Life is a random variable whose probability distribution is a function of our dreams, thoughts and efforts. We may not be able to predict it, but we at least can shape the function well!

University of Alberta

LIFETIME ANALYSIS OF RANDOMLY DEPLOYED WIRELESS SENSOR NETWORKS

by

Moslem Noori ©

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment of the requirements for the degree of **Master of Science**.

Department of Electrical and Computer Engineering

Edmonton, Alberta
Fall 2008

Your file Votre référence
ISBN: 978-0-494-47377-1
Our file Notre référence
ISBN: 978-0-494-47377-1

To My Family

Abstract

Due to the limited energy resource in the network nodes, lifetime is a considerable issue in wireless sensor networks. When the network is randomly deployed, network lifetime exhibits a stochastic behaviour which may be strengthened by a random traffic generation scheme. This dissertation mainly deals with the lifetime study of such networks where a probabilistic approach is proposed to model the network lifetime. First, we perform the study for non-clustered networks and propose an optimal node distribution to prolong the network lifetime. Then, the analysis is extended to clustered networks resulting in a condition which determines whether clustering in the network is beneficial or not. Our results on the lifetime study are helpful in the network design step to adjust the network parameters appropriately. Moreover, we investigate the traffic load distribution over the nodes in a randomly deployed linear network. The outcome of this traffic study specifies where the network hot spot is located.

Acknowledgements

First of all, I owe many thanks to my supervisor, Dr. Masoud Ardakani, for tremendously contributing his effort and expertise to end this dissertation. Fulfilling this research project undoubtedly was impossible without his care, guidance and encouragement. I have not only benefited from Masoud's professionalism in academic issues, but also in the other aspects of life. I just can say that it was my great luck to have him as my supervisor!

I also wish to acknowledge the members of the evaluation committee: Professor Norman C. Beaulieu and Professor Janelle Harms. In addition, I like to thank Nona McDonagh for organizing the thesis defence session.

I desire to cordially acknowledge all my friends in Edmonton for their invaluable friendship, sympathy and help. They have made every moment of my life joyful and hopeful to progress. Specially, I would like to sincerely appreciate Ali Sharifkhani, Ali Sanaei, Alireza Ghaderipoor, Amir Masoud Rabiei, Arash Talebi, Chen Liu, Emma Frontana, Hinako Ishikawa, Mahdi Ramezani, Mohsen Eslami, Payam Dehghani, Raman Yazdani and Wei Zhang.

Moreover, I am grateful to my greatest friends back home in Iran who have never forgotten me and I will never forget them. Among them, I like to name Ali Jafarnia, Ali Nikakhtari, Arash Golabi, Hossein Hosseini, Farid Sahandi, Majid Lari, Mostafa Saboori, Payam Azizi and Reza Modarresi. Our common memories are unforgettable periods of my life and have been etched in my mind.

And my most heartfelt thanks go to my family which I always feel their wholehearted and endless support, even here, far away from them. Without my parents' support, trust and encouragement, none of my accomplishments were achievable. Also, I should recall all the things I have benefited from my sister and brother which have affected my life profoundly.

Unfortunately, I am not able to adequately express my appreciation for all adorable people, who helped me to reach this point, through the words and in this limited space. This formal acknowledgment was just what my mind could think, but my heart forever thanks you all!

Table of Contents

List of Tables

List of Figures

List of Symbols

List of Abbreviations

Abbreviation	Description	First Use
WSN	Wireless sensor network	1
MAC	Media access control	1
LEACH	Low-energy adaptive clustering hierarchy	3
ccdf	Complementary cumulative distribution function	5
CH	Cluster head	5
pdf	Probability distribution function	9
TDMA	Time division multiple access	10
FDMA	Frequency division multiple access	10
CDMA	Code division multiple access	10
i.i.d.	Independent and identically distributed	16
CSMA	Carrier sense multiple access	23
RTS	Request-to-send	23
CTS	Clear-to-send	23

Chapter 1

Introduction

Wireless sensor network (WSN) is an emerging technology used in data collection applications. A WSN consists of a set of tiny and usually battery-powered devices, called sensors. Sensors are in charge of collecting useful data from the field, performing essential data processing and forwarding the processed data toward a central data station, named sink. To make the desired network job done, all sensing, computing and communicating abilities have to be combined in a single sensor. Therefore, WSN technology has opened a new era not only in the field of data communications, dealing with designing proper data transmission schemes, but also in the electronic technologies to compact all the necessary features in a single minute device.

Although WSNs are still in their infancy, they have broken their way through and found several applications in reality. Performing remote monitoring with a low data sampling frequency is a potential application of WSNs. The remote data monitoring ability looks more desirable when the monitored environment is harsh and even not accessible to deploy the network manually. In this case, the network can be deployed in an ad hoc way (for example dropping the sensors from an aircraft) over the area and then sensors form the network and configure themselves.

ZebraNet [1] and Great Duck Island [2] projects are two practical examples where WSNs have been utilized to help the scientist in habitat monitoring and wildlife studies. WSNs also are applicable to surveillance and intrusion detection applications [3]. Another application of the WSNs can be found in road traffic monitoring [4]. Here, just a few practical cases are pointed out, but new applications emerge everyday [5].

Despite the limited capabilities of individual sensors, through collaboration, they can give the network the ability to accomplish more sophisticated tasks in an efficient manner. The cooperation may form in the routing scenario, media access control (MAC) level, data aggregation and compression schemes, etc. Notably, due to the fundamental differences between a WSN and a conventional wireless network, which mainly arise from the sensors limited power resource and the ad hoc topology of the network, the common transmission solutions used in ordinary wireless networks may not directly be applicable to WSNs.

Energy constraint of the network nodes is counted as one of the main issue that distinguishes WSNs from conventional wireless networks. Since data receiving, processing and transmitting all consume energy, the sensor battery can support the sensor to operate for a limited time which is called the sensor lifetime. Death of the nodes in turn defects the network functionality and the network cannot operate desirably. The time duration over which the network can operate satisfyingly is called the network lifetime which is an important concept in WSNs. A more formal definition of the lifetime, which is adopted in this work, will be provided later on.

It is often desirable to increase the network lifetime for cost-benefit purposes. One possible solution to extend the network lifetime is manual battery replacement. This approach, however, may not be always possible, for example in hostile situations. Another solution is to use a rechargeable battery mechanism (e.g. solar battery) to provide the required sensor power. This solution needs more sophisticated sensors and increases the network cost significantly.

Furthermore, to decrease the effect of limited power on the network functionality, it is required to efficiently design the sensors hardware and network protocols in a way that the energy consumption rate is optimized. For instance, Mica Mote [6] and Intel Mote [7] are two types of motes suitably designed for WSN applications (Figure 1.1). These devices are built such that while the necessary sensing, communication and computing abilities are embedded in them, the size, cost and power usage are kept low as well.

In addition to the hardware architecture, network protocol design also matters and influences energy conservation in the network. To design the appropriate network protocol, it is necessary to compromise between different factors (such as data rate, network delay, data throughput, network coverage and connectivity) to efficiently adjust the energy depletion rate in the network and as a consequence, increase the network lifetime. Designing an energy efficient MAC protocol is a promising approach to save energy [8, 9]. An effective technique that MAC scheme might apply is sleep scheduling mechanism which tries to turn off the unnecessary sensors in the network [10]. Applying this technique is feasible when the density of the deployed nodes is more than the density required to have an acceptable coverage over the area. The extra nodes substitute for the dead/sleeping nodes to keep the network performance at the desired level.

Designing an efficient routing scheme is also another way for energy conservation, e.g. [11, 12]. Basically, an energy efficient routing algorithm tries to balance the energy consumption in the network and prevent the early death of nodes. A general review on the routing schemes for WSNs can be found in [13].

In addition, taking a cross layer design approach is common in order to decrease the energy consumption in the network [14]. In a cross layer approach, it is tried to make an adjustment between different network layers such that the energy efficiency in the network increases.

Clustering the network has also been considered in the literature to prolong the network lifetime [15]. For instance, authors in [16] propose the well-known low-energy adaptive clustering hierarchy (LEACH) to improve the energy efficiency in the network. Also, using a mobile data collecting center, instead of a fixed station, can result in energy efficiency [17].

1.1 Thesis Motivation

Although the abovementioned methods are advantageous in the energy efficiency, they do not substantially discuss the achievable network lifetime. This issue becomes more crucial when the network is deployed randomly over the area. In this case, the random position of the sensors gives the network lifetime a random essence. Recall that the sensors positions play an important rule in energy consumption rate. For example, in a single-hop network, nodes far from the sink die earlier than the other nodes. In a multi-hop network, however,

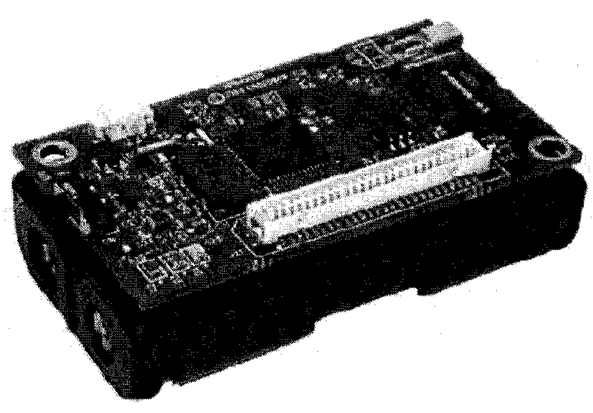


Figure 1.1: Mica mote.

the energy consumption rate is higher in the sensors close to the sink. Therefore, analyzing and estimating the network lifetime, in addition to the efforts spent on increasing it, seems to be necessary. The results of such analysis can be used in the network design step to decrease the network implementation cost by adjusting the network parameters and even choosing the proper protocols. We delay a detailed literature review on the lifetime studies to the following chapter.

In this thesis, we mostly focus on the lifetime analysis of randomly deployed event-driven WSNs. In an event-driven WSN, sensors traffic generation is also stochastic which in turn intensifies the randomness of the lifetime. Random network deployment is common in applications where the network cannot be implemented manually and the sensors have been spread over the field without any prepared structure. Despite the practical importance of the randomly deployed WSNs, their lifetime has not been thoroughly studied in the literature, specifically when the traffic generation in the network is also random or the area size and number of nodes are finite.

Due to the randomness of the network lifetime in a randomly deployed network, the average value of the consumed energy or expected network lifetime may not precisely reflect the network behaviour. Therefore, a stochastic analysis of the network lifetime seems to be necessary. Indeed, the stochastic lifetime analysis reveals the reliability of achieving a desired lifetime by the network which cannot be found through the average analysis.

1.2 Thesis Contributions

Analysis of the lifetime for a randomly deployed event-driven WSN is the main focus of this dissertation. For this purpose, we develop a probabilistic analysis to model the network behaviour. First, the lifetime of a single sensor is investigated and then the study is extended to the network level. The analysis in not limited to a specific network topology and can be applied to single-hop, multi-hop and clustered WSNs.

The following key points distinguish our proposed lifetime analysis from existing results:

- The lifetime of a single node is not assumed to be known. This requires a lifetime analysis at the sensor level first.
- While almost all previous lifetime studies focus on time-driven networks, our proposed analysis investigates the lifetime for any arbitrary traffic model, specially event-driven networks where the traffic generation occurs randomly.

- The analysis is applicable to a network with any arbitrary area size and number of nodes. This also differentiates our work from some of the previous works which study the lifetime in asymptotic situations.
- For clustered networks, we provide a condition indicating whether clustering is beneficial in terms of the energy efficiency in the network or not.

The last part of this thesis, deals with the problem of characterizing the traffic distribution over randomly deployed linear WSNs where the data transmission happens in a multi-hop fashion. This analysis is helpful in lifetime study because the energy consumption and consequently the network lifetime depend on the amount of the sensors traffic load. Our analysis reveals how the traffic load is diffused over the sensors based on their distance from the sink.

1.3 Thesis Organization

In the next chapter, we initially describe the system model used in this work. The system model includes lifetime definition, MAC protocol, energy consumption model in the sensors and data reporting scheme in the network. Then, we proceed with a more detailed review over the existing lifetime studies in the literature. The basic assumptions and the applied lifetime definition in each work are introduced. This is helpful to clarify our contributions in this work.

Chapter 3 explains the lifetime analysis for both single-hop and multi-hop networks when clustering is not applied. It is basically assumed that the network is event-driven and the sensors are spread randomly over the network area. The analysis mostly focuses on the Poisson packet generation model and uniform sensor distribution, however, it will be discussed how the study is extendible to other cases. Based on these assumptions, the complementary cumulative distribution function (ccdf) of the network lifetime is derived. According to the lifetime ccdf, one can determine the probability of achieving a specific lifetime by a network. Moreover, a non-uniform node distribution for multi-hop networks is proposed which can reliably guarantee achieving an arbitrary lifetime by the network.

The lifetime analysis for clustered networks is presented in Chapter 4 where the sensors are partitioned to several groups called clusters. A representative node, called cluster head (CH), is assigned to each cluster which is in charge of the communication between cluster and data sink or other clusters. The random deployment and packet generation model are also considered in the analysis. To avoid the early death of the nodes, it is assumed that the CH role dynamically rotates among the cluster nodes. Similar to Chapter 3, ccdf of the network lifetime is derived. Notably, it is assumed that intra-cluster transmissions is a single-hop scenario but both multi-hop and single-hop cases are studied for inter-cluster transmissions.

Chapter 5 focuses on the traffic load distribution over the nodes within a randomly deployed linear WSN. The shortest path technique is assumed for the routing scheme. Then, the traffic load is determined for an arbitrary point in the network based on its distance from the data sink. The result of the analysis shows that, despite what is widely believed, the traffic distribution does not monotonically increase with the inverse of the distance from the sink. In fact, at a specific point close to the sink, the traffic load starts to fall.

The final chapter concludes the thesis and suggests some research directions for further studies.

Chapter 2

Background

In this chapter, we first describe the basic system model. This includes power consumption model, traffic generation scheme in the network, MAC protocol and the lifetime definition. In addition, a detailed review of the existing work on the lifetime analysis is provided. This can clarify the contributions of this work. A separate literature review for the traffic load distribution over the network is provided in Chapter 5.

2.1 System Model

In the following, we will introduce the system model which is the base of our analysis. We defer the description of the clustering technique until Chapter 4.

2.1.1 Network Model

In this work, it is assumed that N sensors are deployed randomly over an area denoted by $\mathcal{C}$. For clustered networks, $\mathcal{C}$ shows the cluster area. Data sink may be located within the area or out of it (Figure 2.1). While our proposed analysis is applicable to any arbitrary node distribution over $\mathcal{C}$, we consider uniform node distribution to verify our analysis in simulations. Also, some known geometrical shapes, such as circle and different polygons, are used for network (cluster) area in numerical examples.

2.1.2 Power Consumption Model

Sensors consume energy for sensing, receiving, transmitting, data processing and also during the idle mode when no data sensing, processing or exchange happens. Data transmission can be accomplished with a fixed or adjustable power. Utilizing the power adjustment mechanism can benefit the energy conservation in the network [18].

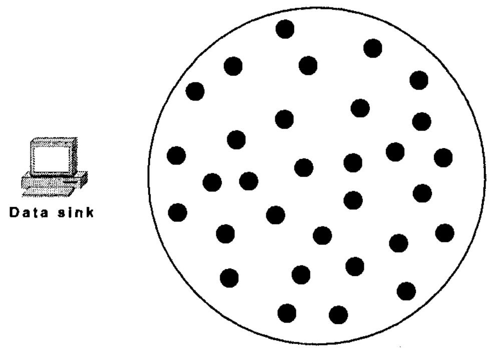


Figure 2.1: A randomly deployed network over a circular area.

In the fixed power scenario, each sensor applies a constant amount of power to perform a packet transmission without considering the distance between itself and the transmission destination. Although this method needs a simpler radio part in the sensor, it is not energy-efficient. On the other hand, the adjustable transmission scheme can improve the energy efficiency in the sensor but needs a more complicated radio part. The adjustable transmission scheme can be implemented in two ways: continuous and discrete adjustment. In the discrete method, there exist several levels, among which the appropriate transmission power is chosen based on the transmission distance and the required signal power at the receiver (e.g. Mica Mote [6] and Intel Mote [7]). In the continuous mode, sensors can adjust their power within a range and the transmission level is not limited to a set of discrete levels. In this thesis, we mostly focus on the continuously adjustable power, however, the analysis is easily extendible to the other two cases.

As proposed in [18], the consumed energy for transmitting one packet follows the model below

$$e(d_i) = l(e_t d_i^{\alpha} + e_o) = md_i^{\alpha} + c \tag{2.1}$$

where l represents the packet length in bits, d_i denotes the transmission distance, α represents the path loss exponent, e_t denotes the loss coefficient for transmitting one bit and e_o is the overhead energy due to the sensing, receiving and processing for the same amount of data. Also, $k = le_t$ and $c = le_o$ represent the loss coefficient and the overhead energy for one packet respectively. Usually, α is considered to be 2 for small distances and 4 for large distances [16]. In Chapter 4, it is assumed that the intra-cluster distances are small and inter-cluster distances are large. Hence, similar to [16] we use $\alpha = 2$ for intra-cluster and $\alpha = 4$ for inter-cluster transmissions. In addition, the idle power, P_{idle}, which is used to keep the radio part on for listening to the channel, is almost fixed and considered in the analysis.

2.1.3 Data Reporting Model in The Network

There exist three main models for reporting data by sensors in WSNs, namely, event-driven, time-driven and query-driven [13]. Based on the network application, data characteristic and the type of data inquiry, usually one of these models is adaptable to the sensors traffic generation.

In a time-driven network, sensors periodically monitor the field and send the data packets toward the sink. This method of data reporting is well-suited to the applications that require continuous data monitoring such as temperature control. Event-driven and query-driven scenarios are used in the time-critical applications where it is required to detect any change in the data field as early as possible. While in a query driven network, sensors reply back to a data collecting request sent by the sink, sensors in an event-driven network report occurrence of an event without waiting for a request from the sink side. Event-driven data reporting scheme is the basic model used for the lifetime analysis in Chapters 3 and 4.

Different models are used to explain the packet transmission in an event-driven network among which Poisson model is the most common. When the events occur independently, totally random in time and with an equal probability, Poisson distribution can accurately model the packet generation in sensors. In this case, if λ shows the average rate of packet generation, the number of transmitted packets between 0 and T, called M, has a probability distribution function (pdf) as follows:

$$P(M = m) = \frac{e^{-\lambda T}(\lambda T)^m}{m!} \quad m = 0, 1, 2, \ldots \quad . \tag{2.2}$$

Also, the time duration between two consequent packet transmissions, t, has an exponential distribution with mean $\frac{1}{\lambda}$, i.e.,

$$f_t(x) = \lambda e^{-\lambda x} u(x) \tag{2.3}$$

where $u(x)$ denotes the unit step function.

In this work, Poisson distribution is mostly considered to model the traffic generated by sensors.

2.1.4 Lifetime Definition

Network lifetime is conceptually the time interval over which the network can operate effectively. Thus, a more specific definition of the lifetime is possible only based on the network application. Hence, different lifetime definitions are used in the literature. For instance, in the applications where the coverage of the area is more crucial compared to other network functionalities (e.g. intrusion detection), the lifetime of the network is defined based on the network coverage [19]. Considering the network connectivity as the lifetime criterion is also common in the literature [20]. In [21], the lifetime of the network is defined based on the quality of service in the network access points.

Another commonly used definition considers the number of dead nodes in the network as the lifetime criterion. In some case, the lifetime is over when the first node dies [22, 23]. This definition is usually over stringent, because in most of the WSNs the remaining nodes can accomplish the task even though a few sensors are dead. An alternative definition is based on the percentage of the dead nodes [24, 25]. To this end, the network lifetime, T_L, is

$$T_L = \max(t) \quad \text{such that} \quad N_L > (1-\beta)N \tag{2.4}$$

where t represents time, N_L is the number of live nodes in the network, β shows the threshold ratio for the number of dead nodes, and N stands for the total number of nodes in the network. This definition also includes the lifetime definition where the network lifetime is the moment of the first node death. Moreover, since the number of dead nodes can nearly reflect the quality of the network coverage and/or connectivity [26], this lifetime definition can also be considered as an approximation of the lifetime based on the network coverage and/or connectivity.

2.1.5 MAC protocol

Owing to the substantial difference of the WSNs with ordinary wireless networks, conventional MAC protocols are not necessarily efficient to be applied in WSNs. Therefore, designing customized MAC protocols for WSNs is essential. Generally, the MAC protocols used in WSNs can be divided to two categories, called scheduled protocols and contention-based protocols [27].

In a scheduled protocol, each sensor has its own communication channel and it does not need to compete with other nodes to obtain the communication resource. Time division multiple access (TDMA), frequency division multiple access (FDMA) and code division multiple access (CDMA) are the main scheduled MAC techniques. Although the scheduled protocols can highly avoid the collision in the network, they suffer from the scalability issue. Among the scheduled MAC protocols, TDMA has attracted more attention for WSNs applications. This is because TDMA sustains low-duty-cycle operations on sensors and has a significant energy efficiency [27]. In this work, it is assumed that sensors utilize the ideal

TDMA protocol. To this end, we are able to neglect the collision occurrence and data retransmission in the network.

Contention-based protocols rely on the fact that all sensors may not have data to transmit, and consequently they do not need to occupy the communication resource. Therefore, the communication channel can be allocated on demand to the sensors and they should compete to gain the access permission. The main benefits of the contention-based protocols are their easy scalability and supporting networks wherein sensors are not fine-grained synchronized. As major disadvantages of these protocols, one may refer to their energy inefficiency and possible delay in delivering data packets [27].

2.2 Related work

In the following, we review the most important existing work on lifetime analysis of WSNs.

In [19], the authors find an upper bound on the lifetime of a sensor network where the lifetime is defined based on the network coverage and the traffic generation is assumed to be time-driven. According to these assumptions and using the convexity of the power consumption model in the sensors, the optimal hop length is derived. Using the optimal hop length, it is shown that the energy consumption lessens to its minimum value when all hops in a routing path have the same length. In addition, the optimal number of hops for delivering a packet from the data source to the sink is found. Using the optimal hop length and the spatial distribution of the data source, the upper bound of the lifetime is determined over several known geometrical shapes such as rectangle or circle sector. Then, a theorem is proven showing how the results are extendible to the shapes which can be partitioned to simpler known sub-regions that the lifetime was studied for.

