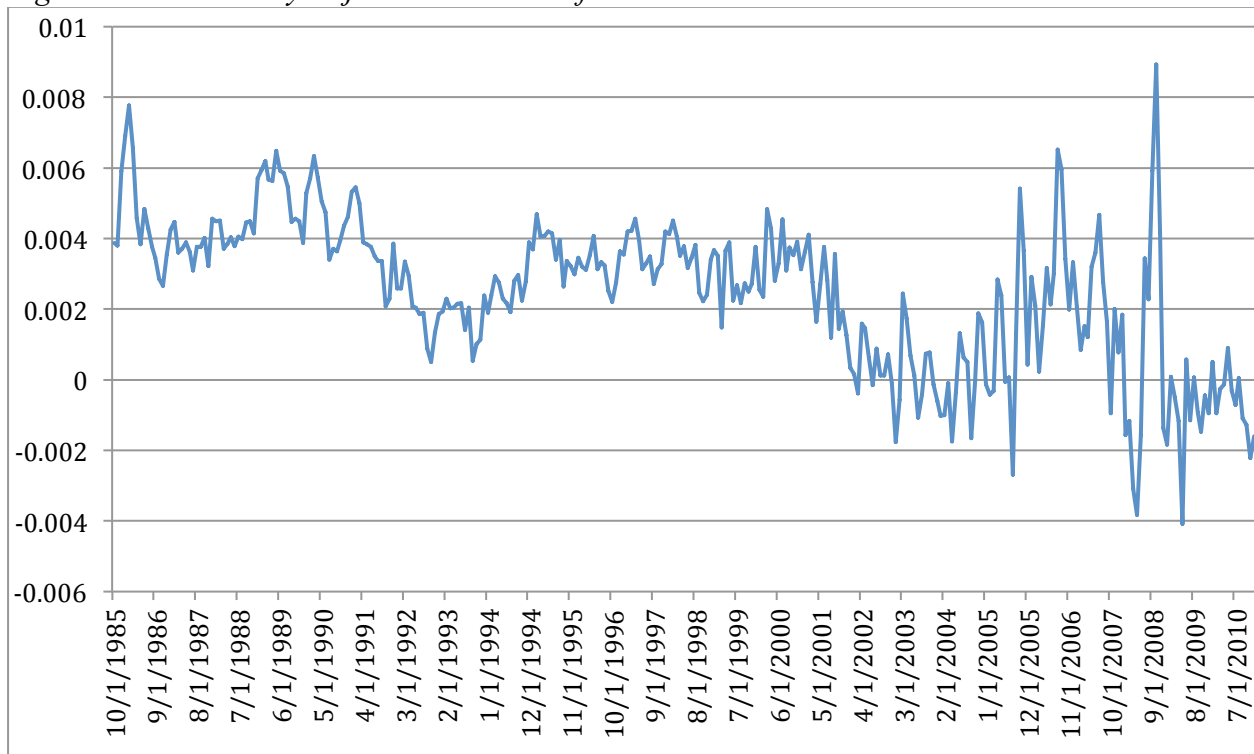
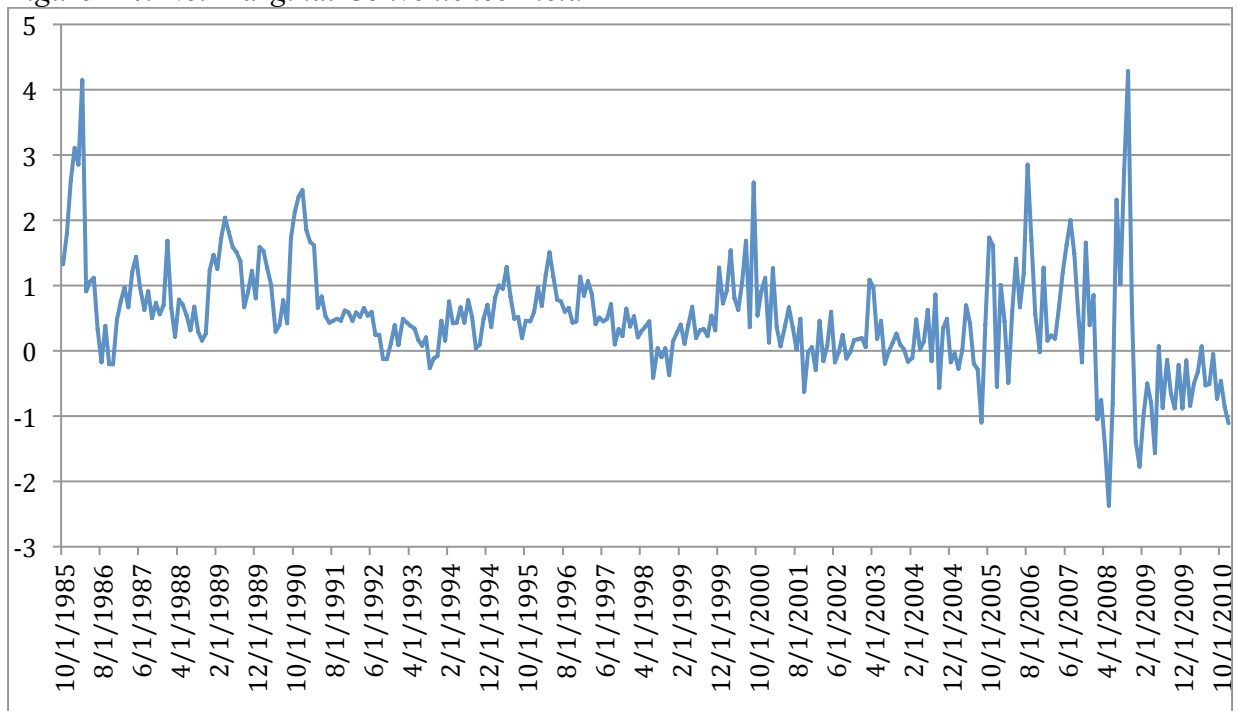


Figure 2-3: Seasonally Adjusted Real Rate of Return on 3-month T-bills



Data Source: Datastream

Figure 2-4: Net Marginal Convenience Yield



The NMCY appears to have a mean above zero for a large part of the sample. It becomes very volatile toward the end of the sample, with a large swing from the minimum value of -\$2.38 to the maximum value of \$4.28. Absent temporary deviations from equilibrium, the NMCY on aggregate inventories is negative theoretically only if speculative inventories are accumulated. Its sustained decline after the summer of 2007 coincides with the oil price shock. The swing in 2008 could be explained by a large accumulation driving the NMCY to its minimum value, followed by a huge sell-off such that the NMCY became positive again. This observation lends credibility to the proposition speculative inventories were accumulated.

I used an ADF test with GLS de-trending to determine the order of integration of the NMCY. I did the test twice: the first time including a constant only, the second time including a constant and a linear trend. Table 2-2 provides the critical values, and Table 2-3 the test statistics.

*Table 2-2: ADF-GLS Critical Values*

<b>Critical Values</b>	<b>0.01</b>	<b>0.05</b>	<b>0.10</b>
<b>Constant Only</b>	-2.58	-1.98	-1.62
<b>Constant &amp; Trend</b>	-3.42	-2.91	-2.62

*Table 2-3: ADF-GLS Test Statistics of NMCY*

	<b>Constant Only</b>	<b>Constant &amp; Trend</b>
<b>No Difference</b>	-0.251	-2.466
<b>First Difference</b>	-0.981	-2.509

As can be seen, the null hypothesis of a unit root cannot be rejected for any of these tests. The NMCY is either integrated of at least order 2, or it is not integrated. This means we cannot assess the likelihood of speculation in inventories by a simple test of the equilibrium value of the NMCY. It is necessary to proceed with a cointegration analysis of the arbitrage conditions.

I first tested for AGSI and then for BGSi, using the following methodology:

1. Test the order of integration of each term in equations (2-4a) and (2-4b).
2. Test for breaks in the unit root processes using the methodology given in Perron (1989).  
Use these breaks to create sub-periods for analysis.
3. Test whether the terms in equation (2-4a) and in (2-4b) are cointegrated over the entire sample period and in each sub-period.
4. Test the arbitrage conditions over the entire sample period and in each sub-period.
5. If the arbitrage condition holds, estimate an error-correction model.

### 3. Results

I first tested the order of integration of both estimates of  $\ln E_t p_{t+1}$ , of  $\ln(1 + r_t)$ , of  $\ln p_t$ , and of  $\ln(p_t - d_t)$  using the augmented Dickey-Fuller test with GLS de-trending. As with the NMCY, the test was done with a constant only and with a constant and linear trend. The critical values are as reported in Table 2-2. Table 2-4 provides the results.

*Table 2-4: ADF-GLS Tests*

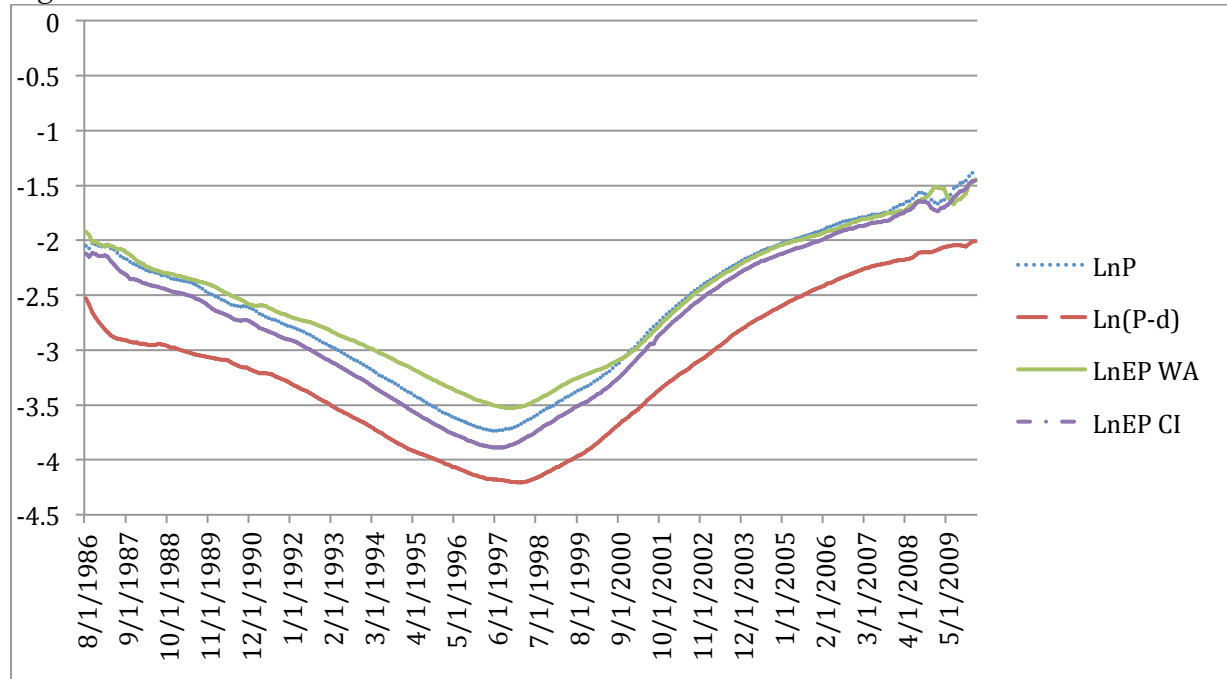
Variable	No Difference		First Difference	
	Constant	Constant & Trend	Constant	Constant & Trend
$\ln E_t p_{t+1}$ (WA)	-1.249	-1.245	-18.382	-18.493
$\ln E_t p_{t+1}$ (CI)	-1.244	-1.207	-14.386	-14.408
$\ln(1 + r_t)$	-0.257	-2.086	-19.105	-19.040
$\ln p_t$	-1.188	-1.146	-3.262	-4.428
$\ln(p_t - d_t)$	-1.537	-1.599	-4.022	-4.835

All five terms are integrated of order one. This means it is possible there is a long-run relationship between the terms, which can be tested for using cointegration tests. I next tested for breaks in the unit root processes using the methodology from Zivot and Andrews (1992). When breaks were identified, I tested each sub-period for additional breaks until any breaks found were at or near the beginning or end of the particular series. In this manner I found one break in each series except  $\ln(1 + r_t)$ , which has no breaks.

The Zivot and Andrews (1992) methodology involves creating a dummy variable for each period in the time series and using the dummy variables in tests for a unit root. The dummy variable that results in the most negative test statistic represents a break point, provided the null hypothesis of

a unit root with break is not rejected by the test statistic. Figure 2-5 graphs the test statistic for each month in the sample period for the four variables that have a break.

Figure 2-5: Break Point Test Statistics



As can be seen, all four series reach their minimum test statistic at almost the same period.  $\ln p_t$  reaches its minimum in June 1997 with the test statistic -3.736;  $\ln(p_t - d_t)$  reaches its minimum in February 1998 with the test statistic -4.205; the weighted average estimate of  $\ln E_t p_{t+1}$  reaches its minimum in November 1997 with the test statistic -3.529; and the cointegration estimate of  $\ln E_t p_{t+1}$  reaches its minimum in June 1997 with the test statistic -3.888. The critical values for the test statistics are -4.89, -4.19, and -3.88 at the 1%, 5% and 10% levels of significance, respectively. The null hypothesis of a unit root with a break is clearly not rejected for the  $\ln p_t$  and weighted average estimate of  $\ln E_t p_{t+1}$  series. The null hypothesis for the cointegration estimate of  $\ln E_t p_{t+1}$  is rejected only at the 5% level of significance, and it is



rejected for  $\ln(p_t - d_t)$  only at the 1% level of significance. Though it is questionable to include this as a break point, I decided to keep it for the purposes of the analysis. Thus I created the sub-periods indicated in Table 2-5.