Authors in [28] derive an upper bound on the lifetime of a multiple-sink network where the nodes are distributed over a rectangle. It is assumed that sinks are located on the sides of the rectangle. To derive the upper bound of the lifetime, the authors apply the result in [19] to find the optimal position of the data sinks. Although for a network with 1 or 2 data sinks the result can be derived analytically, for the networks with higher number of sinks the analysis is very complicated and instead, the authors apply neural network technique to choose the optimal sinks position.

Bounding the network lifetime through optimally assigning the sensors roles in the network is studied in [29]. Authors have extended their work in [19] by considering the topology of the network and the data aggregation capability of sensors in their analysis.

For this purpose, it is assumed that data source is located on a single point and nodes may have three main roles in the network: sensing, relaying and aggregating the data. Then, all possible data routes from the source to the sink, including non-aggregating and aggregating routes, are listed. Having the list of the possible routes in the network, the problem of finding the lifetime bound is formulated as a linear programming problem. In addition, the situation where a selected set of sensors, not necessarily all of them, are active can be accommodated in the proposed method.

Work reported in [20] investigates the lower bound on the lifetime of a cell-based stationary ad hoc network where nodes are randomly deployed over a hypercube area which is divided to cells. Also, the authors assume that the network traffic is distributed evenly over the nodes in the network and the nodes within a cell are routing equivalent. This means that by having merely a single node in active mode within a cell, which is called representative node, the routing tree is connected. The representative node can forward the data packets on behalf of the other nodes in the cell. According to the abovementioned points and assuming the lifetime based on the network connectivity, an upper bound on the network lifetime is derived under the hypothesis of having an almost surely connected network. Notably, this lower bound is based on the given lifetime of a single sensor which is assumed to be known beforehand. To achieve this bound, the authors have taken advantage of the concept of occupancy theory [30] when the size of the network approaches infinity. It is also shown through computer simulation that the cooperative cell-based network strategy can dramatically prolong the network lifetime.

Authors in [31] focus on the lifetime analysis for WSNs where the lifetime is defined based on maintaining k-coverage in the network. k-coverage means that every point in the network area is monitored by at least k nodes in the network. Specifically, they assume the sensors are randomly deployed over a square with a Poisson distribution and the lifetime of a single sensor is known beforehand and is T. To simplify the analysis and remove the boundary effects on the sides of the square, the authors adopt torus convention [32]. According to the assumed torus convention, the coverage area of the sensors (close to the network boundaries) that fall out of the network area, enters the network area from the opposite side. According to these assumptions, first the necessary and sufficient condition on the network density is derived guaranteeing the complete k-coverage in the network when the network size approaches infinity. In addition, it is proved that by having this density over the network, the lifetime of the network is almost surely upper bounded by kT when the network area approaches infinity. Moreover, the upper bound of the lifetime, when the goal is partial area coverage, is derived for a finite network (however large enough) with a given fixed density. Through computer simulation, it is shown in [31] that the upper bound holds even for finite networks.

An asymptotic study on the lifetime of a randomly deployed network is presented in [33]. The authors consider two random models for the network which are based on the percolation theory [34]. In the first model, it is assumed that the sensors are located in a grid structure, while the second case includes the situation where the nodes are deployed according to a two-dimensional Poisson point process. Also, the nodes are connected randomly. A single node may fail at a random time which can model the death of nodes in an ad hoc network. Based on these assumptions, the network can be modeled as a random graph. To this end, authors have applied the Chen-Stein method [35] on the random graph model to study the asymptotic distribution of the number of isolated nodes (i.e. nodes which does not have any active neighbour) in the network. Then, by assuming a common random lifetime distribution for all of the nodes in the network, they derive the moment when the blind spots (disconnected points) start to appear in the network.

In [36], authors analyze the lifetime for a highly dense large-scale cluster-based WSN. Sensors are assumed to be uniformly deployed over a disk area and the data sink is located at the center of the disk. Clusters have a hexagonal shape and tessellate the network area. For each cluster, a cluster head is assigned randomly. In addition, clusters in the network are divided to the tiers where the outer network tiers forward their data through the inner ones. To simplify the analysis, it is assumed that the traffic load is distributed evenly over the sensors within each tier. Moreover, sensors have fixed transmission range. This transmission is chosen such that it guarantees the connectivity of the clusters. Then, the authors derive an upper bound on the network lifetime by an asymptotic analysis over the number of nodes in the network. In addition, it is shown that applying a hybrid transmission protocol in the network, where the data transmission is accomplished in a composition of single-hop and multi-hop schemes, leads to the lifetime extension.

Analysis of the lifetime for a time-driven heterogeneous WSN is studied in [24] where it is assumed that the network consists of two different type of sensors. The sensors are partitioned to the clusters where the first group of the sensors, which are stronger and have more capabilities, are dedicated to being cluster heads. The lifetime of the network is defined as the expected lifetime of any arbitrary node in the network. Hence, to estimate the network lifetime, authors derive the average energy consumption rate for a sensor based on the energy consumption model proposed in [18]. The expected lifetime is easily derived by having the sensors initial energy and the average energy consumption rate. Moreover, the optimal number of clusters in the network is found when the overall network energy, assigned to both type of sensors, is kept fixed.

A lifetime analysis for clustered networks is studied in [21]. The lifetime of the network has been defined based on the quality of service in the network, meaning the maximum tolerable probability of detecting error at the network access point (data sink). The analysis does not go to the sensor level and it is assumed that the sensor lifetime follows a known probability distribution. More specifically, the authors model the sensor lifetime with one of the exponential, uniform, Rayleigh and lognormal distributions. It is also assumed that a binary event occurs in the network field which is contaminated with a Gaussian noise. In addition to the lifetime analysis, authors also show that partitioning the sensors to a few big clusters is a better approach compared to the many small clusters in terms of the network lifetime. Also, reclustering (i.e. reconfiguring the sensors in new clusters after the death of some nodes in the network) is useful in achieving better network functionality.

Chapter 3

Lifetime Analysis for Non-Clustered Networks

In this chapter, we consider the lifetime analysis for randomly-deployed non-clustered WSNs. First, the lifetime of a single-hop network is studied for an arbitrary area shape. To verify the analysis, computer simulation is performed for some known shapes such as circle or polygons. Also, the lifetime of multi-hop networks are investigated. We provide a general framework to model the lifetime of these networks, however, due to the dependance of the lifetime over the routing scheme, the network lifetime is studied for a scenario which is mostly applicable to dense networks. Moreover, an optimal nodes distribution over the network is determined based on the proposed analysis which helps in extending the network lifetime.

3.1 Lifetime Analysis for Single-hop Networks

To start the analysis, we first consider the lifetime at the sensor level. For this purpose, assuming that nodes directly communicate with the sink, the lifetime of each sensor is modeled as a random variable. Based on this lifetime analysis at the sensor level, the ccdf of the network lifetime is derived.

It is assumed here that all of the nodes have the same initial energy, E_{init}, the same packet generation model and distribution over the area. Other cases like nonuniform energy distribution or different packet generation models are studied in Section 3.3. We also delay sleep scheduling considerations until Section 3.3.

First, we find the effective energy that each sensor uses only for packet generation and transmission. Since P_{idle} is fixed, assuming that data packets are transmitted over very short time slots, $T_{\text{thr}} P_{\text{idle}}$ is the idle energy used by each sensor during a desired lifetime period,

T_{thr}. Hence,

$$E_i = E_{\text{init}} - T_{\text{thr}} P_{\text{idle}} \tag{3.1}$$

is the maximum amount of energy available to each sensor for data sensing and transmitting. Now, by defining p_i as

$$p_i = \frac{E_i}{e(d_i)}, \tag{3.2}$$

it is clear that the maximum number of packets that sensor i can transmit during T_{thr} is equal to $\lfloor p_i \rfloor$. Notice that $e(d_i)$ is derived using (2.1)

Lemma 3.1 *If a sensor node with initial energy E_{init} is placed in the area C, having the position of the sensor, the probability of achieving a lifetime more than a threshold T_{thr} is*

$$P(t_i \geq T_{\text{thr}} | p_i) = 1 - \frac{\gamma(\lfloor p_i \rfloor, \lambda T_{\text{thr}})}{\Gamma(\lfloor p_i \rfloor)} \tag{3.3}$$

where $\gamma(\cdot, \cdot)$ denotes the lower incomplete gamma function

$$\gamma(a, x) = \int_0^x t^{a-1} e^{-t}\, dt, \tag{3.4}$$

and $\Gamma(\cdot)$ represents the gamma function

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t}\, dt. \tag{3.5}$$

Proof: The lifetime of sensor i, t_i, depends on the maximum number of packets that can be transmitted by the sensor to the sink. Since the time duration for transmitting a packet is very short, it can be neglected in the analysis of the sensor lifetime. Hence, t_i can be assumed as the summation of time durations between packet transmissions until the last packet is sent by the sensor. Thus,

$$t_i = \sum_{j=1}^{\lfloor p_i \rfloor} t_{ij} \tag{3.6}$$

where t_{ij} denotes the time duration between transmitting packets $j-1$ and j by sensor i, and t_{i1} is defined as the time when the first packet is transmitted. Since a Poisson model is adopted for data packet generation, t_{ij}'s obey an exponential distribution indicated in (2.3). On the other hand, it is known that sum of independent and identically distributed (i.i.d.) exponential random variables has a gamma distribution [37]. It is worthy to note that since the node is deployed randomly in the area, the distance between the node and the sink, and consequently p_i, are random variables. Hence, given p_i, the conditional pdf of t_i can be written as follows:

$$f_{t_i | p_i}(x) = \lambda^{\lfloor p_i \rfloor} \frac{x^{\lfloor p_i \rfloor - 1} e^{-\lambda x}}{\Gamma\left(\lfloor p_i \rfloor\right)} \quad x \geq 0. \tag{3.7}$$

Now

$$P(t_i \geq T_{\text{thr}}|p_i) = 1 - \int_0^{T_{\text{thr}}} \lambda^{\lfloor p_i \rfloor} \frac{x^{\lfloor p_i \rfloor - 1} e^{-\lambda x}}{\Gamma(\lfloor p_i \rfloor)} \, dx$$
$$= 1 - \frac{\gamma(\lfloor p_i \rfloor, \lambda T_{\text{thr}})}{\Gamma(\lfloor p_i \rfloor)}. \tag{3.8}$$

■

Remark 3.1 *Since the fractional part of p_i is usually much smaller than the integer part, $\lfloor p_i \rfloor \simeq p_i$. Thus, (3.8) can be rewritten as*

$$P(t_i \geq T_{\text{thr}}|p_i) = 1 - \frac{\gamma(p_i, \lambda T_{\text{thr}})}{\Gamma(p_i)} \tag{3.9}$$

□

For simplicity, we use (3.9) to analyze the network lifetime in the sequel.

Corollary 3.1 *If sensors apply a fixed transmission range, r,*

$$P(t_i \geq T_{\text{thr}}) = 1 - \frac{\gamma(p_f, \lambda T_{\text{thr}})}{\Gamma(p_f)} \tag{3.10}$$

where

$$p_f = \frac{E_i}{mr^\alpha + c}. \tag{3.11}$$

Proof: In this case, all p_i's have a deterministic value equal to p_f. Therefore, the value of $P(t_i \geq T_{\text{thr}})$ in (3.9) is unconditional and the proof is completed by replacing p_i by p_f in (3.9). □

A simple approximation for (3.9) can be found as bellow.

Remark 3.2 *Since t_i in (3.6) is a sum of i.i.d. random variables, central limit theorem (CLT) [37] indicates that its pdf tends to a Gaussian distribution with mean $\lfloor p_i \rfloor \lambda^{-1}$ and variance $\lfloor p_i \rfloor \lambda^{-2}$. Considering $\lfloor p_i \rfloor \approx p_i$, we have*

$$P(t_i \geq T_{\text{thr}}|p_i) \cong Q\left(\frac{T_{\text{thr}} - p_i\lambda^{-1}}{\sqrt{p_i}\lambda^{-1}}\right). \tag{3.12}$$

where $Q(\cdot)$ is the ccdf of the normal distribution defined as follows:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_\alpha^\infty e^{-\frac{x^2}{2}} dx \tag{3.13}$$

□

To study the lifetime of the network, T_L, we consider the lifetime of all the nodes in the network which necessitates the knowledge of p_i for all of the sensors. When a node is deployed randomly over an area, p_i is a random variable with pdf $f_{p_i}(x)$. In a random network deployment, the p_i's are usually i.i.d. random variables and consequently have the same distribution, $f_p(x)$. This distribution depends on the shape of the area, energy dissipation model and the pdf of node distribution over the area. In Appendix, $f_p(x)$ is derived for some common area shapes assuming a uniform distribution for the node deployment.

Theorem 3.1 *Assuming N equal-energy nodes are distributed independently over the area C, the probability that the network achieves T_{thr} is*

$$P(T_L \geq T_{\text{thr}}) = Q\left(\sqrt{N}\frac{1-\beta-\mu}{\sigma}\right) \tag{3.14}$$

where

$$\mu = \int_C \left(1 - \frac{\gamma(x, \lambda T_{\text{thr}})}{\Gamma(x)}\right) f_p(x)\, dx, \tag{3.15}$$

$$\sigma = \sqrt{\mu - \mu^2}. \tag{3.16}$$

Proof: To find the number of nodes that live more than T_{thr}, we define a Bernoulli random variable l_i indicating the success of achieving the lifetime threshold by sensor i:

$$l_i = \begin{cases} 1 & \text{With probability equal to } s_i, \\ 0 & \text{With probability equal to } 1 - s_i. \end{cases} \tag{3.17}$$

The success probability of l_i, given p_i, is equal to

$$s_i = P(t_i \geq T_{\text{thr}} | p_i) = 1 - \frac{\gamma(p_i, \lambda T_{\text{thr}})}{\Gamma(p_i)} \tag{3.18}$$

which was derived in Lemma 3.1. The number of live nodes after T_{thr} can be found by defining a new random variable, w, that denotes the number of successes in the Bernoulli trials shown by l_i's. Clearly, w has a binomial distribution, however, since the number of nodes in the network is large enough, CLT can be used to accurately approximate the distribution of w. Hence

$$f_w(x) = \frac{1}{\sqrt{2\pi}\sigma_w} \exp -\frac{(x-\mu_w)^2}{2\sigma_w^2} \tag{3.19}$$

where μ_w and σ_w^2 denote the mean and variance of w. Clearly,

$$\mu_w = \sum_{i=1}^{N} \mu_{l_i} \tag{3.20}$$

where μ_{l_i} is the mean of l_i. Also, since the l_i's are independent random variables

$$\sigma_w^2 = \sum_{i=1}^{N} \sigma_{l_i}^2 \tag{3.21}$$

where $\sigma_{l_i}^2$ is the variance of l_i. To find the values of μ_w and σ_w, we need to have the unconditional mean and variance of l_i's using the conditional values. Since l_i's are Bernoulli random variables

$$\mu_{l_i|p_i} = s_i, \quad \sigma_{l_i|p_i}^2 = s_i - s_i^2. \tag{3.22}$$

On the other hand, for two random variables x and z, the unconditional mean and variance of x can be found using its conditional mean and variance as follows [37]:

$$\mu_x = E[\mu_{x|z}], \tag{3.23}$$

$$\sigma_x^2 = E[\sigma_{x|z}^2] + \text{Var}[\mu_{x|z}] \tag{3.24}$$

where $E[\cdot]$ is the expected value and Var$[\cdot]$ denotes the variance of the random variable. Using (3.18), (3.22), (3.23) and (3.24), it can be shown that

$$\mu_{l_i} = E[s_i] = \int_{\mathcal{C}} \left(1 - \frac{\gamma(x, \lambda T_{\text{thr}})}{\Gamma(x)}\right) f_{p_i}(x)\, dx, \tag{3.25}$$

$$\sigma_{l_i}^2 = E[s_i - s_i^2] + \text{Var}[s_i] = E[s_i] - E^2[s_i] = \mu_{l_i} - \mu_{l_i}^2. \tag{3.26}$$

Since p_i's are i.i.d. random variables with pdf $f_p(x)$, we have

$$\mu_{l_i} = \mu = \int_{\mathcal{C}} \left(1 - \frac{\gamma(x, \lambda T_{\text{thr}})}{\Gamma(x)}\right) f_p(x)\, dx \quad \forall i, \tag{3.27}$$

$$\sigma_{l_i}^2 = \sigma^2 = \mu - \mu^2 \quad \forall i. \tag{3.28}$$

Then, by using (3.20) and (3.21)

$$\mu_w = N\mu, \quad \sigma_w^2 = N\sigma^2. \tag{3.29}$$

The probability of achieving the desired lifetime by the network is equal to the probability of achieving the lifetime by at least $(1-\beta)N$ nodes. Hence,

$$\begin{aligned} F_{T_L}^{c}(T_{\text{thr}}) = P(T_L \geq T_{\text{thr}}) &= P(w \geq (1-\beta)N) \\ &= Q\left(\sqrt{N}\frac{1-\beta-\mu}{\sigma}\right) \end{aligned} \tag{3.30}$$

where $F_{T_L}^{c}(T_{\text{thr}})$ represents the ccdf of the network lifetime. ■

Remark 3.3 *Using Remark 3.2, μ can also be calculated as*

$$\mu = \int_{\mathcal{C}} Q\left(\frac{T_{\text{thr}} - x\lambda^{-1}}{\sqrt{x}\lambda^{-1}}\right) f_p(x)\, dx. \tag{3.31}$$

□

3.2 Lifetime Analysis in Multi-hop Networks

In multi-hop networks, position of the nodes in the network plays an important role in its lifetime. In fact, the lifetime of a multi-hop network depends on the distribution of the traffic load over the sensors. In randomly deployed networks, where all the nodes start with an equal initial energy, nodes which are close to the data sink die earlier. This is because they carry the whole network traffic. Therefore, if $\mathcal{H}$ is the set of nodes that are in the vicinity of the sink and directly communicate with it, we define the lifetime based on the ratio of dead nodes within $\mathcal{H}$ to $|\mathcal{H}|$ where $|\cdot|$ denotes the cardinality of the set. Since sensors are deployed randomly, $|\mathcal{H}|$ is itself a random variable. Notice that the number of nodes N was a known constant in single-hop networks.

Suppose that the sensors are distributed randomly over a circular area with radius R. Assuming the maximum transmission range of r for the sensors, the nodes within a circle with radius r centered at the data sink can directly transmit data to the sink. Also, the area can be divided into a number of rings (Figure 3.1). The sensors within a ring send their data to the sensors within the neighboring inner ring. The number of rings, n, can be found as

$$n = \left\lceil \frac{R}{r} \right\rceil \tag{3.32}$$

where $\lceil\cdot\rceil$ denotes the integer ceiling. To ease the notations, let us assume that R is an integer multiple of r.

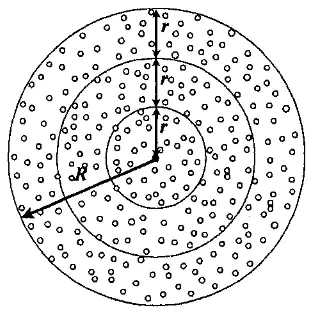


Figure 3.1: Rings within a multi-hop network

Since each ring carries the traffic of all outer rings, the average traffic carried by each ring is different and depends on its relative location to the sink. Here, we omit a detailed traffic distribution analysis, however, the problem is studied in the literature which will be noted in Chapter 5. For simplicity, we illustrate our approach by assuming a uniform load within each ring, i.e. the traffic load is distributed evenly over the nodes within a ring [36, 38]. In addition, we will briefly discuss the case when the traffic load has an arbitrary distribution over the area in Section 3.3. An approximately uniform load distribution can be achieved through a load balancing routing scheme or when the network density is high.

For a uniform load distribution, the average rate of packet transmission by each node in ring i is

$$\lambda_i = \lambda \frac{N - \sum_{j=1}^{i-1} N_j}{N_i} \quad \forall i = 1, 2, \ldots, n, \tag{3.33}$$

where N_i is the number of sensors within ring i. Considering the fact that sum of Poisson random variables is a Poisson random variable, each sensor in ring i has a traffic model obeying the Poisson distribution with mean λ_i.

In addition, if one assumes a uniform node deployment, N_i will have a binomial distribution with mean Nq_i where

$$q_i = \frac{r^2(2i-1)}{R^2} \tag{3.34}$$

represents the probability of a sensor being in ring i. Therefore, the time duration between two consequent transmissions, t, by a node in ring i have a conditional distribution as follows:

$$f_{t|N_i}(x) = \lambda_i e^{-x\lambda_i} u(x). \tag{3.35}$$

Since the network lifetime is mainly effected by the nodes within the first tier, we just consider the probability of achieving the lifetime threshold by the first ring. Nevertheless, the probability of achieving T_{thr} by other rings can similarly be investigated. As discussed earlier, probability of achieving a lifetime threshold depends on the number of nodes within the area. Hence, using Theorem 3.1 and Corollary 3.1, one can find the conditional probability of achieving T_{thr} by the first ring as below:

$$P(T_{L_1} \geq T_{\text{thr}} | N_1) = Q\left(\sqrt{N_1}\frac{1-\beta-\mu}{\sigma}\right) \tag{3.36}$$

where

$$\mu = \int_C \left(1 - \frac{\gamma(x, \lambda_1 T_{\text{thr}})}{\Gamma(x)}\right) f_p(x)\, dx. \tag{3.37}$$

and T_{L_1} shows the lifetime of the first network tier. In (3.37), C shows the circular area with radius r. Therefore, by removing the condition on N_1 in (3.36), we have

$$P(T_L \geq T_{\text{thr}}) = \sum_{j=1}^{N} P(T_{L_1} \geq T_{\text{thr}} | N_1 = j) P(N_1 = j) \tag{3.38}$$

where

$$P(N_1 = j) = \binom{N}{j} q_1^j (1-q_1)^{N-j}. \tag{3.39}$$

The given discussion is not restricted to circular areas and can also be applied to other area shapes. To this end, we just need to recalculate the value of q_1 as follows:

$$q_1 = \frac{S_1}{S} \tag{3.40}$$

where S_1 shows the area of $\mathcal{H}$ (nodes that directly communicate with sink) and S is the network area.

3.3 Some Notes

Here, we provide some discussions on the results of Section 3.1 and Section 3.2 and study the effect of MAC protocol, non-identical energy distribution, and other traffic models.

3.3.1 Effect of MAC Protocol

Here, a discussion is provided on the sleep scheduling mechanism and how the packet transmission rate is effected by MAC protocol.

Assume that a random sleep scheduling mechanism is applied in which a sensor wakes up in each time duration (cycle), T_c, with probability q. The parameter q is determined by the MAC protocol. Hence, the number of cycles that a sensors is in active mode, m, has a binomial distribution. Although these m cycles might not be successive, owing to the memoryless property of exponential distribution, the distribution of transmission instants on the time axis is the same as when the transmissions occur over m successive cycles. In other words, to analyze the probability of achieving the lifetime of T_{thr} by each sensor, one can study the probability of achieving the active lifetime of mT_c. Also, since the transition between active and sleep mode consumes energy, the sensor can utilize the amount of

$$E_i = E_{\text{init}} - mT_c P_{\text{idle}} - mE_t \tag{3.41}$$

for data generation and transmission over mT_c, where E_t shows the required energy for mode transition. Hence, conditioned on m, network achieves the desired lifetime with probability of

$$P(T_L > T_{\text{thr}}|m) = P(T_{L_{\text{eff}}} > mT_c|m) = Q\left(\sqrt{N}\frac{1-\beta-\mu}{\sigma}\right) \tag{3.42}$$

where $T_{L_{\text{eff}}}$ shows the effective lifetime of the network and μ is found by replacing T_{thr} in (3.15) with mT_c. Thus, the unconditional probability of achieving the lifetime can be derived as follows:

$$P(L > T_{\text{thr}}) = \sum_{m=1}^{M=\left\lceil \frac{T_{\text{thr}}}{T_c} \right\rceil} \binom{M}{m} q^m (1-q)^{M-m} P(L > T_{\text{thr}}|m). \tag{3.43}$$

Occurrence of the collision in the network results in an increase in the number of transmissions by the sensors and consequently a shorter lifetime. MAC protocols, scheduled or contention-based, can drastically reduce the number of collisions in the network. Due to the orthogonality of the communication channels in scheduled MAC protocols, e.g. TDMA, they are largely collision free and the sensors packet generation rate does not change [27].

In a network with contention-based MAC protocols, such as carrier sense multiple access (CSMA), transmitted packets experience collision and need to be retransmitted. Here, we use a simple model to consider the effect of the collision in the packet generation rate of a sensor. If α shows the collision rate of a MAC protocol, which can be derived through computer simulation (for example [39]), the effective packet transmission rate by a sensor is approximately $\lambda_e = (1+\alpha)\lambda$. This effective value can be replaced in the derived formulas to modify the analysis in the case of collision occurrence.

In addition, the effect of the request-to-send (RTS) and clear-to-send (CTS) packets on the sensors lifetime can be considered in the proposed analysis. Since transmitting an RTS packet and receiving a CTS packet is performed before sending each data packet, the number of control packets of each type, RTS or CTS, is equal to the number of data packets (or equivalently the number of event occurrence). Also, each sensor usually transmits RTS packets within its maximum transmission range in order to prevent the hidden terminal problem. Moreover, receiving a CTS packet consumes a fixed amount of energy shown by c_c. Therefor, (2.1) can be modified as follows to consider the effect of the control packets on the lifetime analysis

$$e(d_i) = l(e_t d_i^\alpha + e_o) + l_r(e_t r^\alpha + e_o) + c_c$$

where l_r denotes the length of RTS packets.

3.3.2 Asymptotic Analysis

Since the lifetime ccdf in (3.14) depends on the number of nodes distributed over the area, we can study the effect of the node density on the probability of achieving the lifetime threshold.

One interesting scenario is when N is large. Depending on the sign of $a = 1 - \beta - \mu$, two cases can happen. Since $Q(\cdot)$ is a decreasing function, when $a > 0$, the probability of achieving the desired lifetime decreases with N. Using [40],

$$\frac{1}{\sqrt{2\pi}x}\left(1 - \frac{1}{x^2}\right)e^{-\frac{x^2}{2}} < Q(x) < \frac{1}{\sqrt{2\pi}x}e^{-\frac{x^2}{2}} \quad \forall x \geq 0. \tag{3.44}$$

It can be seen that the probability of achieving the desired lifetime decreases proportional to e^{-N}. When N tends to infinity, this probability tends to $Q(\infty) = 0$. In other words, almost surely the desired lifetime, T_{thr}, cannot be achieved. In a similar manner, when $a < 0$, the network almost surely achieves T_{thr}.

Another interesting case occurs when one considers the lifetime of the network based on the first node death. In this case $\beta = \frac{1}{N}$. Hence, when $N \to \infty$, the sign of $a \approx 1 - \mu$ can be used in order to predict the lifetime. For any desired lifetime $T_{\text{thr}} > 0$, we have

$$\mu = \int_C \left(1 - \frac{\gamma(x, \lambda T_{\text{thr}})}{\Gamma(x)}\right) f_p(x)\, dx < \int_C f_p(x)\, dx = 1 \tag{3.45}$$

and consequently $1 - \mu > 0$. Therefore, under this stringent definition of the lifetime, the probability of achieving the lifetime T_{thr} approaches 0 (exponentially with N) as N increases.

3.3.3 Other Traffic Models

In Section 3.1, we considered the case when all the sensors had the same Poisson distribution for their packet generation. Here, we consider two other cases: 1) The average rate of packet generation (the parameter of the Poisson distribution) depends on the position of the sensor. 2) Packet generation distribution is the same for all sensors, but the pdf is not Poisson.

If the average rate of packet generation, λ, varies with the position of the sensor (e.g., lifetime analysis of different rings in a multi-hop network, existence of spatial correlation between data or data aggregation), we have the mean and variance of l_i conditioned on both p and λ. To derive the unconditional mean and variance of l_i, we need to calculate

$$\mu_{l_i} = \mu = \int \int_C \left(1 - \frac{\gamma(x, \lambda y)}{\Gamma(x)}\right) f_{p,\lambda}(x, y)\, dx dy \tag{3.46}$$

where $f_{p,\lambda}(x, y)$ denotes the joint pdf of p and λ. The analysis remains unchanged otherwise.

Theorem 3.1 can be modified to include non-Poisson packet generation distributions. Assume that the pdf of the time duration between two packet transmissions follows a distribution with mean μ_t and variance σ_t^2. Using CLT, t_i can be accurately approximated by a Gaussian distribution with mean $p_i \mu_t$ and variance $p_i \sigma_t^2$. The remainder of the proof remains unchanged.

It is notable that the probability of achieving a lifetime threshold in time-driven networks can also be modeled as a special case of the proposed analysis. Since in time-driven networks, the time duration between two consequent transmissions, T_t, is fixed

$$t_i = \lfloor p_i \rfloor T_t. \quad (3.47)$$

Hence, the probability of achieving T_{thr} by a sensor is

$$P(t_i > T_{\text{thr}}) = P(\lfloor p_i \rfloor > \frac{T_{\text{thr}}}{T_t}) \quad (3.48)$$

which can simply be evaluated from the distribution of p_i. Then, studying the network lifetime is straightforward.

3.3.4 Nonuniform Energy Distribution

Assume that the energy is distributed over the network in a nonuniform way. As a consequence, s_i's in (3.18) are not identically distributed. This may also arise when the sensors generate packets with different rates (i.e. nonidentical Poisson distributions). In this situation, w does not have any standard distribution, however, we can still use CLT to approximate the pdf of w with a Gaussian distribution. Here, we give a brief discussion on the probability of achieving the lifetime threshold by the network.

Lemma 3.2 *Assume that z_i's ($1 \leq i \leq m$) are m independent random variables such that*

$$\sum_{i=1}^{m} \mu_{z_i} = m\overline{\mu}. \quad (3.49)$$

where μ_{z_i} denotes the mean of z_i. Also, X_i's ($1 \leq i \leq m$) are m Bernoulli trials such that

$$P(X_i = 1) = z_i \quad \forall i. \quad (3.50)$$

Now, if X denotes the sum of X_i's, the variance of X is maximum when

$$\mu_{z_i} = \overline{\mu} \quad \forall i = 1, \dots, m \quad (3.51)$$

Proof: see [41]. ■

Corollary 3.2 *For nonidentical distributed s_i's such that*

$$\mu_w = \sum_{i=1}^{N} \mu_{l_i} = \sum_{i=1}^{N} E[s_i] = N\mu, \quad (3.52)$$

(3.14) is an upper bound for the probability of achieving the lifetime when $1 - \beta - \mu > 0$, otherwise it is a lower bound.

Proof: Since we assumed the identical distribution in the proof of Theorem 3.1, Lemma 3.2 indicates that σ_w in (3.29) is the maximum possible variance of w. The proof is completed considering the decreasing property of Q-function. ■

3.3.5 Applications

The results of this work can be used to optimize the network parameters in order to increase the lifetime. The adjustable parameters (e.g. sensors energy distribution and node density over the area) are related to the desired probability through μ. By taking the derivation of $F_L^c(T_{\text{thr}})$ with respect to μ, we have

$$\frac{dF_L^c(T_{\text{thr}})}{d\mu} = \frac{1}{2\pi} \frac{1-\beta+\mu(2\beta-1)}{2(\mu-\mu^2)^{\frac{3}{2}}} e^{-\frac{N(1-\beta-\mu)^2}{\mu-\mu^2}}. \tag{3.53}$$

From (3.53), it can be shown that despite the choice of β, $F_L^c(T_{\text{thr}})$ is always an increasing function of μ. In other words, lifetime optimization is equivalent to maximizing μ. Evidently, this fact is consistent with intuition. Since μ is related to the energy distribution among the nodes and the node distribution over the area, by optimizing these distributions, the network lifetime can be prolonged. An application of our lifetime analysis in the network lifetime extension is provided later.

3.4 Simulation results

In this section, we investigate the accuracy of the proposed analysis through computer simulation. We first study single-hop networks. To investigate the effect of the area shape, simulations are performed over different area shapes with the same area size. The effect of the number of sensors is also studied.

We then perform computer simulation to study the lifetime of multi-hop networks. Also, the effect of transmission range (and consequently the number of hops) on the lifetime of the network is studied.

3.4.1 Single-hop Networks

The parameters of model (2.1) depend on the data rate, antenna height, antenna gain, etc. Typical values of e_t and e_o are given in [18]. For $\alpha = 4$, which we use in our simulations, the values of e_t and e_o are respectively 0.0013 pJ/bit/m^4 and 50 nJ/bit for a 1Mbps data stream. Here, It is assumed that the packets are 1000 bits long, hence, m =1.3 pJ/m^4 and $c = 50$ μJ.

We consider 500 sensors that are deployed according to a uniform distribution. The packet generation model obeys a Poisson distribution with an average package generation rate of 1 packets/time unit. All of the sensors have the same initial energy equal to 11 mJ. Also, to investigate the effect of the area shape on the lifetime, the simulations are carried out over circular, hexagonal, square and triangular areas with equal size of $100\pi m^2$. In all cases, the sink is located at the center of the area and the desired lifetime is 100 time units.

The probability of achieving the desired lifetime as a function of β is obtained and depicted in Figure 3.2. To decrease the final result variance and reach a reasonable confidence interval, the simulations are repeated 10,000 times over each area and the results are averaged. As it can be seen, the curves for circular, hexagonal and square areas are very close. Since in a triangle, the distance of the sensors to the sink is more nonuniform and it has the largest circumcircle compared to the other shapes, triangle has a smaller chance of achieving the desired lifetime.

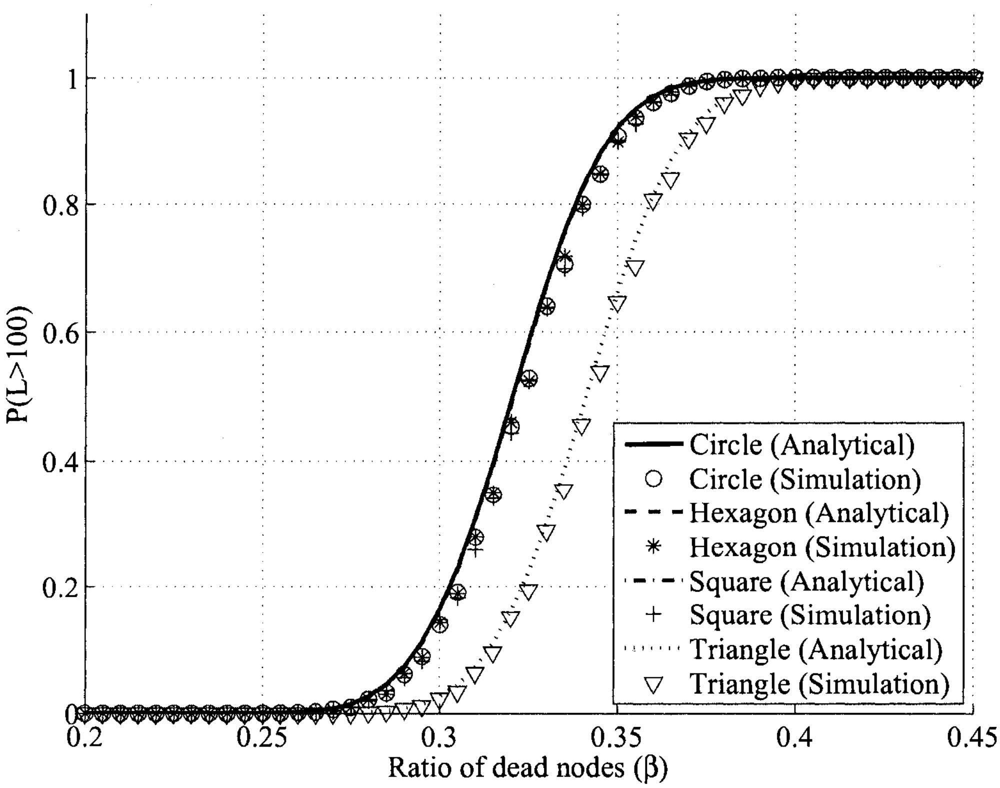


Figure 3.2: Probability of achieving the lifetime threshold vs. the ratio of dead nodes for single-hop networks deployed over different area shapes.

As discussed earlier, depending on the value of $1-\mu-\beta$ and by increasing the number of nodes, it can be almost surely determined whether the network achieves a lifetime threshold or not. The effect of the number of sensors on the lifetime is shown in Figure 3.3. The lifetime of the network is considered as the moment when 30 percent of the nodes in the network die. In the first case, $E_i = 11$ mJ which results in $1 - \beta - \mu > 0$. Hence, as discussed in Section 3.3, the desired probability decreases by increasing N which is verified by simulation. In the second case, the initial energy is equal to 11.8 mJ which causes $1 - \beta - \mu < 0$. As shown in Figure 3.3, the probability of achieving the desired lifetime is an increasing function of N.

3.4.2 Multi-hop Networks

To study the network lifetime in a multi-hop network, it is assumed that 500 nodes are deployed uniformly over a circle with radius 100 m. All of the nodes have the same initial energy equal to $E_{\text{init}} = 100$ mJ. The parameters in (2.1) are kept the same as before. Since the simulation result for the adjustable power is already seen in the previous simulations, here, it is assumed that nodes perform the transmission with a fixed transmission range. The adjustable transmission range would result in observations similar to that shown for single-hop networks.

A greedy routing algorithm is used to balance the network traffic such that data packets are evenly distributed between the nodes within each ring of the network. Considering the fact that all of the nodes use a constant transmission power and the traffic is distributed identically between the first-ring nodes, all of the nodes within $\mathcal{H}$ (the first network ring) have approximately similar lifetime. As a consequence, they die in time moments very close to each other. Therefore, we can say that the desired probability is not significantly effected by the value of β (Figure 3.4).

It is interesting to study the effect of the transmission range and consequently the number of hops on the lifetime. Figure 3.5 depicts the probability of reaching the lifetime threshold vs. the transmission range. The lifetime is considered as the moment when 30% of nodes within $\mathcal{H}$ are dead. By decreasing r, the number of nodes within $\mathcal{H}$ decreases, hence, they carry more packets and will die earlier. Therefore, it is expected that the desired probability decreases by reducing r. Indeed, while the nodes far from the sink still have enough energy to send packets, the nodes within $\mathcal{H}$ cease to function. To overcome this drawback, nonuniform energy or node distribution might be applied [42].

3.5 Optimum Node Distribution

Heavy traffic load on the nodes close to the data sink causes the nodes early death in a multi-hop network. These nodes consequently form the network bottleneck. To surpass this problem, one may assign a higher level of energy to the nodes located in the bottleneck, however, this necessitates implementing a heterogeneous network with a different type of

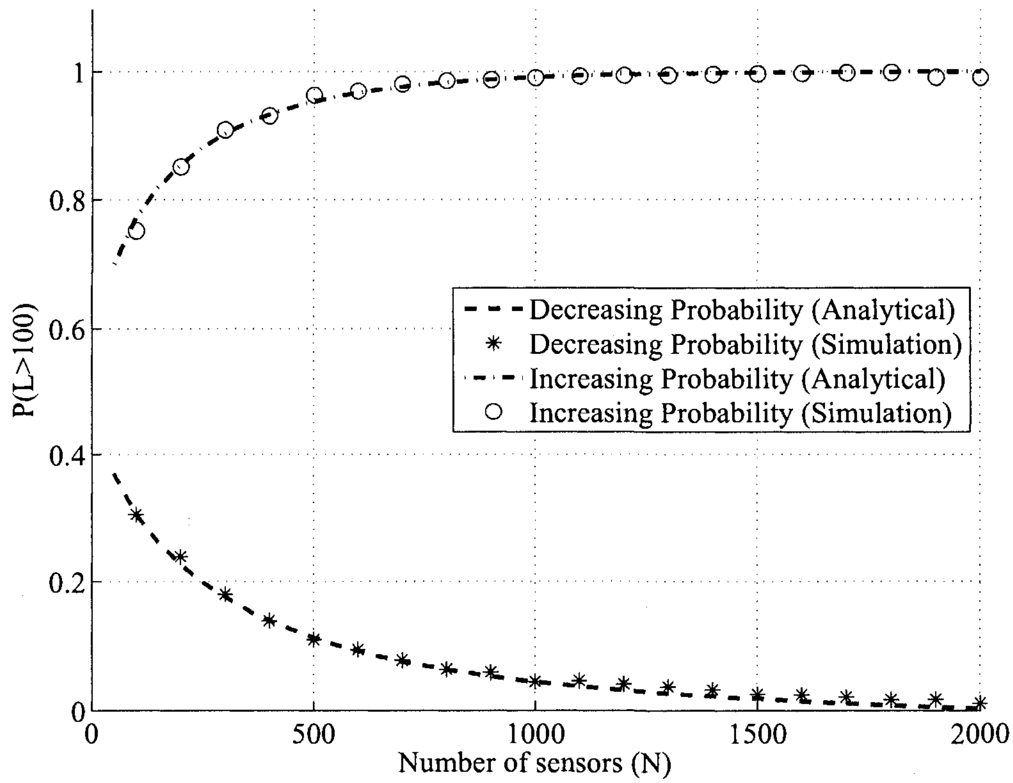


Figure 3.3: Probability of achieving the lifetime threshold vs. the number of sensors in a single-hop network.