*Table 2-5: Sub-Periods*

<b>Sub-Period</b>	<b>Start Date</b>	<b>End Date</b>
Sub-Period 1a	October 1985	June 1997
Sub-Period 1b	July 1997	December 2010
Sub-Period 2a	October 1985	November 1997
Sub-Period 2b	December 1997	December 2010
Sub-Period 3a	October 1985	February 1998
Sub-Period 3b	March 1998	December 2010

I tested each sub-period for additional break points and found none.

To test for cointegration I used the Johansen methodology, and allowed for a constant in the cointegrating relationships to allow for departures due to transportation costs and risk premia.

The Johansen methodology involves first determining whether the terms are cointegrated and how many cointegrating vectors there are. Then the cointegrating vectors are estimated. The Johansen trace test sets the null hypothesis that the number of cointegrating vectors is equal to or less than some integer. The integer is progressively increased until the null hypothesis is not rejected. In this application there is a maximum of two cointegrating vectors. Table 2-6 provides the critical values of the trace test, with  $v$  being the number of cointegrating vectors. The top row gives the levels of significance; the left column gives the null hypotheses.

Table 2-6: Trace Test Critical Values

	1%	5%	10%
$v = 0$	41.07	34.91	32.00
$v \leq 1$	24.60	19.96	17.85
$v \leq 2$	12.97	9.24	7.52

I first tested the cointegration of equation (5-2a), with the cointegration estimate of  $\ln E_t p_{t+1}$  followed by the weighted average estimate. I then tested the cointegration of equation (5-2b) in the same manner. Table 2-7 provides the test statistics for the trace test.

Table 2-7: Trace Test Statistics

Null Hypothesis	Test Statistics			
	Equation (5-2a)		Equation (5-2b)	
	Cointegration	Weighted Average	Cointegration	Weighted Average
$v = 0$	115.47	126.02	92.59	132.00
$v \leq 1$	42.31	41.32	41.90	44.84
$v \leq 2$	3.24	1.92	2.92	2.05

I thus conclude that, in each equation, the terms are cointegrated with two cointegrating vectors. This result is not sensitive to which method of estimating expected price is used. The next step is to determine the cointegrating vectors. These results are given in Table 2-8.

Table 2-8: Cointegrating Vectors and Test Statistics – Full Sample

Term	Equation (5-2a)				Equation (5-2b)			
	Cointegration		Weighted Average		Cointegration		Weighted Average	
$\ln E_t p_{t+1}$	1.000	6.939e-18	1.000	1.518e-18	1.000	1.084e-18	1.000	-2.450e-17
$\ln(1 + r_t)$	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000
$\ln(p_t - d_t)$ or $\ln p_t$	-0.9731	2.254e-03	-0.9268	2.229e-03	-1.009	2.328e-03	-0.9611	2.314e-03
constant	-0.1014	-8.761e-03	-0.2392	-8.689e-03	3.258e-02	-9.047e-03	-0.1116	-9.008e-03
Test Statistic	61.74		39.11		36.56		40.06	

Finally, I tested whether the arbitrage conditions hold. This involves restricting the coefficients on  $\ln(1 + r_t)$ , and  $\ln(p_t - d_t)$  or  $\ln p_t$  to be -1, and the coefficient on  $\ln E_t p_{t+1}$  to be 1. I did not put any restriction on the constant. This is a chi-square test with two degrees of freedom. The critical values are 9.21, 5.99 and 4.61 at the 1%, 5% and 10% levels of significance. The test statistics for each equation are given in Table 2-8. As can be seen, the restricted relationship is rejected in each instance, meaning speculative inventories were not accumulated either above ground or below ground.

Because there are two cointegrating vectors, I cannot test for breaks in the cointegrating relationships. Such tests involve the Engel-Granger method of testing for cointegration, which is not appropriate when there is more than one vector. As a result, I rely on the breaks in the unit root processes to test whether there is a change in behaviour over time. The next stage of the analysis is to follow the above methodology for each sub-period.

I confirmed that, within each sub-period, the terms are still cointegrated with two cointegrating vectors. I then estimated the vectors and tested the restrictions. The results are given in Tables 2-9 through 2-14.

There is some evidence of AGSI up until about 1998, but this result is not robust to both estimations of expected price. The restriction is not rejected only if the weighted average estimate is used. On the other hand, the results consistently find BGSI were accumulated up until about 1998. The tests of the sub-periods after 1998 uniformly reject the restricted relationship.

Table 2-9: Cointegrating Vectors and Test Statistics: Sub-Period 1a

Term	Equation (5-2a)				Equation (5-2b)			
	Cointegration		Weighted Average		Cointegration		Weighted Average	
$\ln E_t p_{t+1}$	1.000	-6.939e-18	1.000	-5.204e-18	1.000	0.000	1.000	4.684e-17
$\ln(1 + r_t)$	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000
$\ln(p_t - d_t)$ or $\ln p_t$	-1.380	-2.840e-02	-1.475	-2.259e-02	-0.9142	-1.660e-02	-1.016	-1.608e-02
constant	0.9200	6.824e-02	1.127	5.352e-02	-0.2078	3.934e-02	2.258e-02	3.797e-02
Test Statistic	24.01		2.1		3.84		2.26	

Table 2-10: Cointegrating Vectors and Test Statistics: Sub-Period 1b

Term	Equation (5-2a)				Equation (5-2b)			
	Cointegration		Weighted Average		Cointegration		Weighted Average	
$\ln E_t p_{t+1}$	1.000	0.000	1.000	-3.686e-18	1.000	6.939e-18	1.000	2.385e-18
$\ln(1 + r_t)$	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000
$\ln(p_t - d_t)$ or $\ln p_t$	-0.9838	1.409e-03	-0.9512	1.588e-03	-1.001	1.486e-03	-0.9710	1.628e-03
constant	-6.452e-02	-5.887e-03	-0.1530	-6.380e-03	2.682e-03	-6.117e-03	-7.867e-02	-6.521e-03
Test Statistic	32.23		34.78		31.46		37.61	

Table 2-11: Cointegrating Vectors and Test Statistics: Sub-Period 2a

Term	Equation (5-2a)				Equation (5-2b)			
	Cointegration		Weighted Average		Cointegration		Weighted Average	
$\ln E_t p_{t+1}$	1.000	-3.686e-18	1.000	5.204e-18	1.000	3.469e-18	1.000	0.000
$\ln(1 + r_t)$	0.000	1.000	0.000	1.000	0.000	1.000	-1.110e-16	1.000
$\ln(p_t - d_t)$ or $\ln p_t$	-0.9512	1.588e-03	-1.471	-2.252e-02	-0.9137	-1.652e-02	-1.011	-1.605e-02
constant	-0.1530	-6.380e-03	1.114	5.3245e-02	-0.2086	3.901e-02	8.863e-03	3.779e-02
Test Statistic	23.6		2.17		3.88		2.26	

Table 2-12: Cointegrating Vectors and Test Statistics: Sub-Period 2b

Term	Equation (5-2a)				Equation (5-2b)			
	Cointegration		Weighted Average		Cointegration		Weighted Average	
$\ln E_t p_{t+1}$	1.000	0.000	1.000	3.469e-18	1.000	1.110e-16	1.000	0.000
$\ln(1 + r_t)$	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000
$\ln(p_t - d_t)$ or $\ln p_t$	-0.9843	1.578e-02	-0.9492	1.791e-02	-1.002	1.682e-02	-0.9681	1.846e-02
constant	-6.2858e-02	-6.674e-02	-0.1602	-7.254e-02	6.459e-03	-6.998e-02	-8.936e-02	-7.451e-02
Test Statistic	30.91		34.47		31.07		36.83	

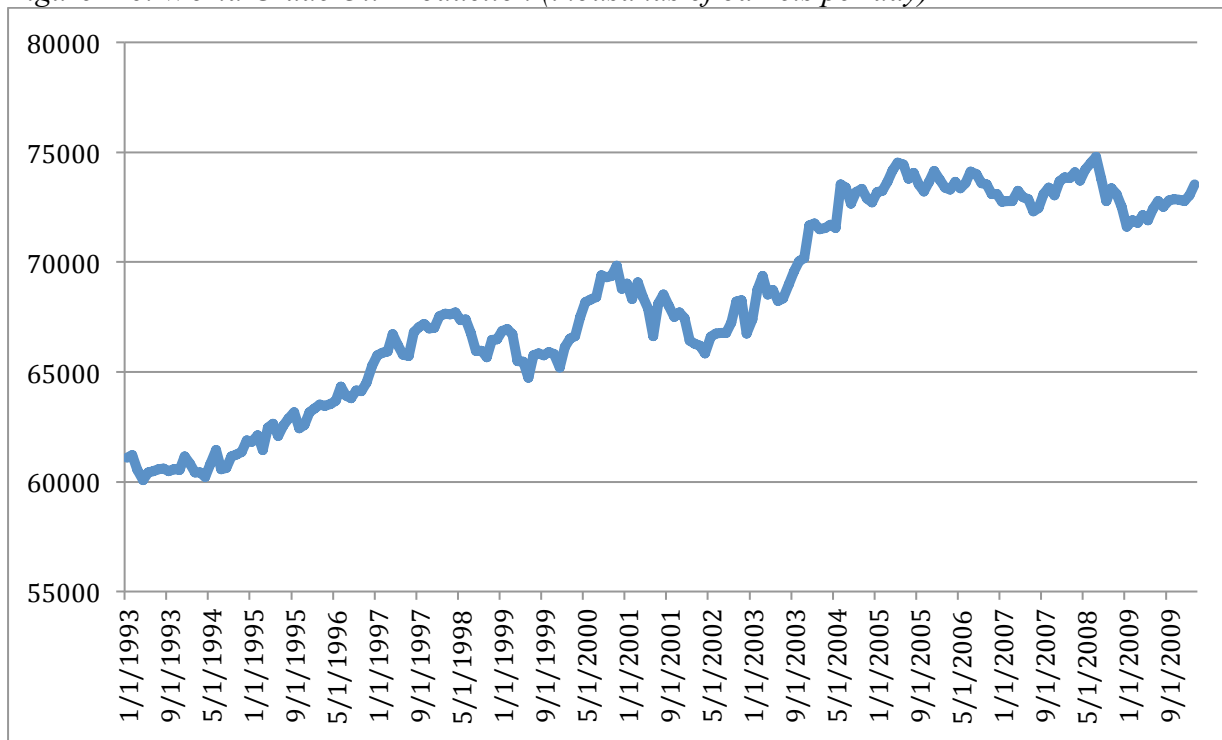
Table 2-13: Cointegrating Vectors and Test Statistics: Sub-Period 3a

Term	Equation (5-2a)				Equation (5-2b)			
	Cointegration		Weighted Average		Cointegration		Weighted Average	
$\ln E_t p_{t+1}$	1.000	8.327e-17	1.000	-5.551e-17	1.000	0.000	1.000	0.000
$\ln(1 + r_t)$	0.000	1.000	-2.220e-16	1.000	0.000	1.000	0.000	1.000
$\ln(p_t - d_t)$ or $\ln p_t$	-1.399	-0.3582	-1.487	-0.2885	-0.9127	-0.2157	-0.9991	-0.2083
constant	0.9639	0.8576	1.149	0.6813	-0.2112	0.5113	-2.039e-02	0.4913
Test Statistic	24.88		1.11		2.47		1.11	

Table 2-14: Cointegrating Vectors and Test Statistics: Sub-Period 3b

Term	Equation (5-2a)				Equation (5-2b)			
	Cointegration		Weighted Average		Cointegration		Weighted Average	
$\ln E_t p_{t+1}$	1.000	0.000	1.000	2.0817e-17	1.000	0.000	1.000	0.000
$\ln(1 + r_t)$	-2.220e-16	1.000	0.000	1.000	0.000	1.000	0.000	1.000
$\ln(p_t - d_t)$ or $\ln p_t$	-0.9851	1.577e-02	-0.9522	1.814e-02	-1.004	1.702e-02	-0.9715	1.867e-02
constant	-5.987e-02	-6.671e-02	-0.1502	-7.331e-02	1.104e-02	-7.067e-02	-7.795e-02	-7.522e-02
Test Statistic	31.27		32.52		29.36		34.77	

Figure 2-6: World Crude Oil Production (thousands of barrels per day)



Data Source: EIA

While at this point the results have no bearing on the 2007-08 oil price shock, they do demonstrate the importance of considering speculative inventories below ground. Such activity could also be reflected in the rate at which crude oil producers extract oil. Accordingly, I obtained monthly data on the world rate of extraction in terms of thousands of barrels extracted per day. The data was obtained from the *Monthly Energy Review* of the Energy Information Administration, and was seasonally adjusted in the same manner as the other data series. The data was available from January 1993 through February 2010. It is illustrated in Figure 2-8.

I again used the ADF test with GLS de-trending to determine the order of integration. The test statistic for the null hypothesis of a unit root is 0.558, indicating a unit root. The test statistic when testing the first difference of the series is -3.119, leading to the conclusion world crude oil

production is  $\sim I(1)$ . As before, I tested for breaks in unit root process and found two breaks: December 2002 and July 2006.

Since the first difference of production is stationary, it has a constant mean. I calculated the mean of the first difference of production for each of the three sub-periods caused by these breaks. From January 1993 to December 2002 the average change in production each month is 47.66; from January 2003 to July 2006 the average change increases dramatically to 159.85. This corresponds to the increasing demand for oil from developing economies such as China and India. Finally, from August 2006 to February 2010 the average change plummets to -10.71.

This decrease in production from 2006 on could be caused by a number of factors, including decreased extraction capacity or the accumulation of BGSI. I therefore tested whether the arbitrage condition holds in the three sub-periods created by the breaks in the unit root process of world crude oil production. Tables 2-15 through 2-17 provide the trace statistics in each period.

*Table 2-15: Trace Test Statistics: January 1993 – December 2002*

Null Hypothesis	Test Statistics			
	Equation (5-2a)		Equation (5-2b)	
	Cointegration	Weighted Average	Cointegration	Weighted Average
$v = 0$	37.01	51.89	41.65	52.36
$v \leq 1$	11.33	14.96	11.69	15.85
$v \leq 2$	2.67	2.98	2.40	3.29

Table 2-16: Trace Test Statistics: January 2003 – July 2006

Null Hypothesis	Test Statistics			
	Equation (5-2a)		Equation (5-2b)	
	Cointegration	Weighted Average	Cointegration	Weighted Average
$v = 0$	57.60	60.48	62.92	64.06
$v \leq 1$	13.81	11.69	24.43	12.11
$v \leq 2$	2.79	2.29	2.94	2.34

Table 2-17: Trace Test Statistics: August 2006 – December 2010

Null Hypothesis	Test Statistics			
	Equation (5-2a)		Equation (5-2b)	
	Cointegration	Weighted Average	Cointegration	Weighted Average
$v = 0$	50.75	40.01	54.18	40.12
$v \leq 1$	21.45	23.30	27.79	23.22
$v \leq 2$	7.39	6.79	10.29	6.97

From January 1993 to December 2002, and from January 2003 to July 2006, each test has only one cointegrating vector. From August 2006 to December 2010 there are two cointegrating vectors. These vectors, and the results from testing the arbitrage equation restrictions, are provided in Tables 2-18 through 2-20.

Table 2-18: Cointegrating Vectors and Test Statistics: January 1993 – December 2002

Term	Equation (5-2a)		Equation (5-2b)	
	Cointegration	Weighted Average	Cointegration	Weighted Average
$\ln E_t p_{t+1}$	1.000	1.000	1.000	1.000
$\ln(1 + r_t)$	-1.056	-3.169	0.5509	-1.360
$\ln(p_t - d_t)$ or $\ln p_t$	-1.003	-1.004	-0.9631	-0.9581
constant	1.391e-02	6.630e-02	-0.1022	-7.017e-02
Test Statistic	0.16	0.73	15.47	0.11



Table 2-19: Cointegrating Vectors and Test Statistics: January 2003 – July 2006

Term	Equation (5-2a)		Equation (5-2b)	
	Cointegration	Weighted Average	Cointegration	Weighted Average
$\ln E_t p_{t+1}$	1.000	1.000	1.000	1.000
$\ln(1 + r_t)$	0.1634	-21.18	5.291	-22.51
$\ln(p_t - d_t)$ or $\ln p_t$	-1.040	-0.1621	-1.205	-0.1120
constant	0.1165	-2.274	0.5887	-2.415
Test Statistic	30.16	40.15	7.92	42.13

Table 2-20: Cointegrating Vectors and Test Statistics: August 2006 – December 2010

Term	Equation (5-2a)				Equation (5-2b)			
	Cointegration		Weighted Average		Cointegration		Weighted Average	
$\ln E_t p_{t+1}$	1.000	0.000	1.000	-6.939e-18	1.000	0.000	1.000	0.000
$\ln(1 + r_t)$	0.000	1.000	0.000	1.000	-1.110e-16	1.000	0.000	1.000
$\ln(p_t - d_t)$ or $\ln p_t$	-0.9755	2.127e-02	-1.249	4.336e-02	-0.7485	-0.5331	-1.240	4.790e-02
constant	-9.113e-02	-8.325e-02	0.9153	-0.1626	-0.9056	1.903	0.8819	-0.1790
Test Statistic	19.27		9.85		12.65		9.78	

Breaking the data into these three sub-periods provides some additional light regarding behaviour over the last two decades. The period in which BGSIs were accumulated is extended through 2002 (though the restriction is rejected using the cointegration estimation of expected price). In addition, it now appears AGSIs were also accumulated for the decade up to 2003. Since that time, no speculative inventories have been accumulated. This makes it clear that speculation did not cause the oil price shock in 2007-08.

## 4. Conclusion

This paper is the first to use cointegration and univariate analysis to test the market for speculative oil inventories, and to empirically distinguish between above ground and below ground inventories. It is also the first to use these techniques to empirically test whether speculative inventories drove the oil price shock in 2007-08. Several papers consider this possibility but do not empirically test it. This research filled this gap.

It also has implications for the asset shortage hypothesis. Caballero et. al. (2008) proposes the oil price shock was the result of the financial market's endogenous response to the bursting of the real estate bubble. A global asset shortage caused investors to look for alternative assets to invest in. Oil inventories were the choice. The speculative accumulation of oil inventories caused the price shock. The results of this research suggest this is not what occurred.

There are two primary reasons this research is important. Firstly, it makes clear that any discussion of speculative inventories needs to distinguish between above ground and below ground. Observation of above ground inventories is not sufficient to answer any question of whether speculative inventories are accumulated. There are periods in which the arbitrage condition for AGSI does not hold, whereas the arbitrage condition for BGSi does hold. Thus, it is possible for extractors to accumulate speculative inventories by withholding production, even though other parties do not accumulate AGSI.

Secondly, it confirms speculation did not drive the 2007-08 oil price shock. The accumulation of speculative inventories of any kind ended by 2003. This is curious, as 2003 marks the beginning of the sustained increase in the real price of oil. It may be that, as global demand for oil increased, the capacity of suppliers to meet that demand stagnated, and this stagnation of supply made the accumulation of speculative inventories impossible.

The alternative explanation for the oil price shock that has the most support is that it was driven by demand and supply fundamentals. There was a long period of time of increasing global demand for oil, driven primarily by emerging economies such as China and India, accompanied by stagnation in the supply of oil. This situation reached a critical point in 2007-08, followed by severe recession in many economies.

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# **Chapter 3: A Cointegration Analysis of the Dynamic Efficiency of the U.S. Economy**

## **1. Introduction**

The dynamic efficiency of an economy concerns its rate of capital accumulation and whether that rate is optimal. An economy is dynamically inefficient if it accumulates capital and too fast a rate, consistently investing more than it earns in profit (Abel et. al., 1989). Diamond (1965) first showed that there can be competitive steady states in which the economy is dynamically inefficient. “Dynamic efficiency is a central issue in analyses of economic growth, the effects of fiscal policies, and the pricing of capital assets” (Abel et. al., 1989). It is a key factor in the determination of whether a given economy is saving too much or too little (Feldstein, 1977). As a result, it is important to be able to measure whether an economy is dynamically efficient.

Since discussions of dynamic efficiency began in the 1960s, several models have been developed to define conditions for dynamically efficient capital accumulation<sup>2</sup>. The actual econometric evaluation of these conditions has progressed at a slower pace. This paper proposes a new method of econometrically testing conditions for dynamic efficiency. It demonstrates how cointegration can be used for two purposes: (1) To test the long-run stability of any of the dynamic efficiency conditions; and, (2) To test for breaks in the long-run relationship.

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<sup>2</sup> See, for example, Tobin (1965), Feldstein (1976), and Abel et. al. (1989)

The advent of such a method is timely, given the upheaval of international financial markets beginning in 2007. The recent crisis is often associated with the bursting of what has been called a real estate bubble. There is some controversy over whether it can be called a bubble, and regarding the exact role it played. Several papers have established that speculative bubbles can arise in a rational expectations equilibrium when an economy is dynamically inefficient (Tirole, 1985; Abel et. al., 1989; Martin & Ventura, 2010; Farhi & Tirole, 2011). If the argument that the persistent increases in the price of real estate in the United States from 1995 to 2007 was a bubble is accepted, one may ask what caused the bubble. Given dynamic inefficiency is one cause of bubbles, it then becomes relevant to ask whether the economy was dynamically inefficient. As explained at the beginning of this paper, dynamic efficiency is an important aspect of the economy.

This analysis provides insight into the possible contributions to the global imbalances over the last decade. Caballero et. al. (2008b) argues dynamic inefficiency was the root of these imbalances and of the financial crisis in 2007-08. Caballero et. al. (2008a, 2008b) propose a global shortage of assets, arising from increasing incomes and financial underdevelopment in emerging economies, caused increasing flows of financial capital into the United States. The increased demand for U.S. assets moved the economy into a dynamically inefficient equilibrium, creating fertile ground for the real estate bubble and subsequent spike in oil prices. Determining whether the U.S. economy was, in fact, dynamically inefficient affects the viability of this argument. Thus, the period from 1990 through 2010 provides a fascinating observation set for

demonstrating this new method of analyzing dynamic efficiency and its effectiveness in explaining economic phenomena.

Though the definition of dynamic inefficiency is clear, several different conditions have been developed for testing whether a given economy meets that definition. Each condition has its advantages and difficulties. Early work proposed estimating the marginal product of capital and comparing it to an estimate of the economy's growth rate (Tobin, 1965; Solow, 1970; Feldstein, 1977; Feldstein et. al., 1977; Abel et. al., 1989). Later work suggesting testing efficiency by comparing an economy's growth rate to the safe real interest rate (Feldstein, 1976; Tirole, 1985; Abel et. al., 1989; Santos & Woodford, 1997). Abel et. al. (1989) asserts a better approach is to compare the safe rate of interest with the growth rate of the market value of the capital stock rather than the economic growth rate.

There are few examples of empirical testing of these methods. Abel et. al. (1989) uses annual data for the United States and six other industrialized countries for the years 1929 through 1985. The approach is simply to observe whether income from capital is consistently greater than, or less than, investment. Feldstein (1977) and Feldstein et. al. (1977) estimate and compare the return on capital and the economic growth rate at a single point in time. They use average rates for the United States over 1946 through 1975. Barbie et. al. (2004) calculates the mean of the series  $\ln[(1 + r_t)/(1 + g_t)]$ , where  $r$  is the interest rate and  $g$  is the growth rate of GDP, for the United States from 1890 to 1999 and statistically tests whether it is smaller than zero.



There are two difficulties with these tests. First, dynamic efficiency/inefficiency is a long-run equilibrium condition (Bullard & Russell, 1999). The condition must hold in all periods and all states of nature (Abel et. al., 1989). Simply observing the condition at a specific point in time, or even annually over a number of years, does not test the long-run relationship between income and investment or the rate of return and the rate of growth. As noted by Abel et. al. (1989), shocks can cause temporary fluctuations in the actual value of any of the above-mentioned variables without affecting their long-run time paths. Visual observation cannot compensate for this. Testing the average difference over a sample period is not robust, either. It cannot inform regarding the relationship in all periods and all states of nature, or whether the economy is even on an equilibrium time path.

Second, dynamic inefficiency can be masked by the development of a bubble. As cited above, several papers have established that bubbles are the natural result of dynamic inefficiency. A bubble increases the asset supply, which affects the interest rate and the return on capital. It can therefore appear as though the economy is efficient, when it is only because of a speculative bubble.

The purpose of this paper is not to argue the merits of one condition over any other. Each condition has been shown to be theoretically valid. This paper contributes to the literature by presenting a much-needed method of testing dynamic efficiency conditions that overcomes these shortcomings. I use cointegration techniques to estimate the long-run equilibrium relationship between the rate of return and the growth rate. If the two series are cointegrated it means there is a long-run relationship between them. The cointegrating vector describes this relationship. It is

therefore more conclusive on whether income from capital is greater than or less than investment in the long-run equilibrium. It also allows for testing whether there was a break in the long-run equilibrium relationship. This makes it possible to determine whether the economy shifted from an efficient equilibrium to an inefficient one, or vice versa. It can also assist in detecting whether bubbles mask an inefficient situation.

In brief, the analysis finds there is a long-run relationship between the interest rate and the economic growth rate. There are breaks in the relationship, which depend on the interest rate used. The results indicate the U.S. economy has been dynamically inefficient since 2000.

The next section briefly explains cointegration and describes the methodology used in this analysis. The following section presents the results. The conclusion ends.

## **2. Methodology**

Cointegration was introduced by Engel and Granger (1987), and has since been developed in many macroeconomic applications. It is used to test and describe the relationships between non-stationary variables. Non-stationary variables are cointegrated if there is a linear combination of the variables that is stationary. In order for such a linear combination to exist, the time paths of the non-stationary variables must be linked. “Equilibrium theories involving non-stationary variables require the existence of a combination of the variables that is stationary” (Enders, 2010).

If a set of non-stationary variables is not cointegrated, the variables can wander arbitrarily far from each other. On the other hand, the time paths of cointegrated variables are influenced by how far they deviate from long-run equilibrium. After a random shock there is a tendency to return to the equilibrium relationship. “The movements of at least some of the variables must respond to the magnitude of the disequilibrium” (Enders, 2010).

In the present application, assuming the variables involved are non-stationary, if an economy’s capital accumulation is in equilibrium, the interest rate and the growth rate of the capital stock will be cointegrated. Estimating the cointegrating relationship will establish whether that equilibrium time path is dynamically efficient. If the interest rate and the growth rate of the capital stock are not cointegrated, each time series will wander without regard to the other, and the economy is in disequilibrium.