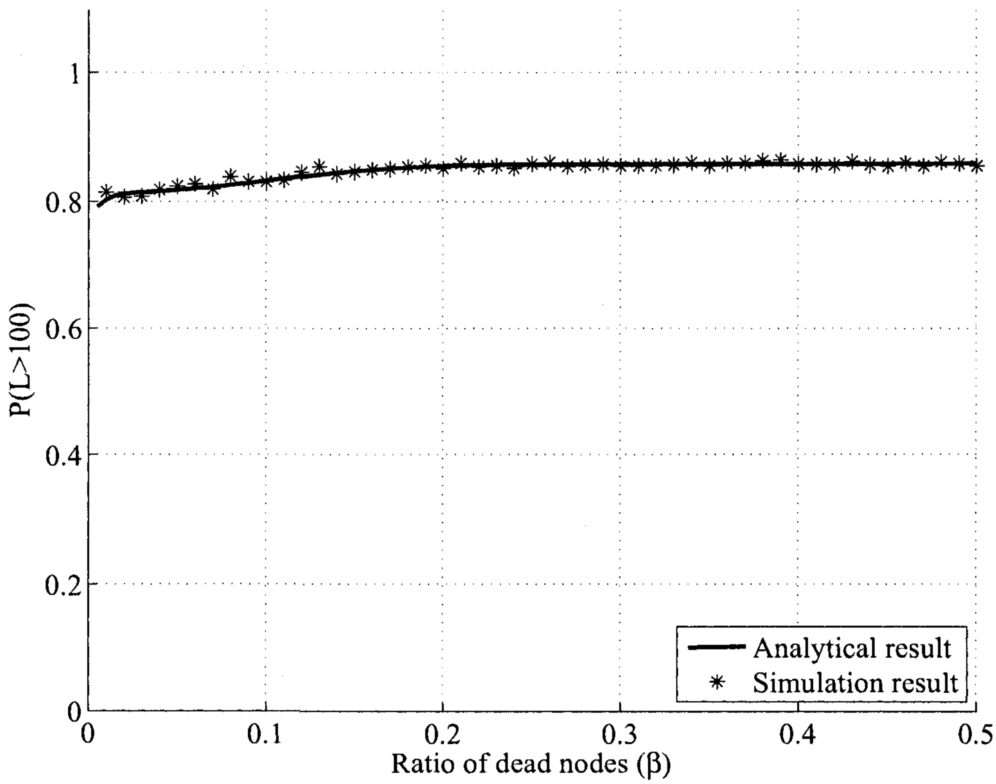


Figure 3.4: Probability of achieving the lifetime threshold vs. the ratio of dead nodes in a multi-hop network

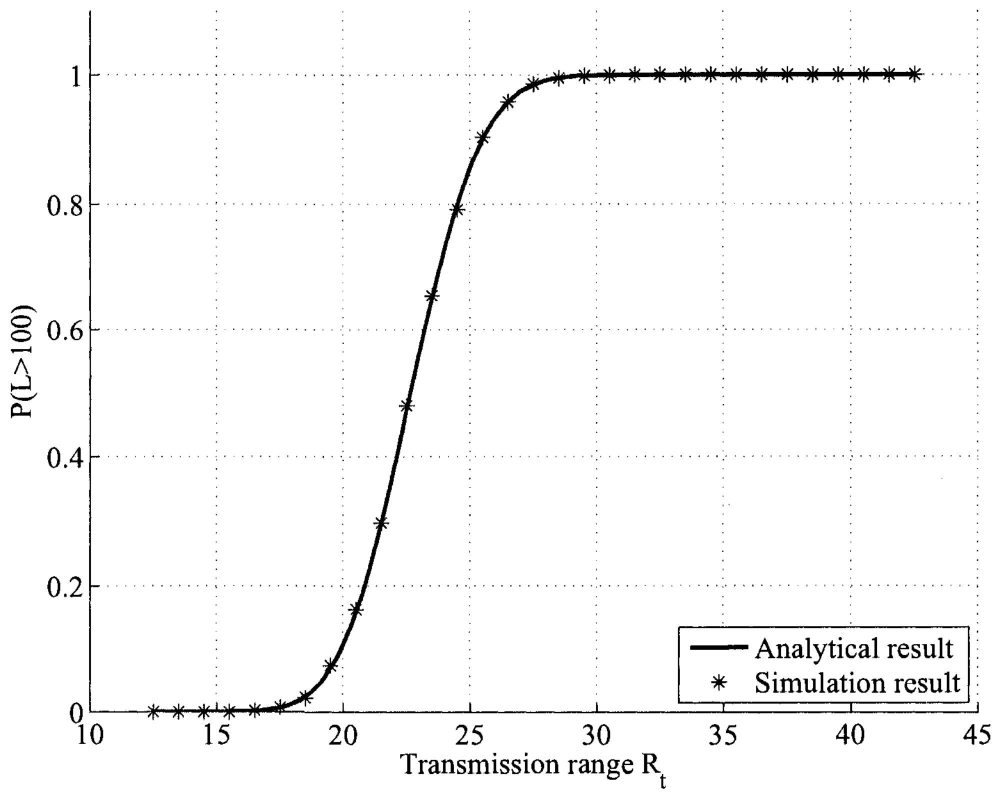


Figure 3.5: Probability of achieving the lifetime threshold vs. the transmission range in a multi-hop network

nodes. Adjusting the node distribution in a homogeneous network is also another approach which we will focus on in the following.

Our goal is to design a network with minimum number of nodes such that while a specified node density is sustained all over the network, the network can achieve a desired lifetime T_{thr} given a reliability value for achieving this lifetime. In fact, the reliability value determines how probable is achieving T_{thr}. The specified threshold for the node density reflects mainly the level of the area coverage in the network. To this end, lifetime is assumed to be over when the density of active nodes in the network drops below the threshold δ_t. Hence, all of the rings should have the density of at least δ_t. As a consequence, ring i will remain alive until T_{thr} if the ratio of the dead nodes in the ring is less than

$$\beta_i = \frac{N_i - (2i-1)\pi r^2 \delta_t}{N_i} \tag{3.54}$$

which is a function of the number of nodes within ring i, N_i.

Here, it is assumed that the sensors are spread over a circle with radius R and data sink is placed at the center of the area. Nevertheless, the methodology can be applied to other area shapes and/or when the sink is not placed at the center which will be discussed later. Assuming a fixed transmission range, r, for each node enables us to divide the circle to some rings each having a thickness of r (Figure 3.1). The sensors within a ring forward their data toward the sink through the neighboring ring. The network desired lifetime is T_{thr} and the sensors are initialized with E_{init} Joules of energy.

Here, we assume that the sleep scheduling can adjust the density of active nodes to a fixed value during the network operation which is less than the original density of the deployed nodes. This helps to conserve the sensors energy and prolong the network lifetime. Owing to the use of sleep scheduling, each sensor has an effective packet generation rate less than λ. If the initial sensor deployment is not very sparse, traffic will be distributed equally over the sensors and all of the sensors in ring i generate packets obeying a Poisson model with the average rate of

$$\hat{\lambda}_i = \frac{\lambda \delta_s \pi r^2 (2i-1)}{N_i} \quad i = 1, 2, \ldots, k \tag{3.55}$$

where k denotes the number of rings within the network and δ_s reflects the node density forced by the sleep scheduling over the network. In addition, by considering that inner rings forward the traffic of the outer ones, the average traffic rate of each sensor within ring i (consists of its own generated traffic and the forwarded traffic of the outer rings) is

$$\lambda_i = \frac{\lambda \delta_s \pi r^2 (k^2 - (i-1)^2)}{N_i} \quad i = 1, 2, \ldots, k. \tag{3.56}$$

Now, by using Corollary 3.1, the probability of achieving the lifetime threshold by a node within ring i is

$$\mu_i = 1 - \frac{\gamma(p_f, \lambda_i T_{\text{thr}})}{\Gamma(p_f)}. \tag{3.57}$$

Therefore, ring i passes T_{thr} with probability

$$P(T_{L_i} \geq T_{\text{thr}}) = Q\left(\sqrt{N_i}\frac{1-\beta_i-\mu_i}{\sigma_i}\right) \tag{3.58}$$

where T_{L_i} represents the lifetime of ring i. In addition,

$$\sigma_i = \sqrt{\mu_i - \mu_i^2}. \tag{3.59}$$

Now, it can be seen how the lifetime of ring i increases with N_i. From (3.56), it is apparent that increasing N_i decreases the average traffic rate of the sensors in ring i. Hence, they send less number of packets and stay alive for a longer time which consequently increases the reliability of achieving the lifetime threshold by ring i. Using (3.56), it is clear that the average traffic load is higher in the inner rings which justifies the application of nonuniform node distribution. On the other hand, the derivative of (3.58) with respect to β,

$$\frac{dP(T_{L_i} \geq T_{\text{thr}})}{d\beta} = \sqrt{\frac{N_i}{2\pi\sigma_i^2}} e^{\frac{N_i(1-\beta_i-\mu_i)^2}{\sigma_i}}, \tag{3.60}$$

is positive, and consequently, $P(L_i \geq T_{\text{thr}})$ is an increasing function of β_i. Therefore, increasing the value of N_i, and consequently β_i, increases the probability of achieving the lifetime threshold. Intuitively, since increasing N_i makes the limit on the ratio of the dead nodes looser, it extends the reliability of achieving the lifetime threshold.

Using the probability of achieving the lifetime by each ring, one can cast the design of node distribution as an optimization problem. The goal of this optimization is to find the minimum number of nodes required to assure a given reliability for achieving T_{thr}. The number of nodes in each ring has to be adjusted in a way such that probability of achieving T_{thr} by each ring is higher than the given reliability for that ring, R_i. Hence, the problem can be formulated as follows:

$$\min \sum_{i=1}^{k} N_i \qquad \text{subject to}$$

$$N_i \geq (2i-1)\pi r^2 \delta_t \qquad i = 1, 2, \ldots, k, \tag{3.61}$$

$$Q\left(\sqrt{N_i}\frac{1-\beta_i-\mu_i}{\sigma_i}\right) \geq R_i \qquad i = 1, 2, \ldots, k. \tag{3.62}$$

Conditions (3.61) and (3.62) respectively shows the restriction on the minimum number of nodes in the ring and the expected lifetime reliability.

Assumed sleep scheduling mechanism results in a traffic load which is independent of the actual density of nodes, because only the active nodes generate traffic. As a consequence, the traffic loaded by ring i on any inner ring is independent of N_i (its actual number of nodes). Therefore, the traffic generated by each ring depends only on the position of the ring with respect to the sink. Notice, however, that according to (3.56), the traffic load λ_i depends on N_i (but as discussed not any other $N_j, j \neq i$). Hence, the number of nodes in each ring does not affect the minimum number of nodes needed within other rings. As a consequence, the optimization problem can be rewritten as follows:

$$\forall i \quad \min N_i \qquad \text{subject to}$$

$$N_i \geq (2i-1)\pi r^2 \delta_t, \tag{3.63}$$

$$Q\left(\sqrt{N_i}\frac{1-\beta_i-\mu_i}{\sigma_i}\right) \geq R_i. \tag{3.64}$$

First, we need to show that the formulated problem has a feasible solution. As mentioned previously, the reliability of achieving T_{thr} increases with N_i. Increasing the value of N_i does not hurt the condition in (3.63) and it still remains satisfied. On the other hand, for sufficiently large N_i, both β_i and μ_i tend to 1. Thus,

$$\lim_{N_i \to \infty} Q\left(\sqrt{N_i}\frac{1-\beta_i-\mu_i}{\sigma_i}\right) = Q(-\infty) = 1 \tag{3.65}$$

which satisfies (3.64), hence, the problem is feasible.

To find the optimal solution, we know that

$$N_i = \lfloor (2i-1)\pi r^2 \delta_t \rfloor + 1 \tag{3.66}$$

satisfies the condition in (3.63) and if (3.64) holds for this value of N_i, the optimal solution is obtained. Otherwise, the optimal solution can be achieved by increasing N_i to the point that (3.64) holds.

Now, the minimum number of sensors within each ring can be easily found. To this end, one can show that the optimal number of nodes in ring i, N_i^*, is

$$N_i^* = \max(\lfloor (2i-1)\pi r^2 \delta_t \rfloor + 1, \mathcal{S}) \tag{3.67}$$

where $\mathcal{S}$ is the smallest integer number satisfying

$$\mu_i N_i + \sigma_i Q^{-1}(R_i)\sqrt{N_i} - (2i-1)\pi r^2 \delta_t \geq 0. \tag{3.68}$$

The solution of the original problem, N^*, which shows the minimum number of sensors in the network is therefore

$$N^* = \sum_{i=1}^{k} N_i^*. \tag{3.69}$$

Remark 3.4 *The failure of network rings are independent of each other. On the other hand, since (3.63) is the active constraint in the optimization, (3.64) may become satisfied with strict inequality for some rings. Therefore, the reliability of achieving a specific lifetime by the network with the proposed density, R, can be lower-bounded by*

$$R \geq \prod_{i=1}^{k} R_i. \tag{3.70}$$

Remark 3.5 *Figure 3.6 shows an arbitrary network where the data sink is not positioned at the center. Unlike the circular shape, the traffic load on the sensors within a ring is not symmetric. Our analysis, however, is still applicable to any sufficient thin sector of a circle which is centered on the sink. For any senor within the intersection of the rings and the sector, the average traffic load can be determined similar to the approach that we took earlier. As a consequence, the optimal density can be obtained for each sector.*

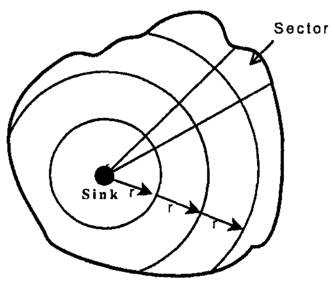


Figure 3.6: An arbitrary network with non-centered sink.

3.6 Simulation Results

Assume a network which covers a circular area with radius $R = 50$ m. Sensors generate packets with a Poisson model having an average of $\lambda = 1$ packet/minute. Also, the nodes density cannot be less than 0.1 nodes/m^2 in order to have satisfying coverage and connectivity. The path loss exponent in (2.1) is $\alpha = 2$. Again, using the power consumption model parameters in [18] and assuming 1000-bit packets, the value of parameters in (2.1) are $m = 10$ nJ/m^2 and $c = 50\mu$J. The sensors have a transmission range of $r = 10$ m. Hence, there exist five rings in the network. We are interested in finding the minimum number of sensors within each ring.

Assuming $T_{\text{thr}} = 9000$ time units and the minimum reliability of $R = 0.95$ for achieving the lifetime by the rings, the optimum numbers of sensors obtained through the proposed method are $N_1^* = 1117$, $N_2^* = 1077$, $N_3^* = 946$, $N_4^* = 724$ and $N_5^* = 411$ when

$E_{\text{init}} = 0.6$ J. To study the performance of the designed network, we compare the lifetime of the designed network with the uniform node distribution. To this end, it is assumed that $N = \sum_{i=1}^{4} N_i^* = 4275$ nodes are distributed uniformly and other parameters are kept unchanged. The simulation has been run 1000 times. The probabilities of achieving the lifetime threshold by each ring are indicated in Table 3.1. In this table, R_i's are the achieved reliability by ring i in the simulations. As it can be seen, uniform distribution cannot achieve the desired reliability for the inner rings, while it is achievable by our proposed node distribution.

Node Distribution	R_1	R_2	R_3	R_4	R_5
Uniform	0	0	0.001	1	1
Optimum	0.987	0.973	0.994	1	0.994

Table 3.1: Comparison between the uniform and optimum distribution.

We also show that the number of nodes obtained by the proposed algorithm is the minimum possible number. For instance, By assuming $N_1^* - 1 = 1116$ sensors over the first ring, the reliability of achieving the lifetime threshold decreases to $R_1 = 0.847$. The same behavior can also be seen in other rings. Hence, no distribution with less nodes can achieve the desired lifetime with the given reliability.

It is also desirable to study the effect of E_{init} on the number of nodes in each ring. It is expected that by decreasing E_{init}, the number of nodes in each ring increases. This increase in the number of nodes tries to reduce the average traffic rate in a way that the lifetime of each ring can achieve T_{thr}. As can be seen in Figure 3.7, initially, by decreasing E_{init}, the values of N_2^*, N_3^*, N_4^* and N_5^* do not change. The reason is that, E_{init} is enough to guarantee the desired lifetime in each ring. Therefore, the condition on connectivity/coverage, i.e. (3.63), is the active constraint in the optimization. But when E_{init} decreases further, the existing number cannot bear the traffic load and increasing the number of nodes is unavoidable to achieve the desired lifetime. As expected, the change in the number of nodes in each ring starts from the inner rings which bear higher traffic load.

In Figure 3.8, the average traffic rate is depicted versus the value of E_{init}. As expected, by decreasing the value of E_{init}, the number of nodes within each ring increases such that the average traffic load on the sensors decreases and the desired lifetime becomes achievable by the network.

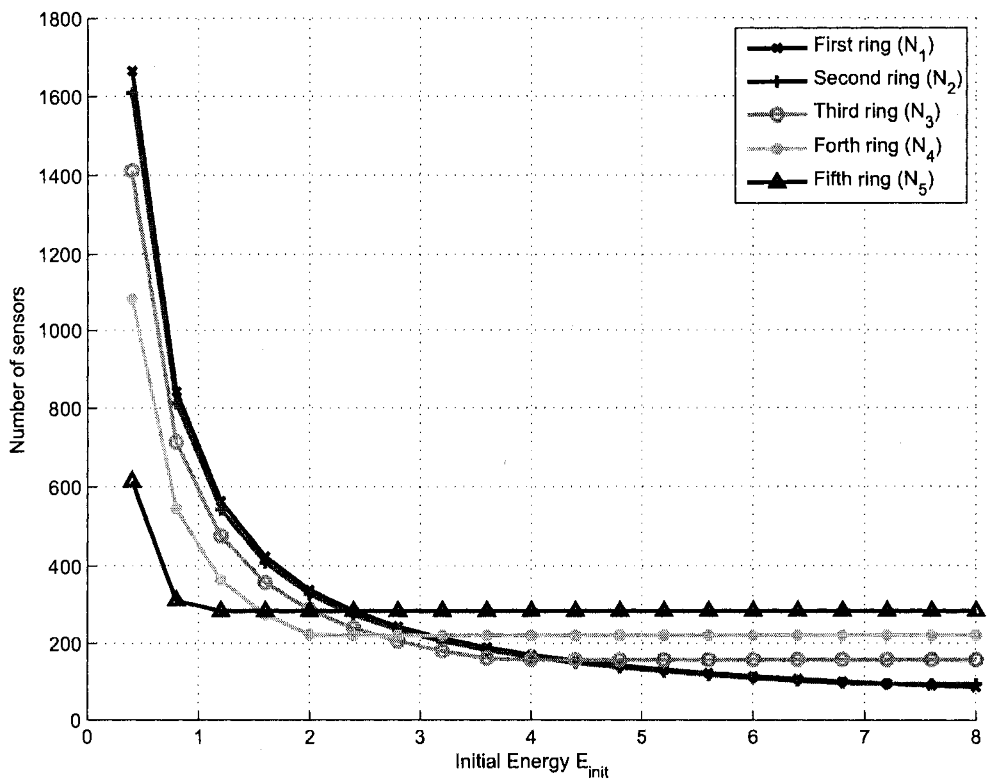


Figure 3.7: Number of nodes within each ring vs. the initial energy.

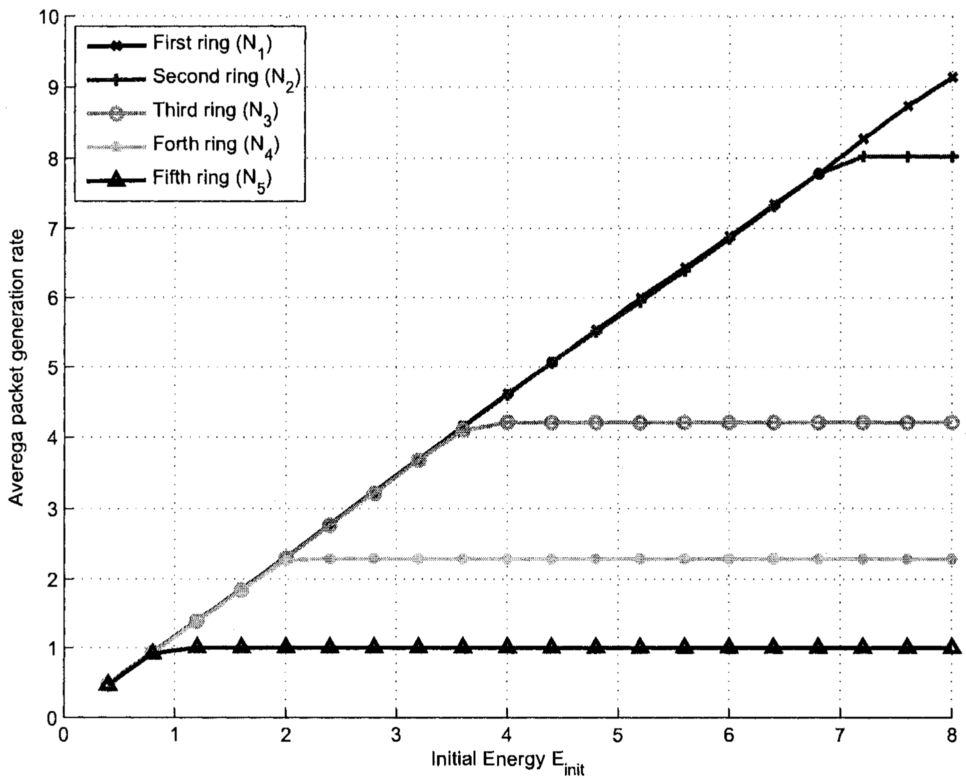


Figure 3.8: The average sensor traffic rate vs. the initial energy.

Chapter 4

Lifetime Analysis for Clustered Networks

In this chapter, a lifetime analysis in provided for clustered networks. First, the applied model for clustering is introduced. Then, the cluster lifetime will be studied for both single-hop and multi-hop networks. Also, a condition will be provided on the efficiency of clustered networks indicating whether a clustered network is beneficial in terms of the energy efficiency or not.

4.1 Clustering Model

Clustering is proposed for WSNs to decrease the energy consumption and ease the network management. Usually, the nodes within a cluster send their data to the CH and then CH performs necessary processing and aggregation before relaying it toward the data sink.

Both intra-cluster (sensor to the CH) and inter-cluster (CH to the data sink) transmissions can be carried out in either single-hop or multi-hop fashion [43, 44]. First, we focus on the case where all transmissions are in single-hop mode, similar to [16]. Later on, the multi-hop mode for inter-cluster transmissions will be discussed. For multi-hop inter-cluster transmissions, it is assumed that a CH forwards its data toward the sink through other CHs, i.e., other nodes are not involved in inter-cluster data communication.

It is also assumed that the network has a static clustering, meaning that the shape of the clusters are fixed during the network operation [45, 46]. While dynamic clustering allows for a more flexible network design, its significant overhead to form the clusters is considered a serious drawback [16], [43]. For many practical situations, therefore, fix clustering is an attractive solution.

Another important issue in the network clustering is to rotate the CH role among the

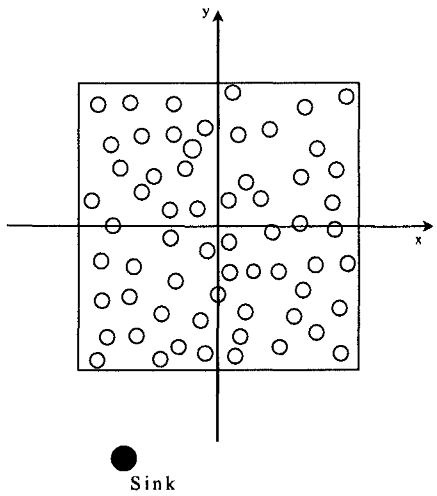


Figure 4.1: Sensor nodes deployed randomly within a cluster.

sensor nodes. Since the inter-cluster transmissions consume a vast amount of energy, rotating the CH role results in the energy consumption balance over all the nodes in the cluster. As a consequence, the early death of the nodes is avoided. Here, we assume that the CH role rotates among the nodes within a cluster based on a pre-scheduled periodic scenario. The pre-scheduled CH assignment reduces/removes the need for the overhead packets [43].

4.2 Lifetime Analysis for Single-hop Clustered Networks

In this section, we propose a probabilistic analysis to model the lifetime of a cluster within a clustered WSN that employs single-hop transmissions for both intra-cluster and inter-cluster communications. More specifically, the analysis determines the probability of achieving a desired lifetime by the cluster when the network parameters (such as the nodes initial energy, number of nodes, data generation rate, etc.) are given.