Let  $r$  be the real interest rate and  $g$  the real economic growth rate (either the growth rate of GDP or the growth rate of the capital stock). The first step of the analysis is to test the order of integration of both  $r$  and  $g$ . To be cointegrated, they must be integrated of the same order. I use the augmented Dickey-Fuller test with GLS detrending for this stage of the analysis. I also test for a unit root with a break in the process.

The second step tests for cointegration. I use both the Engle and Granger (1987) method and the Johansen (1988) method to test whether  $r$  and  $g$  are cointegrated, and what the cointegrating vector is. The cointegrating vector is used to determine whether the equilibrium rate of capital

accumulation is dynamically efficient. I use both methods because the Johansen (1988) method is more powerful, but the Engel and Granger (1987) method is the foundation for the next step.

Gregory and Hansen (1996) presents a method of testing for a shift or break in the cointegrating relationship. This approach is used in the third step to endogenously determine any break point in the relationship between  $r$  and  $g$ .

For this methodology to work there must be reason to believe the interest rate and the growth rate are unit root processes. Regarding the interest rate, there is conflict between empirical and theoretical work. Empirical work routinely finds the interest rate is integrated of order one (see, for example, Rose, 1988; Edison & Pauls, 1993; Evans & Lewis, 1994; Lai, 1997). In theoretical work, “the state-variable which governs short rate behavior is ordinarily assumed to follow a mean-reverting autoregressive process, with the implication that the processes followed by bond yields are also mean-reverting. Unit roots are generally eschewed because they lead to implausible properties for long-horizon forward rates” (Kozicki & Tinsley, 2001). Non-stationary real interest rates are inconsistent with Lucas-type consumption-based asset pricing models (Rose, 1988; Lai, 1997) and the Fisher effect (Lai, 2008).

As this is an empirical paper, I expect tests for stationarity to be similar to previous empirical findings. Explaining why the results of these tests may conflict with theory is outside the scope of this paper.

Finally, a basis for why the growth rate of capital would be non-stationary can be found in the Solow growth model. According to that model, the growth rate of the capital stock changes over time. There is no mean-reversion unless the economy is in the steady state. Given factors such as changes in productivity and technology change the steady state, it can be assumed the growth rate of the capital stock has not, and will not in the near future, settle to zero.

### **3. Results**

For robustness, I start the analysis with three different interest rates and two different growth rates. As interest rates, I use the 3-month U.S. treasury bill, index of AAA-rated U.S. corporate bonds, and U.S. bank prime loan. For growth rates, I use the growth rate of the market value of the U.S. capital stock and the growth rate of U.S. GDP. All series are seasonally adjusted using the weighted-average method, and are put in real terms using the PPI. I chose the PPI because this analysis concerns the producer side of the market rather than the consumer side. The interest rates and growth rates are reported in annualized terms. As some of the series were not available at monthly frequency, I used the quarterly frequency. The sample period is the first quarter of 1990 (1990:1) through the second quarter of 2011 (2011:2). I start in 1990 to avoid the problems that would arise from including the 1980s. Interest rates in the 1980s were abnormally high, and may therefore bias the results of the analysis.

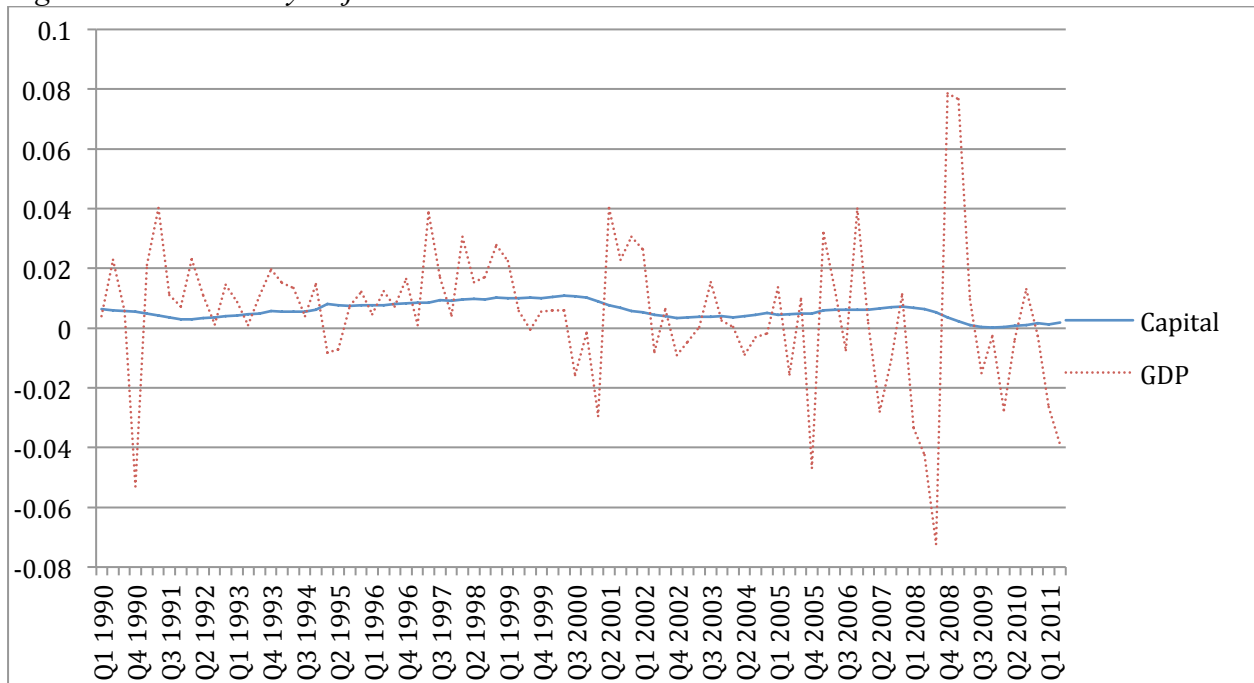
It should be noted the economy is dynamically inefficient provided the condition is met with *at least one* interest rate. As Abel et. al. (1989) noted, the competitive rate of return of any asset can be used.

Figure 3-1: Seasonally Adjusted Real Interest Rates



Data Source: Datastream

Figure 3-2: Seasonally Adjusted Real Growth Rates



Data Source: Datastream

*Table 3-1: Summary Statistics*

	<b>Mean</b>	<b>Variance</b>	<b>Maximum</b>	<b>Minimum</b>
<b>T-Bill</b>	0.696%	0.0000672	2.572%	-1.49%
<b>AAA</b>	1.47%	0.0000494	3.45%	-0.429%
<b>Prime</b>	1.48%	0.0000662	3.31%	-0.478%
<b>Capital</b>	0.576%	0.00000751	1.08%	0.019%
<b>GDP</b>	0.494%	0.000550	7.85%	-7.24%

Figures 3-1 and 3-2 plot the times series over the sample period. Table 3-1 provides some summary statistics. On average, the prime interest rate is the highest of the three; the T-Bill rate is the lowest. Also on average, all interest rates are greater than the growth rates. Simple observation would suggest the economy is dynamically inefficient. But this does not reveal whether it is an equilibrium relationship or whether there are any breaks in that relationship. We now turn to cointegration analysis.

The first step is to determine the order of integration of each series. The null hypothesis of the ADF.GLS test is that the series has a unit root (i.e. is non-stationary). The alternative hypothesis is that the series is stationary and has no unit root. I conducted the ADF.GLS test including a constant only and both a constant and linear trend. I did the same testing on the first difference of each series. Table 3-2 gives the critical values of the tests. Table 3-3 provides the test-statistics.

*Table 3-2: ADF-GLS Critical Values*

<b>Critical Values</b>	<b>0.01</b>	<b>0.05</b>	<b>0.10</b>
<b>Constant Only</b>	-2.58	-1.98	-1.62
<b>Constant &amp; Trend</b>	-3.42	-2.91	-2.62

*Table 3-3: ADF-GLS Test Statistics*

Variable	No Difference		First Difference	
	Constant	Constant & Trend	Constant	Constant & Trend
<b>T-Bill</b>	0.087	-2.536	-11.758	-0.286
<b>AAA</b>	0.333	-7.026	-11.428	-0.098
<b>Prime</b>	-0.596	-2.468	-1.185	-0.207
<b>Capital</b>	-1.688	-1.976	-2.152	-2.426
<b>GDP</b>	-1.495	-1.314	-0.332	-1.112

For the interest rates, there is some conflict depending on whether or not the linear trend is included. If there is no linear trend, the variables are non-stationary. If there is a linear trend, they are stationary. The test statistics for the growth rate of the capital stock and the growth rate of GDP indicate non-stationarity. However, examination of Figure 3-2 suggests they are stationary. Furthermore, testing the first difference of the prime rate and the growth rate of GDP indicate these two series are either at least  $\sim I(2)$  or are not integrated.

There is reason to believe all the series are stationary. If that is the case, cointegration is not appropriate. The analysis will therefore have two parts. The first part will use cointegration techniques to test the dynamic efficiency of the U.S. economy. This analysis will exclude the prime rate and the growth rate of GDP because of the suggestion they are either  $\sim I(2)$  or not integrated. The second part will assume the variables are all stationary. This allows use of ordinary least squares methodology to determine long-run equilibrium relationships.

### *Cointegration Analysis*

I tested for unit roots with breaks as in Zivot and Andrews (1992). I found there are none.



The next step is to test for cointegration between the T-Bill rate and the growth rate of the capital stock, and between the AAA rate and the growth rate of the capital stock. I use the Johansen methodology. The Johansen procedure is essentially a multivariate generalization of the Dickey-Fuller test (Enders, 2010). Let  $x$  be the vector of  $n$  non-stationary variables being tested for cointegration. The Johansen procedure involves arranging these variables in the equation

$$\Delta x_t = \pi x_{t-1} + \varepsilon_t$$

where  $x_t$  and  $\varepsilon_t$  are  $(n \cdot 1)$  vectors and  $\pi$  is a  $(n \cdot n)$  matrix. The rank of the matrix  $\pi$  is equal to the number of cointegrating vectors among the  $n$  variables, and the matrix is used to derive the vectors themselves. The trace test is used to determine the rank of  $\pi$ . If the rank is at least one, the variables are cointegrated.

Because there are only two variables, the rank of the matrix is either zero or one. Table 3-4 provides the critical values for the trace tests.

*Table 3-4: Trace Test Critical Values*

<b>Null Hypothesis</b>	<b>10%</b>	<b>5%</b>	<b>1%</b>
$r \leq 1$	7.52	9.24	12.97
$r = 0$	17.85	19.96	24.60

The test statistics for cointegration between the rate of return on the T-bill and the growth rate are 29.10 for the hypothesis of zero cointegrating vectors and 2.88 for the hypothesis of one cointegrating vector. The test statistics for cointegration between the rate of return on AAA

bonds and the growth rate are 30.54 for the hypothesis of zero cointegrating vectors and 2.91 for the hypothesis of one cointegrating vector. I therefore conclude that each interest rate is cointegrated with the growth rate of the capital stock.

The estimated cointegrating vector when the T-bill is used is (1.000, -1.587, 0.002686) with the variables arranged as  $(r \ g \ c)$ , where  $c$  is a constant term. I next tested the null hypothesis the coefficient on the constant term is equal to zero. This is a chi-squared test with one degree of freedom. The test statistic is 1.03, which has a p-value of 0.31. I do not reject the null hypothesis. I therefore re-estimated the cointegrating vector without a constant term. The vector is (1.000, -1.589). This means that, in equilibrium, the relationship between the interest rate and the growth rate is

$$r_t = 1.589g_t$$

This means that the equilibrium relationship is such that the interest rate is greater than the growth rate of the capital stock. This means the economy is dynamically efficient.

The Engel and Granger (1987) method regresses one variable on the other and tests whether the residual series is stationary. The 10% critical value when there is a constant and no trend is -2.57. It is -3.13 when there is a constant and a trend. When regressing the rate of return on T-bills on the growth rate of the capital stock I again found the constant term is not significant. When the constant term is excluded, the test statistics for the null hypothesis that the residual series has a unit root are -3.304 and -3.803 for without a trend and with a trend, respectively. I

therefore reject the null hypothesis and find the residual series is stationary. This means the interest rate and the growth rate of the capital stock are cointegrated, with cointegrating vector:

$$r_t = 1.2610g_t$$

The Engel and Granger (1987) method also finds the economy is dynamically efficient.

The estimated cointegrating vector from the Johansen (1988) method when the index of AAA-rated bonds is used is (1.000, -0.6927, -0.01028). A test that the coefficient on the constant term is equal to zero has the test statistic 9.9, with a p-value of zero. The null hypothesis is rejected, meaning the constant term is significant. The equilibrium relationship between the AAA interest rate and the growth rate is

$$r_t = 0.6927g_t + 0.01028$$

The economy is dynamically inefficient if  $r < g$ . This condition is met if either the interest rate or the growth rate is greater than 3.345%. This occurs only in 2008:3, when the interest rate reaches 3.45%. I conclude that, using this interest rate, the economy is dynamically efficient.

The Engel and Granger (1987) method also finds these two variables are cointegrated. The cointegrating relationship is:

$$r_t = 0.58536g_t + 0.011301$$

This means the economy is dynamically inefficient if either the interest rate or the growth rate is greater than 2.725%. This occurs in only a handful of observations. I again conclude the economy is dynamically efficient.

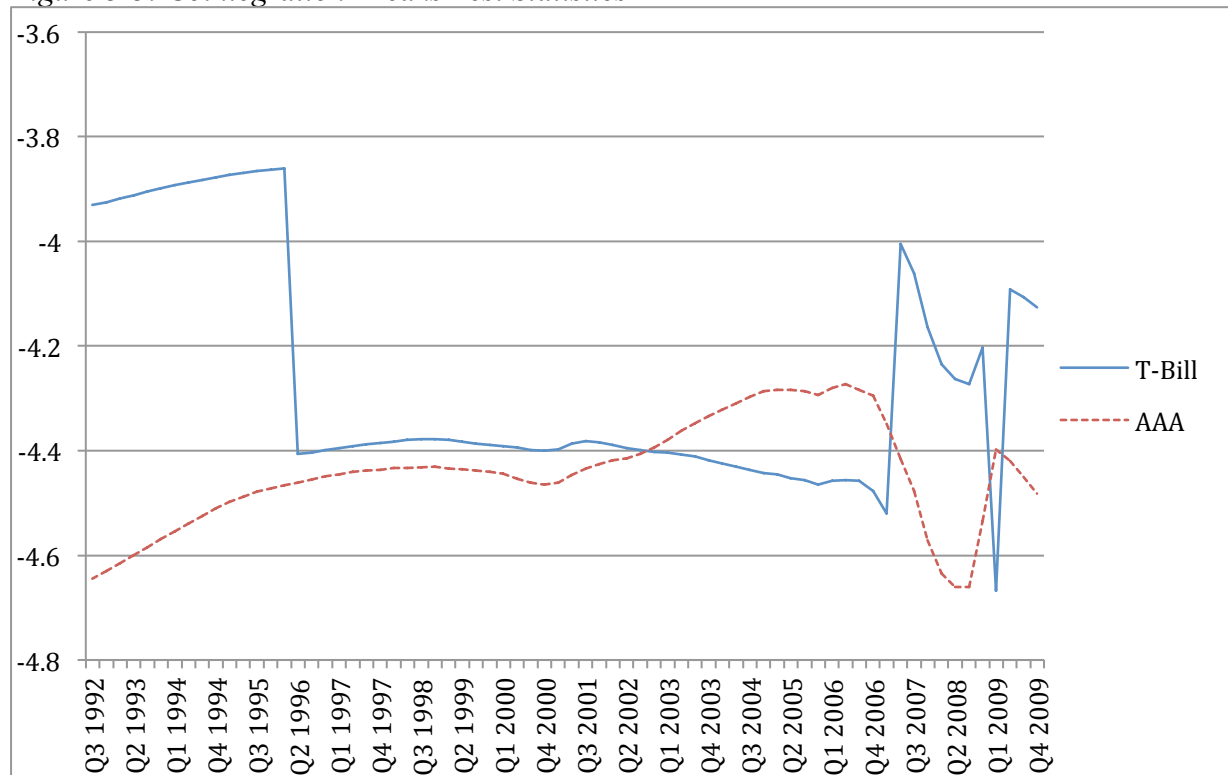
I next tested for breaks in the equilibrium relationship, using the method in Gregory and Hansen (1996). This procedure relies on the Engle-Granger method of testing for cointegration (Engel & Granger, 1987). The Engle-Granger methodology regresses one variable on the other using OLS. One then tests whether the estimated residual series is stationary. If it is stationary, it means the estimated linear combination of the variables is a cointegrating relationship. This test has less power than the Johansen method (Enders, 2010), but is necessary for testing for a break in the cointegrating relationship.

The Gregory-Hansen procedure requires the creation of a set of dummy variables – one for each observation in the sample. A given dummy variable is equal to zero before its associated observation, and equal to one thereafter. One of the variables of the cointegrating relationship is regressed on all other variables of the relationship and those variables multiplied by one of the dummy variables. Equation (3-1) demonstrates. This regression is conducted for each dummy variable, with the estimated residuals being tested for stationarity. The dummy variable that results in the lowest test statistic indicates the break point in the cointegrating relationship.

$$(3-1) \quad r_t = \beta_1 g_t + \beta_2 g_t D_t + \varepsilon_t$$

Figure 3-3 shows the test statistics for both relationships for each dummy variable. There is a break in the cointegrating relationship using the T-bill rate in 2009:1, and a break in the cointegrating relationship using the AAA rate in 2008:3.

*Figure 3-3: Cointegration Breaks Test Statistics*



For each sub-period, I tested again for breaks in the cointegrating relationships. I found no further breaks.

I then tested whether the variables are cointegrated in each sub-period. When the T-bill rate is used, the first sub-period is from 1990:1 to 2009:1. The trace test statistic with the null hypothesis of no cointegrating vectors is 24.55, indicating the variables are cointegrated. The

equilibrium relationship between the T-bill rate and the growth rate of the capital stock, prior to 2009, was

$$r_t = 1.1688 + 0.00041212g_t$$

This means the economy is dynamically inefficient if either  $r$  or  $g$  is less than -0.2441%. The condition is met only in the rare instance. This suggests the economy was dynamically efficient up until 2009:1.

When the AAA rate is used, the first sub-period is from 1990:1 to 2008:3. The trace test statistic with the null hypothesis of no cointegrating vectors is 19.89, indicating the variables are cointegrated. The equilibrium relationship between the T-bill rate and the growth rate of the capital stock, prior to 2009, was

$$r_t = 0.0128199 + 0.3539467g_t$$

This means the economy is dynamically inefficient if either  $r$  or  $g$  is greater than 1.984%. This condition also was met only in the rare instance. I therefore conclude the economy was dynamically efficient during both first sub-periods.

I did not test the second sub-periods because they are not long enough to contain enough data.

The cointegration analysis finds the U.S. economy was dynamically efficient leading up to the financial crisis in 2008. As observed above, however, cointegration likely is not the correct methodology. The next section analyzes the long-run relationships assuming the variables are stationary.

### *Stationary Analysis*

In this section, I use OLS to estimate the long-run relationship between the interest rate and the growth rate (of GDP or of the capital stock), making adjustments for autocorrelation. I test for autocorrelation using the Durbin-Watson method.

I tested for a significant relationship between an interest rate and either the growth rate of the capital stock or GDP growth. Initial tests included a constant. If the constant was not significant, it was dropped from the equation. I found significant relationships between the growth rate of the capital stock and the rate of return on T-bills and the prime rate. The growth rate of GDP was not significantly related to any of the three interest rates. The rate of return on AAA-rated bonds was not significantly related to either growth rate.

In relating the rate of return on T-bills to the growth rate of the capital stock I found that a constant is not significant. The  $R^2$  of the resulting equation is 0.2440. The standard error of the coefficient on  $g$  is 0.1213. The relationship is:

$$r_t = 1.2610g_t$$

The  $R^2$  of the relationship between the prime rate and the growth rate of the capital stock is 0.2904. The standard error of the constant is 0.002002, and the standard error of the coefficient on the growth rate is 0.2738. The relationship is:

$$r_t = 0.0055564 + 1.5993g_t$$

Both of these equations indicate the economy was dynamically efficient.

I tested for breaks in these relationships using the Chow test. I found the test statistic for each period, and accepted as a break the period that gave the most negative statistic. I then tested each sub-period in the same manner. Using this method, I found two breaks. These two breaks were the same for both the T-bill rate and the prime rate. They are 2002:1 and 2008:2. This creates three sub-periods. The results are reported in Tables 3-5 through 3-6.

*Table 3-5: Long-Run Relationship for 1990:1 through 2002:1*

<b>Interest Rate</b>	<b>Constant</b>	<b>Growth Rate</b>
T-bill	0.0096549 (0.003136)	0.19600 (0.4189)
Prime Rate	0.015787 (0.003094)	0.41572 (0.4134)

*Numbers in parentheses are standard errors*

*Table 3-6: Long-Run Relationship for 2002:2 through 2008:2*

<b>Interest Rate</b>	<b>Constant</b>	<b>Growth Rate</b>
T-bill	-0.0069353 (0.006307)	1.9155 (1.206)
Prime Rate	0	2.1270 (0.2427)



In the first sub-period, the coefficient on the growth rate is insignificant. This could be due to lack of sufficient data. If we accept the calculated results, however, they indicate the economy was dynamically *inefficient*. The results for the second sub-period indicate the economy was dynamically efficient.

There is not enough data to test the third sub-period.

Further investigation is necessary. These results indicate the following scenario is possible: the U.S. economy was dynamically inefficient. By 2002, the addition to the asset supply by the housing bubble supported the interest rate sufficiently to make the economy appear dynamically efficient. In 2008, the bubble burst, once again changing the equilibrium relationship.

#### **4. Conclusion**

This paper demonstrates that cointegration techniques enable much more detailed analysis of dynamic efficiency than has previously been conducted. Testing for cointegration of the interest rate and the growth rate estimates the long-run equilibrium relationship between the variables. One can thus draw conclusions on whether the equilibrium accumulation of capital is dynamically efficient or inefficient.

One can also take advantage of methods of testing for breaks in cointegrating relationships. The results indicate whether there has been a change in the equilibrium path. Such a change may be

brought about by external shocks. The ability to check for breaks is important in assessing dynamic efficiency because of changes in the economy.

This paper showed that simply testing the U.S. economy for dynamic efficiency over the entire sample period makes it appear the economy was dynamically efficient. However, there was a break in the equilibrium relationship between the interest rate and the growth rate of the capital stock. Accounting for that break revealed the economy has been dynamically inefficient since 2001.

Cointegration analysis of dynamic efficiency thus provides a more precise picture of economic circumstances and relationships. The results can be used for evaluation of economic theories, the forecasting of future behaviour, and for the formation of economic policies.

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## **Chapter 4: A Trade Model of Global Imbalances and Interest Rates**

### **1. Introduction**

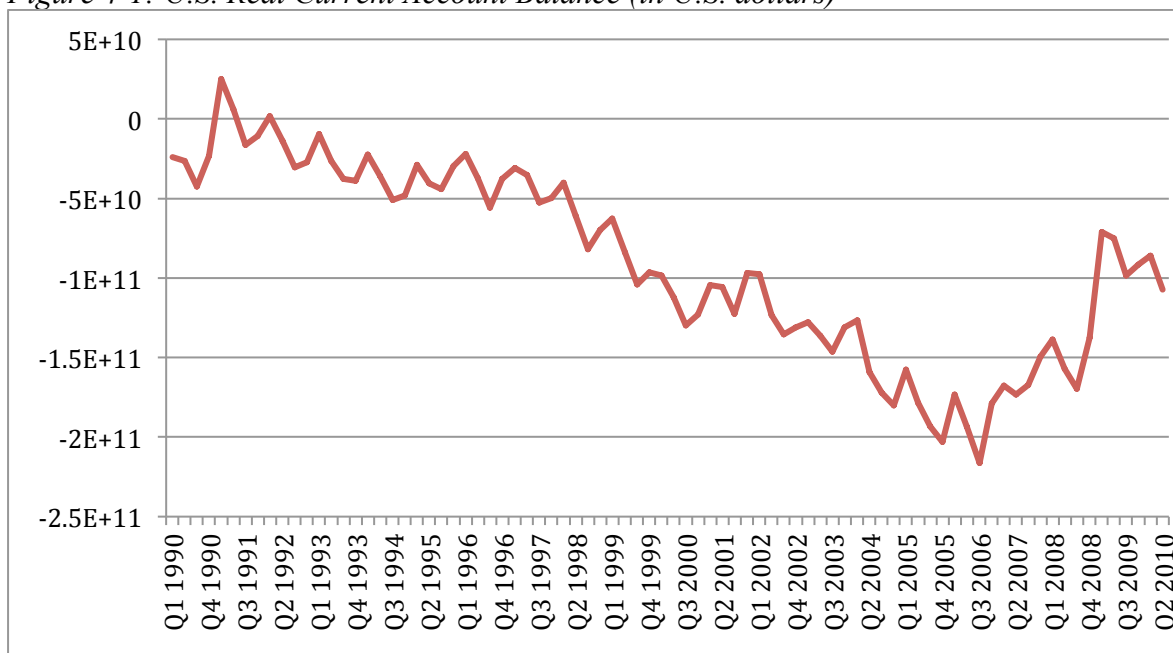
This paper presents a new explanation of how macroeconomic circumstances developed until the 2007-08 financial crisis. I develop a general equilibrium model of goods and assets markets that demonstrates how changes in the relative prices of imports and exports could have caused the U.S. current account deficit, its declining saving rate, the low interest rates, and the dynamic inefficiency and real estate bubble.

Though these four economic phenomena are addressed in the economics literature, it is seldom at the same time. Figure 4-1 illustrates how, after a brief positive balance in 1991, the U.S. real current account balance steadily declined until it reached a record deficit near the end of 2006. The U.S. real interest rate, shown in Figure 4-2, accompanied the current account balance in its downward trajectory, and became significantly more volatile as time progressed. The third chapter of this thesis empirically investigates this phenomenon in the context of dynamic inefficiency. It is shown that, starting in the year 2000, the equilibrium relationship between the interest rate and the growth rate of the value of the capital stock was such that the interest rate was lower – a sure sign of dynamic inefficiency.

There is extensive discussion in the literature about the consequences of dynamic inefficiency, including the creation of *rational* bubbles to increase the asset supply. Many have proposed the boom in U.S. housing markets and related markets for structured credit investments was such a bubble. Though agents on the demand and supply sides of the housing market could have made rational decisions (see Mian & Sufi, 2009), dynamic inefficiency resulted in such decisions creating a rational bubble.