Here, it is assumed that N nodes, each with initial energy of E_{init}, are spread independently and randomly inside the cluster (Figure 4.6) . Also, CH can aggregate l received bits to γl bits, where $0 < \gamma \leq 1$. The smaller the value of γ, the less number of packets will be transmitted toward the sink. In the special case of $\gamma = 1$, no data aggregation is performed.

We start our analysis by finding the probability of achieving a desired lifetime T_{thr} by a single node. For this purpose, we study the consumed energy by a node after T_{thr}, called E_c. It is clear that a sensor is still alive after T_{thr} if its consumed energy does not exceed E_{init}. The probability of achieving the lifetime threshold by the entire cluster can be found based on the derived probability for a single node.

Assume that the data sink is located at $\mathbf{s}_s = (x_s, y_s)$ and T denotes the time duration that a sensor serves as CH in each round. If the desired lifetime is long,

$$k = \left\lfloor \frac{T_{\text{thr}}}{NT} \right\rfloor \approx \frac{T_{\text{thr}}}{NT} \tag{4.1}$$

is the number of rounds that the CH role is assigned to a sensor during the desired lifetime.

The consumed energy by a sensor can be written as $E_c = E_n + E_{\text{CH}}$, where E_n reflects the amount of energy depleted for intra-cluster transmissions and E_{CH} refers to the amount of energy that the sensor uses when the CH role is assigned to it. Since the sensors are deployed randomly and the same traffic generation model is assumed for all of them, without loss of generality, E_c can be found for sensor 1 located at $\mathbf{s}_1 = (x_1, y_1)$.

The energy that sensor 1 consumes to send data to sensor i during T_{thr}, e_i, is

$$e_i = \left(\sum_{j=1}^{k} p_{ij} \right) (m_1 \|\mathbf{s}_1 - \mathbf{s}_i\|^2 + c_1) \quad i = 2, 3, \ldots, N \tag{4.2}$$

where $\|\cdot\|$ denotes the Euclidean distance and $\mathbf{s}_i = (x_i, y_i)$ represents the position of sensor i. Also, m_1 and c_1 are the parameters of transmission model (2.1) when $\alpha = 2$. In addition, p_{ij} represents the number of packets that sensor 1 transmits toward sensor i during the jth round that sensor i functions as CH. Assuming a Poisson model with mean λ for packet generation, p_{ij}'s obey Poisson distribution with mean λT. Since sum of Poisson random variables is another Poisson random variable, $\mathcal{P}_i = \sum_{j=1}^{k} p_{ij}$ has a Poisson distribution with mean $k\lambda T$. The packet generation model has been assumed to be identical for all sensors, thus, all $\mathcal{P}_i$'s have the same distribution and can be denoted with the same random variable $\mathcal{P}_e$. It is notable that by using central limit theorem (CLT), distribution of $\mathcal{P}_e$ can be accurately approximated with a Gaussian distribution in the case that packet generation model is not Poisson. Considering all rounds that sensor 1 does not serve as CH, we have $E_n = \sum_{j=2}^{N} e_j$.

Since sensor 1 serves as CH for k rounds, the value of E_{CH} can be found as follows:

$$E_{\text{CH}} = \gamma \left(\sum_{j=1}^{k} P_j \right) (m_2 \|\mathbf{s}_1 - \mathbf{s}_s\|^4 + c_2) \tag{4.3}$$

where P_j is the number of packets generated by all sensors (including sensor 1) during the jth round of its operation as CH. Also, m_2 and c_2 stand for the value of transmission parameters in (2.1) when $\alpha = 4$. Since P_j is the sum of packets generated by all N sensors within the cluster, it has a Poisson distribution as well. Therefore, $\mathfrak{P} = \sum_{j=1}^{k} P_j$ follows a Poisson distribution with mean $kN\lambda T$. Furthermore, CLT is applicable to accurately approximate the distribution of P_j's when the packet generation has a distribution rather than Poisson.

The entire consumed energy by a sensor, therefore, is

$$E_c = E_n + E_{\mathrm{CH}} = \sum_{j=2}^{N} e_j + E_{\mathrm{CH}}. \tag{4.4}$$

Due to the randomness of the packet generation and placement of the nodes over the area, E_c is a random variable which needs to be characterized. To this end, we first find the distribution of E_c when the position of sensor 1 is given, which is called $E_{c|\mathbf{s}_1}$. Then, the unconditional value can be derived by averaging $E_{c|\mathbf{s}_1}$ over the position of sensor 1 in the cluster.

In a randomly deployed WSN, positions of sensors are independent of each other. Since it is assumed that the sensors packet generation occurs independently as well, e_i's are independent random variables. Apparently, E_{CH} and e_i's are also independent, but E_{CH} does not have the same distribution as e_i's. Thus, we apply generalized CLT to approximate the pdf of E_c in (4.4) with a Gaussian distribution. The accuracy of the approximated Gaussian distribution depends on the number of terms in (4.4) (i.e. the number of sensors). There usually exist enough nodes in a cluster to have an acceptable approximation. Thus, pdf of $E_{c|\mathbf{s}_1}$ can be accurately approximated by a Gaussian distribution with mean

$$\mu_{\mathbf{s}_1} = \sum_{j=2}^{N} \mu_{e_{j|\mathbf{s}_1}} + \mu_{\mathrm{CH}\mathbf{s}_1} \tag{4.5}$$

where $\mu_{e_{j|\mathbf{s}_1}}$ and $\mu_{\mathrm{CH}\mathbf{s}_1}$ represent conditional mean of e_j and E_{CH} given $\mathbf{s}_1$. Assuming the same packet generation and placement model for all sensors, e_j's are i.i.d. random variables. Hence,

$$\mu_{\mathbf{s}_1} = (N-1)\mu_{e_{\mathbf{s}_1}} + \mu_{\mathrm{CH}\mathbf{s}_1} \tag{4.6}$$

where $\mu_{e_{\mathbf{s}_1}}$ denotes the conditional mean of e_j's. In a similar way, one can arrive at

$$\sigma^2_{\mathbf{s}_1} = (N-1)\sigma^2_{e_{\mathbf{s}_1}} + \sigma^2_{\mathrm{CH}\mathbf{s}_1} \tag{4.7}$$

where $\sigma^2_{e_{\mathbf{s}_1}}$ and $\sigma^2_{\mathrm{CH}\mathbf{s}_1}$ are conditional variances of e_j's and E_{CH} respectively.

To evaluate $\mu_{\mathbf{s}_1}$, it is required to find the value of $\mu_{e_{\mathbf{s}_1}} = E\left[\mathcal{P}_e z\right]$ where $E[\cdot]$ denotes the expected value, $z = m_1\|\mathbf{s}_1 - \mathbf{s}\|^2 + c_1$ and $\mathbf{s} = (x, y)$ represents an arbitrary point in the area referring to the position of a sensor within the cluster. Due to the independence of the nodes placement in the cluster and traffic generation, we have

$$\mu_{e_{s_1}} = E[\mathcal{P}_e]E[z] = k\lambda T E[z]. \tag{4.8}$$

One can simplify (4.8) using (4.1) as follows:

$$\mu_{e_{s_1}} = \frac{T_{\text{thr}}\lambda}{N} E[z]. \tag{4.9}$$

Let us denote by $\mathcal{C}$ the cluster region and by $f_{X,Y}(x, y)$ the distribution of sensors over $\mathcal{C}$. Then,

$$E[z] = \int\int_{\mathcal{C}} z f_{X,Y}(x, y)\, dxdy. \tag{4.10}$$

Using (4.8) and (4.10), the value of $\mu_{e_{s_1}}$ can be derived. In addition,

$$\sigma^2_{e_{s_1}} = E[\mathcal{P}_e^2 z^2] - E^2[\mathcal{P}_e z] = \text{VAR}[\mathcal{P}_e]E[z^2] + E^2[\mathcal{P}_e]\text{VAR}[z] \tag{4.11}$$

where VAR$[\cdot]$ denotes the variance of the random variable and

$$E[z^2] = \int\int_{\mathcal{C}} z^2 f_{X,Y}(x, y)\, dxdy. \tag{4.12}$$

Since $\mathcal{P}_e$ has a Poisson distribution, (4.11) can be rewritten as follows:

$$\sigma^2_{e_{s_1}} = k\lambda T E[z^2] + (k\lambda T)^2\text{VAR}[z] = \frac{T_{\text{thr}}\lambda}{N}E[z^2] + \left(\frac{T_{\text{thr}}\lambda}{N}\right)^2 \text{VAR}[z]. \tag{4.13}$$

Consequently, $\sigma^2_{e_{s1}}$ can be evaluated using (4.12) and (4.13).

We now need to find the mean and variance of E_{CH}. Since the sink position is fixed, given $\mathbf{s}_1$, randomness of the E_{CH} is caused only by $\mathfrak{P}$. Thus

$$\mu_{\text{CH}\mathbf{s}_1} = \gamma(m_2\|\mathbf{s}_1 - \mathbf{s}_s\|^4 + c_2)E[\mathfrak{P}] \tag{4.14}$$

and

$$\sigma^2_{\text{CH}\mathbf{s}_1} = \gamma^2(m_2\|\mathbf{s}_1 - \mathbf{s}_s\|^4 + c_2)^2\text{VAR}[\mathfrak{P}]. \tag{4.15}$$

Considering that $\mathfrak{P}$ is a Poisson random variable, we have

$$\mu_{\text{CH}\mathbf{s}_1} = kN\lambda T\gamma(m_2\|\mathbf{s}_1 - \mathbf{s}_s\|^4 + c_2) = \lambda T_{\text{thr}}\gamma(m_2\|\mathbf{s}_1 - \mathbf{s}_s\|^4 + c_2)$$

and

$$\sigma^2_{\text{CH}\mathbf{s}_1} = kN\lambda T\gamma^2(m_2\|\mathbf{s}_1 - \mathbf{s}_s\|^4 + c_2)^2 = \lambda T_{\text{thr}}\gamma^2(m_2\|\mathbf{s}_1 - \mathbf{s}_s\|^4 + c_2)^2.$$

Hence,

$$\mu_{\mathbf{s}_1} = \gamma\lambda T_{\text{thr}}(m_2\|\mathbf{s}_1 - \mathbf{s}_s\|^4 + c_2) + \frac{(N-1)T_{\text{thr}}\lambda}{N}E[z] \tag{4.16}$$

and

$$\sigma_{\mathbf{s}_1} = \gamma^2 \lambda T_{\text{thr}} (m_2 \|\mathbf{s}_1 - \mathbf{s}_s\|^4 + c_2)^2 + (N-1) \left(\frac{T_{\text{thr}} \lambda}{N} E[z^2] + \left(\frac{T_{\text{thr}} \lambda}{N} \right)^2 \text{VAR}[z] \right).$$

Now, the conditional probability of achieving the lifetime threshold by sensor 1, $P_{s|\mathbf{s}_1}$, is

$$P_{s|\mathbf{s}_1} = P(E_{c|\mathbf{s}_1} < E_{\text{init}}). \tag{4.17}$$

Since pdf of $E_{c|\mathbf{s}_1}$ can be approximated by a Gaussian distribution, the above probability can be evaluated as

$$P_{s|\mathbf{s}_1} = 1 - Q\left(\frac{E_{\text{init}} - \mu_{\mathbf{s}_1}}{\sigma_{\mathbf{s}_1}} \right). \tag{4.18}$$

To continue the analysis, it is necessary to derive the unconditional probability of achieving the lifetime threshold by sensor 1, P_s. One can simply find the value of P_s by averaging $P_{s|\mathbf{s}_1}$ over $\mathbf{s}_1$. Hence,

$$P_s = \int \int_{\mathcal{C}} (1 - Q(\frac{E_{\text{init}} - \mu_{\mathbf{s}_1}}{\sigma_{\mathbf{s}_1}})) f_{X_1,Y_1}(x_1, y_1)\, dx_1 dy_1 \tag{4.19}$$

where $f_{X_1,Y_1}(x_1, y_1)$ shows the distribution of x_1 and y_1 over $\mathcal{C}$. Since node 1 was arbitrarily chosen from the set of N nodes within the cluster, P_s represents the probability of achieving the lifetime threshold by other nodes of the cluster as well. Notice that in our probabilistic analysis, we do not have any information about the position of sensors, hence P_s is the same for all sensors. If the locations of the sensors for a network realization is provided, considering the randomness of the node position in the analysis is unnecessary. This simplifies the analysis to considering the randomness of only traffic generation.

According to the lifetime definition, it is required to investigate the probability of achieving T_{thr} by at least $(1-\beta)N$ nodes. To this end, we define Bernoulli random variables b_i's determining whether sensor i can pass T_{thr} or not. It is clear that the probability of success for these Bernoulli random variables is P_s given in (4.19). Sum of b_i's, called w, represents the number of alive nodes after T_{thr}. Since b_i's obey Bernoulli distribution, w has a binomial distribution. Thus, the probability of achieving T_{thr} by the cluster is

$$P_L = P((1-\beta)N < w) = \sum_{j=\lfloor (1-\beta)N \rfloor + 1}^{N} \binom{N}{j} P_s^j (1-P_s)^{N-j} \tag{4.20}$$

which can be written as

$$P_L = I_{P_s}(\lfloor (1-\beta)N \rfloor + 1, N - \lfloor (1-\beta)N \rfloor) \tag{4.21}$$

where $I_z(\cdot,\cdot)$ is the regularized incomplete beta function [47].

In the above analysis, according to (4.1), it is assumed that all the nodes in the cluster serve as CH the same number of times. However, with the death of one node, other sensors has to serve as CH more frequently. While this hurts the assumption made in (4.1), it does not directly affect the analysis. To see this, assume that the number of alive nodes in an arbitrary time during the network operation is $N' < N$. In this case, each sensor becomes CH once after each N' rounds, but it forwards just the traffic of alive nodes. As a result, the traffic load of the CH is scaled by $\frac{N'}{N}$ and the number of times that a node serves as CH is scaled by $\frac{N}{N'}$, keeping the average number of packets transmitted by the CH to the sink unchanged.

Death of nodes in the cluster has an indirect effect on the accuracy of our analysis. Usually, the first nodes to die are those farther from the sink. Thus, the effective cluster region $\mathcal{C}$ would change as nodes die. This influences the equations involving averaging over $\mathcal{C}$, e.g., (4.19). Since the alive nodes are located in a smaller sub-area of the cluster and closer to the sink, both their inter-cluster and intra-cluster energy usage are less than the predictions of our analysis. As a consequence, the predicted lifetime by the analysis becomes a pessimistic approximation. This deficiency in the analysis has a minor effect on results and is observable only when the cluster is located very close to the sink or when the cluster lifetime is very short. In fact, as argued below, in the limit of the distance of the sink from the cluster and also in the limit of the lifetime, the analysis becomes exact.

When the cluster is located very far from the sink, all the sensors have almost the same distance to the sink, hence, the same energy usage for inter-cluster transmission. Thus, death of nodes occurs totally randomly over the cluster region, keeping $\mathcal{C}$ unchanged and making our analysis exact. In the limit of the lifetime, the time duration from the death of the first node to the death of the cluster is negligible compared to the cluster lifetime. Therefore, the time duration over which the analysis is not exact is insignificant compared to the time duration over which the analysis is exact.

The analysis is also applicable to multi-level transmission power models (instead of the continuously adjustable transmission power), e.g., Mica Mote [6] and Intel Mote [7]. In particular, if all the sensors in the cluster use the same power level for intra-cluster and another fixed power-level for inter-cluster transmissions, our analysis provides an exact description of the cluster lifetime. This is because the location of the nodes does not influence the energy consumption and consequently death of nodes occurs randomly over the area. Hence, the effective cluster region is not affected by the nodes death.

Computer simulation presented in Section 5.2 will verify the accuracy of the analysis for various setups.

4.3 Lifetime Analysis for Multi-hop Networks

In the previous section, we assumed that CH relays the data packets to the sink directly. This is not the case in all clustering protocols and data forwarding from CH to the sink may be completed through multi-hop communication [48, 49]. Evidently, the multi-hop communication between CH and data sink does not affect the energy consumption for intra-cluster transmissions among the sensors. Hence, we just need to modify the pdf of E_{CH} accordingly to accommodate the multi-hop inter-cluster transmission.

Suppose that a routing tree, rooted at the data sink, is resulted from applying a routing protocol in the network. We study the lifetime of an arbitrary cluster in this routing tree. The tree vertices are CHs and its depth is D which shows the distance of the farthest node (based on the number of hops) in this routing tree. Also, a CH is in level i, $1 \leq i \leq D$, of the tree if its shortest path to the sink has i hops. Based on the concept of the routing tree, CHs within level i send their data packets to the sink through the CHs in level $i-1$. Members of level 1 transmit their data directly to the sink.

To account for the effect of multi-hop transmission in our analysis, it is required to consider two key points which influence the analysis in the previous section. Firstly, the multi-hop transmission changes the number of packets that a CH has to relay. Indeed, the CH should forward all data packets generated by all clusters located in its routing subtree. In addition, except the CHs in level 1 that send their data to the fixed data sink, the other CHs send their data to the CHs in lower level which do not have a fixed position over the time. In other words, CHs in level i, where $1 < i$, send data packets to a destination which has a random location over the neighboring cluster.

Assume that C_j is an arbitrary cluster in the mentioned routing tree which is located in level i. We study the amount of consumed energy in an arbitrary node in this cluster, called n_1 which is located at $\mathbf{s}_1$. Moreover, it is assumed that C_j has r_{ij} children at level $i+1$ which send their data to C_j. Also, C_j has N_j nodes and its CH compresses data (from its own cluster) with the ratio of γ_j. In addition, C_j forwards data to cluster C_k located in level $i-1$. As mentioned, C_j sends the data to the sink if it is located in level 1. If $\mathbf{s}'$ represents the position of CH in C_k, the consumed energy for inter-cluster transmissions by n_1 is

$$E_{\text{CH}} = \left(\gamma_j \sum_{j=1}^{k_j} P_j + \sum_{j=1}^{r_{ij}} Q_j \right) (m_2 \|\mathbf{s}_1 - \mathbf{s}'\|^4 + c_2). \tag{4.22}$$

In (4.22), Q_j denotes the number of packets sent to n_1 by CHs of C_j's neighbours in level $i+1$ and k_j represents the number of rounds that a sensor serves as CH in C_j.

In addition to the randomness of the packet generation, the average consumed energy by n_1, when it serves as CH, is conditioned on both $\mathbf{s}_1$ and $\mathbf{s}'$. First, one can remove the condition on $\mathbf{s}'$ by averaging the mean of the consumed energy over C_k as follows:

$$\mu_{\text{CH}\mathbf{s}_1} = \big(\gamma_j \lambda T_{\text{thr}} + \sum_{i=1}^{r_{ij}} E[Q_j]\big) \int\int_{C_k} (m_2\|\mathbf{s}_1 - \mathbf{s}'\|^4 + c_2) f_{X',Y'}(x',y')\, dx'dy'. \quad (4.23)$$

Similarly

$$\sigma^2_{\text{CH}\mathbf{s}_1} = \big(\gamma_j^2 \lambda T_{\text{thr}} + \text{VAR}[Q_j]\big) \int\int_{C_k} (m_2\|\mathbf{s}_1 - \mathbf{s}'\|^4 + c_2)^2 f_{X',Y'}(x',y')\, dx'dy'. \quad (4.24)$$

In (4.23) and (4.24), $f_{X',Y'}(x',y')$ shows the distribution of $\mathbf{s}'$ over C_k. The remaining steps of the analysis are the same as the previous section. It is worthy to note that for CHs located in level 1, we have

$$\mu_{\text{CH}\mathbf{s}_1} = \big(\gamma_j \lambda T_{\text{thr}} + \sum_{i=1}^{r_{1j}} E[Q_j]\big)(m_2\|\mathbf{s}_1 - \mathbf{s}_s\|^4 + c_2) \quad (4.25)$$

and

$$\sigma^2_{CH\mathbf{s}_1} = \big(\gamma_j^2 \lambda T_{\text{thr}} + \text{VAR}[Q_j]\big)(m_2\|\mathbf{s}_1 - \mathbf{s}_s\|^4 + c_2)^2. \quad (4.26)$$

4.4 Some Remarks

Remark 4.1 *The effect of the idle power consumption can be considered in the analysis. Assume that P_{idle} shows the idle power consumed to keep the radio part of the sensor on when no transmission occurs. Since the transmission time of each packet is very short compared to the desired lifetime, one can ignore the effect of the transmission time intervals. Thus, a single node can achieve the lifetime when $E_c < E_{\text{init}} - T_{\text{thr}}P_{\text{idle}}$. To accommodate the effect of the idle power in the network, we need to replace E_{init} with $E_{\text{init}} - T_{\text{thr}}P_{\text{idle}}$ in the proposed analysis.*

Remark 4.2 *According to the proposed analysis, one can determine whether clustering results in energy efficiency or not. To this aim, the average consumed energy by a single node in both situations can be studied and used as a measure of energy efficiency. The average consumed energy derived in (4.16) depends on the position of the node. To compare the energy efficiency of the clustered network with the non-clustered one, the unconditional value of the average consumed energy is required which can be obtained by averaging*

μ_{s_1} over the position of sensor 1. If E_d represents the average consumed energy by an arbitrary node in a non-clustered network where the senors send their data directly to the sink, clustered network is more energy-efficient when

$$\int \mu_{s_1} dx_1 dy_1 < E_d. \tag{4.27}$$

The above equation results in the following condition on γ

$$\gamma < 1 - \frac{(N-1)\int\int E[z] f_{X_1,Y_1}(x_1,y_1)\, dx_1 dy_1 dx_1 dy_1}{N\int\int (m_2\|s_1 - s_s\|^4 + c_2) f_{X_1,Y_1}(x_1,y_1)\, dx_1 dy_1 dx_1 dy_1}. \tag{4.28}$$

For greater values of γ, the average energy consumption by a single node is higher in the clustered network and as a consequence, clustering might not be beneficial.