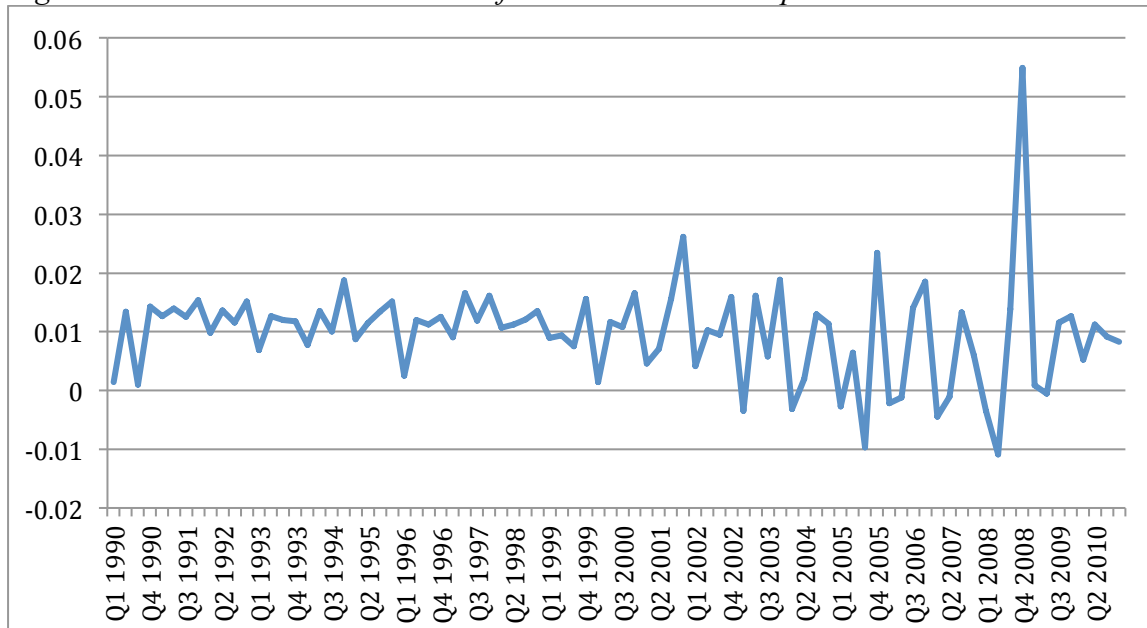
The real saving rate, shown in Figure 4-3, also experienced significant decreases, though its negative trajectory started substantially later than the interest rate and the current account balance. Indeed, the saving rate peaked at 7.4% in the first quarter of 1999, and subsequently fell to -3.0% in 2009.

Figure 4-1: U.S. Real Current Account Balance (in U.S. dollars)



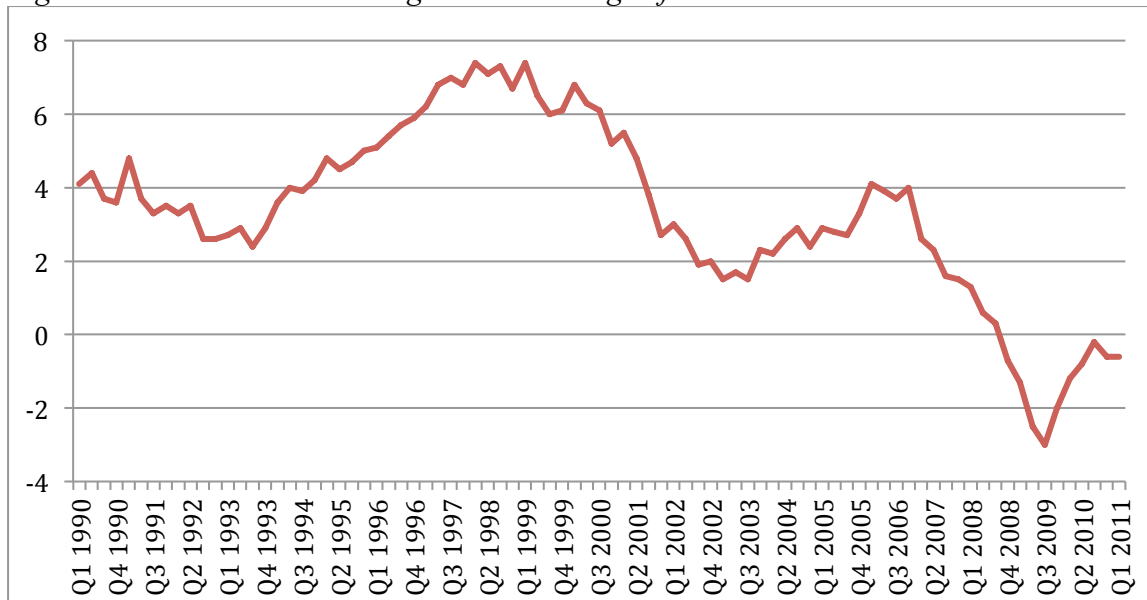
Data Source: Datastream

Figure 4-2: Real Return on an Index of AAA-Rated U.S. Corporate Bonds



Data Source: Datastream

Figure 4-3: Net National Saving as a Percentage of Gross National Income



Data Source: Datastream

This paper addresses the cause of these phenomena – declining current account balance, declining saving rate, declining interest rate, and dynamic inefficiency. It demonstrates that these

can all be explained by negative shocks to the U.S. terms of trade. The changes in the terms of trade contributed to increasing income in emerging markets. Higher income in these countries led to higher saving and demand for assets. Financial underdevelopment and financial crises meant that emerging economies could not create enough assets to meet demand. As a consequence, foreign demand for U.S. assets increased. Higher demand for assets caused the interest rate to decline.

As the interest rate decreased, the U.S. economy became dynamically inefficient in 2000. In response to this new development, the U.S. economy created a bubble asset out of real estate. Interest rates were low and prices for imported goods were now higher, so consumers were enticed into borrowing to finance continued consumption of imported goods. Though the asset supply had increased this increase was located in real estate – a largely unproductive asset. Economic growth did not increase, and expenditures grew at a faster rate than income. The saving rate started to decline.

Consumer borrowing increased demand for loanable funds, which put upward pressure on interest rates. At the same time, however, increased expenditure on imports increased income in foreign countries. Higher income in countries with relatively higher saving rates increased the supply of loanable funds. This put downward pressure on interest rates. The upward and downward forces caused volatility in interest rates.

In the beginning of 2007, the U.S. economy experienced a sharp rise in sub-prime mortgage defaults – the bursting of the real estate bubble. The subsequent financial crisis was due to how



this shock was passed through the financial system. What appears to be the leading argument is that, because financial operators had been packaging risky debt and passing it off as a safe investment – behaviour made possible by the lack of oversight of bond rating agencies and the failure of financial market regulators to keep up with financial developments – the bursting of the bubble resulted in significant negative economic shocks for the U.S. economy and the rest of the world.

This paper does not analyze why the crisis itself was so significant. Rather, it proposes the low current account, low saving rates, low interest rates and dynamic inefficiency experienced until the financial crisis could have been caused by negative terms of trade shocks. There are a number of observations and arguments made elsewhere in the economics literature that support this hypothesis. Debate about the cause of the current account deficit centred around whether it was due to trade patterns or due to foreign demand for U.S. assets. A current account deficit can mean either that the country is living beyond its means (i.e. due to high consumption national saving is less than investment) or that the country is prospering and attracting foreign investment because its economy has higher returns at lower risk than elsewhere (Mann, 2002).

The prosperity argument does not fit the circumstances in the United States over the last two decades. While U.S. assets were seen to have low risk, they did not have high returns. The real interest rate was remarkably low and continued to decrease even as the current account balance worsened. The savings glut argument (Bernanke, 2005; Clarida, 2005a,b) is similarly flawed. Chinn and Ito (2007) demonstrates there was no savings glut. Rather, the evidence suggests the U.S. current account deficit was caused by a saving drought rather than an investment boom.

This supports the first stated cause of current account deficits – spending beyond one’s means.

This view that the current account deficit was caused by insufficient U.S. saving is expressed in Ferguson (2004), Greenspan (2005a,b), Obstfeld and Rogoff (2005), and Chinn and Ito (2007).

According to this view, the current account deficit was American made and could be resolved only by reducing the trade deficit. The U.S. trade balance steadily declined from 1991 to 2006 – the same turning point in the current account. Business demand for investment goods fluctuated with the business cycle, whereas the trade balance for consumer goods was increasingly and persistently negative. That the trade deficit was in consumer goods rather than capital investment supports the assertion made above that foreign investment was funding consumption, not productive assets.

The low interest rates have primarily been attributed to large capital inflows (Caballero & Krishnamurthy, 2009). That begs the question of what caused these inflows – a shortage of assets elsewhere in the world (Caballero, 2006; Caballero et. al., 2008), or the need of American consumers to borrow to finance their consumption of imports? This paper argues the latter position.

There are not any models in the economic literature that explain all four of the economic phenomena the model presented here attempts to explain. However, there are several that are relevant in the area they concern. Blanchard and Giavazzi (2002) also proposes a link between the interest rate and the current account. That paper presents a model to show how the integration of goods markets and of financial markets affects the interest rate such that saving decreases and

investment increases. This results in a larger current account deficit. The model identifies output growth, the rate of change in the terms of trade, and the interest rate as determinants of the current account.

That there is a link between the current account and interest rates was empirically demonstrated in Milesi-Ferretti and Razin (2000). The authors demonstrate that the terms of trade and world interest rates have historically played a role in current account deficits.

Other relevant models involve the current account and exchange rate. Blanchard et. al. (2005) assumes imperfect substitutability between U.S. and foreign goods, and between U.S. and foreign assets. The model demonstrates the importance of the trade balance in the recent U.S. current account deficits. The importance of trade patterns is also demonstrated in Obstfeld and Rogoff (2005), which develops a three-region model with traded and non-traded goods to demonstrate the impact of preference on the terms of trade and exchange rate.

In this paper, I use a similar model to that in Caballero et. al. (2008), but adapted to emphasize the role of trade preferences in the behaviour of the current account, interest rates, and the saving rate. This is the only model to make all three variables endogenous. Unlike the model in Caballero et. al. (2008), each of the two regions produces one aggregate good that is imperfectly substitutable with the good produced in the other region. It also assumes different degrees of financial development in the two regions, and therefore does not use a common interest rate. This requires incorporation of the exchange rate in the model. It is therefore a significant extension of the basic framework from Caballero et. al. (2008).

The next section develops the trade-based model and solves for the equilibrium conditions. The third section discusses the results and their relevance to observed phenomena. The final section concludes.

## 2. A Trade-Based Model

There are two regions in the world economy: the United States (U), and the Rest of the World (R). Agents are born into the world at rate  $\theta$ , and they die at the same rate. Agents receive an endowment at birth, which they save in its entirety until death, at which point they consume their share of aggregate savings. Using  $W_t$  to represent aggregate savings at time  $t$ , and  $C_t$  to represent aggregate consumption, consumption each period is given by:

$$(4-1) \quad C_t = \theta W_t$$

There are two goods in this economy: the  $X$  good, which is produced only in Region U, and the  $M$  good, which is produced only in Region R. The goods are imperfectly substitutable, allowing for consumers to prefer one good to the other. I assume that trade between the two regions is free and costless, thus allowing all agents to consume both goods. Furthermore, the exchange rate,  $E_t$ , is such that the law of one price holds. Intra-temporal preferences are represented by a constant-elasticity-of-substitution utility function:

$$U_t = \left[ C_{X,t}^{(\sigma-1)/\sigma} + \alpha^{1/\sigma} C_{M,t}^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}$$

Here,  $\sigma > 0$  is the elasticity of substitution and  $\alpha > 0$  is a parameter that measures consumer preference for the  $M$  good relative to the  $X$  good. As  $\alpha$  increases consumers increasingly prefer good  $M$  to good  $X$ . In general, this parameter is constant.

The price of the  $X$  good relative to the price of the  $M$  good is equal to Region U's terms of trade, represented by  $p_t$ . We can thus use the price of good  $M$ , measured in Region U's currency, as a numeraire. Given the terms of trade, and using equation (4-1) as the budget constraint, utility maximization results in:

$$(4-2) \quad C_{X,t} = \frac{\alpha^{-1} p_t^{-\sigma} \theta W_t}{1 + \alpha^{-1} p_t^{1-\sigma}} \quad C_{M,t} = \frac{\theta W_t}{1 + \alpha^{-1} p_t^{1-\sigma}}$$

Production is modeled as follows. Production of both goods grows at the same constant rate,  $g > 0$ . Both goods are produced by a stock of tree assets. Region U has the tree assets necessary to produce good  $X$ , and Region R has the tree assets necessary to produce good  $M$ . Each tree asset produces one unit of output, making the number of assets equal to production.

Both goods are capitalizable. Let  $\delta^i$  be the fraction of output that is capitalizable and distributed to current asset owners each period, for  $i = U, M$ . This fraction represents a region's ability to create financial assets out of physical capital. The higher  $\delta^i$  is, the more the asset's production is distributed to the owners of capital as a return on their investment.  $\delta^i$  can therefore be viewed as

an index of the region's level of financial development, a point made in Caballero et. al. (2008). In that paper,  $\delta$  is assumed to be the same in both regions. I maintain a distinction between the financial development of the two regions. The portion of output that is not capitalized is distributed as part of the endowment to new agents.