Remark 4.3 *In the proposed analysis, it is assumed that CH role is evenly assigned to the sensors. This method of assigning CH is not necessarily optimal, since it causes the farther nodes to the sink to die earlier compared to the nodes which are closer. This ends in the loss of coverage in some parts of the network. To balance the energy consumption and keep the coverage at an acceptable level all over the cluster, adaptive CH assignment may be applied. Assigning CH adaptively tries to equalize the energy consumption over the nodes in the cluster and avoids the early death of the farther nodes. However, implementing this scheme requires specific routing scenario and requires more overhead packets in the network [16].*

As an instance of adaptive CH assignment, CH role can be designated to the node within the cluster which has the highest level of energy. This can ensure that all nodes have approximately the same lifetime and prevents the early loss of coverage in the network. Assigning the CH role to the node with maximum energy in the cluster requires all of the nodes to be knowledgable about the residual energy of the others. This demands transmitting extra packets by a single node to inform the other nodes about its remaining energy. Transmitting additional packets causes an increase in the consumed energy by each node. Thus, applying the dynamic CH assignment in a network depends on the energy consumption cost.

Figure 4.2(a) shows the location of the dead nodes in the network when CH rotates periodically among the nodes. On the other hand, Figure 4.2(b) depicts the position of the dead nodes when the adaptive clustering is applied. As it can be seen, adaptive CH assignment distributes the location of the dead nodes over the cluster, therefore, network coverage can be preserved. To show the effect of the adaptive CH assignment on balancing the energy consumption rate, number of alive nodes versus time is plotted in Figure 4.3

when $N = 100$. This figure demonstrates that the adaptive CH assignment makes the lifetime of all sensors almost equal.

Remark 4.4 *Assume that the network is time-driven where sensors generate packet with rate* λ. *In this case, the randomness of the traffic generation on the network lifetime is removed. Hence,* μ_{s_1} *remains unchanged in (4.16), however,* $\sigma^2_{s_1}$ *has to be modified as follows:*

$$\sigma^2_{s_1} = \gamma^2\lambda^2T^2_{\text{thr}}(m_2\|s_1 - s_s\|^4 + c_2)^2 + (N-1)\left(\frac{T_{\text{thr}}\lambda}{N}\right)^2 \left(E[z^2] + VAR[z]\right).$$

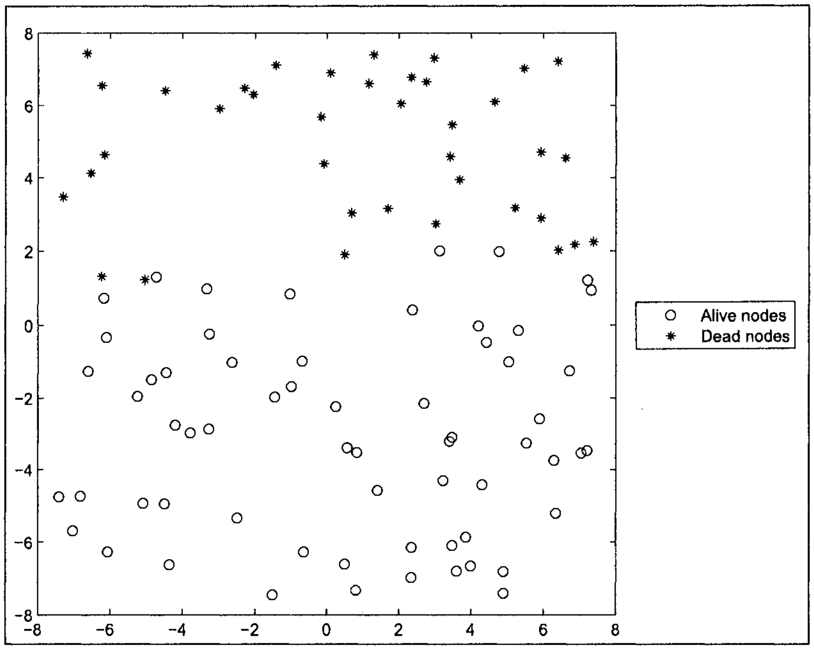
The remaining part of the analysis is the same as event-driven networks.

4.5 Numerical Results

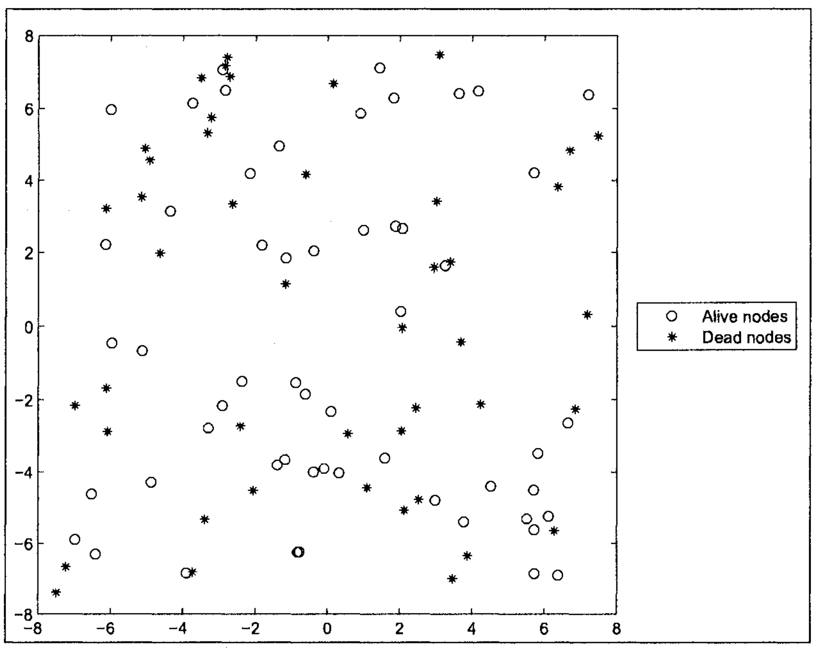
In this section, the analysis is verified through computer simulation. It is assumed that each sensor generates traffic with a Poisson model having the average $\lambda = 1$ packets/time unit and the transmission from the sensors to CH is single-hop. Also, $N = 100$, $T = 10$ time units and $\gamma = 1$ except for the setup where the effect of γ is studied. Having data packets with length 100 bits, the loss coefficient in (2.1) is $m_1 = 0.13$pJ/m^4 when $\alpha = 4$ and $m_2 = 1$nJ/m^2 for $\alpha = 2$. Moreover, the overhead energy for each packet is 5μJ [18]. Here, the CH role is assigned to the sensors periodically based on a predetermined schedule. If the CH assignment schedule reaches a sensor which is already dead, the dead sensor will be removed from the schedule and the next sensor in the list will be assigned as CH.

Figures 4.4 and 4.5 present the analysis and simulation results for different values of T_{thr}. In these figures, the probability of achieving the desired lifetime is plotted versus β. In Figure 4.4, it is assumed that the cluster has a square shape with side length of $L = 10$m and centered at the origin. In addition, the sink is located on $(0, -55)$m. The initial energies for $T_{\text{thr}} = 75000$, $T_{\text{thr}} = 150000$, $T_{\text{thr}} = 300000$ and $T_{\text{thr}} = 600000$ time units are $E_{\text{init}} = 0.875$J, $E_{\text{init}} = 1.75$J, $E_{\text{init}} = 3.5$J and $E_{\text{init}} = 7$J respectively.

Curves show the results of analysis and simulation get closer when T_{thr} increases which is expected based on what discussed in Section 4.2. Also, the gap between the analysis and simulation decreases when the probability of achieving the lifetime by a single sensor and consequently the cluster lifetime increase. This fact does not depend on the desired lifetime and can be seen in all curves. In addition, since the sink is not very close to the cluster in this setup, the distance to the sink does not change significantly from one sensor to



(a)



(b)

Figure 4.2: Position of dead nodes in the cluster: (a) Periodic clustering, and (b) Adaptive clustering.

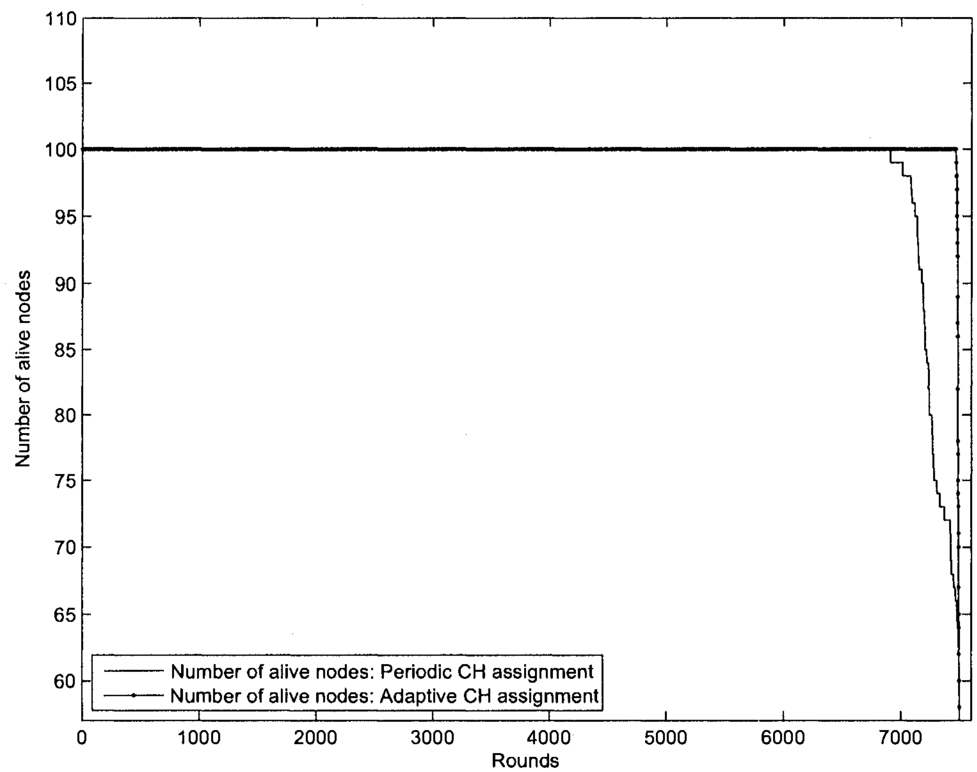


Figure 4.3: Comparison of the number of alive nodes versus time between periodic and adaptive clustering.

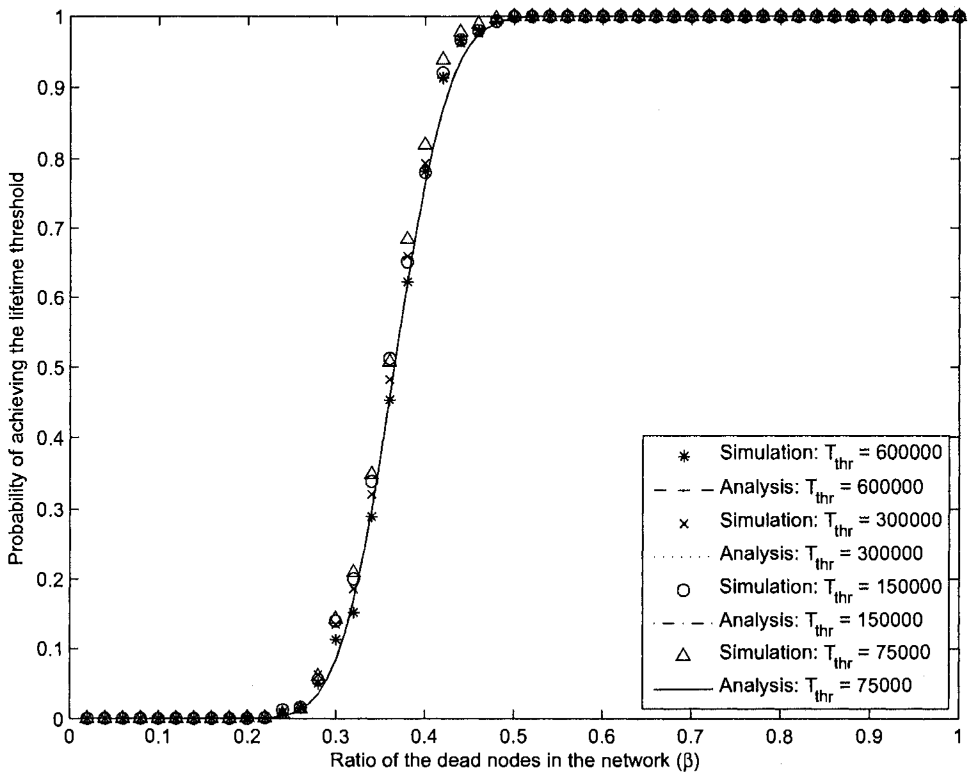


Figure 4.4: Probability of achieving different lifetime thresholds in a single-hop network when the sink is far.

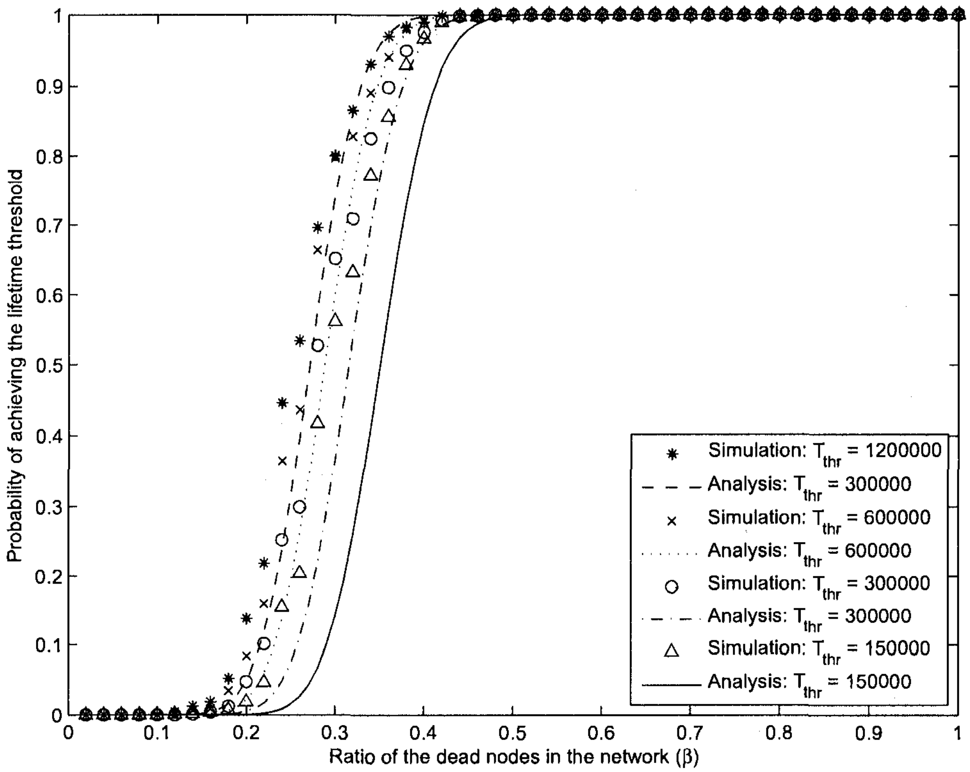


Figure 4.5: Probability of achieving different lifetime thresholds in a single-hop network when the sink is close.

another. Therefore, all the sensors have similar energy consumption rate and lifetime. As a consequence, the effect of the nodes death over the analysis is minor.

Figure 4.5 represents the probability of achieving the lifetime versus β when the sink is close to the cluster. It is assumed that the sink is positioned on $(0, -15)$m and the initial energies are $E_{\text{init}} = 6$J and $E_{\text{init}} = 12$J for $T_{\text{thr}} = 600000$, $T_{\text{thr}} = 1200000$ time units. Here, due to the higher variance of the distances from the sensors to the sink, the energy consumption rate for the sensors is more non-uniform over the cluster. Therefore, the farther nodes die early while the closer nodes are still alive. As explained, this reduces energy consumption in alive nodes and increases the cluster lifetime. In this case, our analysis provides a lower bound of the network lifetime which is shown in Figure 4.5.

Now, we consider the case when the sensors perform data transmission with discrete power levels. It is assumed that the sensors use one low transmission power for the intra-cluster transmissions and one higher level for the inter-cluster transmissions. The transmission power levels are chosen such that the communication between two nodes for intra-cluster transmissions and between a node and data sink in inter-cluster transmissions are assured. The simulation results are shown in Figure 4.6 where the sensors are deployed over the area with the initial energy $E_{\text{init}} = 4.75$J for $T_{\text{thr}} = 400000$ time units and $E_{\text{init}} = 9.5$J for $T_{\text{thr}} = 800000$ time units. Also, the data sink is located on (0,-55)m. As it can be seen, the probability of achieving the desired lifetime changes fast from 0 to 1 by a slight change in the value of β. In other words, the network lifetime can be almost accurately determined. This is due to the fact that the randomness of the network lifetime drastically decreases by using the discrete level transmission which removes sensors location effect on the energy consumption. Therefore, the randomness of the lifetime is resulted just by the traffic generation model in the network.

The effect of the data aggregation ratio on the probability of achieving T_{thr} is shown in Figure 4.7. The desired lifetime is $T_{\text{thr}} = 50000$ time units and $\beta = 0.35$. In addition, the cluster has a square shape with side length of $L = 20$m and the sink is located on $(0, -90)$m. The initial sensors energy is $E_{\text{init}} = 0.7$J. Evidently, with a fixed initial energy in each sensor, the probability of achieving a specific lifetime increases when the sensors have higher ability of data compression. A simulation is also performed for a non-clustered network when sensors send the data directly to the sink. When the initial energy and the size of the area are kept unchanged, the probability of achieving the lifetime by the non-clustered network is $P = 0.08$. A clustered network can achieve a higher probability when γ is smaller than 0.62. This value of γ can also be found using (4.28) which verifies the

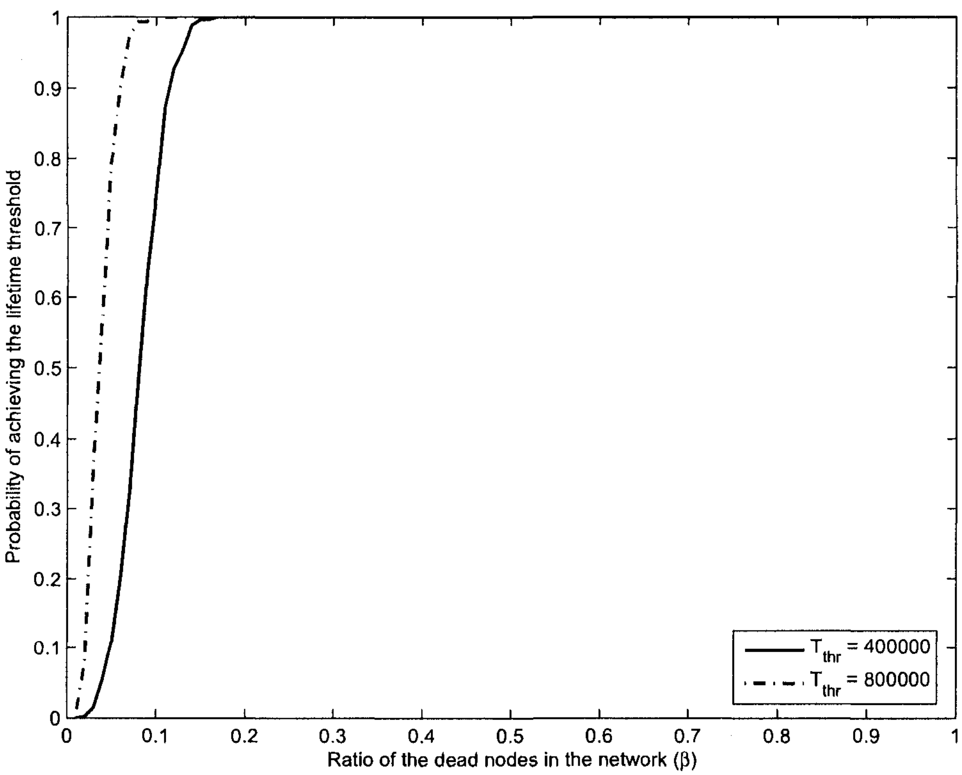


Figure 4.6: Probability of achieving different lifetime thresholds in a single-hop network with two-level transmission power.

accuracy of the clustering efficiency condition.

The probability of achieving the lifetime threshold by a cluster in a multi-hop network is also studied by computer simulation. For this purpose, it is assumed that the clusters have square shape and the inter-cluster transmissions occur between neighboring CHs. Similar to the single-hop case, the analysis is more accurate for the longer lifetimes.

Figure 4.8 shows the probability of achieving the lifetime when the cluster side length is $L = 50$m. The cluster is located on the leaf of the routing tree. The initial energies for $T_{\text{thr}} = 150000$, $T_{\text{thr}} = 300000$, $T_{\text{thr}} = 600000$ and $T_{\text{thr}} = 1200000$ time units are $E_{\text{init}} = 2$J, $E_{\text{init}} = 4$J, $E_{\text{init}} = 8$J and $E_{\text{init}} = 16$J respectively. The simulation and analysis results when the cluster side is $L = 10$m is shown in Figure 4.9. In this case, the initial energies are $E_{\text{init}} = 6$J and $E_{\text{init}} = 12$J for $T_{\text{thr}} = 600000$ and $T_{\text{thr}} = 1200000$ time units. As it can be seen, the analysis is more accurate for the larger cluster size due to the more uniform energy consumption rate in the sensors. The interpretation is similar to what we discussed for the single-hop networks.