The value of the tree assets will change over time. Let  $V_t^U$  represent the aggregate market value of  $X$  trees and  $E_t V_t^R$  the aggregate market value of the  $M$  trees, expressed in Region U's currency. The total value of all tree assets in the global economy is  $V_t = V_t^U + E_t V_t^R$ . Because production grows at rate  $g$ , the number of tree assets in each region must grow at the same rate. Thus, the total value of new trees in each period is equal to  $gV_t^i$ . The new trees each period are distributed to new agents as part of their endowment. The total endowment each period, expressed in Region U's currency, is therefore equal to the output that is not capitalized plus the new trees:

$$(4-3) \quad N_t = (1 - \delta^U)p_t X_t + (1 - \delta^R)E_t M_t + gV_t$$

The return to asset owners each period is equal to the dividend-price ratio,  $\delta^U p_t X_t / V_t^U$  and  $\delta^R M_t / V_t^R$ , plus the capital gain,  $\dot{V}_t^U / V_t^U - g$  and  $\dot{V}_t^R / V_t^R - g$ . In equilibrium, the return to the owner of a region's tree asset must equal that region's interest rate. This condition is represented for each region as:

$$(4-4a) \quad r_t^U V_t^U = \delta^U p_t X_t + \dot{V}_t^U - gV_t^U$$

$$(4-4b) \quad r_t^R V_t^R = \delta^R M_t + \dot{V}_t^R - gV_t^R$$

The interest rates are related to each other by interest rate parity:

$$r_t^U = r_t^R + \frac{\dot{E}_t}{E_t}$$

As in Caballero et. al. (2008), I allow for the development of an unproductive bubble asset with value  $B_t$ . The bubble develops only in Region U, and only if the economy is dynamically inefficient. When there is a bubble, its value must satisfy the arbitrage condition:

$$(4-5) \quad \dot{B}_t = (r_t + \lambda)B_t$$

$\lambda > 0$  is the hazard the bubble will burst the next instant.

Having described the goods markets, the asset market, and income, I now turn to savings and the evolution of savings over time. In equilibrium, the value of aggregate savings must equal the total value of all assets:

$$(4-6) \quad W_t = V_t + B_t$$

I assume an extreme form of home bias. Agents invest in assets of their home region first. Only if the asset supply is insufficient do they buy foreign assets. Because the U.S. is largely regarded as having the most developed financial market, I assume  $\delta^U > \delta^R$ . Region U is more capable of meeting asset demand by its citizens than Region R. Therefore, agents in Region R buy all the

Region R assets and some of the Region U assets. Let  $\pi_t^R$  be the proportion of Region R wealth that is invested in Region U assets. Then:

$$W_t^U + \pi_t^R E_t W_t^R = V_t^U + B_t$$

$$(1 - \pi_t^R) W_t^R = V_t^R$$

Accumulated savings each period changes by the net of consumption, income, and the return on accumulated savings:

$$(4-7a) \quad \dot{W}_t^U = -\theta W_t^U + (1 - \delta^U) p_t X_t + g V_t^U + r_t^U W_t^U$$

$$(4-7b) \quad \dot{W}_t^R = -\theta W_t^R + (1 - \delta^R) M_t + g V_t^R + r_t^R W_t^R + \frac{\dot{E}_t}{E_t} \pi_t^R W_t^R$$

$$(4-7c) \quad \dot{W}_t = -\theta W_t + (1 - \delta^U) p_t X_t + (1 - \delta^R) E_t M_t + g V_t + r_t^U W_t - (1 - \pi_t^R) \dot{E}_t W_t^R$$

In the goods market, equilibrium requires that demand equal supply. This results in the following two equilibrium conditions:

$$(4-8a) \quad \frac{\alpha^{-1} p_t^{-\sigma} \theta W_t}{1 + \alpha^{-1} p_t^{1-\sigma}} = X_t$$

$$(4-8b) \quad \frac{\theta W_t}{1 + \alpha^{-1} p_t^{1-\sigma}} = M_t$$

The terms of trade is the relative price that achieves equilibrium in both markets. This occurs when:



$$(4-9) \quad p = \left( \frac{M_t}{\alpha X_t} \right)^{1/\sigma}$$

Because production of good  $M$  and good  $X$  grows at the same rate,  $p$  is constant over time unless there is a shock to the preference parameter,  $\alpha$ , or the elasticity of substitution,  $\sigma$ . This is a reasonable result for long-run equilibrium, as the historical series of the U.S. terms of trade fits a random walk model with constant mean. Because the terms of trade is influenced only by random shocks, the time subscript will be dropped on all further references.

It is now possible to derive the equilibrium interest rate for Region U<sup>3</sup>. The equilibrium interest rate for Region R can then be found using the interest rate parity condition. The equilibrium interest rate has a similar format to that derived in Caballero et. al. (2008), except here there are no oil inventories, the level of financial development in each country is different, and the exchange rate is included due to each country having a different interest rate. The equilibrium relation is:

$$(4-10) \quad r_t^U = \theta \frac{\left( \delta^U + g \frac{B_t}{pX_t} \right) - \left( \frac{(1-\delta^R)E_t M_t}{pX_t} - \alpha p^{\sigma-1} \right)}{1 + \alpha p^{\sigma-1}} - \dot{E}_t (1 - \pi_t^R) \frac{W_t^R}{W_t}$$

The first bracketed term represents asset supply. The supply of assets is increased if financial development increases in Region U and/or if there is a bubble asset. Increasing the supply of assets increases the interest rate. The second bracketed term concerns trade. If a shock causes the

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<sup>3</sup> See Appendix I for the derivation

terms of trade to decrease (the  $M$  good becomes relatively more expensive), the first effect is to increase the value of the endowment to agents in Region R, which increases asset demand and causes the interest rate to decrease. The second effect is to decrease the effective income of those who consume the  $M$  good, which would decrease asset demand and increase the interest rate. The first effect dominates, so that a decrease in the terms of trade causes a decrease in the interest rate.

The interest rate is also affected by relative consumer preference for the  $M$  good, by financial development in both regions, by changes in the exchange rate, and by the proportion of wealth invested by residents of Region R in Region U's assets.

The next section will review the facts that this model is intended to explain. The model is then used to show how changes in preference for imports over domestic goods may have caused those observations.

### **3. Applying the Model**

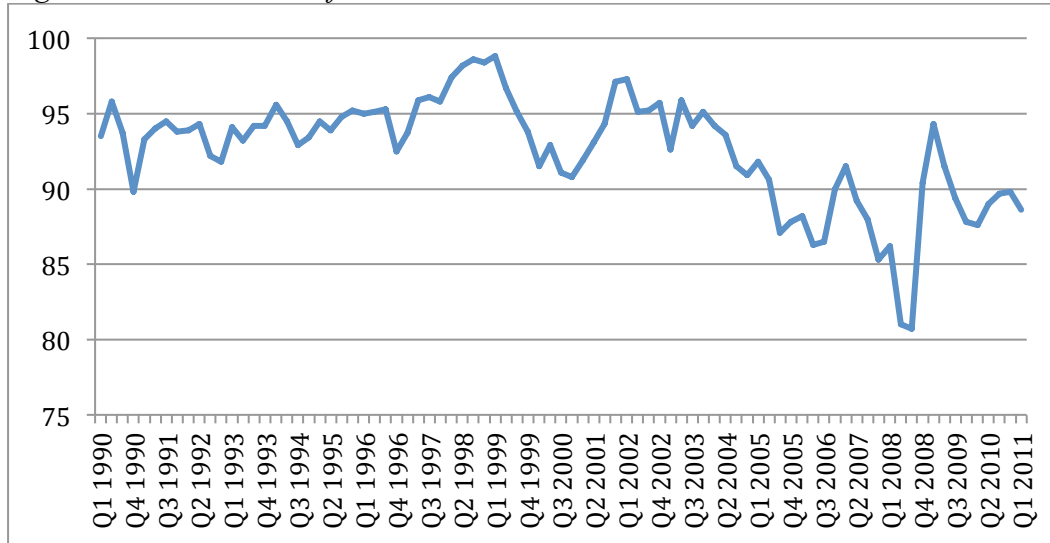
The third chapter found evidence the U.S. economy became dynamically inefficient in 2000. Blanchard et. al. (2005) observed that starting at about the same time there was increased demand in the U.S. for imports. This would be demonstrated in the present model by positive shocks to the parameter  $\alpha$ . At the same time, the U.S. terms of trade followed a cyclical, but overall negative, trajectory (see Figure 4-4). As pointed out in the introduction, the U.S. also had a growing current account deficit.

Though this model does not identify the cause of the shocks to the terms of trade, increased preference for imports is a likely candidate. The effect of an increased preference for good  $M$  is given by the derivative of the terms of trade with respect to  $\alpha$ :

$$(4-11) \quad \frac{\partial p}{\partial \alpha} = -\frac{p}{\alpha\sigma} < 0$$

An increase in preference for good  $M$  relative to good  $X$  would cause a decline in the terms of trade. Alternatively, it could be assumed some other exogenous factor causes the terms of trade to decline.

Figure 4-4: U.S. Terms of Trade Index



Data Source: Datastream

The central point of this model is the effect of changes in the terms of trade on the U.S. interest rate. The declining interest rate made the economy dynamically inefficient, and was therefore

likely responsible for the creation of the real estate bubble. As explained in Chapter 3, there are several conditions that have been proposed as indicative of dynamic inefficiency. The condition relevant for this model is that dynamic inefficiency is indicated by an interest rate lower than the output growth rate (Feldstein, 1976; Tirole, 1985; Abel et. al., 1989; Santos & Woodford, 1997; Caballero et. al., 2008). Rational bubbles emerge when an economy is dynamically inefficient (Tirole, 1985; Abel et. al., 1989; Martin & Ventura, 2010; Farhi & Tirole, 2011). The bubble increases the asset supply and so maintains the interest rate at or just above the economic growth rate.

The results of the model change from the pre-bubble period to the bubble period. Analysis of the interest rate is therefore divided into these two periods.

Without a bubble asset, the equilibrium U.S. interest rate is given by:

$$r_t^{ref} = \theta \frac{\delta^U - \left( \frac{(1-\delta^R)E_t M_t}{pX_t} \alpha p^{\sigma-1} \right)}{1 + \alpha p^{\sigma-1}} - \dot{E}_t (1 - \pi_t^R) \frac{W_t^R}{W_t}$$

I call this the “reference interest rate”, as it provides a baseline against which to compare future changes. If the exchange rate is constant, the reference interest rate is constant. Since the exchange rate is simply the rate at which the law of one price holds for both goods, I treat it as a parameter and therefore constant (absent any shocks). This makes the reference U.S. interest rate given by equation (4-12):

$$(4-12) \quad r^{ref} = \theta \frac{\delta^U - \left( \frac{(1-\delta^R)E_t M_t}{p X_t} \alpha p^{\sigma-1} \right)}{1 + \alpha p^{\sigma-1}}$$

We can determine how a change in the terms of trade affects this reference interest rate by taking the derivative:

$$\frac{dr^{ref}}{dp} = \frac{(1-\sigma)\alpha p^{\sigma-2}}{1 + \alpha p^{\sigma-1}} (\theta E - \theta E \delta^R - \theta + r^{ref})$$

Whether this derivative is positive or negative depends on  $\sigma$ , the elasticity of substitution, and the value of  $r^{ref}$  relative to  $\theta E - \theta E \delta^R - \theta$ . I assume the elasticity of substitution is inelastic and the exchange rate is such that  $r^{ref} > \theta + \theta E \delta^R - \theta E$ . This would mean  $\frac{dr^{ref}}{dp} > 0$ , and negative terms of trade shocks, whether because of increased preference for imports or otherwise, would cause the interest rate to decline.

Assuming inelastic elasticity of substitution means the goods of the two regions are not very substitutable. This is reasonable considering the great difference in the goods produced by a highly developed economy such as the United States and the goods produced by emerging market, commodity-producing economies. The assumption about the reference interest rate is also reasonable.  $\theta$ , the rate of birth and of death in both regions, is a small percentage. Because  $\delta^R < 1$ , I am assuming the reference interest rate is larger than  $\theta$  plus sum negative value.

Thus, until the bubble asset started, repeated negative shocks to the terms of trade caused the interest rate to decline. The terms  $(\theta + \theta E \delta^R - \theta E)$  were a minimum bound for the interest rate. Once the interest rate reached that value, its derivative with respect to  $\alpha$  or with respect to  $p$  would be zero. These shocks would no longer affect the interest rate.

If we assume  $(\theta + \theta E \delta^R - \theta E) < g$ , again a reasonable assumption, then the minimum bound of the reference interest rate is below the economic growth rate. The consequence is that continued shocks would eventually create dynamic inefficiency in the economy. This sets up the economy for the emergence of a bubble asset.

In about 2000, the interest rate became low enough to cause dynamic inefficiency. As a result, the increased interest in real estate became a bubble. With a bubble asset, the equilibrium U.S. interest rate became equation (4-10). To see how it compares to the reference interest rate, we can express it as:

$$(4-13) \quad r_t^U = r^{ref} + \frac{\theta g}{(1 + \alpha p^{\sigma-1})} \frac{B_t}{p X_t}$$

The asset supply is now supplemented by the bubble asset, which has a positive effect on the interest rate. A shock is no longer required to change the interest rate; it can follow a positive or negative time path depending on the growth of the bubble asset relative to the growth of Region U's output. In other words, it depends on the relative strengths of increased borrowing within Region U and higher income (and therefore saving) in Region R. Taking the derivative with respect to time:

$$(4-14) \quad \dot{r}_t = \frac{\theta g}{(1+\alpha p^{\sigma-1})p} \left( \frac{B_t}{X_t} \right) (r_t + \lambda - g)$$

This derivative can be positive or negative. If financial markets react quickly to economic developments, one can assume the bubble started soon after the interest rate became equal to the output growth rate and started falling below it. Assuming the bubble began when the interest rate was marginally below the output growth rate and that the bubble had positive risk, we can assume that, when the bubble began,  $r_t + \lambda > g$ . This would make equation (4-14) positive. The emergence of the bubble would tend to make the interest rate grow over time.