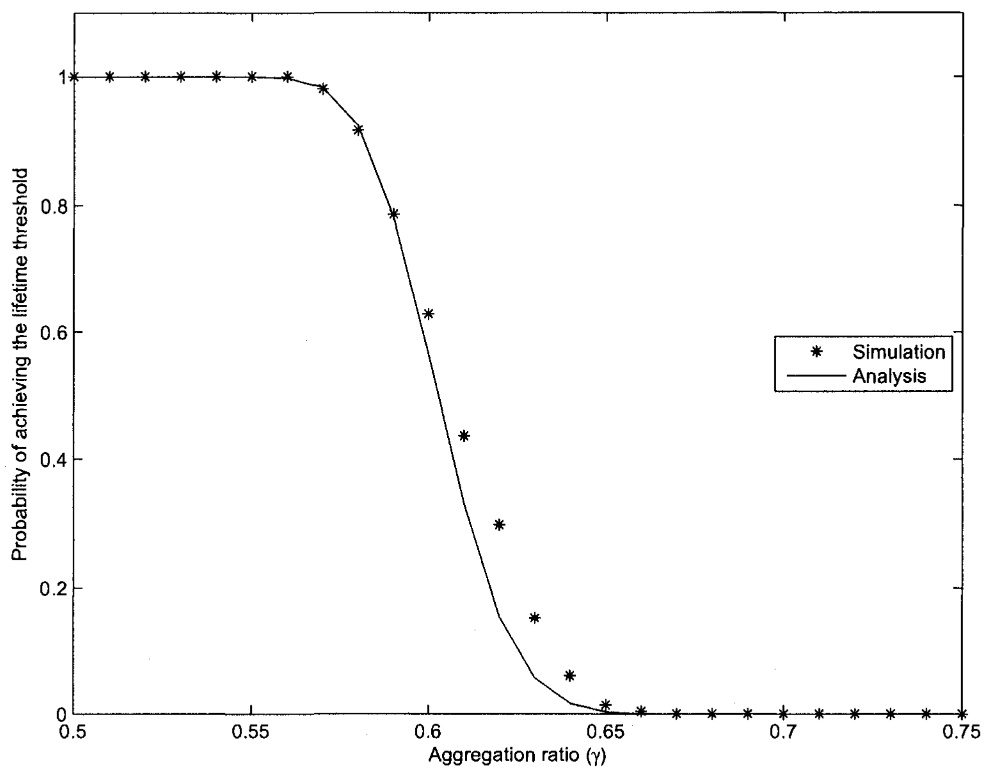


Figure 4.7: Effect of the data aggregation ratio.

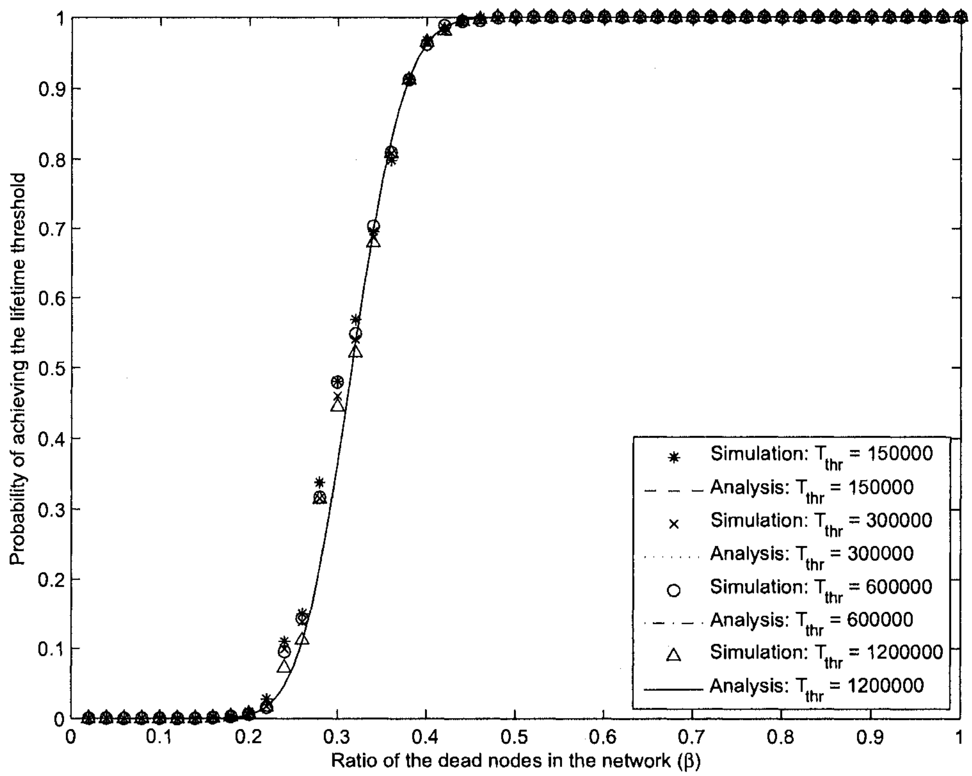


Figure 4.8: Probability of achieving different lifetime thresholds in a multi-hop network with large clusters.

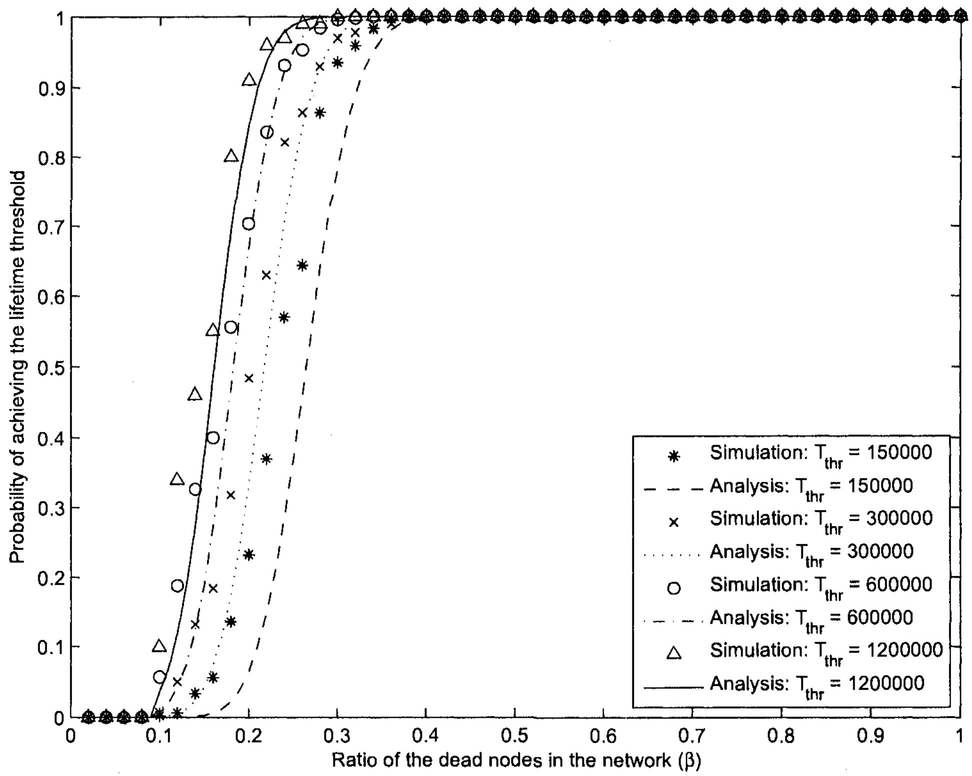


Figure 4.9: Probability of achieving different lifetime thresholds in a multi-hop network with small clusters.

Chapter 5

Characterizing Load Distribution in Linear Wireless Sensor Networks

As mentioned previously, employing routing in a multi-hop network drastically affects the traffic load pattern in the network where the nodes close to the sink experience a heavy traffic load. This imbalanced pattern is more harmful to ad hoc and sensor networks considering the limited power resource in the nodes. In these networks, nodes located in the vicinity of the sink die much earlier compared to the farther nodes which directly influences the network lifetime. Our proposed lifetime analysis for multi-hop networks in Chapter 3 was based on the assumption of a uniform load distribution over the nodes, however, applying our approach to the spares networks necessitates having an analytical model for the network traffic. In this chapter, we investigate the load distribution over linear network, however, extending the analysis to the planar networks is left for a future research activity.

Characterizing the traffic pattern in sensor and ad hoc networks has been studied in the literature e.g., [50, 51, 52]. In [50], the authors evaluate the distribution of the traffic load in an ad hoc network. The proposed analysis shows that the nodes close to the center of a circular ad hoc network carry more traffic compared to the other nodes, and hence, form the network bottleneck. In addition, the study is extended to multi-path routing where the authors assume the traffic load is uniformly spread over the network.

Work reported in [51] provides an analytical comparison between the shortest single-path and multi-path routing in dense ad hoc networks. In their study, the number of routing paths is considered where the authors point out that the multi-path and single-path routings achieve the same load balance when the number of paths is not very large.

The authors in [52] develop a Markov model to evaluate the performance of WSNs. Although the load distribution issue is not directly addressed in their work, an interesting observation is made on the probability of receiving a data unit by a node from its neighboring nodes. The simulation shows that this probability increases when the distance of the receiving node to the sink becomes shorter, however, a drop in the probability occurs when the receiving node is very close to the sink.

In this chapter, we provide an analysis of the traffic load distribution over the network nodes in a randomly deployed linear WSN. To this end, it is assumed that the network is randomly deployed over a line segment and the shortest path routing is adopted. Then, the traffic load distribution is derived based on the distance of the nodes from the data sink. We then show that, despite what is widely believed, the traffic load does not increase monotonically as the node distance from the sink reduces. In fact, at a specific point close to the sink, the traffic load starts to fall. Using our analysis, one can find the position of the nodes which carry the highest level of traffic and hence form the network bottleneck. This result can be helpful in overcoming the early death of bottleneck nodes by applying proper routing schemes, energy distribution or node deployment over the network area.

5.1 Traffic Load Analysis

Linear WSNs are easy to implement and can be effectively used for road traffic monitoring in urban areas or for breach detection [3]. In this section, we find the traffic distribution over the nodes as a function of their distance from the sink for a linear network where the network is deployed randomly.

Here, we assume that N sensors are randomly distributed, with the pdf of $f(x)$, on the x axis from $x = 0$ to $x = \ell$. It is also assumed that the sink is located on $x = \ell$. In addition, it is presumed that a sensor positioned at x, generates traffic with rate $\lambda(x)$. If all sensors generate traffic with the same rate, $\lambda(x)$ is a constant. We also denote the transmission range of sensors with r (Figure 5.1). All the sensors within the transmission range of a sensor can receive its data and may act as an intermediate node to forward the data toward the sink. To simplify the analysis, it is assumed that $\ell = nr$ where n is an integer. Moreover, the shortest path routing scheme is adopted where the path with the minimum number of hops to the sink is selected [13].

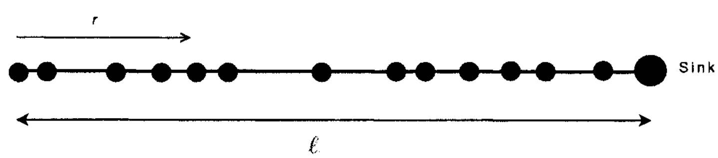


Figure 5.1: Linear network model.

In the following, we do the analysis in three parts based on the position of node, x. The first part is when $x \in [0, r)$, the second part is for $x \in [r, \ell - r)$, and the third part is performed for $x \in [\ell - r, \ell)$. Dividing the network to these parts is because of the boundary effects at the beginning ($x = 0$) and end ($x = \ell$) of the network.

Assume that a node is located on x, $x \in [0, r)$. Let us call this node n_x. Since the shortest path routing protocol is applied, n_x will forward only the traffic of the nodes who are farther from the sink compared to n_x and have no nodes closer to the sink within their transmission range rather than n_x. Considering random deployment of the sensors, n_x relays the traffic of the sensors located in $[y, y + dy)$ [1] with a probability $p(x, y)$. Here, $p(x, y)$ represents the probability of having no other nodes in the transmission range of the nodes in $[y, y + dy)$ which is closer to the sink compared to n_x. This is equivalent to having no nodes in $(x, y + r)$, thus, all of the nodes of the network except n_x have to be located in $[0, x] \cup [y + r, \ell)$. It can easily be shown that a node is out of $(x, y + r)$ with the probability of

$$p_n(x, y) = 1 - F(y + r) + F(x) \tag{5.1}$$

where $F(\cdot)$ represent the cdf of the node distribution in the network. Since the nodes are deployed independently, the probability of having all of the nodes, except n_x, out of $(x, y + r)$ is

$$p(x, y) = p_n^{N-1}(x, y) = \big(1 - F(y + r) + F(x)\big)^{N-1}. \tag{5.2}$$

The average number of nodes within $[y, y + dy)$ is $\delta(y)dy$ where $\delta(y) = Nf(y)$ represents the node density at y. Hence, the amount of traffic, which is related to the nodes in $[y, y + dy)$ and forwarded by n_x, is

$$dL_f(x) = \delta(y)L(y)p(x, y)dy \tag{5.3}$$

where $L(y)$ shows the traffic load on a sensor located on y. Therefore, the total amount of the relayed traffic through n_x is as follows:

$$L_f(x) = \int_0^x dL_f(x) = \int_0^x \delta(y)L(y)\big(1 - F(y + r) + F(x)\big)^{N-1}dy \tag{5.4}$$

where $L_f(x)$ is the traffic on x loaded by its neighbours. The total amount of traffic on n_x, shown by $L(x)$, is the summation of $L_f(x)$ and the traffic generated by n_x itself. Thus,

$$L(x) = \int_0^x \delta(y)L(y)\big(1 - F(y + r) + F(x)\big)^{N-1}dy + \lambda(x). \tag{5.5}$$

[1] $y + dy \leq x$ and dy has a very small value.

Taking a similar approach, one can easily show that when $x \in [r, \ell - r)$, the traffic load is

$$L(x) = \int_{x-r}^{x} \delta(y)L(y)\big(1 - F(y+r) + F(x)\big)^{N-1}dy + \lambda(x). \tag{5.6}$$

To evaluate the traffic pattern over $[\ell - r, \ell)$, it is necessary to consider that the nodes within this interval can directly communicate with the sink and do not forward their traffic to any other node. Then, it is straightforward to show

$$L(x) = \int_{x-r}^{\ell-r} \delta(y)L(y)\big(1 - F(y+r) + F(x)\big)^{N-1}dy + \lambda(x) \tag{5.7}$$

when $x \in [\ell - r, \ell)$.

Equation (5.5) (and similarly (5.6) and (5.7)) can be solved using analytical or numerical approach. Apparently, a node located on $x = 0$ does not forward any traffic and consequently $L(0) = \lambda(0)$. This point is the initial condition for both of the numerical and analytical methods. Here, we use the numerical method to solve (5.5), (5.6), and (5.7), however, the analytical solution can also be applied based on the Leibniz rule which indicates

$$\begin{aligned}\frac{d}{dx}\left(\int_{p(x)}^{q(x)} f(x,t)dt\right) =& f(x,q(x))\dot{q}(x) - f(x,p(x))\dot{p}(x) \\ &+ \int_{p(x)}^{q(x)} \frac{\partial}{\partial x} f(x,t)dt \quad a \le x \le b.\end{aligned} \tag{5.8}$$

Using (5.8), successive differentiation of (5.5) will result in a differential equation of order N. As mentioned, $L(0) = \lambda(0)$ and $L^{(1)}(0), L^{(2)}(0), \cdots, L^{(N-1)}(0)$ can easily be found using (5.5) where $L^{(i)}(\cdot)$ shows the ith derivative of $L(\cdot)$.

5.2 Simulation Results

Here, it is assumed that the sensors are distributed randomly over a line segment of length 30 meters. Each sensor has a maximum transmission range of $r = 10$ meters and the shortest path routing protocol is applied. Moreover, all nodes have the same packet generation rate of $\lambda = 1$ packet per time unit. To simulate the load distribution in different random network implementations, a Monte Carlo approach is used. First, it is assumed that the sensors are spread randomly with a uniform distribution. In this case, it can easily be shown that

$$p_n(x,y) = \frac{\ell + x - y - r}{\ell}. \tag{5.9}$$

Figure 5.2 shows the result for different network densities. When the network density is low (N has a small value), the nodes located in the first tier of the network, i.e. $[0, r)$, may not have a neighbour in the second tier and hence need to first send their traffic to another node within the first tier. This results in the traffic imbalance even within the nodes in the first tier. Apparently, the first-tier nodes close to $x = r$ experience more traffic which can be seen in the simulation results. Since increasing the node density results in a higher chance of having a neighbour in the second tier for the first-tier nodes, the traffic imbalance lessens with increasing the number of nodes in the network. The same explanation is valid for the traffic distribution over the nodes within the second tier. Surprisingly, when a sensor gets closer to the sink, it experiences a reduction in its traffic load. This drop in the traffic is due to the reduction in the sensor receiving range, i.e., the nodes that forward their traffic to the under study sensor. The point, where the traffic load drops, gets farther from the sink when the node density decreases in the network.

Figure 5.3 represents the network traffic distribution when the node density is very high. In a dense network, each sensor almost surely has a neighbour in the next network tier. Thus, the traffic tends to distribute evenly over the nodes within each tier which can be seen in Figure 5.3. When the network density approaches infinity, traffic load distribution looks like a staircase function. This happens because a node in the network, located on x, will have a nighbour on $x + r$. Hence, by considering a uniform node distribution on the line segment, the staircase function is expected for the traffic load distribution.

Now, we investigate the effect of the node distribution over the traffic pattern. As a simple case, it is assumed that the number of sensors does not change compared to the uniform distribution, however, they are distributed with a linearly increasing distribution over the line segment. Also, a minimum density of δ_{min} is maintained everywhere. This density can reflect the level of coverage and/or connectivity in the network [53]. Since the nodes within the area close to the sink need to tolerate more traffic in the network, a higher amount of density is required in this part. To this end, the minimum density of the nodes is assigned to $x = 0$ and the maximum density to $x = \ell$. To satisfy the constraint on the network density, we need to have $Na \geq \delta_{min}$. Choosing $a = \frac{\delta_{min}}{N}$ in Figure 5.4, the value of b can be found as follows:

$$b = \frac{2}{\ell} - a. \tag{5.10}$$

to have the total area under pdf equal to 1. For this setup,

$$p_n(x, y) = 1 - \frac{y + r - x}{2}\big(2a + \frac{b - a}{\ell}(x + y + r)\big). \tag{5.11}$$

Figure 5.5 shows the simulation result for linearly increasing and uniform node distributions in a network with $N = 60$ nodes. In the linearly increasing distribution, it is

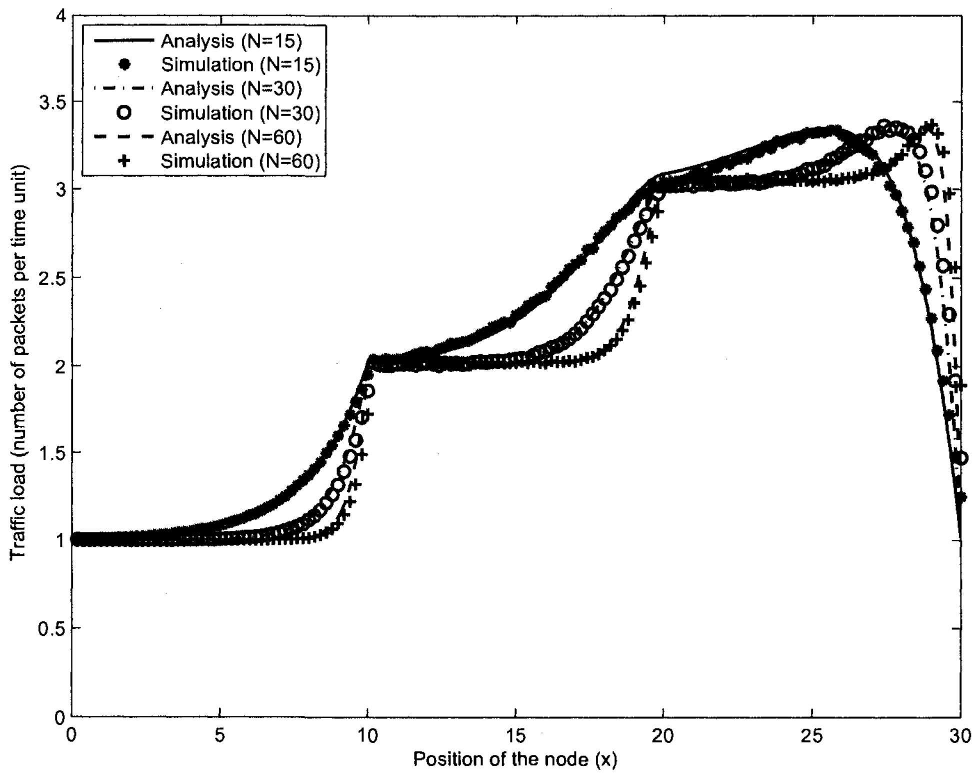


Figure 5.2: Traffic load distribution ($N = 15, 30, 60$).

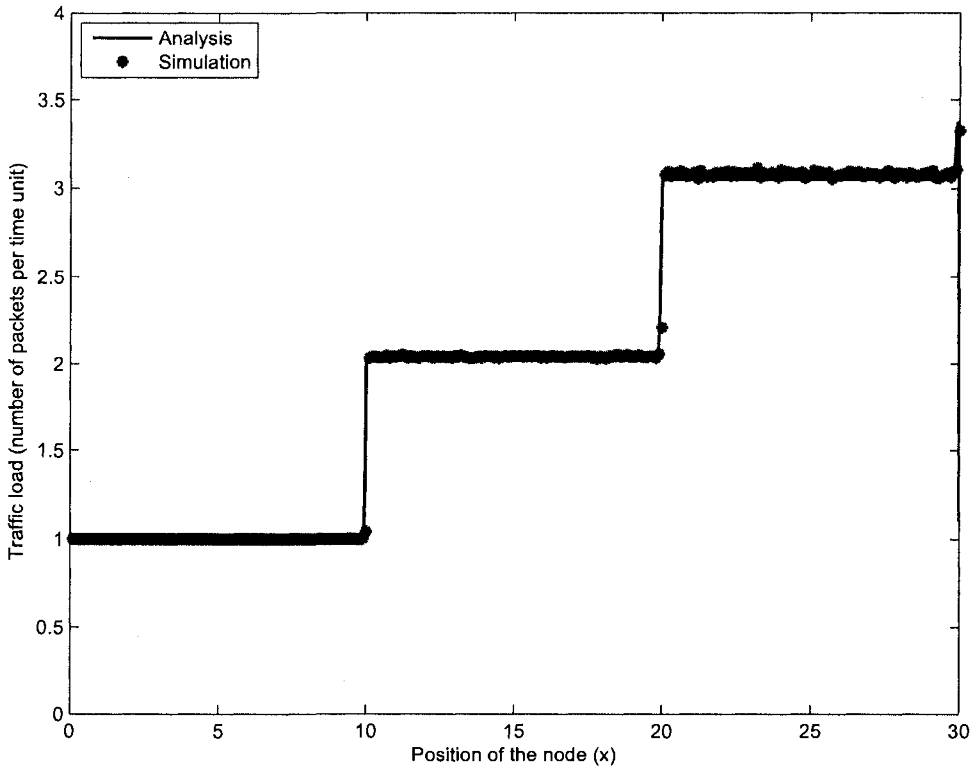


Figure 5.3: Traffic load distribution ($N = 1000$).

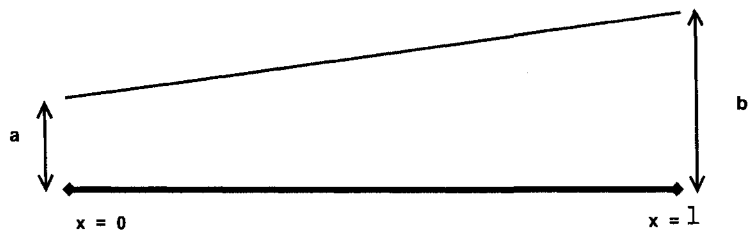


Figure 5.4: Linearly increasing distribution of the nodes over the network.

assumed that $\delta_{min} = 10$. As it can be seen, the nodes within the sink vicinity experience less traffic load compared to the uniform case. Thus, changing the node distribution over the network can result in decreasing the load over the bottleneck nodes and in turn increasing the lifetime of the network. Also, notice that the energy consumption in the sensors is almost a linear function of their traffic load when they use a constant transmission power. Therefore, by distributing the energy over the nodes according to their traffic distribution, one can assure that all sensors have almost the same lifetime.

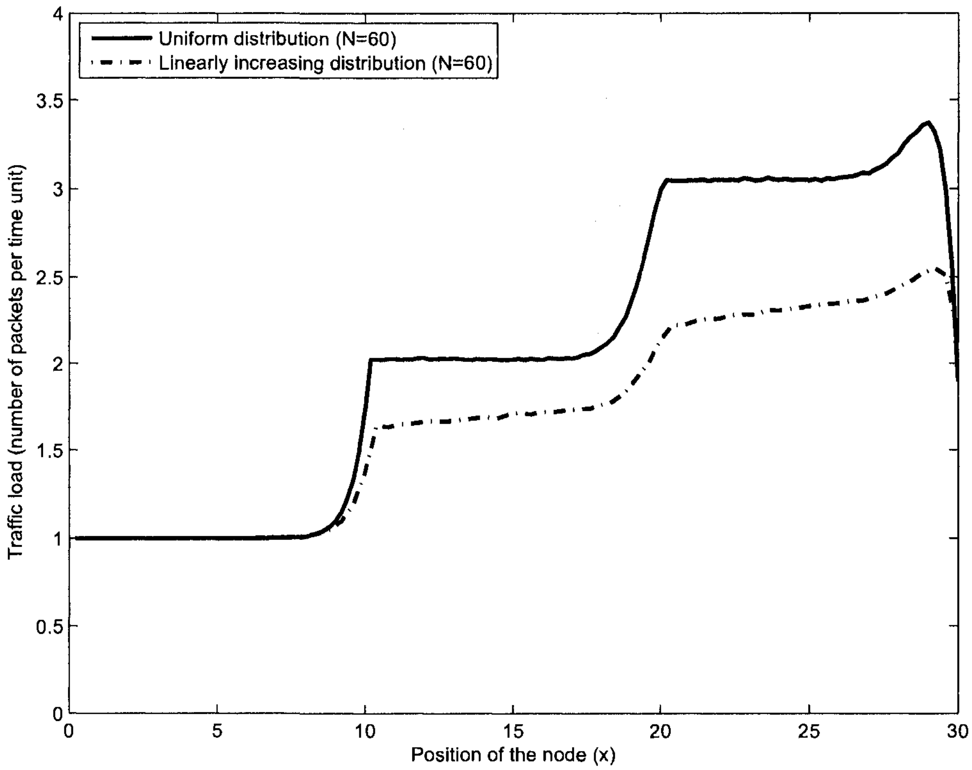


Figure 5.5: Comparison between linearly increasing and uniform distributions.

Chapter 6

Conclusions and Future Works

This chapter summarizes the results of our work presented in this dissertation. Furthermore, we will introduce some issues which can be considered for future studies.

In this work, we studied the lifetime of WSNs where the sensors are deployed randomly over the area. Different parameters affecting the lifetime (such as sensors initial energy, node density, energy consumption model, shape of the area and the data generation model) are included in our analysis. The outcome of our lifetime study may be applied in the network design stage for adjusting the network parameters.

Initially, the lifetime was studied for non-clustered WSNs including both single-hop and multi-hop cases. In fact, we derived the ccdf of the network lifetime indicating how probable is achieving a desired lifetime by the network. Based on the the developed analysis for multi-hop networks, we also proposed an optimal node distribution which guarantees the network lifetime with a given reliability. Through computer simulation, the accuracy of the proposed analysis and the improvement made by applying the suggested node distribution were verified.

In addition, we extended the analysis to clustered WSNs. The analysis considers data aggregating by CH and dynamic CH role rotation among the cluster nodes. Also, we found a condition on the efficiency of clustering in the network. More specifically, we showed that depending on the data aggregating capability of the sensors, clustered networks may or may not outlive non-clustered ones.

We also provided an analytic description of the traffic over a linear randomly deployed WSN. It was shown that the traffic over a node increases when it gets closer to the sink, however, a reduction in the traffic load is expected for sensors that are very close to the sink. The results are useful to find the proper node distribution that can efficiently spread the traffic load over the nodes or for efficient energy distribution across the network.

In our study, we adopted TDMA as the MAC protocol which removes the effect of the collisions on the lifetime of the network. An extension of the analysis, which we are interested in, is for the case when the network MAC protocol is contention-based. Extending the analysis to the contention-based protocols in turn necessitates an analysis on the collision rate and the effect of the collisions on the traffic pattern in the network. To the best of our knowledge, analysis of the collision in random networks has not been thoroughly studied in the literature and can be considered as a future research direction.

Furthermore, our analysis on the lifetime of the multi-hop networks relies on the ideal balanced traffic load over the network which is mostly achievable in the dense networks. For sparse networks, it is essential to first model the traffic load over the network and then apply our proposed framework to estimate the network lifetime. Hence, similar to the study of the traffic distribution for linear networks, it is desirable to characterize the traffic distribution over the planar networks. In addition, the traffic load distribution analysis may result in developing a new routing scheme which distributes the traffic load over the nodes evenly and tackles the problem of hot spot region in the network.

Bibliography

[1] P. Juang, H. Oki, Y. Wang, M. Martonosi, L. S. Peh, and D. Rubenstein, "Energy-efficient computing for wildlife tracking: design tradeoffs and early experiences with zebranet," *SIGOPS Oper. Syst. Rev.*, vol. 36, no. 5, pp. 96–107, 2002.

[2] A. Mainwaring, D. Culler, J. Polastre, R. Szewczyk, and J. Anderson, "Wireless sensor networks for habitat monitoring," in *WSNA '02: Proceedings of the 1st ACM international workshop on Wireless sensor networks and applications.* New York, NY, USA: ACM, 2002, pp. 88–97.

[3] A. Arora, P. Dutta, S. Bapat, V. Kulathumani, H. Zhang, V. Naik, V. Mittal, H. Cao, M. Demirbas, M. Gouda, Y. Choi, T. Herman, S. Kulkarni, U. Arumugam, M. Nesterenko, A. Vora, and M. Miyashita, "A line in the sand: a wireless sensor network for target detection, classification, and tracking," *Comput. Netw.*, vol. 46, no. 5, pp. 605–634, 2004.

[4] M. Zhang, J. Song, and Y. Zhang, "Three-tiered sensor networks architecture for traffic information monitoring and processing," in *Intelligent Robots and Systems, 2005. (IROS 2005). 2005 IEEE/RSJ International Conference on*, Aug. 2005, pp. 2291–2296.

[5] I. F. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci, "Wireless sensor networks: a survey," *Comput. Netw.*, vol. 38, no. 4, pp. 393–422, 2002.

[6] J. L. Hill and D. E. Culler, "Mica: a wireless platform for deeply embedded networks," *IEEE Micro*, vol. 22, no. 6, pp. 12–24, Nov./Dec. 2002.

[7] R. M. Kling, "Intel motes: advanced sensor network platforms and applications," *2005 IEEE MTT-S International Microwave Symposium Digest*, June 2005.

[8] W. Ye, J. Heidemann, and D. Estrin, "An energy-efficient MAC protocol for wireless sensor networks," in *INFOCOM 2002. Twenty-First Annual Joint Conference of the IEEE Computer and Communications Societies. Proceedings. IEEE*, vol. 3, June 2002, pp. 1567–1576.

[9] Y. Nam, T. Kwon, H. Lee, H. Jung, and Y. Choi, "Guaranteeing the network lifetime in wireless sensor networks: A mac layer approach," *Comput. Commun.*, vol. 30, no. 13, pp. 2532–2545, 2007.

[10] L. Wang and Y. Xiao, "A survey of energy-efficient scheduling mechanisms in sensor networks," *Mob. Netw. Appl.*, vol. 11, no. 5, pp. 723–740, 2006.

[11] J.-H. Chang and L. Tassiulas, "Maximum lifetime routing in wireless sensor networks," *IEEE/ACM Transactions on Networking*, vol. 12, no. 4, pp. 609–619, Aug. 2004.

[12] W. Lai and I. C. Paschalidis, "Sensor network minimal energy routing with latency guarantees," in *MobiHoc '07: Proceedings of the 8th ACM international symposium on Mobile ad hoc networking and computing.* New York, NY, USA: ACM, 2007, pp. 199–208.

[13] J. N. Al-Karaki and A. E. Kamal, "Routing techniques in wireless sensor networks: a survey," *IEEE [see also IEEE Personal Communications] Wireless Communications*, vol. 11, no. 6, pp. 6–28, Dec. 2004.

[14] S. Cui, R. Madan, A. J. Goldsmith, and S. Lall, "Cross-layer energy and delay optimization in small-scale sensor networks," *IEEE Transactions on Wireless Communications*, vol. 6, no. 10, pp. 3688–3699, Oct. 2007.

[15] A. Martirosyan, A. Boukerche, and R. Pazzi, "A taxonomy of cluster-based routing protocols for wireless sensor networks," in *Parallel Architectures, Algorithms, and Networks, 2008. I-SPAN 2008. International Symposium on*, Sydney, Australia, May 2008, pp. 247–253.

[16] W. B. Heinzelman, A. P. Chandrakasan, and H. Balakrishnan, "An application-specific protocol architecture for wireless microsensor networks," *IEEE Transactions on Wireless Communications*, vol. 1, no. 4, pp. 660–670, Oct. 2002.

[17] W. Wang, V. Srinivasan, and K.-C. Chua, "Using mobile relays to prolong the lifetime of wireless sensor networks," in *MobiCom '05: Proceedings of the 11th annual international conference on Mobile computing and networking*. New York, NY, USA: ACM Press, 2005, pp. 270–283.

[18] W. B. Heinzelman, "Application-specific protocol for wireless networks," Ph.D. dissertation, Massachusetts Institute of Technology, 2000.

[19] M. Bhardwaj, T. Garnett, and A. P. Chandrakasan, "Upper bounds on the lifetime of sensor networks," in *Communications, 2001. ICC 2001. IEEE International Conference on*, vol. 3, Helsinki, Finland, 2001, pp. 785–790.

[20] D. M. Blough and P. Santi, "Investigating upper bounds on network lifetime extension for cell-based energy conservation techniques in stationary ad hoc networks," in *MobiCom '02: Proceedings of the 8th annual international conference on Mobile computing and networking*. New York, NY, USA: ACM Press, 2002, pp. 183–192.

[21] G. Ferrari and M. Martal, "Extending the lifetime of sensor networks through adaptive reclustering," *EURASIP Journal on Wireless Communications and Networking*, vol. 2007, 2007.

[22] J.-H. Chang and L. Tassiulas, "Energy conserving routing in wireless ad-hoc networks," in *INFOCOM 2000. Nineteenth Annual Joint Conference of the IEEE Computer and Communications Societies. Proceedings. IEEE*, vol. 1, Tel Aviv, Israel, Mar. 2000, pp. 22–31.

[23] R. Madan and S. Lall, "Distributed algorithms for maximum lifetime routing in wireless sensor networks," *IEEE Transactions on Wireless Communications*, vol. 5, no. 8, pp. 2185–2193, Aug. 2006.

[24] E. J. Duarte-Melo and M. Liu, "Analysis of energy consumption and lifetime of heterogeneous wireless sensor networks," in *Global Telecommunications Conference, 2002. GLOBECOM '02. IEEE*, vol. 1, Nov. 2002, pp. 21–25.

[25] W. Ye, J. Heidemann, and D. Estrin, "An energy-efficient MAC protocol for wireless sensor networks," in *INFOCOM 2002. Twenty-First Annual Joint Conference of the IEEE Computer and Communications Societies. Proceedings. IEEE*, vol. 3, 2002, pp. 1567–1576.

[26] S. Adlakha and M. Srivastava, "Critical density thresholds for coverage in wireless sensor networks," *2003. WCNC 2003. 2003 IEEE Wireless Communications and Networking*, vol. 3, pp. 1615–1620, Mar. 2003.

[27] W. Ye and J. Heidemann, *Medium Access Control in Wireless Sensor Networks*. Norwell, MA, USA: Kluwer Academic Publishers, 2004.

[28] A. P. Azad and A. Chockalingam, "WLC12-2: Bounds on the lifetime of wireless sensor networks employing multiple data sinks," in *Global Telecommunications Conference, 2006. GLOBECOM '06. IEEE*, San Francisco, CA, USA, Nov. 2006, pp. 1–5.

[29] M. Bhardwaj and A. P. Chandrakasan, "Bounding the lifetime of sensor networks via optimal role assignments," in *INFOCOM 2002. Twenty-First Annual Joint Conference of the IEEE Computer and Communications Societies. Proceedings. IEEE*, vol. 3, 2002, pp. 1587–1596.

[30] V. Kolcihn, B. Sevast'yanov, and V. Chistyakov, *Random Allocations*, A. V. Balakrishnan, Ed. Washington D.c.: Winston and Sons, 1978.

[31] H. Zhang and J. Hou, "On deriving the upper bound of α-lifetime for large sensor networks," in *MobiHoc '04: Proceedings of the 5th ACM international symposium on Mobile ad hoc networking and computing*. New York, NY, USA: ACM Press, 2004, pp. 121–132.

[32] P. Hall, *Introduction to the Theory of Coverage Process*. John Wiley and Sons, 1988.

[33] M. Franceschetti and R. Meester, "Critical node lifetimes in random networks via the chen-stein method," *IEEE Transactions on Information Theory*, vol. 52, no. 6, pp. 2831–2837, June 2006.

[34] B. Bollobás and O. Riorden, *Percolation*. UK: Cambridge University Press, 2006.

[35] R. Arratia, L. Goldstein, and L. Gordon, "Poisson approximation and the chen-stein method," *Statistical Science*, vol. 5, no. 4, pp. 403–424, 1990.

[36] Q. Xue and A. Ganz, "On the lifetime of large scale sensor networks," *Elsevier Journal on Computer Communications*, vol. 29, pp. 502–510, 2006.

[37] S. M. Ross, *Introduction to Probability Models*, 7th ed. San Diego, US: Academic Press, 2000.

[38] J. Wang and I. Howitt, "Optimal traffic distribution in minimum energy wireless sensor networks," in *Global Telecommunications Conference, 2005. GLOBECOM '05. IEEE*, vol. 6, Nov./Dec. 2005.

[39] S. Yessad, F. Nait-Abdesselam, T. Taleb, and B. Bensaou, "R-mac: Reservation medium access control protocol for wireless sensor networks," in *LCN '07: Proceedings of the 32nd IEEE Conference on Local Computer Networks*. Washington, DC, USA: IEEE Computer Society, 2007, pp. 719–724.

[40] J. G. Proakis and M. Salehi, *Communication Systems Engineering*, 2nd ed. Upper Saddle River, NJ, USA: Prentice-Hall, 2004.

[41] J. Nedelman and T. Wallenius, "Bernoulli trials, Poisson trials, surprising variances, and Jensen's inequality," *Journal of American Statistician Association*, vol. 40, no. 4, pp. 286–289, Nov. 1986.

[42] S. Phoha, T. F. L. Porta, and C. Griffin, Eds., *IEEE Monograph on Sensor Network Operations*, 6th ed. San Diego, US: IEEE Press, 2004.

[43] O. Younis, M. Krunz, and S. Ramasubramanian, "Node clustering in wireless sensor networks: recent developments and deployment challenges," *IEEE Network*, vol. 20, no. 3, pp. 20–25, May/June 2006.

[44] O. Younis and S. Fahmy, "HEED: a hybrid, energy-efficient, distributed clustering approach for ad hoc sensor networks," *IEEE Transactions on Mobile Computing*, vol. 3, no. 4, pp. 366–379, Oct./Dec. 2004.

[45] N. Vlajic and D. Xia, "Wireless sensor networks: to cluster or not to cluster?" in *Proceeding of International Symposium on a World of Wireless, Mobile and Multimedia Networks, WoWMoM.*, June 2006.

[46] S. Soro and W. B. Heinzelman, "Prolonging the lifetime of wireless sensor networks via unequal clustering," in *Proceedings of 19th IEEE International Parallel and Distributed Processing Symposium, 2005*, Apr. 2005.

[47] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 6th ed., D. Zwillinger and A. Jeffery, Eds. San Diego, US: Academic Press, 2000.

[48] Y.-C. Chang, Z.-S. Lin, and J.-L. Chen, "Cluster based self-organization management protocols for wireless sensor networks," *IEEE Transactions on Consumer Electronics*, vol. 52, no. 1, pp. 75–80, Feb. 2006.

[49] X. Du and F. Lin, "Improving routing in sensor networks with heterogeneous sensor nodes," in *Vehicular Technology Conference, 2005. VTC 2005-Spring. 2005 IEEE 61st*, vol. 4, May/June 2005, pp. 2528–2532.

[50] P. P. Pham and S. Perreau, "Performance analysis of reactive shortest path and multipath routing mechanism with load balance," in *INFOCOM 2003*, vol. 1, Mar./Apr. 2003, pp. 251–259.

[51] Y. Ganjali and A. Keshavarzian, "Load balancing in ad hoc networks: single-path routing vs. multi-path routing," in *INFOCOM 2004*, vol. 2, Mar. 2004, pp. 1120–1125.

[52] C. F. Chiasserini and M. Garetto, "Modeling the performance of wireless sensor networks," in *INFOCOM 2004*, vol. 1, Mar. 2004.

[53] P. Balister, B. Bollobas, A. Sarkar, and S. Kumar, "Reliable density estimates for coverage and connectivity in thin strips of finite length," in *MobiCom '07: Proceedings of the 13th annual ACM international conference on Mobile computing and networking*. New York, NY, USA: ACM Press, 2007, pp. 75–86.

Appendix: Distribution of $f_p(x)$ Over Different Area Shapes

The pdf of the network lifetime depends on the distribution of the number of packets, p, that nodes can transmit before death. In this appendix, we find the pdf of p over some known shapes. The pdf of p over a circular area is required for studying the lifetime of multi-hop networks in Section 3.2. The pdf of p over regular polygons is used in simulation results and can also be used for studying the lifetime of some clustered networks.

6.0.1 Network Deployed Over a Circle

Assume that the nodes are deployed uniformly over a circle with radius R and the sink is located at the center. The pdf of the distance between the nodes and sink, d, is

$$f_d(x) = \begin{cases} \frac{2x}{R^2} & 0 < x \leq R \\ 0 & \text{Otherwise} \end{cases} . \tag{6.1}$$

Now, using the energy consumption model (2.1) and Jacobian method for transformation of random variables [37], we have the following expression for the pdf of p

$$f_p(x) = \begin{cases} \frac{2E_i}{R^2 k\alpha x^2} \left[\frac{E_i - cx}{kx}\right]^{\frac{2-\alpha}{\alpha}} & \frac{E_i}{kR^\alpha + c} \leq x < \frac{E_i}{c} \\ 0 & \text{Otherwise} \end{cases} . \tag{6.2}$$

6.0.2 Network Deployed Over a Regular Polygon

Suppose that the sensors are deployed over a regular polygon having n equal sides with length a. Again, we assume that the sink is placed at the center of the area. In this case, it can be shown

$$f_d(x) = \begin{cases} \frac{2\pi x}{S} & 0 < x \leq r_i \\ \frac{2\pi x - 2nx\cos^{-1}\frac{r_i}{x}}{S} & r_i < x \leq R_c \\ 0 & \text{Otherwise} \end{cases} \tag{6.3}$$

where r_i is the radius of the inscribed circle of the polygon, R_c represents the radius of the circumcircle of the polygon and S denotes the polygon area. Now, by applying Jacobi method and using the relation between d and p, we have

$$f_p(x) = \begin{cases} \frac{2E_i}{k\alpha Sx^2}\left[\frac{E_i - cx}{kx}\right]^{\frac{2-\alpha}{\alpha}}\left[\pi - n\cos^{-1}\frac{r_i}{\left(\frac{E_i-cx}{kx}\right)^{\frac{1}{\alpha}}}\right] & \frac{E_i}{kR_c^{\alpha}+c} \leq x < \frac{E_i}{kr_i^{\alpha}+c} \\ f_p(x) = \frac{2\pi E_i}{k\alpha Sx^2}\left[\frac{E_i - cx}{kx}\right]^{\frac{2-\alpha}{\alpha}} & \frac{E_i}{kr_i^{\alpha}+c} \leq x < \frac{E_i}{c} \\ 0 & \text{Otherwise} \end{cases} \quad . \quad (6.4)$$