However, the interest rate is still affected by shocks to the terms of trade. The derivative once the bubble starts is:

$$\frac{dr_t^U}{dp} = \frac{(1-\sigma)\alpha p^{\sigma-2}}{1+\alpha p^{\sigma-1}} \left( \theta E - \theta E \delta^R - \theta + r_t^U - \frac{\theta g B_t}{(1-\sigma)M_t} \right)$$

We still assume  $\sigma < 1$ . The sign of this derivative therefore depends on the value of  $r_t^U$  relative to  $\theta E - \theta E \delta^R - \theta - \frac{\theta g B_t}{(1-\sigma)M_t}$ . The interest rate is now larger than the reference interest rate, but we have added the term  $\frac{\theta g B_t}{(1-\sigma)M_t}$ . This term would be very small, and therefore the derivative is still positive. Thus, there are now two opposing forces acting on the interest rate. The growth of the bubble asset has a positive effect on the interest rate, whereas negative shocks to the terms of trade have a negative effect. Depending on the relative strengths of the two forces, the interest rate could increase, decrease, or stay the same. Referring to Figure 4-2 we see a great deal of

volatility in the interest rate, suggesting that the relative strengths of the two effects frequently changed.

I divide the analysis of the saving rate into the same two time periods. This is necessary because of the saving rate's dependence on the interest rate. In this economy, saving is equal to the difference between the endowment, equation (4-3), and expenditure, equation (4-1). The saving rate is:

$$(4-15) \quad s_t = \frac{S_t}{N_t} = \frac{N_t - \theta W_t}{N_t}$$

Taking the derivative with respect to time, we find:

$$\dot{s}_t = \frac{\theta W_t}{E_t^2} [g(\dot{V}_t - gV_t)] = -\frac{g\theta W_t}{E_t^2} (r_t + g + \lambda)B_t$$

There was no bubble before about 2000, when the U.S. economy became dynamically inefficient. During that period this derivative was zero and the saving rate was constant. Once the bubble began, the saving rate followed a negative time path. The bubble had a negative impact on saving. This could be explained by the assertion made in the introduction that the money borrowed from abroad was used to finance consumption rather than to increase production. The bubble asset enabled higher consumption without increasing income.

The saving rate is also affected by shocks to the terms of trade. Regardless of the time period, the derivative is:



$$\frac{ds_t}{dp} = \frac{X_t}{N_t^2} [(\sigma - 1)N_t X_t + \theta W_t (1 - \delta^U) X_t] < 0$$

Thus, before the bubble started and the saving rate was otherwise constant, negative shocks to  $p$  caused the saving rate to increase. This is what we observe in Figure 4-3. The U.S. saving rate peaked in 1999, right before the economy became dynamically inefficient in 2000 and the real estate bubble put downward pressure on the saving rate.

The current account is the final endogenous variable of interest. From the financial perspective, the U.S. current account is simply the difference between the increase in wealth of U.S. consumers and the increase in U.S. assets:

$$CA_t^U = \dot{W}_t^U - \dot{V}_t^U - \dot{B}_t$$

In equilibrium, this equals:

$$(4-16) \quad CA_t^U = TB_t^U - r_t^U F_t^U - \lambda B_t$$

Here,  $TB_t^U = pX_t - \theta W_t^U$  is Region U's trade balance, and  $F_t^U = V_t^U + B_t - W_t^U$  is Region U's net debt. As would be expected, the current account is positively related to the trade balance. It is the largest determinant of the current account. Region U's current account is also affected by debt payments and the value of the bubble asset. If Region U is a net borrower, an increase in the

interest rate decreases the current account. This is because the higher interest rate increases debt payments.

The creation of a bubble immediately has an increasingly negative effect on the current account. This is because the bubble is in an unproductive asset and does not increase Region U's production. If the money were instead invested in tree assets, it would increase Region U's output and improve the trade balance. Because it is in real estate, the borrowed money is used to finance consumption.

The effect of shocks to the terms of trade on the current account is given by equation (4-17):

$$(4-17) \quad \frac{dCA_t^U}{dp} = X_t - F_t^U \frac{dr_t^U}{dp}$$

Before the bubble started, the interest rate was the reference interest rate and the second term was positive. It is therefore ambiguous as to whether declining terms of trade increased or decreased the current account. On the one hand, declining terms of trade should improve the trade balance and therefore increase the current account balance. On the other hand, if the terms of trade are declining because of increased preference for imports, the trade balance is declining. This would make the current account worse.

A similar result holds after the bubble started. Negative shocks to the terms of trade still have a negative impact on the interest rate. As the net debt grew, this negative effect on the current account would get larger. If it is accepted that the terms of trade shocks were caused by

increasing preference for imports, then the trade balance was declining at the same time. It would therefore be certain these shocks caused declining current account balances.

#### **4. Conclusion**

The model presented in this paper demonstrates how shocks to the U.S. terms of trade can explain the current account deficit, low interest rate, low saving rate, and the dynamic inefficiency that resulted in the real estate bubble. The increased expenditure on imports, and decreased value of exports caused a worsening of the trade balance and subsequently an increase in the current account deficit. The higher prices also contributed to increased income in emerging markets. Higher income increased demand for assets. On the other hand, the higher prices meant decreased purchasing power for U.S. consumers, which would cause a decrease in asset demand. The net effect was positive, such that the interest rate declined.

In about 2000, the U.S. economy became dynamically inefficient. The dynamic inefficiency created a (rational) bubble in real estate, increasing the asset supply. Though the growing bubble put upward pressure on the interest rate, continued shocks to the terms of trade – increasing the demand for assets – kept the interest rate low.

These mechanisms ultimately had a negative effect on the U.S. saving rate. Low interest rates made saving less attractive. Furthermore, the bubble regarded an unproductive asset. Rather than increase U.S. output, and therefore income, it provided additional funding for consumption. This enabled consumers to continue buying imports despite the decline in the terms of trade.

The model presented in this paper makes the current account, interest rate, and saving rate endogenous. Adding the feature of imperfectly substitutable goods and allowing for different interest rates in different regions, it demonstrates how shocks to the terms of trade can explain all the observed phenomena listed above.

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## Chapter 5: Conclusion

The fifteen years previous to the financial crisis in 2007-08 were characterized by several significant economic phenomena. These included a persistent U.S. current account deficit, low long run real interest rates, low saving rates, and the development of a bubble in real estate. These culminated in the crisis and one of the largest shocks to the oil market in history (Hamilton, 2009). This thesis addressed these phenomena.

The first paper, Chapter 2, tested for speculative oil inventories and whether they were linked to the turmoil in financial markets and the oil price shock. Caballero et. al. (2008) proposes a global asset shortage, exacerbated by the bursting of the real estate bubble, motivated investors to invest in oil inventories. There is an arbitrage condition that must hold if such speculative oil inventories were accumulated. This paper made a distinction between above-ground and below-ground inventories. Above-ground inventories, held by firms and investors that do not extract crude oil, incur a storage cost in addition to a convenience yield. Oil extractors can accumulate speculative inventories without a storage cost by leaving the oil underground. The purpose is to delay extraction for an anticipated higher price. Empirical testing established that both above-ground and below-ground inventories were accumulated. However, these inventories did not coincide with the price shock.

The low interest rates and the development of the real estate bubble raise the question of dynamic inefficiency in the U.S. economy. The second paper, Chapter 3, tested for such inefficiency during the period of 1990 to 2011. Previous tests of dynamic efficiency have not

been empirically robust. Dynamic inefficiency is an equilibrium condition, and can be masked by the development of bubbles. Chapter 3 demonstrated how cointegration analysis resolves these problems and enables a deep, effective test of dynamic efficiency. It found the U.S. economy became dynamically inefficient in 2000. This required testing for breaks in the cointegration relationship.

The third paper, Chapter 4, presented a model of international goods markets and asset markets. It explains the behaviour of the U.S. current account, interest rate, and saving rate, as well as the dynamic inefficiency. Its contribution is to demonstrate that negative shocks to the U.S. terms of trade, possibly caused by growing preference for imported goods in the U.S., could be the cause of that behaviour. Because imports were cheaper, the initial effect of the change in preference was to decrease expenditure and increase saving, and to cause a declining trade balance. The trade balance drove the growing current account deficit. As the terms of trade adjusted to continual changes in preferences, expenditure on consumption began increasing. The U.S. saving rate declined and the current account worsened. To finance the increase in consumption, the U.S. needed to borrow from abroad. This created the large capital inflows. The real estate bubble demonstrates the capital was used to finance consumption rather than to invest in productive assets.



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## Appendix I: Derivation of the Equilibrium Interest Rate

From equation (4-8a), equilibrium in the market for the X good can be rearranged as:

$$(A1) \quad \theta W_t = (1 + \alpha p_t^{\sigma-1}) p_t X_t$$

In the asset market, equation (4-6) can be rearranged as:

$$(A2) \quad V_t = W_t - B_t$$

Substituting equations (A1) and (A2) into equation (4-7):

$$\dot{W}_t = -(1 + \alpha p_t^{\sigma-1}) p_t X_t + (1 - \delta^U) p_t X_t + (1 - \delta^R) E_t M_t + g(W_t - B_t) + r_t^U W_t - \dot{E}_t W_t^R$$

$$\dot{W}_t = -\alpha p_t^\sigma X_t - \delta^U p_t X_t + g W_t - g B_t + r_t^U W_t + (1 - \delta^R) E_t M_t - \dot{E}_t W_t^R$$

$$\dot{W}_t = -\alpha p_t^\sigma X_t - \delta^U p_t X_t - g \frac{B_t}{p_t X_t} p_t X_t + g W_t + r_t^U W_t + (1 - \delta^R) \frac{E_t M_t}{p_t X_t} p_t X_t - \dot{E}_t W_t^R$$

$$(A3) \quad \dot{W}_t = -p_t X_t \left( \delta^U + g \frac{B_t}{p_t X_t} \right) + p_t X_t \left( \frac{(1 - \delta^R) E_t M_t}{p_t X_t} - \alpha p_t^{\sigma-1} \right) + g W_t + r_t^U W_t - \dot{E}_t W_t^R$$

From equation (4-8b), equilibrium in the market for the M good can be rearranged as:

$$W_t = \frac{1}{\theta} (1 + \alpha^{-1} p_t^{1-\sigma}) M_t$$

Taking the derivative with respect to time and using equation (4-8a):

$$(A4) \quad \dot{W}_t = \frac{1}{\theta} (1 - \sigma) p_t X_t \tau_t + g W_t$$

Where  $\tau_t$  is the growth rate of the U.S. terms of trade. Substitute equation (A4) into equation

(A3):

$$\begin{aligned} -p_t X_t \left( \delta^U + g \frac{B_t}{p_t X_t} \right) + p_t X_t \left( \frac{(1 - \delta^R) E_t M_t}{p_t X_t} - \alpha p_t^{\sigma-1} \right) + g W_t + r_t^U W_t - \dot{E}_t W_t^R \\ = \frac{1}{\theta} (1 - \sigma) p_t X_t \tau_t + g W_t \end{aligned}$$

Rearranging equation (4-8a) as  $W_t = \frac{1}{\theta} (1 + \alpha p_t^{\sigma-1}) p_t X_t$  and substituting into the equation

above:

$$\begin{aligned} \frac{1}{\theta} (1 + \alpha p_t^{\sigma-1}) p_t X_t r_t^U \\ = p_t X_t \left( \delta^U + g \frac{B_t}{p_t X_t} \right) - p_t X_t \left( \frac{(1 - \delta^R) E_t M_t}{p_t X_t} - \alpha p_t^{\sigma-1} \right) + \frac{1}{\theta} (1 - \sigma) p_t X_t \pi_t \\ + \dot{E}_t W_t^R \end{aligned}$$

$$(1 + \alpha p_t^{\sigma-1})r_t^U = \theta \left[ \left( \delta^U + g \frac{B_t}{p_t X_t} \right) - \left( \frac{(1 - \delta^R)E_t M_t}{p_t X_t} - \alpha p_t^{\sigma-1} \right) \right] + (1 - \sigma)\pi_t + \frac{\dot{E}_t \theta W_t^R}{p_t X_t}$$

$$(A5) \quad r_t^U = \theta \frac{\left( \delta^U + g \frac{B_t}{p_t X_t} \right) - \left( \frac{(1 - \delta^R)E_t M_t}{p_t X_t} - \alpha p_t^{\sigma-1} \right)}{1 + \alpha p_t^{\sigma-1}} - \frac{(1 - \sigma)\pi_t}{1 + \alpha p_t^{\sigma-1}} - \dot{E}_t (1 - \pi_t^R) \frac{W_t^R}{W_t}$$

Because the terms of trade is constant, the final term can be dropped to provide the equilibrium interest rate:

$$(4-10) \quad r_t^U = \theta \frac{\left( \delta^U + g \frac{B_t}{p_t X_t} \right) - \left( \frac{(1 - \delta^R)E_t M_t}{p_t X_t} - \alpha p_t^{\sigma-1} \right)}{1 + \alpha p_t^{\sigma-1}} - \dot{E}_t (1 - \pi_t^R) \frac{W_t^R}{W_t}$$